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New Line Codes for Subscriber Loops

by

© Jorge A. Franco R.

Submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of
Master of Engineering
in
Electrical Engineering

Department of Systems and Computer Engineering
Carleton University
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The undersigned recommend to the Faculty of Graduate
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New Line Codes for Subscriber Loops

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fulfillment of the requirements for the degree of
Master of Engineering in Electrical Engineering.

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Abstract

This thesis reports the results of research undertaken at Carleton University between April 1983 and July 1984 on the feasibility of obtaining new two-level line codes for the purpose of high speed digital data transmission on telephone lines. A new class of line code was devised.

Calculations of power spectra for the new line codes were obtained using computer programs. An analysis of the optimum receiver for sequences with unequal *a priori*, total and transition probabilities was undertaken. An optimum sequence receiver, in the sense of minimum error probability, was obtained. A suboptimum truncated-sequence receiver is proposed in order to reach its practical implementation. Calculation of the bit-error probabilities for truncated sequences was achieved and a theoretical analysis of the synchronization problem was done in a general way in order to apply it, as a special case, to the new line codes. An optimum synchronizer, in the sense of minimum error probability, of the timing and clock was obtained.

The results of this research indicate an acceptable performance for the new line codes. They have favourable characteristics for use on telephone lines as those of being two-level codes, DC free, low high frequency content, transmission by blocks, namely m -ary codes with better performance with increasing m , which in turn have better spectral properties.

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I would like to dedicate this work to my whole family and in a special manner to the memory of my mother.

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Table of contents

Acceptance sheet	ii
Abstract	iii
Acknowledgements	iv
Table of contents	v
List of figures	vii
List of symbols	lx
1 Characteristics of Subscriber Loops	1-1
1.1 Introduction	1-1
1.2 Line codes' requirements	1-4
References	1-6
2 Line codes	2-1
2.1 Introduction	2-1
2.2 Two level codes	2-1
2.3 Power spectra of previous two level codes	2-5
2.4 Three level codes	2-9
References	2-12
3 VEN codes characteristics	3-1
3.1 Introduction	3-1
3.2 VEN_{DA} codes	3-2
3.2.1 Power spectra of VEN_{DA} codes	3-6
3.3 VEN_{DB} codes	3-15
3.3.1 Power spectra of VEN_{DB} codes	3-19
3.3.2 Power spectra of VEN_{DB} codes using relationship with bipolar code	3-21
3.4 VEN_{DC} codes	3-29
3.4.1 Power spectra of VEN_{DC} codes	3-31
Appendix A.3.1.- Spectra of VEN codes. Another method	3-34
Appendix A.3.2.- Program to calculate VEN codes spectra	3-37
References	3-40
4 Detection of VEN codes	4-1
4.1 Introduction	4-1
4.2 Modulation scheme	4-2
4.3 Optimum receiver for VEN codes	4-4
4.4 Structure of the MEPSE receiver	4-7
4.5 A practical (finite memory) MEPSE receiver	4-15
4.6 Probability of error for VEN codes	4-19
4.6.1 Sequence error probability for truncate sequences of ν elements	4-19
4.6.2 Minimum distance for VEN_{DA} codes	4-32
4.6.3 VEN_{DA} codes and their representation of their signals in the space	4-33
Appendix A.4.1.- Program to calculate Bit-error probability	4-37

References	4-40
5 Synchronisation of VEN codes	5-1
5.1 Introduction	5-1
5.2 Minimum error probability for symbol synchronization	5-2
5.3 Minimum error probability for clock synchronization	5-5
5.4 Combining symbol and clock synchronization	5-9
5.5 Equalization and intersymbol interference in VEN_{DA} codes	5-13
References	5-17
6 Conclusions	6-1
6.1 Summary	6-1
6.2 Suggestions for further study	6-2

List of figures

2.2.1	<i>Manchester waveforms and coding state diagram</i>	2-3
2.2.2	<i>WALZ waveforms and coding state diagram</i>	2-3
2.2.3	<i>Modified WALZ or MWALZ waveforms and coding state diagram</i>	2-3
2.2.4	<i>Coded mark inversion or CMI waveforms and coding state diagram</i>	2-4
2.2.5	<i>H-1 waveforms and coding state diagram</i>	2-4
2.2.6	<i>H-2 coding state diagram</i>	2-6
2.2.7	<i>H-3 coding state diagram</i>	2-6
2.2.8	<i>Miller code waveforms and coding state diagram</i>	2-6
2.2.9	<i>Miller-squared coding state diagram</i>	2-7
2.3.1	<i>Comparison of CMI with Manchester differentially encoded</i>	2-7
2.4.1	<i>Bipolar or AMI waveforms and coding state diagram</i>	2-7
2.4.2	<i>Comparison of CHDB₃ and MS - 43 ternary codes.</i>	2-11
3.2.1	<i>Fundamental components of VEN₁A codes</i>	3-3
3.2.2	<i>VEN₁A code waveforms</i>	3-3
3.2.3	<i>Coding state diagram for VEN₁A code</i>	3-3
3.2.4	<i>Example of coding with VEN₁A code</i>	3-4
3.2.5	<i>VEN₂A code waveforms</i>	3-4
3.2.6	<i>VEN₃A code waveforms</i>	3-4
3.2.1.1	<i>Fundamental component of VEN_DA codes</i>	3-8
3.2.1.2	<i>Composition of VEN₂A code waveforms</i>	3-8
3.2.1.3	<i>Representation of VEN_DA waveforms for the calculation of Fourier T.</i>	3-8
3.2.1.4	<i>Power spectra of VEN_DA codes</i>	3-10
3.2.1.5	<i>Comparison of Power spectra of different codes</i>	3-17
3.2.1.6	<i>Fundamental components of CMI</i>	3-18

3.2.1.7	<i>Coding state diagram for different line codes</i>	3-18
3.3.1	<i>Fundamental components of $VEN_D B$ codes</i>	3-18
3.3.2	<i>$VEN_1 B$ coded waveforms</i>	3-20
3.3.3	<i>Coding with $VEN_1 B$</i>	3-20
3.3.4	<i>Waveforms of $VEN_3 B$ code</i>	3-20
3.3.1.1	<i>Fundamental component of $VEN_D B$ codes</i>	3-24
3.3.2.1	<i>State diagram for $VEN_1 B$ code</i>	3-24
3.3.2.2	<i>State diagram for $VEN_2 B$ code</i>	3-24
3.3.2.3	<i>Power spectra of $VEN_D B$ codes</i>	3-27
3.3.2.4	<i>Example of coding with $VEN_3 B$ code for $a=2$</i>	3-30
3.4.1	<i>Fundamental components of $VEN_D C$ codes</i>	3-30
3.4.2	<i>$VEN_3 C$ coded waveforms</i>	3-30
3.4.1.1	<i>$VEN_D C$ fundamental waveform</i>	3-32
3.4.1.2	<i>Power spectra for different values of D in $VEN_D C$ codes</i>	3-32
4.2.1	<i>Communication model</i>	4-3
4.2.2	<i>Reduced model for detection of signals of VEN codes</i>	4-3
4.4.1	<i>Minimum error probability sequence estimation (MEPSE) receiver</i>	4-13
4.4.2	<i>Simplification of the MEPSE receiver for $VEN_1 A$ codes</i>	4-16
4.4.3	<i>MEPSE receiver applied to $VEN_1 A$ code</i>	4-16
4.6.1.1	<i>Bit error probability for $VEN_1 A$ code in a ν-receiver</i>	4-35
4.6.1.2	<i>Bit error probability for $VEN_1 A$, $VEN_2 A$</i>	4-36
5.3.1	<i>Compression or expansion of the signals</i>	5-14
5.4.1	<i>Block diagram for a branch of a symbol and clock synchronizer</i>	5-14
5.4.2	<i>Block diagram of a symbol and clock synchronizer</i>	5-15
5.5.1	<i>$VEN_1 A$ code words and their raised cosine pulses version</i>	5-15

List of symbols

Chapter 2

$S(f)$	Power spectrum, Lindsey & Simon's notation
E_s	Energy per waveform
h	Pulse amplitude
T	Code word duration
f	Frequency

Chapter 3

τ_p	Duration of the fundamental component of <i>VEN</i> codewords
$\omega_p(t)$	p th fundamental component of <i>VEN</i> codewords, $1 \leq p \leq (D + 1)$.
$s_k(t)$	k th <i>VEN</i> codeword, $1 \leq k \leq 2^{D+1}$.
M	Number of codewords of <i>VEN</i> codes, $M = 2^{D+1}$.
D	Number of digits per block.
Π	$2^{D+1} \times 2^{D+1}$ transition probability matrix of <i>VEN</i> codewords.
Π_{one}	Submatrix whose elements are equal to unity.
Π_{zer}	Submatrix whose elements are equal to zero.
$\Omega_p(f)$	Fourier transform of $\omega_p(t)$.
$W_p(f)$	Imaginary part of $\Omega_p(f)$.
$S_k(f)$	Fourier transform of $s_k(t)$.
$A(f)$	Real part of the summation of the $S_k(f)$.
$B(f)$	Imaginary part of the summation of the $S_k(f)$.

c_{kp}	k th element of the $2^D \times (D + 1)$ matrix of all the possible combinations of subindexes of the fundamental components, to form any codeword.
$\phi_z(f)$	Power spectrum, Petrovic's notation.
$G(f)$	Row vector constituted by the $S_k(f)$.
$G_1(f)$	Row vector constituted by the $S_k(f)$ corresponding to those $s_k(t)$ starting with positive amplitude.
$\Pi_1 \& \Pi_2$	Submatrices of Π .
$G_1'(f)$	Conjugate transpose of $G_1(f)$.
ω	Row vector of stationary probabilities.
ω_k	k th element of ω .
I_N	$N \times N$ Identity matrix with $N = 2^D$.
z	Defined as equal to $\exp(-j2\pi fDT)$ for VEN codes.
$\tilde{G}_1(f)$	Defined as the product $G_1(f)\text{diag}(\omega_1)$.
$\text{diag}(\omega_1)$	Matrix with the elements of ω_1 located in its diagonal and the rest of the elements are zeros.
P	$2^D \times 2^D$ matrix.
Q	2^D column vector.
q_j	j th element of Q .
θ	Defined equal to z^{-1} .
Π_{12}	2^D column vector with the first 2^{D-1} elements equal to -1 and the rest equal to 1 .
$H(f)$	Fourier transform of a positive pulse.

Θ_1	Row vector whose elements are the amplitudes of the pulses of VEN_DB codewords.
1_N	N row vector with all its elements equal to unity.
$1_N'$	Transpose of 1_N .
$\tilde{\Theta}_1$	Defined as the product $\varpi_1 \Theta_1$.
Θ_1'	Transpose of Θ_1 .
Θ_{11}' & Θ_{22}'	Submatrices of Θ_1' .
a	Amplitude of the timing pulse.

Chapter 4

s_k	k th codeword of the set s .
s_i	Last transmitted codeword.
$\{s_i\}$	Sequence of transmitted codewords from time <i>zero</i> to time i .
$\{n_i\}$	Noise sequence.
$\{r_i\}$	Received sequence.
s_l	l th transmitted sequence.
$P[E]$	Total error probability.
$P[C]$	Total probability of a correct decision.
$P[C s_l]$	Conditional probability of a correct decision that the sequence s_l was transmitted.
$p_r(\cdot)$	Probability density function of $(\cdot)$.
s_{ml}	Codeword belonging to sequence s_l , transmitted at time m .
r_m	Element of the sequence r .

A_{jm}	Part of M_{jk} .
B_{jk}	Part of M_{jk} .
ν	Memory capacity of the receiver expressed in number of codewords.
α	Number of first elements in a common path.
M_{jk}^ν	Metric used by the truncate-sequence receiver.
d_{it}	Euclidean distance between sequences s_i and s_t .
E_D	Energy of a VEN codeword with D digits per codeword.
E_b	Energy per bit.
s_k	k th codeword of the set, represented as a row vector.
x_k	Row vector whose components are ± 1 .
$P_w[E]$	Word error probability.
$P_b[E]$	Bit error probability.
d_{min}	Minimum Euclidean distance among allowed sequences.

Chapter 5

$r(t)$	Received sequence.
$s_t(t)$	Transmitted sequence.
$n(t)$	Noise sequence.
τ	Delay introduced by the transmission line.
$\hat{\tau}$	Estimate τ at the receiver.
A	Objective function for synchronization.
N	Number of dimensions of the space.
T_0	Agreed duration of the codeword, between transmitter and receiver.

ϵ	Error in synchronization parameters.
$\epsilon_{\hat{r}}$	Error in the estimation of r .
$\epsilon_{\hat{T}}$	Error in the estimation of T .
$\hat{T}_i$	Value of the estimate $\hat{T}$ at time i .
$\hat{r}_i$	Value of the estimate $\hat{r}$ at tie i .
ΔT	Increment on $\hat{T}$ to correct error in the estimation of T .
Δr	Increment on $\hat{r}$ to correct error on the estimation of r .

Chapter 1

Characteristics of subscriber loops

1.1 Introduction

Subscriber loops are the wires and other apparatus connecting each telephone subscriber to his local central office. They are constituted by *the subscriber, the transmitter* and the connecting pair cables to the local exchange.

Originally, telephone lines were intended to be used only for speech transmission. By use of a voice-frequency modulator-demodulator and an acoustic coupler, it is possible to convey data messages via the telephone network.

But, there are differences between the voice and data transmission. Voice transmission allows some kind of deficiencies in the quality of the transmission, because the human ear does not require perfect reception to interpret as a whole phrases and sentences, and moreover, it is insensitive to small changes in the phase of the signal. But data transmission does not tolerate these imperfections. So, constraints in this latter transmission pattern are more rigorous.

Usually, in subscriber loops twisted pair cable is used, whose diameter varies depending on the place where it is located. In general, the main network or the main distribution frame (MDF), between the local exchange and the primary cross connection point (PCCP), uses mainly 0.4mm cable, while the secondary network, between the PCCP and the customer distribution point (CDP), are mainly constituted by 0.6mm diameter pairs.

This cable distribution corresponds to a common design of the subscriber loop's network in most of the telephone administrations in the world. This is so, because its design obeys international conventions about signaling specifications, attenuation requirements and also because the statistical mean value of the distance between subscribers and local network varies not much from one administration to other. Those mentioned variations in the cable diameter constitute a problem with high data transmission rates. But, these variations are not the only aspect to be taken in account, in the problem of data transmission through telephone lines. Also the length of the connection is variable for different communications, because distances between subscriber lines and local exchange are different.

Although diameter differences are not a problem for low transmission rates, and length differences are partially solved by the use of line equalizers, different attenuation patterns are present in transmission at high speed (above 56 *Kbps*) on subscriber loops.

The use of pair cables in subscriber loops is the main factor to consider in the characterization of this channel. Features such as characteristic impedance, capacitive at low frequencies and real and almost constant at high frequencies, with a transition around 100 *KHz*, and the effect of the transformers, mainly defines this channel.

Also it is worth mentioning that bridged taps may be found in both the main and secondary network. These bridged taps, usually left open, have an undesirable effect in digital transmission. A very illustrative and detailed treatment of this theme was done by Ahamed [1] and Groenendaal [2]¹.

In a telephone system, power-carrying current is fed in a local way, by the local exchange to all local subscriber lines. The local exchange uses transformers and capacitors in order to separate DC feeding from alternate signaling. Data transmission over telephone lines has to take these aspects into account, in the sense that line codes must be DC free, in order to

¹ See references at the end of the chapter.

reduce, at a minimum low frequency distortions.

Another impairment to be considered is that of crosstalk. This is due mainly to the arrangement of the conductors as twisted pairs or as groups of four, and to the insulating material used.

In cables made up of a large number of twisted pair lines, crosstalk among neighbours may contribute, at high transmission rates, a dominant source of disturbance. There are two main types of crosstalk: that when it is introduced by a transmitting subscriber into the receiving signal of some neighbour, this is called near end crosstalk (NEXT), and the other one is when crosstalk is introduced by a transmitting subscriber into the transmitting signal of another subscriber. This is called far end crosstalk (FEXT).

There are some types of transmission systems used in order to avoid the effect of crosstalk. Among them we have time compression multiplexing (TCM) and hybrid systems. It is apparent that the former offers good expectations around this problem [1], [3].

One no less important impairment to be included in the analysis of transmitting data over telephone lines is that of the presence of echoes or reflections of the transmitted signal. This problem is bad not only for data transmission but voice transmission also.

This impairment is due to discontinuities in the transmission line, such as those caused by imperfect point connections, variation in the cable diameter, etc. For the case of TCM data transmission system, there are time gaps between transmitted message bursts, used to receive bursts sent in the opposite direction, which can be corrupted by echo, originating errors in the reception.

And finally noise. There are specific or characteristic noise of telephone networks, as for example *click* noise or impulsive interference, generated by switching equipment in the local exchange.

Actually all these impairments make the data transmission over telephone lines a serious problem to solve, imposing restrictions to the transmitted signals and requiring sophisticated equipment to transmit and receive them.

Bylansky [5] says with respect to this topic: "In general, the source digit stream will not conform to these restrictions, and hence some encoding process has to be introduced between the source and the transmission path."

1.2 Line codes' requirements.

As a result of the analysis of these impairments, the choice of the line code to transmit the data stream, becomes one of the most important aspects to be considered.

Among the various features that a transmission line code should exhibit are²:

- 1) Transparency. The line code must not impose any restrictions to the content of the message to be transmitted.
- 2) Unique decodability. No codeword may be confused with another one, in any case.
- 3) Efficiency. The transmitted information should not have much redundancy.
- 4) Ease of generation and detection. This leads to give low complexity in the equipment.
- 5) Favourable energy spectrum. The energy content at high frequencies should be very low, in order to have high tolerance to line distortions. Power spectrum should be concentrated at as low frequencies as possible.
- 6) High timing content. In this way, timing recovery can be easily achieved.
- 7) No DC component. To allow minimum distortion in transmission through transformers.

² See references [1], [3], [4], [5], [6], [7].

- 8) Two level code. A binary double-current signal is desirable in order to allow reception insensitive to signal amplitude. This leads to a maximum eye opening at the decision point.
- 9) No error multiplication.
- 10) Error correction. Some kind of checking the correctness of the received signals is desirable.
- 11) Minimum adjustments on installation. Namely, line code should not be cable dependent.
- 12) Operation over two-wire local lines at high speed (e.g. 144 Kbps). This means over those of conventional length (0 - 5)Km, and preservation of information content characteristics without regeneration.

There are some line codes that meet these requirements in certain degree. In the next chapter we will see some of them.

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Chapter 2

Line codes

2.1 Introduction.

As we have seen in the previous chapter, high speed data transmission through telephone lines is not an easy task. Nevertheless, in the choice of the suitable line code there are approaches that meet in some degree the previous requirements.

By considering those codes that are DC free and with a maximum of three levels, we have the following cases.

2.2 Two level codes.

The most known representatives of these kind of codes are: *Manchester* code, *biphase* or *WAL1* [1], [16], *coded mark inversion* (CMI) [2], *WAL2* and *MWAL2* [3], and *Hedeman* codes [4], [5]¹.

Manchester code

In *Manchester* code a *one* is represented by a half-symbol wide pulse of one polarity, and a *zero* is represented by a half-symbol wide pulse of the opposite polarity. The *Manchester* waveforms and its coding state diagram are shown in the Figure 2.2.1.

WAL2

WAL2 or *hat* code has its waveforms looking like a hat as it is shown in the Figure 2.2.2.

¹ See references at the end of the chapter.

Its coding state diagram is also shown.

Modified WAL2 or MWAL2

In this code the transition of 1/4-period is eliminated from the WAL2 code. This code has four waveforms, instead of two as the preceding ones. Its waveforms and coding state diagram are those of the Figure 2.2.3.

Coded mark inversion

This is a version of the known *bipolar* code, where instead of no pulse for *zero*, here it is represented by a half-symbol wide pulse whose polarity depends on the continuity of the preceding symbol envelope. Its waveforms and coding state diagram are shown in the Figure 2.2.4.

Hedeman codes

There are three versions of Hedeman codes: $H - 1$, $H - 2$ and $H - 3$.

In the $H - 1$ code *one* is represented by a half-symbol wide pulse, and a *zero* by a full-symbol wide pulse. It also requires that successive *zeros* assume alternate levels. We have found that its coding state diagram is that shown in the Figure 2.2.5. There also are shown its waveforms.

In the $H - 2$ code we have the same representation of *ones* and *zeros* as in $H - 1$, but here the former requires that successive *zeros* take alternate levels and successive *ones* also assume alternate polarities. We have found that its coding state diagram is that shown in the Figure 2.2.6.

In the $H - 3$ code the waveform representation is the same as $H - 1$ and $H - 2$ but previous requirements are not present. We have found that the coding state diagram is that of the Figure 2.2.7.

Delay modulation or Miller code [6] [16]

Fig. 2.2.1 Manchester waveforms and coding state diagram.

Fig. 2.2.2 WALS waveforms and coding state diagram.

Fig. 2.2.3 Modified WALS or MWALS waveforms and coding state diagram.

Fig. 2.2.4 Coded mark inversion or CMI waveforms and coding state diagram.

Fig. 2.2.5 H - 1 waveforms and coding state diagram.

This code is not DC free but it has a very ingenious and simple design and its power spectrum is concentrated at very low frequencies. In this code a *one* is represented by a half-symbol wide pulse, and a *zero* by a full-symbol wide pulse, in such a manner that continuity of the preceding signal envelope is preserved at symbol inter-boundaries. Its waveforms and coding state diagram are shown in Figure 2.2.8.

In all the previous coding state diagrams the transition probabilities were $\frac{1}{2}$, namely, each state only has two equally probable transitions to other states, including or not itself. It is worth mentioning here the *Miller-squared* code [6], which is DC free and has a power spectrum very close to the original *Miller* code, but as we already said, without DC component, namely improved. This line code has the same waveforms as single Miller, but its coding state diagram is more elaborate, and as it can be seen in the Figure 2.2.9, the transition probabilities are not the same for all the states. This only means that the occurrence of such transitions have different statistics, namely, the transition probabilities impose some restrictions to the way to do the coding in order to maintain these statistical characteristics.

2.3 Power spectra of previous two level codes.

The power spectrum of each one of the previously mentioned two level codes, for equiprobable data, are:

Manchester code [6] [16]

$$\frac{S(f)}{E_s} = \frac{\sin^4 \pi f \frac{T}{2}}{(\pi f \frac{T}{2})^2} \quad (2.3.1)$$

WALZ

We have found that its power spectrum is given by

$$\frac{S(f)}{E_s} = \left(\frac{\sin^2 \pi f \frac{T}{4}}{\pi f \frac{T}{4}} \sin \pi f \frac{T}{2} \right)^2 \quad (2.3.2)$$

Fig. 2.2.6 *H - 2 coding state diagram.* Fig. 2.2.7 *H - 3 coding state diagram.* Fig. 2.2.8 *Miller code waveforms and coding state diagram.*

Fig. 2.2.9. Miller-squared coding state diagram.

Fig. 2.3.1 Comparison of CMI with Manchester differentially encoded.

Fig. 2.4.1 Bipolar or AMI code waveforms and coding state diagram.

MWAL2 [3]

$$\frac{S(f)}{E_s} = \left(\frac{\sin \pi f \frac{T}{4}}{\pi f \frac{T}{4}} \sin \pi f \frac{T}{2} \sin \pi f \frac{3T}{2} \right) \quad (2.3.3)$$

CMI

We have found that *CMI* can be defined also as using the same waveforms of *Manchester* code, but differentially encoded. In fact, if we define that *one* means a change of polarity and *zeros* no change in the Manchester's signals, we obtain *CMI* code half-symbol shifted. This relationship is shown in the Figure 2.3.1, where a data stream is differentially encoded with *Manchester* signals, and the same data is coded with *CMI*. Because of this fact, *CMI* has the same power spectrum as *Manchester* code, given in equation 2.3.1.

Hedeman codes [4], [5]*H-1*

Authors [4], [5] do not give the closed form of this code, since, in their opinion, the code is of less interest.

H-2

$$\frac{S(f)}{E_s} = \left(\frac{\sin \pi f \frac{T}{2}}{\pi f \frac{T}{2}} \sin \pi f T \right)^2 \quad (2.3.4)$$

H-3

$$\frac{S(f)}{E_s} = \left(\frac{\sin \pi f \frac{T}{2}}{\pi f \frac{T}{2}} \sin \pi f \frac{3T}{2} \right)^2 \quad (2.3.5)$$

Miller code [6] [16]

$$\begin{aligned} \frac{S(f)}{E_s} = \frac{1}{2\theta^2(17 + 8 \cos \theta)} & (23 - 2 \cos \theta - 22 \cos \theta \\ & - 12 \cos 3\theta + 5 \cos 4\theta \\ & + 12 \cos 5\theta + 2 \cos 6\theta \\ & - 8 \cos 7\theta + 2 \cos 8\theta) \end{aligned} \quad (2.3.6)$$

for $\theta = \pi f T$.

A comparison among the different spectra of these line codes can be seen in the Figure 3.2.1.5., in Chapter Three.

A comparison of the performance of these codes are very well treated in the following references: Svensson [2], Lo Cicero et Al. [4], Wood [5], Wescott et Al. [8], Brosio et Al. [9] and Waters [10].

2.4 Three level codes.

Of the different codes to convey digital data over subscriber loops, the *alternate mark inversion* (AMI) [1], [12], [13], [14], [15], or bipolar code may be is the best known. Its coding is really easy: *zero* is represented by an empty time-slot and a *one* by an alternate positive or negative pulse

But AMI has the problem of lacking timing information when long chains of *zeroes* are present in the data stream. This has been considered as a non-transparency feature of AMI code.

However, the timing problem has been solved by modifying the coding state diagram of AMI, in the sense of the violation of AMI coding rules. Among these modified versions of AMI, we have the following ones: BnZS, HDBn and CHDBn, of which *HDB₃* and *B₀ZS₁* have found wide application [13]. The definition of these modified AMI are the following ones.

B_nZS A block of *n* consecutive *zeroes* is replaced by the sequence violating the AMI rule.

It depends on the preceding pulse. One of the best representatives is *B₀ZS* code, where the substitution rule of every block of 6 consecutive *zeroes* by the sequence 0VB0VB, is applied as follows:

<i>Preceding pulse</i>	<i>Substitution sequence</i>
+	0 + -0 - +
-	0 - +0 + -

HDB_n In these codes every block of *n + 1 zeroes* is replaced by the substitution sequence of

the same length: $B \dots V$ or $0 \dots V$. The maximum number of consecutive zeroes allowed in the coded signal is denoted by n . B stands for a normal bipolar pulse and V for a violating pulse. The polarities of V and B are the same. V pulses always alternate in polarity. For example, in HDB_3 code every block of four consecutive zeroes is substituted by the substitution sequence: $B00V$ or $000V$. The reason for using two substitution sequences is in order to maintain, in repeated violations, the DC free characteristic of the code.

$CHDB_n$ Compatible high-density bipolar code. Here every block of $n + 1$ consecutive zeroes are replaced by the substitution sequence of the same length: $\dots B0V$ or $\dots 00V$. B and V with the same meanings as before. They are called compatible because decoding can be performed regardless of the order n of the code. The choice of either sequence is made, such that the number of B pulses between two consecutive V ones is odd. Examples of substitution sequences for HDB_3 and $CHDB_3$ are: $B00V$ or $000V$, and, $0B0V$ or $000V$, respectively.

There are another codes which attempt to provide similar characteristics of the kind of codes previously mentioned. They are called selected ternary codes. These are block codes that map n binary digits into groups of m ternary digits: $nB = mT$. The most popular representative is $4B - 3T$, which has led to the *FOMOT* code.

Good discussions on the performance about the previously mentioned codes are in the following references [1], [10], [11], [12], [13], [14] and [15].

A comparison of the power spectra of bipolar, $CHDB_3$ and $MS - 43$ is presented in the Figure 2.4.2².

² Reference [15], page 299.

Fig. 2.4.2. Comparison of CHDB₃ and MS - 43 ternary codes.

Chapter 2. References

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Chapter 3

VEN codes characteristics

3.1 Introduction

Because the choice of a line code is of great importance, we propose a new family of codes which may have some advantages over the existing ones. We will call them *VEN*¹ codes.

The main features of these codes are the following:

- i) They are very suitable for using through transformers because they have zero DC component in their power spectra.
- ii) They have high tolerance to line distortion because their spectra are concentrated at low frequencies.
- iii) They have no error multiplication

VEN codes are a general family of codes with acceptable power spectra properties, where different pulse shapes can be used. Actually, we will be concerned with two level codes, three level codes and cosinusoidal version of two level codes. It can be realized that any pulse shape that meets with the rules to build *VEN* codes can be used. The specific cases treated here were chosen given either properties in their power spectra or because of their ease of implementation, or both.

The first of them, *VEN_{DA}* code is a two level code with square pulses. The second, *VEN_{DB}* code, is a three level code which is very close to bipolar code, with square pulses too,

¹ *VEN* stands for Venezuela, the author's country.

and the third one, VEN_{DC} code, is a cosinusoidal version of VEN_{DA} code, which has the best spectral properties of all.

3.2 VEN_{DA} codes

The first version of VEN codes defines the generation of **two level block codes of D binary digits**. To construct the waveforms or code words we use fundamental shapes or components which are of the type shown in Figure 3.2.1. Let τ denote the time width of the first component and T the period of the binary transmitted data.

The width of each component will be multiple of τ which in turn is submultiple of the total width of the code word, i.e. $D \cdot T$. Let $\omega_1(t), \omega_2(t), \dots, \omega_p(t), \dots, \omega_{D+1}(t)$ denote the fundamental waveforms, which have constant amplitude h and duration τ_p , where the subindex p goes from 1 to $D + 1$, and let $s_1(t), s_2(t), \dots, s_k(t), \dots, s_M(t)$ denote the code words, where $M = 2^{D+1}$. The M -word code consists of 2^D code words plus their negatives. It can be seen from the same Figure 3.2.1 that $\omega_p(t)$ can be defined as:

$$\omega_p(t) = \begin{cases} h, & \text{if } -\frac{\tau_p}{2} \leq t \leq 0; \\ -h, & \text{if } 0 \leq t \leq \frac{\tau_p}{2}; \\ 0, & \text{if } |t| > \frac{\tau_p}{2}. \end{cases} \quad (3.2.1)$$

The way to build the code words is by joining the fundamental components conveniently such that all the code words have the same duration and so that fundamental components are continuous at their boundaries. The number of fundamental waveforms necessary to build the code words is at most $D + 1$ and the number of code words will be $M = 2^{D+1}$. For example: for blocks of one digit, i.e. $D = 1$, we have to use the first two waveforms shown in Figure 3.2.1, to give those four code words shown in Figure 3.2.2.

As we can see in Figure 3.2.2 the fundamental waveforms are joined such that they are continuous in their boundaries and the total width of the code word equals $D \cdot T$. The coding

Fig. 3.2.1. Fundamental components of VEN_1A codes. Fig. 3.2.2. VEN_1A code waveforms. Fig. 3.2.3. Coding state diagram for VEN_1A code

Fig. 3.2.4. Example of coding with VEN₁A code. Fig. 3.2.5. VEN₂A code waveforms. Fig. 3.2.6. VEN₃A code waveforms.

state diagram for this line code is that of the Figure 3.2.3. From the coding state diagram we can observe that sign continuity between code words must be respected, as it is in Miller code. The transition probabilities are $\frac{1}{2}$ in any case. By defining the $2^{D+1} \times 2^{D+1}$ transition matrix Π as that formed by elements denoted by π_{ij} corresponding to the transition probability from code word $s_i(t)$ to code word $s_j(t)$, the corresponding one for VEN_1A codes is given as follows:

$$\Pi = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \\ 1/2 & 1/2 & 0 & 0 \end{pmatrix}$$

Which is the same as putting:

$$\Pi = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix}$$

An example of the application of this line code can be seen in Figure 3.2.4. To construct the corresponding code words for blocks of $D = 2$ digits we will use the first three fundamental components, i.e. $D + 1 = 3$. Under the same notation the name of this code will be VEN_2A . The number of resultant code words will be *eight*, i.e. $M = 2^3$. And the total width of the code words will be $2 \cdot T$. The transition probability matrix for VEN_2A code is:

$$\Pi = \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

It is important to note that the combination of fundamental components to form the code words of $VEN_D A$ codes is unique. If we denote 1, 2, 3 the subindexes of the fundamental components $\omega_p(t)$, then the different possible combinations, for $D = 2$, are those shown in Figure 3.2.5. In that Figure we also can see the correspondence with the binary blocks, where ones are represented by the union points between fundamental components and zeroes by no union points. In Figure 3.2.6 we can see the similar characteristics for $VEN_1 A$ and $VEN_3 A$ codes. The transition matrix for $VEN_3 A$ is very similar to those of $VEN_1 A$ and $VEN_2 A$. In general we can show that for any D , amount of digits per block, the $2^{D+1} \times 2^{D+1}$ transition matrix Π for $VEN_D A$ codes comes in the characteristic form given below:

$$\Pi = \frac{1}{2^D} \begin{pmatrix} \Pi_{one} & \Pi_{zer} \\ \Pi_{zer} & \Pi_{one} \\ \Pi_{zer} & \Pi_{one} \\ \Pi_{one} & \Pi_{zer} \end{pmatrix} \quad (3.2.2)$$

where Π_{zer} and Π_{one} are $2^{D-1} \times 2^D$ submatrices whose elements are *zeroes*, or *ones*, respectively.

Another property of $VEN_D A$ codes is that each code word has its negative counterpart in the set of code words which leads to:

$$\sum_{k=1}^{2^{D+1}} s_k(t) = 0 \quad (3.2.3)$$

These properties will be useful in the calculation of the power spectra of $VEN_D A$ codes.

3.2.1 Power Spectra of $VEN_D A$ codes

In the calculation of power spectra of $VEN_D A$ codes we will use the method developed by G. Petrovic [1] given that it can be easily implemented by a digital computer with the knowledge of only the transition probability matrix Π and the Fourier transforms of the pulse shape vector of the code words. Also it is important to say that this method reduces the computations because it allows working with vectors and matrices whose size is halved.

In order to achieve our task let us first calculate the Fourier transform of a generic fundamental component with width equal to τ_p .

$$\Omega_p(f) = j2h \frac{\tau_p}{2} \frac{\sin^2 \pi f \frac{\tau_p}{2}}{\pi f \frac{\tau_p}{2}} \quad (3.2.1.1)$$

where $\Omega_p(f)$ is the Fourier transform of $\omega_p(t)$.

We have seen before that any generic fundamental component is a multiple of the first one τ . This relationship is

$$\tau_p = p\tau \quad (3.2.1.2)$$

where, furthermore $\tau_1 = \tau$. Also we have established that the total duration of any code word is DT , thus there exists a relationship between τ and T which is not difficult to arrive at and it is

$$\tau = \frac{DT}{D+1} \quad (3.2.1.3)$$

The obtained relationships in (3.2.1.2) and (3.2.1.3) can be visualized from Figures 3.2.1.1 and 3.2.1.2.

Combining (3.2.1.2) and (3.2.1.3) we obtain

$$\tau_p = p\tau = \frac{DTp}{D+1} \quad (3.2.1.4)$$

Thus, the Fourier transform of $\Omega_p(f)$ becomes:

$$\Omega_p(f) = j \left(2h \frac{DTp}{2(D+1)} \frac{\sin^2 \pi f \frac{DTp}{2(D+1)}}{\pi f \frac{DTp}{2(D+1)}} \right) \quad (3.2.1.5)$$

If we denote $\Omega_p(f)$ as $jW_p(f)$, we can see that $W_p(f)$ is a real expression.

With these results we can obtain the expression for the Fourier transform of any one of the code words $s_k(t)$. Let $S_k(f)$ denote that transform.

Let us do some examples in order to arrive at other necessary relationships. We will calculate the Fourier transform only for the code words starting with positive because the

Fig. 3.2.1.1 Fundamental component of $VEN_D A$ codes Fig. 3.2.1.2. Composition of $VEN_2 A$ code waveforms. Fig. 3.2.1.3 Representation of $VEN_D A$ waveforms for calculation of Fourier Transforms.

other ones are their negative counterpart and their Fourier transforms are the negative of the first ones. With the help of the Figure 3.2.1.3 we can achieve better this task, for example:

For $D = 1$

$$\begin{aligned} S_1(f) &= jW_1(f)e^{j\pi fT/2(1)} - jW_1(f)e^{j\pi fT/2(-1)} \\ S_2(f) &= jW_2(f) \\ S_{k+2}(f) &= -S_k(f) \quad \text{for } k = 1, 2 \end{aligned}$$

For $D = 2$

$$\begin{aligned} S_1(f) &= jW_1(f)e^{j\pi f2T/3(2)} - jW_2(f)e^{j\pi f2T/3(-1)} \\ S_2(f) &= jW_2(f)e^{j\pi f2T/3(1)} - jW_1(f)e^{j\pi f2T/3(-2)} \\ S_3(f) &= jW_1(f)e^{j\pi f2T/3(2)} - jW_1(f)e^{j\pi f2T/3(0)} + jW_1(f)e^{j\pi f2T/3(-2)} \\ S_4(f) &= jW_3(f) \\ S_{k+4}(f) &= -S_k(f) \quad \text{for } k = 1, \dots, 4 \end{aligned}$$

For $D = 3$

$$\begin{aligned} S_1(f) &= jW_1(f) \left(e^{j\pi f3T/4(3)} - e^{j\pi f3T/4(1)} + e^{j\pi f3T/4(-1)} - e^{j\pi f3T/4(-3)} \right) \\ S_2(f) &= jW_1(f)e^{j\pi f3T/4(3)} - jW_3(f)e^{j\pi f3T/4(-1)} \\ S_3(f) &= jW_2(f)e^{j\pi f3T/4(2)} - jW_2(f)e^{j\pi f3T/4(2)} \\ S_4(f) &= jW_3(f)e^{j\pi f3T/4(1)} - jW_1(f)e^{j\pi f3T/4(-2)} \\ S_5(f) &= jW_1(f)e^{j\pi f3T/4(3)} - jW_1(f)e^{j\pi f3T/4(1)} + jW_2(f)e^{j\pi f3T/4(-2)} \\ S_6(f) &= jW_1(f)e^{j\pi f3T/4(3)} - jW_2(f)e^{j\pi f3T/4(0)} + jW_1(f)e^{j\pi f3T/4(-3)} \\ S_7(f) &= jW_2(f)e^{j\pi f3T/4(2)} - jW_1(f)e^{j\pi f3T/4(-1)} + jW_1(f)e^{j\pi f3T/4(-3)} \\ S_8(f) &= jW_4(f) \\ S_{k+8}(f) &= -S_k(f) \quad \text{for } k = 1, \dots, 8 \end{aligned}$$

By observing the first half of Fourier transforms, in each case we can realize that their summation, after some simplifications, gives a real expression and the summation of the second half leads to an imaginary expression. This feature is true for any value of D , namely it is a property of $VEN_D A$ codes. This can be expressed properly in the following manner:

$$\sum_{k=1}^{2^{D-1}} S_k(f) = A(f) \quad (3.2.1.6)$$

and

$$\sum_{k=2^{D-1}+1}^{2^D} S_k(f) = jB(f) \quad (3.2.1.7)$$

where $A(f)$ and $B(f)$ are real expressions.

Another relationship that can be obtained from the above examples is that of the general expression of the Fourier transform of each code word, $S_k(f)$. As we can realize from Figure 3.2.1.3 any combination of subindexes of the fundamental components, must sum to $D + 1$.

Let us state that those combinations constitute a $2^D \times (D + 1)$ matrix and c_{kp} is any one of the elements of such matrix. So, it is not difficult to establish the following expression:

$$S_k(f) = \sum_{p=1}^{D+1} (-1)^{p-1} j W_{c_{kp}}(f) e^{j\pi f \frac{D+1-c_{kp}}{2} \sum_{i=1}^{p-1} c_{ki}} \quad (3.2.1.8)$$

where $\sum_{p=1}^{D+1} c_{kp} = D + 1$.

With these relationships encountered we can achieve more easily the calculation of the power spectra of $VEN_D A$ codes.

As we previously mentioned Petrovic's formula will be used in the power spectrum calculations. The expression for the power spectrum $\Phi_z(f)$ in this formula is as follows:

$$\Phi_z(f) = \frac{1}{T} [2 \operatorname{Re} \{ z \tilde{\mathbf{G}}_1(f) (z \mathbf{I}_N - \mathbf{\Pi}_1 + \mathbf{\Pi}_2)^{-1} \mathbf{G}_1^*(f) \} - \tilde{\mathbf{G}}_1(f) \mathbf{G}_1^*(f)] \quad (3.2.1.9)$$

where, $\mathbf{G}(f)$ is the row vector constituted by $S_k(f)$'s

$$\mathbf{G}(f) \equiv \left(S_1(f) \quad S_2(f) \quad \cdots \quad S_{2^D}(f) \quad ; \quad -S_1(f) \quad -S_2(f) \quad \cdots \quad -S_{2^D}(f) \right) \quad (3.2.1.10)$$

Letting $\mathbf{G}_1(f)$ and $-\mathbf{G}_1(f)$ the respective portions separated by vertical dots, we have

$$\mathbf{G}(f) = \left(\mathbf{G}_1(f) \quad ; \quad -\mathbf{G}_1(f) \right) \quad (3.2.1.11)$$

$\mathbf{\Pi}_1$ and $\mathbf{\Pi}_2$ are $2^D \times 2^D$ submatrices such that

$$\mathbf{\Pi} = \begin{pmatrix} \mathbf{\Pi}_1 & ; & \mathbf{\Pi}_2 \\ \cdots & ; & \cdots \\ \mathbf{\Pi}_2 & ; & \mathbf{\Pi}_1 \end{pmatrix} \quad (3.2.1.12)$$

In this way Π_1 and Π_2 , for VEN_{DA} codes are reduced to

$$\Pi_1 = \begin{pmatrix} \Pi_{one} \\ \Pi_{zer} \end{pmatrix} \quad \text{and} \quad \Pi_2 = \begin{pmatrix} \Pi_{zer} \\ \Pi_{one} \end{pmatrix} \quad (3.2.1.13)$$

G_1^* is the conjugate transpose of G_1 .

By defining ϖ as the row vector whose element ϖ_k denotes the stationary probability of being in the corresponding state to the code word $S_k(f)$, then

$$\tilde{G}_1(f) = G_1(f) \text{diag}(\varpi_1) \quad (3.2.1.14)$$

where $\text{diag}(\varpi_1)$ is a $2^D \times 2^D$ matrix with the elements of ϖ_1 located in its diagonal and the rest of its elements are zeroes. Also ϖ_1 has similar meaning to that explained above, but referred only to the first half of code words.

I_N is a $2^D \times 2^D$ identity matrix, with $N = 2^D$.

And z is defined as²:

$$z \equiv e^{-j2\pi fDT} \quad (3.2.1.15)$$

where DT is the duration of any of the code words.

With these definitions involved in Petrovic's formula we will achieve some simplifications coming from specific characteristics of VEN_{DA} codes.

The row vector whose elements are the stationary probabilities, considered them independent among themselves, is:

$$\varpi = \frac{1}{2^{D+1}} (1 \ 1 \ \dots \ 1 \ ; \ 1 \ 1 \ \dots \ 1) \quad (3.2.1.16)$$

$$\varpi = \frac{1}{2} (\varpi_1 \ ; \ \varpi_1) \quad (3.2.1.17)$$

² This definition is different in the exponent's sign to that defined in Petrovic's paper. We consider that it is a typing error in the referred paper.

which implies that

$$\varpi_1 = \frac{1}{2^D} (1 \quad 1 \quad \cdots \quad 1) \quad (3.2.1.18)$$

The matrix $\text{diag}(\varpi_1)$, needed for our calculations, is easily obtained because all elements have the same value:

$$\text{diag}(\varpi_1) = \frac{1}{2^D} \mathbf{I}_{2^D}$$

thus,

$$\tilde{\mathbf{G}}_1(f) = \frac{1}{2^D} \mathbf{G}_1(f) \quad (3.2.1.19)$$

In order to eliminate matricial operations in Petrovic's formula let us make the following definitions: let $\mathbf{P}$ and $\mathbf{Q}$ be as follows,

$$\mathbf{P} \equiv [\mathbf{I}_N + \vartheta(\Pi_2 - \Pi_1)] \quad (3.2.1.20)$$

as we can see $\mathbf{P}$ is a $N \times N$ matrix and $\mathbf{Q}$ is a column vector with N elements, for $N = 2^D$. Let q_j denote the j th element of $\mathbf{Q}$.

Also in the above equation

$$\vartheta \equiv z^{-1} = e^{j2\pi fDT} \quad (3.2.1.21)$$

$$\mathbf{Q} \equiv \mathbf{P}^{-1} \mathbf{G}_1^*(f) \quad (3.2.1.22)$$

Thus, $\mathbf{P}\mathbf{Q} = \mathbf{G}_1^*(f)$, so,

$$\mathbf{I}_N \mathbf{Q} + \vartheta(\Pi_2 - \Pi_1) \mathbf{Q} = \mathbf{G}_1^*(f) \quad (3.2.1.23)$$

We can observe that

$$(\Pi_2 - \Pi_1)Q = \frac{1}{2^D} \begin{pmatrix} [-1] \\ \dots \\ [1] \end{pmatrix} Q = \frac{1}{2^D} \begin{pmatrix} -1 \\ -1 \\ \vdots \\ -1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \sum_{j=1}^{2^D} q_j \quad (3.2.1.24)$$

where the brackets indicate $2^{D-1} \times 2^D$ matrices with all their elements equal to what is enclosed by them.

After some manipulations we arrive at the following relationship:

$$\sum_{j=1}^{2^D} q_j = \sum_{k=1}^{2^D} S_k^*(f) \quad (3.2.1.25)$$

and also to

$$Q = G_1^*(f) - \frac{\vartheta}{2^D} \left(\sum_{k=1}^{2^D} S_k^*(f) \right) \Pi_{12} \quad (3.2.1.26)$$

where Π_{12} is defined as the following column vector:

$$\Pi_{12} = \begin{pmatrix} -1 \\ -1 \\ \vdots \\ -1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

Thus, substituting the corresponding results in the original formula (3.2.1.9) we have the following expression for the power spectrum $\Phi_x(f)$:

$$\Phi_x(f) = \frac{1}{DT} [2\operatorname{Re}\{\tilde{G}_1(f)Q\} - \tilde{G}_1(f)G_1^*(f)] \quad (3.2.1.27)$$

$$\Phi_x(f) = \frac{1}{DT2^D} [2\operatorname{Re}\{G_1(f)G_1^*(f) - \left(\frac{\vartheta}{2^D} \sum_{k=1}^{2^D} S_k^*(f)\right) G_1(f)\Pi_{12}\} - G_1(f)G_1^*(f)] \quad (3.2.1.28)$$

But, based in previous results:

$$G_1(f)\Pi_{12} = -\sum_{k=1}^{2^{D-1}} S_k(f) + \sum_{k=2^{D-1}+1}^{2^D} S_k(f) = -A(f) + jB(f) \quad (3.2.1.29)$$

also we can observe that

$$\sum_{k=1}^{2^D} S_k^*(f) = A(f) - jB(f) \quad (3.2.1.30)$$

and

$$G_1(f)G_1^*(f) = \sum_{k=1}^{2^D} |S_k(f)|^2 \quad (3.2.1.31)$$

thus,

$$\Phi_x(f) = \frac{1}{DT2^D} \left[\sum_{k=1}^{2^D} |S_k(f)|^2 - \frac{1}{2^{D-1}} \operatorname{Re}\{\vartheta(A(f) - jB(f))(-A(f) + jB(f))\} \right] \quad (3.2.1.32)$$

Substituting $\vartheta = e^{j2\pi fDT}$ in the above expression, we have

$$\Phi_x(f) = \frac{1}{DT2^D} \left(\sum_{k=1}^{2^D} |S_k(f)|^2 + \frac{1}{2^{D-1}} \operatorname{Re}\{e^{j2\pi fDT} [A(f)^2 - B(f)^2 - 2jA(f)B(f)]\} \right) \quad (3.2.1.33)$$

Finally, the power spectrum is obtained as

$$\Phi_x(f) = \frac{1}{DT2^D} \left(\sum_{k=1}^{2^D} |S_k(f)|^2 + \frac{1}{2^{D-1}} \{[A(f)^2 - B(f)^2] \cos(2\pi fDT) + 2A(f)B(f) \sin(2\pi fDT)\} \right) \quad (3.2.1.34)$$

Where, as we already know,

$$A(f) = \operatorname{Re}\left\{\sum_{k=1}^{2^D} S_k(f)\right\} \quad \text{and} \quad B(f) = \operatorname{Im}\left\{\sum_{k=1}^{2^D} S_k(f)\right\} \quad (3.2.1.35)$$

As it will be seen this formula not only is valid for the calculation of VEN_{DA} codes but also for the whole family, VEN_{DB} and VEN_{DC} codes, and also for other known codes that have the same characteristic state diagram of VEN codes. In the Appendix A.3.1 to this chapter this calculation is shown through the method indicated by Lindsey and Simon [2].

In the Figure 3.2.1.4 are some results of the application of the before mentioned formula for the calculation of VEN_{DA} codes spectra. The computer program for achieving these calculations is presented in Appendix A.3.2 of this chapter.

In Figure 3.2.1.5 is presented a power spectra comparison of VEN_{DA} code with *MWAL2* [3], *Delay Modulation* or *Miller* code [4], *OBiph* [5] or *Coded Mark Inversion* (CMI), *biphase*, *Hedeman* [6] and *bipolar* codes.

The power spectra of CMI was calculated treating it as a particular case of VEN codes, where the fundamental components were taken as that of Figure 3.2.1.6.

Some of *Hedeman* codes (H_3), *MWAL2*, *CMI* and *bipolar* are among the codes whose spectra can be calculated through the general formula for power spectra calculations of VEN codes. In Figure 3.2.1.7 can be seen their coding state diagram with the corresponding pulse shapes, put them in the characteristic state diagram of VEN codes.

3.3 VEN_{DB} codes

In the developement of VEN_{DB} codes, fundamental components which have the shape shown in Figure 3.3.1 will be used. As we can see these line codes have three levels instead of two as in VEN_{DA} codes. The reason to be considered is that it is very close to *bipolar* code.

As before, blocks of D digits have to use $D + 1$ components to form their code words and the transition matrices are identically equal to those corresponding to VEN_{DA} codes. For

Fig. 3.2.1.4 Power spectra of VENC codes.

Fig. 3.2.1.5. Comparison of power spectra of different codes.

Fig. 3.2.1.6. *Fundamental components of CMI* Fig. 3.2.1.7. *Coding state diagrams for different line codes* Fig. 3.3.1. *Fundamental components of VENC codes*

example, for $D = 1$ we will have those of the Figure 3.3.2 and an example of the application of coding with a VEN_1B code is shown in Figure 3.3.3.

Let us see for $D = 3$, which is a more illustrative example about the properties of this line codes. In Figure 3.3.4 is shown the set of code words starting with positive.

By choosing the binary blocks to be equal to the code words shown in Figure 3.3.4, we can realize that this line code uses, inside the code words, a three-digit bipolar coding. We can also observe that the last part of the code word, forms, with the first one of the following code word, a new pulse of the same type of the before mentioned internal bipolar coding. This means also that with this code for $D = 3$, or for any other value of D , we can preserve the synchronization content for long chains of zeroes in spite of using a line code very close to the known bipolar code.

3.3.1 Power spectra of $VEN_D B$ codes.

For this calculation we only need to know a Fourier transform of the generic component. For example for the third component, i.e. $p = 3$, $\Omega_3(f)$ could be calculated as follows.

As we can deduce from Figure 3.3.1.1 the expression for the Fourier transform is:

$$\Omega_3 = h \frac{r \sin \pi f \frac{r}{2}}{\pi f \frac{r}{2}} \left(e^{j2\pi f \frac{r}{2}} - e^{j2\pi f \frac{r}{2}(-5)} \right) \quad (3.3.1.1)$$

which, simplifying, becomes:

$$\Omega_3 = 2hj \frac{r \sin \pi f \frac{r}{2}}{\pi f \frac{r}{2}} \sin \pi f \frac{r}{2} (5) \quad (3.3.1.2)$$

For any p , it is easy to see that

$$\Omega_p = 2hj \frac{r \sin \pi f \frac{r}{2}}{\pi f \frac{r}{2}} \sin \left[\pi f \frac{r}{2} (2p - 1) \right] \quad (3.3.1.3)$$

By setting $\Omega_p = jW_p$ and $r = \frac{DT}{D+1}$ we obtain for W_p :

$$W_p = h \frac{DT}{(D+1)} \frac{\sin \pi f \frac{DT}{2(D+1)}}{\pi f \frac{DT}{2(D+1)}} \sin \left[\pi f \frac{DT}{2(D+1)} (2p - 1) \right] \quad (3.3.1.4)$$

Fig. 3.3.2. VEN_1B coded waveforms. Fig. 3.3.3. Coding with VEN_1B . Fig. 3.3.4. Waveforms of VEN_3B code.

With this expression of the generic fundamental component obtained we have all that is necessary to do the power spectrum calculation. Formulas (3.2.1.8) and (3.2.1.34) show us how to calculate each one of the Fourier transforms of the code words $S_k(f)$ and the final calculation of the power spectra of $VEN_D B$ codes.

Recalling the similarity of $VEN_D B$ codes with the bipolar code, in the sense of bipolar coding by blocks, we describe another way to calculate the power spectra of $VEN_D B$ codes, which will give us more insight in their characteristics and which makes clear this similarity.

3.3.2 Power spectra of $VEN_D B$ codes using relationship with bipolar code

Let us do an example, and forget the width of the pulse for the moment, to see clearly the method. Let us do it for $D = 1$ and for $D = 2$. Let us also code a block using bipolar code and generate a pulse *one* to separate this block from the following one and so on. The only condition is that the alternation of $+h$ and $-h$ must be respected, as it is in bipolar coding. The state diagrams for $D = 1$, i.e. $VEN_1 B$, and for $D = 2$, i.e. $VEN_2 B$, can be seen in Figures 3.3.2.1 and 3.3.2.2.

The transition matrices for these two codes are the following ones:

$D = 1$

$$\Pi = \begin{pmatrix} 0 & 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 1/2 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$D = 2$$

$$\Pi = \begin{pmatrix} 0 & 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & 1/2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In general we can see that these matrices are of the before mentioned form:

$$\Pi = \begin{pmatrix} \Pi_1 & \vdots & \Pi_2 \\ \dots & \dots & \dots \\ \Pi_2 & \vdots & \Pi_1 \end{pmatrix} \quad (3.3.2.5)$$

and so Petrovic's formula is also applicable. As it can be noted, transition probabilities are not equal always, as a matter of fact there are three different probabilities: 0, $\frac{1}{2}$ and 1. the reason of this is that we are not considering many code words (S_k 's in VEN_{DA} codes) but only three, whose magnitudes are $+h$, $-h$, and zero, which will lead to a simplification of the power spectrum formula.

Because of having code words with their negative counterpart, $\rho = -1$ is met in Petrovic's conditions for applying his formula.

The transition matrix in general for VEN_{DB} codes is a square matrix with $2^{D+2} - 2$ rows

Taking advantage of having only positive or negative pulses in VEN_{DB} codes let $H(f)$ denote the Fourier transform of the positive pulse, thus, we can say that the row vector constituted by the Fourier transforms of the *code words* or pulse shapes of the line code, can be written in the following manner:

$$\mathbf{G}_1(f) = (T_1(f) \ A_1(f) \ A_2(f) \ B_1(f) \ B_2(f) \ B_3(f) \ B_4(f) \ C_1(f) \) \quad (3.3.2.6)$$

$$\mathbf{G}_1(f) = H(f)(1 \ 0 \ -1 \ 0 \ -1 \ 0 \ 1 \ 0 \ -1 \ . \) \quad (3.3.2.7)$$

The number of elements of $\mathbf{G}_1(f)$ equals $\sum_{r=0}^D 2^r$, in the following manner: 2^0 for $T_1(f)$, 2^1 for $A_1(f)$ and $A_2(f)$, and so on. thus, by defining Θ_1 as

$$\Theta_1 \equiv (1 \ 0 \ -1 \ 0 \ -1 \ 0 \ 1 \ 0 \ -1 \ -1 \ 0 \ 1 \) \quad (3.3.2.8)$$

we have that

$$\mathbf{G}_1(f) \equiv H(f)\Theta_1 \quad (3.3.2.9)$$

The stationary probabilities of each element in the same order as they appear in $\mathbf{G}_1(f)$ were calculated from conditions of Petrovic's paper. They are:

$$i) \ \varpi_1(\Pi_1 + \Pi_2) = \varpi_1, \text{ and}$$

$$ii) \ \varpi_1 \mathbf{1}'_N = 1 \text{ where } \mathbf{1}_N \text{ is a row vector with all } N \text{ elements equal to unity and } \mathbf{1}'_N \text{ is the transpose of } \mathbf{1}_N. \ 1 \text{ is just the number one.}$$

The mentioned stationary probabilities are given by:

$$\varpi_1 = \frac{1}{D+1} (1 \ 1/2 \ 1/2 \ 1/4 \ 1/4 \ 1/4 \ 1/4 \ 1/8 \ \dots) \quad (3.3.2.10)$$

Taken the inner product of $\mathbf{G}_1(f)$ and ϖ_1 we obtain $\tilde{\mathbf{G}}_1(f)$. By defining the product $\varpi_1 \Theta_1$ as $\tilde{\Theta}_1$ we obtain

$$\tilde{\mathbf{G}}_1(f) \equiv H(f)\tilde{\Theta}_1 \quad (3.3.2.11)$$

Fig 3.3.1.1 Fundamental component of $VEN_D B$ codes Fig. 3.3.2.1 State diagram for $VEN_1 B$ code. Fig. 3.3.2.2. State diagram for $VEN_2 B$ code.

where $\tilde{\Theta}_1$ becomes,

$$\tilde{\Theta}_1 = \frac{1}{D+1} (1 \ 0 \ -1/2 \ 0 \ -1/4 \ 0 \ 1/4 \ 0 \ -1/8 \ 0 \ 1/8 \ \dots) \quad (3.3.2.12)$$

With these results we can start to calculate the power spectra of VEN_{DB} codes. Substituting the previous expressions obtained in equation (3.2.1.34), i.e. Petrovic's formula, we have

$$\Phi_x(f) = \frac{H(f)^2}{T} [(2\tilde{\Theta}_1 \operatorname{Re}\{I_N + \vartheta(\Pi_2 - \Pi_1)\})^{-1} \Theta'_1 - \tilde{\Theta}_1 \Theta'_1] \quad (3.3.2.13)$$

where Θ'_1 is the transpose of Θ_1 .

As before, by making the same transformations $P = I_N + \vartheta(\Pi_2 - \Pi_1)$ and $P^{-1} \Theta'_1 \equiv Q$, and observing the characteristics of the defined row vectors, we obtain the following relationships:

$$Q = \Theta'_1 - \frac{\vartheta}{2} \Theta'_{11} - (\vartheta - \frac{\vartheta^2}{2}) \Theta'_{22} \quad (3.3.2.14)$$

where Θ'_{11} is the vector formed by the first half of Θ'_1 and its rest of elements *all zeroes*

Similarly, Θ'_{22} is the vector constituted by the elements of the second half of Θ'_1 , i.e.

$$\text{if } \Theta'_1 = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{pmatrix} \quad \text{then } \Theta'_{11} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{and } \Theta'_{22} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ a_4 \\ a_5 \\ a_6 \end{pmatrix} \quad (3.3.2.15)$$

Thus, substituting previous results in the general formula, we obtain

$$\Phi_x(f) = \frac{H(f)^2}{T} [2\tilde{\Theta}_1 (\Theta'_1 - \frac{\vartheta_r}{2} \Theta'_{11} - (\vartheta_r - \frac{\vartheta_r^2}{2}) \Theta'_{22}) - \tilde{\Theta}_1 \Theta'_1] \quad (3.3.2.16)$$

where $\vartheta_r \equiv \operatorname{Re}(\vartheta) = \cos 2\pi fT$ and $\vartheta_r^2 \equiv \operatorname{Re}(\vartheta^2) = \cos 4\pi fT$.

From the obtained results for $\tilde{\Theta}_1$ and Θ_1 we can calculate their product

$$\tilde{\Theta}_1 = \frac{1}{D+1} \begin{pmatrix} 1 & 0 & 1/2 & 0 & 1/4 & 0 & 1/4 & 0 & 1/8 & 0 & 1/8 & 0 & 1/8 & 0 & 1/16 \end{pmatrix} \quad (3.3.2.17)$$

thus,

$$\tilde{\Theta}_1 \Theta'_1 = \frac{D+2}{2(D+1)} \quad \tilde{\Theta}_1 \Theta'_{11} = \frac{1}{2} \quad \tilde{\Theta}_1 \Theta'_{22} = \frac{1}{2(D+1)} \quad (3.3.2.18)$$

So, substituting properly,

$$\Phi_x(f) = \frac{H(f)^2}{T} \left(\frac{D+2}{2(D+1)} - \vartheta_r \frac{D+3}{2(D+1)} + \frac{\vartheta_r^2}{2(D+1)} \right) \quad (3.3.2.19)$$

At the begining of the calculation we deliberately neglected the width of the pulse and took it as the same as binary. But, the width is less than that of binary pulses and it is equal to $\frac{DT}{D+1}$, where as before T is the duration of the binary pulse.

Changing T by $\frac{DT}{D+1}$ and ϑ_r , ϑ_r^2 by their values, we finally have the expression for the power spectra of $VEN_D B$ codes:

$$\Phi_x(f) = \frac{H(f)^2}{T} \left(\sin^2 \left(\pi f \frac{DT}{D+1} \right) \right) \left(\frac{D-1}{D} + \frac{4}{D} \sin^2 \left(\pi f \frac{DT}{D+1} \right) \right) \quad (3.3.2.20)$$

When rectangular pulses are used the expression obtained by substituting its Fourier transform in $H(f)$, is:

$$\Phi_x(f) = h^2 T \left(\frac{\sin^2 \left(\pi f \frac{DT}{D+1} \right)}{\left(\pi f \frac{DT}{D+1} \right)} \right)^2 \left(\frac{D(D-1)}{(D+1)^2} + \frac{4D}{(D+1)^2} \sin^2 \left(\pi f \frac{DT}{D+1} \right) \right) \quad (3.3.2.21)$$

As it can be seen, this general formula for any pulse shape coded as in $VEN_D B$, has a simple expression. The power spectra of $VEN_D B$ codes can be obtained by either equation (3.2.1.34) or equation (3.3.2.21), but the last formula is easier to handle.

Through equation (3.3.2.20) we can see that when the number of digits per block, D , is taken to be infinite we arrive at:

$$\text{for } VEN_D B \text{ codes} \quad \lim_{D \rightarrow \infty} \Phi_x(f) = \frac{H(f)^2}{T} \sin^2 \pi f T \quad (3.3.2.22)$$

Fig. 3.3.2.3 Power spectra of V/N_{11} codes

which is the expression of the power spectrum of AMI (*alternate mark inversion*) or **bipolar code**, as we should have expected.

$VEN_D B$ codes have the robustness and ease of implementation of AMI code. It does not have the known problem of synchronization of AMI when long chains of zeroes are transmitted, say it has more synchronization content than AMI, but if coded at full T , namely the width of the pulse is of the same size as that of bipolar's pulse, it would mean that one more bit per block will be needed to do the coding. In other words the sacrifice to obtain better synchronization in AMI, is to transmit one more bit for *timing*, through a $VEN_D B$ coding.

Another property of this line code is that the *timing pulse*, T_1 or T_2 in the state diagram of the Figures (3.3.2.1) and (3.3.2.2), can be made bigger than the pulses of the block, and the zero DC characteristic of these line codes is preserved.

The points to be taken into account to arrive at this conclusion are the following ones:

- i) The transition matrix does not change.
- ii) The matrix G_1 changes to

$$G_1 = H(f)(a \ 0 \ -1 \dots) \quad (3.3.2.23)$$

where a is the size of the *timing pulse*.

- iii) The row vector of the stationary probabilities, i.e. ω_1 , does not change.

Because the characteristics of G_1 and ω_1 then

$$\tilde{G}_1(f) = \frac{H(f)}{D+1} (a \ 0 \ -1/2 \ 0 \ -1/4 \ 0 \ \dots) \quad (3.3.2.24)$$

- iv) P and Q are defined with the same characteristics as before. The obtained expression for Q is a little bit different:

$$Q = \Theta_1' - \frac{\vartheta}{2} \Theta_{11}' - (a\vartheta - \frac{\vartheta^2}{2}) \Theta_{22}' \quad (3.3.2.25)$$

The following products give these results.

$$\tilde{\Theta}_1 \Theta_1 = \frac{D+2a}{2(D+1)} \quad \tilde{\Theta}_1 \Theta'_{11} = \frac{2a+D-1}{2(D+1)} \quad \tilde{\Theta}_1 \Theta'_{22} = \frac{1}{2(D+1)}$$

and then

$$\Phi_s(f, a) = \frac{H(f)^2}{2T(D+1)} [(D+2a) - \vartheta_r(2a+D+1) + \vartheta_r^2] \quad (3.3.2.27)$$

We can see that for $f = 0$, $\vartheta_r = 1$ and $\vartheta_r^2 = 1$. Thus,

$$\Phi(0, a) = \frac{H(0)^2}{T} (0) = 0$$

DC component is zero for any value of a .

An example of application of this line code for $D = 3$ and $a = 2$ is shown in Figure 3.3.2.4. In such Figure the darkened pulses are the *timing pulses*.

3.4 VEN_{DC} codes

These line codes are actually a cosinusoidal version of VEN_{DA} codes. The reason we assign a special section to them is because their power spectra have better characteristics than those square-waveform codes previously described.

To construct the waveforms of these line codes we will use fundamental components of cosinusoidal type. In Figure 3.4.1 are shown the different fundamental components and their characteristics. As we have seen, for an amount of D digits per block, we need to form the whole set of code words, $N = D + 1$ fundamental components. The transition matrices for any D are the same as for VEN_{DA} codes and the rules for the combinations are also the same.

In Figure 3.4.2 is presented the different combinations of subindexes of the fundamental components to form each code word and the corresponding equivalent binary block. Also in the same Figure is an example of coding a binary train with the VEN_{DC} code.

Fig 3.3.2.4. Example of coding with VEN_3B code for $n=2$.

Fig. 3.4.1 Fundamental components of VEN_{DC} codes

3.4.1 Spectra of VEN_{DC} codes

For using the general formula (3.2.1.34) previously derived for the power spectrum calculation we only need to have the Fourier transform of a generic fundamental component, in the same way as for VEN_{DA} codes.

From Figure 3.4.1.1 we observe that the function $\omega_1(t)$ for the first fundamental component is given by:

$$\omega_1(t) = \begin{cases} -h \sin \pi \frac{t}{r_1}, & \text{if } |t| \leq \frac{r_1}{2}; \\ 0, & \text{if } |t| > \frac{r_1}{2}. \end{cases} \quad (3.4.1.1)$$

The Fourier transform of $\omega_1(t)$ is given by:

$$\Omega_1(f) = \int_{-\frac{r_1}{2}}^{\frac{r_1}{2}} -h \sin \pi \frac{t}{r_1} \times e^{-j2\pi f t} dt = j \frac{4h f r_1^2 \cos \pi f r_1}{\pi(1 - 4f^2 r_1^2)} \quad (3.4.1.2)$$

Thus, for $\Omega_1(f) = jW_1(f)$

$$W_1(f) = \frac{4h f r_1^2 \cos \pi f r_1}{\pi(1 - 4f^2 r_1^2)} \quad (3.4.1.3)$$

Remembering that $r_p = \frac{DT_p}{D+1}$, and substituting this in the latter expression, we finally have:

$$W_p(f) = \frac{4h f \left(\frac{DT_p}{D+1} \right)^2 \cos \pi f \left(\frac{DT_p}{D+1} \right)}{\pi [1 - 4f^2 \left(\frac{DT_p}{D+1} \right)^2]} \quad (3.4.1.4)$$

Then introducing this result in equations (3.2.1.8) and (3.2.1.34) we reach the complete calculation of the power spectra of VEN_{DC} codes. Characteristic curves for some values of D of VEN_{DC} codes are shown in Figure 3.4.1.2.

Among the most important properties of VEN_{DC} line codes are:

- i) They constitute a complete family of constant envelope modulation technique.
- ii) They present also the continuous phase characteristic.
- iii) They have a favourable power spectrum because of the DC free component and an almost monotonically fall to zero in the high frequencies. This means that its out-of-band content is very low.

Fig 3 4 1 1 VE/N_D fundamental component

Fig 3 4 1 2 Power spectra for different values of D in VE/N_D codes

- iv) This kind of modulation is completely controllable through the number of digits per block, in the sense of that the concentration of low frequency power can be centered at the most convenient frequency of operation.
- v) Due the previously mentioned characteristics these line codes are strongly resistant to distortions.

It can be seen that when $D = 1$, $VEN_D C$ codes coincides with one of the minimum shift keying (MSK) representatives, and of course it has all the properties of MSK modulation: the code words are orthogonal, the modulation coefficient is 0.5, it has a continuous phase characteristic with a change in phase of ± 90 degrees per period, etc.

It is important to note that the minimum values for the frequency pair of MSK modulation, with constant envelope, are those used by $VEN_1 C$ code.

Appendix A.3.1

Spectra of VEN codes. Another method.

We will present the power spectra calculation of VEN codes according to general formula of Lindsey & Simon [2]

The general transition matrix for VEN codes is of the form

$$\Pi = \frac{1}{2^D} \begin{pmatrix} [1] & [0] \\ [0] & [1] \\ \vdots & \vdots \\ [0] & [1] \\ [1] & [0] \end{pmatrix} \quad (A.3.1)$$

where Π is a $2^{D+1} \times 2^{D+1}$ matrix. This comes from the fact that each binary word has two possible versions which are of opposite sign. D is the number of digits per block. It is possible to show that

$$\Pi \cdot \Pi = \Pi^2 = \left(\frac{1}{2^D} \right)^2 2^{D+1} [1] = \frac{1}{2^{D+1}} [1] \quad (A.3.2)$$

where $[1]$ indicates a $2^{D+1} \times 2^{D+1}$ matrix with all the elements equal to unity. We can also show that

$$\Pi^3 = \Pi \cdot \Pi^2 = \Pi^2 \quad \text{and so} \quad \Pi^n = \Pi^2 \quad (A.3.3)$$

The general formula presented in the referred reference is as follows ([2], equation (1-12)):

$$S(f) = \frac{1}{T^2} \sum_{n=-\infty}^{\infty} \left| \sum_{i=1}^N p_i s_i \left(\frac{n}{T} \right) \right|^2 \delta \left(f - \frac{n}{T} \right) + \frac{1}{T} \sum_{i=1}^N p_i |S_i(f)|^2 + \frac{2}{T} \operatorname{Re} \left\{ \sum_{i=1}^N \sum_{k=1}^N p_i S_i^*(f) S_k'(f) p_{ik}(z) \right\} \quad (A.3.4)$$

Because each code word has its opposite one then

$$\sum_{i=1}^N p_i s_i(t) = 0 \quad (A.3.5)$$

thus, the first term of equation (A.3.5) is zero and so, $s_i'(t) = s_i(t)$. The second term is easily calculable and it is: $\frac{1}{T} \sum_{i=1}^N p_i [s_i(t)]^2$, which for the same stationary probabilities simplifies to

$$\frac{1}{T 2^{D+1}} \sum_{i=1}^N |s_i(t)|^2 \quad (\text{A.3.6})$$

Based on the property $\Pi^n = \Pi^2$ we can try the third term by calculating first $p_{ik}(z)$. For $z = e^{-j2\pi fT}$:

$$p_{ik} \equiv \sum_{n=1}^{\infty} p_{ik}^{(n)} e^{j2n\pi fT} \quad (\text{A.3.7})$$

Inside the transition matrix we only find either *ones* or *zeroes*, then in the case of $p_{ik}^{(1)}$, we have:

$$p_{ik}^{(1)} = \frac{1}{2^D} \cdot (\text{ones or zeroes})$$

Let us call

$$\begin{aligned} p_1 &= p_{ik}^{(n)} \quad \text{with only ones} \\ p_0 &= p_{ik}^{(n)} \quad \text{with only zeroes} \end{aligned}$$

Then, we can obtain the following expressions:

$$\begin{aligned} p_1 &= \frac{1}{2^D} e^{-j2\pi fT} + \frac{1}{2^{D+1}} [e^{-2\pi fT^2} + e^{-2\pi fT^3} + \dots] \\ p_0 &= 0 + \frac{1}{2^{D+1}} [e^{-2\pi fT^2} + e^{-2\pi fT^3} + \dots] \end{aligned}$$

and the obtained results are:

$$p_1 = \frac{2e^{-2j\pi fT} - e^{-4j\pi fT}}{2^{D+1}(1 - e^{-2j\pi fT})} \quad p_0 = \frac{e^{-4j\pi fT}}{2^{D+1}(1 - e^{-2j\pi fT})}$$

With these results let us calculate the third term. Let us review the transition matrix of equation (A.3.1). As we can deduce from such matrix, each one of the submatrices has 2^{D-1} rows and 2^D columns. Thus separating conveniently we can write the following term:

$$\sum_{i=1}^{2^{D+1}} \sum_{k=1}^{2^{D+1}} p_i S_i^*(f) S_k(f) p_{ik} (e^{-2j\pi fT})$$

as

$$\begin{aligned}
&= p_1 \left\{ \sum_{i=1}^{2^D-1} \sum_{k=1}^{2^D} () + \sum_{i=-2^D+2^D-1+1}^{2^D-1} \sum_{k=1}^{2^D} () + \sum_{i=-2^D-1+1}^{2^D+2^D-1} \sum_{k=-2^D+1}^{2^D-1} () \right\} + \\
& p_0 \left\{ \sum_{i=1}^{2^D-1} \sum_{k=-2^D+1}^{2^D-1} () + \sum_{i=-2^D+2^D-1+1}^{2^D-1} \sum_{k=-2^D+1}^{2^D-1} () + \sum_{i=-2^D-1+1}^{2^D+2^D-1} \sum_{k=-1}^{2^D} () \right\}
\end{aligned}$$

where $() = S_i^*(f)S_k(f)$. From the characteristics of VEN codes we can show that

$$\begin{aligned}
\sum_{i=1}^{2^D-1} S_i(f) &= A(f) & \sum_{i=-2^D+1}^{2^D+2^D-1} S_i(f) &= -A(f) \\
\sum_{i=-2^D-1+1}^{2^D} S_i(f) &= jB(f) & \sum_{i=-2^D+2^D-1+1}^{2^D-1} S_i(f) &= -jB(f)
\end{aligned}$$

where $A(f)$ and $B(f)$ are real numbers. So, substituting properly we obtain

$$\sum_{i=1}^{2^D-1} \sum_{k=1}^{2^D-1} S_i^*(f)S_k(f)p_{i,k}(z) = 2(A(f) + jB(f))^2(p_1 - p_0) = \frac{1}{2^{D+1}}(A(f) + jB(f))^2 e^{-2j\pi fT}$$

thus, the complete third term is.

$$\begin{aligned}
\frac{2}{T} \operatorname{Re} \left\{ \sum_{i=1}^{2^D-1} \sum_{k=1}^{2^D-1} S_i^*(f)S_k(f)p_{i,k}(z) \right\} &= \frac{2}{T} \frac{1}{2^{D+1}} \frac{1}{2^{D-1}} \operatorname{Re} \left\{ (A(f) + jB(f))^2 e^{-2j\pi fT} \right\} \\
&= \frac{2}{T 2^{2D}} [(A(f)^2 - B(f)^2) \cos 2\pi fT + 2A(f)B(f) \sin 2\pi fT]
\end{aligned}$$

The complete expression $S(f)$ will be:

$$S(f) = \frac{1}{T 2^{D+1}} \sum_{i=1}^{2^D-1} |S_i(f)|^2 + \frac{2}{T 2^{2D}} [(A(f)^2 - B(f)^2) \cos 2\pi fT + 2A(f)B(f) \sin 2\pi fT]$$

Because each code word has its negative counterpart we can simplify to:

$$S(f) = \frac{1}{T 2^D} \left\{ \sum_{i=1}^{2^D} |S_i(f)|^2 + \frac{1}{2^{D-1}} [(A(f)^2 - B(f)^2) \cos 2\pi fT + 2A(f)B(f) \sin 2\pi fT] \right\} \quad (A.3.8)$$

Changing T by DT , given that code words' period is D times the binary's one, we have.

$$S(f) = \frac{1}{DT 2^D} \left\{ \sum_{i=1}^{2^D} |S_i(f)|^2 + \frac{1}{2^{D-1}} [(A(f)^2 - B(f)^2) \cos 2\pi fDT + 2A(f)B(f) \sin 2\pi fDT] \right\} \quad (A.3.9)$$

This is the same result encountered through Petrovic's formula.

Appendix A.3.2

Program to calculate VEN codes spectra.

```

*
*
*
PROGRAM VENSPE
*
*
*
INTEGER DD,MM,RR,PP
REAL PI
COMPLEX J
PARAMETER (DD=4,MM=DD+1,RR=2**DD,J=(0,1),PI=3.14159265,PP=MM*RR)
*
*
Variables
*
INTEGER C(RR,MM)
REAL A,B,X,SUM2,SPECT
COMPLEX S(RR), SUM
DOUBLE PRECISION F(MM)
DOUBLE COMPLEX EULER
*
*
Calculation of the array which gives the subindexes of the
functions used
*
DATA C/PP*0/
MODE=2
*
*
Note: In here the 'MODE' stands by the kind of
code used. For example MODE=1 indicates
that VEN-A code is used, MODE=2 ==> VEN-B
and MODE=3 ==> VEN-C.
*
*
*
C(1,MM)=1
DO 110 L1=DD,1,-1
  L3=1
  L4=0
  L14=NINT(2** (MM-L1))
  L15=NINT(2** (DD-L1))
  DO 110 L5=1,L14
    C(L5,L1)=L3
    IF (L5.EQ.(L15+L4)) THEN
      L3=L3+1
      L4=L4+L15
      IF (L15.GT.1) THEN
        L15=L15/2
      ENDIF
    ENDIF
  110 CONTINUE

```



```

DO 120 L8=MM,1,-1
DO 120 L9=MM,L8,-1
L10=NINT(2*(MM-L8))
DO 120 L10=1,L10
L6=1
L7=L10
DO 120 L11=(DD*2-L8),(DD-1)
L7=L7+NINT(2*L11)
C(L7,(L9-L8))=C(L10,L9)
L6=L6+1
120 CONTINUE

PRINT*, ' '
PRINT*, ' '
PRINT*, ' FOR DD= ',DD,' (DIGITS/SIGNAL)'
IF (DD LT 6) THEN
PRINT*, ' THE TABLE OF SUBINDEXES IS '
PRINT*, ' '
PRINT*(' ',S15)',(C(L0,L2),L2=1,MM),L0=1,RR
ENDIF
PRINT*, ' '
PRINT*, ' '
PRINT*, ' POWER SPECTRUM of the VENIC code'
PRINT*, ' '
PRINT*, ' X=Freq*Time SPECTRA'
PRINT*, ' '

*
* Calculation of the fundamental functions.
*
*
*
DO 1 I=1,40
*
* Note: This DO loop finishes
* at the end of the Program.
*
X=I*0.1

DO 135 M1=1,MM
IF (MODE.EQ.1) THEN
R2=DD*M1*PI/(2*MM)
F(M1)=2*((SIN(R2*X))**2)/(PI*X)
ENDIF
IF (MODE.EQ.2) THEN
R2=DD*M1*X/(2*MM)
IF (ABS(R2-0.25).LT.1E-6) THEN
F(M1)=(16*DD*M1*R2)/(8*MM)
GO TO 135
ENDIF
F(M1)=(16*DD*M1*R2*COS(2*PI*R2))/
(P1*(1-16*(R2**2))*(2*MM))
135 ENDIF
IF (MODE.EQ.3) THEN
R2=PI*DD*X/(2*MM)
F(M1)=2*SIN(R2)*SIN(R2*(2*M1-1))/(PI*X)
ENDIF
130 CONTINUE

```

```

*
*      Calculation of the code words or waveforms
*
DO 140  N1=1,RR
  S(N1)=(0,0)
  DO 140  N2=1,MM
    N3=0
    DO 150  N4=1,(N2-1)
      N3=N3+C(N1,N4)
150    CONTINUE
    IF (F(C(N1,N2)).NE.F(0)) THEN
      EULER=J*CEXP((J*PI*DD*X/MM)*(MM-2*N3-C(N1,N2)))
      S(N1)=S(N1)+((-1)**(N2+1))*F(C(N1,N2))*EULER
    ENDIF
*
140    CONTINUE
*
*      Calculation of the first term of the power spectrum formula.
*
SUM=0*J
DO 10  K1=1,RR
  SUM=SUM+S(K1)
10    CONTINUE
*
A=REAL(SUM)
B=AIMAG(SUM)
SUM2=0.0
DO 20  K2=1,RR
  SUM2=SUM2+(CABS(S(K2)))**2
20    CONTINUE
*
*      Calculation of the POWER SPECTRUM.
*
SPECT=SUM2+((A*A-B*B)*COS(2*PI*DD*X)
             +2*A*B*SIN(2*PI*DD*X))/(2**((DD-1)))
SPECT=SPECT/((2**DD)*DD)
*
PRINT '(F12.2,F30.4)',X,SPECT
*
1    CONTINUE
END

```

Chapter 3. References

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Chapter 4

Detection of *VEN* codes

4.1 Introduction

As we have seen in the previous chapter, in *VEN* codes to the sequence of real numbers to be transmitted corresponds another sequence of *VEN* code words. To each message m , corresponds the transmission of the code word $s_i(t)$ or its negative counterpart $-s_i(t)$, depending on the previously transmitted code word. This reduces the high-frequency content of *VEN* codes.

In order to interpret this situation through a suitable notation, let us assume, for example using *VEN_DA* codes, for $D = 1$, that *ones* can be represented either by the code word s_1 or by $-s_1$, and *zeros* either by s_2 or by $-s_2$, as it is shown in the state diagram of Figure (3.2.3).

By following the state diagram the binary sequence $[1, 0, 1, 0, 0, 1, 1, 1, 0]$ can be put as the new one $[s_1, s_2, -s_1, -s_2, s_2, -s_1, -s_1, -s_1, -s_2]$. After coding the binary sequence $\mathbf{m} = \{m_i\}$ to $\mathbf{s} = \{s_i\}$ by applying the allowed transitions, we now have the complete one-to-one correspondence between a random sequence of messages $\mathbf{m} = \{m_i\}$ and a sequence of transmitted code words $\mathbf{s} = \{s_i\}$, where i is the time index. The same reasoning can be applied to $D = 2$. We can assume also positive and negative code words for 00's, 01's, 10's or 11's as before, and by following the transition rule of *VEN₂A* code, which is summarized through its transition matrix, we arrive at the same one-to-one correspondence model encountered before.

4.2 Modulation scheme

We will consider a synchronous linear data-transmission system with coherent demodulation. In this sense we eliminate the synchronization problem for the moment. Our model is that depicted in the Figure 4.2.1, where the set of transmitted waveforms is $s = \{s_k\}$, where the subindex k goes from 1 to 2^{D+1} .

The one-to-one correspondence between message input and transmitted waveforms implies, at time t , that

$$m_i \longleftrightarrow s_i \text{ and } s_i = s_k \quad 1 \leq k \leq 2^{D+1} \quad (4.2.1)$$

This means that s_i can be any code word s_k of the coding set s .

It is important to mention that, usually, the probability of transmitting any message is equal for all of the set of different messages, but the probability of transmitting any s_k is not the same for all of them. It depends on the previous transmitted code word, i.e. s_{i-1} . In the case of *VEN* codes the number of waveforms of the code is two times the number of different messages, namely 2^{D+1} code words and the number of different messages built from D digits is 2^D . Depending on the previous transmitted code word, half of the s_k are allowed to be transmitted and the other half are not. In chapter Three we can see the transition matrices of *VEN* codes. For the case of *VEN_DB* codes we found three different transition probabilities.

In our model of Figure 4.2.1 we can see that the receiver has to decide which s_k of the set s was transmitted by comparing them with the corrupted received code word. In this sense we can reduce our model to the parts that are really concerned with the detection problem. These are the transmitter, the channel and the receiver. The coder and decoder are not critical devices for this problem.

So, our simplified model becomes as that of the Figure 4.2.2

The specified waveforms $\{s_k\}$ are transmitted over a channel disturbed by additive Gaus-

Fig. 4.2.1 Communication model

Fig. 4.2.2 Reduced model for detection of signals of V/E/N codes

gian noise with zero mean and variance σ^2

As we can see our model is a sequential model. It is constituted by the sequence of transmitted waveforms $\{s_i\}$, the sequence of noise $\{n_i\}$ and the received sequence $\{r_i\}$. At time t , the filtered code word s'_i , given the impulse response of the channel, $h(t)$, for some transmitted waveform s_k , $k = 1, 2, \dots, 2^{D+1}$, is given by

$$s'_i(t) = s_k(t) * h(t) = \int_{-\infty}^{\infty} s_k(\alpha) h(t - \alpha) d\alpha$$

Let us suppose that we have an equalizer that restores the original waveform and the only effect to consider will be that of the white noise. Thus, $\{s'_i(t)\}$ becomes equal to $\{s_i(t)\}$. This assumption leads also to a simplification in the notation. So, at the receiver we will have the received sequence r , which consists of the summation of the transmitted sequence, s_i , and the noise sequence, n , i.e.

$$r = n + s_i$$

The probability of transmitting some sequence from time zero to time t , i.e. s_t , is given by $P[s_t]$, where the subindex t denotes any of the different possible sequences. In the case of VEN codes this subindex goes from 1 to 2^{D+1} .

It is important to note that although the transition probabilities of transmitting code words are not always the same, in general the a priori probabilities of transmitting sequences are the same.

4.3 Optimum receiver for VEN codes

As we can realise VEN codes are representatives of Markov sequences, and a coherent demodulation scheme that exploits this property will be the best one. In this sense a maximum likelihood sequence estimation-based scheme (the Viterbi algorithm) could be very convenient.

Nevertheless, in the selection of the criterion to obtain the structure of the optimum

receiver we will follow a general methodology, which starts with the proper definition of the total probability of error avoiding the use of criteria. This methodology is developed based on the ideas presented by Wozencraft and Jacobs [1] about the use of *decision regions* and that of *Viterbi algorithm* [2], [3], [4], [5], [6], in order to take in account unequal *a priori* probabilities of the sequence $\mathbf{m}$ and unequal *a priori* transition probabilities of the code words of the sequence $\mathbf{s}_i$. With this methodology we will obtain more general results than that of the MLSE-based scheme.

Based on the model of the previous section and in a more general case, let $\mathbf{s}$ denote the set of possible sequences to be transmitted and let $\mathbf{s}_i$ be one of them. The subindex i goes from 1 to 2^{D_i+1} , where i is the time of the last received code word.

The probability of an erroneous decision on the detection of the arriving sequence will be given by

$$P[E] = 1 - P[C] \quad (4.3.1)$$

where $P[E]$ denotes the total probability of error and $P[C]$ the total probability of a correct decision.

According to probability rules, we have

$$P[C] = \sum_{i=1}^L P[C|\mathbf{s}_i]P[\mathbf{s}_i] \quad (4.3.2)$$

But the probability of a correct decision, given that the sequence $\mathbf{s}_i$ was transmitted, $P[C|\mathbf{s}_i]$, is the same as the probability that the received sequence, $\mathbf{r}$, is in the decision region of $\mathbf{s}_i$. This can be put in the following way:

$$P[C|\mathbf{s}_i] = P[\mathbf{r} \text{ in } I_i|\mathbf{s}_i] = \int_{I_i} p_r(\mathbf{r}|\mathbf{s}_i)d\mathbf{r} \quad (4.3.3)$$

where I_i denotes the decision region of $\mathbf{s}_i$. Then,

$$P[C] = \sum_{i=1}^L \int_{I_i} p_r(\mathbf{r}|\mathbf{s}_i)P[\mathbf{s}_i]d\mathbf{r} \quad (4.3.4)$$

where $p(\cdot)$ is the probability density function of $(\cdot)$. By applying mixed Bayes rule to (4.3.4) we have

$$p_r(r|s_l)P[s_l] = p_r(r)P[s_l|r] = p_r(s_l, r) \quad (4.3.5)$$

Then, finally

$$P[C] = \sum_{l=1}^L \int_{I_l} p_r(s_l, r) dr \quad (4.3.6)$$

As we see, by maximizing the total probability of correct decision we obtain the minimization of the total probability of error. In order to maximize $P[C]$ we can say about equation (4.3.6) that because what is under the integration sign is always a positive quantity, maximization of $P[C]$ is indeed the maximization of the joint density function of the random received sequence with the event of transmitting the sequence s_l .

By observing equation (4.3.5) we can find familiar expressions.

- i) The left hand side is just the expression of MLSE multiplied by the *a priori* probabilities of each sequence, which when they are the same, as usually occurs, do not affect the maximization process and hence they could be discarded
- ii) the middle expression is the expression of the maximum *a posteriori* criterion (MAP) multiplied by the density function of the received sequence, which is just a *mean* of all the conditional p.d.f. given that each s_l was transmitted, i.e.

$$p_r(r) = \sum_{l=1}^L p_r(r|s_l)P[s_l] \quad (4.3.7)$$

So, this factor does not affect the maximization process and it can be discarded. It can be said that, MAP is more general than MLSE criterion, since it retains optimality for unequal *a priori* probabilities.

In sum, because of the ease of derivation of the conditions for obtaining minimum probability of error in any optimization process¹ we are going to follow this way in next derivations

¹ The previous derivation was very general.

In order to differentiate this methodology from those referred criteria in the analysis of the optimum receiver we will call it minimum error probability sequence estimation (MEPSE)

Based in the model depicted in Figure 4.2.1, let us establish the following notation: the subindex m will be used to denote any time of transmitting code words. The subindex i will denote the time of the last transmitted code word. The subindex k will be used for denoting any code word of the available set. The number of code words for VEN codes is 2^{D+1} . The subindex l will denote any sequence from time 1 to time i . The total number of allowed sequences for VEN codes is 2^{D+1} .

With this defined notation and the equation (4.2.6) we can obtain the structure of the optimum receiver for VEN codes.

4.4 Structure of the MEPSE receiver.

Our objective function in order to obtain minimum probability of error, as it was established before is:

$$\min P[E] = \max P[C] = \max \sum_{i=1}^{2^{D+1}} \int_{I_i} p_r(r|s_i) P[s_i] \quad (4.4.1)$$

which is the same as the maximisation of that under the integration sign, then the process implied in equation (4.4.1) becomes:

$$\max (p_r(r|s_i) P[s_i]) \quad (4.4.2)$$

The unconditional probability of transmitting the entire sequence s_i implies the transmission of each one of its elements or code words $\{s_{1i}(t), s_{2i}(t), \dots, s_{di}(t)\}$, then

$$P[s_i] = P[s_{1i}(t), s_{2i}(t), \dots, s_{di}(t)] \quad (4.4.3)$$

Although all the allowed paths or sequences are, usually, equally probable, transition probabilities are not equal, and because we will use Viterbi algorithm in our derivation, which

takes relative decisions each time t , transition probabilities will have an important role in the final structure of the optimum receiver for VEN codes. Furthermore, this necessary characteristic makes the final result more general, because it covers the case of *non equally probable sequences*.

As we have pointed out previously the process of transmitting VEN code words is Markov in the sense of that the probability of being $s_u(t) = s_k(t)$, the transmitted waveform, of sequence s_l at time t , depends only on $s_{(t-1)l}(t) = s_j(t)$ the transmitted waveform at time $t-1$, for $1 \leq k, j \leq 2^{D+1}$. This can be written as:

$$P[s_u(t)|s_{1l}(t), s_{2l}(t), \dots, s_{(t-1)l}(t)] = P[s_u(t)|s_{(t-1)l}(t)] \quad (4.4.4)$$

In order to simplify the notation we will take the code words as vectors and so, the code word $s_{ml}(t)$ will be represented by s_{ml} .

Applying Bayes' rule properties and substituting recursively obtained results in equation (4.4.4), we have

$$P[s_l] = P[s_{2l}|s_{1l}]P[s_{1l}] \quad (4.4.5)$$

where $P[s_{ml}|s_{(m-1)l}]$ are the transition probabilities of the transmitted waveforms at time m , and $P[s_{1l}]$ is the unconditional or *a priori* probability of the first transmitted waveform, of the sequence l .

By defining $P[s_{1l}] \equiv P[s_{1l}|s_{0l}]$, we can put (4.4.5) in a shorter form:

$$P[s_l] = \prod_{m=1}^L P[s_{ml}|s_{(m-1)l}] \quad (4.4.6)$$

Thus, our objective function, equation (4.4.2), to maximize becomes

$$\max \left(\prod_{m=1}^L P[s_{ml}|s_{(m-1)l}] \cdot p_r(r|s_l) \right) \quad (4.4.7)$$

The conditional probability density function that the sequence of received code words until time t , given that the transmitted sequence was $\mathbf{s}_t$, is also a Gaussian random variable, function of the Gaussian random variable $\mathbf{n}$ and of the transmitted sequence $\mathbf{s}_t$, which is a deterministic one² during the interval $0 \leq t \leq iT$. Then it is possible to show that

$$p_r(\mathbf{r}|\mathbf{s}_t) = p_r(\mathbf{n} + \mathbf{s}_t|\mathbf{s}_t) = p_n(\mathbf{n}|\mathbf{s}_t) \quad (4.4.8)$$

Assuming that noise sequence is independent of transmitted sequence and noise components independent among them we have that

$$p_r(\mathbf{r}|\mathbf{s}_t) = p_{n_1}(n_1)p_{n_2}(n_2) \cdots p_{n_i}(n_i) \quad (4.4.9)$$

So, we can put equation (4.4.9) in a shorter form:

$$p_r(\mathbf{r}|\mathbf{s}_t) = \prod_{m=1}^i p_{n_m}(n_m) \quad (4.4.10)$$

Substituting this result in equation (4.4.7) we have

$$\max \left(\prod_{m=1}^i P[s_m|s_{(m-1)t}] p_{n_m}(n_m) \right) \quad (4.4.11)$$

The maximization process of this detection problem tells us through equation (4.4.11) that we have to choose that sequence $\mathbf{s}_t$ that maximizes the product indicated by this equation. At this point we invoke Viterbi's idea of taking decisions on all the possible paths or sequences, at every time m , by recognizing that the best sequence to be chosen has to have the minimum Euclidean distance with respect to the received one, which means that it has to have also the minimum Euclidean distance in each element of all the allowed sequences with respect to the corresponding element in the received sequence. Then, if decisions are taken over each element of all the allowed sequences, computations are reduced by converting their growth from an exponential to a linear form [4], because of discarding in each element those Euclidean

² We are assuming that specifically it was transmitted.

distances that are not the least one. This is the essence of the Viterbi algorithm. From this comes the concepts of the surviving path and surviving transitions.

In order to do relative or partial decisions per each element of each allowed sequence and because s_{mi} could be any one of the s_k waveforms of the code, we have to use two more subindexes to identify the current element of the sequence s_i and the previous one. Let j and k be these subindexes. Then our objective function becomes:

$$\max \left(\prod_{m=1}^i P[s_{mj}|s_{(m-1)k}] p_{n_m}(n_m) \right) \quad (4.4.12)$$

We know that the Gaussian p d f of a received signal can be put [1] in the following form

$$p_n(n) = \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} \exp \left(-\frac{1}{2\sigma^2} \int_0^T [r(t) - s(t)]^2 dt \right)$$

where σ^2 is the noise variance, $(0, T)$ is the period of observation and N is the dimension of the space. By taking natural logarithms on the equation (4.4.12) and discarding constant terms we arrive at the following decision function:

$$\max \left(\sum_{m=1}^i \left(\ln P[s_{mj}|s_{(m-1)k}] - \frac{1}{2\sigma^2} \int_{(m-1)T}^{mT} [r_m(t) - s_{mj}(t)]^2 dt \right) \right) \quad (4.4.13)$$

where $k, j = 1, 2, \dots, 2^{D+1}$ and $r_m(t)$ is the received signal during the interval $((m-1)T, mT)$.

At this point it is important to recall the importance of the transition probabilities in this receiver. They allow the receiver to distinguish which are the allowed transitions and which are not. In this sense our receiver does not need to store all the allowed paths but only the transition matrix of the code, in order to implement Viterbi algorithm's structure.

What the equation (4.4.13) means is very important because it says that the *MEPSE* receiver maximizes each time i the whole sequences from time 1 to time i . It uses the summation of all the metrics from the beginning until the state s_{ij} .

Also this equation says that our maximization process is reduced to

- i) discard not-allowed transitions and
- ii) estimate all those elements that have the minimum Euclidean distance in each path, with respect to the received one

So, in this task the work of the receiver is basically directed to what is under the integration sign. After expanding such expression as $r_m^2(t) - 2r_m(t)s_{mj}(t) + s_{mj}^2(t)$, we can see that the first term is independent of the estimated waveform and so, of the subindex j , and we can drop it from the metric to maximize. Also the last term gives us the energy of the j th code word, at time m , in the following way: $E_{jm} = \int_{(m-1)T}^{mT} s_{mj}^2(t)dt$. In the case of VEN_{DA} codes, all the code words have the same energy and this term could be also dropped. But not in general, so let us preserve this term for a general receiver of VEN codes.

So, the total metric to be used by the receiver in this maximization process, can be put as:

$$M_{jk,i} = \sum_{m=1}^i \left(\int_{(m-1)T}^{mT} r_m(t)s_{mj}(t)dt - \frac{1}{2}E_{jm} + B_{jkm} \right) \quad (4.4.14)$$

where

$$B_{jkm} \equiv \sigma^2 \ln P[s_{mj}|s_{(m-1)ik}] \quad (4.4.15)$$

By realizing that the first term of $M_{jk,i}$ does not depend on k and defining A_{jm} as:

$$A_{jm} = \int_{(m-1)T}^{mT} r_m(t)s_{mj}(t)dt - \frac{1}{2}E_{jm} \quad (4.4.16)$$

we have that

$$M_{jk,i} = \sum_{m=1}^i (A_{jm} + B_{jkm}) \quad (4.4.17)$$

By putting this expression as dependent of the previous term, in a recursive form, we have

$$M_{jk,i} = A_{ji} + B_{jki} + \sum_{m=1}^{i-1} (A_{jm} + B_{jkm}) \quad (4.4.18)$$

which leads to

$$M_{jk,i} = A_{ji} + B_{jki} + M_{jk(i-1)} \quad (4.4.19)$$

process in which the last objective function does not have to maximize again what was done in the previous time interval. Thus for maximizing M_{jk} , the receiver would work with the maximization of A_{jk} , which is the detection of the last element, with B_{jk} , which is the factor that discards not allowed transitions, and with $M_{j(k-1)}$ which are the already maximized summations until the previous time interval. Since $M_{j(k-1)}$ was already maximized it does not depend on k but on j . So, the objective function becomes

$$M_{jk} = \int_{(j-1)T}^T r_i(t)s_{i,j}(t)dt - \frac{1}{2}E_{j,m} + \sigma^2 \ln P[s_{i,j}|s_{(j-1)k}] + M_{j(k-1)} \quad (4.4.20)$$

The expression (4.4.20) suggests the configuration for the *MEPSE* receiver shown in Figure 4.4.1.

As we can observe this configuration needs a processor that stores each time m the minimum summation of metrics until previous time interval and takes the final decision at time i , by choosing that sequence with maximum value of metrics. This is simply the *Viterbi algorithm* (MLSE receiver), when the allowed probabilities are equal. Actually this *MEPSE* receiver is more general, since it applies to the case of unequal *a priori* probabilities.

The previous derivation gives us the optimum receiver for Markov sequences of first order. It can be shown that for higher orders of Markov processes the derivation of the optimum receiver according to *MEPSE* methodology is very similar to the previous one. The only difference is that the B_{jk} term will take the general form of equation (4.4.21).

$$B_{jkm} = \sigma^2 \ln P[s_{m,j}|s_{(m-1)k}, s_{(m-2)k}, \dots, s_{(m-q)k}] \quad (4.4.21)$$

This expression does not change the configuration of the receiver. Only that the B_{jkm} 's are different. Because these terms are the conveyers of the information about the transition rules among the waveforms in the sequence, they do not represent any problem because the transition probabilities of any code are predetermined and they can be stored in a transition matrix.

Fig. 4.4.1 Minimum error probability sequence estimation (MEPSE) receiver.

As an extension, whichever is the kind of sequence, the B_{jkm} 's are the conveyers of this information and so, the configuration previously obtained for Markov sequences of first order is indeed general for any sequence, given that the conditional or transition probabilities present in B_{jkm} are known

The implementation of the *MEPSE* receiver can be done through matched filters instead of using cross-correlators due to their equivalence, namely, if the received code word $r_m(t)$ is passed through a filter whose impulse response is $s_j(T-t)$, the output of the filter is the same as that given by the cross-correlators

$$r_m(t) \cdot s_j(T-t)|_{t=T} = \int_{-\infty}^{\infty} r_m(r) s_j(T-T+r) dr = \int_{(m-1)T}^{mT} r_m(t) s_j(t) dt \quad (4.4.22)$$

In sum, in the *MEPSE* receiver there is no restriction for the *a priori* probabilities of the sequences, or the *a priori* unconditional or conditional probabilities of their elements. It can be viewed as a generalization of the Viterbi algorithm (MLSE) receiver

Until now we have applied the properties of *VENDA* codes in a mathematical way. But, the main characteristics of *VENDA* codes of being a two-level codes and that their code words can be decomposed into pulses of equal duration, in order to simplify the structure of the *MEPSE* receiver for their specific case, have not yet invoked.

Fortunately, these characteristics of *VENDA* codes allow to achieve a simple structure of the *MEPSE* receiver.

In fact, instead of taking correlators for full code words, say, during a complete period T , we can think of having a correlator for the pulses that constitute the code words, i.e. during $\frac{T}{2(D+1)}$ seconds, by realizing that,

$$\int_{(m-1)T}^{mT} = \int_{(m-1)T}^{(m-1)T + \frac{T}{2(D+1)}} + \int_{(m-1)T + \frac{T}{2(D+1)}}^{(m-1)T + 2\frac{T}{2(D+1)}} + \dots + \int_{(m-1)T + [2(D+1)-1]\frac{T}{2(D+1)}}^{mT}$$

Then, by taking samples every $\frac{T}{2(D+1)}$ seconds we can reduce the complexity of 2^{D+1} correlators in our receiver to only two correlators one for each positive or negative pulse. Even

it can be reduced to only one correlator, by taking its output with a positive branch and a negative one. This can be visualized in the Figure 4.4.2. The outputs of the correlator can be stored in a shift register, with $2(D + 1)$ cells. This needs to be done to allow each branch of the *MEPSE* receiver to take its own summation of the integrations previously referred, every T seconds.

Let us see an example of this configuration. Let us suppose that VEN_1A code is used. Their code words are constituted by 4 pulses of equal duration, namely each pulse has a duration of $\frac{T}{4}$ seconds. Let us call C_j the output of the correlator at time $\frac{T}{4}$. Then, the configuration of the *MEPSE* receiver for VEN_1A , is that of the Figure 4.4.3.

For other values of D the configuration is similar to that obtained for VEN_1A code. Thus, in this manner we have simplified the structure of the *MEPSE* receiver for the case of $VEN_D A$ codes.

4.5 A practical (finite memory) *MEPSE* receiver

As we have seen, the optimum receiver for sequences is that of *MEPSE* receiver, in the sense of minimum probability of error. But, actually this is an ideal device, because from the practical point of view it needs a processor with a very large memory, for large values of D . It is necessary to think of a practical solution to this problem.

It is possible to think of a suboptimum receiver, which makes decisions over a maximum number of elements³. For example, a receiver which makes decisions on the last ν elements of the transmitted sequence. When the number of elements of the sequence reaches $\nu + 1$, then the first element is accepted and stored, and processing continues over the latest elements. This process leaves out the first element in order to include the last one. It is known that in most of the cases [7] the survivor paths have a common trunk not very far from the head of

³ Elements and code words will be used with the same meaning.

Fig. 4.4.2 Simplification of MEPSE receiver for $VEN_D A$ codes.

Fig. 4.4.3 MEPSE receiver applied to $VEN_1 A$ code.

the sequence. This fact further justifies the intention of approaches like this. It can be thought that instead of a simple shifting of only one element, accepting only one element at the time, it can be generalized to shift α elements out and accepting α new ones. Furthermore, this can be done in an adaptive manner, in order to avoid taking repeated decisions over elements that are in a common trunk. This implies an α variable.

In order to have an idea of the structure of this suboptimum sequence receiver we will use the results obtained for the *MEPSE* receiver, and the configuration of this new approach will be obtained. The decision function for *MEPSE* receiver is rewritten below,

$$M_{jk,i} = \sum_{m=1}^i (A_{jm} + B_{jkm}) \quad (4.5.1)$$

By establishing that the length of the sequence will be less than or equal to ν , the metric for this suboptimum receiver becomes:

$$M_{jk,i}^{\nu} \equiv \sum_{m=1-\nu}^i (A_{jm} + B_{jkm}) \quad (4.5.2)$$

Because the receiver has a processor that stores all the allowed surviving paths, it is easy to obtain the number of the first elements which are the same for all the paths. This number is just α .

Let us see an example of the operation of this suboptimum receiver. Let us establish that it works internally as the *MEPSE* receiver until ν elements have been received. Let us suppose that the reception starts.

- 1) When the last ν element has arrived, say $i = \nu$, this receiver searches for the number of elements in the common trunk, α_1 . At this moment its metric has the following expression:

$$M_{jk,i}^{\nu} = \sum_{m=1}^i A_{jm} + B_{jkm}$$

Then, the first α_1 elements are accepted and the receiver discards those transition metrics corresponding to the accepted α_1 elements.

- 2) The receiver continues receiving elements until $i = \nu + \alpha_1$. In this way ν elements will be in the receiver again. When this number is reached the receiver searches for the current number of elements in the common trunk, α_2 . At this time its metric becomes

$$M_{jk,i}^\nu = \sum_{m=1+\alpha_1}^i A_{jm} + B_{jkm}$$

Then, in the current ν elements, the first α_2 elements are accepted and their transition metrics discarded

- 3) The receiver continues receiving elements until $i = \nu + \alpha_1 + \alpha_2$ to make the searching of the first α_3 in the common trunk, in order to accept them and discard their transition metrics
- 4) And so on

The expression for the metric to be taken in account by the receiver at the last element of the sequence will be

$$M_{jk,i}^\nu = \sum_{1+\sum_{r=1}^R \alpha_r}^i A_{jm} + B_{jkm}$$

Where R is the number α s accepted elements and $i \leq \nu + \sum_{r=1}^R \alpha_r$. The $<$ sign stands for the case in which the number of elements of the sequence do not reach the right hand side number. The structure previously derived, let us call it *adaptive ν - MEPSE* receiver, is exactly the same as that of the *MEPSE* receiver plus an operator that compares the paths and obtains α s.

With this approach, practical Viterbi algorithm-based receivers can be obtained, as that of *MEPSE*. As we have seen, the final structure of this suboptimum receiver was relatively easy to find because of having the *MEPSE* structure.

This suboptimum receiver allows us to think in the minimum memory capacity necessary to give an acceptable performance. To do this it is necessary to work on numerical results of the probability of error. This will be done in the next section.

4.6 Probability of error for VEC codes.

In the calculation of the probability of error of VEC codes, we only will refer to VEC codes, because they have good spectral properties, ease of implementation and are two-level codes

4.6.1 Probability of error for truncated sequences of ν elements.

The calculation of the probability of error for a given sequence s_l with ν elements will be done following the *MEPSE methodology*

We have, in general, that the probability of error for ν -element received sequences can be expressed also as

$$P[E] = \sum_{l=1}^L P[E|s_l] P[s_l] \quad (4.6.1.2)$$

Also $P[E|s_l]$ can be expressed as the probability that the received sequence r is in I_l , where I_l represents all the space except the decision region I_l

In other words, $P[r \text{ in } \bar{I}_l | s_l]$ is the probability that r falls outside the region of correct decision I_l

By considering the decision regions as disjoint ones, and also that they all form the whole space, we have that,

$$P[r \text{ in } \bar{I}_l | s_l] = \sum_{\substack{t=1 \\ t \neq l}}^L P[r \text{ in } I_t | s_l] \quad (4.6.1.3)$$

where the subindex t denotes sequences other than l and L is the number of allowed sequences

We can see that for the case of a number of allowed sequences greater than or equal to two, the decision region of any sequence occupies at most half of the space. We can find an upperbound for the probability of error through this idea. So, we can establish that

$$P[r \text{ in } I_l | s_l] = \int_{I_l} p_n(r - s_l) dr \leq \int_{\text{space} \subset I_l} p_n(r - s_l) dr \quad (4.6.1.4)$$

The integration over the half space is the same as the product of all the integrations over all the dimensions⁴. By locating at $\mathbf{s}_l$ the origin of coordinates, we have

$$\int_{\substack{\text{space} \subset I_l}} p_n(\mathbf{r} - \mathbf{s}_l) d\mathbf{r} = \int_{d_{li}}^{\infty} () \int_{-\infty}^{\infty} () \int_{-\infty}^{\infty} () \int_{-\infty}^{\infty} () \quad (4.6.15)$$

where () is equivalent to $p_n(\mathbf{r} - \mathbf{s}_l)$. And d_{li} is the *Euclidean distance* between the allowed sequences $\mathbf{s}_l$ and $\mathbf{s}_i$. We also know that the integration of a p.d.f. from $-\infty$ to $+\infty$ is unity. Then,

$$\int_{\substack{\text{space} \subset I_l}} p_n(\mathbf{r} - \mathbf{s}_l) d\mathbf{r} = \int_{d_{li}}^{\infty} p_n(\mathbf{r} - \mathbf{s}_l) d\mathbf{r} \equiv Q\left(\frac{d_{li}}{\sqrt{2}N_0}\right) \quad (4.6.16)$$

Where $Q()$ is defined as⁵

$$Q(\alpha) \equiv \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\infty} e^{-\frac{\gamma^2}{2}} d\gamma$$

Substituting (4.6.16), (4.6.13) in (4.6.12), we have

$$P[E] \leq \sum_{i=1}^L P[\mathbf{s}_i] \sum_{\substack{l=1 \\ l \neq i}}^L Q\left(\frac{d_{li}}{\sqrt{2}N_0}\right) \quad (4.6.17)$$

For the case of VEN_{DA} codes we have that

$$L = 2^{D_{\nu}+1} \quad (4.6.18)$$

Also we know that all the *a priori* probabilities of the sequences are equal and have the value $\frac{1}{2^{D_{\nu}+1}}$.

It can be shown that for VEN_{DA} codes all the distances to a determined sequence, from all the other ones, are always the same. Let us take the first sequence as reference, i.e. $l = 1$. So, equation (4.6.17) can be simplified to the following one:

$$P[E] \leq \sum_{i=2}^L Q\left(\frac{d_{i1}}{\sqrt{2}N_0}\right) \quad (4.6.19)$$

⁴ Reference [1], equation (4.80c).

⁵ Reference [1], equation (3.50).

By realizing that when a sequence of binary data is coded through $VEN_D A$ codes, it can be done starting with positive or with negative. This fact is given because each message can be coded either with the code word starting with positive or with that starting with negative polarity. Only the conditional or transition probabilities control which is the corresponding code word to accomplish the coding of any message. Suppose we assume the convention that transmitter starts from the beginning with positive. For the case of truncated sequences, namely the receiver truncates the sequences, the receiver also has the knowledge of the polarity of the first element, because it has recognized and accepted the previous one⁶. In this way any sequence can be confused only with those starting with the same polarity as it. This leads to consider only half of the code words in our calculation of this upperbound of the probability of error of $VEN_D A$ codes. Thus, equation (4.6.1.9) becomes simplified to

$$P[E] \leq \sum_{i=2}^L Q\left(\frac{d_{1i}}{\sqrt{2N_0}}\right) \quad (4.6.1.10)$$

Because the expression (4.6.1.10) contemplates the probability that one sequence is in error, we will now derive the corresponding or equivalent probability of error per word

Word error probability for truncated sequences of ν elements.

In order to obtain the word error probability we will use a simple example to establish the fundamentals of this approach

Let us suppose that we have a sequence error probability of 0.1 in the reception of some sequence. This means that a 10% of the total number of received sequences are expected to be in error. Because probability of error is a dimensionless quantity it can be put as:

$$P_{seq}[E] \approx \frac{\text{Number of sequences in error}}{\text{Total number of sequences}}$$

Let us suppose that the number of words in error per each sequence in error is the same for all of them, let 3 be this number, and the total number of words per sequence is 10. Also let's assume that the probability of a sequence in error to occur is the same for all of them.

⁶ This one corresponds to the last element of the previous accepted sequence

If 100 sequences were transmitted we expect 10 out of them in error, i.e. $P_{seq}[E] = 0.1$. Also, because each sequence in error conveys 3 words in error, we will expect 30 words in error

$$\text{Number of words in error} = \frac{3 \text{ words in error}}{1 \text{ sequence in error}} \times 10 \text{ sequences in error}$$

The total number of transmitted words are $10 \times 100 = 1000$. So, the word error probability will be

$$P_w[E] \approx \frac{\text{Number of words in error}}{\text{Total number of transmitted words}} = \frac{30}{1000} = 0.03$$

Let us now suppose that the number of words in error per sequence are not the same. Let us suppose that 3 out of the 10 sequences in error have 2 words in error and the other 7 have 5 words in error. Let us assume the same error probability. Thus, the word error probability will be $P_w[E] \approx 0.041$.

When we expected that only three sequences out of ten, with two words in error, are present, as erroneous sequences, in the total received number of sequences, i.e. one hundred, we really are assuming that $\frac{3}{100}$ is the conditional probability that they occur, or the probability of receiving them given that the correct sequence was transmitted; and also that the three sequences have the same conditional probability of occurrence, i.e. $\frac{1}{100}$.

In this way, we can put that

$$P_w[E] \approx \frac{3 \times 2 + 7 \times 5}{100 \times 10} = 0.03 \times \frac{2}{10} + 0.07 \times \frac{5}{10}$$

Which can be put also as:

$$P_w[E] \approx 0.01 \frac{2}{10} + 0.01 \frac{2}{10} + 0.01 \frac{2}{10} + 0.01 \frac{5}{10} + 0.01 \frac{5}{10} + 0.01 \frac{5}{10} + 0.01 \frac{5}{10}$$

Because statistical relationships approach to their probabilistic counterpart when involved numbers are very big, we can put, as an exact relationship, that:

$$P_w[E|s_i] = \sum_{t=1}^{10} P[E_t|s_i] \omega_t \quad (4.6.1.11)$$

Where $P[E_t|s_t]$ is the conditional probability that the received sequence falls in the erroneous region I_t , and ω_t is the sequence error fraction defined as:

$$\omega_t = \frac{\text{Number of words in error in sequence } t}{\text{Number of words of sequence } t}$$

Analyzing the last expression we can see that the word error probability, for this example, is given by the summation of the products of the conditional probabilities by the respective fractions of the sequences that have word errors

The previous example is really a very bad example to arrive at the last conclusion, because the made assumptions are very restrictive. In the real life we find sequences that can differ from the correct one in 1, 2, 3, ..., ν words with different probabilities of occurrence. And also that the sequence error fraction, and the conditional probabilities, depend on which is the correct transmitted sequence.

By making a more complete example than this one, taking into account all the referred generalizations we can arrive at similar results.

Let us calculate the conditional word error probability that the sequence s_t was transmitted. Statistical interpretation of this probability requires the occurrence of a very large number events of this type. In order to use the equal sign for equations, let us assume that *many* truncated sequences s_t have been received. Let us assume that the receiver accepts the sequences, one by one⁷. Let L the number of allowed sequences, ν the number of elements per sequence, n_{wt} the number of words in error present in the sequence s_t , n_{st} the number of occurrences of the sequence s_t , N_S the number of received sequences, N_{wt} the total number of words in error contributed by the n_{st} occurrences of s_t , and N_W the total number of transmitted words.

⁷ After we will change our assumption to that the receiver accepts the first elements of the sequence that are in a common trunk, and starts analyzing a new one, without those first elements, but adding equal number to the head of the sequence, keeping constant the total number of elements.

22
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The word error probability, given that s_i was transmitted, can be viewed as the total number of words in error with respect to it, divided by the total number of transmitted words. Which, with the defined notation, becomes.

$$P_w[E|s_i] = \frac{\sum_{t=1}^L N_{wt}}{N_w} = \frac{\sum_{t=1}^L \frac{n_{st}}{N_s} \cdot \frac{n_{wt}}{\nu}}{N_s \cdot \nu}$$

This expression can be put in a more convenient manner:

$$P_w[E|s_i] = \sum_{t=1}^L \frac{n_{st}}{N_s} \cdot \frac{n_{wt}}{\nu}$$

The factor $\frac{n_{st}}{N_s}$ is the expected number of occurrences of the incorrect sequence s_i , divided by the number of received sequences. This gives us the probability that the sequence s_i is received given that s_i was transmitted, and the second factor $\frac{n_{wt}}{\nu}$ is the fraction of the sequence s_i in error. Defining this fraction as ω_i , and thinking of very large numbers, we have:

$$P_w[E|s_i] = \sum_{t=1}^L \omega_i P[r \text{ in } I_i | s_i]$$

By applying probability rules to the previous encountered expression we can reach the general expression for any received sequence. Hence, the total word error probability will be given by:

$$P_w[E] = \sum_{i=1}^L P[E|s_i] P[s_i] = \sum_{i=1}^L P[s_i] \sum_{t=1}^L \omega_i P[r \text{ in } I_i | s_i] \quad (4.5.1.12)$$

So, this expression is completely valid for truncated sequences of ν elements. But this solution of the word error probability is for a receiver that analyzes sequence by sequence, accepting each one as a whole.

When we are in the case of a receiver that accepts the first α elements that have a common path, and shifts its focus over a new ν elements without the first accepted α , it is hoped that the word error probability is reduced.

Let us assume that we have this new receiver. The first thing that we realize is that it re-analyzes those elements that are not in the common path. These re-analysis means

for the receiver only more sequences. In other words, it looks like if the number of received sequences were a bigger number as it actually is, because of the multiplicative effect of shifting the sequences. Also, for this case it is possible to arrive at an expression for the word error probability

Let us assume, in principle, that α is a constant number. Let us assume that the transmitter knows that the receiver has this receiving mode, and in order to ensure that all the accepted elements are always the first α elements, it sends at the end of the transmission enough redundant elements. This assumption is done in order not to complicate the derivation

Then, it can be shown that if the transmitter sends the equivalent to N_S sequences of ν elements, the total number of analyzed and *re-analyzed*⁸ elements in the receiver is

$$N_W = \left(\frac{N_S \cdot \nu}{\alpha} \right) \cdot \nu$$

By considering the total number of analyzed elements as the *the total number of received elements*, the word error probability given that the sequence s_i was transmitted, for this receiver becomes:

$$P[E|s_i] = \frac{\sum_{t=1}^L n_{st} \cdot n_{wt}}{\left(\frac{N_S \cdot \nu}{\alpha} \right) \cdot \nu}$$

which can be expressed more conveniently, as:

$$P[E|s_i] = \frac{\alpha}{\nu} \sum_{t=1}^L \frac{n_{st} \cdot n_{wt}}{N_S \cdot \nu}$$

As we can recognize, the summation factor of the right hand side is the same expression obtained previously. Also, we can see that when $\alpha = \nu$, it equals the conditional error probability of the previous receiver, which is a logical result, because the receiver is accepting $\alpha = \nu$ elements at the time, which is the characteristic of the previous receiver. So,

$$P_w[E] = \frac{\alpha}{\nu} \sum_{i=1}^L P[s_i] \sum_{t=1}^L \omega_t P[r \text{ in } I_t | s_i] \quad (4.6.1.13)$$

⁸ Because this is a Viterbi-based receiver, re-analysis does not mean extra work for the receiver.

Actually, something like this is what occurs in a practical Viterbi-based receiver, because of its limited memory. The word error probability for a variable α will be less or equal to that calculated through equation (4.6.1.13) with a mean value of α , say $\bar{\alpha}$, because the adaptive character of α .

The expression (4.6.1.13) is an exact expression of the word error probability, for the case of truncated sequences, with an α constant.

In order to arrive at the same base of comparison among the different codes it is necessary to calculate the bit error probability.

Bit error probability for truncated sequences of ν elements.

By using a similar analysis as the previous one, we will denote n_{bt} as the number of bits in error in the sequence s_t , n_{st} the number of occurrences of the sequence s_t , b the number of bits, which is the same as D , the number of digits per block, N_{bt} the total number of bits in error in the sequence s_t , N_s the total number of received sequences. Then, the conditional bit-error probability that the sequence s_t was transmitted, for a receiver that receives sequence by sequence, can be put as:

$$P_b[E|s_t] = \frac{\sum_{t=1}^L n_{st} \cdot n_{bt}}{N_s \cdot \nu \cdot b} = \sum_{t=1}^L \frac{n_{st}}{N_s} \frac{n_{bt}}{\nu \cdot b}$$

where, $\frac{n_{st}}{N_s}$ represents $P[r \text{ in } I_t | s_t]$, and the fraction $\frac{n_{bt}}{\nu \cdot b}$ represents the fraction of bits in error of the sequence s_t . By defining this fraction as ω_{bt} we have,

$$P_b[E|s_t] = \sum_{t=1}^L \omega_{bt} P[r \text{ in } I_t | s_t]$$

which is a similar expression to that previously obtained.

Thus, for the case of a receiver that accepts the first α elements, which is our final goal, the total number of analysed and re-analysed bits is:

$$N_B = \left(\frac{N_s \cdot \nu}{\alpha} \right) \cdot \nu \cdot b$$

Then, the conditional bit-error probability that the sequence s_l was transmitted, is given by

$$P_b[E|s_l] = \frac{\alpha}{\nu} \sum_{t=1}^L \omega_{lt} P[r \text{ in } I_t | s_l]$$

and the total bit-error probability becomes

$$P_b[E] = \frac{\alpha}{\nu} \sum_{l=1}^L P[s_l] \sum_{t=1}^L \omega_{lt} P[r \text{ in } I_t | s_l]$$

By associating each code word of VEN_{DA} codes with the corresponding binary block in the data stream, we can know the bit differences among them.

By taking the same way to derive the upperbound, used to derive that of the sequence error probability, we arrived at the following expression for the bit error probability:

$$P_b[E] \leq \frac{\alpha}{\nu} \sum_{l=1}^L P[s_l] \sum_{t=1}^L \omega_{lt} Q\left(\frac{d_{lt}}{\sqrt{2N_0}}\right) \quad (4.6.14)$$

The condition of $t \neq l$ was dropped, because ω_{lt} takes into account the situation when s_l is compared with itself. It gives zero bits in error, eliminating that term from the summation.

It can be shown that, because of the symmetry of VEN_{DA} code words, the summation indicated by the second sum sign are always the same for any value of l . Thus, by taking $l = 1$ the expression (4.6.14) simplifies to:

$$P_b[E] \leq \frac{\alpha}{\nu} \sum_{t=2}^L \omega_{1t} Q\left(\frac{d_{1t}}{\sqrt{2N_0}}\right) \quad (4.6.15)$$

It can be realized that this upperbound is very similar to that of the Union bound [1], [8], [9], [10].

At this point it is important to discuss the significance of d_{lt} . This distance between the sequences l and t , is indeed a measure of the energy of the difference between both sequences. So, it gives us the energy amount of such difference. This is an absolute value. The probability of error is a quantity that depends on this difference energy and on the noise variance, taken them as absolute and independent values.

In this sense, when we talk about the energy of a $VEN_D A$ code word, its amount depends on the number of digits per block D , because the duration of such a code word is D times the duration of the binary digit. Thus, we can establish the following relationship:

$$E_D = DE_b$$

where E_D and E_b are the energies of a code word of a block code with D digits per block, and the corresponding to a binary digit, respectively.

Any code word of a $VEN_D A$ code can be represented by:

$$s_k = \sqrt{\frac{E_D}{2(D+1)}} (x_1, x_2, \dots, x_{2(D+1)}) \quad (4.6.1.16)$$

where $x_n = -1, +1$. By defining the row vector $\mathbf{z}$ as: $\mathbf{z}_k \equiv (x_1, x_2, \dots, x_{2(D+1)})$, then the code word s_k expressed as a row vector becomes:

$$s_k = \sqrt{\frac{E_D}{2(D+1)}} \mathbf{z}_k$$

Thus, Euclidean distance between s_j and s_k of a $VEN_D A$ code, with a number of D digits, or bits, per block, is:

$$d_{jk}^2 = |s_j - s_k|^2 = \frac{E_D}{2(D+1)} |\mathbf{z}_j - \mathbf{z}_k|^2 \quad (4.6.1.17)$$

The same extension can be done for sequences, where the number of components of the vector $\mathbf{z}$ is ν times the number of components per code word, i.e. $2\nu(D+1)$, and the corresponding energy $E_{D\nu} = D\nu E_b$. So, equation (4.6.1.15) can be put as:

$$P_b[E] \leq \frac{\alpha}{\nu} \sum_{i=1}^L \omega_i Q \left(\sqrt{\frac{|\mathbf{z}_i - \mathbf{z}_1|^2 D E_b}{4(D+1) N_0}} \right) \quad (4.6.1.18)$$

Thus, equation (4.6.1.18) allows us to calculate the upperbound for the bit error probability of any $VEN_D A$ code, for any number of elements per sequence and for any number of elements per block. This expression will be used for the representation of the curves of bit error probability with respect to the signal to noise ratio, taken as that given by the signal to noise ratio per

bit. We have found that this is an acceptable base of comparison among the $VEN_D A$ codes for different values of D , and considering also for sequences with different number of elements,

The total probability of error previously calculated is referred to the bit error probability of a sequence s_i of coded data. We can show that this equals the bit error probability of the input sequence of binary data m .

Any sequence of binary data can be coded through $VEN_D A$, as the sequence s_i or as the negative one $-s_i$, with a probability of $\frac{1}{2}$ for each one.

This can be expressed as

$$\begin{aligned} P[C|m_i] &= P[C|s_i]P[s_i] + P[C|-s_i]P[-s_i] \\ &= \frac{1}{2}P[C|s_i] + \frac{1}{2}P[C|-s_i] \end{aligned}$$

We know that

$$\begin{aligned} P[C|s_i] &= \int_{I_i} p_n(r - s_i) dr \\ P[C|-s_i] &= \int_{I_{-i}} p_n(r + s_i) dr \end{aligned}$$

But, r , given that s_i was transmitted, is composed of the summation of the noise plus the transmitted sequence, then we have that

$$r - s_i = n$$

The same can be said for $-s_i$:

$$r + s_i = n$$

Then, because the decision regions are symmetric in the space respect to the origin, and have the same area, we can establish that

$$P[C|s_i] = P[C|-s_i]$$

Thus,

$$P[C] = \sum_{i=1}^{2^M} P[C|m_i]P[m_i] = \sum_{i=1}^{L/2} P[C|s_i]P[s_i] = \sum_{i=1}^{L/2} P[C - s_i]P[s_i] \quad (4.6.1.20)$$

where M is the number of elements of the data sequence m and L , as we know, is related to D and ν as $L = 2^{D\nu+1}$, and $2^M = L/2 = 2^{D\nu}$.

This means that the total probability of error calculated before the decoder, level of sequences s_i , is the same after it, level of decoded sequences m . This is an obvious result because the correspondence one-to-one between $\{m_i\}$ and $\{s_i\}$. This also justifies our simplified model of the Figure 4.2.2.

Example of calculation of $P_b[E]$ for VEN_1A code.

The VEN_1A code words can be represented, according to equation (4.6.1.15), as:

$$\begin{aligned} s_1 &= \sqrt{\frac{E_b}{4}}(1, -1, -1, 1) & s_2 &= \sqrt{\frac{E_b}{4}}(1, 1, -1, -1) \\ s_3 &= \sqrt{\frac{E_b}{4}}(-1, -1, 1, 1) & s_4 &= \sqrt{\frac{E_b}{4}}(-1, 1, 1, -1) \end{aligned}$$

The different distances among code words are the following ones:

$$d^2 = \begin{pmatrix} d_{11}^2 & d_{12}^2 & d_{13}^2 & d_{14}^2 \\ d_{21}^2 & d_{22}^2 & d_{23}^2 & d_{24}^2 \\ d_{31}^2 & d_{32}^2 & d_{33}^2 & d_{34}^2 \\ d_{41}^2 & d_{42}^2 & d_{43}^2 & d_{44}^2 \end{pmatrix} = E_b \begin{pmatrix} 0 & 2 & 2 & 4 \\ 2 & 0 & 4 & 2 \\ 2 & 4 & 0 & 2 \\ 4 & 2 & 2 & 0 \end{pmatrix}$$

Distances among sequences are calculated in the same way. The allowed sequences for different values of ν are:

$\nu = 1$	$\nu = 2$	$\nu = 3$
s_1	$s_1 s_1$	$s_1 s_1 s_1$
s_2	$s_1 s_2$	$s_1 s_1 s_2$
s_3	$s_2 s_3$	$s_1 s_2 s_3$
s_4	$s_2 s_4$	$s_1 s_2 s_4$
—	$s_3 s_1$	$s_2 s_3 s_1$
—	$s_3 s_2$	$s_2 s_3 s_2$
—	$s_4 s_3$	$s_2 s_4 s_3$
—	$s_4 s_4$	$s_2 s_4 s_4$
—	—	$s_3 s_1 s_1$
—	—	$s_3 s_1 s_2$
—	—	$s_3 s_2 s_3$
—	—	$s_3 s_2 s_4$
—	—	$s_4 s_3 s_1$
—	—	$s_4 s_3 s_2$
—	—	$s_4 s_4 s_3$
—	—	$s_4 s_4 s_4$

And so on.

By calculating the distances between each one of the sequences of the first half (starting with positive) and the first sequence using equation (4.6.1.17), and introducing them in equation (4.6.1.18), we get the curves of Figure 4.6.1.1, corresponding to the upperbound. In that Figure we can observe the effect of using a sequence receiver instead of symbol-by-symbol receiver: *the probability of error is reduced*. These curves were taken for $\alpha = \nu$. For different values of α , the curves only are shifted in the vertical axis with less values of the bit error probability. In Figure 4.6.1.2 is represented a comparison among two different codings: $VEN_1 A$

and VEN_2A . We can observe in this Figure the two effects, those of the sequence receiver and the effect of using blocks of D digits instead of single transmission bit-by-bit. We can see that each effect, separately, reduces the probability of error. By using both transmission and receiving modes simultaneously, it leads to achieve a better performance of the receiver.

As we have pointed out, the upperbound calculated previously is not too tight but it allows us to have a some insight of the performance of VEN_DA codes. The reason that it is loose at low values of *signal to noise ratio*, is because of the overlapping of the decision regions, but for values above 5 or more dB, it is reasonably tight for sequences whose code words do not convey much information. When these code words are digital blocks the number of sequences grows exponentially with the number of digits per block. This means more overlapping and it required us to present the comparison among the different VEN_DA codes, in Figure 4.6.1.2 between 4 and 10 dB. The bound of 10 dB was given by high time computing consumption and by the tolerance of a numerical integration used in the computer program.

4.6.2 Minimum distance of VEN_DA codes.

VEN_DA codes have the property of having a general expression for the minimum distance among sequences of ν code words. By representing them in vectorial form we have, according to equation (4.6.1.17), that the Euclidean distance between two sequences is given by:

$$d_{11}^2 = |s_1 - s_2|^2 = \frac{DE_b}{2(D+1)} |z_1 - z_2|^2 \quad (4.6.2.1)$$

We can see that for VEN_1A code the minimum distance among sequences, by considering one correct and the other incorrect starting and ending in a common path, is

$$d_{\min}^2 = \frac{E_b}{4} \cdot (2^2 + 2^2 + 2^2 + 2^2) = 4E_b \quad (4.6.2.2)$$

For VEN_2A and for any other VEN_DA code it is possible to show that the minimum

distance among sequences is given by

$$d_{\min}^2 = 4E_b \frac{D}{D+1} \quad (\text{for } D > 1) \quad (4.6.2.3)$$

From equations (4.6.2.2) and (4.6.2.3) we can realize that the *maximum* minimum distance of $VEN_D A$ sequences occurs for $D = 1$, and the corresponding to other values of D , reach this *maximum* only when D reaches an infinite value. However, we can see from the curves of probability of error that $VEN_2 A$ code presents a lower probability of error than $VEN_1 A$, despite the minimum distance of the former being less than the latter.

The reason for this is that the minimum distance occurs only among a relatively smaller number of sequences.

4.6.3 $VEN_D A$ codes and their representation of their signals in the space.

We have seen that $VEN_D A$ codes' code words can be represented as vectors, through equation (4.6.1.16) where the components of such vectors can only take two values: ± 1 . The space defined by these components has $2(D+1)$ dimensions.

This is a particular case of code words located at the vertices of an *hypercube* [1]. But from the $2^{2(D+1)}$ vertices of the hypercube only 2^{D+1} are occupied; the rest are empty.

Because of the fact that $VEN_D A$ codes' code words occupy some of the vertices of an hypercube it is apparent that it is possible to arrive at tighter upperbounds.

There are some important properties of the cubes that could help in this task. For example, perpendicularity. All axes arriving at a vertex are mutually perpendicular. The number of border axes arriving at any vertex are N , the dimension of the space, or in other words, the number of components of the code word's vectors. The calculation of the probability of a correct decision corresponding to an empty vertex is function of $p = Q(d_{it}/\sqrt{2N_0})$. The empty vertices that contributes to the decision region of any signal, or filled vertex, are those

around the filled vertex. Among these empty vertices are the contiguous ones and those located along the diagonals. The contiguous vertices have the characteristic that their code words differ in only one component from the other contiguous vertex. Diagonals vertices can differ, with respect to a determined vertex, in two, three, ... h components. The number of them are given by $\binom{N}{h} = \frac{N!}{h!(N-h)!}$. We have tried to find an upperbound through this idea but, computing time consumption is very high. Also, because we are trying with a multidimensional space, it is not sure that we are taking in account all the factors involved in the problem. In any case, we think that despite the failed attempts, it is possible to reach tighter upperbounds through this idea.

Fig. 4.6.1.1. Bit-error probability for VEN, A code in a V -element truncate-sequence receiver

Fig. 4.6.1.2. Bit-error probability for VEN_1A , VEN_2A

Appendix A.4.1

Program to calculate the Bit Error Probability

```

DIM A(16,4)
DIM Z(32,8)
REM
REM
REM      "Building VEN codes' signals"
REM
REM
P5=4*ATN(1)
INPUT D
FOR J=1 TO 2^D
FOR G=1 TO 2*(D+1)
INPUT Z(J,G)
Z((2^(D+1) + 1 - J),G)=-Z(J,G)
NEXT G
NEXT J
FOR J1=1 TO 2^(D+1)
FOR G1=1 TO 2*(D+1)
PRINT Z(J1,G1);
NEXT G1
PRINT
NEXT J1
PRINT
FOR J2=1 TO 2^D
FOR G2=1 TO D
INPUT A(J2,G2)
/  A((2^(D+1) + 1 - J2),G2)=A(J2,G2)
NEXT G2
NEXT J2
FOR J3=1 TO 2^(D + 1)
FOR G3=1 TO D
PRINT A(J3,G3);
NEXT G3
PRINT
NEXT J3
PRINT
FOR S9=4 TO 10 STEP 2
PRINT
PRINT
PRINT "      D = ";D;"      SNR(DB) = ";S9
PRINT
PRINT "Time-periods","      Bit Error Prob."
PRINT
FOR I=1 TO 4
PB=0
S8=10^(S9/10)
L=2^(D+I)
N=2*(D+1)
FOR L1=1 TO L
REM

```

```

REM
REM      "Calculation of subindexes of elements
REM      of i-element sequences"
REM
REM
K=0
D2=0
W=0
FOR M=1 TO I
E=-1
600 E=E+1
K=L1 - 2^(M*D + 1)*E
IF K>2^(M*D + 1) THEN 600
S(M)=INT((K - 0.1)/(2^D)^(M-1) + 1)
FOR P=1 TO M
D2=D2 + (Z(S(M),P) - Z(1,P))^2
NEXT P
FOR G4=1 TO D
W=W+ ABS(A(S(M),G4) - A(1,G4))
NEXT G4
NEXT M
IF D2=0 THEN 1150
D2=D*D2/M
Y=SQR(D2*88/2)
IF Y<=3 THEN 770
S6=0.5 - 1/(SQR(2*P5)*Y)*EXP(-Y*Y/2)
GO TO 1130
770 REM
REM
REM      "Calculation of Q(Y) by numerical integration"
REM
REM
DEF FNF(X)=EXP(-X*X/2)/SQR(2*P5)
T9=1.0E-6
P1=2
LO=0
U=Y
D7=(U - LO)/P1
O=FNF(LO + D7)
E8=0
E7=FNF(LO) + FNF(U)
S6=(E7 + 4*O)*D7/3
S1=0
930 IF ABS(S6 - S1)>ABS(T9*S6) THEN 950
IF ABS(S6 - S1)<=ABS(T9*S6) THEN 1130
950 P1=P1*2
S1=S6
D7=(U - LO)/P1
E8=E8 + O
O=0
L2=INT(P1/2)
FOR H=1 TO L2
X=LO + D7*(2*H - 1)
O=O + FNF(X)
NEXT H
REM
REM

```

```

      REM      "Calculation of the Bit Error
      REM      Probability"
      REM
      REM
      S6=(E7 + 4*0 + E8*2)*D7/3
      GO TO 930
1130  S6=(0 5 - S6)*W/(I*D)
      P8=P8 + S6
1150  NEXT L1
      PRINT "      ",I,"      ",P8
      NEXT I
      NEXT S9
      END

```

Chapter 4. References

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Chapter 5

Synchronization of VEN codes.

5.1 Introduction.

In all our previous study we have supposed that the clock reference was completely known, namely we were working on a coherent data transmission system. This a desirable characteristic and in this chapter we will face the synchronization problem. The main aspects of the synchronization between the transmitter and the receiver are:

- i) the lack of knowledge about the starting time of a signal, if T is known.
- ii) the ignorance of the variations of the period T of the signal.

The multiplier effect of the repeaters are one of the causes of these impairments. Also the imperfections of the transmitter oscillator constitutes another of the causes.

The analysis of the synchronization problem will be done to obtain the optimum synchronizer in the sense of the minimum error probability.

We will consider the following assumptions

- 1) The noise is stationary, Gaussian and white with zero mean.
- 2) Data symbols are mutually independent.
- 3) Intersymbol interference is not considered.
- 4) The receiver *knows* almost exactly what is the time-period of the transmitted signals or block of signals.

- 5) The receiver knows exactly the elements of the transmitted sequence.

The previous assumptions are done in order to facilitate the derivations. After the general treatment to synchronization problem, we will apply its results to the synchronization of *VEN* codes.

5.2 Minimum error probability for symbol synchronization.

For this case we will suppose that the period of the transmitted signal, T , is known, but receiver does not know when the signal starts or when it ends.

Let us say that there exists an actual difference r , between the time that the receiver estimates to start the period T , and that at which the transmitter sends the signal. Let us suppose that the receiver estimates a $\hat{r}$, in order to detect correctly the transmitted signal. In this way, the real r , actually is present in the received signal r as

$$r(t) = n(t) + s_i(t - r) \quad (5.2.1)$$

And the receiver's estimate will be

$$\hat{s}_i(t) = s_i(t - \hat{r}) \quad (5.2.2)$$

In this way,

$$r(t) - \hat{s}_i(t) = n(t) + s_i(t - r) - s_i(t - \hat{r})$$

Where we see that when $\hat{r} = r$, receiver's estimation is correct.

In order to simplify the notation, let us assume that the delay introduced by the transmission line is r . Let us assume that r is a continuous random variable taking values between $-T$ and T . Let us consider also that it is slowly time-varying [1]. This last consideration means that there exists a correlation among the values of r . Moreover, about this aspect Stiffler [1] says: "To ignore this correlation is to discard much useful information"¹.

¹ Reference [1], page 124.

By following the same methodology previously applied, and in order to obtain an optimum symbol synchronizer, we have:

$$P[E] = 1 - P[C]$$

Where $P[C]$ depends on the sequence of transmitted symbols and on τ .

Let us assume that τ is independent of the sequence of transmitted symbols, s_i . Then, we have that the total probability of a correct decision will be given by

$$P[C] = \int_{-T}^T P[C|s_i, \tau] p_\tau(\tau) d\tau \quad (5.2.3)$$

Where $p_\tau(\tau)$ is the p.d.f. of τ and $P[C|s_i, \tau]$ is the conditional probability of making a correct decision that a given sequence s_i has been transmitted with a variation in the starting time of τ . Also, this conditional probabilities can be expressed as:

$$P[C|s_i, \tau] = P[r \text{ in } I_i \text{ with } \hat{\tau}|s_i, \tau] \quad (5.2.4)$$

Where I_i is the decision region corresponding to the sequence s_i . Equation (5.2.4) can be put as:

$$P[C|s_i, \tau] = \int_{I_i} p_n(r(t) - s_i(t - \hat{\tau})) dr \quad (5.2.5)$$

Assuming that noise is independent of transmitted symbols, and the elements of the noise sequence independent among them, equation (5.2.5) becomes:

$$P[C|s_i, \tau] = \int_{I_i} \prod_{m=1}^i p_{n_m}(r_m(t) - s_{im}(t - \hat{\tau})) dr \quad (5.2.6)$$

By considering a zero mean, white, Gaussian noise with variance equal to $\frac{N_0}{2}$, we have that

$$P[C|s_i, \tau] = \int_{I_i} \prod_{m=1}^i \frac{1}{(\pi N_0)^{\frac{1}{2}}} \exp\left\{-\frac{1}{N_0} \int_{(m-1)T+\tau}^{mT+\tau} [r_m(t) - s_{im}(t - \hat{\tau})]^2 dt\right\} dr \quad (5.2.7)$$

At this moment it is important to make the following observations. Our time of observation is T for the estimation of the starting time τ . Then, our sample was taken starting at

time $(m-1)T + \hat{\tau}$ until time $mT + \hat{\tau}$, in order to ensure that our sample takes the complete estimated signal $s_{lm}(t)$, starting at the correct time.

Another important observation is referred to the limits of the integration over the region I_{lr} . The integration sign can not be decomposed in a product of integrations because it is not true that $dr = \prod_{m=1}^i dr_m$. This aspect is what introduces the difference between a sequence receiver and a symbol-by-symbol one.

Because our assumption on the slowly time-varying of τ , the decision regions can be considered independent of the estimate starting times. Actually, $\hat{\tau}$ only slightly shifts the decision region, but its area remains the same.

Minimizing the probability of error with respect to $\hat{\tau}$ is the same as maximizing the probability of a correct decision $P[C]$. The maximization of $P[C]$, which is just the mean of $P[C|s_l, \tau]$, with respect to $\hat{\tau}$, is the same as maximizing such conditional probability of correct decision, given our previous assumption that p_r is independent of the values of τ . Then in equation (5.2.3) this task is accomplished, taking into account our previous considerations in order to discard those terms independent of $\hat{\tau}$, by maximizing the conditional probability of making a correct decision that the sequence s_l , with a starting time $\hat{\tau}$, has been transmitted, with respect to $\hat{\tau}$.

By observing that the logarithm is monotonic with respect to its argument, taking the logarithm on the expression of equation (5.2.6), and discarding constant terms, we arrive at the following objective function:

$$A = -\frac{1}{N_0} \sum_{m=1}^i \int_{(m-1)T+\hat{\tau}}^{mT+\hat{\tau}} [r_m(t) - s_{lm}(t - \hat{\tau})]^2 dt \quad (5.2.8)$$

By taking derivatives of A with respect to $\hat{\tau}$, we have

$$\frac{dA}{d\hat{\tau}} = -\frac{1}{N_0} \frac{d}{d\hat{\tau}} \sum_{m=1}^i \int_{(m-1)T+\hat{\tau}}^{mT+\hat{\tau}} [r_m(t) - s_{lm}(t - \hat{\tau})]^2 dt \quad (5.2.9)$$

In order to avoid the derivation of the limits of integration, let us make the following change of variables:

$$t - \hat{t} = \lambda$$

then, we have that:

$$\frac{dA}{d\hat{t}} = -\frac{1}{N_0} \frac{d}{d\hat{t}} \sum_{m=1}^i \int_{(m-1)T}^{mT} [r_m(\lambda + \hat{t}) - s_{tm}(\lambda)]^2 d\lambda \quad (5.2.10)$$

Which, finally gives:

$$\frac{dA}{d\hat{t}} = -\frac{1}{N_0} \sum_{m=1}^i \int_{(m-1)T}^{mT} 2[r_m(\lambda + \hat{t}) - s_{tm}(\lambda)] r'_m(\lambda + \hat{t}) d\lambda \quad (5.2.11)$$

Coming back to the original variables, we have:

$$\frac{dA}{d\hat{t}} = -\frac{1}{N_0} \sum_{m=1}^i \int_{(m-1)T+\hat{t}}^{mT+\hat{t}} 2[r_m(t) - s_{tm}(t - \hat{t})] r'_m(t) dt \quad (5.2.12)$$

This equation gives us the tools for obtaining a synchronizer, because the derivative of A is a good measure of the estimation error.

Because this is a general derivation, without restrictions in the shape of the signals, it is applicable to all the *VEN* codes. We can conclude that this synchronizer is optimum in the sense of *minimum error probability*.

5.3 Minimum error probability for clock synchronization.

As we have pointed out in the previous section, it is necessary to have knowledge of the time period, in order to do all the other things dependent on it.

Until the previous section, inclusive, we supposed a perfect knowledge of the time period T of the signal. The purpose of this section is precisely to face the clock synchronisation problem.

Let us state the problem as follows. Receiver knows exactly the signals it should receive, but it does not know exactly their duration. We will assume that the transmitter can be an imperfect device and the time-period can be time-varying. As before the signals are corrupted with noise which is assumed to be white, zero mean, Gaussian and variance equal to $\frac{N_0}{2}$.

Let us assume that there is an agreement between the receiver and the transmitter that the value of the time-period has a value of T_0 , but because of imperfections of the oscillator in the transmitter and other causes, the real time-period arriving at the receiver is T . The receiver estimates a duration of the time-period $\hat{T}$.

Let us assume also that T is a continuous random variable taking values between $\frac{T}{2}$ and $T + \frac{T}{2}$.

By applying the methodology previously used, we have that in order to achieve minimum error probability

$$\min P[E] = \min(1 - P[C]) = \max P[C] \quad (5.3.1)$$

Where the probability of a correct decision means that our estimation that the transmitted signal is that one of the known set, and has a duration of $\hat{T}$ seconds.

Assuming that the clock signals and the sequence of transmitted waveforms are independent, we can put:

$$P[C] = \int_{\frac{T}{2}}^{T + \frac{T}{2}} P[C|s_t, T] p_r(T) dT \quad (5.3.2)$$

The conditional probability of a correct decision that $\hat{s}_t$, with a period T , was transmitted is given by:

$$\begin{aligned} P[C|s_t, T] &= P[r \text{ in } I_{t\hat{T}}, \text{ with } \hat{T}|s_t, T] \\ &= \int_{I_{t\hat{T}}} p_r(r|s_t, T) dr = \int_{I_{t\hat{T}}} p_n(r - \hat{s}_t) dr \\ &= \int_{I_{t\hat{T}}} \left(\prod_{m=1}^i p_{n_m}(r_m - \hat{s}_{tm}) \right) dr \end{aligned} \quad (5.3.3)$$

At this point it is convenient to say that the decision region is dependent on the length of

the period $\hat{T}$. This effect is over all the dimensions of the space of the decision region. Hence, we imagine that the decision region is affected in an uniform manner. But assuming that the effect on the decision region is negligible, because of similar reasons stated for the previous derivation as that of the period T is also slowly time-varying, we simplify our derivation. Assuming that the probability that the signals have a time-period of duration T , $P[T]$, is not dependent on the values of the random variable, T , we can optimize our clock synchronizer more easily.

By a similar procedure as that used in the previous section, in the maximization process of $P[C|s_i, T]$ with respect to the estimate time-period $\hat{T}$, we can arrive at the following objective function:

$$\max P[C] = \max \left(\int_{t_r} \prod_{m=1}^i p_{n_m}(r_m(t) - \hat{s}_{im}(t)) dr \right) \quad (5.3.4)$$

The previous assumption that the time-period is slowly time-varying creates a dependency among their values, in the sense that it makes possible a slow and smooth variation of the time-period. In this way, the difference among the decision regions is so small, as to consider the decision region constant, and hence independent of $\hat{T}$. By taking derivatives of the logarithm of the objective function, with respect to $\hat{T}$, and discarding constant terms, we obtained

$$\frac{dA}{d\hat{T}} = -\frac{d}{d\hat{T}} \frac{1}{N_0} \sum_{m=1}^i \int_{(m-1)\hat{T}}^{m\hat{T}} [r_m(t) - \hat{s}_{im}(t)]^2 dt \quad (5.3.5)$$

Let us denote that the transmitted sequence with time-period T as $s_i^T(t)$, and the receiver's estimate sequence with time-period $\hat{T}$ as $s_i^{\hat{T}}(t)$. Then,

$$r(t) = n(t) + s_i^T(t)$$

and

$$r(t) - s_i^{\hat{T}}(t) = n(t) + s_i^T(t) - s_i^{\hat{T}}(t)$$

By realizing that the set of available signals have a period T_0 , the agreed time-period, and the transmitted and estimate signals are the same set, but a little bit longer (or shorter), we

can express such signals as an expansion (or compression) of those of the available set. Let us denote the last ones as s_i , which have time-period T_0

The expressions obtained for them are:

$$s_i^T(t) = s_i\left(\frac{T_0}{T}t\right) \quad (5.3.6)$$

$$\hat{s}_i(t) = s_i^{\hat{T}}(t) = s_i\left(\frac{T_0}{\hat{T}}t\right) \quad (5.3.7)$$

The Figure 5.3.1 shows clearly this relationship. For example in the first case, when $t = T$, $s_i^T(T) = s_i(T_0)$; and when $t = 0$, $s_i^T(0) = s_i(0)$. The same check can be done for $s_i^{\hat{T}}(t)$.

Thus, substituting these results in equation (5.3.5), we have:

$$\frac{dA}{d\hat{T}} = -\frac{d}{d\hat{T}} \frac{1}{N_0} \sum_{m=1}^i \int_{(m-1)\hat{T}}^{m\hat{T}} [r_m(t) - s_{im}\left(\frac{T_0}{\hat{T}}t\right)]^2 dt \quad (5.3.7)$$

In order to avoid the derivation of the extremes of integration, let us make the following change of variables:

$$\lambda = \frac{T_0}{\hat{T}}t \quad dt = \frac{\hat{T}}{T_0}d\lambda \quad t = \frac{\hat{T}}{T_0}\lambda \quad (5.3.8)$$

$$\text{for } t = (m-1)\hat{T} \quad \lambda = (m-1)T_0$$

$$\text{for } t = m\hat{T} \quad \lambda = mT_0$$

Then, we have:

$$\frac{dA}{d\hat{T}} = -\frac{d}{d\hat{T}} \frac{1}{N_0} \sum_{m=1}^i \int_{(m-1)T_0}^{mT_0} [r_m\left(\frac{\hat{T}}{T_0}\lambda\right) - s_{im}(\lambda)]^2 \frac{\hat{T}}{T_0} d\lambda \quad (5.3.9)$$

Which becomes,

$$\frac{dA}{d\hat{T}} = -\frac{1}{N_0} \sum_{m=1}^i \int_{(m-1)T_0}^{mT_0} \left(2[r_m\left(\frac{\hat{T}}{T_0}\lambda\right) - s_{im}(\lambda)]r_m'\left(\frac{\hat{T}}{T_0}\lambda\right) \frac{\lambda}{T_0} \frac{\hat{T}}{T_0} + [r_m\left(\frac{\hat{T}}{T_0}\lambda\right) - s_{im}(\lambda)]^2 \frac{1}{T_0} \right) d\lambda \quad (5.3.10)$$

By coming back to the original variables, and defining the estimate error as the derivative of the objective function, we have:

$$\epsilon = -\frac{1}{T_0} \frac{1}{N_0} \sum_{m=1}^i \int_{(m-1)\hat{T}}^{m\hat{T}} \left(2t[r_m(t) - \hat{s}_{lm}(t)]r'_m(t) - [r_m - \hat{s}_{lm}(t)]^2 \right) dt \quad (5.3.11)$$

5.4 Combining symbol and clock synchronization.

Obviously, symbol and clock synchronization have to be done in a simultaneous manner, because both effects can occur at the same time.

By stating the problem again, assuming the transmitted sequence, the time-period and the starting time independent among them, we arrive at the following relationship.

$$\min P[E] = \max P[C] = \max P[C|s_l, T, \tau] \quad (5.4.1)$$

Because $P[C]$ depends on two continuous variables, T and τ , independent between them, the maximization conditions are:

$$\frac{\partial P[C]}{\partial \hat{T}} = 0 \quad \text{and} \quad \frac{\partial P[C]}{\partial \tau} = 0 \quad (5.4.2)$$

As before, our objective function becomes:

$$A = -\frac{1}{N_0} \sum_{m=1}^i \int_{(m-1)\hat{T}+\hat{\tau}}^{m\hat{T}+\hat{\tau}} [r_m(t) - s_{lm}(\frac{T_0}{\hat{T}}t - \hat{\tau})]^2 dt \quad (5.4.3)$$

By making the following change of variables, in order to avoid the derivation of the integration limits, we have:

$$t = \frac{\hat{T}}{T_0} \lambda + \hat{\tau} \quad dt = \frac{\hat{T}}{T_0} d\lambda \quad (5.4.4)$$

$$\text{for } t = (m-1)\hat{T} + \hat{\tau} \quad \lambda = (m-1)T_0$$

$$\text{for } t = m\hat{T} + \hat{\tau} \quad \lambda = mT_0$$

Thus, the objective function is transformed to

$$A = -\frac{1}{N_0} \sum_{m=1}^i \int_{(m-1)T_0}^{mT_0} \left(r_m \left(\frac{\hat{T}}{T_0} \lambda + \hat{\tau} \right) - s_{im} \left[\lambda + \left(\frac{T_0}{\hat{T}} - 1 \right) \hat{\tau} \right] \right)^2 \frac{\hat{T}}{T_0} d\lambda \quad (5.4.5)$$

By taking $\hat{T} \sim T_0$, inside the argument of $s_{im}[\lambda + (\frac{T_0}{\hat{T}} - 1)\hat{\tau}]$, in order to simplify our derivation, we have the results for the errors of $\hat{T}$ and $\hat{\tau}$.

$$\epsilon_{\hat{\tau}} = -\frac{1}{N_0} \sum_{m=1}^i \int_{(m-1)\hat{T}+\hat{\tau}}^{m\hat{T}+\hat{\tau}} 2[r_m(t) - \hat{s}_{im}(t-\tau)]r'_m(t)dt \quad (5.4.6)$$

$$\epsilon_{\hat{T}} = -\frac{1}{T_0} \frac{1}{N_0} \sum_{m=1}^i \int_{(m-1)\hat{T}+\hat{\tau}}^{m\hat{T}+\hat{\tau}} \{2(t-\hat{\tau})[r_m(t) - \hat{s}_{im}(t-\hat{\tau})]r'_m(t) + [r_m(t) - \hat{s}_{im}(t-\hat{\tau})]^2\} dt \quad (5.4.7)$$

Where $\hat{s}_{im}(t-\hat{\tau}) = s_{im}(\frac{T_0}{\hat{T}}t - \hat{\tau})$

As we have seen, in both cases, for the estimation of $\hat{T}$ and τ , we have obtained expressions that really are a summation of the errors on every signal in the received sequence, with respect to $\hat{T}$ and $\hat{\tau}$.

Now, we have to add this results to the *MEPSE* receiver previously obtained in the preceding chapter.

In all this analysis we have always assumed that the receiver had knowledge of the transmitted signal. In this way this synchronizer has to be located after the decision box of the *MEPSE* receiver

With this location, the synchronizer has available the set of signals of the transmitted sequence, decided correct by the *MEPSE* receiver.

Because *MEPSE* receiver gives, every time i , the sequence with the minimum path, namely, every time i it optimizes its reception, then this characteristic can be used for estimating, also every time i , T and τ . This will lead to a best performance of the receiver, in the sense of a quicker recovery from errors.

From a theoretical point of view, it is apparent that the estimation, every time i , of the synchronization parameters is the correct one, although because practical receivers can not

handle too long sequences, they can do the estimation of the synchronization parameters, sequence by sequence. In any case, we will follow this analysis with the consideration of estimating T and τ every time i , which as we know is the time of the last received code word.

From equations (5.4.6) and (5.4.7), we can see that they also need, for estimating the synchronization errors, the received signal, and as we realize, any change in the estimation of T and τ implies a new calculation of all the involved terms in those equations, but it also implies that we have to have, available, the past received signals.

By defining each one of the terms of the referred equations (5.4.6) and (5.4.7) as the error at time m , we will have the following expressions:

$$\epsilon_r = \sum_{m=1}^i \epsilon_{rm} \quad \text{and} \quad \epsilon_T = \sum_{m=1}^i \epsilon_{Tm} \quad (5.4.8)$$

Where, ϵ_{rm} and ϵ_{Tm} are, respectively:

$$\epsilon_{rm} = -\frac{1}{N_0} \int_{(m-1)\hat{T}+\hat{\tau}}^{m\hat{T}+\hat{\tau}} 2[r_m(t) - \hat{s}_{im}(t - \hat{\tau})]r'_m(t)dt \quad (5.4.9)$$

$$\epsilon_{Tm} = -\frac{1}{N_0 T_0} \int_{(m-1)\hat{T}+\hat{\tau}}^{m\hat{T}+\hat{\tau}} \{2(t - \hat{\tau})[r_m(t) - \hat{s}_{im}(t - \hat{\tau})]r'_m(t) + [r_m(t) - \hat{s}_{im}(t - \hat{\tau})]^2\}dt \quad (5.4.10)$$

A block diagram for the expressions (5.4.9) and (5.4.10) is shown in the Figure 5.4.1. Let us assume that the way to have available all the elements of the received sequence, from time 1 to time i , is through delay taps. Without thinking in the complexity of these delay taps, in the sense that they have to vary accordingly, i.e. $Di = \hat{T}_i + \hat{\tau}_i$, in order not to introduce additional shifting. Then, assuming suitable delay taps, we can connect boxes representing the block diagram of the Figure 5.4.1, which at the time are fed by the *MEPSE* receiver with the elements s_{im} , of the *correct* sequence s_i , in order to calculate the components of ϵ_r and ϵ_T for the new estimation of τ and T .

At this point, we can realize that the estimation of the synchronization parameters depend on the decision of the *MEPSE* receiver, and at the time the decision of *MEPSE* receiver

depends on the synchronization parameters. This interdependence implies that the *MEPSE* receiver can not throw the discarded paths arriving to a determined state, because may be in the new estimation of the synchronization parameters, the structure of the survivor paths changes. In this way the estimation of the correct sequence and that of the synchronization parameters helps each other.

A block diagram of this synchronizer is shown in the Figure 5.4.2.

because in this derivation we have not constrained the shape of the codewords to any special one, this synchronizer is applicable to all *VEN* codes.

As we can observe, the previously derived synchronizer takes its decisions, about the estimate errors ϵ_r and ϵ_T , relying on the assumed correct set of signals $\{s_{im}\}$. This is a sort of data aided recovery. Experience has shown that data aided recovery based scheme has the best results² in the solution of the synchronization problem. With respect to the slowly time-varying aspect, it is known that the variation of r and T is of a very low frequency with respect to the transmission rate $\frac{1}{T}$.

One last aspect to consider is that of the use of the error information in the estimation of the synchronization parameters. The estimate values of r and T could be expressed as:

$$\hat{T}_{(i+1)} = \hat{T}_i + \Delta T \quad \text{and} \quad \hat{r}_{(i+1)} = \hat{r}_i + \Delta r \quad (5.4.13)$$

Where ΔT and Δr are the increments to be done in each one of the synchronization parameters and which can be put as proportional to the calculated estimate errors. They can be expressed as:

$$\Delta T = \epsilon_T \delta_T \quad \text{and} \quad \Delta r = \epsilon_r \delta_r \quad (5.4.14)$$

Where δ_T and δ_r are some discrete values depending on the mean value of the variations of T and r , respectively.

² See references [2], [5].

Because our analysis of the synchronization problem has been made by applying the methodology of minimum error probability, This clock-timing synchronizer is optimum in this sense

5.5 Equalization and intersymbol interference in $VEN_D A$ codes.

Given that $VEN_D A$ codes are DC-free component and furthermore their components can be viewed as constituted by $2(D+1)$ pulses of equal duration and equal size, it allows to think in their transmission as a PAM system with only two levels: ± 1 .

Thus, all the solutions or improvements devised until now for binary PAM systems are applicable to the transmission of $VEN_D A$ codes.

For example, by using raised cosine pulses, in order to eliminate intersymbol interference (ISI), in the transmission of $VEN_1 A$ codes, we have the following types of signals, shown in the Figure 5.5.1.

We can see in the referred Figure 5.5.1, the $2(D+1)$ -pulse characteristic of $VEN_D A$ codes. This also suggests the use, at the receiver, of an equalizer that take samples every $\frac{T}{2(D+1)}$ seconds, namely, the delay line taps are spaced at intervals of $\frac{T}{2(D+1)}$ seconds.

With the use of this equalizer we are reducing the impairments introduced by the channel. Although, in principle this is not a fractionally spaced equalizer, with respect to the pulse, it acts as if it were with respect to the signal. As we know, fractionally spaced equalizer have better properties than those spaced T . The choice of an adaptive least mean-square equalizer, in order to minimize the mean square error or the sum of squares of all ISI terms plus noise power, is a simple and robust way to obtain a good reception.

Fig. 5.3.1 Compression or expansion of the signals.

Fig. 5.4.1. Block diagram of a branch of a symbol and clock synchronizer.

Fig. 5.4.1. Block diagram of a symbol and clock synchronizer.

Fig. 5.5.1 VEN_1A code words and their raised cosine pulses version.

Chapter 5. References

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Chapter 6

Conclusions

6.1 Summary

We have presented a set of new digital codes whose major improvement over the existing ones is to have zero DC and moderate high frequency content, be constant envelope codes (VEN_{DA} and VEN_{DC}) and constitute a kind of two level block codes. We also have presented a new kind of three level block codes (VEN_{DB}) very close to bipolar or AMI code, but without the non transparency feature of the latter. One common characteristic of these line codes is that by increasing the number of digits per block, we obtain improvements in bandwidth and in error probability. The cost of these improvements, for larger D , is an increase in the complexity of both the transmitter and receiver equipment. Also, this cost could include a degradation of the synchronization properties of the codes.

Assuming that the main application of VEN codes is on baseband digital loops, where transformers are present, it is apparent that these codes have favourable properties since their signals are DC free, and moreover the fundamental components of their signals are DC free by themselves. This means also that the DC free characteristic is still preserved in the case of unequal *a priori* probabilities of the input messages.

The very low content of power at high frequencies showed by VEN_{DC} codes suggests their application also in *Frequency Division Multiplexing (FDM)* because this means low out-of-band content.

In the analysis of the optimum receiver for *VEN* codes we have presented a general methodology, which allowed us to obtain it without using the criteria of the Estimation Theory. The obtained results are undoubtedly optimum in the sense of minimum error probability. This methodology was applied in the calculation of an upperbound for the bit error probability, which actually is very close to the Union Bound, but we feel that a better insight in the communication problem was gained.

We have applied the mentioned methodology to the synchronization problem, obtaining, theoretically, simple results. We reached a sequence estimation of the synchronization parameters, which matches with the general Viterbi-based optimum receiver obtained, *MEPSE* receiver.

We have presented in this study the analysis of the finite-memory Viterbi-based receiver and we obtained the structure of this practical receiver, with a memory capacity for ν code-words. We obtained an exact expression for the bit error probability, which was used in the further calculation of the upperbound.

6.2 Suggestions for further study

Although in the calculation of the power spectra of the new line codes was done by reducing the original matrix formula given by Petrovic¹ to an algebraic one, this actually can be done directly without going through this unnecessary simplification, and so taking advantage of the halved computations indicated by the author.

The *MEPSE* methodology showed that it can be used to analyze other communication problems, where the minimum error probability objective is involved.

The Synchronization problem was undertaken in a theoretical manner. The obtained synchroniser structure requires the analysis of the feasibility of being implemented.

¹ Reference [1] in Chapter Three.

END

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