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**Two Stage Data
Echo Cancellation
Utilizing Orthonormal Functions
For Echo Tail Cancellation**

by

Clive George Cook, B. Sc.

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of
Master of Engineering.

Ottawa-Carleton Institute for Electrical Engineering,
Faculty of Engineering,
Department of Electronics,
Carleton University,
Ottawa, Ontario,
Canada
February, 1989

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The undersigned hereby recommend to the Faculty of Graduate Studies and Research acceptance of the thesis

**Two Stage Data Echo Cancellation Utilizing
Orthonormal Functions For Echo Tail Cancellation**

submitted by Clive George Cook, B.A.Sc., in partial fulfilment of the requirements for the degree of Master of Engineering.

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February, 1989

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Chapter 1

Introduction

As the telecommunications industry moves toward the realization of the Integrated Services Digital Network (ISDN) a number of problems are posed by full duplex transmission of digital information over existing subscriber loop lines. Of primary concern is the U-interface, the interface necessary for moving from two wire to four wire transmission. Figure 1.1 shows the arrangement of the U and S interfaces in an ISDN transceiver.

ISDN basic access consists of two 64 kbit channels plus one 16 kbit channel ($2B + D$). An additional 16 kbits are added for framing, error correction, maintenance, etc. Thus the transmission rate is 160 kbits/s. The International Telegraph and Telephone Consultative Committee (CCITT) has defined the technical issues at the S interface. The method chosen to obtain full duplex transmission at the U-interface is the echo cancellation technique. Figure 1.2 illustrates this method.

This thesis examines the architecture used for the Echo Canceller (EC) and some additional features which may enhance or degrade the cancellation performance. Tradeoffs between complexity and performance are emphasized. The emphasis is toward a VLSI circuit realization. A novel technique

Figure 1.1: ISDN Interfaces. [Mess86b].

Figure 1.2: Echo Cancellation Technique For Moving From Four Wire To Two Wire Transmission.

based on orthonormal functions is developed toward a solution using echo cancellation at the U-interface. Echo cancellation is done in two stages to achieve a reduction in the complexity required of the succeeding circuitry. The objective is a 20 dB cancellation by the Pre Echo Canceller. A block diagram of the proposed system is illustrated in Figure 1.3 without explanation.

1.1 The Problem Of Echo Cancellation

Figure 1.4 illustrates echo propagation on a two wire twisted pair. Other impairments to transmission are crosstalk [AhBG81], impulse noise [Kapi88], timing jitter [Falc85] and intersymbol interference. The Echo Canceller, shown in Figure 1.2, ideally emulates the echo path, thus allowing a replica of the interfering echo to be subtracted from (*cancel*) the echo. The canceller can consist of a transversal filter, a table-look-up RAM or a combination of the two.

1.1.1 Required Performance

The objective of near error free transmission along the subscriber loop defines the required echo cancellation performance. A received signal to echo noise ratio (SNR) of 20 dB [MMBP86] to 25 dB [LinT88] is necessary. The subscriber loop attenuation of the far end signal can be as much as 40-50 dB [Mess84b]. A best case transhybrid loss of 10 dB [Mess86a] is consistent with previous work. The required cancellation range is given in Table 1.1¹.

This can be translated to bits of resolution using the figure of 6 dB signal to quantization noise per additional bit [OppS75]. The resolution of the echo

¹The far-end signal experiences a coupling action through the hybrid. The near end echo however is attenuated (transhybrid loss).

Lower Performance Bound	Upper Performance Bound
$(-40 \text{ dB}) + (10 \text{ dB}) + x = 20 \text{ dB}$ $x = 50 \text{ dB}$	to $(-50 \text{ dB}) + (0 \text{ dB}) + x = 25 \text{ dB}$ to $x = 75 \text{ dB}$

Table 1.1: Performance Bounds

estimate must be large enough so as not to introduce quantization noise. 12 to 14 bits are required for digitally recovering the far-end signal buried in noise and echo without compromising the receiver performance because of quantization errors (noise). These values agree with those presented previously in the literature [AgMH82] [TKSK87].

1.2 Two Stage Echo Cancellation Utilizing Orthonormal Functions

At the higher bandwidths expected from ISDN implementation considerations are questioning using a single stage Echo Canceller to remove the echo power, both the main echo pulse power and the power in the long echo tail. A logical progression from this approach is to reduce the echo in stages. This thesis examines a two stage approach. The echo impulse response is separated into a main echo pulse covering the first two baud intervals and the echo tail.

A Front End Echo Canceller is examined for analog cancellation of the main echo pulse. An analog Echo Tail Canceller based on approximating the echo tail by a network of simple time responses is examined. The combined Echo Cancellers form the first stage, called the Pre Echo Canceller, of a two stage canceller. The second stage is a digital Echo Canceller, called the Main

Echo Canceller, not unlike those previously reported.

1.3 Two Stage Cancellation

Figure 1.5 shows the structure of a two stage Echo Canceller. The first stage, called the Pre Echo Canceller, performs a *coarse* analog echo cancellation on the far end signal, plus echo, plus noise. The second stage, called the Main Echo Canceller, performs a *fine* digital echo cancellation on the residual signal after cancellation by the Pre Echo Canceller. Both sections may be adaptive.

The basis for examining cancellation in two stages is if sufficient cancellation of the echo can be done in the analog domain before analog-to-digital conversion the requirements on the Analog-to-Digital converter can be relaxed as well as allowing for more accurate operations on the residual echo and far end signal. The Main Echo Canceller can be designed to account for nonlinearities and jitter effects [ACCC86] if need be with increased accuracy. The improvement in performance can ease timing recovery and sequence estimation. Conversely a two stage scheme could be merely adding an unwarranted redundancy to the overall design. Most companies considering ISDN U-interface solutions do not use a two stage structure, Why ? It is implicit in the two stage architecture that the combination should perform equally as well as a single Echo Canceller solution.

A number of factors must be taken into account in the two stage approach which are not present in a single stage realization. The overall convergence time (Pre Echo Canceller plus Main Echo Canceller) must not exceed what would reasonably be expected for the convergence time of a single stage scheme. The convergence time is dependent on the adaptation algorithm and method of realization. For simplicity and as a starting point only the LMS adaptation algorithm will be considered.

Figure 1.4: Echo Propagation. The solid lines indicate the direction of propagation of the near end data signal. The dashed lines indicate the direction of propagation of the echo signal caused by the near end data signal.

Figure 1.5: Two Stage Cancellation

The Pre Echo Cancellor and Main Echo Cancellor should converge in some fraction of the time taken for either Echo Cancellor if they were separate systems. The Main Echo Cancellor will most likely consume a larger fraction of the total convergence time because of the higher performance requirements placed on it.

The problem of when to fix the Pre Echo Cancellor coefficients and allow the Main Echo Cancellor to adapt needs to be examined. The interrogative 'How ?' should also be added to this statement. The location taken for adjustments of either or both cancellers is important. A warm and cold start analysis of each canceller separately and jointly will determine what is needed. A complete analysis will determine how failure of the first or second section, each separately, would influence overall performance.

Another aggregate factor which is to be looked at is VLSI implementation efficiency. Power and size requirements must either be less than or equal to the single canceller scheme if a cost effective VLSI solution is to be embarked upon. The overall tap count (or address space if using a RAM approach) plus any extra circuitry should be as area efficient as a single stage solution. If the two stage approach can achieve appreciably better cancellation then a slight area/power penalty could be tolerated.

The complexity of an additional canceller and the required circuitry accompanying the addition must not render the concept too costly. Circuit elements that can be used by both cancellers might be shifted between the two or multiplexed. New elements, such as an interpolating filter and AGC, may be needed outright. Noise added by additional Echo Cancellor circuitry must not impair system performance.

A concern posed by using two stage cancellation is that the circuit challenges and difficulties of a single stage echo cancellation proposal are being shifted to another area of the architecture (the analog first stage of the two

stage scheme versus the A/D converter of a single stage digital Echo Canceller). The same or worse nonlinearities and noise problems may exist in either scheme as compared to the other. Chapter 6 will examine parameters that must be accounted for when investigating a two stage approach such as explored here.

Figure 1.6 shows a single stage Echo Canceller with A/D converter. A digital approach is assumed for the sake of argument. A comparison with the two stage scheme of Figure 1.5 illustrates that if one method is to be superior to the other (in what sense is determined by the designer and what is being designed for), a shift in resources must occur from the A/D converter to different analog components or vice-versa. An emphasis in design effort toward interface circuitry is already prevalent amongst analog designers [Gray87]. A two stage scheme will require a shift in emphasis.

Previous two stage approaches are presented in Section 1.4.2.

1.4 Echo Canceller Technology Up To The Present

Both technology and echo cancellation schemes which can be used for the U-interface are examined in the following sections.

1.4.1 Present State Of Technology

The determination of whether this proposal is feasible in terms of circuit integration is in large part dependent on the readily available technology. A system is required that can be powered from the local exchange and be integrated onto a single monolithic chip. Future developments may swing the decision one way or another however what is relevant is the technology that

has production capability today. The market window is elusive, relying on what can be used now. A very brief coverage of the technology, and circuit elements depending on the technology, that can be used in a VLSI realization of a two stage data Echo Cancellor will be presented in this section. Typical performance measures are taken from stand alone designs.

A typical state of the art production line CMOS process will have a feature size as small as $0.75\ \mu\text{m}$. Such a process is quoted in [CLCH86]. The feature size can be further reduced to $0.55\ \mu\text{m}$ by scaling. Some interesting characteristics of this process are the relatively high p-channel threshold voltage, $V_{tp} = -1.1\ \text{V}$ ($V_{tn} = 0.7\ \text{V}$); high pressure oxidization is utilized to improve device isolation and thus latch-up resistance; a Lightly Doped Drain (LDD) is used for the n-channel to minimize hot carrier effects; n+ poly is deposited for the gate material; two level metal interconnect is incorporated. A 140 ps stage delay @ 5 V was obtained for a $0.75\ \mu\text{m}$ feature size gate ring oscillator.

Many of the processes used by telecommunications companies have not yet reached this level of sophistication. Telecoms, although not having state of the art processes do produce high quality ICs which utilize both digital and analog circuitry. Northern Telecom's movement to CMOS4 [GMSS88] is a prime example. The IC design environment is also maturing. Standard cell compilers and design tools which incorporate schematic capture, auto place and route, numerous types of simulation and verification along with testing are readily available optimized to a given process [GMSS88]. Many of the tools allow development on mixed analog/digital circuitry as well as providing macro functions for Read-Only-Memory (ROM) and/or Random-Access-Memory (RAM) cell blocks.

Analog IC design has not progressed as far as digital designs. In the submicron era analog IC's cannot be scaled as readily as can digital designs.

General purpose analog IC's, such as operational amplifiers, although having wide application, do not benefit from considerable economies of scale in smaller specific submarket applications due to the solution specific designs [Gray87]. An example would be circuitry which may need to be extremely temperature insensitive yet low bandwidth is sufficient for the application. Analog design tools for the system designer are not yet mature enough for ISDN solutions to be easily generated.

For an ISDN transceiver a few basic circuit components are necessary depending on the particular solution chosen. A survey of Static RAM (SRAM) and Dynamic RAM (DRAM) performance is given in Table 1.2² and Table 1.3 respectively. ISDN applications will not likely use these memory configurations or have available intricate RAM processes to be used along with other circuitry on a single IC, however the survey shows what can be expected in the way of performance and power. Size is application specific. Non volatile RAM [Slat87] is not considered as a viable alternative at present because of the reliability problems and short life span of batteries as compared to semiconductors.

Quality operational amplifiers can dramatically effect what can be accomplished in a continuous or sampled analog design. A wide bandwidth amplifier is necessary for switched capacitor techniques. A likely candidate would be a folded cascode operational amplifier because of its wide and stable closed loop bandwidth. This can be achieved with high capacitive loads and maintain a high Power Supply Rejection Ratio (PSRR).

Operational amplifiers are necessary building blocks for both an Analog-to-Digital converter (sigma-delta) and any switched capacitor analog circuitry likely to be used for echo cancellation. Figure 1.7 shows the cost esti-

² Perf is the abbreviation for Performance Semiconductor

Figure 1.6: Single Stage Digital Echo Cancellor

Manu- facturer	Read Cycle Time t_{RC} (min)	Address to Data Valid t_{AA} (max)	Chip Enable to Data Valid t_{ACE} (max)	Operating Current I_{CC} (max)	Standby Current	Power (Active)
Cypress 4K X 4	25-45 ns	25-45 ns	25-45 ns	70 mA	15 mA	330 mW
Motorola 4K X 4	25-35 ns	25-35 ns	20-30 ns	120 mA	15 mA	N/A
Perf 8K X 8	20-35 ns	20-45 ns	20-45 ns	110 mA	23 mA	605 mW

Table 1.2: SRAM Survey

mate for an Analog-to-Digital converter as a function of bits of resolution³. The necessity for a reduction in bits of resolution is clear. The decimation filter can take up a large proportion of the total A/D integration area. Reducing the filter area may allow for a reduction in the accompanying analog circuitry as well.

A 13 bit two stage 3rd order noise shaping oversampled A/D converter (another name for sigma delta converter) has been reported in [LonC'88]. Another 13 bit sigma delta A/D is reported in [NorP88]. A 3rd order design has also been reported in [KeKC86], although not for ISDN bandwidths. A 16 bit design has been considered in [MUIK87].

Analog cancellation circuitry has been used in [YEWI84]. An analog transversal filter has been realized in [TSFM84], free of the fixed pattern noise that has plagued previous area efficient designs [AhCC78]. The transmission rate used in [TSFM84] is 200 kbits/s.

This brief survey shows that the circuitry is available to meet performance requirements, although some elements will be pushed at their present limit. The decisive factors in terms of circuitry which will influence selection are the effective cancellation per unit area (performance for a given area) and design complexity and the tradeoffs between both.

1.4.2 Echo Canceller Solutions

This section will provide an extensive overview of the various architectures used for data echo cancellation. The emphasis will be on architectures that

³ In Figure 1.7 a provision is made for difficulties associated with both technology and circuit nonlinearities that must be overcome in order to reach the higher resolution and better performance. A 5 % increase in each successive bit is therefore allocated to the cost to include these factors. The cost otherwise is simply the area cost assuming a constant feature size and an increase in bits of output resolution.

Manu- facturer	Access Time From $\overline{RAS}$ t_{RAS}	Access Time From $\overline{CAS}$ t_{CAS}	Read/Write Cycle Time	Operating Current I_{CC} (max)	Power (Active) (Standby)
Hitachi 64K X 1	120-200 ns	60-100 ns	245-345 ns	55 mA	250 mW 18 mW
Motorola 64K X 1	120-200 ns	60-100 ns	255-345 ns	60 mA	302 mW 22 mW

Table 1.3: DRAM Survey

Figure 1.7: Cost Function Estimate For An Oversampled Analog-to-Digital Converter

have application to the U-interface. The advantages and disadvantages of each method will be given, along with any novel ideas or concepts used. Any derivations necessary for comprehension are included. An excellent overview of voice Echo Cancellers and suppressors is found in [SonB80][Mess84b] but will not be examined here. Echo Cancellers are adaptive filters designed for system identification.

Linear Structures

Data echo cancellation over a two-wire circuit in a full duplex system was first considered by [KolW73]. The system utilizes hybrid couplers (differential transformers) and a balance network to reduce the length and precision requirements of the Echo Canceller. The Echo Canceller is a transversal filter. The data rate used is 2 kbits/sec. The output of the transmitter is applied simultaneously to the hybrid and transversal filter. The use of the transmitter output as a reference to the transversal filter requires multipliers. Analog values are multiplied by a coefficient and then summed. A large number of multiplications are required to achieve adequate cancellation since tap spacing must necessarily be closer than $T/2$, where T is the symbol rate. Multiplications are costly when integration is looked at and should be avoided.

A different structure is proposed by [Muel76] where the input to the Echo Canceller is taken before the transmitter. In Muellers' method the near end transmitter is part of the echo path. Adaptation decisions are controlled by the receiver output. This structure is shown in Figure 1.8. The sequence, $a(k)$, is used as the reference to the EC. The signals at each point of the EC in Figure 1.8 are defined and the receiver signal values are shown to be linear combinations of the transmitted signal values [VESG79].

The sampled echo path can be thought of as a finite duration causal

impulse response g .

$$g(n) = 0 \quad \text{if } n < 0 \text{ and } n \geq N \quad (1.1)$$

The sampled transmission path is also a finite duration impulse response, h , of length M . M is not necessarily equal to N .

$$h(n) = 0 \quad \text{if } n < 0 \text{ and } n \geq M \quad (1.2)$$

The scheme proposed by Mueller uses a transversal filter with coefficients c_i for the Echo Canceller. To cancel the entire echo ideally the number of taps (coefficients) should equal N from eq(1.1). An echo replica $\hat{e}(k)$ can be generated from the near end transmit data and the EC coefficients.

In vector notation the near end transmit vector at the k^{th} symbol interval is

$$a_k = (a(k), a(k-1), \dots, a(k-N+1))^T, \quad (1.3)$$

where the $\{a(k)\}$ denotes the data symbols. The echo path response, with sampling at the symbol rate, is denoted

$$g = (g(0), g(1), \dots, g(N-1))^T. \quad (1.4)$$

The coefficient vector at time k is

$$c_k = (c_0(k), c_1(k), \dots, c_{N-1}(k))^T. \quad (1.5)$$

The far end transmit vector at time k is

$$b_k = (b(k), b(k-1), \dots, b(k-M+1))^T, \quad (1.6)$$

where the $\{b(k)\}$ denotes the far end data symbols, and the sampled transmission path response is denoted

$$h = (h(0), h(1), \dots, h(M-1))^T. \quad (1.7)$$

The echo signal and echo replica are

$$e(k) = a_k^T \cdot g \quad (1.8)$$

$$\hat{e}(k) = a_k^T \cdot c_k \quad (1.9)$$

respectively. Taking into account system noise, $n(k)$, the received far end signal is

$$s(k) = b_k^T \cdot h + n. \quad (1.10)$$

The residual signal becomes

$$r(k) = a_k^T \cdot (g - c_k) + s(k). \quad (1.11)$$

Adaptation is done using $r(k)$.

Both the near end transmitted data and the far end transmitted data are scrambled to ensure they are not correlated. The data sequences can then be assumed independent and having equivalent statistical distributions. If the coding and receive and/or transmit filters (this includes the line driver) are linear then the data sequences, $a(k)$ and $b(k)$, are linearly independent. An accurate echo replica can then be derived from the near end data symbols without affecting the far end signals to be recovered. Scrambling assures that the coefficients, c_i , and the far end data values, $b_i(k)$, are uncorrelated. The echo is not correlated with the far end data. Scrambling can also aid in timing recovery.

If $a(k)$ is a binary code the multiplications of eq(1.9) are eliminated. The 2B1Q code proposed by the Committee of the American Standards Association [T1D1] is a linear translation from binary to four levels. Other schemes utilizing nonlinear codes such as alternate mark inversion (AMI) [SzZS86] or bipolar [AgHM82] are not as easily dealt with. The adaptation algorithms must in these cases deal with nonlinear codes.

Adaptive Reference Echo Cancellation (AREC) is compared with Zero Reference Adaptation (ZREC) in [Falc82]. Slow convergence and small step size are characteristic of the zero reference scheme. The principle is the receiver input should ideally be zero if no far end signal is present and echo cancellation has been done. A present far end signal is a source of noise to adaptation. Decision Directed schemes are based on receiver decisions rather than residual echo plus ISI plus far end signal and offer faster convergence.

A technique is given in [KanN87] that adaptively alters the step size. The fact that the echo and error signal are correlated during convergence is utilized to reduce the convergence time. The sign form of the LMS algorithm is considered.

NonLinear Structures

Random Access Memory (RAM) echo cancellation, also called memory compensation echo cancellation, was proposed by [HolS81] for 80 kbits/s transmission. The RAM structure principle for echo cancellation is shown in Figure 1.9. The last N transmitted data symbols form the address to select the correct echo estimate.

$$\hat{e}(k) = RAM_{q(k)}(k) \quad (1.12)$$

$q(k)$ is the RAM address at time k . For a binary sequence this is equivalent to 2^N independent one tap transversal filters. For an M level code M^N locations are needed. Scrambling is necessary in a RAM configuration, as in a transversal, to ensure that all addresses are generated with equal probability.

Holte uses the sign version of the LMS algorithm for adaptation. The coefficients are stored to avoid start up difficulties. In [HolS81] bridged taps are not considered and fixed equalizers are used to cancel ISI.

The adaptation of a RAM scheme is generally slower than the transversal

Figure 1.8: Data Echo Cancellation With Symbol Reference. The Receiver block may incorporate equalization, sequence estimation and timing recovery. Transmit and Receive filters are not necessarily included.

Figure 1.9: Data Echo Cancellation Principle Using A RAM

filter realization due to the increased number of taps necessary to account for nonlinearities. Only one address is updated at each iteration. RAM Echo Cancellers can accomplish nonlinear echo cancellation whereas transversal Echo Cancellers can perform only linear echo cancellation.

Let $f(B(0), B(1), \dots, B(N-1))$ be some arbitrary nonlinear function of N bits. $B(i)$ can assume two values (binary). The function can be rewritten in a finite number of terms as a series,

$$\begin{aligned}
 & f(B(0), B(1), \dots, B(N-1)) \\
 &= f_0 + \sum_{k=0}^{N-1} f_1(k)B(k) + \sum_{k_1 \neq k_2} f_2(k_1, k_2)B(k_1)B(k_2) + \dots \\
 &+ \sum_{k_1 \neq k_2 \dots \neq k_L} f_L(k_1, k_2, \dots, k_L)B(k_1)B(k_2) \dots B(k_L) + \dots \\
 &+ \sum_{k=0}^{N-1} f_{N-1}(k)B(0)B(1) \dots B(k-1)B(k+1) \dots B(N-1) \\
 &+ f_N B(0)B(1) \dots B(N-1).
 \end{aligned} \tag{1.13}$$

The total number of terms is

$$\sum_{L=0}^N \binom{NL}{L} = 2^N. \tag{1.14}$$

or in other words the number of combinations of N bits taken L at a time [AgMH82].

If the function is linear, as in the transversal canceller of Figure 1.11

$$f(B(0), B(1), \dots, B(N-1)) = f_1(0)B(0) + f_1(1)B(1) + \dots + f_1(N-1)B(N-1). \tag{1.15}$$

The reduction in the number of terms as compared with eq(1.13) is clear.

A multi-memory Echo Canceller is given in [MaLS84] and [Pete87]. This is shown in Figure 1.10. The code inputs are grouped to decrease the number

of arithmetic operations and/or reduce some problems such as offset.

The advantage of the multi-RAM structure is the reduction in convergence time over the case of a single RAM of length 2^N . Nonlinearities within each block are cancelled. However nonlinear cancellation between blocks is not possible. Both papers go through the convergence analysis finding the convergence time to be proportional to coefficient memory size. The residual power after cancellation of k symbols

$$R^2(k) = \left(R_0^2 - \frac{m\alpha}{2 - m\alpha} \right) \left(1 - \frac{1}{A} (2\alpha - m\alpha^2) \right)^k + \frac{m\alpha}{2 - m\alpha}, \quad (1.16)$$

where A = RAM size; m = number of RAM's; α = step size; $R_0^2 = R^2(0)$. The number of iterations required to reach a desired residual power, R_d^2 , is

$$k(R_d^2) = \frac{Am}{4R^2} \log_e \frac{R_0^2 - R^2}{R_d^2 - R^2}. \quad (1.17)$$

The latter paper reaches the result through approximation.

A multi-memory compensation structure is again examined in [AdCC85]. A general RAM structure L^{M+N} is broken into ML^N sub RAMs. The residual echo can then be modelled by the summation of the outputs of M memory compensation filters each containing L^N locations. Convergence can be increased by separation into linear and nonlinear components. The linear filter is converged first followed by the nonlinear canceller section. Again nonlinear cancellation between RAM sections cannot be done. An illustrative example of the above technique is given.

A state variable analysis is done in [SmCA88] to explain what linear filter theory does not explain with regard to nonlinearities. A memory compensation architecture is considered. Particular emphasis is placed on the Digital-to-Analog conversion nonlinearities. A general nonlinear canceller is developed based on transpose distributed arithmetic, also called table look-up. The architecture allows for the binary representation of a multilevel

signal. In the first order case the circuit is similar, but not equivalent, to Figure 1.10 with $m = 2$.

The authors show that improved convergence can be obtained by using a linear FIR filter in parallel with a general nonlinear filter. The transversal filter performs simple system identification down to the noise floor⁴. Following this the nonlinear (RAM) canceller would reduce the echo further. The structure is shown in Figure 1.11. The architecture requires modifications for a multilevel data signal. Multiplications in either digital or analog or both will be needed. When to stop adaptation of the linear taps and continue adaptation with the nonlinear taps requires further analysis.

A variation of the single RAM realization is presented in [AgMH82]. This structure compensates for those nonlinearities which have the greatest effect in terms of magnitude on the far end signal. In the limit the structure is either a transversal filter (only linear taps) or a full memory compensation filter (linear taps plus general nonlinear taps).

Multilevel signals are represented in binary form as the canceller address. For a four level code (2B1Q), $3N + 1$ taps are needed. Implementation is simpler for a multilevel code because of the incorporation of the details of the multilevel code into the tap weights and the canceller automatically compensates for uncertainty in the transmitted levels. Adaptation is done with the LMS algorithm and convergence speed is consequently reduced because of less coefficient space. The formation into a binary signal along with a factorization separating linear and nonlinear components is given below, C_k is the transmitted level.

$$C_k = a + bB(k_1) + cB(k_2) + dB(k_1)B(k_2) \quad (1.18)$$

The LMS adaptation algorithm automatically determines a, b, c and d .

⁴ The noise floor as defined here is effectively the nonlinear noise floor or the level where strictly linear cancellation will not result in a further reduction in the echo.

Figure 1.10: Multi RAM EC Configuration. N/m blocks

Figure 1.11: Linear/Nonlinear Echo Canceller Structure

The difficulty of this scheme is in the identification of the most significant nonlinearities. A method given is computationally intensive. A certain amount of coefficient precision (word length) must be available or the nonlinearities cannot be accounted for.

A method proposed in [MoNP86] uses three distinct methods to cancel various parts of the echo. The echo is partitioned into main pulse; transition region; tail. The method is called Mixed Recursive Echo Cancellation. The first part of the echo is cancelled with a memory compensation Echo Canceller. This section is adaptive and accounts for nonlinearities. Most of the convergence time is taken up here. The second section is a non adaptive transversal filter. The final section is the recursive section. The slope of the regular echo tail is modelled from the low frequency cutoff of the hybrid and its balancing impedance.

A data driven Echo Canceller with greatly reduced convergence time is investigated in [CioK85]. The proposed method uses Recursive Least Squares (RLS) procedures for adaptation of a data Echo Canceller. The complexity of this method and those like it (i.e. Kalman filtering techniques [Ale' 85]) , render them unfeasible for production at present.

Timing difficulties are reduced by adding a second echo canceller tap to estimate the gradient of the echo response. These estimates are used to correct for either a positive or negative deviation in the timing phases. This approach is taken in [Mess86b].

A novel approach combining linear and nonlinear echo cancellation as well as incorporating into a single tap jitter compensation, Figure 1.12, is presented in [ACCC86] and [AdCC85]. The approach used in [ACCC86] samples at the baud rate. A training period of 150 ms is triggered on power-up by an activate signal. The LMS algorithm is used during operation. A (1-D) fixed equalizer is used to reduce the echo tail length. A total of only

12 echo canceller taps of the type described were needed to obtain a 98% system reach for the United Kingdom.

A paired memory canceller is investigated [Mess86b]. The principle is to store two canceller tap values. One for each of two relative transmit receive phases. A phase slip of ± 1 is detected and the corresponding receive phase is altered to maintain the same relative phase. The coefficients of the slipped phase are then adapted.

An increase in EC control complexity results from using this technique. Determination of which set of coefficients to use during a phase slip is complicated.

Combining Echo Cancellation and Equalization

Many realizations combine an Echo Canceller and a Decision Feedback Equalizer (DFE). Mueller shows that joint adaptation can be accomplished while still retaining the properties of rapid, channel independent convergence [Muel79]. A DFE is utilized to reduce post cursor ISI. Linear taps for pre cursor ISI removal are not considered. Linear equalization alters significantly the joint convergence properties. The combined DFE/Echo Canceller structure is illustrated in Figure 1.13.

The Echo Canceller is connected to the transmit data symbols and the DFE to the received data symbols. The aggregate structure takes advantage of the benefits given in [Muel76]. The equalizer coefficient vector at time k is ⁵

$$d_k = (d_1(k), d_2(k), \dots, d_{J-1}(k))^T. \quad (1.19)$$

The far end signal estimate is

$$\hat{b}_k = (\hat{b}(k-1), \hat{b}(k-2), \dots, \hat{b}(k-M+1))^T. \quad (1.20)$$

⁵ The same notation is used as in equations 1.1 through 1.11.

Figure 1.12: EC Tap To Cancel Linear And NonLinear Echo As Well As Jitter Compensating [ACCC86]

Figure 1.13: Echo Canceller And Decision Feedback Equalizer Combination

The new residual becomes

$$r(k) = a_k^T \cdot (g - c_k) + s(k) - \hat{b}_k^T \cdot d_k. \quad (1.21)$$

$$r(k) = a_k^T \cdot (g - c_k) + \hat{b}_k^T \cdot (h - d_k) + n. \quad (1.22)$$

The optimum coefficient values minimize the mean square of $(r(k) - \hat{b}(k))$ as $k \rightarrow \infty$. An AGC can be used to normalize $h(0)$ to 1 thus reducing the error signal, $\epsilon_k = r - \hat{b}_k$.

The joint convergence using the stochastic gradient algorithm is analyzed and found to be determined by the sum of N Echo Canceller taps and J Decision Feedback Equalizer taps. If the step size $\alpha = 1/(N + J)$ in the adaptation equation

$$c_{k+1} = c_k - \alpha a_k [a_k^T (c_k - g) + \nu_k], \quad (1.23)$$

the residual mse at iteration k is

$$\epsilon_k = R + \left(1 - \frac{1}{N + J}\right)^k (\epsilon_0 - R), \quad (1.24)$$

where R is the unremovable mean square error after cancellation/equalization. It is the sum of the noise variance, $E[\nu_k^2]$, and any echo beyond the span of the Echo Canceller. ϵ_0 is the initial mean square excess error. Simulations indicate that reduction of echo or ISI is according to which impairment predominates followed by equal reduction. Synchronization requirements are very stringent in this structure.

Consideration of a joint DFE and EC structure has also been done in [GeVC84]. Two separate and one combined RAM approach are considered. Convergence time is reduced by utilizing a nonvolatile memory to store the converged coefficient values between calls. Sampling is done at twice the baud rate to aid timing recovery. An AGC is used after cancellation on the error signal. Gain is normalized to reduce the influence of the AGC on

adaptation. The error signal, taken before a receiver decision, is the same for both EC and DFE. The behavior of the joint EC/DFE structure during different phases is shown to be as follows; Phase I : The DFE functions as a second EC because $\hat{b}(k) = a(k)$. Phase II : The DFE does not work effectively, $\hat{b}(k) \neq b(k) \neq a(k)$. Phase III : $\hat{b}(k) = b(k)$.

Two Stage Echo Cancellers

Few two stage designs have been considered. A voiceband cascadable VLSI echo canceller is presented in [TKGD84]. The integrated realization is designed to cancel 16 ms of echo impulse response and a limit of four can be cascaded. The work is for an 8 kHz sampled Pulse Code Modulation (PCM) system. The cascade idea can be thought of as multi stage cancellation. An important consideration examined is the interconnection of cancellers, Figure 1.14.

The pseudo cascaded realization, Figure 1.14a, uses separate error signals for adaptation to each canceller. The potential for misalignment noise is greater in this case. This is controlled however by lowering the scaling factor in the correction term, but at the expense of convergence speed. The true cascade uses the same error signal to adapt each canceller.

A limit in the number of cascadable taps for the true cascade exists. The echo estimate must correlate the x at each tap position with its corresponding error signal. Thus the time delay when an input sample is used in computing the convolution (echo estimate) and when the corresponding error is received depends on the number of taps. Absolute coefficient divergence is eliminated by a tap leak which bias' the output toward zero (zero forcing algorithm). A dc tap is added to overcome offset problems.

Two stage cancellation for ISDN data rates has been proposed by both [TKSK87] and [FTGH88]. Both perform first stage cancellation in analog,

with digital processing of the echo approximation. This is shown in Figure 1.15

In the first realization a 4B3T code at 120 kbaud is used. The first stage canceller reduces the echo to 1/50th of its former value for the first 5-6 intervals. The residual echo and long echo tail are reduced using a novel vector quantization technique to reduce the number of required taps. An AGC is added to reduce the requirements of the A/D converter and a linear $\sqrt{f}$ equalizer is used between echo cancellation stages. An adaptive DFE is used after cancellation to remove post cursor ISI.

The second echo canceller reduces the remaining error and compensates for the reduced tap count of the first canceller. An infinite echo estimator performs this function following the $\sqrt{f}$ equalizer. Adaptation is done using a self generated transmit signal, with equal time given to both EC's to perform adaptation separately. Adaptation and cancellation is done digitally. The A/D requirements were reduced from 14 to 10 bits. RAM Echo Cancellers are used.

The architecture in [FTGH88] is arranged similarly. A $\sqrt{f}$ equalizer and DFE are used in the same locations. Both Echo Cancellers use analog cancellation and digital processing of the echo as in Figure 1.15. Echo cancellation only in analog may not be sufficiently accurate to meet the performance requirements. D/A converters are multiplexed for simultaneous use by the Echo Cancellor and DFE. As with the previous scheme the tap count is large due to the echo length. This scheme uses an adaptive balance network (to be described later) versus the vector quantization technique proposed in [TKSK87] to reduce the number of taps.

Figure 1.14: Stage Canceller Interconnection

Figure 1.15: First Stage Of A Proposed Two Stage Echo Canceller

Implementation

VLSI designs have been completed using many of the different schemes mentioned. The four possible alternatives for basic implementation are examined thoroughly in [AgHM82], [Mess84b] and [VESG79]. The four alternatives are given in Table 1.4.

Method	Description
1	Full Analog Echo Canceller
2	Full Digital Echo Canceller With Digital Cancellation And A/D Conversion of the Input Signal
3	Digital Echo Canceller With Analog Cancellation Sum in Analog
4	Analog-Digital Echo Canceller With Analog Cancellation Convolution and Sum in Analog

Table 1.4: Implementation Alternatives

The completely analog proposal requires no A/D or D/A conversion and thus no spurious nonlinearities are introduced by them. Adaptation coefficients are stored in integrators. The adaptation algorithm adds a correction term to the coefficients and summing is done with a summing amplifier. The difficulty with an analog design however is the noise introduced by switching, thermal noise, flicker noise (also called $1/f$ noise) and capacitor leakage and mismatch. If a switched capacitor implementation is done aliased broadband noise is also introduced. Synchronous operation can also present problems. Sampling must be synchronous not only with the received data symbols but also with the transmit data symbols. Maintaining synchronization is non trivial considering that the receive data is buried in echo.

A fully digital echo cancellation places stringent requirements on the Analog-to-Digital converter (Section 1.1.1). Nonlinearities affect the input before cancellation and can create problems. These nonlinearities can be present in the A/D converter and D/A converter as well as the line driver. Linearity requirements are also present on the sample and hold (S/H) circuitry.

Selection of the third choice means that nonlinearities introduced by the A/D converter as well as the additional D/A converter must be dealt with.

The final option is least sensitive to conversion nonlinearities [AgHM82]. The analog convolution/summation and adaptation compensate for them. The adaptation algorithm is performed digitally. The architecture is not insensitive to nonlinearities in the echo response however. This scheme may require more complex circuitry. An adaptive transversal filter using switched capacitor ideas is given in [YEWI84]. This is a completely analog implementation.

In all the cases considered which incorporate some analog and digital components isolation is critical. Digital noise caused by the fast edges of digital circuits can be picked up by analog components. Switching noise (switched capacitor design) can be picked up by digital circuitry. In both cases if the noise is not reduced a larger residual can result.

An Interpolating Echo Canceller is given in [VESG79] for cancellation at sampling rates which are integer multiples of the baud rate. An Interpolating Echo Canceller can be thought of as a combination of M noninterpolating sub Echo Cancellers operating independently of each other with respect to both adaptation and output. If M is the interpolation factor then each sub canceller has N/M coefficients. Sampling at M/T aids clock recovery and lessens jitter effects. This is a variation of the fractionally spaced Echo Canceller.

The finite precision requirements are examined for a digital cancellation scheme. The requirements of the Analog-to-Digital converter, the coefficient correction, coefficients, filter accumulator and the input to a Digital-to-Analog converter are all examined in detail. A typical interpolating filter is considered. For a given quantization step of the A/D converter, q , the word length of the Analog-to-Digital Converter should be

$$W_{A/D} = \left\lceil \log_2 \left(\frac{|r_s(k)|_{max}}{q} + 1 \right) \right\rceil + 1. \quad (1.25)$$

Equation 1.25 represents a saturation limit for a mid trend quantizer. $r_s(k)$ is the sampled output error signal at time k . The coefficient correction terms requirements are the same as eq(1.25). The remaining components lose no precision if the word length chosen is

$$W_C = W_{AGC} = W_{\epsilon} = W_{A/D} + \log_2 \frac{1}{2\alpha}, \quad (1.26)$$

where α is an amplification constant.

A judicious choice of q is essential for a sufficient representation of all signal components. A good measure for q relates the mean square value of the receiver signal and that at the hybrid.

Implementation and analysis of an all digital single canceller U-interface circuit has been completed in [SzZS86]. MMS43 coding is used. Error signals for adapting the transversal filter EC are taken from the cancellation point, $r(k)$, in Figure 1.8 and from the DFE. Convergence is improved by using two different error signals. A highpass receive filter is used to reduce low frequency noise and dc offset. An AGC is used before equalization. The chip size is 40 mm^2 using a $2 \text{ }\mu\text{m}$ CMOS feature size. A 12 bit 2nd order sigma delta A/D converter is used.

A hardware realization of a RAM Echo Canceller and DFE combined jointly is given in [GeVC84]. This structure will reduce nonlinearities which

do not significantly influence the cross product of the echo and intersymbol interference.

The analog front end circuitry for an ISDN transceiver has been implemented in [AgaA88]. A 12 bit A/D converter, transmit/receive filters, antialias filter, balance filter, line driver and fixed equalizer are all put on a single monolithic IC. The design is in 4 μm CMOS and has a die size of 46 mm^2 .

Other Solutions

A Pole-Zero adaptive EC is examined in [LoSF87]. The study is for acoustic echo cancellation however the pole-zero concept has some similarities to the method used in this thesis and for this reason will be mentioned. The transfer function of the echo canceller is

$$H(z) = \frac{A(z)}{1 - B(z)}. \quad (1.27)$$

$$A(z) = a_0 + a_1 z^{-1} + \dots + a_n z^{-n}$$

$$1 - B(z) = 1 - b_0 - b_1 z^{-1} - b_2 z^{-2} \dots b_m z^{-m}$$

The LMS algorithm is used for adaptation.

An adaptive IIR realization normally has stability problems. Equation 1.27 is an IIR type representation. The equation error method is utilized to solve the problem of identification of the echo path model parameters. This leads to a well behaved realization and consequently stable operation. Difficulties arise in this realization if the number of poles M and the number of zeroes N do not accurately match the echo.

Korn [Korn84] has examined approximating the hybrid by rational func-

tions. The representation is

$$G_1(s) = \frac{\sum_0^N x_i s^i}{\sum_0^M y_i s^i} = x_N \frac{\prod_1^N (s_i + z_i)}{\prod_1^M (s_i + p_i)} = \sum_1^M \frac{r_i}{s + p_i} + r_0. \quad (1.28)$$

where

$$G_1(s) = \frac{Z_L(s)}{Z_L(s) + Z_T(s)}$$

and the hybrid is perfectly balanced if

$$C_1(s)G_1(s) = C_2(s).$$

$Z_L(s)$ and $Z_T(s)$ are the Line and Terminating impedances of the loop and hybrid respectively. $C_2(s)$ is the echo model in the s domain and $C_1(s)$ is the transmit filter in the s domain. The receiver signal is

$$V_{Rcv}(s) = G(s)V_{Tx}(s) + V_L(s), \quad (1.29)$$

where

$$G(s) = G_1(s)C_1(s) - C_2(s). \quad (1.30)$$

V_{Rcv} is the received signal, V_T is the transmitted signal and V_L is the loop noise.

This method is very similar to the method applied in this thesis with regard to rational functions. In Korn the case of $N = M = 2$ is examined. The results are presented for various single gauge line lengths with no bridge taps and a 100 ohm terminating impedance. Analysis of the method of choice in eq(1.28) to use for implementation is completed. Adaptation as well as practical implementation is considered for each. The choice made is to have

$$C_1(s) = 1 \quad \text{and} \quad (1.31)$$

$$C_2(s) = \sum_1^M \frac{\hat{r}_i}{s + \hat{p}_i} + \hat{r}_0. \quad (1.32)$$

$\hat{r}_i$ and $\hat{p}_i$ are initially arbitrarily chosen and their final values correspond to values obtained after adaptation.

The adaptation of the analog hybrid is done without the presence of a far end signal and convergence time is found to be proportional to the integration gain. In the presence of a far end signal (after convergence) the integration gain must be reduced. [WeiK87] have completed a discrete implementation of eq(1.32) with $i = 1$.

Echo Canceller Burst Mode is proposed in [BrMT85] as a plausible U-interface solution. The method combines hybrid method echo cancellation and Time Division Multiplexing (TDM). The principle is timing recovery as well as echo canceller adaptation can be made easier if compression of the data into bursts interleaved with periods of no data is done. The periods of no data are much shorter than the data transmission lengths.

The burst transmission defines framing information without the need of extra information bits. A strictly adhered to frame organization allows adaptation of the EC and equalizer. A DFE and a linear equalizer are included in this proposal. The frame organization permits selection of which block to train. The CCITT has defined frame organization for ISDN basic access. It may not conform easily with this implementation.

Ideal Hybrid couplers would eliminate the near end echo and the far end echo could be dealt with using equalizers only. In the absence of an ideal hybrid coupler a compromise hybrid termination (balance network) unique to each line can be used. Manually calibrating each line is time inefficient; measuring return loss after sending a transmitting tone is both time consuming (processing time) and requires complex measuring devices (reflectometer); automatic impedance measuring is also a difficult problem.

Automatic adaptive balancing over the two wire subscriber loop for use in speech and low bit rate data transmission is looked at in [Mess80] and [DDHM80]. Both papers utilize variations of the same method for a solution. The method is automatic and continually adapts.

The echo response is modelled as

$$H_L(w) = \frac{Z_L}{R + Z_L}, \quad (1.33)$$

where Z_L is the unknown line impedance and R is the impedance of the hybrid seen from the loop.

$H_L(w)$ is chosen so as to model the impedances of both loaded and non-loaded lines. An estimate of $H_L(w)$ is

$$H(w) = \theta H_1(w) + (1 - \theta) H_2(w), \quad (1.34)$$

where

$$H_j(w) = \frac{Z_j}{R + Z_j} \quad j = 1, 2. \quad (1.35)$$

Z_1 and Z_2 are the impedances chosen to match the loaded and non loaded lines respectively. This is a system identification problem and θ , ranging $0 \leq \theta \leq 1$, is the variable parameter chosen to compromise between the systems. θ is adapted to minimize the residual echo power. Values of Z_j can be found in the literature but these are taken from a average sampling of the loop population. Adequate matching over all lines is not assured.

The first paper goes through analysis while the latter paper examines implementation. A continuum of values for θ are available if an analog realization were used, while a digital realization will depend on the word length of θ . The digital method allows for storage of θ thus eliminating readaptation after initial transmission.

The implementation is highly sensitive to the far end talker spectrum. Adaptation is turned off when simultaneous talking occurs and when the near

end talker is detected. This method has been mentioned here to illustrate that all the components of the U-interface affecting echo are being examined. In this particular case the approach is inadequate for high speed full duplex data transmission under CCITT specifications.

Many of the Echo Canceller prototypes use off chip components for balancing networks to achieve transmission isolation. These are generally resistor capacitor combinations designed to provide some balance over the transmission bandwidth. It is difficult using simple RC networks to compensate over the entire frequency range considered. Off chip component costs are high. A simple structure is shown in Figure 1.16. It can be shown that if a purely resistive balance network is used the echo impulse response will behave approximately as a decaying exponential.

A novel approach was used in [FTGH88] for a balance network, in addition to the other ideas previously attributed to this paper. A selectable balancing network (seven possibilities), which considered seven impedance characteristics each, was used to reduce transhybrid loss. Suppression of the peak value was used as the adaptation measure. An improvement of 18 dB over the single resistor case is reported.

Integration can pose problems. Polysilicon resistors are quite variable, with tolerances not approaching those of quality discrete resistors. The added time and expense of integration to provide the impedance characteristics, as well as selecting the proper impedance characteristics does not warrant its use. Abandoning work on a quality balance network, replaced by more precise cancellation is a reasonable resource allocation choice.

Figure 1.16: Typical RC Off-Chip Balance Network

1.5 Outline

In addition to the previous work already examined and the requirements stated, further chapters will look at; the theory of orthonormal functions in Chapter 2, identifying those functions which are applicable to echo cancellation; the simulation process and scientific method will be presented in Chapter 3; Chapter 4 will give the results of using specific classical orthonormal functions to cancel the echo impulse response; implementation requirements associated with using the two stage method and performing echo tail cancellation using classical orthonormal functions is done in Chapter 5. A discrete orthonormal system is developed; a brief look at the implications of using a two stage approach based on the results obtained in previous chapters will be given last in Chapter 6. Chapter 7 will give final conclusions and recommendations.

Chapter 2

Orthonormal Functions

A novel approach to cancelling the echo tail by modelling the echo response as a linear combination of simple time responses which are orthonormal is presented here. The selection of these functions toward applicability to echo cancellation of the tail is initially based on the similarity of the shape of the responses to the echo tail and the simplicity of realization of the orthonormal functions.

2.1 Definition

A set of real, nonzero, and continuous functions $v_1, v_2, \dots$ are said to be orthonormal in the range (a, b) if

$$\int_a^b v_n(x)v_m(x) dx = \begin{cases} 0 & \text{if } m \neq n \\ 1 & \text{if } m = n. \end{cases} \quad (2.1)$$

The functions $v_i(x)$ are orthogonal if they obey the first condition. The functions are normal if they obey the later condition. The set of functions $\{v(x)\}$ are called orthonormal if both conditions are met[Lec67].

2.2 Use Of Orthogonal Polynomials

The justification for investigating orthonormal polynomials to approximate and cancel the echo response at the U-interface can be developed from the performance measure, Mean Square Error (MSE). MSE is a measure of the residual echo power. Mean Square Error can be written as

$$\int_{\alpha}^{\beta} [S(x) - \sum_{i=0}^n C_i f_i(x)]^2 dx = \epsilon. \quad (2.2)$$

α and β are the interval limits. $S(x)$ is the response to be cancelled. $f_i(x)$ is the i th approximating function and C_i is the i th coefficient associated with the i th approximating function. ϵ is the minimum desired residual after cancellation. The minimum integral square error caused by truncating the series is [Sche71]

$$E(n, x) = \sum_{i=n+1}^{\infty} C_i^2. \quad (2.3)$$

If ϵ is to be a minimum then the finite series

$$\sum_{i=0}^n C_i f_i(x) \quad i = 0, 1, \dots, n$$

must closely approximate the signal, $S(x)$, to within some prescribed accuracy. The logical progression from this statement is the determination of the type of finite series which will yield a minimum in the mean square sense.

Theorem of Weierstrass : A function, continuous in a finite closed interval, can be approximated with a preassigned accuracy by polynomials.

A polynomial function can be written as

$$g(x) = C_0 f_0(x) + C_1 f_1(x) + C_2 f_2(x) + \dots + C_i f_i(x) + \dots + C_m f_m(x). \quad (2.4)$$

A polynomial in x of degree n can be represented as

$$f_n(x) = K_n x^n + K'_n x^{n-1} + \dots \quad n = 0, 1, 2, \dots \quad (2.5)$$

An infinite number of unique polynomials of length $m \leq \infty$ exist which satisfy eq(2.4) and which would approximate the echo signal leaving some residual. The performance measure, MSE, provides a more stringent discriminating selection process. Let ϵ' be the residual after some cancellation using a polynomial of order n , then

$$\min \left[\int_{\alpha}^{\beta} [S(x) - \sum_{i=0}^n C_i f_i(x)]^2 dx \right] = \epsilon'. \quad (2.6)$$

Taking the derivative of eq(2.6) with respect to C_i for the general polynomial case and setting the derivative equal to zero,

$$\frac{d\{MSE\}}{dC_i} = 2 \int_{\alpha}^{\beta} (S(x) - \sum_{i=0}^n C_i f_i(x)) (-f_i(x)) dx = 0. \quad (2.7)$$

$$- \int_{\alpha}^{\beta} (S(x) f_i(x)) dx + \int_{\alpha}^{\beta} \sum_{i=0}^n C_i f_i(x) f_i(x) dx = 0 \quad (2.8)$$

In order to obtain the coefficients which minimize the residual $n + 1$ equations must be solved to obtain the coefficient, C_i . The resulting function $g_1(x)$ (subscript denotes first iteration) must then be checked to verify that the error criterion is met. Subsequent iterations must be performed if $\epsilon' > \epsilon$.

If the polynomials are orthogonal the first condition of eq(2.1) can be utilized to determine the C_i 's. C_i reduces to

$$C_i = \frac{\int_{\alpha}^{\beta} S(x) f_i(x) dx}{\int_{\alpha}^{\beta} f_i^2(x) dx}. \quad (2.9)$$

The error criterion can be met by increasing n such that $\epsilon' \leq \epsilon$. As the order of the orthogonal polynomial expansion is increased the mean square error will be reduced. The coefficients of each approximating polynomial only need to be determined once even if the order of the function needed to be subsequently increased to meet more stringent performance requirements. This

Figure 2.1: Simple Orthogonal Functions

is in contrast to the case of nonorthogonal functions where the coefficients must always be recalculated based on equation 2.8. Orthogonal functions are linearly independent. The coefficients of an orthogonal expansion minimize the mean square error when the series is terminated after a finite number of terms[Beck73]. A simple set of orthogonal functions are illustrated in Figure 2.1[Beau84].

In general an orthogonal system in the interval (a,b) is defined by

$$\int_a^b w(x)f_n(x)f_m(x) dx = \begin{cases} 0 & \text{if } m \neq n \\ h_n^2 & \text{if } m = n. \end{cases} \quad (2.10)$$

$w(x)$ is called the weight function associated with the orthogonal set, unique to that set, and defined in the interval (a,b) . The weight function can be thought of as a probability density function independent of n . h_n is called the norm and is non zero. $f_n(x)$ and $f_m(x)$ are orthogonal functions if they comply with the definition in equation 2.10.

The weight function can be grouped together with $f_n(x)$, $\sqrt{w(x)}f_n(x)$. The resulting set will remain orthogonal¹. The set also remains orthogonal if we multiply by a constant. The only change is to the norm. In order to work with a unique system the orthogonal set is standardized[Beck73]. A specific

¹ An exception of note is the case of the Bessel function. This function does not remain orthogonal when the square root of the weight function is grouped with the orthogonal expansion $f_n(x)$ [Beck73].

type of standardization is normalization. Employing this method of standardization results in a norm equal to unity. The result of these operations yields eq(2.1). Thus $v_n(x) = \frac{\sqrt{w(x)f_n(x)}}{h_n^2}$. A method of orthonormalization using N linearly independent functions is given in [Bing65].

Completeness is the ability of a set of functions to properly represent a given function. Fourier analysis determines the harmonics which are present in a periodic sequence and thus the sequence is completely defined by its harmonics. In a similar manner the orthonormal polynomial set must be complete to adequately replicate the echo signal.

If the orthonormal expansion set is $g(x)$, eq(2.4), then the system is complete if

$$\int g^2(x)dx < \infty. \quad (2.11)$$

The coefficients of $g(x)$ are

$$c_n = \frac{1}{h_n^2} \int_a^b w(x)g(x)f_n(x)dx. \quad (2.12)$$

The law of conservation of energy (Parseval's equation) is then

$$\sum_{n=0}^{\infty} h_n^2 c_n^2 = \int_a^b g^2(x)dx. \quad (2.13)$$

An orthonormal set is uniquely and completely defined by its weight function, orthogonality interval and its normalization.

In addition to the performance measure, MSE, used to discriminate between solutions, a second measure is used to further reduce the possible set of solutions. The second measure is that of realizability. Encompassed in the broad term realizability is the corollary that the orthogonal functions must readily lend themselves to the application at hand. This eliminates any investigation into Walsh functions and functions related to them. The Walsh family of functions are employed in applications for image processing[Beau84].

The use of a sine or a cosine series can also be eliminated from consideration because an examination of the echo signal shows that it will consist of both odd and even harmonics.

2.3 Applicable Functions

The previous section gave a description of the properties of orthonormal functions. The steps required to obtain the definition of orthonormal functions was given. This section will examine those orthonormal functions which may find application to the echo cancellation problem. Two forms of orthonormal functions will be considered, continuous and discrete.

2.3.1 Classical Continuous Polynomials

A class of orthonormal polynomials that are continuous within the interval (a,b) and are polynomials of degree n are known as classical polynomials. The designation '*classical*' originates from the association with the classical mathematicians Laguerre, Gauss, Hermite, Jacobi, etc. These polynomials are solutions generated from solving a differential equation of the Sturm-Liouville type [Szeg68] for f_n .

$$g_2(x)f_n'' + g_1(x)f_n' + a_n f_n = 0 \quad (2.14)$$

$g_2(x)$ and $g_1(x)$ are independent of n and a_n is a constant depending only on n . The solution of eq(2.14) by various techniques is given in [Spie81] and [Simm72]. These polynomials find applications in such areas as quantum mechanics[Simm72], electromagnetic wave propagation[Lebe65] and numerical interpolation[BuFR81] to name a few.

The classical continuous polynomials can be grouped according to their orthogonality interval (a,b). The reasons for grouping the polynomials ac-

Finite (-1, 1)			Semi-Infinite (0, ∞)	Infinite (-∞, ∞)
Jacobi			Laguerre	Hermite
Gegenbauer			Generalized Laguerre	Hermite
Legendre	Chebyshev I	Chebyshev II		
Associated Legendre			Associated Laguerre	

Table 2.1: Classical Orthogonal Polynomials By Orthogonality Interval

cordingly will become clearer once the equations for each group have been developed. Table 2.1 illustrates the groupings². The shifted versions of those polynomials whose orthogonal intervals are (-1,1) are not included as they are simply variations of the unshifted forms. For a complete grouping see [Beck73].

In addition to the orthonormal properties already stated two other properties of these polynomials are particularly useful. The first is known as *Rodrique's Formula*.

$$f_n(x) = \frac{1}{e_n w(x)} \frac{d^n \{w(x)[g(x)]^n\}}{dx^n} \quad (2.15)$$

Using this formula all successive polynomials can be generated from a knowledge of $g(x)$, which is a polynomial in x independent of n , eq(2.4), and from the weight function. e_n is a constant independent of x . A polynomial of

² The English transliteration Chebyshev will be used.

degree n can be determined from the original basis function by taking the derivative n times.

The second relation that is important for this application is the recurrence relation

$$f_{n+1}(x) = (a_n + xb_n)f_n(x) - c_nf_{n-1}(x), \quad (2.16)$$

where

$$b_n = \frac{C_{n+1}}{C_n} \quad (2.17)$$

$$a_n = b_n \left(\frac{C'_{n+1}}{C_{n+1}} - \frac{C'_n}{C_n} \right) \quad (2.18)$$

$$a_n = b_n \left(\frac{C_{n+1}C_{n-1}h_n}{C_n^2h_{n-1}} \right) \quad (2.19)$$

and $f_{-1}(x) = 0$. The C_i and C'_i coefficients originate from eq(2.5). The recurrence relation can be used to generate successive polynomials, $f_{n+1}(x)$, from previous polynomials $f_n(x), f_{n-1}(x)$. More complete coverage of all the properties of classical polynomials is given in [AbSt72].

2.3.2 Polynomials Of A Discrete Variable

A class of orthogonal functions, which may find some application to the echo cancellation problem, are discrete polynomials. As with the case of the classical polynomials, these orthogonal functions are completely defined by their orthogonality interval, weight function and standardization. The set is orthogonal in the interval x_0, x_{N-1} , for $i = (0, 1, 2, \dots, N-1)$, if [Beck73]

$$\sum_{i=0}^{N-1} w(x_i) f_n(x_i) f_m(x_i) = \delta_{nm} h_n^2. \quad (2.20)$$

δ_{nm} is the Kronecker delta. These polynomials are listed in Table 2.2. The interesting thing to note is that the weight function of these polynomials corresponds to the uniform, binomial and Poisson distributions respectively.

Name	Interval	Weight Function
Chebyshev	(0,N-1)	1
Krawthchouk	(0,N-1)	$C_x^n p^x q^{N-1-x}$
Charlier	(0,∞)	$e^{-a} a^x / x!$

Table 2.2: Orthogonal Polynomials Of A Discrete Variable

A set of orthogonal polynomials can be generated using a recurrence relation similar to eq(2.16). The appropriate norm and discrete constants can be generated from the orthogonality sum, as opposed to the orthogonality integral. A particular set of functions would be found for each case with a unique norm and weight function, however these methods are computationally intensive. In addition all the points of the interval x_0, x_{N-1} must be known in order to generate a solution. For continuous on-going cancellation realization of this class of orthogonal polynomials would be a difficult problem.

2.4 Realization Of Orthonormal Functions

The representations given in the previous sections show polynomials that are functions of x (x is equivalent to a time parameter). This does not help in the realization of a practical integrated system. x^n 's are difficult to transform into the frequency domain and generally must be approximated. If used in the time domain they must be multiplied as single powers. This is time consuming and silicon area intensive. A practical alteration of these functions in x is to represent the components as exponential functions. Exponential functions transform into rational functions.

As a starting point the general Jacobi polynomials ($P_n^{(\alpha,\beta)}(x)$) will be

considered first. The Jacobi orthogonal set is defined as

$$\int_{-1}^1 P_n^{(\alpha,\beta)}(x) P_m^{(\alpha,\beta)}(x) dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{2^{\alpha+\beta+1}}{2n+\alpha+\beta+1} \frac{\Gamma(n+\alpha+1)\Gamma(n+\beta+1)}{n! \Gamma(n+\alpha+\beta+1)} & \text{if } m = n. \end{cases} \quad (2.21)$$

Examination of eq(2.21) shows that the limits must be transformed to represent a causal system. The set must also be normalized, however the norm contains the gamma function ($\Gamma(x)$). The gamma function is defined as

$$\Gamma(\alpha) = \int_{-\infty}^{\infty} e^{-t} t^{\alpha} dt. \quad (2.22)$$

The complexity of this realization means the system is impractical in integrated form and can be disregarded from further investigation. The gamma function is present in the norm of the Gegenbauer polynomials as well.

The subclass of the Gegenbauer polynomials, Chebyshev I, Chebyshev II, and Legendre polynomials do not possess a norm of the complexity of the Jacobi polynomial. The Legendre function is defined as [Lee67]

$$\int_{-1}^1 P_n(x) P_m(x) dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{2}{2n+1} & \text{if } m = n. \end{cases} \quad (2.23)$$

The limits of integration must be changed from (-1,1) to (0, ∞). The echo signal is a causal signal of infinite duration. The substitution of $x = 2y - 1$ will shift the limits to (0,1). Conveniently an additional change of variables to $y = e^{-p\tau}$ will shift the limits of integration to (0, ∞). p is called the time scale factor or pole location if the analysis is in the frequency domain. The substitution of the exponential allows a transformation into a rational form.

$$\int_0^{\infty} p e^{-p\tau} P_n(2e^{-2p\tau} - 1) P_m(2e^{-2p\tau} - 1) d\tau = \begin{cases} 0 & \text{if } m \neq n \\ \frac{1}{2n+1} & \text{if } m = n. \end{cases} \quad (2.24)$$

The weight function, $w(x)$, is $p e^{-p\tau}$. Let

$$r_n(\tau) = \sqrt{p(2n+1)} e^{-(p/2)\tau} P_n(2e^{-2p\tau} - 1).$$

Equation(2.23) then reduces exactly to the form of eq(2.1).

$$\int_a^b v_n(x)v_m(x) dx = \begin{cases} 0 & \text{if } m \neq n \\ 1 & \text{if } m = n \end{cases}$$

The function now has orthogonalization intervals commensurate with those of the echo signal. The Legendre set of orthogonal functions in the time domain is given by

$$v_0(\tau) = \sqrt{p}e^{-(p/2)\tau} \quad (2.25)$$

$$v_1(\tau) = \sqrt{3p}(-1 + 2e^{-p\tau})e^{-(p/2)\tau} \quad (2.26)$$

$$v_2(\tau) = \sqrt{5p}(1 - 6e^{-p\tau} + 6e^{-2p\tau})e^{-(p/2)\tau} \quad (2.27)$$

⋮

Legendre polynomials of various order are plotted in Figure 2.2³.

The orthonormal set can be easily presented in the frequency domain.

$$\{V_n(w)\} = \left\{ \frac{\sqrt{(2n+1)p} \left[\frac{p}{2} - jw \right] \left[\frac{3p}{2} - jw \right] \cdots \left[\frac{(2n-1)p}{2} - jw \right]}{2\pi \left[\frac{p}{2} + jw \right] \left[\frac{3p}{2} + jw \right] \cdots \left[\frac{(2n+1)p}{2} + jw \right]} \right\} \quad n = 0, 1, 2, \dots \quad (2.28)$$

A network (Figure 2.3) can be constructed from $\{V_n(w)\}$ to illustrate the manner in which the functions can be implemented using filters. The networks for the Legendre and the Chebyshev polynomials are equivalent except for constants.

The derivations for transforming both the Legendre polynomials and the Chebyshev polynomials into realizable networks are very similar. A complete derivation is performed in Appendix A.3.1 for the Chebyshev polynomial set only. This proof fills a gap in the literature [Lee67] for the derivation of Chebyshev orthonormal polynomials and acts as a guide in this thesis for the derivation of Legendre and Laguerre polynomials.

³ They are plotted individually in Appendix A.1.

Figure 2.2: Legendre Functions : Orders 0 to 5. Normalized to the 0th Order Legendre Function.

Figure 2.3: Legendre Network. The constant $\frac{1}{\pi}$ multiplier is not shown.

Laguerre functions have an orthogonalization interval from $(0, \infty)$. The polynomial sequence can be generated from Rodrigues's formula,

$$L_n(\tau) = \sqrt{2p} \frac{e^{-\tau}}{n!} \frac{d^n \{(-\tau)^n e^{-2p\tau}\}}{d\tau^n}. \quad (2.29)$$

Laguerre polynomials of various order are plotted in Figure 2.4⁴.

The Laguerre functions can be expressed in the frequency domain by applying the Fourier transform to the resulting polynomial set.

$$\{L_n(w)\} = \frac{\sqrt{2p}}{2\pi} \frac{(p - jw)^n}{(p + jw)^{n+1}} \quad n = 0, 1, 2, \dots \quad (2.30)$$

The network equivalent is shown in Figure 2.5.

The final class of orthogonal classical functions are the Hermite polynomials. These polynomials are orthogonal in the interval $(-\infty, \infty)$. The orthogonal relation is

$$\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} H_n(x) H_m(x) dx = \begin{cases} 0 & \text{if } m \neq n \\ n! \sqrt{2\pi} & \text{if } m = n. \end{cases} \quad (2.31)$$

To transform the limits to a causal interval of $(0, \infty)$ only odd ordered functions need be considered. The difficulty however is in the realization of the system. Hermite polynomials do not transform into rational functions in the frequency domain because of their weighting function $e^{-\frac{x^2}{2}}$. A network realization similar to Figure 2.3 or Figure 2.5 cannot be done.

Examination of each network in turn, Figures 2.3 and 2.5, show that for equal order n the Laguerre network contains n sections (a section is defined as a block which performs a filtering operation) and the class which has as its members the Legendre and both Chebyshev networks contain $2n + 1$ sections. The network equations show that all the poles are in the left half of the jw plane. Thus the networks are stable. The networks of Figure 2.3

⁴ They are plotted individually in Appendix A 2

Figure 2.4: Laguerre Functions : Orders 0 to 5. Normalized to the 0th order Laguerre Function.

Figure 2.5: Laguerre Network. The constant $\frac{1}{T}$ multiplier is not shown.

and Figure 2.5 illustrate how a tap, n , contains all the functions, f_0 to f_n inclusive.

2.5 Summary

The general theory of orthonormal functions has been given. Mean square error minimization and the desire to have a unique easily realizable in silicon function set are the motives for choosing the classical orthonormal Legendre and Laguerre functions for echo cancellation as opposed to other orthonormal function sets.

The transformation of the Legendre and Laguerre functions into realizable networks has been shown. The network model is of a set of linear time impulse responses which are orthonormal. The responses are dependent on the time scale factor, p .

Chapter 3

Echo Generation

Generation of near end echo signals is done in stages. Figure 3.1 shows a block diagram representation of the steps involved. Pre (i.e. transmitter) and post (i.e. receiver front end) filtering are not necessarily included or kept the same for all echo signals generated. Gaussian noise and X-talk noise is not introduced into the simulated system. Determination and examination of the echo impulse response is completed.

3.1 Channel And Hybrid

The hybrid and channel combine to produce an echo signal. Impedance mismatch over the frequency band under consideration will produce an echo signal which will corrupt the far end data that is to be received. Figure 3.2 illustrates the impedance mismatch.

The Wheatstone Bridge in Figure 3.3. illustrates clearly the impedance mismatch. If $V_{ab} = V_{ac}$ and $V_{bd} = V_{cd}$ then

$$Z_L = \left(\frac{Z_{Rcv}}{Z_{Tx}} \right) Z_{BAL}. \quad (3.1)$$

Figure 3.1: Steps In Simulated Echo Generation

Figure 3.2: Interface Impedances

Echo can be eliminated entirely if the balance impedance matches the line impedance (channel plus far end hybrid termination impedance) multiplied by the ratio of the transmitter impedance to the receiver impedance. T1D1 specifications call for a 135 ohm nominal driving point impedance at the interface looking toward the Network Terminator[T1D1].

Figure 3.2 can be redrawn as a Wheatstone Bridge, Figure 3.3.

3.1.1 Transformer

The transformer model assumed for work in this thesis is shown in Figure 3.4. Included are typical values for the inductance and capacitance. The value of R is set at 135 ohms for all simulations. A more detailed model of the transformer circuit is not used nor is it needed. Parameter values that are characteristic of a transformer can vary widely¹. For this reason only those parameters which affect the echo response in a marked way are used. The results obtained are independent of transformer selection. A detailed guide of transformer parameters can be found in [Gros67].

The transformer model used contains no balance network. Again the reasoning is to maintain the independence of the results from any circuit configurations. The echo transfer function of the hybrid with no balance network is derived in Appendix B.1 and is stated here in eq(3.2).

$$H_{ec_{nbal}}(s) = \frac{\left(1 + \left(\frac{N_1}{N_2}\right)^2 \left(\frac{1}{sL}\right) \left(Z_L + \frac{1}{sC}\right)\right) R}{R + \left(\frac{N_1}{N_2}\right)^2 \left(Z_L + \frac{1}{sC}\right) \left(\frac{R}{sL} + \frac{1}{2}\right) 2}. \quad (3.2)$$

A balance network which gives adequate attenuation of the echo over the entire ISDN bandwidth over all possible line configurations is extremely difficult to select. Discrete components are more costly than integrated components. A typical off-chip balance network was previously shown in Figure

¹ The echo tail can be affected by the hybrid parameters.

Figure 3.3: Wheatstone Bridge

Figure 3.4: Transformer Circuit Model : V_T is the transmitter source.

1.16. For completeness and comparison value simulations were run for the same transformer model as in Figure 3.4 but with a resistive balance network added. No attempt was made to match the complex impedance of the channel and hybrid. The results using the balance network are reported in Section 3.3.2 for four different resistor values. The transfer function of the transformer incorporating a balance network is developed in Appendix B.1 for completeness.

3.1.2 Line Configurations

As stated previously transmission will be over a two wire copper twisted pair. Subscriber loop configurations are not the same throughout the telephone network population. A number of differences exist which affect the line or render the line inadequate for digital transmission. All variations will be taken into account.

Considering only nonloaded lines² (i.e. those lines not containing inductive coils), the main impairments to echo free transmission over the subscriber loop are bridge taps, gauge changes and temperature variations over the line. The line can be represented by a series of transmission matrices which characterize the behavior of a two port network [Valk74]. The elements of the transmission matrix are as follows

$$\frac{1}{A} = \left. \frac{V_2}{V_1} \right|_{I_2=0}, \quad \frac{-1}{B} = \left. \frac{I_2}{V_1} \right|_{V_2=0}, \quad \frac{1}{C} = \left. \frac{V_2}{I_1} \right|_{I_2=0}, \quad \frac{-1}{D} = \left. \frac{I_2}{I_1} \right|_{V_2=0}. \quad (3.3)$$

Each anomaly in the line can be modelled by a transmission matrix. For a transmission line with no bridge taps

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A = \cosh(\gamma l) & B = Z_o \sinh(\gamma l) \\ C = \sinh(\gamma l)/Z_o & D = \cosh(\gamma l) \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}, \quad (3.4)$$

² Loaded lines make up approximately 23% of the loop population.

where l is the line length, Z_o is the characteristic impedance of the line

$$Z_o = \sqrt{\frac{(R + j2\pi fL)}{(G + j2\pi fC)}} \quad (3.5)$$

and γ is the propagation constant of the line.

$$\gamma = \sqrt{(R + j2\pi fL)(G + j2\pi fC)}. \quad (3.6)$$

R, L, G and C are the resistance, inductance, conductance and capacitance per unit length of the transmission line respectively. The theory behind these definitions is covered thoroughly in [Grid67].

Information taken from [USIT75] provided the characteristic impedance in ohms and the propagation constant in dBs/mile and degrees/mile. Information was available for only 19, 22, 24, and 26 AWG lines at $70^\circ F$ (Appendix C.5). The end-to-end channel response for a line terminated in its characteristic impedance is

$$H(f) = 10^{-\frac{|S|L}{20}} e^{-j\theta L}. \quad (3.7)$$

Figure 3.5 illustrates a subscriber loop line with bridge taps and gauge changes. Transmission matrices are shown for each section. Temperature variations are not considered in this thesis.

The overall line matrix is determined by multiplying each series section together. This is simple for a gauge discontinuity, if however a bridge tap is encountered it must be dealt with differently. Each bridge tap is converted to an equivalent shunt impedance in series with its main line. Appendix C.1 shows how this is done. The line is continually reduced via matrix multiplication until a single transmission matrix results.

A C program, similar to LINEMOD [Mess84a], was written to process any possible line configuration. A sample input routine is shown in Appendix

C.3. The line configuration is read in recursively and is dynamically allocated memory depending upon the line requirements. The routine to process the line utilizes recursion to obtain the overall ABCD matrix and is shown in Appendix C.2. Although this program does not have some of the flexibility of LINEMOD it makes full use of recursion, unlike LINEMOD. A sample output listing is shown in Appendix C.4.

The transfer function of the channel and hybrid as seen from the hybrid on the opposite transmitting end is

$$Z_L = \frac{AZ_T + B}{CZ_T + D}. \quad (3.8)$$

A simplified view of the subscriber line is as shown in Figure 3.6. For the line

$$V_1 = AV_2 + BI_2, \quad (3.9)$$

$$I_1 = CV_2 + DI_2, \quad (3.10)$$

$$V_2 = I_2 Z_T. \quad (3.11)$$

$$\frac{V_1}{I_1} = \frac{A(I_2 Z_T) + BI_2}{C(I_2 Z_T) + DI_2} \quad (3.12)$$

This can be simplified and reduced to the form of eq(3.8).

The hybrid termination impedance seen from the subscriber line is

$$Z_T = \frac{1}{sC} + \left(\frac{N_2}{N_1} \right)^2 \frac{2R}{1 + \frac{2R}{sL}}. \quad (3.13)$$

The derivation for Z_T using the transformer model of Figure 3.4 is given in Appendix B.2.

3.1.3 Coding

An improved tolerance to crosstalk and noise can be obtained by using a multi-level code with reduced sampling rates. The improvement in tolerance

of a multi-level code versus a binary code is given by [AdCG84]

$$I = 10 \log_{10} \left[\frac{3k^\delta}{L^2 - 1} \right] dB, \quad (3.14)$$

where for white noise $\delta = b + 1$, b is a parameter proportional to the loop resistance. $k = B/R_L$ and B is the signaling rate for binary and R_L is the signaling rate for an L -level code. $R_L \geq B/\log_2 L$. The simulation technique used in this thesis is independent of whether a binary code or a multi-level code is used. The line responses are independent of any coding schemes and are thus applicable for any echo cancellation scheme. The particular affect of coding on the echo response can be considered separately.

3.1.4 Filtering

A third order Butterworth filter with $f_{corner} = 50$ kHz is used as a transmit filter for many of the echo signals. The objective of filtering is to reduce crosstalk and Radio Frequency Interference (RFI) during transmission [Mess86b]. The filter frequency response is shown in Figure 3.7 for a sampling rate of $f_s = 610$ kHz. The choice of filter order and corner frequency is made in accordance with T1D1 recommendations to limit the power in the side lobes due to crosstalk disturbers. The simplified near end crosstalk model has a constant slope of 15 dB/decade. The main lobe power is below 50 kHz, yet a significant portion is contained in the side lobes. A deliberate intent of the crosstalk simulation model used in T1D1 specifications is to force implementations to be less sensitive to components greater than 50 kHz. The 50 kHz corner frequency and 60 dB/decade roll off of the 3rd order Butterworth filter meets the requirements of T1D1 without undue complication. A simplified approach is taken although T1D1 specifications must also be met regarding pulse mask and the average NEXT power.

No pulse shaping other than this filter is used in any portion of the work. The integrity of the work with regard to independence from manufacturer or specific design is maintained.

An anti-alias receive filter will be required for a practical implementation. This will remove out of band noise and prevent aliasing in sampling. Computer simulations did not utilize such a filter.

3.2 Line Response

The echo signals generated are for an 80 kbaud channel. The echo responses are completely characterized by the sampling rate, bandwidth and filtering (if any) used. The echo responses are explicitly shown and are not given in terms of loop length [Aham81] or loop resistance [AdCC'85]. The point of this thesis is not to examine the echo response, but to use it for further analysis. Unless stated to the contrary all echo responses are generated for the case of *no* hybrid balance network, resistive or otherwise.

Eight samples per baud is chosen as the sampling rate. This selection allows for high sampling rate implementations to be examined. If lower sampling rates are desired decimation can be easily done. The choice of a high input sampling rate allows flexibility in the choice of the output sampling rate, should a difference in the two rates be warranted. This is more difficult if baud rate sampling is done. All simulations use 8 samples per baud unless stated otherwise.

A 1 volt pulse centered at $T/2$ with a 100% duty cycle³ followed by $n - 1$

³ This large pulse width will obscure any reflections present within the first few baud intervals. A smaller pulse width will show the bridge tap reflections clearly. However the intent is to use a more practical approach where the pulse would be a 100% duty cycle pulse.

zero volt pulses is used for all simulations⁴. The sampled frequency response of this pulse is shown in Figure 3.8.

The effect of gauge discontinuities on the echo response is given in Figure 3.9. The solid line is the echo impulse response of Line 07. Line 07 has no gauge changes or bridge taps. The dashed line corresponds to Line 21 which has a single gauge change within one kilometre and no bridge taps. The dotted line is Line 24. Line 24 has two gauge changes with one and a half kilometres and no bridge taps. The dot-dashed line has two gauge changes within two kilometres and is the echo impulse response of Line 22.

The effect of one or more bridge taps on the echo response is given in Figure 3.10. The solid line is the echo impulse response of Line 07. Line 07 has no gauge changes or bridge taps. The echo impulse response of Line 25 is the dashed line. Line 25 has a single bridge tap within one kilometre. The dotted line is for Line 33 which has two bridge taps within a kilometre. The bridge taps are separated by the length of a single bridge tap. Line 33 has no gauge changes. The individual bridge tap lengths of Line 33 and Line 25 are equal. The dot-dashed line has a bridge tap within the first kilometre of length less than the individual bridge tap lengths of Line 33 and Line 25. The dot-dashed line corresponds to Line 28.

In general the echo response has a low pass filter behavior⁵, attenuating high frequency and dispersing the pulse as cable length changes. The gauge changes appear like an adaptive gain controller (AGC) output with regard to the echo. As the bridge tap length increases, attenuation increases, broadening the pulse. The dispersion and reflections caused by the bridge taps will alter the shape of the echo tail. The effect of a line anomaly greater than one kilometer away from the near end transmitter can be disregarded.

⁴ n is the length of the FFT assuming baud rate sampling.

⁵ For large pulse widths.

Figure 3.7: 3rd Order Butterworth Filter Frequency Response

Figure 3.8: One Volt Pulse Sampled Frequency Response

Figure 3.9. Effect Of Gauge Discontinuity On Echo Response

Figure 3.10: Effect Of Bridge Tap On Echo Response

A single sided transfer function is used. The impulse response, $f(t)$, is

$$f(t) = \text{InvFFT} \ (F(f) + F^*(f)) , \quad (3.15)$$

$$f(t) = \text{InvFFT} \ (F(f) + 0) , \quad (3.16)$$

where $F(f)$ is the complex transfer function and $*$ indicates conjugate. The imaginary time signal has no meaning for the echo impulse response obtained via the inverse FFT.

3.3 Bellcore Loop Survey

Simulation of a wide sample of nonloaded subscriber loop lines is necessary to ensure that adequate echo cancellation is achieved under various line configurations⁶. The T1D1 specifications provide a listing of various line configurations which would be typical of the loop population⁷. The loop configurations are given in Appendix C.6. The C language program written takes these line configurations as input and generates the corresponding echo transfer function for the echo channel and hybrid used.

3.3.1 Unfiltered

The echo responses for all Bellcore lines are given in Figure 3.11 for the case of no pulse shaping (no Butterworth filter).

3.3.2 Filtered

The echo responses for all Bellcore lines are given in Figure 3.12 for the Butterworth filtered case. The echo impulse response for Line 22 with a

⁶ Refer to Appendix C.6 to understand the Line numbering scheme used

⁷ The null loop was not considered

Figure 3.11: Echo Responses With No Transmit Filter : Bell Core Loops; Both Transmission Directions Considered, CO to Subscriber and Subscriber to CO

Figure 3.12: Echo Responses With Transmit Filter : Bell Core Loops; Both Transmission Directions Considered, CO to Subscriber and Subscriber to CO.

resistive balance network is shown in Figure 3.13. The smallest amplitude response corresponds to $R_{bal} = 120$ ohms. The increasing amplitude echo impulse responses are associated with $R_{bal} = 135$ ohms, 270 ohms, 5000 ohms and R_{bal} being infinite, i.e. the case of no resistive balance network. An average of 6.43 dB reduction in echo impulse power is achieved through the use of a resistive balance network, $R_{bal} = 135$ ohms, versus the case of no balance network.

Literature has reported a tail response on the order of forty baud intervals [Mess86b]. The tail responses are seen to last approximately 40 baud intervals in the simulations done for this thesis, see Figure 3.14. After 42 baud intervals (echo main pulse plus echo tail) the echo amplitude is less than 0.3 mV. The Butterworth filter response puts an oscillation in the impulse response at the fourth baud interval. The echo tail response with a resistive balance network is shown in Figure 3.15. The non solid lines correspond to the same lines used in Figure 3.13.

The tail responses are of the same general shape for all lines except Line 31. Line 31 has the longest bridge tap on the shortest line. The bridge tap is very near to the subscriber termination. Large reflections are more likely in this case because the short length of the line will not attenuate the echo as greatly as longer lines would.

3.4 Simulation Accuracy

All computer simulations were run on Sun 3 workstations using the SUN OS 3.5. These are 32 bit (not using double precision) floating point machines. Memory constraints were not limiting. However due to the complexity of the reduction algorithm using orthonormal functions run-time operations were of considerable duration.

Figure 3.13: Echo Responses With Transmit Filter Using a Resistive Balancing Network : Bell Core Loops; Both Transmission Directions Considered, CO to Subscriber and Subscriber to CO.

Figure 3.14: Echo Tail Responses With Transmit Filter : Bell Core Loops; Both Transmission Directions Considered, CO to Subscriber and Subscriber to CO.

3.4.1 Maintaining Orthonormality

Computer simulations were run for the Legendre and Laguerre functions to determine what range of values the time scale factor, p in equations 2.28 and 2.30, can have and still maintain orthonormal properties. The time scale factor affects the limits of orthogonality and consequently the function behavior. The time scale factor is dependent on the portion of the echo impulse response being cancelled.

Romberg integration [BuFR81] is used as the numerical integration procedure in simulations where the explicit function was not known. The interval of integration over N symbol periods ($N=80$)⁸ is a sub interval of one half of a symbol period. Thus an integration was done on every 4 samples (8 samples per baud is the sampling rate). The tolerance threshold for the Romberg integration procedure ($|R_{i,j} - R_{i,j-1}|$) is set at $1.0e-06$. The error is of the order $O(h^4)$ [BuFR81]. Cubic spline interpolation is used within the subintervals to perform evaluation at points other than at the sampled points. The subintervals are summed to complete the integration.

In simulations where the function to be integrated could be explicitly determined numerical integration was foregone, replaced by exact evaluation of the integral. The program MACSYMA is used to perform the explicit integrations. The explicit functions referred to are used in the evaluation of the orthogonal integral for the set of orthonormal functions considered.

3.4.2 FFT Accuracy

A Radix Two Fast Fourier Transform (FFT) was used to go from the frequency domain to the time domain. The FFT assumes that a periodic

⁸ In the computer cancellation algorithm an echo tail of 80 baud intervals in length is used.

function is being transformed and that the function is bandlimited to $f_s/2$. The truncation of the Echo Transfer Function will yield some error in the Echo Impulse Response. This error is illustrated in Figure 3.16.

3.5 Summary

The transformer, line and filter models used to simulate the echo impulse responses were presented. The justification for each choice is developed and worked through. The results pertaining to a variety of selectable line and system models are completed. The Bellcore loop set is used as the basis for the line models. The echo responses generated are independent of coding and specific circuit configurations. The echo responses generated will be used as the echo responses to be cancelled with the orthonormal functions presented in Chapter 2.

Figure 3.15: Echo Tail Responses With Transmit Filter Using a Resistive Balancing Network : Bell Core Loops; Both Transmission Directions Considered, CO to Subscriber and Subscriber to CO.

Figure 3.16: FFT Error In Echo Impulse Response : No Pulse Shaping Filter. The circled region exemplifies the decimation error in the truncated FFT.

Chapter 4

Cancellation Results

4.1 Cancellation Measure

The measure of echo cancellation performance is presented in two forms. Tail Mean Square Error in decibels and Echo Return Loss in decibels.

4.1.1 Tail Mean Square Error

The Tail Mean Square Error is defined as

$$TMSE = \int_{\alpha}^{NT_s} \left[S(t) - \sum_{i=0}^n C_i f_i(t) \right]^2 dt \quad (4.1)$$

$S(t)$ = the echo impulse response

NT_s = echo duration

$\alpha > 0$ = time instant at which the tail of the response
is deemed to start, see Figure 4.1

n = order of the orthogonal approximation

C_i = i th coefficient of the i th approximating polynomial

$f_i(t)$ = i th approximating polynomial

This is the Mean Square residual echo in the tail portion of the response after tail cancellation.

$$\text{Tail Cancellation} = 10 \log_{10} \frac{\int_{\alpha}^{NT_s} S(t)^2 dt}{\int_{\alpha}^{NT_s} \left[S(t) - \sum_{i=0}^n C_i f_i(t) \right]^2 dt} \quad (4.2)$$

All measurements of echo tail cancellation are taken after pre filtering unless stated otherwise.

An alternative approach to measuring the Echo Canceller performance is to determine which expansion minimizes the maximum absolute error (minmax problem) in the interval. This minmax problem is valid only if the points x_i can be freely chosen[Beck73]. This is not the case for this application and thus cannot be used.

4.1.2 Echo Return Loss

Echo Return Loss is defined as

$$\text{ERL} = - \frac{\text{Residual Echo Power After Cancellation}}{\text{Actual Echo Power Before Cancellation}} \quad \text{in dB} \quad (4.3)$$

averaged over a large number of input samples. Time averaging is the same as ensemble averaging for a random ergodic process.

$$\text{ERL} = -10 \log_{10} \frac{\text{NoiseFloor} + \int_{\alpha}^{NT_s} \left[S(t)^2 - \sum_{i=0}^n C_i f_i(t) \right]^2 dt}{\int_0^{NT_s} S(t)^2 dt} \quad (4.4)$$

The noise floor is defined as the cumulative noise power of the analog circuitry in the first stage¹. This consists of the noise of each biquad section needed to reduce the echo within the first few baud intervals.

¹ Four biquads sections are assumed

The normalized echo tail power (ETP) is expressed as

$$\text{ETP} = \frac{\text{Echo Tail Power Before Cancellation}}{\text{Main Echo Power Before Cancellation} + \text{Echo Tail Power Before Cancellation}} \text{ in dB} \quad (4.5)$$

Echo Tail Power will also be expressed as a percentage of the total echo power².

In the Pre Echo Canceller echo cancellation up to the defined echo tail is done by the Front End Echo Canceller. The tail is removed via the Echo Tail Canceller.

4.2 Line Responses : Cancelled

The echo impulse response is divided into main echo pulse and echo tail. The choice of the time instant at which the echo is separated into main pulse and tail is made on the basis of minimizing integration area and conformity to the orthonormal function set. Whether a transversal filter or a RAM echo canceller is used to remove the main pulse power, the length is directly related to the number of baud intervals the cancellation is to be over. The decision point separating main echo pulse and tail is made from observation of where the orthonormal function will most closely resemble the echo tail. The possible decision points are shown in Figure 4.1 for both the Legendre and Laguerre sets. The orthonormal functions are non-zero at their starting points. A fit at positions 2 and 3 most easily conforms to the shape of the orthonormal functions. At position 3 the orthonormal function is inverted, i.e. multiplied by -1.

² See Appendix D 1

The best choices correspond to where the echo amplitude as well as the amplitude of both the Legendre and Laguerre time responses are a maximum in the absolute sense. Different points along the echo tail can be considered but invariably involve lengthening the main pulse cancellation length. This will be advantageous if cancellation is considered from a position that will result in complete removal of the echo tail.

Position 4 could also have been considered. The tail amplitude at position 4 does not change as rapidly from its initial value as opposed to the case where position 3 marks the start of the echo tail. Fewer local maxima and minima are present after the 4th position. Echo tail cancellation at position 4 however was not examined because a decision was made to limit the Front End Echo Canceller to 2 baud intervals.

4.2.1 Full Line

As an initial approximation Legendre and Laguerre cancellation was used to remove the entire echo. The results are poor, ranging from 7 to 10 dB cancellation for the different line configurations. The echo was subsequently divided into two stages, main echo and echo tail, in accordance with the observations of Figure 4.1. The main echo pulse is now assumed to be cancelled to the noise floor up to the point where the echo tail is defined.

4.2.2 Tail

Results for all the Bellcore lines (both transmission directions and various starting points, Figure 4.1, for cancellation) using Laguerre and Legendre functions are given in Tables D.1 to D.12. Results are given for the case of no balance network, Tables D.1 to D.8 and for a resistive balance network of $R_{bal} = 135$ ohms, Tables D.9 to D.12. The conclusion based on these results

is cancellation of the echo tail should be done from point 3 in Figure 4.1 (i.e. the Front End Echo Canceller should cover about 2 baud intervals).

The results in Appendix D.1 show that tail echo cancellation increases (or at least is equal) as the order increases. Echo tail cancellation to any level can be achieved by increasing the order of the particular orthonormal function. This agrees with theory and is clearly illustrated in Figure 4.2. Figure 4.2 shows the diminishing reduction in the residual MSE as the order of the Legendre orthonormal polynomial increases.

As stated at the outset a 3 bit reduction in A/D converter requirements is the goal. The Legendre 4th order and Laguerre 5th order polynomials achieve this. Polynomials of higher order also achieve a greater or at least equal cancellation. The practical limitation of area of integration suggests that for the Echo Tail Canceller the Laguerre function Echo Canceller should be chosen over the Legendre. The Laguerre scheme will require 5 filter sections while the Legendre will require 7 filter sections.

A fixed value of the time scale factor is chosen using a weighted average of the time scale factor determined over all the lines. The time scale factor values for the fifth order Laguerre, position 3, are $p = 43024$, standard deviation = 498. The average tail cancellation, i.e. the reduction in MSE resulting from the subtraction of the approximated echo impulse response from the line echo impulse response, is 18.49 dB, standard deviation = 0.27 dB³. The coefficients for the fifth order Laguerre network and a fixed time scale factor of $p = 43024$ are shown in Table 4.1. These are optimized in the mean square sense using an iterative computer program developed explicitly for this purpose.

³ Line 31 was not used in determining the average value because it was thought to incorrectly bias the results. The results using line 31 are $p = 42620$, standard deviation = 2267. The average tail cancellation is 18.26 dB, standard deviation = 1.30 dB

Figure 4.1: Partitioning Of The Echo Impulse Response Into Main Echo and Tail. The points 1, 2, 3, 4 correspond to sample 8, 17, 26 and 35 respectively.

Figure 4.2: Echo Cancellation Versus Legendre Orthonormal Function Order: Time Scale factor is different for each case. Bellcore lines are considered.

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2

Micred

The time scale factor values for the fourth order Legendre, position 3, are $p = 22745$, standard deviation = 341. The average tail cancellation is 18.10 dB, standard deviation = 0.47 dB⁴.

The cancellation performance for a U-interface containing a resistive balance network of $R_{bal} = 135$ ohms, using position 3 to distinguish main echo and echo tail, is done for both the Laguerre and Legendre functions. The results are in Tables D.9 to D.12. The time scale factor values for the fifth order Laguerre, position 3, are $p = 38717$, standard deviation = 583. The average tail cancellation is 20.53 dB, standard deviation = 0.36 dB⁵. The values for the fourth order Legendre, position 3, are $p = 22745$, standard deviation = 341⁶.

If cancellation to the noise floor within the first two baud intervals is done then an ERL of 16.65 dB is achieved in the Pre Echo Canceller for the fifth order Laguerre network with no resistive balance network. An ERL of 16.42 dB, 18.69 dB and 18.64 dB for the fourth order Legendre network with no resistive balance network, the fifth order Laguerre network with $R_{bal} = 135$ ohms and the fourth order Legendre network with $R_{bal} = 135$ ohms respectively are obtained.

The noise calculations are based on the total rms noise voltage from a simple switched capacitor biquad lowpass filter [Kwan88]. The total rms noise voltage is 0.267 mV. A switched capacitor transversal filter will have a similar rms noise voltage. Additional noise from other circuitry will give a higher noise value. Correlated noise sources will also increase the noise voltage estimate. The noise from each filter (representing a tap) is assumed

⁴ The results using line 31 are $p = 22490$, standard deviation = 1435. The average tail cancellation is 18.09 dB, standard deviation = 1.77 dB.

⁵ The results using line 31 are $p = 38363$, standard deviation = 2021. The average tail cancellation is 20.48 dB, standard deviation = 1.40 dB.

⁶ The results using line 31 are $p = 22490$, standard deviation = 1435.

to be uncorrelated and add in a linear fashion over the 16 samples of the Front End Echo Canceller. The echo tail, echo tail approximation using Laguerre functions and the residual echo after subtraction are shown in Figures 4.3, 4.4 and 4.5 for various line conditions. Similar results are given for Legendre functions in Figures 4.6, 4.7 and 4.8. Both sets of graphical responses are for cancellation from point 3 in Figure 4.1. The tail length is effectively reduced by 280 samples or approximately 35 baud intervals with no amplitude above ± 0.95 mV within these 35 baud intervals. The residual tail amplitude is however more oscillatory in behavior.

4.3 Use Of Orthonormal Functions

4.3.1 Choice Of Pole Location

Essential to the performance of the Pre Echo Canceller is the selection of the time scale factor, p , in equations 2.28 and 2.30. A theoretical analysis of the optimality conditions is given in [KinC69] for Laguerre functions.

The criteria for the optimum pole position value which minimizes the mean square error condition is ⁷.

$$\frac{d}{dp} \sum_0^N C_n^2(p) = \sum_0^N 2C_n(p) \frac{d}{dp} C_n(p) = 0. \quad (4.6)$$

Clowes [Clow65] has shown that the optimum value is one of the roots of

$$C_N(p) = 0 \quad \text{and} \quad C_{N+1}(p) = 0, \quad (4.7)$$

where

$$C_n(p) = \int_0^\infty S(x) f_n(x) dx$$

⁷ C_n has been shown previously in Chapter 2 to be a function of the time scale factor, p

Section n	Coefficients for p average	Coefficients $+3\sigma$ from p average	Coefficients -3σ from p average
0	4.451152e-07	-1.813529e-06	2.881614e-06
1	1.350944e-04	1.338970e-04	1.363036e-04
2	3.488221e-05	3.560761e-05	3.399991e-05
3	7.475305e-05	7.584255e-05	7.347738e-05
4	9.185595e-06	1.126762e-05	6.985085e-06
5	3.514248e-05	3.576502e-05	3.444205e-05

Table 4.1: Laguerre Coefficients : Fifth Order, $p = 43024$. Line 01.

Figure 4.3: Fifth Order Laguerre Cancellation Results For A Single Gauge No Bridge Tap Line. Time Scale Factor $p = 43024$.

Figure 4.4: Fifth Order Laguerre Cancellation Results For A Single Gauge Single Bridge Tap Line. Time Scale Factor $p = 43024$.

Figure 4.5: Fifth Order Laguerre Cancellation Results For A Multiple Gauge Multiple Bridge Tap Line. Time Scale Factor $p = 43024$.

Figure 4.6: Fourth Order Legendre Cancellation Results For A Single Gauge No Bridge Tap Line. Time Scale Factor $p = 22745$.

Figure 4.7: Fourth Order Legendre Cancellation Results For A Single Gauge Single Bridge Tap Line. Time Scale Factor $p = 22745$.

and $C_n(p)$ has been normalized. $S(x)$ is the signal to be operated upon.

The solution of eq(4.7) is tedious as $C_n(p)$ will be a polynomial of at least degree $(N + K)$ for a transfer function of K simple poles. An analytical expression is required for eq(4.7) which is not often available. In [Sche71] the asymptotic time scale factor, $p_\infty = \lim_{n \rightarrow \infty} p_n$, is found.

Parks has shown that two simple measurements are all that is needed to determine the optimum time scale factor for Laguerre polynomials [Park71], the measure of how rapidly the signal decays

$$M_1 \triangleq \frac{\int_0^\infty t S(t)^2 dt}{\int_0^\infty S(t)^2 dt} \quad (4.8)$$

and the measure of how smoothly the signal f decays.

$$M_2 \triangleq \frac{\int_0^\infty t \dot{S}(t)^2 dt}{\int_0^\infty S(t)^2 dt}. \quad (4.9)$$

The choice of p is then

$$p = \left(\frac{M_2}{M_1} \right)^{1/2} \quad (4.10)$$

Two methods are utilized in this thesis to determine the appropriate time scale factor, p . Minimization of the mean square error and maintaining the definition of orthonormality. The latter condition is essential to achieve the former. In the general case the time scale factor is chosen with the objective of obtaining the best approximation with the smallest number of terms.

As stated earlier the choice of where to divide echo main pulse and echo tail is subjective, depending on implementation constraints and performance. For orthonormal functions of the type used here performance is dependent upon the choice of the time scale factor, or stated another way the performance is dependent upon the length of the echo tail to be cancelled. Consider

the exponential term $e^{-p\tau}$ which is in both the Legendre and Laguerre functions.

$$\text{Echo Envelope Length} \propto p\tau = pmT_s, \quad (4.11)$$

where T_s is $1/(80 \text{ kbaud})$, the baud rate for a 2B1Q code. The time scale factor lengthens the decay time τ by the factor p .

4.3.2 Sensitivity Of System To Pole Location

Of practical interest is the sensitivity of cancellation to a variation in the time scale factor. Capacitor sizing or finite word length determination can be affected by the choice of pole location. For simplicity a practical realization would use a fixed value of the time scale factor to avoid the complication of a second adaptive parameter. Figure 4.9 shows the variation from the average value of the time scale factor p in the range $\pm 3\sigma$, where σ is the standard deviation. The average value of the time scale factor is $p = 43024$, chosen from the results using the fourth order Laguerre function. Line 26 is the line considered in Figure 4.9.

4.4 Comparison Of Orthonormal Functions With Other Cancellation Methods

Initial analysis would seem to indicate that the echo tail, with its low frequency components, can be removed via highpass filtering. Figure 4.10 shows the result of highpass filtering the echo impulse response of line 01. The highpass filter is a second order Butterworth filter with a corner frequency of 7.4 kHz. The corner frequency was determined by linear interpolation of results presented in [MMBP86]. The highpass filter removes the low frequency energy of the far-end transmit pulse. In addition the high frequency spectrum

Figure 4.8: Fourth Order Legendre Cancellation Results For A Multiple Gauge Multiple Bridge Tap Line. Time Scale Factor $p = 22745$.

Figure 4.9: Variation From The Optimum MSE Time Scale Factor For A Fifth Order Laguerre Function Used For Echo Cancellation. The echo impulse response is cancelled for time scale factors $\pm 3\sigma = 1494$ from the average time scale factor.. Each value indicates 1 standard deviation

will remain thus compromising the performance of any equalizer later in the signal path. This high frequency distortion is evident in Figure 4.10. A non-trivial tradeoff must be made between highpass filtering and equalization. It is not necessarily the case that the *best* equalizer for the echo signal is also the *best* equalizer for the far-end signal. The use of a simple highpass filter to remove the echo tail is an over simplification to recovering the far-end signal embedded in echo and noise.

A (1-D) filter has been used in both [ACCC86] and [FTGH88] to reduce the tail length. A first order (1-D) filter is used on the echo impulse response of line 03. The residual is shown in Figure 4.11. The echo impulse response is considerably attenuated and shorted. A problem associated with a (1-D) filter is the noise enhancement that results in the presence of near-end crosstalk.

A fixed $\sqrt{f}$ equalizer has also been considered as a solution to echo tail reduction. The hesitation with using a fixed equalizer is its effect on adaptation. The adaptation algorithm complexity and additional circuitry required to implement this equalizer eliminate it from consideration as a plausible solution here. Despite these misgivings the $\sqrt{f}$ equalizer approach has produced good results [Taka86].

A brute force approach can be done by simply increasing the tap length (address space) for the transversal (RAM) filter considered. To cancel an echo tail of 40 baud intervals sampled at eight times the baud rate requires 640 op amps (a switched capacitor design requires two op amps per sample). A RAM approach requires $4^{40} \times 8$ storage addresses. Clearly the increase in area does not justify the exclusive use of either method.

Figure 4.10: Echo Response HighPass Filtered With A 2nd Order Butterworth Filter. The solid line is the echo impulse response and the dashed line is the highpass filtered echo impulse response.

Figure 4.11: Echo Response 1 - D Filtered. Line 01 With Transmit Filter. The solid line is the echo impulse response. The dashed line is the same echo impulse responses 1 - D filtered.

4.5 Adaptation

Convergence time is the usual figure of merit when comparing adaptive filter schemes. A determination of whether or not stable convergence can be achieved using the orthonormal Legendre and Laguerre system is completed.

Adaptation is done according to the LMS algorithm.

$$\underline{c}(n+1) = \underline{c}(n) + \frac{\mu}{2} [-\underline{\nabla}(n)] \quad (4.12)$$

$\underline{c}(n+1)$ and $\underline{c}(n)$ are the coefficient vectors of the $(n+1)$ th and n th iteration respectively. Filter outputs $\underline{u}^T(n)\underline{c}(n)$ and tap coefficient updates take place at T/M second intervals where T is the baud interval, and M is an integer ≥ 1 ; e.g. $M = 8$. μ is the adaptation size or amplification factor and $\underline{\nabla}(n)$ is the gradient vector of the squared error signal for the n th iteration. The gradient vector $\underline{\nabla}(n)$ is

$$\underline{\nabla}(n) = -2e(n)\underline{u}(n), \quad (4.13)$$

where $e(n)$ is the sampled error signal at the n th iteration

$$e(n) = d(n) - \underline{u}^T(n)\underline{c}(n), \quad (4.14)$$

and $\underline{u}^T(n)$ is the transpose of the tap input vector. $d(n)$ is the received signal vector.

$$d(n) = \underline{a}^T(n)\underline{g}(n) + \nu_n, \quad (4.15)$$

where ν_n is noise, echo beyond the cancellers span, and far-end signal. $\underline{g}(n)$ is the sampled echo channel impulse response at the n th iteration.

Taking the expectation and substituting eq(4.13) and eq(4.14) into eq(4.12)

$$\langle \underline{c}(n+1) \rangle = \langle \underline{c}(n) \rangle + \mu \langle \underline{d}(n)\underline{u}(n) \rangle - \mu \langle \underline{u}(n)\underline{u}^T(n)\underline{c}(n) \rangle. \quad (4.16)$$

Simplifying,

$$\langle \underline{c}(n+1) \rangle = \langle \underline{c}(n) \rangle (\mathbf{I} - \mu \mathbf{R}(n)) - \mu \mathbf{P}. \quad (4.17)$$

The matrix $\mathbf{R}(\mathbf{n})$ corresponds to the correlation matrix for the set and $\mathbf{P}$ is the cross correlation vector. $\mathbf{I}$ is the identity matrix.

The output vector for an 8 sample per baud sampling rate is

$$\underline{u}^T(n)\underline{c}(n) = \sum_k c_k(n) \sum_k a_k f_l \left(\left(\frac{n-l}{8} \right) T - kT \right), \quad (4.18)$$

where $f_l(t)$ is the l th orthogonal filter response. The input data symbol a_k is random and assumed uncorrelated. In the correlation matrix $\mathbf{R}(\mathbf{n})$, (e.g. $\mathbf{R}(\mathbf{n}) = E [\underline{u}^T(n)\underline{u}(n)]$) the expectation $E[a^2]$ is a constant. Only the tap function output vector and its transpose need be considered. Each element in $E [\underline{u}^T(n)\underline{u}(n)]$ is

$$\mathbf{R}_{l,m}(n) = \sum_k f_l \left(\left(\frac{n-l}{8} \right) T - kT \right) f_m \left(\left(\frac{n-m}{8} \right) T - kT \right). \quad (4.19)$$

l, m correspond to the l th and m th order equation in the orthonormal function set. $n \bmod T$ is the phase shift. The summation in eq(4.19) is over 80 baud intervals (length of echo tail needed to maintain orthogonality).

The eigenvalues of the correlation matrix illustrate the convergence capabilities. Ideally the diagonal matrix of eigenvalues will be unity. An eigenvalue near zero will imply a very long convergence time. The diagonal elements not including the main diagonal should be zero. The eigenvalues for the 7th order Legendre and 7th order Laguerre polynomials are given in Appendix D.2. The average time scale factor is used for the Laguerre set and the Legendre set of polynomials. The qualitative results show that reasonable convergence and settling times can be expected using the Laguerre orthonormal networks, although one eigenvalue is much smaller than the rest. The Legendre orthonormal network may result however in unreasonably long convergence times since one eigenvalue is zero. The convergence is independent of the phase of the signal as indicated by the eigenvalues of each phase having similar values.

Korn [Korn84] has examined the adaptation of an IIR structure similar to that proposed here, however in Korn's work the pole location is adaptively determined. It is fixed here.

4.6 Summary

Results are presented for echo tail cancellation using Legendre and Laguerre orthonormal systems. The echo is divided into echo main pulse and echo tail, divided at the points of similarity between the echo impulse response and the orthonormal system. Legendre and Laguerre responses are more applicable to cancellation of the echo tail. The choice of an appropriate time scale factor is essential to the cancellation performance. A qualitative analysis of the convergence properties of the Laguerre orthonormal set shows that reasonable convergence times are expected.

Laguerre function echo tail cancellation performs as well as a Legendre type echo tail cancellation but with $n + 1$ less filter sections. For this reason it is a more practical solution. A fifth order Laguerre system is chosen as a compromise between area and performance. The tail is reduced by 17.5 to 18.7 dB for all the Bellcore lines⁸, considering transmission in both directions.

⁸ Excluding Line 31.

Chapter 5

Implementation Considerations

5.1 System Requirements

The overall system to be implemented is shown in Figure 5.1. The area of concern is enclosed in dashed lines.

5.1.1 Requirements : Excluding Echo Canceller

A continuous anti-alias filter is required to bandlimit the received signal. It is desirable that the high frequency noise greater than 500 kHz be eliminated while aliasing at the symbol rate, 80 kHz, be reduced. The reduction of the latter may or may not be desirable at this stage depending on the implementation approach. Specific filter requirements are not outlined in the T1D1 recommendations [T1D1]. An integrated design for an ISDN transceiver has been reported in [AgaA88]. The choice of whether to use an active or passive antialias filter is dependent on area of integration.

Independence of results has been a primary goal of this thesis. However

a modest compromise is the inclusion of a purely resistive balance network in the hybrid. The complexity of impedance matching over the ISDN bandwidth does not warrant using anything other than a resistive off-chip balance network. The merits of the resistive balance network were given previously in Figure 3.13 and the results of section 4.2.2.

A single op-amp sample and hold (S/H) circuit is used. Figure 5.2 illustrates the S/H. When ϕ_1 is closed V_{in} is connected to C_H and the unity gain buffer. When the switch opens at t_0 , $V_{in}(t_0)$ is held on C_H and is available at V_{out} . The estimated silicon area of the S/H is 0.003 mm^2 in 3μ CMOS¹.

After pre-cancellation and prior to echo cancellation by the second stage (Main Echo Canceller), an Adaptive Gain Controller (AGC) is necessary. Before pre-cancellation the far end signal is buried in echo. After initial echo cancellation by the Pre Echo Canceller the far end signal will vary according to the line attenuation. A level adjusting AGC is required so the full resolution of the A/D converter can be utilized regardless of the receive signal level after pre echo cancellation. The output sample rate of the Pre Echo Canceller should be at a rate of greater than or equal to the symbol rate.

An interpolation filter is necessary for pulse shaping after the Pre Echo Canceller. Figure 5.3 illustrates the need for the interpolation filter. The filter spectrum is multiplied by the S/H spectrum, a sinc function. The pulse shaping filter removes the side lobes replicated at $k f_c$ resulting from the introduction of spectral components of the sample and hold operation.

¹ Area estimates are based on standard cells. An analog switched capacitor circuit may include op-amps, a bias circuit, clock driver, transmission gates, switches and capacitors. The exact circuit designed determines which cells are used and the area estimate is done based on the appropriate cells. A 1 pF capacitor is used as the reference capacitor.

Figure 5.2: Sample And Hold Circuit

Figure 5.3: Interpolation Filter Necessity. Solid lines illustrate the signal prior to A/D conversion if no interpolation filter is used. Dashed lines illustrate the signal prior to A/D conversion if an interpolation filter is used. Note : Figure is not to scale.

The interpolation filter² is shown in Figure 5.4. The silicon area of the interpolation filter is 0.162 mm^2 in 3μ CMOS.

5.1.2 Requirements : Main Echo Canceller

In the two stage design considered a RAM architecture is assumed for the Main Canceller. The choice of a RAM canceller versus a more exotic all encompassing design [ACCC86] is done on the grounds of reducing design complexity and area of integration. Minimization of both is essential. A RAM Echo Canceller alone could not be practically large enough to cancel the entire echo including the echo tail. However if the Pre Echo Canceller shortens the echo tail by 35 baud intervals³ the RAM Canceller length can be shortened. The RAM length need only cover 7 baud intervals. With 8 samples/baud the address space required is $4^7 \times 8$. This is still however large although not unreasonable.

A DFE could be combined with the RAM canceller in this section if so desired.

5.2 Pre Echo Canceller

The Front End Echo Canceller will cover 2 baud intervals. The orthonormal network is best suited to cancellation of the echo from the 26th sample or approximately the 2nd baud position. This location has been determined via simulations and the results were presented in Chapter 4. About 97% of the echo power must be removed via a front end cancellation filter (the Front End Echo Canceller) for the first 2 baud intervals with no resistive balance

² The AGC and interpolation filter may be combined [TSFM84].

³ See Chapter 4 and results of Appendix D.1.

network. This is a different network from the orthonormal network proposed to remove the echo tail (the Echo Tail Canceller).

5.2.1 Front End Echo Canceller

Two possible alternatives for implementations exist for reducing the echo within the first 2 baud intervals. A RAM canceller using digital means for adaptation with a D/A converter to allow for cancellation in the analog domain is one alternative. The other possibility is an analog scheme for both cancellation and adaptation. Switched capacitor circuitry is likely to be used.

A digital approach similar to [TKSK87] could be used in the Front End Echo Canceller. A $4^2 \times 8 \times 12$ bit RAM, 9 bit D/A converter and a 1-6 bit A/D converter, depending on the application (adaptation algorithm) is required. These specifications provide sufficient accuracy. Insufficient accuracy of the A/D and D/A converter will cause aliasing [VESG79]. An advantage of the digital approach is that the coefficient values can be stored. For a warm start this is beneficial.

The area requirements for an analog switched capacitor transversal filter and analog multipliers for adaptation can be considerable. 16 op-amps would be required for an analog switched capacitor transversal filter. This is assuming 8 samples/baud. The area required is approximately 1.3 mm^2 in 3μ CMOS. An n-path switched capacitor filter was initially considered however the small dynamic range inherent in this sort of realization would prevent it being chosen.

5.2.2 Echo Tail Canceller

The Echo Tail Canceller is to be an analog system. The likely choice is a switched capacitor filter network. The large value of the time scale factor

needed for both the Legendre and Laguerre networks presents a problem in implementation. Consider the integrator circuit of Figure 5.5 ⁴.

The transfer function of the analog circuit is

$$H_a(s)|_{s=j\omega_a} = \frac{V_{out}}{V_{in}} = -\frac{1/R_1 C_2}{j\omega_a} = -\frac{1/R_1 C_2}{j2\pi f_a} \quad (5.1)$$

The transfer function of the sampled analog switched capacitor filter is

$$H(z) = -\left. \frac{C_1/C_2}{z-1} \right|_{z=e^{j\omega T}} = -\frac{C_1/C_2}{e^{j\omega T}-1} \quad (5.2)$$

or using a Taylor series

$$H(e^{j\omega T}) = -\frac{C_1/(C_2 T)}{j\omega - \omega^2 T/2 - + \dots} \quad (5.3)$$

The two are equivalent if $\omega T \ll 1$ and $C_1 = T/R_1$.

The operation of the switched capacitor integrator is briefly as follows; ϕ_1 is switched on and C_1 rises to a voltage of $V_{in}(t_n)$ (C_1 was previously discharged); ϕ_1 is switched off. C_1 was charged to $C_1 V_{in}(t_n)$, where V_{in} is a function of the switch time t ; ϕ_2 is turned on⁵ at time $t_n + T/2$. T is the clock period. V_{C_1} discharges through ground and the charge across C_2 increases by $C_1 V_{in}(t_n)$. The voltage across C_2 changes by $(C_1/C_2) V_{in}(t_n)$; The change in output voltage is

$$V_{out}(t_{n+1}) - V_{out}(t_n) = -\Delta V_{C_2}(t_n + T/2) = -\frac{C_1}{C_2} V_{in}(t_n) \quad (5.4)$$

The time constant of this integrator is

$$\tau = \frac{C_2}{C_1} \frac{1}{f_s} \quad (5.5)$$

⁴ In a practical realization stray insensitive circuits would be used however Figure 5.5 suffices to illustrate the concept.

⁵ Note that ϕ_1 and ϕ_2 are not necessarily non overlapping.

Figure 5.4: Interpolation Filter. [GreT86].

Figure 5.5: Switched Capacitor Integrator

$f_s = 1/T$, the sampling rate.

In order to integrate the Legendre and Laguerre networks, and their associated large valued time constants, onto silicon four possible alternatives or combinations thereof are appropriate;

1. Ratioed capacitors can be used where the capacitor ratio is the value of the time scale factor.
2. Digital counting can be done to reduce the oversampling rate. The switching rate is reduced by $1/x$, resulting in a time constant x times greater. This is effectively sub switching rate clocking of the output. A count of the switching clock is made and the output is switched after a pre determined count has been reached.
3. Less widely varying ratioed capacitors can be used and subsequently be multiplied together via charge transfer.
4. Combining the first and second alternatives to achieve a time scale factor equivalent.

A capacitor ratio of 43024:1 is difficult to accurately obtain in a monolithic design. Although accuracies of 0.1% are obtainable [GraC85] for ratioed capacitors where the capacitor ratio is small, accuracy cannot be maintained over a large capacitor range. Geometric error will become noticeable. A small *unit* capacitance is desirable for the adaptation algorithm requirements. For a *unit* capacitance of 1 pF, the equivalent area of 43025 capacitors are required to achieve a time scale factor of 43024. This uses approximately 200 mm^2 for a $3\mu m$ CMOS process.

Digital counting can be done to increase the effective time constant, decreasing the switched capacitor switching rate within the passband. The output stage is isolated and clocked at the reduced rate. A reduction of $1/x$

in the switching rate, where x is the time scale factor in the Legendre and Laguerre polynomials, will introduce spectral components at $(1/x)f_s$. These must be eliminated via a bandstop filter. The digital circuitry required to monitor the switching interval will increase the area requirements. Reduction in the switched capacitor sampling rate can lead to charge leakage of the capacitors at nodes of interest. Arithmetic inaccuracies will arise. Problems will also arise as the sampling rate approaches baseband and aliasing occurs. Reduced rate clocking is limited by the Nyquist criteria. Reduced clocking is thus viable only if the highest frequency of interest in the filter is of the same order of magnitude as the filter poles [SanV84].

Switched capacitor circuits have been built to realize large time constants. An effective capacitance ratio of 700 has been realized while the largest physical capacitor ratio was only 25 [SanV84]. A 60 Hz switched capacitor notch filter using a clock frequency of 128 kHz was implemented similarly [Naga88]. The time constant of the Laguerre orthonormal network needed to reduce the echo tail corresponds to approximately 21.5 Hz from equation 4.11.

Capacitance ratio magnification can be achieved by attenuating the input to the integrator by the use of a preceding capacitor network [VisV80]. This preceding network will use smaller ratios of capacitors but will comprise of more than a single capacitor ratio. A large time constant is built up using capacitor ratios which are proportional to the products and/or higher powers of the smaller physical capacitor ratios. The T-cell network is one approach [VanS83][SanV84]. These networks are however parasitic sensitive. The same effect can be produced by decreasing the charge transfer within a switching sub interval [YaTM82]. Offset voltage accumulation and noise problems are inherent to this design. Scaling up the integrating capacitor cannot be done without limiting dynamic range and worsening the slew rate of the integrator.

The fourth alternative is effectively a compromise between the first two

methods. The time constant required is of the order of $n \times 10^4$, where n is an integer. The ratio of $C_2 : C_1$ can be made as large as 100 without geometric inaccuracies becoming prevalent. The duration of ϕ_2 , from Figure 5.5, must be reduced by $n \times 10^4$ to achieve the appropriate time constant.

To achieve a $n \times 10^2$ reduction a digital counting scheme must be added to count $n \times 10^2$. An 8 bit counter is sufficient for this realization. An unwanted spectral component at $(1/(n \times 10^2))f_s$ will corrupt the signal. A bandstop filter will have to be added to remove this component. The bandstop filter is added following the orthonormal network and can also be of the switched capacitor type.

The lower bound on the duration of ϕ_2 is the capacitor leakage of the circuit although it is desirable to have the output switch at a rate at least equal to the symbol rate. For a simple integrator this is of no great consequence however when a biquad structure is used (the fifth order Laguerre implementation can be implemented using biquads) other difficulties will become present. Second order systems can be made more stray-insensitive [BrGM81] and have better dynamic range [Greg80] than single integrators however implementations of long time constants affect the lossy capacitor C_1'' in Figure 5.6. C_1'' will have to be added to realize the allpass network (transmission zeroes outside the unit circle) needed in the Laguerre implementation. The fact that this is a low Q circuit will preclude the use of small capacitor ratios especially for C_4 . A high Q circuit is recommended however because of the high frequency selectivity required. Increasing the time constant via reducing the switching rate of ϕ_2 will also result in considerable leakage through C_4 .

A fifth order Laguerre implementation would require 3 biquads of the type shown in Figure 5.6. The estimated area not including the bandstop filter,

and circuitry associated with it, is 42 mm^2 in 3μ CMOS.⁶ This estimate does not include the digital coefficient adaptation circuitry.

The sample values from the Front End Echo Canceller can be directly input to the Echo Tail Canceller (orthonormal section) without the need of an explicit delay element. If however an implementation was only to remove the echo tail within the first stage a delay is required for the input to the Echo Tail Canceller.

A Laguerre function transmission system has been reported in [Geor73] using continuous filter analysis and a discrete time implementation. Speech transmission is examined using this approach.

5.3 Adaptation

Both the Pre Echo Canceller and the Main Echo Canceller are adaptive. The Pre Echo Canceller will adapt first, followed by adaptation of the Main Echo Canceller. Adaptation of the Pre Echo Canceller will be continuous. To combat spurious signals on the line the Pre Echo Canceller will not be fixed. Spurious signals can be caused by temperature variations, electromagnetic surges and crosstalk by numerous and different disturbers.

Main Echo Canceller adaptation is started after n iterations of the Pre Echo Canceller. The value of n can be determined via simulation. The number of iterations necessary has not been determined here. Main Echo Canceller adaptation will also be continuous.

The stable coefficients for the Laguerre orthonormal network were given in Table 4.1 for cancellation of Line 01. The three sets of coefficients are for the average time scale factor over all lines and the coefficients for $+3\sigma$ and -3σ from the average time scale factor. The range of these coefficients

⁶ The area estimate is based on a standard cell approach

is used to determine the resolution requirements of the coefficients and the amplification factor. The lowest coefficient value is $4.151152\text{e-}07$. The largest coefficient value is $1.350944\text{e-}04$.

The Pre Echo Canceller is an analog system. Multiplication and summation will be done in the analog domain. The adaptation coefficients are however represented digitally. This choice is made because of the large coefficient values that need to be accurately represented. Digital representation also allows for easy coefficient storage. D/A conversion will be done on the coefficients to permit analog multiplication. MDACs can be used if the coefficient length is not large. Quantization at the output will allow A/D conversion for updating the coefficients. Binary ratioed capacitors can be used to represent the digital coefficients or simple registers can be used. Figure 5.7 illustrates the adaptation algorithm.

Each coefficient is represented by at least 3 bits unsigned arithmetic (i.e. 1 bit sign and 3 bits magnitude). This value was chosen such that quantization would not compromise the results. The amplification factor ⁷ required is $\mu = 2^{-24}$. This length (24 bits) is longer than other adaptive implementations [SzZS86]. An amplification factor of 24 bits will require quantization resolution of 8 bits (1 bit sign and 7 bits magnitude) for the coefficient range considered. Each coefficient will require 13 bits (1 bit sign and 12 bits magnitude) for representation.

The circuitry required for such long word lengths will significantly add to silicon area and complexity. The LMS adaptation algorithm (not using the sign algorithm) requires 2 multipliers and a summer. A multiplying digital to analog converter (MDAC) can only, at best, be used for upto 10 bits of resolution. Thus an A/D and D/A are required if analog circuitry are used. Implementation area is significantly increased using an adaptive Laguerre

⁷ See equation 4.12

network.

Both the orthonormal network taps and the main pulse cancelling network taps will have the same adaptation circuitry. No need for different adaptation schemes has been identified for the Front End Echo Canceller versus the Echo Tail Canceller.

5.4 Discrete Implementation Of Orthonormal Functions

Discrete implementation of the continuous orthonormal functions will facilitate the change from integration to summation. However a direct transformation will destroy the property of orthogonality.

$$\int_0^\infty V_i(t)V_j(t)dt = \delta_{ij} \quad (5.6)$$

If $V_i(t)$ is sampled at a rate of kT/M .

$$\text{i.e. } V_i(t) \rightarrow V_i\left(\frac{kT}{M}\right)$$

then

$$\sum_0^k V_i\left(\frac{kT}{M}\right) V_j\left(\frac{kT}{M}\right) dt \neq \delta_{ij} \quad (5.7)$$

This can be proved by using a modified invariant impulse method or realizing that orthonormal exponentials have non zero spectra extending to infinity [YouH62].

The discrete equivalent of the Laguerre orthonormal system which preserves the orthonormal property can also be derived from the Gram-Schmitt orthogonalization theorem. If

$$L_0(t) = e^{-pt} \rightarrow L_0(kT) = e^{-kpT}, \quad (5.8)$$

then the z domain equivalent is

$$\mathcal{Z} \{ \{ L_0(kT) \} \} = \frac{1}{1 - e^{-pT} z^{-1}}. \quad (5.9)$$

Similarly if

$$L_1(t) = (t - \alpha_1) L_0(t) \rightarrow L_1(kT) = (kT - \alpha_1) e^{-kpT} \quad (5.10)$$

then

$$\mathcal{Z} \{ \{ L_1(kT) \} \} = T \frac{1 - \left(\frac{\alpha_1}{T} + 1 \right) (1 - e^{-pT} z^{-1})}{(1 - e^{-pT} z^{-1})^2}. \quad (5.11)$$

This derivation is completed in Appendix E. An approach similar to this is completed in [YouH62] for the derivation of discrete orthonormal exponentials.

The value of the discrete approach is that it shows that the Laguerre orthonormal system can be implemented in a sampled analog system or if quantized correctly in a digital system. The discrete orthonormal network could extend to use in a single stage digital echo canceller, such as the Main Echo Canceller shown in Figure 5.1. The implementation considerations however must be looked at. These would include finite word length effects, word length, multiplications required, etc.

5.5 Summary

A circuit realization using Laguerre orthonormal functions is investigated. The first stage and any necessary function blocks required upto but not including the second stage are examined. The amount of circuitry required for the implementation described is prohibitively large for present die sizes. This is a direct result of the large time constants required. In addition the accuracy requirements resulting from this scheme are strict.

It is well recognized that a fractionally spaced equalizer approach (i.e. more than one sampling point per baud) will aid in timing recovery and jitter compensation [Falc82][Qure85]. The cost of doing this is an increase in area directly proportional to the sampling rate. A reasonable tradeoff is to sample twice per baud. Sampling at eight samples per baud is not prudent for a practical design. Baud rate sampling is an alternative although it presents difficult challenges [VESG79].

Discrete orthonormal equivalents can be derived from the continuous orthonormal functions. This may hold promise in a sampled analog and/or digital design.

Chapter 6

Single Stage Versus Two Stage Cancellation

The justification for the two stage approach is developed through comparison with the single stage approach. The difficulties associated with each are examined and evaluated. The two stage approach is examined further in light of the results of Chapter 4 and Chapter 5. A number of points are subjected to comparison, both in terms of performance and implementation. The first stage is in the analog domain and the second stage is in the digital domain. Two stage cancellation is shown in Figure 6.1. Single stage cancellation is shown in Figure 6.2.

The signals at A are the same for both architectures. Both A/D converters are subjected to the full dynamic range of the incoming signal at B, however both A/D converters are not required to operate with the same resolution and on signals of the same composition.

Figure 6.1: Two Stage Cancellation

Figure 6.2. Single Stage Echo Cancellation Technique.

6.1 Bandwidth

The first consideration is bandwidth. Technology and circuit design techniques support high bandwidth operational amplifiers. Their presence in oversampled A/D converters proves this. If the higher capacity services that are asked for are to be made available wider bandwidth amplifiers capable of use in high sampling rate circuits must be obtainable. The difficulties associated with implementing these amplifiers will present challenges.

In a single stage digital approach the A/D converter is the bottleneck. Once in the digital domain any required arithmetic operations are simple to implement. Resolution, as defined by the A/D converter, is the limiting factor. The A/D converter is subject to the full range of the analog input data signal plus echo plus noise.

In the two stage approach the analog circuitry required to cancel the echo impulse response up to the first few baud intervals is subject to the full range of the analog input data signal plus echo plus noise. The amplifier performance requirements are the same as those required for the single stage A/D converter. The second stage is digital and its performance can be easily predicted.

6.2 Coefficient Storage Implementation

Digital numerical implementations are commonplace and simple to implement. Commercial vendors have developed signal processing chips that have internal 40 bit floating point word lengths.

The difficulty with a two stage approach lies in the accurate analog representation of numerical values. Geometric inaccuracies are low for moderate ratioed capacitor implementations. A wider range representation using

capacitors is not yet mature enough to give accurate results. Geometric inaccuracies lead to numerical inaccuracies. The result is poor accuracy.

6.3 Multiplications And Resolution Requirements

The single stage scheme requires only digital multiplications. n bit multiplication results in a $2n$ bit product. Large word lengths can be accommodated. If a fixed point representation is used values may need to be scaled. Quantization is significant in the updating process required for adaptation. Floating point solutions could be done but do not seem likely as of yet.

Ideally the first stage will perform a *coarse* cancellation with minimum resolution. This will allow for faster convergence over the case requiring higher resolution. The second stage can thus be a higher accuracy Echo Canceller, with a longer adaptation time. The limitation is the number of A/D bits of resolution. The signal resolution cannot be extended further and this limitation will reduce the performance of the system.

A two stage scheme may require both analog and digital multiplication. The first stage multiplication can be done either in analog or digital. Analog multipliers, MDAC's, have only been implemented with 8-10 bits of resolution. The updating process requires a quantizer. This quantizer or comparator needs to be small in area thus only a 2-3 level quantizer can be considered. 2-3 bits is likely to be sufficient for quantizing the error signal for adaptation¹. The 8-10 bits required for producing output of the coarse Pre Echo Canceller is the lower limit of what is required and may be insufficient. Internally 16-20 bits is required for LMS adaptation. Analog multiplications and operations

¹ The sign algorithm could be considered as long as the reduced accuracy will not appreciably degrade performance. Convergence times may also be prohibitively large.

are extremely layout sensitive.

The multiplications in the first stage can be done in digital form but this will require an A/D and a D/A converter. The area of the converters is a concern in an integrated implementation. One of the main reasons for exploring a two stage approach is to avoid the use of these converters.

The number of bits of resolution required in a two stage approach may be quite different for the first stage versus the requirements of the second stage. This fact will complicate the layout and system requirements.

6.4 Noise

Analog circuitry is much more layout sensitive to noise. The first stage canceller in a two stage Echo Canceller design will require analog components. An A/D is required in both the single stage scheme and the two stage scheme. The analog circuitry additively introduces noise.

Figure 6.1 and Figure 6.2 show those components which contribute to noise in both the two stage and single stage schemes. The primary noise sources for the analog circuits will be the low frequency $1/f$ noise and the high frequency thermal noise of the switches and op-amps used.

The line driver linearity is especially critical in both cases. The anti-alias filter must remove sufficient out of band noise in order that proper performance be maintained. The choice of anti-alias filter is critical.

6.5 Summary

This chapter has provided a brief look at the factors which must be taken into account when examining a two stage versus a single stage approach. A qualitative analysis indicates that a two stage approach is only viable when

accurate analog arithmetic operations can be performed over a wide dynamic range.

Chapter 7

Conclusions

7.1 Two Stage

At the present it is arguable as to which scheme is superior, single stage or two stage. Certainly A/D technology can accommodate the bandwidth required for ISDN basic access. At higher bit rates (possibly reaching 800 kbits/s) the resolution required will be at least 2-3 bits greater than the 14 bits @ 160 kbits/s now needed for basic access. It is unlikely that such a dramatic improvement in design and technology will accomplish this in a single stage design. In this context a two stage approach is certainly a viable alternative. In fact a multi-stage approach may be the only plausible solution. Nevertheless the single stage approach is more appropriate for the U-interface at the 160 kbits/s transmission rate.

The strategy of reducing the amplitude of the echo impulse response within the first few symbol periods can be beneficial. The AGC output and A/D resolution are improved if a simple single scheme is realized to reduce the echo amplitude of the first few symbol periods. However the requirements on the circuitry used to reduce the echo is equally as stringent

as the requirements on the A/D. Only the form is different.

7.2 Orthonormal Networks

The classical orthonormal networks considered here for echo cancellation are simple in concept yet are unrealizable in integrated form for this application. Large time constants are difficult to handle with continuous analog circuitry. Superior digital echo cancellation schemes exist which will remove orthonormal functions from consideration as echo channel modelling networks in the analog domain using present technology. It is likely that even a superior analog approach would not be used in designs of limited volume and high complexity because of the inability of analog designs to keep pace with new technology that can much more easily be transferred to digital designs. This is especially so in the competitive telecommunications market.

Orthonormal networks may yield savings at higher bit rates where lengthy transversal filters can be replaced by a small number of discrete orthonormal filters. A comparative savings would likely result as the echo tail length increased and the bit rate is increased.

The Laguerre and Legendre orthonormal networks were the only families of classical orthonormal polynomials which proved appropriate for Echo Canceller applications. Over the Bellcore loops tested a tail cancellation of 18.49 dB was achieved for the 5th order Laguerre network. A tail cancellation of 18.40 dB was achieved for the fourth order Legendre network. The order of each set of functions represents a tradeoff between cancellation performance versus complexity and area. The results indicate that the cancellation performance is dependent on the choice of the time scale factor for both sets of polynomials. A relatively low precision value of the time scale factor is all that is required as the results indicate a relative insensitivity to the time

scale factor. A maximum of 0.25 dB variation over ± 3 standard deviations from the optimum time scale factor was noted for the Laguerre network. A reasonable convergence time for adaptation can only be expected for the Laguerre functions.

Results from generating the echo impulse response from the Bellcore loops indicates that an improvement of greater than 6 dB reduction of the echo can be achieved by using a simple resistive balance network.

The discrete form of the classical orthonormal functions may hold promise in a sampled analog design or digital design.

7.3 Recommendations For Future Work

A quantitative analysis needs to be done to accurately determine whether a multi-stage approach is viable and at what point its use is warranted.

For completeness a detailed simulation of the adaptation of orthonormal functions needs to be completed. This is to determine the exact convergence behaviour and the strategy to be used in an adaptive implementation.

The use of continuous analog orthonormal functions to model the echo channel at the ISDN U-interface for basic access need not be investigated further given today's technology. The results indicate that performance is moderate at best while a realizable design is not a reasonable expectation for the class of orthonormal functions considered. The use of discrete orthonormal functions for handling the echo tail in the Main Echo Canceller needs to be investigated and may in fact prove useful. The orthonormal network may prove very good at removal of the echo tail past position 4 in Figure 4.1. This should be examined further.

A result of this work has been the reaffirmation of using the brute force A/D conversion approach to echo cancellation at the U-interface for ISDN

basic access.

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Appendices

Appendix A

Orthonormal Systems

A.1 Legendre Polynomials

Figure A.1: Legendre Polynomials Order 0 and Order 1

Figure A.2: Legendre Polynomials Order 2 and Order 3

Figure A.3: Legendre Polynomials Order 4 and Order 5

A.2 Laguerre Polynomials

Figure A.4: Laguerre Polynomials Order 0 and Order 1

Figure A.5: Laguerre Polynomials Order 2 and Order 3

Figure A.6: Laguerre Polynomials Order 4 and Order 5

A.3 Orthonormal System Derivation

A.3.1 Transformation Of Chebyshev Functions Into A Set Of Rational Functions

The shifted Chebyshev polynomials of the first kind have the properties

$$\int_0^1 T_n(x)T_m(x) dx = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \quad m, n = 0 \\ \frac{\pi}{2} & \text{if } m = n \quad m, n > 0. \end{cases} \quad (\text{A.1})$$

The range of integration must be shifted from $(0,1)$ to $(0,\infty)$ to meet the causal requirements of the echo signal. A change of variables to produce this is

$$x = e^{-p\tau}$$

hence

$$dx = -pe^{-p\tau} d\tau.$$

At the end points

$$\text{if } x = e^{-p\tau} = 1 \quad \tau = 0$$

$$\text{if } x = e^{-p\tau} = 0 \quad \tau = \infty \quad (\text{in the limit}).$$

The integral, eq(A.1), then becomes

$$\int_{\infty}^0 -(pe^{-p\tau})T_n(e^{-p\tau})T_m(e^{-p\tau}) d\tau = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \quad m, n = 0 \\ \frac{\pi}{2} & \text{if } m = n \quad m, n > 0. \end{cases} \quad (\text{A.2})$$

Using the relation [Spie68]

$$\int_a^b g(x)dx = -\int_b^a g(x)dx \quad (\text{A.3})$$

the limits can be switched from $(\infty, 0)$ to $(0, \infty)$.

$$\int_0^\infty p e^{-p\tau} T_n(e^{-p\tau}) T_m(e^{-p\tau}) d\tau = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \quad m, n = 0 \\ \frac{\pi}{2} & \text{if } m = n \quad m, n > 0 \end{cases} \quad (\text{A.4})$$

Equation (A.4) must be normalized.

$$\int_0^\infty \frac{1}{\pi} p e^{-p\tau} T_n(e^{-p\tau}) T_m(e^{-p\tau}) d\tau = \begin{cases} 0 & \text{if } m \neq n \\ 1 & \text{if } m = n \quad m, n = 0 \end{cases} \quad (\text{A.5})$$

$$\int_0^\infty \frac{2}{\pi} p e^{-p\tau} T_n(e^{-p\tau}) T_m(e^{-p\tau}) d\tau = \begin{cases} 0 & \text{if } m \neq n \\ 1 & \text{if } m = n \quad m, n > 0 \end{cases} \quad (\text{A.6})$$

If

$$v_n(\tau) = \sqrt{p \frac{1}{\pi}} e^{-\frac{p\tau}{2}} T_n(e^{-p\tau}) \quad \text{for } m = n \quad m, n = 0 \quad (\text{A.7})$$

and

$$v_n(\tau) = \sqrt{p \frac{2}{\pi}} e^{-\frac{p\tau}{2}} T_n(e^{-p\tau}) \quad \text{for } m = n \quad m, n > 0. \quad (\text{A.8})$$

The equation then reduces to

$$\int_a^b v_n(\tau) v_m(\tau) d\tau = \begin{cases} 0 & \text{if } m \neq n \\ 1 & \text{if } m = n. \end{cases} \quad (\text{A.9})$$

where $v_n(\tau)$ is defined above in eq (A.8) or eq (A.7) depending on the values of m and n being ≥ 0 .

All the time domain polynomials can be determined from equations A.8 and A.7, plus the value of the Chebyshev polynomial, $T_n(x)$. The first five are listed below.

$$T_0(x) = 1 \quad (\text{A.10})$$

$$T_1(x) = x \quad (\text{A.11})$$

$$T_2(x) = 2x^2 - 1 \quad (\text{A.12})$$

$$T_3(x) = 4x^3 - 3x \quad (\text{A.13})$$

$$T_4(x) = 8x^4 - 8x^2 + 1 \quad (\text{A.14})$$

To obtain the frequency domain equivalent of the set $\{v_n(\tau)\}$ Parseval's Theorem is used in conjunction with equation (A.9).

$$2\pi \int_{-\infty}^{\infty} \overline{U_m(w)} U_n(w) dw = \begin{cases} 0 & \text{if } m \neq n \\ 1 & \text{if } m = n \end{cases} \quad (\text{A.15})$$

First taking the Fourier Transform of $v_0(\tau)$

$$v_0(\tau) = \sqrt{p \frac{1}{\pi}} e^{-\frac{p\tau}{2}} \quad (\text{A.16})$$

$$\mathfrak{F}\{v_0(\tau)\} = V_0(w) = \frac{\sqrt{p \frac{1}{\pi}}}{2\pi} \int_0^{\infty} e^{-\frac{p\tau}{2}} e^{-jw\tau} d\tau \quad (\text{A.17})$$

$$V_0(w) = \frac{\sqrt{p \frac{1}{\pi}}}{2\pi} \int_0^{\infty} e^{-(\frac{p}{2} + jw)\tau} d\tau \quad (\text{A.18})$$

$$V_0(w) = \frac{\sqrt{p \frac{1}{\pi}}}{2\pi} \lim_{a \rightarrow \infty} \left(\frac{-1}{\frac{p}{2} + jw} \right) e^{-(\frac{p}{2} + jw)\tau} \Big|_0^a \quad (\text{A.19})$$

$$V_0(w) = \frac{\sqrt{p \frac{1}{\pi}}}{2\pi} \lim_{a \rightarrow \infty} \left(\frac{1}{\frac{p}{2} + jw} \right) (1 - e^{-ra}) \quad (\text{A.20})$$

$$V_0(w) = \frac{\sqrt{p \frac{1}{\pi}}}{2\pi} \frac{1}{\frac{p}{2} + jw}. \quad (\text{A.21})$$

By the same reasoning $V_1(w)$ can be found

$$V_1(w) = \frac{\sqrt{p \frac{2}{\pi}}}{2\pi} \frac{1}{\frac{3p}{2} + jw}. \quad (\text{A.22})$$

Checking first the normality relation

$$2\pi \int_{-\infty}^{\infty} \frac{\sqrt{p \frac{1}{\pi}}}{2\pi} \left(\frac{1}{\frac{p}{2} + jw} \right) \frac{\sqrt{p \frac{1}{\pi}}}{2\pi} \left(\frac{1}{\frac{p}{2} - jw} \right) dw \quad (\text{A.23})$$

$$= 2\pi \frac{\frac{p}{\pi}}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{1}{\frac{p^2}{4} + w^2} dw \quad (\text{A.24})$$

$$= \frac{\frac{p}{2}}{2\pi} \lim_{\pm a \rightarrow \infty} \int_{-a}^{+a} \frac{1}{\frac{p^2}{4} + w^2} dw \quad (\text{A.25})$$

$$= \frac{\frac{p}{2}}{2\pi} \lim_{\pm a \rightarrow \infty} \left[\frac{1}{p/2} \tan^{-1} \frac{w}{p/2} \right]_{-a}^{+a} \quad (\text{A.26})$$

$$= \frac{\frac{p}{2}}{2\pi} \lim_{\pm a \rightarrow \infty} \left[\frac{1}{p/2} \tan^{-1} \frac{+a}{p/2} - \frac{1}{p/2} \tan^{-1} \frac{-a}{p/2} \right] \quad (\text{A.27})$$

$$= \frac{\frac{p}{2}}{2\pi} \frac{1}{\frac{p}{2}} \left[\frac{\pi}{2} - \frac{-\pi}{2} \right] = \frac{1}{\pi}. \quad (\text{A.28})$$

$V_0(w)$ must be divided by π to maintain normality. The subsequent functions $V_1(w), \dots, V_n(w)$ must be multiplied by $2/\pi$.

Now checking the orthogonality property. For $V_1(w)$

$$2\pi \int_{-\infty}^{\infty} \frac{\sqrt{p\frac{1}{\pi}}}{2\pi} \left(\frac{1}{\frac{p}{2} - jw} \right) \frac{\sqrt{p\frac{2}{\pi}}}{2\pi} \left(\frac{1}{\frac{3p}{2} + jw} \right) dw. \quad (\text{A.29})$$

To evaluate this improper complex integral the easiest method to use is the residue theorem.

If $z = jw$

$$\text{Residue at } \left(z = \frac{p}{2} \right) = \lim_{z \rightarrow \frac{p}{2}} \frac{d}{dz} \left\{ \left(\frac{\frac{p}{2} - z}{\frac{p}{2} + z} \right) \left(\frac{1}{\frac{3p}{2} + z} \right) \right\} \quad (\text{A.30})$$

$$= \lim_{z \rightarrow \frac{p}{2}} \left(\frac{-1}{\frac{3p}{2} + z} \right) = 0. \quad (\text{A.31})$$

From the residue theorem in order to remain orthogonal in the frequency domain the value of $V_{n+1}(w)$ must be multiplied by the reciprocal of the complex conjugate of the poles of $V_n(w)$ and $V_n(w)$ itself. The resulting network is shown in Figure A.7.

Figure A.7: Chebyshev Network

Appendix B

Hybrid Equation Derivations

B.1 Hybrid Transfer Function

Consider the model in Figure B.1 which is without a balance network ¹.

The loop equations are

$$V_T = 2I_1R + 2V_1 \quad \text{Loop } I_1 \quad (\text{B.1})$$

$$2V_2 + I_2 \left(Z_L + \frac{1}{sC} \right) = 0 \quad \text{Loop } I_2 \quad (\text{B.2})$$

$$2V_1 = sL(I_1 - I_3) \quad \text{Loop } I_3. \quad (\text{B.3})$$

The Transformer equations are

$$V_1 = \frac{N_1}{N_2} V_2, \quad (\text{B.4})$$

$$I_2 = \frac{N_1}{N_2} I_3. \quad (\text{B.5})$$

¹ A large resistor was not included to account for an actual implementation in which a small current is applied to reduce oxidization of the copper wire.

Substituting the transformer current equation into eq(B.2)

$$-2V_2 + I_3 \left(\frac{N_1}{N_2} \right) \left(Z_L + \frac{1}{sC} \right) = 0. \quad (\text{B.6})$$

Solving eq(B.3) for I_3 and substituting into eq(B.6)

$$-2V_2 + \left(I_1 - \frac{2V_1}{sL} \right) \left(\frac{N_1}{N_2} \right) \left(Z_L + \frac{1}{sC} \right) = 0. \quad (\text{B.7})$$

Using the transformer voltage relation, eq(B.4),

$$-2V_2 + \left(I_1 - 2 \left(\frac{N_1}{N_2} \right) \left(\frac{V_2}{sL} \right) \right) \left(\frac{N_1}{N_2} \right) \left(Z_L + \frac{1}{sC} \right) = 0. \quad (\text{B.8})$$

Multiplying through and reducing

$$-2V_2 + \left(I_1 \left(\frac{N_1}{N_2} \right) \left(Z_L + \frac{1}{sC} \right) \right) - 2V_2 \left(\frac{N_1}{N_2} \right)^2 \left(\frac{1}{sL} \right) \left(Z_L + \frac{1}{sC} \right) = 0 \quad (\text{B.9})$$

$$-2V_2 \left(1 + \left(\frac{N_1}{N_2} \right)^2 \left(\frac{1}{sL} \right) \left(Z_L + \frac{1}{sC} \right) \right) + I_1 \left(\frac{N_1}{N_2} \right) \left(Z_L + \frac{1}{sC} \right) = 0 \quad (\text{B.10})$$

$$2V_2 \left(1 + \left(\frac{N_1}{N_2} \right)^2 \left(\frac{1}{sL} \right) \left(Z_L + \frac{1}{sC} \right) \right) = I_1 \left(\frac{N_1}{N_2} \right) \left(Z_L + \frac{1}{sC} \right) \quad (\text{B.11})$$

$$2V_2 = \frac{I_1 \left(\frac{N_1}{N_2} \right) \left(Z_L + \frac{1}{sC} \right)}{\left(1 + \left(\frac{N_1}{N_2} \right)^2 \left(\frac{1}{sL} \right) \left(Z_L + \frac{1}{sC} \right) \right)}. \quad (\text{B.12})$$

Substitute into eq(B.3) and using the transformer voltage relation

$$V_T = 2I_1 R + \frac{I_1 \left(\frac{N_1}{N_2} \right) \left(Z_L + \frac{1}{sC} \right)}{\left(1 + \left(\frac{N_1}{N_2} \right)^2 \left(\frac{1}{sL} \right) \left(Z_L + \frac{1}{sC} \right) \right)}. \quad (\text{B.13})$$

$$V_T = I_1 \left(2R + \left(\frac{N_1}{N_2} \right) \left(\frac{\left(Z_L + \frac{1}{sC} \right)}{\left(1 + \left(\frac{N_1}{N_2} \right)^2 \left(\frac{1}{sL} \right) \left(Z_L + \frac{1}{sC} \right) \right)} \right) \right) \quad (\text{B.14})$$

If

$$V_T = I_1 Z_A \quad (\text{B.15})$$

and

$$H_{ec} = R \frac{I_1}{V_T} \quad (\text{B.16})$$

then

$$Z_A = 2R + \frac{\left(\frac{N_1}{N_2}\right) \left(Z_L + \frac{1}{sC}\right)}{\left(1 + \left(\frac{N_1}{N_2}\right)^2 \left(\frac{1}{sL}\right) \left(Z_L + \frac{1}{sC}\right)\right)}. \quad (\text{B.17})$$

This reduces to

$$Z_A = \frac{2R + \left(\frac{N_1}{N_2}\right) \left(Z_L + \frac{1}{sC}\right) \left(\frac{2R}{sL} + 1\right)}{\left(1 + \left(\frac{N_1}{N_2}\right)^2 \left(\frac{1}{sL}\right) \left(Z_L + \frac{1}{sC}\right)\right)}. \quad (\text{B.18})$$

Therefore

$$H_{ec_{no\text{bat}}}(s) = \frac{\left(1 + \left(\frac{N_1}{N_2}\right)^2 \left(\frac{1}{sL}\right) \left(Z_L + \frac{1}{sC}\right)\right) R}{R + \left(\frac{N_1}{N_2}\right)^2 \left(Z_L + \frac{1}{sC}\right) \left(\frac{R}{sL} + \frac{1}{2}\right)} \frac{R}{2}. \quad (\text{B.19})$$

R was chosen as 135 ohms. Z_L is given by the A, B, C, D parameters of the line including the termination impedance of the hybrid (Appendix B.2). C, L would typically be $1\mu\text{F}$ and 50 mH respectively.

If a balance network were used the echo transfer function would be

$$H_{ec_{bat}}(s) = \frac{R(Z_b - Z_m)}{(R + 2Z_m)(R + 2Z_b)}, \quad (\text{B.20})$$

where

$$Z_m = \frac{sL \left(Z_L + \frac{1}{sC}\right)}{4 \left(Z_L + \frac{1}{sC} + \left(\frac{N_2}{N_1}\right)^2 sL\right)}. \quad (\text{B.21})$$

B.2 Hybrid Termination Impedance

The Termination Impedance looking into the Far End Hybrid can be found using Figure B.2.

Figure B.1: Hybrid Model : V_T is the transmitter source.

Figure B.2: Impedance Looking Into Hybrid From Subscriber Loop Line : V_T is the transmitter source.

The termination impedance is found by dividing the open circuit voltage by the short circuit current.

$$Z_T = \frac{V_{OC}}{I_{SC}} = \frac{V_O}{I_2} \quad (\text{B.22})$$

The transmit voltage, V_T , is removed and the Thevenin equivalent resistance is found.

The open circuit voltage for $Z_L = \infty$ is V_O .

$$V_O = 2V_2 - \frac{I_2}{sC}. \quad (\text{B.23})$$

Equation B.1 then becomes

$$2I_1R + 2V_1 = 0. \quad (\text{B.24})$$

The short circuit current is I_2 for $Z_L = 0$. Using the transformer equations B.4 and B.5 and substituting equation B.24 into B.3

$$-2\left(\frac{N_1}{N_2}\right)V_2 + sL\left(-\left(\frac{N_1}{N_2}\right)\frac{V_2}{R} - \left(\frac{N_2}{N_1}\right)I_2\right) = 0. \quad (\text{B.25})$$

Appropriately rearranging

$$-2V_2\left(\frac{N_1}{N_2}\right)\left(1 + \frac{sL}{2R}\right) = sL\left(\frac{N_2}{N_1}\right)I_2, \quad (\text{B.26})$$

$$-2V_2 = \frac{sL\left(\frac{N_2}{N_1}\right)^2 I_2}{1 + \frac{sL}{2R}}. \quad (\text{B.27})$$

Substituting back into eq(B.23)

$$-V_O = \frac{sL\left(\frac{N_2}{N_1}\right)^2 I_2}{1 + \frac{sL}{2R}} + \frac{I_2}{sC}. \quad (\text{B.28})$$

Dividing by I_2 and noting that $I_1 = -I_2$ as seen from the loop

$$Z_T = \frac{1}{sC} + \left(\frac{N_2}{N_1}\right)^2 \frac{2R}{1 + \frac{2R}{sL}}. \quad (\text{B.29})$$

Appendix C

Loop Information

C.1 Derivation Of The ABCD Matrix For A Subscriber Line With A Single Bridge Tap

Figure C.1: Subscriber Loop Line With A Single Bridge Tap

A', B', C', D' are the A, B, C, D parameters for the open circuited bridge tap.

$$\begin{bmatrix} V_1 \\ I_3 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_4 \\ 0 \end{bmatrix} \quad \text{since } I_4 = 0 \quad (\text{C.1})$$

$I_3 = C'V_4 = \frac{C'}{A'}V_1$ and $I_2 = I_1 - I_3 = I_1 - \frac{C'}{A'}V_1$. Since $V_1 = V_2$, the equivalent ABCD parameters of the bridge tap, now at the main line, are

$$\begin{bmatrix} A = 1 & B = 0 \\ C = \frac{C'}{A'} & D = 1 \end{bmatrix}. \quad (\text{C.2})$$

The bridge tap ABCD matrix can also be reduced via another method. The new value of C is equivalent to $Y_o \tanh \gamma$. Y_o is the admittance (i.e., the square root of $C'D$ divided by AB) prior to conversion, and γ is the propagation constant for the entire bridged tap (i.e., the complex log of half the sum of $A, D, B/Z_o$, and $Z_o C$, where $Z_o = 1/Y_o$) prior to conversion [Ahm81]. A and B are 1 and $B = 0$ just as in eq(C.2).

Both methods yield equivalent results. The ABCD matrix of a composite line would be the result of the product of matrix C.2 and the ABCD matrices of sections 1 and 3. The result can be determined for all frequencies of interest.

C.2 Routine To Recursively Reduce A Complex Subscriber Loop To A Line Having A Single ABCD Matrix

The data structures necessary to understand the following routine are added here for completeness. The C programming language is used.


```

typedef struct {
    int      gauge;      /* AWG gauge          */
    int      tempF;      /* temperature in fahrenheit */
    char     linetype;   /* bridge or series lines */
    double   length;     /* line length in kilometers */
    COMPLEX  a[4];       /* abcd matrix         */
} SUBLINE ;

struct lnode {
    SUBLINE  *ldata;     /* line data          */
    struct lnode *left;  /* bridge line        */
    struct lnode *right; /* series line         */
    struct lnode *before; /* backwards pointer   */
};

/* ##### */
/* A recursive traversal of the tree to produce a single */
/* ABCD matrix for the entire compound line. */

oneline(p,freqindex)
struct lnode *p;
int freqindex;
{
    COMPLEX result[SIZE];
    int i;

    if( p->right != NULL){
        oneline(p->right,freqindex);    /* process main line */

        if ( p->left != NULL) {         /* process bridge */
            oneline(p->left,freqindex); /* tap off of any */
            convert(((p->left)->ldata)->a); /* bridge tap */
        }
    }
}

```

```

        cmatmult((p->ldata)->a,((p->left)->ldata)->a,result);
        for( i = 0; i < SIZE; i++)
            (p->ldata)->a[i] = result[i];

    } /* end if */

    cmatmult((p->ldata)->a,((p->right)->ldata)->a,result);
    for( i = 0; i < SIZE; i++)
        (p->ldata)->a[i] = result[i];

} /* end if */

else if (p->left != NULL){          /* process bridge */
    oneline(p->left,freqindex);      /* tap off of      */
    convert(((p->left)->ldata)->a); /* main line      */

    cmatmult((p->ldata)->a,((p->left)->ldata)->a,result);
    for( i = 0; i < SIZE; i++)
        (p->ldata)->a[i] = result[i];

} /* end else if */
} /* end cneline */
/* ##### */

```

C.3 Example Subscriber Loop Definition

A line definition contains the line length, line gauge and temperature of operation. A line can be either a bridge tap or a series line of another line, which in turn can be either bridge tap or series type.

line 03

```
# Bellcore Loop Line 03 ( T1D1 Specifications )
# Central Office to Subscriber ( Forward )
# Main line
line 7500 26 70
# Another series line
series
line 6000 24 70
    # first bridge tap
    bridge
    line 500 24 70
    end
# Another series section
series
line 1500 24 70
    # second bridge tap
    bridge
    line 1000 22 70
    end
# Another series section
series
line 1000 22 70
    # third bridge tap
    bridge
    line 1500 24 70
        # this bridge tap has its own bridge tap
        bridge
        line 1000 24 70
        end
    # continuation of third bridge tap
    series
    line 500 24 70
    end
end
```

C.4 Output Listing From Echo Transfer Function Program

Sun Apr 17 15:50:06 1988

LINE 03

Transformer Parameters

CAPACTOR	1.000000e-06 F
INDUCTOR	5.000000e-02 H
RESISTOR	1.350000e+02 ohms
BALANCE IMPEDANCE	. j
TURNS	N2 1 N1 1

FREQ	MAGNITUDE (dB)	PHASE (deg)	real	imaginary
0.000000	-6.020600e+00	-6.211833e-40	5.000000e-01	5.420847e-42
1.000000	-6.020606e+00	-6.666670e-02	4.999993e-01	-5.817762e-04
5.000000	-6.020747e+00	-3.333379e-01	4.999831e-01	-2.908856e-03
10.000000	-6.021189e+00	-6.667018e-01	4.999322e-01	-5.817545e-03
15.000000	-6.021928e+00	-1.000115e+00	4.998474e-01	-8.725872e-03
20.000000	-6.022965e+00	-1.333598e+00	4.997285e-01	-1.163362e-02
30.000000	-6.025937e+00	-2.000839e+00	4.993882e-01	-1.711634e-02
50.000000	-6.035518e+00	-3.336776e+00	4.982958e-01	-2.905212e-02
70.000000	-6.050028e+00	-4.675009e+00	4.966510e-01	-4.061406e-02
100.000000	-6.081256e+00	-6.686755e+00	4.931430e-01	-5.781542e-02
150.000000	-6.159357e+00	-1.004824e+01	4.845281e-01	-8.585607e-02
200.000000	-6.271247e+00	-1.341005e+01	4.725331e-01	-1.126609e-01
300.000000	-6.600539e+00	-2.006373e+01	4.393218e-01	-1.604537e-01
500.000000	-7.683603e+00	-3.246048e+01	3.483692e-01	-2.215980e-01
700.000000	-9.279724e+00	-4.232329e+01	2.540202e-01	-2.313294e-01
1000.000000	-1.216489e+01	-5.105516e+01	1.549211e-01	-1.916886e-01
1500.000000	-1.711882e+01	-4.750320e+01	9.412732e-02	-1.027335e-01
2000.000000	-2.019793e+01	-2.539234e+01	8.830397e-02	-4.191534e-02
3000.000000	-1.899303e+01	1.965100e+01	1.103572e-01	2.075448e-02
5000.000000	-1.538871e+01	2.236603e+01	1.572531e-01	6.470601e-02
7000.000000	-1.371842e+01	2.265476e+01	1.901983e-01	7.938527e-02
10000.000000	-1.240935e+01	2.122952e+01	2.233636e-01	8.676947e-02
15000.000000	-1.132380e+01	1.870262e+01	2.571875e-01	8.706630e-02
20000.000000	-1.073121e+01	1.658372e+01	2.786045e-01	8.296942e-02
30000.000000	-1.007493e+01	1.357060e+01	3.047586e-01	7.356334e-02
50000.000000	-9.533590e+00	9.975263e+00	3.286283e-01	5.779976e-02
70000.000000	-9.291676e+00	7.848297e+00	3.398827e-01	4.685005e-02
100000.000000	-9.125409e+00	6.002076e+00	3.478102e-01	3.656906e-02
150000.000000	-9.008858e+00	4.385408e+00	3.534140e-01	2.710319e-02
200000.000000	-8.956898e+00	3.547128e+00	3.558953e-01	2.206134e-02
300000.000000	-8.905871e+00	2.705672e+00	3.582796e-01	1.693158e-02
500000.000000	-8.853515e+00	2.015980e+00	3.606246e-01	1.269399e-02
700000.000000	-8.818739e+00	1.697816e+00	3.621366e-01	1.073414e-02
1000000.000000	-8.781281e+00	1.423508e+00	3.637491e-01	9.039167e-03
1500000.000000	-8.743954e+00	1.16093e+00	3.653530e-01	7.423993e-03
2000000.000000	-8.720879e+00	1.009940e+00	3.663436e-01	6.458126e-03

Table C.1: Output Listing For Line 03 (No Balance Network)

C.5 Parameters For Various AWG Lines At 70°F

Only data for polyethylene insulated cable (PIC) lines at 70°F is used. This is in accordance with T1D1 specifications.

Frequency Hz	$Z_o = \Re(Z_o) + j\Im(Z_o)$ ohms		Gamma	
			DB/mi	Deg/mi
0.0	10000.0	-10000.0	0.000001	0.000001
1.0	9132.32	-9127.63	0.041	0.27
5.0	4084.62	-4081.48	0.092	0.61
10.0	2888.76	-2885.55	0.131	0.85
15.0	2359.07	-2355.63	0.160	1.06
20.0	2043.38	-2039.68	0.185	1.22
30.0	1669.00	-1664.81	0.226	1.50
50.0	1293.72	-1288.64	0.292	1.93
70.0	1094.17	-1088.33	0.345	2.29
100.0	916.43	-909.60	0.412	2.73
150.0	749.60	-741.37	0.504	3.36
200.0	650.34	-640.91	0.581	3.89
300.0	532.90	-521.46	0.709	4.75
500.0	415.75	-401.08	0.909	6.21
700.0	353.90	-336.59	1.068	7.40
1000.0	299.30	-278.66	1.263	8.94
1500.0	248.78	-223.59	1.520	11.15
2000.0	219.32	-190.30	1.725	13.10
3000.0	185.51	-150.15	2.041	16.63
5000.0	153.94	-108.88	2.467	22.99
7000.0	183.92	-86.50	2.744	29.05
10000.0	127.32	-66.53	3.015	38.07
15000.0	118.60	-48.35	3.287	53.15
20000.0	114.61	-38.26	3.469	68.49
30000.0	110.69	-27.71	3.770	99.22
50000.0	107.73	-19.14	4.342	160.94
70000.0	106.26	-15.51	4.927	222.24
100000.0	104.74	-12.69	5.761	312.95
150000.0	102.94	-10.52	7.033	461.37
200000.0	101.63	-8.94	8.127	607.35
300000.0	100.02	-7.31	9.968	896.55
500000.0	98.32	-5.67	12.901	1468.89
700000.0	97.38	-4.80	15.292	2036.81
1000000.0	96.52	-4.02	18.315	2893.98
1500000.0	95.69	-3.29	22.482	4282.87
2000000.0	95.18	-2.85	26.002	5638.11
3000000.0	94.56	-2.33	31.919	8476.66
5000000.0	93.92	-1.80	41.333	14031.41

Table C.2: Parameters For 19 Gauge PIC AWG Line

Frequency Hz	$Z_o = R(Z_o) + j\Im(Z_o)$ ohms		Gamma	
			DB/mi	Deg/mi
0.0	14000.0	-14000.0	0.000001	0.000001
1.0	12927.71	-12924.34	0.059	0.39
5.0	5781.80	-5779.59	0.131	0.86
10.0	4088.69	-4086.44	0.185	1.22
15.0	3338.69	-3336.28	0.227	1.50
20.0	2891.65	-2889.05	0.262	1.73
30.0	2361.43	-2358.49	0.321	2.12
50.0	1829.81	-1826.24	0.414	2.73
70.0	1547.02	-1542.91	0.489	3.24
100.0	1295.02	-1290.21	0.585	3.87
150.0	1058.12	-1052.52	0.715	4.74
200.0	917.35	-910.71	0.825	5.48
300.0	750.35	-742.28	1.009	6.73
500.0	583.30	-572.94	1.298	8.71
700.0	494.74	-482.54	1.530	10.35
1000.0	416.16	-401.58	1.819	12.43
1500.0	342.84	-325.02	2.209	15.36
2000.0	299.57	-279.02	2.528	17.90
3000.0	249.00	-223.89	3.043	22.32
5000.0	199.83	-167.56	3.796	29.85
7000.0	174.89	-136.92	4.342	36.58
10000.0	154.00	-109.09	4.942	46.01
15000.0	136.34	-82.53	5.609	61.10
20000.0	127.25	-66.72	6.045	76.04
30000.0	118.37	-48.55	6.600	106.10
50000.0	111.68	-32.21	7.298	166.85
70000.0	109.12	-24.81	7.871	228.24
100000.0	107.24	-19.27	8.732	320.43
150000.0	105.52	-14.99	10.193	472.95
200000.0	104.33	-12.77	11.575	623.45
300000.0	102.56	-10.38	14.124	919.38
500000.0	100.38	-8.05	18.250	1499.65
700000.0	99.16	-6.81	21.617	2074.02
1000000.0	98.04	-5.70	25.873	2929.37
1500000.0	96.95	-4.66	31.742	4345.26
2000000.0	96.28	-4.04	36.698	5753.82
3000000.0	95.48	-3.30	45.021	8558.60
5000000.0	94.64	-2.56	58.244	14139.61

Table C.3: Parameters For 22 Gauge PIC AWG Line

Frequency Hz	$Z_o = \Re(Z_o) + j\Im(Z_o)$ ohms		Gamma	
			DB/mi	Deg/mi
0.0	19000.0	-19000.0	0.0000001	0.0000001
1.0	16306.82	-16297.63	0.074	0.49
5.0	7292.58	-7288.57	0.165	1.09
10.0	5156.83	-5153.61	0.234	1.54
15.0	4210.72	-4207.71	0.286	1.89
20.0	3646.77	-3643.81	0.330	2.18
30.0	2977.88	-2974.85	0.404	2.67
50.0	2307.15	-2303.82	0.522	3.45
70.0	1950.32	-1946.67	0.617	4.05
100.0	1632.29	-1628.16	0.738	4.88
150.0	1333.50	-1328.66	0.903	5.98
200.0	1155.49	-1150.02	1.042	6.90
300.0	944.50	-937.95	1.275	8.45
500.0	733.25	-724.93	1.642	10.95
700.0	621.10	-611.32	1.939	12.99
1000.0	521.40	-509.77	2.310	15.58
1500.0	428.12	-413.93	2.813	19.18
2000.0	372.85	-356.50	3.230	22.28
3000.0	307.87	-287.90	3.913	27.59
5000.0	243.87	-218.18	4.942	36.43
7000.0	210.75	-180.44	5.723	44.07
10000.0	182.25	-146.22	6.625	54.45
15000.0	157.04	-113.38	7.706	70.38
20000.0	143.23	93.49	8.472	85.59
30000.0	128.79	-69.79	9.487	115.44
50000.0	117.22	-46.91	10.628	175.12
70000.0	112.62	-35.78	11.351	235.56
100000.0	109.50	-27.00	12.235	327.15
150000.0	107.21	-20.10	13.665	480.48
200000.0	105.95	-16.69	15.131	633.30
300000.0	104.31	-13.22	17.975	935.61
500000.0	102.12	-10.17	23.048	1525.69
700000.0	100.67	-8.60	27.284	2105.53
1000000.0	99.33	-7.20	32.639	2967.99
1500000.0	98.03	-5.88	40.026	4393.66
2000000.0	97.23	-5.10	46.267	5810.54
3000000.0	96.26	-4.17	56.753	8628.67
5000000.0	95.26	-3.23	73.405	14231.83

Table C.4: Parameters For 24 Gauge PIC AWG Line

Frequency Hz	$Z_c = R(Z_c) + j\Im(Z_c)$ ohms		Gamma	
			DB/mi	Deg/mi
0.0	25000.0	-25000.0	0.000001	0.0000001
1.0	20562.46	-20551.04	0.093	0.61
5.0	9195.60	-9190.92	0.208	1.37
10.0	6502.38	-6498.86	0.294	1.94
15.0	5309.30	-5306.16	0.361	2.38
20.0	4598.12	-4595.14	0.416	2.75
30.0	3754.58	-3751.69	0.510	3.36
50.0	2908.66	-2905.67	0.658	4.34
70.0	2458.60	-2455.41	0.779	5.14
100.0	2057.93	-2053.93	0.931	6.15
150.0	1680.47	-1676.45	1.139	7.53
200.0	1455.83	-1451.35	1.315	8.70
300.0	1189.51	-1184.20	1.610	10.66
500.0	922.69	-916.00	2.075	13.75
700.0	780.91	-733.09	2.452	16.33
1000.0	654.74	-645.46	2.924	19.56
1500.0	536.48	-525.19	3.569	24.04
2000.0	466.25	-453.25	4.107	27.86
3000.0	383.39	-367.51	4.995	34.36
5000.0	301.19	-280.76	6.360	44.99
7000.0	258.16	-234.04	7.422	53.99
10000.0	220.58	-191.85	8.692	65.90
15000.0	186.46	-151.47	10.294	83.56
20000.0	167.08	-126.94	11.503	99.83
30000.0	145.71	-97.35	13.232	130.60
50000.0	127.25	-67.47	15.285	190.10
70000.0	119.21	-52.05	16.509	249.33
100000.0	113.56	-39.12	17.726	339.29
150000.0	109.61	-28.32	19.253	491.26
200000.0	107.80	-22.81	20.676	644.17
300000.0	105.92	-17.35	23.590	949.49
500000.0	103.87	-12.91	29.264	1551.77
700000.0	102.45	-10.86	34.477	2142.83
1000000.0	100.90	-9.09	41.198	3014.78
1500000.0	99.35	-7.42	50.493	4452.71
2000000.0	98.39	-6.43	58.348	5879.86
3000000.0	97.23	-5.26	71.551	8715.35
5000000.0	96.02	-4.08	92.534	14345.97

Table C.5: Parameters For 26 Gauge PIC AWG Line

C.6 Bell Core Loops

The loop responses are different for an impulse travelling from Central Office (NT) to Subscriber (LT) or Subscriber (LT) to Central Office (NT) because of the relative placement of the anomalies in the line. Responses for both directions were generated¹.

Line Numbers 01 to 15 correspond to transmission from the Central Office to Subscriber. Line Numbers 21 to 35 correspond to transmission from the Subscriber to Central Office. Thus line 21 is line 01 with the transmission direction reversed. The same numbering scheme applies for all other lines i.e. line 01 $\Rightarrow$ line 21, line 02 $\Rightarrow$ line 22, etc.

¹ 1 metre = 3 2808 feet

- Note
- 1) AWG means American Wire Gauge
 - 2) Distances are in feet (') 1000' = 0.48 km

Figure 0. LOOPS FOR TESTING RECEIVED SIGNAL PERFORMANCE #1 - #5

Note 1) AWG means American Wire Gauge
 2) Distances are in feet (') 1000' = 3048 km

Figure 8 (CONT) LOOPS FOR TESTING RECEIVED SIGNAL PERFORMANCE #6 - #10

Figure 8 (CONT) LOOPS FOR TESTING RECEIVED SIGNAL PERFORMANCE. #11 - #15

Appendix D

Tail Cancellation Performance Using Orthonormal Functions

D.1 Echo Tail Cancellation

Results for all the Bellcore lines (both transmission directions) using Laguerre and Legendre functions are given in Tables D.1 to D.12. The power in the echo tail is expressed as a percentage of the total echo power. This is calculated over 42 baud intervals.

Bell Core Line Config uration	Power In Echo Tail As A Percent Of Total Echo	Order 0 to 3							
		0		1		2		3	
		Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB
01	47.44	100000	4.36	100000	8.22	77800	11.82	76100	11.79
02	47.47	100000	4.34	100000	8.20	77700	11.78	76100	11.75
03	47.56	100000	4.27	100000	8.17	77600	11.68	76000	11.65
04	47.53	100000	4.29	100000	8.18	77700	11.72	76100	11.69
05	47.55	100000	4.28	100000	8.18	77600	11.70	76100	11.67
06	47.24	100000	4.20	100000	8.18	77800	11.66	76200	11.62
07	47.51	100000	4.31	100000	8.19	77700	11.73	76100	11.70
08	47.63	100000	4.23	100000	8.72	78200	11.83	76200	11.80
09	47.38	100000	4.08	100000	8.26	78800	11.65	76900	11.61
10	47.41	100000	4.19	100000	8.16	77700	11.61	76100	11.58
11	47.54	100000	4.27	100000	8.18	77700	11.68	76100	11.65
12	47.55	100000	4.24	100000	8.18	77700	11.65	76100	11.62
13	47.60	100000	4.22	100000	8.17	77700	11.62	76100	11.59
14	46.61	100000	4.02	100000	8.25	78600	11.54	76800	11.51
15	47.53	100000	4.27	100000	8.19	77700	11.68	76100	11.64
21	48.19	100000	4.43	100000	8.53	77400	11.91	75600	11.88
22	50.22	100000	4.83	100000	9.72	77800	12.41	100000	12.48
23	51.25	100000	4.79	100000	10.16	78400	12.38	100000	12.47
24	46.42	100000	4.16	100000	8.28	78600	11.74	76800	11.71
25	49.87	100000	4.74	100000	9.69	78200	12.35	100000	12.40
26	48.80	100000	4.66	100000	9.66	79300	12.43	100000	12.46
27	47.51	100000	4.31	100000	8.19	77700	11.73	76100	11.70
28	50.01	100000	4.51	100000	8.94	76800	11.89	100000	11.91
29	49.33	100000	4.57	100000	9.51	78200	12.22	100000	12.26
30	50.59	100000	4.60	100000	9.56	75800	12.04	100000	12.11
31	49.01	100000	5.44	100000	11.30	86700	12.88	72700	12.91
32	46.97	100000	4.19	100000	8.35	78400	11.73	76600	11.69
33	49.53	100000	4.18	100000	9.02	78200	11.76	100000	11.75
34	48.92	100000	4.35	100000	9.59	78900	12.04	100000	12.06
35	47.53	100000	4.27	100000	8.19	77700	11.68	76100	11.64

Table D 1: Laguerre Tail Cancellation Results - Order 0,1,2,3. No Balance Network. Echo Tail from sample 17 or position 2 in Figure 4.1. Simulated for Time Scale Factor values of 1000 to 100000 by increments of 100.

Bell Core Line Config uration	Power In Echo Tail As A Percent Of Total Echo	Order 4 to 7							
		4	5	6	7	8	9	10	11
		Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB
01	47.44	52500	13.60	100000	16.29	100000	16.32	88200	20.80
02	47.47	52400	13.59	100000	16.24	100000	16.26	88100	20.75
03	47.56	52200	13.55	100000	16.09	100000	16.09	87800	20.62
04	47.53	52300	13.56	100000	16.15	100000	16.16	87900	20.67
05	47.55	52300	13.55	100000	16.11	100000	16.11	87800	20.63
06	47.24	52300	13.53	100000	16.03	100000	16.03	87800	20.56
07	47.51	52300	13.57	100000	16.15	100000	16.16	87900	20.68
08	47.63	51700	13.51	100000	16.33	100000	16.38	87600	20.78
09	47.38	52400	13.54	100000	15.93	100000	15.93	87700	20.43
10	47.41	52200	13.51	100000	15.97	100000	15.96	87600	20.50
11	47.54	52200	13.55	100000	16.06	100000	16.06	87700	20.59
12	47.55	52200	13.53	100000	16.03	100000	16.03	87700	20.56
13	47.60	52100	13.52	100000	15.97	100000	15.97	87600	20.50
14	46.61	52100	13.49	100000	15.77	100000	15.76	87400	20.28
15	47.53	52200	13.55	100000	16.06	100000	16.06	87700	20.59
21	48.19	52000	13.54	100000	16.54	100000	16.61	88100	21.02
22	50.22	51200	13.57	100000	17.55	100000	17.85	88200	21.94
23	51.25	50400	13.53	100000	17.39	100000	17.70	87900	21.61
24	46.42	52500	13.57	100000	16.10	100000	16.12	88000	20.60
25	49.87	51200	13.57	100000	17.35	100000	17.61	88300	21.72
26	48.80	51800	13.67	100000	17.34	100000	17.62	89100	21.74
27	47.51	52300	13.57	100000	16.15	100000	16.16	87900	20.68
28	50.01	51100	13.36	100000	16.72	100000	16.84	87700	21.15
29	49.33	51400	13.55	100000	17.07	100000	17.29	88200	21.45
30	50.59	50300	13.37	100000	17.00	100000	17.17	87300	21.34
31	49.01	100000	13.78	100000	16.68	100000	17.61	88900	19.29
32	46.97	52300	13.56	100000	16.08	100000	16.09	87800	20.57
33	49.53	51000	13.37	100000	16.24	100000	16.31	87200	20.56
34	48.92	50800	13.18	100000	16.65	100000	16.80	87300	20.93
35	47.53	52200	13.55	100000	16.06	100000	16.06	87700	20.59

Table D.2. Laguerre Tail Cancellation Results . Order 4,5,6,7. No Balance Network. Echo Tail from sample 17 or position 2 in Figure 4.1. Simulated for Time Scale Factor values of 1000 to 100000 by increments of 100.

Bell Core Line Config uration	Power In Echo Tail As A Percent Of Total Echo	Order 0 to 3							
		0		1		2		3	
		Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB
01	47.44	100000	2.26	100000	7.88	52400	12.37	91700	14.27
02	47.47	100000	2.25	100000	7.86	52300	12.34	91700	14.20
03	47.56	100000	2.20	100000	7.81	52100	12.26	91700	14.04
04	47.53	100000	2.21	100000	7.83	52200	12.29	91700	14.11
05	47.55	100000	2.21	100000	7.82	52100	12.27	91700	14.06
06	47.24	100000	2.15	100000	7.81	52100	12.23	91800	14.01
07	47.51	100000	2.22	100000	7.84	52200	12.30	91700	14.11
08	47.63	100000	2.09	100000	8.28	51800	12.29	92700	14.48
09	47.38	100000	2.05	100000	7.85	52400	12.19	92600	13.99
10	47.41	100000	2.14	100000	7.79	52000	12.19	91800	13.94
11	47.54	100000	2.20	100000	7.83	52100	12.25	91700	14.03
12	47.55	100000	2.18	100000	7.81	52000	12.22	91700	13.99
13	47.60	100000	2.16	100000	7.80	52000	12.20	91700	13.94
14	46.61	100000	2.00	100000	7.82	52200	12.10	92500	13.83
15	47.53	100000	2.19	100000	7.83	52100	12.25	91700	14.03
21	48.19	100000	2.26	100000	8.17	51900	12.38	91800	14.58
22	50.22	100000	2.37	100000	9.31	51400	12.64	93400	15.99
23	51.25	100000	2.29	100000	9.69	50900	12.57	94300	16.02
24	46.42	100000	2.10	100000	7.88	52500	12.28	92400	14.15
25	49.87	100000	2.31	100000	9.26	51400	12.60	93600	15.80
26	48.80	100000	2.27	100000	9.22	52200	12.67	94300	15.85
27	47.51	100000	2.22	100000	7.84	52200	12.30	91700	14.11
28	50.01	100000	2.25	100000	8.54	51100	12.28	92100	14.87
29	49.33	100000	2.23	100000	9.06	51700	12.51	93900	15.50
30	50.59	100000	2.23	100000	9.10	50400	12.34	92700	15.33
31	49.01	100000	2.58	100000	11.00	44400	12.71	99500	16.71
32	46.97	100000	2.11	100000	7.95	52300	12.26	92500	14.14
33	49.53	100000	2.01	100000	8.53	51300	12.16	93200	14.52
34	48.92	100000	2.06	100000	9.08	51300	12.34	94100	15.13
35	47.53	100000	2.19	100000	7.83	52100	12.25	91700	14.03

Table D.3: Legendre Tail Cancellation Results : Order 0,1,2,3. No Balance Network. Echo Tail from sample 17 or position 2 in Figure 4.1. Simulated for Time Scale Factor values of 1000 to 100000 by increments of 100.

Bell Core Line Config uration	Power In Echo Tail As A Percent Of Total Echo	Order 4 to 7							
		4		5		6		7	
		Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB
01	47.44	63600	14.67	40700	20.15	31700	20.15	24100	22.42
02	47.47	63400	14.50	40700	20.11	31700	20.42	24100	22.41
03	47.56	63400	14.43	40500	20.00	31600	20.33	24100	22.39
04	47.53	63400	14.51	40600	20.04	31600	20.37	24100	22.39
05	47.55	63400	14.46	40600	20.00	31600	20.34	24100	22.40
06	47.24	63500	14.37	40500	19.94	31600	20.28	24100	22.37
07	47.51	63400	14.51	40600	20.01	31600	20.37	24100	22.41
08	47.63	63700	14.78	40500	20.11	31600	20.43	24100	22.55
09	47.38	63800	14.32	40500	19.81	31700	20.14	24100	22.35
10	47.41	63500	14.32	40500	19.88	31600	20.23	24100	22.36
11	47.54	63300	14.42	40500	19.96	31600	20.31	24100	22.40
12	47.55	63400	14.38	40500	19.91	31600	20.23	24100	22.39
13	47.60	63400	14.33	40500	19.88	31600	20.24	24100	22.38
14	46.61	63600	14.16	40300	19.66	31300	20.05	24100	22.34
15	47.53	63400	14.42	40500	19.96	31600	20.31	24100	22.40
21	48.19	63200	14.95	40600	20.03	31600	20.62	24100	22.53
22	50.22	63600	16.24	40700	21.04	31100	21.23	24200	22.84
23	51.25	63800	16.19	40300	20.76	31100	20.99	24100	22.96
24	46.42	63800	14.49	40700	19.96	31700	20.27	24100	22.37
25	49.87	63700	16.03	40500	20.86	31100	21.07	24000	22.86
26	48.80	64000	16.06	40700	20.87	31100	21.03	24100	22.81
27	47.51	63400	14.51	40600	20.04	31600	20.37	24100	22.41
28	50.01	63400	15.20	40300	20.39	31100	20.70	24100	22.59
29	49.33	64000	15.72	40200	20.62	31100	20.85	24100	22.75
30	50.59	63400	15.58	40200	20.54	31100	20.86	23600	22.79
31	49.01	63800	17.93	41100	18.70	31500	18.75	22800	21.11
32	46.97	63800	14.48	40500	19.94	31600	20.26	24100	22.41
33	49.53	63800	14.76	40100	19.88	31100	20.24	24100	22.54
34	48.92	64000	15.39	40100	20.18	31100	20.49	24100	22.78
35	47.53	63400	14.42	40500	19.96	31600	20.31	24100	22.40

Table D.4: Legendre Tail Cancellation Results : Order 4,5,6,7. No Balance Network. Echo Tail from sample 17 or position 2 in Figure 4.1. Simulated for Time Scale Factor values of 1000 to 100000 by increments of 100.

Bell Core Line Config uration	Power In Echo Tail As A Percent Of Total Echo	Order 1 to 3							
		0		1		2		3	
		Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB
01	3.03	4200	2.83	22100	7.47	22100	7.47	37300	12.26
02	3.06	4200	2.82	21900	7.51	21900	7.51	37100	12.29
03	3.14	4100	2.72	21500	7.56	21500	7.56	36600	12.32
04	3.11	4200	2.74	21700	7.53	21700	7.53	36800	12.30
05	3.13	4100	2.74	21500	7.55	21500	7.55	36600	12.32
06	3.13	4000	2.60	21400	7.54	21400	7.54	36400	12.28
07	3.10	4200	2.77	21600	7.53	21600	7.53	36800	12.30
08	2.78	3600	1.95	21200	7.24	21200	7.24	36200	12.05
09	3.02	3800	2.33	20900	7.42	20900	7.42	36000	12.13
10	3.17	4000	2.58	21200	7.56	21200	7.56	36200	12.30
11	3.13	4100	2.72	21300	7.55	21300	7.55	36400	12.29
12	3.15	4100	2.67	21300	7.56	21300	7.56	36300	12.30
13	3.17	4100	2.65	21100	7.56	21100	7.56	36100	12.29
14	3.11	3700	2.23	20500	7.49	20500	7.49	35400	12.17
15	3.13	4100	2.71	21300	7.55	21300	7.55	36400	12.29
21	2.85	4000	2.44	22300	7.31	22300	7.31	37400	12.21
22	2.06	3500	1.52	22900	6.56	22900	6.56	39000	11.71
23	2.00	70000	1.21	20500	6.47	20500	6.47	36400	11.21
24	2.96	3900	2.42	21500	7.40	21500	7.40	36600	12.12
25	2.11	3400	1.48	22100	6.60	22100	6.60	38000	11.62
26	2.04	3400	1.45	22300	6.51	22300	6.51	38600	11.54
27	3.10	4200	2.77	21600	7.53	21600	7.53	36800	12.30
28	2.69	3700	1.95	21900	7.08	21900	7.08	37200	12.08
29	2.23	3400	1.48	21600	6.66	21600	6.66	37500	11.62
30	2.40	3200	1.40	21100	6.95	21100	6.95	36200	11.88
31	1.16	70000	1.18	9600	3.84	6600	3.87	21800	6.81
32	2.97	3900	2.38	21100	7.39	21100	7.39	36200	12.11
33	2.81	3100	1.56	20200	7.14	20200	7.14	35000	11.78
34	2.39	4400	1.22	19800	6.89	19800	6.89	35000	11.56
35	3.13	4100	2.71	21300	7.55	21360	7.55	36400	12.30

Table D.5: Laguerre Tail Cancellation Results : Order 0,1,2,3. No Balance Network. Echo Tail from sample 26 or position 3 in Figure 4.1. Simulated for Time Scale Factor values of 1000 to 70000 by increments of 100.

Bell Core Line Config uration	Power In Echo Tail As A Percent Of Total Echo	Order 4 to 7							
		4	5	6	7	8	9	10	11
		Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB
01	3.03	37300	12.26	43400	18.53	43200	18.50	42800	22.37
02	3.06	37100	12.29	43300	18.60	43200	18.56	42900	22.45
03	3.14	36600	12.32	43100	18.71	43000	18.67	43000	22.59
04	3.11	36800	12.30	43200	18.64	43100	18.60	42900	22.51
05	3.13	36600	12.32	43200	18.69	43200	18.65	43200	22.56
06	3.13	36400	12.28	43100	18.66	42900	18.62	43000	22.58
07	3.10	36700	12.30	43200	18.66	43100	18.62	42600	22.53
08	2.78	36100	12.05	42700	18.48	42500	18.44	42700	22.27
09	3.02	35900	12.13	43000	18.60	42800	18.60	43300	22.63
10	3.17	36200	12.30	43000	18.72	42800	18.70	43100	22.65
11	3.13	36400	12.30	43100	18.70	42900	18.66	43100	22.60
12	3.15	36300	12.30	43100	18.71	42900	18.67	43100	22.62
13	3.17	36100	12.30	43000	18.71	42800	18.69	43100	22.67
14	3.11	35400	12.17	42600	18.65	42500	18.61	43300	22.72
15	3.13	36400	12.29	43100	18.70	42900	18.66	43100	22.60
21	2.85	37400	12.21	43300	18.56	43100	18.52	42600	22.22
22	2.06	39000	11.71	43900	18.28	43600	18.22	42700	21.40
23	2.00	36400	11.21	42200	17.59	42000	17.55	41900	21.03
24	2.96	36600	12.12	43100	18.46	42900	18.41	43000	22.41
25	2.11	38000	11.62	43400	18.20	43200	18.14	42700	21.55
26	2.04	38600	11.54	44000	18.20	43700	18.14	43200	21.60
27	3.10	36700	12.30	43200	18.66	43100	18.62	42600	22.53
28	2.69	37200	12.08	43100	18.62	42700	18.60	42700	22.06
29	2.23	37500	11.62	43400	18.28	43200	18.22	43100	21.74
30	2.40	36200	11.88	42300	18.39	42100	18.34	42100	21.72
31	1.16	21800	6.81	30900	11.52	30900	11.52	36500	16.70
32	2.97	36200	12.11	43000	18.50	42800	18.46	43000	22.44
33	2.81	34900	11.77	41800	18.01	41600	17.98	42100	21.73
34	2.39	35000	11.56	41800	18.02	41600	17.99	42300	21.79
35	3.13	36400	12.30	43100	18.70	42900	18.66	43100	22.60

Table D.6: Laguerre Tail Cancellation Results : Order 4,5,6,7. No Balance Network. Echo Tail from sample 26 or position 3 in Figure 4.1. Simulated for Time Scale Factor values of 1900 to 70000 by increments of 100.

Bell Core Line Config uration	Power In Echo Tail As A Percent Of Total Echo	Order 0 to 3							
		0		1		2		3	
		Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB
01	3.03	8500	2.83	27400	8.32	22000	9.36	27600	16.54
02	3.06	8400	2.82	27200	8.35	21800	9.38	27500	16.58
03	3.14	8300	2.72	26600	8.39	21400	9.40	27200	16.62
04	3.11	8300	2.74	26900	8.37	21600	9.39	27400	16.59
05	3.13	8300	2.74	26700	8.38	21400	9.39	27200	16.61
06	3.13	8100	2.60	26400	8.36	21300	9.36	27100	16.56
07	3.10	8300	2.77	26800	8.37	21500	9.38	27300	16.59
08	2.78	7200	1.95	25900	8.06	20900	9.01	26700	16.28
09	3.02	7700	2.33	25800	8.22	20800	9.17	26800	16.35
10	3.17	8000	2.58	26200	8.38	21100	9.37	27000	16.58
11	3.13	8200	2.72	26400	8.37	21300	9.37	27100	16.58
12	3.15	8200	2.67	26300	8.38	21200	9.37	27100	16.59
13	3.17	8100	2.65	26100	8.38	21100	9.36	26900	16.57
14	3.11	7500	2.32	25200	8.29	20400	9.21	26500	16.37
15	3.13	8200	2.71	26400	8.37	21300	9.36	27100	16.58
21	2.85	8000	2.44	27500	8.16	22000	9.20	27500	16.52
22	2.06	7000	1.52	28100	7.35	22600	8.30	27800	16.01
23	2.00	5700	1.11	24800	7.17	20100	7.98	26200	15.22
24	2.96	7800	2.42	26500	8.22	21300	9.20	27100	16.35
25	2.11	6800	1.48	27000	7.37	21700	8.27	27300	15.86
26	2.04	6990	1.45	27300	7.26	22100	8.16	27700	15.79
27	3.10	8300	2.77	26800	8.37	21500	9.38	27300	16.59
28	2.69	7400	1.95	26900	7.90	21600	8.87	27200	16.41
29	2.23	6800	1.48	26400	7.41	21300	8.29	27100	15.84
30	2.49	6400	1.40	25600	7.73	20700	8.63	26400	16.09
31	1.16	3800	0.93	11900	3.89	11000	4.01	16700	8.38
32	2.97	7700	2.38	26000	8.20	21600	9.17	26900	16.34
33	2.81	6300	1.56	24500	7.92	19800	8.80	25900	15.82
34	2.39	5800	1.22	24000	7.62	19500	8.42	25700	15.58
35	3.13	8200	2.71	26400	8.37	21300	9.35	27100	16.58

Table D.7: Legendre Tail Cancellation Results : Order 0,1,2,3. No Balance Network. Echo Tail from sample 26 or position 3 in Figure 4.1. Simulated for Time Scale Factor values of 1000 to 70000 by increments of 100.

Bell Core Line Config uration	Power In Echo Tail As A Percent Of Total Echo	Order 4 to 7							
		4		5		6		7	
		Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB
01	3.03	23100	18.72	20900	22.63	14800	22.85	11700	22.70
02	3.06	23000	18.76	20900	22.68	14600	22.88	11700	22.72
03	3.14	22900	18.79	21000	22.74	14500	22.92	11700	22.75
04	3.11	23000	18.77	20900	22.70	14600	22.89	11700	22.73
05	3.13	22900	18.78	21000	22.73	14600	22.91	11700	22.74
06	3.13	22800	18.72	21000	22.73	14500	22.89	11300	22.70
07	3.10	22900	18.76	21000	22.71	14600	22.90	11700	22.73
08	2.78	22500	18.36	20800	22.26	14200	22.38	11700	22.22
09	3.02	22700	18.47	21100	22.66	14500	22.76	11700	22.57
10	3.17	22800	18.74	21100	22.76	14500	22.91	11500	22.71
11	3.13	22900	18.74	21000	22.74	14500	22.90	11500	22.72
12	3.15	22800	18.75	21100	22.74	14500	22.90	11500	22.72
13	3.17	22800	18.72	21100	22.75	14500	22.89	11400	22.70
14	3.11	22500	18.45	21100	22.69	14500	22.75	11500	22.57
15	3.13	22800	18.74	21000	22.73	14500	22.89	11500	22.71
21	2.85	23000	18.68	20700	22.35	14500	22.55	11500	22.33
22	2.06	23200	17.99	20600	21.09	14300	21.22	23200	21.50
23	2.00	22100	17.09	20100	20.91	14000	20.95	11300	20.82
24	2.96	22800	18.50	20800	22.62	14600	22.78	11700	22.57
25	2.11	22900	17.83	20600	21.22	14000	21.34	23200	21.38
26	2.04	23200	17.77	20900	21.24	14100	21.35	23200	21.38
27	3.10	22900	18.76	21090	22.71	14600	22.90	11700	22.73
28	2.69	22800	18.49	20800	21.41	14100	22.05	11300	21.91
29	2.23	22800	17.82	20800	21.41	14000	21.52	11300	21.36
30	2.40	22300	18.06	20400	21.46	13700	21.59	22400	21.57
31	1.16	15100	9.02	16000	14.20	15300	14.17	15100	14.10
32	2.97	22700	18.46	20900	22.55	14300	22.69	11700	22.50
33	2.81	21900	17.78	20400	21.88	14100	21.98	11300	21.85
34	2.39	21800	17.49	20400	21.64	14000	21.64	11300	21.51
35	3.13	22800	18.74	21000	22.73	14500	22.89	11500	22.71

Table D.8: Legendre Tail Cancellation Results : Order 4,5,6,7. No Balance Network. Echo Tail from sample 26 or position 3 in Figure 4.1. Simulated for Time Scale Factor values of 1000 to 70000 by increments of 100.

Bell Core Line Config uration	Power In Echo Tail As A Percent Of Total Echo	Order 1 to 3							
		0		1		2		3	
		Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB
01	12.17	4500	4.37	19200	10.61	19200	10.61	30300	15.51
02	12.34	4500	4.33	19100	10.67	19100	10.67	30200	15.11
03	12.75	4400	4.16	18800	10.73	18800	10.73	30000	15.21
04	12.58	4400	4.20	19000	10.69	19000	10.69	30100	15.15
05	12.66	4400	4.19	18800	10.73	18800	10.73	30000	15.19
06	12.59	4300	3.97	18800	10.69	18800	10.69	29900	15.16
07	12.51	4400	4.24	18900	10.71	18900	10.71	30100	15.16
08	9.87	3800	2.99	18600	10.16	18600	10.16	29900	14.76
09	11.46	4100	3.55	18400	10.50	18400	10.50	29700	14.94
10	12.86	4300	3.94	18600	10.73	18600	10.73	29800	15.21
11	12.67	4400	4.15	18700	10.73	18700	10.73	29900	15.20
12	12.77	4300	4.08	18700	10.74	18700	10.74	29900	15.21
13	12.88	4300	4.03	18600	10.75	18500	10.75	29800	15.23
14	12.05	4000	3.38	18200	10.60	18200	10.60	29400	15.06
15	12.65	4400	4.13	18700	10.73	18700	10.73	29900	15.19
21	10.83	4300	3.78	19400	10.31	19400	10.31	30500	14.94
22	5.40	3700	2.38	19800	8.89	19800	8.89	32400	13.73
23	5.09	3100	1.75	18000	8.85	18000	8.85	30200	13.29
24	11.33	4200	3.72	18800	10.46	18800	10.46	29900	14.88
25	5.57	3600	2.30	19200	8.97	19200	8.97	31700	13.69
26	5.12	3600	2.25	19400	8.76	19400	8.76	32300	13.45
27	12.51	4400	4.24	18900	10.71	18900	10.71	30100	15.16
28	9.04	3900	3.00	19200	9.80	19200	9.80	30900	14.59
29	6.04	3600	2.29	18900	9.06	18900	9.06	31400	13.73
30	7.33	3400	2.17	18500	9.60	18500	9.60	30200	14.39
31	2.83	2000	1.67	8800	5.96	5900	6.22	19200	9.15
32	11.22	4100	3.64	18600	10.45	18600	10.45	29800	14.89
33	10.16	3300	2.41	17900	10.04	17900	10.04	28900	14.48
34	7.02	3100	18.80	17600	9.54	17600	9.54	29300	14.01
35	12.65	4400	4.13	18700	10.73	18700	10.73	29900	15.19

Table D.9: Laguerre Tail Cancellation Results : Order 0,1,2,3. With Resistive Balance Network, $R_{bal} = 135$ ohms. Echo Tail from sample 26 or position 3 in Figure 4.1. Simulated for Time Scale Factor values of 1000 to 70000 by increments of 100.

Bell Core Line Config uration	Power In Echo Tail As A Percent Of Total Echo	Order 4 to 7							
		4	4	5	5	6	6	7	7
		Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB
01	12.17	30300	15.05	38700	20.53	38600	20.51	41200	25.16
02	12.34	30200	15.11	38700	20.63	38600	20.61	41300	25.29
03	12.75	30000	15.20	38600	20.80	38600	20.78	41600	25.52
04	12.58	30100	15.15	38700	20.70	38600	20.68	41200	25.39
05	12.66	30000	15.19	38700	20.79	38600	20.77	41600	25.50
06	12.59	29900	15.15	38600	20.74	38500	20.73	41600	25.49
07	12.51	30100	15.16	38700	20.73	38600	20.71	41500	25.43
08	9.87	29900	14.76	38400	20.58	38300	20.55	41200	25.30
09	11.46	29700	14.94	38700	20.63	38600	20.61	42000	25.57
10	12.86	29800	15.21	38500	20.84	38500	20.82	41700	25.61
11	12.67	29900	15.20	38600	20.81	38600	20.80	41600	25.55
12	12.77	29900	15.20	38600	20.83	38500	20.81	41600	25.58
13	12.88	29800	15.23	38600	20.88	38500	20.86	41700	25.67
14	12.05	29400	15.05	38400	20.75	38300	20.73	42000	25.68
15	12.65	29900	15.19	38600	20.81	38600	20.79	41600	25.55
21	10.82	30500	14.94	38700	20.65	38600	20.65	40900	25.25
22	5.40	32400	13.73	40000	20.19	39900	20.16	41500	24.52
23	5.09	30200	13.29	38300	19.38	38200	19.36	40400	23.74
24	11.33	29900	14.88	38600	20.41	38500	20.39	41500	25.18
25	5.57	31700	13.69	39600	20.09	39500	20.06	41500	24.59
26	5.11	32300	13.45	40400	19.94	40200	19.90	42200	24.67
27	12.51	30100	15.16	38700	20.73	38600	20.71	41500	25.43
28	9.04	30900	14.59	39000	20.78	38900	20.75	41300	25.31
29	6.04	31400	13.73	39700	20.16	39600	20.13	42000	24.88
30	7.33	30200	14.39	38200	20.63	38200	20.60	40600	24.93
31	2.83	19200	9.15	28100	13.11	28100	13.11	35200	17.17
32	11.22	29800	14.88	38600	20.50	38500	20.48	41600	25.27
33	10.16	28900	14.48	37400	19.99	37400	19.98	40600	24.23
34	7.02	29200	14.01	37900	20.03	37800	20.00	40900	24.70
35	12.65	29900	15.19	38600	20.81	38600	20.79	41600	25.55

Table D.10: Laguerre Tail Cancellation Results : Order 4,5,6,7. With Resistive Balance Network, $R_{bal} = 135$ ohms. Echo Tail from sample 26 or position 3 in Figure 4.1. Simulated for Time Scale Factor values of 1000 to 70000 by increments of 100.

Bell Core Line Config uration	Power In Echo Tail As A Percent Of Total Echo	Order 0 to 3							
		0		1		2		3	
		Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB
01	12.17	9000	4.37	23800	11.75	19200	13.04	24400	19.31
02	12.34	9000	4.33	23600	11.80	19100	13.07	24400	19.28
03	12.75	8800	4.16	23300	11.86	18900	13.09	24200	19.38
04	12.58	8800	4.20	23400	11.82	19000	13.07	24300	19.32
05	12.66	8800	4.19	23300	11.85	18900	13.08	24200	19.37
06	12.59	8600	3.97	23100	11.82	18800	13.03	24100	19.32
07	12.51	8800	4.24	23400	11.83	19000	13.07	24300	19.34
08	9.87	7600	2.99	22700	11.26	18500	12.41	23800	18.96
09	11.46	8100	3.54	22600	11.57	18600	12.69	23900	19.03
10	12.86	8500	3.94	23000	11.85	18800	13.04	24000	19.37
11	12.67	8700	4.15	23100	11.85	18800	13.05	24100	19.36
12	12.77	8700	4.08	23100	11.85	18800	13.04	24100	19.38
13	12.88	8600	4.03	22900	11.86	18700	13.03	24000	19.38
14	12.05	7900	3.38	22200	11.67	18300	12.76	23600	19.11
15	12.65	8700	4.13	23100	11.84	18800	13.04	24100	19.36
21	10.82	8600	3.78	23900	11.46	19200	12.75	24400	19.23
22	5.40	7500	2.38	24200	9.88	19500	10.97	25000	18.17
23	5.09	6100	1.75	21600	9.75	17700	10.66	23400	17.35
24	11.33	8300	3.72	23100	11.58	18800	12.78	24100	18.98
25	5.57	7200	2.30	23400	9.93	19000	10.97	24600	18.03
26	5.12	7300	2.25	23700	9.70	19300	10.70	25000	17.76
27	12.51	8800	4.24	23400	11.83	19000	13.07	24300	19.34
28	9.04	7900	3.00	23400	10.88	19100	12.04	24300	19.01
29	6.04	7200	2.29	23000	10.00	18800	11.00	24400	18.01
30	7.32	6800	2.17	22400	10.64	18300	11.71	23600	18.73
31	2.83	4100	1.67	10700	5.79	4000	6.10	14600	9.75
32	11.22	8200	3.64	22800	11.55	18600	12.70	24000	19.00
33	10.16	6700	2.41	21600	11.10	17700	12.16	22900	18.46
34	7.02	6200	1.88	21100	10.48	17500	11.40	23000	18.03
35	12.65	8700	4.13	23100	11.84	18800	13.04	24100	19.36

Table D.11: Legendre Tail Cancellation Results : Order 0,1,2,3. With Resistive Balance Network, $R_{bal} = 135$ ohms. Echo Tail from sample 26 or position 3 in Figure 4.1. Simulated for Time Scale Factor values of '000 to 100000 by increments of 100.

Bell Core Line Config uration	Power In Echo Tail As A Percent Of Total Echo	Order 4 to 7							
		4	4	5	5	6	6	7	7
		Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB	Time Scale Factor p	Tail Cancel lation dB
01	12.17	21100	21.56	21000	26.50	14800	26.64	11900	26.17
02	12.34	21100	21.63	21100	26.57	14600	26.66	11900	26.17
03	12.75	21000	21.69	21200	26.66	14500	26.58	11500	26.41
04	12.56	21000	21.65	21100	26.60	14500	26.61	11500	26.41
05	12.66	21000	21.68	21200	26.65	14500	26.66	11500	26.40
06	12.59	21000	21.60	21100	26.62	14500	26.50	11900	26.32
07	12.51	21000	21.65	21100	26.63	14500	26.60	11700	26.43
08	9.87	20700	21.17	20900	26.04	14000	25.80	11500	25.71
09	11.46	20900	21.20	21300	26.48	14100	26.10	11900	25.98
10	12.86	21000	21.63	21200	26.63	14200	26.46	11900	26.31
11	12.67	21000	21.64	21200	26.67	14500	26.52	11900	26.35
12	12.77	21000	21.65	21200	26.66	14500	26.49	11900	26.34
13	12.88	21000	21.62	21300	26.67	14100	26.42	11700	26.28
14	12.05	20800	21.29	21300	26.41	14000	25.96	11500	25.90
15	12.65	21000	21.64	21200	26.66	14500	26.51	11900	26.34
21	10.82	21000	21.62	20900	26.26	14200	26.36	11500	26.18
22	5.40	21500	20.44	20800	24.39	13600	24.33	11300	24.21
23	5.09	20300	19.39	20100	23.92	13400	23.73	11000	23.66
24	11.33	20900	21.25	21100	26.39	14500	26.31	11900	26.12
25	5.57	21200	20.24	20800	24.52	13600	24.38	11300	24.28
26	5.12	21500	20.00	21100	24.47	13700	24.25	11500	24.13
27	12.51	21000	21.65	21100	26.63	14500	26.61	11700	26.43
28	9.04	21100	21.31	20900	25.56	13600	25.52	11500	25.42
29	6.04	21200	20.16	21000	24.73	13600	24.51	11300	24.44
30	7.33	20600	20.88	20600	24.87	13000	24.81	11000	24.79
31	2.83	13800	10.00	15400	13.61	7900	14.52	7500	14.86
32	11.22	20900	21.21	21100	26.31	14300	26.12	11900	25.97
33	10.16	20100	20.47	20400	25.07	13600	24.96	11300	24.93
34	7.02	20100	19.99	20400	24.86	13200	24.52	11000	24.52
35	12.65	21000	21.63	21200	26.66	14500	26.51	11900	26.34

Table D.12: Legendre Tail Cancellation Results : Order 4,5,6,7. With Resistive Balance Network, $R_{bal} = 135$ ohms. Echo Tail from sample 26 or position 3 in Figure 4.1. Simulated for Time Scale Factor values of 1000 to 100000 by increments of 100

D.2 Eigenvalues

D.2.1 Laguerre Functions

Laguerre Eigenvalues as determined from the Laguerre Functions. The eight phases as shown order 0 to 3 and 4 to 7.

$$\begin{pmatrix} 0.008 \\ 0.008 \\ 0.008 \\ 0.007 \\ 0.007 \\ 0.009 \\ 0.001 \\ 16.681 \end{pmatrix} \begin{pmatrix} 0.0077 \\ 0.0077 \\ 0.0076 \\ 0.0074 \\ 0.0071 \\ 0.0088 \\ 0.0005 \\ 7.9020 \end{pmatrix} \begin{pmatrix} 0.0077 \\ 0.0077 \\ 0.0076 \\ 0.0074 \\ 0.0071 \\ 0.0085 \\ 0.0004 \\ 3.6791 \end{pmatrix} \begin{pmatrix} 0.0077 \\ 0.0077 \\ 0.0076 \\ 0.0073 \\ 0.0080 \\ 0.0071 \\ 0.0003 \\ 1.5986 \end{pmatrix} * 1.0e + 07 \quad (D.1)$$

$$\begin{pmatrix} 0.0771 \\ 0.0773 \\ 0.0762 \\ 0.0759 \\ 0.0702 \\ 0.0713 \\ 0.0031 \\ 6.3329 \end{pmatrix} \begin{pmatrix} 0.0773 \\ 0.0770 \\ 0.0762 \\ 0.0750 \\ 0.0713 \\ 0.0629 \\ 0.0035 \\ 2.2097 \end{pmatrix} \begin{pmatrix} 0.0773 \\ 0.0769 \\ 0.0762 \\ 0.0747 \\ 0.0714 \\ 0.0508 \\ 0.0049 \\ 0.6568 \end{pmatrix} \begin{pmatrix} 0.0769 \\ 0.0773 \\ 0.0762 \\ 0.0745 \\ 0.0716 \\ 0.0315 \\ 0.0092 \\ 0.1928 \end{pmatrix} * 1.0e + 06$$

D.2.2 Legendre Functions

Legendre Eigenvalues as determined from the Legendre Functions. The eight phases as shown order 0 to 3 and 4 to 7.

$$\begin{pmatrix} 0.0755 \\ 0.0731 \\ 0.0781 \\ 0.0709 \\ 0.0603 \\ 0.0002 \\ 0.1439 \\ 36640.6 \end{pmatrix} \begin{pmatrix} 0.0754 \\ 0.0730 \\ 0.0780 \\ 0.0707 \\ 0.0574 \\ 0.1345 \\ 0.0000 \\ 12220.3 \end{pmatrix} \begin{pmatrix} 0.0753 \\ 0.0729 \\ 0.0780 \\ 0.0707 \\ 0.0557 \\ 0.1048 \\ 0.0000 \\ 3775.1 \end{pmatrix} \begin{pmatrix} 0.0750 \\ 0.0776 \\ 0.0726 \\ 0.0707 \\ 0.0799 \\ 0.0501 \\ 0.0000 \\ 1054.2 \end{pmatrix} * 1.0e + 06 \quad (D.2)$$

$$\begin{pmatrix} 0.0709 \\ 0.0708 \\ 0.0734 \\ 0.0756 \\ 0.0781 \\ 0.0340 \\ 0.0000 \\ 255.82 \end{pmatrix} \begin{pmatrix} 0.0756 \\ 0.0739 \\ 0.0780 \\ 0.0722 \\ 0.0704 \\ 0.0183 \\ 0.0000 \\ 50.149 \end{pmatrix} \begin{pmatrix} 0.0726 \\ 0.0751 \\ 0.0774 \\ 0.0785 \\ 0.0701 \\ 0.0066 \\ 0.0000 \\ 6.7875 \end{pmatrix} \begin{pmatrix} 0.0751 \\ 0.0725 \\ 0.0778 \\ 0.0693 \\ 0.0836 \\ 0.0005 \\ 0.0053 \\ 4.7183 \end{pmatrix} * 1.0e + 06$$

3 of/de 3

MicroD

Appendix E

Discrete Equivalent Of The Laguerre Orthonormal System

E.1 Gram-Schmitt Orthogonalization Derivation

Start with

$$L_0(t) = e^{-pt} \rightarrow L_0(kT) = e^{-kpT}. \quad (\text{E.1})$$

Let

$$L_1(t) = (t - \alpha_1) L_0(t) \rightarrow L_1(kT) = (kT - \alpha_1) e^{-kpT}. \quad (\text{E.2})$$

$$\sum_{k=0}^{\infty} (kT - \alpha_1) L_0(kT)^2 = 0 \quad (\text{E.3})$$

So

$$\alpha_1 = \frac{\sum_{k=0}^{\infty} kT L_0(kT)^2}{\sum_{k=0}^{\infty} L_0(kT)^2} \quad (\text{E.4})$$

$$L_2(t) = (t - \alpha_2) L_1(t) - \beta_2 L_0(kT) \quad (\text{E.5})$$

$$\sum_{k=0}^{\infty} (kT - \alpha_2) L_1(kT)^2 = 0 \quad (\text{E.6})$$

Therefore

$$\alpha_2 = \frac{\sum_{k=0}^{\infty} kT L_1(kT)^2}{\sum_{k=0}^{\infty} L_1(kT)^2} \quad (\text{E.7})$$

and

$$\sum_{k=0}^{\infty} \beta_2 L_0(kT)^2 - \sum_{k=0}^{\infty} kT L_0(kT) L_1(kT) = 0. \quad (\text{E.8})$$

Therefore

$$\beta_2 = \frac{\sum_{k=0}^{\infty} kT L_0(kT) L_1(kT)}{\sum_{k=0}^{\infty} L_0(kT)^2} \quad (\text{E.9})$$

Similarly this can be extended to $L_n(t)$

$$L_0(kT) = e^{-kpT}. \quad (\text{E.10})$$

$$L_1(kT) = (kT - \alpha_1) e^{-kpT}. \quad (\text{E.11})$$

$$L_2(kT) = (kT - \alpha_2)(kT - \alpha_1) e^{-kpT} - \beta_2 e^{-kpT}. \quad (\text{E.12})$$

down to $L_n(kT)$

$$L_n(kT) = (kT - \alpha_n) L_{n-1}(kT) - \beta_n L_{n-2}(kT). \quad (\text{E.13})$$

The z domain equivalent is

$$\mathcal{Z} \{ \{ L_0(kT) \} \} = \frac{1}{1 - e^{-pT} z^{-1}}. \quad (\text{E.14})$$

$$\mathcal{Z} \{ \{ L_1(kT) \} \} = T \sum_{k=0}^{\infty} \left(k + 1 - \frac{\alpha_1}{T} \right) e^{-kpT} z^{-k} - T \sum_{k=0}^{\infty} e^{-kpT} z^{-k} \quad (\text{E.15})$$

$$\mathcal{Z} \{ \{ L_1(kT) \} \} = T \frac{1}{(1 - e^{-pT} z^{-1})^2} - T \frac{\left(\frac{\alpha_1}{T} + 1 \right)}{(1 - e^{-pT} z^{-1})}. \quad (\text{E.16})$$

$$\mathcal{Z} \{ \{ L_1(kT) \} \} = T \frac{1 - \left(\frac{21}{T} + 1 \right) (1 - e^{-pT} z^{-1})}{(1 - e^{-pT} z^{-1})^2}. \quad (\text{E.17})$$

etc.

The following identities are needed to obtain this proof

$$\frac{1}{1 - e^{-pT} z^{-1}} = 1 + e^{-pT} z^{-1} + e^{-2pT} z^{-2} + \dots \quad (\text{E.18})$$

$$\left(\frac{1}{1 - e^{-pT} z^{-1}} \right)^2 = 1 + 2e^{-pT} z^{-1} + 3e^{-2pT} z^{-2} + \dots \quad (\text{E.19})$$

$$= \sum_{k=0}^{\infty} (k+1) e^{-kpT} z^{-k} \quad (\text{E.20})$$

$$\left(\frac{1}{1 - e^{-pT} z^{-1}} \right)^3 = 1 + 3e^{-pT} z^{-1} + 6e^{-2pT} z^{-2} + 10e^{-3pT} z^{-3} + \dots \quad (\text{E.21})$$

$$= \sum_{k=0}^{\infty} \frac{(k+2)(k+1)}{2} e^{-kpT} z^{-k} \quad (\text{E.22})$$

END

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