The FEM Analysis of a Heat Pipe

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Abstract

To the author's knowledge the first fully three-dimensional model and analysis of a heat pipe is presented. The model uses a Finite Element Method.

For the operation of a heat pipe, a mathematical model is established by composition of a sequence of physical processes. These processes are a CD thermal model, an interface model, a Darcy flow model and a potential flow model. Corresponding to each mathematical model, one computational solver is implemented in software. Data in-flows and out-flows connect the various mathematical models.

The liquid-vapor interface model is described in detail. Several test problems were solved to test the correctness of the interface solver.

The algorithm of how to solve the heat pipe problem is described. The temperature field in the heat and the velocity pressure field in the wick and in the vapor core of the heat pipe are computed by the solvers and the numerical results are discussed.

To my parents, for their undying support.

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Nomenclature

x, y, z	Cartesian coordinate system.
Ω	Physical domain.
P	Pressure.
ΔP	Pressure difference.
∇P	Pressure gradient.
σ	Surface tension.
α	Contact angle.
R	Principal radius of curvature of the surface.
r	Radius of the pore.
$\mathbf{v}$	Velocity.
K	Permeability of the porous solid.
μ	Kinematic viscosity.
ρ	Density.
k_e	Effective thermal conductivity.
T	Temperature.
k	Thermal conductivity.
q''	Heat flux.
h	Convection coefficient.
h_{fg}	Latent heat.

μ	Viscosity of liquid.
g	Acceleration due to gravity.

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Chapter 1

Introduction

1.1 Background

1.1.1 History of the Heat Pipe

The heat pipe is a device which transfers heat by vaporizing a fluid at one end and condensing it at the other end. So it would be more accurate to call heat pipes heat transfer pipes. However, the original name has already had wide circulation so that it is evidently advisable to preserve it.

The thermosyphon, also named Perkins pipes, was invented by Perkins in 1897 [10]. It was the predecessor of the heat pipe. The thermosyphon is an unwicked conductor in which the heat transfer is achieved through the latent heat of evaporation. But it is different from heat pipes in that the return of liquid to the evaporator is due

to gravity. The liquid vaporizes in the lower region of the cylinder. The vapor goes up and condenses to liquid in the upper region. The liquid is returned by gravity to the boiler or evaporator. Unwicked tubes were used long before the appearance of heat pipes in the construction industry. However, the use of capillary forces in vapor-liquid heat conductors was certainly the important step in their development.

In 1942, Gaugler proposed the idea of constructing a heat conductor in which the heat transfer was achieved using the evaporation and condensation of a working medium [2]. In this device liquid was returned to the evaporator by capillary flow in a porous wick. Gaugler wanted to use this heat conductor in refrigeration engineering [10]. Distinct from the thermosyphon, there is a wick along the wall of the cylinder. The liquid flows from the condenser to the evaporator through the wick. The returning forces are capillary forces instead of gravity. So it is not necessary for the evaporator to be beneath the condenser.

However, this invention did not find an application in technology for more than 20 years. In 1963, Grover and his co-workers, working at Los Alamos, re-invented this concept [10]. Grover's objective was to use these devices in spacecraft, where the effect of gravitational force is negligible. In that case, the return force of the working liquid can not be gravity. The capillary force would be a good force for returning the working fluid. Grover demonstrated the effectiveness of heat pipes as a high-performance heat transmission device and developed several applications. So

the development of high - performance space power systems brought about a rebirth of heat pipes.

Figure 1.1: A Thermosyphon (From [6])

1.1.2 Basic Principle of the Heat Pipe

Heat pipes have a very high thermal conductivity. In fact, heat pipes with liquid metal heat transfer media can have an effective thermal conductivity a thousand times greater than that of the best metallic thermal conductors [10]. Even low-temperature pipes, with heat transfer media of relatively low thermal conductivity and latent heat of the evaporation, can have a thermal resistance tens of times less than the best metallic conductors [10].

Figure 1.2: A Heat Pipe (From [6])

In principle these heat conductors can be used over a very wide temperature range, from low temperature (starting from 1K) to high temperatures (2500K-3000K) [10]. The optimal working fluid is chosen for a given working temperature range: liquefied gases, organic liquids and low-boiling metals. In general each fluid is optimal over only a limited temperature range.

The simplest type of heat pipe is illustrated in Fig. 1.2. It is a sealed case. The inner surface of the case is covered with a layer of the porous material, the wick. Various porous materials can act as the wick (gauzes or sintered porous structures). The wick is saturated with the liquid phase of the working fluid. The liquid flows through the wick from the condensing to evaporating sections under capillary forces.

The basic principle of operation of heat pipes is discussed as follows.

1. Heat is introduced into the pipe as a result of conduction through its casing and through parts of the wick to the working fluid. So the working fluid in the evaporation section will evaporate. The heat flux is transferred from liquid to vapor through the mass flux.
2. The vapor is transferred from the evaporation section to the condensation section under a pressure difference. The heat flux is transferred by the vapor flux.
3. The vapor will condense in the condensation section releasing its latent heat of vaporization.
4. The liquid is transferred from the condensation section to the evaporation section under the action of capillary forces.

The difference between heat pipes and thermosyphons is the force to return the liquid. The thermosyphon uses gravity to return the liquid from the condenser to the evaporator while the heat pipe uses capillary force. So it is important to discuss the capillary force. Fig. 1.3 shows the location of the interface between the liquid phase and the vapor phase for operational conditions.

Under the action of surface tension forces, a capillary pressure leads to a pressure drop ΔP which drives the working fluid from the condenser to the evaporator along the wick.

Figure 1.3: Liquid-Vapor Interface of Heat Pipe (From [6])

Where

$$\Delta P = 2\sigma \left(\frac{\cos\alpha_e}{R_e} \right) \quad (1.1)$$

where ΔP is the pressure difference created by capillary forces

σ is the surface tension

α is the contact angle

R is the principal radius of curvature of the surface of the meniscus.

The subscript e refers to the properties in the evaporator and c refers to the properties in the condenser.

So the capillary pump is used in the heat pipe to achieve the closed circulation of the working fluid.

1.2 Aims and Organization of the Thesis

1.2.1 Aims of the Thesis

This study focuses on the mathematical modeling of heat pipes. Although the heat pipe technology appeared in the early forties, some operational aspects of heat pipe systems are still not well understood and research in this area continues. So this model of a heat pipe can help us analyze the process of operation of a heat pipe and predict the behavior of a heat pipe.

In the operation of a heat pipe, there are several different processes. Corresponding to each of those processes, there are several different mathematical sub-models. For the thermal analysis of the casing and wick in the heat pipe, there is a Convection-Diffusion thermal model. For the liquid flow in the wick, there is a Darcy flow model. For the vapor flow in the vapor part, there is a Potential flow model. Between the liquid in the wick and vapor, the Interface model is designed.

The Interface model is developed to calculate the mass and heat transfer between the liquid-vapor interface due to changes in phase. In this model, the velocity of the liquid, the velocity of the vapor, the mass transfer rate between the liquid-vapor

interface and other parameters can be calculated. This model can be used not only the liquid-vapor interface in a heat pipe but also for the liquid-vapor interface in other devices.

Corresponding to each mathematical sub-model, one computational solver is implemented in software. So the solvers for heat pipe include the Convection-Diffusion thermal solver, the Darcy flow solver, the Potential solver and the Interface solver.

This thesis will focus on the follow aims:

1. Make a mathematical model of the liquid-vapor interface and test its correctness.
2. Use the Convection-Diffusion thermal solver, the Darcy flow solver, the Potential solver and the Interface solver to simulate the operation of a heat pipe.

1.2.2 Organization of the Thesis

This thesis describes how to solve the heat pipe problem using the Finite Element Method.

Chapter 1 is the introduction. This chapter describes a heat pipe and briefly outlines the history and the advantages of heat pipes. This chapter also presents the aims and organization of the thesis.

Chapter 2 presents the basic principles and theory of heat pipes. In this chapter,

the surface tension and surface energy, pressure difference and pressure recovery are presented.

Chapter 3 describes the theory of the Finite Element Method thermal analysis. This chapter presents how to use Finite Element Method to solve the thermal problems.

Chapter 4 present a mathematical model of a heat pipe, which include the Convection-Diffusion thermal model, the Darcy flow model, the Potential flow and the Interface model.

Chapter 5 describes several test problems of the Interface model. The evaporation problem, the Stefan problem and the sucking problem are introduced and solved. The results of these test problems are discussed.

Chapter 6 describes how to use the solvers to solve the heat pipe problem. In this chapter, the mesh of a heat pipe, the materials in a heat pipe and the solver strategy are presented. The results of analyzing a heat pipe problem are discussed.

Chapter 7 presents the conclusions of the thesis.

Chapter 2

Theory of Heat Pipes

This chapter will discuss the basic theory of heat pipes.

2.1 Pressure Balance

During the operation of a heat pipe, the vapor flows from the evaporator section to the condenser section and returns to the evaporator in a liquid phase. As the vapor flows from the evaporator to the condenser, there will be a vapor pressure gradient along the vapor flow. There exists also a pressure gradient when the condensed liquid flows back from the condenser to the evaporator. To operate the heat pipe, there must be another pressure to balance those two pressure gradients. In heat pipes, this

balance pressure is capillary pressure. The equilibrium condition is expressed by

$$\Delta P_v + \Delta P_l + \Delta P_c = 0. \quad (2.1)$$

Where

ΔP_v is the pressure drop causing the vapor to flow from the evaporator to the condenser.

ΔP_l is the pressure drop to return the liquid from the condenser to the evaporator through the wick.

ΔP_c is the capillary pressure across the liquid-vapor interface in the condenser section.

2.2 Surface Tension

In a liquid, molecules attract each other. A molecule in a liquid is attracted by the other molecules around it. The resultant force on a molecule will be in an equilibrium state because the attraction force is occurred in all directions. If the molecule is at or near the surface of the liquid, the resultant force of attraction will no longer balance. In this case, the liquid will tend to keep a shape having a minium area. The shape having a minimum area will be a sphere in vacuum.

There is gravity in nature. So the shape will not actually be a sphere. In order to increase the surface area, the work to balance the force of gravity will need to be done on the liquid. This energy is called free surface energy σ . The force associated

with surface energy is surface tension.

2.3 Capillary Pressure

The pressure on a concave surface is less than that on the convex surface due to the surface tension. This pressure difference is related to the surface tension and to the radius of curvature of the surface. The relation between them is

$$\Delta P = \frac{2\sigma}{R} \quad (2.2)$$

Where

ΔP is the pressure difference across the interface.

σ is the surface tension.

R is the radius of curvature of the surface.

For the pores in the wick, the value of the radius of curvature of the surface R can be determined by the equation

$$R = \frac{r}{\cos\theta} \quad (2.3)$$

Where

r is the radius of the pore.

Figure 2.1: Sketch of Meniscus in Cylindrical Pore

θ is the contact angle.

R is the surface tension.

Substitute Equation 2.3 into Equation 2.2. The expression for the capillary pressure for a cylindrical pore will be

$$\Delta P = \frac{2\sigma \cos\theta}{r} \quad (2.4)$$

For the capillary pressure at the evaporator

$$\Delta P'_e = \frac{2\sigma \cos\theta_e}{r_e} \quad (2.5)$$

Similarly for the condenser, it is

$$\Delta P'_c = \frac{2\sigma \cos\theta_c}{r_c} \quad (2.6)$$

So the resultant capillary pressure will be

$$\Delta P_c = 2\sigma \left(\frac{\cos\theta_e}{r_e} - \frac{\cos\theta_c}{r_c} \right) \quad (2.7)$$

The maximum value of capillary pressure will be obtained when $\cos\theta_e = 1$ and $\cos\theta_c = 0$, that is $\theta_e = 0$ and $\theta_c = \pi/2$.

In that case, Equation 2.7 will be reduced to

$$(\Delta P_c)_{max} = \frac{2\sigma}{r_e} \quad (2.8)$$

2.4 Liquid Pressure Drop

The pressure drop in the liquid flowing in the wick is due to two terms: the frictional drag and dynamic effects. Since the liquid velocity in the wick is generally very low, the pressure drop due the dynamic effect is often neglected. Then the liquid pressure gradient for steady state flow of liquid in the wick is due to the fractional drag.

There are several theories to calculate the pressure drop: the Hagen-Poiseuille equation, the Darcy's equation and the Blake-Kozeny equation [2]. In this thesis, we only discuss Darcy's equation. Darcy's equation describes the relation between the pressure drop ΔP and the velocity field $\mathbf{v}$ when the liquid flows through a porous media. Darcy's equation is written as

$$\mathbf{v} = -\frac{K}{\mu} \nabla P \quad (2.9)$$

Where

$\mathbf{v}$ is the velocity of the liquid in the porous media.

K is the permeability of the porous solid.

μ is the kinematic viscosity of the liquid.

$\nabla P = \frac{\Delta P}{L}$ is the pressure gradient.

ΔP is the pressure drop over a length L .

We rearrange the equation to the form

$$\Delta P = -\frac{L K}{\mu} \mathbf{v} \quad (2.10)$$

So if we know the velocity of the liquid, we can compute the pressure gradient ∇P in the liquid.

2.5 Vapor Pressure Drop

The vapor pressure drop is also due to two terms: frictional drag and dynamic effects. Since the vapor velocity is generally very high, the dynamic pressure drop can not be neglected in this process. In this case, the frictional pressure drop can be neglected because it is very small.

The dynamic pressure drop can be calculated by

$$\Delta P = -\frac{1}{2}\rho(\mathbf{v}^2 - (\mathbf{v}_{stagnation})^2) = -\frac{1}{2}\rho\mathbf{v}^2 \quad (2.11)$$

Where

ΔP is the pressure drop.

ρ is the density of the vapor.

$\mathbf{v}$ is vapor velocity.

$\mathbf{v}_{stagnation}$ is the vapor velocity in the stagnation state. It equals zero.

2.6 Effective Thermal Conductivity of the Saturated Wick Material

Heat conduction in the liquid-saturated wick takes place in two locations in the operation of the heat pipe. In the evaporator section, the heat is conducted across

the casing wall into the liquid-saturated wick to the liquid-vapor interface. In the condenser section, the heat generated by the condensation of vapor on the liquid-saturated wick flows partly by conduction to the casing wall and then to the heat sink. So it is important to calculate the effective thermal conductivity of the wick structure.

Several models are available to calculate the effective thermal conductivity [1]. The simplest form is

$$k_e = \varepsilon k_l + (1 - \varepsilon)k_w \quad (2.12)$$

Where

k_e is the effective thermal conductivity of the liquid-saturated wick.

ε is the volume fraction of liquid, i.e., the volume of the liquid divided by the total volume of the liquid-saturated wick. It is called porosity.

k_l is the thermal conductivity of the liquid.

k_w is the thermal conductivity of the wick material.

Chapter 3

Finite Element Theory for Thermal Analysis

Heat transfer theory aims to predict the energy transfer taking place between material bodies. If there is a temperature difference between material bodies, the heat energy will flow from the body with a high temperature to the body with a low temperature. The science of heat transfer tries to explain how heat energy may be transferred and tries to predict the heat transfer rate and temperature field. The objective of heat transfer theory is different from thermodynamics. Thermodynamics deals with systems in equilibrium and it may be used to predict how much energy is required to change the system from one equilibrium state to another equilibrium state. Heat transfer theory is used to predict how fast a change will occur when the system is not

in equilibrium state during the process.

Exact analytical solutions of the equations of heat transfer can only be obtained for problems which have relatively simple geometry, material properties and boundary conditions. But practical problems often have complex geometries, material properties and boundary conditions. Then there is no option but to seek a numerical method to analyze practical problems. The Finite Element Method is one kind numerical method that can be used to obtain an approximate solution of differential equations for practical problems.

There are three modes of heat transfer: conduction, convection and radiation. In this chapter we will explain the mechanism of these modes and give the basic equations. In addition, the theory of the Finite Element Method will be introduced to analyze the thermal problems.

3.1 Heat Transfer Modes

3.1.1 Conduction Heat Transfer

Conduction is energy transfer across a medium due to a temperature gradient. The physical mechanism of conduction is that of random atomic or molecular activity. In the presence of a temperature gradient, the energy is transferred from molecules with high energy to those with lower energy. So the energy transfer occurs in direction of

decreasing temperature. We may also speak of the transfer of energy by molecular motion as diffusion of energy [4]. Conductive heat transfer can occur in a gas or liquid with no bulk motion, or in solids. So we can define Fourier's law as a conduction rate equation.

$$q''_{cond} = -k\nabla T \quad (3.1)$$

Where:

T is the temperature,

k is the thermal conductivity, which is an important property of the material. ($W/m K$),

q''_{cond} is the heat flux (W/m^2). It is a vector of the same dimension as gradient of temperature.

If we describe the heat flux as a vector quantity, and thermal conductivity is constant in the (x, y, z) directions, the more general conduction rate equation (Fourier's law) is:

$$q''_{cond} = -k\nabla T = -k\left(i\frac{\partial T}{\partial x} + j\frac{\partial T}{\partial y} + k\frac{\partial T}{\partial z}\right) \quad (3.2)$$

Where:

$T(x, y, z)$ is the scalar temperature field.

∇ is the gradient operator and is defined as follows:

$$\nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \quad (3.3)$$

3.1.2 Convection Heat Transfer

Convection is heat transfer between a solid surface and an adjacent moving medium, which is usually a fluid [5]. There are essentially two kinds of mechanism of heat transfer in convection heat transfer. One kind is diffusion in which energy is transferred by random molecular motion. The other kind is advection in which energy is transferred by bulk fluid motion. In nature, both phenomena occur simultaneously.

Convection heat transfer occurs between the surface of a solid and the fluid which may be moving relative to the solid surface. This region is called a boundary layer. There are three different boundary layers: velocity boundary layer, thermal boundary layer and concentration boundary layer. The velocity boundary layer profile begins at the solid surface where the velocity of the liquid is zero. The gradient of velocity gradually increases until the velocity profile is a uniform value, the ambient velocity value u_{∞} . Moreover, the thermal boundary layer occurs in a similar fashion. This kind of layer begins at the solid surface where the temperature is T_s and ends at the outer flow where the temperature is T_{∞} , which is the ambient temperature. Between the two temperatures will be a varying gradient. The concentration boundary layer will develop from the surface, where the concentration of species A is $C_{A,s}$, to the free

stream, where the concentration of species A is $C_{A,\infty}$. A concentration gradient will exist in this region.

There are two main types of convection heat transfer: free convection and forced convection. Free convection occurs in a flow which is induced by buoyancy forces. Buoyancy forces arise from the density difference due to the gradient of temperature. Forced convection occurs when the flow is caused by the external source, such as a fan or pump.

The convection heat transfer rate equation can be expressed as Newton's law if assuming $T_s > T_\infty$.

$$q''_{conv} = h(T_s - T_\infty) \quad (3.4)$$

Where:

q''_{conv} is the convective heat flux (W/m^2),

h is the convection coefficient, ($W/m^2 K$),

T_s is the temperature of the surface,

T_∞ is the temperature of the ambient fluid.

3.1.3 Radiation Heat Transfer

When two objects at different temperature are separated by a finite distance, the energy will be emitted from the high temperature object to the low temperature object

even if there is no media between the two objects. The radiation can be emitted from solid, liquid or gas but primarily we focus on the radiation emitted from a solid. The transport of energy by radiation occurs by electromagnetic waves. The radiation is critical in high temperature or in vacuum such as space-bound applications. But at low temperatures, the heat transferred by radiation often is negligible compared to the heat transferred by conduction and convection. In a vacuum, the only mode of heat transfer is radiation.

The heat transfer rate by radiation to and from a given surface is a complex function. But if we define a black body which absorbs all incident radiant energy, the emission rate is given by Stefan-Boltzmann law:

$$q''_{rad,b} = \sigma T^4 \quad (3.5)$$

Where:

$q''_{rad,b}$ is the emission rate of a black surface (W/m^2),

T is the absolute temperature in Kelvin (K),

σ is the Stefan-Boltzmann constant, whose value is

$$\sigma = 5.669 \times 10^{-8} W/m^2 K^4 \quad (3.6)$$

For a real surface, the energy emitted is lower than that from a black surface at the same temperature. The expression for the emission rate for the actual surface is

$$q''_{rad} = \epsilon \sigma T^4 \quad (3.7)$$

Where:

q''_{rad} is the emission rate of actual surface (W/m^2),

ϵ is the hemispheric emissivity and it depend on the type of the material and surface condition. The value of ϵ is between zero and one. If the body is a black body, then $\epsilon = 1$, which is an ideal case. If the body is a grey body, the value ϵ is between 0 and 1.

3.2 Finite Element Method (FEM)

3.2.1 Basic Theory of FEM

FEM is a numerical technique for solving differential equations of physical problems. The basic idea of FEM is that a continuum domain can be subdivided into an assembly of elements. The approximate solution is computed within the local elements and is then be assembled for the global geometry. There are seven basic steps to the solution of FEM problems.

1. Discretize the domain.

In this step, the continuum domain is divided into a number of discrete elements.

The element shape and mesh design is based on engineering judgment and interpolation functions.

2. Choose suitable interpolation functions.

In this step, we can choose the interpolation functions to approximate the field within a local element. A linear, parabolic or higher order polynomial can be chosen for the interpolation functions. Sufficient nodes are assigned to the element to represent the polynomial.

3. Analyze the element properties.

In this step, the relationship between the field $\{x^e\}$ and the flow $\{F^e\}$ is found for the element properties. This relationship is represented by the following function.

$$\{F^e\} = [K^e]\{x^e\} \quad (3.8)$$

where the superscript “e” indicates the element stiffness matrix, $[K^e]$ and element load vector $\{F^e\}$. This matrix is a function of the material properties, element geometry and the interpolation function.

4. Assemble the relationship

The relationship of each element can be assembled to a global relationship. This relationship is

$$\{F\} = [K]\{x\} \quad (3.9)$$

Where the $[K]$ is the global stiffness matrix, which is assembled from the local stiffness matrix.

$\{x\}$ is the value of the field variable for all the nodes.

$\{F\}$ is value of the flow variable at each node.

The assembly process is carried out by the rules of physics of the problem. They could be based on compatibility theory or on continuity theory for different problems.

5. Apply the Boundary Conditions and Initial Conditions.

The boundary conditions are applied to the global equations in this step. There are two kind of Boundary Conditions: essential and natural. For time dependent or transient problems, initial conditions must be assigned.

6. Solve the global equations.

The global equations are solved after the appropriate Boundary Conditions are applied. The numerical solution is obtained when the solution is computed.

7. Post-processing.

The results can be presented with any format after the solution is obtained.

This is called post-processing. In this step, the results can be re-arranged to a efficient and convenient format.

3.2.2 Differential Equation of Heat Conduction

The differential form of the heat conduction equation is

$$\begin{aligned} \frac{\partial}{\partial x} \left(k_x \frac{\partial T(r, t)}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial T(r, t)}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial T(r, t)}{\partial z} \right) + q'''(r, t) \\ = \rho C_p \frac{\partial T(r, t)}{\partial t} \end{aligned} \quad (3.10)$$

Where:

$q'''(r, t)$ is the rate energy generation per unit volume, per unit time at point $r(x, y, z)$ at time t ,

$T(r, t)$ is the temperature at point $r(x, y, z)$ at time t ,

r is the point in three dimensions (x, y, z) .

If the conductivity is constant in (x, y, z) , then Equation 3.10 will be reduced to

$$k \left(\frac{\partial^2 T(r, t)}{\partial x^2} + \frac{\partial^2 T(r, t)}{\partial y^2} + \frac{\partial^2 T(r, t)}{\partial z^2} \right) + q'''(r, t) = \rho C_p \frac{\partial T(r, t)}{\partial t} \quad (3.11)$$

If we use the operator ∇ in this equation, it becomes

$$k\nabla^2 T(r, t) + q'''(r, t) = \rho C_p \frac{\partial T(r, t)}{\partial t} \quad (3.12)$$

The following integral of this equation times a space of test functions ϕ over a volume v is the integral form equation of heat conduction.

$$\int_v \phi \left[k\nabla^2 T(r, t) + q'''(r, t) - \rho C_p \frac{\partial T(r, t)}{\partial t} \right] dv = 0 \quad (3.13)$$

Separate the two ∇ and substitute Equation 3.1 into Equation 3.13,

$$\int_v \left[-\nabla \phi \cdot q_{cond}(r, t) + \phi q'''(r, t) - \phi \rho C_p \frac{\partial T(r, t)}{\partial t} \right] dv = 0 \quad (3.14)$$

The Equation 3.14 is the differential form of the energy equation. The FEM software will calculate the heat transfer using this equation.

Chapter 4

Computational Model of a Heat Pipe

4.1 Convection-Diffusion Thermal Model for Casing and Wick

4.1.1 Convection-Diffusion Thermal Energy Equation

For the wick and casing of the heat pipe, we will solve the Convection-Diffusion energy equation. The Eulerian form of the Convection-Diffusion equation is

$$\rho \frac{\partial h}{\partial t} + v \cdot \nabla(\rho h) + \nabla \cdot q + \dot{Q} = 0 \quad (4.1)$$

$$q = -k\nabla T \quad (4.2)$$

where:

h is the enthalpy,

$\frac{\partial h}{\partial t} + v \cdot \nabla h$ is the rate of change of the specific enthalpy of a material point,

$\nabla \cdot q$ is the energy diffused in or out of a control volume,

k is the thermal conductivity,

$\dot{Q}$ is the rate at which energy is generated per unit volume of the medium.

If the fluid is viscous, heat will be generated by viscous dissipation. This heat could be represented as $\sigma_{i,j} : \dot{\epsilon}_{i,j}$.

where:

$\sigma_{i,j}$ is the stress tensor,

$\dot{\epsilon}_{i,j}$ is the strain rate tensor.

If there is no energy generated in the medium, which means no viscous flow and no chemical reactions in the medium, then $\dot{Q}$ will be zero. Thus, Equation 4.1 will be reduced to

$$\rho \frac{\partial h}{\partial t} + v \cdot \nabla(\rho h) + \nabla \cdot q = 0 \quad (4.3)$$

4.1.2 Boundary Conditions

There are four boundary conditions applied on the Convection-Diffusion thermal model in the case and the wick.

Flux Neumann BC between the heat pipe and the heat source

Neumann BC flux is applied between the heat pipe and the heat source. The value of the flux is defined by the power of the heat transfer rate q , and the area between the heat pipe and the heat source.

$$q'' = q/A \quad (4.4)$$

Convection BC between the heat pipe and the heat sink

A convection BC is applied between the heat pipe and the heat sink. There are two parameters in the convection BC. One is the ambient temperature T_∞ and the other is the convection coefficient h

Dirichlet BC between the wick and interface

A temperature Dirichlet BC is set between the wick and interface for each iteration. The value is changed dynamically in each iteration and how this value is computed

will be discussed in Chapter 6.

Zero Flux Neumann Boundary Conditions

All other boundary conditions are zero flux Neumann boundary conditions .

4.1.3 Initial Conditions

The initial temperature of all parts in the heat pipe is set to a same value. This value can be the same as the ambient temperature or be a different value from the ambient temperature.

The velocity of all material points are set to zero.

In the first time step, the velocity of fluid in the wick is set to zero. In the following time steps, the velocity of fluid in the wick is set to the value computed by the Darcy flow solver.

4.2 Interface Model between the Wick and the Vapor

4.2.1 Interface Model Equations

1. We obtain the value of the thermal flux q_L from the liquid side of the interface from the Convection-Diffusion thermal solver in the wick. The mass flux at the

interface can be calculated as:

$$\dot{m} = \frac{-k\nabla T}{h_{fg}} = \frac{q_L}{h_{fg}} \quad (4.5)$$

Where

k is the thermal conductivity.

h_{fg} is the latent heat.

q_L is the heat flux transfered from the liquid.

We assume the heat flux from the interface to the vapor q_v is zero because the temperature gradient in the vapor is very small.

2. The Interface solver computes the liquid velocity $\mathbf{v}_L$ and vapor velocity $\mathbf{v}_G$ at the interface.

$$\mathbf{v}_L = \frac{\dot{m} \cdot \mathbf{n}}{\rho_L} + \mathbf{v}_I \quad (4.6)$$

$$\mathbf{v}_G = \frac{\dot{m} \cdot \mathbf{n}}{\rho_G} + \mathbf{v}_I \quad (4.7)$$

Where $\mathbf{v}_I$ is the velocity of the interface.

The above equations are found by rearranging terms of the following equation for conservation of mass across the interface.

$$\dot{m} = \rho_L (\mathbf{v}_L - \mathbf{v}_I) \cdot \mathbf{n} = \rho_G (\mathbf{v}_G - \mathbf{v}_I) \cdot \mathbf{n} \quad (4.8)$$

This Interface model can be used in both the evaporator and the condenser sections.

4.2.2 Boundary Conditions

The velocity of the interface is set to zero. This means the liquid - vapor interface in this model does not move.

4.2.3 Initial Conditions

The thermal flux from the liquid side of the interface is set as an initial condition. The value of thermal flux is obtained by a data flow out from the Convection-Diffusion thermal solver to the Interface solver.

4.3 Darcy Flow Model in the Wick

4.3.1 Equations of Darcy Flow Model

The conservation of momentum for incompressible fluid flow with negligible inertial forces in a porous media is given by Darcy's equation;

$$\rho \mathbf{v} = -\frac{\rho K}{\mu} (\nabla p + \rho g \nabla \eta) \quad (4.9)$$

$$\rho \nabla \cdot \mathbf{v} = 0 \quad (4.10)$$

$$\nabla \cdot \left(-\frac{\rho K}{\mu} (\nabla p + \rho g \nabla \eta) \right) = 0. \quad (4.11)$$

Where

$\mathbf{v}$ is the liquid velocity.

K is the permeability of the wick material.

μ is the viscosity of liquid.

p is the pressure.

g is the acceleration due to gravity.

η is the elevation of the interface.

In this heat pipe model we neglected the effect of gravity.

4.3.2 Boundary Conditions

A Dirichlet pressure boundary condition is prescribed on at least one point on the interface between the wick region and the vapor region.

The boundary condition between the porous region and the heat pipe casing and the end caps is zero flow.

On the interface between the wick and the vapor region, the liquid velocity normal to the interface is prescribed. This is a Neumann boundary condition. The value of this BC is computed by the Interface solver. In the evaporator section, the direction of the liquid velocity is from the liquid to vapor. In the condenser section, the direction of the liquid velocity is from the vapor to liquid.

4.3.3 Initial Conditions

In the first step, the velocity of the fluid in the wick is set to zero. Since we are solving for the steady-state solution, no initial condition is required by the mathematics. However, in the software it is convenient to define an initial condition of zero pressure and zero velocity.

4.4 Potential Flow Model in the Vapor Region

4.4.1 Equations of Potential Flow Model

For inviscid, irrotational fluid flow a potential function Φ exists such that the velocity is the gradient of this potential function $\nabla\Phi = v$. If the flow is steady state then $D\Phi/Dt = 0$. If gravity is neglected and the flow is incompressible, then the divergence of velocity is zero, i.e.,

$$\nabla \cdot \nabla\Phi = 0. \tag{4.12}$$

This is Laplace's equation.

4.4.2 Boundary Conditions

Laplace's equation requires that all points on the boundary have either a Dirichlet boundary condition that prescribes the potential or a Neumann boundary condition that prescribes the velocity. The velocity of vapor can be computed by solving the equation for potential flow with either pressure or velocity prescribed on the interface between the porous region and the vapor region.

We apply velocity (Neumann) boundary on the vapor-liquid interface because it conserves mass exactly. In addition, we apply a Dirichlet BC, i.e., a value of the potential Φ , to one node on the vapor-liquid interface to constrain the null space of the potential. Any value can be chosen because velocity is the gradient of Φ , the velocity is independent of the value chosen.

4.4.3 Initial Conditions

Laplace's equation has no initial conditions. However, because of the way the software is written it is convenient to assign the initial velocity to zero and the initial pressure to the stagnation pressure.

In Section 6.1, we will discuss how to use the above solvers to solve the heat pipe model.

Chapter 5

Interface Test and Results

The liquid-vapor phase change process plays a very important role in many technological applications. The latent heat associated with boiling and condensation can be very high. The Interface solver is designed for this kind of problem. The Interface solver developed in this work can be used not only in heat pipes but also in other technological applications.

For the numerical modeling of heat pipes, it is important to build a liquid-vapor Interface solver. Given the saturation pressure in the vapor and the liquid velocity and thermal gradient in the liquid, this solver can compute the velocity of the vapor and mass flux at the liquid-vapor interface.

In this chapter, we will discuss how to use the liquid-vapor solver and discuss some test results.

5.1 Introduction to MmoFramework

The MmoFramework is a software package designed to perform efficient analysis of mechanical engineering applications that involve heat transfer, stress analysis and fluid flow using the Finite Element Method (FEM). This software is designed by Prof. Goldak and his research group. In this chapter, we will discuss how to build a model that analyzes the operation of a heat pipe using this software.

There are three main modules in the MmoFramework. They are the meshing module, analysis module and visualization module. Using the meshing module, we can create an FEM mesh for the geometry of the object which we want to analyze. In the analysis module, the software can simulate the thermal, stress or fluid flow processes. The visualization module can post-process the result files to be visualized on a computer.

The first step to use this software is to prepare a set of design files. The design files describe the problem and the procedure to solve this problem. The MmoFramework can follow the instructions in the design files to compute the solution.

The material properties are described in the material library, which is a set of particular design files. The MmoFramework will get the material properties from the material library using the name of the material.

5.1.1 Design Files

A design file is a text file that contains the instructions for the MmoFramework to solve a specific problem. The design files have the information needed to create the mesh geometry, the various solvers, the solver strategy, the solver procedure, the visualization strategy and so on. The MmoFramework fetches the information and parameters of mesh and solver from the design files. Then the MmoFramework runs the various solvers following the solving strategy that is defined in design files.

The design files are organized in a parent-child hierarchy. In one project, there must be at least one parent design file. The parent design file defines path to the child design files and some global information. There are zero or several child design files in one project. In the child design files, the data of different analysis tasks can be structured for different solvers. In the parent or a child design file, the data in the design files are grouped in sections.

A section is defined with the following syntax.

@The Section Name

Parameter Value

...

Parameter Value

@ENDOFSECTION

5.1.2 Material Library

The material library is a set of design files that define the properties of a set of material types such as copper, water, steel. In the material library, the density, enthalpy, latent heat, specific heat, thermal conductivity, viscosity, yield stress and so on are defined. In the solver design files, the material name is assigned to each part. Then the MmoFramework can fetch the material properties from the material library for that part. This makes it very convenient to run the same solver strategy with different materials. If we want to run the solver with another material in one part, the only thing to be changed is the material name which is assigned to the part.

There are three representations of properties that can be defined in the material library. These are constant value, table value and polynomial value.

constant value

For some property of the material, it can have only one value. So we can define this property to have a constant value.

The syntax to define a constant value property is:

MaterialPropertyFileName

value

table value

For some complex properties of the material, a table of values will be used. The table lists the range of the independent variable and the value of the dependent variable, e.g., thermal conductivity as a function of temperature or specific enthalpy. The MmoFramework will get the value of a property in a table using interpolation. The independent variable is usually the temperature, but it also can be other properties, such as enthalpy.

We can define a table of values with the following syntax.

```
temp1 value1  
temp2 value2  
...  
tempn valuen
```

polynomial value

If we want a more accurate value of the material properties, a piecewise polynomial representation can be used. In this case, the material property is defined by a set of polynomial functions of an independent variable that is separated by the range of the independent variable, which usually is temperature. An example of a function is shown in Equation 5.1.

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n \quad (5.1)$$

This type of property can be defined as follows.

StartTempearture₁ EndTempearture₁ $a_{0,1}$ $a_{1,1}$ $a_{2,1}$... $a_{n,1}$

StartTemperare₂ EndTemperare₂ $a_{0,2}$ $a_{1,2}$ $a_{2,2}$... $a_{n,2}$

StartTemperare_m EndTemperare_m $a_{0,m}$ $a_{1,m}$ $a_{2,m}$... $a_{n,m}$

5.1.3 Meshing Module

The first step is to run the MmoFramework to generate the mesh that defines the geometry of the object to be analyzed. The information for the MmoFramework to mesh the geometry is defined in the meshing design files. There are several basic kinds of geometry which can be defined in the MmoFramework. There are primitives to define hexahedron, prism, cylinder, sphere and so on. We can use such geometric primitives or a manageable combination of such primitives to define the geometry of parts. There is also a method to make primitives mesh objects in which one can define all the nodes and elements in one file. Then, the MmoFramework can generate the special mesh for this geometry.

A mesh also can be imported into the MmoFramework. In addition, the MmoFramework can import stereolithographic files (STL) files which are usually generated by

CAD software. Then these STL files can be meshed.

5.1.4 Analysis Module

The core of the MmoFramework is the analysis module. In this module, each partial differential equation is discretized and transformed to a set of algebraic equations or ordinary differential equations and then solved. This process is controlled by the data in the design files. The solution is intended to be an accurate prediction of the behavior of the problem.

In one particular project, one or more different solvers can be used in predict the behavior of the problem. There is a list which can define several solvers in the parent design file. The MmoFramework can run the solvers one by one according the solver names in the solver list. Various global parameters are also defined in the parent design file.

For each solver, there is a child design file for it to define the initial conditions, boundary conditions and other parameters. The MmoFramework fetches the initial conditions, boundary conditions and other parameters into memory. Then the MmoFramework makes the set of equations to be solved and solves them. A log file reports progress and any prints any error messages that arise. If the equations converge, the MmoFramework will write the results in result files on disk.

5.1.5 Visualizer Module

When the results of the problem are finished by the analysis module, we can use the visualizer module to generate the visualization files which can be viewed using the visualization software. The visualizer can generate the visualization files which include the geometry of the parts and results of the different field variables such as temperature, pressure, velocity, etc.

There are three kinds of visualization files that can be generated that correspond to two kinds of visualization software. One kind of them is the VTK files, which is used with the VTK visualizer. We can visualize the 3D results in color by part, vector arrow or animation. The other kind of visualization files that can be generated is EnSight files, which is used with the EnSight software. In this software, we can visualize the 3D results in color by part, contour, isosurface, particle trace, vector arrow, animation and so on.

The other kind of visualization files can be generated is gnuplot files, which is used by the gnuplot software.

5.2 Evaporation Test Problem

5.2.1 Description of Evaporation Problem

This test problem follows Juric and Tryggvason [13]. It is simply a horizontal tank, partly filled with liquid, open at the right. A thermal flux is applied to the left of the tank. The velocity in the liquid is assumed to be zero. A temperature gradient that is constant in space and time develops in the liquid. Because the right of the tank is open, the pressure and hence temperature and enthalpy of the vapor are constant in space and time.

The solution for the evaporation test problem is a constant velocity of the vapor and of the liquid-vapor interface. The solution for the interface velocity is an immediate consequence of solving the interface equation with zero temperature gradient in the vapor and the specified thermal flux in the liquid and the specified pressure on the interface.

5.2.2 Numerical Procedure for Solving the Evaporation Test Problem

The numerical procedure for solving the evaporation test problem is described as follows:

1. The given vapor pressure determines the saturation temperature of the vapor,

Figure 5.1: Definition of Domain and Boundary Conditions of the Evaporation Test Problem

vapor density and the latent heat of evaporation.

2. The thermal flux applied to the bottom of the liquid together with the requirement that the liquid velocity be zero, defines the mass flux from the interface.

$$\dot{m} = \frac{-\kappa_G \nabla T_G}{h_{fg}} \quad (5.2)$$

3. The mass flux together with the requirement that the liquid velocity be zero,

defines the interface velocity.

$$\mathbf{v}_I = \left(\mathbf{v}_L - \frac{\dot{m}}{\rho_L} \right) \cdot \mathbf{n} = -\frac{\dot{m}}{\rho_L} \cdot \mathbf{n} \quad (5.3)$$

4. The mass flux and density defines the vapor velocity.

$$\mathbf{v}_G = \left(\frac{\dot{m}}{\rho_G} + \mathbf{v}_I \right) \cdot \mathbf{n} \quad (5.4)$$

5.2.3 Result of Evaporation Problem

We make a three-element mesh to simulate the evaporation problem. The mesh is show in Fig. 5.2. The size of the mesh is $0.1 \times y \times 0.1m$. The y is a variable because the interface position is moving. The heat flux applied on the liquid is $10W$. The temperature of the vapor is $293K$.

Figure 5.2: Mesh for Evaporation Problem

Fig. 5.3 shows the vapor velocity as the function of interface position. Fig. 5.4 shows the interface velocity as the function of interface position. From the results we can see the vapor velocity and liquid-vapor interface velocity are constant.

Figure 5.3: Vapor Velocity for Evaporation Problem

Figure 5.4: Interface Velocity for Evaporation Problem

5.3 Stefan Test Problem

5.3.1 Description and Discussion of Stefan Problem

This test problem follows Wilson and Welch [15]. Gravity is neglected. The domain is simply a horizontal tank, partly filled with liquid, closed at the right end by a movable piston. A Dirichlet temperature is applied to the left end of the tank. The vapor is assumed to be incompressible. The piston is moved to constrain the pressure in the vapor to be a constant value. A temperature gradient develops in the vapor that creates a spatial gradient in density and specific volume. The velocity in the vapor is assumed to be zero and therefore the pressure gradient is zero. The velocity of the interface is not equal to the velocity of the liquid. The velocity in the liquid is constant in space but varies with time.

From the perfect gas law $p = \rho RT$, if pressure is constant, then ρT must also be constant and equal to p/R . Thus the temperature gradient in the vapor does not induce a pressure gradient in the vapor.

Because it is assumed that $\nabla T_L = 0$, the thermal flux in the liquid is zero. Therefore at the interface $-\kappa_G \nabla T_G = \dot{m} h_{fg}$. Because the thermal flux at the vapor-liquid interface from vapor to liquid is known and denoting the volume of vapor $\hat{V}$, we have the following equations.

Figure 5.5: Definition of Domain and Boundary Conditions of the Stefan Problem

$$\rho_{ave} = \frac{p}{RT_{ave}}$$

$$p = RT_{ave} \rho_{G,ave}$$

$$\rho_{ave} = \frac{M + \dot{m}t}{\hat{V} + \hat{V}t} = \frac{M + \dot{m}t}{\hat{V} + V_I t}$$

$$\mathbf{v}_I = \frac{1}{t} \left(\frac{M + \dot{m}t}{\rho_{ave}} - \hat{V} \right)$$

$$\mathbf{v}_L = V_I + \frac{\dot{m}}{\rho_L}.$$

The solution is obtained by first solving the energy equation in the vapor region.

Actually, one could simply assume that the temperature gradient in the vapor region is the Dirichlet wall temperature T_w minus the T_{sat} divided by the length of the vapor region L_G . One might assume that the pressure in the vapor region is constant. Then, we solve the interface equation for the interface velocity. Then, we solve the potential flow in the liquid region or simply assign the liquid velocity computed by the Interface solver to the liquid.

If we solve the transient problem, then the temperature gradient in the vapor decreases with time as the vapor region expands.

Treated as a transient problem but ignoring acoustic or pressure wave effects, we have to solve the coupled conservation of energy and mass and the equation of state. We solve the following steps for each iterative cycle at each time step:

1. Solving the energy equation everywhere changes temperature and specific enthalpy.
2. Solve the interface equation to compute mass flow rate at the interface due to evaporation.
3. The vapor mass flow rate changes the volume of vapor and the total mass and total enthalpy of the vapor.
4. The mass flow rate of the vapor determines the interface velocity. The interface velocity determines the liquid velocity.

5.3.2 Numerical Procedure for Solving the Interface Equations for Stefan Problem

The procedure for solving the interface equation given in the previous section can be summarized in the following steps.

1. The vapor wall temperature is constant for all time.
2. The liquid temperature is constant for all time and equal to the saturation temperature .
3. The interface temperature is fixed to the liquid temperature for all time.
4. The temperature in the vapor varies linearly from the wall to the interface.
5. The mass flux on the interface is given by

$$\dot{m} = \frac{-k\nabla T_G}{h_{fg}} \quad (5.5)$$

6. The interface velocity is given by the

$$\mathbf{v}_I = -\frac{\dot{m}}{\rho_G} + \mathbf{v}_G \cdot \mathbf{n} \quad (5.6)$$

7. The liquid velocity is a function of $\dot{m}$ and the interface velocity.

$$\mathbf{v}_L = \left(\frac{\dot{m}}{\rho_L} + \mathbf{v}_I \right) \mathbf{n} \quad (5.7)$$

8. The interface position as a function of time is obtained by integrating the interface velocity $\mathbf{v}_I$ as a function of time.
9. The interface position $x_{\text{interface}}$ determines the temperature gradient in the vapor

$$\nabla T_G = \frac{T_{\text{wall}} - T_{\text{interface}}}{-(x_{\text{wall}} - x_{\text{interface}})}.$$

5.3.3 Analytical Solution of Stefan Problem

Known:

In the liquid, $T = T_{\text{sat}}, T = T_{\text{sat}}, v_l(t), x = \delta$.

At the interface, $v_s(t), T_{\text{sat}}$.

In the vapor, $v_v = 0, y = 0, T_w$

That is:

$$\begin{aligned} \rho_v C_{pv} \frac{\partial T}{\partial t} &= k_v \frac{\partial^2 T}{\partial y^2} \\ T &= T_{\text{sat}}, \quad x \geq \delta \\ T &= T_w, \quad x = 0 \\ \rho_v \left(\frac{\partial \delta}{\partial t} \right) h_{fg} &= -k_v \frac{\partial T}{\partial x}, \quad x = \delta \end{aligned}$$

Solution:

$$v_s(t) = \frac{\partial \delta}{\partial t} = \frac{k_v}{\rho_v h_{fg}} (T_w - T_{sat}) \frac{2z}{\sqrt{\pi} \operatorname{erf}(z)} \left(\frac{1}{\delta \exp(z^2)} \right)$$

where

$$2e^{z^2} \operatorname{erf}(z) = \frac{c_{pv} (T_w - T_{sat})}{h_{fg}} \frac{1}{\sqrt{\pi}}$$

$$\delta(t) = 2z \sqrt{\left(\frac{k_v}{\rho_v c_{pv}} \right) t}$$

$$T(x, t) = T_w + \frac{T_{sat} - T_w}{\operatorname{erf}(z)} \operatorname{erf} \left(\frac{y}{2 \sqrt{\left(\frac{k_v}{\rho_v c_{pv}} \right) t}} \right)$$

$$v_l = \left(\frac{\partial \delta}{\partial t} \right) \left(1 - \frac{\rho_v}{\rho_l} \right)$$

5.3.4 Result of Stefan Problem

We make a three-element mesh to simulate the Stefan problem. The mesh is shown in Fig. 5.2. The size of the mesh is $0.1 \times y \times 0.1m$. The y is a variable because the interface position is moving.

The parameters of the Stefan problems is:

$$T_{wall} = 125^{\circ}C$$

$$T_{sat} = 100^{\circ}C$$

$$v_g = 0$$

Fig. 5.5 shows the liquid velocity as the function of interface position. The curve in the figure is the analytical solution of Stefan problem. From the results, we can see that the simulation results are in excellent agreement with the analytic solution.

Figure 5.6: Liquid Velocity of Stefan Problem

Liquid Velocity of Stefan Problem is shown vs. interface distance from the wall.

The line represents the analytic solution. The * represent values from the numerical solution.

Chapter 6

Heat Pipe Mathematical Model and its Results

6.1 Meshes and Solvers for the Mathematical Model of Heat Pipes

6.1.1 Description of Heat Pipe Meshes

Before we analyze a heat pipe, the mesh that defines the geometry of the heat pipe must be prepared. The element type chosen for this mesh is an 8-node brick. The size of elements in the mesh depends on how accurate a result is expected.

In FEM analysis, we can mesh the geometry of different parts according to the

material type of each part. Each part is defined by its own part type and part number with an associated material type. In one part, there can be one or several different sections. Each section can have a part number and the part number must be unique. Part numbers can be used to analyze different sections of one particular part in detail. Different boundary conditions and initial conditions can be set for different part numbers.

For the heat pipe mesh, there are five parts in the mesh. These parts are the heat source, casing, wick, liquid-vapor interface, vapor, and heat sink. A graph of the mesh is shown in Fig. (6.1) and Fig. (6.2). The part number and the part type of this mesh is shown in Table (6.1).

Heat Source Part of the Heat Pipe

The heat source is a fake part for the solvers and no fields are solved in it. The purpose of adding this fake part to the whole geometry is to define the boundary condition for the casing adjacent to the heat source. The part number of the heat source is 801.

Casing Part of Heat Pipes

There are several sections in the casing of heat pipe. Part 601 is the connection part between the heat source and the other casing parts. Part 603 is the connection part between the heat sink and the other casing part. If the heat source or heat sink is joined with the other casing part by glue, the materials of part 601 and 603 are glue. If the heat source or heat sink is joined with other casing part by welding, the material of part 601 and 603 are metal. In this case, we choose the material copper for this part. Part 701 and 703 are the two end caps of the heat pipe. Part 501, 502 and 503 are the casing of the heat pipe. We choose the material copper for the end caps and casing.

Wick Part of Heat Pipes

In general, the wick structure consists of a porous material and the working fluid. So the properties of this part must be a combination of properties of the porous material

and working fluid. In Section 2.6, we have discussed how to compute a effective thermal conductivity for the wick part. For other properties, we can use the similar equations as Equation 2.11.

The part numbers of the wick structure are defined as 401, 402 and 403.

Liquid-Vapor Interface Part of Heat Pipes

Because the interface solver must have a mesh to solve, we make a very thin interface mesh between the liquid and the vapor. The part numbers of interface meshes are defined as 301, 302 and 303. The thickness of the interface must not be negative. The material of the interface is the same as the working fluid, which is water in our heat pipe.

Vapor Part of Heat Pipes

The part numbers of the vapor part are defined 201, 202 and 203. The material of the vapor is also the working fluid but the phase is vapor. The MMO material library has functions that define properties of both the liquid and vapor for a specific material. So the solver can get the property of vapor automatically because the temperature of vapor is higher than the saturation temperature.

Heat Sink Part of Heat Pipes

The heat sink is a fake part for the solvers and no fields are solved on the heat pipe. The purpose of meshing this part is to define the boundary conditions with other parts.

6.1.2 Geometric Parameters of Heat Pipe

The geometric parameters of heat pipe are listed in Table (6.2).

6.1.3 Numerical Procedure of the Solvers of Heat Pipes

Composed Solver of Heat Pipe

Four different solvers are composed to solve the heat pipe project. The important thing is to connect the four solvers into one global solver to get the results. The following steps describe the sequence in which the solvers are called and the inputs and outputs of each solver.

1. Initialize the initial conditions and boundary conditions of each solver. The velocity of liquid in the wick is set to zero and set a value for the Dirichlet temperature at the liquid-vapor interface.
2. Solve the CD thermal energy solver in the domain of the casing and the wick. The heat flux from wick to interface will be obtained from this solver.

3. The domain of the interface solver is the liquid-vapor interface. It solves the mass flux between the liquid and vapor, the velocity of liquid at the interface, and the velocity of vapor at the interface will be obtained from this solver. The scale factor, which can be used to force the mass flux of evaporation to be equal the mass flux of condensation, will also be computed by this solver.
4. Solve the Darcy flow solver in the wick domain. This solver computes the pressure and the velocity of liquid in the wick.
5. Solve the potential solver in the vapor domain to compute the velocity of vapor, the pressure of vapor and the temperature of vapor.
6. Go back to Step 2 for the next time step.

The data flow chart of this composed solver is shown in Fig. 6.3. This chart shows the data flows that connect the different solvers.

Constraint Equation of Heat Pipe

For steady-state operation of the heat pipe, the evaporating mass flow rate must equal the condensing mass flow rate according the mass conservation equation. This equation can be applied as a constraint equation by the interface solver.

$$\dot{M}_{eva} = \dot{M}_{con} \quad (6.1)$$

Where $\dot{M}_{eva}$ is the mass flux through the interface of evaporator. $\dot{M}_{con}$ is the mass flux through the interface of condenser. Note that nodes are classified as evaporating or condensing nodes by the direction of the mass flux vector computed in the interface solver.

We define a mass flux factor f_{scale} .

$$f_{scale} = \frac{\dot{M}_{con}}{\dot{M}_{eva}} \quad (6.2)$$

From the Equation 6.1 and 6.2, the mass flux factor must equal 1 for the steady-state operation of the heat pipe.

$$f_{scale} = 1 \quad (6.3)$$

So for steady-state operation of the heat pipe, the mass scale factor must converge to 1 . We can use this constraint equation to justify the Dirichlet boundary condition of temperature at the liquid-vapor interface. We can use the following steps to choose the exact solution of Dirichlet temperature T_{if} .

1. Select a value of Dirichlet temperature T_{if} .
2. Solve the composed solver to obtain the mass flux factor f_{scale} .
3. If the mass flux factor f_{scale} converges to a value smaller than 1. Set a positive value to ΔT .

4. If the mass flux factor f_{scale} converges to a value bigger than 1. Set a negative to value ΔT .
5. Set the new value of T_{if} equal to the old value T_{if} plus ΔT , go back Step 1.
6. Repeat this procedure until the mass scale factor converges to the range $[1 - \text{tolerance}, 1 + \text{tolerance}]$.

The data flow for this procedure is shown in Fig. 6.4.

Part Number	Part Type	Material Type
801	Heat Source	Fake Material
601	Casing-Grease	Copper
603	Casing-Grease	Copper
501	Casing	Copper
502	Casing	Copper
503	Casing	Copper
701	Casing	Copper
703	Casing	Copper
401	Wick	Water
402	Wick	Water
403	Wick	Water
301	Interface	Water
302	Interface	Water
303	Interface	Water
201	Vapor	Water
202	Vapor	Water
203	Vapor	Water
803	Heat Sink	Fake Material

Table 6.1: Heat Pipe Part Types and Part Numbers.

Figure 6.1: Mesh and Part Numbering of the Heat Pipe

The first digit of the part number indicates the part type and the last digit of part number indicates the section. For the last digit, a value of 1 means evaporator section. 2 means adiabatic section and 3 means condenser section.

Figure 6.2: Cross-section of the Mesh of the Heat Pipe

Geometric Parameters	Values (m)
Vapor Core Outside Radius	0.01004
Liquid-Vapor Interface Thickness	0.00001
Wick Thickness	0.00100
Wall Thickness	0.00165
Pipe Length	0.3
Evaporator Length	0.1
Condenser Length	0.1
End-Cap Casing Length	0.01
Heat Source and Sink Length	0.1
Heat Source and Sink Width	0.04
Heat Source and Sink Thickness	0.01

Table 6.2: Geometric Parameters of Heat Pipe.

Figure 6.3: Data Flow Chart for the Solvers in the Heat Pipe Model

Figure 6.4: Data Flow for the Constraint Equation of Heat Pipe

6.2 Material Properties of the Copper and Water

6.2.1 Wick Properties

Wick permeability = 1.8e-11

Wick porosity = 0.625

6.2.2 Fluid Properties for Water (in SI units)

Define $T_f = T/100$

Saturation pressure P_{sat} (Pa)

$$P_{sat} = \exp(-56.346 + 50.859T_f - 16.094T_f^2 + 2.7634T_f^3 - 0.24744T_f^4 + 0.0090907T_f^5)(6.4)$$

Liquid density ρ_L (kg/m^3)

$$\rho_L = 6028.1 - 6460.4T_f + 3269.9T_f^2 - 810.81T_f^3 + 98.069T_f^4 - 4.6678T_f^5 \quad (6.5)$$

Liquid viscosity μ_L ($N \cdot m$)/ s^2

$$\mu_L = \exp(50.668 - 57.958T_f + 23.688T_f^2 - 4.9807T_f^3 + 0.53004T_f^4 - 0.022662T_f^5)(6.6)$$

Vapor density ρ_V (kg/m^3)

$$\rho_V = \exp(-68.62 + 52.894T_f - 17.525T_f^2 + 3.1666T_f^3 - 0.30147T_f^4 + 0.011981T_f^5)(6.7)$$

Vapor viscosity μ_V ($N \cdot m$)/ s^2

$$\begin{aligned} \mu_V = & -0.00052621 + 0.000657797T_f - 0.00032142T_f^2 + 0.000077779T_f^3 \\ & -0.0000092707T_f^4 + 0.00000043559T_f^5 \quad (6.8) \end{aligned}$$

Latent heat of Vaporization h_{fg} (J/kg)

$$h_{fg} = 19664000 - 20880000T_f + 10134000T_f^2 - 2445700T_f^3 + 291010T_f^4 - 13754T_f^5(6.9)$$

6.3 Results and Analysis for the Mathematical Model of Heat Pipes

In this section, we will list the result of the mathematical model of the heat pipes. It is difficult to verify the results of whole heat pipe because this is no analytic solution for whole heat pipes. But we can verify some critical point of heat pipe because we have 1D model of the heat pipe. So in this section, we will discuss the results of the mathematical model and compare the results of mathematical model to 1D model.

6.3.1 Dirichlet Temperature of Interface for the Steady State

Use the algorithm described in first section, The numerical solution will be obtained. At time $t = 0$, all of the parts in the heat pipe are at a uniform temperature $T_{in} = 300K$. At time $t > 0$, a heat flux $q_c = 25W$ is applied to the casing from the heat source, and the evaporation process begins. At the evaporator section, heat is transfered from the casing to wick and to the liquid-vapor interface. So evaporation will take place at the liquid-vapor interface. Then the vapor will flow from the evaporator section to the condenser section. At the condenser section, the vapor is at a temperature higher than its saturation temperature. So condensation of the vapor to the wick will occur. The liquid will return from the condenser section to the evaporator section through the wick driven by capillary forces. The heat pipe

gradually reaches a steady state under these working conditions.

The other parameters are:

wick porosity = 0.594.

wick permeability = $4.0859 \times 10^{-11} (1/m)$.

wick effective thermal conductivity = $1.4316 (W/mK)$, This parameter is computed by Equation 2.12.

Ambient temperature at condenser section = $300K$.

Convection coefficient = $2000 (W/m^2K)$.

For steady state operation of the heat pipe, the mass scale factor f_{scale} must converge to 1. We can use this constraint equation to justify the Dirichlet boundary condition of temperature in the liquid-vapor interface. This problem is solved for several different Dirichlet temperatures at the liquid-vapor interface. From the results in Fig. 6.5, we can see that the scale factor will converge to 1 when $T_{if} = 304.3K$.

6.3.2 CD Thermal Results

From the CD thermal solver, we can obtain the temperature of the casing and wick, Fig. 6.6 shows the temperature gradient of the casing and wick. The figure shows that the part of the casing that is in contact with the heat source has the highest temperature and the part of the casing that is in contact with the heat sink has the lowest temperature. The temperature decreases from the evaporator section to the

Figure 6.5: Scale Factor for Different Dirichlet Temperature at Liquid-vapor Interface

The scale factor is negative because the direction of mass flux in the evaporation section is oppose to the direction of mass flux in the evaporation section condenser section.

Fig. 6.7 shows the temperature in the $x - z$ plane at $y = 0.01$, which is in the evaporator section. There is a small temperature jump between the casing and the wick because there is a contact conductance between these two parts.

6.3.3 Interface Solver Results

The interface solver computes the velocity of vapor and liquid across the interface. Fig. 6.8 shows the 3D liquid velocity vectors at the liquid-vapor interface. Fig. 6.9 shows the value of liquid velocity in the line $x = -0.1005, 0 < y < 0.3, z = 0$ and the line $x = 0.0, 0 < y < 0.3, z = -0.1005$.

Fig. 6.10 shows the 3D vapor velocity vectors at the liquid-vapor interface. Fig. 6.11 shows the value of vapor velocity in the line $x = -0.1005, 0 < y < 0.3, z = 0$ and the line $x = 0.0, 0 < y < 0.3, z = -0.1005$. The profile of the vapor velocity is similar to the profile of liquid velocity. But the value of vapor velocity is higher than the value of liquid velocity.

At the evaporation section, the liquid velocity through the interface is from the wick to the vapor. The liquid velocity through the interface has the highest value at the left side which connects the end-caps and has the lowest value at the right side which connect the adiabatic section.

At the condenser section, the liquid velocity through the interface is from the vapor to the wick, which is opposite to the evaporation section. The liquid velocity through the interface has the highest value at the right side which connects the end-caps and has the lowest value at the left side which connects the adiabatic section.

In the adiabatic section, the liquid velocity through the interface is zero.

Note the evaporator, condenser and adiabatic sections in the mesh refer to the outside section of the heat pipe when the heat is applied. Inside of a heat pipe, whether a node in the interface is evaporating or condensing or adiabatic is determined by the solution quite independently of whether the node is in the evaporator, condenser or adiabatic sections.

For the 1D model,

$$\dot{m}'' = \frac{q''}{h_{fg}} = \frac{q}{Area \cdot h_{fg}} \quad (6.10)$$

So the velocity can be calculated by

$$\bar{v} = \frac{\dot{m}''}{\rho} = \frac{q}{Area \cdot h_{fg} \cdot \rho} \quad (6.11)$$

Using this equation, we can obtain $\bar{v}_{l,1D} = 1.7 \times 10^{-6} m/s$, From Fig. 6.9, we can compute $\bar{v}_{l,num} \approx 1.65 \times 10^{-6} m/s$. The error is

$$err = (1.65 - 1.7)/1.7 = 2.9\% \quad (6.12)$$

In the same way, we can obtain $\bar{v}_{v,1D} = 0.103 m/s$, From Fig. 6.9, we can compute $\bar{v}_{v,num} \approx 0.9 m/s$. The error is

$$err = (0.103 - 0.09)/0.103 = 12.6\% \quad (6.13)$$

6.3.4 Darcy Solver Results

For the Darcy flow solver, the liquid velocity in the wick is computed. Fig. 6.12 shows the 3D liquid velocity vector in the wick. The liquid flow is from the condenser section to the evaporator section. The liquid velocity increases from the condenser section to the adiabatic section and attains its maximum value at the adiabatic section. The liquid velocity decreases from the adiabatic section to the evaporator section.

Fig. 6.13 shows the liquid velocity vector in the wick in the $x - y$ plane at $z = 0$. In this figure, the y direction is enlarged ten times from its original size. Fig. 6.14 magnifies part of figure of Fig. 6.13 in the evaporator section. From these two figures, we can see the detailed flow of liquid in the wick.

Use Equation 6.11, we can obtain the velocity of Darcy flow in the middle of the adiabatic section $\bar{v}_{Darcy,1D} = 1.545 \times 10^{-4} m/s$, From Fig. 6.12, we can see $\bar{v}_{Darcy,num} \approx 1.4 \times 10^{-4} m/s$. The error is

$$err = (1.545 - 1.4)/1.545 = 9.3\% \quad (6.14)$$

Fig. 6.15 shows the liquid pressure in the wick. The liquid pressure has its highest value in the condenser section and decreases following the liquid flow. In the evaporator section, the liquid pressure has its lowest value.

6.3.5 Potential Solver Results

For the potential solver, the vapor velocity, the vapor pressure and the vapor temperature are computed. Fig. 6.16 shows the flow line of vapor velocity in the vapor part. Fig. 6.17 shows the flow line of vapor velocity in the $x - y$ plane at $z = 0$. Fig. 6.18 shows the vapor velocity vector in the $x - y$ plane at $z = 0$. In all of these three figures, the velocity and the mesh in the y direction has been magnified by a factor of ten.

Use Equation 6.11, we can obtain the velocity of potential flow in the middle of the adiabatic section $\bar{v}_{potential,1D} = 1.859m/s$, From Fig. 6.12, we can compute $\bar{v}_{potential,num} \approx 1.78m/s$. The error is

$$err = (1.859 - 1.78)/1.859 = 5\% \quad (6.15)$$

At the evaporation section, the vapor flux flows from the wick part to the vapor part. Then the vapor flux flows from the evaporation section to the adiabatic section. The vapor flow will enter the condenser section. In the condenser section, the vapor condenses to the wick so that the vapor flux flows from the vapor part to the wick part.

Fig. 6.19 shows vapor pressure and vapor pressure drop in the vapor part. The pressure difference ΔP is defined by

$$\Delta P = P - P_{ref} \quad (6.16)$$

Where:

P is the pressure in the vapor.

P_{ref} is the stagnation pressure.

From Fig. 6.19, we can see the vapor pressure and pressure recovery. The pressure drop in the vapor part consists of two terms: the inertial term and viscous forces term. Because the viscous force term was neglected in our model, the pressure change is only due to the inertial term. The pressure recovery should be a full recovery in this model. The result shows that the pressure drops in the evaporation section, maintains a constant value in the adiabatic section and increases to full recovery in the condenser section.

Fig. 6.20 shows the temperature and temperature recovery in the vapor part. The temperature has a full recovery because we neglect the viscous forces.

Figure 6.6: Temperature of the Heat Pipe at the xy plane and $z = 0$

Figure 6.7: Temperature of the Heat Pipe at the xz plane and $y = 0.01$

Figure 6.8: 3D Liquid Velocity at Liquid-vapor Interface

Figure 6.9: Value of Liquid Velocity at Liquid-vapor Interface

Line 1 is the value of liquid velocity in the line $x = -0.1005, 0 < y < 0.3, z = 0$.

Line 2 is the value of liquid velocity in the line $x = 0.0, 0 < y < 0.3, z = -0.1005$

Figure 6.10: 3D Vapor Velocity at Liquid-vapor Interface

Figure 6.11: Value of Vapor Velocity at Liquid-vapor Interface

Line 1 is the value of liquid velocity in the line $x = -0.1005, 0 < y < 0.3, z = 0$.

Line 2 is the value of liquid velocity in the line $x = 0.0, 0 < y < 0.3, z = -0.1005$

Figure 6.12: Liquid Velocity In the Wick

Figure 6.13: Intersection Plane of Liquid Velocity In the Wick

y direction is enlarged ten times from its original size

Figure 6.14: Detailed Figure of Intersection Plane of Liquid Velocity In the Wick

y direction is enlarged ten times from its original size

Figure 6.15: Liquid Pressure Velocity In the Wick

Figure 6.16: 3D Flow Line of Vapor Velocity

y direction is enlarged ten times from its original size

Figure 6.17: Flow Line of Vapor Velocity in the $x - y$ Plane at $z = 0$

y direction is enlarged ten times from its original size

Figure 6.18: Vapor Velocity Vector in the $x - y$ Plane at $z = 0$

y direction is enlarged ten times from its original size

Figure 6.19: Vapor Pressure and Pressure Recovery

Figure 6.20: Vapor Temperature and Temperature Recovery

6.4 Limits of Stable Operation of a Heat Pipe

There are processes that limit the stable operation of a heat pipe. These processes are the vapor flow limit, entrainment limit, capillary limit and boiling limit. In this section we will discuss the limits of the heat pipe model.

6.4.1 Vapor Flow Limit

Because we assume the vapor flow is incompressible, the Mach number must be small enough, say $Ma < 0.3$. If the Mach number increases above this limit, the flow becomes more compressible and the assumption of incompressibility becomes less accurate. A widely accepted limiting value of Mach number for incompressible flow is 0.3.

To calculate Mach number in the vapor we use the following equation

$$Ma = v/\sqrt{\gamma RT}. \quad (6.17)$$

Because we want

$$Ma < 0.3 \quad (6.18)$$

So

$$v < Ma\sqrt{\gamma RT} \quad (6.19)$$

Where $\gamma = 1.3$ [3], $R = 8310/18 = 461 J/kg \cdot K$ [3], If $T = 300K$, Then

$$v < Ma\sqrt{\gamma RT} \quad (6.20)$$

$$= 0.3 \times \sqrt{1.3 \times 461 \times 300} \quad (6.21)$$

$$= 127(m/s) \quad (6.22)$$

The vapor velocity in our heat pipe is 1.8 (m/s). Therefore the assumption of incompressible flow is valid because this value is 70 times smaller than the limit.

6.4.2 Entrainment Limit

A shear force exists at the liquid-vapor interface because the vapor and liquid move in opposite directions on the wick/vapor interface in a heat pipe. If the vapor velocity is very high, the entrainment limit will be reached in the heat pipe. In this case, the liquid will be torn from the surface of the wick and liquid will be entrained in the vapor. The liquid in the evaporator will dry out. We calculate the Weber number to estimate this phenomenon.

The Weber number is defined by

$$We = \frac{2r_{h,s}\rho_v V_v^2}{\sigma}. \quad (6.23)$$

For our heat pipe test, $r_{h,s} = 0.01004m$, $\rho = 0.01729kg/m^3$, $V_v = 1.8m/s$, $\sigma = 1.605 \times 10^{-1}N/m$. So

$$We = 0.07 < 1. \quad (6.24)$$

Our heat pipe test is operating approximately 15 times below the entrainment limitation.

6.4.3 Capillary Limit

The capillary limit of a heat pipe was discussed in Chapter 2. The maximum value of the capillary pressure increment that can be attained is

$$(\Delta P)_{max} = \frac{2\sigma}{r}. \quad (6.25)$$

If $\sigma = 1.605 \times 10^{-1}N/m$, $r = 70 \times 10^{-9}m$, then

$$(\Delta P)_{max} = 2.29 \times 10^6 Pa. \quad (6.26)$$

The pressure drop in our case is 99.8 Pa. This is much less than the capillary limit of 2.29×10^6 Pa.

6.4.4 Boiling Limit

The primary mechanism of heat transfer in the evaporator and condenser sections of the casing is conduction. Conduction of heat across a liquid-saturated wick is accompanied with a radial temperature gradient in the liquid. In the evaporator section, the temperature of the liquid at the casing-wick interface is greater than the temperature of the liquid at the wick-vapor interface. If the difference between these two temperatures is too high, then vapor bubbles will form at the casing/wick interface, i.e., boiling will occur. The formation of vapor bubbles in the wick is undesirable because it will cause hot spots in the liquid and dry-out. This is called the boiling limit.

Vapor bubbles will not form when ΔT is less than the value required for the nucleation at the casing-wick interface.

$$\Delta T = T_L - T_{sat} \quad (6.27)$$

Where

ΔT is superheat.

T_L is the liquid temperature.

T_{sat} is the saturation temperature.

In the practical application of the heat pipe, we must ensure that the temperature

difference between the casing-wick interface and the wick-vapor interface is less than $5^{\circ}C$.

In our heat pipe test, we can see from the Fig 6.6 $\Delta T \approx 0.1^{\circ}C < 5^{\circ}C$. So this heat pipe is operating far from the boiling limit.

Chapter 7

Conclusions

In this thesis, a heat pipe is analyzed using a three-dimensional FEM mathematical model. To the author's knowledge, this is the first true 3D analysis of a heat pipe. There are no adjustable parameters in this model. The errors are believed to be due primarily to errors in data such as the permeability of the wick and the various properties used in the analysis such as thermal conductivity. From the results, we conclude that the FEM technique is an effective method to analyze a heat pipe.

For the heat transfer in the casing and wick of a heat pipe, the CD thermal model is used to solve the heat transfer. For the normal velocity of liquid and vapor at the liquid-vapor interface, the Interface model is designed to solve those variables. A Darcy flow model is used to solve the liquid velocity and pressure in the wick. A potential flow solver computes the vapor velocity and temperature gradient in the

vapor part.

The limits of stable operation of a heat pipe were predicted from the model.

The moving interface problem was also modeled by using an adaptive mesh technique. To demonstrate the validity of this model, two test problems were successfully solved: an evaporation test problem and a Stefan test problem. In a later version, the moving interface model can be incorporated into the heat pipe model to study the start-up and other transit problems. However, in this thesis, only the steady-state behavior of the heat pipe was analyzed.

Although the solution computed by these solvers is believed to quite accurately reflect the operation of a real heat pipe, there is still room for improvement. For the Interface model, a more accurate equation could be used to calculate the mass flux across the liquid-vapor interface. For the vapor flow, we assumed that it is inviscid, incompressible and irrotational, i.e., potential flow. But in nature the vapor flow have viscosity and is compressible. So it would be better to solve the vapor flow using the Navier-Stokes equation for compressible flow. This would also allow analysis of transient effects on the time scale of acoustic waves. However, as long as the heat pipe operates in a stable range, it is expected that these corrections will be quite small.

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