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**Angular Momentum Precession, Acquisition Maneuver
and Attitude Estimation Design for SILA, a University
Micro Satellite Project**

by

Matthew R. Bullock, B.Eng.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Master of Engineering

Ottawa-Carleton Institute for
Mechanical and Aerospace Engineering

Department of Mechanical and Aerospace Engineering
Carleton University
Ottawa, Ontario
December 4th, 1996

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Estimation Design for SILA, a University Micro Satellite Project**
submitted by
Matthew R. Bullock, B. Eng.
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Dr. D.A. Staley

Thesis Supervisor

Dr. R. Bell

Chair, Department of Mechanical and Aerospace Engineering

Carleton University

December 1996

Abstract

Three topics relating to the attitude control and determination for SILA have been addressed:

- steering angular momentum in a precessing orbit
- developing a 'fast' attitude acquisition maneuver
- providing inexpensive yet precise attitude estimation

SILA is a three-axis stabilized momentum-biased microsatellite with aeronomy and technological demonstration payloads being developed at Carleton University. Magnetic (active) and gravity gradient (passive) torque steering techniques have been compared. A non-linear method of estimating the momentum vector inclination angle to the orbit normal for gravity gradient steering has been developed.

Due to power reasons, solar arrays must face the sun within a few orbits of launch vehicle separation. An autonomous acquisition maneuver consisting of three stages has been developed and simulated. The three stages are: the pitch axis alignment with the field line, with the orbit normal, and with the arrays facing the sun, the momentum wheel spin-up.

Attitude estimation using filtered Global Positioning System carrier phase difference and magnetometer measurements has been examined. In the proposed system, the measurements are filtered separately and the estimates are averaged or 'fused' using a weighted averaging technique optimized for estimates from Kalman filters.

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Table of Contents

Acceptance Sheet	ii
Abstract	iii
Acknowledgments	iv
Table of Contents	v
List of Figures	viii
List of Tables	xi
List of Abbreviations	xiii
Nomenclature	xiii
1.0 Introduction	1
1.1 Steering Angular Momentum in a Precessing Orbit	2
1.2 Acquisition Maneuver	3
1.3 Attitude Estimation using GPS and Magnetometers	5
2.0 Background	7
2.1 SILA Bus Design	8
2.2 Aeronomy Mission	10
2.2.1 Pointing Accuracy and Attitude Determination	12
2.3 Attitude Control and Determination System Design	13
2.4 Sun-synchronous Orbit	16
2.5 Global Positioning System	18
2.6 Problems That Require Solutions	22
2.6.1 Momentum Vector Steering in a Precessing Orbit	23
2.6.2 Acquisition Maneuver	24
2.6.3 Attitude Estimation	25
3.0 Momentum Steering in a Precessing Orbit	26
3.1 Required Torque due to Orbital Precession	27
3.2 Steering with Magnetic Torque	29
3.3 Steering with Gravity Gradient Torque	30
3.3.1 Angular Momentum Inclination Angle, Linear Approx.	31
3.3.2 Non-Linear Approximation, Perturbation Analysis	33
3.4 Non-Linear Approx., Technique Trade-off, and Simulations	34
3.5 Non-Linear Approximation, Average Torque	35
3.5.1 Comparison of Non-Linear Approximations	36
3.6 Technique Trade-off	38
3.6.1 Magnetic Torque	38
3.6.2 Gravity Gradient Torque	39
3.6.3 Steering Technique Selection	43
3.7 Back-up Mode	44
3.7.1 Simulations	44
3.7.2 Results	45

3.8 Discussion	45
3.9 Conclusions	46
3.10 Future Research	47
4.0 Attitude Acquisition Maneuver	49
4.1 Magnetic Control Techniques in Attitude Acquisition	50
4.1.1 Energy Reduction using Magnetometer Measurements	52
4.1.2 Gyrocompass Maneuver for Momentum-Biased Satellites	55
4.1.2.1 Application to Acquisition Maneuver	55
4.1.3 Motion of a Magnet	56
4.1.4 Momentum Wheel Acceleration	57
4.2 SILA Acquisition Maneuver	58
4.2.1 Acquisition Power Constraint	59
4.2.2 Proposed Maneuver Using Gyrocompassing	60
4.3 An Alternative 'Fast' Maneuver	61
4.4 The Proposed Acquisition Maneuver	62
4.4.1 Alignment with Field Line	63
4.4.2 Orbit Normal Alignment	63
4.4.3 Wheel Spin-up	65
4.5 Implementation	65
4.5.1 Alignment with Field Line	65
4.5.2 Orbit Normal Alignment	67
4.5.3 Wheel Spin-up	69
4.5.4 Gain Selection	69
4.6 Simulation	71
4.6.1 Simulation Description	72
4.7 Results	73
4.7.1 First Stage	74
4.7.2 Second Stage	75
4.7.3 Final Stage	76
4.8 Discussion	77
4.9 Conclusions	78
4.10 Future Research	79
5.0 Attitude Estimation Using GPS and Magnetometers	92
5.1 Kalman Filtering and Estimate Fusion	94
5.1.1 Kalman Filters	94
5.1.2 Estimate Fusion	96
5.1.2.1 Equal Weighting	97
5.1.2.2 Optimal Weighting	97
5.1.2.3 Scalar Approximation of the Optimal Weighting	98
5.2 Attitude Estimation	99
5.2.1 Attitude Estimation using Magnetometers	99
5.2.1.1 Flight Experience	101
5.2.2 Attitude Determination Using GPS	101
5.2.2.1 Flight Experience	105

5.3 Proposed Attitude Estimation System	106
5.4 Evaluation of Proposed Attitude Estimation System	108
5.5 Simulation	109
5.5.1 Test Case	109
5.5.2 Simulation Design	110
5.5.2.1 Exact Satellite States and Magnetic Field	110
5.5.2.2 Double Difference and Line of Sight	111
5.5.2.3 Measurement and Filter Simulation	112
5.5.2.4 Attitude Determination using GPS and Magnetometers	113
5.5.2.5 Estimate Fusion	114
5.6 Case Studies	114
5.7 Results	115
5.8 Discussion	117
5.8.1 Loosely-Integrated Configuration	117
5.8.2 Performance of the Optimal Weight Averaging	118
5.8.3 Scalar Approximation	119
5.9 Conclusions	120
5.10 Future Research	120
 6.0 Concluding Remarks	 127
6.1 Conclusions	127
6.2 Summary of Original Contributions	129
 References	 131
 Appendix A Component Design Specifications	 135
Appendix B MATLAB Programs	137

List of Figures

Figure 2.1	Three-Sided View of SILA	9
Figure 2.2	Illustration of Aeronomy Mission	11
Figure 2.3	Orientation of SILA Relative to the Direction of Motion	12
Figure 2.4	Illustration of Gyrocompassing	14
Figure 2.5	Orientation Relative to Inertial Space of a Circular Sun-synchronous 800 km Orbit over one Siderial Year	16
Figure 2.6	Regression of the Line of Nodes	17
Figure 2.7	Diagram of the GPS Satellite Constellation	19
Figure 2.8	Angular Momentum Remaining Fixed in a Precessing Orbit	23
Figure 3.1	Plot of Required Precession and Gravity Gradient Torques Against Inclination Angle	37
Figure 3.2	Applying Magnetic Torque at the Line of Nodes	39
Figure 3.3	Plot of Smallest Inclination Angle Against Momentum	40
Figure 3.4	Payload Pointing Direction at Poles and Nodes	41
Figure 3.5	Pointing Direction of Antenna at Poles and Nodes	42
Figure 3.6	Inclination Angle of Pitch Axis Relative to Orbit Normal Against Time	48
Figure 4.1	Motion Damping Using Magnetometer Measurements	53
Figure 4.2	Motion of Magnet and Pendulum	57
Figure 4.3	Momentum Precession	60
Figure 4.4	SILA at Different Points in the Acquisition Maneuver	62

Figure 4.5	Pitch Axis Alignment with Orbit Normal	64
Figure 4.6	Plot of Pitch Axis Angle Relative to Orbit Normal Against Time for 90° Maneuver Using ‘Gyrocompassing’ Technique	80
Figure 4.7	Snapshots of the Satellite Position Relative to the Orbit Reference Frame for the Pitch Axis Alignment with the Orbit Normal	81
Figure 4.8	Plot of the Pitch Axis and the Field Line Angles Relative to Orbit Normal for the Reverse Launch Case using the Fast Acquisition Maneuver	82
Figure 4.9	Plots of the Angular Velocities for the Body Roll, Yaw and Pitch Axes for the Reverse Launch Case using the Fast Acquisition Maneuver	83
Figure 4.10	Plots of the Wheel Angular Momentum and the Roll, Yaw and Pitch Axes Coil Moments for the Reverse Launch Case using the Fast Acquisition Maneuver	84
Figure 4.11	Plots of the Magnetic Field Strength and the Roll, Pitch and Yaw Axis Magnetometer Measurements for the Fast Acquisition Maneuver	85
Figure 4.12	Plots of the Body Kinetic Energy and the Angular Momentum for the First Stage of the Fast Acquisition Maneuver	86
Figure 4.13	Plots of the Field Derivatives and the Pitch Axis Magnetometer Measurements for the First Stage of the Fast Acquisition Maneuver	87
Figure 4.14	Plots of the Pitch Axis Angle Relative to Orbit Normal, Its Position on the Orbit Plane, and the Magnetic Moments for the Second Stage of the Fast Maneuver	88
Figure 4.15	Plots of the Angular Momentum in the Roll, Yaw and Pitch Axes for the Second Stage of the Fast Acquisition Maneuver	89
Figure 4.16	Plots of the Magnetic Torque in the Body-Fixed Reference Frame and for the Final Stage of the Fast Acquisition Maneuver	90

Figure 4.17	Plots of the Body Kinetic Energy, Total Momentum, Pitch Axis Angle Relative to Orbit Normal, and Wheel Momentum for the Final Stage of the Fast Acquisition Maneuver	91
Figure 5.1	Phase Difference as a Function of Baseline and Line of Sight	102
Figure 5.2	Architecture of the Proposed Attitude Estimation System	106
Figure 5.3	GPS Double Difference and Line of Sight Calculation Program, gps3.m	111
Figure 5.4	Sensor and Filter Simulation Program, maggps.m	113
Figure 5.5	Pitch Angle Estimate Error for Accurate GPS Measurements and High Noise Estimate, v_{GPS} (GPS, magn. estimates)	122
Figure 5.6	Pitch Angle Estimate Error for Accurate GPS Measurements and High Noise Estimate, v_{GPS} (fusion methods)	122
Figure 5.7	Pitch Angle Estimate Error for Poor GPS Measurements and High Noise Estimate, v_{GPS} (fusion methods)	123
Figure 5.8	Pitch Angle Estimate Error for Accurate GPS Measurements and Low Noise Estimate, v_{GPS} (GPS, magn. estimates)	123
Figure 5.9	Pitch Angle Estimate Error for Accurate GPS Measurements and Low Noise Estimate, v_{GPS} (fusion methods)	124
Figure 5.10	Pitch Angle Estimate Error for Poor GPS Measurements and Low Noise Estimate, v_{GPS} (GPS, magn. estimates)	124
Figure 5.11	Pitch Angle Estimate Error for Poor GPS Measurements and Low Noise Estimate, v_{GPS} (fusion methods)	125
Figure 5.12	Comparison of Optimally Weighted Averages for Accurate GPS Measurements	125
Figure 5.13	Comparison of Optimally Weighted Averages for Poor GPS Measurements	126

List of Tables

Table 1.1	Attitude Estimating Components on SILA and Their Function	5
Table 2.1	GPS Satellite Constellation Information	19
Table 2.2	Design Parameters of the Trimble Vector Receiver and Antenna System	22
Table 2.3	Charge Capacity Budgeted for Launch and Acquisition	24
Table 3.1	Continuous and Periodic Magnetic Torque	38
Table 3.2	Smallest Inclination Angles	41
Table 3.3	Comparison of Steering Techniques	43
Table 4.1	Results from Gain Estimation	71
Table 4.2	Initial Conditions for Simulations	72
Table 4.3	Parameters for First Stage	73
Table 4.4	Parameters for Second Stage	73
Table 4.5	Acquisition Maneuver Results	73
Table 4.6	Parameters Plotted Against Time for Figures 4.9-4.11	74
Table 4.7	Parameters Plotted Against Time for Figures 4.12-4.13	74
Table 4.8	Parameters Plotted Against Time for Figures 4.14-4.15	75
Table 4.9	Parameters Plotted Against Time for Figures 4.16-4.17	76
Table 4.10	Battery Charge Capacity Required for Fast Acquisition Maneuver	77

Table 5.1	Update Rates and Accuracy Desired for ACDS Functions	106
Table 5.2	Satellite and Orbital Parameters	109
Table 5.3	List of Simulations for the Attitude Estimation System Using GPS and Magnetometers	115
Table 5.4	Outline of Simulation Plots	115
Table A-1	ITHACO T-Reaction Wheel, Type A	135
Table A-2	Design Specifications for the ISTS Earth Sensor	135
Table A-3	ITHACO IM-103 Three-Axis Magnetometer	136
Table A-4	Sample Torque Rod from ITHACO, TR10CFN	136
Table B-1	Listing of MATLAB Files and Their Purpose	137

List of Abbreviations

ACDS	Attitude Control and Determination System
AN	Ascending Node
C/A	Coarse/Acquisition
DOD	Depth of Discharge
EKF	Extended Kalman Filter
FOV	Field of View
GADACS	GPS Attitude Determination and Control Experiment
GDOP	Geometric Dilution of Precision
GPS	Global Positioning System
IGRF	International Geomagnetic Field Model
KF	Kalman Filter
LEO	Low Earth Orbit
P	Precision
PRN	Pseudo Random Noise
TT&C&C	Telemetry, Tracking, and Command and Control
WHECON	Wheel Control

Nomenclature

A	system state matrix
B	input matrix
B_r, B_θ, B_ϕ	magnetic field components in spherical co-ordinates
B_x, B_y, B_z	magnetic field components in body-fixed co-ordinates
C	observability matrix
E_k	total body kinetic energy
G	stochastic process error matrix
G_x, G_y, G_z	gravity gradient torque in the orbital reference frame
K	Kalman gain (Chapter 5.0)
K	magnetic field derivative gain (Chapter 4.0)
H	total angular momentum (body and wheel)
I	identity matrix
I_{xx}, I_{yy}, I_{zz}	principal moments of inertia
J_2	coefficient of oblateness
M_{io}	transformation from the inertial to orbital reference frames
M_{ob}	transformation matrix from the orbital to body-fixed reference frames
M_{so}	transformation matrix from spherical to orbital reference frames
M_x, M_y, M_z	magnetic moments in body-fixed co-ordinates
$\hat{M}$	cross-correlation matrix
$\tilde{M}$	time updated cross-correlation matrix
$\hat{P}$	covariance matrix of error

$\tilde{P}$	time updated covariance matrix of error
R_e	Earth's mean radius
R_v	measurement noise covariance matrix
T	sample time interval
T_{magn}	magnetic torque
T_{prec}	torque required to precess momentum vector with orbit plane
T_x, T_y, T_z	external torques on the satellite body in body-fixed co-ordinates
W	weighting matrix
a	semi-major axis
ant	antenna position (relative to body centre)
b	baseline vector
s	line of sight vector
h	wheel momentum
i	orbital inclination
t_k	time of measurement
u	input vector
q_1, q_2, q_3, q_4	quaternions
w	stochastic process error vector
x	system state vector
$\hat{x}$	system state estimate vector
$\tilde{x}$	time updated system state vector
y	measurement vector
α	angle of pitch axis relative to the orbit normal
β	electrical line bias between the antennas
$\Delta\Phi_m$	GPS signal phase difference
ε	inclination angle of pitch axis relative to the orbit normal
ϕ	roll angle
θ	pitch angle
ψ	yaw angle
λ	angle of field line relative to the orbit plane
μ	Earth's gravitational constant
ω	argument of perigee
ω_n	nutation frequency
ω_o	orbital rate
ω_x	roll axis angular velocity
ω_y	pitch axis angular velocity
ω_z	yaw axis angular velocity
v	stochastic measurement error vector
Ω	right ascension of the ascending node of the orbit plane
$\Lambda_x, \Lambda_y, \Lambda_z$	angular velocity components of the orbit plane

Chapter 1.0

Introduction

A trilogy is defined as three dramas or sometimes writings or musical compositions that are related through a similar theme. This thesis investigation is a 'trilogy in design' consisting of the resolution of three problems that are connected to the theme of attitude control and determination. The object of investigation and hopeful benefactor of this trilogy in attitude control and determination is SILA, the microsatellite being designed at Carleton University. Its purpose is scientific (measuring ozone decomposition), technological (demonstrating new satellite components), and educational (developing student spacecraft engineering skills).

In the 1995-96 academic year, students developed a momentum-biased three-axis stabilized design of SILA. During the design process, three problems relating to attitude control and determination arose, namely:

- steering the angular momentum with the precessing orbit (Chapter 3.0)
- developing an attitude acquisition maneuver (Chapter 4.0)
- providing attitude determination in all three axes (Chapter 5.0)

These three problems are defined in Chapter 2.0 using background information provided in that chapter. Background information is given on the satellite bus configuration, the aeronomy mission, the attitude control system, the orbit, and the Global Positioning System (GPS). The three problems are investigated separately in the chapters indicated in parentheses. An overview of the analyses and results is given in the following sections.

1.1 Steering Angular Momentum in a Precessing Orbit

SILA, being designed for launch into a precessing orbit, is stabilized using a biased angular momentum from a momentum wheel. The wheel spin axis is aligned with the satellite pitch axis and the orbit normal. While the orbit plane precesses, the momentum vector remains inertially fixed, if there are no external torques, resulting in a slow drift away from the orbit normal.

Active magnetic torque may be used to maintain the angular momentum vector (and pitch axis) aligned with the orbit normal. Passive gravity gradient torque may be used to maintain the momentum vector at a small fixed angle with the orbit normal. The existence of the smallest equilibrium angle has been shown through linear analysis. As the angular momentum magnitude increases, the ability of the gravity gradient torque to provide the needed precession torque diminishes. Through a complex non-linear analysis, it has been shown that three angles exist and that at some critical angular momentum magnitude, only one relatively large angle exists.

The three tasks are addressed in Chapter 3.0: a non-linear approximation of the smallest equilibrium angle and the critical angular momentum that is easy to understand and visualize, a trade-off between the magnetic and gravity gradient torque steering methods and a simulation of the gravity gradient torque steering method.

By equating the average non-linear gravity gradient torque to the torque required to precess the momentum with the orbit plane over one orbit, three equilibrium angles and a critical momentum magnitude (where the number of angles goes from three to one relatively large value) have been approximated.

The magnetic torque steering method has been selected and the gravity gradient torque steering method is available as a back-up mode. Despite gaining simplicity by using passive gravity gradient torque, the payload and the antenna pointing becomes more complicated since their reference pointing directions would vary throughout the mission. In the computer simulations of the gravity gradient steering method, the momentum vector and the pitch axis started aligned with the normal, drifted to inclination angles of 1° , 6.7° , and 11° for three momentum magnitudes, 0.1, 0.8, and 1.5 Nms respectively, and drifted back towards the normal.

1.2 Acquisition Maneuver

SILA will be released from the launch vehicle at an undefined orientation and will be tumbling due to unequal spring forces during separation. The

objective of the acquisition maneuver, to be performed by the computer on-board the satellite, is to have the satellite reoriented and prepared for the mission, with the momentum wheel spinning with its spin axis aligned along the orbit normal and the solar arrays facing the sun.

The worst case scenario would have the satellite released with the solar arrays facing the direction opposite to the sun. The main design consideration is the available time for the maneuver. Power, supplied by two secondary batteries, is available for subsystems other than the attitude control and determination subsystem for up to 10 orbits after separation.

A previously designed acquisition maneuver was considered but did not meet the time constraint. In this maneuver, referred to as the gyrocompassing maneuver, the momentum wheel spin axis aligns itself with the magnetic field line spin axis, the orbit normal. It uses only magnetometers and magnetic torque rods and does not require attitude estimation. Since the maneuver takes more than 30 orbits to finish, another maneuver has been designed by the author that fulfills the time constraint.

In the first step of the maneuver, the pitch axis is aligned with the field line by using the pitch axis magnetic coil as a magnet and the other coils in the other axes for damping. In the next step, coils in the axes perpendicular to the pitch axis are used to torque the pitch axis towards the orbit normal and the pitch axis coil is used for damping. The momentum wheel is then accelerated and the body motion is damped until the mission mode can begin. Unlike the gyrocompassing technique, the maneuver requires attitude

estimation. This maneuver was simulated for three cases, including the worst case, in MATLAB. All of the maneuvers took less than 2.4 orbits or 15,000 seconds to complete with the pitch axis within 5° of the orbit normal 9,000 seconds after separation.

1.3 Attitude Estimation Using GPS and Magnetometers

Five components have been selected for various tasks on SILA that can be applied to estimate attitude and are listed with their functions in Table 1.1.

Component	Function
horizon sensor	attitude estimation for normal pointing
star tracker	attitude estimation for mission data analysis
sun sensor	attitude estimation for attitude acquisition
magnetometers (3-axis)	magnetic field measurements
GPS receiver and antenna	on-board orbit determination

Table 1.1: Attitude Estimation Components on SILA and Their Functions

Two components in particular, the magnetometers and the Global Positioning System (GPS) receiver and antenna, are required for tasks other than attitude estimation and are relatively inexpensive in comparison to the star tracker and the Earth sensor.

In a proposed attitude estimation system, estimates are obtained from separately Kalman filtered GPS carrier phase difference and magnetometer measurements and then averaged to give one set of estimates. In Chapter 5.0, this system is examined to determine if it meets the attitude estimation needs for SILA and a weighted averaging method optimized for the separately Kalman-filtered estimates is examined through simulation.

Acquisition, normal pointing control and post-flight mission data analysis require attitude estimation accuracies to within 2° , 0.5° , and 0.02° , respectively, in all axes. The magnetometer attitude estimates are very reliable and accurate to within 1° or 2° , and the GPS attitude estimates are not reliable due to signal blockage and reflection but potentially accurate to within 0.1° for baselines (between antennas) larger than 1 metre [1]. The proposed system could be used to meet the requirements for the acquisition and normal pointing control, but not the mission data analysis.

An optimal weighting method for the estimate averaging was tested in a simulation performed in SIMULINK. The weighting, based on the steady state covariance matrix of error calculated in the Kalman filters, is optimized to put more weight on the estimates that are nominally more accurate. The GPS system attitude estimates are nominally more accurate than the magnetometer attitude estimates, however they are unreliable. The 'optimized' estimate averaging method puts more weight on the GPS Kalman filter estimates whether they are more or less accurate than the magnetometer estimates.

Chapter 2.0

Background

SILA is a senior-level undergraduate aerospace microsatellite project at Carleton University. The goal is to develop a satellite bus for scientific missions, spacecraft technology demonstration and student education in satellite design. In the first two academic years of the project, 1994-95 and 1995-96, student teams designed spin-stabilized and three-axis stabilized versions of the microsatellite with an aeronomy payload. The present student team is modifying the three-axis stabilized version to include an innovative communications system being developed by CAL Corporation, located in Ottawa, Canada. This system requires that a Global Positioning System (GPS) receiver and antenna be included for orbit determination. All versions are designed as a Delta II secondary payload for launch with RADARSAT II, the principal payload, into a circular 800 km sun-synchronous orbit. As a Delta II secondary launch vehicle payload, the satellite is restricted to a rectangular volume of 80 cm by 80 cm by 30 cm and a mass of 80 kg. The target mission life is 2 years.

The motivation behind this thesis is the attitude control and

determination subsystem (ACDS), in particular, for the three-axis stabilized design. Three problems relating to the ACDS that arose during the design are investigated in this thesis. The following sections outline the satellite configuration, the aeronomy mission, the three-axis stabilized attitude control system design, the orbit, and GPS. Using this background information, the three problems are introduced at the end of this chapter.

2.1 SILA Bus Design

The 1995-96 three-axis stabilized version is the design considered in this thesis. Detailed information about system integration, mission analysis, and subsystems is found in the executive report and the individual student reports [2]. This section provides an overview of the design, bringing forth details that are pertinent to the attitude control and determination system.

A three-sided view of the three-axis stabilized version of SILA is shown in Figure 2.1. The satellite body is a flat rectangular box with one corner modified to have an obtuse angle. The body consists of a cover and a component deck with an attachment ring, designed for the Delta II secondary payload attachment assembly, bolted to the centre of the deck. The cover and deck are made of aluminum and are 3/16" and 1/8" thick, respectively.

The obtuse corner has one patch array antenna on each side. The geometry increases the field of view of the antenna system, permitting larger periods of communication with the ground station.

Figure 2.1: Three-Sided View of SILA [3].

The flat rectangular body has one modified corner for patch array antennas. The flat cylindrical attachment ring is shown on the top right and bottom left views. The location and relative size of the payload is shown with the small, long rectangular box in the top and bottom left views.

Solar arrays, made of Gallium Arsenide (GaAs), are located on one of the large body faces and provide an estimated 70 W of power at the end of the target lifetime. Two 2.5 Ahr secondary Nickel-Cadmium batteries are the energy storage devices. Both batteries are used before and after separation from the launch vehicle until the arrays face the sun and one battery supplies power during the eclipse season with the second available in case of a failure. Thermal control is passive and is dependent on the body structure for the heat removal from the solar arrays.

The total satellite mass, including all of the components and attachment fitting is 43 kg, well below the design limit. The moments of inertia are:

$$[I] = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} = \begin{bmatrix} 2.36 & 0.09 & 0.3 \\ 0.09 & 3.9 & 0.06 \\ 0.3 & 0.06 & 2.59 \end{bmatrix} \text{kg} \cdot \text{m}^2$$

The subscripts x, y, and z, correspond to the axes given in Figure 2.1. The satellite is almost symmetric about the y-axis. From this point onward, the x, y, and z axes of the body-fixed reference coordinate frame will be referred to as the roll, pitch, and yaw axes.

2.2 Aeronomy Mission

The aeronomy payload is a linear CCD with a 2° by 0.02° field of view (FOV) that measures energy in the infra-red spectrum emitted during ozone decomposition. The instrument must point at the Earth's limb to avoid interference from the Earth's infra-red energy.

In both spin-stabilized and three-axis stabilized designs, the payload faces the limb in the direction of the satellite motion. In the spin-stabilized design, the satellite rotates once per minute about its pitch axis in the orbital plane, and as a result, the limb is viewed only twice per satellite rotation. In the three-axis stabilized version, the payload is able to track the limb.

The stabilization is achieved with a momentum wheel that is internal to the body, allowing the body to be maneuvered about the pitch axis. The satellite rotates in the orbital plane at an average rate of 1 rotation per orbit

permitting the payload to continually view the limb. Since the Earth is actually asymmetric with an equatorial radius 21 km larger than its polar radius, the pitch motion is not quite uniform. It will need to pitch with a sinusoidal component magnitude of 0.4° in addition to the orbit rotation.

Figure 2.2: Illustration of Aeronomy Mission.

Payload points at the atmosphere in the plane of orbital motion as SILA rotates at a rate of once per orbit about its pitch axis. The pitch axis is into the page. (Not to scale)

The three-axis stabilized bus with payload is presented in Figures 2.2 and 2.3 to show how it is pointed. The payload points towards the atmosphere at the Earth's limb in the orbital plane at an approximate 28° angle from the direction opposite to the direction of satellite motion. Since the corner with the antenna is facing the Earth, the payload is pointed out of one of the flat sides. The satellite body-fixed reference frame is not aligned with the orbital reference frame. As shown in Figure 2.3, the satellite principal moments of

inertia in the roll and yaw axes are at a 45° angle from the axis of the direction of motion and the Earth pointing axis of the orbital reference frame respectively.

Figure 2.3: Orientation of SILA Relative to the Direction of Motion.

Satellite roll and yaw axes at a 45° angle relative to the orbit X-axis and Z-axis. In the orbit reference frame, the X-axis and Z-axis are in the direction of motion and in the Earth pointing direction. The pitch axis points into the page.

2.2.1 Pointing Accuracy and Attitude Determination

The aeronomy mission defines the pointing accuracy and attitude determination requirements. With the 2° FOV in the vertical plane, the aeronomy payload views a 110 km vertical line at the Earth's limb, in the direction of satellite motion, from an 800 km altitude. If the main line of

interest from 10 km to 80 km above the Earth's surface is required to be measured, a 0.5° rotation is allowed about the pitch axis. The maximum allowable pointing errors are defined as 0.3° in pitch to permit a margin of error, and 0.5° in roll and 0.8° in yaw.

The attitude angles are required to determine the measurement location for the analysis of the aeronomy data. The definition of where the satellite is pointing is considered to be relatively more important than the pointing accuracy. The attitude determination goal is set as 1.2 arc minutes or 0.02 degrees in all axes.

2.3 Attitude Control and Determination System Design

In order to meet the pointing requirements, spin stabilization and three axis stabilization have been considered. This section outlines the components and their functions for the three-axis stabilized attitude control system for the aeronomy mission. The components include a momentum wheel, Earth sensor, three magnetic coils, and a 3-axis magnetometer. They are used in a modified version of the WHECON or WHEel CONtrol system. The original version was designed using thrusters for torque control [4].

A momentum wheel provides the biased angular momentum for three-axis stabilization. The wheel spins in the satellite pitch axis, aligned with the orbit normal, providing gyroscopic stiffness against yaw and roll axis disturbances.

In this system, three-axis control is provided through two angle measurements due to the gyrocompassing effect of the momentum wheel. Figure 2.4 shows the yaw error becoming a roll error in one-quarter of an orbit because the angular momentum of the wheel keeps the satellite inertially fixed.

Figure 2.4: Illustration of Gyrocompassing [5].

Angular momentum remains inertially fixed unless modified with torque. Relative to the rotating orbit reference frame, an error initially in the roll axis becomes an error in the yaw axis in a quarter of an orbit.

Through the dynamic coupling in these axes, an Earth sensor can be used to measure the roll and pitch error to provide control in all three axes.

Control torque, T_c , is used to align the momentum vector with the orbit normal. The torque is provided by the magnetic moments from three magnetic coils aligned with each of the satellite's principal axes, M , and the Earth's magnetic field, B , where:

$$\vec{T}_c = \vec{M} \times \vec{B} \quad (2.1)$$

Given the required control torque and the magnetic field, the required magnetic moment is produced in the three magnetic coils, as follows:

$$\vec{M} = \frac{\vec{B} \times \vec{T}_c}{\vec{B}^T \cdot \vec{B}} \quad (2.2)$$

The magnetic coils, also called magnetic torquers, electromagnets or torque rods, can be approximated as magnetic dipoles. A magnetic coil consists of a tightly wound coil that encloses a core of free space or a material with a permeability a few thousand times greater than free space. A magnetic or dipole moment in Am^2 is created on the longitudinal axis of the coil when current flows through it:

$$\vec{M} = \mu N I \vec{A} \quad (2.3)$$

where μ is the permeability of the core material in N/A^2 , N is the number of turns in the coil, I is the current in the loop, A is the cross-sectional area enclosed by the coil. The permeability of free space, μ_0 , is $4\pi \times 10^{-7} \text{ N/A}^2$.

Since the magnetic coils produce a magnetic field when they are operated, they interfere with the measurements of the Earth's field. The magnetometers and magnetic coils have to be operated sequentially. Appendix A gives a summary of size, mass, power, and other specifications

for components that could be used in the attitude control system based on ITHACO magnetometers, magnetic coils, and momentum wheel, and an Earth sensor developed by the Institute for Space and Terrestrial Sciences.

2.4 Sun-synchronous Orbit

The proposed orbit is a circular near polar sun-synchronous orbit at a 98.6° inclination from the equatorial plane and an 800 km altitude.

Figure 2.5: Orientation Relative to Inertial Space of a Circular Sun-synchronous 800 km Orbit over one Siderial Year.

Ω is the right ascension of the ascending node and is projected in the ecliptic plane. The orbit plane is represented by a line on the Earth that is orbiting about the sun.

As a secondary payload, SILA could be released in the circularized 800 km orbit after a time delay following the ejection of RADARSAT II or in a slightly eccentric orbit within 10 km if the launcher performs an evasive maneuver. The orbit is sun-synchronous because it precesses. As shown in Figure 2.5, the sun-synchronous orbital plane rotates around the Earth at the rate that maintains its position relative to the sun. The precession is caused by an approximately uniform ring of mass around the Earth's equator. Its gravitational force creates a torque over each orbit that precesses the orbit plane. The line of nodes, the line of intersection of the orbital and Earth's equatorial planes, precesses as shown in Figure 2.6.

Figure 2.6: Regression of the Line of Nodes.

The average torque on the satellite over one orbit due to the mass bulge at the Earth's equator causes the line of nodes to rotate about the Earth's polar axis. The direction of satellite motion is given for the Winter Solstice condition.

The precession of the plane's line of nodes due to this torque is expressed as the angular rate of the right ascension about the Earth's polar axis:

$$\dot{\Omega} = -1.5J_2\sqrt{\mu}R_e^2a^{-3.5}\cos i \quad (2.4)$$

where μ is the Earth's gravitational constant, R_e , its radius, a , the semi-major axis, i , the orbit inclination, and J_2 , the coefficient of oblateness. For an 800 km orbit, the near-polar 98.6° inclination provides a precession rate of $0.9856^\circ/\text{day}$. The orbit presents an advantage in terms of power. The solar arrays are always pointed in the direction of the sun and do not need to be rotated.

2.5 Global Positioning System

The payloads present different requirements on the satellite design. As previously indicated, the aeronomy mission requires accurate determination of the satellite pointing for data analysis. The new communications system being incorporated into the current design needs the satellite location and velocity to be determined on-board in order to accurately point its antenna beam. GPS can satisfy this requirement.

The NAVSTAR GPS (NAVigation System with Time and Ranging Global Positioning System) is a constellation of satellites that emits radio signals that enables a user, equipped with the appropriate receiver, to accurately determine position, velocity, and time.

Number of Vehicles	21 (and 3 active spares)
Orbit Altitude	20 200 km
Orbital Period	12 hours
Orbit Inclination	55°
Arrangement	6 planes of 4 satellites

Table 2.1: GPS Satellite Constellation Information [1,6]

The system was developed by the United States Department of Defense to provide radio navigation and is usable on the Earth's surface, in the air and in near space. The diagram of the constellation is shown in Figure 2.7 and information about the constellation is given in Table 2.1.

Figure 2.7: Diagram of the GPS Satellite Constellation [6].

The constellation consists of 6 equally spaced planes with 4 satellites in each plane. The orbit altitude and inclination for all of the planes are 20,200 km and 55°.

The GPS satellite emits a pseudo-random noise (PRN) coded signal that includes information on its location and velocity in orbit ephemerides. The coded signal is called pseudo-random because it is a series of binary values

that appear to be random unless the user is equipped with a receiver that has a copy of the signal.

When the receiver detects the PRN code it shifts its code sequence in time until the signals have maximum correlation. Knowing the signal propagation velocity, the distance the signal travelled, the *pseudorange*, is determined. Position is usually determined geometrically using signals from three GPS satellites. A fourth measurement is necessary since the clocks in the GPS satellite and the receiver are not synchronized. Four non-linear equations are iterated to determine the position:

$$\rho_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} + b, i = 1..4 \quad (2.5)$$

where ρ is the pseudorange, i corresponds to the i th GPS spacecraft, and b is the bias between the clocks.

The arrangement of the satellite permits at least five GPS spacecraft to be viewed from anywhere on Earth at one moment. A recent study showed that at least five are in view for a satellite in low Earth orbit [7]. The main difference on the Earth's surface and in LEO is velocity. Since SILA orbits at an approximate rate of 7.4 km/s, the line of sight between the LEO microsatellite and the constellation spacecraft is constantly changing.

The line of sight of the antenna to the GPS satellites determines the accuracy of the location calculation. The arrangement of GPS satellites can lead to a singularity in determining the position. For instance, the satellite position cannot be determined if signals are received from four GPS satellites in the same plane relative to SILA. The Geometric Dilution of Precision

(GDOP) is a value that is calculated in a GPS receiver to qualify the geometry. The GDOP is defined as an rms error for the position and time solutions caused by 1 meter rms error in the measurements [1]. A typical receiver has six channels and is programmed to use the combination of four signals with the lowest GDOP for the position and time calculation.

The signals are also exploited to determine the orbital velocity. It is found using the difference between the Doppler shifted receiver and the known GPS signal frequencies. Another method of position determination is the use of the information transmitted in the signal. Within the PRN code, the GPS spacecraft transmits its broadcast ephemerides, including time parameters, Keplerian elements with additional perturbation information, at a rate of 50 bits per second every 30 seconds.

Two types of PRN code are transmitted: the coarse/acquisition (C/A) and precision (P) codes. The P-code is transmitted on the two carrier frequencies, L1 (1575.4 MHz) and L2 (1227.6 MHz), and the C/A is only on the L1 frequency. The C/A code, used for orbit determination in spacecraft, is susceptible to Selective Availability (SA), the intentional degradation of the GPS signal. Position accuracy with C/A code normally ranges from 50m to 100m, and up to 300m when SA is applied.

To give an idea of a receiver that is being considered for use on a low Earth orbit spacecraft, the design parameters for a Trimble Vector system are given in Table 2.2 [8]. This receiver has not been space qualified and is intended for use on JAWSAT, a university-built satellite.

Number of Channels	6
Code	C/A
Dimensions	127 mm x 241 mm x 56 mm
Weight	1.42 kg
Update Rate	2 Hz
Power	4 W (9-18 V DC)
Number of Antennas	4
Antenna Size	96 mm x 102 mm x 13 mm

Table 2.2: Trimble Vector Receiver and Antenna System Design Parameters

The system has four antennas because it provides attitude information as well as information about position, velocity and time. The possibility of using GPS for accurate attitude determination is investigated in Chapter 5.0.

2.6 Problems That Require Solutions

Background information about the bus, payload, attitude control system, orbit, and GPS have been presented in order to introduce three problems:

- momentum vector steering in a precessing orbit
- attitude acquisition maneuver
- attitude estimation

The following sections introduce these topics. They are the motivation behind this thesis investigation and are presented in three separate chapters. While SILA is hopefully the main benefactor, these solutions can be applied to low Earth orbiting spacecraft in general.

2.6.1 Momentum Vector Steering in a Precessing Orbit

The three-axis control from two angle measurements in the WHECON system design is achieved since the angular momentum tends to remain inertially fixed. As shown in Section 2.4 on the sun-synchronous orbit, torques created from the interaction between the satellite body and the orbital environment precesses the orbital plane. As demonstrated in Figure 2.8, if the wheel angular momentum direction is not changed, it will not precess with the orbit.

Figure 2.8: Angular Momentum Remaining Fixed in a Precessing Orbit.
Over one quarter of a sidereal year, the momentum vector drifts 90° away from the orbit normal if no torque is applied to it.

After a quarter of the year, the momentum direction is in the orbit plane as opposed to being perpendicular to the plane. If the solar arrays were initially facing the sun, they are no longer collecting power after a quarter of a year.

2.6.2 Acquisition Maneuver

Solar panels are only located on one face of the body. The worst case scenario for the release of SILA from the launch vehicle is to have it separate from the launch vehicle with the arrays facing the direction opposite to the sun. After separation, a maneuver must reorient the satellite bus faces so that the solar panels are normal to the orbit plane and pointing at the sun. Since the power is supplied by two secondary batteries charged before the launch, a limited amount of power is available for telemetry, tracking, command and control (TT&C&C), attitude control, and power regulation. For an acquisition maneuver that requires thirty minutes for momentum wheel acceleration and maximum coil torque, 2.02 Ahr of charge capacity is consumed, requiring a depth of discharge of 40%. Table 2.3 shows the charge capacity budgeted for the launch and the acquisition maneuver [9]. The batteries are in danger of being completely discharged if the acquisition maneuver takes more than 8 or 9 orbits without solar power.

Subsystem	Component	Peak Period (W)	Period Req. (Hours)	Rating (Ahr)
TT&C&C	Microprocessor	0.5	36	0.64
	Logic	5	2.62	0.75
	Isotropic Antenna	3		
ACDS	Momentum Wheel	20	0.5	0.36
	Coils	4	0.5	0.07
	Magnetometers	0.3	0.5	0.013
	Sun Sensor	0.2		
Power	Regulation	2	2.62	0.19
	Total	35.5		2.02

Table 2.3: Charge Capacity Budgeted for Launch and Acquisition.

The bus voltage is 28 V and the total charge capacity of the batteries is 5 Ahr. [9]

The rate of battery degradation is dependent on the depth of discharge (DOD). For LEO, the batteries are used thousands of times per year, limiting the DOD to between 20%-30%. Although a 50% discharge is acceptable for acquisition, a lower discharge allows the battery to have a longer life-cycle. A fast attitude acquisition maneuver that minimizes the time the solar arrays face away from the sun is needed. Coarse sun sensors located on both sides of the satellite provide attitude information if required.

2.6.3 Attitude Estimation

Very precise attitude determination is needed in all three satellite axes for the aeronomy data analysis. The Earth sensor measures only the roll and pitch angles and another sensor is required to provide the attitude determination needed for the payload data analysis. A star tracker has been proposed to fulfill this determination requirements. A total of five sensors that are capable of performing attitude estimation have been proposed for different tasks. They are listed in Table 1.1.

The power, mass, size, cost, and complexity of the satellite are dependent on the number of components used to operate the satellite. As a micro satellite and a secondary payload, SILA is restricted in mass and size. The final problem addressed in this thesis investigation is the provision of attitude information for acquisition, pointing control and determination while minimizing the number of components.

Chapter 3.0

Momentum Steering in a Precessing Orbit

As presented in the preceding chapter, the satellite is stabilized with a spinning momentum wheel in its pitch axis. While the orbital plane precesses, the angular momentum initially aligned with the orbit normal remains inertially fixed and slowly drifts away, if there are no external torques (see Figure 2.8). Active magnetic and passive gravity gradient torque control are two steering techniques that are considered for SILA. Magnetic torque coils are used for the WHECON system control torque, and will be used to precess the momentum vector.

Passively steering the vector using gravity gradient torque is an alternative solution. As the momentum vector drifts from the orbit normal, the body becomes inclined with respect to the orbit normal. Depending upon the satellite mass distribution and the orbit, the gravity gradient torque created by the inclined body will steer the momentum vector with the orbit normal. It will be shown through linear and non-linear analysis that equilibrium angles exist where the vector is steered by the gravity gradient torque at the same rate as the orbit precesses. If the momentum vector is

initially inclined at an equilibrium angle from the orbit normal, it will remain at this angle relative to the precessing orbit plane.

In the following section, the angular rate of the orbit plane will be derived in order to identify the torque required to precess the momentum.

3.1 Required Torque due to Orbital Precession

The goal of this section is to define the angular rate of the orbit plane in order to determine the torque necessary to precess the angular momentum. The angular velocity terms of interest are the angular rates of the right ascension and the argument of perigee. In Section 2.4 on the Sun-synchronous orbit, the precession of the plane's line of nodes was introduced in equation 2.4 as the angular rate, $\dot{\Omega}$, of the right ascension about the Earth's y-axis.

The line of apsides is the line that connects the orbit perigee and apogee. The effect of the equatorial bulge causes this line to rotate in the direction opposite to the satellite motion in a polar orbit. The angular rate is expressed as a change in the argument of perigee for a nearly circular orbit:

$$\dot{\omega} = -1.5J_2\sqrt{\mu}R_{\text{ea}}^2a^{-3.5}(2 - 2.5\sin^2 i) \quad (3.1)$$

The rotation is a result of the Earth's asymmetry or the bulge of mass at the equator. The gravitational force on a satellite over a pole due this mass is lower than it would be if it were at the centre of the Earth, causing the satellite to rotate at a slower rate in its orbit. A more detailed derivation is given by Staley [5] on the angular velocity terms for an orbit plane.

The transformation from the Earth to the orbit inertial reference frames, $(i)_z$, is a rotation of the orbit inclination angle around the line of nodes. The orbital angular rate, $\bar{\Lambda}$, expressed in the inertial orbit reference frame is:

$$\bar{\Lambda} = (i)_z \begin{bmatrix} 0 \\ \dot{\Omega} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\omega} \\ 0 \end{bmatrix}$$

and

$$\bar{\Lambda} = \begin{bmatrix} \dot{\Omega} \sin i \\ \dot{\Omega} \cos i + \dot{\omega} \\ 0 \end{bmatrix} \quad (3.2)$$

The magnetic or gravity gradient torque necessary to precess the momentum vector, T_{prec} , is found through Euler's equation:

$$\bar{T}_{\text{prec}} = \dot{\bar{h}} + \bar{\Lambda} \times \bar{h} \quad (3.3)$$

where h is the wheel angular momentum. Two cases are to be examined. For the case where the angular momentum is initially aligned with the orbit normal and constant in magnitude, the torque is found to be:

$$\bar{T}_{\text{prec}} = \begin{bmatrix} \Lambda_x \\ \Lambda_y \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ h \\ 0 \end{bmatrix}$$

and

$$T_z = \dot{\Omega} h \sin i \quad (3.4)$$

The torque in the other axes is null. The second case has the momentum vector, assumed to be constant, inclined from the orbit normal. In the inertial orbit reference frame, the inclination of the momentum vector

is about the Earth pointing axis at the ascending node, $(\epsilon)_z$, and the resulting torque equation is:

$$\bar{T}_{\text{prec}} = \begin{bmatrix} \Lambda_x \\ \Lambda_y \\ 0 \end{bmatrix} \times \begin{bmatrix} h \sin \epsilon \\ h \cos \epsilon \\ 0 \end{bmatrix}$$

As with the previous case, the only torque is about the inertial z-axis:

$$T_z = \dot{\Omega} h \sin i \cos \epsilon - (\dot{\Omega} \cos i + \dot{\omega}) h \sin \epsilon \quad (3.5)$$

Steering techniques using magnetic and gravity gradient torques are examined in the following sections.

3.2 Steering with Magnetic Torque

Magnetic coils are used to provide the control torque needed to counter disturbance torques and to cancel the body torque as the momentum wheel is being accelerated or decelerated. The coils could be used continuously or periodically to precess the momentum vector with the orbital plane. If current is continuously applied such that the momentum vector is precessed with the orbit normal, the required magnetic torque is given by equation 3.4.

Since the magnetic coils and magnetometers are aligned with the body-fixed principal axes, the precessing torque in the orbit inertial z-axis would have to be transformed into the body-fixed co-ordinate system.

Torque could also be applied periodically. As the orbit plane precesses over a fixed time period, t_{prec} , the orbit normal drifts away from the momentum vector. The magnetic torque is applied to realign the momentum vector with the orbit normal. The time necessary to apply the

magnetic torque will depend on the time the orbit plane is left to precess. Assuming the angle of the momentum vector to the orbit normal is small, the equation for the application time, t_{magn} , is:

$$t_{\text{magn}} = \frac{T_{\text{prec}} t_{\text{prec}}}{T_{\text{magn}}} \quad (3.6)$$

where T_{magn} is the magnetic torque magnitude and T_{prec} is given by equation 3.4.

3.3 Steering with Gravity Gradient Torque

Gravity gradient torque may be used as a passive technique that presents an alternative steering method to the magnetic torque control. In the mid-1960s, the most common satellite configurations were spin-stabilized, partly to achieve a thermal balance on the body. Eversman [10] analyzed the dynamics of a slowly spinning body, symmetrical about one axis, in a regressing orbit. Through linearized equations of motion, he showed the existence of an equilibrium angle. At this particular angle, the gravity gradient torque precesses the momentum vector at the same rate as the orbit precesses and the vector remains at the equilibrium angular momentum inclination angle. The equilibrium angle from the orbit normal is a function determined by the angular momentum of the spinning satellite or wheel, the body mass distribution and the regression rate.

This passive method is well suited for a spacecraft or space platform such as the International Space Station that is required to be functional for

decades. An active control system with software and components that need maintenance or replacement is not required. McIntyre [11] was aware of this and patented the technique in 1988.

3.3.1 Angular Momentum Inclination Angle, Linear Approximation

Using the small angle approximation and assuming the principal moments of inertia are aligned with the body-fixed axes, the average gravity gradient and precession torques over one orbit can be equated to find the angular momentum inclination angle, ϵ . The satellite is assumed to be symmetrical, $I_x = I_y = I_z$, and is assumed to rotate once per orbit about the pitch axis such that its pitch angle remains constant. The linearized gravity gradient torque, G , in the body-fixed reference frame is:

$$\begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix} = 3\omega_0^2 \begin{bmatrix} (I_{zz} - I_{yy})\phi \\ (I_{zz} - I_{xx})\theta \\ 0 \end{bmatrix} \quad (3.7)$$

where ω_0 is the orbit rate.

The gravity gradient torque is limited in magnitude by the difference in principal moments of inertia and the magnitude of the roll and pitch angles. For background information on equation 3.7, Chobotov [4] and Staley [5] provide complete derivations for the gravity gradient torque acting on a spacecraft.

The angular momentum inclination angle does not rotate with the orbit reference frame. As shown in the illustration of gyrocompassing (see Figure 2.4), the angle goes from the roll axis to yaw axis of the orbit reference

frame. Through a transformation of the angular momentum inclination angle about the orbit normal axis, the roll angle and yaw angles are found to be:

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = (\omega_0 t)_y \begin{bmatrix} 0 \\ 0 \\ \epsilon \end{bmatrix}$$

and

$$\begin{aligned} \phi &= \epsilon \sin \omega_0 t \\ \psi &= \epsilon \cos \omega_0 t \end{aligned} \quad (3.8)$$

The gravity gradient torque equation in the rotating orbit reference frame is:

$$\begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix} = 3\omega_0^2 \begin{bmatrix} (I_{zz} - I_{yy})\epsilon \sin \omega_0 t \\ 0 \\ 0 \end{bmatrix} \quad (3.9)$$

Transforming the torques into the inertial orbit reference frame:

$$\begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix} = 3\omega_0^2 (-\omega_0 t)_y \begin{bmatrix} (I_{zz} - I_{yy})\epsilon \sin \omega_0 t \\ 0 \\ 0 \end{bmatrix} \quad (3.10)$$

The average gravity gradient torque over one orbit is:

$$\begin{bmatrix} \overline{G_x} \\ \overline{G_y} \\ \overline{G_z} \end{bmatrix} = \int_{\tau=0}^{\tau=2\pi} \frac{3}{2\pi} \omega_0^2 (I_{zz} - I_{yy}) \epsilon \begin{bmatrix} 0.5 \sin 2\tau \\ 0 \\ -\sin^2 \tau \end{bmatrix} d\tau \quad (3.11)$$

and

$$\overline{G_z} = \frac{3}{2} \omega_0^2 (I_{yy} - I_{zz}) \epsilon \quad (3.12)$$

The average gravity gradient torque in one orbit occurs only about the inertial orbit z axis. Assuming the angle of inclination is small, this torque can be equated to the function for the required precession torque (see equation 3.4)

averaged over one orbit. For this approximation, the angular momentum inclination angle is found to be:

$$\epsilon = \frac{2\dot{\Omega}h\sin i}{3\omega_0^2(I_{yy} - I_{zz})} \quad (3.13)$$

Angular momentum, aligned with the satellite pitch axis, initially inclined at this angle, ϵ , about the satellite roll axis at the orbit apogee, will remain inclined at this angle as the satellite orbits. The gravity gradient torque is dependent on fixed parameters such as the body geometry and one variable parameter, the angular momentum inclination angle. As the precession torque increases due to larger momentum, the equilibrium angle becomes too large for the linearized approximation.

3.3.2 Non-Linear Approximation, Perturbation Analysis

In the early 1970s, Pringle [12] performed a complete non-linear analysis of a body spinning about its symmetrical axis in a precessing orbit. Through perturbation analysis using Hamiltonians and canonical transformations, he showed the existence of three equilibrium points. He also derived a limiting equation for the estimation of the angular momentum and spin rate for a spinning satellite where the three real equilibrium angles become one very large angle. The critical nutation frequency was estimated at:

$$\omega_n \cong 2^{5/3} \frac{(I_y - I)\omega_0^{5/3}}{I(\dot{\Omega}\sin i)^{2/3}} \quad (3.14)$$

The approximation is directly related to the angular momentum of a spinning momentum wheel. The nutation frequency for a momentum-biased configuration where the body is symmetrical about the spin axis is:

$$\omega_n = \frac{h}{I} \quad (3.15)$$

where h is the wheel angular momentum and I is the moment of inertia of the transverse axis, $I = I_{xx} = I_{zz}$.

3.4 Non-Linear Approximation, Trade-Off Study and Simulations

The motivation for this chapter of the thesis is threefold:

- to show a more intuitive method of estimating equilibrium angles and the critical angular momentum,
- to perform a trade-off between the magnetic and gravity gradient steering techniques, and
- to simulate the gravity gradient steering technique as a back-up mode.

With regards to the first goal, Pringle's solution is very thorough but difficult to comprehend and visualize. In the back-up mode, the momentum vector is initially aligned with the orbit normal and the magnitude of its angular momentum inclination angle relative to the orbit normal is observed over three hundred and fifty orbits through simulation. The non-linear analysis, trade-off and simulations of the gravity gradient technique used in a secondary mode are presented in the following sections.

3.5 Non-Linear Approximation, Average Torque

The author will provide in this section a simple technique to estimate the three equilibrium points and to show where the three real angles converge to a single large angle. This analysis is similar to that for the linear approximation. It will be done for a momentum-biased satellite where the pitch angle is approximately constant, as in the case of SILA. The body is also assumed to be symmetrical about its pitch axis.

The non-linear form of the gravity gradient torque is [4,5]:

$$\begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix} = 3\omega_0^2 \begin{bmatrix} (I_{zz} - I_{yy}) \sin \phi \cos \phi \\ 0 \\ 0 \end{bmatrix}$$

and

$$G_x = 3\omega_0^2 (I_{zz} - I_{yy}) 0.5 \sin(2\epsilon \sin \omega_0 t) \quad (3.16)$$

As in the linear analysis, the objective is find the average torque over one orbit. To integrate the gravity gradient torque, the sinusoidal function is approximated with a Taylor series expansion. The first two terms are:

$$\sin a = a - \frac{a^3}{3!} + \dots \quad (3.17)$$

and equation 3.16 becomes:

$$G_x = 3\omega_0^2 (I_{zz} - I_{yy}) \left(\epsilon \sin \omega_0 t - \frac{2}{3} \epsilon^3 \sin^3 \omega_0 t \right) \quad (3.18)$$

Transforming into the non-rotating orbit reference frame using $(-\omega_0 t)_y$, the torque is found to be:

$$\begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix} = \begin{bmatrix} 3\omega_0^2(I_{zz} - I_{yy})\cos\omega_0 t(\epsilon\sin\omega_0 t - \frac{2}{3}\epsilon^3\sin^3\omega_0 t) \\ 0 \\ -3\omega_0^2(I_{zz} - I_{yy})\sin\omega_0 t(\epsilon\sin\omega_0 t - \frac{2}{3}\epsilon^3\sin^3\omega_0 t) \end{bmatrix} \quad (3.19)$$

Integrating and averaging over one orbit, the final gravity gradient torque is:

$$\overline{G}_x = 0 \text{ and } \overline{G}_z = \frac{3}{2}\omega_0^2(I_{yy} - I_{zz})(\epsilon - \frac{1}{2}\epsilon^3) \quad (3.20)$$

The equilibrium angles can be determined by equating the torque with the average required precession torque over one orbit (see equation 3.5). Since the gravity gradient torque is a cubic, three real solutions may exist. The equation is:

$$2(\dot{\Omega}h\sin i\cos\epsilon - (\dot{\Omega}\cos i + \dot{\omega})h\sin\epsilon) = 3\omega_0^2(I_{yy} - I_{zz})(\epsilon - \frac{1}{2}\epsilon^3) \quad (3.21)$$

The equilibrium angles and the critical angle can be determined by plotting both functions versus the angle and determining the points of intersection between the curves.

3.5.1 Comparison of Non-Linear Approximation Techniques

The approximations of the critical angular momentum determined through Pringle's analysis (see equation 3.14) and the average torque analysis (see equation 3.21) are presented for a satellite with a spinning momentum wheel in its axis of symmetry.

The orbit inclination and altitude are 98.6° and 800 km and the symmetric and transverse principal moments of inertia are 3.9 kgm² and 2.5 kgm². Figure 3.1 shows the plot of gravity gradient torque and the required

precession torque versus angle for three different angular momentum magnitudes.

The number of angular momentum inclination angles converges to a single large value at an approximate momentum magnitude of 2.1 Nms. The critical angular momentum using Pringle's equation was determined to be 1.3 Nms. It should be noted that both values are approximations and that the critical nutation frequency found using equation 3.14 was 80% of the value found through numerical simulation in a test case in the paper published by Pringle [12].

Figure 3.1: Plot of Required Precession and Gravity Gradient Torques Against Angular momentum inclination angle.

The required precession torque is graphed for three momentum magnitudes. The equilibrium angles are the angles at which the curves intersect.

3.6 Technique Trade-off

This section consists of a trade-off study between the two steering techniques for SILA.

3.6.1 Magnetic Torque

With the magnetic torque steering technique, the pitch axis is always very close to the orbit normal. By precessing the momentum vector continuously or every half orbit, it remains within a small fraction of a degree with the orbit normal.

Momentum	Moment (Continuous)	Time (Half Orbit)
0.5 Nms	0.0094 Am ²	5.86 s
0.8 Nms	0.015 Am ²	9.37 s
1.1 Nms	0.019 Am ²	11.7 s
2 Nms	0.0378 Am ²	23.4 s

Table 3.1: Continuous and Periodic Magnetic Torque.

The magnetic moment necessary for continuous precession and the time necessary for torque every half orbit are given for different angular momentum values.

Table 3.1 shows the moment if applied continuously or the required application time, if the torque is applied every half orbit, for different momentum sizes. The application time assumes a magnetic moment of 5 Am² and the average field strength is 20,000 nT.

A practical point in the orbit to apply the periodic torque is where the satellite crosses the line of nodes, the line of intersection between the orbit and equatorial planes. As shown in Figure 3.2, the magnetic field line is in the plane perpendicular to the z-axis of the inertial orbit reference frame at

this point in the orbit. The moment would be applied in this plane perpendicular to the field line.

Figure 3.2: Applying Magnetic Torque at the Line of Nodes.

A two-sided view of the orbit plane, the magnetic field lines and the Earth. The field line at the ascending and descending nodes is in the plane perpendicular to the z-axis in the non-rotating orbit reference system. Since the torque required for precession is about this axis, the magnetic moment can be applied at the nodes in the plane shown on the left.

3.6.2 Gravity Gradient Torque

The derivation of the equilibrium angles was performed for a body symmetrical about the spin axis. As shown in Section 2.1, SILA is slightly asymmetrical and its principal moments of inertia in its roll and yaw axes are at a 45° angle from the direction of motion and the Earth pointing axis of the rotating orbit frame. However, for this trade-off, the angles found through the linear analysis can be used as an approximation.

With the magnetic torque steering technique, the pitch axis is always very close to the orbit normal. The gravity gradient inclination would have

the pitch axis and the angular momentum inclined at an angle that is fixed relative to the orbit plane.

If this technique were to be applied, the goal of the acquisition sequence would be to align the pitch axis at this inclination angle and have the controller remove pointing errors relative to this inclination angle and point the payload at the limb. Figure 3.3 shows the smallest inclination angle from the linear and non-linear approximation.

Figure 3.3: Plot of Smallest Equilibrium Angle Against Momentum.

The angle is plotted for the linear approximation (see equation 3.14) and for the non-linear approximation (determined from plots of equation 3.21).

As the momentum magnitude increases, the smallest angle increases.

Table 3.2 gives the inclination angles for different angular momentum magnitudes, where 0.8 Nms is the selected value for the mission.

Wheel Momentum	Equilibrium Angle
0.1 Nms	0.5°
0.8 Nms	5.3°
1.5 Nms	13.7°

Table 3.2: Smallest Equilibrium Angles.

Angles are given for the non-linear approximation.

This technique has an impact on the pointing direction of various components. The centre line of the field of view of the payload and the

Figure 3.4: Payload Pointing Direction at Poles and Nodes.

The satellite is inclined at the equilibrium angle, ϵ , about the orbit x-axis at the North and South Poles and about the orbit z-axis at the ascending and descending nodes. The payload field of view (FOV) at the Earth's limb relative to the vertical line in the plane of satellite motion is illustrated for the both cases. If there is no inclination, the FOV is aligned to the vertical line.

Relative to the orbit reference frame, the equilibrium angle rotates from the axis of the motion to the Earth pointing direction as illustrated in Figure 2.4 (see Section 2.3). Thus the linear field of view from the CCD imager will not be perpendicular to the Earth's surface, but tilted slightly from the vertical direction. At the poles, the error is in the direction of motion and at the ascending and descending nodes, the error is in the Earth pointing direction.

Figure 3.4 illustrates the orientation of the satellite due to these errors and the projection of the payload FOV above the Earth's surface. The antenna has similar variation of its 'footprint' on the Earth. Nominally, it is facing the Earth with total 'look' angles of 120° in the direction of motion and 60° normal to the motion. The 120° angle is achieved by placing electronically steerable patches on the obtuse corner.

Figure 3.5: Pointing Direction of the Antenna at Poles and Nodes.

The pointing direction of the antenna beam rotates through the positions shown in Figure 3.5, at the poles and the nodes. Momentum steering using gravity gradient presents a conflict. By decreasing the momentum magnitude, the gravity gradient is able to steer the momentum vector at a smaller inclination angle. However, the reduced momentum degrades the gyroscopic stiffness against disturbance torques.

If the technique were to be used, a trade-off would be necessary. The angular momentum magnitude must be large enough to provide gyroscopic stiffness against yaw and roll axis disturbances, but small enough to allow the gravity gradient torque to steer the satellite pitch axis within the desired angle of the orbit normal.

3.6.3 Steering Technique Selection

The advantages and disadvantages for both techniques are presented in Table 3.3.

Technique	Advantages	Disadvantage
Gravity Gradient Torque	<ul style="list-style-type: none"> • passive 	<ul style="list-style-type: none"> • payload measurement location varies • limits momentum magnitude • adds complication to controller
Magnetic Torque	<ul style="list-style-type: none"> • payload points in direction of motion • large FOV of antenna is aligned with direction of motion 	<ul style="list-style-type: none"> • additional command sequence for ACS microprocessor

Table 3.3: Comparison of Steering Techniques

The main advantage of the gravity gradient technique, its passive nature, is not required in this situation. Since torque coils are already used in the controller to provide tight pointing accuracy, they are available for steering. With this steering technique, the payload and antenna have a constant pointing direction within the orbit plane and the momentum magnitude is not restricted relative to the gravity gradient technique. Magnetic torque is the ideal method to steer the momentum vector.

3.7 Back-up Mode

The passive steering technique is an ideal back-up to the magnetic steering. If for some reason, the coils, the magnetometers, or the steering command were not operating due to failure or to the satellite holding in a safe-hold mode, the gravity gradient torque can steer the momentum vector. This situation is very convenient. The operator has time to assess the problem as the momentum maintains its position relative to the sun. Otherwise, the power collected by the solar arrays would reduce if the momentum vector drifted from the orbit normal. The change in inclination angle of the pitch axis relative to the orbit normal, beginning with the pitch axis aligned with the orbit normal, is examined through simulation.

3.7.1 Simulations

Non-linear simulations of a momentum-biased three-axis stabilized satellite were obtained using a 4th order Runge-Kutta numerical integration in

MATLAB. An example of the simulation programs is given in Appendix B. The simulations were performed over 350 orbits and the quaternions and angular velocities are saved at each orbit. The SILA momentum wheel is nominally at 0.8 Nms. Three cases were examined, with wheel momentum magnitudes of 0.1 Nms, 0.8 Nms, and 1.5 Nms. For each case, the satellite's pitch axis begins aligned with the orbit plane normal and the body has no initial angular velocities.

3.7.2 Results

The angle of the pitch axis relative to the orbit normal is shown in Figure 3.6 (see p. 48). For each case, the inclination angle drifted to a maximum angle and then slowly drifted back towards the orbit normal. As the angular momentum increased, the maximum angle increased. The maximum inclination angles were 1° , 6.7° , and 11° , for the three selected momentum magnitudes, 0.1 Nms, 0.8 Nms, and 1.5 Nms.

3.8 Discussion

Three topics were investigated: the non-linear analysis, the trade-off and the simulation of the back-up mode.

The three equilibrium angles and the convergence to one large single angle were shown through a simple non-linear analysis. The critical angular momentum magnitudes approximated using Pringle's equation and the non-linear torque analysis were only in rough agreement. In order to have a more

accurate approximation of the critical angular momentum, a more complete non-linear analysis is required. However, the method presented the three angles and convergence to one large angle in an intuitive manner.

Magnetic torque was selected to steer the angular momentum with the precessing orbit. If the passive steering technique were to be applied, the magnitude of the angular momentum is limited by the precession and disturbance torque requirements. Its magnitude must be large enough to provide gyroscopic stiffness against disturbances torques, but small enough to allow the gravity gradient torque to steer the satellite y-axis with the orbit normal. The effect on the pointing direction of the inclination angle would also be a design factor.

The back-up mode was analyzed for a satellite without damping. Since the satellite is most likely moving about its smallest equilibrium point, energy damping, whether passive or active, may cause the axis to align itself to the equilibrium inclination angle.

The most recent version of SILA may include deployable solar arrays, making the body highly asymmetric about the spin axis of the momentum wheel. The analysis presented assumes that the satellite is symmetrical about the spin axis of the wheel or the body. Linear and non-linear analysis should be performed for bodies that are asymmetric about this axis.

3.9 Conclusions

- Equating the non-linear approximation of the gravity gradient torque to the torque required to precess the momentum vector averaged over one

orbit shows the existence of three equilibrium angles in an intuitive manner.

- The critical nutation frequency determined by the author and Pringle's equation are only in rough agreement.
- Magnetic torque should be used to steer the momentum vector with the precessing orbit.

3.10 Future Research

- Determine the critical angular momentum through a more complete non-linear analysis.
- Study the effect of damping in the back-up mode. Perform simulations of a satellite in a precessing orbit with gravity gradient torque and energy dissipation to examine the alignment of the pitch axis with the inclination angle.
- Analyze gravity gradient torque steering for a highly asymmetrical satellite.

Figure 3.6: Inclination Angle of Pitch Axis Relative to Orbit Normal Against Time.

The pitch axis is initially aligned with the orbit normal in a precessing orbit. The gravity gradient torque steers the momentum vector, aligned with the satellite pitch axis, at the angle plotted above relative to the orbit normal. No damping is used.

Chapter 4.0

Attitude Acquisition Maneuver

The attitude acquisition sequence is a series of commands and maneuvers that reconfigure and reorient a satellite from an initial orientation and rotation state after launch vehicle separation to a state where the mission can be performed. The sequence of events is dependent on the orbit and the components that require calibration, activation, and deployment.

They can be orchestrated with full autonomy by a computer on-board the satellite, from a ground station, with semi-autonomy or a combination of the two. Ground station control is performed in a closed loop format using a computer or in an open loop format using a computer to display sensor data and simulate the effect of command sequences [13].

The main acquisition requirements for SILA are the removal of separation velocities, satellite reorientation and momentum wheel spin-up. Separation or tumble velocities are created by unequal spring forces applied to the satellite during launch vehicle separation. The satellite should be prepared for the mission, with the momentum wheel spinning at its mission speed aligned with the orbit normal, the solar arrays facing the sun, and the aeronomy payload pointing at the limb of the Earth in the plane of the satellite's orbital motion.

The main goal is to have these tasks performed autonomously from any initial orientation. The worst case scenario consists of the REVERSE LAUNCH, with the solar arrays facing the direction opposite to the sun. As presented in Section 2.6.2, the main constraint is the time. A maneuver is required to reorient the satellite in a few orbits, so the arrays face the sun before the batteries are over or fully discharged.

The sequence should also be usable for a reacquisition or 'safe-hold' mode. For instance, if the horizon is no longer detected by the horizon sensor, the satellite would automatically go to this mode to either reorient the satellite so the mission can be executed or hold a defined position until the reason for failure is determined on the ground.

Magnetic coils and the momentum wheel are used for control and stabilization in the mission mode, and their use in acquisition maneuvers is examined in the following sections.

4.1 Magnetic Control Techniques in Attitude Acquisition

Since the earliest initiatives in satellite attitude control, the use of Earth's magnetic field for stabilization, despinning, and control torque has been realized. Magnetic devices such as magnets, hysteresis dampers, and magnetic coils have been used in a variety of ways. One of the earliest applications was on the Transit II-A satellite launched June 22, 1960. Long cylinders with highly permeable material were exploited for hysteresis damping torques that reduced the initial spin-rate [14].

A method of particular interest for an attitude acquisition is a motion damping technique proposed by Stickler and Alfried [15]. They presented a method of using the change in magnetometer field measurements to dampen body motion. If the method is used on a satellite with biased momentum, the angular momentum vector aligns itself with the orbit normal.

The tendency of a magnetic coil at constant current or a magnet to align itself with the field line is another application of magnetic control to attitude acquisition. Used in conjunction with a damping method, this technique aligns the magnet or coil axis on the satellite with the field line, giving the satellite a defined state from an originally undefined orientation and rotation speed.

The compass technique and this field property have been exploited for use in gravity gradient stabilized satellites such as UoSAT-2, built at the University of Surrey, to point the extendible boom towards the Earth when it is being deployed [16].

If the boom is deployed at an angle to the Earth-pointing axis, the satellite and the boom would cone about the Earth-pointing axis like a three-degree of freedom pendulum moving about its lowest point. The effectiveness of the gravity gradient stabilizing boom is dependent on the initial orientation during deployment.

The Earth's magnetic field line can be approximated as a dipole with its poles at an 11° inclination from the Earth's North and South Poles. As a result, the field line direction is mainly in the orbital plane and points towards the Earth near its poles in high inclination orbits. Using the field to steer the axis to the

desired position, the boom of UoSAT-2 was deployed through a command from an observer at the ground station, after the satellite passed the North magnetic pole.

The final technique considered is the use of magnetic torque when the momentum wheel is accelerated. As the wheel is accelerated, a torque is applied to the body causing it to spin as well. Magnetic torque is used to cancel this effect. All of these methods are applicable to the acquisition maneuver for SILA. The despinning, gyrocompassing, compassing and wheel acceleration techniques are presented in more detail in the following sections.

4.1.1 Energy Reduction using Magnetometer Measurements

Using the field vector as an inertial reference, torque is created to reduce body motion until the magnetometers have constant measurements. The field line actually changes in magnitude and position, but over a very short time period, such as one second, it can be considered to be fixed. Stickler and Alfriend [15] show how the rate of change of magnetometer measurements can be used to create magnetic torque that reduces the satellite body rotational kinetic energy:

$$\mathbf{M}_i = -K\dot{\mathbf{B}}_i \quad (4.1)$$

where $\mathbf{M}_i$ and $\mathbf{B}_i$ are the magnetic moment and field measurement in the i th axis. (See page 15 for background information on the magnetic moment.)

To illustrate this technique, Figure 4.1 shows a satellite rotating in a positive direction about the axis going out of the page, with the field vector in the plane of the body motion. The magnetometer measurement in the satellite y -axis

decreases. A positive magnetic moment from the coil in the same axis creates torque and angular acceleration that opposes the motion. As the body angular velocity is reduced, the coil moment is reduced until the body stops rotating.

Figure 4.1: Motion Damping Using Magnetometer Measurements.

For a positive rotation about the z-axis (out of the page), the y-axis and x-axis magnetometer measurements decrease and increase respectively. As a result, positive and negative currents are applied in the y-axis and x-axis coils, creating a torque that opposes the motion.

This example demonstrates the technique in one axis. It can be applied to a satellite having angular velocities about the three axes, each with different moments of inertia. The total rotational kinetic energy of the body can be modified using torque:

$$\dot{E}_k = \vec{\omega} \cdot \vec{T} \quad (4.2)$$

The function for magnetic torque is:

$$\vec{T} = \vec{M} \times \vec{B} \quad (4.3)$$

Substituting the magnetic torque into the energy derivative equation and rearranging:

$$\begin{aligned}\dot{E}_K &= \bar{\omega} \cdot (\bar{M} \times \bar{B}) \\ \dot{E}_K &= -(\bar{\omega} \times \bar{B}) \cdot \bar{M}\end{aligned}\tag{4.4}$$

Over a brief period in time, such as a few seconds, the field line can be considered to be inertially fixed. Applying Euler's equation to the inertially fixed field line:

$$\begin{aligned}\dot{\bar{B}} &= \dot{\bar{B}}_{\text{body}} + \bar{\omega} \times \bar{B} \\ \dot{\bar{B}} &= 0\end{aligned}\tag{4.5}$$

where $\bar{B}$ is in the inertial reference frame, and $\bar{B}_{\text{body}}$ is in the body-fixed reference frame. Energy reduction can now be shown as a function of the rate of change of the field measured by the magnetometers:

$$\dot{E}_K = (\dot{\bar{B}}_{\text{body}}) \cdot \bar{M}\tag{4.6}$$

To ensure the energy is reduced, the moment is to be made a linear function of the field derivative. The moment and the energy derivative are:

$$\begin{aligned}\bar{M} &= -K\dot{\bar{B}}_{\text{body}} \\ \dot{E}_K &= -K(\dot{\bar{B}}_{\text{body}})^2\end{aligned}\tag{4.7}$$

where K is a diagonal gain matrix. The torque size is determined by K , the gain matrix. The gain is a function of the body principal moments of inertia, the coil torque strength and the worst case angular velocity. The method will reduce the angular velocities to zero or will stop when one of the magnetometers is aligned with the field line and the satellite is rotating about that line.

Over a short period of time, the field line is considered to be stationary but it is actually rotating at twice the orbit rate about the orbit normal. This rotation has a beneficial effect for momentum-biased three-axis stabilized satellites.

4.1.2 Gyrocompass Maneuver for Momentum-Biased Satellites

In the same paper outlining the field derivative technique to remove body rotations, Stickler and Alfrend [15] present a gyrocompassing technique to orient the momentum bias with the orbit normal without the need of a body-fixed torque and attitude determination.

The proposed technique is analogous to the operation of a gyrocompass used on a ship. Despite any rolling and pitching motions of the ship, the gyrocompass senses the Earth's rotation about its poles. Using an internal damping mechanism such as a viscous fluid, the spin axis of the gyrocompass tends to align with the Earth's spin axis.

The gyrocompass effect of a momentum wheel remaining inertially fixed as the satellite orbits is exploited in the WHECON system to perform attitude control about all three axes without a yaw angle measurement. The inclination angle between the orbit normal and the momentum vector moves from a roll to a yaw angle in a quarter of an orbit. Relative to an orbit reference frame, the angular momentum vector is rotating about the orbit normal at the described inclination angle. If the body motion relative to the orbital reference frame is damped, the momentum vector would align itself with the orbit normal.

4.1.2.1 Application to Acquisition Maneuver

As shown in Section 4.1.1, the energy reduction technique using magnetic moment based on magnetometer measurement derivatives causes the satellite to rotate about the orbit normal at the twice the orbit rate.

On a momentum-biased satellite, the energy reduction technique allows the spin axis of the momentum wheel to align with the spin axis of the magnetic field line, the orbital normal. The technique is advantageous as an acquisition maneuver for SILA since attitude estimation and large control torques are unnecessary.

4.1.3 Motion of a Magnet

A magnetic coil or magnet on a satellite will attempt to align itself with the field line like a needle in a compass pointing towards the magnetic poles on the Earth's surface. If the motion is damped, this part of the satellite would coincide with the field, otherwise the magnet will oscillate around or through the field line.

Figure 4.2: Motion of a Magnet and a Pendulum.

The magnet motion through the field line is analogous to pendular motion through the lowest point.

As shown in Figure 4.2, the motion is analogous to the motion of a pendulum moving through its lowest point. When they are aligned, the torque is zero and the angular velocity is a maximum. In the case of a pendulum, this energy state occurs when the pendulum mass is at its lowest point. When the coil has completed its rotation, the torque is maximum and the angular velocity is zero. Since the satellite has different moments of inertia and angular velocity, the coil in a satellite will move about the field line. Assuming that the principal moments of inertia are aligned with the body fixed frame, Euler's equations are:

$$\begin{aligned} I_x \ddot{\omega}_x + (I_y - I_z) \omega_y \omega_z &= T_x \\ I_z \ddot{\omega}_z + (I_x - I_y) \omega_x \omega_z &= T_y \\ I_y \ddot{\omega}_y + (I_x - I_z) \omega_x \omega_z &= T_z \end{aligned} \quad (4.8)$$

As shown in equation 4.8, a torque applied about one of the axes, would create torque in the other axes due to the difference in moment of inertia or initial angular velocities. However, by combining this technique with the energy reduction technique, the coil or magnet will move towards the field line until they coincide.

4.1.4 Momentum Wheel Acceleration

The acceleration of the pitch axis momentum wheel causes the satellite to spin about this axis in the opposite direction. Magnetic torque can be applied to create acceleration that opposes the wheel acceleration.

This method is often applied during mission operation, such as the HETE mission [17], when the wheel reaches its saturation speed and needs to be

decelerated. If the momentum wheel is in the satellite pitch axis, a moment is created with its yaw and roll coils to provide the opposite torque in the pitch axis such that:

$$T_y = \dot{h} \quad (4.9)$$

where h is the pitch axis wheel momentum. The required magnetic moments are:

$$\bar{M} = \frac{\dot{h}}{B_x^2 + B_z^2} \begin{bmatrix} -B_z \\ 0 \\ B_x \end{bmatrix} \quad (4.10)$$

The torque and thus the wheel acceleration are limited by the size of the magnetic coils. Another point of interest is that torque is created in the other axes:

$$\begin{bmatrix} T_x \\ T_z \end{bmatrix} = \frac{1}{B_x^2 + B_z^2} \begin{bmatrix} -B_x B_y \dot{h} \\ -B_z B_y \dot{h} \end{bmatrix} \quad (4.11)$$

For high inclination orbits where the momentum wheel spin axis is aligned to the orbit normal, these torques are relatively small since the magnetic field in the orbit normal direction is very weak.

4.2 SILA Acquisition Maneuver

The main requirement for SILA's attitude acquisition sequence is to bring the satellite to a state where the mission mode can be started. The desired state has the momentum wheel spinning at mission speed with its spin axis coinciding with the orbit normal, the bus face with the solar arrays in the direction of the

sun, the aeronomy payload pointing at the limb of the Earth in the plane of the satellite's orbital motion.

An initial consideration is whether the wheel should be spinning before or after launch vehicle separation.

4.2.1 Acquisition Power Constraint

The power during acquisition is supplied by two secondary batteries charged before the launch. As described more thoroughly in Section 2.6.2, power is required by various subsystems, limiting the battery power available to approximately eight orbits without solar power.

The budgeted charge capacity of 2.02 Ahr to be used during the launch and acquisition phases allows the magnetic coils to operate at maximum power and the momentum wheel to accelerate to mission speed during a thirty minute period. In this scenario, the charge capacity used for attitude control and determination was approximately 0.45 Ahr. If the wheel could be accelerated when the arrays are facing the sun, 0.3 Ahr could be conserved. In fact, if the wheel is at its mission spin rate before separation, the acquisition maneuver may take too long.

The satellite momentum wheel operates around a nominal spin rate that provides 0.8 Nms during the mission mode. To test the feasibility of spinning up the wheel before separation, a calculation is performed for the worst case situation, a 180° maneuver required because the solar arrays are facing the direction opposite to the sun.

Figure 4.3: Momentum Precession.

The momentum vector, H , is precessed by applying a perpendicular torque, T , over a small period of time, t .

The satellite reorientation has to be performed by applying torque perpendicular to the wheel angular momentum, as shown in Figure 4.3. The equation to estimate the time necessary to rotate the momentum vector using constant torque is:

$$t = \frac{\epsilon H}{T} \quad (4.12)$$

where ϵ is the rotation angle in radians, H is the angular momentum and T is the applied torque. Assuming the maximum magnetic torque available is approximately 4×10^{-4} Nm, the 180° rotation takes slightly more than two hours for the satellite with the wheel at the nominal speed. In this scenario, the batteries would be discharged by approximately 60%.

4.2.2 Proposed Maneuver Using Gyrocompassing

The technique presented by Stickler and Alfrend has been used or proposed for momentum-biased three axis stabilized satellites such as the HCMM [13] and is an ideal candidate maneuver for SILA. It can be performed autonomously, not

requiring attitude acquisition or any large torques. However, the main restraint is the time necessary to complete the maneuver.

A simulation of the technique was performed with an initial wheel momentum magnitude of 0.2 Nms. Figure 4.6 shows the pitch axis angle relative to the orbit normal. (Note: Figures 4.6-4.17 are located at the end of the chapter.) The maneuver takes over 30 orbits to bring the spin axis within 10° of the orbit normal. A transition phase would be necessary to add momentum to the wheel and to point the horizon sensor so the mission control system can be implemented. If the satellite did not have the power restriction, this technique would be very attractive, but a faster alternative maneuver should be developed.

4.3 An Alternative 'Fast' Acquisition Maneuver

Due to power reasons, the attitude acquisition maneuver is limited by time. As shown in Section 4.2.1, the battery will be deeply drained if the maximum torque available is used to precess the wheel momentum at the mission spin rate for the reverse launch case. As shown in Section 4.2.2, the battery could be completely drained if the gyrocompassing technique is used. Since the battery degradation is a function of the depth of discharge, a maneuver is required that has the pitch axis aligned to the orbit normal and the solar arrays continuously facing the sun within a few orbits, using minimal power from the batteries. The maneuver should work from any orientation and be performed autonomously.

4.4 The Proposed Acquisition Maneuver

The following maneuver has been designed by the author so the pitch axis and the orbit normal coincide quickly. It has been developed in a three phase strategy

- separation velocity removal and pitch axis alignment with the field line
- pitch axis alignment with the orbit normal
- wheel acceleration and body rotation rate removal

Figure 4.4: SILA at Different Points in the Acquisition Maneuver.

The satellite pitch axis is shown at an undefined orientation (left), aligned with the magnetic field line (centre), and within a few degrees to the orbit normal (right).

Figure 4.4 shows the satellite at different steps in the acquisition sequence, at an arbitrary initial orientation, with the y-axis aligned with the magnetic field line, and then at the orbit normal when the wheel is initially accelerated. When these phases are completed, the pitch axis is a few degrees from the normal. The mission control system is activated to align the pitch axis with the orbit normal and to point the payload at the limb of the Earth.

4.4.1 Alignment of Pitch Axis to the Field Line

The goal of the first phase is to bring the satellite to a known position with minimal body motion from an unknown orientation without the need for attitude estimation. This task is accomplished using the compass (see Section 4.1.3) and field derivative energy reduction (see Section 4.1.1) techniques. Using the satellite's pitch axis coil as a magnet and the other coils to reduce separation velocities and damp the oscillatory motion of the magnet, the pitch axis aligns itself with field line. When aligned to the field line, the body will be stationary or rotating about the line since the magnetometers measurement are constant in either situation.

4.4.2 Alignment of Pitch Axis with the Orbit Normal

As opposed to the gyrocompassing technique using the momentum bias to align the pitch axis to the normal, this sequence relies on a controlled torque maneuver. The maneuver uses the magnetic coils in all three axes to torque the pitch axis towards the orbit normal. Applying the technique outlined in Section 4.1.1, the pitch axis coil is also used for damping.

As shown in Figure 4.5, the moment is created in the plane containing the orbit normal and the pitch axis using the coils in all three axes. When the pitch axis coincides with the orbit normal, the magnetic moment and the magnetic field line are at the same angle relative to the orbit plane.

Figure 4.5: Pitch Axis Alignment with Orbit Normal.

The magnetic moment is created in the plane of the pitch axis and the orbit normal, causing the pitch axis to rotate towards the orbit normal. The moments in the pitch and perpendicular axes are selected such that the moment is aligned to the field line in the final state (right).

In a situation where the body is symmetrical about the pitch axis and initially stationary, the constant moment will align itself with the field line as the pitch axis coincides with the orbital normal.

In most situations, the body will be rotating about the pitch axis since the magnetometers in the other axes are normal to the field line. As a result, the pitch axis momentum and the body asymmetry make the axis cone about the normal. As the maneuver progresses, the direction of the magnetic moment is changed so that it always remains in the same plane as the orbit normal and the pitch axis.

Two torques result because the field line is no longer in the plane of the pitch axis and the orbit normal. The primary magnetic torque moves the pitch axis towards the orbit normal and the secondary torque accelerates the body

about its pitch axis. The moment in the pitch axis is applied as a function of the field derivative to reduce the roll and yaw axis angular velocities.

As the pitch axis approaches the normal, the body angular momentum is almost in the pitch axis. Snapshots of the satellite motion in this stage in Figure 4.7 (Note: Figure 4.7 is located at the end of the chapter) show the spiral motion of the pitch axis about the orbit normal.

4.4.3 Wheel Spin-up

In the final stage, the wheel spin-up begins with the magnetic torque strategy, presented in the Section 4.1.4 applied, to counter the angular velocity created in the pitch axis. When the wheel spin-up is completed, the magnetic torque is applied to remove the body angular velocity created during the pitch axis alignment to the orbit normal.

4.5 Implementation

4.5.1 Alignment with Field Line

In the first maneuver, the pitch axis coil is operated at a constant magnitude, M , and the other coils are used for energy reduction, such that:

$$\begin{aligned} M_x &= -K\dot{B}_x \\ M_y &= M \\ M_z &= -K\dot{B}_z \end{aligned} \tag{4.13}$$

where K is the field derivative gain. The gain is dependent on the size of the moments of inertia, separation velocities, torque caused by magnet aligning with

the field line and the coil strength and the desired sensitivity to the field derivative. The gain selection is outlined in Section 4.6.

The derivative is approximated with a lead-lag filter using a bilinear approximation. This method is implemented in a digital form to smooth a series of discrete measurements [18].

The equation for the approximation of the field derivative is:

$$\dot{B}(n) = \beta \dot{B}(n-1) + k[B(n) - \alpha B(n-1)] \quad (4.14)$$

where $\alpha = e^{-aT}$, $\beta = e^{-bT}$, $B(n)$ and $B(n-1)$ are the immediate and previous measurements, and T is the sample time interval. The exponential time constants would be determined through root-locus plots.

The magnetic coil is a series of loops tightly wound about a core material with a few thousand times the permeability of free space. When the current passes through the coils, it produces field lines characteristic of a dipole, that interfere with the magnetometer measurement of the Earth's field. The coils and the magnetometers have to be operated sequentially.

The goal is to reduce the body rates, usually ranging from $1^\circ/\text{s}$ to $0.1^\circ/\text{s}$ after separation to a state where the coil is following the field line and the body rates are less than $0.03^\circ/\text{s}$ with the alignment of the vector. The sensitivity to the body rate depends on the noise as well as the extraneous magnetic fields produced in the spacecraft.

The magnetometer maximum noise level, 5 nT , is very small relative to the Earth's field ranging from 20 to $50 \text{ } \mu\text{T}$. By applying the approximation technique

on a set of measurements over a one second period, derivatives of 10 nT/s or body rates less than $0.03^\circ/\text{s}$ can be detected with the assumed noise.

Magnetic torque is then applied over 9 seconds. The calculated value would be increased to compensate for the approximate 1 second 'off' period without torque. The rise time to the desired moment level is a function of the inductance of the coil and in the case of the ITHACO TR10CRF coils, about 50 μs are necessary to achieve 99% of the desired value. Information about these coils is presented in Appendix A.

The next stage occurs when the calculated field derivatives are below a minimum value, such as 15 nT/s and the field measured in the pitch axis exceeds a nominal field value, such as 30 000 nT.

4.5.2 Orbit Normal Alignment

The second stage uses the energy reduction technique outlined in Section 4.1.1 as well as a torque that moves the pitch axis towards the orbit normal. The disadvantage of this maneuver with respect to the gyrocompassing method is the need for attitude estimation to determine the location of the orbit normal with respect to the field line. Using attitude information from instruments such as the sun sensor or a star tracker, the location of the orbit normal with respect to the body-fixed reference frame and its angle from the field line are determined.

The torque direction is determined using the pitch axis and orbit normal direction vectors in body-fixed coordinates:

$$\begin{aligned} |\vec{T}| &= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times [A] \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ |\vec{T}| &= \begin{bmatrix} A_{23} \\ 0 \\ -A_{13} \end{bmatrix} \end{aligned} \quad (4.15)$$

where A is the coordinate transformation matrix between the body-fixed reference frame and the orbital reference frame. The angle between the field line and the orbit normal must also be determined. Since the magnetometers measure in the body-fixed axes and the orbital normal vector is known in these axes, the angle can be determined:

$$\cos \lambda = \frac{1}{\sqrt{B_x^2 + B_y^2 + B_z^2}} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} \cdot [A] \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (4.16)$$

This angle, λ , is used to create a moment that is inclined at the same angle as the field line with respect to the orbit normal when the pitch axis is almost aligned with the orbit normal (see Figure 4.5). The equations for the moments are:

$$\begin{aligned} M_y &= M \cos \lambda \\ M_t &= M \sin \lambda \end{aligned} \quad (4.17)$$

where M_t is the moment in the axes perpendicular to the pitch axis. The moment components of M_t in the roll and yaw axis coils are calculated by finding the direction of the required moment in body-fixed coordinates:

$$\vec{M}_t = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \frac{1}{\sqrt{A_{23}^2 + A_{21}^2}} \begin{bmatrix} -A_{23} \\ 0 \\ A_{21} \end{bmatrix} \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} \quad (4.18)$$

Including the energy reduction method used to remove body angular velocities, the final equation for the magnetic moments is:

$$\begin{aligned}
M_y &= M \cos \lambda - K_y \dot{B}_y \\
M_x &= M \frac{A_{21}}{\sqrt{(A_{21})^2 + (A_{23})^2}} \sin \lambda \\
M_z &= M \frac{A_{23}}{\sqrt{(A_{21})^2 + (A_{23})^2}} \sin \lambda
\end{aligned} \tag{4.19}$$

As in the first stage, the magnetometers take measurements and the magnetic coils are operated over 1 and 9 second intervals, respectively.

4.5.3 Wheel Spin-up

The wheel spin-up begins with magnetic torque applied to counter spin axis acceleration using equation 4.10. When the wheel acceleration is finished, this magnetic moment is applied until the pitch axis rate approaches one rotation per orbit. Since the field derivative is not being calculated as in the two previous stages, the magnetic field measurement rate is increased to 0.1 Hz. The mission mode is implemented when the body motion is null and the horizon sensor is able to measure the horizon.

4.5.4 Gain Selection

The gain for the magnetic moment as a function of the magnetometer measurement derivative, K , is estimated through the analysis and simulation of a simplified maneuver. The pitch axis of a stationary satellite is required to be rotated 90° using a non-rotating magnetic field line.

The satellite roll, pitch and yaw axes, denoted by the x, y, z subscripts and the orbital axes, denoted by 1,2,3 are assumed to coincide initially. The principal

moments of inertia are aligned with the body-fixed axes and the satellite is symmetrical about the pitch axis, where I_{zz} and I_{xx} are equal. In this case, the pitch axis is rotated 90° about the yaw axis. The initial roll, yaw, and pitch angles are:

$$\phi_0 = 0^\circ, \theta_0 = 0^\circ, \text{ and } \psi_0 = 90^\circ$$

Since the field line is on the orbital x-axis, $[B_1 B_2 B_3] = [B 0 0]$, the pitch axis is to be realigned with the orbit normal by applying a constant moment, M , in the roll axis coil and a moment based on the field derivative in the pitch axis coil (see equation 4.1). The magnetic field is transformed into body-fixed coordinates:

$$\bar{B}_{\text{body}} = (\phi)_x (\theta)_y (\psi)_x \bar{B} \quad (4.20)$$

Since the magnetic field is in the orbital x-axis and the magnet is the satellite y-axis, rotation will occur around the yaw axis. The field line is measured by the roll and pitch axis magnetometers. The field measurement and its derivative are:

$$\begin{aligned} B_x &= B \cos \psi & \dot{B}_x &= -B \dot{\psi} \sin \psi \\ B_y &= -B \sin \psi & \dot{B}_y &= B \dot{\psi} \cos \psi \\ B_z &= 0 \end{aligned}$$

Using Euler's equation including constant magnetic torque, the equations of motion are:

$$\begin{aligned} I_x \dot{\omega}_x + (I_y - I_z) \omega_y \omega_z &= 0 \\ I_z \dot{\omega}_z + (I_x - I_y) \omega_y \omega_x &= MB_y \\ I_y \dot{\omega}_y &= 0 \end{aligned} \quad (4.21)$$

Since the satellite is symmetrical about the pitch axis, its angular acceleration is null and there is no rotation about the roll axis. The angular acceleration in the roll and yaw axes are:

$$\dot{\omega}_x = 0 \text{ and } \dot{\omega}_z = \ddot{\psi} = \frac{-MB \cos \psi}{I} \quad (4.22)$$

The final equation is similar to that for a simple pendulum at the same angle from its position of rest:

$$\ddot{\psi} - \frac{-M_y B \cos \psi}{I_z} = 0 \quad (4.23)$$

Applying the energy reduction technique with the pitch axis coil, the final equation for motion in the z-axis plane is:

$$\ddot{\psi} + \frac{KB^2}{I} \psi (\cos^2 \psi) - \frac{MB}{I} \sin \psi = 0 \quad (4.24)$$

M (Am ²)	Gain	Time (s), $\psi < 5^\circ$	$\omega_{z,\max}$ (rad/s)	Comments
6	1×10^9	1750 s	-0.006 rad/s	No overshoot
6	1×10^8	660 s	-0.008 rad/s	Slight Overshoot
6	5×10^7	1350 s	-0.0087 rad/s	Oscillation
3	5×10^8	3500 s	-0.0035 rad/s	No Overshoot
3	1×10^8	440 s	-0.0053 rad/s	Slight Overshoot
3	5×10^7	1350 s	-0.0060 rad/s	Some Oscillation

Table 4.1: Results from Gain Estimation

Magnetic moments of 3 and 6 Am² were selected for the first two stages of the maneuver. Different gains were selected to estimate the gains for the simulation of the maneuver. The time at which the yaw angle was below 5°, the maximum yaw rate and a description of the motion are given in Table 4.1.

4.6 Simulation

A computer simulation of the maneuver has been performed using the numerical integration function, ode45, in MATLAB. The main program, att.m calls acq.m

through the fourth and fifth order Runge-Kutta numerical integration technique, specifying the start and finish time, and the error tolerance. These programs are listed in Appendix B.

4.6.1 Simulation Description

The satellite equations of motion are based on Euler's equations. The body angular rates in the body-fixed reference frame are related to the reference frame through quaternions and their derivatives. Background information about quaternions can be found in the noted references [4,5, and 13].

The International Geomagnetic Field Model (IGRF) model is used to determine the magnetic field values. The magnetic field values are transformed into the body-fixed reference frame and used as the magnetometer measurements and in the magnetic torque calculation. The magnetometer measurement derivative is calculated using the body angular velocities and the magnetic field components in equation 4.5.

Case	ω_x	ω_y	ω_z	q_1	q_2	q_3	q_4	α
1	0.006	-0.008	0.01	0.5	0.707	0.5	0	90°
2	0.01	0.008	0.01	1	0	0	0	180°
3	0.01	-0.008	0.01	0.5	-0.707	0.5	0	90°

Table 4.2: Initial Conditions for Simulations.

The variable, α , represents the pitch axis angle relative to the orbit normal. The angular velocities are in rad/s.

Simulations were performed for three different initial orientations and angular velocities. The initial angular velocities, quaternions, and pitch axis

angle relative to the orbit normal are given in Table 4.2. The constant moments, gains and end conditions for the first two stages are given in Tables 4.3 and 4.4.

	M_y	K_x	K_z	End Conditions
First Stage	3 Am^2	1×10^7	1×10^7	$\Delta B / \Delta t < 20 \text{ nT/s}$, $B_y > 30 \text{ } \mu\text{T}$

Table 4.3: Parameters for First Stage.

The constant moment is applied in the pitch axis coils and the other coils are used for energy reduction (see equation 4.13). The final section shows the end conditions for the transition into the next stage.

	K_y	M	Wheel Torque	Conditions for Wheel Spin-up
Second Stage	3×10^8	6 Am^2	-0.0002 Nm	$\Delta B_y / \Delta t < 20 \text{ nT/s}$

Table 4.4: Parameters for Second Stage.

For the pitch axis alignment to the orbit normal, the pitch axis coil is used for energy reduction and all coils are used to torque the pitch towards the orbit normal (see equation 4.19). The wheel spin-up torque and the condition for the transition into the spin-up stage are also given.

4.7 Results

The results for the three simulations are given in Table 4.5. The table shows the time after separation when:

- the second phase begins, (the pitch axis is aligned with the field line)
- the pitch axis is within 5° of the orbit normal,
- the body has stopped rotating

and the final inclination angle when the body has stopped rotating. The transition time is the time after launch at which the second stage begins.

Case	Transition Time	Time, $\alpha < 5^\circ$	Total Time	Final α
1	5000 s	8 500 s	15 200 s	5°
2	5200 s	8 700 s	15 500 s	5°
3	4500 s	8 000 s	13 500 s	7°

Table 4.5: Acquisition Maneuver Results.

The second stage, the pitch axis alignment with orbit normal, begins at the transition time.

More detailed results for the second case, the REVERSE LAUNCH, are presented in Figures 4.8 through 4.17. The pitch axis angle and the field line angle relative to the orbit normal are shown in Figure 4.8. At the transition point, the pitch axis aligns with the field line which is within 10° of the orbit plane throughout the simulation. After the transition, the pitch axis takes approximately 3,300 seconds to be within 5° of the normal. Table 4.6 outlines the parameters plotted for the overall maneuver.

Figure 4.8	pitch axis angle from orbit normal axis
Figure 4.9	angular velocities
Figure 4.10	wheel momentum and coil moments
Figure 4.11	field and magnetometer measurements

Table 4.6: Parameters Plotted Against Time for Figures 4.8-4.11 (overall maneuver)

Figures 4.9-4.11 show plots of the angular velocities, the wheel momentum, total angular momentum, the coil moments and the magnetic field and the magnetometer measurements for the entire maneuver. The results for the three stages of the maneuver are presented in the following sections.

4.7.1 First Stage

The objective of the first stage was to reduce the body velocities and align the pitch axis with the field line. Table 4.7 shows the parameters plotted for the first stage.

Figure 4.12	Total Angular Momentum and Energy
Figure 4.13	Total Field and Derivatives (per axis)

Table 4.7: Parameters Plotted Against Time for Figures 4.12-4.13 (First Stage)

As shown in Figure 4.12, the energy and momentum were reduced over the 5,000 second period with momentum remaining only in the pitch axis. This was a result of the yaw and roll axis magnetometers rotating in the plane approximately perpendicular to the field vector. Since the field changes were small, the energy reduction was very slow. Figure 4.13 shows the field derivatives in each axis decreased, and the pitch axis magnetometer reading nearly equal to the overall field in magnitude as the pitch axis magnetometer coincided with the magnetic field line.

4.7.2 Second Stage

The objective of the second stage was to align the pitch axis to the orbit normal.

Table 4.8 shows the parameters that are plotted for the second stage.

Figure 4.14	Pitch Axis Position and Magnetic Moments
Figure 4.15	Angular Momentum

Table 4.8: Parameters Plotted Against Time for Figures 4.14-4.15
(Second Stage)

The pitch axis was within 5° of the orbit normal approximately 3,300 seconds after the transition. Figure 4.14 shows the pitch axis angle relative to the orbit normal and its location on the orbit plane. As the axis moved towards the orbit normal, it swung around the normal. The plot of the pitch axis magnetic moment, a function of the field derivative, shows that it decreased exponentially. The increase in the frequency between the peak yaw and roll axis moments shows that the pitch rate was increasing. The pitch axis field derivative decrease and the pitch rate increase indicate that the momentum was increasing in the

pitch axis as it decreased in the roll and yaw axes. This trend is verified through the angular momentum plots in Figure 4.15.

4.7.3 Final Stage

The objective of the final stage was to spin-up the momentum and reduce the body angular rates to zero. Table 4.9 shows the parameters plotted for the final stage.

Figure 4.16	Energy, Momentum, Pitch Axis Angle
Figure 4.17	Magnetic Torque and Field

Table 4.9: Parameters Plotted Against Time for Figures 4.16-4.17
(Final Stage)

Figure 4.16 shows the body kinetic energy, total and wheel angular momentum and the pitch axis angle relative to the orbit normal. At approximately 9,500 seconds, the wheel was accelerated and the total angular momentum began to increase. The body kinetic energy remained constant as a result of the magnetic torque cancelling the wheel torque on the body. After the wheel was accelerated, the total momentum was the wheel momentum and the body energy was reduced to zero.

During this procedure, the pitch axis angle relative to the orbit normal increased after the wheel acceleration finished. This increase was due to magnetic torque in the roll and yaw axes created to reduce the pitch axis angular velocity (see equation 4.11). As shown in Figures 4.16 and 4.17, this torque slowly increased after the wheel spin-up and precessed the momentum vector a few degrees from the orbit normal.

4.8 Discussion

The objective of the fast maneuver was to have:

- the solar arrays face the sun without a large depth of discharge
- an autonomous maneuver
- a 'safe-hold' mode in case the horizon sensor fails

In these simulations, the pitch axis was within 5° of the orbit normal 8,500 seconds after separation, requiring approximately a 49% depth of discharge. Table 4.10 shows the battery charge capacity required for the first 8,800 seconds assuming no solar power was available.

This DOD is high relative to the eclipse period discharge range of 20%-30%. If it is desirable to reduce the DOD, the time necessary to complete the maneuver can be reduced by increasing the constant moments applied and by optimizing the gain selection for the damping moments accordingly in both the first and second stage.

Subsystem	Component	Peak Period (W)	Period Req. (Hours)	Rating (Ahr)
TT&C&C	Microprocessor	0.5	37.9	0.68
	Logic	5	4.52	1.29
	Isotropic Antenna	3		
ACDS	Momentum Wheel	20	0	0
	Coils (Average)	1.05	2.4	0.09
	Magnetometers	0.3	2.4	0.064
	Sun Sensor	0.2		
Power	Regulation	2	4.52	0.32
	Total	35.5		2.44

Table 4.10: Battery Charge Capacity Required for Fast Acquisition Maneuver.

The total time period for the battery charge capacity required is 8,800 seconds. The bus voltage is assumed to be 28 Volts.

The attitude acquisition maneuver used a small amount of power relative to the other components. The maneuver required a charge capacity of 0.09 Ahr since

only the torque rods, magnetometers, and sun sensor are used and the wheel is accelerated after the arrays are facing the sun.

The maneuver could be orchestrated by an on-board computer. The coil, magnetometer operation, the calculation of the orbit normal and field line position relative to the satellite axes, and the transitions between the stages could be performed on-board. To model the sensor errors, operation of the coils and the magnetometers, the implementation procedure should be examined using real-time computer and hardware simulations.

An advantage of the designed maneuver is the first stage. The pitch axis alignment with the field line works from any orientation and initial velocity and with stored angular momentum. It could be performed as a safe hold mode, in case the horizon sensor was to lose the horizon, assuming the problem or failure does not involve the coils and the magnetometers.

The disadvantage of this maneuver is the fact that attitude estimation is necessary during the second phase. The gyrocompassing technique is more robust since it only requires magnetometers and the coils without attitude estimation. The problem with the gyrocompassing maneuver is related to time. Since the power is consumed by the other subsystems, the batteries would be drained before the arrays face the sun.

4.9 Conclusions

- The 'gyrocompass' acquisition maneuver is the ideal technique for a momentum-biased satellite in a high inclination orbit. The maneuver, taking approximately 30 orbits to complete, cannot be applied due to power reasons.

- An acquisition maneuver that takes approximately 1.5 orbits to reorient the solar arrays to face the sun has been developed. It can be performed autonomously for any initial orientation and for large body rates. The maneuver should be tested using real-time computer and hardware simulations.

4.10 Future Research

- Simulate the maneuver using real-time simulation software so attitude estimation and magnetic control can be simulated with errors.
- In the simulations, evaluate performance with different sensors at various sample rates and accuracies.
- The design of SILA is currently being modified. Both maneuvers should be re-evaluated given the new configuration and power consumption.
- Modify the maneuver for a satellite with deployable solar arrays, evaluating whether the deployment occurs before or after the acquisition maneuver.
- Perform hardware simulation of coils and magnetometer operation.

Figure 4.6: Plot of the Pitch Axis Angle Relative to the Orbit Normal for a 90° Maneuver using the 'Gyrocompassing' Technique.

The angle plotted above is represented by α in the diagram on the right. The pitch axis is attached to the satellite body and the three orthogonal axes represent the rotating orbit reference frame.

The gyrocompassing technique is described in Section 4.1.2. For the simulation, the wheel momentum, h , is constant (-0.2 Nms), and the three magnetic coils are pulsed at 0.25 Am^2 . For each axis, the sign of the moment, M , is opposite to the sign of field measurement derivative, $B\dot{\text{dot}}$.

Figure 4.7: Snapshots of the Satellite Position Relative to the Orbit Reference Frame for the Pitch Axis Alignment with the Orbit Normal.

In the first picture in the top left hand corner, the satellite pitch axis is initially aligned with the field line in the orbit plane, approximately 90° from the orbit normal. Following the subsequent snapshots in rows from left to right starting from the top row, the pitch axis moves towards the orbit normal as it spirals around it.

The axes on the right of this explanation represent the ones shown in the snapshots. The three orthogonal axes are the orbit x-axis, z-axis and y-axis (the orbit normal). The pitch axis is an axis in the body-fixed reference frame.

Figure 4.8: Plot of the Pitch Axis and the Field Line Angles Relative to the Orbit Normal for the Reverse Launch Case using the Fast Acquisition Maneuver.

The parameters are obtained from the simulation of the 180° maneuver, case 2, described in Section 4.6.1. The angle plotted above is represented by α in the diagram on the right. The pitch axis is attached to the satellite body and the three orthogonal axes represent the rotating orbit reference frame.

After 5,000 seconds, the pitch axis is approximately aligned with the field line. At this point, the pitch axis alignment to the orbit normal begins. The pitch axis is within 5° of the orbit normal after 8,700 seconds.

Figure 4.9: Plots of the Angular Velocities for the Body Roll, Yaw, and Pitch Axes for the Reverse Launch Case using the Fast Acquisition Maneuver.

The body has initial angular velocities in all axes from launch vehicle separation. In the first stage, the pitch axis alignment with the magnetic field line, magnetic torque is used to reduce angular velocity.

In the second stage, the pitch axis alignment to the orbit normal, the magnetic torque increases the pitch axis angular velocity and reduces the others to zero. The second stage begins 5,200 seconds into the maneuver. The pitch axis angular velocity reduction occurs after the wheel spin-up, which starts 9,500 seconds into the maneuver.

Figure 4.10: Plots of the Wheel Momentum and the Roll, Yaw and Pitch Axis Coil Moments for the Reverse Launch Case using the Fast Acquisition Maneuver.

The wheel spin-up, shown by the increase in momentum (top, left), begins 9,500 seconds in the maneuver. At this point in time, the arrays are facing the sun and the pitch axis is within 5° of the orbit normal (see Figure 4.8).

The magnetic coil moments are given in Am². In the first stage, the pitch axis moment is constant and the roll and yaw axis coil moments are based on the magnetometer measurement derivatives to reduce the body angular velocity (see equation 4.1). Plots of and explanations about the magnetic moments for the second stage and magnetic torques for the third stage are given in Figures 4.14 and 4.16 respectively.

Figure 4.11: Plots of the Magnetic Field Strength and the Roll, Pitch, and Yaw Axis Magnetometer Measurements for the Fast Acquisition Maneuver.

As shown in the top left-hand plot and in Figure 4.8, the magnetic field line is primarily in the orbit plane. The field strength values are given in the rotating orbit reference frame. The magnetometer measurement locations coincide with the principal axes of the satellite body. Strengths are given in μT .

Figure 4.12: Plot of the Body Kinetic Energy and the Angular Momentum for the First Stage of the Fast Acquisition Maneuver.

In the first stage, magnetic torque is used to reduce the body angular rates created during launch vehicle separation and to align the pitch axis with the field line. The body angular rate reduction is shown through the reduction in body kinetic energy (top) and the angular momentum (bottom). Both energy and momentum remain almost only in the pitch axis when the pitch axis is aligned with the field line.

Figure 4.13: Plots of the Field Derivatives and the Pitch Axis Magnetometer Measurements for the First Stage of the Fast Acquisition Maneuver.

The field derivatives are calculated in each axis using equation 4.5. They represent the rate of change of the magnetometer measurements and are used to reduce the body angular rates in the first stage using equation 4.1. The reduction of the body angular rates, created during the launch vehicle separation, are shown through the reduction of the field derivatives.

The comparison of the total magnetic field strength with the field measured in the pitch axis (bottom, right) shows that the pitch axis is aligned with the field line at the end of the first stage.

Figure 4.14: Plots of the Pitch Axis Angle Relative to the Orbit Normal, Its Component in the Orbit Plane, and the Magnetic Moments for the Second Stage of the Fast Maneuver.

The angle plotted above (top, left) is represented by α in the diagram on the right. The pitch axis is attached to the satellite body and the three orthogonal axes represent the rotating orbit reference frame.

The plot of the pitch axis vector components on the orbit plane (top, right) shows the motion of the pitch axis as it aligns with the orbit normal.

The magnetic moments in the satellite roll and yaw axes are used to torque the pitch axis towards the normal and the pitch axis coil is used for motion damping (see equation 4.19).

Figure 4.15: Plots of the Angular Momentum in the Roll, Yaw, and Pitch Axes for the Second Stage of the Fast Acquisition Maneuver.

During the pitch axis alignment to the orbit normal, the second stage, the magnetic moments used to torque the pitch axis towards the orbit normal also torque the body about its pitch axis. The pitch axis coil is also used for motion damping. As a result, the angular momentum in the roll and yaw axes (top) is reduced, and almost all of the angular momentum is in the pitch axis (bottom).

Figure 4.16: Plots of the Magnetic Torque in the Body-Fixed Reference Frame for the Final Stage of the Fast Acquisition Maneuver.

In the final stage, magnetic torque is used to cancel the wheel spin-up torque created in the pitch axis (bottom). Torque is also created in the roll and yaw axes (top) by the moments used for the torque cancellation (see equation 4.10). These torques precess the pitch axis angular momentum (see Figure 4.17).

Figure 4.17: Plots of the Body Kinetic Energy, Total Momentum, Pitch Axis Angle Relative to the Orbit Normal, and Wheel Momentum for the Final Stage of the Fast Acquisition Maneuver.

The results for the final stage are described in Section 4.7.3. The purpose of this stage is the momentum wheel spin-up and body angular rate reduction.

The body kinetic energy plot (top, left) shows the angular rate reduction and the total and wheel momentum plots (right) show that the remaining momentum is stored in the momentum wheel at end of the stage.

The angle plotted above (bottom, left) is represented by α in the diagram on the right. The pitch axis is precessed by a few degrees due to the roll and yaw axis torques.

Chapter 5.0

Attitude Estimation Using GPS and Magnetometers

The third and final topic in the trilogy involves attitude estimation. As shown in Section 2.6.3, five components that are capable of estimating attitude are serving various functions on the satellite. Two components with attitude estimation capability that are necessary for other functions are the Global Positioning System (GPS) receiver and the 3-axis magnetometers. Relative to Earth and star sensors that cost in the order of hundreds of thousands of dollars, both systems are inexpensive. If one or both of these components could fulfill the attitude estimation needs for SILA, the other sensors would not be required.

Attitude is estimated using magnetometers by comparing their magnetic field measurements to a field model. The measurement system is reliable since the field is measurable from any satellite orientation. It can be performed at a high update rate but its estimation accuracy is limited to a few degrees in each axis due to the accuracy of the model.

An attitude estimation accuracy of less than 1° is obtainable using the received GPS signal phase difference for antennas at different locations on a

satellite. While potentially being more accurate than magnetometer measurements, GPS phase difference measurements are not as reliable. Signal blockage and a low update rate limit its capability to provide attitude information during high angular rate motions. Signal reflections on the satellite body may cause a large variation in estimation error.

Attitude information can be provided for both measurement systems using Kalman filtering. Estimates are calculated in the filter using a model of the satellite attitude dynamics and the measurements. An autonomous attitude estimation system for satellites with similar pointing requirements to SILA has been proposed by the thesis author and the principal author of the paper by Chu et al [6].

In this system, GPS and magnetometer measurements are filtered individually and the two sets of attitude estimates are then averaged or 'fused' in order to have one set of estimates. Estimate fusion is a term often used to describe the weighted averaging of two or more sets of estimates. An optimal method of weighting the estimates from separate Kalman filters, designed and simulated for a lunar rendezvous, is being considered for the proposed attitude estimation system.

The following sections provide background information on Kalman filtering, the optimal weighting method and its scalar approximation, and attitude estimation using magnetometers and GPS. The proposed attitude estimation system is described and SILA's attitude estimation requirements are outlined to examine if the proposed system can fulfill these requirements.

The proposed attitude estimation system is simulated in SIMULINK in order to examine the performance of the optimal fusion method and its scalar approximation.

5.1 Kalman Filtering and Estimate Fusion

A detailed explanation of extended and linear Kalman filters is given in the noted references [19,20]. The fusion method being considered in this chapter is optimized based on the error information provided by the Kalman filters. This section outlines the equations and the variables for a linear Kalman filter that are essential to the simulation and the fusion method.

5.1.1 Kalman Filters

The general state space form of a continuous system with measurements at discrete moments in time can be written as:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t) + \mathbf{G}(t)\mathbf{w}(t); \mathbf{x}(t_0) = \mathbf{x}_0 \\ \mathbf{y} &= \mathbf{C}(k)\mathbf{x}(k) + \mathbf{v}(k); k = 1, 2, \dots, N\end{aligned}\tag{5.1}$$

The first equation is called the *system equation*. The state vector, $\mathbf{x}(t)$, is composed of the attitude estimates.

The attitude estimates used in this chapter consist of the roll, pitch, and yaw angles, ϕ , θ , and, ψ , used in the Euler angle transformation between the orbital and body-fixed reference frames, and the angular velocities, ω_x , ω_y , and ω_z in the body-fixed pitch, roll, and yaw axes.

The system state matrix, $A(t)$, consists of the satellite equations of motion and contains sinusoidal functions of the attitude angles. $B(t)$ and $u(t)$, the input matrix and vector, are used to represent functions for any known external torques, while $G(t)$ and $w(t)$, the stochastic process error matrix and vector, are used to estimate unknown system noise.

The second equation is known as the *observability equation*. The observation or measurement vector, $y(k)$, is related to the state vector through the observability matrix, $C(x)$. The relationship between the state vector and the magnetometer and GPS system measurements is outlined in the upcoming sections. Measurement noise is estimated using $v(k)$, the stochastic measurement error vector.

The Kalman filter updates the state estimate vector, $\hat{x}$, at t_k , the measurement time, using the equation:

$$\hat{x}(k) = \tilde{x}(k) + K(k)(y(k) - C(k)\tilde{x}(k)) \quad (5.2)$$

where $K(k)$ is the Kalman filter gain. The estimate vector calculated at the measurement time, t_{k-1} , is updated to give $\tilde{x}(k)$ by numerically integrating the *system equation* over the time increment, $t_k - t_{k-1}$. This step is known as the *time update*. The second step consists of adding the scaled difference between the measurement and the measurement estimate. It is known as the *measurement update*. The Kalman filter gain, $K(k)$, is updated at each measurement using the equation:

$$K(k) = \tilde{P}(k)C(k)(C(k)\tilde{P}(k)C(k) + R_v)^{-1} \quad (5.3)$$

where $\tilde{P}(k)$ is the steady state covariance matrix of the error and R_v is the measurement noise covariance matrix. By increasing R_v , the gain is decreased and the *measurement update* is de-emphasized. The weight of the update is dependent on the designer since R_v is a function of v , a vector selected to estimate unknown measurement noise. For each measurement, the *time update* and *measurement update* equations of the steady state covariance matrix of error, $\tilde{P}(k)$ and $\hat{P}(k)$, are:

$$\begin{aligned}\tilde{P}(k) &= \Phi(k)\hat{P}(k-1)\Phi(k)^T + \Gamma(k)V_{ww}\Gamma(k)^T \\ \hat{P}(k) &= (I - K(k)C(k))\tilde{P}(k) \\ \text{where } \Phi(k) &= e^{A(k)(t_k - t_{k-1})} \text{ and } \Gamma(k) = \int_{t_{k-1}}^{t_k} \Phi(t_k - \tau)G(\tau)d\tau\end{aligned}\tag{5.4}$$

As shown in equation 5.4, the measurement update covariance matrix of error is a function of the Kalman gain. As it will be shown in the next section, the optimized fusion filter performance is determined by $K(k)$ and $\hat{P}(k)$, a function of $K(k)$, from each Kalman filter.

5.1.2 Estimate Fusion

The proposed system architecture (see Section 5.3) will have the measurements filtered separately. A method is required to fuse the Kalman filter estimates to give one set of state estimates. The final set of estimates, $\hat{x}_{\text{final}}$, is a weighted average of the two sets of estimates:

$$\hat{x}_{\text{final}} = (I - W)\hat{x}_{\text{gps}} + W\hat{x}_{\text{mag}}\tag{5.5}$$

where I is the identity matrix and W is a weighting matrix.

The ideal matrix would place most or all of the weight on the more accurate estimates. Three methods of calculating the weighting matrix are outlined:

- equal weighting
- optimal weighting
- the scalar approximation of the optimal weighting

5.1.2.1 Equal Weighting

A simple fusion method takes an equal weighting of the estimates by placing 0.5 along the diagonal of the weighting matrix. This method is computationally simple but does not take advantage of the error information provided by the Kalman filters. The technique is used as a comparison for the optimized method during the simulation.

5.1.2.2 Optimal Weighting

An algorithm that fuses estimates using the error information from the Kalman filters has been derived and simulated for a lunar rendezvous in Carpenter and Bishop [21]. The weighting matrix, W , is found by minimizing the trace of a covariance matrix of error, $\hat{P}_f$, where:

$$\hat{P}_f = \hat{P}_{gps} - W_{opt}(\hat{P}_{gps} - \hat{M}) \quad (5.6)$$

and $\hat{M}$ is the cross-correlation matrix. Similarly to the covariance matrix of error in the Kalman filter, the cross-correlation matrix has the following time and measurement update equations:

$$\begin{aligned}\tilde{M}(k) &= \Phi(k)\hat{M}(k-1)\Phi(k)^T + \Gamma(k)V_{ww}\Gamma(k)^T \\ \hat{M}(k) &= (I - K_{\text{gps}}C_{\text{gps}}^T)\tilde{M}(k)(I - K_{\text{mag}}C_{\text{mag}}^T)\end{aligned}\quad (5.7)$$

Since the Kalman filters use the same equations of motion and system noise matrices and vectors, the estimates are correlated. By not accounting for the correlation during the fusion, the filter could underestimate the covariance matrix, resulting in large estimate errors and possibly divergence [21]. The optimal weighting matrix equation is:

$$W_{\text{opt}} = (\hat{P}_{\text{gps}} - \hat{M})(\hat{P}_{\text{gps}} - \hat{M} - \hat{M}^T + \hat{P}_{\text{mag}}) \quad (5.8)$$

5.1.2.3 Scalar Approximation of the Optimal Weighting

In the above mentioned publication [21], the following equation is given as the *scalar approximation* of the optimal weighting matrix:

$$W_{\text{opt}} = \frac{\text{tr}\hat{P}_{\text{gps}} - \text{tr}\hat{M}}{\text{tr}\hat{P}_{\text{gps}} + \text{tr}\hat{P}_{\text{mag}} - 2\text{tr}\hat{M}} \quad (5.9)$$

The advantage of the approximation is the reduction of the information transferred and the number of calculations. By neglecting the cross-covariance, the only information transferred would be the trace of the Kalman filter covariance matrices. The weighting matrix equation ignoring the cross-covariance is:

$$W_{\text{opt}} = \frac{\text{tr}\hat{P}_{\text{gps}}}{\text{tr}\hat{P}_{\text{gps}} + \text{tr}\hat{P}_{\text{mag}}} \quad (5.10)$$

5.2 Attitude Estimation

This section outlines how the attitude angles and the angular velocities are calculated based on the magnetometer and GPS system measurements. The Kalman filter observability equations for both measurement systems are also defined.

5.2.1 Attitude Estimation using Magnetometers

Magnetometers are reliable, lightweight, low power-consuming sensors that measure both the direction and the size of the magnetic field. With knowledge of the Earth's magnetic field, their measurements can be used to determine the attitude. However, the field shifts position and fluctuates in magnitude making it difficult to model accurately.

For low Earth orbits, the International Geomagnetic Field Model (IGRF) provides a harmonic model of the Earth's field in spherical co-ordinates (r, θ, ϕ) . A higher accuracy requires that a higher order of the spherical harmonics series expansion to be used. However, the order selected is limited by the computing time which becomes the deciding factor in the order used. As demonstrated by Landriech [22], the 10th order model rms error in field orientation of 0.11° is smaller than the 4th order model rms error of 2.5° yet

the computing time is four times longer. The relationship between the measured and modelled fields is:

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = M_{ob} M_{so} \begin{bmatrix} B_r \\ B_\theta \\ B_\phi \end{bmatrix} \quad (5.11)$$

where B_r , B_θ , and B_ϕ are the modeled magnetic field components in spherical co-ordinates and B_x , B_y , and B_z are the measured components in body-fixed co-ordinates. M_{ob} and M_{so} are the transformation matrices from the orbital to body-fixed reference frames and spherical to orbital reference frames respectively. Using the 321 Euler angle sequence, the transformation matrix, M_{ob} , is found to be:

$$M_{ob} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi - \cos\phi\cos\psi & \cos\theta\sin\phi \\ \cos\phi\sin\theta\cos\psi - \sin\phi\sin\psi & \sin\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\theta\cos\phi \end{bmatrix} \quad (5.12)$$

Attitude angles can be determined by solving for M_{ob} , and in the case of the extended Kalman filter, equation 5.11 is used as the non-linear *observation equation*.

Error results mainly from the model inaccuracy. Other sources of error include the misalignment of the magnetometers relative to each other, the measurement of secondary or induced field from devices such as the magnetic coils, and the misalignment of the magnetometers with the body-fixed reference frame axes.

5.2.1.1 Flight Experience

Very few attitude determination systems have been developed using magnetometer data. Autonomous attitude determination systems using only the Kalman filtering of magnetometer measurements have been developed by Psiaki and by Hodgart [23] for gravity gradient stabilized satellites. The system developed by Psiaki was flown on LACE, a 1400 kg gravity gradient stabilized spacecraft. The extended Kalman filter produced estimates based on the satellite attitude dynamic model that uses Euler's equation and quaternion kinematics, and measurements from a 3-axis magnetometer. The system was estimated to have a 3σ accuracy capability of 1.8° in roll, 2.4° in pitch, and 2.8° in yaw [24].

5.2.2 Attitude Determination Using GPS

As described in Section 2.5, The NAVSTAR Global Positioning System (GPS) is a satellite constellation that enables a user equipped with the appropriate receiver to determine position, velocity, and time. Attitude determination using GPS has been shown to be possible using the signal strength and the received carrier phase difference. Signal strength measurements, requiring only one antenna, are a function of the azimuth and elevation of the GPS satellite and the target satellite. The azimuth and elevation of the target satellite is dependent on its attitude, making the calculation of attitude possible. This method is not accurate as a result of the non-sphericity and flat beam of the antenna used to receive the GPS signal [6].

Carrier phase difference, a more accurate system, is the method used in the proposed system. Attitude information can be obtained from the phase difference of the GPS signal received by two antennas located on the target spacecraft. The phase difference is related to the range difference through the signal frequency, f_s , and the speed of light, c :

$$\Delta\Phi_m(k) = -\frac{f_s}{c} \Delta\rho_m \quad (5.13)$$

As illustrated in Figure 5.1, the range difference, $\Delta\rho_m$, is the same as the projection of the baseline, b , the line connecting the antennas expressed in satellite *body-fixed co-ordinates*, onto the line of sight unit vector, s :

$$\Delta\rho_m = b \cdot s \quad (5.14)$$

The line of sight vector determined from the locations of the GPS and target satellite, and is expressed in *inertial co-ordinates*.

Figure 5.1: Phase Difference as a Function of Baseline and Line of Sight.

Attitude angles are found by solving the transformations from inertial to orbital, M_{io} , and orbital to the satellite body-fixed co-ordinates, M_{ob} , if the line of sight and baselines are known. In order to determine three attitude

angles, a plane must be formed by the antennas, requiring at least three antennas.

Since the phase difference only accounts for the difference between the phase of the final wavelength, the difference in full integer wavelengths between the received signals, the integer ambiguity, ΔN , must also be determined. Including the electrical line bias between the antennas, β , and the measurement noise, v , the final equation is written as:

$$\Delta\Phi_m(k) = -\frac{f_s}{c} \mathbf{b}_b^T \mathbf{M}_{ob} \mathbf{M}_{io} \mathbf{s}_i + \Delta N + \beta + v \quad (5.15)$$

The unknown line bias can be eliminated by taking the difference between two independent single phase differences. This function is known as the *double difference*:

$$[\Delta\Phi_{m,2}(t_r) - \Delta\Phi_{m,1}(t_r)] = -\mathbf{b}_b^T \mathbf{M}_{ob} \mathbf{M}_{io} (\mathbf{s}_{i,2} - \mathbf{s}_{i,1}) + (\Delta N_2 - \Delta N_1) + (v_2 - v_1) \quad (5.16)$$

In the proposed system, the magnetometer attitude estimates can be used to help solve the integer ambiguity. The following equation is used for the simulation:

$$[\Delta\Phi_{m,2}(t_r) - \Delta\Phi_{m,1}(t_r)] = -\mathbf{b}_b^T \mathbf{M}_{ob} \mathbf{M}_{io} (\mathbf{s}_{i,2} - \mathbf{s}_{i,1}) + v \quad (5.17)$$

In the Kalman filter, this equation is used as the non-linear *observation equation*. In the proposed system, the *system equation* is linearized using the magnetometer estimates. By linearizing the observation equation, a linear Kalman filter can be implemented. The linearization can be performed by either using the magnetometer estimates or, assuming the angle variation is small, by using a reference phase difference, $\Delta\Phi_o$:

$$[\Delta\Phi_{o,2} - \Delta\Phi_{o,1}] + [\delta\Delta\Phi_2 - \delta\Delta\Phi_1] = -b_b^T(\delta M_{ob})(M_{ob})M_{io}(s_{i,2}-s_{i,1}) \quad (5.18)$$

For a perturbation for the nominal position, the small angle approximation can be applied to M_{ob} giving:

$$\delta M_{ob} = \begin{bmatrix} 1 & \delta\psi & -\delta\theta \\ -\delta\psi & 1 & \delta\phi \\ \delta\theta & -\delta\phi & 1 \end{bmatrix} \quad (5.19)$$

The right hand side can be separated into the reference component and the measurement component:

$$[\Delta\Phi_{o,2} - \Delta\Phi_{o,1}] + [\delta\Delta\Phi_2 - \delta\Delta\Phi_1] = -b_b^T M_{ob} M_{io}(s_{i,2}-s_{i,1}) - b_b^T \delta M_{ob} M_{io}(s_{i,2}-s_{i,1}) \quad (5.20)$$

The observability equation, $y = Cx$, for the linear Kalman filter is:

$$[\delta\Delta\Phi_2 - \delta\Delta\Phi_1] = \begin{bmatrix} b_y(s_{z,1}-s_{z,2}) - b_z(s_{y,1}-s_{y,2}) \\ b_z(s_{x,1}-s_{x,2}) - b_x(s_{z,1}-s_{z,2}) \\ b_x(s_{y,1}-s_{y,2}) - b_y(s_{x,1}-s_{x,2}) \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \begin{bmatrix} \delta\phi \\ \delta\theta \\ \delta\psi \\ \delta\omega_x \\ \delta\omega_y \\ \delta\omega_z \end{bmatrix} \quad (5.21)$$

External sources of error in GPS double phase difference measurements are due to:

- multipath effects (reflections)
- change in antenna baseline length
- antenna pattern
- receiver noise
- line of sight vector error

A discussion on these error sources is given in Lightsey, Cohen and Parkinson [25]. The most critical source of error is the multipath effect. A GPS signal reflecting off a surface of the satellite takes longer to reach the antenna than the direct signal, resulting in a larger phase value. The phase

error is variable since and the reflection paths are dependent on the line of sight between the GPS and target satellite and its attitude.

5.2.2.1 Flight Experience

Flight experience of attitude estimation systems using GPS is very limited. The capability of the system was demonstrated on RADCAL, US Air Force-sponsored satellite launched in June of 1993. The carrier phase difference measurements were obtained during the mission and processed on the ground. In their preliminary results, rms attitude estimation errors from 0.5° to 1° have been estimated for unfiltered measurements. However, the true accuracy is not known since magnetometers were the only other source of attitude estimation. Based on filtered data reduction, an rms attitude error of better than 0.1° has been estimated for a one meter baseline if the measurements are Kalman filtered.

Currently, there are many experiments using GPS such as GADACS (GPS Attitude Determination and Control System) [1]. The GPS experiment was flown on the SPARTAN 'OAST' Flyer, a satellite deployed on the Space Shuttle mission STS-69 in early 1996.

The intent of the mission is to quantify the attitude estimation capability of the GPS system with gyroscope measurements. Star tracker updates to calibrate the gyroscope are performed every other orbit. GADACS is also to be used autonomously in the closed-loop attitude control system for the final portion of the mission.

5.3 Proposed Attitude Estimation System

The purpose of this section is to outline the proposed attitude estimation system that uses GPS system and magnetometers and examine if the system could fulfill the acquisition, pointing control, and determination goals for SILA. The accuracy and update rate requirements are given in Table 5.1.

Task	Requirements
Acquisition	<ul style="list-style-type: none"> • reasonable accuracy (a few degrees) • low update rate (1 or 2 Hz)
Pointing Control	<ul style="list-style-type: none"> • pointing accuracy (less than 0.5°) • low update rate (1 or 2 Hz)
Attitude Determination	<ul style="list-style-type: none"> • less than 0.02° in all axes

Table 5.1: Update Rates and Accuracy Requirements for ACDS Functions

Due to the difference in reliability and update rates of the measurement systems, a loosely-integrated attitude estimation system has been developed by the author and the principal author of the paper by Chu et al. [6].

Figure 5.2: Architecture of the Proposed Attitude Estimation System.

Estimates from the magnetometer Kalman filter are used to linearize the GPS Kalman filter. A weighted average is taken of both sets of attitude estimates in the 'fusion filter' to give one set of estimates.

The proposed attitude determination system is outlined in Figure 5.2.

Since the system used in the magnetometer Kalman filter, the satellite

equations of motion, is non-linear, an extended Kalman filter (EKF) is necessary.

By using the state estimates from this filter in the GPS system filter, the equations of motion are linearized and a linear Kalman filter (KF) can be applied. This filter is more stable and requires a small number of calculations than the EKF. The magnetometer estimates would also be used to solve the integer ambiguity, ΔN , and to reinitialize the GPS system Kalman filter. By performing these tasks, the update rate and the reliability of the GPS measurement Kalman filter is increased.

The system could fulfill the requirements for acquisition and normal pointing but not for attitude determination. The determination accuracy of 0.02° in all axes necessary after the mission for the payload data analysis is too stringent for Kalman filtered GPS carrier phase difference measurements.

During the attitude acquisition maneuver, the satellite will experience large angular velocities. Filtered magnetometer measurements would be used independently because of their reliability and high update rate. They are always capable of providing measurements since the field is measurable from any orientation.

During the aeronomy phase, a pitch axis rotation rate of once per orbit and small attitude angle corrections are required for limb tracking and disturbance torque corrections. Attitude estimation update rates of 1 or 2 Hz are acceptable, and a pointing accuracy of less than 0.5° is necessary in each axis.

Using the proposed system, the estimates from the GPS system and magnetometer measurements are averaged to give one set of state estimates. The role of the magnetometer measurements is to provide the coarse attitude estimates while the GPS system provides the focus in order to meet the desired accuracy.

For this system to have the attitude estimation accuracy that is small enough to achieve the required pointing accuracy in each axis, the averaging technique should put more weight on the more accurate estimates.

5.4 Evaluation of Proposed Attitude Estimation System

The purpose of the third and final section of this trilogy is to simulate the proposed attitude estimation system to compare the equal, optimal, and scalar approximated optimal weight matrices outlined in Section 5.1.2.

Ideally, all of the weight should be placed on the most accurate estimates. In the proposed attitude estimation system, the GPS system estimates are more nominally accurate, and they should have a greater weighting in the final set of estimates.

However, in cases of large error in the GPS measurements, greater emphasis should be put on the magnetometer estimates. The performance of the three weighting methods should be compared for different GPS measurement accuracies.

5.5 Simulation

An outline of the test case and the simulation is given in this section.

5.5.1 Test Case

The attitude estimation system is modelled in the simulation using a three-axis stabilized momentum-biased microsatellite in a polar, circular, low Earth orbit. A disturbance torque, a sinusoidal function of the pitch angle, is included about the pitch axis. Since the pitch axis angular rate is independent of the motion in the other axes, the satellite will experience sinusoidal motion only in the pitch axis. The satellite and orbital parameters and amplitude and frequency of the pitch angular motion are given in Table 5.2.

mass, m	40 kg
principal moments of inertia (I_x, I_y, I_z)	(2,3,2) kgm^2
wheel momentum, h	1 Nms
orbit radius, a and rate, ω_o	7175 km, 0.001 rad/s
orbit inclination, i	98.6°
frequency of pitch angular motion	0.314 Hz
amplitude of pitch angular motion	0.05 radians

Table 5.2: Satellite and Orbital Parameters for Simulation

The satellite is assumed to be a rigid body and to have principal moments of inertia aligned with the body-fixed co-ordinate axes. Its equations of motion are used for the system state equations for the Kalman filter. Using a 321 Euler sequence (yaw, theta, roll), the system equations are:

$$\begin{aligned}
\dot{\phi} &= \omega_x + (\omega_y \sin \phi + \omega_z \cos \phi) \tan \theta + \omega_0 \frac{\sin \psi}{\cos \theta} \\
\dot{\theta} &= \omega_y \cos \phi - \omega_z \sin \phi + \omega_0 \cos \psi \\
\dot{\psi} &= \frac{\omega_z \cos \phi + \omega_y \sin \phi}{\cos \theta} + \omega_0 \sin \psi \tan \theta \\
\dot{\omega}_x &= -((I_z - I_y)\omega_0 + h) \frac{\omega_z}{I_x} \\
\dot{\omega}_y &= -0.157^2 \sin \theta \\
\dot{\omega}_z &= ((I_x - I_y)\omega_0 + h) \frac{\omega_x}{I_z}
\end{aligned} \tag{5.22}$$

5.5.2 Simulation Design

The simulation is composed of three steps:

- the exact state and magnetic field vector calculations,
- the exact GPS double difference and line of sight calculations, and
- the final attitude estimate error calculations.

5.5.2.1 Exact Satellite States and Magnetic Field

Using the MATLAB file, `traj.m`, the exact attitude angles and angular velocities in the body-fixed reference frame and the magnetic field in the inertial reference frame are calculated at a defined time interval, T_s , for a defined number of steps, n , and saved into the data file, `data1.mat`. The satellite dynamics are modelled using `mot.m` in the fourth and fifth order Runge-Kutta numerical integration algorithm, `ode45`. The initial attitude angles in radians and angular velocities in rad/s are:

$$[\phi, \theta, \psi, \omega_x, \omega_y, \omega_z] = [0.05, 0.05, 0, 0, 0, 0.005]$$

The magnetic field model, written by CB van de Kolk, uses the IGRF 1990 constants for g and h , the time dependent Gaussian coefficients [19]. The files, **magn10.m** and **magn5.m** correspond to 10th and 5th order models.

5.5.2.2 Double Phase Difference and Line of Sight

Double phase difference and line of sight difference calculations are performed in the SIMULINK file, **gps3.m**. The SIMULINK simulation configuration is given in Figure 5.3.

Figure 5.3: GPS Double Difference and Line of Sight Calculation Program, **gps3.m**

The positions of four orbiting GPS and target satellites are calculated in the inertial reference frame at each time interval. Using the exact attitude angles and the target satellite position, the three antenna positions are

calculated in the inertial reference frame in `ant.m`. Three antennas are required to form a plane in order to solve the three attitude angles. The antennas are placed on the satellite face perpendicular to the y-axis in the body-fixed co-ordinates. Their position relative to the body center (in metres) and the three baselines in body-fixed co-ordinates are:

$$\begin{bmatrix} \text{ant}_1 \\ \text{ant}_2 \\ \text{ant}_3 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0.5 \\ -0.5 & 0 & 0.5 \\ 0.5 & 0 & -0.5 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} \text{ant}_1 - \text{ant}_2 \\ \text{ant}_2 - \text{ant}_3 \\ \text{ant}_3 - \text{ant}_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

Using the antenna and GPS satellite positions, the exact double phase difference and line of sight unit vector differences are calculated using `exact.m` and `unit.m` respectively. Two sets of double phase differences are calculated the pseudoranges for GPS satellites 1 and 3, and GPS satellites 2 and 4. The double phase difference and line of sight unit vector differences are then stored with the exact attitude angle, magnetic field values, and measurement time into the data file, `r_mat.mat`.

5.5.2.3 Measurement and Filter Simulation

The measurement and filter simulation are conducted in `maggps.m` using the data supplied by `r_mat.mat`. The SIMULINK simulation configuration is given in Figure 5.4.

Figure 5.4: Sensor and Filter Simulation Program, `maggps.m`

5.3.2.4 Attitude Determination using GPS and Magnetometers

Double difference measurements with uniformly distributed noise and the line of sight values are inputs for the GPS linear Kalman filter, `filtdd.m`. The magnetometer estimates for the most recent measurement time are used in the satellite equations of motion. The linearized observability matrix found in Lightsey, Cohen and Parkinson [25] is implemented in the filter. One set of state estimates is calculated based on the two sets of double difference measurements.

The magnetometer measurements have two error sources. In order to simulate the modeling error, the magnetometer field values are obtained from the tenth order model while the filter field values, used for the

measurement estimation, are obtained from the fifth order model. In the file **magnet.m**, the magnetometer simulation, the field values are transformed from the orbit reference frame into the body-fixed reference frame using the attitude angles *with added noise* to simulate measurement error. The field measurements are sent to the extended Kalman filter in the file, **mag_ekfo.m**.

5.3.2 Estimate Fusion

The function, **fusion.m**, determines the final estimates based on three methods and outputs the following values to **theta.mat**:

- time
- pitch estimate error
 - GPS Kalman filter
 - magnetometer Kalman filter
 - equal weighting
 - scalar approximation of the optimal weighting
 - optimal weighting

5.6 Case Studies

Four 1000 second simulations performed for the satellite with the pitch angle oscillation are listed in Table 5.3. To test the performance of the filter, simulations were performed with highly accurate and poor GPS system measurements. These simulations were repeated with a different measurement noise value, v_{GPS} , in the GPS Kalman filter.

Simulation	GPS Accuracy	Noise Value, v_{GPS}	Time
1	high	0.025	1000 s
2	high, low ($t > 150$ s)	0.025	1000 s
3	high	0.0125	1000 s
4	high, low ($t > 250$ s)	0.0125	1000 s

Table 5.3: List of Simulations for the Attitude Estimation System Using GPS and Magnetometers.

5.7 Results

The ten figures for the four simulation are described in Table 5.4. Each simulation consists of a satellite oscillating (amplitude = 0.05 radians, frequency = 0.314 Hz) about its pitch axis in a low Earth polar orbit ($h = 800$ km, $i = 98.6^\circ$) for 1,000 seconds. In each figure, the difference between the pitch angle estimate and the true pitch angle, the estimate error, is plotted versus time.

Figure	Sim.	GPS System Accuracy	v (GPS)	Estimate Error
5.5	1	high	0.025	GPS, Magn., OW
5.6	1	high	0.025	OW, Scalar App., Equal
5.7	2	high, low ($t > 150$ s)	0.025	OW, Scalar App., Equal
5.8	3	high	0.0125	GPS, Magn., Opt. Gain
5.9	3	high	0.0125	OW, Scalar App., Equal
5.10	4	high, low ($t > 250$ s)	0.0125	GPS, Magn., Opt. Gain
5.11	4	high, low ($t > 250$ s)	0.0125	OW, Scalar App., Equal
5.12	1,3	high	0.025, 0.0125	Optimal Weighting
5.13	2,4	high, low	0.025, 0.0125	Optimal Weighting

Table 5.4: Outline of Simulation Plots. (OW = Optimal Weighting)

Figures 5.5 and 5.6 show the pitch angle estimate error for the first simulation, when the GPS system estimate was more accurate than

magnetometer estimate. For the given GPS measurement error value, v_{GPS} , of 0.025, the optimally weighted estimates and its scalar approximation were slightly more accurate than the equally weighted estimate.

In the second simulation, the GPS system pitch angle estimates were less accurate than the magnetometer estimates 150 seconds into the simulation. As shown in Figure 5.7, the error for the optimally weighted estimates was slightly smaller than the error for the equally weighted estimate with very accurate GPS measurements. With degraded measurements, the two optimally weighted estimate errors were slightly greater than the magnetometer estimate error, and less than the average.

The two simulations were repeated for v_{GPS} at 0.0125 and the results are presented in Figures 5.8 through 5.11. In general, the optimal and scalar weighting estimates followed the GPS estimate. These estimates were more accurate than the equally weighted estimate with very accurate GPS system measurements but less accurate with very noisy GPS system measurements.

The optimally weighted pitch estimate error for the two simulations are presented in Figures 5.12 and 5.13 to compare the effect of different noise estimate values. The fusion estimates were closer to the GPS estimate when v_{GPS} at 0.0125. When the GPS estimate was more accurate than the magnetometer estimate, the optimal fusion estimates for v_{GPS} at 0.0125 had the best performance. However, with a more accurate magnetometer estimate, the fusion estimates with v_{GPS} at 0.025 were more accurate.

5.8 Discussion

5.8.1 Loosely-Integrated Configuration

Highly accurate attitude determination, within 1° , with GPS system measurements is feasible, but signal blockage, periods of large measurement error due to multipath effects and low update rates make the system unreliable. Magnetometers are low-cost measurement sensors that can solve the problem of reliability. Since they can provide reasonably accurate measurements at high update rate for any satellite orientation, the GPS antenna placement can be selected to provide highly accurate measurements when required, i.e. payload operation.

A loosely-integrated configuration is ideal for a combination of a reliable sensor with a high update rate and an unreliable sensor with a low update rate. The 'fusion' filter can use only the magnetometer attitude estimates if too many GPS signals are blocked. When the GPS Kalman filter attitude estimates are available, they can be fused with the magnetometer attitude estimates.

By linearizing the GPS Kalman filter, the magnetometer estimates stabilize and simplify the filter and increase the update rate. The estimates act as a coarse attitude estimation that help in solving the integer ambiguity and are available to reinitialize the filter. After a long period of signal blockage, the magnetometer estimates and Kalman filter error information can be used for the initial conditions. Inversely, the availability of GPS measurements allows a lower order model to be used in the magnetometer filter. Despite

having a lower accuracy, the model computing time decreases and permits a faster sample rate.

The benefits of each system can be capitalized upon in the loosely-integrated system. As shown in the simulations, an improvement in the estimate fusion is necessary to get the best performance out of this configuration.

5.8.2 Performance of the Optimal Weight Averaging

The performance of the optimal weight technique is dependent on the selection of v , the stochastic measurement noise value for the Kalman filters. It is selected based on how noisy the measurements are expected to be.

For highly accurate GPS measurements, v can be decreased so that Kalman gain and the fusion filter put more weight on the difference between the measurement and the measurement estimation on the GPS system estimates respectively. This relationship is shown in the scalar gain approximation equation since a smaller trace value of $\hat{P}_{gps}$, a function of $K(k)$ and v places the fusion estimate closer to the GPS estimate.

The 'optimal' fusion method performances are not optimal since the GPS estimate error is variable. The fusion estimate error was identical to the good or poor GPS system estimate error when v_{GPS} was 0.0125 and similar to the estimate error of the average estimation technique when v_{GPS} was at 0.025.

If GPS measurement noise due to reflections on the body could be analyzed for different line of sights, the measurement noise estimate value,

v_{GPS} , could be set accordingly. If the GPS signal is very accurate, v_{GPS} can be set so the fusion filter will follow the estimates based on the GPS system measurements. If the measurement is poor, v_{GPS} can be increased so the fusion filter estimates will follow the estimates based on the magnetometer measurements.

5.8.3 Scalar Approximation

The optimal gain fusion is based on the minimum of the fusion filter covariance matrix or error. It accounts for the cross-covariance between the covariance matrices of the two Kalman filters, and requires two 6 by 6 matrices to be sent from the two filters, and a feedback of the cross-covariance matrix, another 6 by 6 matrix.

The scalar gain requires one value, the trace of the covariance matrix of error, to be sent from each filter. Its performance was almost identical to the performance of the complete optimal gain fusion filter. For these simulations, the cross-correlation did not affect the fusion filter performance. If, in more detailed study, the scalar gain performs as well, it should be implemented to reduce the number of calculations and transferred information to be able to increase the sample rate.

5.9 Conclusions

- The combination of filtered magnetometer and GPS measurements could provide an inexpensive and highly accurate autonomous attitude determination system.
- Nevertheless, the attitude estimation requirements for SILA cannot be met using the two measurement systems. The attitude determination required for the aeronomy mission data analysis is too stringent.
- The optimal weighting technique is dependent on the noise estimate vector in the Kalman filters.
- GPS measurements may be too inconsistent for the proposed weighting technique. The accuracy of the GPS phase difference measurements need to be estimated in the Kalman filter.
- The scalar approximation of the optimal weighting matrix gave the same estimate in the simulation. It should be used since only the estimates and trace of the covariance matrix are required to be transferred to the 'fusion filter', where only two calculations are required to be performed.

5.10 Future Research

- Update literature review with results from current attitude estimation experiments using either magnetometers and GPS.
- Perform a real-time simulation for different test conditions. A large angle maneuver would be an example.

- Examine the fusion of the GPS Kalman filter estimates with attitude estimates from other sensors.
- Determine how the optimal weighting method and the Kalman filter can be adapted to short periods of degraded GPS measurements.

Figure 5.5: Pitch Angle Estimate Error for Accurate GPS Measurements and High Noise Estimate, v_{GPS} .

The estimate errors for the GPS and magnetometer Kalman filter estimates and the optimally weighted average are plotted. (Simulation 1 in Table 5.3)

Figure 5.6: Pitch Angle Estimate Error for Accurate GPS Measurements and High Noise Estimate, v_{GPS} .

The estimate errors for the optimally weighted average, its scalar approximation, and the equally weighting average are plotted. For this GPS noise estimate, v_{GPS} , all three weighted averages are approximately the same. (Simulation 1 in Table 5.3)

Figure 5.7: Pitch Angle Estimate Error for Poor GPS Measurements and High Noise Estimate, v_{GPS}

The degraded GPS measurements start 150 seconds into the simulation. The estimate errors for the optimally weighted average, its scalar approximation, and the equally weighted average are plotted. For this GPS noise estimate, v_{GPS} , all three weighted averages are approximately the same. (Simulation 2 in Table 5.3)

Figure 5.8: Pitch Angle Estimate Error for Accurate GPS Measurements and Low Noise Estimate, v_{GPS}

The estimate errors for the GPS and magnetometer Kalman filter estimates and the optimally weighted average are plotted. (Simulation 3 in Table 5.3)

Figure 5.9: Pitch Angle Estimate Error for Accurate GPS Measurements and Low Noise Estimate, v_{GPS} .

The estimate errors for the optimally weighted average, its scalar approximation, and the equally weighted average are plotted. For this GPS noise estimate, v_{GPS} , all the optimally weighted averages are approximately the same and more accurate than the equally weighted average. (Simulation 3 in Table 5.3)

Figure 5.10: Pitch Angle Estimate Error for Poor GPS Measurements and Low Noise Estimate, v_{GPS} .

The degraded GPS measurements start approximately 250 seconds into the simulation. The estimate errors for the GPS and magnetometer Kalman filter estimates and the optimally weighted average are plotted. (Simulation 4 in Table 5.3)

Figure 5.11: Pitch Angle Estimate Error for Poor GPS Measurements and Low Noise Estimate, v_{GPS}

The degraded GPS measurements start approximately 250 seconds into the simulation. The estimate errors for the optimally weighted average, its scalar approximation, and the equally weighted average are plotted. The equally weighted estimates are more accurate than the optimally weighted averages when the GPS signal is degraded. (Simulation 4 in Table 5.3)

Figure 5.12: Comparison of Optimally Weighted Averages for Accurate GPS Measurements.

The estimate errors of the optimally weighted averages for both noise estimates are plotted. (Simulations 1 and 3 in Table 5.3)

Figure 5.13: Comparison of Optimally Weighted Averages for Poor GPS Measurements.

The estimate errors of the optimally weighted averages for both noise estimates are plotted. (Simulations 2 and 4 in Table 5.3)

Chapter 6.0

Concluding Remarks

Three topics relating to the attitude control and determination for SILA have been addressed:

- steering angular momentum in a precessing orbit (Chapter 3.0)
- developing a 'fast' attitude acquisition maneuver (Chapter 4.0)
- providing inexpensive yet precise attitude estimation (Chapter 5.0)

The main conclusions from the investigated topics (see also Sections 3.9, 4.9, and 5.9) are listed in the following section. Suggestions for future research are given in Sections 3.10, 4.10, and 5.10. The final section of this chapter summarizes the original contributions.

6.1 Conclusions

- By equating the non-linear approximation of the gravity gradient torque to the torque required to precess the momentum, three equilibrium angles and a critical angular momentum at which the three angles converge to one large angle were identified (see Section 3.5).

- Magnetic torque should be used to steer the momentum vector with the precessing orbit (see Section 3.6).
- If magnetic torque cannot be used, gravity gradient torque can steer the momentum vector with the precessing orbit plane. Steering with gravity gradient torque was simulated for three angular momentum magnitudes for SILA. In each case, the momentum vector drifted from the orbit normal to a maximum inclination angle and returned to the orbit normal (see Section 3.7).
- The 'gyrocompass' acquisition maneuver is the ideal technique for a momentum-biased satellite in a high inclination orbit (see Section 4.1.2). The maneuver, taking approximately 30 orbits to complete, cannot be used for SILA due to power reasons (see Section 4.2.2).
- An acquisition maneuver that reorients the satellite such that the solar arrays face the sun in approximately 1.5 orbits has been developed. It can be performed autonomously for any initial orientation and for large body angular rates. The maneuver is outlined in Section 4.4.
- The maneuver should be tested using real-time computer and hardware simulations to investigate the effect of attitude estimation and magnetic field measurement errors.

- The loosely-configured integration of filtered magnetometer and GPS phase difference measurements could provide an inexpensive yet accurate (to within 1° in each axes) autonomous attitude determination system (see Section 5.3).
- The attitude estimation requirements for SILA cannot be met using the magnetometer and GPS phase difference measurements. The attitude determination required for the aeronomy mission data analysis is too stringent (see Section 5.3).
- The weighting method optimized for separately Kalman filtered estimates was examined through simulation. It was found to be dependent on the noise estimates in the Kalman filters. The attitude estimates from GPS phase difference measurements may be too inconsistent for this proposed weighting method.
- Varying the noise estimate with the noise in the GPS signal may improve the performance of the weighting technique (see Section 5.8.2).

6.2 Summary of Original Contributions

- The existence of three equilibrium inclination angles for the gravity gradient steering in a precessing orbit was shown in a simple non-linear analysis. The critical momentum value where only a single large angle exists was identified (see Section 3.5).

- A trade-off of magnetic and gravity gradient steering methods was performed for SILA, a low Earth orbit microsatellite (see Sections 3.6).
- Non-linear simulations of SILA in a precessing orbit were performed to investigate the motion of the momentum vector relative to the orbit normal (see Section 3.7).
- A fast acquisition maneuver has been designed and simulated for SILA. In this autonomous maneuver, the pitch axis is aligned with the field line and then with the orbit normal for any initial orientation and large angular velocities (see Section 4.4).
- Application of a proposed attitude estimation system using GPS carrier phase difference and magnetometer measurements for SILA was investigated (see Section 5.3).
- The use of an optimal fusion method for an attitude estimation system consisting of separately Kalman filtered GPS carrier phase difference and magnetometer measurements was investigated through simulation (see Section 5.5).

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Appendix A

Component Design Specifications

Tables A-1 through A-4 outline the design specifications for a reaction/momentum wheel, a 3-axis magnetometer, a torque rod, and an Earth horizon sensor. These components could be used in the WHECON control system outlined in Section 2.3 and are used in the calculations for the implementation of the 'fast' attitude acquisition maneuver (see Section 4.5).

Parameters	Design Specifications
Angular Momentum (at 1,000 rpm)	0.75 Nms
Angular Momentum (at 5,100 rpm)	4.0 Nms
Steady-State Power (at 1,000 rpm)	6.5 Watts
Steady-State Power (at 5,100 rpm)	9.0 Watts
Torque	0.020 Nm
Torque Offset	< 1% max. torq.
Mass	2.55 kg
Height	63.5 mm
Diameter	205.0 mm
Alignment	+/- 0.05°

Table A-1: ITHACO T-Reaction Wheel, Type A [26].

Parameters	Design Specifications
Mass	0.5 kg
Power	3.0 Watts
Volume	1 L
Lifetime	3 years
Accuracy (Pitch & Roll)	0.05°
Update Rate	10 Hz
Operating Temperature	-20°C to 70°C
Cost	\$30,000 (CDN)

Table A-2: Design Specifications for the ISTS Earth horizon sensor [27]. (Institute for Space and Terrestrial Science)

Parameters	Design Specifications
Range	+/- 0.6 Gauss
Linearity	< 0.5% of full scale
RMS Noise	< 5 microgauss
Frequency Response	-3 dB , > 100 Hz
Operating Temperature	-40°C to 80°C
Number of Axes	Three Orthogonal
Dimensions	15.5 x 4.2 x 3.6 cm
Mass	< 0.227 kg
Output Scale Factor	8.3 Volts/Gauss

Table A-3: ITHACO IM-103 Three-Axis Magnetometer [26].

Parameters	Design Specifications
Linear Moment (minimum)	12 Am ²
Saturation Moment (minimum)	15 Am ²
Residual Moment (minimum)	0.1 Am ²
Linear Voltage (nominal)	10.6 Volts
Linear Power (nominal)	0.7 Watts
Saturation Voltage (nominal)	14.1 Volts
Saturation Power (nominal)	1.3 Watts
Resistance of a Single Coil at 25°C	150 ohms
Scale Factor	0.17 Am ² /mA
Mass (including mounting block)	0.4 kg
Length	40 cm
Diameter (not including mounting feet)	1.8 cm
Number of Coils	1

Table A-4: Sample Torque Rod from ITHACO, TR10CFN [26].

Appendix B

MATLAB File Listings

The appendix consists of a listing of MATLAB files used in the simulations.

Table B-1 is the table of contents, giving the file name and the purpose of the file, the topic the file was used for, and the page number.

File	Function	Topic	Page
GGPREC.M	Define initial parameters, call QUAT.M through ode45 function, calculate inclination angle.	MP	138
QUAT.M	Attitude Dynamics in a Precessing Orbit	MP	138
ATT.M	Define initial parameters, call ACQ.M through ode45 function, calculate inclination angle.	AM	139
ACQ.M	Attitude Dynamics, Magnetic Torque and Magnetic Field Calculations for Maneuver	AM	141
TRAJ.M	Exact Attitude Angles, Angular Velocity and Magnetic Field Strength Calculations	AE	145
MOT.M	Satellite Attitude Dynamics	AE	146
ANT3.M	Antenna Position in Inertial Coordinates	AE	146
UNIT.M	Line of Sight Vector Calculations	AE	147
EXACT.M	Exact Double Phase Difference Calculations	AE	147
MAGNET.M	Magnetometer Measurements	AE	148
MAGN5.M	Magnetic Field Model (in Orbit Reference Frame)	AE/AM	148
MAG_EKF.M	Extended Kalman Filter for Attitude Estimation using Magnetometer Measurements	AE	149
FILTDD.M	Linear Kalman Filter for Attitude Estimation using GPS double phase difference measurements	AE	151
FUSION.M	Weighted Averaging of Attitude Estimates	AE	153

Table B-1: Listing of the MATLAB Files and Their Purpose.

(MP = Momentum Precession, AM = Acquisition Maneuver, AE = Attitude Estimation)

GGPREC.M

```

% This function calls quat4.m through the ode45 function.
% Its purpose is to simulate a satellite with angular momentum in its pitch axis
% in a precessing orbit with gravity gradient torque.
% Clear variables
ykk=[0 0 0 0 0 0];
al=0;
w=0;
v=0;

% Define number of intervals and interval length.
n=350;
t0=0;
tf=6300;
Ts=tf-t0;

% Set initial conditions for angular velocities (x,y,z) and quaternions (q1,q2,q3,q4).
ykk(1,:)=[0 0 0 0 0 1];

% Call Runge-Kutta numerical integration program.
for i=1:n
[t,y]=ode45('quat4d',t0,tf,ykk(i,:));
c=length(t);
% Store velocities and quaternions.
ykk(i+1,:)=y(length(t,:));
% Calculate angle of pitch axis to orbit normal using middle term of transformation matrix.
al(i)=acos((y(length(t),7)^2-y(length(t),4)^2+y(length(t),5)^2-y(length(t),6)^2))*180/pi;

% Update time intervals.
t0=tf;
tf=tf+Ts;
orbit=tf/Ts;
end;

```

QUAT4.M

```

function yp = quat4(t,y)
% Orbital angular velocity
n=-0.001;

% Define precession rates a,b for the x and z axes of the orbit reference frame,
% and the rotation of the line of apsides for the orbit normal axis.
a=-1.88e-7*cos(n*i);
b=1.88e-7*sin(n*i);
e=-5.7e-7;

% Define principal moments of inertia of the satellite and wheel momentum.
Iz=2.5;
Ix=2.3;
Iy=3.9;
h=0.1;

```

```

% Define coordinate transformation matrix in quaternions.
qu=[(y(7)^2+y(4)^2-y(5)^2-y(6)^2) 2*(y(4)*y(5)+y(6)*y(7)) 2*(y(4)*y(6)-y(5)*y(7));...
    2*(y(4)*y(5)-y(6)*y(7)) (y(7)^2-y(4)^2+y(5)^2-y(6)^2) 2*(y(5)*y(6)+y(4)*y(7));...
    2*(y(4)*y(6)+y(5)*y(7)) 2*(y(6)*y(5)-y(4)*y(7)) (y(7)^2-y(4)^2-y(5)^2+y(6)^2)];

% Define angular velocity of body relative to precessing orbit frame of reference.
w(:)= [y(1);y(2);y(3)] + qu*[a; n+e; b];

% Define gravity gradient torque (divided by principal moi).
g(1) = 3*n^2*(Iz-Iy)/Ix * 2*(y(5)*y(6)+y(4)*y(7)) * (y(7)^2-y(4)^2-y(5)^2+y(6)^2);
g(2) = 3*n^2*(Ix-Iz)/Iy * 2*(y(4)*y(6)-y(5)*y(7)) * (y(7)^2-y(4)^2-y(5)^2+y(6)^2);
g(3) = 3*n^2*(Iy-Ix)/Iz * 2*(y(5)*y(6)+y(4)*y(7)) * 2*(y(4)*y(6)-y(5)*y(7));

% Update equations of motion.
yp(1)=(Iy-Iz)/Ix*y(2)*y(3)-h/Ix*y(3)+t(1)/Ix+g(1);
yp(2)=-(Ix-Iz)/Iy*y(1)*y(3)+g(2);
yp(3)=-(Iy-Ix)/Iz*y(1)*y(2)+h/Iz*y(1)+t(3)/Iz+g(3);
% Update quaternion derivatives.
yp(4)=(w(3)*y(5)-w(2)*y(6)+w(1)*y(7))/2;
yp(5)=(-w(3)*y(4)+w(1)*y(6)+w(2)*y(7))/2;
yp(6)=(w(2)*y(4)-w(1)*y(5)+w(3)*y(7))/2;
yp(7)=(-w(1)*y(4)-w(2)*y(5)-w(3)*y(6))/2;

```

ATT8.M

```

% This m-file calls the 4th and 5th order Runge -Kutta differential equation function
% through function ACQ11.m. It is used to simulate the fast acquisition maneuver.
global g90 h90

```

```

%Load 1990 IGRF coefficients.
load h
load g
g90=g;
h90=h;

```

```

% Zero all matrices.
br=[0;0;0];
x2=0;x1=0;x3=0;
y1=0;y2=0;y3=0;
z1=0;z2=0;z3=0;
b1=0;b2=0;b3=0;
mg=[0;0;0];
% Set time of simulation (in seconds).
t0=0;
tf=15500;

```

```

% initial values for angular velocities (x,y,z), quaternions (q1,q2,q3,q4),
% wheel momentum (h)
y0=[0.01 0.008 0.01 1 0 0 0 0]';

```

```

% numerical integration using 4th and 5th order Runge-Kutta
[t,y]=ode45('acq11',t0,tf,y0);

% Recalculate magnetic field and coil moments for entire simulation.
%Orbit Rate
n=0.001;
c=length(y);
for i=1:c
%Calculate co-ordinate transformation matrix.
x2(i)=2*(y(i,5)*y(i,4)+y(i,6)*y(i,7));
x1(i)=y(i,7)^2+y(i,4)^2-y(i,5)^2-y(i,6)^2;
x3(i)=2*(y(i,6)*y(i,4)-y(i,5)*y(i,7));

y3(i)=2*(y(i,5)*y(i,6)+y(i,4)*y(i,7));
y2(i)=y(i,7)^2+y(i,5)^2-y(i,4)^2-y(i,6)^2;
y1(i)=2*(y(i,5)*y(i,4)-y(i,6)*y(i,7));

z1(i)=2*(y(i,6)*y(i,4)+y(i,5)*y(i,7));
z3(i)=y(i,7)^2+y(i,6)^2-y(i,4)^2-y(i,5)^2;
z2(i)=2*(y(i,6)*y(i,5)-y(i,4)*y(i,7));

qu=[x1(i) x2(i) x3(i);y1(i) y2(i) y3(i);z1(i) z2(i) z3(i)];

% Calculate magnetic field in orbit reference frame (approximation of 2nd order IGRF).
b1(i)=2.06e-5*[(1.41*cos(2*n*t(i))+0.14*sin(2*n*t(i))-0.47)*cos(n*t(i))-(0.1401*cos(2*n*t(i))-1.41*sin(2*n*t(i))+0.0467)*sin(n*t(i))];
b2(i)=-2e-6*(cos(n*(t(i)+1000)));
b3(i)=-2.06e-5*[(1.41*cos(2*n*t(i))+0.14*sin(2*n*t(i))-0.47)*sin(n*t(i))+(0.1401*cos(2*n*t(i))-1.41*sin(2*n*t(i))+0.0467)*cos(n*t(i))];

% Calculate magnetic field in body reference frame (magnetometer measurements).
br(:,i)=qu*[b1(i);b2(i);b3(i)];
% Calculate magnetic torques during each stage.
if t(i) < 5201
m(:,i)=cross(y(i,1:3)',br(:,i));
mg(1,i)=-1e7*m(1,i);
mg(2,i)=3;
mg(3,i)=-1e7*m(3,i);
elseif t(i) < 10000
alpha(i)=atan(b2(i)/(b1(i)^2+b3(i)^2)^0.5);
m(:,i)=cross(y(i,1:3)',br(:,i));
mg(1,i)=6*cos(alpha(i))*(-x2(i))/((-x2(i))^2+z2(i)^2)^0.5;
mg(3,i)=-6*cos(alpha(i))*z2(i)/((-x2(i))^2+z2(i)^2)^0.5;
mg(2,i)=6*sin(alpha(i))-3e8*m(2,i);
if mg(2,i)<-12
mg(2,i)=-12;
end;
else
if y(i,2) > (-0.001)
m(:,i)=cross(y(i,1:3)',br(:,i));
mg(1,i)=br(3,i)*0.0002/(br(3,i)^2+br(1,i)^2);
mg(3,i)=-br(1,i)*0.0002/(br(3,i)^2+br(1,i)^2);
mg(2,i)=-7e6*m(2,i);
else

```

```

mg(1,i)=0;
mg(3,i)=0;
mg(2,i)=0;
end;
end;
end;

```

ACQ11.M

```

function yp = acq11(t,y)
%SIMULATION OF ACQUISITION MANEUVER, principal moments of inertia, 2nd order
field model
% Orbital angular velocity
n=0.00104;

% maximum torque available to de-spin satellite
torq = -0.0002;
% Define time of wheel start-up.
if t >=9500
    if t <=13500
        dh=-0.0002;
    else
        dh=0;
    end;
else
    dh=0;
end;

% Define principal moments of inertia of the satellite.
Ix=2.4;
Iy=3.9;
Iz=2.6;

% Calculate angular velocities relative to the orbit reference frame.
w(1)=y(1)-2*(y(4)*y(5)+y(6)*y(7))*n;
w(2)=y(2)-(y(7)^2-y(4)^2+y(5)^2-y(6)^2)*n;
w(3)=y(3)-2*(y(6)*y(5)-y(4)*y(7))*n;

% Calculate second order approximation of magnetic field.
b(1)=2.06e-5*[(1.41*cos(2*n*t)+0.14*sin(2*n*t)-0.47)*cos(n*t)-(0.1401*cos(2*n*t)-
1.41*sin(2*n*t)+0.0467)*sin(n*t)];
b(2)=-2e-6*cos(n*(t+1000));
b(3)=-2.06e-5*[(1.41*cos(2*n*t)+0.14*sin(2*n*t)-0.47)*sin(n*t)+(0.1401*cos(2*n*t)-
1.41*sin(2*n*t)+0.0467)*cos(n*t)];

% Coordinate Transformation Matrix
x1=y(7)^2+y(4)^2-y(5)^2-y(6)^2;
x2=2*(y(4)*y(5)+y(6)*y(7));
x3=2*(y(6)*y(4)-y(5)*y(7));
y2=y(7)^2-y(4)^2+y(5)^2-y(6)^2;
y3=(y(6)*y(5)+y(4)*y(7))*2;

```

```

y1=2*(y(4)*y(5)-y(6)*y(7));
z2=2*(y(6)*y(5)-y(4)*y(7));
z1=2*(y(4)*y(6)+y(5)*y(7));
z3=(y(7)^2-y(4)^2-y(5)^2+y(6)^2)

br(1)=b(1)*x1+b(2)*x2+b(3)*x3;
br(2)=b(2)*y2+b(3)*y3+b(1)*y1;
br(3)=b(3)*z3+b(1)*z1+b(2)*z2;

% Calculate field derivative.
db(1)=-(y(2)*br(3)-y(3)*br(2));
db(2)=-(y(3)*br(1)-y(1)*br(3));
db(3)=-(y(1)*br(2)-y(2)*br(1));

%Coil operation First Stage
if t < 5201
    m(1)=-1e7*db(1);
    m(2)=3;
    m(3)=-1e7*db(3);

% Second Stage
elseif t < 10000
    mp(1)=2*(y(6)*y(5)-y(4)*y(7));
    mp(2)=0;
    mp(3)=-2*(y(4)*y(5)+y(6)*y(7));
    alpha=atan(b(2)/((b(1)^2+b(3)^2)^0.5));
    si=6;
    m(1)=si*cos(alpha)*mp(3)/(mp(3)^2+mp(1)^2)^0.5-1e4*db(1);
    m(3)=-si*cos(alpha)*mp(1)/(mp(3)^2+mp(1)^2)^0.5-1e4*db(3);
    m(2)=si*sin(alpha)-3e8*db(2);
% Apply maximum available if moment requested exceeds this value.
    if m(2)<-12
        m(2)=-12
    end;
    else
% Third Stage
        if y(2) > (-0.001)
            m(1)=-br(3)*torq/(br(3)^2+br(1)^2);
            m(3)=br(1)*torq/(br(3)^2+br(1)^2);
            m(2)=-7e6*db(2);
        else
            m(1)=0;
            m(3)=0;
            m(2)=0;
        end
    end;

end;

% Define magnetic torques in the satellite principal axes.
magx=m(2)*br(3)-m(3)*br(2);
magy=m(3)*br(1)-m(1)*br(3);
magz=m(1)*br(2)-m(2)*br(1);

% Define gravity gradient torques.
g(1) = 3*n^2*(Iz-Iy)/Ix * 2*(y(5)*y(6)+y(4)*y(7)) * (y(7)^2-y(4)^2-y(5)^2+y(6)^2);

```

```

g(2) = 3*n^2*(Ix-Iz)/Iy * 2*(y(4)*y(6)-y(5)*y(7)) * (y(7)^2-y(4)^2-y(5)^2+y(6)^2);
g(3) = 3*n^2*(Iy-Ix)/Iz * 2*(y(5)*y(6)+y(4)*y(7)) * 2*(y(4)*y(6)-y(5)*y(7));

```

```

% Update equations of motion, quaternions and wheel momentum.

```

```

yp(1)=(Iy-Iz)/Ix*y(2)*y(3)-y(8)*y(3)/Ix + g(1)/Ix+magx/Ix;
yp(2)=-dh/Iy + ((Iz-Ix)/Iy)*y(1)*y(3)+ g(2)/Iy+ magy/Iy ;
yp(3)=(Ix-Iy)/Iz*y(1)*y(2)+y(8)*y(1)/Iz + g(3)/Iz + magz/Iz;

```

```

yp(4)=(w(3)*y(5)-w(2)*y(6)+w(1)*y(7))/2;
yp(5)=(-w(3)*y(4)+w(1)*y(6)+w(2)*y(7))/2;
yp(6)=(w(2)*y(4)-w(1)*y(5)+w(3)*y(7))/2;
yp(7)=(-w(1)*y(4)-w(2)*y(5)-w(3)*y(6))/2;
yp(8)=dh;

```

ACQ12.m

```

function yp = acq12(t,y)
% SIMULATION OF ACQUISITION MANEUVER, 5th order field model, full inertia matrix
% Orbital angular velocity
global g90 h90
n=0.001;

% magnetic torque available to de-spin satellite and to equal wheel acceleration
torq = -0.0002;
% Define time of wheel acceleration.
if t >=9500
    if t <=13500
        dh=-0.002;
    else
        dh=0;
    end;
else
    dh=0;
end;

% Define moments of inertia of the satellite.
Ixx=2.4; Iyy=3.9; Izz=2.6;Ixy=0.06; Iyz=0.03; Ixz=0.3;
I=[Ix Ixy Ixz;Ixy Iy Iyz;Ixz Iyz Iz];
Iinv=inv(I);

% Define angular velocities relative to the x,y,z axes.
w(1)=y(1)-2*(y(4)*y(5)+y(6)*y(7))*n;
w(2)=y(2)-(y(7)^2-y(4)^2+y(5)^2-y(6)^2)*n;
w(3)=y(3)-2*(y(6)*y(5)-y(4)*y(7))*n;

% Call IGRF90 model (field determined in rotating orbit reference frame).
[Hx Hy Hz]=magn(t,g90,h90);
b(1)=Hx;
b(2)=Hy;
b(3)=Hz;

```



```

% 3-axis magnetometer, field transformed into body-fixed axes
br(1)=b(1)*(y(7)^2+y(4)^2-y(5)^2-y(6)^2)+b(2)*2*(y(4)*y(5)+y(6)*y(7))+b(3)*2*(y(6)*y(4)-
y(5)*y(7));
br(2)=b(2)*(y(7)^2-y(4)^2+y(5)^2-y(6)^2)+b(3)*2*(y(6)*y(5)+y(4)*y(7))+b(1)*2*(y(4)*y(5)-
y(6)*y(7));
br(3)=b(3)*(y(7)^2-y(4)^2-y(5)^2+y(6)^2)+b(1)*2*(y(4)*y(6)+y(5)*y(7))+b(2)*2*(y(6)*y(5)-
y(4)*y(7));

% Calculate field derivatives.
db(1)=-(y(2)*br(3)-y(3)*br(2));
db(2)=-(y(3)*br(1)-y(1)*br(3));
db(3)=-(y(1)*br(2)-y(2)*br(1));

% First stage. Pitch axis coil current is constant and other coils are a function of derivative.
if t < 5200
m(1)=-1e7*db(1);
m(2)=-1e7*db(2)+1;
m(3)=-1e7*db(3);
elseif t < 10000          % Second Stage
% Coil Moments in all axes used to torque pitch axis towards orbit normal.
% Pitch axis coil also used for motion damping.
mp(1)=2*(y(6)*y(5)-y(4)*y(7));
mp(2)=0;
mp(3)=-2*(y(4)*y(5)+y(6)*y(7));
alpha=atan(abs(b(2))/((b(1)^2+b(3)^2)^0.5));
m(1)=7*cos(alpha)*mp(3)/(mp(3)^2+mp(1)^2)^0.5-1e3*db(1);
m(3)=-7*cos(alpha)*mp(1)/(mp(3)^2+mp(1)^2)^0.5-1e3*db(3);
m(2)=7*sin(alpha)-2e8*db(2);

else          %Third Stage
% Apply during wheel acceleration and body angular rate removal
m(1)=-br(3)*torq/(br(3)^2+br(1)^2);
m(3)=br(1)*torq/(br(3)^2+br(1)^2);
m(2)=-1e8*db(2);
end;

% Limit coil moment to maximum allowable moment.
if m(1)^2 > 225 m(1)=m(1)/abs(m(1))*15; end;
if m(2)^2 > 225 m(2)=m(2)/abs(m(2))*15; end;
if m(3)^2 > 225 m(3)=m(3)/abs(m(3))*15; end;

% Define magnetic torque.
magx=m(2)*br(3)-m(3)*br(2);
magy=m(3)*br(1)-m(1)*br(3);
magz=m(1)*br(2)-m(2)*br(1);
M=[magx,magy,magz];

% Define gravity gradient torque.
g(1) = 3*n^2*(Iz-Iy)/Ix * 2*(y(5)*y(6)+y(4)*y(7)) * (y(7)^2-y(4)^2-y(5)^2+y(6)^2);
g(2) = 3*n^2*(Ix-Iz)/Iy * 2*(y(4)*y(6)-y(5)*y(7)) * (y(7)^2-y(4)^2-y(5)^2+y(6)^2);
g(3) = 3*n^2*(Iy-Ix)/Iz * 2*(y(5)*y(6)+y(4)*y(7)) * 2*(y(4)*y(6)-y(5)*y(7));
G=[g(1);g(2);g(3)];

% Define angular momentum.

```

```

ytot=[y(1);y(2);y(3)];
H=I*ytot;
% Calculate angular accelerations.
Tor=M+G-cross(ytot,H)-cross(ytot,[0;y(8);0])-[0;dh;0];
x(:)=linv*Tor;

% Update equations of motion, quaternions, and wheel acceleration
yp(1)=x(1);
yp(2)=x(2);
yp(3)=x(3);
yp(4)=(w(3)*y(5)-w(2)*y(6)+w(1)*y(7))/2;
yp(5)=(-w(3)*y(4)+w(1)*y(6)+w(2)*y(7))/2;
yp(6)=(w(2)*y(4)-w(1)*y(5)+w(3)*y(7))/2;
yp(7)=(-w(1)*y(4)-w(2)*y(5)-w(3)*y(6))/2;
yp(8)=dh;

```

TRAJ.M

% Attitude Estimation. Calculate exact angles, angular rates, and magnetic field
% for test case.

```
load data1
```

```
global xl0 pert.omO Hx_ti Hy_ti Hz_ti Hx_tf Hy_tf Hz_tf incl OM nmax
```

% total simulation time

```
N=100;
```

% Define initial conditions.

```
xl0=[0.05; 0.05; 0; 0; 0; 0.00005];
```

```
xkk=xl0;
```

```
n=length(xkk);
```

```
ti=0;
```

```
tf=1;
```

% interval time

```
Ts=1;
```

```
for i=1:N,
```

% magnetic field calculation, 10th order

```
[Hx Hy Hz]=magn10(ti);
```

```
H(:,i)=[Hx Hy Hz]';
```

%%%% angle and angular velocity time update &&

```
[t,x]=ode45('mot',ti,tf,xkk);
```

```
xkk1(:,i)=x(length(t),:);
```

```
%%%%%
```

```
ti=tf;
```

```
tf=tf+Ts;
```

```
xkk=xkk1(:,i);
```

```
end;
```

MOTION.M

```
% Called by TRAJ.M through 4th and 5th Runge-Kutta numerical integration
% function. Simulates attitude dynamics.
% define orbit rate
om0=0.0010;
% define wheel momentum and satellite moments of inertias
h=1;
Ix=2;
Iz=2;
Iy=3;

ph=x(1);
th=x(2);
ps=x(3);
p=x(4);
q=x(5);
r=x(6);

% 321 Euler sequence (yaw, theta, roll)
% Update angles and angular rates
xdot(1)=p+(q*sin(ph)+r*cos(ph))*tan(th)+om0*sin(ps)/cos(th);
xdot(2)=q*cos(ph)-r*sin(ph)+om0*cos(ps);
xdot(3)=(q*sin(ph)+r*cos(ph))/cos(th)+om0*sin(ps)*tan(th);
xdot(4)=-((Iz-Iy)/Ix*om0 + h/Ix)*r;
xdot(5)=-((0.157^2)*th;
xdot(6)=-((Ix-Iy)/Iz*om0 + h/Iz)*p;
```

ANT3.M

```
function y=ant3(x);
% This function determines the position of THREE antennas
% in INERTIAL CO-ORDINATES
% The satellite is oscillating about its centre of mass
% in the z axis (pitch angle is varying sinusoidally to a maximum of 0.05).
% Define satellite position in the inertial reference frame.
satx=x(1);
saty=x(2);
satz=x(3);
% Define body position using a linearized
% transformation matrix (A).
ph=x(4);
th=x(5);
ps=x(6);
p=x(7);
q=x(8);
r=x(9);

A=[1 -ps th;ps 1 -ph;-th ph 1];
% Calculate antenna position relative to satellite centre.
del1=A*[0.5;0;0.5];
```

```

del2=A*[-0.5;0;0.5];
del3=A*[0.5;0;-0.5];
%Calculate antenna position in inertial space.
ant1=[satx;saty;satz]+del1;
ant2=[satx;saty;satz]+del2;
ant3=[satx;saty;satz]+del3;
y=[ant1; ant2; ant3]';
clear;

```

UNIT.M

```

function un=unit(x)
% This function computes the LINE OF SIGHT to four GPS satellites
% in INERTIAL CO-ORDINATES (x,y,z).

satpos=zeros(4,3);
% Define antenna positions.
recpos=x(1:9);
% Define satellite position matrix.
for j=1:4
    satpos(j,:)=x(3*(j+1)+4:3*(j+3))';
% Compute exact pseudorange to each satellite.
ra(j)=sqrt((recpos(1)-satpos(j,1))^2+(recpos(2)-satpos(j,2))^2+(recpos(3)-satpos(j,3))^2);
% Calculate unit vector [x,y,z].
sx(j)=(recpos(1)-satpos(j,1))/ra(j);
sy(j)=(recpos(2)-satpos(j,2))/ra(j);
sz(j)=(recpos(3)-satpos(j,3))/ra(j);
end
% Calculate line of sight differences (GPS satellites 1 - 3 and 2 -4).
dsx(1)=sx(1)-sx(3);
dsy(1)=sy(1)-sy(3);
dsz(1)=sz(1)-sz(3);
dsx(2)=sx(2)-sx(4);
dsy(2)=sy(2)-sy(4);
dsz(2)=sz(2)-sz(4);
un=[dsx(1);dsy(1);dsz(1);dsx(2);dsy(2);dsz(2)];

```

EXACT.M

```

function r=exact(x)
% This function computes the exact range to four GPS satellites.
% Define the antenna positions.
satpos=zeros(4,3);
% Define receiver position.
recpos=x(1:9);
% Define satellite position matrix.
for j=1:4
    satpos(j,:)=x(3*(j+2)+1:3*(j+3))';

% Compute ranges from each antenna to each GPS satellite.

ra(j)=sqrt((recpos(1)-satpos(j,1))^2+(recpos(2)-satpos(j,2))^2+(recpos(3)-satpos(j,3))^2);

```

```

ra(j+4)=sqrt((recpos(4)-satpos(j,1))^2+(recpos(5)-satpos(j,2))^2+(recpos(6)-satpos(j,3))^2);
ra(j+8)=sqrt((recpos(7)-satpos(j,1))^2+(recpos(8)-satpos(j,2))^2+(recpos(9)-satpos(j,3))^2);

% Calculate difference in pseudoranges for the antennas (1-2, 2-3, 3-1)
d1(j)=ra(j)-ra(j+4);
d2(j)=ra(j+4)-ra(j+8);
d3(j)=ra(j+8)-ra(j);
end;
%Calculate double difference (GPS satellites 1-3, 2-4)
for i=1:2
dd1(i)=d1(i)-d1(i+2);
dd2(i)=d2(i)-d2(i+2);
dd3(i)=d3(i)-d3(i+2);
end;

r=[dd1(1);dd2(1);dd3(1);dd1(2);dd2(2);dd3(2)];

```

MAGNET.M

```

function y=magnet(x)
% Simulates errors in magnetometer measurements by placing
% error on the transformation angles (from ORBITAL to BODY-FIXED reference frames).

xatt=[x(1);x(2);x(3);x(7);x(8);x(9)];
xerr=[x(4);x(5);x(6);x(7);x(8);x(9)];
mat_orb1=transotb(xatt);
mat_orb2=transotb(xerr);
H=mat_orb2*[x(10);x(11);x(12)];
y=[H;xatt(2);xerr(2)];

```

MAGN5.M

```

% program written by C.D. van de Kolk [19]
function [Hx,Hy,HZ]=magn(t)
incl=98.6/180*pi;
om0=0.0010;
ome=7.77220522e-5;
OM=0;
% Define ORDER (nmax) and 1990 IGRF constants.
nmax=5;
g=g*1e-9;
h=h*1e-9;
% Calculate orbit position of satellite.
a=6375;
R=7175;

THETA=om0*5;
z=THETA/(2*pi);
j=-1;

```

```

while j <= (z-1)
    j=j+1;
end;

THETAh=THETA-j*2*pi;
delta=asin(sin(incl)*sin(THETA));
alpha=acos(cos(THETA)/cos(delta))+OM;
if THETAh > pi
    alpha=2*pi-alpha;
end;

alpha=alpha+j*2*pi;
phi=alpha-ome*t;
theta=pi/2-delta;
Hr=0;
Ht=0;
Hp=0;

%%% berekening van de componenten van het magn. veld %%%%
for n=1:nmax
    sumr1=0;
    sumt1=0;
    sump1=0;
    for m=0:n
        [P,dP,Pst]=schmitn(n,m,theta);
        sumr1=sumr1+(g((n+1),(m+1))*cos(m*phi)+h((n+1),(m+1))*sin(m*phi))*P;
        sumt1=sumt1+(g((n+1),(m+1))*cos(m*phi)+h((n+1),(m+1))*sin(m*phi))*dP;
        sump1=sump1+(m*g((n+1),(m+1))*sin(m*phi)-m*h((n+1),(m+1))*cos(m*phi))*Pst; end; % 2e
    for
        Hr=Hr+(n+1)*(a/R)^(n+2)*sumr1;
        Ht=Ht-(a/R)^(n+2)*sumt1;
        Hp=Hp+(a/R)^(n+2)*sump1;
    end;
mat_sto=transsto(alpha,delta);
Hx=mat_sto(1,:)*[Hr; Ht; Hp];
Hy=mat_sto(2,:)*[Hr; Ht; Hp];
Hz=mat_sto(3,:)*[Hr; Ht; Hp];
H=[Hx Hy Hz];

```

MAG_EFK.M

```
function xe=mag_ekfo(y)
```

```
% this function is the Kalman filter for the magnetometer data
```

```

y=y(:);
t=y(1);
ti=y(1);
t2=y(5);

```

```

% These values are the initial conditions for the states and the covariance
% matrix of error, P.
% The states are the roll, pitch and yaw angles and the corresponding angular
% velocities.

x0=[0.05; 0.05; 0; 0; 0; 0.00005];
xs=[0.05; 0.05; 0.05; 0.0005; 0.0005; 0.0005];
n=length(x0);
unit=eye(n);
P0=diag([0.05^2*2; 0.05^2*2; 0.05^2*2; 0.0005^2*2;0.0005^2*2;0.0005^2*2]);

if ti==0
    xa=x0';
    Pkk=reshape(P0,1,36);
    Pkkb=Pkk;
    [Hx_ti,Hy_ti,Hx_ti]=magn5(ti);
    mat_orb=transotb(x0);
    Bmeas=(mat_orb*[Hx_ti,Hy_ti,Hx_ti]')';

elseif y(1)==y(5)
    xa=[y(9:14)]';
    Pkk=[y(15:50)]';
    Bmeas=[y(6:8)]';
    Pkkb=[y(51:86)]';
else
    tf=y(1);
    ti=y(5);
    xa=y(9:14);

%%%%% TIME UPDATE
    [t,x]=ode45('mot',ti,tf,xa);
    xa1=x(length(x),:);

%%%%%%%%%%

% CALCULATE THE SYSTEM ERROR AND SENSOR ERROR COVARIANCE MATRICES, Q
AND R
W=1e-5;
V=1e-7;

Q=W^2*eye(6);
R=V^2*eye(3);

B=eye(n);
pert=0.0001;

% FOR A NON-LINEAR SYSTEM, PHI AND GAMMA ARE CALCULATED USING
% A LINEARIZATED SYSTEM OVER A SHORT PERIOD OF TIME
for z=1:n
    if xa(z) >= 0
        dx=xs(z)*pert;
    else
        dx=-xs(z)*pert;
    end;

```

```

    xp=xa+unit(:,z)*dx;
    fpl=mot(ti,xp);
    xm=xa-unit(:,z)*dx;
    fm=mot(ti,xm);
    F(:,z)=(fpl'-fm')/(2*dx);
end;
Ts=1;
[Phi,Gamma]=c2d(F,B,Ts);

% MEASUREMENT UPDATE

[Hx_tf,Hy_tf,Hx_tf]=magn5(tf);
mat_orb=transotb(xa1);
Best=(mat_orb*[Hx_tf,Hy_tf,Hx_tf]);
Bmeas=y(2:4);
Bdiff=Bmeas-Best;
for z=1:n
    if xa1(z) >= 0
        dx=xs(z)*0.0001;
    else
        dx=-xs(z)*0.0001;
    end;
    xp=xa1+unit(:,z)*dx;
    mat_orb=transotb(xp);
    zp=(mat_orb*[Hx_tf,Hy_tf,Hx_tf]); xm=xa1-
    unit(:,z)*dx;
    mat_orb=transotb(xm);
    zm=(mat_orb*[Hx_tf,Hy_tf,Hx_tf]); H(:,z)=(zp-
    zm)/(2*dx);
end;

P=reshape(y(15:50),6,6);
Pkk1=Phi*P*Phi'+Gamma*Q*Gamma';
Lam=H*Pkk1*H'+R;
Kk=Pkk1*H'*inv(Lam);
xe1=xa1+Kk*(Bdiff); Pkk2m=(eye(n)-
Kk*H)*Pkk1*(eye(n)-Kk*H)'+Kk*R*Kk';
Pkk=reshape(Pkk2m,1,36); Pkkb=reshape(Pkk1,1,36);
xa=xe1';
Bmeas=Bmeas';
end

xe=[y(1) Bmeas xa Pkk Pkkb];

```

FILTDD.M

```

function xe=filtdd(y)
% GPS linear Kalman filter
% y(1) is the time, t

```



```

xi=[0.05 0.05 0 0 0 0.00005];
W=1e-5;
V=1e-2;
Q=W^2*eye(6);
R=V^2*eye(6);
B=eye(6);
pert=0.0001;
Ts=1;
if y(1)==0
    xa=xi;
    Pkk1=diag([0.05^2*2 0.05^2*2 0.05^2*2 0.0005^2*2 0.0005^2*2 0.0005^2*2]);
    Pkkb=reshape(Pkk1,1,36);
    Pkk=reshape(Pkk1,1,36);
    ymeas=[1.2094 -1.6607 0.4513 -0.1347 -0.1729 0.3077];
elseif y(1)==y(26)
    xa=[y(20:25)]';
    Pkk=[y(27:62)]';
    ymeas=[y(63:68)]';
    Pkkb=[y(69:104)]';
else
    tf=y(1);
    xo=[y(20:25)]';
    ti=y(26);
    [t,x]=ode45('mot',ti,tf,xo);
    xa=x(length(x),:);
% calculation of phi and gamma

% Use the magnetometer data to linearize the filter.
ph=y(14);
th=y(15);
ps=y(16);
Al=[0 0 0 1 -ps th;0 0 0 ps 1 -ph;0 0 0 -th ph 1;...
    0 0 0 0 0.001;0 -0.157^2 0 0 0 0;0 0 0 -0.001 0 0];

[Phi,Gamma]=c2d(Al,B,Ts);

A=[1 xa(3) -xa(2);-xa(3) 1 xa(1);xa(2) -xa(1) 1];
base=[1 0 0;-1 0 1;0 0 -1];

n=length(xo);

H1=cross(base',[y(2);y(3);y(4)]);
H1=H1';
H2=cross(base',[y(8);y(9);y(10)]);
H2=H2';

ex=zeros(3,3);
C=[H1 ex;H2 ex];

yest1=base*A*[y(2);y(3);y(4)];
yest2=base*A*[y(8);y(9);y(10)];
ymeal=[y(5);y(6);y(7)];
ymeal2=[y(11);y(12);y(13)];

```

```

P=reshape(y(27:62),6,6);
Pkk1=Phi*P*Phi'+Gamma*Q*Gamma';
Lam=C*Pkk1*C'+R;
Kk=Pkk1*C'*inv(Lam);
xe1=xa+Kk*[(ymea1-yest1);(ymea2-yest2)];
Pkk2=(eye(n)-Kk*C)*Pkk1*(eye(n)-Kk*C)'+Kk*R*Kk';

Pkk=reshape(Pkk2,1,36);
Pkkb=reshape(Pkk1,1,36);
xa=xe1';
ymear=[ymea1;ymea2]';
end

xe=[xa y(1) Pkk ymeas Pkkb];

```

FUSION.M

```

function y=fusion(x)

% Weighted Averaging Function.
% Calculate estimate error
% for filter estimates and weighted averages.
% The estimate fusion is based on the values from
% covariance matrices of error from the Kalman filters.

% rl is the actual pitch angle, theta
rl=x(13);

if x(206)==0
    ytemp=zeros(1,41);
else
    % W is the weighting (0 < W < 1)
    P1=reshape(x(15:50),6,6);
    P2=reshape(x(93:128),6,6);
    Pb1=reshape(x(57:92),6,6);
    Pb2=reshape(x(129:164),6,6);
    % scalar approximation
    tr1=trace(P1);
    tr2=trace(P2);
    if tr1==0
        if tr2==0
            w=0.5;
        else
            w=tr1/(tr1+tr2);
        end
    else
        w=tr1/(tr1+tr2);
    end
    f=(1-w)*x(2)+w*x(8)-rl;
    err1=x(2)-rl;
    err2=x(8)-rl;

```

```

err3=x(14)-rl;

% optimal fusion
ph=x(7);
th=x(8);
ps=x(9);
A1=[0 0 0 1 -ps th;0 0 0 ps 1 -ph;0 0 0 -th ph 1;...
0 0 0 0 0.001;0 -0.157^2 0 0 0 0;0 0 0 -0.001 0 0];
B=eye(6);
Ts=1;
[Phi,Gamma]=c2d(A1,B,Ts);
Mh=reshape(x(165:200),6,6);
W=1e-5;
Q=W^2*eye(6);
Mb=Phi*Mh*Phi'+Gamma*Q*Gamma';
M=P1*inv(Pb1)*Mb*inv(Pb2)*P2;
wopt=(P1-M)*inv(P1-M-M'+P2);
h=(eye(6)-wopt)*x(1:6)+wopt*x(7:12);
theta=h(2)-rl;
M2=reshape(M,1,36);
ytemp=[M2 err1 err2 f g theta];
end;
y=ytemp;

```