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Optimization of High-Speed VLSI Interconnects

by

Ruolong Liu, M.Eng

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Master of Engineering

**Ottawa-Carleton Institute for Electrical Engineering
Department of Electronics
Faculty of Engineering
Carleton University
Ottawa, Canada
May, 1993**

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for the degree of Master of Engineering

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Abstract

To meet the increasing demand for a design tool for transient analysis and optimization of high-speed VLSI interconnects, this thesis first derives an algorithm for the efficient computation of group delay, gain slope and their sensitivities with respect to design parameters in multiconductor transmission line networks. A systematic formulation of optimization for interconnect networks is developed by using frequency-domain information such as group delay and gain slope. By combining this method with minimax optimization, an efficient frequency-domain optimization technique is then presented to minimize transient delay, distortion and crosstalk in high-speed VLSI interconnects. Examples of interconnect optimization demonstrate simultaneous reductions of signal propagation delay, distortion and crosstalk at several vital connection ports. Compared with direct time domain optimization, the proposed frequency domain approach is faster. Finally, as a initial step towards the optimization of nonlinear interconnect networks, a new efficient technique is developed for the transient analysis of the overall nonlinear networks. The recent Asymptotic Waveform Evaluation (AWE) technique is extended from analyzing linear circuits to nonlinear ones. By successively applying AWE to the nonlinear network equation, we converted the overall simulation into the solution of a few linear network problems, thus achieving significant CPU speedup over conventional simulators such as SPICE.

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List of Symbols

$\mathbf{A}_k$	modified nodal admittance matrix for the linear subnetwork k
$\mathbf{C}$	symmetric matrix representing the capacitance per unit length of the transmission line
$\mathbf{C}_\pi$	constant matrix with entries determined by the lumped linear components in the network π
$\mathbf{D}_k$	selector matrix that maps the terminal currents of subnetwork k $\mathbf{i}_k(t)$ into the N_π -node space of network π
$\mathbf{e}_\pi$	the vector of source waveforms in the network π
$\mathbf{G}$	symmetric matrix representing the conductance per unit length of the transmission line
G_L	gain slope
$\mathbf{G}_\pi$	constant matrix with entries determined by the lumped linear components in the network π
$\mathbf{\Gamma}$	diagonal matrix containing the square-roots of the eigenvalues of the transmission line matrix $\mathbf{Z}_L\mathbf{Y}_L$
γ_i^2	eigenvalues of the transmission line matrix $\mathbf{Z}_L\mathbf{Y}_L$
H	transfer function of the network
$\mathbf{i}_k(t), \mathbf{I}_k$	vector of currents entering the linear subnetwork k
k_i	complex residues

$\mathbf{L}$	symmetric matrix representing the inductor per unit length of the transmission line
l	length of the transmission line
m_i	moments of the output
N_π	number of elements in the vector $\mathbf{V}_\pi$
ϕ, Φ	design variable and vector of design variables
p_i	complex poles
S_c	specification on the magnitude of spectrum of crosstalk waveform
$S_{G,u}, S_{G,l}$	upper and lower specifications on group delay T_G
$S_{H,l}$	lower specifications on the magnitude of transfer function H
S_L	upper specification on the magnitude of gain slope G_L
$\mathbf{S}_v$	a matrix whose columns contain the eigenvectors $\mathbf{x}_i$ of the transmission line matrix $\mathbf{Z}_L \mathbf{Y}_L$
s	complex frequency
T_G	group delay
$\mathbf{U}$	identity matrix
u	unit step function
$\mathbf{V}_\pi, \mathbf{v}_\pi$	vector of node voltages appended by independent voltage source currents and inductor currents of the network π
W_G, W_L	weighting factors for group delay and gain slope specifications
ω	angular frequency
$\mathbf{Y}_L$	complex admittance per unit length of the transmission line
$\mathbf{Y}_\pi$	modified nodal admittance matrix for the network π
$\mathbf{Z}_L$	complex impedance per unit length of the transmission line

Chapter 1

Introduction

1.1 Problem Description

Multiconductor transmission lines are frequently encountered in digital computers and communication systems. The rapid advances in VLSI technologies and packaging techniques are yielding larger chips with smaller and faster devices. As feature sizes have decreased, the density of devices on a chip has increased resulting in a greater number of interconnections both on the IC and between ICs. As interconnection densities and switching speeds increase, the electrical length of interconnects becomes a significant fraction of a wavelength. Finer and longer interconnections means an increase in losses which can affect signal integrity. All of these factors have made the analysis and design of interconnections a primary concern in the development of high-speed digital computers and communication systems. Improperly designed interconnects can result in increased signal delay, ringing, and inadvertent and false switching. This phenomenon can be observed at chip level as well as the system level, and the interconnected blocks can be analog, digital or mixed.

The conventional lumped impedance interconnect model is not adequate for high-speed applications where the analysis of delay and crosstalk between coupled lines is required. In addition, due to small line geometries, losses in the interconnections can be significant. As a result, a distributed lossy coupled transmission line model should be used.

There is a recent thrust of research in the time-domain analysis of such interconnect effects. Several techniques have been proposed in the literature [1]-[20]. These analysis or simulation tools allow a circuit designer to determine the response of a given network. Human expertise is still required to study the effect of variance of network parameters on system performance. An intuitive solution to the problem of delay and distortion is to decrease the length of the interconnections by increasing the system density. The trend towards greater density, however, fosters another problem; that of crosstalk between adjacent transmission lines. A designer must make proper trade-offs among various conflicting factors. For large circuits, the relation between a large number of circuit parameters and design criteria becomes extremely complicated. There is an increasing need of a design optimization approach that uses interconnect parameters explicitly in overall network design, a problem only addressed very recently in the literature [21]-[23].

In [21]-[23] a time-domain optimization method has been presented for design optimization of the time responses of high-speed VLSI interconnects modeled by distributed coupled transmission line networks, where the problem of simultaneous minimization of crosstalk, delay and reflection is directly formulated as a minimax optimization. By imposing design specifications directly on transient waveforms, the method is a straightforward and accurate approach. It is an effective method, especially for minimizing crosstalk. However, it is difficult, in general, to efficiently

minimize propagation delay by imposing design specifications directly on transient waveforms. On the other hand, the time-domain optimization approach is also computationally expensive. Other techniques have to be investigated.

Practical VLSI interconnects are usually terminated by nonlinear elements. Since nonlinear simulation and optimization are both iterative processes, the CPU time requirement for design optimization of VLSI interconnects with nonlinear terminations, such as line drivers, will be extremely high. An important step is the development of an efficient simulator for the nonlinear networks. Since the electromagnetic propagation modes in lossy transmission lines are defined in the frequency domain, conventional nonlinear simulators based on numerical integration, such as SPICE, can not be directly used. Recently, several techniques for the analysis of nonlinear networks with frequency dependent parameters have been developed such as macromodeling [11], [12], [13], convolution [14], [15], [16] or recursive convolution [17], piece-wise decomposition [18], or waveform relaxation [19]. All these approaches take the entire nonlinearity into account and therefore are computationally expensive. It is desirable for a circuit designer to have a computationally less expensive method which can approximate interconnection delay and crosstalk in an efficient way by capturing only the dominant nonlinear waveform.

1.2 Thesis Objectives

The main objective of this thesis is to present an alternative approach for optimization of transient responses such as delay, distortion and crosstalk by using frequency domain information such as group delay and gain slope [24], [25]. Firstly, a technique for efficient computation of exact group delay, gain slope and their sensi-

tivities is derived for multiconductor transmission line networks. Secondly, the group delay information is combined with gradient based minimax optimization to indirectly optimize transient responses of the interconnects such as signal propagation delay, distortion, crosstalk and reflection. Compared with the direct time-domain optimization, the proposed frequency-domain approach is faster.

As an initial step towards design optimization of nonlinear interconnect networks, the objective of the remainder of the thesis is devoted to deriving a new technique SAWE (Successive Asymptotic Waveform Evaluation) as an alternative approach to the solution of the overall nonlinear network equations. SAWE is a waveform approximation technique which expands the original AWE (asymptotic waveform evaluation) [26] principle directly to the analysis of nonlinear networks. Instead of taking the entire nonlinearity into account, SAWE has the ability to capture the dominant nonlinear waveform and therefore is faster.

1.3 Outline of the Thesis

Chapter 2. Background and Literature Review provides the necessary background information and literature review of existing transient analysis and optimization approaches for high-speed VLSI interconnects.

Chapter 3. Group Delay and its Sensitivity Analysis derives a technique for efficient computation of exact group delay, gain slope and their sensitivities for multiconductor transmission line networks. A numerical example is provided to illustrate the accuracy and efficiency of the proposed technique compared to the perturbation method.

Chapter 4. Formulation of VLSI interconnect Optimization presents a frequency domain formulation for minimization of delay, distortion and crosstalk in VLSI interconnects. Three numerical examples are presented to illustrate the principles of frequency domain interconnect optimization.

Chapter 5. Analysis of Circuits with Nonlinear Components proposes a new technique SAWE as an alternative approach to the solution of the overall nonlinear network equations. Three numerical examples are provided to illustrate the accuracy and efficiency of the proposed technique compared to conventional simulators.

Chapter 6. Conclusions and Future Research summarizes the results of the thesis and provides suggestions for further research.

Chapter 2

Background and Literature Review

This chapter introduces the necessary background information and literature review of existing transient analysis and optimization approaches for high-speed VLSI interconnects. The modified nodal admittance (MNA) representation of a lossy multiconductor transmission line and the formulation of the high-speed interconnect network equations are described. The time domain solution obtained from the frequency domain solution by using the numerical inverse of Laplace transform (NILT) algorithm and asymptotic waveform evaluation (AWE) technique are described. Also reviewed are group delay and existing transient optimization approaches.

2.1 MNA Representation of a Lossy Multiconductor Transmission Line

In this section the MNA representation of a lossy multiconductor transmission line is derived. The resulting admittance matrix is used in the formulation of the frequency domain equations of the linear subnetworks as described in the following section.

The transmission line is assumed to be uniform along its length with an arbitrary cross-section. The reference directions for the voltage and current signals on a transmission line are shown in Figure 2.1. The cross-section of a transmission line with N signal conductors can be represented by the following $N \times N$ matrices of line parameters: the resistance per unit length $\mathbf{R}$, the inductance per unit length $\mathbf{L}$, the capacitance per unit length $\mathbf{C}$ and the conductance per unit length $\mathbf{G}$. These matrices can be computed from physical/geometrical parameters of the transmission line through quasi-static analysis [27], [28] or empirical formulas [29]. Let x represent the distance along the transmission line with $x = 0$ corresponding to the near end and $x = l$ corresponding to the far end as shown in Figure 2.1.

In the frequency domain the transmission line can be described by the differential equations [1]

$$\frac{d\mathbf{V}(x)}{dx} = -\mathbf{Z}_L \mathbf{I}(x) . \quad (2.1)$$

$$\frac{d\mathbf{I}(x)}{dx} = -\mathbf{Y}_L \mathbf{V}(x) , \quad (2.2)$$

where $\mathbf{V}(x)$, $\mathbf{I}(x)$ are the line voltages and currents,

$$\mathbf{Z}_L = \mathbf{R} + s\mathbf{L} , \quad (2.3)$$

$$\mathbf{Y}_L = \mathbf{G} + s\mathbf{C} , \quad (2.4)$$

Figure 2.1: A multiconductor transmission line.

and s is the Laplace operator.

The differential equations can be combined into the single wave equation

$$\frac{d^2 \mathbf{V}(x)}{dx^2} = -\mathbf{Z}_L \mathbf{Y}_L \mathbf{V}(x) , \quad (2.5)$$

which will have the solutions of the form

$$\mathbf{V}_i(x) = \mathbf{V}_i(0) e^{\pm \gamma_i x} , \quad (2.6)$$

$$\mathbf{I}_i(x) = \mathbf{I}_i(0) e^{\pm \gamma_i x} , \quad (2.7)$$

where γ_i describes the wave propagating along the conductors with the amplitude and phase on a particular conductor set by the complex vectors $\mathbf{V}_i(0)$ and $\mathbf{I}_i(0)$.

Substituting the solution into the wave equation (2.5) yields

$$(\gamma_i^2 \mathbf{U} - \mathbf{Z}_L \mathbf{Y}_L) \mathbf{V}_i(0) = 0 , \quad (2.8)$$

where $\mathbf{U}$ is the identity matrix. This equation has nontrivial solutions when γ_i^2

satisfies the eigenvalue problem

$$\det(\gamma_i^2 \mathbf{U} - \mathbf{Z}_L \mathbf{Y}_L) = 0 . \quad (2.9)$$

There will be N solutions γ_i , $i = 1, \dots, N$ to the eigenvalue problem with associated eigenvectors $\mathbf{x}_i$, $i = 1, \dots, N$. Let $\mathbf{\Gamma}$ be a diagonal matrix containing $\{\gamma_1, \gamma_2, \dots, \gamma_N\}$, obtained by taking the square roots of the eigenvalues, and $\mathbf{S}_v$ be a matrix with the eigenvectors $\mathbf{x}_i$ in the columns. Solutions for the voltages can now be written as

$$\mathbf{V}(x) = \mathbf{S}_v \mathbf{E}(x) \mathbf{C}_1 + \mathbf{S}_v \mathbf{E}^{-1}(x) \mathbf{C}_2 , \quad (2.10)$$

where $\mathbf{E}(x)$ is the diagonal matrix $\text{diag}\{e^{-\gamma_1 x}, e^{-\gamma_2 x}, \dots, e^{-\gamma_N x}\}$ and $\mathbf{C}_1$ and $\mathbf{C}_2$ are constant vectors.

Substituting (2.10) into (2.5) produces

$$\mathbf{I}(x) = \mathbf{S}_i \mathbf{E}(x) \mathbf{C}_1 + \mathbf{S}_i \mathbf{E}^{-1}(x) \mathbf{C}_2 , \quad (2.11)$$

where $\mathbf{S}_i = \mathbf{Z}_L^{-1} \mathbf{S}_v \mathbf{\Gamma}$.

From (2.10) the relationship between the near and far end voltages can be written as

$$\begin{bmatrix} \mathbf{V}(0) \\ \mathbf{V}(l) \end{bmatrix} = \begin{bmatrix} \mathbf{S}_v & \mathbf{S}_v \\ \mathbf{S}_v \mathbf{E}(l) & \mathbf{S}_v \mathbf{E}^{-1}(l) \end{bmatrix} \begin{bmatrix} \mathbf{C}(1) \\ \mathbf{C}(2) \end{bmatrix} . \quad (2.12)$$

Similarly, from (2.11) the relationship between the near end and far end currents can be written as

$$\begin{bmatrix} \mathbf{I}(0) \\ \mathbf{I}(l) \end{bmatrix} = \begin{bmatrix} \mathbf{S}_i & \mathbf{S}_i \\ \mathbf{S}_i \mathbf{E}(l) & \mathbf{S}_i \mathbf{E}^{-1}(l) \end{bmatrix} \begin{bmatrix} \mathbf{C}(1) \\ \mathbf{C}(2) \end{bmatrix} . \quad (2.13)$$

Solving (2.12) for the constants and substituting into (2.13), we have

$$\begin{bmatrix} \mathbf{I}(0) \\ \mathbf{I}(l) \end{bmatrix} = \begin{bmatrix} \mathbf{S}_i & \mathbf{S}_i \\ \mathbf{S}_i \mathbf{E}(l) & \mathbf{S}_i \mathbf{E}^{-1}(l) \end{bmatrix} \times \begin{bmatrix} \mathbf{S}_v & \mathbf{S}_v \\ \mathbf{S}_v \mathbf{E}(l) & \mathbf{S}_v \mathbf{E}^{-1}(l) \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{V}(0) \\ \mathbf{V}(l) \end{bmatrix}. \quad (2.14)$$

This equation represents the transmission line in admittance form. $\mathbf{A}\mathbf{V} = \mathbf{I}$, where

$$\mathbf{I} = \begin{bmatrix} \mathbf{I}(0) \\ \mathbf{I}(l) \end{bmatrix} \quad (2.15)$$

is a vector of the terminal currents into the transmission line,

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}(0) \\ \mathbf{V}(l) \end{bmatrix} \quad (2.16)$$

is a vector of the terminal voltages of the transmission line and

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} \mathbf{S}_i & \mathbf{S}_i \\ \mathbf{S}_i \mathbf{E}(l) & \mathbf{S}_i \mathbf{E}^{-1}(l) \end{bmatrix} \begin{bmatrix} \mathbf{S}_v & \mathbf{S}_v \\ \mathbf{S}_v \mathbf{E}(l) & \mathbf{S}_v \mathbf{E}^{-1}(l) \end{bmatrix}^{-1} \\ &= \begin{bmatrix} \mathbf{S}_i \mathbf{E}_1 \mathbf{S}_v^{-1} & \mathbf{S}_i \mathbf{E}_2 \mathbf{S}_v^{-1} \\ \mathbf{S}_i \mathbf{E}_2 \mathbf{S}_v^{-1} & \mathbf{S}_i \mathbf{E}_1 \mathbf{S}_v^{-1} \end{bmatrix} \end{aligned} \quad (2.17)$$

is an admittance matrix for the transmission line, where $\mathbf{E}_1$ and $\mathbf{E}_2$ are diagonal matrices

$$\mathbf{E}_1 = \text{diag.} \left\{ \frac{(1 + e^{-2\gamma_i l})}{(1 - e^{-2\gamma_i l})}, \quad i = 1, 2, \dots, N_k \right\}, \quad (2.18)$$

$$\mathbf{E}_2 = \text{diag.} \left\{ \frac{2}{(e^{-\gamma_i l} - e^{\gamma_i l})}, \quad i = 1, 2, \dots, N_k \right\}. \quad (2.19)$$

2.2 Formulation of the Network Equations

Suppose the network π consists of lumped elements and N_s multiconductor transmission lines. The modified nodal equations [30] for the overall network π are

$$\mathbf{C}_\pi \frac{d\mathbf{v}_\pi(t)}{dt} + \mathbf{G}_\pi \mathbf{v}_\pi(t) + \sum_{k=1}^{N_s} \mathbf{D}_k \mathbf{i}_k(t) = \mathbf{e}_\pi(t) , \quad (2.20)$$

where $\mathbf{C}_\pi$ and $\mathbf{G}_\pi$ are N_π by N_π matrices determined by lumped elements in the network. $\mathbf{v}_\pi(t)$ is the vector of node voltage waveforms appended by independent voltage source current and inductor current waveforms. $\mathbf{D}_k$ is an incidence matrix containing 1's and 0's which maps $\mathbf{i}_k(t)$, the terminal current waveforms of the k th distributed transmission line, into the N_π -node space of network π . $\mathbf{e}_\pi(t)$ is the vector of source waveforms.

The s-domain equation is obtained by taking Laplace transform of (2.20)

$$\mathbf{Y}_\pi \mathbf{V}_\pi(s) = \mathbf{E}_\pi(s) + \mathbf{C}_\pi \mathbf{v}_\pi(0) , \quad (2.21)$$

where

$$\mathbf{Y}_\pi = \mathbf{G}_\pi + s\mathbf{C}_\pi + \sum_{k=1}^{N_s} \mathbf{D}_k \mathbf{A}_k \mathbf{D}_k^T . \quad (2.22)$$

T denotes transpose and $\mathbf{A}_k$ is the nodal admittance matrix of the k th distributed transmission line defined by (2.17).

Example The following matrices illustrate the formulation for the network shown in Figure 2.2.

Figure 2.2: A simple transmission line network.

$$G_{\pi} = \begin{bmatrix} G_1 & -G_1 & 0 & 0 & 0 & 1 \\ -G_1 & G_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_3 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad C_{\pi} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$V_{\pi} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ I_1 \end{bmatrix} \quad D_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad I_1 = \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \end{bmatrix} \quad E_{\pi} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E \end{bmatrix}$$

2.3 Time-domain Analysis of the Linear Network

2.3.1 Numerical Inversion of Laplace Transform Algorithm

The method for the NILT (Numerical Inversion of Laplace Transform) [1] is based on the numerical evaluation of the Laplace transform inversion integral [31]

$$\begin{aligned} v(t) &= \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} V(s)e^{st} ds \\ &= \frac{1}{2\pi j t} \int_{\sigma'-j\infty}^{\sigma'+j\infty} V\left(\frac{z}{t}\right) e^z dz , \end{aligned} \quad (2.23)$$

where $z = st$.

Approximate the function e^z in (2.23) by a Padé rational function

$$\xi_{N,M}(z) = \frac{P_N(z)}{Q_M(z)} = \frac{\sum_{i=0}^N (M+N-i)! \binom{N}{i} z^i}{\sum_{i=0}^M (-1)^i (M+N-i)! \binom{M}{i} z^i} , \quad (2.24)$$

which has the first $M+N+1$ terms of its Taylor expansion equal to those of the Taylor expansion of $e^z = \sum_{i=0}^{\infty} z^i/i!$. By inserting (2.24) into (2.23), an approximation $\hat{v}(t)$ to $v(t)$ is obtained:

$$\hat{v}(t) = \frac{1}{2\pi j t} \int_{\sigma'-j\infty}^{\sigma'+j\infty} V\left(\frac{z}{t}\right) \xi_{N,M}(z) dz . \quad (2.25)$$

The integral can be evaluated by residue calculus by closing the path of integration along an infinite arc either to the right or to the left. In order that the path

along the infinite arc does not contribute to the integral, take N, M such that the function

$$F(z) = V\left(\frac{z}{t}\right)\xi_{N,M}(z) \quad (2.26)$$

has at least two more finite poles than zeros. Then

$$\oint F(z)dz = \pm 2\pi j(\sum \text{residue at poles inside the closed path}) , \quad (2.27)$$

where the sign depends on the direction in which the path is closed.

Select any $N < M$. Then

$$\xi_{N,M}(z) = \sum_{i=1}^M \frac{K_i}{z - z_i} , \quad (2.28)$$

where K_i are the residues and z_i are the poles of $\xi_{N,M}(z)$. Closing the path of integration around the poles in the right half plane produces an equation of the form

$$\hat{v}(t) = -\frac{1}{t} \sum_{i=1}^M K_i V\left(\frac{z_i}{t}\right) ; \quad t > 0 , \quad (2.29)$$

which is the formula for the calculation of the time-domain response. The real-time function can be evaluated using only the poles z_i in the upper half plane. When M is even,

$$\hat{v}(t) = -\frac{1}{t} \sum_{i=1}^{M'} K'_i V\left(\frac{z_i}{t}\right) ; \quad t > 0 , \quad (2.30)$$

where $M' = M/2$, $K'_i = 2K_i$. When M is odd, $M' = (M + 1)/2$ and $K'_i = K_i$ for the residues corresponding to the real pole.

2.3.2 Application of the NILT to Linear Networks

Given a Laplace domain MNA representation for a linear network π containing lossy multiconductor transmission lines, $\mathbf{Y}_\pi(s)\mathbf{V}_\pi(s) = \mathbf{E}_\pi(s) + \mathbf{C}_\pi \mathbf{v}_\pi(0)$, where $\mathbf{V}_\pi(s)$

is the circuit response as defined in (2.21), the time domain response can be calculated using (2.30) as

$$\hat{\mathbf{v}}(t) = -\frac{1}{t} \sum_{i=1}^{M'} \operatorname{Re}\{K_i' \mathbf{Y}_\pi^{-1}(\frac{z_i}{t}) [\mathbf{E}_\pi(\frac{z_i}{t}) + \mathbf{C}_\pi \mathbf{v}_\pi(0)]\} . \quad (2.31)$$

The response at each time point is obtained from the M' solutions to the network equation evaluated at the complex frequencies $s = z_i/t$.

Several points of interest should be mentioned when comparing (2.31) with inverse FFT (Fast Fourier Transform) methods [15]. First, the numerical inversion of the Laplace transform does not suffer from aliasing effects. Second, the number of frequency points defined by $M' < 15$ is usually much less than the number of frequency points required by the inverse FFT algorithm [15] for the same accuracy in the output waveforms. This leads to considerable savings in the CPU requirements. Third, from (2.31), it is obvious that the solution at time t is completely independent of solutions at all other time points.

2.4 Asymptotic Waveform Evaluation Technique

The Asymptotic Waveform Evaluation (AWE) [26] is a variable refinement waveform estimator for linear RLC approximate interconnect models. In general, AWE can evaluate circuits with non-monotone details, such as undershoot, overshoot and glitches due to charge sharing, by increasing the order of the approximation as required. Higher order approximations are obtained recursively, at an incremental CPU time. The AWE technique is a very efficient method for transient analysis of RLC circuits. Because of this outstanding uniqueness, it has been expanded to the analysis of networks containing lossy coupled transmission lines by Tang and Nakhla [18]. However, the concept of AWE is at present only applicable to linear circuits or the

linear subcircuit from a nonlinear network. In this thesis, we will extend the AWE principle directly to the solution of nonlinear network equations. The details of the AWE technique will be presented in Chapter 5.

2.5 Group Delay and Gain Slope

Group delay, gain slope and their sensitivities have been an attractive vehicle for circuit design such as design of filters [32], [33], [34], [35]. Consider a network that has an input/output transfer function $H(j\omega)$. Group delay can be defined as [31]:

$$T_G \triangleq -\frac{\partial \theta}{\partial \omega} = -\text{Imag}\left\{\frac{1}{H} \frac{\partial H}{\partial \omega}\right\} \quad (2.32)$$

and gain slope (amplitude sensitivity) can be defined as [31]:

$$G_L \triangleq \frac{\partial 20\log|H|}{\partial \omega} = \frac{20}{\ln 10} \text{Real}\left\{\frac{1}{H} \frac{\partial H}{\partial \omega}\right\}, \quad (2.33)$$

where θ is the phase of transfer function $H(j\omega)$.

Group delay is a frequency-domain characteristic that can be used to obtain insight into a system's transient responses. It is well known that there is a close relationship between group delay and signal propagation delay. For RC networks, Vlach *et al.* recently verified group delay at frequency zero as being exactly equal to Elmore delay and observed a simple empirical relationship between group delay and signal delay of responses corresponding to a step excitation [36]. In general, over a frequency range of interest, the smaller the group delay, the smaller the signal propagation delay. And furthermore, if an output is a delayed replica of the input, flat group delay and zero gain slope should hold across the entire frequency range.

Computation of group delay, gain slope and their sensitivities requires first- and second-order sensitivity analysis of the network. These sensitivities can be obtained

by means of the adjoint network approach [37], [38], [39]. The applicability of the adjoint network concept to the evaluation of second-order network sensitivities [40], [41] has been known for many years. The computation of group delay [32], [42] and its sensitivities with respect to component parameters using these ideas has also been studied [35]. For lossy coupled multiconductor transmission line networks, however, group delay and gain slope sensitivity analysis is relatively more involved and requires the second-order derivatives of the eigenvalues and eigenvectors of the propagation matrix. The detailed algorithm for group delay and gain slope sensitivity analysis for transmission line networks will be derived in Chapter 3.

2.6 Existing Optimization Approaches

Computer-aided circuit optimization is certainly one of the most active areas of interest. The studies in the past two decades on the theoretical and algorithmic aspects of optimization techniques have produced a great number of results. Optimization methods have evolved from simple, low-dimension-oriented algorithms into sophisticated and powerful ones, e.g., [42], [43], [44]. Gradient-based optimization methods have gained increasing popularity in recent years for their effectiveness and efficiency. In particular, a class of two-stage algorithms, suggested by Hald and Madsen in [45], [46] and Bandler *et al.* in [47], [48], provided effective and efficient solutions to the popular minimax and nonlinear l_1 optimization problems. A first-order method of the Gauss-Newton type is employed in stage 1 to provide global convergence to a neighborhood of a solution. When the solution is singular, the first-order method suffers from a very slow rate of convergence and a switch is made to a quasi-Newton method (stage 2), i.e., a second-order method, which has superlinear final conver-

gence. Several switches between the two stages may take place and the switching criteria ensure the global convergence of the combined algorithm.

The application of gradient-based l_p optimization techniques requires the first-order derivatives of the error functions with respect to the design variables. The well-known adjoint network approach, developed by Director and Rohrer [37], has been a classical vehicle and remains a powerful tool for sensitivity analysis of linear and nonlinear circuits [49]. The exact adjoint sensitivity technique (EAST) enjoys high computational efficiency, but suffers from implementation complexity. On the other hand, the conventional perturbation approximate sensitivity technique (PAST) is conceptually simple and easy to implement. However, it may not be accurate enough and the computational effort involved, especially for large-scale problems, may be prohibitive. Several novel, efficient methods for sensitivity analysis have been presented in the literature such as the integrated gradient approximation technique (IGAT) [50] and the feasible adjoint sensitivity technique (FAST) [51]. IGAT utilizes the Broyden formula to update the approximate gradients using only information from function evaluation. FAST combines the efficiency and accuracy of the adjoint sensitivity technique with the simplicity of the perturbation technique.

The development of large-scale network optimization techniques have been motivated by the requirements of the VLSI era. Realistic VLSI circuits are usually very large, containing many elements, many design parameters and design specifications. Direct design optimization will fail in such cases due to high computation effort and the increased likelihood of stopping at an undesired local minimum. Large-scale optimization has been successfully approached by decomposing the large numbers of design variables and error functions [52] and by taking advantage of network topological decomposition [53].

Another important application of circuit optimization is statistical design. There are many uncertainties in realistic circuit design, for example, due to physical (manufacturing, operating) tolerances, model uncertainties as well as to parasitic effects. Statistical circuit design which takes into account the uncertainties has been recognized as an indispensable tool for modern CAD of integrated circuits and has received more and more serious attention from IC designers. During the past two decades, a number of algorithms for yield optimization have been developed, such as the simplicial approximation [54], the gravity method [55] and parametric sampling [56]. One of the recent algorithms, proposed by Bandler *et al.* [43], [51], is to formulate the yield-driven design as a two level one-sided l_p optimization problem, where the lower level l_q deals with the error functions associated with each circuit, and the higher level l_p deals with the multicircuit outcomes. With different q and p , it leads to a variety of algorithms for yield enhancement.

Various applications of optimization techniques for modeling, diagnosis and tuning (MDT) of electrical circuits have also been found in the literature [44], [57]-[60]. Unlike the typical circuit design problems, whose objectives are to satisfy or to exceed design specifications as much as possible, the MDT problems are usually oriented either toward (response) data fitting or toward “parameter fitting” or a combination of both. Many optimization algorithms and techniques, such as the efficient l_1 algorithm [43], [57]-[59], the l_2 algorithm [59], [60] and large-scale optimization technique [58], have been successfully used for model parameter extraction. The principles and general formulations of optimization techniques for MDT problems have been summarized by Bandler and Zhang in [44].

Although optimization has been used to solve design, modeling, tuning, diagnosis and yield optimization problems for many years, optimization of high-speed VLSI

interconnect networks has not been well exploited. Recently Zhang *et al.* presented an approach for optimal design of time responses of high-speed interconnects modeled by distributed coupled transmission line networks [21]-[23]. A brief review of the transient optimization technique developed in [21]-[23] will be presented in the rest of this section.

[21] derived a systematic formulation for optimization of transient responses such as delay, crosstalk and reflection, as required in high-speed VLSI interconnects design. Let Φ be a vector of design variables which include physical/geometrical parameters of the interconnects and parameters in terminating/matching networks. Let $v_j(\Phi, t)$ be the response signal at node j and time t . Suppose the signal propagation delay is described by the time at which the transient signal reaches a threshold value V_T . Specifications on crosstalk, delay, and reflection at node j are denoted by $S_{cj}(t)$, τ_j and $S_{rj}(t)$, respectively. Error functions for crosstalk, delay and reflection optimization $e_k(\Phi)$ are defined respectively as the following:

$$\begin{aligned} &W_c(v_j(\Phi, t) - S_{cj}(t)) \quad \text{and} \\ &-W_c(v_j(\Phi, t) + S_{cj}(t)) \quad \text{for } j \in \mathbf{J}_c \text{ and } 0 < t < \infty, \end{aligned} \quad (2.34)$$

$$\begin{aligned} &W_d(v_j(\Phi, \tau_j) - V_T) \quad \text{and} \\ &-W_d(v_j(\Phi, \tau_j) - V_T) \quad \text{for } j \in \mathbf{J}_d, \end{aligned} \quad (2.35)$$

$$\begin{aligned} &W_r(v_j(\Phi, t) - S_{rj}(t)) \quad \text{and} \\ &-W_r(v_j(\Phi, t) + S_{rj}(t)) \quad \text{for } j \in \mathbf{J}_r \text{ and } T + \tau_j + \delta < t < \infty, \end{aligned} \quad (2.36)$$

where index sets $\mathbf{J}_c$, $\mathbf{J}_d$ and $\mathbf{J}_r$ contain circuit nodes where crosstalk, delay and reflections, respectively, are the responses of interest. W_c , W_d and W_r are weighting

factors. T is the duration of the excitation signal and δ is a small value not exceeding the signal's fall time. The design optimization problem is to find Φ such that

$$U(\Phi) \triangleq \max_j \{c_j(\Phi), j = 1, 2, \dots, m\} \quad (2.37)$$

is minimized subject to electrical and physical constraints $\mathbf{g}(\Phi) \leq \mathbf{0}$ and $\mathbf{h}(\Phi) = \mathbf{0}$. This optimization problem is solved by combining a powerful two stage minimax algorithm [47] and the recently available technique for simulation and sensitivity analysis based on numerical inversion of Laplace transform (NILT) for lossy coupled transmission lines [2]. At the solution, crosstalk, signal delay and reflections are simultaneously minimized. The following example illustrates the principle of the time-domain optimization approach. This example was solved by R. Liu and presented in [61], [23].

Example: A circuit which contains 35 transmission lines is shown in Figure 2.3. The circuit is obtained by cascading the interconnect model shown in Figure 2.4. The objective is to reduce the propagation delay and rise time of v_{out} without increasing the delay of v_{sub} in each of the five subcircuits. Design variables in each subcircuit includes lengths of the 7 transmission lines, resistors R_1, R_2, R_3, R_4 , capacitors C_1, C_2 , and inductor L_1 resulting in a total of 70 variables for optimization. The constraint for optimization is that the total length of the 7 transmission lines in each subcircuit remain unchanged. Design specifications on the transient response v_{out} as plotted in Figure 2.5 are violated before optimization. After optimization, the response met the specifications exactly as shown in figure 2.5. Figure 2.6 shows the response v_{sub} from a representative subcircuit (subcircuit 1) where the delay of the signal has not been increased.

Figure 2.3: An interconnect network with 35 transmission lines.

Figure 2.4: A subnetwork of Figure 2.3. There are 7 transmission lines in this interconnect model.

Figure 2.5: Response v_{out} of Figure 2.3 before (solid line) and after (dashed line) optimization.

Figure 2.6: Response v_{sub} of subcircuit 1 in Figure 2.3 before (solid line) and after (dashed line) optimization.

By imposing design specifications directly on transient waveforms, the time domain optimization method is a straightforward and accurate approach. It is an effective method, especially for minimizing crosstalk and ringing. However it is difficult, in general, to efficiently minimize propagation delay by imposing design specification directly on transient waveform. A new and more efficient frequency domain optimization approach is derived in Chapter 4.

2.7 Conclusions

In this Chapter, we have provided the necessary background information and literature review of existing transient analysis and optimization approaches for high speed VLSI interconnects. Optimization methods are considered important tools and found a large number of applications in solving circuit design problems. However for high-speed VLSI interconnect networks, fairly treated in the literature are the simulation and sensitivity analysis. Design optimization of such circuits has only been addressed very recently and is limited to the time-domain optimization technique. It is the main objective of this thesis to further address such a design optimization problem. A frequency domain optimization approach will be developed in Chapter 4. The formulation of interconnect network equations and group delay and gain slope concepts presented in this Chapter will be used throughout the thesis.

Chapter 3

Group Delay and its Sensitivity Analysis

3.1 Introduction

Group delay and its sensitivities have been an attractive vehicle for circuit design such as filters [32], [33], [34], [35] and IC digital cells [36]. Evaluation of group delay sensitivities with respect to design variables, which requires second order sensitivity analysis of the network, is necessary for gradient based optimization. However for lossy multiconductor transmission line networks, calculation of group delay sensitivity is relatively more involved and has not been previously presented in the literature. In this chapter, a technique for efficient computation of exact group delay and its sensitivities is derived for multiconductor transmission line networks. It is based on our recent work in [24].

3.2 Computation of Group Delay and Gain Slope

Consider a multiconductor transmission line network which can be formulated as (2.21). Let

$$V_{out}(s) = \mathbf{u}^T \mathbf{V}_\pi(s) \quad (3.1)$$

be the frequency response of interest, where $\mathbf{u}$ is a constant N_π -vector and $s = j\omega$. Let θ be the phase of V_{out} . We redefine the group delay as [31]

$$T_G \triangleq -\frac{\partial \theta}{\partial \omega} = -\text{Real}\left\{\frac{1}{V_{out}} \frac{\partial V_{out}}{\partial s}\right\} \quad (3.2)$$

and gain slope as [31]

$$G_L \triangleq \frac{\partial 20 \log |V_{out}|}{\partial \omega} = -\frac{20}{\ln 10} \text{Imag}\left\{\frac{1}{V_{out}} \frac{\partial V_{out}}{\partial s}\right\}. \quad (3.3)$$

Computation of group delay and gain slope requires first-order sensitivity analysis of the network. To compute $\frac{\partial V_{out}}{\partial s}$, we use the adjoint sensitivity approach [37], [38], [39],

$$\begin{aligned} \frac{\partial V_{out}}{\partial s} &= (\mathbf{V}_\pi^a)^T \left[\frac{\partial \mathbf{Y}_\pi}{\partial s} \mathbf{V}_\pi - \frac{\partial \mathbf{E}_\pi}{\partial s} \right] \\ &= (\mathbf{V}_\pi^a)^T \left[(\mathbf{C}_\pi + \sum_{k=1}^{N_s} \mathbf{D}_k \frac{\partial \mathbf{A}_k}{\partial s} \mathbf{D}_k^T) \mathbf{V}_\pi - \frac{\partial \mathbf{E}_\pi}{\partial s} \right], \end{aligned} \quad (3.4)$$

where $\mathbf{Y}_\pi$ is defined in (2.22), and $\mathbf{V}_\pi^a$ is solved from the adjoint equation:

$$\mathbf{Y}_\pi^T \mathbf{V}_\pi^a = -\mathbf{u}. \quad (3.5)$$

To compute $\frac{\partial \mathbf{A}_k}{\partial s}$, we first compute the sensitivities of eigenvalues γ_i^L and eigenvectors $\mathbf{x}_i$ w.r.t. frequency s by solving the linear equation:

$$\mathbf{B} \begin{bmatrix} \frac{\partial \mathbf{x}_i}{\partial s} \\ \frac{\partial \gamma_i^L}{\partial s} \end{bmatrix} = \begin{bmatrix} \frac{\partial (\mathbf{Z}_L \mathbf{Y}_L)}{\partial s} \mathbf{x}_i \\ 0 \end{bmatrix}, \quad (3.6)$$

where:

$$\mathbf{B} = \begin{bmatrix} \gamma_i^2 \mathbf{U} - \mathbf{Z}_L \mathbf{Y}_L & \mathbf{x}_i \\ \mathbf{x}_i^T & 0 \end{bmatrix}. \quad (3.7)$$

The solution of (3.6) forms $\frac{\partial \mathbf{S}_v}{\partial s}$ and $\frac{\partial \Gamma}{\partial s}$. The sensitivity $\frac{\partial \mathbf{A}_k}{\partial s}$ is then obtained by differentiating (2.17):

$$\begin{aligned} \frac{\partial \mathbf{A}_k}{\partial s} \begin{bmatrix} \mathbf{S}_v & 0 \\ 0 & \mathbf{S}_v \end{bmatrix} &= \begin{bmatrix} \frac{\partial \mathbf{S}_i}{\partial s} & 0 \\ 0 & \frac{\partial \mathbf{S}_i}{\partial s} \end{bmatrix} \begin{bmatrix} \mathbf{E}_1 & \mathbf{E}_2 \\ \mathbf{E}_2 & \mathbf{E}_1 \end{bmatrix} \\ &+ \begin{bmatrix} \mathbf{S}_i & 0 \\ 0 & \mathbf{S}_i \end{bmatrix} \begin{bmatrix} \frac{\partial \mathbf{E}_1}{\partial s} & \frac{\partial \mathbf{E}_2}{\partial s} \\ \frac{\partial \mathbf{E}_2}{\partial s} & \frac{\partial \mathbf{E}_1}{\partial s} \end{bmatrix} - \mathbf{A}_k \begin{bmatrix} \frac{\partial \mathbf{S}_v}{\partial s} & 0 \\ 0 & \frac{\partial \mathbf{S}_v}{\partial s} \end{bmatrix}, \end{aligned} \quad (3.8)$$

where $\frac{\partial \mathbf{S}_i}{\partial s}$ is obtained from

$$\mathbf{Z}_L \frac{\partial \mathbf{S}_i}{\partial s} = \frac{\partial \mathbf{S}_v}{\partial s} \Gamma + \mathbf{S}_v \frac{\partial \Gamma}{\partial s} - \frac{\partial \mathbf{Z}_L}{\partial s} \mathbf{S}_i. \quad (3.9)$$

3.3 Computation of Group Delay and Gain Slope Sensitivities

Let ϕ be a design variable. In order to obtain the sensitivity of group delay, we differentiate (3.2)

$$\frac{\partial T_G}{\partial \phi} = \text{Real} \left\{ \frac{1}{V_{out}^2} \frac{\partial V_{out}}{\partial \phi} \frac{\partial V_{out}}{\partial s} - \frac{1}{V_{out}} \frac{\partial^2 V_{out}}{\partial s \partial \phi} \right\}. \quad (3.10)$$

Similarly, by differentiating (3.3), we have

$$\frac{\partial G_L}{\partial \phi} = \frac{20}{\ln 10} \text{Imag} \left\{ \frac{1}{V_{out}^2} \frac{\partial V_{out}}{\partial \phi} \frac{\partial V_{out}}{\partial s} - \frac{1}{V_{out}} \frac{\partial^2 V_{out}}{\partial s \partial \phi} \right\}. \quad (3.11)$$

Therefore both group delay sensitivity and gain slope sensitivity require first- and second-order derivative information $\frac{\partial V_{out}}{\partial s}$, $\frac{\partial V_{out}}{\partial \phi}$ and $\frac{\partial^2 V_{out}}{\partial s \partial \phi}$.

Similar to computing $\frac{\partial V_{out}}{\partial s}$, the sensitivity $\frac{\partial V_{out}}{\partial \phi}$ can be calculated from [2]

$$\frac{\partial V_{out}}{\partial \phi} = (\mathbf{V}_\pi^a)^T \left[\left(\frac{\partial \mathbf{G}_\pi}{\partial \phi} + s \frac{\partial \mathbf{C}_\pi}{\partial \phi} + \sum_{k=1}^{N_s} \mathbf{D}_k \frac{\partial \mathbf{A}_k}{\partial \phi} \mathbf{D}_k^T \right) \mathbf{V}_\pi - \frac{\partial \mathbf{C}_k}{\partial \phi} \mathbf{v}_\pi(0) \right], \quad (3.12)$$

where

$$\begin{aligned} \frac{\partial \mathbf{A}_k}{\partial \phi} \begin{bmatrix} \mathbf{S}_v & 0 \\ 0 & \mathbf{S}_v \end{bmatrix} &= \begin{bmatrix} \frac{\partial \mathbf{S}_1}{\partial \phi} & 0 \\ 0 & \frac{\partial \mathbf{S}_1}{\partial \phi} \end{bmatrix} \begin{bmatrix} \mathbf{E}_1 & \mathbf{E}_2 \\ \mathbf{E}_2 & \mathbf{E}_1 \end{bmatrix} \\ &+ \begin{bmatrix} \mathbf{S}_i & 0 \\ 0 & \mathbf{S}_i \end{bmatrix} \begin{bmatrix} \frac{\partial \mathbf{E}_1}{\partial \phi} & \frac{\partial \mathbf{E}_2}{\partial \phi} \\ \frac{\partial \mathbf{E}_2}{\partial \phi} & \frac{\partial \mathbf{E}_1}{\partial \phi} \end{bmatrix} - \mathbf{A}_k \begin{bmatrix} \frac{\partial \mathbf{S}_v}{\partial \phi} & 0 \\ 0 & \frac{\partial \mathbf{S}_v}{\partial \phi} \end{bmatrix}. \end{aligned} \quad (3.13)$$

$$\mathbf{Z}_L \frac{\partial \mathbf{S}_i}{\partial \phi} = \frac{\partial \mathbf{S}_v}{\partial \phi} \mathbf{\Gamma} + \mathbf{S}_v \frac{\partial \mathbf{\Gamma}}{\partial \phi} - \frac{\partial \mathbf{Z}_L}{\partial \phi} \mathbf{S}_i. \quad (3.14)$$

$$\mathbf{B} \begin{bmatrix} \frac{\partial \mathbf{x}_1}{\partial \phi} \\ \frac{\partial \gamma_1^2}{\partial \phi} \end{bmatrix} = \begin{bmatrix} \frac{\partial (\mathbf{Z}_L \mathbf{Y}_L)}{\partial \phi} \mathbf{x}_1 \\ 0 \end{bmatrix}. \quad (3.15)$$

Now the problem is to find the second-order derivative $\frac{\partial^2 V_{out}}{\partial s \partial \phi}$. Differentiating (3.4), we have

$$\begin{aligned} \frac{\partial^2 V_{out}}{\partial s \partial \phi} &= \frac{\partial (\mathbf{V}_\pi^a)^T}{\partial \phi} \left[\frac{\partial \mathbf{Y}_\pi}{\partial s} \mathbf{V}_\pi - \frac{\partial \mathbf{E}_\pi}{\partial s} \right] \\ &+ (\mathbf{V}_\pi^a)^T \left[\frac{\partial^2 \mathbf{Y}_\pi}{\partial s \partial \phi} \mathbf{V}_\pi + \frac{\partial \mathbf{Y}_\pi}{\partial s} \frac{\partial \mathbf{V}_\pi}{\partial \phi} \right], \end{aligned} \quad (3.16)$$

where the sensitivity of the adjoint response is solved from

$$\mathbf{Y}_\pi^T \frac{\partial \mathbf{V}_\pi^a}{\partial \phi} = - \frac{\partial (\mathbf{Y}_\pi^T)}{\partial \phi} \mathbf{V}_\pi^a \quad (3.17)$$

and the second-order sensitivity of $\mathbf{Y}_\pi$ is

$$\frac{\partial^2 \mathbf{Y}_\pi}{\partial s \partial \phi} = \frac{\partial \mathbf{C}_\pi}{\partial \phi} + \sum_{k=1}^{N_s} \mathbf{D}_k \frac{\partial^2 \mathbf{A}_k}{\partial s \partial \phi} \mathbf{D}_k^T. \quad (3.18)$$

Now it is necessary to compute the second-order derivative $\frac{\partial^2 \mathbf{A}_k}{\partial s \partial \phi}$ for multiconductor transmission lines. For this purpose, the calculation of the second order derivatives

of eigenvalues and eigenvectors is required. From (3.6), we have

$$\mathbf{B} \begin{bmatrix} \frac{\partial^2 \mathbf{x}_i}{\partial s \partial \phi} \\ \frac{\partial^2 \gamma_i^2}{\partial s \partial \phi} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 (\mathbf{Z}_L \mathbf{Y}_L)}{\partial s \partial \phi} \mathbf{x}_i + \frac{\partial (\mathbf{Z}_L \mathbf{Y}_L)}{\partial s} \frac{\partial \mathbf{x}_i}{\partial \phi} \\ 0 \end{bmatrix} - \frac{\partial \mathbf{B}}{\partial \phi} \begin{bmatrix} \frac{\partial \mathbf{x}_i}{\partial s} \\ \frac{\partial \gamma_i^2}{\partial s} \end{bmatrix}, \quad (3.19)$$

where $\frac{\partial \mathbf{B}}{\partial \phi}$ contains first-order sensitivities $\frac{\partial \gamma_i^2}{\partial \phi}$, $\frac{\partial \mathbf{x}_i}{\partial \phi}$ and $\frac{\partial (\mathbf{Z}_L \mathbf{Y}_L)}{\partial \phi}$ which have already been solved in calculating $\frac{\partial V_{\text{opt}}}{\partial \phi}$. The solution of the linear equations (3.19) needs only forward and backward substitutions since the LU factors of $\mathbf{B}$ are available from solving (3.6).

Once we have $\frac{\partial^2 \mathbf{x}_i}{\partial s \partial \phi}$ and $\frac{\partial^2 \gamma_i^2}{\partial s \partial \phi}$, we can easily obtain $\frac{\partial^2 \mathbf{S}_v}{\partial s \partial \phi}$ and $\frac{\partial^2 \Gamma}{\partial s \partial \phi}$. Finally from (3.8), $\frac{\partial^2 \mathbf{A}_k}{\partial s \partial \phi}$ can be solved from

$$\begin{aligned} \frac{\partial^2 \mathbf{A}_k}{\partial s \partial \phi} \begin{bmatrix} \mathbf{S}_v & 0 \\ 0 & \mathbf{S}_v \end{bmatrix} &= \begin{bmatrix} \frac{\partial^2 \mathbf{S}_i}{\partial s \partial \phi} & 0 \\ 0 & \frac{\partial^2 \mathbf{S}_i}{\partial s \partial \phi} \end{bmatrix} \begin{bmatrix} \mathbf{E}_1 & \mathbf{E}_2 \\ \mathbf{E}_2 & \mathbf{E}_1 \end{bmatrix} \\ &+ \begin{bmatrix} \frac{\partial \mathbf{S}_i}{\partial s} & 0 \\ 0 & \frac{\partial \mathbf{S}_i}{\partial s} \end{bmatrix} \begin{bmatrix} \frac{\partial \mathbf{E}_1}{\partial \phi} & \frac{\partial \mathbf{E}_2}{\partial \phi} \\ \frac{\partial \mathbf{E}_2}{\partial \phi} & \frac{\partial \mathbf{E}_1}{\partial \phi} \end{bmatrix} + \begin{bmatrix} \frac{\partial \mathbf{S}_i}{\partial \phi} & 0 \\ 0 & \frac{\partial \mathbf{S}_i}{\partial \phi} \end{bmatrix} \begin{bmatrix} \frac{\partial \mathbf{E}_1}{\partial s} & \frac{\partial \mathbf{E}_2}{\partial s} \\ \frac{\partial \mathbf{E}_2}{\partial s} & \frac{\partial \mathbf{E}_1}{\partial s} \end{bmatrix} \\ &+ \begin{bmatrix} \mathbf{S}_i & 0 \\ 0 & \mathbf{S}_i \end{bmatrix} \begin{bmatrix} \frac{\partial^2 \mathbf{E}_1}{\partial s \partial \phi} & \frac{\partial^2 \mathbf{E}_2}{\partial s \partial \phi} \\ \frac{\partial^2 \mathbf{E}_2}{\partial s \partial \phi} & \frac{\partial^2 \mathbf{E}_1}{\partial s \partial \phi} \end{bmatrix} - \frac{\partial \mathbf{A}_k}{\partial \phi} \begin{bmatrix} \frac{\partial \mathbf{S}_v}{\partial s} & 0 \\ 0 & \frac{\partial \mathbf{S}_v}{\partial s} \end{bmatrix} \\ &- \mathbf{A}_k \begin{bmatrix} \frac{\partial^2 \mathbf{S}_v}{\partial s \partial \phi} & 0 \\ 0 & \frac{\partial^2 \mathbf{S}_v}{\partial s \partial \phi} \end{bmatrix} - \frac{\partial \mathbf{A}_k}{\partial s} \begin{bmatrix} \frac{\partial \mathbf{S}_v}{\partial \phi} & 0 \\ 0 & \frac{\partial \mathbf{S}_v}{\partial \phi} \end{bmatrix}, \end{aligned} \quad (3.20)$$

where $\frac{\partial^2 \mathbf{S}_i}{\partial s \partial \phi}$ is obtained from

$$\begin{aligned} \mathbf{Z}_i \frac{\partial^2 \mathbf{S}_i}{\partial s \partial \phi} &= \frac{\partial^2 \mathbf{S}_v}{\partial s \partial \phi} \Gamma + \frac{\partial \mathbf{S}_v}{\partial s} \frac{\partial \Gamma}{\partial \phi} + \frac{\partial \mathbf{S}_v}{\partial \phi} \frac{\partial \Gamma}{\partial s} \\ &+ \mathbf{S}_v \frac{\partial^2 \Gamma}{\partial s \partial \phi} - \frac{\partial^2 \mathbf{Z}_L}{\partial s \partial \phi} \mathbf{S}_i - \frac{\partial \mathbf{Z}_L}{\partial s} \frac{\partial \mathbf{S}_i}{\partial \phi} - \frac{\partial \mathbf{Z}_L}{\partial \phi} \frac{\partial \mathbf{S}_i}{\partial s}. \end{aligned} \quad (3.21)$$

3.4 Verification of Group Delay Sensitivity Formulas

The formulas for calculating group delay, gain slope and their sensitivities are implemented in a program for the design of distributed networks with lossy coupled transmission lines. Consider the 3 transmission line example of Figure 3.1. The lengths of the 3 transmission lines are 0.05m, 0.04m and 0.03m, respectively. The cross-sectional physical parameters are conductor width $w = 0.00058m$, circuit board thickness $h = 0.00117m$, distance between the two conductors $s = 0.00249m$ and relative dielectric constant $\epsilon_r = 4.54$. The empirical formulas from [29] are used to compute the $\mathbf{R}$, $\mathbf{L}$, $\mathbf{G}$ and $\mathbf{C}$ matrices for the transmission lines. Group delay sensitivities computed from the proposed method are compared with those from perturbation in Figure 3.2. Excellent agreement is observed. Computation speed by the new formulas is faster than by perturbation. In the perturbation method frequency ω and variable ϕ are perturbed by a small amount, $\Delta\omega$ and $\Delta\phi$, respectively. The network equations are resolved at perturbed points of ω and ϕ . The group delay sensitivity is found using a finite difference approximation. The perturbation method requires at least three analyses of the original circuit for each variable under consideration. However by using the proposed method, only one circuit analysis plus a small additional computational cost is needed to get the sensitivities with respect to all variables. Notice that no LU factorization is required for solving (3.5) and (3.17), only forward/backward substitutions being needed. Similarly only forward/backward substitutions are required for solving (3.15) and (3.19). Since $\mathbf{A}_k$, $\mathbf{B}$, $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{S}_1$ and $\mathbf{S}_v$ are all relatively small matrices, the computational effort for solving (3.6), (3.8), (3.13), (3.19) and (3.20) is small compared with a large circuit simulation.

Figure 3.1: Circuit schematic for the 3 transmission line network example.

Figure 3.2: The 3 transmission line example. Sensitivity of group delay at v_{out1} vs. angular frequency by proposed method (solid line) and by perturbation method (the stars). (a). sensitivity w.r.t. R_9 , (b). sensitivity w.r.t. C_2 , (c). sensitivity w.r.t. the length of transmission line 1 and (d). sensitivity w.r.t. the conductor width of all transmission lines.

3.5 Conclusions

Based on the adjoint sensitivity analysis technique, an algorithm for efficient computation of exact group delay, gain slope and their sensitivities for multiconductor transmission line networks has been derived. A numerical example is provided to illustrate the accuracy and efficiency of the proposed technique compared to the perturbation method. The group delay and gain slope information will be used for the formulation of a frequency domain optimization approach in Chapter 4.

Chapter 4

Formulation of VLSI Interconnect Optimization

4.1 Introduction

It is well known that group delay contains information about the signal propagation delay. For RC networks Vlach *et al.* recently verified group delay at frequency zero as being exactly equal to Elmore delay and observed a simple empirical relation between group delay and signal delay of responses corresponding to a step excitation [36]. In general, over a frequency range of interest, the smaller the group delay, the smaller the signal propagation delay. Furthermore, if an output is a delayed replica of the input, flat group delay and zero gain slope should hold across the entire frequency range. We take advantage of group delay and gain slope information for reduction of the transient signal propagation delay and distortion in a distributed network environment. The actual optimization is performed in the frequency domain. This chapter is based on our work in [25].

4.2 Error Functions

Let Φ denote all design variables including physical/geometrical parameters of the transmission lines and parameters in termination/matching networks. Let $S_{G,u}(s)$ and $S_{G,l}(s)$ be upper and lower specifications on the group delay T_G , respectively. Let W_G be a positive weighting factor. The minimization of signal delay can be achieved by minimizing

$$W_G(T_G(\Phi, s_k) - S_{G,u}(s_k)) \quad (4.1)$$

over a range of frequencies, $s_k = j\omega_k$, $k = 1, 2, \dots, K$. By imposing a lower specification on group delay, we have an error function

$$-W_G(T_G(\Phi, s_k) - S_{G,l}(s_k)) . \quad (4.2)$$

A typical lower specification is $S_{G,l}(s) = 0$. Simultaneous minimization of (4.1) and (4.2) over a frequency range reduces the group delay and improves its flatness.

Let $H(\Phi, s)$ be the transfer function of the network. Let $S_{H,l}(s)$ be the lower specification on the magnitude of $H(\Phi, s)$ and $S_L(s)$ be the upper specification on the magnitude of gain slope. Let W_H and W_L be positive weighting factors. Minimizing the following error functions

$$-W_H[|H(\Phi, s_k)| - S_{H,l}(s_k)] \quad (4.3)$$

$$W_L[G_L(\Phi, s_k) - S_L(s_k)] \quad (4.4)$$

$$-W_L[G_L(\Phi, s_k) + S_L(s_k)] \quad (4.5)$$

over a frequency range increases signal level and flatness of the transfer function magnitude. A flat group delay and flat magnitude of the transfer function reduces signal distortion.

Let $V(\Phi, s)$ be the spectrum of the crosstalk waveform. Reduction of crosstalk is realized by minimizing the magnitude of $V(\Phi, s)$ using

$$W_C(|V(\Phi, s_k)| - S_C(s_k)) , \quad (4.6)$$

where S_C is the specification on the magnitude of spectrum $V(\Phi, s)$ and W_C is a positive weighting factor.

4.3 Selection of Frequencies

The range of frequencies is determined by the fundamental frequency and its significant harmonics of the input signal. Consider a periodic symmetrical trapezoidal signal, which is characterized by four quantities: the signal period T_P , pulse width T_W , rise/fall times T_R and maximum amplitude (taken to be 1 volt here), as shown in Figure 4.1. The Fourier series for this signal is

$$v_i(t) = \sum_{n=1}^N a_n e^{j\omega_n t} , \quad (4.7)$$

where a_n is Fourier coefficient of the n th harmonic defined as

$$\begin{aligned} a_n &= 2a_v \frac{\sin(n\pi a_1)}{n\pi a_1} \frac{\sin(n\pi a_v)}{n\pi a_v} , \\ a_v &= (1.25T_R + T_W)/T_P , \\ a_1 &= 1.25T_R/T_P , \end{aligned}$$

and $\omega_n = 2n\pi/T_P$ is the angular frequency of the n th harmonic. Obviously the fundamental frequency is determined by the signal period. The number of significant harmonics is determined by the rise time and, to a lesser degree, the pulse width.

To find an approximate fundamental frequency for a trapezoidal signal excitation, we first select a time interval T long enough such that the transient will

Figure 4.1: (a) periodic trapezoid signal v_i ($T_P = 75$ ns, $T_R = 1.5$ ns, $T_W = 4.5$ ns) and (b) spectrum of v_i ($\text{abs}(a_n)$).

effectively vanish within T , and then choose $\frac{2\pi}{T}$ as the fundamental angular frequency ω_1 . To cover the dominant transient, generally 20-25 harmonics need to be considered[6]. This is a conservative estimate since interconnects are in general equivalent to low pass filters which will further attenuate the high frequency components.

It should be pointed out that by optimizing only the dominant components of the transient responses, we achieve CPU speedup over time domain optimization. Final transient responses can be accurately obtained by using time-domain simulators

after optimization converges.

4.4 Selection of Optimization Variables

The overall circuit performance is affected by parameters in both the interconnections and their terminations. The terminations typically represent basic circuit blocks and matching networks. There are two stages of interconnect design. In the first stage, the length of the transmission line l , the distance between the coupled conductors d , and the width of the conductors w can be optimized. The thickness of PCB layer h is typically selected from a set of standard values. In the second design stage only the length l is optimizable. A different design scenario is to find optimal termination parameters for a given interconnect configuration. In this way, the interconnect network can be optimally matched and signal reflections and distortions are effectively reduced.

4.5 Formulation of Optimization

Let $\mathbf{e}(\Phi)$ be an m -vector containing all necessary error functions as defined by (4.1)-(4.6). The optimization problem is to find Φ such that

$$U(\Phi) \triangleq \max_j \{e_j(\Phi), j = 1, 2, \dots, m\} \quad (4.8)$$

is minimized subject to electrical and physical constraints

$$\mathbf{g}(\Phi) \leq \mathbf{0} \ ,$$

$$\mathbf{h}(\Phi) = \mathbf{0} \ .$$

The constraints represent design rules. For example the total length of several interconnect lines must be constrained by the physical dimensions of the circuit board. The total separation between several coupled conductors must be limited by the geometrical space available to them. The minimax optimization of (4.8) is solved by a gradient-based two stage minimax algorithm [47]. The derivatives of $\mathbf{e}$ with respect to Φ required by the optimizer are obtained by the new approach of first- and second-order sensitivity analysis described in Chapter 3.

4.6 Examples

4.6.1 Example 1: A 3 Transmission Line Network

The circuit of Figure 4.2 is excited by the 6ns trapezoidal signal shown in Figure 4.3. The time responses before optimization are plotted in Figure 4.4. The objective is to reduce signal delay of V_{out1} and V_{out2} , and the crosstalk voltages V_{cross1} and V_{cross2} , respectively. Design variables Φ include capacitors C_2 and C_3 , resistor R_9 and width, length and spacing parameters of the 2-conductor transmission lines as defined in Table 4.1. The cross-sectional parameters are common among the 3 transmission lines. We require that the total length of the three transmission lines be fixed at 0.12m and the width of each conductor plus the spacing between them be fixed at 0.00307m. This results in constraints

$$h_1(\Phi) = l_1 + l_2 + l_3 - 0.12 = 0,$$

$$h_2(\Phi) = w + d - 0.00307 = 0.$$

Figure 4.5 shows frequency domain specifications and responses for group delay at two outputs, namely, $T_{G,out1}$ and $T_{G,out2}$ and crosstalk spectrums V_{cross1} and V_{cross2} .

The error functions used in (4.8) are

$$\mathbf{e} = \begin{bmatrix} e_1 \\ \vdots \\ e_{20} \\ e_{21} \\ \vdots \\ e_{40} \\ e_{41} \\ \vdots \\ e_{60} \\ e_{61} \\ \vdots \\ e_{80} \end{bmatrix} = \begin{bmatrix} T_{G,out1}(\Phi, s_1) - 0.5 \\ \vdots \\ T_{G,out1}(\Phi, s_{20}) - 0.5 \\ T_{G,out2}(\Phi, s_1) - 0.5 \\ \vdots \\ T_{G,out2}(\Phi, s_{20}) - 0.5 \\ 1000(|V_{cross1}(\Phi, s_1)| - 0.00028) \\ \vdots \\ 1000(|V_{cross1}(\Phi, s_{20})| - 0.00028) \\ 1000(|V_{cross2}(\Phi, s_1)| - 0.00028) \\ \vdots \\ 1000(|V_{cross2}(\Phi, s_{20})| - 0.00028) \end{bmatrix},$$

where $s_k = j\omega_k = jk\omega_1$, $k = 1, 2, \dots, 20$ and $\omega_1 = 0.0837758 \times 10^9 \text{rad/sec}$. The total number of error functions is 80. The total number of linear constraints is 10, including bounds on the design variables. After optimization, the objective function (4.8) is reduced from 1.0134 to -0.0386, indicating that all specifications are satisfied. The variables before and after optimization are listed in Table 4.1. The group delay after optimization, as plotted in Figure 4.5, is much lower and flatter than before optimization. Time responses after optimization are plotted against those before optimization in Figure 4.4. The propagation delay times for v_{out1} and v_{out2} are both reduced from 1.6ns to 1.2ns. The magnitude of crosstalk signals v_{cross1} and v_{cross2} is reduced by more than 55%.

Figure 4.3: Trapezoidal signal used as excitation for the 3 transmission line network.

Figure 4.4: The 3 transmission line example. Transient responses (volts) vs. time (nanoseconds) before (solid line) and after (dashed line) frequency domain optimization. (a). v_{out1} , (b). v_{out2} , (c). v_{cross1} and (d). v_{cross2} .

Table 4.1: Optimization Variables for the 3 Transmission Line Example

Variable	Description	Before Optimization	After Optimization
l_1	length of transmission line # 1	0.05m	0.09838m
l_2	length of transmission line # 2	0.04m	0.01162m
l_3	length of transmission line # 3	0.03m	0.01m
d	distance between conductors 1 & 2	2.49mm	2.97mm
w	width of conductors 1 & 2	0.58mm	0.1mm
C_2	capacitor	2pF	0.1pF
C_3	capacitor	1pF	8.884pF
R_9	resistor	50 Ω	197.8 Ω

Figure 4.5: The 3 transmission line example. Frequency domain specifications and group delay and spectrum responses vs. angular frequency before (solid line) and after (dashed line) optimization. (a). T_G for v_{out1} , (b). T_G for v_{out2} , (c). spectrum for v_{cross1} and (d). spectrum for v_{cross2} .

4.6.2 Example 2: 25 Transmission Line Network

The circuit of Figure 4.6 represents a 4-bit bus structure in which the excitation signals propagate through the bus lines to various circuit blocks. The 13 4-conductor transmission lines all have the same cross-sectional geometry. Both excitations are 5V pulses with 1ns rise/fall time and 5ns duration as shown in Figure 4.7. The responses of interest are v_a , v_b , v_c , and v_d , as shown in Figure 4.8.

Design specifications, as listed in Table 4.2, include upper specifications on the group delay and lower specifications on the magnitudes of transfer functions, respectively, for both output v_b and v_d . Upper specifications are also placed on the spectrums of crosstalk signals of v_a and v_c . The optimization frequency points are $s_k = j\omega_k = jk\omega_1$, where $k = 1, 2, \dots, 20$ and $\omega_1 = 0.0897598 \times 10^9 \text{ rad/sec}$. The total number of error functions is 120. There are 27 design variables in Φ consisting of parameters in both the transmission lines and the termination networks, the lengths of the 13 4-conductor transmission lines, the distance between the conductors, 7 terminating resistances and 4 terminating capacitances, as listed in Table 4.3. The minimum distance between adjacent conductors and the maximum separation between conductors 1 and 4 were constrained at 0.8mm and 5mm, respectively. The total length of the 13 transmission lines was fixed at 40cm. The total number of linear constraints is 43, including bounds on the design variables.

After 12 iterations of optimization, the objective function (4.8) was reduced from 22.908 to -1.177. The group delay after optimization is lower and flatter as shown in Figure 4.9. The parameters after optimization are listed in Table 4.3. The output responses v_a , v_b , v_c , and v_d after optimization are plotted against those before optimization in Figure 4.6. The propagation delay times of v_b and v_d are reduced from 2.5 and 3.9ns down to 1.5ns and 2.1ns, respectively based on 2V threshold criteria.

Table 4.2: Design Specifications for the 25 Transmission Line Network

Response of Interest	Design Specification
group delay for v_b	≤ 1.2 ns
group delay for v_d	≤ 2.0 ns
magnitude of transfer function for v_b	≥ 0.6
magnitude of transfer function for v_d	≥ 0.6
spectrum for v_a	≤ 0.001 v
spectrum for v_c	≤ 0.001 v

The peaks of crosstalk voltages v_a and v_c are reduced to 0.09 and 0.05V, respectively. In addition, the reflection in the output response v_d is significantly smaller than before optimization. The rise/fall time of signal v_b is also improved.

In order to compare our new approach with direct time-domain optimization [21], we also performed optimization explicitly on the time waveform of this example. Comparable results were obtained. However, The CPU times for our approach and the direct time domain approach are 4519 and 15410 seconds (on SPARCstation 2), respectively. Our proposed approach is 3.4 times faster than direct time-domain optimization.

We also performed optimization with only termination parameters as design variables. After optimization, the crosstalk and the distortion of the signals were significantly improved. There was no substantial change in signal propagation delay because the transmission lines were fixed.

Figure 4.6: Circuit diagram for the 25 transmission line network example. There are 13 4-conductor coupled transmission lines and 12 single conductor transmission lines. Two excitations are simultaneously applied.

Figure 1.7: Trapezoidal signal used as excitation for the 25 transmission line circuit.

Figure 4.8: The 25 transmission line example. Transient responses (volts) vs time (nanoseconds) before (solid line) and after (dashed line) frequency domain optimization. (a). v_a , (b). v_b , (c). v_c and (d). v_d .

Table 4.3: Optimization Variables for the the 25 Transmission Line Network

Variable	Before Optimization	After Optimization	Variable	Before Optimization	After Optimization
l_1	0.025m	0.026788m	d_2	1.0mm	1.325mm
l_2	0.035m	0.049477m	d_3	1.0mm	0.8mm
l_3	0.025m	0.01m	R_2	25 Ω	18.9 Ω
l_4	0.01m	0.010296m	R_4	25 Ω	20.4 Ω
l_5	0.04m	0.033452m	R_7	200 Ω	1067.3 Ω
l_6	0.025m	0.01m	R_{10}	200 Ω	10.0 Ω
l_7	0.035m	0.012302m	R_{14}	100 Ω	353.2 Ω
l_8	0.035m	0.031221m	R_{18}	100 Ω	87.1 Ω
l_9	0.025m	0.017584m	R_{20}	100 Ω	78.7 Ω
l_{10}	0.04m	0.045879m	C_1	5pF	0.1pF
l_{11}	0.025m	0.01m	C_2	5pF	0.1pF
l_{12}	0.025m	0.07m	C_3	5pF	0.1pF
l_{13}	0.025m	0.07m	C_5	10pF	0.1pF
d_1	1.0mm	2.875mm

Figure 4.9: The 25 transmission line example. Frequency domain specifications and group delay and spectrum responses vs. angular frequency before (solid line) and after (dashed line) optimization. (a). spectrum for v_a , (b). T_G for v_b , (c). spectrum for v_c and (d). T_G for v_d .

4.6.3 Example 3: Multi-Circuit Optimization

The simultaneous optimization of the circuit in Figure 4.6, namely Circuit 1, and its modified version as shown in Figure 4.19, namely Circuit 2, is performed. Circuit 2 is identical to Circuit 1 except that the excitations are applied through R_1 and R_3 . This multi-circuit optimization example represents optimal design of such an interconnect structure for two different and conflicting cases of signal propagation.

As listed in Table 4.1, both upper and lower specifications are applied to group delay and gain slope responses. Lower specifications are also applied to the magnitudes of transfer functions and upper specifications to the spectrums of crosstalk signals in both Circuits 1 and 2. The frequency sample points used in optimization are the same as Example 2. The total number of error functions is 380.

The 31 optimization variables as listed in Table 4.5 are common between the two circuits. Design constraints are the same as in Example 2 except that 7 more constraints are introduced for bounds on the design variables. The total number of linear constraints is 50. The time-domain and the frequency domain responses before and after optimization are plotted in Figure 4.11 and Figure 4.12, respectively. After optimization both group delay and gain slope responses become flatter. The magnitude of gain slope responses becomes lower. The propagation delay and distortion for both circuits are reduced. As shown in Figure 4.11 the signal levels are increased and the rise/fall times of all the four signals are improved.

Figure 4.10: A 25 transmission line network identical to that in Figure 4.6 except the source excitations.

Table 4.4: Design Specifications for the Multicircuit Optimization Example

Circuit	Response of Interest	Design Specification
Circuit 1	group delay for v_b	≤ 1.5 ns and ≥ 0.8 ns
	group delay for v_d	≤ 2.4 ns
	gain slope for v_d	≤ 1.5 ns/rad and ≥ -1.5 ns/rad
	magnitude of transfer function for v_b	≥ 0.5
	magnitude of transfer function for v_d	≥ 0.5
	spectrum for v_a	≤ 0.004 v
	spectrum for v_c	≤ 0.004 v
Circuit 2	group delay for v_a	≤ 1.1 ns and ≥ 0.4 ns
	gain slope for v_a	≤ 1.5 ns/rad and ≥ -1.5 ns/rad
	group delay for v_c	≤ 2.0 ns and ≥ 1.0 ns
	magnitude of transfer function for v_a	≥ 0.5
	magnitude of transfer function for v_c	≥ 0.5
	spectrum for v_b	≤ 0.004 v
	spectrum for v_d	≤ 0.001 v

Table 4.5: Variables for the Multicircuit Optimization Example

Variable	Before Optimization	After Optimization	Variable	Before Optimization	After Optimization
l_1	0.025m	0.07m	R_2	100 Ω	50.9 Ω
l_2	0.035m	0.07m	R_3	100 Ω	38.8 Ω
l_3	0.025m	0.012365m	R_4	100 Ω	110.0 Ω
l_4	0.01m	0.014719m	R_7	200 Ω	199.4 Ω
l_5	0.04m	0.015807m	R_8	50 Ω	1245.7 Ω
l_6	0.025m	0.012618m	R_{10}	200 Ω	977.5 Ω
l_7	0.035m	0.019749m	R_{11}	100 Ω	817.8 Ω
l_8	0.035m	0.022156m	R_{15}	50 Ω	112.0 Ω
l_9	0.025m	0.010761m	R_{18}	100 Ω	94.1 Ω
l_{10}	0.01m	0.036799m	R_{20}	100 Ω	139.4 Ω
l_{11}	0.025m	0.028267m	C_1	5pF	1.36pF
l_{12}	0.025m	0.028331m	C_2	5pF	1.721pF
l_{13}	0.025m	0.058425m	C_3	5pF	0.872pF
d_1	1.0mm	1.076mm	C_4	10pF	0.1pF
d_2	1.0mm	1.116mm	C_5	10pF	3.0pF
d_3	1.0mm	1.577mm	L_1	5nH	5.413nH
R_1	100 Ω	39.9 Ω	L_2	10nH	31.6nH

Figure 1.11: The multi circuit optimization example. Transient responses (volts) vs. time (nanoseconds) before (solid line) and after (dashed line) frequency domain optimization. (a). v_b of Circuit 1, (b). v_d of Circuit 1, (c). v_a of Circuit 2 and (d). v_c of Circuit 2.

Figure 4.12: The multi-circuit optimization example. Representative group delay and gain slope responses vs. angular frequency before (solid line) and after (dashed line) optimization. (a). T_G for v_d of Circuit 1, (b). G_L for v_d of Circuit 1, (c). T_G for v_a of Circuit 2 and (d). G_L for v_a of Circuit 2.

4.7 Conclusions

By combining the algorithm for the efficient computation of group delay, gain slope and their sensitivities derived in Chapter 3, with minimax optimization, a frequency-domain approach is developed to indirectly minimize delay, distortion and crosstalk of transient responses in high-speed VLSI interconnects. Compared to direct time domain optimization, the new approach is approximate and therefore faster. It can be used as an efficient method for the design of interconnects in high-speed VLSI systems.

Chapter 5

Analysis of Circuits with Nonlinear Components

5.1 Introduction

Practical VLSI interconnects are usually terminated by nonlinear elements. Since nonlinear simulation and optimization are both iterative processes, the CPU time requirement for design optimization of VLSI interconnects with nonlinear terminations, such as line drivers, will be extremely high. An important step is the development of an efficient simulator for the nonlinear networks. Since the electromagnetic propagation modes in lossy transmission line are defined in the frequency domain, conventional nonlinear simulators such as SPICE can not be directly used. Recently several techniques for the analysis of nonlinear networks with frequency dependent parameters have been developed, such as macromodeling [11], [12], [13], convolution [14], [15], [16] or recursive convolution [17], piece-wise decomposition [18], or waveform relaxation [19]. All these approaches take the entire nonlinearity into

account and therefore are computationally expensive.

Recently an Asymptotic Waveform Evaluation (AWE) technique [26], [18] was developed where the entire time-domain waveform can be extracted from mainly one DC simulation and sensitivity analysis of the network. This is in sharp contrast to conventional methods such as FFT [15] or numerical inversion of Laplace transform [1] where the network has to be solved at many frequency points. The AWE method is extremely fast and has recently gained serious attention from the CAD community [62]. However the concept of AWE at present is only applicable to linear circuits or the linear portion of a nonlinear network [26] [18] [17].

As an initial step towards design optimization of nonlinear interconnect networks, the objective of this chapter is to extend the AWE principle directly to the solution of nonlinear network equations. A new technique SAWE (Successive Asymptotic Waveform Evaluation) is developed as an alternative approach for fast analysis of VLSI interconnects with nonlinear terminations [63].

5.2 The Existing AWE Method for Linear Networks

Details of the AWE method are described in [26], [18]. A summary of the basic principles as applied to the step response is given in this section.

The Modified Nodal Admittance (MNA) matrix equations of a linear network with a step excitation can be written as

$$\mathbf{Y}_\pi(s)\mathbf{V}_\pi(s) = \frac{\mathbf{E}_\pi}{s}, \quad (5.1)$$

where $\mathbf{Y}_\pi(s)$ is an $N_\pi \times N_\pi$ MNA matrix for the network and $\mathbf{E}_\pi$ is the excitation

vector.

To approximate the solution of (5.1), $\mathbf{V}_\pi(s)$ is expanded in a Maclaurin series

$$\mathbf{V}_\pi(s) = \frac{\mathbf{m}_{-1}}{s} + \mathbf{m}_0 + \mathbf{m}_1 s + \dots + \mathbf{m}_{2q-2} s^{2q-2}, \quad (5.2)$$

where

$$\begin{aligned} \mathbf{m}_{-1} &= \mathbf{Y}_\pi^{-1}(0) \mathbf{E}_r, \\ \mathbf{m}_i &= \mathbf{Y}_\pi^{-1}(0) \sum_{j=1}^{i+1} \left[\frac{1}{j!} \frac{\partial^j \mathbf{Y}_\pi(s)}{\partial s^j} \right]_{s=0} \mathbf{m}_{i-j} \end{aligned} \quad (5.3)$$

for $i = 0, 1, \dots, 2q-2$, and q is the number of the poles. These moments are matched to a lower order frequency-domain Padé function of the form

$$\mathbf{V}_\pi(s) = \frac{\mathbf{v}_{\pi s}}{s} + \mathbf{K} \left[\frac{1}{s-p_1} \frac{1}{s-p_2} \dots \frac{1}{s-p_q} \right]^T, \quad (5.4)$$

where $\mathbf{v}_{\pi s}$ is the vector of steady states, $\mathbf{p} = [p_1 \ p_2 \ \dots \ p_q]^T$ is a q vector of approximating complex poles and $\mathbf{K}$ is an $N_\pi \times q$ matrix of the corresponding residues. The time-domain response is

$$\mathbf{v}_\pi(t) = \mathbf{v}_{\pi s} + \mathbf{K} [e^{p_1 t} \ e^{p_2 t} \ \dots \ e^{p_q t}]^T. \quad (5.5)$$

Notice that equation (5.3) represents the DC simulation and sensitivity analysis.

5.3 The New SAWE Method for Nonlinear Networks

Suppose a network π consists of linear lumped elements, N_s distributed transmission lines and nonlinear terminations. The modified nodal equations for the overall network π are

$$\mathbf{C}_\pi \frac{d\mathbf{v}(t)}{dt} + \mathbf{G}_\pi \mathbf{v}_\pi(t) + \sum_{k=1}^{N_s} \mathbf{D}_k \mathbf{i}_k(t) + \mathbf{f}(\mathbf{v}_\pi) = \mathbf{e}_\pi(t), \quad (5.6)$$

where $\mathbf{C}_\pi$ and $\mathbf{G}_\pi$ are $N_\pi \times N_\pi$ matrices determined by lumped elements in the network. $\mathbf{v}_\pi(t)$ is the vector of node voltage waveforms appended by independent voltage source currents and inductor current waveforms. $\mathbf{D}_k$ is an incidence matrix containing 1's and 0's which maps $\mathbf{i}_k(t)$, the terminal current waveform of the k th distributed transmission line, into the N_π -node space of network π . $\mathbf{f}(\mathbf{v}_\pi)$ is the vector which contains the current waveforms of all nonlinear terminations and $\mathbf{e}_\pi(t)$ is the vector of source waveforms.

Expanding the nonlinear term $\mathbf{f}(\mathbf{v}_\pi)$ into Taylor series at $\mathbf{v}_0 = [v_{01} \ v_{02} \ \dots \ v_{0N_\pi}]^T$, we have

$$\mathbf{f}(\mathbf{v}_\pi) = \mathbf{f}(\mathbf{v}_0) + \sum_{l=1}^L \frac{1}{l!} \sum_{i_1=1}^{N_\pi} \dots \sum_{i_l=1}^{N_\pi} \frac{\partial^l \mathbf{f}(\mathbf{v}_0)}{\partial v_{i_1} \dots \partial v_{i_l}} x_{i_1} \dots x_{i_l}, \quad (5.7)$$

where $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_{N_\pi}]^T = \mathbf{v}_\pi - \mathbf{v}_0$.

Step 1

Approximating $\mathbf{f}(\mathbf{v})$ by its linear part, that is setting $L = 1$ in (5.7), we have

$$\mathbf{C}_\pi \frac{d\mathbf{x}(t)}{dt} + [\mathbf{G}_\pi + \mathbf{A}_1]\mathbf{x}(t) + \sum_{k=1}^{N_s} \mathbf{D}_k \mathbf{i}_k(t) + \mathbf{A}_0 + \mathbf{G}_\pi \mathbf{v}_0 = \mathbf{e}_\pi(t), \quad (5.8)$$

where

$$\mathbf{A}_0 = \mathbf{f}(\mathbf{v}_0),$$

$$\mathbf{A}_1 = \frac{\partial \mathbf{f}(\mathbf{v}_0)}{\partial \mathbf{v}}.$$

We can solve the linearized system of (5.8) using AWE [18] to obtain steady state, residues and poles for *Step 1*, i.e., $\mathbf{x}_s^1$, $\mathbf{K}^1$ and $\mathbf{p}^1$.

Step 2

Now we take the second-order term into account, that is setting $L = 2$ in (5.7). Assume this will only cause a small amount of perturbation $\Delta \mathbf{x}$ on $\mathbf{x}$ in (5.8). Ignoring

the second-order term of $\Delta \mathbf{x}$, we have

$$\mathbf{C}_\pi \frac{d\Delta \mathbf{x}(t)}{dt} + [\mathbf{G}_\pi + \mathbf{A}_1 + \mathbf{A}_2(t)]\Delta \mathbf{x}(t) + \sum_{k=1}^{N_\pi} \mathbf{D}_k \Delta \mathbf{i}_k(t) = \mathbf{b}_2(t) , \quad (5.9)$$

where $\mathbf{b}_2(t)$ is an N_π -vector defined as

$$\mathbf{b}_2(t) = \frac{1}{2!} \sum_{i_1=1}^{N_\pi} \sum_{i_2=2}^{N_\pi} \frac{\partial^2 \mathbf{f}(\mathbf{v}_0)}{\partial v_{i_1} \partial v_{i_2}} x_{i_1}(t) x_{i_2}(t) ,$$

and $\mathbf{A}_2(t)$ is an $N_\pi \times N_\pi$ matrix whose j th column is defined as

$$[\mathbf{A}_2]_j = \sum_{i_1=1}^{N_\pi} \frac{\partial^2 \mathbf{f}(\mathbf{v}_0)}{\partial v_j \partial v_{i_1}} x_{i_1} .$$

Generally, at *Step L* ($L \geq 2$), we have

$$\mathbf{C}_\pi \frac{d\Delta \mathbf{x}(t)}{dt} + \sum_{k=1}^{N_\pi} \mathbf{D}_k \Delta \mathbf{i}_k(t) + [\mathbf{G}_\pi + \sum_{l=1}^L \mathbf{A}_l(t)]\Delta \mathbf{x}(t) = \mathbf{b}_L(t) , \quad (5.10)$$

where $\mathbf{b}_L(t)$ is an N_π -vector defined as

$$\mathbf{b}_L = \frac{1}{L!} \sum_{i_1=1}^{N_\pi} \cdots \sum_{i_L=1}^{N_\pi} \frac{\partial^L \mathbf{f}(\mathbf{v}_0)}{\partial v_{i_1} \cdots \partial v_{i_L}} x_{i_1}(t) \cdots x_{i_L}(t) ,$$

and $\mathbf{A}_L(t)$ is an $N_\pi \times N_\pi$ matrix whose j th column is defined as

$$[\mathbf{A}_L]_j = \frac{1}{(L-1)!} \sum_{i_1=1}^{N_\pi} \cdots \sum_{i_{L-1}=1}^{N_\pi} \frac{\partial^L \mathbf{f}(\mathbf{v}_0)}{\partial v_j \partial v_{i_1} \cdots \partial v_{i_{L-1}}} x_{i_1}(t) \cdots x_{i_{L-1}}(t) .$$

Now we need to solve the linear time-variant problem of (5.10). Since the AWE technique has been used, the $\mathbf{x}(t)$ has the following form

$$\mathbf{x}(t) = \mathbf{x}_s^{L-1} + \sum_{l=1}^{L-1} \mathbf{K}^l \{e^{p_1^l t} e^{p_2^l t} \cdots e^{p_q^l t}\}^T , \quad (5.11)$$

where $\mathbf{x}_s^{L-1}$ is the steady state of $\mathbf{x}(t)$ at *Step L-1*, $\mathbf{K}^l$ and $\mathbf{p}^l$ are the corresponding residues and poles obtained at *Step L*. This particular form of $\mathbf{b}_L$ and $\mathbf{A}_L$ allows us

to convert the linear time-variant system into algebraic linear equations by Laplace transform. In fact, take Laplace transform of (5.10):

$$(\mathbf{Y}_\pi(s) + \mathbf{A}_1)\Delta\mathbf{X}(s) + \sum_{l=2}^L \mathbf{H}_l(s) = -\mathbf{B}_L(s), \quad (5.12)$$

where $\mathbf{Y}_\pi(s)$ is the MNA matrix formed from the linear part of the network, and $\mathbf{H}_l(s)$ and $\mathbf{B}_L(s)$ are given by the Laplace transforms of $\mathbf{A}_l(t)\Delta\mathbf{x}(t)$ and $\mathbf{b}_L(t)$, respectively. For $L = 2$ as an example, we have

$$\mathbf{H}_2(s) = \sum_{i=1}^{N_\pi} \sum_{j=1}^{N_\pi} \frac{\partial^2 \mathbf{f}(\mathbf{v}_0)}{\partial v_i \partial v_j} x_{s,j}^1 \Delta\mathbf{X}_i(s) + \sum_{i=1}^{N_\pi} \sum_{j=1}^{N_\pi} \frac{\partial^2 \mathbf{f}(\mathbf{v}_0)}{\partial v_i \partial v_j} \sum_{r=1}^q k_{j,r}^1 \Delta\mathbf{X}_i(s - p_r^1) \quad (5.13)$$

and

$$\begin{aligned} \mathbf{B}_2(s) = & \frac{1}{2!} \sum_{i=1}^{N_\pi} \sum_{j=1}^{N_\pi} \frac{\partial^2 \mathbf{f}(\mathbf{v}_0)}{\partial v_i \partial v_j} \left[\sum_{r_1=1}^q \sum_{r_2=1}^q k_{i,r_1}^1 k_{j,r_2}^1 \frac{1}{s - p_{r_1}^1 - p_{r_2}^1} \right. \\ & \left. + x_{s,i}^1 \sum_{r_1=1}^q k_{j,r_1}^1 \frac{1}{s - p_{r_1}^1} + x_{s,j}^1 \sum_{r_1=1}^q k_{i,r_1}^1 \frac{1}{s - p_{r_1}^1} + \frac{x_{s,i}^1 x_{s,j}^1}{s} \right], \quad (5.14) \end{aligned}$$

where $k_{i,j}^1$ is the (i, j) th component of residue matrix $\mathbf{K}^1$ and p_i^1 is the i th component of the pole vector $\mathbf{p}^1$. Extension to higher order terms is possible. In fact for $L > 2$, $\mathbf{H}_L$ and $\mathbf{B}_L$ have similar patterns to (5.13) and (5.14), respectively. Applying the AWE technique to (5.12), we can obtain the corresponding steady state, residues and poles for $\Delta\mathbf{x}(t)$, i.e., $\Delta\mathbf{x}_s^L$, $\mathbf{K}^L$ and $\mathbf{p}^L$. Then the updated solution is

$$\mathbf{x}(t) = \mathbf{x}_s^L + \sum_{l=1}^L \mathbf{K}^l [\epsilon^{p_1^l t} \epsilon^{p_2^l t} \dots \epsilon^{p_s^l t}]^T, \quad (5.15)$$

where $\mathbf{x}_s^L = \mathbf{x}_s^{L-1} + \Delta\mathbf{x}_s^L$.

The technique presented in this section is most suitable for mildly nonlinear networks. For highly nonlinear networks, an additional loop within *Step L* is required. The solution at *Step L* can be then refined by this loop where the solution from (5.15) is fed back to $\mathbf{A}_l(t)$ and $\mathbf{b}_L(t)$ of (5.10).

5.4 Examples

Example 1

A RC circuit with widely varying time constants is shown in Figure 5.1. The applied voltage is a unit step excitation. There is a nonlinear component between the output node and the ground. The nonlinear function is defined as $i = 0.01v_{out} + 0.02v_{out}^2 + 0.06v_{out}^3$. The approximation from the **Successive** Asymptotic Waveform Evaluation is compared with the HSPICE response in Figure 5.2.

Example 2

An interconnect circuit which contains 7 transmission lines, linear and nonlinear lumped elements is shown in Figure 5.3. The nonlinear functions is defined as: $i = 0.02v_{out}^2$. The SAWE response is compared with the HSPICE response in Figure 5.4.

Example 3

A circuit which contains 35 transmission lines is shown in Figure 5.5. The circuit is obtained by cascading the interconnect model shown in Figure 5.3. The nonlinear function is defined as: $i = 0.001v_{out} + 0.2v_{out}^2$. The SAWE response is compared with the HSPICE response in Figure 5.6. Very good approximation and 16 times CPU speedup are achieved.

Figure 5.1: RC circuit with widely varying time constants.

Figure 5.2: Comparison of voltage responses of the RC circuit

Figure 5.3: Seven transmission line network.

Figure 5.4: Comparison of voltage responses of the Seven transmission line network

Figure 5.5: Thirty-five transmission line network.

Figure 5.6: Comparison of voltage responses of the Thirty-five transmission line network.

5.5 Conclusions

In this Chapter we have proposed a new technique SAWE (Successive Asymptotic Waveform Evaluation) as an alternative approach to the solution of the overall nonlinear network equations. SAWE is a waveform approximation technique with the ability to capture the dominant nonlinear waveform. The nonlinear network is decomposed into a sequence of first-order (linearized), second-order, and higher order subproblems. By successively applying the Asymptotic Waveform Evaluation to the sequence of subproblems, each subproblem is converted into a linear network. Within each subproblem, only a linear DC simulation and sensitivity analysis of the network is carried out, yielding information of the entire nonlinear waveform. Significant CPU speedup over conventional simulators is achieved.

Chapter 6

Conclusions and Further Research

6.1 Summary

This research was motivated by the increasing demand for an automated design tool to solve the interconnect problems in high-speed VLSI systems where the interconnects are modeled as lossy coupled transmission lines. Several contributions have been presented in this thesis to address the design of interconnections:

- 1 An efficient method has been presented for the computation of group delay, gain slope and their sensitivities with respect to design parameters in multi-conductor transmission line networks. The group delay and gain slope sensitivities are evaluated in terms of the second order derivatives of eigenvalues and eigenvectors of the propagation matrix.
- 2 A systematic formulation of optimization for interconnect networks has been developed based on frequency-domain information such as group delay and gain slope. By reducing group delay and crosstalk energy, and improving the shape

of group delay and gain slope responses, the transient signal delay, crosstalk and distortion can be simultaneously optimized.

- 3 By combining this method with minimax optimization, efficient frequency-domain optimization was carried out to address the optimal design problem of the high-speed VLSI interconnections. Numerical examples of interconnects optimization have demonstrated simultaneous reductions of signal propagation delay, distortion and crosstalk at several vital connection ports. Compared with existing time-domain optimization, the proposed frequency-domain approach is faster.
- 4 A new technique, namely SAWE, has been introduced to simulate high-speed interconnect networks with nonlinear terminations, where the principle of the highly efficient AWE method is expended to directly apply to the nonlinear circuit. Significant CPU speedup over conventional simulators is achieved.

6.2 Suggestions for Further Research

- 1 As mentioned, both the time-domain optimization approach and the frequency-domain optimization approach have their own advantages and disadvantages. In the time domain approach, crosstalk can be effectively reduced by imposing design specifications directly on transient waveforms, but it is difficult to do so for signal delay and distortion. In the frequency-domain approach, on the other hand, signal delay and distortion can be conveniently reduced by imposing design specifications on the corresponding group delay and gain slope of the signal. One of the possible ways to further improve the accuracy and efficiency

of the optimization method is to develop a mixed frequency domain and time domain approach, where signal delay and distortion will be reduced by the frequency-domain approach and crosstalk will be reduced by the time domain approach.

- 2 It is well known that group delay contains information of signal propagation delay. For RC networks, Vlach *et al.* recently verified group delay at frequency zero as being exactly equal to Elmore delay and observed a simple empirical relationship between group delay and signal delay of responses corresponding to a step excitation [36]. It is desirable to have more research effort on investigating the relationship between propagation delay and group delay in high speed VLSI interconnect networks. Then we will be able to determine group delay specifications from the given propagation delay specifications. This will make our approach more efficient and effective.
- 3 The possibility of applying our frequency domain optimization approach to other applications, such as optimal design of networks with non uniform transmission lines [20], should also be explored. An important step in this direction is to derive an efficient algorithm for the computation of group delay sensitivities for nonuniform transmission line models.
- 4 The accuracy of our Successive Asymptotic Waveform Evaluation technique can be further improved through iterative refinement of the solution at *Step L* ($L \geq 2$), which can be done by adding a loop within the step to capture the non-dominant nonlinear waveform.

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Appendix A

Design optimization of high-speed VLSI interconnect networks presented in this thesis is realized through our CAD system [22]. The system contains various simulation modules for simulation, optimization and design specifications, and supports run time data communication between various modules. To meet the different analysis requirements of this thesis, several modules (simulators) are implemented in the system, including frequency-domain simulators: **LCKT** and **DLCKT** (with sensitivity analysis); time-domain simulators: **NILT** and **DNILT** (with sensitivity analysis); simulators for the analysis of group delay and gain slope: **LCKTGD** and **DLCKTGD**(with sensitivity analysis). In this Appendix a brief description of these modules is presented. As an example, the representative input files for the three transmission line example in Chapter 4 are given to illustrate the use of these modules.

MODULE 1

LCKT

simulator for frequency-domain analysis of high-speed VLSI interconnect networks.

Module Type

function(n, m, x, y, k, s, ierror)

Definition of Input

$x(0)$, $x(1)$

contain the real and imaginary parts of the complex frequency. For simple frequency-domain analysis, $x(0) = 0$ and $x(1)$ contain the angular frequency ω at which the circuit is simulated.

$x(.)$

The rest of $x(.)$ contains circuit parameters. Detailed definition of the input array $x(.)$ will be given through the input files which will be described later.

n

dimension of input array $x(.)$.

m

dimension of output array $y(.)$. Let N_π be the number of nodes, N_s be the number of voltage sources and N_l be the number of inductors, then

$$m = 2(N_\pi + N_s + N_l) .$$

The first $2N_\pi$ elements in $y(.)$ contains nodal responses and the rest contains the currents of voltage sources and inductors.

k

length of the string s , $k \geq 32$

s

name of the input file which defines the topology of the circuit.

Description of Input File

A unified approach based on the modified nodal admittance matrix is used in this module to formulate the equations describing the transmission line system and the equations describing the terminal and interconnecting networks, as described in (2.21) in chapter 2.

An input file, as shown in Figure A.1, is presented here for the following three transmission line network of Figure A.2. The format of the input file is

```
#include "lckt.h"
element1
element2
:
elementn
```

Note that the source to network should be defined as one of the elements. Also, the last element should be the type of response definition.

The general syntax for each element in the input file is

```
type_of_element    node1    node2    ...    nodek    idres
```

where the *type_of_element* is defined in the head file *lckt.h*, as shown in Figure A.3, and *idres* indicates the starting address in the input array **x** where parameters for this element are stored. For example, line 2 of the input file in Figure A.1 means that there is a resistor between node 1 and node 2. The resistor's value is stored as the third element of the input array **x**, that is **x**(2) (Notice the first element is **x**(0)).

As another example, a two conductor transmission line with physical parameters is represented by *L2*. The six physical parameters for a two conductor transmission line are stored in the following order in **x** (start from *x(idres)*): length, relative dielectric constant, relative permeability, conductor width, circuit board thickness and distance between the two conductors. *T_L2* represents a two conductor transmission line whose cross-sectional physical parameters are the same as the previous transmission line and only the length is different. By grouping these transmission lines together, only one eigenvalue evaluation is required for each group.

A detailed description for each element is given in Table A.1.

Definition of Output

y(.)	This module supports seven different output formats defined by the user: 1) real and imaginary format: y(2i-1) contains the real part and y(2i) the imaginary part of the output at node i ; 2) magnitude and phase format: y(2i-1) contains the magnitude and y(2i) the phase of the output at node i ; 3) attenuation format: y(i) contains the attenuation at node i ; 4) insertion loss format: y(i) contains the insertion loss at node i ;

- 5) reflection coefficient format: $y(2i-1)$ contains the magnitude and $y(2i)$ the phase of the reflection coefficient at node i ;
 - 6) return loss format: $y(i)$ contains the return loss at node i ;
 - 7) group delay format (by perturbation): $y(i)$ contains the group delay at node i .
- ierror* error number
s error message

Remarks

An input file which describes the topology of the circuit is required. The file name is given by string s . Requirements in the input file are

- (a). source excitation assume to be unit pulse, i.e., parameters a and b for the source in Table A.1 should be 1 and 0, respectively;
- (b). response type should be defined by the user. Treated as a special element in the input file, each response type has the following syntax:

$$type_of_element \quad idres$$

where *type_of_element* is one of the 7 output formats defined in the head file *lckt.h* in Figure A.3. Notice that for output formats such as real and imaginary, magnitude and phase, attenuation, and group delay, *idres* is dummy since no parameter is required for these output formats. On the other hand, the parameters required for insertion loss format, reflection coefficient format, and return loss format are source and load resistances $\{R_g, R_L\}$, source resistance $\{R_g\}$, and source resistance $\{R_g\}$, respectively. In this case, *idres* indicates the starting address in the input array $\mathbf{x}$ where R_g and/or R_L are stored.

```

#include "lckt.h"

res 1 2 2
res 3 0 3
res 5 0 4
res 4 6 5
res 4 10 6
res 7 0 7
res 11 0 8
res 9 0 9
res 12 14 10
res 13 0 11
cap 4 0 12
cap 8 0 13
cap 15 0 14
ind 14 15 15
vsource 1 0 16
t_l2 2 3 4 5 18
T_L2 6 7 8 9 24
T_L2 10 11 12 13 25
resp_real_imag 26

```

Figure A.1: For simulator **LCKT**: input file of the three transmission line network.

Figure A.2: Three transmission line network.

```

#define res          10100200      /* resistor */
#define ind          10200200      /* inductor */
#define cap          10300200      /* capacitor */
/* transmission line with electrical parameters */
#define tl1          13100200      /* 1 conductor */
#define tl2          13100400      /* 2 conductors */
#define tl3          13100600      /* 3 conductors */
#define tl4          13100800      /* 4 conductors */
/* transmission line whose electrical parameters */
/*
are the same as previous one */
#define TL1          13200200      /* 1 conductor */
#define TL2          13200400      /* 2 conductors */
#define TL3          13200600      /* 3 conductors */
#define TL4          13200800      /* 4 conductors */
/* transmission line with physical parameters */
#define t_l1         13300200      /* 1 conductor */
#define t_l2         13300400      /* 2 conductors */
#define t_l3         13300600      /* 3 conductors */
#define t_l4         13300800      /* 4 conductors */
/* transmission line whose physical parameters */
/*
are the same as previous one */
#define T_L1         13400200      /* 1 conductor */
#define T_L2         13400400      /* 2 conductors */
#define T_L3         13400600      /* 3 conductors */
#define T_L4         13400800      /* 4 conductors */

#define isource      14100200      /* current source */
#define vsource      14200200      /* voltage source */
/* opt means sensitivity analysis is required */
#define optres       10100201
#define optind       10200201
#define optcap       10300201

#define optt_l1      13300201
#define optt_l2      13300402
#define optt_l3      13300403
#define optt_l2      13300404

#define optT_L1      13400201
#define optT_L2      13400402
#define optT_L2      13400403
#define optT_L2      13400404
/* definition of the output format */
#define resp_real_imag 20100000    /* real and imaginary */
#define resp_mag_phase 20200000    /* magnitude and phase */
#define resp_attenuation 20300000   /* attenuation */
#define resp_insertion_loss 20400000 /* insertion loss */
#define resp_refle_coeff 20500000   /* reflection coefficient */
#define resp_return_loss 20600000   /* return loss */
#define resp_group_delay 20700000   /* group delay */

```

Figure A.3: For simulator **LCKT**: head file *lckt.h*.

Table A.1: The format of element definition

Format of the Input File			Topology	Parameters of Element Stored in Input array x (Start from $x(idres)$)
Type	Nodes	Idres		
res	n1 n2	idres		R
cap	n1 n2	idres		C
ind	n1 n2	idres		L
tl2	n1 n2 n3 n4	idres		$l \begin{matrix} R_{11} & R_{21} & R_{12} & R_{22} & L_{11} & L_{21} & L_{12} & L_{22} \\ G_{11} & G_{21} & G_{12} & G_{22} & C_{11} & C_{21} & C_{12} & C_{22} \end{matrix}$
tlk	n1 n2 ... nk n(k+1) ... n2k	idres		$l \begin{matrix} R_{11} \dots R_{k1} \dots R_{kk} & L_{11} \dots L_{k1} \dots L_{kk} \\ G_{11} \dots G_{k1} \dots G_{kk} & C_{11} \dots C_{k1} \dots C_{kk} \end{matrix}$
TLk	n1 n2 ... nk n(k+1) ... n2k	idres		l
t_l2	n1 n2 n3 n4	idres		$l \epsilon_r \mu_r w h d$
t_lk	n1 n2 ... nk n(k+1) ... n2k	idres		$l \epsilon_r \mu_r w h d_1 d_2 \dots d_{k-1}$
T_Lk	n1 n2 ... nk n(k+1) ... n2k	idres		l
isource	n1 n2	idres		$a \ b$ (s-domain input is a/s^b)
vsorce	n1 n2	idres		$a \ b$ (s-domain input is a/s^b)
<p>where: l is the transmission line's length; w is the conductor width; h is the circuit board thickness; d is the distance between the two conductors; ϵ_r is the relative dielectric constant and μ_r is the relative permeability.</p>				

MODULE 2

DLCKT

frequency-domain simulator with sensitivity analysis

Module Type

dfunction(n, m, x, y, dy, k, s, ierror)

Definition of Input

The same as **LCKT** module

Definition of Output

y(.)	y(2i-1) contains the magnitude and y(2i) the phase of the output response at node i
dy(.)	dy is $n \times m$ matrix which contains sensitivities. In Fortran language format dy(i, j) (dy(j, i) in C format) contains y(j)'s derivative with respect to x(i).
ierror	error number
s	error message

Remarks

Requirements in the **LCKT** circuit file are the same as **LCKT** module except the output type is defaulted as magnitude and phase. Design parameters which require sensitivity analysis is also defined by user in the input file.

Description

Adjoint sensitivity technique is used to perform sensitivity analysis.

An input file, as shown in Figure A.3, is presented here for the previous example. It is similar to the input file shown in Figure A.1 except *opt* is added to the *type_of_element*, indicating this element requires sensitivity analysis.

```
#include "lckt.h"

optres 1 2 2
res 3 0 3
res 5 0 4
res 4 6 5
optres 4 10 6
res 7 0 7
res 11 0 8
res 9 0 9
res 12 14 10
res 13 0 11
cap 4 0 12
cap 8 0 13
cap 15 0 14
ind 14 15 15
vsource 1 0 16
optt_l2 2 3 4 5 18
optT_L2 6 7 8 9 24
optT_L2 10 11 12 13 25
```

Figure A.4: For simulator **DLCKT**: input file of the three transmission line network.

MODULE 3

NILT

simulator for time-domain analysis by using numerical inversion of Laplace transform of the nodal voltages from the **LCKT** simulator.

Module Type

function(n, m, x, y, k, s, ierror)

Definition of Input

$x(0) = t$ time

$x(1) = \text{idres}$ starting address in $x(\cdot)$ where the excitation signal is defined, address range for $x(\cdot)$ is: $[0, n - 1]$.

$x(\text{idres}) = N_p$ the number of pieces in piece wise linear excitation defined as:

$$\sum_i^{N_p} [S_i u(t - t_i)(t - t_i)],$$

where $u(\cdot)$ is a unit step function and S_1, t_1, S_2, t_2 are defined as follows:

$x(\text{idres} + 1) = S_1$

$x(\text{idres} + 2) = t_1$

$x(\text{idres} + 3) = S_2$

$x(\text{idres} + 4) = t_2$

...

$x(\text{idres} + 2N_p) = t_{N_p}$

$x(\cdot)$ the rest of $x(\cdot)$ contains circuit element parameters in exactly the same way as that of the simulator **LCKT**. see **LCKT**'s definition of the $x(\cdot)$'s.

n dimension of input vector $x(\cdot)$

m dimension of output vector $y(\cdot)$. Let N_n be the number of nodes, N_s be the number of voltage sources and N_l be the number of inductors, then

$$m = N_n + N_s + N_l .$$

k length of the string s , $k \geq 32$

$s = \text{filename}$ file name of the input file

Definition of Output

$y(\cdot)$ The first N_n elements in $y(\cdot)$ contains nodal responses at time t and the rest contains the currents of voltage sources and inductors.

$ierror$ error number

s error message

Remarks

An input file is required. The file name is given by string s . The file is consistent with **LCKT** circuit description file except for the following requirements:

- (a). source excitation should be $\frac{1}{s^2}$, where s is the complex frequency, i.e., the parameters of the source excitation are $\{1 \ 2\}$
- (b). response type is default (real and imaginary).

Description

The method of numerical inversion of Laplace transform implemented in this module is according to [1].

The module is internally linked to the circuit simulator **LCKT**.

MODULE 4

DNILT

time-domain simulator with sensitivity analysis

Module Type

dfunction(n, m, x, y, dy, k, s, ierror)

Definition of Input

the same as **NILT** module

Definition of Output

y(.) the same as **NILT** module

dy(.) **dy** is $n \times m$ matrix which contains sensitivities. In Fortran language format dy(i, j) (dy(j, i) in C format) contains y(j)'s derivative with respect to x(i).

ierror error number

s error message

Remarks

Requirements in the circuit input file are the same as in **NILT** module except that the design parameters which require sensitivity analysis are also defined by the user in the input file.

Description

Adjoint sensitivity technique is used to perform sensitivity analysis.

The module is internally linked to the circuit simulator **LCKT**.

MODULE 5

LCKTGD

simulator for group delay computation

Module Type

function(n, m, x, y, k, s, ierror)

Definition of Input

the same as in the **LCKT** module

Definition of Output

$y(.)$	$y(2i-1)$ and $y(2i)$ contain gain slope and group delay at output node i .
ierror	error number
s	error message

Remarks

Requirements in the input file are the same as **LCKT** module except that the response type is default as real and imaginary.

MODULE 6

DLCKTGD

group delay simulator with sensitivity analysis.

Module Type

dfunction(n, m, x, y, dy, k, s, ierror)

Definition of Input

the same as the **LCKT** module

Definition of Output

y(.) **y**(2i-1) and **y**(2i) contain gain slope and group delay at output node i.

dy(.) **dy** is $n \times m$ matrix which contains group delay and gain slope sensitivities. In Fortran language format **dy**(i, j) (**dy**(j, i) in C format) contains **y**(j)'s derivative with respect to **x**(i).

ierror error number

s error message

Remarks

Requirements in the input file are the same as in the **DLCKT** module.

END

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FIN