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Development of an Object Oriented Handbook and a Numerical Methods Application for Steel Structures

by

Ian James Fuller

**A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of
Master of Engineering**

**Department of Civil and Environmental Engineering
Carleton University
Ottawa, Ontario
September, 1999**

**The Master of Engineering in the Civil and Environmental Engineering
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administered by the Ottawa-Carleton Institute for Civil Engineering**

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Department of Civil and Environmental Engineering

The Undersigned recommend to the Faculty of Graduate Studies
and Research acceptance of the thesis

**Development of an Object Oriented Handbook and a Numerical Methods
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submitted by

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in partial fulfilment of the requirements for
the degree of Master of Engineering

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September, 1999

Abstract

Advances in programming languages, architecture, hardware and internet technology make the development of complex systems, like an *Integrated System for the Analysis-Design-Fabrication-Erection of Steel Structures*, for environments with evolving technologies, possible and practical. An object oriented architecture has been developed for the *Integrated System* that will generate a platform independent program that is readily updateable, managed and integrated. An Object Oriented Handbook and a prototype numerical methods application were developed to illustrate the functionality and versatility of the system architecture. The Object Oriented Handbook includes the development of objects and classes for the Object Oriented Standard, Dynamic Databases, Geometric Tools and an Extended Library for all clauses in CAN/CSA-S16.1-94 related to the stability of beams and beam columns and all referred clauses from CAN/CSA-S136-94. To extend the scope of application of the Standard, a prototype object oriented numerical methods application was developed to determine the resistance of steel members subject to stability failure. Comparisons between the results of the analytical model and existing test data were used to verify the model.

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List of Symbols

A	=	area of the cross section; area of the triangular element
$[\bar{B}]$	=	strain displacement matrix including both the infinitesimal and large strain components.
$[B^b]$	=	strain-displacement matrix for degrees of freedom associated with bending forces
$[B^m]$	=	strain-displacement matrix for degrees of freedom associated with membrane forces
$[B_o]$	=	strain displacement matrix used in linear infinitesimal strain analysis
$[B_L]$	=	strain displacement matrix used in large displacement analysis where the coefficients of the matrix are functions of the displacements
C	=	cope length.
C_w	=	warping torsional constant
$[D]$	=	elastic constitutive matrix
$[\bar{D}^b]$	=	average constitutive matrix associated with bending stresses
$[\bar{D}^m]$	=	average constitutive matrix associated with membrane stresses
$[D_T]$	=	tangential constitutive matrix
E	=	modulus of elasticity
E_{st}	=	strain hardening modulus
F	=	geometric coefficient relating the length of the back span to the length of the cantilevered span; von Mises failure stress
F_y	=	yield strength of steel
G	=	elastic shear modulus

H_k	=	Gaussian integration weight
I_T	=	moment of inertia of the joist's top chord about the horizontal centroidal axis
I_x	=	moment of inertia about the x axis
I_{xy}	=	product of inertia
I_y	=	moment of inertia about the y axis
I_{yf}	=	moment of inertia of the top flange about the cross sectional weak axis
I_{wx}	=	warping product of inertia about the x-axis
I_{wy}	=	warping product of inertia about the y-axis
J	=	St. Venant's torsional constant
J^*	=	modified torsional constant accounting for the distortional resistance of the cross section
K	=	effective length factor
$[K]$	=	elemental local structural stiffness matrix
K_b	=	in-plane bending stiffness of the brace divided by the spacing
$[K^b]$	=	stiffness matrix for those degrees of freedom associated with bending forces
K_e	=	effective continuous torsional restraint stiffness
$[K_e]$	=	elemental stiffness matrix including structural and geometric effects
K_f	=	effective torsional stiffness of the flange
$[K_G]$	=	elemental geometric stiffness matrix
$[K^m]$	=	stiffness matrix for those degrees of freedom associated with membrane forces
$[K_T]$	=	global tangent stiffness matrix
K_w	=	web stiffness against distortional effects

L	=	length of the member; length of the back span
L_1	=	area co-ordinate associated with node 1
L_2	=	area co-ordinate associated with node 2
L_3	=	area co-ordinate associated with node 3
L_b	=	unbraced length
L_c	=	length of the cantilevered span
L_e	=	effective segment length
L_i	=	area co-ordinate associated with node i
L_j	=	area co-ordinate associated with node j
L_k	=	area co-ordinate associated with node k
L_p	=	length of the end panel of the joist
L'	=	equivalent design length
M_o	=	magnitude of the moment field at the beginning of the unbraced length
M_1	=	magnitude of the moment field at $L_b/4$
M_2	=	magnitude of the moment field at $L_b/2$
M_3	=	magnitude of the moment field at $3L_b/4$
M_4	=	magnitude of the moment field at the end of the unbraced length
M_{Bx}	=	moment about the x axis on the bottom of the beam column (illustrated in Figure 3.1)
M_{By}	=	moment about the y axis on the bottom of the beam column (illustrated in Figure 3.1)
M_b	=	moment resistance of the back span
M_c	=	moment resistance of the cantilevered span
$M_{uncoped}$	=	elastic critical buckling capacity of the gross member cross section

M_{coped}	=	elastic critical buckling capacity of the effective monosymmetric cross section
M_{cr}	=	critical elastic buckling moment of the beam specimen
M_L	=	larger of the magnitudes of the end moments
M_m	=	maximum magnitude of the moment field
M_r	=	moment resistance of the cross section
M_S	=	smaller of the magnitudes of the end moments
M_x	=	moment at any point along the length of the member
$M_{y,p}$	=	yield or plastic moment capacity of the cross section
M_{Tx}	=	moment about the x axis on the top of the beam column (illustrated in Figure 3.1)
M_{Ty}	=	moment about the y axis on the top of the beam column (illustrated in Figure 3.2)
M_u	=	critical elastic buckling moment of the beam specimen
M_x	=	moment about the x axis at any point along the specimen
$[N^m]$	=	shape function matrix associated with the membrane degrees of freedom
ND_1	=	shape function describing the u and v translations due to displacement at node 1
ND_2	=	shape function describing the u and v translations due to displacement at node 2
ND_3	=	shape function describing the u and v translations due to displacement at node 3
NU_1	=	shape function describing the u translations due to in-plane rotation of node 1
NU_2	=	shape function describing the u translations due to in-plane rotation of node 2

NU_3	=	shape function describing the u translations due to in-plane rotation of node 3
NV_1	=	shape function describing the v translations due to in-plane rotation of node 1
NV_2	=	shape function describing the v translations due to in-plane rotation of node 2
NV_3	=	shape function describing the v translations due to in-plane rotation of node 3
P	=	axial load; magnitude of in span concentrated load; arbitrary constant load in a single degree of freedom system; point on a surface
R	=	torsional parameter used in the resistance calculations for cantilevered and overhanging beams
R_x	=	reaction force along the x axis (illustrated in Figure 3.1)
R_y	=	reaction force along the y axis (illustrated in Figure 3.1)
S	=	elastic section modulus
S_e	=	effective elastic section modulus
$[T]$	=	transformation matrix
V	=	volume
W	=	geometric torsional parameter
W_n	=	normalised unit warping term
X	=	torsional parameter used in the resistance calculations for cantilevered and overhanging beams
Z	=	plastic section modulus
a	=	arbitrary displacement in a single degree of freedom system; Gauss point
a^i	=	cumulative displacement in a single degree of freedom system, illustrated in Figure 4.1

$\{\Delta a\}$	=	incremental displacement vector
b	=	flange width; Gauss point
b_1	=	width of the top flange (illustrated in Figure 2.3)
b_2	=	width of the bottom flange (illustrated in Figure 2.3)
b_i	=	difference in y components of the co-ordinates the triangular elements nodes
c	=	centroid; Gauss point
c_i	=	difference in x components of the co-ordinates the triangular elements nodes
d	=	depth of the cross section; Gauss point
d'	=	centre-to-centre distance between flanges
e	=	eccentricity of the load placement with respect to the shear centre
$\{f\}$	=	externally applied force vector
$\{f_o\}$	=	applied load vector
h	=	centre-to-centre flange distance; height above the shear centre
k	=	effective buckling coefficient
k_s	=	effective length factor associated with the top flange loading condition
k_t	=	effective length factor associated with the shear centre loading
l_i	=	radius of the n-dimensional 'sphere'
n	=	number of segments; number of load steps
r	=	interpolating factor accounting for load placement above the shear centre
s	=	shear centre; joist spacing along the length of the beam
ds	=	differential distance

t	=	plate thickness; flange thickness
t_1	=	thickness of the top flange (illustrated in Figure 2.3)
t_2	=	thickness of the bottom flange (illustrated in Figure 2.3)
t_w	=	web thickness
u	=	translation of the shear centre in the x direction; lateral displacements; translation of the node in the local x direction
u_i	=	translation of node i in the local x direction
v	=	translation of the shear centre in the y direction; translation of the node in the local y direction
v_i	=	translation of node i in the local y direction
w	=	web thickness; intensity of distributed loading; translation of the node in the local z direction,
x_1	=	x co-ordinate associated with node 1
x_2	=	x co-ordinate associated with node 2
x_3	=	x co-ordinate associated with node 3
x_o	=	distance of the shear centre with respect to the centroid of the cross section in the x direction
y	=	distance from the mid-height on a singly symmetric I-beam cross section (illustrated in Figure 3.6)
y_1	=	y co-ordinate associated with node 1
y_2	=	y co-ordinate associated with node 2
y_3	=	y co-ordinate associated with node 3
$\bar{y}$	=	the magnitude of the distance from the centroid to the centre of the top flange (illustrated in Figure 2.3)
y_{LB}	=	distance from the critical vertical deflection to the centre of rotation of a member subjected to lateral-torsional buckling.

y_o	=	distance of the shear centre with respect to the centroid of the cross section in the y direction
z	=	distance along the z-axis
β_x	=	monosymmetric bending coefficient
δ_R	=	residual displacements
η	=	natural co-ordinate axis
ε	=	uniaxial strain
Φ	=	angle of twist of the cross section about the shear centre
κ	=	ratio of the smaller to larger of the end moments
λ	=	direction cosines of the local axis with respect to the global axis used in the transformation matrix
$\{\Delta\lambda\}$	=	incremental load step factor
ν	=	Poisson's ratio
σ	=	uniaxial stress; residual stress
σ_x	=	normal stress in the x direction
$\overline{\sigma}_x$	=	average normal stress through the thickness in the x direction
σ_{xy}	=	shear stress in the xy plane
σ_y	=	normal stress in the y direction
$\overline{\sigma}_y$	=	average normal stress through the thickness in the y direction
θ_x	=	rotation of the node about the local x axis
θ_y	=	rotation of the node about the local y axis
θ_z	=	rotation of the node about the local z axis
θ_{zi}	=	rotation about the local z axis at node i
ω_2	=	equivalent moment factor

- ω_2^* = equivalent moment factor relating the height of load placement for monosymmetric sections
- ρ_f = measure of the degree of monosymmetry
- $\{\Psi\}$ = out-of-balance (residual) force vector
- $\bar{\tau}_{xy}$ = average shear stress through the thickness
- ξ = natural co-ordinate axis

Chapter 1

Introduction

1.1 An Object Oriented Handbook for an Integrated System for Steel Structures

Advances in programming languages, architecture, hardware and internet technology facilitate the development of complex systems, like an *Integrated System for the Analysis-Design-Fabrication-Erection of Steel Structures*, possible and practical. The advantages that would accrue to all parties involved in the design and construction of steel structures (if computing technology is applied appropriately), discussed by Kennedy (1996), Trahair (1996) and Kennedy and Murray (1999), are:

- increased communication between all parties involved and a reduction in the duplication of work.
- minimisation of errors associated with data manipulation.
- the production of an informative and comprehensive environment which is instructive, allowing for better engineered and more cost effective structures.
- the reduction of computational effort and time, allowing structural engineers the time necessary to investigate alternative structural forms (optimising the design).
- allowances are provided to the fabricators to modify designs accounting for the availability of materials and the local fabrication practices.

- allowances are provided to both fabricators and erectors to simulate the erection procedures and modify the design accordingly.
- the availability of tools, to the structural engineer that properly evaluate resistances of members susceptible to complex stability problems.
- modifications to the Standard may be incorporated into the *Integrated System* as they are approved (in real time) by the regulatory body rather than with every edition of the published document (approximately every 5 years).
- the provision of a mechanism by which the Canadian Institute of Steel Construction can provide the steel industry with instruction or information that may be relevant to the design of steel structures.

Research related to the development of a series of independent modules for the *Integrated System* included specific developments on the description of building geometry, the determination and attribution of loads to members, member analysis and design, and connection analysis and design, resulted in:

- the development of comprehensive design algorithms.
- the development of graphical user interface structures to maximise clarity and understanding, to minimise input and to emulate the design process making it intuitive to the structural engineer.
- the identification of methods and state required by all objects that would need to be defined in the building process.

While this work was strategic and contributed to the understanding of the requirements of an *Integrated System*, the presentation of the *Integrated System* through these modules became dated as they were dependent on the computing

technology of the time of the development. Continued investigation of evolving computing technology lead to the development of a hypertext document entitled "*The Hyper-Handbook of Steel Construction*" that is delivered using web based technology (Frattaroli, 1999). This document provides an interactive, easily navigable, electronic version of the *Handbook of Steel Construction* (CISC, 1997) with some limited computational functionality.

Further to this and the previous research, it was concluded that the natural extension of this research was the development of a system architecture using an object oriented programming philosophy that would provide a structured program that would resolve issues regarding longevity in the evolving computing technology world. Subsets towards the development of this architecture include the development of core applications like an Object Oriented Handbook and an advanced numerical methods program. Note, the Object Oriented Handbook houses and Object Oriented Standard and Extended Libraries which would be utilised by all design applications, rendering them easily updateable.

1.2 Objectives and Scope

For the meaningful development of a prototype system architecture, including applications and objects, for an *Integrated System* that illustrates the functionality and versatility of that architecture, a substantial amount of work is required. As a result this thesis was conducted in parallel with Raby (1999) with similar general objectives but separate development and research tasks. The objectives include:

- conception and development of a prototype system architecture that will accommodate future changes and advancements in computing technology and changes in Standards and Codes which reflect new advances in the understanding of structural behaviour.
- development of applications and objects for a prototype Object Oriented Handbook and a numerical methods program, illustrating the functionality and versatility of the prototype architecture as applied to structural engineering.
- investigation of the state-of-the-art resistance determination for the stability of beams, identifying the extent and limitation of existing engineering knowledge, clearly establishing the usefulness of implementing numerical analysis in the design of members that are susceptible to stability failures.
- development of a prototype numerical methods application and corresponding objects required to model stability problems that arise in the design of structural steel members with validation of this application against existing test data

The scope of these prototype developments are limited to the Object Oriented Handbook (encompassing the development of clauses related to stability problems for beams and beam columns, the development of the corresponding tools and extended libraries) as well as the development of the numerical methods program to determine the resistance of structural steel members susceptible to stability failures.

1.3 Summary

Chapter 1 establishes the need for the development of an Object Oriented Handbook as part of the system architecture for an *Integrated System for Steel Structures* that can be maintained and expanded as computing technology and Standards change. Outlined are the objectives and scope associated with the corresponding prototype developments, which illustrate the versatility and functionality. A detailed description of the functionality and structure of the Object Oriented Handbook within the proposed architecture for the *Integrated System* is given in Chapter 2. The state-of-the-art for the stability of flexural members subject to lateral-torsional buckling failure is presented in Chapter 3. The merits and limitations of these resistance formulations are described and the need for the development of a numerical methods application is established. Chapter 4 describes the numerical methods application with the corresponding element objects and supplemental objects that are required. Comparisons of the analytical results obtained from models developed with this numerical methods application are made with test data for beams and beam columns subject to stability problems to verify the application. Execution of this numerical methods application verifies the corresponding system architecture. Chapter 5 summarises the thesis and states the relevant conclusions. Recommendations associated with continuing this work are given.

Chapter 2

The Object Oriented Handbook

2.1 Integrated System Development

A series of independent, comprehensive, stand alone modules were developed in an attempt to define an *Integrated System for the Design of Steel Structures* that emulates the conception-analysis-design-fabrication-erection process (Kennedy, 1996). Completed modules include a pre-processing module for load determination and structural definition along with several modules for the design of various members and connections. A description of these and the corresponding references are given in Table 2.1. The major objectives for this body of research were:

- To explore the use of graphical user interface (GUI) systems to simplify the manipulation, interpretation and representation of large sets of data associated with structural design so that it is intuitive to the structural engineer. The development of an interactive GUI system allows for the detection and minimisation of gross errors (blunders) at intermediate stages of the design process.
- To develop comprehensive design algorithms, for connections and members, that encompassed current advances in research, guidelines,

material Standards, specifications, numerical methods and rational analyses.

- To explore the capability of the existing computing technologies.
- To develop optimisation algorithms, particular to the industry involved, which are based not only on weight but also on availability and cost.
- To determine loads and load combinations calculated according to the *National Building Code of Canada 1990* (NRCC, 1990) and attributing these to the associated structural members appropriately.
- To investigate the process advantages and disadvantages of integrating independent modules.

A significant body of knowledge was synthesised to enable the development of the algorithms for the design of connections and members. A better understanding of the data required in the design process has been acquired. Integrated tools that contain calibrated cost algorithms will allow Code and Standard writing bodies to calibrate changes and evaluate the cost associated with those changes. These modules used existing computing technologies, dating from 1990 onwards and are of limited functionality as the programming languages (FORTRAN and C) and graphical packages essentially dictated a top down programming approach that is unsuitable for the overall *Integrated System*.

Table 2.1: Analysis and Design Modules

Reference	Module and Description
Devitt, 1992. The Design, Analysis and Detailing of Bolted and Welded Simple Steel Connections	<p>Steel Connection Analysis Design and Detailing (SCADD version 1.0)</p> <ul style="list-style-type: none"> Analyses, designs and details simple single and double angles beam-to-beam and beam-to-column type connections using CAN/CSA-S16.1-M89 provisions along with the available fabrication practices outlined in the relevant version of the <i>Handbook of Steel Construction</i> CISC, 1997). Provides additional data files on relevant connection information. Produces engineering type reports including a 3-D graphical representation of the final design.
He, 1993. Interactive Design of Flexural Steel Members Subjected to Bending, and Combined Bending and Torsion	<p>Interactive Design of Flexural Steel Members (IDFSM)</p> <ul style="list-style-type: none"> Analyses and designs a member subject to flexure, torsion or a combination of both, accounting for lateral support, location of load application, coping and influences of interactive buckling. Optimisation algorithm based on weight is incorporated for the purposes of member selection. Facilitates the review of intermediate calculations and the performance of design checks. Provides a comprehensive graphical representation of the generated design.
Jiang, 1995. Interactive Design of Continuous Flexural Steel Members	<p>Interactive Design of Continuous Steel Members (IDCSM)</p> <ul style="list-style-type: none"> Analyses and designs prismatic and non-prismatic continuous flexural members accounting for various realistic boundary conditions, lateral restraint and load combinations. Employs simple finite elements in conjunction with the governing differential equations to model the analysis correctly. Implements an algorithm for the resistance determination of eccentrically loaded bolt groups (symmetric or unsymmetric) Facilitates design checks and generates engineering reports with comprehensive graphical representation of the generated design.
Sma, 1995. Development of and Interactive Design and Analysis Program for Steel Beam-Columns	<p>Beam-Column Design Tool (BeCol)</p> <ul style="list-style-type: none"> Analyses and designs beam-columns using the prevalent equations from the CAN/CSA S16.1-M89, S16.1-94 and S136-M89 along with providing both elastic and inelastic solutions as generated by advanced analysis techniques. Generates interaction curves (moment-thrust curvature, equivalent moment factors, column) for user defined cross sections allowing for a comparison of the influences of residual stress patterns and varying moment fields. Facilitates a review of appropriate intermediate information and provides a comprehensive graphical representation of the deformed member.

Table 2.1: Analysis and Design Modules (cont'd)

References	Module and Description
Sumajouw, 1995. The Computer Aided Analysis, Design and Detailing for Column Base Plate Steel Connections	Steel Connection Analysis Design and Detailing (SCADD version 2.0) <ul style="list-style-type: none"> • Supplements SCADD version 1.0 by providing the additional capacity to analyse, design and detail column base plates of varying geometry according to CAN/CSA-S16.1-M89 and CAN/CSA-A23.3-M84. • Facilitates intermediate design checks and generates engineering reports including a 3-D graphical representation of the design.
Bao, 1996. Analysis, Design, and Detailing of Flexible and Extended End Plate Connections	End Plate Connection Design (EPCD) <ul style="list-style-type: none"> • Analyses and designs flexible and extended end plate connections according to CAN/CSA-S16.1-M89 allowing for various input such as material types and even costing information. • Facilitates intermediate design checks, review of failure modes and access to supplemental information. • Provides a comprehensive graphical representation of the design.
Chmelicek, 1993 and 1996. Determination of Loads Acting on Surfaces of Structural Steel Buildings in Canada	Load Determination for Structural Steel Buildings in Canada (LDFSSBC) (LOADS 95) <ul style="list-style-type: none"> • Attributes various loads and load combinations to building surfaces and members, according to the <i>National Building Code of Canada 1990</i> (NRCC 1990). • Building surfaces and geometry are defined by the user (allows for curved and irregular surfaces with any orientation). • Provides comprehensive graphical descriptions allowing for the intermediate review of load applications. • Provides supplemental information and generates engineering type reports of loads to be attributed to structural components.
Martino, 1997. Development of an Integrated System for the Geometry Definition, Load Determination and Analysis of Steel Structures	Data Transfer LOADS 95 to CU-STAND 4.0 (Cornell University - static analysis and design) <ul style="list-style-type: none"> • Provides the appropriate transfer of complex loading information, as defined in LOADS 95, to an analysis package, CU-STAND 4.0, that performs an elastic-plastic analysis on three dimensional structures.
Frataroli, 1999. The Development of an Interactive Web Based version of CAN/CSA-S16.1-94	Hyper-Handbook <ul style="list-style-type: none"> • Provides an interactive, easily navigable version of CAN/CSA-S16.1-94 displayed as web based document. • Functionality and educational value has been increased through the addition of calculator tools, customised table generators, section tables along with descriptive photos.

Note: There is a logical progression from connection and component design, to analysis tools, to an integration attempt, to interactive 'Help' document

2.2 Continued Development

To develop an *Integrated System* for structural engineering in an environment in which the computing technologies (languages and platforms), design Codes (e.g. *National Building Code of Canada 1995* (NRCC, 1995), material Standards (e.g. CAN/CSA-S16.1-94) and research knowledge (e.g. SSRC Guide (Galambos, 1988)) are continuously evolving, the system must be portable and easily updateable.

One possibility that was investigated was the development and delivery of an *Integrated System* over the internet using web technology. As of the beginning of 1998, time constraints (running computationally intensive analysis-design tools) and security protocols associated with web technology presented serious limitations that precluded it as an option. However, web technology is still in a relatively infantile stage and it may become a viable platform for the execution of large comprehensive programs.

As part of this research, in conjunction with Raby (1999), a system architecture that utilises an object oriented programming philosophy has been mapped, with the development of prototype sections.

Object oriented programming is a programming philosophy whereby computer representations of physical or conceptual objects can be created (instantiated) and remain active (maintain state) throughout the duration of execution. These computer representations of objects can have certain attributes (state variables) and functionality (methods) that can be accessed by either objects or by algorithms (applications). The instantiation of an object is performed by passing appropriate

data to a specific method (the constructor method) contained within the object. The constructor method will then manipulate the input and use the input to set the object's state variables appropriately. Several different objects of the same type can be instantiated from the same framework (class), each having different state variables which are necessary to describe the particular attributes of the specific object. Depending on the particular object, the state can be accessed either through methods or by accessing the state variables directly. The user objects or applications need not know how the accessed object functions but just that the method call will return the appropriate information. This type of "ignorance" (encapsulation) is beneficial as it allows changes to be made to the rest of the program without affecting specific parts. Object oriented programming philosophy also supports inheritance, where the child objects can inherit changes to the common state variables and methods made to the parent objects. The inheritance principle is a powerful option when several objects have methods and state which are the same but also have different methods and state. By pushing the commonalties up the hierarchy to the parent object changes will affect all child objects as these child objects will still have those methods and state that have been pushed up the hierarchical chain. Update methods can be used to modify state variables of associated objects and must be encoded within the object. Note, reference may be made to the Glossary for a concise definition of the object oriented terms.

Java is being used as the programming language as it is powerful, expanding, platform independent and supports the object oriented programming philosophy (Horstmann and Cornell 1997). The combination of the proposed architecture and

programming language will meet the criteria of providing a portable and easily updateable *Integrated System for Steel Structures*. Note, the logic from the existing analysis-design modules can be readily incorporated. This program structure will keep the option open for delivery via the web as that technology develops.

Platform independence is achieved by Java by compiling its code to bytecode rather than machine code, which is machine dependant. Provided that the user's machine has the "Java Virtual Machine" (JVM) installed, which can be downloaded via the internet, this bytecode is then compiled by the JVM to machine code and then executed. Most current internet browsers have a JVM which allows for the execution of small contained programs (Java Applets) using the browser as the platform and the graphical user interface.

2.3 Development of the Object Oriented Handbook

Reviewing past modules has led to the conclusion that rather than beginning with developing design algorithms for a structure or structural components, that it would be more efficient to develop a library of Clause objects based on the material Standard from which design modules can then be developed. This library of Clause objects also allows for the development of other applications. These Clause objects combined with dynamic databases (**SectionProperties**), extended libraries (Appendices) and a geometric tool library forms the basis of what can be entitled the Object Oriented Handbook (OOH). Note, **bolded text indicates objects**, which have been assigned names, throughout the thesis and reference may be made to the Glossary for concise definitions.

The OOH should demonstrate its usefulness as a dynamic document by facilitating the updating (reflecting anticipated changes in CAN/CSA-S16.1-2000) and publication of the *Handbook of Steel Construction* (CISC, 1997) as a printed document, as a document available on a CDROM, or as a web based document. Publishing of this document in any one of these forms would be accomplished by instantiating each Clause object and then manipulating its state variables appropriately.

To demonstrate the applicability and functionality of the current prototype, the development of Clauses 13 and 15 of CAN/CSA-S16.1-94 along with all supporting clauses and supplemental objects have been completed. The selection of these clauses will allow for a comprehensive review of all technical (both computing and engineering) issues that may arise with further development of the OOH. With these developments and the logic from the previously developed modules, the higher level objects (**DESIGN ELEMENT** objects) can be developed without needing knowledge of the functional aspects of these Clause objects.

2.3.1 CSA Material Standards

The scope of the prototype version of the OOH is limited to the development of clauses within CAN/CSA-S16.1-94 that are related to the behaviour of beams and beam columns. These clauses are the more complex in behaviour and have significant references involving subject matter not specifically contained within the Standard. Table 2.2 lists the clauses, reference clauses, appendices and externally referenced clauses from both CAN/CSA-S16.1-94 and CAN/CSA-S136-94.

Table 2.2: Clauses, Referenced Clauses and Appendices, and Externally Referenced Clauses Incorporated into the Prototype OOH

Applicability	Clause and Title
CAN/CSA-S16.1-94, Primary Prototype Clauses	Clause 13: Member and Connection Resistance
	Clause 15: Beams and Girders
CAN/CSA-S16.1-94, Referenced Clauses	Clause 10: Slenderness Ratios
	Clause 11: Width-Thickness Ratios: Elements in Compression
	Clause 12: Gross and Net Areas
CAN/CSA-S16.1-94, Referenced Appendices	Appendix D: Torsional-Flexural Buckling of Compression Members
	Appendix F: Columns Subject to Biaxial Bending
CAN/CSA-S136-94, External Referenced Clauses	Clause 5.6: Properties of Sections
	Clause 6.2: Resistance Factors for Strength Analysis
	Clause 6.4: Members in Bending

Commonalties amongst these clauses have been identified and are listed in Table 2.3 under the categories of common state, common methods, or clause specific identities. The common state variables associated with the reading and retrieval of text are; Title, References, ChildrenName, ParentName, and BodyLocation.

Table 2.3: Commonalties in the Development of CSA Code Provisions

Common State	Common Methods	Clause Specific
<ul style="list-style-type: none"> Title References ChildrenName ParentName BodyLocation Date 	<ul style="list-style-type: none"> GetTitle() GetReference() GetChildren() GetParent() GetBody() GetDate() 	<ul style="list-style-type: none"> Boolean tests Restrictions Resistances Logical Operators

Note: Date variable and GetDate() method could be easily incorporated.

These variables can only be accessed through the corresponding common methods. Note, all clauses contain text but can be further defined by the clause specific attributes; Boolean Tests, Restrictions, Resistances, and Logical Operators.

Standardisation was accomplished by creating a super class 'Clause', from which all sub classes inherited these common abstract methods. The inheritance of these methods forces the child clause to overwrite these abstract methods with it's own method of the same name. With standardisation, the design algorithms only need to know that a particular Clause object has a method for determining certain values. Provided it receives the appropriate information, the Clause object will respond accordingly. These Clause objects generally have static methods that are similar to subroutine calls and would generally be used by the higher level design applications or interrelations between other clauses. The state variables associated with the display of text, titles and equations, can also be accessed by higher level applications that instantiate a Clause object, allowing for interactive use of this information.

Although not currently included, an additional state variable which identifies the applicable Standard to which the clause applies would increase the functionality of the OOH as it would allow engineers to apply different versions of the Standard, as may be required.

2.3.2 Dynamic Databases

In general, static databases are searched line by line for specific information and tend to slow down the computing process. Dynamic databases not only overcome this, to some extent, but also preserve the encapsulation principle

associated with object oriented programming. Five dynamic databases that have been included in the prototype OOH are; **SectionProperties**, **MemberProperties**, **LoadingConditions**, **ConnectionProperties**, and **RestraintConditions**. These are described in Table 2.4. These databases have been named according to the type of information it is intended to store. All databases store variables as a "label-value" pair so that the information is easily retrievable by the Clause objects. The higher level objects, **DESIGN ELEMENT** objects, will be encoded in a fashion that allows for the "communication" or data transfer between certain dynamic database objects so that information may stored appropriately. Cloning functions have been developed for **SectionProperties**, **MemberProperties**, and **LoadingConditions**. These functions allow for the possibility of having multiple **DESIGN ELEMENT** objects with one or more of the same dynamic databases without giving rise to the possibility of changing the properties of another **DESIGN ELEMENT**.

Table 2.4: Dynamic Databases Included in the Prototype OOH

Dynamic Databases	Description
SectionProperties	<ul style="list-style-type: none"> • Stores all cross section dimensions, as input by user of the object (higher level object), as "label-value" pairs. Allows for custom sections. • Stores all geometric properties, as computed by appropriate algorithms, as "label-value" pairs. • Initialises a series of rectangular plate objects associated with the cross sectional dimensions, allowing for retrieval of one or all of the plates so that appropriate properties or effective properties can be set. • To be used in conjunction with an appropriate MemberProperties object to fully describe the structural element. • For different DESIGN ELEMENT objects with the same cross section type, the SectionProperties object would be cloned.
MemberProperties	<ul style="list-style-type: none"> • Stores all relevant member information (length, coping details, location of openings, etc...), as input by the user of the object, as "label-value" pairs. • To be used in conjunction with the appropriate SectionProperties object to fully describe the structural element. • For different DESIGN ELEMENT objects with the same member information, the MemberProperties object would be cloned.
LoadingConditions	<ul style="list-style-type: none"> • Stores all relevant loading information (internal force distribution, load type, load placement, etc...), as input by the user, in the "label-value" pair format, for discrete and continuous loading conditions. • For different DESIGN ELEMENT objects with the same loading conditions, the LoadingConditions object would be cloned.
ConnectionProperties	<ul style="list-style-type: none"> • Stores all connection information, as input by user of the object, in "label-value" pairs. • Must have the capacity to "communicate" with the SectionProperties object, to store the appropriate net cross sectional properties.
RestraintConditions	<ul style="list-style-type: none"> • Stores all information associated with the restraint conditions (continuously or discretely laterally braced, torsional restraint or not, etc...), as input by the user of the object. • Must have the capacity to "communicate" with the MemberProperties to store the appropriate effective properties.

Notes: (1) Labels are expected to be consistent with nomenclature of CAN/CSA-S16.1-94 and CAN/CSA-S136-94.
(2) Cloning function can also be developed for the **ConnectionProperties** and the **RestraintConditions** objects.

2.3.3 Tools Library

The tools library may contain educational tools, computational tools and tools for the generation of design tables. Of these, several classes have been developed with methods for the computation of geometric properties for both standard shapes and the series of shapes illustrated in Figure 2.1.

Figure 2.1: Built-up Sections for Which Specific Determination Algorithms Were Developed.

Note, the acronyms attributed to these cross sections are consistent with Srna (1995).

The geometric properties for which algorithms were developed include; moments of inertia of the cross section, section modulus of the cross section (whole

or partial), radius of gyration, St. Venant's torsional constant, centroidal location, shear centre location, warping torsional constant, and the monosymmetric bending coefficient. These properties were calculated on the assumption that the section is comprised of thin rectangular plates that are orthogonally placed and the effects of the fillets and rounds may be neglected. Of these the more difficult calculations for the monosymmetric bending coefficient, the shear centre location, and the warping torsional constant are presented.

The monosymmetric bending coefficient, β_x , is a geometric property that is used in the determination of the critical moment on the cross section accounting for an induced torsional moment arising due to unbalanced compressive and tensile stresses that occur during twist buckling, Kitipornchai and Trahair (1980). The general equation of β_x for an asymmetric section, is expressed as:

$$\beta_x = \frac{I}{I_x} \left[\int_A y(x^2 + y^2) dA - 2x_o I_{xy} - 2y_o I_x \right] \quad (2.1)$$

where, x_o = x co-ordinate of the shear centre with respect to the centroid of the cross section.

y_o = y co-ordinate of the shear centre with respect to the centroid of the cross section.

I_x = moment of inertia about the x-axis.

I_{xy} = product of inertia.

y , x , and dA are defined in Figure 2.2

Figure 2.2: Integration Variables Involved in the Computation of β_x For An Asymmetric Section

For monosymmetric sections, assuming the y-axis is the plane of symmetry, the values of I_{xy} and x_o are equal to zero. Equation 2.1 can be reduced to the following form:

$$\beta_x = \frac{I}{I_x} \left[\int_A y(x^2 + y^2) dA - 2y_o I_x \right] \quad (2.2)$$

A negative sign is attributed to the value of β_x when the moments on the section place the smaller of the two flanges in compression, the antithesis is true if the larger of the flanges is in compression. Note, for monosymmetric sections with unequal flanges, the value of β_x can be calculated from Eq. 2.3.

$$\beta_x = \frac{1}{I_x} \left\{ \begin{aligned} & \left(d' - \bar{y} \left[\frac{b_2^3 t_2}{12} + b_2 t_2 (d' - \bar{y})^2 \right] - \bar{y} \left(\frac{b_1^3 t_1}{12} + b_1 t_1 (\bar{y})^2 \right) \right) \\ & + \frac{w}{4} \left[\left(d' - \bar{y} - \frac{t_2}{2} \right)^4 - \left(\bar{y} - \frac{t_1}{2} \right)^4 \right] \end{aligned} \right\} - 2y_o \quad (2.3)$$

where the variables are illustrated in Figure 2.3

$$\begin{aligned} \bar{y} &= \text{ABS}(y) \\ d' &= d - (t_1 + t_2)/2 \end{aligned}$$

Figure 2.3: Torsional Properties For Monosymmetric I-beams, after Galambos (1988)

This expression is the correct form of the equation that was incorrectly reproduced in Galambos (1988).

The location of the shear centre was calculated using Equations (2.4a) and (2.4b), Galambos (1968) and Heins (1975).

$$x_o = \frac{I_{xy}I_{wx} - I_yI_{wy}}{I_{xy}^2 - I_xI_y} \quad (2.4a)$$

$$y_o = \frac{I_xI_{wx} - I_{xy}I_{wy}}{I_{xy}^2 - I_xI_y} \quad (2.4b)$$

where, I_y = moment of inertia about the y-axis.

I_{wx} = warping product of inertia about the x-axis.

I_{wy} = warping product of inertia about the y-axis.

The warping torsional constant, C_w , was calculated using similar algorithms as the shear centre, Heins (1975) and is given by:

$$C_w = \int_0^b W_n^2 t ds \quad (2.5)$$

where, W_n = normalised unit warping term.

The procedure for calculating the shear centre and the warping torsional constant is general. The method calls within the classes associated with each calculation are section dependant.

Comparisons have been made with published values, available in the *Handbook of Steel Construction* (CISC, 1997), for standard sections and built-up sections. Each set of calculations has a set of assumptions that may contribute to the relatively small differences between the calculated and the published values, making it difficult to ascertain which is better. The small variances could be attributed to integration techniques that effectively double the area of the flange web junctions due to the use of centreline dimensions or these methods have accounted for the effects of

the fillets and the rounds. The greatest variances were found with Channel-Capped W section in which published values are based on Kitipornchai and Trahair (1980). Again there is no definitive method of quantifying the merits of one set of assumptions versus the other.

2.3.4 Extended Libraries and Tools

Appendices D, Torsional-Flexural Buckling of Compression Members, and F, Columns Subject to Biaxial Bending, of CAN/CSA-S16.1-94 have been developed as objects as they are both specifically referenced in Clause 13.

Appendix D, as referred to in Clause 13.3.2, is used to calculate the compressive resistance for singly symmetric, asymmetric, cruciform and bisymmetric sections that are subject to torsional-flexural buckling failures. Functions were developed to give the first root for the cubic buckling expression for asymmetric sections so that the buckling resistance can be determined. Note, for class 4 sections, the clauses given in CAN/CSA-S136-94 are identical in content to the formulations given in Appendix D. Using the object which describes Appendix D with the appropriate net section properties as defined by Clause 5.6 of CAN/CSA-S136-94 instead of the gross cross sectional properties will yield the correct compressive resistance of a class 4 section.

Appendix F accounts for the beneficial effects of residual stress distributions associated with the design of class 1 or 2 section beam-columns with WWF or square HSS cross sections. The methods contained within this object can be exercised in lieu or in conjunction with those described in Clause 13.8 if it is properly triggered during member design.

For certain load and support boundary conditions the resistance equations in CAN/CSA-S16.1-94 may not be applicable. The Standard explicitly recognises this and refers to additional references, like the *SSRC Guide to Stability Design Criteria for Metal Structures* (Galambos, 1988), for information and rational methods that the engineer may use to calculate the appropriate resistance. An example of this sort of reference is from Clause 13.6 (d):

"For monosymmetric shapes, a rational method of analysis such as that given in the Structural Stability Research Council's *SSRC Guide to Stability Design Criteria for Metal Structures* should be used."

These type of references are usually made when the problem being addressed by the Standard is a complex stability issue where the formulations provided in the Standard have limited applicability, such as those involving frame analysis, beam and beam column design. The OOH is well suited to include supplemental guidance for these types of calculations.

Similar to the inclusion of the Appendices previously discussed, the system architecture has the capacity to allow the user to determine the resistance of an element with a solution strategy with increasing sophistication and minimising the limitations. The first level of resistance formulation that must be satisfied are the requirements directly stated in the Standard and explicitly defined by the OOH. Successive levels of sophisticated solutions, such as monosymmetric flexural resistance formulae (Galambos, 1988), an interactive buckling solution strategy (Schmitke and Kennedy, 1985) and finite element solutions, will be implemented in the form of extended libraries or tools. It may be possible to automatically identify

design situations in which more sophisticated solutions would be required and instruct the user to do so, along with providing the appropriate solution methodology.

International standards, such as the Australian, AS4100-1990, and the Eurocode, EC3 (1990), have provisions that allow for the use of advanced analysis in the design of steel structures. CAN/CSA-S16.1-94 has similar provisions as stated in Clause 1.4:

"A rational design based on theory, analysis, and engineering practice acceptable to the regulatory authority may be used in lieu of the formulae provided in this Standard. In such cases, the design shall provide nominal margins (or factors) of safety at least equal to those intended in the provisions of this Standard (see Appendix E)."

In addition, the Standard explicitly recognises in Clause 8.6 that advanced second order inelastic analyses must be used for steel structures that meet certain criteria and are susceptible to overall stability problems. Note, numerical methods suitable for stability problems associated with frame analysis, beams, and beam-columns have been developed as tools and included in the development of the prototype system architecture.

2.4 Relationship Between the Object Oriented Handbook and the System Architecture

The relationship of the OOH within the system architecture is represented by the schematic diagram illustrated in Figure 2.4. Design modules within the *Integrated System*, highlighted in light pink, contain logic to trigger the actions of the higher

Figure 2.4: Schematic Diagram Illustrating the Processes of the System Architecture

level objects (purple). The OOH and the publishing tools or previously published material are highlighted in light green and light blue, respectively. Processes between design modules, objects, tools and libraries can be categorised as preliminary, intermediate or final processes and are shown as blue, red, or black arrows, respectively. These processes may pass data or objects and have yet to be fully defined.

The controlling module of the *Integrated System* will be the Graphical User Interface (GUI) which instantiates one or more **BUILDING STRUCTURE** objects and the corresponding **FRAME** objects that each one contains. The GUI passes appropriate geometric information about the **BUILDING STRUCTURE** to the Load Definition module which in turn instantiates **LOADING ELEMENT** objects that can be used, directly or indirectly, to update the state of the **BUILDING STRUCTURE** and **FRAME**. **FRAME** contains the loading information that is passed to the Structural Analysis module which instantiates **ANALYSIS ELEMENT** objects. Similarly, the **ANALYSIS ELEMENT** objects can be used, directly or indirectly, to update the **BUILDING STRUCTURE** and **FRAME**. Elements that are determined to be susceptible to stability problems and that lie outside the scope of normal design algorithms can be reanalysed, to determine their resistance, with the appropriate finite element analysis by passing the appropriate information back to the Structural Analysis module.

FRAME can also interface with the Member Design module to instantiate **DESIGN ELEMENT** objects. **FRAME** will contain the methods to access the **SectionProperties** object, once the appropriate information has been defined. The

DESIGN ELEMENT will then contain the logic necessary to perform a comprehensive design check against the provisions of the OOH. If the design check is unsatisfactory then methods would be employed to modify the **DESIGN ELEMENT** and cause it to interact with **FRAME** and **BUILDING STRUCTURE**, repeating the design process. A consequence of a universal electronic document like the OOH, which was conceived as a result of *The Hyper-Handbook* (Frattaroli, 1999), is that it could be used (if properly designed) to produce the *Handbook of Steel Construction* (CISC, 1997) in different media (a printed version, a CDrom version, or an electronic version that is accessed via the web). These documents would be assembled through the accessing methods contained within the Publishing Tool.

A detailed schematic diagram illustrating the interaction between **FRAME**, **DESIGN ELEMENT** and the OOH, which includes the Object Oriented Standard, the Geometric Tools, and the Extended Library, is presented in Figure 2.5. The colour scheme highlighting objects and processes has the same interpretation as that used in Figure 2.4. The processing of data between the objects is managed by using the Dynamic Databases.

As previously discussed **FRAME** has been instantiated by **BUILDING STRUCTURE**. **DESIGN ELEMENT** objects instantiate the **SectionProperties** object and store either calculated properties obtained from the Geometric Tools functions for customised sections, or published properties from the Structural Steel Section Tables. **FRAME** contains methods to access the **SectionProperties** object once they are defined. Note, for trivial calculations it may be faster to calculate certain cross sectional properties rather than search a static database. Triggering

Figure 2.5: Schematic Diagram of the Interaction between FRAME, DESIGN ELEMENT and the Object Oriented Handbook

methods should be included to recognise the circumstances and to invoke this option. For members in which the Object Oriented Standard or the Extended Library are applicable for design checks, **FRAME** instantiates the dynamic databases (**MemberProperties**, **ConnectionProperties**, **LoadingConditions**, **RestraintConditions**) contained in the **DESIGN ELEMENT**. Subsequently, the logic within the **DESIGN ELEMENT** objects will perform comprehensive design checks.

Chapter 3

Lateral-Torsional Buckling Behaviour of Flexural Members

The flexural resistance of beams and girders for in-plane strength and lateral-torsional buckling are given in Clauses 13.5, 13.6, 13.8 and 15 of CAN/CSA-S16.1. The resistance formulations for lateral-torsional buckling are based on a solution of the governing second order differential equations for singly symmetric and doubly symmetric beams subject to uniform bending. Provisions are made to include the effects of varying moment fields through the equivalent moment factor and references are made to the *SSRC Guide to Stability Design Criteria For Metal Structures* (Galambos, 1988) to further broaden the scope of the applicability of these formulations. The *SSRC Guide* (Galambos, 1988) provides a number of empirical expressions for equivalent moment factor to include the effects of varying; moment fields, type and placement of load, and restraint conditions. In addition, modifications to the elastic lateral-torsional buckling moment resistance equation are given for monosymmetric sections and for cantilevered and continuous beam configurations.

The application of the information detailed in the *SSRC Guide* (Galambos, 1988) is complex and could easily be misapplied by most structural engineers unless they have an intimate knowledge of the corresponding research. The application of

this information is well suited for inclusion within an Object Oriented Standard through Extended Libraries where the methods are well documented and verified. These libraries could also be used to apply rational mathematical expressions developed in recent refereed published research that has not yet been incorporated into a material Standard.

The original solution, for the critical moment, to the governing second order differential equation for elastic lateral-torsional buckling and all modifications and extensions to that expression for other applications, as cited in CAN/CSA-S16.1-94 are described in detail in this chapter.

3.1 Solution to the Governing Second Order Differential Equations

3.1.1 Second Order Differential Equations for Beam Columns with an Arbitrary Cross Section

The second order governing differential equations for beam columns with an arbitrary cross section (Galambos, 1968) subject to biaxial bending and axial load, illustrated in Figure 3.1, have been formulated on the deformed configuration assuming linear elastic behaviour, small deflections, negligible shear deformations and undistorted cross section. The equations for strong axis bending, weak axis bending and twisting involve coupled displacements expressed by Eq.(3.1a), (3.1b) and (3.1c), respectively.

$$EI_x v'' + Pv - \Phi \left[M_{By} - \frac{z}{L} (M_{Ty} + M_{By}) + Px_o \right] = M_{Bx} - \frac{z}{L} (M_{Tx} + M_{Bx}) \quad (3.1a)$$

$$EI_y u'' + Pu - \Phi \left[M_{Bx} - \frac{z}{L} (M_{Tx} + M_{Bx}) - Py_o \right] = M_{By} - \frac{z}{L} (M_{Ty} + M_{By}) \quad (3.1b)$$

$$EC_w \Phi'' - (C_T + \bar{K}) \Phi' + u \left[-M_{Bx} + \frac{z}{L} (M_{Bx} + M_{Tx}) + Py_o \right] - v \left[M_{By} - \frac{z}{L} (M_{By} + M_{Ty}) + Px_o \right] - \frac{v}{L} (M_{Ty} + M_{By}) - \frac{u}{L} (M_{Tx} + M_{Bx}) = 0 \quad (3.1c)$$

where, E = modulus of elasticity

C_w = warping torsional constant

C_T = GJ

G = elastic shear modulus

J = St. Venant's torsional constant

$\bar{K}$ = $\beta_x M_x$

u = translation of the shear centre in the x direction

v = translation of the shear centre in the y direction

Φ = angle of twist of the cross section about the shear centre

$M_{Tx}, M_{Bx}, M_{Ty}, M_{By}$ = moments at top and bottom of the beam column about the principal x and y axis

R_x and R_y = reaction forces in the x and y directions

P = axial load

Figure 3.1: Beam Column with Arbitrary Cross Section Subject to Bending and Axial Load

3.1.2 Homogeneous Differential Equations and Solutions for Lateral-Torsional Buckling

Simplifying the general second order differential equations for singly symmetric beams with bending about the strong axis ($M_{Ty} = M_{By} = R_x = P = 0$) gives the homogeneous equations for lateral-torsional buckling:

$$EI_x v'''' = 0 \quad (3.2a)$$

$$EI_y u^{iv} + M_x \Phi'' + 2M'_x \Phi' = 0 \quad (3.2b)$$

$$EC_w \Phi^{iv} - (C_T + \bar{K}) \Phi'' - M'_x \Phi' \beta_x + M_x u'' = 0 \quad (3.2c)$$

where, M_x = moment at any point along the z axis

$$= \frac{z}{L} (M_{Tx} + M_{Bx}) - M_{Bx}$$

The relationship between u and Φ can be determined by evaluating the constants of integration for Eq (3.2b) for a beam with pinned boundary conditions and a constant moment field. Substituting this relationship into Eq. (3.2c) yields a homogeneous 4th order differential equation in terms of Φ only, from which the solution yields the critical moment to cause lateral-torsional buckling. Lateral-torsional buckling is the bifurcation phenomenon that occurs when the assumed initially straight member is loaded in such a manner that the predominant deflections switch from in-plane to a combination of lateral and twisting (Galambos 1988) and can be characterised by the events illustrated in Figure 3.2.

3.1.3 Lateral-Torsional Buckling Solutions for Singly Symmetric and Doubly Symmetric Beams with Constant Moment

The solutions for the differential equation for singly symmetric and doubly symmetric cross sections yields the elastic critical moment for lateral-torsional buckling and are given by Eq. (3.3) and (3.4) respectively.

$$M_{cr} = \frac{\pi^2 EI_y \beta_x}{2L^2} \left\{ 1 + \sqrt{1 + \frac{4}{\beta_x^2} \left[\frac{GJL^2}{\pi^2 EI_y} - \frac{C_w}{I_y} \right]} \right\} \quad (3.3)$$

where, β_x = monosymmetric bending coefficient

$$M_{cr} = \frac{\pi}{L} \sqrt{EI, GJ} \sqrt{1 + \frac{\pi^2 EC_w}{GJL^2}} \quad (3.4)$$

Figure 3.2: The Deflected Positions at Midspan for Beams Subject to Lateral-Torsional Buckling

Clause 13.6 of CAN/CSA-S16.1-94 presents the corresponding critical moment for doubly symmetric sections as Eq. (3.5) where an equivalent moment factor, ω_2 , has been introduced to account for varying moment fields.

$$M_u = \frac{\omega_2 \pi}{L} \sqrt{EI, GJ + \left(\frac{\pi E}{L}\right)^2 I, C_w} \quad (3.5)$$

where, ω_2 = equivalent moment factor

3.2 Expanded Application

The solution of this particular differential equation is limited to either singly symmetric or doubly symmetric sections subject to constant moment field with pinned end boundary conditions. To extend the scope of application to encompass a wide range of real conditions CAN/CSA-S16.1-94 includes provisions and reference to other provisions accounting for varying moment fields, type and placement of load, and restraint conditions. Additional factors that influence the lateral-torsional buckling capacity of a beam are continuity, presence of stiffeners, residual stress and initial imperfections, cross sectional distortion and local instabilities, and coping (Galambos, 1988).

3.2.1 Equivalent Moment Factor

Varying Moment Field

In most practical situations, the moment field is likely to vary (non-uniform) due to either unequal end moments or transverse loading between support points, or both. Empirical equations have been developed from numerical and test data for the equivalent moment factor, ω_2 , for varying loading conditions. In Clause 13.6 of CAN/CSA-S16.1-94 the equivalent moment factor for varying moment fields along an unbraced length of a member with no transverse loading and loaded through the shear centre is given by:

$$\omega_2 = 1.75 + 1.05\kappa + 0.3\kappa^2 \leq 2.5 \quad (3.6)$$

where,

$$\kappa = \frac{M_S}{M_L} \text{ and positive for double curvature and negative for single curvature, as illustrated in Figure 3.3}$$

M_S = smaller of the magnitudes of the end moments.

M_L = larger of the magnitudes of the end moments.

Figure 3.3: Bending Moment Diagrams For Beams Subject To Arbitrary End Moments

Transverse Loading and/or Singly Symmetric Subject to Single Curvature

Helwig et. al (1997) found the application of Eq (3.6) for singly symmetric sections to be satisfactory for beams subject to end moments that place it in single curvature only. For singly symmetric beams subject to transverse loading, Helwig et. al (1997) suggest, when the member exhibits single curvature, using a minor variation of the Kirby and Nethercot (1979) expression, developed for doubly symmetric sections subject to an arbitrary moment field, given in the *Load & Resistance Factor Design Specification* (AISC, 1994) (expressed by Eq.(3.7) for terms illustrated in Figure 3.4):

$$\omega_2 = \frac{12.5M_m}{2.5M_m + 3M_1 + 4M_2 + 3M_3} \quad (3.7)$$

where, M_m = maximum magnitude of the moment field
 M_1 = magnitude of the moment field at $L_b/4$
 M_2 = magnitude of the moment field at $L_b/2$
 M_3 = magnitude of the moment field at $3L_b/4$
 L_b = unbraced length of the member

Figure 3.4: Definition of Variables Required For Evaluating the Equivalent Moment Factor For a Beam Section With an Arbitrary Bending Moment Diagram

Doubly or Singly Symmetric Subject to Double Curvature

For double curvature, the use of Eq. (3.6) to predict the critical moment capacity of singly symmetric sections was found to be unsatisfactory for both end moment and transverse load application. Helwig et. al (1997) proposed the following modification to Eq. (3.7) for singly symmetric I-beams with the following restriction, $0.1 < \rho_f < 0.9$:

$$\omega_2 = \frac{12.5M_m}{2.5M_m + 3M_1 + 4M_2 + 3M_3} (0.5 + 2\rho_f^2) \leq 3.0 \quad (3.8)$$

where,

$$\rho_f = \frac{I_{yf}}{I_y}$$

I_{yf} = moment of inertia of the top flange about the cross sectional weak axis

Transverse Load and Location of Application

For many practical cases loading may occur above or below the shear centre as illustrated by the schematic diagram in Figure 3.5. For top flange and bottom flange loading, above and below the shear centre, the load will have either a destabilising or restoring effect, increasing or decreasing the lateral-torsional buckling capacity.

(a) Top-Flange Loading (b) Shear Centre Loading (c) Bottom-Flange Loading

Figure 3.5: Load Placement on the Cross Section

A series of equivalent moment factors for three specific transverse load cases and three locations of application on the cross section are given in Table 3.1 (Galambos, 1988).

As beams become stockier or the unbraced length becomes shorter, the warping stiffness increases, hence the equivalent moment factor and lateral-torsional buckling capacity increase. However, for values of W greater than 1.75 the

Table 3.1: Equivalent Moment Factors for Transverse Loads Acting at Different Locations on the Cross Section, after Galambos (1988)

Loading	BMD	M	A	B
$\frac{PL}{4}$	1.35	$1 - 0.180W^2 + 0.649W$
$\frac{wL^2}{8}$	1.12	$1 - 0.154W^2 + 0.535W$
PL_1	$1 + \left(\frac{L_1}{2L_1 + L_2} \right)^2$	$1 - 0.465W^2 + 1.636W$

Loading Condition:

Top-flange loading: $\omega_2 = A/B$

Load at shear centre: $\omega_2 = A$

Bottom-flange loading: $\omega_2 = AB$

where, $W = \frac{\pi}{L_p} \sqrt{\frac{EC_w}{GJ}}$

Effect of Restraint

The previous expressions for calculating the equivalent moment factor were developed for a single span simply supported beam. The corresponding coefficients, A and B , for single span beams with varying end restraint, illustrated in Table 3.2, or with any end restraint and midspan lateral support, illustrated in Figure 3.7, are given in Table 3.3.

3.2.2 Tension Flange Continuously Braced

The critical elastic lateral-torsional buckling capacity of a doubly symmetric I-section with the tension flange continuously braced is given as:

$$M_{cr} = \frac{GJ}{d'} + \frac{\pi^2}{d'L^2} \left(EC_w + \frac{EI_y d'^2}{4} \right) \quad (3.10)$$

where, d' = centre-to-centre distance between the flanges.

This value can be modified by the equivalent moment factors given in section 3.2.1 to account for varying moment fields, transverse load types and location of the application of load.

Table 3.2: Plan View Illustrating Idealised End Restraints, after Galambos (1988)

Restraint Type	Idealised End Restraint
<p>I</p> <p>Simply Supported</p>	<p>$u = \phi = u' = \phi' = 0$</p>
<p>II</p> <p>Warping Prevented</p>	<p>$u = \phi = u' = \phi' = 0$</p>
<p>III</p> <p>Lateral Bending Prevented</p>	<p>$u = \phi = u' = \phi' = 0$</p>
<p>IV</p> <p>Fixed End</p>	<p>$u = \phi = u' = \phi' = 0$</p>

Notes: (1) u - lateral displacements
 ϕ - angle of twist about the centroid
 (2) In all cases the lateral displacements of the flange tips are restrained.

Note: end restraint types must be identical at both ends

Figure 3.7: Restraint Condition V, after Galambos (1988)

Table 3.3: Restraint Categories, after Galambos (1988)

Loading	Restraint	A	B
	I	1.35	$1 - 0.180W^2 + 0.649W$
	II	$1.43 - 0.485W^2 + 0.463W$	$1 - 0.317W^2 + 0.619W$
	III	$2.0 - 0.074W^2 + 0.304W$	$1 - 0.207W^2 + 1.047W$
	IV	$1.916 - 0.424W^2 + 1.851W$	$1 - 0.466W^2 + 0.923W$
	V	$2.95 - 1.143W^2 + 4.0701W$	1
	I	1.12	$1 - 0.154W^2 + 0.535W$
	II	$1.2 - 0.416W^2 + 0.402W$	$1 - 0.225W^2 + 0.571W$
	III	$1.9 - 0.120W^2 + 0.006W$	$1 - 0.100W^2 + 0.806W$
	IV	$1.643 - 0.405W^2 + 1.771W$	$1 - 0.339W^2 + 0.625W$
	V	$2.093 - 0.947W^2 + 3.117W$	$1.073 + 0.044W$

3.2.3 Flange Coping

Cheng (1992) developed the following two interaction equations to calculate the reduced elastic lateral-torsional buckling capacity for beams with copes at both ends and at one end of the unbraced length. The elastic lateral-torsional buckling capacities, $M_{uncoped}$ and M_{coped} , are calculated according to Eq.(3.4) and (3.3), with the cross sectional dimensions for the entire beam associated with the full cross section and the coped cross section, respectively.

- Copes at both ends of the unbraced length

$$\left[\frac{1}{M_{uncoped}} \right] + \left[\frac{1}{(L_b/2C)M_{coped}} \right] = \left[\frac{1}{M_{cr}} \right] \quad (3.11a)$$

- Coped at one end of the unbraced length

$$\left[\frac{1}{M_{uncoped}} \right]^2 + \left[\frac{1}{(L_b/2C)M_{coped}} \right]^2 = \left[\frac{1}{M_{cr}} \right]^2 \quad (3.11b)$$

where, L_b = the unbraced length

C = the cope length

These interaction equations only apply to situations where the cope length is less than twice the beam depth and where the cope depth is less than one fifth the beam depth.

3.2.4 Cantilevers, Continuous Beams and Cantilever Suspended Spans

Notwithstanding a significant number of modifications that have been proposed, there are still special restraint conditions associated with cantilevers, continuous beams and cantilever suspended span systems that require specifically derived formula for elastic critical buckling as they are susceptible to this failure.

Cantilevers

Clause 13.6 (c) of CAN/CSA-S16.1-94 specifically states that a rational method of analysis be used to determine the lateral-torsional buckling capacity of cantilevers and that it should account for the lateral support conditions at the support and at the tip of the cantilever. The definitive reference which provides a series of rational methods is the *SSRC Guide to Stability Design Criteria for Metal Structures*

(Galambos, 1988). The equation given for the elastic lateral-torsional buckling capacity for cantilevers, Eq.(3.12), is a modified version of Eq.(3.4) and incorporates an effective length concept in which the effective length factor is a function of the lateral support condition.

$$M_{cr} = \frac{\pi}{KL} \sqrt{EI, GJ} \sqrt{1 + \frac{\pi^2 EC_w}{(KL)^2 GJ}} \quad (3.12)$$

where, K = effective length factors are given in Table 3.4

Graphic representation of the support conditions at the root and tip, and the corresponding effective length factors are given in Table 3.4. These recommended effective length factors have been published without indication to the geometric limits of applicability. The factors are only applicable to relatively long span cantilevers where $R > 4$ and $0 < X < 1.57$, Essa and Kennedy (1993).

$$X = \sqrt{\frac{\pi^2 EC_w}{GJL^2}} = \frac{\pi}{R} \quad (3.13)$$

where, R = torsional parameter

**Table 3.4: Effective Length Factors For Cantilevers,
after Galambos (1988)**

Restraint Conditions		Effective Length Factor	
At Root (over or at support)	At Tip	Top Flange Loading	All Other Cases
Built-in Cantilever		1.4	0.8
	1.4	0.7
	0.6	0.6
Overhanging Segment		2.5	1.0
	2.5	0.9
	1.5	0.8
Overhanging Segment		7.5	3.0
	7.5	2.7
	4.5	2.4

- Based on Kirby and Nethercot (1979)

It was also noted that the graphical representations of the restraint conditions at the tip were not the same as those for which the factors were developed, as illustrated in Table 3.5. In addition, the notation "All Other Cases" under the effective length factor title is misleading as the corresponding effective length factors were developed for loading applied to the shear centre of the cross section.

Table 3.5: Comparison of Restraint Conditions at the Cantilever Tip, after Essa and Kennedy (1993)

SSRC guide (Galambos, 1988)	Kirby and Nethercot (1979)

The effective length factors given in Table 3.4, (Galambos, 1988), yield inaccurate results as they do not properly account for the restraint conditions and the length of the back span nor the limitations cited.

To avoid unconservative results, the effective length factors for the overhanging segment shouldn't be taken as less than that of the associated back span (Essa and Kennedy, 1993). Furthermore, Essa and Kennedy (1993), developed the appropriate effective length factors for the built-in cantilever case with the tip

restraints specified in the *SSRC Guide to Stability Design Criteria for Metal Structures* (Galambos, 1988). These are listed in Table 3.6.

Table 3.6: Effective Length Factors for Built-in Cantilevers, after Essa and Kennedy (1993)

Restraint Conditions		Effective Length Factor	
At Root	At Tip	Top Flange Loading	Shear Centre Loading
$0.57 + 0.71X - 0.1X^2$	0.75
	$1.2 - \frac{0.161}{X} - \frac{0.184}{X^2} \geq 0.85$	0.5
	0.44	0.44

For cases where loading is between the top flange and the shear centre, the following interpolation expression can be applied to determine the appropriate effective length factor that should be used:

$$K = rk_t + (1-r)k_s \quad (3.14)$$

where,

$$r = \frac{h}{d} \left(1.5 + \frac{h}{d} \right)$$

k_t = effective length factor associated with the top flange loading condition.

k_s = effective length factor associated with the shear centre loading condition

d = depth of the cross section

h = height above the shear centre

For continuous members or simply supported members with overhanging segments, the elastic lateral-torsional buckling capacity must first be evaluated for each segment independently. Since the member is continuous, the critical span or segment (most susceptible to lateral-torsional buckling) will be elastically restrained by the other. If the back span is critical, then the overall resistance of the member is taken to be the elastic lateral-torsional buckling capacity of the back span, since the overhanging portion offers little torsional or lateral restraint. If the overhanging segment is critical then the back span provides additional restraint and the elastic lateral-torsional buckling capacity can be calculated as:

$$M_{cr} = M_c + F(M_b - M_c) \quad (3.15)$$

where, M_b = elastic lateral-torsional buckling resistance for the back span as calculated according to Eq.(3.5)

M_c = elastic lateral-torsional buckling resistance of the overhanging segment calculated according to equations, restraint and loading conditions given in Table 3.7.

F = geometric coefficient factor relating the length of the back span, L , to the length of the cantilevered span, L_c , as given in Table 3.8 for the corresponding restraint conditions.

Table 3.7: Moment Resistance of Overhanging Beams Free to Warp at the Root and Subject to Specific Tip Loading and Restraint Conditions (Essa and Kennedy, 1993)

Restraint Conditions		Tip Loading	
At Root	At Tip	Top Flange	Shear Centre
$M_c = 1.5 \frac{GJ}{d}$	$M_c = \frac{4}{L_c} \sqrt{EI_y GJ}$

The formulations are only applicable when the back span is loaded through the shear centre. If the back span is loaded through the top flange, Essa and Kennedy (1993) recommend that the moment capacity be determined using finite element analysis.

Table 3.8: Determination of the Coefficient F for Specific Restraint Conditions (Essa and Kennedy, 1993)

Restraint Conditions		Top Flange Tip Loading
At Root	At Tip	Coefficient F
$F = -0.08 + 0.18 \frac{L}{L_c} - 0.009 \left(\frac{L}{L_c} \right)^2$
	$F = 0.064 + 0.162 \frac{L}{L_c} - 0.009 \left(\frac{L}{L_c} \right)^2$

For the remaining restraint condition illustrated in Table 3.9, the interactive buckling concept (Schmitke and Kennedy, 1985) for continuous spans can be applied.

Table 3.9: Restraint Conditions For Which an Interactive Buckling Analysis is Applicable (Essa and Kennedy, 1993)

At Root	At Tip

Continuous Beams

Schmitke and Kennedy (1985) have developed an equivalent beam method, which properly determines the lateral-torsional buckling capacity of members (continuous or single span) with discrete lateral support that must buckle interactively. Interactive buckling must occur since critical segments (susceptible to lateral-torsional buckling) of the flexural member are restrained laterally and are continuous with adjacent elements. The resultant buckled shape is sinusoidal (smooth lateral deflected shape). The equivalent beam method asserts that:

- the entire member must buckle simultaneously.
- the equivalent moment concept can be applied to give effective segment lengths, L_e , with a constant moment that are equivalent in capacity determination to the segment being considered with a varying moment field.
- for a beam to buckle simultaneously, the equivalent design length, L' , of all segments, acted upon by uniform moments, must be the same.

The effective length of each segment can be calculated from Eq. (3.16):

$$L_e = \frac{\pi\sqrt{EI, GJ}}{\sqrt{2}M_u} \sqrt{\left\{1 + \sqrt{1 + \frac{4M_u^2 C_w}{I, (GJ)^2}}\right\}} \quad (3.16)$$

where, M_u = elastic lateral-torsional buckling capacity evaluated for the segment with the appropriate equivalent moment factors.

The equivalent beam length for all segments with equivalent uniform moments is equal to the sum of the effective lengths of every segment. The equivalent design length, L' , which is subsequently used to calculate the elastic lateral-torsional buckling moment associated with simultaneous buckling is simply equal to the equivalent beam length divided by the number of beam segments, as given by Eq. (3.17):

$$L' = \frac{\sum_{i=1}^n L_e}{n} \quad (3.17)$$

where, n = number of segments.

Cantilever Suspended Spans

Double overhanging cantilever suspended span construction (interior section) is frequently used in low rise steel construction and is susceptible to lateral-torsional buckling failures. Application of existing design equations and referenced guidelines for determining the moment capacity for this type of construction yield unsatisfactory results. As a result, Essa and Kennedy (1993) have developed a specific equation to calculate the critical elastic moment capacity, Eq. (3.18), given the following limitations which are consistent with current construction practices.

- the member must have symmetric geometry and loading conditions.

- the ratio of the back span to the cantilever span, L/L_c , must lie within the range of 4 to 6.
- load transmitted from the suspended span to the tip of the cantilever must be within 15% of the section depth above the shear centre.
- load along the length of the double overhanging cantilevered beam results from joists framed and welded to the top flange.
- all joists have the same in-plane stiffness.
- the torsional parameter, X , based on the back span length, must lie within the range of 0.4 to 1.7.

For situations that fit these criteria, the critical elastic buckling moment can be approximated by (Essa and Kennedy, 1993):

$$M_{cr} = \frac{k}{L} \sqrt{EI, GJ^*} \quad (3.18)$$

where, k = effective buckling coefficient obtained by finite element analysis for which curves have been presented for specific situations in Essa and Kennedy (1993)

$$J^* = J + \frac{K_e L^2}{\pi^2 G}$$

K_e = effective continuous torsional restraint stiffness given by Eq. (3.19)

$$\frac{1}{K_e} = \frac{1}{K_b} + \frac{1}{K_w} + \frac{1}{K_f} \quad (3.19)$$

where, K_b = in-plane bending stiffness of the brace divided by the brace spacing.

$$K_b = \frac{EI_T}{L_p s} \quad (3.20)$$

where, I_T = moment of inertia of the joist's top chord about the horizontal centroidal axis

L_p = length of the end panel

s = joist spacing along length of the beam

and, K_w = web stiffness to against distortional effects

$$K_w = \frac{Et_w^3}{4(1-\nu^2)d} \quad (3.21)$$

where, t_w = web thickness

ν = Poisson's ratio

d = depth of the section

and, K_f = effective torsional stiffness of the flange

$$K_f = \frac{7.29Gbt^3}{s^2} \quad (3.22)$$

where, b = flange width

t = flange thickness

3.2.5 Residual Stresses, Web Distortion, Local Buckling and Holes

It is clearly recognised that residual stresses, web distortion, local buckling of class 4 elements and the presence of holes may influence the lateral-torsional buckling capacity of beams. Essa and Kennedy (1993) and Trahair (1996) describe, in general, the probable influence of these but give no specific guidelines for design as rational mathematical models would become too complex. Lateral-torsional buckling problems for which these parameters are key for determining the strength are best solved using appropriate numerical analyses.

3.2.6 Inelastic Lateral-Torsional Buckling

The limit to elastic lateral-torsional buckling occurs when any portion of the cross section starts to yield which is consistent with the maximum residual stresses. Currently, Clause 13.6 of CAN/CSA-16.1-94 limits the elastic range to moments less than 0.67 of the cross sectional strength. This is consistent with an assumed maximum residual compressive stress equal to one third of yield. Note, Essa and Kennedy (1993) found the flanges of most beams to be in residual tension, which would delay the onset of inelastic lateral-torsional buckling. The inelastic lateral-torsional buckling capacity given by Eq. (3.23) is based on results from tests and numerical analyses and is a straight line statistical fit to that data with an upper limit equal to the cross sectional strength.

$$M_r = 1.15M_{y,p} \left(1 - \frac{0.28M_{y,p}}{M_{cr}} \right) \quad (3.23)$$

where, $M_{y,p}$ = either the yield moment or the plastic moment capacity given by Clause 13.5 as denoted by the subscript y or p , respectively

M_{cr} = elastic lateral-torsional buckling capacity

3.2.7 Cross Sectional Capacity

Flexural members that are continuously laterally braced or loaded about their weak axis (inherently stable) can obtain a maximum moment equal to their cross sectional moment capacity. The moment resistance for class 1 or 2, class 3 and class 4 sections are given by Eq. (3.24), Eq.(3.25) and Eq. (3.26), respectively.

- class 1,2 $M_r = ZF_y$ (3.24)

- class 3 $M_r = SF_y$ (3.25)

- class 4 $M_r = S_e F_y$ (3.26)

where, M_r = moment resistance of the cross section

Z = plastic section modulus

S = elastic section modulus

S_e = effective section modulus

F_y = yield stress

Many practical loading conditions such as those illustrated in Figure 3.8 may induce both torsion and flexure into a beam.

Figure 3.8: Typical Eccentric Loading Conditions

For these cases the capacity of the member is limited by maximum tolerable distortion (twisting), or by deflection, or by the maximum combined normal stress arising from warping torsion and bending, or by cross sectional strength evaluated for the combination of torsion and flexure calculated in accordance with Clause 15.11.

Driver and Kennedy (1989) provide guidance for the determination of moment torque interaction diagrams for class 1, 2 or 3 I-shaped sections only. Once again numerical methods can be used effectively to determine the appropriate behaviour and resistance.

3.3 Summary

CAN/CSA-S16.1-94 provides a series of clauses and references to guidelines on how to compute the resistance of beams with a variety of load conditions, restraint conditions, geometry and configurations that may be susceptible to lateral-torsional buckling. It is apparent by the number of references, equations and specifically cited modifications that the broad spectrum of flexural stability problems confronted by structural engineers cannot be resolved by any singular rational mathematical expression. The best means of evaluation must be a well calibrated numerical method which is developed to the point in which it can be used as a practical design tool, as currently allowed in accordance with Clause 1.4.

Chapter 4

Numerical Methods Applied to Stability Problems

4.1 Introduction

Numerical methods allow the engineer to solve complex stability problems for which there are no closed form solutions or the solutions have a multiplicity of modifications that can be misinterpreted. Although numerical methods are computationally intensive, the use of computers allows solutions to be readily developed for most practical problems. Graphical user interfaces can render the output such that interpretation of the data is clear and complex behaviour can be easily visualised through load deflection curves, deformed shapes and internal stress distributions.

Numerical method algorithms are well suited to the object oriented programming philosophy in that the solution strategy need only interact with the common methods of each element object. Element object types are independent and have standardised methods that return common data. Attributes of object oriented programming allow for the decoupling of the graphical user interface and permit the incorporation of new elements, specific to steel structures and construction, with minor modifications.

This chapter presents the solution strategy, material model and element types implemented into the objected oriented numerical methods program. Also included is a description of the verification of the program against test data collected for beams and beam columns subject to stability problems.

4.2 Solution Strategy and Material Model

Solution Strategy

The solution strategy is an application which is housed within a class entitled "FEAMain" and is modelled after an existing Visual Basic program (Albert, 1998a). The solution strategy is an incremental direct (Euler) iteration scheme (Zienkiewicz, 1977), illustrated in Figure 4.1.

Figure 4.1: Incremental Direct Iteration Scheme for a Single Degree of Freedom System

Starting from its original position, a^0 , which generally includes the initial imperfections, an incremental load, $\Delta\lambda_1\{f_o\}$, is applied and the corresponding incremental displacements, $\{\Delta a_n\}$, associated with the nodal degrees of freedom are calculated according to Eq.(4.1):

$$\{\Delta a_n\} = [K_{T(n-1)}]^{-1} \Delta\lambda_n \{f_o\} \quad (4.1)$$

where, $\{\Delta a_n\}$ = incremental displacement vector evaluated for the current load step

$[K_{T(n-1)}]$ = global tangent structural stiffness matrix evaluated based on displacements computed in the last load step

$\Delta\lambda_n$ = current incremental load step factor

$\{f_o\}$ = applied load vector

n = current load step

The incremental displacement is added to the previous position, a^0 , to give the current position, a^1 . Successive load increments, $\Delta\lambda_n$, are applied using a global tangent stiffness matrix, $[K_{T(n-1)}]$, based on the total displacements at the end of the previous load step, a^{n-1} , to calculate each successive incremental displacement vector, $\{\Delta a_n\}$.

The global tangent stiffness matrix is assembled from the elemental transformed structural stiffness matrices including geometric effects as given by:

$$[K_T] = \sum [T]^T ([K] + [K_G]) [T] \quad (4.2)$$

where, $[K]$ = elemental local structural stiffness matrix

$[K_G]$ = elemental geometric stiffness matrix

$[T]$ = transformation matrix

The global tangent stiffness matrix is a banded square matrix and only requires the upper half bandwidth to be retained to describe all of the matrix coefficients.

The Crout reduction algorithm, Mondkar and Powell (1974), is used to solve the set of linear equations described by Eq.(4.1). Note, recent references such as Ayyub and McCuen (1996) refer to this algorithm as Cholesky decomposition which is a form of the LU decomposition method that reduces the number of repetitive computations for symmetrical banded matrices.

The incremental method is guaranteed to converge provided sufficiently small load steps are applied, Zienkiewicz (1977). With larger load steps an upper bound solution will be achieved as "drift off errors" (Chen and Lui, 1991), illustrated in Figure 4.1, will occur. Obviously the load step size must be reduced with decreasing stiffness to minimise the "drift off errors" and to give the best description of behaviour.

Material Model

Element objects employ methods that determine the current stress-strain state throughout the element. These methods use a bilinear material model, as illustrated in Figure 4.2 for a uniaxial stress condition, that is coupled with the von Mises yield criterion, presented in Appendix A.1, and isotropic strain hardening and the associated flow rule, presented in Appendix A.2, to describe plasticity. This combination provides an appropriate model for material behaviour that is associated with the instability of steel beams and beam columns.

Figure 4.2: Bilinear Material Model Illustrated For a Uniaxial Stress Condition

4.3 Element Object Descriptions

4.3.1 Beam Column Element Objects

Three 2 node beam column objects, with 6 or 7 degrees of freedom per node (illustrated in Figure 4.3), have been developed for use within the object oriented numerical methods program.

Figure 4.3: Two Node Beam Column Element

These objects are capable of modelling the specific cross sections and residual stress patterns illustrated in Figure 4.4.

Notes: (1) the maximum residual stress, σ , can be specified by the user.
 (2) element object names are indicated in bold print

Figure 4.4: Beam Column Element Objects

The beam column element objects allow for partial plastification of the cross section and the development of plastic zones, if the member is discretised into a series of these elements. Beams or beam columns that are discretised by a series of these elements result in a reasonably accurate description of overall member behaviour associated with stability problems. A complete description of the derivation of these element objects, their functionality, and verification of their ability to predict strength and stability behaviour is given in Raby (1999).

4.3.2 Triangular Shell Element Object (TShell1)

Lee and Yoo (1988) developed a triangular shell element, illustrated in Figure 4.5, which combines the in-plane (membrane) stiffness with the bending stiffness of a non-conforming triangular plate element, creating 6 degrees of freedom per node, that satisfies C^0 continuity.

Figure 4.5: Triangular Shell Element Object

The addition of the 6th degree of freedom provides a complete description of the element deformation and avoids potential singularity or near singularity that may occur for elements in which only 5 degrees of freedom per node are defined. Note, optimum performance of the element is achieved when the aspect ratio of the sides is 1:1:1. Lee and Yoo (1988) used this element to model steel members with a large variety of load and boundary conditions (simply supported beam with midspan load, cantilever beam with tip loading, a shell structure, a plate subject to in-plane torsion) and found that the behaviour was well modelled. Given the element's attributes and numeric stability and the success that the researchers have had in modelling the behaviour and stability of steel members, this element was selected to be included in the development. Another attribute of the element, as encoded in the numerical methods program, is that it allows for the input of stresses, which can approximate residual stress patterns.

Membrane Stiffness Matrix

Since the shape functions describing the out-of-plane displacements and rotations for the non-conforming plate element are well documented in Zienkiewicz (1977), only those shape functions associated with the membrane degrees of freedom will be presented. The membrane stiffness matrix, $[K^m]$, can be decoupled from the bending stiffness matrix, $[K^b]$, and derived explicitly. Lee and Yoo (1988) derived shape functions that describe the horizontal and vertical displacements associated with the in-plane rotations, assuming:

- The total subtended angle at any node of the element does not change after in-plane rotation, as illustrated in Figure 4.6.
- A point (x,y) within an element will move in the direction normal to a line connecting the point and a nodal point, subjected to the in-plane rotation.
- A cubic polynomial describes the displacement field associated with any line on the cross section.

The shape functions, NU_i and NV_i for displacements in the x and y directions (u , v) for each node i , expressed in terms of area co-ordinates (described in Appendix A.3) are:

$$NU_1 = -y_3 L_3 L_1^2 \quad (4.3a)$$

$$NV_1 = x_2 L_2 L_1^2 + x_3 L_3 L_1^2 \quad (4.3b)$$

$$NU_2 = -y_3 L_2 L_3^2 \quad (4.4a)$$

$$NV_2 = -x_2 (1 - L_2) L_2^2 + x_3 L_2 L_3 \quad (4.4b)$$

$$NU_3 = y_3 (1 - L_3) L_3^2 \quad (4.5a)$$

$$NV_3 = x_2 L_2 L_3^2 - x_3 (1 - L_3) L_3^2 \quad (4.5b)$$

in which, L_1, L_2 , and L_3 are the area co-ordinates associated with the nodes:

$$\text{where, } L_1 = 1 - \xi - \eta$$

$$L_2 = \xi$$

$$L_3 = \eta$$

Figure 4.6: Derivation of the Shape Functions Associated with the In-plane Rotational Degree of Freedom

The shape functions associated with horizontal and vertical translations are the same for relative displacements in the u and v directions for each particular node and are also expressed in terms of area co-ordinates:

$$ND_1 = L_1 + L_1^2 L_2 + L_1^2 L_3 - L_1 L_2^2 - L_1 L_3^2 \quad (4.6)$$

$$ND_2 = L_2 + L_2^2 L_3 + L_2^2 L_1 - L_2 L_3^2 - L_2 L_1^2 \quad (4.7)$$

$$ND_3 = L_3 + L_3^2 L_1 + L_3^2 L_2 - L_3 L_1^2 - L_3 L_2^2 \quad (4.8)$$

The translational displacements at any point within the element can be expressed as:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \sum_{i=1}^3 ND_i \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} + \sum_{i=1}^3 \begin{Bmatrix} NU_i \\ NV_i \end{Bmatrix} \theta_{\bar{x}} \quad (4.9)$$

where, i = node number

Therefore the membrane shape function matrix, $[N^m]$, is defined as:

$$[N^m] = \begin{bmatrix} ND_1 & 0 & NU_1 & ND_2 & 0 & NU_2 & ND_3 & 0 & NU_3 \\ 0 & ND_1 & NV_1 & 0 & ND_2 & NV_2 & 0 & ND_3 & NV_3 \end{bmatrix} \quad (4.10)$$

The relevant strain displacement matrix for the membrane action is defined as the partial derivative of this and is given as:

$$[B^m] = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} [N^m] \quad (4.11)$$

The evaluation of Eq.(4.11) gives $[B^m]$, which is expressed as $[B^m]^T$ to fit the page:

$$[B^m]^T = \begin{bmatrix} \frac{\partial ND_1}{\partial x} & 0 & \frac{\partial ND_1}{\partial y} \\ 0 & \frac{\partial ND_1}{\partial y} & \frac{\partial ND_1}{\partial x} \\ \frac{\partial NU_1}{\partial x} & \frac{\partial NV_1}{\partial y} & \frac{\partial NU_1}{\partial y} + \frac{\partial NV_1}{\partial x} \\ \frac{\partial ND_2}{\partial x} & 0 & \frac{\partial ND_2}{\partial y} \\ 0 & \frac{\partial ND_2}{\partial y} & \frac{\partial ND_2}{\partial x} \\ \frac{\partial NU_2}{\partial x} & \frac{\partial NV_2}{\partial y} & \frac{\partial NU_2}{\partial y} + \frac{\partial NV_2}{\partial x} \\ \frac{\partial ND_3}{\partial x} & 0 & \frac{\partial ND_3}{\partial y} \\ 0 & \frac{\partial ND_3}{\partial y} & \frac{\partial ND_3}{\partial x} \\ \frac{\partial NU_3}{\partial x} & \frac{\partial NV_3}{\partial y} & \frac{\partial NU_3}{\partial y} + \frac{\partial NV_3}{\partial x} \end{bmatrix} \quad (4.12)$$

The membrane stiffness matrix is determined from Eq.(4.13):

$$[K^m] = \int_A [B^m]^T [\bar{D}^m] [B^m] dA \quad (4.13)$$

where, $[\bar{D}^m]$ = average constitutive matrix through the thickness associated with membrane stresses

Structural Stiffness Matrix

The complete structural stiffness matrix (18x18) is composed of both the membrane and bending stiffness matrices, as given by Eq. (4.14)

$$[K] = [K^m] + [K^b] \quad (4.14)$$

where, $[K^b]$ = stiffness matrix associated with the bending degrees of freedom

$$[K^b] = \int_A [B^b]^T [\bar{D}^b] [B^b] dA$$

$[B^b]$ = strain displacement matrix associated with bending stresses

$[\overline{D}^b]$ = average constitutive matrix through the thickness associated with bending stresses

The introduction of the in-plane rotational degrees of freedom in the element causes the shape functions to be cubic and the corresponding strains to vary quadratically. This may potentially result in membrane locking. To minimise this possibility, Lee and Yoo (1988), recommend the use of a four point integration technique in place of the exact six point integration. A three point integration technique was used for the determination of the bending stiffness (Albert, 1998b). Definition of the Gauss points and weights are given in Appendix A.4. The corresponding membrane and bending stiffness matrices expressed as a function of area co-ordinates and in terms of a Gaussian summation are given in Eq.(4.15) and (4.16), respectively.

$$[K^{m,b}] = \int_0^{1-L_1} \int_0^{L_1} [B^{m,b}]^T [\overline{D}^{m,b}] [B^{m,b}] dL_2 dL_1 \quad (4.15)$$

$$[K^{m,b}] = \sum_k H_k [B^{m,b}]^T [\overline{D}^{m,b}] [B^{m,b}] A \quad (4.16)$$

where, H_k = Gaussian integration weight

A = surface area of the element

m,b = membrane and bending respectively

As an aside, the stresses and strains are evaluated using a single point Gaussian integration technique.

Element and Geometric Stiffness Matrices

The effect of membrane stresses acting on the deformed configuration of the element (similar to an amplification factor for beam columns) is accounted for by the addition of the geometric matrix, $[K_G]$ (Akay et. al, 1977). The element stiffness matrix, $[K_e]$, given in Eq.(4.17) is equal to the sum of the structural stiffness matrix and the geometric stiffness matrix.

$$[K_e] = [K] + [K_G] \quad (4.17a)$$

where,

$$[K_G] = \begin{bmatrix} [k_s] & 0 & 0 \\ 0 & [k_s] & 0 \\ 0 & 0 & [k_s] \end{bmatrix} \quad (4.17b)$$

$$[k_s] = \int_A [G]^T [M] [G] dA \quad (4.17c)$$

$$[G] = \begin{bmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \quad (4.17d)$$

$$\text{where, } b_i = y_j - y_l$$

$$c_i = x_l - x_j$$

$$\text{where, } i = 1, 2, 3$$

$$j = 2, 3, 1$$

$$l = 3, 1, 2$$

$$[M] = \begin{bmatrix} \bar{\sigma}_x & \bar{\tau}_{xy} \\ \bar{\tau}_{xy} & \bar{\sigma}_y \end{bmatrix} \quad (4.17e)$$

$$\begin{Bmatrix} \bar{\sigma}_x \\ \bar{\sigma}_y \\ \bar{\tau}_{xy} \end{Bmatrix} = \text{average membrane stresses taken through the thickness}$$

Transformation Matrix

The transformation matrix for transforming the element stiffness matrix and local displacements into the global co-ordinate system, Zienkiewicz (1977), is given in Eq.(4.18).

$$[T] = \begin{bmatrix} [L] & 0 & 0 \\ 0 & [L] & 0 \\ 0 & 0 & [L] \end{bmatrix} \quad (4.18)$$

where,

$$[L] = \begin{bmatrix} [\lambda] & 0 \\ 0 & [\lambda] \end{bmatrix}$$

$$[\lambda] = \begin{bmatrix} \lambda_{xx} & \lambda_{xy} & \lambda_{xz} \\ \lambda_{yx} & \lambda_{yy} & \lambda_{yz} \\ \lambda_{zx} & \lambda_{zy} & \lambda_{zz} \end{bmatrix}$$

in which, the λ values correspond to the direction cosines associated with the local axes (x', y', z') and the global axes (x, y, z)

4.4 Object Oriented Numerical Methods Program

An object oriented numerical methods program was developed for structural steel strength and stability problems which incorporates the direct iteration strategy and features a triangular shell element and 3 beam column elements that are specific to shape and residual stress pattern.

4.4.1 Supplemental Objects

Supplemental objects have been developed to separate functional tasks, to minimise computations and to simplify data manipulation and control. The supplemental objects **Matrix**, **Node**, **Material**, **MatrixOperations**, **NodalLoad**, and

Restraint are presented in Table 4.1 along with their corresponding lists of public methods and state variables. The parentheses, (), contain the necessary variables taken as input to the particular method and the syntax [] and [][] following the state variables indicates a single and double dimension array, respectively.

The **Matrix** object contains its dimensions (number of rows and number of columns), the logic to extract these numbers and all of the coefficients. Storage and extraction of coefficients is simply addressed in a row-column format. The **MatrixOperations** class contains all relevant methods used to manipulate matrices. The **Crout1** and **Crout2** methods within **MatrixOperations** reduce the global tangent stiffness matrix and incremental loading vector, respectively. **Crout2** also calculates the incremental displacement vector using the reduced stiffness matrix and the reduced incremental loading vector.

The **Material** object allows for multiple material definitions or characteristics. The **Node** object contains a node number, the number of degrees of freedom and the corresponding co-ordinates. Separating the nodal state variables from the element minimises the data manipulation and computations associated with updating both the node locations and element state. The **NodalLoad** object contains the loads corresponding to the degrees of freedom for any given node and simplifies the formation of the applied incremental load vector. The **Restraint** object contains the restraints attributed to the degrees of freedom for any given node and computes the corresponding reactions.

Table 4.1: Public Methods and State Variables Associated with the Supplemental Objects

Object	Public Methods	State Variables
Matrix	<ul style="list-style-type: none"> Matrix(int Rows, int Columns) SetElement(int RowNum, int ColumnNum, double Value) GetElement(int RowNum, int ColumnNum) NumRows() NumColumns() 	<ul style="list-style-type: none"> double ThisMatrix[][] int NumberOfRows int NumberOfColumns
Node	<ul style="list-style-type: none"> Node(int num, double xcoord, double ycoord, double zcoord) number() setZ(double zcoord), getZ() setY(double ycoord), get Y() setX(double xcoord), get X() getDof() setDof(int tempdof) 	<ul style="list-style-type: none"> int z int y int x int Number int dof
Material	<ul style="list-style-type: none"> Material(byte number, double E1, double E2, double n, double fy) GetNum() 	<ul style="list-style-type: none"> byte Number double Fy double Gp double Gy double nu double Ep double Ey
MatrixOperations	<ul style="list-style-type: none"> Transpose(Matrix OldMatrix) Multiply(Matrix AA, Matrix BB) Multiply(Matrix AA, double[] b) Crout1(Matrix OldMatrix) Crout2(Matrix ReducedMatrix, double Loads[]) Determinant(Matrix ReducedMatrix) NodalLoad(int nodenum, double loading[]) 	<ul style="list-style-type: none"> N/A
NodalLoad	<ul style="list-style-type: none"> NodalLoad(int nodenum, double loading[]) 	<ul style="list-style-type: none"> double loads[] int NodeNum
Restraint	<ul style="list-style-type: none"> Restraint(int nodenum, double res[]) UpdateReactions(double rigidity, double inclDisp[]) 	<ul style="list-style-type: none"> double restraints[] double reactions[] int NodeNum

4.4.2 FEA ELEMENT Objects

The proposed element hierarchy associated with the **ANALYSIS ELEMENT** object given in Figure 2.4 is illustrated in Figure 4.7.

Figure 4.7: Element Hierarchy

The highlighted objects that have been developed contain the common methods and state variables listed in Table 4.2.

Table 4.2: Common Methods and State Variables For All FEA ELEMENT Objects

Common methods	Common State Variables
<ul style="list-style-type: none"> • getType() • getGeometric() • getStructural() • getNodes() • getNumNodes() • setDisplacements(double incdisp[][]) • update() • getStresses() • getVonMises() 	<ul style="list-style-type: none"> • Material material • Vector nodes • Byte Numberofnodes • Byte numberofdof • Matrix Geometric • Matrix Structural • Matrix LocalGeo • Matrix LocalStr • Matrix Transformation • String type

Note, the numerical methods program need only know that it is accessing an **FEA ELEMENT** object (the particular type is not required) since the methods and state variables are common and the encapsulation principle is upheld.

4.4.3 Methodology

The first step prior to executing the numerical methods program is to create and "in.dat" data file which houses all the necessary information to describe the structure, the loads, the restraint and the material characteristics. This may be created more simply through a graphical user interface (not yet developed) or with macros. The first activity of the program is to initialise the following objects; **Material** objects, **Node** objects, **FEA ELEMENT** objects, the global tangent stiffness **Matrix**, **Restraint** objects, **NodalLoad** objects and the load increment vector.

Subsequently, **FEAMain** performs the numerical analyses for the series of specified load increments. The procedure has the following steps:

- the global tangent stiffness matrix is assembled by accessing the methods for the appropriate structural and geometric matrices of each **FEA ELEMENT** object.
- the **Restraint** objects are accessed. In doing so the appropriate stiffness terms are inserted along the diagonal of the global tangent stiffness matrix,
- the **NodalLoad** objects are accessed to form the loading vector.
- the system of linear equations is solved by accessing a series of methods from **MatrixOperations** to determine the nodal displacements.
- this data is passed to the **FEA ELEMENT** objects which update their stress-strain state for the load increment.
- the node objects are accessed to update their co-ordinates.

- the **FEA ELEMENT** objects are accessed to reformulate the structural and geometric matrices based on the current set of data.
- the appropriate output is printed to the data file "out.dat".
- the process is repeated for each subsequent load step.

Note, as initial input, the user can specify a load increment where the program will pause allowing for a review of the load deflection curve through a simple graphical interface. This interface allows for viewing of the load deflection curve, the modification of subsequent load increments, the addition of more load increments, the insertion of a new intermediate stop increment and the termination of the program, which allows the engineer to use judgement in tracing the equilibrium path. As an aside, it is conceivable that unforeseen numeric instability could cause premature termination of this program.

4.5 Program Verification

The information and data required to model experimental tests numerically, listed in Table 4.3, includes geometry, boundary conditions, material characteristics, connection behaviour (displacements and rotations), loading description and characteristic test data.

Table 4.3: Required Information and Data For Modelling

Category	Information/Data
Geometry and boundary conditions	<ul style="list-style-type: none"> • Centreline dimensions • Joint numbers, member numbers, co-ordinates of joints • Member sizes, designation, cross sectional properties (nominal and/or measured) • Initial imperfections (local and global) • Connection details • Points of lateral support and types of restraint provided
Material characteristics and connection response	<ul style="list-style-type: none"> • Stress-strain curves (tension coupons/stub columns) • Residual stresses • Moment-Rotation characteristics of the connections
Loading	<ul style="list-style-type: none"> • Location, direction, type (proportional/non-proportional), load history • Load application (load or stroke control)
Test data	<ul style="list-style-type: none"> • Load deformation response (global rotations and displacements) • Deformed shape • Strains/internal forces/reactions • Failure mode; development of plastic hinges; mechanism; out of plane stability • Observations (unique to any given configuration) spread of yielding

Based on a review of existing literature, three sets of data were selected to verify the numerical model (Essa and Kennedy (1993), Kitipornchai and Trahair (1975), Van Kuren and Galambos (1964)), as these were the most complete and represented a variety of lateral-torsional buckling and strength problems. Generally, complete data sets are difficult to obtain as it is time consuming and costly to produce

this information and if obtained, it is often poorly presented (difficult to retrieve meaningful information) or not reported due to space limitations in refereed journals. Hence, there are very few good sets of data for comparison and even for the ones selected, some assumptions had to be made. Note, assumptions that are made due to poorly defined boundary conditions can greatly influence the predicted results generated numerically.

All of the test specimens, used in the numerical verification, had class 1 or 2 flanges and are adequately modelled with **R_BeCol** element objects as local flange buckling didn't govern their capacity. **R_BeCol** element objects were also used to model stiffeners and to properly locate load and restraint points relative to the cross section. **TShell1** element objects were used to model all webs.

4.5.1 Essa and Kennedy (1993)

Essa and Kennedy (1993) conducted an experimental and analytical investigation on the affects of loading, restraint and boundary conditions on single propped cantilevers that are susceptible to lateral-torsional buckling.

A mesh density study was conducted on one of the specimens that failed by lateral-torsional buckling, test 7, to determine an appropriate number of web elements which would adequately model the behaviour without being overly complex and time consuming. Lateral-torsional buckling problems are generally controlled by web distortion, which in turn is controlled by the discretisation of the web. Three mesh densities were established on the basis of discretising the web depth into 2, 4 and 8 rows of elements while maintaining the aspect ratio of the element as close to 1 as possible. The meshed beam models and corresponding test-to-predicted ratios for

these models with these mesh densities (240, 960 and 3840 total web elements) are illustrated in Figure 4.8.

(a) 240 Web Elements

$$\frac{t}{p} = 0.816$$

(b) 960 Web Elements

$$\frac{t}{p} = 0.993$$

(c) 3840 Web Elements

$$\frac{t}{p} = 1.041$$

**Figure 4.8: Varying Mesh Densities For Test Specimen 7,
Essa and Kennedy (1993)**

The corresponding Load vs. Euclidean Norm of the Displacements plot, which characterises the load displacement behaviour, is illustrated in Figure 4.9.

Figure 4.9: Load Displacement Behaviour For Varying Mesh Densities

With increasing mesh density, the stiffness of the member degrades and the predicted load decreases. The relative change in test-to-predicted ratios between 240 and 960 web elements and 960 and 3840 web elements was 17.8% and 4.6%, respectively. Based on this limited comparison it appears that there is little improvement in the model prediction beyond discretising the web into 960 web elements.

Of the thirty-three tests reported, twenty-seven were modelled with both 240 and 960 web elements, as three were eliminated by the researchers (identified yet unquantified additional restraint) and three were duplicates. The predicted loads and test-to-predicted ratio (t/p) for each test is listed in Table 4.4.

Table 4.4: Predicted Loads For Two Different Mesh Densities, Essa and Kennedy (1993)

Test Specimen Designation	240 Web Elements			960 Web Elements		
	Test Load (kN)	Predicted Load (kN)	v/p	Test Load (kN)	Predicted Load (kN)	v/p
2	78.9	67.8	1.163	78.9	67.1	1.176
3	77.1	61.7	1.25	77.1	67.1	1.149
4	104.9	93.3	1.124	104.9	91.3	1.149
5	125.8	107.0	1.176	125.8	107.0	1.176
7	76.1	93.3	0.816	76.1	76.6	0.993
9	35.5	34.8	1.02	35.5	33.7	1.053
10	41.1	36.6	1.124	41.1	35.8	1.149
12	46.4	45.5	1.02	46.4	45.0	1.031
13	48.2	48.8	0.988	48.2	44.8	1.075
14	132.4	123.2	1.075	132.4	117.8	1.124
15	136.8	127.3	1.075	136.8	124.5	1.0989
16	84	88.2	0.952	84	87.3	0.962
17	77.6	88.5	0.877	77.6	76.1	1.02
18	118.9	108.1	1.1	118.9	110.6	1.075
19	152.6	160.3	0.952	152.6	157.2	0.971
20	52.7	50.0	1.053	52.7	44.8	1.176
21	154.5	153.0	1.01	154.5	153.0	1.01
22	133.3	126.6	1.053	133.3	126.6	1.053
23	45.9	33.0	1.389	45.9	36.7	1.25
24	40.7	30.5	1.333	40.7	30.5	1.333
25	55.9	38.0	1.471	55.9	33.5	1.667
26	46.7	35.0	1.333	46.7	35.0	1.333
27	127.5	136.4	0.935	127.5	142.6	0.894
28	128.8	112.1	1.149	128.8	112.1	1.149
29	76.3	66.4	1.149	76.3	64.9	1.176
32	72.3	62.9	1.149	72.3	57.8	1.25
33	73.4	83.7	0.877	73.4	86.6	0.847

- Notes:
- 1) Tests 1, 6, and 8 were excluded due to identified yet unquantified restraint.
 - 2) Tests 10 and 11, 24 and 30, 27 and 31 were identical pairs. Test loads for 11, 30, 31 are 44.1 kN, 42.2 kN, 127.8 kN, respectively.
 - 3) Tests 7, 21 and 25 were used in a parametric study to determine the influence of initial out-of-straightness

The mean test-to-predicted ratio and coefficient of variation for beams discretised into 240 web elements and 960 web elements were 1.097 and 0.145, and 1.124 and 0.143, respectively.

The test loads greater than the predicted load may be attributed to:

- excessive initial out-of-straightness (sweep) in the model which are greater than those experienced in the tests,
- a more favourable residual stress pattern in the test specimen over that induced in the model,
- small additional unaccounted restraint associated with the load application and the reaction devices (idealised modelling of boundary and load conditions),
- the characteristic behaviour of the elements used to model the beam (too flexible),
- or some combination of any of the above.

A parametric study was conducted, using the 960 web element model, on beams 7, 21 and 25 to determine the effects of initial sweep, varying from $L/100$ to $L/5000$, on the failure load. With the exception of the largest imperfection for beam 25, the effects of initial imperfections on the predicted load were found to be insignificant, hence no real changes in the test-to-predicted mean values.

The residual stress patterns for the beams tested were favourable for lateral-torsional buckling capacity in that the flanges were essentially in residual tension. To model this pattern, statically equivalent nodal forces (compression and tension) were applied to the line of nodes on the web at each end. The thickness of the first two rows of web elements was increased to 1.5 times, to minimise the effects of high localised stresses.

Having eliminated the contribution of the modelling components dealing with initial sweep and residual stresses in predicting the ultimate load, the scatter in the test-to-predicted data is most likely related to variances in testing (alignment and undetectable restraint). The consistently higher test-to-predicted ratio is probably attributed to the performance of the elements used in the model and the idealised modelling of the boundary conditions.

4.5.2 Kitipornchai and Trahair (1975)

Kitipornchai and Trahair (1975) tested six simply supported beams, with the same nominal cross section and with varying length, subjected to concentrated midspan loading, to determine the effects of residual stresses on the lateral-torsional buckling capacity. Beams were tested in the as-rolled condition (with built-in residual stresses) and an annealed condition (stress relieved). Note, initial imperfections were not presented. Boundary conditions were described as preventing or permitting rotations or translations about or along certain axes. Without photographs or drawings illustrating the load or reaction devices, it is difficult to assess whether or not the physical devices provided the boundary conditions cited by the authors. Notwithstanding, the beams were modelled with the boundary conditions described by the authors and with a mesh density consistent with four elements through the depth. Analyses were run on all four members with and without residual stresses. The test load, the predicted load and the test-to-predicted ratio for each of the tests with residual stresses is given in Table 4.5.

**Table 4.5: Test-to-Predicted Load Ratios,
Kitipornchai and Trahair (1975)**

Specimen Designation	Test Load (kN)	Predicted Load (kN)	t/p
S1-20	56.96	52.40	1.111
S2-10	185.12	177.71	1.042
S3-12	145.07	133.46	1.111
S4-8	234.96	225.56	1.063

Specimen S3-12 with Typical Mesh Density

The mean test-to-predicted ratio and coefficient of variation are 1.082 and 0.032, respectively. Note, the predicted loads and corresponding test-to-predicted load ratios with and without residual stresses were essentially the same. This confirms the conclusion by Kitipornchai and Trahair (1975) that the residual stress patterns for these beams had a negligible effect on their inelastic lateral-torsional buckling capacity. The mean test-to-predicted ratio is similar to that for the Essa and Kennedy (1993) test series, reinforcing previous concluding statements. This test series is less sensitive to set up conditions (alignment and undetectable restraint) as compared to the previous, which may account for the smaller coefficient of variation.

4.5.3 Van Kuren and Galambos (1964)

Van Kuren and Galambos (1964) present the results of 42 tests on beam columns subject to varying restraint, loading and boundary conditions, of which 17 had failures which were classified as lateral-torsional buckling (possibly flexural torsional buckling).

These beam columns were discretised with a mesh density consistent with four elements through the depth. The following data (assumed or given as required) was used for the analyses:

- Imperfections were not presented and were modelled using a small transverse load.
- Nominal values were used for cross sectional dimensions.
- Van Kuren and Galambos (1964) stated that the maximum average residual compressive stress was equal to $0.3F_y$. The residual stress distribution was assumed to be linear as per the description attributed to the **R_BeCol** element object.
- With the exception of out-of-plane rotational restraint, the cited boundary conditions were used. Note, the ends of the loading apparatus were not fixed against out-of-plane rotations but had some flexibility which was calculated to give an effective length factor of approximately 0.6 out-of-plane (Van Kuren and Galambos, 1964). Idealised boundary conditions associated with the model provide complete rotational fixity, which is equivalent to an effective out-of-plane length factor of 0.5.
- Cited material properties and loading were used.

The loading sequence, for both the experiment and the numerical analyses, consisted of the application of a constant axial load or moment followed by the corresponding monotonically increasing axial load or moment causing failure. The test specimen designations, specific loading conditions (as illustrated in Figure 4.10), test loads, test-to-predicted ratios are given in Table 4.6:

Table 4.6: Test-to-Predicted Ratios, Van Kuren and Galambos (1964)

Test Specimen Designation	Loading Condition	Test Loads		Predicted Loads		t/p
		Axial (kN)	Moment (kN.m)	Axial (kN)	Moment (kN.m)	
T-1	d	211.82	143	211.82	128.7	1.111
T-6	b	178	16.046	178	17.330	0.926
T-7	b	178	18.193	178	20.740	0.877
T-8	c	956.75	21.47	1148.1	21.47	0.833
T-11	e	1412.875	0	1356.36	0	1.042
T-12	c	199.805	106	199.805	114.48	0.926
T-15	e	1379.5	0	1627.81	0	0.847
T-16	c	199.805	104	199.805	122.72	0.847
T-18	e	1468.5	0	1321.65	0	1.111
T-19	c	199.805	109	199.805	126.44	0.862
T-20	c	81.88	22.487	81.88	25.635	0.877
T-22	b	418.3	11.074	468.496	11.074	0.893
T-23	d	81.88	27.798	81.88	27.798	1.000
T-29	a	81.88	31.414	81.88	30.157	1.042
T-30	a	81.88	27.346	81.88	28.440	0.962
T-31	d	81.88	23.504	81.88	27.735	0.847
T-32	c	81.88	18.08	81.88	19.165	0.943

Typical Mesh Density Illustrated on a 8WF31 Section

Note: Highlighted values are predicted values.

Figure 4.10: Loading and Boundary Conditions Applied to Beam Columns, after Van Kuren and Galambos (1964)

The highlighted values are the predicted values corresponding to the constant axial load or moments that were initially applied. The mean test-to-predicted ratio and coefficient of variation for this test series are 0.938 and 0.099, respectively. The mean test-to-predicted ratio less than 1.0 is mainly influenced by the differences in the out-of-plane rotational fixity between the test and that assumed in the model. Obviously, the other assumptions may also contribute to the differences in predicted values. Notwithstanding, the complex failure modes associated with this wide range of boundary conditions, the failure loads are well predicted by this numerical method and elements.

Chapter 5

Summary, Conclusions and Recommendations for Future Work

5.1 Summary and Conclusions

The development of an *Integrated System for Steel Structures* (simply because of the magnitude and scope) must be done in a modular approach. An object oriented system architecture that separates graphical tasks from numerical computations has been proposed because it will generate a platform independent program (using Java) that is readily updateable, managed and integrated. Trends in computing languages and technology development support the decision to use this system architecture and programming language. Two prototype developments illustrate the functionality and versatility (ease in which additional information can be added or changes made) of the system.

The first prototype development was an Object Oriented Handbook (OOH), which consisted of the Object Oriented Standard, Dynamic Databases, Geometric Tools and the Extended Library. The development of the Object Oriented Standard forms the core of all design programs within the *Integrated System* as it provides access to the resistances, rules and guidelines specified within the material Standard, CAN/CSA-S16.1 "*Limit States Design of Steel Structures*" and other Standards that are referred to. The structure of the clauses within the Object Oriented Standard has

been standardised with common methods and common state variables. Clause specific methods have also been implemented. The Clause objects that have been developed for this prototype include all clauses related to the stability and strength of beams and beam columns and in particular are Clauses 10, 11, 12, 13 and 15 of CAN/CSA-S16.1-94 and Clauses 5.6, 6.2 and 6.4 of CAN/CSA-S136-94. Appendices D and F of CAN/CSA-S16.1-94 are included as extended libraries in the prototype OOH.

To facilitate the use of the Clause objects without knowledge of the functionality within the object, dynamic database objects (**SectionProperties**, **MemberProperties**, **LoadingConditions**, **ConnectionProperties**, **RestraintConditions**) have been developed. The Clause objects have the logic to manipulate these to extract the appropriate stored data or information. Often this data consists of geometric properties and as a result a Geometric Tools Library was developed to determine cross sectional properties (monosymmetric bending coefficient, shear centre location, warping torsional constant) for standard and built-up sections.

The resistances given within CAN/CSA-S16.1-94 are limited in their application to stability problems. The prototype architecture for the Object Oriented Handbook has been developed to allow for the addition of a series of Extended Libraries which contain the additional resistance formulations for specific stability problems, referred to by the Standard, in the *SSRC Guide to Stability Design Criteria for Metal Structures* (Galambos, 1988). These methods are detailed in Chapter 3 and have yet to be included.

These Extended Libraries could contain a large number of suggested resistance equations to account for every conceivable permutation and combination of loading, restraint and boundary conditions associated with steel structures. These would require a complicated logical algorithm to search for and to apply the appropriate Extended Libraries. These resistance equations were generally derived from experimental and analytical data. Realising that these are in part based on numerical analyses and considering that Clause 1.4 of CAN/CSA-S16.1-94 allows for the use of numerical analyses in the design of steel structures, it made sense to directly implement a calibrated, specifically designed numerical methods program for stability problems as an additional or alternate Extended Library.

The second prototype development was an object oriented numerical methods program. A properly calibrated and robust numerical methods program would be constructed with tools to model the structure appropriately (mesh density, loading, in-plane restraint and boundary conditions) to prevent misapplication and to ease approvals by regulatory bodies. As previously demonstrated (He, 1993 and Srna, 1995), graphical input and output routines can be developed to be simple and intuitive making the modelling of complex stability problems and analysis of the data easy.

The methods for the object oriented program have been standardised and are based on the algorithm developed by Albert (1998a). Two types of elements (4 FEA ELEMENT objects) were developed; three 2 node beam column elements (R_BeCol, HR_BeCol, I_BeCol) and a triangular shell element (TShell1). Supplemental objects (Matrix, Node, Material, MatrixOperations, NodalLoad,

Restraint) were developed for the manipulation, transfer and extraction of data within the program.

The initial calibrations of the program and appropriate modelling techniques were conducted by a comparison against experimental data obtained by Essa and Kennedy (1993), Kitipornchai and Trahair(1975), and Van Kuren and Galambos (1964). A summary of the mean test-to-predicted ratio (t/p) and the coefficient of variation (COV) for each are listed in Table 5.1.

Table 5.1: Calibration Summary

Researchers	Test Objectives	t/p	COV
Essa and Kennedy (1993)	Determination of the lateral-torsional buckling capacity and strength of single propped cantilevers subject to a variety of load, restraint and boundary conditions.	1.124	0.143
Kitipornchai and Trahair (1975)	Determination of the lateral-torsional buckling capacity of simply supported beams of varying lengths subject to a concentrated midspan load.	1.082	0.032
Van Kuren and Galambos (1964)	Determination of the stability and strength capacities associated with beam columns of varying lengths subject to various loading, restraint and boundary conditions.	0.938	0.099

The higher test-to-predicted ratios obtained from the first two tests is likely associated with the relatively flexible out-of-plane stability associated with the triangular shell element and undetectable restraint (physical vs. idealised analytical restraint). The lower test-to-predicted ratios associated with the last test is directly attributed to the idealised out-of-plane fixity attributed to the members ends in the analytical model (greater than that present in the test). The scatter of the test-to-predicted ratios is associated with the sensitivity of the specimen's ultimate load to initial alignment conditions and variations in restraint.

5.2 Recommendations and Future Work

5.2.1 Continued Development of an Object Oriented Integrated System for Steel Structures

The development work required to complete the *Integrated System for Steel Structures* is significant and requires the expertise of both structural engineers and of software developers. The remaining steps include:

- Assimilation of the logic from one of the past modules into a **DESIGN ELEMENT** object to demonstrate the usefulness and functionality of the OOH.
- Completion of the development of the OOH, including programming of the remaining clauses of CAN/CSA-S16.1-94 and developing the appropriate Extended Libraries associated with the resistances for the sort of stability problems described in Chapter 3.
- Development of the **FRAME** object and validation of its interaction with the Structural Analysis module and the Member Design module.
- Development of the numerical methods program:
 - Expand the number of elements available for use
 - Develop a specific graphical user interface
 - Incorporate semi-rigid connection responses for frame analysis
 - Incorporate typical non-linear incremental-iterative schemes given in Appendix A.5.
- Development of the Graphical User Interface (GUI) that instantiates the **BUILDING STRUCTURE** object and controls the application.
- Assimilation of the logic from all previously developed modules.

- Development of the outstanding modules to complete the system.

5.2.2 Publishing Documents

The *Handbook of Steel Construction* (CISC, 1997) should be electronically produced in an XML (eXtensible Markup Language) format. This would allow publishers to "write once and publish everywhere" as the document is tagged in a manner so that it can be reproduced for different media (Bosak and Bray, 1999) such as the Object Oriented Standard, *The Hyper-Handbook* (Frattaroli, 1999) and as a printed document.

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Glossary

The following computing terminology presented is consistent with definitions provided by Horstmann and Cornell (1997) and Lewis and Loftus (1998).

Term	Definition
Abstract	A Java reserved word that modifies classes, interfaces, and methods. An abstract class cannot be instantiated however it forces classes which extend this class to implement the methods and variables.
Access	The ability to reference a variable or invoke a method from outside of from other classes, objects or applications.
Application	<ul style="list-style-type: none">• A general term describing any program.• A standalone Java program (run without the use of the web browser)
Applet	A small Java program that is linked into an HTML document, then retrieved and executed using a web browser for the graphical user interface
Child	The extension of a class located at a higher level on the hierarchical structure.
Class	The framework of an object defining methods and variables to be implemented.
Class hierarchy	The hierarchical structure defining the location of a class with respect to parent, sibling, and child classes.
Class library	A set of classes which implement functional services for a programmer.
Cloning	Creating an instance of an object with the exact state of another so that future reference or change to the cloned object will not affect the other object and it's references.
Constructor	A special method in a class that is invoked when an object is initialised (instantiated) from the class.

Term	Definition
Encapsulation	The characteristic of an object that limits access to the variables and methods as all interactions must occur through a well defined interface. Permits "ignorance" of the user to the inner workings of the object.
Extends	A Java modifier used to specify the parent class from which a child class is derived.
Hierarchy	A programming technique allowing functionality to be pushed up to parent classes thus affecting all child classes.
HTML	An acronym for Hyper Text Markup Language. Current application of HTML is the presentation electronic documents via a web browser.
Inheritance	The derivation of a new class from an existing one thus inheriting all variables and methods.
Instance	The creation of one object from a class is said to be an instance of that object.
Instantiation	Creating an object from a class.
Method	A group programming statements that can be invoked when accessed by a call from another method, either within the same class or not.
Object	An instance of a class.
Parent	The class from which another class is derived.
Private	Java reserved word used to modify methods and state variables so that may only be accessed by the methods of the class to which it belongs.
Public	Java reserved word used to modify methods and state variables so that may be accessed by the methods of any class, object or application.
SGML	An acronym for Standard Generalised Markup Language. The markup language from which both HTML and XML were derived.
Sibling	Two classes within the hierarchy that have the same parent are said to be siblings.

Term	Definition
State	The values of the variables within an objects at any given time is said to be the state of the object.
Static	Java reserved word used to modify a class and a method such that the methods may be accessed without instantiating the class.
Sub	See 'Child'
Super	See 'Parent'
System Architecture	A 'map' of the organisational structure of the respective developments
XML	An acronym for Extensible Markup Language, a new markup language. Has increased application over HTML in that the language can be customized for each specific job and is starting to be adopted for publishing and document engineering applications.

The following are concise definitions of the objects which were presented in bold text throughout the thesis.

Object	Brief Description
ANALYSIS ELEMENT	An element that is instantiated in the Structural Analysis module and used in the numerical analysis routine.
BUILDING STRUCTURE	The highest level object that is defined by the user through the Graphical User Interface. Acts as a controlling object which is constantly updated as a result of interaction with the lower level modules.
ConnectionProperties	Dynamic database object used for the transfer of information concerning connection properties to the clause objects.
DESIGN ELEMENT	An object that could represent a beam, a beam-column or a connection. Instantiated in the Member Design modules and uses the Object Oriented Handbook to perform a design check. Has the logic necessary to interact with the FRAME object.

Object	Brief Description
FEA ELEMENT	Objects used to represent finite element types and likely to inherit from ANALYSIS ELEMENT so that methods and state variables will be standardised.
FRAME	An object contained by a BUILDING STRUCTURE object. Used for the interaction with the Structural Analysis module and the Member Design module.
HR_BeCol	The FEA ELEMENT that describes the square hollow structural section element.
I_BeCol	The FEA ELEMENT that describes the I-Shaped section element developed by.
LOADING ELEMENT	Instantiated within the Load Definition module defining a load type applied to the surfaces of the BUILDING STRUCTURE .
LoadingConditions	Dynamic database object used for the transfer of information concerning loading conditions to the clause objects.
Material	Supplemental object that defines material properties referred to by FEA ELEMENT objects.
Matrix	Supplemental object developed for usage within the numerical analysis algorithm.
MATRIX ELEMENT	Likely to inherit methods and state from ANALYSIS ELEMENT . Would be used in a structural analysis to such as a plane frame analysis program.
MatrixOperations	Supplemental object containing the static methods necessary for the manipulation of the Matrix objects used within the numerical analysis routine.
MemberProperties	Dynamic database object used for the transfer of information concerning member properties to the clause objects.
Node	Supplemental object developed as a representation of a node used within the finite element analysis program.

Object	Brief Description
NodalLoad	Supplemental object used to store the individual nodal loading information within the Structural Analysis module
R_BeCol	The FEA ELEMENT that describes the solid rectangular element.
Restraint	Supplemental object used to store the individual nodal restraint information along with the computation of restraint information
RestraintConditions	Dynamic database object used for the transfer of information concerning restraint conditions to the clause objects.
SectionProperties	Dynamic database object used for the transfer of information concerning cross section and geometric properties to the clause objects.
TShell1	The FEA ELEMENT that describes the triangular shell element developed by Lee and Yoo (1988) as implemented in the developed numerical methods program.

Appendix A

Numerical Methods - Miscellaneous Definitions and Descriptions

A.1 Von Mises Failure Criterion

The elements currently implemented into the numerical methods program define yield based on the two dimensional von Mises elliptical yield surface, described by Eq. (A.1.1):

$$F = \sqrt{\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2} - F_y \quad (\text{A.1.1})$$

where, σ_x = normal stress in the x direction
 σ_y = normal stress in the y direction
 σ_{xy} = shear stress in the xy plane

Note, failure is predicted when the calculated value of F is positive.

A.2 Isotropic Strain Hardening and the Associated Flow Rule

The material model implemented in the numerical methods program, illustrated in Figure 4.2, assumes that a small non-zero value, E_{st} , is assigned to the post yield stiffness to avoid numeric instability. The inclusion of this non-zero value

leads to a uniform expansion of the yield surface, isotropic strain hardening, illustrated in Figure A.1.

Figure A.1: Isotropic Strain Hardening of the von Mises Yield Surface, after Owen and Hinton (1980)

Using the associated flow rule (assuming that the plastic strain increment is equal to the plastic stress increment) and the normality principle (the expansion of the surface is perpendicular to the previous surface) to describe the uniform expansion, the tangent constitutive matrix, $[D_T]$, may be defined (Zienkiewicz, 1977):

$$[D_T] = [D] - \frac{[D] \left\{ \frac{\partial F}{\partial \sigma} \right\} \left\{ \frac{\partial F}{\partial \sigma} \right\}^T [D]}{E_\pi + \left\{ \frac{\partial F}{\partial \sigma} \right\}^T [D] \left\{ \frac{\partial F}{\partial \sigma} \right\}} \quad (\text{A.2.1})$$

where, $[D]$ = elastic constitutive matrix

A.3 Area Co-ordinate System

The area co-ordinate system is useful in describing displacement fields (shape functions) within a triangular area. For a simple 3 node triangular element (illustrated in Figure A.2) the area co-ordinate system relates the ratio of areas associated with each node (Stasa, 1985), given by Eq.(A.3.1a), (A.3.1b) and (A.3.1c):

Figure A.2: Three Node Triangular Element with Interior Point P, after Stasa (1985)

$$L_i = \frac{\text{area } pjk}{\text{area } ijk} \quad (\text{A.3.1a})$$

$$L_j = \frac{\text{area } pki}{\text{area } ijk} \quad (\text{A.3.1b})$$

$$L_k = \frac{\text{area } pij}{\text{area } ijk} \quad (\text{A.3.1c})$$

where, $L_i + L_j + L_k = 1$

Lines can be drawn on the element surface for which a point P could coincide giving a constant value of nodal area co-ordinate, L_i , illustrated in Figure A.3.

Figure A.3: Lines of Constant L_i , after Stasa (1985)

A chain rule has been developed (Tottenham and Brebbia, 1970) describing the partial differentiation of functions expressed in area co-ordinates, given by Eq.(A.3.2a) and (A.3.2b):

$$\frac{\partial}{\partial x} = \left(\frac{1}{2A} \right) \sum_{i=1}^3 \left[b_i \frac{\partial f}{\partial L_i} \right] \quad (\text{A.3.2a})$$

$$\frac{\partial}{\partial y} = \left(\frac{1}{2A} \right) \sum_{i=1}^3 \left[c_i \frac{\partial f}{\partial L_i} \right] \quad (\text{A.3.2b})$$

where, A = area of the triangle

$$b_i = y_j - y_l$$

$$c_i = x_l - x_j$$

where, $i = 1,2,3$

$$j = 2,3,1$$

$$l = 3,1,2$$

A.4 Gauss Integration Points and Weights

Table A.1 presents the Gaussian integration points (Zienkiewicz, 1977) and weights used for various calculations within the numerical methods program.

Table A.1: Gaussian Integration Points and Weights

Four point integration coefficients
	L_1	L_2	L_3	H_k	
a	1/3	1/3	1/3	-27/48	
b	0.6	0.2	0.2	25/48	
c	0.2	0.6	0.2	25/48	
d	0.2	0.2	0.6	25/48	
Three point integration coefficients
a	0.5	0.5	0	1/3	
b	0	0.5	0.5	1/3	
c	0.5	0	0.5	1/3	
One point integration coefficients
a	1/3	1/3	1/3	1	

A.5 Incremental-Iterative Strategies to Trace Non-Linear Responses

Prediction of the non-linear response of a structure can be performed using incremental-iterative strategies that use convergence criteria to dictate whether the structure has satisfied equilibrium at each load step. Since numerous strategies are available only a select few are presented below.

Convergence Criteria

All strategies are based on convergence criteria that provide a close approximation of equilibrium based on a user specified tolerance. Specified tolerances that are too strict could result in a predicted divergence for a case where the system has actually converged, however loose tolerances can result in erroneous conclusions. Since the true solution is rarely known, convergence is normally based on the result of the previous iteration. Convergence must be evaluated for each element within the system and can be based on the out-of-balance displacements, out-of-balance load vector or the out-of-balance work increment.

Bathe (1982) suggests that convergence based on displacement can be misleading and generally not applicable to elastic-plastic analysis because the convergence may be predicted when in fact displacements may continue to change for several iterations. Zienkiewicz (1977) suggests that convergence should be based on the equilibrium of the internally and externally applied forces. Bathe (1982) favours convergence based on the work done by the out-of-balance load vector as it travels through the incremental displacements, as it provides an indication of equilibrium for both the displacements and the load vector.

The out-of-balance forces can be described by the following equation (Zienkiewicz, 1977):

$$\{\psi\} = \int_V [\bar{B}]^T \{\sigma\} dV - \{f\} \quad (\text{A.5.1})$$

where, $[\bar{B}] = [B_o] + [B_L]$

$[B_o]$ = strain displacement matrix used in linear infinitesimal strain analysis

$[B_L]$ = strain displacement matrix where the terms vary with displacements. General formulation presented in Zienkiewicz (1977).

$\{f\}$ = externally applied force vectors

Equilibrium is satisfied when the out-of-balance forces, $\{\psi\}$, are equal to zero.

Solution Strategies

The Newton-Raphson or the modified Newton-Raphson methods are commonly used to trace non-linear response (Clarke and Hancock, 1990). The Newton-Raphson method re-solves the stiffness of the system after each iteration, within a specified load increment, until convergence is achieved (illustrated in Figure A.4).

Figure A.4: Newton-Raphson Method as Applied to the Load Control Method

The modified Newton-Raphson method uses a constant stiffness, based on the converged condition of the previous load increment, for each iteration within the current load increment (illustrated in Figure A.5).

Figure A.5: Modified Newton-Raphson Method as Applied to the Load Control Method

Both the full Newton-Raphson method and the Modified Newton-Raphson method will achieve convergence within a load step, however the modified method requires

more iterations within a given load step, whereas the full method requires the assembly and reduction of the matrix at each iteration. The modified version is generally accepted as the preferred option, as it is less computationally intensive. Both methods will fail due to singularities in the stiffness matrix at limit points when the load increments are kept constant.

A method that remains stable beyond these limit points is termed the "constant arc length method". The method was first presented by Riks (1979) where both load and displacement were to be taken as variables in the iterative algorithm. Riks (1979) proposed that in every load step, an initial incremental load and incremental displacement be specified defining a plane in n -dimensional space. Then a plane that is both normal to the stiffness plane and tangent to the equilibrium path is specified as the incrementing limit. This concept is illustrated for a one degree of freedom system in Figure A.6:

Figure A.6: Riks' Method as Applied to a One Degree of Freedom System

Crisfield (1981) determined that the Riks' Method had difficulties traversing paths with sharp changes in the equilibrium path and proposed using a n -dimensional

'sphere' rather than an n-dimensional plane as the incrementing limit (illustrated in Figure A.7).

Figure A.7: Constant Arc Length Method as Applied to a One Degree of Freedom System

The constant arc length method is optimal in describing the whole structural response and as such, this method is formulated below, Eq.(A.5.2) to (A.5.9), using the modified Newton-Raphson methodology (Clarke and Hancock, 1990). Figure A.8 illustrates a typical load step (i) with the typical convergence iterations within. The initial incremental displacements, $\{\Delta a\}_i^1$, within the load step (i) are solved based on the tangent stiffness, $[K_T]_i$, calculated using the converged displacements, a_{i-1} , from the previous load step, as given in Eq.(A.5.2).

$$[K_T]_i \{\Delta a\}_i^1 = \Delta \lambda_i^1 \{f_o\} \quad (A.5.2)$$

where, $[K_T]_i$ = the tangent stiffness based on the converged state of the load step (i-1) to be used throughout load step (i)

$\{\Delta a\}_i^1$ = the incremental displacements calculated in the first iteration within the (i) load step

$\Delta \lambda_i^1$ = the incremental load factor initially applied to the first iteration within the increment

$\{f_o\}$ = load to be applied incrementally

Figure A.8: Constant Arc Length Method and Variables as Applied to a Single Degree of Freedom System, after Clarke and Hancock (1990)

For subsequent iterations (j), the cumulative displacement and load factor within load step (i) are expressed by Eq. (A.5.3) and (A.5.4), respectively.

$$\{a\}_i^j = \{a\}_i^{j-1} + \{\Delta a\}_i^j \quad (\text{A.5.3})$$

$$\lambda_i^j = \lambda_i^{j-1} + \Delta \lambda_i^j \quad (\text{A.5.4})$$

The solution of Eq.(A.5.5) for the incremental displacements for each iteration (j) within each load step (i) can be manipulated and expressed as Eq.(A.5.6).

$$[K_T]_i \{\Delta a\}_i^j = \Delta \lambda_i^j \{f_o\} - \{\psi\}_i^{j-1} \quad (\text{A.5.5})$$

$$\{\Delta a\}_i^j = \Delta \lambda_i^j \{\Delta a\}_i^1 + \{\delta_R\}_i^j \quad (\text{A.5.6})$$

where, $\{\delta_R\}_i^j$ = residual displacements that can be defined by Eq (A.5.7)

$$[K_T]_i \{\delta_R\}_i^j = -\{\psi\}_i^{j-1} \quad (\text{A.5.7})$$

The constant arc length method then defines a n-dimensional 'sphere' with radius, l_i , to be used as the iterative path and defined by:

$$(l_i)^2 = \{a\}_i^{jT} \{a\}_i^j \quad (\text{A.5.8})$$

The load factor, $\Delta \lambda_i^j$, to be applied in the next iteration is determined by solving the following quadratic:

$$0 = A(\Delta \lambda_i^j)^2 + B\Delta \lambda_i^j + C \quad (\text{A.5.9})$$

$$\begin{aligned} \text{where, } A &= \{\Delta a\}_i^{1T} \{\Delta a\}_i^1 \\ B &= 2[\{a\}_i^{j-1} + \{\delta_R\}_i^j]^T \{\Delta a\}_i^1 \\ C &= [\{a\}_i^{j-1} + \{\delta_R\}_i^j]^T [\{a\}_i^{j-1} + \{\delta_R\}_i^j] - (l_i)^2 \end{aligned}$$

The root to be used is that which gives a positive 'angle' between the incremental displacement before the iteration and the incremental displacement after the iteration. If both give positive use the one that approximates the linear solution best. If an imaginary root is obtained then the load increment is too large to capture the multiple instability points within (Clarke and Hancock, 1990).