INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

**ProQuest Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600**

UMI[®]

Modeling the Effects of Particle Clustering on Ductile Failure

by

Cheryl I.A. Thomson

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Doctor of Philosophy

Ottawa-Carleton Institute for
Mechanical and Aerospace Engineering

Department of
Mechanical and Aerospace Engineering
Carleton University
Ottawa, Ontario
May 14, 2001

© Copyright
2001 - Cheryl I.A. Thomson

**National Library
of Canada**

**Acquisitions and
Bibliographic Services**

**395 Wellington Street
Ottawa ON K1A 0N4
Canada**

**Bibliothèque nationale
du Canada**

**Acquisitions et
services bibliographiques**

**395, rue Wellington
Ottawa ON K1A 0N4
Canada**

Your file Votre référence

Our file Notre référence

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-60970-7

Canada

**The undersigned hereby recommend to
the Faculty of Graduate Studies and Research
acceptance of the thesis,**

Modeling the Effects of Particle Clustering on Ductile Failure

**submitted by
Cheryl Ilona Alicia Thomson, B.A.Sc.**

**in partial fulfilment of
the requirements for the degree of
Doctor of Philosophy**

**Chair, Department of Mechanical and Aerospace Engineering
and Thesis Co-supervisor**

Thesis Co-supervisor

Thesis Co-supervisor

External Examiner

**Carleton University
March 30, 2001**

Abstract

The effect of particle clustering on the nucleation, growth and coalescence of voids is studied through modeling of discrete spherical particles in a cubic, three dimensional unit cell using finite element analysis. Void nucleation at discrete particles develops at a traction-separation interface and is stress-controlled. Voids formed at the discrete particle-matrix interface coalesce through a void sheeting mechanism where matrix damage originating at a secondary population of small void-nucleating particles is modeled using the Gurson-Tvergaard-Needleman (GTN) constitutive equations with strain-controlled nucleation. The effect of various model parameters on the nucleation, growth and coalescence behaviour of the unit cells are studied including cluster type, cluster orientation, cluster density, strain state, stress triaxiality, particle matrix interface characteristics, and GTN nucleation and coalescence parameters. The study represented by this thesis takes a unique approach to modeling particle clusters in a unit cell and accounts for all stages of ductile damage development. The model also incorporates a comparative length scale in the continuum analysis characterized by the ratio of distance between particles within a cluster to the distance between neighboring clusters.

This thesis demonstrates that a cluster of particles cannot be equated to a single

particle with similar dimensions to the cluster but that stresses and strains within inter-particle ligaments play an important role in damage progression. The orientation of an anisotropic cluster is also an important factor in void nucleation and growth with clusters elongated in the direction of major principal strain developing lower damage rates than particle clusters elongated transverse to the major principal strain. In a dual population of particles, the parameters in the GTN constitutive equations which have the greatest effect on strain-to-failure are the volume fraction of void nucleating secondary particles, f_N , and the critical void volume fraction for void coalescence, f_c . It is also shown that coalescence of voids within a particle cluster is a stable event whereas coalescence of voids from one cluster with those of a neighboring cluster leads to imminent material failure.

Acknowledgements

First of all, I would like to thank my supervisor Mike Worswick for his guidance and patience throughout my graduate studies.

I also appreciate the conversations I had with my second supervisor, Keith Pilkey, which offered good advice and insights into the material behaviour.

I am grateful for financial support provided by the Natural Science and Engineering Research Council (NSERC), ALCAN International through the Kingston Research and Development Centre (KRDC), the Carleton University Department of Mechanical and Aerospace Engineering and the Zonta Foundation through an Amelia Earhart scholarship.

To Sandy,
whose love and support is at the core of everything I accomplish

and to my parents,
whose example and encouragement is the foundation of my goals

Contents

Table of Contents vii

List of Tables xii

List of Figures xiii

List of Symbols xxi

1 Introduction 1

 1.0.1 Length Scale Considerations in Continuum Mechanics 4

 1.1 Void Nucleation 5

 1.1.1 Experimental Measures of Interfacial Shear Strength 12

 1.2 Void Growth 13

 1.2.1 Gurson’s Constitutive Equation 13

 1.2.2 Modifications of Gurson’s Constitutive Equation 14

 1.2.3 Unit Cell Models 17

 1.2.4 Random Void Distribution 22

 1.2.5 Experimental Studies of Void Interaction 23

 1.3 Void Coalescence 25

1.3.1	Critical Void Size Models	26
1.3.2	Applications of Gurson's Constitutive Model	27
1.3.3	Plastic Instability	30
1.4	Current Research	39
1.4.1	Thesis Overview	40
2	Model Description	42
2.1	Unit Cell Geometries	44
2.1.1	Particle Cluster Types	44
2.1.2	Particle and Unit Cell Dimensions	45
2.1.3	Cluster Density	46
2.2	Finite Element Analysis	46
2.2.1	Finite Element Solver	46
2.2.2	Meshing	47
2.2.3	Material Properties	47
2.2.4	Boundary Conditions	50
2.3	Load Cases	51
2.3.1	Aligned vs. Transverse Loading	51
2.3.2	High Hydrostatic Stress	53
2.3.3	Sheet Forming Biaxial Strain States	54
2.3.4	Control of Inertial Effects	56
2.4	Nucleation Model	57
2.5	Void Growth Calculations	59
2.6	Coalescence Model	59

2.7	Summary	62
3	Model Validation	63
3.1	Mesh Discretization Studies	63
3.2	Void Growth Rates	67
3.3	Rigid Particles	70
3.4	Density Reduction	72
3.5	PID Controller	74
4	Void Growth in the Absence of Particles	76
4.1	Cluster Type	77
4.1.1	Uniaxial tension	78
4.1.2	Plane Strain	86
4.1.3	Equi-biaxial Stretch ($\Sigma_{11} = \Sigma_{22}$)	91
4.1.4	Deep Draw ($\epsilon_{22}^{\infty} = -\epsilon_{11}^{\infty}$)	93
4.1.5	High Triaxiality	97
4.2	Cluster Density	100
4.3	Aspect Ratio	103
4.4	Summary	105
5	Void Nucleation and Growth in the Presence of Rigid Particles	108
5.1	Particle-Matrix Interfacial Strength	109
5.1.1	Magnitude of Normal Traction, σ_{max}	111
5.1.2	Relative Magnitude of Shear Traction, α	111
5.1.3	Interface separation distance, δ	112

5.2	Nucleation within Clusters	114
5.2.1	Uniaxial Tension	114
5.2.2	Biaxial Strain States	127
5.2.3	High Triaxiality Strain States	136
5.2.4	Cluster Density	144
5.3	Damage Rates in Clusters of Particles	147
5.3.1	Equivalent Ellipsoidal Particles	150
5.4	Summary	154
6	Void Coalescence in a Dual Particle Population	156
6.1	Coalescence in the Unit Cell Model	157
6.2	GTN Constitutive Equation Parameters	164
6.2.1	Primary Void Nucleation	171
6.3	Cluster Orientation	173
6.4	Stress Field Effects	177
6.5	Cluster Density	181
6.6	Summary	183
7	Discussion	185
7.1	Model Geometry	187
7.2	Meshing	188
7.3	Stress Fields	189
7.4	Other Simulation Issues	189
7.5	Alternative Coalescence Strategies	190

8 Conclusions and Recommendations 192

8.1 Conclusions 192

8.2 Recommendations for Future Work 195

References 197

List of Tables

- 1.1 Some values for f_c and f_f in the Gurson-Tvergaard constitutive equation 28
- 1.2 Some parameters used in the Gurson-Tvergaard in modeling dual
populations of voids 31
- 2.1 The dimensions for cubical unit cells with different cluster types . . . 46
- 2.2 Aluminum matrix and silicon particle material properties 49
- 2.3 Aligned and transverse boundary conditions 52
- 2.4 The ratio of primary stress to secondary stresses to maintain a re-
quired component of hydrostatic tension. 53
- 2.5 Sheet metal forming strain ratios 55
- 3.1 Meshing for the single void model discretization study 65
- 3.2 Meshing for the linear cluster model discretization study 67
- 3.3 Model parameters of benchmark studies 68
- 3.4 Aluminum matrix, rigid and elastic silicon particle material properties 71

List of Figures

- 1.1 Failure modes in a ductile round bar specimen in uniaxial tension
(Worswick, 1988) 2
- 1.2 Influence of a particle on void shape in shear stress (Fleck *et al.*, 1989). 6
- 1.3 Interfacial traction as a function of separation (Needleman, 1987). . . 10
- 1.4 A unit cell representing a periodic array of voids 17
- 1.5 (a) square and (b) rhombic periodic arrangements of voids (Kitagawa
and Honke, 1988) 18
- 1.6 The unit cells representing three different periodic void distributions
(Kuna and Sun, 1996). 19
- 1.7 An axisymmetric unit cell representing an ordered distribution of two
different sized voids (Tvergaard, 1996). 21
- 1.8 A non-periodic void distribution (Thomason, 1993) 21
- 1.9 Some of the periodic unit cells containing a random distribution of
voids used by Needleman and Kushner (1990) 21
- 1.10 Experimental schematic for simulating void interaction and ligament
thinning (Goto and Koss, 1996) 23
- 1.11 Comparison of experimental and predicted inter-void ligament thinning 24

1.12	Experimental schematic for creating artificial neighboring voids in a tensile specimen (Forero and Koss, 1994)	25
1.13	Three theories of void coalescence	25
1.14	A 2-D unit cell model with sliplines	32
1.15	The limiting stress for plastic flow as a function of geometry in 2-D .	33
1.16	A 3-D unit cell with an upper bound plastic flow field	34
1.17	2-D non-orthogonal boundary conditions	39
2.1	Regular periodic array of linear clusters or “stringers”	43
2.2	A unit cell representing a periodic array of linear clusters	43
2.3	One-eighth unit cell representations of different clusters	45
2.4	Typical finite element discretizations.	48
2.5	The uniaxial stress-strain curve.	49
2.6	Stress and strain referencing with the unit cell.	52
2.7	A flow chart for the operation of the PID controller	54
2.8	A forming limit diagram	55
2.9	Comparison of damage at two different strain rates	56
2.10	The traction-separation nucleation curve	58
3.1	A single void model with 135 elements	64
3.2	A single void model with 1764 elements	64
3.3	Results of the single void model discretization study	65
3.4	Results of the linear cluster model discretization study	66
3.5	The linear cluster model in 373 elements	67
3.6	The linear cluster model in 2117 elements	68

3.7	Comparison of damage rates with benchmark studies	69
3.8	Comparison of stress and strain data with benchmark data	70
3.9	Comparison of damage rates in models with rigid vs. elastic particles	71
3.10	Comparison of void shape in models with rigid vs. elastic particles . .	72
3.11	Validity of material density reduction with stress-strain curves	73
3.12	Validity of material density reduction with void growth rates	74
3.13	The effectiveness of the PID controller in following a prescribed displacement.	75
3.14	The effectiveness of the PID controller in controlling minor principal stresses	75
3.15	The effectiveness of the PID controller in controlling stress ratios . . .	75
4.1	Void growth rates within various cluster types under uniaxial tension	78
4.2	Contours of plastic strain in a single void model	79
4.3	Contours of plastic strain in an aligned linear cluster model	79
4.4	Contours of plastic strain in a transverse linear cluster model	80
4.5	Contours of plastic strain in an aligned planar cluster model	80
4.6	Contours of plastic strain in a transverse planar cluster model	81
4.7	Contours of plastic strain in a spherical cluster model	81
4.8	Void shape evolution in aligned uniaxial tension	82
4.9	Void shape evolution in transverse uniaxial tension	85
4.10	Void growth rates within various clusters under plane strain deformation	86
4.11	Void shape evolution in aligned uniaxial tension	87
4.12	Void shape evolution in transverse plane strain	90

4.13 Void growth rates within various clusters under equi-biaxial stretch deformation	91
4.14 Void shape evolution in an equi-biaxial stretch	92
4.15 Void growth rates within various clusters under deep draw deformation	93
4.16 Void shape evolution in an aligned deep draw	94
4.17 Contours of plastic strain in a spherical cluster model deformed in an aligned deep draw	95
4.18 Void shape evolution in a transverse deep draw	96
4.19 Void growth rates within various cluster types under high hydrostatic stress	97
4.20 Void shape evolution at moderate levels of triaxiality	98
4.21 Contours of plastic strain in a spherical cluster model deformed at moderate triaxiality	99
4.22 Void growth rates within various clusters under high hydrostatic stress	100
4.23 Contours of plastic strain in a spherical cluster model deformed at high triaxiality	101
4.24 Effect of density on void growth	102
4.25 Element mesh of a linear ellipsoidal void	104
4.26 Element mesh of a planar ellipsoidal void	104
4.27 Void growth rates of ellipsoidal voids	105
4.28 Void shape change for ellipsoidal voids in aligned uniaxial tension . .	106
4.29 Void shape change for ellipsoidal voids in transverse uniaxial tension .	106
5.1 Damage rates in a single particle model deformed in plane strain . . .	110

5.2	Void shape evolution with and without a particle	111
5.3	Effect of shear to normal traction ratio	112
5.4	Effect of shear to normal traction ratio at high χ	113
5.5	Void growth in a linear cluster of particles deformed in uniaxial tension	115
5.6	Nucleation progression of an aligned linear cluster in uniaxial tension	116
5.7	Mesh plots of nucleation progression in an aligned linear cluster . . .	118
5.8	Void shape evolution of an aligned linear cluster in uniaxial tension .	118
5.9	Mesh plot of nucleation in a transverse linear cluster	119
5.10	Void shape evolution of a transverse linear cluster in uniaxial tension	119
5.11	Void growth in a planar cluster of particles deformed in uniaxial tension	120
5.12	Nucleation progression of an aligned planar cluster in uniaxial tension	121
5.13	Void shape evolution of an aligned planar cluster in uniaxial tension .	122
5.14	Void shape evolution of a transverse planar cluster in uniaxial tension	123
5.15	Void growth in a spherical cluster of particles deformed in uniaxial tension	123
5.16	Nucleation progression of a spherical cluster in uniaxial tension	125
5.17	Mesh plot of nucleation in a spherical cluster	126
5.18	Void shape evolution of a spherical cluster in uniaxial tension	127
5.19	Nucleation progression in an aligned linear cluster	128
5.20	Nucleation progression in a transverse linear cluster	130
5.21	Nucleation progression in an aligned planar cluster	132
5.22	Nucleation progression in a transverse planar cluster	133
5.23	Nucleation progression in a spherical cluster	135
5.24	Nucleation progression in an aligned linear cluster at various triaxialities	137

5.25	Nucleation progression in a transverse linear cluster at various triaxialities	138
5.26	Nucleation progression in an aligned planar cluster at various triaxialities	140
5.27	Nucleation progression in a transverse planar cluster at various triaxialities	141
5.28	Nucleation progression in a spherical cluster at various triaxialities . .	143
5.29	Nucleation progression in linear clusters at various cluster densities .	145
5.30	Nucleation progression in planar clusters at various cluster densities .	146
5.31	Nucleation progression in spherical clusters at various cluster densities	148
5.32	Damage rates of different particle clusters in uniaxial tension	149
5.33	Damage rates of different particle clusters in plane strain	150
5.34	Damage rates of different particle clusters in equi-biaxial stretch . . .	151
5.35	Damage rates of different particle clusters in deep drawing	151
5.36	Damage rates of linear and planar ellipsoidal particles	152
5.37	Mesh plots of an aligned linear cluster and prolate ellipsoid	153
5.38	Mesh plots of a transverse linear cluster and prolate ellipsoid	153
6.1	Void nucleation at primary particles by element deletion	159
6.2	Inter-particle ligament collapse due to element deletion	160
6.3	Inter-cluster stress field interaction	160
6.4	Inter-cluster coalescence.	161
6.5	Unit cell failure.	161
6.6	Stress strain response of a unit cell with the GTN model	163

6.7	Comparison of stress-strain curves with a variety of f_N values	165
6.8	Comparison of stress-strain curves with a variety of ϵ_N values	166
6.9	Comparison of stress-strain curves with a variety of S_N values	167
6.10	Comparison of stress-strain curves with a variety of f_c values	168
6.11	Comparison of stress-strain curves with a variety of f_f/f_c ratios . . .	169
6.12	Comparison of stress-strain curves with two orders of magnitude of f_N	170
6.13	Comparison of stress-strain curves with different σ_{max} in dual particle population material	171
6.14	Mesh plots of matrix coalescence for various bond strengths	172
6.15	Void nucleation at primary particles by element deletion in an aligned linear cluster.	173
6.16	Extensive element deletion at primary outer particle boundary in an aligned linear cluster.	174
6.17	Localization of matrix damage at the outer particle in an aligned linear cluster.	174
6.18	Complete unit cell failure at the outer particle in an aligned linear cluster.	175
6.19	Stress-strain response of an aligned unit cell with the GTN model . .	176
6.20	Comparison of stress-strain curves of linear cluster materials deformed in uniaxial tension and plane strain	178
6.21	Comparison of stress-strain curves for linear clusters deformed in dif- ferent strain fields	179
6.22	Comparison of stress-strain curves for linear clusters deformed at dif- ferent levels of hydrostatic stress, Σ_{hyd}	180

6.23 Comparison of stress-strain curves for linear clusters of different cluster densities deformed in uniaxial tension 181

List of Symbols

A, B	Weighting coefficients
A_n	The normal area
D	The inter-cluster ligament thickness
E	Young's modulus
$\Delta E_{elastic}$	The change in elastic energy
$\Delta E_{surface}$	The change in surface energy
$\dot{E}$	The rate of work of external loads
G	The shear modulus
$\dot{I}$	The rate of internal energy dissipation
K_p	The proportional gain of a PID controller
$\mathcal{R}_A$	The ratio of inter-void cross-sectional area to global cross-sectional area
S	The surface area of the rigid-plastic interface

S_N	The standard deviation of void nucleating strain
T	The total running time of the model
T_D	The differential gain of a PID controller
T_I	The integral gain of a PID controller
T_n	The normal traction at the particle-matrix interface
T_s	The shear traction at the particle-matrix interface
V	Volume
$\dot{W}$	The major principal deformation velocity
$\mathfrak{S}$	The major principal traction applied at the previous time step
$\mathfrak{S}'$	The major principal traction applied at the next time step
Λ	The number of stress waves passing through the model per unit strain
Φ	Gurson's constitutive equation variable
Ψ	A constant
Σ_{eq}	The equivalent global stress
Σ_{hyd}	The hydrostatic component of global stress
Σ_{ij}	The global stress tensor

Σ'_{ij}	The deviatoric components of global stress
α	A scaling factor to relate the normal traction amplitude to the shear traction amplitude
χ	The ratio of hydrostatic stress to equivalent stress
δ	the separation distance for permanent void nucleation
δ_{ij}	The Kronecker delta
ℓ	The unit cell dimension
ℓ_0	The initial unit cell dimension
$\dot{\epsilon}$	The rate of change of flow strain
ϵ_{eq}^∞	The equivalent global strain
ϵ_{hyd}^∞	The hydrostatic component of the global strain
ϵ^p	Local plastic strain
ϵ_f^∞	The final major principal strain
ϵ_N	The mean strain for void nucleation
ϵ_c	The critical global nucleation strain
ϵ_{ij}	Local strain tensor
ϵ_{ij}^∞	Global strain tensor

ϵ_{ij}^{poo}	The plastic component of the global strain tensor
$\bar{\epsilon}_g^p$	Global effective plastic strain
$\bar{\epsilon}_g$	Global effective total strain
γ_{ij}	The shear strain tensor
κ	A comparative microstructural length scale.
ν	Poisson's ratio
ω	The ligament thickness
ρ_d	The dislocation density
ρ	The density of aluminum
$\bar{\sigma}_1, \bar{\sigma}_2$	The hydrostatic and equivalent components of flow stress, page 16
σ_N	The mean void nucleating stress
σ_n	The incipient limit load failure stress, page 32
σ_y	The yield stress
σ_{ij}	The local stress tensor
σ_{max}	The maximum stress of the particle-matrix interface
$\bar{\sigma}$	The flow stress

a, b	Ellipsoid axes lengths
b	The Burger's vector
c	The speed of sound in aluminum
d	The inter-particle ligament thickness
$d(t)$	The displacement error as a function of time
d_{actual}	The actual major principal displacement
$d_{desired}$	The required major principal displacement
e_1, e_2	Ellipsoid eccentricities
e_i	Engineering strain
$\dot{f}_{growth}$	The component of void volume fraction growth contributed by growth of existing voids
$\dot{f}_{nucleation}$	The component of void volume fraction growth contributed by nucleation of new voids
f	The void volume fraction
f^*	Gurson's modified void volume fraction, see equation (1.23)
f_N	The volume fraction of void nucleating particles
f_c	The critical void volume fraction to initiate void coalescence
f_f	The final void volume fraction to complete void coalescence

f_o	The initial void volume fraction
f_p	The volume fraction of particles
f_u^*	A coalescence constant, see equation (1.23)
h	The equivalent plastic hardening modulus of the matrix
n	The power-law, material work hardening parameter
n_p	The number of voids or particles within an eighth of a unit cell
p	The distance between nearest neighbor voids or particles as measured from center to center
q_i	Curve fitting variables
r	Radius
r_c	Critical void radius for void coalescence, page 26
r_i	The length of the semi-axes of a particle
$\dot{s}$	Shear velocity
$\dot{u}, \dot{v}, \dot{w}$	The cartesian components of material velocity
u_i	The displacement vector
u_i^*	The displacement vector of a master node
u_n	The normal particle-matrix interface separation
u_s	The shear particle-matrix interface separation

Chapter 1

Introduction

This research stems from the requirement to reduce weight in automotive structural and skin components in order to reduce the fuel consumption of cars and small trucks. Various candidate materials have been proposed as lightweight replacements for mild steel including high strength steel, plastics and aluminum alloy sheet, the latter being the focus of the current research. The sheet formability of aluminum is generally lower than steel so a great deal of effort is required to gain a better understanding of aluminum alloy behaviour and better control of aluminum forming processes. To further complicate the issue, increased demand for recycling of automotive materials will introduce a higher volume of contaminants such as iron and silicon. In aluminum, these elements promote formation of intermetallic particles which are brittle and/or poorly bonded to the aluminum matrix and so further reduce the formability of the recycled material.

There are three types of failure in ductile materials according to Ashby *et al.* (1985). In the absence of void formation and/or coalescence, a tensile specimen can experience fully plastic failure by thinning down to a point or chisel edge (Fig-

Figure 1.1: Failure modes in a ductile round bar specimen in uniaxial tension (Worswick, 1988)

ure 1.1a). More commonly, damage in the form of voids nucleates prior to plastic failure. If void development leads to the formation of a macroscopic shear band, shear band localization and a shear fracture can result (Figure 1.1b). If no shear localization takes place, the voids will continue to grow until void coalescence occurs which leads to development of a central crack. As the crack grows close to the boundaries of the specimen, the remaining material shears due to material instability creating the cup-cone ductile fracture depicted in Figure 1.1c. The type of fracture that occurs in a particular case depends on the geometry of the part, the forming conditions, and the material microstructure.

Ductile fracture (Figure 1.1c), is a common problem during sheet aluminum forming processes such as stamping, bending and stretch flanging operations and is the focus of the current research. It should be noted that the coalescence of

voids prior to crack formation under relatively uniform strain states is different from coalescence of a void with a growing crack due to the singular stress state and high stress gradients at a crack tip. This thesis is concerned with the initial coalescence of voids under low stress gradients, typical of sheet forming conditions.

Pilkey (1997) performed experiments to observe the effect of particle clustering on ductile failure of sheet aluminum under various loading conditions. Detailed statistical measurements were made from two-dimensional micrographs of undamaged and damaged Al-Si model materials, establishing the level and degree of second phase particle clustering in these alloys and the effect of clustering on formability. In effect, the statistical data identifies various levels of microstructural length scales in the material. This experimental work observed a strong dependence of particle clustering on material response and theorized that the degree of second order clustering (or clusters of clusters) determines the failure strain of a ductile material. The current thesis is a compliment to the work of Pilkey (1997) in that effects of second phase particle clustering on the nucleation, growth and coalescence of voids is studied numerically using finite element analysis.

There exists an extensive collection of modeling research in the literature that has similar aspects to the investigations of this thesis. The following literature review will summarize the work done on this topic by others and is divided into the three stages of ductile fracture: nucleation, growth, and coalescence. These stages are not mutually exclusive, however, and will often occur simultaneously (ie. as one void is nucleating, voids in a neighboring particle cluster may be coalescing).

Material damage occurs within the plastic range of material behaviour on a microstructural scale. Some researchers such as Asaro (1983) and Haddad (1990) contend that a crystal plasticity model which deals with the statistical dislocation motion events during material plasticity would best represent the material behaviour.

In this study, particles are on the order of 1-5 μm in diameter. A particle of this size would contain approximately 25 billion silicon atoms and presumably is several orders of magnitude larger than an average dislocation cell. As a result, the particles do not so much serve to impede individual dislocation motion as to obstruct the flow of the softer matrix around the hard particle. Hence it is felt that the statistical dislocation behaviour in this case can be adequately represented by continuum mechanics. In some models a secondary population of smaller particles is assumed to exist within the matrix with diameters on the order of 10 nm which would contain approximately 25,000 silicon atoms. This secondary population of particles is possibly small enough to impede individual dislocation motion but it is not modeled explicitly. Instead, it is represented through a constitutive equation which averages out the dislocation interactions and therefore is also based on continuum mechanics (see Section 1.2.1). It follows from these arguments that the literature review should focus on continuum mechanics models.

1.0.1 Length Scale Considerations in Continuum Mechanics

In recent years, continuum mechanics modeling of deformation processes on the mesoscale (microstructural sizes ranging from a fraction of a micron to 100 μm , Needleman (2000)) have been focusing on inclusion of microstructural length scales in the continuum formulation. Traditional continuum mechanics has no associated length scale although calculations through finite element methods introduces an undesirable mesh-dependent length scale. For example, the cup-cone fracture modeling study by Tvergaard and Needleman (1984), which demonstrates changes in fracture path at the fracture surface, makes use of traditional continuum mechanics. Although the stresses and strains are not significantly altered by mesh refinement (beyond a certain point), the fracture path is highly dependent on the local mesh

size.

Inclusion of a length scale into a continuum model can take many forms. Fleck *et al.* (1994) include a description of the local dislocation density and a measure of the associated crystal lattice mismatch in the flow stress calculation based on local strain gradients. Tvergaard and Needleman (1995) introduce a material length scale related to an integral condition on the rate of increase of void volume fraction, which represents a spatial average void spacing in the material. At an even larger scale, Needleman and Tvergaard (1991) directly specified the size and spacing of larger voids in a study on ductile crack growth. This thesis makes use of a comparative length scale relating the spacing between particles in a cluster to the spacing between clusters of particles, but does not explicitly adopt a non-local formulation.

1.1 Void Nucleation

Voids usually nucleate at inclusions or second phase particles within the matrix through particle cracking or decohesion. For equiaxed particles, nucleation usually occurs through particle decohesion (Argon *et al.*, 1975; Barlat and Jalinier, 1985). This thesis concentrates on the particle decohesion mechanism of void nucleation.

The interface between a particle and the surrounding matrix has been modeled using assumptions ranging from frictionless and tractionless to an attraction-repulsion model similar to interatomic forces. Fleck *et al.* (1989) showed that the presence of a particle, even with a frictionless and tractionless interface, significantly affects the post-nucleation void growth in a shear state by propping open the void (Figure 1.2).

Keer *et al.* (1973) also considered the effects of a particle without interfacial strength on void growth, in this case under various biaxial stress fields. These

Figure 1.2: Influence of a particle on void shape in shear stress (Fleck *et al.*, 1989). (a) undeformed, (b) deformation without a particle, (c) deformation with a particle.

tractionless interface models are overly simplified but the significance of including a particle is to increase void growth by propping open the void.

In general, nucleation models are based on an energy criterion and a critical stress or critical strain criterion. In order for a void to nucleate, the elastic energy relieved must be greater than the surface energy created:

$$\Delta E_{elastic} - \Delta E_{surface} \leq 0. \quad (1.1)$$

It is generally recognized, however, that this negative energy balance is a necessary but not a sufficient condition in that the interfacial strength must also be exceeded by the local stresses (Argon *et al.*, 1975). Tanaka *et al.* (1970) have shown that for any particle larger than $250\text{\AA}$, the energy criterion is automatically satisfied just prior to yield.

The critical strain criterion is developed from crystal plasticity theory. McClintock (1968*b*) observed high strain concentrations at non-deforming particles and theorized that these concentrations would be composed of dislocation pile-ups. Given that voids can form at an interface through plastic relaxation, it was suggested that nucleation follows a strain criterion.

A critical stress criterion by Brown and Stobbs (1976) is based on the stresses at the particle-matrix boundary being related to the dislocation density, ρ_d by,

$$\sigma_{max} = \frac{Gb(\rho_d)^{\frac{1}{2}}}{2\pi} \quad (1.2)$$

where σ_{max} is the maximum stress at the particle-matrix boundary, G is the shear modulus of the matrix, and b is the Burgers vector. The dislocation density is related to strain by,

$$\rho_d = \frac{5.1\epsilon_c}{2\pi b} \quad (1.3)$$

where ϵ_c is the critical global nucleation strain and r is the radius of a spherical precipitate. The critical strain to nucleation based on the maximum particle-matrix interfacial stress is then given by,

$$\epsilon_c = \frac{8\pi^2\sigma_{max}^2 r}{5.1G^2b} \quad (1.4)$$

Koenigsmann *et al.* (1996) and Koenigsmann and Starke (1996) compared this theory to experimental measures of strains to nucleation in an Al-Si-Ge alloy as a function of different particle diameters and found good correlation. Another approach to the critical stress theory is that there is a particle-matrix interface with a finite strength that must be exceeded prior to void nucleation (Needleman, 1987; Levy, 1994, 1997; Su *et al.*, 1999).

Gurson (1975) attempted to use both criteria in a nucleation formula,

$$\dot{f}_{nucleation} = A\dot{\epsilon} + B\frac{\partial}{\partial t}\left(\frac{\Sigma_{hyd}}{1-f}\right) \quad (1.5)$$

where $\dot{f}_{nucleation}$ is the rate of void nucleation, $\dot{\epsilon}$ is the flow strain, Σ_{hyd} is the hydrostatic component of stress, f is the void volume fraction and A and B are weighting

coefficients used to combine the critical stress and critical strain theories. A similar equation from Chu and Needleman (1980) is used more often, given by

$$\dot{f}_{nucleation} = A\dot{\bar{\sigma}} + B\dot{\Sigma}_{hyd} \quad (1.6)$$

where $\bar{\sigma}$ is the flow stress. When $B = 0$, Equation 1.6 reduces to a strain controlled nucleation equation since

$$\dot{\bar{\sigma}} = h\dot{\epsilon} \quad (1.7)$$

where h is the equivalent plastic hardening modulus of the matrix. For stress controlled nucleation, $A = B$. From experimental measurements by Gurland (1972) on spheroidized steels, Gurson (1975) found that the rate of void nucleation is proportional to the volume fraction of un-nucleated particles, such that

$$A = \Psi \left(f_p - \int_0^t \dot{f}_{nucleation} dt \right) / h \quad (1.8)$$

where f_p is the volume fraction of particles, and Ψ is a constant. Sometimes, A is taken as a constant (Zhang and Niemi, 1994). The most commonly used equation for A was introduced by Chu and Needleman (1980) and is a normal distribution. For nucleation at a critical strain,

$$A = \frac{f_N}{S_N\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\bar{\epsilon}^p - \epsilon_N}{S_N} \right)^2 \right] \quad (1.9)$$

where f_N is the volume fraction of void nucleating particles, ϵ_N is the mean strain for nucleation and S_N is the standard deviation. For a critical stress criterion,

$$B = A = \frac{f_N}{S_N\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\bar{\sigma} + \sigma_{hyd} - \sigma_N}{S_N} \right)^2 \right] \quad (1.10)$$

where σ_N is the mean nucleation stress.

Huang *et al.* (1998) compared the effects of adopting critical stress vs. critical strain criteria for nucleation using equations 1.6 and 1.8. Nucleation rates were accelerated by introducing a disk shaped region of high particle volume in an infinite region of standard particle volume fraction. Huang *et al.* (1998) found that the critical stress criterion nucleates particles at lower critical strains and at higher rate than the critical strain criterion. In fact, it was concluded that the stress criterion leads to localization dominated by void nucleation events while the strain criterion facilitates localization controlled by void growth. Koenigsmann and Starke (1996) also found that higher nucleation strains are required at larger particles and yet the larger particles lead to lower ductility. The void growth mechanism was dominant, thereby causing lower ductilities in Al-Si-Ge alloys with longer aging times (ie. larger particles). The debate over the legitimacy of the critical strain and critical stress theories has not yet been resolved. In general, it is believed that larger particles nucleate voids under stress controlled nucleation, while for small particles, ($< 1\mu m$), nucleation is strain controlled.

Most nucleation studies are based on spherical/circular inclusions. However, Tszeng (1993) examined the differences in the local stress fields for spherical and ellipsoidal particles and found the stress distributions were significantly different. In sheet metal, particles are usually not equiaxed and the effect of void shape becomes an important consideration. Harik and Cairncross (2000) used a critical shear strain at a two dimensional particle-matrix interface to govern void nucleation in a compressive strain state. Other models focus on the nature of the traction forces acting across the interface. Levy (1994, 1997) used plasticity calculations to determine the separation around a circular particle by assuming a nonlinear interfacial traction. Needleman (1987) has proposed a very similar traction curve to model nucleation

Figure 1.3: Interfacial traction as a function of separation (Needleman, 1987).

in a periodic, axisymmetric finite element model of spherical voids (Figure 1.3).

The specific interfacial strengths are somewhat arbitrary as there have been few accurate measurements from experiments (Section 1.1.1). Needleman's interfacial strength model appears to be thorough though some consideration should be given to the extent to which this interface model can be used within a finite element model. In the case of strong interfacial strengths, the crack-like nature of the particle-matrix separation may require the use of elements that can model stress singularities such as quarter-point elements.

Gall *et al.* (2000) modeled an incoherent aluminum(FCC)-silicon(diamond cubic) interface on an atom-by-atom basis, accounting for inter-atomic forces and atomic density. As such, the strength of the interface was not known *a priori*. The shape of the local average stress-strain curves during normal interface separation was found to be consistent with the traction separation model of Needleman (1987).

Some references to modeling nucleation at cracked particles are present in the literature. In one study, the material of the cracked particle is replaced by an anisotropic material which then approximates the presence of a void (Favier *et al.*, 1995). A study by Brockenbrough and Zok (1994) examines the matrix behaviour

after particle cracking but does not attempt to simulate the evolution of a crack within the inclusion. Clearly, modeling of nucleation through particle cracking is difficult and the existing studies have not considered the phenomenon adequately. Søvik (1996) mapped the maximum cross-sectional stress of a particle prior to nucleation as a function of the stress state using finite element analysis. Gurson's constitutive equation was then used to predict the experimental material behaviour of tensile specimens with varying notch radii. The nucleation parameters, exclusive of the particle cracking stress, were matched to micromechanical measurements. The coalescence criterion, f_c was controlled by the plastic instability theory developed by Thomason (1985a,b) and implemented by Zhang and Niemi (1994) (Section 1.3.3). This approach left the critical particle fracture stress as the only parameter to be varied in order to match the material response observed by experiment.

The statistical nature of particle populations has been recognized as being an important factor in the nucleation mechanism (Thompson, 1987; Argon *et al.*, 1975; Goods and Brown, 1979; Becker, 1987). The most common void interaction model uses a periodic distribution of voids and therefore neglects the localization effects of void clustering (Needleman, 1987). Watt *et al.* (1996) does examine the strain distributions around particles with varying sizes and spacings by including more than one particle in each unit cell. Prior to void nucleation at the particles, a major principal stress that is oriented parallel to the direction of closest particle spacing (referred to as aligned loading herein) leads to higher inter-particle ligament stresses than a major principal stress oriented perpendicular to the line of closest particle spacing (so-called transverse loading). A more thorough examination of particle interactions uses void spacing statistics to calculate the percentage of voids nucleated from particles based on the critical stress criterion (Argon *et al.*, 1975). However, such averaging techniques do not closely examine the nucleation event.

1.1.1 Experimental Measures of Interfacial Shear Strength

A few studies have been performed that consider experimental values for interfacial bond strengths in aluminum using approximate testing techniques. In general, experimental measurements of interfacial bond strengths between different materials are measured in a fibre pullout or pushout test, where a single fibre or a rod of fibres is pulled from or pushed through a cylinder of matrix material in tension. Ziv *et al.* (1999) performed pushout tests with alumina rods in pure aluminum, A356 commercial casting alloy (Al-0.35Mg-7Si-0.2Cu), and commercial wrought alloy 6061 (Al-1Mg-0.6Si-0.3Cu). In pure aluminum, the interfacial shear strength was 12 MPa whereas the alloys displayed bond strengths with the alumina rod ranging from 30 to 62 MPa in shear. An alternative experimental method (Kim and Yoon, 1995) pushes a rectangular block of hard material, which is cast into the surface of a sample of matrix material, until the interface at the sides of the block fail and the harder material is sunk into the softer matrix. Kim and Yoon (1995) measured interfacial shear strengths by pushing a block of Al_2O_3 with varying levels of SiO_2 into a matrix of pure aluminum or aluminum alloy (Al-12Si-1Cu-1Mg-2Ni). The measured interfacial shear strengths ranged from 30 to 32 MPa in the pure aluminum matrix and from 38 to 60 MPa in the aluminum alloy. A third method due to Lu and Hirohashi (1997) calculates the shear stress of a fibre-matrix interface by counting the number of fractures in a composite fibre composed of a fibre of hard material surrounded by a coating of matrix material. Lu and Hirohashi (1997) measured interfacial shear strengths ranging from 19 to 54 MPa between SiC fibre and aluminum (A1050) in a multiple broken fibre test. In all cases, the geometry of the interface and the loading are simplified compared to the actual case of embedded particles.

1.2 Void Growth

Void growth is the least complicated stage of ductile failure and is relatively well understood. The main criticism of void growth models to date is that the effects of void interaction and void clustering are inadequately modeled. The papers most often quoted as the pioneering work in modeling void growth were written by McClintock (1968a) and Rice and Tracey (1969) and simply consider a void within an infinite continuum. These analytical solutions lead to the important conclusion that a state of high triaxiality causes an exponential increase in the predicted void growth rate. Budiansky *et al.* (1981) considered a similar geometry with a different material model. The surprising observation made by Budiansky *et al.* (1981) is that for a strongly hardening material under appreciable negative hydrostatic stress the void becomes oblate even though the axial stress is higher than the transverse stress. These early studies led to some interesting observations but did not incorporate other ductile failure events such as nucleation, void interaction, and coalescence. For compressive or highly deviatoric stress fields, the presence of a particle would significantly alter the local strain field. Favier *et al.* (1995) modeled a nucleated void as an anisotropic particle. This approach does account for the particle but the selection of the degree and direction of anisotropy is arbitrary.

1.2.1 Gurson's Constitutive Equation

Gurson (1975) modeled the effect of porosity in a rigid-plastic material during plastic flow to obtain a yield function for porous materials .

$$\Phi = \left(\frac{\Sigma_{eq}}{\bar{\sigma}} \right)^2 + 2f q_1 \cosh \left(q_2 \frac{3\Sigma_{hyd}}{2\bar{\sigma}} \right) - 1 - q_3 f^2 = 0 \quad (1.11)$$

where

$$\Sigma_{eq}^2 = \frac{3}{2} \Sigma'_{ij} \Sigma'_{ij} \quad \text{and} \quad \Sigma_{hyd} = \frac{1}{3} \Sigma_{ij} \delta_{ij}. \quad (1.12)$$

The coefficients q_1 , q_2 and q_3 were introduced by Tvergaard (1981, 1982*b*) in order to bring the void growth rates into agreement with numerical studies of void growth. Other constitutive equations for porous materials have been developed (Michel and Suquet, 1992; Lee and Dawson, 1993) but are not presented in a closed form and have not received as much general use. Gurson's approach to ductile failure is only slightly more sophisticated than the early studies of McClintock and Rice and Tracey but the characterization of porosity with a single parameter and the resultant closed form yield function represent a valuable and much-cited contribution.

Gurson's constitutive model has met with criticism from a number of sources. Thomason (1985*a,b*) has stated that Gurson's model significantly over estimates void growth prior to coalescence. Thomason also states that localization is based not so much on void impingement as on ligament instability. These concerns are examined more closely in Section 1.3.2.

1.2.2 Modifications of Gurson's Constitutive Equation

Although Gurson's constitutive model is based on a hollow sphere, many researchers have based more complex studies of the effects of non-uniform void distributions (Narasimhan, 1994; Tvergaard and Needleman, 1995) on Gurson's equation. Lee and Mear (1992) and Søvik and Thaulow (1997) studied initially ellipsoidal voids rather than spherical voids and compared the void growth rates to Gurson predictions. They argued that the porosity cannot be adequately characterized by the void volume fraction alone but that the void aspect ratios are also significant. Corrections

to Gurson's constitutive equation were suggested which take the form of variations in q_2 as a function of hardening rate, level of hydrostatic stress and void aspect ratio. However, the function for q_2 is given in graphical form. Some other complications to this corrective approach are that the value of q_2 changes as the void grows and the aspect ratio evolves, and a population of ellipsoidal voids would require some statistical application of the data for q_2 . Gologanu *et al.* (1993) developed a constitutive equation (1.13) for ellipsoidal voids which reduces to Gurson's equation for spherical voids. For a prolate ellipsoid with axes a , b , and c ($a > b = c$),

$$\Phi = \left(\frac{\Sigma_{eq}}{\bar{\sigma}} \right)^2 + 2f q_1 \cosh \left(\kappa \frac{\Sigma_{hyd}}{\bar{\sigma}} \right) - 1 - q_3 f^2 = 0 \quad (1.13)$$

where

$$\kappa = \left[\frac{1}{\sqrt{3}} + (\sqrt{3} - 2) \frac{e_a/e_b}{\ln f} \right]^{-1} \quad (1.14)$$

$$\Sigma_{hyd} = \alpha_2 (\sigma_{11} + \sigma_{22}) + (1 - 2\alpha_2) \sigma_{33} \quad (1.15)$$

and

$$S = \ln \frac{a}{b}; \quad e_a = \sqrt{1 - e^{-2S}}; \quad (1.16)$$

$$\frac{e_b^3}{1 - e_b^2} = f \frac{e_a^3}{1 - e_a^2}; \quad \alpha_2 = \frac{1}{2e_b^2} - \frac{1 - e_b^2}{2e_b^3} \tanh^{-1} e_b \quad (1.17)$$

where f is the void volume fraction, S is the void shape parameter, e_a and e_b are eccentricities, q_1 , q_2 , and q_3 are curve fit constants, σ_{11} and σ_{22} are the principal stresses in the b and c direction, σ_{33} is the principal stress in the a direction, Σ_{hyd} is the equivalent hydrostatic component of stress, Σ_{eq} is the equivalent deviatoric component of stress, and $\bar{\sigma}$ is the current flow stress of the material. Although Gologanu's constitutive equation for ellipsoidal voids does not predict void growth

as accurately as the corrections suggested by Lee and Mear (1992) and Søvik and Thaulow (1997), it is easier to use.

In addition to adding modifying coefficients and a coalescence criterion, Tvergaard and Needleman (1995, 1997) pointed out that the constitutive model of Gurson (1975) has no characteristic length associated with it. As a result, any use of Gurson's yield function within a numerical solution procedure would give results dependent on the element size. This is particularly true during void coalescence. Tvergaard and Needleman (1995) and Leblond *et al.* (1994) propose a characteristic length adjustment to the Gurson constitutive equation but early studies show that the choice of characteristic length is case specific.

Sun and Wang (1989) have proposed an alternative to Gurson's constitutive equation which is simply a lower bound approximation in contrast to Gurson's upper bound approximation. Sun (1995) has also developed a lower bound constitutive equation based on a dual population of large and small voids. Cortés and Elices (1993) studied the adiabatic growth of voids in a thick hollow sphere similar to Gurson's model but with a superimposed shear stress, as would be encountered during the dynamic formation of a shear band.

Leblond *et al.* (1995) suggested a modification to the Gurson equation (1.11) based on a correction in the analytical model. Rather than using a single flow stress, $\bar{\sigma}$, there are two flow stresses, $\bar{\sigma}_2$ related to the hydrostatic component of stress, Σ_{hyd} , and $\bar{\sigma}_1$ for the equivalent stress component, Σ_{eq} . The revised Gurson constitutive equation is then,

$$\Phi = \frac{\Sigma_{eq}^2}{\bar{\sigma}_1^2} + 2fq_1 \cosh\left(q_2 \frac{3\Sigma_{hyd}}{2\bar{\sigma}_2}\right) - 1 - q_3 f^2 = 0 \quad (1.18)$$

where $\bar{\sigma}_1$ and $\bar{\sigma}_2$ are functions of the void volume fraction, f , and the global com-

Figure 1.4: A unit cell representing a periodic array of voids

ponents of strain, ϵ_{hyd}^{∞} and ϵ_{eq}^{∞} .

Leblond and Perrin (1999) proposed another alternative to the Gurson constitutive equation which takes into account effects of localized void clustering. The constitutive equation derivation begins with void growth calculations of a porous sphere inside an infinite medium of lower porosity. From there, the theory can be expanded to map a number of regions, each of different porosity throughout the sample.

Zheng *et al.* (1996) have raised the issue that the porosity of a material is reversible and as such should not be a ductile damage parameter at all. They argue that since material damage is irreversible, even under complicated strain histories, an irreversible damage parameter should be used such as “ductility dissipation” based on the principles of thermodynamics.

1.2.3 Unit Cell Models

A very common approach for modeling void growth, while accounting for void interaction, is the periodic unit cell (Figure 1.4). In contrast to studies of a void in an infinite medium (McClintock, 1968a; Rice and Tracey, 1969; Tracey, 1971) or even Gurson’s hollow sphere model (Gurson, 1975), a periodic unit cell models strain field interaction and fully accounts for stress concentrations within the inter-void

Figure 1.5: (a) square and (b) rhombic periodic arrangements of voids (Kitagawa and Honke, 1988)

ligament. As long as the boundaries of the model are maintained such that similar cells could be stacked in all directions, the effect of the interaction of the modeled void with an infinite array of similar voids can be simulated.

Needleman (1972) was one of the first to use periodic unit cells in studying void growth. He used finite element methods to solve a two-dimensional, large-deformation, elastic-plastic model. Kitagawa and Honke (1988) compared Needleman's square periodic array to an alternative rhombic void array (Figure 1.5). Interestingly, Kitagawa and Honke found that the rhombic array leads to shear band development between voids much more quickly than the square void array. Koplik and Needleman (1988) developed an axisymmetric three-dimensional periodic unit cell.

A square-prismatic three-dimensional unit cell was introduced by both Hom and McMeeking (1989) and Worswick and Pick (1990). The unit cell of Hom and McMeeking (1989), with an initial porosity of $f^o = 0.82\%$ and a hardening exponent of $n = 0.1$, was deformed under uniaxial tension, pure shear and high triaxial stress. Under these conditions, there was a significant degree of softening due to a regular distribution of porosity but Hom and McMeeking (1989) comment that their models do not display adequate softening as compared to the experiments of Bourcier

Figure 1.6: The unit cells representing three different periodic void distributions (Kuna and Sun, 1996). (a) cubic, (b) body centered cubic, (c) hexagonal.

et al. (1986). Hom and McMeeking (1989) anticipate that a greater degree of softening would be achieved if the void distributions were random rather than periodic. Worswick and Pick (1990) varied the hardening exponent ($n = 0.0, 0.1, 0.2$) and the initial porosity ($f^o = 0.01, 0.025, 0.1$) in their unit cell models. Under uniaxial deformation with varying degrees of hydrostatic stress, Worswick and Pick (1990) found that the constitutive softening in a periodic array of voids is primarily a function of the level of hydrostatic tension and the initial void volume fraction.

Kuna and Sun (1996) compared the void growth rates of three different void distributions based on cubic, body-centered cubic and hexagonal arrangements. The three corresponding unit cells are depicted in Figure 1.6. The unit cells were deformed under different levels of hydrostatic tension and in some cases the matrix material was modeled as a Gurson material, thus simulating a second population of smaller voids. Void nucleation was not considered and the initial void volumes were quite high (0.114) which accentuated the void interactions. Kuna and Sun (1996) observed that the often used axisymmetric unit cell model exaggerated void growth in comparison to the three dimensional models. The body centered cubic unit cell which is analogous to the rhombic void array in Figure 1.5 showed significantly higher levels of void interaction similar to that of the two dimensional rhombic array. Zhang *et al.* (1999) conducted a comparable study of axisymmetric

and three-dimensional unit cells, varying void spacing and size with an initial void volume fraction of $f_o = 0.077$. They also found that the axisymmetric unit cell overemphasized void spacing and size effects with respect to the fully three-dimensional model.

Many researchers have studied specific variables within the unit cell model. Fleck *et al.* (1989) included a void nucleating particle within a unit cell. The interfacial traction treatment utilizes a simple model that fails at a set value of normal stress. The purpose of their study was to compare the growth rates of simple voids to the growth of voids with particle contact or particle traction. As expected, the void growth at low or negative triaxialities is overestimated by models that do not contain a particle such as Gurson's model. Growth rates of initially ellipsoidal voids have been studied by Becker *et al.* (1989) and Søvik and Thaulow (1997). Becker *et al.* (1989) considered an axisymmetric model of oblate ellipsoidal voids under axisymmetric tension and various degrees of hydrostatic tension. Under low triaxialities, the void growth rate is similar to a spherical void with a radius equal to the ellipse dimension perpendicular to the tension axis. Under high triaxialities, the initial shape of the void is less important. Søvik and Thaulow (1997) also studied ellipsoidal voids using axisymmetric models. Tvergaard (1996) investigated the relative growth rates of an ordered array of two different sized voids in an axisymmetric unit cell (see Figure 1.7). At very small void volume fractions, the voids grow at the same rate, while at higher void volume fractions, the larger void grows more quickly. At even higher void volume fractions, the smaller void grows more quickly. When the model reaches coalescence strains, the growth of one of the voids dominates.

A significant problem with the periodic unit cell model is that it does not take void clustering into effect. The random nature of void distribution is likely to account

Figure 1.7: An axisymmetric unit cell representing an ordered distribution of two different sized voids (Tvergaard, 1996).

Figure 1.8: A non-periodic void distribution (Thomason, 1993)

for the lower strains-to-failure observed in experiments compared to unit cell predictions. Thomason (1993) attempts to simulate random size and spacing through a non-periodic but ordered model containing several voids of varying sizes (Figure 1.8). The stresses in the ligaments are averaged over the entire width of the group of unit cells, however, so the cascade effect of ligament collapse is not represented.

Needleman and Kushner (1990) also modeled the random distribution of voids by assigning six randomly spaced voids to a 2D periodic unit cell (Figure 1.9). The stress strain curves for the different void arrangements show that the void distribu-

Figure 1.9: Some of the periodic unit cells containing a random distribution of voids used by Needleman and Kushner (1990)

tion is important but the differences are not linked to a quantitative characteristic of the void distribution.

Another criticism of the periodic unit cell is the creation of planes of high area volume fraction. If such planes exist, then void growth will be concentrated within these planes which will become preferred paths for void sheeting. A random particle distribution will not form planes of voids but will cause more localized softening effects.

1.2.4 Random Void Distribution

Becker (1987) examined microstructural inhomogeneity in two dimensions. From a micrograph, Becker characterized the porosity distribution and mapped it onto a plane of elements, each element having a uniform porosity. For comparison with their numerical prediction, Becker and Smelser (1994) conducted a simple experiment involving a steel sheet with a random distribution of drilled holes. Using triangular elements and very approximate boundaries, the numerical model accurately predicted the failure path based solely on geometric instability. Magnusen *et al.* (1988) also drilled holes into plate metal to examine the effect of hole distribution and material hardenability on ductility. They concluded that a random hole distribution significantly reduced the ductility of sheet metal with the nearest neighbor spacing being the critical factor (ie. material length scale).

Worswick *et al.* (1998) represented the particle size, aspect ratio and distribution data from tessellated particle fields as ellipses and simulated the nucleation, growth and coalescence of these voids, as well as their interaction with nearest neighbors, under various two-dimensional strain states. Nucleation was strain controlled and dependent on particle size. Void growth was based on three-dimensional FEM calculations of unit cells and was dependent on void aspect ratio and the degree of

Figure 1.10: Experimental schematic for simulating void interaction and ligament thinning (Goto and Koss, 1996)

clustering. Void coalescence followed a modified version of the ligament to void size ratio criterion theorized by Brown and Embury (1973). Although the nucleation and void interaction criteria used in this study are not necessarily representative of actual events, the progression of damage within the model demonstrates a tendency to jump from one particle cluster to the next, indicating a strong dependence on microstructural length scale.

1.2.5 Experimental Studies of Void Interaction

Goto and Koss (1996) simulated void interaction and ligament thinning experimentally by drilling concentric holes with spherical ends from opposite directions into a round bar which was then deformed in uniaxial tension (Figure 1.10). The thinning of the ligament was directly measured at intervals until ligament failure occurred. The thinning of the ligament was also predicted using a model based on the Rice and Tracey (1969) void growth model which does not account for void interaction. Figure 1.11 summarizes the results from Goto and Koss (1996). It is clear that the ligament thinning trend for different ligament thicknesses is opposite to the trends predicted when void interactions are not taken into consideration. In other words, void interaction effects cause thinner ligaments (with respect to void radius) to thin

Figure 1.11: Comparison of experimental and predicted inter-void ligament thinning under uniaxial tension where predictions do not account for void interactions (Goto and Koss, 1996). ω is the ligament thickness and r is the void radius.

more quickly than thicker ligaments.

Forero and Koss (1994) also studied interactions between growing voids experimentally by monitoring the growth of artificial voids in titanium. Two holes with spherical ends were drilled into the polished end of a cylinder of titanium. A second cylinder with a polished end was butted up against the drilled holes and held there at 695° and 30 MPa of compressive force for 4 hours. The specimen was then annealed and diffusion bonded, resulting in a pair of approximately spherical voids embedded in a homogeneous titanium matrix (Figure 1.12). The inter-void ligament was approximately one diameter thick. The dimensions of the cavities were then monitored during uniaxial tension using ultrasonic imaging. Forero and Koss (1994) found that both the longitudinal and transverse axis dimensions of the voids increased whereas for a single void, the transverse axis dimension decreased. It was also observed that the transverse expansion on the voids was completely at the expense of the inter-void ligament thickness.

Figure 1.12: Experimental schematic for creating artificial neighboring voids in a tensile specimen (Forero and Koss, 1994)

Figure 1.13: Three theories of void coalescence: (a) stable coalescence through void impingement, (b) unstable coalescence as in local shearing between voids and (c) impingement of voids, nucleated at a secondary population of smaller inclusions, across the inter-void ligament (Schlüter *et al.*, 1996)

1.3 Void Coalescence

Three mechanisms of void coalescence have been proposed: (a) stable coalescence through impingement of primary voids, (b) unstable coalescence through shearing of the primary inter-void ligament and (c) impingement of secondary voids, nucleated at a population of smaller inclusions, across the primary inter-void ligament, also known as “void sheeting” (Figure 1.13). It is generally agreed that voids usually coalesce through a combination of mechanisms (b) and (c), as is indicated by the low strain-to-failure values observed in experiments and the sparse population of relatively small voids in the immediate vicinity of a ductile fracture surface. In at-

tempts to predict the onset of void coalescence and crack initiation, some researchers have focused on a critical void size (Rice and Tracey, 1969), whereas others have considered a critical void volume fraction (Gurson, 1975).

1.3.1 Critical Void Size Models

Critical void size coalescence models are an attempt to extend the void growth models of single voids in an infinite medium beyond the void growth stage and into void coalescence. The premise is in itself a contradiction in that a single void in an infinite matrix has no neighboring voids with which to coalesce. The void growth model by Rice and Tracey (1969) is as follows:

$$\frac{dr}{r_o} = 0.283 \exp \left(\frac{3}{2} \frac{\sigma_{hyd}}{\sigma_{eq}} \right) d\bar{\epsilon}_g^p \quad (1.19)$$

where r is the void radius, r_o is the initial void radius, σ_{hyd} is the hydrostatic component of stress, σ_{eq} is the Von Mises stress and $\bar{\epsilon}_g^p$ is the effective plastic strain. Integrating and assuming a constant hydrostatic stress and a negligible void nucleation strain gives the following equation for critical void radius, r_c , at coalescence.

$$\bar{\epsilon}_g^p = \frac{\ln(r_c/r_o)}{0.283} \exp \left(-\frac{3}{2} \frac{\sigma_{hyd}}{\sigma_{eq}} \right). \quad (1.20)$$

This equation leaves no room for material variability however so an alternate form is suggested

$$\bar{\epsilon}_g^p = \frac{\ln(r_c/r_o)}{A} \exp \left(-B \frac{\sigma_{hyd}}{\sigma_{eq}} \right) \quad (1.21)$$

where A and B are fit to experimental data from a given material. Some other critical void radius models have been developed by Tai (1990) and Shockey *et al.* (1980)

Schlüter *et al.* (1996) conducted experiments on a structural steel at different levels of hydrostatic tension. They found that the modified form of the Rice and Tracey critical void radius equation fit their experimental data well and that the level of triaxiality had little effect on the critical void radius.

1.3.2 Applications of Gurson's Constitutive Model

Gurson's model (Gurson, 1975) has been found to predict much higher strain to fracture than revealed by experiments. In an attempt to achieve more realistic coalescence strains, Needleman and Triantafyllidis (1978) induced necking using Gurson's model by introducing a region of initially high void volume fraction. The failure strain was greatly reduced but the degree of porosity required to develop a bifurcation was unrealistically high. Yamamoto (1978) did a similar study, simulating shear bands with comparable results. Hutchinson and Tvergaard (1981) and Saje *et al.* (1982) also found that the initial inhomogeneity in void volume fraction had to be significantly high, before a shear band would develop at a reasonable level of strain. Ohno and Hutchinson (1984) and Huang *et al.* (1998) examined a disk-like region of initially high porosity rather than an infinite band. In contrast, the lower bound constitutive equation by Sun and Wang (1989) was used to model a planar imperfection and the strains to failure were found to be low. All told, none of these studies considering inhomogeneous porosity distributions adequately captured strains-to-failure at reasonable porosity levels. These results suggest that the random nature of particle distributions cannot be modeled as a number of large regions, each with a homogeneous void volume fraction.

As Gurson's original formulation (Equation 1.11) only considered an approximate form of stable void growth, Tvergaard (1982a) and Tvergaard and Needleman (1984) replaced the void volume fraction parameter, f , with a parameter dependent on a

	f^o	f_c/f^o	$f'_f f^o$
Becker (1987)	0.0268	5.60	9.33
Brocks <i>et al.</i> (1996)	0.002	16.5	75.0
Kuna and Sun (1996)	0.114	1.67-2.63	2.06-3.86
Kuna and Sun (1996)	0.0114	4.82	11.4
Steglich and Brocks (1998)	0.114	1.40-1.67	-
Zhang and Niemi (1994)	0.04	2.68	-
Zhang (1996)	0.002	7.25	-
Zhang (1996)	0.0005	3.80	-

Table 1.1: Some values for f_c and f_f as fit by various researchers to the Gurson-Tvergaard constitutive equation (Equation 1.23)

critical void volume fraction at which voids coalesce, f^* , such that

$$\Phi = \frac{\Sigma_{eq}^2}{\bar{\sigma}} + 2q_1 f^* \cosh\left(\frac{3q_2 \Sigma_{hyd}}{2\bar{\sigma}}\right) - 1 - q_3 f^{*2} = 0 \quad (1.22)$$

$$\text{where } f^* = \begin{cases} f & f \leq f_c \\ f_c + \frac{f_u^* - f_c}{f_f - f_c} (f - f_c) & f \geq f_c \end{cases} \quad (1.23)$$

where f_c , f_u^* , and f_f control the rate of coalescence. Becker (1987) supported this formulation and used the same values of $f_c = 0.15$ and $f_f = 0.25$ for an initial void volume fraction of $f^o = 0.0268$. This coalescence model is an empirical curve fit, however, and does not examine the phenomenon of void coalescence. A common approach to finding the value of f_c for a given model involves deforming a unit cell under a particular degree of triaxiality to failure. The void volume fraction at failure then simply becomes the critical volume fraction at coalescence f_c . Many researchers have calibrated f_c and f_f to unit cell results. Some of their findings are summarized in Table 1.1

Koplik and Needleman (1988) found that f_c is not a constant as its value depends on the stress state, the unit cell geometry and the initial void volume fraction. However, the unit cell models employed by Koplik and Needleman (1988) did not

include void nucleation, which may account for the variation in f_c with initial void volume fraction. Richelsen and Tvergaard (1994) also concluded that f_c is a function of initial void volume fraction. Steglich and Brocks (1998) found f_c to be a function of void shape and particle hardness at low triaxialities. He *et al.* (1998) and Zhang *et al.* (1999) discovered that a different set of nucleation and coalescence parameters are required for the same material under different loading conditions (such as tension and bending). The non-unique nature of f_c has been observed experimentally (Shi *et al.*, 1991; Thomson and Hancock, 1984). On the other hand, Kuna and Sun (1996) and Brocks *et al.* (1996) concluded that f_c is independent of the level of stress triaxiality. He *et al.* (1998) also considered the Gurson model to be limiting as it did not predict fracture in bend specimens.

Steglich and Brocks (1997, 1998) observed a significant difference in the compressibility predictions of round bar tensile specimens using Gurson's constitutive equation. The reduction in diameter of the bar as a ratio of the elongation of the bar is 1/3 in experiments but fixed at 0.5 in the Gurson model due to the assumption of matrix incompressibility. They concluded that modeling the voids as spheres throughout the deformation history leads to poor localization predictions, especially at low triaxialities.

Characteristic lengths are required to characterize particle size and spacing. Gurson's constitutive equation does not model these characteristics but some attempt has been made to include these effects through the application of the finite element analysis. The particle spacing can be accounted for by the size of mesoscopic unit cells used to calculate f_c , and the particle size could be approximated by a linear relationship to size of the elements in the macroscopic models (Steglich and Brocks, 1998).

Until the validity of the Gurson parameters, such as f_c , are verified by exper-

iment, the Gurson constitutive equations should be used with the understanding that the rate of damage progression associated with the model is suspect.

Tvergaard (1982a) was one of the first researchers to model coalescence of primary voids in two dimensions as the failure of the inter-void ligament due to coalescence of small secondary particles. The secondary particles are modeled using Gurson's constitutive equation. Brocks *et al.* (1996) also took this approach to describe the behaviour of a dual population of void nucleating particles. They analyzed the respective void growths in a unit cell model containing large voids ($f^o = 0.002$) and a matrix material with a secondary population of particles, represented by the Gurson model, which also nucleate voids ($f_N = 0.004$, $f_c = 0.15$, $f_f = 0.25$). Kuna and Sun (1996) also introduced a second population of void nucleating particles into a unit cell model using Gurson's constitutive equation in the matrix material. Kuna and Sun (1996) used a primary initial void volume fraction of $f^o = 0.114$ and a secondary volume of nucleating voids with $f_N = 0.008$, $f_c = 0.04$ and $f_f = 0.195$. Mathur *et al.* (1996), Narasimhan (1994) and Needleman and Tvergaard (1991) modeled a dual population of voids using Gurson's constitutive equation. Rather than physically representing the larger voids, they are simulated as regularly spaced islands of material that nucleate voids at a low stress level, which is characteristic of large particles. The remainder of the material is governed by strain controlled nucleation. The nucleation and coalescence parameters used in the dual particle population studies above are summarized in Table 1.2. It should be emphasized that many experiments have revealed that larger particles nucleate voids at much lower global strains than smaller particles (Koenigsmann and Starke, 1996).

1.3.3 Plastic Instability

Brown and Embury (1973) hypothesized that coalescence would take place when

	Large Particles			Small Particles		
	f^o	f_N	ϵ_N	f_N	ϵ_N	f_f
Brocks <i>et al.</i> (1996)	0.002	-	0.3	0.004	0.8	0.15
Kuna and Sun (1996)	0.114	-	(FEM)	0.008	0.3	0.04
Mathur <i>et al.</i> (1996)	-	0.12	(σ_N)	0.02	0.1,0.3	0.12
Needleman and Tvergaard (1991)	-	0.04	(σ_N)	0.02	0.3	0.12
Narasimhan (1994)	-	0.166	(σ_N)	0.008	0.3	0.15
Tvergaard (1982a)	0.0	0.03	(FEM)	0.04	0.3	0.15
						0.75

Table 1.2: Some values for nucleation and coalescence parameters used by various researchers in the Gurson-Tvergaard constitutive equation (Equation 1.23) in order to model dual populations of voids.

Figure 1.14: A unit cell model illustrating the proposed sliplines that would lead to plastic ligament collapse at sufficient stress levels where $\dot{W}$ is the global velocity during plane strain deformation, $2b + w$ is the unit cell width, $2a$ and $2b$ are the vertical and horizontal dimensions of the void and x and z are the coordinate axes (Thomason, 1981).

the height of a row of voids became equal to the spacing between them. In a more rigorous analysis, Thomason (1985a,b) developed two and three-dimensional plastic limit load coalescence criteria. According to plasticity theory, a plastic material will deform homogeneously until a stability limit is reached within a given inter-void ligament, or until the local maximum principal stress reaches a plastic limit load.

The two dimensional formulation begins by assuming that voids in a periodic distribution are sufficiently far apart to avoid local stress field interactions during stable uniform plastic flow. The Rice and Tracey (1969) void growth model is utilized to estimate geometry changes in the unit-cell model during plane strain deformation at each step in the calculation (Figure 1.14).

The major principal stress at each new step is then compared with the stress limit for uniform plastic flow for the current geometry based on a slipline field calculation (Figure 1.15). If the stable stress limit is exceeded, the material outside of the slipline field is considered to be rigid and all strain is concentrated in rapid dilation of the void leading to imminent ductile fracture through failure of the inter-void ligament.

Figure 1.15: The limiting major principal stress for stable plastic flow in a two dimensional unit cell containing an ellipsoidal void based on slipline plasticity theory where a is the height of the void and ω is the thickness of the ligament from Thomason (1993) (see Figure 1.14).

Thomason (1985a,b) also developed a three-dimensional plastic limit load theory for a periodic distribution of voids using upper bound plasticity solutions rather than slipline fields. In developing the upperbound limit stress for plastic flow as a function of unit cell geometry the voids are assumed to maintain a rectangular prismatic shape to facilitate a tractable solution. Like a slipline solution, the velocity field within the flow region of an upperbound solution must maintain a constant volume and be consistent with the boundary conditions. Unlike slipline solutions, however, stress equilibrium is not required in an upperbound solution.

The emphasized region in Figure 1.16b is one of the regions of concentrated plastic flow during ligament collapse proposed by Thomason (1985a) where $\dot{u}$, $\dot{v}$ and $\dot{w}$ take the form

$$\dot{u} = \frac{\dot{W}}{2ax} \left[\left(b + \frac{\omega}{2} \right)^2 - x^2 \right] \quad (1.24a)$$

Figure 1.16: The geometry of an upperbound velocity field during plastic collapse of an intervold ligament where the void is approximated as a rectangular prism with a width and depth of $2b$ and a height of $2a$ (Thomason, 1985a). d is the ligament thickness, and $\dot{W}$ is the rate at which the height of the unit cell is increasing. (a) a unit cell containing a single ellipsoidal void, reduced by symmetry, (b) the full unit cell with a prismatic void, (c) detail of the plastic flow region in the inter-void ligament.

$$\dot{v} = \frac{\dot{W}y}{2ax^2} \left[\left(b + \frac{\omega}{2} \right)^2 - x^2 \right] \quad (1.24b)$$

$$\dot{w} = \frac{\dot{W}z}{a} \quad (1.24c)$$

where $2b$ is the width and depth of the void, $2a$ is the void height, ω is the ligament thickness and $\dot{W}$ is the rate at which the height of the unit cell is increasing (See Figure 1.16). At the interface of the plastic flow region and the rest of the unit cell, there is a velocity discontinuity, $\dot{s}$, defined as

$$\dot{s} = \sqrt{\dot{u}^2 + \dot{v}^2}. \quad (1.25)$$

The limit load, σ_n for any given state of void deformation can be found by equating the rate of internal energy dissipation, $\dot{I}$, in the region of plastic flow to the rate of work of the external loads, $\dot{E}$.

$$\dot{I} = \sqrt{\frac{2}{3}} \int_V \left[\dot{\epsilon}_{xx}^2 + \dot{\epsilon}_{yy}^2 + \dot{\epsilon}_{zz}^2 + \frac{1}{2} (\dot{\gamma}_{xy}^2 + \dot{\gamma}_{xz}^2 + \dot{\gamma}_{yz}^2) \right]^{\frac{1}{2}} dV + \frac{\sigma_y}{\sqrt{3}} \int_S \dot{s} dS \quad (1.26)$$

$$\dot{E} = \sigma_n A_n \dot{W} \quad (1.27)$$

where σ_y is the uniaxial yield stress, σ_n is the mean stress on the surface area A_n (subject to velocity $\dot{W}$), V is the the volume of the region undergoing plastic deformation, S is the surface area of the rigid-plastic interface and the ϵ and γ terms are the normal and shear strains, respectively.

Thomason (1985a) summarized the critical normal stress as a function of unit

cell geometry in the form of an empirical curve fit as follows (see Figure 1.16):

$$\frac{\sigma_n}{\sigma_y} = \frac{0.1}{\left(\frac{2a}{\omega}\right)^2} + \frac{1.2}{\left(\frac{2b}{2b+\omega}\right)^{0.5}} \quad (1.28)$$

Zhang and Niemi (1994) adopted this dual dilational constitutive response for unstable coalescence of voids developed by Thomason (1985a,b) and adapted it to be compatible with the Gurson (1975) constitutive equation (Equation 1.11) for the nucleation and growth of voids. To simplify the application, the void is assumed to grow spherically, ie. $a = b = r$. The Gurson nucleation and growth equations were employed to calculate r and $\ell = b + \omega/2$:

$$r = \left(\frac{3f}{4\pi}V\right)^{\frac{1}{3}} \quad (1.29)$$

$$\ell = \ell_o \epsilon_2 \quad (1.30)$$

$$\mathcal{R}_A = \frac{\ell^2 - \pi r^2}{\ell^2} \quad (1.31)$$

where f is the void volume fraction as determined by the Gurson equations, V is the normalized volume of the element, ℓ_o is the initial unit cell dimension in the x and y directions, $\mathcal{R}_A$ is the ratio of inter-void cross-sectional area to global cross-sectional area and ϵ_2 is the second principal strain for the element. Using these geometry values from the finite element calculations, Thomason's limit load equation (1.28) becomes:

$$\frac{\sigma_n}{\sigma_y} = \mathcal{R}_A \frac{0.1}{\left(\frac{r}{\ell-r}\right)^2} + \frac{1.2}{\left(\frac{r}{\ell}\right)^{0.5}} \quad (1.32)$$

Once the instability condition of Equation 1.32 is reached, f becomes f_c and the void growth can be accelerated as in Equation 1.23. Zhang and Niemi (1994) made no

suggestions as to the calculation of f_f in the Tvergaard coalescence equation. The main attraction to this coalescence criterion is that f_c does not need to be calibrated for the specific modeling case, rather the nucleation parameters can be set instead.

Pardoen and Delannay (1998) conducted an experiment with notched round tensile copper bars and compared the predictive capabilities of the Gurson coalescence criterion and the Thomason limit load model based on measured strain-to-fracture results. The initial void volume fraction of voids was varied by deforming the specimens to a range of pre-strain values followed by a full anneal which removed any work hardening effects but did not alter the size and distribution of voids or second phase particles. Similar to findings by Koplik and Needleman (1988), Pardoen and Delannay (1998) found that the coalescence parameter, f_c , used in the Gurson constitutive equation is dependent on the initial void volume fraction, the work hardening parameter, n , and the level of hydrostatic tension. Thomason's limit load failure model (Equation 1.28), however, is able to predict the strain to coalescence in all cases provided that the voids are modeled as ellipsoids. In a follow-up to this experimental study, and independently from a similar study by Gologanu (1997), Pardoen and Hutchinson (2000) developed an adaptation of the Gurson (1975) void growth model that incorporates the initial and changing void aspect ratios as well as Thomason (1985a,b) plastic limit load coalescence model. Not only does this model account for changes in void shape but also incorporates the dependence of the critical coalescence porosity on the material hardening rate (within a limited range, $0 \leq n \leq 0.3$) and the degree of stress triaxiality. Pardoen and Hutchinson (2000) has calibrated these equations to axisymmetric unit cell models containing single voids without inclusions or a nucleation model and coalescence is through void impingement.

In keeping with Hill (1950) bifurcation theory, Rudnicki and Rice (1975) achieved

instability in a two-dimensional model assuming the possibility of non-normality of the yield surface within a narrow band of material parallel to the major principal strain. Needleman (1979) used a similar approach to model material instability in an arbitrary direction within a two-dimensional rectangular regime. Hutchinson and Tvergaard (1981) studied the development of a shear band beyond the initial bifurcation and found that the shear band development was strongly dependent on the shape of the vertex on the yield surface. Plasticity theory is complex and intractable even for some simple geometries. As a result, large deformation plasticity equations have been adapted to finite element methods in order to accommodate more complex arrangements.

Hill (1950) bifurcation theory is based on instability arising from loss of normality on a yield surface, which in turn stems from dislocation theory. Instability can also arise from the geometry of the microstructure, as when a critical number of inter-void ligaments have coalesced within void clusters, leading to a loss of load carrying capacity. The development of microscopic shear bands between voids has been studied by Kitagawa and Honke (1988) and Tvergaard (1981, 1982a). As mentioned previously, a rhombic periodic array of voids is more likely to produce shear bands than a square periodic array of voids, as opposed to the traditional approach of maintaining planar symmetry between unit cells, which overly constrains the deformation. As long as the cells can be stacked in all directions, periodicity can be maintained. Therefore, the boundaries do not need to be straight and orthogonal (Figure 1.17). This type of flexible symmetry boundary condition may lead to geometric instabilities, such as shear bands, more quickly than the usual orthogonal symmetry plane models.

Figure 1.17: An illustration of non-orthogonal unit cell boundaries which are more likely to allow for geometric instabilities than standard orthogonal boundaries (Tvergaard, 1981).

1.4 Current Research

Previous research has seen a wide range of approaches to model the three stages of ductile failure. Although a few studies have included all three stages in a single model, none have incorporated detailed models of all three stages. Plastic instability-based approaches to predict coalescence (Thomason, 1985*a,b*) have not been applied in the context of particle clustering. Geometric instability has occasionally been studied in two dimensions, but not in three dimensions. Any attempt at modeling clustering has placed particles at the vertices of unit cells or in such a way that the particle distribution is still regular. There have been a few non-periodic particle distributions considered, but they are patterned and/or two-dimensional. The statistical nature of particle distributions, to a large degree, has not been addressed. The issue of microstructural length-scale in a void growth model has been discussed but only superficially considered.

This thesis presents a model that captures the influence of particle clustering on all three stages of ductile fracture. In doing so, three of the issues listed above are addressed in a unique way:

- All three stages of damage are modeled in a detailed manner and the effects of a wide range of parameters are considered.
- True three-dimensional particle clustering is modeled such that only the clusters of particles are periodically spaced. The anisotropic nature of some clusters leads to a global anisotropic response which has been observed by Pilkey (1997) experimentally.
- The critical length scale of damage is also addressed through the effects of cluster density and through observation of void coalescence stages.

...
A finite element model of particle clusters is developed utilizing a periodic unit cell structure deformed through typical sheet forming strains and various levels of hydrostatic tension. Discrete second phase particles are introduced within the unit cell at various inter-particle spacings; the degree of clustering controlled by the particle spacing. Void nucleation is treated using a traction-separation interface model at the particle-matrix interface similar to that of Needleman (1987). The matrix material is modeled using a Gurson-Tvergaard-Needleman damage-based model (Gurson, 1975; Tvergaard and Needleman, 1984). Void coalescence occurs through nucleation, growth and coalescence of a secondary population of particles which leads to primary ligament collapse. This approach allows both inter-particle coalescence within clusters and inter-cluster coalescence to be modeled.

1.4.1 Thesis Overview

The finite element model, along with the assumed material properties and the post-processing calculations, is described in Chapter 2. Evidence relating to the validity of the model and the assumptions made is outlined in Chapter 3. Chapter 4 contains the results from a number of studies of void growth within void clusters without the

influence of particles and nucleation. The influence of particles and void nucleation at the particle-matrix interface is considered in Chapter 5. Chapter 6 examines the factors affecting coalescence in the presence of a secondary population of particles and the issue of length-scale. The results are discussed in the light of other work and suggestions for future work are made in Chapter 7. The conclusions and recommendations stemming from this research are given in Chapter 8.

Chapter 2

Model Description

The particle field in a dispersion strengthened ductile material can often be described as a random distribution displaying a certain level of particle clustering (Pilkey *et al.*, 1995). In the interests of computing efficiency, rather than modeling every particle and its position for a given sample of undeformed ductile material, the material can be represented as an infinitely repeated pattern of unit cells each containing one or more spherical particles (Figure 2.1). Assuming that the boundaries of the unit cell are maintained such that they can be stacked with perfect contact in all directions, the model of the material can be reduced to a single unit cell by taking advantage of the periodic symmetry of the model material. The obvious choice for unit cell boundaries, and the simplest to model, is a rectangular prism containing a single particle cluster. If the particle cluster is symmetric about the principal axes of the rectangular prism, the model can be further reduced to one-eighth of the unit cell. Most of the models in this thesis have cubic unit cells and the particle clusters are symmetric about the three principal axes. Figure 2.2 depicts a linear, three particle cluster in a cubic unit cell and outlines the one-eighth segment that is modeled. For

Figure 2.1: Regular periodic array of linear clusters or “stringers”

Figure 2.2: A unit cell representing a periodic array of linear clusters

comparison purposes, unclustered particle dispersions are also modeled, corresponding to one central particle within each unit cell. Some consideration is also given to ellipsoidal particles within cubical unit cells.

In representing the material as one-eighth of a unit cell, any particle that is centered in the unit cell is constrained to remain at the center of the unit cell through symmetric boundary conditions. In a real material, this central particle could drift from the geometric center of the cluster after void nucleation has occurred at its particle-matrix interface. However, it is felt that the effect of this extra constraint is small.

2.1 Unit Cell Geometries

2.1.1 Particle Cluster Types

Three cluster types are modeled, each with variation in particle cluster density. Figure 2.3 illustrates the one-eighth unit cell models of a linear particle cluster or a “stringer” (Figure 2.3a), a planar or “plate-like” particle cluster (Figure 2.3b), and a quasi-spherical particle cluster (Figure 2.3c). By mirroring the one-eighth unit cell models about their symmetry planes, it can be seen that the full unit cell models contain a linear cluster of three particles, a planar cluster of five particles or a three dimensional cluster of seven particles where each cluster type contains a central particle. Note that all of the ligaments between particles in a given cluster have equal thicknesses.

Figure 2.3: One-eighth unit cell representations of (a) a linear cluster or “stringer”, (b) a planar or “plate-like” cluster and (c) a quasi-spherical cluster.

2.1.2 Particle and Unit Cell Dimensions

In both the clustered and unclustered models, the initial particle radius, r , is held constant and the initial unit cell dimensions, ℓ_i^o , are varied to achieve the desired initial void volume fraction, f^o . In all cases, the unit cell is cubical ($\ell_1^o = \ell_2^o = \ell_3^o$) and the initial void volume fraction is set to 0.01. The dimension ℓ_i^o is calculated as follows:

$$\ell_i^o = \sqrt[3]{\frac{4\pi}{3} \frac{n_p}{8} \frac{r_1 r_2 r_3}{f^o}} \quad (2.1)$$

where n_p is the number of particles within the unit cell and r_i are the lengths of the particle semi-axes. The unit cell dimensions for each of the cluster types are summarized in Table 2.1

cluster type	ℓ_i^0/r
single particle	3.8
linear	5.4
planar	6.4
quasi-spherical	7.15

Table 2.1: The dimensions for cubical unit cells with different cluster types

2.1.3 Cluster Density

The cluster density or microstructural length scale, κ , of a cluster is adopted as a measure of the degree of particle clustering and is defined as the ratio of the size of the inter-cluster ligament between nearest neighbor clusters, D , to the inter-particle ligament size between nearest neighbor particles, d , (see Figure 2.1):

$$\kappa = \frac{D}{d} \quad (2.2)$$

The following four cluster densities, in descending order of cluster severity, are considered:

$$\kappa = 22, \kappa = 10, \kappa = 6, \kappa = 3$$

In an unclustered periodic unit cell (containing a single central void), the cluster density, $\kappa = 1$.

2.2 Finite Element Analysis

2.2.1 Finite Element Solver

Calculations are performed using an explicit, dynamic finite element code, VEC-DYNA (Hallquist, 1988). An explicit code was chosen over an implicit formulation because an explicit solver is typically less expensive for large models since a large

stiffness matrix is not required. The use of an explicit solver also allows calculations to continue well into the strain softening regime induced by damage.

2.2.2 Meshing

The matrix and particle materials were discretized using eight node linear hexahedron bricks with single point integration and a Flanagan and Belytschko (1981) hourglassing control scheme. Examples of the meshes for each of the linear, planar and quasi-spherical cluster models are shown in Figure 2.4. Mesh convergence studies were performed and are discussed in Section 3.1.

2.2.3 Material Properties

For the purposes of Chapters 3, 4 and 5, the matrix material is modeled as an elastic-plastic material with a von Mises isotropic yield function and a piecewise material hardening curve similar to an aluminum 2% silicon alloy (Figure 2.5). The measured softening in the Al -2%Si stress-strain data was not included in the material model. Material softening was modeled through simulating inter-void ligament collapse. This material damage was invoked within the inter-particle ligaments by modeling the matrix material as a Gurson yield function as described in Section 2.6 and implemented in Chapter 6. The particles are modeled as rigid spheres to reduce computation time (the material response with a rigid particle is compared to that with an elastic particle in Section 3.3). Values adopted for the material parameters, E , ν , σ_y , and ρ are given in Table 2.2. The particle-matrix interface properties are described in Section 2.4.

For a dynamic solver such as VECDYNA, the material density is required. In initial studies, the matrix density was that of aluminum, 2.7112×10^{-3} g/mm³ and the particle density was that of silicon, 7.1765×10^{-3} g/mm³. In an effort to

Figure 2.4: Typical finite element discretization for the matrix of the one-eighth unit cell models. (a) a linear cluster ($\lambda = 0.2$), (b) a planar cluster ($\lambda = 0.2$) and (c) a quasi-spherical cluster ($\lambda = 0.2$).

Figure 2.5: The experimental uniaxial stress-strain curve (Pilkey, 1997) and the piecewise material hardening curve used for the matrix material

	Matrix	Particle
E	71,000 MPa	(rigid)
ν	0.33	
σ_y	42 MPa	
ρ	$2.7112 \times 10^{-5} \text{ g/mm}^3$	
		$7.1765 \times 10^{-5} \text{ g/mm}^3$

Table 2.2: Aluminum matrix and silicon particle material properties

reduce computing time further, it was determined that the applied velocities could be increased if the material density is decreased such that a constant wave speed, Λ , is maintained (see Section 2.3.4). Although the decrease in density increases the calculated time step, the inertial effects are reduced, thereby maintaining a static analysis at faster analysis times. A two order of magnitude reduction in density allows a factor of ten decrease in analysis time. The validity of this modification is verified in Section 3.4.

2.2.4 Boundary Conditions

The inner faces of the unit cell are constrained as symmetry planes.

$$u_i(x_i = 0) = 0 \quad (2.3)$$

where u_i is the displacement in the i^{th} direction and x_i is the Cartesian coordinate in the i^{th} direction (Figure 2.2). The outer faces of the unit cell were forced to remain plane and orthogonal through the use of multi-point constraints. These multi-point constraint equations treat all of the nodes on an outer surface of the unit cell as “slave” nodes, except for one which is designated the “master” node. As a result, the normal component of displacement of each slave node, u_i , is then equated to the normal displacement of its master node, u_i^* , such that

$$u_i(x_i = \ell_i) = u_i^* \quad (2.4)$$

where ℓ_i is the current position of the outer face of the cell (Figure 2.2). Global nodal velocities were then applied to each of the three outer unit cell faces to achieve a desired strain or stress state.

2.3 Load Cases

A variety of strain and stress states are considered in order to study damage evolution under a range of sheet forming conditions. Each stress state develops a different degree of constraint within the model, and hence, a different level of stress triaxiality. The triaxiality of the global stress, χ , is measured as a ratio of the hydrostatic stress to the von Mises equivalent stress

$$\chi = \Sigma_{hyd} / \Sigma_{eq}, \quad (2.5)$$

where

$$\Sigma_{hyd} = \frac{1}{3} \sum_{i=1}^3 \Sigma_{ii} \quad (2.6)$$

and

$$\Sigma_{eq} = \sqrt{\frac{3}{2} \sigma'_{ij} \sigma'_{ij}} \quad (2.7)$$

2.3.1 Aligned vs. Transverse Loading

Note that the linear and planar clusters are not symmetrical about the $x_1 = x_2$ plane which implies that a non-spherical stress state, such as uniaxial tension, can be applied in two different directions with respect to the unit cell representing two different load cases. Consider Figure 2.6 which depicts a linear cluster labeled with the global stresses and strains. For the purposes of this research, an aligned uniaxial stress state is imposed utilizing a strain rate in the x_1 direction and with the stresses Σ_2 and Σ_3 set to zero (Table 2.3). A transverse uniaxial stress state incorporates an applied strain rate in the x_2 direction and with the stresses Σ_1 and Σ_3 set to zero. These two different orientations of the same stress state can result in markedly different damage rates.

Figure 2.6: A unit cell representing a periodic array of linear particle clusters with illustrations of stress and strain references

	ϵ_1^∞	ϵ_2^∞	Σ_1	Σ_2	Σ_3
Aligned	ϵ_1^∞	-	-	0	0
Transverse	-	ϵ_2^∞	0	-	0

Table 2.3: The boundary condition settings for aligned and transverse plane stress states on a linear or planar particle cluster.

χ	$\Sigma_2/\Sigma_1, \Sigma_3/\Sigma_1$
0.33	0.0
1.25	0.478
3.00	0.727

Table 2.4: The ratio of primary stress to secondary stresses to maintain a required component of hydrostatic tension.

2.3.2 High Hydrostatic Stress

A uniaxial stress or plane stress state has a triaxiality of $\chi = 0.33$ (see Equations 2.5, 2.6, and 2.7) which is relatively low. For this loading case, a velocity corresponding to a strain rate of $0.417 \times 10^{-3} s^{-1}$ is specified in the major principal strain direction while the stresses on the other two faces are set to zero. The degree of stress triaxiality can be elevated by imposing an additional component of hydrostatic stress. This is achieved by prescribing a strain rate $\dot{\epsilon}_1^\infty$, and then forcing Σ_2 and Σ_3 to maintain a level proportional to Σ_1 . For example, if a triaxiality level of $\chi = 1.25$ is desired, then $\Sigma_2 = \Sigma_3 = 0.478\Sigma_1$. Table 2.4 lists the triaxiality levels considered and the corresponding ratios of Σ_2/Σ_1 and Σ_3/Σ_1 .

Enforcing an elevated triaxiality level requires a prescribed load on the x_2 and x_3 face of the unit cell. For dynamic simulations, load-controlled boundary conditions often result in excessive oscillations unless loading is applied carefully (slowly). In addition, it is difficult to simultaneously impose a prescribed strain rate ($\dot{\epsilon}^\infty$) and triaxiality level using conventional FEM techniques. To achieve this required loading, the stresses are imposed directly to the faces of the unit cell, however a proportional integral derivative (PID) feedback loop is used to control the strain rate, $\dot{\epsilon}^\infty$, and eliminate dynamic oscillations. To initiate the deformation, an arbitrary tensile traction is applied to all three faces of the unit cell. At each subsequent time step in the finite element calculations, the displacement of the major principal outer face of the unit cell is compared to the displacement required to maintain the desired

Figure 2.7: A flow chart for the operation of the PID controller

strain rate of $0.417 \times 10^{-3} s^{-1}$ ($d_{actual} - d_{desired} = error = d(t)$). The derivative of the error, the integral of the error, and the error itself are then used to calculate the traction that needs to be applied at that face to reduce the error (see Figure 2.7). The PID feedback control equation, as taken from Franklin *et al.* (1994) is expressed in the time domain equation as:

$$\mathfrak{S}' = K_p \left(d(t) + \frac{1}{T_I} \int_0^t d(\tau) \partial\tau + T_D \dot{d}(\tau) \right) \mathfrak{S} \quad (2.8)$$

where $\mathfrak{S}$ is the traction applied at the previous time step, $\mathfrak{S}'$ is the traction to be applied at the next time step, $d(t)$ is the displacement error, K_p is the proportional gain, T_I is the integral gain and T_D is the differential gain of the feedback controller. The tractions applied to the other faces of the unit cell are then taken to be a function of the traction on the major principal face such that the χ ratio is maintained (Table 2.4). The three gains of the controller ($K_p = 2 \times 10^4$, $T_I = 1 \times 10^{-2}$, $T_D = 1 \times 10^{-6}$) have been optimized through an iterative procedure and give stable, well-controlled results for all of the different models analyzed (see Section 3.5).

2.3.3 Sheet Forming Biaxial Strain States

Sheet metal formability of materials with particle clustering is of particular interest. There are three characteristic stress states in sheet metal forming: biaxial stretching, plane strain, and deep drawing. These stress states are characteristic lines on a forming limit diagram (Keeler, 1965; Goodwin, 1968) as depicted in Figure 2.8.

Figure 2.8: A forming limit diagram showing the three characteristic strain paths for sheet metal forming and some typical forming limit data.

	$\epsilon_2^\infty / \epsilon_1^\infty$	Σ_3
Biaxial stretch	1	0
Plane strain	0	0
Deep draw	-1	0

Table 2.5: The strain ratios for the three principal stress states used in sheet metal forming.

In order to maintain the required biaxial stress state, the principal strains in the plane of the sheet are kept at a specific ratio (Table 2.5). The stress on the third face of the unit cell, corresponding to the sheet thickness direction, is set to zero.

It should be noted that the plane strain and deep drawing stress states are not symmetrical about the $x_1 = x_2$ plane; that is, the major principal load can be applied in two different directions with respect to the non-symmetrical cluster models (ie. aligned and transverse cases, see Section 2.3.1).

Figure 2.9: A comparison of damage rates at two different strain rates in a model with a single particle model under plane strain conditions ($\chi = 0.333$, no interfacial strength).

2.3.4 Control of Inertial Effects

A static analysis can be approximated with a dynamic solver by using applied strain states which are slow enough to prevent inertia from significantly affecting the solution. In order to examine the effect of inertia on the solution, a series of plane strain calculations were performed with two different strain rates, $\dot{\epsilon}_f^\infty = 0.417 \times 10^{-3} s^{-1}$, and $\dot{\epsilon}_f^\infty = 0.0417 \times 10^{-3} s^{-1}$. Figure 2.9 illustrates that these two strain rates give identical results which implies that the inertial affects are small for strain rates up to at least $\dot{\epsilon}^\infty = 0.417 \times 10^{-3} s^{-1}$.

One of the loading characteristics which can define whether the strain rate is dynamic or static is related to the number of times the stress wave passes through the material during a unit of global strain, referred to here as Λ . If this rate is kept constant, the degree of dynamic stress response in the model should remain constant regardless of the model size. For the model represented in Figure 2.9 which was run

at a strain rate of 0.417×10^{-3} , Λ can be approximated by

$$\Lambda = \frac{c\ell_i}{T\epsilon_f^\infty} \approx 2700 \quad (2.9)$$

where ℓ_i is the length of the model in the direction of major principal strain application, T is the total analysis time (real time, not CPU time), ϵ_f^∞ is the final major principal strain and $c = \sqrt{E/\rho} = 5117 \text{ m/s}$ is the speed of sound in aluminum. The solution times and corresponding loading rates were limited to enforce this number of stress wave transits per unit strain for all of the low triaxiality models considered in this thesis. The rates for the models with a high level of hydrostatic stress were further reduced in order to maintain a static response.

2.4 Nucleation Model

Void nucleation is modeled through a traction-separation relationship at the particle-matrix interface, similar to that used by Needleman (1987). Modeling of nucleation due to particle cracking was not considered and is left for future work. Along the particle-matrix interface, each matrix node is matched to a particle node and the distance between these nodes, in the normal and shear directions, is monitored at each calculation step. The traction-separation relationship is then used to apply equivalent nodal attractive forces between the node pairs. The normal equation (2.10) is identical to that used by Needleman (1987). The shear treatment differs in that the original relationship proposed by Needleman utilized a linear relationship between shear separation and traction. This form was considered in the current model, however, the stresses in the elastic particles of the low triaxiality models became very high and caused model instabilities. It was felt that the shear interface strength had to reduce to zero at some value of shear separation rather than

Figure 2.10: The traction separation behaviour of the particle matrix interface for different ratios of shear to normal separation.

continuing to infinity. The form of the revised shear traction was chosen to mimic the normal traction curve, such that

$$T_n = \frac{27}{4} \sigma_{max} \left\{ \left(\frac{u_n}{\delta} \right) \left[1 - 2 \left(\frac{u_n}{\delta} \right) + \left(\frac{u_n}{\delta} \right)^2 \right] + \alpha \left(\frac{u_s}{\delta} \right)^2 \left[\left(\frac{u_n}{\delta} \right) - 1 \right] \right\} \quad (2.10)$$

$$T_s = \frac{27}{4} \sigma_{max} \left\{ \alpha \left(\frac{u_s}{\delta} \right) \left[1 - 2 \left(\frac{u_s}{\delta} \right) + \left(\frac{u_s}{\delta} \right)^2 \right] + \left(\frac{u_n}{\delta} \right)^2 \left[\left(\frac{u_s}{\delta} \right) - 1 \right] \right\} \quad (2.11)$$

where T_n is the normal traction, T_s is the shear traction, u_n is the normal separation, u_s is the shear separation, δ is the separation distance for permanent separation, σ_{max} is the amplitude of the traction curve, and α is a parameter to relate the shear traction amplitude to the normal traction amplitude. The effects of a variety of particle matrix interface parameters were considered ie. σ_{max} was selected as either 3, 4 or $5\sigma_y$, $\delta = 0.01r$, $0.02r$ or $0.03r$ where r is the particle radius and $\alpha = 0.5$, 1.0 , 3.0 or 5.0 . For the most part, $\alpha = 1$ in this study and $T_s(u_s, u_n) = T_n(u_n, u_s)$. Figure 2.10 illustrates the traction separation behaviour of the particle-matrix interface for different ratios of normal and shear separation.

In the event of closure of the particle-matrix interface, a penalty function-based contact interface (Hallquist, 1988) was used to prevent the particle and matrix ma-

terial from overlapping.

Note that nucleation of a void within the unit cell cluster is defined here as the complete separation of the first node pair within a particle-matrix interface.

2.5 Void Growth Calculations

A post-processor was developed in order to evaluate the void growth parameters, including void volume fraction, global effective plastic strain, and flow stress in the matrix.

The void volume is calculated by numerically integrating the volume enclosed by the element surfaces of the matrix which lie on the the particle-matrix interface. The global effective plastic strain rate in the matrix is defined by Gurson (1975):

$$\dot{\epsilon}_g^p = \frac{\sum_{ij} \dot{\epsilon}_{ij}^{p\infty}}{(1-f)\bar{\sigma}} \quad (2.12)$$

where $\dot{\epsilon}_g^p$ is the effective plastic strain rate in the matrix, Σ_{ij} are the nominal global stresses acting on the outer surfaces of the unit cell, $\dot{\epsilon}_{ij}^{p\infty}$ is the global plastic strain rate, f is the current void volume fraction of the unit cell and $\bar{\sigma}$ is the flow stress. The average flow stress rate within the unit cell is specified from the hardening rate, h , and the increment in effective plastic strain rate,

$$\dot{\bar{\sigma}} = h \dot{\epsilon}_m^p \quad (2.13)$$

2.6 Coalescence Model

In order to introduce material softening due to ligament collapse, the matrix material is assumed to contain a secondary population of small particles which are the site

of strain controlled void nucleation and growth (Basu and Narasimhan, 1999). The particles which make up the second population are much smaller than the smallest elements and their effect on the constitutive behaviour of the matrix is represented by the Gurson (1977) constitutive equation as modified by Tvergaard (1981) and Tvergaard and Needleman (1984).

$$\Phi = \frac{\Sigma_{eq}^2}{\bar{\sigma}} + 2f^* q_1 \cosh \left(q_2 \frac{3\Sigma_{hyd}}{2\bar{\sigma}} \right) - 1 - q_3 f^{*2} = 0 \quad (2.14)$$

The coefficients q_1 , q_2 and q_3 are calibration constants introduced by Tvergaard (1981) in order to more accurately represent the effects of void interaction on material softening due to void nucleation and growth. As in Worswick and Pelletier (1998), the values of the coefficients are taken to be $q_1=1.25$, $q_2 = 0.95$ and $q_3 = q_1^2$.

A modified form of the void volume fraction, f^* , accounts for the effects of void coalescence (Tvergaard and Needleman, 1984).

$$f^* = \begin{cases} f & \text{if } f \leq f_c \\ f_c + \frac{f_u - f_c}{f_f - f_c} (f - f_c) & \text{if } f \geq f_c \end{cases} \quad (2.15)$$

where f is the void volume fraction, f_c is the critical value for the initiation of void coalescence, f_f is the porosity at which the coalescence of voids is complete and f_u^* is the value of f^* at which the material strength disappears. Usually, the value of f_c is taken to be 0.10 and $f_f = 0.12$.

The change in material porosity can be attributed to the growth of existing voids and the nucleation of new voids.

$$\dot{f} = \dot{f}_{nucleation} + \dot{f}_{growth} \quad (2.16)$$

The growth of voids is due to the volumetric dilation of the material since the matrix itself is considered to be incompressible.

$$\dot{f}_{growth} = (1 - f)\dot{\epsilon}_{ii}^{p\infty} \quad (2.17)$$

The nucleation of small secondary voids is assumed to be a strain controlled event with a normal distribution where a fraction f_N of particles nucleate voids during deformation at a mean strain of ϵ_N and a standard deviation of S_N .

$$\dot{f}_{nucleation} = A\dot{\epsilon}^p \quad (2.18)$$

where

$$A = \frac{f_N}{S_N\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{\bar{\epsilon}_g^p - \epsilon_N}{S_N} \right)^2 \right] \quad (2.19)$$

The effective plastic strain within an element is calculated according to the principle of work equivalence,

$$\dot{\epsilon}_g^p = \frac{\Sigma_{ij}\dot{\epsilon}_{ij}^{p\infty}}{(1 - f)\bar{\sigma}} \quad (2.20)$$

where $\dot{\epsilon}^p$ is the effective plastic strain rate for the element, Σ_{ij} is the element stress, $\dot{\epsilon}_{ij}^{p\infty}$ is the rate of elemental plastic strain, f is the void volume fraction nucleated at the secondary population of particles and $\bar{\sigma}$ is the flow stress in the element. The plastic flow in the element is taken from the uniaxial stress versus effective plastic strain curve (Figure 2.5).

$$\dot{\bar{\sigma}} = h\dot{\epsilon}_g^p \quad (2.21)$$

where h is the slope of the uniaxial stress-strain curve.

Using this approach, two particle populations are modeled, one consisting of larger particles that are modeled discretely and the other, much smaller set of par-

ticles being treated using the foregoing Gurson damage model. Damage progresses through decohesion of the larger particles as well as distributed damage within the matrix. As the conditions for void coalescence in the matrix are met (Equation 2.15), the matrix elements are deleted thus allowing coalescence to occur. The advantage of adopting a Gurson-based damage model for the matrix lies in its sensitivity to triaxiality in predicted void growth rates. Thus the effect of constraint on the inter-particle ligament caused by two or more clustered large particles and the interaction with the remote stress triaxiality can be captured.

2.7 Summary

The various aspects of these models represent a unique contribution to the development of ductile fracture predictive models. The effects of particle clustering on damage during all three stages of ductile failure can be studied, including void nucleation, growth and coalescence. The introduction of particle clusters within a unit cell approach allows consideration of both inter-particle and inter-cluster coalescence processes. Many strain and stress states are applied (uniaxial, biaxial and triaxial) providing a wide range of hydrostatic stress components. Parametric studies of the nucleation and coalescence coefficients of the Gurson-Tvergaard-Needleman constitutive equations will give insight into how these values affect predicted strain-to-failure in a clustered particle population.

Chapter 3

Model Validation

Prior to examining the results of the models described in Chapter 2 evidence will be presented concerning the validity of the models. Some predictions will also be compared to those of other researchers.

3.1 Mesh Discretization Studies

In the light of the qualitative nature of this research and the limitation in available computer resources, a mesh reduction study was conducted rather than a rigorous convergence study. The first study examines the quality of mesh refinement in unit cells containing a single central void. Figure 3.1 illustrates the coarsest mesh in the study which contains 135 elements. The surface of the void is discretized into 27 elements (3 patches, each with 3 by 3 elements) and there are 5 layers of elements between the surface of the void and the outer boundaries of the unit cell. Figure 3.2 illustrates the finest mesh in the study which contains 1764 elements. The surface of the void is discretized into 147 elements (3 patches, each with 7 by 7 elements)

Figure 3.1: The discretization of one-eighth of a unit cell containing a single central void into 135 elements.

Figure 3.2: The discretization of one-eighth of a unit cell containing a single central void into 1764 elements.

and there are 12 layers of elements between the surface of the void and the outer boundaries of the unit cell. The discretization of the other models are summarized in Table 3.1.

Figure 3.3 compares the void growth rates in the model subject to uniaxial tensile loading for different levels of discretization. The finest mesh which uses 1764 elements is still not refined enough to obtain a converged result but it is clear that the solution is becoming more accurate. In this thesis, since the studies are, to

number of elements	elements on void surface	number of element layers
135	27 (3×3×3)	5
336	48 (4×4×3)	7
600	75 (5×5×3)	8
1080	108 (6×6×3)	10
1764	147 (7×7×3)	12

Table 3.1: The discretization of unit cell elements containing a single void used in the meshing study

Figure 3.3: A comparison of void growth rates for different levels of mesh refinement in a unit cell containing a single void (uniaxial tension).

Figure 3.4: A comparison of void growth rates for different levels of mesh refinement in a unit cell containing a linear cluster of voids (uniaxial tension, transverse loading).

some degree, qualitative rather than quantitative, the 1080 element discretization is considered to be sufficient for reliable results and to capture trends associated with cluster geometry and loading.

In the case of a model containing a void cluster, a separate discretization study was carried out to verify the accuracy of the stress-strain fields in the inter-void ligaments. The number of elements in the inter-void ligament will become particularly important when examining void coalescence and macroscopic material failure. Figure 3.4 compares total void growth in a linear particle cluster under uniaxial tensile loading which is oriented transverse to the cluster orientation. The inter-void ligament has a initial thickness that is 20% of the initial void radius which corresponds to a cluster density of $\kappa = 22$. A transverse major principle strain leads to higher inter-void ligament strains as will be further discussed in Section 4.1.1.

Table 3.2 summarizes the number of elements that constitute each void surface

number of elements	elements on void surface	number of elements through ligament thickness
373	27 ($3 \times 3 \times 3$)	4
897	48 ($4 \times 4 \times 3$)	6
1227	75 ($5 \times 5 \times 3$)	6
2117	108 ($6 \times 6 \times 3$)	8
2903	147 ($7 \times 7 \times 3$)	8

Table 3.2: The discretization of unit cell elements, containing a linear cluster of voids, used in the linear cluster meshing study

Figure 3.5: The discretization of one-eighth of a unit cell, containing a linear cluster of voids, into 373 elements.

and the number of elements through the inter-void ligament. The discretization of the linear cluster model with 373 elements is depicted in Figure 3.5. The discretization of the linear cluster model with 2117 elements is depicted in Figure 3.6. As in the discretization study for the single void model, the analyses shown in Figure 3.4 have not converged but the 2117 element solution is considered to be adequate for the purposes of this parametric study.

3.2 Void Growth Rates

In order to benchmark the models used in this thesis, some published research has been reproduced using the current model. Both Worswick and Pick (1990) and Kuna

Figure 3.6: The discretization of one-eighth of a unit cell, containing a linear cluster of voids, into 2117 elements.

	Worswick and Pick	Kuna and Sun
f_o	0.01	0.0114
E	30 GPa	195 GPa
σ_y	100 MPa	366 MPa
n	0.2	≈ 0.13 *
ν	0.3	0.3
elements	192 20-node bricks	540 20-node bricks
nodes	1560	3521

* approximate power law equivalent to a piece-wise material model

Table 3.3: The model parameters from similar studies by Worswick and Pick (1990) and Kuna and Sun (1996) used to benchmark the current work.

and Sun (1996) reported void growth rates for three-dimensional unit cell models with low initial void volume fractions subject to uniaxial tension and various levels of hydrostatic stress. Both of these studies utilized the implicit finite element code, ABAQUS, and the model parameters are listed in Table 3.3. VEC DYNA only supports 8-node brick elements and single point integration with hourglass control. As a result, the VEC DYNA models were given an equivalent number of nodes, rather than an equivalent number of elements, in order to obtain a similar degree of mesh convergence as in the published results. All other parameters including piece-wise

Figure 3.7: Comparison of the void growth rates in the current work with growth rates reported by Worswick and Pick (1990) and Kuna and Sun (1996). These models are unit cells containing a single spherical void under uniaxial tensile deformation with superimposed hydrostatic tension.

material models were reproduced from the published data.

Figure 3.7 indicates good agreement in predicted void growth rates between the current FEM calculations and those made by Worswick and Pick (1990) and Kuna and Sun (1996). Note that the Worswick and Pick (1990) data was published in terms of effective plastic strain in the unit cell, $\bar{\epsilon}_g^p$, as is done in the current work but Kuna and Sun (1996) report void growth in terms of equivalent total strain in the unit cell, $\bar{\epsilon}_g$. Since the plastic strains are so large compared to the elastic strains in these cases, there is only a very slight discrepancy between the data of Kuna and Sun (1996) and the data reported from the current FEM calculations.

Detailed data from the Worswick and Pick (1990) calculations was available and each component, such as effective plastic strain vs. major principal strain ($\bar{\epsilon}_g^p$ vs. ϵ_1) and major principal stress vs. major principal strain (Σ_1 vs. ϵ_1), has been compared

Figure 3.8: Comparison of other stress and strain data obtained from the study by Worswick and Pick (1990) illustrating the similarity of model performance.

in detail with the corresponding values in the current model and have been found to have nearly identical values (Figure 3.8). Only the predicted void growth rates are somewhat different and that is attributed to differences in mesh convergence levels. Given the good agreement with other void growth studies as shown in Figures 3.7 and 3.8, the current models are judged to perform well.

3.3 Rigid Particles

In all models presented in this thesis, the discrete particles are assigned a rigid material description in order to reduce computational costs. This section presents a brief study comparing unit cell behaviour with elastic silicon particles, to that with rigid particles. Table 3.4 presents the pertinent data for the rigid and elastic silicon material models along with the Aluminum matrix material model. Figure 3.9 illustrates the difference that the rigidity of the particle material model makes on the damage rate in an aligned linear cluster deformed under uniaxial tension. There is a slight difference in the void growth rates but this is primarily due to the progression of nucleation. In the model with a rigid particle description, the first nucleation event occurs within the inter-particle ligament at the outer particle edge. In contrast, the

	Aluminum Matrix	Rigid Silicon Particle	Elastic Silicon Particle
E	71,000 MPa	∞	177,700 MPa
ν	0.33	-	0.2
σ_y	42 MPa	-	-
ρ	2.7112×10^{-5} g/mm ³	7.1765×10^{-5} g/mm ³	7.1765×10^{-5} g/mm ³

Table 3.4: Aluminum matrix, rigid and elastic silicon particle material properties

Figure 3.9: Comparison of the void growth rates in linear clusters of elastic particles as opposed to those with rigid particles. Nucleation events are located in (a) and strain levels are indicated in (b). Models are deformed in aligned uniaxial tension with $\kappa = 22$, $\sigma_{max} = 5\sigma_y$, $\delta = 2.0r$ and $\alpha = 1.0$

Figure 3.10: Comparison of void shape in linear clusters of elastic particles as opposed to those with rigid particles. Models are deformed in aligned uniaxial tension to $e_{11} \approx 0.35$ with $\kappa = 22$, $\sigma_{max} = 5\sigma_y$, $\delta = 2.0r$ and $\alpha = 1.0$

model with elastic particles also first nucleates a void at the inter-particle ligament, but next to the central particle. There is no detectable deformation in the elastic particles, as seen in Figure 3.10, which illustrates the void shapes in both models at a major principal engineering strain of $e_1 \approx 0.35$. However, there is enough flexibility in the elastic particle to change the nucleation progression. Apparently, a void nucleated at the outer particle (point 2) grows faster than a void nucleated at the central particle (point 1). At higher strains ($e_1 \approx 0.4$), the level of damage and the damage rates are almost identical. Overall, the slight reduction in particle flexibility with a rigid particle is assumed to have an acceptably small influence on the damage progression in these models.

3.4 Density Reduction

In order to reduce computing time, an effort was made to run the models with as high a strain rate as possible. The strain rate is limited, however, by the manifestation of dynamic effects such as stress lag due to the momentum of the model and oscillations due to stress waves traveling through the material. The strain rate can be increased

Figure 3.11: Comparison of the stress-strain curves of three models, one with the typical density, ρ , of aluminum with a low strain rate, $\dot{\epsilon}_1^\infty$, one with a standard density but a high strain rate, and the third with a much lower density and a high strain rate. These models are of single voids under uniaxial tension.

beyond these levels, however, by reducing the density of the material. This increases the speed of sound in the material (and so the velocity of stress waves) and it reduces the momentum of the material, thereby eliminating stress lag.

Figure 3.11 illustrates the effect of increasing the strain rate from $\dot{\epsilon}_1^\infty = 0.417\text{ms}^{-1}$ to $\dot{\epsilon}_1^\infty = 4.167\text{ms}^{-1}$ with and without reducing the density. At this strain rate, the model with a regular density, $\rho = 2.7 \times 10^{-3}\text{g/mm}^3$, reaches plastic stresses at higher strains and the amplitude of oscillations from traveling stress waves is a significant percentage of the flow stress. The standard stress-strain response can be maintained at the higher strain rate if the density of the material is mathematically reduced by two orders of magnitude ($\rho = 2.7 \times 10^{-5}\text{g/mm}^3$).

In this thesis the reduction in density is directly related to the increase in strain rate such that the number of stress waves passing through the material per unit strain remains constant as explained in Section 2.3.4.

As a further confirmation of the validity of the changes in the density and in the strain rates, Figure 3.12 shows that the void growth rates are unaffected. Increasing the strain rate alone does increase void growth, which is seen as a sudden void

Figure 3.12: Comparison of the void growth rates of three models, one with the typical density, ρ , of aluminum with a low strain rate, one with a standard density but a high strain rate, and the third with a much lower density and a high strain rate. These models are of single voids under uniaxial tension.

growth in the early stages of deformation.

3.5 PID Controller

The proportional-integral-differential controller allows the displacement rates and the stress ratios to be controlled at the same time in a dynamic finite element solver such as VECDYNA (see Section 2.3.2). Figure 3.13 demonstrate how a well-tuned controller can follow a prescribed displacement very closely.

A required level of hydrostatic stress is achieved by setting the global stress on the two minor faces to be a prescribed ratio of the major principal stress (see Section 2.3.2). The principal stresses of a model under moderate hydrostatic stresses ($\chi = 1.25$) are depicted in Figure 3.14. Note that the stress-strain curves for the principal faces all have the same shape. Figure 3.15 illustrates how consistently the principal stress ratios are maintained for the same model. These ratios of hydrostatic stress to equivalent stress are calculated from the stresses on the outer faces of the unit cell in the FEM results and follow the prescribed values very closely.

Figure 3.13: The effectiveness of the PID controller in following a prescribed displacement.

Figure 3.14: The effectiveness of the PID controller in controlling minor principal stresses in a high hydrostatic stress model ($\chi = 1.25$).

Figure 3.15: The effectiveness of the PID controller in controlling stress ratios in a high hydrostatic stress model ($\chi = 1.25$).

Chapter 4

Void Growth in the Absence of Particles

The prediction of void growth from the models which do not contain particles is the focus of this chapter. There are a number of parameters that can be examined on their own without the further complication of particle-matrix interaction and the presence of a secondary population of voids. Such complicating issues are examined in Chapters 5 and 6. The parameters that will be discussed here are the effects of void cluster type, cluster orientation, cluster density and applied strain state.

Ductile material damage is often measured in terms of void volume fraction, f . As such, the primary basis for evaluation of parameter effects will be to compare the void growth, normalized with respect to initial void volume fraction ($\frac{f}{f_0}$), as a function of global effective plastic strain in the unit cell ($\bar{\epsilon}_g^p$). In some cases, the changes in void shape will also be examined in order to characterize void growth. Severe levels of damage will often be observed as softening in the stress-strain curve.

For the low levels of void volume fraction considered in this thesis, however, softening of the material due to void growth is minimal. In the cases involving high levels of superimposed hydrostatic tension ($\chi > 1$), void growth accelerates very quickly and material softening can be observed.

Some of the void shape evolution and contour plots in this section show an unrealistic degree of ligament thinning since there is no model for material failure and element deletion in these models. The over-stretched ligaments will add to the stiffness of the model. In Chapter 6, there is an element deletion model that allows these ligaments to fail before unrealistic strains are reached.

4.1 Cluster Type

As described in Section 2.1.1, the void growth behaviour of three void cluster types (see Figure 2.3) is compared to the growth of a single void in a unit cell. The linear cluster comprises three voids arranged in a line parallel to what could be considered the rolling axis of a sheet of metal. The planar cluster consists of five voids arranged in a plane, the normal of which is transverse to the rolling axis. The spherical cluster is made up of seven voids, six outer voids distributed on the three axes around a central void. The linear and planar clusters are anisotropic which implies that deformation in the aligned (rolling) and transverse directions will yield different void growth results. For examination of the effects of cluster type, only the void clusters with high density, $\kappa = 22$, are considered since the effect of cluster type becomes more pronounced with elevated cluster density.

Figure 4.1: Void growth rates within various cluster types under uniaxial tension ($\kappa = 22$, $\chi = 0.33$)

4.1.1 Uniaxial tension

Figure 4.1 displays the void growth rates of the four cluster types under uniaxial tension where the major stress is applied parallel to the plane of the sheet (plane stress). The distributions of local effective plastic strain in each of the cases considered in Figure 4.1 are given in Figures 4.2 through 4.7. All of these images are taken at the same level of major principal engineering strain ($e_1 \approx 0.35$), and, for plotting purposes, consider the same range of plastic strain ($0.13 < \epsilon^p < 0.8$) and the same level of magnification. Note that in the case of uniaxial tension, the level of hydrostatic tension is low ($\chi = 0.33$) and void growth is primarily extensional.

Aligned Uniaxial Tension ($\Sigma_{22} = \Sigma_{33} = 0$)

Three of the clusters, the linear, planar and spherical clusters under aligned uniaxial tension, contain a row of voids aligned with the major principal strain direction. The voids within this row that are not the central void have been termed outer aligned voids and any voids that are not part of the row are labeled outer transverse voids. When there are a row of voids parallel to the major principal stress, the ligaments

Figure 4.2: Contours of plastic strain in a single void model at $e_1 \approx 0.35$ of uniaxial deformation. Dark blue contours indicate $\epsilon^p = 0.13$ and red contours indicate $\epsilon^p = 0.8$.

Figure 4.3: Contours of plastic strain and in a linear cluster model at $e_1 \approx 0.35$ of aligned uniaxial deformation. Dark blue contours indicate $\epsilon^p = 0.13$ and red contours indicate $\epsilon^p = 0.8$.

Figure 4.4: Contours of plastic strain in a linear cluster model at $e_1 \approx 0.35$ of transverse uniaxial deformation. Dark blue contours indicate $\epsilon^p = 0.13$ and red contours indicate $\epsilon^p = 0.8$.

Figure 4.5: Contours of plastic strain in a planar cluster model at $e_1 \approx 0.35$ of aligned uniaxial deformation. Dark blue contours indicate $\epsilon^p = 0.13$ and red contours indicate $\epsilon^p = 0.8$.

Figure 4.6: Contours of plastic strain in a planar cluster model at $e_1 \approx 0.35$ of transverse uniaxial deformation. Dark blue contours indicate $\epsilon^p = 0.13$ and red contours indicate $\epsilon^p = 0.8$.

Figure 4.7: Contours of plastic strain in a spherical cluster model at $e_1 \approx 0.35$ of uniaxial deformation. Dark blue contours indicate $\epsilon^p = 0.13$ and red contours indicate $\epsilon^p = 0.8$.

Figure 4.8: Evolution of the void shape in each void of the single void, linear, planar, and spherical cluster models at a strain of $e_1 \approx 0.35$ (aligned uniaxial tension, $\kappa = 22$). Note that the void shape evolution changes as the number of voids in the cluster increases due to changes in local flow fields.

between the voids are shielded leading to low plastic strain in the ligaments as seen by the dark blue color in the contour plots of Figures 4.3, 4.5 and 4.7. Figure 4.8 examines the shape change of the various voids in a cluster more closely.

When comparing the linear cluster void growth to the single void growth in Figure 4.1, it can be seen that the elongated nature of the linear cluster leads to lower void growth rates. When the shape changes of the voids are also examined, the relative transverse void contraction is the same in both cases but the extensional growth of all the voids in the linear cluster is lower than in the single void model

(Figure 4.8). Note that the extensional growth of the central void of the linear cluster is lower than that of the outer aligned voids since the central void is shielded from the major principal stress in the same way as the inter-void ligaments are shielded. In brief, an increase in the number of aligned outer voids decreases void growth rates in a clustered model under uniaxial tension.

When comparing the three clustered models in Figure 4.8, the first observation is that the void contraction in the two minor principal strain directions is the same for all voids in all clusters and the cluster type has no effect on lateral contraction of individual voids under uniaxial tension. This is somewhat surprising as the inter-void ligaments in the transverse direction show a high level of plastic strain as indicated by the yellow and red contours in Figures 4.5 and 4.7. In effect, the major principal strain, ϵ_{11}^{∞} , that is local to the cluster is concentrated in the transverse ligaments. The more transverse ligaments there are, the less dissipation there is of major strain, ϵ_{11}^{∞} in the vicinity of the central void and the longer the central void becomes. The outer transverse void elongation does not reflect the high transverse ligament strains to the same degree because the principal strains around the majority of the surface of the void are dissipated. Figure 4.8 illustrates the equal elongation rate of the outer transverse voids in the major principal strain direction for both the planar and spherical clusters.

As the number of outer transverse voids in a cluster increases from 0 (linear) to 2 (planar) to 4 (spherical), the outer aligned voids show a very slight decrease in elongation rate. It is the central void that is most affected by the change in the number of outer transverse voids and the high transverse inter-void ligament strains. The central void is experiencing a higher level of extensional growth, overwhelming the tendency of the outer aligned voids to shield the central void, partially at the expense of the the growth of the outer aligned voids. The overall result is that, under

uniaxial tension, the total void growth rates in the model increase as the number of outer transverse voids increase.

Transverse Uniaxial Tension ($\Sigma_{11} = \Sigma_{33} = 0$)

There are two void cluster types considered in this thesis that do not contain a row of voids parallel to the major principal strain axis, that is the linear cluster and the planar cluster under transverse loading. Under this loading condition, the linear cluster contains two outer transverse voids and the planar model contains four outer transverse voids. Figure 4.9 illustrates the void shape change for each void in the single void, linear, and planar cluster models deformed under transverse uniaxial tension.

As with the aligned loading cases of Figure 4.8, it can be seen that the void contraction in the two minor principal strain directions is consistent for all voids in all cluster types. In fact, void contraction in the i^{th} direction is proportional to the unit cell contraction in the i^{th} direction whenever $\Sigma_i = 0$. The outer transverse void elongation in the direction of major principal strain, x_2 , only sees a slight increase as the number of outer transverse voids increase from 0 (single void) to 2 (transverse linear cluster) to 4 (transverse planar cluster). Once again, as with the aligned loading cases, it is the central void that has the greatest increase in extensional growth as the number of outer transverse voids increase.

Looking again at Figure 4.1, the inclusion of aligned outer voids reduces the void growth rate within a cluster significantly (single void model versus linear cluster). Conversely, if outer transverse voids are added to the model, the void growth increases. The models without aligned outer voids have the highest void growth rates.

Figure 4.9: Evolution of the void shape in each void of the single void, linear, and planar cluster models at a strain of $e_1 \approx 0.35$ (transverse uniaxial tension, $\kappa = 22$).

Figure 4.10: Void growth rates within various cluster types under plane strain deformation ($\kappa = 22$, $\chi \approx 0.58$)

4.1.2 Plane Strain

One of the main differences between uniaxial tension and plane strain deformation is that there are three distinct principal stresses in plane strain deformation whereas uniaxial tension has only two. In addition, the component of hydrostatic tension in plane strain deformation, $\chi \approx 0.58$, is higher than that of uniaxial tension, $\chi = 0.33$. Figure 4.10 compares the void growth rate of the four cluster types under aligned and transverse plane strain deformation.

In order to gain further insight into the void growth behaviour of the different cluster types, the growth of each void in the three symmetry planes is considered in the following subsections. Since there are now three distinct principal stresses to consider, there are three types of outer voids in the clusters. As in the uniaxial tension cases, there are outer aligned voids. The outer transverse voids are now taken to mean those voids which are transverse to the major principal stress direction but still in the plane of the sheet (ie. parallel to Σ_{22}). The other outer transverse voids are now termed through-thickness voids since they are stacked normal to the plane of the sheet or in the x_3 direction. As with the examination of void growth under

Figure 4.11: Evolution of the void shape in each void of the single void, linear, planar, and spherical cluster models at a strain of $e_1 \approx 0.35$ (aligned plane strain, $\kappa = 22$).

uniaxial tension, the clusters are grouped according to whether or not they contain outer aligned voids.

Aligned Plane Strain Loading ($\epsilon_{22}^\infty = 0$)

Figure 4.11 compares the change of shape of each void in the single void, aligned linear cluster, aligned planar cluster, and the spherical cluster under plane strain deformation. All three cluster models contain a row of three voids aligned with the major principal strain direction.

In some ways, the trends in void shape changes in aligned clusters deformed in

plane strain are very similar to those seen in the aligned clusters deformed in uniaxial tension (Figure 4.8). As in uniaxial deformation, the voids in aligned clusters all show the same degree of void contraction in the x_3 direction, the central void growth is limited due to shielding by the outer aligned voids, and the more outer transverse voids there are in a cluster, the more strain concentration in the transverse ligaments and the more central void growth occurs in the direction of major principal strain.

There are also differences due to the change in applied strain state. In plane strain loading, the aligned clusters develop more pronounced differences in elongation in the major strain direction as the number of transverse voids increases than with uniaxial tensile loading. This is likely a result of the positive stress in the x_2 direction which compounds the strain concentration in the transverse ligaments. There is also void elongation in the medial principal strain direction (x_2 direction) since the applied medial stress, Σ_{22} is greater than zero. The elongation rate increases as the number of voids in the $x_1 - x_3$ plane increases. It is interesting to note that though the medial stress, Σ_{22} , is a fraction of the major principal stress, Σ_{11} , in plane strain loading, there is some degree of ligament and void shielding in the x_2 direction in a spherical cluster. A third difference is the more pronounced “squaring” of the the voids towards their nearest neighboring void. The central void becomes less ellipsoidal and more rectangular in all three symmetry planes (Figure 4.11) as the number of voids in the cluster increase. This “squaring” effect is also evident in the aligned and transverse voids at the poles next to the central void. Void squaring would have as strong an influence on damage rate increases as void elongation. The degree of void squaring is likely a function of the component of hydrostatic tension, a theory supported in later examinations of other applied strain states.

Referring back to Figure 4.10, the shielding effect of the outer aligned voids in the aligned linear cluster reduces void growth as with uniaxial tensile deformation.

The addition of transverse voids concentrates strains in the transverse ligaments and therefore increases the void growth, an effect which is compounded by additional strain concentration in the x_2 direction due to the positive medial stress, Σ_{22} . An increased component of hydrostatic stress also contributes to increased void growth through void “squaring” as the number of voids in the cluster increases.

Transverse Plane Strain Loading ($\epsilon_{11}^\infty = 0$)

Figure 4.12 compares the change in void shape, under plane strain deformation, of every void in the clusters that do not contain outer aligned voids, that is the single void model, the transverse linear cluster and the transverse planar cluster. The void shape changes illustrated in this figure follow the same trend as that described for the case of transverse uniaxial tension; more voids in the $x_1 - x_3$ plane lead to a higher degree of central void extension in the major principal strain direction. As in aligned plane strain loading, the positive medial principal stress, this time in the x_1 direction, serves to compound the transverse ligament strains in the major stress (x_2) and medial stress (x_1) directions. This stress compounding leads to higher levels of central void extension in the x_1 and, to some degree, the x_2 directions as the number of voids transverse to the two principal stresses increases. It was also illustrated in the aligned plane strain case that the higher component of hydrostatic stress, as compared to that of uniaxial tension, leads to void “squaring” in clusters of voids (more voids in a cluster results in more void squaring) and increased void growth. Overall, these effects cause significantly higher void growth in the transverse planar cluster than in the transverse linear cluster (Figure 4.10).

Figure 4.12: Evolution of the void shape in each void of the single void, linear, and planar cluster models at a strain of $e_1 \approx 0.35$ (transverse plane strain deformation, $\kappa = 22$).

Figure 4.13: Void growth rates within various cluster types under equi-biaxial stretch deformation ($\kappa = 22$, $\chi \approx 0.67$)

4.1.3 Equi-biaxial Stretch ($\Sigma_{11} = \Sigma_{22}$)

In the case of an equi-biaxial stretch deformation state, there are two non-zero global stresses as with the plane strain deformation but the two major principal stresses are equal. Since the stress state is symmetrical, there is no distinction between aligned and transverse loading of the anisotropic clusters. The component of hydrostatic stress is also high at $\chi \approx 0.67$. Figure 4.13 compares the void growth rates of the various clusters under an equi-biaxial stretch.

Once again, the shape change of the individual voids within the clusters is examined in detail in Figure 4.14. The symmetrical stress field leads to counteracting effects in the in-plane ligaments; a ligament along the x_1 axis is sheltered from the major principal strain in the x_1 direction but at the same time, strains in the x_2 direction are concentrated in the same ligament since there is also a major principal strain in the x_2 direction. In the linear cluster, the increased growth due to strain concentration in the x_2 direction is not enough to counteract the reduced void growth due to ligament shielding in the x_1 direction, thus the overall void growth is less than in a single void model (Figure 4.13). In the planar cluster, however, the

Figure 4.14: Evolution of the void shape in each void of the single void, linear, planar, and spherical cluster models at a strain of $e_1 \approx 0.35$ (equi-biaxial stretch deformation, $\kappa = 22$).

Figure 4.15: Void growth rates within various cluster types under deep draw deformation ($\kappa = 22$, $\chi \approx 0.0$)

four outer voids transverse to Σ_{22} tip the balance and the strain concentration in the transverse ligaments (in the x_2 direction) has a stronger effect than the ligament shielding in the x_1 direction, resulting in increased void overall void growth as compared to the single void model. The greater number of voids in the planar cluster and the relatively high component of hydrostatic stress also combine to increase the degree of void squaring and contribute to the increased overall void growth within the planar cluster.

4.1.4 Deep Draw ($\epsilon_{22}^\infty = -\epsilon_{11}^\infty$)

The deep draw sheet forming strain state has two non-zero principal stresses as in the plane strain and equi-biaxial strain states, but one is negative. As a result the voids tend to collapse in the minor principal stress direction, as reflected by the decreasing void volume fractions in Figure 4.15. The component of hydrostatic stress in deep draw deformation is around zero ($\chi \approx 0$).

Figure 4.16 compares the shape change of the single void to growth of individual voids in the aligned clusters, that is the aligned linear cluster, the aligned planar

Figure 4.16: Evolution of the void shape in each void of the single void, linear, planar, and spherical cluster models at a strain of $e_1 \approx 0.35$ (aligned deep draw deformation, $\kappa = 22$).

cluster and the spherical cluster. The aligned clusters all contain a row of voids parallel to the major principal stress direction.

The changes in void shape for different clusters are smaller under deep drawing deformation than under the other strain states considered in this thesis. There is also no evidence of squaring since the hydrostatic component of stress, $\Sigma_{hyd} = 0$. Figure 4.17 depicts contours of local effective plastic strain in a spherical cluster under deep draw deformation to $e_1 = 0.35$. It can be seen that the maximum local plastic strains under deep draw deformation are not as high as in the uniaxial deformation case of Figure 4.7 ($\bar{\epsilon}_{max}^p \approx 0.57$ in a deep draw vs. $\bar{\epsilon}_{max}^p \approx 0.71$ in uniaxial

Figure 4.17: Contours of plastic strain in a spherical cluster model at $\epsilon_1 \approx 0.35$ of aligned deep draw deformation. Dark blue contours indicate $\epsilon^p = 0.13$ and red contours indicate $\epsilon^p = 0.8$.

tension). On the other hand, the inter-void ligaments in the row of aligned voids are not shielded in the same way as they were in uniaxial tension ($\bar{\epsilon}_{ligament}^p \approx 0.4 - 0.54$ in a deep draw vs. $\bar{\epsilon}_{ligament}^p \approx 0.13 - 0.19$ in uniaxial tension). The compressive negative principal stress changes the plastic flow behaviour in the aligned ligaments, reducing the shielding effect.

To further examine this effect, consider the change in the void contraction behaviour in the x_2 direction with the addition of an outer void in the x_2 direction in the spherical cluster (Figure 4.16). It is interesting to note that the outer transverse voids offer a shield in the x_2 direction and so the contraction of the central void in the x_2 direction is reduced in the spherical cluster.

Figure 4.18 serves to further illustrate the behaviour of transverse void clusters deformed in deep drawing. As in aligned deep drawing, the differences in void shape between voids in the cluster are minimal in transverse deep drawing.

Overall, the negative principal stress in deep draw deformation has two effects on

Figure 4.18: Evolution of the void shape in each void of the single void, linear, and planar cluster models at a strain of $e_1 \approx 0.35$ (transverse deep draw deformation, $\kappa = 22$).

Figure 4.19: Void growth rates within various cluster types under high hydrostatic stress deformation ($\kappa = 22$, $\chi = 1.25$)

the shape change of individual voids in a cluster. The first is to reduce the shielding effect in rows of voids parallel to positive principal stress directions and the other is to collapse voids in the negative principal stress direction, an effect which can be reduced if there is a row of voids parallel to the negative principal stress.

4.1.5 High Triaxiality

Material at the root of a notch or at a crack tip experiences elevated levels of hydrostatic stress since it is constrained by the un-stressed material around it. As established in early void growth studies by Rice and Tracey (1969) and McClintock (1968a), this high level of triaxiality leads to exponential increases in void growth rate. Thus it is of interest in this thesis to study void cluster growth behaviour under high hydrostatic stress conditions.

The high triaxiality load cases are equivalent to uniaxial tension, superimposed with a large value of hydrostatic stress. Figure 4.19 illustrates the effect of uniaxial deformation with a triaxiality of $\chi = 1.25$, rather than $\chi = 0.33$ for uniaxial tension, on the growth of various void clusters. The most obvious difference between

Figure 4.20: Evolution of the void shape in each void of the single void, linear, planar, and spherical cluster models at a strain of $e_1 \approx 0.35$ (aligned uniaxial tension with superimposed high triaxiality ($\chi = 1.25$), $\kappa \leq 22$).

Figure 4.19 and Figure 4.1 is the much higher levels of void expansion under high triaxiality, as expected. Note that the void growth rate increases under high hydrostatic stress, whereas the void growth rate under uniaxial tension decreases with strain (Figure 4.1).

Figure 4.20 shows the shape change of the aligned clusters under high hydrostatic stress. Examination of the aligned linear cluster shapes reveals an increase in the shielding of the central void under high hydrostatic tension, as seen by the increased difference in void growth in the x_1 direction of the outer aligned void and the central

Figure 4.21: Contours of plastic strain in a spherical cluster model at $e_1 \approx 0.35$ of high hydrostatic stress deformation ($\chi = 1.25$). Dark blue contours indicate $\epsilon^p = 0$ and red contours indicate $\epsilon^p \geq 2$.

void, compared to that of an aligned linear cluster in uniaxial tension (Figure 4.8). The increased component of hydrostatic stress also leads to increased concentration of ϵ_{11} strain in the transverse ligaments of the planar and spherical clusters as well as extreme void squaring in all cluster models.

Overall, the void growth increases under high triaxiality as expected but the void interactions also become more pronounced. It should be noted that the transverse ligaments show a significant degree of thinning and plastic strain ($\epsilon^p \approx 1.8$ at $e_1 = 0.35$). At this level of deformation, the void clusters that contain transverse voids take on the appearance of a single void spanned by thin membranes as seen in Figure 4.21, which depicts a spherical cluster deformed under a high component of hydrostatic stress ($\chi = 1.25$). If these models were equipped with a material failure criterion, the transverse ligaments would have torn, leading to coalescence of the central and outer transverse voids.

The void interactions at a triaxiality of $\chi = 3.00$, which may be found at a crack

Figure 4.22: Void growth rates within various cluster types under high hydrostatic stress deformation ($\kappa = 22$, $\chi = 3.00$)

tip, are even more pronounced than at $\chi = 1.25$. At this extreme level of triaxiality, the deformation is almost completely hydrostatic. Figure 4.22 shows the rapid rates of void growth. Note that these models are only run to global effective plastic strains of $\bar{\epsilon}_g^p \approx 0.15$. The high level of hydrostatic stress increases dynamic effects in these models and as a result, they must be run very slowly. The shielding of ligaments in one principal strain direction is now overwhelmed by the concentration of transverse strains in that same ligament. The aligned clusters now display higher void growth rates than the single void model. The orientation of the cluster still affects the void growth rates but to a lesser degree. Figure 4.23 illustrates the tendency of a spherical cluster of voids to rapidly coalesce under triaxialities of $\chi = 3.00$.

4.2 Cluster Density

The density of a void cluster is expected to affect void interaction during growth. In this thesis, the cluster density is compared to the void radius which gives the model a form of length scale. Figure 4.24 illustrates the effect of cluster density on void growth for all of the cluster types under uniaxial deformation. Figure 4.24(a), which

Figure 4.23: Contours of plastic strain in a spherical cluster model at $e_1 \approx 0.2$ of high hydrostatic stress deformation ($\chi = 3.00$). Dark blue contours indicate $\epsilon^P = 0$ and red contours indicate $\epsilon^P = 2$.

shows void growth for the linear clusters, demonstrates a gradual reduction in the void interaction effects as the voids get further apart. The shielding of the intervoid ligaments is gradually reduced in the aligned loading case and the concentration of strain in the transverse ligaments is gradually reduced for transverse loading. As expected, the void growth curves are moving towards that of the single void model as the voids move further apart.

In the planar clusters of Figure 4.24(b), the void growth curves move closer to that of a single void as the cluster becomes less dense but the change is not gradual. The void interaction mechanism, in this case, is not a linear function of inter-void spacing. Under transverse uniaxial tension, there is a significant drop in void growth rates between a void cluster with a density of $\kappa = 22$ and that with a density of $\kappa = 10$. Examining the same transverse planar cluster, this time with densities of $\kappa = 10$ and $\kappa = 4$, only a slight difference in void growth rates is observed.

Note that the void interaction effects dissipate quickly as the voids move further

Figure 4.24: Effect of cluster density on void growth in each of the (a) linear, (b) planar, and (c) spherical cluster models as a function of effective plastic strain under uniaxial deformation ($\chi = 0.33$).

apart. In the linear cluster of Figure 4.24(a), a change from $\kappa = 22$ to $\kappa = 4$ has reduced the void growth rates half-way to that of the single void model which has a density of $\kappa = 1$. In the transverse planar cluster, the void interaction will likely persist for greater ligament spacings.

4.3 Aspect Ratio

The previous sections have shown that there are interactions within void clusters that strongly affect damage rates. There remains a question as to whether a void cluster can be effectively represented by a single void with an equivalent initial aspect ratio. In other words, would a single void with an initial aspect ratio of $r_1 : r_2 : r_3 = 3 : 1 : 1$ result in the same damage rates as a linear cluster of three spherical voids in a unit cell? Figure 4.25 depicts the equivalent single prolate ellipsoidal void used to replace the linear cluster within the unit cell and Figure 4.26 shows the single oblate ellipsoidal void used to replace the planar cluster within the unit cell. Note that an equivalent single void replacing a spherical cluster would revert back to the spherical single void model. It has already been shown that the void interactions within the spherical clusters cause more severe damage than that found in single void model. Figure 4.27 compares the void growth rates of the linear and planar clusters to that of their equivalent ellipsoidal clusters under uniaxial deformation. As with the spherical cluster of voids, the linear and planar clusters do not develop damage at the same rate as single ellipsoidal voids with initial aspect ratios that are similar to the cluster shape. Aligned clusters of voids have higher damage rates than aligned ellipsoids while transverse void clusters have lower damage rates than the transverse ellipsoids. Figures 4.28 and 4.29 illustrate that the change in overall aspect ratio for the clusters and their corresponding ellipsoids

Figure 4.25: Finite element discretization of an ellipsoidal single void that compares with the initial shape of the linear cluster.

Figure 4.26: Finite element discretization of an ellipsoidal single void that compares with the initial shape of the planar cluster.

Figure 4.27: Void growth rates in linear and planar clusters and ellipsoidal single voids with equivalent initial aspect ratios under uniaxial tension ($\kappa = 22$, $\chi = 0.33$).

are similar. The difference lies in the effective distribution of void extent. In the aligned case of Figure 4.28, the cluster damage is higher since each void sees the same growth in the x_3 direction whereas the ellipsoidal void tapers down more quickly. In the transverse case of Figure 4.29, each void has high void extension in the x_2 direction while the ellipsoidal void extension is high at the symmetry plane but tapers off in each direction.

From these comparisons, it can be seen that the interaction of local strain fields within void clusters cannot be approximated by an equivalent single ellipsoidal void.

4.4 Summary

Void clusters develop significantly different growth rates than single, more isolated voids with the same initial void volume fraction. Furthermore, these differences in void growth can not be accounted for by differences in the initial effective aspect ratio of the cluster. Rows of voids that are aligned with a positive principal strain

Figure 4.28: Void shape change for the linear cluster and an ellipsoid with an equivalent initial aspect ratio in the x_1 - x_3 plane deformed under aligned uniaxial tension to $e_1 \approx 0.35$ ($\kappa = 22$, $\chi = 0.33$).

Figure 4.29: Void shape change for the linear cluster and an ellipsoid with an equivalent initial aspect ratio in the x_2 - x_3 plane deformed under transverse uniaxial tension to $e_2 \approx 0.35$ ($\kappa = 22$, $\chi = 0.33$).

shield each other and reduce overall void growth rates. Rows or planes of voids that are transverse to a positive principal strain develop strain concentrations within the transverse ligaments that accelerate void growth. Introduction of a negative principal strain transverse to a row of aligned voids reduces void shielding effects. Void interaction effects are also gradually lessened as the voids within a cluster are moved further apart. Elevated levels of hydrostatic tension rapidly increases void growth rates, as is the case with single voids, but the void interaction effects are also increased including non-ellipsoidal void geometries such as void "squaring".

Chapter 5

Void Nucleation and Growth in the Presence of Rigid Particles

The previous chapter presented the interaction and growth of voids in a variety of cluster arrangements deformed by a number of stress and strain states. This chapter examines the same void interactions with the addition of studying void nucleation effects at the particle-matrix interface and the presence of a hard particle. First of all, the particle-matrix interface characteristics of Equations 2.10 and 2.11, that is, the maximum strength of the interface, σ_{max} , the interface separation distance, δ , and a parameter α which scales the magnitude of shear separation strength to the normal separation strength, are examined to determine the sensitivity of the void nucleation behaviour to these characteristics. Secondly, void nucleation in particle clusters under various deformation strain states are studied. The growth of the nucleated voids is then compared to that presented in the previous chapter.

As is Chapter 4, some of the void shape evolution and contour plots in this

section show an unrealistic degree of ligament thinning since there is no model for material failure and element deletion in these models. The over-stretched ligaments will add to the stiffness of the model. In Chapter 6, there is an element deletion model that allows these ligaments to fail before unrealistic strains are reached.

It should be noted that the void volume fractions reported for the models which contain particles includes the volume of the rigid particle.

5.1 Particle-Matrix Interfacial Strength

Examination of void nucleation at the particle-matrix interface will be considered independently from clustering effects in this section, based on single-void models (Figure 3.2). Figure 5.1 shows the different levels of void growth obtained when the interfacial strength, interface separation distance, and shear stress to normal stress ratio are varied in a single particle model deformed in plane strain.

Firstly, it is apparent that the presence of a particle within a void, even without strength ($\sigma_{max} = 0$), significantly increases void growth rates, as reported by Needleman (1987) and Fleck *et al.* (1989) in two dimensions. The particle props open the void, primarily preventing contraction in the negative strain direction (x_3 direction) but also allowing slightly higher void growth in positive and neutral strain directions (x_1 and x_2 directions respectively, see Figure 5.2).

The second effect is associated with the strength and ductility of the interface. In general, an increase in strength (σ_{max}) or ductility (δ) will delay void nucleation (Figure 5.1a,c). Once nucleated, the growth rates for all interface conditions matched those for a “weak” interface ($\sigma_{max} = 0$).

Figure 5.1: Void nucleation and growth rates within a unit cell containing a single particle under plane strain ($\chi = 0.58$). Unless stated otherwise, the interface parameters are $\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$ and $\delta = 0.02r$ where the green curve represents the same model throughout. Note that r is the particle radius.

Figure 5.2: Comparison of void shape changes in a single void model with and without a particle, independent of interfacial strength, under plane strain deformation ($\chi = 0.58$)

5.1.1 Magnitude of Normal Traction, σ_{max}

The magnitude of the traction separation interface strength is specified by σ_{max} which is arbitrarily set as a scalar function of σ_y for the purposes of this study. Figure 5.1(a) demonstrates an increase in the nucleation strain as the magnitude of the interface traction, σ_{max} is increased.

5.1.2 Relative Magnitude of Shear Traction, α

The parameter α scales the magnitude of the shear traction to the magnitude of the normal traction in the particle matrix interface as specified in Equations 2.10 and 2.11. Figure 5.1(b) indicates that nucleation of the void is significantly delayed as the value of α is increased from $\alpha = 0.5$ to 5.0. Figure 5.3 shows the change in void shape as the magnitude of the shear effect is altered. The single void model is deformed to $e_1 \approx 0.35$ in plane strain and the other interface parameters are $\sigma_{max} = 5\sigma_y$ and $\delta = 0.02r$. The inset in Figure 5.3 provides a closer look at the change in position of one of the matrix nodes on the interface with changes in shear traction. The degree of tangential displacement of the node decreases as the shear strength increases. Thus α acts like a friction coefficient, reducing tangential motion.

Figure 5.3: Comparison of void shape changes in a single particle model with variation in the shear to normal traction ratio, α , of the particle-matrix interface. Results shown for a single particle model deformed in plane strain to $e_1 \approx 0.35$ ($\chi = 0.58$, $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$).

Note that for larger values ($\alpha \geq 3$), there is only slight change in void growth response. It is likely that the shear traction on the particle and matrix nodes which are still in contact at e_1^∞ is not high enough to separate the interface at that location when $\alpha \geq 3$.

As the magnitude of the hydrostatic component of stress increases, the particle-matrix interface separation and void growth becomes increasingly spherical and the parameter α plays less of a role (Figure 5.4).

5.1.3 Interface separation distance, δ

The interface separation distance, δ , is the distance at which all interfacial tractions drop to zero. This parameter has a small effect on the rate of void nucleation for the range of separation distances considered in Figure 5.1(c).

Figure 5.4: Comparison of void shape changes in a single particle model with variation in the shear to normal traction ratio, α , of the particle-matrix interface and in deformations with a high component of hydrostatic stress. Results shown for a single particle model deformed to $e_1 \approx 0.35$ ($\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$).

Summary

The interface parameters control the nucleation strain, but the rate of void growth after nucleation is not affected in single void models. The presence of a particle in the void dramatically increases the slope of the void growth curve in low triaxiality deformation states. The balance of the nucleation and void growth studies presented in this chapter will use $\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$ and $\delta = 0.02r$, where r is the radius of the spherical particles.

5.2 Nucleation within Clusters

Of particular interest is the presence of the particles and the particle-matrix interface on void nucleation and growth within clusters of particles, as distinct from clusters of pre-existing voids. It is anticipated that the particles will prop open the voids at low triaxialities, leading to increased void growth rates within particle clusters, as is the case with void growth from single particles. It is less clear what effects the stress field interactions within clusters will have on void nucleation and growth. In this section, void nucleation will be studied in linear, planar and spherical clusters. The damage rates will be compared with rates from similar models with no interfacial strength ($\sigma_{max} = 0$).

5.2.1 Uniaxial Tension

Linear Cluster Nucleation

Figure 5.5 shows the void growth rates within a linear cluster and a single void model under aligned and transverse uniaxial tension in order to compare the effects of clustering and cluster orientation on damage rates. As expected, the voids are

Figure 5.5: Void growth rates within linear clusters of particles with and without a traction-separation nucleation model, deformed under uniaxial tension ($\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $\kappa = 22$, $\chi = 0.33$).

propped open in the minor principal strain directions and the damage rates are higher than in the models that do not contain particles (Figure 4.1). The void growth rates are also dependent on the orientation of the anisotropic cluster, as is the case in the models without particles, with the transverse orientation leading to higher void growth rates, due to strain concentration in the inter-void ligament, in comparison to the aligned loading case, where the ligament is shielded from the major principal stress.

It is evident that the nucleation event is also strongly affected by the orientation of the cluster relative to the major principal strain. The strength of the particle-matrix interface is quickly exceeded in the aligned loading case whereas the transverse loading case requires significant levels of remote strain to initiate void nucleation (strains almost as high as in the single particle model). This difference is related to the location on the particle-matrix interface at which nucleation initiates. Figure 5.6(a) pinpoints the nucleation events on the void growth curve for the aligned and transverse linear clusters deformed in uniaxial tension. The physical location of each of the events, on the particle-matrix boundary, is identified in Figure 5.6(b). Numbers 1, 2 and 3 indicate nucleation events and locations within the

(a)

(b)

Figure 5.6: (a) Global effective plastic strain level and (b) location of void nucleation in a linear cluster under aligned and transverse uniaxial tension ($\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$, $\delta = 0.02r$, $\kappa = 22$, and $\chi = 0.33$).

aligned linear cluster and number 4 corresponds to the transverse cluster nucleation event.

The first nucleation event in the aligned linear cluster is at the inter-particle ligament end of the outer aligned void (Figure 5.7(a)). Prior to nucleation, a stress concentration develops in the inter-particle ligament, contrary to the void shielding seen in the aligned voids of the models without particles (Section 4.1.1). Watt *et al.* (1996) also observed higher stresses in ligaments between aligned particles than in ligaments between transverse particles, prior to nucleation. The stress builds up quickly in the aligned ligaments as the particles are being pulled apart. Once nucleation initiates at point 1 ($\bar{\epsilon}_g^p \approx 0.025$), the stress in the aligned ligament drops and the ligament becomes shielded from the major principal strain by the growing void, as indicated by the lower void growth rates. At this stage, all of the void growth takes place at the inner side of the outer aligned void.

The second and third nucleation events within the aligned linear cluster (Figure 5.7(b) and (c)) take place at high levels of strain ($\bar{\epsilon}_g^p \approx 0.25, 0.3$) and have very little influence on the rate of void growth in the cluster. The second void nucleates at the higher stress points (number 2 in Figure 5.6) which are slightly removed from the shielded ligament. This nucleation of the central void quickly spreads down to the inter-particle ligament, allowing the central void to grow in the x_1 direction. The third nucleation event occurs at the outer edge of the outer aligned particle, allowing growth of the outer void on both sides of the outer aligned particle. Figure 5.8 shows the different resulting void shapes for the aligned linear clusters, with and without interfacial strength, at a major principal engineering strain of $e_1 \approx 0.35$.

The transverse linear cluster experiences a single nucleation event at location 4 on both particles (Figure 5.9). This nucleation location is removed from the ligament. As a result the stresses are not significantly different from those of the single void

Figure 5.7: Mesh plots of nucleation progression in an aligned linear particle cluster deformed in uniaxial tension. Part (a) illustrates the first stage of nucleation with separation of the outer void at the inter-particle ligament. Parts (b) and (c) show the subsequent nucleation events at the outer edge of the outer particle and at the central particle respectively.

Figure 5.8: Void shape changes within a linear cluster of particles with and without a traction-separation nucleation model, deformed under aligned uniaxial tension ($\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $\kappa = 22$, $\chi = 0.33$).

Figure 5.9: Mesh plot of nucleation in a transverse linear particle cluster deformed in uniaxial tension.

Figure 5.10: Void shape changes within a linear cluster of particles with and without a traction-separation nucleation model, deformed under transverse uniaxial tension ($\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $\kappa = 22$, $\chi = 0.33$).

model and nucleation occurs at a major principal strain within 8% of that for a single particle model; the difference is attributed to a slight change in the plastic flow field due to the transverse ligament, leading to slightly higher stresses at the nucleation points for a given level of global strain.

Figure 5.10 shows the resulting void shapes in the transverse linear cluster, with and without interface strength, at a major principal engineering strain of $e_2 \approx 0.35$.

Planar Cluster Nucleation

Growth of voids from the particle-matrix interfaces of a planar cluster are characterized in Figure 5.11. As with the linear and single particle models, void growth

Figure 5.11: Void growth rates within planar clusters of particles with and without a traction-separation nucleation model, deformed under uniaxial tension ($\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $\kappa = 22$, $\chi = 0.33$).

rates are higher with particles than without at low levels of stress triaxiality due to void propping (see Figure 4.1 for comparison). Similar to the observed behaviour for linear clusters, the void nucleation and growth rates with planar clusters are a function of cluster orientation.

Figure 5.12(a) illustrates the levels of strain at which various nucleation events occur for both aligned and transverse planar clusters deformed in uniaxial tension. Figure 5.12(b) pinpoints the location of the nucleation events on the particle matrix interfaces. As with the linear aligned cluster, initial nucleation occurs at the inter-particle ligament at low levels of global plastic strain ($\bar{\epsilon}_g^p \approx 0.03$). The row of aligned voids leads to a rapid build-up of stress in the ligament which is relieved by the first nucleation event. Unlike the aligned linear cluster, nucleation starts at the central particle in the aligned planar cluster. This behaviour is attributed to stress concentration induced by the transverse ligament. The outer transverse void nucleates next ($\bar{\epsilon}_g^p \approx 0.07$), followed by the outer aligned void ($\bar{\epsilon}_g^p \approx 0.09$). Since the aligned ligament is now shielded by the growing central void, the outer aligned void nucleates at the outer edge of the particle, where stresses are higher. The second and third nucleation events, though not contained within either inter-particle ligament,

Figure 5.12: (a) Global effective plastic strain level and (b) location of void nucleation in a planar cluster under aligned and transverse uniaxial tension ($\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$, $\delta = 0.02r$, $\kappa = 22$, and $\chi = 0.33$).

Figure 5.13: Void shape changes within a planar cluster of particles with and without a traction-separation nucleation model, deformed under aligned uniaxial tension ($\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $\kappa = 22$, $\chi = 0.33$).

occur at strains lower than those seen in an un-clustered model ($\bar{\epsilon}_y^p \approx 0.12$), which suggests that these nucleation locations are also affected by the complex stress field interactions within the planar cluster. Figure 5.13 illustrates the different resulting void shapes at $e_1 \approx 0.35$, with and without interface strength.

Void nucleation within the transverse planar cluster is also influenced by stress field interactions. The transverse voids nucleate first (points labeled by a 4 in Figure 5.12(b)) followed by nucleation of the central void (point 5). Further nucleation occurs at points 6 and 7 due to strain concentration not within the nearest neighbor ligaments but within the larger ligament between transverse particles. Figure 5.14 illustrates the different void shapes formed at $e_1 \approx 0.35$ of transverse uniaxial tension, with and without interfacial strength.

Spherical Cluster Nucleation

Figure 5.15 displays the nucleation and growth rates of voids formed within a spherical cluster of particles deformed in uniaxial tension. Although the spherical cluster

Figure 5.14: Void shape changes within a planar cluster of particles with and without a traction-separation nucleation model, deformed under transverse uniaxial tension ($\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $\kappa = 22$, $\chi = 0.33$).

Figure 5.15: Void growth rates within spherical clusters of particles with and without a traction separation nucleation model, deformed under uniaxial tension ($\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $\kappa = 22$, $\chi = 0.33$).

contains a row of particles aligned with the major principal stress, nucleation does not occur at low values of strain like the other aligned clusters. Either the stress build-up in the aligned ligament does not occur in this cluster or the first nucleation event has a minor effect on the overall void growth rate of the cluster. Figure 5.16 demonstrates that the latter assumption is correct; there is early nucleation at the aligned ligament ($\bar{\epsilon}_g^p \approx 0.03$, Figure 5.17(a)) but the void grows very slowly until the second nucleation event occurs at the transverse particles ($\bar{\epsilon}_g^p \approx 0.095$, Figure 5.17(b)). Apparently, the central void is prevented from growing significantly due to the constraint created by the surrounding transverse voids which have not yet nucleated. Once these transverse voids nucleate, the central void is free to grow and the central void does indeed grow more than the transverse voids, as demonstrated in Figure 5.17(c). The nucleation of the outer aligned void occurs later ($\bar{\epsilon}_g^p \approx 0.145$), and any further void nucleation events in the transverse direction have little effect of the overall damage rates in the spherical cluster.

The modeling results show that there are two factors which govern nucleation rates in particle clusters. The first is the rapid stress build-up that develops in ligaments between particles aligned with the major principal stress. In most cases, this stress riser leads to early void nucleation in aligned particle cluster models. When that row is surrounded by more than two transverse particles, there are sufficient constraints on that central void to prevent significant void growth, as seen in the spherical cluster. The transverse constraints are removed and void growth rates become significant once the transverse voids have also nucleated at higher levels of strain.

The second, and less important consideration, is the slight change in plastic flow that occurs due to the transverse ligaments. The transverse ligaments allow

(a)

(b)

Figure 5.16: (a) Global effective plastic strain level and (b) location of void nucleation in a spherical cluster deformed in uniaxial tension ($\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$, $\delta = 0.02r$, $\kappa = 22$, and $\chi = 0.33$).

(a) point 1 on Figure 5.16

(b) point 2 on Figure 5.16

(c) point 4 on Figure 5.16

Figure 5.17: Mesh plot of nucleation in a spherical particle cluster deformed in uniaxial tension.

Figure 5.18: Void shape changes within a spherical cluster of particles with and without a traction-separation nucleation model, deformed in uniaxial tension ($\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $\kappa = 22$, $\chi = 0.33$).

a greater degree of plastic flow through the cluster, and hence, somewhat higher stresses at the nucleation points and slightly earlier void nucleation.

5.2.2 Biaxial Strain States

Section 5.2.1 considered the effects of particle clustering on the strain to nucleation of voids at a particle matrix interface, when the unit cell is deformed in uniaxial tension. The three biaxial strain states characteristic of sheet metal forming, referring to plane strain, equi-biaxial stretch, and deep drawing, have also been studied to determine their effects on void nucleation in particle clusters. Only models with a particle-matrix interfacial traction strength, $\sigma_{max} = 5\sigma_y$ will be presented here.

There are two primary observations when examining nucleation behaviour in clustered models deformed under biaxial strain states. Firstly, a row of voids aligned with the major principal strain leads to a stress concentration and very early void nucleation at the aligned inter-particle ligament. Secondly, a void which is nucleated at a low value of effective plastic strain and has neighboring particles located transverse to the major principal strain, will not grow significantly until one or both of these transverse particles nucleates a void. Nucleation of the transverse voids

Figure 5.19: (a) Global effective plastic strain level and (b) location of void nucleation in an aligned linear cluster deformed under a variety of biaxial strain states ($\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$, $\delta = 0.02r$, and $\kappa = 22$).

releases a level of constraint that prevents the initial void from growing.

Aligned Linear Clusters

Figure 5.19 details the location and sequence of the particle matrix debonding in an aligned linear cluster deformed in various biaxial strain fields. In all cases, the outer aligned void nucleates first (point 1 in Figure 5.19(b)), at a major principal strain of $\bar{\epsilon}_g^p \approx 0.03$. This is a direct result of the stress concentration developed in the ligament prior to void nucleation.

Figure 5.19(a) indicates that there is a delay in void growth in the equi-biaxial strain case, even though the outer aligned void has nucleated at low strains. There

are no particles transverse to the row of aligned voids to cause this delay but an equi-biaxial strain state has two major principal strains, and there must be voids nucleated in both directions (ie. at 1, 4, and 5) before significant void growth can occur. This growth delay is similar to the behaviour of a spherical cluster subject to uniaxial tension.

It is interesting to note that the damage rates in the linear particle cluster deformed in deep drawing are positive (Figure 5.19) whereas, the damage rates in a linear cluster without a particle propping open the void is negative since the void is able to collapse (Figure 4.15).

Transverse Linear Clusters

Figure 5.20 depicts the void nucleation progression in a transverse linear cluster of particles deformed under uniaxial tension, plane strain and deep draw deformation. As noted previously for uniaxial deformation, there is no stress concentration developed in the inter-particle ligament when the cluster is transverse to the major principal strain. As a result, initial nucleation occurs at a higher level of global strain, $\bar{\epsilon}_g^p \approx 0.11$. Both transverse voids nucleate in rapid succession at the points marked 4 and 5, allowing for immediate void growth. Once these transverse voids have nucleated, further interface separation does not change the overall rate of void growth.

Aligned Planar Clusters

In the case of an aligned planar cluster, there is a row of particles aligned with the major principal strain, thereby leading to a stress concentration in the ligament and early nucleation. There is also a particle transverse to the major principal strain that restricts void growth at the ligament until the transverse void has nucleated.

Figure 5.20: (a) Global effective plastic strain level and (b) location of void nucleation in a transverse linear cluster deformed under a variety of biaxial strain states ($\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$, $\delta = 0.02r$, and $\kappa = 22$).

Figure 5.21 depicts the void nucleation progression in an aligned planar cluster of particles deformed in uniaxial tension, plane strain, deep drawing and an equi-biaxial stretch. It is evident that deep drawing, uniaxial tension and plane strain conditions lead to early nucleation of the central void ($\bar{\epsilon}_g^p \approx 0.03$) and the growth of that void is delayed until the void along the x_3 axis has also nucleated at point 2. In an aligned planar cluster, nucleation of the outer aligned void at point 3 occurs shortly thereafter and may also contribute to the acceleration in overall void growth.

In the case of equi-biaxial deformation, the central void nucleates at the aligned ligament at a low value of effective plastic strain, $\bar{\epsilon}_g^p \approx 0.01$, but the through-thickness void nucleates simultaneously from point 2 circumferentially about the x_3 axis to point 6. The through-thickness void is then able to grow in both major principal strain directions, even though the other two un-nucleated particles provide some constraint to that growth. The overall void growth accelerates further when the central and outer aligned voids nucleate in both major principal strain directions (points 5, 4, and 7).

Transverse Planar Clusters

A transverse planar cluster is expected to have comparatively high strains prior to nucleation since there is no row of aligned particles. Figure 5.22 illustrates the void nucleation progression in a transverse planar cluster of particles deformed in uniaxial tension, plane strain, and deep drawing. As expected, the uniaxial tension and plane strain deformation conditions lead to high strains-to-nucleation ($\bar{\epsilon}_g^p \approx 0.10$), with all three voids nucleating in quick succession. In deep drawing, the through-thickness void nucleates at a low value of global strain, $\bar{\epsilon}_g^p \approx 0.03$ at point 6, which does not follow the pattern for the other two loading conditions in Figure 5.22. This early nucleation is the result of a combination of a positive stress in the x_2 direction and

Figure 5.21: (a) Global effective plastic strain level and (b) location of void nucleation in a aligned planar cluster deformed under a variety of biaxial strain states ($\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$, $\delta = 0.02r$, and $\kappa = 22$).

Figure 5.22: (a) Global effective plastic strain level and (b) location of void nucleation in a transverse planar cluster deformed under a variety of biaxial strain states ($\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$, $\delta = 0.02r$, and $\kappa = 22$).

a highly negative stress in the x_1 direction. This early void is subject to low void growth rates prior to the nucleation of the other voids in the cluster, as are the early-nucleated voids in aligned clusters.

Spherical Clusters

The spherical cluster should show a combination of aligned and transverse planar cluster behaviour. Figure 5.23 depicts the void nucleation progression in a spherical cluster of particles deformed in uniaxial tension, plane strain, deep drawing and equi-biaxial stretch.

For all four deformation cases, there is early void nucleation at the aligned inter-particle ligament, the growth of which is delayed until one or both of the transverse voids has nucleated. In the case of a deep draw, the void along the x_1 axis nucleates first and subsequent void nucleation at point 2 removes enough constraint to allow a moderate amount of void growth. The final void growth rate is not reached until the central and through-thickness voids have also nucleated at points 3 and 1. With equi-biaxial deformation, the early nucleation of the central void at both aligned ligaments does not lead to significant void growth until the through-thickness void nucleates around its circumference in the plane of the stretch. Nucleation of the outer aligned voids completes the constraint relief process, triggering a substantial rate of overall void growth.

Summary

The effect of cluster orientation on the strain-to-nucleation within particle clusters is contrary to the trend observed for void growth in the absence of particles (Chapter 4). If there is a row of particles aligned with the major principal stress, a stress concentration develops in the ligament and early void nucleation occurs. This early

Figure 5.23: (a) Global effective plastic strain level and (b) location of void nucleation in a spherical cluster deformed under a variety of biaxial strain states ($\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$, $\delta = 0.02r$, and $\kappa = 22$).

nucleation leads to higher damage in aligned clusters than in transverse clusters at low strain levels. Once a void has nucleated at the aligned ligament however, the stress concentration is relieved and the new void shields the aligned ligament, reducing the stress level. Therefore, void growth rates are low in an aligned cluster, compared to a transverse cluster, and overall damage levels become higher in the transverse cluster at higher deformation strains.

If there are un-nucleated transverse particles surrounding an early developed void, the void is constrained and experiences very little growth until the transverse voids have also nucleated in the direction of major principal strain. This constraint field is also in effect during equi-biaxial stretching, though in a slightly different sense. If voids nucleate at all of the transverse particles in one of the major principal strain directions, the void growth is minimal until voids are also nucleated at all of the particles transverse to the other major principal strain direction.

5.2.3 High Triaxiality Strain States

The effect of varying degrees of superimposed hydrostatic tension on the nucleation of voids in a particle cluster is studied in this section, where the hydrostatic tension is superimposed on a uniaxial deformation state.

Aligned Linear Cluster

Figure 5.24 illustrates the level of global strain and the location of the nucleation events within an aligned linear cluster deformed at various levels of hydrostatic tension.

As observed during biaxial deformation, there is an early stress concentration developed in the aligned ligament which leads to void nucleation at global strains of $\bar{\epsilon}_g \approx 0.03$. A high level of hydrostatic tension ($\chi = 3.0$) elevates stress levels and so

Figure 5.24: (a) Global effective plastic strain level and (b) location of void nucleation in an aligned linear cluster deformed under a variety of degrees of superimposed hydrostatic stress ($\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$, $\delta = 0.02r$, and $\kappa = 22$).

Figure 5.25: (a) Global effective plastic strain level and (b) location of void nucleation in a transverse linear cluster deformed under a variety of degrees of superimposed hydrostatic stress ($\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$, $\delta = 0.02r$, and $\kappa = 22$).

void nucleation occurs at even lower global strains ($\bar{\epsilon}_g^p \approx 0.01$). Increased levels of hydrostatic tension also cause separation of the particle-matrix interface transverse to the major principal strain direction. The higher the value of triaxiality, the earlier the separation.

Transverse Linear Cluster

Figure 5.25 illustrates the level of global strain and the location of the nucleation events within a transverse linear cluster deformed at various levels of hydrostatic tension. At all of the levels of hydrostatic stress considered, both the central and outer particles nucleate voids in the direction of major principal stress (ie. the x_2

direction) almost simultaneously, as was observed under sheet forming strain states. High hydrostatic stress ($\chi = 3.0$) causes particle-matrix separation transverse to the major principal strain at the same level of strain as for separation in the major principal strain direction, meaning there is no delay in void growth due to transverse constraints.

Aligned Planar Cluster

Figure 5.26 illustrates the level of global strain and the location of the nucleation events within an aligned planar cluster deformed at various levels of hydrostatic tension. Again, the same nucleation and growth mechanisms are observed here as in the biaxial strain states; the first void nucleates at the aligned ligament at low strain but there is little growth until the transverse particles also nucleate voids in the major principal strain direction (in this case at point 2). Particle matrix separation occurs in the minor principal strain directions at the higher levels of triaxiality.

Transverse Planar Cluster

Figure 5.27 illustrates the level of global strain and the location of the nucleation events within a transverse planar cluster deformed at various levels of hydrostatic tension. As is the case for linear clusters, nucleation in the direction of major principal strain at points 5 and 6, is virtually simultaneous. Furthermore, increased triaxiality leads to transverse particle-matrix separation, while high triaxiality ($\chi = 3.0$) leads to transverse constraints and lower void growth prior to nucleation in the minor principal strain directions at points 1, 2, and 3.

Figure 5.26: (a) Global effective plastic strain level and (b) location of void nucleation in an aligned planar cluster deformed under a variety of degrees of superimposed hydrostatic stress ($\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$, $\delta = 0.02r$, and $\kappa = 22$).

(a)

(b)

Figure 5.27: (a) Global effective plastic strain level and (b) location of void nucleation in a transverse planar cluster deformed under a variety of degrees of superimposed hydrostatic stress ($\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$, $\delta = 0.02r$, and $\kappa = 22$).

Spherical Cluster

Figure 5.28 illustrates the level of global strain and the location of the nucleation events within a spherical cluster deformed at various levels of hydrostatic tension. This spherical cluster follows the same nucleation mechanisms as described for spherical clusters subject to biaxial sheet forming conditions. These consistent trends include, early nucleation occurs at the aligned ligament (point 1), there is limited growth prior to nucleation at transverse particles in the major principal strain direction, increased particle-matrix separation in the minor principal strain directions with increasing triaxiality, and void growth constraints prior to void separation in the minor principal strain directions at high levels of triaxiality ($\chi = 3.0$).

Summary

The nucleation mechanisms at higher components of hydrostatic stress are similar to those of the biaxial strain states. Early nucleation occurs at a ligament between particles aligned with the major principal strain due to a stress concentration in the ligament. Growth of a void nucleated at an aligned ligament is delayed if there are unseparated transverse particles. The higher component of hydrostatic stress causes an increased level of particle-matrix separation in the directions of minor principal stress and the very high triaxiality case ($\chi = 3.0$) leads to another constraint component, where particle-matrix separation in the minor principal strain directions is required to achieve full void growth rates. However, this constraint is not as strong as that created by the presence of transverse particles.

Figure 5.28: (a) Global effective plastic strain level and (b) location of void nucleation in a spherical cluster deformed under a variety of degrees of superimposed hydrostatic stress ($\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$, $\delta = 0.02r$, and $\kappa = 22$).

5.2.4 Cluster Density

So far, this chapter has considered nucleation behaviour in particle clusters with a relatively dense arrangement ($\kappa = 22$). A closer look at the effect of particle cluster density (ie. the thickness of the ligaments between nearest neighbor particles) on damage rates reveals that within a range of $22 \geq \kappa \geq 4$, there is usually only a slight change in nucleation rates.

Linear Clusters

Figure 5.29 illustrates the progression of void nucleation within linear particle clusters of various densities deformed in aligned and transverse uniaxial tension. An increase in inter-particle ligament thickness has the expected result of reducing the effects of cluster orientation. The aligned cluster nucleates at progressively higher strains as the ligament thickens, while the transverse cluster nucleates a void earlier. In effect, the degree of particle clustering is reduced. There is also a tendency, with reduced cluster density, for all the voids in a given cluster to nucleate within a narrower range of strain.

Planar Clusters

Figure 5.30 illustrates the progression of void nucleation within planar particle clusters of various densities deformed under aligned and transverse uniaxial tension. As with the linear clusters, the transverse planar clusters show very little change in strain-to-nucleation as the cluster density is varied. The aligned planar cluster, on the other hand, shows a marked difference in nucleation rates for the different cluster densities. A possible difference lies in the effect of ligament thickness on the constraint field, which prevents growth of the centrally nucleated void prior to nucleation of the transverse void. Since the aligned planar cluster creates a constraint

Figure 5.29: Global effective plastic strain level in (a) aligned and (b) transverse linear clusters and (c) the location of void nucleation at various levels of cluster density deformed in uniaxial tension ($\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$, $\delta = 0.02r$, and $\chi = 0.33$).

Figure 5.30: Global effective plastic strain level in (a) aligned and (b) transverse planar clusters and (c) the location of void nucleation at various levels of cluster density deformed in uniaxial tension ($\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$, $\delta = 0.02r$, and $\chi = 0.33$).

on void growth in only one direction (the x_3 direction), an increase in inter-particle ligament thickness increases the flexibility of the cluster enough to allow the unit cell to deform to higher strains without significant void growth. As a result nucleation of the transverse void occurs at higher levels of global strain when $\kappa \leq 10$.

At a ligament thickness described by $\kappa = 4$ there is a change in the initial void nucleation location and the outer aligned void nucleates first. This void is not as subject to the constraint on transverse void growth, hence, there is more immediate void growth than in the $\kappa = 10$ case.

Spherical Clusters

The growth of the central void in the spherical cluster is constrained by the un-nucleated transverse particles in the x_2 and the x_3 directions (see Figure 5.31). There is a small increase in void nucleation strain as the degree of clustering is reduced (κ decreased).

5.3 Damage Rates in Clusters of Particles

In this section, the void growth rates are compared to those of Chapter 4 to identify how the combination of a void nucleation delay and particle propping during void growth affects the rate of damage in clusters of particles at higher levels of strain ($\bar{\epsilon}_g^p \approx 0.5$).

Figure 5.32 compares the void growth rates within the different particle cluster types when deformed in uniaxial tension. Figure 5.32 is comparable to Figure 4.1, which presents the void growth rates for the same type of clusters, but in the absence of embedded particles. The first observation is that the voids nucleated at a particle-matrix interface have significantly higher growth rates than the voids that do not

(a)

(b)

Figure 5.31: (a) Global effective plastic strain level and (b) the location of void nucleation in a spherical cluster at various levels of cluster density deformed in uniaxial tension ($\sigma_{max} = 5\sigma_y$, $\alpha = 1.0$, $\delta = 0.02r$, and $\chi = 0.33$).

Figure 5.32: Comparison of void growth rates for different cluster types deformed in uniaxial tension as a function of global effective plastic strain ($\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $\kappa = 22$, $\chi = 0.33$).

contain a particle. This trend can simply be attributed to the fact that the particle props the void open, thereby preventing transverse void contraction.

In Figure 5.32 all of the clustered models develop higher levels of damage than the unclustered model whereas in Figure 4.1 the damage rate of the single-void models is higher than for some of the clustered void models. This is largely a nucleation effect, as the single void model exhibits a longer delay in nucleation than that of the clustered models in Figure 5.32. Furthermore, the nucleation delay has a strong effect on the eventual degree of void damage in each of the clustered models though at 50% engineering strain ($e_1 \approx 0.5$), the void growth rates have counteracted the effects of the nucleation delay to some degree. Although the aligned clusters have larger voids than the transverse clusters at $\epsilon_g^p \approx 0.12$, the shielding of the aligned voids and the strain concentrations in the transverse ligaments described in Chapter 4 take effect once the voids have nucleated. As a result, the transverse models experience higher damage rates than the aligned models, as reported in Chapters 4 and 5. Figure 5.32 demonstrates that at large accumulated strains ($e_1 \approx 0.5$), the transverse cluster damage levels have significantly exceeded those

Figure 5.33: Comparison of void growth rates for different cluster types deformed in plane strain as a function of global effective plastic strain ($\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $\kappa = 22$, $\chi = 0.58$).

of the aligned clusters. If the single void model is omitted in Figures 4.1 and 5.32, then the order of the cluster models, with regards to the level of damage at $e_1 \approx 0.5$, is very similar. All of these observations hold for the biaxial strain states as well, referring to Figures 5.33, 5.34, and 5.35 in comparison to Figures 4.10, 4.13, and 4.15, respectively.

A further point of interest is that the alignment of an anisotropic cluster (relative to the direction of major loading) becomes less important as the component of hydrostatic stress increases from $\chi = 0$ in deep draw to $\chi = 0.67$ in an equi-biaxial stretch. This trend makes sense as an increase in triaxiality renders the major and medial stress levels less distinct.

5.3.1 Equivalent Ellipsoidal Particles

In Section 4.3 the growth of linear and planar clusters are compared to single ellipsoidal voids with the same initial aspect ratios as the overall cluster. In the models which do not contain particles, the growth of the single ellipsoid is quite different from the growth of the corresponding clustered spherical voids (see Figure 4.27).

Figure 5.34: Comparison of void growth rates for different cluster types deformed in equi-biaxial stretch as a function of global effective plastic strain ($\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $\kappa = 22$, $\chi = 0.67$).

Figure 5.35: Comparison of void growth rates for different cluster types deformed in deep drawing as a function of global effective plastic strain ($\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $\kappa = 22$, $\chi = 0.0$).

Figure 5.36: Comparison of void nucleation and growth rates of linear and planar clusters to their equivalent ellipsoidal single voids ($\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $\kappa = 22$, $\chi = 0.33$).

Figure 5.36 again compares the clustered models to equivalent single-ellipsoid models but in this case, there is an embedded particle that is strongly bonded to the matrix. It is evident that the propping of voids by embedded particles during void growth does not induce similar damage rates in the ellipsoidal void models to that of the corresponding clustered model. Mesh plots of the aligned linear cluster and the aligned prolate ellipsoid deformed to $e_1 \approx 0.44$ are shown in Figure 5.37. Mesh plots of the transverse linear cluster and the transverse prolate ellipsoid deformed to $e_1 \approx 0.44$ are shown in Figure 5.38. The transverse linear cluster does display an overall damage level that is consistent with its equivalent ellipsoidal void, but the other clustered models have very different damage rates when compared to their ellipsoidal counterparts. As was concluded for the models without particles, the models of particle clusters cannot be approximated by a single ellipsoidal void with a similar cluster aspect ratio.

In Figure 5.36, it can be seen that the strains-to-nucleation in the transverse cluster models are much lower than in the single-ellipsoid models. In fact, the transverse ellipsoidal models nucleate voids at similar strains to the single spheri-

Figure 5.37: Mesh plots of void damage in the (a) aligned linear cluster and the (b) aligned single prolate ellipsoid models.

Figure 5.38: Mesh plots of void damage in the (a) transverse linear cluster and the (b) transverse single prolate ellipsoid models.

cal particle model of Figure 5.35. This outcome implies that the areal fraction of particles transverse to the major principal strain is a minor factor in determining the strain-to-nucleation of a cluster. It is the transverse ligaments of the clustered models which play an important role in accelerating nucleation within the transverse models.

In contrast, the aligned cluster models and ellipsoidal models nucleate voids at common strains. Based on this result, it seems that the depth of rigid material in an aligned row, and not just the existence of an aligned inter-particle ligament, plays a significant role in the rapid build-up of stress at the particle pole aligned with the major principal stress. As an aside, the aligned single ellipse begins to resemble a fiber. As such, this elongated particle would likely fracture and become an aligned cluster of particles.

5.4 Summary

Overall, the principal factor affecting damage levels in clusters of particles, as opposed to clusters of voids, is the propping open of the voids by the particles, which prevents void contraction in the minor strain directions. This leads to much higher damage levels in the models containing particles. This principal holds for lower levels of stress triaxiality, χ , including most sheet forming deformations.

Other significant factors include the nucleation delay created by the particle-matrix interfacial traction-separation model. Nucleation occurs at relatively low strains in clusters consisting primarily of a row of particles aligned with the major principal strain direction. However, early nucleation in an aligned row of particles can be counteracted by the presence of additional particles that are transverse to the aligned row. These transverse particles create a constraint field which effectively

prevents the growth of voids in the aligned row until voids have also been nucleated around the transverse particles. The voids then all grow simultaneously.

A similar constraint field exists if the medial and/or minor principal strains are a significant fraction of the major principal strain ($\epsilon_{II} \geq 0.4\epsilon_I$). In this case, particle-matrix separation must occur in the major and medial strain directions before obtaining significant void growth in either direction.

The main result of the nucleation delay is that the cluster models all develop higher damage levels than the single void models. Without the nucleation model, the single void model has an average level of damage compared to the clustered models. Cluster alignment has opposing effects on nucleation versus void growth rate after nucleation. Namely, an aligned cluster will nucleate a void at low strains but the void grows slowly, whereas a transverse cluster will show delayed void nucleation but faster void growth rates. At strain levels of $\bar{\epsilon}_g^p \approx 0.5$, the nucleation delays are overwhelmed by the void growth rates and transverse clusters have higher overall levels of damage than aligned clusters, at least for the interface modeled herein.

Chapter 6

Void Coalescence in a Dual Particle Population

This chapter is primarily concerned with the third stage of ductile fracture, that is, void coalescence. The material is treated as having a primary second phase consisting of large particles with a finely dispersed, much smaller secondary particle population. Following the approach of Tvergaard (1982*a*), coalescence is modeled through ductile failure of the matrix due to the nucleation, growth and coalescence of the small secondary particle population (ie. void sheeting) which leads to element deletion and the eventual collapse of the unit cell. The damage originating at the secondary population of particles is modeled using the Gurson-Tvergaard-Needleman (GTN) constitutive equations as outlined in Section 2.6. Using this approach, the primary particle population can be modeled using the clustered particle geometries in Chapter 5; thus, the effect of clustering on coalescence can be quantified.

The balance of the models presented in this thesis consider linear particle cluster

geometries. This geometry corresponds to that in rolled sheet materials and is of interest to the sheet metal pressing industry. The effect of cluster orientation is captured through consideration of aligned and transverse loading cases.

Section 6.1 introduces the behaviour of unit cells containing a transverse linear cluster with the inclusion of the GTN model and void coalescence through element deletion. The effect of varying some of the GTN constitutive equation parameters on the coalescence behaviour of transverse linear cluster deformed in uniaxial tension is presented in Section 6.2. The progression of damage in an aligned linear cluster is compared to that of a transverse linear cluster in Section 6.3. Section 6.4 then considers the effect of various stress fields on damage progression in linear particle clusters. Finally, Section 6.5 examines a range of linear particle cluster densities and the comparative coalescence behaviour of these models.

6.1 Coalescence in the Unit Cell Model

There are a number of material parameters that can be varied in the GTN constitutive equations (Equations 2.14 to 2.19). The q parameters in Equation 2.14 (q_1 , q_2 , and q_3) are designed to fit the constitutive equation to experimental or numerical data. In this research, the q parameters were set to $q_1 = 1.25$, $q_2 = 0.95$, and $q_3 = q_1^2$ (Worswick and Pick, 1990).

The rest of the GTN parameters are related to the nucleation and growth behaviour of the voids originating at the secondary particles. It is assumed that there are no initial voids, thus the initial porosity within the matrix between the large primary particles is zero. It is also assumed that the rate of secondary void nucleation is controlled by the level of strain in the matrix since the secondary particles are assumed to be small. The voids nucleate following a statistically normal dis-

tribution around a mean value of strain, ϵ_N , with a specified standard deviation, S_N , and a specified volume fraction of secondary void nucleating-particles, f_N . The coalescence behaviour of the secondary voids is characterized by two parameters: the volume fraction of voids in the matrix at the beginning of coalescence, f_c , and the volume fraction of voids at local matrix failure, f_f .

In order to familiarize the reader with the general behaviour of a unit cell containing a cluster of large or primary particles and a secondary population of very small particles modeled by the GTN constitutive equation, a thorough examination of a transverse linear cluster with $\kappa = 10$ deformed in uniaxial tension is laid out in this section. As in previous sections, the primary particles are spherical and explicitly discretized. Nucleation of the primary particles is modeled using the same particle-matrix interface presented in Chapter 5, with $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, and $\alpha = 1$. Thus, nucleation of the primary particle population can be described as stress-controlled. The secondary voids follow strain controlled nucleation ($f_N = 0.005$, $\epsilon_N = 0.1$, and $S_N = 0.02$) and the secondary void coalescence parameters are $f_c = 0.01$ and $f_f = 1.2f_c$. When the local void volume fraction of a finite element reaches $f^* = 1$, as calculated by the GTN model, the element loses its load bearing capacity and is deleted.

Figures 6.1 to 6.5 illustrate the progression of matrix damage in the unit cell during deformation. The contours indicate the level of local damage in the matrix where blue contours represent an undamaged element ($f^* = 0$) and red contours indicate severely damaged regions and imminent element deletion ($f^* \approx 1$). The primary particles were included in the FEM model but have only been shown in Figure 6.1(a) for clarity.

Figure 6.1 shows the early stages of matrix failure due to coalescence of secondary voids at $\epsilon_1 \approx 0.1$. The first deleted elements are at the particle surface in the region

Figure 6.1: Void nucleation at primary particles by nucleation, growth and coalescence of voids at a secondary population of particles shown (a) with and (b) without primary particles. This linear cluster ($\kappa = 10$) has been deformed in transverse uniaxial tension to $\epsilon_1 = 0.1$. For the primary particle-matrix interface model, $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, and $\alpha = 1$. For the secondary particle population, represented by the GTN model, the contours correspond to the local void volume fraction (blue: $f^* = 0$ and red: $f^* = 1$). The GTN parameters are $f_N = 0.005$, $\epsilon_N = 0.1$, $S_N = 0.02$ and $f_c = 0.01$, $f_f = 1.2f_c$.

Figure 6.2: Inter-particle ligament collapse due to coalescence of secondary voids under transverse uniaxial tension to $\epsilon_1 \approx 0.2$. Primary particle-matrix interface model and GTN model parameters as in Figure 6.1. The contours correspond to the local void volume fraction of the secondary void population (blue: $f^* = 0$ and red: $f^* = 1$).

Figure 6.3: Beginning of inter-cluster interaction during coalescence of secondary voids under transverse uniaxial tension ($\epsilon_1 \approx 0.39$). Primary particle-matrix interface model and GTN model parameters as in Figure 6.1. The contours represent the local void volume fraction of the secondary void population (blue: $f^* = 0$ and red: $f^* = 1$).

Figure 6.4: Inter-cluster coalescence under transverse uniaxial tension ($\epsilon_1 \approx 0.49$). Primary particle-matrix interface model and GTN model parameters as in Figure 6.1. The contours correspond to the local void volume fraction of the secondary void population (blue: $f^* = 0$ and red: $f^* = 1$).

Figure 6.5: Unit cell failure under transverse uniaxial tension ($\epsilon_1 \approx 0.52$). Primary particle-matrix interface model and GTN model parameters as in Figure 6.1. The contours correspond to the local void volume fraction of the secondary void population (blue: $f^* = 0$ and red: $f^* = 1$).

of expected primary void nucleation. Although it may not be evident from the diagram, the stress controlled nucleation of primary voids does not occur due to the strength of the particle-matrix interface. Rather, the matrix surrounding the primary particles experiences sufficient strain to initiate strain controlled matrix failure leading to an alternate form of primary void nucleation. An increase in the nucleation strain within the matrix or a decrease in the interfacial strength of the particle-matrix boundary leads to stress controlled nucleation and prevention of element deletion at the particle-matrix interface (see Section 6.2.1).

Figure 6.2 shows the first stage of primary void coalescence in the unit cell which signifies the beginning of the inter-particle ligament collapse at $\epsilon_1 \approx 0.2$. The inter-particle ligament continues to break down at the transverse symmetry plane causing the linear cluster to merge into a single ellipsoidal void. This void also continues to grow through element deletion at its boundary in the transverse symmetry plane until a bifurcation point is reached leading to void growth in the inter-cluster ligament rather than growth of the primary void itself (Figure 6.3, $\epsilon \approx 0.39$). Inter-cluster damage then starts to accelerate, leading to the eventual collapse of the inter-cluster ligament as depicted in Figure 6.4 at $\epsilon_1 \approx 0.49$. Once the inter-cluster ligament has collapsed, damage progresses rapidly in the remaining elements on the transverse symmetry plane and final failure of the unit cell occurs at $\epsilon \approx 0.52$ (Figure 6.5).

The stress-strain response of this model is compared to that of similar model with a single particle population in Figure 6.6 along with markers showing the strain levels of the various stages of coalescence shown in Figures 6.2 through 6.5. The stress-strain curves of the two models are the same until a strain level of $\epsilon_1 \approx 0.21$. This region of equivalency extends beyond deletion of the first elements at the particle-matrix interface and beyond the initial failure of the inter-particle ligament

Figure 6.6: Stress Strain response of a transverse linear cluster ($\kappa = 10$) with and without a secondary population of small particles represented by the GTN model and deformed in uniaxial tension. $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $f_N = 0.005$, $\epsilon_N = 0.1$, $S_N = 0.02$, $f_c = 0.01$, and $f_f = 1.2f_c$.

failure. It is not until the inter-particle ligament has completely failed that there is some softening observed in the dual-particle population model. This material softening does not lead to imminent material failure, however, and the now-single void continues to grow steadily until there is a second stage (or length scale) of coalescence, bringing about failure of the inter-cluster ligaments.

There are two material length scales exhibited by these models. The first length scale relates to the distance between nearest neighbor particles, referred to as first-order clusters (Pilkey, 1997). A second length scale exists between clusters of first-order clusters. In the case of a linear cluster, the nearest neighbor clusters lie along the x_1 -axis.

It is evident from the stress-strain behaviour in Figure 6.6 that coalescence within first-order clusters does not constitute material failure. It is coalescence at the second level of clustering or between first-order clusters that leads to ultimate material collapse. Unfortunately, the unit cell geometries studied in this thesis lead to a limited analysis of the geometric parameters affecting this second level of particle

clustering; however, they do establish the requirement that catastrophic void linkage must occur on a cluster-to-cluster length scale.

6.2 GTN Constitutive Equation Parameters

The parameters of the Gurson-Tvergaard-Needleman constitutive equation can be categorized into three groups: the curve fit parameters, q_i (which will not be varied here); the nucleation parameters, f_N , ϵ_N , and S_N which are varied; and the coalescence parameters, f_c , and f_f which are also examined. In addition to the GTN model nucleation parameters, the effect of the strength of the interface bonding the primary particle to the matrix, σ_{max} , will be studied. The nucleation parameters of the GTN model will be examined first. As in the previous section, the unit cell studied here is a transverse linear cluster of particles ($\kappa = 10$) deformed in uniaxial tension.

Markers on the following stress-strain plots are used to indicate the strain level at various stages of damage in the unit cell. “Primary void nucleation” indicates the first sign of nucleation at the particle-matrix interface either through element deletion at the interface (which should be assumed unless stated otherwise), or through failure of the traction-separation nucleation model, as described in Chapter 5. “Inter-particle coalescence” refers to the complete failure, through element deletion, of the ligament between nearest neighbor primary particles. Since the damage in the inter-particle ligament can progress in several different ways, the most consistent measure of strain is at complete ligament failure rather than at the development of a hole in the ligament. “Inter-cluster coalescence” occurs when all of the elements between the cluster and the outer symmetry plane along the x_1 axis, or some parallel line, have been deleted.

Figure 6.7: Stress strain responses of a transverse linear cluster ($\kappa = 10$) deformed in uniaxial tension. The parameter, f_N in the GTN nucleation model is varied. $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $\epsilon_N = 0.1$, $S_N = 0.02$, $f_c = 0.01$, and $f_f = 1.2f_c$. The “no coalescence” curve corresponds to a model for which element deletion due to matrix void coalescence has been disabled, as in the models of Chapter 5.

Figure 6.7 depicts the change in stress-strain response of a transverse linear cluster deformed in uniaxial tension when the volume fraction of secondary particles capable of nucleating a void, f_N , is varied. The other GTN model parameters are $\epsilon_N = 0.1$, $S_N = 0.02$, $f_c = 0.01$, and $f_f = 1.2f_c$ and the stress controlled nucleation parameters are $\sigma_{max} = 5\sigma_y$, $\delta = 0.2r$, and $\alpha = 1$. It is evident from Figure 6.7 that the volume fraction of nucleating secondary voids has a strong impact on the strain-to-failure of the material. For all of the stages of damage listed (ie. primary void nucleation, inter-particle coalescence, and inter-cluster coalescence), the damage event occurs at lower strains when the value of f_N increases. Note that for these low levels of secondary void nucleating particles, given by f_N , the material exhibits very little softening.

Figure 6.8: Stress strain responses of a transverse linear cluster ($\kappa = 10$) deformed in uniaxial tension. The parameter, ϵ_N in the GTN nucleation model is varied. $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $f_N = 0.005$, $f_c = 0.01$, and $f_f = 1.2f_c$. The “no coalescence” curve corresponds to a model for which element deletion due to matrix void coalescence has been disabled, as in the models of Chapter 5.

Figure 6.8 compares the effects of varying the mean strain to nucleation of the secondary voids, ϵ_N , on the stress-strain response of a transverse linear cluster deformed in uniaxial tension. There are two studies displayed on this graph. The first is a collection of lower mean nucleation strains with a very narrow nucleation strain distribution, $S_N = 0.01$. In this case, for the range of mean nucleation strains considered ($0.05 \leq \epsilon_N \leq 0.1$), there is very little change in the stress-strain behaviour of the model. The second study on mean nucleation strain has higher mean strains ($0.1 \leq \epsilon_N \leq 0.2$) and a wider nucleation strain standard deviation, $S_N = 0.03$. These models develop a slightly larger range of strains-to-failure but the differences are still small compared to those seen in Figure 6.7.

Figure 6.9 further examines the role of the standard deviation of the nucleation strain for the secondary voids, S_N , in the stress-strain response of a transverse linear cluster deformed in uniaxial tension. The effect of the nucleation strain distribution on the rate of material damage is small for this range of standard deviations ($0.01 \leq$

Figure 6.9: Stress strain responses of a transverse linear cluster ($\kappa = 10$) deformed in uniaxial tension. The S_N parameter in the GTN nucleation model is varied. $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $f_N = 0.005$, $\epsilon_N = 0.10$, $f_c = 0.01$, and $f_f = 1.2f_c$. The “no coalescence” curve corresponds to a model for which element deletion due to matrix void coalescence has been disabled, as in the models of Chapter 5.

$S_N \leq 0.03$). A larger S_N causes earlier nucleation events in the secondary particle population, and hence, earlier element deletion during primary nucleation. These models are also more prone to earlier element deletion within the particle clusters because the hydrostatic stresses are higher and the local damage is primarily through void growth. In contrast, the inter-cluster coalescence events are delayed by the wider distributions of secondary nucleation strains. The elements in these regions are subject to lower hydrostatic stresses and the nucleation stage of the secondary voids plays a larger role in the evolution of damage.

The effect of the critical void volume fraction for coalescence in the matrix, f_c , on the stress-strain response of a linear cluster is shown in Figure 6.10 for transverse uniaxial tension. It is evident that the critical coalescence parameter plays a significant role in the damage behaviour of the unit cell. The range of strain-to-nucleation is not large since the local hydrostatic stresses are large, so the secondary void growth is rapid (note that in the case of $f_c = 0.1$, primary void nucleation

Figure 6.10: Stress strain responses of a transverse linear cluster ($\kappa = 10$) deformed in uniaxial tension. The parameter, f_c in the GTN nucleation model is varied. $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $f_N = 0.010$, $\epsilon_N = 0.10$, $S_N = 0.02$, and $f_f = 1.2f_c$. The “no coalescence” curve corresponds to a model for which element deletion due to matrix void coalescence has been disabled, as in the models of Chapter 5.

Figure 6.11: Stress strain responses of a transverse linear cluster ($\kappa = 10$) deformed in uniaxial tension. The parameter, f_f in the GTN nucleation model is varied. $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $f_N = 0.07$, $\epsilon_N = 0.10$, $S_N = 0.02$, and $f_c = 0.1$. The “no coalescence” curve corresponds to a model for which element deletion due to matrix void coalescence has been disabled, as in the models of Chapter 5.

occurs through particle-matrix separation). However, the differences in strain at both the onset of inter-particle and inter-cluster coalescence are very large. The secondary void growth in the majority of the unit cell is gradual, meaning that the point at which void growth is accelerated, f_c , makes a significant difference in the strain-to-failure of most elements.

Figure 6.11 depicts the variation in stress-strain response for different levels of secondary void volume fraction at the completion of local coalescence, f_f . In this study, the term f_f is presented as a function of f_c . The term f_f does not have a strong effect on the damage rates in the unit cell. The damage acceleration equation (Equation 2.15) of the GTN constitutive equations is quite effective at accelerating damage; consequentially the range of f_f/f_c considered plays a minor role.

Figure 6.12 considers the relationship between the two terms that have the most profound effect on the rate of damage progression in the unit cell, f_N and

Figure 6.12: Stress strain responses of a transverse linear cluster ($\kappa = 10$) deformed in uniaxial tension. The parameter, f_N in the GTN nucleation model is varied within two different orders of magnitude such that the coalescence parameter, f_c is also changed by the same order of magnitude. $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $\epsilon_N = 0.10$, $S_N = 0.02$, and $f_f = 1.2f_c$. The “no coalescence” curve corresponds to a model for which element deletion due to matrix void coalescence has been disabled, as in the models of Chapter 5.

f_c . Figure 6.12 shows the stress-strain response for various ratios of f_c/f_N for both $f_N = 0.005$ and $f_N = 0.05$. Interestingly, the final failure strains are similar for this ten-fold increase in f_N , provided that f_c/f_N is held constant. The increase in f_N does lead to an increased degree of material softening, as well as higher strains-to-failure.

To summarize the effects of the GTN constitutive equation parameters on damage progression, there are two parameters that are of particular interest, that is, f_N and f_c . The remaining parameters (ϵ_N , S_N , and f_f) have a relatively minor effect on void growth in the secondary particle phase. It has also been shown that the ratio of f_c/f_N has a nearly linear relationship on the strain-to-failure (within the ranges considered). A range of ratios of $2.00 \geq f_c/f_N \geq 1.0$ produces a range of strain-to-failure for the unit cell of $0.55 \geq \epsilon_1 \geq 0.12$, respectively. An order-of-magnitude

Figure 6.13: Stress strain responses of a transverse linear cluster ($\kappa = 10$) deformed in uniaxial tension. The interfacial strength of the particle-matrix interface is varied between $0 \leq \sigma_{max} \leq 5\sigma_y$. $\delta = 0.02r$, $\alpha = 1$, $\epsilon_N = 0.10$, $S_N = 0.02$, $f_N = 0.006$, $f_c = 0.01$ and $f_f = 1.2f_c$. The “no coalescence” curve corresponds to a model for which element deletion due to matrix void coalescence has been disabled, as in the models of Chapter 5.

increase in the parameter f_N also increases both the degree of material softening and the strain-to-failure values.

6.2.1 Primary Void Nucleation

In the GTN parametric studies of the previous section, the primary voids were nucleated at the particle-matrix interface through element deletion rather than by separation of the particle-matrix interface. A brief study has been done to determine the effects of the strength of the particle-matrix interface model on the damage rate. Figure 6.13 shows the stress-strain curves for five levels of interfacial strength in a unit cell containing a transverse linear particle cluster and a GTN matrix, deformed in uniaxial tension. The models with the three lowest interfacial strengths ($\sigma_{max} = 0, 2$ and $3\sigma_y$) all develop particle-matrix separation while the higher interfacial

Figure 6.14: Mesh plots of matrix coalescence progression in a transverse linear particle cluster deformed in uniaxial tension to $e_1 \approx 0.175$. The particle-matrix bond strengths are (a) $\sigma_{max} = 0$, (b) $\sigma_{max} = 2\sigma_y$, (c) $\sigma_{max} = 3\sigma_y$, (d) $\sigma_{max} = 4\sigma_y$ and (e) $\sigma_{max} = 5\sigma_y$. $\delta = 0.02r$, $\alpha = 1$, $\epsilon_N = 0.10$, $S_N = 0.02$, $f_N = 0.006$, $f_c = 0.01$ and $f_f = 1.2f_c$.

strengths ($\sigma_{max} = 4$ and $5\sigma_y$) lead to strain controlled failure of the matrix elements at the particle-matrix boundary before separation can occur. Mesh plots showing the different damage patterns for the various particle-matrix bond strengths are given in Figure 6.14. When nucleation occurs through particle-matrix interface separation, the stress-strain curves are almost identical, meaning that the element deletion sequence is the same in each case. Although a higher interfacial strength changes the element deletion sequence in the model somewhat, the overall strain-to

Figure 6.15: Void nucleation at primary particles by nucleation, growth and coalescence of voids at a secondary population of particles. This linear cluster ($\kappa = 10$) has been deformed in aligned uniaxial tension to $\epsilon_1 \approx 0.038$. For the primary particle-matrix interface model, $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, and $\alpha = 1$. For the secondary particle population, represented by the GTN model, the contours correspond to the local void volume fraction (blue: $f^* = 0$ and red: $f^* = 1$). The GTN parameters are $f_N = 0.006$, $\epsilon_N = 0.1$, $S_N = 0.02$ and $f_c = 0.01$, $f_f = 1.2f_c$.

-failure is not significantly altered.

6.3 Cluster Orientation

So far, coalescence has only been examined for the case of transverse linear clusters. This section will examine the damage progression within an aligned linear cluster ($\kappa = 10$) deformed in uniaxial tension, with matrix material containing a secondary particle population. The nucleation and coalescence parameters of the model are $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1.0$, $f_N = 0.006$, $\epsilon_N = 0.1$, $S_N = 0.02$, $f_c = 0.01$, and $f_f = 0.012$.

Figures 6.15 to 6.18 illustrate the progression of matrix damage in the unit cell during aligned uniaxial tension. The contours indicate the level of local damage in the matrix where blue contours represent an undamaged element ($f^* = 0$) and red contours indicate severely damaged regions and imminent element deletion ($f^* \approx 1$).

Figure 6.16: Sudden extensive element deletion at primary outer particle surface. This linear cluster ($\kappa = 10$) has been deformed in aligned uniaxial tension to $\epsilon_1 \approx 0.138$. For the primary particle-matrix interface model, $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, and $\alpha = 1$. For the secondary particle population, represented by the GTN model, the contours correspond to the local void volume fraction (blue: $f^* = 0$ and red: $f^* = 1$). The GTN parameters are $f_N = 0.006$, $\epsilon_N = 0.1$, $S_N = 0.02$ and $f_c = 0.01$, $f_f = 1.2f_c$.

Figure 6.17: Localization of matrix damage at the mid-point of the outer primary particle. This linear cluster ($\kappa = 10$) has been deformed in aligned uniaxial tension to $\epsilon_1 \approx 0.306$. For the primary particle-matrix interface model, $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, and $\alpha = 1$. For the secondary particle population, represented by the GTN model, the contours correspond to the local void volume fraction (blue: $f^* = 0$ and red: $f^* = 1$). The GTN parameters are $f_N = 0.006$, $\epsilon_N = 0.1$, $S_N = 0.02$ and $f_c = 0.01$, $f_f = 1.2f_c$.

Figure 6.18: Complete unit cell failure at the mid-point of the outer primary particle. This linear cluster ($\kappa = 10$) has been deformed in aligned uniaxial tension to $\epsilon_1 \approx 0.531$. For the primary particle-matrix interface model, $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, and $\alpha = 1$. For the secondary particle population, represented by the GTN model, the contours correspond to the local void volume fraction (blue: $f^* = 0$ and red: $f^* = 1$). The GTN parameters are $f_N = 0.006$, $\epsilon_N = 0.1$, $S_N = 0.02$ and $f_c = 0.01$, $f_f = 1.2f_c$.

The primary particles were included in the FEM model but have not been shown here for clarity.

Figure 6.15 shows the first step in primary void nucleation occurring at the central particle at a major principal strain of $\epsilon_1 \approx 0.038$, thereby relieving the stress in the inter-particle ligament. As observed in the transverse cluster, the particle-matrix interfacial strength is high enough to cause element deletion through GTN constitutive softening within the matrix adjacent to the particle surface prior to particle-matrix separation. Primary voids also nucleate at the outer and inner edges of the outer particle at strains of $\epsilon_1 \approx 0.088$ and 0.094 , respectively. These nucleation events also occur through element deletion and are manifested in the stress-strain curve in Figure 6.19 as local discontinuities in stress.

A particularly large dip in the stress-strain curve of Figure 6.19 occurs at a strain of $\epsilon_1 \approx 0.138$, corresponding to a sudden and extensive deletion of elements that contact the surface of the outer primary particle. Figure 6.16 illustrates the state

Figure 6.19: Stress Strain response of an aligned linear cluster ($\kappa = 10$) with and without a secondary population of small particles represented by the GTN model and deformed in uniaxial tension. $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1$, $f_N = 0.005$, $\epsilon_N = 0.1$, $S_N = 0.02$, $f_c = 0.01$, and $f_f = 1.2f_c$.

of damage in the unit cell following this sudden element deletion in the aligned cluster. Contrary to the transverse cluster case, the inter-particle ligament does not see significant damage in the aligned cluster. As such, this sudden element deletion event will substitute as a first coalescence event in the aligned cluster, corresponding to collapse of the inter-particle ligament in the transverse cluster.

Once primary void nucleation is complete, there is steady damage evolution and occasional element deletion feeding the growth of primary voids until the major principal strain reaches $\epsilon_1 \approx 0.306$. At this point, the damage localizes on a plane that is perpendicular to the major principal strain and intersects the mid-point of the outer void (see Figure 6.17). This localization initiates a decrease in the stress-strain curve (Figure 6.19) as element deletion occurs almost exclusively on this plane. Final unit cell failure occurs at $\epsilon_1 \approx 0.531$, as depicted in Figure 6.18.

Similar to the behaviour of the transverse linear cluster, extensive damage within the particle cluster does not cause catastrophic failure in the unit cell. It is not until the damage has localized between clusters of clusters (ie. between first-order

clusters) that imminent failure becomes evident in the stress-strain response. This behaviour is very significant because it demonstrates that the model contains two significant length scales in relation to damage and final failure. Furthermore, these two material length scales, corresponding to first- and second-order clusters, respectively, delineate the change-over from stable to unstable levels of damage.

6.4 Stress Field Effects

Previous sections have discussed the stages of failure in aligned and transverse linear clusters deformed in uniaxial tension, along with the effect of varying the GTN nucleation and coalescence parameters. This section examines the influence of different applied stress states on the deformation of unit cells containing a linear cluster, still considering only the $\kappa = 10$ cases. Of particular interest is the effect of stress state on coalescence behaviour.

Figure 6.20 compares the stress-strain responses of the unit cell deformed in both aligned and transverse plane strain to those deformed in uniaxial tension. As surmised by observing the directional effects in Chapters 4 and 5, the orientation of this anisotropic cluster with respect to the major principal strain has a significant effect on the strain-to-failure or the ductility of the material. A transverse linear cluster has a lower strain-to-failure than an aligned linear cluster. This same orientation effect has been measured in experimental studies by Pilkey (1997).

Figure 6.21 depicts the linear cluster response when deformed in the four main biaxial strain states. The response of the same unit cell without a GTN matrix damage model is also plotted for reference. In most cases, a strain state that leads to a higher component of hydrostatic stress causes higher stresses and lower strains-to-failure. The exception is with the equi-biaxial stretch where the stresses do not

Figure 6.20: Stress strain responses of linear clusters ($\kappa = 10$) deformed in aligned and transverse uniaxial tension and plane strain. The first coalescence event refers to inter-particle ligament failure in transverse clusters and sudden extensive element removal from a primary particle boundary in aligned clusters. $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1.0$, $\epsilon_N = 0.10$, $S_N = 0.02$, $f_N = 0.006$, $f_c = 0.01$ and $f_f = 1.2f_c$.

Figure 6.21: Stress strain responses of linear clusters ($\kappa = 10$) deformed in four biaxial strain fields. Each case is compared with the stress-strain response of the same model without a GTN model matrix. $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1.0$, $\epsilon_N = 0.10$, $S_N = 0.02$, $f_N = 0.006$, $f_c = 0.01$ and $f_f = 1.2f_c$.

Figure 6.22: Stress strain responses of linear clusters ($\kappa = 10$) deformed at three levels of hydrostatic tension. The first is deformed in uniaxial tension ($\chi = 0.33$), the second in plane strain ($\chi = 0.58$) and the third in uniaxial tension with extra superimposed hydrostatic tension ($\chi = 1.25$). $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1.0$, $\epsilon_N = 0.10$, $S_N = 0.02$, $f_N = 0.006$, $f_c = 0.01$ and $f_f = 1.2f_c$.

exceed those of the plane strain test case. Presumably, the stress increase is limited in the biaxial strain fields by the low constraint in the third direction. Figure 6.22 considers the triaxial stress states comprised of uniaxial tension with superimposed hydrostatic stress ($\chi = 0.33, 1.25$ and 3.00). In the high triaxiality cases, the stresses do exceed those of the plane strain cases and the unit cell fails at even lower strains, as expected. Conversely, the deep draw strain case, which has minimal stress triaxiality, leads to a very low level of damage, even at very high strains.

Figure 6.23: Stress strain responses of linear clusters of different cluster densities deformed in uniaxial tension ($\chi = 0.33$). $\sigma_{max} = 5\sigma_y$, $\delta = 0.02r$, $\alpha = 1.0$, $\epsilon_N = 0.10$, $S_N = 0.02$, $f_N = 0.006$, $f_c = 0.01$ and $f_f = 1.2f_c$.

6.5 Cluster Density

This section examines the comparative length scales in the unit cell, referring to the distance between clusters compared to the distance between nearest neighbor particles. This ratio, κ , which is defined in Equation 2.2, characterizes the spatial distribution of the particle population in an array of unit cells. Figure 6.23 illustrates the differences in material response for various linear cluster densities deformed under uniaxial tension. Evidently, a stronger degree of clustering ($\kappa = 22$) leads to a lower strain-to-failure for both the aligned and transverse cases. This trend is seen throughout the damage evolution, where a high value of κ leads to earlier primary void nucleation, an earlier first coalescence event and earlier failure. Note that as the value of κ is reduced, the early coalescence events occur over a larger range of strain and are more difficult to pinpoint. Again, the experimental work

of Pilkey (1997) has also observed early failure strains in more clustered materials. Furthermore, it should be noted that the parameter κ in the current study can be directly related to inter-particle dilational spacing (IPDS) measures employed by Pilkey (1997) to establish length scales associated with particle clustering.

It is somewhat surprising that there is not more evidence of mesh sensitivity, especially near model failure. In the previous sections of this chapter, the mesh has been the same for all of the test cases. For this study, however, the different unit cell geometries require unique element discretizations. It was assumed that these mesh differences would lead to irregularities in the failure trends but such anomalies are not exhibited. It should not be assumed that mesh effects are not present, however. Further consideration of mesh sensitivity was not considered due to computational expense.

It was also expected that the aligned and transverse strains-to-failure would begin to converge as the particles became more evenly distributed. A cluster distribution of $\kappa = 3$ is still highly anisotropic however, in fact there appears to be some divergence. In the case with $\kappa = 1$, in a single void model, the model symmetry implies convergence and the failure strain is half way between the failure strains of the least clustered material ($\kappa = 3$). As such, values of $\kappa < 3$, in a unit cell with a linear cluster, should lead to less and less anisotropy in formability. These observations may also be strongly affected by mesh sensitivity issues since the mesh of the single void model has a significantly different design from the clustered model meshes.

6.6 Summary

Inclusion of a damage-based constitutive model to simulate the local softening and failure associated with a secondary population of small voids is an effective approach to predict coalescence in a unit cell containing a cluster of discrete particles. There are many nucleation and coalescence parameters associated with such a model but the ones that produced the greatest effect on the strain-to-failure were the volume fraction of void nucleating secondary particles, f_N , and the volume fraction of secondary voids at local coalescence, f_c . In addition, the ratio, f_c/f_N has been shown to be almost linear with respect to the predicted failure strains, while a significantly higher value of f_N leads to increases in both material softening and ductility. The other GTN parameters and the nucleation parameters associated with the primary particle-matrix interface have a minor effect on the progression of damage in the models considered.

As observed in Chapters 4 and 5, the orientation of an anisotropic cluster is a very important factor for damage evolution within a clustered material. A transverse cluster displays higher damage rates than an aligned cluster. Although a transverse cluster develops inter-particle ligament collapse whereas an aligned cluster does not, both orientations exhibit stable deformation well beyond their first coalescence events. As such, it can be stated that coalescence within first-order clusters is a stable event and does not constitute material failure. Rather, it is the coalescence between first-order clusters (ie. within second order clusters) that leads to ultimate failure of the unit cell.

In general, an increase in the component of hydrostatic tension leads to higher stresses and lower material failure strains. In the case of a deep draw, where $\chi = 0$, there is very little damage even at high strains. A triaxial strain field leads to major

principal stresses exceeding those of the biaxial strain states with associated high damage rates.

The microstructural length scale of a particle population has a significant effect on failure, in that a highly clustered material will develop critical damage at lower strains than a less clustered material. In the case of linear clusters, the orientation of the clusters remains an important factor in relation to damage rate, even at low ratios of cluster distance to nearest neighbor particle distance, κ .

Chapter 7

Discussion

With the aid of increasingly powerful computers, microstructural modeling of ductile fracture becomes more and more predictive. The work represented in this thesis further advances the development of these models both with the inclusion of particle clustering effects and the integration of all three stages of ductile fracture, that is the nucleation, growth and coalescence of voids, in a three-dimensional unit-cell finite element study.

Previous analyses of particle clustering are either two-dimensional or the particles are limited to placement at the vertices. The clusters considered in this thesis are discrete three dimensional entities. As such, the particle interactions within the cluster are largely independent of other cluster effects during the early stages of deformation which is more representative of real damage evolution.

The cluster geometries considered in this work have the added feature of introducing a microstructural length scale into the unit cell model. The comparative distance between particles in a cluster to the distance between clusters establishes the length scale. Other forms of clustering used in previous studies do not model

this feature and the microstructural length scale has been shown to be an important parameter in damage percolation studies (Worswick *et al.*, 1998; Pilkey, 1997).

Nucleation through particle-matrix debonding has been introduced within a three dimensional formulation. Previous traction-separation models have been limited to two dimensions. The three dimensional nucleation model, coupled with particle clustering and a matrix damage model, has allowed detailed examination of nucleation progression and the effects of geometry and strain field.

Addition of a PID controller to the finite element source code allows for a unique approach to modeling high components of hydrostatic stress in an explicit solver. As a result, models can be both strain and stress controlled simultaneously and strains well-beyond the mechanical instability point can be considered. The effects of particle clustering on damage in regions of high hydrostatic stress is particularly relevant in predictive models of ductile failure.

Of the studies of coalescence in three-dimensional unit cell models, most rely on void impingement after stable void growth to achieve coalescence which results in high failure strains. A few studies have introduced a dual population of particles but the distribution of larger voids was unclustered and these microstructures were studied in the context of dynamic crack growth. In this work, the dual population of particles facilitates progression of damage in a clustered microstructure under static forming conditions and a variety of stress fields.

There are two trends from the results that compare favourably with the experimental results of Pilkey (1997). In this thesis, the anisotropy of particle clustering has been shown to have an effect on the damage rates and failure strain of clustered materials. Pilkey (1997) has shown experimentally that a more clustered Al2%Si alloy has superior formability in the longitudinal direction than in the direction transverse to rolling. When strontium is added to the Al2%Si alloy, thereby at-

taining a more evenly distributed second phase, the sheet metal behaviour becomes more isotropic.

This thesis has also demonstrated that void coalescence within first-order clusters, or between voids within a cluster, is a stable event and that significant deformation and damage progression occurs in the material beyond this localized coalescence. This supports microstructural observations made of interrupted material testing performed by Pilkey (1997). Many observations of coalescence within first-order clusters has been observed in micrographs of failed or almost failed material but Pilkey (1997) did not capture any examples of coalescence between neighboring clusters, not even within a neck. these experimental observations also imply that the coalescence within first-order clusters is stable but that the coalescence of voids from neighboring clusters is catastrophic.

7.1 Model Geometry

Within the unit-cell structure, it was assumed that the central void or particle would remain at the center during deformation. This assumption permits reduction of the discretization domain and model size through symmetry, but adds extra constraint to the model. In addition, the arrangement of clusters within the unit cell always placed particles on the model axes. These arrangements are much easier to mesh with three dimensional bricks than more random placements, particularly when the particles are very close together. Note, however, that the cluster orientation leads to uncharacteristically high concentrations of voids on the symmetry planes of the model, which leads to premature softening and failure. On the other hand, these cluster orientations can be considered to represent a lower limit of material behaviour.

The adopted unit cell approach exerts a major influence on the concentration of voids on particular planes in these models. With sufficient computing resources, it would be very interesting to examine a less-periodic distribution of randomly oriented clusters of particles. This would reduce planar void concentration effects and would increase the number of potential levels of clustering. The current model has two levels of clustering: clusters of particles and clusters of clusters. If a third level were added, perhaps through variations in inter-cluster spacing, there would be sufficient geometrical flexibility in the model to determine which level of coalescence leads to imminent material failure. Certainly, damage percolation approaches suggest that more than two neighboring clusters must coalesce prior to profuse damage and fracture (Worswick *et al.*, 1998). Such models could further establish the conditions at instability.

7.2 Meshing

As discussed in Chapter 3, there are discretization errors associated with the finite element meshing scheme chosen for this study. A finer mesh would increase the accuracy to a small degree but would also require more computing resources. The discretization is particularly coarse in regions removed from the particles which should lead to mesh effects in the coalescence calculations. One possible improvement would be to refine the mesh in the planes of anticipated fracture. Not all stress fields lead to fracture in the same plane, however, and there may be some degree of trial-and-error required to achieve the desired results. Another possibility is to incorporate a non-local version of the Gurson-Tvergaard-Needleman constitutive equation into the finite element calculations (Tvergaard and Needleman, 1997), which would tend to reduce or control mesh effects.

7.3 Stress Fields

Although an effort was made to minimize dynamic effects in the analyses contained in this thesis, the models subject to a high component of hydrostatic tension are particularly susceptible to these effects and, in some cases, the data from these models does not represent true static deformation. Despite this disadvantage, the adopted load control approach permitted high triaxiality calculations to be performed well into the catastrophic void coalescence stage.

The PID controller that facilitated superimposing a high component of hydrostatic stress on a uniaxial stress field could be easily applied to biaxial stress fields as well, though this loading was not examined in this thesis. The biaxial fields considered were exclusively strain controlled.

7.4 Other Simulation Issues

During the finite element calculations, element and nodal data were only output at selected time intervals. As a result, the time integrations performed in post-processing are subject to errors related to time step size. For more accurate results, subroutines were added directly to the finite element solver which would reduce the integration time step to that of the solver. Comparing the two methods, however, it was found that the error associated with time step was small and the more flexible and manageable post-processing tool was chosen as the preferred method.

In Chapter 5 it was concluded that material damage rates are significantly increased by a particle propping open the growing void in the minor principal strain direction. During the gradual element deletion associated with coalescence in Chapter 6, the elements that were used to define the particle-matrix interface which prevents overlapping of the two materials, were also deleted and the interface was

lost. As a result, the particle was no longer able to prop open the void and damage rates were reduced. A newer version of the finite element code, LS-DYNA features an updated contact algorithm that would prevent the matrix from overlapping the particle and should be considered in future work.

7.5 Alternative Coalescence Strategies

In this thesis, coalescence is achieved through element softening and deletion which assumes a secondary population of small particles in the matrix. It may be possible to achieve void coalescence through geometric instability and shear banding without need of a matrix damage model.

The first strategy is related to the shear band instability study by Tvergaard (1981) in two dimensions. Rather than using orthogonal boundary conditions, the symmetry conditions on a unit cell could be enforced through more flexible, non-orthogonal boundary conditions. As long as opposite faces of the unit cell have the same shape such that the unit cell can be stacked in all directions, the model should still behave as a small unit of a much larger microstructure. The anticipated advantage of non-orthogonal boundary conditions is that the added geometric flexibility could lead to earlier development of shear bands and void coalescence.

The second strategy is that proposed by Thomason (1985a) which calculates approximate slipline fields in three dimensions. When the global stress reaches the pre-calculated stress required to initiate slipline flow around the particles, the plastic flow localizes and the voids in the cluster coalesce. The first difficulty with this approach is finding a flow field that satisfies the slipline flow rules and minimizes

the initiation stress. The second difficulty arises after the clustered voids coalesce; that is, is it possible to continue the slip-line field finite element calculation from that point?

Chapter 8

Conclusions and Recommendations

8.1 Conclusions

This research has demonstrated that material damage rates in clustered microstructures are different than in unclustered microstructures. The difference is not only due to the non-spherical shape of the cluster since damage rates associated with a single ellipsoidal void will be different than those of a cluster with equivalent aspect ratio. The stresses and strains in the inter-particle ligament play an important role in the damage progression. Based on the results of this research, the following conclusions can be drawn:

- i) Material anisotropy has an important effect in all stages of ductile damage progression. In terms of nucleation behaviour, with the introduction of a particle-matrix interface, there is an orientation-dependent delay in the initi-

ation of damage. In an aligned cluster, the inter-particle ligament develops higher stress levels than the surrounding material, leading to early void nucleation. The inter-particle ligament in a transverse cluster of particles shows no sign of stress concentration until after void nucleation has occurred at a much higher level of strain. In addition, complex particle clusters under multi-axial strain fields experience additional constraint on void growth after nucleation; voids must nucleate at all particles in a plane or in all major principal strain directions before any significant void growth can develop.

- ii) Orientation of an anisotropic cluster of nucleated voids has a significant impact on the void growth rates in ductile materials. In an orientation with the major axis of a cluster of voids aligned with the major principal stress direction, the inter-particle ligament is sheltered from the major principal stress and damage rates are lower than in an unclustered material. In a transverse cluster arrangement, the ligament develops higher strains than in the surrounding materials and damage is accelerated beyond those in an unclustered voided material.
- iii) Although the aligned particle cluster models nucleate voids earlier than the transverse models and have higher initial damage rates, the voids in the transverse case grow more rapidly than in the aligned case. By the time the material has reached typical failure strains, void growth has become dominant and transverse models have higher levels of damage than the aligned models.
- iv) The presence of particles within a nucleated void prevents void contraction in the minor principal strain direction resulting in significantly higher damage rates compared to a voided material that does not contain particles.
- v) In contrast to the models without a particle, that is in initially voided ma-

terials, the clustered particle models display higher damage rates in all load orientations compared to the corresponding unit-cell models that contain only a single particle at the center.

- vi) Incorporation of the Gurson-Tvergaard-Needleman constitutive equations to simulate damage surrounding a secondary population of smaller particles has successfully reduced the failure strains of the unit cell to levels comparable with experimental results (Pilkey, 1997).
- vii) Of the six nucleation and coalescence parameters studied pertaining to the GTN model, the following parameters had a major impact on the strain-to-failure within the unit cells:
 - (a) the volume fraction of void nucleating secondary particles, f_N ; an order-of-magnitude increase in the value of f_N also leads to increased material softening.
 - (b) the secondary void volume fraction at the beginning of coalescence, f_c ; the strain-to-failure was found to have an almost linear relationship with f_c/f_N .

The following had a minor impact on damage rate and strain-to-failure:

- (a) the primary void nucleation stress, σ_{max}
 - (b) the secondary void nucleation strain, ϵ_N , and standard deviation, S_N
 - (c) the void volume fraction for final coalescence, f_f
- viii) At the coalescence stage of damage, the orientation of anisotropic particle clusters continues to affect the progression of damage with transverse clusters experiencing inter-particle ligament collapse but not aligned clusters. The

higher damage rates in the transverse clusters also lead to fracture at lower strains than in aligned models. If the degree of clustering in the material is lessened, failure strains are increased but cluster orientation still has a strong influence.

- ix) The behaviour with respect to the microstructural length scales of this model has served to confirm that first order coalescence in a clustered material, that is coalescence within a cluster of particles, is a stable event and that the material continues to support loads and harden beyond this point. It is the second stage of coalescence, that between clusters of clusters, that initiates unit cell failure. This observation is consistent with post-fracture micrographs (Pilkey, 1997) that reveal localized damage within particle clusters but little coalescence between clusters except for those adjacent to the fracture surface.

8.2 Recommendations for Future Work

There are two principal investigations that would improve the predictive capability of this ductile failure model: (i) an increase in the number of levels of particle clustering; and, (ii) exploration of geometric instability-based coalescence.

Increased levels of clustering would extend the study of critical coalescence and fracture path, leading to a clearer understanding of the effects of particle clustering on ductile fracture strain under various deformation conditions. If the particles are to be modeled as discrete entities, increasing the number of levels of particle clustering would require a significant increase in computing resources. A unit cell would have to contain more than one particle cluster arranged with irregular spacing and alignment, thus reduction of model size by symmetry would no longer be viable. The number of geometric parameters would also increase dramatically.

In many cases, ductile fracture has been shown to occur as much through shear banding as through void coalescence. A failure mechanism that facilitates geometric instability would allow failure to occur through a combination of void growth and shear banding. Geometric instability may be possible with non-orthogonal, non-planar unit cell boundary conditions. Again, reduction of the model size through symmetry would no longer be an option in this case and more computer resources would be required.

References

- Argon, A.S., J. Im and R. Safoglu (1975), 'Cavity formation from inclusions in ductile fracture', *Metallurgical Transactions A* **6A**, 825–837.
- Asaro, R.J. (1983), 'Crystal plasticity', *Journal of Applied Mechanics* **50**, 921–934.
- Ashby, M.F., J.D. Embury, S.H. Cooksley and D. Teirlinck (1985), 'Fracture maps with pressure as a variable', *Scripta Metallurgica* **19**, 385–390.
- Barlat, F. and J.M. Jalinier (1985), 'Formability of sheet metal with heterogeneous damage', *Journal of Materials Science* **20**, 3385–3399.
- Basu, S. and R. Narasimhan (1999), 'A finite element study of the effects of material characteristics and crack tip constraint on dynamic ductile fracture initiation', *Journal of the Mechanics and Physics of Solids* **47**, 325–350.
- Becker, R. (1987), 'The effect of porosity distribution on ductile failure', *Journal of the Mechanics and Physics of Solids* **35**, 577–599.
- Becker, R. and R.E. Smelser (1994), 'Simulation of strain localization and fracture between holes in an aluminum sheet', *Journal of the Mechanics and Physics of Solids* **42**, 773–796.
- Becker, R., R.E. Smelser, O. Richmond and E.J. Appleby (1989), 'The effect of void

- shape on void growth and ductility in axisymmetric tension tests', *Metallurgical Transactions A* **20A**, 853–861.
- Bourcier, R.J., D.A. Koss, R.E. Smelser and O. Richmond (1986), 'The influence of porosity on the deformation and fracture of alloys', *Acta Metallurgica* **34**, 2443–2453.
- Brockenbrough, J.R. and F.W. Zok (1994), 'On the role of particle cracking in flow and fracture of metal matrix composites', *Acta Metallurgica et Materialia* **43**, 11–20.
- Brocks, W., D.Z. Sun and A. Hönl (1996), 'Verification of micromechanical models for ductile fracture by cell model calculations', *Computational Materials Science* **7**, 235–241.
- Brown, L.M. and J.D. Embury (1973), The initiation and growth of voids at second phase particles, in 'Proceedings of the 3rd International Conference on the Strength of Metals and Alloys', Institute of Metals, London, England, pp. 164–169.
- Brown, L.M. and W.M. Stobbs (1976), 'Work-hardening of copper-silica-5. equilibrium plastic relaxation by secondary dislocations', *Philosophical Magazine* **34**, 351–372.
- Budiansky, B., J.W. Hutchinson and S. Slutsky (1981), Void growth and collapse in viscous solids, in H. Hopkins and M. Sewell, eds, 'Mechanics of solids: Rodney Hill 60th Anniversary Edition', Pergamon Press, England, pp. 13–45.
- Chu, C.C. and A. Needleman (1980), 'Void nucleation effects in biaxially stretched sheets', *Journal of Engineering Materials and Technology* **102**, 249–256.

- Cortés, R. and M. Elices (1993), 'The plastic growth of a cavity nucleated at a shear band', *International Journal of Solids and Structures* **30**, 2971–2981.
- Favier, G.R., G.R Canova and B. Baudalet (1995), 'A micromechanical incremental modeling of damage in materials containing second phase particles', *Acta Metallurgica et Materialia* **43**, 1249–1258.
- Flanagan, D.P. and T. Belytschko (1981), 'A uniform strain hexahedron and quadrilateral and orthogonal hourglass control.', *International Journal of Numerical Methods in Engineering* **17**, 679–706.
- Fleck, N.A., G.M. Muller, M.F. Ashby and J.W. Hutchinson (1994), 'Strain gradient plasticity: theory and experiment', *Acta Metallurgica et Materialia* **42**, 475.
- Fleck, N.A., J.W. Hutchinson and V. Tvergaard (1989), 'Softening by void nucleation and growth in tension and shear', *Journal of the Mechanics and Physics of Solids* **37**, 515–540.
- Forero, L.E. and D.A. Koss (1994), 'On the strain-induced growth of neighboring voids', *Scripta metallurgica et materialia* **31**, 419–422.
- Franklin, G.F., J.D. Powell and A. Emami-Naeini (1994), *Feedback Control of Dynamic Systems*, 3rd ed., Addison-Wesley Publishing Company.
- Gall, K., M.F. Horstemeyer, M. Van Schilfgaarde and M.I. Baskes (2000), 'Atomistic simulations on the tensile debonding of an aluminum-silicon interface', *Journal of the Mechanics and Physics of Solids* **48**, 2183–2212.
- Gologanu, M. (1997), Étude de Quelques Problèmes de Rupture Ductile des Métaux, PhD thesis, l'Université Paris VI.

- Gologanu, M., J.B. Leblond and J. Devaux (1993), 'Approximate models for ductile metals containing non-spherical voids: Case of axisymmetric prolate ellipsoidal cavities', *Journal of the Mechanics and Physics of Solids* **41**, 1723–1754.
- Goods, S.H. and L.M. Brown (1979), 'The nucleation of cavities by plastic deformation', *Acta Metallurgica* **27**, 1–15.
- Goodwin, G.M. (1968), 'Application of strain analysis to sheet metal forming problems in the press shop.', *SAE paper No. 680093*.
- Goto, D.M. and D.A. Koss (1996), 'An experimental model of the growth of neighboring voids during ductile fracture', *Scripta Materialia*.
- Gurland, J. (1972), 'Observations on the fracture of cementite particles in a spheroidized 1.05% C steel deformed at room temperature', *Acta Metallurgica* **20**, 735.
- Gurson, A.L. (1975), Plastic flow and fracture behavior of ductile materials incorporating void nucleation, growth and interaction, PhD thesis, Brown University.
- Gurson, A.L. (1977), 'Continuum theory of ductile rupture by void nucleation and growth: Part 1 - yield criteria and flow rules for porous ductile media', *Journal of Engineering Materials and Technology* **99**, 2–15.
- Haddad, Y.M. (1990), 'Microstructural approach to the mechanical response of a class of polycrystalline systems: I. Theoretical analysis', *International Journal of Structural Mechanics and Materials Science* **29**, 177–196.
- Hallquist, J.O. (1988), *VECDYNA*, Livermore Software Technology Corporation.

- Harik, V.M. and R.A. Cairncross (2000), 'Formation of interfacial voids in composites with a weakly bonded viscoplastic matrix', *Mechanics of Materials* pp. 807–820.
- He, R., D. Steglich, J. Heerens, G.X. Wang, W. Brocks and M. Dahms (1998), 'Influence of particle size and volume fraction on damage and fracture in Al-Al₃Ti composites and micromechanical modelling using the GTN model', *Fatigue and Fracture of Engineering Materials and Structures* **21**, 1189–1201.
- Hill, R. (1950), *The Mathematical Theory of Plasticity*, Oxford University Press.
- Hom, C.L. and R.M. McMeeking (1989), 'Void growth in elastic-plastic materials', *Journal of Applied Mechanics* **56**, 309–317.
- Huang, Y., A. Chandra and N.Y. Li (1998), 'Void-nucleation vs. void-growth controlled plastic flow localization in materials with non-uniform particle distributions', *International Journal of Solids and Structures* **35**, 2475–2486.
- Hutchinson, J.W. and V. Tvergaard (1981), 'Shear band formation in plane strain', *International Journal of Solids and Structures* **17**, 451–470.
- Keeler, S.P. (1965), 'Determination of forming limits in automotive stampings.', *Sheet Met. Ind.* **42**, 683–691.
- Keer, L.M., J. Dundurs and K. Kiattikomol (1973), 'Separation of a smooth circular inclusion from a matrix', *International Journal of Engineering Science* **11**, 1221.
- Kim, K.B. and E.P. Yoon (1995), 'Effect of SiO₂ content in Al₂O₃ on interfacial bonding strength of aluminum/Al₂O₃ composite', *Materials Science and Technology* **11**(9), 629–632.

- Kitagawa, H. and K. Honke (1988), 'Anisotropy and softening induced by void growth: A numerical simulation of micro-structural process', *Computers and Structures* **30**(4), 909–913.
- Koenigsmann, H.J. and E.A. Starke (1996), 'Cavity nucleation and growth in an Al-Si-Ge alloy', *Scripta Materialia* **35**, 1205–1209.
- Koenigsmann, H.J., E.A. Starke and P.E. Allaire (1996), 'Finite element/experimental analysis of cavity nucleation in an Al-Si-Ge alloy', *Acta Materialia* **44**, 3069–3075.
- Koplik, J. and A. Needleman (1988), 'Void growth and coalescence in porous plastic solids', *International Journal of Solids and Structures* **24**, 835–853.
- Kuna, M. and D.Z. Sun (1996), 'Three dimensional cell model analyses of void growth in ductile materials', *International Journal of Fracture* **81**, 235–258.
- Leblond, J.B. and G. Perrin (1999), 'A self-consistent approach to coalescence of cavities in inhomogeneously voided ductile solids', *Journal of the Mechanics and Physics of Solids* **47**, 1823–1841.
- Leblond, J.B., G. Perrin and J. Devaux (1995), 'An improved Gurson-type model for hardenable ductile materials', *European Journal of Mechanics, A/Solids* **14**(4), 499–527.
- Leblond, J.B., G. Perrin and P. Suquet (1994), 'Exact results and approximate models for porous viscoplastic solids', *International Journal of Plasticity* **10**, 213–235.
- Lee, B.J. and M.E. Mear (1992), 'Axisymmetric deformation of power-law solids containing a dilute concentration of aligned spheroidal voids', *Journal of the Mechanics and Physics of Solids* **40**, 1805–1836.

- Lee, Y.S. and P.R. Dawson (1993), 'Modeling ductile void growth in viscoplastic materials. Part I: Void growth model', *Mechanics of Materials* 15(1), 21–34.
- Levy, A.J. (1994), 'Separation at a circular interface under biaxial load', *Journal of the Mechanics and Physics of Solids* 42, 1087–1104.
- Levy, A.J. (1997), 'On the nucleation of cavities in planar elasticity', *Phil. Trans. R. Soc. Lond. A* 355, 2417–2458.
- Lu, Y. and M. Hirohashi (1997), 'Analysis of interfacial shear strength of SiC/Al composite by acoustic emission method', *Scripta materialia* 38, 273–278.
- Magnusen, P.E, E.M. Dubenski and D.A. Koss (1988), 'The effect of void arrays on void linking during ductile fracture.', *Acta Metallurgica* 36(6), 1503–1509.
- Mathur, K.K, A. Needleman and V. Tvergaard (1996), 'Three dimensional analysis of dynamic ductile crack growth in a thin plate', *Journal of the Mechanics and Physics of Solids* 44, 439–464.
- McClintock, F.A. (1968a), 'A criterion for ductile fracture by the growth of holes', *Journal of Applied Mechanics* 35, 363–371.
- McClintock, F.A. (1968b), On the mechanics of fracture from inclusions, in 'Ductility', ASM, Metals Park, Ohio, chapter 9, pp. 255–277.
- Michel, J.C. and P. Suquet (1992), 'The constitutive law of nonlinear viscous and porous materials', *Journal of the Mechanics and Physics of Solids* 40, 783–812.
- Narasimhan, R. (1994), 'A numerical study of fracture initiation in a ductile material containing a dual population of second-phase particles: I. Static loading', *Engineering Fracture Mechanics* 47, 919–934.

- Needleman, A. (1972), 'Void growth in an elastic-plastic medium', *Journal of Applied Mechanics* **39**, 964–970.
- Needleman, A. (1979), 'Non-normality and bifurcation in plane strain tension and compression', *Journal of the Mechanics and Physics of Solids* **27**, 231–254.
- Needleman, A. (1987), 'A continuum model for void nucleation by inclusion debonding', *Journal of Applied Mechanics* **54**, 525–531.
- Needleman, A. (2000), 'Computational mechanics at the mesoscale', *Acta Materialia* **48**, 105–124.
- Needleman, A. and A.S. Kushner (1990), 'An analysis of void distribution effects on plastic flow in porous solids', *European Journal of Mechanics, A/Solids* **9**(3), 193–206.
- Needleman, A. and N. Triantafyllidis (1978), 'Void growth and local necking in biaxially stretched sheets', *Journal of Engineering Materials and Technology* **100**, 165–169.
- Needleman, A. and V. Tvergaard (1991), 'An analysis of dynamic, ductile crack growth in a double edge cracked specimen', *International Journal of Fracture* **49**, 41–67.
- Ohno, N. and J.W. Hutchinson (1984), 'Plastic flow localization due to non-uniform void distribution', *Journal of the Mechanics and Physics of Solids* **32**, 63–85.
- Pardoen, T. and F. Delannay (1998), 'The coalescence of voids in prestrained notched round copper bars', *Fatigue and Fracture of Engineering Materials and Structures* **21**, 1459–1472.

- Pardoen, T. and J.W. Hutchinson (2000), 'An extended model for void growth and coalescence', *Journal of the Mechanics and Physics of Solids* **48**, 2467–2512.
- Pilkey, A.K. (1997), Effect of second-phase particle clustering on aluminum-silicon alloy sheet formability, PhD thesis, Carleton University.
- Pilkey, A.K., J.F. Fowler, M.J. Worswick, G. Burger and D. Lloyd (1995), 'Characterizing particle distributions in model aluminum systems', *Microstructural Science* **22**, 329–336.
- Rice, J.R. and D.M. Tracey (1969), 'On the ductile enlargement of voids in triaxial stress fields', *Journal of the Mechanics and Physics of Solids* **17**, 201–217.
- Richelsen, A.B. and V. Tvergaard (1994), 'Dilatant plasticity or upper bound estimates for porous ductile solids', *Acta Metallurgica et Materialia* **42**, 2561–2577.
- Rudnicki, J.W. and J.R. Rice (1975), 'Conditions for the localization of deformation in pressure-sensitive dilatant materials', *Journal of the Mechanics and Physics of Solids* **23**, 371–394.
- Saje, M., J. Pan and A. Needleman (1982), 'Void nucleation effects on shear localization in porous plastic solids', *International Journal of Fracture* **19**, 163–182.
- Schlüter, N., W. Grimpe and W. Dahl (1996), 'Modelling of the damage in ductile steels', *Computational Materials Science* **7**, 27–33.
- Shi, Y.W., J.T. Barnby and A.S. Nadkarni (1991), 'Void growth at ductile crack initiation of a structural steel', *Engineering Fracture Mechanics* **39**, 37–44.
- Shockey, D.A., L. Seaman, K.C. Dao and D.R. Curran (1980), 'Kinetics of void development in fracturing a533b tensile bars', *Journal of Pressure Vessel Technology* **102**, 14–21.

- Søvik, O.P. (1996), 'Experimental and numerical investigation of void nucleation in an AlMgSi alloy', *Journal de Physique IV C6*, 155–164.
- Søvik, O.P. and C. Thaulow (1997), 'Growth of spheroidal voids in elastic-plastic solids', *Fatigue and Fracture of Engineering Materials and Structures* **20**, 1731–1744.
- Steglich, D. and W. Brocks (1997), 'Micromechanical modelling of the behaviour of ductile materials including particles', *Computational Materials Science* **9**, 7–17.
- Steglich, D. and W. Brocks (1998), 'Micromechanical modelling of damage and fracture of ductile materials', *Fatigue and Fracture of Engineering Materials and Structures* **21**, 1175–1188.
- Su, X.F., H.R. Chen, D. Kennedy and F.W. Williams (1999), 'Effects of interphase strength on the damage modes and mechanical behaviour of metal-matrix composites', *Composites: Part A* **30**, 257–266.
- Sun, Y. (1995), 'Influence of void nucleation and growth on deformation localization in tensile sheet specimen', *Engineering Fracture Mechanics* **51**, 381–389.
- Sun, Y. and D. Wang (1989), 'A lower bound approach to the yield loci of porous materials', *Acta Mechanica Sinica* **5**, 237–243.
- Tai, W.H. (1990), 'Plastic damage and ductile fracture in mild steels', *Engineering Fracture Mechanics* **37**, 853–880.
- Tanaka, K., T. Mori and T. Nakamura (1970), 'Cavity formation at the interface of a spherical inclusion in a plastically deformed matrix', *Philosophical Magazine* **21**, 267–279.

- Thomason, P.F. (1981), 'Ductile fracture and the stability of incompressible plasticity in the presence of microvoids', *Acta Metallurgica* **29**, 763–777.
- Thomason, P.F. (1985a), 'A three-dimensional model for ductile fracture by the growth and coalescence of microvoids', *Acta Metallurgica* **33**, 1087–1095.
- Thomason, P.F. (1985b), 'Three-dimensional models for the plastic limit-loads at incipient failure of the intervoid matrix in ductile porous solids', *Acta Metallurgica* **33**, 1079–1085.
- Thomason, P.F. (1993), 'Ductile fracture by the growth and coalescence of microvoids of non-uniform size and spacing', *Acta Metallurgica et Materialia* **41**, 2127–2134.
- Thompson, A.W. (1987), 'Modeling of local strains in ductile fracture', *Metallurgical Transactions A* **18A**, 1877–1886.
- Thomson, R.D. and J.W. Hancock (1984), 'Ductile failure by void nucleation, growth and coalescence', *International Journal of Fracture* **26**, 99–112.
- Tracey, D.M. (1971), 'Strain-hardening and interaction effects on the growth of voids in ductile fracture', *Engineering Fracture Mechanics* **3**, 301–315.
- Tszeng, T.C. (1993), 'A model of void nucleation from ellipsoidal inclusions in ductile fracture', *Scripta Metallurgica et Materialia* **28**, 1065–1070.
- Tvergaard, V. (1981), 'Influence of voids on shear band instabilities under plane strain conditions', *International Journal of Fracture* **17**, 389–407.
- Tvergaard, V. (1982a), 'Material failure by void coalescence in localized shear bands', *International Journal of Solids and Structures* **18**, 659–672.

- Tvergaard, V. (1982b), 'On localization in ductile materials containing spherical voids', *International Journal of Fracture* **18**(4), 237–252.
- Tvergaard, V. (1996), 'Effect of void size difference on growth and cavitation instabilities', *Journal of the Mechanics and Physics of Solids* **44**, 1237–1253.
- Tvergaard, V. and A. Needleman (1984), 'Analysis of the cup-cone fracture in a round tensile bar', *Acta Metallurgica* **32**(1), 157–169.
- Tvergaard, V. and A. Needleman (1995), 'Effects of nonlocal damage in porous plastic solids', *International Journal of Solids and Structures* **32**, 1063–1077.
- Tvergaard, V. and A. Needleman (1997), 'Nonlocal effects on localization in a void-sheet', *International Journal of Solids and Structures* **34**, 2221–2238.
- Watt, D.F., X.Q. Xu and D.J. Lloyd (1996), 'Effects of particle morphology and spacing on the strain fields in a plastically deforming matrix', *Acta Materialia* **44**, 789–799.
- Worswick, M.J. (1988), Cavity growth and constitutive softening in ductile solids, PhD thesis, University of Waterloo.
- Worswick, M.J., A.K. Pilkey, C.I.A. Thomson, D. Lloyd and G. Berger (1998), Percolation damage predictions based on measured second phase particle distributions, in 'Proceedings for the IMS '98 conference at Ottawa, Canada', p. ?
- Worswick, M.J. and P. Pelletier (1998), 'Numerical simulation of ductile fracture during high strain rate deformation', *The European Physical Journal; Applied Physics* **4**, 257–267.
- Worswick, M.J. and R.J. Pick (1990), 'Void growth and constitutive softening in a

- periodically voided solid', *Journal of the Mechanics and Physics of Solids* **38**, 601–625.
- Yamamoto, H. (1978), 'Conditions for shear localization in the ductile fracture of void-containing materials', *International Journal of Fracture* **14**, 347–365.
- Zhang, K.S., J.B. Bai and D. François (1999), 'Ductile fracture of materials with high void volume fraction', *International Journal of Solids and Structures* **36**, 3407–3425.
- Zhang, Z.L. (1996), 'A sensitivity analysis of material parameters for the Gurson constitutive model', *Fatigue and Fracture in Engineering Materials and Structures* **19**, 561–570.
- Zhang, Z.L. and E. Niemi (1994), 'Analyzing ductile fracture using dual dilational constitutive equations', *Fatigue and Fracture of Engineering Materials and Structures* **17**(6), 695–707.
- Zheng, M., C. Hu, Z.J. Luo and X. Zheng (1996), 'A ductile damage model corresponding to the dissipation of ductility of metal', *Engineering Fracture Mechanics* **53**, 653–659.
- Ziv, I., F. Weinberg and W.J. Poole (1999), 'The shear strength of aluminum/alluminum alloy interfaces', *Scripta Materialia* **40**, 1243–1248.