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Occupant-Induced Vibration in Wood Floors: A Serviceability Approach

By

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*A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of the
requirements for the degree of*

DOCTOR OF PHILOSOPHY

in

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Faculty of Graduate Studies and Research
acceptance of the thesis titled:

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A SERVICEABILITY APPROACH**

submitted by

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in partial fulfillment of the requirements for
the degree of

Doctor of Philosophy in Engineering

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Abstract

Lightweight wood floors for residential and light industrial applications have been known at times to exhibit annoying levels of vibration even under normal occupancy loading. This unsatisfactory performance is mainly attributed to the availability and the use of higher strength materials of reduced weight and size. Most existing design methods dealing with this serviceability problem have been developed primarily with respect to the dynamic characteristics of the floor. Available experimental data, however, suggest alternative design methods which recognize the behavioural characteristics of the floor with its occupants as a coupled problem.

This study presents a numeric tool for analyzing the frequencies, and the dynamic response of a light wood floor system under occupants' footfall loadings. In particular, the occupant's interaction with the floor is found to be sufficiently modelled by a measured forcing function applied in conjunction with a human oscillator. The time and positional characteristics of the footstep forces from occupants' activities are also accounted for. Principally, the modal synthesis analysis techniques are used to assemble the mass and stiffness matrices of the floor system, and are able to account for the various complexities common to wood floor construction: orthotropic plate cover, semi-rigid subfloor-joist connections, joists' end supports rigidity, joist bridging, and gaps in subfloor sheathing. Since analytical mode shapes are used to express the floor system's displacement fields, the size of each of the system's dynamic parameters (mass, stiffness, and damping) is correspondingly reduced which makes this method even adaptable to microcomputers. A comparison with existing experimental results confirms the adequacy of this numerical tool.

The rationale behind this study is towards a provision of a realistic tool for evaluating a lightweight floor system's dynamic response under occupancy-induced loading, which is a necessary step in the development of vibrational-based serviceability design criteria. Using the tool developed for this thesis, a new vibration

serviceability design guideline for lightweight wood floors of short to medium span has been developed which is not only cost effective, but also requires simple input parameters of the floor which are readily available to the designer at the design stage. A more achievable feature of the proposed model is its uniqueness in accounting simultaneously for both the essential features of the occupant-induced forces and those of the floor in a simplistic manner.

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Table of Contents

Abstract	iii
Acknowledgements	v
Dedication	vi
Table of Contents	vii
List of Figures	xi
List of Tables	xv
Notation	xvi
1 Introduction	1
1.1 Problem Definition	1
1.2 Literature Review	4
1.2.1 <i>Scope of Review</i>	4
1.2.2 <i>Measuring Human Response to Annoying Floor Vibrations</i>	4
1.2.3 <i>Estimating Floor Vibrational Parameters</i>	7
1.2.3.1 <i>Studies Based on Occupant-Induced Impulsive Load Model</i>	8
1.2.3.2 <i>Studies Based on Moving-Load Model</i>	10
1.2.3.3 <i>Studies Based on In-Place Mass Model</i>	11
1.2.3.4 <i>Characteristics of Damping in Wood Floor Construction</i>	13
1.2.4 <i>Vibration Serviceability Design Criteria</i>	14
1.3 Scope and Objectives of the Research	19
1.4 Structure of the Thesis	20
2 Model Development	22
2.1 General	22
2.1.1 <i>Structure of A Wood-Joist Floor System</i>	23
2.1.2 <i>Application of Component Mode Synthesis Method</i>	24
2.1.3 <i>Governing Assumptions</i>	26
2.2 Energy Development in Sheathing Substructure	28

2.2.1	<i>The simply Supported Floor</i>	30
2.2.2	<i>Floor With One Pair of Opposite Edges Simply Other Pair Free</i>	33
2.3	Energy Development in Stiffeners' Substructure	34
2.3.1	<i>Modelling Wood Floor Design Features</i>	38
2.3.1.1	<i>Strain Energy in Sheathing-Joist Connectors</i>	39
2.3.2	<i>System Compatibility Conditions</i>	41
3	Frequency Analysis	43
3.1	<i>General</i>	43
3.2	<i>Equations of Motion</i>	43
3.3	<i>Natural Frequencies and Mode Shapes</i>	47
3.4	<i>Solution Convergence</i>	47
3.5	<i>Model Validation</i>	49
3.5.1	<i>Comparison with Long (1971) Analytical Model</i>	49
3.5.1	<i>Comparison with Ohlsson (1982) Test Results</i>	49
3.5.1	<i>Comparison with Chui (1987) Test Results</i>	50
3.5.1	<i>Comparison with Hu (1992) Test Results</i>	50
3.6	<i>Discussion on Model Validity</i>	51
4	Model Applications	60
4.1	<i>General</i>	60
4.2	<i>Modelling Transverse Gravity Loads</i>	61
4.3	<i>Effect of Joist End Support Conditions</i>	63
4.4	<i>Effect of Material Discontinuities in Sheathing</i>	65
4.5	<i>Modelling the Behaviour of Between-Joists Strutting</i>	66
4.5.1	<i>Effect Due to Bridging on Natural Frequencies</i>	70
5	Occupant-Floor System Vibrational Response	73
5.1	<i>General</i>	72
5.2	<i>Human Body As A Mechanical System</i>	74
5.3	<i>Characteristics of Human-Induced Footfall Excitations</i>	76
5.3.1	<i>Forces with Both Temporal and Spatial Variations</i>	77
5.3.2	<i>Forces with Only Temporal Variation</i>	79
5.3.3	<i>Forces Due to Impact Activities</i>	80

5.4 Development of General Equations of Motion of Coupled Floor-Occupant(s) System Under Induced Excitations	81
5.4.1 <i>Applicable Assumption</i>	83
5.4.2 <i>Equations of Motion</i>	83
5.4.2.1 <i>Equations of Motion—Subsystem 1</i>	85
5.4.2.2 <i>Equations of Motion—Subsystem 2</i>	86
5.4.2.3 <i>Equations of Motion—Subsystem 3</i>	87
5.5 Method of Solution	88
6 Response Prediction: Model Verification & Applications	92
6.1 General	92
6.2 Model Verification: Heel Drop Impact	92
6.2.1 <i>Equations of Motion: Heel Drop Impact</i>	93
6.2.2 <i>Convergence Characteristics</i>	94
6.2.3 <i>Accuracy</i>	96
6.3 Modelling Human-Floor Dynamic Interaction	97
6.4 Response of Lightweight Floors to Walking or Running Impacts	100
6.4.1 <i>Influence of Occupant Presence on Floor Dynamic Response</i>	102
6.4.2 <i>Verification of the Implementation of Spatial and Temporal Variations in the Footfall Forces Associated Walking</i>	104
6.4.2.1 <i>Main Roads Department of Western Australia Footbridge</i>	104
6.4.2.2 <i>University of New Brunswick Wood Science and Technology Centre (WSTC) Wood I-Joist Floor</i>	106
6.4.3 <i>Sensitivity of Response Time History to the Shape of Forcing Function</i>	108
6.5 Response of Lightweight Floors to Rhythmic Activities In-Place	109
7 Probabilistic Vibrational Serviceability Criterion	135
7.1 General	135
7.2 Effects on Floor Response of Floor Parameters	135
7.3 Effects on Floor Response of Variations in Occupant Dynamic Characteristics	138
7.4 Development of A New Vibrational-Based Serviceability Limit State Criterion	138
7.4.1 <i>Background</i>	138
7.4.2 <i>Criterion Formulation</i>	140

7.5 Discussion	143
8 Summary, Conclusions & Recommendations	154
8.1 General	154
8.2 Summary	154
8.3 Conclusions	155
8.3 Recommendations	157
References	158
Appendix	162

List of Figures

Figure	Page
1.1 Equal Sensation Contours for Standing Position with Vertical Vibration (<i>After Reiher and Meister 1934</i>)	5
1.2 Preliminary Proposal for Classification of the Response of A Light Floor to Impact Load (<i>Ohlsson 1988</i>)	16
2.1 Light Wood-Joist Floor Assembly	24
2.2 Displacement Fields at Centroid of A Generic Floor joist	35
2.3 Deformation at Flooring-Stiffener Interface (a) <i>x</i> -direction (b) <i>y</i> -direction	39
3.1 Comparison of Predicted Mode Shape Functions in the Across-Joists Direction of Long (1971) Stiffened Plate (a) This Study (b) Long (1971) (c) Hu 1992	54
4.1 Imposed Gravity Loads (a) Line Load in Along-Joists Direction (b) Concentrated Point Loads (c) Localized Distributed Loads	62
4.2 Representation of the More Common Bridging Types (<i>After Onysko 1988</i>)	67
5.1 Idealized Lumped Parameter Vibratory Human Models (a) 2-DOF ISO Model (b) 1-DOF Model (c) 11-DOF Model	74
5.2 Approximated Versus Measured Footfall Forces (<i>Mouring and Ellingwood, 1994</i>)	77
5.3 Spatial Distribution of Forces on A Floor Due to One Person Walking From Location <i>a</i> to <i>b</i>	79
5.4 Forces Caused by Rhythmic Activities In Place (<i>Ellingwood and Tallin, 1984</i>)	80
5.5 Forces Due to Impact Activities (<i>Ellingwood and Tallin, 1984</i>) . .	81
5.6 2 Degree-of-Freedom ISO Human Model (a) Dynamic Forces on Mass, m_1 (b) Dynamic Forces on Mass, m_2 (c) External Dynamic Forces on Floor Plate at Point of Footstep	84
5.7 Walker Loading Function	90

6.1	<i>Convergence: Displacement Response to Bag Drop Impact (Floor Joists: SPF No.2 Grade)</i>	111
6.2	<i>Convergence: Velocity Response to Bag Drop Impact (Floor Joists: SPF No.2 Grade)</i>	111
6.3	<i>Convergence: Acceleration Response to Bag Drop Impact (Floor Joists: SPF No.2 Grade)</i>	112
6.4	<i>Accuracy: Comparison of DYFAP and Thesis Predicted Displacement Time Histories (Floor Joists: SPF No.2 Grade)</i>	113
6.5	<i>Accuracy: Comparison of DYFAP and Thesis Predicted Velocity Time Histories (Floor Joists: SPF No.2 Grade)</i>	113
6.6	<i>Accuracy: Acceleration Time History (Floor Joists: SPF No.2 Grade) (a) Thesis Prediction (b) DYFAP Prediction (c) Comparison of DYFAP and Thesis Predictions</i>	114
6.7	<i>Accuracy Study: Comparison of DYFAP and Thesis Predicted Displacement Time Histories (Wood I-Joist Floor)</i>	115
6.8	<i>Accuracy Study: Comparison of DYFAP and Thesis Predicted Velocity Time Histories (Wood I-Joist Floor)</i>	115
6.9	<i>Accuracy Study: Comparison of DYFAP and Thesis Predicted Acceleration Time Histories (Wood I-Joist Floor)</i>	116
6.10	<i>Comparing Coupled Responses Associated with Three Forms of Viscous Damping Matrices: Heel Drop Excitation is Modelled as Forcing Function + 75 kg Mass of Human</i>	117
6.11	<i>Comparing Coupled Responses Associated with Three Forms of Viscous Damping Matrices: Heel Drop Excitation is Modelled as Forcing Function + 2-DOF ISO Human Model</i>	118
6.12	<i>Comparing Coupled Responses Associated with Three Forms of Viscous Damping Matrices: Heel Drop Excitation is Modelled as 2-DOF ISO Human Model with Assigned Initial Velocities</i>	119
6.13	<i>Comparing (a) Displacement, and (b) Velocity Time Histories of Three Excitation Types: Floor has 38 × 184 mm SPF Joists</i>	120
6.14	<i>Comparing Acceleration Time Histories From Three Excitation Types: Viscous Damping Matrix is $[C] = \alpha[M]$; Floor has 38 × 184 mm SPF Joists</i>	121
6.15	<i>Comparing Acceleration Time Histories From Three Excitation Types: Viscous Damping Matrix is $[C] = \alpha[M] + \beta[K]$; Floor has 2 × 8 SPF Joists</i>	122
6.16	<i>Comparing Displacement, Velocity and Acceleration Time Histories From Three Excitation Types: Viscous Damping Matrix is $[C] = \alpha[M]$; Floor has Composite Wood I-Joists</i>	123

6.17	Comparing Displacement, Velocity and Acceleration Time Histories From Three Excitation Types: Viscous Damping Matrix is $[C] = \alpha[M] + \beta[K]$; Floor has Composite Wood I-Joists . . .	124
6.18	Wood Floor Assembly Showing Location <i>O</i> of Passive Occupant, and Models Simulating A Walker Activity: (a) Forcing Function + 2-DOF ISO Human Oscillator (b) Forcing Function + 75-kg Mass of Walker (c) Forcing Function only	125
6.19	Comparison of Response Histories from Reference Heavy Floor (Composite Steel-Concrete Floor) Due to WALKER-Induced Excitations: (a) Displacement Time History (b) Velocity Time History (c) Acceleration Time History	126
6.20	Comparison of Response Histories from Light Wood I-Joist Floor Due to WALKER-Induced Excitations (a) Displacement Time History (b) Velocity Time History (c) Acceleration Time History	127
6.21	Comparison of Response Histories from Reference Heavy Floor (Composite Steel-Concrete Floor) Due to RUNNER-Induced Excitations: (a) Displacement Time History (b) Velocity Time History (c) Acceleration Time History	128
6.22	Comparison of Response Histories from Light Wood I-Joist Floor Due to RUNNER-Induced Excitations (a) Displacement Time History (b) Velocity Time History (c) Acceleration Time History	129
6.23	Comparison of Acceleration Histories Obtained From Reference Light Wood I-Joist Floor Due to RUNNER-Induced Excitations Simulated as: (a) 3.2 Hz Forcing Function + 2-DOF ISO Human Oscillator (b) 3.2 Hz Forcing Function + 75-kg Mass of Runner (b) 3.2 Hz Forcing Function Only	130
6.24	Curtin Avenue Footbridge (<i>Wheeler 1982; Mouring & Ellingwood 1994</i>) (a) Bridge Elevation and Section (b) Structural Modelling and Mode Shape	131
6.25	Time History Acceleration Due to Walking (University of New Brunswick-WSTC Wood I-Joist Floor) (a) Test Results (b) Numerical Results for No Loading (c) Numerical Results for Imposed Loading	132
6.26	Single Footfall Force functions	133

6.27	Sensitivity of Acceleration Time History to Shape of Footfall Pulse (a) A Walker at Pacing Frequency of 2.0 Hz (b) A Runner at Pacing Frequency of 3.2 Hz	134
7.1	Effect on Floor Response of Joist Spacing	145
7.2	Effect on Floor Response of Interface Connector Spacing	145
7.3	Effect on Floor Response of Joist Depth	145
7.4	Effect on Floor Response of Joist Grade (Elasticity Modulus)	146
7.5	Effect on Floor Response of Sheathing Thickness	146
7.6	Effect on Floor Response of Joist Bridging	146
7.7	Effect on Floor Response of Damping	147
7.8	Effect on Floor Response of Joist Span	148
7.9	Effect on Floor Response of Walker Stride (Pace) Length	149
7.10	Disposition of Two Human Occupants For Formulation of Vibration Acceptance Criterion	150
7.11	Distribution of RMS-Velocity For Different Spans of Basic Vibrational Floor	151
7.12a	Normalized RMS-Velocities With Respect To Changing Floor (a) Span and Nature of Joist Bridging (b) Joist Sectional Rigidity (c) Thickness of Sheathing	152
7.12b	Normalized RMS-Velocities With Respect To Changing Floor (a) Width (b) Joist Spacing (c) Subfloor-Joist Rigidity	153

List of Tables

Table	Page
3.1 Model Convergence	55
3.2 Comparison of Natural Frequencies (Hz) in Long (1971) With Predictions From Other Numerical Models	56
3.3 Comparison of Natural Frequencies (Hz) in Ohlsson (1982) With Predictions From Other Numerical Models	56
3.4 Comparison of Test Natural Frequencies (Hz) (Chui, 1987) With Predictions From Other Numerical Models	57
3.5a Comparison of Test Natural Frequencies (Hz) (Hu, 1992) With Predictions From Other Numerical Models (S-S-S-S Floors)	58
3.5b Comparison of Test Natural Frequencies (Hz) (Hu, 1992) With Predictions From Other Numerical Models (S-F-S-F Floors)	58
3.5c Comparison of Predicted vs. Test Natural Frequencies (Hz) (Hu, 1992) for S-F-S-F Floor with Static Masses (Span 5.9m)	59
4.1 Effect of Joist End Fixity on Lower Six Natural Frequencies	64
4.2 Effect of Material Discontinuities on Lower Eight Natural Frequencies	64
4.3 Effect of Joist Bridging on Lower Natural Frequencies (<i>Only Constraints at Bridging Positions Considered</i>)	71
4.4 Test vs. Predicted Lower Natural Frequencies (Chui and Hu, 1990) (<i>Consideration of Between-Joist Bridging Effect; Both Supplemental Dy Stiffness and All Constraints at Bridging Positions</i>)	72
5.1 Parameters For Oscillators Modelling a Human	76
6.1 Effect of the Number of Across-Joist Modes N on Computed Response Parameters	95
6.2 Material and Configuration Data for Composite Steel-Concrete Floor (Heavy floor) and Wood I-Joist Floor (Light Floor)	105
6.3 Comparison of Results of Curtin Avenue Footbridge (Fundamental Natural Frequency, Hz)	106
6.4 Comparison of Results of Curtin Avenue Footbridge (Total Peak Displacement Response (mm); Pacing Frequency 2.0 Hz)	106
7.1 Material and Configuration Data of Reference Wood Joist Floor	135

Notation

a	Floor span.
a_i	Span of i th joist.
A_i	Cross-sectional area of the i th joist.
A_{mn}	Modal coordinate function associated with displacement w .
$[\hat{\mathbf{A}}]$	Constraint matrix.
B_{mn}	Modal coordinate function associated with displacement u .
a	Floor width.
c_1, c_2	Damping values associated with the 2 degree-of-freedom ISO model.
C_{mn}	Modal coordinate function associated with displacement v .
d_i	Sectional depth of the i joist.
$\{\mathbf{D}\}$	Vector of generalized coordinates of floor system.
$\{\mathbf{D}\}^{(\tau)}$	Eigenvector of the r th natural mode.
D_x, D_y	In-plane stiffnesses of orthotropic plate.
DOF_w	Degrees-of-freedom of subfloor substructure due to flexure.
DOF_{uv}	Degrees-of-freedom of subfloor substructure due to in-plane actions.
e	Centre-to-centre spacing of connectors.
E_{J_i}	Modulus of elasticity of the i th joist.
E_x, E_y	Moduli of elasticity for specified directions for an orthotropic material.
g	Number of constrained coordinates of the system.

G	Modulus of rigidity (shear modulus) in the x - y plane.
$[G]$	Matrix of specialized shape functions defining positions of occupants whose activities show both time and positional characteristics.
G_{J_i}	Shear Modulus of the i th joist.
G_{eij}	Equivalent shear modulus of j th row bridging element in-between the i th and $i + 1$ th joists.
J_i	Torsional moment of inertia of the i th joist.
I_{y_i}, I_{z_i}	Moment of inertia of the i th joist with respect to x and y axes.
<i>ISO</i>	International Organization for Standardizations.
k_1, k_2	Stiffnesses associated with the 2 degree-of-freedom ISO model.
$k_{\parallel}, k_{\perp}$	Connector stiffnesses in the parallel and perpendicular grain direction.
k_{θ}	Connector rotational stiffness.
K_s	Spring constant of joist end connection computed at 0.4 mm slip.
K_x, K_y	Flexure stiffnesses of orthotropic plate.
$[K]$	System constrained stiffness matrix.
$[K_{wj}^{(i)}]$	Submatrix of $[\hat{K}]$ associated with the coupling associated with the sheathing-joists connections.
$[\hat{K}]$	System unconstrained or generalized stiffness matrix.
m	Mass of a floor occupant.
m, n	Mode shape number referenced to the along- and across-joists directions.
m_1, m_2	Masses associated with the 2 degree-of-freedom ISO human model.
$\mathbf{m}_f, \mathbf{c}_f, \mathbf{k}_f$	Dynamic parameters associated with all occupants whose activities show time-dependent characteristics only.
$\mathbf{m}_v, \mathbf{c}_v, \mathbf{k}_v$	Dynamic parameters associated with all occupants whose activities show both time and positional characteristics.

M, N	Total number of mode shapes used in the characterization of displacements in the along- and across-joists direction.
$[\mathbf{M}]$	System constrained mass matrix.
$[\mathbf{M}_{uv}], [\mathbf{K}_{uv}]$	Submatrices of $[\hat{\mathbf{M}}]$ and $[\hat{\mathbf{K}}]$ associated with sheathing in-plane displacement.
$[\mathbf{M}_w], [\mathbf{K}_w]$	Submatrices of $[\hat{\mathbf{M}}]$ and $[\hat{\mathbf{K}}]$ associated with sheathing transverse displacements.
$[\mathbf{M}_j^{(i)}], [\mathbf{K}_j^{(i)}]$	Submatrices of $[\hat{\mathbf{M}}]$ and $[\hat{\mathbf{K}}]$ associated with joists' displacements.
$[\hat{\mathbf{M}}]$	System unconstrained or generalized mass matrix.
N	Number of harmonics in footstep force.
$[\mathbf{N}]$	Matrix of specialized shape functions defining positions of occupants whose activities show time-dependent characteristics only.
NC_i	Number of connectors on the i th joist.
NG	Number of across-joists open gaps in floor sheathing.
n_j, NJ	Number of floor joists.
NSB	Number of rows of joist bridging.
$\{\mathbf{p}\}$	Vector of constrained coordinates of the system.
$P_i(t)$	Force applied by the foot of the i th occupant at time t .
$\mathbf{Q}_u, \mathbf{Q}_v, \mathbf{Q}_\theta$	Matrices that relate the displacement vector $\{\delta\}$ to horizontal and rotational slippages.
RMS	Root-mean-square.
s	Number of constraint equations derived from the equality of interface vertical displacements.
s^*	Local coordinate with respect to bridging orientation.
t_p	Subfloor sheathing thickness.
T	Total kinetic energy of the system.

T_{J_i}	Kinetic energy of the j th joist.
u	Sheathing in-plane displacement along the direction of joists.
u_i, v_i, w_i	Displacements of i th joist in directions x , y and z .
ut_i, vt_i	Finite number of mode shapes used in defining u_i and v_i .
U	Total strain energy stored in a system.
U_c	Contributory strain energy derived from sheathing-joist connectors.
U_{J_i}	Strain energy stored in the j th joist.
U_{mn}	Mode shape function describing displacement u .
U_p	Total strain energy stored in the subfloor.
U_p^w, U_p^{uv}	Contribution of U_p due to transverse and in-plane deformations.
v	Sheathing in-plane displacement across the direction of joists.
V_{mn}	Mode shape function describing displacement v .
w	Sheathing out of plane displacement.
w_{b_i}	Contribution of pure bending in w_i .
$wt_i, \theta t_i$	Finite number of mode shapes used in defining w_i and θ_i .
W_{mn}	Mode shape function describing displacement w
W_{sij}	Vertical deflection of the j th element in the i th row of bridging.
$x_o(k), x_1(k)$	x -coordinates at the beginning and end of the k th gap.
x, y, z	Global or material coordinate axes.
X_k, Y_k	k th Mode shape function in the definition of u_i and v_i .
$[z]$	Vector of dofs of the ISO human oscillator.
Z_k, Θ_k	k th Mode shape function in the definition of w_i and θ_i .
α, β	Proportional damping constants.

α_p	Non-sheathing material equivalent factor.
$\{\delta\}$	Vector of unknown parameters associated with all substructures.
$\delta()$	Dirac delta function.
Δs	Natural stride of an occupant.
Δt	Time step in numerical integration.
$\gamma_k, k = 1, \dots, 6$	Serviceability factors on floor configuration and material.
$[\Gamma]$	Transformation matrix.
$\kappa_{ik}, \vartheta_{ik}$	k th Modal coordinate function associated with u_i and v_i .
ν_{xy}, ν_{yx}	Poisson's ratio in specified directions for orthotropic material.
ω	Angular frequency.
ϕ_{ik}, τ_{ik}	k th Modal coordinate function associated with θ_i and w_i .
Φ_r	r th natural mode shape.
θ_i	Cross-sectional rotation of the i th joist.
ρ_{J_i}	Mass density of the i th joist.
ρ_p	Mass density of subfloor sheathing.
φ_{ik}	k th Modal coordinate function associated with w_{b_i} .

1.1 Problem Definition

Recent technical advancement in wood engineering has greatly increased the availability of wood and wood-based products for both residential and light industrial construction. One achievement in this direction has been the availability of high-strength materials of significantly reduced weight and structural size. In certain respect, however, this advantage has a drawback in that some wood floor systems built from these materials exhibit objectionably high vibrational response (Folz and Foschi 1991; Filiatrault *et al.* 1990; Ohlsson 1988). Traditionally, wood floors are designed for strength and some form of static stiffness. Design with respect to static stiffness has thus acted as a kind of guarantee by proxy that vibrations due to dynamic loads will be held in check. It is noted, however, that commonly existing criteria regarding static stiffness have not performed this function satisfactorily [Onysko and Bellosillo (1978)].

Human beings are surprisingly sensitive to vibrations, and are usually disturbed by intensities that are well below those required to overstress the structures they inhabit (Smith 1988). Therefore, perceptible vibrations existing in such structures may be seen as a serviceability limit state problem related to human discomfort. In lightweight floors, for instance, perceptible vibrations have been known to be induced by a variety of normal activities which include walking (or running), dancing, children playing, domestic appliances, and also door slam. Polensek (1970) observed that footfall loadings due to a single person walking was not only the most frequently occurring vibratory activity in lightweight floors, but also one activity that gave rise to the greatest nuisance. Previous studies [eg. Polensek (1970); Pernica and Allen (1982)] indicated that wood joist floors and long-span lightweight concrete floors on steel joists have been found to be among those structural systems

most susceptible to perceptible vibrations.

Currently, there are few standard annoyance criteria in domestic dwellings for quantifying perceptible vibrations in wood-joint floors due to human activities such as walking. Research efforts are still fragmented on the choice of pertinent floor dynamic response parameter with which an acceptable standard criterion could be formulated. Whereas various predictive models have been attempted, the requisite input excitations, in most cases, have been based on the heel drop impact tests since the transient characteristics associated with the ensuing time history response are thought to closely resemble those due to human activity [eg. Smith and Chui 1988]. In principle, therefore, the determination of the dynamic response of a typical wood floor follows the *heel-drop* test recommended by Lensen and Murray (1969), Polensek (1970), and Wiss and Parmelee (1974). This consists of a person of average weight rising up on the toes and then dropping suddenly on the heels at a location near the midspan (or the most flexible position) of the test floor. In this manner, explicit response history for any design situation could be obtained by applying a simulated triangular impulse of the force-time curve associated with single heel drop.

Allen and Rainer (1989), on the other hand, noted that the heel-impact design criterion is, at best, only a convenient means of linking the observed acceptability of a floor with the dynamic properties of the floor assuming the floor vibrates at its lowest dynamic frequency. Besides, transient vibrations resulting from single heel drop impact are usually characterized by larger amplitudes which are readily dissipated. These amplitudes (or intensities) have been observed to be much more tolerated by humans than those of near continuous vibrations of footfalls due to walking (Ellingwood and Tallin 1984).

A rather subtle situation in the evaluation of the dynamic performance of lightweight floors is the neglect of mass and other dynamic characteristics of the human occupants. In certain design situations, particularly wood floors found in domestic environment, the total mass of the occupants (disturbing and disturbed occupants) could amount to as high as 30% of the entire mass of the system. Admit-

tedly, current numerical models abound for predicting the vibrational performance of light-wood floors; for example FEAFL0 (Thompson *et al.* 1977), PERF (Onysko 1984), DYFAP (Folz and Foschi 1991), the modal synthesis model by Hu (1992), and the Rayleigh-Ritz model by Chui (1987). They are, however, with the sole exception of DYFAP (Folz and Foschi 1991), mostly applicable for the estimation of the natural frequencies of floor systems, and also for obtaining the time history response based on single heel drop excitation without the due consideration of the inherent physical interaction that exists between the occupant(s) and the floor system.

In evaluating the acceptability of floors in respect of perceptible vibrations, current numerical models such as those mentioned above may not at all be capable of modelling simultaneously such features as the composite effect of the spatial as well as the temporal variation of footfall excitations associated with walking; the dynamic characteristics (mass, stiffness and damping) of both the disturbed and disturbing occupant(s); and the number and disposition of such occupants. Folz and Foschi (1991), using DYFAP, sought to model the complex dynamic behaviour of floor-occupant interactive behaviour by considering the strong influence on the combined dynamical system of the contributory mass, damping, and stiffness of the occupants. However, only the temporal variations associated with heel-drop impact was studied. Although, the spatial and temporal variations of human footfall excitations have been studied by Ellingwood and Tallin (1984) for a single-degree-of-freedom beam system, and also by Mouring and Ellingwood (1992) for relatively long-span lightweight floors with steel joists, the floor-occupant interaction was assumed negligible in both studies. Neglecting floor-occupant interaction, in part, facilitates analysis as the parameters of both the floor and its occupants become uncoupled. Also, neglecting the spatial and the temporal variation characterizing footsteps excitation ensure that the mass, damping and stiffness matrices of the combined system are time invariant which results in linear analysis. However, for the wooden floor, serious errors might occur should above factors be neglected.

In light of the foregoing therefore, current research emphasis is toward the development of analytic or numeric tool capable of handling vibrational input which

directly simulates near continuous footfall vibrations in a specified time span. Such a tool must provide versatility for the incorporation of all the appropriate dynamic characteristics of the floor occupants as well as any coupled floor-occupant behaviour.

1.2 Literature Review

1.2.1 Scope of Review

There exists a great amount of literature on human response to vibration—both experimental and numerical studies have been well documented. Paramount among these studies is the sensitivity of humans to vibrations generated in lightweight structures by activities such as walking. Although, studies governing excitations other than those induced by humans have also been well documented, in line with the topic of this study, however, only an overview of studies pertaining to human-generated vertical excitations on light floors is presented.

The initial section is devoted to a summary of some research efforts aimed at understanding as well as measuring human response to perceptible floor vibrations. Specifics on some human tolerance scales based on both transient and steady-state excitations are given. Furthermore, mathematical models for quantifying the salient vibrational parameters of frequency, amplitude, and damping in the case of light wood floor construction are given. Finally, practical feasibilities of various proposed serviceability vibrational criteria for lightweight wood floors in normal human occupancies such as residences, offices, and classrooms are also given.

1.2.2 Measuring Human Response to Annoying Floor Vibrations

The experimental work by Reiher and Meister (1931) pioneered the study of human response to annoying floor vibrations. This work consisted of subjecting separately 10 persons of different background, aged 27–29 years, to vertical and horizontal vibrations of frequencies ranging from 5 Hz to 70 Hz, and having single amplitude displacements from 0.0254 mm to 1.02 mm. Each subject, after exposure to vibrations for approximately 5 min, was asked to classify the vibration in one

of the following categories: (1) slightly perceptible; (2) distinctly perceptible; (3) strongly perceptible; (4) disturbing; and (5) very disturbing. Based on their results, a human tolerance scale (Fig. 1.1) was constructed based on peak deflection and frequency referenced to the input steady-state vibrations.

Fig. 1.1 *Equal sensation contours for standing position with vertical vibration (After Reiher and Meister 1934)*

To incorporate the effect of damping in the evaluation of human response to vibrations, Wiss and Parmelee (1974) studied experimentally human response to transient vibrations. The input wave-form was produced from a mathematical model designed to simulate vibrations compatible with heel drop impact. The scale for rating had the following characteristics: (1) Imperceptible ($R = 1$); (2) Barely perceptible ($R = 2$); (3) Distinctly perceptible ($R = 3$); (4) Strongly perceptible ($R = 4$); and (5) Severe ($R = 5$). Based on observations made by 40 subjects, a mathematical model was proposed which related response rating, R , to frequency (F), in Hz, displacement (A), in inches, and damping ratio (D) as

$$R = 5.08 \left(\frac{FA}{D^{0.217}} \right)^{0.265} \quad [1.1]$$

One significant result from this study was that as damping increased, vibrations of a given frequency and peak deflections were less perceptible. Also, in an effort to provide a basis for comparison of the results of their investigation with those of Reiher and Meister (1931), Wiss and Parmelee (1974) also studied the case of zero damping using steady-state signals of 5 sec duration. The limits of their scale were in general found to be lower than those obtained by Reiher and Meister (1931). These discrepancies were attributed to the difference in duration time of the vibration, and also to dissimilarities in data evaluation.

Further study of human response to objectionable floor vibrations, particularly those induced in light wood-joint floor construction, have been carried out by Ather-ton *et al.* (1976) who expanded on an earlier work by Polensek (1970). To determine which characteristics of the wood floor system were the best indicators of human tolerance to vibrations, 1222 impacts and 278 walking displacement-time traces obtained experimentally from 24 full-size floors were evaluated. From the results obtained, a tolerance scale was constructed with rating 1-5 (greater disturbance to smaller disturbance). Marginal differences were found to exist when this scale was compared with that by Reiher and Meister (1931). As with the study by Wiss and Parmelee (1974), these differences were ascribed to the variations in both the input excitations and perception evaluation.

One general finding from these tests is that human response to floor vibration is dependent on frequency, amplitude and damping. Besides, larger amplitudes characterizing transient motion are known to dissipate within a few cycles and are much more tolerated than steady-state motion which occurs over an extended period of time. The specific findings in these studies, however, are not always consistent with each other due to the rather diverse purposes and methods in these experiments. One important principle underlying these differences being that the psychological factors inherent in structural motion perception are difficult to duplicate in a laboratory experiment. The willingness of test subjects to tolerate higher motion levels when impending motion is known suggests that tolerance and comfort levels reported from controlled experiments may be upper bounds on the levels that may actually be acceptable in reality (Ellingwood and Tallin 1984).

1.2.3 Estimating Floor Vibrational Parameters

Significant advances have been made towards the quantification of vibrational parameters of frequency, amplitude, and damping for the establishment of serviceability limit state design criteria on perceptible vibrations existing in lightweight floors. Floor vibrational parameters, particularly damping and frequencies, have normally been obtained by physical means such as tests in the laboratory. One principal drawback of the physical methods is that, though, they furnish accurate results, test results are only applicable to floors with the same configuration as floors tested, and are therefore not amenable for the prediction of parameters of other floors. More limited still, the number of test replications required to obtain adequate data for statistical analyses makes the associated cost of experimentation very prohibitive.

By far, analytical or numerical methods have provided the most versatility for estimating floor vibrational parameters of frequency and amplitude. In using these methods, we generally seek to simulate the floor actual behaviour by employing a simple set of equations which, subjected to certain assumptions, describe the behaviour of the floor system under any excitation (Hu 1992; Dowell 1972). As a departure from the complete use of the more conventional finite element method, most numerical models for dynamic analysis of lightweight wood-joint floors aim at achieving versatility and computational efficiency by reducing significantly the system unknown parameters. This factor becomes crucial more especially in codification where simplicity and accuracy may be warranted. One major concern, though, with this approach is that the inherent calculations are often so complex that most studies have been based on the more simplified formulations which inevitably result in lower accuracy (Kalkert *et al.* 1993).

In the following, a discussion is given on the more current mathematical models available for free- and forced-vibrational analysis of light wood-joint floors.

A review of the literature indicates that a large number of available mathematical models may have the capability of handling frequency analysis. As a more

comparative study, the ability of these models to handle occupant-floor interaction in a positional as well as time varying fashion in time-history response analysis is reviewed below. Studies discussed involve floors of rectangular plan with joists (joists), if any, aligned parallel to one pair of the edges. It is noted that current research emphasis has been on the rectangular-shaped floors ostensibly to provide the primary focus on which the data appropriate to other floor plans could well be extrapolated.

1.2.3.1 Studies Based on Occupant-Induced Impulsive Load Model

Almost all of the studies reported under this section either assume negligible effect on the time-history response of the dynamic characteristics of the occupants, or in an implicit manner, the floor damping coefficient is assumed to be estimated considering the composite effect of the floor system with its occupants. In general, occupant-induced excitations are thus modelled as acting impulsive forces.

Hu (1992) presented a model based on the modal synthesis method which sought to attain a balance between computational efficiency and accuracy of response prediction. Essentially, the model employs the Rayleigh-Ritz method to achieve computational efficiency, with added flexibility in the selection of suitable mode shape functions. Other salient features of the model included the influence on the natural frequencies of shear deformation and rotary inertia of the joists, and also the semi-rigid connection of joists to floor plate. One unique feature of this model is that it could be extended to cope with nonrectangular floors with or without openings and at the same time maintaining its computational efficiency. Such a procedure necessitates the use of rectangular elements each of which may be analysed separately, after which constraint requirements at the elements' boundaries are accounted for. The Hu model (1992) assumes the dominance of transverse deformation of the floor and as such does not account for in-plane deformation of the floor sheathing. Also, neither the axial deformation nor the cross-sectional distortion of the joists is considered. Excellent results are obtained particularly for prediction of the lower natural frequencies of wood floors stiffened with I-joists.

Based on the Rayleigh-Ritz method, Smith and Chui (1988) provided a model

for the estimation of the natural frequencies of wood-joist floors. Using as the primary input the mode shape functions of a rectangular plate with all edges simply supported, an analytical expression was developed for the estimation of the frequencies. Regarding time-history response, the model assumed the dominance of the fundamental mode, and the floor system was thus modelled as a single-degree-of-freedom spring dashpot system. In this way, an explicit expression of the displacement time-response was obtained using Duhamel's integral. One setback with this model is that the assumption of full composite action between the floor plate and the joists may underestimate the floor frequencies. Also, the assumption of fundamental mode dominance implies that the associated floor frequencies are well separated. Olhsson (1982), however, was of the view that the initial peak velocity due to impulsive force was also highly dependent on higher modes.

Chui (1987) presented a model based on the Rayleigh-Ritz method, assuming implicitly in the model that the mode shape function of wood-joist floor with joists having low ratio of flexural to shear rigidity could be approximated by that of the floor plate without joists. Though this model is computationally efficient in that it requires a relatively smaller computer memory and less running time, it can at best predict only the first lower natural frequencies with some degree of accuracy.

Chui and Hu (1990) conducted a series of testing on laboratory-built wood I-joist floors to examine the effect on the dynamic response of some pertinent construction parameters such as span, edge support, bridging, and number and disposition of static masses. From the results obtained from the study, it was observed, among other things, that owing to their relatively lightweight, the presence of static masses significantly influenced the vibratory behaviour of wooden floors.

Filiatrault *et al.* (1990), and Foschi and Gupta (1987) developed, respectively, the computer programs *Natural Frequency Floor Analysis Program* (NAFFAP) and *Dynamic Floor Analysis Program* (DFAP) for dynamic analysis of lightweight floors. Whereas the former was developed exclusively for frequency analysis, the latter has an added feature of carrying out forced vibrational analysis. Both NAFFAP and DFAP were extensions to the finite-strip model (Cheung 1976) reformulated

by Foschi (1982) for static analysis of wood-joist floor systems. In essence, a floor system is modelled as an assemblage of T-beam finite strips each consisting of a joist and the associated portion of the flooring. The deformational modes of a generic T-beam are defined by Fourier series in the direction parallel to the joist, and by a finite element discretization in the direction perpendicular to the joist. This method of analysis permits numerical modelling of a typical residential wood floor by a relatively small number of equations. Accordingly, both NAFFAP and DFAP could be implemented in a microcomputer environment and at the same time accounting for the various complexities in wood floor systems such as orthotropic property of the floor sheathing, variability in joist-to-joist stiffness, and the semirigid sheathing-to-joist connections. Though, both NAFFAP and DFAP provide efficient numerical tools for a comprehensive study of perceptible vibrations existing in light wooden floors, in the case of floors with adequate joist bridging any significant increase in the across-joist stiffness due to the bridging might not be adequately characterized by these models. It is noted that, in-between joist bridging is accounted for in both NAFFAP and DFAP by restricting the cross-sectional rotation of the joists.

1.2.3.2 Studies Based On Moving-Force Model

Studies based on the moving-force model recognize the spatial variation in occupant-induced excitation but assume negligible occupant dynamic characteristics. This approach has been used extensively in the serviceability limit state evaluations of pedestrian footbridges (Smith 1988).

Ellingwood and Tallin (1984) compared the predicted responses of simply supported one-way floor system under (1) excitation due to normal walking; (2) sinusoidal excitation; and (3) excitation resulting from a heel drop impact. It was observed that calculated peak accelerations at floor midspan due to sinusoidal excitation, corresponding to *walking in place*, were approximately $\frac{1}{3}$ the accelerations caused by normal pedestrian movement, while the accelerations from a heel drop at midspan ranged from 8 to 100 times larger. They therefore pointed out the importance of the use of realistic forcing functions in assessing the sensitivity of floors to disturbing motion. In another development, results from parametric studies using

normal walking excitation as primary input indicated the sensitivity of the floor response to the shape of the footfall force pulses. Particularly, the influence of higher modes on the response was only significant in situations where the natural frequencies of these modes were comparable to the contributory harmonics of the forcing functions.

Using the *moving-force* approach, Mouring and Ellingwood (1992) sought to incorporate the characteristics of crowd-induced loading in response analysis of light floor systems in a commercial setting. Induced load characteristics considered included crowd density, randomness of crowd movement, crowd activity, and temporal interaction between individuals. Probabilistic methods were employed in the evaluation of these characteristics. Two methods were developed for response analysis of occupant-induced excitations. The first was based on the simulation in the time domain of forces due to individuals and groups of people on a typical floor system. Such a system was modelled as a linear, classically damped multidegree of freedom system, and the finite element package ABAQUS was used. The second involved a frequency-domain solution using random vibration theory. For this method, the generalized forces associated with occupant activity were transformed into a frequency domain. It was observed, based on the results from these studies, that the common assumptions that human-induced loading can be modelled as fixed in position, that a footfall can be modelled as a half-sine function, or further still, that group activities are fully coherent, and that only loads due to an individuals need be considered may be oversimplifications.

1.2.3.3 Studies Based on In-Place Mass Model

Contrary to studies based on the moving force model, those based on the *in-place* mass model account for only the temporal variations in the occupant-induced footsteps forces as well as any possible dynamic interaction between the floor and its occupants. The complexity of the ensuing dynamic response analysis, in a large measure, depends on the degree of the occupant-floor interaction. For instance, in dynamic problems necessitating the recognition of the mass, damping and stiffness of the occupant, nonlinear behaviour may result. However, if only the mass of the

occupant is considered, first order linear behaviour is obtained and the analysis is greatly simplified.

Folz and Foschi (1991), developed a dynamic model, the *Dynamic Floor Analysis Program* (DYFAP) based on the finite-strip model for static analysis of wood-joint floor systems developed by Foschi (1982). Unlike DFAP (Foschi and Gupta, 1987), the dynamic characteristics of human occupants were not ignored—in fact in DYFAP it was assumed the wooden floor was excited dynamically by, and interacted with its occupant(s). Floor occupants were modelled as lumped parameter oscillators whose complexities varied from the 2-DOF International Organization for Standardization (ISO) human model, to a more detailed 11-DOF oscillatory system. The general time-history of the floor response due to footfall excitation was found to be influenced by the damping and the stiffness characteristics of the occupants; this being dependent on the occupant to floor-weight ratio. This study concluded that for light wood floor construction such as those found in private dwellings and offices, the coupling between the occupants and the floor cannot be ignored in human perception evaluation. In fact such is the strong dependence of the response on the occupants' dynamic behaviour that this study recommended that any formulation of vibration acceptance criteria ought to be investigated experimentally with human subjects. Specifically, the characteristics of the vibration used in any regulatory experiment must furnish a range of frequencies, amplitudes, and damping that would provide results applicable to other floor construction. Notwithstanding, however, this model restricts the position of the test subject on the floor to that directly above a joist.

Olhsson (1982, 1988) employed a structural analysis grillage model for the response prediction of wooden floors. Though, the simplicity of the model for dynamic analysis was superb, in most cases, it does not lend itself easily to the incorporation of certain floor system characteristics such as variability in joist stiffness and the semirigid sheathing-to-joist connections. Besides, only the transverse deformation of the floor (displacement normal to the plane of the floor plate) was considered of significance. One significant contribution of this research was the explicit formulation of footstep forces generated by people in motion. Neglecting the spatial nature

of footstep forces, frequency analysis of such forces indicated the predominance of low frequency components while for relatively higher frequencies, intensity of the force was found to be inversely proportional.

1.2.3.4 Characteristics of Damping in Wood Floor Construction

Structural damping in wood systems occurs in two modes: slip (*interlayer or hysteretic*) damping, and material or *internal* damping (Yeh *et al.* 1971; Polensek 1988). Principally, interlayer damping existing at surfaces in contact at joints, and yielding of mechanical fasteners constitute the most significant damping of wood systems (Ungar 1973). The fastener-type at the interface between a floor plate and the joists, therefore, plays a dominant role in determining the amount of damping present in the floor. Material damping as a secondary source of energy dissipation is due to the basic energy-loss mechanism characterizing a non-yielding structural system by virtue of its vibratory state (Filiatrault 1990). In practical structures, however, this energy-loss behaviour is seldom fully understood and is therefore conveniently represented by an equivalent viscous damping. Besides structural damping, human occupants also provide an additional source of damping which could singly influence human perception to annoying floor vibrations (Polensek 1988; Foschi and Gupta 1987).

In the dynamic analysis of wood-joist floors, damping in sheathing-joist connectors is modelled in a linear fashion [eg. Smith and Chui (1988); Hu (1992); Foschi and Gupta (1987); Ohlsson (1988)], as perceptible floor vibration is only seen as a serviceability limit state problem. Kalkert *et al.* (1993), however, pointed out that the linearity assumption may be valid under low amplitude vibrations where slippages are minimal and also in all cases of material damping. On the other hand, for amplitudes where sufficient slippage occurs, the resulting frictional damping may be significant to warrant nonlinear analysis.

Generally, damping in wood floors has been evaluated using the simplified design criterion wherein the entire floor is modelled as a single degree-of-freedom system. The resulting time-amplitude traces under free-vibration when only the fundamental mode is considered is then used to obtain the damping which is as-

sumed viscous in nature. The viscous damping obtained thus is based primarily on the fact that peak deflection and maximum damping occur during the first cycle. Polensek (1975), however, noted that since the response history of a lightweight floor system is also influenced by the second and higher modes of vibration, viscous damping assumption may be inaccurate when applied to an entire floor system.

1.2.4 Vibration Serviceability Design Criteria

Nearly all the existing vibrational-based criteria for light floor construction have been based on either frequency, amplitude, or damping, among other non-structural factors. Salient aspects of frequency usually considered are natural frequencies and mode separation of adjacent frequencies. Any stipulation on floor natural frequencies not only ensures that these frequencies are well above the 4–8 Hz frequency band that humans are particularly sensitive to, but also the possibility of resonance due to human activities is obviated. A minimum separation of adjacent frequencies also help offset any interaction of closely spaced vibration modes to produce high amplitudes. In a current study by Kalkert *et al.* (1993), it was observed that vibrational amplitudes are most frequently expressed in three forms: deflection, velocity and acceleration. Atherton *et al.* (1976) stated that deflection provides the single best indicator for both single impact and footsteps vibrations. A drawback in using this indicator as a measure of amplitude is that for smaller deflections, inherent limitations in accuracy of measuring devices make test data somewhat unreliable (Kalkert *et al.* 1993). Velocity, as a parameter, is known to best describe human disturbance above 8 Hz (Ohlsson 1988; ISO 1987). Although, it is seldom measured directly in the literature, Ohlsson (1982, 1984, 1988, 1991) utilized velocity to explicitly quantify amplitude. By far, acceleration is the parameter commonly used by most researchers. Not only is it easy to measure, it simplifies data analysis, and has been found to correlate well with human tolerance to vibration (Kalkert *et al.* 1993). Curves, indicating human tolerance levels, constructed on the basis of acceleration are often horizontal with marked reduction at the major resonant frequencies of the human body. More still, vibration exposures measured in terms of acceleration are easily harmonized with other units. In reality, however, vibra-

tion is usually either random or of complex waveform (Smith 1988) and tolerance measures in the form of root mean square amplitude are much more realistic.

Given below are some of the more current human comfort criteria applicable to lightweight wood-joist floors.

Foschi and Gupta (1987), developed a quantitative reliability-based design criterion based on single joist stiffness. The criterion stipulates that for a concentrated load of $P = 1 \text{ kN}$ acting at the midspan of the joist with the average modulus of elasticity, $\bar{E}$, of the system, irrespective of the span, the measured static deflection, δ , is not to exceed an absolute value of 1 mm. Thus,

$$\delta = \frac{PL^3}{48\bar{E}I} \quad [1.2]$$

where I and L are the respective moment of inertia and span of a single *bare* joist.

Ohlsson (1991) proposed a general set of serviceability requirements encompassing one static and two dynamic criteria. The static criterion requires the maximum limiting static deflection of 1.5 mm for a concentrated static load of 1 kN applied at the centre of a floor to account for the composite effect of joists and sheathing. Another requirement of this criterion is that the floor undamped fundamental frequency, f_1 , be greater than 8 Hz, where f_1 for a rectangular floor of length L and width B , is computed as

$$f_1 = \frac{\pi}{2} \sqrt{\left(\frac{D_x}{gL^4}\right) \left\{ 1 + \left(\frac{L}{B}\right)^2 \left[2 + \left(\frac{L}{B}\right)^2 \right] \left(\frac{D_x}{D_y}\right) \right\}} \quad [1.3]$$

in which g = mass per unit area of floor (kg/m^2); and D_x , D_y = respective equivalent plate bending rigidity parallel and perpendicular to joist length (Nm^2/m).

The first of the two dynamic criteria provides guidance for estimating the highest attainable initial impulse velocity response, $h'_{\max}$, which is calculated at the most *flexible* point on the wood floor. In conjunction with the floor damping coefficient, σ_o , $h'_{\max}$ is considered acceptable in accordance with a classification chart (Fig. 1.2). The computed $h'_{\max}$ is also acceptable if is not greater than a threshold value expressed as $100^{f\xi-1}$, in $[(\text{m/s})/\text{Ns}]$, where f is taken as the fundamental frequency, and the modal damping ratio $\xi_n = (c/c_{\text{cr}})_n$ may be taken as 1% for

Fig. 1.2 Preliminary Proposal for Classification of the Response Light Floor to Impact Load (After Ohlsson 1988)

ordinary wood-based floors. The general equation defining $h'_{\max}$ is given as

$$h'_{\max} = \sum_{n=1}^{N_{40}} \frac{\Phi(x_o, y_o)}{M_n} \quad \left[\frac{\text{m/s}}{\text{N}} \right] \quad [1.4]$$

where N_{40} is the number of normal modes with natural frequencies greater than 40 Hz; (x_o, y_o) being the coordinate of the most flexible point; $\Phi(x, y)$ and M_n are, respectively, the mode function and modal mass in the n th normal mode. For rectangular-shaped floors with a standard addition of 50 kg to the modal mass denoting the vibratory portion of the human body, Eq. 1.3 takes the simplified form

$$h'_{\max} = \frac{4(.4 + .6N_{40})}{gBL + 200} \quad \left[\frac{\text{m/s}}{\text{N}} \right] \quad [1.5]$$

The second dynamic criterion has application only in floors of large span (typically spans greater than 4 m), and also in premises with long unobstructed passages for pedestrian traffic. In such cases, the root mean square (RMS) value of the vertical velocity, w'_{RMS} , must be comparable with those of other similar floor construction which have been found to have performed satisfactorily under normal

occupancy. Particularly for the simply supported rectangular-shaped orthotropic wood-joint floors, w'_{RMS} , could be estimated as

$$w'_{\text{RMS}} = \frac{100}{gBL\sqrt{\xi}} \sqrt{\left(\frac{N_{1.2}^2 + 1}{2f_1^3}\right)} \quad [(\text{m/s})_{\text{RMS}}] \quad [1.6]$$

in which, again, ξ = relative damping; $N_{1.2}$ = number of first order normal modes which have resonant frequencies $< 1.2f_1$. The calculated w'_{RMS} is thus the RMS velocity at the point of application of the load due to one person marking time at the midpoint of the floor. The Ohlsson (1991) criteria is the basis for the draft Eurocode 5 on serviceability prediction of light wood floor construction.

Smith and Chui (1988) also proposed a design criterion for rectangular-shaped floors based on the heel-drop or the impulse test. As with the Ohlsson (1991) proposal, one aspect of this criterion requires the undamped natural fundamental frequency, f_o , to be above 8 Hz, and under the assumption of zero composite action between sheathing and joists, f_o is estimated, for ordinary nailed floors, as

$$f_o = \frac{\pi}{2a^2} \sqrt{E_J I_J (n-1) / (\rho_s h b + \rho_J c d (n-1))} \quad [\text{Hz}] \quad [1.7]$$

where E_J , I_J , ρ_J , c , and d are the average physical properties for the joists; ρ_s and h are the properties associated with the sheathing; n is the number of joists; and a , b are the length and width of the floor, respectively. An additional stipulation to this criterion is that the estimated root-mean-square (rms) acceleration, A_r , should be such that

$$A_r = \frac{2000 K_{\delta, \omega_n, t_1}}{m \pi f_o^2} < 0.45 \text{ m/s}^2 \quad [1.8]$$

in which

$$K_{\delta, \omega_n, t_1} = \sqrt{\frac{\omega_n [1 - e^{-2\delta \omega_n t_1} + (1 - e^{-2\delta \omega_n (1-t_1)}) (1 - 2e^{-\delta \omega_n t_1} \cos \omega_n t_1 + e^{-2\delta \omega_n t_1})]}{4\delta}}$$

and m , the mass in one-degree-of-freedom analogue, is obtained as

$$m = \frac{a}{2} \left[\rho_s h b + \rho_J c d (n-1) + \frac{4W_o}{a} \right]$$

In computing A_r ; $\omega_n = 2\pi f_o$; the viscous damping ratio, $\delta = 0.03$, is chosen to account for the presence of human bodies on the floor; the duration of the impulsive

force, t_1 , is also selected in the range 0.05–0.07 s with W_o being the mass of the observer, in kg, applying the heel-drop impact.

Empirical serviceability criteria using results of field study of consumer response (Onysko 1985), and its later revision (Onysko 1988) have gained much attention. The criteria are acceptability-based, and for that matter provide assessment based on real-life situation as opposed to perceptibility-based which aims at examining specific effects only, often with such evaluations happening in a controlled environment. Onysko (1985) assessment was based on a field study of 107 in-situ lightweight floors chosen from five metropolitan areas in Canada. For each selected floor, in-house testing was conducted and occupants were asked some strategic questions concerning their acceptability of the floor. The revised 1988 criteria provide for a more general application because of their unique formulation. Assuming a static response dependency, the limiting static deflection, Δ , based on a concentrated load of 1 kN applied at the centre of the floor takes the form

$$\Delta = \frac{7.217}{\text{SPAN}^{1.274}} \quad 3.0 \text{ m} \leq \text{SPAN} \leq 6.0 \text{ m} \quad [1.9]$$

and if *in-grade* material values are accounted for

$$\Delta = \frac{8.0}{\text{SPAN}^{1.3}} \quad 3.0 \text{ m} \leq \text{SPAN} \leq 6.0 \text{ m} \quad [1.10]$$

The Canadian Wood Council (1990) also stipulates an additional requirement for the design of floor joists based on Part 9 of the National Building Code of Canada (NBCC). By this criterion, the *vibration-controlled* span for an entire floor could be estimated via the deformability of a single joist (span = S_i) supporting a concentrated load of 1 kN at the midspan. The vibration-controlled span, VCS , is thus estimated as

$$VCS = K S_i \quad [1.11]$$

in which

$$\log_e K = A - B \log_e \frac{S_i}{S_{184}} + G$$

and A = factor for subfloor sheathing thickness; B = factor for bridging of joists; G = factor for connection of subfloor to joists; S_i = span that results in a 2 mm

deflection of the joist under a 1 kN concentrated midpoint load; and S_{184} = span that results in a 2 mm deflection of a 38×184 mm joist of the same species and grade and under a 1 kN concentrated midpoint load. We note that the criteria set out in Onysko (1988), provided the basis for the development of the vibrational-based serviceability limit state provisions detailed in the NBCC.

1.3 Scope and Objectives of the Research

This study considers only the dynamic loadings whose cause and nature may be regarded as serviceability limit state behaviour. Also, emphasis is placed on the more common human-induced footfall vibrations which have been observed to be major source of perceptible vibration in domestic or in light commercial/industrial environments. In this regard, the general dynamic loading (forcing function), wherever mentioned, and or used in this work, refers to the excitations due to human footstep forces applied in the vertical manner. In understanding human response to objectionable vibrations, this work attempts to evaluate only the associated physical component which is primarily the measure, in a quantitative manner, of the respective (or the combined) dynamic behaviour of the human occupant and the defined structural system. The manner by which this component is measured may somewhat differ from that devised for the psychological (subjective) component, which, though, of much overriding importance is extremely difficult to quantify analytically.

An equally important aspect of this work is the incorporation of salient features arising from current design practices into a constitutive model capable of achieving a balance between computational efficiency and ease of general application. Particularly, the influence of joists' support conditions—typically for designs using joist hangers—on the dynamical behaviour of a typical wood-joist floor is examined.

The numerical model carried out under this study incorporates certain characteristic behaviour of wood-joist floors such as the orthotropic nature of the subfloor, membrane action of a deformed plate, the semirigidity of the sheathing-joist connections, joist bridging as improvement of the floor transverse flexural stiffness, and

shear deformation as well as rotary inertia effects of the joists on the overall floor vibration performance.

Current results on the study of perceptible vibrations generated in light floors indicate that human reaction to such vibrations not only depends on the use and occupancy of the floor concerned, but also on the proximity of the vibration source to the floor centre and the distance between the source and the annoyed individual. The cumulative effect of these factors is considered in this work through the examination of the spatial and temporal variations of footfall forces associated with occupant(s) activity such as walking or running, and the walker-floor dynamic interaction.

Although, emphasis in this work is on wood-joist floor systems, the methodology outlined could be employed in other light floor construction such as light steel and concrete floors ribbed with light steel joists.

Specifically, this study was carried out to achieve the following:

- To develop a comprehensive model for the analysis of light floor systems under footfall excitations which explicitly accounts for the positional and time variations associated with footfall forces.
- To provide a model with ease of incorporation of various wood floor construction features so as to study the interdependency of such characteristics in affecting the overall floor vibrational response.
- To assess the influence of joist support conditions on floor frequencies.

1.4 Structure of the Thesis

The research work carried out in this thesis is structured in three phases. The first phase comprises of Chapters 1 and 2. Chapter 1 introduces the serviceability limit state problem of human discomfort arising from perceptible vibrations in lightweight floors, and succinctly provides a state-of-the-art review of literature on the topic. Chapter 2 sets out the theoretical work leading to the development of the comprehensive model for the thesis. The general assumptions underlying the model are also detailed.

Chapters 3 and 4 form the second phase of this study which essentially outlines the solution strategy required for the prediction of undamped natural frequencies of lightweight joist floors. More specific still, Chapter 3 discusses the development of system dynamic equations of motion as well as the procedure for obtaining a few of the lower natural frequencies alongside their respective normal modes. Since the vibrational response of a floor system is determined in large part by the knowledge of its natural frequencies, Chapter 4 looks at the effect of salient construction features of the floor on its natural frequencies. Particularly, the effect on the frequencies of joist support conditions, between-joist bridgings, and the inclusion of shear deformation and rotary inertia effects in the joists' subsystem analysis are examined.

The last phase focuses on the various methodologies for predicting coupled human-floor dynamic responses resulting from human activity. The development of the governing equations of motion using walking as the requisite input activity is fully given in Chapter 5. Chapter 6 validates the methodology described in Chapter 5 using some results from other studies. As well, this Chapter provides the requirements for specializing the general response equations in Chapter 5 for other human activity such as heel drop impact, running, jogging, and walking or running in place.

To calibrate or measure human tolerance for annoying vibration in light wood floors due to human activities, Chapter 7 uses the results based on a walker activity to formulate a serviceability-based annoyance criteria. Lastly, Chapter 8 draws some conclusions from this study, and also recommends some future supplementary studies.

2.1 General

Overall, the approach used in developing the numerical model for the thesis is the method of *component mode synthesis* wherein the main structural components (the sub-floor sheathing and the stiffening joists) are analyzed separately. In adopting this method as the analysis tool, it becomes uniquely expedient to adopt for each system component different analysis techniques in such a manner as to achieve an optimum reduction in the total number of degree-of-freedom (dof) for the combined system. It must, however, be remarked that due to the rather complex nature of occupant-floor dynamic interaction, the general forced vibration analysis is performed by considering the entire system as a single physical entity. Therefore, the use of the method of component mode synthesis is in this work used mainly in the development of mass and stiffness matrices of the various system components.

The overriding objective in the formulation of a numerical or analytical model for the implementation of a vibrational serviceability criterion is to provide one having versatility and computational efficiency. As in other research studies [eg., Filiatrault *et al.* (1990), Ohlsson (1988), Smith and Chui (1988)], a complete analytical (or semi-analytical) method is adopted in this case, though unlike those studies cited in Chapter 1, a more general approach is used here to facilitate the incorporation of certain wood construction features which are generally ignored or difficult to introduce in currently available models. In the development of the model for this thesis, the computational efficiency of the modal synthesis approach (Hu 1992) and the versatility of DYFAP (Filiatrault *et al.* 1990) are recognized. The nature of the ensuing model being that the various characteristic features of wood-joist floors could each, or jointly, be considered in an analytical manner. Additional benefit is the more broader application of the current model.

The dynamic analysis of a typical subsystem is studied by employing classical beam (or plate) equations (mode shapes) which govern both the vibration modes of the subsystem, and which also is appropriate to its support (boundary) conditions. The seeming difficulty in manipulating the complex mathematical expressions of mode shapes for nonregular boundary conditions is performed using symbolic computations and the results converted directly to FORTRAN code for numeric processing—use is made of standard functions in *Mathematica* libraries in this regard.

A brief outline of the following is presented as an introduction to the methodology of the work in this thesis.

- Structure or design features of a typical wood-joist floor in light wood-frame construction;
- Application of component mode synthesis in the dynamic evaluation of light wooden floors; and
- Main assumptions underlying the model.

Fig. 2.1 Light Wood-Joist Floor Assembly

2.1.1 Structure of A Wood-Joist Floor System

The basic structural configuration of a typical wood-joist floor in a domestic, commercial or light industrial application, such as shown in Fig. 2.1, consists of a number of wood-based panels built into a rectangular or nonrectangular floor plan and supported on fixed perimeter wood beams, or *headers*. The floor is supported lengthwise by a number of joists whose ends are often connected to the supporting walls by means of light-gauge metal framing connectors such as joist hangers. The wood-based subfloor-to-joist connections are fabricated using mechanical fasteners such as common nails and wood screws, or to ensure good bonding, by a combination of mechanical fasteners and elastomeric adhesive material.

In more recent wood-frame construction, particularly those for use in commercial or light industrial dwellings, the subfloor may have an overlay of a thin layer of nominal reinforced concrete. This method of construction may result in a partial or no composite action between the wood-based sheathing and the concrete.

2.1.2 Application of Component Mode Synthesis Method

The component mode synthesis, or the modal synthesis method, as a computational efficient tool in most practical dynamic analysis applications, provide a more explicit approach for predicting the characteristics of a somewhat complex system via the study of the behaviour of its constituent components (Hurty 1960; Crag and Bampton 1968; Hou 1969; Dowell 1972). An important application of the modal synthesis method is that if a particular component is difficult to model numerically or analytically, its modal representation can be determined experimentally (Ewins 1984). Such method involves dividing a system up into a number of separate components (or elements) with mode shape functions of the assembled structure obtained as the synthesis of some selected mode shape functions which are ordinarily a subset of several sets of admissible functions obtained at substructuring level. The closer the substructure mode shapes are made conformable to the general system's boundary and constraint conditions, the more accurate the estimated mode shape

functions.

An important step in the successful application of the modal synthesis method is the choice of the substructures modes. Two major documented approaches to deal with this concern are the *fixed* and *free* interface methods. By the fixed interface method, the element mode shape functions are chosen to satisfy both the prescribed external boundary conditions of the global structure and also any existing interface conditions between the substructures. In numerical analysis, this method leads to faster convergence in most cases. On the other hand, it may potentially introduce a large number of unknown variables into the system equations if too many substructures are used. Hu (1992) pointed out that for stiffened plate systems with several semirigid connections between the plate and the joists, it becomes rather difficult to define explicit mode functions to account for all the essential physical characteristics of the connections. The free interface method (Hou 1969), on the other hand, assumes that all restrictions on the substructure motions are released during the selection of a substructure's mode shape functions, and are only applied when the selected mode shapes are later synthesized. Substructure restrictions are usually the compatibility of displacements and forces at the substructures' interfaces. Unlike the fixed interface method, however, convergence of the free interface method is rather weak and the accuracy of the solution somewhat jeopardized. This method, though leads to a much reduced eigenproblem, computed system frequencies are, however, upperbounds to those obtained using the original system equations.

For simplicity, the work carried out in this study describes a generic wood-joist floor as having two principal sub-systems or substructures—*flooring* (such as plywood and Oriented Strand Board (OSB) sheathing), and *joists*. Since, as noted above, a typical wood-joist floor can have rather complicated interface connections, comprising of countless flexible connections, defining appropriate mode shape functions for any one substructure based on the conditions at both the external boundary supports and the interfaces may be too impractical. The need for a computational efficient analysis tool, therefore, may necessitate the use of the free interface method in this case. Accordingly, in the respective analysis for the sheathing and joists' subsystems, the mode shape functions are defined on the basis

of the external (support) conditions pertaining to each sub-system only and not on the interface conditions at the connections.

2.1.3 Governing Assumptions

An active principle underlying the development of the model is the fact that for the serviceability problem under consideration, application of small deflection theories for both the analysis of the flooring and the joists substructure is valid. In other words, the principle of superposition may hold true for the nature of the vibration problem in question. Considering the material properties of the structural components and also the design techniques available for wood-frame construction, the following assumptions therefore underscore the theoretical development of the model.

- Analysis is restricted to motions of small amplitude such that all possible deformational types, i.e. longitudinal, transverse or twisting motion associated with each component are uncoupled. Also, since small amplitude of displacement is considered, deformation normal to the plane of the sheathing (plate) is small in comparison with the flooring thickness.
- The model is formulated with exclusive reference to rectangular-shaped floors with, or without, any internal support system. (The model could also be applied to floors built from rectangular panels but are generally non-rectangular in plan.)
- Floor sheathing is thin, uniform and orthotropic with axes of material symmetry parallel to the global x , y , and z axes (Fig. 2.1).
- Within the range of loading considered, both the sheathing and joists behave as perfectly elastic material.
- Each joist has uniform stiffness throughout its length, however, variability in joist-to-joist stiffness is permissible. Joists are arranged such that they are parallel to one another and to the appropriate perimeter (edge) beam with which they are referenced.
- Shear deformation and torsional inertia effects are accounted for in all joist

types particularly those with relatively high ratios of flexure to shear rigidity.

- Effect of gaps in the subfloor parallel to the joists is neglected. This assumption is valid for the reason that only butt joints are formed at the interface of the floor cover and the joists, and structural continuity is ensured due to the presence of in-plane forces which are readily transmitted across joints. Also, due to the rather staggering nature of construction gaps, generally, any resulting butt joint may not extend over the full length of a joist. The cumulative effect of this, therefore, is to impart a degree of continuity in curvature at the locations of these gaps (Hu 1992). The experimental work carried out by Foschi (1982) on the static response of wooden floors with butt joints clearly reinforces this assertion.
- The analysis tool developed in this thesis does not consider the case of gap closure (particularly for gaps perpendicular to the joists) during the deformation of a floor. Under normal service load, the assumption of only the existence of open gaps is appropriate. Where a gap is accounted for, and for the purpose of analysis, the actual gap dimension is adjusted to the separation between adjoining fasteners. Design may, however, dictate that perpendicularly-installed solid wood strutting, or its equivalent, be located at all positions of these gaps. In such a situation, the entire floor is considered as one continuous plate.
- No vertical separation occurs between the sheathing and joists when floor becomes subject to occupant-induced vibration. Relative rotation and horizontal slippages, however, can occur at these interfaces.
- At all points defining the floor sheathing to joist junctions, shear transfer is continuous, and sheathing-to-joist connections behave linearly. The linearity assumption is justified in that vibration of wooden floors, as a serviceability limit state problem, is only realized at low load levels—usually due to normal occupancy. This linear behaviour is also used in deriving the characteristics of both the material and friction damping of the system.

Though, the above constitutes the basic assumptions underlying the work in the thesis, certain supporting assumptions, where appropriate, are also stated through-

out this work.

2.2 Energy Development in Sheathing Substructure

Relevant to this work, the term ‘floor plate’ is used in the general sense to denote either subfloor consisting of sheathing only, or sheathing with an overlay of nominal reinforced concrete or mortar. In particular, where parquet flooring is used in conjunction with plywood sheathing, or with any other wood-based product, full composite action is assumed to be provided by the bonding between the parquet and the sheathing. Accordingly, the parquet flooring is considered as a layer of sheathing-like material equal in thickness to some fraction, α_p , of the parquet thickness (Onysko 1988).

Under the assumption of small deflections and orthotropic plate theory, the strain energy per unit area stored in the sheathing is of the form

$$U_p^e = \frac{K_x}{2} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + \frac{K_y}{2} \left(\frac{\partial^2 w}{\partial y^2} \right)^2 + K_v \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + 2K_G \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 + \frac{D_x}{2} \left(\frac{\partial u}{\partial x} \right)^2 + \frac{D_y}{2} \left(\frac{\partial v}{\partial y} \right)^2 + D_v \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} + \frac{D_G}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \quad [2.1]$$

in which for sheathing of thickness t_p , the basic stiffness quantities K_x , K_y , K_v , K_G , D_x , D_y , D_v , and D_G are given by

$$\begin{aligned} K_x &= \frac{E_x t_p^3}{12(1 - \nu_{xy}\nu_{yx})} & K_y &= K_x \left(\frac{E_y}{E_x} \right) & K_v &= \nu_{xy} K_x & K_G &= G \left(\frac{t_p^3}{12} \right) \\ D_x &= \frac{E_x t_p}{1 - \nu_{xy}\nu_{yx}} & D_y &= D_x \left(\frac{E_y}{E_x} \right) & D_v &= \nu_{xy} D_x & D_G &= G t_p \end{aligned} \quad [2.2]$$

in which E_x ; E_y are, respectively, the modulus of elasticity in the x - and y -direction; ν_{xy} = Poisson’s ratio, strain in the x -direction when stress applied in the y -direction; ν_{yx} = Poisson’s ratio, strain in the y -direction when stress applied in the x -direction; and G = modulus of rigidity (shear modulus) in the x - y plane.

Neglecting any gap in the flooring, the associated kinetic energy, T_p , is of general form

$$T_p = \frac{1}{2} \rho_p t_p \int_0^a \int_0^b (\dot{w}^2 + \dot{u}^2 + \dot{v}^2) dx dy + \frac{1}{24} \rho_p t_p^3 \int_0^a \int_0^b \left[\left(\frac{\partial \dot{w}}{\partial x} \right)^2 + \left(\frac{\partial \dot{w}}{\partial y} \right)^2 \right] dx dy \quad [2.3]$$

with ρ_p being the mass density of the flooring, and a superposed dot denoting differentiation with respect to time. We note from Eq. 2.3 that the second integral represents the contribution from rotatory inertia effects.

As indicated earlier, a complete semi-analytical method is employed in the development of mass and stiffness matrices of the subfloor system. In the discussions which follow, attention is focused on floors simply supported at two opposite edges, corresponding to lines $x = 0$ and $x = a$ (Fig. 2.1), and with the other two opposite edges all simply supported or all free. This assumption is consistent with prevailing construction detailing of light wood-frame structures and as such does not place any undue restriction on the general application of the model developed under this thesis. This design situation thus facilitates the use of the Lévy method for separation of variables. Furthermore, based on the free-interface method, and also on the Ritz method, the transverse or bending displacement at time t occurring at the middle surface of the flooring, simply supported at all the four edges, takes the form

$$w(x, y, t) = \sum_{n=1}^N \sum_{m=1}^M A_{mn}(t) W_{mn}(x, y); \quad W_{mn}(x, y) = \sin\left(\frac{n\pi y}{b}\right) \sin\left(\frac{m\pi x}{a}\right) \quad [2.4]$$

In defining in-plane deformations, $u(x, y, t)$ and $v(x, y, t)$, we note that alternate simple boundary conditions are completely constrained and have zero constraint. Thus, the harmonic functions with similarities to those usually established for comparable boundary conditions for axial and torsional vibrations of beams become the more obvious candidates. The following, therefore, is defined

$$u(x, y, t) = \sum_{n=1}^N \sum_{m=1}^M B_{mn}(t) U_{mn}(x, y); \quad U_{mn}(x, y) = \sin\left(\frac{n\pi y}{b}\right) \cos\left(\frac{m\pi x}{a}\right) \quad [2.5]$$

$$v(x, y, t) = \sum_{n=1}^N \sum_{m=1}^M C_{mn}(t) V_{mn}(x, y); \quad V_{mn}(x, y) = \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{m\pi x}{a}\right) \quad [2.6]$$

In Eqs. 2.4–2.6, the following are defined:

a, b = span and width of floor, respectively.

W_{mn}, U_{mn}, V_{mn} = mode shape functions used in the definition of subfloor's displacement field w, u and v , respectively.

M, N = number of mode shape functions used in the characterization

of displacement fields along and across the directions of the joists.

A_{mn}, B_{mn}, C_{mn} = modal coordinate function used in the definition of plate substructure displacement field w, u and v , respectively.

We also note from Eqs. 2.4, 2.5 and 2.6 that the defined mode shape functions are actually a combination of equivalent beam functions in the defined principal axes.

For the class of joist floor arrangement with edges containing ends of joists simply supported and all edge joists free to vibrate, we employ equivalent beam functions for the across-rib direction, and thus express $W_{mn}(x, y)$ in the form (Chui 1987)

$$W_{mn}(x, y) = \begin{cases} \frac{1}{\sqrt{2}} \sin\left(\frac{m\pi x}{a}\right) & n = 1 \\ \cos\left[\frac{(n-1)\pi y}{b}\right] \sin\left(\frac{m\pi x}{a}\right) & n = 2, 3, \dots, N \end{cases} \quad [2.7]$$

Following similar argument advanced for the plate with all edges simply supported, the modal functions $U_{mn}(x, y)$ and $V_{mn}(x, y)$ are, respectively, defined as

$$U_{mn}(x, y) = \begin{cases} \frac{1}{\sqrt{2}} \cos\left(\frac{m\pi x}{a}\right) & n = 1 \\ \cos\left[\frac{(n-1)\pi y}{b}\right] \cos\left(\frac{m\pi x}{a}\right) & n = 2, 3, \dots, N \end{cases} \quad [2.8]$$

$$V_{mn}(x, y) = \begin{cases} \frac{1}{\sqrt{2}} \sin\left(\frac{m\pi x}{a}\right) & n = 1 \\ \sin\left[\frac{(n-1)\pi y}{b}\right] \sin\left(\frac{m\pi x}{a}\right) & n = 2, 3, \dots, N \end{cases} \quad [2.9]$$

2.2.1 The Simply Supported Floor

In this section, we develop the pertinent strain and kinetic energies of the flooring with all four edges simply supported. It is noted that the detailed procedure given below is valid for the other class of support condition mentioned earlier.

The strain energy stored in the plate, U_p , taking into account the existence of gaps at across-joist direction, could be expressed functionally as

$$\begin{aligned} U_p &= U_p^w + U_p^{uv} \\ &= \int_0^a \int_0^b U_p^e dx dy - \sum_{k=1}^{NG} \int_{x_0(k)}^{x_1(k)} \int_0^b U_p^e dx dy \end{aligned} \quad [2.10]$$

in which NG is the number of open gaps in the sheathing; and $x_0(k)$ and $x_1(k) =$ the respective x -coordinates corresponding to the beginning and end of the k th gap. Implicit in the above formulation is the assumption that all perpendicular to joist gaps extend from one edge of the plate to the other. U_p^w and U_p^{uv} , respectively, are the contributions due to transverse and in-plane deformations.

To estimate the portion of the strain energy due to transverse deformation, in Eq. 2.1 we isolate the transverse deformation contributory terms, and upon substitution of $w(x, y, t)$ from Eq. 2.1, we obtain

$$U_p^w = \int_0^a \int_0^b U_p^{ew_1} dx dy - \sum_{k=1}^{NG} \int_{x_0(k)}^{x_1(k)} \int_0^b U_p^{ew_2} dx dy \quad [2.11]$$

where

$$\begin{aligned} U_p^{ew_1} = & \frac{1}{2} \sum_{m=1}^M \sum_{n=1}^N A_{mn}^2 \int_0^a \int_0^b \left[K_x \left(\frac{\partial^2 W_{mn}}{\partial x^2} \right)^2 + K_y \left(\frac{\partial^2 W_{mn}}{\partial y^2} \right)^2 + 2K_v \frac{\partial^2 W_{mn}}{\partial x^2} \frac{\partial^2 W_{mn}}{\partial y^2} + \right. \\ & \left. + 4K_G \left(\frac{\partial^2 W_{mn}}{\partial x \partial y} \right)^2 \right] dx dy + \frac{1}{2} \sum_{m=1}^M \sum_{n=1}^N \sum_{\substack{i=1 \\ i \neq m}}^M \sum_{\substack{j=1 \\ j \neq n}}^N A_{mn} A_{ij} \int_0^a \int_0^b \left[K_x \frac{\partial^2 W_{mn}}{\partial x^2} \frac{\partial^2 W_{ij}}{\partial x^2} \right. \\ & \left. + K_y \frac{\partial^2 W_{mn}}{\partial y^2} \frac{\partial^2 W_{ij}}{\partial y^2} + K_v \left(\frac{\partial^2 W_{mn}}{\partial x^2} \frac{\partial^2 W_{ij}}{\partial y^2} + \frac{\partial^2 W_{mn}}{\partial y^2} \frac{\partial^2 W_{ij}}{\partial x^2} \right) + 4K_G \frac{\partial^2 W_{mn}}{\partial x \partial y} \frac{\partial^2 W_{ij}}{\partial x \partial y} \right] dx dy \end{aligned}$$

and in which, the pair $(i, j) \neq (m, n)$. $U_p^{ew_2}$ is of the same form as $U_p^{ew_1}$ but the limit of integration with respect to x in the former definition runs from $x_0(k)$ to $x_1(k)$ instead of from 0 to a in the latter. We further employ the orthogonal properties of the mode shapes, and Eq. 2.11 then takes the explicit form

$$\begin{aligned} U_p^w = & \frac{1}{2} \sum_{m=1}^M \sum_{n=1}^N \frac{\pi^4 A_{mn}^2}{4a^3 b^3} \left[K_x m^4 b^4 + K_y n^4 a^4 + 2K_v m^2 n^2 a^2 b^2 + 4K_G m^2 n^2 a^2 b^2 \right] \\ & - \frac{1}{2} \sum_{m=1}^M \sum_{i=1}^M \sum_{n=1}^N \frac{\pi^4 A_{mn} A_{in}}{2a^4 b^3} \left\{ \left[K_x m^2 i^2 b^4 + K_y n^4 a^4 + K_v n^2 a^2 b^2 (m^2 + i^2) \right] \cdot \sum_{k=1}^{NG} G_k(x) \right. \\ & \left. + 4K_G i m n^2 a^2 b^2 \cdot \sum_{k=1}^{NG} H_k(x) \right\} \quad [2.12] \end{aligned}$$

where

$$G_k(x) = \begin{cases} \frac{a}{\pi(i^2 - m^2)} \left[m \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{i\pi x}{a}\right) - i \cos\left(\frac{i\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right) \right]_{x_0(k)}^{x_1(k)} & \text{for } m \neq i \\ \frac{1}{4m\pi} \left[2m\pi x - a \sin\left(\frac{2m\pi x}{a}\right) \right]_{x_0(k)}^{x_1(k)} & \text{for } m = i \end{cases}$$

and

$$H_k(x) = \begin{cases} \frac{a}{\pi(i^2 - m^2)} \left[i \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{i\pi x}{a}\right) - m \cos\left(\frac{i\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right) \right]_{x_0(k)}^{x_1(k)} & \text{for } m \neq i \\ \frac{1}{4m\pi} \left[2m\pi x + a \sin\left(\frac{2m\pi x}{a}\right) \right]_{x_0(k)}^{x_1(k)} & \text{for } m = i \end{cases}$$

The plate strain energy contributed by in-plane deformation may also be expressed in the form

$$U_p^{uv} = \int_0^a \int_0^b U_p^{euv_1} dx dy - \sum_{k=1}^{NG} \int_{x_0(k)}^{x_1(k)} \int_0^b U_p^{euv_2} dx dy \quad [2.13]$$

where

$$\begin{aligned} U_p^{euv_1} = & \frac{1}{2} \sum_{m=1}^M \sum_{n=1}^N \int_0^a \int_0^b \left[D_x B_{mn}^2 \left(\frac{\partial U_{mn}}{\partial x} \right)^2 + D_y C_{mn}^2 \left(\frac{\partial V_{mn}}{\partial y} \right)^2 + 2D_v B_{mn} C_{mn} \frac{\partial U_{mn}}{\partial x} \frac{\partial V_{mn}}{\partial y} \right. \\ & \left. + D_G \left(B_{mn} \frac{\partial U_{mn}}{\partial y} + C_{mn} \frac{\partial V_{mn}}{\partial x} \right)^2 \right] dx dy + \frac{1}{2} \sum_{m=1}^M \sum_{n=1}^N \sum_{\substack{i=1 \\ i \neq m}}^M \sum_{\substack{j=1 \\ j \neq n}}^N \int_0^a \int_0^b \left[B_{mn} B_{ij} D_x \frac{\partial U_{mn}}{\partial x} \frac{\partial U_{ij}}{\partial x} \right. \\ & \left. + C_{mn} C_{ij} D_y \frac{\partial V_{mn}}{\partial y} \frac{\partial V_{ij}}{\partial y} + D_v \left(B_{mn} C_{ij} \frac{\partial U_{mn}}{\partial x} \frac{\partial V_{ij}}{\partial y} + B_{ij} C_{mn} \frac{\partial U_{ij}}{\partial x} \frac{\partial V_{mn}}{\partial y} \right) \right. \\ & \left. + D_G \left(B_{mn} \frac{\partial U_{mn}}{\partial y} + C_{mn} \frac{\partial V_{mn}}{\partial x} \right) \left(B_{ij} \frac{\partial U_{ij}}{\partial y} + C_{ij} \frac{\partial V_{ij}}{\partial x} \right) \right] dx dy \end{aligned}$$

As in Eq. 2.11, $U_p^{euv_2}$ is of the same form as $U_p^{euv_1}$ but the limit of integration with respect to x in the former definition runs from $x_0(k)$ to $x_1(k)$.

The principle behind the format expressed by Eqs. 2.11 and 2.13 is to make use of the orthogonality properties of the modal functions which for pairs $(m, n) \neq (i, j)$ we have most terms evaluating to zero. On this basis, U_p^{euv} , given by Eq. 2.13, can be simplified thus as

$$\begin{aligned} U_p^{uv} = & \frac{1}{2} \sum_{m=1}^M \sum_{n=1}^N \frac{\pi^2}{4ab} \left[D_x B_{mn}^2 m^2 b^2 + D_y C_{mn}^2 n^2 a^2 + 2D_v B_{mn} C_{mn} mnab \right. \\ & \left. + D_g \left(B_{mn}^2 n^2 a^2 + C_{mn}^2 m^2 b^2 + 2B_{mn} C_{mn} mnab \right) \right] - \frac{1}{2} \sum_{m=1}^M \sum_{i=1}^M \sum_{n=1}^N \frac{\pi^2}{2a^2 b} \left\{ \left[D_x B_{mn} B_{in} imb^2 \right. \right. \\ & \left. \left. + D_y C_{mn} C_{in} n^2 a^2 + D_v \left(B_{mn} C_{in} mnab + B_{in} C_{mn} inab \right) \right] \cdot \sum_{k=1}^{NG} G_k(x) + D_g \left(B_{mn} B_{in} n^2 a^2 \right. \right. \\ & \left. \left. + B_{mn} C_{in} inab + B_{in} C_{mn} mnab + C_{mn} C_{in} imb^2 \right) \cdot \sum_{k=1}^{NG} H_k(x) \right\} \quad [2.14] \end{aligned}$$

As previously mentioned, the kinetic energy expression given by Eq. 2.3 represents

contributions due to both displacement and torsional inertia effects. The contribution due to displacement, T_{p1} , could be expressed, using Eq. 2.4, as

$$T_{p1} = \frac{1}{2} \rho_p t_p \sum_{m=1}^M \sum_{n=1}^N \int_0^a \int_0^b \left(\dot{A}_{mn}^2 W_{mn}^2 + \dot{B}_{mn}^2 U_{mn}^2 + \dot{C}_{mn}^2 V_{mn}^2 \right) dx dy \\ + \frac{1}{2} \rho_p t_p \sum_{m=1}^M \sum_{n=1}^N \sum_{i=1}^M \sum_{j=1}^N \left(\dot{A}_{mn} \dot{A}_{ij} W_{mn} W_{ij} + \dot{B}_{mn} \dot{B}_{ij} U_{mn} U_{ij} + \dot{C}_{mn} \dot{C}_{ij} V_{mn} V_{ij} \right) dx dy \quad [2.15]$$

and that due to inertia effects, T_{p2} , as

$$T_{p1} = \frac{1}{24} \rho_p t_p^3 \sum_{m=1}^M \sum_{n=1}^N \int_0^a \int_0^b \dot{A}_{mn}^2 \left[\left(\frac{\partial W_{mn}}{\partial x} \right)^2 + \left(\frac{\partial W_{mn}}{\partial y} \right)^2 \right] dx dy \\ + \frac{1}{24} \rho_p t_p^3 \sum_{m=1}^M \sum_{n=1}^N \sum_{\substack{i=1 \\ i \neq m}}^M \sum_{\substack{j=1 \\ j \neq n}}^N \dot{A}_{mn} \dot{A}_{ij} \left[\frac{\partial W_{mn}}{\partial x} \frac{\partial W_{ij}}{\partial x} + \frac{\partial W_{mn}}{\partial y} \frac{\partial W_{ij}}{\partial y} \right] dx dy \quad [2.16]$$

On simplifying Eqs. 2.15 and 2.16, the total kinetic energy of the plate assumes the form

$$T_p = \frac{1}{2} \sum_{m=1}^M \sum_{n=1}^N \left\{ \frac{\rho_p t_p ab}{4} \left(A_{mn}^2 + B_{mn}^2 + C_{mn}^2 \right) + \frac{\rho_p t_p^3 \pi^2}{48 ab} \left(b^2 m^2 + a^2 n^2 \right) \right\} \quad [2.17]$$

where we recognize that $T_p = T_{p1} + T_{p2}$.

2.2.2 Floor With One Pair Opposite Edges Simply Supported Other Pair Free

This is a Lévy-type rectangular floor with the edges at the joists supports simply supported and the other opposite edges *free*. As pointed out earlier, the general formulations presented for both the strain and the kinetic energies of the floor with all four edges simply supported are also appropriate in this case. Employing the same notations as before, the pertinent explicit expressions for the kinetic and strain energy are as follows:

• Plate Kinetic Energy

$$T_p = \frac{\rho_p t_p ab}{8} \sum_{m=1}^M \sum_{n=1}^N \left(A_{mn}^2 + B_{mn}^2 + C_{mn}^2 \right) \\ + \sum_{m=1}^M \left[\frac{\rho_p t_p^3 \pi^2}{96 ab} \left(\sum_{n=1}^N A_{mn}^2 m^2 b^2 + \sum_{n=2}^N A_{mn}^2 (n-1)^2 a^2 \right) \right] \quad [2.18]$$

• Strain energy contribution due to transverse deformation

$$\begin{aligned}
 U_p^w = & \frac{1}{2} \sum_{m=1}^M \frac{K_x m^4 \pi^4 b A_{m1}^2}{4a^3} - \frac{1}{2} \sum_{m=1}^M \sum_{i=1}^M \frac{K_x m^2 i^2 \pi^4 b A_{m1} A_{i1}}{2a^4} \cdot \sum_{k=1}^{NG} G_k(x) \\
 & + \frac{1}{2} \sum_{m=1}^M \sum_{n=2}^N \left\{ \frac{\pi^4 A_{mn}^2}{4a^3 b^3} \left[K_x m^4 b^4 + K_y (n-1)^4 a^4 + 2K_v m^2 (n-1)^2 a^2 b^2 + 4K_G m^2 (n-1)^2 a^2 b^2 \right] \right. \\
 & \quad - \sum_{i=1}^M \frac{\pi^4 A_{mn} A_{in}}{2a^4 b^3} \left\{ \left[K_x i^2 m^2 b^4 + K_y (n-1)^4 a^4 + K_v (n-1)^2 a^2 b^2 (m^2 + i^2) \right] \cdot \sum_{k=1}^{NG} G_k(x) \right. \\
 & \quad \quad \left. \left. + 4K_G (n-1)^2 i m a^2 b^2 \cdot \sum_{k=1}^{NG} H_k(x) \right\} \right\} \quad [2.19]
 \end{aligned}$$

• Strain energy contribution due to in-plane deformation

$$\begin{aligned}
 U_p^{uv} = & \frac{1}{2} \sum_{m=1}^M \frac{m^2 \pi^2 b}{4a} (D_x B_{m1}^2 + D_G C_{m1}^2) + \frac{1}{2} \sum_{m=1}^M \sum_{n=2}^N \frac{\pi^2}{4ab} \left[D_x B_{mn}^2 m^2 b^2 \right. \\
 & \quad + D_y C_{mn}^2 (n-1)^2 a^2 - 2D_v B_{mn} C_{mn} m(n-1)ab + D_G (C_{mn}^2 m^2 b^2 - 2B_{mn} C_{mn} m(n-1)ab \\
 & \quad \left. + B_{mn}^2 (n-1)^2 a^2) \right] - \frac{1}{2} \sum_{m=1}^M \sum_{i=1}^M \sum_{k=1}^{NG} \frac{m i \pi^2 b}{2a^2} \left[B_{m1} B_{i1} D_x G_k(x) + C_{m1} C_{i1} D_G H_k(x) \right] \\
 & \quad - \frac{1}{2} \sum_{m=1}^M \sum_{i=1}^M \sum_{n=2}^N \sum_{k=1}^{NG} \frac{\pi^2}{2a^2 b} \left\{ \left[B_{mn} B_{in} D_x m i b^2 + C_{mn} C_{in} D_y (n-1)^2 a^2 \right. \right. \\
 & \quad \left. - D_v (n-1)ab (B_{mn} C_{in} m + B_{in} C_{mn} i) \right] \cdot G_k(x) + D_G \left[B_{mn} B_{in} (n-1)^2 a^2 \right. \\
 & \quad \left. \left. - B_{mn} C_{in} i (n-1)ab - B_{in} C_{mn} m (n-1)ab + C_{mn} C_{in} m i b^2 \right] \cdot H_k(x) \right\} \quad [2.20]
 \end{aligned}$$

2.3 Energy Development in Stiffeners' Substructure

A semi-analytical method is again employed in the response analysis of the floor joists. As in the analysis of the subfloor, greater accuracy is ensured, and fewer degrees-of-freedom (dofs) are needed as well to completely describe the joists' sub-system dynamic behaviour. The complex mathematical computations associated with this approach are handled using symbolic computations. Again, following detailing practices for wood frame construction, joists end connections are assumed simply or spring supported. Spring-supported joists are an idealization of floor joists having attachment with perimeter headers usually by means of joists hangers, or by

any other appropriate light metal-framing connectors. It is noted that in previous research efforts floor joists are modelled as simply supported. In this work, the appropriateness of this assumption as applied to the case of support attachments using joist hangers is examined.

Fig. 2.2 Displacement field at centroid of a generic floor joist
(a) Sawn lumber (b) Composite wood I-joist

Figure 2.2 shows the centroidal deformations, $u_i(x, t)$, $v_i(x, t)$ and $w_i(x, t)$, of a generic i th joist of span a_i in the respective directions of the principal axes x , y and z together with the cross-sectional rotation $\theta_i(x, t)$. In this work, the transverse deformation, $w_i(x, t)$, is made up of contributions due to both bending and shearing effects. As before, we employ both the Ritz method and separation of variable, and thus define each displacement type as the superposition of appropriately selected lower mode shapes which at the same time satisfy the joist support geometrical conditions. The following are therefore defined:

$$u_i(x, t) = \sum_{k=1}^{u_i} \kappa_{ik}(t) X_k(x); \quad X_k(x) = \cos \frac{k\pi x}{a_i} \quad [2.21]$$

$$v_i(x, t) = \sum_{k=1}^{vt_i} \vartheta_{ik}(t) Y_k(x); \quad Y_k(x) = \sin \frac{k\pi x}{a_i} \quad [2.22]$$

$$\theta_i(x, t) = \sum_{k=1}^{\theta t_i} \phi_{ik}(t) \Theta_k(x); \quad \Theta_k(x) = \sin \frac{k\pi x}{a_i} \quad [2.23]$$

If any vertical slip of the joist-header connection is accounted for

$$w_i(x, t) = \sum_{k=1}^{wt_i} \tau_{ik}(t) Z_k(x); \quad Z_k(x) = \sin \beta_k x + \frac{\sin \beta_k a_i}{\sinh \beta_k a_i} \sinh \beta_k x + \gamma_k \left(\cos \beta_k x + \cosh \beta_k x - \frac{(\cosh \beta_k a_i - \cos \beta_k a_i)}{\sinh \beta_k a_i} \sinh \beta_k x \right) \quad [2.24]$$

On the other hand, for simply support conditions (Hu 1992; Smith and Chui 1988; Foschi and Gupta 1987; Ohlsson 1984)

$$w_i(x, t) = \sum_{k=1}^{wt_i} \tau_{ik}(t) Z_k(x); \quad Z_k(x) = \sin \frac{k\pi x}{a_i} \quad [2.25]$$

The uniqueness of the selection of the normal mode shape function, $Z_k(x)$, in Eqs. 2.24 and 2.25 is explained in the following. Typically, we define $w_i = w_{b_i} + w_{s_i}$, where w_{b_i} and w_{s_i} are deflections due to bending and shear, respectively. Using the Rayleigh-Ritz approach, one can readily choose consistent modal functions to define w_{b_i} . This cannot, however, be said of w_{s_i} when support conditions other than the simply supported is the case (generally, the shape of the shear displacement is not known *a priori*). Jacobsen and Ayre (1959), on the other hand, asserted that in many cases, bending and shear deflection curves do not differ greatly. This becomes all the more accurate in situations where bending action is the more significant and only a few lower natural frequencies are of interest. This condition gives the guarantee that the behaviour between bending and shear are uncoupled. It is noted that this work adopts the above proposal for the functional representation of both the bending displacement, w_{b_i} , and the shear displacement, w_{s_i} irrespective of the joist support conditions.

Reference to the i th joist, in Eqs. 2.21–2.25 ut_i , vt_i , wt_i and θt_i are, respectively, the finite number of mode shapes which define $u_i(x, t)$, $v_i(x, t)$, $w_i(x, t)$ and $\theta_i(x, t)$; κ_{ik} , ϑ_{ik} , τ_{ik} and ϕ_{ik} gives the constrained coordinates associated with the k th mode

shape. The constant, γ_k , appearing in Eq. 2.24 is given as (Gorman 1975)

$$\gamma_k = \frac{\sinh \beta_k a_i - \sin \beta_k a_i}{2K_s a_i^3 \sinh \beta_k a_i / (E_{J_i} I_{y_i} \beta_k a_i) + \cos \beta_k a_i - \cosh \beta_k a_i} \quad [2.26]$$

and in which β_k is obtained from the frequency equation

$$\begin{aligned} \frac{K_s a_i^3}{E_{J_i} I_{y_i}} \left[\left(\frac{K_s a_i^3}{E_{J_i} I_{y_i}} \right) + \frac{2(\beta_k a_i)^3 (\sinh \beta_k a_i \cos \beta_k a_i - \sin \beta_k a_i \cosh \beta_k a_i)}{2 \sin \beta_k a_i \sinh \beta_k a_i} \right] \\ + \frac{(\beta_k a_i)^6 (1 - \cos \beta_k a_i \cosh \beta_k a_i)}{2 \sin \beta_k a_i \sinh \beta_k a_i} = 0 \quad [2.27] \end{aligned}$$

where, again, K_s is the spring constant of the joist end connection computed at 0.4 mm slip of tests conducted in accordance with the ASTM D1761-88; E_{J_i} and I_{y_i} being respectively, the modulus of elasticity and moment of inertia of the i -th joist. Implicit in Eq. 2.24 is the assumption that for each joist the two end connections have the same stiffness. In Eq. 2.27, the parameter β_k associated with the k th mode shape could be computed using the *secant method*; the initial trial values to solve the unconstrained function selected as $\frac{k\pi}{a_i}$ and $\lambda \frac{k\pi}{a_i}$, with λ chosen very close to unity for the first few lower modes.

As stated earlier, if the joist deformational types are uncoupled, the strain energy, U_{J_i} , of the i -th joist when shear deformation effects are considered may be put as

$$\begin{aligned} U_{J_i} = \frac{E_{J_i} I_{y_i}}{2} \int_0^{a_i} \left(\frac{d^2 w_{b_i}}{dx^2} \right)^2 dx + \frac{\alpha G_{J_i} A_{J_i}}{2} \int_0^{a_i} \left(\frac{dw_i}{dx} - \frac{dw_{b_i}}{dx} \right)^2 dx \\ + \frac{E_{J_i} I_{z_i}}{2} \int_0^{a_i} \left(\frac{d^2 v_i}{dx^2} \right)^2 dx + \frac{E_{J_i} A_i}{2} \int_0^{a_i} \left(\frac{du_i}{dx} \right)^2 dx + \frac{G_{J_i} J_i}{2} \int_0^{a_i} \left(\frac{d\theta_i}{dx} \right)^2 dx \quad [2.28] \end{aligned}$$

where, $w_{b_i} = \sum_{k=1}^{w_{t_i}} \psi_{ik} Z_k(x)$. The corresponding kinetic energy, T_{J_i} , if the torsional inertia effect is also accounted for is given as

$$\begin{aligned} T_{J_i} = \frac{\rho_{J_i} A_i}{2} \int_0^{a_i} (\dot{w}_i^2 + \dot{v}_i^2 + \dot{u}_i^2) dx + \frac{\rho_{J_i} I_{z_i}}{2} \int_0^{a_i} \left[\left(\frac{d\dot{v}_i}{dx} \right)^2 + \dot{\theta}_i^2 \right] dx \\ + \frac{\rho_{J_i} I_{y_i}}{2} \int_0^{a_i} \left[\left(\frac{d\dot{w}_{b_i}}{dx} \right)^2 + \dot{\theta}_i^2 \right] dx \quad [2.29] \end{aligned}$$

Substituting Eqs. 2.21–2.25 into Eqs. 2.28 and 2.29, and employing the orthogonality properties of beam mode shapes, the strain energy expression of Eq. 2.28 takes

the explicit form

$$\begin{aligned}
U_{J_i} = & \frac{\pi^2}{4a_i} E_{J_i} A_i \sum_{m=1}^{ut_i} m^2 \kappa_{im}^2 + \frac{\pi^4}{4a_i^3} E_{J_i} I_{z_i} \sum_{m=1}^{vt_i} m^4 \vartheta_{im}^2 + \frac{\pi^2}{2a_i} G_{J_i} J_i \sum_{m=1}^{\theta t_i} m^2 \phi_{im}^2 \\
& + \frac{E_{J_i} I_{y_i}}{2} \sum_{m=1}^{wt_i} \psi_{im}^2 \Lambda_m \int_0^{a_i} Z_m^2 dx + \frac{\alpha G_J A_J}{2} \left[\sum_{j=1}^{wt_i} (\tau_{ij} - \psi_{ij})^2 \int_0^{a_i} \left(\frac{dZ_j}{dx} \right)^2 dx \right. \\
& \left. + \sum_{j=1}^{wt_i} \sum_{\substack{k=1 \\ k \neq j}}^{wt_i} (\tau_{ij} - \psi_{ij})(\tau_{ik} - \psi_{ik}) \left(\frac{dZ_j}{dx} \right) \left(\frac{dZ_k}{dx} \right) dx \right] \quad [2.30]
\end{aligned}$$

in which

$$\Lambda_m = \begin{cases} \left(\frac{m\pi}{a_i} \right)^4 & \bullet \text{ hinged-hinged condition} \\ \beta_m^4 & \bullet \text{ spring-spring condition} \end{cases}$$

The kinetic energy expression in Eqs. 2.29 also simplifies as

$$\begin{aligned}
T_{J_i} = & \frac{1}{2} \rho_{J_i} A_i \left[\frac{a_i}{2} \sum_{n=1}^{ut_i} \dot{\kappa}_{in}^2 + \frac{a_i}{2} \sum_{n=1}^{vt_i} \dot{\vartheta}_{in}^2 + \sum_{n=1}^{wt_i} \dot{\tau}_{in}^2 \int_0^{a_i} Z_n^2 dx \right] \\
& + \frac{1}{2} \rho_{J_i} I_{z_i} \left[\frac{\pi^2}{2a_i} \sum_{n=1}^{vt_i} n^2 \dot{\vartheta}_{in}^2 + \frac{\pi^2}{2a_i} \sum_{n=1}^{\theta t_i} \dot{\phi}_{in}^2 \right] + \frac{1}{2} \rho_{J_i} I_{y_i} \left[\sum_{j=1}^{wt_i} \dot{\psi}_{ij}^2 \int_0^{a_i} \left(\frac{dZ_j}{dx} \right)^2 dx \right. \\
& \left. + \sum_{j=1}^{wt_i} \sum_{\substack{k=1 \\ k \neq j}}^{wt_i} \dot{\psi}_{ij} \dot{\psi}_{ik} \left(\frac{dZ_j}{dx} \right) \left(\frac{dZ_k}{dx} \right) dx + \frac{\pi^2}{2a_i} \sum_{n=1}^{\theta t_i} \dot{\phi}_{in}^2 \right] \quad [2.31]
\end{aligned}$$

where, ρ_{J_i} is the mass density, A_i the cross-sectional area, G_{J_i} and J_i are the shear modulus and torsional moment of inertia, respectively. We note in Eqs. 2.30 and 2.31 that for the case of the simply supported joist, terms involving the product of $\frac{dZ_j}{dx}$ and $\frac{dZ_k}{dx}$ drop out.

2.3.1 Modelling Wood Floor Design Features

Among the principal design features particularly unique to wood floor construction are the sheathing-joist connections, the presence of material discontinuities (or gaps) in the flooring, between-joist struttings, and the use of light-gauge metal connectors, if any, to effect the connections between the joists and the perimeter beams, or the headers. Unlike the analyses governing the two principal substructures which may only follow after standard procedures, the requisite formulations modelling the actual behaviour of some of the above features are rather complex.

and are often approximated by simpler methods whose accuracies have been validated through experimentation. Notice that both the strain and kinetic energy formulations for the two principal substructures, where applicable, incorporate the effects of the springiness of the joists' end connections, and material discontinuities. The qualitative effects of these two features are later discussed in Sections 3.2 and 3.3. A thorough discussion on the behaviour of between-joist strutting is also given both quantitatively and qualitatively in Section 3.4.

2.3.1.1 Strain Energy in Sheathing-Joist Connectors

It is assumed in this work that the floor plate is semirigidly attached to the joists by means of uniformly spaced nail fasteners running along the centerline of the top surface of the joists. Considering the serviceability nature of the problem at hand, we assume linear load-slip characteristics of the connectors (Foschi 1982; Filiatrault *et al.* 1990). For lightweight wooden floors, this assumption has been validated by tests on static response of wood-joist floors (Foschi 1985) where the contribution due to the actual nonlinear load-slip characteristics of the nail fasteners to the overall floor behaviour has been found to be negligibly small under normal service loads. Therefore, the strain energy, U_c , stored in the connectors due to horizontal slippages as well as any rotational slippage resulting from connector withdrawal takes the form (Foschi 1982)

$$U_c = \sum_{i=1}^{nj} \sum_{j=1}^{NC_i} \left[\frac{k_{\parallel}}{2} (\Delta u)_{ij}^2 + \frac{k_{\perp}}{2} (\Delta v)_{ij}^2 + \frac{k_{\theta}}{2} (\Delta \theta)_{ij}^2 \right] \quad [2.32]$$

in which NC_i is the number of connectors on the i th joist. $k_{\parallel}$ and $k_{\perp}$ are, respectively, connector stiffnesses in the parallel and perpendicular to grain directions, and k_{θ} is the rotational stiffness. In developing U_c , it is assumed that $k_{\parallel}$ and $k_{\perp}$ are of the same order of magnitude. Thus, the strain energy expression which should customarily be based on the stiffness corresponding to the actual direction of slip is thus formulated in terms of the components of slip along the x and y -directions.

Eq. 2.32 considers the connectors in a discrete fashion. We make a further approximation that the connectors are assumed smeared out along the entire length

Fig. 2.3 Deformation at Flooring-Stiffener Interface
(a) x-direction (b) y-direction

of any one joist. We thus express Eq. 2.32 as

$$U_c = \int_0^{a_i} \left[\frac{k_{\parallel}}{2e} (\Delta u)^2 + \frac{k_{\perp}}{2e} (\Delta v)^2 + \frac{k_{\theta}}{2e} (\Delta \theta)^2 \right] \quad [2.33]$$

in which e is the spacing of any two successive connectors.

From Fig. 2.3, slippages of the j th connector on the i th joist having coordinate (x, y) can be defined as follows:

$$\bullet \quad \Delta u_{ij}(x, y) = u(x, y) - \frac{t_p}{2} \left[\frac{\partial w(x, y)}{\partial x} \right] - \left[u_i(x) - \frac{d_i}{2} \frac{dw_i}{dx} \right] \quad [2.34a]$$

$$\bullet \quad \Delta v_{ij}(x, y) = v(x, y) - \frac{t_p}{2} \left[\frac{\partial w(x, y)}{\partial y} \right] - \left[v_i(x) - \frac{d_i}{2} \theta_i(x) \right] \quad [2.34b]$$

$$\bullet \quad \Delta \theta_{ij}(x, y) = \frac{\partial w(x, y)}{\partial y} - \theta_i(x) \quad [2.34c]$$

where d_i is the sectional depth of the i th joist. The uniqueness of the contribution from the connections to the global dynamic equation is explained using the

contribution of the ij th connector. In the following, the subscript ij is dropped for clarity. Firstly, from Eq. 2.33, we define $\Delta u \equiv \mathbf{Q}_u^T \{\delta\}$; $\Delta v \equiv \mathbf{Q}_v^T \{\delta\}$; and $\Delta_\theta \equiv \mathbf{Q}_\theta^T \{\delta\}$, with $\{\delta\}$ being a vector of unknown parameters associated with the flooring and joists substructures. Following standard procedure, we form a vector of the derivatives of U_c with respect to elements in $\{\delta\}$, δU_c , and thus write the contribution of the ij th connector to the global stiffness matrix as

$$\delta U_c = \mathbf{K}_c \bar{\delta} \quad [2.35]$$

with the matrix $\mathbf{K}_c$ given by

$$\mathbf{K}_c = k_{\parallel} \mathbf{Q}_u \mathbf{Q}_u^T + k_{\perp} \mathbf{Q}_v \mathbf{Q}_v^T + k_{\theta} \mathbf{Q}_\theta \mathbf{Q}_\theta^T$$

2.3.2 System Compatibility Conditions

Considering the structure of the wood joist floor, we assume identical vertical displacements for both the joists and the floor plate at the sheathing-joist boundaries. This continuity constraint may be presented as

$$f_i \equiv w(x, y_i) - \sum_{k=1}^{wt_i} \tau_{ik} Z_k(x) = 0 \quad i = 1, \dots, nj \quad [2.38]$$

where, y_i is the y -coordinate at the location of the i th joist. It is noted that the form of constraint f_i given by Eq. 2.38 is generally suitable for the analysis employing a discretization scheme such as the finite element method. Similar to the derivation given by Dowell (1972), we seek to present an alternative form of f_i which is amenable to the semi-analytical method employed in this thesis. In effect, the plate modes given by either Eq. 2.4 or Eq. 2.7 is expanded in terms of the beam transverse modes by the approximation,

$$W_{mn}(x, y_i) = \sum_{u=1}^{wt_i} E_{mnu}^{(i)} Z_u(x) \quad [2.39]$$

Furthermore, by employing the orthogonality properties of the beam modal functions, we obtain

$$E_{mnu}^{(i)} \equiv \int_0^a W_{mn}(x, y_i) Z_u(x) dx / \int_0^a [Z_u(x)]^2 dx$$

and the constraint equations, f_i , take the form

$$f_i \equiv \sum_m \sum_n \sum_u A_{mn} E_{mnu}^{(i)} Z_u(x) - \sum_u \tau_{iu} Z_u(x) = 0 \quad [2.40]$$

from which we obtain $f_i^{(u)}$ as

$$f_i^{(u)} \equiv \sum_m \sum_n A_{mn} E_{mnu}^{(i)} - \tau_{iu} = 0 \quad \begin{array}{l} u = 1, \dots, wt_i \\ i = 1, \dots, nj \end{array} \quad [2.41]$$

The above formulations have been based on the assumption that any spanwise plate transverse mode is well approximated by wt_i of the beam modes. This has been validated by results from previous experimental tests on orthotropic wood-joint floors [eg. Hu 1992, Ohlsson 1984]. For now, if the system compatibility relations are expressed only in terms of the constraints depicting the vertical displacements of sheathing and the joists, the number of constraint equations of the floor system, s , is given according to the expression

$$s = \sum_{i=1}^{nj} wt_i \quad [2.42]$$

3.1 General

This chapter discusses the procedure for obtaining lower and higher natural frequencies alongside their respective normal modes for a typical wood-joint floor. The results provide a basis for the response analysis techniques discussed at length in Chapter 5. A knowledge of the floor highest frequency also provides a useful guide as to the choice of the time step which is essential for numerical stability control in response history analysis.

As a comparative study, results from both experimental work (Hu 1992; Smith and Chui 1988) and analytical work (Folz and Foschi 1990) are used as benchmarks to check on the accuracy of floor frequency prediction by the current model. A convergence study is also performed to obtain the finite number of parameters in the Rayleigh-Ritz formulation needed for accuracy.

3.2 Equations of Motion

The application of Hamilton's principle to the general dynamic behaviour of an undamped wood-joint floor system is of the following form by the methods of variational calculus:

$$\int_{t_1}^{t_2} \sum_{i=1}^g \left[\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{p}_i} \right) - \frac{\partial T}{\partial p_i} + \frac{\partial U}{\partial p_i} \right] \delta p_i dt = 0 \quad [3.1]$$

in which T and U , respectively, are the total kinetic and strain energy of the system; and $\{\mathbf{p}\} = [p_1 \ p_2 \ \dots \ p_g]^T$ is the vector of g constrained coordinates of the system. Notice that the second term of Eq. 3.1 is zero since the kinetic energy is a function of the velocities of the constrained coordinates $\dot{p}_i$ only. We evaluate g as

$$g = \text{DOF}_w + \text{DOF}_{uv} + \sum_{k=1}^{n_j} (ut_k + vt_k + 2wt_k + \theta t_k) \quad [3.2]$$

where, $\text{DOF}_w = \text{total d.o.f. (or unknown parameters) of sheathing substructure due to bending}$; $\text{DOF}_{uv} = \text{total d.o.f. (or unknown parameters) of sheathing substructure due to membrane action}$. It is noted from Eq. 3.1 that since the $\mathbf{p}$ coordinates are constrained by means of the general system compatibility conditions set out in Section 2.3.2, any choice of the virtual displacements $\{\delta \mathbf{p}\}$ cannot be arbitrary in the Lagrangian sense (Hurty 1960). We therefore transform the $\{\mathbf{p}\}$ coordinates to $\{\mathbf{D}\}$ independent generalized coordinates, of size $(g - s) \times 1$, using the relationship

$$\{\mathbf{p}\} = [\Gamma]\{\mathbf{D}\} \quad [3.3a]$$

or,

$$\{\delta \mathbf{p}\} = [\Gamma]\{\delta \mathbf{D}\} \quad [3.3b]$$

in which $[\Gamma]$, of size $g \times (g - s)$, is the requisite transformation matrix, and $s = \text{constraint equations of the system defined by Eq. 2.42}$. On substituting Eq. 3.3b into Eq. 3.1 we obtain

$$\int_{t_1}^{t_2} \sum_{j=1}^g \sum_{k=1}^{g-s} \Gamma_{jk} \left[\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{p}_i} \right) + \frac{\partial U}{\partial p_i} \right] \delta D_k dt = 0 \quad [3.4]$$

Next, taking into account the independence of the variables in vector $\{\delta \mathbf{D}\}$, we can select a set of values in which δD_k , is zero for all j except $i = k$. We note that since δD_k is arbitrary, the integral in Eq. 3.4 can be zero only when the integrand is zero. Furthermore, since $k = 1, 2, 3, \dots, (g - s)$ the above reasoning leads to $g - s$ Euler-Lagrange equations of the general form

$$\sum_{i=1}^g \Gamma_{ik} \left[\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{p}_i} \right) + \frac{\partial U}{\partial p_i} \right] = 0 \quad k = 1, 2, 3, \dots, (g - s) \quad [3.5]$$

For small amplitude vibrations of a conservative system, the following can be defined:

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{p}_i} \right) &= \sum_{i=1}^g m_{ij} \ddot{p} \\ \frac{\partial U}{\partial p_i} &= \sum_{i=1}^g k_{ij} p \end{aligned} \quad [3.6]$$

where, m_{ij} and k_{ij} are, respectively, elements of constrained mass and stiffness matrices. Substituting Eq. 3.6 into Eq. 3.5, the $g - s$ Euler-Lagrange equations

may be put in the matrix form

$$\Gamma^T \mathbf{M} \ddot{\mathbf{p}} + \Gamma^T \mathbf{K} \mathbf{p} = 0 \quad [3.7]$$

where, $\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrices, respectively, of the constrained system. Again, substituting Eq. 3.3a into Eq. 3.7, and noting that for harmonic vibrations $\ddot{\mathbf{D}} = -\omega^2 \mathbf{D}$, the equations of motion of the undamped wood joist-floor system under unconstrained conditions take the format

$$(\hat{\mathbf{K}} - \omega^2 \hat{\mathbf{M}}) \mathbf{D} = 0 \quad [3.8]$$

in which $\hat{\mathbf{K}} = \Gamma^T \mathbf{K} \Gamma$ and $\hat{\mathbf{M}} = \Gamma^T \mathbf{M} \Gamma$. $\hat{\mathbf{K}}$ and $\hat{\mathbf{M}}$ are square matrices of size $(g - s) \times (g - s)$. The transformation matrix, Γ , is derived in accordance with standard procedures [eg. Gallagher 1976] as follows:

In general, the constraint conditions discussed in Section 2.3.2 can be put in the form

$$\underset{(s \times g)}{[\mathbf{A}]} \underset{(g \times 1)}{\{\mathbf{p}\}} = 0 \quad [3.9]$$

Following standard procedures, matrix $\mathbf{A}$ is then partitioned as $\mathbf{A} = [\mathbf{A}_1 \quad \mathbf{A}_2]$, where $\mathbf{A}_1$, of size $s \times s$, is constructed such that it is nonsingular. In this work, elements of $\mathbf{A}_1$ are chosen such that all diagonal elements are nonzero. Vector $\mathbf{p}$ is correspondingly rearranged as $\mathbf{p} = [\mathbf{p}_1 \quad \mathbf{p}_2]$, and the transformation matrix Γ readily obtained as

$$\Gamma = \begin{bmatrix} -\mathbf{A}_1^{-1} \mathbf{A}_2 \\ \mathbf{I} \end{bmatrix} \quad [3.10]$$

$\begin{matrix} (s \times (g - s)) \\ ((g - s) \times (g - s)) \end{matrix}$

In FORTRAN coding, however, Γ is reconstructed to follow the assemblage of matrices $\mathbf{M}$ and $\mathbf{K}$.

The forms of $\mathbf{M}$ and $\mathbf{K}$ in Eq. 3.7 are given schematically as

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_w & & & & \\ & \mathbf{M}_{uv} & & & \\ & & \mathbf{M}_j^{(1)} & & \\ & & & \ddots & \\ & & & & \mathbf{M}_j^{(1)} \end{bmatrix} \quad \mathbf{K} = \begin{bmatrix} \mathbf{K}_w & 0 & \mathbf{K}_{wj}^{(1)} & \dots & \mathbf{K}_{wj}^{(n_j)} \\ & \mathbf{K}_{uv} & \mathbf{K}_{uvj}^{(1)} & \dots & \mathbf{K}_{uvj}^{(n_j)} \\ & & \mathbf{K}_j^{(1)} & & \\ & & & \ddots & \\ & & & & \mathbf{K}_j^{(1)} \end{bmatrix} \quad [3.11]$$

The submatrices M_w , M_{uv} and $M_j^{(i)}$ of the global constrained mass matrix M are the respective mass matrices associated with the sheathing transverse displacement w ; sheathing in-plane deformations u and v ; and the i th joist vibrational modes w_i , u_i , v_i and θ_i . The submatrices of K are similarly defined.

The explicit formulation of the kinetic and strain energy expressions given in Chapter 2 allows for the formation of these submatrices without much difficulty. For example, using notations as before and considering the particular case of $A_k(t)$ ($k = 1, 2, \dots, MN | k = N(m-1) + n$) as being the time dependent generalized coordinates characterizing the transverse vibrational mode of the plate (flooring), we form the elements of the associated submatrix M_w , from either Eq. 2.17 or 2.18 as

$$M_{w_{ij}} = \frac{\partial^2 T_p}{\partial A_j \partial A_i} \quad [3.12]$$

We notice from Eq. 3.11 that, the stiffness matrix K is not well banded and for that matter either the upper or lower triangular elements are all to be stored. In most practical analysis, however, the applicable system unknown variables g may not be more than 80. Thus the ease of implementation of this model in micro-computer environment is not at all jeopardized.

The dynamic behaviour of the i th joist is characterized by a total of $(ut_i + vt_i + 2wt_i + \theta_i)$ time dependent parameters with the wt_i of the parameters arising from the consideration of shear deformational effects. For the simple joists, further reduction in the unknown system parameters is possible by combining respective parameters associated with bending and shear deformation as follows. Since the joist total vertical deflection is the sum of the deflections due separately to bending and shear deformation, and based on an earlier discussion on the selection of appropriate mode shapes each for bending and shear, we can express the parameters associated with bending (and indirectly shear deformations) in terms of the total vertical deflection (Eq. 2.25):

$$\sum_{(k=1)}^{wt_i} \phi_{ik}(t) Z_k(x) = \sum_{(k=1)}^{wt_i} \frac{1}{(1 + \alpha_{ik})} \tau_{ik} Z_k(x) \quad [3.13]$$

or

$$\phi_{ik} = \frac{1}{(1 + \alpha_{ik})} \tau_{ik} \quad [3.14]$$

We estimate α_{ik} as

$$\alpha_{ik} = \frac{k^2 \pi^2 EI}{\mu G A a_i^2} \quad [3.15]$$

3.3 Natural Frequencies and Mode Shapes

Equation 3.8 expresses the desired eigenvalue problem whose solution yields the natural frequencies $\omega_1, \omega_2, \dots, \omega_{(g-s)}$ and the eigenvectors $\{\mathbf{D}\}^{(1)}, \{\mathbf{D}\}^{(2)}, \dots, \{\mathbf{D}\}^{(g-s)}$ of the generalized coordinates. By the substitution of the appropriate components of these eigenvectors into any one of the vibratory expressions derived in Chapter 2, the pertinent natural modes of the wood-joint floor corresponding to a defined boundary conditions can be obtained. For instance, the r th natural mode $\Phi_r(x, y)$ of the floor transverse vibrational mode considering a typical rectangular floor with all edges simply supported is approximated by

$$\Phi_r(x, y) = \sum_{m=1}^M \sum_{n=1}^N \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) D_{N(m-1)+n}^{(r)} \quad [3.16]$$

in which, $D_{N(m-1)+n}^{(r)}$ is the $[N(m-1) + n]$ th component of the eigenvector $\{\mathbf{D}\}^{(r)}$ with its first MN components describing the floor transverse displacement.

The subspace iteration method (Bathe 1982) is implemented in the computer program written to determine the lower eigenvalues and their corresponding eigenvectors for the desired frequency range. The iteration scheme used ensures that the poor bandedness and sparsity of the generalized stiffness matrix is taken into account in the solution techniques. Also, as a guarantee that all eigenvalues of interest are extracted, a Sturm sequence check is incorporated in the eigensolution.

3.4 Solution Convergence

To a large extent, the computational efficiency of the current model in analyzing the vibrational behaviour of lightweight floors is assessed by way of its convergence

characteristics. As in previous models [eg. Chui (1987) and Hu (1992)], the efficiency of this model, is largely determined by the number of parameters (both along- and across-joists direction) employed in the Rayleigh-Ritz analysis to obtain accurate solution. Fortunately, experimental results from tests on orthotropic wood-joist floors indicate that the primary lower modes, which largely control human discomfort levels, are dominated by across-joists deformations. This means that the number of half sine waves in the across-joists' direction mostly determines the lower natural frequencies. We thus have the pertinent benchmark against which the model's efficiency in predicting floor frequencies can be measured. Toward this direction, these convergence properties are studied using the results of a typical wood floor with the following data:

- Sheathing:-

Tongued and grooved (T&G) 15.9 mm Douglas-Fir Plywood (D-Fir)

Mass density = 500 kg/m³

$K_x = 1.04 \text{ kN.m}$ $K_y = 3.23 \text{ kN.m}$

$K_v = 70.0 \text{ N.m}$ $K_G = 210 \text{ N.m}$

$D_x = 60.72 \text{ MN/m}$ $D_y = 76.79 \text{ MN/m}$

$D_v = 3.21 \text{ MN/m}$ $D_G = 10.79 \text{ MN/m}$

- Joist:-

depth = 184 mm

width = 38 mm

Modulus of elasticity for all joists = $1.07 \times 10^4 \text{ MPa}$

Mass density = 500 N.s²/m⁴

- Connector:-

Connector horizontal slip stiffnesses, k_x and $k_y = 1200 \text{ N/mm}$

Spacing of connectors = 200 mm

- Floor span = 4000 mm
- Number of joists = 9
- No fastener withdrawal effect, ie. $k_\theta \rightarrow \infty$
- Floor is simply supported on all sides
- No bridging nor across-joist gaps in sheathing

In the following discussion, M corresponds to the number of half sine waves in the direction of the joists, and N the number in the across-joists direction which would normally exist if the plate were unstiffened. Notice that M and N are terms referenced to the subfloor substructure analysis. For the joists possible modes of

vibration as described in Section 2.3, ut , vt , wt , and θ_t , are the number of half-wave terms in the modal expansion. Table 3.1 show the results for $M = 1, 2$, and 3 , respectively, for varying N . It is noted that convergence is obtained readily for only a few terms in the modal expansion. In particular, in respect of frequencies obtained for $M = 1$ and $N = 33$, the first, second, third, fourth, fifth, sixth, and seventh natural frequencies using $M = 1$ and $N = 7$ terms are only 0.14%, 0.12%, 0.07%, 0.06%, 0.16%, 0.44%, and 0.99%, respectively, greater than their counterpart values for $N = 33$. Also, with the possible exclusion of additional frequencies associated with higher M terms, frequencies obtained each for $M = 1, 2$, and 3 do not differ significantly. Similar deductions could also be made for characteristic terms associated with the joists.

3.5 Model Validation and Performance

The accuracy of the results from frequency analysis using the model developed in this thesis is verified against some analytical and experimental data available in the literature. For brevity, only one analytical and three experimental data sets are used for comparison. Where not explicitly stated, the results from this study are based on modal expansion involving two and seven terms, along and across the joists directions, respectively.

3.5.1 Comparison with Long (1971) Analytical Model

Long (1971) used, respectively, the stiffness method of structural analysis and the composite beam-plate method (CBP)—the latter method developed in an earlier work (Long 1968)—to analyse a rectangular steel panel spanning 2440 mm, and stiffened in this direction by 3 centrally placed steel $\perp$ -beams at 915 mm on centres. The joists were assumed to be integral with the plating, and consideration of in-plane deformations were restricted to spanwise displacements only. Table 3.2 compares the result of this study with those of Long (1971), and also with similar predictions in Hu (1992). Recognizing the assumptions inherent in Long (1971), the author's results have been obtained by neglecting contribution to the energy

formulations of terms due to in-plane displacements normal to the direction of the joists, and also by making $k_\theta \rightarrow \infty$.

3.5.2 Comparison with Ohlsson (1982) Test Results

Ohlsson (1982) carried out series of dynamic testing on laboratory-built wood floors to establish their vibrational characteristics such as natural frequencies and damping. The floors tested measured 5×2.4 m and had subfloors, or sheathings, consisting of 22 mm particleboard. Each floor was stiffened by five equally spaced 45×220 mm sawn lumber joists simply supported and spanning in the 5 m direction. In all floors, the joists were assumed rigidly connected to the sheathing. Table 3.3 gives a comparison of natural frequencies predicted by various numerical models and Ohlsson (1982) test results. Notice that tabulated results are for floors with edge joists free to vibrate in all modes.

3.5.3 Comparison with Chui (1987) Test Results

Similar to the objectives of the vibration tests reported in Ohlsson (1982), Chui (1987) tests were conducted on wood floors of typical dimensions 3.92×2.4 m, each floor ribbed by 47×197 mm Hem-Fir joists spaced at 600 mm on centres, and subfloor constructed from 15 mm tongue and groove (T&G) Douglas-Fir plywood panels. The fasteners for the sheathing to joist connections were 3.0×50 mm ring shank nails spaced at 300 mm.

The test result used for this comparative study corresponded to the floor type with all edge joists unrestrained from vibrating vertically. Table 3.4 compares the test natural frequencies with those predicted by some currently available numerical models and the one outlined in this thesis. Notice that the results of the Rayleigh-Ritz numerical model (Chui 1987) assumed rigid sheathing-joist connections. However, appropriate load-slip modulus for all connectors were allowed for in obtaining the results given in this study.

3.5.4 Comparison with Hu (1992) Test Results

Hu (1992), using the modal test technique, tested series of rectangular-shaped

floors having constant width 3.46 m and spans 6.8, 5.9 and 4.45 m, respectively. Floors consisted of 15.5 mm tongue and groove Type 0-1 Oriented Strand Board (OSB) flooring stiffened in the span direction by a total of eight wood-I joists spaced approximately at 481 mm on centres. Structurally, each I-joist had flanges made of 38×89 mm machine-stress rated (MSR) Douglas-Fir lumber and an 11.0 mm thick OSB web. Flooring to joist attachments were by means of No. 8 (4.37 mm diameter) wood screws placed at average centre-to-centre spacing of 160 mm.

Table 3.5a and 3.5b, respectively, indicate test and accompanying predicted natural frequencies for floors simply supported at all edges (designated S-S-S-S), and for floors with the two opposite edge free to vibrate in all modes (designated S-F-S-F). Table 3.5c, on the other hand, shows the unique case of a test floor (span 5.9 m; S-F-S-F floor) under imposed concentrated masses comprising of bags of peat at positions E_1 , K_1 , E_4 , and K_4 . Reference to the floor global X and Y axes, with origin at one corner of the floor and the X -axis denoting the direction of the joists, the coordinates of these positions are $E_1(1.180, 1.035)$, $K_1(1.180, 2.45)$, $E_4(4.720, 1.035)$ and $K_4(4.720, 2.45)$. Note that the requisite input material data for the various computations are either derived from or quoted directly from Hu (1992). This is presented as follows:

• *Joist:-*

	ρA (kg/m)	ρI_y (10^{-2} kg.m)	ρI_z (10^{-3} kg.m)	EI_y (10^5 N.m ²)	EI_z (10^4 N.m ²)	EA (10^8 N ²)	GJ (10^2 N.m ²)	μGA (10^6 N ²)
Joist #1	4.25	3.574	1.3641	9.00	3.43	1.07	10.22	1.34
Joist #2	4.43	3.725	1.4219	9.25	3.53	1.10	9.54	1.25
Joist #3	4.60	3.839	1.1637	9.83	2.98	1.18	8.99	1.35
Joist #4	4.48	3.846	1.1686	10.31	3.13	1.20	8.41	1.24
Joist #5	4.28	3.526	1.1804	9.07	3.04	1.10	8.47	1.21
Joist #6	4.70	4.088	1.2690	11.45	3.55	1.32	9.27	1.34
Joist #7	4.38	3.852	1.0887	8.64	2.44	9.82	9.69	1.47
Joist #8	4.22	3.711	1.0489	8.69	2.46	9.88	8.83	1.34

• *Sheathing:-*

$$\text{Mass density} = 642 \text{ kg/m}^3$$

$$K_x = 747.66 \text{ N.m} \quad K_y = 2338.85 \text{ N.m}$$

$$K_v = 71.70 \text{ N.m} \quad K_G = 428.25 \text{ N.m}$$

$$D_x = 37.34 \text{ MN/m} \quad D_y = 116.82 \text{ MN/m}$$

$$D_v = 3.58 \text{ MN/m} \quad D_G = 21.39 \text{ MN/m}$$

- Connector slip stiffnesses: $k_x = k_y = 1800 \text{ N/mm}$

- No connector withdrawal effect, ie. $k_\theta \rightarrow \infty$

- No between-joint bridging, and across-joint gaps in sheathing neglected
- Imposed masses are 79.4 kg per point

3.6 Discussion on Model Validity

As previously pointed out, the modelling techniques employed in this thesis are directed at developing a unified numeric tool for the serviceability evaluation of wood floors in such a manner that the influence on the floor dynamic behaviour of the more commonly construction or end-use features are systematically accounted for. In achieving this, the model has been formulated to achieve accuracy, simplicity, and the computational efficiency known of some currently available numerical models.

In this section, the results found through comparative studies done in Section 3.5 are discussed to help establish the attainment, or otherwise, of the above objectives. Underlying this discussion is the recognition of some basic inherent shortcomings associated with all numerical models including truncation of infinite series, discretization of a continuum problem as well as inaccuracies in obtaining material data experimentally which may jeopardize the accuracy of all predictive models. Note also that the limitations and experimental errors found under the various experimental programs used in this study are discussed elsewhere. Implicitly, these limitations may also help explain differences between test results and those obtained using the unified model proposed in this thesis. It must also be pointed out that the tabulated values under the various predictive models other than those from this study have been obtained from Hu (1992).

From the results in Tables 3.2–3.5, it is evident that the modelling techniques in this thesis can lead to accurate predictions of the first few natural frequencies whose modal behaviour also play a decisive role in human perception evaluation of lightweight floors. Specifically, the accuracy of this model in predicting lower frequencies may, in some situations, improve the predictive behaviour presented by Hu (1992) modal synthesis model. In fact the seeming similarity in the modelling techniques adopted in these two models may explain this uniqueness.

It is worthy of observing that, whereas predicted higher modes frequencies are relatively on the conservative side due in part to the neglect of in-plane deformation by the models, the account of this in this study has ensured that predicted frequencies are not unduly overestimated. This becomes all the more apparent given the results in Table 3.2 and the predicted mode shapes in Fig. 3.1. Since also the basic methodological developments in NAFFAP (Filiatrault *et al.*, 1990), particularly as pertain to the formulation of the interface behaviour between the sheathing and the joists, the inclusion of in-plane displacements in the flooring as well as rotary inertia effects have been adopted in this work, one may expect very marginal differences in the predicted higher modes. The discrepancies in the tabulated values may be attributed to the neglect of the joists shear deformation effects as well as the use of the finite-element polynomial discretization for the across-joists behaviour in NAFFAP. The use of trigonometric functions such as those in this thesis seem more appropriate.

Fig. 3.1 Comparison of predicted mode shape functions in the across-joists direction of Long (1971) stiffened plate
 (a) *This study* (b) *Long (1971)* (c) *Hu 1992*

Table 3.1 Model Convergence of Lower Frequencies

<i>mode</i>	$M = 1 \quad ut = 1 \quad vt = 1 \quad wt = 1 \quad \theta t = 1$						
	$N = 3$	$N = 5$	$N = 7$	$N = 10$	$N = 14$	$N = 22$	$N = 33$
$f_{1,1}$	18.6412	18.6411	18.6410	18.6410	18.6410	18.6183	18.6141
$f_{2,1}$	20.1038	20.1034	20.1032	20.1031	20.0914	20.0914	20.0813
$f_{3,1}$	24.5312	24.5310	24.5310	24.5309	24.5205	24.5148	24.5126
$f_{4,1}$		32.9586	32.9585	32.9584	32.9443	32.9387	32.9369
$f_{5,1}$		45.1020	45.0990	45.0977	45.0578	45.0283	45.0261
$f_{6,1}$			60.2051	60.0728	60.0636	59.9504	59.9434
$f_{7,1}$			77.4506	77.0551	77.0161	76.9903	76.6932
<hr/>							
<i>mode</i>	$M = 2 \quad ut = 2 \quad vt = 2 \quad wt = 2 \quad \theta t = 2$						
	$N = 3$	$N = 5$	$N = 7$	$N = 10$	$N = 14$	$N = 22$	$N = 33$
$f_{1,2}$	18.6412	18.6411	18.6410	18.6410	18.6410	18.6183	18.6141
$f_{2,2}$	20.1038	20.1034	20.1032	20.1031	20.0914	20.0813	20.0790
$f_{3,2}$	24.5312	24.5310	24.5310	24.5309	24.5205	24.5148	24.5126
$f_{4,2}$	<i>65.8289</i>	32.9586	32.9585	32.9584	32.9443	32.9387	32.9369
$f_{5,2}$	<i>66.1450</i>	45.1020	45.0990	45.0977	45.0587	45.0283	45.0261
$f_{6,2}$	<i>67.4607</i>	<i>65.8288</i>	60.2051	60.0728	60.0636	59.9504	59.9434
$f_{7,2}$		<i>66.1444</i>	<i>65.8287</i>	<i>65.8287</i>	<i>65.7800</i>	<i>65.4083</i>	<i>65.3789</i>
$f_{8,2}$		<i>67.4595</i>	<i>66.1441</i>	<i>66.1438</i>	<i>65.8284</i>	<i>65.6754</i>	<i>65.6539</i>
$f_{9,2}$		<i>70.7365</i>	<i>67.4590</i>	<i>67.4585</i>	<i>66.9922</i>	<i>66.9270</i>	<i>66.9016</i>
$f_{10,2}$		<i>76.8231</i>	<i>70.7335</i>	<i>70.7296</i>	<i>70.1641</i>	<i>70.1369</i>	<i>70.1060</i>
$f_{11,2}$			<i>76.8142</i>	<i>76.8058</i>	<i>76.1484</i>	<i>76.1384</i>	<i>76.0936</i>
$f_{12,2}$			77.4506	77.0551	77.0161	76.9903	76.6932
<hr/>							
<i>mode</i>	$M = 3 \quad ut = 3 \quad vt = 3 \quad wt = 3 \quad \theta t = 3$						
	$N = 3$	$N = 5$	$N = 7$	$N = 10$	$N = 14$	$N = 22$	$N = 33$
$f_{1,3}$	18.6412	18.6411	18.6410	18.6410	18.6410	18.6183	18.6141
$f_{2,3}$	20.1038	20.1034	20.1032	20.1031	20.0914	20.0813	20.0790
$f_{3,3}$	24.5312	24.5310	24.5310	24.5309	24.5205	24.5148	24.5125
$f_{4,3}$	<i>65.8289</i>	32.9586	32.9585	32.9584	32.9443	32.9387	32.9369
$f_{5,3}$	<i>66.1450</i>	45.1020	45.0990	45.0977	45.0587	45.0283	45.0261
$f_{6,3}$	<i>67.4607</i>	<i>65.8288</i>	60.2051	60.0728	60.0636	59.9504	59.9433
$f_{7,3}$		<i>66.1444</i>	<i>65.8286</i>	<i>65.8287</i>	<i>65.7800</i>	<i>65.4083</i>	<i>65.3789</i>
$f_{8,3}$		<i>67.4595</i>	<i>66.1442</i>	<i>66.1438</i>	<i>65.8241</i>	<i>65.6754</i>	<i>65.6539</i>
$f_{9,3}$		<i>70.7365</i>	<i>67.4590</i>	<i>67.4585</i>	<i>66.9922</i>	<i>66.9270</i>	<i>66.9016</i>
$f_{10,3}$		<i>76.8231</i>	<i>70.7335</i>	<i>70.7276</i>	<i>70.1614</i>	<i>70.1369</i>	<i>70.1060</i>
$f_{11,3}$			<i>76.8142</i>	<i>76.8058</i>	<i>76.1484</i>	<i>76.1384</i>	<i>76.0936</i>
$f_{12,3}$			77.0551	77.0551	77.0161	76.9903	76.6932

Note: Quantities in *italics* are associated with $M \geq 2$

Table 3.2 Comparison of Natural frequencies (Hz) in Long (1971) with Predictions from Other Numerical Models

	<i>Mode No.</i>	<i>Stiffness Model (Long, 1971)</i>	<i>CPB (Long, 1968)</i>	<i>Modal Synthesis Hu (1992)</i>	<i>This Study</i>
<i>M</i> = 1	1	34.69	34.36	34.53	34.22
	2	36.00	35.76	35.00	35.48
	3	39.32	39.20	39.28	38.90
	4	42.92	42.78	44.12	42.46
	7	70.62	70.32	70.10	69.85
	10	90.59	90.06	91.04	89.70
<i>M</i> = 2	6	66.75	63.40	67.02	63.35
	8	71.60	71.95	76.00	73.47
	9	80.77	80.76	89.60	86.43
	5	59.71	60.90	—	60.23

• *Notes:-* mass density = 7820 kg/m³; modulus of elasticity, E_x or E_y = 207.0 GPa;
Poisson's ratio, ν_{xy} or ν_{yx} = 0.3; width = 3.66 m; span = 2.44 m

Table 3.3 Comparison of Natural frequencies (Hz) in Ohlsson (1982) with Predictions from Other Models

<i>Mode No.</i>	<i>Test (Ohlsson) 1982</i>	<i>Grillage Model (Ohlsson) 1982</i>	<i>Rayleigh-Ritz Model (Chui, 1987)</i>	<i>Modal Synthesis (Hu, 1992)</i>	<i>This Study</i>
1	12.2	13.1	13.9	13.7	14.8
2	15.3	14.6	14.8	14.6	17.4
3	21.9	18.3	17.5	17.5	23.2
4	28.0	25.1	25.3	25.4	29.7
5	34.8	33.4	35.4	36.4	41.0

Material Data

- mass density of plate, ρ_p = 600 kg/m³
- average mass density of all joists ρ_J = 500 kg/m³
- modulus of elasticity of the particleboard, $E_x = E_y$ = 1.8 GPa
- shear modulus of particleboard, G_{xy} = 1.12 GPa
- *modulus of elasticity of joists*
 - Joist # 1 11.0 GPa
 - Joist # 2 11.7 GPa
 - Joist # 3 16.2 GPa
 - Joist # 4 14.0 GPa
 - Joist # 5 11.9 GPa
- *Connector:-* $k_x = k_y = k_\theta \rightarrow \infty$

Table 3.4 Comparison of Test Natural frequencies (Hz) (Chui, 1987) with Predictions from Other Numerical Models

<i>Mode No.</i>	<i>Test (Chui) 1987</i>	<i>Rayleigh-Ritz Model (Chui, 1987)</i>	<i>Modal Synthesis (Hu, 1992)</i>	<i>NAFFAP (Filiatrault et. al, 1990)</i>	<i>This Study</i>
1	21.1	21.4	20.6	20.7	22.7
2	23.8	23.1	22.4	21.6	24.6
3	30.4	26.7	26.4	26.1	29.3
4	38.3	39.1	38.9	36.3	41.1
5	48.7	54.4	53.9	48.6	57.0

Material Data

- mass density of plate, $\rho_p = 631 \text{ kg/m}^3$
- moduli of elasticity of plywood sheathing, $E_x = 2.1 \text{ GPa}$ $E_y = 10.8 \text{ GPa}$
- shear modulus of sheathing, $G_{xy} = 6.74 \text{ GPa}$
- modulus of elasticity of joists and mass density of joist,

Joist # 1	12.3 GPa	520 kg/m ³
Joist # 2	11.0 GPa	463 kg/m ³
Joist # 3	10.8 GPa	489 kg/m ³
Joist # 4	14.0 GPa	536 kg/m ³
Joist # 5	9.2 GPa	432 kg/m ³
- Connector slip stiffnesses: $k_x = k_y = 1500 \text{ N/mm}$, $k_\theta \rightarrow \infty$

Table 3.5a Comparison of Test Natural frequencies (Hz) (Hu, 1992) with Predictions from Other Numerical Models (S-S-S-S Floors)

	<i>Mode No.</i>	<i>Test (Hu) 1992</i>	<i>This Study</i>	<i>Modal Synthesis (Hu, 1992)</i>	<i>Rayleigh-Ritz Model (Chui, 1987)</i>	<i>NAFFAP (Filiatrault et. al, 1990)</i>
span (6.8 m)	1	11.15	11.78	10.93	12.18	15.69
	2	15.44	14.55	12.24	12.86	18.72
	3	21.65	19.04	16.84	16.98	23.25
	4	28.64	26.59	25.57	24.98	30.24
	5	32.71	29.28	36.08	36.34	38.69
	6	39.06	33.13	38.01	46.64	42.85
span (5.9 m)	1	13.11	14.57	14.13	16.16	19.77
	2	18.36	17.52	15.15	16.44	22.37
	3	24.14	21.74	19.00	19.80	26.24
	4	29.72	28.81	27.06	27.07	32.36
	5	39.22	33.30	39.06	37.83	40.00
span (4.45 m)	1	17.06	21.51	22.01	27.55	31.44
	2	23.69	24.94	22.78	27.90	33.10
	3	28.43	29.02	25.40	29.71	35.17
	4	34.02	35.27	32.08	35.24	38.88
	5	42.96	39.59	43.00	44.06	43.76

Table 3.5b Comparison of Test Natural frequencies (Hz) (Hu, 1992) with Predictions from Numerical Models (S-F-S-F Floors)

	<i>Mode No.</i>	<i>Test (Hu) 1992</i>	<i>This Study</i>	<i>Modal Synthesis (Hu, 1992)</i>	<i>Rayleigh-Ritz Model (Chui, 1987)</i>	<i>NAFFAP (Filiatrault et. al, 1990)</i>
span (6.8 m)	1	10.27	10.46	10.93	11.52	13.28
	2	11.50	11.59	11.14	11.61	12.96
	3	14.90	13.55	12.44	12.82	16.06
	4	17.40	17.95	16.77	17.12	19.82
	5	21.90	25.49	24.71	25.08	24.36
	6	37.10	34.20	36.12	44.70	36.91
span (5.9 m)	1	13.93	13.21	14.18	15.39	17.56
	2	15.22	14.40	14.43	15.30	17.08
	3	18.74	16.20	15.50	16.40	19.95
	4	21.42	20.21	19.22	20.03	22.91
	5	25.21	27.30	26.43	27.14	26.70
span (4.45 m)	1	17.44	20.72	22.24	26.51	29.99
	2	19.74	22.65	22.67	26.58	27.18
	3	27.07	25.80	26.31	27.45	30.57
	4	34.39	30.32	32.13	30.12	32.67
	5	39.75	36.33	41.69	35.25	34.44

Table 3.5c Comparison of Predicted vs. Test Natural Frequencies (Hz) (Hu, 1992) for S-F-S-F Floor with Static Masses (Span 5.9 m)

<i>Position(s) of Mass(es)</i>	<i>Mode No.</i>	<i>Test (Hu) 1992</i>	<i>This Study</i>	<i>Modal Synthesis (Hu, 1992)</i>	<i>Rayleigh-Ritz Model (Chui, 1987)</i>
<i>E₁, E₄, K₁, K₄</i>	1	10.63	10.82	11.05	11.46
	2	11.39	11.41	11.25	11.37
	3	18.08	16.59	15.92	16.37
	4	21.37	17.67	17.08	17.50
	5	26.51	20.42	22.87	23.29
<i>K₁, E₄</i>	1	11.89	12.32	12.63	12.90
	2	13.52	13.13	12.96	12.98
	3	20.36	16.62	15.96	16.38
	4	24.47	18.12	17.53	17.77
	5	26.46	23.72	23.93	23.74
<i>K₁</i>	1	12.67	12.68	12.89	12.98
	2	15.05	14.84	14.79	15.35
	3	19.17	16.65	15.95	16.38
	4	23.71	18.89	18.36	18.67
	5	26.43	25.14	18.67	25.85
<i>Centre of Floor</i>	1	9.59	9.89	9.85	10.02
	2	15.34	15.21	14.94	15.30
	3	20.34	15.52	15.32	15.99
	4	23.91	20.09	19.61	20.03
	5	26.22	23.75	23.71	24.17

4.1 General

Dynamic-based serviceability studies on end-use conditions such as the presence of dead or occupancy loads on wood floors have generally been restricted to static loading from imposed masses and for a specified disposition of these loads, ie. the location(s) of the loads are assumed fixed in space and are thus time invariant. This assumption may suffice for modelling loadings from furniture, fixed fixtures, etc, but may not provide for the spatial and temporal nature usually characterizing human activities. Due to their light weight, wood-joint floors may have their dynamic behaviour very influenced by the presence of imposed loads—in fact, since a zero situation hardly occurs in practice, any estimation of floor natural frequencies at design stage ought to be based on the loaded condition. This chapter thus aims, among other things, at providing the necessary formulation for the incorporation of dynamic mass contribution due to dead and other time-invariant loads in the estimation of natural frequencies. Gravity loads due to occupancy loads such as associated with humans are however not time invariant, and being also spatially distributed, certain modelling techniques such as described in Chapter 5 may be required for frequencies estimation.

This chapter also examines certain design features in light-wood frame construction and their possible effects on the vibrational performance of floors found in this type of construction. Previous research efforts have helped provide comprehensive documentation on some of the more common design features such as the rigidity and fastener spacing at the sheathing-joint boundaries, and the overall configuration of the floor. In addition, the influence due to the dimensions and mechanical properties of the principal components has equally been documented [eg. Filiatrault *et al.* (1990); Smith and Chui (1989); Chui (1987)]. In recognition

to the foregoing, in this study, the focus is mainly on three equally important features which have either previously been studied only in qualitative terms or have not received significant research attention. These features include (1) the nature of joists' end support conditions, (2) the presence of material discontinuities in the sheathing, and (3) between-joist bridgings.

4.2 Modelling Transverse Gravity Loads

Wood-joist floors are rarely without imposed loads and for that matter any serviceability evaluation criteria designed for these floors must consider a standard imposed gravity load acting in conjunction with the self-weight. The development in this section is concerned mainly with imposed loads with a fixed disposition defined for a specified time. Therefore, such loads are assumed to provide only additional dynamic masses.

In general, imposed dead loads such as static loading from furniture, partitions, and other fixed fixtures may be modelled either as concentrated point loads, line loads, or as localized distributed loads uniformly or nonuniformly distributed. In a more simplistic manner, the dynamic mass increase obtained from complete uniformly loaded floor may be incorporated in the general system dynamic mass matrix by modifying the density term for the subflooring. Other loading types may warrant additional computations from the point of view of system energy development.

Figure 4.1(a), 4(b) and 4(c) give, respectively, the diagrammatic representations of a line loading in the along-joists direction at $y = y_o$, concentrated point loads P_1 , P_2 , and P_3 , at respective positions (x_1, y_1) , (x_2, y_2) and (x_3, y_3) , and a localized distributed loading. For these loading types, we require to generate additional terms in the constrained mass matrix, $\mathbf{M}$. Notice that the general expression for the transverse displacement of the Lévi-type floor systems under consideration may be reformulated as

$$w(x, y) = \sum_m \sum_n F_n(y) \sin\left(\frac{m\pi x}{a}\right) \quad (4.1)$$

Fig. 4.1 Imposed Gravity Loads
 (a) *Line Load in along-joists direction*
 (b) *Concentrated Point Loads*
 (c) *Localized distributed loads*

Then, the additional kinetic energy due to a line loading of unit mass distribution $\bar{\rho}(x)$ in the x -axis at location $y = y_o$ may be written from first principles as

$$T_{ll} = \frac{1}{2} \sum_m \sum_{i^*} A_{mn} \left\{ \int_0^a \bar{\rho}(x) \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{i^*\pi x}{a}\right) dx \right\} \sum_n \sum_{j^*} F_n(y_o) F_{j^*}(y_o) \quad (4.2)$$

and that due to a concentrated load W_d acting at coordinate (x_d, y_d) as

$$T_{cl} = \frac{1}{2} \sum_m \sum_{i^*} \sum_n \sum_{j^*} A_{mn} A_{i^*j^*} \sin\left(\frac{m\pi x_d}{a}\right) \sin\left(\frac{i^*\pi x_d}{a}\right) F_n(y_d) F_{j^*}(y_d) \quad (4.3)$$

A localized distributed load having dynamic mass per unit area, $\bar{m}(x, y)$, and encompassing a total area A^* provides a contributory kinetic energy of

$$T_{dl} = \frac{1}{2} \sum_m \sum_{i^*} \sum_n \sum_{j^*} A_{mn} A_{i^*j^*} \int_{A^*} \bar{m}(x, y) \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{i^*\pi x}{a}\right) F_n(y) F_{j^*}(y) dA \quad (4.4)$$

Since Eqs. 4.2–4.4 introduce no additional system degree of freedom (dof) the resulting mass matrix may be incorporated directly into the system constrained mass matrix given by Eq. 3.11. We note also that the tabulated results given in Table 3.5 for acting concentrated loads case examined previously have been obtained using the formulation given by Eq. 4.3.

4.3 Effect of Joist End Support Conditions

Light-gauge metal connectors such as joist hangers have found increased use in the light wood-frame constructions. By this practice, the two ends of a typical joist in a floor assembly, are connected to fixed perimeter beams (or headers) by means of the metal connectors. Though this practice is on the increase, the appropriateness, or otherwise, of using the simply supported beam analogy in modelling this condition in a general dynamic analysis of the floor incorporating this feature has not been established. In reality, the vertical slip occurring at each end of the joist due to the presence of end connection must be accounted for and thus spring-supported beam analogy is seemingly much more applicable. Accordingly, Eq. 2.24 then becomes the choice for estimating the i th joist vertical displacement, w_i .

The results of sensitivity analysis obtained by varying only the value of the connection slip modulus K_s , are given in Table 4.1. We note that $K_s = \infty$ is an idealization of a simply supported condition. As evident from the results, a weaker joist end connection typified by a reduced K_s value, furnishes comparatively lower floor natural frequencies, particularly the first three frequencies. The results of tests designed to estimate the load-carrying capacity of *face-mount* joist hanger by (Berkoh, 1990) used 38×235 mm SPF No.2 joists with average moisture content 11%, and gauge 18 joist hanger (3.25 mm), the average K_s values 13016 N/mm and 21200 N/mm, respectively. These were obtained for full-scale testing using the ASTM D1761 method, and indicate that for the more common wood-frame construction, the simply supported analogy used for joist modelling is adequate. It is worthwhile noting that the prototype tests were carried out on 3600 mm span aimed ostensibly at accounting for possible joist end rotation effects. Computed K_s values were obtained at 0.4 mm slip. Further information concerning detailed tests procedure and manner of testing is fully reported in Berkoh (1990).

Table 4.1 Effect of Joist End fixity on Lower Six Natural Frequencies

Connection Slip Modulus, K_s (N/mm)	NATURAL FREQUENCIES (Hz)					
	Mode #1	Mode #2	Mode #3	Mode #4	Mode #5	Mode #6
1	18.22	20.25	26.35	37.26	52.15	69.99
1000	20.01	21.91	27.65	38.16	52.77	70.47
5000	21.85	23.62	29.01	39.13	53.44	70.89
10,000	22.12	23.86	29.21	39.27	53.54	70.96
50,000	22.37	24.10	29.40	39.41	53.63	71.02
100,000	22.41	24.14	29.43	39.43	53.65	71.03
∞	22.45	24.17	29.46	39.45	53.66	71.04

• Construction data: span = 3.66 m, width = 3.20 m; joist spacing = 400 mm, joist width = 38 mm, joist depth = 184 mm (S-P-F, mean MOE 10,700 MPa); sheathing is 15.5-mm Douglas fir plywood; connectors are ordinary round nails at 200 mm centres; no bridging; no nail withdrawal; floor simply supported on all sides.

Table 4.2 Effect of Material Discontinuities on Lower Eight Natural Frequencies

Width Span	† Gap Location	NATURAL FREQUENCIES (Hz)							
		Mode #1	Mode #2	Mode #3	Mode #4	Mode #5	Mode #6	Mode #7	Mode #8
1.14	No Gaps	36.62	37.68	41.25	48.85	60.63	76.48	94.52	125.70
	Midspan only	36.55	37.52	40.66	47.39	58.12	72.33	88.94	125.67
	Code Design	36.54	37.48	40.55	47.14	57.69	71.69	88.09	125.40
0.80	No Gaps	19.12	21.10	26.97	37.61	52.31	67.58	67.93	69.93
	Midspan only	19.07	20.72	25.68	34.84	47.68	63.29	67.56	67.86
	Code Design	19.01	20.72	25.68	34.84	47.68	63.29	67.56	67.86
0.72	No Gaps	15.79	18.09	24.64	35.94	51.11	55.70	56.28	58.54
	Midspan only	15.74	17.91	24.03	34.68	49.07	55.70	56.27	58.52
	Code Design	15.69	17.73	23.47	33.54	47.21	55.64	56.13	58.02
0.54	No Gaps	9.62	12.89	20.97	33.28	33.43	34.46	38.35	46.46
	Midspan only	9.58	12.71	20.43	32.41	33.28	34.45	38.33	46.44
	Code Design	9.51	12.41	19.60	30.85	33.20	34.25	37.59	44.61

• Width of floor = 3.2 m

† The number and gap locations based on Code design (CAN/CSA-086.1-M94) depend on floor panel type and overall floor span. For instance, for a floor of span 4.45 m built from 1200 × 2400 mm panels, gap locations are at distances 1.2, 2.4 and 3.6 m along the span.

4.4 Effect of Material Discontinuities in Sheathing

In general, the presence of material discontinuities, or gaps, in a vibrating floor introduces zones of reduced stiffness which, in theory, will cause it to exhibit greater amplitudes. However, for the floor partially loaded, or under normal service loads, full stress transfer between floor panels may not occur, therefore, any assignment of flexibility properties to these zones cannot be done *a priori*. As partial contact or gap closure may only take place at relatively high loads, previous studies [eg. Foschi (1985)] on the analysis of wood floor under static loading in the elastic range have considered all continuous gaps, such as gaps in the direction perpendicular to the joists, to exist as open gaps prior and after deformation. In this case, relevant portions of the floor sheathing in the neighbourhood of these joints are permanently free of stress, and in analysis, this may be accounted for by considering the actual gap dimension as being the distance between the adjoining fasteners. In wood-frame construction, gaps in the direction of the joists are usually staggered (Fig. 2.1), and the joists are also placed at the positions of these gaps. In this way, effective transfer of in-plane stresses across gaps, even at low load levels, can be realized and thus the effects of material discontinuities may be neglected in along-joists direction. More appropriate still, where flooring panels are tongued and grooved to form butt joints in the across-joist direction to facilitate the transfer of loads across panels and thus eliminate any differential deflection between adjoining panel edges, material discontinuities may be discarded in this direction as well.

The analysis tool discussed above is used in this thesis to study the influence of open gaps in across-joists direction on floor natural frequencies. Explicit development of the necessary theoretical formulations has been given in Chapter 2. For the purpose of this study, as stated above, butt joints such as obtained from tongued and grooved panels are not considered as open gaps, however, gaps formed from square-edged panels are.

Gap effect on floor dynamic characteristics such as the natural frequencies is

studied qualitatively using the results of a reference floor with material data given as notes in Table 4.1. The tabulated results in Table 4.2 are for various width/span ratios, and consideration of open gaps are either for gaps located at the midspan, or gaps resulting from the Canadian Code CSA-086.1-M94 design. It is evident from Table 4.2 that floor fundamental frequencies are not significantly affected by the presence of gaps in the flooring. On the other hand, marginal error may occur for the prediction of some frequencies other than the fundamental if material discontinuities effects are ignored entirely. For instance, in the results reported, as high as +9.7% error was incurred for the prediction of the fifth mode frequency for width/span ratio of 0.8. Notice, however, that in human perception evaluation for which only a few of the lower modes are of significance, the results indicate that modelling based on the assumption of 'no gaps' effects may not err appreciably.

4.5 Modelling the Behaviour of Between-Joists Strutting

Generally, joist bridgings function primarily to provide lateral stability and also to offset any distortion of the joists' spacing. In performing these functions, they provide supplementary stiffness in the across-joists direction. Two of the principal joist bridging types in current use are the wooden strap usually fastened to the underside of floor joists, and between-joist cross bridgings. A combination of both types are also common. Figure 4.2 details some of the more common bridging types.

In a floor assembly, a wooden strap acts primarily as a bending element which deforms to conform to the transverse deflection profile of the floor (Onysko 1988). The bending stiffness obtained this way, being of the same type as that associated with the subfloor sheathing, is assumed to add to the stiffness of the floor plate at the location of installation. Under loading, the strain energy stored in the strap is easily deducible from the modal function giving by either Eq. 2.4 or 2.7 depending on the nature of the floor support.

Between-joist strutting may be either solid wood blocking (perpendicularly or skewly installed between two adjacent joists), or cross bridging (herring-bone strut-

Figure 4.2: Representation of the More Common Bridging Types
(After Onysko 1988)

ting) type. Functionally, herring-bone struttings help create a laced structure with the sheathing acting as the top flange and some parts of the bridging, in compression, as the lacing bars (Ohlsson, 1988). Tests, however, show a minimal increase in stiffness with this technique. One reason advanced for this being that normal forces in the lacing bars transmitted to the joists by contact may be limited by the perpendicular to grain strength of the joist. Since the stiffness in this direction is rather small, the contact areas necessary for the transmission of these forces are correspondingly small, hence the rather low stiffness of this notional laced structure. Also, the poor performance of the cross bridging can be attributed to the

shrinkage of the timber pieces vis-a-vis the expected loading during the service life of the floor. On the other hand, properly-installed solid bridgings may result in an increase in the transverse stiffness, and can even outperform the efficiency of cross bridging. Theoretically, they perform this function by bringing about beam action in the transverse direction which presupposes the existence of rigid connections between the bridging and the adjoining structural elements to effect the transfer of the moment M_y across all joists. In reality, however, it is rather difficult to achieve complete rigidity more especially when bridging is installed in-situ in the field. In this case, the estimation of the supplementary K_y stiffness due to any beam action in the across-joists direction for any one floor, may be computed using equivalent material properties obtained as extrapolations from test results on floors with joist bridging (Onysko 1988). This approach may appear restrictive in its application as the calibrated equivalent material properties are as accurate as the selected floor analysis model.

Onysko (1988) modelled solid or cross bridging as shear beam elements with the beam properties extrapolated from results of actual floor tests (Onysko 1988). By this approach, the behaviour of both the cross bridging type and the skewly-installed blockings is modelled by a perpendicularly-installed solid rectangular strut whose stiffness properties are assigned based on the type of bridging. Thus, for a strut of equivalent length l_{eij} , cross-sectional area A_{eij} , the total strain energy contribution due to either herring-bone or solid wood strutting, U_{se} , may take the form

$$U_{se} = \sum_{i=1}^{NSB} \sum_{j=1}^{(NJ-1)} \mu G_{eij} A_{eij} \int_0^{l_{eij}} \left(\frac{dW_{sij}^*}{dx} \right)^2 dy \quad [4.1]$$

where, G_{eij} is the equivalent shear modulus of the ij th element; and NSB being the number of lines of joist bridging. As stated above, G_{eij} is not known *a priori* as the effectiveness of the bridging element depends not only on the manner of installation and fit, or on connectors' effectiveness, or more still, on the displacement-resisting mechanism of the element, but also on whether bridging is installed using *green* or *dry* wood (Onysko 1983).

Considering the deflection compatibility condition structurally imposed at the ends of the bridging element, the displacement, W_{sij}^* , for the Onysko's shear beam

may be defined as

$$W_{s_{ij}^*} = W_i^j + \left(\frac{W_i^{j+1} - W_i^j}{Y^{j+1} - Y^j} \right) s^* \quad [4.2]$$

where W_i^j at coordinate Y^j and W_i^{j+1} at coordinate Y^{j+1} are, respectively, the vertical deflections of the i th and $(i+1)$ th joists y -coordinate. s^* is a local coordinate with respect to the direction of the bridging element. One may note that unlike Onysko (1988), the above formulation introduces no additional system restraint.

Filiatrault *et al.* (1990) simulated the effect of joist bridging in their computer program NAFFAP by restricting the torsional deformation of the joists, ie. $\theta(x, t) = 0$. Olhsson (1988) also considered the effect of between-joists bridging by estimating the contributory K_y stiffness from beam action. Unlike Onysko (1988), complete rigidity was assumed for all connections formed by these beam elements whose properties and dimensions were the same bridging elements.

In this work, the effect of joist bridging is studied by considering simultaneously the corresponding increase in the K_y stiffness and the imposition of constraints introduced at the positions of these bridgings. Notice that corresponding increase in floor mass due to the joist bridging is also considered. The supplementary K_y stiffness is estimated using either Onysko (1988) approach whereby between-joists strappings are modelled as shear beams or from beam action by presupposing complete rigidity of connections formed by the bridging element. Since a typical between-joists element, m th element say, properly installed between the k th and $(k+1)$ th joists restricts the torsional deformation of these joists as well as influence their mode of bending about their respective minor axes, we therefore have the following constraint conditions depicting each of the two joists.

$$\begin{aligned} f_k^{(m)} &\equiv \theta_m(x, t) - \frac{\partial w(x, y, t)}{\partial y} = 0 \\ s_k^{(m)} &\equiv V_{J_m}(x, t) - v(x, y, t) + \frac{t_p}{2} \frac{\partial w(x, y, t)}{\partial y} - \frac{d_i}{2} \theta_m(x, t) = 0 \end{aligned} \quad m = 1, \dots, NSB_k \quad [4.3]$$

where NSB_k , being the number of bridgings installed between the k th joist and the $(k+1)$ th joist. We thus have the number of system constraint equations, s redefining

as

$$s = \sum_{i=1}^{n_j} wt_i + \sum_{k=1}^{n_j} [NSB_k + NSB_{k-1}] \quad [4.4]$$

where $NSB_0 = 0$. Equation 4.4 is formed on the implicit assumption that bridging elements having the same coordinate in the along-joists direction are of the same type and are thus counted as contributing to one element type in the relation $NSB_k + NSB_{k-1}$. Note that the above formulation particularly facilitates the analysis of floors with isolated bridgings.

4.5.1 Effect Due to Bridging on Natural frequencies

As pointed out above, between-joists strutting (bridging) when properly constructed in a typical light-frame wooden floor serves the dual purpose of raising the natural frequencies as well as increasing the spacing between adjacent frequencies. In a vibrating floor, therefore, amplitudes of response are correspondingly reduced which better enhance the acceptability of such floor judged in terms of human perception to unwanted floor vibrations. To study the effects that bridging has on floor response, two of the more common proposals existing in the literature are examined. These proposals include (1) methods that restrain only the cross-sectional rotation of the joists (eg. Filiatrault *et al.*, 1990), and (2) proposals that consider the contribution to the K_y stiffness as well as any constraint impositions (Onysko, 1988). It is worthwhile noting that the formulation of the constraints at the positions of the struttings as in Eq. 4.3 gives a more realistic situation of the constraint conditions introduced by the blockings, and thus a more enhanced form of that given by Filiatrault *et al.* (1990).

Table 4.3 gives the results of the first twelve natural frequencies for various lines of strutting based exclusively on the constrain conditions given in this work, and also on the data in Table 3 of Filiatrault *et al.* (1990). The results indicate the insensitivity of the proposal by Filiatrault *et al.* (1990) to variations in the number of rows of blocking. For example, using one or more lines of blocking, a restriction on the torsional deformation of the joists always furnishes the lower five frequencies as 22.49, 24.31, 29.97, 41.03, and 57.48 Hz, respectively. Also, in

Table 4.3 Effect of Joist Bridging on Lower Natural Frequencies
(Only Constraints at Bridging Positions Considered)

Mode #	NATURAL FREQUENCIES (Hz)				
	NB = 0		NB = 1		NB ≥ 2
	$k_{\theta} = 180$	$k_{\theta} = \infty$	$k_{\theta} = 180$	$k_{\theta} = \infty$	$k_{\theta} = 180$ or $k_{\theta} = \infty$
1	22.44	22.46	22.46	22.46	22.46
2	24.11	24.18	24.19	24.19	24.19
3	29.46	29.47	29.48	29.48	29.48
4	36.70	39.46	39.51	39.51	39.51
5	37.48	53.66	53.85	53.85	53.85
6	37.61	71.04	64.46	71.65	71.65
7	37.66	79.30	64.67	79.30	79.33
8	37.71	79.46	64.71	79.46	79.56
9	37.77	80.68	64.73	80.68	80.91
10	37.84	84.13	64.74	84.12	84.58
11	38.08	90.38	64.75	90.71	91.61
12	41.88	90.72	64.77	92.11	92.11

Notes:

1. NB = number of lines of *equivalent* solid wood strutting.
2. Construction and material data are same as in Table 3. of Filiatrault *et al.* (1990).
3. $k_{\theta} = \infty$ corresponds to the case of no withdrawal of sheathing-joists connectors.

the serviceability limit state dynamic performance evaluations of wooden floors where fastener withdrawal effect may be insignificant, the benefit of between-joists strutting assessed in accordance with only the enforcement of the constraints due to blocking elements may not be very apparent. Tabulated frequencies for the floor without bridging are not significantly different from even those obtained for floors with two or more lines of blockings. In reality, notwithstanding the value for the fastener rotational resistance, effecting only the constraints due to blocking may underestimate the lower frequencies of the floor even for well constructed blockings. Using an effective shear modulus of 12 MPa and cross-sectional area 7020 mm² for a herring-bone blocking installed and tested in the same condition, the assumption of a shear beam behaviour of a typical block element together with the constraints conditions associated the blockings in a floor structure with one line of blocking gives the frequencies 22.44, 26.86, 34.56, 45.62, and 59.85 Hz.

Table 4.4 compares natural frequencies obtained from dynamic testing of floors constructed under laboratory conditions (Chui and Hu, 1990) with those predicted

Table 4.4 Tests* vs. Predicted Lower Natural Frequencies (Chui & Hu, 1990)
(Consideration of between-joists bridging effect: Both supplemental D_y stiffness and all constraints at bridging positions)

Mode #	NATURAL FREQUENCIES (Hz)			
	Floor Type A† with $NB = 0$	Floor Type A with $NB = 1$	Floor Type B‡ with $NB = 0$	Floor Type B with $NB = 1$
1	13.9 (14.3)¶	13.5 (13.7)	13.1 (14.7)	13.5 (19.6)
2	15.2 (15.2)	14.8 (16.7)	18.3 (16.2)	23.3 (21.6)
3	18.7 (16.7)	21.0 (20.1)	24.1 (19.7)	29.6 (26.1)
6	29.2 (35.1)	40.1 (35.2)	39.2 (43.3)	49.6 (44.0)

* Floor span = 5.9 m; width = 3.413 m

¶ Predicted values in parentheses are by author's model. All connections between bridging and adjoining floor elements are assumed rigid.

† Floor has only one pair opposite edges supported.

‡ Floor is simply supported on all sides.

using author's unified model. For this comparison, the various block elements are assumed to interact completely with the sheathing in accordance with beam action in the across-joists direction and also all connections with the block elements are modelled as rigid. The predicted values are for a single line of between-joist blocking and have all been obtained based on the behaviour of blockings as providing a supplementary K_y stiffness as well as effecting constrain conditions defined by Eq. 4.3. Both the test and predicted frequencies variously substantiate the author's assertion that consideration of the constrain conditions (Eq. 4.3) alone may underestimate floor frequencies. Evidently, the assumption of complete rigidity of all connections with the blockings may also err for the predictions of certain frequencies. Limited though the data presented in Table 4.4, the floor with all edges simply supported (Floor Type B) is found to benefit substantially from joist bridging.

OCCUPANT-FLOOR SYSTEM VIBRATIONAL RESPONSE

5

5.1 General

Most current design criteria for assessing annoyance vibrations caused by occupant(s) activities in lightweight floors have been developed exclusively from test data, or through analytical studies which rarely recognize the dynamic characteristics of the occupant(s). Admittedly, neglecting the occupants dynamic parameters greatly facilitates analysis in that the resulting dynamic behaviour is without any coupling effect. However, for the typical lightweight floor such as a wood-joint floor in domestic dwellings for which the total mass of occupants could amount to as much as 30% of the system mass, a comprehensive analytical approach for the system dynamic analysis which considers jointly the mass, stiffness, and the damping properties of each occupant is seemingly much more appropriate. An equally significant factor is the forces induced on a floor due to occupant activity, eg. walking, the positional as well as time-dependent characteristics of which may further compound such dynamic analysis.

Essentially, this chapter attempts to present a numerical tool which could be used to predict real time-history response resulting from human footfall-induced excitations, and which may thus pave the way for the development of realistic annoyance criteria for lightweight floors such as wood-joint floors. The efficiency of the numerical tool is established using the results of previous experimental studies. The versatility of this numerical model facilitates the incorporation of certain input excitations caused by occupant activities such as running, jumping, and heel drop impacts. Availability of such a model will help provide the possibility of predicting behaviour during the design stage of a light floor subjected to any one of the aforementioned excitation types.

In developing the numerical tool, an examination is made of certain plausible mechanics of human footfalls which could be candidates for modelling actual footfall behaviour.

Fig. 5.1 Idealized Lumped Parameter Vibratory Human Models:
(a) 2-DOF ISO Model (b) 1-DOF Model
(c) 11-DOF Model (Modified Nigam & Malik (1987) 15-DOF Model)

5.2 Human Body as a Mechanical System

The human body is a complex mechanical system, and it is often modelled dynamically as a lumped parameter vibratory model consisting of an assemblage of discrete masses, linear springs, and viscous dashpots. Such a model may vary from a simple single-degree-of-freedom (1-DOF) model to a more complex fifteen-degree-of-freedom (15-DOF) model, the choice of which may be determined in terms of the effects of frequency and posture when the human body is subjected to certain form of vibration. Through transmissibility tests on test subjects in standing position,

resonance frequencies below 10 Hz have been found to contribute in large part to human discomfort. This work considers human models having dynamic parameters calibrated with respect to a human body in standing position responding to vertical excitations. Three such models are shown in Fig. 5.1 with the accompanying model parameters given in Table 5.1.

The two-degree-of-freedom (2-DOF or ISO) vibratory model (Fig. 5.1a), first proposed by Coermann (1962), is the standard for simulating a human body in the standing position adopted by the International Organization for Standardization (ISO 1989). As reported by Folz and Foschi (1991), this model was developed by correlating the model parameters to an experimental study of the mechanical impedance of a male human body. In Table 5.1, the two natural frequencies of this model are given as 5.03 Hz and 12.49 Hz.

A single-degree-of-freedom (1-DOF) oscillator (Fig. 5.1b) has been used by Foschi *et al.* (1995) to study the coupled oscillator-floor system dynamic behaviour when the combined system vibrates freely under a single heel drop impact. The dynamic parameters of this model were calibrated from tests results of heel drop impact produced by a 91-kg test subject. Foschi *et al.* (1995) are of the opinion that satisfactory response history may equally be obtained with 1-DOF model since the ISO model's second frequency does not significantly influence such response.

The 11-DOF vibratory human model (Fig. 5.1c) is derived from the parent 15-DOF spring mass system first proposed by Nigam and Malik (1987). According to Folz and Foschi (1991), the reduced 11-DOF model provides a single point attachment at the base which thus simplifies any analysis. Like the parent model, the 11-DOF model does not include any damping capability which limits its general usage.

In the development of the numerical tool, this work places a greater emphasis on the 2-DOF ISO model since it offers a more uniform standard against which the results of similar studies could be compared with those of this thesis. Again, the general recognition of the ISO vibratory model ensures that the numerical tool developed herein could be employed in other applications.

Table 5.1 Parameters for Oscillators Modelling a Human
(Foschi et al. 1995; Folz and Foschi 1991)

Model	Element	Mass (Kg)	Stiffness (N/mm)	Damping (N.s/mm)
2-DOF (ISO)	1	62.0	62.0	1.46
	2	13.0	80.0	0.93
1-DOF	1	91.0	40.0	1.35
11-DOF	1	9.286	1.9160	-
	2	15.650	0.1378	-
	3	12.590	0.2610	-
	4	16.550	0.5510	-
	5	9.105	0.5842	-
	6,8	2.322	0.1521	-
	7,9	1.910	0.0981	-
	10	0.207	1.3920	-
	11	3.044	0.6805	-

5.3 Characteristics of Human-Induced Footfall Excitations

Footfall forces induced on a floor due to normal human activities may arise from such activities as walking, running, jumping, dancing, jogging, and playing. For the purpose of this study, only vertically applied footfall forces are considered. Accordingly, this work restricts occupant activities to ones such as slow walking, normal walking, brisk walking, running, jogging, jumping, and single heel drop impacts. In later analysis, emphasis is placed on the activity due to a single person since this provides the basis for the development of vibrational-based serviceability criteria. Alternatively, where consideration is given to activities due to two or more persons, it is assumed that the respective excitations are coherent.

In general, depending on the type of occupant activity, the footfall forces may vary either temporally or spatially. Temporal varying forces are functionally time dependent whereas the spatially varying ones are uniquely defined with respect to the positions of the footfalls.

Fig. 5.2 Approximated Versus Measured Footfall Forces
(Mouring and Ellingwood, 1994)

5.3.1 Forces With Both Temporal and Spatial Variations

Forces due to human activities such as walking and running for which the activity may move from one position to another fall in this category. The functional representations of these forces with respect to time (temporal variation) are illustrated in Fig. 5.2 *a-d* which show the vertical component of foot impacts with time of a human. Typically, the pacing frequency due to an individual walking may range between about 1.5 Hz and 3.0 Hz, whereas frequencies above 3.2 Hz are representative of running or jogging. As in Fig. 5.2 *a-c*, a force function resulting from walking impact show two distinct peaks, the first corresponding to *heel strike* and the second *toe liftoff* contact.

A Fourier analysis of the force pulses (eg. Rainer and Pernica, 1986; Ellingwood and Tallin, 1984; Jacobs *et al.* 1972) shows the low frequency contributions as the

most significant with the first three to four harmonics being the essential ones. Jacobs *et al.* (1972) distinctly pointed out the dominance of the first and third harmonics of the pulses due to walking with the average relative magnitudes of the second and third harmonics being approximately 20% and 45% of the first harmonic, respectively, while the higher harmonics contribute only less than 10%. Rainer and Penica (1986), on the basis of above, proposed force functions representing the force pulses obtained from walking and running activities using Fourier series expansions. Such characterizations were based on test time records due to human footstep loading. Mouring (1992) used this procedure to obtain approximate curves to the actual forcing functions (Fig. 5.2). In general, therefore, the dynamic component of the walking or running force may be represented functionally with respect to time t as

$$f(t) = w_p \sum_{n=1}^N \alpha_n \sin(2n\pi ft + \phi_n) \quad [5.1]$$

where, w_p is the static weight of the walker or runner; α is the Fourier amplitude or dynamic load factor; n being the order of harmonic of the walking rate; f the rate of walking; ϕ the relative phase angle; and N the total number of harmonics involved. By measuring at a fixed point the induced forces due to continuous series of footsteps applied by a test subject traversing an experimental floor strip (Rainer and Penica 1986), the maximum dynamic load factors corresponding to the first four harmonics, of respective frequencies 2.4, 5.6, 5.8 and 7.6, for the case when the subject is walking were calibrated as 0.52, 0.24, 0.06 and 0.05. Notice that this procedure establishes a force-time history the direct use of it as input forcing function in the dynamic analysis of lightweight floors may not effectively model the coupled walker-floor system since overall system dynamic characteristics (mass, stiffness and damping) are assumed positional invariant.

The nature of occupant activity on the floor may greatly dictate the functional representation of the associated footfall forces. For instance, in Fig. 5.2, it is seen that changing occupant activity from walking to running results in a single peak (Fig. 5.2d) with the peak force running in the neighbourhood of 2.75 times the body weight. Generally, walking is distinct from running in that during walking the

person always has one foot in contact with the floor whereas during running, contact is lost during those periods that the runner is airborne. This distinguishing feature is used in the numerical model to introduce the coupling between the disturbing occupant and the entire floor dynamic system in the case of running.

The footfall forces due to one person walking from location a to b on the same floor is shown schematically in Fig. 5.3 with respect to both time and positions of footfall. Notice that the force pulses due to successive footsteps will usually overlap for some human activities.

Fig. 5.3 *Spatial distribution of forces on a floor due to one person walking from location a to b*

5.3.2 Forces with only Temporal Variation

The associated force pulses are generally assumed to have negligible spatial variation. Activities such as walking or running in place, dancing, jumping in place, and other rhythmic activities result in induced forces having this behaviour. Owing to the fact that these forces, in general, display similar characteristics, the actual functional representation of the force pulses due to any one of the above-mentioned activities is readily obtained by applying a factor α to an applicable

standard dynamic/static ($F(t)/F_s$) vs. time curve. Figure 5.4 provides two cases of such standard curves for $\alpha > 1$ and $\alpha \leq 1$.

The assumption of negligible positional variation in the walker footfall impacts have been used to estimate peak response displacement, velocity or acceleration (eg. Hu 1992; Ohlsson 1991). However, in the development of vibrational-based serviceability criteria using excitations from walking impacts, this approach may fail to provide actual response history with which any realistic human perception evaluation could be made.

Fig. 5.4 Forces caused by rhythmic activities in place (Ellingwood and Tallin, 1984)

5.3.3 Forces Due to Impact Activities

As noted by Ellingwood and Tallin (1984), the most distinct feature of the forces created due to impact activities is that their duration is usually short compared with the fundamental period of vibration of the floor system. Though, they create high impulsive loads, they are easily tolerated due to their shorter duration. Activities such as people jumping off furniture or ladders, or dropping heavy objects, heel drop impacts may generate such forces. Figure 5.5 shows typical forces generated

from heel drop, and the general force-time relations obtained from tests results of some more common impact events.

Notice that almost all studies aimed at producing annoyance-based criteria for lightweight floor assemblies have been based on the heel drop impact test. Such initial high impulsive force is thought to provide a basis for screening some potentially troublesome floors particularly at the design stage. However, forces of this nature damped out readily thus indicating that criteria based on the heel drop tests may only show stronger dependence of the acceptability of the floor on damping.

Fig. 5.5 Forces due to impact activities
(Ellingwood and Tallin, 1984)

5.4 Development of General Equations of Motion of Coupled Floor-Occupant(s) System under Induced Excitations

Since the principal objective of this work is towards the provision of a realistic means for evaluating floor systems under occupancy-induced loading—a tool very essential for the establishment of vibrational-based serviceability design criteria,

the numerical model in this thesis has been formulated in terms of the reaction of one or more passive (disturbed) occupants experiencing vibrations generated by a single disturbing occupant. The seeming unimportant case of one or more passive occupants reacting to excitations caused simultaneously by several disturbing occupants is best studied using random vibration theories. The discretization scheme adopted in this thesis ensures that at any time step in the time domain analysis all induced forces are considered at once—a procedure which does not require any spurious data storage especially in situations where the forces have different pacing frequencies. The implementation of this scheme, however, becomes rather unwieldy for incoherent excitations caused simultaneously by three or more occupants.

The basic principle governing the formulation of the system's equations of motion is the fact that a human occupant excites the floor and at the same time acts dynamically with it. The coupling behaviour of the above could be modelled in an independent manner since data representing the corresponding induced forces is readily obtained. However, to correct for the impacts to which the wooden floor is actually subjected in service, the analysis described subsequently models human interaction with a given floor system as an application of a prescribed forcing function acting in conjunction with a human dynamic model. Notice that the implementation of such a coupling behaviour will invariably be dictated by the occupant activity type. It must be remarked that, the analysis in which occupant interaction is directly implemented via the dynamic system of the human model may provide more realistic response histories. Such an analysis method was considered by Foschi et al. (1995) for heel drop impact studies. For other excitation types, it is indeed difficult to assign *a priori* realistic time-dependent relative velocities or displacements to the human oscillator. For that matter, the approach such as used in this work may provide an alternative means. Rigorous validation of this is given later in Chapter 6.

Though, the system dynamic characteristics derived in this Chapter are based on a walker exciting a floor and dynamically interacting with it, in Chapter 6, these characteristics are modified to account for excitation types associated with other occupancy loadings such as due to heel drop impact, running, jumping or running

in place.

5.4.1 Applicable Assumptions

In addition to the general assumptions given in section 2.1.3, this section discusses some pertinent assumptions used specifically in the development of the governing equations of motion for time history response analysis.

- *Damping of the wood-joist floor is viscous in nature, and is dependent on either the mass or stiffness, or a combination of the two.* The main source of energy dissipation in a non-yielding structural system such as the one under study is the basic energy-loss mechanism inherent in the system undergoing small vibrations. This phenomenon, however, is hardly fully understood thus in most applications, this is usually represented by an equivalent viscous damping for which damping coefficients can be estimated experimentally.
- *The selected dynamic model of the human has time-invariant parameters (Table 5.1).* In reality, these parameters are not constant with time and may depend to a large extent on how the body is excited. Human body rather adapts a posture by muscle tuning in reaction to the intensity of the motion (Foschi et al. (1995). At present, data, if any, does not abound for the time-dependent parameters characterizing the human dynamic model. In any case, the nonlinear incremental version of the solution scheme used in this work to solve the system's equations of motion could be employed to capture any time-dependent properties of the system dynamic parameters.
- *Equal footfall intensity is applied at each step location.* This assumption is not necessarily correct even for the case where the force intensity is measured with reference to a rigid base. For instance, the phenomenological heel strike and toe liftoff contact associated with normal walking may each differ in significance from one step to the other.

5.4.2 Equations of Motion

In considering the coupled floor-occupant interaction problem, three subsystems

are identified:

- *Subsystem 1* refers to the dynamic behaviour of occupant(s) at rest—*disturbed occupant(s)*. Such a passive occupant is assumed to influence the general system dynamics by virtue of the contributory dynamic characteristics (mass, stiffness, and damping).
- *Subsystem 2* identifies the occupant whose activities result in induced footfall force pulses with considerable variation in time as well as in position. This occupant, as well, interacts dynamically with the entire system.
- *Subsystem 3* refers to the dynamic behaviour of the wood-joint floor assembly.

Fig. 5.6 (a) 2 degree-of-freedom ISO human model
(b) Dynamic forces on mass, m_1
(c) Dynamic forces on mass, m_2
(d) External dynamic forces on floor plate at point of footstep

The equations of motion describing the above subsystems can be presented in the following:

5.4.2.1 Equations of Motion—Subsystem 1

Figure 5.6 *b* and *c* show the dynamic forces acting on a 2-DOF ISO human model (Fig. 5.5*a*). Considering one particular disposition of the occupants, the equations of motion describing the dynamic behaviour of $R1$ of such occupants located, respectively, at positions (x_{R_i}, y_{R_i}) , $i = 1, 2, \dots, R$ on the floor are

$$\begin{aligned} m_1^{(i)} \ddot{z}_1^{(i)} + c_1^{(i)} \dot{z}_1^{(i)} + k_1^{(i)} z_1^{(i)} &= -m_1^{(i)} \ddot{w}_{R_i} \\ m_2^{(i)} \ddot{z}_2^{(i)} + c_2^{(i)} \dot{z}_2^{(i)} + k_2^{(i)} z_2^{(i)} &= -m_2^{(i)} \ddot{w}_{R_i} \end{aligned} \quad i = 1, \dots, R1 \quad [5.2]$$

where, $\ddot{w}_{R_i}$ is the floor plate vertical acceleration at the location (x_{R_i}, y_{R_i}) of the i th occupant. By the principle of virtual work, we can express $\ddot{w}_{R_i}$ in terms of the generalized coordinates (dofs) of the floor, we thus have

$$\ddot{w}_{R_i} = \langle \mathbf{N}_i \rangle \ddot{\mathbf{D}} \quad [5.3]$$

in which $\langle \mathbf{N}_i \rangle$ is a $1 \times (g - s)$ row matrix of some specialized shape functions; $\mathbf{D}$ being the vector of the generalized coordinates. In constructing $\langle \mathbf{N}_i \rangle$, only terms associated with the floor sheathing transverse mode of deformation, and which uniquely define the position of the i th occupant is nonzero. In particular, for the rectangular-shaped floor simply supported at all edges, if M and N terms (Eq. 2.4) are used in the characterization of the displacement w , then in constructing $\langle \mathbf{N}_i \rangle$ only the first MN terms are nonzero; the k th nonzero term is of the form $\sin \frac{k\pi x_{R_i}}{a} \sin \frac{k\pi y_{R_i}}{b}$. Notice that Eq. 5.2 could be presented in the matrix form

$$\mathbf{m}_f \ddot{\mathbf{z}}_f + \mathbf{c}_f \dot{\mathbf{z}}_f + \mathbf{k}_f \mathbf{z}_f = -\mathbf{m}_f \mathbf{N} \ddot{\mathbf{D}} \quad [5.4]$$

where

$$\begin{aligned} [\mathbf{N}] &= \begin{bmatrix} \langle \mathbf{N}_1 \rangle \\ \langle \mathbf{N}_1 \rangle \\ \vdots \\ \langle \mathbf{N}_{R1} \rangle \\ \langle \mathbf{N}_{R1} \rangle \end{bmatrix} & [\mathbf{k}_f] &= \begin{bmatrix} k_1^{(1)} & & & \\ & k_2^{(1)} & & \\ & & \ddots & \\ & & & k_1^{(R1)} \\ & & & & k_2^{(R1)} \end{bmatrix} & \{\mathbf{z}\} &= \begin{Bmatrix} z_1^{(1)} \\ z_2^{(1)} \\ \vdots \\ z_1^{(R1)} \\ z_2^{(R1)} \end{Bmatrix} \end{aligned}$$

Notice also that the mass and damping matrices, $[\mathbf{m}_f]$ and $[\mathbf{c}_f]$, are all diagonal matrices, and are of the same form as $[\mathbf{k}_f]$.

5.4.2.2 Equations of Motion—Subsystem 2

The formation of the dynamic equations of *subsystem 2* necessitates some careful consideration. The occupant applies footstep force (due to walking) in a consistent manner and at the same time interact dynamically with the floor at the step location. Furthermore, it is assumed the positional variation in the footfall forces is along an arbitrary straight path on the floor. Thus, if the occupant walks across the floor taking a stride length Δ_s in time t_v , with the first footstep occurring at (x_1, y_1) , or reference to the line of travel, at s_1 , then the occupant's location at time t after the first step is given by

$$s_t = s_1 + \frac{\Delta_s}{t_v} t$$

and

$$\dot{s}_t = \frac{\Delta_s}{t_v} \quad [5.5]$$

In general, since the locations of the footsteps are time-dependent, we formulate the subsystem dynamic equation as

$$\mathbf{m}_v \ddot{\mathbf{z}}_v + \mathbf{c}_v \dot{\mathbf{z}}_v + \mathbf{k}_v \mathbf{z}_v = \mathbf{P}_v; \quad \mathbf{P}_v^T = \begin{bmatrix} P_v^{(1)} & P_v^{(2)} \end{bmatrix} \quad [5.6]$$

Notice that $\mathbf{m}_v$, $\mathbf{c}_v$ and $\mathbf{k}_v$ are all 2×2 diagonal matrices. If the direction of travel is assumed θ to the positive global X -axis (+ve counterclockwise), the components of the travel distance along the X and Y axes are $x = s \cos \theta$ and $y = s \sin \theta$, respectively. The vertical displacement of the floor at the walker location can be expressed as

$$w_v = \langle \mathbf{G}(x, y, t) \rangle \ddot{\mathbf{D}} \quad [5.7a]$$

and the time derivative of the displacement w_v at that location is then

$$\dot{w}_v = \frac{\partial \langle \mathbf{G} \rangle}{\partial x} \mathbf{D} \dot{x} + \frac{\partial \langle \mathbf{G} \rangle}{\partial y} \mathbf{D} \dot{y} + \langle \mathbf{G} \rangle \dot{\mathbf{D}} \quad [5.7b]$$

$$\begin{aligned} \ddot{w}_v = & \frac{\partial^2 \langle \mathbf{G} \rangle}{\partial x^2} \mathbf{D} \dot{x}^2 + \frac{\partial^2 \langle \mathbf{G} \rangle}{\partial y^2} \mathbf{D} \dot{y}^2 + 2 \dot{x} \dot{y} \frac{\partial^2 \langle \mathbf{G} \rangle}{\partial y \partial x} \mathbf{D} \\ & + 2 \frac{\partial \langle \mathbf{G} \rangle}{\partial x} \dot{\mathbf{D}} \dot{x} + 2 \frac{\partial \langle \mathbf{G} \rangle}{\partial y} \dot{\mathbf{D}} \dot{y} + \langle \mathbf{G} \rangle \ddot{\mathbf{D}} \end{aligned} \quad [5.7c]$$

Therefore, the elements of load vector $\{\mathbf{P}_v\}$ in Eq. 5.6 are

$$P_v^{(i)} = -m_i \left(\frac{\partial^2 \langle \mathbf{G} \rangle}{\partial x^2} \mathbf{D} \dot{x}^2 + \frac{\partial^2 \langle \mathbf{G} \rangle}{\partial y^2} \mathbf{D} \dot{y}^2 + 2 \frac{\partial^2 \langle \mathbf{G} \rangle}{\partial y \partial x} \mathbf{D} \dot{x} \dot{y} \right. \\ \left. + 2 \frac{\partial \langle \mathbf{G} \rangle}{\partial x} \dot{\mathbf{D}} \dot{x} + 2 \frac{\partial \langle \mathbf{G} \rangle}{\partial y} \dot{\mathbf{D}} \dot{y} + \langle \mathbf{G} \rangle \ddot{\mathbf{D}} \right) \quad i = 1, 2 \quad [5.8]$$

The generic row matrix, $\langle \mathbf{G} \rangle$, is formed on the same basis as $\langle \mathbf{N}_i \rangle$ in Eq. 5.3. However, $\langle \mathbf{G} \rangle$ is in addition position dependent—in fact the temporal variation in the footfall pulses is expressed through $\langle \mathbf{G} \rangle$. Notice further that since for a selected, or a specified force-time function, the characteristic forcing (pacing) frequency, $\frac{1}{t_v}$, is known, the location of each footstep is also known *a priori*, we can therefore evaluate the expressions in Eq. 5.8 at times when foot contacts are made with the floor.

5.4.2.3 Equations of Motion—Subsystem 3

In formulating the equations of motion of the floor, we seek to satisfy the compatibility and equilibrium conditions at the various locations of the occupants. Therefore, when all loads are converted to equivalent forces using the principle of virtual work, we write for subsystem 3

$$\hat{\mathbf{M}} \ddot{\mathbf{D}} + \hat{\mathbf{C}} \dot{\mathbf{D}} + \hat{\mathbf{K}} \mathbf{D} = \sum_{i=1}^{R1} \langle \mathbf{N}_i \rangle^T \left[k_{f1}^{(i)} z_{f1}^{(i)} + c_{f1}^{(i)} \dot{z}_{f1}^{(i)} + k_{f2}^{(i)} z_{f2}^{(i)} + c_{f2}^{(i)} \dot{z}_{f2}^{(i)} \right] \\ + \langle \mathbf{G}(x, y, t) \rangle^T \left[k_{v1} z_{v1} + c_{v1} \dot{z}_{v1} + k_{v2} z_{v2} + c_{v2} \dot{z}_{v2} \right] + \langle \mathbf{G}(x, y, t) \rangle P(t) \quad [5.9]$$

Overall, the general equations of motion of the coupled floor-occupant problem could be put in a matrix form

$$\mathbf{M}^* \ddot{\mathbf{S}} + \mathbf{C}^* \dot{\mathbf{S}} + \mathbf{K}^* \mathbf{S} = \mathbf{P}^* \quad [5.10a]$$

or, more specifically, as

$$\begin{bmatrix} \mathbf{m}_f & 0 & \mathbf{m}_f \mathbf{N} \\ 0 & \mathbf{m}_v & \mathbf{m}_v \mathbf{G} \\ 0 & 0 & \hat{\mathbf{M}} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{z}}_f \\ \ddot{\mathbf{z}}_v \\ \ddot{\mathbf{D}} \end{bmatrix} + \begin{bmatrix} \mathbf{c}_f & 0 & 0 \\ 0 & \mathbf{c}_v & 2\mathbf{m}_v \left(\dot{x} \frac{\partial \mathbf{G}}{\partial x} + \dot{y} \frac{\partial \mathbf{G}}{\partial y} \right) \\ -\mathbf{N}^T \mathbf{c}_f & -\mathbf{G}^T \mathbf{c}_v & \hat{\mathbf{C}} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{z}}_f \\ \dot{\mathbf{z}}_v \\ \dot{\mathbf{D}} \end{bmatrix} \\ + \begin{bmatrix} \mathbf{k}_f & 0 & 0 \\ 0 & \mathbf{k}_v & \mathbf{m}_v \left(\dot{x}^2 \frac{\partial^2 \mathbf{G}}{\partial x^2} + \dot{y}^2 \frac{\partial^2 \mathbf{G}}{\partial y^2} + 2\dot{x}\dot{y} \frac{\partial^2 \mathbf{G}}{\partial y \partial x} \right) \\ -\mathbf{N}^T \mathbf{k}_f & -\mathbf{G}^T \mathbf{k}_v & \hat{\mathbf{K}} \end{bmatrix} \begin{bmatrix} \mathbf{z}_f \\ \mathbf{z}_v \\ \mathbf{D} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \mathbf{G}^T \mathbf{P} \end{bmatrix} \quad [5.10b]$$

where for simplicity we define $\mathbf{G} = \langle \mathbf{G} \rangle$.

It is apparent from Eq. 5.10 that the matrices of the dynamic problem are neither banded nor symmetric, hence a special analysis technique is adopted here to (1) increase computational efficiency, (2) minimize storage requirement, and (3) to facilitate rapid convergence. It may suffice to note from Eq. 5.10, that the submatrices, $\mathbf{m}_f$, $\mathbf{m}_v$, $\hat{\mathbf{M}}$, $\mathbf{c}_f$, $\mathbf{c}_v$, $\hat{\mathbf{C}}$, $\mathbf{k}_f$, $\mathbf{k}_v$, and $\hat{\mathbf{K}}$ are all time invariant symmetric matrices. The coupling between the occupants and floor is realized through the time invariant submatrices $\mathbf{m}_f \mathbf{N}$, $\mathbf{N}_T \mathbf{c}_f$, and $\mathbf{N}_T \mathbf{k}_f$, as well as time and position dependent submatrices $\mathbf{m}_v \mathbf{G}$, $\mathbf{G}_T \mathbf{c}_f$, $\mathbf{G}_T \mathbf{k}_f$, $2\mathbf{m}_v \left(\dot{x} \frac{\partial \mathbf{G}}{\partial x} + \dot{y} \frac{\partial \mathbf{G}}{\partial y} \right)$, and $\mathbf{m}_v \left(\dot{x}^2 \frac{\partial^2 \mathbf{G}}{\partial x^2} + \dot{y}^2 \frac{\partial^2 \mathbf{G}}{\partial y^2} + 2\dot{x}\dot{y} \frac{\partial^2 \mathbf{G}}{\partial y \partial x} \right)$

5.5 Method of Solution

The procedure used in this work to perform the nonlinear equations of motion given by Eq. 5.10 is the direct or step-by-step integration procedure. A comprehensive outline of this procedure, which has been used by many researchers and engineers in diverse computational efforts pertaining to dynamic loading, has been reviewed independently by Newmark (1959), and Clough and Penzien (1975). A specific derivative of this procedure, commonly referred to as the Newmark's β method, is employed for the solution of Eq. 5.10. Notice that though the nonlinear nature of Eq. 5.10 may structurally warrant the use of the incremental version of the Newmark's direct integration scheme, as in Fig. 5.7, the discretization procedure used in implementing the force-time footfall forces in the solution algorithm ensures that in any one time step, the dynamic parameters of mass, damping and stiffness of the overall occupant(s)-floor system given by Eq. 5.10 are all constant. We therefore use the displacement vectors $\{\mathbf{z}\}$ and $\{\mathbf{D}\}$, or $\{\mathbf{S}\}$, as the basic variable tool for the analysis. Since the step-by-step numerical integration procedure is well documented, only brief summaries on the applicable method used in this study are given herein.

In the Newmark's direct integration scheme the following expressions are used

to relate velocities and displacements at time $t + \Delta t$ to their historic values

$$\begin{aligned}\dot{\mathbf{S}}_{t+\Delta t} &= \dot{\mathbf{S}} + [(1 - \epsilon)\ddot{\mathbf{S}}_t + \epsilon\ddot{\mathbf{S}}_{t+\Delta t}]\Delta t \\ \mathbf{S}_{t+\Delta t} &= \dot{\mathbf{S}}\Delta t + \left[\left(\frac{1}{2} - \nu\right)\ddot{\mathbf{S}}_t + \nu\ddot{\mathbf{S}}_{t+\Delta t}\right]\Delta t^2\end{aligned}$$

in which the parameters ϵ and ν are selected to obtain integration accuracy and stability. In this work we choose parameters $\epsilon = \frac{1}{2}$ and $\nu = \frac{1}{4}$ which correspond to the *Constant Average Acceleration method*. The accuracy and unconditional stability of this scheme is well documented in the literature. The method is based on the assumption that the accelerations at all points during any small interval of time, Δt , are constant, and are given as

$$\ddot{\mathbf{S}}(t) = \frac{1}{2}(\ddot{\mathbf{S}}_n + \ddot{\mathbf{S}}_{n+1}) \quad t_n < t < t_{n+1} \quad [5.11]$$

where $\{\mathbf{S}\} = [\{\mathbf{z}\}_f \{\mathbf{z}\}_v \{\mathbf{D}\}]^T$, and $\ddot{\mathbf{S}}_n$ and $\ddot{\mathbf{S}}_{n+1}$ are, respectively, the point accelerations at times t_n and t_{n+1} . Integration of Eq. 5.11 with respect to time yields expressions for velocities and displacements, which have the following values at the end of the interval:

$$\ddot{\mathbf{S}}_{n+1} = \frac{4}{\Delta t^2}(\mathbf{S}_{n+1} - \mathbf{S}_n - \Delta t \dot{\mathbf{S}}_n) - \ddot{\mathbf{S}}_n \quad [5.12]$$

$$\dot{\mathbf{S}}_{n+1} = \frac{2}{\Delta t}(\mathbf{S}_{n+1} - \mathbf{S}_n) - \dot{\mathbf{S}}_n \quad [5.13]$$

Upon substituting the above expressions for time t_{n+1} into Eq. 5.10 an alternative form of the equations of motion is obtained.

$$\begin{aligned}\left(\frac{4}{\Delta t^2}\mathbf{M}^* + \frac{2}{\Delta t}\mathbf{C}^* + \mathbf{K}^*\right)\ddot{\mathbf{S}}_{n+1} &= \mathbf{P}_{n+1} \\ &+ \mathbf{M}^*\left(\frac{4}{\Delta t^2}\mathbf{S}_n + \frac{4}{\Delta t}\dot{\mathbf{S}} + \ddot{\mathbf{S}}\right) + \mathbf{C}^*\left(\frac{2}{\Delta t}\mathbf{S}_n + \dot{\mathbf{S}}\right)\end{aligned} \quad [5.14]$$

We note from the above expression that since all terms on the right-hand side are known at time t_{n+1} , the point accelerations, $\ddot{\mathbf{S}}$, are readily solved. Eqs. 5.12 and 5.13 are then used to obtain the corresponding velocities and displacements, respectively.

As noted before, though, the matrices in Eq. 5.10 are all unsymmetric, the submatrices defining them are however symmetric, thus in writing the program

code, any advantage accruing from the bandedness of the submatrices are utilized to minimize the amount of storage requirement. It may be noted that though the solution strategy used in this work could well be combined with the modal superposition method to greatly reduce the overall system unknown variables (dof), and thus facilitate the solution of the governing equations of motions, this approach is not pursued in this work since from experience the size of matrices M^* , C^* and K^* required for accuracy may not be larger than 80×80 .

Fig. 5.7 Walker Loading Function

In conclusion, in implementing the solution algorithm, the following may be noted:

- (1) In the time domain analysis, $t = 0$ corresponds to the instant when the first foot contact is made. Similarly, $t = 0$ in the analysis involving heel drop impact is when the heel of the foot just makes contact with the floor.
- (2) All displacements are relative to equilibrium position under self weight, and initial displacements and velocities are zero at time $t = 0$. This is truly applicable

when the human-floor interaction is represented by a joint forcing function and either a concentrated mass of the human or an appropriate human oscillator. In any case, the initial accelerations is obtained from the equations of motion for which the shape of the forcing function is a determining factor.

- (3) As indicated in Fig 5.7, each footfall force-time curve is divided into 3 zones based on the occupant walking rate, stride length and the forward speed. Since a different small time interval may be specified for each zone, there would normally be N_1 , N_2 and N_1 integration steps for zone 1, 2, and 3, respectively. Recognizing the overlaps in the walking paces, at the i th foot contact, for instance, a total of $N_1 + N_2$ integration steps is performed with N_2 of these steps representing zone 2, and N_1 for zone 3 and zone 1 of the $(i + 1)$ th foot contact. Exception to this are the *first* foot contact when $2N_1 + N_2$ integration steps are made, and the *last* foot contact when $N_1 + N_2$ integration steps are performed for zones 2 and 3 only. Notice that this time discretization, considers any temporal variations in the foot forces. The spatial variation in the forces is also indirectly considered in that during the performance of the N_1 integration steps for zone 3 of the i th foot contact and zone 1 of the $(i + 1)$ th contact, the equivalent forces due to the two footsteps are computed at the respective coordinates of the foot contacts.
- (4) At the new time step, since all terms on the right hand side of Eq. 5.14 are known the acceleration vector $\ddot{\mathbf{S}}_{n+1}$ can be obtained, so are the velocity and displacement vectors $\dot{\mathbf{S}}_{n+1}$ and $\mathbf{S}_{n+1}$. In solving for $\ddot{\mathbf{S}}_{n+1}$, Gaussian-elimination procedure with partial pivoting is employed. The quantities $\ddot{\mathbf{S}}_{n+1}$, $\dot{\mathbf{S}}_{n+1}$ and $\mathbf{S}_{n+1}$ are then considered as $\ddot{\mathbf{S}}_n$, $\dot{\mathbf{S}}_n$ and $\mathbf{S}_n$ for the next step integration.
- (5) The incorporation of the walker dynamic characteristics may only be seen to damp out the ensuing floor response. As a result, occupant interaction with the floor comes into play only during the period of foot contact.

6.1 General

This chapter provides not only some results for verifying the accuracy and suitability of the response analysis extension of the numerical model to predict response histories, but also attempts to illustrate its unique applications. Since available test results on wood floors under walking-induced excitations are rather limited, the verification studies conducted in this chapter are based on experimental and theoretical studies on heel drop impacts for which comprehensive tests and analytical results are readily available. This chapter also presents the techniques by which the general equations of motion (Eq. 5.10), pertaining to walking-induced excitations, are used for applications such as heel drop impact, running, jogging, and *in-place* walking or running.

6.2 Model Verification: Heel Drop Impact

An important step in the verification process is to assess the accuracy and convergence characteristics of the proposed numerical model for dynamic response predictions. Specifically, the accuracy study seeks to examine the response prediction capability of the model proposed in this study by comparing its predictions with those given in the literature. The convergence study also aims at verifying the implementation of the constant-average integration scheme adopted here to solve the equations of motion of the combined human oscillator-floor system under human-induced excitations. In this regard various predicted response histories (displacements, velocity or acceleration), obtained as a result of variations in the total number of the system's parameters (or dofs), are compared. Material data from two experimental floors are used as input to the computer program which is a codification of the numerical model presented herein. Floors analyzed are all

rectangular-shaped, and consist of an assemblage of plywood/OSB panels stiffened spanwise by either the traditional solid wood joists or composite wood I-joists.

6.2.1 Equations of motion: Heel Drop Impact

Vibration of this nature in a given floor system is initiated by a human occupant who, in a raised position, suddenly drops his/her weight via the heels on the floor and, at the same time, acts dynamically with the floor. As mentioned before, the most appropriate manner to incorporate this in a numerical model is to assign a consistent initial velocity to a human oscillator situated at the point of impact. An alternative representation, which is an approach investigated later in this work, is to employ a standard forcing function representing heel impact drop in addition to a damper consisting of either a human oscillator or an acting mass of the occupant. Therefore, for a floor with R human oscillators, the applicable equations of motion assuming the heel drop activity due to the active occupant is modelled by a forcing function and 2-DOF human oscillator (damper), are of the form

$$\begin{aligned} \begin{bmatrix} \mathbf{m}_f & \mathbf{m}_f \mathbf{N} \\ \mathbf{0} & \hat{\mathbf{M}} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{z}}_f \\ \ddot{\mathbf{D}} \end{bmatrix} + \begin{bmatrix} \mathbf{c}_f & \mathbf{0} \\ -\mathbf{N}^T \mathbf{c}_f & \hat{\mathbf{C}} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{z}}_f \\ \dot{\mathbf{D}} \end{bmatrix} \\ + \begin{bmatrix} \mathbf{k}_f & \mathbf{0} \\ -\mathbf{N}^T \mathbf{k}_f & \hat{\mathbf{K}} \end{bmatrix} \begin{bmatrix} \mathbf{z}_f \\ \mathbf{D} \end{bmatrix} = - \sum_{i=1}^R \delta(x - x_i) \delta(y - y_i) \langle \mathbf{N}_i \rangle^T P_i(t) \end{aligned} \quad [6.1]$$

in which (x_i, y_i) is the position of the i th occupant, and $\delta(\cdot)$ is the Dirac delta function. Note that for the purpose of this study, the total of R occupants is made up of one *active* occupant and $R - 1$ *passive* occupants. The right-hand side of Eq. 6.1 accordingly becomes

$$\mathbf{P}_o^* = \begin{Bmatrix} \mathbf{0} \\ -\mathbf{N}_o^T P_o(t) \end{Bmatrix} \quad [6.2a]$$

On the other hand, where the heel drop action of the *active* occupant is introduced via an assumed initial velocity of the requisite human oscillator, we have

$$\mathbf{P}_o^* = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \quad [6.2b]$$

in which this velocity is given as $\dot{\mathbf{z}}_{f_k} = \sqrt{2gh}$; $k = 1, 2$ with h defining the height in which the heel is initially raised. Note again that $\mathbf{N}_o$ is evaluated at the position

(x_o, y_o) of the impact. It may also suffice to note that for the case where the above situation is modelled as an acting forcing function only Eq. 6.1 assumes the linear form

$$\hat{\mathbf{M}}\ddot{\mathbf{D}} + \hat{\mathbf{C}}\dot{\mathbf{D}} + \hat{\mathbf{K}}\mathbf{D} = -\mathbf{N}_o^T P_o \quad [6.3]$$

The dynamic quantities describing the human oscillators, $\mathbf{m}_f$, $\mathbf{c}_f$ and $\mathbf{k}_f$ are all diagonal matrices each of R elements.

6.2.2 Convergence Characteristics

The convergence features of the numerical model related to the response analysis is studied by examining the predicted displacement, velocity and acceleration responses for various terms used in the modal expansions to describe the displacement fields for the plate and joists. To be more accurate still, greater emphasis is placed on the number of terms M , N and wt required for the characterization of the dominant vertical displacement fields.

The wood-joint floor used for this study is the 38×184 mm S-P-F floor the material and construction data of which are given in Section 6.2.3. For the purpose of this study, we assume floor damping to be viscous in nature with a 4% damping ratio for the first mode of vibration. Notice that this value has been selected based on the results of experimental work by Foschi *et al.* (1995) on a similar floor configuration. Overall, the floor damping matrix is obtained using stiffness proportional damping.

Computed responses at floor position (0.775, 1.600) m are due to an impulsive force function obtained by dropping a 6.8-kg sand bag from a height of 27 mm. After the drop the sand bag remains on top of the floor at coordinates (1.500, 1.600) m. Since the influence of the low mass sand bag on the dynamic response of the system is small (Foschi *et al.*, 1995), the computed responses are easily obtained using Eq. 6.3. It is worthwhile pointing out that, since, in respect of the selected integration scheme (Newmark's β method: $\epsilon = 0.25$; $\nu = 0.5$), the overriding objective for examining the convergence behaviour is to study the effects of the number of substructure modes selected in the modal expansion, we elect to minimize any ef-

fect on the computed responses due to the size of the time step by choosing a relatively small time step value—a preliminary study in this work led to a choice of $\Delta t = 0.002$ s.

Figures 6.1, 6.2 and 6.3 show the computed responses using a function with M terms for the numerical model. For simplicity and uniformity, these responses are calculated using $N = 10$ with ut , vt , wt , and θ_t each equal to M . One may note the rapid convergence of the solution procedure. The results thus indicate that for problems of this nature, accurate results could be obtained for a relatively small value of M ($M = 3$ in this case). To obtain the optimum number of N terms required in the modal expansion, we calculate as function of N the displacement, velocity and acceleration root-mean squares of the responses. Table 6.1 gives the results for such analysis. It is seen that sufficient accuracy could be obtained for response computations using only $N = 10$ terms. Therefore, in subsequent calculations the terms employed in the modal expansions are as follows: $M = 3$, $N = 10$, $ut = 3$, $vt = 3$, $wt = 3$ and $\theta_t = 3$.

Table 6.1 Effect of the number of across-joist modes N on computed response parameters

N	$M = 1$			$M = 3$		
	w_{rms} (mm)	$\dot{w}_{rms}$ m/s	$\ddot{w}_{rms}$ m/s ²	w_{rms} (mm)	$\dot{w}_{rms}$ m/s	$\ddot{w}_{rms}$ m/s ²
3	0.0893	0.0133	2.4760	0.0883	0.0133	2.4809
5	0.0946	0.0133	2.4785	0.0863	0.0135	2.6057
7	0.0961	0.0133	2.4934	0.0851	0.0135	2.6954
9	0.0981	0.0134	2.4933	0.0842	0.0135	2.7175
10	0.0985	0.0134	2.4942	0.0837	0.0135	2.7408
11	0.0986	0.0134	2.4947	0.0836	0.0135	2.7356
14	0.0988	0.0134	2.4983	0.0837	0.0135	2.7206
15	0.0990	0.0134	2.5077	0.0831	0.0136	2.7821
19	0.0995	0.0134	2.4978	0.0841	0.0136	2.7668

- Notes:- $w_{rms} = \sqrt{\left[\frac{1}{K} \sum_{i=1}^K w^2(x, y, t_i)\right]}$; $\dot{w}_{rms} = \sqrt{\left[\frac{1}{K} \sum_{i=1}^K \dot{w}^2(x, y, t_i)\right]}$;
 $\ddot{w}_{rms} = \sqrt{\left[\frac{1}{K} \sum_{i=1}^K \ddot{w}^2(x, y, t_i)\right]}$; K = number of equal time steps used in the time domain analysis; $w(x, y, t_i)$, $\dot{w}(x, y, t_i)$ and $\ddot{w}(x, y, t_i)$, respectively, are the displacement, velocity and acceleration at the end of the i th time step.

6.2.3 Accuracy

The accuracy study seeks to compare and contrast the characteristics of the computed responses with those obtained using other numerical models. Often, a good match between standard/experimental values and predicted ones may provide an indicator denoting good accuracy of response. For excitation due to heel drop impact, floor response characteristics considered include: frequency, rate of decay and peak value.

The material and construction data from two rectangular plan wood floor types constructed in accordance with guidelines from the National Building Code of Canada are used for this study. The floors are all simply supported on all four sides, each having a 18.5 mm plywood cover fastened to the joists by 8d common nails spaced at 300 mm on centres. One floor, of span 3.1 m, is 3.6 m wide and has joists consisting of 38×184 mm No. 2 S-P-F sawn lumber. The other has composite wood-I joists with Oriented Strand Board (OSB) webs and 2100f-1.8E machine-stress rated (MSR) SPF 38×89 mm flanges. The latter floor spans 4.2 m in the direction of the joists and 4.8 m across. The wood I-joists are spaced at 600 mm on centres (depth of each joist is 241 mm), while the 38×184 mm S-P-F joists are spaced 400 mm centre to centre. Response decay observations from hammer impacts (Foschi *et al.* 1995) estimated the viscous damping ratio of the floor with S-P-F joists as 4%, and that of the wood I-joist floor as 2.5%. Other construction and material data for these floors are given in Foschi *et al.* (1995).

The required input excitation is due to a lone occupant who applies a heel impact impulse to the floor system. Predicted responses are obtained at position (1.550, 2.325)m for the SPF floor, and (2.140, 3.150)m in the case of the I-Joist floor. With the simplicity of the input excitation, the coupled floor-occupant problem is solved by assuming the oscillator at height $h = 27$ mm comes into contact with the floor surface with an initial velocity of its mass given as $\sqrt{2gh}$.

Figures 6.4 through 6.9 compare the predicted responses from the present numerical model with those obtained by the computer program **DYFAP** (*DY*namical *F*loor *A*pproximation *P*rogram).

Floor Analysis Program). In these predictions, parameters M and N which define the number of modal expansion terms depicting the floor sheathing vibratory modes are assigned $M = 3$ and $N = 10$, respectively. In the same way, for terms defining joists' modes of vibration we have $ut = 3$, $vt = 3$, $wt = 3$, and $\theta t = 3$. Furthermore, in order to minimize the influence on the predicted responses of the choice of the time step Δt , we set the latter as $\Delta t = 0.002$ s. One may note from the predicted responses that despite the methodological differences between the model in this thesis and **DYFAP**, there is a good agreement of the responses in terms of frequency (peak-to-peak interval), peak response and the overall decay of the vibrations.

6.3 Modelling Human-Floor Dynamic Interaction

In the analysis of the lightweight floor subject to occupant(s)-induced excitations, one fundamental modelling difficulty often encountered is the consideration of the occupant(s) dynamic characteristics in coupled situation with those of the floor. Whereas in heavier floors, the resulting coupled human-floor interaction may only be governed by the dynamic parameters of the floor alone, the same cannot be said of the light floor system. Therefore, for any human activity type, standard or applied forcing functions usually obtained from heavier floor experimentations, when used directly in the light floor environment, may not adequately reflect the actual occupant(s)-floor dynamic interaction. To circumvent this difficulty, a more appropriate procedure involves analytical simulation of the occupant's activity such as that used in most heel-drop impact studies. In such studies, the human dynamic oscillator model is assigned initial velocities corresponding to the heel fall. Though, this measure could easily be employed in lightweight floor applications, it cannot be applied in the general sense because human activity such as walking cannot be easily modelled by this means.

The approach used in this work to model occupant activity in a coupled floor-occupant(s) system is through the use of a calibrated forcing function, applied in conjunction with either a human oscillator or an acting mass representing the

occupant. This approach recognizes the fact that, for a given floor, the standard or calibrated forcing function rarely reflects the true coupling behaviour which would normally exist between the floor and the interacting occupant(s). The additional human oscillator (or mass) therefore adjusts this forcing function to actual service conditions.

To establish the feasibility of the implementation of human dynamic interaction with light floors as used in this work, a comparison is made of predicted responses (displacements, velocities, and acceleration) using (1) a 2-DOF ISO human oscillator with assigned initial velocities, (2) a heel drop forcing function with 2-DOF ISO human oscillator (damper) at the position of impact, and (3) a heel drop forcing function and 75 kg mass at the position of impact representing the mass of the impacter. The above methods for simulating a heel drop impact in a coupled dynamical system are henceforth referred to as *Method 1*, *Method 2* and *Method 3*, respectively. Notice that the procedure involved in *Method 1* is the basis of the computer program **DYFAP** developed by Folz and Foschi (1991), and which was later used by Foschi *et al.* (1995).

Displacement time-histories obtained for the above input excitations are either referenced to the 38 × 184 mm SPF floor, or to the composite wood I-Joist floor described previously. To account also for the sensitivity of the response time histories to the form of the proportional damping matrix $[\hat{\mathbf{C}}]$, and thus to facilitate the interpretation of such responses, the analytical predictions have been based on the following:

$$[\hat{\mathbf{C}}] = \alpha[\hat{\mathbf{M}}]$$

$$[\hat{\mathbf{C}}] = \beta[\hat{\mathbf{K}}]$$

$$[\hat{\mathbf{C}}] = \alpha[\hat{\mathbf{M}}] + \beta[\hat{\mathbf{K}}]$$

Notice that the parameters α and β for each form of the damping matrix are determined as in the Appendix.

Figures 6.10, 6.11 and 6.12 compare the responses associated with the above viscous damping matrices obtained using data from the SPF floor. These response time histories have been obtained for an excitation applied at the midfloor position and responses measured at (2.325, 1.600) m in the case of the SPF floor, and

(3.150, 2.400) m for the wood I-joist floor. A time step of 0.002 s has been used in the response analysis. Predictions were determined for input excitation given by *Method 1*, *Method 2* and *Method 3*, respectively. It is evident from these results that predicted displacement and velocity time histories show little or no sensitivity to the form of the damping matrix used in the analysis. Damping sensitivity only comes into play in the case of acceleration time histories prediction. Notice, however, that damping matrices $[\hat{C}] = \alpha[\hat{M}] + \beta[\hat{K}]$ and $[\hat{C}] = \beta[\hat{K}]$ give practically the same results. Subsequent comparative studies are either based on $[\hat{C}] = \alpha[\hat{M}] + \beta[\hat{K}]$ or $[\hat{C}] = \alpha[\hat{M}]$ only.

In another development, Figs. 6.13 through 6.17 compare the time histories obtained for the above three excitation types, using the data on both the 38 × 184 mm SPF and the I-joist floors. Assuming the method of assigning initial velocities to the 2-DOF human oscillator (*Method 1*) provides the most realistic results, it is apparent from Fig. 6.10 that the displacement response by *Method 2* is overall better than by *Method 3*. The low frequency of the *true* displacement response is seen to be dominated by the natural frequencies of the human oscillator—particularly the 5.03 Hz fundamental frequency. The displacement responses associated with *Method 2* or *Method 3* are, however, dominated by the lower frequencies of the floor.

Of all the predicted response histories (Fig. 6.13, 6.16 and 6.17), the velocity responses show the best correlation between *Method 1* and *Method 2*. The decay rate and frequencies of the associated responses are better matched, with the responses now heavily dominated by the frequencies of the floor. From the results of both the two floor types, a good agreement in terms rate of decay and frequency is also found to exist between acceleration responses by *Method 1* and *Method 2* when the viscous damping matrix is of the form $[\hat{C}] = \alpha[\hat{M}] + \beta[\hat{K}]$. It must be remarked, however, that the form of the damping matrix do not appreciably affect the frequency or the peak-to-peak interval of the acceleration responses—in fact only the amplitudes of the response are affected. Notice also that the response histories by *Method 3* are modified forms of the responses given by heel drop forcing function without *modifiers*. We thus see a poor agreement in the predictions given

by *Method 3* in comparison with those by *Method 1*. The errors involved in the prediction of the response amplitudes by each the 3 methods may be attributed to the intrinsic difficulty in assigning realistic stiffness and damping to the human oscillator since these parameters are realistically time-dependent, an effect which may more influence the results given by *Method 1* than by *Method 2* or *Method 3*. For instance, in the comparatively heavier floor systems where either *Method 1* or the use of forcing function alone may provide acceptable results, predicted initial acceleration peak amplitude by *Method 1* may often be as high as 8 times that of the latter (Folz and Foschi (1991)). The results given by Fig. 6.14 through 6.17 reinforces this fact even for the light floor systems analyzed herein.

The results given above indicate that *Method 2* may help provide the basis for incorporating floor-occupant interaction in a numerical tool usable for the development of vibrational-based serviceability criteria with either velocity or acceleration as principal control parameter. Also, the reliability of *Method 2* in the prediction of the occurrence as well as the amplitude of the peak displacement indicates that for studies involving floors under multiple impacts such as those produced during walking activity, the method may facilitate the introduction of coupled floor-occupant(s) dynamic interaction. Though, the results of the heel drop single impact study indicate the suitability of *Method 2* over *Method 3*, the two methods are further considered in subsequent studies involving multiple impacts or repetitive loadings.

6.4 Response of lightweight floors to walking or running impacts

This section considers a further justification of the numerical model developed in this work to predict the response histories of lightweight floors under multiple impacts having both positional and time variations. Throughout this section, it is assumed that the requisite input excitation is produced by a single occupant, and the responses computed at the position of a passive occupant who also interacts dynamically with the floor system.

As stated in section 5.3.1, walking may be distinguished from running in that

in the latter only one foot is in contact with the floor at a time. Whereas walking may be characterized by the heel and toe contact peaks and also by the overlapping of succeeding paces, running activity creates discrete force-time shapes each with a distinct single peak value. Therefore, the forces associated with running are simulated analytically, without undue loss in accuracy, by combining duplicates of the forces due to a single step separated by time delays of the running rate. The same procedure is also used for simulating forces due to walking but the difficulty in this application involves having to deal with the overlaps in the footstep forces. Rainer and Pernica (1986) sought to avoid this difficulty by measuring at a fixed location the dynamic forces produced from walking or running steps—the result of this work is the basis for Eq. 5.1. Such a forcing function has a limited application in that the positional and time variations in the induced footfall forces considered for any one floor may not be adequately simulated. On the other hand, the method of solution given in section 5.5 for solving Eq. 5.10 involves duplicating forces due to a single footstep. This ensures that in the time domain of the analysis, the coupled action of the walker/runner-floor as well as the positional and time variations in the footstep forces are considered simultaneously.

Equation 5.10 has been formulated for multiple impact applications such as walking or running; the walker/runner-floor interaction is simulated by applying an appropriate forcing function acting in conjunction with a 2-DOF ISO human oscillator. When such an interaction is represented by a forcing function and a concentrated mass of the occupant, however, the applicable equations of motion are obtained as follows:

The inertia force, P_v , associated with the moving human of mass m based on Newton's second law is expressed as

$$P_v = -m \left(\frac{\partial^2 \langle G \rangle}{\partial x^2} \mathbf{D}\dot{x}^2 + \frac{\partial^2 \langle G \rangle}{\partial y^2} \mathbf{D}\dot{y}^2 + 2 \frac{\partial^2 \langle G \rangle}{\partial y \partial x} \mathbf{D}\dot{x}\dot{y} + 2 \frac{\partial G}{\partial x} \mathbf{D}\dot{x} + 2 \frac{\partial \langle G \rangle}{\partial y} \mathbf{D}\dot{y} + \langle G \rangle \mathbf{D}\ddot{} \right) \quad [6.4]$$

and the equations of motion of the floor may then be given as

$$\hat{\mathbf{M}}\ddot{\mathbf{D}} + \hat{\mathbf{C}}\dot{\mathbf{D}} + \hat{\mathbf{K}}\mathbf{D} = (k_{f_1}z_{f_1} + c_{f_1}\dot{z}_{f_1} + k_{f_2}z_{f_2} + c_{f_2}\dot{z}_{f_2})\langle \mathbf{N}(x_o, y_o, t) \rangle^T$$

$$+ \langle \mathbf{G}(x, y, t) \rangle P_v \quad [6.5]$$

in which (x_o, y_o) is the location of the *disturbed* occupant. As in Eq. 5.10, if we define $\mathbf{G} = \langle \mathbf{G}(x, y, t) \rangle$ the equations of motion of the coupled occupant(s)-floor system take the form

$$\begin{bmatrix} \mathbf{m}_f & \mathbf{m}_f \mathbf{N} \\ 0 & m \mathbf{G}^T \mathbf{G} + \hat{\mathbf{M}} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{z}}_f \\ \ddot{\mathbf{D}} \end{Bmatrix} + \begin{bmatrix} \mathbf{c}_f & 0 \\ -\mathbf{N}^T \mathbf{c}_f & 2m \mathbf{G}^T \left(\dot{x} \frac{\partial \mathbf{G}}{\partial x} + \dot{y} \frac{\partial \mathbf{G}}{\partial y} \right) + \hat{\mathbf{C}} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{z}}_f \\ \dot{\mathbf{D}} \end{Bmatrix} \\ + \begin{bmatrix} \mathbf{k}_f & 0 \\ -\mathbf{N}^T \mathbf{k}_f & m \mathbf{G}^T \left(\dot{x}^2 \frac{\partial^2 \mathbf{G}}{\partial x^2} + \dot{y}^2 \frac{\partial^2 \mathbf{G}}{\partial y^2} + 2\dot{x}\dot{y} \frac{\partial^2 \mathbf{G}}{\partial y \partial x} \right) + \hat{\mathbf{K}} \end{bmatrix} \begin{Bmatrix} \mathbf{z}_f \\ \mathbf{D} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \mathbf{G}^T \mathbf{P} \end{Bmatrix} \quad [6.6]$$

In solving Eq. 6.6, the time dependent row matrix $\langle \mathbf{G} \rangle$ is evaluated using the coordinates defining the foot contacts. For a coupled runner-floor dynamic problem, the associated force envelopes do not overlap, as a result in the time domain analysis, the centre of mass of the runner can always be assumed to coincide with the location of the foot contact, thus $\langle \mathbf{G} \rangle$ is always estimated with good accuracy. In the case of coupled walker-floor problem, the period during which two successive force envelopes overlap presents a challenge! This is so in that the location of the centre of mass of the moving occupant during this period is not known *a priori*—the locus of the mass centre being a straight line joining the locations of the n th and $(n+1)$ th steps, say. Preliminary investigation using as the centre of mass: (1) coordinates at the n th step; (2) coordinates at the $(n+1)$ th step and (3) coordinates at midpoint of line joining n th and $(n+1)$ th steps, however, indicated the insensitivity of the predicted response histories to the choice of the mass centre location.

One may note further that Eq. 5.10 in the analysis involving running activity is only applicable when the runner is in contact with the floor; during the time that the runner is airborne the pertinent equations of motion may be given simply as

$$\begin{bmatrix} \mathbf{m}_f & \mathbf{m}_f \mathbf{N} \\ 0 & \hat{\mathbf{M}} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{z}}_f \\ \ddot{\mathbf{D}} \end{Bmatrix} + \begin{bmatrix} \mathbf{c}_f & 0 \\ -\mathbf{N}^T \mathbf{c}_f & \hat{\mathbf{C}} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{z}}_f \\ \dot{\mathbf{D}} \end{Bmatrix} \\ + \begin{bmatrix} \mathbf{k}_f & 0 \\ -\mathbf{N}^T \mathbf{k}_f & \hat{\mathbf{K}} \end{bmatrix} \begin{Bmatrix} \mathbf{z}_f \\ \mathbf{D} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad [6.7]$$

6.4.1 Influence of occupant presence on floor dynamic response

As a justification of the appropriateness of incorporating the coupling dynamic

characteristics of the floor and its occupants in the response analysis, we investigate the influence that the physical presence of two 75-kg occupants have on the responses of two typical floor types—one *heavy* floor, and one *light* floor. The heavier floor system is an 8-m long-span composite floor system consisting of concrete slab (subfloor) and steel I-beams as the main joists with the connections between the slab and the beams realized by means of shear connectors (steel studs). The construction data for this floor has been taken from Folz and Foschi (1991). The light floor is a wood-I joist floor having plywood sheathing subfloor and composite wood I-joists as joists in the direction of the span. Unlike the heavier floor, connections between the flooring and the joists use wood screws. Other detailed construction data on this floor is obtained from Hu (1992).

For the purpose of comparison, identical floor configuration data is assumed for each floor, ie. the overall floor width is maintained at 12.0 m with joists spaced at 1.2 m on centres. Table 6.1 lists both the material and configuration data as well as the parameters of the walker/runner used in the response analysis. In carrying out the analysis, the 75-kg walker/runner moves from position *A* to *C* (Fig. 6.18), and the ensuing response calculated at location *O* of the passive occupant who also has a mass of 75 kg. Notice from Fig. 6.18 that for each floor type, the dynamic analysis is performed using, in turn, the three loading simulations of the walker/runner-floor interaction.

The response histories obtained for the above two reference floors are given in Fig 6.19–6.23. It is noted that the 150 kg combined mass of both the walker/runner (*active occupant*) and the passive occupant only amounts to 0.5% of the total mass of the heavier composite steel/concrete floor system, and 13% of that of the light wooden floor. From Figs. 6.19 and 6.21 it is observed that for the relatively heavy floor for which the occupants' contributory mass is insignificant, either of the previously mentioned three methods may provide reliable results, as the floor response histories are mainly influenced by the dynamic parameters of the floor alone. Response histories (Fig. 6.20, 6.22 and 6.23) obtained under the light wooden floor, on the contrary, show greater sensitivity to the occupants dynamic parameters. This indicates that for light floors, the use of calibrated forcing functions alone to sim-

ulate floor-walker/runner interaction may not give realistic results. Furthermore, considering the computed response histories, the acceleration time histories show greater sensitivity to the presence of occupants on the floor, and as well, time histories obtained under walking excitations are generally more sensitive to occupants dynamic characteristics than those obtained under running excitations. The latter statement may be attributed to the fact that during running the periods of foot contacts are comparatively shorter than in walking.

6.4.2 Verification of the Implementation of Spatial and Temporal Variations in the Footfall Forces Associated with Walking

Among the principal objectives of this study is the incorporation in a computer program of the occupant(s) dynamic characteristics as well as the temporal and spatial variations in the footfall forces resulting from activities such as walking, running or jumping. Since such a computer program offers the capability for evaluating floor response histories, it is essential the above features be verified. In this regard, the results from two experimental floors—one heavy floor and one light floor—are studied.

6.4.2.1 Main Roads Department of Western Australia Footbridge

Data on the Curtin Avenue footbridge representing test data on heavy floor construction was studied. This footbridge formed part of the 21 footbridges in Perth, Australia, used in both experimental and analytical study by the Main Roads Department of Western Australia (Wheeler 1982) of objectionable vibrations existing in pedestrian walkways. It may be noted that test data on this footbridge had been used by Mouring and Ellingwood (1994) in a similar study. As indicated in Fig. 6.24, the Curtin Avenue footbridge is a two-span steel structure consisting of a 16-mm steel plate overlay with 50-mm thick asphalt and stiffened longitudinally by I-steel girders whose top flanges are welded to the steel deck to confer a composite structure. Notice that cantilever construction was used indirectly in this structure by the introduction of a hinge of approximately 2 m to the right of the centre support and in the 19.38-m span.

Table 6.2 Material and configuration data for composite steel/concrete floor (Heavy floor) and wood-I joist floor (Light floor)

• **Steel I-beams (All beams)**

ρA (kg/m)	ρI_y (10^{-2} kg.m)	ρI_z (10^{-3} kg.m)	EI_y (10^6 N.m ²)	EI_z (10^4 N.m ²)	EA (10^8 N)	GJ (10^2 N.m ²)	μGA (10^8 N)
32.59	63.498	22.827	16.178	58.157	8.302	56.481	1.526
Mass density of concrete = 2400 kg/m ³							

• **Concrete flooring:**

K_x (10^3 N.m)	K_y (10^3 N.m)	K_v (10^3 N.m)	K_G (10^3 N.m)	D_x (10^9 N/m)	D_y (10^9 N/m)	D_v (10^9 N/m)	D_G (10^9 N/m)
1800.0	1800.0	550.0	640.0	2.20	2.20	0.66	0.77
Mass density of beam = 7850.0 kg/m ³							

• **Composite wood I-Joists:**

	ρA (kg/m)	ρI_y (10^{-2} kg.m)	ρI_z (10^{-3} kg.m)	EI_y (10^6 N.m ²)	EI_z (10^4 N.m ²)	EA (10^7 N)	GJ (10^2 N.m ²)	μGA (10^5 N)
joist #1	4.87	9.57	2.096	1.931	4.2307	9.827	6.0952	7.0578
joist #2	5.06	9.94	2.179	1.931	4.2307	9.827	6.4659	7.4871
joist #3	5.09	10.01	2.192	1.944	4.2583	9.891	5.8993	6.8310
joist #4	5.10	10.02	2.195	1.949	4.2702	9.919	5.9110	6.8445
joist #5	5.24	10.30	2.256	1.960	4.2939	9.974	5.9319	6.8688
joist #6	5.16	10.10	2.222	1.973	4.3216	1.004	6.7201	7.7814
joist #7	5.17	10.16	2.225	1.976	4.3295	1.006	7.8603	9.1017
joist #8	5.11	10.04	2.200	1.980	4.3374	1.007	6.1325	7.1010
joist #9	5.32	10.46	2.291	1.986	4.3493	1.010	7.9955	9.2583
joist #10	5.38	10.58	2.317	1.987	4.3532	1.011	5.3840	6.2343
joist #11	5.09	10.01	2.192	2.020	4.4244	1.028	7.5269	8.7156

Mass density of all joists = 425.0 kg/m³

• **Wood floor sheathing:**

K_x (N.m)	K_y (N.m)	K_v (N.m)	K_G (N.m)	D_x (10^7 N/m)	D_y (10^7 N/m)	D_v (10^7 N/m)	D_G (10^7 N/m)
1365.33	4096.0	136.0	682.67	0.533	1.6	0.053	3.2

Mass density of sheathing = 425.0 kg/m³

- Connectors slip stiffnesses (*steel/concrete floor*): $k_x = k_y = 1.0$ MN/mm, $k_\theta = 1.0 \times 10^6$ MNmm; spacing is 300 mm on centres.
- Connectors slip stiffnesses (*wood floor*): $k_x = k_y = 3.0$ MN/m; no connector withdrawal, ie. $k_\theta \rightarrow \infty$; connectors are spaced at 160 mm on centres.
- For wood floor, no between-joist bridging, and across-joist gaps in sheathing have negligible effect.

	<u>Parameters of walker</u>	<u>Parameters of runner</u>
• Pacing frequency, f_z	2.3 Hz	3.2 Hz
• Number of steps, N_s	16	10
• Forward velocity, v	1.45 m/s	3.2 m/s
• Stride	0.63 m	1.00 m

Experimental work carried out by Wheeler (1982) included free-vibration excitation of the footbridge to determine the fundamental natural frequency of the structure. Also, the displacement responses of the footbridge due to a 77-kg pedestrian walking across the entire length of the bridge at pacing frequency of 2.0 Hz (normal walk), and at resonant frequency corresponding to the natural frequency of the bridge were measured. Notice that for lack of adequate data on the latter pacing frequency, in this study, only the results obtained with the 2.0 Hz pacing frequency had been considered for comparison purposes. Wheeler (1982) analysed the dynamic response of the footbridge under pedestrian-induced loading by modelling the ensuing footfall force as a half-sine function varying in time along the length of the footbridge. Mouring and Ellingwood (1994) solved the same problem as a moving point load for which the finite element package **ABAQUS** was used. Since the semi-analytical approach employed in this thesis assumes availability of analytical mode shape function(s), the requisite mode shape was obtained as the deflected shape produced by the application of prescribed static load on the structure. Table 6.3 and 6.4, respectively, compare the fundamental frequency and the total peak displacement responses obtained experimentally and through theoretical models. In the response analysis, the time steps for zones 1, 2, and 3 (Fig. 5.7) was each 0.0125 s.

6.4.2.2 University of New Brunswick Wood Science and Technology Centre (WSTC) Wood I-joist Floor

Whereas the data on the previous heavy floor structure could only be suitable for verifying the implementation of the spatial and temporal variations in the footstep forces, data on experimental wooden floor affords the opportunity of verifying as well the implementation of human dynamic characteristics in the model developed in this work.

The particular wood I-joist floor selected for this study formed part of a series of full-scale wooden floors built and tested at the Wood Science and Technology Centre (WSTC), University of New Brunswick, Canada to examine the influence on wood-joist floors dynamic responses of shear deformation and rotatory inertia associated with vibrating wood I-joist floors. Essentially, the material and con-

**Table 6.3 Comparison of results of Curtin Avenue footbridge
(Fundamental natural frequency, Hz)**

Calculated (Wheeler)	Experimental (Wheeler)	Mouring & Ellingwood	This study
3.20	3.70	3.28	3.38

**Table 6.4 Comparison of results of Curtin Avenue footbridge†
(Total peak displacement response (mm); Pacing 2.0 Hz)**

Calculated (Wheeler)	Experimental (Wheeler)	Mouring & Ellingwood	This study
0.70	0.60	0.59	0.51

† damping ratio, δ (based on log decrement) is 0.055

figuration data of the selected floor are as given in both Table 6.2 and Fig. 6.18 with minor variations. The test floor measured 5.18×6.83 m with 15 wood-I joist spaced approximately at 449 mm on centres. All joists had the ends simply supported with the edge joists free to vibrate (S-F-S-F floor classification), and viscous damping ratio of 1.4% was measured. The floor also had an imposed load of 15 psf (718 N/m²). As in Fig. 6.18, the test walker of mass 60 kg at initial position (1.708, 2.590) m walked to position (5.122, 2.590) m at a pacing frequency of 2.4 Hz using an approximate forward speed of 1.0 m/s.

As a preliminary check, the fundamental natural frequency of 20.46 Hz obtained using the computer program for this study was found to compare well with the experimental value of 19.93 Hz. To account for the imposed uniformly distributed load, the plate density term (Eq. 2.3) was modified as $\rho^* = \rho_p + \frac{718}{t_p g}$, where g represents acceleration due to gravity and t_p being the thickness of the sheathing.

Figure 6.25 compares the test acceleration time histories with predicted results with and without the 718 N/m² imposed structural loads. Notice that time step for both zones 1 and 3 (Fig. 5.7 was 0.013 s and that for zone 2 was 0.0185 s. Though, the predicted response histories compare fairly with the test data in terms

of peak-to-peak interval and the overall decay of vibration between the intervals, the maximum peak values do not agree. The peak value obtained in the unloaded case of 4.43 m/s^2 and 2.21 m/s^2 for the loaded condition are both less than the 6.00 m/s^2 test peak value. The seeming differences in the test results may be attributed to the following.

- (1) The test data did not provide the actual number of footsteps involved, and in implementing the requisite input data in the numerical model, this value is calculated based on assumed forward speed of 1.0 m/s which was also not measured experimentally.
- (2) In the author's computer program, the forcing function corresponding to the test 2.4 Hz pacing frequency was substituted by that associated with 2.3 Hz which is readily available in the literature. In actual fact, the use of alternative forcing function should be assessed vis-a-vis the general sensitivity of floor response to the shape (amplitude and contact time) of footfall forcing function.
- (3) A situation which may have further influenced the results is the combined effect of the difference between the experimental walker mass (60 kg) and the total mass of the ISO oscillator of 75 kg .

6.4.3 Sensitivity of Response Time Histories to the Shape of Forcing Function

An underlying assumption assumed throughout this work is the availability of data bank from which forcing functions representing various pacing frequencies may be drawn. Since, in general, no two persons walk or run exactly alike, any force-time history required as input in a serviceability vibrational-based criteria for light floors may need to be calibrated against a *standard* human model such as the 2-DOF ISO human model. Such a standardization may alleviate the difficulty of correlating the results of several experimental studies on objectionable vibrations in light floors. The need for such standardization is illustrated using two sets of force pulses (Fig. 6.26); one set denoting the footstep force produced by a person walking *normally* at a pacing frequency of 2 Hz , and the other, a runner taking steps at

3.2 steps/s. The assumed forcing functions given in Fig. 6.26 have been scaled to describe the force exerted on the base (floor) structure by a single footfall produced by a walker of mass 70 kg, and also by a runner of mass 75 kg. Notice that for the purpose of comparing predicted floor responses, all forces have been discretized at 0.005 s for force pulses due to the runner, and at 0.0125 s for the walker-induced force pulses. The material data in Table 6.2 and configuration data given in Fig. 6.23 have been used to perform the analysis using as input data the force pulses associated with the runner, and the University of New Brunswick (WSTC) wood I-joist floor data is also used for the analysis dealing with the walker foot pulses.

Figure 6.27 gives the predicted results for each of the above case studies. The sensitivity of the acceleration time-history response to the shape of the footfall pulse is particularly more evident in Fig. 6.27a which shows the initial peak acceleration values being largely influenced by the gradient of the rising limb of the force pulses. On the other hand, predicted results given in Fig. 6.27b indicate that the half-sine approximation often used to model single footfall pulse associated with higher pacing frequencies may be justified.

6.5 Response of Lightweight Floors to Rhythmic activities In Place

As mentioned elsewhere, eg. Ellingwood and Tallin (1984), induced forces on the lightweight floors may arise from the response of a group of people to some external stimulus such as music or spectator event. Since these forces have little spatial variation, they may only be modelled as time varying forces which thus greatly facilitate any response analysis. Though, dynamic forces from group activities would cause an increase in floor static deflection, however, if the resulting excitation lacks any coherence, the dynamic component of motion would be small and any serviceability problem may then be minimal.

Therefore, the governing equations of motion of a typical light floor under group activity is developed by assuming group activities are in step such as during aerobics classes. Such equations take the same general format given in Eq. 6.1. For activities

such as dancing or walking in place for which the occupant is always in contact with the base structure, Eq. 6.1 is applicable throughout the time-domain of the analysis. On the contrary, for activities such as jumping or jogging in place, Eq. 6.1 is applied during the times that foot contacts with the base structure are made, for all other periods the base (floor) structure vibrates freely as a coupled passive occupant(s)-floor system and Eq. 6.7 is applicable.

Fig. 6.1 Convergence: Displacement Response to Bag Drop Impact (Floor Joists: SPF No.2 Grade)

Fig. 6.2 Convergence: Velocity Response to Bag Drop Impact (Floor Joists: SPF No.2 Grade)

Fig. 6.3 Convergence: Acceleration Response to Bag Drop Impact (Floor Joists: SPF No.2 Grade)

Fig. 6.4 Accuracy: Comparison of DYFAP and Thesis Predicted Displacement Time Histories (Floor Joists: SPF No.2 Grade)

Fig. 6.5 Accuracy: Comparison of DYFAP and Thesis Predicted Velocity Time Histories (Floor Joists: SPF No.2 Grade)

Fig. 6.6 Accuracy: Acceleration Time Histories (Floor Joists: SPF No.2 Grade) (a) Thesis Prediction (b) DYFAP Prediction (c) Comparison of Thesis and DYFAP Predictions

Fig. 6.7 Accuracy: Comparison of DYFAP and Thesis Predicted Displacement Time Histories (Floor Joists: Wood I-Joist)

Fig. 6.8 Accuracy: Comparison of DYFAP and Thesis Predicted Velocity Time Histories (Floor Joists: Wood I-Joist)

Fig. 6.9 Accuracy: Comparison of DYFAP and Thesis Predicted Acceleration Time Histories (Floor Joists: Wood I-Joist)

Fig. 6.10 Comparing Coupled Responses Associated with Three Damping Matrices: Input Heel Drop Excitation is Modelled as Forcing Function + 75 kg mass of Impacter

Fig. 6.11 Comparing Coupled Responses Associated with Three Damping Matrices: Input Heel Drop Excitation is Modelled as Forcing Function + 2DOF ISO Human model

Fig. 6.12 Comparing Coupled Responses Associated with Three Damping Matrices: Input Heel Drop excitation is Modelled as 2-DOF ISO Human Model with Assigned Initial Velocities

Fig. 6.13 Comparing (a) Displacement, and (b) Velocity Time histories From Three Excitation Types: Floor has 38×184 mm SPF Joists

**Fig. 6.14 Comparing Acceleration Time Histories From three Excitation Types: Viscous Damping Matrix is $[C] = \alpha[M]$:
Floor has $38 \times 184 \text{ mm}$ SPF Joists**

Fig. 6.15 Comparing Acceleration Time Histories From Three Excitation Types: Viscous damping matrix is $[C] = \alpha[M] + \beta[K]$: Floor has 38×184 mm SPF joists

Fig. 6.16 Comparing Displacement, Velocity and Acceleration Time Histories From Three Excitation Types: Viscous Damping Matrix is $[C] = \alpha[M]$:
 Floor has Composite Wood I-Joists

Fig. 6.17 Comparing Displacement, Velocity and Acceleration Time Histories From Three Excitation Types: Viscous Damping Matrix is $[C] = \alpha[M] + \beta[K]$: Floor has Composite Wood I-Joists

Fig. 6.18 Wood Floor Assembly Showing Location O of Passive Occupant :
Models Simulating Walker Activity:
 (a) Forcing Function + 2-DOF Human Oscillator
 (b) Forcing Function + 75-kg Mass of Occupant
 (c) Forcing Function only

Fig. 6.19 Comparison of Response Histories From Reference Heavy Floor (Composite Steel/Concrete Floor) due to WALKER-Induced Excitations:
 (a) Displacement Time History (b) Velocity Time History
 (c) Acceleration Time History

Fig. 6.20 Comparison of Response Histories From Reference Light Wood I-Joist Floor due to WALKER-Induced Excitation:
 (a) Displacement Time History (b) Velocity Time History
 (c) Acceleration Time History

Fig. 6.21 Comparison of Response Histories From Reference Heavy Floor (Composite Steel/Concrete Floor) due to RUNNER-Induced Excitations: (a) Displacement Time History (b) Velocity Time History (c) Acceleration Time History

Fig. 6.22 Comparison of Response Histories From Reference Light Wood I-Joist Floor due to RUNNER-Induced Excitations:
(a) Displacement Time History (b) Velocity Time History

Fig. 6.23 Comparison of Acceleration Histories From Reference Light Wood I-Joist Floor due to RUNNER-Induced Excitation Simulated as:
 (a) 3.2 Hz Forcing Function+ 2-DOF Human Oscillator (b) 3.2 Hz Forcing Function +75-kg Mass of Runner (c) 3.2 Hz Forcing Function only

(a)

Mode shape function:

$$\varphi_1(x) = \Delta(x) - \delta(x)$$

$$\varphi_1(z) = \Delta(e) + \delta(e) - \frac{(b-e) \cdot z}{(b-e)} \delta(z)$$

$$\varphi_1(y) = \Delta(y) + \delta(y)$$

(b)

Fig 6-24 Curtin Avenue Footbridge (Wheeler 1982; Mouring & Ellingwood 1994)
(a) Bridge Elevation and Section; (b) Structural Modelling and Mode Shape

**Fig. 6-25 :-Time History of Acceleration due to Walking (University of New Brunswick-WSTC Wood I-Joist Floor) (a) Test Results
(b) Numerical Results for Case of NO Loading
(c) Numerical Results for Imposed loading**

Fig 6.26 Typical Single Footfall Force Functions

(a)

(b)

Fig. 6.27 Sensitivity of Acceleration Time-History to Shape of Footfall Pulse
 (a) Walker at Pacing Frequency of 2.0 Hz (b) Runner at Pacing Frequency of 3.2 Hz

7.1 General

In this chapter, attention is focused on the development of a probabilistic-based vibrational serviceability criterion appropriate for the evaluation of perceptible vibrations existing in light floors under normal walking footsteps. Although, various forms of such criteria exist in the literature [eg. Foschi and Gupta 1985; Chui 1987; Smith and Chui 1988; Ohlsson 1991; Foschi *et al.* 1995], they have nearly all been developed with respect to transient vibrations resulting from single heel drop impact studies. The inadequacy of some of these criteria to address frequent concerns of annoying vibrations from walking has often been expressed [eg. Allen and Rainer 1989].

As a prime objective of this study, the essential features of the analysis leading to the development of a vibration-based criterion may be expressed as follows. The root-mean-square velocity of floor velocity response at the midspan position of a passive occupant receiving the vibrations generated by a walker is used in conjunction with data on acceptable floors (Onysko 1988) to randomly determine the floor dynamic acceptability. This acceptability is obtained as a function of the randomness in both the floor construction parameters and occupants' dynamic characteristics.

7.2 Effects on Floor Response of Floor Parameters

Since any appropriate vibration criterion for light floors must be developed in consideration of floor construction features, as a preliminary step, the sensitivity of floor response to different floor parameters was investigated. Features of a typical wood floor system considered included joist span and spacing, viscous damping

Table 7.1 Material and configuration data of reference wood joist floor

• **General characteristics:**

- Joist spacing = 400 mm
- Span = 3.66 m
- Joist depth = 184 mm
- Joist width = 38 mm
- Joist species and grade = Canadian SPF, Select Structural
- Flooring is 15.5 mm Tongued and Grooved (T&G) Douglas Fir Plywood
- Number of joist = 10
- Mass density of all joists = 500.0 kg/m³
- Mass density of sheathing = 500.0 kg/m³
- Viscous damping ratio in all modes = 4%
- Connectors slip stiffnesses: $k_x = k_y = 1.2 \text{ MN/m}$; no connector withdrawal effect, i.e. $k_\theta \rightarrow \infty$; connectors are spaced at 200 mm on centres.
- No between-joist bridging, and across-joist gaps in sheathing have negligible effect.
- Disturbing occupant (walker) dynamic model is the 2-DOF ISO human oscillator.
- Disturbed occupant dynamic model is the 2-DOF ISO human oscillator.
- Shape of 2 Hz forcing function is as given in Fig. 5.2(a)

Parameters of walker

◦ Pacing frequency, f_z	2.0 Hz
◦ Number of steps, N_s	5
◦ Forward velocity, v	1.40 m/s
◦ Stride length	0.70 m

• **Parameters of all Joists:**

ρA	ρI_y	ρI_z	$E I_y$	$E I_z$	EA	GJ	μGA
(kg/m)	(10 ⁻³ kg.m)	(10 ⁻⁴ kg.m)	(10 ⁵ N.m ²)	(10 ³ N.m ²)	(10 ⁷ N)	(10 ³ N.m ²)	(10 ⁶ N)
3.496	9.86338	4.20685	2.11076	9.00267	7.48144	1.84271	3.66737

• **Parameters of Wood floor sheathing:**

K_x	K_y	K_v	K_G	D_x	D_y	D_v	D_G
(N.m)	(N.m)	(N.m)	(N.m)	(10 ⁷ N/m)	(10 ⁷ N/m)	(10 ⁶ N/m)	(10 ⁶ N/m)
630.0	3000.0	29.4306	140.1458	4.7	9.5	1.47	7.0

ratio, in-between joist bridging, joist depth, sheathing-joist connector spacing and thickness of floor sheathing. In all cases, a walker, modelled as a 2-DOF ISO oscillator, was assumed to walk along the median line of the floor and in the direction of the joists, and the ensuing response computed at the midfloor position of a passive receiver, also simulated as a 2-DOF ISO oscillator. The results given in Fig. 7.1 through 7.8 are based on a *reference* floor whose dimensions and properties are as shown in Table 7.1.

Of all the floor construction parameters considered, variations in joist span (Fig. 7.8) affect the calculated response the most—in fact changes in floor span not only affect the relative magnitudes of the peak response parameters, they also influence in part the duration of any perceptible vibrations. Floor span ranging from 2.40 m to 4.88 m had been considered. Figure 7.1 illustrates the effect of joist spacing on the calculated response. It is seen that increasing in-between joist spacing increases the peak displacement values. On the contrary, within the practical range of connector spacing possible (i.e. 200–400 mm range), any change in fastener spacing has only a negligible effect (Fig. 7.2). Another parameter also of significant influence on floor response is the changes in the depth of the floor joists. Since the use of higher sectional depth results in increased floor stiffness, corresponding floor response amplitudes are greatly reduced (Fig. 7.3). It must be pointed out that though higher sectional depth has corresponding increase in mass and damping, their effects are only secondary. Increased floor stiffness may also be achieved with the use of joist bridging either by using straps or, more appropriate still, by carefully-installed between-joist struts. As seen in Fig. 7.6, the use of bottom straps alone does not provide adequate supplemental floor stiffness. Considering the three bridging types in current use, between-joist bridging alone or between-joist bridging in conjunction with bottom strap are found to be the more effective. Though, the effect of varying sheathing thickness (Fig. 7.5) on floor response is not too apparent from this study, floor stiffness in the across-joist direction may however increase which may help control perceptible vibrations in floors with low span/width ratios.

The effect on floor response of variations in joist modulus of elasticity was also studied by considering two grades of sawn lumber (Select Structural and No.1/No.2 Grades) of the reference SPF joists (Table 7.1). Figure 7.4 indicates the sensitivity of the floor response to changes in the material grade. One inference could be drawn from this result: considering the same joist species the use of different grade materials have only a minimal effect on overall floor response. The effect of damping on the floor response is studied for viscous damping ratios ranging from 0.01–0.05. As detailed in Fig. 7.7, variations in damping in this range have little influence on the computed displacement time histories. Damping effect is slightly noticeable

when amplitudes of response in the intervals between peak acceleration values are considered.

7.3 Effects on Floor Response of Variations in Occupants' Dynamic Characteristics

In lightweight floor-occupant(s) systems, the sensitivity of floor response to variations in the dynamic characteristics defining a walker/runner activity may be studied by considering changes in such features as activity pacing frequency, shape of associated forcing function, individual stride length, and parameters of all human oscillatory systems involved. One may note that in the development of vibration serviceability criterion for which the governing criterion may be based on a selected activity, characteristics such as activity pacing frequency and those of the human oscillatory system may be preassigned. This implies that the shape of the forcing function representing the chosen activity as well as the parameters of the human oscillator model are known *a priori*. Such guidelines are used later in formulating a new vibrational-based serviceability limit state for assessing the performance of wooden floors. It should however be pointed out that variations in the parameters of the human oscillator have been known to greatly influence certain floor response characteristics (Foschi and Gupta, 1985).

To examine the sensitivity of floor response to changes in one aspect of human dynamic features, Figure 7.9 shows the effect of changing stride length on the response of the reference floor. It is noted that though variations in individual pace or stride do not significantly affect floor response particularly in terms of peak response and peak-to-peak interval, smaller strides produce perceptible vibrations of longer duration.

7.4 Development of A New Vibrational-Based Serviceability Limit State Criterion

7.4.1 Background

Human discomfort resulting from footfall-induced and machine-induced vibra-

tions have been identified as the two most important internal sources of vibrations in lightweight floors (Ohlsson 1991). The discomfort associated with the latter is thought to be caused by single frequency steady-state sinusoidal vibrations due to rotating machinery. The design criterion given later in this work, however, is related to footfall-induced loadings. It may also be pointed out that severe dynamic loading associated with activities such as dancing and other rhythmic activities may require a more stringent criterion other than the one developed in this work.

Characteristically, footfall-induced forces obtained from activities such as walking consist of two major vibrational components—*low-frequency* components (0–8 Hz) associated with the walking rate or step frequency and its harmonics, and *high frequency components* (> 8 Hz) which are due to impacts on the floor when the heel makes its contact with the subfloor. As pointed out earlier, perceptible vibration may be expressed in terms of the sensation or *springiness* experienced during the time of contact between the foot and the floor surface, or in terms of the degree of vibration perceived by a passive subject reacting to vibrations set up by another subject—the latter commonly referred to as *vibrational disturbance*.

The floor deflection on foot contact and the ensuing vibration, or *springiness*, is governed by the low-frequency, quasi-static, forced motion of the resulting footstep coupled with the relatively high frequency decaying vibration. The low-frequency response is mass or damping independent, and some form of static stiffness criterion with respect to point load can be applied. The high frequency decaying vibration is also the occurrence of high rate of force increase, or impulse, created during the initial part of the footstep. The degree of floor *vibration disturbance* is due in part to the steady state nature of resulting forced oscillations of resonant character and and in part to the strong and varying transient impulsive elements. It is noted that for heavy long span floors, for which frequency band of below 8 Hz may dominate floor response, the RMS-value of the vibration acceleration provides a good measure of the disturbing effect. On the other hand, for light wood floors of short to medium spans, the RMS-value of the vibration velocity is the more appropriate measure of human disturbance (Ohlsson 1991).

On the basis of the above and for design purposes, a suitable vibrational criterion for use in light commercial or residential environments must be formulated to meet the following objectives:

- (1) To satisfy any limitation with respect to floor springiness.
- (2) The calculated RMS-value of the vibration velocity obtained for a specified human activity must not exceed a threshold value.
- (3) The criterion must include well defined quantities that are possible to calculate or measure at the design stage.
- (4) The stipulations governing such criterion must be such that the designer must have the option of changing floor design to achieve cost effectiveness.

7.4.2 Criterion Formulation

The objectives (1) and (2) in the foregoing reflect the degree of vibrational serviceability. Since design guidelines in most existing building codes are expressed by simple and measurable quantities, it is therefore appropriate that any proposed vibrational serviceability criterion has inputs that are readily quantified, hence objective (3). The need to provide optimum design is expressed by objective (4).

The basic requirement of the present criterion is that the root-mean-square velocity of any light wood floor, of short to medium span, calculated on the basis of the activities of two standard occupants should not exceed a threshold value. As given in Fig. 7.10, the activities of these occupants are described by one *active* occupant walking normally at 2 steps/sec and who in consequence disturbs a passive occupant situated at the midfloor. Both occupants are modelled by the 2 degree-of-freedom ISO human model, and the direction of travel of the active occupant or the walker is such that both the duration of exposure to perceptible vibrations and the RMS-velocity value of the response at midfloor are maximized.

The threshold RMS-velocity value is established on the basis of a class of floor that has been considered to have satisfactory vibrational acceptability in the past (Onysko 1988). The floor type consists of 15.5 mm Canadian Softwood Plywood, nailed at an average spacing of 200 mm to No.2 grade 38 × 184 mm Spruce-Pine-Fir

(SPF) joists spaced at 400 mm on centres, and with a 19×89 mm bridging strap nailed to the underside of the joists at midspan. Onysko (1988) indicates that the permissible span for this floor under 1.9 kN/m^2 uniform design live load is 3.45 m . To satisfy the requirements of floor springiness and impulsive load dissipation resulting from footfalls, Onysko (1988) acceptability-based criterion (Eq. 1.9) is used to configure the basic floor for the calculation of threshold RMS-velocity value. The number of joists, 13, for the basic floor is thus obtained from the limiting static deflection of 1.6 mm, Onysko (1988), being the deflection resulting from the application of 1 kN concentrated load at midfloor.

Mathematically, the acceptability of any floor in terms of its vibrational behaviour is defined by the relation

$$G = \bar{V}_{\text{rms}} - V_{\text{rms}} \quad [7.1]$$

where $\bar{V}_{\text{rms}}$ is the threshold value (maximum RMS-velocity for the basic floor) and V_{rms} , the maximum calculated RMS-velocity for the floor concerned. It is obvious from Eq. 7.1 the floor is only acceptable if $G \geq 0$. Notice that Eq. 7.1, though theoretically straightforward, may not be useful for design purposes since the input parameters are often random in nature and the computation of RMS-velocity in the coupled floor-occupant system may be prohibitive.

The above difficulty is circumvented by incorporating in the calibration of Eq. 7.1, the randomness in certain floor parameters and occupant activity characteristics that are rather difficult to quantify at the design stage. The randomness in parameters such as floor damping characteristics, expressed in this work as viscous damping ratio ξ , and also walker pacing or stride length Δs are identified in this work. Based on data in the literature, a 0.8% to 4% range of viscous damping for light wood floors, and 0.4 to 0.9 m range of stride length are considered in this work. Also, in accordance with the provisions in Part 9 of the National Building Code of Canada (NBCC) and the Canadian Wood Design Code CAN/CSA-086.1-M94, the calibration process makes use of the following floor properties:

- Sheathing thickness: 12.5, 15.5, 18.5, 20.5 mm
- Joist sectional rigidity, EIs: 82.5×10^9 – $926 \times 10^9 \text{ N/mm}^2$.

- Bridging types: Strapping only, In-between joist, In-between joist + Strapping.
- Span: 2.40 – 7.0 m.
- Joist spacing: 300, 400, 600 mm
- Floor width: 2.40, 3.60, 4.80, 6.0, 7.2 m
- Sheathing-joist connection type: Nailed, Nailed + Elastomeric glue, Full composite

For any floor designed on the basis of the above data, a non-parametric distribution of the RMS-velocity was obtained. Typical results for floors having same configuration and material data as the basic floor with only variation in span is given by Fig. 7.11. Note that in this example, the number of rows of wood straps required for each span is chosen such that the provisions in the NBCC relating to installation of wood straps in light structures are met.

The 95% percentile non-parametric value obtained from 30 simulations, representing 5 different viscous damping values 0.8%, 1%, 2%, 3%, 4% and 6 stride lengths 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 m, is chosen as the representative RMS-velocity for any one floor analyzed. Notice also that a similar statistical procedure is used to establish the threshold value of the basic floor. Normalization of all calculated RMS-velocities with respect to the threshold value leads to the construction of design curves given in Figs. 7.12a and 7.12b. For instance, in Fig. 7.12a, γ_1 obtained for span L using *strapping only* design is defined as

$$\gamma_1 = \frac{\text{RMS-velocity (Span } L \text{ with strapping only)}}{\text{RMS-velocity (Basic floor)}} \quad [7.2]$$

For the vibrational serviceability problem in question for which consideration is only given to structural behaviour in the elastic range, it is always possible to extrapolate from γ -curves values corresponding to any given floor parameter such that the 95% upper confidence limit on the RMS-velocity mean is guaranteed.

From the foregoing therefore, a unified vibrational-based serviceability limit state criteria for applications in lightweight wood joist floors would take the format

$$1 \geq \prod_{k=1}^6 \gamma_k \quad [7.3]$$

where,

γ_1 = factor for floor span L and blocking type (Fig. 7.12a)

γ_2 = factor for joist sectional rigidity EI_s (Fig. 7.12a)

γ_3 = sheathing thickness factor (Fig. 7.12a)

γ_4 = factor for floor width (Fig. 7.12b)

γ_5 = Joist spacing factor (Fig. 7.12b)

γ_6 = sheathing-joist connector type (Fig. 7.12b). For the design using nails with spacing meeting code specifications $\gamma_6 = 1.00$; $\gamma_6 = 0.85$ if nailing is used in conjunction with elastomeric glue. Also, where the nature of the fasteners and spacing used is such that full composite action is achieved γ_6 may be taken as 0.81.

7.5 Discussion

The criterion given by Eq. 7.3 is simple to implement in any building code. This is so because simple design parameters such as sheathing thickness and joist sectional rigidity which constitute the requisite input parameters are easily quantified at the design stage. Functionally, the criterion has been formulated to satisfy important floor vibrational characteristics such as springiness and impulsive load dissipation. It also incorporates all aspects of floor properties considered in the current serviceability limit state vibrational criteria for light wood floor designs in NBCC Part 9. The advantage of the criterion set out in this work over the one in current use lies in its capability to incorporate data pertaining to an entire floor as against the consideration of those of an average bare floor joist more emphasized by the code criterion. For instance, by considering a floor property such as floor width, the advantage of having a heavier floor mass of comparatively low RMS-velocity is accounted for in this work. Since the existing NBCC code criterion has been developed on the basis of Eq. 1.9 only, occupant activity effects in terms of any duration of perceptible vibration is not fully accounted for.

The criterion given in this work has been based on *in-grade* material data in use

in the Canadian Wood Design Code CAN/CSA-086.1-M94, for that matter it also shares in certain design constraints stipulated in the code. Finally, the proposed criterion in its interactive form gives the designer the flexibility of adjusting design for cost effectiveness.

Fig. 7.1 Effect on Floor Response of Joist Spacing

Fig. 7.2 Effect on Floor Response of Interface Connectors' Spacing

Fig. 7.3 Effect on Floor Response of Joist Depth

Fig. 7.4 Effect on Floor Response of Joist Grade (Elasticity Modulus)

Fig. 7.5 Effect on Floor Response of Sheathing Thickness

Fig. 7.6 Effect on Floor Response of Joist Bridging

Fig. 7.7 Effect on floor Response of Damping

Fig. 7.8 Effect on Floor Response of Joist Span

Fig. 7.9 Effect on Floor Response of Walker Stride (Pace) Length

Fig . 7. 10 Disposition of Two Human Occupants for Formulation of Vibration Acceptance Criterion

Fig. 7.11 Distribution of RMS-Velocity for Different Spans of Basic Vibrational Floor

Fig. 7.12a Normalized RMS-Velocities With Respect To Changing Floor:
 (a) Span and Nature of Joist Bridging (b) Joist Sectional Rigidity (c) Thickness of Sheathing

Fig. 7.12b Normalized RMS-Velocities With Respect To Changing Floor: (a) Width (b) Joist Spacing (c) Subfloor Sheathing -Joist Rigidity

SUMMARY, CONCLUSIONS & RECOMMENDATIONS

8

8.1 General

Lightweight floors for residential or commercial applications may exhibit annoying levels of vibration even under normal occupancy loading, and extensive reports abound of consumer dissatisfaction in this regard. The availability of higher strength materials, refined engineering analysis, and improved construction detailing in recent times have all resulted in lighter and longer spanning floor systems. Admittedly, while this technological gain has been made without any significant loss in strength performance, existing stiffness-based serviceability design guidelines in most building codes have not been adequate to deal with perceptible vibrations resulting from occupants activities.

Against this background, therefore, studies have been carried out ostensibly to develop predictive models for evaluating floor vibrational performance and thus formulate appropriate vibrational serviceability criteria for use in design.

8.2 Summary

An extensive analytical program was undertaken to study the behaviour of lightweight wooden floors under human-induced footfall excitations.

A detailed literature review was conducted to establish the current state of the art. It was found out that in evaluating the vibrational performance of lightweight floors under normal occupancy, there is the need for adequate analytical tools for predicting the dynamic behaviour of coupled floor-occupant systems particularly under discrete forces exhibiting considerable spatial and temporal characteristics, hence the work described in this thesis.

Because of its computational efficiency, the semi-analytical method proposed

here was the preferred method of analysis. Firstly, with the component mode synthesis method, the use of analytical definitions for substructures' modes resulted in system mass and stiffness matrices of comparatively smaller size. Subsequent frequency and response analyses were thus greatly facilitated in terms of computer running time and storage requirements.

In the forced response analysis, human interaction with a typical wood joist floor was modelled by an appropriate forcing function acting in conjunction with a two-degree-of-freedom ISO human model. The ensuing equations of motion, of nonlinear form, were solved by the step-by-step integration scheme for which the average velocity method was used.

Results from frequency analyses of selected floors compared favourably with those from similar experimental or analytical studies. In the absence in the literature of comprehensive experimental or analytical results on wooden floors under excitations ascribable to actual walking or running, the correct implementation of the coupled floor-occupant characteristics in the time domain analysis was corroborated through results from studies on pedestrian steel or concrete footbridges, and also on heel drop impact studies on wooden floors.

With the analytical tool developed in this thesis, the root-mean-square velocity of the response of the coupled floor-occupant system was used as a basis for the formulation of a vibrational-based serviceability criterion. Through a normalization scheme, a design guideline is developed which uses as input certain floor configuration and measurable material properties, and which makes its adoption in building codes much easier.

It should be pointed out that the work done in this thesis has been focused on rectangular-shaped floors to provide the first step from which data appropriate to other floor plans could be extrapolated.

8.3 Conclusions

Conclusions drawn from this study could be stated as follows:

- A semi-analytical tool for studying the dynamic behaviour of a coupled floor-occupant system has been presented. The numeric tool not only performs frequency analysis, but also response analysis under time-dependent excitations, or excitations characterized by both time and position.
- The model presented is also capable of dealing with pertinent features of the wood joist floor such as the springiness of joists' end supports associated with connections by light gauge metal framing connectors, between-joist bridging and across-joists gaps in sheathing which have been inherently difficult to introduce in most currently available numeric models.
- Joist end fixity conditions influence natural frequencies in higher modes. For the floor having metal frame connector such as joist hanger, the simply supported analogy modelling of the joist was found adequate.
- At width/span ratio greater than 1, floors having across-joists gaps in the sub-floor (the number and location of gaps are based on the Canadian Wood Code (CAN/CSA-086.1-M89) provisions) may have the higher natural frequencies overestimated under the assumption of no across-joist gaps.
- In between-joist bridging in the light floor, when modelled as a supplemental stiffness in the across-joist direction and introduction of additional constraints at the locations of the bridging elements result in frequencies that are more separated than the model based on the restriction of the joist cross-sectional rotations.
- Human dynamic interaction with light floors could be sufficiently modelled by a forcing function (of type related to the human activity and measured with respect to a rigid base) applied in conjunction with an appropriately calibrated human oscillator such as the International Standardization Organization (ISO) 2 degrees-of-freedom human model. Such an interaction involving human activity like running could as well be modelled by a forcing function and an isolated mass of the human.
- For short to medium lightweight floors, consideration must be given to the coupled system of the floor and its occupants since the dynamic characteristics

of the latter have a decisive influence on the overall floor response.

- Probability-based design guideline for minimizing the effects of floor springiness and objectionable vibrations resulting from human activities in lightweight floors has been developed based solely on simple and measurable parameters of the floor. The design guideline which is limited to lightweight floors, of short to medium span, has the format

$$1 \geq \prod_{k=1}^6 \gamma_k$$

in which, $\gamma_k, 1, \dots, 6$ are parameters derived solely from the floor material and configuration data.

8.4 Recommendations

The study has indicated the need for future research in the following areas:

- Extensive experimental work is required to gather data on the response of wood floors under footfall-induced excitations exhibiting time and positional dependent characteristics. Such response measurements must be made with respect to some number of both active and passive occupants. The effects of occupants biomechanical characteristics such as stride length and mass on floor response must also be investigated. Such data would further establish the accuracy of the modelling of the coupled occupant-floor problem such as carried out in this work, or any future related work.
- Development of a general purpose numeric tool which could handle, among other things, floors with large openings and or non-rectangular in plan. Such a tool would require the use of the finite element method to model the plate substructure, and an analytical method such as the one adopted in this work to model the joists. Facet shell element consisting of a combination of suitable plate and in-plane elements may be a good candidate for modelling the plate substructure. To model the complexities associated with the partial rigidity of the subfloor-joist connections and at the same time preserve the computational efficiency currently achieved by some numerical models for analysis of

rectangular-shaped floors, the selected shell element should not only possess fewer nodal degrees-of-freedom, but should also be defined within its interior to minimize the number of elements used in the discretization. The response analysis part of the model may follow the formulation set out in this thesis.

- Establishment of statistical procedures for calibrating the dynamic parameters of any human given the mass and the oscillator type, and also the development of extensive statistical data on footstep forcing functions derived from various human activities. In the latter case, the forcing function could be referenced to a rigid surface, the results of which could be used, in conjunction with appropriate human oscillator, for analysis of concrete, wood or steel floor under human-induced excitations.
- The vibrational serviceability design guideline formulated in this work has been based on a forcing function and human oscillator parameters both of which are assumed deterministic. In other words, the non-parametric 95 percentile RMS velocity obtained from the distribution of RMS velocities representing 30 floor replications, has been based only on the randomness in floor damping and human stride. A more thorough reliability analysis, however, could also incorporate the randomness in both the specified forcing function and the parameters of the human oscillator.

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Appendix

Determination of α and β in Floor Damping Matrix

- Mass proportional damping: $[\hat{\mathbf{C}}] = \alpha[\hat{\mathbf{M}}]$

Given the damping ratio for the i th mode to be ξ_i , we have

$$\{\mathbf{D}^{(i)}\}^T [\hat{\mathbf{C}}] \{\mathbf{D}^{(i)}\} = c_i = 2\xi_i \omega_i \quad [\text{A.1a}]$$

or,

$$\alpha \{\mathbf{D}^{(i)}\}^T [\hat{\mathbf{M}}] \{\mathbf{D}^{(i)}\} = 2\xi_i \omega_i \quad [\text{A.1b}]$$

$$\alpha = 2\xi_i \omega_i \quad [\text{A.1c}]$$

- Stiffness proportional damping: $[\hat{\mathbf{C}}] = \beta[\hat{\mathbf{K}}]$

Given the damping ratio for the i th mode to be ξ_i , we have

$$\{\mathbf{D}^{(i)}\}^T [\hat{\mathbf{C}}] \{\mathbf{D}^{(i)}\} = c_i = 2\xi_i \omega_i \quad [\text{A.2a}]$$

or,

$$\beta \{\mathbf{D}^{(i)}\}^T [\hat{\mathbf{K}}] \{\mathbf{D}^{(i)}\} = \beta \omega_i^2 \quad [\text{A.2b}]$$

which gives the following value for β :

$$\beta = \frac{2\xi_i}{\omega_i} \quad [\text{A.2c}]$$

- Rayleigh damping: $[\hat{\mathbf{C}}] = \alpha[\hat{\mathbf{M}}] + \beta[\hat{\mathbf{K}}]$

Given the damping ratio for the i th and j th modes to be ξ_i and ξ_j , respectively, we have

$$\{\mathbf{D}^{(i)}\}^T [\hat{\mathbf{C}}] \{\mathbf{D}^{(i)}\} = c_i = 2\xi_i \omega_i \quad [\text{A.3a}]$$

or,

$$\alpha \{\mathbf{D}^{(i)}\}^T [\hat{\mathbf{M}}] \{\mathbf{D}^{(i)}\} + \beta \{\mathbf{D}^{(i)}\}^T [\hat{\mathbf{K}}] \{\mathbf{D}^{(i)}\} = \alpha + \beta \omega_i^2 \quad [\text{A.3b}]$$

Similarly,

$$\{\mathbf{D}^{(j)}\}^T [\hat{\mathbf{C}}] \{\mathbf{D}^{(j)}\} = \alpha + \beta \omega_j^2 \quad [\text{A.3c}]$$

from [A.3b] and [A.3c], we obtain α and β as:

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = 2 \begin{bmatrix} \frac{1}{\omega_i} & \omega_i \\ \frac{1}{\omega_j} & \omega_j \end{bmatrix}^{-1} \begin{bmatrix} \xi_i \\ \xi_j \end{bmatrix} \quad [\text{A.3d}]$$