Effects of a Number Game and a Spatial Game on Number Line Estimation

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Abstract

The goal of the present study was to examine whether young children benefited from having an explicit midpoint reference on the number line after training on ordinal knowledge. Preschool-aged children (N = 84; M = 53 months old) were exposed to one session of ordinal training or spatial training. The results show that children who received ordinal training showed near transfer to an ordinal task and far transfer to a number line task. The results also show that children showed more improvements in the version of the number line task where an explicit midpoint reference was presented than no midpoint was presented. In contrast, children who received spatial training only showed near transfer to a 2-D mental rotation task. These results indicate that the ordinal knowledge of the number system is crucial for successful estimation on the number line task.

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Considerable research has been focused on the development of children's number representations in the early grades of elementary school (e.g., Ashcraft & Moore, 2012; Barth & Paladino, 2011; Booth & Siegler, 2006; Laski & Siegler, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003). One typical method of assessing children's number representations is to examine accuracy on a number line task, which requires children to estimate the position of a number on a visual number line in which a fixed number at each end is labeled (e.g., Whyte & Bull, 2008, for 0-10 number range; Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Bouwmeester & Verkoeijen, 2012, for a 0-20 number range; Siegler & Booth, 2004, for a 0-100 number range; and Opfer & Siegler, 2007, for 0-1000 number range). The number line task is a practical and useful tool that can provide insights into children's and adults' underlying representations of numbers (Bouwmeester & Verkoeijen, 2012; Schneider et al., 2008).

There is controversy about how children represent numbers and in how that representation is reflected in children's performance on the number line task. Three different kinds of explanations have been proposed about how children do the number line task and these make different assumptions about the relation between task performance and children's internal representations of quantity or number. First, according to Siegler and colleagues' **multiple representation model**, when children perform the number line task, they map the symbolic numerals (e.g., 4) onto a pre-existing non-symbolic mental magnitude (e.g., the quantity of the "4"), and number line performance reflects their mental representations of quantity (e.g., Booth & Siegler, 2006; Siegler & Opfer, 2003). These researchers reported a consistent age-related progression such that children's number line performance (as indexed by the relation between their estimates and the actual numbers) initially increased logarithmically with numerical magnitude but as they mastered the number system in the specific range, estimates increased linearly with numerical magnitude (Booth & Siegler, 2006; Laski & Siegler, 2007; Siegler & Booth, 2004; Siegler & Opfer, 2003). For example, estimates made by American kindergarteners on a 0-10 number line and of first graders on a 0-100 number line fit a logarithmic function such that children tended to overestimate the magnitude of small numbers and underestimate the magnitude of larger numbers (Siegler & Booth, 2004). In contrast, the estimates made by American second graders on a 0-100 number line fit a linear function, and the degree of linearity was related to children's arithmetic skills (Booth & Siegler, 2006; Siegler & Booth, 2004).

According to the **multiple representation model** proposed by Siegler (1996), young children lack knowledge of the mappings between symbolic and non-symbolic representation of numbers (Booth & Siegler, 2006), and their mental representations are therefore assumed to reflect the characteristics of their internal (non-symbolic) quantity system, a system which has been called the "approximate number system" (Dehaene, 2009). This assumption is based on the findings that infants and other mammals show evidence of quantity discriminations (Xu & Spelke, 2000; Ansari, 2008).

However, the assumption that symbolic representations of numerals are mapped onto pre-existing non-symbolic representations has been challenged (Ansari, 2008). Alternatively, different representations of symbolic and non-symbolic numerals might exist, and they might be mapped onto different internal representations (Ansari, 2008). Therefore, there is no clear evidence to support the view that children's internal representations of number change from logarithmic ones to linear ones, even though their number line task performance shows that progression.

Barth and Paladino (2011) proposed a different explanation for developmental changes in children's number line performance: the **proportional judgment model**. They claim that children of all age groups use a *proportional strategy* in estimating the location of numbers for any number line task that provides numbers marking both endpoints of the line and a value to be estimated (Barth & Paladino, 2011). For example, on a 0-100 number line task, participants need to estimate the magnitude of 30 relative to the magnitude of 100 as a whole. The endpoints on the number line function as explicit reference points to guide participants' estimates (Bouwmeester & Verkoeijen, 2012). A pattern of logarithmic performance may occur because numbers that are close to the reference points are estimated more accurately than those that are farther from the reference points (Bouwmeester &Verkoeijen, 2012; Siegler & Opfer, 2003).

With increasing experience, children are more likely to use the estimated midpoint of the presented number line as an implicit reference point and the accuracy of their estimates increases accordingly (Ashcraft & Moore, 2012; Petitto, 1990; Schneider et al., 2008). Implicit reference points refer to certain specific locations that participants can identify on the number line (often at locations of symmetry: e.g., halved and quartered; Hollands & Dyre, 2000). Ashcraft and Moore (2012) found some evidence of use of the *proportional strategy* for children from Grade 3 to Grade 5 through the investigation of estimating errors and latencies on the number line task (0-100). Children were faster and made fewer errors when the location to be estimated was closer to the endpoints and the middle of the line. As a result, a 'M-shaped' pattern was observed, suggesting that children used the implicit midpoint reference on the number line task (Ashcraft & Moore, 2012). Further, Bouwmeester and Verkoeijen (2012) found more accurate estimation performance for numbers that were close to the two endpoints and the midpoint for 7-year-old children, suggesting that children used the implicit midpoint reference. The observed improvements in children's number line performance might occur as they use more reference points, for example, from two (endpoints) to using three or more reference points (e.g., two endpoints and an estimated midpoint; Hollands & Dyre, 2000).

The **proportional judgment model** reviewed above, however, may not apply to young children because they probably have not acquired the concepts related to proportions such as 'half' and 'double' given that the formal teaching of these concepts starts from Grade 2 (White & Szucs, 2012). Prior to Grade 2, Petitto (1990) found that children rely on a *counting strategy* when they perform the number line task. When children use a *counting strategy*, they start at one of the endpoints and count up or down until they reach the target number (Petitto, 1990). Use of the *counting strategy* suggests that children need to know the counting sequence and recognize the symbolic number words to be estimated. However, knowing the counting sequence and symbolic number words does not imply that they understand the relationship between number words and the cardinal values (Petitto, 1990). For example, children who can fluently count to 10 might not be able to produce, label, or point to a specific quantity (Wynn, 1990). After Grade 2, children use another strategy when performing the number line task, the *midpoint strategy,* which both counting and proportional judgments are required (Petitto, 1990). That is, when the target number is closer to the estimated midpoint of the number line than to one of the endpoints, children start at the middle of the line and count on from there toward the target number, which may lead to more accurate estimates (Petitto, 1990).

The gradual shift from only using a counting strategy to using counting and midpoint strategies coincide with improved accuracy in number line performance (Ashcraft & Moore, 2012; Schneider et al., 2008; White & Szucs, 2012). Schneider et al. (2004) found an increasingly frequent use of the midpoint strategy from Grade 1 to Grade 3 that was reflected in children's eye fixation frequencies to locations on the number line. More specifically, children from Grade 3 started to count up from the left endpoint or from the midpoint until they reached the target number and then stopped moving their eyes further to the right (Schneider et al, 2004). Ashcraft and Moore (2012) also found that children from Grade 1 and 2 used a counting strategy whereas starting from Grade 3, children used both counting and midpoint strategies based on the 'M-shaped' pattern of the performance (Ashcraft & Moore, 2012). In summary, children's performance on the number line task can be improved if they use a midpoint strategy, and an understanding of proportional reasoning helps children to find the estimated midpoint (Ashcraft & Moore, 2012; Petitto, 1990; Schneider et al., 2008). However, children before Grade 2 are less likely to use the midpoint strategy because they lack knowledge of proportions. In the present study, I was interested in whether young children would benefit from the inclusion of an explicit midpoint reference (e.g., showing the location of 5 on the 0 – 10 number range). Even without an understanding of proportional reasoning, it seemed possible that children might use the additional reference point to refine their counting strategy.

No published study to date has investigated whether children benefit from an explicit midpoint reference on the number line task. As a first step I conducted a study to determine how preschool age children benefited from having an explicit midpoint reference (*N* = 94; mean age = 4 years: 3 months; Xu, 2012). I compared two groups of children who were expected to differ in their numerical knowledge based on Siegler and Mu (2008). One group of children were bilingual Chinese-speaking children with Chinese-educated parents, and the other group were English-speaking children (most were educated in Canada). All of the children had been born in Canada and were attending Kindergarten, preschool, or daycare in English.

I found that the Chinese-speaking children performed better on the numeracy measures (e.g., counting, and number recognition) than the English-speaking children, suggesting that the numeracy knowledge of the Chinese-speaking children was more advanced than that of English-speaking children. This finding was consistent with other research for children with Chinese-educated parents (Siegler & Mu, 2008). Most importantly, only Chinese-speaking children benefited from having the midpoint reference on the number line task. The pattern of performance is shown in Figure 1. The accuracy of estimates was measured by percent of absolute error (PAE): | (Estimate – Presented Number) / Scale of the Estimate | x 100. For example, if a child marked the location of 7 on a 0 -10 number line at the position that corresponded to 6.5, the PAE would be 5% [| (6.5 – 7) / 10 | x 100)]. As shown in Figure 1, Chinese-speaking children made fewer errors in the midpoint condition compared to the standard condition whereas English-speaking children were not helped by the midpoint. The Chinese-speaking children showed the "M" pattern of performance identified by Ashcraft and Moore

(2012) for older children on the 0-100 number line, suggesting that they were using the midpoint to refine and improve their solution strategy.

I explained the results as follows (Xu, 2012). Because Chinese-speaking children had relatively better number system knowledge than English-speaking children, the explicit midpoint reference helped them to keep their estimates closer to the actual number – in particular, make fewer errors around the number 5. Based on the pattern of performance, Chinese-speaking children possibly used both counting and midpoint strategies when an explicit midpoint reference was presented. More specifically, when the location of estimated number was closer to number 5, Chinese-speaking children might start counting from number 5, which resulted in more accurate performance. However, English-speaking children had relatively weaker number system knowledge, particularly of numbers larger than number 5 (see Figure 1). They did not count from number 5 for numbers that were closer to the explicit midpoint reference, and therefore did not benefit from being shown the location of number 5 on the number line.

Based on the pattern of results in Xu (2012), I concluded that children's basic numeracy knowledge plays a crucial role in the early understanding of number line estimation. More specifically, it is possible that mental representations for children who have limited numeracy knowledge consist of a list of counting words and that they do not have much further understanding of *ordinal* knowledge on numbers that are captured by those words (Petitto, 1990). *Ordinal knowledge* is defined as the knowledge and understanding of the ordered relations between quantities (e.g., judge whether 6 comes after 5; Brannon & Van de Walle, 2001). For example, if children know that number 6 comes after 5, then they would estimate the location of 6 close to number 5, which leads to accurate estimates. In contrast, magnitude knowledge may not provide help in locating the relative position of numbers: Knowing that 6 is larger than 5 does not by itself indicate how closely the numbers are located. As a result, I hypothesized that children need to have ordinal knowledge to benefit from having a midpoint reference on a number line (Xu, 2012). However, in Xu (2012) I did not explicitly assess children's ordinal knowledge of the numbers from 1 to 10. Therefore, the goal of the present study was to examine whether children benefited from having an explicit midpoint reference on the number line after a short-term training designed to enhance their ordinal knowledge.

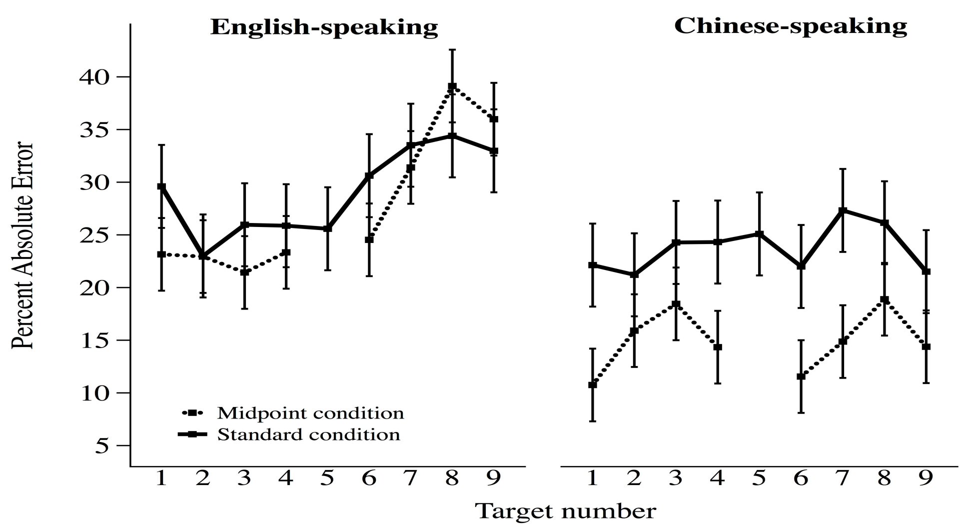


Figure 1. Percent absolute error for each target number across conditions for Chinese- and English-speaking children in Xu (2012); controlling for mother's education. Error bars are 95% confidence intervals based on the *MSE* values for the interaction.

The ability to process ordinal knowledge has been suggested as the foundation of number representations, which is distinct from the approximate number system that is used to process quantity information (Rubinsten & Sury, 2011). Research on braindamaged patients with a quantity deficit showed that they were able to process the ordinal knowledge of numbers (e.g., which number comes next; Delazer & Butterworth, 1997), suggesting two separate systems for ordinal processing and quantity processing. Further, Lyons and Beilock (2009, 2011) claimed that the understanding of ordinal relations among numbers is fundamental such that it forms the building blocks of symbolic numerals as a larger associative system. For adults, the ability to quickly and accurately judge ordinality of a set of digits (e.g., whether 3-7-1 is in an ascending order) is highly correlated with mental arithmetic skills on multi-digit numbers (e.g., 76 x 4; Lyons & Beilock, 2011). Moreover, the ability to order numbers fully mediates the relation between non-symbolic number estimation (e.g., arrays of dots) and complex arithmetic skills, suggesting that the representation of ordinal relations among numbers serves as an important precursor to more complex arithmetic skills (Lyons & Beilock, 2011). The present study used a similar task as in Lyons and Beilock (2011) to measure the understanding of ordinal knowledge for young children. I predicted that the ability to order numbers would be relate to accurate estimation on the complex number line task for preschool children.

No study to date has focused on examining the role of ordinal knowledge in performance on the number line task. However, studies have explored the effects of numerical training on improvement in number line skills. Of these, most have focused on training numerical skills using number board games where children are required to move a game piece along a number line with each square labeled (e.g., 1-10; Mix & Cheng, 2012).

Playing number board games enhance children's basic numerical knowledge (Ramani & Siegler, 2008; Siegler & Ramani, 2009). Through four 15-minute sessions of playing, preschoolers showed improvements not only on number line skills, but also on three other types of numerical skills - counting, number recognition, and numerical magnitude comparison (Ramani & Siegler, 2008). Furthermore, Whyte and Bull (2008) found that providing preschoolers with experience playing a board game with a linear arrangement of numbers lead to improvements in children's number line performance (0-10). They also found that children who played the linear number game improved on basic numerical skills (i.e., counting, number recognition, and numerical magnitude comparison; Whyte & Bull, 2008). However, children who were exposed to a non-linear number game only showed improvements in basic numerical skills but not in number line estimation (Whyte & Bull, 2008). These studies suggest that experience with a linear arrangement of numbers can lead to better performance on the number line task. However, none of these studies allows a separation of improvements that might be due to specific numerical skills or to related abilities and knowledge such as spatial skills as potential causes of number line task improvements.

Space and math are related very early in development such that spatial skill supports learning that in turn supports math gains (Mix & Cheng, 2012). Numerous studies have shown that people who perform better on spatial tasks also perform better on mathematical tasks across a variety of ages (e.g., Casey, Nuttall, Pezaris, & Benbow, 1995; Deglado & Prieto, 2004; Kurdek & Sinclair, 2001; McLean & Hitch, 1999). Gunderson, Ramirez, Beilock, and Levine (2012) showed that children's mental rotation ability (i.e., mentally rotating a 2-D or 3-D objects) at the beginning of Grade 1 and

Grade 2 predicted improvement on the number line task at the end of the school year. Given the large amount of empirical evidence of a close relationship between numbers and spatial skills, it seems that the latter may be crucial when young children perform the number line task. Thus, improvement in the number line task could be a consequence of improved spatial skill occurring with development.

Because playing a number game involves practice of a wide variety of skills, the specific skill that causes improvements on the number line task remain unclear. For example, the spatial skills being practiced in linear games may be similar to those involved in the number line task (Gunderson et al., 2012) because the number line task involves spatial representations in its presentation and response format. Thus, the improvement in number line performance might be facilitated by the spatial cues provided by the linear number board games, where the relative positions of the numbers are stressed (Whyte & Bull, 2008). Further, the spatial cues (i.e., evenly spaced squares with numbers presented in a counting sequence) provided in the linear board game might also help young children understand the mapping between numerical magnitudes, ordinal relations and the presented numerals. To explore the effect of spatial training on math gains, the training should be focused on pure spatial ability, such as mental rotation, not training on math in a spatial context (Newcombe, 2012; Mix & Cheng, 2012).

Spatial skills are not the only ones that may be affected by number game play. Children are also required to name the number on each square through which they move (e.g., Ramani & Siegler, 2008; Whyte & Bull, 2008). As a result, board games are expected to improve counting and number recognition skills, which might lead to better understanding of numerical magnitude. In summary, because there are so many different cues provided in the number board game, it is impossible to tell whether spatial or numerical knowledge, or some combination, are more important in helping children perform better on the number line task.

Present study

The goal of the present study was to explore the role of basic numerical knowledge in the acquisition of number line estimation among preschool children. In particular, I was interested in whether the ordinal aspect of numerical knowledge is a crucial piece of knowledge that preschoolers are lacking for accurate estimation. Given that the educational interventions in past research involve training numerical knowledge in a spatial context, it is impossible to tease out the role of early numeracy ability in the development of numerical line skills. Therefore, the present study aimed to directly address the question of the independent impact of number skills and spatial skills in the number line task by comparing the improvement in estimation performance, in two groups of children, one trained on pure spatial skills, and the other trained on ordinal knowledge.

With respect to the number line task, I used a small number range (i.e., 0 – 10 number line) that was familiar to the young children. Because the results of Xu (2012) showed that the inclusion of an explicit midpoint reference on the number line is helpful for children who have relatively better understanding of number system knowledge, in the present study I included both standard and midpoint reference versions of the number line task. Children were also tested on basic numerical knowledge and spatial knowledge to investigate how these skills would be related to their ability to estimate.

The study consisted of three sessions, including a pretest, a training, and a posttest session. In the pretest, both groups of children performed tasks to measure their number skills and spatial skills. The numerical measures included four tasks: a number line task, a counting task, a number recognition task, and an ordinal task. The spatial measures include three tasks: a spatial span task, a 2-D mental rotation task, and a 3-D mental rotation task. During the training phase, in the number group, children were trained on the understanding of ordinal knowledge among numbers from 1 to 10. In the spatial group, children were trained on spatial mapping of 2-D shapes, which was inspired by the classic mental rotation task on spatial development (Levine, Huttenlocher, Taylor, & Langrock, 1999). Numerical skills and spatial skills were evaluated at posttest.

My hypotheses in relation to the training conditions were discussed as followed. I expected to see different types of *transfer of training* in the number game condition and spatial game condition accordingly. *Transfer of training* refers to the extent to which participants can apply what was learned in one context to another context (Baldwin & Fold, 1988). Based on the extent of similarity between the training and the task content, *transfer of training* can be categorized into two ways: near and far transfer (Royer, 1979). Near transfer requires a close match between the training and the task, and emphasizes specific concepts and skills, whereas far transfer requires an approximate match between the training and the task, and emphasizes general concepts and skills (Royer, 1979).

In the present study, I hypothesized that children in the number training group would show near transfer of training on the ordinal task and far transfer of training on the number line task. More specifically, because the number training was focused on enhancing the understanding of ordinal knowledge between numbers, I hypothesized that children would show improvements in the ordinal task after training. If the ordinal aspect of the numerical knowledge is the crucial skill to improve children's performance on the number line task, then children would benefit from training on ordinal knowledge by showing improvements in the number line task. Particularly, children would improve more in the midpoint version than the standard version, because children would count from the midpoint for numbers that are close to 5.

In contrast, I hypothesized that children in the spatial training group would show near transfer of training on a 2-D mental rotation task, given that the training was very similar to the 2-D mental rotation task used in the pre- and posttest. Moreover, I included a 3-D mental rotation task as indicative of children's sophisticated mental rotation skill. Given that there was little research on how preschool age children perform on 3-D mental rotation, the present study aimed to explore whether a simplified 3-D mental rotation task was appropriate for young children. I hypothesized that children in the spatial group would show far transfer of training on the 3-D mental rotation task. In summary, the expected results are showed in Figure 2.

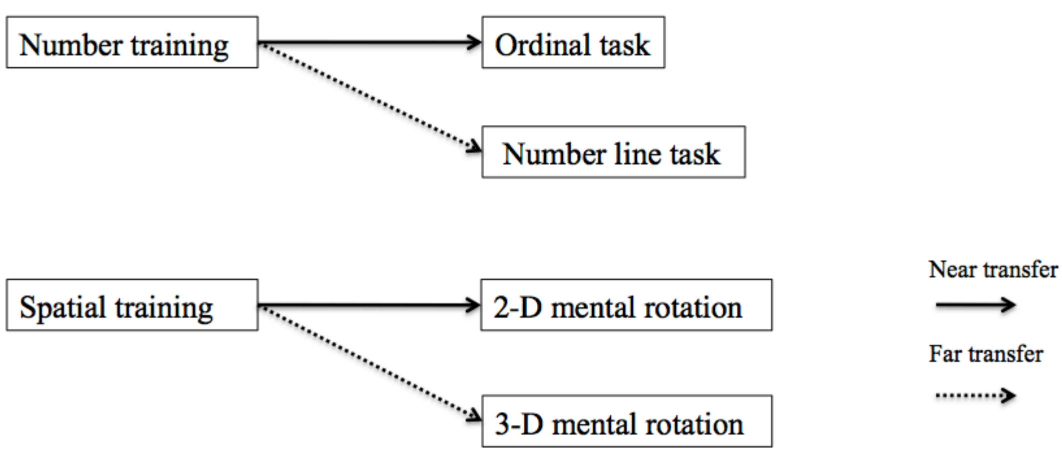


Figure 2. Expected results with respect to the training of transfer.

Method

Participants

A pilot study was conducted with seven children (comparable in the age range and daycare background to the participants in the present study) to ensure that all measures were appropriate and that time requirement for each task was reasonable.

In the present study, consent letters were provided to the parents/guardians of each child. Participation was completely voluntary. Eighty-four children (*Mean* age = 53 months, *SD* = 8 months; range from 3:4 to 6:1 years:months; 50% female) were recruited from 12 local daycares in Ottawa.

All children spoke English fluently. However, 37% of the children also spoke another language at home in addition to English. The most frequent two secondary languages were Chinese (10%) and French (8%).

Information about mother's highest education level was collected from the consent form for 79 participants: 33% had received a post-graduate degree, 35% had received an undergraduate degree, 20% had received a community college degree, 6% had received a high school degree, and 1% had received a less than high school degree (*Mdn* = Undergraduate degree). After the first session, all 84 children were randomly assigned to one of the training sessions: 42 children were assigned to the number training group (*mean age* = 54 months, *SD* = 8 months; 50% female) and 42 children were assigned to the spatial training group (*mean age* = 53 months, *SD* = 8 months; 50% female). The highest education that mothers received did not differ across groups, $t(77) = .23$, $p > .05$.

Procedure

The study included three 15- to 20- min sessions within a 3-week period. Sessions were held in the children's classroom at a quiet place in the daycare. During Session 1, children were administered pretest measures of numerical and spatial skills. During Session 2, children played an educational training game with the experimenter. The same measures of numerical knowledge and similar measures of spatial skills as in Session 1 were presented as the posttest in Session 3.

At the beginning of each session, children were told that they were going to play a game named 'Collecting Stamps". For each session, children were given a stamp-collecting card, with seven circles presented in two rows. They were told that they were going to play seven different games (Session 1 and 3) or similar games (Session 2) with the experimenter, and that after they completed each game, they got a stamp. Their goal was to collect all stamps on the card in order to get a cartoon sticker. Two children did not complete all of the tasks in the first session due to illness and thus were not included in the study. However, they were given a sticker.

Pre- and posttest Measures

All children completed four tasks that measured their numerical knowledge and three tasks that measured their spatial knowledge in both pre- and posttest. Half of the children were exposed to the set of numerical tasks first, and the other half of the children were exposed to the set of spatial tasks first. The order of which set of tasks children was exposed to was predetermined randomly across children.

Numerical Measures. Children were presented four tasks – a counting task, a number recognition task, an ordinal task, and a number line task – in the same order on the pretest and posttest.

Counting. Children were asked to count from 1 through 20. There were two reasons why children were stopped when they reached 20. First, I was most interested in whether children could count up to 10 as a requisite skill to do the 0 – 10 number line task. Second, the English number naming system after number 20 is relatively regular (e.g., twenty-one, twenty-two, twenty-three) compared to numbers in the teens (e.g., eleven, twelve, thirteen); thus, children who can count up to 20 are more likely to generate numbers names beyond 20 based on the regularity of the number naming system.

In the task, the experimenter used a puppet to engage the child’s interest claiming that the puppet did not know how to count beyond 3 and wanted to learn how from him/her. If the child stopped, two types of prompting were used. For example, if the child stopped at 7, (1) the experimenter said, "What is the number after 7?" If this type of prompting did not elicit more counting, (2) the experimenter repeated the last three numbers counted in a rising tone, “5, 6, 7?” Counting was coded as correct up to the point of the first error.

Number recognition. The task involved 20 randomly presented cards, each with a number from 1 to 20 (inclusive) printed on it. On each trial, the experimenter held up a card and asked the child to identify the numeral. The stimuli were presented in two sets. The small set included numbers from 1 to 10 (inclusive), and the large set included the numbers from 11 to 20 (inclusive). The order of the presentation was random, but the small set was introduced first. Children who were able to identify all or most (more than 7 numbers) in the small set were asked to continue identifying the numbers in the large set. No feedback was given regarding the accuracy of the identification of the numbers. The accuracy of number recognition performance was measured. Each number that children correctly recognized was scored as 1 such that the minimum score is 0 and maximum score is 20.

Ordinal task. The ordinal task was similar to the one used by Lyons and Beilock (2009). In order to make it child-friendly, instead of judging whether a set three numbers (e.g., 4-7-9) was in an ascending order or not, children were asked to point the numbers in order from the smallest to the biggest. The experimenter said: “On each card, there are three numbers. Can you point to the numbers from the smallest to the biggest?” Two types of sequences were presented: *counting sequences* and *neutral sequences*. The numbers in a *counting sequence* were sequential (e.g., 1-2-3 or 5-3-4) whereas the numbers in a *neutral sequence* had no identifiable relationship between numbers (e.g., 4-7-9 or 1-8-3). To eliminate the possibility of any effect resulted from memorizing simple arithmetic fact, no set of numbers resembled an arithmetic operation (e.g., 1-3-4, given that $1 + 3 = 4$) or an arithmetic pattern (e.g., 2-4-6 or 3-5-7). All numbers in each set were between 1 and 10. A total of 10 questions were asked to each child in a random order (see full stimuli in Appendix 1). Each question that children correctly answered was scored as 1 such that the minimum score is 0 and maximum score is 10. The results of pretest on the ordinal task revealed a high reliability, Cronbach’s $\alpha = .86$.

Number line task. Children were asked to complete a standard version and a midpoint version of a number line task on an iPad application, with “0” at the left end and “10” at the right end. The only difference between the two versions was that an explicit midpoint (5) was included in the midpoint version, whereas no midpoint was included on the number line in the standard version. To control for practice effects, half of the children did the standard version first, whereas the other half did the midpoint version first. The order of the version for the pre- and posttest was the same for each child.

Standard version. At the beginning of the task, children were told that they were going to play a number game. When the number line appeared on the screen, the experimenter firstly explained the endpoints of the number line: "We call this black line 'a number line' because there are numbers on it! At one end (pointing to "0"), we have number 0; at the other end (pointing to "10"), we have number 10. There are some other numbers hidden in the line. Those numbers are 1, 2, 3, 4, 5, 6, 7, 8 and 9. Your job is to find those numbers out one by one."

After the instructions, children were asked to complete three practice trials to get familiar with the procedure. Children were not required to make actual estimates in the practice trials. Instead, the target location was clearly marked on the number line, and children were asked to touch that mark on the screen. The experimenter said: "Before the game starts, let's do some practices. Can you find the green mark on the number line and touch it?" Positive feedback was provided for each practice trial children successfully completed.

Followed by the practice trials, children were asked to complete the experimental trials. Eight numbers to be located on the line (i.e., 1, 2, 3, 4, 6, 7, 8, 9) were randomly presented one at a time on the top of the screen. Number 5 was not included in the stimuli set to be consistent with the stimuli set of the midpoint version.

Children were firstly asked to identify the target number N (e.g., Number 4) at the top of the screen. If the child erred or failed to identify the numbers, the experimenter correctly named the number and then asked the child to repeat it. Then, children were asked to find out the estimated location of the target number on the number line: “Can you show me where number 4 goes on this number line?” No feedback was given regarding the accuracy of their marks. The iPad recorded responses automatically.

Midpoint reference version. The stimuli for the midpoint reference condition also consisted of eight numbers (i.e., 1, 2, 3, 4, 6, 7, 8, 9). The procedure was identical to the standard version, except that a brief instruction on the midpoint was given. That is, when the number line appeared on the screen, after the experimenter explained the endpoints of the number line, the experimenter said: “We have a little hint for you! Look, right in the middle (pointing to “5”), number 5 is already marked for you! ”

Strategy coding. The strategies that children used to estimate each target number were systematically recorded. The experimenter carefully observed the type of strategy children used for each number. If no visible strategy was observed, then the experimenter asked: “How do you know number 4 goes here?” Children who answered ‘I do not know” or failed to provide an answer were categorized into the ‘Guess’ strategy category. Given the young age range in my sample, I did not expect any complex estimation strategies such as using implicit reference points based on proportional judgments. Thus, only three types of strategies were coded: guess, counting, and midpoint strategy (see descriptions in Table 1). No other visible strategies were observed. However, some children did not need to explicitly rely on counting using their fingers to locate the estimated number after a few trials. For example, if they just estimated the location of number 2, and they were asked to estimate the location of number 3 in a subsequent trial, then they directly located it as a bit further away from number 2 without counting. This was also considered as a counting strategy.

Table 1

Descriptions of each strategy category

Strategy	Descriptions
Guess	Children have difficulty understanding the goal of the task and thus randomly locate numbers on the line (i.e., no visible strategy).
Counting	Children count forwards from the left endpoint (i.e., 0) to a target or backwards from the right endpoint (i.e., 10) to a target.
Midpoint	Children count either forwards or backwards from the midpoint (i.e., 5) to a target.

Spatial Measures. Children were presented three tasks to measure their spatial knowledge – a 2-D mental rotation task, a 3-D mental rotation task and a spatial span task – in the same order on the pretest and posttest.

2-D mental rotation task*.* Children were asked to complete a subset of items from a 2-D mental rotation task developed by Levine et al. (1999). The stimuli consisted of 12 trials (2 for practice trials, and 10 for experimental trials). Each trial consisted of two halves of a shape and a 2 x 2 choice array that could be formed from the halves (Levine et al., 1999). Children's task was to select the whole shape from among four choices that was formed by the two pieces presented in different orientations (see Figure 3). The target pieces in each trial were displayed on a white sheet of paper, and the four choice shapes were displayed in a 2 x 2 arrays on a separate sheet of paper (Levine et al., 1999). The task is suggested to be appropriate for children ranged from 4 to 7 years of age (Levine et al., 1999).

At the beginning of the task, children completed two practice trials to understand the 2-D mental rotation task. The experimenter demonstrated the first practice trial to children: "Look at these two pieces (pointing to the target pieces). If I put these two pieces (pointing to the target pieces again) together, they will make one of the pictures (pointing to the correct picture these pieces make)". On the second practice trial, children were asked to try to find the correct picture in the choice array that the two target pieces formed under guidance. Feedback was given if children made any mistakes on this item.

On subsequent trials, children were asked to complete a total of 10 experimental trials without any feedback. The procedure was identical to the practice trials. The order of the presentation was random. Each trial that children correctly selected the picture that was matching the target pieces was scored as 1 such that the minimum score is 0 and the maximum score is 10. The reliability analysis based the 10 items showed a Cronbach's α of .61. Given that the value of α depends on the number of items on the scale, the relatively low reliability was possibly due to the reduced item number from the 32 items of the original 2-D mental rotation task by Levine et al. (1999). No outlier was detected through the inspection of *z*-scores.

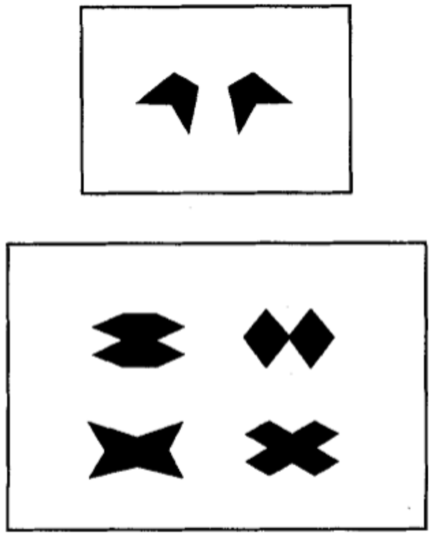


Figure 3. Example of a 2-D mental rotation task.

3-D mental rotation task. The 3-D mental rotation task was measured by having children mentally rotate 2-D representations of 3-D objects or real 3-D objects, based on classic measures of mental rotation (Shepard & Metzler, 1971). Children were presented one target item and three alternatives (one identical item, one mirror item, and one structurally distinct item) and their task was to select the one item that was identical to the target item after mental rotation. In the present study, two types of 3-D mental rotation task were used: real 3-D mental rotation task and pictures of 3-D mental rotation task (see Figure 4). The only difference between the two types was that whether the stimuli consisted of real objects constructed of wooden blocks or pictures of those objects. I was interested in whether this hard task was easier for young children when real objects were used. Half of the children were exposed to the real 3-D version in the pretest and pictures of 3-D version in the posttest, and the other half were exposed to the pictures of 3-D version in the pretest and real 3-D version in the posttest. The order of which type of task children were exposed to was predetermined randomly across children.

Four demonstration trials were completed together by the child and experimenter prior to the experimental trials to ensure understanding of the task. The first three demonstration trials required the child to make judgments about whether the two objects or pictures of the objects were the same or different. At the beginning, the experimenter said: “we are going to play a game with some shapes. First I am going to teach you how to play the game.” The first demonstration trial included a pair of *same* 3-D objects/pictures of the objects: “These shapes are the same. Do you see how they are same?” Children were allowed to respond. The experimenter explained how the two shapes are the same if necessary. The second demonstration trial included a pair of *different* 3-D objects/pictures of the objects (presented as mirror images): “These shapes are different. Do you see how they are the different? Children were allowed to respond. The experimenter explained how the two shapes were different if necessary “They look very similar, but they go different directions (pointing)”. The third demonstration trial included a pair of *same* 3-D objects/pictures of the objects *with one in 90 degrees clockwise*: “These shapes are the same, but they go a different way (flip left-hand shape to the left 90 degrees). If we turn one of these, they can go the same way. So these shapes are the same”. In the real objects version, children were asked to turn one of the objects and to make it go the same way. If the child erred or failed to do so, the experimenter demonstrated.

The last demonstration trial included a sample item. Children were shown a target object and a choice of three objects, with the choice objects closet to the child and target object directly above it. “Now, you have to look at each of these shapes carefully without touching them (pointing to each one) and choose the one that is the same and could go the same way as the special shape (pointing to the target object). You want to be sure of your answer, so don't go fast. When you are ready you will just point to the shape you think as the same and can go the same way as this one (pointing to the target object)". If the child erred or failed to provide an answer, feedback was given: "I chose this shape. Could you put that shape here and show me how it can go the same way?"

The 3-D mental rotation task consisted of 12 experimental trials. The procedure was identical to the last sample except that no feedback was provided. For the real object version, children were asked to close their eyes while the experimenter set up the stimuli for each trial: "From now on, the game starts! I want you to close your eyes." Then the experimenter placed the real blocks on the table based on a booklet with pictures of the blocks. Children were then asked to open their eyes and to select the one object that was the same and could go the same way as the target object.

The stimuli differed in the degrees of orientation relative to the target object and were ordered from the easiest to the hardest. The task stopped when children made three consecutive errors. Each trial that children correctly selected the object/picture of the object that was matching the target shape was scored as one such that the minimum score is 0 and the maximum score is 12. The reliability analysis based the 12 items for both pre- and posttest showed a Cronbach's α of .27, suggesting that the 3-D mental rotation task was not very reliable and possibly too difficult for young children.

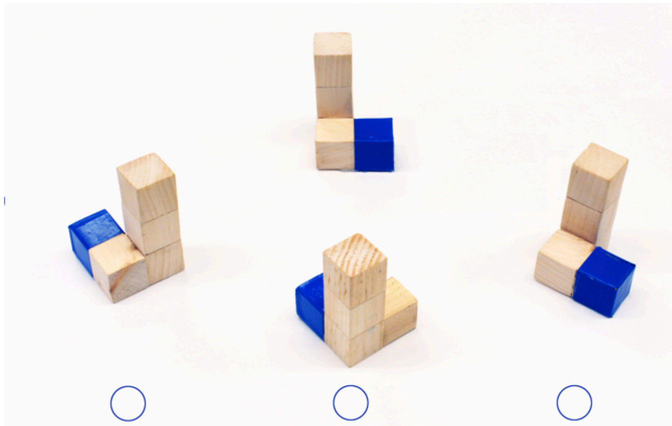


Figure 4. Example of the 3-D mental rotation task.

Spatial span task. A Spatial Span Task was used to measure children's spatial attention and working memory ability (McLean & Hitch, 1999) using an iPad application. This was similar to the corsi-span task, which is used frequently in similar research (LeFevre et al., 2010; Rasmussen, Ho, & Bisanz, 2003).

At the beginning of the task, a demonstration trial was utilized to explain the procedure to the children: "On the screen a set of green circles will appear. After I press the button 'play', some of the green circles will light up one by one. At first, only two circles light up. I need to copy the sequence that the circles light up by touching the circles on the screen". In the experimental trials, each sequence included three trials. The stimuli consisted of three trials at each of sequence lengths 2 through 8. The task automatically stopped when the child erred in all three trials within a sequence. The sequences of pattern followed by the child were automatically marked each time by the iPad for each child to determine the maximum sequence length. The minimum score was 1 (from the practice trial) and maximum score was 22.

Training session

In the training session, all children participated in a 15-minute intervention session in which they played their allocated game. Children were randomly assigned to one of the two sessions: *Number game condition* and *spatial game condition*.

Number game condition. The number game consisted of 7 target numbers (2 through 8). Children were also provided eight numbers between 1 and 10 other than the target number for them to choose from randomly predetermined (see full stimuli set and an example of the number training in Appendix 2). Children were trained on selecting the number comes before the target number and the number comes after the target number. This training aimed to guide children to use their current knowledge on numbers (i.e., counting and recognizing numbers) to understand the ordinal relations among numbers.

The experimenter first showed the sheet of paper with the target number in the centre and children were asked to name the target number. Help was provided if the child was unfamiliar with the numbers. After they identified the number, children were guided to find the number comes after the target number first, and then to find out the number comes before the target number. For example, the experimenter said: “Do you know what is the number after 3 when we are counting?” If the child correctly answered the question, then they were shown a separate sheet of paper with eight numbers presented in a row in a predetermined randomized order: “Good job! Number 4 comes after 3. Now, can you find number 4 from here (referring to the separate sheet of paper with eight randomly presented numbers on it)?” However, if the children erred or had difficulty finding out the number comes after the target number, help was provided: “We can count to find out the answer! Let's count together: 1, 2, 3…? (in a rising tone)". Children were encouraged to say the number themselves. Then, the experimenter said, "Good job! Can you find number 4 from here ".

A similar procedure was used for children to select the number before the target number. After children found the number comes after the target number, the experimenter asked: "Now, do you know what is number before 3 when we are counting?" If they erred or had difficulty, help was provided: "Again, let's count to find out the answer! 1, 2, 3 (in a falling tone)! What is the number before 3? If they still had difficulty, children were asked to count slowly with the experimenter again: "Let's count again. This time, listen really carefully to find the number before 3. 1, 2, 3 (emphasized on number 2)". Each child was asked to complete seven training sets, with numbers between 2 and 8 inclusive for each target number. Overall, children had more difficulty finding the number that comes before the target number.

Spatial game condition. The spatial game consisted of seven sets of 2-D shapes. Each set consisted of six pieces that comes from three different shapes (three pairs of the two halves) and a target shape that could be formed from one pair of the two halves (see full stimuli set and an example of the spatial training in Appendix 3). The stimuli were made by vertically cutting each of the whole shape into halves from the four choices in Levine et al.'s (1999) task. The target shape in the training session was different from the target shape in the 2-D mental rotation task in the pre- and posttest to eliminate practice effect. Children were asked to find the two halves of the shape to match with the target shape. No mental rotation was required for this training.

First, they were given a set of six pieces and a target shape in separate sheets of paper. Their task was to find the two pieces from the set of that could form the target shape. They were allowed to try as many times as needed: “Look at this shape carefully (pointing to target shape). Two of these shapes (pointing to the six pieces) can make the same shape as this one (pointing to the target shape). Can you find out which two pieces can go together and make up this one (pointing to the target shape)?” Praise was given if the child correctly found out the two halves of the target shape. However, if the children erred or had difficulty finding out the correct pieces, help was provided: “Let’s find out together! When I cover half of this shape (using a piece of white card to cover half of the target shape), I see this piece. Then we need to find the matching piece from here (pointing to the six pieces)! Which one looks exactly same?” The procedure was repeated if the child still had difficulty finding the correct pieces. As a result, each child was asked to find out a total of seven sets of matching pairs.

Results

No significant differences were observed at the pretest between the children assigned to each of the two training conditions in any of the numerical measures (counting, number recognition, ordinal task, and number line task) and spatial measures (spatial attention, 2-D mental rotation, and 3-D mental rotation), *p*s > .05 (see Table 2). No effect of gender was found for all measures, *p*s > .05, except for the 3-D mental rotation task.

Table 2

Results from pre- and posttest for all measures across training conditions

Measures	Pretest Mean (*SD*)		Posttest Mean (*SD*)	
	Number Group	Spatial Group	Number Group	Spatial Group
Numerical measures				
Counting (max 20)	15.00 (5.23)	14.33 (4.92)	15.50 (4.41)	14.71 (4.97)
Number Recognition (max 10 each set)				
1 - 10	7.88 (2.66)	7.54 (3.49)	8.19 (2.59)	8.00 (2.97)
11 - 20	2.81 (4.28)	3.62 (4.37)	3.43 (4.61)	3.86 (4.46)
Ordinal Task (max 10)	3.67 (3.12)	3.57 (3.19)	5.93 (3.06)	4.07 (3.50)
Number Line (Standard)[1]	21.32 (11.87)	21.99 (13.46)	16.61 (9.55)	21.95 (10.44)
Number Line (Midpoint)[1]	21.03 (12.71)	21.66 (13.64)	11.15 (6.23)	22.30 (12.29)
Spatial measures				
2-D Task (max 10)	4.95 (2.27)	4.48 (2.20)	5.48 (2.37)	6.79 (2.18)
3-D Task (max 12)	2.31 (2.01)	2.61 (2.77)	2.57 (2.23)	2.60 (1.94)
Spatial Span (max 22)	4.48 (3.16)	4.12 (2.58)	4.48 (2.92)	4.52 (2.65)

[1]The accuracy of estimates for each trial was measured by percent of absolute error (PAE).

The partial correlations among the various numerical measures and spatial measures after controlling for age (in months) are shown in Table 3. All measures were significantly correlated except the 3-D mental rotation task. Further analyses were conducted using analyses of covariance (ANCOVA) and analyses of variance (ANOVA), to examine whether children benefited differentially on each of the task as a function of training conditions.

Numerical Measures. To account for varying pretest scores, the posttest scores were compared using an ANCOVA with pretest score as the covariate for each measure, and training condition (number vs. spatial) as a between-subject factor to examine the training effect for the counting, number recognition and ordinal task.

Counting. Results of the counting task revealed no significant effect of training condition, indicating no improvement after training for both number and spatial group, $p > .05$ (see Table 2). Moreover, the majority of children (88%) could count up to 10.

Number recognition. Results of number recognition task were examined separately for the small set (1 – 10) and the large set (11 – 20). For both sets, no significant effect of training condition was observed, *ps* > .05, indicating no improvements after training for both number and spatial group (see Table 2).

Table 3

Partial correlations Among Children's Performance on Numerical and Spatial Measures in pretest (controlling for age)

Measures	1	2	3	4	5	6	7
1. Counting	-						
2. Number Recognition	.66***	-					
3. Ordinal Task	.35**	.41***	-				
4. Number Line (Standard)	-.39***	-.44***	-.33**	-			
5. Number Line (Midpoint)	-.47***	-.51***	-.39***	.73***	-		
6. Spatial Span	.47***	.47***	.38***	-.42***	-.58***	-	
7. 2-D Task	.32**	.43***	.44***	-.46***	-.49***	.54***	-
8. 3-D Task	.13	.20	-.04	-.10	-.04	.22*	.08

* $p < .05$; ** $p < .01$; *** $p < .001$

Ordinal task. The ordinal task required children to indicate the sequence of three numbers from the smallest to the largest. Results of the pretest scores showed no difference in performance between counting sequences ($M = 1.80$) and neutral sequences ($M = 1.81$), $p > .05$. Results also showed a significant effect of training, $F(1, 81) = 13.44$, $MSE = 4.97$, $p < .001$, $\eta^2 = .14$. Children in the number group performed better than children in the spatial group after training, suggesting that children in the number group showed improvements on the ordinal task whereas children in the spatial group did not (see Table 2).

Number line task. First, the mean PAE for both standard version and midpoint version across training conditions at pre- and posttest were examined using a 2 (condition: number vs. spatial) x 2 (version: standard and midpoint) x 2 (time: pre- and posttest) mixed ANOVA with repeated measure on the last 2 factors. No order effect of version (standard version first vs. midpoint version first) was found for the pretest, $p >$ .05.

The main effect of time was significant, $F(1, 82) = 14.36$, $MSE = 71.49$, $p < .001$, $\eta^2 = .15$. Children's estimates were more accurate at the posttest ($M = 18.00$) than at the pretest ($M = 21.50$). The main effect of version was also significant, $F(1, 82) = 5.57$, $MSE = 31.05$, $p = .021$, $\eta^2 = .06$. Children's estimates were more accurate in the version where an explicit midpoint reference was presented ($M = 19.03$) than the version where no explicit midpoint reference was presented ($M = 20.47$).

The interaction between time x condition was significant, $F(1, 82) = 16.93$, $MSE = 71.49$, $p < .001$, $\eta^2 = .17$. Children in the number group showed improvements from pre- to posttest ($M = 21.18$ vs. 13.88) whereas children in the spatial group did not show any improvements (M = 21.82 vs. 22.12). The interaction between version and condition was also significant, $F(1, 82) = 5.61$, $MSE = 31.05$, $p = .02$, $\eta^2 = .06$. Children in the number group were more accurate in the midpoint version than the standard version (M = 18.97 vs. 16.09) whereas no difference between versions was found in the mean PAE for children in the spatial group (M = 21.97 vs. 21.98). The interaction between time and version was not significant, $p > .05$.

The 3-way interaction between time, version and condition was significant, $F(1, 82) = 5.60$, $MSE = 32.05$, $p = .02$, $\eta^2 = .06$. As shown in Figure 5, children in the spatial group did not show any improvements from pre- to posttest in either standard or midpoint version. In contrast, children in the number group showed significant improvements from pre- to posttest in both standard and midpoint version. Moreover, children showed more improvements in the midpoint version than the standard version. These results suggest that training on ordinal relations among numbers supported children's numerical estimation skills, and that children benefited more from having an explicit midpoint reference presented than no midpoint presented on the number line.

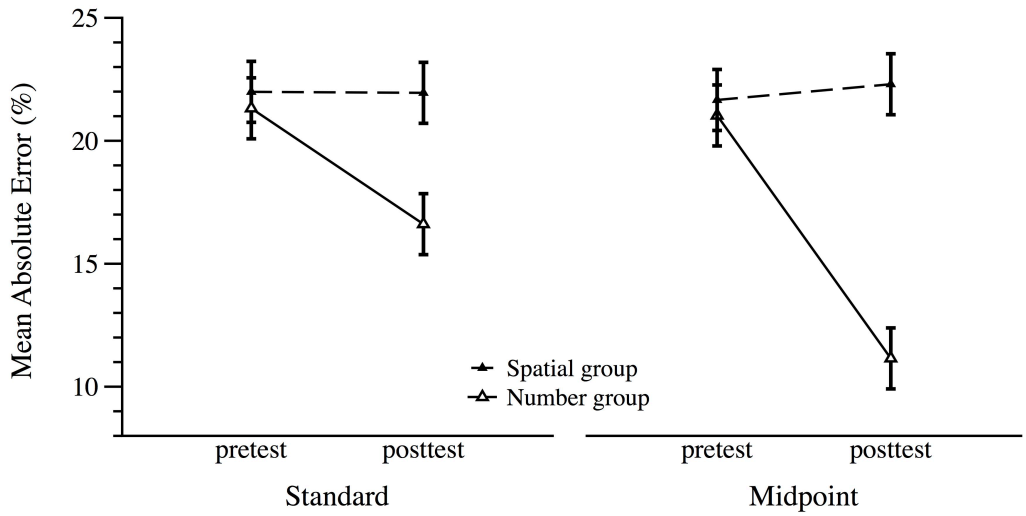


Figure 5. Mean PAEs in the standard and midpoint version over time across training conditions. Error bars are 95% confidence intervals based on the *MSE* values for the 3-way interaction.

Next, the pattern of errors for each target number was examined. The mean PAE for both standard version and midpoint versions across training conditions at pre- and posttest were analyzed in a 2 (condition: number vs. spatial) x 2 (version: standard and midpoint) x 2 (time: pre- and posttest) x 8 (target number: 1, 2, 3, 4, 6, 7, 8, 9) mixed ANOVA with repeated measure on the last 2 factors[1].

As shown in Figure 6a, the patterns of errors for the number group were different for the standard version and the midpoint version. In the standard version of the number line task, children's estimates improved significantly for target numbers 1, 8 and 9 after training only. The results indicate that children used the left endpoint as a reference point to locate number 1 and used the right endpoint to locate number 8 and 9. In the midpoint

[1] Detailed statistics are not reported here. Confidence intervals based on the *MSE* values for the 4-way interaction were used to examine the patterns of errors across target number (Figure 6).

version, children's estimates improved significantly for all target numbers (the mean difference for number 4 was approaching significance; see Figure 6a).

In contrast, the pattern of errors for the spatial group was similar across version of the number line task (See Figure 6b). Children did not show any improvements for any of the target number except for number 8 in the midpoint version. The results suggest that training on mapping of 2-D shapes did not help children to improve their estimation performance.

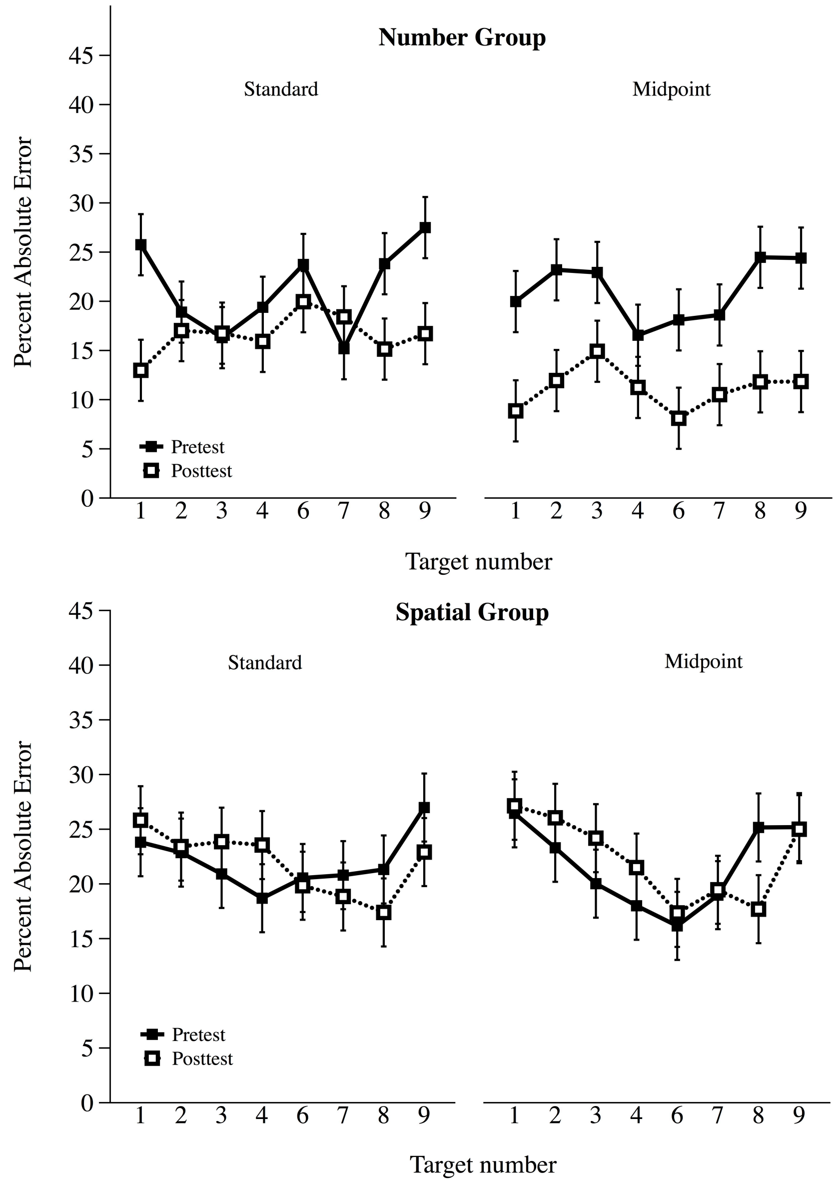


Figure 6. Mean PAE for each target number for standard and midpoint version over time for number (a) group and spatial group (b). Error bars are 95% confidence intervals based on the *MSE* values for the interaction. Number 5 is not shown on the x-axis because it was not included in the stimuli set in either standard or midpoint version.

Lastly, children's strategy used for each target number was analyzed for the number group. I was interested in whether children shifted their strategy after the ordinal training. In the standard version of the number line task, the percentage of children who shifted from guessing to counting strategy from pre- to posttest[2] is shown in Figure 7a. No midpoint strategy was observed because no explicit midpoint reference was presented in the standard version. As shown in Figure 7a, children shifted their strategies from guessing to counting for all target numbers. The results show that training on ordinal knowledge helped children use counting strategy in the number line task.

In the midpoint version of the number line task, the percentage of children who shifted from guessing to counting strategy, guessing to midpoint strategy, and counting to midpoint strategy from pre- to posttest is shown in Figure 7b. The results show that children differentially shifted their strategies for different target numbers. More specifically, for numbers 1, 2, and 3 that are closer to the left endpoint, children shifted their strategies from guessing to counting strategy. Starting from number 4, children shifted their strategies from guessing to midpoint strategy after the training. In addition, for numbers 6 and 7, some children shifted their strategies from counting to midpoint strategy after the training. The results indicate that children used both counting and midpoint strategies after the number training when an explicit midpoint reference was presented. For numbers larger than 5, children started from the explicit midpoint reference and counted up toward the target number, and therefore their estimation performance improved significantly.

[2] For example, for number 1, in the pretest, 43% of the children guessed, and 57% used counting strategy; in the posttest, 17% guessed, and 83% used counted strategy. Thus, 26% (83% - 57%) of the children shifted from guessing to counting strategy.

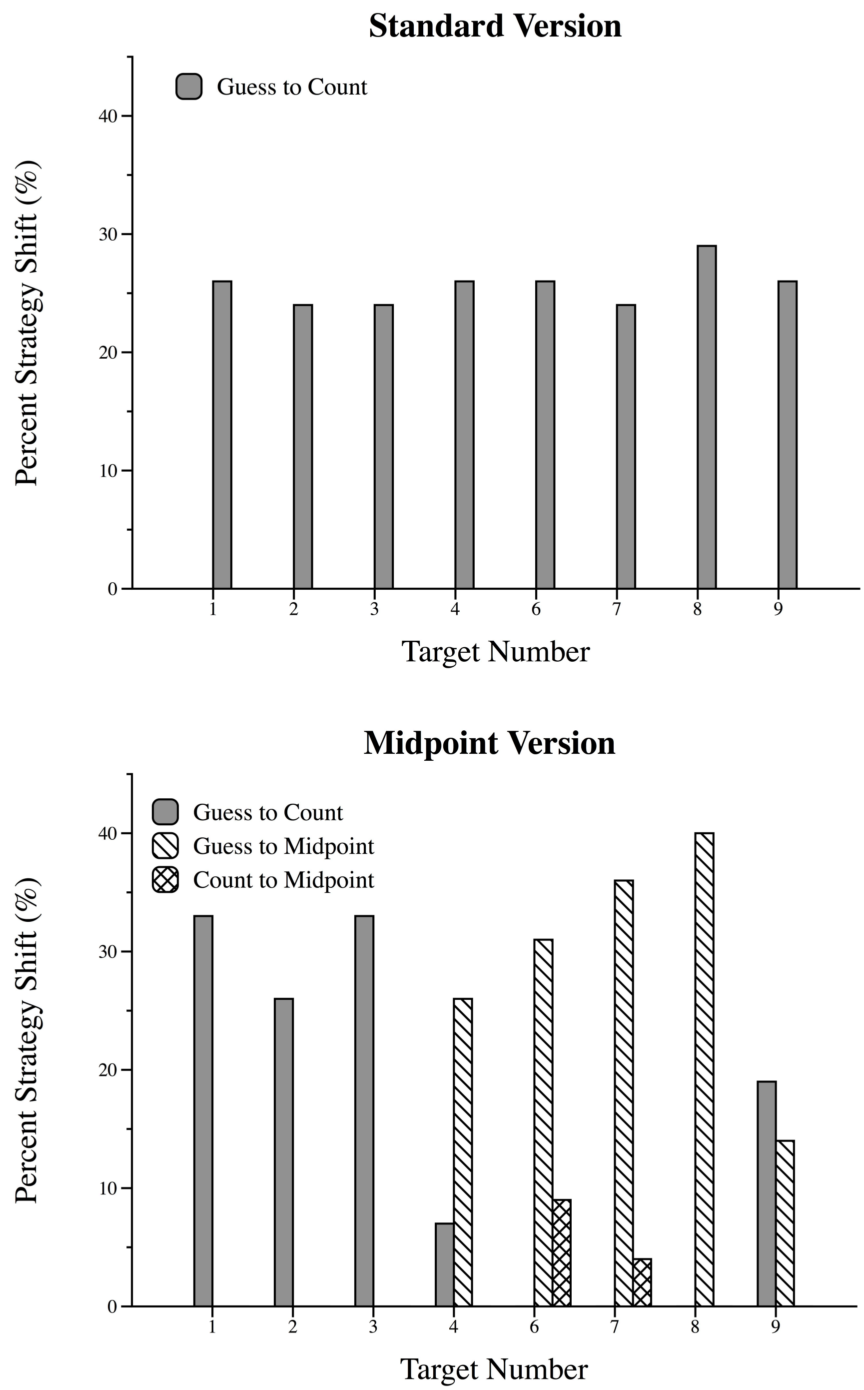


Figure 7. Percentage of children who shifted their strategy from guess to counting strategy in the standard version (a), and who shifted their strategy from guess to counting, guess to midpoint, and count to midpoint strategy in the midpoint version (b) for number group.

Spatial measures. For the 2-D mental rotation task, 3-D mental rotation task, and spatial span task, the effect of training conditions was examined by ANCOVA with pretest score as the covariate for each measure, and training condition (number vs. spatial) as a between-subject factor.

2-D Mental rotation task. Results of the 2-D mental rotation task showed a significant effect of training condition, $F(1, 81) = 20.53$, $MSE = 2.74$, $p < .001$, $\eta^2 = .20$. After the training, children in the spatial group performed better than children in the number group, suggesting that training on spatial mapping of various shapes helped children improve on the 2-D mental rotation task (see Table 2).

3-D Mental rotation task**.** Lastly, children completed the 3-D mental rotation task using real 3-D objects and pictures of the objects (in pre- and posttest; the order was randomly predetermined). Results showed no effect of training condition, $p > .05$ (see Table 2). In addition, children performed poorly across all age groups (see detailed information Appendix 4). These results suggest that the 3-D mental rotation task was not appropriate for preschool age children.

An ANOVA was then performed to examine whether children performed better when real objects were used and whether there was a gender effect while controlling for age (in months). The main effect of stimuli type (real, picture) approached significant, $F(1, 80) = 3.49$, $MSE = 3.65$, $p = .06$, $\eta^2 = .04$. Children performed marginally better using the real objects ($M = 2.81$) than with pictures of the objects ($M = 2.22$). The advantage of using real objects appeared in children who were older than 5 years of age (see Appendix 4). The main effect of gender was also significant, $F(1, 80) = 9.94$, *MSE*

$= 4.88$, $p = .002$, $\eta^2 = .11$. Girls ($M = 3.07$) performed better than boys ($M = 1.94$). The interaction between type and gender was not significant, $p > .05$.

Spatial span task. Results of the spatial attention task showed no significant effect of training condition, indicating no improvements for children of both groups, $p > .05$ (see Table 2).

Discussion

The present study focused on examining the role of ordinal knowledge in performance on the number line task for preschool-aged children. The goal was to examine whether children benefited from having an explicit midpoint reference on the number line (i.e., showing the location of 5 on a 0-10 number line) after short-term training designed to enhance their ordinal knowledge (e.g., what number comes before and after 4?). The results show that children improved on both an ordinal task and a number line task after the ordinal training, indicating the effectiveness of the training. The results also show that children showed more improvement in the version of the number line task where an explicit midpoint reference was presented than when no midpoint was presented, suggesting that the understanding of ordinal relations among numbers helped children to use the midpoint to refine and improve their solution strategies.

The results show that children shifted their strategies from guessing (i.e., no visible strategy) to counting (i.e., counting from endpoints and/or counting from the midpoint) after the ordinal training. More specifically, in the standard version where no midpoint was presented, children shifted from guessing to counting from the endpoints (i.e., 0 and 10) to locate the estimated number, which resulted in more accurate estimates for numbers 1, 8 and 9 after the training. This was consistent with the finding by Ashcraft and Moore (2012) that children always estimate most accurately on numbers that are closer to the two endpoints of the line (i.e., 0 and 100). Despite the overall improvements in the standard version after the ordinal training (see Figure 5), no significant improvements were observed for numbers that were further from the endpoints. Based on observations in the present study, when children used the counting strategy, they moved their fingers along the number line until they counted up to target number. For numbers that are farther from the endpoints (e.g., number 6), the physical distance they moved while counting might be too small or too large (e.g., their counting increments underestimated or overestimated the actual size of each unit) compared to the actual magnitude. This inaccuracy might be because preschool-aged children lack knowledge of proportions, for example, that the line is divided into equal-sized units. Thus, they were not very accurate when estimating numbers that are farther from the endpoints when no midpoint was presented.

In contrast, in the midpoint version where an additional midpoint reference was presented, children shifted from guessing to both counting from the endpoints and counting from the midpoint to locate the target number. When the target number is closer to the explicit midpoint reference of the number line than to one of the endpoints (e.g., numbers 4, 6, and 7), children adjusted their strategies in two ways. First, they counted directly from the midpoint toward the number, which resulted in more accurate estimates for all numbers after the ordinal training. Alternatively, they firstly counted from the one of the endpoints, and then made adjustments based on the location of the explicit midpoint reference. For example, when children were asked to locate number 6 on the number line, they started to count from 0. As they counted and moved the finger along the number line, they might find that the location of ‘5’ they estimated did not match with the actual location of ‘5’ presented on the number line (i.e., the midpoint reference). Then, they counted from 0 again and adjusted the physical distance for each number as they moved their fingers, which led to more accurate estimates.

In summary, the results of the present study indicate the crucial role of ordinal knowledge on children’s performance on the number line task. On a number line with a smaller range (0 – 10), children as young as preschool age improved with one session of training focused on ordinal relations among numbers. In addition, with appropriate training on ordinal knowledge, children improved more in the midpoint version of the number line task than in the standard version because they used the explicit midpoint as a reference to estimate nearby numbers. These results indicate that children need to understand the ordinal relations among numbers to benefit from having an additional reference point on a number line.

The results of the present study did not support the assumption that different patterns of performance on the number line task for preschool children (i.e., linear vs. logarithmic) reflect the mappings between symbolic (e.g., 4) and pre-existing non-symbolic representations of number or quantity (e.g., the magnitude of 4; e.g., Booth & Siegler, 2006). Alternatively, different patterns of performance may reflect different levels of understanding of ordinal relations among numbers. The present study suggests that children need to have the ordinal piece of the number system knowledge to perform well on the number line task. These results support the view that the representation of ordinal relations among numbers is fundamental for the acquisition of more complex arithmetic skills (Lyons & Beilock, 2011), such as number line estimation.

In the present study, children showed significant improvements on the number line task after brief 15-minute training on ordinal knowledge. In contrast, other training studies required four sessions of practice on a number game (i.e., total of about 60 minutes; e.g., Ramani & Siegler, 2008; Whyte & Bull, 2008). The reason that one session of training could be as effective as four sessions of training may be because that the traditional interventions involve training on multiple aspects of numerical and spatial knowledge whereas the present study was focused on training a relatively specific numerical skill. All of the interventions in past research involved practice on a wide range of skills. As a result, children improve not only on performance of the number line task, but also on counting, number recognition, and numerical magnitude comparison. Moreover, the content and format of linear board games (i.e., a linear arrangement of numbers presented on a line) and the number line task are very similar (Mix & Cheng, 2012). When children are exposed to linear number board game four times during training, they may remember the layout of the number line. Thus, the improvement on performance of the number line task might be facilitated by the spatial cues of the linear board game as well as by other aspects of the experience with numbers and counting. Given the various types of spatial skills and math skills that children are trained on from these number board games, the source of the improvements on the number line task remained unclear.

In contrast, in the present study, children were trained on a specific numerical skill - the ordinal understanding among numbers. During the brief training, children were asked to find the numbers that came before and after a number. In this way, children were trained to apply their previous knowledge of numbers (i.e., knowledge of counting and number recognition) to determine the ordinal relations among numbers of 1 to 10 using a counting strategy. They improved on the ordinal training task and were able to generalize the knowledge they acquired to the number line task. In contrast, training on 2-D mental rotation did not facilitate number line performance. The present results suggest that providing children with increased ordinal knowledge and/or a relevant counting strategy in a very brief session is sufficient for most to show improved number line performance. It is possible that this skill is also acquired in the number board game interventions used by Siegler and colleagues but those interventions may influence many aspects of children's early numeracy skill. The present research provides a much more focused link between the children's learning in the training and their improved number line performance.

In contrast, training on spatial rotation did not improve number line performance. Although the spatial arrangement of the number board games in previous studies may have been helpful for children's performance on the number line, it was probably a very specific transfer given that the number board game arrangement is very similar to that of the number line task. Further research is need to determine whether specific spatial cues that link the number board game to the number line contribute to improved number line performance.

Limitations

Several limitations of the present study should be noted. First, it is unclear whether children's improvements on performance on the number line task are only a result of enhanced ordinal knowledge after training. During the training phase, children were explicitly trained to use counting as a strategy to find out the number before and after a target number. As a result, they were more likely to shift their strategies from guessing to counting after the ordinal training, which resulted in more accurate estimates. However, it is hard to tell whether children fully acquire the understanding of the ordinal relations among numbers after the training, or they just learned to use counting as a strategy as opposed to guessing. Children improved more in the midpoint version of the number line task than the standard version might be because they learned to count from 5 to estimate numbers that are larger than 5 instead of counting from 1 or 10. Moreover, despite the significant improvements on performance of the number line task after the ordinal training, the results from the ordinal task show that children in the number group still had difficulty in this task (M = from 3.7 to 5.9; SD = 3.1; max = 10). Thus, it is possible that children did not fully understand the ordinal relations among numbers. Therefore, the source of improvements on the number line task is still unclear.

A second limitation of the present study is regarding the design of the spatial training. The spatial skills being trained in the spatial group seem fairly close to the skills required for the 2-D mental rotation task used in the pre- and posttest. Children were trained on spatial mapping of 2-D shapes (presented horizontally or vertically; see Appendix 3), which was adapted from the 2-D mental rotation task developed by Levine et al. (1999). Given the similarity between the training and the 2-D mental rotation task, children were expected to show near transfer of training. As a result, whether children could generalize the mental mapping skills to other mental rotation tasks remains an open question.

The last limitation of the present study is that the inclusion of the 3-D mental rotation task is not appropriate and too difficult for preschool age children. The results of the present study show a floor effect such that children across all age groups performed poorly on the 3-D mental rotation task. In addition, children's performance in 3-D the mental rotation were not correlated with any numerical measures or spatial measures except the spatial span task, whereas children's performance in the 2-D mental rotation task were highly correlated with all measures (see Table 2). These results suggest that the 2-D mental rotation task is more likely to reflect children's true mental rotation ability than the 3-D mental rotation task. Hoyek, Collet, Fargier, and Guillot (2012) suggest that 3-D mental rotation tasks are too cognitively demanding for children in early grades of elementary school. Even for adults, the 2-D mental rotation task seems easier than 3-D mental rotation (Shepard & Metzler, 1988). Therefore, I concluded that the 3-D mental rotation task was not appropriate to assess preschool-aged children's mental rotation ability. Instead, the 2-D mental rotation task developed by Levine et al.'s (1999) was more suitable.

Future research

The results of the present study show that children's spatial skills (i.e., 2-D mental rotation and spatial span) were correlated with their performance on the number line task, suggesting one type of link between math and spatial skills. Past research shows that training on spatial ability can improve children's mathematics learning (Mix & Cheng, 2011). However, the present study was not focused on examining the effect of spatial training on children's performance on the number line task. Thus, the possibility that improvement in the number line task is a consequence of improved spatial skill occurring with development still remains an open question.

Children in the present study were trained on spatial mapping of 2-D shapes (i.e., related to mental rotation skill) and showed no improvements on performance of the number line task. It is possible that training on some other type of spatial skill would be helpful for children to perform better in the number line task. According to Mix and Cheng (2012), to measure spatial ability, two other types of spatial skills, spatial orientation and spatial visualization, should be considered. Spatial orientation refers to the ability to perceive the positions of various objects in pace, relative to each other and to the viewer, whereas spatial visualization refers to the ability to perceive complex spatial patterns and comprehend imaginary movements in space (Mix & Cheng, 2012). Further research could examine whether training on spatial skills related to spatial orientation or spatial visualization would improve children's performance in the number line task.

Implications

Understanding the core skills required for the number line task is an important first step in designing interventions and educational curricula that are intended to improve children's number line skill. The results from the present study indicate an educational importance of developing numerical activities that emphasize enhancing the understanding of ordinal knowledge for preschool-aged children. Thus, it may be helpful to develop a similar game that emphasizes the relative relationship of numerical orders as in the present study (i.e., what comes before and after a number) in the preschool classroom. The number game used in the present study requires little instructions for parents or teachers and also requires a short period of time to generate positive effects. Taken together, parents and teachers should provide children with rich experiences of number relationships to help them understand the meaning of numbers, which is the foundation for developing more sophisticated number representations (Jung, 2011).

Conclusion

The purpose of the present study was to explore the effects of short-term training on number line estimation performance. Preschool-aged children were exposed to a 15-minute training on ordinal knowledge or on 2-D spatial mapping. The results show that training on the ordinal relations among numbers improved number line performance. In addition, with appropriate training on ordinal knowledge, children improved more in the version of the number line task where an explicit midpoint was presented than the version where no midpoint was presented. These results indicate that children need to understand the ordinal relations among numbers to benefit from having an additional reference point on a number line. I conclude that the ordinal knowledge of the number system is crucial for successful estimation on the number line task.

References

Ansari, D. (2008). Effects of development and enculturation on number representation in the brain. *Nature Review Neuroscience, 9*, 278-291.

Ashcraft, M. H., & Moore, A. M. (2012). Cognitive processes of numerical estimation in children. *Journal of Experimental Child Psychology, 111*, 246-267.

Baldwin, T. T., & Ford, J. K. (1988). Transfer of training: A review and directions for future research. *Personnel Psychology, 41*, 63-105.

Barth, H. C., & Paladino, A. M. (2011). The development of numerical estimation: evidence against a representational shift. *Developmental Science, 14*, 125-135.

Berteletti, H., Lucangeli, D., Piazza, M., Dehaene, S., & Zorzi, M. (2010). Numerical estimation in preschoolers. *Developmental psychology, 46,* 545-551.

Booth, J. L, & Siegler, R. S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental psychology, 41*, 189-201.

Bouwmeester, S., & Verkoeijen, P. P. J. L. (2012). Multiple representations in number line estimation: A developmental shift or classes of representations? *Cognition and Instructions, 30,* 246-26.

Brannon, E. M., & Van de Walle, G. A. (2001). The development of ordinal numerical competence in young children. *Cognitive Psychology, 43*, 53-81.

Casey, M. B., Nuttall, R. L., Pezaris, E., & Benbow, C. P. (1995). The influence of spatial ability on gender differences in mathematics college entrance test scores across diverse samples. *Developmental Psychology, 31*, 697-705.

Dehaene, S. (2009). Origins of mathematical intuitions: The case of arithmetic. *Annals of the New York Academy, 1156,* 232-259

Delazer, M., & Butterworth, B. (1997). A dissociation of number meanings. *Cognitive Neuropsychology, 14,* 613-636.

Delgado, A. R., & Prieto, G. (2004). Cognitive mediators and sex-related differences in mathematics. *Intelligence, 32*, 25-32.

Gunderson, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2012). The relation between spatial skills and early number knowledge: The role of the linear number line. *Developmental Psychology*. Advance online publication. doi: 10.1037/a0027433

Hollands, J. G., & Dyre, B. P. (2000). Bias in proportion judgments: The cyclical power model. *Psychological Review, 107*, 500-524.

Hoyek, N., Collet, C., Fargier, P., & Guillot, A. (2012). The use of the Vandenberg and Kuse mental rotation test in children. *Journal of Individual Differences, 33,* 62-67.

Kurdek, L. A., & Sinclair, R. J. (2001). Predicting reading and mathematics achievement in fourth-grade children from kindergarten readiness scores. *Journal of Educational Psychology, 93*, 451-455.

Laski, E. V., & Siegler, R. S. (2007). Is 27 a big number? Correlational and causal connections among numerical categorization, number line estimation, and numerical magnitude comparison. *Child Development, 76*, 1723-1743.

Levine, S. C., Huttenlocher, J., Taylor, A., & Langrock, A. (1999). Early sex differences in spatial ability. *Developmental Psychology, 35*, 940-949.

LeFevre, J., Fast, L., Skwarchuk, S. L., Smith-Chant, B.L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child Development, 81*, 1753-1767.

Lyons, I. M., & Beilock, S. L. (2009). Beyond quantity: Individual differences in working memory and the ordinal understanding of numerical symbols. *Cognition, 113*, 189-204.

Lyons, I. M., & Beilock, S. L. (2011). Numerical ordering ability mediates the relation between number-sense and arithmetic competence. *Cognition, 121*, 256-261.

McLean, J. F., & Hitch, G. J. (1999). Working memory impairments in children with specific arithmetic learning disabilities. *Journal of Experimental Child Psychology*, *74*, 240–260.

Mix, K. S., & Cheng, Y. L. (2012). The relation between space and math: Development and educational implications, *Advances in Child Development and Behavior*, 197-243.

Newcombe, N. S. (2010). Picture this: Increasing math and science learning by improving spatial thinking. *American Educator, Summer*, 29-43.

Opfer, J. E., & Siegler, R. S. (2007). Representational change and children's numerical estimation. *Cognitive Psychology, 55*, 169-195.

Petitto, A. L. (1990). Development of numberline and measurement concepts. *Cognition and instruction, 7*, 55-78.

Royer, J. M. (1979). Theories of transfer of learning. *Educational Psychologist, 14*, 53-69.

Schneider, M., Heine, A., Thaler, V., Torbeyns, J., De Smet, B., Verschaffel, L., et al (2008). A validation of eye movements as a measure of elementary school children's developing number sense. *Cognitive Development, 23*, 409-422.

Siegler, R. S. (1996). Emerging minds: The process of change in children's thinking. New York: Oxford University Press.

Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development, 75*, 428-444.

Siegler, R. S., & Mu, Y. (2008). Chinese children excel on novel mathematics problems even before elementary school. *Psychological Science, 19*, 759-763.

Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: evidence for multiple representations of numerical quantity. *Psychological Science, 14*, 237-243.

Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games -- but not circular ones -- improves low-income preschoolers' numerical understanding. *Journal of Educational Psychology, 101*, 545-560.

Shepard, R. N., & Metzler, J. (1971). Mental rotation of three-dimensional objects. *American Association for the Advancement of Science, 171*, 701-703.

Ramani, G. B., Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development, 79*, 375-394.

Rasmussen, C., Ho, E., & Bisanz, J. (2003). Use of the mathematical principle of inversion in young children. *Journal of Experimental Child Psychology*, *85*, 89-102.

Rubinsten, O., & Sury, D. (2011). Processing ordinality and quantity: The case of developmental dyscalculia. *PLos One, 9*, e24079.

Whyte, J. C., & Bull, R. (2008). Number games, magnitude representation, and basic number skills in preschoolers. *Developmental Psychology, 44 (2)*, 588-596.

White, S. L. J., & Szucs, D. (2012). Representational change and strategy use in children's number line estimation during the first years of primary school, *Behavioral and Brain Functions, 8,* doi: 10.1186/1744-9081-8-1

Wynn, K. (1990). Children's understanding of counting. *Cognition, 36*, 155-193.

Xu, C. (2012). Effects of an Explicit Midpoint Reference on Number Line Estimation, *Manuscript in preparation.*

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition, 74*, 1-11.

Appendix 1

Full stimuli for the ordinal task

Stimuli Set	Target Number	Sequence Type	In order or not (From smallest to largest)
Set 1	1-2-3	Counting	Yes
Set 2	5-6-7	Counting	Yes
Set 3	4-6-5	Counting	No
Set 4	2-3-1	Counting	No
Set 5	8-6-7	Counting	No
Set 6	4-7-9	Neutral	Yes
Set 7	3-4-8	Neutral	Yes
Set 8	5-8-9	Neutral	Yes
Set 9	1-8-3	Neutral	No
Set 10	1-9-7	Neutral	No

Appendix 2

Full stimuli set of the number training

Target Number	Numbers to choose from
2	5-**3**-6-4-**1**-8-7-9
3	1-8-**2**-5-7-**4**-6-9
4	2-6-8-**3**-1-**5**-9-7
5	3-1-7-2-**6**-8-**4**-9
6	**7**-2-8-**5**-1-4-9-3
7	4-1-3-**6**-9-2-5-**8**
8	6-3-5-2-4-**7**-1-**9**

An example of the number training: Target number = 3

3

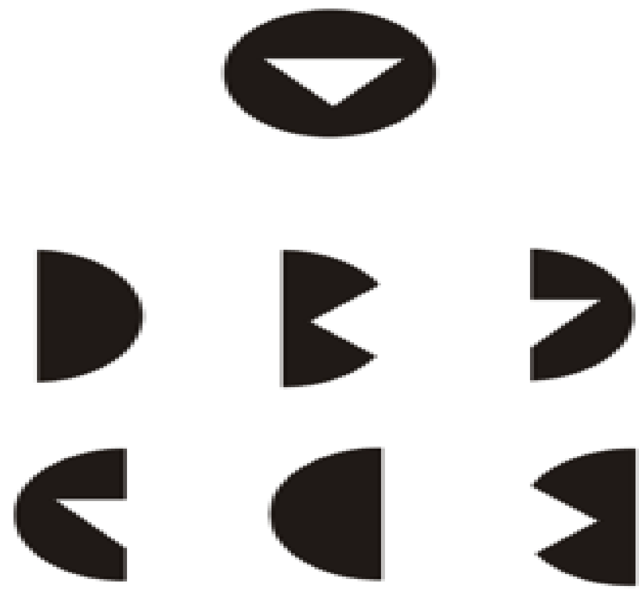
1 8 2 5 7 4 6 9

Appendix 3

Full stimuli of the spatial training

Stimuli	Target shape
Set 1	
Set 2	
Set 3	
Set 4	
Set 5	
Set 6	
Set 7	

An example of the spatial training: Set 1



Appendix 4: 3-D mental rotation

Children were divided into four age groups to explore whether the advantage of using real objects varied with age: Group 1 (41 – 47 months; n = 24), Group 2 (48 – 53 months; n = 23), Group 3 (54 – 59 months; n = 17), and Group 4 (60 – 73 months; n = 18). As shown in Figure 4.1., children who were younger than 4.5 years of age performed equally poorly in both versions. Although children of 4.5 to 5 years of age performed better using real objects compared to pictures of the objects, the overall performance remained poor. Finally, children who were 5 years of age and older showed an advantage of using the real objects compared to the pictures of the objects but their performance was still quite poor.

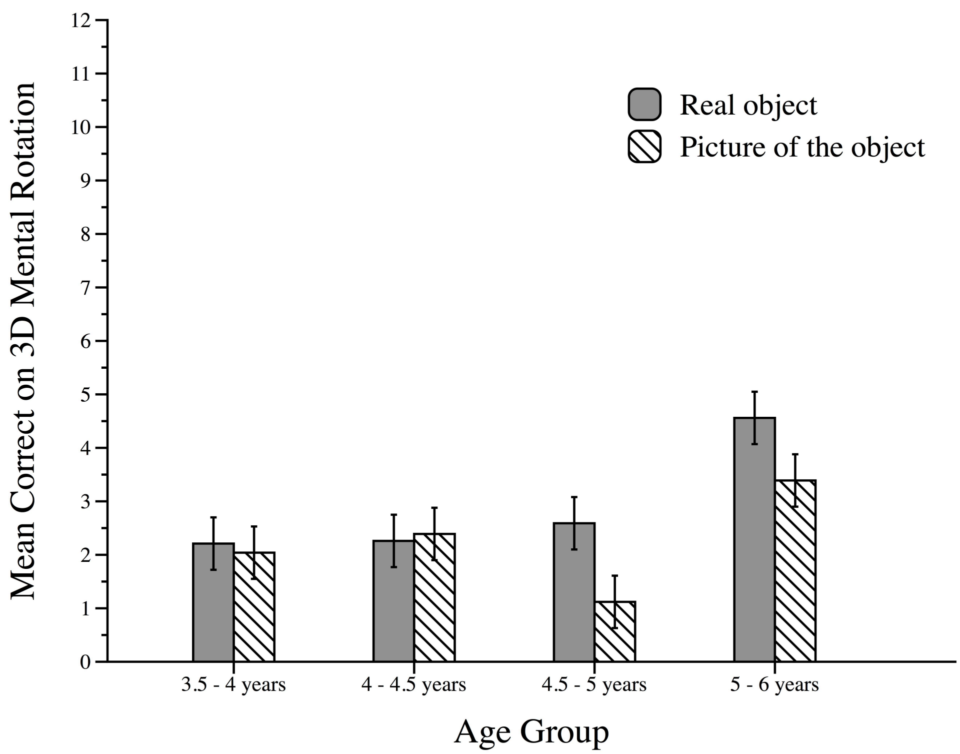


Figure 4.1. Performance on the 3-D mental rotation task by age group. Error bars are 95% confidence intervals based on the *MSE* values for the interaction.