Performance Characterization and Control of a Smart Spring Device for Indirect-Active Vibration Suppression

by

Tim Harold,
Bachelor of Engineering (Mechanical)
Carleton University

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of
Master of Applied Science

Department of Mechanical and Aerospace Engineering

Ottawa-Carleton Institute

For Mechanical and Aerospace Engineering

Carleton University
Ottawa, Ontario
2004

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Abstract

The Smart Spring concept is a semi-active approach to vibration suppression that makes use of an actively tunable impedance device. The large bandwidth and force generating capabilities of the piezoelectric actuators are exploited while the relatively large power requirement often associated with other active approaches is circumvented. A numerical model was developed, as well as 2 proof-of-concept hardware prototypes, for the purpose of numerical and experimental investigations. It was demonstrated that the Smart Spring device is capable of controlling all aspects of the complex impedance of a dynamic system. The Smart Spring is therefore capable of controlling the damping and resonance frequencies exhibited by such a system. An active feedback controller was implemented to control a Smart Spring device incorporated into a base-excited system. Numerical simulations demonstrated that the Smart Spring, in conjunction with the feedback controller, achieved significant vibration suppression; particularly in frequency bands near the system resonance frequencies.

Acknowledgements

The author would like to thank his supervisor, Dr. F. Nitzsche, for his valuable guidance and financial assistance, National Research Council of Canada for the facilitation of this research, Mr. V.K. Wickramhasinghe and Mr. Chen Yong for their technical support during experimentation.

The author would like to give special thanks to his dear friend and wife, Mrs. Jennifer Czaban, for her encouragement and support during this research.

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1 Introduction

Suppression of undesired vibration generated by mechanical systems has been a challenging problem in many applications. Conventional passive vibration control techniques using stiffeners, dampers, and isolators are usually effective for suppressing vibration in narrow frequency bands only [1]. Furthermore, passive approaches can only provide moderate vibration suppression and such passive approaches usually incur significant weight penalties. In contrast, active approaches can be used to significantly suppress vibration in relatively wider frequency bands due to their controllable nature [2]. Development of various integrated systems incorporating devices and control schemes capable of performing active vibration suppression has been the focus of a multitude of past and present research. Current active vibration suppression schemes can be classified into two broad categories including direct-active approaches, and indirect-active (commonly referred to as semi-active) approaches. In direct-active concepts, vibration is controlled using direct, forced action against the vibratory forces. There are a number of direct-active schemes using hydraulic and smart material actuators being developed for various vibration applications. In contrast, indirect-active approaches perform vibration suppression using controlled variations of impedance characteristics of dynamic systems.

1.1 Direct-active Vibration Suppression

Many of the current concepts are under investigation for use in rotorcraft to suppress vibratory hub loads. In general, these concepts prescribe the use of smart material actuators to provide sufficient force and displacements to various rotor blade components to produce new aerodynamic loads that are used to mitigate vibratory loads.

One of the concepts in the advanced development phase is a trailing edge flap powered by a piezoceramic actuator in a helicopter rotor blade to suppress rotor vibration [3]. The trailing edge flap is designed to generate sufficient forces to counteract the vibratory force of the blade. Open and closed-loop control of a Mach-scaled rotor model with trailing-edge flaps was investigated using wind tunnel tests [4]. The tests showed that the trailing-edge flap system was capable of achieving sufficient control authority to significantly suppress vibratory hub loads. Full-scale flight hardware implementing this concept on a MD-900 rotor blade is scheduled for flight tests [5].

Figure 1: Schematic of a Trailing-edge Flap [4]

A schematic of the trailing-edge flap system used by Koratkar and Chopra is shown in Figure 1. In this design, the displacements of the piezoelectric bending-type actuator are amplified to achieve sufficient flap deflections. As a result of this amplification, the actuator force required to oppose the aerodynamic loads generated by the flap are relatively high.

Other similar concepts include the use of leading-edge flaps, active blade tips, controllable blade camber/twist mechanisms using piezoelectric elements and integral blade twist using embedded piezoelectric fibers. Like the trailing-edge flap, these concepts use smart-material actuators to develop sufficient force and deflection to produce relatively high aerodynamic forces. In addition, these systems operate at relatively high rotational velocities, which result in large centripetal forces, dynamic pressure and frictional moments. Thus, performance of these systems has been observed to rapidly degrade with respect to increasing rotational velocity [6].

Another development in direct-active smart structures is an amplified active cargo floor mount using piezoelectric actuators to mitigate transfer of vibratory loads from the fuselage to the floor of a tilt-rotor aircraft. Smith et al. investigated the use of a dynamically tailored X-frame actuator for this application [7].

Figure 2: Active Floor Mount using X-frame Actuator

A physical system that is dynamically similar to the schematic diagram in Figure 2 was used to test this concept. Optimal control strategy was used to drive the X-frame actuator. In this concept, the displacements of the piezoceramic material in the actuator are amplified using the X-frame geometry. This amplification is achieved at the expense of lowered output force capabilities. The results of tests using this actuator were compared to identical tests using a hydraulic actuator instead of the X-frame actuator. The overall attenuation was improved using the piezoelectric X-frame actuator. This is due to the larger over-all bandwidth of the piezoelectric actuators compared to hydraulic actuators. However, it was found that use of hydraulic actuators in such a system may be more attractive in cases where there is a need for relatively high-energy control; for example, when large actuation displacements must be supplied in conjunction with large forces.

A fundamental problem with direct-active approaches is that the piezoelectric actuators are required to generate sufficient forces as well as displacement to directly counteract the excitation forces. In general, smart material actuators are capable of producing relatively high forces, but possess extremely low stroke capabilities; typically on the order of 50 μm . This restricted deformation capability results in a need for complex displacement amplification mechanisms or application of extremely high voltage to achieve the required power to effectively suppress vibrations. The high power requirements of these applications using piezoelectric actuators have hindered the successful implementation of these concepts.

1.2 *Indirect-active Vibration Suppression*

In indirect-active concepts, the vibration is suppressed by modulating the structural properties of a dynamic system such as stiffness, damping or mass. In these approaches smart materials can be used to perform such modulations in dynamic impedance to suppress vibrations. In contrast to direct-active approaches, the power requirements would be relatively low because the actuator forces are not generated to directly counteract the vibratory forces. In fact, in most devices designed for indirect-active control the actuator forces are orthogonal to the vibration modes that are target for control. Implementation of indirect-active approaches has been performed predominantly using controllable orifice dampers, electrorheological (ER) or magnetorheological (MR) fluids, and to a lesser extent, variable stiffness elements.

1.2.1 Damping-based Indirect-active Control

For example, a magnetorheological damper concept was implemented to suppress the

Figure 3: Schematic of an Indirect-active MR Fluid Damper System for Earthquake Protection (a), and MR Fluid Damper (b) [8]

response of a three story building due to seismic ground movement [8]. Dyke et al. demonstrated that a considerable reduction in the RMS and peak vibration response of such a building compared to traditional passive systems could be achieved using the MR damper system shown schematically in Figure 3(a). A linear quadratic optimal control strategy was used to optimize the damper force to reduce the structural response to seismic ground movements. The variations in damper force were achieved through control of the magnetic coil (Figure 3(b)) which, in turn, resulted in controlled variations in the damping coefficient associated with the MR damper.

A similar approach making use of controlled variable damping to suppress vibration was investigated by Anusanti-Inthra et al. This analytical investigation demonstrated that significant reduction of helicopter rotor hub loads could be

achieved through cyclic variations of damping coefficients associated with rotor blade lead-lag and flap modes [9].

1.2.2 Stiffness-based Indirect-active Control

The capability of MR and ER fluid dampers and variable orifice dampers to achieve significant control of structural damping characteristics has been well established and such devices are fairly readily available. However, somewhat less attention has been given to the development of mechanical devices capable of achieving effective control of stiffness characteristics to suppress vibration. It is worthwhile to note that the possibility of using controlled variations in stiffness to effectively suppress vibrations has been demonstrated. It has been shown that controlled variation of the stiffness characteristics associated with dynamic systems including helicopter rotor blades can result in significant suppression of vibrations. Nitzsche demonstrated, using analytical development and numerical simulations, that controlled modulation of the stiffness at the root of a helicopter blade could result in as much as three orders of magnitude of reduction of vibration energy at target frequencies [10]. Similarly, Anusanti-Inthra et al. analytically demonstrated that significant vibration suppression could be achieved through cyclic variation of helicopter blade-root stiffness, as well as damping [11]. Thus, development of a device capable of achieving such controlled stiffness variations in dynamic systems will become an enabler in the application of the above-mentioned developments.

One such device was used in an approach to suppress vibrations of a tilt-rotor aircraft cargo bay floor. To validate this approach, testing was done using a similar set-up to the amplified active floor mount (Figure 2). Smith et al. used an adaptive-

stiffness element in place of the X-frame actuator [7]. In the adaptive stiffness element, rotation of a bi-directional elastomer was used to achieve the stiffness variation between 1 and 20 times the baseline nominal stiffness. A *simulated annealing* control strategy that is a random optimization method similar to genetic algorithms was used to control the stiffness of the device to suppress vibration of the cargo floor in a quasi-static manner. Actuation was provided using a stepper motor in conjunction with a worm gear to rotate the elastomer bearing up to 90 degrees. It was demonstrated that this indirect-active system achieved significant reduction in the response of the cargo floor. However, optimal performance was not achieved for several minutes after initiating the controller [7]. This leads to the conclusion that the above-mentioned controller would achieve less suppression in increasingly dynamic conditions. Therefore, the variable stiffness device would be limited to quasi-static applications as the frequency of the cyclic stiffness variations was limited by the capability of the stepper motor. Use of a smart material actuator could improve this limited bandwidth. However, limited stroke capabilities of these materials would require an impractical amount of amplification.

Another example of a device capable of controlled stiffness variations was discussed by Cunefare. A piezoelectric element can be used in a series spring arrangement. Since the stiffness of a piezoelectric is different when it is short-circuited as opposed to open circuited, the stiffness state of the series spring arrangement can be “switched” by this action. Cunefare developed an indirect-active control strategy for such a state-switchable spring when incorporated into an otherwise ordinary classical vibration absorber [12]. The result was an actively

tunable vibration absorber. The control strategy was based on maximum energy extraction principles that led to a control law that is relatively easy to implement. A practical drawback to the state-switchable spring device is the onset of undesirable mechanical transients due to impulse-like forces introduced to the system by the piezoelectric element when a switch is initiated at times when there is potential energy stored in the spring. Thus, the state-switching control law was limited to initiating switches between stiffness states only at times of zero potential energy. No other specific practical matters concerning the state-switchable device were discussed. The investigation demonstrated, using numerical simulations, that the actively tunable vibration absorber achieved significantly better suppression compared to a tuned classical vibration absorber when applied to single DOF lumped-mass systems and point-excited beams and plates [12].

It is important to note that most of the indirect-active approaches are capable of actively altering a single mechanical property, such as the stiffness or damping or inertia but not the combination of all three properties. In contrast, the Smart Spring is an adaptive impedance control device that can be designed to preferentially control combinations of the structural stiffness, damping and mass. It is a unique approach that overcomes several difficulties in other active vibration control approaches. It was developed by Nitzsche et al. and its use in an integrated system to suppress airfoil vibrations due to unsteady aerodynamic excitation has already been demonstrated [13]. The tests included open-loop and XLMS closed-loop control tests.

1.3 Focus of Present Research

This thesis describes the Smart Spring concept in detail and establishes how a Smart Spring device can be used to control the dynamic impedance characteristics of a mechanical system. Capability of preferentially controlled variations of stiffness, damping and effective mass using a Smart Spring device is further demonstrated using numerical simulations. Open-loop control of the device is used in the simulations. Further demonstration of this capability is presented using bench tests. The bench tests were conducted using proof of concept hardware developed in collaboration with the Canadian National Research Council (NRC) and the results show that the simulations adequately predict the trends in dynamic behavior of a Smart Spring device.

Development of a closed-loop feedback controller is presented. Although it was established that the Smart Spring is capable of continuously varying the dynamic impedance characteristics of a dynamic system, state-switch-type control was chosen for this investigation for the sake of demonstration. The state-switch controller developed is based on principles of maximum energy extraction and predominantly makes use of controlled damping effects. Numerical simulations using the closed-loop feedback controller are presented to demonstrate the viability of controlling a Smart Spring device to suppress vibrations in a base-excited lumped-mass system.

The simulation results will show that controlled variation in dynamic impedance of the Smart Spring system can be used to achieve significant vibration suppression. This suppression occurs over multiple, relatively wide frequency bands and the peak

suppression occurs at the baseline resonance frequencies where anti-resonance peaks are produced.

2 Smart Spring Concept

The Smart Spring is a patented concept based on an actively variable impedance device using piezoceramic actuators to adaptively vary the stiffness, damping, and mass-coupling of a dynamic system [13]. The mechanism shown in the conceptual drawing in Figure 4 exploits the large stiffness and bandwidth of piezoelectric materials to modulate the structural properties to perform indirect-active vibration control.

2.1 System Overview

Figure 4: Smart Spring Concept

Referring to Figure 4, it is seen that the device consists of two mass-spring systems. The primary mass-spring system consists of the primary structural mass and stiffness, designated by m_1 and k_1 , respectively. In this schematic, the primary structural mass motion is targeted for control. The secondary active system consists of the secondary mass and stiffness designated by m_2 and k_2 , respectively. The excitation force applied to the primary mass is designated by $F(t)$. The piezoelectric actuators are located on m_2 , which is depicted as a plunger, and is attached to the free end of the secondary spring. The plunger (m_2)

is contained in a sleeve that is integral to the primary mass. The control force is designated by $N(t)$. This control force is generated by the active material actuators and results in the frictional force, $vN(t)$. The frictional force acts on the primary and secondary masses in an equal and opposite fashion. It can be seen that the frictional force is the only cause in the system depicted for coupling between the primary and secondary systems. Since the frictional force is directly related to $N(t)$, this coupling is controlled by the actuators.

2.2 Actuator Configuration

The purpose of the actuators in the Smart Spring concept is to affect the amount of dynamic coupling between the primary and secondary mass-spring systems. This is done by taking advantage of the so-called piezoelectric converse effect exhibited by the actuators. The actuators considered for this application are piezoceramic stacks. Applying a positive voltage across a piezoceramic stack results in an elongation in its neutral length. In a Smart Spring device, this elongation is partially blocked since the plunger walls in m_2 are in contact with the sleeve that is integral to m_1 . This results in a statically indeterminate strain that is shared between the piezoceramic stack and the sleeve. Thus, the control force is a function of the control voltage, $V(t)$, supplied to the actuators, and the relative stiffness between the actuators and the sleeve structure. It is worthwhile to note that the sensitivity of $N(t)$ with respect to $V(t)$ can be greatly affected through a carefully integrated design of the actuators and the sleeve structure. Furthermore, minimizing the allowable strain in the piezoelectric stack by

ensuring the sleeve walls are relatively stiff will result in reduced power requirements for actuation.

The system in Figure 4 is a 2 DOF system consisting of the motion of m_1 and m_2 in the “x” direction. It can be seen that $N(t)$ is orthogonal to these modes of vibration because the virtual work performed by this force is independent of the motion of m_1 and m_2 , and $F(t)$. As such, the piezoelectric actuators do not directly perform work against the excitation or internal vibratory loads. As stated in the preceding sections, the Smart Spring concept is an indirect-active approach. An interesting phenomenon exists in a Smart Spring system, and some other indirect-active systems, due to this orthogonality between the control force and target vibration modes. No direct transfer function between the control force and modal displacement output exists in such a system. For example, if m_1 and m_2 are motionless, application of a non-zero $N(t)$ will not result in a displacement of either masses. Due to this phenomenon, many classical control approaches are not applicable to this system.

2.3 Smart Spring for Impedance Control

The Smart Spring is an indirect-active vibration suppression device. By design, the Smart Spring is used to actively vary the dynamic impedance of a system. This is achieved by controlling the frictional force, which in turn, controls the contribution of the secondary mass-spring system to the dynamic impedance of the system. If the control force is zero, there is no contact at the frictional interface. Therefore, in this case, there is no coupling between the primary and secondary mass-spring systems. Thus, there is no contribution to the

dynamic impedance of the system due to the secondary mass-spring system. As such, when the control force is zero, the system is said to be fully uncoupled. It has one degree of freedom consisting of the primary mass-spring system and has the natural frequency as defined below:

$$\omega_0 = \sqrt{\frac{k_1}{m_1}} \quad (1)$$

If the control force is sufficiently high, the resulting frictional force can cause the primary and secondary masses to lock together. In this fully coupled condition the primary and secondary masses move as one mass and the primary and secondary springs act in parallel. As such, the natural frequency of the fully coupled system is defined as follows:

$$\omega_c = \sqrt{\frac{k_1 + k_2}{m_1 + m_2}} \quad (2)$$

It can be seen from observing Equation 1 and Equation 2 that the mass and stiffness seen by the controlled structure can be shifted between the two extreme cases corresponding to the natural frequencies ω_0 and ω_c .

Thus far, consideration has been paid only to the two extreme input force conditions such that the system is either fully coupled or fully uncoupled. It is worthwhile to note that the ability for the actuators to produce sufficient force to cause the system to become fully coupled is highly dependant on the magnitude

of the excitation force, $F(t)$, and the instantaneous dynamic conditions such as the relative positions and speeds. It is also worthwhile to note that it is not possible for the frictional force to instantaneously lock the primary and secondary masses together at times when the relative velocity is non-zero.

In order to illustrate the expected qualitative effects on the dynamic impedance seen by the controlled structure due to real-time changes in the control force, the following scenario is presented. Consider the system depicted in Figure 4 when the primary system exhibits a steady-state response to the excitation force $F(t)=F\sin(\omega t)$, and when $N(t)=0$ at time $t=t_0$. In this condition, the secondary system is not in motion and does not contribute to the response of the primary system. After an arbitrary period of time, Δt , a constant non-zero control force is supplied by the actuator. In practice, this can be achieved by supplying the actuator with a step increase in input voltage at $t=t_0+\Delta t$. The introduction of the control force will result in a dynamic friction force that is applied equally and oppositely between the primary and secondary masses. As such, mechanical energy will be exchanged between the primary and secondary systems as the degree of coupling between them increases and the secondary system will be set into motion. Frictional losses will occur at the contact interface due to the frictional force and the relative sliding motion between m_1 and m_2 . These frictional losses result in damping seen by the controlled structure and the secondary mass. The damping will be present as long as there is (1) contact at the frictional interface and (2) a non-zero relative velocity between m_1 and m_2 . After a transient relaxation period, a new steady state response will be reached.

If the frictional force is sufficient to fully couple the system, first the amplitude of the relative velocity, and thus the frictional losses, will reach a maximum value, then they will tend to approach zero due to the increasing coupling effect. In addition, the response of the controlled mass will approach the new steady-state response due to a change in the system natural frequency seen by the controlled structure. As previously discussed, the change in natural frequency (from ω_0 to ω_c) is expected due to the introduction of the additional mass and stiffness associated with the secondary system. In summary, the load path between the structure and the base that was originally carried by m_1 & k_1 is now divided between m_1 & k_1 , and m_2 & k_2 .

During the transient period, a second mode of vibration is expected to become apparent. The second mode of vibration consists of the motion of the secondary system. The natural frequency, ω_2 , corresponding to this mode is simply that of the secondary mass-spring system:

$$\omega_2 = \sqrt{\frac{k_2}{m_2}} \quad (3)$$

For the system depicted in Figure 4, it can be seen that ω_2 can only be apparent at times when $N(t)$ is non-zero. This is so because the frictional force is the only source of coupling between the primary and secondary systems. However, it is also expected that the ability to observe both ω_0 and ω_2 shall

diminish and the ability to observe ω_c shall increase as the system approaches the fully coupled condition.

3 Practical Implementation of the Smart Spring Concept

In this chapter, the physical realization of a Smart Spring device is discussed. An overview of two prototype designs is presented. The first prototype design that is discussed is the torsional Smart Spring prototype. This prototype was designed in part by this author and tested at NRC. Results of this testing are discussed in this thesis and in previous publications [10,13,14,15]. The second prototype design that is discussed is the axial Smart Spring prototype. The axial Smart Spring prototype was also designed in part by this author. Test results using this prototype are not presented in the present research, however a brief overview of the design is presented in order to demonstrate alternative configurations of a Smart Spring device.

Any device that can be dynamically modeled using a schematic analogous to that shown in Figure 4 is a Smart Spring device. The working characteristic of a Smart Spring device requires that the dynamic coupling between two or more mass-spring systems can be affected in a controlled manner. In the prototype designs discussed below, this is achieved using the frictional control force that is generated by the piezoelectric actuators. The practical difference between the torsional and the axial Smart Spring prototype designs is that the motion in the torsional prototype is rotational rather than axial.

3.1 Torsional Prototype Design Overview

Figure 5: Conceptual Schematic of Torsional Smart Spring Configuration

The isometric and side views of the torsional Smart Spring prototype are shown in Figure 6 (a) and (b), respectively. The dynamic behavior of this prototype can be modeled using the schematic in Figure 5. Since the motion of the torsional Smart Spring prototype consists of angular displacements whereas the schematic in Figure 4 depicts axial displacements, the elements shown in Figure 4 are re-defined in Figure 5. Table 1 illustrates this redefinition of parameters.

Configuration	Axial	Torsional
Inertia:	m_1, m_2 [kg]	I_{1o}, I_{2o} [kg-m ²]
Spring Stiffness:	k_1, k_2 [N-m]	K_1, K_2 [N-m/rad]
Excitation Load:	$F(t)$ [N]	$T(t)$ [N-m]
Frictional Control Load:	$f(t)=\nu N$ [N]	$T(t)=\nu Nd$ [N-m/rad]

Table 1: Comparison Between Dynamic Parameters of Axial and Torsional Smart Spring Configurations

Figure 6: Torsional Smart Spring Prototype

The controlled mass assembly consists of the polar moments of inertia associated with the upper cylindrical plate and the excitation/sensor bar depicted in Figure 6. The actuator mass assembly consists of the polar moments of inertia of the lower square plate and the actuator cans. Excitation is supplied to the torsional prototype by the mechanical shaker that can be seen in the upper right-hand corner of Figure 6(b). The shaker is connected to the outer end of the excitation/sensor bar by a stinger. Therefore, the force supplied by the shaker results in the excitation torque about the vertical central axis of the device. This torque can be represented by the excitation force, $T(t)$ in Figure 5. The angular position of the primary controlled mass and the secondary actuator mass assemblies are schematically expressed as the displacements θ_1 and θ_2 in Figure 5, respectively. When a positive voltage signal is supplied to actuators, they are caused to elongate inside the actuator can. This causes the upper surface of the actuator can to press against the lower surface of the primary mass,

resulting in the coupling frictional torque that is applied equally and oppositely to the primary and secondary masses.

3.1.1 Torsional Prototype Spring Arrangement

The primary and secondary spring arrangements are shown in Figure 7. In the torsional Smart Spring prototype, both the primary and secondary torsion springs are attached to the rigid base.

Figure 7: Torsional Prototype Spring Arrangement

The primary spring is a computer numerically controlled (CNC) machined torsion tube. It is rigidly bolted to the primary mass as seen in Figure 7(a), and to the rigid base as indicated in Figure 7(b). As such, small rotations of the primary mass cause elastic twist in the torsion tube. It is important to note that the primary spring has no attachment to the secondary mass. The secondary mass is inserted around the cylindrical protrusion on the rigid base so that it rests on the thrust bearing support

shown in Figure 7(b). As such, there is no cause for coupling between the primary and secondary systems other than the frictional control load.

The secondary spring arrangement is shown in Figure 7(b). This configuration consists of four cantilevered beam-type leaf springs. It is important to note that in the photograph shown, one of four leaf springs has been removed. This can be seen in the lower portion of Figure 7(b). The four symmetrically configured cantilevered leaf springs are bolted to the rigid base such that they are fully constrained at their root. The outer ends of the leaf springs are pinned at locations close to the outer edges. Thus, small rotation of the secondary mass causes elastic deformation in the four leaf springs.

3.1.2 Actuator Can Assembly

Figure 8: Actuator Can Assembly (Torsional Prototype)

As seen in Figure 6(b), there are three actuator cans incorporated into the secondary mass. They are arranged symmetrically at an equal radial distance from the central axis of the device at 120° intervals. An exploded schematic of the actuator can design is shown in Figure 8. The purpose of the actuator can is to provide a uniform frictional surface in order to transfer the frictional control forces and to

protect the relatively brittle piezoceramic material from transverse loading. Each actuator can is fitted with a load cell so that during testing, the control forces can be quantified and, more importantly, equalized between the three cans.

There are three stacked piezoceramic actuators in each actuator can assembly. These actuators are electrically connected in parallel and mechanically assembled in series in order to maximize the force generated for a given input voltage signal. A drawback to this arrangement is that the electrical capacitance of the piezoelectric elements is increased in comparison to a series configuration. As such, the increased force capabilities of the arrangement is achieved at the cost of slightly heightened power requirements due to the relatively large capacitance associated with the parallel electrical configuration.

Upon receiving a positive control voltage, the actuators are made to elongate along the axis of the actuator can. This causes the frictional surface to press outward against the frictional surface on the primary mass. The frictional surface is attached to the shell of the actuator can by the flex-rings. It can be seen that the frictional surface is integral to the outer flex-ring. The cylindrical protrusion extending from the back of the frictional surface is attached to the inner flex-ring. These flex-rings are designed to deflect elastically in a uniform manner in the direction of the actuator force. This allows for the required axial deflections of the frictional surface while the transverse loads caused by the resulting frictional forces are carried radially through the flex-rings to the stiff outer shell of the can. As such, the stacked actuators are made to carry the axial actuation forces while the outer shell of the actuator can

carries the bulk of the transverse force. A drawback to this design is that the force required to elastically deflect the flex-rings is directly counter to the actuation force.

3.1.3 Torsional Prototype Actuator Pre-load

Figure 9: View of Torsional Prototype Showing Actuator Pre-load Bolt

In the course of testing, it is generally necessary to adjust the baseline normal forces exerted on the actuator cans. This is necessary due to the limited displacement capabilities of the stacked piezoceramic actuators. The maximum free deflection that can be achieved by the stacks is approximately $30\text{ }\mu\text{m}$. The normal forces that result in the coupling frictional force are generated by blocking these displacements. As such, in order to maximize the range of the normal force, there should be contact at the frictional interfaces whenever the input voltage to the actuators is greater than zero. It may also be desired to increase the maximum attainable normal force. This

can be done by pressing the lower surface of m_1 on to the frictional surfaces, thereby preloading the actuator cans.

In the torsional prototype, this adjustability is achieved using the actuator preload bolt. The actuator preload bolt is depicted in Figure 9 and Figure 7(b). The actuator preload bolt is a through-bolt that is inserted through the bottom of the rigid base, as seen in Figure 7(b), and extends through to the top of m_1 , as seen in Figure 9. A thrust bearing is inserted around the preload bolt such that it rests on the upper surface of m_1 ; then a nut is threaded on to the preload bolt and torqued against the thrust bearing. This causes the primary torsion spring to be axially compressed, thus preloading the actuator cans.

An additional benefit to using the preload bolt is the increased axial stiffness introduced by the bolt. In the torsional prototype, the actuator forces are carried by the primary torsion tube and the actuator preload bolt. As discussed in Section 2.2, the stiffness of these load-bearing components greatly affects the range and magnitude of the normal force. Increased stiffness of the device in the axial direction limits the actuator strain, thus increasing the efficiency of the actuators to produce the normal forces.

3.2 Axial Prototype Design Overview

The axial Smart Spring prototype was designed to investigate and characterize the behavior of a Smart Spring system undergoing axial vibrations. In designing the axial prototype, it was desired that the device be as compact as possible and versatile for installation into various experimental set-ups. It was also desired that the design

should make use of state-of-the-art piezoceramic actuators that have relatively low electrical capacitance. The axial Smart Spring prototype is depicted Figure 10.

Figure 10: Axial Smart Spring Prototype and Exploded Schematic

3.2.1 Axial Smart Spring Prototype Schematic Overview

The axial Smart Spring prototype was designed so that the motion of the device is directly analogous to the schematic in Figure 4. The device consists of four major sub-assemblies including the actuator can housing assembly, the actuator can, and the primary and secondary spring assemblies. The actuator can housing assembly is designed to be attached to a mass, or structure, that is targeted for control. This attachment is made using the actuator can housing bracket.

The lower secondary and primary spring attachments are fastened rigidly together at the base of the device. Once bolted together, the lower spring attachments can be fastened to a rigid base as schematically depicted in Figure 4. Alternatively, they can be fastened to a mass, making a more complex dynamic system than that shown in Figure 4. Thus, the primary mass-spring system consists of the primary

spring, k_1 , and the mass of the actuator can housing plus the mass of the structure attached to it, m_1 .

The upper end of the guide shaft is threaded securely into a tapped bore in the actuator can. The upper secondary spring attachment is fastened to the top of the guide shaft as shown in Figure 10. As such, the secondary mass-spring system consists of the secondary spring stiffness, k_2 , and the mass of the actuator can and the guide shaft, m_2 . It can be seen in the left-hand portion of Figure 10 that the guide was designed to extend through the bottom of the device. This was done so that additional mass could be added to the shaft, thereby increasing the magnitude of the secondary mass.

In Figure 4, the actuator mass is contained within a sleeve that is integral to m_1 . The frictional surfaces on the actuator mass press against the sleeve walls, generating the frictional control force. In the axial Smart Spring prototype, the sleeve walls are the friction plates. The friction plates are bolted to the inside walls of the actuator can housing as shown in Figure 10. The actuator can housing is a thick walled aluminum structure designed to provide adequate stiffness for the sleeve walls.

3.2.2 Axial Prototype Actuator Can

As mentioned in the preceding section, it was desired that the axial Smart Spring prototype design be as compact as possible. It was also desired that the design make use of a state-of-the-art piezoceramic actuator with relatively low capacitance. Since the prototype is to be used as proof-of-concept hardware, the power requirements to operate the device to suppress vibrations should be as low as possible. This is because the Smart Spring concept is an indirect-active vibration

suppression technique and one of the prime advantages to this is that the power requirements should be relatively low in comparison to direct-active systems. Since the actuator displacements in a Smart Spring device are relatively low at times when the actuator forces are applied, little mechanical work is being done by the actuator. However, electrical work is done in generating the forces. The magnitude of the actuator capacitance is directly related to the power consumed in its operation. As such, when selecting an appropriate actuator for a Smart Spring device, the blocked force generating capabilities and the actuator capacitance should be considered. Additionally, the actuator selected needed to be as small as possible in order to achieve a compact prototype design.

A solid model of the axial prototype actuator can assembly is depicted in Figure 11. The actuator selected for this design has an electrical capacitance of 14.5 μF and is capable of producing up to 7000 N of force when fully blocked. It is approximately 5.5 cm in length. Only one actuator is needed to produce sufficient actuation forces.

Figure 11: Axial Prototype Actuator Can Assembly

As previously discussed, the purpose of the actuator can is to protect the brittle piezoceramic material in the actuator from the off-axis loads due to the frictional forces. In the axial prototype actuator can design, the actuator is fitted into the precise inner bore of the hollow cylindrical plunger. The outer surface of the plunger is similarly fitted into the inner bore of the actuator can. As such, the transverse friction forces are transferred directly into the actuator can through the plunger.

The load cell pre-load screw is used to press the back face of the actuator firmly against the load cell, thereby increasing the nominal load on the load cell. Control of the pre-load on the actuator is achieved by inserting shims behind the friction plates on the actuator can housing prior to assembly.

4 Dynamics of a Smart Spring Device

In the present chapter, dynamic models are introduced to predict the nature of the dynamic behavior of a system fitted with a Smart Spring device. The equations of motion are developed for two Smart Spring systems. The sets of non-linear equations obtained are then expressed in state-space form. These equations are used to produce numerical models that will be used in later sections to investigate and characterize the performance of the Smart Spring device during open and closed-loop control. The first system to be investigated is the two-degree-of-freedom (2 D.O.F.) system depicted in Figure 4. The numerical model obtained from this analysis will be used to demonstrate and characterize the ability of the Smart Spring to affect the dynamic impedance of the system. For the second analysis, a base-excited, 3 D.O.F. system is investigated. In a later section, a closed loop control algorithm is developed for this system.

4.1 *Equations of Motion of a 2 D.O.F. Smart Spring System*

In this section, the equations of motion are obtained for the dynamic system shown schematically in Figure 4. For convenience, this figure is presented below. Lagrange's formulation is used and the generalized forces are obtained through inspection.

Figure 12: 2 D.O.F. Smart Spring System

Using Lagrange's Formulation:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{x}_i} \right) - \frac{\partial T}{\partial x_i} + \frac{\partial V}{\partial x_i} = Q_{x_i} \quad (4)$$

where: T is the system kinetic energy

V is the system Potential energy

Q_{x_i} are the system generalized applied forces

The kinetic and potential energy are:

$$\left. \begin{aligned} T &= \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 \\ V &= \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2 \end{aligned} \right\} \quad (5)$$

The first term in Equation 4 is obtained by taking the time-derivative of the partial derivatives of the kinetic energy in Equation 5.

$$\left. \begin{aligned} \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{x}_1} \right) &= m_1 \ddot{x}_1 \\ \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{x}_2} \right) &= m_2 \ddot{x}_2 \end{aligned} \right\} \quad (6)$$

The kinetic energy in the system is not a function of position; therefore the second term in Equation 4 is zero.

$$\frac{\partial T}{\partial x_1} = \frac{\partial T}{\partial x_2} = 0$$

The third term in Equation 4 is obtained by taking the partial derivatives of the potential energy in Equation 5.

$$\left. \begin{aligned} \frac{\partial V}{\partial x_1} &= k_1 x_1 \\ \frac{\partial V}{\partial x_2} &= k_2 x_2 \end{aligned} \right\} \quad (7)$$

Combining the terms developed above, the equations of motion are as follows:

$$\left. \begin{aligned} m_1 \ddot{x}_1 + k_1 x_1 &= Q_{x_1} \\ m_2 \ddot{x}_2 + k_2 x_2 &= Q_{x_2} \end{aligned} \right\} \quad (8)$$

The generalized forces can be determined through inspection of the free body diagram shown in Figure 13. The only forces that are unrelated to the kinetic and potential energy are the excitation force, $F(t)$, the relatively small viscous damping

forces introduced by the springs, and the coupling frictional control force, denoted as $f(t)$.

Figure 13: Free Body Diagram Showing Generalized Forces

The excitation force is applied to m_1 . The frictional control force is applied equally and oppositely to both masses. Thus, the generalized forces are expressed as follows.

$$\left. \begin{aligned} Q_1 &= F(t) + f(t) - c\dot{x}_1 \\ Q_2 &= -f(t) - c\dot{x}_2 \end{aligned} \right\} \quad (9)$$

Combining Equation 8 and Equation 9, the equations of motion are as follows.

$$\left. \begin{aligned} m_1 \ddot{x}_1 + c\dot{x}_1 + k_1 x_1 &= F(t) + f(t) \\ m_2 \ddot{x}_2 + c\dot{x}_2 + k_2 x_2 &= -f(t) \end{aligned} \right\} \quad (10)$$

The non-linear nature of the friction force, $f(t)$, will require that the equations be solved numerically. To this end, the two second-order differential equations will

be expressed as a set of four first-order equations using state-space formulation. This is done by making the following substitutions.

$$\left. \begin{aligned} x_1 &= X_1 \\ \dot{x}_1 &= X_2 \\ x_2 &= X_3 \\ \dot{x}_2 &= X_4 \end{aligned} \right\} \quad (11)$$

Noting that:

$$\begin{aligned} \dot{\{X\}} &= [A]\{X\} + [B]\{u\} + [E]\{F(t)\} \\ \{y\} &= [C]\{X\} \end{aligned} \quad (12)$$

where:

$$[A] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1}{m_1} & \frac{-c_1}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-k_2}{m_2} & \frac{-c_2}{m_2} \end{bmatrix}; [B] = \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ \frac{-1}{m_2} \end{bmatrix}; u = f(t); \quad (13)$$

$$[E] = \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix}; [C] = [X_1; X_3; X_4 - X_2]$$

4.2 Base-excited Smart Spring System

Figure 14: Base-excited Smart Spring System

A schematic of the base-excited Smart Spring system is shown in Figure 14. In this system, the Smart Spring device is installed between the controlled mass, m_c , and the base mass, m_b . The primary and secondary springs are denoted as k_1 and k_2 , respectively. In this system, m_c is the primary mass, and m_a denotes the secondary mass. The disturbance force is applied to m_b . This particular dynamic system is analogous to many practical engineering applications. For example, this model can be used to approximate the interaction between an aircraft fuselage and cargo-bay floor, where m_b , and k_3 represent the mass and stiffness of an element of the aircraft fuselage, and m_c denotes the cargo mass. In a later section, a closed-loop control algorithm is developed for this system, and numerical simulations are performed to investigate its use for suppressing vibrations.

4.2.1 Equations of Motion for the Base-excited System

The kinetic and potential energy in this system are described as follows.

$$\left. \begin{aligned} T &= \frac{1}{2}m_a \left(\dot{x}_a \right)^2 + \frac{1}{2}m_b \left(\dot{x}_b \right)^2 + \frac{1}{2}m_c \left(\dot{x}_c \right)^2 \\ V &= \frac{1}{2}k_1 (x_c - x_b)^2 + \frac{1}{2}k_2 (x_a - x_b)^2 + \frac{1}{2}k_3 x_b^2 \end{aligned} \right\} \quad (14)$$

Referring to Equation 4, and noting that the generalized forces can be determined using Figure 14, the equations of motion are developed as shown in Equation 15.

$$\left. \begin{aligned} m_a \ddot{x}_a + k_2 x_a - k_2 x_b &= -f(t) \\ m_b \ddot{x}_b + (k_1 + k_2 + k_3)x_b - k_1 x_c - k_2 x_a &= F(t) \\ m_c \ddot{x}_c - k_1 x_b + k_1 x_c &= f(t) \end{aligned} \right\} \quad (15)$$

Making the following substitutions, the state-space equations can be formulated.

$$\begin{array}{lll} x_b = x_1 & x_c = x_3 & x_a = x_5 \\ \dot{x}_b = x_2 & \dot{x}_c = x_4 & \dot{x}_a = x_6 \\ \ddot{x}_b = x_7 & \ddot{x}_c = x_8 & \ddot{x}_a = x_9 \end{array} \quad (16)$$

Referring to Equation 12 and Equation 16, the coefficients of the state-space formulation are as shown in Equation 17.

$$\left. \begin{aligned}
[A] &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -(k_1+k_2+k_3) & 0 & \frac{k_1}{m_b} & 0 & \frac{k_1}{m_b} & 0 \\ \frac{k_1}{m_c} & 0 & \frac{-k_1}{m_c} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{k_2}{m_a} & 0 & 0 & 0 & -\frac{k_2}{m_a} & 0 \end{bmatrix}; [B] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_c} \\ 0 \\ -\frac{1}{m_a} \end{bmatrix}; [E] = \begin{bmatrix} 0 \\ \frac{1}{m_a} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \\
\{u\} &= f(t); \quad \{F\} = F(t); [C] = [X_3; X_5; X_6 - X_4]
\end{aligned} \right\} \quad (17)$$

4.3 Numerical Model

In order to solve the equations of motion, integration of Equation 12 will be performed. Numerical techniques will be used to integrate the set of first-order equations in time. Numerical solution is used due to the non-linear behavior of the frictional control force, and its dependency on the orthogonal input actuation force, $N(t)$.

4.3.1 Friction Forces

In general, the excitation force is of known quantity and direction. It is the response of the system due to this force that is the object of investigation. The nature of the friction force is highly non-linear. At times when the relative velocity, $v_{rel} = (\dot{x}_2 - \dot{x}_1)$ is non-zero, the friction force, $f(t)$ is equal to the dynamic friction force. The magnitude of this force can be estimated as the product of the dynamic friction coefficient, v_k , and the input actuation normal force, $N(t)$. Observing Figure 13, it can be seen that the direction of the dynamic friction force must be the same as

the direction of v_{rel} . Thus, at times when $v_{rel} \neq 0$, the following equation can be used to determine $f(t)$.

$$f(t) = \begin{cases} \nu_k N(t); & (v_{rel} > 0) \\ -\nu_k N(t); & (v_{rel} < 0) \end{cases} \quad (18)$$

It is generally accepted that friction coefficient values for a steel on steel interface increase with decreasing sliding speeds, and a singularity exists for the case where the sliding speed is zero [16]. The singularity exists due to the instantaneous increase in the friction coefficient from the dynamic friction coefficient, ν_k , to the static friction coefficient, ν_s . If the dynamic friction coefficients are assigned a negative value for cases where the sliding speed is negative and non-zero, Equation 18 can be used. The general plot of the resulting friction coefficient values against sliding speed is shown in Figure 15.

Figure 15: Dynamic Friction Coefficient vs. Sliding Speed for Steel-steel Interface

Quantitative values for the static and dynamic friction coefficients were obtained from a standard engineering handbook [16]. The range of dynamic friction coefficients corresponding to sliding speeds between 2.54×10^{-5} m/s and 2.54 m/s at one order-of-magnitude increments was formatted into a friction coefficient look-up table. The relative sliding speed of 2.54×10^{-6} m/s was selected to represent the speed at which the static friction coefficient will be used.

Figure 16: Friction Coefficient Look-up Table

The values used in the friction coefficient look-up table are plotted in Figure 16. The tabulated points are denoted by the “*” symbol. When integrating the equations of motion, v_{rel} is to be evaluated at each time step. The friction coefficients will be linearly interpolated between the nearest tabulated values. The highest sliding speed in the domain of the friction values given was 2.54 m/s, corresponding to a friction coefficient of 0.18. In theory, the friction coefficient should approach zero as v_{rel} tends toward infinity. It can be seen that the slope of the Friction Coefficient vs. Sliding Speed plot decreases with increasing sliding speed quite rapidly; therefore the

friction coefficient value for absolute sliding speeds greater than 2.54 m/s is reasonably approximated as constant at $\mu=0.18$.

In the numerical approximation described above, the highly non-linear singularity shown in Figure 15 is not entirely captured. In fact, for $0 \leq |v_{rel}| \leq 2.54 \times 10^{-6} \text{ m/s}$, the friction coefficients are interpolated from between 0 and $|v_s|$. Thus, this numerical model will not exhibit the phenomenon often termed as “static friction lock-up”. However, it is expected that the behavior of the numerical model will approach this dynamic condition given a sufficiently high input actuation force.

4.3.2 Overview of Integration Technique

A block diagram depicting the integration technique used to solve Equation 12 is shown in Figure 17. It can be seen that a feedback loop is necessary in order to calculate the on-line friction force.

Figure 17: Block diagram of Open-loop Smart Spring System

In the feedback loop, v_{rel} is used to evaluate the on-line friction coefficient. The friction force is then calculated according to the product: $f(t) = \nu N(t)$. At the summation junction, the sum in Equation 12 is calculated, producing the $\{\dot{x}\}$ vector. The latter vector is then numerically integrated producing the state vector output, $\{Y\}$. This integration process is repeated each time step in order to form the output signal, $\{y(t)\}$. The above-described numerical analysis can be used to integrate the governing equations of any Smart Spring system, given that the appropriate matrix coefficients in Equation 12 are provided. In the present research, a built-in adaptive time step integration scheme provided by MATLAB was used to implement the numerical model.

5 Characterization of Smart Spring for Active Impedance Control

In the present chapter, numerical simulation results are presented to illustrate the controllable nature of the dynamic impedance of a Smart Spring device. The numerical integration technique outlined in Chapter 4 is implemented using Simulink. Simulink is a MATLAB graphical user interface (GUI) that is convenient for discretizing and numerically integrating sets of first-order equations in time, and for the design of control systems. The software provides a number of adaptive-time step integration schemes that can be chosen from to ensure convergence and acceptable levels of error, while reducing the time required to perform the integration. A block diagram similar to Figure 17 is created using the GUI. Prior to performing the numerical integration, the initial conditions pertaining to the problem and state-space matrix coefficients must be provided. This is done by executing an initialization file that assigns the appropriate numerical values.

5.1 *Scope of Simulations*

The purpose of the simulations is to investigate and characterize the controllable nature of the dynamic impedance characteristics of a system fitted with a Smart Spring device. The numerical analysis was also used to guide the design of a test matrix that implements an actual hardware prototype. In a later section, the behavior predicted by the numerical model is qualitatively correlated with test results using the torsional prototype. Prior to performing the analysis, examination of

simulation results pertaining to two key input control force conditions is discussed. The results of these simulations are compared against theoretical results in order to validate the numerical model.

5.2 *Validation of the Numerical Model*

To verify if the numerical model produces accurate results, simulations designed to reproduce dynamic conditions that can be modeled in closed mathematical form were studied. These simulations include the situation where there is zero input actuation, and where the input force is sufficiently high to fully couple the two mass-spring systems. The numerical model is not expected to produce error other than numerical for the case where $N(t)=0$ because when the frictional force loop is open, no approximation exists other than that associated with the numerical integration. Since there is no closed-form solution for all $N(t)\neq 0$ when the system is not fully coupled, the validation does not apply in that case. However it seems sufficient to show that the model exhibits acceptable error levels for the fully coupled and uncoupled cases because one can extrapolate from the results of the validation for these extreme cases. This can be said because the numerically approximated behavior traits that can potentially produce significant error are associated with the treatment of the discontinuity at $v_{rel}=0$. This portion of the friction coefficient range resides over a very small portion of the integration domain, which can be made smaller as needed. For the general case where $v_{rel}\neq 0$, v_{rel} passes through this portion of the integration domain over the course of a very few number of time steps.

Therefore, any error produced in comparison to a real system for the case where $v_{rel} \neq 0$ other than the numerical error will be due solely to the friction coefficient data.

In Figure 17, a feedback loop was used to form the frictional control force. This loop is equivalent to a closed-loop feedback control system. If this loop is opened, the remaining system consists of two completely independent mass-spring systems, namely: the primary and secondary systems. In this case, the system is linear, and the only force applied is the excitation force. Thus, the response of the system when the loop is open is that of the single primary mass-spring system. The natural frequencies, ω_0 and ω_1 of the primary and secondary systems were defined previously in Equation 1 and Equation 3.

The response of the primary system with the frictional control force loop open is easily obtained in closed mathematical form. It is a 1 D.O.F. system having mass m_1 , stiffness k_1 , and the structural damping ratio ζ_1 . The response of such a system is the superposition of the transient and the steady-state solutions. The steady-state response, $x_1(t)$, in closed form, is defined as follows.

$$x_1(t) = \frac{F e^{i\omega t}}{m_1 \omega Z_1 e^{i\phi}};$$

where:

$$Z_1 e^{i\phi} = 2\zeta_1 \omega_0 + i\omega \left(1 - \frac{\omega_0^2}{\omega^2}\right)$$

$$Z_1 = \sqrt{(2\zeta_1\omega_0)^2 + (\omega(1 - \frac{\omega_0^2}{\omega^2}))^2}$$

$$\phi = \tan^{-1} \left(\frac{\omega(1 - \frac{\omega_0^2}{\omega^2})}{2\zeta_1\omega_0} \right)$$

$$\therefore x(t) = \frac{Fe^{i(\omega t - \phi)}}{m_1\omega \sqrt{(2\zeta_1\omega_0)^2 + (\omega(1 - \frac{\omega_0^2}{\omega^2}))^2}} \quad (19)$$

The MATLAB script in Appendix A1.1 entitled `ini_multi_sim.m` was used to initialize the simulation. The coefficients under the subtitle “*stiffness*” were used in the state matrices. These parameters are shown in Table 2.

Property	Primary System	Secondary System
Mass [kg-m ²]	m ₁ =0.0686	m ₂ =0.05
Stiffness [N-m/rad]	1864	6933
Natural Frequency [Hz]	ω ₀ =26.24	ω ₂ =59.27
Damping Ratio	0.05	0.05

Table 2: Properties of 2 D.O.F Torsional Model used for Validation

The numerical simulations were performed with a 10 N-m sinusoidal excitation force at 5 Hz, 25 Hz, and 45 Hz. These excitation frequencies correspond to values far below, near, and far above the primary system natural frequency, ω₀. The results of these simulations are plotted in Figure 18.

Figure 18: Numerical Response of Torsional Smart Spring at Various Excitation Frequencies:

$$F=10 \sin (\omega t) \text{ Nm; } N(t)=0.$$

These plots were obtained using the MATLAB script file `fft_SS_sim.m` located in Appendix A1.2. It can be seen that the steady-state response is reached in all three cases well before $t=1$ s. The effect of the transient response can be clearly noted by observing the time responses during first 0.3s, and is clearly visible by the peaks near ω_0 in the corresponding frequency responses. Otherwise, as expected, the steady-state response is dominated by the excitation frequency.

The theoretical steady-state response amplitude for each excitation frequency was calculated using Equation 19, and compared against the numerical results. The results of this analysis are shown in Table 3.

ω/ω_0	0.19	0.95	1.72
Numerical Results	0.320°	2.320°	0.158°
Theoretical Results	0.320°	2.320°	0.158°
% Error	0.00 %	0.00 %	0.00 %

Table 3: Error Between Numerically and Theoretically Calculated Response Amplitude for Torsional Smart Spring at Various Excitation Frequencies: $F=10 \sin (\omega t) \text{ Nm; } N(t)=0$.

It can be seen that the Numerical results for the simple case where $N(t)=0$ are very accurate, as expected.

The case where $N(t)$ is sufficient to fully couple the two systems can also be modeled according to Equation 19. In this case, the primary and secondary springs act in parallel with stiffness $k=(k_1+k_2)$ and the mass of the system is the total mass $m=(m_1+m_2)$. The natural frequency of the fully coupled system, ω_c , is defined by Equation 2. In the numerical model, the fully coupled condition is approximated when $|v_{rel}| < 2.54 \times 10^{-6}$, and remains so for a significant time. In the simulation used for validation of the model in the fully coupled condition, a requirement that was imposed was that $|v_{rel}| < 2.54 \times 10^{-6}$ for the entire simulation. The fully coupled damping ratio, ζ_c , is defined as follows.

$$\begin{aligned} \frac{c}{m} &= 2\zeta\omega_n \quad (1 \text{ D.O.F. system}) \\ \zeta_c &= \frac{c_c}{2m_c\omega_c} \end{aligned} \quad (20)$$

where:

$$\left. \begin{aligned} c_c &= c_1 + c_2 = 2m_1\zeta_1\omega_0 + 2m_2\zeta_2\omega_1; \\ m_c &= m_1 + m_2; \\ \omega_c &= \sqrt{\frac{k_1 + k_2}{m_1 + m_2}}; \end{aligned} \right\} \quad (21)$$

The following substitutions are made to simplify the results:

$$\left. \begin{aligned} a &= \frac{\zeta_2}{\zeta_1} \\ b &= \frac{m_2}{m_1} \\ c &= \frac{k_2}{k_1} \end{aligned} \right\} \quad (22)$$

Substitution of Equation 21 and Equation 22 into Equation 20 leads to the non-dimensional form shown in Equation 23, applicable to all 2 D.O.F. Smart Spring systems:

$$\zeta_c = \zeta_1 \frac{1 + ab\sqrt{\frac{c}{b}}}{(1+b)\sqrt{\frac{1+c}{1+b}}} \quad (23)$$

The relationship in Equation 23 is not required for the numerical calculations because the model treats the primary and secondary systems separately during the calculation; therefore the numerical model approximates the result of Equation 23.

Using Equation 22, Equation 23 and the system parameters in Table 2, the fully coupled damping ratio is $\zeta_c=0.0463$ and the fully coupled natural frequency, defined by Equation 2, is $\omega_c= 43.35$ Hz.

The numerical time and frequency response of the system was calculated using a sinusoidal excitation force of 10 Nm at frequencies of 10 Hz, 41 Hz, and 75 Hz. The actuators were made to supply a constant D.C. input force in order to fully couple the system. It was determined that the minimum value for $N(t)$ to ensure that the system remained fully coupled throughout the three presented simulations was approximately 2 kN. In order to achieve this, $N(t)$ was incremented until

$v_{rel} \ll 2.54 \times 10^{-6} \frac{rad}{s}$ (peak-to-peak) throughout the entire simulation. The results of these simulations are shown in Figure 19. The numerical results are compared to the theoretically calculated results obtained using Equation 19.

Figure 19: Numerical Response of Torsional Smart Spring at Various Excitation Frequencies:
 $F=10 \sin(\omega t)$ Nm; $N(t)=300$ N-m (Fully Coupled).

ω/ω_c	0.19	0.95	1.72
Numerical Results	0.069°	0.476°	0.325×10^{-1} °
Theoretical Results	0.069°	0.476°	0.325×10^{-1} °
% Error	0.00 %	0.00 %	0.00 %

Table 4: Error Between Numerically and Theoretically Calculated Response Amplitude for Torsional Smart Spring at Various Excitation Frequencies: $F=10 \sin(\omega t)$ Nm; $N(t)=2kN \rightarrow$ (Fully Coupled).

As shown in Table 4, there is again no significant error produced by the numerical model when the system is fully coupled.

5.3 Adaptive Impedance Characterization

The adaptive impedance capability of a Smart Spring device is derived from the ability to control the amount of coupling between the primary and secondary mass-spring systems. In doing so, the transfer function of the system is affected in real time. This can be seen by observing the frequency response plots in Figure 18 and Figure 19. The complex impedance, $\bar{Z}(\omega)$, shapes the frequency response. For example, Equation 19 defines the frequency response of a 1 D.O.F. mass-spring system. Letting $f = F/m$, and rearranging the equation, the transfer function is obtained.

$$\begin{aligned} \bar{Z} &= 2\zeta_1\omega_n + i\omega\left(1 - \frac{\omega_n^2}{\omega^2}\right) \\ \frac{x(\omega)}{f} &= \frac{1}{\omega\sqrt{(2\zeta_1\omega_n)^2 + \left(\omega\left(1 - \frac{\omega_n^2}{\omega^2}\right)\right)^2}} \quad \text{(magnitude)} \\ \therefore \phi(\omega) &= \tan^{-1}\left(\frac{\omega\left(1 - \frac{\omega_n^2}{\omega^2}\right)}{2\zeta_1\omega_n}\right) \quad \text{(phase)} \end{aligned} \quad (24)$$

Further examination of the frequency response of a Smart Spring device during uncoupled, fully coupled, and slightly coupled conditions using D.C. input actuation is presented next to demonstrate that a Smart Spring device can be configured to preferentially affect the on-line stiffness, effective mass, and damping of a dynamic system. The frequency responses of two slightly different Smart Spring configurations are presented. All three configurations have identical primary system

parameters. Thus, when the friction force feedback loop is open ($N(t) = 0$), the resulting primary system response is identical for both configurations. The secondary system in each configuration studied has mass and stiffness parameters selected to enable the Smart Spring device to affect the dynamic impedance in a preferential manner. These parameters are listed in Table 5. The frequency response plots in Figure 20 were created using an excitation frequency with an amplitude of 10 N-m at a linearly increasing frequency from 5 Hz-80 Hz where $\frac{d\omega}{dt} = 1.25 \frac{\text{Hz}}{\text{s}}$. Thus, the model is integrated over a time interval of 60 s. This was done to ensure that the response is a close approximation to the steady-state solution. In these simulations, the response of the system is gauged by the primary mass response. The fully uncoupled response is denoted as the *reference response*.

Configuration	Mass System	Stiffness System
m_2/m_1	4	0.75
k_2/k_1	0.5	3.75
ω_c	14 Hz	43 Hz

Table 5: Secondary System Parameters and Fully Coupled Natural Frequencies for Primary System: $m_1=0.0686 \text{ kg-m}^2$; $k_1=1864 \text{ N-m/rad}$

Stiffness Configuration

Observing Figure 20, it is seen that a stiffness-configured Smart Spring device is capable of causing the frequency response to exhibit an increased fully coupled natural frequency and reduced low-frequency response amplitude. This can be achieved by choosing the secondary system parameters such that $\omega_c > \omega_0$ by setting

$\frac{k_2}{k_1} > 1$, and/or $\frac{m_2}{m_1} < 1$; either choice will result in an increased fully coupled natural

frequency. In a stiffness-configured system, the higher frequency response amplitude is not greatly affected.

Mass Configuration

A mass-configured Smart Spring device is capable of exhibiting a fully coupled natural frequency that is much lower than the primary natural frequency. This is achieved by configuring the secondary system parameters such that $\frac{k_2}{k_1} < 1$, and/or $\frac{m_2}{m_1} > 1$. In the fully coupled condition this configuration exhibits lower high frequency response amplitudes in comparison to the reference response.

5.3.1 Smart Spring Damping Control

In the simulations discussed so far, a D.C. open-loop control force was used to cause the system to be fully coupled, or uncoupled in order to demonstrate that an appropriately configured Smart Spring device is capable of affecting both mass, and stiffness characteristics of the dynamic impedance. Any Smart Spring system is capable of preferentially achieving a response according to either the fully coupled, or the uncoupled response. Damping in a Smart Spring system is achieved through sliding frictional losses. As such, it is necessary to supply sufficient actuation force to produce relatively high frictional loads while avoiding the onset of total coupling between the primary and secondary systems. This is because the controllable frictional damping that is seen by the controlled mass is a product of the work that is done by the former on the secondary system. It is important to note that this work can be either positive or negative, depending on the relative phase and amplitude of the primary and secondary responses. In the case that this work is negative, the response of the primary mass will be increased.

In order to further characterize the damping, the simulation described above was repeated for both configurations using an open-loop D.C. control force of 200 N. These damped frequency responses are shown in Figure 20. In both cases the damped response has a propensity to follow either the reference response, or the fully coupled response pertaining to the configuration that it is associated with. In the case of the stiffness-configured system, the damped response shows a low propensity for coupling in the $0 < \omega < \omega_0$ frequency band; thus, it follows the response of the reference system. A relatively low, yet significant reduction in the response is achieved in this band. Generally, the response of oscillating systems is not greatly affected by damping in bands far below the natural frequencies. The impedance is dominated by the stiffness in these bands, causing the magnitude of the reactive component of the dynamic impedance to far outweigh the real component. It is worthwhile to note that even though the system was configured for enhanced stiffness, the stiffness seen by the primary mass is not significantly affected in this band for the case where $N = 200\text{N}$ because the systems are not significantly coupled. In the band near ω_0 , the response amplitude reduction is significantly more pronounced because damping is the prominent impedance component for systems oscillating near the natural frequency (see Equation 24). The prominence of the damping in this band occurs because as $\omega \rightarrow \omega_n$, the reactive component of the impedance: $\text{Im}(\bar{Z}) \rightarrow 0$. When $\omega \gg \omega_0$, the stiffness-configured system becomes nearly fully coupled; thus the response begins to follow the fully coupled frequency response.

The response of the mass-configured system is already approaching the resonance peak at $\omega = 1$ Hz; however the response does not become significantly damped in this band. This is because the secondary stiffness is much lower than the primary stiffness, and the secondary system, though relatively massive, offers little inertial resistance at such low frequencies. As such, the system has a high propensity for coupling at low frequencies. As the coupled mass-configured system approaches resonance, the propensity for coupling is reduced and the damping effects begin to dominate the response. The primary system is beginning to resonate in the band preceded by the fully coupled resonance, thus the propensity for coupling is further reduced and the magnitude of the damping is increased. During the transition from the fully coupled resonance and the primary resonance, the response of the system is increased with respect to the fully coupled and uncoupled responses. This occurs because the secondary system is in a stiffness dominated frequency band and $k_2 < k_1$; thus, kinetic energy is imparted to the primary system by the secondary system. As $\omega \rightarrow \omega_0$, the damped mass-configured system follows the reference frequency response. The damping levels achieved in this band are quite similar to the damped stiffness-configured system. As the excitation frequency is increased beyond the primary resonance, the coupled condition persists once again.

From the observations made concerning the frequency response curves in Figure 20, some very interesting conclusions can be drawn about the controllable nature of the dynamic impedance of a Smart Spring device. These frequency response curves are a qualitative approximation of the general shape of the system

transfer function when open-loop D.C. control is used. However, it is important to note that the curves generated for a Smart Spring system are not a general system identification. These curves are case-specific; they pertain only to the response of the system described in Table 5 generated using an excitation force of 10 N-m. The transfer function can be shifted between the fully coupled and uncoupled states. The *present state* of the system is hereby defined as the transfer function that describes the response of the primary system due to a specific combination of excitation and input actuation forces. Thus, the present dynamic impedance exhibited by the system can quantitatively describe the present state. The fully coupled and uncoupled states are defined as *reference states*.

An advantage to the Smart Spring that is not achieved in other active impedance devices is that all three of the dynamic characteristics that define the state of the system can be controlled in a preferential manner. A Smart Spring device can be configured for mass control, or stiffness control as depicted in Figure 20. The mass control configuration enables the controller to shift the transfer function leftward from the reference response to the fully coupled mass configuration response, and the stiffness configuration shifts the transfer function to the right. Additionally, the damping levels in the system can be preferentially controlled to affect the response amplitude near the resonance of the on-line state.

Figure 21 depicts the present states of the stiffness configuration system for various levels of open-loop D.C. input actuation. It can be seen that the state of the system is continuously variable between the two reference states. The transfer

function can be affected to reside anywhere in the two-dimensional surface that is loosely defined by the fully coupled and uncoupled states in the frequency domain. The surface, in which the present states can reside, is denoted as the *state surface*. This state surface is made up of the infinite number of possible present states. The states shown in Figure 22 are plotted with respect to input actuation level in order to visualize the state surface more clearly.

Figure 21: Frequency Response of a Stiffness Configured Smart Spring Using Incremental Open-loop D.C. Control

Figure 22: Three Dimensional Plot Depicting State Surface

Interestingly, other active impedance devices can produce similar state surfaces. For example, a 1 D.O.F. mass-spring system fitted with a continuously variable damper will exhibit a state surface that is bounded between the two states defined by Equation 24, where the upper and lower bounds of the possible damping ratios are used. In fact, if the piezoelectric actuators in the Smart Spring system were replaced by an active damper that is attached from m_1 to m_2 , it would be expected that similar state surfaces to Figure 22 could be observed. The main difference between the state surfaces observed using such a system and the Smart Spring system is that the fully coupled state should not be observable for all ω since this would entail an unbounded damping ratio.

6 Experimentation

In the present Chapter, details of tests performed using the torsional Smart Spring prototype are presented. The purpose of the tests was to verify if the proposed numerical model appropriately represents the dynamic impedance characteristics and the overall dynamics of an actual Smart Spring device. In order to achieve this, a test matrix was developed to investigate the characteristics of the dynamic impedance behavior of the prototype using the state surface approach discussed in the previous section. The results of this investigation are compared with numerical results obtained using simulations designed to emulate the experimental test matrix.

6.1 Experimental Set-up

The test apparatus used in the investigation is the torsional Smart Spring prototype described in Section 3.1. A schematic of the experimental set-up is depicted in Figure 23.

Figure 23: Schematic of Test Set-up using Torsional Smart Spring Prototype

The sensors used to collect the relevant data included a tension/compression load cell, the actuator can load cells, and a micro-displacement probe. The tension/compression load cell was installed between the shaker stinger and the excitation/sensor bar to collect the excitation loads provided to the primary mass by the shaker. The load cell was calibrated in tension/compression from 0N to 1500N with accuracy of $\pm 1\%$ (full-scale output). The displacement probe was a linear variable differential transformer (LVDT) transducer with maximum non-repeatability of 6 micro-inches and maximum non-linearity of $\pm 0.25\%$ (full-scale output). The data acquisition system was configured to record the sensor outputs in an A.C. coupled format at a rate of 10 kHz. The actuators and mechanical shaker were controlled using Simulink and a MATLAB P.C./hardware interface.

The relevant system parameters were measured and are presented in Table 6.

Property	Primary System	Secondary System	Fully Coupled
Mass [$\text{kg}\cdot\text{m}^2$]	$m_1=0.0686$	$m_2=0.170$	$(m_1+m_2)=0.239$
Stiffness [$\text{N}\cdot\text{m}/\text{rad}$]	$k_1=1864$	$k_2=10275$	$(k_1+k_2)=12139$
Natural Frequency [Hz]	26	39	35

Table 6: System parameters of the Torsional Smart Spring Prototype

6.2 Overview of Test Procedure

The test matrix was designed to produce data that could be used to form curves similar to Figure 22 in order to characterize the dynamic impedance of the hardware device at various D.C. actuator input voltage levels. The input excitation

force provided by the shaker was a chirp signal as in the case of the numerical analysis discussed in Chapter 5. However, it was found that the peak-to-peak amplitude of the excitation force could not be maintained reliably throughout the frequency sweep. As such, the shaker was instead controlled to provide an excitation that approximated white noise. The use of this type of excitation entails less stringent demands regarding the precision of the shaker performance. However, the response of the system to white noise does not approximate the steady-state solution in the frequency domain. In fact, the system response to such an excitation is transient at all times. Due to this fact, it is expected that the frequency response will be dominated by the resonance frequency associated with the on-line state.

Before testing commenced, the actuator pre-load bolt was adjusted so that there was no significant contact pressure at the frictional interfaces when the D.C. actuator input was zero, and that sufficient coupling occurred when the input was a maximum of 100 V. As such, the range of states expected to be observable included the uncoupled and fully coupled conditions.

For each test run, the D.C. actuator input voltage was adjusted to the desired level. The actuator can load cell signals were monitored at this time to ensure that the actuation load was evenly distributed among the three actuator cans. When the shaker excitation was initiated, the load cell and displacement probe output signals were recorded for a period of 30 seconds. During this period, the actuator can load cell signals were monitored in real time to ensure that no significant drift in the actuation force occurred. This process was repeated using voltage levels ranging from zero to 100 V D.C. at 10 V increments.

Upon completion of the test matrix, the data was transformed into the frequency domain and the frequency response functions were plotted.

6.3 *Experimental Results*

Some results obtained from the experimental procedure are shown in Figure 24. As expected, the response was found to be dominated by the resonance frequency. This can be seen in the magnification window in Figure 24(B).

Figure 24: Example of Experimental Time Domain Excitation and Displacement Response of Torsional Smart Spring Prototype

The time-domain response in Figure 24(B) is shown in the frequency domain in Figure 25. It can be seen that the results contain significant scatter when presented in the frequency domain. This is because the excitation consisted of noise, which was carried in the response. The scatter was avoided, in part, by measuring the transfer function rather than the frequency-domain output of the displacement probe. In this case, the transfer function is the ratio between the angular response and the excitation

torque calculated using the excitation force in the frequency domain, $F(\omega)$. The transfer function associated with the 0V input actuation response is shown in Figure 26. As it is commonly done with transient data, the scatter was further removed using rectangular windowing [17].

Figure 25: Frequency Response of Torsional Prototype using White Noise Excitation with Zero Actuator Input

Figure 26: Transfer Function of Torsional Prototype for 0V Input Actuation

Note that the window-averaged curve is truncated at both ends by half the length of the window.

After processing the remaining data associated with the test matrix as described above, the state surface plot in Figure 27 was created.

Figure 27: Experimentally Obtained Frequency Response of Torsional Prototype using Random Excitation and Various Actuator Input Voltage Levels

Observing Figure 27, the resonance peak associated with the uncoupled response is measured at approximately 26 Hz, corresponding to the primary system natural frequency in Table 6. This peak was significantly reduced as the actuation voltage was increased from 0 to 60 V. This corresponds to the expected increased damping of the primary resonance as the states were incremented to those associated with larger input forces. At 70 V the fully coupled resonance peak is visible in the

data. As the states were further incremented to 100 V, the primary and secondary systems became fully coupled.

6.4 Comparison to Numerical Results

The trends mentioned above are representative of the previously obtained numerical results. However a more direct comparison can be made in order to consolidate the correlation between the experimental and numerical results. The numerical model was used to simulate the test matrix in order to make a direct comparison. To this end, the numerical model was provided the system parameters listed in Table 6. The signals collected from the tension/compression load cell that was used to measure the excitation force were used as the excitation force in the numerical calculations. As such, the experimental procedure was followed as closely as possible.

Figure 28: Numerically Obtained Frequency Response of Torsional Prototype using Experimental Excitation and Various Actuator Input Voltage Levels

The results of the numerical calculations are shown in Figure 28. In the figure, the actuator input levels are denoted as the percentage of the force required to cause the model to become fully coupled throughout the simulation.

Observing Figure 28, it can be noted that the states are quite well approximated by the numerical model. The main differences between the experimental and numerically obtained data are observed at the low frequency end of the states. The experimentally obtained states begin to decline rapidly as the response frequency approaches zero. This is because the sensor signals were A.C. coupled, as previously stated. As such, the data acquisition system was configured to reduce the sensitivity of relatively low frequency signals in the recorded data.

From these results, it is concluded that the controllable dynamic impedance model described in Chapter 5 presents an accurate description of the actual Smart Spring device. The nature of the control authority that is brought to bear on the dynamic system by the frictional control force is such that (1) the damping level that affects the motion of the controlled system is continuously variable within the bounds illustrated by the reference states, and (2) the resonance frequency exhibited by the controlled mass is affected in a “step-wise” fashion. Thus, the resonance frequency is not continuously variable.

7 Control Law for a Base-excited Smart Spring System

In the present Chapter, a control law is developed for the base-excited Smart Spring system depicted in Figure 29. The objective of the control law is to use the controllable nature of the system's dynamic impedance to reduce the motion of the controlled mass, m_c .

Figure 29: Base-excited Smart Spring System

It is worthwhile to note that the system in Figure 29 is analogous to many practical engineering applications. For example, a cargo payload in an aircraft, or space launch vehicle can be analyzed using this simplified schematic. In such an analogy, the relatively light and stiffly sprung base mass denoted by m_b and k_3 can be used to represent the mass and stiffness of an element of the vehicle fuselage. The relatively heavy cargo payload is denoted by m_c . The Smart Spring system is used to mount the payload to the fuselage.

A control law developed to reduce the motion of m_c should generally be required not to cause the base mass motion to be amplified. Such amplification could result in increased unwanted vibration elsewhere in an integrated system. As such, the controllable nature of the dynamic impedance introduced by the Smart Spring device was used to extract kinetic energy from the system.

7.1 Development of Control Law

This energy can be extracted from the system through carefully controlled frictional coupling. If the system is controlled to approach the fully coupled state from the uncoupled state, or vice versa, during selective dynamic conditions, kinetic energy that would have otherwise been returned to the system in a reactive manner can be extracted. A similar concept was investigated by Cunefare using a stiffness-based state-switchable device [12]. In addition, frictional losses will occur due to the sliding contact between m_c and m_a . It will be demonstrated that in order to evaluate the potential of the device to achieve this dual goal at a given time, the only state information needed is the absolute velocity of the controlled structure, m_c , and the speed of the structure relative to that of the actuators, m_a . To demonstrate this, an analogy is presented that is useful for visualizing the frictional interaction in the Smart Spring system. Figure 30 relates the effects of the frictional interaction to a controllable damper. This analogy was discussed briefly in Chapter 5. Since the Smart Spring assumed to be used with this controller uses surface contact friction, the damper in Figure 30 can be turned off, corresponding to $N(t)=0$, or be turned on ($N(t)\neq 0$) and cause the coupling and frictional losses corresponding to the states

attainable. These states range continuously between the uncoupled and fully coupled states. The present control law shall switch the actuator either *on* or *off*. The *on* condition shall correspond to $N(t)=N_{high}$, where N_{high} is a positive constant.

Figure 30: Switchable Damper Analogy

If the actuator can be either *on* or *off*, the decision of when to do so is the only relevant consideration for the controller to discern. As previously stated, this decision needs only to be based on the absolute velocity of the controlled structure, m_c , and the speed of the structure relative to that of the actuators, m_a . As such, Equation 25 contains all of the necessary information.

$$u = \dot{x}_c (\dot{x}_a - \dot{x}_c) \quad (25)$$

The decision-making process is described in the following four possible cases defining the sign of u in Equation 25.

Case 1:

$\dot{x}_c$ is positive and $\left(\dot{x}_a - \dot{x}_c\right)$ is positive.

$\therefore u = \dot{x}_c \left(\dot{x}_a - \dot{x}_c\right)$ is positive

The controlled mass is moving in the positive direction.

The relative speed is in the positive direction.

Thus, the resulting damper force will promote the motion of the controlled structure and the damper should be switched off.

Case 2:

$\dot{x}_c$ is positive and $\left(\dot{x}_a - \dot{x}_c\right)$ is negative.

$\therefore u = \dot{x}_c \left(\dot{x}_a - \dot{x}_c\right)$ is negative

The controlled mass is moving in the positive direction.

The relative speed is in the negative direction.

Thus, the resulting damper force will oppose the motion of the controlled structure and the damper should be switched on.

Case 3:

$\dot{x}_c$ is negative and $\left(\dot{x}_a - \dot{x}_c\right)$ is negative.

$\therefore u = \dot{x}_c \left(\dot{x}_a - \dot{x}_c\right)$ is positive

The controlled mass is moving in the negative direction.

The relative speed is in the negative direction.

Thus, the resulting damper force will promote the motion of the controlled structure and the damper should be switched off.

Case 4:

$\dot{x}_c$ is negative and $\left(\dot{x}_a - \dot{x}_c\right)$ is positive.

$\therefore u = \dot{x}_c \left(\dot{x}_a - \dot{x}_c\right)$ is negative

The controlled mass is moving in the negative direction.

The relative speed is in the positive direction.

Thus, the resulting damper force will oppose the motion of the controlled structure and the damper should be switched on.

It can be seen from the four cases presented, that whenever u is negative, the damper should be turned on and when u is positive, the damper should be turned off. This enables the system to provide mechanical resistance to the motion of the controlled mass only at times when the resulting damper force does not cause energy to be pumped to the controlled mass from the remainder of the system, and vice versa. When $u = 0$, either the controlled mass velocity and/or the relative velocity is zero. If the controlled mass velocity is zero, no change in the motion is desired. Furthermore, if the relative velocity is zero, no change in the damper force will result from a switch at that instant. Thus, the case $u = 0$ is associated with no switch. The switching rules can be summarized as follows:

$$\text{Input force: } N = \begin{cases} N_{high} & \text{if } u < 0 \\ N = 0 & \text{if } u > 0 \\ N_{last} & \text{if } u = 0 \end{cases} \quad \text{Equation 26}$$

Implementation of the control law described by Equation 26 is shown in Figure 31. The term *state-switch* controller was discussed by Cunefare and is retained in this work. However, it is worthwhile to note that the states, as referred to

in the present research, are not necessarily “switched” by the control action. It is the actuation force that is switched.

Figure 31: Implementation of the Closed-loop State-switch Feedback Controller

Although the control force N_{high} is constant, the actuation conditions are not the same as in the investigation presented in Chapters 5 and 6, where $N(t)$ was kept constant at all times. The uncoupled state is only attained with certainty when $N(t)=0$, as in the previous work. However, the discontinuous step-wise increase from $N(t)=0$, to $N(t)=N_{high}$ can result dynamic conditions that were not attainable using constant D.C actuation. For example, the control force necessary to fully couple the stiffness-configured system studied in Chapter 5 was 1300 N when the excitation

force used was $10\sin(\omega t)$ N in the frequency range of $5 < \omega < 80$ Hz. If at a given time during the simulation, $N(t)$ was switched from 0 to 1300 N instantaneously, the system would not lock immediately since this would be in defiance of Newton's 2nd Law. Thus, the system must progress from the uncoupled state, to the fully coupled state over a finite period of time.

In Figure 31, the appropriate control input is deduced using Equation 25 and the three inequalities in Equation 26. Since the inequalities are mutually exclusive, the input to the summation junction is either N_{high} or $N_{low}=0$. For the case where $u=0$, the previous input value is circulated back through the junction and the decision is deferred until the next sample arrives.

7.2 Performance of the State-switch Controller

In the present section, numerical simulations are performed to investigate the performance of the state-switch controller. The parameters used during the simulations are listed in Table 7.

m_a (kg)	m_b (kg)	m_c (kg)
1.5	1.0	11.5
K_1 (N/m)	K_2 (N/m)	K_3 (N/m)
13135	26270	175126

Table 7: Parameters of the Base-excited System

The natural frequencies and eigenvectors associated with this system are dependent on the input actuation force that is supplied by the actuators. As discussed in previous chapters, the natural frequencies are expected to switch between the uncoupled state and the fully coupled state. In the uncoupled state, the system will respond with three degrees of freedom. When the actuator force is sufficient to fully

couple the system, it responds with only two degrees of freedom. These two input conditions represent two possible extremes in the range of dynamic characteristics that can be achieved in the system by the controller. The natural frequencies and respective modal displacements exhibited by the system in the fully coupled and uncoupled states are shown in Table 8.

Input Condition		1 st Resonance	2 nd Resonance	3 rd Resonance
Uncoupled	Natural Frequency (Hz)	7.13	17.61	73.91
	x_a	0.17	1.00	-0.06
	x_b	0.13	0.06	1.00
	x_c	1.00	-0.015	-0.011
Fully Coupled	Natural Frequency (Hz)	8.11	73.81	N/A
	$x_a = x_c$	1.00	-0.02	N/A
	x_b	0.19	1.00	N/A

Table 8: Natural Frequencies and Modal Displacements of the Numerical Model for Fully Coupled and Uncoupled States

The motion of the controlled mass dominates during the first resonance in the case of both input conditions. It can be seen that fully coupling the system results in an exclusion of the second resonance condition. For example, the motion of the base mass dominates the third resonance in the uncoupled state. The motion of the actuator mass dominates the second resonance in this condition. Furthermore, when the system is fully coupled, there is no resonance associated with the actuator mass alone. The motion of the base mass in this case dominates the second resonance, whereas the first resonance is dominated by the motion of the controlled mass in conjunction with the actuator mass because they are moving as one mass.

7.2.1 Overview of Simulations

The first set of simulations was used to investigate the performance of the controller in the time domain. To this end, a constant sinusoidal excitation force was provided

to the system, using various constant excitation frequencies for each simulation. During the simulations, the actuator was controlled as follows. The control force was held at zero for the first 3 seconds. At $t=3$ seconds, the control force was increased to 1000N for a period of 5 seconds in order to examine the response of the fully coupled state. Then the actuation force was reduced back to zero for a period of 2 seconds. Subsequently, at $t=10$ seconds the state-switch controller was activated for a period of 5 seconds.

The above-described simulation procedure was used to investigate the performance of the state-switch controller when the first two resonance frequencies are targeted for control. Finally, a slow frequency sweep similar to that used during the investigation discussed in Chapter 5 was used to examine the performance at all relevant frequencies. During this simulation, the state-switch controller is on at all times.

7.3 Simulation Results

In the first simulation presented, the first resonance frequency was targeted for control. The results of this simulation are displayed in Figure 32. Since the excitation frequency is at the first resonance of the system, the controlled mass begins to resonate when there is no actuation force supplied. Note that no structural damping is used during these simulations. At $t=3$ seconds, a 1000N control force was supplied. This resulted in a temporary increase in damping as the response of the controlled mass relaxed toward that of the fully coupled state. This period of damping is due to the fact that the 1000N actuation force did not immediately initiate

the fully coupled state. After $t=5$ seconds the Smart Spring system is fully coupled. During the above-described period of damped oscillations, the resonance characteristics changed from those of the uncoupled state to the fully coupled state (Table 8). The system's first resonance frequency has changed from the original 7.13 Hz to 8.11 Hz. As such, the excitation frequency is now slightly lower than the resonance and the resulting response is reduced and stable. At $t=8$ seconds, the control force is reduced to zero. Thus, the resonance frequency is returned back to that of the uncoupled state and m_c again begins to resonate. At $t=10$ seconds, the state-switch controller is engaged. At this point the response of the controlled mass is drastically reduced. There is a period of relaxation, as in the case with the DC actuation. After this relaxation period, the steady response of the controlled mass has amplitude of approximately 1mm, whereas the response during the DC actuation has peak amplitude (highest point of the cycle) of 20mm. Thus, the maximum response with the state-switch control is approximately 5% of that with the DC actuation. This proves that the active feedback controller works to suppress vibrations at the first resonance quite well in comparison to the simple shift in the system stiffness associated with changing the on-line state.

Figure 32: Numerically Obtained Results using State-switch Control at 1st Resonance

This simulation was repeated using the second resonance and the results are shown in Figure 33. It can be seen that similar results were obtained in these conditions.

Figure 33: Numerically Obtained Results using State-switch Control at 2nd Resonance

In the final simulation, the state-switch controller was used in conjunction with a slowly sweeping excitation frequency. The results are presented in the frequency domain in Figure 34. Referring to the plot, it can be seen that there are five distinct frequency regions between 0 and 50Hz in which the performance of the state-switch controller can be evaluated. The band between 0 and 4Hz is a stiffness-dominated region. The performance of the controller ranges from moderate to insignificant in this region. This is because the stiffness of the secondary spring is relatively low.

Figure 34: Numerically Obtained Frequency Response of the Base-excited System using State-Switch Control

The band between 4Hz and 13Hz encompasses the first resonance frequency for the no actuation and locked input conditions. As such, for this band the response of the system is best controlled through damping. The performance of the state-switch

controller for vibration suppression in this band is excellent. In the region between 13Hz and 16Hz, the vibration reduction achieved by the state-switch controller was actually less than that for the DC input conditions. This band is both a mass and stiffness controlled region for vibration suppression. The band between 16 Hz and 24Hz encompasses the second resonance frequency for the uncoupled system. Thus, it is a damping controlled region. Once again, the performance of the state-switch controller is vastly increased. In the band between 24Hz and 50Hz the state-switch controller gives slightly better results than those achieved by the Smart Spring system in the fully locked condition. It is worthwhile to point out that the use of the Smart Spring in conjunction with the state-switch controller actually produced an anti-resonance peak at the second uncoupled resonance.

8 Conclusions and Future Work

The Smart Spring is an indirect-active vibration suppression device that is capable of preferentially altering the impedance characteristics of a dynamic system. The Smart Spring device employs piezoelectric actuators to generate friction forces that are used to control the dynamic coupling between the primary and secondary mass-spring systems. Such control over this coupling effect enables the Smart Spring device to affect the real and reactive components of the dynamic impedance of a system such that (1) the resonance frequencies and (2) the damping levels can be dynamically controlled. This ability to control all aspects of the complex impedance of a dynamic system is unique to the Smart Spring device.

The nature of the control authority that is brought to bear on a system's complex impedance by the Smart Spring device was investigated. It was found that appropriate configuration of the Smart Spring device can facilitate either an increase or a reduction in the system's fully coupled resonance frequencies, and that significant control of damping can be achieved using such configurations. It was found that this shift in resonance frequency does not occur in a continuous fashion, but rather occurs in a step-wise fashion as the actuation voltage is varied between the fully coupled and uncoupled states. These characteristics were observed in Chapters 5 and 6.

Use of the Smart Spring device to provide sufficient cyclic variations in the complex impedance associated with the root of a helicopter rotor blade to suppress vibratory rotor hub loads should be investigated. The work of Nitzsche and Anusanti-

Inthra et al. suggests that cyclically varying the damping and stiffness at the blade root can result in such reductions. In the above-mentioned research, continuous sinusoidal variations in individual impedance characteristics such as stiffness or damping were used. The Smart Spring device is not capable of such continuous variations in stiffness, as this would result in a continuous variation in the system resonance frequency between ω_0 and ω_c , which has been shown not to be the case. However, in the general case where $N(t) \neq 0$, the reactive component of the system's complex impedance is affected as the motion of m_2 results in the storage of mechanical energy in the secondary spring. The influence of these changes in reactive impedance was observed in Chapter 5 (Figure 21). Variation in input force between 0 and 250 N resulted in a continuous change in the state of the system from the uncoupled to the fully coupled states in the low frequency band between 5 Hz and 15 Hz. As such, even though the resonance frequencies exhibited by the system can consist only of those that are defined by the system eigenvalues (Equation 1, Equation 2 and Equation 3), the states exhibited a seemingly continuous change in the "effective stiffness" due to the Smart Spring's control over the reactive component of the complex impedance. Numerical and experimental investigations should be conducted to verify whether the Smart Spring's control authority over the complex impedance can be used to produce results similar to those of Nitzsche and Anusanti-Inthra et al. For this investigation, it is recommended that the numerical model and the experimental prototype be dynamically similar to a real helicopter blade. The

numerical and experimental procedures should be performed such that the results can be directly compared with results of the above-mentioned research.

Numerical simulations using the state-switch controller implemented for the base-excited Smart Spring system demonstrated that such control of the complex impedance of a system could result in significant vibration suppression. The viability of integrated systems using a Smart Spring device in conjunction with the state-switch control approach should be further evaluated using numerical and experimental testing. It is recommended that these tests be done using models and hardware that are dynamically similar to real applications. One of the major focal points in recent smart structures research has concentrated on suppression of vibration in helicopter rotor systems. The Smart Spring/state-switch control approach should be investigated for this application. The approach is well suited for increasingly dynamic conditions because the controller only uses estimates of the real-time primary and secondary mass absolute velocities and does not depend on estimates of the oscillation amplitudes and frequencies. Due to the simplicity of the control law, computation speed is not expected to be a limiting factor associated with the performance of the system for the above-mentioned application.

Appendix

A1 Utility Code

A1.1

ini_multi_sim.m

```

Declare System Parameters
%%%%reference%%excitationForce[N-m]
F=10;
m1=.0686;
m2=.170;
k1=288*4.44822*.0254*180/pi
k2=1071*4.44822*.0254*180/pi
zeta=.05;
c1=2*zeta*sqrt(k1/m1)*m1;
c2=2*zeta*sqrt(k2/m2)*m2;
%%%%damping%%
m1_d=m1;
m2_d=1*m2;
k1_d=k1
k2_d=2*k2
c1_d=2*zeta*sqrt(k1_d/m1_d);
c2_d=2*zeta*sqrt(k2_d/m2_d);
%%%%mass%%
m1_m=m1; m2_m=4*m1;
k1_m=k1
k2_m=.5*k1

```

```
c1_m=2*zeta*sqrt(k1_m/m1_m);
c2_m=2*zeta*sqrt(k2_m/m2_m);
%%%Stiffness%%
m1_s=m1;
m2_s=.75*m1;
k1_s=k1;
k2_s=k2;
c1_s=2*zeta*sqrt(k1_s/m1_s);
c2_s=2*zeta*sqrt(k2_s/m2_s);
%%%reference%%
%
%
% Declare state matrix parameters
%
%
A=[0,1,0,0;-k1/m1,-c1/m1,0,0;0,0,1,0,0,-k2/m2,-c2/m2];
B=[0;1/m1;0;-1/m2];
E=[0;1/m1;0;0];
%%%damping%%
A_d=[0,1,0,0;-k1_d/m1_d,-c1/m1_d,0,0,0,0;-k2_d/m2_d,-c2_d/m2_d];
B_d=[0;1/m1_d;0;-1/m2_d];
E_d=[0;1/m1_d;0;0];
%%%mass%%
A_m=[0,1,0,0;-k1_m/m1_m,-c1/m1_m,0,0,0,0;-k2_m/m2_m,-c2_m/m2_m];
B_m=[0;1/m1_m;0;-1/m2_m];
E_m=[0;1/m1_m;0;0];
%%%stiffness%%
A_s=[0,1,0,0;-k1_s/m1_s,-c1/m1_s,0,0,0,0;-k2_s/m2_s,-c2_s/m2_s];
B_s=[0;1/m1_s;0;-1/m2_s];
E_s=[0;1/m1_s;0;0];
```

```

% State output matrix
%
%
x1=[1,0,0,0];x2=[0,0,1,0];v1=[0,1,0,0];v2=[0,0,0,1];
C=[x1;x2;v2-v1];

A1.2

fft_SS_sim.m

% FILE FOR FFT ANALYSIS of Simulation Results
% Use with Simulink models ref_sim, stiff_sim, damp_sim mass_sim
%
%
% data sample rate
% sample time
% length of time for data to be analyzed

Fs=1e4;
dt=1/Fs;
t2=25;
% time=0:dt:t2;
% length(time);

%Plot text%
lttext='m_1 Time Response for F=100 N-m, N=0 V';
tttext='10-50 Hz Chirp Response Torsional Smart Spring: Numerical; N=40 ';
```

```

Y_LABEL1='Position [degrees]';
%%%%%%%%%%LOAD FILE WITH DATA TO BE ANALYZED: [Comment to exclude] %%%%%%%%%%%%%%%
%Reference system%

load simtheta_ref;
load simtheta_ref2;
% DATA_IN = simtheta_ref(2,1:length(time))*180/pi;
DATA_IN = simtheta_ref(2,:)*180/pi;
X_2=simtheta_ref2(2,:)*180/pi;
time = simtheta_ref(1,:);

%%mass system%
% load simtheta_mass;
% load simtheta_mass2;
% % DATA_IN = simtheta_mass(2,1:length(time))*180/pi;
% DATA_IN = simtheta_mass(2,:)*180/pi;
% X_2=simtheta_mass2(2,:)*180/pi;
% time = simtheta_mass(1,:)*180/pi;

%stiffness system%
% load simtheta_stiff;
% load simtheta_stiff2;
% % DATA_IN = simtheta_stiff(2,1:length(time))*180/pi;
% DATA_IN = simtheta_stiff(2,:)*180/pi;
% X_2=simtheta_stiff2(2,:)*180/pi;
% time = simtheta_stiff(1,:)*180/pi;

%damping system%
% load simtheta_damp;
% load simtheta_damp2;

```

% VECTORS OF DATA TO BE ANALYZED

%% shorten length of data if necessary {or comment}
% assign data

% and time vectors

%% VECTOR OF DATA TO BE ANALYZED

%% shorten length of data if necessary {or comment}
% assign data

% and time vectors

%% VECTOR OF DATA TO BE ANALYZED

%% VECTOR OF DATA TO BE ANALYZED
%% shorten length of data if necessary {or comment}
% assign data

% and time vectors

%% VECTOR OF DATA TO BE ANALYZED


```

%
% ylabel('Position [dB]')

figure(1),plot(time,DATA_IN,'m'),grid on
hold on
Xmax=max(DATA_IN(length(DATA_IN)-1*Fs:length(DATA_IN)))*ones(1,length(time))
plot(time,Xmax,'r--')
xlabel('TIME(s)')
ylabel(Y_LABEL1)

figure(2),plot(f,log10(MX),'m'), zoom on, grid on,xlim([0 65])
xlabel('FREQUENCY (Hz)')
ylabel('Position [dB]')
title(tttext)
hold on

%figure(1), subplot(3,1,3), plot(f(1:NumUniquePts-1),PH(1:NumUniquePts-1)), zoom on, grid on

%xlabel('FREQUENCY (Hz)')

%ylabel('Phase (degrees)')

%legend(INPUT_DATA_NAME)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%END%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

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