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AN INTEGRATED VIBRATION-BASED STRUCTURAL HEALTH MONITORING SYSTEM

by

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A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

Department of Civil and Environmental Engineering
Carleton University, Ottawa, Canada
March 2002

The Doctor of Philosophy in Civil Engineering is a joint program with the University of
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Department of Civil and Environmental Engineering

The undersigned recommend to the Faculty of Graduate Studies and Research acceptance
of the thesis

An Integrated Vibration Based Structural Health Monitoring System

Submitted by

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B.Sc., M.Sc. M.T.C., Cairo, Egypt.

In partial fulfillment of the requirements for the degree of
Doctor of Philosophy

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March 2002

Abstract

Continuous monitoring of the health of a structure is a requirement desirable for many engineering structures, to obtain early warning of impending disasters, which is important in both civil and aerospace applications. However, the complexity of large structures and the difficulty in accessing them makes the use of traditional non-destructive evaluation, such as visual inspection and instrument evaluation methods, impractical. An effective alternative is the use of methods that depend on vibration-based damage identification. These methods are based on measurement of changes in the modal characteristics of the structure, that is, its frequencies and mode shapes, to identify changes in its physical properties. However, vibration-based damage identification (VBDI) still faces a number of challenges that have not been fully resolved. The incompleteness of measured mode shapes, the challenges posed by the selection of the set of identification modes and ensuring the fidelity of the finite element model (FEM) of the monitored structure, and the mathematical complexity of the identification algorithm are the issues that must be addressed to ensure satisfactory performance of the VBDI techniques.

The objective of this work is to develop an improved approach to damage detection that may significantly enhance the ability of the optimal matrix update detection algorithms in identifying damage in complex structures, using limited number of measured degrees of freedom.

A new set of criteria is developed for an optimal selection of the response points. The selection criteria are used to determine the minimum number of measurement

degrees of freedom, which would constitute an adequate modal database for model-based identification algorithms and at the same time preserve the identity of the set of identification modes. An existing mode selection criterion is modified for use in damage identification and for the selection of a FEM reduction/expansion technique that would have the ability of preserving the modal properties of the original model for a selected set of modes. A new combination of two simple mathematical algorithms, modal residual and nonlinear optimization, is used to identify damage location and extent.

The theoretical background of the proposed VBDI approach is derived. Computer simulation studies are presented. They show the feasibility of the proposed VBDI approach as well as its limitations. Some of the challenges are highlighted through modal tests designed to provide estimates of damage in a three dimensional eight-bay free-free frame used for the experimental validation of the proposed approach. The construction of a finite element model of the structure and its correlation with measured test data are described. Details of tests on a healthy structure as well as on a structure in which several predetermined damage scenarios have been introduced are presented. Test data and damage identification results are presented. The proposed VBDI approach is found to be successful in predicting single and multiple damage sites using a limited amount of modal data of a truncated set of incomplete identification modes, provided the damage is significant.

*To the memory of my mother to whom I owe everything in my life
and to my father whose guidance and support made my dreams come true*

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Notations

M	An $n \times n$ mass matrix
u	A vector of nodal displacements
K	An $n \times n$ stiffness matrix
f(t)	the vector of externally applied dynamic loads
K_a	The $n \times n$ adjusted stiffness matrix
K_j^e	The elemental stiffness matrix of the j^{th} element
M_R	Reduced mass matrices
K_R	Reduced stiffness matrices
C	The damping matrix
I	The identity matrix
λ_i	The square of the natural frequency
ω_i	The i^{th} frequency of the system
ϕ_i	The i^{th} mode shape of the system
δ_{ij}	The Kronecker delta
u_n	The full set of physical FEM displacement
u_a	The active set of DOFs
u_c	The condensed or deleted set of DOFs
T	The transformation matrix
$\ \cdot \ _F$	Frobenius norm
A	The original $n \times n$ system property matrix
A_a	The $n \times n$ updated system property matrix
S and Y	Are $n \times p$ matrices that define constraints with the measured data
n	The number of degrees of freedom of the model
p	The number of matrix constraint equations
Ω^2	The $p \times p$ diagonal matrix of corresponding measured eigenvalues
D	An $m \times n_e$ matrix of constraints coefficients whose elements $d_{ij} = \phi_i^T K_j^e \phi_i$
m	The number of modes used in the identification procedure
n_e	Vector of unknown changes in the stiffness δk_j

$\delta\lambda$	The m vector of changes in the eigenvalues $\delta\lambda_j$.
$K_r^e(i, j)$	The ij^{th} coefficient of matrix K_r^e .
Q	A symmetric positive definite matrix
δk	The vector of the unknown damage parameters
R_i	Modal Residual Vector
ϕ_{di}	The damaged i^{th} mode shape
$\{\phi_{ij}\}$	a collection matrix, containing the j^{th} mode shape components associated with the degree of freedom of the element i
λ_{di}	The associated eigenvalue
FRF	Frequency Response Functions
DDI	Damage Detection Index
FI_i	The amplitude of the FRFs of the undamaged structure
FD_i	The amplitude of the FRFs of the damaged structure
U_j	The total strain energy
U_{ij}	The modal strain energy contribution of the element i in the j^{th} mode
MAC	Modal Assurance Criterion
SEREP	The system equivalent reduction expansion process
FIM	Fisher Information Matrix
E	An idempotent matrix (i.e. $E^2 = E$)
$\Delta\omega_i$	Error in natural frequencies
ω_{im}	The i^{th} measured natural frequency
ω_{ia}	The i^{th} calculated natural frequency
Δq_i	Errors in mode shapes
q_{ij}^m	The measured components at the j^{th} measured DOF for the i^{th} mode
q_{ij}^a	The analytical components at the j^{th} measured DOF for the i^{th} mode
EA_s	The calculated tube axial stiffness
EA_j	The experimentally measured joint stiffness
EA_e	The effective axial stiffness of the joint subassembly
ρ	The mass density

A	The cross section area
E	The elastic modulus
L	The beam element length
I	The moment of inertia
I_p	The polar moment of inertia
T_G	The matrix static transformation for Guyan technique
y	The modal coordinates
Φ_a^g	The generalized inverse of the modal matrix
T_S	The transformation matrix for SEREP technique
Φ_{ma}	The incomplete mode shapes measured in the monitored structure
Φ_{fem}	The matrix of mass normalized analytical modal vectors
Φ_{exp}	The matrix of mass normalized measured modal vectors
SE	Strain energy distribution
ω_{ud}	The frequency of the undamaged structure
ω_d	The frequency of the damaged structure

Chapter 1

Introduction

1.1 General

Health monitoring of structures and the detection of damage at the earliest possible stage have become important issues in almost all areas of application, ranging from aerospace to civil engineering. Continuous monitoring allows evaluation of in-service structural integrity. Such evaluation can optimize the structure usage, avoid catastrophic failure and minimize the downtime and cost. Many new diagnostic technologies that can be used for monitoring have been developed during the last two decades, spurred by recent advances in the sensing technologies, communication and computational algorithms. These technologies have led to the emergence of two main techniques: Non-Destructive Evaluation methods (NDE), such as ultrasonic and acoustic methods, and Vibration Based Damaged Identification methods (VBDI).

The conventional (NDE) methods depend on direct measurements to determine the physical condition of the structure and to evaluate its reliability. In addition they require that the vicinity of the damage be known in advance and the portion of the structure being inspected be easily accessible. The need for more generalized damage identification methods that do not rely on prior knowledge of the damage location and

can be applied to large and complex structures has led to the development of Structural Health Monitoring systems that use the VBDI methods. The VBDI techniques rely on the measurement of changes in the vibration characteristics of the structure and periodic measurement of these characteristics while the structure is in service. The accuracy of the data acquired from the vibration testing is crucial for the success of the VBDI technique. Success in damage identification depends mainly on the sensitivity of sensors used, the accuracy of the test-based modal parameters and the reliability of interpretation algorithms.

The efficiency and reliability of VBDI has become an important issue in almost all structural applications ranging from aerospace to civil. Ongoing research on VBDI has led to the development of various methods of damage identification and health monitoring that are based on changes in the measured modal parameters. These parameters (frequencies, mode shapes and modal damping) are functions of the physical properties of the structure (stiffness, mass and damping matrices). Consequently, any change in these parameters can be used as an indicator of the overall integrity of the structure and a means of finding the location and extent of damage. The VBDI methods are categorized based on the type of measured data and /or the mathematical algorithm used to identify the damage from the measured data. In the following sections literature relevant to the VBDI methods will be introduced, the theoretical background of different methods will be presented, and the advantages and drawbacks associated with the application of different methods in the health monitoring of different types of structures will be pointed out.

1.2 Conventional NDE Methods

A brief discussion of the conventional NDE methods is considered useful at this point, so as to review the advantages and drawbacks of both NDE and VBDI in various applications. The conventional NDE methods are dependent on direct measurements to determine the physical conditions of the structure at any given time and to evaluate its reliability. They require that the region of the damage be known in advance and the portion of the structure being inspected is easily accessible. Many conventional methods are used for the non-destructive evaluation of structural reliability; among them are visual inspection, static field tests and instrumental evaluation.

1.2.1 Visual inspection

For civil engineering applications, the success of visual inspection is mainly dependent on the qualification and experience of the inspector in identifying the vicinity of damage, irrespective of the type of damage. The damage might be the spalling of concrete, steel corrosion and cracking in reinforced concrete structures, or the loosening of bolts or development of cracks in connection welds in steel structures. Visual inspection requires that all portions of the structure be accessible. This might be possible for simple structures, but would be impractical for large complex ones, where it is impossible to inspect the whole structure. Sometimes even if it is possible to identify the vicinity of damage by visual inspection, it would be difficult to evaluate the damage extent without obtaining samples from the structure, such as for example cores from concrete, and testing them in the laboratory. In addition, some types of damage cannot be

identified by visual inspection, such as fibre fracture and internal delamination in composites.

In aerospace applications, visual inspection has often been employed as the preferred means of inspection of ground-based structures due to its simplicity and relatively low cost. However, in space the use of such methods would require extensive extra-vehicular activity. Also, not all of the elements under consideration can be accessed using robotic inspection or extra vehicular activity (EVA) by astronauts.

1.2.2 Static Field Tests

The static tests can be divided into proof tests, diagnostic tests and behaviour tests. The diagnostic and behaviour tests are not meant for damage detection. The proof tests can, however, identify damage in a structure. Proof tests are recommended when there is prior warning that damage has occurred somewhere in the structure and it is required to evaluate the residual strength of the structure. Obviously, such tests cannot be relied upon to provide prior warning of a decrease in the strength of the structure. Generally these tests are expensive and time consuming and require the structure being evacuated or at least a portion of it being evacuated during the test. Also, the method is impractical for aerospace applications, such as aerospace support and transportation systems, and for space structures, such as space shuttles and orbiting spacecraft.

1.2.3 Instrumental Evaluation Methods

Many NDE damage identification methods are available that can be used to identify damage in structures not easily noticed during a visual inspection. Among them

are the magnetic field methods, radiography, and acoustic methods. Successful implementation of such methods requires that the likely damage sites be known in advance and can be easily accessed, so that the sensors can be installed on the critical elements or positions to measure stresses or strains. The measurements acquired during the use of these methods do not give a firm evaluation of the variation in the strength of the structure, but do provide an early warning of possible damage, at the positions where the sensors are installed. These types of damage identification techniques are considered expensive and impractical, as it is not possible to install sensors at all critical elements and components of the structure. In space applications, use of NDE methods may be impractical because of the complexity of some large space structures, such as the main supporting structure of the International Space Station (ISS).

1.3 Classification of Damage

Damage is considered as an undesirable weakening of the structure that has a negative effect on its performance, and entails a risk to the safety of the structural system. Damage may also be defined as any deviation in the structure original geometric or materials properties, which may cause undesirable stresses, displacements or vibrations in the structure. The effect of damage in a structure can be linear or non-linear. A linear damage situation is defined as the case in which the initially linear-elastic structure remains linear elastic after damage. The changes in modal properties are a result of changes in the geometry and/or the material properties of the structure, but the structural response can still be modelled using linear equations of motion.

Another classification system defines four levels of damage identification (Rytter, 1993):

- Level 1: Determination that damage is present in the structure.
- Level 2: Level 1 plus determination of the geometric location of the damage.
- Level 3: Level 2 plus quantification of the severity of the damage.
- Level 4: Level 3 plus prediction of the remaining service life of the structure, or in other words how reliable the structure is in serving its proposed function.

The damages in civil structures can be in the form of cracks, reinforcement fracture, delamination, broken welds, loose bolts, corrosion, etc. Long-duration space and aerospace support and transportation systems are susceptible to many possible damage scenarios. One of these possibilities is the impact of meteoroids and collisions with orbital debris at hypervelocity. These impacts occur at extremely high speeds and can damage important components, which in turn might significantly affect the functional operation of these systems or lead to a catastrophic failure, either instantaneously or after a period of time. Accidental damage caused to flexible links and revolute joints of a flexible manipulating system during docking and berthing of the space shuttle to the space station can be identified as another possible damage scenario. Long exposure to the space environment can also cause partial damage in space structures.

1.4 Vibration Based Damage Identification (VBDI)

The vibration characteristics of a structure (frequencies, mode shapes and modal damping) are functions of the structure physical parameters, which include mass, stiffness and damping. The VBDI technique is based on the measurement of the changes

in the vibration characteristics caused by damage. As damage accumulates in the structure, the physical parameters of the structure change. The changes in structural parameters, if properly identified and classified, can be used as quantitative measures that provide the means of assessing the state of damage of the structure. Vibration measurements are conducted to determine the changes in the vibration characteristics of the structure. The problem then boils down to determining the location and magnitude of damage from a knowledge of the differences between the vibration characteristics measured before and after damage. A number of techniques of VBDI have been proposed in recent years, each of which has its own advantages and shortcomings depending on the particular assumptions made. However, it can be stated that in general VBDI has many advantages over the other NDE methods. One of these advantages is that the location of the damage does not need to be known in advance. Also, the sensors required for measuring the vibration characteristics need not to be placed in the vicinity of the damage. Often, a limited number of sensors can provide vital information that would permit identification of damage, even in large complex structures.

Structural Health Monitoring (SHM) using VBDI procedures depend on an optimal integration of experimental, analytical and information system components. These components include:

- Experimental identification of the structural modal characteristics through modal testing.
- An efficient damage identification algorithm to relate sensor measurements to the physical changes.

Each of these parts has its strengths and limitations, which strongly affect the overall performance of the SHM system based on VBDI methods. Better understanding of the characteristics of each part that influences the SHM would lead to an optimum SHM system. In the following sections, the role of each part in the SHM system will be briefly discussed to investigate the requirements for an optimum SHM system that uses VBDI.

1.4.1 Measurement of Modal Data

The modal testing of structures to obtain the modal characteristics is significantly affected by many factors: the excitation method used to excite the modes of interest, the sensing technology used, and signal processing of the data retrieved directly from sensors.

The methods of excitation used to measure the modal characteristics of a structure fall into two main categories: (1) controlled input and (2) ambient excitation. Generally, controlled-input tests are conducted on structures, which have relatively small dimensions. For large structures, ambient forces are the only practical means of exciting the structures.

Sensing Technology

Advancements in the electronic and communication technology have led to the availability of a wide variety of sensors. Among them are the optical fiber sensors, piezoelectric sensors and dielectric measurement sensors. Optical fiber sensors and peak

strain sensors have been used for damage assessment in bridges. Following are some of the important issues related to sensors that need to be considered:

- (1) The sensor used should be reliable under the various loading and environmental conditions encountered during operation. The long-term behaviour of sensors with respect to the monitored structure needs to be considered. The failure of sensors will result in faulty signals, which will make the system produce a false positive damage indication sign. Consequently, calibration of the sensors is a must for ensuring a greater degree of confidence in the measurements, particularly because the sensor behaviour may be affected by the interface between the host structure and the sensor.
- (2) An optimum distribution of sensors network is desirable, especially in large complex structures, to ensure that the response of segments between sensors can be estimated with reasonable accuracy and to reduce the number of sensors used, which accordingly will reduce the overall cost of the SHM system.
- (3) As the number of sensors increases with the size of the structure, a wireless communication system will be required to remotely gather the data.

Signal Processing

The data gathered from the sensor network contains large amount of information, much of which is irrelevant to the actual response of the structure. In addition, the data is

always corrupted by the environmental noise. Such noise may cause difficulties in parameters identification. Consequently, measurements need to be processed and refined before extracting the modal characteristics. Use of efficient processing technique would ensure an accurate identification of the modal characteristics that will be used later in the damage identification. This issue is particularly important when ambient measurements are obtained.

1.4.2 Identification Algorithms

A great deal of research has been carried out in the past decade or two on the detection, location and characterization of structural damage on the basis of measured changes in vibration properties. A large volume of literature therefore exists on this topic. An excellent and extensive review of the available literature has been provided by Doebling et al (1996) and Shen et al (1998). Farrar et al (1994) have reviewed the literature on vibration testing and damage detection in bridges. Fu-Kuo Chang (1999) has introduced numerous technical papers in the field of structural health monitoring. It is sufficient here to cite these rich sources of literature review. However, some specific references are included in the following sections on damage identification methods. Notable work in the area of civil engineering has been reported by Aktan et al (1999), Sikorsky (1999) and Abe et al (1999).

A summary of the methods used in VBDI and damage identification for structures based on the changes in their dynamic characteristics is presented in this section. In the methods presented here only a few vibration modes are required to identify the damage.

In practice, this is an important issue because during continuous monitoring of the structure, the number of modes that can be measured is limited. In fact, only the lower modes can be measured with acceptable accuracy. The algorithms are categorized on the basis of the type of measured data used and/or the mathematical techniques used to detect the damage from the measured data. The methods presented here use the change in frequencies, change in the strain energy content of the structure, the change in mode shapes and mode shape curvatures and the modification of the structural model matrices (matrix update algorithms) such as mass, stiffness and damping.

Frequency Changes

Early work in the damage identification algorithms for SHM mainly depended on relating variation in the modal frequencies to damage (Cawley and Adams 1979). The volume of literature related to damage identification using shifts in frequencies is quite large. An excellent review on the use of modal frequency change for damage identification can be found in Salawu (1997). It should be noted that frequency shifts have significant practical limitations for applications to the type of structures considered. The somewhat low sensitivity of frequency shifts to damage requires either very precise measurements or large levels of damage. However, recent studies have shown that resonant frequencies have much less statistical variation resulting from random error sources than do other modal parameters (Farrar et al. 1997; Doebling et al. 1997).

Mode Shape and Strain Energy Changes

West (1984) has presented a method for the systematic use of mode shape information for the location of structural damage without the use of a prior FEM. West

used the modal assurance criteria (MAC) to determine the degree of correlation between modes from the test of an undamaged space shuttle orbiter body flap and the modes from the test of the flap after it had been exposed to acoustic loading. Fox (1992) showed that single-number measures of mode shapes changes, such as MAC, are relatively insensitive to damage in a beam with a saw cut. Node line MAC, a MAC based on measurement points close to a node point for a particular mode, was found to be a more sensitive indicator of the changes in the mode shape caused by damage.

An alternative to the use of mode shapes to obtain information about sources of vibration changes is to use mode shape derivatives, such as curvature. This method proposed by Pandey et al. (1991) is based on the observation that the damage in beam type structures would affect the flexural rigidity of the beam. The pre- and post-damage mode shapes are first extracted from an experimental analysis. Curvature of the mode shapes for the undamaged and damaged conditions can then be estimated numerically from the displacement mode shapes using any of the mathematical means of differentiation. The curvature is inversely proportional to the flexural stiffness, EI . Thus a reduction of stiffness associated with damage will, in turn, lead to an increase in the curvature. Theoretically, the difference between the pre- and post-damaged curvature mode shapes will be largest in the damaged region. Chance et al. (1994) found that numerically calculating the curvature from measured mode shapes results in unacceptable errors. They used measured strains directly instead of measuring curvature.

Stubbs and Kim (1994) introduced an algorithm called Damage Index to identify damage in a beam-type structure. They defined the damage index, β , in terms of the change in the strain energy content in the structure when it deforms in its particular mode shapes. This requires the determination of more complete mode shapes than a limited number of sensors can provide. To determine mode shape values between sensor locations, an interpolation using cubic polynomials can be used. Statistical methods are then used to examine changes in the damage index and associate these changes with possible damage locations. This algorithm has the advantage that it can be used with ambient vibration measurements, as is the case for most of the monitored bridges.

There are two common disadvantages of the methods that use the mode shape curvature or the decrease in modal strain energy to identify damage. First, the methods depend on interpolation to determine the complete mode shapes. This interpolation sometimes can lead to false predictions. Second, measurement at many points and very precise measurements are required to identify damage using the measured changes in mode shapes. However, mode shapes measurements are difficult to make with acceptable accuracy. These limitations make the methods impractical for use in damage identification of complex large-scale structures.

Methods Based on Updating Physical Property Matrices

These methods rely on updating the physical property matrices, usually the stiffness matrix, so that the frequencies and mode shapes of the modified system closely match the measured characteristics of the damaged structure. Different mathematical algorithms are used to solve for the update matrices (or perturbations to the nominal model that produce

the updated matrices). Most methods rely on the solution of a nonlinear optimization problem based on the structure equations of motion, properties of the nominal model, and the measured dynamic characteristics. The methods use a common basic set of equations, and the difference in the various algorithms can be related to the choice made for the following:

1. Objective function to be minimized
2. Constraints placed on the problem
3. Numerical scheme used to implement the optimization

Most of the objective functions are formulated to minimize the Frobenius norm of the changes in the property matrices or the perturbations to the nominal model that produce the updated matrices. The associated equality and inequality constraints are derived from the measured data and aim to preserve the symmetry, sparsity and connectivity of the stiffness matrix. Constraining the sparsity of the updated matrix to be the same as the original matrix ensures that the update model generates no new load paths. Finally, a number of alternative numerical schemes are available for solving the optimization problem. Some of the matrix update methods are described in the following paragraphs.

Optimal Matrix Update Methods

Methods that use a closed-form direct solution to compute the damage model matrices or the perturbation matrices are commonly referred to as optimal matrix update

methods. Zimmerman and Smith (1992), Hemez (1993), and Kaouk (1993) have published reviews of these methods. Some of these methods are discussed in Section 2.4. Baruch and Bar Itzhack (1978), Kabe (1985), and Berman and Nagy (1983) provide a common formulation of the optimal update problem that is essentially a minimization of the Frobenius norm of the global parameter matrix perturbations using modal force error and property matrix symmetry as constraints. Chen and Garba (1988) present a method for minimizing the norm of the model property perturbations with a zero modal force error constraint. They also enforce a connectivity constraint to impose a known set of load paths onto the allowable perturbations. The updates are obtained at the element parameter level rather than at the matrix level. Another approach to this problem, used by Kammer (1988), is to carry out a minimization of the modal force error with a property matrix symmetry constraint. The symmetry constraint preserves the reciprocity condition in the updated structural model. Smith and Beattie (1991) have extended the formulation of Kabe (1985) to include a sparsity preservation constraint, and have formulated the problem as minimization of both the perturbation matrix norm and the modal force error norm subject to the symmetry and sparsity constraint.

Another type of approach to the optimal matrix update problem involves the minimization of the rank of the perturbation matrix (MRPT) rather than the norm of the perturbation matrix. This approach is motivated by the observation that damage will tend to be concentrated in a few structural elements rather than distributed throughout a large number of structural elements. Thus, the perturbation matrices will tend to be of small rank. Zimmerman and Kaouk (1994) have developed this approach. Doebling (1996)

presents a method of computing a minimum rank update for the elemental parameter vector rather than dealing with global or elemental stiffness matrices. Another method uses the same basic formulation as the MRPT but constrains the global stiffness matrix perturbation to be an explicit function of the diagonal elemental stiffness parameter perturbation matrix that preserves the finite element strain-displacement relations. A limitation of this method, as with all minimum rank procedures, is that the rank of the perturbation is always equal to the number of modes used in the computation of the modal force error and consequently limits the ability of the technique.

Sensitivity-Based Update Methods

The basic theory here is the determination of a modified model parameter vector that contains material and / or geometric parameters. The parameter perturbation vector is computed from the Newton-Raphson iteration solution for minimizing an error function. The main difference between the various sensitivity-based update schemes lies in the method used to estimate the sensitivity matrix. Basically, either the experimental or the analytical quantities can be used in the differentiation. For experimental sensitivity, the orthogonality relations can be used to compute the modal parameter derivatives. Norris and Meirovitch (1989), Haug and Choi (1984), and Chen and Garba (1980) have proposed such an approach. Analytical sensitivity methods usually require the evaluation of the stiffness and mass matrix derivatives, which are less sensitive than experimental sensitivity matrices to noise in the data and to large perturbations of the parameters. Hemez and Farhat (1995) have presented a sensitivity-based matrix update procedure that

formulates the sensitivities at the elemental level. This has the advantage of being computationally more efficient than forming the sensitivities at the global matrix level.

Hybrid Matrix Update methods

A number of researchers have presented hybrid approaches for damage identification in structures monitored with limited numbers of sensors. Li and Smith (1994, 1995) presented a hybrid model update technique for damage identification that uses a combination of the sensitivity and optimal update approaches. This method constraints the stiffness matrix perturbation to preserve the connectivity of the FEM, and the solution minimizes the magnitude of the vector of perturbation in the elemental stiffness parameters. Dos Santos and Zimmerman (1996) applied a hybrid stiffness matrix update algorithm to the data from an eight-bay truss. The hybrid method was compared to the basic MRPT algorithm for its ability to precisely locate the damage in the truss. Numerical results showed that there was little difference between the two algorithms except in the case of high levels of measurement noise. The results obtained from the application of the hybrid technique to the eight-bay truss were consistent with the numerical results obtained from other methods. The hybrid method is shown to reduce the amount of spurious damage indications that may be obtained through MPRT by producing a perturbed stiffness matrix estimate that is consistent with the connectivity of the original FEM. The authors also pointed out that the quality of the results from the hybrid method was strongly dependent on the accuracy of the estimated stiffness matrix perturbation obtained from the MRPT algorithm.

1.5 Difficulties Associated With VBDI Techniques

As stated earlier VBDI techniques offer several advantages over the other methods of damage identification. However, in practice there are a number of limitations associated with vibration-based damage assessment.

A vibration-based damage identification technique would be successful only if damage results in a degradation of the stiffness of the structure. There are, however, instances where damage may not cause a change in stiffness. For example, failure of strands in a prestressed concrete element may still leave the section uncracked with no change in stiffness. Also, structures loaded into the inelastic range may regain their initial stiffness when the load that caused inelasticity has ceased to act.

The currently available VBDI techniques are also not fully successful when damage results in nonlinearity in the structure, for example, through opening and closing of cracks in a concrete structure and slippage along bolted joints in a steel structure.

There exist other factors that may place limitations on the success of VBDI techniques. These are summarized in the following sections:

1.5.1 Sensitivity of the global response to damage

One of the confounding factors in VBDI is the fact that damage typically is a local phenomenon. Local response is captured by higher frequency modes, whereas lower frequency modes tend to capture the global response of the structure and are relatively

less sensitive to local changes in a structure. On the other hand, from the standpoint of testing, it is more difficult to excite the higher order frequency response of a structure, since more energy is required to produce measurable response at higher frequencies than at lower frequencies. This is especially true if the excitation source is distant from the response locations. Damage detection is feasible for members that contribute significantly to the strain energy of the modes included in the identification. It can be argued from physical considerations that damage in structural members that translate as rigid members in the lower modes and can be activated only in the higher modes will not be detected if the first few modes are measured and used in the identification process. On the other hand, only the first few modes can be measured reliably and with acceptable accuracy. Even if the damage in members with low level of stress impacts the modal properties of the lower modes, this impact must exceed the level of uncertainty present in modelling or testing, so that the damaged element can be identified. Examination of the distribution of modal strain energy may assist in identifying whether the damage is detectable or not based on the contribution of the damaged element to the modal strain energy of the modes included.

1.5.2 Optimal Selection of Response and Excitation Placement

Another important issue is the selection of the type and location of the sensors that would best capture the response of the structure. When dealing with complex structures, only a limited number of sensors and exciters would in practice be used because of constraints such as the cost and physical accessibility. It is therefore important to optimize the sensor location. The location of the exciters and level of excitation should

be properly determined so as to be able to excite almost all of the lower global modes of the structure. The optimum locations for sensors and excitation points are not always easy to determine. Some mathematical approaches have been developed to define the optimal sensor placements for modal testing. However, none of these techniques has been adopted so far for the purpose of damage identification. In summary a robust identification technique requires careful pre-test planning, which typically involves optimal placement of sensors and exciters to provide sufficient information for damage identification.

1.5.3 Finite Element Modeling

A finite element modelling of the undamaged structure is an essential component of the optimal matrix update method. Modelling errors have a significant impact on the performance of the overall technique. Accordingly, the minimization of modelling errors would improve the optimization problem output. This is usually achieved by updating FEM physical parameters to produce the measured modal parameters. FEM updating is considered as a field by itself and this process is affected by many factors. It should be noted that in general the number of degrees of freedom (DOFs) used in a finite element model would be considerably larger than the number of degrees of freedom along which measurements are made. Consequently, the finite element model should be appropriately reduced to match with the measured DOFs. The technique used to reduce the finite element model has a significant effect on the dynamic properties of the reduced model with respect to the original models. Consequently the performance of the optimal matrix update methods is indirectly affected by the technique used to reduce the FEM.

1.5.4 Complexity of Mathematical Identification Algorithm

Many of the VBDI methods have evolved from model updating procedures. Among them is the optimal matrix update, which has proven to be the most widely used, although it has its limitations. A major shortcoming of the existing techniques that depend on model update procedures is that comparison of the post-damage structural modes with those of the pre-damage model often requires the solution of a non-linear optimization problem. A direct implementation of the optimal matrix update methods would perform well when used with simple structures. However when applied to complex structures, these methods may sometimes lead to ambiguous results. Any one method by itself would normally fail to account for the complexity of the modal analysis and testing methods and the mathematical algorithms used to identify damage and may be unable to generate physically acceptable results.

The problem of identifying the damage location and extent on the basis of changes in the global vibration characteristics determined from measurements at a limited number of response locations can be formulated in terms of an under-determinate system of non-linear equations implying that there are more unknowns than the number of equations. The solution of such a mathematical problem is non-unique. Sophisticated and complex mathematical techniques including non-linear optimization need to be employed to obtain the most probable solution. Optimization aims to obtain a solution that would minimize an objective function. The formulation and solution of the identification problem, posed as a non-linear optimization problem, is currently an area of ongoing research. Sometimes due to the type of physical data used in the optimization process

(FEM and measured modal parameters) or the mathematical formulation of the optimization problem, the obtained solution could give a non-unique minimum. It should be noted that in general many local minima would exist. A local minimum does not represent the true solution, which is associated with a global minimum. In physical terms a local minimum solution is often a false prediction of the damage site and extent. A false prediction due to the poor performance of the optimizer would invalidate the entire structural health monitoring system. In fact, most of the available software optimizers do not ensure the identification of a global minimum and need a long time for the completion of the process, especially when the objective function to be minimized contains a large number of design variables. The problem of having a large number of design variables in the objective function evolves directly from the large size of a physical FE model in which all structural elements of a complex structure are considered candidates in the optimization problem.

Another source of uncertainty is associated with the non-linear optimization numerical scheme used to solve for the global minimum. Such schemes are affected by the mathematical parameters used in defining the solution. Sometimes due to the mathematical configuration of the optimizer itself (method of search, mathematical operations), the obtained solution could be a local minimum solution. The solution to an optimization problem that uses measured modal parameters can be obtained using different numerical optimization techniques. Some of these techniques are derivative-based, others are probabilistic. A good understanding of the mathematical bases of these algorithms is crucial in the selection of the optimal scheme in a particular application.

1.6 Objectives and Scope of the Present Study

The main objective of this work is to develop an integrated structural health monitoring system procedure that includes an optimal pre-test planning and uses a combination of various analytical approaches to enhance the performance of the optimal matrix update methods. The approach adopted in this study is expected to mitigate some of the difficulties outlined in the previous section and improve the efficiency of the detection algorithm in accurately identifying possible damage location and severity at single and multiple sites. The approach will be validated by applying it to a realistic structure. The study will focus on a critical evaluation of the current methods of vibration-based damage identification, particularly those that use the optimal matrix update, and on the refinement and further development of such methods. The evaluation and refinements will be based on both analytical studies and experimental validation. For the experimental part, a space frame type structure representing a large space structure, such as the main supporting system of the International Space Station ISS, will be tested to extract the modal parameters of both the undamaged and damaged structure. The analytical part will focus on two main issues. A simple strategy will first be formulated for developing and updating the FEM of the structure used in the validation of the approach. More than one technique will then be combined to improve the efficiency of damage identification algorithms in the presence of modelling and measurement errors. The possibility of adopting some of the techniques for application to large-scale structures will be discussed.

The original contributions of this work can be summarized as follows

- (1) A criterion for optimal selection of response points is developed in this study and justified for the purpose of generating adequate information for damage identification algorithms based on experimental investigation and analytical study.
- (2) Criteria are developed for selection of modes to be used in damage identification and for the selection of a FEM reduction/expansion technique that has the ability of preserving accurately the modal properties of the original model for a selected set of modes.
- (3) A combination of two simple mathematical algorithms is used to identify damage location and extent accurately. The first algorithm is meant to identify the damaged region so as to minimize the number of suspected damaged elements. The second algorithm depends on a simple mathematical formulation of the optimization problem that assures the globalization and feasibility of the solution and is computationally inexpensive.

The proposed approach includes the following main steps.

Step 1: This step involves the construction of a Finite Element Model of the structure using measured physical parameters such as modulus of elasticity, mass density and geometry and the specified boundary conditions. The FEM is used to calculate analytical modal parameters, which are then compared with the acquired experimental data from modal testing to evaluate the discrepancy

between the two. The FE model is then updated by suitably adjusting the physical parameters in the model so that the agreement between the numerical predictions and test results is improved. Minimization of modelling errors would have a significant impact on the ability of the optimal matrix update methods used to identify damage location and extent.

Step 2: This step involves the selection of dynamic modes that will be used in the identification process using the correlated FEM. This goal is achieved by using an optimum selection criterion that is based on the stored strain energy in dynamic modes and on the suitability of the selected modes for the FEM reduction/expansion technique. The contribution of each structural element to the strain energy associated with the selected modes will be calculated to identify the elements that do not contribute significantly to such strain energy.

Step3: Recognizing the limitation on the allowable number of measured points, an optimal selection will be made for both the excitation and sensor locations. Also the FEM will be reduced using one of the available condensation algorithms to obtain a match between the number of analytical degrees of freedom and the measurement degrees of freedom while preserving the modal properties of the original FEM for the selected modes.

Step 4: Different damage scenarios will be experimentally simulated to validate the proposed approach. For each scenario, the modal parameters of the damaged

structure will be acquired through modal testing. Then each damaged mode will be paired with the same undamaged mode using the orthogonality check.

Step5: The modal parameters of the undamaged structure and each damage case are completed using the refined data from steps 2, 3 and 4. These data is used to identify the candidate elements to be included in the optimization problem. Such identification will be based on the calculation of the modal residual vectors or differences between the frequency response functions (FRFs) of the damaged and undamaged structure.

Step 6: Step 5 permits a reduction in the number of candidate elements for the optimization problem. In the present step the optimization problem will be formulated using a simple optimality criterion to identify damage location and extent using the Sequential Quadratic Programming SQP optimization technique that ensures the globalization of the obtained solution. The success of the optimization technique is attributed to the mathematical formulation of the optimization problem. This issue is mathematically justified in Section 2.8.

1.7 Layout of the Thesis

An outline of the research thesis is presented in the following.

Chapter (1): Includes reviews of the existing Vibration-Based Damage Identification (VBDI) methods and the difficulties associated with such methods. Such reviews are of help in developing the enhancement approach.

Chapter (2): Consists of the theoretical development of the proposed selection of algorithms that enhances the ability of the optimal matrix update algorithms to detect damage in complex and large-scale structures.

Chapter (3): Describes the design and building of an eight-bay space frame that is used as a test article. The design and manufacturing of the test hardware as well as the modal testing and analysis procedures used in the experimental validation of the proposed approach are also described. The modal parameters of the undamaged structure as obtained from modal testing are presented in this chapter.

Chapter (4): Discusses all the issues related to the development and updating of the FEM of the test article that is used in the experimental validation of the proposed approach. A discussion of different model reduction techniques is also included to help in the selection of an appropriate technique that may be applied in the reduction of the large FEM to about 5 % of the original model DOFs that match the measured DOFs.

Chapter (5): Includes a description of the computer software codes developed for the numerical implementation of the proposed mathematical approach. These codes are then used in a numerical simulation designed to investigate the feasibility of the proposed approach. Description of the subroutines needed for data transfer between programs and data preparation and analysis for the VBDI enhancement approach is also included.

Chapter (6): Presents the experimental verification of the proposed approach through modal testing of the test article in which predetermined damage has been introduced and estimation of the damage by the approach proposed in this work using the acquired modal data from the undamaged and damaged structures. The software codes presented in Chapter 5 are used for this purpose.

Chapter (7): Includes discussion, conclusions and suggestions for possible future work.

Chapter 2

Theoretical Background to the Proposed Approach

2.1 General

The objective of this work is to develop a new approach for damage detection that may significantly enhance the ability of the optimal matrix update detection algorithms in identifying damage in complex structures using a limited number of measured DOFs. The analytical part includes a combination of more than one technique to process the finite element model and the measured modal data. The experimental part will be used in conjunction with the analytical one to identify the optimal excitation and pickup points, and to validate the proposed approach through experimental simulation of several different possible damage scenarios. In this chapter, the theoretical background to the main procedures used in the proposed approach will be discussed. Also, the mathematical methods that may be used to formulate and process the data required in different steps of the proposed approach will be studied and discussed, so as to select the most appropriate algorithms for the present study.

2.2 Eigenvalue Sensitivity to Perturbations in Model Matrices

It is assumed that the damage is quantified by a local decrease of the stiffness, which affects the modal parameters (frequencies and mode shapes) of the structure. The equations of motion (EOM) governing the free vibrations of an undamped n -degree of freedom system are

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = 0 \quad (2.1)$$

where $\mathbf{M}$ is an $n \times n$ mass matrix, $\mathbf{K}$ is an $n \times n$ stiffness matrix and $\mathbf{u}$ is a vector of nodal displacements. $\mathbf{M}$ and $\mathbf{K}$ are positive definite. The associated eigenvalue problem is given by

$$(\mathbf{K} - \lambda_i \mathbf{M})\boldsymbol{\phi}_i = 0 \quad (2.2)$$

where λ_i is the square of the natural frequency, ω_i , and $\boldsymbol{\phi}_i$ is the corresponding mode shape of the system. Normalization of the mode shapes can be obtained by

$$\boldsymbol{\phi}_i^T \mathbf{M} \boldsymbol{\phi}_j = \delta_{ij} \quad (2.3)$$

where δ_{ij} is the Kronecker delta. Damage in the original structure causes a change in the stiffness matrix by an amount $\delta\mathbf{K}$. As mentioned earlier, damage is assumed not to cause any changes in the system mass matrix. The change in the system stiffness matrix leads to

change $\delta\lambda_i$ in the i^{th} eigenvalue and $\delta\phi_i$ in the i^{th} eigenvector. The eigenvalue problem of the damaged structure is then given by

$$[\mathbf{K} + \delta\mathbf{K} - (\lambda_i + \delta\lambda_i)\mathbf{M}][\phi_i + \delta\phi_i] = 0 \quad (2.4)$$

Expansion of Equation (2.4) gives

$$(\mathbf{K} - \lambda_i\mathbf{M})\phi_i + (\delta\mathbf{K} - \delta\lambda_i\mathbf{M})\phi_i + (\mathbf{K} - \lambda_i\mathbf{M})\delta\phi_i + (\delta\mathbf{K} - \delta\lambda_i\mathbf{M})\delta\phi_i = 0 \quad (2.5)$$

The first and third terms of Equation (2.5) represents the eigenvalue problem and are equal to zero. Pre-multiplication of Equation (2.5) by ϕ_i^T yields

$$\phi_i^T (\delta\mathbf{K} - \delta\lambda_i\mathbf{M})\phi_i + \phi_i^T (\mathbf{K} - \lambda_i\mathbf{M})\delta\phi_i + \phi_i^T (\delta\mathbf{K} - \delta\lambda_i\mathbf{M})\delta\phi_i = 0 \quad (2.6)$$

The second set of parentheses is zero as it also represents the transpose of the eigenvalue problem. This simplifies Equation (2.6) to the form

$$\phi_i^T (\delta\mathbf{K} - \delta\lambda_i\mathbf{M})\phi_i + \phi_i^T (\delta\mathbf{K} - \delta\lambda_i\mathbf{M})\delta\phi_i = 0 \quad (2.7)$$

Expanding Equation (2.7), assuming that ϕ_i represents a mass orthonormalized mode shape, simplifying and noting that $\delta\phi_i = (\phi_{id} - \phi_i)$, where ϕ_{id} is the i^{th} mode shape of the damaged structure, we get

$$\boldsymbol{\varphi}_i^T \delta \mathbf{K} \boldsymbol{\varphi}_{id} = \delta \lambda_i \boldsymbol{\varphi}_i^T \mathbf{M} \boldsymbol{\varphi}_{id} \quad (2.8)$$

Equations (2.7) and (2.8) relate the unknown changes in stiffness matrix ($\delta \mathbf{K}$) to the changes in modal parameters $\delta \lambda_i$ and $\delta \boldsymbol{\varphi}_i$ since $\mathbf{M}$ is the original mass matrix and is known in advance. In a case in which the damage would result in a small ($\delta \mathbf{K}$), the higher order terms in Equation (2.7) can be neglected to relate the changes in the stiffness matrix to the changes in the eigenvalues only as follows

$$\delta \lambda_i = \boldsymbol{\varphi}_i^T \delta \mathbf{K} \boldsymbol{\varphi}_i \quad (2.9)$$

Damage identification follows directly from the identification of the perturbation in the stiffness matrix. Equations (2.8) and (2.9) form the bases for this process since they can be used as constraints in a non-linear optimization problem when the number of modes used is less than the number of candidate elements that contribute to changes in the structural stiffness matrix. In Equation (2.8) the perturbations in the stiffness matrix are related to the changes in both frequencies and mode shapes.

In Equation (2.9), due to the assumption that only a small change in the stiffness matrix is considered, the perturbation in the stiffness matrix is related to the changes in only the frequencies. This system of equations has been used and numerically implemented by Hassiotis, S. and Jeong, G. D. (1995) as a constraint in the optimization problem to identify damage at a single site of a ten-story, two-bay steel frame. In this

formulation, it was assumed that an analytical model existed that correctly described the system before damage and that all of the full DOFs were considered. All elements of the structure were considered as candidates for damage identification. The influence of different parameters, such as modelling and measurement errors, the problem of mode shape incompleteness and the number and type of modes included in the formulation of the constraint system of equations was not considered in the study. The success of such an approach is related to the difference between the changes caused by damage and those resulting from FEM assumptions and measurement errors. On the other hand, the difference in eigenvalues due to small changes in the stiffness matrix may be smaller than the differences due to modelling and measurement errors. In practice, this would by itself spoil the damage identification procedures.

In the following sections, the use of the relationships given by Equation (2.8) and (2.9) will be discussed with respect to the different mathematical formulations of the optimization problem, illustrating the different aspects of the use of these equations to identify damage using the measured modal parameters.

2.3 Finite Element Model

2.3.1 Role of FEM in VBDI Techniques

Finite Element Model of the monitored structure is considered as the backbone of the SHM techniques that use VBDI methods, especially the optimal matrix update methods. Since the finite element solution technique is a numerical procedure that analyses only the selected mathematical model, all the assumptions made in constructing

the model are reflected in the predicted response. Minimizing the modelling errors would significantly enhance the overall performance of the proposed approach. Consequently, it is necessary to construct as accurate a model as possible. The updated model would be very useful in many steps later on in building an enhanced VBDI algorithm. FEM is used in the following activities in the identification process:

- In finding optimal sensor placement for monitoring the structure. The optimal sensor placement methods typically utilize the information obtained from a nominal or baseline FEM to find ways in which only a limited number of sensors may be used for the monitoring of a large and complex structure.
- In selecting the subset of identified lower structural vibration modes that may be used for SHM and damage identification. The methods of selecting such modes are based on an evaluation of the modal strain energy, which depends on the FEM stiffness matrices.
- In reducing the number of degrees of freedom in the FEM to match the experimentally measured DOF. This reduction is crucial for the optimization problem. Model reduction procedures cause loss of some information. Accordingly the modelling errors should be minimized to prevent the compounding of such errors with the errors resulting from the model reduction procedures.
- In the numerical simulation and experimental verification for damage identification using optimal matrix update as described in the theoretical development and formulation of the optimization problem. The minimization of

modelling errors would significantly improve the ability of the optimizer to detect the damaged element as well as the severity of damage.

In this work, a three-step strategy will be used to construct and update the FEM of the test article so that a well-correlated FEM is obtained. The main steps of this strategy and the theoretical and practical issues regarding FEM will be discussed in Chapter 4.

2.3.2 FEM Reduction Techniques

Vibration-based damage detection depends on the measurement of the frequencies and mode shapes of both the undamaged and the damaged structures. Measurements on the undamaged structure are required for the development of a correlated finite element model. Measurements on the damaged structure are required to detect changes in the vibration properties caused by damage. A real structure will usually possess a large number of degrees of freedom and hence a large number of mode shapes. However, the higher frequencies and mode shapes can rarely be measured with sufficient accuracy. Vibration-based damage detection must therefore depend on the measurement of a limited number of the lowest vibration frequencies, or such frequencies and the associated mode shapes. It is often very difficult or impracticable to measure the response along all of the FEM DOFs necessary for the complete definition of a given mode shape. Two different strategies are available to address this problem. Thus, the FEM used in the damage detection may be reduced to correspond to the DOFs along which the measurement of mode shapes is made by using any one of the standard methods of reduction. Alternatively, the partial mode shapes obtained from measurements may be

expanded to the full set of the DOFs in the finite element model by using a scheme of extrapolation or model expansion.

For the reasons cited above, FEM reduction is crucial in the mathematical formulation of the optimization problem meant to identify damage. A number of techniques have been developed for model reduction or expansion. Among them are Guyan or static reduction, dynamic reduction, improved reduced system (IRS) and system equivalent reduction expansion process (SEREP). For all model reduction / expansion techniques, there is a relationship between the active DOFs and the condensed DOFs, which can be written in general form as

$$\{\mathbf{u}_n\} = \begin{Bmatrix} \mathbf{u}_a \\ \mathbf{u}_c \end{Bmatrix} = [\mathbf{T}] \{\mathbf{u}_a\} \quad (2.10)$$

where $\mathbf{u}_n$ is the full set of physical FEM displacements, $\mathbf{u}_a$ is the active set of DOFs, $\mathbf{u}_c$ is the condensed or deleted set of DOFs and $\mathbf{T}$ is the transformation matrix. The transformation matrix $[\mathbf{T}]$ will take on various forms depending on the reduction technique employed.

Based on their studies Avitabile et al (1989), found that all the Guyan reduced frequencies were higher than those of the full model. In fact, as the mode number increases, the discrepancy increases. This is due to the fact that significant inertia terms that are critical to the accuracy of higher modes of the system are discarded. Dynamic

condensation is an excellent technique for reduction of large system matrices when only one mode is of concern. The IRS technique is a continuation of the Guyan condensation technique. IRS makes an adjustment to account for the inertia terms that are not retained in the Guyan reduction process and therefore generally produces a better representation of the original model in the reduced state. SEREP process by the nature of its formulation retains all of the dynamics of the full model in the reduced state for the modes selected in the reduction process and by definition contains no approximation. In this study, two of the reduction techniques will be used to match the number of measured and analytically calculated degrees of freedom. Complete detail of the theoretical background of these reduction techniques and how they would be employed is described in Chapter 4.

2.4 Basic Mathematical Formulation of the Optimal Matrix Update

Different mathematical algorithms are used to solve for the update in the physical property matrices based on the measured changes in modal characteristics. One development has its basis in the field of optimization theory. Optimal update identification is an approach to produce, through the solution of a constrained non-linear optimization problem, updated property matrices (mass, stiffness and damping) that will yield a close match between the analytical and measured modal responses. The formulation of the problem is dependent on the structure's equation of motion (EOM), represented by Equations (2.8) and (2.9), the nominal model, and the measured data. Matching of the measured modal data, preservation of symmetry, closeness to the initial model, positive definiteness, and preservation of structure connectivity are some of the equality constraints imposed in various optimization algorithms. The early work in the

field of optimal matrix update is summarized in two different formulations. Baruch and Bar Itzhack (1978), Kabe (1985), and Smith and Beatti (1991) used the first mathematical formulation that can be stated as

$$\begin{aligned}
 & \min \| \mathbf{A}_s - \mathbf{A} \|_F \\
 & \text{subject to} \\
 & \quad \mathbf{A}_s \mathbf{S} = \mathbf{Y} \\
 & \quad \mathbf{A}_s = \mathbf{A}_s^T
 \end{aligned} \tag{2.11}$$

where $\| \cdot \|_F$: is Frobenius norm,

$\mathbf{A}$ is the original $n \times n$ property matrix,

$\mathbf{A}_s$ is the $n \times n$ updated property matrix,

$\mathbf{S}$ and $\mathbf{Y}$ are $n \times p$ matrices that define constraints on the measured data,

n is the number of degrees of freedom of the model,

p is the number of matrix constraint equations, and

T is the matrix transpose

A matrix that is closest to the original model matrix is found. It should however, be symmetric and should satisfy the constraints related to the measured data. Additional constraints may be imposed, including sparsity and positive definiteness. Weighting matrices may be included in the objective function of the minimization problem. The algorithms represented by Equation (2.11) exhibit several characteristics that make them suitable for FEM updating. However, the FEM connectivity is lost in the update. To preserve model connectivity in the updated model matrix, an additional constraint should be imposed to preserve the sparsity of the original matrix. The inclusion of too many constraints makes the solution of this formulation computationally expensive and may

mislead the optimizer, steering it away from the global solution. In addition, the updating is done to the global parameters, which does not give a direct identification of the source of the discrepancy and its magnitude.

The second formulation for the optimal matrix update problem has a different philosophy. Kammer (1988) has adopted this formulation. It starts with symmetric minimizer for the measured data constraints and imposes further constraints from there on. In general terms, the formulation can be represented by

$$\begin{aligned} \min \quad & \| \mathbf{A}_s \mathbf{S} - \mathbf{Y} \|_F \\ \text{subjected to} \quad & \\ & \mathbf{A}_s = \mathbf{A}_s^T \end{aligned} \tag{2.12}$$

Stiffness matrix Update

Although optimal matrix update algorithms can be used to update mass, damping and stiffness property matrices, stiffness matrix updating has been the most widely explored method for damage identification. This is so because damage is most likely to affect the stiffness matrix. For both FEM updating and damage identification, the stiffness matrix is considered the most important candidate for updating based on the measured modal data. Baruch and Bar Itzhack (1978) presented an optimization technique for stiffness matrix adjustment using measured frequencies and mode shapes. This technique resulted in the following closed form solution involving only matrix multiplications

$$\mathbf{K}_a = \mathbf{K} + \mathbf{M}\mathbf{S}\mathbf{\Omega}^2\mathbf{S}^T\mathbf{M} - \mathbf{K}\mathbf{S}\mathbf{S}^T\mathbf{M} - \mathbf{M}\mathbf{S}\mathbf{S}^T\mathbf{K} + \mathbf{M}\mathbf{S}\mathbf{S}^T\mathbf{K}\mathbf{S}\mathbf{S}^T\mathbf{M} \quad (2.13)$$

where $\mathbf{K}$, $\mathbf{M}$ are the original stiffness and mass matrices, respectively.

$\mathbf{K}_a$ is the $n \times n$ adjusted stiffness matrix.

$\mathbf{S}$ is the $n \times p$ matrix of measured mode shapes, and

$\mathbf{\Omega}^2$ is the $p \times p$ diagonal matrix of corresponding measured eigenvalues.

Positive definiteness is guaranteed in the adjusted stiffness matrix if the original stiffness matrix is positive definite. However, the sparsity pattern of the original model is lost in the update. Smith and Beattie (1991) presented a sparsity preserving update, which is an analogue to Kabe's formulation (1985). They formulated the objective function for perturbation stiffness matrix as follows;

$$\min \sum_{i,j=1}^n \frac{(\mathbf{K}_{ij}^a - \mathbf{K}_{ij})^2}{\mathbf{K}_{ii}\mathbf{K}_{jj}} = \min \|\mathbf{D}^{-1}(\mathbf{K}_a - \mathbf{K})\mathbf{D}^{-1}\|_F^2$$

subjected to

$$\mathbf{K}_a \mathbf{S} = \mathbf{M}\mathbf{S}\mathbf{\Omega}^2$$

$$\mathbf{K}_a = \mathbf{K}_a^T \quad (2.14)$$

$$\text{sparse}(\mathbf{K}_a) = \text{sparse}(\mathbf{K})$$

where

$$\mathbf{D} = \text{diag}(\mathbf{d}_i) = \text{diag}(\sqrt{\mathbf{K}_{ii}})$$

The symmetry, sparsity and system connectivity is guaranteed in the adjusted stiffness matrix ($\mathbf{K}_a$). This technique requires the solution of an auxiliary problem to produce the update. Lagrange multipliers incorporate the constraints into an extended objective function. Minimization of the Lagrangian function produces a system of linear equations to solve for the Lagrange multipliers and an update equation for the stiffness matrix ($\mathbf{K}$). This auxiliary problem has the dimension ($np \times np$), where p is the number of the measured modes and n is the number of original model DOF, and has the property of symmetry and positive semi-definiteness. Complete details of the auxiliary problem can be found in (Zimmerman and Smith 1992) from which it can be seen that the dimension of the auxiliary problem can exceed computer storage capabilities if a large structure is considered; and that there are additional difficulties associated with the approach due to the numerical implementation of the type of constraints used to preserve the sparsity of the global stiffness matrix, which physically means the preservation of the FEM connectivity.

A majority of the early work in optimal matrix update has used the minimum norm perturbation of the global stiffness matrix. The correction matrices are usually constructed at the global level through the constrained minimization of a given weighted functional. The motivation of using this objective function is that the desired perturbation is the one, which is the “smallest” in overall magnitude. A common drawback of the methods is that the computed perturbations are made to stiffness matrix values at the structural DOF, rather than at the element stiffness parameter level. Such an optimization may yield updated matrices where the symmetry and orthogonality conditions as well as

the original connectivity are destroyed. Lagrangian multipliers are often required to enforce these constraints, which undoubtedly increase the computational effort. In order to preserve symmetry, positive definiteness and connectivity properties, while keeping the load paths undisturbed, Hassiotis et al (1995) used an element – by – element parameter based updating method. If $\mathbf{K}_j^e$ is the contribution of the j^{th} element to the global stiffness matrix of the structure, the stiffness matrix of the damaged structure can be written as

$$(\mathbf{K} + \delta\mathbf{K}) = \sum_{j=1}^{n_e} \mathbf{K}_j^e (1 + \delta k_j) \quad (2.15)$$

where: n_e is the number of elements in the n-DOFs structure and δk_j is the proportional change in the stiffness of element j . Consequently, $\delta\mathbf{K}$ can be written as

$$\delta\mathbf{K} = \sum_{j=1}^{n_e} \mathbf{K}_j^e \delta k_j \quad (2.16)$$

This formulation has several advantages because perturbations are made to the element stiffness parameters rather than to the global stiffness parameters. These advantages are:

- The resulting updates have direct physical relevance, and thus can be easily interpreted in terms of structural damage or errors in the FEM.

- The connectivity of the FEM is preserved, so that the resulting updated FEM has the same load path set as the original.
- A single parameter, which affects a large number of structural elements, can be varied independently.

Substitution of Equation (2.16) into Equation (2.8) gives a system of simultaneous equations

$$\sum_{j=1}^n \boldsymbol{\varphi}_i^T \mathbf{K}_j^e \boldsymbol{\varphi}_{id} \delta k_j = \delta \lambda_i \boldsymbol{\varphi}_i^T \mathbf{M} \boldsymbol{\varphi}_{id} \quad i = 1, 2, \dots, m \quad (2.17)$$

where m is the number of modes used in the identification procedure. The system represented by Equation (2.17) can be simplified to the form

$$\mathbf{D} \delta \mathbf{k} = \delta \boldsymbol{\lambda} \quad (2.18)$$

where $\mathbf{D}$ is an $m \times n_e$ matrix of elements $d_{ij} = \boldsymbol{\varphi}_i^T \mathbf{K}_j^e \boldsymbol{\varphi}_{id} / \boldsymbol{\varphi}_i^T \mathbf{M} \boldsymbol{\varphi}_{id}$, $\delta \mathbf{k}$ is the n_e vector of unknown changes in the stiffness δk_j , and $\delta \boldsymbol{\lambda}$ is the m vector of changes in the eigenvalues $\delta \lambda_j$. When the number m of the available measured changes in the eigenvalues equals the number of unknown changes in the element stiffness matrices, Equations (2.8) and (2.9) can be solved to provide a unique solution. In the case when not all the structural modes can be measured, but reliable measurements can be obtained for only the lower modes, the system of the simultaneous Equations in (2.8) and (2.9) is

underdetermined, which means that an infinite number of solutions can satisfy this system of equations. A unique solution can be obtained only by placing Equation (2.8) or (2.9) as a constraint on an appropriate objective function through the solution of an optimization problem.

2.5 Combined Damage Identification Algorithm

As stated earlier, the major part of the objectives of the present work is to formulate a mathematical algorithm that includes a combination of more than one technique to process the FEM and the measured modal data. The mathematical components of this algorithm are meant to be simple and computationally inexpensive. The mathematical algorithm consists of two steps. First, the damaged region is identified using the modal residual vector or the FRF differences. The objective of this step is to identify a subset of the structure candidate elements to be included in the optimization problem. In the second step, a simple formulation of the optimization problem that represents the optimal matrix update method is used to identify damage location and extent.

2.5.1 Mathematical Formulation of the Optimization Problem

As discussed earlier in Chapter 1, the various damage identification algorithms based on optimal matrix update differ in the objective function used, constraints placed on the problem, and the numerical scheme used to implement the optimization problem. In this section, a simple form of the optimization problem used in the present work will be presented. The objective function and the constraints are formulated from existing

schemes. The objective function used is the square of the Frobenius norm of perturbations to the stiffness matrix. Minimization of the squared norm of the change in the stiffness $\delta\mathbf{K}$ matrix is given by

$$\min \|\delta\mathbf{K}\|^2 \quad (2.19)$$

The square of the Frobenius norm of $\delta\mathbf{K}$ is given by

$$\|\delta\mathbf{K}\|^2 = \sum_{i=1}^n \sum_{j=1}^n \delta K_{ij}^2 \quad (2.20)$$

In order to preserve the symmetry, positive definiteness and connectivity, thereby keeping the load paths undisturbed, an element – by – element parameter-based updating method is used as shown in Equation (2.16). Substitution of (2.16) and (2.20) in (2.19) gives

$$\min \sum_{i=1}^n \sum_{j=1}^n \left(\sum_{r=1}^{n_r} K_r^e(i, j) \delta k_r \right)^2 \quad (2.21)$$

where $K_r^e(i, j)$ is the ij^{th} coefficient of matrix $\mathbf{K}^e_r$. Equation (2.21) can be expressed as

$$\min \sum_{i=1}^n \sum_{j=1}^n \sum_{r=1}^{n_r} \sum_{s=1}^{n_s} K_r^e(i, j) K_s^e(i, j) \delta k_r \delta k_s \quad (2.22)$$

Equation (2.22) is an expanded form of the quadratic programming problem defined by the following equation

$$\min \quad \delta \mathbf{k}^T \mathbf{Q} \delta \mathbf{k} \quad (2.23)$$

where $q_{rs} = \sum_{i=1}^n \sum_{j=1}^n K_r^e(i, j) K_s^e(i, j)$ and $\delta \mathbf{k}$ is the vector of the unknown damage parameters. It is evident from the expression for q_{rs} that $\mathbf{Q}(n_e \times n_e)$ is a symmetric positive definite matrix. A detailed discussion of the quadratic programming is presented in subsequent sections.

Equality and inequality constraints are imposed in the minimization of the quadratic objective function defined by Equation (2.22) or (2.23). To relate the changes in the modal parameters to the changes in elemental stiffness matrices, Equation (2.17) simplified in Equation (2.18), is used to define equality constraints. Considering that damage cannot result in a positive change in the element stiffness matrix, an additional constraint can be used by which the upper bound to the changes in the elemental stiffness matrices is set to zero while the lower bound is set to -1 . The optimization problem using this formulation can be summarized as follows

$$\min \sum_{i=1}^n \sum_{j=1}^n \sum_{r=1}^{n_r} \sum_{s=1}^{n_s} K_r^e(i, j) K_s^e(i, j) \delta k_r \delta k_s,$$

$$\text{or } \min \delta \mathbf{k}^T \mathbf{Q} \delta \mathbf{k}$$

subject to

$$\sum_{j=1}^{n_r} \boldsymbol{\varphi}_i^T \mathbf{K}_j^e \boldsymbol{\varphi}_{id} \delta k_j = \delta \lambda_i \boldsymbol{\varphi}_i^T \mathbf{M} \boldsymbol{\varphi}_{id} \quad (2.24)$$

or

$$\mathbf{D} \delta \mathbf{k} = \delta \boldsymbol{\lambda}$$

and

$$-1 \leq \delta \mathbf{k} \leq 0$$

2.5.2 Identification of the Damaged Region

Many identification algorithms have been proposed to identify damage location and extent. The success of these techniques is dependent on many parameters; however, they do perform relatively well in identifying the location of damage. Among the techniques for identifying damage location are the modal residual force method, FRF differences, and weighted modal strain energy differences. Each of these is described in the following paragraphs. For the present work the modal residual vector has been used to identify the region of damage.

Modal Residual Vector (R_i)

If the frequencies and mode shapes of the damaged structure are measured through a modal test, this method can be used to identify what is called the modal

residual (Humar and Amin (2001)). Substituting for the change in the stiffness matrix, eigenvectors and eigenvalues in Equation (2.2) and simplifying, we get

$$(\mathbf{K} + \delta\mathbf{K})\boldsymbol{\varphi}_{di} - \lambda_{di}\mathbf{M}\boldsymbol{\varphi}_{di} = \mathbf{0} \quad (2.25)$$

where $\boldsymbol{\varphi}_{di} = \boldsymbol{\varphi}_i + \delta\boldsymbol{\varphi}_i$ is the i^{th} damaged mode shape and λ_{di} is the associated eigenvalue.

On expanding Equation (2.25) the modal residual vector is obtained as follows

$$\mathbf{K}\boldsymbol{\varphi}_{di} - \lambda_{di}\mathbf{M}\boldsymbol{\varphi}_{di} = -\delta\mathbf{K}\boldsymbol{\varphi}_{di} = \mathbf{R}_i \quad (2.26)$$

The left hand side of Equation (2.26), and hence $\mathbf{R}_i$, is known. The damage factors $\delta\mathbf{k}_i$, which are the unknowns in the problem, define the severity of damage in the damaged structural elements. This can be observed by substituting Equation (2.16) in (2.26), which gives

$$\left(\sum_{j=1}^{n_e} \delta k_j \mathbf{K}_j^e \right) \boldsymbol{\varphi}_{di} = -\mathbf{R}_i \quad (2.27)$$

Equation (2.27) can be summed over all of the measured modes to give

$$\sum_i \left[\left(\sum_{j=1}^{n_e} \delta k_j \mathbf{K}_j^e \right) \boldsymbol{\varphi}_{di} \right] = \sum_i \mathbf{R}_i \quad (2.28)$$

Regions within the structure that are potentially damaged correspond to the degrees of freedom that have large magnitudes in $\mathbf{R}_i$. Knowledge of the affected DOF and connectivity of the structure allows the identification of the elements that may possibly be damaged. These elements can then be treated as the candidate elements in an optimization problem. Equation (2.27) can be solved to obtain at least some approximate estimates of the parameters in $\delta\mathbf{K}$, which gives an indication of both the location and severity of damage.

FRF Differences

This technique is based on the analysis of experimentally evaluated Frequency Response Functions (FRFs) and consists of a comparison of the FRFs of the undamaged structure, which are taken as reference, and the FRFs of the damaged structure. If damage is introduced in any of the structure's elements, the FRFs of the sensor close to this damage experience significant effect in terms of amplitudes. Napolitano et al (1998) have used a scalar index called Damage Detection Index (DDI) to localize damage in a typical aeronautical structure. DDIs interpret the differences between the FRFs of the damaged and the undamaged structure for each sensor. A DDI is given by the following expression

$$DDI = \sum_{j=1}^N \left(\frac{\sum_{i=1}^n FI_i - FD_i}{\sum_{i=1}^n FI_i} \right) \times 100 \quad (2.29)$$

where FI_i and FD_i are the amplitudes of the FRFs of the undamaged and the damaged structure respectively, at sample point i . The FRF is divided into N sections covering the

range of interest. Each section is then sampled at n equidistant frequency points. In their study Napolitano et al found that the DDI always had its maximum value for the sensor which was nearest to the region where damage had occurred. This technique has the basic feature that makes it an appropriate component of non-model based VBDI and offers the possibility of application in real time monitoring.

Weighted Modal Strain Energy Differences

This approach is basically dependant on the modal deformations in the vicinity of the damaged element caused by the degradation of the stiffness of that element. The basic formulation consists of the computation of the total strain energy for each of the identified modes using normalized mode shapes. The total strain energy U_i in the i^{th} mode is the sum of the modal strain energy contributed by all elements in the system. The contribution of the j^{th} element in mode i is defined by U_{ji} and is given by

$$U_{ji} = \frac{1}{2} \{\phi_{ji}\}^T [K_j^e] \{\phi_{ji}\} \quad (2.30)$$

where $[K_j^e]$ is the stiffness matrix of the j^{th} element and $\{\phi_{ji}\}$ is a collection matrix, containing the components of the i^{th} mode shapes associated with the degrees of freedom of the element j . The total modal strain energy in the i^{th} mode is then defined as

$$U_i = \sum_{j=1}^{n_e} U_{ji} \quad (2.31)$$

The damage can be localized based on the weighted changes in the energy distribution between the undamaged and damaged cases, obtained from the following equation

$$\Delta U_{ji} = (U_{u,j} - U_{d,j}) U_{u,j} \quad (2.32)$$

where subscripts u and d indicate undamaged and damage cases, respectively and $U_{u,j}$ is used as the weighting factor. To avoid false positive detection of elements adjacent to the damaged elements, a better approach would be to lump the element contribution to the modal strain energies at the end nodes of the element so that two peaks would identify damage, one at each of the end nodes of the damaged element. This method has been used by Carrasco et al (1997) to localize damage in a typical hexagonal boom truss. The method was able to detect damage at single locations, consisting of a 100 % cut, however it was not able to detect even single damage location when the severity of damage was low.

2.6 Mode Selection for Damage Identification

Regardless of what model update algorithm is employed to locate damage in a structure, if identification of damage location is to be accomplished through the use of modal parameters measured in a modal test on the structure, the damage must affect the response of the structure and the effect must lead to an observable and measurable change in the modal parameters. The structural health monitoring systems that use VBDI techniques generally require that the user select a subset of the measured modes to be used in damage identification. The first few modes are usually used in FEM updating and

damage identification, since they are generally the modes that can best be measured reliably and with acceptable accuracy. However, in some situations the higher frequency modes are more sensitive to the location of the structural damage. Even if the damage level in any of the structural elements impacts the modal properties of the selected lower modes, this impact must exceed the level of uncertainty, which is present in modelling or measurements, so that the damaged element can be identified.

Previous studies by Kashangaki et al (1992) and Chen and Garba (1988) have indicated the importance of strain energy in the identification of structural behaviour and location of structural damage. This importance can be described in terms of the structural load paths. When a particular mode stores a large amount of strain energy in a particular structural load path, the frequency and displacement shape of that mode are highly sensitive to changes in the stiffness of that load path. Thus, strain energy is a logical criterion in damage identification mode selection. In case of a cantilever beam, the lowest frequency modes may also contain the highest overall distributions of structural strain energy. However, for structures that are dominated by local behaviour, such as a free-free space frame with distributed mass, the strain energy distribution is not as concentrated in the lowest frequencies. In many cases, the modes that contain vital information about the damage occur at higher frequencies and some of these modes should be included in the selected subset of modal modes, as discussed by Kashangaki, et al. (1992). The modal strain energy stored in mode j can be determined by deflecting the structure according to the modal displacement (mode shape) to obtain Equation (2.31) for each measured mode ϕ_j in the selected modal set.

Doebling et al. (1993) studied a series of strategies for selecting the subset of the identified modes that could be used in FEM updating and to identify damage. These strategies include the selection of the lowest frequency modes and the modes that have maximum modal strain energy for both the undamaged and damaged structures. It is found that the strategies that use the maximum modal strain energy as criteria provide more accurate update results than those that use the minimum modal frequency. An examination of the form of strain energy in Equation (2.31) shows the fundamental significance of using modal strain energy as an indicator of the relative importance of modes. The modes that yield the highest value of the strain energy are the modes that have the most significant contribution to the stiffness matrix itself, and thus provide the best information about changes to the elemental stiffness parameters of the structure. Sometimes the modes with highest overall strain energy are lower frequency modes, but this is not always the case.

The modes used in the update are actually the damaged modes that correspond to the selected-undamaged modes. To select these corresponding modes, equivalence must be established between the modes of the undamaged structure and the modes of the damaged structure. This can be achieved through the use of the Pseudo Orthogonality check PORTH to pair corresponding experimental mode shapes.

$$\text{PORTH} = \Phi_{u_{exp}}^T \mathbf{M} \Phi_{d_{exp}} \quad (2.33)$$

For each undamaged mode selected a corresponding damaged mode is selected that produces the highest PORTH with that undamaged mode.

Another issue that has not been addressed earlier is the suitability of the selected modes for use in the reduction / expansion technique employed to match the FEM coordinates with the measured coordinates. Some of these techniques depend on the preservation of the model dynamic properties through the preservation of some of the selected modes. Techniques such as dynamic reduction and the SEREP technique are sensitive to the selection of modes. A detailed discussion of these techniques will be presented in Chapter 4. In the present work, this issue has been considered by applying another criterion for mode selection besides the modal strain energy criterion. This criterion requires that the set of selected modes do not affect the performance of the reduction / expansion technique used. This criterion would ensure that the selected modes are appropriate for both FEM reduction and damage identification.

2.7 Optimal Selection of Sensors Locations

Sensor location is an important issue in the field of modal testing and model correlation. Since VBDI methods depend mainly on the measured data collected from modal testing, optimal selection of excitation and response points is an essential component of a well-designed monitoring system. When dealing with a complex and large structure, sensors and actuators can be placed at only a limited number of degrees of freedom. The most common sensor placement methods use various scales to measure the significance of candidate sensor locations with respect to the observability of the target modes. In damage identification another important issue should be addressed, which is

that the measured points should be selected in a such a way as to provide the information required for identification algorithms. Also the excitation points should be selected in a way that ensures optimal excitation of all target modes.

In practice, the measurement locations are determined as follows. A FEM is developed, from which the system coordinates are defined. As a first step, before implementing any of the sensor placement methods, coordinates in FEM that cannot be measured are excluded. These normally include rotational coordinates and translational coordinates that are not accessible. The selection procedure that satisfies the essential requirements is then used to reduce the number of measurement points. The existing selection methods that are commonly used in the field of modal testing were studied and will be briefly discussed in the following paragraphs. In the present study, an appropriate method is adopted and modified to include an additional criterion for the selection of response points that has the ability to provide an adequate modal database for damage identification.

Schedlinski and Link (1996) introduced a two-step approach to select the optimal pick-up and exciter placement for FEM correlation with modal test results. This approach depends on the evaluation of the QR decomposition of the modal matrix and the QR decomposition of the product of the mass matrix and modal matrix. The final outcome is the determination of a set of measurement points that maximize the linear independence of the measured mode shapes. Zimmerman et al (2000) have reviewed some of these methods for the purpose of FEM correlation and compared their relative performance. In

their study they recommended the Sensor Elimination Algorithm that uses MAC (SEA-MAC). Each term in MAC ψ matrix measures the squared cosine of the angle between two mode shapes, i.e. $\psi_{ij} = \cos^2 \theta_{ij}$. For a set of measured mode shapes, ψ is always symmetric with unity diagonal terms. Ideally the i^{th} and j^{th} mode shapes are orthogonal to each other when $\psi_{ij} = 0$. Hence the mode shapes are completely linearly independent when all off-diagonal terms of ψ are zeros. The SEA-MAC algorithm measures the effect of removing each candidate sensor DOF. The removal that results in the minimizing the largest off-diagonal term of MAC is then carried out, and the process is repeated for the remaining candidate sensor DOFs.

Effective Independence

In this method the identification of the measured coordinates is based on the estimation theory. A set of measured coordinates is selected to produce mode shapes of interest that are as linearly independent as possible, and at the same time retain as much information as possible about the selected modal responses in the measurement data. Kammer (1991, 1992) has introduced an effective independence procedure to remove the coordinates of little significance. In this procedure Fisher Information Matrix FIM is formed first as follows

$$\mathbf{A} = \Phi^T \Phi \quad (2.34)$$

where Φ is the reduced modal matrix that contains only the modes of interest with realistically measured coordinates. Then an idempotent matrix $\mathbf{E}$ (i.e. $\mathbf{E}^2 = \mathbf{E}$) with the property that its trace equals its rank is formed by using the relationship

$$\mathbf{E} = \Phi \mathbf{A}^{-1} \Phi^T \quad (2.35)$$

Terms on the diagonal of $\mathbf{E}$ represent the fractional contribution of each measurement coordinate to the rank of $\mathbf{E}$, and hence to the independence of the chosen modes. Matrix $\mathbf{E}$ is full rank only if the modes are linearly independent. The selection procedure relies on an estimation of the elements of the diagonal of $\mathbf{E}$. Since the smallest element relates to the coordinate that contributes least to the independence of the chosen modes, that coordinate is excluded. The matrix $\mathbf{E}$ is then recomputed and the process is repeated, excluding coordinates iteratively.

It is highly desirable that a criterion be established in order to assess the ability of any of the prescribed methods to identify a set of candidate sensor locations for the purpose of damage identification. If identification of damage location is to be accomplished through the use of measured modal parameters, it must affect the measured response of the structure so that the effect on modal parameters is observable and measurable. The implementation of any of the methods described above would provide a set of candidate coordinates, which could be used for modal identification, FEM correlation and FEM updating. However, they are not guaranteed to be the best selection for the purpose of constructing a modal database for damage identification. In the present

work, another constraint is invoked on the selection procedures. This constraint is based on Equation (2.30), and requires that the modal strain energy contribution of the element i in the target mode j is preserved after the exclusion of any of the global coordinates. A numerical investigation of this criterion will follow in Chapter 5.

2.8 Mathematical Aspects of the Optimization Techniques

2.8.1 Basic Terminology

Optimization problem: The constrained optimization problem is commonly expressed in the following notation (Gill et al (1989) and Fletcher (1987)).

$$\begin{aligned}
 &\min F_0(x) \\
 &\text{subject to} \\
 &\quad f_i(x) \leq 0, \quad i = 1, \dots, m \\
 &\quad h_i(x) = 0, \quad i = 1, \dots, p
 \end{aligned} \tag{2.36}$$

The objective is to find an x that minimizes $F_0(x)$ among all other x that satisfy the conditions $f_i(x) \leq 0, \quad i = 1, \dots, m$ and $h_i(x) = 0, \quad i = 1, \dots, p$. Variable $x \in \mathbb{R}^n$ are defined as the optimization design variables and the function $F_0: \mathbb{R}^n \rightarrow \mathbb{R}$ as the objective function. Inequalities $f_i(x) \leq 0$ represent the inequality constraints. Equations $h_i(x) = 0$ represent the equality constraints. If there are no constraints, Equation (2.36) defines an unconstrained optimization problem. A point x is said to be feasible if it satisfies the constraints and the optimization problem represented by Equation (2.36) is said to be feasible if there exists at least one feasible point, but is infeasible otherwise.

Convexity: The subject of convexity is often treated quite extensively in the field of optimization. Convexity is expressed in terms of a so-called convex optimization problem. Into this category come certain quadratic programming problems and some more general problems, more often with linear constraints. A *convex set* K in R^n is defined by the property that for all $x_0, x_1 \in K$, it follows that $x_\theta \in K$ where

$$x_\theta = (1 - \theta)x_0 + \theta x_1 \quad \forall \theta \in [0,1] \quad (2.37)$$

It follows from this that K can have no corners as shown in Figure (2-1). Examples of convex sets are many and include the whole of R^n , a line, hyperplanes (or linear equations) $A^T x = b$, the half space (or linear inequalities) $A^T x \geq b$.

The other fundamental definition is that of a *convex function*. Essentially a convex function $F(x)$ is one for which the epigraph is a convex set. The epigraph is the set of points in $R \times R^n$ that lie on or above the graph of $F(x)$ as shown in Figure (2-1). Thus a convex function is defined by the condition that for any $x_0, x_1 \in K$ the following is satisfied

$$f_\theta \leq (1 - \theta)f_0 + \theta f_1 \quad \forall \theta \in [0,1] \quad (2.38)$$

where f_θ refers to $f(x_\theta)$, and where x_θ is given by Equation (2.37). The right hand side of Equation (2.38) is the chord joining (x_0, f_0) to (x_1, f_1) on the graph of $F(x)$ shown in Figure (2-1), and the inequality expresses the fact that the graph of a convex function

always lies below or along the chord. A linear function or a quadratic function is convex if its Hessian is positive semi-definite and strictly convex if the Hessian is positive definite. Another convex function is $\|x\|$ (for any norm (x)). The problem of minimizing a convex function on a convex set is defined as a convex optimization problem. Every local solution x^* to a convex optimization problem is a global solution, and if $F(x)$ is strictly convex on K any global solution is unique. Another attraction of convexity is that it provides a framework within which the first order (Kuhn-Tucker) conditions are sufficient for the global solution, so that if $F(x)$ is at least differentiable (C^1) function on K then x^* is a global unique solution.

Figure (2-1) convex combination

The optimization techniques are mostly numerical search techniques. These techniques start from an initial guess for the design variable and proceed in small steps to improve the value for the objective function, or the degree of compliance with the constraints, or both. The search is terminated when no progress can be made in improving the objective function without violating some of the constraints. Some optimization methods terminate when progress in the objective function becomes very slow. Others check for optimality by employing the necessary conditions, called the *Kuhn-Tucker conditions* that must be satisfied at a global minimum. Non-linear optimization numerical schemes, which solve for the global minimum, are affected by some mathematical parameters such as the mathematical formulation of the objective function and the constraints, which is reflected in the definition of such solution. Sometimes due to the nature of the data involved in the optimization process or the configuration of the optimizer itself (method of search, mathematical operations), the obtained solution could be a non-unique minimum (many local minima may exist). The physical implication of finding a local minimum solution is that a false prediction of the damage site and extent has been obtained. In VBDI techniques, the solution of the optimization problem is the last step in the damage identification algorithms. Getting false prediction due to poor performance of the optimizer used would spoil the whole identification system. The presence of a large number of design variables in the objective function is another factor that affects the performance of the optimization technique evolving directly from the size of the physical model. In other words when identifying damage in large complex structures, if all the structural elements represented in the FEM are considered as candidates in the optimization problem, the number of design variables will be large.

The solution of the optimization problem using measured modal parameters can be implemented using different numerical optimization techniques. Some of these techniques are derivative-based algorithms such as Quadratic Programming (QP), Sequential Quadratic Programming (SQP) and Linear Matrix Inequality (LMI). Others are probabilistic-based techniques such as Genetic Algorithms (GA) (Friswell et al (1998)) and Simulated Annealing. A better understanding of the mathematical bases of these algorithms is crucial in the selection of the technique to be used in the proposed mathematical methodology. It is undesirable to have a false prediction of damage location and severity due to poor performance of the optimization technique used. Each technique has its own advantages and drawbacks. In the following subsections, the mathematical background of some optimization techniques will be briefly discussed in order to select the appropriate technique to be used in the present work.

2.8.2 Sequential Quadratic Programming (SQP)

In constrained optimization, the general aim is to transform the problem into an easier sub-problem that can be solved and used as the basis of an iterative process. Recently, efficient methods have been developed that depend on the solution of Kuhn-Tucker (KT) equations. The KT equations are necessary conditions for optimality for a constrained optimization problem. If the problem is a so-called convex programming problem, that is if the objective function and constraints are convex functions, the KT equations are both necessary and sufficient for a global solution point. The solution of the KT equations forms the basis of many non-linear optimization algorithms (Coleman (1999)). These algorithms attempt to compute directly the Lagrange multipliers. However

methods that employ constrained quasi-Newton updating procedure guarantee super-linear convergence by accumulating second order information regarding the KT equations. These methods are commonly referred to as Sequential Quadratic Programming (SQP), since a quadratic programming sub-problem is solved at each major iteration. SQP is also considered as the state of the art in the non-linear optimization field. This method allows the user to closely minimize Newton's method for constrained optimization just as is done for unconstrained optimization. At each major iteration, an approximation is made of the Hessian of the Lagrangian function using a quasi-Newton updating method. This is then used to generate a QP sub-problem whose solution is used to form a search direction for a line search procedure.

2.8.3 Linear Matrix Inequality LMI

LMI-based computational optimization methods have been developed recently in the field of control system design taking advantage of the specialized matrix inequality that can be solved very efficiently using newly developed interior point methods. A linear matrix inequality (LMI) is unlike other derivative based optimization techniques, since it is dependent on a different search technique for identification of the optimal solution. In the search technique called the interior point method, the initial guess is evaluated as a vector pointing from an interior point in the design space to interior surface of the design space to check for optimal solution. The main advantage of this method over other derivative-based methods is that it is much faster in identifying the optimal solution. The novel Linear Matrix Inequality (LMI) methods have been used by Abdalla et al (1999, 2000) to provide improved and computationally efficient optimal matrix update methods

for model updating and damage identification. The damage identification problem is formulated as a standard convex optimization problem with LMI constraints.

Based on the previous discussion of the mathematical aspects of the formulation of the optimization problem and the numerical techniques used to solve for the optimum solution, it can be concluded that whatever the optimization technique used, the optimization problem that is formulated for damage identification would have a global unique solution (damage location and extent) if and only if the objective function is strictly convex and that the equality and inequality constraints are linear. In this study, the optimization problem is formulated in such a way as to guarantee that the obtained solution is representative and to avoid any uncertainty in the solution of the optimization technique that could be attributed to the performance of the optimization techniques. The optimization problem formulated in Equation (2.24) was found to satisfy these requirements.

2.9 Summary

In this chapter, the mathematical algorithms used in the formulation of the proposed approach are presented. The selection of these algorithms has been made on the basis of an extensive search of the literature that was briefly introduced in the previous chapter. Each block of this approach has been studied and employed in a way to simplify the procedures and maximize the ability of the optimal matrix update methods to identify damage in unsymmetrical complex structures in the presence of the common modelling and measurement errors and the other VBDI limitations. It is useful at this point to

summarize the main steps of the proposed approach and to show how each block in the approach is employed. Figure (2-2) illustrates the essential steps in the proposed approach, from which it can be seen that the analytical and experimental parts are executed in parallel in simple procedures that can be summarized as follows

1. A FEM of the structure is developed using the experimentally identified physical parameters of the structure components.
2. The developed FEM is used to identify the optimal excitation and response points based on the criteria developed in this study and described in Section 2.7.
3. SEREP reduction / expansion technique that preserves the full model dynamic properties is employed to match the limited measured degrees of freedom with the large number of FEM degrees of freedom.
4. Robust modal testing procedures are developed and applied to the undamaged structure to confirm the selection of the optimal excitation points and to extract the modal parameters (frequencies and mode shapes). These procedures would be used in modal testing of the damaged structure as well.
5. A correlation scheme is applied to compare and pair the measured and calculated modal properties of the structure. A simple systematic procedure is followed to physically update the FEM without using any of the complicated updating techniques that are usually employed in such cases.
6. Mode selection procedures that are described in Section 2.6 are implemented to identify those modes that would be used for damage identification as well as in reduction techniques.

7. A damage location method such as the modal residual vector is used to identify the damaged region as has been described in Section 2.5.2 from which a reduced number of suspected elements is selected for use as design variables in the optimization problem.
8. A simple mathematical formulation of the optimal matrix update methods is developed that is represented by Equation (2.24), in which the objective function is formulated to be strictly convex and the equality and inequality constraints strictly bound the search region for the optimizer and possess the advantage of being linear. This mathematical formulation ensures that there will be no false predictions due to the performance of the optimization technique used, as has been discussed in Section 2.8, and permits the use of any of the commercially available optimizers. The identification procedure in this step is an element-by-element based detection process, given that only a limited number of candidates are included as a result of the damage location described in the previous step. This formulation minimizes the solution time and ensures that a solution would be obtained.

Figure (2-2) Flow chart of the proposed approach

Chapter 3

Modal Testing and Analysis of Undamaged Structure

3.1 General

In this chapter the criteria used in the design of test article employed in the present study are presented. The different aspects of modal testing of the eight-bay or panel space frame are discussed. Among the objectives of the modal tests that were carried out on the undamaged space frame were to develop a robust test procedure and to help in determining the optimal excitation and response points. These test objectives and details of modal tests are presented and the results obtained from such tests are discussed.

3.2 Test Article

The experimental test article used in this work is an erectable aluminum space frame built from commercially available hardware (Meroform M12) produced by MERO. Figures (3-1) and (3-2) show details of the frame nodes and elements that are used to build a frame. The nodal joint design allows the frame to be assembled into numerous configurations in any of three orthogonal directions, thereby providing structures with

varying complexity. The elements are aluminum tubes with screwed solid steel end connectors, which when tightened into the node also clamp the tube by means of an internal compression fitting. These features allow any of the frame elements to be replaced without disassembly of the whole frame, which is very useful in introducing damage in any of the frame elements. The space frame consists of eight bays. Each assembled frame bay is a cube shaped structure with 707mm sides. This test article represents a large space structure such as the main supporting structure of the international space station (ISS).

Preliminary Design

It was important to predict vibration characteristics of the test article in order to design the suspension system and for pre-test analysis. The most important issue was to design a test article, which from the point of view of complexity and symmetry represents a real life structure. In addition, a free-free boundary condition had to be produced to provide a representation of a structure in space. The test article and the suspension system had to satisfy the following: (1) the natural frequency of the suspension system should be no more than one-tenth the fundamental natural frequency of the test article for a free-free boundary condition and be well separated and away from the first lower natural frequency of the frame; and (2) the stiffness of the frame in the two orthogonal sides should be unequal, since the majority of real life structures are likely to be unsymmetrical. Unsymmetry of stiffness in the two orthogonal directions would, at the same time, achieve a separation of the bending modes into two families in the y and z directions. A preliminary design for the eight-bay space frame was performed using the

nominal values of the physical properties of Meroform M12 components to help in selecting the cross sectional areas for frame components. A FE software package (COSMOS) was used to predict the modal parameters of the selected frame configurations. Since the node joint is neither fully pinned nor completely rigid, two different types of elements were used to model the frame joints subassembly. In the first model, a 3D truss element, which has six translational degrees of freedom was used. In the second model, the elements were modelled using a 3D beam element, which has six translations and six rotational degrees of freedom. The nodes were modeled as pin joints in the 3D truss model and as rigid joints in the 3D beam model. For both models the masses of the node, end-fitting connectors connected to the node and the connected elements were lumped at the joint. Analysis was performed to determine the undamped free vibration properties of the system, for different boundary conditions. After considering the aforementioned design limits and the results of FE analysis, the element cross-section selected for the vertical and horizontal members oriented along the frame Y and X-axes, respectively, and the diagonals in the vertical (x - y) planes was (30×1.5 mm). The element cross-section selected for members along the Z-axis and the diagonals in the horizontal (x - z) planes was (22×1.0 mm). Figure (3-3) shows the complete geometric configuration of the eight-bay space frame. The nominal physical properties of the frame components are listed in Table (3-1).

3.3 Determination of Consistent Procedures for Assembling the Frame

At this stage of the study, static characterization tests were performed on the frame components to develop an optimum assemblage procedure. This was found

necessary because of the behaviour of the joint subassembly, and the fact that this type of joint exhibits some level of flexibility. It became evident that the level of joint flexibility depends on the level of the tightening torque. Accordingly, to have a consistent assemblage procedure and to reduce the joint modelling errors, it was considered important to experimentally determine the level of tightening torque that would be applied in assembling the frame components.

A set of tests was carried out to measure the flexural stiffness of the four different sizes of aluminum tube-node connections. In this test a tube was tightened to a node at one end, fixing the node into an aluminum block tightened to a rigid vertical support of a closed steel frame. The tube was then loaded vertically at the free end, as shown in Figure (3-4). Deflections were measured along the length of the tube at different applied tightening torques (5.7, 11.3, 17.0, 22.6 and 28.3 N.m), which correspond to (50, 100, 150, 200 and 250 lb.inch), respectively. The joint between the tube and the spherical node becomes more rigid as the attachment bolt torque is increased to 200 lb.inch (22.6 N.m), as shown in Figure (3-5). Based on these static tests the optimum tightening torque to be used in assembling the space frame was selected as (22.6 N.m).

3.4 Modal Testing

Modal testing is one form of vibration testing. The objective of modal testing is to acquire sets of frequency response functions (FRF) that are sufficiently extensive and accurate in both the frequency and spatial domains to permit the analysis and extraction of modal parameters (e.g. frequencies and mode shapes). In the last two decades the

modal testing field has grown rapidly due to advances in the technology related to test equipment, computer hardware and software and the development of reliable parameter estimation techniques. Extensive literature has been published covering many aspects of the modal testing and analysis (Ewins 2001, Maia et al 1997).

Structural Health Monitoring that uses VBDI procedures provides an integrates of experimental, analytical and information system components. The experimental procedures involve the extraction of modal parameters of the structural modes that contain vital information about the structure dynamic behaviour. This task is achieved in the VBDI through modal testing and analysis. This section provides description of the test hardware and the procedures used in the testing of the eight-bay space frame used in this work.

A modal test may be divided into two phases: data acquisition and modal analysis. The data acquisition phase consists of exciting dynamically the test article and of measuring the excitation force(s) as well as the structural response at various locations on the test article. This phase includes as a final step the estimation of the frequency response functions and the coherence functions. The modal analysis phase consists of processing the test data, usually through some form of curve fitting, to extract the estimated modal parameters of the test article. The most commonly sought modal parameters are the natural frequencies, damping values and mode shapes. For VBDI algorithms, only the natural frequencies and mode shapes would be of interest.

3.4.1 Basic Measurement System and Test Equipment

The data acquisition phase of modal testing requires various hardware and software components such as those shown schematically in Figure (3-6), which presents a typical layout for a simple measurement system. Basically, there are three main measurement components: 1) excitation mechanism, 2) transducers and 3) data acquisition and signal processing system. Clear understanding of the basic components would lead to appropriate selection of the ingredients for the structure to be tested, since every structure presents its own special problems in modal testing. The main test hardware units encountered in modal test set-up, other than the analysis software, relate to the data acquisition phase of the process. A brief description of these units follows:

Exciter: The exciter or shaker is used to physically vibrate the structure. The use of an exciter allows the application of a variety of dynamic disturbances to the test article. Present technology permits the simultaneous application of several exciters for certain types of excitation.

Transducers: These usually comprise force sensors for measuring the force applied to the structure and accelerometers for measuring the structural responses at various locations. Both devices are manufactured in a wide variety of size, weight, sensitivity and other characteristics.

Generator: This device creates the electric signal, which drives the exciter. Various types of excitation (including sine sweep, random and burst random) may be produced. Each

type of excitation has its own advantages and disadvantages (SDRL 1992). Although frequently occurring as a separate device, it can be incorporated into the analyser as shown in the figure.

Power Amplifier: This component amplifies the electric signal created by the generator before it is fed to the exciter.

Analyzer: The main function of this device is to collect and measure the various signals developed by the transducers in order to assess the magnitude of the force and the responses, as well as the phase difference between the two. In the case of multiple input excitations, advanced signal processing algorithms are used to separate the responses produced by individual exciters. As mentioned above, the analyser often provides the excitation signal. It also triggers the data collection, and filters and zooms the transducer signals. Finally, it produces various time-domain and frequency-domain functions (including FRFs, coherence, auto-spectrum, and cross-spectrum functions).

Analysis Software: This item is an essential part of the system. Its purpose is to estimate the modal parameters of the test article by curve fitting appropriate mathematical equations to the acquired test-based FRF's (or other frequency or time-domain functions). It may be located inside the analyser or on a separate computer. Several curve-fitting techniques have been and continue to be developed (e.g. SDRL 1994). Some procedures (such as the single-degree-of-freedom methods) are very simple and can be implemented on low cost analyzers. No one technique is best for all tests. In some cases, the simple

procedures will give as good results as the more complex methods; while in other cases (e.g. for closely-spaced modes), only the most advanced procedures will be able to produce satisfactory estimation of modal parameters.

3.4.2 Test Equipment Used for the Present Study

The entire testing phase of this research was performed at the Modal Analysis Laboratory of the Canadian Space Agency, in Saint-Hubert, Quebec, which also supplied the equipment described below.

Modal System

The modal test facility used in this project runs under the control of the LMS International (Leuven Measurement & Systems) CADA-X program on an HP 715/50 Unix workstation. An HP 3565 measurement hardware is added as a front-end unit to the workstation. These items constitute an integrated modal testing system having the following relevant characteristics:

- 32 measurement channels with sampling rates up to 12.8 kHz per channel;
- 2 source modules for driving independently 2 exciters with random type excitation;
- Data acquisition and averaging software for MIMO (multiple-input-multiple-output) testing, limited to two inputs because of the source modules. The software also generates the time and frequency domain functions (including auto-power spectra, cross-power spectra, coherence functions and frequency response functions)

- Modal analysis software for parameter estimation. The software includes a variety of curve fitting algorithms for SDOF (single-degree-of-freedom), MDOF (multiple-degree-of-freedom), SIMO (single-input-multiple-output) and MIMO; a facility for animation of the estimated mode shapes, and validation tools such as the MAC (Modal Assurance Criteria) technique;
- Ability to create various types of Universal Files which are industry standard well documented ASCII files for transfer of data or modal parameters to other software programs or systems; and
- Command mode allowing some mathematical manipulation of time and frequency-domain functions.

Excitation mechanism

In the present study, two MODAL 50A exciters from MB Dynamics were used to provide the dynamic excitation to the test article. These exciters are especially designed for modal testing in order to eliminate problems that may be encountered when using environmental shakers. The main characteristics of this exciter model are: minimum set-up time and simplicity in set-up procedures, portability with a weight of only 25 kg for a unit providing 222N (50 lb) dynamic force, 25.4 mm peak to peak stroke, and broad usable frequency range 1-4000 Hz. These features make the exciters quite suitable for the modal tests to be conducted in this study. The exciters come with their own solid-state power amplifiers (model SS250VCF). The dynamic force from each exciter is measured with a quartz force sensor manufactured by PCB (model 208B). One of the important issues is how the shaker should be supported or mounted. The most satisfactory

arrangement is one in which the shaker is fixed to the ground while the test article is supported by a soft suspension (Ewins 2001). This configuration was used in the present work and the test article was suspended by flexible cords to simulate free-free boundary conditions. It is important that at a point of excitation, the force is applied along only one direction without any other secondary excitation forces. This is achieved by connecting the exciter to the force sensor attached to the test article through a stringer. Stringers are designed so that they are axially stiff, but flexurally weak so as to minimize the transmission of any bending moments to the sensors. For the modal tests in this work, the stringers consisted of a stainless steel rod with a diameter of 1.6mm, with length varying between 80 to 120mm.

Transducers

The response of the test article was measured simultaneously at many points with two types of accelerometers manufactured by PCB Piezotronics and ENDEVCO. The 32 channel modal system allowed the simultaneous measurement of up to 30 accelerations since the two force sensors occupied two of the 32 channels. The accelerometers were connected to two 16-channel programmable signal conditioner units. The two signal conditioner units were then connected to the front end of the modal system. The PCB Flexcel accelerometers used in the present work have several characteristics that make them suitable for the present application: relatively low mass of 4.5 g, small dimensions of 16x13 mm, excellent sensitivity 1000 mV/g and an acceptable measurement range of ± 5 g. Their usable frequency ranges are: (1-2000 Hz) for a maximum of 5% sensitivity

deviation, and 4-2000 Hz for a maximum of 5% error on the phase shift. The 4-2000 Hz frequency range was sufficient for the present application.

3.5 Test Procedures

The procedures described in this section were used in all of the modal tests for both the undamaged and damaged space frame. These procedures lead to the determination of the FRFs of the space frame in a free-free configuration. The translational FRFs are the only ones that can be readily measured directly because appropriate measurement have not yet been fully developed. The rotational FRFs are hard to measure directly. In this work measurements were made of only the translational FRFs. The following guidelines were used in the selection of test procedures.

1. In order to allow for reciprocity check, FRFs with respect to at least two driving points were required. Therefore the exciters were applied at two different locations simultaneously so as to minimize the number of runs required for generating the data corresponding to all excitation locations. This procedure also had the effect of minimizing the measurement errors.
2. In order to minimize the time required for data acquisition, measurement at all the translation response points had to be carried out in a minimum number of runs. The space frame consists of 36 spherical nodes with two free flat faces. Since three translational DOFs must be measured at each node, a total of 108 acceleration measurements need to be made. Because two channels of the modal system were required for force measurements, a total of 30 accelerometer

locations could be specified for each run. The preliminary plan was to install the available accelerometers along the space frame during a pilot test conducted for the following purposes:

- a. To find the number of global modes existing in the frequency range of interest.
- b. To validate the selection of optimal excitation points, which had been defined using the FEM to excite as many modes as possible
- c. To investigate the possibility of measuring the 108 response points consistently without any significant measurement discrepancies, given that maximum available number of accelerometers was 60. This was a difficult step, since precise measurements of mode shapes is hard to achieve.
- d. To determine how far the lower global modes were separated and how far away they were from the local modes. It should be noted that only the measured data related to the global modes would be used in the damage identification process.

The various components of the selected test procedures that were applied to all modal tests of both the undamaged and damaged structures are described in the following paragraphs.

Test Article Support System

As stated earlier, the modal tests in this study were to be performed on the eight-bay space frame described in Section 3.2. During testing the frame was supported by a flexible suspension system to simulate a free-free boundary condition. This configuration

allows the simulation of expected on-orbit boundary conditions of a large space structure such as the international space station (ISS). The frame is supported by a suspension system, which does not restrain the structure at any point, so that the test article is freely suspended in space and exhibits rigid body motion. Two aluminium portal frames were manufactured to support two longitudinal I-beams from which the space frame was suspended by means of rubber bungee cords as shown in Figure (3-7). Theoretically, the frequencies of rigid body motion should be zero. In practice, the achievement of such an ideal boundary condition is impossible, since the suspension system provides some minimal restraint. However, dynamic coupling between the suspension system and the space frame can be ignored as long as the highest frequency of the six rigid body modes of the suspension system is at least an order of magnitude smaller than the lowest natural frequency of the frame. The preliminary design of the space frame that has been described in Section 3.2 showed that the frame was relatively rigid, having all natural frequencies above 50 Hz. The separation between the suspension system natural frequencies and the lowest structural modes was achieved by suspending the space frame through long bungee cords with calculated natural frequency of 2.5 Hz. The mass of the suspension system was very small compared to the test article mass. This avoided mass coupling (Cooley 1993).

Selection of frequency bandwidth for the FRFs

The selection of the frequency bandwidth to be used in testing is directly related to the requirements of the vibration based damage identification process. The preliminary FEM of the test article was used to approximately determine the number of modes that

would be of interest for damage identification. In order to assess the impact of truncation of modes, simulation studies were carried out first with a reasonably large number of retained modes, and the results obtained were compared with those obtained from a smaller number of modes. This approach led to the conclusion that a frequency bandwidth of (0-256 Hz) would be adequate. Testing was carried out to span the entire range starting from 0 Hz as opposed to zooming on frequency lines around the desired frequencies in order to increase the frequency resolution. This approach was selected to minimize the number of runs required to capture all the modes of interest, and was justified by the fact that the modal frequencies were fairly distributed in the frequency range of interest. The front end of the modal system accepts only power of 2 as maximum frequency. Consequently, the selected upper limit of frequency bandwidth was the minimum power of 2 for which the bandwidth still contained a reasonable number of modes that could be used reliably for damage identification.

Selection of response locations

As discussed earlier, the test space frame had 36 nodes. Only those positions that corresponded to a DOF in the reduced FEM located at the aluminium spherical nodes were selected to be the primary response points (108 DOFs). The original FEM had 2928 DOFs, a full detail of the model will follow in Chapter 4. This selection would allow for subsequent comparison and correlation with the data calculated from the FEM and would be helpful in solving the optimization problem for identifying damage using the measured modal parameters and different sizes of mode shape vectors. In general, response measurements were made at all of the selected response points. The mode shape values

became available for 108 DOF. This allowed the option of using some or all of these measured values in damage identification, so that the impact of using only some of the 108 measured DOF could be assessed. The theoretical background of the criteria developed in the present work for the optimal selection of minimum pick up points for damage identification has been discussed in Chapter 2 and implemented in Chapter 5 and Chapter 6.

Selection of the excitation points

Selection of the excitation or driving points was based on the FEM predictions. The ideal excitation point is the one that produces significant displacement and is sufficiently far from the nodal points of all the modes of interest (Farra et al (1999)). If the exciter is placed at a nodal point of a particular mode it will not excite that mode. Generally, the criteria for selecting the driving points are: (1) the excitation must produce a reasonable displacements for most of the modes of interest, (2) driving points must be far away from each other to allow for a meaningful reciprocity check. In the preliminary modal tests that were carried out to investigate different issues, two sets of candidate driving points were selected based on results of the preliminary FEM of the space frame. The detail of the experimental investigation of the optimum excitation points is described in details later in this chapter.

Data acquisition

The space frame was set vibrating by means of burst random excitation applied independently by both exciters. With burst random excitation, the force is applied for

only a certain initial fraction of the time window during which data sampling is performed. In the present case, the burst signal was applied for the first 50% of the time. For the second half of the sampling period, no excitation was applied, and the measured force and acceleration signals quickly died down to almost zero before the end of the acquisition window. Burst random excitation has significant advantages over other types of excitation, since it minimizes leakage error that may occur during signal processing and removes signal distortion that may result from the application of weighting functions used to reduce such leakage. A total of 8192 samples were taken within each time window leading to 4096 frequency lines for the FRFs. For each run of measurements, the FRFs were measured 40 times and averages were taken. This was found to be sufficient to average out the uncorrelated noise in most FRFs, except for those corresponding to locations exhibiting small motion.

Data Verification

At the end of each data acquisition run, the following frequency domain functions were available for verifying the quality of data: FRFs and coherence functions. Using these data sets the following verifications were performed:

(1) Reciprocity check

Perfect reciprocity means that the FRF with response at point i due to the excitation at point j (H_{ij}) is equal to the FRF of the response at point j due to the excitation at point i (H_{ji}). Full reciprocity is possible only in linear structure. In practice, small differences are expected in comparing the two FRF's H_{ij} and H_{ji} . These differences

could be caused by different factors such as slight non-linearity in the test article and misalignment of the accelerometers and force sensors at any of the two measured DOF. However, in a carefully conducted test these differences are expected to be small. In the present study after each run, a reciprocity check was performed to verify the acquired data.

(2) Quality of FRF's

Each measured FRF was examined to verify its quality. The main factor was the noise level in the data. The FRF's are expected to have some noise at anti-resonance. A very noisy FRF throughout the frequency bandwidth could be an indication of cable or connection problems, or lack of rigidity at the attachment of an accelerometer to the test article.

(3) Coherence function

A coherence function is a frequency-domain function measuring how well a response relates to the input. In case of dual input, as in the present situation, the multiple-coherence function defines how much of the response is related to both excitations. The coherence lies in the range (0-1). Ideally the coherence function corresponding to every response DOF should be unity. Coherence functions that are at or near unity at resonance provide reassurance that the measurements were well taken. Multiple-coherence functions were observed together with the corresponding FRF's in an attempt to detect any measurement problems.

3.6 Modal Analysis

The LMS CADA-X modal analysis software offers several methods for estimating the modal parameters, from the simplest to the most advanced. However, the use of only one of these methods was found to be adequate for the entire experimental phase of the work. Modal parameters are in fact estimated in two steps: 1) frequencies and damping values, 2) residues or mode shape coefficients.

Step 1: The method that was used in this work is called the multiple inputs Least Square Complex Exponential (LSCE), also referred to as the polyreference technique. It is an advanced MIMO (multiple-input-multiple-output) estimation technique (Void et al. 1982). The polyreference technique uses all data measured in a structure to obtain global estimates of the frequencies and damping of several modes, as well as modal participation factors, which express the participation of modes in the structure response as a function of the reference DOF. The fact that these data FRFs may be with respect to several inputs (or references) makes the technique very powerful, especially for difficult cases such as closely spaced modes. An overview of the mathematical background of the technique and how it is implemented in the LMS software are given in LMS International (1992).

Step 2: In this step the parameter estimation process was performed with the Least Square Frequency-Domain method. This technique allows simultaneous estimation of all residues, which are related to the mode shape coefficients, within a specific frequency bandwidth, one FRF at the time. The implementation of the method in CADA-X allows

the estimation of either real or complex mode shape coefficients, as well as the lower and upper residuals. The upper residual corresponds to the residual flexibility of the structure. By applying standard curve fitting procedures, available in commercial modal analysis software, to the acquired frequency response functions, the required modal parameters of the modes of interest may be estimated. These modal parameters are the frequencies, mode shapes and residual flexibility. The parameter estimation process also generates an estimate of damping values of the modes. Most structures exhibit complex modes, rather than real normal modes, except in the rare case when the structures possess proportional damping. However, most of these non-proportionally damped systems exhibit modes that are almost real, the magnitudes of the complex parts being dependent on the system characteristics rather than the actual damping values (Imregun and Ewins 1995). According to the same authors, it may be difficult to find systems with significant complex modes simply because of a higher damping unless close or double modes exist. However, they also mention that several other factors may give rise to complex modes, such as analysis and/or measurement limitations (e.g. measurement noise, nonlinearities) and asymmetric system matrices (e.g. gyroscopic effects).

For lightly damped structures, the damping effects are usually neglected and a set of real modes is extracted from the measured data, using one of the several existing techniques for converting complex mode shapes into real ones (Ahmadian et al. 1995). In fact, as mentioned by Ibrahim and Sestieri (1995), normal mode identification of non-proportionally damped systems remains the basis for the majority of modal analysis applications such as modelling, structure modification, component mode synthesis,

sensitivity analysis, modelling error localization and model updating. These two authors mention that such transformation (from complex to normal modes) becomes an acceptable approximation when the applications in which the results are intended to be used, are relatively insensitive to the errors introduced, and that in modal analysis such applications are numerous. On the other hand, such approximation might lead to erroneous and misleading results when the measured mode shapes are used 1) to update analytical models, and 2) for structural integrity monitoring. Complex modes are representative of the structural modal parameters, however, dealing with complex modes is a field by itself. Consequently, in this study, it is crucial to investigate the differences created due to the transformation from complex to real mode shapes of the corresponding modes in terms of amplitude magnitudes. By examining the phase angle of all the modes of interest in the specified frequency range (0-256Hz), it is found that the phase angles are in the order of 90° . This means that the errors created due to the transformation from complex to real modes would have an insignificant influence on the damage identification algorithms used in this study. The acquired modal parameters of the undamaged space frame shown in Table (3-2) indicate that the system has very light damping confirming that the damping effects in this structure can be ignored without creating a recognizable effect on damage identification.

3.7 Experimental Estimation of the Modal Parameters of Undamaged Frame

As stated earlier, the space frame was assembled by applying a torque of (22.6 N.m) to tighten the aluminium tubes to the spherical nodes. As soon as the space frame was assembled, a series of modal tests was performed on it in order to understand and clarify the factors affecting the modal parameters and the dynamic response of the space frame and to determine the means of minimizing the possible measurement errors. The main objective of these tests was to arrive at a standard test procedure that gave consistent measured data with minimum measurement errors and inconsistency. Several test runs were performed using different test configurations. A parameter was changed in each run. These parameters included the length of the stringer connecting the force sensors to the shakers; different strategies to move the accelerometers to cover the specified space frame response locations (108 DOFs) since only 60 accelerometers were available. In the following subsections, significant details of this set of tests will be briefly discussed and the conclusions arrived at will be presented. The general test procedures prescribed in the previous section were followed in each of the modal tests reported here.

3.7.1 Modal Test of Unloaded Partially Instrumented Space Frame

The objective of this test was to determine the extent of discrepancy between the measured modal parameters and their analytical counterparts, the number of real global modes that were well separated and not mixed with the local ones and that could reasonably be identified, and the required number of DOFs that need to be instrumented

to construct the database of the modal parameters of the undamaged structure needed for FEM updating and damage identification. Only 30 of the 108 DOF were instrumented as response points. These DOFs were selected to cover the whole frame. Since the bending and torsional motions of the frame were found to be of interest at this stage of measurements according to the FEM, 14 accelerometers were installed in the Y direction at nodes (8, 10, 11, 13, 16, 17, 18, 19, 20, 22, 23, 25, 28 and 31) see Figure (3-10), and 14 accelerometers were installed in the Z direction at nodes (5, 10, 11, 13, 16, 17, 18, 19, 20, 23, 25, 28, 30 and 31). In addition to the accelerometers located in the Z and Y directions, two others were fixed nodes 12 and 34 in the X-direction to measure the axial acceleration.

Two force sensors were fixed at node 5 in Z-direction and at node 22 in Y-direction. The two exciters were positioned so as to be perpendicular to the force sensor at DOF's (5-Z) and (22-Y) and were attached through stringers as shown in Figure (3-8). Data acquisition was performed with two different frequency bandwidths: 0-512 Hz and 0-256 Hz. Because of the higher resolution, only the later bandwidth was finally used, particularly since it contained enough global real modes for the purpose of damage identification. The space frame was tested without adding to its nodes any of the lumped masses that would later be used to simulate the payloads. Since 32 channels were available for measurements, it was possible to collect all the FRFs of the instrumented DOFs in a single run.

All FRFs with respect to the two driving points were of good quality with each of the first three global modes well separated. Figure (3-9) shows FRF at DOF (22-Y) due to

excitation at DOF (5-Z). An examination of the peaks of the FRF plot shows that the lowest three modes are well separated. Visual animation of the measured modes superimposed on their analytical counterparts was used at this stage to study the correlation of modes. The lowest three measured modes were paired with their analytical counterparts derived from the (G1) FEM, which will be described in details later in Chapter 4. The comparison showed that the correlation was reasonably good as shown in Table (3-3). However, the level of discrepancy between the measured and calculated frequencies may negatively influence the performance of the damage identification algorithm used in this study. Also the first mode frequency was found to be relatively high as expected since the space frame was not loaded by any of its payload, which is not the case in real life. Identification of complete mode shapes of modes higher than the first three from the measurements was not possible as only a limited number of the DOFs were measured. In addition there existed a number of local modes that were mixed with or closely spaced to the global ones.

3.7.2 Modal Test of Fully Instrumented Loaded Space Frame

This test was planned on the basis of results obtained from the preliminary modal test described in Section 3.7.1. The main objective was to eliminate the problems associated with the previous modal test and to find solutions for issues that had not been addressed in the previous test. In this test, eight lumped masses were attached to the space frame nodes as shown in Figures (3-10) and (3-11). These masses were added to simulate a large space structure with attached instrumentation, such as the International Space Station ISS. The larger three masses were 2.75 kg each and were attached to nodes

(6, 17 and 30) and the smaller five masses were 1.75 kg each and were attached to nodes (4, 9, 25, 28 and 36). The location of the masses was decided on the basis of FEM studies with the objective of obtaining well-separated modes and lowering the frequency of some of the global modes that were mixed with the local ones. Since the primary objective of this study is to develop a damage identification technique that would work at least with well-separated modes, this step was justified. The total of the attached masses is about 40% of the total structural mass. Increasing the mass without increasing the stiffness decreases the frequencies. Consequently the lower global modes become well separated from the local modes. It was crucial to measure all the available DOFs to allow for the identification of the global modes and to have a robust curve fitting of the data. The available accelerometers could cover only half of the space frame nodes, since each node needs three accelerometers to measure the response in the X, Y and Z directions. A 45 gm lumped mass was attached to each of the remaining nodes to simulate the instrumentation masses for the unmeasured DOFs. As mentioned earlier, only 30 channels were available to record the responses during each run. Accordingly, for each half of the space frame, the responses at the selected DOFs were measured during two runs. After recording the response of the first half and saving all the corresponding FRFs, the accelerometers were removed from the DOFs of that half and moved to the second half to be exchanged with the small lumped masses that simulate the instrumentation masses. The test was then repeated to obtain the response at the DOFs of the second half of the frame.

Figure (3-12) shows the reciprocity check between driving points for one of the measured halves, from which it can be seen that many more global modes have been separated from the local modes due to the attachment of the eight lumped masses. By measuring the response at all DOFs of the structure, the well-separated global modes are identified reasonably well, also the match between the set of curves is very good. However, when reciprocity check was carried out between measurements of the two halves some differences in frequencies and amplitudes were always found, even when the tests were repeated. These differences can be attributed to the exchanging of accelerometers and the small lumped masses that simulate the instrumentation masses for the unmeasured half, causing non-stationarity in the structure. Some of the modes in the range of frequencies above 120 HZ were found to be global bending and axial. Animation of modes in this range revealed that those global modes were mixed with local modes.

3.7.3 Experimental Investigation of the Optimum Excitation Points

Two sets of driving points were selected based on the criteria discussed in Section 3.5. Two modal tests were carried out to identify the optimum excitation points. In the first set-up the exciters were placed at DOFs (22-Y) and (5-Z) as shown in Figure (3-8). These points satisfy the excitation points criteria mentioned in Section 3.5. In the second set-up two other candidate points were selected to be the excitation points. These were located at DOFs (2-Y) and (33-Z) as shown in Figure (3-13). Figures (3-12) and (3-14) show the reciprocity check for the two sets of driving points (22-Y) and (5-Z) and (2-Y) and (33-Z) respectively. On comparing the peaks of the FRFs plots for the lower modes,

it is seen that the driving points placed at DOFs (2-Y) and (33-Z) optimally excite the modes in the frequency range of interest. Figure (3-15) shows the coherence function of the modes in the range (0-260 Hz) for driving points (2-Y) and (33-Z), it can be seen that most of the response is due to the applied excitation forces.

3.7.4 Modal Test of Partially Instrumented Loaded Space Frame

Based on the experience obtained during the tests described in the previous sections, several conclusions were reached that helped in the refinement of test procedures. As stated earlier, it was crucial to measure all of the 108 available translational DOFs to have the maximum possible size of mode shape vectors. At the same time, only 60 accelerometers were available for making the measurements. The measurement strategy described in the previous section in which the accelerometers were moved and exchanged with compensating masses was not quite satisfactory because of the non-stationarity introduced by the switching of accelerometers. A different measurement strategy was therefore developed to avoid the identified non-stationary problem. In this strategy a light plastic block was glued to one of the flat faces of each spherical node to carry one accelerometer at a time. Three accelerometers were glued permanently to each one of the spherical nodes that served as the driving point, nodes 2 and 33, since each of these two spherical nodes had three free faces. This allowed the measurement of response in the three directions simultaneously at both nodes for subsequent comparisons. For the other 34 nodes, the accelerometers were glued to a plastic blocks at corresponding node and were oriented to measure the response in one direction as shown in Figure (3-16). This meant that only 40 accelerometers were used at

a time and three runs were required to measure the (X, Y and Z) responses at all nodes. In fact because only 32 channels of measurement were available, only half of the measurements in a run would be made at one time, however, the other half were measured simply by shifting the channels. After measuring all the FRFs in one of the directions, the test set-up was disassembled and the plastic blocks were switched to direct the accelerometers in another direction to measure the response in that direction. As stated above, the complete test required six steps during the three runs, at two steps in a run. Carrying out the two steps in any one run required only a switching of the cable connection at the front end of the modal system without having to remove any of the accelerometer cables running from the frame to the data acquisition system.

Although, the procedure described in the foregoing paragraph took much longer time and more effort, it was robust and was successful in resolving the problem caused by the non-stationarity. This conclusion was reached based on a comparison of FRFs on the two nodes of the driving points in the three directions for the three test set-ups. Examination of these plots showed that measuring the 108 DOFs during three test set-ups did not cause any frequency shifts or significant differences in the amplitudes of the modes in the frequency range of interest. Measurement of the response at all of the selected translational DOFs resulted in precise identification of the lower global modes, as can be seen from the peak values in Figure (3-14). Also the acquired FRF's were of good quality as seen from the coherence values for these modes shown in Figure (3-15). The robustness in the test measurements can be attributed to the strategy followed to define the sources of errors and to minimize the measurements errors.

3.7.5 Identification of Global Modes

Based on the modal analysis procedures discussed in Section 3.6 and since only real normal modes are required for the VBDI method presented herein, estimates of real normal modes are obtained from measured data. This is readily done since real normal mode estimation is normally provided, as an option, by the used modal analysis software LMS CADA-X.

Due to the fact that numerous local modes were extracted during the curve fit process, it was crucial to eliminate local and mixed modes, since only global real modes would be used in damage identification later on. This was carried out by the following procedures:

- (1) A set of global mode shapes was generated to serve as the baseline modal parameter set of undamaged space frame. Global mode extraction was carried out by examining the FRF peaks and phase shift and by visually displaying each of the modes in the frequency range of interest (0-256 Hz) so as to eliminate the local and mixed modes.
- (2) A Modal Assurance Criterion (MAC) analysis was performed between all the modes in each of the baseline files and the reference mode shapes to identify the modes that match the global modes of the reference measurements. The modes that produce significant off diagonal terms in MAC matrix were removed and the curve fitting process was repeated. The addition and removal of the modes was made to eliminate the mixed and local modes that were

closely spaced with the global ones. This process generated files that contained only the global modes.

The procedure outlined above resulted in the identification of eighteen global modes of the undamaged space frame in the frequency range of (0-256 Hz). The frequencies of the eighteen modes are listed in Table (3-3). As will be described later, the same procedure was repeated for the mode shapes corresponding to the tests on a space frame in which damage had been introduced for the experimental verification of the damage identification algorithm.

At a certain stage during the analysis for extracting the modal parameters of the undamaged space frame, there was some doubt that one of the real global modes at 99.16 Hz was not well excited and more excitation points would be needed to excite that mode. Two additional points at DOFs (9-Y) and (18-Z) were added as shown in Figure (3-17) to excite the space frame at four different points to ensure a proper excitation of the modes of interest including the mode at 99.16 Hz. Comparing the FRF of any selected point showed that the peak of this mode (99.16 Hz) was well excited due to the additional two-excitation points as shown in Figure (3-18). These FRFs were curve-fitted to extract the modal parameters. Unfortunately, it was found that the residues of this mode did not improve significantly by increasing the number of excitation points from two to four. It was therefore decided to use only the two driving points at DOFs (2-Y) and (33-Z).

It was noticed that the eighteen identified modes were bending and torsion modes. It was suspected that due to the density of the modes and the direction of excitation, the first axial mode may not have been well excited and an additional driving point in the X-direction was required. Accordingly, the frame was excited at DOF (34-X) and the response was measured in the X-direction at the end nodes at DOFs (1, 2, 3, 4, 33, 34, 35 and 36). Figure (3-19) shows the response at the driving point (34-X), from which it was found that the first axial mode is at 161.125 Hz. It was not possible to update the FEM to match this mode with a relatively high frequency. Consequently, it was decided not to excite the frame in the X-direction.

Table (3-1) Physical properties of the frame components

Component	Dimension (mm)	Cross- sectional area (mm ²)	Inertia (mm ⁴)	Weight (gm)
Aluminum node	φ 46	-	-	80
Elements in (x-y) planes	707×30×1.5	134.3	13673.7	350
Elements in (x-z) planes	707×22×1.0	65.97	3645.0	230
Diagonals in (x-y) planes	1000×30×1.5	134.3	13673.7	450
Diagonals in (x-z) planes	1000×22×1.0	65.97	3645.0	275
Diagonals in (y-z) planes	1000×22×1.0	65.97	3645.0	275

Table (3-2) Experimentally identified global normal modes for the loaded frame

Mode Number	Description	Mode Frequency (Hz)	Mode Damping (%)
1	First Torsion	50.98	0.172
2	First Bending in Z-Direction	51.15	0.172
3	First Bending in Y-Direction	57.51	0.220
4	Second Bending in Z-Direction	90.13	0.059
5	Second Torsion	92.41	0.153
6	Third Bending in Z-Direction	99.167	0.118
7	Normal Global	101.63	0.099
8	Normal Global	106.17	0.276
9	Second Bending in Y-Direction	109.92	0.163
10	Normal Global	111.11	0.180
11	Normal Global	115.56	0.146
12	Normal Global	123.41	0.180
13	Normal Global	129.86	0.166
14	Normal Global	134.69	0.139
15	Normal Global	135.70	0.137
16	Normal Global	147.73	0.118
17	Normal Global	149.87	0.096
18	Normal Global	153.64	0.119

Table (3-3) Comparison of 1st three measured modes with the FEM results of the unloaded frame

Mode No.	Description	Measured (Hz)	Analytical (Hz)	Discrepancy %
1	First torsion mode	62.12	61.24	1.42
2	First bending in Z - direction	66.75	65.52	1.84
3	First bending in Y - direction	68.00	66.80	1.77

Figure (3-1) Details of a typical aluminum node and element assembly
(Dimensions in mm)

Figure (3-2) Details of a typical space frame spherical node connecting seven elements

Figure (3-3) Assembled eight-bay space frame

Figure (3-4) Test set-up for the static tests

Figure (3-5)(a) Performance of the joint with respect to the applied torque for tube (1000x30) for 30 N applied load

Figure (3-5)(b) Performance of the joint with respect to the applied torque for tube (707x30) for 39.24 N applied load

Figure (3-6) Basic measurement system

Figure (3-7) Space frame suspension system

Figure (3-8) Shaker placements at degrees of freedom (22-Y) and (5-Z)

Figure (3-9) FRF at (22-Y) due to excitation at (5-Z) for unloaded partially instrumented eight-bay frame.

Figure (3-10) Distribution of the eight lumped masses

Figure (3-11) Typical connection details for the attachment of the lumped masses to a spherical node

Figure (3-12) Reciprocity check for the driving points: FRF measured at DOF (22-Y) due to the excitation at DOF (5-Z), solid line and at DOF (5-Z) due to the excitation at DOF (22-Y), dashed line

Figure (3-13) Shaker placements at degrees of freedom (2-Y) and (33-Z)

Figure (3-14) Reciprocity check for the driving points: FRF measured at DOF (2-Y) due to the excitation at DOF (33-Z), solid line and at DOF (33-Z) due to the excitation at DOF (2-Y), dashed line.

Figure (3-15) Coherence function of (2-Y) due to excitation at (33-Z)

Figure (3-16) Accelerometer installations for measuring the response in X, Y and Z directions at a typical spherical node.

Figure (3-17) Additional excitation at degrees of freedom (9-Y) and (18-Z)

Figure (3-18) Comparison of FRF at (19-Z) due to excitation at (33-Z) and (19-Z) due to excitation at (18-Z) for the
Identification of mode 99.16 Hz.

Figure (3-19) FRF at (34-X) due to the excitation at (34-X) for the identification of the first axial mode at 161.125 Hz.

Chapter 4

Finite Element Modelling and Updating

4.1 General

The theoretical development presented in Chapter 2 shows that the proposed VBDI structural health monitoring system is sensitive to the finite element model of the structure, especially for a complex type of structure such as the one used in this study. The first stage in validating the proposed approach is the construction of the FEM of the eight-bay space frame. The present chapter concentrates on the characteristics of the finite element modelling, correlation and updating techniques. The first part of the chapter presents the modelling of elements, the parameters involved and calculation of the sensitivities. Subsequent parts of the chapter describe the techniques used to correlate the analytical modal data with its experimental counterpart. A comprehensive study of the available updating techniques is presented. Also, a combined methodology is proposed and applied to update the FEM of the eight-bay space frame. This methodology includes the use of both static and dynamic responses of the structure. The results show that the proposed strategy resulted in a well-correlated FEM of the eight-bay space frame that can be used with confidence in subsequent steps of the proposed VBDI approach.

4.2 Influence of FE Modelling Errors in Damage Identification

Success of the damage identification algorithm in the present study is mainly dependent on the fidelity of the FEM of the monitored structure. Because of the assumptions and simplifications made during the modelling process, discrepancies between measurements and the simulation results of the structure model are unavoidable and may lead to poor performance of the damage identification algorithm. The errors in modal parameters are defined to quantify the discrepancy between the measured structure response and the model predictions.

Error in natural frequencies

This error is defined by

$$\Delta\omega_i = \left| \frac{\omega_{im} - \omega_{ia}}{\omega_{im}} \right|, \quad i = 1, 2, \dots, m \quad (4-1)$$

where ω_{im} and ω_{ia} are the i^{th} measured and calculated natural frequencies, respectively. and m is the number of modes used in the identification.

Error in mode shapes

This error measure is given by

$$\Delta q_i \equiv \sqrt{\sum_{j=1}^n (q_{ij}^m - q_{ij}^a)^2}, \quad i = 1, 2, \dots, m \quad (4-2)$$

where q_{ij}^m and q_{ij}^a are the measured and analytical components at the j^{th} measured DOF for the i^{th} mode, respectively, and n is the number of measurement points. If modelling and/or experimental errors exists, the modal parameter errors defined above will be

nonzero and may influence the modal data used in damage identification algorithms. The minimization of the modelling errors would significantly enhance the performance of the damage identification algorithms. The impact of simulated errors in modal parameters on the performance of the damage detection approach used in the present work has been studied in the computer simulation described in Chapter 5.

4.3 Finite Element Model Parameterization

The construction of FE models usually includes some simplifying assumptions. These assumptions ignore detailed features, that include accurate geometric representation of the structural elements, and boundary conditions, precise values of material properties (modulus of elasticity, Poisson's ratio, mass density, etc.) and realistic modelling of the connections between components such as bolted joints and welds. Since the finite element solution technique is a numerical procedure that analyzes only the selected mathematical model, all the assumptions made in constructing the model are reflected in the predicted dynamic response.

A number of different model updating techniques and approaches are available and have been introduced and discussed in many published papers. These techniques depend on the adjustment of a selected set of parameters to improve the FEM prediction of the measured modal parameters (Avitable (2000)). Some of these parameters have a very strong influence in the success of the updating process. Taking this into consideration Sun et al (2000) introduced a model correlation scheme using a sensitivity based updating method, which instead of relying on updating the mass and stiffness

matrix elements used the physical parameters as design variables. In general, the FEM update is based on the solution of an optimization problem. The formulation and solution of the optimization problem focuses on minimizing an objective function that is based on the weighted sum of deviations from measured eigenvalues and eigenvectors. The objective function and the constraint inequalities are functions of the updating parameters, such as cross-sectional area, material properties and joint stiffness. Mottershead et al (2000) studied the updating of a FEM of a three-story frame that had been built using Meroform components of 22 mm tubes. The frame was modelled using 140 cubic Euler-Bernoulli beam elements and a total of 708 DOFs. As described earlier, Meroform tubes have steel connectors at their ends, which screw into the nodes. In the study by Mottershead et al these connectors together with the spherical nodes were considered to be perfectly rigid. Five model-updating experiments were carried out using different sets of updating parameters. For each set of parameters, the FEM was updated using eigenvalue sensitivity analysis. The authors concluded that the convergence of FE predictions to measured natural frequencies could be obtained by using many different sets of updating parameters. However, not all the parameter sets (that resulted in improved predictions) could be justified on physical ground. Only one parameter set was found to provide a physically motivated improvement to the modelling of the joint. It is evident from this study that the success of such a method is mainly dependant on the selection of the parameters set, which has no rules and is also computationally expensive even for a relatively simple structure.

There exist other techniques (Friswell et al 1995) that can be used to update a FEM using the measured modal parameters. Many of these updating techniques would

work for simple structures, which include a relatively small number of updating parameters. For complex large-scale structures, the number of updating parameters may increase dramatically and the solution of the optimization problem for model updating may result in unrealistic values of the updated parameters.

Usually the updating of large-size FEMs, such as that of the test article of this study, requires the use of one of the updating methods to adjust the model physical parameters (Horton et al (2000)). Different methods for updating the FEM have been presented in the previous paragraph. However the use of these methods for complex structures, such as the test article used for this work, may at times result in unrealistic values of the updated physical parameters. This is due to the numerous uncertainties in the physical parameters of the components. At the same time, the FEM of the monitored structure is considered as the backbone of the SHM techniques that use model-based VBDI methods. Minimizing the modelling errors would significantly enhance the overall performance of the proposed approach. Consequently, a better strategy to update a FE model of the space frame considered here is to carefully identify the physical parameters and adjust them appropriately to provide as faithful a representation of the structure as possible.

4.3.1 Identification of Model Parameters that May Generate Uncertainty

The parameters listed below were found to be the main sources of uncertainty in the model. Experimental and analytical sensitivity studies were performed to evaluate the

influence of each parameter in the structural response for both individual elements and the assembled frame

- The idealization assumption in modelling the spherical node connection shown in Figure (3-1) in which it is modelled either as true pin or as fully rigid joint.
- Material properties (modulus of elasticity) and dimensions of the elements. Since the tubes used in manufacturing the frame elements are extruded, the thickness of the tubes may not conform exactly to the design value and is therefore a source of uncertainty.
- Discretization in the generation of the FE model that is generally associated with mesh coarseness.
- Representation of the structural mass matrix. The use of lumped mass is not realistic and produces higher values of the mode frequencies.
- Influence of the mathematical representation of the actual boundary conditions of the frame supporting system.
- Selection of the parameters to be included in the updating process. The selection of the set of parameters and their characteristics, which are allowed to change, is considered to have the greatest influence on the updating process.

If all of the previous uncertainties are included as design variables in the updating procedure the resulting updated model may not be realistic and this may have an adverse effect on the performance of the damage identification algorithm. In this study, a step-by-step approach has been developed to accurately parameterize the FEM of the eight-bay

space frame using simple procedures. It is believed that this approach would be able to produce a FE model that is representative of the physical nature of the structure.

4.3.2 FE Model Parameterization and Error Minimization Strategy

The localization of the modelling uncertainty sources and the characteristics of each parameter have shaped the strategy used in this work for constructing and updating the FEM of the space frame. A four-step strategy is used to develop and update the FEM. Model updating is carried out at the level of individual element models rather than at the global level. This procedure has the advantage of minimizing the computational burden of the formulation and solution of the optimization problem to update the FEM of the whole frame and ensures that the model parameters are physically representative. The four steps are:

1. The first step includes accurate measurement of the physical parameters of the components through static tests. The physical parameters include the modulus of elasticity of the material used to manufacture the aluminum tubes, and the effective flexural rigidity of the element-joint subassembly.
2. A FEM for the individual tube-joint subassembly is constructed and updated using the static tests results.
3. The updated element-joint subassembly models are used to construct the space frame FEM. The modal parameters obtained from the FEM are compared with the modal parameters obtained from modal testing of the assembled eight-bay space frame to measure the discrepancy.
4. To minimize the discrepancy, adjustments are made to a set of selected element parameters using a systematic sensitivity updating technique.

4.3.3 Determination of Physical Parameters Values through Static Tests

(1) *Material Properties:*

The space frame component's material properties were determined by carrying out the static test procedures described in Section 3.3. In this set of tests, the modulus of elasticity of the aluminium tube was determined by attaching strain gauges at different points along the tube and measuring the strains due to applied loads. The relationship between the stress calculated from the applied bending moment and the measured strain provided a value for the modulus of elasticity. Figure (4-1) shows the experimental results obtained from the test. The average modulus of elasticity, at the predetermined consistent torque (22.6 N.m), was found to be (0.67E+11Pa), 2% below its nominal value. The thicknesses of the tubes used in the model were also measured. The average thickness of the 30 mm diameter tubes was found to be 1.57 mm compared to its published value of 1.5 mm. The thickness of the 22 mm diameter tubes was measured as 1.05 mm compared to the published value of 1.0 mm. Published value was used for the Poisson's ratio ν .

(2) *Joint Modelling:*

One idealization error that is inherent in modelling a structure, such as the one used in this study, is in modelling of the joints. The conventional procedure for modelling the member-end joint, such as that shown in Figure (3-1), assumes that it behaves as a pure pin or, as a perfectly rigid joint. A rigid connection assumption may be reasonable for predicting the lower modes, but may be incorrect for higher order modes. From a dynamic point of view, representation of the joint is of interest because the modal

characteristics of the beam are often found to be quite sensitive to the stiffness of the joints. Experimental studies have shown that the joints in Meroform typically exhibit some flexibility in bending and are therefore flexible (El-Sheikh (1992) and Goto and Miyashita (1998)). Li (2000) has presented a structural dynamic model updating method that focuses on identification of joint parameters. Specifically, for this type of joints that is not sufficiently rigid, the relative motions between the element and the joint have to be taken into consideration. The method depends on a Reduced-Order Characteristic Polynomial (ROCP) defined in terms of joint stiffness represented by linear and rotational springs, and the measured natural frequencies and the modal properties determined from an initial FE model. In Li's study a numerical example of a cantilevered beam elastically constrained at one end was used to demonstrate the feasibility of this method. The ROCP method was implemented to update the rotational and linear stiffness of the end springs. Although, the updated FEM was able to match the measured frequencies of the cantilevered beam, it was computationally expensive, even for the introduced simple model, in which only one beam was connected to the joint.

It should be noted that the effective cross-sectional axial stiffness (EA_e) of the element-joint subassembly is higher than that of the tube, as has been reported by McGowan and Lake (1987). Consideration of the flexural and axial rigidity of joint subassembly is expected to enhance the accuracy of numerical predictions obtained for the space frame modal parameters (frequencies and mode shapes) since it makes the mathematical model more representative of the physical structure.

A set of static tests was carried out to identify the joint flexural rigidity. The test set-up used in the identification of the optimum tightening torque and shown in Figure (3-4) was used in this set of tests as well. For each of the four different types of element, the element was loaded vertically at the free end and the deflections produced by the application of different load levels were recorded along the element. A direct approach to have a generalized determination of the joint flexibility is to develop FEM of the individual elements. It was not easy to develop one typical model configuration that can be used to model different types of elements, and would be able to validate the measured static response at different joint stiffness, corresponding to different tightening torque levels.

To meet the requirements outlined above, different FE models were tried using FE software (COSMOS), The only model that succeeded was the one in which five 3-D beam elements were used to model each element, as shown in Figure (4-2) with a total of 60 DOFs. The nominal physical properties of each of the five elements are listed in Table (4-1). A sensitivity study was performed to select the parameters for updating these models to validate the measured static deflections. These parameters were found to be, the length of element (EL1), which represents the spherical node, the size of the mesh and the flexural rigidity of element (EL3), which represents the aluminum tube. The results of this study showed that the length of element (EL1) was 23 mm for elements that were 1000 mm long and 18.5 mm for elements that were 707mm long. The optimum refinement mesh of element (EL3) was found to be five elements, however when the element FE model was condensed to the end DOFs, the refinement of the tube mesh did

not contribute to the improvements of the model predictions and it was decided to model the tube element (EL3) as one element.

For each element, the experimentally measured deflections were compared to the ones calculated from the FE model as shown in Figures (4-3) to (4-6). It was evident from the results of the different groups of static tests that the joint subassembly exhibited significant flexibility; and that the FEM developed for the individual elements by correlating the measured static flexibilities had proven to be representative.

The identification of the joint model would positively influence the correlation between the modal parameters obtained from a FEM and those measured through modal testing of the space frame assembly. It was planned to measure the axial stiffness of the joint subassembly, but the available equipment for conducting such a test was not appropriate for testing of the joint subassembly (Meroform M12). McGowan and Lake, (1987) have determined the axial stiffness of the joint subassembly. In their study two different types of specimens were tested: a joint subassembly consisting of a node with a half length of a tube (22×1.0 mm) mounted on either side, and a segment of a tube having the same length as the joint subassembly as shown in Figure (4-7). In their experimental study, effective axial stiffness of the joint subassembly was measured and found to be 124 percent that of the aluminum tube. The effective axial stiffness was calculated using the following expression (Timoshenko and Young, 1965).

$$EA_e = \frac{L(EA_s)(EA_j)}{L_s(EA_j) + L_j(EA_s)} \quad (4.3)$$

where (EA_s) is the calculated tube axial stiffness, (EA_j) is the experimentally measured joint stiffness and (EA_e) is the effective axial stiffness of the joint subassembly. In the present study the effective stiffness given by Equation (4.3) was used in constructing the finite element model of the individual tube and hence of the assembled frame.

(3) *Boundary Conditions:*

As has been described in Section 3.5, the vibration characteristics of the suspension system were experimentally identified. The values of the axial stiffness of the bungee cord obtained from these experiments were used in modelling the suspension system by four spring elements in all of the FEM models developed for the eight-bay space frame.

4.4 Finite Element Modelling

The finite element method is widely used to solve physical problems in structural analysis and design. The physical problem typically involves a structure or structural components subjected to certain loads. The idealization of the physical problem by a mathematical model requires certain assumptions (Bathe (1996)). Using these assumptions a system of differential equations governing the mathematical model is formed. Equation (4.4) represents the equations of equilibrium governing the linear dynamic response of a system of finite elements.

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}(\mathbf{t}) \quad (4.4)$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices, respectively and $\mathbf{f}(t)$ is the vector of externally applied dynamic loads. For structures having relatively low damping, such as the eight-bay space frame used, damping term in Equation (4.4) can be neglected and the equation of motion becomes

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}(t) \quad (4.5)$$

The finite element analysis solves this mathematical model. Since the finite element solution technique is a numerical procedure that solves only the selected mathematical model, all the assumptions made in the construction of the model will be reflected in the predicted response. The VBDI structural health monitoring system of this study uses a combination of model-based methods. These methods would be more robust when the model has the ability to precisely predict the measured dynamic response of the structure. To meet the accuracy criteria, the mathematical model system matrices should be properly identified to represent the actual physical parameters of the model. As described in the previous chapter, the experimental test article is an erectable space frame consisting of aluminum elements. These elements are aluminum tubes with screwed solid steel end connectors. The stiffness and consistent mass matrices for a slender three-dimensional frame element shown in Figure (4-8) are described in Equation (4.6) and (4.7) (Craig (1981)).

$$K = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 & u_9 & u_{10} & u_{11} & u_{12} \end{bmatrix} \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} & 0 & -\frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} \\ 0 & 0 & \frac{12EI_y}{L^3} & 0 & -\frac{6EI_y}{L^2} & 0 & 0 & 0 & -\frac{12EI_y}{L^3} & 0 & -\frac{6EI_y}{L^2} & 0 \\ 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & 0 & -\frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 \\ 0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} \\ -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI_z}{L^3} & 0 & 0 & 0 & -\frac{6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & -\frac{6EI_z}{L^2} \\ 0 & 0 & -\frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 \\ 0 & 0 & 0 & -\frac{GJ}{L} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & 0 & -\frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 \\ 0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} \end{bmatrix} \quad (4.6)$$

$$M = \frac{\rho AL}{420} \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 & u_9 & u_{10} & u_{11} & u_{12} \end{bmatrix} \begin{bmatrix} 140 & 0 & 0 & 0 & 0 & 0 & 70 & 0 & 0 & 0 & 0 & 0 \\ 0 & 156 & 0 & 0 & 0 & 22L & 0 & 54 & 0 & 0 & 0 & -13L \\ 0 & 0 & 156 & 0 & -22L & 0 & 0 & 0 & 54 & 0 & 13L & 0 \\ 0 & 0 & 0 & \frac{140I_p}{A} & 0 & 0 & 0 & 0 & 0 & \frac{70I_p}{A} & 0 & 0 \\ 0 & 0 & -22L & 0 & 4L^2 & 0 & 0 & 0 & -13L & 0 & -3L^2 & 0 \\ 0 & 22L & 0 & 0 & 0 & 4L^2 & 0 & 13L & 0 & 0 & 0 & -3L^2 \\ 70 & 0 & 0 & 0 & 0 & 0 & 140 & 0 & 0 & 0 & 0 & 0 \\ 0 & 54 & 0 & 0 & 0 & 13L & 0 & 156 & 0 & 0 & 0 & -22L \\ 0 & 0 & 54 & 0 & -13L & 0 & 0 & 0 & 156 & 0 & 22L & 0 \\ 0 & 0 & 0 & \frac{70I_p}{A} & 0 & 0 & 0 & 0 & 0 & \frac{140I_p}{A} & 0 & 0 \\ 0 & 0 & 13L & 0 & -3L^2 & 0 & 0 & 0 & 22L & 0 & 4L^2 & 0 \\ 0 & -13L & 0 & 0 & 0 & -3L^2 & 0 & -22L & 0 & 0 & 0 & 4L^2 \end{bmatrix} \quad (4.7)$$

where ρ is the mass density, A is the cross section area, E is the elastic modulus, L is the beam element length, I is the moment of inertia and I_p is the polar moment of inertia. Using the physical parameters obtained from the static tests described earlier, four different finite element models were constructed for the eight-bay space frame. The modal parameters (frequencies and mode shapes) of each of these models were first calculated using subspace iteration method implemented in FE software COSMOS. Because element stiffness matrices, in the global coordinates, were needed for the formulation of the optimization problem, MATLAB codes were developed to build the FEM of each of the models described below and to solve the eigenvalue problem. A full description of these codes will follow in the subsequent chapters. The description of the configuration of each FE model is presented below:

Model (G1): In this model, the elements were modelled using a uniform 3D beam element, which had six translational and six rotational degrees of freedom. The joints were modelled as being rigid. The masses of the spherical nodes, the difference between the actual mass of a element and that derived on the basis of a uniform mass density, and the mass of instrumentation attached to each node were accounted for by attaching appropriate lumped masses at the joints. The inertia effects of the eight lumped masses, meant to simulate the payload on the space frame, were considered in the model. The model had a total of 312 DOFs. In calculating the modal parameters (frequencies and mode shapes) a consistent mass formulation was used.

Model (G2): In this model, the elements were modelled using a uniform 3D truss element, which had six translational degrees of freedom. The joints were modelled as pure pin joints. Again, the masses of the spherical nodes, the difference between the actual mass of a element and that derived on the basis of a uniform mass density, and the mass of instrumentation attached to each node were accounted for by attaching appropriate lumped masses at the joints. The inertia effects of the eight lumped masses, meant to simulate the payload on the space frame, were considered in the model. The model had a total of 108 DOFs. Again, in calculating the modal parameters (frequencies and mode shapes) a consistent mass formulation was used.

Model (G3): It was recognized from the static tests described in Section 4.3.3 that the connection of the aluminium tube to the spherical node through a bolt and pressure fitting would result in significant flexibility in the joint. A refined model was therefore constructed in which each element was represented by five 3-D beam elements as shown in Figure (4-2), giving a total of 36 DOFs per element. The bolt element at each end of the element provided a representation of the flexibility of the joint subassembly. It was found that the sensitivity of the analytically predicted lower frequencies with respect to mesh refinement of the tube part was insignificant in the frequency range of interest. Accordingly, the tube part of the element was modelled using one element to minimize the model size. The physical properties of each of the five elements of the elements were derived from measurements and static tests referred to earlier and listed in Table (4-1). The five-element element model was used to construct the FEM of the space frame, which includes 109 elements and 36 nodes as shown in Figure (4-9). The inertia effects

of the eight lumped masses, meant to simulate the payload on the space frame, were considered in the model. Each of the masses was modelled as a beam element, which was found to be more representative. The space frame model had a total of 2928 DOFs. Again, a consistent mass formulation was used in calculating the modal parameters.

Model (G4): The modal parameters prediction of FE model (G3) was found to be very close to the measured parameters of the undamaged frame; yet, this model could be updated. A sensitivity study was performed to investigate the parameters that could be adjusted physically and would have an impact on reducing the discrepancy between the analytical and measured modal parameters. The flexural inertia of the element that represented the node was found to be the best candidate for the updating process. Actually, the representation of the geometry of this element in model (G3) was based on the assumption that the element had constant (EI) and (EA) throughout its length, which is not the case since it has 18 holes. Figure (4-10) shows the convergence of the lower frequencies with variation in the inertia of the node element having a constant (EI/EA) ratio. Table (4-2) shows comparisons between the measured frequencies and the analytically calculated ones produced by each of the developed FEM generations.

4.5 Correlation of Finite Element Model

One major step of VBDI techniques is to provide a comparison between analytical predictions of the dynamic behaviour of the structure and those measured experimentally. This, in fact, constitutes the first step in these techniques and is meant to validate the baseline finite element model of the structure through a correlation with the

corresponding modal data. This step can be executed only if there is a match between the corresponding coordinates. Because the number of DOFs along which measurements are made is usually significantly smaller than the DOFs in the FEM, a model reduction or expansion technique has to be used to reduce the model degrees of freedom to the measured ones or to expand the measured mode shapes to the size of their analytical counterparts. In the following subsections, the mathematical techniques used for the achievement of this part of the study are described.

4.5.1 Model Reduction and Expansion Techniques

Large finite element models are often generated to characterize complicated structural systems. For example, the use of a rather large number of degrees of freedoms in FE models (G3) and (G4) is necessary in order to properly discretize the system. Dimension mismatch arises when comparing measured experimental data with numerically generated data because of incompleteness of the measured degrees of freedom. The damage identification approach presented in this study depends on pairing the analytical modal parameters with their experimental counterparts. Consequently, model reduction is crucial for the mathematical formulation of the optimization problem meant to solve for damage identification. In this study we were able to measure response at only a limited number of locations, 108 degrees of freedom out of a total of 2928 analytical degrees of freedom included in the FEM. Also only a limited number of lower modes were measured. The modes thus represented a small number of mode shape vectors with a reduced number of degrees of freedom. Different techniques that have been described in Chapter 2 can be employed to execute the task of model reduction. As

stated earlier, model reduction is needed in checking the correlation and orthogonality between analytical and experimental modal vectors, updating the analytical model using the experimental data, and in the formulation of the constraint system of equations of the optimization problem meant to be solved for damage identification.

In this study two techniques, Guyan and SEREP, were employed to match the measured and analytically-calculated degrees of freedom

Guyan Static Condensation

This method introduced by Guyan (1965) is possibly the most popular and the simplest. The basic assumption of static condensation in the calculation of frequencies and mode shapes is that the mass of the structure can be lumped at only some specific translational degrees of freedom that are preserved in the reduced model and the remaining rotational degrees of freedom can be condensed out from the mass matrix because the kinetic energy component corresponding to those degrees of freedom is negligible in comparison with the included translational degrees of freedom. As the number of DOFs at which the mass is lumped reduces, the computer effort is also reduced. However, at the same time it becomes more probable that the frequencies and mode shapes are predicted with less accuracy. The displacement vector is partitioned so that u_a represents the retained degrees of freedom and u_c represents the condensed degrees of freedom, and the stiffness and mass matrices are also partitioned accordingly. The equation of undamped free vibrations can then be written as

$$\begin{bmatrix} \mathbf{M}_{aa} & \mathbf{M}_{ac} \\ \mathbf{M}_{ca} & \mathbf{M}_{cc} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}_a \\ \ddot{\mathbf{u}}_c \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{aa} & \mathbf{K}_{ac} \\ \mathbf{K}_{ca} & \mathbf{K}_{cc} \end{bmatrix} \begin{bmatrix} \mathbf{u}_a \\ \mathbf{u}_c \end{bmatrix} = \begin{bmatrix} \mathbf{f}_t \\ 0 \end{bmatrix} \quad (4.8)$$

For the lumped mass matrix, in which the mass is lumped only along the retained DOFs, the sub-matrices $\mathbf{M}_{ac}$, $\mathbf{M}_{ca}$ and $\mathbf{M}_{cc}$ are equal to zero, and accordingly the second set of equations can be written as

$$\mathbf{K}_{ca}\mathbf{u}_a + \mathbf{K}_{cc}\mathbf{u}_c = 0 \quad (4.9)$$

Equation (4.9) may be used to eliminate the condensed degrees of freedom from both mass and stiffness matrices so that

$$\begin{bmatrix} \mathbf{u}_a \\ \mathbf{u}_c \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ -\mathbf{K}_{cc}^{-1}\mathbf{K}_{ca} \end{bmatrix} \begin{bmatrix} \mathbf{u}_a \end{bmatrix} = \mathbf{T}_G \begin{bmatrix} \mathbf{u}_a \end{bmatrix} \quad (4.10)$$

where $\mathbf{T}_G$ is the static transformation between the full displacement vector and the retained degrees of freedom. The reduced mass and stiffness matrices are then given by

$$\mathbf{M}_R = \mathbf{T}_G^T \mathbf{M} \mathbf{T}_G \quad \mathbf{K}_R = \mathbf{T}_G^T \mathbf{K} \mathbf{T}_G \quad (4.11)$$

where $\mathbf{M}_R$ and $\mathbf{K}_R$ are the reduced mass and stiffness matrices, respectively. If a consistent mass matrix is considered for the system, the sub-matrices $\mathbf{M}_{ac}$, $\mathbf{M}_{ca}$ and $\mathbf{M}_{cc}$ will have nonzero values and the inertia terms would be more significant. Using Guyan

static transformation is only accurate for stiffness transformation; inertia forces are not preserved in this process. All the frequencies of the reduced model will be higher than those of the original full model.

System Equivalent Reduction Expansion Process (SEREP).

Most reduction techniques affect the dynamic characteristics contained in the full analytical model. Generally the estimated frequencies in the reduced model are higher than those of the original model as has described in the previous section. Avitabile et al (1989) have provided a comparison of the various techniques. However, a reduction technique has been developed that produces reduced models (containing arbitrary selected degrees of freedom), which preserves the model characteristic of the full system for selected modes of interest. This technique is called the System Equivalent Reduction / Expansion Process (SEREP). A brief description of the method follows. For a more detailed description reference may be made to O'Callahan et al (1989). The free vibration problem given by Equation (4.8) can be solved by using m modal vectors, in which the displacements $\mathbf{u}$ is expressed in terms of the m measured mode shapes and the corresponding modal coordinates as

$$\begin{bmatrix} \mathbf{u}_a \\ \mathbf{u}_c \end{bmatrix} = \begin{bmatrix} \Phi_a \\ \Phi_c \end{bmatrix} \mathbf{y} \quad (4.12)$$

where the displacements as well as the mode shapes have been partitioned along the active or measured DOF a and the condensed or unmeasured DOF c , and $\mathbf{y}$ are the modal coordinates. It is assumed that the complete analytical mode shapes included in Equation (4.12) are available. The first partition in Equation (4.12) gives

$$\mathbf{u}_a = \Phi_a \mathbf{y} \quad (4.13)$$

Matrix Φ_a is a rectangular matrix of size a by m , in which a is in general significantly larger than m . Solving Equation (4.13) for the modal vector $\mathbf{y}$ requires the inverse of Φ_a which does not exist. However by taking a generalized inverse Φ_a^g the modal coordinates $\mathbf{y}$ can be expressed in terms of $\mathbf{u}_a$.

$$\mathbf{y} = [\Phi_a^T \Phi_a]^{-1} \Phi_a^T \mathbf{u}_a = \Phi_a^g \mathbf{u}_a \quad (4.14)$$

Using Equation (4.14) the condensed DOF can be expressed in terms of the active DOF

$$\mathbf{u}_c = \Phi_c \mathbf{y} = \Phi_c \Phi_a^g \mathbf{u}_a \quad (4.15)$$

The complete displacement vector is now given by

$$\begin{bmatrix} \mathbf{u}_a \\ \mathbf{u}_c \end{bmatrix} = \begin{bmatrix} \mathbf{I}_a \\ \Phi_c \Phi_a^g \end{bmatrix} \mathbf{u}_a = \mathbf{T}_s \mathbf{u}_a \quad (4.16)$$

where $\mathbf{I}_a$ is an identity matrix of size a by a , and $\mathbf{T}_s$ is a transformation matrix. By using $\mathbf{T}_s$ the incomplete mode shapes Φ_{ma} measured in the monitored structure can be expanded to their full size.

$$\begin{bmatrix} \Phi_{ma} \\ \Phi_{mc} \end{bmatrix} = T_s \Phi_{ma} \quad (4.17)$$

Alternatively, the matrices **M** and **K** can be reduced by using Equation (4.11). This technique provides features that other reduction techniques do not, such as

- It is possible to arbitrarily select modes that are to be preserved in the reduced model.
- The quality of the reduced model is not dependent upon the location of the selected active degrees of freedom.
- The frequencies and mode shapes of the reduced system are exactly equal to the frequencies and mode shapes (for the selected modes) of the full system model.

The reduction/expansion process is reversible; expanding the reduced system's mode shapes back to the full system's space develops mode shapes that are exactly the same as the original mode shapes of the full system model. This technique was first developed to extract rotational degrees of freedom for modal test data. Since then, however, SEREP has been used in different applications such as correlation of experimental and analytical modal vectors, linear and non-linear forced response vectors and analytical model updating.

To match the measured 108 DOF modes with their 312 DOF analytical counterparts produced by FEM (G1) for correlation purposes and subsequent use in damage identification, SEREP was used in the present study to reduce the full model to the corresponding measured 108 DOFs. The selection of modes used in the formulation of the transformation matrix **T_s**, was made on the basis of mode selection criteria

described in Chapter 2 and identified using the visual animation option offered in COSMOS and implemented in MATLAB codes. A full description of these codes will follow in the subsequent chapters.

Substructure Reduction Methodology

Since the FEM (G4) produces the best predictions it was decided to use it as the baseline model for the damage identification algorithm in the subsequent chapters. Given that only 108 DOFs of the 2928 DOFs were measured, it was crucial to reduce the FEM DOFs or to expand the measured DOFs to the full model DOFs. The SEREP technique was first used to execute this task. As mentioned earlier, unlike other reduction processes, which develop reduced systems whose modal parameters approximate the modal parameters of the full model, SEREP develops reduced models whose frequencies and mode shapes are exactly the same as the full system model for the selected modes of interest. The criteria of mode selection described in Chapter 2 was applied to pick up the modes that would be preserved in the reduced model formulation. These modes have been identified by visual animation of the mode shapes using COSMOS software. It is evident that the number of DOFs in the reduced model is greater than the number of modes that are used to develop the T_s transformation matrix (i.e. $a \gg m$). The rank of the mass and stiffness matrices for the reduced model equals m . The orders of the mass and stiffness matrices of the reduced system are of order a , therefore, the mass and stiffness matrices are rank deficient. A MATLAB code was developed for the numerical implementation of this part of the study. Unfortunately, the large size of the full model stiffness and mass matrices (2928 DOFs), the ill-conditioning and numerical instability of

the original consistent mass matrix and the numerical truncation errors prevented the built-in eigenvalue solver of MATLAB from converging to the same solution as obtained from COSMOS. Consequently it was found necessary to develop an alternative methodology of reducing the model, which offered advantages similar to those offered by SEREP.

A two-step reduction methodology based on the concept of sub-structuring was developed for this study in order to reduce the FEM (G4) from 2928 DOFs to the measured 108 DOFs. In the first step, at the elemental level, the individual element model stiffness and mass matrices (60 DOFs) were reduced to (12 DOFs) at the end nodes using Guyan condensation. In order to reduce the effect of the reduction technique on the calculated modal parameters of the reduced model, a consistent mass formulation was used in building the full individual element models. The reduced individual models were used to build the FEM of the eight-bay space frame, which had 264 DOFs. In the second step, the full 264 DOFs FEM was reduced to 108 DOFs using SEREP, preserving the first lowest seven non rigid-body global modes that had a maximum modal strain energy content. Table (4-3) shows a comparison of the analytical frequencies of the first seven modes obtained from the full FEM (G4) and the reduced FEM (GR4). It is evident that the reduction strategy used has reduced the effect of reduction techniques, since the increase in the modal frequencies due to reduction is insignificant.

It is worth mentioning at this point that when SEREP was used to reduce the individual element models, preserving some of the element modes at the global level, the

resulting 264 DOFs FEM was not able to develop, even approximately, modal parameters of the full model. It has not been established that the use of SEREP at the substructure level offers any particular advantage.

4.5.2 Correlation of Analytical and Experimental Modal Vectors

The analytical and experimental modal vectors are paired at different steps in the procedures meant to identify damage in the test article used in this study. The problem of experimental modal vector incompleteness has been addressed in the previous section. Correlation techniques are to a great extent based on the orthogonality conditions. In this study, two correlation techniques have been used to carry out the following tasks:

1. Correlate experimental modal vectors of the undamaged structure with their analytical counterparts developed from the FEM for updating the baseline analytical model.
2. Pair the experimental modal vectors of the damaged structure with the experimental undamaged ones in the modal domain to help in picking a subset of damaged modal vectors that are the best candidates for damage identification.
3. Correlate the subset of the damaged modal vectors selected in the previous step with the analytical undamaged ones developed from the updated FEM. The damaged modes that have acceptable correlation are paired with their analytical counterparts for the formulation of the different mathematical steps of the proposed approach for damage identification.

An orthogonality check is performed in this study for the correlation purposes described above. A great deal of research has been done in the area of modal vectors correlation and several orthogonality tools have been developed. Among them are the modal assurance criterion MAC and its modifications, normalized MAC, frequency scaled FMAC and coordinate COMAC. Mass weighted orthogonality, Pseudo orthogonality check PORTH and cross orthogonality check XORTH are additional tools that are commonly used in the field of modal analysis. A detailed discussion of the characteristics of each method is presented in Ewins (2001). In the present study two of the following orthogonality equations were used;

Mass-Weighted Orthogonality

$$\Phi_{\text{fem}} \mathbf{M} \Phi_{\text{fem}} = \mathbf{I} \quad (4.18)$$

where Φ_{fem} is the matrix of mass normalized analytical modal vectors, $\mathbf{M}$ is the analytical mass matrix and $\mathbf{I}$ is the identity matrix. Equation (4.18) is, of course, satisfied.

Pseudo Orthogonality Check

$$\Phi_{\text{exp}} \mathbf{M} \Phi_{\text{exp}} = \text{PORTH} \quad (4.19)$$

The pseudo-orthogonality check correlates the corresponding mass normalized experimental modal vectors. When the experimental results are good, PORTH should be a near identity matrix.

Cross Orthogonality Check

$$\Phi_{\text{fem}} \mathbf{M} \Phi_{\text{exp}} = \mathbf{XORTH} \quad (4.20)$$

As seen from Equation (4.20) the cross orthogonality check is between analytical and experimental modal vectors. The correlation is ideal, if the XORTH is an identity matrix, which means that the experimental modal vectors are orthogonal to the analytical ones when weighted by the analytical mass matrix. As mentioned by Mains and Vold (1995), it is common practice in modal testing to require that elements of the orthogonality matrices not differ by more than 0.1 from the corresponding elements of an identity matrix. This means that the absolute value of diagonal terms must be between 0.9 and 1.1 and the off-diagonal terms must be between -0.1 and 0.1 .

In the present work, a pseudo orthogonality check was performed with the normalized experimental mode shapes of the undamaged structure using the reduced analytical mass matrix. The reduced mass matrix was derived using the procedures described in Section 4.5.1. The elements of the resulting PORTH matrix are given in Table (4-4). It will be noted that the absolute values of all diagonal terms are equal to 1.0. This is expected because the experimental mode shapes have been orthonormalized with respect to the mass matrix. What is important is that most off-diagonal terms for the lower modes, up to mode no. 5 are less than 0.1. The condition is violated by mode nos. 6 through 9, implying that these mode shapes have not been well identified

Table (4-5) and Table (4-6) shows cross orthogonality check for the first few modes between undamaged measured modes and the analytical predictions of FEM (G1) and FEM (G4), respectively. FEM (G1) shows that a good correlation exists for the first three to four modes. The refined model FEM (G4) gives good correlation for six or seven out of the first nine modes.

The pseudo orthogonality and the cross orthogonality check were also performed for the experimental modal vectors matrix for each damaged case. The PORTH and XORTH matrix for each damaged case will be presented in Chapter 6.

Table (4-1) Physical parameters of the finite elements of the individual element models

Element Number	Young's Modulus (Pa)	Density (kg/m ³)	Area (m ²)	Inertia I _z , I _y (m ⁴)
EL (1)	0.067E+12	2.7E+03	0.955E-03	0.18E-06
EL (2)	0.200E+12	7.8E+03	0.1179E-03	1.105E-09
EL (3) tube (30×1.5)	0.067E+12	2.7E+03	0.1343E-03	13.67E-09
EL (3) tube (22×1.0)	0.067E+12	2.7E+03	65.97E-06	3.645E-09

Table (4-2) Comparison between the measured and analytical frequencies

Mode No.	Measured Frequencies (Hz)	FEM (G1)		FEM (G2)		FEM (G3)		FEM (G4)	
		ω (Hz)	% $\Delta\omega$	ω (Hz)	% $\Delta\omega$	ω (Hz)	% $\Delta\omega$	ω (Hz)	% $\Delta\omega$
1	50.98	51.45	0.94	56.55	10.9	52.11	2.22	50.98	0.0
2	52.15	53.14	1.89	54.93	5.33	53.64	2.88	52.31	0.0
3	57.51	58.99	2.57	68.38	18.9	59.26	3.04	57.49	0.0
4	90.13	96.30	6.85	107.05	18.7	93.49	3.72	91.67	1.7
5	92.41	97.56	5.57	109.39	18.4	94.70	2.48	92.26	0.16
6	99.17	109.5	10.42	-	-	102.81	3.7	100.5	1.3
7	101.63	-	-	-	-	104.69	3.01	102.4	0.76
8	106.17	-	-	-	-	-	-	113.6	7.00
9	109.91	-	-	140.61	27.9	-	-	114.9	4.54

Table (4-3) Comparison between the analytical frequencies obtained from the full and reduced FEM

Mode No.	Measured Frequencies (Hz)	Full FEM (G4) (2928 DOFs)		Reduced FEM (GR4) (108DOFs)	
		ω (Hz)	% $\Delta\omega$	ω (Hz)	% $\Delta\omega$
1	50.98	50.98	0.0	51.16	0.35
2	52.15	52.31	0.0	52.56	0.79
3	57.51	57.49	0.0	57.82	0.54
4	90.13	91.67	1.7	91.97	2.04
5	92.41	92.26	0.16	90.47	-2.1
6	99.17	100.5	1.3	98.52	-0.66
7	101.63	102.4	0.76	99.48	-2.12

Table (4-4) Pseudo orthogonality check of the measured undamaged modes

Modes	50.98	52.15	57.51	90.13	92.41	99.17	101.63	106.17	109.91
50.98	1.000	0.0174	0.0009	0.0671	0.0070	0.0029	0.1419	0.1064	0.0027
52.15	0.0174	1.000	0.0075	0.0067	0.0115	0.1652	0.0894	0.2835	0.0069
57.51	0.0009	0.0075	1.000	0.0291	0.0075	0.2247	0.0385	0.1362	0.0677
90.13	0.0671	0.0067	0.0291	1.000	0.0708	0.1520	0.6615	0.2180	0.3593
92.41	0.0070	0.0115	0.0075	0.0708	1.000	0.8627	0.4969	0.8618	0.1383
99.17	0.0029	0.1652	0.2247	0.1520	0.8627	1.000	0.5558	0.8451	0.1388
101.63	0.1419	0.0894	0.0385	0.6615	0.4969	0.5558	1.000	0.5743	0.2133
106.17	0.1064	0.2835	0.1362	0.2180	0.8618	0.8451	0.5743	1.000	0.0512
109.91	0.0027	0.0069	0.0677	0.3593	0.1383	0.1388	0.2133	0.0521	1.000

Table (4-5) Cross-orthogonality check between the measured undamaged modes and their analytical counterparts from FEM (G1)

Modes	51.45	53.14	58.99	96.30	97.56
50.98	-0.9971	0.065	0.0048	0.0098	0.0051
52.15	-0.0575	-0.9972	-0.0134	0.0283	-0.1204
57.51	-0.0025	-0.0054	-0.9989	0.0036	0.0022
90.13	0.0201	-0.009	0.0074	-0.8755	0.4817
92.41	0.008	-0.0002	-0.0079	-0.4688	-0.8587

Table (4-6) Cross-orthogonality check between the measured undamaged modes and their analytical counterparts from FEM (GR4)

Modes	51.16	52.56	57.82	91.97	90.47	98.52	99.48
50.95	-0.9953	0.0589	0.0137	0.021	0.0353	0.0036	0.0238
52.15	0.0407	0.9966	-0.0022	-0.0148	0.0006	-0.1278	-0.0138
57.51	0.0087	0.0086	0.9955	0.0172	-0.0196	0	-0.018
90.13	-0.0302	-0.0129	-0.047	-0.8546	0.3296	0.0552	-0.0082
92.41	0.0452	-0.0224	-0.0035	-0.2837	-0.9323	0.0121	-0.1897
99.17	0.0083	0.0325	0.0592	-0.4151	0.0624	-0.9695	0.0132
101.63	0.0152	-0.0113	-0.0254	-0.0646	-0.0796	-0.1609	-0.9792

Figure (4-1) Stress-strain relationship for selected point along the length of the aluminium tube

Figure (4-2) Finite element model of the individual elements

Figure (4-3) Experimental and analytical deflections at different points along the tube for different load levels for element (1000x22).

Figure (4-4) Experimental and analytical deflections at different points along the tube for different load levels for element (1000x30)

Figure (4-5) Experimental and analytical deflections at different points along the tube for different load levels for element (707x22)

Figure (4-6) Experimental and analytical deflections at different points along the tube for different load levels for element (707x30)

Figure (4-7) Joint subassembly for the axial stiffness test

Figure (4-8) Degrees of freedom associated with each node of the 3-D beam element

Figure (4-9) Finite elements connectivity of the FEM (G3) and (G4)

Figure (4-10) Convergence of the analytical frequencies of FEM (G4) compared to the measured ones with respect to the updated parameter $(EI)_{node}$

Chapter 5

COMPUTER SIMULATION AND SENSITIVITY STUDIES

5.1 General

In order to investigate the feasibility of the integrated VBDI approach, for which the theoretical background was presented in Chapter 2, a series of computer simulation and sensitivity studies were performed. The approach is used to identify different levels of damage at single and at multiple sites. The ability of the approach to detect damage in the presence of simulated random errors is tested through comparisons with cases without errors. Samples of results of the computer simulation studies, which demonstrate the feasibility of different steps of the proposed approach, are presented in this chapter.

5.2 FEM used for Simulation

In this study, the numerical simulation has been applied to a simplified FEM of the eight-bay space frame used for the experimental validation of the proposed approach. The full description of the space frame is presented in Section 3.2 and the physical properties of its components are listed in Table (3-1). The FEM used for the purpose of computer simulation is based on nominal values of the physical properties of the components of the frame. The elements are modelled using a 3D beam element, which has six translational and six rotational DOFs. The joints are modelled as being rigid. The masses of the spherical nodes, the difference between the actual mass of the element and that derived on the basis of uniform mass density, and the mass of the instrumentation attached to each node are accounted for by attaching equivalent lumped masses at the joints. To simulate the effect of payload, added lumped masses, each having mass of 1.75 kg are added to nodes 4, 9, 25, 28 and 36 while masses of 2.75 kg are added at nodes 6, 17 and 30. For simplicity, rotational inertia effects of the eight lumped masses are ignored and the masses are modelled as point masses. The FEM connectivity is shown in Figure (4-9). Table (5-1) provides a list of numbers assigned to the elements and specifies their connectivity. The model has a total of 216 DOFs. For simplicity, the mass of the elements is accounted for by using a lumped mass formulation. A MATLAB code developed for the purpose is used to generate the element matrices, construct the FEM through assembly of elements and solve the eigenvalue problem. The solution of the eigenvalue problem is verified by comparing it with similar results obtained from COSMOS, FE software (COSMOS: user guide 1995). Table (5-2) lists the first eight non-

rigid body modes. In computer simulation presented in this chapter, the first six non-rigid body modes are considered as being of interest.

5.3 Strain Energy Distribution

Many researchers have used strain energy distribution as an important indicator in damage identification algorithms. Among those researchers are Carrasco et al (1997) and Hajela and Soiero (1990). It is presumed that modal data will contain sufficient information to identify damage only if the damaged member contributes a significant part to the total strain energy in the modes being included in the damage identification procedures. A detailed study of the strain energy distribution is therefore carried out to provide an indication of the relative participation of member j in mode i . The strain energy contribution of member j in the i^{th} mode is calculated as

$$SE_{ji} = \frac{\{\Phi\}_i^T [K_j] \{\Phi\}_i}{\sum_{j=1}^n \{\Phi\}_i^T [K_j] \{\Phi\}_i} \times 100\% \quad (5.1)$$

where: $[K_j]$ is the stiffness matrix of the j^{th} element, and n is the number of elements in the frame. The contribution of each element to each of the first lower six modes is shown in Figures (5-1) to (5-6). Only members that contribute above 1 % are considered as having significant participation in the strain energy content of that mode. On examining Figures (5-1) to (5-6) it is observed that for each mode, the number of members that contribute significantly to the total strain energy is different. Higher modes include

participation of a larger number of elements, but may be more difficult to identify from modal testing. Only 56 of the 109 elements contribute more than 1 % to the strain energy of any of the first six non-rigid body modes. In this study such elements are referred to as detectable elements. The elements that contribute less than 1 % are: vertical elements numbering (1 to 18), chord elements (19 to 22 and 47 to 50), cross-chord elements (51 to 68) and cross diagonal elements (85, 88, 91, 94, 97, 100, 103, 106 and 109). It will be difficult to identify any level of damage in these elements by using only the first six modes, since the elements have an insignificant effect on the modal parameters of these modes. In this study, such elements are called undetectable elements.

5.4 Effect of Damage on Natural Frequencies

It is evident that if identification of damage location is to be accomplished through the use of modal parameters measured in a modal test on the structure, the damage must affect the response of the structure and the effect must lead to an observable and measurable change in the modal parameters. An examination of the strain energy distribution described in the preceding section indicates that 53 of the 109 space frame elements make insignificant contribution to the strain energy of the target modes. Accordingly, if any of these elements is damaged, it will be difficult to identify such damage using the targeted lower modes. To verify the conclusions reached on the basis of an examination of the strain energy distribution, a severe damage, defined as being an 80% decrease in the element stiffness matrix, is introduced in each of the undetectable 53 elements, one at a time, and the changes in the natural frequencies of the first six modes are calculated. Sample results are shown in Table (5-3) in which ω_{ud} is the frequency of

the healthy structure and ω_d the corresponding frequency of the damaged structure. The results show that the change in natural frequencies due to a simulated severe damage in any of the 53 elements is insignificant and cannot be distinguished from modelling and measuring errors. These results confirm the predictions made earlier on the basis of strain energy distribution. For comparison, the same damage severity is applied to 56 detectable elements, one at a time, and again the changes in the natural frequencies of the first six non-rigid body modes are calculated. A sample of results is presented in Table (5-4). This time the amount of change due to the same damage severity is observable and is likely to be significantly larger than modeling and measurement errors. Consequently damage in these elements can be identified if it is severe enough to produce measurable changes to the modal parameters of the targeted modes.

5.5 Selection of Identification Modes

Examination of the strain energy distribution can also help in the selection of the set of modes that should be included in the damage identification process. The theoretical background to the mode selection criterion that depends on strain energy content of the selected modes has been discussed in Section 2.5.2 and 2.6. That criterion is applied to the set of identified lower modes listed in Table (5-2). Table (5-5) lists the elements that contribute more than 1 % to the strain energy of each mode. It can be seen that on average 87 % of the strain energy of each mode is represented by the elements included in the lists. These elements are also the detectable elements referred to earlier. Accordingly the six lower modes are considered as a credible set of identification modes, and can be used to identify damage to any of the 56 detectable elements. It will be seen

that a majority of the detectable elements (51 elements) contribute significant strain energy to more than one of the first three modes. Thus these modes may be enough to identify damage in such elements.

5.6 Optimal Selection of Response Points

Different criteria for optimal selection of response points have been discussed in Section 2.7. These criteria were originally developed for use in modal identification, FEM correlation and FEM updating. Kammer (1991) used the *effective independence method* for sensor placement for the purpose of on-orbit modal identification and FEM - measurement correlation. The theoretical background to this method is described in Section 2.7. This method proceeds in an iterative manner, in which locations that do not contribute significantly to the independence of the targeted modes are removed. Based on studies reported in the literature, this method is considered as the most reliable method for selecting sensors placement. However, this method like other methods has not been used to provide the sensor location scheme for damage identification. In practical damage identification, DOFs that affect the strain energy contribution should also be retained. Thus a DOF can be removed only if its removal matches the criterion of the effective independence and at the same time if it makes minimal contribution to the strain energy. In the present work this is achieved by adding an additional constraint on the procedures used in the effective independence method. In each iteration the strain energy contribution of all the detectable elements is checked and only those DOFs that do not significantly affect the strain energy contribution to any of the 56 detectable elements are removed. This additional condition allows a reduction in the DOF for response

measurement and at the same time ensures that the removal of these DOFs would not affect the formulation of the modal database required for identification of damage in any of the detectable elements. A MATLAB code has been developed to implement these criteria. In the first step the change in the strain energy contribution of the detectable elements caused by the removal of all of the 108 rotational DOFs is checked, the effect is found to be insignificant. In the second step, the iterative procedures described earlier are applied to the translational DOFs. Results show that 79 translation DOFs have to be measured to formulate an adequate modal database for identification of damage in any of the detectable elements. The number of selected DOFs for measurements represents 36 % of the total DOFs. Table (5-6) lists the selected 79 DOFs.

5.7 Damage Identification

To test the feasibility of the integrated damage identification approach described in Chapter 2, different damage cases were considered in the computer simulation. Table (5-7) illustrates the DOFs numbers corresponding to each node and Table (5-8) lists the simulated damage cases, which include single site and multiple site damage cases in which up to four elements are damaged at the same time and also included detectable and undetectable elements. A MATLAB code has been developed to act as computer simulator for the implementation of the different procedures of the damage identification approach. Figure (5-7) illustrates the flow chart of the code including all subroutines. The main steps are described in the following subsections.

5.7.1 Identification of Damage Without Simulated Errors

The VBDI approach is first tested assuming that there are no errors due to modelling or measurements. This assumption is made to study two main aspects of the proposed approach: (1) the ability of the selected response points (79 DOFs) to provide an adequate modal database for the damage identification, (2) the influence of including two different sets of identification modes and verifying that the inclusion of more modes is not always beneficial. The main steps in the procedure can be summarized as follows:

1. Generate the modal parameters (frequencies and mode shapes) of the undamaged space frame through the solution of the eigenvalue problem formed from the model matrices described in Section 5.2. The frequencies of the first eight modes are listed in Table (5-2). These modal data are considered as the baseline measured data of the undamaged structure.
2. Generate the modal parameters of the damaged structure by reducing the element stiffness matrix by the specified amount for each damaged element as listed in Table (5-8). It should be noted that only 79 DOFs that correspond to the selected response points are used to describe the measured mode shapes of the damaged structure. These DOFs are also used in the simulation studies. Table (5- 6) lists the selected 79 DOFs.
3. Select the set of modes to be used in damage identification. Two sets of modes are tested for damage identification. In the first set only the first three lower modes are included, in the second set the first six lower modes are included. Both sets

have been selected based on the strain energy content of the undamaged modes as mentioned in Section 5.5.

4. Reduce the undamaged mode shapes to the number of measured points (79 DOFs) or expand the number of the measured points to the size of the full model (216 DOFs). The SEREP technique is used in this step. This technique has been selected based on the theoretical background on the relative performance of the model reduction / expansion techniques presented in Section 2.3.2 and detailed in Section 4.5.1. The SEREP process retains all of the dynamics of the full model corresponding to the modes used; no error is therefore involved in the reduction process.
5. Pair the damaged and undamaged set of identification modes using cross-orthogonality check, in which the modes that have a correlation factor close to one are paired.
6. Formulate the non-linear optimization problem to solve for damage location and severity using all the elements of the structure as design variables. The objective function and the linearity and inequality constraints are formulated based on Equation (2.24). This formulation guarantees the globalization of the solution. This step helps in evaluating the performance of optimal matrix update in detecting different possible damage scenarios considering all elements of the structure as candidates for damage identification.
7. Identify the damaged region using modal residual approach described in Section 2.5.2 and represented by Equation (2.26). The regions within the structure that are potentially damaged correspond to the DOFs that have large magnitudes in the

modal residual vector. The identified damaged DOFs are used to locate a set of possible damaged elements that are connected to these DOFs.

8. Formulate the non-linear optimization problem to solve for damage location and severity using a limited number of design variables represented by the elements defined in the previous step. Again, the objective function and the linearity and inequality constraints are formulated based on Equation (2.24).

5.7.2 Identification of Damage with Simulated Random Errors in Modal Parameters

In a practical situation, the modal database is likely to be corrupted by modelling and measurement errors. To study the influence of such errors, the procedure outlined in Section 5.7.1 is repeated for all the damaged cases with simulated modelling and measurement errors applied to the modal parameters of the damaged structure. A vector of random errors varying between (0.5 % – 2 %) of the exact values is considered for the frequencies. For the mode shapes, a vector of random errors varying between (1 % – 10 %) of the original values of the mode shape elements is used. These random vectors are generated by using the RAND generator in MATLAB, which produces a set of pseudo-random numbers chosen from a uniform distribution between (0.0 and 1.0). Introduction of random errors in the modal parameters allows testing of the performance of VBDI approach in the presence of modelling and measurement errors.

5.8 Simulation Results and Discussion

The results of computer simulation are shown in Figure (5-8) through Figure (5-21). Figures that have labels, (a, b, c, d) present the results of the identification process in which all 109 elements are considered as possible candidates for damage. Figures with label (e) present values of the modal residual vector for each damaged case, derived using the first three modes with simulated random errors. Figures with label (f) present the results of the identification process in which the modal residual has been used in the first step to identify the potentially affected DOFs and elements that are connected to those DOFs are considered as possible candidates for damage in the formulation of the optimization problem.

The results presented in Figures (5-8) to (5-17) are used to assess the role of each step in the proposed approach to damage identification and the ability of the enhanced optimal matrix update to identify single and multiple damage scenarios. The computer simulation study presented in this chapter is also used to investigate the limitations of the VBDI methods that arise from the presence of modelling and measurement errors.

Identification of Damage in Undetectable Elements

The study of the strain energy distribution described in Section 5.3 showed that 53 of the 109 frame elements make insignificant contribution to the strain energy content of the first six modes. In Section 5.4, it was shown that frequency changes in the first six modes were insignificant even when severe damage was introduced in one of the

undetectable elements. In order to investigate the ability of the VBDI methods to identify damage in any of these elements, two damage cases (D1 and D2) are studied. In damage cases (D1) and (D2) severe damage is simulated by introducing 80 % reduction in the element stiffness matrix, assuming that there are no modelling or measurement errors. All the 108 translational DOFs are used as response points and the first six lower modes are used in the identification process. Figures (5-8) and (5-9) show the results of damage identification for the case in which damage is introduced in elements Nos. 4 and 100, respectively. It can be seen that, even though no measurement errors are supposed to exist and all 108 translational DOF are measured, it is not possible to identify damage in these elements. This confirms the prediction made earlier that VBDI would be unable to detect damage in those elements, which contribute little to the strain energy of modes included in the identification process.

Selection of Minimum Response Points for Damage Identification

In this study, a new approach was developed to determine the minimum number of response points that must be measured to obtain a modal database that is adequate for VBDI. This approach has been numerically implemented as described in Section 5.6 and resulted in the determination of a set of 79 translational DOFs as listed in Table (5-6). The computer simulation results shown in Figures (5-10) through (5-21) are based on incomplete mode shapes that have been defined only along the 79 translational DOFs. Results presented in Figures bearing labels (a, b and f) show that the selected set of DOFs can provide an adequate modal database for identifying two damage levels (50 % and

80%) for different single and multiple damage scenarios even in the presence of simulated random errors in the measured modal parameters of the damaged structure.

Selection of Mode Set Used in Identification

In Section 5.5, it was shown that most of the strain energy in the first six modes was contributed by the 56 detectable elements. However the set of elements out of the 56 contributing most to the strain energy was different for each mode. It was also shown that 51 of the 56 elements made substantial contribution to the strain energy content of the first three modes. In order to investigate the number of modes that should be used to identify damage in any of the detectable elements, two different sets of identification modes are used. Initially, it is assumed that no measurement errors are present, so that the focus is on a study of the influence of the number of modes in the identification process. The first set includes the first six lower modes. The results corresponding to this set are shown in Figures (5-10) through (5-21) and labeled (a). In the second set, only the first three modes are used for identification and the results obtained are as shown in Figures (5-10) through (5-21) labeled (b). On comparing the two sets of simulation results, it will be observed that the use of an increased number of identification modes is not always beneficial, since in most cases (case 4 is an exception) the use of only three modes provides more accurate damage identification than the use of six modes. This can be explained by the fact that most of the elements make strain energy contribution to the first three modes and at most one other mode out of the fourth, fifth and the sixth mode. The inclusion of additional modes that do not contain remarkable information about the

damaged elements adds constraint equations to the optimization problem and steers the solution of the optimization problem away from a global minimum.

The damage identification studies are repeated for all simulated damage cases but in the presence of random measurement and modelling errors. In the first instance all 109 elements are considered as being candidates for possible damage. The results are presented in Figures (5-10) through (5-21) carrying labels c and d. It is seen that in general VBDI is unable to predict damage in the presence of errors. The use of six rather than three modes offers no advantage. In fact, the results obtained with three modes may be considered as being better. In any case, it is evident that either the measurements have to be very precise, or further refinement must be made in the identification process. As shown in the following sections, identification of damaged regions using the modal residual improves the ability of VBDI procedures to identify damage.

Advantages of Damaged Region Identification

To improve the results of damage identification in the presence of measurement and modelling errors, a modal residual vector is calculated to identify the damaged region. Modal residual vectors for all the damage cases are calculated using the first three modes with simulated random errors in damaged modal parameters. Figures (5-10) through (5-21) carrying label (e) show the affected DOFs in each of the modal residual vectors. It can be seen that the higher values in the modal residual vector correspond to the DOFs connected to the damaged elements. The identification of the damaged region plays an important role in damage identification, especially for those damage cases,

which could not be identified by including all of the 109 elements in the optimization problem. This is proven by comparing figures with labels c and f for all of the damage cases. By isolating the damaged region it is possible to improve the ability of the VBDI to identify damage. However, in some cases the identification of damage severity is not precise. This can be attributed to the fact that damage has caused insignificant change in the modal parameters, which is easily overshadowed by the introduced random errors. Such is for example the case where damage is 50 % reduction in the stiffness of one element. However, if two or more elements with 50 % damage are connected to the same DOF all such elements can be detected, possibly with imprecise identification of damage severity, since the common DOF is affected by both damaged elements and the changes in modal parameters are measurable. Consider for example damage case D5L. Elements 34 and 95 are not connected to the same DOFs. Only element 95 is detected using region identification as can be seen from Figures (5-15) (c and f). In a similar manner in damage case D7L only element 34 is identified as seen from Figures (5-19) (c and f). For damage case DL6, the DOFs at nodes 17 and 23 are affected by two 50 % damaged elements, hence both damaged elements are identified when optimization is applied after isolation of the damaged region using modal residual. In the case of multiple damage sites such as in damage case D8L shown in Figure (5-21) labels (c and f), where four elements are damaged each with 50 % damage, the changes in modal parameters are relatively significant and measurable. However, even here all the damaged elements could not be identified without isolating the damaged region. When the damaged region was isolated using modal residual, the locations of the damaged elements are correctly identified even in the presence of measurement and modelling errors.

5.9 Summary

The following conclusions can be drawn on the basis of previous discussions and the different simulated damage cases presented in this chapter.

- (1) Study of the strain energy distribution gives a good indication about the elements that can be detected and those which cannot be. The strain energy distribution is also useful in identifying the set of modes that should be included in the identification process.
- (2) A novel technique for selecting a minimum number of response points is found to be successful. It allows the selection of a set of 79 DOFs out of a total of 216 DOFs along which the response needs to be measured. The set of selected DOFs was able to constitute an adequate modal database for damage identification purposes and provides information that allows the identification of single site and multiple site damage cases for two damage levels (50 % and 80 %).
- (3) A study of the appropriate number of identification modes has led to the conclusion that the use of an increased number of modes increases the number of equality constraints in the optimization problem and may in some cases reduce the accuracy of identification. In the frame being studied the detectable elements are found to make a significant contribution to the strain energy content of only some out of the first six identification modes. Consequently inclusion of all six modes increases the number of equality constraints in the optimization problem without supplying significant information about the damaged elements and consequently steers the optimizer away from the right solution.

- (4) A two-step identification approach is developed to first identify the damaged region using a modal residual vector and then solves an optimization problem with a reduced number of variables. This approach is found to be relatively successful in some of the damage cases, especially when 50 % damage is induced at single and multiple sites with up to four sites damaged at a time. Searching of all of the 109 structural elements in the optimization problem is not successful in identifying damage in the presence of measurement and modelling errors.
- (5) The approach outlined in this work is successful in the selection of modes to be included in the identification process, in the selection of a reduced set of DOFs, along which response measurement will be made, and in the selection of a combined FEM reduction technique. The location and severity of damage are correctly identified in almost all cases when measurements are assured to be accurate. With measurement errors of (0 to 2 %) in the frequencies, and (0 to 10%) in the mode shapes, identification of damage becomes difficult. Isolation of the damage region by using the modal residual helps in identifying the location of damage. When a reduced number of candidate elements are specified in the optimization on the basis of the results of modal residual, the assessment of the severity is improved in some cases, but not always.

Table (5-1) Space frame element numbers and connectivity

ELE No.	Nodes	ELE No.	Nodes	ELE No.	Nodes	ELE No.	Nodes	ELE No.	Nodes	ELE No.	Nodes
1	1-3	21	3-7	41	23-27	61	21-22	81	25-31	101	21-26
2	2-4	22	4-8	42	24-28	62	23-24	82	28-30	102	24-27
3	5-7	23	5-9	43	25-29	63	25-26	83	31-33	103	25-28
4	6-8	24	6-10	44	26-30	64	27-28	84	30-36	104	26-29
5	9-11	25	7-11	45	27-31	65	29-30	85	1-4	105	27-32
6	10-12	26	8-12	46	28-32	66	31-32	86	2-5	106	30-31
7	13-15	27	9-13	47	29-33	67	33-34	87	3-8	107	29-34
8	14-16	28	10-14	48	30-34	68	35-36	88	6-7	108	32-35
9	17-19	29	11-15	49	31-35	69	1-7	89	5-10	109	33-36
10	18-20	30	12-16	50	32-36	70	4-6	90	8-11		
11	21-23	31	13-17	51	1-2	71	7-9	91	9-12		
12	22-24	32	14-18	52	3-4	72	6-12	92	10-13		
13	25-27	33	15-19	53	5-6	73	9-15	93	11-16		
14	26-28	34	16-20	54	7-8	74	12-14	94	14-15		
15	29-31	35	17-21	55	9-10	75	15-17	95	13-18		
16	30-32	36	18-22	56	11-12	76	14-20	96	16-19		
17	33-35	37	19-23	57	13-14	77	17-23	97	17-20		
18	34-36	38	20-24	58	15-16	78	20-22	98	18-21		
19	1-5	39	21-25	59	17-18	79	23-25	99	19-24		
20	2-6	40	22-26	60	19-20	80	22-28	100	22-23		

Table (5-2) Description of the analytical modes used in computer simulation

Mode number	Frequency (Hz)	Identification
1	49.397	First torsion
2	52.960	First bending in Z - direction
3	58.098	First bending in Y - direction
4	96.789	Second torsion
5	100.52	Second bending in Z – direction
6	120.21	Second bending in Y- direction
7	131.44	Third bending in Z-direction
8	148.67	Third torsion

Table (5-3) Frequency changes caused by an 80 % decrease in element stiffness in a sample of the undetectable elements.

ω_{nd} (Hz)	Element 12		Element 16		Element 20		Element 48		Element 53		Element 61		Element 91		Element 103	
	ω_d (Hz)	% $\Delta\omega$	ω_d (Hz)	% $\Delta\omega$	ω_d (Hz)	% $\Delta\omega$	ω_d (Hz)	% $\Delta\omega$	ω_d (Hz)	% $\Delta\omega$	ω_d (Hz)	% $\Delta\omega$	ω_d (Hz)	% $\Delta\omega$	ω_d (Hz)	% $\Delta\omega$
49.397	49.39	0.01	49.39	0.01	49.36	0.06	49.36	0.07	49.21	0.37	49.38	0.02	49.39	0.01	49.38	0.02
52.960	52.96	0.00	52.96	0.00	52.77	0.36	52.93	0.05	52.82	0.27	52.93	0.06	52.96	0.00	52.96	0.00
58.098	58.06	0.06	58.09	0.02	58.09	0.00	57.84	0.44	58.09	0.00	58.09	0.00	58.09	0.01	58.09	0.00
96.789	96.71	0.08	96.77	0.02	96.41	0.39	96.28	0.53	95.65	1.17	96.74	0.05	96.78	0.00	96.78	0.00
100.52	100.48	0.03	100.51	0.00	99.75	0.76	100.5	0.01	100.3	0.21	100.21	0.31	100.32	0.19	100.51	0.01
120.21	119.92	0.24	120.15	0.05	120.20	0.01	118.47	1.45	120.1	0.05	120.21	0.00	120.13	0.06	120.21	0.01

Table (5-4) Frequency changes caused by an 80 % decrease in element stiffness in a sample of the detectable elements.

ω_{nd} (Hz)	Element 29		Element 32		Element 39		Element 42		Element 74		Element 87		Element 90		Element 95	
	ω_d (Hz)	% $\Delta\omega$	ω_d (Hz)	% $\Delta\omega$	ω_d (Hz)	% $\Delta\omega$	ω_d (Hz)	% $\Delta\omega$	ω_d (Hz)	% $\Delta\omega$	ω_d (Hz)	% $\Delta\omega$	ω_d (Hz)	% $\Delta\omega$	ω_d (Hz)	% $\Delta\omega$
49.397	46.80	5.25	46.11	6.66	497.79	3.26	47.69	3.46	47.37	4.09	47.94	2.95	46.67	5.51	45.11	8.68
52.960	51.36	3.03	50.89	3.92	50.78	4.12	50.54	4.57	52.92	0.08	52.19	1.46	51.89	2.01	52.64	0.61
58.098	56.83	2.18	55.39	4.65	56.09	3.46	55.96	3.67	57.28	1.41	58.07	0.05	58.06	0.06	58.09	0.01
96.789	96.54	0.25	96.65	0.14	91.30	5.67	94.56	2.30	94.01	2.87	88.79	8.25	92.13	4.81	90.82	6.17
100.52	98.38	2.11	98.82	1.69	99.94	0.58	96.89	3.60	100.27	0.24	97.77	2.72	98.54	1.96	97.19	3.31
120.21	114.24	4.96	120.13	0.07	117.71	2.09	117.92	1.91	118.42	1.49	119.38	0.69	120.14	0.06	120.19	0.02

* ω_{nd} and ω_d are the calculated undamaged and damaged natural frequencies respectively and % $\Delta\omega$ is the percentage change in the natural frequencies

Table (5-5) Percentage strain energy contribution of each element to each of the six lower modes

Mode (1)				Mode (2)				Mode (3)				Mode (4)				Mode (5)				Mode (6)			
EL	SE	EL	SE	EL	SE	EL	SE	EL	SE	EL	SE	EL	SE	EL	SE	EL	SE	EL	SE	EL	SE		
26	0.92	80	2.38	23	1.87	42	2.75	24	2.57	40	3.22	23	1.19	79	2.42	23	1.48	64	1.61	22	2.33		
29	1.75	82	1.16	26	1.72	43	1.72	25	1.84	41	2.96	33	2.12	80	1.10	24	1.72	79	0.92	23	1.19		
31	1.10	87	1.46	27	2.05	46	2.15	27	4.05	42	4.38	39	3.69	81	2.47	26	3.44	86	1.58	24	4.86		
32	1.52	89	2.20	28	3.71	86	0.92	28	3.07	44	3.07	41	0.92	82	2.63	27	4.45	87	5.40	25	2.98		
34	1.12	90	2.77	29	2.74	87	1.26	29	2.53	45	1.88	44	3.19	87	1.46	29	1.27	90	3.07	26	1.72		
35	1.43	92	4.80	30	2.76	89	3.52	30	4.06	71	1.42	46	3.36	89	6.13	30	1.81	92	3.07	27	1.50		
37	1.17	93	3.93	31	6.53	90	2.49	31	3.68	72	1.77	49	1.36	90	1.66	32	1.47	95	6.62	28	4.51		
40	1.13	95	5.98	32	2.77	92	2.66	32	6.90	73	1.16	59	1.98	92	3.64	37	2.30	96	5.56	29	3.62		
41	1.06	96	4.68	33	3.26	93	1.60	33	7.11	74	1.06	68	1.87	93	2.09	39	1.04	98	2.78	30	1.77		
72	1.53	98	6.30	34	5.91	95	1.00	34	3.91	80	1.08	71	2.25	96	1.35	40	1.42	99	8.63	31	1.49		
73	2.50	99	4.50	35	5.89	101	1.75	35	4.04	81	1.28	72	2.07	98	5.2	42	4.15	102	4.92	34	2.29		
74	2.30	101	5.27	36	3.28	102	1.95	36	7.72	82	2.25	73	2.64	101	7.37	43	3.39	104	5.26	35	1.25		
75	3.2	102	3.56	37	3.02	104	2.96	37	6.88	84	0.98	74	1.51	104	2.35	46	1.32	107	2.87	38	2.20		
76	2.67	104	2.77	38	6.48	105	3.35	38	3.85			75	1.34	105	5.58	52	1.11	108	1.02	39	1.84		
77	3.41	105	1.75	39	2.48	108	1.78	39	3.87			77	0.98	108	7.32	63	1.46			40	5.76		
78	2.72	108	1.04	40	3.43															41	3.19		
79	2.73			41	3.29																		
SE _{Total} = 86.81 %				SE _{Total} = 93.05 %				SE _{Total} = 89.1 %				SE _{Total} = 83.24 %				SE _{Total} = 85.14 %				SE _{Total} = 83.92 %			

* EL is the element number and SE is the percentage strain energy contribution of each element.

Table (5-6) Selected 79 measured DOFs to identify damage in the 56 detectable elements

Measured X-DOF at nodes			Measured Y-DOF at nodes			Measured Z-DOF at nodes		
3	15	26	6	17	29	2	13	24
5	16	27	7	20	30	3	14	25
6	17	28	9	22	31	4	15	26
7	18	29	12	23	35	5	16	27
8	19	30	14	25		6	17	28
9	20	31	15	28		7	18	29
10	21	32				8	19	30
11	22	33				9	20	31
12	23					10	21	32
13	24					11	22	33
14	25					12	23	34

Table (5-7) Correspondences between DOF's numbering and node's numbering

Node No	Corresponding DOFs numbers	Node No	Corresponding DOFs numbers	Node No	Corresponding DOFs numbers
1	1-6	13	73-78	25	145-150
2	7-12	14	79-84	26	151-156
3	13-18	15	85-90	27	157-162
4	19-24	16	91-96	28	163-168
5	25-30	17	97-102	29	169-174
6	31-36	18	103-108	30	175-180
7	37-42	19	109-114	31	181-186
8	43-48	20	115-120	32	187-192
9	49-54	21	121-126	33	193-198
10	55-60	22	127-132	34	199-204
11	61-66	23	133-138	35	205-210
12	67-72	24	139-144	36	211-216

Table (5-8) Simulated damage cases

Damage Case ID	Damaged Elements	Elements Connectivity	Damage Severity %
D1	4	6-8	80
D2	100	22-23	80
D3S	38	20-24	80
D3L	38	Same as above	50
D4S	74	12-14	80
D4L	74	Same as above	50
D5S	34	16-20	80
	95	13-18	80
D5L	34 & 95	Same as above	50
D6S	35	17-21	80
	37	19-23	80
	77	17-23	80
D6L	35, 37 & 77	Same as above	50
D7S	33	15-19	80
	34	16-20	80
D7L	33 & 34	Same as above	50
D8S	31	13-17	80
	32	14-18	80
	34	16-20	80
	35	17-21	80
D8L	31, 32, 34 & 35	Same as above	50

Figure (5-1) Contributions of the elements of space frame to the strain energy in the first mode (49.397 Hz)

Figure (5-2) Contributions of the elements of space frame to the strain energy in the second mode (52.96 Hz)

Figure (5-3) Contributions of the elements of space frame to the strain energy in the third mode (58.09 Hz)

Figure (5-4) Contributions of the elements of space frame to the strain energy in the fourth mode (96.79 Hz)

Figure (5-5) Contributions of the elements of space frame to the strain energy in the fifth mode (100.52 Hz)

Figure (5-6) Contributions of the elements of space frame to the strain energy in the sixth mode (120.21 Hz)

Figure (5-7) Computer simulation flow chart

Figure (5-8) Damage identification for case D1 without simulated measurement errors and using 6 modes

Figure (5-9) Damage identification for case D2 without simulated measurement errors and using 6 modes

Figure (5-10-a) Damage identification for case (D3S) without errors using 6 modes

Figure (5-10-b) Damage identification for (D3S) without errors using 3 modes

Figure (5-10-c) Damage identification for (D3S) with errors using 3 modes

Figure (5-10-d) Damage identification for (D3S) with errors using 6 modes

Figure (5-10-e) Modal residual vector using 3 modes with errors for case (D3S)

Figure (5-10-f) Damage Identification of case (D3S) based on region identification using modal residual with errors

Figure (5-11-a) Damage identification for case (D3L) without errors using 6 modes

Figure (5-11-b) Damage identification for case (D3L) without errors for 3 modes

Figure (5-11-c) Damage identification for (D3L) with errors using 3 modes

Figure (5-11-d) Damage identification for case (D3L) with errors using 6 modes

Figure (5-11-e) Modal residual vector using 3 modes with errors for case (D3L)

Figure (5-11-f) Damage identification of case (D3L) based on region identification using modal residual with errors

Figure (5-12-a) Damage identification for case (D4S) without errors using 6 modes

Figure (5-12-b) Damage identification for cases (D4S) without errors using 3 modes

Figure (5-12-c) Damage identification for case (D4S) with errors using 3 modes

Figure (5-12-d) Damage identification for case (D4S) with errors using 6 modes

Figure (5-12-e) Modal residual vector using 3 modes with errors for case (D4S)

Figure (5-12-f) Damage identification of case (D4S) based on region identification using modal residual with errors

Figures (5-13-a) Damage identification for case (D4L) without errors using 6 modes

Figure (5-13-b) Damage identification for case (D4L) without errors using 3 modes

Figure (5-13-c) Damage identification for case (D4L) with errors using 3 modes

Figure (5-13-d) Damage identification for case (D4L) with errors using 6 modes

Figure (5-13-e) Modal residual vector using 3 modes with errors for case (D4L)

Figure (5-13-f) Damage identification of case (D4L) based on region identification using modal residual with errors

Figure (5-14-a) Damage identification for case (D5S) without errors using 6 modes

Figure (5-14-b) Damage identification for case (D5S) without errors using 3 modes

Figure (5-14-c) Damage identification for case (D5S) with errors using 3 modes

Figure (5-14-d) Damage identification for case (D5S) with errors using 6 modes

Figure (5-14-e) Modal residual vector using 3 modes with errors for case (D5S)

Figure (5-14-f) Damage identification of case (D5S) based on region identification using modal residual with errors

Figure (5-15-a) Damage identification for case (D5L) without errors using 6 modes

Figure (5-15-b) Damage identification for case (D5L) without errors using 3 modes

Figure (5-15-c) Damage identification for case (D5L) with errors using 3 modes

Figure (5-15-d) Damage identification for case (D5L) with errors using 6 modes

Figure (5-15-e) Modal residual vector using 3 modes with errors for case (D5L)

Figure (5-15-f) Damage identification of case (D5L) based on region identification using modal residual with errors

Figure (5-16-a) Damage identification for case (D6S) without errors using 6 modes

Figure (5-16-b) Damage identification for case (D6S) without errors using 3 modes

Figure (5-16-c) Damage identification for case (D6S) with errors using 3 modes

Figure (5-16-d) Damage identification for case (D6S) with errors using 6 modes

Figure (5-16-e) Modal residual vector using 3 modes with errors for case (D6S)

Figure (5-16-f) Damage identification for case (D6S) based on region identification using modal residual with errors

Figure (5-17-a) Damage identification for case (D6L) without errors using 6 modes

Figure (5-17-b) Damage identification for case (D6L) without errors using 3 modes

Figure (5-17-c) Damage identification for case (D6L) with errors using 3 modes

Figure (5-17-d) Damage identification for case (D6L) with errors using 6 modes

Figure (5-17-e) Modal residual vector using 3 modes with errors for case (D6L)

Figure (5-17-f) Damage identification of case (D6L) based on region identification using modal residual with errors

Figure (5-18-a) Damage identification for case (D7S) without errors using 6 modes

Figure (5-18-b) Damage identification for case (D7S) without errors using 3 modes

Figure (5-18-c) Damage identification for case (D7S) with errors using 3 modes

Figure (5-18-d) Damage identification for case (D7S) with errors using 6 modes

Figure (5-18-e) Modal residual vector using 3 modes with errors for case (D7S)

Figure (5-18-f) Damage identification of case (D7S) based on region identification using modal residual with errors

Figure (5-19-a) Damage identification for case (D7L) without errors using 6 modes

Figure (5-19-b) Damage identification for case (D7L) without errors using 3 modes

Figure (5-19-c) Damage identification for (D7L) with errors using 3 modes

Figure (5-19-d) Damage identification for case (D7L) with errors using 6 modes

Figure (5-19-e) Modal residual vector using 3 modes with errors for case (D7L)

Figure (5-19-f) Damage identification of case (D7L) based on region identification using modal residual

Figure (5-20-a) Damage identification for case (D8S) without errors using 6 modes

Figure (5-20-b) Damage identification for case (D8S) without errors using 3 modes

Figure (5-20-c) Damage identification for case (D8S) with errors using 3 modes

Figure (5-20-d) Damage identification for case (D8S) with errors using 6 modes

Figure (5-20-e) Modal residual vector using 3 modes with errors for case (D8S)

Figure (5-20-f) Damage identification of case (D8S) based on region identification using modal residual with errors

Figure (5-21-a) Damage identification for case (D8L) without errors using 6 modes

Figure (5-21-b) Damage identification for case (D8L) without errors using 3 modes

Figure (5-21-c) Damage identification for case (D8L) with errors using 3 modes

Figure (5-21-d) Damage identification for case (D8L) with errors using 6 modes

Figure (5-21-e) Modal residual vector using 3 modes for case (D8L)

Figure (5-21-f) Damage identification for case (D8L) based on region identification using modal residual with errors

Chapter 6

Experimental Verification of the Damage Identification Approach

6.1 General

For experimental verification of the proposed VBDI approach specified levels of damage were introduced in the eight-bay space frame described in Section 3.2. Modal tests were then carried out to determine the vibration characteristics of the damaged structure. The damage scenarios investigated included single and multiple damage sites, severe damage scenarios, light damage scenarios and mixed damage scenarios. The scenarios are grouped according to the amount of changes introduced in the damaged elements. In the present chapter, details of the modal testing are presented for each damage case. In order to formulate the modal database needed for different steps in the VBDI approach, the measured modal parameters related to each damage case need to be correlated to the measured modal parameters of the undamaged space frame, so as to pair the corresponding modes. Details of such correlation are presented. The computer modules developed to implement the different procedures of the proposed VBDI approach are described. The results of the application of the proposed VBDI approach to the identification of damage are presented, and the level of success achieved in predicting damage is discussed.

6.2 Predetermined Damage Simulation

As stated earlier, in the space frame assembled from Mero components it is relatively easy to replace any single component without disassembly of the whole frame, which is very useful in simulating damage in any of the frame struts. One possible means of achieving this is to replace a candidate member consisting of a (30×1.5 mm) tube by a (22×1.0 mm) tube, which represents a reduction of about 50% in the cross-sectional area. Another alternative is to replace any of the tubes with another one that has a complete saw cut to simulate 100% damage. Damage identification algorithms used in the study assume that any change in the modal parameters is caused solely by a reduction in the stiffness of one or more elements keeping the structural masses unchanged. In order to ensure this, two lumped masses were glued to the damaged tube ends to compensate for any difference in mass that may have been caused by replacement of a tube. The compensating masses are shown in Figure (6-1). It was suspected that the process of disassembly and assembly might introduce other uncertain changes in the structure. To ascertain that this was unlikely, a modal test of the undamaged frame was repeated after disassembling one of the frame bays completely and assembling it again to evaluate the modal parameter sensitivity. Figure (6-2) shows a comparison of a sample FRF of one of the driving points (2-Y) before and after disassembly. It can be seen that the peaks and amplitudes have not change in the frequency range of interest. The only significant difference appears at the anti-resonance between some modes. This test was important to assure that any measured discrepancy in the modal parameters later on could be attributed completely to the simulated damage.

6.3 Modal Testing and Analysis of Damaged Structure

The modal testing procedures developed and described in Chapter 3 were followed in carrying out the modal tests on a damaged structure. The excitations were applied at DOFs (2-Y) and (33-Z) and measurements of response made at all 108 translational DOFs as stated earlier. At the end of each modal test on the damaged structure, the FRFs and coherence functions were available. As described earlier in Section 3.5, one step in the modal tests was to verify the test data. For each modal test, the reciprocity, FRFs quality, and the coherence functions were used to check the quality of the measured modal parameters. Plots showing the reciprocity check and coherence values in the frequency range of interest are included in this chapter. Also, two samples of FRF plots are shown for each damage case. The first FRF plot is for one of the driving points and the other for a DOF connected to one of the damaged elements. Examination of the FRF samples confirms the quality of the acquired modal parameters and illustrates the effect of introducing damage evidenced by the frequency shifts and changes in the amplitudes.

The computer simulation study presented in Chapter 5 had shown that the measurements needed to be made along a minimum of 79 response points. However, in estimating the frequencies and mode shapes all 108 acquired measurements were used. The LMS CADA-X modal analysis software offers several types of output files, in which the estimated modal parameters are stored in a universal file format. A computer code was developed and implemented in LABVIEW software to transfer the modal parameters of each damaged case from the universal file format to a MATLAB file that contained a

vector of measured frequencies and a matrix of measured mode shapes. This was necessary because all of the computer codes developed for the VBDI approach are MATLAB codes. In some cases, the order of modes for the damaged structure was not the same as in the undamaged structure, due to the introduced damage and the resulting degradation in the global stiffness matrix. In order to pair the corresponding mode shapes of the measured undamaged and damaged structure, the mode shapes were first mass normalized using the mass matrix developed for the FEM (G4), then a Pseudo orthogonality check (PORTH) was applied to correlate the damaged mode shapes to their undamaged counterparts.

6.4 Computer Code Developed for the Validation of the VBDI Approach

A MATLAB code that was developed and used for the computer simulation studies to investigate the feasibility of the proposed VBDI approach has been described in Chapter 5. The code used analytical modal parameters of the undamaged and damaged structures respectively, calculated from an approximate FEM. In the results presented in this chapter the modal parameters of the undamaged and damaged structure are obtained from response measurements during modal tests, followed by a modal analysis. A FEM has been constructed and updated to closely match the measured modal parameters of the undamaged structure, so that it can be used as a baseline model in the formulation of the optimization problem. Details of FEM updating have been presented in Chapter 4. The MATLAB code used in Chapter 5 was modified to combine the corresponding experimental and analytical modal parameters and to implement the different steps in the

proposed VBDI approach that uses experimental modal parameters. Figure (6-3) illustrates the flow chart of the main subroutines that constitute the code.

6.5 Damage Scenarios

Based on the results of computer simulation studies presented in Chapter 5, different damage scenarios were experimentally simulated to verify the reliability of the proposed VBDI approach. It is evident from the strain energy distribution presented earlier in Section 5.3, that it would be difficult to identify damage to any of the 53 undetectable elements. Therefore the damage cases studied included elements only from the list of 56 detectable elements. As stated in Section 6.2, two different damage severities were studied: 50 % and 100 % decrease in the elemental cross-sectional area. Three groups of damage scenarios were tested. These groups were named severe damage group (GR1), mixed damage group (GR2), and light damage group (GR3). To verify the results of the criteria developed in this work for selection of the measured DOFs that would constitute an adequate description of the modes for damage identification and used in the computer simulation in Chapter 5, two different sets of measured DOFs were used. The first set included all of the 108 measured translational DOFs (Amin et al (2002)), while the second set included only the selected 79 DOFs listed in Table (5-6). The set of modes used in the damage identification process included the first three lower modes based on the conclusions reached in the computer simulation studies. The effect of isolating the damaged region on a possible improvement in damage detection was also studied. The different steps in the proposed VBDI approach shown in Figure (6-3) were applied for the identification of damage in the damage cases studied.

6.5.1 Severe Damage Scenarios in Group (GR1)

In the first group (GR1), named severe damage, four different damage scenarios were studied. In each case selected elements were completely damaged by replacing an element tube with another one that had a complete saw cut to simulate 100% damage in that tube. Table (6-1) lists the simulated damage scenarios in this group. Figure (6-4) shows damage scenario EDS3, in which elements 38 and 102 have been damaged by means of a saw cut.

Figure (6-5) and Table (6-2) present some results from modal tests on severely damaged structure. Figure (6-5) shows an almost perfect reciprocity in the frequency range of interest between the driving point responses in damage scenario EDS1. It will be observed that the acquired FRFs are of good quality as seen from the coherence values presented in the same figure for the modes in the frequency range of interest. The quality of the measured modal data obtained in the other tests for this group was as good as the modal data for damage scenario EDS1. Table (6-2) compares the measured frequencies of the first six lower modes for the damage scenarios of group (GR1) with the corresponding measured modes of the undamaged structure. It can be seen that the percentage differences in frequencies are quite significant.

Damage Scenario EDS1

This damage scenario is a sample of a single damaged site in which damage is in one of the chord elements of the eight-bay space frame. Figures (6-6) and (6-7) show a comparison between the undamaged and damaged FRFs at driving point (2-Y) and at node (24-Z), respectively, due to excitation at (33-Z). It can be seen that the damage has

most affected the second and the third modes. This can be attributed to the fact that this element makes a significant contribution to the strain energy content of these modes, as seen from Table (5-5). The shift in the frequencies of these two modes is most significant, as can be seen from Table (6-2). Table (6-3-a) and (6-3-b) show the results of PORTH and XORTH orthogonality checks that were used to pair the measured undamaged and condensed analytical modes respectively with measured damaged modes.

Damage Scenario EDS2

This damage scenario is a sample of single damaged site, in which damage is in one of the diagonal elements of the eight-bay space frame. Figures (6-8) and (6-9) show a comparison between the undamaged and damaged FRFs at driving point (2-Y) and at node (22-Z), respectively, due to excitation at (33-Z). It can be seen that the damage has most affected the first mode. This can be attributed to the fact that this element makes a significant contribution to the strain energy content of this mode, as seen from Table (5-5). The shift in the frequency of this mode is quite significant as can be seen from Table (6-2). Tables (6-4-a) and (6-4-b) show the results of PORTH and XORTH used to pair the measured undamaged and condensed analytical modes, respectively, with the measured damaged modes.

Damage Scenario EDS3

This damage scenario is a sample of multiple damage sites, in which a cross-diagonal element No. 102 and a chord element No. 38 of the eight-bay space frame are completely damaged. Figures (6-10) and (6-11) show a comparison between the

undamaged and damaged FRFs at driving point (2-Y) and at node (24-Y), respectively, due to excitation at (33-Z). It can be seen that the damage has most affected the first three modes. This can be attributed to the fact that the damaged elements make a significant contribution to the strain energy content of these modes, as seen from Table (5-5). Element No. 38 contributes to the second and third modes, while element No. 102 contributes to the first and second modes. The shift in the frequencies of these modes is quite significant as can be seen from Table (6-2). Tables (6-5-a) and (6-5-b) show the results of PORTH and XORTH used to pair the measured undamaged and condensed analytical modes, respectively, with the measured damaged modes.

Damage Scenario EDS4

This damage scenario is a sample of multiple damaged sites, in which two cross-diagonal elements Nos. 98 and 99 of the eight-bay space frame are completely damaged. Figures (6-12) and (6-13) show a comparison between the undamaged and damaged FRFs at driving point (2-Y) and at node (19-Z), respectively, due to excitation at (33-Z). It can be seen that the damage has most affected the first mode. This can be attributed to the fact that the damaged elements make a significant contribution to the strain energy content of this mode, as seen from Table (5-5). Element Nos. 98 and 102 both contribute to the first mode. The shift in the frequency of this mode is quite significant as can be seen from Table (6-2). Tables (6-6-a) and (6-6-b) show the results of PORTH and XORTH used to pair the measured undamaged and condensed analytical modes, respectively, with the measured damaged modes.

6.5.2 Results of Damage Identification for Damage Scenarios of Group (GR1)

Due to the significant changes in mode shapes caused by damage, the PORTH matrix elements are not ideal, since the absolute value of many of the diagonal terms is less than 1.0, and the off diagonal terms are more than 0.1. In two of the damage scenarios in this group (EDS3 and EDS4) only four modes can be correlated. The modelling errors and the significant changes in the measured mode shapes in each of the damage scenarios resulted in similar results for the XORTH, in which the measured damaged mode shapes are paired with their analytical counterparts calculated from an updated FEM.

As stated earlier, only the first three lower modes are used in the identification process. The results of damage identification for damage scenario EDS1, EDS2, EDS3 and EDS4 using 108 measured DOFs, with all of the 109 frame elements included in the optimization problem, are shown in Figures (6-14) through (6-17) with label (a). The results of damage identification using the selected 79 DOFs are shown in Figures (6-14) through (6-17) with label (b). Comparison of the identification results in figures with label (a) to those in figures with label (b) indicates that the use of 79 DOFs is as effective as that of the full 108 DOFs. Although the damaged region has not been isolated prior to carrying out the optimization, the proposed VBDI approach is able to successfully identify damage in all of the damaged scenarios in this group. This validates the different proposed criteria used in this work, including selection of the set of identification modes and the minimum number of measured DOFs. Although the modal residual method was successful in locating the potentially affected DOFs in the presence of random errors

throughout the computer simulation presented in Chapter 5, the same was not true for any of the damage scenarios in this group. This could be due to the errors introduced when the measured mode shapes of the damaged structure are combined with the condensed analytical model matrices for calculating the modal residual vector. This would also explain the imperfect correlation results seen in the PORTH and XORTH matrices.

6.5.3 Mixed Damage Scenarios of Group (GR2)

In the second group (GR2), named mixed damage, more than one element was damaged in each damage scenario but with different damage severity namely, 50 % and 100 % decrease in the element cross-sectional areas. Table (6-7) lists the damage scenarios in this group. The computer simulation in Chapter 5 showed that the proposed VBDI approach was not successful in clearly identifying elements that had been damaged by 50% in the presence of simulated random errors in the measured modal parameters. The introduced random errors were up to 2 % in frequencies and up to 10 % in mode shapes. In fact the calculated changes in frequencies due to 50 % decrease in the element cross sectional area, in single and multiple damage sites, were found to be less than the introduced random errors. The damage scenarios in group (GR2) were selected with a view to investigate the ability of the proposed VBDI approach to detect a relatively light damaged element (50 % decrease in the cross-sectional area) when such damage occurred in combination with 100 % damage in other elements. Combinations of different damage severity were not been studied in the computer simulation presented in Chapter 5. Figure (6-18) shows damage scenario EDM2, in which elements 42 and 78 have been damaged by means of a saw cut and element No. 38 has been damaged by replacing the 30x1.5

mm tube by a smaller tube 22x1.0 mm. Figure (6-19) and Table (6-3) show some results from modal tests. Figure (6-19) shows an almost perfect reciprocity between the responses of driving points in damage scenario EDM2. Also the acquired FRFs are of good quality as seen from the coherence values for the modes in the frequency range of interest shown in the same figure. The quality of the measured modal data of the other tests of this group was as good as the modal data for damage scenario EDS1. Table (6-8) compares the measured frequencies of the first six lower modes for the damage scenarios in group (GR2) with the corresponding measured modes of the undamaged structure. It can be seen that the percentage differences in frequencies are significant.

Damage Scenario EDM1

This scenario incorporates multiple damage sites, with two damaged chord elements as listed in Table (6-7). Figures (6-20) and (6-21) show a comparison between the undamaged and damaged FRFs at driving point (2-Y) and at node (22-Y), respectively, due to excitation at (33-Z). It can be seen that the damage has most affected the second mode and a major amount of change in the frequency of this mode is due to the 100 % damage in element No. 38. This can be confirmed by comparing the frequency change in damage scenario EDS1, in which the same element is damaged by 100 %. The 50 % damage of element No. 37 in the present scenario causes only small changes in the frequencies of the second and third modes. However, both elements make a significant contribution to the strain energy content of the second and third modes, as can be seen from Table (5-5). Tables (6-9-a) and (6-9-b) show the results of PORTH and XORTH

used to pair the measured undamaged and condensed analytical modes, respectively, with the measured damaged modes.

Damage Scenario EDM2

In this scenario three sites were damaged at the same time. A chord element and diagonal were damaged by 100 % and another chord element was damaged by 50 % as listed in Table (6-7). Figures (6-22) and (6-23) show a comparison between the undamaged and damaged FRFs at driving point (2-Y) and at node (28-Z), respectively, due to excitation at (33-Z). It can be seen that the damage has most affected the first four modes as shown in Table (6-8). This is to be expected since the three damaged elements make a significant contribution to the strain energy content of the set of identification modes as shown in Table (5-5). Tables (6-10-a) and (6-10-b) show the results of PORTH and XORTH used to pair the measured undamaged and condensed analytical modes, respectively, with measured damaged modes.

Damage Scenario EDM3

In this scenario four chord elements, all in the same bay, were damaged at the same time. A 50 % damage was introduced in three elements while the fourth element had 100 % damage, as shown in Table (6-7). Figures (6-24) and (6-25) show a comparison between the undamaged and damaged FRFs at driving point (2-Y) and at node (21-Y), respectively, due to excitation at (33-Z). It can be seen that the damage has most affected the first three modes as shown in Table (6-8). Each of the four damaged elements makes a significant contribution to the strain energy content of the first three

modes, as shown in Table (5-5). Tables (6-11-a) and (6-11-b) show the results of PORTH and XORTH used to pair the measured undamaged and condensed analytical modes, respectively, with measured damaged modes.

6.5.4 Results of Damage Identification for Damage Scenarios in Group (GR2)

In each damage scenario in this group, at least one element was damaged by 100%. As a result, the measured modal parameters experienced significant changes, as in the damage scenarios of group (GR1). The diagonal terms of the PORTH and XORTH matrices for the damage scenarios in this group were not ideal being different from 1.0. Also, the off-diagonal terms were nonzero. The absolute values of the correlation factors for the first three modes in the XORTH matrix were in the range of (0.5 to 0.9) for all of the damage scenarios in this group. The weak correlation can be attributed to modelling errors and to the significant changes in the measured mode shapes.

The results of damage identification for damage scenarios EDM1, EDM2 and EDM3 using 108 measured DOFs, and all of the 109 frame elements included in the optimization problem are shown in Figures (6-26) through (6-28) with label (a). The results of damage identification using the selected 79 DOFs are shown in Figures (6-26) through (6-28) with label (b). Comparison of the identification results presented in figures with label (a) with those in figures with label (b) indicates that the use of 79 DOFs is as effective as that of the full 108 DOFs. In damage scenario EDM1, the proposed VBDI approach is able to identify the 100 % damage in element No. 38 but the 50 % damage in element No. 37 is not identified. Despite the poor correlation of the first

three modes used in the identification in damage scenario EDM2, as can be seen from Table (6-10-b), the proposed VBDI approach is able to identify the 100 % damage in elements No. 42 and No. 78. However, the 50 % damage in element No. 38 is not identified. In damage scenario EDM3, the 100 % damage in element No. 38 and the 50 % damage in element Nos. 35 and 36 are clearly identified but the 50 % damage in element No. 37 is not identified.

In summary, the proposed VBDI approach is able to successfully identify damage in all the elements that have 100 % damage. However, it is not able to identify the element with 50 % damage in scenarios that include severely damaged elements along with lightly damaged elements as in EDM1 and EDM2. The change in modal parameters due to the 50 % damage in one element is insignificant when compared to the changes caused by even one severely damaged element in the same damage scenario. As a result, when lightly damaged members occur in combination with severely damaged elements, the formers are not identified. In damage scenario EDM3, the changes caused by 50 % damage in three elements is more significant, so that even in combinations with severely damaged element No. 38, two of the lightly damaged elements have been identified. Again, the modal residual method was unable to identify the potentially affected DOFs, presumably for reasons similar to those cited in Section 6.5.3.

6.5.5 Light Damage Scenarios of Group (GR3)

In the third group (GR3), named light damage, four different damage scenarios were included, in each of which the elements were partially damaged by replacing a

candidate member consisting of a (30×1.5 mm) tube by a (22×1.0 mm) tube, representing a reduction of about 50% in the cross-sectional area. Table (6-12) lists the damage scenarios in the group. The different damage scenarios in this group were designed to include an increasing number of lightly damaged elements. Thus one element was damaged in damage scenario EDL1, two elements were damaged in damage scenario EDL2, three elements were damaged in damage scenario EDL3, and four elements were damaged in damage scenario EDL4. The objective was to investigate if it would be possible to detect damage, even when a number of elements suffered light damage at the same time. Figure (6-29) shows the damaged scenario EDL1 in which element No. 78 was damaged by replacing the original (30×1.5 mm) strut by a smaller strut (22×1.0 mm), keeping the mass unchanged by gluing two lumped masses at the ends of the struts.

Figure (6-30) and Table (6-13) present some of the results from modal tests. Figure(6-30) shows an almost perfect reciprocity between the responses at driving points of damage scenario EDL1. Also the acquired FRFs are of good quality, as seen from the coherence values for the modes in the frequency range of interest. The quality of the measured modal data for other tests of this group was as good as the modal data obtained for damage scenario EDL1. Table (6-13) compares the measured frequencies of the first six lower modes of the damaged scenarios of group (GR3) with those of the corresponding measured modes of the undamaged structure. It can be seen that the percentage differences in frequencies in most of the damage scenarios of this group are almost insignificant.

Damage Scenario EDL1

This damage scenario is a sample of a single lightly damaged site in which the damage is in one of the diagonal elements of the eight-bay space frame. Figures (6-31) and (6-32) show a comparison between the undamaged and damaged FRFs at driving point (2-Y) and at node (22-Y), respectively, due to excitation at (33-Z). It can be seen that the damage is insignificant, affecting only the first mode. A similar conclusion can be drawn from the results presented in Table (6-13). The first mode is affected most because the element under consideration makes a significant contribution to the strain energy content of that mode, as can be seen from Table (5-5). Evidently, it would be difficult to detect damage in this scenario because the changes in modal parameters are quite insignificant. Tables (6-14-a) and (6-14-b) show the results of PORTH and XORTH used to pair the measured undamaged and condensed analytical modes, respectively, with the measured damaged modes.

Damage Scenario EDL2

This damage scenario is a sample of multiple lightly damaged sites in which two of the chord elements of the eight-bay space frame are damaged. Figures (6-33) and (6-34) show a comparison between the undamaged and damaged FRFs at driving point (2-Y) and at node (28-Z), respectively, due to excitation at (33-Z). It can be seen that the damage has an effect on the second and third modes, even though it is quite small. This is also seen from Table (6-13), and is attributed to the fact that these elements make a significant contribution to the strain energy content of these modes as can be seen from

Table (5-5). A change in frequencies of the second and third modes is noticeable, however, it is still within the expected range of modelling and measurement errors. Tables (6-15-a) and (6-15-b) show the results of PORTH and XORTH used to pair the measured undamaged and condensed analytical modes, respectively, with the measured damaged modes.

Damage Scenario EDL3

In this damage scenario three chord elements of the eight-bay space frame are lightly damaged. Figures (6-35) and (6-36) show a comparison between the undamaged and damaged FRFs at driving point (2-Y) and at node (23-Z), respectively, due to excitation at (33-Z). It can be seen that the damage most affects the second and third modes, as shown in Table (6-13). This can be attributed to the fact that these elements make a significant contribution to the strain energy content of these modes, as can be seen from Table (5-5). The percentage change in frequencies of the second and third modes is noticeable and somewhat beyond the level of modelling and measurement errors. Tables (6-16-a) and (6-16-b) show the results of PORTH and XORTH used to pair the measured undamaged and condensed analytical modes, respectively, with the measured damaged modes.

Damage Scenario EDL4

In this damage scenario three chord elements and one diagonal element of the eight-bay space frame are lightly damaged. Figures (6-37) and (6-38) show a comparison between the undamaged and damaged FRFs at driving point (2-Y) and at node (28-Z),

respectively, due to excitation at (33-Z). It can be seen that the damage most affects the first four modes as shown in Table (6-13). This can be attributed to the fact that each of these elements makes a significant contribution to the strain energy content of these modes, as can be seen from Table (5-5). The percentage change in frequencies of the first three modes is quite significant reaching 5% in the second mode. These changes are expected to be larger than the modelling and measurement errors. Tables (6-17-a) and (6-17-b) show the results of PORTH and XORTH used to pair the measured undamaged and condensed analytical modes, respectively, with the measured damaged modes.

6.5.6 Results of Damage Identification for Damage Scenarios in Group (GR3)

The change in modal parameters caused by the introduced damage in the damage scenarios of this group is relatively insignificant. Accordingly, the absolute values of the diagonal terms in the PORTH and XORTH matrices are quite close to (1.0), except in damage scenario EDL3, in which the first and second modes have been mixed due to the introduced damage. The correlation results of the damage scenarios of this group are clearer than those of the two previous damage scenario groups since the introduced damage did not significantly affect the mode shapes.

The results of damage identification of damage scenarios EDL1, EDL2, EDL3 and EDL4, using 108 measured DOFs and all of the 109 elements included in the optimization problem, are shown in Figures (6-39) through (6-42) with label (a). The results of damage identification using the selected 79 DOFs are shown in Figures (6-39) through (6-42) with label (b). Comparison of the identification results presented in the

figures with label (a) with those in figures with label (b) indicates that the use of 79 DOFs is as effective as that of the full 108 DOFs. However, in either case a clear identification of the damage location and severity is not available. It is noted that the recognized elements are in the same region in all of the damage scenarios of this group. The effect of 50 % damage is smeared and not localized, as was the case in simulating 100 % damage. It is of interest to note that a single element with 100 % damage, as in scenario EDS1, produces greater changes in the frequencies than four elements each with 50 % damage.

The modal residual were calculated for the damage scenarios in this group. For damage scenario EDL1, the modal residual vector failed to identify any of the damaged DOFs. In damage scenarios, EDL2, EDL3 and ELD4, the modal residual vector was able to identify only a limited number of the affected DOFs. The optimization problem for damage identification was solved again after restricting the number of candidates based on the results of modal residuals for scenarios EDL2, EDL3 and EDL4. The results are shown in Figures (6-40) through (6-42) with label (c). In each case only one or two elements are identified, along with a few other false predictions.

6.6 Summary

The objective of the work presented in this chapter is the experimental verification of the proposed VBDI approach. The damage scenarios investigated included single and multiple damage sites, severe damage scenarios, light damage scenarios and mixed damage scenarios. This combination provides a clear insight about the advantages

and the limitations of the proposed approach. MATLAB computer modules were developed to implement the different procedures of the proposed VBDI approach. The results of the application of the proposed VBDI approach to the identification of damage are presented. The following conclusions focus on the evaluation of the level of success achieved in predicting damage. The conclusions are drawn on the basis of damage identification results presented in Section 6.5.2, Section 6.5.4, and Section 6.5.6.

1. Results of damage identification scenarios in group (GR1) showed that, although the damaged region has not been isolated prior to carrying out the optimization and imperfect correlation exists in the PORTH and XORTH matrices, the proposed VBDI approach is able to successfully identify damage in all of the four severe damaged scenarios in this group. This validates the different proposed criteria used in this work.
2. For mixed damage scenarios in group (GR2), each damage case included a mix of severe (100%) and light (50%) damage. The proposed VBDI approach is found to be successful in identifying damage in all the elements that have 100 % damage. However, it is not able to identify the elements with 50 % damage in scenarios that include severely damaged elements along with lightly damaged elements as in EDM1 and EDM2. The change in modal parameters due to the 50 % damage in one or more elements is insignificant when compared to the changes caused by even one severely damage element in the same damage scenario. In damage scenario EDM3, however, the changes caused by 50 % damage in three elements is more significant, so that even in combinations with severely

damaged element No. 38, two of the lightly damaged elements have been identified. Again, the identification results obtained were based on the inclusion of all of the 109 frame elements in the optimization problem.

3. The change in modal parameters caused by the introduced damage in the light damage scenarios in group (GR3) is relatively insignificant. Accordingly, the VBDI approach is only able to identify the damaged region without accurately identifying the damaged elements. In damage scenarios, EDL2, EDL3 and ELD4, the modal residual vector is able to identify only a limited number of the affected DOFs. The optimization problem for damage identification was solved again after restricting the number of candidates based on the results of modal residuals for scenarios EDL2, EDL3 and EDL4. In each case only one or two elements are identified along with a few other false predictions.
4. A novel technique for selecting a minimum number of response points is found to be successful when it is combined with efficient model reduction technique such as SEREP that is used in this work. It allows the selection of a set of 79 DOFs out of a total of 2928 DOFs along which the response needs to be measured. The set of selected DOFs constitutes an adequate modal database for damage identification purposes and provides information that allows the identification of single site and multiple site damage cases.
5. In previous chapters, a study of the appropriate number of identification modes has led to the conclusion that a set of the first three lower modes

would satisfy the criterion developed and detailed in Chapter 2. This selection has been verified throughout by the use of the selected measured modes in damage identification in all of the damage cases studied in this chapter. However, in light damage cases, the insignificant changes in modal parameters prevented a clear identification of the damaged elements.

6. Although, the modal residual method was successful in locating the potentially affected DOFs in the presence of random errors throughout the computer simulation presented in Chapter 5, the same was not true for most of the experimental damage scenarios presented in this chapter. This could be due to the errors introduced when the measured mode shapes of the damaged structure are combined with the condensed analytical model matrices for calculating the modal residual vector. However, the method did identify some of the affected DOFs in multiple light damage cases, which helped in identifying accurately some of the damaged elements in each case.

Table (6-1) Damage scenarios in Group GR1

Damage Case ID	Damaged Elements	Element Connectivity	Damage Severity %
EDS1	38	20-24	100
EDS2	78	20-22	100
EDS3	38	20-24	100
	102	24-27	100
EDS4	98	18-21	100
	99	19-24	100

Table (6-2) Comparison of measured frequencies of undamaged and damaged structure, in group (GR1)

Undamaged Frequencies (Hz)	EDS1		EDS2		EDS3		EDS4	
	ω (Hz)	$\delta\omega\%$	ω (Hz)	$\delta\omega\%$	ω (Hz)	$\delta\omega\%$	ω (Hz)	$\delta\omega\%$
50.98	50.98	0.00	40.17	21.2	41.83	18.0	42.05	17.52
52.16	32.83	37.1	52.05	0.21	32.69	37.3	54.40	-4.3
57.50	55.96	2.70	57.16	0.60	53.72	6.60	57.38	0.21
90.14	90.09	0.10	90.32	-0.20	90.97	-0.90	--	--
92.41	92.18	0.25	93.17	-0.82	--	--	80.84	12.52
99.16	98.95	0.21	99.31	-0.15	--	--	--	--

Table (6-3-a) PORTH between measured modes of undamaged and damaged frame for EDS1

Undamaged modes (Hz)	Damaged modes (Hz)					
	50.98	32.83	55.96	90.09	92.18	98.95
50.98	0.9829	-0.1826	0.067	-0.0574	0.0033	0.0045
52.16	0.1524	0.8371	-0.4859	0.0203	0.0049	-0.0677
57.50	0.0281	0.4306	0.8653	-0.0244	0.0274	-0.0328
90.14	-0.0614	0.0262	-0.0299	0.9985	-0.0854	-0.0359
92.41	0.0005	-0.0206	0.0218	-0.0541	0.9982	0.2783
99.16	0.0132	-0.3316	0.038	-0.1081	0.3258	0.9781

Table (6-3-b) XORTH between the measured damaged modes of EDS1 and the corresponding analytical undamaged modes

FEM modes	Measured damaged modes (Hz)					
	50.98	32.83	55.96	90.09	92.18	98.95
51.16	0.9695	-0.2389	0.0894	-0.0078	-0.0168	0.018
52.56	-0.2309	-0.817	0.4744	0.0004	-0.0047	0.0291
57.82	0.0362	0.4039	0.8733	0.0055	-0.0108	0.03
91.97	0.0138	0.0635	0.0189	0.884	-0.3106	-0.1323
90.47	0.0316	-0.046	0.0084	-0.238	-0.9436	0.0134
98.52	0.0321	-0.2204	-0.0257	0.3804	-0.0556	0.9865

Table (6-4-a) PORTH between measured modes of undamaged and damaged frame for EDS2

Undamaged modes (Hz)	Damaged modes (Hz)					
	40.17	52.05	57.16	90.32	93.17	99.31
50.98	0.9014	0.1264	-0.2061	-0.0596	0.0115	0.0012
52.16	-0.1499	0.9888	0.0407	0.0236	0.0045	-0.0413
57.50	0.1685	-0.0083	0.9762	-0.0374	0.0433	-0.0648
90.14	-0.0994	0.0029	-0.0232	0.9959	-0.0264	0.0134
92.41	0.1292	-0.0115	0.0516	-0.1041	0.9903	0.1967
99.16	-0.0366	-0.1621	-0.0336	-0.184	0.3858	0.9587

Table (6-4-b) XORTH between the measured damaged modes of EDS2 and the corresponding analytical undamaged modes

FEM modes	Measured damaged modes (Hz)					
	40.17	52.05	57.16	90.32	93.17	99.31
51.16	0.9012	0.065	-0.2165	-0.2956	-0.1703	0.2281
52.56	0.0839	-0.9962	-0.0297	-0.019	0.3747	-0.0949
57.82	0.1654	-0.0242	0.9718	-0.1431	0.0573	-0.0725
91.97	0.1882	0.0152	0.0514	0.578	0.0973	0.0557
90.47	0.03	-0.0118	-0.003	-0.0258	-0.6029	-0.41
98.52	-0.0583	-0.034	-0.0292	-0.1286	0.5696	0.1233

Table (6-5-a) PORTH between measured modes of undamaged and damaged frame for EDS3

Undamaged modes (Hz)	Damaged modes (Hz)			
	41.83	32.69	53.72	90.97
50.98	0.5500	-0.1027	0.5253	0.0380
52.16	-0.1499	0.7926	0.4192	-0.0046
57.50	0.2385	0.4703	-0.6511	0.0171
90.14	0.1726	0.0631	-0.0956	-0.7809

Table (6-5-b) XORTH between the measured damaged modes of EDS3 and the corresponding analytical undamaged modes

FEM modes	Measured damaged modes (Hz)			
	41.83	32.69	53.72	90.97
51.16	0.5541	-0.1568	0.5041	-0.0008
52.56	0.1369	-0.7733	-0.4694	0.0043
57.82	0.194	0.4358	-0.6314	-0.0189
91.97	0.4732	0.1616	-0.2009	0.8299

Table (6-6-a) PORTH between modes of measured undamaged and damaged frame for EDS4

Undamaged modes (Hz)	Damaged modes (Hz)			
	42.05	54.40	57.38	80.84
50.98	0.9281	0.1077	-0.0444	-0.0613
52.16	-0.2352	0.7894	-0.04	-0.0293
57.50	0.0172	0.011	0.9836	0.0256
92.41	0.1393	-0.019	0.0186	0.9804

Table (6-6-b) XORTH between the measured damaged modes of EDS4 and the corresponding analytical undamaged modes

FEM modes	Measured damaged modes (Hz)			
	42.05	54.40	57.38	80.84
51.16	0.9422	0.0737	-0.0506	-0.0641
52.56	0.1692	-0.7759	0.0315	0.0323
57.82	0.0207	-0.0191	0.9916	0.0181
91.97	0.1364	0.0091	-0.0169	0.7756

Table (6-7) Damage scenarios in Group GR2

Damage Case ID	Damaged Elements	Element Connectivity	Damage Severity %
EDM1	37	19-23	50
	38	20-24	100
EDM2	38	20-24	50
	42	24-28	100
	78	20-22	100
EDM3	35	17-21	50
	36	18-22	50
	37	19-23	50
	38	20-24	100

Table (6-8) Comparison of measured frequencies of the undamaged and damage structure in group (GR2)

Undamaged Frequencies (Hz)	EDM1		EDM2		EDM3	
	ω (Hz)	$\delta\omega\%$	ω (Hz)	$\delta\omega\%$	ω (Hz)	$\delta\omega\%$
50.98	50.4	1.14	28.48	44.14	49.63	2.65
52.16	32.17	38.3	44.43	14.82	30.00	42.5
57.50	54.25	5.65	54.56	5.11	52.0	9.56
90.14	89.76	0.42	83.7	7.14	89.47	0.74
92.41	91.88	0.57	92.08	0.36	91.15	1.38
99.16	98.54	0.63	98.36	0.81	97.91	1.26

Table (6-9-a) PORTH between measured modes of the undamaged and damaged frame for EDM1

Undamaged modes (Hz)	Damaged modes (Hz)					
	50.40	32.17	54.25	89.76	91.88	98.54
50.98	0.9331	-0.2114	0.3221	-0.0659	0.0042	-0.0119
52.16	0.2995	0.8037	-0.4616	0.0152	-0.0071	0.0397
57.50	-0.1579	0.4843	0.8194	0.0017	0.0495	0.0318
90.14	-0.0292	0.0133	-0.0714	0.9815	-0.2131	0.0043
92.41	0.0057	-0.0308	-0.0219	0.0997	0.9874	-0.2106
99.16	-0.0102	-0.3276	-0.0141	0.0055	0.41	-0.9546

Table (6-9-b) XORTH between the measured damaged modes of EDM1 and the corresponding analytical undamaged modes

FEM modes	Measured damaged modes (Hz)					
	50.40	32.17	54.25	89.76	91.88	98.54
51.16	0.9134	-0.2662	0.3406	-0.0103	-0.0258	-0.0268
52.56	-0.3722	-0.7817	0.4312	0.0077	-0.001	0.0002
57.82	-0.1527	0.4581	0.8268	0.032	0.0164	-0.0325
91.97	0.0161	0.0558	0.0103	0.8766	-0.2003	0.2005
90.47	0.0012	-0.0298	0.0521	-0.1055	-0.9712	-0.0019
98.52	0.03	-0.223	-0.0774	0.4542	0.0653	-0.9681

Table (6-10-a) PORTH between measured modes of the undamaged and damaged frame for EDM2

Undamaged modes (Hz)	Damaged modes (Hz)					
	28.48	44.43	54.56	83.70	92.08	98.36
50.98	0.5076	-0.564	0.3172	-0.1012	0.0164	0.0283
52.16	0.2759	0.6928	0.5173	0.0978	-0.0083	0.0432
57.50	0.3657	0.2456	-0.7675	0.1829	0.021	-0.0417
90.14	0.2495	0.2496	-0.1122	-0.6686	-0.5006	0.1556
92.41	0.241	0.022	-0.0737	-0.0389	0.8815	0.2004
99.16	-0.5406	-0.3065	0.0149	0.4102	0.2231	0.901

Table (6-10-b) XORTH between the measured damaged modes of EDM2 and the corresponding analytical undamaged modes

FEM modes	Measured damaged modes (Hz)					
	28.48	44.43	54.56	83.70	92.08	98.36
51.16	0.4889	-0.5997	0.2876	-0.0112	-0.151	0.0447
52.56	-0.2878	-0.6389	-0.5412	0.0117	0.1354	-0.0819
57.82	0.3217	0.22	-0.7686	-0.0115	-0.1723	0.0213
91.97	0.3902	0.0392	-0.0514	0.9415	0.4103	-0.2421
90.47	-0.382	-0.2656	0.139	0.188	-0.8248	-0.1496
98.52	-0.4834	-0.2009	0.0742	0.2461	-0.0534	0.9376

Table (6-11-a) PORTH between measured modes of the undamaged and damaged frame for EDM3

Undamaged modes (Hz)	Damaged modes (Hz)					
	49.63	30.00	52.0	89.47	91.15	97.91
50.98	0.9351	-0.1729	0.2918	-0.0635	0.0054	0.0028
52.16	0.2686	0.8291	-0.4508	0.0089	0.0021	0.0264
57.50	-0.1539	0.4696	0.8354	0.0116	0.0574	0.0323
90.14	-0.0328	0.024	-0.1037	0.9753	-0.2755	0.0271
92.41	0.0515	-0.0282	-0.0006	0.1404	0.9708	-0.1934
99.16	0.0298	-0.2632	0.0632	-0.0041	0.445	-0.9492

Table (6-11-b) XORTH between the measured damaged modes of EDM3 and the corresponding analytical undamaged modes

FEM modes	Measured damaged modes (Hz)					
	49.63	30.00	52.0	89.47	91.15	97.91
51.16	0.9147	-0.2276	0.3089	-0.012	-0.0241	-0.0104
52.56	-0.3424	-0.8134	0.4193	-0.0038	0.0062	0.0129
57.82	-0.1467	0.4494	0.8471	0.042	0.0241	-0.0311
91.97	0.0463	0.0253	0.0076	0.8649	-0.1535	0.2076
90.47	-0.0071	-0.0365	0.0823	-0.0376	-0.9809	-0.0303
98.52	0.0731	-0.1511	-0.0008	0.4824	0.0571	-0.9624

Table (6-12) Damage scenarios in Group GR3

Damage Case ID	Damaged Elements	Element Connectivity	Damage Severity %
EDL1	78	20-22	50
EDL2	41	23-27	50
	42	24-28	50
EDL3	39	21-25	50
	40	22-26	50
	41	23-27	50
EDL4	38	20-24	50
	42	24-28	50
	46	28-32	50
	78	20-22	50

Table (6-13) Comparison of measured frequencies of the undamaged and damaged structure in group (GR3)

Undamaged Frequencies (Hz)	EDL1		EDL2		EDL3		EDL4	
	ω (Hz)	$\delta\omega\%$	ω (Hz)	$\delta\omega\%$	ω (Hz)	$\delta\omega\%$	ω (Hz)	$\delta\omega\%$
50.98	50.46	1.02	50.52	0.90	50.82	0.30	50.14	1.65
52.16	52.19	0.00	51.05	2.13	49.80	4.53	49.50	5.10
57.50	57.51	0.00	56.03	2.56	55.26	3.89	56.24	2.20
90.14	90.11	0.00	89.32	0.88	88.32	2.02	88.58	1.73
92.41	92.20	0.20	91.87	0.58	91.80	0.66	91.90	0.55
99.16	99.09	0.00	98.79	0.37	98.55	0.62	98.49	0.68

Table (6-14-a) PORTH between measured modes of the undamaged and damaged frame for EDL1

Undamaged modes (Hz)	Damaged modes (Hz)					
	50.46	51.19	57.51	90.11	92.2	99.09
50.98	0.9961	0.0248	-0.0191	-0.0565	-0.0021	-0.0156
52.16	-0.0666	0.998	0.0232	0.019	0.014	-0.0907
57.50	0.0175	-0.0022	0.9989	-0.0282	0.0403	-0.0282
90.14	-0.0609	0.0146	-0.0315	0.9987	-0.0895	-0.0154
92.41	0.0042	0.0006	0.0295	-0.0518	0.9989	0.2736
99.16	-0.0112	-0.1798	-0.0863	-0.122	0.3486	0.9789

Table (6-14-b) XORTH between the measured damaged modes of EDL1 and the corresponding analytical undamaged modes

FEM modes	Measured damaged modes (Hz)					
	50.46	51.19	57.51	90.11	92.2	99.09
51.16	0.997	-0.0373	-0.0296	-0.0134	-0.0159	-0.0003
52.56	-0.0111	-0.9965	-0.025	-0.0091	-0.0027	0.054
57.82	0.025	-0.0202	0.9929	0.0196	-0.0157	0.0358
91.97	0.0388	0.0306	0.0566	0.8761	-0.3022	-0.1446
90.47	0.0272	-0.0265	0.0024	-0.2306	-0.9461	-0.0033
98.52	-0.0211	-0.05	-0.0844	0.4029	-0.0696	0.9847

Table (6-15-a) PORTH between measured modes of undamaged and damaged frame for EDL2

Undamaged modes (Hz)	Damaged modes (Hz)					
	50.52	51.05	56.03	89.32	91.87	98.79
50.98	0.9854	-0.1497	-0.1584	-0.0653	-0.0012	0.0083
52.16	0.0528	0.9858	0.0013	0.0125	0.0092	-0.0739
57.50	0.1363	-0.0279	0.9831	-0.0433	0.0223	-0.0113
90.14	-0.0266	0.048	0.0128	0.9831	-0.1763	-0.0606
92.41	0.0176	-0.0094	0.073	0.0605	0.9862	0.1899
99.16	-0.0202	-0.2255	-0.086	-0.1779	0.257	0.9675

Table (6-15-b) XORTH between the measured damaged modes of EDL2 and the corresponding analytical undamaged modes

FEM modes	Measured damaged modes (Hz)					
	50.52	51.05	56.03	89.32	91.87	98.79
51.16	0.9784	-0.2099	-0.1656	-0.0171	-0.0269	0.0215
52.56	-0.1301	-0.9682	0.0086	-0.0012	0.0095	0.0335
57.82	0.1427	-0.0508	0.9751	-0.0064	-0.0408	0.0548
91.97	0.0369	0.0305	0.0855	0.9324	-0.1591	-0.2116
90.47	-0.0131	-0.0518	-0.0531	-0.1535	-0.974	0.0678
98.52	-0.0108	-0.0964	-0.0815	0.3048	-0.121	0.969

Table (6-16-a) PORTH between measured modes of undamaged and damaged frame for EDL3

Undamaged modes (Hz)	Damaged modes (Hz)					
	50.82	49.80	55.26	88.32	91.80	98.55
50.98	0.7014	-0.6974	0.0858	-0.0667	-0.0052	0.0015
52.16	0.6841	0.7032	-0.1787	0.0079	0.0067	-0.0318
57.50	0.0454	0.175	0.9702	-0.0544	0.0403	-0.0484
90.14	0.0421	0.1121	0.0249	0.9851	-0.1331	0.0731
92.41	0.0185	-0.0178	0.0076	-0.0114	0.9913	0.2016
99.16	-0.2163	-0.1334	0.0329	-0.2312	0.2592	0.9457

Table (6-16-b) XORTH between the measured damaged modes of EDL3 and the corresponding analytical undamaged modes

FEM modes	Measured damaged modes (Hz)					
	50.82	49.80	55.26	88.32	91.80	98.55
51.16	-0.7355	0.657	0.0917	-0.0195	-0.0277	0.0186
52.56	-0.6471	-0.7323	0.164	0.002	0.0152	-0.0064
57.82	0.1586	0.0315	0.9787	0.0116	-0.0528	0.0177
91.97	-0.0348	0.0751	-0.0258	0.9233	-0.2087	-0.2314
90.47	-0.0927	-0.0824	-0.0418	-0.1938	-0.9554	-0.0684
98.52	-0.0384	-0.1229	0.0121	0.3088	-0.1782	0.9637

Table (6-17-a) PORTH between measured modes of undamaged and damaged frame for EDL4

Undamaged modes (Hz)	Damaged modes (Hz)					
	50.14	49.50	56.24	88.58	91.90	98.49
50.98	0.9699	-0.2383	-0.0393	-0.0602	-0.0019	-0.011
52.16	0.1831	0.9492	-0.1914	0.0208	0.0077	-0.0575
57.50	0.1203	0.2019	0.9701	-0.0497	0.0308	-0.0368
90.14	-0.0383	0.0882	0.0164	0.978	-0.1379	0.0614
92.41	0.0161	-0.0097	0.0648	-0.0127	0.9959	0.2357
99.16	-0.0608	-0.2196	-0.0948	-0.2666	0.3213	0.9611

Table (6-17-b) XORTH between the measured damaged modes of EDL4 and the corresponding analytical undamaged modes

FEM modes	Measured damaged modes (Hz)					
	50.14	49.50	56.24	88.58	91.90	98.49
51.16	0.9546	-0.2953	-0.0346	-0.0153	-0.0228	0.006
52.56	-0.258	-0.9268	0.1933	-0.0019	0.003	0.021
57.82	0.1223	0.1816	0.9657	0.0067	-0.0503	0.028
91.97	0.052	0.0097	0.0911	0.9008	-0.1934	-0.1987
90.47	-0.0029	-0.0931	-0.059	-0.1855	-0.9527	-0.0641
98.52	-0.0369	-0.0922	-0.1164	0.3725	-0.2124	0.9734

Figure (6-1) Lumped masses attached to compensate for the difference of mass of the exchanged struts.

Figure (6-2) Comparison of FRFs of the driving point (2-Y) due to excitation at (33-Z) before and after disassembly and reassembly of one of the frame bays

Figure (6-3) Flow chart of the main subroutines of the computer code that implements the proposed VBDI approach

Figure (6-4) Damage scenario EDS3

Figure (6-5) Reciprocity check for driving point responses for of damage scenario EDS1: FRF measured at DOF (2-Y) due to the excitation at DOF (33-Z) and at DOF (2-Y). The lower bay shows the coherence values

Figure (6-6) Comparison of FRF before and after damage for damage scenario EDS1: FRF measured at DOF (2-Y) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-7) Comparison of FRF before and after damage for damage scenario EDS1: FRF measured at DOF (24-Z) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-8) Comparison of FRF before and after damage for damage scenario EDS2: FRF measured at DOF (2-Y) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-9) Comparison of FRF before and after damage for damage scenario EDS2: FRF measured at DOF (22-Z) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-10) Comparison of FRF before and after damage for damage scenario EDS3: FRF measured at DOF (2-Y) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-11) Comparison of FRF before and after damage for damage scenario EDS3; FRF measured at DOF (24-Y) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-12) Comparison of FRF before and after damage for damage scenario EDS4; FRF measured at DOF (2-Y) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-13) Comparison of FRF before and after damage for damage scenario EDS4: FRF measured at DOF (19-Z) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-14-a) Damage identification results for damage scenario EDS1 using 108 measured DOFs

Figure (6-14-b) Damage identification results for damage scenario EDS1 using 79 measured DOFs

Figure (6-15-a) Damage identification results for damage scenario EDS2 using 108 measured DOFs

Figure (6-15-b) Damage identification results for damage scenario EDS2 using 79 measured DOFs

Figure (6-16-a) Damage identification results for damage scenario EDS3 using 108 measured DOFs

Figure (6-16-b) Damage identification results for damage scenario EDS3 using 79 measured DOFs

Figure (6-17-a) Damage identification results for damage scenario EDS4 using 108 measured DOFs

Figure (6-17-b) Damage identification results for damage scenario EDS4 using 79 measured DOFs

Figure (6-18) Damage scenario EDM2

Figure (6-19) Reciprocity check for the driving points in damage scenario EDM2: FRF measured at DOF (2-Y) due to the excitation at DOF (33-Z) and at DOF (33-Z) due to the excitation at DOF (2-Y)

Figure (6-20) Comparison of FRFs before and after damage for damage scenario EDM1: FRF measured at DOF (2-Y) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-21) Comparison of FRFs before and after damage for damage scenario EDM1 : FRF measured at DOF (22-Y) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-22) Comparison of FRFs before and after damage for damage scenario EDM2: FRFs measured at DOF (2-Y) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-23) Comparison of FRFs before and after damage for damage scenario EDM2: FRFs measured at DOF (28-Z) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-24) Comparison of FRFs before and after damage for damage scenario EDM3: FRFs measured at DOF (2-Y) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-25) Comparison of FRFs before and after damage for damage scenario EDM3: FRFs measured at DOF (21-Y) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-26-a) Damage identification results for damage scenario EDM1 using 108 measured DOFs

Figure (6-26-b) Damage identification results for damage scenario EDM1 using 79 measured DOFs

Figure (6-27-a) Damage identification results for damage scenario EDM2 using 108 measured DOFs

Figure (6-27-b) Damage identification results for damage scenario EDM2 using 79 measured DOFs

Figure (6-28-a) Damage identification results for damage scenario EDM3 using 108 measured DOFs

Figure (6-28-b) Damage identification results for damage scenario EDM3 using 79 measured DOFs

Figure (6-29) Damage scenario EDL1

Figure (6-30) Reciprocity check for the driving points in damage scenario EDL1: FRF measured at DOF (2-Y) due to the excitation at DOF (33-Z) and at DOF (33-Z) due to the excitation at DOF (2-Y)

Figure (6-31) Comparison of FRFs before and after damage for damage scenario EDL1: FRFs measured at DOF (2-Y) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-32) Comparison of FRFs before and after damage for damage scenario EDL1: FRFs measured at DOF (22-Y) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-33) Comparison of FRFs before and after damage for damage scenario EDL2: FRFs measured at DOF (2-Y) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-34) Comparison of FRFs before and after damage of damage scenario EDL2: FRFs measured at DOF (28-Z) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-35) Comparison of FRFs before and after damage of damage scenario EDL3: FRFs measured at DOF (2-Y) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-36) Comparison of FRFs before and after damage of damage scenario EDL3: FRFs measured at DOF (23-Z) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-37) Comparison of FRFs before and after damage for damage scenario EDL4: FRFs measured at DOF (2-Y) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-38) Comparison of FRFs before and after damage of damage scenario EDL4: FRFs measured at DOF (28-Z) due to excitation at DOF (33-Z), solid line shows FRFs before damage, dashed line shows FRFs after damage

Figure (6-39-a) Damage identification results for damage scenario EDL1 using 108 measured DOFs

Figure (6-39-b) Damage identification results for damage scenario EDL1 using 79 measured DOFs

Figure (6-40-a) Damage identification results for damage scenario EDL2 using 108 measured DOFs

Figure (6-40-b) Damage identification results for damage scenario EDL2 using 79 measured DOFs

Figure (6-40-c) Damage identification results for damage scenario EDL2 using 79 measured DOFs based on the isolation of the damaged region using modal residual

Figure (6-41-a) Damage identification results for damage scenario EDL3 using 108 measured DOFs

Figure (6-41-b) Damage identification results for damage scenario EDL3 using 79 measured DOFs

Figure (6-41-c) Damage identification results for damage scenario EDL3 using 79 measured DOFs based on the isolation of the damaged region using modal residual

Figure (6-42-a) Damage identification results for damage scenario EDL4 using 108 measured DOFs

Figure (6-42-b) Damage identification results for damage scenario EDL4 using 79 measured DOFs

Figure (6-42-c) Damage identification results for damage scenario EDL4 using 79 measured DOFs based on the isolation of the damaged region using modal residual

Chapter 7

Summary, Conclusions and Recommendations

7.1 Summary

Vibration-based damage identification, which was the focus of the present work depends on an optimal integration of careful testing, information system components and analytical identification algorithms. The analytical approach used in the present work is based on optimal matrix update. The technique is a model-based technique. A direct implementation of the optimal matrix update methods would perform well when used with simple structures. However when applied to complex structures, these methods may sometimes lead to ambiguous results. In fact, any one method by itself would normally fail to account for the complexity of the modal analysis and testing methods and the mathematical algorithms used to identify damage and may be unable to generate physically acceptable results. The incompleteness of measured mode shapes, the selection of the set of identification modes, the ability of the baseline FEM to produce the measured modal parameters and the mathematical complexity of the identification algorithm, are factors that most affect the overall performance of the optimal matrix update methods.

A number of novel approaches or optimum integration of existing approaches were developed in this work to improve the performance of VBDI. One novelty is the development of a set of new criteria for the selection of response points. The selection criteria are designed to determine the minimum number of measurement DOFs that would constitute an adequate modal database for model-based identification algorithms and at the same time preserve the identity of the set of identification modes. Also, an existing mode selection criterion is modified for damage identification and FEM reduction/expansion. The SEREP technique that has the ability of preserving accurately the modal properties of the original model for a selected set of modes is used in comparison with Guyan reduction. A combination of two simple mathematical algorithms is used to identify damage location and extent. A computationally inexpensive mathematical formulation of the optimization problem is used to assure the globalization and feasibility of the solution.

The theoretical background to the main mathematical methods used in the proposed approach is derived. Computer simulation studies are presented. They show the feasibility of the proposed VBDI approach along with some of its limitations. The experimental validation based on modal tests designed to provide estimates of damage in a 3D eight-bay free-free frame has highlighted the important issues and some of the challenges. The construction of a finite element model of the structure, two-step reduction methodology and model correlation with measured test data is described. Details of tests on an undamaged structure as well as on a structure in which several predetermined damage scenarios have been introduced are presented. Test data and

damage identification results are presented. In all severe damage cases involving single and multiple damage sites, the proposed VBDI approach was found to be successful in predicting both the location and severity of damage even with incomplete and truncated mode shapes and in the presence of measurement errors. Estimating of light damage was not as successful, however in most cases it was possible to at least recognize the region of damage.

7.2 Conclusions

The following important conclusions can be drawn from the analytical and experimental studies undertaken in the present research:

- (1) The proposed VBDI approach has been demonstrated as being reliable in identifying all severe damage cases using a minimum modal database even when a structure is somewhat complex. The combination of the mathematical procedures of the VBDI approach used in the present study represent a simple robust sequential scheme of VBDI techniques. The damage scenarios investigated included single and multiple damage sites, severe damage scenarios, light damage scenarios and mixed damage scenarios. This combination provides clear insight about the advantages and the limitations of the proposed approach.
- (2) Study of the strain energy distribution gives a good indication of the elements that can be detected and those, which cannot be. Strain energy distribution is also a useful first step in identifying the set of modes that should be included in the identification process.

- (3) In order to reduce the effect of reduction techniques on the calculated modal parameters two-step reduction methodology based on the concept of substructuring was developed for the present research in order to reduce the FEM from 2928 DOFs to the monitored 79 DOFs. In the first step Guyan's reduction was used to reduce the DOFs of one element from 36 to 12. In the second step reduction was applied at the global level using the SEREP technique to reduce the DOFs from 216 to 79. The reduction strategy used has minimized the effect of reduction, as evidenced by the fact that the increase in the modal frequencies caused by reduction was found to be insignificant.
- (4) A novel technique for selecting the minimum number of response points was found to be successful when it was combined with efficient model reduction technique such as SEREP or the two-step reduction technique used in this work. It has allowed the selection of a set of 79 DOFs out of a total of 2928 DOFs along which the response needs to be measured. The set of selected DOFs has proven to constitute an adequate modal database for damage identification purposes and provides information that allows the identification of single site and multiple site damage cases.
- (5) A study of the appropriate number of identification modes has led to the conclusion that the use of an increased number of modes increases the number of equality constraints in the optimization problem and may in some cases reduce the accuracy of identification. In the frame being studied the detectable elements are found to make a significant contribution to the strain energy content of only some out of the first six identification modes. Consequently inclusion of all six modes

increases the number of equality constraints in the optimization problem without supplying significant information about the damaged elements and consequently steers the optimizer away from the right solution.

- (6) In the computer simulation studies, a two-step identification approach was developed to first identify the damaged region using a modal residual vector and then to solve an optimization problem with a reduced number of variables. This approach is found to be relatively successful in many of the damage cases, especially when a light 50 % damage is induced at single and multiple sites with up to four sites damaged at a time. Isolation of the damage region by using modal residual helps in identifying the location of damage. When a reduced number of candidate elements are specified in the optimization on the basis of the results of modal residual, the assessment of the severity is improved in some cases, but not always.
- (7) Although the damaged region has not been isolated prior to carrying out the optimization and imperfect correlation results seen in the PORTH and XORTH matrices, the proposed VBDI approach was able to successfully identify damage in all of the four severe damaged scenarios carried out under the Group (GR1) in the experimental studies.
- (8) For mixed damage scenarios in group (GR2), each damage case included a mix of severe (100%) and light (50%) damage. The proposed VBDI approach was found to be successful in identifying damage in all the elements that had 100 % damage. However, it was not able to identify the elements with 50 % damage. In most cases, the change in modal parameters due to the 50 % damage in one element is

insignificant when compared to the changes caused by even one severely damage element in the same damage scenario when combined effects of light damage in several elements is more significant as in scenario EDM3. Some of the lightly damaged elements can be identified.

(9) The change in modal parameters caused by the introduced damage in the light damage scenarios in group (GR3) is relatively insignificant. Accordingly, the VBDI approach is able to identify the damaged region but not the specific elements that were damaged. The use of modal residual improved the identification but does not give purer results. The solution of the optimization problem for damage identification with a reduced number of candidate elements based on the results of modal residuals does not lead to any significant improvements

(10) Although, the modal residual method was successful in locating the potentially affected DOFs in the presence of random errors throughout the computer simulation presented in Chapter 5, the same was not true for most of the experimental damage scenarios presented in Chapter 6. This could be due to the errors introduced when the measured mode shapes of the damaged structure were combined with the condensed analytical model matrices for calculating the modal residual vector. However, it did identify some of the affected DOFs in multiple light damage cases, which helped in identifying accurately some of the damaged elements in each case.

(11) Results of damage identification in light damage scenarios of Group (GR2) showed that if the damage in members impacts the modal properties of the

lower modes, this impact must exceed by a significant margin the level of uncertainty present in modelling or testing, for the damaged element to be identified.

7.3 Recommendations for the Future Work

- (1) Ambient excitation is defined as the excitation experienced by a structure under its normal operating conditions due to wind, traffic etc. For large structures, such as large bridges and on orbit space structures the ability of mechanical devices to input significant energy into the structure, particularly at higher frequencies is severely limited and ambient excitation remains the only means of excitation of such structures. Because excitation input cannot be measured in ambient excitation the extraction of modal parameters is more challenging and errors are larger. The proposed VBDI approach should be developed further for application to structures excited by ambient excitation
- (2) Although, in the present study, the proposed VBDI approach has been applied to a fairly complex space-type structure, it should be validated by application to different types of structures such as bridges and vessels to investigate different modelling challenges.
- (3) Considering the success achieved in constructing and updating the FEM of the test article that was able to produce exactly the first few modes, it will be useful to investigate the application of other more recent identification algorithms such as neural network- based methods.

- (4) In this work a deterministic optimization technique (SQP) has been used to solve the optimization problem meant to identify damage. Genetic algorithms are believed to be a reliable probabilistic optimization technique that can be used to execute the same task. This technique is expected to perform better in eliminating the spurious elements appearing in the damage identification charts.
- (5) In the computer simulation studies carried out as a part of this work, the modal residual vector was able to identify the damaged region even in the presence of simulated random errors. However, it did not perform so well in the experimental damaged scenarios perhaps because of modelling and measurement errors in mode shapes. The use of FRF differences to isolate the damaged region should be explored. This technique is based on the comparison of FRFs of the undamaged, with those of the damaged structure. This method has the advantage of using only measured mode shapes.
- (6) It is evident that measured changes in modal parameters may be caused by factors other than damage. For redundant structures, temperature variation could alter the vibration characteristics and produce changes in modal parameters. Civil and aerospace structures are both exposed to such environmental effects. Further studies need to be carried out to adopt the proposed VBDI approach to account for such effects and to test its ability to identify damage in the presence of environmental effects.

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