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**Effects of Instruction on Basic Multiplication Performance
Implications of Speeded Response and Self-Report Methods**

by

Brenda L. Smith

**A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of**

Master of Arts

Department of Psychology

Carleton University

Ottawa, Ontario

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**THE EFFECTS OF INSTRUCTIONS ON MULTIPLICATION PERFORMANCE
IMPLICATIONS OF SPEEDED RESPONSE AND SELF-REPORT METHODS**

submitted by Brenda L. Smith, B. Sc.

In partial fulfillment of the requirements for the degree of

Master of Arts

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Thesis Supervisor

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September, 1996

Abstract

LeFevre et al (in press) have suggested that adults may maintain non-retrieval procedures (e.g., counting or rules) to solve single-digit multiplication problems that are usually associated with children's performance. Criticism of the online self-report technique to determine the solution procedures in single-digit multiplication suggests that on-line self-report procedures may reflect knowledge unrelated to the direct retrieval of an answer. This contention is supported by the observation of longer latencies in studies utilizing on-line self-report methods. LeFevre et al suggest that latency differences between studies may be more attributable to differences in the emphasis placed on speed or accuracy instructions given to participants than to the report requirements. This study compares performance of two groups of participants in a single-digit multiplication production task. Participants performed the task once with a speed emphasis in the instructions and once with an accuracy emphasis. Half of the participants were also required to report solution procedures. Results support LeFevre et al (1996, in press) that instructional emphasis affects both latencies and errors. Once groups had been equated for arithmetic skill, requirements to report solution procedures did not significantly affect performance. Implications of non-retrieval performance for existing models of multiplication are discussed.

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Introduction

Most researchers examining the cognitive process of arithmetic assume that by adulthood, basic addition and multiplication facts are directly retrieved from memory (Ashcraft, 1992, 1995). Direct retrieval of an answer to a basic fact develops as a result of paired-associate learning. Initially, children learn basic arithmetic facts by using a variety of procedures, such as counting (Siegler, 1988). Presumably, with practice the connection between a basic fact and its answer becomes stronger until eventually the answer is directly available upon presentation of a fact (Siegler, 1988). Thus, most researchers assume that the time taken to retrieve an arithmetic answer reflects direct accessing of arithmetic facts from memory (Ashcraft, 1992). Similarly, errors made when solving basic arithmetic problems are presumed to arise from the organization of facts in a retrieval network and from the processes involved in the direct retrieval of an answer (Ashcraft, 1992). However, the assumption that adults rely solely on direct retrieval in basic arithmetic has recently been challenged by the suggestion that adults may use non-retrieval solution procedures (Geary & Brown, 1991, Geary, Brown & Samaranavake, 1991, LeFevre et al., in press, LeFevre, Sadesky & Bisanz, 1996, Svenson, 1985).

LeFevre et al. (1996) asked university students to solve basic addition problems and to describe how they arrived at the answer. Participants reported direct retrieval 71% of the time. On the remaining 29% of the trials, participants reported using a variety of non-retrieval methods, such as repeated addition. Similarly, LeFevre et al. (in press) found that university students solving basic multiplication problems reported using non-retrieval solution procedures on approximately 20% of the trials. LeFevre et al. (1996, in

press) suggested that assuming adults always directly retrieve the answers to basic arithmetic facts is questionable. It seems more plausible that adult basic arithmetic processing involves a variety of solution methods. Thus, latency and error patterns do not necessarily reflect only direct retrieval processes (LeFevre et al., 1996, in press).

The idea that even university educated adults regularly use non-retrieval procedures to solve basic arithmetic facts has not been received without scepticism (see Ashcraft, 1995 for a review). The main criticism of LeFevre et al. (1996, in press) focuses on the issue of introspection to determine solution methods. In particular, requiring participants to self-report the solution method they use may qualitatively change the arithmetic task (Ashcraft, 1992, Campbell, 1995). In this view, reports of non-retrieval solution procedures may be artifacts caused by the task demands and may not reflect actual solution processes (Campbell, 1995; LeFevre et al., 1996). LeFevre et al. (in press) report longer response times than many other studies, which may indicate that participants perform differently when self-reporting solution methods than when they are not asked to self-report solutions (Campbell, 1994). LeFevre et al. (in press) have justified the longer latencies found in their study by suggesting that other differences in procedures, such as speed instructions, may influence latencies. They note that researchers differ in the degree, or manner in which speed and accuracy are emphasized in the instructions given to participants. As a result of the methodological inconsistencies between studies, the influence of self-reporting solution procedures on performance is not clear.

One goal of the present study was to further test the suggestion by LeFevre et al. (in press) that adults use non-retrieval procedures to solve basic multiplication facts. In

particular, the validity of the self-report method for determining the use of a non-retrieval procedure will be examined by comparing the effects of self-report requirements to no self-report requirements on participant performance in solving multiplication facts. Also, the effect of either speed or accuracy emphasis in the participants' instructions on latency and error patterns will be compared. I expected to find that longer latencies in solving multiplication are not associated with self-report requirements, but rather are associated with less emphasis placed on speeded response in instructions.

Non-Retrieval Procedures in Solving Basic Multiplication Facts

Much of the evidence for non-retrieval solution methods in basic multiplication is derived from examinations of children's performance (Cooney & Ladd, 1992, Lemaire & Siegler, 1995, Siegler, 1987, 1988, Siegler & Jenkins, 1989). For example, Siegler (1988) observed that non-retrieval methods used by children, such as counting, are available even after a child has developed an association between the problem and the answer. During learning, children use procedures to solve basic multiplication problems, such as repeated addition (e.g., $3 \times 4 = 4, 8, 12$). Over trials, each fact is associated with both correct and incorrect responses. Although the strongest association develops between the problem and the correct answer, other related answers (i.e., mistakes) also develop associative strength. Smaller problems (i.e., 3×4) develop very strong, or peaked, associations with the answer because children are less likely to make errors when solving these problems using counting strings (Siegler, 1988). Larger problems have less peaked associations with the answer, because more errors are made when solving them. Whether a non-retrieval method or direct retrieval is used to solve a fact depends upon the degree of

perceived certainty with which an answer is correctly associated with a fact. The perception of certainty the participant has with the answer is termed a "confidence criterion" (Siegler, 1988). When there is a high need for accuracy (perceived or actual) or a weak direct association between an answer and a fact, a child may choose to rely on non-retrieval solution procedures to solve problems instead of relying on direct retrieval. Thus, children have multiple solution routes available. However, Siegler (1988) has suggested that associative strength develops such that, eventually, retrieval is the solution method used by adults.

Contrary to Siegler's suggestion of a developing reliance on retrieval as a solution method, LeFevre et al. (in press) have found evidence that some adults continue to rely on non-retrieval procedures to solve basic multiplication facts. Some adults reported solving 2-operand problems (i.e., 2×3) by first converting to a tie addition problem (i.e., $3 + 3 = 6$). Five-operand problems were sometimes solved via the counting-by-fives string (i.e., $5 \times 3 = 5, 10, 15$). Participants frequently reported using rules to solve 0- and 1-operand problems (i.e., anything $\times 1 = \text{itself}$, and anything $\times 0 = 0$). The answers to large problems were sometimes derived using well-learned facts (i.e., 7×8 : $7 \times 7 = 49$, $49 + 7 = 56$). The procedural method used to solve a problem was related to latencies. For example, response times to problems solved with derived fact procedures (most common with larger problems) were the slowest. Alternatively, problems solved with rules (i.e., 0-rule, 1-rule) were as fast as those solved using direct retrieval. LeFevre et al. (1996) suggested that such non-retrieval procedures may provide an explanation for a feature of arithmetic performance known as the problem size effect in the empirical literature.

The Problem Size Effect

The fundamental feature of basic mathematical cognition is the problem size effect (Groen & Parkman, 1972, Ashcraft, 1992). As problem size increases, so does the reaction time to produce an answer (Groen & Parkman, 1972). In other words, latencies are positively correlated with the size of the product and operands (Campbell, 1995). The robust and stable nature of the problem size effect has led researchers to develop theoretical frameworks that incorporate some sort of mechanism or organization that can explain the basis of the problem size effect (Ashcraft, 1992, 1995). For models that assume that all basic problems are solved via direct retrieval, the implementation of such an explanatory feature is complicated by inconsistencies in the problem size effect among certain problem types and operand families (Ashcraft, 1992, 1995).

Generally, as problem size increases, latencies to produce an answer increase accordingly. However, tie problems (e.g., 4×4 , 8×8) are answered more quickly than other problems of comparable size (Miller, Perlmuter, & Keating, 1984; LeFevre et al., in press). As well, problems from the 5-operand family are solved more quickly than other problems of corresponding size (Campbell & Graham, 1985; Campbell & Oliphant, 1992). The exception of the tie and 5-operand family problems to the general pattern of the problem-size effect must be accommodated in conceptualizations of the cognitive mechanisms or in the organization of facts responsible for producing the problem-size effect. Direct retrieval models must allow the retrieval process to be different for these problems, or alternatively, have features that account for these exceptions (Ashcraft, 1995; Campbell, 1995; LeFevre et al., in press; Siegler, 1988). As well, direct retrieval models

must also explain the patterns in errors noticed in basic arithmetic performance (Ashcraft, 1995; Campbell, 1995; LeFevre et al., 1995; Siegler, 1988).

Within-Operation Errors in Basic Multiplication

Generally, errors to basic multiplication facts are highly related to correct answers. Errors are typically “operand related” or are the correct answer if either of the operands was changed by +1 or -1 (Ashcraft, 1991; Campbell, 1994). For example, participants may respond with the answer 42 to the problem 6×8 , which is the correct answer to the neighbouring fact, 6×7 . Direct retrieval models explain these results by suggesting that families of problems (i.e., the $6x$ ’s family or the $9x$ ’s family) are highly associated in memory (Ashcraft, 1992; Campbell & Oliphant, 1992). Association between members of a family is also suggested by the types of errors made in responding. Within an operand-family, problems with operands that are similar in size (i.e., 6×7 and 6×8) are more closely associated than facts that involve operands that are less similar (i.e., 6×7 and 6×2 ; Campbell, 1994). Thus, errors are usually the correct answers to a family-related fact of a similar magnitude. However, some errors are not operand-related but are answers to a different family of problems. For example, participants may incorrectly respond 14 (which is the correct answer to 2×7) to the problem 3×4 . These errors, called “operand-unrelated” errors, are considered to reflect the close association among all of the basic facts in memory.

Non-table errors

Models of basic arithmetic processing must also explain why participants occasionally produce answers that are not correct answers for any of the basic

multiplication facts (i.e., responding 13 to 6×2 ; Ashcraft, 1992; Campbell, 1995). These errors are relatively rare answers. Sometimes, these errors are the answer to a corresponding addition fact (i.e., $3 \times 4 = 7$), which suggests that addition and multiplication facts may be weakly associated in memory (Ashcraft, 1992). If the non-table errors are +1 or -1 away from the true answer (i.e., $2 \times 6 = 13$ or $2 \times 6 = 11$), they are called "close miss" errors (Ashcraft, 1992). There are many reasons why a person might respond to a presented fact with a non-table value. Siegler (1988) suggests that such close-miss answers may become associated a given problem due to common errors made when learning. For example, during acquisition, children develop associations not only to the correct answer, but also to incorrect answers, due to errors made when solving basic problems using repeated addition (Siegler, 1988). Alternatively, Campbell (1994, 1995) has suggested that numbers of similar magnitude may also be associated in memory. Thus, the answer to 6×7 (i.e., 42) is associated with other numbers of the same size (i.e., 40, 41, 43 etc). Campbell (1995) has also suggested that some non-table errors may be naming errors. One or both of the operands in the fact may accidentally be stated as the answer (e.g., $4 \times 7 = 47$ or $6 \times 4 = 6$). Termed operand-naming errors, such mistakes result from the involvement of reading processes in encoding the presented problem. Campbell's explanation of naming errors shifts away from a purely retrieval-based explanation for error patterns toward more non-retrieval based mechanisms such that reading-based associations are involved in performance patterns. Other researchers have also implicated non-retrieval procedures to explain the problem-size effect (LeFevre et al, 1996, in press).

A framework that allows for non-retrieval processes in basic mathematical performance can be used to interpret the problem-size effect. LeFevre et al. (in press) found that small problems are often retrieved, or solved, using very fast procedures (i.e., 0-rule) resulting in fast latencies. Larger problems are often solved using slower procedures, such as repeated addition. The 5-operand problems are solved more quickly than problems of a similar size because they are often directly retrieved or rely on a fast and well-learned counting string. LeFevre et al. also found that ties are usually retrieved, and are rarely solved using non-retrieval procedures. Also, five-operand and tie problems are often used as a preliminary step to solve other problems. For example, to solve 8×6 , participants often reported that they first recalled 8×5 and then added 8 to derive the proper answer. If ties and 5-operand problems are used to derive the answers to larger problems, tie and 5-operand problems would be more practised and experienced than larger problems. An increase in practice would lead to stronger problem/answer association for tie and 5-operand problems, resulting in relatively faster latencies and fewer errors than less-practised larger problems. In fact, because non-retrieval procedures (e.g., derived facts or counting strings) usually involve more than one step, facts solved in this manner may be more error prone than problems solved by direct retrieval.

At each step of a derived fact or counting string procedure, there is the potential for error. For example, a participant who solves 7×8 by using a derived fact procedure (i.e., $7 \times 7 = 49$, $49 + 7 = 56$) can make a variety of errors. Initially, the answer to 7×7 must be correctly retrieved. Then, participants may prematurely state the answer to the base fact (i.e., 49) instead of progressing with the derivation process. Even if no error is made at

this step, mistakes can occur when 7 is added to the interim answer of 49 to get the final solution (i.e., 56). If a participant always relies on a non-retrieval process to solve a particular fact, he or she may not develop a strong direct answer/fact association. Instead, the non-retrieval procedure itself may become automatic (Baroody, 1984, LeFevre et al., in press). Thus, the latencies and errors made on non-retrieval trials would not inevitably reflect errors made in the direct retrieval of an answer. Instead, performance patterns would sometimes reflect the implementation of non-retrieval procedures.

LeFevre et al. (in press) found that the problem size effect and error patterns made by participants are consistent with reports stating which type of solution procedure was used. However, LeFevre et al.'s reliance on self-reports to detect non-retrieval procedures has been criticized for potentially altering the nature of the production task. Requiring participants to constantly indicate what they are doing may intrinsically change the task (Cooney & Ladd, 1992; Russo, Johnson, & Stephens, 1989). For example, LeFevre et al. (in press) reported mean latencies of 1335 ms in a self-report condition. This is more than 500 ms longer than the mean latencies of 826 ms found by Campbell and Graham (1985) who did not require participants to self-report. The large latency difference between self-report and no self-report studies may be the result of an extra processing step(s) or due to accessing of information that is unrelated to direct retrieval of a multiplication fact. Current theories of basic multiplication processing differ in how they incorporate alternative sources of information in solving basic multiplication problems with patterns that influence performance in self-report conditions.

Models of Direct Retrieval of a Multiplication Fact

Generally, retrieval based theories of basic mathematical cognition can be classified into two types (Ashcraft, 1992, 1995). Structural retrieval models implicate features of the cognitive system to account for latencies and error patterns (Campbell, 1995; Campbell & Oliphant, 1992). Other models suggest that experiential factors influence the cognitive organization of mathematical facts to account for the same effects

Structural-based retrieval models

A prominent theory in this category is Campbell's Network Interference Model (Ashcraft, 1995; Campbell, 1994; 1995). In this model, when a problem is presented, memory codes (called nodes) corresponding to all the addition and multiplication facts are activated to varying degrees. A combination of activation and inhibition among these nodes is used to explain how a particular node eventually reaches a critical threshold and is made available to output mechanisms.

The arithmetic memory codes involve two components: a magnitude code and a physical code. The magnitude code represents the approximate size of the answer to a basic fact. This magnitude code primes physical codes with answers of a corresponding size. Physical codes can be conceptualized as a node that contains all of the relevant information concerning a given fact, consisting of the operand pair, operation sign and the answer. For example, 6×7 is represented as $[(6,7)(\times)(42)]$. Each node receives excitatory input determined by the similarity of the presented fact to the physical representation in each node. This excitatory input is combined with excitatory input from the magnitude code. For example, presenting the fact 7×8 causes physical codes for all multiplication

problems and all 7-operand and 8-operand nodes to receive activation. Excitatory input from the magnitude code also causes all facts with answers close to the size of the answer (i.e., 56) to be activated as well. While a particular node receives excitatory input, it subsequently inhibits the activation of other nodes. Over time, the strength of excitatory and inhibitory inputs gradually approach equilibrium for most of the nodes. A response is generated when one node reaches a critical threshold level of activation and is available for output.

In Campbell's computer simulation of this model, the problem size effect primarily results from the influence of magnitude codes. Larger problems are more similar in magnitude than are smaller numbers. So more multiplication nodes of a similar magnitude are activated in response to a presented fact with a large answer (i.e., 56) than are activated in response to a fact with a smaller answer (i.e., 6). To support this notion, Campbell (1995) refers to research demonstrating that choosing the larger number of a presented pair is harder for bigger numbers (i.e., 54 versus 56) than for smaller numbers (i.e., 3 versus 5; Dehaene, 1989). Thus, larger numbers are less distinct in magnitude and activate other problems with answers of a similar size more strongly than do smaller numbers. This process increases the inter-node interference on larger problems. Resolving such interference requires more input, consequently increasing the time taken for any one node to reach the critical threshold for larger problems. Subsequently, latencies and errors increase for larger problems, resulting in the problem size effect. However, although magnitude effects serve as explanations for the problem size effect, other latency and error patterns are explained using different features of direct retrieval.

In Campbell's (1994, 1995) model, tie problems are answered more quickly than corresponding non-tie problems because they are intrinsically easier. Tie problems have stronger associations to other tie problems (i.e., 6×6 , 7×7) than to other members of the same operand family (i.e., 6×7). Therefore, tie problems constitute a distinct family of problems. Because the number of tie problems is small (i.e., only 10 problems in all), the interference generated within the family is less than the interference generated by the families of non-tie problems (i.e., 20 problems, 10 for each operand). It is not clear in Campbell's model how the tie problems become so closely associated. No experiential factor or processing mechanism is proposed as an explanation for why 6×6 is more closely associated with 7×7 , than it is to 6×7 . Campbell is more clear, however, in identifying a source for the strong associations among 5-operand problems. The 5-operand problems are highly associated with each other, and less so with other problems because of experiential factors. Five-operand answers are a common counting string (e.g., 5, 10, 15, 20). Early and pervasive exposure to the 5-operand table creates strong bonds between members of the operand family. This notion is supported by empirical evidence that suggests that frequent errors to 5-operand facts are the answers to other 5-operand facts (i.e., $5 \times 6 = 35$, Campbell & Graham, 1985). Thus, interference and association are considered to be potential factors in patterns of latencies and errors.

In Campbell's model, errors made during the production of an answer are not only due to activation patterns among the nodes but also reflect format specific associations at the encoding stage. For example, operand-naming errors are a result of reading-based associations. Campbell notes that operand position is usually preserved in operand

intrusion errors (i.e., $6 \times 9 = 64$ and not $6 \times 9 = 46$). He suggests that at some level of encoding, the problem is read as a two-digit number, with the first number representing the ten's value, and the second digit representing the one's. Thus, the operands for 6×9 are encoded as the number "sixty-nine". Operand-naming errors occur when the format specific encoding of the operands as digits intrudes into the answer. As mentioned before, this is a departure from a retrieval based (i.e., activation and inhibition) explanation for naming errors, deviating from a purely structural account of error and latency patterns. That is, errors and latencies are not necessarily a reflection of the organization of nodes in long term memory alone, but also may be due to encoding and production components (Ashcraft, 1995).

Although Campbell's (1995) network-interference model can account for many latency and error patterns in basic mathematical fact performance, some aspects of the model are limited. For example, the model is based on a computer simulation designed to mimic human performance and although the simulation can produce some patterns typical to people, it overestimates and underestimates certain types of effects and errors (Campbell, 1995). Specifically, the model underestimates operand-related errors related to both operands (i.e., $4 \times 8 = 24$ occurs because $4 \times 6 = 24$ and $3 \times 8 = 24$). The model especially underestimates these double-related errors when they are not of the same magnitude as the correct answer. This underestimation may indicate that although the magnitude code calculation in the model is required to replicate some aspects of human behaviour, it is a mathematical device that may not be analogous to actual cognitive functions (LeFevre et al., in press). The model is also limited in its ability to bridge the

gap between child and adult performance. No mechanisms in the model explain how the memory codes are acquired throughout development (LeFevre et al., in press). For example, children learn basic facts by relying on strategies such as counting (Siegler, 1988). However, Campbell's model is a retrieval-based theory and does not include a means by which non-retrieval information can activate a node, or an explanation of how non-retrieved facts are eventually retrieved (Ashcraft, 1992, LeFevre et al., in press). Other retrieval models have attempted to reflect a degree of continuity in child-to-adult performance that is missing in structurally-based retrieval models (Ashcraft, 1987, 1992, 1995).

Learning-based retrieval models

Ashcraft's (1987) Network Retrieval Model is typical of learning based-retrieval models. Ashcraft proposed that basic addition and multiplication facts are organized in memory in an interconnected network. Information about a fact, including the answer, is accessed and retrieved by spreading activation. Problems and their answers are associated, but the associative strength differs across problems. The strength of association between any given problem and its answer is a function of the frequency with which the problem has been experienced and practised. Hence, the smaller problems (i.e., 2×3 , 3×5) are experienced first, and often are most strongly associated with their answer (Ashcraft, 1987, Siegler, 1988). Problems learned later, or practised less often are less strongly connected to their correct answer. In this case, retrieval involves spreading activation "triggered" by the operands and the answer provided (in the case of a verification task). The most highly activated node in memory is the response selected as

correct. Stronger connections between the answer node and the problem often result in faster retrieval of the answer, thus causing the problem size effect (Ashcraft, 1987)

In Ashcraft's model, neighbour problems (i.e., 6×7 , 6×8) are more strongly associated than more distant problems (i.e., 6×2). Errors result when an incorrect answer node is activated, revealing the associations between facts. For example, problems that are closely associated may activate the wrong answer on occasion, especially if the correct answer is not strongly associated with the fact. Tie and 5-operand problems are less error prone and faster than problems of a similar magnitude because they are experienced more often in learning and, therefore, develop strong connections with their answers (Ashcraft, 1987). The 5-operand problems also develop strong associations among each other because of the experience with the common counting-by-fives string. Such an association results in the common error pattern noticed for 5-operand problems, in which the errors are often operand-related errors from the 5-operand family.

Ashcraft's model has been criticized because it is unable to explain why frequency effects observed during acquisition of mathematical facts are still a factor in adult performance (Ashcraft, 1995). Also, little empirical support has been found to support the effects of frequency (Ashcraft, 1995). In children's textbooks, smaller problems are always more frequent than larger problems, but ties and 5-operand problems are not more common than facts of a similar size (Ashcraft & Christy, 1995). It could be argued that 5-operand problems are distinct in that components of the answer (i.e., all have either a 5 or a zero in the one's position of the answer) are similar, creating commonality among the problems that manifests as an operand-related familiarity effect. However, this

explanation fails to account for the close association among tie problems. Regardless, Ashcraft has incorporated experiential and developmental factors as considerations in the cognitive structure of basic arithmetic facts that is lacking in purely structural models such as Campbell's. Such consideration for the continuity of arithmetic experience from childhood to adulthood encompasses important development issues and has influenced other theories of basic mathematic fact performance (Campbell & Graham, 1985, LeFevre et al., 1996).

Summary

Direct retrieval models of basic arithmetic processing can account for most major empirical patterns of performance such as the problem-size effect and error patterns. However, when participants are asked how they solved a problem, they indicate non-retrieval solutions on some trials (LeFevre et al., 1996, in press). The validity of self-report methods as indices of actual cognitive processes involved in answering basic problems is important to consider because the perception of non-retrieval procedures is the only indication that a fact is not directly retrieved (but see Smith, LeFevre, & Hiscock, 1995, Thibodeau, 1993). Potentially, a person's perception of solving a problem using non-retrieval processing may be an artifact of the requirement to report a solution method (Campbell, 1995). Self-report methods may bias participants to report solution procedures that are not related to the actual retrieval of the answer (LeFevre et al., 1996, in press). A participant who perceives a need to be accurate in their response may choose to rely on non-retrieval procedures to solve problems, particularly for problems that have weak associations with the corresponding answer (Siegler, 1988). Also, participants may

check directly retrieved answers for more difficult problems by comparing them with answers derived using other methods (LeFevre et al., 1996, in press). Thus, longer latencies could result from the participant comparing the answers from various sources (Siegler, 1988). Alternatively, participants in self-report conditions may be reporting information unrelated to how they actually solved the problem (Campbell, 1995; Dehaene, 1992). Instead of indicating how the answer to a presented problem was actually retrieved, participants could be reporting information related to what they know about a fact. This semantically related information would be activated upon presentation of a fact but would be unrelated to the retrieval of an answer in long term memory. Longer latencies could reflect the additional processing required to semantically access related information about the presented fact. Thus, self-report methods may influence the participant to consider very different information from that which they might consider without such a requirement.

LeFevre et al. (in press) contend that the longer latencies in their study, compared to studies that did not have a self-report requirement, may be a result of differences in a speed versus accuracy emphasis in the task instructions and not because of self-report methods. They note that longer latencies are common in the production task literature where experimenters have not placed an emphasis on speed. For example, Zbrodoff and Logan (1991) did not emphasize speeded response in the participants' instructions and they reported mean latencies of 1323 ms. Although these researchers used an unusual combination of verbal response and button press in their response format, it is unlikely that this difference was the sole contributor to the increase in latencies (LeFevre et al., in

press) Rather, participants who were instructed to respond quickly, but also accurately, may be slower and less error prone than participants who are instructed to respond quickly, but with less emphasis on accuracy requirements. For example, LeFevre et al. note that they asked participants to respond as quickly as possible, but to be accurate as well. These instructions may have been more similar to those used by Zbrodoff and Logan than to instructions given by other researchers, such as Campbell (1994, Campbell & Graham, 1985, Campbell & Oliphant, 1992) who placed more emphasis on speed.

LeFevre et al. (in press) also suggest that factors such as sample choice and outlier deletion may contribute to the faster latencies reported in some studies. For example, Miller et al. (1984) drew their sample from a graduate student population. These participants likely have had more mathematical experience and ability than the undergraduates used in experiments with slower latencies such as Zbrodoff and Logan (1991) and LeFevre et al. (in press). Furthermore, Miller et al. (1984) and Campbell and Graham (1985) statistically deleted outliers from their data. The deletion of trials with long latencies would reduce the overall mean latencies reported. Neither LeFevre et al. (in press), nor Zbrodoff and Logan (1991) deleted outliers. This combination of instruction differences, the experience of the participants and outlier deletion provides alternative explanations for the notion that self-report alone was responsible for the latency discrepancy between these studies.

Speed/Accuracy Tradeoffs

The issue of speed and accuracy tradeoffs in the instructions given to participants is the crux of LeFevre et al.'s defence of the validity in self-report procedures. Speed of

response is important because the task enables an attempt to examine a highly practised and well-learned process (Ashcraft, 1992). Thus, direct retrieval of an answer to a presented problem should be a fast process in comparison to a less-direct solution method. If participants are relatively slow, then the researcher cannot conclude that other unrelated cognitive processes have not intruded on performance (Campbell, 1995). However, there is a trade-off in terms of speed because accuracy is another important consideration in basic arithmetic performance. A researcher must be certain that a participant is involving the arithmetic solution process and not merely guessing at the answer. Guessing may result in a large number of errors that may be a result of different mechanisms than those responsible for errors made in retrieval attempts. The empirical literature offers different perspectives on how the demands for both speed and accuracy are balanced by researchers according to their particular theoretical interests.

The studies by Campbell (1995; Campbell & Graham, 1985; Campbell & Oliphant, 1992) tend to emphasize speed of response over the accuracy of response. In this body of research, participants are instructed to respond as quickly as possible and are assured that it is understandable that errors will be made. The presentation of stimuli is controlled by the experimenter and encourage rapid responses. Also, the participant is periodically reminded of the need for speed throughout the experiment. Campbell states that a major consideration in his decision to emphasize speed over accuracy is to prevent opportunity for the participant to access information other than a directly retrieved answer. By forcing the participant to go fast, he argues that the participant is biased to using only information gathered from a fast, direct retrieval method. For this assumption to be correct, all

educated adults must be able to directly retrieve an answer to multiplication facts. This view is consistent with many retrieval and multiple procedure models of basic arithmetic cognitive processing but is challenged by LeFevre et al. (in press).

As speed increases, the error rate increases as well. In studies with slower latencies, such as LeFevre et al. (in press), error rates are in the 3-5% range. For studies with faster latencies, error rates were in the 8-10% range (Campbell & Oliphant, 1992; Campbell & Graham, 1985; Campbell, 1994). A high error rate is desired by some experimenters interested in the association among basic facts in long term memory (Ashcraft, 1992; Campbell, 1995) as errors are assumed to reveal those items that are closely related (Ashcraft, 1992, 1995). Accordingly, if participants are not encouraged to go fast, error rates may be so low as to hinder observations of the association among facts (Ashcraft, 1992). Again, the assumption is that errors made by participants are a result of the interconnected organization of the retrieval network and the nature of the direct retrieval process (Campbell, 1995, Campbell & Graham, 1985). However, in speeded conditions, a high error rate is predicted by non-retrieval theories as well (LeFevre et al., in press). Many non-retrieval procedures, such as derived facts and multiple counting, are slower than direct retrieval (LeFevre et al., in press). In speeded conditions a participant who occasionally relies on a non-retrieval process to solve a multiplication fact may be more likely to err when carrying out the steps of the procedure. The resulting increase in errors may reflect premature responses and flawed procedures and not only weaknesses in the associations between answers and facts. Consistent with this view, LeFevre et al. (in

press) found that more errors were made on problems in which participants reported using derived facts or multiple counting.

The Need for Online Reports of Procedure Use

The assumption that participants have knowledge of their non-retrieval solution procedures raises an interesting point concerning the need for self-reporting after the presentation of each fact. If participants are aware of their solution procedures, then one way to avoid the criticism that self-reports influence performance is to ask about procedure use immediately after the production task is finished. However, immediate (or on-line) reports are needed to address the criticism that participants may be reporting knowledge about arithmetic that is unrelated to the direct retrieval of a fact (Campbell, 1995). By further removing the reports of solution procedure from the actual solving process, the possibility of intruding information from sources not involved in the actual solution process may increase. Also, it is possible that participants cannot reliably report procedures accessed automatically (Cooney & Ladd, 1992; LeFevre et al., in press; Russo et al., 1989). That is, a lasting perception of non-retrieval procedures may not be available to participants, especially for fast procedures (Cooney & Ladd, 1992; Russo et al., 1989). For example, Russo et al. (1989) found that participants tended to forget aspects of their behaviour when retrospective reports were compared with on-line verbal reports. Thus, participants may forget more of the non-retrieval procedures they used if asked post-task than they might if asked during the task (LeFevre et al., in press).

Present Research

In this study, two main issues in performance on the production task in basic multiplication were considered: First, the effects of task instructions on latencies and errors were examined. Participants were asked to complete two production tasks in different conditions. In one condition, instructions emphasized accuracy of response. In another condition, an emphasis was placed on speed of response in the instructions. I hypothesized that speeded instructions would cause participants to respond more quickly, but also to make more errors relative to performance when instructions emphasized accuracy. Second, the effects of self-reports of procedure use on latencies and errors were examined. Participants were divided into two groups. One group was required to self-report their procedure use during both of the production task presentations and the other group did not have a self-report requirement. The performance of participants who must self-report procedure use in the two instructional conditions was compared with that of participants who did not have such a requirement. The self-report requirements were not expected to affect latencies and error patterns. Thus, slower latencies in self-report studies may be due to the type of instruction and not to the self-report procedure itself (LeFevre et al., in press)

Method

Materials

Mathematics Skills and Experience Questionnaire. Participants completed a brief questionnaire concerning their attitudes toward math. A modified Mathematics Skills Questionnaire used by LeFevre, Kulak, and Heymans (1992) was used (see Appendix A).

Participants rated their mathematical skill on a seven-point Likert scale where 1 indicates negative affect, or low perceived skill, and 7 indicates positive affect or high perceived skill. Sample questions include ratings of skill in basic and complex mathematics (e.g., calculus, algebra), degree of avoidance of situations involving math, and nervousness in situations involving math. The ratings were summed to yield a total affective score where larger scores indicate more positive affect toward mathematics. In addition, participants answered questions concerning their age, sex, educational background and previous educational training in mathematics. For example, participants were asked to indicate where they received their education, the type of arithmetic instruction they received, and their university major. Total time to complete this survey was about 5 minutes.

The Production Task. The set of multiplication problems included all possible combinations of single-digit multiplication problems from 2×2 to 9×9 presented twice in each presentation ($n=128$). The 0- and 1-operand problems were excluded from the production task to make this study comparable to Campbell (1995). Campbell omits 0- and 1-operand problems because empirical evidence suggests that these problems are solved using fast and efficient rules (Ashcraft, 1992). Because of the potential confound of fast rule use in a paradigm attempting to examine answer retrieval, researchers often exclude these problems (Campbell, 1995; Campbell & Oliphant, 1992; Campbell & Graham, 1985). Order of items within each list was randomized, with each of the two presentations of a problem appearing only once in each half of the list. Each participant completed the production task twice, once with instructions that emphasized speed and once with instructions that emphasize accuracy. Thus, two separate stimulus lists were

created. The order of presentation of the speed and accuracy conditions was counterbalanced across participants. As well, half the participants completed the production tasks with the additional requirement to self-report their solution method.

Skill test. Participants completed the Addition and the Subtraction-Multiplication subtests of the French Kit as a measure of arithmetic fluency (French, Ekstrom, & Price, 1963). Each subtest of this pencil-and-paper task consists of two pages of problems, for a total of four pages. Participants were required to solve as many problems on a page as they could in a two minute limit. The addition subtest consisted of three 2-digit numbers arranged in a column. The subtraction and multiplication subtest required participants to complete a row of subtraction alternating with a row of multiplication. The subtraction problems consisted of two 2-digit problems arranged in a column. The multiplication problems consisted of a 2-digit problem to be multiplied by a single digit. An addition subscore was combined with a subtraction/multiplication subscore to get a total fluency score. Total time to complete the entire French Kit was about 10 minutes.

Survey of Non-retrieval Strategy Use. Participants indicated how frequently they use non-retrieval processes when solving basic multiplication, addition and subtraction facts (see Appendix B). Participants were asked to indicate the extent to which they use non-retrieval procedures to solve basic multiplication, addition and subtraction problems. As well, participants were asked to indicate the degree to which they use procedures such as derived facts, counting, rules, tricks (e.g., songs, rhymes, etc...) and checking to solve basic arithmetic problems. Participants were also asked to indicate other procedures they

may use that were not mentioned in the survey. Total time to complete the survey was 10 minutes.

Procedure

Participants were tested individually in a single session lasting approximately one hour. Prior to the participant's arrival, he or she was randomly assigned to a particular condition. Each participant was then seated in front of a computer monitor. After the completion of the Math Attitudes and Experience Survey, participants were given either the speed or accuracy instructions for the first presentation of the production task.

In the speeded condition, participants were given the following instructions which were similar to those used by Campbell (1995):

"You are being tested on how quickly you can solve simple multiplication problems. First, you will see an asterisk in the centre of the computer screen. The asterisk will begin to flash. This signals that the asterisk will be replaced by a single digit multiplication problem like '3x4'

I would like you to say the answer as quickly as you possibly can, without making any mistakes. Occasional mistakes are normal when people go fast, so do not be too concerned if you make a mistake. It is important that you respond as quickly as possible."

In the accuracy condition, instructions comparable to LeFevre et al. (in press) were used. Participants were told:

"You are being tested on how accurately you can solve multiplication problems. First, you will see an asterisk in the centre of the computer screen. The

asterisk will begin to flash. This signals that the asterisk will be replaced by a single-digit multiplication problem, like '3x4'

I would like you to say the correct answer as quickly as you can.

Occasional errors are normal, but please try to avoid making mistakes. It is important that you respond as accurately as possible."

All participants, regardless of instruction condition, were given a short practice session before beginning each of the production task presentations. Halfway through each block, the experimenter reminded participants of the speed (or accuracy) requirements of that particular presentation. The experimenter initiated all trials according to a similar procedure used by Campbell (1994). An asterisk was presented for one second to indicate the fixation point for the stimuli. This asterisk flashed off and on over a one second SOA. The problem appeared on what would have been the third flash, with the operation sign appearing at the fixation point. The problem remained on the screen until the participant responded. Upon the detection of a verbal response, the problem disappeared and the computer recorded the response latency. The experimenter then recorded the response as an answer or a failure of the voice activated relay.

Half the participants (n=32) completed the two tasks with the additional requirement to self-report the solution procedure used to solve each problem. Participants were given either the speed or accuracy condition instructions with the following addition to the initial instructions:

"After you say the answer, the words 'How did you solve the problem?' will appear on the screen. I would like you to explain how you arrived at your

answer. For example, you might 'just know' or 'just remember' the answer. If so, tell me you remembered or retrieved the answer.

Remembering is just one way to solve a multiplication problem.

Sometimes people figure out the answer by changing, or simplifying the problem.

Please tell me exactly how you solved each problem. If you used more than one solution approach, please tell me everything that you did."

Participants were then reminded of the speed (or accuracy) requirements of the task and the trials began according to the procedure outlined above. Once an answer response was made and recorded, a cue stating "How did you solve the problem?" appeared at the fixation point. Participants were encouraged to report as much as possible about how they solved each problem and the experimenter recorded their responses. Once the solution procedure was recorded, the next trial began according to the description above.

In all conditions, the stimuli were presented on a monochrome monitor using white digits on a black background. Participants wore a set of headphones. A microphone connected to a computer recorded verbal latencies to the millisecond. Before donning the headphones, participants were instructed to say each answer clearly, and to avoid non-answer vocalizations (e.g., um, er) because the computer cannot distinguish between an answer response and other noises. The experiment was controlled by an 80286 IBM type computer. Participants were presented with 10 practice problems to exemplify the particular condition before the experiment began. The next trial was initiated as soon as the experimenter pressed the enter key. The average inter-trial interval was 2.6 seconds.

for the no self-report condition and 4.6 seconds for the self-report condition. The longer time for the self-report condition reflects reporting by the participant and recording by the experimenter of a solution procedure. No feedback about accuracy or response time was provided to the participants. Total time to complete each block of trials was about 10 minutes.

The Mathematics Skills and Experiences Questionnaire was given before the mathematical tasks to prevent experiences with the arithmetic used in the study from influencing ratings of mathematical affect and skill. The French Kit was given between each presentation of the production task. The Survey of Non-retrieval Strategy Use was given last, to avoid biasing the participant's self-reports during the production task.

Coding of Solution Procedures

Initially, solution procedures were coded by the experimenter into three categories: retrieval, strategy and invalid. A solution procedure was classified as retrieval if the participant said "I just knew that" or "memory" and did not report some other solution procedure. A solution procedure was classified as a strategy if the participant responded with a response that did not fit the requirements for a retrieval classification. For example, a response was classified as a strategy if the participant said "I used the nine-times rule" or "I remember 6×6 and added 6". Also included in this category were responses of guessing the answer. Guessing does not necessarily reflect a retrieval of an answer, but may indicate the failure of a strategy solution procedure. If a response was classified as a strategy, the experimenter recorded all the details concerning how the answer was obtained, to allow a more detailed classification of responses later. A solution

procedure was classified as invalid if the answer response was a non-answer vocalization, or a failure of the vocalization to be registered by the computer

Participants

Sixty-six introductory students at Carleton University (32 males, 34 females) participated in this study. Two participants (both female) did not complete the study because they were unable to finish the production task. All participants received credit toward partial fulfilment of a course requirement, or a \$12 honorarium. Participants ranged in age from 19 to 41 years of age, with a median age of 21. Participants were all high-school graduates and were mainly educated in Canada (see Table 1). Overall, most participants were Arts majors. The two groups (self-report and no self-report) were equivalent on these demographic measures. Participants were recruited for this study in a slightly different manner than has been done in previous studies. Usually, the recruiting poster states that participants will be required to complete a "simple arithmetic (or single-digit multiplication) task". To prevent inadvertent screening of participants, the recruiting notice for this study stated that participants would be required to complete a "simple task". No specific mention of arithmetic or multiplication was made until the participants were fully informed by the consent form. Participants were free to discontinue the experiment at any time. However, only two participants failed to finish the study. Both were females (ages 19 and 20), who attempted the first presentation of the multiplication production task, but were unable to answer even basic multiplication problems. Both participants agreed to take part in a separate study to develop a screening battery for dyscalculia. The presence of two individuals unable to complete a series of single-digit

multiplication problems in a single study suggests that the removal of the term "arithmetic" from the recruiting notice may have been successful in preventing less-mathematically inclined students from participating

Results and Discussion

Participant Skill

The composite total score on the French Kit Measure of Arithmetic Skill was computed for the 64 participants who completed the experiment. The total was computed by adding the total number of correct problems completed for both the addition and subtraction/multiplication subtests. The mean score of participants in this study was higher than the mean found in a separate study recruiting from the same participant pool (Daley, 1996). Furthermore, comparison of scores of the two report groups (no self-report, self-report) indicated a potential between-group difference was noted. The average arithmetic fluency score for the no self-report group was higher than the mean for the self-report group (see Table 2). A t-test comparing the difference between the means failed to reach significance, $t(62)=1.57$, $p=.12$. However, when the groups are compared on an individual-to-individual basis, the differences between the two report groups becomes apparent. The ranges for each group are truncated (see Appendix C). No one in the no self-report group scored below 50, whereas 3 participants scored below 50 in the self-report group. Nine individuals scored above 100 in the no self-report group, whereas only 4 scored in that range in the self-report group. Between-group differences in arithmetic skill may have important implications for the results of this study. In this

particular study, measures of skill are highly correlated with all features of arithmetic performance such as latencies, errors and percent use of retrieval (see Table 3)

The Mathematics Skills and Experiences Questionnaire was used to calculate two composite scores for each participant. The first was a self-evaluated measure of arithmetic skill. There was no difference between the no self-report and self-report groups in terms of self-rated skill (see Table 2). The second composite score reflected the participants' ratings of the degree to which arithmetic was emphasized in both primary and secondary schooling. Participants' ratings of educational emphasis were similar for both report groups. However, self-rated measures are not as highly correlated with actual performance measures as the more objective French Kit scores (see Table 3).

Report requirement (self-report, no self-report) was a between-subjects measure, and although all attempts were made to ensure complete random assignment to conditions it is still possible that the two groups were not equivalent in terms of arithmetic skill. Thus, skill as measured by the French Kit was used as a covariate on subsequent analyses of errors and latencies of the production task.

The Production Task

Each group (no self-report and self-report) had 4096 trials per instruction condition. Of the total 16,384 trials, 569 (3.5%) were invalid due to failure of the voice key, non-answer vocalizations by the participant (e.g., um, er, pfff) or failure of the participant to respond within the 15 second time limit allowed. Also, a trial was scored as invalid if the participant started to say one answer, but changed their answer to another number before they finished stating the first. If a complete number was stated, it was

recorded as the stated answer regardless of whether the participant stated another afterwards. Only 15 trials were lost because the participant failed to respond within the 15 seconds allowed for an answer. Three of these trials were in the no self-report condition and 12 in the self-report condition. The remaining trials were available for analysis. Unless otherwise noted p values were less than .05.

Because participants completed the multiplication task twice, it is important to determine whether there was an effect of presentation order on latencies and errors. Half the participants in each report group had the speed instruction condition followed by the accuracy instruction condition. The other half of the participants had the accuracy instruction condition followed by the speed condition. It is generally expected that recent exposure to the problems in the first task should facilitate performance in the second task. Fewer errors and faster latencies should be observed when a particular instruction condition (speed or accuracy) is second in order compared to when the same condition is presented first. Thus, order of presentation was used as a control variable for subsequent analysis of errors and latencies.

Errors

Percentage. The percentage of errors made by participants was analysed in a 2 (group: no self-report, self-report) by 2 (order: speed/accuracy or accuracy/speed) by 2 (instruction: speed, accuracy) ANOVA, with repeated measures on the last factor. There was no significant difference between the two report conditions for percentage of errors made, $F(1,62)=.61$, $MSe=.013$ (8.8% and 6.4% for no self report and self-report respectively). There was a significant effect of instruction, $F(1,62)=49.7$, $MSe=.00133$.

Thus, participants made more errors when the instructions emphasized speed (9.6%) than when accuracy was emphasized (5.6%). Figure 1 presents the percent errors according to the group and instruction conditions. Overall, the error rate in the accuracy condition in this study was comparable to the error rates of 3-5% obtained by LeFevre et al (1996). The overall error rate for the speed condition is only slightly higher than the error rates of approximately 7.6% that are reported in Campbell (1996) and the 8.4% error rate reported in Campbell (1994).

Order effects and errors. A significant interaction between presentation order and instruction condition, $E(1, 60)=22.6$, $MSe=.013$ was obtained. Figure 2 shows the mean percentage of errors for both the speed-then-accuracy and accuracy-then-speed presentation orders. Overall, if the speed condition was presented first, the percentage of errors made by participants in that condition was larger than the percentage of errors made when the speed condition was presented second. However, if the accuracy condition was presented first, the percentage of errors made in that condition was only slightly larger than when the accuracy condition was presented second.

Errors and operand-family. Figure 3 shows the percentage of errors according to operand-family for each report group and instruction condition. These graphs indicate that the typical problem-size effect for the percentage of errors is replicated in each condition. Generally, there is a rapid increase in errors as operand increases. Five-operand problems are an exception to the trend for the percentage of errors to increase over operand. The percentage of errors made on 5-operand problems was relatively low compared to the 4- and 6-operand problems. The pattern of results is similar for both

report conditions (no self-report and self-report) in each of the instruction conditions (speed or accuracy). However, there is a difference between the patterns of errors between instruction types. Not only is the percentage of errors higher in the speed condition than in the accuracy condition, but the increase in errors over operand-family is much more dramatic in the speed condition. This pattern is consistent with the ANOVA results that indicated a main effect of instruction, but not of report condition for errors. Error patterns were consistent whether participants were required to self-report solution procedures or not.

The speed condition for the self-report group is unique from the other conditions in terms of the number of invalid trials (see Table 4a and 4b). The number of invalid trials was not consistent across operand family for this group as it is for the other three conditions. Rather, the number of invalid trials increases in a pattern reflective of the problem-size effect. The invalid trials reflect non-answer vocalizations (e.g., um, er) and failures of the voice key. An increase in invalid trials over operand family could possibly be due to the fact that, generally, participants take longer to respond to larger problems (Ashcraft, 1992). As a result, there would be more opportunity to have a non-answer vocalization ruin the trial for larger, slower problems. However, the increase in the number of invalid trials is not apparent for the accuracy trials, which have much longer average latencies than the self-report speed group. This suggests that the increase in invalid trials is not simply a matter of having more of an opportunity for a non-answer vocalization to ruin a trial. If invalid trials are included with errors across operand family, the percentage of errors made by participants in the no-self report and self-report groups

are almost identical in the speed condition (see Figure 4). The differences between the two report groups in the accuracy condition are slightly reduced as well. This between-group difference in the pattern of invalid trials for the speed condition may be related to between-group differences in participant skill. If the self-report group was slightly lower skilled than the no self-report group, the requirement to go fast may result in prematurely stating an answer, only to change it mid-stream. This tendency would be greatest for more difficult problems, especially for those that the participant usually solved with a relatively slow procedure (e.g., derived fact), or problems with a weak problem-answer association. If the answer was incomplete before the change, this would be recorded as an invalid trial. Participants may also be more likely to state non-answer vocalizations (e.g., um, err) for the more difficult problems when they feel pressure to respond quickly. Alternatively, the between-group difference in the number of invalid trials may be related to the self-report requirement. Possibly, self-reporting solution procedures may encourage participants to make non-answer vocalizations when the instructions emphasize speed.

Types of errors. Errors were classified into mutually exclusive categories as either operand-related, operand-unrelated, operation, non-table and operand-naming. An error was classified as an operation error when the answer would be correct if the operation had been addition. An error was classified as operand-related if it was the correct answer to a problem in the same operand family for either of the two operands. Operand-unrelated errors referred to an answer that was a potential answer to a multiplication problem, but was not related to either operand. An operand-naming error occurred if one, or both of

the operands were stated in the answer (and it was not a operand-related error). Finally, if the error did not fit in any of the previous categories it was classified as a non-table error

The proportion of error types were generally similar to that of other studies (see Table 5) The majority of errors were operand-related errors. Across conditions, an average of 70-77% of errors were operand-related, which is similar to the 72% recorded by Campbell (1995) and the 76% observed by LeFevre et al. (experiment 1, in press). In general, errors made by participants in all conditions were highly related to the operands, supporting the view that multiplication knowledge is interconnected. The percentage of errors was similar to that observed by other researchers, except there was a smaller percentage of operation errors made by the participants in this study compared to comparable studies. More importantly, however, are the similar types of errors made in the speed and accuracy condition, suggesting that participants in the speed condition are not differing in the type of errors made in the speed condition. Rather, participants just make more errors in the speed condition compared to the accuracy conditions. As well, requiring participants to report solution procedures does not yield a markedly different pattern of errors than observed when participants do not have a self-report requirement.

Summary. The pattern of errors made by participants in this study is similar to the pattern of errors reported in other studies. In particular, the pattern of errors made by participants when they were in the accuracy condition was similar in the pattern of errors made by participants in the study by LeFevre et al. (in press). However, the speed instruction condition was similar to Campbell (1994, 1995) in the percentage of errors. This supports the hypothesis that instructional differences may be a factor in the

discrepancy in error rates between LeFevre et al. (in press) and studies such as Campbell (1995). Error rates and types are similar across report condition supporting the hypothesis that self-reporting solution procedures does not result in significant differences between-groups in the number or type of errors. However, instructional emphasis (speed versus accuracy) has a considerable impact on error rates. When participants are instructed to answer quickly, they make significantly more mistakes than when they are asked to place an emphasis on accuracy. As well, there was a smaller difference in the percentage of errors made in the speed and accuracy instruction conditions if the accuracy condition was first. The change of instructional emphasis does not seem to have a dramatic effect on the types of errors made by participants. Fewer errors are made when accuracy is emphasized, but the general proportions of the types of errors is consistent whether participants are required to respond fast or to respond accurately.

Latencies

Only correct trials were included for analysis of latencies. The average latency for each instruction condition (speed, accuracy) was calculated for each participant, and then averaged across report groups and instruction conditions.

Order effect and latencies A 2 (report: no self-report, self-report) by 2 (order: speed/accuracy, accuracy/speed) by 2 (instruction: speed, accuracy) ANOVA revealed a significant difference in latencies between the speed and accuracy conditions, $F(1, 60)=53.7$, $MSe=27988$. Thus, participants were, on average, 219 ms faster in the speed condition than they were in the accuracy condition (1091 ms and 1310 ms, respectively). As well, participants were significantly faster in the no self-report condition than in the

self-report condition, $F(1, 60)=4.39$, $MSe=448360$ (1083 ms and 1200 ms, respectively).

Figure 5 presents the mean latencies according to report and instruction conditions.

Latencies in the speed condition are longer relative to those obtained by researchers who emphasize speed, such as Campbell and Graham (1984; 826 ms). The latencies in the present study may be longer because no outlier deletion protocol was applied to this data, whereas Campbell and Graham eliminated long trials. On average, latencies in the accuracy condition are comparable to that of LeFevre et al. (1996; 1335 ms) and Zbrodoff and Logan (1991; 1323 ms). However, the participants in the self-report, accuracy condition are 100 ms slower than LeFevre et al.'s participants who were similarly reporting their solution procedures (1996, experiment 2). No outlier deletion was performed on the LeFevre et al.'s data, so the discrepancy between latencies may reflect more fundamental differences between the two subject pools, such as participant skill.

Order effects and latencies. There was a significant interaction between presentation order and instruction condition, $F(1, 60)=12.2$, $MSe=341394$. Participants were significantly faster in the speed condition if the speed condition was second in order (1002 ms and 1217 ms respectively, see Figure 6). The accuracy condition latencies were not affected by presentation order

Summary. Average latencies in the self-report condition were over 200 ms higher than that of the corresponding no self-report condition. As well, participants were significantly faster in the speed condition. There were no higher-order interactions between report, instruction or order factors. At first glance, these observations support the hypothesis that self-report requirements result in significantly increased latencies.

However, it is surprising that there is not a corresponding reduction in the rate of errors. Presumably, the longer (203 ms) time taken by participants in the self-report speed condition should result in significantly more accurate performance than participants in the no self-report speed condition. Although the self-reporting participants in the speed condition took significantly longer than their no self-reporting counterparts, they were just as error prone as the faster group. If the two groups were indeed equivalent in terms of skill, the extra time taken by the self-reporting group in both conditions should result in a drop in the error rate, such as the drop noticed when participants are asked to be accurate. In both the no self-report and self-report group, participants required an average of 221 ms longer to respond in the accuracy condition and corresponding error rates for each group dropped compared to the speed condition. Longer latencies in the self-report condition, with a speed emphasis that does not result in better performance, could be a result of differences in participant skill.

Skill and Latencies. Skill (French Kit Scores) was partialled out of a 2 (order speed/accuracy, accuracy/speed) by 2 (report, no self-report, self-report) by 2 (instruction speed, accuracy) comparison. Skill accounted for a significant proportion of variance, $F(1, 61)=37.13$, $MSe=278281$. Once skill had been partialled out from the total variance in latencies, the main effect of report was no longer significant, $F(1, 61)=2.00$, $p=.16$, but the main effect of instruction was still significant $F(1, 62)=45.19$, $MSe=33229$. The absence of a significant difference between the self-report and no self-report groups, once the variance due to skill has been removed, supports the notion that report requirements do not influence latencies. Rather, it is the speed and accuracy emphasis in the

instructions that consistently impacts on both latencies and error patterns. Thus, how participants are instructed to balance speed and accuracy requirements is an important consideration when making cross-experimental comparisons.

Problem-size effects The mean latencies for each operand-family was computed to determine the problem-size effect for each of the report groups in each instruction condition (see Figure 7). Generally, latencies are expected to increase as operand size increases, with exception to the 5-operand and tie problems (Ashcraft, 1992). Interpreting the graph reveals the problem-size effect is replicated for the latencies. Latencies increase as operand-family increases. The exception to the trend occurs at the 5-operand family. The two report groups show similar patterns in both conditions. Generally, the increase in latencies is more extreme for the accuracy condition than the speed condition. This is consistent with the observations made by LeFevre et al. (in press) who found that the slope of the problem-size effect in their experiment using a self-report procedure was larger than that of studies emphasizing speed, such as Campbell (1994, 1996). Thus, the increase in the slope of the problem-size effect may be related to the speed/accuracy emphasis of the instructions. However, it also appears that, overall, the self-report group has a much steeper increase in latencies than the no self-report group. LeFevre et al. (1996) has suggested that the increase in latencies with operand size may reflect the use of non-retrieval solution procedures, a factor which is also related to skill. Possibly the steeper curve for the self-report group may reflect a skill-related difference, such as a higher proportion of non-retrieval use in the self-report group. The difference within the groups according to instruction, however, may reflect the reaction of participants to the

specific response demands of the task (i.e., to be accurate or to be fast). Possibly, participants could be using more, or different non-retrieval procedures in the accuracy condition, resulting in the larger slope for latencies across problem-size for that condition.

In order to examine problem-size, ties, 5-operand and differential slopes for the ties and 5-operand problems, regression coefficients predicting latencies from these variables were calculated for each individual. The regression approach predicts a particular individual's performance from a set of variables (e.g., problem-size, ties, 5-operand, tie and 5-operand slopes). As a result, the coefficients representing a number of effects can be compared across groups to determine if there is a systematic difference between report groups and instruction conditions.

Regression Analysis. Individual regressions were calculated for each participant on all correct problems using the following variables: product, ties (coded as -0.5 and 0.5 for tie and non-tie problems, respectively), fives (coded as -0.5 and 0.5 for five-operand and non-five operand problems, respectively), product x ties (to test for differential slopes on tie and non-tie problems), and product x five-operand (to test for differential slopes on five-operand and non-five-operand problems). For each participant, all five variables were entered simultaneously into a regression to predict his or her latencies for the speed and accuracy condition. The mean of the resulting coefficients was tested for significance for each group and then compared across groups. Mean coefficients and significance values for both report groups and both conditions are shown in Table 6.

Overall, the coefficients reflect a significant slope over product, a tie and 5-operand advantage, and differential slopes for ties (product x tie interaction) for all four

conditions. The slope of the 5-operand problems (product $\times$ fives interaction) was significantly different than the slope for non-five problems for all groups except the no self-report group in the speed condition. In all groups, R-squared values were significantly different from zero, indicating that the variables account for a significant proportion of variance in latencies.

The self-report group had significantly larger coefficients for product, product $\times$ ties and product $\times$ fives than the no self-report group (see Table 7). Specifically, the product coefficient was larger for the self-report group than for the no self-report group, reflecting that the problem-size effect for the self-report group was larger than for the no self-report group. As well, the difference between the slope of tie and non-tie problems was larger for the self-report group. Similarly, the difference between the slope of 5-operand and non-fives was larger for the self-report group than the no self-report group. LeFevre et al. (in press) observed that longer latencies are associated with the use of non-retrieval procedures. Non-retrieval procedures are most often used to solve problems with larger products resulting in the problem-size effect. Ties and 5-operand problems are less often solved using non-retrieval procedures and consequentially have smaller slopes over product than non-tie and non-five operand problems. Thus, larger coefficients in the self-report group may indicate a higher proportion of non-retrieval solution procedure use for non-tie and non-five operand problems among the participants in that group. Because non-retrieval procedure use is associated with skill, between-group differences in skill could be a factor in the differences between the coefficients for the self-report and no self-report group.

In order to determine if participant skill was a factor contributing to the between-group difference in coefficients, scores on the French Kit Measure of Arithmetic Fluency were partialled out of a 2 (group: self-report, no self-report) by 2 (order: speed/accuracy, accuracy/speed) by 2 (instruction: speed, accuracy) ANCOVA. In all cases, skill accounted for a significant proportion of variance. Once variance due to skill was partialled out, the differences between the no self-report and self-report groups on all coefficients was no longer significant. Thus, once the skill differences between the self-report and no self-report groups was controlled for, the patterns of performance were similar.

The ANOVA revealed that the accuracy instruction condition had significantly larger coefficients than the speed instruction condition for all variables except the 5-operand coefficient. Specifically, participants had a larger problem-size effect for the accuracy condition than the speed condition. LeFevre et al. (in press) observed a larger problem-size coefficient than observed in studies that did not have a self-report requirement (Campbell, 1995; Campbell & Oliphant, 1984). The larger product coefficient for the accuracy condition than the speed condition suggests that the instruction emphasis may account for much of the differences in the problem-size effect between studies. As well as a larger product coefficient, the ties advantage over non-ties is larger for the accuracy conditions than the speed condition. Also, the difference between the slope of tie problems and non-tie problems and the difference between the slope of 5-operand and non-fives is larger for the accuracy condition. Since tie and 5-operand problems are usually directly retrieved they are solved faster than corresponding

problems of the same magnitude. Consequently, tie and 5-operand problems show an overall advantage in mean latencies and less of an increase in latencies over problem-size which is larger in the accuracy condition than the speed condition. The coefficient variables also accounted for more variance in the accuracy condition than the speed condition (R-squared). Partialling out the variance due to skill does not affect the significance of the difference between coefficients for the speed and accuracy conditions. Thus, instructional emphasis on accuracy results in a different pattern of latencies than an instructional emphasis on speed.

There was a significant interaction between presentation order and instruction for coefficient reflecting the differential slope of the 5-operand problems. The difference between the slope of the 5-operand and non-five problems is smaller for the speed condition when the speed condition was second in presentation (see Appendix D). This observation suggests that the difference over problem-size between 5-operand and non-fives is reduced in the speed condition when preceded by an accuracy condition. Thus the 5-operand advantage in the speed condition is reduced when participants have been exposed to an accuracy condition. As well, there was a significant three-way interaction between order, report and instruction for the product, product by ties and product by fives coefficients. In all cases, there seems to be a larger reduction in the difference between the slopes of the product, tie and 5-operand problems in the speed condition when speed was preceded by an accuracy condition. This tendency was most apparent for the self-report group. The three-way interactions are not reduced when the between-group differences in skill are controlled. Potentially, the effects of prior exposure to accuracy

practice on subsequent performance may have different implications for self-report and no self-report groups.

Summary The problem-size effect pattern typically found in mathematics research was replicated in this study. Speed instructions generally result in a smaller problem-size effect than accuracy instructions, with a corresponding decrease in a tie and 5-operand advantage and a decrease in the difference between the slope for tie and non-tie, 5-operand and non-five problems. Once the difference attributable to skill had been controlled, there was no significant difference between the two report conditions for any of the coefficients.

In the speed condition, latencies over operand-family do not show the dramatic increase that occurs in the accuracy condition. However, errors increase in the speed conditions over operand family. In other words, participants sacrifice being correct when asked to go fast. Campbell (1994) suggests that forcing individuals to respond quickly prevents them from relying on information unrelated to the actual retrieval of an answer (i.e., how a problem was once solved, or learned initially). LeFevre et al. (in press) suggest that participants may have more than one solution procedure for a given fact, including both retrieval and non-retrieval procedures. Depending on various response requirements, individuals may choose different solution procedures, or be unable to finish the procedures they have initiated. Thus, Campbell's model predicts that when accuracy and speed trials are compared, the speed condition would reveal significantly less reporting of non-retrieval solution procedures than the speed condition. LeFevre et al. (in press) suggest that solution procedures may change from slower to faster procedures

(such as retrieval), but participants (especially lower-skilled) may still rely on non-retrieval solution procedures. Reliance on non-retrieval would be most likely among lesser-skilled individuals who rely almost exclusively on non-retrieval use, and who may not have strong, direct access to a particular answer.

Procedure Use. To determine the types of procedures used by participants in the self-report condition, the recorded accounts of solution methods for all trials were coded into mutually exclusive categories. If the participant reported “just remembering” or “knew it”, the response was recorded as retrieval. A response was recorded as a conversion to an addition tie fact, if the respondent stated that he or she added for a 2-operand problem. If the participant stated that he or she repeatedly added one of the operands, initiating the count at the operand value (e.g., $3 \times 6 = 6, 12, 18$) and the problem did not belong to the 2-operand family, the response was recorded as a number series procedure (LeFevre et al., in press). The counting string that was used was also recorded. A solution procedure was coded as a derived fact if the answer was obtained by first recalling another multiplication fact and then adding the correct amount to get the answer to the original problem (LeFevre et al., in press). The number of times the operand was added to the base problem to get the correct answer was recorded as well. For example, participants reported using a derived fact and a one count increment (e.g., $6 \times 8 = 5 \times 8, +8$), two count increments (e.g., $7 \times 8 = 5 \times 8, +8, +8$), and three or more count increments. Participants also reported using two derived facts and adding (e.g., $7 \times 6 = 7 \times 3, +7 \times 3$). These were recorded as combinations. If the participant reported using a special procedure for the nine’s based on a rule, then the solution procedure was recorded as a

specific nine's rule (LeFevre et al., in press). Guesses were recorded separately. Finally, if the participant reported using rhymes, songs, visual tricks or other such methods, they were recorded in a miscellaneous category. The percentage of solution strategies for the self-report group in both the speed and accuracy conditions are recorded in Appendix E and summarized in Table 8.

Overall, participants used retrieval on the majority of trials. LeFevre et al. (in press, experiment 1) also found that participants reported direct retrieval on approximately 80% of trials. Derived fact procedures were the second most common procedure type for the accuracy condition. LeFevre et al. (1996) found that 6.3% of trials were reportedly solved using this procedure which corresponds to the 6.1% found in this experiment in the accuracy condition. In the speed condition, the percentage of problems solved using derived facts is somewhat lower than in the accuracy condition, suggesting that participants do not use derived fact solution procedures when required to respond quickly as they do when accuracy is emphasized. In this study, guesses were recorded as a separate procedure. LeFevre et al. (1996) included guesses in the "other" category because they were relatively rare responses. Participants in the LeFevre et al. study were not given guessing as a solution option in the instructions which outlined potential solution procedures. Also, guessing as a solution procedure was reportedly used on 5.7% of the speed condition trials and only 2.1% of the accuracy trials in the present study. If the LeFevre et al. (in press) study had instructions which were comparable to the accuracy condition in this study, guesses would not necessarily be a common solution procedure.

Reported use of non-retrieval solutions dropped slightly from the accuracy to speed trials. However, the number of guesses increased, suggesting that participants may have abandoned some of the longer non-retrieval procedures and guessed instead. Surprisingly, the percentage of retrieval trials did not differ between the speed and accuracy condition. Corresponding to the high degree of similarity between the number of retrieval trials between the speed and accuracy conditions, the correlation between the percentage of retrieval in both conditions was very high, $r(32) = .92$, $p < .001$.

Participants may have abandoned slower non-retrieval procedures for faster non-retrieval procedures when asked to go fast, or were more likely to err. Overall, participants take more time for all solution procedure types in the accuracy condition, with the notable exception of the derived fact with one-count increment, procedures (Appendix F). This supports the notion that participants may be aborting slower procedures for guesses when asked to respond quickly. The similar latencies between the derived fact latencies in both the speed and accuracy conditions, especially for the 1-count procedures, suggests that participants do not always stop using longer derived facts in the speed condition. Possibly, for some individuals, a derived fact procedure may be so automatic that the individual is unable, or unwilling to abandon the procedure to guess. As with the 0- and 1-rule procedures observed in other experiments, some non-retrieval procedures are faster than retrieval when speed is emphasized. Reverting to the tie addition fact is a fast and accurate procedure, as is repeated addition by 5's (Appendix G). In fact, the number-series procedures that are most likely to be abandoned in the speed condition are the slower and error prone 4-, 6-, and 9-counting strings. Derived fact, one-count

procedures are similarly highly accurate (86.9% and 91.3% in the accuracy and speed condition, respectively). This may account for why participants in the speed condition maintain the use of a fairly slow procedure. Overall, the largest change in the speed and accuracy conditions is the tendency for participants to guess much more often in the speed condition and at faster latencies.

Guesses. Guesses are second only to direct retrieval solution procedures in the speed condition. Participants guess much more often and more quickly in the speed condition than in the accuracy condition. However, do guesses reflect the failure, or abandonment of a procedure, or are they a form of direct retrieval? Arguably, it could be said that a guess reflects a form of direct accessing of an answer, albeit an incomplete one. Only 43% of the guesses made are correct in the speed condition, suggesting that the association between the problem and answer is not strong for guesses. Even when participants take much longer to make guesses in the accuracy condition, the percentage of correct guesses is only 55%. This suggests that Campbell's (1994) notion that more cycles (i.e., time) will allow the pattern of excitation and inhibition to settle on the correct node may not be true for these problems. Rather, the high degree of errors among the guesses may reflect the abandonment or aborting of a slow procedure when time requirements are perceived to be extended to the limit by participants who don't know the answer via direct retrieval. This idea is supported by the following observations. First, the percentage of retrieved solution procedures for both speed and accuracy conditions is consistent. This suggests that individuals who can retrieve usually use this method in

both conditions. Participants allow more time for direct retrieval in the accuracy condition, thus improving the percentage of correct responses. Second, the percentage of non-retrieval solutions reported by participants decreased in the speed condition compared to the accuracy condition. Participants were especially less likely to report using procedures that were slow and error-prone in the accuracy condition when the task required speed. The rate of guessing and errors increased proportionate to the number of non-retrieval trials that were dropped in the speed condition. Third, guessing is moderately fast compared to other non-retrieval procedures in the speed condition. However, the percentage of incorrect guessed answers was very high. Less than half of the guesses in the speed condition resulted in correct answers although guesses produced mean latencies almost three times longer than retrieval trials.

Summary. The percentage of retrieval trials is consistent across speed and accuracy conditions. However, the number of guesses increased in the speed condition compared to the accuracy condition. These results indicate that participants may be abandoning longer and more error-prone procedures to guess when required to respond quickly. However, these guesses were often wrong, suggesting that for guessed answers, there was only a weak association between the problem and answer. However, there is no on-line information concerning procedure use to determine the pattern of solution methods for the no self-report group. Instead, the Survey of Non-retrieval Strategy Use may offer insights into the patterns of non-retrieval procedure use for this group.

Survey of Non-retrieval Strategy Use. The actual percentage of retrieval trials was computed for all individuals who were in the self-report group. The percentage of

retrieval trials was correlated with the remaining questions from the Survey of Non-retrieval Strategy Use and the French Kit Measure of Arithmetic Fluency to develop a measure of the retrieval use that is independent of actual on-line reports

Of all of the questions concerning multiplication solution procedures from the survey, three were highly correlated ($p < .05$) with the online measures of retrieval for both the accuracy and speed conditions in the multiplication production task. These questions included M1 (What proportion of the time do you automatically remember the answers to simple multiplication problems?), M5 (How often do you use repeated addition or counting on multiplication problems with 3 in them?), and M7 (How often do you use repeated addition or counting on other multiplication problems?). These three questions and the scores from the French Kit were entered in a hierarchical regression to predict the percentage of retrieval for the accuracy condition and then for the speed condition. The French Kit scores were entered on step one of the regression, followed by M1, M5 and M7.

For the accuracy condition, the regression coefficient for the composite of questions was significantly different from zero, $R(31) = .64$. Similarly, in the speed condition the regression coefficient was significant, $R(31) = .62$. However, once the French Kit and M1 were entered into the regression, neither M5 or M7 accounted for any additional variance.

Summary Self-reporting participants were consistent in their solution procedures across instruction conditions. In the speed condition, it appears that some slow and error-prone non-retrieval procedures used in the accuracy condition are abandoned for faster processes

or guesses. The correlation between the combination of the French Kit scores and M1 with the reported percentage of retrieval supports the notion that the two questions reflect different aspects of skill. The French Kit is a measure of speed of arithmetic processing and M1 is a measure of direct retrieval use.

General Discussion

In this study, two main issues in performance on the production task in basic multiplication were considered. First, this study found that there was a significant effect on latencies and errors related to whether instructions emphasized speed or accuracy. Second, this study found that once between-group differences in skill were controlled, there was no significant effect of self-reporting solution procedures on latencies and errors. Thus, this body of research has important implications for the methodology of arithmetic performance and the validity of self-reports.

Instructions that emphasize speed result in faster latencies and more errors than studies that emphasize accuracy. However, there does not seem to be support for the notion that speed prevents, or deters, participants from considering non-retrieval information (Campbell, 1994). Rather, the reported percentages of retrieval use are highly consistent across instruction types. When participants are asked to place an emphasis on speed they do not report switching to direct retrieval to solve problems. Instead, participants use faster procedures or guess more often than they do when accuracy is emphasized, resulting in a reduction of the problem-size effect for latencies and an increase in the problem-size effect for errors.

A larger problem-size effect for errors was observed in the speed condition than in the accuracy condition. Participants report much more guessing to solve problems, especially those usually solved with slower non-retrieval procedures. Guesses are often wrong and consequently, the percentage of errors increases more dramatically for errors in the speed condition than the accuracy condition. However, the self-reports of solution procedures does not support Campbell's suggestion that speed prevents participants from accessing non-retrieval information. Fast and accurate non-retrieval procedures are maintained in the speed condition. Most of the other non-retrieval solution procedures are still not as fast as direct retrieval. Therefore, the problem-size effect in the speed condition still reflects non-retrieval procedure use as suggested by LeFevre et al. (in press). As well, the high percentage of errors made when guessing suggests that these problems may have a weak direct problem/answer association.

In the speed condition, participants were faster for all solution methods including direct retrieval. This reduction suggests that instructions may influence participants to allow less time for a solution procedure to settle on a particular answer, resulting in a corresponding rise in errors for all solution types. However, do subsequent errors made in the speed condition necessarily reflect processes involved in the direct retrieval of an answer (Ashcraft, 1992), or do the errors reflect associations related to a particular solution method? For example, Campbell (1995) suggests that errors reflect patterns of inhibition and activation directly related to the presented problem. Thus, the errors reflect direct retrieval processes. However, if an individual normally solves a problem using non-retrieval, it is possible that the errors they make are related to the solution procedure. For

example, if a participant reports they solve 6×8 by deriving the answer from 6×6 and adding by 6's, then the errors made may reflect errors related to this process

Although guesses, on average, produce mean latencies that are three times as long as direct retrieval, there seems to be no benefit of this extra time in terms of correctness. This pattern of results is compatible with LeFevre et al.'s (1996) suggestion that there may be multiple solution modes available, each associated with a particular 'confidence criterion'. Participants may use more comfortable and reliable non-retrieval procedures when they feel they have the time to complete them. When the speed requirements change, participants may switch to faster, less accurate procedures, such as guessing. This would be particularly true for very slow, and less reliable (in terms of attaining a correct answer) procedures. For example, there was considerably less reported use of derived facts with two counts ($6 \times 8 = 36$, $+6$, $+6$) in the speed condition than the accuracy condition, but this procedure was very slow (mean = 6043 ms) and was only correct 79% of the time in the accuracy condition. Possibly, individuals who usually normally rely on non-retrieval solution procedures have a weak direct connection between a given problem and answer.

The problem-size effect for latencies were similarly affected by the emphasis on speed or accuracy in instructions. Participants are much faster in the speed condition than they are in the accuracy condition. In the speed condition, latencies maintain the general problem-size effect pattern noticed in the literature, but latencies do not show the same rate of increase over operand that is noted in the accuracy condition. The self-reports of solution procedures suggests that when participants switch to guessing in the speed

condition, they are often wrong. The latencies of error trials are not typically included in calculations of mean latencies. Consequently, many problems usually solved using slower non-retrieval procedures are excluded from analysis, reducing the problem-size effect for latencies in the speed condition.

The differences in the latency and error patterns in performance suggests that participants are sensitive to the manner and degree that speed/accuracy trade-offs are emphasized in instructions. The effect of instructional factors supports the hypothesis that between-study differences in latency and error patterns could be attributable to instructions. Future comparisons between studies will have to consider how participants were instructed within each study. Also, the assumption that errors reflect aspects of the processes involved in direct retrieval processes may not be true. The types of errors made by participants may reflect associations related to their accustomed solution procedures. Although the percentage of errors differs according to instructional emphasis, the types of errors made are consistent in both the speed and accuracy conditions. Potentially, such consistency implies the similar processes are responsible for the types of errors made and suggests that participants are not using dramatically different solution processes in the speed condition than in the accuracy condition. As well, the consistency in the patterns of latencies and errors between the self-report and no self-report groups suggest that report requirements are not influencing performance patterns.

Compared to instructional factors, requirements to self-report solution procedures had little impact on performance on the production task. Both the no self-report and self-report groups made similar percentages and types of errors in each instruction conditions.

There was a tendency for self-reporting participants to make more invalid responses in the speed condition than the other report group, which may reflect that self-reports encourage non-answer vocalizations under speeded conditions. Alternatively, the difference in the percentage of invalid trials for the self-report group may reflect differences in participant skill between report groups. Lower skilled individuals may make more non-answer vocalizations when pressured to go fast, compared to higher skilled individuals. The truncated ranges for the French Kit scores indicates that the self-report group had more low skilled individuals than the no self-report group.

Overall, the self-report group was slower in responding than the no self-report group. However, once between-group differences attributable to skill (French Kit scores) were controlled, the differences between the no self-report and self-report groups were no longer significant. The lack of an effect of report requirements is consistent with the observations by Daley (1996). Daley used participants from the same participant pool as the present study to examine the effects of report requirements on simple subtraction performance. Daley reported that there was no difference in latencies between the self-report and no self-report conditions.

The absence of an effect of self-reports on latencies and errors supports the hypothesis that self-reporting solution procedures was not a factor in the longer latencies noted by LeFevre et al (1996) compared to studies such as Campbell (1994) and Campbell and Graham (1984). Rather, instructional emphasis are more plausibly the sources for the differences between these studies. However, the differential effects of presentation order for the self-report conditions were not reduced when skill was

controlled. Thus the consequences of presentation order may have different implications for self-report conditions than for no self-report conditions

This study found that the order of presentation of the speed and accuracy conditions had a significant effect on performance. Generally, previous exposure to multiplication tends to improve subsequent performance in terms of lowering latencies and reducing errors. However, the effect of practice is differential. Performance in a speeded instruction condition after a previous accuracy condition was faster and more accurate than when a speed condition was completed first. However, performance in an accuracy condition is not similarly facilitated when preceded by a speeded instruction trial. This suggests that the adage "perfect practice makes perfect" is an apt descriptor for the order of presentation effect observed here. The accuracy trial may facilitate subsequent performance because participants are able to fully complete non-retrieval procedures. Guessing is reduced and participants are much more accurate, regardless of solution procedure type. Potentially, the connection between a problem and answer, or a problem and solution procedure may become stronger in accuracy trials. With speed trials, however, the participants may not be able to strengthen associations between the problem and answer, or problem and solution procedure because time pressure inclines participants to abandon or abort solution procedure and guess. Thus, there is not the same degree of facilitation for trials subsequent to a speeded trial. The facilitation of the initial accuracy condition seemed to be greater for the self-report group. Potentially, the longer time between trials, or the introspection required when reporting the solution procedures in the

accuracy condition allowed the participants to gather more feedback on the accuracy of a stated answer, and to self-correct for the subsequent exposures to the speeded condition.

The presence of a differential practice effect according to instruction type may have further implications for arithmetic studies, particularly ones with multiple or blocked presentations. Specifically, the effect of speeded versus accuracy practice on performance in basic multiplication needs to be explored further. For example, this study does not address the duration of the facilitation of previous exposure to multiplication problems. As well, the implications of the practice type (speeded performance or accuracy emphasis) for subsequent instruction types needs to be more fully examined (i.e., Does speeded practice facilitate speeded trials?). Also, the potential differences between report groups in potential feedback concerning answer accuracy from introspective sources should be examined. The implications of practice may have implications for solution procedures. For example, do many presentations (with or without feedback) strengthen direct associations between a problem and answer, or does repeated presentation strengthen associations between a problem and a solution procedure?

If self-reports are accurate indications of how participants are actually solving multiplication problems, then theories of basic arithmetic will have to incorporate the concept that not all adults directly retrieve the answers to basic problems. Rather, latency and error patterns may in part reflect the use of non-retrieval and retrieval solution procedures. Since non-retrieval procedure use is associated with skill, any methodology which deters lower-skilled individuals from participating may yield different results than methodologies that do not deter lower skilled individuals. Also, the educational

background of the participants may impact on performance patterns. For example, LeFevre and Liu (in press) found that graduate students educated in China used different solution procedures than graduate students educated in Canada. Chinese students had error and latency patterns that reflected procedures specific to how they learned multiplication. Thus, previous educational experiences and participant skill influences latencies and errors and must be considered in cross-experimental comparisons.

The diversity in how participants solve multiplication studies allows for continuity between child and adult performance. In Canada, children learn to solve multiplication by using counting strings (Siegler, 1988). For simple problems, a child quickly associates the problem with the answer. However, for larger problems, counting strings are more error-prone. Consequently, the association between the problem and answer is reduced because errors are also associated to the problem. Possibly, to reduce errors and improve latencies, some children may eventually use procedures such as derived facts to solve larger problems. For example, instead of repeatedly adding 8's to solve 8×6 , a child may learn to retrieve an answer to problem close to the presented problem that they already know (e.g., $5 \times 8 = 40$) and repeatedly add only enough to reach the correct answer (i.e., $40 + 8 = 48$). If children break down problems into smaller components, then aspects of the frequency and primacy of exposure during learning would be maintained.

Ashcraft (1992) suggested that faster latencies for smaller problems, ties, and 5-operand problems were related to the higher frequency of, early exposure to these problems in textbooks. If some children use problems they already know to solve larger problems, then the higher frequency of smaller problems, ties and 5-operand problems

would be maintained. Specifically, a child who used the derived fact procedure of " $6 \times 8 = 5 \times 8, + 8$ " would experience directly retrieving the answer to " 5×8 " every time " 6×8 " was experienced. LeFevre et al. (in press) reported that participants usually directly retrieve the answers to 4- and 5-operand problems and that the 4- and 5-operand problems are often used in derived fact procedures. Eventually, a child may directly associate a procedure with a given problem, and may maintain a reliance on non-retrieval solutions into adulthood. Thus, experiential factors and individual abilities may influence the types of solution procedures that are used and maintained and account for the diversity among adult performance on basic multiplication.

If adults use non-retrieval solution procedures, the assumption that latencies and errors solely reflect processes involved in the direct retrieval of an answer is violated. Although research into developing alternatives to introspective measures of non-retrieval use is ongoing (see Thibodeau, 1993; Smith et al., 1995), the best indication that an individual does not directly retrieve an answer to a multiplication fact is the perception that they did something different. The observation in this study that instructional factors and not self-report requirements may influence latency and error patterns supports the validity of the self-report method. Thus, models of direct retrieval of multiplication must consider methods by which non-retrieval solution procedures are incorporated in basic multiplication knowledge.

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Table 1

Demographic Characteristics of Participants for Each Report Condition

	No Self-report	Self-report	Overall
N size	32	32	64
Age: Median	22	20	21
Range	19-41	19-27	19-41
Major (%): Psychology	18.8	21.9	20.3
Arts	65.6	43.8	54.7
Sciences	3.1	25.0	14.1
Undeclared	12.5	9.4	10.4
Education (%):			
Elementary			
Canada	81.3	84.4	82.8
Asia	18.7	9.4	14.0
Other	--	6.4	3.2
Secondary (%):			
Canada	87.5	90.6	89.1
Asia	12.5	6.3	9.3
Other	--	3.1	1.6

Table 2

Comparison of Means and Standard Deviations for Each Report Condition on the French
Kit Measure of Arithmetic Fluency and Self-rated Measures

	No Self-report	Self-report	Daley (1996)
Addition Subtest			
Mean	35.1	32.1	
Std. Deviation	8.7	7.9	
Subtraction/Multiplication Subtest			
Mean	51.2	45.3	
Std. Deviation	17.1	13.9	
Composite Total			
Mean	86.2	77.4	75.7
Std. Deviation	24.9	20.2	27.1
Self-rated Skill (Maximum Score 42)			
Mean	27.0	27.0	
Std. Deviation	6.7	7.1	
Educational Emphasis (Maximum Score 14)			
Mean	9.0	10.0	
Std. Deviation	2.9	2.1	

Table 3a

Correlations Between Measures of Arithmetic Skill and Performance Measures for the Accuracy Condition (n=64)

	F.Kit	S R Skill	Ed Emph	A Latencies	A Errors
French Kit	--				
Self-rated Skill	58*	--			
Educational Emphasis	19	31*	--		
Accuracy Latencies	- 63*	- 48*	- 09	--	
Accuracy % Errors	56*	49*	23	- 33*	--

* $p < .01$

Note: F.Kit=French Kit, S.R.Skill=Self-rated Skill, Ed Emph =Educational Emphasis,
A.Latencies=Accuracy Latencies, and A Errors=Accuracy Percentage of Errors

Table 3b

Correlations Between Measures of Arithmetic Skill and Performance Measures for the
Speed Condition (n=64)

	F Kit	S.R Skill	Ed Emph.	S.Latencies	S.Errors
French Kit	--				
Self-rated Skill	58*	--			
Educational Emphasis	19	31*	--		
Speed Latencies	- 60*	- 49*	- 14	--	
Speed % Errors	50*	51*	.07	- 49*	--

*p < .01

Note F Kit=French Kit, S R.Skill=Self-rated Skill, Ed.Emph =Educational Emphasis,
 S Latencies=Speed Latencies, and S Errors=Speed Percentage of Errors.

Table 4a

The Percentage of Errors and Invalid Trials Across Operand Family In The Speed
Condition

Operand Family:	No Self Report			Self-Report		
	Errors	Invalid	Total	Errors	Invalid	Total
2	2.3	4.2	6.5	1.2	3.9	5.1
3	8.3	3.2	11.5	5.7	3.9	9.6
4	9.6	2.7	12.3	9.3	3.8	13.1
5	4.2	1.4	5.6	3.8	2.6	6.4
6	13.1	3.1	16.2	10.1	4.6	14.7
7	15.1	3.1	18.2	12.2	4.8	17.0
8	16.5	3.0	19.5	13.0	7.1	20.1
9	19.3	3.6	22.9	16.1	6.8	22.9

Table 4b

The Percentage of Errors and Invalid Trials For Each Operand Family In The Accuracy Condition

Operand Family	No Self Report			Self-Report		
	Errors	Invalid	Total	Errors	Invalid	Total
2	1.2	3.2	4.4	3	3.3	3.6
3	3.4	2.9	6.3	2.3	2.7	5.0
4	5.8	2.1	7.9	3.6	3.4	7.0
5	2.9	2.3	5.2	1.9	2.9	4.8
6	9.6	3.6	13.2	5.5	3.3	8.8
7	10.0	2.6	12.6	5.2	3.5	8.7
8	12.4	2.8	15.2	6.0	4.3	10.3
9	12.6	3.1	15.7	8.5	4.7	13.2

Table 5

Percentages of Types of Errors for Each Report Group in Each Instruction Condition

	No Self-report		Self-Report	
	Speed	Accuracy	Speed	Accuracy
	(n=429)	(n=283)	(n=353)	(n=165)
<u>Error Type:</u>				
Operand-related	76.7	76.7	74.8	69.7
Operand-naming	2.6	2.8	2.5	4.8
Operand-unrelated	12.6	11.3	15.3	15.8
Operation	2.1	1.1	1.1	3.0
Non-table	6.1	8.1	6.2	6.7

Table 6

Mean Coefficients and Standard Deviations for Regression Analysis of Latencies

		No Self-report		Self-report	
Variable		Speed	Accuracy	Speed	Accuracy
Product	Mean	8**	12**	17**	24**
	SD	6	9	21	24
Tie	Mean	213**	258**	327**	426**
	SD	272	306	484	402
5-operand	Mean	270**	253**	362**	539**
	SD	617	246	482	711
Product x Tie					
	Mean	8**	14**	20**	25**
	SD	9	15	31	26
Product x Fives					
	Mean	3	5*	12*	19*
	SD	10	11	29	36
R squared	Mean	21**	25**	23**	27**
	SD	46	50	48	52

*p< .05

**p< .01

Table 7

2 (Report Group: Self-report, No Self-report) x 2 (Instruction: speed, accuracy) ANOVA
and ANCOVA (Partialling Skill) F Values for Main Effects of Order, Report and
Instruction

Main Effect	ANOVA			ANCOVA		
	<u>Order</u>	<u>Report</u>	<u>Instruction</u>	<u>Order</u>	<u>Report</u>	<u>Instruction</u>
R squared	00	74	4.92**	31	13	4.29**
Product	81	6.32**	27.25**	3.46	3.72	27.25**
Ties	1.38	3.38	5.20**	4.31*	1.44	5.20*
5-operand	07	2.31	2.74	1.22	.68	1.88
Prod. x Ties	1.59	5.71*	6.09**	5.22*	3.18	6.09**
Prod. x Fives	.8	4.00*	4.50*	1.29	2.16	4.46*

*p < .05

**p < .01

Table 8

Summary of Reported Solution Procedure Types, Latencies and Error Rates For Speed and Accuracy Conditions (n=32)

	Speed			Accuracy		
	<u>Percent</u>	<u>Latency</u>	<u>Correct</u>	<u>Percent</u>	<u>Latency</u>	<u>Correct</u>
Retrieval	82.0	1023	94.5	82.5	1168	97.5
Number Series	1.3	885	100.0	1.0	1332	100.0
Repeated Addition	1.0	1747	79.5	1.5	3700	91.0
Derived Facts	4.0	3730	83.1	6.1	4146	88.8
Nines Rule	0.4	2642	100.0	1.0	2782	100.0
Guesses	5.7	2886	43.2	2.1	4021	55.3
Miscellaneous	0.8	1096	97.1	1.6	1642	98.4
Invalid	4.7	--	--	3.7	--	--

Note Percent refers to the percentage of trials in which the procedure was reported out of the total 4096 trials. Latency refers to the mean latencies (ms) for all trials solved using a procedure. Correct refers to the percentage of trials in which the answer was correct.

Figure 1. Percent errors for each report group in speed and accuracy conditions.

Figure 2. Percent errors for speed and accuracy conditions according to presentation order.

Note: Order 1 is Speed then Accuracy
Order 2 is Accuracy then Speed

Figure 3. Percent errors for report groups in speed and accuracy conditions over operand family.

Figure 4. Percent errors and invalids for report groups in speed and accuracy conditions over operand family

Figure 5. Mean latencies (ms) for report groups in speed and accuracy conditions.

Figure 6. Mean latencies (ms) for speed and accuracy conditions according to presentation order.

Note: Order 1 is Speed then Accuracy
Order 2 is Accuracy then Speed

Figure 7. Mean latencies (ms) for report groups in speed and accuracy conditions over operand family.

Appendix A

Mathematics Skills and Experience Survey

Subject # _____

Age _____

Gender _____

Major in _____

University _____

Please indicate the country where you received your

a) elementary school education: Canada Other: _____

b) high school education Canada Other: _____

For each of the following questions, please circle a number on the scale provided

1 How would you rate your level of basic arithmetic skill (e.g., skill at calculations)?

1 2 3 4 5 6 7

very low skill

moderate skill

very high skill

2 How would you rate your level of mathematical skill in more complex areas of mathematics (e.g., calculus, algebra)?

1 2 3 4 5 6 7

very low skill

moderate skill

very high skill

3 Do you ever actively AVOID situations involving mathematics?

1 2 3 4 5 6 7

always try

sometimes try

never try

4 How often do you find that situations involving mathematics make you nervous?

1 2 3 4 5 6 7

almost always

sometimes

never

5 How important do you feel that mathematics will be in the development of your career?

1 2 3 4 5 6 7

not important

moderately

very important

6 How difficult was mathematics for you in high school?

1 2 3 4 5 6 7

extremely

moderately

not at all

7 How much emphasis was placed on learning basic arithmetic facts (e.g., $3+4$, 5×9) by your teachers in elementary school?

1 2 3 4 5 6 7

little

moderate

strong

8 How much emphasis was placed on rote learning (or memorizing) basic arithmetic facts by your teachers in elementary school?

1 2 3 4 5 6 7

little

moderate

strong

Appendix B
Questionnaire on Procedure Use

Questionnaire on Procedure Use

We are using this questionnaire to learn about how people solve simple arithmetic problems (e.g., 3×2 , $5 + 4$, $6 - 1$). Please estimate your answers to the following questions as accurately as possible and circle the responses that best fits your estimate

Multiplication

1 What proportion of the time do you automatically remember the answers to simple multiplication problems?

never rarely sometimes often always

2. Some people use rules to solve multiplication problems. For example, on the problem 1×8 , some people see the 1 and just say "8" because of the 1s rule (any number times one is that number). Or for example, on the problem 0×7 , some people see the 0 and say "0" every time because of the 0s rule (anything times zero is zero).

How often do you use the 1s rule?

never rarely sometimes often always

How often do you use the 0s rule?

never rarely sometimes often always

3. Some people use repeated addition or counting to solve multiplication problems. For example, on the problem 3×2 , some people think "3 + 3", or on the problem 5×4 some people think "5, 10, 15, 20".

How often do you use repeated addition or counting on multiplication problems with a 2 in them?

never rarely sometimes often always

How often do you use repeated addition or counting on multiplication problems with a 3 in them?

never rarely sometimes often always

How often do you use repeated addition or counting on multiplication problems with a 5 in them?

never rarely sometimes often always

How often do you use repeated addition or counting on other multiplication problems?

never rarely sometimes often always

Which problems? _____

4 Some people use solutions called derived facts where they use a problem they know to figure out the answer for another problem. For example, on the problem 6×7 , some people think " $6 \times 6 = 36$, then $36 + 6 = 42$." Or, on the problem 9×6 , some people think " $10 \times 6 = 60$, then $60 - 6 = 54$." How often do you use derived fact solutions?

never rarely sometimes often always

Which derived facts do you use and which problems do you tend to use derived-fact solutions on?

5 Some people use special tricks to solve problems. For example, on the problem 9×5 , some people use this trick: for the 10s digit, they go down one from 5 (to get 4) and for the 1s digit, they subtract $9 - 4 = 5$. They put these numbers together for the answer "45." How often do you use special tricks on problems with a 9 in them?

never rarely sometimes often always

Please describe the special tricks you use _____

6 Overall, please estimate what proportion of the time you solve simple multiplication problems in ways *other than* by automatically remembering the answers?

never rarely sometimes often always

Addition

1 What proportion of the time do you automatically remember the answers to simple addition problems?

never rarely sometimes often always

2 Some people use rules to solve addition problem. For example, on the problem $1+8$, some people see the one and just say "9" because of the 1s rule (any number plus one is the next number). Or for example, on the problem $0+7$, some people see the 0 and say the other number every time because of the 0s rule (anything plus zero is that number).

How often do you use the 1s rule?

never rarely sometimes often always

How often do you use the 0s rule?

never rarely sometimes often always

3 Some people use counting to solve addition problems. For example, for the problem 7×2 , some people count up "8, 9". How often do you use counting to solve addition problems?

never rarely sometimes often always

4 Some people use derived facts to solve addition problems. For example some people use ties (e.g., $7 \times 7 = 14$) to figure out answers to other problems like $7+8$ ($7 \times 7 = 14$, $14+1=15$). How often do you use ties to figure out addition problems?

never rarely sometimes often always

2 OF 2

PM-1 3 1/2 x 4 PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

PRECISIONSM RESOLUTION TARGETS

Some people use other kinds of derived facts. For example, some people use tens facts (e.g., $6+4=10$, $8+2=10$, $7+3=10$) to figure out addition problems. For the problem $6+5$, some people might solve $6+4+1$ instead. How often do you use tens to figure out addition problems?

never rarely sometimes often always

5 Overall, please estimate what proportion of the time you solve simple addition problems in ways *other than* by automatically remembering the answers?

never rarely sometimes often always

Subtraction

1 What proportion of the time do you automatically remember the answers to subtraction problems?

never rarely sometimes often always

2 Some people use known addition facts to solve subtraction. For example, on the problem $16-9$, some people think " $9+7=16$ so the answer is 7." How often do you use addition facts to get the answers to subtraction problems?

never rarely sometimes often always

3 Some people count when they solve subtraction problems. For example, on the problem $8-3$, some people might count "7, 6, 5." How often do you count to get the answers to subtraction problems?

never rarely sometimes often always

4 Some people use derived-facts to solve subtraction problems. Some people use tie problems (e.g., $14-7$) to get the answer. For example, for the problem $13-7$, some people solve it by getting $14-7=7$ then taking one off the 14 and one off the 7 to get the answer 6. How often do you use ties to get the answers to subtraction problems?

never rarely sometimes often always

Some people use other kinds of derived-facts to solve subtraction problems. Some people break the numbers down to make the numbers smaller. For example, for the problem $12-8$, some people take 2 off the 12 to make it 10, then take 2 off the 8 so they get the problem $10-6=4$. How often do you break numbers up to make subtraction problems easier to solve?

never rarely sometimes often always

5 Some people use other special tricks to solve subtraction problems. If you use other solutions for subtraction, how often do you use them?

never rarely sometimes often always

Please describe these solutions and the problems you use them for _____

6 Overall, please estimate what proportion of the time you solve subtraction problems in ways other than by automatically remembering the answers?

never rarely sometimes often always

Is there anything else you often use or other tricks you might have? Please describe them and the problems you use them for

Appendix C

Frequencies Distribution of Participant Scores on the French Kit Measure of Arithmetic Fluency

Score	No Self-report (n=32)	Self-report (n=32)
<50	0	3
50-59	4	2
60-69	6	7
70-79	6	6
80-89	3	7
90-99	4	3
100+	9	4

Appendix D

Means for Regression Coefficients for Product, Product x Ties and Product x Fives to show Two-way and Three-way Interactions for ANOVA

1 Significant Order by Instruction Interaction for Product x Fives Coefficient. $F(1,$

60) 5.4, Mse= 113

	<u>Speed/Accuracy</u>	<u>Accuracy/Speed</u>
Speed	11	4
Accuracy	10	12

Appendix F

Percentages of Correct Solution Strategy Trials for Each Self-Report Group Condition

Solution Type		Speed	Accuracy
Retrieval		94.5	97.5
Addition Tie Fact		100.0	100.0
Number Series Count by	4's	75.0	80.0
	5's	100.0	100.0
	6's	71.4	90.0
	7's	100.0	100.0
	8's	81.8	100.0
	9's	66.7	81.3
Derived Facts	1 count	86.9	91.5
	2 counts	62.5	78.8
	3+ counts	100.0	---
	Combination	64.3	84.8
Nines Rule		100.0	100.0
Guesses		43.2	55.3
Miscellaneous		97.1	98.4

3 Significant Order by Report by Instruction Interaction for Product x Ties Coefficient.

$F(1,60) = 7.41$, $Mse = 181$

	No Self-Report		Self-report	
	Speed/Acc	Acc/Speed	Speed/Acc	Acc/Speed
Speed	10	5	27	13
Accuracy	19	9	23	28

4 Significant Order by Report by Instruction Interaction for Product x Fives Coefficient.

$F(1,60)=5.40$, $Mse=113$

	No Self-Report		Self-report	
	Speed Acc	Acc Speed	Speed Acc	Acc Speed
Speed	7	15	4	9
Accuracy	8	13	0.4	24

Appendix E

Percentages of Solution Procedure Types for Each Self-Report Group (N=4096)

Solution Type		Speed	Accuracy
Retrieval		82.0	82.5
Addition Tie Fact		1.3	1.0
NumberSeries Count by	1's	0.1	0.2
	5's	0.1	0.1
	6's	0.2	0.2
	7's	0.1	0.2
	8's	0.3	0.4
	9's	0.2	0.4
Derived Facts	1 count	3.2	4.5
	2 counts	0.4	0.8
	3+ counts	0.1	---
	Combination	0.3	0.8
Nines Rule		0.4	1.0
Guesses		5.7	2.1
Miscellaneous		0.8	1.6
Invalids		4.7	3.7

Appendix F

Percentages of Correct Solution Strategy Trials for Each Self-Report Group Condition

Solution Type		Speed	Accuracy
Retrieval		94.5	97.5
Addition Tie Fact		100.0	100.0
Number Series Count by	4's	75.0	80.0
	5's	100.0	100.0
	6's	71.4	90.0
	7's	100.0	100.0
	8's	81.8	100.0
	9's	66.7	81.3
Derived Facts	1 count	86.9	91.5
	2 counts	62.5	78.8
	3+ counts	100.0	---
	Combination	64.3	84.8
Nines Rule		100.0	100.0
Guesses		43.2	55.3
Miscellaneous		97.1	98.4

Appendix G

Mean Latencies For Each Solution Strategy Type According to Instruction Condition

Solution Type		Speed	Accuracy
Retrieval		1023	1168
Addition Tie Fact		885	1332
Number Series Count by	4's	1994	3101
	5's	744	1660
	6's	2319	2085
	7's	1183	1364
	8's	1953	4944
	9's	1528	5245
Derived Facts	1 count	3228	3944
	2 counts	7386	6043
	3+ counts	6772	---
	Combination	3319	3376
Nines Rule		2642	2782
Guesses		2886	4021
Miscellaneous		1096	1642

END

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FIN