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FRAMEWORK ANALOGY FOR LATERALLY LOADED
BUILDING FRAMES

by

© Ahmed Mahmoud Abdel-Akher, B.Sc.

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of
Master of Engineering

FACULTY OF ENGINEERING

Carleton University

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BUILDING FRAMES"

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ABSTRACT

In this research study, a 3-dimensional Framework Model has been developed to simulate the elastic behaviour of flat plate structures. The Framework Model consists of columns, and grid systems modelling the slabs. The connection between the columns and the grid systems are considered semi-rigid. Each of the grid systems consists of non-prismatic members, aligned with each row of columns in two orthogonal directions, and prismatic crossing diagonal members in every panel.

The case of square slabs supported by square columns has been considered. The effect of column to slab stiffness on the Framework Model stiffness was investigated by varying the column dimension to slab length ratio, in a range from .05 to 0.20.

The stiffnesses of the model frame elements have been obtained by calibrating the model with a Finite Element Model. Formulas have been developed to compute the element stiffnesses.

The model has been used to analyze 3-dimensional building frames subjected to lateral loads, including eccentric lateral loads. The results obtained for building sways and column moments, are compared to those of the Finite Element Model. For all cases considered, the results are shown to be in excellent agreement.

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CHAPTER 1

INTRODUCTION

1.1 General

Reinforced concrete slabs are among the most widely used structural elements. In many structures, the slab provides a versatile and economical method of supporting vertical and lateral loads (wind and earthquake loads). Slabs spanning in more than one direction fall into two categories: "two-way slabs" which are supported on beams and girders, and "flat slabs" which are directly supported on columns. In the design for relatively light loads, "flat plates" are convenient. These may be defined as flat slabs with no column capitals or drop panels (Fig. 1.1).

Initially, flat plates were mainly used for buildings requiring large open spaces, such as warehouses and factories. Later, flat plates were used for offices and residential buildings. They have the advantage of simple formwork and complete freedom of internal architectural design.

As flat plates have become extensively popular, much research results have been published concerning their behaviour. Only in recent years has the analysis of the complete real structure become possible through the use of finite elements and computers. However, some theoretical and practical difficulties have prevented the general application of this approach in engineering practice. The most common approach to the analysis of such structures has been to idealize the

slab floors into equivalent beams and then use a suitable method of analysis, such as the stiffness method, for the resulting equivalent frames. The success of the idealization depends on the evaluation of realistic effective stiffnesses of the equivalent beams so that the behaviour of the equivalent frame matches that of the real structure.

All of the attempts, carried out to model the slab system, recommend the reduction of the 3-dimensional structure into series of 2-dimensional frames in two orthogonal directions [1, 2, 3, 4, 5, 6]. The general approach does not address directly the coupling between the actions in the two orthogonal directions, so that its adequacy depends on whether the 3-dimensional effect can be ignored or possibly accounted for using an artifice. Some of the 2-dimensional models are suitable only for the analysis of certain types of vertical or lateral loading, but not both.

Recently El-Kafrawy[7] proposed an elastic Finite Element Model to analyze the building as a full 3-dimensional structure. The main idea of this approach has been to obtain a stiffness matrix for the slab system related to the column-slab joint rotations. This matrix is integrated with the column stiffness matrices to generate the overall stiffness matrix of the building. This approach is obviously applicable to any type of loading. It has a significant cost advantage over the full Finite Element analysis. However it still can not be generally used in engineering practice because data preparation is time consuming relative to current procedures.

The work carried out in the course of this thesis is a stage in an attempt to idealize flat plate structures by a 3-dimensional Framework Model applicable to vertical and lateral load analyses. The stiffnesses of the frame elements in the model have been calibrated using the stiffness matrices obtained by the Finite Element study mentioned earlier.

1.2 Review of Previous Work

Hrennikoff [24] idealized the elastic plate by a framework. According to his method, the plate is divided into finite cells (Fig. 1.2.a). Each of these cells consists of prismatic beams arranged in lattice pattern (Fig. 1.2.b). The torsional rigidity of these beams are neglected. The method introduced by Hrennikoff is essentially a finite element method in which the element stiffness coefficients are derived from the lattice analogy.

The Equivalent Frame method recommended by the ACI code has been mainly developed for the analysis of vertical loads. The equivalent frames consist of slab members and equivalent column members. Equivalent columns are further divided into actual column members and attached torsional members (Fig. 1.3) introduced to estimate the actual moment distribution between the adjacent panels in the 3-dimensional column-slab system. The use of the Equivalent Frame method in lateral load analysis is questionable since the code does not recommend nor reject it. Since 1971 several studies, concerned with the validity of the equivalent frame in sidesway analysis, have been published.

Pecknold [1] has suggested that the 3-dimensional building frame be modelled as a series of typical plane frames aligned in the direction of the applied load. The slab is idealized as a beam having a depth equal to that of the slab, and its width is the slab panel width multiplied by a reduction coefficient. To derive these coefficients, a typical interior panel was considered and the column moment was replaced by a load distributed over the area of contact. Different load distributions were selected to simulate the upper and the lower bounds of the stiffness. Allan and Darvall[2,3] followed the same procedure to find the reduction coefficient experimentally. Tables of the coefficients for varying slab aspect ratio and column size were presented.

Recently, Elias [4,5] proposed a 2-dimensional frame model consisting of slab members and unmodified column members to analyze flat plate structures subjected to either horizontal or vertical loads. The analysis of the stiffness was based on the separation of the general state of loading into symmetric and anti-symmetric states. A Finite Element Model of a single panel was used. Compatible deformation of all frames in the direction of bending was assumed.

1.3 Finite Element Model

The Finite Element Model used in the study carried out by El-kafrawy[7] consists of a flat plate floor supported by columns framing in from above and below. The columns framing into the floor are assumed to be fixed at the remote ends, at adjacent storeys. In the analysis, the flat plate floor was divided into plate bending

elements (Fig. 1.4.a) while the columns were frame type elements (Fig. 1.4.b). The material was assumed to be linearly elastic, homogeneous and isotropic. In the analysis, the column cross section was assumed to remain plane after bending and no axial deformation in the column was considered. The slab was rigidly connected to the column. Accordingly, the column-slab intersection was assumed to rotate as a rigid patch around the intersection centerpoint about the X and the Y axes (Fig. 1.5). The objective of the finite element investigation was to develop stiffness matrices of slab systems related to column-slab joint rotations (Fig. 1.6), and to study the parameters affecting the slab rotational stiffnesses. These stiffnesses were found to depend on ratios of the dimensions of the slab panels, $L1/L2$; the column dimensions, $C1/C2$; and the column to slab stiffness ratio, $C1/L1$. The values of the coefficients were proportional to

$$\frac{E t^3}{(1 - \nu)^2}$$

where

E = the modulus of elasticity

ν = Poisson's ratio

t = the slab thickness

The floor stiffness matrices were used to analyze full 3-dimensional building frames. The frame degrees of freedom were selected as two translations and one rotation per floor, Δx , Δy & θ_z (Fig. 1.7), in addition to two rotations per slab-column intersection, θ_x & θ_y .

El-Kafrawy has concluded that there are fundamental differences in the analysis results between his approach and the equivalent beam models cited earlier. Stiffness coupling takes place, theoretically, between every degree of freedom at a floor level. In other words, in the joint locked mode, transfer can take place around a degree of freedom to remote degrees of freedom. A second important distinction between the two methods arises from the fact that the plane frame approaches ignore the three-dimensional effect. Only slabs under unrealistic circumstances, will deform in a one way mode. When a building frame is subjected to a panel loading, the response of the frame is in both directions, and when a joint in this frame is forced to rotate a restraint must be applied in the orthogonal direction at all joints to prevent their rotations.

To illustrate the errors resulting from neglecting the 3-dimensional effect, El-kafrawy presented a numerical example of a 10-storey building (Fig. 1.8) analyzed for lateral and vertical loads. The applied loads are shown in Figs. 1.9 and 1.10. For the lateral load analysis, the lateral sway was compared with the results obtained by the Equivalent Frame method, and other methods provided in the literature, for the typical internal frame indicated in Fig. 1.8. The comparison is summarized in Fig. 1.11 and Table 1.1. For the vertical load analysis, the moments in the slab and the columns were compared with those of the Equivalent Frame method. The comparison is presented in Tables 1.2 and 1.3.

The Finite Element Model was used to provide the necessary data for developing the 3-dimensional framework model proposed in this report.

1.4 Objective and Scope

Apart from El-Kafrawy's research, the body of work relating to flat plate behaviour has not dealt directly with the analysis of 3-dimensional building frames. It is clear to all structural engineers that the idealization of slab systems by 2-dimensional frames is not generally applicable; for example, in the torsional response of the structure to an earthquake. Also, the concept in present codes (American and Canadian) of the Equivalent Frame method is only applicable to vertical load analysis. Recent studies have attempted to idealize the slab system by 2-dimensional models applicable to lateral load analysis. However, these 2-dimensional models do not represent properly the 3-dimensional behaviour of the building frames. Thus, there is nothing available, except the Finite Element method, for engineers to analyze full building frames subjected to wind and seismic loads. The work done in this thesis will enable engineers to use the familiar concept of frame analysis to solve such a problem. The author is in full agreement with the general approach adopted by Hrennikoff, however, it still involves too many degrees of freedom to be of significant practical use. The framework analogy as applied in this thesis is more simple and direct, and potentially more easily applied in practice since it involves fewer members. Thus the building frame will be modelled here using non-prismatic or orthogonal members between the columns, and diagonals across the slab panels. These members are connected to the columns through torsional springs. The stiffnesses of the framework elements will be obtained by direct matching of their general stiffness terms

with those contained in a full finite element stiffness model.

The objective of this study is to develop a 3-dimensional Framework Model, which is applicable to the analysis of building frames for vertical and lateral loads. The work presented here is only a stage to achieve this objective. This study addresses only the flat plate system. Also, the only parameter that has been considered is the column to slab stiffness. Several other parameters are of vital importance such as the aspect ratios of the slab and the column, column offset and slab holes. However, it has been necessary to limit the study in order to test the model feasibility in a reasonable time. Extensions to this work are obvious, and are planned. This will be elaborated upon in the closing chapter of this thesis.

In all of the examples presented in this study, only lateral loads have been applied. The work can be extended to vertical load analysis and this is commented upon later. The author believes that the main importance of full building frame analysis is associated with the wind and seismic load analyses. It is for this reason that the author regards an appropriate design procedure as comprising the following stages:

- (i) Analysis for vertical loads employing the Direct Design or Equivalent Frame methods to obtain slab thickness and reinforcement layouts
- (ii) Column loads using tributary loadings
- (iii) Check for sway and torsional effects and possibly dynamic effects using a Framework Analogy.

Flat Plate with Beams

Flat Slab Structure

Flat Plate Structure

Fig. 1.1: Types of Slabs

Fig. 1.2.a: Framework analogy (Hrennikoff)

Fig. 1.2.b: Finite cell

Fig. 1.3: The Equivalent Frame Model

$$\Delta = \begin{bmatrix} \omega \\ \omega_{,x} \\ \omega_{,y} \\ \omega_{,xx} \\ \omega_{,xy} \\ \omega_{,yy} \end{bmatrix}_i$$

$i=1,2,3$

$$\Delta = \begin{bmatrix} \omega, n \end{bmatrix}_j$$

$j=4,5,6$

Fig. 1.4.a: Triangular Bending Element.

(a) Slab System

(b) Column Stiffness

Fig. 1.4.b: Column Element

Fig. 1.5: Column Displacement Mode

Fig. 1.6: Floor Degrees of Freedom (El-Kafrawy)

Fig. 1.7: Building Frame D.O.F (El-Kafrawy)

Floor Plan

Fig. 1.8: Layout of 10-Storey Building Frame

Fig. 1.9: Building Frame Model Subjected to Lateral Loads

$$q = 10 \text{ KN/m}^2$$

Fig. 1.10: Building Frame Model Subjected to Vertical Loads

Fig. 1.11 : Storey Sway of Model Under Lateral Loading

Method	El-Kafrawy	Pecknold (1975)	Elias (1979)	Allen & Darvall (1977)	Equivalent Frame Method
ratio	1.00	1.12	1.21	1.31	1.62

Table 1.1: Ratios of roof sway

γ	Method	Section							
		1	2	3	4	5	6	7	8
1	El-Kafrawy	-240	-254	-234	-210	-210	-234	-254	-240
	equivalent frame*	-102	-247	-227	-201	-201	-227	-247	-102
2	El-Kafrawy	-200	-320	-485	-453	-249	-260	-486	-460
	equivalent frame*	-78	-335	-390	-346	-258	-290	-425	-212

*Moments are calculated at the column face

Moments in KN.m

Table 1.2: Slab Moments in Interior Frame

γ	Method	Section				
		E	F	G	H	K
1	El-Kafrawy	-120	+7.1	0	-7.1	120
	equivalent frame	-86	13.1	0	-13.1	86
2	El-Kafrawy	-100	-82.5	102	-113	230
	equivalent frame	-71	-61	84	-99	190

Table 1.3: Column Moments* in Interior Frame

*Moments transferred to the column above and below

Moments in KN.m

CHAPTER 2

THE FRAMEWORK MODEL

2.1 Description of Model

The Framework Model consists of the columns and equivalent grid systems representing the slabs at each floor. One floor showing the different types of elements, constituting the model, is presented in Fig. 2.1. The connection between the columns and the grid system are considered semi-rigid. In other words the "column joints" and the "grid joints" are connected by rotational springs. The grid consists of orthogonal beams and diagonal members. The orthogonal beams are aligned with each row of columns in the X and the Y directions. The diagonal members connect the grid joints at the corners of each panel. Each panel has four diagonal members which intersect at the "panel joint".

The model geometry has been developed by taking the structural behaviour of the flat plate system into consideration and the means by which a framework with one-dimensional elements could be made to simulate this behaviour. For instance, the springs allow differences between the "column-joint" rotations and the "grid-joint" rotations similar to the differences between the column rotations and the slab effective rotations. The springs are classified as corner springs, edge springs and internal springs. Each of these categories has basic stiffness properties (Fig. 2.2).

In the slab system, cross coupling takes place diagonally across the slab as well as in the orthogonal directions. The decision to model the slab using both orthogonal and diagonal members is quite natural.

The orthogonal beams are assumed to be non-prismatic to account for the fact that carry over factors are not 0.5 in such a structure. These beams are classified as internal beams which are aligned with the internal rows of columns, and external beams which are aligned with the external rows of columns. These classifications have been required to treat members with basically different stiffness properties.

The diagonal members are assumed to be prismatic. It has been determined during the early stages of this investigation that precise control of carry over factors across diagonals is not necessary. These members are classified as the corner diagonal, the edge diagonal and the internal diagonal which connect the panel joint with the corner joint, the edge joint and the internal joint respectively (Fig. 2.3). This classification has been found essential to have good agreement between the Finite Element Model [7] and the Framework Model.

The grid members are considered to be inextensible. The columns also are considered inextensible, that is, the grid joints are restrained against vertical translation.

2.2 Member Stiffness Characteristics

2.2.1 Column-Grid Joint Connecting Springs

The spring is assumed to have two rotational stiffnesses about the X and the Y axes respectively. The spring degrees of freedom are considered as the rotations at its ends, $\theta_{Ly}, \theta_{Lx}, \theta_{Ry}$ and θ_{Rx} , (Fig. 2.4). The stiffness coefficients due to unit displacements of each degree of freedom are illustrated in Fig. 2.5. Thus, the spring stiffness matrix can be written as follows:

$$[S_s] = \begin{bmatrix} K_{sy} & 0 & -K_{sy} & 0 \\ 0 & K_{sx} & 0 & -K_{sx} \\ -K_{sy} & 0 & K_{sy} & 0 \\ 0 & -K_{sx} & 0 & K_{sx} \end{bmatrix} \quad (2.1)$$

where

K_{sy} = the rotational stiffness about the Y axis; and

K_{sx} = the rotational stiffness about the X axis.

Some of the general stiffness analysis programs do not have the capability of generating directly the spring stiffness matrix. However, such inability can be overcome by introducing fictitious members which lead to the same stiffness matrix as that of the spring. For example, one may choose the two orthogonal torsional members shown in Fig. (2.6).

2.2.2 Orthogonal Beam

The degrees of freedom of these beams are taken as the end

rotations, $\theta_{Ly}, \theta_{Lx}, \theta_{Ry}$ and θ_{Rx} , (Fig. 2.7). The stiffness coefficients corresponding to each degree of freedom are illustrated in Fig. 2.8. These coefficients can be put in matrix form,

$$[S_b] = \begin{bmatrix} K_{fb} & 0 & \alpha_b K_{fb} & 0 \\ 0 & K_{tb} & 0 & -K_{tb} \\ \alpha_b K_{fb} & 0 & K_{fb} & 0 \\ 0 & -K_{tb} & 0 & K_{tb} \end{bmatrix} \quad (2.2)$$

where

K_{fb} = the flexural stiffness of the beam;

α_b = the carry over factor; and

K_{tb} = the torsional stiffness.

2.2.3 Diagonal Member

The diagonal member degrees of freedom related to the member axes, x_m & y_m , are shown in Fig. 2.9. In Fig. 2.10, the stiffness coefficients related to each degree of freedom are illustrated. Arranging these coefficients in a matrix form, gives

$$[S_{dm}] = \begin{bmatrix} K_{fd} & 0 & 0.5K_{fd} & 0 & \frac{1.5K_{fd}}{L_1} \\ 0 & K_{td} & 0 & -K_{td} & 0 \\ 0.5K_{fd} & 0 & K_{fd} & 0 & \frac{1.5K_{fd}}{L_1} \\ 0 & -K_{td} & 0 & K_{td} & 0 \\ \frac{1.5K_{fd}}{L_1} & 0 & \frac{1.5K_{fd}}{L_1} & 0 & \frac{3K_{fd}}{2L_1} \end{bmatrix} \quad (2.3)$$

where

K_{fd} = the flexural stiffness of the diagonal member;

K_{td} = the torsional stiffness; and

L_1 = the length of the diagonal member

The diagonal member is inclined to the X axis in the floor plane by an angle ϕ (Fig. 2.11). The stiffness matrix in Eqn.(2.3) is transformed to obtain a stiffness matrix related to the floor axes, X & Y. The matrix transformation is done by applying the following equation ,

$$[S_d] = [T]^T [S_{dm}] [T] \quad (2.4)$$

The matrix $[T]$ is a transformation matrix which can be written as:

$$[T] = \begin{bmatrix} cx & -cy & 0 \\ cy & cx & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.5)$$

where

$cx = \cos\phi$; and

$cy = \sin\phi$

Matrix $[S_d]$ is the stiffness matrix related to floor axes and can be expressed as follows:

[Sd] =

$$\begin{bmatrix} cx^2 \cdot Kfd + cy^2 \cdot Ktd & cx \cdot cy (Ktd - Kfd) & \frac{cx^2 \cdot Kfd}{2} - cy \cdot Ktd & -cx \cdot cy \left(\frac{Kfd}{2} + Ktd \right) & \frac{3 \cdot cx \cdot Kfd}{2L} \\ cx \cdot cy (Ktd - Kfd) & cy^2 \cdot Kfd + cx^2 \cdot Ktd & -cx \cdot cy \left(\frac{Kfd}{2} + Ktd \right) & \frac{cy^2 \cdot Kfd}{2} + cx \cdot Ktd & -\frac{3 \cdot cy \cdot Kfd}{2L} \\ \frac{cx^2 \cdot Kfd}{2} - cy \cdot Ktd & -cx \cdot cy \left(\frac{Kfd}{2} + Ktd \right) & cx^2 \cdot Kfd + cy^2 \cdot Ktd & cx \cdot cy (Ktd - Kfd) & \frac{3 \cdot cx \cdot Kfd}{2L} \\ -cx \cdot cy \left(\frac{Kfd}{2} + Ktd \right) & \frac{cy^2 \cdot Kfd}{2} + cx \cdot Ktd & cx \cdot cy (Ktd - Kfd) & cy^2 \cdot Kfd + cx^2 \cdot Ktd & -\frac{3 \cdot cy \cdot Kfd}{2L} \\ \frac{3 \cdot cx \cdot Kfd}{2L} & -\frac{3 \cdot cy \cdot Kfd}{2L} & \frac{3 \cdot cx \cdot Kfd}{2L} & -\frac{3 \cdot cy \cdot Kfd}{2L} & \frac{3 \cdot Kfd}{L^2} \end{bmatrix}$$

(2.6)

2.2.4 Column

The column is also modelled as a conventional frame-type element. The column height is taken from the center line of the top floor slab to the center line of the bottom floor slab. The portion of the column between the upper slab face and the lower slab face is assumed to have an infinite stiffness (Fig. 2.12). The column is considered to have 10 degrees of freedom. At each node, there are two translations, Δx & Δy , the rotation (θ_z) about the Z axis, and two rotations, θ_x & θ_y , about two axes passing through the centroid of the column cross section (Fig. 2.13). The stiffness coefficients due to a unit displacement of each degree of freedom are shown in Figs. 2.14 to 2.16. The column stiffness matrix is presented in Eqn. 2.7, where

K_{fcy} = the flexural stiffness of the column about the Y axis;

K_{fcx} = the flexural stiffness of the column about the X axis;

K_{tc} = the torsional stiffness of the column;

α_c = the carry over factor of the column about the X and the Y axes;

x = the distance in the X direction between the column and the Z axis; and

y = the distance in the Y direction between the column and the Z axis.

A derivation of the column flexural and torsional stiffness is provided in Appendix B.

$[K_c] =$

$$\begin{bmatrix}
 K_{fcy} & 0 & -D_x & 0 & D_x.y & \alpha c.K_{fcy} & 0 & D_x & 0 & -D_x.y \\
 0 & K_{fcx} & 0 & D_y & D_y.x & 0 & \alpha c.K_{fcx} & 0 & -D_y & -D_y.x \\
 -D_x & 0 & F_x & 0 & -F_x.y & -D_x & 0 & -F_x & 0 & F_x.y \\
 0 & D_y & 0 & F_y & F_y.x & 0 & D_y & 0 & -F_y & -F_y.x \\
 D_x.y & D_y.x & -F_x.y & F_y.x & T_z & D_x.y & D_y.x & F_x.y & -F_y.x & -T_z \\
 \alpha c.K_{fcy} & 0 & -D_x & 0 & D_x.y & K_{fcy} & 0 & D_x & 0 & -D_x.y \\
 0 & \alpha c.K_{fcx} & 0 & D_y & D_y.x & 0 & K_{fcx} & 0 & -D_y & -D_y.x \\
 D_x & 0 & -F_x & 0 & F_x.y & D_x & 0 & F_x & 0 & -F_x.y \\
 0 & -D_y & 0 & -F_y & -F_y.x & 0 & -D_y & 0 & F_y & F_y.x \\
 -D_x.y & -D_y.x & F_x.y & -F_y.x & T_z & -D_x.y & -D_y.x & -F_x.y & F_y.x & T_z
 \end{bmatrix}$$

(2.7)

2.3 Model Stiffness Matrix

2.3.1 General

Having determined the stiffness characteristics of each of the model elements, it is required to compile a matrix which gives the stiffness of the complete structure. In Fig. 2.17, the d.o.f of a floor of a building frame are presented. Upper case Δ is used to designate generalized displacements, or translations and rotations, which are primary d.o.f, i.e. which will exist in the final set of building frame stiffness equations. The delta symbol, Δ or δ , will designate translational type displacements when they are referred to specifically. When rotational type displacements are referred to specifically, Θ and θ will be used. Lower case symbols, δ and θ refer to non-primary d.o.f. Thus, the "column-joint" rotations, Θ_x & Θ_y , the "grid-joint" rotations, θ_x & θ_y , and the "panel-joint" displacement, δ_x , δ_y & δ_z , are shown in Fig. 2.17. In addition, each floor translates and rotates as a stiff diaphragm in its plane, Δ_x , Δ_y & Θ_z as illustrated in Fig. 2.18.

Only the primary d.o.f, consisting of two translations and one rotation per storey and two rotations per column joint (Fig. 2.19), actually form a stiffness system for full building frame analysis. Other d.o.f are eliminated in the course of building up the final stiffness relationship. The primary d.o.f are similar to those of the Finite Element Model, discussed in chapter 1 (Figs. 1.6 & 1.7). Hence, it possible to use the Framework model stiffness matrix to match that of the Finite Element Model. Also, this procedure reduces the number

of the mathematical equations involved in the analysis of the building frame.

The non-primary d.o.f are categorized as secondary and tertiary d.o.f. The "grid-joint" rotations, θ_x & θ_y , are considered as the secondary d.o.f as they are interconnected to the primary d.o.f through the "springs". The tertiary type relate to the "panel-joint" displacements and rotations, δ_x , δ_y , δ_z .

In order to derive the model overall stiffness matrix, each floor system can be considered independently as a substructure. The column stiffnesses can be incorporated subsequently, since, in the joint locked mode for stiffness development and force processing, each floor is isolated from the others.

To condense the floor stiffness matrix, it has been found that it is convenient to eliminate each set of the non-primary d.o.f in turn. The grid system, which consists of the orthogonal beams and the diagonal members, is first considered. This system contains the secondary and tertiary d.o.f. In the first stage, tertiary type d.o.f are eliminated. In the next step, the spring stiffnesses are incorporated with the grid stiffness to generate the floor stiffness matrix. subsequently, the secondary d.o.f in this matrix are condensed out. This procedure will be illustrated in the discussion which follows.

2.3.2 Elimination of Tertiary D.O.F

The grid system without the "springs" will be considered first.

The grid stiffness matrix is derived for the secondary d.o.f shown in Fig. 2.20. In order to simplify the derivation of the grid stiffness matrix, the grid has been considered to be a combination of two superimposed grids: the diagonal grid consisting of the diagonal members and containing tertiary d.o.f (Fig. 2.21) ; and the orthogonal grid, consisting of the orthogonal beams (Fig. 2.22). Hence,

$$[K_g] = [K_{go}] + [K_{gd}] \quad (2.8)$$

where

$[K_g]$ = the stiffness matrix of the floor grid;

$[K_{go}]$ = the stiffness matrix of the orthogonal grid; and

$[K_{gd}]$ = the stiffness matrix of the diagonal grid.

The stiffness matrix of the orthogonal grid can be derived directly with respect to the secondary d.o.f. However, the diagonal grid requires some mathematical manipulation to derive its stiffness matrix in terms of the secondary d.o.f alone. The stiffness matrix of the diagonal grid is an assembly of the stiffness matrices of each of the four diagonal members in each panel.

For the four diagonal members shown in Fig. 2.23, the displacement equations are given in the matrix form,

$$\begin{bmatrix} [S\delta\delta] & [S\delta\delta'] \\ [S\delta'\delta] & [S\delta'\delta'] \end{bmatrix} \begin{bmatrix} \{\delta\} \\ \{\delta'\} \end{bmatrix} = \begin{bmatrix} \{F\delta\} \\ \{F\delta'\} \end{bmatrix} \quad (2.9)$$

where

$\{\delta\}$ = the generalized displacements of the grid joints (A, B, C, D), consisting of the X and Y rotations at these locations;

$\{\delta'\}$ = the generalized displacements of the panel joint (F), consisting of the vertical translation and the X and Y rotations;

$\{F\delta\}$ = the end moments corresponding to $\{\delta\}$; and

$\{F\delta'\}$ = the joint moments and the vertical force corresponding to $\{\delta'\}$.

The stiffness matrix, in Eqn. (2.9), is condensed to eliminate $\{\delta'\}$ using the following procedure:

$$[S\delta\delta]\{\delta\} + [S\delta\delta']\{\delta'\} = \{F\delta\} \quad (2.10)$$

$$[S\delta'\delta]\{\delta\} + [S\delta'\delta']\{\delta'\} = \{F\delta'\} \quad (2.11)$$

$$[S\delta'\delta']\{\delta'\} = \{F\delta'\} - [S\delta'\delta]\{\delta\} \quad (2.11.a)$$

$$\{\delta'\} = [S\delta'\delta']^{-1} [\{F\delta'\} - [S\delta'\delta]\{\delta\}] \quad (2.11.b)$$

Substituting Eqn.(2.11.b) into Eqn.(2.10):

$$[S\delta\delta]\{\delta\} + [S\delta\delta'] [S\delta'\delta']^{-1} [\{F\delta'\} - [S\delta'\delta]\{\delta\}] = \{F\delta\} \quad (2.12)$$

$$[[S\delta\delta] - [S\delta\delta'] [S\delta'\delta']^{-1} [S\delta'\delta]]\{\delta\} = \{F\delta\} - [S\delta\delta'] [S\delta'\delta']^{-1} \{F\delta'\} \quad (2.12.a)$$

Eqn.(2.12.a) can be written in the form:

$$[Sdp] \{\delta\} = \{F\}$$

thus:

$$[Sdp] = [S\delta\delta] - [S\delta\delta'] [S\delta\delta']^{-1} [S\delta\delta] \quad (2.13)$$

where $[Sdp]$ is the condensed stiffness matrix of the four diagonal members corresponding to the secondary degrees of freedom.

Now, the diagonal grid stiffness matrix can be obtained by assembling the $[Sdp]$ matrices. Applying Eqn. 2.8, the condensed grid stiffness matrix, related only to the secondary d.o.f, can be calculated.

2.3.3 Floor Stiffness Matrix

In this stage, the whole floor is considered. The floor stiffness matrix is an assembly of the grid stiffness matrix and the spring stiffness matrices. The primary degrees of freedom of the floor are the "column-joint" rotations (Fig. 2.24). The "grid-joint" rotations are secondary degrees of freedom. In the following discussion, the secondary d.o.f are condensed out.

The displacement equations of the floor can be written as follows:

$$\begin{bmatrix} [K\Delta\Delta] & [K\Delta\delta] \\ [K\delta\Delta] & [K\delta\delta] \end{bmatrix} \begin{bmatrix} \{\Delta\} \\ \{\delta\} \end{bmatrix} = \begin{bmatrix} \{M\Delta\} \\ \{M\delta\} \end{bmatrix} \quad (2.14)$$

where

$\{\Delta\}$ = the primary d.o.f or column joint rotations

$\{\delta\}$ = the secondary d.o.f or grid joint rotations

$\{M\Delta\}$ = the "column-joint" moments corresponding to $\{\Delta\}$

$\{M\delta\}$ = the "grid-joint moments" corresponding to $\{\delta\}$

and where $[K_{\Delta\Delta}]$ is the submatrix of stiffness coefficients related directly to the primary d.o.f. The column stiffnesses have not, at this point, been incorporated so that the matrix $[K_{\Delta\Delta}]$ is comprised entirely of "spring" stiffnesses in a diagonal pattern. The matrices $[K_{\Delta\delta}]$, $[K_{\delta\Delta}]$ are also in diagonal patterns, and they are related to the matrix, $[K_{\Delta\Delta}]$, by the following equations:

$$[K_{\delta\Delta}] = [K_{\Delta\delta}] = -[K_{\Delta\Delta}] \quad (2.16)$$

The matrix $[K_{\delta\delta}]$ can be obviously expressed as:

$$[K_{\delta\delta}] = [K_g] + [K_{\Delta\Delta}] \quad (2.17)$$

The stiffness matrix in Eqn.(2.14) is condensed to eliminate $\{\delta\}$ as follows:

$$[K_{\Delta\Delta}]\{\Delta\} + [K_{\Delta\delta}]\{\delta\} = \{M\Delta\} \quad (2.18)$$

$$[K_{\delta\Delta}]\{\Delta\} + [K_{\delta\delta}]\{\delta\} = \{M\delta\} \quad (2.19)$$

$$[K_{\delta\delta}]\{\delta\} = \{M\delta\} - [K_{\delta\Delta}]\{\Delta\} \quad (2.19.a)$$

$$\{\delta\} = [K_{\delta\delta}]^{-1} \{\{M\delta\} - [K_{\delta\Delta}]\{\Delta\}\} \quad (2.19.b)$$

substituting Eqn.(2.19.b) into Eqn.(2.18):

$$[K_{\Delta\Delta}]\{\Delta\} + [K_{\Delta\delta}][K_{\delta\delta}]^{-1}\{M\delta\} - [K_{\delta\Delta}]\{\Delta\} = \{M\Delta\} \quad (2.20)$$

$$[[K_{\Delta\Delta}] - [K_{\Delta\delta}][K_{\delta\delta}]^{-1}[K_{\delta\Delta}]]\{\Delta\} = \{M\Delta\} - [K_{\Delta\delta}][K_{\delta\delta}]^{-1}\{M\delta\} \quad (2.20.b)$$

thus:

$$[K_s] = [K_{\Delta\Delta}] - [K_{\Delta\delta}][K_{\delta\delta}]^{-1}[K_{\delta\Delta}] \quad (2.21)$$

where $[K_s]$ is the condensed stiffness matrix of the floor. By substituting Eqn.(2.16) and Eqn.(2.17) into Eqn.(2.21), we get

$$[K_s] = [K_{\Delta\Delta}] - [K_{\Delta\Delta}][[K_g] + [K_{\Delta\Delta}]]^{-1}[K_{\Delta\Delta}] \quad (2.22)$$

The stiffness matrix $[K_s]$ is related to the d.o.f corresponding to those of the Finite Element Model. Hence, it is possible to calibrate the Framework Model elements by direct matching of the floor stiffness matrix produced by the Finite Element Model.

Fig. 2.1: The Framework Model

Fig. 2.2: Types of Springs

Fig. 2.3: Types of Diagonal Members ;

Fig. 2.4: Spring Degrees of Freedom

Fig. 2.5: Spring Stiffness Coefficients

Fig. 2.6: Fictitious Members

Fig. 2.7: Orthogonal Beam D.O.F

Fig. 2.8: Orthogonal Beam Stiffness Coefficients

Fig. 2.9: Diagonal Member D.O.F

Fig. 2.10: Diagonal Member Stiffness Coefficients

Fig. 2.11: Diagonal Member Inclination

Fig. 2.12: Column Inertia

Fig. 2.13: Column D.O.F

Fig. 2.14: Column Stiffness Coefficients

Fig. 2.15: Column Stiffness Coefficients(cont.)

$$D_x = \frac{(1+\alpha_c) K_{fcy}}{H}, \quad D_y = \frac{(1+\alpha_c) K_{fcx}}{H}, \quad F_x = \frac{2 \cdot D_x}{H}$$

$$F_y = \frac{2 \cdot D_y}{H}, \quad T_z = K_{tc} + F_{x,y}^2 + F_{y,x}^2$$

Fig. 2.16: Column Stiffness Coefficients (cont.)

Fig. 2.17: Floor Degrees of Freedom

Fig. 2.18: Model Degrees of Freedom

Fig. 2.19: Model Primary D.O.F

Fig. 2.20: Grid D.O.F (Floor Secondary D.O.F)

Fig. 2.21: The Diagonal Grid

Fig. 2.24: Floor Primary D.O.F

CHAPTER 3

EVALUATION OF MEMBER STIFFNESSES

3.1 Introduction

The stiffnesses of the frame elements in the Framework Model were obtained by calibrating the model against the Finite Element Model [7]. In order to make the solution tractable, a one panel model was first investigated. This model involved only corner springs, external beams and corner diagonal members (Fig. 3.1). The work done towards developing the stiffness of the one panel model, was then modified and extended into a study of a four panel model. This model involved the additional member types (Fig. 3.2).

The framework member stiffnesses were computed for a range of slab to column stiffness by varying the C/L (column dimension to slab length) ratio. The slab aspect ratio and the column aspect ratio were taken equal to 1. The slab was assumed to have no openings and a regular arrangement of the columns was adopted (no column offset). A range of the column dimension to the span length ratio (C/L), between 0.05 and 0.2, was considered. The author believes that use of the model can be stretched moderately beyond the limits of parametric variability presented here. For example, there is no particular reason to believe that column aspect ratio can not be varied. Also as the slab aspect ratio is varied, the framework Model being geometrically similar to the actual structure, should be able to follow the general change in geometry within moderate variations; and

this might also be true for offset columns. However, such use must be validated by comparison with further finite element study.

The results obtained for the framework member stiffnesses were plotted and fitted to polynomial curves using the least squares technique [23].

3.2 One Panel Model

3.2.1 General

For the case of a square slab supported on square columns, each corner spring has equal rotational stiffnesses about the X and the Y axes, "Ksc". The external beam has a flexural stiffness, "Kfbe", and a torsional stiffness, "Ktbe", and a carry over factor, "Cbe". The corner diagonal member has a flexural stiffness, "Kfdc", and a torsional stiffness, "Ktdc".

In Fig. 3.3, the floor d.o.f of the Finite Element Model [7] are shown. In Fig. 3.4, the corresponding floor d.o.f of the Framework Model are presented. These are primary d.o.f of the Framework Model. The floor stiffness matrix [Ks] is obtained, in terms of these primary d.o.f, by applying Eqn.(2.22), repeated below.

$$[Ks] = [K\Delta\Delta] - [K\Delta\Delta][[Kg] + [K\Delta\Delta]]^{-1}[K\Delta\Delta] \quad (2.22)$$

Due to the symmetry of the model, the four corner springs have equal rotational stiffnesses. Hence, the diagonal matrix [K $\Delta\Delta$] can be written as:

$$[K\Delta\Delta] = K_{sc} [I] \quad (3.1)$$

where $[I]$ is a unit matrix. The matrix $[K_g]$, in Eqn. (2.22), is the stiffness matrix of the grid system without the springs. This matrix is related to the secondary d.o.f shown in Fig. 3.5. For the one panel model, it has been possible to derive the stiffness matrix $[K_g]$, algebraically, in terms of the diagonal member and the orthogonal beam stiffnesses, K_{fdc} , K_{tdc} , K_{fbe} , α_{be} & K_{tbe} . Each of the matrix elements is a function of these stiffnesses. Matrix $[K_g]$ has been derived using the procedure discussed earlier in Section 2.3.2. The derivation of $[K_g]$ is shown in detail in Appendix A. Here, the final form of the matrix is given in the following equation:

$$[K_g] = \begin{bmatrix} K_{g1} & K_{g2} & K_{g3} & K_{g4} & K_{g5} & K_{g6} & K_{g7} & K_{g8} \\ K_{g2} & K_{g1} & -K_{g4} & K_{g5} & -K_{g6} & K_{g3} & K_{g8} & K_{g7} \\ K_{g3} & -K_{g4} & K_{g1} & -K_{g2} & K_{g7} & -K_{g8} & K_{g5} & -K_{g6} \\ K_{g4} & K_{g5} & -K_{g2} & K_{g1} & -K_{g8} & K_{g7} & -K_{g4} & K_{g3} \\ K_{g5} & -K_{g6} & K_{g7} & -K_{g8} & K_{g1} & -K_{g2} & K_{g3} & -K_{g4} \\ K_{g6} & K_{g3} & -K_{g8} & K_{g7} & -K_{g2} & K_{g1} & -K_{g6} & K_{g5} \\ K_{g7} & K_{g8} & K_{g5} & -K_{g4} & K_{g3} & -K_{g6} & K_{g1} & K_{g2} \\ K_{g8} & K_{g7} & -K_{g6} & K_{g3} & -K_{g4} & K_{g5} & K_{g2} & K_{g1} \end{bmatrix} \quad (3.2)$$

where

$$K_{g1} = \frac{3.25}{8} K_{fdc} + 0.5 K_{tdc} - \frac{0.25 K_{fdc}^2 + K_{tdc}^2}{4(K_{fdc} + K_{tdc})} + K_{fbe} + K_{tbe} \quad (3.2.a)$$

$$Kg2 = \frac{-3.25}{8} Kfdc + 0.5 Ktde + \frac{0.25 Kfdc^2 - Ktde^2}{4(Kfdc + Ktde)} \quad (3.2.b)$$

$$Kg3 = \frac{Kfdc Ktde}{4(Kfdc + Ktde)} - \frac{0.75}{8} Kfdc - Ktbe \quad (3.2.c)$$

$$Kg4 = \frac{-0.75}{8} Kfdc \quad (3.2.d)$$

$$Kg5 = \frac{Kfdc Ktde}{4(Kfdc + Ktde)} + \frac{0.75}{8} Kfdc + \alpha_{be} Kfbe \quad (3.2.e)$$

$$Kg6 = \frac{0.75}{8} Kfdc \quad (3.2.f)$$

$$Kg7 = \frac{0.75}{8} Kfdc - \frac{0.25 Kfdc^2 + Ktde^2}{4(Kfdc + Ktde)} \quad (3.2.g)$$

$$Kg8 = \frac{-0.75}{8} Kfdc + \frac{0.25 Kfdc^2 - Ktde^2}{4(Kfdc + Ktde)} \quad (3.2.h)$$

substituting Eqn.(3.1) into Eqn.(2.22):

$$[Ks] = Ksc[I] - Ksc \frac{2}{[[Kg] + Ksc[I]]} \quad (3.3)$$

$$Ksc \frac{2}{[[Kg] + Ksc[I]]} = Ksc[I] - [Ks] \quad (3.3.a)$$

$$[[Kg] + Ksc[I]] = \frac{-1}{Ksc} [[I] - \frac{1}{Ksc} [Ks]] \quad (3.3.b)$$

$$[Kg] + Ksc[I] = Ksc[[I] - \frac{1}{Ksc} [Ks]] \quad (3.3.c)$$

$$[Kg] = Ksc[[I] - \frac{1}{Ksc} [Ks]] - Ksc[I] \quad (3.3.d)$$

3.2.2 Determination of Member Stiffnesses

A trial and error procedure was used to solve Eqn.(3.3.d) for the unknown values, "Ksc", "Ktbe", "Kfbe", "Obe", "Kfde" & "Ktde". This procedure can be summarized as follows. For each value of C/L, the floor stiffness matrix [ks] was calculated using the Finite Element Model. A reasonable initial value of the spring stiffness was substituted in Eqn.(3.3.d) to calculate the grid stiffness matrix [Kg]. Solving Eqns.(3.2.a) to (3.2.e) the member stiffnesses were obtained. New values for Kg7 and Kg8 were calculated from Eqns.(3.2.g) and (3.2.h). A modified [Kg] matrix was then assembled and a modified [Ks] matrix was obtained by applying Eqn.(3.3). The value of Ksc was adjusted so that the modified floor stiffness matrix produced by the Framework Model was close to that produced by the Finite Element Model. The C/L ratio was varied from 0.05 to 0.2. Member stiffnesses were determined corresponding to each of these values.

Having obtained the member stiffnesses, they were then formulated in terms of the C/L ratio. The formulas were chosen as :

$$K_{sc} = \frac{E t^3}{(1-\nu)^2} P1(C/L) \quad (3.4)$$

$$K_{fbe} = \frac{E t^3}{(1-\nu)^2} P2(C/L) \quad (3.5)$$

$$\alpha_{be} = P3(C/L) \quad (3.6)$$

$$K_{tbe} = \frac{E t^3}{(1-\nu)^2} P4(C/L) \quad (3.7)$$

$$K_{fdc} = \frac{E t^3}{(1-\nu)^2} P5(C/L) \quad (3.8)$$

$$K_{tdc} = \frac{E t^3}{(1-\nu)^2} P6(C/L) \quad (3.9)$$

The above formulas are proportional to $\frac{Et^3}{(1-\nu)^2}$ as indicated by

the finite element results, discussed earlier in chapter 1. The terms, $P_1, P_2, \dots, P_6$ are functions of the C/L ratio. These functions were approximated by polynomial functions using the least squares technique [23]. These polynomials are presented below:

$$\frac{1}{P_1} = 5.0 - 122.15(C/L) + 1183.34(C/L)^2 - 5351.3(C/L)^3 + 9420.83(C/L)^4 \quad (C/L \leq 0.16)$$

$$\frac{1}{P_1} = 0.0093(1 - C/L)^4 \quad (C/L \geq 0.16)$$

$$P_2 = \frac{1}{(1-C/L)^2} (0.0482 + 0.272(C/L) + 1.148(C/L)^2 - 10.652(C/L)^3 + 19.252(C/L)^4)$$

$$P_3 = 0.367 + 0.597(C/L) + 4.818(C/L)^2 \quad (C/L \leq 0.16)$$

$$P_3 = 0.106 + 3.403(C/L) - 2.509(C/L)^2 \quad (C/L \geq 0.16)$$

$$P_4 = \frac{1}{(1-C/L)^2} (0.0034 + 0.0356(C/L) - 0.206(C/L)^2 + 3.071(C/L)^3 - 5.85(C/L)^4)$$

$$P_5 = (1-C/L)^{0.5} (0.0761 + 0.30(C/L) + 3.481(C/L)^2 + 8.042(C/L)^3)$$

$$P_6 = (1-C/L) \left(0.0279 + 0.116(C/L) - 1.58(C/L)^2 + 14.21(C/L)^3 \right)$$

The polynomial curves are illustrated in Figs. 3.9 through 3.18. These polynomials satisfy the conditions at the extreme case when the C/L ratio approaches 1. For example, the spring stiffness, K_{sc} , approaches infinity indicating no difference between the slab rotations and the column rotations. The diagonal member stiffnesses, K_{fde} & K_{tde} , vanish indicating no coupling between the moments about the X and The Y axes, i.e. the slab has one way action. The orthogonal beam stiffnesses, K_{fbe} & K_{tbe} , approach infinity and its carry over factor, α_{be} , approaches 1.

The floor stiffness matrix of the Framework Model generated using the formulas has shown good agreement with that produced by the Finite Element Model. In Table (3.1), the floor stiffness coefficients due to a unit rotation of the d.o.f Θ_1 shown in Fig. 3.4 are presented for C/L ratios equal to 0.05, 0.16 and 0.2.

3.3 Four Panel Model

3.3.1 General

In Fig. 3.2, the different element types of the four panel model are shown. For the case of a square slab supported on square columns, the internal spring has equal rotational stiffness about the X and the Y axes, " K_{si} ". However, the edge spring has different rotational stiffnesses in the two directions. In other words, the edge spring has a rotational stiffness about an axis parallel to the edge, " K_{sep} ",

and a rotational stiffness about an axis normal to the edge, "Ksen". In Fig. 3.6, a and b are edge joints of the Framework Model. At joint a, the Y axis is normal to the edge, and the X axis is parallel to the edge. Hence,

$$K_{say} = K_{sen} \quad (3.10)$$

$$K_{sax} = K_{sep} \quad (3.11)$$

where

K_{say} = the rotational stiffness about the Y axis at (a)

K_{sax} = the rotational stiffness about the X axis at (a)

At joint b, the Y axis is parallel to the edge, and the X axis is normal to the edge. Hence,

$$K_{sby} = K_{sep} \quad (3.12)$$

$$K_{sbx} = K_{sen} \quad (3.13)$$

K_{sby} = the rotational stiffness about the Y axis at (b)

K_{sbx} = the rotational stiffness about the X axis at (b)

This difference in the edge spring stiffnesses accounts for the actual slab behaviour at an edge column-slab joint. At the edge joint, the slab has different rotational stiffnesses about the parallel and the normal axes.

For the orthogonal beams, the external beam stiffnesses have been determined using the one panel model. The internal beam is assumed to have a flexural stiffness, "Kfbi", carry over factor, "Cbi", and a torsional stiffness, "Ktbi".

The corner diagonal stiffnesses have been obtained using the one panel model. The internal diagonal member has a flexural stiffness, "Kfdi", and a torsional stiffness, "Ktdi". The edge diagonal member has a flexural stiffness, "Kfde", and a torsional stiffness, "Ktde".

There are many parameters influencing the behaviour of the four panel model. In order to make the mathematical solution possible, it has been necessary to reduce the number of these parameters. Some assumptions have been included which relate some of the parameters to the others. Although not all of these assumptions have strong theoretical justification, they made it possible to achieve the objective of this study, which has been to match the stiffness terms of the Finite Element Model. Since this work started with a one panel model, the member stiffness terms obtained by this one panel model are used as a basis in the discussion which follows.

The spring stiffness terms, "Ksi", "Ksep" and "Ksen", are assumed to have the following relations with the corner spring stiffness, "Ksc".

$$K_{sen} = \{1 K_{sc} \quad (3.14)$$

$$K_{sep} = \{2 K_{sc} \quad (3.15)$$

$$K_{si} = \{3 K_{sc} \quad (3.16)$$

the preliminary investigation of the four panel model has shown that $\{3$ can be expressed approximately as:

$$\{3 = \{1 \{2 \quad (3.17)$$

thus:

$$K_{s1} = \zeta_1 \zeta_2 K_{sc} \quad (3.18)$$

where ζ_1 and ζ_2 are functions of the C/L ratio.

The torsional stiffness of the internal beam, "Ktbi", is assumed to be equal to that of the external beam, "Ktbe". i.e.,

$$K_{tbi} = K_{tbe} \quad (3.19)$$

The diagonal members are assumed to have equal torsional stiffness. i.e.,

$$K_{tdi} = K_{tde} = K_{tdc} \quad (3.20)$$

The flexural stiffness of the edge and internal diagonal members, "Kfde" & "Kfdi", are assumed to be related to that of the corner diagonal member, "Kfdc", by the following equations:

$$K_{fdi} = \beta K_{fdc} \quad (3.21)$$

$$K_{fde} = \beta_1 K_{fdc} \quad (3.22)$$

The preliminary study has shown that β_1 can be expressed approximately as:

$$\beta_1 = \frac{\beta + 1}{2} \quad (3.23)$$

thus:

$$K_{fde} = \frac{\beta + 1}{2} K_{fdc} \quad (3.24)$$

where β is a function of the C/L ratio.

To simplify the solution for the unknown values, " β ", " α_1 ", " α_2 ", " β ", " K_{fbi} " & " α_{bi} ", the grid system was separated into two grids superimposed upon each other. These were the orthogonal grid which consisted of the orthogonal beams, and the diagonal grid which contained the diagonal members. This procedure was discussed earlier in section (2.3). Referring to Eqn.(2.8),

$$[K_g] = [K_{go}] + [K_{gd}] \quad (2.8)$$

where

$[K_g]$ = the stiffness matrix of the floor grid;

$[K_{go}]$ = the stiffness matrix of the orthogonal grid; and

$[K_{gd}]$ = the stiffness matrix of the diagonal grid

The stiffness coefficients in the matrix $[K_{go}]$ are functions of the unknown variables, K_{fbi} and α_{bi} . The matrix $[K_{gd}]$ are functions of the variable β .

In Fig. 3.7, the numbering system of the grid d.o.f are shown. Referring to Eqn. 2.8, the following stiffness expressions have been obtained:

$$K_g(7,9) = K_{go}(7,9) + K_{gd}(7,9) \quad (3.25)$$

$$K_g(9,9) = K_{go}(9,9) + K_{gd}(9,9) \quad (3.26)$$

$$K_g(3,9) = K_{go}(3,9) + K_{gd}(3,9) \quad (3.27)$$

Hence:

$$K_g(7,9) = -K_{tb1} + K_{gd}(7,9) \quad (3.28)$$

$$K_g(9,9) = 2K_{fb1} + 2K_{tb1} + K_{gd}(9,9) \quad (3.29)$$

$$K_g(3,9) = \alpha_{b1} K_{fb1} + K_{gd}(3,9) \quad (3.30)$$

thus:

$$K_{tb1} = K_{gd}(7,9) - K_g(7,9) \quad (3.28.a)$$

$$K_{fb1} = K_g(9,9) - K_{gd}(9,9) - 2K_{tb1} \quad (3.29.a)$$

$$\alpha_{b1} = \frac{1}{K_{fb1}} (K_g(3,9) - K_{gd}(3,9)) \quad (3.30.a)$$

Substituting Eqn.(3.19) in Eqns.(3.28.a) and (3.29.a):

$$K_{tbe} = K_{gd}(7,9) - K_g(7,9) \quad (3.31)$$

$$K_{fb1} = K_g(9,9) - K_{gd}(9,9) - 2K_{tbe} \quad (3.32)$$

Eqn.(3.15) can be rewritten in general form as follows:

$$[K_g] = [K_{\Delta\Delta}] \begin{bmatrix} -1 & -1 \\ [I] & [K_s] \end{bmatrix} - [K_{\Delta\Delta}] \quad (3.33)$$

3.3.2 Determination of the Member Stiffnesses

The unknown values were solved on the computer using a trial and error procedure. This procedure can be summarized as follows. For each value of the C/L ratio, the floor stiffness matrix $[K_s]$ was

generated by the Finite Element Model. Initial values of ξ_1 and ξ_2 were substituted in Eqn.(3.33) to calculate the grid stiffness matrix $[Kg]$. A value of β was assumed. Then, the matrix $[Kgd]$ was generated. The value of β was adjusted so that Eqn.(3.31) was satisfied. Applying Eqns.(3.30.a) and (3.32), the values of α_{bi} and K_{fbi} were calculated. Hence, a modified $[Kg]$ matrix was assembled and a modified $[Ks]$ matrix was calculated by applying Eqn.(2.22). The values of ξ_1 and ξ_2 were altered so that the modified floor stiffness matrix produced by the Framework Model was close to that produced by the Finite Element Model.

The values obtained for ξ_1 , ξ_2 , K_{fbi} , α_{bi} & β , have been approximated to the following formulas using the least squares technique.

$$\xi_1 = 1.843 - 8.375(C/L) - 17.759(C/L)^2 \geq 0.0484 \quad (3.34)$$

$$\xi_2 = 5.63 - 61.329(C/L) + 197.52(C/L)^2 \geq 0.874 \quad (3.35)$$

$$K_{fbi} = \frac{E t^3}{(1-\nu)^2} P7(C/L) \quad (3.36)$$

where

$$P7 = \frac{1}{(1 - C/L)^2} \left(0.07 + 0.383(C/L) + 4.1(C/L)^2 - 43.5(C/L)^3 + 105.41(C/L)^4 \right)$$

$$\alpha_{b1} = 0.246 + 1.374(C/L) + 4.642(C/L)^2 \quad (C/L \leq 0.16)$$

$$\alpha_{b1} = \alpha_{be} \quad (C/L \geq 0.16)$$

$$\beta = 1.534 - 12.96(C/L)^2 \geq 1.0$$

the above formulas are illustrated in Figs. 3.9 through 3.18.

The stiffness matrix of the floor generated using the formulas has shown good agreement with the stiffness matrix of the floor produced by the Finite Element Model. In Tables (3.2) and (3.3), the coefficients of both methods due to unit rotations in the direction of the degrees of freedom 1, 3, 4 and 9 (Fig. 3.8) are presented for C/L ratios equal to 0.05 and 0.2.

Fig. 3.1: The One Panel Model

Fig. 3.2: The Four Panel Model

Fig. 3.3: The D.O.F of the Finite Element Floor

Fig. 3.4: Floor D.O.F of One Panel Model

Fig. 3.5: Grid D.O.F of One Panel Model

Fig. 3.6: The Edge Spring

Fig. 3.7: Numbering System of Grid D.O.F

Fig. 3.8: Numbering System of Floor D.O.F

Fig. 3-9: Corner Spring Stiffness

Fig. 3.10: Factors ξ_1 & ξ_2

Fig. 3.11: Internal and Edge Springs

Fig. 3.12: Flexural Stiffness of Orthogonal Beams

Fig. 3.13: Carry Over Factor (α_b)

Fig. 3.14: Torsional Stiffness of Orthogonal Beams

Fig. 3.15: Flexural Stiffness of Corner Diagonal Member

Fig. 3.16: Factor β

Fig. 3.17: Flexural Stiffness of Internal and Edge Diagonal Members

Fig. 3.18: Torsional Stiffness of Diagonal Members

d.o.f	C/L = 0.05		C/L = 0.16		C/L = 0.20	
	F.W.M	F.E.M	F.W.M	F.E.M	F.W.M	F.E.M
1	0.1050	0.1053	0.2459	0.2454	0.3118	0.3126
2	0.0162	0.0163	0.0590	0.0586	0.0780	0.0786
3	-0.0070	-0.0072	-0.0294	-0.0294	-0.0426	-0.0426
4	0.0063	0.0063	0.0210	0.0210	0.0284	0.0286
5	0.0327	0.0327	0.1051	0.1053	0.1428	0.1440
6	-0.0063	-0.0063	-0.0210	-0.0210	-0.0284	-0.0286
7	0.0028	0.0050	0.0074	0.0133	0.0102	0.0106
8	0.0052	0.0027	0.0111	0.0090	0.0159	0.0116

Table (3.1) Floor stiffness coefficients(one panel model)($\bar{K}_s$)

$$K_s = K_s^* E t^3 / (1 - \nu^2)$$

d.o.f	1		3		4		9	
	F.W.M	F.E.M	F.W.M	F.E.M	F.W.M.	F.E.M	F.W.M	F.E.M
1	0.1070	0.1090	-0.0080	-0.0084	-0.0064	-0.0056	0.0034	0.0065
2	-0.0171	-0.0194	0.0081	0.0110	0.0311	0.0330	-0.0051	-0.0040
3	-0.0080	-0.0084	0.2160	0.2130	0.0000	0.0000	0.0570	0.0560
4	-0.0064	-0.0056	0.0000	0.0000	0.2100	0.2100	0.0000	0.0000
5	0.0000	0.0003	-0.0080	-0.0084	0.0064	0.0056	0.0034	0.0065
6	0.0002	0.0007	-0.0081	-0.0110	0.0311	0.0330	0.0051	0.0040
7	0.0311	0.0330	0.0037	0.0057	0.0056	0.0013	-0.0143	-0.0145
8	0.0081	0.0110	0.0051	0.0068	0.0037	0.0057	0.0000	0.0000
9	0.0034	0.0065	0.0570	0.0560	0.0000	0.0000	0.4350	0.4290
10	-0.0051	-0.0040	0.0000	0.0000	-0.0143	-0.0145	0.0000	0.0000
11	-0.0001	-0.0002	0.0037	0.0057	-0.0056	-0.0013	-0.0143	-0.0145
12	0.0001	0.0006	-0.0051	-0.0068	0.0037	0.0057	0.0000	0.0000
13	0.0010	0.0014	-0.0002	-0.0011	-0.0002	-0.0005	0.0034	0.0065
14	-0.0002	-0.0007	-0.0001	-0.0006	0.0000	-0.0002	-0.0051	-0.0040
15	-0.0002	-0.0011	-0.0011	-0.0003	0.0000	0.0000	0.0570	0.0560
16	0.0002	0.0006	0.0000	0.0000	0.0000	0.0003	0.0000	0.0000
17	0.0000	0.0000	-0.0002	-0.0011	0.0002	0.0005	0.0034	0.0056
18	0.0000	0.0000	0.0001	0.0006	0.0000	-0.0002	-0.0051	-0.0040

Table 3.2: Floor Stiffness Coefficients (four panel model)(Ks*)

$$C/L = 0.05$$

$$K_s = K_s^* Et^3 / (1 - \nu^2)$$

d.o.f	1		3		4		9	
	F.W.M	F.E.M	F.W.M	F.E.M	F.W.M	F.E.M	F.W.M	F.E.M
1	0.3111	0.3178	-0.0426	-0.0454	-0.0281	-0.0288	0.0100	0.0171
2	-0.0779	-0.0836	0.0287	0.0326	0.1399	0.1459	-0.0152	-0.0129
3	-0.0426	-0.0454	0.5362	0.5275	0.0000	0.0000	0.2174	0.2105
4	-0.0281	-0.0288	0.0000	0.0000	0.5859	0.6002	0.0000	0.0000
5	0.0000	0.0007	-0.0426	-0.0454	0.0281	0.0288	0.0100	0.0171
6	0.0001	0.0009	-0.0287	-0.0326	0.1399	0.1459	-0.0152	-0.0129
7	0.1399	0.1458	0.0104	0.0168	0.0152	0.0095	-0.0548	-0.0585
8	0.0287	0.0327	0.0157	0.0163	0.0104	0.0169	0.0000	0.0000
9	0.0100	0.0171	0.2174	0.2105	0.0000	0.0000	0.9855	0.9855
10	-0.0152	-0.0129	0.0000	0.0000	-0.0548	-0.0583	0.0000	0.0000
11	0.0000	-0.0007	0.0104	0.0168	-0.0152	-0.0095	-0.0548	-0.0583
12	0.0001	0.0009	-0.0157	-0.0163	0.0104	0.0169	0.0000	0.0000
13	-0.0007	-0.0009	-0.0001	-0.0010	0.0000	-0.0008	0.0100	0.0171
14	-0.0001	-0.0009	-0.0001	-0.0008	0.0000	-0.0006	0.0152	0.0131
15	-0.0003	-0.0010	-0.0020	-0.0019	0.0000	0.0000	0.2174	0.2105
16	0.0000	0.0008	0.0000	0.0000	0.0001	0.0011	0.0000	0.0000
17	0.0000	0.0000	-0.0001	-0.0010	0.0000	0.0008	0.0100	0.0171
18	0.0000	-0.0001	0.0001	0.0009	0.0000	-0.0007	-0.0152	-0.0129

Table 3.3: Floor Stiffness Coefficients(four panel model)(Ks)

$$C/L = 0.20$$

$$K_s = K_s^* \cdot Et^3 / (1 - \nu^2)$$

CHAPTER 4

SIDE SWAY ANALYSIS OF BUILDING FRAME

4.1 Introduction

The stiffness matrix of the flat plate floor which has been obtained in chapter 3 can be used in the analysis of models of full 3-dimensional building frames. In this chapter, the lateral load analysis of the frame is discussed.

In order to examine the capability of the framework model to simulate the behaviour of flat plate structures due to the effect of horizontal forces, a number of building frames have been analyzed using the Framework Model and the Finite Element Model. The lateral displacements of the frame and the moments in the columns obtained by both models are compared in this chapter. Comparison between the Finite Element results and those of the code method (the Equivalent Frame) and other methods provided in the literature, mentioned earlier in chapter 1, was presented by El-Kafrawy[7].

The analysis using the Framework Model was done by a stiffness analysis program written specially for this task.

4.2 Analysis of Building Frame

A full three-dimensional building frame showing the degrees of freedom considered for the Finite Element Model is presented in Fig. 4.1. In Fig. 4.2, the corresponding degrees of freedom, considered for the model, are shown. These degrees of freedom consist of two translations and one rotation per floor, $\Delta x, \Delta y$ & θ_z , in addition to two rotational degrees of freedom per joint, at each connection

• The program is available in Carleton University, Civil Eng. Dept.

between columns and slab, Θ_x & Θ_y .

The global stiffness matrix of the building frame is an assembly of the floor stiffness matrices and the column stiffness matrices. Each floor can be considered as a substructure. The floor stiffness matrix has been derived by the procedure discussed in chapters 2 and 3. Then, the column stiffnesses are incorporated to construct the building frame stiffness matrix.

The displacement equations of the building frame can be expressed as:

$$[K] \{\Delta\} = \{F\} \quad (4.1)$$

where

$[K]$ = the global stiffness matrix;

$\{\Delta\}$ = the displacements corresponding to the d.o.f; and

$\{F\}$ = the forces corresponding to the d.o.f

For the case of the horizontal forces, the vector $\{F\}$ contains forces corresponding to the degrees of freedom Δ_x , Δ_y and Θ_z at each floor. There are generally no forces corresponding to the degrees of freedom, Θ_x & Θ_y , at column-slab connections. Referring to Fig. 4.3, the forces P_x and P_y are the applied lateral loads on the frame. The position of the reference axes, X , Y & Z , can be taken arbitrarily. The distances X and Y are the eccentricities of P_y and P_x , measured from the origin of the reference axes. The forces F_x , F_y and M_z , corresponding to Δ_x , Δ_y and Θ_z , can be calculated as follows:

$$P_x = P_x \quad (4.2)$$

$$P_y = P_y \quad (4.3)$$

$$M_z = P_y \cdot X - P_x \cdot Y \quad (4.4)$$

Having obtained the vector $\{F\}$, Eqn.(4.1) can be solved to calculate the unknown values of the displacements.

4.2.1 Column Moments

In order to calculate the column moments, the displacements of each individual column must be determined. Referring to Fig. 4.4, x and y are the coordinates of the column with respect to the reference axes (X & Y). The lateral displacements of the column at each floor are calculated from the following equations.

$$\Delta_{cx} = \Delta_x - \theta_z \cdot y \quad (4.5)$$

$$\Delta_{cy} = \Delta_y + \theta_z \cdot x \quad (4.6)$$

$$\theta_{cz} = \theta_z \quad (4.7)$$

where Δ_x , Δ_y and θ_z are the lateral displacements at each floor of the center O . The flexural rotations of the column nodes, θ_{cy} & θ_{cx} , are obtained directly from the vector $\{\Delta\}$.

The column end moments (Fig. 4.5) are calculated from the following equations.

$$M_{yu} = K_{cy} \cdot (\theta_{yu} + \alpha_c \cdot \theta_{y1}) - D_x \cdot (\Delta_{xu} - \Delta_{x1}) \quad (4.8)$$

$$M_{xu} = K_{fcx} (\theta_{cxu} + \alpha_c \theta_{cxl}) + D_y (\Delta_{cyu} - \Delta_{cyl}) \quad (4.9)$$

$$M_{yl} = K_{fcy} (\theta_{cyl} + \alpha_c \theta_{cyu}) - D_x (\Delta_{cxu} - \Delta_{cxl}) \quad (4.10)$$

$$M_{xl} = K_{fcx} (\theta_{cxl} + \alpha_c \theta_{cxu}) + D_y (\Delta_{cyu} - \Delta_{cyl}) \quad (4.11)$$

$$M_t = K_{tc} (\theta_{czu} - \theta_{czl}) \quad (4.12)$$

The stiffness terms in the above equations are defined in section 2.2.

4.2.2 Slab Moments

The total slab-joint moments at the floor level (Fig. 4.6) can be obtained by applying the conditions of equilibrium of the moments at each joint. These moments must be distributed to the right and the left of the column. Referring to Fig. 4.7, lines T-T and T'-T' are the line of zero torsional moments. MR is the total moment at the right of the centerline of the column and ML is the total moment at its left. The internal moments and the external moment are in equilibrium.

$$M = M_R + M_L \quad (4.13)$$

To obtain MR and ML, reference[7] recommends "50%-50% distribution" of the joint moment, M, for equal-span slabs. This approach can be generalized after additional study but is beyond the scope of this thesis

4.3 Examples and Comparison with Finite Element Method

To examine the capability of the model to simulate the behaviour of

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the flat plate structure subjected to lateral loads, two cases were analyzed, and were compared with Finite Element results.

Case 1

The structure analyzed was a 5-storey building. Each storey was 2 bays by 2 bays. All of the panels were 10.0 metres a span and the slab thickness was 0.313 metres. the storey heights were 4.0 metres. the columns were assumed to be fixed at the base of the frame. the modulus of elasticity was 25000 MPa and poisson's ratio was 0.2. The building frame was analyzed for C/L ratio varying between .05 and 0.2. In Fig. 4.8, the building frame showing the applied loads and the position of the reference axes, X, Y, Z, is presented.

Due to the symmetry of the frame and the symmetry of loading, both Y-translations and Z-rotations were equal to zero. In Fig. 4.9, the variation of X-translations at each floor is presented. The maximum difference in these translations between the Finite Element results and the results computed by the Framework Model was found at the top of the frame to be equal to 3.94%.

In Tables (4.1) to (4.4), column joint moments about the y axis (M_y) are shown for the four columns, A, B, C, D, indicated in Fig. 4.8. The maximum difference in design moment of columns between the two methods were found in the range of -3.4 % to 2.8 %.

Case 2

A 10-storey building was analyzed for the C/L ratio equal 0.10.

Each floor was 4 bays by 4 bays. The building was subjected to lateral forces applied in X-direction with eccentricity, e , equal to 0.0 and 0.10. In Fig. 4.10, the building showing the applied loads and the position of the reference axes is presented.

The same X-translations were obtained for the both values of eccentricity (Fig. 4.11). The maximum difference in these translations was found equal to 3.16 % . For e equals 0.10, the maximum difference in Z-rotations (Fig. 4.12) was less than 1.2%.

In Tables (4.5) and (4.6), the column joint moments, M_y , M_x & M_t , of the column A indicated in Fig. 4.10 are presented . They indicate differences in the design moments between the two methods in the range of -10.5 % to 12.5 % .

Fig. 4.1: Finite Element Model D.O.F

Fig. 4.3: The Applied Lateral Forces

Fig. 4.4: Column Displacements

Fig. 4.5: Column End Moments

Fig. 4.6: Slab Joint Moments

Fig. 4.7: Slab Moment Diagram

$$P = 100 \text{ KN}$$

$$t = .313 \text{ M}$$

$$L = 10 \text{ M}$$

$$H = 4 \text{ M}$$

Fig. 4.8: Case 1

Fig. 4.9: X-translation (case 1)

$$P = 100 \text{ kN}$$

$$t = .313 \text{ M}$$

$$L = 10 \text{ M}$$

$$H = 4 \text{ M}$$

Fig. 4.10: Case 2

Fig. 4.11: X-translation (case 2)

Fig. 4.12: Z-rotation (case 2)

No. of Storey	C/L 0.05			C/L 0.10			C/L 0.20			
	F.E.	Model	%diff.	F.E.	Model	%diff.	F.E.	Model	%diff.	
5	U.node	-11.60	-11.50	-0.86	-24.10	-24.20	0.40	-19.10	-18.60	-2.60
	L.node	00.91	1.17	30.00	27.80	28.50	2.50	15.20	15.1	-0.65
4	U.node	-31.40	-31.20	-0.60	-60.50	-60.90	0.66	-35.00	-34.30	-2.00
	L.node	-12.40	-12.00	-3.33	16.40	16.90	3.00	-22.10	-22.60	2.26
3	U.node	-45.80	-45.40	-0.87	-61.10	-60.70	-0.65	2.06	3.20	55.00
	L.node	-28.10	-27.40	-2.50	-17.50	-17.90	2.20	-101.00	-102.00	1.00
2	U.node	-56.40	-55.80	-1.00	-34.20	-32.30	-5.50	83.10	84.60	1.80
	L.node	-46.60	-45.90	-1.50	-87.90	-90.10	2.50	-229.00	-231.00	0.87
1	U.node	-51.00	-50.20	-1.56	44.80	48.50	8.20	217.00	219.00	0.92
	L.node	-118.00	-118.00	0.00	-228.00	-232.00	1.70	-413.00	-415.00	0.48

Table 4.1: Moments My in Column A (Case 1)

No. of Storey	C/L 0.05				C/L 0.10				C/L 0.20			
	F.E	Model	%diff		F.E	Model	%diff		F.E	Model	%diff	
5	U.node	-20.40	-20.30	-00.50	-47.20	-48.0	01.70	-38.10	-36.80	-03.40		
	L.node	-09.04	-08.77	-03.00	10.50	10.90	03.80	02.62	02.89	01.03		
4	U.node	-47.80	-47.50	-00.62	-75.40	-76.00	00.70	-42.20	-41.20	-02.30		
	L.node	-32.70	-32.10	-01.80	-02.67	-02.32	-13.1	-31.30	-31.50	00.64		
3	U.node	-74.30	-73.80	-00.67	-85.80	-85.60	00.23	-08.69	-07.16	-17.60		
	L.node	-59.20	-58.50	-01.18	-42.50	-43.00	01.10	-111.0	-112.0	00.90		
2	U.node	-97.00	-96.10	-00.92	-59.70	-57.80	-03.2	75.00	76.80	02.40		
	L.node	-91.90	-91.00	-01.00	-113.0	-115.0	01.80	-237.0	-238.0	00.42		
1	U.node	-84.00	-82.90	-01.30	28.30	32.20	13.70	213.0	215.00	00.94		
	L.node	-137.0	-137.0	00.00	-237.0	-241.0	01.70	-416.0	-417.0	00.23		

Table 4.2: Moments My in Column B (Case 1)

No. of Storey	C/L 0.05				C/L 0.10				C/L 0.20			
	F.E	Model	%diff		F.E	Model	%diff		F.E	Model	%diff	
5	U.node	-17.80	-18.30	02.80	-40.00	-41.00	02.50		-29.60	-30.40	02.70	
	L.node	- 6.25	-06.72	07.50	15.50	15.60	00.64		08.19	07.20	-12.10	
4	U.node	-43.30	-44.30	02.25	-71.60	-72.40	01.11		-39.10	-38.80	00.77	
	L.node	-27.00	-28.00	03.70	02.58	02.70	04.60		-27.20	-28.40	04.40	
3	U.node	-66.30	-68.00	02.60	-78.80	-78.90	00.13		03.94	-03.49	11.40	
	L.node	-50.60	-52.20	03.20	-35.50	-36.30	02.23		-107.0	-108.0	00.93	
2	U.node	-85.30	-87.40	02.50	-52.60	-51.10	-2.85		78.50	79.50	01.27	
	L.node	-78.40	-80.80	03.00	-106.0	-109.0	02.83		-234.0	-236.0	00.89	
I	U.node	-73.80	-75.10	01.76	33.00	36.60	11.00		215.0	216.0	00.46	
	L.node	-131.0	-132.0	00.76	-235.0	-239.0	01.70		-415.0	-417.0	00.48	

Table 4.3: Moments My in Column C (Case 1)

No. of Storey	C/L 0.05			C/L 0.10			C/L 0.20			
	F.E	Model	%diff	F.E	Model	%diff	F.E	Model	%diff	
5	U.node	-29.60	-29.70	00.33	-78.10	-78.60	00.64	-59.20	-59.40	00.33
	L.node	-20.50	-20.50	00.00	-14.20	-13.50	-04.90	-11.50	-12.20	06.00
4	U.node	-67.20	-67.40	00.30	-98.60	-98.50	-00.10	-50.30	-49.90	-00.80
	L.node	-55.90	-55.90	00.00	-30.80	-29.50	-04.20	-41.70	-42.60	-00.21
3	U.node	-107.0	-108.0	00.93	-121.0	-120.0	-00.83	-20.80	-20.00	-03.80
	L.node	-95.90	-96.00	00.10	-79.00	-78.10	-01.10	-122.0	-123.0	00.82
2	U.node	-142.0	-142.0	00.00	-97.10	-93.50	-03.70	65.80	67.66	01.82
	L.node	-140.0	-140.0	00.00	-151.0	-151.0	00.00	-246.6	-247.0	00.16
1	U.node	-117.0	-117.0	00.00	04.71	09.75	107.00	208.0	210.0	00.96
	L.node	-155.0	-156.0	00.64	-250.0	-254.0	01.60	-419.0	-420.0	00.23

Table 4.4: Moments My in Column D (case 1)

No. of Storey	e 0.0			e 0.1		
	F.E	Model	%diff	F.E	Model	%diff
10 U.node	-11.30	-13.60	20.30	-14.20	-16.80	18.31
L.node	27.30	26.50	-03.00	33.90	32.80	-03.24
09 U.node	-43.60	-46.20	06.00	-54.10	-56.90	05.10
L.node	28.10	27.10	-03.50	35.40	34.00	-03.95
08 U.node	-53.60	-57.70	07.60	-66.70	-71.10	06.60
L.node	26.90	25.10	06.70	34.10	31.80	-06.70
07 U.node	-62.70	-67.90	08.30	-78.00	-83.60	07.20
L.node	21.40	18.50	-13.60	27.40	23.90	-12.77
06 U.node	-68.00	-74.30	09.20	-84.40	-91.10	08.00
L.node	13.50	09.44	-30.00	17.50	12.70	-27.40
05 U.node	-70.60	77.60	10.00	-87.50	-94.70	08.20
L.node	-01.32	-04.19	217.00	01.96	-04.34	321.40
04 U.node	-67.70	-74.90	10.63	-82.70	-90.70	09.70
L.node	-19.30	-26.50	37.30	-24.40	-32.30	32.37
03 U.node	-53.20	-59.80	12.40	-63.70	-71.20	11.80
L.node	-60.40	-67.90	12.50	-76.10	-85.40	12.20
02 U.node	-11.20	-15.70	40.00	-10.50	-16.10	53.33
L.node	-143.0	-153.0	07.00	-179.0	-189.0	05.60
01 U.node	88.30	88.90	00.68	113.00	113.0	00.00
L.node	-337.0	-345.0	02.40	-415.0	-422.0	01.70

Table 4.5: Moments M_y in Column A (Case 2)

No. of Storey	Mx			Mt		
	F.E	Model	%diff	F.E	Model	%diff
10 U.node	00.96	00.70	-27.20	01.92	01.88	-01.90
10 L.node	-08.09	-08.18	01.11	01.92	01.88	
09 U.node	09.10	08.82	-03.08	02.86	02.75	-03.85
09 L.node	-09.17	-09.23	00.65	02.86	02.75	
08 U.node	10.50	10.00	-04.70	04.04	03.94	-02.47
08 L.node	-10.20	-10.30	00.98	04.04	03.94	
07 U.node	11.80	11.20	-05.10	05.37	05.26	-02.05
07 L.node	-09.85	-10.10	02.54	05.37	05.26	
06 U.node	11.90	11.20	-05.90	06.65	06.55	-01.50
06 L.node	-08.80	-09.19	04.43	06.65	06.55	
05 U.node	11.10	10.40	-06.30	07.87	07.72	-01.97
05 L.node	-06.33	-06.89	08.85	07.87	07.72	
04 U.node	08.84	08.12	-08.10	08.74	08.59	-01.72
04 L.node	-01.37	-02.13	55.40	08.74	08.59	
03 U.node	03.84	03.14	-18.23	08.94	08.80	-01.60
03 L.node	09.07	08.12	-10.47	08.94	08.80	
02 U.node	-06.99	-07.34	05.00	07.73	07.63	-01.30
02 L.node	29.70	28.50	-04.04	07.73	07.63	
01 U.node	-28.40	-28.20	-00.70	03.61	03.58	-00.97
01 L.node	75.50	74.60	-01.20	03.61	03.58	

Table 4.6: Moments Mx&Mt in Column A (Case 2)

CHAPTER 5

CONCLUSIONS AND FUTURE WORK5.1 Summary

The work presented in this thesis is concerned with the modelling of flat plate structures. The model has been developed to simulate the 3-dimensional elastic stiffness of a Finite Element Model. In this study, parameter variation was limited to the effect of slab to column stiffness ratio, while other parameters affecting stiffness were kept constant. Formulas have been derived for the model member stiffnesses as functions of the column to slab dimension ratio. The model was used to analyze building frames subjected to lateral loads. The model behaviour showed good agreement with that of the Finite Element Model.

5.2 Conclusions

On the basis of the results obtained in this study the following conclusions have been reached:

1- The formulas presented in this work allow accurate evaluation of elastic flat plate stiffness coefficients.

2- The Framework Model has obvious advantage over the Finite Element Model with respect to the computing cost and the preparation of input data.

3- The Framework Model is a full 3-dimensional model which, especially after further development, can be used for dynamic analysis, complex loading cases, and interaction with foundation and other structures in contrast to 2-dimensional models.

4- For lateral load analysis, the Framework Model is considered a proper analogy of the flat plate structures.

5- Any frame analysis program can thus be used to analyze building frames. Some of those available do not permit the direct modelling of connecting springs, however, this can be done easily using artificial members.

5.3 Recommendations for Future Work

The following topics are suggested for future study:

1- In this thesis, the stiffness of the model members have been evaluated as functions of column to slab stiffness ratio. The study of the other parameters such as the slab aspect ratio, the column aspect ratio, the slab holes and the column offset, is essential for complete development of the Framework Model

2- This work can also be extended to include flat slabs and flat plates with beams.

3- The Framework Model was used in this thesis for lateral load analysis of flat plate structures. The use of the model for vertical load analysis is an extension to the work presented here. The vertical load analysis involves the calculation of consistent forces,

for the applied uniform or concentrated load, related to the model primary d.o.f (Fig. 5.1). To calculate these consistent forces, the author suggests that the applied vertical load may be represented by applied moments shown in Fig. 5.2. These moments are calibrated so that the consistent forces match those calculated by the finite element study [7].

4- As more general approach for the analysis of flat plate structures, it is suggested to subdivide the slab into slab panels (Fig. 5.3). The slab panel stiffness matrix may be derived in terms of rotational degrees of freedom at column-slab joints (Fig. 5.4). Due to different boundary conditions at interior and exterior edges, the elements are classified as corner elements, exterior elements and interior elements (Fig. 5.5). These three types of elements are new, and they are developed especially for this kind of buildings. The building frame stiffness matrix can be obtained by combining the contributions of the panel stiffness.

Fig. 5.1: Consistent Moments

Fig. 5.2: Equivalent Moments

Fig. 5.3: Subdivision system

Fig. 5.4: Panel Degrees of Freedom

a) Corner Panel

—— Exterior Edge

- - - Interior Edge

b) Exterior Panel

c) Interior Panel

Fig. 5.5: Panel Boundary Conditions

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APPENDIX A: STIFFNESS MATRIX OF A ONE-PANEL GRID

In Fig. A-1, an one-panel grid showing the numbering system of the degrees of freedom is presented.

Symbols:

K_{fb} = the flexural stiffness of the orthogonal beam

K_{tb} = the torsional stiffness of the orthogonal beam

α_b = the carry over factor of the orthogonal beam

K_{fd} = the flexural stiffness of the diagonal member

K_{td} = the torsional stiffness of the diagonal member

L = the length of the orthogonal beam

L_1 = the length of the diagonal member

A.1 The diagonal-grid stiffness matrix

For the diagonal grid shown in Fig.(A.2), the degrees of freedom are the rotations at points (A,B,C,D) while the displacements at point (F) are considered to be released. Using the diagonal member stiffness matrix described in Eqn. 2.6, the matrices $[S\delta\delta]$, $[S\delta\alpha]$, $[S\alpha\delta]$ and $[S\alpha\alpha]$, defined in section 2.3, can be assembled.

member	Cx	Cy
1	$1/\sqrt{2}$	$1/\sqrt{2}$
2	$1/\sqrt{2}$	$-1/\sqrt{2}$
3	$-1/\sqrt{2}$	$1/\sqrt{2}$
4	$-1/\sqrt{2}$	$-1/\sqrt{2}$

$$[s\delta\delta] = \begin{bmatrix} G & -Q & & & & & & \\ -Q & G & & & & & & \\ & & G & Q & & & & \\ & & Q & G & & & & \\ & & & & G & Q & & \\ & & & & Q & G & & \\ & & & & & & G & -Q \\ & & & & & & -Q & G \end{bmatrix} \quad (A.1)$$

where

$$G = 0.5 (K_{fd} + K_{td})$$

$$Q = 0.5 (K_{fd} - K_{td})$$

$$[s\delta\delta] = \begin{bmatrix} A & -B & A & B & A & B & A & -B \\ -B & A & B & A & B & A & -B & A \\ C & -C & C & C & -C & -C & -C & C \end{bmatrix} \quad (A.2)$$

where

$$A = 0.5 (0.5K_{fd} - K_{td})$$

$$B = 0.5 (0.5K_{fd} + K_{td})$$

$$C = \frac{1.5K_{fd}}{\sqrt{2} L_1}$$

$$[S_{\delta\delta}'] = \begin{bmatrix} 4Q & & \\ & 4Q & \\ & & 4D \end{bmatrix} \quad (A.3)$$

where

$$D = \frac{3Kfd}{L_1^2}$$

substituting the above matrices in Eqn. 2.16

$$[S_{\delta\delta}']^{-1} = \begin{bmatrix} \frac{1}{2(Kfd+Ktd)} & & \\ & \frac{1}{2(Kfd+Ktd)} & \\ & & \frac{L_1^2}{12Kfd} \end{bmatrix} \quad (A.4)$$

$$[s\delta\delta][s\delta\delta]^{-1} =$$

$\frac{0.5Kfd-Ktd}{4(Kfd+Ktd)}$	$\frac{-0.5Kfd-Ktd}{4(Kfd+Ktd)}$	$\frac{L1}{8\sqrt{2}}$
$\frac{-0.5Kfd-Ktd}{4(Kfd+Ktd)}$	$\frac{0.5Kfd-Ktd}{4(Kfd+Ktd)}$	$\frac{-L1}{8\sqrt{2}}$
$\frac{0.5Kfd-Ktd}{4(Kfd+Ktd)}$	$\frac{0.5Kfd+Ktd}{4(Kfd+Ktd)}$	$\frac{L1}{8\sqrt{2}}$
$\frac{0.5Kfd+Ktd}{4(Kfd+Ktd)}$	$\frac{0.5Kfd-Ktd}{4(Kfd+Ktd)}$	$\frac{L1}{8\sqrt{2}}$
$\frac{0.5Kfd-Ktd}{4(Kfd+Ktd)}$	$\frac{0.5Kfd+Ktd}{4(Kfd+Ktd)}$	$\frac{-L1}{8\sqrt{2}}$
$\frac{0.5Kfd+Ktd}{4(Kfd+Ktd)}$	$\frac{0.5Kfd-Ktd}{4(Kfd+Ktd)}$	$\frac{-L1}{8\sqrt{2}}$
$\frac{0.5Kfd-Ktd}{4(Kfd+Ktd)}$	$\frac{-0.5Kfd-Ktd}{4(Kfd+Ktd)}$	$\frac{-L1}{8\sqrt{2}}$
$\frac{-0.5Kfd-Ktd}{4(Kfd+Ktd)}$	$\frac{0.5Kfd-Ktd}{4(Kfd+Ktd)}$	$\frac{L1}{8\sqrt{2}}$

(A.5)

$$[Kn] = [s\delta\delta][s\delta\delta]^{-1}[s\delta\delta]$$

(A.6)

$$[K_n] = \begin{bmatrix} K_{n1} & & & & & & & \\ & K_{n2} & & & & & & \\ & & K_{n3} & & K_{n6} & & K_{n1} & \\ & & & K_{n4} & & & & \\ & & & & K_{n5} & & -K_{n2} & \\ & & & & & K_{n7} & & \\ & & & & & & -K_{n8} & \\ & & & & & & & K_{n1} \\ & & & & & & & & K_{n1} \\ & & & & & & & & & K_{n1} \\ & & & & & & & & & & K_{n1} \\ & & & & & & & & & & & K_{n1} \\ & & & & & & & & & & & & K_{n1} \\ & & & & & & & & & & & & & K_{n1} \end{bmatrix} \quad (A.7)$$

Where

$$K_{n1} = \frac{0.75}{8} K_{fd} + \frac{0.25 K_{fd}^2 + K_{td}^2}{4(K_{fd} + K_{td})}$$

$$K_{n2} = \frac{-0.75}{8} K_{fd} - \frac{0.25 K_{fd}^2 - K_{td}^2}{4(K_{fd} + K_{td})}$$

$$K_{n3} = \frac{-K_{fd} K_{td}}{4(K_{fd} + K_{td})} + \frac{0.75}{8} K_{fd}$$

$$K_{n4} = \frac{0.75}{8} K_{fd}$$

$$K_{n5} = \frac{-K_{fd} K_{td}}{4(K_{fd} + K_{td})} + \frac{0.75}{8} K_{fd}$$

$$K_{n6} = \frac{-0.75}{8} K_{fd}$$

$$K_{n7} = \frac{-0.75}{8} K_{fd} + \frac{0.25K_{fd}^2 + K_{td}^2}{4(K_{fd} + K_{td})}$$

$$K_{n8} = \frac{0.75}{8} K_{fd} - \frac{0.25K_{fd}^2 - K_{td}^2}{4(K_{fd} + K_{td})}$$

$$[K_d] = [K_{dd}] - [K_n] \quad (A.8)$$

$$[K_d] = \begin{bmatrix} K_{d1} \\ K_{d2} & K_{d1} \\ K_{d3} & K_{d6} & K_{d1} & \text{SYMETRIC} \\ K_{d4} & K_{d5} & -K_{d2} & K_{d1} \\ K_{d5} & K_{d4} & K_{d7} & -K_{d8} & K_{d1} \\ K_{d6} & K_{d3} & -K_{d8} & K_{d7} & -K_{d2} & K_{d1} \\ K_{d7} & K_{d8} & K_{d5} & -K_{d4} & K_{d3} & -K_{d6} & K_{d1} \\ K_{d8} & K_{d7} & -K_{d6} & K_{d3} & -K_{d4} & K_{d5} & K_{d2} & K_{d1} \end{bmatrix} \quad (A.9)$$

Where

$$K_{d1} = \frac{3.25}{8} K_{fd} + 0.5K_{td} - \frac{0.25K_{fd}^2 + K_{td}^2}{4(K_{fd} + K_{td})}$$

$$K_{d2} = \frac{-3.25}{8} K_{fd} + 0.5K_{td} + \frac{0.25K_{fd}^2 - K_{td}^2}{4(K_{fd} + K_{td})}$$

$$K_{d3} = \frac{K_{fd} K_{td}}{4(K_{fd} + K_{td})} - \frac{0.75}{8} K_{fd}$$

$$Kd4 = \frac{-0.75}{8} Kfd$$

$$Kd5 = \frac{Kfd \quad Ktd}{4(Kfd + Ktd)} + \frac{0.75}{8} Kfd$$

$$Kd6 = \frac{0.75}{8} Kfd$$

$$Kd7 = \frac{0.75}{8} Kfd - \frac{0.25Kfd^2 + Ktd^2}{4(Kfd + Ktd)}$$

$$Kd8 = \frac{-0.75}{8} Kfd + \frac{0.25Kfd^2 - Ktd^2}{4(Kfd + Ktd)}$$

A.2 The Orthogonal grid stiffness matrix

For the orthogonal grid shown in Fig. A.3, the stiffness matrix can be expressed as

$$[Kgo] = \begin{bmatrix} Kfb+Ktb & 0 & -Ktb & 0 & \alpha_b Kfb & 0 & 0 & 0 \\ 0 & Kfb+Ktb & 0 & \alpha_b Kfb & 0 & -Ktb & 0 & 0 \\ -Ktb & 0 & Kfb+Ktb & 0 & 0 & 0 & \alpha_b Kfb & 0 \\ 0 & \alpha_b Kfb & 0 & Kfb+Ktb & 0 & 0 & 0 & -Ktb \\ \alpha_b Kfb & 0 & 0 & 0 & Kfb+Ktb & 0 & -Ktb & 0 \\ 0 & -Ktb & 0 & 0 & 0 & Kfb+Ktb & 0 & \alpha_b Kfb \\ 0 & 0 & \alpha_b Kfb & 0 & -Ktb & 0 & Kfb+Ktb & 0 \\ 0 & 0 & 0 & -Ktb & 0 & \alpha_b Kfb & 0 & Kfb+Ktb \end{bmatrix}$$

(A.10)

A.3 The grid stiffness matrix

$$[K_g] = [K_{go}] + [K_d] \quad (2.8)$$

$$[K_g] = \begin{bmatrix} K_{g1} & & & & & & & \\ K_{g2} & K_{g1} & & & & & & \\ K_{g3} & K_{g6} & K_{g1} & \text{SYMETRIC} & & & & \\ K_{g4} & K_{g5} & -K_{g2} & K_{g1} & & & & \\ K_{g5} & K_{g4} & K_{g7} & -K_{g8} & K_{g1} & & & \\ K_{g6} & K_{g3} & -K_{g8} & K_{g7} & -K_{g2} & K_{g1} & & \\ K_{g7} & K_{g8} & K_{g5} & -K_{g4} & K_{g3} & -K_{g6} & K_{g1} & \\ K_{g8} & K_{g7} & -K_{g6} & K_{g3} & -K_{g4} & K_{g5} & K_{g2} & K_{g1} \end{bmatrix} \quad (A.11)$$

Where

$$K_{g1} = \frac{3.25}{8} K_{fd} + 0.5 K_{td} - \frac{0.25 K_{fd}^2 + K_{td}^2}{4(K_{fd} + K_{td})} + K_{fbe} + K_{tbe}$$

$$K_{g2} = \frac{-3.25}{8} K_{fd} + 0.5 K_{td} + \frac{0.25 K_{fd}^2 - K_{td}^2}{4(K_{fd} + K_{td})}$$

$$K_{g3} = \frac{K_{fd} K_{td}}{4(K_{fd} + K_{td})} - \frac{0.75}{8} K_{fd} - K_{tbe}$$

$$K_{g4} = \frac{-0.75}{8} K_{fd}$$

$$K_{g5} = \frac{K_{fd} K_{td}}{4(K_{fd} + K_{td})} + \frac{0.75}{8} K_{fd} + \alpha_{be} K_{fbe}$$

$$K_{g6} = \frac{0.75}{8} K_{fd}$$

$$K_{g7} = \frac{0.75}{8} K_{fd} - \frac{0.25K_{fd}^2 + K_{td}^2}{4(K_{fd} + K_{td})}$$

$$K_{g8} = \frac{-0.75}{8} K_{fd} + \frac{0.25K_{fd}^2 - K_{td}^2}{4(K_{fd} + K_{td})}$$

Fig. A.1: One Panel Grid

Fig. A.2: Diagonal Grid D.O.F

Fig. A.3: Orthogonal Grid D.O.F

APPENDIX B: FLEXURAL AND TORSIONAL STIFFNESSES OF COLUMN

In deriving the column stiffness, the slab thickness at the top and the bottom ends of the column, are considered to be equal. The column analogy method is used to determine the flexural stiffness and the carry over factor. The column stiffness characteristic and the analogous section are shown in Fig. B.1

The properties of the analogous section are:

$$A = \frac{H_c}{E I_c}$$

$$I = \frac{1}{E I_c} \left(\frac{H_c^3}{12} \right)$$

The flexural stiffness of the column is:

$$K_{fc} = \frac{1}{A} + \left(\frac{H_c + t}{2} \right) \frac{1}{I}$$

$$= E I_c \left(\frac{1}{H_c} + \left(\frac{H_c + t}{2} \right) \frac{12}{H_c^3} \right)$$

$$= \frac{E I_c}{H_c} \left(1 + 3 \left(1 + \frac{t^2}{H_c} \right) \right) \quad (B.1)$$

The carry over factor is:

$$\alpha_c = \frac{(3W - 1)}{(1 + 3W)} \quad (B.2)$$

where

$$W = \left(1 + \frac{t^2}{H_c} \right)$$

The torsional stiffness is

$$K_{tc} = \frac{G J}{H_c} \quad (B.3)$$

where

G = the shear modulus

J = torsion constant

$$J = \left(1 - 0.63 \frac{C_1}{C_2} \right) \frac{C_1^3 C_2}{3} \quad C_1 < C_2 \quad (B.4)$$

where C_1 and C_2 are the dimensions of the column.

a) Elevation

b) Column Stiffness Diagram

c) Column X-section

d) Analogous section

Fig. B.1: Column Stiffness

END

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FIN