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ANALYSIS OF BUILDING FRAMES

by

© Moustafa Fouad El-Kafrawy
B.Sc.Eng., M.Eng.

A Thesis

Presented to the Faculty of Graduate Studies
of Carleton University in partial
fulfillment of the degree of
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March 28, 1983

To my wife and my parents

1

ABSTRACT

Flat plate structural building frames are studied in this thesis. Two numerical models are used in the investigation. The first is a finite element model of a single storey of the structure, with the floor divided into plate bending elements, and the columns framing into that floor represented by conventional frame elements. The second is a three-dimensional model of the full building frame. The degrees of freedom in this model consist of two translational and one rotational degrees of freedom per storey at the storey level; and two rotational per slab-column joint. A third structural model was initiated in the course of the present work but is now being carried forward in another investigation. This work will be briefly reported herein.

The finite element floor model has been used to determine the distribution of slab moments and shears and the moments transferred to columns in a uniformly loaded flat plate floor, under varying parameters. Of particular interest here was the effect of column elongation on the moments. Column offsets, column sizes, slab aspect ratio and slab openings were also studied.

The variations of slab and column stiffnesses are also reported and a finite element analysis of the elastic behaviour of flat plate full building frames is presented. The analysis is carried out using a two-stage finite element and stiffness method. Although the analysis process involves somewhat more data preparation than design

office methods require, it does involve techniques which are considerably less expensive than full finite element analysis both in terms of CPU costs and data preparation, and furthermore it is amenable to fairly routine procedures which could be made automatic. The advantage is a rational analysis, which can take care of sidesway analysis in a realistic way, and can handle such complexities as holes for stairways, and offset columns. Dynamic analysis and stability analysis are also practicable extensions. The results of an analysis of a 10-storey building frame are presented and compared to earlier studies.

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LIST OF SYMBOLS

[A]	square matrix derived from {P} evaluated at the nodes
[B]	matrix of strain shape functions (derived from [N])
C	cross section dimension of a square column
{C}	vector of plate curvatures, $\omega_{,xx}$, $\omega_{,yy}$ and $\omega_{,xy}$
C_1, C_2	cross sectional dimensions of rectangular column
D	flexural rigidity of the plate ($D = \frac{Et^3}{12(1-\nu^2)}$)
[D]	element elastic property matrix
e	column eccentricity
E	Young's modulus
{F}	vector of applied forces
{f ^(e) }	element force vector
h	column or storey height
h_c	column clear height
I_s	slab moment of inertia away from the columns
I_{sc}	slab moment of inertia over the support
K	slab panel stiffness
[K]	stiffness matrix of the flat plate floor
K_x	slab panel stiffness in the x direction
K'	stiffness coefficient ($K' = K \frac{L_1}{L_2 D}$)
K'_x	stiffness coefficient ($K'_x = K_x \frac{L_1}{L_2 D}$)
[k ^(e)]	element stiffness matrix
K_{cx}	column stiffness in the x direction

K_{cy}	column stiffness in the y direction
L	dimension of square slab panel
L_1, L_2	dimensions of rectangular slab panel
L_0	slab opening dimension
$\{m\}$	vector of the plate moments M_{xx} , M_{yy} and M_{xy}
$\{M\}$	vector of direct joint forces (moments)
$\{M^F\}$	vector of fixed end forces (moments)
M_x, M_y	slab bending moments in the x and y directions
M_{cx}, M_{cy}	column moments in the x and y directions
$[N]$	matrix of displacement shape functions
$\{P\}$	vector contains the assumed deflection functions
q	uniform load intensity
$\{q\}$	vector of applied surface traction
t	plate thickness
U_s	strain energy
V_x, V_y	plate vertical shear forces per unit length in the x and y directions
V_p	punching shear forces
Z	vertical distance measured from the plate neutral surface
Π_p	total potential energy
$\{\alpha\}$	vector contains constants determined from the element geometry
β	bending moment factor
$\{\epsilon\}$	element strain vector (ϵ_{xx} , ϵ_{yy} and ϵ_{xy})
ν	Poisson's ratio
θ_x, θ_y	the columns rotations in the x and y directions
$\{\Delta\}$	vector of nodal displacements
$\{\sigma\}$	element stress vector (σ_{xx} , σ_{yy} and σ_{xy})
w	plate lateral displacement

CHAPTER 1

INTRODUCTION

1.1 General

A building frame in this thesis is defined as a reinforced concrete structure consisting of floor slabs with columns framing into the slabs at right angles, an example of which is shown in Figure 1.1. The design and analysis of these structures has occupied a great deal of design office activity and yet this is an area which has lacked sufficient attention by researchers.

Reinforced concrete floor slabs spanning in more than one direction fall into two different categories: 'two-way slabs' which are supported on beams and girders, and 'flat slabs' with or without drop panels which are directly supported on columns which may have capitals. In designs for relatively light loads, 'flat plates' are commonly used. These may be defined as flat slabs with no column capitals or drop panels.

In early applications, flat slabs and flat plates were used chiefly for buildings like warehouses and factories which required large open floor-spaces. Their main advantage was that they permitted a given minimum clear height to be maintained with a reduced overall story height; though at the expense of calling for more material in the floors because of the reduction in stiffness of horizontal members. Today, two further advantages have been gained which have made flat slabs and flat plates attractive also for offices and residential buildings.

These are simplicity of formwork and the complete freedom with which internal partitions can be placed beneath an unbroken level soffit.

As flat slabs and flat plates have become extensively used, their correct design is of primary interest to ensure safety and, at the same time, to provide economical design. On the other hand, obtaining the exact stress distribution in these slabs is not an easy task due to the complexity of the plate behaviour and the large number of variables involved in the analysis. The work carried out in the course of this thesis is a stage in an attempt to establish the design of such a slab structure in a path toward a more theoretical analysis.

1.2 Development of the Code Design Methods

1.2.1 Construction of Early Slabs

The use of reinforced concrete flat slab structures began at the turn of this century. In 1903, C. A. P. Turner received a patent for a "mushroom" slab, that is, a slab supported directly by columns with no beams spanning between columns. Two years later he built the first such slab in Minneapolis but, since there was no rigorous mathematical basis for the design, a performance load test was required. The performance of the flat slab structure in the load test was considered satisfactory and by 1913 over 1000 flat-slab buildings had been built around the world.

Although, the early proof tests satisfied the immediate requirements, they were not adequate to form a basis for design recommendations. In

1910, A. R. Lord reported the strain reading he had made during a load test. With the help of such measurements the moment was computed adopting the Kirchhoff assumption of straight line normal sections.

1.2.2 Empirical Design Methods

In 1914, Nichols presented a paper in which he applied the principles of elementary statics to a flat plate in order to determine the total moment in one panel [1].* In his original paper, Nichols determined an expression for the 'total static moment' (the net sum of positive center span and average absolute negative end moment) in a panel with circular column capitals. He assumed that the panel was one central panel in an infinite grid of identical panels, all uniformly loaded. The shear was assumed uniformly distributed around the perimeter of each column capital.

The moments which had been computed from steel strains, obtained from the early tests, indicated much lower moments than the Nichols expression. The 1917 edition of the ACI Building Code permitted an 'Empirical Method' of design of $0.09 \cdot qL^2$ for the 'total moment', which is approximately 72% of the total static moment ($0.125 qL^2$) in a panel.

In 1921, Westergaard and Slater published a paper on the analysis and design of slabs [2]. In this paper they showed that the moments calculated from steel strains in flat slab tests were not correct

* Numbers in square parentheses refer to the cited references on page 156.

because they did not properly take into consideration the tension carried by the concrete. Modified moments based on this study averaged about 90% of those given by Nichols, decreasing the discrepancy between theoretical and measured moments.

More recent work [3] showed that there were at least two other sources of error in the early interpretation of the flat slab tests which had not been recognized and consequently not considered. These were the neglect of the twisting moment around the columns; and the neglect of moments carried by adjacent unloaded panels.

The amount of the twisting moment carried by the concrete in the vicinity of the columns depends upon the geometry of the supports, the loading patterns, the amount of cracking, and the material properties. For slabs with circular capitals, the twisting moments are quite small although they may still be important. For other shapes of capitals they become more and more important until they reach a maximum for square or rectangular columns. Results of solutions for the three by three panel slabs in [3] indicate that, for the extreme case of one strip of panels loaded, twisting moments at the columns may be 15% of the total static moment in one panel. This large moment would drop once the concrete began to crack, however, a portion of it would exist until the slab was cracked through completely.

The error of neglecting the moments carried by adjacent unloaded panels can be quite large. The analysis of the three by three panel slab in [3] indicates that, for the extreme case of the middle row of the panels loaded, the moments carried by the adjacent unloaded panels

may be 25% of the total static moment. Figure 1.2 shows the computed moments at various sections with the middle row of panels loaded. It can be seen that the sum of the positive and negative moments in the centre panel is only about 75% of the sum of the positive and negative moments across the entire width of the structure. Although this is a much more severe case than those of the early slab tests, it indicates that neglecting the effect of adjacent unloaded panels may have substantial effect on the total moment in a span.

In the early transactions of the ASCE, the discussions relating to total static moment versus load test are fascinating reading for the modern student of Civil Engineering. The statements made by Nichols now seem quite trivial, and of course, academically perfectly correct. On the basis of the information presented above, it appears that the lack of agreement between theory (i.e. Nichols) and the results of early tests (i.e. Turner), is due to an improper consideration of the amount of tension carried by the concrete, neglect of twisting moment at the column capitals, and neglect of the effects of unloaded panels adjacent to the loaded spans. The most important of these appears to be the error in the amount of tension carried by the concrete.

The 1956 ACI Building Code introduced a new factor, F , out of concern that the old tests on flat slabs, which were designed with large column sizes, might not apply to modern flat plates having somewhat smaller column sizes. The value of this factor varied between 1.0 and 1.15 depending on column size. Thus, the total moment permitted by the code ranged between $0.09 qL^2$ to $0.103 qL^2$ (72% to 82.8% of the total static moment in a panel).

In 1959, Siess [4] pointed out that the total static moment procedure can be extended to cases of rectangular panels with square or rectangular column capitals. Assuming different distributions of shear around the column capital, he arrived at expressions for the total static moment in an interior panel with different column shapes.

In order to make provision for the ultimate strength design of flat slabs, the 1963 ACI Building Code increased the total moment permitted from $0.09F_qL^2$ to $0.1F_qL^2$, which was to be adopted only when ultimate strength theory was used. It was also thought that the use of high strength steel might result in a thinner slab section and hence, larger deflection. In order to offset this, the minimum limit on the thickness of the slab was also increased.

To obtain reliable data on failure and to serve as the basis for evaluating several analytical approaches which were under investigation, a series of slabs were tested at the University of Illinois in the 1960's [5,6,7]. These tests were the basis of the 1971 edition of the ACI Building Code.

The Direct Design Method of the current ACI Building Code [8] is similar to the Empirical Method for flat slabs included in preceding editions of the ACI Building Code, in that a bending analysis is never actually carried out. However, the current code stipulates the use of the full static moment.

On the basis of the results obtained from the tests at the University of Illinois and a three-quarter scale test by the Portland Cement

Association [9], the distribution of moments in the current code, ACI 318-77 are slightly different from the 1963 values.

1.2.3 Elastic Frame Analysis

In early ACI Building Codes (1917 to 1941) the Empirical Method, was the only method provided for flat slab design. The need for a more rational analysis method was felt, primarily because of the anticipated effects of pattern loadings on the slab and bending moments in the columns. Also, the use of the Empirical Method was restricted to certain slab and column dimensions. This led eventually to the development of a codified method of analysis which could consider any dimension, shape or loading pattern.

One of the first attempts to treat a reinforced concrete flat slab structure as a system of two-dimensional frames was presented by Taylor, et al. [10] in 1925. The concept of frame analysis was also used by Westergaard in 1921 to study critical loading cases in flat slabs [2].

In 1929, a committee working on the California Building Code carried out an investigation to determine the applicability of the Empirical Method; as well as to try to extend it [11]. From this study, a procedure evolved for computing moments in flat slabs by means of elastic frame analysis. This method consisted of dividing the structure into a system of bents one bay wide. Stiffnesses of the members were found by taking into account all variations in moments of inertia of the members.

At about the same time, an investigation was carried out under the direction of R. L. Bertin to incorporate frame analysis into the ACI Building Code [12]. This investigation was initiated to determine a method of frame analysis which would give results similar to the empirical analysis. In 1939, Peabody published a paper on frame analysis of flat slabs in which he used essentially the same method [13]. This method was eventually incorporated in the 1941 ACI Code. In this procedure the structure was again divided into a system of frames, each consisting of a row of columns and a strip of the slab one bay wide. The joints between the slab and the columns in these frames were considered to be rigid. The moments were then determined and the negative moments were reduced to the value at a distance X_L from the centerline of the columns. As originally developed, the distance X_L was to be determined such that the total moment in a panel was the same as that computed using the Empirical Method. With some additional modifications, the same procedure was used in the 1963 ACI Building Code [14].

The procedure as outlined above does not represent the two-way action of the slab; and it leads to positive moments which are too low, and negative moments which are unrealistically high, before the reduction is applied. Zweig [15] has shown that, in the case of low live load to dead load ratio, the negative moments can be 70% less than those found for the Empirical Method and the positive moments can be 25% less. Studies at the University of Illinois in the 1960's [16,17] have shown that, the positive moment in a slab increases under pattern loads even if rigid columns are used. However, if a two-dimensional frame analysis is applied to a structure with infinite column stiffness, pattern loads

will have no effect. This is because there is a difference in behaviour between frames and slab structures. In slab structures, the part of the slab across the column can rotate though the column is infinitely stiff. In the case of edge beams, the behaviour mechanism is easily visualized. Some of the moment is transferred from the slab directly to the column while the remainder is transferred first to the beams, then to the column. It can be seen that a rigid column does not prevent rotation of the beams with respect to the column. Assuming rigid slab-column joints in these frames precludes the consideration of the torsional resistance of these edge beams.

The Equivalent Frame Method introduced in the 1971 edition of the ACI Building Code is similar to the elastic design method previously used. However, the definitions of the frame member stiffness have been modified substantially to take into account the torsional rotation of the slab [18].

1.3 The Code Design Methods

The two methods recommended by the code reduce the three-dimensional problem into a series of two-dimensional problems in two orthogonal directions. Each direction involves the consideration of rigid frames consisting of the columns above and below the slab, and the slab system bounded laterally between the centerlines of the two panels adjacent to the line of columns.

1.3.1 Direct Design Method

The direct design method is basically a fairly heavily parameterized tabulation scheme for obtaining the distribution of moments directly, without any need for detailed analysis. It is restricted by limitations on continuity, dimensions and live-load to dead-load ratios to ensure regular stress patterns in the slab system. The distribution of the moments in the longitudinal direction is found by proportioning the total static design moments according to a set of coefficients prescribed by the code. Then, these moments are distributed over the entire width of the slab strip according to another set of coefficients provided by the code.

1.3.2 Equivalent Frame Method

The equivalent frame method does involve a structural analysis and therefore is capable of providing a more realistic distribution of moments. It differs from the direct design method only in the way in which the longitudinal moments (i.e. total moments in the plane of the frame) along the spans of the equivalent frame are obtained. In the equivalent frame method the elastic analysis of the equivalent rigid frame is carried out and the maximum positive moments within the span and negative moments at the supports are determined.

The equivalent frame consists of slab-beam members and equivalent column members. Equivalent columns are further divided into actual column members and attached torsional members (Figure 1.3) introduced to estimate the actual moment distribution between the adjacent slab

3.2.2 Force Terms

The elements in the force vector consist of lateral forces applied at each storey level, which can be implemented in the force vector directly; and vertical panel loads. In numerical examples given later only the lateral forces were applied in order to provide direct comparisons with previous work. The panel loads require some processing before equivalent values corresponding to degrees of freedom can be assembled with the force vector.

The "fixed end forces" for a single loaded panel, as shown in Figure 3.3, consist of the negatives of the joint locking forces which are induced when that load is applied. When the slab is loaded as in Figure 3.3, the stiffness equations are

$$[K] \{\Delta\} = \{M - M^F\} \quad (3.2)$$

where $\{M\}$ comprises only direct joint forces, and is equal to zero, and $\{M^F\}$ are the fixed end forces. The fixed end moments for individual panels for implementation in equation (3.1) are computed using equation (3.2) as follows:

- (i) The analysis of the finite element model, with a single panel loaded, provides the vector $\{\Delta\}$ in equation (3.2).
- (ii) The inversion of the flexibility matrix discussed previously provides $[K]$.

The reader may thus be surprised to note that "fixed end forces" for uniformly loaded panels are not generated using locked joints at all, and are in fact calculated from additional load cases used in the finite

of these torsional members should be increased in proportion to the thickness of the plate to increase their stiffness, which in effect would increase the stiffness of the equivalent column members.

Sharan, et al. [21,22] investigated the elastic behaviour of idealized flat plates under vertical loading using the finite element method. The effect of horizontal loads was studied by applying moments at the column slab joints. The results of the finite element analysis were compared with experimental results obtained by Liu [23] on a three-span flat plate aluminum model. It was concluded in the study that the equivalent member stiffness depends primarily on whether the slab or the column is loaded. Therefore, it was suggested that, for vertical loads on the slab, the full slab flexure stiffness and an equivalent column stiffness be used; however, for horizontal loads on the column, the full column flexure stiffness and an equivalent slab stiffness would be used. It is interesting to note that Sharan's work has not been assimilated into the Canadian Standard, nor has any similar study been incorporated in any code for lateral load analysis of building frames. In a later chapter the author will present results of a lateral load analysis using the Equivalent Frame Method and compare these to a full building frame analysis. The codes do not suggest using the Equivalent Frame Method for sidesway analysis but neither do they say not to, nor offer an alternative.

Pecknold [24] has suggested that, for sidesway analysis, the three dimensional building frame be modelled as a series of typical plane frames aligned in the direction of the applied loads. In these frames, the slab is modelled as a beam equal in depth to that of the slab, and its width is the slab panel width multiplied by an effective width

coefficient having a value less than one. In deriving these effective width coefficients a typical interior panel was considered, and a different load distribution in the column area was selected to simulate rigid and flexible columns. Allen and Darvall [25,26] used a similar procedure to determine the width of the equivalent beams. The theoretical results were compared with experimental ones using one meter span steel plates of various aspect ratio, and a single microconcrete (1/5 scale) rectangular plate with square rigid columns. Tables of the values of the equivalent width coefficients for varying aspect ratio and column size were presented.

The equivalent beam approach provides a simple method of analysis; however, the method is only applicable to building frames subjected to lateral loads, also in irregular building frames it is difficult to predict the values of the equivalent beam coefficients. Results from the equivalent beam approach will be scrutinized in more detail later in the thesis.

Recently Elias [27,28] proposed an equivalent frame model consisting of slab members and unmodified column members to analyze flat plate structures subjected to either horizontal or vertical loads. The analysis of slab stiffness was based on separating the general state of loading into symmetric and antisymmetric states. A finite element model of a single panel was used. Compatible deformation of all frames in the direction of bending was assumed.

1.5 Objective and Scope

The work that has been done on flat plate structures has not dealt directly with the analysis of three-dimensional building frames. It is clear to all structural engineers that the lateral displacement of a building frame results in transverse bending effects, except on lines of perfect symmetry. The concept in the present codes (American and Canadian) of the equivalent column stiffness with torsional edge beam is the closest we have come to a rational design procedure. However, this concept cannot be applied in analysis for lateral loads. Recent studies have attempted to provide more accurate analyses of a slab-column structural system subjected to lateral loads. In all of these studies, the three-dimensional building frame was modelled as a series of typical plane frames aligned in the direction of the applied lateral loads. This leads to two different methods for the analysis of the same structure; one for vertical loads and one for horizontal.

It could be argued that by consideration, piecemeal, of each element of a system, some totality of the strength requirements of a structure are satisfied and some totality of adequacy will be provided in the design. This does not necessarily ensure the integrity of every element as part of the system. Also, it is an anomaly that neither complete building frames, nor even two dimensional floor systems are analyzed in the current state of this industry.

A rational three-dimensional analysis of a building frame can be carried out using the finite element method, however this is considered too costly by the profession in terms of the time spent in preparing a

data deck and the cost of the finite element run itself. In addition, the output from a typical general purpose finite element program contains a minuita of information of no use to the designer.

One objective of this study is to develop a method of analysis to be used here as a research tool based on the finite element method, and which considers a full three-dimensional model of the structure, including lateral and vertical loads. The work in this thesis addresses mainly the flat plate system (i.e. no column capitals or drop panels). However, the extension to flat slabs is a minor one.

The current building code provisions are based on the series of slab tests which were undertaken at the University of Illinois. It is not the intent of the present work to change the code design methods. The author's objective is to investigate, in a more detailed way the effect of geometric structural changes on the distribution of moments in flat plate systems. It is not feasible to carry out a study of this nature by using physical model tests, and for this reason, this work must be measured, or validated, against the Illinois study.

The work in this thesis provides the basic information needed for a parallel research, which is now under way, to develop a stiffness model for the three-dimensional building frames eliminating the need to do finite element building frame analysis. Part of the author's contribution was in the planning and supervision of this next phase of the effort. Since it now appears that this will require at least two years of additional research, the work is being carried forward by another student. Some of the early results will be reported in the closing chapter of this thesis.

The objective of this research can be summarized as follows:

- 1 - To develop a finite element program for elastic analysis of single storeys of flat plate systems with various geometrical configurations and subjected to vertical loads.
- 2 - To develop a method of analysis of the full three-dimensional building frame subjected to lateral and vertical loads.
- 3 - To examine the stiffness characteristics of the building frame to provide enough information needed for a parallel research to develop a stiffness model for the three-dimensional building frame, capable of representing the actual behaviour of the building frame in a more rational way.
- 4 - To investigate the distribution of moments in a flat plate floor under varying parameters (column sizes, slab aspect ratios and column aspect ratios) using the finite element method and to compare the finite element results with the equivalent frame method recommended by the code.
- 5 - To study the effect of column offsets and the effect of slab holes on the shear and bending moments in a flat plate floor.

Figure 1.1: Three-Dimensional Building Frame

Figure 1.2: Moments in 3 by 3 Panel Flat Plate With Middle Row of Panels Loaded ($C/L=0.1$) (Reference 3)

$$* M_0 = q L^3$$

Figure 1.3: Model of Slab-Column Joint

CHAPTER 2

FINITE ELEMENT ANALYSIS2.1 Introduction

A finite element model for elastic analysis of flat plate systems is presented in this chapter. The model consists of a flat plate floor supported directly on columns. The application of the finite element method to the present model does not involve any new fundamental principles for the finite element method but does involve the use of the existing principles in a new way. In the idealization, the material was assumed to be linearly elastic, homogeneous and isotropic.

2.2 Plate Bending Elements

The derivation of plate bending elements are often, and most simply, based on the classical plate theory [29], where the straining in the plate is completely defined by the lateral deflection w ; therefore, w and its derivatives are taken as degrees of freedom. The theoretical discussion is not long, and, although it is not original, the author has chosen to include it since its use in reinforced concrete applications is somewhat controversial.

The lateral deflection, w of the finite element is defined by mathematical polynomial functions with generalized constants $\{\alpha\}$:

$$w = \{P\}^T \{\alpha\} \quad (2.1)$$

where $\{P\}$ is a vector which contains the assumed deflection functions.

Applying equation (2.1) at the element nodal points

$$\{\Delta\} = [A] \{\alpha\} \quad (2.2)$$

where $\{\Delta\}$ is a vector which contains the nodal degrees of freedom, and $[A]$ is a square matrix derived from $\{P\}$ evaluated at the nodal points. Substituting equation (2.2) into equation (2.1) the deflection can be expressed as,

$$\{\omega\} = \{P\}^T [A]^{-1} \{\Delta\}$$

which can be written as,

$$\{\omega\} = [N] \{\Delta\} \quad (2.3)$$

where $[N]$ is a matrix of finite element shape functions.

In the thin plate theory it is assumed that, provided that the lateral deflections of the plate are small in comparison with the plate thickness, the strains and stresses vary linearly from the neutral surface, which is assumed to be equidistant from the upper and lower plate surfaces. Hence the strains can be expressed as:

$$\begin{aligned} \epsilon_{xx} &= -\omega_{,xx} z \\ \epsilon_{yy} &= -\omega_{,yy} z \\ \epsilon_{xy} &= -\omega_{,xy} z \end{aligned} \quad (2.4)$$

where z is the distance measured from the neutral surface.

The relationship between the stresses and moments per unit length is given by:

$$\begin{aligned}\sigma_{xx} &= \frac{12 M_{xx}}{t^3} \cdot z \\ \sigma_{yy} &= \frac{12 M_{yy}}{t^3} \cdot z \\ \sigma_{xy} &= \frac{12 M_{xy}}{t^3} \cdot z\end{aligned}\tag{2.5}$$

where t is the plate thickness.

Dealing with the strains and stresses to determine the plate strain energy would require integration through the plate thickness; therefore, it is more convenient to deal with the curvatures and the moments. Once the curvatures and the moments are known the stresses and strains can be found from equations (2.4) and (2.5).

Considering the plate element shown in Figure (2.1), the strain energy can be expressed as:

$$dU_s = -\frac{1}{2} M_{xx} \omega_{,xx} - \frac{1}{2} M_{yy} \omega_{,yy} - M_{xy} \omega_{,xy}$$

The strain energy can be written as:

$$U_s = -\frac{1}{2} \int \{m\}^T \{c\} dA\tag{2.6}$$

where $\{m\}$ is the vector of bending moments

$$\{m\} = \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}\tag{2.7}$$

and $\{c\}$ is the vector of curvatures

$$\{c\} = \begin{Bmatrix} \omega_{,xx} \\ \omega_{,yy} \\ 2\omega_{,xy} \end{Bmatrix} \quad (2.8)$$

The relationship between the distributed bending moments and the curvatures for an elastic isotropic plate is given by:

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = - \frac{E t^3}{12(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ 1 & \nu & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix} \begin{Bmatrix} \omega_{,xx} \\ \omega_{,yy} \\ 2\omega_{,xy} \end{Bmatrix} \quad (2.9)$$

where E is Young's modulus of elasticity and ν is Poisson's ratio.

This relationship may be written as:

$$\{m\} = - [D] \{c\} \quad (2.10)$$

Substituting equation (2.10) into equation (2.6), the strain energy can be written as:

$$U_s = \int_A \frac{1}{2} \{c\}^T [D] \{c\} dA \quad (2.11)$$

Using equation (2.3) the curvature-modal displacement relationship can be expressed as

$$\{c\} = [B] \{\Delta\} \quad (2.12)$$

where $[B]$ is derived by differentiating corresponding elements of $[N]$.

Substituting equation (2.12) into equation (2.11),

$$U_s = \int_A \frac{1}{2} \{\Delta\}^T [B]^T [D] [B] \{\Delta\} dA \quad (2.13)$$

Assuming body forces are negligible, the potential energy II_p may be written as,

$$II_p = U_s - \int_A \{\omega\}^T \{q\} dA \quad (2.14)$$

where $\{q\}$ is the vector of the applied surface tractions. Substituting equations (2.13) and (2.3) into equation (2.14) and noting that the plate is subdivided into a number of elements, the potential energy for the entire plate may be written as

$$II_p = \sum_{e=1}^E \left[\int_{A(e)} \frac{1}{2} \{\Delta\}^T [B^{(e)}]^T [D^{(e)}] [B^{(e)}] \{\Delta\} dA - \int_{A(e)} \{\Delta\}^T [N^{(e)}]^T \{q^{(e)}\} dA \right] \quad (2.15)$$

The solution is achieved by minimizing the total potential energy of the plate with respect to the nodal displacements $\{\Delta\}$, as follows:

$$\frac{\partial II_p}{\partial \{\Delta\}} = \sum_{e=1}^N \left[\int_{A(e)} [B^{(e)}]^T [D^{(e)}] [B^{(e)}] dA \{\Delta\} - \int_{A(e)} [N^{(e)}]^T \{q^{(e)}\} dA \right] = 0 \quad (2.16)$$

which can be written as

$$\sum_{e=1}^E [K^{(e)}] \{\Delta\} - \sum_{e=1}^E \{f^{(e)}\} = 0 \quad (2.17)$$

where $[K^{(e)}]$ is the element stiffness matrix and $\{f^{(e)}\}$ is the element force vector.

2.3 Finite Element Model

The finite element model used in this study consists of a flat plate floor supported by columns framing in from above and below. The columns framing into the floor are assumed to be fixed at the remote ends, at adjacent storeys. In the analysis, the flat plate floor was divided into plate bending elements while the columns were frame type elements assumed to extend from the centerline of the floor above to the centerline of the floor below. The portion of the column between the upper slab face to the lower slab face was assumed to have an infinite stiffness (Figure 2.2).

Two column-slab interface models were considered. In both models the part of the slab outside the column area was divided into plate bending elements. In the first model, or "constrained interface model", to include the column stiffness in the global stiffness matrix, a constraint method at the slab column interface patch was used. In the second model, or "element interface model", the slab inside the column patch was modelled by plate bending elements and appropriate boundary conditions were imposed to simulate the column action. The two approaches will be described in more detail later in this chapter.

Since there are a wide variety of plate bending elements available in the literature, it was essential to choose the appropriate one which could represent the elastic behaviour of the flat plate structure including the column slab interface.

2.4 Finite Element Analysis

2.4.1 Choice of Element

Various plate bending elements with varying shapes, nodal constraints and displacement functions have been developed since the appearance of the first plate bending element in 1959 [30]. However, the results obtained by using each of these elements are not equally satisfactory and depend upon various factors such as element shape, nodal variables, displacement functions, etc. In general, a finite element based on a low order polynomial displacement function, results in only a crude curvature approximation, and yields discontinuous bending moments between elements. On the other hand, those elements which use higher order polynomial distribution functions and consequently higher order derivatives of the deflection such as $\frac{\delta^2 w}{\delta x^2}$, $\frac{\delta^2 w}{\delta x \delta y}$ and $\frac{\delta^2 w}{\delta y^2}$ as nodal variables, in addition to the deflections and the slopes, usually provide a better approximation and hence, are superior in the analysis of flexural problems [30]. If the polynomials are of too high an order, a mathematically unstable numerical model will result in any problems with non-smooth geometry, loadings, or boundary conditions; in other words, most practical engineering problems. So that some reasonable balance of mathematical sophistication and engineering acceptability is important, not only to achieve economy, but also correct results.

According to the previous discussion plate bending elements could be broadly classified as: elements which do not use curvatures, or second derivatives of lateral displacement, as nodal variables; and elements which do incorporate curvatures as nodal variables. In using the former,

the flexural rigidity of the plate could be increased arbitrarily to simulate the infinite rigidities at the ends of the columns. However, no direct constraints could be imposed on the curvatures since only the plate lateral displacement and slopes are used as nodal variables. Furthermore, the selection of a suitable value for the flexural rigidity of the plate within the column-slab interface is crucial, but not obvious. In the latter case, where curvatures are used as variables, the curvatures could be assumed to be zero at all nodal points inside the column boundary. However, due to the imposed continuity of the curvatures in the derivations of these elements, they are not capable of handling problems with abrupt change in the plate flexural rigidity. Therefore, the flexural rigidity of the plate within the column-slab interface must be a continuation of that for the actual slab. The use of such elements results in an error in the finite element presentation at the column boundary where a discontinuity of the curvature occurs. This error decreases as the size of the elements inside the column boundary decreases.

In the present work only flat plate systems were considered. In this type of structure a constant slab thickness is maintained throughout the floor. The use of plate bending elements with curvatures as nodal variables does not create any difficulty, since no abrupt change in the plate flexural rigidity occurs in such slabs. Further, these elements ensure the desired continuity of the internal moments in the slab. In addition, the existence of higher derivatives makes it relatively easy to incorporate the boundary conditions at the slab edges.

In light of the above discussion and since most of the high precision bending elements fall under the second category, elements incorporating curvatures were selected. It was found that satisfactory and consistent results were obtained using this alternative, and as shown later in this chapter, the various assumptions, made with respect to the modelling of the slab joint, were to have small effect on the accuracy of the results.

2.4.2 Properties of 18d.o.f. Plate Bending Element

The triangular element used here is described in Holand and Bell [30], and Cowper et.al.[38]. It is also described in Bowes and Russel [31], and a computer program by Bowes at Carleton University was modified and used in the course of the present work (Figure 2.3).

This finite element makes use of six generalized deflection parameters at each vertex, namely, the transverse displacement, w , its first derivatives $w_{,x}$ and $w_{,y}$, and the three second derivatives $w_{,xx}$, $w_{,xy}$ and $w_{,yy}$. The displacement function for the element is assumed to be a complete fifth order polynomial in x and y . The variation of deflection along any edge is then a fifth order polynomial in the edgewise co-ordinate. The six coefficients of this polynomial are uniquely determined by six quantities: the deflection, the edgewise slope and edgewise curvature, at each of the terminal vertices. Since the general fifth order polynomial in two variables depends on 21 constants, three additional conditions could be stipulated. As a result, three mid-side nodes are introduced and the slopes normal to each side of the triangle at these nodes are considered as additional degrees of freedom. These mid-side nodes are then eliminated by assuming that the variation of the slope, normal to each side, is a cubic function of the edgewise co-ordinates. This cubic variation is uniquely defined by four quantities, the normal slope and the twist at each of the two terminal vertices. Thus, continuity of displacement and slope are ensured between two adjacent elements. The element appears to have 21 degrees of freedom, however, the condition of cubic variation of edge normal slope implies 18 independent d.o.f. at the vertices. Since the curvatures are introduced

as degrees of freedoms, the element ensures continuity of the internal moments between elements at the corner nodes provided that the plate thickness is continuous. Furthermore, in this formulation, the approximate solution converges to the exact one as the mesh size is decreased [30].

The derivation of the stiffness matrix and the consistent load vector are obtained by using the classical plate theory and the principle of minimum potential energy as discussed earlier in section 2.2 and can be found in [30].

2.4.3 Column-Slab Interaction

The following assumptions pertain to the column-slab joint:

- (i) the column cross section remains plane after bending;
- (ii) no axial deformations occur in the columns; and
- (iii) the slab is rigidly connected to the columns.

According to the first two assumptions, the column slab intersection rotates as a rigid patch around the intersection center point with an angle θ_x and θ_y in the x and the y directions (Figure 2.4). Hence, the vertical displacement w at every point within the column perimeter can be expressed in terms of θ_x and θ_y as follows:

$$w = \theta_x x + \theta_y y \quad (2.18)$$

where x and y are the cartesian coordinates of a point measured from the column center.

Since the plate is assumed to be rigidly connected to the columns, the slopes of the slab at the column-slab interface are equal to the slopes of the column patch (Figure 2.4).

In order to incorporate these assumptions in the finite element model, two column-slab interface models, introduced in section 2.3, were considered. In the constrained interface model, only a part of the slab outside the column patch was divided into elements. According to the assumptions, the nodes lying on the column boundary should follow the relationship in equation (2.18). Therefore, the degrees of freedom in terms of w at these nodal points could be expressed in terms of nodal point rotations. Afterwards, these rotations, in both the x and y directions are combined into two degrees of freedom, namely θ_x and θ_y , where the column stiffness in two directions can be applied.

The method was introduced by El-Kafrawy and Hartley in 1979, and details of the analysis can be found in [32]. The method was found to give accurate results. It ensures that the column patch rotates rigidly. It has the advantage of eliminating the need to model the column region with new elements and therefore no additional degrees of freedom have to be considered. However, combining all of the rotations of the nodal points along the column perimeter into only two rotations θ_x and θ_y , results in increasing the bandwidth in the stiffness matrix. This makes the method unsuitable in large floor problems.

In the element interface model, the column region was divided into plate bending elements. Making use of the previous assumptions, the two rotational degrees of freedom, and lateral displacement, w , at every

nodal point within the column region were combined as follows:

$$\begin{Bmatrix} \omega_i \\ \omega_{i,x} \\ \omega_{i,y} \end{Bmatrix} = \begin{bmatrix} x_i & y_i \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \omega'_{i,x} \\ \omega'_{i,y} \end{Bmatrix} \quad (2.19)$$

where ω_i is the deflection at the i^{th} nodal point,

$\omega_{i,x}$, $\omega_{i,y}$, $\omega'_{i,x}$ and $\omega'_{i,y}$ are the rotations of the i^{th} nodal point in the x and the y directions before and after transformation;
 x_i and y_i are the cartesian coordinates of the i^{th} nodal point measured from the column center.

The column rotational stiffness K_{cx} and K_{cy} were then divided equally between the rotational degrees of freedom ω'_{ix} and ω'_{iy} at the nodal points in the column region.

In order to model the column patch rigid rotation, all the curvatures ($\omega_{,xx}$, $\omega_{,xy}$ and $\omega_{,yy}$) for the nodal points inside the column region were assumed to be zero. The flexural rigidity of the plate modelling the column interface was assumed to be the same as that for the slab outside of the interface.

The model as outlined above is approximate and does not ensure that the column patch rotates perfectly rigidly, however the error was found to be small, as will be shown later in section 2.5. The method has the advantage that the bandwidth does not increase as it did in the constrained interface model. The model is thus suitable for large floor problems.

2.4.4 Calculation of Moments Transferred to Columns

The moments transferred to the columns were calculated by multiplying the column patch rotation by the stiffness of the column. In the element interface model, the column patch rotation was defined as the average rotation of all the nodal points within the column patch.

2.4.5 Computation of Shear in the Slab

The distribution of the shear force per unit length in the slab was calculated using the following equations:

$$V_x = -D \frac{\partial}{\partial x} \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) \quad (2.20)$$

$$V_y = -D \frac{\partial}{\partial y} \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) \quad (2.21)$$

where V_x and V_y are the shear forces per unit length in the x and the y directions, and D is the flexural rigidity of the plate.

The partial derivatives of the sum of the curvatures in the x and the y directions were calculated by numerical differentiation. The total shear along a particular line was obtained by integrating the shear distribution.

2.4.6 Boundary Conditions

In order to represent the actual behaviour of the flat plate structure under the application of different load conditions, the proper boundary conditions must be modelled as closely as possible. In the

element interface model, all of the curvatures for the nodal points inside the column region were assumed to be zero. Along the column-slab interface, a discontinuity in the curvature in the direction normal to the interface exists in the physical model. It was not possible to incorporate this condition in the finite element model (Figure 2.5). This results in finite curvatures inside the column region, however these curvatures are limited to a zone near the column boundary. The width of this zone decreases as the size of the elements inside the column region decrease. This approximation was found to have an effect on the slab negative moments especially at the column corners.

The finite element boundary conditions for the other edges were more easily treated. For an edge in an axis of symmetry, the slope normal to the line of symmetry and the warping of the plate $\omega_{,xy}$ along the edge were set to zero. For an edge on an axis of antisymmetry, the deflection, the slope along the edge, the two curvatures $\omega_{,xx}$ and $\omega_{,yy}$ were set to zero. In the case of fixed end support, the deflection, the slope along the support, the warping $\omega_{,xy}$, and the curvature along the edge were set to zero.

The free edge support involves two conditions. These conditions are zero bending moment, and zero vertical shear, along the plate edge. The first condition was treated by transforming the nodal parameters to a modified set of nodal parameters as follows:

$$\begin{Bmatrix} w \\ w_{,x} \\ w_{,y} \\ w_{,xx} \\ w_{,xy} \\ w_{,yy} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{1-\nu^2} & 0 & \frac{-\nu}{1-\nu^2} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{-\nu}{1-\nu^2} & 0 & \frac{1}{1-\nu^2} \end{bmatrix} \begin{Bmatrix} w \\ w_{,x} \\ w_{,y} \\ w_{,xx} + \nu w_{,yy} \\ w_{,xy} \\ w_{,yy} + \nu w_{,xx} \end{Bmatrix} \quad (2.22)$$

The fourth and the sixth degrees of freedom in this modified set of nodal parameters are directly related to the slab bending moments as follows:

$$\begin{aligned} M_{xx} &= -D (w_{,xx} + \nu w_{,yy}) \\ M_{yy} &= -D (w_{,yy} + \nu w_{,xx}) \end{aligned} \quad (2.23)$$

which allow the condition of zero bending moments to be incorporated.

The second condition of zero vertical shear cannot be prescribed since it was not possible to express the vertical shear in terms of the nodal parameters. This results in an error in the finite element presentation. However, this error was found to be insignificant, and is limited to a small zone near the free edge [30].

2.5 Comparison Between the Two Models

In order to verify the assumptions used to model the column-slab joint in the element interface model of analysis, a comparison with the constrained interface model was made. The comparison involved the analysis of an interior flat plate floor panel supported on square columns and subjected to vertical loads as shown in Figure 2.6. A number of values of C/L (ratio of column size to panel dimensions) ranging from 0.025 to 0.15 were examined. The analysis was made by combining symmetrical and antisymmetrical cases of loadings (Figure 2.7). Making use of the floor symmetry, an analysis of only one-quarter of the panel was necessary. Figure 2.8 shows the finite element mesh used in the analysis.

In Figure 2.9 the bending moments transferred to the columns (above and below the floor) are plotted against increasing column size. It can be seen that the column moments obtained from the constrained interface model are higher. The difference between the two models increases as the column increases in size; however, that difference is within about 5% in the most extreme case investigated.

Tables 2.1 and 2.2 show the bending moments, in the x and y directions, at four different points in the slab panel. The locations of these points are shown in Figure 2.6. These results show good agreement in the bending moments obtained from both models at the panel center. Near the column, the difference between the two methods increases, especially in the case of small columns. That difference is maximum at the column corner (10.7% for $C/L=0.025$ and 5.7% for $C/L=0.1$)

as shown in Tables 2.1 and 2.2. In general, it was found that the constrained interface model gives higher negative moments near the column because of the perfectly rigid rotation of the column.

The total positive and negative moments in the slab panel are shown in Tables 2.3 and 2.4. These moments were calculated by integrating the laterally distributed moments. It can be seen that the constrained interface model gives higher negative moments and lower positive moments. For all the examined column sizes, the difference in the results is less than 4.5%.

As a result of the above comparison it is clear that the approximations used in the element interface model to model the column-joint have a limited effect on the results, and the error is tolerable.

It should be pointed out that both models were used in the course of the thesis. In Chapters 3 and 4 it was only required to determine the column patch rotations in flat plate floors. It was found that these rotations could be obtained with reasonable accuracy using a relatively coarse mesh which allowed the use of the constrained interface model. Also, the analyses in these chapters involved small rotations; therefore, based on the concern that the error in calculating these rotations by the second method would be high, the constrained interface model was preferred. The constrained interface model is not categorically more correct. It is however well established, in that its assumptions are clear; and it does satisfy geometrical compatibility as given in the assumptions for flexural behaviour in the code [33].

Chapters 5 and 6 describe the analysis of a large flat plate floor. The main objective in these two chapters was to determine the bending moment and the shear distribution in a five panel by four flat plate floor. Such analysis requires a relatively fine mesh precluding use of the constrained interface model. Therefore, the element interface model was used in these two chapters.

2.6 Comparison With Experimental Results

In order to test the validity of the finite element method, the results were compared with existing experimental results obtained on a perspex (plexiglass) model [34]. The model is a typical two-storey three bay by three bay flat plate structure with rectangular 14 inches by 16 inches (356 mm by 406 mm) panels and supported by one inch (25.4 mm) circular columns (Figure 2.10). In order to reduce the storage and the time required by the computer, only one quarter of the flat plate floor was analyzed by making use of the symmetry of the model. Figure 2.11 shows the plan of the structure analyzed. All of the interior panels were loaded with a uniformly distributed load of 200 lb./in^2 (1.38 MPa). The elastic properties were taken equal to those of the model with Poisson's ratio = 0.35; and Young modulus = $4.67 \times 10^5 \text{ lb./in}^2$ (3200 MPa).

The results of the finite element analysis were compared with the experimental results obtained for the first level of the two storey experimental model. The comparison is presented in Figures 2.12 and 2.13. These results show good agreement in the nature of the distribution of the moments in the slab. However, a difference of 15% was observed

in some locations. The difference may be attributed to several factors involved with the experimental modelling and the finite element modelling.

Figure 2.1: Slab Moments

(a) Slab System

(b) Column Stiffness

Figure 2.2: Column Element

Figure 2.3: Nodal Point System and Nodal Parameters for 18 d.o.f.
Triangular Bending Element

Figure 2.4: Column Displacement Mode

Points	ω	$\omega_{,x}$	$\omega_{,y}$	$\omega_{,xx}$	$\omega_{,xy}$	$\omega_{,yy}$
a	0			0	0	0
b	**			0	0	0
c	**			*	0	0
d	**			0	0	*
e	**			*	*	*

Figure 2.5: Boundary Conditions for the Nodal Points in the Column Region (element interface model)

- * No boundary condition imposed because discontinuity in curvature existed
- ** Degrees of freedoms were eliminated

Figure 2.6: Interior Flat Plate Panel
Subjected to Vertical Loads

Figure 2.7: Combination of Symmetrical and Anti-symmetrical Loading

Figure 2.8: Finite Element Mesh

Figure 2. 9: Moments Transferred to the Columns

Figure 2.10: Layout of the Experimental Model (Ref. 3 4)

Figure 2.11: Layout of the Finite Element Model

Figure 2.12: Comparison of Experimental and Finite Element Moments (M_x Distribution Across Middle Panel)

Figure 2.13: Comparison of Experimental and Finite Element Moments (M_y distribution across middle panel)

Table 2.1: Bending Moment ($M_x = 8q L^2$) in Interior Flat Plate Panel

C/L	Point A			Point B			Point C			Point D		
	C.I.M.*	E.I.M.**	Diff. %	C.I.M.*	E.I.M.	Diff. %	C.I.M.	E.I.M.	Diff. %	C.I.M.	E.I.M.	Diff. %
0.025	0.06903	0.06894	0.13%	0.08605	0.08680	0.87%	-0.0184	-0.0183	0.54%	-0.233	-0.208	10.73%
0.050	0.05851	0.05872	0.36	0.0758	0.07711	0.17	-0.0177	-0.0178	0.56	-0.23102	-0.2079	10.01
0.075	0.05018	0.05030	0.24	0.06764	0.06836	0.11	-0.017	-0.0173	1.76	-0.20818	-0.1874	9.98
0.100	0.04574	0.04577	0.11	0.06147	0.06261	0.18	-0.0164	-0.0167	1.83	-0.1724	-0.1625	5.74
0.150	0.0401	0.04003	0.17	0.0532	0.05364	0.83	-0.0149	-0.0151	1.34	-0.1226	-0.117	4.56

* Constrained interface model

** Element interface model

Table 2.2: Bending Moment ($M_y = \rho q L^2$) in Interior Flat Plate Panel

C/L	Point A			Point B			Point C			Point D		
	C.I.M.*	E.I.M.**	% Diff.	C.I.M.	E.I.M.	% Diff.	C.I.M.	E.I.M.	% Diff.	C.I.M.	E.I.M.	% Diff.
0.025	0.04032	0.04033	0.02%	-0.01149	-0.01138	0.96%	0.0497	0.0505	1.61%	-0.0467	-0.0417	10.7%
0.050	0.03954	0.04008	1.36	-0.01492	-0.01515	1.54	0.0481	0.0494	2.7	-0.04613	-0.04151	10.01
0.075	0.03878	0.03934	1.44	-0.01649	-0.01683	2.06	0.0461	0.0477	3.47	-0.0417	-0.0376	9.83
0.100	0.03765	0.03815	1.33	-0.01831	-0.01863	1.75	0.0439	0.0452	2.96	-0.03453	-0.0326	5.59
0.150	0.03515	0.03531	0.46	-0.01781	-0.01805	1.35	0.0392	0.0397	1.27	-0.02456	-0.02339	7.46

* Constrained interface model

** Element interface model

Table 2.3: Total Bending Moment ($M_x = \beta q L^3$)

C/L	Positive Moment			Negative Moment		
	C.I.M.*	E.I.M.**	% Diff.	C.I.M.	E.I.M.	% Diff.
0.025	0.07722	0.07599	1.59%	-0.07574	-0.07493	1.07%
0.05	0.06697	0.06747	0.75%	-0.07485	-0.07299	2.48%
0.075	0.05825	0.05893	1.17%	-0.07368	-0.07145	3.03%
0.10	0.05336	0.05391	1.03%	-0.07155	-0.06849	4.27%
0.15	0.04656	0.04678	0.47%	-0.06419	-0.06233	2.89%

Table 2.4: Total Bending Moment ($M_y = \beta q L^3$)

C/L	Positive Moment			Negative Moment		
	C.I.M.*	E.I.M.**	% Diff.	C.I.M.	E.I.M.	% Diff.
0.025	0.046	0.04617	0.37%	-0.07286	-0.07198	1.21%
0.05	0.04527	0.04601	1.63%	-0.07277	-0.07166	1.53%
0.075	0.04419	0.04516	2.2%	-0.07234	-0.07004	3.18%
0.10	0.04273	0.04354	1.9%	-0.06904	-0.06776	1.85%
0.15	0.03928	0.03971	1.09%	-0.06116	-0.05955	2.63%

* Constrained interface model

** Element interface model

CHAPTER 3

STIFFNESS MODEL OF BUILDING FRAMES3.1 Introduction

A method of analysis of building frames is carried out using a two-stage finite element process, which considers a full three-dimensional model of the structure, including lateral and vertical loadings. Although the analysis process requires somewhat more data preparation than current design office methods, it does involve techniques which are considerably less expensive than full finite element analysis both in terms of CPU costs and data preparation, and it is amenable to fairly routine procedures which could be made automatic. The advantage is a rational analysis, which can take care of both vertical loadings and sidesway analysis in a realistic way, and can handle such complexities as holes for stairways, and offset columns. Dynamic analysis and stability analysis are also practicable extensions, since the displacement equations which must be solved are formulated for the model with reduced degrees of freedom, in a manner similar to the familiar substructure approach in matrix analysis.

3.2 Finite Element Building Model

A single floor of the building frame is shown in Figure 3.1. The shaded patches are finite size areas to which column stiffness can be applied. A finite element model of this floor is used to provide flexibility coefficients corresponding to orthogonal rotations at each

rigid patch; and also the displacements corresponding to individual panel loads, used to calculate the so-called fixed end moments. Both of these concepts, although relatively well-known in frame analysis, must now be rethought out in the present context, and are discussed more fully later in this section. Finite element analysis as applied herein, addresses only single floors as in Figure 3.1.

A full three-dimensional model of a building structure is shown in Figure 3.2. The degrees of freedom of this model consist of 2 translational and one rotational degree of freedom per storey at the slab levels; and 2 rotational degrees of freedom per joint (at the intersection points of column and slab). A stiffness analysis of this structure is carried out by assembling and solving a set of equations,

$$[K] \{\Delta\} = \{F\} \quad (3.1)$$

where $[K]$ is the stiffness matrix; $\{\Delta\}$ is the vector of unknown degrees of freedom shown in Figure 3.2; and $\{F\}$ is a vector of applied forces, and each element in this vector is associated with a particular degree of freedom in the $\{\Delta\}$ vector. The coefficients in the stiffness matrix associated with the three d.o.f. at each storey level are given by the columns framing into that storey. The coefficients corresponding to the two degrees of freedom at each joint are obtained from the finite element model. The means of obtaining these stiffness terms and the elements in the force vector will now be described.

3.2.1 Joint Stiffness Terms

In Figure 3.1 a direct procedure for obtaining the stiffness coefficients for the slab, would require a separate computer run for each joint or patch rotational degree of freedom, since a different set of boundary conditions is associated with each run; that is, a particular degree of freedom is released and forced while all other degrees of freedom are fixed. The inverse problem, which would obtain the flexibility coefficients, requires only a single computer run, but with multiple right hand sides. In this approach the column patches are not restrained with respect to rotation, and a single non zero load is applied corresponding to a degree of freedom in each load case. Rotations in both directions are solved for each column patch, for each load case. These rotations are divided by the arbitrary values of the couples which produce them. Thus each load case gives rise to a displacement vector which is, in fact, a column of the flexibility matrix. The flexibility matrix must now be inverted giving the stiffness matrix. Note that the matrix to be inverted is of a low order equal to twice the number of patches.

It should also be noted that a direct method of obtaining stiffness terms is not, in fact, workable. A direct procedure requires determination of the distributed bending moments in the slab, which then must be further processed to develop the stiffness terms. Near the columns these moments increase very rapidly and extremely fine meshes are required to give reasonable accuracy in the stiffness terms. In the flexibility procedure, the rotations (inverse stiffnesses) are computed with no further processing, and the accuracy is thus reasonable with relatively coarse meshes.

3.2.2 Force Terms

The elements in the force vector consist of lateral forces applied at each storey level, which can be implemented in the force vector directly; and vertical panel loads. In numerical examples given later only the lateral forces were applied in order to provide direct comparisons with previous work. The panel loads require some processing before equivalent values corresponding to degrees of freedom can be assembled with the force vector.

The "fixed end forces" for a single loaded panel, as shown in Figure 3.3, consist of the negatives of the joint locking forces which are induced when that load is applied. When the slab is loaded as in Figure 3.3, the stiffness equations are

$$[K] \{\Delta\} = \{M - M^F\} \quad (3.2)$$

where $\{M\}$ comprises only direct joint forces, and is equal to zero, and $\{M^F\}$ are the fixed end forces. The fixed end moments for individual panels for implementation in equation (3.1) are computed using equation (3.2) as follows:

- (i) The analysis of the finite element model, with a single panel loaded, provides the vector $\{\Delta\}$ in equation (3.2).
- (ii) The inversion of the flexibility matrix discussed previously provides $[K]$.

The reader may thus be surprised to note that "fixed end forces" for uniformly loaded panels are not generated using locked joints at all, and are in fact calculated from additional load cases used in the finite

element analysis of the floor (Figure 3.1) with the multiple load cases used to generate the stiffness terms.

3.2.3 Stiffness Analysis

Stiffness coupling between stories is confined to the far end of columns framing into the storey in equation (3.1), since when the far end of the column is locked, no distortion of the adjacent slab can occur. Equation (3.1) is assembled and solved using an entirely different computer program from the one which produces the stiffness coefficients and forcing terms.

There are fundamental differences between the approach and equivalent beam models cited earlier. Stiffness and 'fixed-end moment' coupling take place, theoretically, between every degree of freedom at a floor level. In other words, in the joint locked modes, transfer can take place around a degree of freedom to remote degrees of freedom. A second important distinction between methods arises from the fact that the planar approaches totally ignore three-dimensional effects. Slabs, only under unrealistic circumstances, will deform in a one-way mode. When a building frame is subjected to a panel loading, the response of the frame is in both directions, and when a joint in this frame (not on a line of symmetry in the bending direction) is forced to rotate, a restraint must be applied in the orthogonal direction at this joint, and at others, in order to prevent these additional rotations.

The stiffness analysis yields the end moments in slabs and columns in addition to storey sways and joint rotations. A number of finite element single storey floor models have been analyzed and archived by the author. The broader aspects of this study also include the development of a more practically usable stiffness model of a building frame. A finite element model of the floor shown in Figure 3.4, costs approximately \$380 of computer time for a single run with multiple load cases. The cost in terms of data preparation time is also fairly high, although the author did find that this time reduces considerably with experience, and the development of simple mesh generators. By handling each storey one at a time followed by more detailed analyses of the full building model, a considerably reduced number of degrees of freedom is brought into play at the point at which multiple load combinations are considered, and also possible refinements such as length or large displacement effects, and dynamic analysis.

3.3 Joint Stiffness

Results from the analysis of the finite element model provide the direct "diagonal" joint stiffnesses, and coupling or "off-diagonal" stiffnesses. One structure which was investigated is shown in Figure 3.4. This model provided interior, exterior and corner panels, and interior, first interior, exterior and corner columns. In the following discussion, primary bending takes place in the L_1 direction.

In Figure 3.5, the stiffness of interior, exterior and corner panels are plotted against increasing column size for a slab system with slab

and column aspect ratios of one. The vertical intercepts represent the panel stiffness for a column patch which has shrunk to a point. The cross over point at $c/L=0.04$ where the exterior panel joint appears to get stiffer than the interior is a result of the layout adopted in Figure 3.4. Exterior columns here are of the same dimensions as interior, and the exterior panels are actually larger and contain a full column plus a half column. Interior panels are limited to the distances between column centers, and only contain two half columns. At the vertical intercept in Figure 3.5 where column sizes are zero the exterior panel will have a lower stiffness due to the free edge. As the columns grow in size, the extra column area in the exterior panel dominates the stiffness term.

In Figure 3.6 a stiffness factor has been plotted for panels of varying aspect ratio L_1/L_2 maintaining square columns of constant area. The stiffness is calculated by multiplying the factor plotted by L_2/L_1 and slab rigidity D .

In Figure 3.7 and 3.8 stiffness factors are plotted for varying column aspect ratio. In Figure 3.7 the ratio c_2/L_2 is constant while the columns are elongated in the direction of bending. The panel stiffness increases rapidly as the c_1/c_2 is increased. In Figure 3.8 the column is elongated transverse to the direction of bending. The limiting value corresponds to one-way action in the direction of bending.

Finally, in Table 3.1, carry over factors for five values of column to slab stiffness ratios are presented maintaining an aspect ratio of one.

Only very limited results have been given in this section. More complete data has been archived and is being used in an extension to this study.

A significant previous work was presented in [26]. A comparison of results already presented in Figure 3.5 for interior panels, with that of Elias is given in Figure 3.9. Elias developed his coefficients from a flat plate finite element model of a single panel shown shaded in Figure 3.10(a). Compatible deformation of all frames in the direction of bending was assumed so that a row of joints, transverse to the direction of bending, is assumed to be subjected to stiffness forcing terms. The symmetric and anti-symmetric superimposed cases in Figure 3.10(b) do indicate a discrepancy in the Elias model since after superposition, not only is the transverse row of joints indicated in Figure 3.10(a) being forced, but also every second transverse row of joints beyond. The author shows in Figure 3.9, three different cases. In one, a single joint is subjected to a forcing stiffness term. For direct comparison, however, the forced row of joints should be used and it is seen that these results are within about 7% of the Elias results. The author has also carried out an analysis corresponding to the superimposed results of Figure 3.10(b) giving results within 2.5% of those given in the reference. This difference can be attributed to differences in the numerical models used in the two studies. In the analysis of complete building frames discussed in the next chapter, somewhat more substantial differences in results will be found. Given the above discussion, these differences will not be attributable to the frame model alone, thus far presented.

It is not completely clear whether or not equivalent width coefficients can (or should) be extracted from the author's results. First, it is difficult to define these equivalent frames and the amount of load carried by each one especially for irregular building frames. Also, it is questionable that these coefficients which were developed for rectangular panels can be used for other shapes of panels. Secondly, it is clear that this approach can not take care of three-dimensional action which, as will be shown in the next chapter, is significant for all sizes and shapes of building frames.

Figure 3.1: Floor Degrees of Freedom

Figure 3.2: Building Frame Degrees of Freedom

Figure 3.3: Uniformly Loaded Panel

Figure 3.4: Floor Layout

Figure 3.5: Effect of Column Size on the Panel Stiffness ($L_1=L_2$, $C_1=C_2$)

Figure 3.6: Effect of Slab Aspect Ratio on Stiffness Coefficients for Interior Panel ($C_1 = C_2$)

Figure 3.7: Effect of Column Length on Stiffness Coefficients for Interior Panel ($L_1 = L_2$)

Figure 3.8: Effect of Column Width on Stiffness Coefficients for Interior Panel ($L_1 = L_2$)

Figure 3.9: Comparison of Equivalent Frame and Three-dimensional Joint Stiffness

(a) Multiple Joint Loading

Symmetrical Loading

Anti-Symmetrical Loading

Total Loading

(b) Combination of Symmetrical and Anti-Symmetrical Loading

(c) Single Joint Loading

Figure 3.10: Degree of Freedom Forcing
Terms for Sidesway Models

Table 3.1: Stiffness Coefficients for Interior Panels
 $(L_1=L_2, C_1=C_3)$

C/L	K'	C.O.F.
0.03	2.27	0.275
0.05	2.55	0.31
0.10	3.41	0.33
0.15	4.48	0.36
0.20	5.70	0.37

$$K = K' \frac{L_2 D}{L_1}$$

C.O.F. = Carry Over Factor

CHAPTER 4

ANALYSIS OF 10-STOREY BUILDING FRAME4.1 Introduction

To illustrate the use of the method that was developed in Chapter 3, an analysis of a 10-storey building frame is given here. The analysis involves both lateral and vertical loads; and a comparison with other methods from the literature is provided.

4.2 Structural Model

The structure used in the analysis is a 10-storey flat plate building. Each storey consists of four panels in the x-direction and four in the y-direction. All of the panels are 7.0 meters square. A floor thickness of 0.28 meters, satisfying a serviceability condition in the national building code [33], was adopted. The structure has prismatic columns, 0.7 meters square. All storey heights are 4.0 meters and the columns are assumed to be fixed at the base of the frame. A Poisson's ratio of 0.2 was adopted as an average value for reinforcement and concrete, and Young's modulus was taken as 25000 MPa (Figure 4.1).

4.3 Sidesway Analysis

In the sidesway analysis, a constant force of 100 kN is assumed to act at each storey level (Figure 4.2). The analysis addresses the full three-dimensional frame. The code method and other methods used for

comparison relate to a planar frame indicated by the shaded section in Figure 4.2. A comparison of the methods shown in Figure 4.3 and Table 4.1 indicates that the Equivalent Frame Method exhibits much more flexibility, having lateral displacements 62% higher at the top storey. The Equivalent Frame Method models the structure using full slab widths and 'equivalent columns'. There is no theoretical justification for doing this in the analysis of a multistorey model. The equivalent column connection was developed to correctly transfer end moments in a horizontal member into a joint. The model is correct in the analysis of slabs or continuous slabs for vertical load analysis. It has been used here, for comparison, to point out an error in assuming that the concept can be extended to sidesway analysis.

The Australian SAA Concrete Structure Code, AS 1480-1974 [35], models the structure as a series of typical plane frames aligned in the direction of the applied lateral load. Each frame consists of unreduced column sections and full slab panel widths, hereafter referred to as a 'basic frame'. The results shown in Figure 4.3 and Table 4.1 indicate that the basic frame method, as expected, overestimates the building frame stiffness and therefore results in smaller lateral displacements. This method, in this example, is much more accurate than the Equivalent Frame Method but errs on the unconservative side.

Some of the other approaches from the literature are also given. In the approach adopted in [24,26], the horizontal slab element is modelled using 'equivalent beams' and full columns. The equivalent beam is of narrower width than the original panel but has a rotational

stiffness of such a magnitude that a proper distribution of a directly applied joint moment, between the horizontal and vertical elements will result. The actual structures whose analyses are being compared in Figure 4.3 are identical. For in-plane lateral load analysis of building frames, some of the basic theoretical differences, pointed out earlier regarding such approaches, do not apply, and the results in Figure 4.3 show reasonable agreement. There is a 20% difference in the sway at the top storey between the present work and Elias [27]. There are three sources to account for this disagreement. The first is that of modelling a three dimensional frame as planar frame. It was noted earlier in section (3.3) that the Elias model is basically correct if a two-dimensional displacement mode can be assumed to take place. If the slabs are relatively flexible, three-dimensional behaviour will take place in a uniform lateral loading situation. Also, closely related to slab stiffness, is the issue of force prorating, or lumping. The scheme shown in Figure 4.2 is based on engineering judgement, and is really the only judgement which could be applied on intuitive grounds. If the slabs are flexible relative to the columns, however, the typical frames shown in Figure 4.2 are not subjected to the loadings shown. This is also not the case if the slabs are infinitely stiff. If consideration is given to prorating a uniformly distributed load to a storey of a building frame, both infinitely stiff and infinitely flexible slabs both lead directly to the conclusion that each column line is of equal importance in resisting the lateral force, and that $1/5$ of the total load should be applied at each column line; and that only this load applied to an equivalent frame will lead to correct results. If the slab is finitely flexible, however, the consideration of frame stiffness becomes important. One might expect that equivalent interior frames are approximately twice as stiff as

exterior ones and therefore should be subjected to approximately twice the load. This argument, however is somewhat too simplistic. The actual distributions for the example given here are shown in Table 4.2. The actual force carried by an equivalent interior frame is approximately 22% of the total, rather than 25%, for the example given. Finally it was noted earlier that the Elias stiffness is approximately 2.5% more flexible based strictly on the numerical methods used. Fully, 14.5% of the difference in the results are directly attributable to the three-dimensional effects.

4.3.1 Bending Analysis

Having carried out the stiffness analysis of the full building frame model, slab and column bending moments are required for design. In this section, the further processing of the data to obtain this information is discussed. Numerous examples have been worked out, but only one example is presented for illustration.

The column joint moments and slab joint moments in the x (major direction of bending) and y directions are shown in Tables 4.3 and 4.4 for interior, exterior, and corner joints. These results clearly indicate the importance of the three-dimensional effects in columns not close to a line of geometrical and loading symmetry.

Knowing the storey sway and the rotations of the column-slab joints, the column moments can be obtained directly, since the individual stiffness terms are available for these elements. The total slab moment (right and left of column) due to building frame displacements is also available as the negative total column moment. This total slab moment is for the combined horizontal elements and must subsequently be

distributed to the two elements.

In Table 4.4, the distribution of moments between slabs, right and left of column, is shown for one example, namely, the sidesway analysis discussed in the previous section. A 50/50 distribution between horizontal elements meeting at interior columns is to be expected, when slab stiffnesses on both sides of the column are the same. It was also found that this distribution is approximately correct at first interior columns in Table 4.4. It can be observed that the finite element analysis for panel loaded, can be added to the slab moments caused by displacements, giving the slab end moments.

One may wonder what exactly these slab end moments represent since they are formulated based on a single point at the column center. They do represent some totality of moment along a line roughly in the direction of the transverse column line, but the slab moments in the column patch are theoretically zero. The author has found consistently that these moments are almost identical to the slab moments aligned with the column face, which the engineer uses in designing the slab. It is recommended that these moments would be laterally distributed according to code procedures and used directly as the negative design moments, with no further adjustment for column dimension.

The total positive bending moments at slab center spans can be calculated from the total static moment on the slab, and the slab end moments by a simple summation.

4.4 Analysis for Vertical Loads

The analysis of the 10-storey building frame for vertical loads involves several possible cases of loading, however, only one case of loading is reported here. The case of loading shown in Figure 4.4 was selected. The slab and the column moments in the x-direction are shown in Tables 4.5 and 4.6 for the middle equivalent frame. Excluding the first and the last two storeys, the slab and the column moments are almost the same in all floors. The difference in the moments in the first two storeys is due to the effect of fixing the columns at the base of the frame. At the roof, the slab-column joints consist of the slab and lower columns only unlike those of the other floors. This results in having more flexible joints at the top and consequently higher joint rotations under the loads. These high rotations reduce the slab moments in the roof and increase the column moments in the tenth storey. It can be seen in Tables 4.5 and 4.6 that, these end conditions at the base and the roof have higher effects on the column moments than the slab moments.

In order to make a direct comparison between the present method and the equivalent frame method, an analysis of one storey of a building frame with columns fixed at remote ends was carried out (Figure 4.5). Tables 4.7 and 4.8 shows the slab and the column moments obtained from the two methods for the equivalent frame, F1, shown in Figure 4.5. These results indicate that, for uniformly loaded floor, the present method gives higher negative moments than the equivalent frame method; and the difference between the two methods increases in the exterior panels (Table 4.7). When the floor panels are loaded with unequal uniform loads,

the difference in the results increases (Table 4.7). The moments transferred to the columns are higher by the present method (Table 4.8).

The results of the slab moments obtained by analyzing the full 10-storey building frame (Table 4.5), and one storey (Table 4.7), by the present method, indicate that, in general, the slab moments resulting from the analysis of the full building frame are higher. Similarly by comparing the column moments obtained from the analysis of the full building frame (Table 4.6) with those obtained from the analysis of one storey (Table 4.8), it can be seen that the analysis of the full building frame results in higher column moments.

4.5 General Remarks

This chapter has dealt with the analysis of a 10-storey building frame using a method that was developed in Chapter 3. The floor stiffness matrix and the fixed end moments were developed using the finite element method. Since all the floors have an identical configuration, only one floor stiffness matrix was developed. While the finite element analysis of the floor costs approximately \$380 of computer time, the stiffness analysis of the 10-storey building frame (having obtained the stiffness coefficients by the finite elements) for lateral and vertical loads costs approximately \$10. This total cost is considerably less than that of a full finite element analysis of the building frame.

It is to be reaffirmed that the cost includes the analysis of multiple load cases. The method is particularly advantageous if other refinements of analysis, such as geometrical or length effects ($P-\Delta$ effects)

or dynamic analysis are to be included since these analysis refinements are applied at the system with reduced degrees of freedom.

The sidesway analysis presented in this chapter indicates the importance of the three-dimensional effects on column moments. In irregular floor layouts, it is to be expected that these effects are even higher.

In terms of overall cost of design, however, including the costs of training staff, preparing data decks, and the initial outlay to substantially increase computer resources, the procedure presented here is of very doubtful practical use. The foregoing two chapters have been included here in the same spirit as those in the following chapters of the thesis; that is, so that the author and the reader may observe and compare, either critically or favourably, the currently applied code procedures and a full homogeneous elastic building frame analysis.

The structural system of interest here (as a generic structural type) does appear in modified form in other applications (parking garages, underground facilities, and so on). It is feasible that the approach as outlined here for dealing with a structure made up of complex elements (floors of various geometries) connected by simple elements (columns) may be of more practical importance in as yet unforeseen applications.

Figure 4.1: Layout of 10-Storey Building Frame

Floor Plan

Figure 4.2: Building Frame Model Subjected To Lateral Loads

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OF / DE

Figure 4.3: Storey Sway of Model Under Lateral Loading

$$q = 10 \text{ kN/m}^2$$

Figure 4.4: Building Frame Model Subjected to Vertical Loads

Figure 4.5: One Storey of a Building Frame

Table 4.1: Ratios of Roof Sways

Method	Basic Frame	Present Work	Pecknold (1975)	Elias (1979)	Allan & Darvall (1977)	Equivalent Frame Method
ratio	0.86	1.00	1.12	1.21	1.31	1.62

Table 4.2: Percentage of Load Carried by the Columns

Columns along (1-1)	Columns along (2-2)	Columns along (3-3)
17.2%	22.0%	21.6%

Table 4.3: Column Joints Moments Due to Lateral Loads

Floor Level	Section	Column A		Column B		Column C	
		M_x	M_y	M_x	M_y	M_x	M_y
Base		117.0	1.66	128.2	0.43	142.3	0.30
1	Below	3.38	3.0	23.6	0.77	49.0	0.55
	Above	40.5	4.5	71.3	1.17	11.03	0.84
2	Below	26.0	4.4	56.5	1.15	95.0	0.83
	Above	23.3	4.18	52.5	1.1	90.0	0.80
3	Below	31.0	4.15	60.0	1.09	96.75	0.79
	Above	15.3	3.9	42.3	1.02	76.8	0.74
4	Below	29.8	3.73	55.8	0.98	88.8	0.71
	Above	11.2	3.35	34.5	0.88	64.3	0.64
5	Below	27.0	3.2	49.3	0.84	77.8	0.61
	Above	7.73	2.83	27.3	0.74	52.3	0.54
6	Below	23.9	2.68	42.5	0.70	66.0	0.51
	Above	4.53	2.27	20.4	0.60	40.5	0.43
7	Below	20.9	2.1	35.5	0.55	54.3	0.40
	Above	1.34	1.74	13.4	0.46	28.8	0.33
8	Below	17.3	1.61	28.5	0.42	42.5	0.40
	Above	1.32	1.16	6.83	0.31	17.2	0.23
9	Below	14.4	0.93	21.2	0.25	40.0	0.19
	Above	4.5	0.82	1.08	0.21	8.13	0.15
10	Below	5.9	0.95	11.9	0.24	19.3	0.16

Moments in, KN.m

Table 4.4: Slab Joint Moments (M_x) Due to Lateral Loads

Storey	Joint B	Joint C		Joint D	
		Right	Left	Right	Left
1	94.9	78.03	81.23	80.75	80.75
2	109.0	90.65	94.35	93.88	93.88
3	102.3	85.03	88.48	88.00	88.00
4	90.3	74.98	78.03	77.63	77.63
5	76.6	63.70	66.30	66.00	66.00
6	62.4	52.18	54.33	54.13	54.13
7	48.9	40.68	42.33	42.05	42.05
8	35.33	29.28	30.45	30.33	30.33
9	22.28	18.68	19.45	19.33	19.33
10	11.9	9.48	9.85	9.73	9.73

Moments in KN.m

Table 4.5: Slab Joint Moments (M_x) Due to Vertical Loads

Storey	Joint E	Joint F		Joint G		Joint H		Joint K
		Right	Left	Right	Left	Right	Left	
1	-222	-305	-499	-445	-249	-260	-507	-472
2	-234	-308	-517	-445	-249	-255	-513	-483
3	-232	-307	-515	-448	-250	-255	-513	-478
4	-233	-307	-515	-447	-250	-255	-513	-479
5	-232	-307	-515	-447	-250	-255	-513	-479
6	-233	-307	-515	-447	-250	-255	-513	-479
7	-233	-307	-515	-447	-250	-255	-513	-479
8	-232	-307	-514	-446	-251	-254	-512	-476
9	-242	-311	-531	-449	-248	-252	-519	-491
10	-196	-290	-459	-419	-240	-228	-460	-423

Moments in KN.m

Table 4.6: Column Joint Moments (M_x) Due to Vertical Loads

Floor Level	Section	Column E	Column F	Column G	Column H	Column K
Base		-53.5	-47.6	33.8	-59.5	92
1	Below	-89.1	-76.8	87.6	-99	202
	Above	-133	-117	111	-148	270
2	Below	-120.6	-107	104	-133	251
	Above	-113	-102	95.0	-125	232
3	Below	-115	-103	97.8	-127	237
	Above	-117	-105	98.9	-130	241
4	Below	-117	-104	98.6	-129	240
	Above	-116	-104	98.1	-129	239
5	Below	-116	-104	92.2	-129	239
	Above	-116	-104	98.3	-129	240
6	Below	-117	-104	98.5	-129	240
	Above	-116	-104	98.0	-129	239
7	Below	-115	-103	97.5	-128	237
	Above	-118	-105	99.2	-131	242
8	Below	-121	-107	102	-135	250
	Above	-111	-99.9	93.3	-122	226
9	Below	-98.1	-91.6	82.8	-104	198
	Above	-144	-128	118	-163	293
10	Below	-196	-169	169	-232	423

Moments in KN.m

Table 4.7: Slab Moments in the Equivalent Frame F1

q_1/q_2	Method	Section							
		1°	2	3	4	5	6	7	8
1	present work	-240	-254	-234	-210	-210	-234	-254	-240
	equivalent frame*	-102	-247	-227	-201	-201	-227	-247	-102
2	present work	-200	-320	-485	-453	-249	-260	-486	-460
	equivalent frame*	-78	-335	-390	-346	-258	-290	-425	-212

* Moments are calculated at the column's face

Moments in KN.m

Table 4.8: Column Moments* in the Equivalent Frame F1

q_2/q_1	Method	Section				
		E	F	G	H	K
1	present work	-120	+7.1	0	-7.1	120
	equivalent frame	-86	13.1	0	-13.1	86
2	present work	-100	-82.5	102	-113	230
	equivalent frame	-71	-61	84	-99	190

* Moments transferred the column above or below

Moments in KN.m

CHAPTER 5

EFFECT OF COLUMN SIZE AND EFFECT OF SLAB AND
COLUMN ASPECT RATIOS ON BENDING MOMENTS

5.1 Introduction

The distribution of bending moments in flat plate systems depends on a number of geometric properties of the slab and column elements, and the material properties of these elements. The current building code provisions are based on a series of slab tests which were undertaken at the University of Illinois as mentioned earlier in Chapter 1. These tests are, and will continue to be, one of the more important contributions to American and Canadian building codes.

In this chapter and the next the objective is to investigate, in a more detailed way the effect of geometric structural changes on the distribution of moments in flat plate systems.

This chapter constitutes a part of this parametric study, wherein the effects of changing the column size, slab aspect ratio, and column aspect ratio on the distribution of moments in flat plate systems are presented.

5.2 Structural Model

The structure modelled in this study is a single storey of a flat plate floor system, consisting of five panels in the x-direction and four in the y-direction (Figure 5.1). The columns framing into the storey are assumed fixed where they frame into adjacent storeys at their remote ends. Only vertical loads were treated in this section.

The structure was analyzed using the finite element model described in Chapter 2. In each analysis symmetry about the x and y axes, shown in Figure 5.1 is assumed. The finite element grid consists of 30 divisions in the x-direction and 20 divisions in the y-direction, so that 1059 elements have been used (Figure 5.2).

A homogeneous and elastic model may differ markedly from a real reinforced concrete structure under certain fairly usual conditions of loading, however, since the research addresses a comparative study of structural behaviour as geometrical parameters change, the model does provide results which could be of use to designers.

In each analysis, the floor system was fully loaded with uniform panel loads. The flat plate thickness was constant throughout the study at 0.033 of the panel length satisfying the serviceability conditions in the current code. Poisson's ratio was again taken as 0.2.

5.3 Column Size

In analysis relating to effect of column size, all the floor panels and the interior columns were kept square while varying the C/L ratio (ratio of the column size to the panel length). The columns above and below the floor were assumed to have the same height h , equal to half the panel length. Different values of C/L ranging from 0.04 to 0.24 were selected. These values were considered to be close to the lower and the upper limits in common practical cases.

The lateral distribution of both the positive and the negative moments (M_x) in an interior and an exterior panel for C/L ratios equal to 0.05, 0.10 and 0.15 are shown in Figures 5.3 and 5.4. The overall shapes of the distribution curves in interior panels are similar to the distribution curves in exterior panels; however, in the lateral distribution curves of the negative moments at the inner face of the exterior columns, positive moments are observed in Figure 5.4(b). The magnitudes of these positive moments decrease with the increase in the column size.

Figure 5.5 shows the total positive moments and Figure 5.6 shows the negative moments (M_x), in an interior slab strip obtained from the finite element method and the equivalent frame method, for different C/L ratios. From these results, it can be seen that the total positive moments, in an interior slab strip, decrease with the increase in column size (Figure 5.5). The effect of the column size is more pronounced for the positive moments in exterior panels, where a decrease of 47% in the positive moment is noted when the relative column size is increased from 0.05 to 0.2.

Figure 5.6 shows that, the total negative moments, in an interior slab strip, along the faces of the interior columns decrease with the increase in column size. For exterior columns, the increase in column size results in an increase in the negative moments. This effect becomes less significant for columns with a C/L ratio greater than 0.1.

Figure 5.7 shows the total moments transferred to the columns above and below the floor level, for different C/L ratios. Figure 5.7(a) shows the moments transferred to the exterior columns for a uniformly loaded flat plate floor. For interior columns, the moments were too small to draw a conclusion therefore the analysis of a partially loaded floor was made. The floor loading and the moments transferred to the columns are shown in Figure 5.7(b). It can be seen that the moments transferred to the columns increase with the increase in the column size. The effect of the column size is more pronounced for small columns with C/L ratios less than 0.1.

Figures 5.5, 5.6 and 5.7 also indicate comparisons between the finite element analysis and the equivalent frame method. The shapes of the curves obtained from these methods are essentially the same. However, there are differences in the magnitudes of the bending moments. The differences in the results will now be summarized.

1 - The slab positive moments obtained from the finite element analysis are lower than those obtained from the equivalent frame method. The difference increases with the increase in C/L ratio.

2 - However the finite element method gives higher negative moments, tending to offset this disparity with respect to the total static moment. The difference is more pronounced at the inner face of the exterior columns where the finite element moments are higher than the equivalent frame moments by 68% for $C/L = 0.07$ and by 46% for $C/L = 0.1$.

3 - The moments transferred to the columns are relatively higher in the finite element method.

4 - It was found that the total static panel moments obtained from the finite element analysis differ from the total static moment by less than 4%.

All of these results are consistent with a less rigid connection between column and slab in the equivalent frame model. For example, the equivalent frame provides higher positive slab moments, lower negative moments and a smaller transfer to the columns, while maintaining the same total static moment, when vertical loads are considered. The equivalent frame model could thus be defended on the basis of an inelastic joint. However, the author feels that the difference in the results in the exterior panels, especially the negative moments at the inner face of the exterior columns, is due to the inability of the equivalent frame method to represent the actual behaviour of the column-slab exterior joints rather than a consideration of inelastic behaviour.

The basis of the equivalent column used in the equivalent frame is described in [16,18]. It is noted that there is no consideration given to inelastic behaviour in the development. Also, the basis of the equivalent column method is a "flexibility analysis" which tacitly assumes zero stiffness in the connection at zero torsional stiffness of the edge member with an assumed linear distribution of edge torsion. This is clearly not a completely rigorous theoretical basis.

In conclusion, the negative design moments obtained from the equivalent frame method are generally lower than those obtained from the finite element method and, correspondingly, the mid-span positive moments obtained from equivalent frame method are higher. Column moments obtained from the equivalent frame method are lower. From a design point of view, the significant differences between the two methods occur for column moments in pattern loading and also for exterior panels. The same conclusion was reached by Elias [28]. In order to have results more agreeable with the finite element analysis, it is recommended that the stiffnesses of the attached torsional members be increased. This increase will result in increasing the equivalent column stiffness giving higher negative moments. By studying square interior panels supported on square columns, Mehrain and Aalami [20] made a similar recommendation.

The effect of the column size on the slab and column moments is due to factors which occur simultaneously as the column increases in size. These factors are the reduction of the clear span, and the increased column stiffness. The first factor has the effect of reducing

the total static moment in the slab. For example, when C/L increases from 0.05 to 0.1, the total static moment in the slab panel decreases by 10%.

To study the effect of the column stiffness, the column heights were varied while the column sizes were kept constant. A number of values of the h/L ratios (ratio of column height to the panel length) ranging from 0.25 to 1.5 were examined. The results of this study are shown in Figures 5.8, 5.9, 5.10 and 5.11. The results indicate that, the effect of column stiffness is more significant for lower ranges of C/L and h/L ratios. On the other hand, it is of less significance and almost linear for higher ranges of C/L and h/L ratios. The column stiffness has a larger influence on the column moments than on the slab moments.

On the basis of the above information it is clear that, the effect of the column size on the slab moments is caused mainly by the reduction in the clear span resulting from larger column size.

5.4 Slab Aspect Ratio

The slab aspect ratio (ratio of largest dimension of the slab panel to the smallest dimension) has a significant effect on the behaviour of the slab. To study this, the panel dimensions in the x-direction (L_1) were maintained constant while varying the panel dimensions in the y-direction (L_2) to give slab aspect ratio ranging from 1 to 2. The columns used in the analysis were square and their sizes were kept constant with a c/L ratio of 0.1.

The change in the slab aspect ratio was found to alter the shape of the moment distribution curves considerably, and as the aspect ratio increases from 1 to 2, the lateral distribution of the moments in the long direction becomes almost uniform indicating one-way action (Figure 5.12). From these curves the percentage of the moments, in the long direction, resisted by the column strip and the middle strip were calculated. Table 5.1 shows a comparison between these results and the code coefficients for the lateral distribution of the moments, for different values of slab aspect ratio. The coefficients obtained from the finite element analysis clearly indicate the one-way behaviour mentioned above, and for flat plate panels with a slab aspect ratio equal to 2 the total positive moment in the panel, in the long direction, is divided equally between the column and the middle strips. However, since there are no beams present in flat plate systems, the code coefficients for distributing the moments laterally do not change as the slab aspect ratio is increased. Thus, the code coefficients do not reflect the transfer in behaviour of the flat plate from two-way to one-way, as the slab aspect ratio increases from 1 to 2. In the transverse direction, the short direction, the same conclusion can be reached. Therefore, it is recommended that these coefficients should be changed to a new set of coefficients capable of representing the actual behaviour of the slab.

Figures 5.13 and 5.14 show the total positive and negative moments in an interior panel, obtained from the equivalent frame method and the finite element method for different slab aspect ratios. In these figures the areas of the slab panels were kept constant while the slab aspect

ratios were varied. It can be seen that, as the aspect ratio of the slab increases, the moments in the long direction (M_x) increase, while the moments in the short direction (M_y) decrease. There is good agreement between the two methods regarding the effect of the aspect ratio on the magnitude of the moments.

5.5 Column Aspect Ratio

In multistorey flat plate apartment buildings, the use of rectangular columns with cross sections elongated in one direction is becoming increasingly popular as they can be used as short walls to resist lateral loads in addition to vertical loads. To evaluate the effect of the column aspect ratio (ratio of largest lateral dimension of the column cross section to the smallest lateral dimension) on the moments and the behaviour of the slab, solutions were obtained for square panels with a $\frac{c_1}{L}$ ratio in the x-direction equal to 0.06. Different values of the column aspect ratio were considered by varying the $\frac{c_2}{L}$ ratio in the y-direction from 0.06 to 0.5 (Figure 5.15).

Figures 5.16 and 5.17 show the total positive and negative moments, in an interior panel, in both the x and y directions. It can be seen that, the positive and the negative moments in the y-direction of bending (direction of the increase in the column dimension) decrease rapidly with the increase in the column aspect ratio while there is no significant change in the moments in the x-direction. When $\frac{c_2}{L}$ increases from 0.06 to 0.35 the positive and the negative moments, in the y-direction, decrease by 44% and 60% respectively.

Figure 5.18 shows the lateral distribution of the positive and the negative moments, in the x-direction, for different values of $\frac{c_2}{L}$. These results indicate that, the lateral distribution of the moments becomes more uniform as $\frac{c_2}{L}$ ratio increases, and the slab tends to behave as a one way slab in the direction transverse to the direction of elongating the columns.

A comparison between the equivalent frame method and the finite element method was made and is presented in Figures 5.19 and 5.20. These results show good agreement between the finite element method and the equivalent frame method in the direction transverse to elongating the columns. However, as expected, the negative moments obtained from the finite element analysis are slightly higher. In the direction of elongating the columns, the total positive moments obtained from the finite element analysis decrease as the column aspect ratio increases, while the positive moments obtained from the equivalent frame method remain almost constant. As a result, the difference between the two methods increases with the increase in $\frac{c_2}{L}$. For example, the positive moments obtained from the equivalent frame method are higher than those obtained from the finite element by 27% for $\frac{c_2}{L} = 0.2$ and by 100% for $\frac{c_2}{L} = 0.5$.

The negative moments obtained from the finite element method, in the direction of elongating the columns, are higher than those obtained from the equivalent frame method. The difference increases with the increase in the $\frac{c_2}{L}$ ratio (Figure 5.20). It can be observed that a corner

in the equivalent frame curve results at $\frac{c_2}{L} = 0.35$. This corner is due to clause 11.5.2 in the building code [33], which requires the critical section for negative moment to be taken at the face of the support but in no case at a distance greater than $0.175L$ from the center of the column. Despite this requirement, the results remain significantly lower than those obtained from the finite element method. For example, the code results are only 65%, 47% and 54% of the finite element moments for $\frac{c_2}{L} = 0.3, 0.35$ and 0.4 respectively.

Table 5.2 shows a comparison between the percentage of the moments, in the direction transverse to elongating the columns, resisted by the column strip and the middle strip obtained from the finite element analysis and the code method of distributing the moments laterally, for different $\frac{c_2}{L}$ ratios. The results in the table show that the code does not recognize the effect of the column aspect ratio in altering the lateral distribution of the moments.

In general it can be concluded that for large values of column aspect ratio the code method of analysis leads to an irrational design procedure. The author has attempted to be realistic in all of his comparisons so far. The present discussion of column elongating is, however, an area of the code requiring further scrutiny. The critical reader, who would take issue with these comments, is advised to consider code provisions relating particularly to column face moments in elongated columns (shown in Figure 5.21); and the transfer of behaviour from the two-way to one-way, as columns are elongated.

Figure 5.22 illustrates the code assumptions for determining the slab stiffness, carry over factor, and fixed end moments. To account for the increase in the slab stiffness due to the columns, the code recommends that the moment of inertia I_{sc} of the slab over the support (from the face of the column to the column center line) be increased as follows:

$$I_{sc} = \frac{I_s}{\left(1 - \frac{C_2^2}{L_2^2}\right)} \quad (5.1)$$

where I_s is the slab moment of inertia in the center span area, C_2 is the column dimension measured transverse to the direction moments are being determined, and L_2 is the slab panel lateral dimension.

The use of equation (5.1) is based on studies at the University of Illinois in the 1960's [16,17]. This research indicated that these assumptions give results in agreement with those found by flat plate theory and experimental results. However, these investigations involved flat plates supported on square columns only. It is clear that by elongating the columns in one direction the slab flexural stiffness increases rapidly in the direction of the elongation. In the extreme case where the ratio $\frac{C_1}{L_1}$ reaches one, the columns become a continuous wall (Figure 5.23(a)), and the slab flexural stiffness in the direction of the wall becomes infinite. This indicates one-way action in the direction transverse to the wall. However, according to the code assumptions the slab stiffness remains finite and yet for small $\frac{C_2}{L_2}$ ratio the increase is too small (Figure 5.23(b)). Therefore, it is more appropriate to include the ratio $\frac{C_1}{L_1}$ in equation (5.1), as follows:

$$I_{sc} = \frac{I_s}{\left(1 - \frac{C_2}{L_2}\right)^2 \left(1 - \frac{C_1}{L_1}\right)^2} \quad (5.2)$$

Figure 5.24 shows a comparison between the results obtained by using equation (5.2) and the finite element analysis. It should be noted that all the negative design moments shown in Figure 5.24 (b) using equation (5.2), were calculated at the column face.

The effect of column size, slab ratio and the column aspect ratio on the bending moments in a uniformly loaded flat plate floor have been investigated. In summary, for different C/L ratios, the finite element method gives lower positive moments and higher negative moments than those obtained from the equivalent frame method. Also, the moments transferred to the columns are higher in the finite element method. The difference between the two methods is more significant in the exterior panels. Generally, the differences increase with the increase in the C/L ratio. To improve the equivalent frame method results, it is recommended that the stiffness of the attached torsional members be increased.

In flat plate floors supported by wall columns the equivalent frame method underestimates the slab negative moments and overestimates the positive moments in the direction of elongating the columns.

As the slab aspect ratio increases the slab tends to behave as a one way slab in the long direction. Also, as the column aspect ratio increases the slab behaviour tends to one-way action in the direction transverse to the direction of elongating the columns.

The slab aspect ratio and the column aspect ratio have a significant effect on the lateral distribution of the moments. This effect is not recognized by current codes in distributing the slab moments laterally. Therefore, it is recommended that the code coefficients should be changed to a new set of coefficients capable of representing the effect of the slab aspect ratio and the column aspect ratio on the behaviour of the slab.

Figure 5.1: Finite Element Model

Figure 5.2: Finite Element Mesh

(a) Lateral Distribution of M_x at Section 1 (b) Lateral Distribution of M_x at Section 5

Figure 5.3: Effect of Column Size on the Lateral Distribution of the Positive Moments

(a) Lateral Distribution of M_x at Section 2 (b) Lateral Distribution of M_x at Section 6

Figure 5.4: Effect of Column Size on the Lateral Distribution of the Negative Moments

Figure 5.5: Effect of Column Size on the Total Positive Moment in an Interior Slab Strip

Figure 5.6: Effect of the Column Size on the Total Negative Moment in an Interior Slab Strip

(a) Moments transferred to exterior columns

(b) Moments transferred to interior columns

Figure 5.7: Effect of Column Size on Moments Transferred to Columns

Figure 5.8: Effect of Column Height on the Total Positive Moments in an Interior Slab Strip

Figure 5.9: Effect of Column Height on the Total Negative Moments (in an interior slab strip) at the Inner Face of Exterior Columns

Figure 5.10: Effect of Column Height on the Total Negative Moments in an Interior Panel of Interior-Slab Strip

Figure 5.11: Effect of Column Height on the Moments Transferred to Exterior Columns

(a). Lateral Distribution of the M_x at Section 1 (b) Lateral Distribution of M_x at Section 2

Figure 5.12: Effect of Slab Aspect Ratio on the Lateral Distribution of the Bending Moments

Figure 5.13: Effect of Slab Aspect Ratio on the Total Positive Moments in an Interior Panel

Figure 5.14: Effect of Slab Aspect Ratio on the Total Negative Moments in an Interior Panel

Figure 5.15: Flat Plate Floor Plan
With Rectangular Columns

Figure 5.16: Effect of Column Aspect Ratio on the Total Positive Moments in an Interior Panel

Figure 5.17: Effect of Column Aspect Ratio on the Total Negative Moments in an Interior Panel

Figure 5.18: Effect of Column Aspect Ratio on the Lateral Distribution of the Moments

Figure 5.19: Total Positive Moment in an Interior Panel Supported on Elongated Columns

Figure 5.20: Total Negative Moment in an Interior Panel Supported on Elongated Columns

Figure 5.21: Flat Plate With Wall Columns

(a) An Interior Flat Plate Panel

(b) Moment of Inertia Diagram (CSA 1977)

Figure 5.22: Code Assumptions for Calculating the Slab Stiffness

(a) Flat Plate Panel Supported by Wall

$$I_{sc} = \frac{I_s}{(1 - c_2/L_2)^2}$$

(b) Moment of Inertia Diagram (CSA 1977)

Figure 5.23: Moment of Inertia for Calculating Slab Stiffness for Slab Panel Supported by Wall

(a) Total Positive Moment in an Interior Panel

(b) Total Negative Moment in an Interior Panel

Figure 5.24: Comparison Between the Finite Element and the Modified Equivalent Frame

Table 5.1: Percentage of Moments* Resisted by Column Strip and Middle Strip in an Interior Panel for Different Slab Aspect Ratios

Positive Moments				Negative Moments			
Finite Element		Code Method		Finite Element		Code Method	
L_2/L_1	Column Strip	Middle Strip	Column Strip	Middle Strip	Column Strip	Middle Strip	Middle Strip
1.0	57	43	60	40	80	20	75
1.2	55	45	60	40	75	25	75
1.4	53	47	60	40	71	29	75
1.6	52	48	60	40	69	31	75
1.8	51	49	60	40	67	33	75
2.0	50	50	60	40	66	34	75

* Moments in the long direction

$C/L = 0.1$

Table 5.2: Percentage of M_y Resisted by Column Strip
and Middle Strip in an Interior Panel for Different Column Aspect Ratios

C_2/L	Positive Moments				Negative Moments			
	Finite Element		Code Method		Finite Element		Code Method	
	Column Strip	Middle Strip	Column Strip	Middle Strip	Column Strip	Middle Strip	Column Strip	Middle Strip
0.06	57	43	60	40	81	19	75	25
0.10	56	44	60	40	79	21	75	25
0.20	54.3	45.7	60	40	76	24	75	25
0.30	52.9	47.1	60	40	72	28	75	25
0.40	51.8	48.2	60	40	67	33	75	25
0.50	50.6	49.4	60	40	59	41	75	25

* $C_1/L = 0.06$

CHAPTER 6

EFFECT OF COLUMN OFFSETS AND EFFECT OF
SLAB OPENING ON SHEARS AND BENDING MOMENTS

6.1 Introduction

This chapter constitutes the second part of the parametric study begun in Chapter 5. In this chapter the effects of column offsets and slab openings on the shears and the bending moments in a single floor of a slab-column building frame are investigated.

The literature devoted to this particular area is sparse; although the ASCE-ACI Joint Task Committee [36] has summarized the available information regarding slab openings. Additional data related to slab openings is provided in [37]. These studies address mainly the shear capacity of the column-slab joint.

6.2 Column Offsets

In the flat plate floor system shown in Figure 5.1, columns are arranged on a rectangular modules, with a regular grid of column axes in the x and y directions. A column offset is defined as the translation of a single column off the module, such that it is eccentric to a grid point. In using the direct design method, column offsets up to 10% of the span in the offset direction are permitted. In the

equivalent frame method, in general terms, either one-storey analysis or multi-storey analysis may be carried out, however, the code does not mention non-straight column line axes in the equivalent frame model. The code has no specific provision for treating a column offset of the order of a half-span for example. Column offsets often may not be avoided, and these do effect the moments in the slabs. This problem was investigated by analyzing the one-storey model for different ratios of eccentricity, e to span length, L ranging from 0.1 to 0.5 for the column in the center of the one quarter structure shown in Figure 6.1. In each case, the interior columns and all slabs, were square, and the C/L ratio was equal to 0.1.

In the definition of the equivalent frame method, the lateral boundaries of the equivalent frame are the panel centerlines on each side of the centerline of the columns. The interpretation of this is shown in Figure 6.2 for the structure under consideration, where the equivalent frame boundaries are shown for frames aligned opposite the direction of offset. In equivalent frame, F1, the offset, e of the column results in an offset $e/2$ of the panel centerlines on either frame edges, resulting in a frame, F1, which has a constant transverse width,

In equivalent frame, F2, a variable width is shown in Figure 6.2. Less attention will be paid to frame, F2, since the most significant increases in moments and shears will take place in frame, F1. The increases which do occur, are levelled somewhat by distributing these across increased widths. Also, a punching shear increase in F2 at the

column adjacent to the offset column is dealt with in the analysis of the transverse equivalent frame, which has an increased span in that region. Equivalent frames in the transverse direction, or direction, of offset are not variable in width; and positive and negative moments M_y , take due account of slab span increases and decreases. Moments in the transverse direction will not be considered.

In frame, F1, Table 6.1 shows that the maximum positive moments, in the two spans adjacent to the offset column, in sections 4 and 7 (Figure 6.2) remain substantially the same. The negative moments along the offset column faces, in sections 5 and 6 (Figure 6.2) decrease approximately uniformly with the increase in offset by about 20% in the interior and the exterior spans. The negative moments along the faces of adjacent columns, in sections 2, 3, and 8 increase in absolute value, and by adding, for example, the moment at section 7 to the average absolute moments at sections 6 and 8, it is noted that the total static moment remains constant.

In Table 6.2 the moments transferred to adjacent columns A and C increase substantially while the bending moment in offset column B is slightly lower.

In Tables 6.3 and 6.4 a similar set of results for frame, F2, is given, only to point out that the major influence of the column offset is felt in the equivalent frame containing the offset. One may note, for example, that regarding panel moments, a 50% increase in equivalent frame width, results in a corresponding negative total section increase of only 29%; and a 25% increase in equivalent frame

width results in a corresponding positive total section moment increase of 30%.

The lateral distribution of the moments (M_x) for $e/L = 0.0, 0.3$, and 0.5 at sections 4 and 5 is shown in Figures 6.3 and 6.4. It is evident that a substantial change in the lateral distribution of the moments in sections near the shifted column occurs. Positive moment curve "amplitudes" do not increase with offset, but rather shift and decrease; and the zone width, or curve "period", increases. Also, a positive moment in negative region section 5 is evident from Figure 6.4 in the middle strip.

In the results, so far, the column dimensions, and slab dimensions have been kept constant, and the only varying parameters have related to a single column offset. Varying aspect ratios of column and slab, in addition to offset, will not be examined here; however, it was considered to be of fundamental importance to present results for varying ratios of column to slab stiffness. Additional results are thus presented for three values of column dimension to slab length (C/L). Thus, Table 6.5 presents the panel moments in frame, F1 at the 8 sections indicated in Figure 6.2, for values of C/L equal to 0.05 and 0.15, in addition to those presented for $C/L = 0.10$.

In Table 6.5 there is a general tendency for the increasing column stiffness to reduce the effect of the column offset on the panel moments.

The results presented in this section have indicated a number of trends for an equivalent frame F1, with column offset transverse to the frame direction, when the offset e/L increases.

1 - The negative moments in the slab strip at the offset column lines decrease by roughly between 15% and 21% while the negative moments at an adjacent column face increase by 16% to 27% at interior spans.

2 - No significant changes in positive moments occur.

3 - Moments transferred to adjacent columns increase, and only a slight decrease occurs in moments transferred to offset columns.

4 - For large or medium columns the effect of offset is only felt in the span adjacent to the offset.

5 - Slab systems with small columns are more sensitive to offset.

The effects of offsets and column size on punching shear was investigated in the course of this work. The findings are presented very briefly in Table 6.6. It is well known that in any uniformly loaded span, as one of the supports is reduced from full fixity to a simple support, while the other remains fixed, the shear will increase at the fixed support and decrease at the partially fixed support. The importance of this trite example, is that the punching shear at column A and C in frame F1, does increase as expected, but no increase is indicated in equivalent frame method calculations.

It is important to note that the code restricts the use of 'direct design' for columns offsets up to 10% of span, and contains no special provisions for large offsets in the equivalent frame method. The results of the finite element analysis are compared with those of the equivalent frame method in Table 6.7 for $e/L=0.0$ and 0.5 in frame, F1. Percentage differences exist between the two methods regardless of offset. The most significant results relating to column offset are the large increases in the negative moments at the columns adjacent to the offset column in frame, F1. The equivalent frame method calculations are not identical in frame, F1 when a column is offset. There is a change in the calculation of the stiffness of attached torsional members. However, this is the only change relating to F1, and the change has a small effect on the moment calculations.

In summary, a basic weakness in the code provisions exists in the calculation of the slab stiffness and the fixed end moments in the two spans adjacent to the offset column, in the equivalent frame F1. It is clear that, due to the column offset, a change in behaviour in these two spans occurs. In order to improve the code results, the author recommends that the slab moment of inertia I_{sc} at the offset column be reduced as the column offset increases. This can be achieved by modifying the code expression as follows:

$$I_{sc} = \frac{I_s \left[1 - \frac{e}{L_2} \left(1 - 2 \frac{C_1}{L_1} \right) \right]^2}{\left(1 - \frac{C_2}{L_2} \right)^2} \quad (6.1)$$

where I_s is the slab moment of inertia in the center span area.

The additional term in equation (6.1) has no effect in the case of zero column offset ($\frac{e}{L_2} = 0.0$). However, the effect of this term increases as the ratio $\frac{e}{L_2}$ increases and the ratio $\frac{c_1}{L_1}$ decreases. It was found that, in all of the cases examined, the use of equation (6.1) adjusts the results satisfactorily, and the negative moments in the equivalent frame F1, at the offset column decrease and the negative moments at the adjacent columns increase as the offset increases. This is similar to the trend that was found by the finite element analysis.

6.3 Slab Openings

Openings in the floor slabs accommodate stairways, elevators, ducts, and so on. The presence of these openings affects the distribution of the moments as well as the shear forces, and the severity of the effect is dependent not only upon size, but also upon location. Openings are least harmful where shear stresses are small and bending moments are below maximum. The code provides that the total amount of reinforcement calculated without the opening must be maintained. Openings in an all column strip area, as shown in Figure 6.5(b) must be limited in size. Also, where openings occur in areas which are column strips in one direction and middle strips in the other, the interruption or displacement of the quantity of reinforcement is limited.

Openings in areas common to two column strips degrade the building frame system more significantly than those in other locations. The structure will lose lateral and bending stiffness rapidly if the column to slab connection is weakened. Alternatively, stiffness is lost as

the moment transfer mechanism between slabs and columns becomes less effective. Since these openings are located in areas adjacent to columns where the maximum negative moment and maximum shear stresses occur, they reduce, in a direct way, the load carrying capacity of the system. It was felt that the severe impact of such openings is well understood by structural designers. In addition, studying this type of opening requires a very refined finite element mesh; therefore it was decided to study only the other two opening locations.

A floor plan with an opening H1 in a column strip-middle strip area is shown in Figure 6.6; and an opening H2 in a middle strip-middle strip area in Figure 6.7. The floor panels were square and were supported on square columns with $\frac{c}{L} = 0.1$. In each case examined, the floor was fully loaded with a uniformly distributed load q , and the opening was square. Two opening sizes $L_0/L = 1/8$ and $1/4$ were examined.

Figures 6.8 and 6.9 show the distribution of the bending moments M_x and M_y along section (A-A₁) near the hole H1, while Figures 6.10 and 6.11 show the distribution of the bending moments M_x along section (C-C₁) and M_y along section (B-B₁) near the hole H2. These results indicate that high local bending moments occur in the area around the openings in a direction parallel to the opening edge (Figure 6.9, 6.10 and 6.11). Near the sharp edge corner of the opening these moments result in high principal moments. It was found that these principal moments, near the edge corner, are the maximum moments in the area around the opening. In general, the magnitudes of these moments are higher for small openings with $L_0/L = 1/8$ (Figures 6.9, 6.10 and 6.11). However, these

moments of a localized character and, moving away from the opening, they decrease more rapidly in the case of small openings (Figures 6.9, 6.10 and 6.11). These high bending moments would drop once concrete began to crack. However, additional reinforcement around the opening should be added to control the width of these cracks.

In the floor with an opening, H2 (middle strip-middle strip area) the change in the total moments in the equivalent frames containing the opening was found to be small as shown in Table 6.8. This is due to the fact that this type of opening does not interrupt the normal flexural action of the slab; and also that each equivalent frame contains only one half of an opening. Since $L_0/L = 1/4$ is the maximum configuration that a middle strip-middle strip can accommodate, it is concluded that the code requirement governing this type of opening is justified, that is, no restrictions on size provided the total amount of reinforcement for a no-opening situation is maintained.

In the floor having an opening, H1 in a column strip-middle strip area, with $L_0/L = 1/8$, the change in the total moments in the equivalent frame containing the opening was found to be small (Table 6.9). This size is the maximum opening size allowed by the code without special analysis. In the case of the opening with $L_0/L = 1/4$, a reduction of 24% in the positive moment in the middle span, which contains the opening, occurs, while there is no significant reduction in the negative moments. The reduction in the total static moment in the span containing the opening was expected because of the reduction in the total panel load due to the presence of the opening. This reduction affects only the positive moment because of the loss in the stiffness in this area.

The results in Table 6.9 show that there is no change in the moments in the other spans.

It was also found that high local shear stresses occur in the area around the openings in a direction parallel to the opening edge. The magnitude of these shears are higher for small openings with $L_0/L = 1/8$. At the opening corner, discontinuity in the shear distribution curves occurs, and the shear stresses are maximum. Moving away from the opening, these high shears decrease rapidly especially in the case of small openings. In reinforced concrete slabs these high stresses would exist only until the concrete began to crack.

The author has used the finite element method to study the elastic behaviour of a flat plate floor with column offsets and openings. It was found that column offsets have a significant effect on the magnitudes and the lateral distribution of the slab moments especially in the slab panels adjacent to the shifted column. These effects increase as the column size decreases. In a flat plate supported by small columns high shear stresses are observed at the columns adjacent to the offset column. These high shear stresses may cause punching shear failure, so that particular attention should be paid to column offsets involving small columns.

None of the two methods recommended by the code is capable of handling a flat plate floor with large column offsets in a realistic manner.

The effect of floor openings on slab moments is one of a local nature where openings are not common to two column strips; that is, in areas far from the openings no significant change in the distribution of the slab moments occurs. Near the openings high shears and bending moments occur. Moving away from the opening, these effects decrease rapidly especially in the case of small openings.

The results of the study on openings in middle strip-middle strip areas and column strip-middle strip areas are in substantial agreement with the code requirements for these types of openings.

Figure 6.1: Flat Plate Floor With Column Offset

Figure 6.2: Layout of the Equivalent Frames F1 and F2

Figure 6.3: Lateral Distribution of M_x Along Section 4

Figure 6.4: Lateral Distribution of M_x Along Section 5

Figure 6.5: Types of Openings

Figure 6.6: Floor Plan With Hole H1

Figure 6.7: Floor Plan With Hole H2

Figure 6.8: M_y Distribution Along Section (A-A₁)

Figure 6.9: M_x Distribution Along Section (A-A₁)

Figure 6.10: M_x Distribution Along Section (C-C₁) Figure 6.11: M_y Distribution Along Section (B-B₁)

Table 6.1: Effect of Column Offsets on the Bending Moments
Coefficients (β)* in the Equivalent Frame F1

e/L Sec.	1	2	3	4	5	6	7	8
0.0	+38.4	-65.2	-64.9	+37.8	-71.3	-79.8	+44.8	-42.0
0.1	+38.4	-65.4	-66.8	+37.8	-69.4	-76.9	+44.9	-42.8
0.2	+38.3	-65.6	-69.4	+37.9	-66.5	-73.8	+45.0	-45.6
0.3	+38.2	-65.9	-73.1	+37.9	-64.4	-72.3	+45.2	-48.5
0.5	+37.6	-67.5	-79.7	+38.1	-58.5	-66.7	+45.5	-59.2

* $M=10^{-3} \beta q L^3$

Table 6.2: Effect of Column Offsets on the Bending Moments
Transferred to the Columns in the Equivalent Frame F1

e/L	$M_c/10^{-3} q L^3$		
	Col. A	B	C
0.0	+0.7	-9.7	+66.1
0.1	-0.1	-9.5	+66.7
0.2	-4.4	-9.1	+68.1
0.3	-12.0	-8.6	+71.5
0.5	-23.5	-7.2	+87.1

Table 6.3: Effect of Column Offsets on the Bending Moment
Coefficients (β)* in the Equivalent Frame F2

Sec. e/L	1	2	3	4	5	6	7	8
0.0	+37.2	-63.9	-63.6	+36.7	-70.6	-77.9	+46.0	-40.0
0.1	+36.8	-64.7	-65.9	+39.0	-77.0	-83.7	+48.7	-40.1
0.2	+36.5	-65.2	-68.0	+41.1	-83.5	-89.4	+51.7	-41.3
0.3	+36.3	-65.3	-70.1	+43.2	-87.5	-93.9	+55.0	-44.8
0.5	+34.9	-65.9	-73.4	+47.0	-91.2	-98.4	+61.4	-46.0

* $M=10^{-3} \beta q L^3$

Table 6.4: Effect of Column Offsets on the Bending Moments
Transferred to the Columns in the Equivalent Frame F2

$M_c/10^{-3} q L^3$				
e/L	Col.	A	B	C
0.0		+0.7	-9.9	+63.0
0.1		+0.4	-10.1	+63.3
0.2		-1.4	-10.4	+64.0
0.3		-4.5	-10.8	+65.6
0.5		-9.3	-11.9	+72.5

Table 6.5: Effect of Column Offsets on the Bending Moment Coefficients
(β^*) in the Equivalent Frame F1 for Different $\frac{C}{L}$ Ratios

C/L	Section	1	2	3	4	5	6	7	8
0.05	$\frac{e}{L} = 0.0$	+43.3	-76.8	-75.8	+39.2	-85.9	-98.5	+68.2	-19.0
	$\frac{e}{L} = 0.5$	+38.9	-83.9	-96.2	-38.6	-68.0	-78.0	+70.9	-35.0
	% change	10.2	9.2	26.9	1.5	20.8	20.8	4.0	84.2
0.10	$\frac{e}{L} = 0.0$	+38.4	-65.2	-64.9	+37.8	-71.3	-79.8	+44.8	-42.0
	$\frac{e}{L} = 0.5$	+37.6	-67.5	-79.7	+38.1	-58.5	-66.7	+45.5	59.2
	% change	2.1	3.5	22.8	0.8	18.0	16.4	1.6	41.0
0.15	$\frac{e}{L} = 0.0$	+34.8	-55.1	-54.8	+34.5	-58.6	-66.3	+49.3	-46.0
	$\frac{e}{L} = 0.5$	+34.4	-56.5	-63.7	+33.4	-49.7	-56.8	+50.5	-55.6
	% change	1.1	2.5	16.2	3.2	15.2	14.3	2.4	20.9

* $M=10^{-3} \beta q L^3$

Table 6.6: Effect of Column Offsets on the Punching Shear Forces
 V_p Around Column A and C in the Equivalent Frame F1

$V_p/q L^2$						
Column	A			C		
C/L	0.05	0.10	0.15	0.05	0.10	0.15
$e/L = 0.0$	1.03	0.982	0.973	0.45	0.48	0.504
$e/L = 0.5$	1.07	1.01	0.99	0.483	0.506	0.521
% increase	3.9	2.9	1.7	7.3	5.4	3.4

Table 6.7: Comparison Between the Bending Moment
Coefficients (β)* Obtained in the Equivalent Frame
FI by the Equivalent Frame Method and the
Finite Element ($\frac{C}{L} = 0.1$)

e/L	Section	1	2	3	4	5	6	7	8
0.0	equivalent frame	+40.8	-60.5	-59.1	+38.6	-66.3	-74.8	+54.5	-28.8
	finite element	+38.4	-65.2	-64.9	+37.8	-71.3	-79.8	+44.8	-42.0
	% difference	-5.9	+7.8	+9.8	-2.1	+7.5	+6.7	-17.8	+45.8
0.5	equivalent frame	+40.7	-60.6	-59.6	+39.0	-65.1	-76.0	+54.1	-28.5
	finite element	+37.6	-67.5	-79.7	+38.1	-58.5	-66.7	+45.5	-59.2
	% difference	-7.2	+11.6	+33.7	-2.3	-10.1	-12.2	-15.9	+105.5

$M/10^{-3} q L^3$

section	1	2	3	4	5	6	7	8
$\frac{L_0}{L} = 0$	+38.4	-65.2	-64.9	+37.8	-71.3	-79.8	+44.8	-42.0
$\frac{L_0}{L} = \frac{1}{8}$	+37.6	-65.3	-64.9	+37.8	-71.3	-79.8	+44.8	-42.0
$\frac{L_0}{L} = \frac{1}{4}$	+34.0	-64.2	-64.7	+37.8	-71.3	-79.8	+44.8	-42.0

Table 6.8: Effect of the Opening H2 on the Bending Moments in an Equivalent Frame

$M/10^{-3} q L^3$

section	1	2	3	4	5	6	7	8
$\frac{L_0}{L} = 0$	+37.2	-63.9	-63.6	+36.7	-70.6	-77.9	+46.0	-40.0
$\frac{L_0}{L} = \frac{1}{8}$	+34.4	-66.5	-62.6	+36.8	-70.6	-77.9	+46.0	-40.0
$\frac{L_0}{L} = \frac{1}{4}$	+28.3	-62.5	-62.3	+36.8	-70.6	-77.9	+46.0	-40.0

Table 6.9: Effect of the Opening H1 on the Bending Moments in an Equivalent Frame

CHAPTER 7

CONCLUSIONS AND FUTURE WORK7.1 Conclusions

In Chapters 3 and 4, the author has used the finite element method in the analysis of building frame systems. The application of the finite elements in these two chapters is unique to this particular type of frame. The expenditure of effort in this limited area is felt to be justified considering the importance of the building frame as a design office activity; and also considering the state of building analysis, versus numerical analysis in the general field of structural mechanics. The method proposed is considerably more manageable than a full finite element analysis of the building. It is also a more efficient use of computer resources, since the complex behaviour attributable to building frames is confined to single storeys and only a single computer run is required to provide all necessary information to construct the storey stiffness matrix and the storey fixed end forces. Although, it will seem to many readers that the analysis is cumbersome, the author did in fact find that with experience it can be done quite expeditiously, even though at this point no attempt has been made to fully automate the procedures. In addition one of the usual handicaps of the finite element procedure, that of interpreting large quantities of output data, is avoided here, since the analysis ultimately produces slab and column moments. The method is particularly advantageous if multiple load cases are involved, or other refinements of analysis, such as

geometrical or length effects, or dynamic analysis are to be included. The analysis refinements are applied to the system with reduced degrees of freedom. These extensions are already on sound theoretical ground, well known to practising structural engineers.

The method can also be extended to more complex situations such as flat plate systems with shear walls. The method as described here is valid and provides an appropriate tool to study the behaviour of such systems in a rational way.

An analysis of a 10-storey building frame has been carried out. The analysis shows the importance of the three-dimensional action of building frames. Also, it was pointed out that the equivalent column concept used in the equivalent frame method cannot be extended to sidesway analysis.

The author has used the finite element method to study the effect of geometric structural changes on the distribution of moments. Some of the more important results obtained in the study give rise to the following general conclusions.

For different column sizes, the finite element method gives lower positive slab moments, higher negative moments and higher column moments than the equivalent frame method. The difference between the two methods is more significant in the exterior panels and increases with the increase in the column size. The equivalent frame results are consistent with a more flexible joint. Thus, it could be defended on the basis of an inelastic joint. However, the author feels that the difference in the results in the exterior panels, especially the negative moments at the

inner face of the exterior columns, is due to the inability of the equivalent frame method to represent the actual behaviour of the column-slab exterior joints rather than a consideration of inelastic behaviour. To improve the equivalent frame method results it is recommended that the stiffness of the attached torsional members be increased.

As the column aspect ratio increases the slab tends to behave as a one-way slab in the direction perpendicular to the direction of elongating the columns. Also, as the slab aspect ratio increases the slab behaviour tends to one-way in the long direction. This behaviour is not recognized by the code design method in distributing the slab moments laterally.

In a flat plate floor supported by wall columns the equivalent frame method underestimates the slab negative moments and overestimates the positive moments in the direction of elongating the columns. Keeping in mind the code's inability to recognize the one-way action mentioned above, it is concluded that the code design methods of analysis of flat plates with wall columns lead to irrational design procedures. Recommendations to improve the code results were presented in section 5.5.

Column offsets have significant effects on the magnitudes and the lateral distribution of slab moments especially in the slab panels adjacent to the shifted column. These effects increase as the column size decreases. In a flat plate system supported by small columns, high shear stresses at the columns adjacent to the offset column occur.

These high shear stresses may cause punching shear failure, so that particular attention should be paid to column offsets involving small columns. Based on the comparison between the results obtained by the finite element method and the equivalent frame method for a flat plate floor with column offsets, it was found that the equivalent frame method is not capable of handling a flat plate floor with large column offsets in a realistic manner. Recommendations to improve the code results were presented in section 6.2.

7.2 Future Work and Development

The work in this thesis provides the basic information needed for a parallel study, which is now under way, to develop a stiffness model for the three-dimensional building frame. The stiffness matrix for different flat plate floor layouts and the fixed end moments for uniformly loaded panels have been derived using the method presented in Chapter 3. In the extended work, several floor models have been considered. The preliminary results have indicated that the model shown in Figure 7.1 gives results close to those obtained from the finite element analysis. The grid system shown in the figure is designed to model the slab two-way action. The "joint (connecting) springs" are introduced to allow a difference between the column rotation and the slab average rotation. The model has two orthogonal rotational degrees of freedom at each column joint (joints A, B, ... and I in Figure 7.1). The model was designed so that when a particular d.o.f. is released and forced, while all the other degrees of freedom are fixed, the moments are transferred throughout the whole system, similar to the actual behaviour of the slab.

The diagonal members in the grid system are used to model the coupling stiffnesses between the slab degrees of freedoms in the x and y directions. The cross members are introduced to transfer the slab load from a point load at the intersection point in the middle of the slab panels to the supporting edge beams.

The use of the finite element analysis in this thesis has been directed toward the study of the elastic behaviour of flat plate structural systems. The work can be extended to flat slab systems with drop panels and column capitals. The extension to flat slabs with column capitals is a minor one since the columns have been treated here as stiffnesses applied to patch areas. Also, the finite element plate bending approach can easily handle slabs of nonuniform thickness, and thus can model flat slabs with drop panels.

As previously indicated, in reinforced concrete slabs, the high negative moments occurring at the column corners is limited by cracking of the concrete and yielding of the steel reinforcement. This results in a change in the slab mechanism accompanied by re-distribution in the slab moments. The inelastic analysis of the selected model can help in studying the slab behaviour in a more realistic way.

Figure 7.1: Stiffness Model of 2 x 2 Panel Flat Plate Floor

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