INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

**Bell & Howell Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600**

UMI[®]

An Experimental and Computational Study of the Aerodynamics of Turbine Blades with Damage

by

Alamgir M.T. Islam

M.Eng. (Mechanical)

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of

Doctor of Philosophy

in

Mechanical Engineering

**Ottawa-Carleton Institute for Mechanical
and Aerospace Engineering**

**Department of Mechanical and Aerospace Engineering
Carleton University
Ottawa, Ontario
September, 1999**

Copyright © 1999

Alamgir M.T. Islam

**National Library
of Canada**

**Acquisitions and
Bibliographic Services**

395 Wellington Street
Ottawa ON K1A 0N4
Canada

**Bibliothèque nationale
du Canada**

**Acquisitions et
services bibliographiques**

395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file Votre référence

Our file Notre référence

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-48331-2

Canada

The undersigned hereby recommend to the
Faculty of Graduate Studies and Research
acceptance of the thesis

**An Experimental and Computational Study of the Aerodynamics of
Turbine Blades with Damage**

submitted by

Alamgir M.T. Islam, M.Eng.

in partial fulfilment of the requirements for
the degree of Doctor of Philosophy

Chairman, Department of Mechanical and Aerospace Engineering

Thesis Supervisor

External Examiner

Carleton University
September, 1999

Abstract

Investigations have been made of the aerodynamic effects of in-service damage on the performance of axial turbine blades. Two aspects of blade damage were considered: surface roughening and trailing edge damage. The work is related to gas turbine engine health monitoring.

Correlations for the effects of surface roughness were developed based on a database obtained from Kind et al. (1998). The correlations account for the effects of the roughness height as well as the location and extent of the roughness patch on the blade surface.

The effect of trailing edge damage at transonic flow conditions was investigated both experimentally and computationally. The blade damage was idealized as a circular cutout at the midspan in the trailing edge. Measurements have been made in a planar cascade in a high speed wind tunnel for both undamaged and damaged blades. Two damage sizes have been considered: 10% and 20% of chord cutout at the trailing edge. The measurements were conducted for exit Mach numbers ranging from 0.7 to 1.2 and for the incidence values of 0° and 4.5° . Detailed flow field measurements downstream of the cascade include profile losses, deviation angles and trailing edge base pressure. Computational investigation was conducted for only trailing-edge damage using a three-dimensional Navier-Stokes solver developed by Dawes (1988). The computations with trailing edge damage represent a novel application of the code and the wind tunnel measurements were therefore used to validate the computations.

Results showed that surface roughening and trailing edge damage produced significantly different aerodynamic behavior of the flow. Surface roughening largely influences the profile losses and trailing edge damage has a considerable effect on the flow deviation. The effect of trailing edge damage on the loss characteristics of the blades was found to be fairly small over the full range of flow conditions. In fact, the overall measured profile losses were actually lower for 20% damage than for the undamaged blade. The physics of the flow and the reasons for the

observed reduction in losses are discussed, particularly the contribution of base pressure to the reduction in the profile loss.

The measured flow deviation increased with the increase in damage size as well as cascade exit Mach number. Computational investigations were made to identify the parameters that influence flow deviation in turbines with both undamaged and damaged blades so that correlations could be developed. The existing correlations for the prediction of subsonic deviation for undamaged blades were found to be unsatisfactory. It was found that the deviation is primarily determined by the blade loading (pressure difference) towards the trailing edge. The blade row parameters which influence this pressure difference were identified. New correlations for undamaged and damaged blades were developed using a genetic algorithm to optimize the coefficients based on a database of known values of the deviation. The deviation correlation for undamaged blades is applicable for subsonic flow cases and that for damaged blades is applicable for the full range of Mach numbers studied. The new subsonic deviation correlation for undamaged blades appears to be significantly more successful than existing correlations.

The three dimensional Navier-Stokes solver of Dawes predicted many of the measured results for damaged blades. Particularly, predictions of deviation for different undamaged and damaged blade geometries were found to be quite satisfactory and the code was extensively used to generate deviation data to supplement experimental measurements.

Acknowledgments

I would like to express my profound gratitude to my thesis supervisor Professor S.A. Sjolander for his invaluable guidance and advice throughout the course of this research. Discussions and solutions relating to many experimental and computational problems are greatly appreciated.

I wish to express my sincere thanks to Michael Saroch and Michael Benner for helping me get started with my computational work. I would also like to thank Bruce Johnston for providing much assistance with the resources of the Computer Laboratory.

My experimental work could not have been completed without the assistance of Dhafer Jouini who showed me how to use the high speed wind tunnel. His constant guidance and cooperation throughout the whole phase of the experimental work is greatly appreciated. In this regard, the valuable suggestions provided by Micheal Jeffries are also acknowledged. I would also like to thank Dhafer for his contribution to the preparation of this manuscript.

My thanks to Larry Boissonneault and Peter Serjak of the Science and Technology Centre whose technical and practical advice helped me design three pressure measuring probes.

Initial financial support for this study provided by the Government of Canada (under the Commonwealth Scholarship Program) and the support provided by GasTOPS Ltd., Ottawa during the later phase of the work are gratefully acknowledged. I also wish to express my sincere thanks to Professor H.I.H. Saravanamuttoo for being kind enough to provide additional financial support during a brief period of the research work.

Finally, I would like to thank my wife and family for their support and patience during the course of my long period of research.

Table of Contents

Abstract	Page i
Acknowledgments	iii
Table of Contents	iv
List of Figures	viii
List of Tables	xiv
Nomenclature	xv
1. Introduction	1
2. Review of Literature	4
2.1 Introduction	4
2.2 Loss Correlations for Undamaged Blades	5
2.2.1 General Approach	5
2.2.2 Profile Loss	6
2.2.3 Secondary Loss	12
2.2.4 Tip Clearance Loss	16
2.3 Flow Deviation Correlations	21
2.4 Turbine Blade Damage	25
2.4.1 Engine Health Monitoring	25
2.4.2 Surface Roughening Damage	25
2.4.3 Trailing Edge Damage	26
2.4.4 Tip Damage	26
3. Computational Model	30
3.1 Introduction	30
3.2 Three Dimensional Compressible Navier-Stokes Solver (BTOB3D)	31
3.2.1 Governing Equations	31
3.2.2 Computational Mesh	33
3.2.3 Discretization of the Governing Equations	35

3.2.3.1	Finite Volume Formulation	35
3.2.3.2	Solution Algorithm	35
3.2.3.3	Artificial Viscosity	36
3.2.3.4	Boundary Conditions	36
3.2.4	Multigrid Acceleration	38
3.2.5	Turbulence Model	39
3.3	Modifications of the Navier-Stokes Code	41
3.3.1	Baldwin-Lomax Turbulence Model Implementation	41
3.3.2	Inviscid and Viscous Endwalls	45
3.4	Simulation of Cascade Flow	45
3.4.1	Selection of Computational Grid	45
3.4.1.1	Grid Boundary	45
3.4.1.2	Grid Orientation at Inlet and Exit Plane	46
3.4.1.3	Grid Quality	50
3.4.1.4	Grid Density and Refinement	57
3.4.1.5	Grid Modification and Refinement for Damaged Blade	61
3.4.2	Convergence Criteria	64
3.4.2.1	The Residual and Percent Mass Error	64
3.4.2.2	Number of Timesteps	67
3.4.3	Influence of Mach Number	67
3.4.4	Influence of Artificial Viscosity	70
4.	Development of Correlations for Blade Roughness Damage	72
4.1	Introduction	72
4.2	Effect of Surface Roughness in Incompressible Flow	73
4.2.1	Sources of Data	73
4.2.2	Effect of Extent of Roughness Patch	73
4.2.3	Effect of Roughness Size on Losses	78
4.2.4	Effect of Incidence on Losses	80
4.3	Application of the Correlations to F404 Turbine	84
4.4	Effect of Surface Roughness at Transonic Flow	86
5.	Experimental Apparatus	89
5.1	Introduction	89
5.2	High Speed Wind Tunnel	89
5.3	Cascade Test Section	91

5.4	Probe Traversing Mechanism	93
5.5	Cascade Geometry for Undamaged and Damaged Blades	93
5.6	Miniature Fast Response Probes	96
5.6.1	General Overview of Selection of Probes for High Speed Flow Measurements	96
5.6.2	Carleton University Static and Pitot Probes	98
5.6.3	Carleton University Three Hole Probe	100
5.7	Data Acquisition System	102
6	Measurement Procedure	104
6.1	Introduction	104
6.2	Three Hole Probe Performance	104
6.2.1	Probe Calibration and Accuracy	104
6.2.2	Probe Blockage Effects	105
6.3	Transducer Calibration	106
6.4	Data Acquisition	108
6.4.1	Procedure	108
6.4.2	Data Sampling Times and Rates	110
6.5	Flow Conditions	110
6.5.1	Reynolds Number and Turbulence Intensity	110
6.5.2	Inlet Flow Uniformity	111
6.5.3	Outlet Flow Periodicity	111
6.6	Probe Measurements	116
6.6.1	Total Pressure Measurements	116
6.6.2	Base Pressure Measurements	118
6.6.3	Axial Velocity Density Ratio (AVDR)	118
6.6.4	Blade Surface Pressure Measurements	121
6.7	Calculation of Mixed-Out Values	123
7.	Experimental Results for Blades with Trailing Edge Damage	125
7.1	Introduction	125
7.2	Effect of Damage on Losses	126
7.3	Effect of Damage on Deviation	131

8.	Development of Deviation Correlations	135
8.1	Introduction	135
8.2	Factors Influencing Deviation	136
8.3	Correlations for Subsonic Flow	145
8.3.1	Sources of Data	145
8.3.1.1	Experimentally Measured Data	145
8.3.1.2	Experimental Data for Different Values of AVDR	145
8.3.1.3	Computations for Existing Blades	146
8.3.1.4	Computations for Hypothetical Blades	146
8.3.1.5	Summary of Ranges of Data	148
8.3.2	Using Genetic Algorithms for Optimization	148
8.3.2.1	Basic Description of Genetic Algorithms	148
8.3.2.2	Selection of Basic GA Parameters	154
8.3.2.3	Evaluation of the Fitness of Solutions	158
8.3.3	Deviation Correlation for Undamaged Blades	159
8.3.4	Deviation Correlation for Damaged Blades	168
8.4	Deviation Correlations for High Speed Flow	172
8.4.1	Deviation Correlation for Damaged Blades	172
8.4.2	Outlet Flow Angle Predictions by Ainley & Mathieson Method	175
8.5	Predictions of Deviation for HS1A Blade	179
9.	Conclusions and Recommendations	199
9.1	Conclusions	199
9.2	Recommendations	202
	References	204
	Appendices	
	Appendix A: Blade Geometry for Static and Pitot Probe	213
	Appendix B: Blade Geometry of Three Hole Probe	225

List of Figures

		Page
Figure 2.1	Profile Loss Coefficients for Conventional Section Blades at Zero Incidence (reproduced from Ainley & Mathieson, 1951).	7
Figure 2.2	Profile Incidence Loss Correlation (reproduced from Benner et al., 1997).	11
Figure 2.3	The Loading Factor Z as a Function of α_1 and α_2 (reproduced from Sieverding, 1985).	14
Figure 2.4	Secondary Incidence Loss (reproduced from Moustapha et al., 1990).	17
Figure 2.5	Schematic Breakdown of the Losses in the End Region (excluding profile losses)(reproduced from Yaras & Sjolander, 1992).	19
Figure 2.6	(a) Relationship between gas outlet angles and $\cos^{-1} \alpha/s$ for straight-backed blades operating at low Mach numbers; (b) function $f(s/e)$ for determining difference between α_2 and $\cos^{-1} \alpha/s$ when $M_t=1.0$ (reproduced from Horlock, 1973).	22
Figure 2.7	Relationship between constant m and stagger angle (reproduced from Horlock, 1973).	24
Figure 2.8	Comparison Between Measured and Calculated Loss and Deviation (reproduced from Sjolander et al., 1993).	27
Figure 2.9	End Losses as a Function of Clearance (reproduced from De Cecco et al., 1997).	29
Figure 3.1	Structured H-mesh used by the Dawes code (nodes in pitchwise x axial x spanwise are 25x75x25)	34
Figure 3.2	Relationship between C_{cp} and $C_{k_{leb}}$ (reproduced from Granville, 1987).	44
Figure 3.3	Upstream Grid Orientation	47

Figure 3.4	Downstream Grid Orientation.	48
Figure 3.5	Grid Section Terminology.	52
Figure 3.6	Grid Geometry for Grid A.	53
Figure 3.7	Grid Geometry for Grid B.	54
Figure 3.8	Grid Geometry for Grid C.	54
Figure 3.9	Grid Geometry for Grid D.	55
Figure 3.10	Grid Geometry for Grid E.	55
Figure 3.11	Grid Geometry for Grid F.	56
Figure 3.12	Predicted Loss Coefficient with Grid Density.	59
Figure 3.13	Meridional Plane of Trailing Edge Damaged Blade.	62
Figure 3.14	Rounded Trailing Edge Viewed from Streamwise Surface at Midspan.	62
Figure 3.15	Effect of Number of Spanwise Nodes on Loss Prediction.	63
Figure 3.16	Composition of the Residual.	64
Figure 3.17	Nature of Residual and Mass Error for a Bad Solution.	65
Figure 3.18	Relationship of Mass Error and Loss Coefficient	66
Figure 3.19	A Typical Solution Convergence History.	68
Figure 3.20	Appropriate Number of Timesteps.	68
Figure 3.21	Mass-Averaged Total Pressure Loss vs. Exit Mach Number (reproduced from Isaacs, 1994).	69
Figure 3.22	Mach Contours with Recommended Values of Coefficients.	71
Figure 3.23	Mach Contours with Altered Values of Coefficients.	71

Figure 4.1	Locations and Roughness Band Widths on the Blade Profile as used by (a) Abbott and (b) Serjak (reproduced from Abbott, 1993; and Serjak, 1995).	74
Figure 4.2	Effect of Location of the Roughness Patch.	75
Figure 4.3	Variation of the Loss Ratio K_L with Roughness Height (k/c) at Design Incidence (roughness patches begin at $x/c=0.05$)	79
Figure 4.4	Effect of Surface Roughness at Off-design incidence for a Fully Roughened Blade.	82
Figure 4.5	Effect of Partially Roughened Surface ($\Delta s/c=0.50$) on the Losses at Off-design incidence.	83
Figure 4.6	Center-line average Roughness Heights versus Service Time (reproduced from Tarada & Suzuki, 1993).	84
Figure 4.7	Variation of Incremental Profile Loss due to Roughness with Mach Number (reproduced from Jouini et al., 1998).	87
Figure 5.1	High Speed Wind Tunnel (reproduced from Jeffries, 1999).	90
Figure 5.2	A Schematic Diagram of the Cascade Test Section (reproduced from Jouini, 1999).	92
Figure 5.3	Summary of Cascade Geometry for HS1A Blade.	95
Figure 5.4	Damaged Blade.	96
Figure 5.5	Geometry of Carleton University Static Probe.	99
Figure 5.6	Geometry of Carleton University Pitot Probe.	100
Figure 5.7	Geometry of Carleton University Three Hole Probe.	101
Figure 5.8	Data Acquisition System (reproduced from Jouini, 1999).	103
Figure 6.1	Kulite Transducer Calibration Curve (Model XCQ-62).	107
Figure 6.2	Cascade Blade Row Measurement Locations (reproduced from Jouini, 1999).	109

Figure 6.3	Downstream Total Pressure Traverses at Design Incidence	112
Figure 6.4	Downstream Total Pressure Traverses at an Incidence of 4.5 Degrees.	113
Figure 6.5	Downstream Flow Angle Traverses at Design Incidence.	114
Figure 6.6	Downstream Static Pressure Traverses at Design Incidence.	115
Figure 6.7	Comparison of Downstream Three Hole and Total Pressure Probe Traverses.	116
Figure 6.8	Flow in the Vicinity of the Total Probe Mouth.	117
Figure 6.9	Axial Velocity Density Ratio Nomenclature (reproduced from Jouini, 1999).	119
Figure 6.10	HS1A Blade Loading at Design Flow Conditions.	122
Figure 6.11	A Comparison Between the Mixed-out and the Pitchwise Variation of the Flow Quantities (reproduced from Jouini, 1999).	124
Figure 7.1	Variation of Total Pressure Losses with Exit Mach Number for Undamaged and Damaged Blades at mid-span.	126
Figure 7.2	Variation of Base Pressure Coefficient with Exit Mach Number for Undamaged and Damaged Blades.	128
Figure 7.3	Effect of Damage Size on the Trailing Edge Loss Component at $M_{2is}=0.95$	129
Figure 7.4	Variation of Losses with Damage Size at Different Exit Mach Numbers	130
Figure 7.5	Effect of Damage Size on Deviation.	132
Figure 7.6	Variation of Incremental Deviation with Damage Size.	133
Figure 8.1	Surface Pressure Distributions of Blades with Different s/c.	139
Figure 8.2	Surface Pressure Distribution of Blades with Similar Leading Edge Pressure Distribution.	139
Figure 8.3	Surface Pressure Distributions of Cascades with Stagger.	141

Figure 8.4	Surface Pressure Distributions of Blades with Varying Maximum Thickness.	142
Figure 8.5	Effect of AVDR on Deviation.	144
Figure 8.6	Profile Generator Parameter List (reproduced from Trigg et al.,1997).	147
Figure 8.7	Gene Cross Over.	153
Figure 8.8	Influence of Number of Generations.	157
Figure 8.9	Evaluation of Solutions with Maximum and Minimum Fitness.	160
Figure 8.10	Evaluation of Solution with Average Fitness.	160
Figure 8.11	Evaluation of Deviation Correlation of Carter & Hughes.	162
Figure 8.12	Evaluation of Correlation with β_1 Excluded.	164
Figure 8.13	Evaluation of Subsonic Deviation Correlation.	167
Figure 8.14	Evaluation of Subsonic Damaged Blade Deviation Correlation (Equation 8.11).	171
Figure 8.15	Variation of Non-dimensional Deviation Parameter R_d with Exit Mach Number.	173
Figure 8.16	Evaluation of Transonic Damaged Blade Deviation Correlation (Equation 8.13).	174
Figure 8.17	Schematic Diagram of Cascade Exit Showing Throat and Suction Surface Curvature Parameters.	177
Figure 8.18	Evaluation of the Predicted Outlet Flow Angle by Ainley & Mathieson.	179
Figure 8.19	Evaluation of the Present Subsonic Deviation Correlation for Cases with Different AVDR Values.	180
Figure 8.20	Predicted Deviation Based on a Linear Variation in AVDR.	181

Figure A.1	The Static Probe and Various Parts	214
Figure A.2	Static Probe Tip	215
Figure A.3	Measuring Tube with Static Holes	216
Figure A.4	Probe Leading End Support	217
Figure A.5	Assembly of Probe Stem	218
Figure A.6	Leading End of Probe Stem	219
Figure A.7	Probe Stem Body	220
Figure A.8	Rear End of Probe Stem	221
Figure A.9	The Probe Stem Cover Ring	222
Figure A.10	The Pitot Probe	223
Figure A.11	The Pitot Probe Measuring Tube	224
Figure B.1	The Three Hole Probe	226
Figure B.2	The Three Hole Probe Stem	227
Figure B.3	The Three Measuring Tubes	228
Figure B.4	Arrangement of Measuring Tubes in Slot and Solder	229
Figure B.5	Rear End of Three Hole Probe Stem	230

List of Tables

Table 3.1	Effect of Grid Orientation and Inlet and Exit Plane.	49
Table 3.2	Parameters for Determination of Grid Quality.	56
Table 3.3	Grid Refinement.	58
Table 3.4	Calculation of Grid Convergence Index	61
Table 4.1	Determination of Loss for 70% Blade Suction Surface.	76
Table 4.2	Ratios of Suction to Pressure Surface Incremental Losses.	77
Table 4.3	Ratios of k_s/k for various values of the spacing parameter λ	79
Table 4.4	Design-point Loss Ratio (K_L) at Different Roughness Conditions for the F404 Engine (roughness patches starting at $x/c=0.05$).	86
Table 5.1	Blade Aerodynamic Parameters for Design Flow Condition.	94
Table 6.1	Estimates of Uncertainties Associated with the Measurements.	105
Table 8.1	Experimental and Computed Deviation for Subsonic Flow in Axial Turbines.	182
Table 8.2	Range of Blade Parameters.	186
Table 8.3	Influence of Population Size for Genetic Algorithm Runs.	156
Table 8.4	Comparison of Different Solutions.	158
Table 8.5	Determination of Correlation Coefficients.	166
Table 8.6	Experimental and Computational Deviation for Damaged Blades.	187
Table 8.7	Deviation Data for High Speed Flow.	189
Table 8.8	Predicted Outlet Flow Angle by Ainley & Mathieson Correlation.	197
Table 8.9	Experimental Subsonic Deviation for Cases with Different Values of AVDR.	198

Nomenclature

English Symbols

AVDR	$= \rho_2 V_{x2} / \rho_1 V_{x1}$	= axial velocity density ratio
A^+		= constant used in the van-Driest damping function (=26)
c		= blade chord
C_p	$= (P - P_{CL}) / \frac{1}{2} \rho V_{CL}^2$	= static-pressure coefficient
C_{bp}		= base pressure coefficient, $C_{bp} = (P_b - P_{s2})/q_2$
C_x		= blade axial chord length
D		= diameter; damage size as percent of chord penetration (e.g. 0.1,0.2)
d		= leading edge diameter
e		= suction side radius of curvature from throat to trailing edge
E		= total internal energy
H		= total enthalpy
h		= span; enthalpy
$\hat{i}_r, \hat{i}_\theta, \hat{i}_x$		= unit vectors in r, θ, x directions
i	$= \alpha_1 - \alpha_{ides}$	= incidence
i_{des}		= design incidence
o		= throat opening
I		= rothalpy
I, J, K		= indices for computational grid
k		= actual roughness height
k_s		= standard-sand roughness height
K_L		= correction factor for influence of roughness on profile loss (Equation 4.2)
K_M		= correction factor for influence of Mach on the incremental loss due to roughness (Equation 4.5)
L		= lift force

l	= mixing length
M	= Mach number
P_s	= static pressure
P_o	= total pressure
q	= dynamic pressure, $q = \frac{1}{2}\rho V^2$
r	= radial coordinate
Re	= Reynolds number based on blade chord and downstream flow quantities, $Re = \rho V c / \mu$
s	= blade pitch or spacing
Δs	= surface length of roughness patch
t_m	= blade maximum thickness
T	= absolute temperature
u, v, w	= components of velocity in x, y, z directions
u^+	= non-dimensional velocity, $u^+ = u/u_\tau$
u_τ	= skin-friction velocity, $u_\tau = \sqrt{(\tau_{wall}/\rho)}$
V	= velocity; volume
W_e	= leading edge wedge angle
x, y, z	= coordinates in the axial, pitchwise and spanwise directions
X	= non-dimensional roughness height
y	= normal distance from the wall
y^+	= non-dimensional distance from the wall, $y^+ = y u_\tau / \nu$
Y	= loss coefficient, $Y = (P_{01} - P_{02}) / (P_{02} - P_{s2})$
ΔY_R	= incremental loss due to roughness

Greek Symbols

α	= flow angle, measured from the axial direction
β	= blade metal angle, measured from the axial direction
Γ	= circulation
γ	= specific heat ratio
δ	= flow deviation
$\Delta\delta$	= incremental flow deviation
θ_u	= unguided turning angle

κ	= von Karman constant (=0.41)
λ	= roughness spacing parameter (area of roughened blade surface / total frontal area of roughness elements)
μ	=dynamic viscosity
ν	= kinematic viscosity
ξ	= blade stagger angle
ρ	= fluid density
τ_{wall}	= wall shear stress, $\tau_{\text{wall}} = \mu (du/dy)_{\text{wall}}$
χ	= off-design incidence parameter
Ω	= rotational speed
ω	= vorticity

Subscripts

1,2	= cascade inlet and outlet
CL	= centerline value
CLA	= centerline average
des	= design
o	= stagnation condition
PS	= pressure surface; smooth surface profile loss
PR	= rough surface profile loss
R	= rough
ref	= reference value
r, θ , x	= components in radial, tangential and axial directions
S	= smooth
SS	= suction surface;

Abbreviations

AVDR	= axial velocity density ratio
CFL	= Courant-Friedrichs-Lewy number
EHM	= engine health monitoring
GCI	= grid convergence index

1 Introduction

During the past years gas turbines have received considerable attention regarding improvements in design and efficiency. The increasing cost of the manufacture and maintenance of gas turbines has also encouraged the development of more effective engine health monitoring (EHM) systems to increase the life of the engines and decrease the maintenance costs. Currently most civilian and military users of gas turbines routinely use engine health monitoring systems. Gas turbine engines used for aircraft propulsion particularly require close monitoring due to the serious consequences of engine failure. The EHM systems continuously monitor a number of engine performance parameters and activate cautionary signals when necessary (e.g. Cue & Muir, 1991). Also, the systems routinely record engine condition data that are analyzed by maintenance personnel to detect gradual or sudden deterioration of the engine. As a result, repair can be initiated in an orderly way before the problem becomes severe or, for example, leads to an in-flight shut down of an engine. Though engine operation is greatly enhanced by this on-condition maintenance policy, there still remains the problem of promptly identifying the specific cause or nature of a deterioration from the available data. In fact, considerable maintenance time is typically required simply to diagnose the specific nature of the problem. At the present stage of development, engine health monitoring (EHM) systems can be used only to detect the presence of a problem and the general area of the engine where it exists. To overcome this limitation, the goal of future EHM systems will be to determine and distinguish various types of damage and thus minimize the required diagnostic time.

Gradual or abrupt deterioration of engine performance can occur due to various forms of in-service damage. For example, roughening of blade surfaces can occur due to fouling, erosion or corrosion. Also, damage to blade tips, leading or trailing edges can occur due to foreign-object ingestion, over-temperatures, vibration problems, or blade cooling problems. One starting point

for providing the EHM systems with the ability to distinguish different types of damage is to develop an understanding, at the blade level, of the aerothermodynamic effects of each of the damage modes mentioned. In particular, a sound understanding must be obtained about the effect of a particular damage mode on the loss characteristics and flow-turning ability of the blades. From such information, correlations may be developed and used to simulate the performance of a stage with specified damage. The characteristics of the damage stage can then be incorporated in an engine model to predict the effect of a particular type of damage on the performance of the engine as a whole.

Some of this information will be of interest to the manufacturer as well as the user of gas turbine engines. For example, data are available for the range of surface roughening which can be expected in different operating environments after various lengths of time in service. In the turbine section of the engine, these changes in surface finish affect both the losses and the convective heat transfer. Thus, an understanding of the aerothermodynamic effects of typical levels of in-service roughening will allow the manufacturer to predict (and guarantee) more reliably the engine performance retention over time, as well as the life of the turbine blades.

A joint program involving GasTOPS Ltd. and Carleton University has been conducted to investigate the effects of specific types of in-service damage on the performance of axial-flow turbines. The Carleton University contribution has focused on studies of individual damaged blades and the development of prediction methods and correlations for estimating the effects of the damage. The present thesis is a contribution to this program.

The present work focused on two aspects of blade damage: surface roughening and trailing-edge damage.

Deterioration of engine performance due to blade roughening is one of the most frequently diagnosed forms of damage to gas turbines. As part of the program at Carleton, Kind et al.(1998) examined experimentally the effect of surface roughness on profile losses for turbine blades at subsonic flow conditions. Chapter 4 describes the development of correlations and graphs based on the data collected for design and off-design conditions by Kind et al. These correlations may be easily implemented in meanline or throughflow predictions of turbine stage performance.

Sjolander et al. (1993) have pioneered the work on damaged turbine blade performance by making a fundamental blade level study of the effects of trailing-edge damage. This study was carried out both experimentally and computationally at low-speed flow conditions and it produced encouraging results. Gas turbine blades today operate in the high subsonic or transonic flow regime and the present undertaking pursued the work further by studying the influence of high speed flow past a blade with trailing-edge damage.

The investigation was carried out again both experimentally and computationally. The computations were carried out using the three-dimensional Navier-Stokes solver developed by Dawes (1988). A description of the code is provided in Chapter 3. A high-speed wind tunnel was used to gather experimental data for the present work. Chapters 5 and 6 provides a brief description of the wind tunnel, the cascade test section and the data acquisition system. Trailing-edge damage was modeled by a semi-circular cutout located at the midspan of the blade trailing edge. Two different damage sizes and several exit Mach numbers were examined. The experimental and computational results, presented in Chapter 7, provide insights into the physics of the flow, including the changes in losses and deviation due to damage.

Chapter 8 is devoted to the development of deviation correlations for both undamaged and damaged blades based on the data collected in the present study as well as from other researchers. The available deviation correlations for axial flow turbine blades proved to be inadequate for the purposes of the present study and therefore, with the objective to develop an improved correlation, an investigation was made to re-examine the factors that influence subsonic deviation. This study provides a broader understanding of the mechanism of flow deviation in axial turbines. The new improved deviation correlation will be of particular interest to turbine designers and researchers since empirical deviation correlations continue to play an important role in the early stages of blade design and there is an ongoing need to review and improve these correlations.

Finally, correlations were developed for the prediction of deviation for damaged blades at both low-speed and high-speed flows. These correlations can be used to simulate engine performance due to damage.

Lastly, conclusions and recommendations for future work are presented in Chapter 9.

2 Review of Literature

2.1 Introduction

Studying the effects of damaged blades of turbines is an immature field and very little directly related research has been performed to date. Information available in the open literature regarding the effects of turbine blade damage is mostly due to the fundamental research performed by Sjolander et al. (1993) and De Cecco et al. (1995). These studies are the results of ongoing efforts to improve the performance of Engine Health Monitoring (EHM) systems. This chapter concludes with a review of work related to the aerodynamic performance of turbine blades degraded by effects such as, surface roughening, trailing-edge damage and tip damage.

As mentioned in the previous chapter, the present research is primarily directed towards the understanding of flow characteristics of blades with damage.. It is believed that the parameters that affect loss and flow turning ability of an undamaged blade will also largely determine the expected changes in loss and flow turning ability due to damage. For example, the correlation for deviation (the difference between the metal angle and flow angle at the trailing edge) for an undamaged blade may be extended to predict deviation for a blade with trailing-edge damage. It is therefore pertinent to look into the loss and deviation models for undamaged turbines before attempting to discuss the performance of damaged ones.

The following sections present a review of correlations for predicting the performance of undamaged turbines. These correlations form the basis for developing correlations for the additional losses and deviation due to blade damage. This discussion is followed (in Section 2.4) by a review of studies related to the aerodynamic effects of turbine blade damage.

2.2 Loss Correlations for Undamaged Blades

2.2.1 General Approach

The flow through the turbine blade channel is so complex that both analytical and experimental treatment of the flow present a formidable challenge. Much effort has been devoted to experiments to gain physical insights into the nature of the flows and losses. Based on simplifying assumptions, researchers like Soderberg (1949), Ainley & Mathieson (1951), and Hawthorne (1955) formulated loss prediction methods which have formed the basis of subsequent turbine design. In recent years much attention is being given to the study of transonic flows in turbines due to the high efficiency gained at high inlet temperature and overall pressure ratio. The number of loss correlations relating to turbine performance is extensive. During the last few decades turbine cascade tests have led to a notable increase in the number of proposed loss correlations (e.g. Dunham & Came, 1970; Craig & Cox, 1971; Traupel, 1977; Hultsch & Sauer, 1978; Kacker & Okapuu, 1982; Martelli & Boretti, 1987). For purposes of the present investigation, mainly those correlations which have proved useful for extension to include the effects of damage are reviewed.

Losses occur when useful mechanical energy is degraded to internal energy through irreversible processes such as the viscous effects in boundary layers and mixing processes. For adiabatic flows, entropy production is a direct measure of these irreversibilities, and consequently, a measure of loss (Denton, 1993). Unfortunately entropy is a rather abstract quantity that cannot be measured directly and therefore, is rarely used to represent losses in turbomachines. However, the total-pressure loss coefficient is a more convenient loss parameter which gives an indirect measure of the entropy production for stationary blade rows. This parameter is easily obtained from cascade measurements. The loss coefficient, γ , in a turbine is defined as the loss in the mass-averaged total pressure through the passage normalized with the outlet dynamic pressure:

$$\gamma = \frac{P_{01} - P_{02}}{P_{02} - P_2}$$

where, P_{01} = mass averaged total pressure at inlet

- P_{02} = mass averaged total pressure at outlet
 P_2 = static pressure at outlet

It is assumed that the overall loss in a machine blade row is the sum of a number of basic loss components. It is usually assumed that these can be evaluated independently and then combined linearly to obtain the total loss. The basic loss components are usually listed as follows:

1. **Profile loss (Y_p):** loss due to the boundary layer growth on the blade pressure and suction surface in a uniform two-dimensional flow plus the trailing-edge loss.
2. **Secondary loss (Y_s):** loss due to viscous effects on the endwalls within the blade row.
3. **Tip clearance loss (Y_{tp}):** loss due to the leakage of fluid around the shrouded or unshrouded blade ends.

All the correlations express losses as a function of key geometric and flow parameters. Beginning with Ainley & Mathieson (1951), the following sections give a description of recent developments in the correlation of each loss component.

2.2.2 Profile Loss

The two-dimensional profile loss correlation due to Ainley & Mathieson is based on cascade tests made on a variety of blade sections designed prior to 1950. The tests were performed at a Reynolds number of 2×10^5 (based on chord and outlet velocity) and a Mach number less than 0.6. The profile losses are correlated with the blade inlet angle (β_1), pitch-to-chord ratio (s/c), maximum thickness-to-chord ratio (t_m/c), and fluid outlet angle (α_2). These are represented by a series of graphs of profile loss coefficient (Y_p) versus pitch-to-chord ratio (s/c) for nozzle blades ($\beta_1 = 0$) and impulse blades ($\beta_1 = \alpha_2$) as shown in Figure 2.1. Profile loss coefficients of blades having intermediate angles can be interpolated using the following expression:

Figure 2.1 Profile Loss Coefficients for Conventional Section Blades at Zero Incidence (reproduced from Ainley & Mathieson, 1951)

$$Y_p = \left\{ Y_{p(\beta_1=0)} + \left(\frac{\beta_1}{\alpha_2} \right)^2 \left[Y_{p(\beta_1=\alpha_2)} - Y_{p(\beta_1=0)} \right] \right\} \left(\frac{t_m/c}{0.2} \right)^{\beta_1/\alpha_2}$$

where $Y_{p(\beta_1=0)}$ is the profile loss coefficient of blades having $\beta_1 = 0$ and the same α_2 and s/c as the actual blade and $Y_{p(\beta_1=\alpha_2)}$ is the profile loss coefficient of a blade having $\beta_1 = \alpha_2$ and same α_2 and s/c as the actual blade. The above equation includes a correction for the effect of thickness-to-chord ratio and is valid in the range $0.15 \leq t_m/c \leq 0.25$. If the blade has a t_m/c outside this range, the loss should be taken as the value resulting from $t_m/c = 0.25$ or 0.15 .

After 20 years of turbine development, Ainley & Mathieson's (AM) correlation proved inadequate to predict performance of modern turbines, particularly small ones. Dunham & Came (1970) then recommended a Reynolds number correction factor to the AM correlation to compensate for the discrepancy between the original correlation's prediction and recent data. What is known as the Ainley, Mathieson, Dunham & Came (AM-DC) correlation can be written as follows:

$$Y_p = \left\{ Y_{p(\beta_1=0)} + \left(\frac{\beta_1}{\alpha_2} \right)^2 \left[Y_{p(\beta_1=\alpha_2)} - Y_{p(\beta_1=0)} \right] \right\} \left(\frac{t_m/c}{0.2} \right)^{\beta_1/\alpha_2} \left(\frac{R_e}{2 \times 10^5} \right)^{-0.02}$$

Since 1970 advances in aerodynamic design of turbines led to a reduction in profile losses and so Kacker & Okapuu (1982) suggest that a factor of 2/3 should be applied to the loss correlation to reflect recent trends in turbine performance. They also showed that flow compressibility can affect Y_p even in subsonic flows, by affecting the flow acceleration within blade channels. Subsequently they modified the AM-DC correlation to predict turbine efficiency of recent design by introducing a compressibility factor. Their correlation for subsonic operation of a cascade of blades is represented by:

$$Y_p = 0.914 \left(\frac{2}{3} Y_{p,AMDC} K_p + Y_{shock} \right)$$

where $Y_{p,AMDC}$ = AM-DC profile loss prediction and K_p is a correction for exit Mach number and channel acceleration,

$$K_p = 1 - K_2(1 - K_1)$$

and K_1 and K_2 are given by

$$K_1 = 1 - 1.25(M_2 - 0.2), \quad \text{for } M_2 > 0.2$$

$$K_2 = (M_1 / M_2)^2$$

$$Y_{shock} = \frac{1 - \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma - 1}}}{1 - \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{\gamma}{\gamma - 1}}}$$

The correction for Reynolds number is

$$\begin{aligned} f_{(Re)} &= \left(\frac{Re}{2 \times 10^5}\right)^{-0.4} & \text{for } Re \leq 2 \times 10^5 \\ &= 1.0 & \text{for } 2 \times 10^5 < Re < 10^6 \\ &= \left(\frac{Re}{10^6}\right)^{-0.2} & \text{for } Re > 10^6 \end{aligned}$$

So far improvements to Ainley & Mathieson's profile loss correlation have been confined to design conditions. Based on their off-design correlations and recent cascade data, Moustapha et al. (1990) proposed a new prediction method at off-design conditions which included the effects of leading edge diameter on profile losses. In the Ainley & Mathieson's method the stalling incidence is shown to be independent of the leading edge shape. But Muktarov & Krichakin (1969) demonstrated that it affected the incidence characteristics of turbine blades. This is mainly because

stalling of a blade is due to boundary layer separation, which results from peaks in the velocity distribution in the leading edge region. The formation of these peaks is dependent, among other factors, on the leading edge geometry. Benner et al. (1997) further improved the correlation developed by Moustapha et al. by including the leading-edge wedge angle as an additional and more influential correlating parameter. It was noted that the off-design loss behaviour of the blade is influenced by the magnitude of the discontinuity in curvature at the points where the leading-edge circle meets the rest of the blade profile. The wedge angle appeared to be an approximate and convenient measure of the discontinuity in curvature at those blend points. Taking into account the influence of leading edge wedge angle (We) and leading edge diameter, defined as diameter-to-pitch ratio (d/s), Benner et al. derived a functional relationship that produced a significantly improved correlation from recent experimental data, as shown in Figure 2.2. The improved correlation is represented according to the following equation:

$$\Delta\Phi_p^2 = a_8\chi^8 + a_7\chi^7 + a_6\chi^6 + a_5\chi^5 + a_4\chi^4 + a_3\chi^3 + a_2\chi^2 + a_1\chi^1$$

where,

$$\chi = \left(\frac{d}{s}\right)^{-0.05} (We)^{-0.2} \left(\frac{\cos\beta_1}{\cos\beta_2}\right)^{-1.4} (\alpha_1 - \alpha_{1(des)})$$

$$\begin{aligned} a_8 &= +3.711 \times 10^{-7}, & a_7 &= -5.318 \times 10^{-6}, \\ a_6 &= +1.106 \times 10^{-5}, & a_5 &= +9.017 \times 10^{-5}, \\ a_4 &= -1.542 \times 10^{-4}, & a_3 &= -2.506 \times 10^{-4}, \\ a_2 &= +1.327 \times 10^{-3}, & a_1 &= -6.149 \times 10^{-5}, \end{aligned}$$

for $\chi \geq 0$, and

$$\Delta\Phi_p^2 = 1.358 \times 10^{-4} \chi^2 - 8.720 \times 10^{-4} \chi$$

for $\chi < 0$.

Figure 2.2 Profile Incidence Loss Correlation (reproduced from Benner et al., 1997)

The kinetic energy loss coefficient ϕ^2 is defined as:

$$\phi^2 = 1 - V_2^2/V_{2is}^2$$

where V_2 is the actual outlet velocity and V_{2is} is the isentropic outlet velocity. The pressure loss coefficient (Y) is converted to kinetic energy loss coefficient by means of the following equation:

$$Y = \frac{\left[1 - \frac{\gamma-1}{2} M_2^2 \left(\frac{1}{\phi^2} - 1 \right) \right]^{-\frac{\gamma}{\gamma-1}} - 1}{1 - \left(1 + \frac{\gamma-1}{2} M_2^2 \right)^{-\frac{\gamma}{\gamma-1}}}$$

where M_2 is the Mach number at the outlet of the blade row.

In the above correlation the effect of compressibility and Reynolds number were deliberately neglected due to the lack of incidence loss data related to the aforementioned factors. However, the authors felt that it would be sufficient to include their effects, by using the corrections of Kacker & Okapuu (1982).

2.2.3 Secondary Loss

The secondary losses arise from complex three-dimensional flows set up as a result of the rolling up of the endwall boundary layer, and associated streamwise vorticity and separation of the flow. This interaction between the flow through the blade passage and the end wall boundary layer results in a total pressure loss which is often a significant fraction of the total loss for low aspect ratio turbines. Ainley & Mathieson (1951) simplified the complex situation by taking into account the effect of convergence and blade loading only, and proposed the following secondary loss correlation:

$$Y_s = \lambda Z$$

where,

$$\lambda = f\left(\frac{A_2^2/A_1^2}{1+D_H/D_T}\right)$$

$$Z = \frac{C_L \cos^2 \alpha_2}{s/c \cos^3 \alpha_m}$$

$$\frac{C_L}{s/c} = 2(\tan \alpha_1 + \tan \alpha_2) \cos \alpha_m,$$

$$\alpha_m = \tan^{-1} \left[\frac{1}{2} (\tan \alpha_1 + \tan \alpha_2) \right]$$

where A_1 and A_2 = areas normal to the entering and leaving flow respectively
 D_H and D_T = hub diameter and tip diameter respectively
 C_L = lift coefficient
 α_1 and α_2 = inlet and outlet flow angle respectively
 α_m = mean flow angle

The function $Z = f(\alpha_1, \alpha_2)$ is shown in Figure 2.3 and it can be seen that the loading factor Z depends on α_1 and α_2 and not on the pitch-to-chord ratio.

Subsequently Dunham and Came (1970) found that this equation was not correct for blades of low aspect ratios, as in small turbines. They suggested that the secondary loss depends on the blade loading, the blade shape and thickness of the end wall boundary layer. With the introduction of a new blade loading parameter which better fitted available cascade data, they modified Ainley's correlation to include the influence of an aspect ratio (h/c) and end wall boundary layer thickness (δ_1/c). The Ainley, Mathieson, Dunham and Came (AM-DC) secondary loss correlation is of the form

$$Y_s = \left(\frac{c}{h} \right) \left(\frac{\cos \alpha_2}{\cos \beta_1} \right) Z f \left(\frac{\delta_1}{c} \right)$$

Figure 2.3 The Loading Factor Z as a Function of α_1 and α_2 (reproduced from Sieverding, 1985)

However, from a comparison with overall efficiency data of 25 turbines Dunham & Came adopted a single numerical constant in place of the function of wall boundary layer thickness: that is $f(\delta_1/c)$ is replaced by 0.0334.

The above correlation shows that Y_s varies as the reciprocal of the blade aspect ratio. However, Kacker & Okapuu (1982) showed that for smaller aspect ratios the Dunham & Came method overpredicted the losses and therefore they included a restriction on the influence of aspect ratio (h/c) to the secondary loss term as follows:

$$Y_s = 0.0334 f_{(AR)} \left(\frac{\cos \alpha_2}{\cos \beta_1} \right) Z$$

where

$$\begin{aligned} f_{(AR)} &= \frac{1 - 0.25\sqrt{2-h/c}}{h/c} \quad \text{for } h/c \leq 2, \\ &= \frac{1}{h/c} \quad \text{for } h/c > 2 \end{aligned}$$

In addition they suggest that compressibility also has an effect on the acceleration of flow next to the endwalls, and hence the endwall losses. To account for this Kacker & Okapuu applied a subsonic Mach number correction factor as follows:

$$K_s = 1 - K_3 \left[\left(\frac{M_1}{M_2} \right)^2 (1 - K_1) \right]$$

where $K_3 = (C_x/h)^2$ (where C_x is axial chord) and K_1 decreases linearly from $K_1 = 1$ at $M_2 = 0.2$ to $K_1 = 0$ at $M_2 = 0$.

Kacker & Okapuu introduced improvements only for the design point predictions. As with profile loss, Moustapha et al. (1990) supplement Kacker & Okapuu by giving the off-design

behaviour of turbines. They proposed a secondary loss correlation taking into account channel convergence and leading edge diameter for off-design conditions. They felt that the leading edge diameter had an influence on the secondary losses by affecting the developing horseshoe vortex at the leading edge. As shown in the Figure 2.4, their relation provides a good collapse of experimental data and it can be mathematically expressed as follows:

$$\left(\frac{Y}{Y_{(des)}} \right)_s = \exp(0.9\chi'') + 13\chi''^2 + 400\chi''^4$$

$$\text{for } 0.3 > \chi'' > 0$$

$$\left(\frac{Y}{Y_{(des)}} \right)_s = \exp(0.9\chi'')$$

$$\text{for } 0 > \chi'' > -0.4$$

where

$$\chi'' = \frac{\alpha_1 - \beta_1}{180 - (\beta_1 + \beta_2)} \left(\frac{\cos\beta_1}{\cos\beta_2} \right)^{-1.5} \left(\frac{d}{c} \right)^{-0.3}$$

2.2.4 Tip Clearance Loss

The tip clearance loss arises due to the required gap between the end of a rotor blade and the adjacent wall. For the two types of blades generally encountered, i.e. unshrouded and shrouded blades, the loss generating mechanisms are quite different. In the case of shrouded rotor blades, the clearance flow is separated from the flow through the blading. Losses arise then from energy dissipation across the shroud and from mixing of the clearance flow with the main flow downstream of the blade row. For unshrouded rotor blades, a clear separation between secondary and clearance losses is not possible. In this case the secondary loss obtained at zero clearance is generally assumed to remain unaltered as the clearance gap is increased. The tip clearance loss is then taken as the difference between total losses with and without clearance. However, it is obvious that secondary loss will be affected somewhat by the clearance.

Figure 2.4 Secondary Incidence Loss (reproduced from Moustapha et al., 1990)

Ainley & Mathieson (1951) established that the tip clearance loss coefficient Y_{tip} depends on the blade loading and the size and nature of the clearance. They assumed a linear dependence of tip clearance loss on clearance size (τ) and it is expressed as follows:

$$Y_{tip} = C_{D_t} \left(\frac{c}{s} \right) \left(\frac{\cos^2 \alpha_2}{\cos^3 \alpha_m} \right)$$

$$C_{D_t} = B C_L^2 \left(\frac{\tau}{s} \right) \left(\frac{c}{h} \right)$$

where B is 0.5 for an unshrouded blade and 0.25 for a shrouded blade.

Dunham and Came (1970) modified Ainley & Mathieson's model in the light of some later cascade data, which suggested a nonlinear variation of the tip leakage losses with clearance size. The resultant expression is as follows:

$$Y_{tip} = B \frac{c}{h} \left(\frac{\tau}{c} \right)^{0.78} Z$$

where Z is Ainley's blade loading parameter and B = 0.47 for unshrouded and 0.37 for shrouded blades respectively.

However, Kacker & Okapuu (1982) confirmed that this model overestimated the losses of recent turbines and Yaras & Sjolander (1992) came to the same conclusion. Yaras & Sjolander made a detailed study of tip clearance flow and arrived at an improved tip leakage loss model for unshrouded blades consistent with available data. They essentially made a breakdown of the losses as shown in Figure 2.5 and formulated expressions for each component in accordance with the respective loss generating mechanisms. They found that the losses increased substantially downstream of the trailing edge but that most of the additional loss production occurred over about the first axial chord length. The losses inside the gap itself were found to be a relatively small fraction of the total losses. The loss identifiably associated with the tip-leakage flow was referred to as 'end loss' and was defined as the total loss that occurs in the endwall region, apart from the profile loss.

Figure 2.5 Schematic Breakdown of the Losses in the End Region (excluding profile losses)(reproduced from Yaras & Sjolander, 1990)

In the Yaras & Sjolander approach the total tip-leakage loss is given by

$$(Y_{\text{end}})_{\text{TP}} = Y_{\text{gap}} + k_s Y_{s,0} + Y_{\text{tip}}$$

where $Y_{s,0}$ is the secondary loss coefficient at the tip for zero clearance. The factor k_s is the fraction of the secondary loss that remains identifiably associated with a secondary flow structure, principally the passage vortex. In Ainley & Mathieson's loss model k_s is implicitly taken to be 1.0 but Yaras & Sjolander speculated that k_s would be much less than 1.0. Chan et al. (1994) found that the value of k_s was generally on the order of 0.4 - 0.6 for clearances below 5.5% of chord. The losses generated during the mixing of the tip-leakage flow with the mainstream is given by:

$$Y_{\text{tip}} = 2 K_E \sigma \frac{\tau}{h} C_D \frac{\cos^2(\alpha_2)}{\cos^3(\alpha_m)} C_L^{1.5}$$

where $\sigma = c/s$ is the solidity of the blade row, C_D is the discharge coefficient; and $K_E = 0.566$ for a front or aft-loaded blade and 0.5 for a mid-loaded blade. The losses generated in the tip gap are given by the following expression:

$$Y_{\text{gap}} = C K_G \frac{\sigma C_D C_L^{\frac{1}{2}}}{\cos \alpha_m} \left(\frac{c}{h} \right)$$

where $K_G = 0.943$ for front or aft-loaded blades and 1.0 for mid-loaded blades and $C = 0.007$.

2.3 Flow Deviation Correlations

Unlike the numerous loss correlations available for predicting turbine blade performance, there seems to be a lack of sufficiently accurate deviation correlations. The prediction of blade outlet flow angle is an important step in the preliminary design phase of a turbine. The estimation of optimum work output and inlet flow angle of successive stages of a turbine are intrinsically related to the accurate prediction of the deviation, the difference between the metal angle and the flow angle at the trailing edge. Unfortunately, relatively little attention seems to have been paid to this aspect of turbine blade design.

The lack of agreement among several published correlations for turbine blade deviation was reported by Riess et al. (1986). The authors concluded that most of the correlations were valid only for particular profile forms and narrow ranges of cascade geometry and operating conditions.

The most reasonable among the alternatives and perhaps the most widely used is that due to Ainley & Mathieson (1951). The correlation shows the outlet flow angle to be a function of the gauge angle, $\cos^{-1}(o/s)$, where o is the throat opening. The Ainley & Mathieson correlation is usually provided graphically for subsonic operating conditions, as shown in Figure 2.6(a) for conventional 'straight-backed' blades. They suggest that at the choking condition (throat Mach number, $M_t = 1.0$), the outlet flow angle α_2 is equal to the gauge angle and also that α_2 varies linearly with throat Mach number between the low speed condition ($M_t = 0.5$) and the choking condition ($M_t = 1.0$). Corrections are provided for 'curved-back' blades with radius of curvature e between throat and suction surface trailing edge shown in Figure 2.6(b). At $M_t = 0.5$ the outlet flow angle is reduced by $4(s/e)$ from the value obtained from Figure 2.6(a). At the choking condition the outlet flow angle for 'curved-back' blade is reduced by $f \sin^{-1} o/s$, where f is given in Figure 2.6(b). They further suggest that for 'curved-back' blading the outlet flow angle also varies linearly with throat Mach number between low speed condition and choking condition.

Wilson (1984) has translated these graphical representations into three analytical expressions for different Mach number operating conditions. These are as follows:

Figure 2.6 (a) Relationship between gas outlet angles and $\cos^{-1} o/s$ for straight-backed blades operating at low Mach numbers; (b) function $f(s/e)$ for determining difference between α_2 and $\cos^{-1} o/s$ when $M_t=1.0$ (reproduced from Horlock, 1973)

1. Predicted outlet angle for exit Mach numbers up to 0.5:

$$|\alpha_2|_{0 < M_t < 0.5} = \left[\frac{7}{6} \left(\cos^{-1} \left(\frac{o}{s} \right) - 10^\circ \right) + 4^\circ \left(\frac{s}{e} \right) \right],$$

where, o = diameter of throat opening,
 s = blade pitch,
 e = radius of curvature of blade suction surface downstream of throat,
 M_t = Mach number at throat.

2. Predicted outlet flow angle for throat Mach number between 0.5 and 1.0:

$$|\alpha_2|_{0.5 < M_t < 1.0} = |\alpha_2|_{0 < M_t < 0.5} - (2 M_t - 1) (|\alpha_2|_{0 < M_t < 0.5} - |\alpha_2|_{M_t = 1.0})$$

3. Predicted outlet flow angle for turbines with sonic flow at the throat ($M_t = 1.0$):

$$|\alpha_2|_{M_t = 1.0} = \left| \cos^{-1} \left(\frac{o}{s} \right) - \left(\frac{s}{e} \right)^{(1.786 + 4.128 (s/e))} \left| \sin^{-1} \left(\frac{o}{s} \right) \right| \right|$$

What is known as the Cosine Rule, has been extensively used in steam turbine design and is given by $\alpha_2 = \cos^{-1}(o/s)$. This simple empirical rule is frequently used to obtain a first approximation of outlet flow angle. Though this gives a good approximation to the outlet flow angle for gas turbines when the exit Mach number is close to unity, substantial variations have been observed between predicted values and measured values of flow angle for low-speed flow cases.

As already evident, the above correlations provided flow outlet angle rather than deviation. Another widely used correlation that gives the deviation as a function of cascade geometry is that due to Carter & Hughes (1950). They correlate the deviation with the blade stagger angle, blade camber, and the solidity. The correlation is usually expressed as:

$$\delta = m \theta \left(\frac{s}{c} \right)^n$$

where δ = deviation
 θ = blade camber
 s/c = blade spacing-to-chord ratio

The multiplier m is a function of the stagger angle (ξ) and the shape of the blade camber line as indicated in Figure 2.7. It can be seen that the value of m depends on whether the blade is of parabolic arc camber line or circular arc camber line. The Carter & Hughes correlation was originally developed for compressors. The authors recommended a value of $\frac{1}{2}$ for the power n for general compressor blades and a value of 1 for inlet guide vanes, which are essentially nozzle vanes giving accelerating flow. When applying the correlation to turbine blade rows it is appropriate to use $n=1$ since they have accelerating flows also (e.g. see Dixon, 1989).

Figure 2.7 Relationship between constant m and stagger angle (reproduced from Horlock, 1973)

2.4 Turbine Blade Damage

2.4.1 Engine Health Monitoring

Engine health monitoring (EHM) is an evolving technology and in recent years the advancement of modern computer technology is allowing much more complex EHM systems to be developed. However, as already mentioned, there is very little published material regarding the ongoing development of EHM systems. A review of some basic methods employed to monitor engine components can be found in Schofield et al. (1988) and Turyk (1989). Cue & Muir (1991) also sheds some light on the successful maintenance program using EHM Systems on the Canadian Forces CF-18 aircraft. Further discussion of Engine Health Monitoring can be found in Klein (1993) and Isaacs (1994).

The ultimate goal of EHM is to be able to predict not only when engine performance is suffering, but to what degree and why. With this end in view some fundamental research has been accomplished in the area of gas turbine performance with degraded components under a joint program involving GasTOPS Ltd. and Carleton University. A brief summary of this work is provided in the following sections.

2.4.2 Surface Roughening Damage

Kind et al. (1998) examined experimentally the effect of surface roughness on profile losses for turbine blades at subsonic flow conditions. They investigated different roughness configurations by varying the roughness height, the location of the roughness strip, and the width of the roughness strip on the blade suction and pressure surface. All three of these parameters were found to have a substantial effect on the losses. In general, increasing the roughness height and the width of the roughness strip on the suction surface caused a significant increase in the losses but only a small decrease in the outlet flow angle. In fact, suction surface roughness strips located at the leading 25% of chord produced significant increase in the losses while the flow was relatively insensitive to roughness near the trailing edge.

Jouini et al. (1998) developed correlations for the effects of surface roughness on profile losses for both subsonic and transonic operating conditions of turbines. The subsonic correlation is based on the experimental data generated by Kind et al. (1998) and the transonic correlation is based on computational data generated by Navier-Stokes computations with surface roughness. The correlations are intended for use in meanline or throughflow predictions of turbine stage performance. The correlations account for the effects of the roughness height as well as the location and extent of the roughness patch on the blade surface. A detailed discussion of the correlations is included in Chapter 4.

2.4.3 Trailing-Edge Damage

Sjolander et al. (1993) examined the effect of trailing-edge damage on the aerodynamics of turbine cascade blades and found the losses are not very influenced by the damage in a low speed cascade. However, significant underturning of the flow in the region of the cutout is observed (see Figure 2.8). Semi-circular trailing-edge cutouts with diameters of 15 and 25 percent of chord were used for the experiment. The measurements were made at a Reynolds number of 3×10^5 and turbulence intensity of 0.3 percent. The losses are slightly lower in the cutout region and also at higher values of incidence with respect to the undamaged blade. Computation of the flow using the three dimensional Navier-Stokes code of Dawes (1988) showed a negligible change in overall losses with the presence of the damage. The apparent reasons for this unusual outcome is thought to be due to a reduction of skin friction contribution to the observed losses as well as reduction of trailing-edge loss contribution due to change of base pressure.

2.4.4 Tip Damage

De Cecco et al. (1997) made measurements in a turbine cascade with unusually large tip clearances to examine the aerodynamic effects of in-service damage to blade tips. It was found that the losses (i.e. secondary and tip-leakage losses) in the blade-end region reached a maximum at a clearance of about 7% of the blade chord. A substantial reduction in the losses occurred as the

**Figure 2.8 Comparison Between Measured and Calculated Loss and Deviation
(reproduced from Sjolander et al., 1993)**

clearance was increased further to 15% (see Figure 2.9). At this clearance, the end losses were comparable with the secondary losses measured at zero clearance. It is believed that the reduction in losses is due in large part to a significant reduction in the gap pressure difference towards the endwall where the majority of the tip leakage normally occurs. Also it is thought that at the largest clearance, the interaction between the tip-leakage and endwall flows was much diminished and this probably contributed to the observed reduction in losses.

At the larger clearances, the streamtube in the tip gap was also significantly underturned. At 15% clearance the gap flow was turned only about 10° , compared with a turning of about 50° in the main flow. Also at 15% clearance, the midspan axial velocity was reduced by about 20% compared with its undisturbed value further upstream. Both of these effects would need to be taken into account when predicting the performance of a stage with tip damage.

Figure 2.9 End Losses as a Function of Clearance (reproduced from De Cecco et al., 1997)

3 Computational Model

3.1 Introduction

Recently significant progress has been made in the development of Navier-Stokes solvers for turbomachinery flows. Among the codes capable of computing the flow field for a wide range of blade geometries and flow conditions, the code developed by Dawes (1986) is comparatively simple in structure and particularly applicable to the present damaged blade geometry. It has been used with apparent success by Sjolander et al.(1993) and De Cecco et al. (1996) to calculate the subsonic flow through a turbine cascade with trailing-edge damage and tip damage respectively. In numerous papers (e.g. Dawes 1987, 1988) Dawes demonstrated the capability of the code to deal with various turbomachinery flow situations, notably, loss prediction in the V2 transonic compressor cascade and development of secondary flow in a high-speed radial inflow turbine with and without tip clearance flow. Storer & Cumpsty (1993) used the Dawes three-dimensional Navier-Stokes code to formulate a model for loss created by the tip clearance flow in an axial compressor. Trigg et al. (1997) used the code to determine profile losses for the purpose of optimizing two-dimensional steam turbine blade profiles.

The Dawes code was viewed as a useful tool to study the flow characteristics at both low-speed and high-speed flow conditions for turbine blades with trailing-edge damage. The results are discussed in Chapter 7. It was also used to predict subsonic deviation for a number of different blade geometries to supplement experimental results for the purpose of compiling a database of deviation values for turbine blades of recent design.

First a brief description of the formulation of the three-dimensional Navier-Stokes solver (BTOB3D) is provided. Previous users of the code (e.g. Isaacs, 1994; Saroch, 1995; DeCecco, 1995) present excellent descriptions of the code and much of the information provided here is

drawn from their description. For purposes of the present investigation a few modifications were made to the code in the areas of turbulence modeling and inviscid endwall calculations. These are discussed in Section 3.3. This is followed by a study of the computational mesh structure required to obtain reasonable predictions. The chapter concludes with a discussion of some of the features and limitations of the code.

3.2 Three-Dimensional Compressible Navier-Stokes Solver (BTOB3D)

3.2.1 Governing Equations

The three-dimensional compressible Navier-Stokes (NS) equations are expressed in a frame rotating with the blade in an orthogonal cylindrical co-ordinate system (r, θ, x) as follows:

$$\frac{\partial}{\partial t} \oint_V U dV = \oint H dA + \oint \rho S dV \quad (3.1)$$

where U represents the flow variables, H represents the various fluxes, S contains the body forces, and A and V are the areas and volumes of the control volumes respectively. U , H and S are expressed as:

$$U = \begin{bmatrix} \rho \\ \rho V_x \\ r \rho V_\theta \\ \rho V_r \\ \rho E \end{bmatrix}, \quad H = \begin{bmatrix} \rho \bar{V} \\ \rho V_x \bar{V} + \bar{\tau} i_x \\ r \rho V_\theta \bar{V} + \bar{\tau} i_\theta \\ \rho V_r \bar{V} + \bar{\tau} i_r \\ \rho I \bar{V} \end{bmatrix}, \quad S = \begin{bmatrix} 0 \\ 0 \\ -2r\Omega V_r \\ \frac{1}{r} V_\theta^2 + r\Omega^2 + 2\Omega V_\theta \\ 0 \end{bmatrix}$$

where the resultant velocity is, $\vec{V} = V_x i_x + V_\theta i_\theta + V_r i_r$;

the rotational speed is, Ω ;

the rothalpy is, $I = h_{0_{rel}} - \frac{1}{2}(r\Omega)^2 = \gamma E - \frac{1}{2}(\gamma - 1)[\vec{V} \cdot \vec{V} - (r\Omega)^2]$;

and $h_{0_{rel}}$ and E are the relative reference total enthalpy and total internal energy respectively.

The stress tensor is given by:
$$\tau = \begin{vmatrix} -p + \tau_{xx} & \tau_{rx} & \tau_{\theta x} \\ \tau_{rx} & -p + \tau_{rr} & \tau_{\theta r} \\ \tau_{\theta x} & \tau_{\theta r} & -p + \tau_{\theta\theta} \end{vmatrix}$$

The above system (Equation 3.1) represents five equations which are in order: conservation of mass, streamwise-momentum, tangential-momentum, radial-momentum and energy equations respectively. Dawes simplifies the energy equation by eliminating the viscous stress and heat conduction terms. This is a reasonable assumption for high-speed flows with no external heat addition. The NS equations are further simplified with the thin-layer approximation. This is a widely used approximation for the computation of viscous flows at high Reynolds numbers. The thin-layer approximation simplifies the stress tensor components by neglecting the viscous stress terms involving the streamwise derivatives. This assumption is valid if the diffusion in the direction normal to the surface is locally dominant compared to the diffusion in the streamwise direction. Thus, without the viscous stress terms involving streamwise derivatives ($\partial/\partial x$), the resulting stresses are then expressed as:

$$\tau_{xx} = -\frac{2}{3}\mu\left(\frac{\partial v_r}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}\right), \quad \tau_{rx} = \mu\frac{\partial v_r}{\partial r}, \quad \tau_{\theta x} = \frac{\mu}{r}\frac{\partial v_r}{\partial \theta}$$

$$\tau_{rx} = \tau_{rx} \quad \tau_{rr} = -\frac{2}{3}\mu\left(\frac{2\partial v_r}{\partial r} - \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}\right) \quad \tau_{\theta r} = \mu\left(\frac{1}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r} + \frac{\partial v_\theta}{\partial r}\right)$$

$$\tau_{\theta x} = \tau_{\theta x} \quad \tau_{r\theta} = \tau_{\theta r} \quad \tau_{\theta\theta} = \frac{2}{3}\mu\left(\frac{2}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{3v_r}{r} - \frac{\partial v_r}{\partial r}\right)$$

The equations are time averaged which produces the Reynolds-Averaged Navier-Stokes equations. The system of Reynolds-averaged NS equations is closed by an equation of state and a two-layer mixing length turbulence model patterned after Baldwin and Lomax (1978). The equation of state is expressed in terms of the total internal energy (E) as follows:

$$p = \rho(\gamma - 1) \left(E - \frac{1}{2} [\bar{V} \cdot \bar{V} - (\Omega r)^2] \right)$$

The turbulence model is further described in Section 3.2.5.

3.2.2 Computational Mesh

Among the popular computational meshes (e.g. O-mesh, C-mesh, H-mesh etc.), Dawes prefers the simplicity and flexibility of the structured H-mesh. The mesh is formed by the intersection of three families of surfaces: streamwise (x, θ), quasi-orthogonal (r, θ) and meridional (x, r) as seen in Figure 3.1. The streamwise surfaces are taken to be surfaces of revolution, the first coinciding with the hub and the last with the casing. The first and last meridional surfaces coincide with the blade suction and pressure surfaces respectively. The resulting grid consists of a three-dimensional network of hexahedral cells; the geometry is defined by the cell nodes, and the flow variables are associated with the cell centres. A mesh of this form is simple to generate and flexible enough to apply to a wide range of blade geometries. As will be shown later, the H-mesh is satisfactory for meshing blade with trailing-edge damage.

Figure 3.1 Structured H-mesh used by the Dawes code (nodes in pitchwise x axial x spanwise are $25 \times 75 \times 25$)

3.2.3 Discretization of the Governing Equations

3.2.3.1 Finite Volume Formulation

A finite-volume approach is applied to discretize the equations of motion. The computational domain is divided into hexahedral cells and the variables $U(\rho, \rho V_x, \rho V_\theta, \rho E)$ are stored at centres (ijk) of the hexahedral cells. Each integral in the governing Equation 3.1 is replaced by a discrete summation around the faces of the cells as follows:

$$\frac{\Delta U_{ijk}}{\Delta t} \Delta V_{ijk} = \sum_{cell(ijk)} (H \Delta A + \rho S_{ijk} \Delta V_{ijk} + D_{ijk}) = R_{ijk} \quad (3.2)$$

where R_{ijk} is the residual at cell (ijk) . The additional term (D_{ijk}) is an adaptive artificial viscosity term which will be further discussed in Section 3.2.3.3.

The fluxes through the cell faces are obtained by simple averaging of adjacent cell-centre values of the dependent variables multiplied by appropriate components of the face area. The resulting flux sum is second order accurate in space, on smoothly varying meshes and, since the flux out of one cell face is identical to the flux into the adjacent cell face, both local and global conservation is guaranteed. However, for the viscous terms the present version of the code uses a mesh transformation which is only first-order accurate. Dawes (1986) suggests that this should not cause a problem provided the spacing between successive grid points does not differ by more than $\pm 30\%$.

3.2.3.2 Solution Algorithm

The system of differential equations is solved using time marching. In this method, an initial solution is guessed and it is then advanced in time towards a steady-state solution using a two step explicit and one step implicit method. The method is similar to the Beam & Warming (1978) implicit scheme but makes use of a preprocessing matrix in order to ensure diagonal dominance; this allows for an efficient inversion of the resulting tridiagonal matrices. Convergence

is achieved by imposing the Courant-Friedrichs-Lewy (CFL) criterion. Dawes has shown that optimum convergence is achieved for CFL values in the range of 1 - 3.

3.2.3.3 Artificial Viscosity

Artificial diffusion (i.e. D_{ijk} in Equation 3.2) is added to the discretized finite-volume equations in order to prevent odd-even point solution decoupling and to suppress oscillations in regions with strong pressure gradients (e.g. shock waves). The artificial viscosity scheme adopted by Dawes is based on the method developed by Jameson & Baker (1984). In this method a fourth-order dissipation term is used to inhibit odd-even point decoupling of the solutions and a more rigorous second-order term is used to permit shock capture without oscillations. To apply the artificial viscosity in an adaptive manner a shock-sensing pressure parameter is used. The current formulation of the artificial viscosity algorithm incorporates three constants and Dawes provides recommended values for these constants. However, in the present study different values of the constants were examined in an effort to improve shock capture (e.g. the smoothness of Mach contours) in transonic flows. This is discussed further in Section 3.4.4. The artificial viscosity is set to zero in directions normal to solid boundaries for cells adjacent to the boundaries to avoid masking the physical viscosity.

3.2.3.4 Boundary Conditions

Several boundary conditions are imposed at the inflow and outflow boundaries and at solid boundaries. At subsonic inflow boundary, total temperature, total pressure and flow angle are held fixed throughout the calculations. The inlet flow angle consists of two components: the pitch angle and the swirl angle. The pitch angle is usually set to zero to denote zero radial component of velocity, while the swirl angle is adjusted to set the incidence of the inlet flow. The upstream boundary must be placed sufficiently far upstream of the leading-edge plane, where no viscous phenomena occur. Dawes recommends that the inflow and outflow boundaries should be located at least half an axial chord upstream and downstream of the blades, respectively. In the present work,

the boundaries were generally located farther away than the minimum recommended values (see Section 3.4.1.1).

For a subsonic axial outlet, the average static pressure corresponding to the desired isentropic exit Mach number is specified. If the blade passage is an annular sector then the outlet pressure is specified at the hub and the radial distribution is determined from a simple radial equilibrium equation as follows:

$$\frac{\partial p}{\partial r} = \rho \frac{V_\theta^2}{r}$$

All cell faces within the computational domain which align with the solid walls (hub, casing, and blades) are subject to the no slip and no flux (mass, momentum and energy) conditions. Also, cells adjacent to periodic boundaries are treated as if they were interior cells and the flux across periodic boundaries is calculated by linear interpolation between cells at either side of the boundary.

Wall static pressure is found by setting the gradients of pressure normal to the wall to zero, which is a reasonable assumption for high Reynolds number flows.

The known velocity at the cell centre adjacent to the wall and the known zero velocity on the surface is used to determine the wall shear stress, τ_w . In the original version of the code this is accomplished with the aid of a skin friction correlation based on the log-law. The skin-friction coefficient is given by,

$$C_f = -0.001767 + \frac{0.03177}{\log(Re)} - \frac{0.25614}{[\log(Re)]^2} \quad (3.3)$$

where the Reynolds number (Re) is based on local conditions and normal distance to the wall. The skin-friction coefficient (C_f) is related to the wall shear stress by

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho \bar{V} \cdot \bar{V}}$$

The log-law is a well known approximation for the variation of velocity in a two-dimensional, turbulent boundary layers. The log-law is generally valid in the range $30 < y^+ < 500$, where

$$y^+ = \frac{y \sqrt{\tau_w / \rho}}{\nu}$$

and ν is the kinematic viscosity. For the approach outlined to be valid, the centers of the grid cells adjacent to solid boundaries must be in the range of applicability of the log-law. The original version of the code does not calculate y^+ . However, in the present study, the code was modified to calculate y^+ which is further discussed in Section 3.3.1.

To simulate planes of symmetry, such as at blade midspan, or to achieve effectively a two-dimensional blade-to-blade flow field, it is possible to set the endwall shear stress to zero and thus achieve inviscid endwalls. Unfortunately, this option was not implemented properly in the original version of the code. The code was subsequently modified to implement the inviscid endwalls correctly. This is further discussed in Section 3.3.2. The present study primarily focused on the profile losses and deviation for both undamaged and damaged turbine blades at midspan and hence endwall effects were not considered. Therefore, inviscid endwalls were used for all undamaged and damaged blade computations for the present study. The advantage of using inviscid endwalls is that a smaller number of grid nodes is required in the spanwise direction for both undamaged and damaged blades, thereby reducing computational time.

3.2.4 Multigrid Acceleration

The Dawes code employs one level of multigrid to accelerate convergence of steady state solutions. The basic idea is to use coarser grids to speed up the propagation of the fine grid corrections. This numerical scheme is based on the Full Approximation Storage concept developed by Brandt (1981). The coarse grids are generated by eliminating every second surface (streamwise, quasi-orthogonal and meridional) in each coordinate direction. By allowing the overall flow pattern information to propagate rapidly over a coarse grid, while resolving details on a fine grid, the multigrid procedure is capable of roughly halving the required computational time

(Dawes, 1986). The process involves sweeps that transfer residuals from the fine grid to the coarse grid and then interpolate corrections from the coarse grid to the fine grid. The multigrid process cycles back and forth between fine and coarse grids until final steady state convergence is achieved. The viscous stress terms are only calculated on the fine mesh.

Multigrid acceleration was routinely turned on during all computations in the present study.

3.2.5 Turbulence Model

The closure of the Reynolds-averaged Navier-Stokes equations in the Dawes code is obtained via the algebraic turbulence model developed by Baldwin & Lomax (1978). Dawes favored this two-layer model, based on the Prandtl mixing-length concept, due to its simplicity and computational efficiency. Dawes (1990) also showed that the loss predicted using a mixing-length model was of similar accuracy to that of a higher-order turbulent kinetic energy transport model. The results of the study of losses in a transonic compressor indicated that the majority of the losses were generated deep within the boundary layer where both of the models were tuned to reproduce the log-law portion of the law-of-the wall. The major advantage of the Baldwin-Lomax turbulence model, as compared to other mixing length models (e.g. Cebeci & Smith, 1974) is that it avoids the necessity of finding the outer edge of the boundary layer. For turbomachinery flows, it is difficult to clearly identify the edge of the boundary layer, especially since the free stream is often a shear flow. The Baldwin-Lomax model determines the length scales from the vorticity distribution in the boundary layer, thus alleviating the need to define an outer edge.

In the Baldwin-Lomax model, the eddy viscosity is divided into separate parts for the inner and outer layers. In the inner layer the eddy viscosity is obtained from:

$$\mu_t = \text{the minimum of} \begin{bmatrix} (\mu_t)_{inner} \\ (\mu_t)_{outer} \end{bmatrix}$$

The Prandtl mixing length formulation is used in the inner region,

$$(\mu_t)_{inner} = \rho l^2 |\omega|$$

where the mixing-length is given by,

$$l = ky$$

The van Driest damping function $1 - e^{-y^+/A^+}$ associated with the above equation is omitted in the original version of the code, thereby eliminating the need to calculate y^+ . The damping function only becomes significant very close to the wall ($y^+ < 30$). Dawes neglected the function because he used coarse mesh densities and law of the wall approach ($y^+ > 30$) for his computations. The magnitude of the vorticity is given by,

$$|\omega| = \sqrt{\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right)^2}$$

For the outer region, which is used in wakes as well as in attached and separated boundary layers, the eddy viscosity is given by:

$$(\mu_t)_{outer} = K C_{cp} \rho F_{wake} F_{Kleb}$$

where $K = 0.0168$, $C_{cp} = 1.6$

$$F_{wake} = y_{max} F_{max}$$

The quantities F_{max} and y_{max} are determined from the function,

$$F(y) = y|\omega|$$

where F_{max} is the maximum value of $F(y)$ in a given profile and y_{max} is the value of y at which the maximum occurs. The van Driest damping function which appears in the original Baldwin-Lomax

version of $F(y)$, is again omitted for reasons already mentioned. The Klebanoff intermittency factor is given by:

$$F_{kleb} = \frac{1}{1 + 5.5 \left(\frac{C_{kleb} y}{y_{max}} \right)^6}$$

where $C_{kleb} = 0.3$.

To apply the Baldwin-Lomax formulation to three dimensions, Dawes code finds the distance normal to each solid boundary (hub, casing or blade) for each node in the blade passage and determines the eddy viscosity for a given node based on the length scale associated with the nearest wall. Upstream of the blade passage, only the hub and casing are considered for determination of the length scale. Downstream of the blade passage (in the wakes), the outer formulation is used and the terms F_{max} and y_{max} are determined based on the length scales associated with the blade suction surface in the trailing-edge plane. This is not the standard Baldwin-Lomax modeling of the wake.

The Dawes code includes no method of predicting the boundary-layer transition point on the blade surfaces. Dawes assumes that a turbulent boundary layer exists over the full blade surface right from the leading edge. This does not affect the predictions significantly in the present study since the blades are assumed to operate at fairly high values of Reynolds number. Previous studies (e.g. Isaacs, 1994; De Cecco, 1995) have shown that computational predictions of losses are quite satisfactory and further such comparisons are presented in Chapter 7.

3.3 Modifications of the Navier-Stokes Code

3.3.1 Baldwin-Lomax Turbulence Model Implementation

As already mentioned, Dawes omitted the van Driest damping function from the Baldwin-Lomax model, thus eliminating the need to calculate y^* . The problem with this simplification is that it is impossible to know whether the predictions are within the allowable range of y^* values for

a particular mesh density. The skin-friction correlation (Equation 3.3) used by Dawes to calculate wall shear stress is valid for $y^+ > 30$. In this study the van Driest damping function was implemented and y^+ was calculated to check whether a given cell centre lies in the log-law region. Also, the wall function was extended in a manner similar to that of Turner & Jennions (1993) so that it can be applied for values of y^+ less than 30.

First a relationship between cell Reynolds number and y^+ is derived which is used to determine the viscous sub-layer, buffer layer and log-law based on the values of cell Reynolds number which is known from the computations. From the log-law (see White, 1991):

$$\frac{u}{u_\tau} = \frac{1}{k} \ln(y^+) + B \quad (3.4)$$

$$\text{or, } \frac{uy}{\nu} \frac{\nu}{yu_\tau} = \frac{1}{k} \ln(y^+) + B$$

$$\text{or, } Re_y = y^+ \left(\frac{1}{k} \ln(y^+) + B \right) \quad (3.5)$$

where the wall friction velocity $u_\tau = \sqrt{(\tau_w/\rho)}$, y is the distance of cell centre from surface, $k = 0.41$ and $B = 5.5$.

In the computations, to check whether a given cell lies in the log-law region it is only necessary to check the value of $Re = yu/\nu$, which is known from the computations. If the log-law region is taken to start at $y^+ = 30$, then from Equation 3.5 the corresponding value of Re_y is 414. Comparing Equations 3.4 and 3.5 the following relation between u_τ and cell Reynolds number can be deduced:

$$\frac{u}{u_\tau} = \frac{Re}{y^+} \quad (3.6)$$

In the viscous sub-layer $u/u_\tau = y^+$. Again comparing this relation with Equation 3.6 shows that in the viscous sub-layer $Re_y = (y^+)^2$. Based on the calculated Reynolds number for each cell on the blade surface, a wall function appropriate to the laminar sub-layer, buffer layer or log-law region is applied to obtain the wall shear stress as follows:

For the viscous sub-layer, when $0 < y^+ < 4.0$ (i.e. $Re < 16.0$):

$$u_\tau = \frac{u}{\sqrt{Re}}$$

For the buffer layer, when $4 \leq y^+ \leq 30$ (i.e. $16 \leq Re \leq 414$) a curve fit is obtained from the Thomson profile (1965) given by:

$$y^+ = 55.17 - 57.76 (\ln Re) + 23.50 (\ln Re)^2 - 4.16 (\ln Re)^3 + 0.28 (\ln Re)^4$$

For the log-law region, when $y^+ > 30$ (i.e. $Re > 414$) the original skin-friction correlation (Equation 3.3) used by Dawes is used to calculate the wall shear stress.

As previously mentioned, Dawes used the original recommended values for the constants C_{cp} and C_{kleb} in the Baldwin-Lomax turbulence model. York & Knight (1985) suggested slightly different values of these constants in their work related to incompressible (low Mach number) turbulent flows. Granville (1987) also concluded that for boundary layers in favorable or adverse pressure gradients, the Baldwin-Lomax values of C_{cp} and C_{kleb} were not satisfactory. Based on the outer similarity law for turbulent boundary layers in pressure gradients, Granville derived a formula for C_{cp} as a function of C_{kleb} as follows:

$$C_{cp} = \frac{3 - 4C_{kleb}}{2C_{kleb}(2 - 3C_{kleb} + C_{kleb}^3)}$$

Figure 3.2 shows the relationship between C_{cp} and C_{kleb} for different pressure gradients based on the above function. The original values $C_{cp} = 1.6$ and $C_{kleb} = 0.3$ lie far outside the plotted curve. The new values of C_{cp} and C_{kleb} adopted for the present work are 1.4 and 0.545 respectively as reasonable mean values for favorable pressure gradients based on Granville's curve. It was found that the change did not noticeably affect the predicted losses or Mach contours for high Mach number flows (i.e. exit Mach number range 0.7 to 1.2). Chima et al. (1993) noted that C_{cp} is a strong function of pressure gradient, ranging from $C_{cp} = 1.548$ for highly favorable pressure

gradient to $C_{cp} = 0.851$ for very adverse pressure gradient. They concluded that problems with the Baldwin-Lomax model are most likely to occur for low Reynolds number flows.

Though no noticeable change was observed with the new values of C_{cp} and C_{kleb} they were retained as a measure of updating the Baldwin-Lomax turbulence model.

Figure 3.2 Relationship between C_{cp} and C_{kleb} (based on Granville, 1987)

3.3.2 Inviscid and Viscous Endwalls

The original version of the Dawes code worked only with viscous endwalls. The inviscid endwall option was not coded properly. As a result, on his calculations for blades with trailing-edge damage Isaacs (1994) had to increase the aspect ratio of the blades to move the endwall effects away from the midspan region. This also increased the number of spanwise nodes needed to model the trailing-edge damage and consequently increased the computational time. To eliminate this problem the implementation of the inviscid endwall model was corrected.

The problem with the inviscid endwall computation was that the boundary condition at the endwall was incorrectly coded. By appropriately setting the shear stress to zero for all cell faces adjacent to the endwall, the inviscid endwall option in the Dawes code was restored.

A number of additional studies were conducted to establish criteria for judging whether satisfactory solutions have been obtained with the code. These studies are described in the following sections. They include investigations of grid quality, the influence of grid density and convergence criteria.

3.4 Simulation of Cascade Flow

3.4.1 Selection of Computational Grid

3.4.1.1 Grid Boundary

Before the Dawes code can be used as an analysis tool, it is necessary to establish the required environment such as, the appropriate computational mesh, the number of time-steps, the allowable inlet Mach numbers, to produce satisfactory predicted results. Initial examination of the code was carried out using the undamaged blade section (LS3) used by Isaacs (1994). The geometry of the blade is given in Table 8.1. This is a well documented blade with experimental results available for use as a benchmark to establish the performance of the Dawes code. It is believed that the criteria established for satisfactory solutions using the LS3 blade would be generally applicable to the blade geometries investigated in the rest of the study.

Since the code was developed in cylindrical coordinates the blade passages are represented as annular sections, whereas the experimental data were mainly obtained from linear cascades. To approximately model the blade passage as a linear section, similar to a turbine cascade, a very large hub radius of 100000mm was used.

The grid inlet and outlet boundaries were selected slightly larger than the minimum requirement suggested by Dawes. The inlet boundary was placed about 60% of an axial chord length upstream of the blade leading-edge plane. The outlet boundary was placed 110% chord length downstream of the trailing-edge plane. This was considered adequate to allow a uniform outlet static pressure to be assumed.

3.4.1.2 Grid Orientation at Inlet and Exit Plane

As previously mentioned, the computational grid used by Dawes is the structured H-mesh. The grid orientation shown in Figure 3.3(a) is the one normally used by Dawes. It can be seen that the blade upstream passage is aligned in the axial direction and the downstream passage is aligned with the blade outlet metal angle. This arrangement of the grid is designated here as the 'axial-skewed' grid. Another common practice of grid alignment is the 'skewed-skewed' grid where both the upstream and downstream passages are aligned with the blade metal angles as shown in Figure 3.3(b). It is noticeable that this arrangement results in a somewhat skewed mesh for both the inlet and outlet flows. It is generally desirable to construct the grid in such a way that the coordinate lines are orthogonal as much as possible throughout the computational domain. This is beneficial in terms of numerical accuracy and stability. Further discussion on the quality of grid is presented in the following Section 3.4.1.3. To improve grid orthogonality in the upstream and downstream passages, other combinations of grid orientation at inlet and exit sections were tested. These are shown in Figure 3.4: (a) with axial upstream and downstream grid (axial-axial) and (b) with axial upstream and curved downstream grid (axial-curved). The downstream curvature

(a) Axial-Skewed Grid

(b) Skewed-Skewed Grid

Figure 3.3 Upstream Grid Orientation

(a) Axial-Axial Grid

(b) Axial-Curved Grid

Figure 3.4 Downstream Grid Orientation

produces orthogonal cells near the exit plane. The grids shown are rather coarse only to show the finite volume cells more clearly. The actual grids used for performance predictions have a much higher density (e.g. pitchwise nodes (33) x axial nodes (97) x spanwise nodes (17)).

The influence of the grid orientations was examined based on the predicted mass-averaged total pressure loss coefficients (Y) at a plane $0.4c_x$ downstream of the blade trailing edge. Table 3.1 shows the loss predictions obtained from the different grid orientations. It is evident that the axial-skewed grid and skewed-skewed grid produce similar loss results and are closer to the experimental results. However, the axial-axial grid results in a much higher loss prediction with the axial-curved grid producing comparatively better results. Though the axial downstream grid produced orthogonal cells, by extending the downstream grid in the axial direction the blades wakes quickly moved into coarser grid regions, which is the likely reason for the apparently poorer predictions. It seems that better wake prediction is obtained when the mesh is extended in the approximate flow direction. The axial-curved grid produced reasonable results also. However, a slightly higher loss prediction seems to suggest that problems associated with wakes is still present to a lesser extent.

The axial-skewed grid configuration was used for all computations in the present study. It was felt that for reasons of grid orthogonality, an axial inlet would be the better choice.

Table 3.1
Effect of Grid Orientation and Inlet and Exit Plane

Experimental Result (Isaacs, 1994):	
Total Pressure Loss Coefficient, $Y = 0.028$	
Grid Orientation	Predicted Loss
Axial-Skewed	0.0316
Skewed-Skewed	0.0315
Axial-Axial	0.0386
Axial-Curved	0.0326

3.4.1.3 Grid Quality

The appropriate distribution of grid coordinates is very important for the quality of the solution in terms of numerical accuracy and stability. The primary requirements of the grid are accurate representation of geometric boundaries, clustering of grid points in regions of rapid changes, smooth distribution of step sizes, and orthogonality. Generally all requirements cannot be satisfied and compromises have to be made. For example, the H-grid used by the Dawes code contains considerable skew at the blade leading and trailing edges (see Figure 3.6) though the other requirements for a quality grid are somewhat satisfactory. It is preferable to have orthogonal grids at the boundary because it makes the application of boundary conditions and algebraic turbulence models (e.g. Baldwin-Lomax) more straightforward and reliable (see Thompson, 1984). Fortunately, the finite-volume discretization used in the Dawes code is relatively tolerant of less than orthogonal grids.

The appropriate distribution of the grid coordinates for the H-mesh is achieved by using grid clustering functions. Two different clustering functions are used for different sections of the H-mesh as shown in Figure 3.5. The grid lines are spaced in each of the streamwise (blade section), pitchwise and spanwise directions using a trigonometric algorithm (see Isaacs, 1994). A geometric progression function is used for the leading grid (upstream) and trailing grid (downstream) clustering in the streamwise direction. At the common blade leading edge plane where the leading grid section and blade grid section meet, the adjacent grid spacing on either side are kept the same so that there is no abrupt change in step size from one clustering method to the other. A similar condition is enforced at the blade trailing edge plane. The relative spacing between mesh lines is kept below $\pm 30\%$ in compliance with the recommendation of Dawes and this also ensures smooth variation of the local grid spacing.

An effort was made to develop a guideline to identify an appropriate H-mesh for computations using the Dawes code. The process involved identifying grid structural characteristics that produced satisfactory solutions. The grid dimension used for the study was $33 \times 97 \times 17$ (pitchwise $\times$ axial $\times$ spanwise). Several cases were studied with grid spacing variations in the blade-to-blade plane. Inviscid endwalls and uniform clustering in the spanwise direction was used for all the cases.

One of the main sources of problems for the H-mesh is the grid skewness at the blade leading and trailing edges (see Turner et al., 1993). The grid clustering should be done in such a way that it minimizes the grid skewness and at the same time provides good definition of blade geometry and blade surface viscosity effects. The shape and size of the cells at the blade leading- and trailing-edge change considerably with varying degrees of clustering. An effort was made to determine a quantitative relationship between this property and the corresponding solutions obtained. However, a definite relationship between cell length ratios and acceptable solution could be established only for the cells at the blade leading-edge plane. Six cases are listed in Table 3.2 for the purpose of illustrating the procedure which is described below.

Different enlarged grids for the LS3 blade leading- and trailing-edge region are shown in Figures 3.6 through 3.11. These Figures illustrate the effects of varying degrees of clustering.

Figure 3.6 shows the blade leading- and trailing-edge with no clustering, i.e. uniform distribution of grid nodes. It is evident that the blade leading and trailing edges are not well defined and the blade-surface cells are likely to be large compared with the blade-surface boundary layers. The grid cells used for the quantitative analysis of the grid quality shaded and the cell dimensions used are designated as a, b and c. The quality of the grid is assessed based on the predicted total pressure loss coefficient (γ).

Figure 3.5 Grid Section Terminology

The parameters d and e show the relative proportions of the lengths of the cell sides where $d = a/b$ and $e = b/c$. Leading edge cells with high aspect ratios are produced in grids B and C (Figures 3.7 and 3.8) which can be recognized by the large values of e and d respectively shown in Table 3.2. It is evident from Figures 3.7 and 3.8 that the leading-edge cells are also highly skewed. The use of these grids results in high losses. Grids A and D (Figures 3.6 and 3.9) give cells with more moderate aspect ratios. However, they also result in high losses apparently due to the poor resolution of leading- and trailing-edge blade geometry.

Figure 3.6 Grid Geometry for Grid A

(a) Blade Leading Edge

(b) Blade Trailing Edge

Figure 3.7 Grid Geometry for Grid B

(a) Blade Trailing Edge

(b) Blade Trailing Edge

Figure 3.8 Grid Geometry for Grid C

(a) Blade Leading Edge

(b) Blade Trailing Edge

Figure 3.9 Grid Geometry for Grid D

(a) Blade Leading Edge

(b) Blade Trailing Edge

Figure 3.10 Grid Geometry for Grid E

Figure 3.11 Grid Geometry for Grid F

Table 3.2

Parameters for Determination of Grid Quality

Grid	Clustering Description and Effects on Blade	$d = \frac{a}{b}$	$e = \frac{b}{c}$	$r = \frac{d}{e}$	Loss (%)
A	<i>no clustering</i> : leading- and trailing-edge blade geometry ill-defined and large cells at blade surface.	2.22	1.15	1.93	0.107
B	<i>streamwise clustering</i> : leading- and trailing-edge blade geometry well-defined.	0.67	16.5	0.04	0.051
C	<i>pitchwise clustering</i> : fine cells at blade surface.	10.84	4.25	2.55	0.097
D	<i>little clustering in both directions</i> : moderately good definition of blade geometry and cell size at blade surface.	2.75	2.16	1.27	0.041
E	<i>extreme clustering in both directions</i> : blade geometry well defined and fine cells at blade surface.	3.3	3.4	0.97	0.030
F	<i>moderate clustering in both directions</i> : blade geometry well defined and fine cells at blade surface.	3.1	3.0	1.03	0.031

Based on the loss coefficients, reasonable solutions are obtained using Grids E and F (Figures 3.10 and 3.11). The parameters d and e show moderate aspect ratios for the leading-edge cells. The parameter r defined as the ratio of d and e seems to distinguish between 'bad clustering' and 'good clustering'. A value of r close to unity represents balanced cell sizes and grid node distributions and seems to result in good predictions. It was found that grids having the value of d and e less than 3.5 and the value of r in the range 0.60 to 1.20 generally produced good solutions. Lower values of r indicate good definition of blade geometry (see Grid B) and this seems to be the predominant factor for achieving reasonable predictions.

The HS1A blade (see Section 5.5) was used in the present study to investigate the effects of trailing-edge damage. The grid parameters used for all computations for undamaged and damaged blades are: $d = 2.0$, $e = 3.1$ and $r = 0.65$. For a different blade geometry a similar study should probably be conducted to identify suitable values of the parameters.

3.4.1.4 Grid Density and Refinement

A mesh refinement study was performed to evaluate the grid density required to establish essentially grid-independent solutions. Grid-independence is achieved when no change in results is obtained with additional mesh refinement.

First mesh refinement computations were performed for an undamaged blade. After establishing the grid dimension that produced acceptable results for an undamaged blade, additional computations were performed with increased number of spanwise nodes to determine the adequate grid size for a trailing-edge damaged blade. This is described in Section 3.4.1.5.

Four grid densities were investigated. The finest grid dimension was generated by roughly doubling of the coarsest grid in each coordinate direction. The boundary-layer resolution was estimated by calculating the average y^+ values for the cells adjacent to the blade suction surface. Table 3.3 shows the grid dimensions with the maximum number of nodes in the pitchwise (I_m), axial (J_m) and spanwise (K_m) direction and the corresponding average y^+ values. The dependence of the solutions on the grid densities was examined based on the mass-averaged total pressure loss

coefficients (Y) at a plane $0.4c_x$ downstream of the blade trailing edge. Table 3.3 shows the loss coefficients obtained for the different grid sizes.

Table 3.3
Grid Refinement

Grid No.	Grid Dimension ($I_m \times J_m \times K_m$)	y^+	Loss Coefficient Y
1	17 x 49 x 17	121	0.0821
2	25 x 75 x 25	54	0.0527
3	33 x 97 x 33	32	0.0316
4	37 x 107 x 37	26	0.0313

Figure 3.12 shows the loss coefficients predicted with increased grid density. The higher losses predicted for the coarse Grids 1 and 2 are believed to be mostly due to the poor definition of leading- and trailing-edge blade geometries. The predicted losses for Grids 3 and 4 seems to indicate that they are approaching the asymptotic range: a nearly 1.4 times (or 40%) increase in grid size resulted in only about 1% change in loss prediction. These results also suggest that the solution is approaching grid-independence. Relative to memory constraints and computation time involved with dense grids, Grid 3 seems to be the optimum grid density for reasonable prediction of losses. This grid size was used for all computations for undamaged blades.

Figure 3.12 Predicted Loss Coefficient with Grid Density

Roache (1994) proposed that the results of grid refinement studies should be uniformly reported using a Grid Convergence Index (GCI), which is based upon a grid refinement error estimator derived from the theory of generalized Richardson Extrapolation. Inconsistent and often confusing reporting of grid refinement studies in the literature prompted Roache to develop this method so that at least a uniform interpretation of the studies could be made. This method relates the error estimation obtained for an arbitrary grid refinement to one which would be expected from a grid refinement study of the same problem but using a grid doubling and a second-order spatially accurate scheme. In other words, the error estimate obtained from an arbitrary grid refinement is

normalized to give an equivalent error estimate, which Roache refers to as the Grid Convergence Index (GCI).

The Grid Convergence Index for a fine grid solution is expressed as:

$$GCI(fine\ grid) = \frac{3|\epsilon|}{r^p - 1}$$

with

$$\epsilon = (f_2 - f_1) / f_1$$

- where, f_1 = result (such as total pressure loss coefficient, Y) of fine grid solution
 f_2 = result of coarse grid solution
 ϵ = relative error between fine grid solution and coarse grid solution
 r = grid refinement ratio (e.g. $r = 2$ is grid doubling in each coordinate direction)
 p = spatial accuracy (e.g. $p = 2$ for a second-order accurate method)

The method applies to the point-by-point solution values as well as the solution functionals (e.g. loss coefficient).

Table 3.4 presents the calculation of GCI values for the different grid densities already shown based on the solution of mass-averaged total pressure loss coefficient (Y). The loss predicted for the coarse Grid 3 is within 1.0% of the fine Grid 4 value. This relative error estimation is commonly quoted by authors. However, Roache noted that this does not take into account the amount of grid refinement (i.e. r) and the spatial accuracy of the solver (i.e. p). Often this relative error estimation can be misleading if the grid refinement is made sufficiently small. The grid refinement between Grid 3 and 4 represents an approximately 10% increase ($r = 1.1$) in the number of nodes in each coordinate direction. The spatial accuracy of the present code is given by $p = 2$ and therefore the GCI is 13.6 %. Evidently this is a more conservative estimate of the error. A doubling of the coarse Grid 1 in each coordinate direction ($r = 2$) results in GCI value of 159% which is equal to the relative error estimate ϵ .

Table 3.4
Calculation of Grid Convergence Index

Grid No.	Grid Dimension	Loss Coefficient γ	r	ϵ (%)	GCI (%)
1	17 x 49 x 17	0.0821			
3	33 x 97 x 33	0.0316	2	159	159
3	33 x 97 x 33	0.0316			
4	37 x 107 x 37	0.0313	1.1	1	13.6

3.4.1.5 Grid Modification and Refinement for Damaged Blade

Isaacs (1994) developed a method of including trailing edge damage in the blade profile description specified in the Dawes code input file. Damage was added at the trailing edge in the form of a semi-circular cutout at midspan. For streamwise surfaces outside of the damage, the undamaged profile description is used. Figure 3.13 shows the meridional plane with a cutout of 20% of blade chord at midspan. The pressure side corner inside the damage is slightly rounded as shown in Figure 3.14. This grid modification was found to be necessary to maintain stability of the solution.

The grid definition developed by Isaacs could be used for only two damage sizes, namely 15% and 25% of chord penetration. Also the location of the cutout was fixed at midspan. In the present study, the code was modified so that arbitrary damage size and damage location could be used.

Figure 3.13 Meridional Plane of Trailing-Edge Damaged Blade

Figure 3.14 Rounded Trailing Edge Viewed from Streamwise Surface at Midspan

The optimum number of spanwise nodes required to produce a good resolution of the flow in the cutout region was determined using three different grid sizes. Solutions were obtained for grids with 13, 19 and 23 nodes distributed across a 20% cutout. The corresponding grid dimensions used for the HS1A blade were : 33 x 97 x 37, 33 x 97 x 45 and 33 x 97 x 53. Figure 3.15 shows the predicted distributions of loss coefficient. Loss peaks are observed at the edges of the cutout and they are better defined by the denser grids. However, similar losses are predicted at the center of the cutout by all the grids. For the present work, the grid size 33 x 97 x 45 was selected since it shows essentially the same resolution of the flow as that of the finest grid. This grid also saves about 11 hours of computation time relative to the finest grid.

Figure 3.15 Effect of Number of Spanwise Nodes on Loss Prediction

3.4.2 Convergence Criteria

3.4.2.1 The Residual and Percent Mass Error

The convergence of the Dawes code solution is basically judged by two parameters: the residual and percent mass error. The residual is a measure of the average imbalance of mass, momentum, and energy over the entire computational grid. The percent mass error term represents the maximum error in the mass flux through the computational domain compared with the specified mass flux at the grid inlet plane. The Dawes code provides an output file for these parameters.

As the steady-state solution is approached, the residual converges to some low value (typically order of 10^{-5}). The overall residual is expressed as the sum of the residuals of mass, momentum and energy terms in the code and their convergence pattern is shown in Figure 3.16. It is evident that the energy terms are several orders of magnitude larger than the other terms and the convergence to steady-state is essentially governed by the energy terms. In case a solution is unstable or diverging the residual tends to produce large oscillations or diverge. However, it was

Figure 3.16 Composition of the Residual

found that in some cases the residual achieves typically low values even for unacceptable solutions. Therefore, low residual value is a necessary but not a sufficient requirement for acceptable predictions.

The percent mass error term was found to be a sensitive indicator of the convergence and quality of a solution. The mass error must level off to some low value before a steady-state solution is achieved. This is linked to the grid density and exit Mach number of the flow: denser grids have lower mass error and mass error slightly increases with higher exit Mach number. For a stable solution the residual usually asymptotically reduces to some low value, but if the percent mass error does not reduce simultaneously then the steady-state solution is usually not satisfactory. Figure 3.17 shows an example of a case where the residual and mass error appear to be well behaved but is not associated with an acceptable solution. The mass error value is unusually high

Figure 3.17 Nature of Residual and Mass Error for a Bad Solution

It was found that the percent mass error usually assumes larger values for unacceptable solutions, as judged from the predicted total pressure loss coefficients.

Figure 3.18 shows the mass error values for a few good and 'bad' solutions obtained with a grid density of $33 \times 97 \times 33$, and exit Mach number 0.71. It can be concluded from the Figure that the percent mass error value should be less than 0.3 for an acceptable solution.

Figure 3.18 Relationship of Mass Error and Loss Coefficient

3.4.2.2 Number of Time Steps

As described earlier, the Dawes code used the time-marching procedure. Therefore, the user must select the number of time steps to be run by the code. The sufficient number of timesteps necessary for convergence depends on the grid density and Mach number. Generally, for higher Mach numbers and/or finer grids, more time steps are necessary to achieve convergence. The residual and the mass error provide a good indication of the approximate number of timesteps required for a steady-state solution. Figure 3.19 shows a typical convergence history for a grid size $33 \times 97 \times 33$ and exit Mach number 0.71. It can be seen from the Figure that a rapid decrease in the residual occurs up to about 2000 timesteps and then it starts to level off. Likewise, the mass error is initially quite oscillatory, but attains a steady value after about 2000 timesteps. However, the sufficient timestep has to be judged from the prediction of the flow variables, namely the loss coefficient. Dawes (1988) notes that the loss coefficient is the last parameter to converge. Figure 3.20 shows the trend in the mass-averaged total pressure loss at 1000, 2000, 3000, and 4000 timesteps obtained from the computation shown in Figure 3.19. It is evident that there is little change in the predicted solution beyond 3000 timesteps. Also it can be seen from the corresponding convergence history for this solution shown in Figure 3.19, that the residual has approached the asymptotic range and the mass error has leveled off. It is important that a sufficient number of time steps is selected to ensure convergence.

3.4.3 Influence of Mach Number

The Dawes code was developed for compressible flows and is not applicable for incompressible predictions. Isaacs (1994) tested the code for low Mach number flows and found that a stable solution could be achieved for an inlet Mach number of 0.29 for his particular geometry; the corresponding exit Mach number was 0.41. However, it was found that for exit Mach numbers less than about 0.6, the predicted losses were unrealistic, as shown in Figure 3.21. In light of the above observations, care must be taken whenever low Mach number predictions are

Figure 3.19 A Typical Solution Convergence History

Figure 3.20 Appropriate Number of Timesteps

**Figure 3.21 Mass-Averaged Total Pressure Loss vs. Exit Mach Number
(reproduced from Isaacs, 1994)**

being performed. In the present study, the low-speed results were obtained for exit Mach numbers of about 0.7.

3.4.4 Influence of Artificial Viscosity

As previously mentioned in Section 3.2.3.3 , the artificial viscosity scheme incorporates three coefficients and Dawes (1986) provides recommended values for these coefficients. These coefficients are designated as AV2 associated with the second-order dissipation terms, AV4 associated with the fourth-order dissipation terms and a blending coefficient AVA. The recommended values are:

$$AV2 = 1.0; \quad AV4 = 0.01; \quad AVA = 2.0.$$

In the present study different values of the coefficients were examined in an effort to improve the smoothness of Mach contours in transonic flows.

It was found that larger values of AV2 and AV4 effectively reduced oscillations near the blade leading- and trailing-edge and produced smoother Mach contours. Results were relatively insensitive to changes in the value of AVA. The altered values of the coefficients AV2 and AV4 that produced smooth Mach contours were 2.0 and 0.02 respectively. Figures 3.22 and 3.23 shows the Mach contours for a solution with the recommended coefficients and the altered coefficients respectively. The blade trailing-edge exit Mach number is 1.20. The appearance of shock waves at the trailing edge is noticeable. Evidently Mach contours are much smoother with larger values of the coefficients.

Unfortunately, it was also found that this modification led to a more than 10% increase in the predicted loss. If the primary interest is the evaluation of Mach contours then the higher artificial viscosity values may be used to produce smoother contours.

In the present study the original coefficients were used in favor of the new ones. This is because the present focus is on the evaluation of losses.

Figure 3.22 Mach Contours with Recommended Values of Coefficients

Figure 3.23 Mach Contours with Altered Values of Coefficients

4 Development of Correlations for Blade Roughness Damage

4.1 Introduction

The investigation of the effects of surface roughness on profile losses is part of a fundamental study of the aerodynamic behavior of damaged turbine blades being carried out at Carleton University. Particularly, the research program aims to determine the effect of specific types of damage on the blade's losses and flow-turning ability. As mentioned earlier, blade damage can take a number of forms, such as blade surface roughening, leading and trailing-edge damage, tip damage and a few other types of damage. Blade surface roughening is a very common type of in-service blade damage caused by erosion, corrosion and deposits. As part of this study of damaged blades, Kind et al. (1998) examined experimentally the effect of surface roughness on profile losses for turbine blades at subsonic flow conditions. They collected data for design and off-design conditions, with varying configurations of the surface roughness using a single cascade geometry. The present goal is to express these results in the form of correlations and graphs that may be easily implemented in meanline or throughflow predictions of turbine stage performance. The correlations account for the effects of the roughness height as well as the location and extent of the roughness patch on the blade surface.

Jouini et al. (1998) further extended the results computationally to include the effect of surface roughness on profile losses at transonic flow conditions. No experimental data are available at compressible flow conditions. Jouini et al. implemented modified wall functions in the three-dimensional Navier-Stokes code of Dawes (1988) to predict the aerodynamic effects of surface roughness. The correlations developed by Jouini et al. are summarized here to provide a complete treatment of the effects of surface roughness.

4.2 Effect of Surface Roughness in Incompressible Flow

4.2.1 Sources of Data

Based on the experimental data obtained by Abbott (1993), Serjak (1995) and Kind et al. (1998), an attempt has been made to develop correlations for the effect of surface roughness on profile losses of turbine blades at subsonic flow conditions. They investigated different roughness configurations by varying the roughness height, the location of the roughness strip, and the width of the roughness strip. However, the available data were not sufficient to develop the correlations for a complete range of roughness configurations and therefore, reasonable extrapolations are made from the data, as described below.

Randomly distributed sand grains were used to generate rough surfaces. Three roughness heights (k) were used: 338, 722, and 1020 μm nominal diameter, corresponding to k/c values of 0.0021, 0.0045, and 0.0063. The spacing between the roughness elements is characterized by the spacing parameter λ (area of roughened blade surface / total frontal area of roughness elements) which ranged from 7 to 16 with most tests for a value near 10. Measurements were made with spanwise strips of sand grain roughness applied to portions of the suction surface and pressure surface of the blade. Figure 4.1 illustrates the locations of the roughness strips on the blade surface as used by Abbott and Serjak. As can be seen from the Figure, measured data for a maximum of 47 percent of the blade suction surface and 71 percent of the blade pressure surface are available. As a result, extrapolations were made from the available experimental data to estimate the losses for roughening of the whole of the blade surface.

4.2.2 Effect of Extent of Roughness Patch

Figure 4.2 shows the effect of the locations of the roughness on the losses. The data shown in this Figure are for the same width of roughness strips ($\Delta s/c = 0.235$) for different roughness heights and chordwise locations. From the Figure it is clear that as the roughness strip is advanced toward the leading edge, losses increase considerably. For larger roughness heights,

Figure 4.1 Locations and Roughness Band Widths on the Blade Profile as used by (a) Abbott and (b) Serjak (reproduced from Abbott, 1993; and Serjak, 1995)

Figure 4.2 Effect of Location of the Roughness Patch

the losses are proportionately greater. Three experimental data points are shown for the roughness height $k/c = 0.00445$. It may be assumed that all the curves will follow the same trend of increase in loss as that for the roughness height $k/c = 0.00445$. Therefore, all the curves can be reasonably extended toward the leading edge to obtain an estimate of loss for the same roughness strip located near the leading edge. The contribution to the loss from the leading edge stagnation point is negligible and so an extrapolation of the data is made toward the chordwise position $x/c = 0.05$ to avoid overestimation of the result. The extrapolated losses are shown in the Figure 4.2.

The measurements for different roughness patches can also be combined approximately to estimate the losses which would be obtained by larger patches. The incremental loss (ΔY_R), defined as the additional loss due to surface roughness over that of the smooth one, is used for the purpose of obtaining total losses for different combinations of roughness patches. An examination

of the relative locations of the roughness strips shown in Figure 4.1(a) indicates that pairs of strips starting at 0.25c, 0.534c and 0.645c can be combined approximately to cover 70% of the blade suction surface. Consequently, the corresponding incremental losses (ΔY_R) due to these strips can be added to give the total loss for roughening of 70% of the blade suction surface. Table 4.1 shows that combining data from two different sets of measurements leads to almost the same result, suggesting that individual contribution to losses by the strips can be added to give the total loss for the whole surface involved. The table includes, in the k/c column, the trial number used by Abbott (1993) to tabulate his experimental results. Overall loss due to roughness covering 95% of the blade suction surface can be obtained by adding the extrapolated loss from Figure 4.2 to the total incremental loss for 70% of the blade suction surface.

Table 4.1
Determination of Loss for 70% Blade Suction Surface

k/c	s/c	$\Delta s/c$	Incremental Loss ΔY_R	Incremental Loss for 70% Blade Suction Surface
0.00445 (5a)	0.250	0.470	0.0243	0.0243 + 0.0010 = 0.0253
0.00445 (3a)	0.645	0.235	0.0010	
0.00445 (6a)	0.250	0.235	0.0177	0.0177 + 0.0067 = 0.0244
0.00445 (4a)	0.534	0.470	0.0067	

Serjak (1995) obtained roughness data for more than 70% of the pressure surface of the blade with the roughened patch starting from 25% of the axial chord as shown in Figure 4.1(b). Also, as discussed earlier, an estimated loss for roughening of 70% of the blade suction surface is obtained. Therefore, the incremental loss due to the roughening of 70% of each blade surface, can be compared to provide an estimate of the relative contribution of the suction and pressure surface

roughness on the total loss. It may be mentioned that the suction surface is dominant in producing loss (Denton, 1993) since the high surface velocity on the suction surface results in much higher entropy generation. In fact, Forster (1966) showed that for steam turbine nozzle blades roughness over the suction surface is three times as detrimental as roughness over the pressure surface. Table 4.2 lists the ratio of suction surface incremental losses (ΔY_{ss}) to pressure surface incremental losses (ΔY_{ps}) for two different roughness heights (k/c) and various angles of incidences, as estimated from the data of Serjak (1995).

Table 4.2
Ratios of Suction to Pressure Surface Incremental Losses

k/c	Incidence	$\Delta Y_{ss}/\Delta Y_{ps}$
0.00445	0	4.03
0.00445	0	3.61
0.00445	10	4.35
0.00445	15	4.70
0.00628	0	4.13
0.00628	10	5.71
0.00628	15	2.01

An inspection of the ratios suggests that a suction to pressure surface loss ratio of 4:1 is a reasonable average value. Therefore, the equation used to convert suction surface roughness data to total surface loss is as follows:

$$\Delta Y_R = 1.25\Delta Y_{ss} \quad (4.1)$$

4.2.3 Effect of Roughness Size on Losses

The effects of the roughness on the profile losses were correlated as the ratio of the rough losses (Y_{PR}) to the smooth ones (Y_{PS}):

$$K_L = \frac{Y_{PR}}{Y_{PS}} = \frac{Y_{PS} + \Delta Y_R}{Y_{PS}} \quad (4.2)$$

where ΔY_R is the incremental profile loss due to roughness. Kind et al. (1998) showed that the profile losses increased with the roughness height. For the purpose of establishing a general relationship between the loss parameter K_L and roughness size, the present roughness heights (k) are converted to equivalent standard-sand roughness heights (k_s). If k and λ are known for any roughness, it is possible to determine an equivalent standard-sand roughness height (Kind et al., 1998). Table 4.3 gives values of the ratio k_s/k for λ values used by Serjak and Abbott. However,

Table 4.3
Ratios of k_s/k for various values of the spacing parameter λ

λ	k_s/k
7	6.1
10	4.0
16	2.4

most of the results are based on a λ value of 10. Figure 4.3 shows the variation of K_L with standard roughness height (k_s/c) for particular roughness configurations at design incidence and a λ value of 10. The values of roughness height considered span the range from mild to severe roughening as measured on actual turbine blades (Tarada & Suzuki, 1993). It is observed that there is a rapid change in loss from the smooth blade to the first roughness of $k_s/c = 0.0083$. Above this, there is a more gradual increase in the loss.

Figure 4.3 Variation of the Loss Ratio K_L with Standard Roughness Height (k_s/c) at Design Incidence

For design incidence, the profile loss ratio, K_L , is then expressed as:

$$(K_L)_{des} = \left(\frac{Y_{PR}}{Y_{PS}} \right)_{des} = f(X) \quad (4.3)$$

where, $X = (k_s/c)10^3$ and $f(X)$ is approximated as follows for all roughness patches starting from the chordwise position $x/c = 0.05$:

(1) For 25% covered suction surface:

$$f(X) = 1.0 + 0.058X - 0.0005X^2, \quad 0 \leq X \leq 25$$

(2) For 50% covered suction surface:

$$f(X) = 1.0 + 0.112X - 0.0012X^2, \quad 0 \leq X \leq 25$$

(3) For 95% covered suction surface:

$$f(X) = 1.0 + 0.151X - 0.0022X^2, \quad 0 \leq X \leq 25$$

(4) For both blade surfaces covered:

$$f(X) = 1.0 + 0.192X - 0.0028X^2, \quad 0 \leq X \leq 25$$

The correlations can be used for any combination of actual roughness height (k) and spacing parameter λ after determining the equivalent values of k_s using Table 4.3. If the roughness height is expressed in terms of the 'centerline average' (CLA) value, it is converted to equivalent sand grain roughness height using $k_s = 6.2k_{CLA}$ (see Koch & Smith, 1976).

As mentioned, the correlation presented is based on results for a single cascade geometry. It is assumed that the amplification of the profile losses by roughness, as expressed by K_L , is the same for other geometries and loadings. The influence of the specific geometry will be present in the profile loss for the smooth blade, Y_{PS} , to which K_L will be applied.

4.2.4 Effect of Incidence on Losses

The effects of off-design incidences are correlated using the incidence parameter χ , defined as follows:

$$\chi = \left(\frac{d}{s}\right)^{-0.05} (We)^{-0.2} \left(\frac{\cos \beta_1}{\cos \beta_2}\right)^{-1.4} (\alpha_1 - \alpha_{1(des)}) \quad (4.4)$$

where d = blade leading edge diameter
 s = blade spacing

- We = blade wedge angle (degrees)
 α_1 = incidence angle (degrees)
 $\alpha_{1(des)}$ = design incidence angle (degrees)
 β_1 = blade inlet metal angle (degrees)
 β_2 = blade outlet metal angle (degrees)

A similar incidence parameter was used in the improved correlation for the turbine blade profile losses at off-design incidences developed by Moustapha et al. (1990). The parameter was later modified by Benner et al. (1997) to the form given in Equation 4.4. The relationship between K_L and χ , derived from the measured and extrapolated data for a fully roughened blade, is shown in Figure 4.4. From Equation 4.2, the incremental loss due to roughness, at design incidence, can be expressed as:

$$\left(\frac{\Delta Y_r}{Y_{PS}} \right)_{des} = (K_L)_{des} - 1$$

Consequently, the functional relationship between K_L , $(K_L)_{des}$ and the off-design incidence parameter χ can be written as:

$$\frac{(K_L - 1)}{((K_L)_{des} - 1)} = f(\chi)$$

or,

$$K_L = ((K_L)_{des} - 1)f(\chi) + 1 \quad (4.4)$$

where, from Figure 4.4,

$$f(\chi) = 1 + a\chi + b\chi^2 + c\chi^3, \quad \text{for } -13 \leq \chi \leq 6.5$$

and,

$$a = -4.40.10^{-2}$$

$$b = -1.46.10^{-2}$$

$$c = -4.06.10^{-4}$$

For $6.5 \leq \chi \leq -13$ the value of K_L is taken as 1.0.

It is noticeable from Figure 4.4 that the effect of roughness diminishes rapidly with the increase in off-design incidence. At large values of off-design incidences the losses are dominated by the effects of boundary layer separation and the incremental losses due to roughness become relatively small by comparison. As would be expected, this effect is larger at positive values of incidence.

A similar relationship between K_L and χ is shown in Figure 4.5 for a partially covered blade with a roughness patch on the suction side originating at $x/c = 0.05$ and of length $\Delta s/c = 0.50$. In this case $f(\chi)$ is approximated as follows:

Figure 4.4 Effect of Surface Roughness at Off-design incidence for a Fully Roughened Blade

Figure 4.5 Effect of Partially Roughened Surface ($\Delta s/c=0.50$) on the Losses at Off-design incidence

$$f(\chi) = 1 + a\chi + b\chi^2 + c\chi^3, \quad \text{for } -10 \leq \chi \leq 5$$

and,

$$a = -7.60 \cdot 10^{-2}$$

$$b = -1.61 \cdot 10^{-2}$$

$$c = -1.47 \cdot 10^{-4}$$

For $5 \leq \chi \leq -10$ the value of K_L is taken as 1.0.

Based on the data presented in Figure 4.6, the roughness condition is divided into three levels according to its magnitude as shown below:

Low roughness	0 - 50 μm
Moderate roughness	51 - 100 μm
Severe roughness	101 - above μm

The chord length of the blades of the F404 engine varies from 2.16 inches to 1.23 inches. From Figure 4.3, the corresponding non-dimensional standard roughness heights for the different surface conditions are:

Low roughness	$k_s/c = 0 - 0.0072$
Moderate roughness	$k_s/c = 0.0073 - 0.0144$
Severe roughness	$k_s/c = 0.0145 - 0.025$

For this range of k_s/c values the corresponding average values of K_L are determined from Figure 4.3. Table 4.4 summarizes the multiplying factors developed.

Inspection of the loss ratios suggested in the table reveals that for an initial 25% covered suction surface, losses will increase considerably for any roughness condition. In fact it will be about twice as much for severely roughened blade surface. However, it is notable that for a low roughness condition, even when the whole of the blade surface is roughened, losses will increase by about 40% only. But in case of moderate or severe roughness this is increased by about 80% . It can be concluded that the performance of the F404 engine will deteriorate considerably for any amount of moderate or severe roughness conditions.

Table 4.4
Design-point Loss Ratio (K_L) at Different Roughness Conditions
for the F404 Engine (roughness patches starting at $x/c=0.05$)

Roughness Condition	K_L			
	25% Suction Surface	50% Suction Surface	95% Suction Surface	95% Both Surfaces
Low	1.2	1.4	1.5	1.7
Moderate	1.6	2.1	2.4	2.8
Severe	2.0	2.8	3.2	3.7

4.4 Effect of Surface Roughness at Transonic Flow

There are no experimental data available for the effect of surface roughness at transonic flow conditions. However, Jouini et al. (1998) used computational methods to simulate the effects of surface roughness at compressible flow conditions. The computations included only design incidence. They introduced a new correction factor K_M to correlate the influence of Mach number on the incremental profile loss due to roughness. It was found that the incremental losses due to roughness decreases gradually with Mach number. Their results supplement the subsonic correlations and therefore, the transonic correlations developed by Jouini et al. are briefly described here to give a complete picture of the effect of surface roughness on profile losses. The author wishes to make it explicitly clear that the transonic correlations are borrowed from Jouini's work and he has no contribution, whatsoever, in its development.

Jouini used only one roughness height (k/c) of 0.00628 and one roughness patch configuration for his computations. The leading edge of the patch was located at $x/c = 0.25$ and the patch covered 47% of the suction surface. Calculations were carried out at design incidence with the exit Mach number varying from 0.71 to 1.31. Figure 4.7 depicts the variation of incremental profile loss due to roughness with Mach number.

The incremental losses due to surface roughness shown in Figure 4.7 may be approximated by the following curve fit:

$$K_M = \frac{\Delta Y_{PR}(M)}{\Delta Y_{PR}(M=0)} = \frac{ab + cM^d}{b + M^d} \quad (4.5)$$

where:

$$a = 0.1692 \quad b = 0.5559 \quad c = 1.0069 \quad d = -8.3828$$

$\Delta Y_{PR}(M=0)$ is the incremental rough surface profile loss at incompressible conditions, as presented in Section 4.2.2. $\Delta Y_{PR}(M)$ is the incremental rough surface profile loss at the required outlet Mach number. Equation 4.5 applies for exit Mach numbers from about 0.55 to 1.5.

Figure 4.7 Variation of Incremental Profile Loss due to Roughness with Mach Number (reproduced from Jouini et al., 1998)

Equation 4.5 is to be used with the low-speed design incidence correlations (Equation 4.3) to compute the profile losses for rough turbine blades at transonic flow conditions. From Equations 4.4 and 4.5 then:

$$\frac{\Delta Y_{PR}(M)}{\Delta Y_{PR}(M=0)} = \frac{\Delta Y_{PR}(M)}{Y_{PR}(M=0) - Y_{PS}(M=0)} = K_M$$

or,

$$K_M = \frac{\Delta Y_{PR}(M)}{Y_{PS}(M=0)(K_L - 1)} \quad (4.6)$$

or,

$$\Delta Y_{PR}(M) = K_M(K_L - 1)Y_{PS}(M=0) \quad (4.7)$$

Thus, the incremental loss due to roughness at compressible flow conditions is obtained from the baseline (smooth) profile loss coefficient before the compressibility correction is applied. The final profile loss coefficient, with roughness, for the outlet Mach number of interest is then:

$$Y_{PR}(M) = Y_{PS}(M) + K_M(K_L - 1)Y_{PS}(M=0) \quad (4.8)$$

where $Y_{PS}(M)$ is the smooth-surface profile loss coefficient as corrected for Mach number. The approach outlined appears reasonable for outlet Mach numbers from about 0.55 to 1.50.

The modified profile loss corrections can be used with the Kacker & Okapuu (1982), or similar, loss correlations to estimate the overall losses in the turbine.

5 Experimental Apparatus

5.1 Introduction

The test facility used in the present investigation is the Carleton University High-Speed Wind Tunnel. The wind tunnel was originally designed and constructed at McMaster University, Hamilton, Ontario. It was later purchased by Carleton University and reinstalled in the High Speed Aerodynamics Laboratory located in the Minto Centre. Three graduate students have been involved in the recommissioning of this wind tunnel. Jeffries (1994, 1999) was primarily involved in the overall set-up and testing of the facility including making a number of improvements to the performance of the facility, particularly the pressure control system. Jouini (1999) was primarily involved in the design and fabrication of cascades of turbine blades and probe traverse mechanisms. The author was involved in the design and fabrication of fast response pressure probes required for flow measurements in the test section. The High Speed Wind Tunnel was used for the present experimental work on blade trailing-edge damage. The following sections provide a brief description of the new High Speed Wind Tunnel and instruments used in the present investigation. Detailed description of the various components and operation of the wind tunnel can be found in Jeffries (1999) and Jouini (1999).

5.2 High Speed Wind Tunnel

Figure 5.1 shows the installation of the high speed wind tunnel in the laboratory, including the cascade test section and ejector-diffuser system. It is a blow down intermittent type high speed wind tunnel. The operation of the wind tunnel is initiated by first filling the large storage tanks

Figure 5.1 High Speed Wind Tunnel (reproduced from Jeffries, 1999)

with high pressure air using a compressor. The compressor is a 100 HP (75 KW) two-stage reciprocating type located in the basement of the Minto Centre. The air is passed through an air drier prior to filling the tanks. This lowers the dew point of the air to about -45°C . The compressor takes about 15 minutes to pressurize the four storage tanks of the wind tunnel to 100 psig (690 KPa). These tanks have a total volume of about 26 m^3 . After the storage tanks are filled, the control valve connected between the test section and the reservoir is opened and the air is discharged through the test section at high velocity. The air mass flow rate can be as high as 3 kg/sec, and the cascade outlet Mach number can be as high as 1.5. The pressure control system quickly establishes the desired blowing pressure in the test section and maintains it approximately constant during a run. A typical run is about 30s in duration. As a result, this necessitates the use of fast response pressure probes for flow measurements and a fast data acquisition system. The control system is operated using a 486DX2 microcomputer. The pressure control system, developed by Jeffries (1999), uses the feed-back Proportional-Integral-Derivative (PID) type algorithm to maintain the test section blowing pressure. The purpose of the ejector-diffuser assembly is to allow independent control of both the Reynolds and Mach numbers in the test section by controlling the pressure at the outlet of the cascade. The ejector-diffuser system was not used during the measurements in the present study.

5.3 Cascade Test Section

As can be seen in Figure 5.1 the upper wall of the test section has steel ribs which provides structural rigidity to the wall. This reinforcement is necessary because of the high pressures in the wind tunnel during a run. Figure 5.2 shows a schematic diagram of the interior of the test section including the measuring probe and traverse gear in plan view. The turntable is used to vary the incidence of the inlet flow to the cascade blades. The turntable is supported on a worm and gear mechanism and can be rotated to vary the values of the incidence from about -20° to $+20^{\circ}$. A protractor mounted on the periphery of the turntable allows the incidence angle to be set to within $\pm 0.2^{\circ}$.

Figure 5.2 A Schematic Diagram of the Cascade Test Section (reproduced from Jouini, 1999)

The cascade inlet side walls are attached to the turntable by a parallelogram linkage. This linkage system allows the inlet side walls to move parallel to each other for all turntable angular positions. Since the cascade inlet side walls adjust position with the angle of the turntable, the cascade inlet area varies from a maximum width of approximately 280 mm to a minimum of approximately 120 mm. The height of the internal passage of the test section is 61 mm. The turntable accommodates the cascade of blades. For the present investigation seven blades were mounted on a base plate giving a total of 8 complete blade passages.

5.4 Probe Traversing Mechanism

The probe traversing mechanism is mounted to the downstream transition side wall as shown in Figure 5.2. The measuring probe is mounted inside a probe holder which is in turn mounted at the end of the probe stem. The stem protrudes through the right side transition wall through an opening that allows spanwise and pitchwise adjustment of the probe. The vertical traversing gear was used to adjust the probe at midspan and it was held fixed at that position for all measurements. The transverse traversing gear was used for pitchwise movement of the probe. The traversing mechanism is mounted to the cascade test section using extension arms. These arms allow the independent motion of the ejector-diffuser system for different values of incidence without affecting the position of the probe holder. A high speed stepper motor drives the transverse traversing gear. A Pentium 90 computer is used to control the motor. The traversing mechanism was calibrated for the pitchwise motion (see Jeffries, 1999). From the calibration data, an estimated accuracy of better than ± 0.05 mm for three blade pitches traversed was obtained.

5.5 Cascade Geometry for Undamaged and Damaged Blades

The turbine blade used for the present investigation is designated as HS1A. HS1A is one of two cascades used by Jouini (1999) in a study of incidence effects in transonic turbines which is being conducted for Pratt & Whitney Canada. Spare blades from this blade set were modified with simulated trailing-edge damage for use in the present study. This blade represents a mid-section profile of a high pressure rotor of a modern Pratt & Whitney gas turbine engine. The blade coordinates were provided by Pratt & Whitney Canada. In order to maintain the accuracy in the blade coordinates, the blades were machined to the highest possible tolerance that can be obtained with the numerical machining process. After polishing the tolerance on all the blade coordinates were within ± 0.0005 inches. A summary of the main geometric specifications of the HS1A blade

are presented in Figure 5.3. Table 5.1 shows all the aerodynamic design flow conditions for the blade where the flow angle is measured from the axial direction.

Two blades were instrumented with static pressure taps to obtain static pressure distribution. One blade was instrumented with 19 static pressure taps on the pressure surface and the other blade was instrumented with 22 static pressure taps on the suction surface including one base pressure tap at the trailing edge. These blades were placed adjacent to each other during measurements to form the pressure surface and suction surface of the central blade passage.

As mentioned, two blades were used to simulate trailing-edge damage. One blade had a semi-circular cutout with a radius of 20% of the blade true chord machined from the trailing edge. The second blade had a cutout of 10% of the blade chord and also instrumented with a pressure tapping at the centre of the cutout to measure base pressure. This blade was again used to measure trailing-edge base pressure for larger damage size by enlarging the cutout to 20% chord. One damaged blade at a time was used in the cascade passage during measurements. Figure 5.4 shows a schematic diagram of a damaged blade.

Table 5.1
Blade Aerodynamic Parameters for Design Flow Condition

Parameter	Value
Inlet Mach Number, M_1	0.53
Outlet Mach Number, M_2	1.045
Inlet Flow Angle, α_1 (degrees)	46
Outlet Flow Angle, α_2 (degrees)	58.49

Chord Length, c	40.0 mm	Trailing Edge Wedge Angle, We_t	6.0°
Blade Span, h	61.0 mm	Leading Edge Wedge Angle, We_l	38.0°
Blade Spacing, s	29.14 mm	Design Incidence, i_{des}	-4.5°
Aspect Ratio, h/c	1.525	Stagger Angle, ζ	25.1°
Leading Edge Metal Angle, β_1	50.5°	Throat opening, o	15.3mm
Trailing Edge Metal Angle, β_2	59.0°	Unguided Turning Angle, θ_u	11.5°

Figure 5.3 Summary of Cascade Geometry for HS1A Blade

Case 1 : $R = 10\%$ of Chord
Case 2 : $R = 20\%$ of Chord

Figure 5.4 Damaged Blade

5.6 Miniature Fast Response Probes

5.6.1 General Overview of Selection of Probes for High Speed Flow Measurements

An appropriate probe is required for the present objective of measuring the flow field at the cascade outlet. The choice of the appropriate probe largely depends on the cascade flow characteristics and the research objective. Certain types of probes are better suited for a particular task than others. For the present blow-down type intermittent test facility a fast response probe is essential because of the limited measuring time. The runtime for the High Speed Wind Tunnel is typically about 30 seconds. The Mach number of the flow is in the range 0.5 to 1.2. For high-speed flow measurements the probe size should be as small as possible because the majority of errors are directly related to the probe size (Sieverding, 1975, 1994); Humm et al., 1994). On the

other hand, if the probe and pneumatic tubing are too small the pneumatic response becomes slow. Therefore, a compromise must be reached on the size.

A three port probe, a pitot probe and a static probe have been designed for use in the present wind tunnel facility. Though some work has been done on the development of the popular five-hole and wedge-type probes in terms of probe geometry, sensor placement, transducer mounting, and aerodynamic performance (Ainsworth, et al., 1994; Humm, et al., 1994; Naughton, et al., 1993) limited work has been reported so far in the literature concerning the design of fast response three port cobra type probes. Sieverding et al.(1994) used a miniature fast response three hole probe for surveying the outlet flow field of a transonic turbine guide vane with apparent success. Sieverding showed that the three hole probe is adequate for the resolution of pitchwise pressure gradients provided the probe tip dimension is less than about 6% of the blade pitch.

A simplified treatment of the response of pneumatic pressure measuring systems due to a change in pressure has been described in many papers and textbooks (e.g., Ducoffe, 1953; Benedict, 1960; Kinsler & Frey, 1962). The analysis shows that the resultant time constant of the system is inversely proportional to d^4 (where d is the smallest diameter tubing in the system), and proportional to the effective length of the tubing and the volume of the system. Minimizing the probe head dimension to obtain good spatial resolution adversely affects the response time of the measuring system since this results in a small value of d . This can be compensated by placing miniature fast response transducers close to the probe tip thereby reducing both the effective length and volume of the system (see Naughton et al., 1993; Humm et al., 1994). For the purposes of the present investigation, probes were initially designed with a very small probe head and Kulite miniature transducers (0.064 inches diameter) placed inside of the probe stem. However, during initial testing of the probes in the high speed wind tunnel test section, it was found that the miniature transducers inside the probe body were sensitive to the changes in the operating temperature. Subsequently measurements were made with the miniature transducer placed in a Scanivalve pressure scanner which was located a short distance outside the test section. Tests showed that this arrangement did in fact give acceptable time response.

The following sections describe the geometry of the pitot probe, the static probe and the three port cobra probe designed for the present work. The calibration and performance of the probes (e.g. accuracy, blockage etc.) are discussed in Chapter 6.

5.6.2 Carleton University Static and Pitot Probes

The flow over the tip of the probe causes pressure variations such that it is some distance along the probe before the static pressure is equal to free stream static pressure. In the case of transonic flow the effect of the pressure field of the tube support also influences the pressure at the static holes. The correct position of the static orifices depends partly on the shape of the probe head and partly on the position of the stem or support. Literature shows that for compressible flow measurements (Mach number < 1.6) the static holes can be placed more than 10 tube diameters (smallest diameter tube containing the measuring holes) downstream from the tip for cone angles less than 30 degrees (Ower, 1977; Sieverding, 1975). The distance between the probe stem and the measuring plane has to be a maximum without sacrificing probe rigidity. A stem distance of 18 diameters from the measuring plane is sufficient for accurate measurements in transonic flows (Ower, 1977). Probe tip dimensions are typically not more than 6% of the blade pitch when used for midspan traverses in transonic flows (see Kiock et al., 1986; Mee et al., 1992).

The static pressure probe which has been designed is shown in Figure 5.5. Detailed drawings of the probe can be found in Appendix A. Concentric stainless steel hypodermic tubes were used to form the main portion of the probe. The diameter of the tube with the measuring holes is 0.04 inch (1.02 mm) outside diameter (OD). This diameter is about 3.5 percent of the blade pitch used in the present experiment. The downstream end of the measuring tube is soldered to a short 0.04 inch OD bulged tube. The bulged tube is used to connect the measuring tube to the vinyl tubing. Two static holes with 0.3 mm diameter and separated by 180 degrees are used to measure the static pressure. The length between the conical probe tip and the measuring holes is about 12 times the tube diameter which is slightly over the recommended limit of 10 tube diameters. The probe tip has a cone angle of 15 degrees to minimize flow disturbance. The hypodermic tube is supported coaxially by a larger tube of outside diameter 0.066 inch (1.7 mm) with its leading edge 30 mm downstream from the measuring holes. This is in effect nearly 30 tube diameters downstream from the measuring holes which is well over the recommended limit of 18 tube diameters. This tube within a tube assembly is again connected to the probe stem 55 mm downstream from the measuring holes. The probe stem is a hollow tube with an outside diameter of 0.275 inches (7 mm). It was originally used to house the Kulite transducer but as previously mentioned the transducer was later moved from the probe body to outside the test section. The

Dimensions are in mm unless otherwise specified

Figure 5.5 Geometry of Carleton University Static Probe

circular shoulder plate and square block at the rear end of the probe is used to align the probe in the probe holder.

The construction of the total pressure probe is quite similar to that of the static pressure probe, as shown in Figure 5.6. The tubular dimensions of the probe head and body are identical to that of the static probe. Detailed drawings of the pitot probe can be found in Appendix A.

Dimensions are in mm unless otherwise specified

Figure 5.6 Geometry of Carleton University Pitot Probe

5.6.3 Carleton University Three Hole Probe

A three hole pressure probe was initially designed and manufactured with a chamber in the stem to hold three miniature Kulite pressure transducers in the same way as for the pitot and static probes. Unfortunately, this probe was destroyed during testing due to a fatigue failure in the probe holder. Since it had subsequently been found that it was not necessary or even desirable to have the pressure transducers inside the test section, the replacement three hole probe was manufactured without the transducer cavity. The new three hole probe is shown in Figure 5.7. Detailed drawings of the probe can be found in Appendix B.

The probe consists of a 165 mm long solid stainless steel rod and three stainless steel hypodermic tubes with outside diameters of 0.018 inches (0.46 mm). The steel rod is machined to 0.275 inches (7 mm) OD near the probe stem and tapered down to 0.07 inch (1.78 mm) OD near the probe head. The three pressure measuring orifices at the probe tip are formed by the three hypodermic tubes laid side by side inside a slot made in the probe body. The three tubes protrude 5 mm outside the supporting steel body and the exterior portion of the tubes are soldered together. The interior portion of the tubes extend up to the probe stem where they are connected to vinyl tubing. The two outer hypodermic tubes are machined to form a 30 degree semivertex angle. The open end of the slot is filled with solder and blended smoothly with the outside surface. The total width of the probe tip is 0.054 inches (1.37 mm) which is approximately 4.7% of the blade pitch in the present measurements. As previously noted probe heads should not be more than 6% of the

Figure 5.7 Geometry of Carleton University Three Hole Probe

blade pitch when used for downstream measurements in transonic flows (Kiock et al., 1986; Mee et al., 1992) to obtain reasonably accurate results. The miniature three hole probe used by Sieverding et al. (1994) for downstream flow measurements at an exit Mach number of 1.05 had a maximum tip dimension of approximately 4.4% of the blade pitch.

5.7 Data Acquisition System

Figure 5.8 shows schematically the data acquisition system used to collect all the measurements in the test section. The system basically consists of the following components:

- (a) A HP3852A Data Acquisition Control Unit;
- (b) A HP44702B 13 bit high speed voltmeter;
- (c) SSS-48C 48 port Scanivalve Pressure Scanner;
- (d) A HP44708A 20-channel Relay Multiplexer with Thermocouple Compensation (MUX/TC);
- (e) Kulite miniature transducer, Model XCQ-062;
- (f) And a Pentium 90 desktop computer.

The Pentium 90 computer was used to control the main frame 3852A Data Acquisition Unit. The Data Acquisition Unit uses the HP44702B high speed voltmeter to measure all voltage values. The high speed voltmeter can make DC voltage, resistance, strain, and temperature measurements at speeds up to 100,000 readings per second. The number of readings per second is dependent on the voltmeter programming mode, the type of measurement performed, and the type of controller used with the voltmeter. The voltmeter resolution is 13 bits (12 bits and sign bit). The voltmeter uses the successive approximation technique for analog-to-digital conversion. The voltmeter has four voltage ranges; 40 mV, 320 mV, 2.56 V and 10.24 V. The effective range for DC voltage measurements is from about $\pm 10 \mu\text{V}$ to $\pm 10.24 \text{ V}$ peak.

A Kulite miniature transducer (model XCQ-062) was used to convert the pressure signals measured in the cascade into analog signals. This 25 psia transducer was mounted in the

transducer housing of the single 48 port Scanivalve module. The Scanivalve System is capable of scan rates of 15 ports per second. The signal output from the transducer is fed to the rear panel of the HP44702B high speed voltmeter.

For temperature measurements, the HP44708A 20-Channel Relay Multiplexer unit with Thermocouple Compensation (MUX/TC) is used. Scanning rates of 450 channels/second can be achieved using this multiplexer unit. The multiplexer consists of a 20-channel terminal module with an isothermal connector block and a relay MUX component module.

The sampling rates and sampling times used during data acquisition is discussed in Chapter 6.

Figure 5.8 Data Acquisition System (reproduced from Jouini, 1999)

6 Measurement Procedure

6.1 Introduction

Measurements were made for both damaged and undamaged blades for cascade HS1A for a wide range of Mach numbers, at design and off-design values of incidence. Measurement data for the undamaged blades were jointly collected by the author and Jouini (1999) since both persons projects employ the same cascade. Jouini was also involved in the development of the data reduction programs PROCESS and MIXLOSS, which were used to process the collected data to obtain the flow angle, dynamic pressure, total pressure loss etc. The work load was managed in such a way that the present author was primarily responsible for collection of data for both damaged and undamaged blades, while Jouini mostly put his efforts to processing the data. As a result, some of the Figures relating to the present measurements will also appear in the thesis by Jouini (1999). More details on data reduction methods used in the present measurements can be found in Jouini (1999) while merely a summary is given here.

The following sections describe the data acquisition and sampling procedures used. A brief description of the performance of the three hole probe is presented first.

6.2 Three Hole Probe Performance

6.2.1 Probe Calibration and Accuracy

The three hole probe was the main instrumentation used to measure most of the data. The probe was calibrated for yaw angles from -10° to $+10^\circ$ with a pitch angle of zero degrees and a

Mach number range of 0.5 to 1.0. The calibration procedure follows the method of Lewis (1966) and detailed description of the procedure can be found in Jeffries (1999). Though the probe was not calibrated for Mach numbers greater than unity it is assumed that the same yaw calibration applies for Mach numbers only slightly larger than 1.0 (maximum Mach number 1.2). However, deviation values obtained ⁶⁻²⁰ ~~this~~ yaw calibration should be treated with caution. The total pressures measured with the centre port are corrected for shock effects for Mach numbers greater than 1.0. Jeffries showed that for pitch angles within $\pm 10^\circ$, the maximum error in the measured angle can be ± 0.5 degrees.

The estimates of uncertainties for the probe measurements are summarized in Table 6.1 along with the uncertainties for some other quantities. The uncertainty estimates include both accuracy and repeatability, as well as transducer calibration errors. The uncertainties were obtained by consideration of all test cases over the whole range of Mach number (0.7 to 1.2) and the whole period of measurements (over two months).

Table 6.1
Estimates of Uncertainties Associated with the Measurements

Quantity	Maximum Estimated Error
Total and Static Pressures	$\pm 2\%$ of local mixed-out dynamic pressure
Flow Angle	± 0.5 degrees
Mach Number	± 0.02

6.2.2 Probe Blockage Effects

The presence of a probe in transonic flow inevitably alters the local flow conditions due to the blockage of the probe, particularly if the stem is too close to the probe tip. Ideally the probe should be so small that the flow in the measurement plane is not affected by its presence. As the size of the probe is increased relative to the flow area, blockage effects increase and erroneous

results will be obtained from the probe. When the blockage area of the probe is sufficiently small compared to the flow area, the flowfield is not significantly affected by the presence of the probe.

As reported by Sieverding (1993), one of the most comprehensive studies on probe blockage effects is the one due to Wyler (1975). Wyler quantified the blockage effects as a change in both the downstream Mach number and total pressure. These changes in the downstream flow quantities are expressed as a function of the ratio of the probe head diameter (d) to the jet flow diameter (D). The graphical representation of Wyler's data shows that for a diameter ratio (d/D) of 0.05, the probe blockage effects result in a maximum change of less than 2.0% of the Mach number itself. In the present case, the jet flow diameter is taken as the hydraulic diameter of one blade passage area at outlet which is about 48 mm. All the probes used in the present measurements have a maximum probe blockage area less than 0.03 which implies a percentage change in Mach number of less than 1.0%. In addition, the probe stem/holder assembly is located at a downstream distance of 3.75 times the blade true chord length. The results presented by Sieverding show that probe stem effects become significant only when the probe stem is located at a distance less than one true chord length. Sieverding (1994) also suggested that if the wall static pressure in the measurement plane is not significantly disturbed with the presence of the probe compared to that in absence of the probe then probe blockage effects are negligible. Measurements made in the present experiment showed that for all operating conditions the change in the wall static pressure in the measurement plane due to the presence of the probes was less than 1% of the dynamic pressure.

6.3 Transducer Calibration

The transducer used for the pressure measurements was calibrated prior to each session of collecting data. This was done to account for the day to day variation in atmospheric conditions and any changes in the transducer characteristics. A mercury manometer was used for the purpose of calibration. During calibration the pressure was applied gradually in both directions in order to

detect any hysteresis. Figure 6.1 shows the collective calibration points for most of the calibrations conducted in a period of over two months. The data exhibit good linearity characteristics.

Figure 6.1 Kulite Transducer Calibration Curve (Model XCQ-62)

6.4 Data Acquisition

6.4.1 Procedure

The probe measurements were conducted at a plane located 40% of true chord (45% of axial chord) downstream of the cascade row. Mid-span traverses in the measurement plane were made using the three hole probe to obtain the total pressure and the flow angle. The static probe was used to measure the static pressure. A total pressure probe was used to monitor the blowing pressure at all operating conditions. This total pressure was used as the reference total pressure (P_{0ir}). A static tap located on the upper endwall at a distance of one blade true chord upstream of the blade row was used to measure the reference inlet static pressure (P_{ir}). Both the reference pressures were used to monitor the cascade inlet isentropic Mach number. Before conducting any downstream traverses a pitot-static probe placed at a distance of one true chord upstream of the blade row was used to measure the inlet Mach number. The measurement of the pitot-static probe was referenced to the inlet total pressure (P_{0ir}) and the static pressure (P_{ir}). The pitot-static probe was removed from the test section when downstream traverses were made. The inlet Mach number was monitored during each run based on the relationship established between the upstream pressure measurements (i.e. P_{0ir} and P_{ir}) and the ones obtained when using the pitot-static probe. Due to the short duration of the wind tunnel run time, each blade pitch traversed in the downstream measuring plane was divided into four runs. The procedure for downstream flow measurements is described next.

Figure 6.2 shows the upstream and downstream measuring planes including the upstream and downstream static tap locations. A downstream traverse over one blade pitch is typically accomplished in four runs. Figure 6.2 shows schematically the distribution of measurement points for each run. Each run takes about 15 seconds to complete. In each run there are 10 measuring points equally spaced. The spatial resolution for passes 2 and 3 is half of the one used for passes 1 and 4. This was chosen in order to have a better definition of the downstream wake. Before collecting any data, the probe was first centered on the blade wake. The wake was located by trial and error during preliminary runs. This procedure determines the starting location of the downstream traverse. A ruler placed along the traverse direction on the side of the traversing mechanism was used to monitor the starting position for all subsequent wake traverses. Using this

Figure 6.2 Cascade Blade Row Measurement Locations (reproduced from Jouini, 1999)

procedure, the data collected at each traversing point using the static probe were assigned to the corresponding measuring point for the three hole probe. In addition, a downstream static tap located at the measuring plane on the cascade base plate, as indicated in Figure 6.2, was used to monitor the pressure ratio across the cascade blade row during each test run. This is the reference downstream static pressure. This pressure ratio is used to match the same operating conditions during the four runs.

Once the wake measurements were completed, blade loading as well as trailing edge base pressure data were collected. When taking loading measurements, all reference pressures (i.e. the two upstream and the one downstream) were also collected in order to match the operating conditions with that used for the wake traverses.

6.4.2 Data Sampling Times and Rates

Jeffries et al. (1997) conducted an experimental investigation of the effects of both the sampling times and the sampling rates on the accuracy of the mean values of the flow quantities. The investigation was for both total and static pressures. In the investigation both low and high frequency contents of the pressure signal were examined. They found that the wind tunnel test section flow contained fluctuations with a frequency of the order of 50 Hz. These fluctuations were traced to the wind tunnel control system. In addition to these low frequencies, the pressure signal contained high frequency fluctuations due to the turbulence of the flow. For all flow conditions the authors found that satisfactory mean pressure measurements could be obtained by sampling for 0.1 second at a minimum frequency of 2000 Hz. In addition, to eliminate the low frequency content, they recommended normalizing the pressure values by the test section reference inlet total pressure. The test section reference pressure is used to monitor the test section blowing pressure. The integration time of 0.1 seconds gave errors less than 1% of the mean value obtained over a long period of integration. In the present experiments, the data sampling procedure recommended by Jeffries et al. (1997) was adopted.

6.5 Flow Conditions

6.5.1 Reynolds Number and Turbulence Intensity

In the present experiment the test section flow discharged to a constant outlet pressure. As a result both Mach number and Reynolds number increased with increasing blowing pressure. The Reynolds number based on true chord and downstream flow quantities ranged from 4×10^5 to 10^6 for different exit Mach numbers varying from 0.5 to 1.2. The turbulence intensity in the test section was around 4% (Corriveau, 1999).

6.5.2 Inlet Flow Uniformity

The uniformity of the cascade inlet flow was determined by Jeffries (1999) as part of the wind tunnel commissioning. For the present experiment it was assumed that the inlet flow uniformity remained unchanged. The boundary layers were found to extend less than 20% of span.

6.5.3 Outlet Flow Periodicity

Flow periodicity measurements were made using the three hole probe at blade midspan. No downstream control surfaces were used. Figure 6.3 and 6.4 shows the downstream total pressure traverses at 40% chord length downstream of the blade row for incidence values of 0° and 4.5° respectively and at three representative values of outlet Mach number. For the design incidence two wakes near the centre of the cascade are shown while at 4.5 degrees three adjacent wakes are presented. It is evident from the Figures that for all isentropic exit Mach numbers the wake profiles are very similar, indicating that there is good periodicity in the cascade flow.

Figure 6.5 shows the downstream flow angle traverses for two blade pitches at design incidence. Some variation in flow angle is expected in the blade wake since the wake is filling through the mixing process. Figure 6.5 shows that some angle variation was detected with the probe. However, it should be noted that there is some uncertainty about the flow angles inferred from the three hole probe measurements in the wake due to the shear in the velocity. This shear leads to a difference in the pressures sensed by the side holes of the probe and this difference will be erroneously interpreted as flow angularity. As mentioned in Section 5.6.3, the width of the head of the three hole probe is about 4.7% of the blade spacing or about 6% of a typical wake width. While there may be some errors due to shear effects, the flow angles shown in Figure 6.5 again confirm that there is good periodicity in the flow from blade to blade.

Figure 6.6 shows the downstream static pressure traverses for two blade pitches at design incidence. The static pressure distribution is quite uniform at exit Mach number of 0.85. The observed rise in static pressure at exit Mach numbers of 1.05 and 1.15 can be attributed to the crossing of a shock wave by the probe.

Figure 6.3 Downstream Total Pressure Traverses at Design Incidence

Figure 6.4 Downstream Total Pressure Traverses at an Incidence of 4.5 Degrees

Figure 6.5 Downstream Flow Angle Traverses at Design Incidence

Figure 6.6 Downstream Static Pressure Traverses at Design Incidence

6.6 Probe Measurements

6.6.1 Total Pressure Measurements

Downstream total pressure measurements were obtained using the three hole probe, and for comparison purposes additional traverses were also conducted using the total pressure probe. Figure 6.7 shows a comparison of the measured total pressures obtained from the two probes. The agreement is quite satisfactory.

Figure 6.7 Comparison of Downstream Three Hole and Total Pressure Probe Traverses

For outlet Mach numbers greater than unity, a bow shock was assumed to exist in front of the probe mouth. Therefore, the total pressure was corrected to obtain the pressure upstream of the shock using the following equation:

$$\frac{P_{02c}}{P_{02}} = \left[\frac{\frac{\gamma+1}{2} M_2^2}{1 + \frac{\gamma-1}{2} M_2^2} \right]^{\frac{\gamma}{\gamma-1}} \left[\frac{\gamma+1}{2\gamma M_2^2 - (\gamma-1)} \right]^{\frac{1}{\gamma-1}} \quad (6.1)$$

Figure 6.8 shows the nomenclature adopted in the above equation. The total pressure correction was performed for each traversing point along the pitch wise direction. The corrected total pressure value is the one used in the mixed-out loss calculations.

Figure 6.8 Flow in the Vicinity of the Total Probe Mouth

6.6.2 Base Pressure Measurements

Base pressure measurements were performed using the HS1A blade which is instrumented with static taps on the suction surface. This blade has a static tap at the base of the trailing edge. The base pressure (P_b) results were referenced to the mixed-out downstream static pressure P_{s2} and non-dimensionalized by the downstream dynamic pressure q_2 , to give the base pressure coefficient as follows:

$$C_{bp} = \frac{P_b - P_{s2}}{q_2} \quad (6.2)$$

where $q_2 = \frac{1}{2} \rho V_2^2$

Based on repeatability measurements, the estimated uncertainty on the measured base pressure is less than $\pm 2\%$ of the downstream mixed-out dynamic pressure.

6.6.3 Axial Velocity Density Ratio (AVDR)

The value of the axial velocity density ratio (AVDR) is an indication of the two dimensionality of the flow through the cascade. AVDR is the ratio of mass flow rate per unit area at the outlet of the cascade to that at the inlet. A value of unity for AVDR, as evaluated at midspan, indicates that on average there is no contraction or expansion of the streamtube passage through the cascade. In the present facility, there was no control over the value of the AVDR. Such control is usually achieved through bleeding of the sidewall boundary layers since the growth of these boundary layers is the main source of streamtube contraction.

Using the nomenclature shown in Figure 6.9, the AVDR can be calculated using the following equation:

$$AVDR = \frac{\int_0^1 (\rho_2 C_{ax2})_{MS} d\left(\frac{y}{s}\right)}{\int_0^1 (\rho_1 C_{ax1})_{MS} d\left(\frac{y}{s}\right)} \quad (6.3)$$

where, $\rho_{1,2}$ = air densities upstream and downstream of blade row respectively,
 $C_{(ax)1,2}$ = upstream and downstream axial flow velocities respectively,
 y = distance along pitchwise direction,
 s = blade pitch,
 MS = midspan flow quantities.

Figure 6.9 Axial Velocity Density Ratio Nomenclature
 (reproduced from Jouini, 1999)

Using the isentropic relations and assuming air to behave as an ideal gas, the AVDR given in Equation 6.3 is rearranged to give the following expression:

$$AVDR = \frac{\int_0^1 \left[\frac{P_2}{P_{01}} M_2 \sqrt{\frac{T_{01}}{T_{02}}} (\cos \alpha_2) \left(1 + \frac{\gamma-1}{2} M_2^2 \right)^{\frac{1}{2}} \right] d\left(\frac{y}{s}\right)}{\int_0^1 M_1 (\cos \alpha_1) \left(1 + \frac{\gamma-1}{2} M_1^2 \right)^{\frac{-(1+\gamma)}{2(\gamma-1)}} d\left(\frac{y}{s}\right)} \quad (6.4)$$

Equation 6.4 shows that the AVDR is a function of the upstream and downstream total temperature ratio. In the present experimental study, no total temperature measurements were conducted downstream of the blade row. In an investigation on the total temperature field downstream of a stationary turbine cascade, Carscallen & Oosthuizen (1989) showed that for an adiabatic flow there can be a redistribution of the total temperature. Their measurements showed a deficit in total temperature in the wake region, but an increase in total temperature at the boundaries of the wake. This behavior was a function of the downstream isentropic Mach number. However, they concluded that the area averaged downstream total temperature is the same as the upstream one as should be the case in adiabatic flow. Consequently, it seemed a reasonable assumption to neglect the effect of the downstream total temperature on the AVDR. This assumption leads to the following simplified expression for AVDR:

$$AVDR = \frac{\int_0^1 \left[\frac{P_2}{P_{01}} M_2 (\cos \alpha_2) \left(1 + \frac{\gamma-1}{2} M_2^2 \right)^{\frac{1}{2}} \right] d\left(\frac{y}{s}\right)}{\int_0^1 M_1 (\cos \alpha_1) \left(1 + \frac{\gamma-1}{2} M_1^2 \right)^{\frac{-(1+\gamma)}{2(\gamma-1)}} d\left(\frac{y}{s}\right)} \quad (6.5)$$

The estimated uncertainty in the value of AVDR is less than ± 0.02 .

6.6.4 Blade Surface Pressure Measurements

The blade surface pressure measurements were referenced to the upstream static pressure P_1 and non-dimensionalized by the upstream dynamic pressure q_1 . Therefore, the results are presented as a surface static pressure coefficient given by the following equation:

$$C_{p_s} = \frac{(P_s)_x - P_{1r}}{q_{1r}} \quad (6.6)$$

where, $(P_s)_x$ = measured pressure at static tap x ,
 P_{1r} = reference upstream measured static pressure,
 q_{1r} = reference inlet dynamic pressure, $0.5\rho_1 V_1^2$.

Also, the results are expressed as surface isentropic Mach number distributions. These are given by the following equation:

$$(M_{is})_x = \left\{ \left(\frac{2}{\gamma - 1} \right) \left[\left(\frac{P_{01r}}{(P_s)_x} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \right\}^{\frac{1}{2}} \quad (6.7)$$

where P_{01r} is the upstream reference total pressure. Figure 6.10 shows typical blade loading measurements for HS1A at design flow conditions for which the outlet isentropic Mach number is 1.03.

Figure 6.10 HS1A Blade Loading at Design Flow Conditions

6.7 Calculation of Mixed-Out Values

The measured downstream flow quantities were processed using the data reduction programs PROCESS and MIXLOSS to obtain the corresponding mixed-out values. Details of the data reduction procedure can be found in Jouini (1999). It is briefly summarized here.

All input pressure data are normalized with the corresponding upstream reference total pressure (P_{01r}). The total pressure obtained from the three hole probe and the corresponding static pressure obtained from the static probe were used in the program. The resulting output file from the PROCESS program includes tabulated values of the downstream total pressure, flow angle, Mach number and static pressure.

The output file from the PROCESS program is used as input file to the MIXLOSS program to calculate the mixed-out values of the downstream flow quantities. The calculation procedure followed in the MIXLOSS program is based on that of Amecke & Šafařík (1995) to calculate the mixed values. The loss coefficient based on the mixed-out values is defined as follows:

$$Y = \frac{P_{01} - P_{02}}{P_{02} - P_{s2}} \quad (6.8)$$

where P_{01} = total pressure upstream of the blade row,
 P_{02} = mixed-out value of the downstream total pressure,
 P_{s2} = mixed-out value of the downstream static pressure.

Figure 6.11 shows an example of the resulting mixed-out flow quantities and the corresponding measured downstream quantities at exit Mach number 1.03 and 4.5 degrees of incidence.

Figure 6.11 A Comparison Between the Mixed-out and the Pitchwise Variation of the Flow Quantities (reproduced from Jouini, 1999)

7 Experimental Results for Blades with Trailing-Edge Damage

7.1 Introduction

This chapter examines experimentally the effect of trailing-edge damage on the losses and flow turning ability of axial turbine blades. A broader understanding, at the blade level, of the aerodynamic effects of trailing-edge damage is necessary to improve the capabilities of EHM systems to distinguish the presence of such a damage. Aerodynamic parameters such as total pressure loss, flow deviation, base pressure, etc. have been measured at high-speed flow conditions for both undamaged and damaged blades to study the effects of damage. Measurements include design and off-design cases in the exit Mach number range of 0.70 to 1.20. The damaged blade losses and trailing-edge deviation were measured for cutouts of 10% and 20% of the blade chord. The experimental apparatus and procedures used to obtain these results were described in the previous Chapters 5 and 6 respectively. Computational results were used to supplement the experimental measurements. It was found that the computational predictions were quite satisfactory and they were used extensively to study the flow characteristics for different damage sizes and Mach numbers which were otherwise not possible to obtain experimentally within the scope of the present study.

As previously mentioned, the present study is an extension of the work completed by Isaacs (1994) on the effects of trailing-edge damage at low-speed flow conditions. Data collected by Isaacs are also used here in conjunction with the present results to establish the relationships between the flow parameters.

7.2 Effect of Damage on Losses

Figure 7.1 shows the measured and computed total pressure losses at mid-span for both undamaged and damaged blades in the exit Mach number range of 0.7 to 1.2. The computations were made using the Dawes code as described in Chapter 3. Initial computational predictions showed that there is no relative increase in the losses due to blade damage. As can be seen from the Figure, the code tends to underpredict the losses for both undamaged and damaged blades. The reason for this is not clear but may be attributed to the Baldwin-Lomax turbulence model which has difficulty predicting the wake development with downstream distance. As expected, the measured undamaged blade loss coefficient rises with the increase in exit Mach number due to the formation of expansion and compression waves at the trailing edge associated with transonic flows.

Figure 7.1 Variation of Total Pressure Losses with Exit Mach Number for Undamaged and Damaged Blades at mid-span

However, the measurements also show that there is no relative increase in the losses due to blade damage, even at high exit Mach numbers. In fact the midspan losses due to 20% blade damage are lower than the undamaged blade. Isaacs (1994) also showed that for the low-speed flow cases losses slightly decreased with blade damage. The same effect seems to persist for high-speed flows also. The computational predictions also show a slight decrease in the losses due to 20% blade damage at high exit Mach numbers.

The trailing-edge cutout is definitely altering the boundary layer thickness and the trailing edge shape and this may be influencing the base pressure. In this context, the relationship between the total pressure loss, base pressure and mixed out loss behind the trailing edge derived by Denton (1993) may be useful in understanding the strange behavior of the losses due to blade damage. Denton derived an expression for stagnation pressure loss coefficient by analyzing the entropy created due to the mixing out of a wake behind a blunt trailing edge in a constant area passage with incompressible flow. The analysis includes the boundary layers on the blade surface immediately upstream of the trailing edge and also the base pressure acting on the trailing edge. The resulting expression is given by:

$$\frac{\Delta P_o}{0.5\rho V_2^2} = -\frac{C_b t}{o} + \frac{2\theta}{o} + \left(\frac{\delta' + t}{o} \right)^2 \quad (7.1)$$

- Where, ΔP_o = stagnation pressure loss
 ρ = density
 V_2 = exit velocity
 C_b = base pressure coefficient
 t = trailing edge thickness
 o = throat opening
 θ = boundary layer momentum thickness
 δ' = boundary layer displacement thickness

The first term on the right-hand side of Equation 7.1 is the loss due to the low base pressure acting on the trailing edge, the second term is the mixed out loss of the boundary layers on the blade

surface just before the trailing edge and the third term arises from the combined blockage of the trailing edge and the boundary layers.

Increasing damage size increases the trailing edge thickness and therefore this might be expected to increase the trailing-edge losses and thus an overall increase in profile losses. The fact that the losses do not increase must then be related to changes in the base pressure due to damage.

To obtain further insight into the behavior of the losses due to damage, the base pressure was measured in the middle of the cutout for both 10% and 20% chord penetration. Figure 7.2 shows the variation of the measured base pressure coefficient for undamaged and damaged blades. The well known trend of decreasing base pressure as the isentropic exit Mach number is increased toward unity (Denton, 1993) is observed for all the cases in Figure 7.2. Near Mach 0.95 it can be seen that the base pressure for the 10% damaged blade falls lower than that of the undamaged

Figure 7.2 Variation of Base Pressure Coefficient with Exit Mach Number for Undamaged and Damaged Blades

blade and conversely the base pressure for the 20% damaged blade rises higher than that of the undamaged blade. Lower base pressure is generally associated with higher losses and this is reflected in Figure 7.1 where, in the neighborhood of Mach 0.95, higher losses are observed for 10% damage compared to the 20% damage. Though the behavior of base pressure at transonic speeds is poorly understood, it is generally believed that it mostly depends on the state of the boundary layer, the shape of the trailing edge and the ratio of trailing edge thickness to boundary layer thickness (Denton, 1993).

For the purpose of the present discussion, the measured loss and base pressure coefficients are taken near Mach 0.95 where it can be seen from Figure 7.1 and 7.2 that the respective values are comparatively widely separated. In Equation 7.1 only the boundary layer components are unknown. The variation of the boundary layer loss components calculated from Equation 7.1 using the measured total and base pressure losses is plotted for each damage size in Figure 7.3. It

Figure 7.3 Effect of Damage Size on the Trailing Edge Loss Component at $M_{21a}=0.95$

is evident that with the increase in damage size the combined effect of trailing edge blockage and mixed out boundary layer losses decrease. It is understandable that the reduction in blade surface length due to damage reduces the boundary layer thickness resulting in a slight decrease in the associated losses. Apparently, the 20% damage is altering the base pressure in such a way that the trailing edge contribution to the measured loss is considerably reduced resulting in lower loss than the undamaged blade.

It appears that the change in losses cannot be used as a parameter to detect the presence of blade trailing-edge damage. This is further illustrated in Figure 7.4 which shows the effect of blade damage on the measured and computed losses at different exit Mach numbers. It is evident that there is no appreciable change in the losses due to damage. Though the Dawes code under predicts the amount of loss, it does capture the general trend in the behavior of loss with damage.

Figure 7.4 Variation of Losses with Damage Size at Different Exit Mach Numbers

The reasons are not clear for the increase in base pressure, and the resulting reduction in base pressure losses, as the damage approaches 20% of chord. However, the trailing edge flow is distinctly different in the region of the cutout compared with an undamaged trailing edge. At the upper and lower edges of the cutout, fluid leaks from the pressure to the suction side of the blade in a similar way to the tip leakage flow at the tip of a blade with clearance. In fact, Klein (1993) observed experimentally two counter-rotating vortices trailing from the edges of the cutout. These structures are also seen in the computations. It is possible that the interaction between these vortices and the flow in the base region of the cutout has the effect of raising the base pressure. It is reasonable that this effect, if present, would be strongest for the largest cutout. In any case, the net effect is that the overall profile losses are actually lower with the 20% cutout compared with the undamaged blade, as seen in Figure 7.3. This raises the intriguing possibility that profile losses in turbines might be reduced by using blades with scalloped trailing edges. Of course, the blade geometry would have to be modified to compensate for the underturning of the flow which is associated with the cutouts. This underturning is discussed next.

7.3 Effect of Damage on Deviation

Figure 7.5 shows the influence of trailing-edge damage size on flow deviation at exit Mach numbers of 0.7 and 1.2. Both measured and computed deviation at midspan, which is the region of the cutout, are shown in the Figure. Unlike pressure loss, blade damage has considerable effect on the flow turning ability as can be seen in the Figure. The flow underturning produced by the damage is more severe for higher Mach number. It is interesting to note that the computed deviation is reasonably close to the measured deviation for an undamaged blade but they diverge considerably with increasing damage size. The most likely reason for this is that the experimental and computed cascade geometries are not identical. The measured deviation is obtained from one damaged blade in a cascade of undamaged blades whereas in the computational grid, all the blades are damaged. It is believed that in the experimental setup, the blades adjacent to the damaged blade help guide the flow thereby reducing the amount of underturning.

Figure 7.5 Effect of Damage Size on Deviation

Isaacs (1994) also showed that for subsonic flow the Dawes code predicted higher deviation for the LS3 damaged blade compared to the value measured for a single damaged blade in a cascade.

Figure 7.6 shows the variation of incremental deviation ($\Delta\delta$), defined as the additional increase in deviation over that of an undamaged blade, with damage size for both the HS1A and LS3 blades for subsonic flow conditions. It is interesting to note that the incremental deviation varies approximately linearly with damage size for all the cases. Also it is evident from the Figure that the computed incremental deviation for a particular damage size is approximately twice that of the measured incremental deviation. Though these relationships are based on limited data, they do provide a useful means to correlate the effects of damage on deviation. These simple relationships

Figure 7.6 Variation of Incremental Deviation with Damage Size

are in fact used in the following chapter in the development of correlations based on the present results. The correlations are based on the assumption that the calculations provide accurate values of deviation for a blade row in which all blades have experienced the same damage. The differences between the computations and the measurements are then taken as the differences in incremental deviation between the two extremes of a single damaged blade and all blades damaged.

As noted, blade damage does not affect the losses. However, it produces considerable under turning of the flow in the area of the cutout. This implies that a turbine stage that has a rotor with trailing-edge damaged blades will produce less power output for a given flow rate and rotor speed but with little deterioration in efficiency. This is in contrast to the effects of surface roughening, discussed in Chapter 4, for which significant changes in losses occur but the flow

turning ability is largely unaffected. The distinctly different aerodynamic effects of the two types of damage may provide the basis for identifying the type of damage present from engine health monitoring measurements.

The following chapter describes the development of deviation correlations based on the present results.

8 Development of Deviation Correlations

8.1 Introduction

In the previous chapter the effect of trailing-edge damage on the aerodynamics of turbine blades was examined experimentally and computationally. It was found that there is considerable underturning or increase in deviation in the region of blade damage. It was also shown that the incremental deviation ($\Delta\delta$), defined as the additional increase in deviation over that of an undamaged blade, is a linear function of the chordwise penetration of the cutout or blade damage. This simple relationship was utilized to develop an empirical correlation to predict the incremental deviation expected due to a certain amount of trailing edge damage. However, to obtain the total deviation in the region of blade trailing edge damage, an accurate prediction of deviation for the undamaged blade is required. Disappointingly, there are no satisfactory correlations available for turbine blade deviation. The turbine performance correlations due to Ainley & Mathieson (1951) are expressed in terms of flow outlet angle rather than deviation. The well-established deviation rule due to Carter & Hughes (1950) is primarily for low speed compressors and has questionable applicability to turbine cascades. Moreover, the correlations mentioned were developed using data for blade designs of the 1940's and 50's and therefore do not account for recent improvements in airfoil design. It will be shown later that, when tested against 45 available experimental and computational results for deviation in turbines, the Carter & Hughes correlation does not predict the deviation satisfactorily.

An experimental and computational study was made to identify the parameters that influence flow deviation in a turbine cascade at high subsonic flow conditions. It is believed that the parameters that affect deviation for an undamaged blade will also largely determine the

incremental deviation due to damage. The deviation correlation for the damaged blade takes into account an additional parameter: the chordwise extent of the damage. Thus, to obtain a correlation for the effects of damage, an improved correlation was developed for deviation in undamaged turbines. This was accomplished by first identifying the functional relationship between deviation and the various geometric and flow parameters of a turbine cascade. Then the unknown coefficients in the function were determined based on a database of known values of deviation. The evaluation of the empirical coefficients of the correlation was treated as an optimization problem. The optimization was performed using a genetic algorithm which proved very effective and robust. Also, the procedure allows continuous updating of the deviation correlation as new experimental and computational data become available.

The result of the study is described in the following section. This is followed by a brief description of the working principle of genetic algorithms. Finally, the development of the deviation correlation for undamaged blade and damaged blade with the help of genetic algorithms is presented.

8.2 Factors Influencing Deviation

Development of a deviation correlation calls for an understanding of the relationship between flow deviation and the cascade geometric and flow parameters. A modern turbine cascade is defined by numerous geometric and flow parameters of which the blade chord, stagger angle, pitch, inlet and outlet metal angles, leading and trailing edge diameters, throat opening, inlet and outlet flow angles are just a few. In a subsonic two-dimensional flow field through a cascade the outlet flow angle invariably tends to be less than the blade exit metal angle and most of the mentioned parameters have some influence in producing this difference. The problem is to determine what flow and blade parameters are most important in determining the deviation.

The shortcomings of some of the widely used correlations (e.g. Ainley & Mathieson, Carter & Hughes, Cosine Rule, etc.) were discussed in Section 2.2. It was noted that these correlations primarily correlate the outlet flow angle to specific details of the cascade geometry. For example, Carter & Hughes correlate the deviation with the blade stagger angle, blade camber,

and the solidity. In a review of some of the simplified methods used in the turbine industry for determining deviation it was suggested by Riess et al. (1986) that the application of those methods are limited to narrow ranges of cascade geometry because they do not take into account sufficiently the real flow field at the outlet area of a cascade. In view of the shortcomings of the existing correlations, an investigation was made to re-examine the factors that influence subsonic deviation and, if possible, to develop an improved correlation. Based on new experimental and computational results it was found that additional parameters have significant effects on deviation compared with those used in the traditional correlations.

The study revealed that one aspect of the flow through the blade passage largely determined the magnitude of deviation: the pressure difference towards the trailing edge of the blade. This pressure difference is influenced by the total blade loading (the total lift generated) and how that loading is distributed.

The total blade loading is primarily determined by two parameters: the total amount of flow turning and the spacing between blades in the row (or more precisely by the spacing-to-chord ratio, s/c). The closer the blade spacing the smaller the loading since the total tangential force is carried by a larger number of blades. The total flow turning in the blade passage is the sum of the inlet and outlet flow angles, measured from the axial direction. In the deviation correlation of Carter & Hughes (1950) the flow turning is approximated by the blade camber angle. This neglects the effect of both incidence and deviation on the flow turning. Since the inlet flow direction is usually known, there seems no reason not to use the inlet flow angle, as opposed to the metal angle, in specifying the flow turning. Also, this makes the correlation potentially valid for off-design values of incidence. The outlet flow angle is itself a function of the deviation. Thus, it would require iteration to apply a deviation correlation which uses the true flow turning. In the present work, the total flow turning is approximated by the sum of the inlet flow angle and the outlet metal angle. Since the deviation is generally small compared with the total flow turning in turbines, this approximation introduces relatively little error. It would be less satisfactory in the case of compressor flows.

Figure 8.1 shows the surface pressure distributions predicted for two similar blades with spacing-to-chord ratios of $s/c = 0.76$ and $s/c = 0.60$. The area enclosed between the two surface pressure distributions represents the tangential blade loading. It is apparent that the blade loading

is smaller for the spacing $s/c = 0.60$ since the enclosed area is smaller by comparison. As expected, and as indicated on the Figure, the cascade with the smaller trailing edge loading gave a lower value of predicted deviation.

Another example of surface pressure distributions predicted for two cascades with different solidity, flow turning and stagger but with similar loading distribution on the forward half of the blade is shown in Figure 8.2. Near the trailing end the suction side pressure distribution is different. The suction side pressure distribution largely determines the magnitude and distribution of loading on the blade because the blade pressure side pressure distribution cannot rise above the leading edge stagnation pressure while the suction side static pressure can drop to significant amounts before rising again to the outlet static pressure of the cascade. Figure 8.2 represents moderately aft-loaded blades which can be recognized by the continuous drop in suction side pressure toward the rear end of the blade. A closer look at the pressure distribution at the trailing end reveals that the blade with lower deviation has a smaller pressure difference between the suction surface and pressure surface. As the suction and pressure surface flows meet at the trailing edge and adjust to their common downstream static pressure, there will be a tendency for the pressure side flow, as it expands, to turn towards the suction side. The effects of suction surface pressure distribution on deviation was also observed by Denton & Xu (1990). Using inviscid flow calculations they showed that a blade must have low average suction surface pressure downstream of the throat to produce large deviation. Again Roberts & Denton (1996) suggested that an increase in the loading of a blade corresponds to an increase in the deviation. Similar conclusions can be drawn from Figures 8.1 and 8.2 also. The difference in magnitudes of pressure gradients past the throat opening definitely affects the development of the pressure and suction surface boundary layers. However, it seems that there is no correlation between the thicknesses of boundary layers and change in deviation. There is some support for this assumption in the measurements of Kind et al. (1998). The authors measured the losses and deviation for the flow through a turbine cascade in which patches of roughness of varying extent and severity had been applied to the surfaces. The roughness resulted in a wide range of boundary layer thicknesses at the trailing edge while the blade loading was essentially constant. The variation in the boundary layer thickness had very little effect on the measured deviation. Of course, if the blade loading is such that the suction surface boundary layer separates near the trailing edge then deviation will be largely determined by

Figure 8.1 Surface Pressure Distributions of Blades with Different s/c

Figure 8.2 Surface Pressure Distribution of Blades with Similar Leading Edge Pressure Distribution

viscous effects. The present discussion is intended to apply only for fully-attached blade-surface boundary layers.

The chordwise distribution of the loading is determined by other geometric parameters. Korakianitis & Papagiannidis (1993) have noted the effect of stagger angle and suction-side curvature (Korakianitis, 1993) on the loading distribution. The effect of stagger angle is considered first.

Figure 8.3 shows the surface pressure distributions for two cascades with stagger angles of 18.8 and 28.8 degrees but the same inlet and outlet blade metal angles. The pressure distributions were calculated using the Navier-Stokes code of Dawes (1988). As stagger angle is increased, the thickness of the blade has to be reduced to maintain the same throat opening. More importantly, to obtain the same metal angles at inlet and outlet, the higher stagger blade must produce more of its turning in the forward part of the passage. As a result, as stagger angle increases the blade tends to become more forward loaded, as confirmed by the predicted pressure distributions. It is noticeable from the Figures that the pressure difference at the trailing end of the front-loaded blade is smaller compared to the aft-loaded blade. As previously noted, this should lead to higher deviation for the aft-loaded blade. In fact, this is what was predicted by the code, as indicated on the Figures. Therefore, an increase in stagger angle tends to produce a decrease in the deviation.

The effect of suction side curvature is considered next. The suction side curvature would be an inconvenient parameter to use in a correlation since it will not be known in the early stages of design. However, the increased curvature tends to be obtained by increasing the maximum thickness of the blade. Therefore, in the present work the maximum-thickness-to-chord ratio (t_m/c) is used as an indirect measure of the suction-side curvature. In Figure 8.4 three blades with maximum blade thickness varying from $t_m/c = 0.20$ to 0.26 are compared. The corresponding blade surface pressure distributions are also shown in the Figure. As shown in the Figure, increasing the suction-side curvature has the effect of increasing the strength of the suction peak on the suction side and thus making the blade apparently more forward loaded. However, it is seen that this has less effect on the pressure difference towards the trailing edge than might have been expected, due to the adjustment of the pressure on the pressure side of the blade. This suggests that suction-side curvature (or t_m/c) may have a somewhat weak influence on the deviation. This was confirmed by the final deviation correlation but t_m/c is retained as a correlating parameter.

(a) Stagger Angle of 18.8 Degrees

(b) Stagger Angle of 28.8 Degrees

Figure 8.3 Surface Pressure Distributions of Cascades with Stagger

Figure 8.4 Surface Pressure Distributions of Blades with Varying Maximum Thickness

The previous discussion leads to the conclusion that the geometric and flow parameters that influence deviation most are the total flow turning, the solidity, the stagger angle and the thickness-to-chord ratio. An initial attempt was made to correlate the deviation on these parameters. While the correlation appeared somewhat better than the existing ones, there was sufficient scatter to suggest that one or more significant parameters were missing. It was subsequently found that including the blade inlet metal angle as a correlating parameter substantially improved the quality of the correlation.

The effect of inlet metal angle on deviation is not immediately obvious. However, it is noteworthy that Ainley & Mathieson (1951) correlated profile losses in turbines with the blade inlet angle because of its effect on the blade loading. For a given total flow turning, two extremes exist: an inlet angle of zero (a nozzle row) gives maximum flow acceleration while an impulse row gives essentially zero acceleration. Ainley & Mathieson found that the resulting differences in the pressure distributions led to different profile losses. In their loss system, the profile losses for a given blade row are obtained as a weighted average of the values at the two extremes of nozzle and impulse rows with the same flow turning. Since the inlet metal angle affects the pressure distribution around the blade, it is plausible that it would affect the deviation at the trailing edge as well as the losses. It could be argued that the inlet flow angle, rather than the metal angle, should be used to correlate this effect. However, the inlet metal angle is convenient and the effect of incidence is already included in the total flow turning.

One final parameter was included in the deviation correlation: the axial velocity density ratio (AVDR). The AVDR is an average measure of the flow acceleration in the axial direction at midspan. AVDR imposes on the flow in the passage an overall acceleration or deceleration which alters the blade pressure distribution in an effect somewhat analogous to the influence of the inlet metal angle. Over the past years much work has been done on the effects of AVDR on losses and deviation in compressor blades. Reports on the effects of AVDR on the aerodynamic parameters of turbine blades are scanty. Kiock et al. (1985) concluded that the AVDR does not appear to affect the exit flow angle for turbine blades as much as it does for compressor blades. Rodger (1992) observed that for turbine blades, an increase in the AVDR increased the pressure differential across the trailing edge and resulted in a higher deviation. Rodger also suggested a linear trend in deviation with AVDR. An inverse linear variation in deviation with AVDR had been observed by Pollard & Gostelow (1967) for compressors. In developing the present correlation, the power on the term involving AVDR was left open and it was found that in fact an approximately cubic variation gave the best fit. This was based on the data of Rodger as well as a few other experimental cases from the literature. Figure 8.5 compares the trends in deviation as observed by Rodger with linear and cubic functional relationships given by:

$$\delta = \delta_0 + 14(\text{AVDR} - 1) \quad (8.1)$$

$$\text{and } \delta = (\text{AVDR})^3 \delta_0 \quad (8.2)$$

where δ_0 is the deviation at $\text{AVDR} = 1.0$. It can be seen from the Figure that both the equations predict deviation very well within the AVDR range of 0.95 to 1.10. It can be seen from the Figure that the two curves are difficult to distinguish from each other because the AVDR varies over a very narrow range centred on 1.0. However, the cubic dependence was retained since it gave the best fit for all of the available data.

Figure 8.5 Effect of AVDR on Deviation

Finally, six geometric and flow parameters were selected to correlate with deviation, namely, the blade stagger angle, solidity, inlet metal angle, thickness-to-chord ratio, flow turning and axial velocity density ratio. The procedure for developing the deviation correlation for subsonic flow cases is discussed in the following sections. This is followed by the development of deviation correlation for damaged blades.

8.3 Correlations for Subsonic Flow

8.3.1 Sources of Data

8.3.1.1 Experimentally Measured Data

The deviation correlation was developed using a database of 45 cases, as summarized in Table 8.1. shown at the end of this chapter. The sources of the data are indicated in the second column. A total of 16 experimental cases were identified. A number of these were obtained in-house during low-speed cascade studies using three different blade geometries. A number of other cases were located in the open literature for which the required geometric parameters were available, or could be estimated with acceptable accuracy. It should be noted that the experimental cases include several measurements made at two different times for the identical cascades: cases (3)-(4), (13)-(15) and (14)-(16). The results are seen to agree within about ± 0.5 degrees. This is indicative of the uncertainty present in measured values of deviation.

Each category of data are assigned a different symbol so that it can be easily recognized in the graphical representation which will be shown later in Section 8.3.3. The design and off-design experimental cases have different symbols as can be seen from the Table 8.1.

8.3.1.2 Experimental Data for Different Values of AVDR

As previously mentioned in Section 8.2, there is limited work on the effects of AVDR on the aerodynamic parameters of turbine blades. Rodger (1992) generated deviation data for AVDR values of 0.96 and 1.08 at midspan using an in-house blade. It was believed that developing a

correlation based on this limited data for a particular blade profile may not be applicable to other cascade configurations. Extensive search in the literature provided three additional data that were useful for the present objective. The five deviation data for different values of AVDR are grouped together in Table 8.1 as case nos. 8, 9, 10, 11 and 12.

8.3.1.3 Computations for Existing Blades

All computational data were generated using the three-dimensional Navier-Stokes code solver of Dawes (1988). As already shown in Section 7.3, the code predicted the deviations reasonably well for both low-speed and high-speed flows. Additional computations were performed for three of the geometries for which in-house experimental results were available: in Table 8.1 the companion cases are (1)-(34), (2)-(17), (3)-(18), (15)-(44) and (16)-(45). It is seen that the measured and computed values of deviation agree to within, at worst, 1.0 degree, and are usually considerably closer. In view of the estimated ± 0.5 degree uncertainty in the measured values, it was felt that the computed values were sufficiently accurate to use as an additional basis for developing the correlation. Therefore, it was extensively used to predict deviation for all in-house blades at design and off-design flow conditions.

8.3.1.4 Computations for Hypothetical Blades

The available experimental deviation data did not cover wide ranges of all six parameters which had been identified to influence the deviation. Therefore, the Navier-Stokes solver was used to calculate the flow through a series of hypothetical turbine blade rows which broadened the range of the parameters for which deviation values were available. Hypothetical blades were constructed using an in-house 2D profile generator. The geometry of each hypothetical blade row was defined uniquely by specifying 17 geometric parameters (see Figure 8.6). Leading- and trailing-edge circles were used and the remainder of the pressure and suction surfaces were defined by Beziercubics. A very similar procedure has been used by Trigg et al. (1997) to generate steam-turbine blade rows. The present procedure differs slightly in that continuous surface curvature was

No.	Parameter
1.	Number of blades
2.	Aerofoil radius
3.	Axial chord
4.	Tangential chord
5.	Unguided turning angle
6.	Blade inlet angle
7.	Blade inlet wedge angle
8.	Leading edge circle radius
9.	Blade exit angle
10.	Trailing edge circle radius
11.	Throat/pitch
12.	Tangent proportion (Pt 3-2)
13.	Tangent proportion (Pt 2-3)
14.	Tangent proportion (Pt 2-1)
15.	Tangent proportion (Pt 1-2)
16.	Tangent proportion (Pt 4-5)
17.	Tangent proportion (Pt 5-4)

Figure 8.6 Profile Generator Parameter List (reproduced from Trigg et al.,1997)

enforced on both sides of the blade from the leading- to the trailing-edge blend points. The additional hypothetical blade rows are given as cases (19) through (45) in Table 8.1. Four different nozzle blade profiles were generated with zero inlet metal angle shown as cases (38) through (41). All the hypothetical blades were designed for exit Mach number about 0.7.

8.3.1.5 Summary of Ranges of Data

Table 8.2 shows the ranges of the available geometric and flow parameters. A wide range of the various geometric and flow parameters were gathered from various authors, computations and experiments. No experimental data could be found for cascade geometry with zero stagger angle. Zero stagger angle is usually observed in steam turbine impulse blades and has rarely been used for gas turbines. Hence further attempts to obtain computational data for the special case of zero stagger angle were not made. However, zero inlet metal angle is commonly used in gas turbine nozzle blades and though no experimental data were available, a few computational data were generated. Though the number of cases for different AVDR values are limited to five only, the range of AVDR from 0.91 to 1.08 is believed to be within the usual range of AVDR values found in turbine cascades.

8.3.2 Using Genetic Algorithms for Optimization

8.3.2.1 Basic Description of Genetic Algorithms

Genetic Algorithms are powerful and broadly applicable stochastic search and optimization techniques based on principles derived from genetics and evolution. They emulate the process of natural selection in nature to find good solutions to a problem. Because genetic algorithms are rooted in both natural genetics and computer science, the terminologies used in genetic algorithm literature are borrowed from both subjects.

Recently, genetic algorithms have received considerable attention regarding their potential as an optimization technique for complex problems and have been successfully applied in the areas of the physical sciences, social sciences and engineering (e.g. see Goldberg, 1989). Genetic

algorithms differ from conventional optimization and search procedures in several fundamental ways. Goldberg has summarized them as follows:

- a. Genetic algorithms work with a coding of solution set, not the solutions themselves.
- b. Genetic algorithms search from a population of solutions, not a single solution.
- c. Genetic algorithms use payoff information (fitness function), not derivatives or other auxiliary knowledge.
- d. Genetic algorithms use probabilistic transition rules, not deterministic rules.

Genetic algorithms appear to be remarkably robust compared with other optimization techniques. Due to their evolutionary nature, genetic algorithms will search for solutions without regard to the specific inner workings of the problem. Particularly in discontinuous and multimodal search spaces they can effectively reject local maxima (or minima) and home in on the global optimum. These characteristics of genetic algorithms seemed to make them well suited for optimizing the coefficients in empirical correlations such as for deviation in axial turbines. The basic working principles of the genetic algorithm (GA) are described here with reference to the parameters used to obtain the turbine blade deviation correlation.

Suppose that the deviation at the trailing edge of a turbine blade row is assumed to vary as follows:

$$\delta = A^x B^y C^z D^v E^w \quad (8.3)$$

where δ is the deviation, A, B,E are geometric parameters of the blade row (such as the stagger angle, the solidity, the maximum thickness-to-chord ratio, etc.) and x, y, ... w are the (unknown) coefficients of the correlation. The goal then is to determine the optimum set of the correlation parameters which gives the best agreement between the predicted deviation and that observed experimentally, or predicted computationally, for the same values of the geometric parameters. The genetic algorithm provides a useful tool for determining this optimum set of the correlation parameters.

The GA starts with a collection of many randomly generated individuals called a *population*. Each individual in the population is called a *chromosome* and each chromosome represents a trial set of the correlation coefficients, x , y , z , etc. The chromosome is made up of *genes*. The value of each gene is encoded as a binary bit string. Each of the binary digits making up the gene is referred to as an *allele*. Shown below is a chromosome composed of five genes representing the first trial of coefficients $x_1, y_1, \dots, w_1$:

Gene1	Gene2	Gene3		Gene5
100110100011	011101011111	110100100010		101100100011010
x_1	y_1	z_1		w_1

The length of the encoded bit string depends on the required precision. For example, if the coefficient of the stagger angle in the deviation correlation is allowed to vary from 0.0 to 3.0 and the desired precision is four decimal places, the required number of alleles for the variable is calculated as follows:

$$2^{12} \geq (3.0 - 0.0) \times 1000$$

Therefore, a 12 bit string (e.g. Gene1) was used to encode the coefficient for stagger angle.

The chromosomes evolve through successive iterations, called *generations*. During each generation, the chromosomes are decoded and *evaluated*, using some measure of *fitness*. This is done with a fitness function, or objective function, which returns some positive number for a given chromosome. The greater the number, the greater the fitness. For the present case, for each individual in the population at each iteration (i) the objective function (F_{obj}) is evaluated as follows:

$$F_{obj_i} = \frac{1}{\sum_{j=1}^k (\delta_i - \delta_i^p)^2} \quad (8.4)$$

where δ_i is the given deviation, δ_i^p is the predicted deviation with the trial coefficients and k is the total number of data points against which the trial correlation is compared. The objective function will return larger positive numbers for the set of coefficients that predict deviations closer to the known values. The drawback of such an inverse function is that it might become indeterminate if the denominator becomes infinitely small or zero. However, this is effectively impossible because a perfect solution does not exist in the search domain and there is sufficient scatter in comparison data. There is also a 'scaling mechanism' to deal with a few extra-ordinarily large or small fitness values. This will be discussed later in the appropriate section.

Using the fitness information, the genetic algorithm performs the following three kinds of operations to improve the average fitness of the next generation:

1. Selection
2. Crossover
3. Mutation

Selection is the process by which individuals are chosen for the next generation or new population. A simple sampling technique, commonly referred to as Roulette Wheel Selection, is used where each member of the old population is assigned a slot on the wheel, the size of which is proportional to their fitness. Thus, population members with high fitness have wider slots on the wheel than members with low fitness. The selection process is based on 'spinning' the roulette wheel by a random number generator N times, where N is the population size. Since fit members occupy larger fractions of the wheel, they are expected to be chosen many times more than the low fit members during N selections. In successive generations the relatively good solutions are reproduced, while the relatively bad solutions ultimately die. In this way, "survival of the fittest" plays a part in the functioning of the genetic algorithm.

This simple operation of fitness-proportionate selection may show some undesirable properties. For example, in early generations, there is a tendency for a few 'super' chromosomes to dominate the selection process while the rejected low-fitness members may nevertheless carry some useful genetic information. Also, after many generations all the members of the population will tend to have very similar values of fitness. Therefore, they all stand very similar chances of passing their genes on to the next generation. This tends to slow down the convergence toward the

optimum. To solve these problems, GAs often use 'fitness scaling'. Among the different scaling methods available, (e.g. linear scaling, power law scaling, logarithmic scaling, etc.) the present GA uses linear fitness scaling. Linear scaling adjusts the raw fitness values of all chromosomes such that the high fitness values are scaled down and low values are scaled up while maintaining the same average fitness. In other words, the scaling method maps raw objective function values to some positive real values, and the survival probability of each chromosome is determined according to these values. The scaled fitness f' from the raw fitness f for each chromosome can be expressed as follows:

$$f' = af + b$$

The coefficients a and b are normally selected such that each average population member contributes one expected offspring to the next generation and the best chromosome provides a fixed number of expected offspring. This helps prevent the extra-ordinary individuals from reproducing too many in the early stages of evolution. Conversely, when the peak fitness is close to the average, the scaling amplifies the differences from the average and thus speeds convergence in later generations.

Crossover is the main genetic operator. It operates on two chromosomes at a time and generates offspring by combining features from the chromosomes of both parents. The newly selected population members are candidates for the crossover operation. The random number generator is used to decide whether a crossover will actually occur, subject to a probability or crossover rate which is specified by the user. This ratio controls the expected number of genes to undergo the crossover operation. For example, if the probability of crossover is specified as 0.7 then it is expected that on average, 70% of the genes will undergo crossover. The crossover operation allows many new individuals to be created and new points in the search space to be tested. If crossover is to occur, then a random cut-point is selected next. For example, if the crossover location in a particular gene is between the 8th and 9th alleles, as shown in Figure 8.7 then the crossover process proceeds as follows:

Figure 8.7 Gene Cross Over

- (a) Up to the crossover location, Child 1 receives all of Parent1's alleles and Child 2 receives all of Parent 2's.
- (b) From the crossover location to the end of the gene, Child 1 receives alleles from Parent 2 and Child 2 receives them from Parent 1.

Mutation is a secondary genetic operator which produces spontaneous random changes in various chromosomes. A probability of mutation is specified by the user. For example, a probability of 0.001 means that on average 1 in every 1000 alleles will have its value changed. In genetic algorithms, mutation serves the role of either (a) replacing the genes lost from the population during the selection process, so that they can be tried in a new context or (b) providing genes that were not present in the initial population. As in nature, mutation can introduce chance improvements in fitness which would not otherwise have occurred through the selection and crossover processes. However, mutation is used sparingly in genetic algorithms. If the value of mutation rate is too high, the mutation process will randomly destroy too much useful genetic information and impair the progress toward individuals with optimum fitness. For example, if the probability of mutation is set to 0.5, the genetic algorithm reduces to a completely random search for the most fit individual. Again, if it is too low, genes that might have been useful but did not appear in the initial population, are never tried out.

The termination criterion is usually specified in terms of a maximum number of generations to be run. A method of selecting the total number of generations is described in the next section.

8.3.2.2 Selection of Basic GA Parameters

The genetic algorithm used in the optimization process was originally adapted from Goldberg (1989). It was modified to suit the requirements of the present objective but the simple structure of the various operators and functions were retained. Complex problems may require alternative selection, genetic operation, scaling function, etc. (e.g. see Goldberg) but the simple GA proved to be extremely effective in optimizing the set of coefficients used in the deviation correlation for turbine blades.

The major parameters that have to be specified by the user during a GA run are the population size, probability of crossover, probability of mutation and the number of generations. Each run represents the evolution of the population starting from a different, randomly generated, initial population of the specified size. It is essential to choose suitable values of these parameters to obtain good solutions. However, no general criteria exist for choosing the appropriate values for these quantities and the user has to exercise additional judgement in selecting them.

The correlation equation and related coefficients used in the initial stage of development of the undamaged blade deviation correlation are quoted here for the sake of an understandable description of the procedure. The correlation parameters or coefficients to be optimized are x , y , z , v , and w . These coefficients are related to the cascade geometric and flow parameters by the following equation:

$$\delta = \zeta^{-1} (t/c)^{-y} (s/c)^z (\Delta\alpha)^v \beta_1^{-w} \quad (8.5)$$

where δ	= flow deviation (degrees)
s/c	= blade space-to-chord ratio
$\Delta\alpha$	= total flow turning (degrees)
ζ	= stagger angle (degrees)
t/c	= blade maximum thickness-to-chord ratio
β_1	= blade inlet metal angle (degrees)

Table 8.3 shows the results obtained from several GA runs. Each run consists of results obtained from a different randomly selection population. The population size, probability of crossover (P_c), probability of mutation (P_m) and the number of generations for each run are also shown in the table. Typical values of the probability of crossover and the probability of mutation of 0.70 and 0.002 are used respectively. Unless these values are inappropriately too large or too small, they do not seem to have a strong effect on the end results. For example, a mutation probability of 0.005 produced essentially no change in the present results. The underlying principle in selecting these two quantities is that they should promote the evolution of more fit individuals. For example, a crossover probability of 0.10 inhibits information exchange in successive generations. This tends to limit full exploitation of the search space. A mutation probability of 0.5 reduces the genetic algorithm to a random search process.

The selection of population size and the number of generations depends on the problem. Table 8.3 illustrates the influence of population size in the present optimization process. The population size has to be sufficiently large so that the search environment includes individuals from all possible parts of the solution space. The table shows an initial execution of ten GA runs with a population size of 350 and a final set of iterations with a population size of 700. Due to the stochastic nature of the search it is necessary to test the consistency of the fitness evaluation with several runs each starting from a different randomly chosen population. For each set of ten GA runs the average value of the coefficients and their maximum range of variation is also shown. It can be seen that the range of variations from the average values are considerably larger for the population size of 350 compared to the population size of 700. Fitness values range from 0.18 (run no. 2) to 0.42 (run no. 4) when a population size of 350 is used. This wide variation implies that the population size is too small to explore the solution space adequately and thus allow the procedure to reach a global optimum. When a larger population size of 700 was used the average fitness changed from 0.3415 to 0.4720 and each run consistently achieved high fitness values. The range of variation of each correlation parameter is also small. The significance of the range of variation of the fitness value is discussed further in Section 8.3.2.3.

Terminating a GA run with the appropriate number of generations is important in terms of developing high fitness solutions while minimizing the computing time. Figure 8.8 shows a GA run with an initial population of 700 and the progression of the solutions to higher fitness values in

Table 8.3
Influence of Population Size for Genetic Algorithm Runs

Number of Generations = 200,				First Trial					
Run No.	Pop. Size	P_c	P_m	Correlation Parameters					Fitness Value
				x	y	z	v	w	
1	350	0.70	0.002	1.6155	0.2912	1.1197	2.2692	1.1580	0.3862
2	350	0.70	0.002	1.5702	0.3499	1.1240	1.9589	0.8237	0.1835
3	350	0.70	0.002	1.4923	0.2678	1.0794	2.4816	1.5539	0.3495
4	350	0.70	0.002	1.5765	0.4272	1.0780	2.2840	1.2883	0.4211
5	350	0.70	0.002	1.6162	0.2955	1.1050	2.4752	1.4392	0.4090
6	350	0.70	0.002	1.6636	0.3209	1.1920	2.4965	1.4285	0.3877
7	350	0.70	0.002	1.6141	0.4130	1.1382	2.4547	1.4604	0.2831
8	350	0.70	0.002	1.6332	0.4314	1.1686	2.3328	1.2869	0.4156
9	350	0.70	0.002	1.7826	0.2084	1.1736	2.6452	1.4633	0.2993
10	350	0.70	0.002	1.7968	0.4335	1.1934	2.6452	1.5588	0.2797
Average Value				1.6361	0.3439	1.1372	2.4043	1.3461	0.3415
Range				+0.1607	+0.0896	+0.0560	+0.2409	+0.2127	+0.0796
				-0.1438	-0.1355	-0.0592	-0.4454	-0.5224	-0.1580
Number of Generations = 200,				Final Iteration					
Run No.	Pop. Size	P_c	P_m	Correlation Parameters					Fitness Value
				x	y	z	v	w	
1	700	0.70	0.002	1.5741	0.2990	1.1051	2.3422	1.3001	0.4702
2	700	0.70	0.002	1.5653	0.3971	1.1563	2.3194	1.3211	0.4532
3	700	0.70	0.002	1.5601	0.3080	1.1261	2.3233	1.2901	0.4755
4	700	0.70	0.002	1.5928	0.3393	1.0595	2.2691	1.2132	0.4226
5	700	0.70	0.002	1.5504	0.3323	1.0928	2.3095	1.2968	0.4765
6	700	0.70	0.002	1.5695	0.3209	1.1134	2.3300	1.2975	0.4718
7	700	0.70	0.002	1.4385	0.2869	1.0752	2.2089	1.2458	0.4879
8	700	0.70	0.002	1.3372	0.2536	1.0794	2.1991	1.3124	0.4806
9	700	0.70	0.002	1.3676	0.2508	1.1085	2.2373	1.3308	0.4845
10	700	0.70	0.002	1.4123	0.2622	1.0999	2.2387	1.2954	0.4977
Average Value				1.4968	0.3050	1.1016	2.2777	1.2903	0.4720
Range				+0.0960	+0.0921	+0.0547	+0.0652	+0.0405	+0.0257
				-0.1596	-0.0542	-0.0423	-0.0786	-0.0771	-0.0495

Abbreviations: P_c - Probability of Crossover P_m - Probability of Mutation

Pop. - Population

500 generations.. The fitness value shown represents the fitness of the best individual found up to that generation. Jumps occur when a more fit individual turns up in that particular generation. It can be seen that the first 100 generations rapidly evolve towards high fitness values and after about 150 generations no individual with higher fitness is found.

Based on these investigations the population size was set at 700 and 200 generations were used for the final calculations.

When suitable input values are finalized for the GA run it is then possible to obtain close to the optimum solution with one GA run. Any one of the ten results shown in Table 8.3 with the population size of 700 could have been taken as a reasonable optimum solution.

Figure 8.8 Influence of Number of Generations

8.3.2.3 Evaluation of the Fitness of Solutions

Table 8.4 shows three sets of coefficients obtained from the results for a population of 700 as shown in Table 8.3. They are the average values of each coefficients obtained from the ten runs and the set of coefficients with the maximum and minimum fitness. The coefficients are rounded to three significant digits and the corresponding fitness are recalculated. The best way to visualize the significance of these different fitness values is to plot the predicted deviations as obtained from Equation 8.3 using these coefficients. The database used to compare the predicted deviation comprises experimental and computational deviation results obtained from different sources as already mentioned in Section 8.3.1.

Table 8.4
Comparison of Different Solutions

	Correlation Parameters					Fitness Value
	x	y	z	v	w	
Maximum Fitness	1.41	0.26	1.10	2.24	1.30	0.4975
Minimum Fitness	1.59	0.34	1.06	2.27	1.21	0.4148
Average Fitness	1.50	0.30	1.10	2.28	1.29	0.4900

An evaluation of the deviation results obtained with the two sets of coefficients with maximum and minimum fitness are shown in Figure 8.9. This diagram graphically compares the actual and predicted deviations. The solid diagonal line represents perfect agreement between the predicted and actual deviation. The two dashed lines on either side of the diagonal represent a ± 0.50 degree difference between the two. It is apparent from the Figure that both the maximum and minimum fitness solutions produced very similar prediction of deviation and they are well within acceptable boundaries. Therefore, the differences in fitness values quoted in Table 8.4 do not result in significant changes in the correlation coefficients.

Figure 8.10 shows the evaluation based on the averaged coefficients. Evidently, these coefficients also predicted satisfactory deviations. In fact all the coefficients derived from the appropriate GA runs are apparently in the vicinity of the global optimum. Though any of these solutions are acceptable, the average reflects a balance between the extreme ranges of coefficients obtainable and seems the best choice. Collectively all the coefficients may produce a good fitness for a particular solution, but all the individual coefficients may not be the best. Some very good individuals may offset the poor quality of one or two of the other individuals. For example, in the final iteration shown in Table 8.3, the coefficient (x) for stagger angle varies from 1.3372 (run no. 8) to 1.5928 (run no. 4). The average value turns out to be about 1.50 which demonstrates a moderate influence of stagger angle in the overall deviation compared to the other two values. Taking the average of each coefficient ensures that all the coefficients are in good standing and this is reflected in the above average fitness value of 0.490. The averaged coefficients obtained from multiple runs are used in the development of the final form of the deviation correlation discussed in the following Section 8.3.3.

The use of multiple runs is feasible here since the fitness function can be evaluated very quickly. In other optimization problems this would not necessarily be the case. The present results indicate that an acceptable approximation to the optimum can be found from a single run provided that the initial population has been carefully established to be large enough.

8.3.3 Deviation Correlation for Undamaged Blades

As mentioned earlier, the two most widely used correlations for subsonic deviation in axial turbines appear to be those due to Ainley & Mathieson (1951) and Carter & Hughes (1950).

As noted in Section 2.3, the Ainley & Mathieson correlation requires a knowledge of the throat opening and the radius of curvature of the suction side aft of the throat. The throat opening is not available for many of the experimental cases in Table 8.1, although it is known for the hypothetical blade rows. The suction-side radius of curvature is not known for any of the experimental cases. While it might be possible to estimate an average value of the radius of curvature from the dimensions of the blade and the unguided turning (where known), this value

Figure 8.9 Evaluation of Solutions with Maximum and Minimum Fitness

Figure 8.10 Evaluation of Solution with Average Fitness

would be approximate at best. For these reasons, no attempt was made to compare the predictions of the Ainley & Mathieson correlation with the deviation values for the blade rows in the database.

The Carter & Hughes correlation is usually expressed as (e.g. see Dixon, 1989):

$$\delta = m \theta \left(\frac{s}{c} \right)^n \quad (8.6)$$

where, δ = deviation (degrees)
 θ = blade camber (degrees)
 s/c = blade spacing-to-chord ratio

The multiplier m depends on the stagger angle (ξ) and the shape of the blade camber line: namely whether it is circular-arc and parabolic-arc. Modern turbine blade profiles have mean lines which are neither circular nor parabolic. However, they are probably closer to parabolic than circular. Therefore, the correlation for m for parabolic-arc camber lines has been used for the comparisons here. Based on the graphical presentation of m for the parabolic-arc camber provided by Horlock (1973), the correlation for m can be approximated by:

$$m = 0.12 - 2.36 \times 10^{-3} (\xi) + 3.64 \times 10^{-5} (\xi)^2 - 2.27 \times 10^{-7} (\xi)^3$$

As noted, the Carter & Hughes correlation was originally developed for compressors. The authors recommended a value of 1/2 for the power n for general compressor blades and a value of 1 for inlet guide vanes. Dixon (1989) has suggested also using $n=1$ when applying the correlation to turbine blade rows since they, like compressor inlet guide vanes, have accelerating flows. This suggestion has been adopted in the present work.

Figure 8.11 compares the values of deviation predicted by the Carter & Hughes correlation with the reference values for all the cases in Table 8.1. The solid diagonal line represents perfect agreement between the predicted and reference deviations. It is seen that for modern turbine profiles the Carter & Hughes correlation almost invariably overpredicts the deviation, and often by

Figure 8.11 Evaluation of Deviation Correlation of Carter & Hughes

a substantial amount. It should be mentioned that the results are even poorer if $n=1/2$ is used in Equation 8.6. It will be noted that the Carter & Hughes correlation uses several of the parameters which were identified earlier as influencing deviation. The poor agreement shown in Figure 8.11 can be attributed partly to the fact that the correlation was based on old cascade data. However, a more important reason is probably the fact that it omits one or more important parameters.

It is apparent that the Carter & Hughes correlation is not suitable as a starting point for developing a deviation model for damaged blades which would require the predicted deviation

values of the corresponding undamaged blade. Therefore, steps were taken to develop an improved correlation.

In the initial steps toward development of a new deviation correlation, it was found that the blade spacing-to-chord ratio, stagger angle, maximum thickness-to-chord ratio and the total flow turning are the important geometric and flow parameters influencing deviation. Hence, as previously discussed, the genetic algorithm was utilized to optimize the coefficients of the hypothesized correlation. The database used in the optimization process comprised all the experimental and computational cases shown in Table 8.1 with AVDR of unity and blade inlet metal angles greater than 10 degrees. The resulting deviation correlation had the following form:

$$\delta = \frac{\left(\frac{s}{c}\right)^{1.05} (\alpha_1 + \beta_2)^{1.2}}{\zeta^{1.33} \left(\frac{t_m}{c}\right)^{0.3}} \quad (8.7)$$

where

δ	= flow deviation (in degrees)
α_1	= inlet flow angle (in degrees)
β_2	= outlet metal angle (in degrees)
s/c	= blade space-to-chord ratio
t_m/c	= blade maximum thickness-to-chord ratio
ζ	= stagger angle (in degrees)

An evaluation of the results obtained with Equation 8.7 is shown in Figure 8.12. Comparison with Figure 8.11 shows that this correlation is more effective than Carter's correlation, although it still noticeably over or underpredicts the deviations for some cascades.

Figure 8.12 Evaluation of Correlation with β_i Excluded

While encouraging, the results suggested that one or more additional geometric or flow parameters are needed to improve the correlation. Therefore, for reasons already discussed in Section 8.2, the blade inlet metal angle was introduced into the correlation. The optimization process was carried out again with the same initial data set. The process of obtaining the optimum coefficients for this case was described in the previous section. The evaluation diagram for the resulting correlation is already shown in Figure 8.9 in the previous section. It is evident that there is further improvement in the predicted deviation.

The next step was to make the correlation compatible with zero inlet metal angle and AVDR values other than unity. Cases with zero stagger angle were not considered for reasons already mentioned in Section 8.3.2.3. The blade inlet metal angle parameter, $(\beta_1)^w$, was replaced by the following function to allow the correlation to apply for $\beta_1 = 0$:

$$a + b\beta_1^w$$

where a and b are constants. Using this function, an initial estimate of the constant a was determined by applying the Equation 8.5 to the four cases of nozzle blades with zero inlet metal angle (case nos. 38 through 41 in Table 8.1). It is obvious that the constant b has to assume some fractional value to accommodate the constant a . The optimization of the revised correlation was finally carried out with the additional parameter AVDR and constants a and b . However, in the final optimization process the range of search space for each coefficient was narrowed down to $\pm 10\%$ of the already known values (except that the power on the term β_1 was left open). As a result a smaller population size of 500 could be used to evolve through 100 generations only. Besides the fact that this saved computational time, it was believed that the slight adjustments to the original form of the correlation would not affect the values of the coefficients significantly. Table 8.5 shows the optimization process and the finally selected coefficients. The coefficients x , y , z ...etc. represent the same geometric and flow parameters already defined in the previous section. As anticipated, the final coefficients are not significantly different from the earlier values. The following equation gives the final form of the deviation correlation for undamaged blade:

$$\delta = \frac{(AVDR)^3 \left(\frac{s}{c}\right)^{1.1} (\alpha_1 + \beta_2)^{2.25}}{\zeta^{1.45} \left(\frac{t_m}{c}\right)^{0.3} (22 + 0.22\beta_1^{1.64})} \quad (8.8)$$

Table 8.5
Determination of Correlation Coefficients

Final Iteration											
Number of Generations = 100, Population Size = 500											
Run No.	P_c	P_m	Correlation Parameters								Fitness Value
			av	x	y	z	v	w	a	b	
1	0.7	0.002	2.99	1.4331	0.3134	1.0677	2.2219	1.6759	21.71	0.1831	0.1484
2	0.7	0.002	3.05	1.5120	0.3056	1.1134	2.2962	1.6240	21.71	0.2413	0.1464
3	0.7	0.002	2.91	1.4157	0.2600	1.1102	2.2502	1.6807	23.43	0.1988	0.1486
4	0.7	0.002	3.02	1.4835	0.2900	1.1244	2.2834	1.5972	21.71	0.2649	0.1459
5	0.7	0.002	2.91	1.4015	0.3024	1.0913	2.2043	1.6728	22.28	0.1830	0.1501
6	0.7	0.002	2.99	1.4378	0.2708	1.0945	2.2639	1.6507	23.42	0.2240	0.1504
7	0.7	0.002	2.99	1.4315	0.2535	1.0661	2.2727	1.6335	24.00	0.2508	0.1474
8	0.7	0.002	3.01	1.4535	0.3024	1.0930	2.2658	1.6161	23.42	0.2586	0.1475
9	0.7	0.002	3.05	1.5153	0.2840	1.1129	2.2921	1.6590	20.57	0.1986	0.1452
10	0.7	0.002	2.95	1.4227	0.3153	1.1114	2.2056	1.6292	20.57	0.2065	0.1460
Average Value			2.98	1.4506	0.2897	1.0985	2.2555	1.6439	22.28	0.2209	0.1478
Selection of Correlation Coefficients											
Average Value of Each Coefficient to Three Significant Digits			Correlation Parameters								Fitness Value
			av	x	y	z	v	w	a	b	
			3.00	1.45	0.30	1.10	2.25	1.64	22.0	0.22	

Abbreviations: P_c - Probability of Crossover, P_m - Probability of Mutation

The symbols are already defined in Equation 8.7. Figure 8.13 compares the deviations predicted by Equation 8.8 with the reference values for all the cases in Table 8.1. Bearing in mind the estimated uncertainty of ± 0.5 degrees in both the measured and computed values of deviation, the agreement is seen to be very satisfactory. The improvement over the Carter & Hughes correlation (Figure 8.11) is evident. The largest discrepancies occur mainly for off-design cases or values of AVDR significantly different from 1.0. In the former, viscous effects in the form of trailing-edge separation may be starting to influence the deviation. The influence of AVDR has been determined from rather sparse data and may therefore be somewhat inaccurately resolved.

Figure 8.13 Evaluation of Subsonic Deviation Correlation

The new deviation correlation for undamaged blades represents a single equation which predicts deviation based on the general characteristics of the cascade (e.g. metal angles, stagger, chord, pitch etc.). This is a simple correlation that can be easily used as the basis for determining the total deviation for a damaged blade. This is discussed in the following section. Finally, this is followed by the discussion of deviation correlations for high-speed flow.

8.3.4 Deviation Correlation for Damaged Blades

In Chapter 7 it was shown that the experimental and computational incremental deviation ($\Delta\delta$), defined as the additional increase in deviation over that of an undamaged blade, is approximately a linear function of the chordwise penetration of the blade damage. It was also shown that the computed incremental deviation was approximately twice the magnitude of measured incremental deviation for both the HS1A and LS3 blades due to the respective difference in the number of blades damaged. These relationships are utilized to develop an empirical correlation to predict the incremental deviation due to different amounts of damage. Thus the actual predicted deviation due to damage is obtained by adding the predicted undamaged deviation to the corresponding predicted incremental deviation. As already mentioned, in addition to the extent of damage size, it is believed that the parameters that affect deviation for an undamaged blade will also largely determine the incremental deviation due to damage. Again the genetic algorithm was used to determine the empirical coefficients of the correlation based on a database of known values of damaged blade deviation.

Ten different blade geometries were selected from Table 8.1 to use in computing the additional deviation due to damage. The selected blades represent a reasonable range of the various geometric and flow parameters such as stagger, inlet metal angle and total turning. For the damaged blade computations, only cases at design incidence and with AVDR of unity were considered. For each selected blade, computational data were generated for damage sizes of 5% , 10%, 15% and 20% chord penetration.

Unlike the cases for undamaged blades, there is a very limited number of measured cases for blades with damage. As already mentioned, experimental data were collected for two damage

sizes, namely 10% and 20% chord penetration. Table 8.6 summarizes the damaged blade database for low-speed flow conditions and can be found at the end of this chapter. The table also includes the deviation for the same blades without damage. It may be noted that the measured AVDR values are considerably larger than unity, namely about 1.1 for the undamaged blade and 1.2 for the damaged blades. Only the deviation data are listed in Table 8.6; the corresponding blade geometries are given in Table 8.1.

The genetic algorithm was used to determine the correlation coefficients in the same manner as previously discussed for undamaged blades. The database used in the optimization process comprised of only the computational incremental deviation (see Table 8.6) with 20% damage size. As already noted in the previous chapter, the computed deviation represents the expected deviation due to all the blades being damaged in the rotor. On the other hand, the measured deviation is due to a single blade damaged. Therefore, a correction factor is applied to the maximum deviation predicted by the correlation, to account for the number of blades damaged in the blade row. Based on the assumed linear relationship between incremental deviation and damage size, the values of incremental deviation for other than 20% damage size are obtained by applying a simple linear scaling factor. The correlation function was assumed to take the following form:

$$\Delta\delta_d = \frac{k\zeta^x \left(\frac{t_m}{c}\right)^y \left(\frac{s}{c}\right)^z (\alpha_1 + \beta_2)^v}{a + b\beta_1^p (1 + c\beta_1^q)} \quad (8.9)$$

where,

- $\Delta\delta_d$ = incremental flow deviation for 20% damaged blade (in degrees)
- α_1 = inlet flow angle (in degrees)
- β_2 = outlet metal angle (in degrees)
- s/c = blade space-to-chord ratio
- t_m/c = blade maximum thickness-to-chord ratio
- ζ = stagger angle (in degrees)
- $a, b, c, k, p, q, x, y, z, v$ = correlation coefficients

The above equation differs slightly from that used for undamaged blades (Equation 8.8) because the blade inlet metal angle parameter used in the denominator of the above function was found to give better results. The following equation gives the final form of the incremental deviation correlation for blades with 20% damage size:

$$\Delta\delta_d = \frac{2\sqrt{\left(\frac{s}{c}\right)}(\alpha_1 + \beta_2)^{1.3}}{\zeta\sqrt{\left(\frac{t_m}{c}\right)}(20 - 0.85\beta_1(1 - 0.015\beta_1))} \quad (8.10)$$

The above correlation predicts the maximum incremental deviation when all blades in the rotor are damaged. Based on the fact that the maximum incremental deviation is reduced by a factor of two when only a single blade is damaged (see Section 7.3), a correction factor is applied to this correlation to account for the number of blades damaged. The actual deviation can be obtained by adding the undamaged blade deviation to the incremental one which is as follows:

$$\delta_d = \delta + 5D \Delta\delta_d \left[1 - 0.5 \left(1 - \frac{N_D}{N_T} \right) \right] \quad (8.11)$$

where

δ_d	= deviation due to damage (in degrees)
δ	= deviation for undamaged blade (in degrees)
D	= damage size as a fraction of chord penetration (e.g. 0.1, 0.2 etc.)
$\Delta\delta_d$	= incremental deviation (Equation 8.10)
N_D	= number of damaged blades
N_T	= total number of blades in rotor

The above correlation provides the incremental deviation for per blade damaged in a blade row. In a mean line calculation, the average value of incremental deviation for the entire row would be needed. Since the damage would normally affect only part of the blade span, the deviation given by Equation 8.11 would be taken to apply to the damaged portion only. The average deviation for

the blade row would then be taken as a weighted average of damaged and undamaged deviation, based on the spanwise extent of the damage.

The correlation is based on a database which includes a maximum damage size of 20% chord penetration. It should be used with caution for larger damage sizes. Figure 8.14 compares the deviation predicted by Equation 8.11 with the reference deviation values listed in Table 8.6. The agreement is quite satisfactory. However, for the experimental cases, it should be noted that some AVDR values used in the prediction of undamaged blade deviation (δ) are outside the limits used in the development of the new correlation (Equation 8.8). The effect of large AVDR on the predictions of deviation for the HS1A blade is discussed further in Section 8.5.

This low-speed deviation correlation for damaged blades was further extended to include the influence of compressibility. The development of the high-speed deviation correlation for damaged blades is discussed in the following sections.

Figure 8.14 Evaluation of Subsonic Damaged Blade Deviation Correlation (Equation 8.11)

8.4 Deviation Correlations for High-Speed Flow

8.4.1 Deviation Correlation for Damaged Blades

It has been shown that the use of the damaged blade deviation correlation for subsonic flow requires the knowledge of the corresponding undamaged blade deviation. However, in the case of high-speed flows, it was found that a suitable correlation could be developed without the use of undamaged blade deviation.

The correlation is based on both experimental and computed results, as before. The experimental data comprised of 10% and 20% chord penetration with exit Mach numbers 0.95, 1.05 and 1.20 for each damage size. Additional computed deviation for damaged blades were obtained for damage sizes of 5%, 10%, 15% and 20% chord penetration using the same set of ten blades that had been used for subsonic damaged blade calculations. For each damage size computed deviation was obtained for exit Mach numbers 0.85, 1.05 and 1.20. The complete database of all the experimental and computational deviation results is given in Table 8.7 which can be found at the end of this chapter.

The subsonic deviation correlation was extended to include the effects of compressibility by introducing a non-dimensional deviation parameter R_d which is defined as follows:

$$R_d = \frac{\delta_{d,7} + \Delta\delta_{d,M}}{\delta_{d,7}}$$

where $\delta_{d,7}$ = damaged blade deviation at exit Mach number 0.7

$\Delta\delta_{d,M}$ = change in damaged blade deviation due to increase in Mach number above 0.7

Figure 8.15 shows the variation of the non-dimensional damaged blade deviation parameter R_d with Mach number for five different computed damaged blade cases. The variation of the parameter R_d for all the blades appears to follow a similar pattern. The nature of the curves suggest that a polynomial equation based on the exit Mach number and damage size will reasonably follow the variation in deviation. Again the genetic algorithm was used to determine the optimum coefficients

for the polynomial equation based on the available database. The final form of the equation is given by:

$$R_d = a + \frac{bM}{\sqrt{1+D}} + \frac{cM^2}{1+D} + \frac{dM^3}{1+D} \quad (8.12)$$

where $a = 3.1$ $b = -3.25$ $c = -1.8$ $d = 2.8$

M = exit Mach number

D = damage size as percent of chord penetration (e.g. 0.1, 0.2)

Figure 8.15 Variation of Non-dimensional Deviation Parameter R_d with Exit Mach Number

Finally the deviation for a damaged blade at any Mach number ranging from subsonic exit conditions to Mach 1.2 is obtained from the equation:

$$\delta_d = \delta_{d_s} \times R_d \quad (8.13)$$

where δ_d = resultant deviation for damaged blade

δ_{d_s} = subsonic damaged blade deviation (Equation 8.11)

Figure 8.16 compares the deviation predicted by Equation 8.13 with the reference deviation values, for different exit Mach numbers listed in Table 8.7. The agreement is quite satisfactory for the measured deviation values. Most of the values of deviation predicted by the

Figure 8.16 Evaluation of Transonic Damaged Blade Deviation Correlation (Equation 8.13)

correlation are within ± 1 degree of the reference computed deviation. The largest discrepancies are observed for an exit Mach number of 1.2. A few of the poorly predicted deviation values at Mach number 1.2 are for nozzle blades with 20% damage. Evidently the predicted non-dimensional deviation parameter R_d (Equation 8.12) for nozzle blades does not work well for high Mach numbers and large damage size.

Unfortunately, it was not possible to develop an improved deviation correlation for undamaged blades at high-speed flows within the scope of the present study. However, to provide a complete treatment of the subject of deviation for undamaged blades, the widely used prediction methods of Ainley & Mathieson (1951) at both low and high-speed flows are discussed in the following sections.

8.4.2 Outlet Flow Angle Predictions by Ainley & Mathieson

Method

As already mentioned previously in connection with the new subsonic deviation correlation, the outlet flow angle prediction method by Ainley & Mathieson (1951) requires the knowledge of the throat opening and the radius of curvature of the suction side aft of the throat. Though the throat opening is known for most of the in-house blades and hypothetical blades, the suction surface radius of curvature aft of the throat is not available for any of the blades. In fact, for modern turbine blades the radius of curvature in this region is generally not constant. As a result, an evaluation of the deviation predictions by Ainley & Mathieson for subsonic flow was not presented previously. However, for a few of the blade geometries in the present database it was possible to obtain an approximate value of the mean radius of curvature based on the known values of the unguided turning angle. These are used to evaluate the predictions of outlet flow angle by Ainley & Mathieson.

The Ainley & Mathieson correlation is usually provided graphically for various operating conditions, as already shown in Figure 2.6 in Chapter 2. Wilson (1984) has fitted these graphical representations with analytical expressions, as also shown in Chapter 2. They are as follows:

1. Predicted outlet angle for exit Mach numbers up to 0.5:

$$|\alpha_2|_{0 < M_t < 0.5} = \left[\frac{7}{6} \left(\cos^{-1} \left(\frac{o}{s} \right) - 10^\circ \right) + 4^\circ \left(\frac{s}{e} \right) \right],$$

where, o = throat opening,

s = blade pitch,

e = radius of curvature of blade suction surface downstream of throat,

M_t = Mach number at throat.

2. Predicted outlet flow angle for turbines with sonic flow at the throat ($M_t=1.0$):

$$|\alpha_2|_{M_t=1.0} = \left| \cos^{-1} \left(\frac{o}{s} \right) - \left(\frac{s}{e} \right)^{(1.786 + 4.128(s/e))} \sin^{-1} \left(\frac{o}{s} \right) \right|$$

Both the equations require the value of s/e . It is possible to obtain an approximate value of s/e from the throat geometry and unguided turning angle. Figure 8.17 illustrates the various geometric parameters involved in the above correlation. From the right angle triangle formed by the lines showing the throat (o), pitch (s) and the tangent to the suction surface (L), the following relationship is evident:

$$\left(\frac{o}{s} \right)^2 + \left(\frac{L}{s} \right)^2 = 1$$

or,
$$L = s \sqrt{1 - \left(\frac{o}{s} \right)^2}$$

The arc length from aft of the throat to the trailing edge can be approximated by L . Also, the approximate arc length L can be related to the radius of curvature e as follows:

$$L = e \theta_u \frac{\pi}{180}, \quad \text{where } \theta_u \text{ is the unguided turning angle in degrees.}$$

Figure 8.17 Schematic Diagram of Cascade Exit Showing Throat and Suction Surface Curvature Parameters

Finally, equating the above two relationships for L , an approximate value of s/e can be obtained as follows:

$$\frac{s}{e} = \frac{\theta_u}{\sqrt{1 - \left(\frac{o}{s}\right)^2}} \frac{\pi}{180}$$

Eight blade geometries were selected for the purpose of evaluation of the Ainley & Mathieson prediction method. Outlet flow angles were obtained using the appropriate subsonic and sonic analytic expressions derived by Wilson. Since the predictions are given in terms of outlet flow angles, they are compared with the computed and measured outlet flow angles. The database used for the present evaluation is summarized in Table 8.8, at the end of the chapter.

Figure 8.18 compares the outlet flow angles predicted by the Ainley & Mathieson method with the reference values for the different subsonic and sonic exit Mach numbers listed in Table 8.8. Most of the predictions for outlet flow angles for subsonic flow are within ± 1 degree of the reference values. This helps to explain why the Ainley & Mathieson correlation has continued to be used with reasonably satisfactory results. However, it is also seen that there is one rogue point for which the outlet flow angle is significantly over predicted. Comparisons with a wider range of cases would presumably reveal additional such points. By contrast the new subsonic deviation correlation developed in Section 8.3.3 did not give any rogue points for a much larger database of cases. The results in Figure 8.18 indicate that the gauge angle and suction surface radius of curvature are insufficient to capture the geometric influences on the outlet flow angle for modern turbine blade geometries.

The evaluation diagram shows reasonable predictions of outlet flow angles for most of the cases with exit Mach number of unity although there is again one rogue point. No attempt has been made to arrive at an improved correlation for the sonic outlet condition.

Figure 8.18 Evaluation of the Predicted Outlet Flow Angle by Ainley & Mathieson

8.5 Predictions of Deviation for HS1A Blade

Experimental deviation data for the HS1A blade was obtained after the new subsonic deviation correlation for undamaged blades had been developed. This section re-examines the deviation correlation based on measured design and off-design cases at exit Mach number 0.7 for the HS1A blade. It has already been noted that the present deviation correlation has the potential for off-design applications also. Measured deviations have been obtained for the undamaged HS1A blade at zero and 4.5 degrees of incidence only. Unfortunately, it was not possible to obtain other off-design cases due to limited time. The AVDR for the design incidence is 1.10 and it is 1.02 for off-design incidence. The design incidence AVDR for the HS1A blade is larger than the AVDR

values that had been used to derive the new correlation. Since the influence of AVDR in the new correlation is based on rather sparse data, the predictions of deviation for the HS1A blade are also compared with the other five cases with non-unity AVDR values. Table 8.9 summarizes the subsonic deviation data for the HS1A blade as well as the five additional cases with different AVDR values. Table 8.9 is located at the end of this chapter.

Figure 8.19 shows the deviations predicted by the subsonic undamaged blade deviation correlation for all the cases listed in Table 8.9. The AVDR is raised to the power 3 as indicated in Equation 8.8. It is evident that the discrepancies are slightly over 1 degree for the HS1A blade at design incidence and also for two other blades. The off-design prediction for the HS1A blade is quite satisfactory.

Figure 8.19 Evaluation of the Present Subsonic Deviation Correlation for Cases with Different AVDR Values

As previously mentioned, Rodger (1992) suggested a linear trend in deviation with AVDR while the genetic algorithm optimization procedure gave a cubic variation. An attempt was made to see if the overall predictions improved with a power of unity on the AVDR. Figure 8.20 shows the predicted deviation based on this linear variation in AVDR. Though the predictions for the HS1A blade improved, there is a larger discrepancy for one of the other cases. It seems the seven cases are still insufficient to provide a conclusive value of the coefficient on AVDR in the new subsonic deviation correlation. As a result, the original value of the coefficient was retained. It may be noted that Saroch (1995) investigated the influence of AVDR on deviation computationally with the Dawes code and his results were also inconclusive. More experimental data are required to determine the appropriate relationship between deviation and AVDR at subsonic flow conditions.

Figure 8.20 Predicted Deviation Based on a Linear Variation in AVDR

Table 8.1

Experimental and Computed Deviation for Subsonic Flow in Axial Turbines

S. No.	Source	Designation	Incidence (I)	Experimental Cases							Deviation		Symbol
				ζ	L_w/C	S/C	α_1	β_2	β_1	AVDR	Measured	Present Correlation (Eqn. 8.8)	
1	De Cecco (19995)	LS1	0.0	37.20	0.090	0.600	0.0	49.60	11.10	1.00	1.8	1.21	■
2	Rodger (1992)	LS2	0.0	23.06	0.182	0.682	28.4	57.50	29.29	1.00	4.4	3.33	
3	Isaacs (1994)	LS3	0.0	21.60	0.196	0.682	28.4	57.50	25.35	1.00	4.5	4.22	
4	Serjack (1995)	LS3	0.0	21.60	0.196	0.682	28.4	57.50	25.35	1.00	4.2	4.22	
5	Mee & Baines (1992)		0.0	42.00	0.250	0.840	42.8	70.00	45.00	1.00	2.0	1.70	●
6	Haller & Camus (1984)		0.0	29.60	0.375	0.842	56.0	64.50	56.00	1.00	2.1	2.14	
7	Moustapha et al. (1993)		0.0	62.10	0.250	0.840	0.0	76.00	10.00	1.00	2.0	1.70	
8	Klock et al.(1986)		0.0	33.30	0.240	0.71	30.0	67.03	33.50	0.929	1.6	1.68	
9	Hodson & Dominy (1990)		0.0	19.60	0.125	0.564	38.8	53.60	42.80	0.91	1.9	2.09	◆
10	Hohelsel (1990)		0.0	30.72	0.100	0.799	37.7	63.20	48.80	1.014	2.4	2.41	
11	Rodger (1992)	LS2	0.0	23.06	0.182	0.682	28.4	57.50	29.29	0.96	3.8	2.94	
12	Rodger (1992)	LS2	0.0	23.06	0.182	0.682	28.4	57.50	29.29	1.08	5.5	4.20	

Abbreviations: Carleton Cascade

- LS1 - Low Speed Blade No. 1
 LS2 - Low Speed Blade No. 2
 LS3 - Low Speed Blade No. 3

 ζ - Stagger ($^\circ$) α_1 - Inlet flow Angle ($^\circ$) β_2 - Outlet Metal Angle ($^\circ$) β_1 - Inlet Metal Angle ($^\circ$) i - Incidence Angle ($^\circ$) t_{tr}/C - Maximum Thickness to Chord Ratio S/C - Space to Chord Ratio

AVDR - Axial Velocity Density Ratio

Table 8.1(continued)

Experimental and Computed Deviation for Subsonic Flow in Axial Turbines

Case No.	Source	Designation	Off-Design (I)	Experimental Cases							Deviation		Symbol
				ζ	t_w/C	S/C	α_1	β_2	β_1	AVDR	Measured	Present Correlation (Eqn. 8.8)	
13	Abbott (1993)	LS3	-10.0	21.60	0.196	0.682	18.4	57.50	25.35	1.00	4.0	3.19	▲
14	Abbott (1993)	LS3	+10.0	21.60	0.196	0.682	38.4	57.50	25.35	1.00	4.5	5.40	
15	Serjak (1995)	LS3	-10.0	21.60	0.196	0.682	18.4	57.50	25.35	1.00	4.6	3.19	
16	Serjak (1995)	LS3	+10.0	21.60	0.196	0.682	38.4	57.50	25.35	1.00	4.9	5.40	
		Designation	Incidence (I)	Computational Cases							Computed	Present Correlation	Symbol
17	3D Navier-Stokes Solver	HS1A	0.0	25.10	0.203	0.728	46.00	59.00	50.50	1.00	1.88	2.37	□
18		HS3	0.0	18.80	0.250	0.760	46.0	59.00	50.50	1.00	3.46	3.54	
19		LS1	0.0	37.20	0.090	0.600	0.0	49.60	11.10	1.00	1.00	1.21	
20		LS2	0.0	23.06	0.182	0.682	28.4	57.50	29.29	1.00	3.57	3.33	
21		LS3	0.0	21.60	0.196	0.682	28.4	57.50	25.35	1.00	4.53	4.22	
22		HypTB	0.0	25.10	0.200	0.728	46.0	59.00	50.50	1.00	2.34	2.37	
23		HypTB	0.0	18.80	0.250	0.600	46.0	59.00	50.50	1.00	2.38	2.67	

Abbreviations: Carleton Cascade

HS1A - High Speed Blade No. 1A
 HS3 - High Speed Blade No. 3
 LS1 - Low Speed Blade No. 1
 LS2 - Low Speed Blade No. 2
 LS3 - Low Speed Blade No. 3

 ζ - Slagger ($^\circ$) α_1 - Inlet flow Angle ($^\circ$) α_2 - Outlet flow Angle ($^\circ$) β_1 - Inlet Metal Angle ($^\circ$) I - Incidence Angle ($^\circ$) t_w/C - Maximum Thickness to Chord Ratio

S/C - Space to Chord Ratio

AVDR - Axial Velocity Density Ratio

HypTB - Hypothetical Turbine Blade

Table 8.1(continued)
Experimental and Computed Deviation for Subsonic Flow in Axial Turbines

Case	Source	Designation	Incidence (I)	Computational Cases							Deviation		Symbol
				ζ	t_w/C	S/C	α_1	β_2	β_1	AVDR	Computed	Present Correlation (Eqn. 8.8)	
24	3D Navier-Stokes Solver	HypTB	0.0	25.10	0.231	0.728	46.0	59.00	50.50	1.00	2.03	2.27	□
25		HypTB	.0.0	25.10	0.195	0.728	36.0	59.00	40.50	1.00	2.58	2.59	
26		HypTB	0.0	25.10	0.202	0.728	25.5	59.00	25.50	1.00	3.00	3.46	
27		HypTB	0.0	25.10	0.221	0.682	26.9	57.50	25.35	1.00	3.62	3.14	
28		HypTB	0.0	18.80	0.235	0.682	28.4	57.50	25.35	1.00	5.20	4.90	
29		HypTB	0.0	18.80	0.260	0.700	28.4	57.50	25.35	1.00	4.76	4.90	
30		HypTB	0.0	21.60	0.240	0.689	30.0	57.50	33.00	1.00	3.76	3.10	
31		HypTB	0.0	28.80	0.200	0.760	46.0	59.00	50.50	1.00	2.06	2.04	
32		HypTB	0.0	30.10	0.190	0.728	46.0	59.00	50.50	1.00	2.27	1.85	
33		HypTB	0.0	30.10	0.191	0.728	33.0	59.00	37.50	1.00	1.95	2.06	
34		HypTB	0.0	30.10	0.195	0.728	25.5	59.00	25.50	1.00	2.31	2.70	

Abbreviations:

HypTB - Hypothetical Turbine Blade

 ζ - Stagger ($^\circ$) α_1 - Inlet flow Angle ($^\circ$) α_2 - Outlet flow Angle ($^\circ$) β_1 - Inlet Metal Angle ($^\circ$) I - Incidence Angle ($^\circ$) t_w/C - Maximum Thickness to Chord Ratio

S/C - Space to Chord Ratio

AVDR - Axial Velocity Density Ratio

Table 8.1(continued)

Experimental and Computed Deviation for Subsonic Flow in Axial Turbines

Case No.	Source	Designation	Incidence (i)	Computational Cases							Deviation		Symbol
				ζ	t_w/C	S/C	α_1	β_2	β_1	AVDR	Computed	Present Correlation (Eqn. 8.8)	
35	3D Navier-Stokes Solver	HypNB	0.0	27.60	0.200	0.780	10.0	49.00	10.00	1.00	2.48	3.06	○
36		HypNB	0.0	32.10	0.186	0.746	5.0	51.00	5.00	1.00	2.20	2.66	
37		HypNB	0.0	32.10	0.186	0.746	2.5	51.00	5.00	1.00	2.21	2.40	
38		HypNB	0.0	27.60	0.226	0.780	0.0	49.00	0.00	1.00	2.64	2.78	
39		HypNB	0.0	26.90	0.242	0.786	0.0	49.00	0.00	1.00	2.62	2.85	
40		HypNB	0.0	29.50	0.200	0.749	0.0	51.00	0.00	1.00	3.35	2.75	
41		HypNB	0.0	26.90	0.235	0.780	0.0	49.90	0.00	1.00	3.18	3.00	
			Off-Design										
42		HS3	-10.0	18.80	0.250	0.760	36.0	59.00	50.50	1.00	3.00	2.82	Δ
43		HS3	+10.0	18.80	0.250	0.760	56.0	59.00	50.50	1.00	3.89	4.34	
44		LS3	-10.0	21.60	0.196	0.682	18.4	57.50	25.35	1.00	4.40	3.19	
45		LS3	+10.0	21.60	0.196	0.682	38.4	57.50	25.35	1.00	4.80	5.40	

Abbreviations: Carleton Cascade

HS3 - High Speed Blade No. 3

LS3 - Low Speed Blade No. 3

HypNB - Hypothetical Nozzle Blade

ζ - Stagger ($^{\circ}$)

α_1 - Inlet flow Angle ($^{\circ}$)

α_2 - Outlet flow Angle ($^{\circ}$)

β_1 - Inlet Metal Angle ($^{\circ}$)

I - Incidence Angle ($^{\circ}$)

t_w/C - Maximum Thickness to Chord Ratio

S/C - Space to Chord Ratio

AVDR - Axial Velocity Density Ratio

Table 8.2**Range of Blade Parameters**

Blade Parameters	Maximum	Minimum
Stagger (ζ)	62.10	18.80
Maximum Thickness-to-Chord Ratio (t_m/C)	0.375	0.09
Space-to-Chord Ratio (S/C)	0.842	0.564
Inlet Metal Angle (β_1)	56.00	0.0
Inlet Flow Angle (α_1)	56.00	0.0
Outlet Metal Angle (β_2)	76.00	49.00
Axial Velocity Density Ratio (AVDR)	1.08	0.91
Deviation (δ)	5.50	1.60

Table 8.6
Experimental and Computational Deviation for Damaged Blades

Case No.	Blade Geometry Reference (Table 8.1)	Experimental Cases					Symbol
		Damage Size (% Chord)	Total Deviation		Incremental Deviation		
			Measured	Present Correlation (Eqn. 8.11)	Measured	Present Correlation (Eqn. 8.10)	
1	8.1/17	0	1.91	2.37	0.0	0.0	
2	8.1/17	10	4.30	4.81	2.39	2.44	▲
3	8.1/17	20	5.61	6.47	3.70	4.10	■
Computational Cases							
4	8.1/17	0	1.88	2.37	0	0	
5	8.1/18	0	3.46	3.54	0	0	
6	8.1/26	0	3.00	3.46	0	0	
7	8.1/34	0	2.31	2.70	0	0	
8	8.1/3	0	4.53	4.22	0	0	
9	8.1/29	0	4.76	4.90	0	0	
10	8.1/30	0	3.76	3.10	0	0	
11	8.1/41	0	3.18	3.00	0	0	
12	8.1/40	0	3.35	2.75	0	0	
13	8.1/36	0	2.20	2.66	0	0	
14	8.1/17	5	3.64	4.04	1.76	1.67	Δ
15	8.1/18	5	4.73	5.60	1.27	2.06	
16	8.1/26	5	4.50	5.28	1.5	1.82	
17	8.1/34	5	3.15	4.25	0.84	1.55	
18	8.1/3	5	6.10	6.34	1.57	2.12	
19	8.1/29	5	6.27	7.04	1.51	2.14	
20	8.1/30	5	4.60	5.35	0.84	2.25	
21	8.1/41	5	3.93	3.27	0.75	0.27	
22	8.1/40	5	4.12	3.02	0.77	0.27	
23	8.1/36	5	2.99	3.02	0.79	0.36	
24	8.1/17	10	5.84	5.71	3.96	3.34	□
25	8.1/18	10	6.85	7.64	3.39	4.10	

Table 8.6 (continued)
Experimental and Computational Deviation for Damaged Blades

Case No.	Blade Geometry Reference (Table 8.1)	Computational Cases					Symbol
		Damage Size (% Chord)	Total Deviation		Incremental Deviation		
			Computed	Present Correlation (Eqn. 8.11)	Computed	Present Correlation (Eqn. 8.10)	
27	8.1/26	10	5.30	7.11	2.30	3.65	=
28	8.1/34	10	4.84	5.80	2.53	3.10	
29	8.1/3	10	9.33	8.47	4.80	4.25	
30	8.1/29	10	9.18	9.19	4.42	4.29	
31	8.1/30	10	7.44	7.60	3.68	4.50	
32	8.1/41	10	4.18	3.55	1.00	0.55	
33	8.1/40	10	4.36	3.30	1.01	0.55	
34	8.1/36	10	3.51	3.39	1.35	0.73	
35	8.1/17	15	7.71	7.38	5.83	5.01	=
36	8.1/18	15	8.80	9.69	5.34	6.15	
37	8.1/26	15	7.50	8.94	4.50	5.48	
38	8.1/34	15	6.51	7.35	4.20	4.65	
39	8.1/3	15	11.60	10.60	7.07	6.38	
40	8.1/29	15	11.66	11.34	6.90	6.44	
41	8.1/30	15	9.54	9.85	5.78	6.75	
42	8.1/41	15	4.71	3.82	1.53	0.82	
43	8.1/40	15	4.69	3.56	1.34	0.81	o
44	8.1/36	15	4.16	3.75	2.00	1.09	
45	8.1/17	20	9.28	9.04	7.40	6.67	
46	8.1/18	20	10.49	11.74	7.03	8.20	
47	8.1/26	20	9.00	10.77	6.00	7.31	
48	8.1/34	20	8.21	8.90	5.90	6.20	
49	8.1/3	20	14.32	12.71	9.79	8.49	
50	8.1/29	20	13.79	13.48	9.03	8.58	
51	8.1/30	20	11.87	12.10	8.11	9.00	
52	8.1/41	20	5.07	4.09	1.89	1.09	
53	8.1/40	20	5.16	3.84	1.81	1.09	
54	8.1/36	20	4.76	4.11	2.60	1.45	

Table 8.7
Deviation Data for High Speed Flow

Case No.	Blade Geometry Reference (Table 8.1)	Mach Number	Damage Size (% Chord)	Total Deviation		Symbol
				Measured	Correlation for damaged blade (Equation 8.13)	
1	8.1/17	0.72	0	1.91		
2			10	4.31	4.81	▲
3			20	5.61	6.47	
4		0.95	0	1.76		
5			10	4.85	4.00	■
6			20	6.61	5.89	
7		1.05	0	2.19		
8			10	5.20	4.52	▼
9			20	7.50	6.34	
10		1.20	0	3.46		
11			10	5.50	6.44	◆
12			20	8.40	8.66	
				Computed	Correlation for damaged blade	
13	8.1/17	0.70	0	1.88		Δ
14			5	3.64	4.04	
15			10	5.84	5.71	
16			15	7.71	7.38	
17			20	9.28	9.04	
18	8.1/18	0.70	0	3.46		
19			5	4.73	5.60	
20			10	6.85	7.64	
21			15	8.80	9.69	
22			20	10.49	11.74	
23	8.1/26	0.7	0	3.00		
24			5	4.50	5.28	
25			10	5.30	7.11	
26			15	7.50	8.94	
27			20	9.00	10.77	

Table 8.7 (continued)
Deviation Data for High Speed Flow

Case No.	Blade Geometry Reference (Table 8.1)	Mach Number	Damage Size (% Chord)	Total Deviation		Symbol
				Computed	Correlation for damaged blade (Equation 8.13)	
28	8.1/34	0.7	0	2.31		Δ
29			5	3.15	4.25	
30			10	4.84	5.80	
31			15	6.51	7.35	
32			20	8.21	8.90	
33	8.1/3	0.7	0	4.53		
34			5	6.10	6.34	
35			10	9.33	8.47	
36			15	11.60	10.60	
37			20	14.32	12.71	
38	8.1/29	0.7	0	4.76		
39			5	6.27	7.04	
40			10	9.18	9.19	
41			15	11.66	11.34	
42			20	13.79	13.48	
43	8.1/30	0.7	0	3.76		
44			5	4.60	5.35	
45			10	7.44	7.60	
46			15	9.54	9.85	
47			20	11.87	12.10	
48	8.1/41	0.7	0	3.18		
49			5	3.93	3.27	
50			10	4.18	3.55	
51			15	4.71	3.82	
52			20	5.07	4.09	
53	8.1/40	0.7	0	3.35		
54			5	4.12	3.02	
55			10	4.36	3.30	

Table 8.7 (continued)
Deviation Data for High Speed Flow

Case No.	Blade Geometry Reference (Table 8.1)	Mach Number	Damage Size (% Chord)	Total Deviation		Symbol
				Computed	Correlation for damaged blade (Equation 8.13)	
56			15	4.69	3.56	Δ
57			20	5.16	3.84	
58	8.1/36	0.7	0	2.20		
59			5	2.99	3.02	
60			10	3.51	3.39	
61			15	4.16	3.75	
62			20	4.76	4.11	
63	8.1/17	0.85	0	1.10		□
64			5	3.61	3.24	
65			10	5.22	4.84	
66			15	6.78	6.56	
67			20	8.63	8.38	
68	8.1/18	0.85	0	2.25		
69			5	4.04	4.50	
70			10	6.36	6.47	
71			15	8.44	8.61	
72			20	10.24	10.89	
73	8.1/26	0.85	0	1.81		
74			5	3.91	4.24	
75			10	5.04	6.02	
76			15	6.68	7.94	
77			20	8.73	9.99	
78	8.1/34	0.85	0	1.79		
79			5	2.67	3.41	
80			10	4.60	4.91	
81			15	6.30	6.53	
82			20	7.96	8.25	
84	8.1/3	0.85	0	3.65		

Table 8.7 (continued)
Deviation Data for High Speed Flow

Case No.	Blade Geometry Reference (Table 8.1)	Mach Number	Damage Size (% Chord)	Total Deviation		Symbol
				Computed	Correlation for damaged blade (Equation 8.13)	
85	8.1/3	0.85	5	5.50	5.09	□
86			10	8.91	7.17	
87			15	10.93	9.42	
88			20	12.92	11.79	
89	8.1/29	0.85	0	3.25		
90			5	5.61	5.65	
91			10	8.77	7.78	
92			15	10.58	10.07	
93			20	12.54	12.50	
94	8.1/30	0.85	0	3.19		
95			5	4.36	4.30	
96			10	7.07	6.44	
97			15	9.35	8.75	
98			20	11.51	11.22	
99	8.1/41	0.85	0	1.28		
100			5	2.41	2.63	
101			10	3.06	3.01	
102			15	3.89	3.39	
103			20	4.71	3.79	
104	8.1/40	0.85	0	1.44		
105			5	2.39	2.43	
106			10	3.61	2.80	
107			15	4.30	3.16	
108			20	4.98	3.56	
109	8.1/36	0.85	0	1.22		
110			5	2.20	2.43	
111			10	3.28	2.87	
112			15	4.07	3.33	

Table 8.7 (continued)
Deviation Data for High Speed Flow

Case No.	Blade Geometry Reference (Table 8.1)	Mach Number	Damage Size (% Chord)	Total Deviation		Symbol
				Computed	Correlation for damaged blade (Equation 8.13)	
113		0.85	20	4.81	3.81	□
114	8.1/17	1.05	0	1.32		▽
115			5	3.87	3.91	
116			10	6.69	5.65	
117			15	8.36	7.46	
118			20	9.86	9.33	
119	8.1/18	1.05	0	1.85		
120			5	4.30	5.41	
121			10	7.62	7.56	
122			15	9.51	9.79	
123			20	11.43	12.12	
124	8.1/26	1.05	0	1.56		
125			5	4.05	5.10	
126			10	5.99	7.03	
127			15	8.07	9.04	
128			20	9.98	11.12	
129	8.1/34	1.05	0	1.72		
130			5	3.80	4.11	
131			10	6.49	5.74	
132			15	8.66	7.43	
133			20	10.65	9.19	
134	8.1/3	1.05	0	3.76		
135			5	6.02	6.13	
136			10	10.28	8.38	
137			15	12.84	10.71	
138			20	15.02	13.12	
139	8.1/29	1.05	0	3.19		
140			5	5.99	6.81	

Table 8.7 (continued)
Deviation Data for High Speed Flow

Case No.	Blade Geometry Reference (Table 8.1)	Mach Number	Damage Size (% Chord)	Total Deviation		Symbol
				Computed	Correlation for damaged blade (Equation 8.13)	
141		1.05	10	9.80	9.09	C
142			15	11.61	11.46	
143			20	13.48	13.91	
144	8.1/30	1.05	0	3.02		
145			5	5.88	5.17	
146			10	9.20	7.52	
147			15	11.61	9.96	
148			20	13.94	12.49	
149	8.1/41	1.05	0	0.50		
150			5	1.65	3.16	
151			10	2.99	3.51	
152			15	4.43	3.86	
153			20	5.73	4.22	
154	8.1/40	1.05	0	0.85		
155			5	2.18	2.92	
156			10	3.50	3.26	
157			15	5.00	3.60	
158			20	6.45	3.96	
159	8.1/36	1.05	0	0.94		
160			5	2.55	2.92	
161			10	3.98	3.35	
162			15	5.74	3.79	
163			20	7.05	4.24	
164	8.1/17	1.20	0	2.71		D
165			5	6.23	5.79	
166			10	8.69	8.13	
167			15	10.70	10.45	
168			20	12.97	12.76	

Table 8.7 (continued)
Deviation Data for High Speed Flow

Case No.	Blade Geometry Reference (Table 8.1)	Mach Number	Damage Size (% Chord)	Total Deviation		Symbol
				Computed	Correlation for damaged blade (Equation 8.13)	
169	8.1/18	1.20	0	3.63		C
170			5	7.02	8.03	
171			10	9.96	10.88	
172			15	12.68	13.73	
173			20	14.88	16.57	
174	8.1/26	1.20	0	3.46		
175			5	5.96	7.7.57	
176			10	8.71	10.12	
177			15	11.42	12.66	
178			20	13.46	15.21	
179	8.1/34	1.20	0	3.57		
180			5	6.52	6.09	
181			10	9.34	8.26	
182			15	12.02	10.41	
183			20	13.98	12.57	
184	8.1/3	1.20	0	6.30		
185			5	9.64	9.09	
186			10	13.49	12.06	
187			15	17.08	15.02	
188			20	19.67	17.94	
189	8.1/29	1.20	0	5.80		
190			5	9.43	10.09	
191			10	12.08	13.08	
192			15	15.12	16.06	
193			20	16.81	19.03	
194	8.1/30	1.20	0	5.60		
195			5	8.62	7.67	
196			10	12.33	10.82	

Table 8.7 (continued)
Deviation Data for High Speed Flow

Case No.	Blade Geometry Reference (Table 8.1)	Mach Number	Damage Size (% Chord)	Total Deviation		Symbol
				Computed	Correlation for damaged blade (Equation 8.13)	
197		1.20	15	15.18	13.95	
198			20	17.62	17.08	
199	8.1/41	1.20	0	2.20		
200			5	3.45	4.69	
201			10	5.60	5.05	
202			15	7.84	5.41	
203			20	9.82	5.77	
204	8.1/40	1.20	0	2.60		
205			5	4.85	4.33	
206			10	7.00	4.70	
207			15	8.80	5.04	
208			20	10.38	5.42	
209	8.1/36	1.20	0	2.88		
210			5	5.27	4.33	
211			10	7.67	4.83	
212			15	9.65	5.31	
213			20	11.25	5.80	

Table 8.8**Predicted Outlet Flow Angle by Ainley & Mathieson Correlation**

Case No.	Blade Geometry Reference (Table 8.1)	Unguided Turning Angle θ_u degrees	Gauge Angle $\cos^{-1}(a/s)$	Exit Mach Number	Outlet Flow Angle		Symbol
					Computed	Ainley & Mathieson Correlation	
1	8.1/17	11.5	57.96	0.7	57.12	56.90	■
2	8.1/18	15.0	57.79	0.7	55.54	57.00	
3	8.1/26	11.5	57.96	0.7	56.00	56.90	
4	8.1/34	11.5	57.96	0.7	56.70	56.90	
5	8.1/3	16.0	55.10	0.7	52.97	53.99	
6	8.1/29	14.5	55.10	0.7	52.74	53.75	
7	8.1/30	14.5	55.10	0.7	53.74	53.75	
8	8.1/36	6.0	55.20	0.7	48.80	53.18	
9	8.1/17	11.5	57.96	1.0	58.00	57.37	Δ
10	8.1/18	15.0	57.79	1.0	57.20	56.91	
11	8.1/26	11.5	57.96	1.0	57.50	57.37	
12	8.1/34	11.5	57.96	1.0	57.30	57.37	
13	8.1/3	16.0	55.10	1.0	53.90	54.00	
14	8.1/29	14.5	55.10	1.0	54.50	54.05	
15	8.1/30	14.5	55.10	1.0	54.50	54.05	
16	8.1/36	6.0	55.20	1.0	50.00	54.85	

Table 8.9**Experimental Subsonic Deviation for Cases with Different Values of AVDR**

Case No.	Blade Geometry Reference (Table 8.1)	Incidence	AVDR	Deviation		Symbol
				Measured	Correlation for undamaged blade (Equation 8.11)	
1	8.1/17	0	1.02	1.91	3.14	■
2	8.1/17	4.5	1.20	2.50	3.00	▲
3	8.1/8	0	0.929	1.60	1.68	Δ
4	8.1/9	0	0.91	1.90	2.09	
5	8.1/10	0	1.014	2.40	2.41	
6	8.1/11	0	0.96	3.80	2.94	
8	8.1/12	0	1.08	5.50	4.20	

9 Conclusions and Recommendations

9.1 Conclusions

Investigations have been made of the aerodynamic effects of in-service damage on the performance of axial turbine blades. Two aspects of blade damage were considered: surface roughening and trailing edge damage. The work was motivated by gas turbine engine health monitoring. The goal was to provide the physical understanding and data correlations which can be used in the development of future, more capable health monitoring systems. Although the work was motivated by a rather specialized area of technology, a number of the results and observed flow features will be of interest to turbine designers generally.

Correlations for the effects of surface roughness were developed based on a database obtained from Kind et al. (1998). The authors measured the losses and deviation for the flow through a turbine cascade in which patches of roughness of varying extent and severity had been applied to the surfaces. They showed that the patches of roughness applied to the blade surfaces resulted in an increase in profile losses but had very little effect on the deviation. In the present work, these results were expressed in the form of correlations and graphs that may be easily implemented in meanline or throughflow predictions of turbine stage performance. The correlations account for the effects of the roughness height as well as the location and extent of the roughness patch on the blade surface. The correlations for design incidence show that for an initial 25% covered suction surface from the leading edge of the blade, profile losses increase considerably for all roughness heights. The correlations for off-design incidence show that the effect of roughness diminishes rapidly with the increase in off-design incidence. It is believed that

at large values of off-design incidences the losses are dominated by the effects of boundary layer separation and the incremental losses due to roughness become relatively small by comparison.

The effect of trailing edge damage was investigated both computationally and experimentally. The damage was idealized as a circular cutout in the trailing edge. The computations were performed using the Navier-Stokes code of Dawes (1988). This represents an unusual application of the code and special grid generation procedures had to be devised to mesh the trailing edge region. The computations were subsequently validated with measurements conducted in the Carleton University High Speed Wind Tunnel. The measurements were made in a single cascade of turbine blades for a wide range of Mach numbers, at design and off-design values of incidence, and for two cutout sizes. These measurements are unique.

The effect of trailing edge damage on the loss characteristics of the blades was found to be fairly small over the full range of flow conditions. In fact, the mid-cutout profile losses were actually found to be lower for 20% damage than for the undamaged blade. This was attributed to a combination of reduced boundary layer losses, due to a shorter surface length, and a favourable interaction between the cutout flow and the trailing edge flow. The latter interaction apparently raised the base pressure and thus reduced the base pressure contribution to the profile loss. This somewhat unexpected result raises the interesting possibility that profile losses in turbines could be reduced by the use of scalloped trailing edges.

Trailing edge damage produced considerable under turning of the flow in the region of the cutout. The measured flow deviation increased with the increase in damage size as well as cascade exit Mach number. At subsonic flow conditions, an approximately linear relationship was observed between the incremental deviation ($\Delta\delta$) and the chordwise penetration of the cutout or blade damage.

Computational investigations were made to identify the parameters that influence flow deviation in turbines with both undamaged and damaged blades so that correlations could be developed. The existing correlations for the prediction of subsonic deviation for undamaged blades were found to be unsatisfactory. Therefore a new correlation was developed using a genetic algorithm to optimize the coefficients. It was found that the deviation is primarily determined by the blade loading (pressure difference) towards the trailing edge. The blade row parameters which

influence this pressure difference are the blade stagger angle, total flow turning angle, inlet metal angle, pitch, thickness-to-chord ratio and the axial velocity density ratio (AVDR). Many of these parameters do not appear in earlier correlations for deviation.

A broader understanding of the mechanism of deviation led to the successful development of deviation correlations for damaged blades for both subsonic and high speed flow cases. The genetic algorithm was again used for this purpose and it proved to be a very effective and robust optimization tool. In addition to the parameters that affect deviation for an undamaged blade the deviation correlation for the damaged blade includes an additional parameter: the chordwise extent of the damage. It is suggested that this correlation should be used with caution for damage size greater than 20% chord penetration and exit Mach numbers greater than 1.2.

The successful use of the genetic algorithm to the present problem has again demonstrated its broad applicability to engineering problems (e.g. see Goldberg, 1989, Trigg et al., 1997). The genetic algorithm can be easily used to update the present deviation correlations as new experimental and computational data become available.

Though the deviation correlations for damaged blades have specialized applicability, the new correlation for undamaged blades will be of interest to turbine designers. Empirical correlations continue to play an important role in the early stages of blade design and there is an ongoing need to review and improve these correlations. This is because the prediction of blade outlet flow angle is an important step in the preliminary design phase of a turbine. The estimation of optimum work output and inlet flow angle of successive stages of a turbine are intrinsically related to the accurate prediction of the deviation. In this regard the new subsonic deviation correlation for axial turbine blades appears to be substantially better than existing ones.

Apparently surface roughening and trailing edge damage produce significantly different aerodynamic behavior of the flow. Surface roughening largely influences the profile losses and trailing edge damage has a considerable effect on the flow deviation. These observations may form the basis for identifying these two types of damage in engine health monitoring data.

The 3D Navier-Stokes solver of Dawes (1988) was found to be a powerful tool for investigating the aerodynamics of damaged turbine blades. The code predicted many of the measured results for damaged blades: the small change in overall losses even at transonic flow, the

relative reduction in mid-span losses for 20% damaged blade and the significant under turning of the flow in the area of the cutout. Though the predicted losses were lower than the measured ones, the calculations nevertheless captured many of the flow features mentioned above. Particularly, predictions of deviation for different undamaged and damaged blade geometries were found to be quite satisfactory and the code was extensively used to generate deviation data to supplement experimental measurements.

9.2 Recommendations

In the present study several areas requiring additional work or improvement became apparent. A few problems could not be solved because of time constraints. The following suggestions are made regarding future work on the present topic.

The current database of surface roughness results should be expanded to include more roughness sizes, roughness locations and different blade geometries and loading. These results may be used to improve the prediction capabilities of the present subsonic roughness correlations. Experimental and computational work for the effects of surface roughness at transonic flow conditions is needed.

The genetic algorithm may be used to develop an improved correlation for the prediction of deviation at a Mach number of unity. At present, only the Ainley & Mathieson correlation is available for the sonic condition. While it produces reasonable results for some blades, the presence of rogue points suggests that one or more important geometric parameters are missing from the correlation. This will enable the development of a complete set of equations to predict the deviation for a turbine blade at any exit Mach number.

In the present subsonic deviation correlation, the power on the AVDR remains uncertain because of the lack of sufficient data. More experimental data are required to determine the appropriate relationship between deviation and AVDR at subsonic flow conditions. It may be possible to generate such results computationally.

It is presumed that the discrepancy in the measured and computed deviation for damaged blades is due to the fact that the experimental and computed geometries were not identical for damaged blades. Either computations should be done with one damaged blade in a cascade of undamaged blades or measurement of deviation should be done with at least three adjacent blades damaged in a cascade. This is necessary to provide an accurate correction factor to the damaged blade correlation to account for the number of blades damaged in a row.

The reasons are not clear for the increase in base pressure, and resulting reduction in base pressure losses, as the damage approaches 20% of chord. Experiments with some form of flow visualization should be performed to assist in interpreting the measurements. As already suggested, further understanding of the loss mechanism with trailing edge cutout may lead to turbine blade designs with reduced profile losses.

References

Abbot, M.W.P.	1993	"The Aerodynamic Effects of Roughness on the Performance of a Turbine Cascade," M.S.Thesis, Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa, 1993
Ainley, D.G.
Mathieson, G.C.R.	1951	"A Method of Performance Estimation for Axial-Flow Turbines," Aeronautical Research Council of Great Britain, Reports and Memoranda No.2974, December, 1951
Ainsworth, R.W.
Allen, J.
Batt, J.J.M.	1994	"The Development of Fast Response Aerodynamic Probes for Flow Measurements in Turbomachinery," ASME Paper No. 94-GT-23.
Amecke, J.
Safärík, P.	1995	"Data Reduction of Wake Flow Measurements with Injection of an Other Gas," DLR-FB 95-32, DLR, Köln, Germany.
Baldwin, B.
Lomax, H.	1978	"Thin Layer Approximation and Algebraic Model for Separated Turbulent Flows," AIAA Paper 78-257.
Beam, R.M.
Warming, R.F.	1978	"An Implicit Factored Scheme for the Compressible Navier-Stokes Equations," AIAA Journal, Vol. 16, No. 4, pp. 393-402.
Benedict, R.P.	1960	"The Response of a Pressure-Sensing System," ASME Journal of Basic Engineering, Vol. 82, pp. 482-488.
Benner, M.W.
Sjolander, S.A.
Moustapha, S.H.	1997	"Influence of Leading-Edge Geometry on Profile Losses in Turbines at Off-Design Incidence: Experimental Results and an Improved Correlation," Transactions of the ASME, Journal of Turbomachinery, 1997, Vol. 119, pp.193-199.
Brandt, A.	1981	"Guide to Multigrid Development," Lecture Notes in Mathematics, Vol. 960, pp. 220-312, Springer Verlag Berlin.
Carter, A.D.S.
Hughes, H.P.	1950	"A Theoretical Investigation into the Effect of Profile Shape on the Performance of Aerofoils in Cascade," Aeronautical Research Council, R&M 2384

Carscallen, W.E.
Oosthuizen, P.H.	1989	"The effect of Secondary Flow on the Redistribution of the Total Temperature Field Downstream of a Stationary Turbine Cascade," AGARD CP-469
Cebeci, T.
Smith, A.M.O.	1974	<i>Analysis of Turbulent Boundary Layers</i> , Academic Press, New York.
Corriveau, D.	1999	Private Communication, Carleton University, Ottawa.
Chan, J.K.
Yaras, M.I.
Sjolander, S.A.	1994	Interaction Between Inlet Boundary Layer, Tip-Leakage and Secondary Flows in a Low-Speed Turbine Cascade," ASME Paper 94-GT-250.
Chima, R.V.
Giel, P.W.
Boyle, R.J.	1993	"An Algebraic Turbulence Model for Three-Dimensional Viscous Flows," NASA Technical Memorandum 105931.
Craig, H.R.M.
Cox, J.J.A.	1971	"Performance Estimation of Axial Flow Turbines," Proceedings of the Institution of Mechanical Engineers, Vol.85, No.32, pp.407-424.
Cue, R.W.
Muir, D.E.	1991	"Engine Performance Monitoring and Troubleshooting Techniques for the CF-18 Aircraft," Transactions of the ASME, Journal of Engineering for Power, January 1991, Vol.113, pp.11-19.
Dawes, W.N.	1993	"The Extension of a Solution-Adaptive Three-Dimensional Navier-Stokes Solver Toward Geometries of Arbitrary Complexity," ASME Transactions, Journal of Turbomachinery, Vol.115, pp.283-295.
Dawes, W.N.	1992	"Towards Improved Through Flow Capability: The Use of Three Dimensional Viscous Flow Solvers in a Multistage Environment," ASME Journal of Turbomachinery, Vol. 114, pp. 8-17.
Dawes, W.N.	1990	"A Comparison of Zero and One Equation Turbulence Modelling for Turbomachinery Calculations," ASME Paper 90-GT-70.
Dawes, W.N.	1988	"Development of a 3D Navier Stokes Solver for Application to all Types of Turbomachinery," ASME Paper 88-GT-70.

Dawes, W.N.	1987	"A Numerical Analysis of the Three-Dimensional Viscous Flow in a Transonic Compressor Rotor and Comparison With Experiment," ASME Transactions, Journal of Turbomachinery, Vol.109, No.1, pp.83-90.
Dawes, W.N.	1986	"Application of Full Navier-Stokes Solvers to Turbomachinery Flow Problems," Numerical Techniques for Viscous Flow Calculation in Turbomachinery Blading, V.K.I. Lecture Series 2.
De Cecco, S.F.	1995	"The Aerodynamics of Turbine Blades with Tip Damage," M.Eng. Thesis, Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa.
De Cecco, S.
Yaras, M.I.
Sjolander, S.A.	1995	"Measurements of the Tip-Leakage Flow in a Turbine Cascade with Large Clearances," ASME Paper No. 95-GT-77.
Denton, J.D.	1993	"Loss Mechanisms in Turbomachines," ASME paper 93-GT-140.
Denton, J.D.
Xu, L.	1990	"The Trailing Edge Loss of Transonic Turbine Blades," ASME Transactions, Journal of Turbomachinery, Vol.112, pp.277-285.
Dixon, S.L.	1989	<i>Fluid Mechanics, Thermodynamics of Turbomachinery</i> , 3 rd Edition, Pergamon Press, Oxford.
Ducoffe, A.L.	1953	"Pressure Response in Supersonic Wind-Tunnel Pressure Instrumentation," Journal of Applied Physics, Vol. 24, Number 11, pp. 1343-1354.
Dunham, J.
Came, P.M.	1970	"Improvements to the Ainley-Mathieson Method of Turbine Performance Prediction," ASME Transactions, Journal of Engineering for Power, pp.252-256.
Forster, V.T.	1966	"Performance Loss of Modern Steam-Turbine Plant Due to Surface Roughness," Proceedings of the Institution of Mechanical Engineers, Vol. 181, Part 1, pp.391-405.
Goldberg, D.E.	1989	<i>Genetic Algorithms in Search, Optimization, and Machine Learning</i> , Addison-Wesley Publishing Company, Inc., Massachusetts.

Granville, P.S.	1987	"Baldwin-Lomax Factors for Turbulent Boundary Layers in Pressure Gradients," AIAA Journal, Vol.25, 1987.
Haller, B.R. and Camus, J-J	1984	"Aerodynamic Loss Penalty Produced by Film Cooling Transonic Turbine Blades," ASME Journal of Engineering for Gas Turbines and Power, Vol. 106, pp. 198-202.
Hawthorne	1955	"Some Formulae for the Calculation of Secondary Flow in Cascades," Aeronautical Research Council, Rep. 17519.
Hodson, H.P. and Dominy, R.G.	1990	"Test Cases for Computation of Internal Flows in Aero Engine Components," AGARD Advisory Report No. 275, pp.124-138.
Hoheisel, H.	1990	"Test Cases for Computation of Internal Flows in Aero Engine Components," AGARD Advisory Report No. 275, pp. 112-123.
Horlock, J.H.,	1973	<i>Axial Flow Turbines</i> , Robert E. Krieger Publishing Co. Inc., New York.
Humm, H.J.
Gossweller, C.R.
Gyarmathy, G.	1994	"On Fast-Response Probes: Part 2 - Aerodynamic Probe Design Studies," ASME Paper No. 94-GT-27.
Hultsch, M.
Sauer, H.	1979	"Sekundärströmungen in Beschaufelungen axialer Turbomaschinen," Maschinenbautechnik, BD. 28, Heft 1.
Isaacs, D.	1993	"Aerodynamics of Turbine Blades with Trailing Edge Damage: Measurements and Computations," M.Eng. Thesis, Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa.
Jameson, A.
Baker, T.J.	1984	"Multigrid Solution of the Euler Equations for Aircraft Configurations," AIAA Paper 84-93.
Jeffries, M.S.	1994	"The Commissioning and Enhancement of a Blow Down Wind Tunnel," M.Eng. Thesis, Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa.
Jeffries, M.S.
Jouini, D.B.
Sjolander, S.A.	1997	"Determining the Sampling Rates and Times in a High Speed Wind Tunnel," CASI 6 th Symposium on Aerodynamics, Toronto, April, 1997.

Jeffries, M.S.	1999	"Initial Investigations of Transonic Turbine Aerodynamics Using the Carleton University High Speed Wind Tunnel," Ph.D. Thesis, Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa.
Jouini, D.B.
Islam, A.M.T.
Sjolander, S.A.	1998	"Effect of Surface Roughness on Profile Losses in Axial Turbines at Transonic Mach Numbers," Canadian Aeronautics and Space Journal, Vol. 44, No. 4, December, 1998.
Jouini, D.B.	1999	"Effects of Leading Edge Geometry on Profile Losses at Transonic Mach Numbers in Axial Flow Turbines," Ph.D. Thesis, Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa.
Kacker, S.C.
Okapuu, U.	1982	"A Mean Line Prediction Method for Axial Flow Turbine Efficiency," ASME Journal of Engineering for Power, January, 1982, Vol. 104, p.111.
Kind, R.J.
Serjak, P.J.
Abbott, M.W.P.	1998	"Measurements and Prediction of the Effects of Surface Roughness on Profile Losses and Deviation in a Turbine Cascade," ASME Journal of Turbomachinery, Vol. 120, pp. 20-27.
Kinsler, L.E.
Frey, A.R.	1962	<i>Fundamentals of Acoustics</i> , 2 nd edition, J.Wiley & Sons, New York
Kiock,R., Lehthaus,F.,
Baines, N.C., and
Sieverding, C.	1986	"The Transonic Flow Through a Plane Turbine Cascade as Measured in Four European Wind Tunnels," ASME Journal of Engineering for Gas Turbines and Power, Vol. 108, pp. 277-284.
Klein, W.A.	1993	"Contributions to the Study of the Aerodynamics of Damaged Turbine Blades," M.Eng. Thesis, Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa
Koch, C.C.
Smith, L.H.	1976	"Loss Sources and Magnitudes in Axial-Flow Compressors," Journal of Engineering for Power, Vol.98.
Korakiantitis, T.	1993	"Hierarchical Development of Three Direct-Design Methods for Two-Dimensional Axial-Turbomachinery Cascades," Transactions of the ASME, Journal of Turbomachinery, 1993, Vol.115, pp.314-324.

Korakiantitis, T.	1993	"Prescribed-Curvature-Distribution Airfoils for the Preliminary Geometric Design of Axial-Turbomachinery Cascades," Transactions of the ASME, Journal of Turbomachinery, 1993, Vol.115, pp.325-333.
Korakiantitis, T.
Papagiannidis, P.	1993	"Surface-Curvature-Distribution Effects on Turbine-Cascade Performance," Transactions of the ASME, Journal of Turbomachinery, 1993, Vol. 115, pp.334-341.
Lewis, W.E.	1966	"Fixed-Direction Probes for Aerodynamic Measurements," Proc. Inst. Mech. Engrs., Vol. 180, pp. 141-152.
Martelli, F.
Boretti, A.	1987	"Development of an Experimental Correlation for Transonic Turbine Flow," Transactions of the ASME, Journal of Turbomachinery, Vol. 109, pp.246-250.
Mee, D.J., Baines,
N.C., Oldfield, M.L.G.
and Dickens, T.E.	1992	"An Examination of the Contributions to Loss on a Transonic Turbine Blade in Cascade," ASME Journal of Turbomachinery, Vol. 114, pp. 155-162
Miller, D.C.
Wasdell, D.L.	1987	"Off-design Prediction of Compressor Blade Losses," AGARD Report No. C279.
Moustapha, S.H.
Kacker, S.C.
Tremblay, B.	1990	"An Improved Incidence Losses Prediction Method for Turbine Airfoils," Transactions of the ASME, Journal of Turbomachinery, Vol. 112, pp. 267-276.
Mukhtarov, M.K.
Krichakin, V.I.	1969	"Procedure for Estimating Flow Section Losses in Axial Flow Turbines when Calculating their Characteristics," Teploenergetika, 16(7) 76-79, pp.115-119.
Naughton, J.W.
Catafesta, L.N.
Settles, G.S.	1993	"Miniature Fast-Response Five-Hole Conical Probe for Supersonic Flowfield Measurements
Ower, E.	1977	<i>Measurement of Air Flow</i> , 5 th edition, Oxford, Pergamon Press.
Pollard, D. and
Gostelow, J.P.	1967	"Some Experiments at Low Speed on Compressor Cascades," ASME Journal of Engineering for Gas Turbines and Power, Vol. 89, pp. 427-436.

Riess, W., Giess, P.A.,
Dalbert, P. and
Heinemann, H.J.	1986	"Downstream Flow Angle Correlations for Turbine Cascades in Subsonic and Transonic Flow Conditions," AGARD-CP-401. <i>Transonic and Supersonic Phenomena in Turbomachines</i> ,
Roache, P.J.	1994	"Perspective: A Method for Uniform Reporting of Grid Refinement Studies," Transactions of the ASME, Journal of Fluids Engineering, 1994, Vol. 116, pp.405-413.
Roberts, Q.D.
Denton, J.D.	1996	"Loss Production in the Wake of a Simulated Subsonic Turbine Blade," Transactions of the ASME, Paper No. 96-GT-421
Rodger, P.	1992	"Establishing Two-Dimensional Flow in a Large Scale Planar Turbine Cascade," M.Eng. Thesis, Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa.
Saroch, M.F.	1996	"Contributions to the Study of Turbomachinery Aerodynamics; Part I: Design of a Fish-tail Diffuser Test Section, Part II: Computations of the Effects of AVDR on Transonic Turbine Cascades," M.Eng. Thesis, Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa.
Schofield, C.
La Grandeur, R.
Dubé, F.
Harris, T.
Cue, R.
LeBlanc, A.	1988	"Canadian Forces Aircraft Engine Condition/Health Monitoring - Policy, Plans and Experience," AGARD Conference Proceedings, Engine Condition Monitoring - Technology and Experience, Paper No. 7, AGARD-CP-448.
Serjak, P.J.	1995	"An Investigation of the Effects of Roughness on the Performance of a Turbine Cascade," M.Eng. Thesis, Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa.
Sieverding, C.	1975	"Modern Methods of Testing Rotating Components of Turbomachines (Instrumentation)," North Atlantic Treaty Organization Advisory Group for Aerospace Research and Development, AGARD-AG-207.

Sieverding, C.	1993	"Advanced Methods for Cascade Testing," AGARD-AG-328
Sieverding, C.H.
Arts, T.
Denos, R.
Martelli, F.	1994	"Investigation of the Flow Field Downstream of a Turbine Trailing Edge Cooled Nozzle Guide Vane," Transactions of the ASME, 94-GT-209.
Soderberg,	1949	Unpublished Notes, Gas Turbine Laboratory, MIT.
Storer, J.A. and
Cumpsty, N.A.	1993	"An Approximate Analysis and Prediction Method for Tip Clearance Loss in Axial Compressors," ASME Paper 93-GT-140.
Sjolander, S.A.
Isaacs, D.
Klein, W.A.	1993	"Aerodynamics of Turbine Blades with Trailing-Edge Damage: Measurements and Computations," Proceedings, XI International Symposium on Air Breathing Engines, Vol. II, pp. 1327-1334.
Thompson, J.F.	1984	"Grid Generation Techniques in Computational Fluid Dynamics," AIAA Journal, Vol. 22, No. 11, pp.1505-1523.
Traupel, W.	1977	"Thermische Turbomaschinen" BD I, Springer Verlag, 8b-Ausgabe.
Trigg, M.A.
Tubby, G.R.
Sheard, A.G.	1997	"Automatic Genetic Optimization Approach to 2D Blade Profile Design for Steam Turbines," Transactions of the ASME, ASME Paper 97-GT-392.
Turner, M.G.
Jennions, I.K.	1993	"An Investigation of Turbulence Modeling in Transonic Fans Including a Novel Implementation of an Implicit k- ϵ Turbulence Model," Transactions of the ASME, Journal of Turbomachinery, 1993, Vol. 115, pp.249-260.
Turner, M.G.
Liang, T
Beauchamp, P.P.
Jennions, I.K.	1993	"The Use of Orthogonal Grids in Turbine CFD Computations," ASME Paper 93-GT-38.
Turyk, P.J.	1989	"Compressor Fault Modelling for Engine Health Monitoring : Review and Recommendations," NRC Technical Report TR-ENG-004, NRC No. 31204

Wilson, D.G.	1984	<i>The Design of High-efficiency Turbomachinery and Gas Turbines</i> , MIT Press, Massachussetts.
White, F.M.	1991	<i>Viscous Fluid Flow</i> , Second Edition, McGraw-Hill Inc., New York.
Wyler, J.S.	1975	"Probe Blockage Effects in Free Jets and Closed Tunnels," <i>Journal of Engineering for Power</i> , Vol. 97, No. 4, pp.509-515.
Xu,L.
Denton, J.D.	1988	"The Base Pressure and Loss of a Family of Four Turbine Blades," <i>Transactions of the ASME, Journal of Turbomachinery</i> , 1988, Vol. 110, pp.9-17.
Yaras, M.I.
Sjolander, S.A.	1992	"Prediction of Tip-Leakage Losses in Axial Turbines," <i>Transactions of the ASME, Journal of Turbomachinery</i> , Vol. 114, pp. 204-210.
Yaras, M.
Sjolander, S.A.	1990	"Development of the Tip-Leakage Flow Downstream of a Planar Cascade of Turbine Blades: Vorticity Field," <i>Transactions of the ASME, Journal of Turbomachinery</i> , Vol. 112, pp.609-617.
York, B.
Knight, D.	1985	"Calculations of Two-Dimensional Turbulent Boundary Layers Using the Baldwin-Lomax Model," <i>AIAA Journal</i> , Vol. 23, pp. 1849-1850.

Appendix A

This appendix shows the detailed drawings of the static and pitot pressure probes which has been designed in the course of this research. As previously mentioned in Chapter 5 both the probes have identical stem design. Figures A.1 through A.9 shows the drawings of the static probe. Figures A.10 and A.11 show the drawings of the pitot probe.

Figure A.1 The Static Probe and Various Parts

Figure A.2 Static Probe Tip

Figure A.3 Measuring Tube with Static Holes

Figure A.5 Assembly of Probe Stem

Figure A.6 Leading End of Probe Stem

Figure A.8 Rear End of Probe Stem

Figure A.9 The Probe Stem Cover Ring

Figure A.10 The Pitot Probe

Figure A.11 The Pitot Probe Measuring Tube

Appendix B

This appendix shows the detailed drawings of the three hole probe which have been designed in the course of this research. These drawings are shown through Figures B.1 through B.4.

Figure B.1 The Three Hole Probe

Figure B.2 The Three Hole Probe Stem

Figure B.3 The Three Measuring Tubes

Figure B.4 Arrangement of Measuring Tubes in Slot and Solder

Figure B.5 Rear End of Three Hole Probe Stem