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DEVELOPMENT OF A BAYESIAN DECISION THEORY FRAMEWORK TO ENHANCE THE DESIGN OF REAR-END COLLISION WARNING SYSTEMS

by

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A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the Degree of
Doctor of Philosophy

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Abstract

Collision warning systems, like any in-vehicle technology, must be useful to the driver and not interfere with normal driving. Warnings given to the driver must result in a minimum load on driver attention. Therefore, two critical requirements govern their success. First, the collision warning system must promote a timely and appropriate driver response. Second, annoyance associated with nuisance warnings must be minimized to promote driver acceptance.

Hence, research was conducted to contribute to the improvement of such systems. This involved developing a car-following model for off-line testing. Once validated, a rear-end collision warning algorithm was integrated along with a Bayesian statistical decision theory framework. This framework enables the combination of intelligent technology with human factors to address the limitations of the existing systems.

The developed car-following model assumes that drivers adjust their speed based on the current spacing and relative velocity to the lead vehicle. By using data from the System for Assessment of Vehicle Motion Environment (SAVME) database, desired and relative vehicle movements were obtained. This was conducted rather than using the assumptions about a driver's ability to ascertain the exact absolute speed and acceleration/deceleration rate of the lead vehicle.

The rear-end collision warning algorithm combines the principles of vehicle dynamics and driver response to ascertain a critical distance. While integrated into the car-following model, it uses the information pertaining to the current velocity and driver alertness from the following vehicle as input. In addition, to account for driver behaviour, a framework that utilizes the Bayesian statistical decision theory is incorporated. Prior conditions of driver reliability are combined with conditional probabilities of driver alertness to determine when and what type of warning should be given to the driver.

Overall, the results show a reduction in the number of warnings outputted while the driver is alert. This indicates a potential decrease in distractions for the driver, hence demonstrating the effectiveness of the Bayesian statistical decision theory framework. With further research and development, the proposed rear-end collision warning algorithm and framework can be embedded in driving simulators to examine the benefits with human operators. In addition, a prototype may be built and installed in an actual vehicle under controlled conditions in order to develop a commercial product.

Table of Contents

	Page
Acknowledgements.....	i
Abstract.....	iii
Table of Contents.....	v
List of Figures	ix
List of Tables.....	xi
List of Appendices.....	xii
Glossary of Terms.....	xiii
 CHAPTER 1: PROBLEM IDENTIFICATION AND METHODOLOGY FRAMEWORK	 1
1.1 Introduction	1
1.2 Background.....	2
1.3 Problem Identification.....	6
1.4 Objectives and Scope	7
1.5 Study Methodology	10
1.6 Thesis Organization	13
 CHAPTER 2: LITERATURE REVIEW	 15
2.1 Background on Collision Warning Systems	15
2.1.1 Time-Headway Criterion.....	16
2.1.2 Time-to-Collision Criterion	20
2.1.3 Kinematic Constraints Criterion.....	22

2.2 Functional Operation.....	26
2.3 Driver-Vehicle Interaction.....	29
CHAPTER 3: SOFTWARE AND DATABASE DESCRIPTION...	34
3.1 Software Utilized	34
3.2 SAVME Database Description	41
3.3 Data Analysis Process	46
CHAPTER 4: CAR-FOLLOWING MODEL AND REAR-END COLLISION WARNING ALGORITHM METHODOLOGY	48
4.1 Human Perception	48
4.2 Application of Human Perception and Reaction.....	50
4.3 Car-Following Models	56
4.3.1 Introduction.....	56
4.3.2 Limitations of Car-Following Models.....	57
4.4 Car-Following Methodology	60
4.5 Stopping Sight Distance Analogy	63
4.6 Deceleration Rate Criterion.....	66
4.7 Rear-End Collision Warning System Methodology	68
4.7.1 Warning Initiation.....	69
4.7.2 Critical Distance Computation	71
CHAPTER 5: SYSTEM DESIGN AND INTEGRATION	74
5.1 Methodology	74

5.2 Framework Development.....	75
5.2.1 Introduction.....	75
5.2.2 Bayesian Analysis	77
5.3 Bayesian Framework Methodology	81
5.3.1 Warning Distances	81
5.3.2 Framework Description.....	83
5.4 Algorithm and Framework Integration	88
 CHAPTER 6: DEVELOPMENT OF THE SIMULATION ENVIRONMENT AND ITS VALIDATION	 91
6.1 Introduction	91
6.2 SIMULINK Model	91
6.2.1 Fundamental Aspects of the Car-Following Model	93
6.2.2 Velocity and Acceleration/Deceleration Rate.....	99
6.2.3 Reaction time of Following Vehicle Driver	103
6.2.4 Desired Following Distance	105
6.3 Car-Following Model Validation	106
6.3.1 Qualitative Validation.....	107
6.3.2 Quantitative Validation.....	107
6.4 User Interface Design	110
6.5 Summary.....	112

CHAPTER 7: VALIDATION OF THE WARNING ALGORITHM AND BAYESIAN STATISTICAL DECISION THEORY FRAMEWORK	114
7.1 Parameter Validation and Sensitivity Analysis	114
7.2 Simulation Performance.....	118
7.3 Bayesian Framework Sensitivity Analysis	124
7.4 Bayesian Framework Verification.....	130
CHAPTER 8: EXAMPLE APPLICATION	140
8.1 Example Time Step.....	140
8.2 Further Time Steps	145
CHAPTER 9: CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS	147
9.1 Thesis Contributions	147
9.2 Conclusions	148
9.3 Future Research Directions	150
REFERENCES	154

List of Figures

	Page
Figure 1.1: Vehicle Safety	4
Figure 1.2: System Development.....	9
Figure 1.3: Study Methodology	12
Figure 2.1: Collision Warning Vehicle Mechanization	28
Figure 3.1: Example of a SIMULINK Block Diagram, Coding, and Graphical Outputs.....	35
Figure 3.2: Example of SIMULINK Block Libraries.....	36
Figure 3.3: Signal and State Variables.....	37
Figure 3.4: Sample Equation Blocks	38
Figure 3.5: Example of a Subsystem	39
Figure 3.6: Sample Tabular and Graphical Output.....	40
Figure 3.7: Diagram of Refugee Road-Winchester Pike Intersection.....	42
Figure 3.8: Block Diagram of Data Collection, Processing, and Analysis	44
Figure 4.1: Collision Scenario and Contributing Variables	51
Figure 4.2: Perception-Reaction Process	52
Figure 4.3: Analysis of car-following behaviour.....	61
Figure 4.4: Structure of the Car-following Model.....	62
Figure 4.5: Warning System Logic.....	68
Figure 5.1: Major Functions of the Rear-End Collision Warning System	74
Figure 5.2: Early and Late Warning Analogy.....	82
Figure 5.3: Bayesian Framework Methodology.....	84

Figure 5.4: Flow Diagram of Integration Process	89
Figure 6.1a: SIMULINK Diagram for Car-following Portion	97
Figure 6.1b: SIMULINK Diagram Integrating the Warning Algorithm	98
Figure 6.2: Calibration of Desired Velocity and Spacing	101
Figure 6.3: Initialized User-Interface	111
Figure 7.1: Vehicle Braking Distribution	115
Figure 7.2: Critical Distance Comparison.....	116
Figure 7.3: Comparison of Critical (Warning) Distances versus Velocity	118
Figure 7.4: Example Result of a Collision Avoided	121
Figure 7.5: Example Result with a Collision	122
Figure 7.6: ΔV Versus Reaction Time	123
Figure 7.7: Conditional Probabilities vs. Posterior Probabilities (Constant Prior Probabilities).....	127
Figure 8.1: Example Application Decision Tree: Preposterior Analysis.....	144
Figure 8.2: Real Time Operation of the Collision Warning System	146

List of Tables

	Page
Table 7.1: Critical Distances	117
Table 7.2: ΔV Given Restricted Warning Ranges	124
Table 7.3: Gain Matrix.....	125
Table 7.4: Probabilities Used for Gain Matrix Sensitivity Analysis	126
Table 7.5: Sensitivity Analysis: Constant Conditional Probabilities	128
Table 7.6: Sensitivity Analysis: Changes in Conditional Probabilities	130
Table 7.7: Comparison of the Results Using the Bayesian Statistical Decision Theory Framework.....	131
Table 7.8: Comparison of the Results Using Other Warning Algorithms and the Bayesian Statistical Decision Theory Framework.....	135
Table 8.1: Conditional Probabilities.....	141
Table 8.2: Marginal Probabilities.....	141
Table 8.3: Posterior Probabilities	141
Table 8.4: Data for the Calculation of Value of Information.....	143

List of Appendices

	Page
Appendix A: Sample Database	167
Appendix B: Bayesian Statistical Decision Theory.....	168
Appendix C: Generated MATLAB Syntax	179
Appendix D: Qualitative Validation of Car-Following Model	201
Appendix E: Quantitative Validation of Car-Following Model	231

Glossary of Terms

AL : Alertness Level defined within the simulation

A_0 : No alert issued

A_A : Amber alert

a_F : Following Vehicle Acceleration

a_L : Lead Vehicle Acceleration

A_R : Red alert

d_c : Critical distance—distance that could lead to a collision

dS/dt : Relative Velocity/Rate of Change in Spacing

e_0 : Warning Issued Early

e_w : Warning Issued Later—Wait for Further Information

f : Coefficient of Friction

F : Following Vehicle

G : Gain Matrix

h : Headway

L : Lead vehicle

NHTSA: National Highway Transportation Safety Administration

OECD: Organisation for Economic Co-operation and Development

OEM: Original Equipment Manufacturers (e.g., Ford, General Motors)

PIEV: Perception-Reaction (Perception Identification Emotion Volition)

r_0 : No New Reading, when e_0 is selected

$r_{1.25c}$: Reading Corresponding to $d_{1.25c}$

$r_{1.5c}$: Reading Corresponding to $d_{1.5c}$

r_c : Reading Corresponding to d_c

RT : Reaction Time

S : Following Distance—Spacing

S_0 : Initial Spacing at time = 0

SAVME: System for Assessment of Vehicle Motion Environment

S_d : Desired Following Distance

S_F : Assumed Safe Stopped Distance within Critical/Warning Distance Equation

SSD : Stopping Sight Distance

S_W : Warning Distance

t : Simulation Time

T : Time lag,

T_d : Calculated Reaction Time

TTC : Time-to-collision, spacing over relative velocity

V_d : Following Vehicle Desired Non-Steady-State Velocity

V_{dss} : Following Vehicle Desired Steady-State Velocity

V_F : Following Vehicle Current Velocity

V_L : Lead Vehicle Velocity

V_{max} : Maximum Velocity

$V_t^*(e)$: Value of Information—Pertaining to When to Issue an Alert

X : Position

X_n : Lead Vehicle Position

X_{n+1} : Following Vehicle Position

CHAPTER 1: PROBLEM IDENTIFICATION AND METHODOLOGY FRAMEWORK

1.1 Introduction

Canadians are among the most mobile people on earth where in the year 2000, there were 17.9 million motor vehicles registered along with 21 million licensed drivers on the road (TC, 2001). As compared with other Organisation for Economic Co-operation and Development (OECD) countries, Canada is one of the safest nations ranking fifth with a death rate of 9.37 per billion kilometres travelled (TC, 2001). However, traffic collisions are still a leading cause of human death and injury in this country. Statistics also indicate that a high proportion of the traffic collisions are rear-end collisions. Hence, they place a significant and continuous burden on societal resources.

Increasingly, drivers are regularly engaging in a wide variety of multi-tasking activities when they are behind the wheel. As the average time spent commuting increases, there is a growing interest in trying to make the time spent on the roadway more productive. Unfortunately, due to the inherent limited capacity of human attention, engaging in these multi-tasking activities often comes at a cost of diverting attention away from the primary task of driving.

Although, there is a number of more traditional sources of driver distraction like talking to passengers, eating, drinking, or listening to the radio, over the

last decade many new electronic devices have been developed and are making their way into the vehicle. In most cases, these new technologies are engaging, interactive information delivery systems that can be more distracting than the traditional sources.

Therefore, continued efforts must be made to promote successful strategies and introduce targeted initiatives in the field of intelligent in-vehicle technology. One such is the subject of road safety in the context of impact of new technologies. Although most of the initiatives focus on road users, considerable effort is also underway to make vehicles and roads safer (Sharon, 2003).

1.2 Background

Current efforts in technological advancements that make vehicles safer to operate and help motorists avoid rear-end collisions are underway. Intelligent Systems that adjust steering, throttle and brakes to compensate for driver error or inaction are now a reality. Advanced systems such as adaptive cruise control, stability control, and night vision are currently available on some new vehicle models. These developments should help reduce the number of motor vehicle collisions in the future (Sharon, 2003).

Figure 1.1 illustrates a more scientific perspective of vehicle safety. It is divided into two main areas: active safety measures (collision avoidance), and

passive safety measures (mitigation of injuries). A collision warning system is an active safety measure, where research is focused in three separate areas. These are human factors, technical features of the car, and the environment.

Each area has been considered individually in many studies wherein humans (behaviour) are cited as the main cause of collisions. However, the driver's health and physical condition, driving education, and experience do influence their performance. More recently, studies show that drivers tend to adapt their driving, including distraction behaviour and maintenance of speed and headway, based on road conditions either consciously or unconsciously (Smith and Zhang, 2004). This indicates that drivers are influenced by their environment and therefore may shed non-driving tasks (e.g. ending a cell phone conversation) when driving under unfavourable weather conditions (i.e. heavy rain or snow). When collisions do occur under these conditions, it is the vehicle's ability to brake effectively, not the driver's ability to perceive a potential collision situation that is the cause (TC, 2003).

It has also been realized that there are strong influences from other areas such as technologies related to vehicle navigation. Recent literature suggests that the improper design and use of in-vehicle telematics can impair driving performance (TC 2003, ITS America 2002). In-vehicle telematics are a form of Intelligent Transportation Systems (ITS), which incorporate wireless communi-

Figure 1.1: Automotive Safety (Selfert and Wech, 2003)

cations technologies for providing information services, vehicle automation and other functions. A study by the Texas Transportation Institute evaluated the impacts of ITS from a behavioural perspective (Lomax, Vadali and Eisele, 2002). The authors acknowledge that some of the in-vehicle technologies may lead to more “risky” behaviour, and that there is a need to reduce driver workload and fatigue through better design.

As well, drivers alter their behaviour based on the conditions within their vehicle. For example, the general handling characteristics, driver support systems (e.g., anti-lock brakes, cruise control, and information systems), field of view, lighting, and comfort level (e.g. noise and vibration conditions, heating and air conditioning levels) also influence the driver (Seiffert and Wech, 2003). Any control button or switch found on an information panel (dash board) can take the attention of a driver away from driving and if sufficiently distracting, can potentially have the driver looking away from the road for extended periods of time.

Therefore, it is no wonder that in the majority of rear-end collisions, driver behaviour in relation to distraction and inattentiveness has been identified as major components in the cause of the collision (Knipling et al, 1993 cited in Brown et al, 2000, Treat et al., 1977 cited in Noy, 2002, Granet et al, 2003). Hence, actions that serve to reduce these collision occurrences due to

numerous outside influences that modify driver behaviour need continued research to improve their success.

1.3 Problem Identification

As new and innovative technologies are added into vehicles, there is a greater likelihood of them causing a distraction for the driver and in turn increasing the risk of a collision. Since driver distraction is such a powerful contributor to rear-end collisions, many such collisions can be avoided if measures are taken to resolve or lessen this problem. Minimizing driver distraction should be the key factor in designing products that can be used while a vehicle is in motion.

Many systems have been developed and tested to determine the best possible method for warning drivers of potential rear-end collisions (Brown et al., 2000). While such in-vehicle technology offers great potential to improve automobile safety, beneficial effects depend on the joint performance of the system and the driver (Dravidam and Tosunoglu, 2000a).

Many in-vehicle technologically sophisticated systems fail, not because of technical shortcomings, but because designers failed to consider the role of the human operator. This is especially important to consider for rear-end collision warning systems since user acceptance and performance are integral to its success (Dravidam and Tosunoglu, 2000b). In-vehicle technology,

integrated properly with the understanding of human factors, has the greatest potential to mitigate rear-end collisions.

As well, in order to be effective, drivers must accept in-vehicle technology. In general, this means that the system must be useful to the driver and must not interfere with normal driving habits. The system should result in a minimum load on driver attention.

In addition, it is not enough to simply rely on distance measuring technology for warning drivers of decreasing distance between vehicles (McGehee et al., 1992). An attentive driver may well be aware of the situation and has already taken corrective action. Providing a warning in all such cases may give the impression of a false alarm and become a nuisance. Hence, at this stage of development, the performance needs to be improved in terms of reliability and optimal time for providing the warning messages.

1.4 Objectives and Scope

The objective of this thesis is to develop a new approach to enhance the design of a rear-end collision warning system. Such a system has the capability of warning the driver of a potential collision with another vehicle in the forward path of the subject vehicle.

The process to develop this system is shown in Figure 1.2. It involves the development of a simulation environment (car-following model) that integrates a rear-end collision warning algorithm. The source codes for commercially available micro-simulators were not available so as to adopt and modify and others reported in the literature with mathematical descriptions are not reported with full details, therefore, an original model that will have the flexibility of use in this research is being developed.

To enable an increased accuracy, the rear-end collision warning algorithm combines the principles of vehicle dynamics and driver response to ascertain a critical distance. The following vehicle braking rate and velocity and driver reaction time will be combined to offer a more comprehensive model.

In addition, a framework that uses the Bayesian statistical decision theory is integrated into the simulation environment to determine when and what type of a warning should be given to the driver. This improves upon current systems that mainly rely upon the critical distance or time headway to warn the driver of an imminent collision.

Figure 1.2: System Development

However, in such a system, a common limitation is that of predicting abrupt merging operations or offering advice on lane changing manoeuvres. Since human control is continually maintained, the rear-end collision warning system is only intended for use in warning the driver of an imminent collision ahead.

1.5 Study Methodology

In detailing the tasks required to conduct this thesis, a flow chart was developed. It outlines the approach taken to carry out the overall study methodology. This is illustrated in Figure 1.3 and explained below.

- Problem identification and definition of the various aspects of the problem to be solved.
- Development of the methodological framework and identification of the essential elements.
- Literature reviews, covering the relationships between simulation modelling, car-following logic, rear-end collision warning system methodologies and human factors.

- Definitions of variables necessary for use in the rear-end collision warning algorithm and car-following model ensuring the principles of vehicle dynamics and appropriate warning logic are followed.
- Analysis and determination of appropriate variables from the SAVME database to calibrate and validate the car-following model.
- Development of the car-following model using MATLAB-SIMULINK.
- Integration of the rear-end collision warning algorithm into the car-following model and development of the Bayesian statistical decision theory framework.
- Validation of the rear-end collision warning system and Bayesian statistical decision theory framework.
- Compilation of the summary, conclusions, and recommendations.

Figure 1.3: Research Study Methodology

1.6 Thesis Organization

This thesis is organized into nine chapters. The first chapter introduces the reader to the background information regarding driver-vehicle interactions with respect to in-vehicle technologies and the potential consequences due to poor design or driver characteristics. Research objectives, study methodology, and scope of research are also described. Chapter 2 is devoted to literature review. This chapter describes the basic functionality of current collision warning systems and technology behind them, along with potential problems when the driver-vehicle interactions are not fully considered in the design. In chapter 3, the software utilized to develop the simulation environment is described along with the SAVME database. The model development process and the influential variables and parameters used are outlined in chapter 4. The methodology used to design the car-following model and the rear-end collision warning algorithm are also described. Chapter 5 outlines the system design process including key factors and variables. It also describes the methodology used in the development of the Bayesian statistical decision theory framework. This leads into the model development process using MATLAB described in chapter 6. The car-following model calibration and validation that includes both qualitative and quantitative analysis is also detailed. This follows with the results obtained within the simulation environment in Chapter 7. Verification and validation of the rear-end collision warning algorithm are described along with sensitivity analyses of the parameters used within the Bayesian statistical decision theory framework.

Chapter 8 outlines an example application to describe the process of evaluation using the Bayesian statistical decision theory framework within the simulation environment. Finally, a summary, conclusions, and recommendations along with future work are presented in Chapter 9.

This thesis also contains five Appendices. In Appendix A, a sample of the SAVME database is given, illustrating the variables in the model that were used to develop the simulation model. The Bayesian statistical decision theory framework, including the variables, basic formulation, and the process involved in solving the decision problem are described in Appendix B. Appendix C gives the syntax used within MATLAB to build the simulation model within SIMULINK. Appendix D and E give the calibration and validation results respectively used for the car-following portion of the simulation model.

CHAPTER 2: LITERATURE REVIEW

This chapter describes the basic functionality of current collision warning systems and technology behind them, along with potential problems when the driver-vehicle interactions are not fully considered in the design. This understanding is required in order to establish an algorithm that will simulate the logic behind a collision warning system that integrates both the driver and vehicle.

2.1 Background on Collision Warning Systems

Automotive engineers have developed and refined in-vehicle systems to mitigate against rear-end collisions. These systems either are in development, in the testing stages or are available on the market based on either radar- or laser-based sensors. Among the collision-warning systems that exist, as the technology to develop these systems improves, so have the accuracy and, in some cases, complexity of the algorithms used to trigger a warning to the driver and/or an automatic vehicle response. A collision warning system generally operates by using a sensor installed at the front end of a vehicle that constantly scans the road ahead for vehicles or obstacles. When found, the system determines whether the vehicle is in imminent danger of colliding, and if so, warns the driver. The sensor systems employed can measure clearance and relative velocity of the lead car and the acceleration is estimated through the coordinated systems using a radio to transmit the lead car velocity and

acceleration data (Drew, 1994). Proper signal processing methods must be employed for good controller performance.

Industry, academia, and governments have proposed several collision warning algorithms. In general, the algorithms can be classified according to their use of time-headway, time-to-collision, or the underlying kinematic constraints. Whereas time-headway algorithms offer simplicity and are consistent with current driving-manual recommendations for safe driving, they are indirectly influenced by relative velocity. They have been designed to be consistent with current driving manual recommendations for safe driving (i.e. 2-sec gap). Time-to-collision algorithms are based on perception of contact time and are sensitive to relative velocity. Algorithms based on kinematic constraints offer increased accuracy by calculating the moment that the driver must initiate braking, given an assumed reaction time and following vehicle deceleration response. Because this class of algorithm considers both reaction time and the capacity of the following vehicle to decelerate, it offers more comprehensive model than the other two categories.

2.1.1 Time-Headway Criterion

Wheatly and Hurwitz (2001) described an algorithm using time-headway as a criterion for collision warning that they were investigating at Motorola. Time-headway (T_H), expressed in seconds, is defined as the range between the

front bumper of the following vehicle and the rear bumper of the lead vehicle (*Range*), divided by the velocity of the following vehicle (V_H).

$$T_H = \text{Range} / V_H \quad (2.1)$$

Most Ministries of Transportation recommend that drivers adopt a minimum two-second time-headway for safe driving practice. Time-headway is a useful metric because it corresponds to the amount of time the driver of a following vehicle has to match the braking profile of the lead vehicle. Assuming that both vehicles are travelling at the same speed and that the following vehicle perfectly matches the average braking rate of the lead vehicle, time-headway is precisely equivalent to the amount of time required for the following vehicle to initiate braking in order to avoid collision.

To illustrate this relationship, the range required for collision avoidance (R_{CA}) is equal to the braking distance for the following vehicle (D_{HS}) minus the braking distance for the lead vehicle (D_{LS}). This can be expressed in terms of the following vehicle velocity (V_H), the average deceleration of the following vehicle (A_H), the lead vehicle velocity (V_L), the average deceleration of the lead vehicle (A_L), and the response time of the following driver (RT_H):

$$R_{CA} = \left[\sqrt{\frac{2V_H^2}{A_H}} + V_H RT_H \right] - \left[\sqrt{\frac{2V_L^2}{A_L}} \right] \quad (2.2)$$

Assuming $V_H = V_L$, $A_H = A_L$, and therefore that the quantities $\sqrt{\frac{2V_H^2}{A_H}}$ and $\sqrt{\frac{2V_L^2}{A_L}}$ are equivalent, the expression can be reduced to:

$$RT_H = R_{CA} / V_H \quad (2.3)$$

This set of equations illustrates that time-headway (T_H) is equal to reaction time (RT_H) when the range to the lead vehicle is equal to the collision avoidance range (R_{CA}). As a general rule of thumb, drivers should drive within the constraints of their reaction time. As General Motors implemented for the Advanced Collision Avoidance System Field Operational Test (ACAS FOT), a time-headway criterion can be used to trigger cautionary warnings, in order to make the boundaries on the safe field of travel more explicit. A smaller time-headway criterion could also serve as an imminent warning (e.g., Wheatley & Hurwitz, 2001).

There are several limitations of the time-headway criterion for an imminent criterion. Firstly, the time-headway rationale assumes that the two vehicles are initially travelling at the same speed. Horowitz and Dingus (1992)

estimated that in 75% of rear-end collisions, the lead and following vehicles are not in a continuous car-following state before the rear-end collision occurs. Furthermore, Najm et al. (2003) estimated that the lead vehicle is travelling at a constant and slower speed before 10% of rear-end collisions occur. Consequently, it appears that the assumption of equivalent lead and following vehicle speeds may be violated in a significant number of cases.

Another assumption that may be limited is that the following vehicle matches the average braking rate of the lead vehicle. While it seems likely that following-vehicle braking is somewhat modulated as a function of the lead vehicle activity, perceptual limitations suggest that drivers may perceive rates poorly. Given that the human visual system is insensitive to the rate of deceleration, it is likely that higher lead-vehicle braking rates may (at least initially) be perceived poorly. If the lead vehicle produces an immediate maximal braking response, the following vehicle may significantly lag behind the braking profile of the lead vehicle. Drivers may also overreact to lower braking rates when the lead-vehicle brake lamps are observed. Time-headway may be an overly simplistic algorithm to capture all rear-end pre-collision scenarios. However, if it is used only in cases where the two vehicles are coupled with similar velocities, time-headway may be a useful criterion, especially for cautionary (lower severity) warnings.

2.1.2 Time-to-Collision Criterion

Using a time-to-collision criterion for triggering rear-end collision warning alerts is based on the theory that humans directly perceive time-to-collision (Lee et al., 1999). Several European researchers have recommended using time-to-collision as the criterion for rear-end collision warning systems (e.g., Van der Horst and Hogema, 1993; Graham & Hirst, 1994). Graham and Hirst (1994) reported that drivers tend to initiate braking at 4 seconds before collision, and therefore a criterion of around 5 seconds should be used to allow for driver's brake reaction time.

Time-to-collision is equal to the range between the front bumper of the following vehicle and the rear bumper of the lead vehicle, divided by the relative velocity or range rate (expressed as a positive value).

$$T_c = \text{Range} / \text{Range Rate} \quad (2.4)$$

The difference between time-to-collision and time-headway is that closure speed rather than following vehicle speed is used in the denominator and therefore the time-to-collision represents the amount of time before the headway will be reduced to zero. When the following and lead vehicles are travelling at similar speeds, time-to-contact approaches infinity, and when the

following vehicle is travelling at a slower speed the time-to-collision does not logically exist.

Like time-headway, one advantage of using time-to-collision is its simplicity. If one accepts the theory that drivers directly perceive time-to-collision, using this as the criterion for rear-end collision warning algorithm seems reasonable. Unlike time-headway, a time-to-collision-based algorithm is extremely sensitive to relative velocity. However, this may be a weakness because if the lead vehicle is travelling at a speed greater than or equal to the following vehicle, the algorithm will detect no threat, even in the extremely dangerous situation where bumpers are mere inches apart. Another potential limitation is that the time-to-collision criterion does not take into account instantaneous vehicle accelerations. Thus, a lead vehicle that is decelerating at maximum rate is treated as being equivalent to a vehicle that is travelling at a constant rate, or even accelerating.

Hirst and Graham (1997) proposed that rear-end collision warning systems adopt a time-to-collision criterion that assesses a speed penalty so that the time-to-collision threshold is higher for larger speeds. Although the speed penalty addresses some limitations, after modelling various collision warning algorithms, Brown, Lee, and McGehee (2001) demonstrated the counterintuitive result that this formula has the potential of leading to more severe collisions at larger initial headways.

The basic time-to-collision formula offers simplicity as an advantage, however, as more modifications are made to the basic formula to accommodate the limitations, the algorithm begins to lose this advantage. The various adjustments used to compensate for initial over-simplification become burdensome, perhaps suggesting that a more comprehensive model be used as a starting point. In this regard, algorithms based on the inertial braking constraints may be a more effective alternative. However, this formula could be used within cautionary warning criteria.

2.1.3 Kinematic Constraints Criterion

Lee, McGehee, and Brown (2000) used the term “kinematic constraints” to refer to the algorithms that are based on the inertial constraints of the vehicles. The algorithms assume the lead vehicle will continue its current rate of acceleration/deceleration until it stops and the following vehicle will respond after a specified brake reaction time, by braking at a specified constant deceleration rate.

Using the instantaneous lead and following vehicle speeds and accelerations, these algorithms calculate the minimum collision avoidance range that will result in the following vehicle missing the lead vehicle by a specified margin (Knipling et al., 1993; Wilson et al., 1994; Seiler et al., 1998; Burgett et al.,

1998; Krishnan et al., 2001, 2002, Lee, McGehee, and Brown, 2000; Lee et al., 2002a; 2002b).

Burgett et al. (1998) proposed one of the first versions of this algorithm. Their algorithm assumed that the following vehicle will brake at a rate of $-0.75g$ after a brake reaction time of 1.5 seconds, and calculated the minimum collision-avoidance range to allow the following vehicle to miss the lead vehicle by 2m. Unlike other algorithms, this algorithm assumed that the following and lead vehicles were initially travelling at the same velocity.

Following this approach, the Collision Avoidance Metrics Partnership (CAMP) between several Original Equipment Manufacturers (OEMs) developed a collision avoidance algorithm, based on human factors research (Kiefer et al., 1999). Kiefer et al. (1999) instructed drivers on a test track to brake at the last moment to a stationary or braking lead vehicle target. Based on the variability of the moment of braking they concluded that drivers responded at relatively consistent levels of required deceleration. For this reason and because required deceleration is connected to the fundamental kinematic variable underlying collision avoidance, they recommended that the algorithm should use required deceleration as a criterion for rear-end collision warning.

The CAMP algorithm differed from the Burgett et al. (1998) algorithm in two respects: the assumption of equivalent lead and following vehicle speeds was

removed and the deceleration was modelled on last-minute braking data (Brunson et al, 2002). This helped to mimic the performance limitations of the driver, rather than maximally enhancing the detection of threatening scenarios. This resulted in a fixed following vehicle braking response of $-0.55g$.

Overall, the algorithms that are based on kinematic constraints appear to comprehensively model the range of pre-collision scenarios. Lead vehicle deceleration rate, following vehicle braking limitations, and driver reaction time are represented in the equations. Not only does this type of algorithm model the reality of collision-avoidance with the greatest fidelity of the three classes of algorithms, it also appears to directly map onto the underlying nature of what represents a collision threat (Kiefer et al., 1999).

However, one potential limitation of this category of algorithm is that, like the time-to-collision algorithms, it does not detect any level of threat until either range-rate or range-acceleration is negative. Thus, if one assumes no minimum collision-avoidance range, the algorithm may register no threat even when the vehicles are mere inches apart if there is no relative closure. To fix this, one could adopt a minimum 'miss' distance, such as the 2 meters used by Burgett et al. (1998). As well, by providing a cautionary warning when drivers become too close to the lead vehicle, drivers are warned before an imminent warning becomes necessary. This minimizes the chance that drivers will move beyond their brake reaction time capability.

Most algorithms appear to adopt a brake reaction time of approximately 1.5 seconds; however, response times vary depending on a driver's state: alerted, surprised, and distracted. This variable should be dependent on an approximation of alertness to reduce an excessive number of nuisance warnings.

In addition, each algorithm differs with the choice of following vehicle deceleration rate. Krishnan et al. (2001) used simulation techniques to design and test a collision-warning system for the lead vehicle not moving case. The system was designed to maximize its capability of preventing collisions, to minimize the severity of collisions, and to reduce the frequency of nuisance alarms; this resulted in a mean deceleration of 5.5m/s^2 . However, the limitations of a vehicle and driver comfort will determine how the driver will choose to decelerate.

Nevertheless, one parameter that does affect a vehicle's potential to decelerate is the tire-road coefficient of friction. Assuming a 20% level of wheel-slip, the coefficient of friction can vary between 0.1 (wet ice) and 0.8 (bare and dry surface) due to weather conditions (NPRA, 2000). Depending on a warning system's capacity to detect or estimate the coefficient of friction of a roadway, the algorithm could adapt the brake rate criterion as a function of the conditions. However, when large changes to the algorithm parameters

are based on unreliable values from sensors that read temperature and windshield wiper movements, a large number of nuisance warnings may result. This was recently observed and contributed to an almost three-fold increase in the number of nuisance warnings in the Stage 3 pilot testing of the Advanced Collision Avoidance System Field Operational Test (ACAS FOT) program (Ervin, Sayer, & LeBlanc, 2003).

2.2 Functional Operation

Most of the new generation of assistance devices are derivatives, in one form or another, of the adaptive cruise-control system (Smith, 2001; ENG, 2001). They work by detecting the distance and velocity of the vehicle ahead. The easiest way to do this is to use a small onboard sensor system (ENG, 2001; Granet et al., 2003). These systems will use different optical, electromagnetic radar, or ultrasonics techniques. Researchers are considering combinations of several of these technologies (sensor fusion), to overcome some of the disadvantages (Smith, 2001; Granet et al., 2003).

Overall, these systems measure the distance between the two vehicles, to within a few centimetres, using the time that it takes for pulses of the radar beam to travel to the car ahead and bounce back (Granet et al., 2003). By contrast, the velocity of the car ahead is determined by measuring how the Doppler Effect has altered the frequency of the radar beam that has echoed back. The difference in frequency between the transmitted and received

signals gives a measure of how much faster or slower the vehicle ahead is travelling (ENG, 2001). In some cases, a small digital camera keeps an eye on the road ahead, processing continuously to track the white lines making the lane. Such examples are present on the Fiat, Nissan, Daimler-Chrysler, Volvo, BMW, Subaru, Honda, and Mitsubishi (ENG, 2001).

Each vehicle that has been tested had slightly different features and functionalities. However, for the NHTSA system, the design approach follows a common architectural process, which is shown in Figure 2.1. The core of the system is the collision avoidance processor, which takes the inputs from the sensor suite (active and vehicle sensors), processes the sensor information using the collision warning processing suites and provides the appropriate driver-vehicle warning response. A data acquisition system is used to provide diagnostic features.

Figure 2.1: Collision Warning Vehicle Mechanization (NHTSA, 2002)

In 1999, the Chinese Academy of Sciences' Intelligent Control and Systems Engineering Centre started the Intelligent Vehicle Platforms project. Supported by the Vehicular Embedded Computing Platform project, the Xi'an Jiaotong University Institute of Artificial Intelligence and Robotics collaborated to develop intelligent driver-assistance and safety warning systems for passenger vehicles, particularly GPS- and vision-based systems (Zheng et al., 2004). These systems provide appropriate just-in-time information regarding the vehicle, road, and surroundings for safer and better driving. The system architecture contains three layers: the sensory layer, decision-making layer, and the human-interface layer (Zheng et al., 2004). It warns the driver passively when his or her vehicle is in a hazardous situation and proactively

about possible hazardous situations on the basis of the vehicle's current position, orientation, and velocity, and the road conditions or surrounding environment.

2.3 Driver-Vehicle Interaction

Slowly an emphasis is now being placed on developing efficient algorithms for collision detection and keeping the number of false alarms to a minimum. Lerner et al. (1996b) proposed a set of human factors guidelines for the design of collision warning systems. After reviewing the human factors literature in a range of different fields (e.g., aviation, military, nuclear power plant), they recommended that all warning systems should be capable of producing at least two levels of warning. An imminent warning should be activated when immediate evasive actions are needed to avoid a collision. A cautionary warning should be initiated when there is the possibility of a collision and the driver needs to be alerted, but in which the driver does not need to take immediate evasive action (Dingus, 1996; Wilson et al., 1996). One problem with cautionary warnings is that they are activated more often than imminent warnings, which can potentially annoy the driver and cause him/her to ignore the warning (Horowitz and Dingus, 1992). Nuisance warnings may not only decrease driver acceptance of the system, but could potentially undermine the effectiveness of the warning, by reducing the credibility of the system (Lerner et al., 1996a; Lee, Caven, Haake, and Brown, 2000, Horowitz and Dingus, 1992). If the technology is not reliable, the system will be deemed useless

and lose its effectiveness. The inherent trade-off between a more sensitive system and more nuisance warnings should be to integrate the driver into the process of a warning system.

Studies conducted at Transport Canada indicate that changes in drivers' visual behaviour are observed when they perform demanding cognitive tasks while driving (Harbluk and Noy, 2002). Hence, warnings should be issued only when the driver is not focusing on the road or when a dangerous change occurs rapidly in the position and velocity of a vehicle in front (Horowitz and Dingus, 1992). However, as long as a collision warning system does not have information about the driver's state of attention, warnings will likely be issued even when the driver is fully aware of the danger. If this warning startles the driver, this adds to the cognitive load and potentially leads to stress, delay in action and incorrect responses (Horowitz and Dingus, 1992).

The success of collision warning systems also depends on how well the algorithm and driver interface are tailored to driver capabilities and preferences. Driver warnings may increase the attention and information-processing load of drivers and divert driver attention if they are not properly designed.

Although many evaluations have examined driver response to various display and algorithm alternatives, little research has considered creating a

quantitative model of driver performance to aid in these evaluations. Most use assumptions to simplify the problem and allow for some analyses of the algorithms, but they do not provide a comprehensive explanation of how the algorithm and driver interact (Brown et al., 2000).

Research on specific combinations of the various modalities is also abundant. McGehee et al. (1994) and Dingus et al. (1997) determined that the addition of auditory warnings to visual warnings served to improve driver reactions times. This reduction in driver reaction time has further been qualified as a reduction in the time in accelerator release (Lee et al., 1997, 2002a, 2002b; Lee, Ries, McGehee and Brown, 2000; Brown et al., 2001). Similar results were obtained by Belz (1997) and Belz et al. (1998, 1999) using auditory icons as the display. Shirkey, Mayhew, and Casella (1996) determined that a multi-modal system using auditory and visual displays was more effective than each modality on its own, due to a decrease in the participant's ability to identify warnings that were offered using a single modality. These benefits can be maximized by using graded and dynamic warnings (General Motors Corp. and Delphi-Delco Electronic Systems, 2002a, 2002b).

Combinations of auditory and haptic (e.g. seat vibrations) modalities have also received some attention, especially for their use in 'automatic' braking systems (Shefer and Klensch, 1973; SAE, 1974; Troll, 1974). Studies on this combination of modalities have shown that haptic warnings, in the form of a

brake pulse, can reduce the number of collisions, likely because the vehicle is already braking while the warning is provided (Shutko, 1999). Shutko (1999) also determined that auditory warnings result in faster reaction times than do haptic warnings.

Combinations of the three warning modalities have also been studied. For studies of this nature, the determination of a 'best' modality has been attempted, but various researchers reach different conclusions. For example, Janssen and Nilsson (1992, 1993) determined that the haptic modality was the most effective in reducing headways while avoiding the development of other risky behaviours (e.g. increases in speed). Perhaps the most visible effort with respect to the design of tri-modal collision-warning devices is being performed by the group CAMP (Kiefer et al., 1999; Kiefer, 2000). After considerable testing of numerous participants, this partnership developed an interface of a single-stage collision warning consisting of the non-speech tone combined with a flashing high head-down display of the visual icon with the word "WARNING." Recent reports by other groups, however, suggest that graded warnings, combined with head-up displays, are a better warning mechanism than this original alternative (General Motors Corp. and Delphi-Delco Electronic Systems, 2002a). A similar conclusion on graded warnings has also been reached by Pierowicz et al. (2000). Head-up displays were also effectively used by Zador, et al. (2000), especially when assisted by a haptic display. Zador et al. (2000) found that auditory warnings were generally

considered the most annoying; however, they were very effective in attracting participants' attention.

Finally, the accelerated development of collision-warning systems, evidenced by the substantial amount of research available on them, has prompted the International Standards Organization (ISO) to develop a standard for the design of these systems (ISO, 2002). The main recommendations in the standard include:

- Should include at least two separate (i.e., graded) warnings, such as a preliminary collision warning and a collision warning.
- Audible and/or tactile modalities should be the primary modalities; visual warnings may be used only in addition to one of these modalities.
- When a vehicle has more than one warning (e.g., rear and side collision warning), each warning should be clearly distinguishable to the driver.

Overall, in the design of a rear-end collision warning system, sensors fulfill the tasks of headway control and obstacle detection. However, consideration and techniques are required to import the driver into the analysis. Without the knowledge of how the driver is performing, the driver in response to the warning could undertake continued poor reactions. By incorporating driver awareness and a statistical decision theory, driver behaviour can be incorporated into an algorithm to determine the best possible type of warning and when to inform the driver.

CHAPTER 3: SOFTWARE AND DATABASE DESCRIPTION

3.1 Software Utilized

Traditional approaches to system design typically include building a prototype followed by extensive testing and revision. This method can be both time-consuming and expensive. As an effective and widely accepted alternative, simulation is now the preferred approach to engineering design.

SIMULINK, by The Mathworks, is a companion program to MATLAB and is a powerful simulation software tool that enables one to quickly build and test virtual systems at any level of detail. By using SIMULINK to iterate and refine designs before building the first prototype, designers can benefit from a faster, more efficient design process. Hence, SIMULINK was utilized for this thesis.

This software package enables modelling, simulating, and analyzing dynamic systems. As shown in Figure 3.1, it provides an interactive graphical environment that enables one to accurately design, implement, and test time-varying systems (The Mathworks, 2004; RTI, 2002). It permits complex feedback systems while allowing continuous interactions among many variables and numerous cause-and-effect relationships found in this analysis.

Figure 3.1: Example of a SIMULINK Block Diagram, Coding, and Graphical Outputs

Simulink comprises of a customizable set of block libraries (see Figure 3.2) used within the simulation environment (The Mathworks, 2004). These blocks give a graphical representation of a mathematical model of a system. Each block consists of a set of equations (block methods) that define a relationship between the input signals, output signals and the state variables that vary over time (The Mathworks, 2004).

Figure 3.2: Example of SIMULINK Block Libraries

Figure 3.3 represents a “block” functionality. Here a signal u is used as input into state variable x , which then determines a block's output y (signal into another state block). The solution of the block diagram is obtained by evaluating these relationships over time, where time starts at a user specified “start time” and ends at a user specified “stop time.” Each evaluation of these relationships is referred to as a time step.

Figure 3.3: Signal and State Variables

A more detailed example is illustrated in Figure 3.4. Here “ $y = ax + b$ ” can be solved using five different blocks representing input variable and equation blocks. A constant value a is multiplied by a variable x that varies with time. This product is then added to a value b , which is outputted as y to the user.

Figure3.4: Sample Equation Blocks

A Simulink block diagram can also consist of layers (The Mathworks, 2004). Each layer is defined by a subsystem. A subsystem is part of the overall block diagram and consists of a subset of equations used in “if...else...” situational logic. They are provided primarily to help in the organizational aspects of a block diagram. Figure 3.5 illustrates an example of an “if...else...” subsystem where specific actions are taken depending on the input signal conditions in the main block.

Figure 3.5: Example of a Subsystem

The SIMULINK blocks are integrated with MATLAB, providing immediate access to an extensive range of tools for algorithm development, data visualization, data analysis and access, and numerical computation. With SIMULINK, one can incorporate coding directly into a model. This enables the inclusion of user-defined code and the ability to create custom blocks and provide an alternative way to represent algorithms in a model (The Mathworks, 2004).

After building a model in SIMULINK, one can simulate its behaviour and view the results live. SIMULINK includes several tools for analyzing a system, visualizing results, and testing, validating, and documenting models (The Mathworks, 2004). Figure 3.6 gives an example of output data in a tabular format to enable separate statistical analysis along with graphical representations of system results.

Figure 3.6: Sample Tabular and Graphical Output

Custom-built displays can also enable visualization within the MATLAB environment or through a graphic user interface development tools. Model analysis tools can be accessed from the MATLAB command line, plus the many tools in MATLAB and its application toolboxes. Since MATLAB and SIMULINK are integrated, you can also simulate, analyze, and revise models in either environment at any point.

3.2 SAVME Database Description

There is very little information that is both quantitatively and statistically representative of the headways, time spacing between vehicles, or the correspondence between these inter-vehicular variables and acceleration, which are driver induced in response to these actions (Ervin et al., 2000). Hence, the pre-collision environment has lacked quantification in dynamic temporal/spatial terms by which vehicle motions and clearances are determined. Since inter-vehicular relationships vary stochastically and since true collision-bound trajectories appear only with great rarity, it is very difficult to take a system prototype and try it against a distribution of kinematic exposure variables. In addition, attempts to simulate the application environment tend to lack validation until some robust form of data is brought forward through a direct-measurement characterization (Ervin et al, 2000). As well, with the advancements of active safety packages, there is a lack of definitive data for these complex application environments. However, the SAVME database was developed to help in the process of refining collision warning and intervention systems (Ervin et al., 2000).

The data being used for the purposes of this research was obtained through the Transportation Research Centre under the National Highway Transportation Safety Administration (NHTSA) in East Liberty, Ohio. It contains a 53-minute segment of data collected in Columbus, Ohio near a dangerous intersection during the late afternoon rush hour. A sample of the

database is found in Appendix A. This database provides microscopic records at a time resolution of 0.1 seconds in a real traffic stream. It contains trajectory data representing 3,284 vehicles traversing two arterial streets through an intersection. All data were collected for purposes of studying pre-collision conditions during weekday trips and during normal weather conditions. The figure below illustrates the layout of the two arterial streets and their intersection. The street edges are marked in bold, where Refugee Road runs east west and Winchester Pike runs north south.

Figure 3.7: Diagram of Refugee Road-Winchester Pike Intersection (Smith, 2004)

As shown in Figure 3.8, preparation of the SAVME database involved three major steps (Ervin et al., 2000; Mehmood, 2003; Smith, 2004):

- 1) A data collection system includes three cameras mounted on three telescoping towers (100ft) installed approximately 65ft from the roadside and roughly 200ft apart to collect the images of each individual vehicle passing through the site. The images were taken at a rate of 10 images per second on a removable 100GB hard disk. The removable drives were then downloaded to network drives and then off-loaded to magnetic tape until they could be brought back on-line for processing.
- 2) These magnetic tapes were then processed to capture individual vehicle images and track their motions as they moved across the data collection site, yielding raw track-files or a trajectory for each individual vehicle. The track-file data contains batch information, vehicle header, and the vehicle location data-packet. Batch information describes the date and time the data was collected. Vehicle header information describes each vehicle in the track-file. The following data are stored in the header for each vehicle: an identification number, the number of location samples, the first frame in which the vehicle appears on the road, the last frame in which the vehicle appears, the vehicle's length, and the vehicle's width. The vehicle location data-packets contain data about the vehicle from the time it enters the viewing area to the time it exits. The packets contain the X and Y position of the vehicle in road coordinates.

Figure 3.8: Block Diagram of Data Collection, Processing, and Analysis
(Ervin et al., 2000)

- 3) In the next step raw track-files were processed using Kalman filtering to improve the estimates of X and Y locations for each vehicle and then used to estimate inter-vehicular variables such as forward and lateral velocity, longitudinal acceleration/ deceleration rate, and vehicle steering angle for each individual vehicle. The Kalman filter utilizes a simple vehicle dynamics model scaled to the size of each vehicle. The inter-vehicular information for each individual vehicle is then stored in Microsoft Access tables.

After converting the track-file data into a Microsoft Access database, a series of queries were run to classify the tracks contained in the database by each vehicle's point of origin and its destination. An Inter-vehicle Variables Table was created to give the relationships between all vehicles that co-exist on the road at any given time. The fields are identical to those in the main track-file table. Classifications were confirmed visually using the VME_Animator software, and a text field named "Type", containing a code for this information, was added to the database (Smith, 2004). An additional field was added to the tables to distinguish between vehicles entering the view on Winchester Pike ("NB" (northbound) or "SB" (southbound)), and similarly vehicles entering the view on Refugee Road ("EB" (eastbound) or "WB" (westbound)). The classifications are further defined by whether they make a left turn, make a right turn, or go straight through the intersection.

Once the data are in a Microsoft Access database, analysis is straightforward. Queries are used to select scenarios of interest. The results of these queries can be graphed by exporting the data to Microsoft Excel or viewed using the VME_Animator application.

3.3 Data Analysis Process

Based on previous research (Ervin et al., 2000; 2001; Mehmood, 2003; Mehmood et al., 2003) it is proposed that an approach to determine the relationships between vehicle movements and following distances using the SAVME database be used to conduct the analysis of the car-following model and the collision warning system. Hence, trajectory data for a random sample of vehicle pairs was utilized to determine spacing, relative velocities, and acceleration of a following vehicle for use in the model. Driving behaviour is normally represented in simulation models by the desired vehicle movements; therefore, for each pair of vehicles, the velocity of the following vehicle and the spacing were extracted. These were then used to compare with current conditions calculated by differential equations within the utilized software. The difference between them will in turn be used to determine the acceleration/ deceleration rate of the following vehicle.

The SAVME database reflects driver behaviour on an arterial roadway. Consequently, the calibration and validation of the model was conducted using this criterion. However, it is anticipated that a similar relationship would be obtained for other roadway types given that the evaluation of this model considers both steady state and non-steady state conditions found on most roadways. It is assumed that roadway conditions (i.e. weather) are ideal since the data was collected during clement weather. Nevertheless, research has shown (TC, 2003; Smith and Zhang, 2004) that drivers tend to adapt their

driving based on road conditions and it is the vehicle's ability to brake effectively that is the cause of collisions in poor conditions. As well, almost all of the sensors (temperature and wiper gages) used on rear-end collision warning systems described in the literature have reduced operational range and accuracy in poor weather conditions, with laser sensors suffering particularly badly.

Upon further analysis of the data, it was discovered on the southeast corner of the intersection, a collision occurred near the end of the data collection period and blocked traffic moving northbound on Winchester Pike. Hence, this direction was not considered in the data analysis. As well, while further examining the data, it was found that Refugee Road has two lanes per direction for the 'ST' designation and the database does not explicitly distinguish between each of these lanes. Therefore, if vehicles were merging, diverging, passing, or travelling within other driver's blind spots, the database analysis would yield largely aggressive following distances and unsteady relative velocities. Hence, it was decided that only the 'SB' direction would be utilized to capture realistic car-following behaviour for the calibration. This resulted in 151 vehicle pairs to analyse.

CHAPTER 4: CAR-FOLLOWING MODEL AND REAR-END COLLISION WARNING ALGORITHM METHODOLOGY

This chapter contains an introduction to the car-following model and rear-end collision warning algorithm development processes and the definition of some of the influential variables and parameters used. The first two sections outline the theory behind human perception and reaction as it relates to driving and car following. The next section describes background information concerning types and limitations of existing car-following models. Following this, the car-following methodology that is used for this thesis is described. Next, key information pertaining to the development of the critical (warning) distance calculation are given. This is in the form of outlining stopping sight distance and deceleration rate criteria. The final section explains the methodology used to develop the rear-end collision warning algorithm.

4.1 Human Perception

In examining human perception, rear-end collisions are of particular relevance because they also appear to be the category of collisions that are most rooted to driver inattention and distraction. In addition to driver inattention, some researchers have suggested that rear-end collisions may be partially attributable to limitations of the human perceptual system (Mortimer, 1990; Hoffman, 1968).

Although the human visual system is extremely sophisticated, it has several limitations. Psychophysicists have a long history of measuring perceptual thresholds, and have combined a converging body of research that documents the limitations of the perception of motion. Mortimer (1990) examined the perceptual limitations relating to rear-end collisions. He states that drivers are able to make accurate estimates on the distance to the car ahead of them (within approximately 20%) and are reasonably sensitive in determining a change in the headway between their vehicle and one ahead of them (within an approximately 12% change). This means that the magnitude of relative velocity is very difficult to scale by drivers especially in the time available before a collision occurs. Hoffman (1968) claimed that even at high relative-velocity situations, drivers are unable to discriminate relative velocity until just a short distance apart, proposing that driving performance might be improved if drivers have access to a display of relative speed information.

Psychophysical evidence also suggests that the human visual system is insensitive to the acceleration of objects. Although acceleration can potentially be inferred if the visual system perceives a large change in velocity, there are no direct perceptual mechanisms to support the perception of acceleration (Watamaniuk and Duchon, 1992). This evidence suggests that the human visual system temporally integrates velocity information in order to filter out noise, directly counteracting the perception of acceleration (Watamaniuk and

Heinen, 1999). Watamaniuk and Heinen (1999) observed that pursuit eye movements lag behind accelerating targets, intermittently requiring saccades (rapid eye movement while performing a task) to catch up with the target. This suggests that the visual system is insensitive to acceleration.

Insensitivity to visual acceleration has clear implications for rear-end collisions. Just as the human eye lags behind an accelerating target, drivers' braking responses to a rapidly decelerating lead vehicle may also lag behind the deceleration requirement to avoid collision. Although deceleration may be inferred (e.g., from the presence of brake lights), the driver of a following vehicle has little information regarding the magnitude of the lead vehicle braking action.

Considering the imprecision of visual motion perception and the inability to perceive acceleration, it is likely that through experience, drivers have learned the patterns of information (such as angle, expansion rate, and brake lights) that correspond with the requirement for full braking.

4.2 Application of Human Perception and Reaction

In order to mimic real world decisions, models must be calibrated so that they are applicable in a variety of contexts. Simulations may use simple rules based on driver behaviour. These decision rules are based on estimates made with discrete choice models or on heuristic and algorithmic procedures

derived from behavioural research. Figure 4.1 describes the logical steps behind such models. By combining perception variables in addition to those used in conventional engineering, the formulation of a coherent model of vehicle and driver behaviour can be achieved (Wilson, 1994).

Figure 4.1: Collision Scenario and Contributing Variables (adapted from Wilson, 1994)

Overall, drivers attempt to gather information about the behaviour of the vehicle ahead to determine the criticality of the situation. After evaluating the information gathered about the lead vehicle, the driver must decide the appropriate action to be taken. This will ultimately determine whether the collision will be avoided. The final step is to execute the action that has been decided. This step will often consist of moving the foot from the accelerator to the brake pedal and pressing it. This process is known as the perception reaction time or distance. This is illustrated in Figure 4.2.

Figure 4.2: Perception-Reaction Process

A great deal of research has been conducted in the area wherein the following sequential components have been defined.

1. **Mental Processing Time:** This is the time it takes for the responder to perceive that a signal has occurred and to decide upon a response. Mental processing time is itself a composite of three sub-stages (Garber et al., 1999; Green, 2000; SDT, 2004):
 - **Perception:** the time it takes to detect the sensory input from an object.

- Identification: the time needed to recognize the meaning of the sensation.
 - Emotion: the time necessary to decide which if any response to make and to mentally program the movement.
2. Movement Time: Once a response is selected, the responder must perform the required movement (Garber et al., 1999). In general, movements that are more complex require longer movement times.
 3. Device Response Time: Mechanical devices take time to actuate, even after the responder has acted (SDT, 2004). For example, a driver stepping on the brake pedal does not stop the car immediately. Instead, the stopping is a function of physical forces, gravity, and friction.

Two information-processing strategies are available to the driver according to human cognitive dynamics (Green, 2000). The first strategy, called automatic response, is either biologically spontaneous (an innate, immediate reaction) or is so well learned that it is functionally spontaneous (as if the processing skips response selection altogether to go directly from sensation to movement). An example of this learned automatic response is hitting the brake pedal immediately when the lead vehicle's brake lights flash. The second strategy is attentive or controlled processing and occurs whenever novel events require thought to understand or react appropriately. Attentive processing takes

longer than automatic processing because it requires cognitive assessment and problem-solving synthesis. Drivers use both strategies not discretely but on a continuum dependent on the scenario. Alexander (1989) recommends that this fact be taken into account when computing response times for the development of algorithms to be used in both complex and simple situations.

For years, the value of the perception-reaction (PIEV) time has been defined through the 85th percentile of the population. One second has been used for the brake response time since 1940 and 2.5 seconds for the perception reaction time in highway design (AASHTO, 2001; Fambro, 1998).

However, depending on the complexity of the situation and the degree of driver alertness different values of PIEV should be used in the analysis. Whether the driver is attentive to his/her surroundings will determine how quickly they will respond to a warning. The alertness criteria may be divided into 3 classes for analysis (Green, 2000):

- Expected: Driver is alert and aware of the possibility that braking will be necessary.
- Unexpected: Reaction time is slower due to the increase in perception time from distractions.
- Surprised: While alert, the driver encounters a sudden event. Extra time is needed to interpret the event and to decide upon a response where the reaction time depends to some extent on the distance to the obstacle.

In driving situations, people tend to brake faster when there is great urgency, when the time to collision is briefer (SDT, 2004), or when the driver is travelling faster and/or the obstacle is close when first seen. When other driving or non-driving matters consume the driver's attention, then brake time becomes longer (SDT, 2004). Studies have shown that when people are not aware of the forthcoming braking, the distance and braking conditions had major effects on reaction time (Liebermann et al. 1995).

Fambro et al. (1998) have provided a summary of previous studies on the PIEV in surprise conditions and anticipated conditions. The PIEV in a surprise condition using a signal ranged from 1.12 to 1.30 seconds (1.27 second mean). In the same instance, while using an unexpected object, the surprise condition ranged from 1.06 to 1.50 seconds (1.28 seconds mean). As for anticipated conditions, PIEV ranges from 0.31 to 0.84 seconds (0.56 sec. mean) in driving simulator studies, and 0.72 to 0.75 seconds (0.73 sec. mean) in an actual driving condition.

Capparella et al. (2003) stated that studies have found that the average human reaction time to a single stimulus is between 0.45 and 0.6 seconds. That same person's reaction time increases to 1.5 seconds while driving due to driver distraction or inattentiveness and lack of visual cues that warrant a hazard situation.

Green (2000) examined the following variables: age of the driver, gender of the driver, urgency of the situation, and the driver's cognitive load at the time of stimulus presentation. Surprisingly, age was found to lack a robust effect, although there is a slight tendency to find increases in response time, as participants get older. The lack of a clear effect may be due to sample bias since healthier older people are more likely to drive and, thus, to volunteer to participate in a study. Another possible explanation is that the extra experience an older driver has accumulated compensates for the slowing of perception or movement, which is likely since they also recognize dangerous situations more quickly than younger drivers do. The research data on the effects of gender on response time were mixed (Green, 2000). While some of his studies found faster responses by men, others found no detectable difference between men and women. Essentially, the results show that drivers do not have a fixed reaction time but rather exhibit different reaction times while they are driving. As cognitive load changes with task complexity and increased information-processing demands, it influences the driver reaction time.

4.3 Car-Following Models

4.3.1 Introduction

Driving an automobile is an example of man-machine feedback control systems. When a driver is faced with a difficult situation; they must adjust to other vehicles in the traffic stream. It may be observed that whenever a driver

approaches fairly close to the vehicle ahead and is unable to pass, in most cases, the driver strives to maintain a safe following distance. This is the basis behind car-following models.

The models determine the acceleration (or deceleration) rate of the following vehicle in a given time interval based on the actions of the lead vehicle. Once the acceleration or deceleration rate of the following vehicle is determined, equations of motion are used to compute the velocity and the position of the following vehicle for any given time interval (Lieberman and Rathi, 2002; Mehmood et al., 2003).

Since the 1950s, the car-following logic has been studied and analysed, resulting in various models and algorithms (Lee and Peng, 2002; Hatipkarasulu et al., 2002). Understanding car-following contributes significantly to an understanding of traffic flow (Rothery, 2002), since it is a task that is commonly practiced on roadways when passing becomes difficult or when traffic is restrained to a single lane.

4.3.2 Limitations of Car-Following Models

Literature suggests four main types of car-following models. They are: stimulus–response car-following models, safety-distance or behavioural car-following models, psychophysical or action-point car-following models, and fuzzy logic-based car-following models. However, a number of limitations and

assumptions of these car-following models that inhibit their use in representing driver behaviour are reported in literature.

For example, Winsum (1999) has pointed out that stimulus-response, safety-distance, and action-point models were developed based on assumptions for which there is no empirical evidence. For example, these models assume particular mathematical form, but no explanation is provided as to why the mathematical form is chosen. Further, the parameter values of these models do not provide an obvious connection with identifiable driver and/or vehicle's characteristics that explain the dynamics of driver behaviour. Instead, parameter values represent an aggregated measure of the characteristics of driver, vehicle, and environment that limits the analysis for individual characteristics.

Stimulus-response car-following models assume symmetrical driver responses to a lead vehicle. In other words, they assume the magnitude of the acceleration and deceleration rate to be the same. In addition, they assume a zero acceleration/deceleration rate for the following vehicle when the relative speed between the lead and following vehicle approaches zero, regardless of the spacing between the vehicles (Chakroborty and Kikuchi, 1999).

The safety-distance and action-point models assume that the following vehicle driver can accurately ascertain the absolute speed and acceleration/

deceleration rate of the lead vehicle (Mehmood, 2003). In a simulation environment the speed, acceleration/deceleration, and position of each vehicle is known at any point in time. However, the objective of the car-following model is to simulate driver behaviour in terms of not only a driver's decision but also the process by which this decision is made. This is important when the car-following model is used to evaluate technologies such as rear-end collision warning system that influences the driver's perception-reaction time.

Safety-distance models are based on an assumed safe following distance. However, this distance is fixed at every speed and does not represent that, which drivers actually maintain in real traffic conditions. A car-following model should reflect the actual following distance observed in real traffic situations for different speeds (Boer, 1999; Winsum, 1999).

The major problem with the fuzzy logic-based car-following models is that the calibration of their functions lacks credibility and reliability in their application. These models have not been calibrated for different following vehicle driver situations. (Mehmood et al, 2001)

Almost all of the existing car-following models assume the perception-reaction time of drivers is a constant for all traffic conditions; however, drivers are continually scanning the environment around them or being distracted by stimuli outside the vehicle or through in-vehicle devices. Be it two seconds or

ten seconds, this in turn influences the alertness level that the following vehicle driver may have at any one time, affecting the perception-reaction time in the event of a potential rear-end collision. (Johansson and Rumer, 1971; Olson and Sivak, 1986; Ozaki, 1993; Lerner et al., 1995)

Therefore, the car-following model established for this work needs to overcome these limitations within the development of the simulation model. The following sections discuss the points of interest involved in the process of developing both the car-following model and the rear-end collision warning system integration.

4.4 Car-Following Methodology

In developing a car-following model, the relationship between spacing and velocity needs to be considered. It is assumed that spacing is a function of velocity and that spacing between vehicles should be sufficient to avoid a rear-end collision when the vehicle ahead stops quickly. The roadway surface conditions are not explicitly considered, however are reflected in the desired speed selected by the drivers based on the information discussed in section 3.3.

For this analysis, the car-following model considers two vehicles moving at certain velocities in the same direction. Both vehicles (*F*-following and *L*-leading) are assumed to initially be travelling with a velocity, V_{OL} and V_{OF} , and

separated by a distance of S_0 . The relationship of the two vehicles and the corresponding dynamic variables are shown in Figure 4.3. The position (X) of both vehicles is continually monitored.

It is assumed that in determining the acceleration/deceleration rate for the following vehicle, the following vehicle driver would use the information obtained from the lead vehicle travelling within his/her desired following distance. This distance will vary based on the current velocity of the following vehicle and is defined by the longitudinal distance measured from the lead vehicle's back bumper to the following vehicle's front bumper (X_n).

Figure 4.3: Analysis of car-following behaviour

Figure 4.4 illustrates the car-following model assumptions. The following vehicle's movements (i.e. acceleration/deceleration) are influenced directly or

indirectly by each of the blocks. Overall, the acceleration/deceleration rate of the following vehicle driver will depend on his/her current velocity (V_F), the desired velocity (V_d), rate of change in spacing (dS/dt), and the reaction time of that driver (T_d). However, with the research expressed by Mortimer (1990) and Brown et al (2000), the car-following model is developed on the assumption that the following vehicle driver can ascertain only the spacing and rate of change in spacing between the following vehicle and the lead vehicle.

Figure 4.4: Structure of the Car-following Model (adapted from Mehmood, 2003)

Therefore, it is assumed that the following vehicle driver attempts to match his/her current velocity with his/her desired velocity. If the desired velocity is less than the current velocity and spacing between the following and lead vehicle is decreasing in length at an increasing rate, the following vehicle driver will decelerate to increase the spacing. Conversely, if the desired velocity is higher than the current velocity, the following vehicle driver will accelerate to achieve this velocity.

As well, the car-following model assumes that the alertness of the following vehicle driver affects the reaction time component of the acceleration/deceleration rate. As stated earlier, when a driver is alert, less time is needed to perceive and react to a given situation. It is assumed that the following vehicle driver's reaction time will be modified based on the relationship between the position of the lead vehicle with respect to the desired following distance and that of the current following distance defined by an alertness factor.

4.5 Stopping Sight Distance Analogy

In controlling braking and vehicle-following behaviour, the driver has to monitor visually, more or less continuously, the motion relative to the lead vehicle or obstacle, in relation to distance (Lee, 1976). An understanding of how a driver does this, what visual information is used, and what visual factors affect performance is clearly important for improving road safety.

A fundamental requirement is to ensure that driver's have sufficient road to perceive and react appropriately to a situation. In order to provide sufficient sight distance, engineers have used stopping sight distance models since the 1940's. A sight distance model is simply a mathematical representation of key driver and vehicle characteristics together with a procedure and parameters that relate to a given situation. They include the following key parameters: vehicle speed, perception-reaction time, braking or manoeuvring distance, driver eye height, and object height. They form two components: brake reaction distance (distance travelled from the instant of object detection to the instant the brakes are applied) and braking distance (distance travelled from the instant the brakes are applied to when the vehicle is decelerated to a stop).

Traditionally, sight distance depended on the pavement friction value but it varies depending on speed, tire, and pavement conditions. Deceleration can be simplified for practical usage, assuming that aerodynamic resistance, rolling resistance, moments of inertia, and braking efficiency is ignored (Mannering and Kilareski, 1998). However, then these parameters are not representative of the driving environment or of safe driving behaviour. Therefore, a relatively simple driver-performance-based model was developed (AASHTO, 2001). This model uses the following equation for stopping sight distance:

$$SSD = 0.278PV + \frac{V^2}{254\left(\frac{a}{g} \pm G\right)} \quad (4.1)$$

Where,

SSD: Stopping Sight Distance (m)

P: Perception-Reaction time (sec)

V: Initial velocity (km/h)

a: Driver deceleration (m/s²)

g: Acceleration due to gravity (m/s²)

G: Grade of slope (%)

The parameters for the above model have been validated with field data and represent safe driving behaviour. They also reflect driver, vehicle, and roadway limitations related to stopping sight distance situations that reflect the 90th percentile values.

Overall, stopping sight distance allows alert, competent drivers to come to a quick stop under ordinary circumstances. Therefore, characteristics of this equation and perception-reaction time along with kinematic constraint criteria discussed in the literature review are used in the development of the critical distance used in the rear-end collision warning algorithm.

4.6 Deceleration Rate Criterion

One way to conceptualize the “kinematic constraints” algorithm is that as the situation becomes more severe the deceleration rate required for avoiding a collision increases. Brown et al. (2000) reviewed previous work to verify that realistic deceleration for an emergency braking situation ranges between 0.4g and 0.85g. Their analysis revealed a mean of 0.5g, a mean maximum of 0.75g and an overall maximum of 0.86g. If a deceleration rate is defined as the average rate of the period of time starting with the initial brake depression, then clearly 0.85g is an excessively large value. Even 0.75g would likely be too large for an average deceleration rate if the goal of the algorithm were collision avoidance. However, Burgett et al. (1998) proposed an algorithm adopting a specified deceleration of 0.75g. Using this rate, Lee et al (2002a,b) observed early and late criteria using a driving simulator. The late criterion (0.75g) resulted in 50 percent reduction in collisions. This was then compared to an early criterion (0.4g), which resulted in an 80 percent reduction in collisions. Although the early collision warning resulted in fewer collisions than the late collision warning, in an on-road functioning system, the earlier warning would result in a greater number of nuisance alerts. This trade-off between reducing collisions and reducing nuisance alerts is an important consideration for selecting a deceleration criterion.

The 1999 CAMP project selected the deceleration rate criterion based on the results of an experiment that they conducted. The experiment was conducted

on a test-track and instructed participants to “brake with hard braking intensity or pressure” at the latest possible moment in response to a lead vehicle. In this study, the average required decelerations varied between 0.15g and 0.45g, with larger required decelerations for greater lead vehicle deceleration rates and faster initial following vehicle speeds. Average decelerations followed a similar pattern as the required decelerations, but were greater, with averages varying between 0.18g and 0.54g. The average peak deceleration values varied between 0.75g and 0.9g. This study provides a good indication of the absolute magnitudes of deceleration in a real vehicle, as opposed to a driving simulator. In particular, it demonstrated that in the most severe condition, the average actual deceleration rate of participants was 0.55g. This provides a useful indication of how hard drivers are likely to brake in an emergency rear-end collision situation on dry pavement.

The NHTSA algorithm that was developed for the ACAS FOT program adopted a fixed deceleration rate of 0.55g. The combination of a 0.55g deceleration rate and a 1.5 second reaction time was selected based on a Monte Carlo simulation that compared false alarm and correct positive rates across different parameter values. Therefore, it would seem that a simple but effective solution would be to adopt a fixed deceleration rate criterion across all kinematic conditions.

4.7 Rear-End Collision Warning System Methodology

In developing the warning system algorithm, it is essential to identify the independent functions that need to be performed and to separate each function into unique steps or components. Each step is logical rather than computational in context. This property reflects the fact that traffic flow is largely the outcome of driver decisions, which are logical in context. As the vehicles are processed, they transition from one state to the next. Figure 4.5 illustrates the general methodology used in developing the rear-end collision warning system.

Figure 4.5: Warning System Logic

The collision warning algorithm is linked to the car-following algorithm through “IF...ELSE” subroutines. When the following vehicle falls within the critical distance, the warning algorithm initiates the warning process advising of a potential collision with the vehicle ahead. Bayesian statistical decision theory will be applied, wherein prior and posterior information will be used to test any improvement with the warning system. Along side, the probabilities will be utilized to judge the threat assessment. This will determine the type of warning and when it should be given to the driver.

4.7.1 Warning Initiation

The time of the warning is based on the assumption that it will take the following driver a certain reaction time to apply the brakes and decelerate at a certain constant rate. Conditions surrounding driver alertness are used to execute this portion of the algorithm.

The time at which the following vehicle’s brakes are applied is the summation of the warning time and the reaction time. At all times, the following distance—spacing (S), headway (h), and the relative velocity (dS/dt) have the following relationships that are based on the kinematics of motion (Burgett et al., 1998; Klee, 2003):

$$S = X_L - X_F \quad (4.2)$$

$$h = (X_L - X_F)/V_F \quad (4.3)$$

$$dS/dt = V_L - V_F \quad (4.4)$$

The pattern of relative motion between the two vehicles is completely determined by the initial conditions at the onset of braking by the lead vehicle, at the time $t = 0$.

There have been three cases defined that may result in a rear-end collision. These are as follows (Lee, 1976):

1. Closing at a constant or accelerating rate. This is where a driver is cruising along and comes upon a slower vehicle, which is either cruising, decelerating, or is stopped.
2. Closing at an inadequate decelerating rate. This is where a driver is cruising along and comes upon a slower vehicle, and brakes inadequately behind a slower vehicle.
3. Receding at a decelerating rate. This is where a driver is cruising along and comes upon a faster vehicle, which is decelerating or when accelerating towards a faster vehicle, which is decelerating, cruising or accelerating less.

However, in evaluating these expressions in relation to the time-to-collision (TTC) or time-headway, the lead vehicle information (i.e. velocity, acceleration/ deceleration) may not be known discretely. Therefore, determining an expression for the critical distance in relation to the TTC is being considered that does not include the lead vehicle information but instead uses other measurable quantities related to the following vehicle movements. This is discussed in the following section.

4.7.2 Critical Distance Computation

By taking the concept of stopping sight distance and the knowledge of kinematics of motion, a critical distance can be calculated. This critical distance is formulated by dividing it into three parts: these being the final spacing between vehicles, the distance travelled during the following vehicle's response time, plus the distance travelled while stopping at constant maximum deceleration. This can be expressed as follows:

The final spacing of vehicles:

$$X_1 = S_F \quad (4.5)$$

Distance travelled during the driver's reaction time:

$$X_2 = V_0 t_x \quad (4.6)$$

Distance travelled during following vehicle braking:

$$(V_2^2 - V_1^2) = 2ax \quad (4.7)$$

$$X_3 = \frac{V^2}{2a} \quad (4.8)$$

In combining these, they form the following equation:

$$d_C = S_F + t_X V + \frac{V^2}{2a} \quad (4.9)$$

However, the analysis should also include incidences of braking on an incline/decline like that defined by AASHTO (2001). The gradient can be measured using a Global Positioning System (GPS) that uses satellites, receivers and software to allow for exact geographic position. While hooked up to the vehicle, calculations can yield the x and y positions of the vehicle but also the height above mean sea level (z-direction) (Fraczek, 2003). Any change in the z-direction can be determined through geometry and stored in a GPS receiver for further analysis. When a substantial change is observed over time, this gradient would be inputted into the critical distance calculation, increasing or decreasing the required length. Therefore, the third component of equation (4.9) is modified to result in the following critical distance equation used in the warning algorithm.

$$d_C = S_F + t_x V + \frac{V^2}{2g \left(\frac{a}{g} \pm G \right)} \quad (4.10)$$

Where,

d_C : Critical distance (m)

S_F : Safe stopped distance between vehicles (assume 2m, but may be altered by the user)

t_x : Perception-reaction time of following vehicle driver (sec)

V : Velocity of following vehicle (m/s)

a : Deceleration of following vehicle (m/s²)

g : Acceleration due to gravity (9.81m/s²)

G : Grade in slope (%)

In using this analysis, the principles of vehicle dynamics contribute to the development of the warning algorithm. This approach to warning implementation avoids the difficult problem of estimating the lead vehicle deceleration using instead other measurable quantities from the following vehicle. It determines an adequate stopping distance to avoid a collision based on current conditions.

CHAPTER 5: SYSTEM DESIGN AND INTEGRATION

5.1 Methodology

The system design identifies the potential for a rear-end collision using following distance estimations determined in the car-following model. In addition, with the integration of the Bayesian statistical decision theory framework, the system determines the optimal timing (i.e. immediately or wait) for the warning and suitable type of alert message (e.g., no alert message, amber alert, red alert). Figure 5.1 illustrates these major functions of the rear-end collision warning system.

Figure 5.1: Major Functions of the Rear-End Collision Warning System

The following sections outline the major component of the application of the Bayesian statistical decision theory framework. A description of the theorem and the methodological approach taken for this thesis is given.

5.2 Framework Development

5.2.1 Introduction

Bayesian methods are attracting increasing attention because they provide a principled approach that can account for real-life experiences by combining the predicted information with observed data to estimate probable outcomes that avoid erroneous approximations (Brownstone, 2000; Microsoft, 2003). Bayesian analysis is also inductive and in practice, this is true when observing driver behaviour. There is uncertainty in driver behaviour and until one can predict this behaviour with confidence, an estimate of the probable behaviour may be used, which can be obtained through probabilities. This type of analysis can account for the occurrence of a random event rather than always using the same pattern recognition to determine the outcome.

For this application, continual examinations of driver alertness conditions are required to inform and update the rear-end collision warning algorithm. By applying Bayesian techniques, driver alertness can be updated based on observations of driver actions. This analysis of the driver's ability to perceive

an imminent collision ahead enables a human element to be added to the technology-based system.

The Bayesian approach to decision making asserts that it is better for the analyst to use all the knowledge that can be brought to a problem. Therefore, as a starting point, probabilities should be estimated for all chance events. Given these estimates, the Bayesian methodology enables the calculation of gains/utilities, which is used to select a course of action based on the maximum expected gain.

Since probabilities are to be used, which can be assigned subjectively, there is a need to refine these estimates as well as possible. Bayes' theorem is a mechanism for revising prior estimates of probabilities on the basis of new information. That is, the use of the Bayes' theorem enables the mathematical operation to revise probabilities. In philosophical and analytical terms, the Bayesian approach, if used, leads to the commitment to on-going information acquisition or learning process. Therefore, the analyst is in a position to acquire information that can be used to improve estimate of the future states.

Appendix B provides a brief description of the Bayesian statistical decision theory. In the following section, basic variables and selected formulations are presented.

5.2.2 Bayesian Analysis

The principles of statistical decision theory enable a decision maker (or a decision system) to identify the optimal course of action in situations when the outcomes are not known with certainty. Decision making under uncertainty is a specialized subject, invoking probabilistic states of nature and gains/payoffs that have to be defined for the applicable actions and states of nature combinations. Bayesian analysis is a fundamental statistical approach that allows one to assess the probability of an event given prior information. It adds much flexibility to decision analysis owing to the ability to update probabilities as a result of new information.

The elements of this approach are as follows (Raiffa and Schlaifer, 1968):

1. A set of alternative terminal actions wherein for this work they are the types of possible alerts that may be given to the driver (A_0 (no alert), A_A (amber alert), A_R (red alert)).
2. The possible states of nature that have a possibility of occurring at the time that the selection of an action is being made. The distance between the following and the leading vehicle defines these: critical distance (d_c), 1.25 times the critical distance ($d_{1.25c}$), and 1.5 times the critical distance ($d_{1.5c}$).
3. The set of alternative information acquisition tactical measures (called experiments in the jargons of the decision theory) in order to obtain more

information about the actual state of nature prior to the selection of an act. These being an early warning (e_0) or a wait and observe (i.e., suspended warning) (e_w).

4. The set of outcomes/results of the information acquisition tactic (i.e., experimental outcomes) that relates to the options of driver alerts. These are the possible reading on the states of driving condition (no new information (r_0), critical distance reading (r_c), 1.25 times the critical distance reading ($r_{1.25c}$), and 1.5 times the critical distance reading ($r_{1.5c}$)).
5. The gain $G(e, r, A, d)$ represent the value for the course of action defined by e, r, A, d combinations. In accordance with the principles of utility theory, it is assumed that the gains $G(e, r)$ and $G(A, d)$ are additive. In this research, a $G(A, d)$ matrix is used and $G(e, r)$ is a variable that is internal to the computation process since there is no cost associated with and the (e, r) combination. For further information on gain, please see Appendix B.

The objective is to find the best course of action. This involves determining the expected value of information using the probability distributions over the possible states of driving.

However, to determine this value of information, a joint probability $P(d, r|e)$ must be assigned to the joint distribution of d, r , for each state of driving. This

requires the reliability of each possible outcome to be defined in order to predict the true state of driving. In doing so, four other probability measures are determined. These are: prior probability $\{P'(d)\}$, conditional probability $\{P(r/d,e)\}$, marginal probability $\{P(r/e)\}$, and posterior probability $\{P''(d/r,e)\}$.

Given the prior probability and the conditional probability, the marginal probability can be computed as follows:

$$P(r | e) = \sum P'(d) P(r | d, e) \quad (5.1)$$

The posterior probability is computed using Bayes's theorem:

$$P''(d | r, e) = \frac{P'(d) P(r | d, e)}{\sum P'(d) P(r | d, e)} \quad (5.2)$$

The assumption is that each warning time (e_o, e_w) can be characterized by a conditional probability of $P(r/d,e)$, such that the relationship between the prior and posterior probabilities is as given by Bayes Theorem.

In order to find the optimal course of action, an extension of this analysis is required. This involves determining a relative value of gain with each combination (i.e. $G(e, r, A, d)$). For each combination, an optimal act (maximum value) and its associated gain are found.

$$G^*(r,e) = \text{Max}_A G^*(A,r,e) \quad (5.3)$$

For each e , the expected gain can be found:

$$G^*(e) = \sum_r [P(r|e) G^*(r,e)] \quad (5.4)$$

$$G^*(e^*) = \text{Max}_A G^*(e) \quad (5.5)$$

If the optimal experiment turns out to be any result other than the 'null' one, it could be of much interest to know the value of information acquired. This involves determining the increase in the optimal gain which would result for each r if the prior choice of terminal action A , was altered after posterior information was attained.

This involves taking the optimal alternative under posterior information (A_r) and subtracting it from the optimal alternative under prior probabilities (A'). Or,

$$V_t^*(e) = \sum_r P(r|e) [\sum_d P''(d|r,e) G_t(A_r,d) - \sum_d P''(d|r,e) G_t(A',d)] \quad (5.6)$$

This equation (5.6) is a combination of equations (B.15) and (B.16) presented in Appendix B. The subscript " t ", refers to the terminal gain in terms of the optimal course of action. For further details, please refer to Appendix B.

5.3 Bayesian Framework Methodology

The methodology combines information on following distance and driver alertness level (*AL*) using Bayesian statistical decision theory. The inputs to the framework are obtained from the simulation environment based on real conditions. The framework methodology used is described in section 5.3.2.

5.3.1 Warning Distances

In examining current collision warning systems, early and late warnings may be given to the driver. Cases have been identified (NHTSA, 2002; Ervin et al., 2003; Mobileye, 2004) and they define a late warning between 0.7-1 second headway and an early warning between 2-2.5 second headway. However, choosing a late warning at less than one second headway is very dangerous since this may leave little room for a successful stop at high speeds, especially if no early warning is given to the driver. Therefore, using this as a basis, a sufficient range was chosen to enable a degree of warning types to the driver, but still enable safe stopping distances. Figure 5.2 illustrates the comparison used to develop sufficient ranges for the Bayesian statistical decision theory.

Figure 5.2: Early and Late Warning Analogy

As illustrated above, the critical distance (d_c) measured by the rear-end collision warning algorithm falls in the range of time headway of 1 to 2 seconds. This critical distance is that outlined by many Ministries of Transportation as that which drivers should follow to ensure safety. However, for a distracted driver, this distance is insufficient. Therefore, an early warning that gives an adequate length to allow a driver to respond and react was chosen as 1.5 times that of the critical distance. At lower velocities, it yields results similar to a 2-2.5 second headway, but increases as speed increases allowing extra room for driver manoeuvring.

5.3.2 Framework Description

The framework methodology is illustrated in Figure 5.3. The driving condition (i.e. obtained from the car-following model) is used to initiate the process. To begin, a check is made on current following distance and if this distance is less than or equal to 1.5 times critical distance, the algorithm launches the Bayesian statistical decision theory. In essence, this is deemed as the stage for early warning but if the user wishes, may change this to suit his/her own investigations. A late warning is given when the driver reaches the critical distances.

The spacing between the following and the leading vehicles defines the states of driving. Three driving states are defined for this analysis: d_c (critical distance, i.e. late warning), $d_{1.25c}$, and $d_{1.5c}$ (1.25 and 1.5 times the critical distance, i.e. early warnings) that correspond to when the warning signal may be given to the distracted driver. However, the user has the ability to modify these values to determine other potential results for different situations.

Figure 5.3: Bayesian Framework Methodology

Following this, the prior probabilities are obtained and are expressed as follows: $P'(d_c)$, $P'(d_{1.25c})$, $P'(d_{1.5c})$, wherein they must sum to 1.0. In theory, prior probabilities are estimated from past measurements or presupposed by experience. For the purposes of this framework as used, in the simulation testbed, they can either be assigned subjectively by the designer or based on driver's vigilance. In the latter case, for initialization, an alertness variable (AL) is used that is based on the relationship between current and desired movements of the following vehicle. Each iteration that follows uses the posterior probabilities in relation to the current following distance to update the prior probabilities.

Driver reliability of perceiving the current driving condition is expressed as the conditional probability ($P(r/d,e)$). That is, given a distance state ' d ', the probability that the driver will perceive it accordingly (i.e. ' r '). If the driver is fully attentive and has excellent perception-reaction characteristics, the driver reliability would be very high. If the driver is distracted or inattentive, and/or has relatively poor perception-reaction attributes, driver reliability would be low.

In order to characterize this, information concerning the driver's alertness and monitoring characteristics is incorporated. In recent years, eye tracking systems have been made to measure eye glance behaviour and determine the percentage of time a test subject looks forward while driving and performing

tasks in a simulator (Smith and Zhang, 2004; Harbluk et al, 2002; Smith et al, 2000). Hence, to show a change in forward glances for this analysis, driver monitoring is done using a random number generated from a normal distribution whose mean reflects the variable AL . This establishes a greater likelihood that glance behaviour and alertness are similar but does not rule out the possibility of a driver performing a random act changing their level of alertness. To determine the alertness level, it is defined by comparing current and desired movements of the following vehicle. This information is updated after every iteration, changing the conditional probabilities to reflect the current condition.

Corresponding to the prior and conditional probabilities are the possible readings on the state of driving condition. These are used to determine the marginal probabilities used within the calculation of posterior probabilities:

- r_0 (no new reading, if e_0 is selected),
- r_c (corresponds to d_c),
- $r_{1.25c}$ (corresponds to $d_{1.25c}$),
- $r_{1.5c}$ (corresponds to $d_{1.5c}$).

Following this, a check is made on the current condition of the warning. The warning can have one of the following three levels:

- A_0 (no alert issued),
- A_A (amber alert—higher than normal deceleration to avoid a collision),

- A_R (red alert—emergency deceleration to avoid a collision).

In addition to the types of warnings given, they may be issued now (e_o) or later (e_w) after acquiring conditional probabilities and determining posterior probabilities. The posterior probabilities (i.e., revised probability on the basis of obtaining further information before issuing a warning) are calculated by using the Bayes' Theorem (see equation 5.2).

A Gain (G) Matrix is added to determine the value or gain of providing a driver warning. It is added to determine the optimal alert. This gain matrix is based on the statistical analysis of results obtained from Lee et al. (2000) on a study conducted to test the avoidance of collisions using early and late warnings. This study, conducted in a driving simulator, indicated only 8.8% of the rear-end collision potential situation scenarios ended in a collision when an early warning was given to the driver. When a late warning was given, this increased to 22.5% of the simulations resulting in a collision. These percentages were normalized to the base case scenario in the study to yield the values used for the Gain Matrix. However, the user may also subjectively assign these using different values and scales.

Now, the framework can be solved. For each (e, r) combination, the optimal action " A " and its associated gain can be found (see equation (5.3)). The highest expected gains are calculated based on posterior probabilities. The

result is $G^*(r,e)$. From these, one can find the expected gain for each e , $G^*(e)$, by using equation (5.4).

The next step is to find the optimal e^* by comparing the results (see equation (5.5)). In the case where e_w is identified as the optimal option for the information acquisition strategy, the value of information can be found by using Equation (5.6). This information, if positive, is a confirmation that e_w is the choice. An example application of the framework within the simulation environment is presented in Chapter 8.

5.4 Algorithm and Framework Integration

The rear-end collision warning algorithm and the Bayesian statistical decision theory framework, as mentioned previously, are integrated into the car-following model. This has been accomplished through SIMULINK blocks and subsystems. Current following vehicle conditions (V_f , a , T_d) from the car-following model are used as input into mathematical blocks (+, -, ×, ÷) to calculate the critical distance. The result of these blocks is compared to the car-following model output of the current following distance using an “If...else” subsystem. If the result of the comparison indicates that the current following distance is greater than 1.5 times the critical distance, then the rear-end collision warning algorithm stops here and loops back into the car-following model to execute the next time step.

However, if the comparison indicates that the current following distance is less than 1.5 times the critical distance, then the Bayesian statistical decision theory framework initiates. At this point, another subsystem links to a MATLAB function (coding) that runs the Bayesian statistical decision theory framework. The result from the framework, as described earlier, is the type of warning and when to give it to the driver. This information is outputted (looped) back into the car-following model to control the response of the following vehicle before the next time step is executed. Figure 5.4, below, illustrates this integration process through a flow diagram.

Figure 5.4: Flow Diagram of Integration Process

If the Bayesian statistical decision theory framework is not utilized within the rear-end collision warning algorithm, a direct comparison of following distance and critical distance is used to indicate the type of warning that should be given to the driver. Here, a separate subsystem is linked to the car-following model before this portion is executed and results are looped back into the following vehicle portion of the model. Details of this process are given in the next chapter.

CHAPTER 6: DEVELOPMENT OF THE SIMULATION ENVIRONMENT AND ITS VALIDATION

6.1 Introduction

As stated previously, a rear-end collision warning algorithm and a car-following model are being incorporated into a simulation environment. This simulation environment provides an ideal setting from which controlled, repeatable evaluation of system performance, and sensitivity to alternative configurations, scenarios and error sources can be performed without having to deal with real-time vehicle implementation considerations. This also allows for both qualitative and quantitative analysis that will indicate the accuracy of the system.

6.2 SIMULINK Model

In this section, the development of the SIMULINK blocks to formulate a mathematical model suitable for describing the characteristics of a driver/vehicle attempting to follow a lead vehicle in a single lane of traffic is shown.

As defined earlier, kinematics of motion are used to calculate the current position, velocity, and acceleration. Within SIMULINK, predefined functions are used to obtain these parameters. These are defined below:

$$\left. \begin{aligned}
 a(t) &= c \\
 V(t) &= \int a(t)dt = \int cdt = ct + V_0 \\
 x(t) &= \int V(t)dt = \int (ct + V_0)dt = \frac{1}{2}ct^2 + V_0t + x_0
 \end{aligned} \right\} \quad (6.1)$$

The system comprises of lead and following vehicles and is driven by the velocity of the lead vehicle (V_L). The outputs include the position of the lead and following vehicles, the velocity and acceleration of the following vehicle $\{X_n, X_{n+1}, V_F, a_F\}$ as well as the following distance (S), headway (h), and relative velocity (rate of change in spacing) (dS/dt). These relate directly to the combination of lead and following vehicle movements, as defined earlier.

Standard practice is to assume an equation for the acceleration of the following vehicle in response to certain stimuli. In this case, the following vehicle driver's desired and current velocities as well as his/her cognition and reaction time are incorporated, resulting in the following:

$$a_F(t+T) = f(V_F(t), V_d(t), T_d(t)) \quad (6.2)$$

The following vehicle velocities will influence the following distances, which are also dependent on the lead vehicle movement, while the reaction time will be a function of the driver alertness level. The acceleration response is delayed by an amount T , which represents the vehicle response time.

6.2.1 Fundamental Aspects of the Car-Following Model

Within the simulation environment, the velocity profile of the lead vehicle is defined externally, while the acceleration/deceleration rate, velocity, and spacing profiles for the following vehicle are determined internally using the car-following model. As illustrated in Figures 6.1a and 6.1b, variables of the model interact with each other through feedback.

An initialization file called 'cfparams.m' (Appendix C) is used to store the system parameters referenced in a number of the blocks. The use of user interface enables automatic assignment of these factors and parameters. The main SIMULINK blocks and their functions are described briefly here.

1. 'Clock': Outputs the simulation time variable " t " for the 'Look-Up Table' block.
2. 'Look-Up Table': Linearly interpolates between specified data points to generate the lead car's velocity profile.
3. 'MATLAB Function': Passes the inputs ' $clock$ ', ' V_F ', ' V_L ', ' $X_n - X_{n+1}$ ', ' X_n ', ' X_{n+1} ', ' $warning$ ', and ' T_d ' to the MATLAB function 'acc2.m' (Appendix C), which computes the vehicle acceleration response.
 - a. ' $clock$ ' indicates the time step required for internal calculations

- b. ' V_F ' and ' V_L ' represent the following and lead vehicle velocities respectively
 - c. ' X_n ' and ' X_{n+1} ' represent the position of the lead and following vehicle respectively, while the difference yields the current headway.
 - d. '*Warning*' is passed when the system determines if the warning should be visible to the driver of the following vehicle
 - e. ' T_d ' represents the calculated reaction time
4. 'Acceleration Limiter': Sets limits for minimum and maximum vehicle acceleration defined in the 'Look-Up Table'.
 5. 'Transport Delay': Delays vehicle acceleration by T_r , the driver/vehicle reaction time. The acceleration is passed to calculate one factor of the critical following distance.
 6. 'Limited Integrator': An integrator configured to limit vehicle velocity between a minimum and a maximum value. The velocity is passed to calculate another factor of the critical following distance.
 7. 'If Block/Else Subsystem': Logical block that passes the current positions of the lead and following vehicles but ensures the simulation only executes if

their difference is greater than zero. The simulation stops when a collision occurs.

8. 'MATLAB Function': used to determine the desired following distance of the following vehicle driver, passes the inputs ' V_L ' and ' V_F ' (defined above) to the MATLAB Function 'Sd.m' (Appendix C) that is then used within the reaction time calculations. Additional internal variables defined in cfparams.m are also used within the function.
9. 'MATLAB Function': used to determine the reaction time (T_d). It passes the inputs ' G ', ' G_d ', and 'Random Number' to the MATLAB Function 'Td.m' (Appendix C) that is then used within the critical following distance calculations. Additional internal variables defined in cfparams.m are also used within the function.
10. The critical following distance ($S_w = V_f^2/a_f + T_d V_f + S_F$) is calculated, multiplied by 1.5 then compared to the actual spacing. While velocity (V_f) and driver reaction time (T_d) are taken directly from the simulation, a_f is calculated using the following: $(2g((a/g) + Gradient))$. Vehicle deceleration (a) and gravity (g) are set through internal definitions assigned in cfparams.m and the gradient is generated through a 'Look-up Table'.

11. 'MATLAB Function': critical distance calculations are passed to 'Bayes3.m' (Appendix C) that implements the Bayesian statistical decision theory algorithm and outputs the type of warning to the 'acc2.m' function.
12. 'MATLAB Function': critical distance calculations are passed to 'noBayes.m' (Appendix C) that implements the warning algorithm without the application of the Bayesian statistical decision theory then outputs the type of warning to the 'acc2.m' function.
13. 'MATLAB Function': used to check if the actual spacing and critical calculation in the previous step are zero, passes the parameters to 'checkZero.m' (Appendix C) to ensure that future calculated values are defined.
14. 'Gain Blocks': Situated throughout the diagram as triangular blocks that convert system functions from meters to kilometres, hours to seconds, etc.

There are 11-output blocks that store the results in tabular format for further analysis. These contain data pertaining to the type of warning, the lead and following vehicle velocities and positions, following distance, headway, and acceleration of the following vehicle and the desired velocity and spacing. Further visual analysis may be done using SIMULINK while the algorithm is running.

Figure 6.1a: SIMULINK Diagram for Car-following Portion

In order to show that the Bayesian statistical decision theory framework will improve the design of a rear-end collision warning system, the simulation model is also run without the framework yielding an alert (early or late) based only on the critical distance. The set-up is similar to the existing system described previously. However, as stated in the last chapter, instead of executing the Bayesian MATLAB function, the logic passes to a separate subsystem and function and the result is then outputted to the following vehicle driver. This can be seen in Figure 6.1b.

6.2.2 Velocity and Acceleration/Deceleration Rate

As mentioned previously, it is assumed that the following vehicle driver does not have the required depth perception to accurately ascertain the instantaneous absolute velocity and/or acceleration/deceleration rate of the lead vehicle. Hence, the absolute velocity and/or acceleration of the lead vehicle is not required as input in determining the acceleration/deceleration of the following vehicle driver within the model.

It is assumed that for every time step (0.1sec) the following vehicle driver decides to adjust the difference, if any, between his/her current velocity and the maximum (desired) velocity at which he/she would like to be travelling. The desired velocity will be computed to represent the situations when relative velocity between the following and the lead vehicle is close to zero (steady

state) and those when relative velocity between the following and the lead vehicle is larger.

The desired velocity in steady state conditions at time t ($V_{dss}(t)$) is assumed to depend on the current distance between the following and the lead vehicle (S). The relationship between V_{dss} and S has been calibrated based on observed individual vehicle tracking data obtained from the SAVME database. Trajectory data for a sample of 151 vehicle pairs were extracted from the database. The samples of vehicles that were selected from the database are all travelling within headway of less than three seconds to ensure car-following behaviour. As well, each vehicle pair is travelling with relative velocities close to zero. Using stopping distance calculations and a maximum velocity obtained from the database, a headway of 3.0 seconds ensures adequate data to obtain a range of car-following distance values. The headway was calculated by dividing the spacing of two successive vehicles by the current velocity of the following vehicle. For each observed speed, the mean headway from all vehicles observed to travel at that speed was computed.

Both parametric and non-parametric regression analysis was conducted to determine an ideal relationship between these variables. While parametric regression analysis traces the dependence of the distribution of a response variable to see if it bears a particular relationship to one or more of the predictors, nonparametric regression analysis makes minimal assumptions

about the form of relationship between the average response and the predictors. In conducting the analysis, the goodness-of-fit in both cases gave good results. However, a non-linear parametric relationship yielded better results for each relationship. The parameter estimates that define the relationship between the velocities and observed mean spacing are shown in Figure 6.2.

Figure 6.2: Calibration of Desired Velocity and Spacing

To ensure a realistic behaviour for calibration and validation purposes, constraints are incorporated such that the desired speed in steady state must not be negative and not greater than the maximum speed of 60km/h, which corresponds to the associated posted speed limit (35mph) of the arterial road

of the SAVME database. Based on the observed data, it is assumed for spacing greater than 28m, the desired velocity is 60km/h, and for spacing less than 4m, the velocity is 0km/h (see equation 6.3). However, for other locations and roadway types, these can be extended to reflect their conditions.

$$V_{dss}(t) = \begin{cases} 60\text{km/h} & S(t) \geq 28\text{m} \\ 27.737\text{Ln}(S) - 33.178 & 4\text{m} < S(t) < 28\text{m} \\ 0 & S(t) \leq 4\text{m} \end{cases} \quad (6.3)$$

The spacing is obtained through the calculations defined earlier through the simulation procedures. The calibration indicates analogous results as compared to the results obtained from Mehmood (2003), using a similar SAVME database, where fairly aggressive car-following behaviour is observed at short spacing and less aggressive car-following behaviour at longer spacing.

The relationship between the variables defined above is based on average values and differs between individuals due to differences in the tendency for risk-taking, skills, and vehicle performance characteristics. However, for the purposes of this model, the behaviour is realistic in order to integrate the rear-end collision warning system algorithm.

It is assumed that a change in the relative velocity between the following and the lead vehicle affects the desired velocity in any state. However, it has been determined that when the rate of change in spacing between the lead and following vehicles is negative, a new relationship should be used to state the appropriate desired velocity. For an instant in time where the rate is negative (non-steady state conditions) and the headway is less than 3.0sec, mean values of following vehicle velocity and spacing were determined and plotted. Based on graphical analysis, non-steady state conditions occurred where acceleration/deceleration was steeper between the lead and following vehicles. Hence, the conditions for non-steady state are slightly different. Analogous processes were conducted to determine an estimate for the desired spacing.

$$V_d(t) = \begin{cases} 60\text{km/h} & S(t) \geq 24\text{m} \\ -0.0023(S)^3 + 0.1023(S)^2 + 1.1041(S) & 0\text{m} < S(t) < 26\text{m} \\ 0 & S(t) = 0\text{m} \end{cases} \quad (6.4)$$

6.2.3 Reaction time of Following Vehicle Driver

The majority of existing car-following models make a simplifying assumption that the perception-reaction behaviour of drivers is constant. Given the variation in estimate of the perception-reaction time of the following vehicle drivers in different situations, the car-following model incorporates the process by which the driver's reaction time can change. This is illustrated in equation

(6.5) where a base reaction time (RT) and an alertness level (AL) are multiplied with each other to yield the following vehicle driver's reaction time (T_d).

$$T_d = RT * AL \quad (6.5)$$

It is assumed that ' RT ' is equal to 1.5 seconds (as assumed by most algorithms) and is influenced by a driver's alertness level. From the response time, individual drivers can be assigned different response time based on their age, risk-taking propensity, and skills. This will introduce a stochastic process for selecting the response time of the driver of the following vehicle.

' AL ' is a unit-less quantity defined as the ratio of the current to desired following distance. For example, when the desired following distance is smaller than the actual spacing, the resulting ratio is greater than one, increasing the base reaction time indicating a lower driver alertness. However, if the current spacing were smaller than the desired, then this ratio would decrease the base reaction time required indicating higher driver alertness. This falls under the assumption that the driver becomes more alert with reduced reaction time when the following distance is within the following vehicle driver's desired range. The length of the desired following distance is a function of the current speed of the vehicle.

The ratios produced are able to vary the reaction time up to a value of 3 seconds and down to a value of 0.5 seconds. Based on the research, this range of values for the reaction time are appropriate since this range satisfies the extreme limits of a driver's perception-reaction time in different situations (anticipated/ unanticipated situations).

This alertness level is also used within the algorithm for the collision warning system. Since, the system requirements are to further avoid distractions and annoyances, as discussed previously; this value will also alter the Bayesian calculations to determine when a warning should be given to the driver.

6.2.4 Desired Following Distance

The length of the desired following distance is assumed to depend on the current velocity of the following vehicle. It is also assumed that the relationship between desired following distance and current velocity of the following vehicle would yield a similar relationship to that between desired velocity and current spacing. Hence, the calibration of this relationship was conducted based on the same sample used for the calibration of the relationship defined for the desired velocity.

The results are shown next for steady state conditions:

$$S(t) = \begin{cases} 3.376 \exp^{0.0354 V_{dss}} & V_{dss} \leq 60 \text{ km/h} \\ 28 \text{ m} & V_{dss} > 60 \text{ km/h} \end{cases} \quad (6.6)$$

and non-steady state conditions:

$$S(t) = \begin{cases} 0.0002(V_d)^3 - 0.0147(V_d)^2 + 0.7736(V_d) & V_d \leq 60 \text{ km/h} \\ 26 \text{ m} & V_d > 60 \text{ km/h} \end{cases} \quad (6.7)$$

The following section describes the car-following model validation process.

6.3 Car-Following Model Validation

The validation of the car-following model is conducted by comparing model estimates of the velocity and spacing for the following vehicle to those observed in the SAVME database. 30 samples of lead and following vehicle trajectories were selected travelling in the 'SBST' lane. The speed of the lead vehicle was used as input for the car-following model. The model was then used to estimate the behaviour of the following vehicle in response to the known behaviour of the lead vehicle. The estimated behaviour of the following vehicle driver in 30 simulations was compared qualitatively and quantitatively with the observed values in the SAVME database.

6.3.1 Qualitative Validation

Qualitative validation of the car-following model consists of comparing estimated behaviour of the following vehicle to the observed dynamic pattern. Appendix D illustrates the model results and the observed trajectories of the samples selected for the car-following model. The number of observations in each of the samples is not the same due to the different observed periods. Each observation is taken at 0.1-second intervals. For example if 100 observations are recorded it is equivalent to 10 seconds * 10 observations/second. The sample results indicate that the velocity and spacing profiles estimated by the car-following model follow those observed in the SAVME database.

6.3.2 Quantitative Validation

For each of the samples, the Root-Mean-Squared (*RMS*) Error associated with the prediction of speed and spacing of the following vehicle was estimated. Equations (6.8) and (6.9) are used to calculate these respectively. The *RMS Error* is the most commonly used measure of success of numeric prediction, and represents the square root of the mean-squared-error taken to give it the same dimensions as the predicted values themselves. This method exaggerates the prediction error (the difference between prediction and actual values) for a particular case. If this number is significantly greater than the mean absolute error, it signifies that there are cases in which the prediction

error is significantly greater than the average prediction error. For a perfect fit, the *RMS Error* equals 0.

$$RMS\ Error\ (V_F) = \sqrt{\frac{1}{n} \sum_{t=1}^n (V_F(t)_{sim} - V_F(t)_{obs})^2} \quad (6.8)$$

Where,

RMS Error (V_F) = Root-Mean-Squared Error associated with speed of the following vehicle (km/h)

N = Number of observations ($t = 1, 2, 3 \dots n$)

$V_F(t)_{sim}$ = Speed of the following vehicle estimated from the model at time t (km/h)

$V_F(t)_{obs}$ = Speed of the following vehicle observed in the SAVME database at time t (km/h)

$$RMS\ Error\ (S_F) = \sqrt{\frac{1}{n} \sum_{t=1}^n (S_F(t)_{sim} - S_F(t)_{obs})^2} \quad (6.9)$$

Where,

RMS Error (S_F) = Root-Mean-Squared Error associated with spacing of the following vehicle (m)

$S_F(t)_{sim}$ = Spacing of the following vehicle estimated from the model at time t (m)

$S_F(t)_{obs}$ = Spacing of the following vehicle observed in the SAVME database at time t (m)

The *RMS Error* associated with the estimation of speed and spacing of the following vehicle using 30 samples are given in Appendix D. The average observed speed and spacing of the following vehicle for each sample were also computed. The car-following model was tested over a wide range of observed speed. The average *RMS Error* associated with the estimation of the following vehicle speed and spacing of the 30 samples was found to be 1.25km/h and 1.26m, respectively.

A scatter plot of estimated and observed mean speed and mean spacing of the following vehicle was carried out for the 30 samples. The results are shown in Appendix E. They show the plot of estimated versus observed mean spacing and speed of the following vehicle. The results indicate an agreement between the estimated output from the model and the observed field data with a correlation coefficient (r) of 0.99 for speed and 0.98 for spacing. Based on the comparison between the car-following estimates and observed SAVME data, it is suggested that the model can closely reflect observed speed and spacing profiles.

6.4 User Interface Design

Figure 6.3 shows the user interface that integrates the car-following model and collision warning algorithm. The interface can be used to analyse the behaviour of a following vehicle and the resultant warnings under different driving scenarios. In the design of the user interface, the key objective is to make it user-friendly and to visualize as much important information about the simulation as possible. The algorithm called 'car_following.m' (Appendix C) is used to open the user-interface as shown in Figure 6.3.

The button called "INITIALIZE" (top left hand corner of Figure 6.3) allows the user to define inputs to the model. The inputs to the model include the lead vehicle's velocity profile, maximum/minimum acceleration/deceleration rates of the following vehicle, along with roadway characteristics such as speed limit and gradient. In response to these inputs, the car-following model estimates the acceleration/deceleration, velocity, and spacing profiles of the following vehicle.

Figure 6.3: Initialized User-Interface

Once the variables have been changed, the user selects the start button, which runs the simulation model. The simulation may also be paused or stopped at any point. The top area illustrates the movement of two vehicles along with the type of warning being given to the driver of the following vehicle. Below this area are four graphs showing the acceleration, velocity, position, and spacing between the lead and following vehicles during the simulation. Each is labelled to indicate clearly which vehicle is moving and how. Closer details can be obtained by clicking on each graph, which will allow the user to zoom into specific time steps. The results may also be saved in tabular form, for further analysis, by accessing the button labelled "SIMULATION".

6.5 Summary

In this chapter, a car-following model is developed to simulate the behaviour of the driver of the following vehicle travelling in a passenger car on a straight section of an arterial roadway. The proposed model is calibrated and validated using an observed individual vehicle-tracking database (SAVME). The structure of the proposed model can be used for any roadway type however; re-calibration of the model parameters would be required for a different road environment.

To ensure stability (i.e. localized behaviour of two vehicles), the model parameters were altered to determine their range of acceptability. This is an important determination with respect to understanding the applicability of the car following model. If the model is realistic, this range should be consistent with measured values of these parameters. Hence, the responses of the following vehicle to a fluctuation in the motion of the lead vehicle were observed. The acceleration and speed of the following vehicle, as well as the inter-vehicle spacing between the two vehicles were determined to be non-oscillatory or dampened.

Additionally, the performance of the proposed model was not assessed in comparison to existing car-following models using observed databases because the major purpose was not to develop a model better than the

existing models. The major purpose was to develop a car-following model for incorporating the rear-end collision warning algorithm and Bayesian statistical decision theory framework. The model considers different behavioural aspects of the drivers such as adjustment in response time and desired movements. The validation results suggest that the proposed car-following model yields speed and spacing profiles for vehicles that are consistent with those observed empirically.

CHAPTER 7: VALIDATION OF THE WARNING ALGORITHM AND BAYESIAN STATISTICAL DECISION THEORY FRAMEWORK

7.1 Parameter Validation and Sensitivity Analysis

The warning algorithm is set up to acquire information pertaining to following vehicle velocity and deceleration, driver reaction time, and a roadway gradient. The following vehicle velocity and roadway gradient are dynamically inputted based on current vehicle readings. However, unlike other kinematic constraint algorithms (Burgett, 1998; NHTSA, 2002) that assume a fixed value of 1.5 seconds for a driver reaction time, this algorithm allows it to vary from a minimum of 0.5 seconds up to a maximum of 3.0 seconds and is based on current driver alertness. This enables an increased variability in the warning distance, while the set limits reflect the conducted studies described previously in section 4.4.

On the other hand, from the previous work, this algorithm also assumes a maximum deceleration rate. This value is set at 6m/s^2 (0.6g) and is based on research discussed in section 4.6, but more specifically using statistical data based on a truncated Gaussian distribution (see Figure 7.1) with a mean of 6m/s^2 (0.6g), a standard deviation of 1m/s^2 (0.1g), a maximum of 8m/s^2 (0.82g), and a minimum of 3m/s^2 (0.3g).

Figure 7.1: Vehicle Braking Distribution (Najm and Smith, 2004)

However, since the rear-end collision warning algorithm pulls variables from the stopping sight distance outlined by AASHTO (2001), a closer look at their deceleration rate was considered. They suggest a deceleration rate of 3.4m/s^2 ($0.35g$) to be a comfortable deceleration within the driver's ability to stay within his/her lane and maintain steering control during braking manoeuvres on *wet* pavements. An emergency deceleration of 4.5m/s^2 ($0.46g$) can also be used (AASHTO, 2001). Therefore, as described later, different deceleration rates were used to determine the effectiveness of using 6m/s^2 as the maximum deceleration rate within the rear-end collision warning algorithm.

Sensitivity analysis is used to ascertain how a given system output depends upon the input parameters. This is an important method for checking the

quality of a given system, as well as a powerful tool for checking the robustness and reliability of the analysis. The warning algorithm is dependent on the current conditions of the following vehicle along with a check on the current following distance. Hence, Figure 7.2 presents the algorithm performance for the following vehicle from a stopped position to a speed of 100km/h. As expected, at higher velocities, the following vehicle requires a longer warning range to stop the vehicle before a collision will occur. The warning distance increases to 150m with a reaction time of 3 seconds at 100km/h.

Figure 7.2: Critical Distance Comparison

Along with this analysis, a comparison between the critical distances of this algorithm was contrasted against those of Burgett et al (1998) and NHTSA (2002) described in the literature that utilizes kinematic constraint analysis

(see Table 7.1). The length of each critical distance was compared as velocity changes.

Table 7.1: Critical Distances

	Critical Distance
Burgett et al. (1998)	$d = v^2/1.5g + vt + 2$
NHTSA (2002)	$d = v^2/1.1g + vt + 2$

The results, illustrated in Figure 7.3, indicate the effectiveness of the variability of the algorithm used for this analysis. Having a fixed value for the driver reaction time limits the range of critical distances. It also increases the likelihood that the value determined to be that of the critical distance might in fact not be sufficient to stop if the driver reaction time is longer. Given the variability in driver response, this may either result in nuisance warnings or increase the likelihood of a collision with the lead vehicle. It is also important to note that when such a system is installed in a vehicle, the information that is collected from sensors will have measurement noise. This will also add to the error associated in estimating a critical warning distance.

Figure 7.3: Comparison of Critical (Warning) Distances versus Velocity

7.2 Simulation Performance

This section presents the results of simulations of the performance of the warning algorithm. This is done for three scenarios representing the pre-collision conditions for many of the rear-end collisions that the system is meant to help prevent. For each scenario the potential for a collision depends on three basic factors: 1) current speed, 2) difference in speed, and 3) following distance. Hence, scenarios were designed to have a following vehicle respond to a lead vehicle moving at different rates. These are as follows:

1. Stopped lead vehicle – The following vehicle is driving at a constant velocity and encounters a stopped lead vehicle in its path of travel. According to the 2000 General Estimates System (GES) (Wassim et al.,

2003), 59.1% of rear-end collisions occur when a constant speed following vehicle collides into a stopped lead vehicle.

2. Slower lead vehicle – The following vehicle is driving at a constant velocity and encounters a lead vehicle driving at a constant but slower velocity in its lane of travel. The GES data indicate that 9.5% of rear-end collisions occur under these conditions.
3. Braking lead vehicle – The following and lead vehicles are driving at the same velocity, and then the lead vehicle suddenly brakes. According to the GES data, 26.5% of rear-end collisions occur when two vehicles are driving at the same velocity and the following vehicle strikes the lead vehicle as it is braking.

As stated previously, weather was not considered in this system. This was due to the fact that in unfavourable weather drivers tend to follow the conditions of the roadway (Smith and Zhang, 2004) and it is the roadway conditions themselves that will influence the occurrence of a rear-end collision and not the driver's distraction level. As well, almost all of the sensors (temperature and wiper gages) used on rear-end collision warning systems described in the literature have reduced operational range and accuracy in poor weather conditions, with laser sensors suffering particularly badly.

The objective of the simulations is to determine how well the rear-end collision warning algorithm (without the Bayesian statistical decision theory framework) performs in providing enough time for the driver to avoid a collision given set velocities, deceleration rates, and reaction times. Three different following vehicle velocities are used (100km/h, 80km/h, 60km/h) and the driver of the following vehicle is assumed inattentive and will not initiate braking until a set reaction time in seconds after a warning is issued.

Figure 7.4 shows the results of a single simulation run where the velocity is 100km/h, level terrain, and the deceleration rate is 6m/s^2 . Here, the warning is issued when the following vehicle is about 130 m from a stopped lead vehicle. In this particular case, the driver reaction time is 2.2 seconds. Thus, the driver did not begin braking until the range was around 67 m. The following vehicle comes to a full stop 7 m away from the lead vehicle.

Figure 7.4: Example Result of a Collision Avoided

A second simulation run, with a slower reaction time of 3.5 seconds, is shown in Figure 7.5. Here the driver begins braking too late (55 m from the lead vehicle) to avoid a collision. In this case, the collision occurs with a difference in velocity (ΔV) between the following and lead vehicles at the moment of impact of 11km/h. This occurrence of a collision results because of the range of allowable driver reaction time within the algorithm. However, if this warning algorithm is used in conjunction with the Bayesian statistical decision theory framework or based on early and late warnings, it is most likely this would have been given as a “late” warning to the driver. In all likelihood, an earlier warning would have been given before reaching the critical distance and appropriate action would have already started.

Figure 7.5: Example Result with a Collision

Simulations were repeated for different values of reaction time and for different values of deceleration. Figure 7.6 shows the ΔV , when a collision does occur, versus reaction time. The results indicate that for the algorithm (deceleration = 6 m/s^2), the driver has 3.5 seconds to react. However, any driver reaction times greater than 3 seconds have an increased probability of a collision due to the algorithm parameters, especially if the deceleration rate is lower than 6 m/s^2 . Nevertheless, as discussed in section 4.4, most driver reaction times are much less than 3 seconds.

Figure 7.6: ΔV versus Reaction Time

In each scenario, it is assumed that they are supported by the operation of collision warning system radar. But, it is possible, for example that the required warning at a distance of 130m for a vehicle driven at 100km/h would not be given because of the lead vehicle being outside the detection range of the collision radar, even when on a straight, flat road (Brunson et al., 2002). Therefore, each scenario was also conducted using a range of initial following distances (25m to 250m) to determine if there is still adequate distance for a following vehicle to come to a complete stop behind a lead vehicle (see Table 7.2). As expected, at higher velocities and shorter headways there is an increased risk in the possibility of a collision. At lower velocities and shorter headways, the risk decreases. Overall, the greatest risk occurs when the headway is less than 50m at velocities greater than 80km/h. Nevertheless,

collision warning radar ranges are good for headways of 120m (Brunson et al. 2000) allowing such circumstances to be avoided.

Table 7.2: ΔV Given Restricted Warning Ranges

Initial Following Vehicle Velocity	Reaction Time (sec)	Initial Following Distance (Radar Range)					
		250m	200m	150m	100m	50m	25m
		ΔV at Impact (km/h)					
100km/h	0.5	0	0	0	0	62	87
	1.5	0	0	0	12	90	100
	3.0	0	0	0	90	100	100
	4.5	0	0	0	100	100	100
80km/h	0.5	0	0	0	0	0	62
	1.5	0	0	0	0	57	80
	3.0	0	0	0	24	80	80
	4.5	0	0	0	80	80	80
60km/h	0.5	0	0	0	0	0	24k
	1.5	0	0	0	0	0	54
	3.0	0	0	0	0	24	60
	4.5	0	0	0	0	60	60
40km/h	0.5	0	0	0	0	0	0
	1.5	0	0	0	0	0	0
	3.0	0	0	0	0	0	40
	4.5	0	0	0	0	0	40

7.3 Bayesian Framework Sensitivity Analysis

To obtain an understanding of the logic of the framework, sensitivity analyses were conducted. They consist of the outcomes when the Gain Matrix is altered and when the prior and conditional probabilities are changed. This is

to ensure that the values obtained are reasonable and reliable and to illustrate the effectiveness of the Bayesian statistical decision theory framework.

a) Changes in the Gain Matrix

Within the framework, the value or gain of providing a driver warning is specified in the form of a Gain Matrix, shown in Table 7.3. However, the values can also be specified on a 0 to 1 scale or even a negative interval.

Table 7.3: Gain Matrix

System Action	d_c	$d_{1.25c}$	$d_{1.5c}$
A_0	0.2	0.5	1.0
A_A	0.5	1.0	0.2
A_R	1.0	0.5	0.2

In order to test the effect of changing $G(A_0, d_c)$, calculations were conducted using prior and conditional probabilities given in Table 7.4 and then repeated changing $G(A_0, d_c) = -1$. The results from the analysis indicated a drop in value of information $V_t^*(e)$ from 0.014 to 0. This implies the slightly higher penalty assigned to the case of "no alert message" even though there is a 0.2 probability of critical distance d_c . However, the optimal answers remain unchanged (i.e., e_w & A_A).

Table 7.4: Probabilities Used for Gain Matrix Sensitivity Analysis

Prior Probabilities	$P'(d_c) = 0.2$	$P'(d_{1.25c}) = 0.7$	$P'(d_{1.5c}) = 0.1$
	$P(r_c d_c) = 0.7$	$P(r_c d_{1.25c}) = 0.2$	$P(r_c d_{1.5c}) = 0.1$
Conditional Probabilities	$P(r_{1.25c} d_c) = 0.2$	$P(r_{1.25c} d_{1.25c}) = 0.7$	$P(r_{1.25c} d_{1.5c}) = 0.2$
	$P(r_{1.5c} d_c) = 0.1$	$P(r_{1.5c} d_{1.25c}) = 0.1$	$P(r_{1.5c} d_{1.5c}) = 0.7$
	1.0	1.0	1.0

The same was also carried out by using prior probabilities: $P'(d_c) = 0.3$, $P'(d_{1.25c}) = 0.4$, $P'(d_{1.5c}) = 0.3$ as well as $P'(d_c) = 0.7$, $P'(d_{1.25c}) = 0.2$, $P'(d_{1.5c}) = 0.1$. Similar results were attained, where there was a drop in the value of information (0.2 to 0.17 and 0.024 to 0 respectively) but the optimal answers remained unchanged.

b) Changes in Conditional Probabilities vs. Posterior Probabilities

Figure 7.7 presents eleven cases with different conditional and resulting posterior probabilities. The $P(r_c|d_c, e_w)$ could be viewed as an indicator of reliability of the driver to relatively estimate critical distance correctly. Therefore, $1 - P(r_c|d_c, e_w)$ may be regarded as an index of driver distraction or inattentiveness. The posterior probability $P''(d_c|r_c, e_w)$ presents driver's perception of unsafe distance. Now, as the figure suggests, as the reliability increases, the driver's perception of unsafe distance increases. Therefore, the driver has the incentive to take corrective action. On the other hand, as the driver distraction or inattentiveness increases, the

driver's perception of unsafe distance decreases. In this case, the driver's ability to take corrective action drops.

Figure 7.7: Conditional Probabilities vs. Posterior Probabilities (Constant Prior Probabilities)

c) Changes in Prior Probabilities

The sensitivity of the rear-end collision warning algorithm results to changes in prior probabilities can be seen in Table 7.5. Here, conditional probabilities and the Gain Matrix are held constant. The first set of conditional probabilities shows a rather low probability of critical distance between vehicles. In the second set, this probability is well pronounced. Since the conditional probabilities and other factors are held constant, a change in the results obtained from the framework reflects changes to prior probabilities.

Table 7.5: Sensitivity Analysis: Constant Conditional Probabilities

Prior Probabilities	Conditional Probabilities	Results
$P(d_c) = 0.3$ $P(d_{1.25c}) = 0.4$ $P(d_{1.5c}) = 0.3$	$P(r_c d_c, e_w) = 0.4$ $P(r_c d_{1.25c}, e_w) = 0.3$ $P(r_c d_{1.5c}, e_w) = 0.3$ $P(r_{1.25c} d_c, e_w) = 0.3$ $P(r_{1.25c} d_{1.25c}, e_w) = 0.4$ $P(r_{1.25c} d_{1.5c}, e_w) = 0.3$ $P(r_{1.5c} d_c, e_w) = 0.3$ $P(r_{1.5c} d_{1.25c}, e_w) = 0.3$ $P(r_{1.5c} d_{1.5c}, e_w) = 0.4$	e_w, A_A
$P(d_c) = 0.7$ $P(d_{1.25c}) = 0.2$ $P(d_{1.5c}) = 0.1$	$P(r_c d_c, e_w) = 0.4$ $P(r_c d_{1.25c}, e_w) = 0.3$ $P(r_c d_{1.5c}, e_w) = 0.3$ $P(r_{1.25c} d_c, e_w) = 0.3$ $P(r_{1.25c} d_{1.25c}, e_w) = 0.4$ $P(r_{1.25c} d_{1.5c}, e_w) = 0.3$ $P(r_{1.5c} d_c, e_w) = 0.3$ $P(r_{1.5c} d_{1.25c}, e_w) = 0.3$ $P(r_{1.5c} d_{1.5c}, e_w) = 0.4$	e_w, A_R

In the case of low prior probability of critical distance, the results show that there is nothing to be gained by waiting and making further observations before giving the amber alert. The e_0 answer may in part be due to the conditional probabilities used as well as the Gain Matrix structure. However, given the inputs, the system gives the answers shown.

In the second case, the prior probabilities are changed in such a way that the probability of critical distance increases. The results reflect this change.

Here, a red alert message is given to the driver without waiting for additional information acquisition.

d) Changes in Conditional Probabilities

The effect of changes in conditional probabilities can be seen in Table 7.6. Here, the prior probabilities and the Gain Matrix are held constant. In the first case, perfect reliability of driver's reading of distance is assumed. This means that the driver is fully attentive. Although the probability of critical distance is set at 0.7, the best course of action, as computed by the framework, is to wait and get more information and for now no alert message is warranted. It should be noted here that the assignment of a perfect conditional probability might be constrained by the designer. Therefore, this example is intended to test the limits of the system.

In the second case, the driver is not perfectly attentive and there is some distraction. Therefore, the best course of action is to wait for additional information and then give a red alert message to the driver.

Table 7.6: Sensitivity Analysis: Changes in Conditional Probabilities

Prior Probabilities	Conditional Probabilities	Results
$P(d_c) = 0.7$	$P(r_c d_c, e_w) = 1.0$	e_w, A_o
$P(d_{1.25c}) = 0.2$	$P(r_c d_{1.25c}, e_w) = 0.0$	
$P(d_{1.5c}) = 0.1$	$P(r_c d_{1.5c}, e_w) = 0.0$	
	$P(r_{1.25c} d_c, e_w) = 0.0$	
	$P(r_{1.25c} d_{1.25c}, e_w) = 1.0$	
	$P(r_{1.25c} d_{1.5c}, e_w) = 0.0$	
	$P(r_{1.5c} d_c, e_w) = 0.0$	
	$P(r_{1.5c} d_{1.25c}, e_w) = 0.0$	
	$P(r_{1.5c} d_{1.5c}, e_w) = 1.0$	
$P(d_c) = 0.7$	$P(r_c d_c, e_w) = 0.7$	e_w, A_R
$P(d_{1.25c}) = 0.2$	$P(r_c d_{1.25c}, e_w) = 0.2$	
$P(d_{1.5c}) = 0.1$	$P(r_c d_{1.5c}, e_w) = 0.1$	
	$P(r_{1.25c} d_c, e_w) = 0.2$	
	$P(r_{1.25c} d_{1.25c}, e_w) = 0.7$	
	$P(r_{1.25c} d_{1.5c}, e_w) = 0.2$	
	$P(r_{1.5c} d_c, e_w) = 0.1$	
	$P(r_{1.5c} d_{1.25c}, e_w) = 0.1$	
	$P(r_{1.5c} d_{1.5c}, e_w) = 0.7$	

7.4 Bayesian Framework Verification

This section describes the verification of the Bayesian statistical decision theory framework. Each scenario, described in section 7.2, was conducted without the use of the Bayesian statistical decision theory framework and then repeated with its insertion. Different probabilities indicating various degrees of driving alertness and reliability were used in the analysis (see Table 7.7).

Table 7.7: Comparison of the Results Using the Bayesian Statistical Decision Theory Framework

Prior Probabilities	Conditional Probabilities			% Match to Non-Bayesian Simulations	Favourable Results	
Shorter Headway	Alert	$P(r_c d_c)=0.7$ $P(r_{1.25c} d_c)=0.2$ $P(r_{1.5c} d_c)=0.1$ $Sum = 1.0$	$P(r_c d_{1.25c})=0.2$ $P(r_{1.25c} d_{1.25c})=0.7$ $P(r_{1.5c} d_{1.25c})=0.1$ $Sum = 1.0$	$P(r_c d_{1.5c})=0.1$ $P(r_{1.25c} d_{1.5c})=0.2$ $P(r_{1.5c} d_{1.5c})=0.7$ $Sum = 1.0$	1.2-2.5%	$e_w A_R$
$P'(d_c)=0.7,$ $P'(d_{1.25c})=0.2,$ $P'(d_{1.5c})=0.1$	Somewhat Alert	$P(r_c d_c)=0.5$ $P(r_{1.25c} d_c)=0.3$ $P(r_{1.5c} d_c)=0.2$ $Sum = 1.0$	$P(r_c d_{1.25c})=0.3$ $P(r_{1.25c} d_{1.25c})=0.5$ $P(r_{1.5c} d_{1.25c})=0.2$ $Sum = 1.0$	$P(r_c d_{1.5c})=0.2$ $P(r_{1.25c} d_{1.5c})=0.3$ $P(r_{1.5c} d_{1.5c})=0.5$ $Sum = 1.0$	2.7-3.7%	$e_w A_R$
	Distracted	$P(r_c d_c)=0.1$ $P(r_{1.25c} d_c)=0.2$ $P(r_{1.5c} d_c)=0.7$ $Sum = 1.0$	$P(r_c d_{1.25c})=0.2$ $P(r_{1.25c} d_{1.25c})=0.1$ $P(r_{1.5c} d_{1.25c})=0.7$ $Sum = 1.0$	$P(r_c d_{1.5c})=0.7$ $P(r_{1.25c} d_{1.5c})=0.2$ $P(r_{1.5c} d_{1.5c})=0.1$ $Sum = 1.0$	98.5-99.5%	$e_0 A_R$

Table 7.7: Comparison of the Results Using the Bayesian Statistical Decision Theory Framework (cont'd)

Prior Probabilities	Conditional Probabilities				% Match to Non-Bayesian Simulations	Favourable Results
Medium Headway	Alert	$P(r_c d_c)=0.7$ $P(r_{1.25c} d_c)=0.2$ $P(r_{1.5c} d_c)=0.1$ $Sum = 1.0$	$P(r_c d_{1.25c})=0.2$ $P(r_{1.25c} d_{1.25c})=0.7$ $P(r_{1.5c} d_{1.25c})=0.1$ $Sum = 1.0$	$P(r_c d_{1.5c})=0.1$ $P(r_{1.25c} d_{1.5c})=0.2$ $P(r_{1.5c} d_{1.5c})=0.7$ $Sum = 1.0$	1.6-2.6%	$e_w A_A$
$P'(d_c)=0.1,$ $P'(d_{1.25c})=0.7,$ $P'(d_{1.5c})=0.2$	Somewhat Alert	$P(r_c d_c)=0.5$ $P(r_{1.25c} d_c)=0.3$ $P(r_{1.5c} d_c)=0.2$ $Sum = 1.0$	$P(r_c d_{1.25c})=0.3$ $P(r_{1.25c} d_{1.25c})=0.5$ $P(r_{1.5c} d_{1.25c})=0.2$ $Sum = 1.0$	$P(r_c d_{1.5c})=0.2$ $P(r_{1.25c} d_{1.5c})=0.3$ $P(r_{1.5c} d_{1.5c})=0.5$ $Sum = 1.0$	2.3-3.2%	$e_w A_A$
	Distracted	$P(r_c d_c)=0.1$ $P(r_{1.25c} d_c)=0.2$ $P(r_{1.5c} d_c)=0.7$ $Sum = 1.0$	$P(r_c d_{1.25c})=0.2$ $P(r_{1.25c} d_{1.25c})=0.1$ $P(r_{1.5c} d_{1.25c})=0.7$ $Sum = 1.0$	$P(r_c d_{1.5c})=0.7$ $P(r_{1.25c} d_{1.5c})=0.2$ $P(r_{1.5c} d_{1.5c})=0.1$ $Sum = 1.0$	98.2-99.3%	$e_0 A_A$

Table 7.7: Comparison of the Results Using the Bayesian Statistical Decision Theory Framework (cont'd)

Prior Probabilities	Conditional Probabilities					% Match to Non-Bayesian Simulations	Favourable Results
Longer Headway	Alert	$P(r_c d_c)=0.7$ $P(r_{1.25c} d_c)=0.2$ $P(r_{1.5c} d_c)=0.1$ $Sum = 1.0$	$P(r_c d_{1.25c})=0.2$ $P(r_{1.25c} d_{1.25c})=0.7$ $P(r_{1.5c} d_{1.25c})=0.1$ $Sum = 1.0$	$P(r_c d_{1.5c})=0.1$ $P(r_{1.25c} d_{1.5c})=0.2$ $P(r_{1.5c} d_{1.5c})=0.7$ $Sum = 1.0$		2.1-2.7%	$e_w A_0$
$P'(d_c)=0.1,$ $P'(d_{1.25c})=0.2,$ $P'(d_{1.5c})=0.7$	Somewhat Alert	$P(r_c d_c)=0.5$ $P(r_{1.25c} d_c)=0.3$ $P(r_{1.5c} d_c)=0.2$ $Sum = 1.0$	$P(r_c d_{1.25c})=0.3$ $P(r_{1.25c} d_{1.25c})=0.5$ $P(r_{1.5c} d_{1.25c})=0.2$ $Sum = 1.0$	$P(r_c d_{1.5c})=0.2$ $P(r_{1.25c} d_{1.5c})=0.3$ $P(r_{1.5c} d_{1.5c})=0.5$ $Sum = 1.0$		2.2-2.8%	$e_w A_0$
	Distracted	$P(r_c d_c)=0.1$ $P(r_{1.25c} d_c)=0.2$ $P(r_{1.5c} d_c)=0.7$ $Sum = 1.0$	$P(r_c d_{1.25c})=0.2$ $P(r_{1.25c} d_{1.25c})=0.1$ $P(r_{1.5c} d_{1.25c})=0.7$ $Sum = 1.0$	$P(r_c d_{1.5c})=0.7$ $P(r_{1.25c} d_{1.5c})=0.2$ $P(r_{1.5c} d_{1.5c})=0.1$ $Sum = 1.0$		98.2-99.2%	$e_0 A_0$

Results show that when the probabilities specified a fair degree of alertness, there was a significant drop in the number of warnings given to the driver when the Bayesian statistical decision theory framework was utilized. As the probabilities were set to indicate a rather distracted driver, the results of using the framework resembled that where only the warning algorithm was used within the rear-end collision warning system. A third comparison of a moderately alert driver with normal reaction times yielded warnings that fell close to those of an alert driver when using the framework.

However, as shown in Table 7.7, in conducting each scenario, there was slight variability in the occurrence of the warning (shown as “% Match to Non-Bayesian Simulations”). This was due to the fact that the rear-end collision warning algorithm used in this thesis allows for a change in driver reaction time. For each simulation, the driver reaction time variable was altered from 0.5 seconds to 3 seconds. These in turn represented examples of an alert driver with slower reaction times or a distracted driver with quick reactions. For each case, the framework determined a slight inconsistency in the number of times the warning should be given to the driver. However, this mainly occurred at slower speeds (< 5 km/h) during the simulation and in the future the designer may automatically suppress these alert messages given at lower speeds, as performed on a few existing collision warning systems today (NHTSA, 2002).

Table 7.8: Comparison of the Results Using Other Collision Warning Algorithms and the Bayesian Statistical Decision Theory Framework

Prior Probabilities	Conditional Probabilities				% Match to Non-Bayesian Simulations		Favourable Results
					Burgett et al. (1998)	NHTSA (2002)	
Shorter Headway	Alert	$P(r_c/d_c)=0.7$ $P(r_{1.25c}/d_c)=0.2$ $P(r_{1.5c}/d_c)=0.1$ $Sum = 1.0$	$P(r_c/d_{1.25c})=0.2$ $P(r_{1.25c}/d_{1.25c})=0.7$ $P(r_{1.5c}/d_{1.25c})=0.1$ $Sum = 1.0$	$P(r_c/d_{1.5c})=0.1$ $P(r_{1.25c}/d_{1.5c})=0.2$ $P(r_{1.5c}/d_{1.5c})=0.7$ $Sum = 1.0$	1.8%	1.8%	$e_w A_R$
$P'(d_c)=0.7,$ $P'(d_{1.25c})=0.2,$ $P'(d_{1.5c})=0.1$	Somewhat Alert	$P(r_c/d_c)=0.5$ $P(r_{1.25c}/d_c)=0.3$ $P(r_{1.5c}/d_c)=0.2$ $Sum = 1.0$	$P(r_c/d_{1.25c})=0.3$ $P(r_{1.25c}/d_{1.25c})=0.5$ $P(r_{1.5c}/d_{1.25c})=0.2$ $Sum = 1.0$	$P(r_c/d_{1.5c})=0.2$ $P(r_{1.25c}/d_{1.5c})=0.3$ $P(r_{1.5c}/d_{1.5c})=0.5$ $Sum = 1.0$	2.8%	2.8%	$e_w A_R$
	Distracted	$P(r_c/d_c)=0.1$ $P(r_{1.25c}/d_c)=0.2$ $P(r_{1.5c}/d_c)=0.7$ $Sum = 1.0$	$P(r_c/d_{1.25c})=0.2$ $P(r_{1.25c}/d_{1.25c})=0.1$ $P(r_{1.5c}/d_{1.25c})=0.7$ $Sum = 1.0$	$P(r_c/d_{1.5c})=0.7$ $P(r_{1.25c}/d_{1.5c})=0.2$ $P(r_{1.5c}/d_{1.5c})=0.1$ $Sum = 1.0$	97.6%	97.8%	$e_0 A_R$

Table 7.8: Comparison of the Results Using Other Collision Warning Algorithms and the Bayesian Statistical Decision Theory Framework (cont'd)

Prior Probabilities	Conditional Probabilities				% Match to Non-Bayesian Simulations		Favourable Results
					Burgett et al. (1998)	NHTSA (2002)	
Medium Headway	Alert	$P(r_c/d_c)=0.7$ $P(r_{1.25c}/d_c)=0.2$ $P(r_{1.5c}/d_c)=0.1$ Sum = 1.0	$P(r_c/d_{1.25c})=0.2$ $P(r_{1.25c}/d_{1.25c})=0.7$ $P(r_{1.5c}/d_{1.25c})=0.1$ Sum = 1.0	$P(r_c/d_{1.5c})=0.1$ $P(r_{1.25c}/d_{1.5c})=0.2$ $P(r_{1.5c}/d_{1.5c})=0.7$ Sum = 1.0	2.2%	2.2%	$e_w A_A$
$P'(d_c)=0.1$, $P'(d_{1.25c})=0.7$, $P'(d_{1.5c})=0.2$	Somewhat Alert	$P(r_c/d_c)=0.5$ $P(r_{1.25c}/d_c)=0.3$ $P(r_{1.5c}/d_c)=0.2$ Sum = 1.0	$P(r_c/d_{1.25c})=0.3$ $P(r_{1.25c}/d_{1.25c})=0.5$ $P(r_{1.5c}/d_{1.25c})=0.2$ Sum = 1.0	$P(r_c/d_{1.5c})=0.2$ $P(r_{1.25c}/d_{1.5c})=0.3$ $P(r_{1.5c}/d_{1.5c})=0.5$ Sum = 1.0	3.2%	3.2%	$e_w A_A$
	Distracted	$P(r_c/d_c)=0.1$ $P(r_{1.25c}/d_c)=0.2$ $P(r_{1.5c}/d_c)=0.7$ Sum = 1.0	$P(r_c/d_{1.25c})=0.2$ $P(r_{1.25c}/d_{1.25c})=0.1$ $P(r_{1.5c}/d_{1.25c})=0.7$ Sum = 1.0	$P(r_c/d_{1.5c})=0.7$ $P(r_{1.25c}/d_{1.5c})=0.2$ $P(r_{1.5c}/d_{1.5c})=0.1$ Sum = 1.0	97.4%	97.5%	$e_0 A_A$

Table 7.8: Comparison of the Results Using Other Collision Warning Algorithms and the Bayesian Statistical Decision Theory Framework (cont'd)

Prior Probabilities	Conditional Probabilities					% Match to Non-Bayesian Simulations		Favourable Results
						Burgett et al. (1998)	NHTSA (2002)	
Longer Headway	Alert	$P(r_c d_c)=0.7$ $P(r_{1.25c} d_c)=0.2$ $P(r_{1.5c} d_c)=0.1$ $Sum = 1.0$	$P(r_c d_{1.25c})=0.2$ $P(r_{1.25c} d_{1.25c})=0.7$ $P(r_{1.5c} d_{1.25c})=0.1$ $Sum = 1.0$	$P(r_c d_{1.5c})=0.1$ $P(r_{1.25c} d_{1.5c})=0.2$ $P(r_{1.5c} d_{1.5c})=0.7$ $Sum = 1.0$		2.3%	2.3%	$e_w A_0$
$P'(d_c)=0.1$, $P'(d_{1.25c})=0.2$, $P'(d_{1.5c})=0.7$	Somewhat Alert	$P(r_c d_c)=0.5$ $P(r_{1.25c} d_c)=0.3$ $P(r_{1.5c} d_c)=0.2$ $Sum = 1.0$	$P(r_c d_{1.25c})=0.3$ $P(r_{1.25c} d_{1.25c})=0.5$ $P(r_{1.5c} d_{1.25c})=0.2$ $Sum = 1.0$	$P(r_c d_{1.5c})=0.2$ $P(r_{1.25c} d_{1.5c})=0.3$ $P(r_{1.5c} d_{1.5c})=0.5$ $Sum = 1.0$		2.6%	2.7%	$e_w A_0$
	Distracted	$P(r_c d_c)=0.1$ $P(r_{1.25c} d_c)=0.2$ $P(r_{1.5c} d_c)=0.7$ $Sum = 1.0$	$P(r_c d_{1.25c})=0.2$ $P(r_{1.25c} d_{1.25c})=0.1$ $P(r_{1.5c} d_{1.25c})=0.7$ $Sum = 1.0$	$P(r_c d_{1.5c})=0.7$ $P(r_{1.25c} d_{1.5c})=0.2$ $P(r_{1.5c} d_{1.5c})=0.1$ $Sum = 1.0$		97.4%	97.4%	$e_0 A_0$

To determine the flexibility of the Bayesian statistical decision theory framework, the algorithms listed in Table 7.1 were also used and results were compared (see Table 7.8). The Bayesian statistical decision theory framework proved to be effective. The rear-end collision warning remained suppressed when the driver had a fair degree of alertness and only indicated a warning should be given when the driver became increasingly distracted.

The only difference in the comparison was reflected in the “% Match to Non-Bayesian Simulations”. Since these two rear-end collision warning algorithms have a constant driver reaction time (1.5 seconds), the results could only represent alert to distracted drivers using an average driver reaction time. Hence, the results remained constant and did not differ as seen in the previous table. As well, since each warning algorithm yields distinct critical distances, the Bayesian statistical decision theory framework initiated at different times leading to minute changes in the “% Match to Non-Bayesian Simulations” column of the table. However, the results from both rear-end collision warning algorithms fall within those obtained from the algorithm used in this thesis.

Overall, the results indicate that while the driver’s alertness was high and the reaction time was good, the driver did not receive a warning and the signal remained as a “wait for more information”. However, as the driver became distracted, the signal changed indicating that a warning should be given to the

driver. As well, the framework uses different types of warnings that correspond to different driver following distances. As following distance changes from longer to shorter distances, the warning type also changes from a “green” (no alert) to “amber” and then to a “red” alert for the driver.

By in large, the Bayesian statistical decision theory framework yields promising results when implemented into a rear-end collision warning algorithm. It effectively and logically determines an appropriate action to take based on the knowledge of perceived driver perception. It useful in determining different degrees of warning types for a driver as recommended by the ISO and as long as the Bayesian statistical decision theory framework determines the driver to have an increased probability of being alert, the warning is not given to the driver. Hence, it aids in determining when and what type of warning should be given to the driver.

CHAPTER 8: EXAMPLE APPLICATION

This chapter contains results from simulations conducted within SIMULINK. Firstly, a detailed example of the results obtained from one time step during a simulation process of the model is shown. This includes the results obtained from the Bayesian statistical decision theory framework. Following this, examples of additional time steps are given. This illustrates the process of updating the system and replacing prior conditions to ensure reliable information is used to determine an effective warning.

8.1 Example Time Step

For this example application, the driver has a tendency to follow at relatively safe headways. Hence, the following prior probabilities are used. $P'(d_c) = 0.2$, $P'(d_{1.25c}) = 0.7$, $P'(d_{1.5c}) = 0.1$. The sum of priors is equal to 1.0.

As discussed previously, driver reliability of perceiving the driving condition is expressed as the conditional probability $(P(r/d,e))$. For this example, they have been assigned subjectively and are shown in Table 8.1. These imply a fair degree of driver alertness.

Table 8.1: Conditional Probabilities

	$P(r d, e)$		
	d_c	$d_{1.25c}$	$d_{1.5c}$
r_c	0.7	0.2	0.1
$r_{1.25c}$	0.2	0.7	0.2
$r_{1.5}$	0.1	0.1	0.7
Sum	1.0	1.0	1.0

The marginal probabilities are calculated as shown in the Table 8.2.

Table 8.2: Marginal Probabilities

$P(r e)$			
r_c	$r_{1.25c}$	$r_{1.5}$	Sum
0.29	0.55	0.16	1.00

Note: $P(r_0|e_0) = 1.0$

The posterior probabilities (i.e., revised probability on the basis of obtaining further information before issuing a driver warning) are calculated as shown below in Table 8.3:

Table 8.3: Posterior Probabilities

	$P''(d r,e)$			Sum
	d_c	$d_{1.25c}$	$d_{1.5c}$	
r_c	0.48	0.48	0.04	1.00
$r_{1.25c}$	0.07	0.89	0.04	1.00
$r_{1.5c}$	0.12	0.44	0.44	1.00

Note: For (e_0, r_0) combination, $P''(d|r, e_0) = P'(d)$

The framework can now be solved by integrating the Gain Matrix (given in Table 7.3). For each (e, r) combination, the optimal action "A" and its associated gain are found.

$$G^*(r, e) = \text{Max}_A G^*(A, r, e) \quad (8.1)$$

The highest expected gains are calculated next. These are shown below.

$$\left. \begin{aligned} G^*(A_R, r_c, e_w) &= (0.48)(1.0) + (0.48)(0.5) + (0.04)(0.2) = 0.728 \\ G^*(A_A, r_{1.25c}, e_w) &= (0.07)(0.5) + (0.89)(1.0) + (0.04)(0.2) = 0.933 \\ G^*(A_0, r_{1.5c}, e_w) &= (0.12)(0.2) + (0.44)(0.5) + (0.44)(1.0) = 0.684 \end{aligned} \right\} \quad (8.2)$$

The optimal alternative under posterior information is A_A .

The expected value of information for waiting on giving the warning can be found by comparing e_0 with e_w . This is done by calculating the optimal alternative, A' , using the prior probabilities. This value is used to yield:

$$A' = \text{Max}_A P'(d).G(A, d) \quad (8.3)$$

$$\left. \begin{aligned} G^*(A_0) &= (0.2)(0.2) + (0.7)(0.5) + (0.1)(1.0) = 0.490 \\ G^*(A_A) &= (0.2)(0.5) + (0.7)(1.0) + (0.1)(0.2) = 0.820 \\ G^*(A_R) &= (0.2)(1.0) + (0.7)(0.5) + (0.1)(0.2) = 0.570 \end{aligned} \right\} \quad (8.4)$$

The optimal alternative under prior probability distribution is A_A .

Next, to determine when to issue the warning, the value of information (V_i^*) is calculated based on data shown in Table 8.4.

Table 8.4: Data for the Calculation of Value of Information

	r_c	$r_{1.25c}$	$r_{1.5c}$
A_r	0.728	0.933	0.684
A'	0.82	0.82	0.82
$A_r - A'$	-0.092	0.113	-0.136
$P(r e)$	0.29	0.55	0.16

$$V_i^*(e) = (0.29)(-0.092) + (0.55)(0.113) + (0.16)(-0.136) = 0.014 \quad (8.5)$$

Since $V_i^*(e)$ is ≥ 0 , the optimal strategy is e_w (i.e. wait for additional information before giving the warning message) and the optimal warning message is A_A (amber alert). Figure 8.1 illustrates the above process (i.e. preposterior analysis) in a decision tree. Please note that the numbers shown by ' r ' and ' d ' are probabilities and those shown next to ' A ' are expected gains.

Figure 8.1: Example Application Decision Tree: Preposterior Analysis

8.2 Further Time Steps

The role of the collision warning system in a dynamic driving environment is portrayed in Figure 8.2. This illustration shows situations following the conditions given in the previous example. No warning is given to the driver at time t_1 based on updates of probabilities that indicate safer following distances ($P'(d_{1.25c} = 0.7)$) and a fair degree of alertness ($P(r_{1.25c}/d_{1.25c}, e_w) = 0.7$). Following this, the driver maintains this distance, but his/her alertness level decreases ($P_2(r_{1.25c}/d_{1.25c}, e_w) = 0.2$). Therefore, at time t_2 , the collision warning system provides the following optimal answers: e_0 & A_A to the driver. However, at time t_3 , the driver has not responded. The probability of the distance between the lead and following vehicles to be equal to the critical distance has now increased to 0.7 (i.e. $P_3'(d_c) = 0.7$) and the following vehicle's driver maintains a lower level of alertness ($P_3(r_c/d_c, e_w) = 0.2$). Therefore, the collision warning system provides the following optimal answers: e_0 & A_R . At time t_4 , the driver has taken appropriate action (i.e. decelerated) and the following distance has increased to more than $1.5d_c$. Therefore, the collision warning system takes on the driving monitoring role (i.e., measuring changes in relative velocities and distances) and no warning is required to be issued to the driver.

Figure 8.2: Real Time Operation of the Collision Warning System

CHAPTER 9: CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

This chapter provides a summary of the contributions made by this thesis and presents suggestions for future research work.

9.1 Thesis Contributions

This thesis presents a framework for improving the design of a rear-end collision warning system. It is unique in considering Bayesian statistical decision theory to integrate the driver into the analysis. Overall, the following can be stated:

- The use of the Bayesian statistical decision theory framework enhances the science and engineering of active safety measures. Rather than separately studying human factors and applying it to the design of collision warning algorithms, the framework directly integrates the driver into decision process to help avoid potential rear-end collisions.
- With reference to rear-end collisions; the framework is realistic since it treats the stochastic nature of headway. It can take into account driver distraction or inattentiveness that affect perception-reaction times and the ability to perceive a potential collision.

- The application of the Bayesian statistical decision theory framework enables driver reliability to be continually updated leading to warnings given the driver only when distracted, lessening the occurrence of nuisance warnings.
- The Bayesian statistical decision theory framework can aid in the design of the collision warning system since it can answer two most important questions: what type of warning message to provide and when to provide it using driver reliability rather than just using critical distance or time headway criterion.
- The Bayesian statistical decision theory framework is generic in that it may be used with differing critical distance algorithms, hence it can potentially be of immediate use in designing a rear-end collision warning system.

9.2 Conclusions

A framework for modelling driver behaviour in a car-following situation is developed along with an algorithm to determine when and what type of warning should be given to a driver in a rear-end collision potential scenario. Both the car-following model and warning algorithm have been validated and the system has been verified to function as a user-friendly model. Overall, the results of the system are encouraging and demonstrate the effectiveness of the modelling framework.

The car-following model is tested qualitatively as well as quantitatively. The speed and spacing profiles for the following vehicles estimated from the model were compared to observed speed and spacing profiles. These comparisons suggest that the car-following model yields realistic results in replicating the behaviour of the following vehicle driver from an observed vehicle-tracking database.

The rear-end collision warning algorithm issues alerts designed to allow a driver to stop or approach no closer than a designated distance (2 m) behind a stopped or slower moving vehicle in its lane of travel. It is variable on the current following vehicle velocity and unlike other rear-end collision warning systems, considers driver alertness within the design. The algorithm was subjected to analysis and verification to determine its performance. Overall, it helps to reduce the probability of a collision when issued to a distracted driver.

The Bayesian statistical decision theory framework for the rear-end collision warning algorithm was verified by running different scenarios to compare results. Sensitivity analyses were done to determine influential variables within the algorithm. As well, comparisons were conducted to view the results when the Bayesian statistical decision theory framework was applied to multiple algorithms. Results show that by integrating the Bayesian statistical decision theory, the warning can be suppressed if the driver is alert. The

simulations conducted indicate an improvement as to when and what type of warning should be given to the driver indicating that human factors are important in the development of a rear-end collision warning system.

Overall, the safety concerns about driver distraction caused by in-vehicle components of intelligent transportation systems or other causes can be addressed by a collision warning system, if such a system is designed by accommodating driver behaviour.

9.3 Future Research Directions

Further development and refinement of the system can be carried out along the following directions.

- Data is only as good as how well it was collected, recorded, and analysed. It would be beneficial in the future to be involved in the analysis process to prevent the limitations that resulted in the data used for this thesis. In the future, each roadway lane should be identified along with sections outside the view of an intersection. Comparisons of driver behaviour in different weather conditions should also be conducted to determine any discrepancies to compare more precisely the driver behaviour that may occur between vehicles within the SAVME database.

- Although the rear-end collision warning algorithm and Bayesian statistical decision theory framework were verified within a simulation environment, it would be beneficial to incorporate these into a driving simulator to obtain driver behaviour and responses in different cognitive scenarios. For example, different gain utility functions could be applied based on specific behavioural factors and relationships to the conditional probabilities enabling the framework to recognize influential patterns.
- While on-line within a vehicle or a driving simulator, this framework can be implemented using sensors and cameras. Data obtained from a camera monitoring system and sensors that use different radar or ultrasonic techniques to measure relative distances and velocities would be used as input into the system. Observations of driver eye and head movement would be used to indicate the degree of alertness for the conditional probabilities in relation to current driving following distance. Initial prior probabilities would be subjectively inputted or based on a researcher's expertise to be used in calculating the posterior probabilities.
- However, effective integration of technology within the Bayesian statistical decision theory framework may present some limitations. This is in regards to the ability to successfully estimate the probabilities from the present applicable technology (e.g. cameras, sensors) to observe the following vehicle driver. Hence, further research would be necessary to

determine methods of evaluating the technology to find feasible alternatives to integrate sensors and feedback systems to measure probabilities within the framework.

- The variable *AL*, for this research is only meant to indicate the alertness requirement and not alertness itself. Since, alertness is difficult to assess in a simulation environment, it would be beneficial to further develop this variable before adding it into a driving simulator. With the addition of sensors and cameras, the alertness requirement may be expanded to link facial expressions with desired movements to yield an effective alertness factor.
- In regards to the following vehicle reaction time, it is assumed to vary over time. This may be simply conducted within a simulation environment, however technology may limit this criterion within the rear-end collision warning system. Since technology has yet to read minds, which are inevitably linked to a driver's alertness level to estimate reaction time, it would be beneficial to determine how effectively driver reaction time could be estimated while driving in car-following situations. Currently, sensors and cameras are used to monitor a driver's movement, hence this may lead to further development to estimate a driver's reaction over time based on perceived driver conditions.

- Sensor fusion could be incorporated using a Bayesian algorithm to enable an integrated approach to collect, distil, and display information to the user within a driving simulator or vehicle. The core of the system would take inputs from the sensor suite, process the information and provide an appropriate driver warning response.

- The Bayesian statistical decision theory framework follows the recommendations by the ISO that at least two separate warnings be available to the driver. However, to ensure future integration that considers the human-computer interface, studies should be conducted to determine the ideal modalities (audible, visual and/or tactile) to be used alongside the rear-end collision warning algorithm and Bayesian statistical decision theory framework.

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Appendix A: Sample of SAVME Database

BATCH ID	FOLL VEH	TARG VEH	TIME	FILTERX	FILTER Y	FOR VEL	LAT VEL	YAW RATE	ACCEL	TYPE	RANGE	ALPHA	RANGE RATE
1	79	82	1226.2	201.718	50.938	-22.334	2.339	17.061	-5.836	SBST	23.303	-175.998	-16.315
1	79	82	1226.3	200.044	49.287	-22.328	2.781	20.627	-4.977	SBST	24.484	178.043	-20.116
1	79	82	1226.1	203.374	52.541	-22.15	1.933	13.814	-6.53	SBST	22.043	-171.692	-14.552
1	79	82	1226.4	198.367	47.593	-22.077	3.246	24.469	-3.899	SBST	25.489	170.484	-22.682
1	79	82	1226	205.002	54.095	-21.815	1.564	10.9	-7.097	SBST	20.873	-169.183	-16.126
1	79	82	1226.5	196.698	45.867	-21.508	3.716	28.485	-2.52	SBST	26.372	161.337	-21.963
1	79	82	1225.9	206.593	55.602	-21.358	1.234	8.32	-7.558	SBST	19.936	-168.409	-19.938
1	79	82	1225.8	208.136	57.064	-20.804	0.947	6.07	-7.938	SBST	19.305	-168.881	-24.496
1	79	82	1226.6	195.048	44.131	-20.51	4.159	32.489	-0.705	SBST	27.351	150.786	-16.634
1	79	82	1225.7	209.626	58.486	-20.155	0.702	4.14	-8.217	SBST	19.034	-169.849	-28.266
1	79	82	1225.6	211.061	59.867	-19.416	0.498	2.504	-8.386	SBST	19.082	-170.475	-28.306
1	79	82	1226.2	195.801	28.398	-19.047	0.212	-37.51	-44.459	SBST	51.318	-96.604	-6.853
1	79	82	1226.7	193.423	42.421	-18.964	4.516	36.13	1.693	SBST	28.741	139.267	-6.554
1	79	82	1225.5	212.44	61.204	-18.605	0.329	1.127	-8.453	SBST	19.293	-170.099	-23.586
1	79	82	1225.4	213.76	62.49	-17.741	0.191	-0.035	-8.434	SBST	19.491	-168.418	-15.63
1	79	82	1226.1	197.965	31.172	-17.694	-0.907	-38.739	-51.169	SBST	49.687	-88.313	-3.978
1	79	82	1226.3	193.202	25.779	-17.015	0.503	-37.747	-32.877	SBST	52.836	-105.895	-11.179
1	79	82	1225.3	215.023	63.722	-16.826	0.077	-1.022	-8.318	SBST	19.53	-165.343	-6.178
1	79	82	1226.8	191.823	40.783	-16.741	4.692	38.75	4.846	SBST	30.805	127.511	6.963
1	79	82	1225.2	216.227	64.891	-15.893	-0.02	-1.863	-8.146	SBST	19.282	-160.775	2.814
1	79	82	1225.1	217.371	65.997	-14.949	-0.101	-2.589	-7.919	SBST	18.681	-154.636	9.854
1	79	82	1225	218.454	67.038	-14.008	-0.172	-3.204	-7.654	SBST	17.749	-147.025	14.322
1	79	82	1226.9	190.238	39.278	-13.738	4.569	39.356	8.907	SBST	33.616	116.264	21.602
1	79	82	1226	199.623	33.927	-13.122	-2.55	-49.213	-52.338	SBST	48.2	-82.075	-3.801
1	79	82	1224.9	219.474	68.012	-13.081	-0.231	-3.728	-7.365	SBST	16.612	-138.293	16.078
1	79	82	1224.8	220.428	68.922	-12.185	-0.281	-4.161	-7.078	SBST	15.432	-129.016	15.417
1	79	82	1224.7	221.316	69.768	-11.325	-0.322	-4.511	-6.804	SBST	14.305	-120.019	13.24
1	79	82	1224.6	222.138	70.554	-10.511	-0.356	-4.762	-6.56	SBST	13.257	-112.186	10.312
1	79	82	1227	188.636	37.975	-9.888	4.038	36.73	13.998	SBST	37.025	106.039	35.042
1	79	82	1224.5	222.895	71.284	-9.737	-0.379	-4.925	-6.343	SBST	12.309	-106.041	7.193
1	79	82	1223.4	238.851	72.196	-9.485	-2.168	-21.951	-3.952	SBST	27.702	44.741	7.875
1	79	82	1223.3	239.734	72.892	-9.407	-1.93	-19.613	-4.557	SBST	26.913	40.67	8.086
1	79	82	1223.5	237.94	71.477	-9.378	-2.387	-24.15	-3.14	SBST	28.478	49.275	7.423
1	79	82	1223.2	240.583	73.559	-9.172	-1.689	-17.279	-4.978	SBST	26.125	37.065	8.089
1	79	82	1223.6	237.01	70.739	-9.058	-2.571	-26.066	-2.093	SBST	29.228	54.268	6.704
1	79	82	1224.4	223.589	71.964	-8.994	-0.39	-4.955	-6.14	SBST	11.509	-101.553	4.156
1	79	82	1223.1	241.396	74.193	-8.8	-1.456	-15.036	-5.223	SBST	25.348	33.91	7.912
1	79	82	1223.7	236.076	69.985	-8.473	-2.693	-27.463	-0.744	SBST	29.93	59.697	5.689
1	79	82	1223	242.167	74.788	-8.322	-1.24	-12.94	-5.315	SBST	24.592	31.189	7.591
1	79	82	1224.3	224.224	72.6	-8.271	-0.393	-4.904	-5.936	SBST	10.889	-98.296	1.303

Appendix B: Bayesian Statistical Decision Theory

B.1 Variables and Basic Formulation

The Bayesian approach is a common sense approach. It quantifies the tradeoffs between various decisions using probabilities and costs that accompany such decisions. It involves a set of techniques for orderly expression and revision of opinions with due regard for internal consistency among their various aspects and for the data. When a Bayesian procedure violates your intuition, reflection is likely to show the procedure to have been incorrectly applied.

The basic data required to solve a decision problem as defined by Raiffa and Schlaifer (1968):

1. A set of alternative terminal actions: $A_0, A_1, \dots \in a$, wherein the decision is to be made by selecting a single act A , from some domain a of potential acts.
2. The possible states of nature: $d_1, d_2, \dots \in D$. The states of nature are probabilistic. That is, probabilities have to be assigned to each state in the set of states of nature. The decision maker or the decision making system believes that the selection of an alternative A from the set a is dependent upon the actual state of nature which is not known with certainty.

3. The set of alternative information acquisition tactical measures (termed experiments in the decision theory terminology): $e_1, e_2, \dots \in E$. The decision maker (or system) may elect to use one tactical measure from the set E for the purpose of obtaining more information about the actual state of nature prior to selection of an action.
4. The sample space or the set of outcomes or results of information acquisition tactic: $r_0, r_1, \dots \in R$. For each tactic (experiment) e , there is a space of possible outcomes for that tactic. The space of possible outcomes R , is defined to encompass any outcome of any e in E .
5. The values or utilities or gains $G(e, r, A, d)$ represent the decision maker's (or system's) preferences for all e, r, A, d combinations. The sequence of a course of action is as follows. The decision maker (or system) selects an information acquisition tactic e , observes a result r , selects a particular A , and then a particular state of nature, d , occurs. The space of all possible combinations of (e, r, A, d) is $E \times R \times A \times D$.

It is assumed that a single valued gain function (utility function) is defined over this space, expressed as $G(e, r, A, d)$, and that in accordance with the principles of utility theory, the gains $G(e, r)$ and $G(A, d)$ are additive.

The gain functions should reflect the value structure of the decision maker. For this reason, it is not possible to generalize the characteristics of these functions. In some cases, the diminishing marginal gain property may be applicable. In other cases, step functions (with well defined threshold values) may be relevant. Owing to much diversity in the estimation of gain functions, it is left up to the designer or analyst to estimate or acquire the applicable gain values.

6. The probability distributions of $P(d)$ and $P(r/d, e)$, portray a joint probability measure $P(d, r/e)$ over the space $D \times R$ for each information acquisition tactic. The $P(d)$ represents the decision maker's judgment about the relative likelihood of values of d , and $P(r/d, e)$ characterizes each information acquisition tactic. It is the probability that the outcome r , will be observed if the experiment e , is performed and d is the value of the state of nature variable. Further information on probability measures is provided in section B.2.

B.2 Probability Measures

For solving the Bayesian decision problem, it is required to assign, either directly or indirectly, a joint probability $P(d, r/e)$ to the joint distribution of d, r , over the space $D \times R$ for each information acquisition tactic e . This implies that the decision maker (or the decision system) should define the reliability of

each possible information outcome r in predicting the true state of nature d , for each information acquisition tactic e .

From the joint probability measure, four other probability measures are defined, as described below.

1. The prior measure $P'(d)$ on the states of nature that the decision system would assign to d prior to observing the outcome r of tactic (experiment) e .
2. The conditional measure $P(r/d, e)$ on the space R – the probability that the outcome r will be observed if the tactic (experiment) e is performed and d is the true value of the state of nature.
3. The marginal measure $P(r/e)$ on the space R for all d or the probability of observing outcome r from tactic e . It is computed using the following:

$$P(r/e) = \sum P'(d) P(r/d, e) \quad (B.1)$$

4. The posterior measure $P''(d/r, e)$ on the space D , wherein this is the likelihood of different states d , given r and e . The decision maker assigns this probability measure to the space D after knowing the outcome r of the experiment e . It is computed using the following equation:

$$P''(d/r, e) = \frac{P'(d) P(r/d, e)}{\sum P'(d) P(r/d, e)} \quad (B.2)$$

The Bayesian philosophy is that each e can be characterized by a conditional probability distribution $P(r/d, e)$, such that the relationship between the prior and posterior distribution is given as Bayesian Theorem.

B.3 Solving the Decision Problem

A decision tree is an appropriate representation of a decision problem. See Figure B.1. Any sequence of (e, r, A, d) can be represented through this tree. The desirability of a course of action (i.e. the sequence) is represented by the gain $G(e, r, A, d)$.

The decision maker (or decision system) wishes to choose an information acquisition tactic (including the “null” tactic) and therefore has to evaluate all the possible sequences or courses of action.

The method of analysis that involves the evaluation of alternative courses of action to determine the most desirable information acquisition tactic e is known as preposterior analysis. In this thesis, to determine if additional information should be obtained, the preposterior analysis mode is used. However, if it is intended to find the optimal e and the purpose of the analysis is to compare results of analysis under prior and posterior probability distribution, then the mode of analysis can be termed posterior analysis.

Figure B.1: Decision Tree

B.3.1 Preposterior Analysis

The decision maker is interested in choosing an experiment that includes the 'null' experiment and therefore has to evaluate all the possible courses of action. The mode of analysis concerned with the evaluation of alternative courses of action for the purpose of determining the most appropriate is known as preposterior analysis.

Preposterior analysis is a sequence of operations required in making a decision. These are as follows:

1. The likelihood of different states of nature, d , is expressed in the form of prior probability distribution $P'(d)$.
2. The conditional probability characteristics $P(r|d, e)$, is determined for each experiment.
3. Compute the marginal measures $P(r|e)$, for each experiment as given above, where the null is equal to one.
4. Compute the posterior probability distribution $P''(d|r, e)$ for each combination as defined above wherein for the null, the prior and posterior probabilities are equal.
5. Find for each combination of e, r, A, d , its relative gain: $G(e, r, A, d)$.

6. Compute the expected gain for each action A , for each combination (e, r) combination:

$$G^*(A, r, e) = \sum_d P''(d|r, e) G(e, r, A, d) \quad (B.3)$$

However, for the prior branch, where no new experiment is acquired,

$$G^*(A, r_0, e_0) = \sum_d P''(d|r_0, e_0) G(e_0, r_0, A, d) \quad (B.4)$$

7. For each (e, r) combination, determine the optimal action and its associated gain:

$$G^*(r, e) = \text{Max}_A G^*(A, r, e) \quad (B.5)$$

8. For each information acquisition tactic e , compute the expected return:

$$G^*(e) = \sum_r P(r|e) G^*(r, e) \quad (B.6)$$

9. The optimal experiment e^* is that e for which $G^*(e)$ is a maximum. That is:

$$G^*(e^*) = \text{Max}_e G^*(e) \quad (B.7)$$

B.3.2 Value of Information

In the preposterior analysis, if the optimal experiment is determined not to be the 'null', the value of information acquired from sampling should be calculated. In this analysis, an assumption is made that the sampling and terminal utilities are additive. Then, instead of calculating the gain $G(e, r)$ for each possible experimental outcome, it is possible to compute the increase in gain, which would result for each r if the prior choice of a terminal action after learning about it was changed. A weighted average of these gain increases can be calculated as follows:

$$G(e, r, A, d) = G_s(e, r) + G_t(A, d) \quad (\text{B.8})$$

Where subscript s is used for "sampling" gain and t is used for "terminal" gain.

The expected gain of an information acquisition tactic e as defined by equation 6 is:

$$G^*(e) = \sum_r P(r|e) G^*(r, e) = \sum_r P(r|e) [\text{Max}_A \sum P''(d|r, e) G(e, r, A, d)] \quad (\text{B.9})$$

From equations 8 and 9:

$$\begin{aligned} G^*(e) &= \sum_r P(r|e) [\text{Max}_A \sum P''(d|z, e) [G_s(e, r) + G_t(A, d)]] \\ &= \sum_r P(r|e) [G_s(e, r) + \text{Max}_A \sum P''(d|r, e) G_t(A, d)] \end{aligned} \quad (\text{B.10})$$

Or

$$G_s^*(e) = \sum P(r|e) G_s(e, r) \quad (B.11)$$

and

$$G_t^*(e) = \sum P(r|e) [\text{Max}_A \sum P''(d|r, e) G_t(A, d)] \quad (B.12)$$

If A' is an optimal action under prior conditions of d :

$$\sum_d P'(d) G_t(A', d) \geq \sum_d P'(d) G_t(A, d) \quad (B.13)$$

and letting A_r be defined as the optimal action under the posterior conditions of d is determined by:

$$\sum_d P''(d|r, e) G_t(A_r, d) = \text{Max}_A \sum_d P''(d|r, e) G_t(A, d) \quad (B.14)$$

If instead of choosing the action A directly (i.e., based on based on prior probabilities), the decision maker (or decision system) performs tactic e , observes r , and then chooses A_r , the terminal gain increases by:

$$V_t(e, r) = \sum_d P''(d|r, e) G_t(A_r, d) - \sum_d P''(d|r, e) G_t(A', d) \quad (B.15)$$

The term $V_i(e, z)$ is known as the conditional value of sample information r . However, before r is observed, the expected value of sample information is given by:

$$V_i^*(e) = \sum_r P(r|e) V_i(e, r) \quad (\text{B.16})$$

Therefore, the expected terminal gain of a particular experiment is the expected terminal gain of an immediate terminal action plus the expected value of sample information. Written in equation form, gives the following:

$$G_i^*(e) = G_i^*(e_0) + V_i^*(e) \quad (\text{B.17})$$

Now, the expected net gain of information acquisition is therefore determined to be the expected value of sample information less the expected cost of obtaining it. In equation form:

$$v^*(e) = V_i^*(e) + c_s^*(e) \quad (\text{B.18})$$

Where,

$$c_s^*(e) = G_s^*(e) = \sum_r P(r|e) G_s(e, r) \quad (\text{B.19})$$

Appendix C: Generated MATLAB Syntax

Content	Page
car_following.m	180
cfparams.m	186
acc2.m	188
Td.m.....	191
Sd.m	192
Bayes3.m.....	193
NoBayes.m	199
checkZero.m	200

car_following.m

```
function varargout = car_following(varargin)
% CAR_FOLLOWING M-file for car_following.fig
%   CAR_FOLLOWING, by itself, creates a new CAR_FOLLOWING or
%   raises the existing
%   singleton*.
%
%   H = CAR_FOLLOWING returns the handle to a new CAR_FOLLOWING
%   or the handle to
%   the existing singleton*.
%
%   CAR_FOLLOWING('CALLBACK',hObject,eventData,handles,...) calls
%   the local
%   function named CALLBACK in CAR_FOLLOWING.M with the given
%   input arguments.
%
%   CAR_FOLLOWING('Property','Value',...) creates a new
%   CAR_FOLLOWING or raises the
%   existing singleton*. Starting from the left, property value pairs are
%   applied to the GUI before car_following_OpeningFunction gets called.
%   An
%   unrecognized property name or invalid value makes property application
%   stop. All inputs are passed to car_following_OpeningFcn via varargin.
%
%   *See GUI Options on GUIDE's Tools menu. Choose "GUI allows only
%   one
%   instance to initial (singleton)".
%
% See also: GUIDE, GUIDATA, GUIHANDLES

% Edit the above text to modify the response to help car_following

% Last Modified by GUIDE v2.5 12-Apr-2005 10:21:18

% Begin initialization code - DO NOT EDIT
gui_Singleton = 1;
gui_State = struct('gui_Name',    mfilename, ...
                  'gui_Singleton', gui_Singleton, ...
                  'gui_OpeningFcn', @car_following_OpeningFcn, ...
                  'gui_OutputFcn', @car_following_OutputFcn, ...
                  'gui_LayoutFcn', [], ...
                  'gui_Callback', []);
if nargin & isstr(varargin{1})
    gui_State.gui_Callback = str2func(varargin{1});
```

```

end

if nargout
    [varargout{1:nargout}] = gui_mainfcn(gui_State, varargin{:});
else
    gui_mainfcn(gui_State, varargin{:});
end
% End initialization code - DO NOT EDIT

% --- Executes just before car_following is made visible.
function car_following_OpeningFcn(hObject, eventdata, handles, varargin)
% This function has no output args, see OutputFcn.
% hObject    handle to figure
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
% varargin   command line arguments to car_following (see VARARGIN)

% Choose default command line output for car_following
handles.output = hObject;

% Update handles structure
guidata(hObject, handles);

% UIWAIT makes car_following wait for user response (see UIRESUME)
% uiwait(handles.figure1);
set(handles.play,'cdata',playbutton);
set(handles.stop,'cdata',stopbutton);
set(handles.pause,'cdata',pausebutton);
% set(handles.faster,'cdata',fasterbutton);
% set(handles.slower,'cdata',slowerbutton);

% --- Outputs from this function are returned to the command line.
function varargout = car_following_OutputFcn(hObject, eventdata, handles)
% varargout  cell array for returning output args (see VARARGOUT);
% hObject    handle to figure
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Get default command line output from handles structure
varargout{1} = handles.output;

% --- Executes on button press in simulation.
function simulation_Callback(hObject, eventdata, handles)

```



```

% hObject    handle to simulation (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
open('C:\MATLAB6p5\Car-following-Klee\car-following-try5\car_following8.mdl');

% --- Executes on button press in initial.
function initial_Callback(hObject, eventdata, handles)
% hObject    handle to initial (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
open('C:\MATLAB6p5\Car-following-Klee\car-following-try5\cfparams');

% --- Executes on button press in play.
function play_Callback(hObject, eventdata, handles)
% hObject    handle to play (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% cd('C:\MATLAB6p5\Car-following-Klee\car-following-try5')
% run cfparams;

% y=4;
% for x=0:0.01:10
% axes(handles.axes6);
% plot (x,y);
% hold on;
% % set(handles.view_axes,'XMinorTick','off');
% % grid on
% end
% axes(handles.axes6);
% hold on;plot(y,'o');
% hold on;plot(x-7,y,'o');
% hold off;
pause on;
axes(handles.view_axes);

% modelname = get(handles.modelname, 'String');
% stoptime = str2num(get(handles.simstoptime, 'String'));

[time,state,output]=sim('car_following8',[0,300]);
zoom yon;
axes(handles.position_axes);

```

```

plot(time,output(:,1),time,output(:,2));
zoom on;
axes(handles.velocity_axes);
plot(time,output(:,3),time,output(:,4));
zoom on;
axes(handles.accelerate_axes);
plot(time,output(:,5));
zoom on;
axes(handles.followingDistance_axes);
plot(time,output(:,6));
zoom on;

```

```

%----Execute on play
function play = playbutton

```

```

play = iconize(imread('play.jpg'));
play(play==255) = .8*255;

```

```

function stop = stopbutton

```

```

stop = iconize(imread('stop.jpg'));
stop(stop==255) = .8*255;

```

```

function pause = pausebutton

```

```

pause = iconize(imread('pause.jpg'));
pause(pause==255)=.8*255;

```

```

% --- Executes on button press in stop.
function stop_Callback(hObject, eventdata, handles)
% hObject    handle to stop (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
% set_param(car_following7.mdl, 'Simulationmode', 'stop');
% stop('car_following7');
% stop;
global GUIStopFlag;
GUIStopFlag = 1;

```

```

function out = iconize(a)

```

```

% Find the size of the acquired image and determine how much data will need

```

```

% to be lost in order to form a 18x18 icon
[r,c,d] = size(a);
r_skip = ceil(r/18);
c_skip = ceil(c/18);

% Create the 18x18 icon (RGB data)
out = a(1:r_skip:end,1:c_skip:end,:);

% %--- Executes during object creation, after setting all properties.
% function popupmenu1_CreateFcn(hObject, eventdata, handles)
% % hObject    handle to popupmenu1 (see GCBO)
% % eventdata  reserved - to be defined in a future version of MATLAB
% % handles    empty - handles not created until after all CreateFcns called
%
% % Hint: popupmenu controls usually have a white background on Windows.
% %       See ISPC and COMPUTER.
% if ispc
%     set(hObject,'BackgroundColor','white');
% else
%     set(hObject,'BackgroundColor',get(0,'defaultUicontrolBackgroundColor'));
% end
%
%
% % --- Executes on selection change in popupmenu1.
% function popupmenu1_Callback(hObject, eventdata, handles)
% % hObject    handle to popupmenu1 (see GCBO)
% % eventdata  reserved - to be defined in a future version of MATLAB
% % handles    structure with handles and user data (see GUIDATA)
%
% % Hints: contents = get(hObject,'String') returns popupmenu1 contents as
% cell array
% %       contents{get(hObject,'Value')} returns selected item from
% popupmenu1
%
%
% % --- Executes during object creation, after setting all properties.
% function popupmenu2_CreateFcn(hObject, eventdata, handles)
% % hObject    handle to popupmenu2 (see GCBO)
% % eventdata  reserved - to be defined in a future version of MATLAB
% % handles    empty - handles not created until after all CreateFcns called
%
% % Hint: popupmenu controls usually have a white background on Windows.
% %       See ISPC and COMPUTER.
% if ispc

```

```

% set(hObject,'BackgroundColor','white');
% else
% set(hObject,'BackgroundColor',get(0,'defaultUicontrolBackgroundColor'));
% end
%
%
% % --- Executes on selection change in popupmenu2.
% function popupmenu2_Callback(hObject, eventdata, handles)
% % hObject    handle to popupmenu2 (see GCBO)
% % eventdata  reserved - to be defined in a future version of MATLAB
% % handles    structure with handles and user data (see GUIDATA)
%
% % Hints: contents = get(hObject,'String') returns popupmenu2 contents as
cell array
% %    contents{get(hObject,'Value')} returns selected item from
popupmenu2

% --- Executes on button press in pause.
function pause_Callback(hObject, eventdata, handles)
% hObject    handle to pause (see GCBO)
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
set(handles.play,'value',0);
set(handles.play,'enable','on');
pause;

```

cfparams.m

```
% script file cfparams.m
% initialization file that must be run before car_following.mdl
clc; clear all; close all
% Sets the values of the following parameters

% L: Vehicle Length (m)
L=3.66;

% amin: Min vehicle acceleration, i.e. max deceleration (m/s^2)
amin=-7;

% amax: Maximum vehicle acceleration (m/s^2)
amax=3;

% SL: Speed limit
SL=50; %km/h
SL=SL*1000/3600; %m/s

% delta: Excess above speed limit
delta=10; %k/hr
delta=delta*1000/3600; %m/s

%acceleration due to gravity--used in critical distance calculation
g=9.81; %(m/s^2)

% reaction time-->defined within system
NTd = 1.5; %(sec)

% safe stopped distance
Sf = 2; %(m)

% vmax: maximum velocity (m/s)
vmax=15.65; %(m/s)
vmin=0;

% T0: zero
% T1: Time when lead vehicle begins to decelerate, (sec)
% T2: Time when lead vehicle reaches speed 'SL-10' k/hr, (sec)
% T3: Time when lead vehicle begins to acccelerate, sec
% T4: Time when lead vehicle reaches speed 'SL+10' k/hr, (sec)
% T5: Time when lead vehicle begins to decelerate, (sec)
% T6: Final time for Look-Up table, (sec)
% v0: Lead vehicle speed at time T0, (k/hr)
```

% v1: Lead vehicle speed at time T1, (k/hr)
% v2: Lead vehicle speed at time T2, (k/hr)
% v3: Lead vehicle speed at time T3, (k/hr)
% v4: Lead vehicle speed at time T4, (k/hr)
% v5: Lead vehicle speed at time T5, (k/hr)
% v6: Lead vehicle speed at time T6, (k/hr)

% Gradient of the road
% G0: Gradient at time T0, (%*100)
% G1: Gradient at time T1, (%*100)
% G2: Gradient at time T2, (%*100)
% G3: Gradient at time T3, (%*100)
% G4: Gradient at time T4, (%*100)
% G5: Gradient at time T5, (%*100)
% G6: Gradient at time T6, (%*100)

n=1;
save n n;

acc2.m

```
% function acc2.m
% Computes the temporary commanded acceleration x1dd_temp

function a=acc2(clock,x1d,x0d,S,warning,x0,x1,Td)

Gap=S; %+3.66;

if (x0d==0)&&(Gap<5)
    a=-7;
    if (x1d==x0d)
        a=0;
    else
        a=a;
    end
else
    if ((x0d-x1d)>0) %(steady state)
        if(Gap>=28) %m G-actual gap, Gd-desired gap
            Vdss=56.33*0.278; %km/h
        elseif (Gap<=5) %m
            Vdss=0;
        else
            Vdss=(27.737*(ln(Gap)) - 33.178)*0.278;
        end

        a=(Vdss-x1d)/Td;

        %plot results
        plot(150,2,'-rs','LineWidth',5,...
            'MarkerEdgeColor','k',...
            'MarkerFaceColor','k',...
            'MarkerSize',30);

    elseif((x0d-x1d)<0) %(non-steady state)
        if(Gap>=26)
            Vdss=56.33*0.278;
        elseif (Gap<=0)
            Vdss=0;
        else
            Vdss=((1.1041*Gap)+(0.1023*Gap*Gap)-(0.0023*Gap*Gap*Gap))*0.278;
        end
        a=(Vdss-x1d)/Td;
```

```

else
    a=0;
end
end

%plot results
plot(150,2,'--rs','LineWidth',5,...
     'MarkerEdgeColor','k',...
     'MarkerFaceColor','k',...
     'MarkerSize',30);

%acceleration for each of the warnings will be that defined above
if warning == 1

    plot(150,3,'--rs','LineWidth',5,...
         'MarkerEdgeColor','g',...
         'MarkerFaceColor','g',...
         'MarkerSize',20);

elseif warning == 2

    plot(150,3,'--rs','LineWidth',5,...
         'MarkerEdgeColor','y',...
         'MarkerFaceColor','y',...
         'MarkerSize',20);

elseif warning == 3

    plot(150,3,'--rs','LineWidth',5,...
         'MarkerEdgeColor','r',...
         'MarkerFaceColor','r',...
         'MarkerSize',20);

else

    plot(150,3,'--rs','LineWidth',5,...
         'MarkerEdgeColor','g',...
         'MarkerFaceColor','g',...
         'MarkerSize',20);

end

end

if a>=4

```



```

a=4;
elseif a<=-7
a=-7;
else
a=a;
end
a=a;

axis([0 1000 0 3.5]);
g=[0 1000 2500];
h=[1.5 1.5 1.5];
line(g,h);
hold on;

% lateral position of the vehicle
y=1;
z=2;
%lead vehicle
plot(x0,y,'-rs','LineWidth',2,...
      'MarkerEdgeColor','b',... 'MarkerFaceColor','b',...
      'MarkerSize',15);
%following vehicle
plot(x1,y,'-rs','LineWidth',2,...
      '%MarkerEdgeColor','m',... 'MarkerFaceColor','m',...
      '%MarkerSize',15);

hold off;

```

Td.m

% function Td.m

function Td=TD(Sd,S,NTd)

if S<=0;
Td=0.5;

else%ratio of desired to actual spacing
A = Sd/S;

%actual reaction time
Td=NTd*A;

%check on actual reaction time criteria
if (Td<=0.5)
Td=0.5;
elseif (Td>=3)
Td=3;
else
Td=Td;
end
end

Sd.m

% function Sd.m

function Sd=SD(x0d,x1d)

%random>=0.6 % use alertness 'A' instead--no if statements

```
if((x0d-x1d)>0)
    if (x1d<=15.65)
        V=x1d*3.6;
        Sd=3.376*(exp(0.0354));
    else
        Sd=28;
    end
else
    if (x1d<=15.65)
        V=x1d*3.6;
        Sd=(0.0002*V*V*V)-(0.0147*V*V)+(0.7736*V);
    else
        Sd=26;
    end
end
```

bayes3.m

```
% bayes3.m
% generation of algorithm for collision warning system

% algorithm requires inputs to generate the warning output
function warning=bayes(A,critical,actual)

xdiff=critical-actual;

%if the following vehicle is not within the critical zone
% the output will indicate a green signal
if xdiff<0
    plot(150,2,'--rs','LineWidth',5,...
        'MarkerEdgeColor','g',...
        'MarkerFaceColor','g',...
        'MarkerSize',30);
    warning=0;

% all other cases require the application of Bayes
else
    % priors use alertness value and assume an equal likelihood for the
    % initial iteration
    load n n;
    save n n;

    if n<2
        x=0.33;
        y=0.33;
        z=0.34;

        % transform into probabilities
        X=x/(x+y+z);
        Y=y/(x+y+z);
        Z=z/(x+y+z);
        P1=X;
        P2=Y;
        P3=Z;
        P4=P1+P2+P3;

    else
        load rc1dc1 P11; load rc1dc2 P21; load rc1dc3 P31;
        load rc2dc1 P12; load rc2dc2 P22; load rc2dc3 P32;
        load rc3dc1 P13; load rc3dc2 P23; load rc3dc3 P33;
```

```

    if (actual<=(critical/1.5))
        P1=P11;
        P2=P12;
        P3=P13;

    elseif (actual<=((critical/1.5)*1.25)&(actual>=(critical/1.5)))
        P1=P21;
        P2=P22;
        P3=P23;
    else
        P1=P31;
        P2=P32;
        P3=P33;
    end
end

% define conditional probabilities
% use alertness, random number, and gain
% P(r|d,e)
%      dc  d1.25c d1.5c
% rc
% r1.25c
% r1.5c
% sum   1.0  1.0   1.0

% conditional probabilities
% combine alertness and camera information (i.e. random number)

% need a value for the area being looked at
% the value should be between between 75% and 80% straight ahead or
% else the driver is fixated on something or is looking elsewhere
lookaheadarea=randn*sqrt(0.09)+0.7; %random number between 0 and 1
if (lookaheadarea >= 1)
    lookaheadarea=1;
elseif (lookaheadarea < 0)
    lookaheadarea=0.001;
else
    lookaheadarea=lookaheadarea;
end

% however lookahead area is only good between 75-85%
% need to add a factor to change results
if (lookaheadarea < 0.75)
    lookaheadarea=lookaheadarea*1.33;
elseif (lookaheadarea > 0.85)

```

```

    lookaheadarea=(-1.33*lookaheadarea)+2.13;
else
    lookaheadarea=1;
end

conditionals=lookaheadarea; %*alertness;
c=conditionals;

% conditional probabilities
rc1dc1=(1-c)/2; rc1dc2=(1-c)/2; rc1dc3=c;
rc2dc1=(1-c)/2; rc2dc2=c; rc2dc3=(1-c)/2;
rc3dc1=c; rc3dc2=(1-c)/2; rc3dc3=(1-c)/2;

%Marginal Probabilities (Prior * Conditional)
Marg1=((P1*rc1dc1)+(P2*rc1dc2)+(P3*rc1dc3));
Marg2=((P1*rc2dc1)+(P2*rc2dc2)+(P3*rc2dc3));
Marg3=((P1*rc3dc1)+(P2*rc3dc2)+(P3*rc3dc3));
sum = Marg1+Marg2+Marg3;

% display
% Prior Strategy: i.e. e0, ew —from type =1 or 2
% Optimal Strategy: i.e. e0, ew (Do not wait) OR (Wait)
% Optimal Alert: i.e. a0, aa, ar (No alert) OR (Amber Alert) OR (Red Alert)

%Gain Matrix
a0dc1=0.2; a0dc2=0.5; a0dc3=1.0;
aadc1=0.5; aadc2=1.0; aadc3=0.2;
ardc1=1.0; ardc2=0.5; ardc3=0.2;

%Posterior Probabilities
P11=(P1*rc1dc1)/Marg1; P12=(P2*rc1dc2)/Marg1; P13=(P3*rc1dc3)/Marg1;
G11=P11*a0dc1; G12=P12*a0dc2; G13=P13*a0dc3;
G111=P11*aadc1; G112=P12*aadc2; G113=P13*aadc3;
G1111=P11*ardc1; G1112=P12*ardc2; G1113=P13*ardc3;

P21=(P1*rc2dc1)/Marg2; P22=(P2*rc2dc2)/Marg2; P23=(P3*rc2dc3)/Marg2;
G21=P21*a0dc1; G22=P22*a0dc2; G23=P23*a0dc3;
G221=P21*aadc1; G222=P22*aadc2; G223=P23*aadc3;
G2221=P21*ardc1; G2222=P22*ardc2; G2223=P23*ardc3;

P31=(P1*rc3dc1)/Marg3; P32=(P2*rc3dc2)/Marg3; P33=(P3*rc3dc3)/Marg3;
G31=P31*a0dc1; G32=P32*a0dc2; G33=P33*a0dc3;
G331=P31*aadc1; G332=P32*aadc2; G333=P33*aadc3;
G3331=P31*ardc1; G3332=P32*ardc2; G3333=P33*ardc3;

```

```

%gain application
g1=G11 + G12 + G13;
g11=G111 + G112 + G113;
g111=G1111 + G1112 + G1113;

if(g1==g11)&(g11==g111)
    G1=g1;
elseif g111<=g1>=g11
    G1=g1;
elseif g111<=g11>=g1;
    G1=g11;
else
    G1=g111;
end

g2=G21 + G22 + G23;
g22=G221 + G222 + G223;
g222=G2221 + G2222 + G2223;

if(g2==g22)&(g22==g222)
    G2=g2;
elseif g222<=g2>=g22
    G2=g2;
elseif g222<=g22>=g2;
    G2=g22;
else
    G2=g222;
end

g3=G31 + G32 + G33;
g33=G331 + G332 + G333;
g333=G3331 + G3332 + G3333;

if(g3==g33)&(g33==g333)
    G3=g3;
elseif g333<=g3>=g33
    G3=g3;
elseif g333<=g33>=g3;
    G3=g33;
else
    G3=g333;
end

%alert types
a0=(P1*a0dc1)+(P2*a0dc2)+(P3*a0dc3);

```

```

aa=(P1*aadc1)+(P2*aadc2)+(P3*aadc3);
ar=(P1*ardc1)+(P2*ardc2)+(P3*ardc3);

```

```

if (ar==aa)&(aa==a0)
    aprime = a0;
    alert = 0;
elseif ar <= a0 >= aa
    aprime = a0;
    alert = 0;
elseif ar <= aa >= a0
    aprime = aa;
    alert = 1;
else
    aprime = ar;
    alert = 2;
end

```

```

%terminal value
rc1=aprime-G1;
rc2=aprime-G2;
rc3=aprime-G3;

```

```

V=(Marg1*rc1)+(Marg2*rc2)+(Marg3*rc3);

```

```

if V > 0.009
    action = 0; %ew
else %V >= 0.009
    action = 1; %e0
end

```

```

warning=action+alert;

```

```

%for other iterations
rc1dc1=P11; save rc1dc1 P11;
rc1dc2=P21; save rc1dc2 P21;
rc1dc3=P31; save rc1dc3 P31;
rc2dc1=P12; save rc2dc1 P12;
rc2dc2=P22; save rc2dc2 P22;
rc2dc3=P32; save rc2dc3 P32;
rc3dc1=P13; save rc3dc1 P13;
rc3dc2=P23; save rc3dc2 P23;
rc3dc3=P33; save rc3dc3 P33;
%end
load n n;
n=n+1;

```



```
    save n n;  
end  
  
warning=warning;
```

nobayes.m

```
% function nobayes.m  
% outputs the warning to the user when bayes is not used
```

```
function warning=nobayes(a,actual,critical,Vf)
```

```
xdiff=critical-actual;  
crit1=Vf;  
crit2=Vf*2;
```

```
if xdiff<0  
    plot(150,2,'-rs','LineWidth',5,...  
         'MarkerEdgeColor','g',...  
         'MarkerFaceColor','g',...  
         'MarkerSize',30);  
    warning=0;
```

```
elseif (actual<crit1)  
    warning = 3; %red
```

```
elseif (actual<=crit2)&(actual>=crit1)  
    warning = 2; %yellow
```

```
else  
    warning = 1; %green
```

```
end
```

```
warning=warning;
```

zeroCheck.m

```
% function zeroCheck.m  
% ensures the gap is passed as a real number
```

```
function Gap=zeroCheck(G)
```

```
Z=isinf(G);  
Y=isnan(G);
```

```
if Z==1  
    Gap=0;  
elseif Y==1  
    Gap=0;  
else  
    Gap=G;  
end
```

Appendix D: Qualitative Validation of Car-Following Model

Sample #1

Sample #2

Sample #3

Sample #4

Sample #5

Sample #6

Sample #7

Sample #8

Sample #9

Sample #10

Sample #11

Sample #12

Sample #13

Sample #14

Sample #15

Sample #16

Sample #17

Sample #18

Sample #19

Sample #20

Sample #21

Sample #22

Sample #23

Sample #24

Sample #25

Sample #26

Sample #27

Sample #28

Sample #29

Sample #30

Appendix E: Quantitative Validation of Car-Following Model

Sample	Avg Obs Speed (km/h) (V_F)	Estimated Speed (km/h)	RMS Error	Avg Obs Spacing (m) (S_F)	Estimated Spacing (m)	RMS Error	n
1	48.73	49.54	1.63	25.56	25.99	1.85	53
2	39	39.83	2.02	17.01	17.01	1.02	145
3	43.43	43.64	1.22	18.94	18.94	2.08	148
4	44.09	43.31	1.33	21.84	21.73	0.2	123
5	52.13	52.2	0.51	24.18	24.18	0.45	78
6	41.16	41.22	1.73	15.07	15.4	1.79	148
7	28.48	28.76	1.42	14.75	15.73	1.98	127
8	33.69	33.87	0.78	18.43	18.88	1.45	125
9	40.93	40.75	1.12	15.03	15.27	0.7	58
10	38.79	39.06	1.69	15.48	15.48	0.99	104
11	53.66	52.92	0.97	20.22	20.6	1.49	30
12	40.8	40.69	0.9	19.6	18.76	0.96	133
13	40.38	40.61	1.8	24.2	22.72	1.7	62
14	39.17	39.26	0.9	22.06	20.95	1.57	133
15	45.89	45.25	0.69	22.58	22.78	0.64	85
16	41.43	41.04	1.74	16.56	16.38	1.08	97
17	39.9	39.54	1.46	16.23	16.09	1.67	169
18	35.01	34.51	1.83	13.17	11.73	2.23	96
19	33.79	33.02	1.05	15.21	16.34	1.19	11
20	39.7	39.67	2.23	17.57	15.92	1.88	82
21	50.39	49.9	0.602	26.16	26.79	0.66	9
22	40.3	40.55	0.68	18.95	19.23	0.91	147
23	46.21	45.68	1.42	18.61	18.68	1.15	136
24	41.64	41.4	1.11	17.19	17.42	0.3	12
25	46.07	45.32	1.2	20.62	20.02	1.24	93
26	42.79	42.86	1.1	22.88	21.93	0.98	16
27	47.89	48.27	0.95	22.09	22.37	1.96	88
28	40.93	39.94	1.26	18.91	18.27	1.48	114
29	41.49	42.1	1.07	20.12	20.26	0.68	84
30	36.19	35.82	1.17	19.18	19.32	1.59	82
AVG	41.80	41.68	1.25	19.28	19.17	1.26	92.93

Comparison of Average Observed and Simulated Following Vehicle Speed

Comparison of Average Observed and Simulated Vehicle Spacing