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**LA THÈSE A ÉTÉ
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A Study of:

Flat Oval Bourdon Tube Stress Behaviour

Peter Slinn, B. Eng

A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the
Degree of Master of Engineering

DEPARTMENT OF MECHANICAL AND AERONAUTICAL ENGINEERING
CARLETON UNIVERSITY
OTTAWA, ONTARIO
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JANUARY 1986

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Abstract

Stress and deflection behaviour in flat oval bourdon tubes are studied.

Theories and finite element models dealing with two dimensional cross-sectional, two dimensional thin wall, and general three dimensional cases are described. Their assumptions are examined and their results compared.

Two dimensional cross-sectional theories agree with the two dimensional cross-sectional finite element model. The general three dimensional finite element model does not completely agree with published theories and results.

Cross-sectional deflection predictions are found to be similar regardless of the analytical procedure used.

The three dimensional model indicates that end effects and annular stresses can be ignored only for some geometries.

Empirical equations are considered. Equations developed have limitations imposed by assumptions made and data available.

Recommendations are made with respect to
improvements and future development work.

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Nomenclature

- 'a' = semimajor axis
- 'b' = semiminor axis
- 'CP' = relative circumferential position
- 'dR' = change in radius of curvature
- 'E' = Young's Modulus
- 'EKPO' = observed experimental change in radius of curvature
- 'EKPW' = Wolf's calculated change in radius of curvature
- 'EKPP' = Hamada's calculated change in radius of curvature
- 'SPW' = Wolf's stress solutions
- 'SPP' = Hamada's stress solutions
- 'IAS' = inner surface annular stress
- 'ICS' = inner surface circumferential stress
- 'IES' = inner surface effective stress
- 'IRS' = inner surface radial stress
- 'l' = tie deflection
- 'mew' = Poisson's Ratio
- 'OAS' = outer surface annular stress
- 'OCS' = outer surface circumferential stress
- 'OES' = outer surface effective stress
- 'ORS' = outer surface radial stress
- 'P' = applied internal pressure

'Phi0' = sector angle of finite element model

'R0' = radius of curvature

'Sa' = stress on the inner surface of the major axis

'Sb' = stress on the outer surface of the minor axis

't' = wall thickness

Glossary

TERM	DEFINITION
------	------------

Analysis of Variance - this is a method of testing for relationships between variables.

Annular Stress - this is the stress induced by the radius of curvature of the bourdon tubes length. Since the longitudinal geometry of the bourdon tube is in the shape of a 'C', we can consider this portion of the geometry to be a part of a ring. This ring like geometry then will induce ring stresses as the tube inflates, hence the term annular stress.

Circumferential Stress - when considering the stresses of the cross-section, only, we will have two stresses, the radial stress and the circumferential stress. Note: the annular stress is perpendicular to the cross section. The circumferential stress is then the stress that goes along the perimeter of the cross section.

Curve - the curved portion of the cross section.

Effective Stress - the Von Mises or Distortion Energy Theory yield stress

Flat - the flat portion of the cross section.

Flat Oval Cross Section - a bourdon tube that has a cross section shaped with a flat top and bottom, and semicircular ends.

Large Radius - this relates to the radius of curvature of the bourdon tube. A large radius of curvature was taken as being ten or more times the semimajor axis.

Long Form - this is a finite element model which was used to gather all the three dimensional data for this paper. For a detailed explanation see the theory section.

Radial Stress - this is the cross-sectional stress that goes through the wall thickness perpendicular to the cross-sectional perimeter.

Radius of Curvature - this affects the bourdon tubes longitudinal geometry. The bourdon tubes chosen is in a 'C' shape and the radius of curvature affects the tightness of the 'C'.

Short Form - this is a finite element model that did not work out. It is explained in detail in the

finite element theory section.

Short Radius - this refers to a bourdon tube having a small radius of curvature, eq. about three times the semimajor axis.

Chapter 1: Introduction

The indicating needle of a pressure gauge is driven by a bourdon tube which is receiving an internal pressure. Some characteristics of bourdon tubes are: ease of use, ease of manufacture, reliability, and low cost. These features have resulted in wide use and extensive interest in the performance of bourdon tubes. While they are generally reliable devices, bourdon tubes have failed by fatigue frequently enough to warrant studies of the stresses associated with the large deflections that are needed for useful operation.

Flat oval bourdon tubes are curved pressure vessels having flat oval cross sections, figures 1 and 2. Applying an internal pressure deforms the cross section, figures 3, and 4, causing the bourdon tube's free end to deflect, figure 5. This free end deflection is referred to as tip deflection.

This study examines the flat oval bourdon tube since this type of bourdon tube is the most widely used and analysed. The flat oval cross section has received so much attention since this shape possesses a desirable combination of high sensitivity and low stresses, when compared to other cross-sectional shapes, such as elliptical, circular, dogbone, etc.

To design flat oval bourdon tubes ('tubes'): tip deflection; tip force; and stress characteristics must be understood. Even though these characteristics have been studied in numerous papers, fatigue failure continues to be a problem.

This project is intended to improve the understanding of stress behaviour of flat oval bourdon tubes ("tubes") so that their designs will give lower maximum stress levels, reducing the probability of fatigue failure. The project, at the same time, seeks to develop simple empirical equations for routine design. Specific objectives are:

- to investigate the classical theories of stress behaviour and deflections of tubes;

- to develop a two dimensional cross-sectional finite element model of tubes;

- to develop a general three dimensional finite element model of tubes;

- to compare the results obtained from the finite element models with those obtained from the classical theories;

- to develop empirical equations for routine design, which will give an improved estimate of maximum stresses and which will be simpler to use than the classical equations;

- to develop recommendations for subsequent

research which would develop a general design solution simple enough for the routine design of flat oval bourdon tubes.

In order to simplify the analysis while improving the understanding of these characteristics, previous authors have studied restricted cases. Some authors have studied the two dimensional cross-sectional case while other authors have studied the two dimensional thin wall case.

The two dimensional cross-sectional case neglected annular stresses, considering the stresses in the cross section only. This approach is valid for thick wall, large radius bourdon tubes.

The two dimensional thin wall case assumed stresses and strains varied linearly through the wall thickness and that the annular and circumferential stresses were the only important stresses. These assumptions permitted a biaxial stress state to be considered. This approach is valid for thin wall, large radius bourdon tubes.

In all instances it has been necessary to make approximations in arriving at solutions, to make the problems more manageable.

The term 'classical theory' is used here to refer

to methodologies and knowledge which are long established in analysis of stress and strain and in this paper it sets apart finite element analysis, a more recent analytical technique, from those long established methods. This distinction is maintained for convenience of exposition even though the derivation of finite element theory is based on classical theory. The finite element method has been used by only a few of the previous authors. In attempting to reach the objectives, the finite element method is featured in this study since it offers considerable flexibility for analysing stresses and displacements.

Annular stresses are ring-like stresses which appear in an object having a circular geometry. Therefore annular stress will be found along the sector length of the flat oval bourdon tube.

Circumferential stresses are the tangential stresses which occur on the cross section. These stresses are usually by far the largest stresses present.

Radial stresses are the stresses that are perpendicular to the circumferential stresses. These stresses go through the wall thickness.

Effective stresses are stresses calculated through the use of Von Mises or Distortion Energy Theory. This results in a yield stress or "Effective Stress". Effective stress is a term used by some of the referenced authors and therefore it will be used here for consistency.

Plane stress means there are only two principal stresses present. The third principal stress is zero.

Plane strain means there are only two principal strains present. The third principal strain is zero.

Angular strain refers to the strain related to a rotational distortion of the body. This strain leads to an angular deformation of a cross section when the axes of symmetry are permitted to rotate.

A cross section is a planar slice of a bourdon tube, which is perpendicular to the longitudinal axis of the tube. Since bourdon tubes have curved longitudinal axes, the cross sections are not parallel to each other.

Cross-sectional stresses are the stresses of the cross section, i.e. the circumferential and radial stresses.

Tip deflections result from annular stresses set

up as a result of the internal pressure deforming the cross section. Applying an internal pressure produces tensile annular stresses at the upper minor axis, and compressive annular stresses at the lower minor axis, of the flat oval bourdon tube's cross section. These annular stresses behave like an applied moment on the tube's free end, causing the free end to deflect proportionally to the applied pressure. The tip deflection and accompanying force operate the mechanism of a pressure gauge, figure 6.

Chapter 2: Theory and Published Experimental Data

2.1 Classical Cross-Sectional Theories

In the development of classical theory for bourdon tube stress analysis, previous authors have greatly reduced the complexity of the problem through assumptions chosen to avoid the need to consider annular stresses and end effects. Annular stresses can be considered negligible when the radius of curvature of a flat oval bourdon tube is very large. When the radius of curvature approaches infinity the flat oval bourdon tube becomes straight.

Weydert and Wuest used two different approaches to analyse the stresses of a straight tube. Weydert considered the plane stress condition, while Wuest considered the plane strain condition.

Even though Weydert did not particularly study flat oval bourdon tubes, the conditions of his analysis are directly applicable to them.

2.1.1 Thick-Wall Cylinder Theory [9]

This theory is quite well established, therefore it is used as a reference. No discussion of this.

theory will be presented here.

2.1.2 A Study of Stresses in Thick-Walled Elliptical Pressure Tubes, J.C. Weydert [55][56][61]

In his paper Weydert studied the stress behaviour of internally pressurized thick wall elliptical and flat oval pressure tubes, using two techniques to examine results. Initially, strength of materials methods were applied to study the stresses of the tube cross section. To verify these theoretical results, photoelastic techniques were used on these same tube cross sections.

The analysis was started by analysing the elliptical cross section, which was approximated by two curved beams, for simplification. To represent the flat oval cross section, one beam was considered as a straight beam and the other beam was curved, figure 7.

Weydert's study is good only for near circular cross sections, since the approximations made in using the beam equations of strength of materials, are not as accurate as they should be.

Weydert used the Winkler-Bach curved beam theory, for the curved beam, which disregards radial stresses as well as lateral deformations, and assumes pure

bending. The radial stresses present in the curved beam are small with relation to the other stresses and can be acceptably ignored. Lateral deformation, or change in the wall thickness, is most likely quite small and can also be acceptably ignored. Pure bending, on the other hand, is not a good approximation to make for wider cross sections since shear stresses become very pronounced, making this approximation of the Winkler-Bach theory inappropriate, and significant error occurs.

For the straight beam, standard flexure theory was used. Flexure theory assumes:

- a beam is prismatic (has parallel sides);
- the beam has a length at least ten times its depth;
- the external forces are all at right angles to the axis of the beam and in a plane of symmetry;
- flexure is slight;
- the material is homogeneous;
- Hooke's law is obeyed;
- stresses are within the elastic limit;
- every layer of material is free to expand and contract longitudinally and laterally as if separate from other layers;
- the beam's cross section remains a planar surface, this being true only if shear is constant or

zero over the cross section and when the shear is constant throughout the length of the beam.

Of these assumptions, of flexure theory, wide cross sections violate: the slight flexure requirement; the constant shear throughout the length of the beam, for the beam's cross section to remain planar; and the requirement of every layer of material being free to expand and contract as if separate from other layers. For near circular tube cross sections the requirement of the beam's length being at least ten times the beam's depth, is violated.

These violations result in significant errors in the use of Weydert's theory. The computed values for narrow and circular cross sections are still in fairly good agreement with photoelastic and thick-wall cylinder theory. Weydert's theory is not applicable to wide cross sections since the errors become prominent, due to these violations.

The major and minor axes, the axes of symmetry of the quarter section, were selected to help establish the boundary conditions of the solution. At the axes of symmetry, a moment and a tangential force were found, while at the interface of the two beams a constant shearing force was assumed.

To determine these moments and forces, and thus the stresses, the moment equation of the cross section was developed. Each beam had to be studied separately, with its boundary conditions, to develop the equilibrium equations. Then the angular deformations of the cross section were considered, to find the angular strain equations. Since the axes of symmetry remained perpendicular to each other, the total angular strain for the cross section was zero and the angular strain equations for the two beams of the cross section could be set equal to each other. The general moment equation for the cross section could then be derived and the cross-sectional stresses found.

Weydert's results indicate that the maximum stresses can occur at either the major or minor axes. This finding is important since not all analyses predict it.

For ease of description the stresses involved in the cross-sectional case, figure #8 is presented showing one of the photoelastic results. In this figure the locations of stress concentrations are evident.

2.1.3 Theory of High Pressure Bourdon Tubes,
W. Wuest [46]

In his paper, Wuest studied the stress behaviour of internally pressurized thick wall flat oval bourdon tubes, through the application of elasticity theory.

Initially several restrictions and approximations were made which limit the applicability of this approach. The flat oval bourdon tube cross section was approximated and idealized by using a central rectangular section with two end arcs.

The stresses and strains of the rectangular section are assumed to equal those of the end arcs, at their interface. To simplify the solution a stress function was chosen which satisfied the end conditions at the major and minor axes while at the interface only the integrals of stress, moment, and strain are satisfied. This stress function assumed uniform transverse shear through the wall thickness, at the interface.

Wuest assumed any disturbance by these approximations at the interface would diminish rapidly, within the distance of one wall thickness, by the use of St. Venant's principle. This would leave undisturbed the stress states at the major and minor

axes. St. Venant's principle could not be used if the wall thickness was very large, since the disturbances set up by the approximations at the interface would not diminish rapidly enough.

For low pressure bourdon tubes a further approximation was assumed in order to consider annular stresses to be negligible:

$$\frac{a^2}{R_0 t} < 1$$

This condition is always met in high pressure bourdon tubes. When this condition is met the stresses of the flat oval bourdon tube can be found by considering the stresses in an equivalent straight tube, reducing the problem to one which deals only with cross-sectional distortion. A unit length of this straight tube was analysed, the unit length being bound by two cross-sectional planes which were not permitted to distort and were far enough away from the ends of the tube for the end effects to be negligible.

This analysis determined the stresses at the inner major axis, this being the location where the maximum stress was considered, by Wuest, to always occur. Results of his equation appear in figures 9 to 11.

2.2 Classical Two Dimensional Thin Wall Theories

2.2.1 Tubular Manometric Springs, L.E. Andreeva [30]

In her study, Andreeva analysed elliptical and flat oval bourdon tubes by using thin shell theory and Ritz energy methods.

Throughout the analysis, numerous approximations were made in the model of bourdon tubes. The boundary conditions of the model were based on the concepts that one end was clamped and the other end was free to deflect. The assumptions used were as follows: the end effects are negligible; shell theory applies; the longitudinal axis of the tube does not elongate; the wall thickness is small relative to the minor axis; the minor axis is small relative to the radius of curvature; the bourdon tube cross section will deform in approximately the same fashion as that of a similar straight tube; the mean contour of the cross section does not stretch; the tube cross section is symmetrical with respect to the major and minor axes; and the axial load from the end plug is considered. The approximations limit the analysis to large radius, large sector angle, thin wall bourdon tubes, but it permits the study of cross sections which range from nearly circular to very wide.

The approximations permitted the examination of a model containing a biaxial stress state, the stresses considered being the annular and circumferential stresses. The values of the resulting equations are represented in figures 12 to 25.

2.2.2 Deformation and Stresses of Flat-Oval

Bourdon Tubes, Minoru Hamada, Katsuhisa Fujita,
and Kiyoshi Asahina [28]

Many of the derivations of this paper were done in another paper, by these authors, that has not been translated to English. Therefore the results can be used only in a general way, for comparison.

To study flat oval bourdon tubes these authors used thin shell theory and numerical difference methods. The model represented a full length, half section, open ended, flat oval bourdon tube. The end boundary conditions of the model were that one end was clamped while the other end was unconstrained.

While the assumptions made in this study were not stated in the English translation, upon analysis of this translation it seemed that end effects were assumed to be negligible, that shell theory approximations applied, that a large radius of

curvature was necessary, that there was no axial load due to internal pressure on the end cap, and that only wide cross sections could be analysed.

A set of approximate equations and graphs which contain results of this study were given. The results were shown to be in good agreement with other theoretical solutions and experimental results, table 12.

2.2.3 Finite Element Analysis of Bourdon Tubes,

R.L. Davis and W.D. Webster [11]

In this paper Davis and Webster present the results of their finite element program. The program used sixteen node isoparametric thick shell elements to analyse both elliptical and flat oval bourdon tubes. To determine accuracy, the results were compared with experimental and theoretical data.

Vertical half section, full length bourdon tubes having open ends were modelled. Since the boundary conditions and limitations of the models were not specified, direct comparison of results requires that assumptions be made. The boundary conditions will be assumed to be as follows: one end of the bourdon tube is clamped while the other end is completely free to

deflect; there is no axial load from an end cap; and the minor axis movement is permitted only on the minor axis plane, for the vertical half section boundary condition. It also will be assumed that only relatively thin wall tubes may be considered.

To check the validity of their results, Davis and Webster selected a straight tube as an example, simulating a two dimensional cross-sectional case, and compared results against those of Weydert. The two deflection calculations were compared with experimental data and the stress results with Motherway's strain gauge data.

The deviation between their results and those of Motherway, was thought to be due to the idealization of their bourdon tube model. Actual bourdon tubes deviate from the idealized shape, by varying degrees, depending on the manufacturing process used.

2.3 Finite Element Theory

The finite element method is a generally accepted method for static, elastic analysis of the stresses and displacements of loaded objects.

The approximations made in the finite element

theory vary with the type and order of element used.

Classical elasticity theory and classical plate and shell theory, along with their corresponding approximations, are used in the derivations of the types of elements.

The order of the element determines the type of stress and strain variation the element will permit through its body, i.e. linear, quadratic, cubic, etc. The more general the elements (the ones with the fewer approximations) and the higher the order of the elements, the better the finite element method will depict the stress and strain characteristics of an object. The greatest drawback of this method is that the fewer the approximations and the more comprehensive the finite element model the greater the demand for computer resources, resulting in higher costs.

This study attempts to use the finite element method results to verify the previously stated classical theories.

2.4: Shell Theory - End Effect Predictions

The publications that have been considered in this paper stated that end effects are negligible. For an economical and thorough finite element analysis of flat

oval bourdon tubes knowledge of the exact extent of the end effects is required, since the end effects have been found to have a significant effect on bourdon tube behaviour.

Thin circular tube shell theory can give an approximate estimate of the duration of the end effects as set out below.

Even though thin circular tube shell theory cannot be directly applied to flat oval bourdon tube analysis, using it as an analogy helps to indicate whether or not a particular model of flat oval bourdon tube behaviour is reasonable.

This topic was brought forth during a discussion with Dr. Cowper of the NRC.

End effects for thin shell circular tubes can be separated into two types. The first type of end effect is the uniform cross-sectional expansion (or contraction) of the circular tube end. The duration of this end effect is roughly predicted by:

$$(R_0 t)^{1/2}$$

which suggests a short disturbance length.

The second type of end effect is the nonuniform cross-sectional distortion of the circular tube. This

end effects duration is roughly predicted by:

$$R_0 \left(\frac{R_0}{t} \right)^{\frac{1}{2}}$$

which suggests a long disturbance length.

This study has found that the second type of end effect appears to be the type that is most prominent in flat oval bourdon tubes. Since this end effect has a very long disturbance length in thin circular tubes, this same end effect may have a fairly long disturbance length in flat oval bourdon tubes.

2.5: Dimensional Analysis [60][33][49][25]

The Buckingham Pi Theory was used in this study for dimensional analysis. This theory generalizes experimental results, reducing the number of experimental runs required, and permits an approximate solution to be found even when the problem is not fully understood.

Physical parameters are organized by the Buckingham Pi theorem into nondimensional groups. Each nondimensional group can then be taken as an independent parameter.

Before studying the nondimensional groups an

understanding of the physical parameters, and how they relate to stress and the change in radius of curvature is important.

The cross-sectional deflections can be characterized by the deflection of the minor axis 'db'. Since cross-sectional and annular deflections are strongly interdependent, anything that affects 'db' will greatly affect 'dR' and 'S'.

The parameters that affect 'db' are 'a', 'b', 't', and 'R0' and the physical significance of each of them is as follows. If only one variable is changed, the cross section becomes more flexible and 'db' increases with:

- an increase of the major axis;
- a decrease of the minor axis;
- a decrease of the wall thickness.

As the radius of curvature increases the cross section becomes less constrained by the annular stiffness and 'db' increases. These relationships can be represented as:

$$db = \frac{f(a) f(R_0)}{f(b) f(t)}$$

Since 'db' is directly related to 'dR' and 'S',

therefore:

$$dR = \frac{f(a)}{f(b)} \frac{f(R_0)}{f(t)}$$

and

$$S = \frac{f(a)}{f(b)} \frac{f(R_0)}{f(t)}$$

If 'E' and 'P' are included in this description, 'db' is found to increase with:

- an increase in 'P';
- a decrease in 'E'.

Therefore the relationship for 'db' becomes:

$$db = \frac{f(a)}{f(b)} \frac{f(R_0)}{f(t)} \frac{f(P)}{f(E)}$$

and

$$dR = \frac{f(a)}{f(b)} \frac{f(R_0)}{f(t)} \frac{f(P)}{f(E)}$$

$$S = \frac{f(a)}{f(b)} \frac{f(R_0)}{f(t)} \frac{f(P)}{f(E)}$$

This understanding of the significance of the physical parameters and how they affect stress and deflection, helps in the understanding of the empirical

equations that are developed by dimensional analysis.

The authors reviewed by this paper have selected several nondimensional groups, such as:

$$b/a, \quad \frac{R_0 t}{a^2}, \quad \frac{dRE}{R_0 P}, \quad dR/R_0, \quad E/P,$$

$$t/a, \quad R_0/a, \quad S/P, \quad R_0/t, \quad \frac{dREbt^3}{R_0 Pa^4}$$

These nondimensional groups are meant to combine related physical parameters into physically understandable ratios. The simpler ratios can be developed through the use of the Buckingham Pi Theorem, while the more complicated ratios are just recombinations of the simpler ratios. The more complicated ratios are meant to reduce the number of independent parameters in the empirical relations developed. When studying these ratios, the easiest to understand in terms of physical significance are the simple ratios: 'b/a', 't/a', 'R₀/a', 'dR/R₀', 'S/P', 'E/P', and 'R₀/t'.

The ratio 'b/a' describes the aspect ratio of the cross section, i.e. the cross-sectional shape.

The ratio 't/a' describes the flexibility of the cross section.

The ratios ' R_0/a ' and ' R_0/t ' describe the flexibility of the sector length of the tube. These ratios relate the sector geometry to the cross-sectional geometry. The Buckingham Pi theorem resolves the dilemma of which of these ratios to use. Since the ratios ' b/a ' and ' t/a ' have already been chosen, the Buckingham Pi theory dictates that the ratio ' R_0/a ' be used to describe the sector flexibility.

Using all three ratios, ' b/a ', ' t/a ', and ' R_0/a ' together in an empirical equation would describe the geometric behaviour of the tube.

The ratio ' dR/R_0 ' describes the change in tube radius of curvature; i.e. the degree of unwinding of the tube, and can be related to tip deflection.

The ratio ' S/P ' represents the stress concentration factor of the tube geometry.

Finally the ratio ' E/P ' follows from the Buckingham Pi theory. When this ratio is grouped with ' S/P ' the resulting ratio ' S/E ' represents strain. The inversion of ' E/P ' to ' P/E ' is done since ' S/P ' is a response variable, ' E/P ' is a predictor variable, and this is a linear elastic problem. Thus ' E/P ' is brought over to the other side of the equation to be

paired with 'S/P'.

The referenced authors recognized the difficulty of correlating all these ratios and therefore they sought to simplify the equations by combining the nondimensional groups. These combined groups are:

$$\frac{R_0 t}{a^2}, \quad \frac{dRE}{R_0 F}, \quad \text{and} \quad \frac{dRE b t^3}{R_0 Pa^4}$$

The main difficulty that has arisen in using these combinations is that certain portions of a combination will dominate the entire ratio, i.e. 'E' and 'a⁴', making that ratio insensitive to any variations of other portions of that ratio. This has led to some difficulties in correlating data, particularly for the more complicated ratios such as

$$\frac{dRE b t^3}{R_0 Pa^4}$$

The ratio $\frac{R_0 t}{a^2}$ was considered to be the combination of the most important geometric parameters, for a constant value of 'b'. Therefore this ratio was called the geometric parameter.

The ratio $\frac{dRE b t^3}{R_0 Pa^4}$ was considered to be combinations of all the physical parameters related to 'dR'. This ratio was called the sensitivity ratio. When combined with $\frac{R_0 t}{a^2}$ these ratios would produce a family of curves for constant 'b/a'.

Since there was a direct relationship between pressure and deflection the ratios dR/R_0 and E/p were combined to give the simpler relation $\frac{dRE}{R_0 P}$. This was used with $\frac{R_0 t}{a^2}$ and b/a .

It was found that the simpler the relation the easier it is to interpret. With the advent of computers it has become easy to analyse many simple parameters in an empirical equation.

2.6: Published Experimental Data

2.6.1 Tests on Deflection of Flat Oval Bourdon Tubes, G. Kardos [42]

This paper was intended to add further experimental data to that already published by other authors, as well as to compare this experimental data against some classical theories and to develop an empirical approach.

The classical theories used in this paper's comparisons were by Wolf and Wuest. Wolf's theory assumed that the radius of curvature approached infinity, while Wuest's theory assumed the axis ratio, b/a , approached zero.

The theory by Wuest used in Kardos' paper is not

the same theory by Wuest presented earlier in this paper.

Seventy flat oval bourdon tubes were tested and the experimental data on tip deflections were correlated against the classical solutions. The experimental data were tabulated, table 15. The test data were found to correlate fairly well with both classical theories.

The experimental data were also plotted on a log/log graph to establish if there were any correlations between dimensionless ratios. Two correlations were found.

While manipulating the data, Kardos found that end effects, caused by different end fittings, had a significant influence on the experimental results and were believed to have caused some of the scatter present in his graphs.

2.6.2. Correlation and Analysis of Bourdon Tube Test Data, G. Kardos [25].

This paper correlates previously published data by using dimensional and correlation analyses.

Kardos was looking for empirical relationships between the physical parameters and the change in

radius of curvature. His approach was to establish the relation of the change in radius of curvature to the nondimensional groups for geometrically similar tubes using power functions. The analysis concentrated on the flat oval bourdon tube, though the elliptical tube was briefly investigated as well.

The physical parameters used were: 'a', 'b', 't', 'R₀', 'P', 'E'. These parameters encompass only two dimensions, length and force. Dimensional analysis therefore requires five dimensionless parameters to be used. From other publications four of the five dimensionless parameters were already in common usage. These dimensionless parameters were: 'dR/R₀', 'P/E', 'a/b', and $\frac{R_0 t}{a^2}$.

Since tip deflection is proportional to pressure, 'dRE/ROP' was used as a dimensionless parameter.

The objective of this paper was to establish values for the power function's coefficients and to establish the form of all the nondimensional parameters.

One dimensionless parameter at a time was incorporated into the empirical equation. Initially, since the coefficients of the empirical equation had not yet been proven to be independent of the other geometric parameters. Once it was shown that there was

only a low dependence of the coefficients on the geometric parameters, correlation analysis was used to develop the empirical equation containing all four unknown dimensionless parameters. Results obtained through application of this empirical equation were found to correlate to within 10% of the experimental data. This was a great improvement over previous empirical equations, which could only predict results within 50% of the experimental data.

Finally Kardos searched for the correct form of the last nondimensional parameter. Previous authors had suggested ' a/R_0 '. This ratio was found to give good results, but it still had a significant error. The nondimensional parameter ' P/R_0 ' was tried and found to be more successful, where ' P ' was the outer perimeter of the tube's cross section. This nondimensional parameter was then used in a polynomial series of the fourth order, to get the best result. However it did not prove possible to conclusively establish this relationship.

2.6.3 Bourdon Tube Deflection Characteristics,

P.G. Exline [39]

The purpose of this paper was to establish an accurate testing procedure for the change in radius of curvature and to develop reliable data for flat oval bourdon tubes to compare with theoretical predictions.

Once all the experimental data was collected, Exline compared the various relationships suggested by Wuest and Kardos. The experimental data is tabulated in table 14. From the results there was found to be considerable scatter except for the power function approach. This scatter was felt to be due to end effects and to the cross section deviating from the standard because of manufacturing variances.

2.7 Statistics [14][15][26]

2.7.1: Stress Model - Solution Convergence

The mesh of the long form finite element model used in this study can be varied in four ways. These are: the number of elements on the flat of the cross-section; the number of elements on the curve of the cross-section; the number of elements through the wall thickness; and the number of elements along the

sector length.

In order to analyse the effects of the finite element model variables on solution convergence, an analysis of variance approach with these model variables can be done. The analysis of variance will indicate the most significant variables and will give an error estimate for each solution. This approach would also supplement experience in obtaining an accurate solution, quickly.

Although this is a desirable approach, it was not used in the research described in this paper since it was recognized after the data were gathered.

2.7.2: Multilinear Regression

Regression analysis fits a mathematical model to experimental data. When a linear relationship appears to correlate two or more variables a linear regression model is used. This model contains two types of variables, a predictor variable and a response variable. The existence of more than one predictor variable for a given response variable requires the use of a multilinear regression model.

Regression model errors occur when the model does not exactly describe the relationship in the data. In

order to get an estimate of the model's error, each parameter must have at least two more values than the total number of parameters in the model. The greater the number of values, though, the better the estimate. Therefore for the two dimensional case where the parameters are 'a', 'b', 't' we need to look at a minimum of five independent solutions having five independent values for each parameter.

To make the data fall in an approximate straight line a log-log data transformation had to be done before a multilinear regression analysis could be undertaken. This data transformation affects the importance of the individual data points with respect to each other. This effect of the transformation is most pronounced in a region of rapid change of the response variable. In this region, when the data is plotted on a regular linear graph, there is a drastic change in slope. When the data in this region has its influence reduced by the transformation, the linear regression routine has difficulty estimating the proper values for the slope and constant of the regression equation.

To solve this problem more data has to be taken throughout this region to increase its influence.

2.7.3: Nonlinear Regression

Another approach to getting the regression models' coefficients is to use nonlinear regression analysis. In this approach the data is taken in its original form, thus there is no change of the data's importance.

This approach is particularly useful when the data cannot be easily transformed to produce a linear relationship.

2.8 Scaling of Geometric Dimensions

The dimensions of real bourdon tubes can be quite small. If these small dimensions are used to develop a finite element model, rounding and truncation errors may occur with the finite element routine. Therefore if these dimensions could be expanded, this type of error would be reduced.

Upon examination of the classical equations the desired magnification of the geometric dimensions was found to be possible, as long as ν , E , and Poisson's ratio remained at their initial values, without affecting the stress behaviour of the tube. The displacement behaviour of the tube would be magnified, though, an equivalent amount with respect to

the geometric dimensions. This means that magnification of the geometric dimensions results in a geometrically similar tube.

Chapter 3: Procedure

3.1 Outline

The sequence of steps followed while analysing the flat oval bourdon tube was: the analysis of the two dimensional cross-sectional case; the analysis of the general three dimensional case; and the development of empirical equations. Since the finite element method was used, mesh generators were constructed for the finite element models to facilitate the creation of the input data for the finite element routine (a commercial finite element program called "ASAS" was used).

The first mesh generator was developed in conjunction with the two dimensional finite element cross-sectional model ("cross-sectional model"). The cross-sectional model was tested by comparing its stress and deflection results against those of the classical theories by Wuest, Weydert, and the thick cylinder theory. Such comparisons were necessary since the cross-sectional model made fewer assumptions than the works of either Weydert or Wuest.

Studying the cross-sectional case helped in the understanding of the basic concepts involved in the general three dimensional case.

The second mesh generator was developed in conjunction with the general three dimensional finite element short form model ("short form model"). As this model was being developed, it was tested against the results from the two dimensional thin wall classical theories ("thin wall theories"). While comparing results, the short form model was found to be inadequate, due to the unexpected influence and the inability of the FEM routine to meet the boundary conditions. To overcome the boundary condition problem of the short form model, a long form model was evolved, along with appropriate modifications to the mesh generator.

The long form model was also tested against the classical thin wall results. Here the long form model behaved considerably better than the short form model, but after an indepth review of all its stress and deflection results the long form model was also found to be inadequate. Again it was found that boundary conditions and the assumptions about them were invalid.

The development of the long form model was halted since acquiring the data was excessively costly. This left the data from the long form model insufficient for extensive empirical equation development.

An attempt was made to develop empirical

equations from published results of experimental tests and classical theories. To get the form of these equations, regression analysis was used with nondimensional parameters. This approach differs from that of Kardos in that these were meant to be global equations, while Kardos developed empirical equations applicable only to geometrically similar tubes.

3.2 Cross-Sectional Model

The first step undertaken was development of the cross-sectional case, figure 7. This case was dealt with first since it was simple, easy to understand, and covered all the basic concepts of stress effects in flat oval bourdon tubes. To help in the understanding of this case, there were several classical theories available for direct comparison.

The cross-sectional model was developed, based on the classical theory assumption of no annular stress. This assumption greatly simplified the analysis and the cross-sectional model, and is valid for straight tubes as well as very large radius of curvature tubes.

Constructing the cross-sectional model was an introductory step toward developing a finite element model for the general three dimensional approach. The cross-sectional model: increased the understanding of

the computerized finite element technique; helped check the understanding of this case; helped check the correlation among the classical theories; indicated whether or not the finite element model could predict the same stresses and displacements as the classical theories; made fewer assumptions; and set a base on which the general three dimensional finite element model could be started.

To represent the cross-sectional case, the finite element model was made to represent the cross section closely. Based on findings from the classical theories, modelling a quarter section was found to be sufficient, due to symmetry about the major and minor axes.

Since the assumption of zero annular stress was used, the cross-sectional model used the highest order membrane element available, which was the eight node membrane element.

The boundary conditions of this model were simplified by the two axes of symmetry. At the major axis the cross section has a tangential force and a bending moment acting on it, with no radial shear present. These loads then require that the boundary condition at the major axis permit only radial displacement. At the minor axis the loads present are

similar to those at the major axis. Therefore also at the minor axis the boundary condition must permit only radial displacements. Since only a cross-sectional slice of a straight tube was analysed in this model, no out of plane movement could be permitted. This required that the boundary conditions permit only in-plane motion.

The finite element routine produced the cross-sectional displacements and stresses for this model. These values were then passed to a post processor to get the principal and effective stresses.

The results obtained from the classical theories and the cross-sectional model were compared. This helped check the cross-sectional model to see if it was performing as it should.

The results of the model were compared with those of Wuest, Weydert, and thick-wall cylinder theory. Calculations were done using a wide range of values for the physical parameters for each approach, allowing direct comparison of results. All three classical theories were used, but particularly the thick-wall cylinder theory, to check the model's results for circular cross sections. For the noncircular cross sections the results from both Wuest and Weydert were used. With these results the location of, as well as,

values for both the effective and principal stresses were checked. This was done since the different classical theories predicted different maximum values and locations for these stresses.

These comparisons were made to make sure the cross-sectional model was behaving as it should, since the cross-sectional model involves fewer approximations. The classical theories, by Wuest and Weydert, both make an approximation of the stress behaviour. Since the cross-sectional model did not make all these approximations, it was necessary to make sure that the stress behaviours were similar.

The results that were compared were the stresses and cross-sectional deflections. The stress values were compared at both the major and minor axes, for the maximum value locations and magnitudes. The cross-sectional displacements were checked since these displacements could also give an idea if the model was behaving correctly.

Basic concepts developed at this stage were: stress patterns; cross-sectional distortion behaviour; stress and distortion variation due to different values for the physical parameters; and the type of assumptions that could be made.

3.3 Short Form Model

Once the cross-sectional model had been satisfactorily developed and checked, the general three dimensional case ("general case") was undertaken. The general case was to be applicable to any tube geometry. As the general case is a very complicated case, the basic construction of its model was facilitated by beginning with the cross-sectional model.

To develop the general case, again the finite element method was used, resulting in general models. These models were based on many of the assumptions made in both the classical cross-sectional and thin wall theories. The assumptions of the thin wall case were included since this approach recognized annular stress. Both these sets of assumptions could be incorporated into the general models because they are special cases of the general case.

The short form model was conceived to be an inexpensive and easy to run three dimensional approach, based on the assumptions that end effects are negligible and that all sections of the tube are similarly constrained, figure 26.

To model the short form a vertical half section of the cross section was used, since the only axis of

symmetry for the general case is about the minor axis.

The finite element used in the general case was the twenty node brick. This element was used since: it was the most general element available; it has a corresponding wedge element in the finite element package; and its front face is identical to that of the eight node membrane element, permitting direct mesh comparisons between the cross-sectional and general mesh models.

The boundary conditions for the short form model were: one cross-sectional face was placed on a fixed plane and on this plane the cross section was allowed to expand; the other (second) cross-sectional face was placed on a movable plane of controllable motion. On this second plane the cross section was allowed to expand, while the movement of this plane was dictated by classical calculations of tip deflection and planar cross-sectional rotation.

While developing the short form model, the finite element routine was found to be unable to represent the required boundary conditions.

To accommodate the boundary condition problems of the short form, an extended model using different boundary conditions was developed.

3.4 Long Form Model

This extended model, the long form model, used boundary conditions similar to those used in the classical thin wall theories while at the same time modelling a sector length long enough, it was hoped, to incorporate St. Venant's principle. While the long form model was based on the short form model, it deviated from the latter in the length of the modelled sector and boundary conditions used. The long form model was made to represent only a segment of the total sector length of the tube, in the expectation that the end effects would be negligible and diminish rapidly, figures 27 and 28.

The boundary conditions of the long form model were: one end of the sector was completely fixed, unable to expand or move; the other end of the sector was completely free to reflect and distort. These boundary conditions were similar to the ones used in many of the classical thin wall theories.

The results from the long form model were then compared to the results from thin wall and cross-sectional theories and approaches. Upon comparison of results it was found that though the stresses and deflections of the long form were of the

same magnitude as those of the classical theories and experimental results, they were not in very close agreement. This lack of agreement was due to the fact that the short sector lengths and the boundary conditions of the long form model were inadequate. It also appeared that the fundamental assumptions used in thin wall theory, such as those concerning end effects, were inadequate for the general model. Because of the large costs of computer runs for the long form model, it was decided to stop work on this approach and to limit further efforts on this case to utilizing the obtained results as well as possible.

The limited long form results were compared against the results of the classical and experimental thin wall and cross-sectional theories. These comparisons were made to check the assumptions and behaviour of the theories and long form model against each other to identify weaknesses in both the theories and the long form model and where long form model improvements could be made. These comparisons were made by analysing the stress and displacement patterns as well as magnitudes in all three principal directions over a broad range of physical parameter values.

3.5 Empirical Equations

To complete the design approach, empirical equations were developed based on the results of the finite element models, classical theories, and published experimental data. To develop these equations regression analysis was used with nondimensional parameters.

In using the regression approach it was discovered that certain statistical requirements must be met. These requirements are: each physical parameter must have a sufficient number of values to allow a good error estimate to be computed; each physical parameter must have a broad range of values; and each physical parameter must have its values randomized to create a case, before solving for stresses and displacements. These requirements were realized only after all the finite element runs had been completed, thus restricting the usefulness of the results obtained from these models.

To overcome this restriction the classical theories and published experimental data were also used to develop empirical equations. Even here there were limitations. The published experimental data were found to have the same shortcomings, but to a lesser

extent, as the finite element results. The restrictions of the published experimental data were that not all the physical parameters had values over a broad enough range.

Upon examination of the empirical equation results obtained from the classical theories, the data was found to deviate somewhat from the hypothesized linear regression model. The transformed data had a slight but significant nonlinearity present. To simulate this nonlinearity, the linear regression model had to be changed to a nonlinear regression model. While studying the behaviour of the nonlinear model, a continued disagreement with the hypothesized regression model was found.

To improve the model further would require an indepth study of the data behaviour. Therefore development of the empirical equations was halted and the data obtained so far was used for recommendations on further empirical equation development.

With all these restrictions on the data available, the empirical equations that were developed give only an indication of what the equations should be.

Chapter 4: Results

4.1 Finite Element Models

4.1.1 Cross-Sectional Model

This model was processed into a useable format for the finite element routine, by the mesh generator P6. The finite element mesh is illustrated in figure 29. Through analysis, this model and program have been found to function correctly.

4.1.2 Short Form Model

The finite element routine was found to be unable to represent the required boundary conditions, therefore this approach was stopped.

4.1.3 Long Form Model

This model's mesh generator is shown as P5, and its finite element mesh is illustrated in figures 30 and 31. After comparison with other results, this model was found to be inadequate.

The long form results indicate that the pronounced end effects were causing the stress and deflection

magnitudes to be too low.

4.2 Finite Element Results

4.2.1 Cross-Sectional Model Results

The results from the cross-sectional model can be separated into four parts, the stress and deflections for both the circular and noncircular cross sections.

For the circular cross sections the maximum values of the circumferential, radial, and effective stresses are compared with the classical results in figures 32 to 34.

For the noncircular cross sections, circumferential, radial, and effective stresses are plotted versus circumferential position in figures 35 to 40. The maximum values of these stresses are plotted on bar charts with other results, figures 41 to 46.

For a detailed reference, all the values used in these graphs are tabulated in tables 16 to 24.

4.2.2 Long Form Model Results

The results from the long form model can be separated into two parts, stresses and deflections, for the noncircular cross section.

The effective, circumferential, and annular stresses are plotted with respect to circumferential position in figures 47 to 54. The maximum values for these stresses were then plotted on barcharts with other results, figures 55 to 62. The annular stress versus circumferential position for one case has also been plotted without the Poisson effect, figure 63. The other stresses were also plotted, for one case, versus annular position, figures 64 to 66.

The deflections of the long form model can be separated into the categories of tip and cross-sectional deflections. Cross-sectional deflections and cross-sectional distortion plots versus annular position, for long and short radii of curvature, are shown in figures 3 and 4. The results for change in radius of curvature are correlated with Andreeva's data in figure 67. The change in radius of curvature, for one case, was also plotted versus annular position, figure 68. The distortion of the cross-sectional plane, for one case, was sketched with

reference to two annular locations, figures 69 and 70. For a detailed reference, all values used in these graphs are in tables 3 to 5, 11, 13, and 25 to 32.

4.3 Classical Theory Results

To facilitate classical theory calculations, four programs were written. P1 is for individual calculations while P2 is for many calculations using Wuest's theory. P3 is for individual calculations while P4 is for many calculations using Andreeva's theory.

4.4 Empirical Equations

4.4.1 - Cross-Sectional Model Empirical Equations

Empirical equations obtained from Weydert's results depicting effective stresses at the major and minor axes are equations 7, to 8 from Appendix C:

$$\begin{array}{l} \text{Major Axis} \\ \text{Effective Stress} \end{array} \quad \left(\frac{S}{P}\right) = 2.1119 \left(\frac{a}{t}\right)^{1.35258} \left(\frac{a}{b}\right)^{1.23862} \quad (7)$$

$$\begin{array}{l} \text{Minor Axis} \\ \text{Effective Stress} \end{array} \quad \left(\frac{S}{P}\right) = 0.61129 \left(\frac{a}{t}\right)^{1.79045} \left(\frac{a}{b}\right)^{2.73102} \quad (8)$$

Empirical equations obtained from Wuest's results

for circumferential, annular, and effective stresses at the major axis are represented in equations 9 to 11 from Appendix C:

$$\text{Major Axis Circumferential Stress} \quad \left(\frac{S}{P}\right) = 7.1708 \left(\frac{a}{t}\right)^{0.35527} \left(\frac{a}{b}\right)^{0.95450} \quad (9)$$

$$\text{Major Axis Annular Stress} \quad \left(\frac{S}{P}\right) = 0.81001 \left(\frac{a}{t}\right)^{0.57965} \left(\frac{a}{b}\right)^{1.38015} \quad (10)$$

$$\text{Major Axis Effective Stress} \quad \left(\frac{S}{P}\right) = 7.6952 \left(\frac{a}{t}\right)^{0.31816} \left(\frac{a}{b}\right)^{0.89175} \quad (11)$$

The finite element cross-sectional empirical equations for circumferential and effective stresses on the major and minor axes are equations 12 to 15 from Appendix C:

$$\text{Major Axis Circumferential Stress} \quad \left(\frac{S}{P}\right) = 1.11283 \left(\frac{a}{t}\right)^{1.14998} \left(\frac{a}{b}\right)^{4.10097} \quad (12)$$

$$\text{Major Axis Effective Stress} \quad \left(\frac{S}{P}\right) = 1.46090 \left(\frac{a}{t}\right)^{1.05418} \left(\frac{a}{b}\right)^{3.80277} \quad (13)$$

$$\text{Minor Axis Circumferential Stress} \quad \left(\frac{S}{P}\right) = 0.93065 \left(\frac{a}{t}\right)^{1.52688} \left(\frac{a}{b}\right)^{5.12012} \quad (14)$$

$$\text{Minor Axis Effective Stress} \quad \left(\frac{S}{P}\right) = 0.48466 \left(\frac{a}{t}\right)^{1.52912} \left(\frac{a}{b}\right)^{5.11861} \quad (15)$$

4.4.2 - Published Experimental Data Empirical Equations

The empirical equation from Smith's deflection data is equation 16 from Appendix C. Kardos' deflection data has been reduced to the empirical equation 17 from Appendix C:

$$\text{Change in Radius of Curvature} \quad \left(\frac{\Delta R}{R_0 P}\right) = 1.19248 \left(\frac{R_0}{a}\right)^{0.28492} \left(\frac{a}{t}\right)^{2.37325} \left(\frac{a}{b}\right)^{0.59711} \quad (16)$$

$$\text{Change in Radius of Curvature} \quad \left(\frac{\Delta R}{R_0 P}\right) = 0.039972 \left(\frac{R_0}{a}\right)^{1.63425} \left(\frac{a}{t}\right)^{2.71647} \left(\frac{a}{b}\right)^{0.78437} \quad (17)$$

4.4.3 Thin Wall Empirical Equations

Empirical equations developed from Hamada's results represent 'dR' and effective stresses at the major axis, in equations 18 and 19 from Appendix C:

$$\text{Change in Radius of Curvature} \quad \left(\frac{\Delta RE}{R_0 P}\right) = 1.3384 \left(\frac{R_0}{a}\right)^{0.32421} \left(\frac{a}{t}\right)^{2.11735} \left(\frac{a}{b}\right)^{0.87222} \quad (18)$$

$$\text{Change in Radius of Curvature} \quad \left(\frac{\Delta RE}{R_0 P}\right) = 2.2205 \left(\frac{R_0}{a}\right)^{0.21894} \left(\frac{a}{t}\right)^{2.15597} \left(\frac{a}{b}\right)^{0.48003} \quad (19)$$

Circumferential, annular, and effective stresses based on Andreeva's equations are represented in empirical equations 20 to 25 from Appendix C. These equations depict stresses at both axes of symmetry. Empirical equations for 'dR', based on Andreeva's results, are represented in equations 26 and 27 from

Appendix C:

$$\text{Major Axis Circumferential Stress} \quad \left(\frac{S}{P}\right) = 0.53010 \left(\frac{R_0}{a}\right)^{0.26923} \left(\frac{a}{t}\right)^{1.84320} \left(\frac{a}{b}\right)^{0.43496} \quad (20)$$

$$\text{Major Axis Annular Stress} \quad \left(\frac{S}{P}\right) = 0.15905 \left(\frac{R_0}{a}\right)^{0.26919} \left(\frac{a}{t}\right)^{1.84319} \left(\frac{a}{b}\right)^{0.43496} \quad (21)$$

$$\text{Major Axis Effective Stress} \quad \left(\frac{S}{P}\right) = 0.47121 \left(\frac{R_0}{a}\right)^{0.26921} \left(\frac{a}{t}\right)^{1.84320} \left(\frac{a}{b}\right)^{0.43494} \quad (22)$$

$$\text{Minor Axis Circumferential Stress} \quad \left(\frac{S}{P}\right) = 0.77784 \left(\frac{R_0}{a}\right)^{0.20308} \left(\frac{a}{t}\right)^{1.89066} \left(\frac{a}{b}\right)^{0.05415} \quad (23)$$

$$\text{Minor Axis Annular Stress} \quad \left(\frac{S}{P}\right) = 0.42037 \left(\frac{R_0}{a}\right)^{-0.13413} \left(\frac{a}{t}\right)^{2.16759} \left(\frac{a}{b}\right)^{0.09034} \quad (24)$$

$$\text{Minor Axis Effective Stress} \quad \left(\frac{S}{P}\right) = 0.69668 \left(\frac{R_0}{a}\right)^{0.18792} \left(\frac{a}{t}\right)^{1.89176} \left(\frac{a}{b}\right)^{0.05563} \quad (25)$$

$$\text{Change in Radius of Curvature} \quad \left(\frac{\Delta R}{R_0}\right) = 1.224 \cdot 10^{-12} \left(\frac{R_0}{a}\right)^{0.26919} \left(\frac{a}{t}\right)^{2.84320} \left(\frac{a}{b}\right)^{0.96116} \quad (26)$$

$$\text{Change in Radius of Curvature} \quad \left(\frac{\Delta RE}{R_0 P}\right) = 0.24534 \left(\frac{R_0}{a}\right)^{0.26587} \left(\frac{a}{t}\right)^{2.84468} \left(\frac{a}{b}\right)^{0.96118} \quad (27)$$

The critical geometry empirical equations for Andreeva's theory are equations 28 to 30 from Appendix C:

$$\text{Circumferential Stress} \quad 1.46735 = \left(\frac{R_0}{a}\right)^{0.06615} \left(\frac{a}{t}\right)^{-0.04746} \left(\frac{a}{b}\right)^{0.3808} \quad (28)$$

$$\text{Annular Stress} \quad 2.64301 = \left(\frac{R_0}{a}\right)^{0.40332} \left(\frac{a}{t}\right)^{-0.3244} \left(\frac{a}{b}\right)^{0.34462} \quad (29)$$

$$\text{Effective Stress} \quad 1.47849 = \left(\frac{R_0}{a}\right)^{0.08129} \left(\frac{a}{t}\right)^{-0.04856} \left(\frac{a}{b}\right)^{0.37931} \quad (30)$$

4.4.4 Long Form Model Empirical Equations

Equations for 'dR' are equations 31 and 32 from

Appendix C:

$$\text{Change in Radius of Curvature} \quad \left(\frac{\Delta R}{R_0}\right) = 0.016291 \left(\frac{R_0}{a}\right)^{0.89157} \left(\frac{a}{t}\right)^{3.43370} \left(\frac{a}{b}\right)^{0.12449} \quad (31)$$

$$\text{Minor Axis Effective Stress} \quad \left(\frac{S}{P}\right) = 0.013258 \left(\frac{R_0}{a}\right)^{1.33845} \left(\frac{a}{t}\right)^{2.33847} \left(\frac{a}{b}\right)^{-0.42565} \quad (32)$$

Circumferential, annular and effective stresses at the minor and major axes are represented in equations 33 to 37 from Appendix C:

$$\text{Major Axis Effective Stress} \quad \left(\frac{S}{P}\right) = 0.15033 \left(\frac{R_0}{a}\right)^{0.57339} \left(\frac{a}{t}\right)^{1.82211} \left(\frac{a}{b}\right)^{0.57891} \quad (33)$$

$$\text{Minor Axis Circumferential Stress} \quad \left(\frac{S}{P}\right) = 7.3911 \cdot 10^{-5} \left(\frac{R_0}{a}\right)^{2.98531} \left(\frac{a}{t}\right)^{3.18287} \left(\frac{a}{b}\right)^{-0.60282} \quad (34)$$

$$\text{Minor Axis Annular Stress} \quad \left(\frac{S}{P}\right) = 0.016465 \left(\frac{R_0}{a}\right)^{1.02829} \left(\frac{a}{t}\right)^{2.45286} \left(\frac{a}{b}\right)^{-0.13902} \quad (35)$$

$$\text{Major Axis Circumferential Stress} \quad \left(\frac{S}{P}\right) = 0.29846 \left(\frac{R_0}{a}\right)^{0.56779} \left(\frac{a}{t}\right)^{1.85274} \left(\frac{a}{b}\right)^{0.52005} \quad (36)$$

$$\text{Major Axis Annular Stress} \quad \left(\frac{S}{P}\right) = 0.077192 \left(\frac{R_0}{a}\right)^{0.69949} \left(\frac{a}{t}\right)^{1.40420} \left(\frac{a}{b}\right)^{1.27883} \quad (37)$$

Chapter 5: Discussion of Results

5.1: Cross-Sectional Model

The cross-sectional model, figures 7 and 29, performs as expected and its results agree with the classical cross-sectional theories, figures 32 to 46.

For circular cross sections all four approaches Cross-Sectional Model; Thick-Wall Cylinder Theory; Wuest's Theory; and Weydert's Theory agree very closely, figures 32 to 34.

For noncircular cross sections the remaining three approaches: Cross-Sectional Model; Wuest's Theory; and Weydert's Theory agree well, figures 41 to 46.

For these cross sections Weydert's Theory agreed with the Cross-Sectional Model more closely than Wuest's Theory did. Weydert's Theory consistently predicted the same maximum effective stress values and locations as the Cross-Sectional Model.

Wuest's theory predicted the same effective stress values as the Cross-Sectional Model, but only at the inner surface of the major axis. Wuest's theory predictions of the circumferential stress, at the major

axis, were slightly higher than the Cross-Sectional Model's predictions at times. These differences were due to Wuest's Theory being used on cross sections outside its applicable region.

These bar charts show very close agreement amongst the theories, for all cross sections studied. Therefore a high level of confidence in the Cross-Sectional Model's behaviour can be maintained.

These bar charts also illustrate that maximum stresses can occur at more than one location.

The objective of this part of the study was to develop a two dimensional finite element model and this has been achieved satisfactorily. The model gives stresses and displacements at every node in the model, giving a more descriptive analysis of the cross-sectional behaviour than is done in the classical theories.

5.2: Short Form Model

This three dimensional approach was conceived to be an improved method for finding stresses after tip deflections and planar rotations have been estimated through classical thin-wall theory. Using classical theory would restrict this approach to thin wall large

radius tubes, since classical techniques are accurate for this type of tube shape/.

This model is meant to simulate a very short sector-length (for economy of computation) of the real tube, figure 26. Since this model is so short and since a steady state condition is desired within it, the rest of the tube length, which is not modelled, must be simulated in the boundary conditions of the model. To simulate the behaviour of the unmodelled portion of the tube the short form model must be provided with tip and planar rotational displacements that are the result of the classical procedures. The remaining movement of the model, the cross-sectional expansion, was meant to be derived by the finite element routine.

The boundary conditions of the short form model were intended to be as follows:

- the model would have had two planes on which the ~~cross section~~ could expand symmetrically about the major axis and the cross section could travel with any movement of these planes;

- the planes would rotate and move and these movements would have been input into the model, from classical calculations;

- one plane would have been rotated and displaced

while the other plane would have remained fixed.

However the boundary conditions that the finite element routine could accept were found to be quite different from what was desired. The finite element routine could accept predetermined nodal displacement and it could also simulate a fixed plane. But the finite element routine could not accept planar movements.

An attempt to circumvent this difficulty was to describe completely all the nodal movements. However this approach relied too heavily on the classical theories, since everything had to be known and predicted about the tube's displacement behaviour before the finite element analysis was started. The results from this type of analysis would then be very similar to the classical theories results. This would mean that the short form model would have no independence from the classical models and therefore it would be no better.

While this approach was being developed, the extent of the end effects was not known or even suspected. Since end effects, as predicted by the long form model, appear to be a very substantial portion of the stress field in tubes, any further work on the short form would not be fruitful.

5.3: Long Form Model

The long form, figures 27, 28, 30 and 31, evolved from the short form to overcome its boundary condition problems. However the boundary conditions used in the long form, though they had been found useful for classical theories, were not suitable for the finite element approach. These assumed conditions did not take into consideration sufficiently, the end conditions resulting in a not completely representative general model. No further investigation was carried out on this model since it became too expensive to run. Considerable reworking of the model appeared to be necessary, and sufficient findings - already had been obtained for this study.

When comparing Wuest's predictions against Andreeva's and the Long Form's predictions, Wuest's predictions always were higher than the others. Particular, for the very short radius of curvature tubes, figures 55 to 62. This difference appears since these tubes are well outside Wuest's region of applicability. The annular stiffness which he does not consider, comes into play. When Wuest's theory is applied properly, figure 62, the results are found to agree well with the other approaches.

The Long Form's results are always higher than Andreeva's results. If Andreeva's results are taken to be the closest to the correct solution, then the difference noticed between the two results can be attributed to the end effects, figures 55 to 62. These approaches agree closely for the cases having large R_0/a and b/a values, ie low annular and high cross-sectional stiffnesses, figures 55 to 62. For these cases it would be expected that the steady state condition is being approached more quickly and closely than the other cases.

For two cases Andreeva's theory predicts a change in the location of the maximum stresses, figures 55. This shift in location is not reflected in the Long Form's results. Since the Long Form is not entirely accurate, no conclusion can be made about this shift in location. Considering Weydert's theory, since this case has a cross section that is close to circular and a large radii of curvature, therefore Weydert's theory suggests this shift in location may be valid. If so a refined Long Form would also reflect this change in location.

These bar charts, figures 55 to 62, do indicate that the Long Form is not all that far off and that it is approaching a valid solution. Examining the stress

patterns developed by Hamada and others also indicates that the concepts used in the Long Form Model are sound. Therefore with the suggested refinements to the Long Form, the results would be expected to agree more closely to the published theories and data, while depicting accurate solutions.

The objective in developing the long form was to develop an accurate general model. The main drawback of the long form model is that it does not model the tube accurately enough, because of the unexpected behaviour of the end effects.

In order to examine the end effects and boundary conditions more thoroughly, while making direct comparisons with the thin wall classical theories and while keeping costs down, a shell model could have been made before going into the general model.

5.4 Programs

5.4.1 Mesh Generators

The mesh generators, programs P5 and P6, were developed as a research tool to facilitate creating input files for the finite element routine. To use these programs the operator must be familiar with their inner workings.

These programs are crude and inefficient. They are initial prototypes which were meant to work and not necessarily be economical. Even though these are prototypes, they do their jobs well.

The mesh generators can be improved to be more effective by incorporating the triangular and wedge elements; by eliminating the large arrays used to store coordinate data; and by changing the mesh to concentrate about the points of maximum stress.

The use of the triangular and wedge elements would permit greater flexibility for a variable density mesh. These elements would potentially make the mesh more cost effective.

Eliminating the large arrays used for storing coordinate data can be done since the output file and the data being generated are both sequential. The coordinate data can therefore be written directly to the output file, upon computation.

The mesh can be concentrated about the points of maximum stress, at the inner surface of the major and at the outer surface of the minor axes. This approach would only be valid if these locations were conclusively established to be the only points of maximum stress.

Any mesh that is generated should be established in a systematic fashion so that the analysis of variance method can be readily incorporated to determine optimum mesh refinement.

5.4.2 Classical Theory Programs

The classical theory programs, programs P1 to P4, were developed as research tools to facilitate the acquisition of large amounts of data, based on these theories. The use of the programs requires the operator to have a good understanding of programs themselves and the applicable theories. These programs have been found to work well.

Weydert's theory was not programmed since his equations, as published, contained typographical errors and were therefore unusable. Data could be obtained from his graphs, a tedious process, so only a limited amount of data from Weydert was obtained.

5.5: Cross-Sectional Deflections

Studying cross-sectional deflections with both the two and three dimensional approaches has given the following results:

- for circular cross sections, the results of the cross-sectional model were verified by thick cylinder

theory;

- for both the two and three dimensional models the noncircular cross sections tend toward a circular shape when inflated;

- for both types of models, the wider the cross section the greater the deflection at the minor axis;

- for both model types, the cross-sectional stiffness is made up of two stiffnesses, the stiffness of the flat and the stiffness of the curve;

- for the three dimensional model, the cross-sectional stiffness affects the annular stress;

- for the three dimensional model, annular stresses distort the cross-sectional plane;

- for the three dimensional model, annular stiffness constrains the cross-sectional distortion.

The deflections of the cross-sectional model, for circular cross sections, were similar to those estimated by thick-wall cylinder theory, a reassuring verification that the cross-sectional model is working correctly. The results indicate that the cross-sectional model behaved as it should, that the circular cross section would uniformly expand.

This verification should have been done also for the long form model, since this verification may have illustrated the model's problem with its boundary

conditions and the extent of the end effects.

Noncircular cross sections, for both the cross-sectional and the long form models, tend toward a circular shape when inflated, figure 3 and 4. This result previously has been predicted by the referenced authors and, it can therefore be used as another check on the behaviour of the models.

As the cross section is inflated, its nonuniformity permits maximum effective and principal stresses and deflections to occur at unique locations on the cross section. These locations have been predicted by previous authors, but no agreement as yet has been reached on where the maximum stresses will be located. The determination of this was beyond the scope of this study.

Wide cross sections have larger cross-sectional deflections than narrow ones, for both model types. This trend has also been illustrated by previous authors and therefore can be used to verify the behaviour of these models.

From the results of both models, the cross-sectional deflections, for noncircular cross sections, can be seen to be nonuniform and as the cross section becomes wider the cross-sectional distortion

takes on a distinctly warped appearance in the shape of an 'S', figure 3. This nonuniform expansion is not commented upon by any of the previous investigators, who considered only uniform cross-sectional deflections. This nonuniformity would increase the difficulty of making deflection or stress predictions.

The wider cross sections obtain the warped 'S' appearance because of the larger difference between the stiffness of the flat and the curve. These wider cross sections also have larger annular stresses and larger circumferential stresses at the axes of symmetry, figures 50, 52, and 53. The latter tendencies, from the model results, are in agreement with the predictions of the previous authors.

The cross section contains two types of stiffness, one stiffness for the flat and one stiffness for the curve. These stiffnesses are not entirely independent of each other. The stiffness of the curve can be considered as an elastic constraint on the stiffness of the flat, with most of the cross-sectional deflections occurring on the flat. As would be expected, any variation in the cross-sectional stiffnesses would result in a variation in the cross-sectional deflection.

Cross sections that have longer flats than other

cross sections also have more flexible flats and thus more flexible cross sections. This results in a higher degree of distortion for this cross section.

There would be a less pronounced effect on the cross sectional distortion if the curve's stiffness were reduced.

These cross-sectional deflections create annular stresses, as predicted by both the long form model and the previous authors. These annular stresses increase when the cross-sectional deflections increase. When the cross-sectional stiffness is high the deflections are low, producing only small strains in the annular direction. For low cross-sectional stiffnesses the cross-sectional distortion is high and large annular strains are induced. This behaviour is predicted by previous authors, and acts as a good check on the long form's behaviour.

The cross-sectional plane is distorted by the annular stresses because of the way these annular stresses are created, i.e. the annular stress is not created uniformly. In a wide and thin-wall tube, as the cross section distorts, the material on the flat near the minor axis deflects outward more than the material on the flat near the curve. Since the material on the flat was initially of the same length,

in the annular direction, and since the material on and close to the curve stays nearly its initial length, the material near the minor axis is strained to a greater extent than the material on and near the curve. This difference in strain occurs since there is considerably more material near and on the curve than there is material near the minor axis. Previous authors consider this annular strain to be enough to keep the cross-sectional plane planar. The results from the long form suggests, but does not conclusively prove, that the cross-sectional plane may actually warp, figures 69 and 70, due to these uneven annular strains. The long form model would have to be improved before this phenomenon could be proven either way.

If the cross-sectional planes don't actually remain plane, then the relationship $\phi/\phi_0 = R/R_0$, used by the referenced authors, cannot be used except on the free end of the tube.

A further result of cross-sectional distortions is that the annular stiffness constrains the cross-sectional distortion. This means that for smaller radius tubes, the annular stress will be relatively higher and the cross-sectional distortion and the circumferential stress will be lower, when compared to a similar large radius tube, figures 3, 4,

50, 52, 53, and 54. This result is not mentioned by any of the previous authors and could be important since the maximum stresses and deflections can be changed by changing the radius of curvature. This indicates that ' R_0 ' could play an important role in stress and deflection prediction. This cannot be verified with the present published data because there has been insufficient variation of ' R_0 ' and ' R_0/a ' in this data. This finding seems to be quite reasonable, though, when placed in the light of the previously mentioned physical interpretation of the geometric parameters.

5.6: End Effects and Boundary Conditions

The results obtained from the long form model indicate that:

- the boundary conditions assumed for the long form model are inappropriate;
- using natural boundary conditions would result in the stresses and deflections peaking in the central portion of the sector;
- end effects are much more pronounced than expected;
- end effects may complicate the empirical prediction equations.

The boundary conditions used in the long form model are inappropriate.

One of the boundary conditions used in the long form model was: one end of the tube was restrained from moving, therefore the cross section at that location was restrained from expanding. The effect of this clamped end has been found to persist for a considerable distance along the tube sector length, figures 64, 65, 66, and 68, which causes stress and tin deflection values to be too low when compared to the estimated steady state condition. The cross section has not expanded to this state.

At the free end, the end effects are taken to be caused by the unconstrained condition represented in the long form model. Of the boundary conditions used in the long form model it is this open ended condition which is least realistic. Since there is no constraint at the end, and thus no load, this portion of the tube sector is calculated to expand and deflect more than it really does, with a corresponding excessive estimate of stresses and deflections, figures 64 to 66.

Real tubes have a plug at the free end which constrains the cross section from expanding and also closes off this end of the tube, while leaving the end free to deflect. Using a plug on the long form model

would make this model much more realistic, permitting a more accurate study of the extent of the end effects. The end effects that would arise from using the plug would be similar to those that arise from the clamped end. These end effects would result in a peaking of cross-sectional deflections and stresses in the central portion of the tube's sector, figures 64 to 66. This type of behaviour has been found by experiment, since fatigue failure has been noted to initiate in this area. These boundary conditions would permit a realistic analysis of any tube length since the tube behaviour would always be accurately modelled.

Previous authors have used the initially assumed boundary conditions, but they have also modelled the complete length of the tube, in which they assumed that a steady state condition was obtained for a major portion of this length. Since the long form model represented only a segment of the tube the end effects became very prominent. Only a segment of the tube was modelled because the end effects were initially considered to be transmitted only a short distance and because of the need to limit the computer costs. A rough estimation of five times the length of the semimajor axis, following from the use of St. Venant's principle, was taken as the length of the long form model sector. In contrast, shell theory predictions

suggest that the end effects could be transmitted through the entire length of the tube and therefore any modelling should incorporate the actual tube length and the natural boundary conditions. However, the shell theory predictions of the end effects were not known until after all the long form data had been gathered.

End effects may complicate prediction of the tube behaviour by causing this behaviour to deviate from the power function relationships which are used for steady state behaviour prediction. These end effects could then create scatter in the data for global tube behaviour predictions. They could even restrict the prediction capability to geometrically similar tubes, such as Kardos has suggested.

5.7: Tip Deflections

The tip deflection results that have been obtained from Andreeva's classical work and the long form model are:

- Andreeva's tip deflection (or change in radius of curvature) calculations agree well with the published experimental and other classical data;
- the long form's tip deflections do not agree well with Andreeva's calculations;
- trends are confirmed in which larger radius

and/or wider tubes have larger tip deflections while smaller radius and/or more circular tubes have smaller tip deflections.

Andreeva's tip deflection calculations agree well with previously published experimental and other classical data. This indicates that Andreeva's methodology is valid for the case she dealt with and her results can be used for comparison with the long form.

The long form tip deflections are less than those estimated by Andreeva and others, figure 67, because the end effects are more pronounced than expected. In general the other techniques are based on the assumption that end effects are negligible.

To avoid the error caused by the fact that the long form model is not realistic from about midsector to the free end, the results for the tip deflection were taken from the midsector length. This location would give the model's best estimate of tip deflection, but since a steady state condition was not yet reached, the values for tip deflection were too low. The interaction of the two end effects and how it would affect these tip deflection values is unknown. It would seem, though, that the clamped end effect would be the most prominent here.

To obtain better values for tip deflection a more realistic model and a more complete understanding of the extent of the end effects are required.

Tip deflection trends found while analysing the data are that large radius and/or wider tubes have larger tip deflections. This trend appears since larger radii and wider tubes have more cross-sectional distortion. The reasons for this were described in the previous section. These trends are also suggested, in a general way, by the previous authors and therefore give a good criterion for further checking the model's performance.

5.8: Principal Stresses

5.8.1: Circumferential Stress

The results obtained from analysis of circumferential stress are:

- the cross-sectional model's stress results agree with thick-wall cylinder theory;
- the maximum circumferential stress is found to occur, for both the cross-sectional and the long form models, at either the major or minor axis;
- circumferential stress patterns are approximately sinusoidal;
- a unique 'bulge' occurs, in the circumferential

stress pattern, for tubes having low cross-sectional and annular stiffnesses.

The cross-sectional model circumferential stress results agree with thick-wall cylinder theory, figures 32 to 34. This result helps verify that the cross-sectional model is actually working properly. This check should have been done also with the long form, but was not. This check with the long form might have highlighted at an early stage some of this model's shortcomings.

The maximum circumferential stress is found to occur, for both the two and three dimensional models, at either the inner surface of the major axis or on the outer surface of the minor axis, figures 32 to 34, 41 to 46, and 55 to 62. This finding is in agreement with some of the classical theories. Wuest predicts the maximum stress to occur only at the inner surface of the major axis, Weydert predicts the maximum stress to occur at either the inner surface of the major axis or the outer surface of the minor axis, while Andreeva predicts the maximum stress to occur anywhere on the cross section. These predictions have not been proven conclusively in this study since they are beyond this study's scope, even though the maximum stresses have been found at both the axes of symmetry.

This study found that the maximum stress occurred at the inner surface of the major axis when the cross-sectional stiffness was low. The maximum stress occurred on the outer surface of the minor axis when the cross-sectional stiffness was high. The critical geometry for which the maximum stress changes location has been suggested only by Weydert. The determination of the critical geometry from the other data is beyond the scope of this study.

Circumferential stress patterns are approximately sinusoidal, figures 35 to 40, and 47 to 54. This finding is in complete agreement with the published data; helping to verify the proper behaviour of the models. A unique 'bulge' occurs, in the circumferential stress pattern, for tubes having low cross-sectional and annular stiffnesses, figure 50. For this tube geometry the cross-sectional distortion is high and the distorted 'S' shape of the cross section, becomes very pronounced. This 'S' shape forces the material of the flat on the convex sides of the 'S' to go into tension while the material of the flat on the concave side of the 'S' is forced into compression. The material nearer to the major-axis must then operate at a higher stress level, increasing the chance of material failure here. This behaviour has not been mentioned by the previous authors and may

warrant further study.

5.8.2: Radial Stress

The results obtained for radial stress, from both models are:

- the cross-sectional model radial stress results agree with thick-wall cylinder theory;
- the long form radial stress variation through the wall thickness, agrees with the approximations of the shell theory;
- radial stress acts as a good check on the model estimations of the other stresses.

The cross-sectional model radial stress results agree with thick-wall cylinder theory. This theory states that the radial stress on the inner surface of the tube must be equal to the internally applied pressure while the radial stress on the outer surface of the tube must be equal to the externally applied pressure. This agreement is another check on how well the cross-sectional model is behaving.

The long form radial stress variation through the wall thickness agrees with shell theory approximations. This finding is true for the thin wall tubes modelled, which helps to verify the long form model behaviour. For the thicker wall tubes modelled by the long form,

the radial stress variation through the wall thickness was not linear but was closer to quadratic. This is the sort of result to be expected from thick-wall cylinder theory and helps to indicate if the model is working correctly.

The radial stress acts as a good check on the model estimations of the other stresses. If the model is behaving correctly the radial stresses on the surfaces would be equal to the applied pressures. Any variation of radial stress from that expected would be due to the model's finite element mesh not being refined enough, thus giving an estimation of how close the model is to a good solution.

5.8.3: Annular Stress

The results found from the long form model on annular stress are:

- annular stress is greatly influenced by the circumferential stress, through the Poisson effect;
- when the Poisson effect is not considered the annular stress appears to be sinusoidal.

Annular stress is greatly influenced by the circumferential stress, through the Poisson effect, figures 50, 52, and 53. With the circumferential stress affecting the annular stress, prediction of this

stress becomes dependent on the cross-sectional geometry. Any empirical equations developed for annular stress can be expected to be very similar to the circumferential stress empirical equations, unless the annular stress is very pronounced on its own, as would be the case for small radius tubes, figure 53.

When the Poisson effect is not considered, the annular stress appears to be sinusoidal, figure 63. This behaviour has been predicted by Andreeva and thus helps verify the performance of the long form model.

5.9: Effective Stresses

The main result obtained concerning effective stresses for both models is, the circumferential stress usually dominates the effective stress pattern, figures 35 to 40, and 47 to 54.

However, while circumferential stress is usually by far the largest of the three stresses, for short radius tubes the annular stresses may be of the same order of magnitude, greatly altering the effective stress pattern, figure 53. Radial stresses, which can normally be neglected, may have to be considered for very large radii tubes of nearly circular cross section, figure 54. For these tubes all three

principal stresses can be of the same order of magnitude, figure 62.

The effective stress is used for tube design purposes when failure by plastic yielding is being considered. When fatigue analysis is being done a simple fatigue failure analysis can be done by using S-N curves and the yield stress (or effective stress). For more complex and accurate fatigue failure analysis fracture mechanics can be used where the principal stresses are of importance.

Since the circumferential stress usually dominates the effective stress pattern, preliminary tube analysis may be made considering this stress only, if the tube's geometry is appropriate, greatly speeding up any design procedure.

5.10 Data Correlations

To show the behaviour of tube data on a log/log plot, Wuest's and Andreeva's data are illustrated in figures 9 to 25. The data shows a distinctive nonlinear behaviour on these plots.

The extrapolations appearing in these plots were taken as linear, but this does not have to be so. Nonlinear extrapolations may be more realistic. The

linearity illustrated, and which is essential for the empirical power function relationships, may only be applicable to a region of cross sections that are not close to circular and not excessively wide. This region would be where neither the effect from the flat or the curve dominates. For near circular cross sections the curve will dominate while for very wide cross sections the flat will dominate.

Both Wuest's and Andreeva's theories illustrate similar nonlinearities, therefore the finite element approaches would also be expected to reflect this nonlinear behaviour. Therefore power functions can be expected to be inappropriate for global empirical equations. For tubes with values within the limited range and for geometrically similar tubes the use of power functions in the empirical equations may be valid.

5.11. Empirical Equations

The statistical data, which accompanies the empirical equations, in appendix C, contains correlation coefficients, equation coefficients and standard error estimates of these coefficients.

The standard error estimates give a good

indication of how well a particular parameter fits the power function relationship being used in these empirical equations.

The correlation coefficient and its related standard error estimate give a very good indication of how well the whole equation fits the data.

The equations used here contained nondimensional groups. The nondimensional groups are useful for developing equations which are both easy to interpret and which can be readily related to the manufacture of tubes.

The general forms of the empirical equations obtained from this analysis were very similar to what would be expected from the physical understanding of the geometric parameters. Any deviation could be attributed to restrictions in theoretical analysis or the inadequacy of the data used.

The form and coefficient values of the empirical equations for different stresses and different locations did not change a great deal while the equations representing effective stress were found to be a composite of the principal stress equations.

While developing the empirical equations the nondimensional groups were found to mask problems with

the data and to give misleading results. This occurs if the data available for physical parameters does not contain sufficient variation and if the values used are not independent.

The empirical equations developed from the classical theories are only as useful as their corresponding theory. Therefore the empirical equations developed from classical theories are no better and definitely not as good as the classical equations. The empirical equations are not as good since the classical data did not lend itself to the power relationships, figures 9 to 25. These equations were developed so that their form could be compared to the other empirical equations developed from experimental and FEM data.

The empirical equations developed using Weydert's data indicate that the use of power functions are acceptable, equations 7 and 8.

For the effective stress on the inner surface of the major axis, the physical parameter that has the most significance is the major axis with the wall thickness being the next most significant, equation 7.

For the effective stress on the outer surface of the minor axis, the physical parameter that has the

most significance is the major axis, with the minor axis being the next most important, equation 8.

The equations developed from Wuest's data indicate that using a power function relationship in the empirical equations is acceptable, equations 9 to 11. From these equations the major axis can be seen to be the most important geometric parameter with the wall thickness being the next most important. The minor axis turns out to be the least important. This relationship is in agreement with the understanding of the physical significance of the geometric parameters and the empirical equations from Weydert's data.

The annular stresses calculated by Wuest are due to the Poisson effect, solely, making them only a slight variation from the circumferential stress, the predominant stress. This dependence is illustrated in the annular stress empirical equations. In these annular stress equations 'a', 'b', and 't' are all considered to be significant, which is not the case for other empirical annular stress equations developed from other data.

The correlations of the cross-sectional model empirical equations are very poor, due to inappropriate data for statistical purposes. Therefore these equations, equations 12 to 15, are not of much value.

Through a study of Hamada's data, high correlations between physical parameters and small variation ranges for these same parameters, were found. These conditions in Hamada's data make his data very difficult to analyse statistically. Even with these restrictions on the data, Hamada's empirical equations still are comparable to Andreeva's. Hamada's empirical equations place most of their emphasis on 'a' and 't', for either stress or change in radius of curvature, equations 18 and 19.

As a source of data, Smith's paper was the best for statistical analysis. Almost all the geometric parameters had a sufficient variation range, except for 'R0'. Since 'R0' was not varied enough, the empirical equations developed give 'R0' no significance, equation 16. It was found as well that 'R0' had a high correlation with 'b' making any relationship developed with 'b' dependent on 'R0' as well.

These results suggest that 'a' and 't' are of near equal magnitude and therefore of nearly equal importance.

Smith's empirical equations also suggest that the power function employed is quite appropriate.

Using Kardos' data, the empirical equations

developed in this paper are similar to the others, equation 17. Empirical equations developed from his data suggest that the major axis is not as important as is implied by the other empirical equations. These equations state that 't' is the most important parameter with 'a' being the next most important and 'b' being the least. The minor axis is given more significance here than elsewhere though. The empirical equations developed from Kardos' data are not all that similar to Kardos' own published empirical equations, since Kardos' own equations were for geometrically similar tubes and these equations are for a global set of tube geometries.

When using all the tubes any variation due to manufacture will cause scatter in the data for the global equations. Tubes that are similar would also have similar fabrication variation and therefore data for empirical equations of geometrically similar tubes would have a smaller degree of error. This results in Kardos' own equations correlating very well for his approach.

Andreeva's empirical equations agree with an intuitive understanding of the physical behaviour of the tube, equations 20 to 30.

As in the other empirical equations 'a' and 't'

are paramount in significance. It would seem that these two geometric parameters describe the tube flexibility the best. This also suggests that annular stress anywhere in the cross section is mostly affected by the cross-sectional geometry and not by the radius of curvature. Andreeva's analytical equations also suggest that the annular stress is largely but not entirely derived from the circumferential stress, through the Poisson effect. Long Form cases, equations 31 to 37, indicate that this is true only for large radius tubes. The physical understanding of the parameters could not differentiate between 'a' and 't' as which one is the more important. This requires further study.

When comparing all the empirical equations dealing with the change in radius of curvature the coefficients are very similar. This suggests that the general behaviour of the data is the same and that the Long Form is behaving approximately correctly. The variation in the empirical equations amongst the previously published data illustrates the effect of an inappropriate empirical model and insufficient data.

When comparing all the empirical equations developed from Andreeva's, Smith's, and Hamada's data the equation's coefficients are found to vary for

different locations and different stresses. This variation is expected since the different stresses at different locations are created by a unique interplay of the physical parameters. This then supports the impression that, to describe most completely the behaviour of the tube, a whole set of empirical equations would be required.

The empirical equations developed are not directly applicable now, but once the models have been improved the corresponding equations could very well be more accurate and more useful than any of the other available equations. With improved models the main advantage of these empirical equations could be exploited, which is their ease of use.

Chapter 6: Conclusions

6.1: Published Theories and Data

Each of the studies considered in this paper highlighted a specific area illustrating some difficulty which must be considered. The combination of all the studies and topics covered give a solid background of what has been done and why, and what can be done and how, to develop predictive equations. A base was then established for:

- guiding the development of this paper's approach;
- for verifying this paper's results;
- for determining the significance of the work done;
- for developing recommendations for improvements to this work.

6.2: Finite Element Models

The cross-sectional model has been found to work satisfactorily. Its results behave as predicted by thick cylinder theory and cross-sectional analytical theory.

The short form model should only be pursued if another FEM routine could be found which would simulate

the required boundary conditions. The boundary conditions chosen for this model have been found to be inappropriate for analysing three dimensional stresses in tubes, with the present FEM routine.

The long form model has been found to work partially correctly, but the boundary conditions used at its free end were not appropriate for this approach. This model should have used a plug at the free end, in order to perform suitably. The use of St. Venant's principle was also found to be unsuitable since the end effects were much more pronounced than expected and occupied the entire length of the modelled sector, therefore the entire length of the tube should be modelled. Lastly the long form model was very expensive to run and, it should be altered to make it more economical.

6.3: Cross-Sectional Deflections

The two and three dimensional cross-sectional deflection trends are very similar.

Wider cross-sections have more intricate cross-sectional deflection patterns, due to the more pronounced cross-sectional stiffness difference between the flat and the curve.

6.4: Tip Deflections

Andreeva's tip deflection calculations agreed very well with previous experimental and analytical results. The tip deflections predicted by the three dimensional model were low.

These tip deflections were too low because the end effects made the finite element model stiffer than expected.

6.5: Stresses

The trends of the two and three dimensional stresses were very similar. The biggest difference between the two approaches was caused by the annular stresses. This third stress affected the cross-sectional distortion, which in turn affected the other two principal stresses. The effect of the annular stress was to decrease the magnitude of the cross-sectional displacements and the cross-sectional stresses.

The effective stresses are greatly affected by the 'circumferential principal stresses.' For many geometries the annular stress also plays a very significant role in determining the effective stress. The annular stress has a most pronounced effect on the effective stress for small ' R_0/a ' ratios and large

'a/b' ratios.

What complicates the three dimensional problem the most is the interaction of the principal stresses. This interaction is affected by the cross-sectional distortion, the variation in the cross-sectional stiffness, the annular stiffness, and the end effects. In order to analyse the three dimensional problem accurately it is necessary to use the natural boundary conditions and to make no assumptions about the tubes behaviour.

Further work will have to be done to get quantitative results.

6.6: End Effects and Boundary Conditions

Previous authors assumed that the end effects were negligible. The assumption that end effects diminish quickly is not appropriate for the long form model. It has been found that the end effects occupy a major portion of the tubes sector length.

Cross-sectional planes remaining planar is another assumption which appears to be in question. This question has not been settled conclusively though.

6.7: Applicability of Two and Three Dimensional Theories

The two dimensional method is a good approach for many useful bourdon tube shapes. For a first try at a tube's design, the two dimensional method is cheap, easy to use and easy to understand.

The three dimensional method would be a good method to use for the final analysis. This method could give accurate results for any configuration, since it would use no approximations.

6.8: Empirical Equations

The form of the empirical equations developed with the nondimensional groups does agree with the intuitive understanding of the tubes behaviour.

Of the four geometric parameters, 'a' and 'r' have been found to be the most significant. These two parameters also appear to be of nearly equal importance.

The data obtained from the finite element models was unsuitable for empirical equation development. Once this data is improved, potentially very representative global equations can be obtained.

The slight variations in geometry of actual

tubes, arising from manufacturing may hinder the experimental development of global equations, since they cause a scattering of global equation data. This scatter could restrict empirical equations to geometrically similar tubes, where this scatter is reduced.

- The empirical equations can also be improved through the use of a better empirical model.

Chapter 7: Recommendations

The following recommendations have been determined for future research on the development of a general design solution simple enough for routine design of tubes. These recommendations are:

- to produce more data from the cross-sectional model to complete the development of its global empirical equations;

- to develop a shell model to study, further, end effects (in a cost effective manner) and to duplicate thin wall classical theory results;

- to develop empirical equations for the shell model for comparison against the other empirical equations that have been developed;

- to rework the long form to include all the natural boundary conditions and to also make it more cost effective;

- to study end effects, and stress and deflection behaviour of the entire tube;

- to produce data for global general three dimensional empirical equations;

- to use all the physical parameters to develop as complete a set of global equations as possible, i.e.:

'a', 'b', 't', 'RD', 'E', 'P', 'Mew';

- to use an improved empirical model to better

fit the curvilinear data.

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Appendix A: Figures

A

Figure 1: Physical Parameters of Flat Oval Cross Section

Figure 2: Physical Parameters of Flat Oval Bourdon Tube,
Side View

Figure 3: Large Radius of Curvature Cross-Sectional Distortion
(Case 13)

1

Figure 4: Short Radius of Curvature Cross-Sectional Distortion
(Case 16)

Figure 5: Tip Deflection and Change in the Radius of Curvature

Figure 6: Bourdon Tube Gauge

Figure 7: Free Body Diagram of Flat Oval Quadrant

Figure 8: Weydert's Photoelastic Result for a Flat Oval Pressure Vessel

Figure 9 , Wueat's Data, Effective Stress

Figure 10 : Wuest's Data; Circumferential Stress

Figure 11 : West's Data; Annular Stress

Figure 12 : Andreeva's Data; Change in Radius of Curvature $\frac{\Delta R_E}{R_0^P}$

$$\frac{R_0}{a} = 5$$

Figure 13 : Andreeva's Data; Effective Stress at the Major Axis

Figure 14 : Andreeva's Data; Circumferential Stress at the Major Axis

Figure 15 : Andreeva's Data; Annular Stress at the Major Axis

$$\frac{R_0}{a} = 5$$

Figure 16 : Andreeva's Data; Effective Stress at the Minor Axis

Figure 17: Andreeva's Data; Circumferential Stress at the Minor Axis

Figure 18 : Andreeva's Data; Annular Stress at the Minor Axis

$$\frac{R_0}{a} = 5$$

Figure 19 : Andreeva's Data; Change in Radius of Curvature $\frac{\Delta R}{R_0}$

$$\frac{R_0}{a} = 10$$

Figure 20 : Andreeva's Data, Effective Stress at the Major Axis

Figure 21: Andreaga's Data: Circumferential Stress at the Major Axis

Figure 22 : Andreeva's Data; Annular Stress at the Major Axis

$$\frac{R_0}{a} = 10$$

Figure 23 : Andreeva's Data; Effective Stress at the Minor Axis

Figure 24 : Andreeva's Data; Circumferential Stress at the Minor Axis

$$\frac{R_0}{a} = 10$$

Figure 25 : Andreeva's Data: Annular Stress at the Minor Axis

Figure 26: Free Body Diagram of Short Form Model

Figure 27: Free Body Diagram of Flat Oval Half Section

Figure 28: Free Body Diagram of Long Form Model

Figure 29: Cross-Sectional Finite Element Model

Figure 30: Long Form Finite Element Model, Cross Section

Figure 31: Long Form Finite Element Model, Side View

Figure 32: Stress/Pressure

Case 1

Figure 33: Stress/Pressure

Case 2

Figure 34: Stress/Pressure

Cross-Sectional Model

Figure 35 Case 4 a)

Effective and Principal Stresses
vs Circumferential Position

Physical Parameters

$$a = 100 \text{ mm}$$

$$b = 83.3 \text{ mm}$$

$$t = 91.3 \text{ mm}$$

$$E = 2.0 \times 10^{11} \text{ Pa}$$

$$P = 1.0 \text{ Pa}$$

$$a/b = 1.2$$

$$a/t = 1.1$$

Circumferential Positions

(cont.)

Figure 35 (cont.) b)

- (cont.)

Figure 35 (cont.) c)

(cont.)

Figure 35 (cont.) d)

Cross-Sectional Model

Figure 36: Case 5 a)

Effective and Principal Stresses
vs Circumferential Position

Physical Parameters

$$a = .100 \text{ mm}$$

$$b = 83.3 \text{ mm}$$

$$t = 18.3 \text{ mm}$$

$$E = 2.0 \times 10^{11} \text{ Pa}$$

$$P = 1.0 \text{ Pa}$$

$$a/b = 1.2$$

$$a/t = 5.46$$

Circumferential Positions

(cont.)

Figure 36 (cont.) b)

(cont.)

Figure 36 (cont.) c)

(cont.)

Figure 36 (cont.) d)

Cross-Sectional Model

Figure 37: Case 6 a)

Effective and Principal Stresses
vs Circumferential Position

Physical Parameters

$a = 100 \text{ mm}$

$b = 83.3 \text{ mm}$

$t = 10.1 \text{ mm}$

$E = 2.0 \times 10^{11} \text{ Pa}$

$P = 1.0 \text{ Pa}$

$a/b = 1.2$

$a/t = 9.9$

Circumferential Positions

(cont.)

Figure 37 (cont.) b)

(cont.)

Figure 37 (cont.) c)

-(cont.)

Figure 37 (cont.) d)

Cross-Sectional Model

Figure 38: Case 7 a)

Effective and Principal Stresses
vs Circumferential Position

Physical Parameters

$a = 100 \text{ mm}$

$b = 71.4 \text{ mm}$

$t = 84.5 \text{ mm}$

$E = 2.0 \times 10^{11} \text{ Pa}$

$P = 1.0 \text{ Pa}$

$a/b = 1.4$

$a/t = 1.18$

Circumferential Positions

(cont.)

Figure 38 (cont.) b)

(cont.)

Figure 38 (cont.) c)

(cont.)

Figure 38 (cont.) d)

Cross-Sectional Model

Figure 39: Case 8 a)

Effective and Principal Stresses
vs Circumferential Position

Physical Parameters

$$a = 100 \text{ mm}$$

$$b = 71.4 \text{ mm}$$

$$t = 16.9 \text{ mm}$$

$$E = 2.0 \times 10^{11} \text{ Pa}$$

$$P = 1.0 \text{ Pa}$$

$$a/b = 1.4$$

$$a/t = 5.92$$

Circumferential Positions

(cont.)

Figure 39 (cont.)

b)

(cont.)

Figure 39 (cont.) c)

(cont.)

Figure 39 (cont.) d)

Cross-Sectional Model

Figure 40: Case 9 a)

Effective and Principal Stresses
vs. Circumferential Position

Physical Parameters

- $a = 100 \text{ mm}$
- $b = 71.4 \text{ mm}$
- $t = 9.4 \text{ mm}$
- $E = 2.0 \times 10^{11} \text{ Pa}$
- $P = 1.0 \text{ Pa}$
- $a/b = 1.4$
- $a/t = 10.6$

Circumferential Positions

(cont.)

Figure 40 (cont.)

b)

(cont.)

Figure 40 (cont.) c)

(cont.)

Figure 40 (cont.) d)

Figure 41: Stress/Pressure

Case 4

Figure 42: Stress/Pressure

Case 5

Figure 43: Stress/Pressure

Case 6

Figure 44: Stress/Pressure

Case 7

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Figure 45: Stress/Pressure

Case 8

Figure 46: Stress/Pressure

Case 9

Long Form Model

Figure 47: Case 10 a)

Effective and Principle Stresses
vs Circumferential Position

Physical Parameters

$$a = 100 \text{ mm}$$

$$b = 50 \text{ mm}$$

$$t = 7.9 \text{ mm}$$

$$R_0 = 1000 \text{ mm}$$

$$E = 2.0 \times 10^{11} \text{ Pa}$$

$$P = 1.0 \text{ Pa}$$

$$a/b = 2.0$$

$$a/t = 12.7$$

$$R_0/a = 10$$

Circumferential Positions

(cont.)

Figure 47: (cont.) b)

(cont.)

Figure 47: (cont.) c)

(cont.)

Figure 47: (cont.) d)

-(cont.)

Figure 47: (cont.) . e)

(cont.)

Figure 47: (cont.) f)

(cont.)

Figure 47: (cont.) g)

Long Form Model

Figure 48: Case 11 a)

Effective and Principal Stresses
vs Circumferential Position

Physical Parameters

$$a = 100 \text{ mm}$$

$$b = 33.3 \text{ mm}$$

$$t = 6.4 \text{ mm}$$

$$R_0 = 1000 \text{ mm}$$

$$E = 2.0 \times 10^{11} \text{ Pa}$$

$$P = 1.0 \text{ Pa}$$

$$a/b = 3.0$$

$$a/t = 15.6$$

$$R_0/a = 10.0$$

Circumferential Positions

(cont.)

Figure 48: (cont.) b)

(cont.)

Figure 48: (cont.) c)

.(cont.)

Figure 48: (cont.) d)

(cont.)

Figure 48: (cont.) e)

(cont.)

Figure 48: (cont.) f)

(cont.)

Figure 48: (cont.) g)

Long Form Model

Figure 49: Case 12 a)

Effective and Principal Stresses
vs Circumferential Position

Physical Parameters

$$a = 100 \text{ mm}$$

$$b = 25 \text{ mm}$$

$$t = 5.6 \text{ mm}$$

$$R_0 = 1000 \text{ mm}$$

$$E = 2.0 \times 10^{11} \text{ Pa}$$

$$P = 1.0 \text{ Pa}$$

$$a/b = 4.0$$

$$a/t = 17.9$$

$$R_0/a = 10.0$$

Circumferential Positions

(cont.)

Figure 49: (cont.) b)

(cont.)

Figure 49: (cont.) c)

(cont.)

Figure 49: (cont.) d)

(cont.)

Figure 49: (cont.) e)

(cont.)

Figure 49: (cont.) f)

(cont.)

Figure 49: (cont.) g)

Long Form Model

Figure 50: Case 13 a)

Effective and Principal Stresses
vs Circumferential Position

Physical Parameters

$$a = .100 \text{ mm}$$

$$b = 20 \text{ mm}$$

$$t = 5 \text{ mm}$$

$$R_0 = 1000 \text{ mm}$$

$$E = 2.0 \times 10^{11} \text{ Pa}$$

$$P = 1.0 \text{ Pa}$$

$$a/b = 5.0$$

$$a/t = 20.0$$

$$R_0/a = 10.0$$

Circumferential Positions

(cont.)

Figure 50: (cont.) b)

(cont.)

Figure 50: (cont.) - c)

(cont.)

Figure 50: (cont.) d)

(cont.)

Figure 50: (cont.) e)

(cont.)

Figure 50: (cont.) f)

(cont.)

Figure 50: (cont.) g)

Long Form Model

Figure 51: Case 14 a)

Effective and Principal Stresses
vs Circumferential Position

Physical Parameters

$$a = 70 \text{ mm}$$

$$b = 20 \text{ mm}$$

$$t = 5 \text{ mm}$$

$$R_0 = 700 \text{ mm}$$

$$E = 2.0 \times 10^{11} \text{ Pa}$$

$$P = 1.0 \text{ Pa}$$

$$a/b = 3.5$$

$$a/t = 14$$

$$R_0/a = 10$$

Circumferential Positions

.(cont.)

Figure 51: (cont.) b)

(cont.)

Figure 51: (cont.) c)

(cont.)

Figure 51: (cont.) d)

(cont.)

Figure 51: (cont.) e)

(cont.)

Figure 51: (cont.) f)

(cont.)

Figure 51: (cont.) g)

Long Form Model

Figure 52: Case 15 a)

Effective and Principal Stresses
vs Circumferential Position

Physical Parameters

$$a = 100 \text{ mm}$$

$$b = 20 \text{ mm}$$

$$t = 5 \text{ mm}$$

$$R_0 = 500 \text{ mm}$$

$$E = 2.0 \times 10^{11} \text{ Pa}$$

$$P = 1.0 \text{ Pa}$$

$$a/b = 5$$

$$a/t = 20$$

$$R_0/a = 5$$

Circumferential Positions

(cont.)

Figure 52: (cont.) b)

(cont.)

Figure 52: (cont.) c)

(cont.)

Figure 52: (cont.) d)

(cont.)

Figure 52: (cont.) e)

(cont.)

Figure 52: (cont.) f)

(cont.)

Figure 52: (cont.) g)

Long Form Model

Figure 53: Case 16 a)
Effective and Principal Stresses
vs Circumferential Position

Physical Parameters

$$a = 100 \text{ mm}$$

$$b = 20 \text{ mm}$$

$$t = 5 \text{ mm}$$

$$R_0 = 300 \text{ mm}$$

$$E = 2.0 \times 10^{11} \text{ Pa}$$

$$P = 1.0 \text{ Pa}$$

$$a/b = 5$$

$$a/t = 20$$

$$R_0/a = 3$$

Circumferential Positions

(cont.)

Figure 53: (cont.) b)

(cont.)

Figure 53: (cont.) c)

(cont.)

Figure 53: (cont.) d)

(cont.)

Figure 53: (cont.) e)

(cont.)

Figure 53: (cont.) f)

(cont.)

Figure 53: (cont.) g)

Long Form Model

Figure 54: Case 17 a)
Effective and Principal Stresses
vs Circumferential Position

Physical Parameters

$$a = 45 \text{ mm}$$

$$b = 20 \text{ mm}$$

$$t = 7.5 \text{ mm}$$

$$R_0 = 700 \text{ mm}$$

$$E = 2.0 \times 10^{11} \text{ Pa}$$

$$P = 1.0 \text{ Pa}$$

$$a/b = 2.25$$

$$a/t = 6$$

$$R_0/a = 15.6$$

Circumferential Positions

(cont.)

Figure 54: (cont.) b)

(cont.)

Figure 54: (cont.) c)

(cont.)

Figure 54: (cont.) d)

(cont.)

Figure 54: (cont.) e)

(cont.)

Figure 54: (cont.) f)

(cont.)

Figure 54: (cont.) g)

Figure 55: Stress/Pressure

Case 10

Figure 56: Stress/Pressure

Case 11

Figure 57: Stress/Pressure

Case 12

Figure 58: Stress/Pressure

Case 13

S_a - maximum stress on inner surface of major axis
 S_b - maximum stress on outer surface of minor axis

Figure 59: Stress/Pressure

Case 14

Figure 60: Stress/Pressure

Case 15

Figure 61: Stress/Pressure

Case 16

Figure 62: Stress/Pressure

Case 17

Figure 63: Inner Annular Stress with Poisson's Effect Removed vs Circumferential Position, for a Large Radius of Curvature (Case 17)

Figure 64: Inner Circumferential Stress vs Annular Position,
Case 17

Figure 65: Inner Radial Stress vs Annular Position,
Case 17

Figure 66. Inner Annular Stress vs Annular Position.
Case 17.

Figure 67: $\frac{\Delta RE}{R_0 P}$

Figure 68: Change in Radius of Curvature vs. Annular Position,
Case 13

Figure 69: Planar Cross-Sectional Distortion at 15° Plane
From Fixed Tube End

Notes: - longitudinal distortion of the cross-sectional plane shown.
- cross-sectional expansion due to inflation ignored here for simplification and clarity.

Figure 70, Planar Cross-Sectional Distortion at 6° Plane
From Fixed Tube End

Notes: - longitudinal distortion of the cross-sectional plane shown
- cross-sectional expansion due to inflation ignored here for
simplification and clarity.

Appendix 8: Tabulated Results

Table 1: Physical Parameters

Case	a (mm)	b (mm)	t (mm)	R0 (mm)	Phi0 (degrees)
1	100.0	100.0	100.0	-	-
2	100.0	100.0	20.0	-	-
3	100.0	100.0	11.1	-	-
4	100.0	33.3	21.3	-	-
5	100.0	83.3	18.3	-	-
6	100.0	83.3	10.1	-	-
7	100.0	71.4	84.5	-	-
8	100.0	71.4	16.9	-	-
9	100.0	71.4	9.4	-	-
10	100.0	50.0	7.9	1000.0	30.0
11	100.0	33.3	6.4	1000.0	30.0
12	100.0	25.0	5.6	1000.0	30.0
13	100.0	20.0	5.0	1000.0	30.0
14	70.0	20.0	5.0	700.0	42.0
15	100.0	20.0	5.0	500.0	60.0
16	100.0	20.0	5.0	300.0	100.0
17	45.0	20.0	7.5	700.0	20.0

$$E = 2.0E+11 \text{ Pa}$$

$$P = 1 \text{ Pa}$$

$$MEW = .3$$

Table 2: Dimensionless Ratios

Case	b/a	t/a	R0/a	R0/t
1	1.0	1.0	-	-
2	1.0	0.2	-	-
3	1.0	0.11	-	-
4	0.83	0.01	-	-
5	0.83	0.18	-	-
6	0.83	0.10	-	-
7	0.71	0.84	-	-
8	0.71	0.17	-	-
9	0.71	0.09	-	-
10	0.50	0.08	10.0	126.6
11	0.33	0.06	10.0	156.3
12	0.25	0.06	10.0	178.6
13	0.20	0.05	10.0	200.0
14	0.29	0.07	10.0	140.0
15	0.20	0.05	5.0	100.0
16	0.20	0.05	3.0	60.0
17	0.44	0.17	15.6	93.3

Table 3: Long Form Tip Deflections

Case	(*E-07) (mm)	(*E-07) (mm)	(*E-07) (mm)	DRE/ROP	DR/RO
10	1.4	2.4	41.	819.	0.20
11	3.4	5.8	100.	1997.	0.18
12	4.8	8.1	140.	2807.	0.12
13	7.7	1300.	230.	4504.	0.11
14	2.4	300.	51.	1462.	0.15
15	8.7	370.	64.	2551.	0.06
16	8.7	130.	23.	1549.	0.04
17	0.052	20.	3.4	98.	0.20

Table 4: Andreeva's Tip Deflection Results

Case	(*E-07) (mm)	(*E-07) (mm)	DRE/ROP	DR/RO
10	380.	65.	1300.	0.32
11	990.	170.	3410.	0.30
12	1800.	310.	6257.	0.27
13	2800.	490.	9748.	0.24
14	610.	110.	3023.	0.31
15	710.	120.	4903.	0.12
16	200.	34.	2254.	0.06
17	38.	6.6	189.	0.38

Table 5: Wuest's Stress Results (Pa); Case Study

Case	Circumferential Sa	Radial Sa	Annular Sa	Effective Sa
1	1.67	-1.0	0.33	2.3
2	5.55	-1.0	2.3	5.7
3	9.53	-1.0	4.3	9.1
4	2.68	-1.0	0.64	3.2
5	18.95	-1.0	6.3	17.5
6	50.04	-1.0	16.45	44.5
7	3.68	-1.0	0.94	4.1
8	32.6	-1.0	10.4	29.5
9	89.9	-1.0	28.4	80.4
10	208.1	-1.0	63.8	185.4
11	407.9	-1.0	123.6	363.0
12	589.5	-1.0	178.0	524.4
13	780.5	-1.0	235.2	694.2
14	356.3	-1.0	107.8	317.2
15	780.5	-1.0	235.2	694.2
16	780.5	-1.0	235.2	694.2
17	60.62	-1.0	18.6	54.5

Table 6: Wuest's Data for the Empirical Equations

a (mm)	b (mm)	t (mm)	P (Pa)	S2 (Pa)	S3 (Pa)	Sv (Pa)
54.0	26.0	16.4	2.32	49.51	15.03	45.70
58.0	23.6	10.4	3.64	207.9	63.42	187.3
68.0	30.8	5.6	1.96	412.0	126.0	367.1
66.0	34.4	12.2	1.72	75.84	23.43	68.54
76.0	20.0	19.4	1.48	60.76	18.21	55.10
62.0	46.4	15.2	3.28	53.29	17.02	49.63
52.0	33.2	9.2	3.52	129.7	40.76	117.5
72.0	47.6	20.0	1.6	27.06	8.456	25.18
96.0	44.0	7.4	1.36	320.0	97.80	285.0
50.0	42.8	11.6	2.56	30.90	10.35	29.23
100.0	27.2	6.2	3.4	1607.	485.8	1430.
64.0	50.0	14.0	4.0	69.80	22.68	64.72
56.0	28.4	17.6	1.12	21.63	6.574	20.05
60.0	32.0	11.0	3.04	132.7	41.06	119.9
82.0	21.2	18.8	1.84	90.21	27.10	81.52
86.0	24.8	5.0	1.0	520.9	157.5	463.5
90.0	48.8	15.8	2.92	134.7	41.75	121.6
74.0	41.6	18.2	2.08	52.88	16.34	48.45
94.0	29.6	6.8	2.2	733.9	222.4	653.4
88.0	40.4	12.8	3.16	236.2	72.51	211.9
78.0	35.6	8.6	3.88	476.2	146.0	425.4
98.0	39.2	14.6	2.68	212.8	65.00	190.9
92.0	36.8	8.0	3.76	782.6	238.5	697.5
70.0	45.2	13.4	2.9	88.89	27.97	80.82
80.0	22.4	9.8	1.24	168.1	50.92	150.2
84.0	38.0	17.0	2.44	105.1	32.18	95.07

Table 7: Weydert's Stress Results (Pa); Case Study

Case	Effective	
	Sa	Sb
1	1.8	-
2	5.6	-
3	9.0	-
4	-	3.0
5	21.8	-
6	82.0	-
7	-	4.2
8	34.0	-
9	104.0	-

Table 8: Weydert's Results for the Empirical Equations

a (mm)	b (mm)	t (mm)	Sa (Pa)
98.	44.	30.	29.
94.	40.	10.	132.
96.	41.	18.	57.
92.	50.	42.	13.
100.	47.	34.	23.
80.	49.	14.	41.
86.	43.	38.	14.
84.	45.	50.	9.
90.	46.	46.	14.
88.	42.	26.	26.
82.	48.	22.	24.

a (mm)	b (mm)	t (mm)	Sb (Pa)
84.	60.	12.5	44.
90.	64.	11.	64.
98.	70.	14.	48.
100.	76.	12.	60.
80.	62.	13.	33.
82.	72.	10.	35.
94.	74.	14.5	34.
92.	80.	15.	22.
86.	68.	11.5	45.
96.	78.	13.5	39.
88.	66.	10.5	62.

Table 9: Cross-Sectional Finite Element Stress Results (Pa)

Case	Circumferential		Radial		Effective	
	Sa	Sb	Sa	Sb	Sa	Sb
1	1.68	0.67	-0.97	0.0	2.32	0.67
2	5.55	4.55	-0.98	0.01	6.10	4.54
3	9.54	8.54	-0.96	0.04	10.50	8.52
4	2.68	1.58	-0.94	0.03	3.26	1.56
5	18.30	22.59	-0.92	0.14	18.78	22.52
6	45.69	64.41	-0.23	0.06	45.90	64.38
7	3.68	2.34	-0.91	0.03	4.21	2.33
8	31.50	36.41	0.42	0.03	31.29	36.40
9	82.81	107.70	0.74	-0.06	82.44	107.99

Table 10: Thick Cylinder Theory Stress Results (Pa)

Case	Circumferential		Radial		Effective	
	Sa	Sb	Sa	Sb	Sa	Sb
1	1.667	0.667	-1.0	0.0	2.33	0.667
2	5.55	4.55	-1.0	0.0	6.11	4.55
3	9.54	8.54	-1.0	0.0	10.07	8.54

Table 11: Andreeva's Stress Results (Pa)

Case	Circumferential		Annular		Effective	
	Sa	Sb	Sa	Sb	Sa	Sb
10	150.7	161.9	45.2	87.5	133.9	148.4
11	276.4	250.2	82.9	147.9	245.7	217.9
12	372.1	308.9	111.6	195.7	330.7	270.7
13	461.9	362.5	138.6	159.2	410.5	314.7
14	248.8	210.4	74.6	118.7	221.1	182.7
15	233.5	205.6	70.0	197.4	207.5	201.6
16	107.5	108.4	32.2	136.6	95.5	124.9
17	44.3	41.9	13.3	15.7	39.4	36.7

Table 12: Hamada's Data

Case	E (kg/mm)	mew	RO (mm)	a (mm)	b (mm)	s (mm)
1	1.12E+04	0.28	41.15	10.34	2.75	0.31
2	1.12E+04	0.28	52.3	10.34	3.05	0.38
3	1.12E+04	0.28	41.15	10.37	3.03	0.43
4	1.12E+04	0.28	52.3	10.39	3.18	0.53
5	1.12E+04	0.28	41.15	10.47	3.13	0.53
6	1.12E+04	0.28	52.3	10.49	3.23	0.66
7	2.0E+04	0.30	41.80	8.50	2.33	0.782
8	2.0E+04	0.30	41.28	8.55	1.91	0.935
9	2.0E+04	0.30	41.17	8.96	1.75	0.810
10	2.0E+04	0.30	41.12	8.73	2.00	0.671
11	2.0E+04	0.30	41.40	11.41	1.70	0.582
12	2.0E+04	0.30	42.77	13.99	1.85	0.241
13	2.0E+04	0.30	41.05	11.30	1.97	0.434
14	2.0E+04	0.30	41.55	8.21	2.10	1.016
15	2.0E+04	0.30	41.86	8.10	2.20	1.171
16	2.0E+04	0.30	41.17	8.02	2.05	1.265
17	1.96E+04	0.30	62.10	7.55	3.18	1.008
18	1.96E+04	0.30	63.09	7.33	3.0	1.521
19	1.96E+04	0.30	63.3	7.55	2.88	1.328
20	1.96E+04	0.30	62.38	10.3	3.28	0.94
21	1.96E+04	0.30	62.61	10.42	3.21	0.617
22	1.96E+04	0.30	63.75	10.66	2.89	0.782
23	1.96E+04	0.30	63.88	10.80	3.25	1.179
24	1.96E+04	0.30	63.12	9.23	2.78	1.709
25	1.96E+04	0.30	63.45	10.49	2.87	0.615
26	1.96E+04	0.30	63.45	11.15	2.84	1.646

Case	EKPO	EKPW	EKPP	SPW	SPP
1	8500.	6236.	8915.4	190.	605.5
2	7200.	6264.	7529.1	271.	565.4
3	4000.	3633.	4573.1	176.	420.81
4	3800.	3537.	3888.7	217.	348.75
5	3400.	2607.	3067.2	157.	311.95
6	2300.	2388.	2501.5	180.	255.96
7	1390.	1123.	1154.3	119.	149.12
8	1062.	879.	873.04	99.2	117.89
9	1738.	1382.	1476.8	125.	159.53
10	2565.	1877.	1938.0	151.	200.72
11	5990.	4055.	4613.8	147.	335.76
12	40600.	12150.	28538.	157.	1547.3
13	9260.	5287.	6938.5	179.	408.34
14	633.	598.	589.27	92.5	94.933
15	458.	377.	375.00	62.8	72.260
16	343.	315.	308.18	55.6	63.028
17	420.	346.0	355.49	65.9	74.836
18	112.	102.	102.73	29.4	34.235
19	196.	178.	178.05	41.9	47.946
20	1400.	1157.	1175.5	130.	155.23
21	3500.	3144.	3329.3	227.	307.23
22	2330.	2203.	2266.0	186.	230.22

23	775.	767.	769.71	100.	116.79
24	174.	187.	183.47	41.4	47.235
25	3490.	3592.	3771.1	243.	324.38
26	349.	389.	382.93	63.9	72.261

Table 13: Long Form Stress Results (Pa)

Case	Circum.		-Radial		Annular		Effective	
	Sa	Sb	Sa	Sb	Sa	Sb	Sa	Sb
10	170.0	155.0	5.0	-3.0	30.0	80.0	154.0	136.9
11	325.0	230.0	20.0	0.0	88.0	130.0	277.3	200.0
12	480.0	260.0	5.0	0.0	125.0	170.0	346.4	226.7
13	625.0	295.0	10.0	0.0	180.0	205.0	550.1	261.9
14	318.0	185.0	5.0	0.0	90.0	105.0	280.3	160.7
15	440.0	95.0	10.0	0.0	135.0	121.0	383.1	110.3
16	335.0	7.0	5.0	0.0	84.0	60.0	298.4	56.8
17	59.0	48.0	1.0	0.0	18.0	20.0	51.6	41.8

Table 14: Smith's Data

P (psi)	E (psi)	a (in)	b (in)	t (in)	R ₀ (in)	dR (in)
110.	28.5E+06	0.3346	0.0916	0.0308	1.646	0.008
210.	28.5E+06	0.3346	0.0916	0.0308	1.646	0.017
310.	28.5E+06	0.3346	0.0916	0.0308	1.646	0.024
410.	28.5E+06	0.3346	0.0916	0.0308	1.646	0.034
110.	28.5E+06	0.3366	0.0751	0.0368	1.625	0.006
210.	28.5E+06	0.3366	0.0751	0.0368	1.625	0.012
310.	28.5E+06	0.3366	0.0751	0.0368	1.625	0.018
410.	28.5E+06	0.3366	0.0751	0.0368	1.625	0.025
510.	28.5E+06	0.3366	0.0751	0.0368	1.625	0.031
110.	28.5E+06	0.3420	0.0690	0.0319	1.621	0.011
210.	28.5E+06	0.3420	0.0690	0.0319	1.621	0.020
310.	28.5E+06	0.3420	0.0690	0.0319	1.621	0.031
410.	28.5E+06	0.3420	0.0690	0.0319	1.621	0.041
50.	28.5E+06	0.3436	0.0788	0.0264	1.619	0.007
100.	28.5E+06	0.3436	0.0788	0.0264	1.619	0.014
150.	28.5E+06	0.3436	0.0788	0.0264	1.619	0.022
200.	28.5E+06	0.3436	0.0788	0.0264	1.619	0.029
20.	28.5E+06	0.4491	0.0670	0.0299	1.630	0.006
40.	28.5E+06	0.4491	0.0670	0.0299	1.630	0.013
40.	28.5E+06	0.5507	0.0727	0.0095	1.684	0.097
30.	28.5E+06	0.4448	0.0775	0.0171	1.616	0.015
60.	28.5E+06	0.4448	0.0775	0.0171	1.616	0.030
90.	28.5E+06	0.4448	0.0775	0.0171	1.616	0.047
120.	28.5E+06	0.4448	0.0775	0.0171	1.616	0.063
210.	28.5E+06	0.3234	0.0826	0.0400	1.636	0.007
410.	28.5E+06	0.3234	0.0826	0.0400	1.636	0.015
610.	28.5E+06	0.3234	0.0826	0.0400	1.636	0.022
810.	28.5E+06	0.3234	0.0826	0.0400	1.636	0.029
210.	28.5E+06	0.3189	0.0864	0.0461	1.648	0.006
410.	28.5E+06	0.3189	0.0864	0.0461	1.648	0.011
610.	28.5E+06	0.3189	0.0864	0.0461	1.648	0.016
810.	28.5E+06	0.3189	0.0864	0.0461	1.648	0.022
250.	28.5E+06	0.3156	0.0806	0.0498	1.621	0.005
500.	28.5E+06	0.3156	0.0806	0.0498	1.621	0.010
750.	28.5E+06	0.3156	0.0806	0.0498	1.621	0.014
1000.	28.5E+06	0.3156	0.0806	0.0498	1.621	0.019
10.	15.5E+06	0.4086	0.1151	0.0177	2.513	0.010
20.	15.5E+06	0.4086	0.1151	0.0177	2.513	0.020
30.	15.5E+06	0.4086	0.1151	0.0177	2.513	0.031
40.	15.5E+06	0.4086	0.1151	0.0177	2.513	0.042
50.	15.5E+06	0.4086	0.1151	0.0177	2.513	0.053
30.	15.5E+06	0.4127	0.1337	0.0285	2.538	0.011
60.	15.5E+06	0.4127	0.1337	0.0285	2.538	0.022
90.	15.5E+06	0.4127	0.1337	0.0285	2.538	0.033
130.	15.5E+06	0.4127	0.1337	0.0285	2.538	0.048
30.	15.5E+06	0.4124	0.1279	0.0251	2.547	0.015
60.	15.5E+06	0.4124	0.1279	0.0251	2.547	0.030
90.	15.5E+06	0.4124	0.1279	0.0251	2.547	0.046
120.	15.5E+06	0.4124	0.1279	0.0251	2.547	0.062
20.	15.5E+06	0.4133	0.1248	0.0224	2.532	0.013

40.	15.5E+06	0.4133	0.1248	0.0224	2.532	0.026
60.	15.5E+06	0.4133	0.1248	0.0224	2.532	0.039
80.	15.5E+06	0.4133	0.1248	0.0224	2.532	0.052
60.	15.5E+06	0.4187	0.1357	0.0385	2.523	0.010
120.	15.5E+06	0.4187	0.1357	0.0385	2.523	0.021
180.	15.5E+06	0.4187	0.1357	0.0385	2.523	0.032
240.	15.5E+06	0.4187	0.1357	0.0385	2.523	0.044
40.	15.5E+06	0.4163	0.1328	0.0304	2.563	0.015
80.	15.5E+06	0.4163	0.1328	0.0304	2.563	0.029
120.	15.5E+06	0.4163	0.1328	0.0304	2.563	0.042
160.	15.5E+06	0.4163	0.1328	0.0304	2.563	0.056
50.	15.5E+06	0.4203	0.1313	0.0363	2.540	0.011
100.	15.5E+06	0.4203	0.1313	0.0363	2.540	0.022
150.	15.5E+06	0.4203	0.1313	0.0363	2.540	0.034
200.	15.5E+06	0.4203	0.1313	0.0363	2.540	0.045
40.	15.5E+06	0.4171	0.1356	0.0327	2.545	0.011
80.	15.5E+06	0.4171	0.1356	0.0327	2.545	0.022
120.	15.5E+06	0.4171	0.1356	0.0327	2.545	0.034
160.	15.5E+06	0.4171	0.1356	0.0327	2.545	0.045
10.	15.5E+06	0.4047	0.1247	0.0156	2.539	0.022
20.	15.5E+06	0.4047	0.1247	0.0156	2.539	0.037
30.	15.5E+06	0.4047	0.1247	0.0156	2.539	0.051
40.	15.5E+06	0.4047	0.1247	0.0156	2.539	0.064
20.	15.5E+06	0.4082	0.1267	0.0186	2.543	0.019
40.	15.5E+06	0.4082	0.1267	0.0186	2.543	0.039
60.	15.5E+06	0.4082	0.1267	0.0186	2.543	0.059
80.	15.5E+06	0.4082	0.1267	0.0186	2.543	0.078
200.	28.0E+06	0.2971	0.1251	0.0397	2.445	0.007
400.	28.0E+06	0.2971	0.1251	0.0397	2.445	0.014
600.	28.0E+06	0.2971	0.1251	0.0397	2.445	0.021
800.	28.0E+06	0.2971	0.1251	0.0397	2.445	0.028
500.	28.0E+06	0.2885	0.1180	0.0599	2.484	0.005
750.	28.0E+06	0.2885	0.1180	0.0599	2.484	0.007
1000.	28.0E+06	0.2885	0.1180	0.0599	2.484	0.010
250.	28.0E+06	0.2973	0.1133	0.0523	2.492	0.004
500.	28.0E+06	0.2973	0.1133	0.0523	2.492	0.009
750.	28.0E+06	0.2973	0.1133	0.0523	2.492	0.011
1000.	28.0E+06	0.2973	0.1133	0.0523	2.492	0.017
50.	28.0E+06	0.4055	0.1290	0.0370	2.456	0.006
100.	28.0E+06	0.4055	0.1290	0.0370	2.456	0.012
150.	28.0E+06	0.4055	0.1290	0.0370	2.456	0.018
200.	28.0E+06	0.4055	0.1290	0.0370	2.456	0.024
20.	28.0E+06	0.4103	0.1263	0.0243	2.465	0.006
40.	28.0E+06	0.4103	0.1263	0.0243	2.465	0.013
60.	28.0E+06	0.4103	0.1263	0.0243	2.465	0.019
80.	28.0E+06	0.4103	0.1263	0.0243	2.465	0.026
30.	15.5E+06	0.4196	0.1136	0.0308	2.510	0.013
60.	15.5E+06	0.4196	0.1136	0.0308	2.510	0.024
90.	15.5E+06	0.4196	0.1136	0.0308	2.510	0.034
120.	15.5E+06	0.4196	0.1136	0.0308	2.510	0.045
50.	15.5E+06	0.4253	0.1278	0.0464	2.515	0.006
100.	15.5E+06	0.4253	0.1278	0.0464	2.515	0.012
150.	15.5E+06	0.4253	0.1278	0.0464	2.515	0.018
200.	15.5E+06	0.4253	0.1278	0.0464	2.515	0.024
200.	15.5E+06	0.3633	0.1093	0.0673	2.485	0.006
400.	15.5E+06	0.3633	0.1093	0.0673	2.485	0.011

600.	15.5E+06	0.3633	0.1093	0.0673	2.485	0.017
800.	15.5E+06	0.3633	0.1093	0.0673	2.485	0.023
10.	15.5E+06	0.4129	0.1129	0.0242	2.498	0.005
20.	15.5E+06	0.4129	0.1129	0.0242	2.498	0.010
30.	15.5E+06	0.4129	0.1129	0.0242	2.498	0.016
40.	15.5E+06	0.4129	0.1129	0.0242	2.498	0.022
100.	15.5E+06	0.4391	0.1116	0.0648	2.514	0.006
200.	15.5E+06	0.4391	0.1116	0.0648	2.514	0.012
300.	15.5E+06	0.4391	0.1116	0.0648	2.514	0.017
400.	15.5E+06	0.4391	0.1116	0.0648	2.514	0.023
30.	28.5E+06	0.3875	0.0490	0.0159	1.552	0.018
60.	28.5E+06	0.3875	0.0490	0.0159	1.552	0.036
90.	28.5E+06	0.3875	0.0490	0.0159	1.552	0.056
120.	28.5E+06	0.3875	0.0490	0.0159	1.552	0.076
30.	28.5E+06	0.3879	0.0479	0.0162	1.540	0.018
60.	28.5E+06	0.3879	0.0479	0.0162	1.540	0.036
90.	28.5E+06	0.3879	0.0479	0.0162	1.540	0.054
120.	28.5E+06	0.3879	0.0479	0.0162	1.540	0.074
50.	28.5E+06	0.3949	0.0269	0.0182	1.035	0.016
100.	28.5E+06	0.3949	0.0269	0.0182	1.035	0.032
150.	28.5E+06	0.3949	0.0269	0.0182	1.035	0.050
200.	28.5E+06	0.3949	0.0269	0.0182	1.035	0.070
50.	28.5E+06	0.3948	0.0298	0.0174	1.047	0.017
100.	28.5E+06	0.3948	0.0298	0.0174	1.047	0.035
150.	28.5E+06	0.3948	0.0298	0.0174	1.047	0.054
200.	28.5E+06	0.3948	0.0298	0.0174	1.047	0.074
60.	28.5E+06	0.2475	0.0500	0.0189	1.495	0.008
120.	28.5E+06	0.2475	0.0500	0.0189	1.495	0.017
180.	28.5E+06	0.2475	0.0500	0.0189	1.495	0.026
240.	28.5E+06	0.2475	0.0500	0.0189	1.495	0.035
60.	28.5E+06	0.2480	0.0490	0.0189	1.511	0.010
120.	28.5E+06	0.2480	0.0490	0.0189	1.511	0.015
180.	28.5E+06	0.2480	0.0490	0.0189	1.511	0.029
240.	28.5E+06	0.2480	0.0490	0.0189	1.511	0.039
80.	28.5E+06	0.2547	0.0297	0.0196	1.098	0.010
160.	28.5E+06	0.2547	0.0297	0.0196	1.098	0.021
240.	28.5E+06	0.2547	0.0297	0.0196	1.098	0.032
320.	28.5E+06	0.2547	0.0297	0.0196	1.098	0.044
80.	28.5E+06	0.2546	0.0291	0.0198	1.067	0.009
160.	28.5E+06	0.2546	0.0291	0.0198	1.067	0.019
240.	28.5E+06	0.2546	0.0291	0.0198	1.067	0.030
320.	28.5E+06	0.2546	0.0291	0.0198	1.067	0.041
200.	28.5E+06	0.3013	0.0528	0.0384	1.571	0.007
400.	28.5E+06	0.3013	0.0528	0.0384	1.571	0.015
600.	28.5E+06	0.3013	0.0528	0.0384	1.571	0.023
800.	28.5E+06	0.3013	0.0528	0.0384	1.571	0.034
200.	28.5E+06	0.3001	0.0576	0.0387	1.583	0.008
400.	28.5E+06	0.3001	0.0576	0.0387	1.583	0.015
600.	28.5E+06	0.3001	0.0576	0.0387	1.583	0.023
800.	28.5E+06	0.3001	0.0576	0.0387	1.583	0.031
130.	15.5E+06	0.2981	0.1001	0.0347	1.617	0.007
260.	15.5E+06	0.2981	0.1001	0.0347	1.617	0.015
390.	15.5E+06	0.2981	0.1001	0.0347	1.617	0.022
520.	15.5E+06	0.2981	0.1001	0.0347	1.617	0.029
130.	15.5E+06	0.2986	0.1001	0.0348	1.623	0.008
260.	15.5E+06	0.2986	0.1001	0.0348	1.623	0.015

390.	15.5E+06	0.2986	0.1001	0.0348	1.623	0.023
520.	15.5E+06	0.2986	0.1001	0.0348	1.623	0.030
130.	15.5E+06	0.2986	0.1001	0.0348	1.620	0.008

Table 15: Kardos' Data

t (in)	a (in)	b (in)	RO (in)	DRRQEP
0.034	0.125	0.031	0.798	70.5
0.034	0.127	0.0265	0.792	86.4
0.032	0.125	0.0315	0.795	83.5
0.032	0.128	0.025	0.790	122.5
0.032	0.123	0.0355	0.801	87.1
0.019	0.128	0.049	0.805	342.
0.019	0.131	0.0455	0.807	379.
0.019	0.137	0.0315	0.791	602.
0.019	0.139	0.0255	0.792	743.
0.021	0.114	0.030	0.793	256.
0.021	0.116	0.0255	0.786	312.
0.025	0.112	0.027	0.791	159.
0.025	0.110	0.032	0.797	126.
0.030	0.123	0.0355	0.799	94.8
0.030	0.125	0.031	0.795	110.
0.008	0.137	0.044	0.797	3570.
0.008	0.144	0.0335	0.806	5400.
0.0145	0.137	0.037	0.798	939.
0.0145	0.139	0.0327	0.794	1210.
0.0115	0.140	0.0353	0.792	1840.
0.0115	0.138	0.0417	0.792	1468.
0.01	0.143	0.035	0.790	3000.
0.01	0.148	0.025	0.783	4900.
0.01	0.140	0.04	0.797	2890.
0.01	0.145	0.0305	0.81	3415.
0.01	0.152	0.019	0.791	5610.
0.01	0.136	0.0445	0.792	1992.
0.0135	0.156	0.0135	0.782	21100.
0.009	0.195	0.0195	0.806	12100.
0.009	0.197	0.0145	0.806	14300.
0.022	0.132	0.026	0.831	309.
0.022	0.132	0.031	0.834	256.
0.015	0.135	0.0275	0.807	911.
0.015	0.137	0.023	0.822	1112.
0.018	0.139	0.0195	0.825	793.
0.018	0.137	0.0285	0.822	493.
0.010	0.142	0.0265	0.840	3175.
0.010	0.136	0.035	0.832	2560.
0.010	0.146	0.0155	0.834	4990.
0.010	0.148	0.0105	0.847	6130.
0.0125	0.141	0.021	0.818	2500.
0.0125	0.138	0.026	0.825	1838.
0.0235	0.264	0.0337	0.849	1320.
0.0235	0.266	0.0282	0.844	1709.
0.0135	0.352	0.033	0.850	7420.
0.0135	0.353	0.028	0.855	7780.
0.0105	0.351	0.041	0.863	9320.
0.0105	0.354	0.0307	0.875	8960.
0.0105	0.360	0.017	0.867	17650.
0.0285	0.363	0.0637	0.827	1020.
0.0225	0.342	0.0427	0.831	2060.

0.0225	0.346	0.0327	0.834	2330.
0.0225	0.350	0.0217	0.823	2718.
0.0225	0.336	0.054	0.828	1671.
0.019	0.336	0.053	0.826	2060.
0.019	0.344	0.0355	0.819	2790.
0.019	0.347	0.0265	0.825	3115.
0.019	0.351	0.0165	0.825	3880.
0.019	0.341	0.0415	0.824	2465.
0.019	0.346	0.0305	0.727	2710.
0.026	0.436	0.029	0.825	2295.
0.026	0.434	0.0345	0.822	225.
0.045	0.439	0.048	0.814	584.
0.026	0.122	0.047	0.817	102.5
0.026	0.124	0.043	0.816	122.
0.026	0.126	0.037	0.807	144.
0.026	0.128	0.032	0.806	155.
0.026	0.129	0.0295	0.806	192.
0.026	0.133	0.0225	0.807	225.5
0.026	0.119	0.0505	0.816	92.5

Table 16: Case 1 Stress Results(Pa)

CP	IES	OES	ICS	IRS	OCS	ORS
0.0000	2.32	0.67	1.68	-0.97	0.67	0.0
0.0625	2.32	0.67	1.68	-0.97	0.67	0.0
0.1250	2.32	0.67	1.68	-0.97	0.67	0.0
0.1875	2.32	0.67	1.68	-0.97	0.67	0.0
0.2500	2.32	0.67	1.68	-0.97	0.67	0.0
0.3125	2.32	0.67	1.68	-0.97	0.67	0.0
0.3750	2.32	0.67	1.68	-0.97	0.67	0.0
0.4375	2.32	0.67	1.68	-0.97	0.67	0.0
0.5000	2.32	0.67	1.68	-0.97	0.67	0.0
0.5625	2.32	0.67	1.68	-0.97	0.67	0.0
0.6250	2.32	0.67	1.68	-0.97	0.67	0.0
0.6875	2.32	0.67	1.68	-0.97	0.67	0.0
0.7500	2.32	0.67	1.68	-0.97	0.67	0.0
0.8125	2.32	0.67	1.68	-0.97	0.67	0.0
0.8850	2.32	0.67	1.68	-0.97	0.67	0.0
0.9375	2.32	0.67	1.68	-0.97	0.67	0.0
1.0000	2.32	0.67	1.68	-0.97	0.67	0.0

Table 17: Case 2 Stress Results(Pa)

CP	IES	OES	ICS	IRS	OCS	ORS
0.0000	6.10	4.54	5.55	-0.98	4.55	0.01
0.0556	6.10	4.54	5.55	-0.98	4.55	0.01
0.1111	6.10	4.54	5.55	-0.98	4.55	0.01
0.1667	6.10	4.54	5.55	-0.98	4.55	0.01
0.2222	6.10	4.54	5.55	-0.98	4.55	0.01
0.2778	6.10	4.54	5.55	-0.98	4.55	0.01
0.3333	6.10	4.54	5.55	-0.98	4.55	0.01
0.3889	6.10	4.54	5.55	-0.98	4.55	0.01
0.4444	6.10	4.54	5.55	-0.98	4.55	0.01
0.5000	6.10	4.54	5.55	-0.98	4.55	0.01
0.5556	6.10	4.54	5.55	-0.98	4.55	0.01
0.6111	6.10	4.54	5.55	-0.98	4.55	0.01
0.6667	6.10	4.54	5.55	-0.98	4.55	0.01
0.7222	6.10	4.54	5.55	-0.98	4.55	0.01
0.7778	6.10	4.54	5.55	-0.98	4.55	0.01
0.8333	6.10	4.54	5.55	-0.98	4.55	0.01
0.8889	6.10	4.54	5.55	-0.98	4.55	0.01
0.9444	6.10	4.54	5.55	-0.98	4.55	0.01
1.0000	6.10	4.54	5.55	-0.98	4.55	0.01

Table 18: Case 3 Stress Results(Pa)

CP	IES	OES	ICS	IRS	OCS	ORS
0.00	10.05	8.52	9.54	-0.96	8.54	0.04
0.05	10.05	8.52	9.54	-0.96	8.54	0.04
0.10	10.05	8.52	9.54	-0.96	8.54	0.04
0.15	10.05	8.52	9.54	-0.96	8.54	0.04
0.20	10.05	8.52	9.54	-0.96	8.54	0.04
0.25	10.05	8.52	9.54	-0.96	8.54	0.04
0.30	10.05	8.52	9.54	-0.96	8.54	0.04
0.35	10.05	8.52	9.54	-0.96	8.54	0.04
0.40	10.05	8.52	9.54	-0.96	8.54	0.04
0.45	10.05	8.52	9.54	-0.96	8.54	0.04
0.50	10.05	8.52	9.54	-0.96	8.54	0.04
0.55	10.05	8.52	9.54	-0.96	8.54	0.04
0.60	10.05	8.52	9.54	-0.96	8.54	0.04
0.65	10.05	8.52	9.54	-0.96	8.54	0.04
0.70	10.05	8.52	9.54	-0.96	8.54	0.04
0.75	10.05	8.52	9.54	-0.96	8.54	0.04
0.80	10.05	8.52	9.54	-0.96	8.54	0.04
0.85	10.05	8.52	9.54	-0.96	8.54	0.04
0.90	10.05	8.52	9.54	-0.96	8.54	0.04
0.95	10.05	8.52	9.54	-0.96	8.54	0.04
1.00	10.05	8.52	9.54	-0.96	8.54	0.04

Table 19: Case 4 Stress Results(Pa)

CP	IES	OES	ICS	IRS	OCS	ORS
0.00	1.07	1.56	0.05	-1.04	1.58	0.03
0.05	1.08	1.55	0.07	-1.04	1.56	0.02
0.10	1.09	1.53	0.10	-1.04	1.54	0.02
0.15	1.15	1.48	0.12	-1.04	1.49	0.02
0.20	1.22	1.41	0.33	-1.02	1.42	0.01
0.25	1.40	1.23	0.60	-1.00	1.23	-0.01
0.30	1.60	1.11	0.86	-0.99	1.11	0.00
0.35	1.77	1.00	1.07	-0.98	1.00	0.00
0.40	1.96	0.91	1.29	-0.97	0.91	0.00
0.45	2.14	0.81	1.48	-0.97	0.81	0.00
0.50	2.32	0.72	1.68	-0.96	0.72	0.00
0.55	2.48	0.63	1.86	-0.96	0.63	0.00
0.60	2.63	0.55	2.02	-0.96	0.55	0.00
0.65	2.77	0.49	2.18	-0.95	0.49	0.00
0.70	2.90	0.41	2.30	-0.95	0.41	0.00
0.75	3.01	0.37	2.43	-0.94	0.37	0.00
0.80	3.10	0.31	2.51	-0.95	0.30	0.00
0.85	3.17	0.29	2.60	-0.93	0.29	0.00
0.90	3.22	0.24	2.64	-0.94	-0.24	0.00
0.95	3.26	0.25	2.69	-0.93	0.25	0.00
1.00	3.26	0.22	2.68	-0.94	0.22	0.00

Table 20: Case 5 Stress Results(Pa)

CP	IES	OES	ICS	IRS	OCS	ORS
0.00	12.87	22.52	-13.40	-1.11	22.59	0.14
0.05	12.56	22.23	-13.08	-1.10	22.28	0.11
0.10	12.23	21.93	-12.77	-1.15	21.96	0.07
0.15	11.48	20.80	-11.99	-1.10	20.87	0.14
0.20	10.36	19.63	-10.83	-1.01	19.69	0.13
0.25	8.51	15.98	-9.11	-1.37	15.85	-0.27
0.30	4.72	14.48	-5.11	-0.91	14.50	0.05
0.35	2.67	11.28	-3.06	-1.14	11.23	-0.10
0.40	1.51	9.53	0.56	-1.15	9.52	-0.03
0.45	3.33	6.90	2.69	-1.04	6.88	-0.04
0.50	6.03	4.89	5.46	-1.00	4.89	-0.04
0.55	8.33	2.95	7.84	-0.91	2.97	0.04
0.60	10.35	0.98	9.83	-0.97	0.89	-0.17
0.65	12.64	0.42	12.22	-0.80	-0.35	0.11
0.70	13.93	2.64	13.43	-0.95	-2.84	0.00
0.75	16.04	3.05	15.67	-0.72	-2.96	0.16
0.80	16.59	5.09	16.10	-0.93	-5.11	-0.04
0.85	18.38	4.86	18.04	-0.66	-4.76	0.19
0.90	18.22	6.61	17.75	-0.92	-6.64	-0.05
0.95	19.57	5.79	19.25	-0.63	-5.68	0.21
1.00	18.78	7.13	18.30	-0.92	-7.15	-0.05

Table 21: Case 6 Stress Results (Pa)

CP	IES	OES	ICS	IRS	OCS	ORS
0.00	47.39	64.38	-47.89	-1.02	64.41	0.06
0.05	46.44	63.45	-46.92	-0.96	63.46	0.02
0.10	45.54	62.34	-46.01	-0.97	62.26	-0.17
0.15	42.43	59.26	-43.05	-1.28	59.24	-0.03
0.20	37.17	56.89	-37.79	-1.29	56.65	-0.48
0.25	33.59	44.06	-34.86	-2.70	43.29	-1.49
0.30	14.15	42.03	-20.45	-1.10	41.73	-0.59
0.35	15.60	29.37	-16.47	-1.91	28.93	-0.86
0.40	6.14	25.26	-6.15	0.00	24.97	-0.56
0.45	2.47	15.59	1.30	-1.56	15.46	-0.27
0.50	10.61	10.15	9.78	-1.50	9.81	-0.65
0.55	17.03	3.21	16.67	-0.70	3.32	0.24
0.60	22.33	5.94	21.87	-0.91	-5.31	1.10
0.65	30.03	7.44	29.95	-0.17	-7.05	0.72
0.70	32.25	16.11	31.95	-0.59	-15.80	0.61
0.75	40.26	15.72	40.39	0.24	-15.16	1.06
0.80	39.66	24.29	39.47	-0.38	-23.97	0.63
0.85	47.32	21.44	47.58	0.52	-20.76	1.30
0.90	44.23	29.37	44.10	-0.25	-29.03	0.67
0.95	50.91	24.35	51.24	0.67	-23.61	1.43
1.00	45.80	31.09	45.69	-0.23	-30.74	0.68

Table 22: Case 7 Stress Results (Pa)

CP	IES	OES	ICS	IRS	OCS	ORS
0.00	0.95	2.33	-0.84	-1.04	2.34	0.03
0.05	0.93	2.28	-0.79	-1.03	2.30	0.03
0.10	0.93	2.23	-0.76	-1.02	2.24	0.02
0.15	0.91	2.08	-0.59	-1.05	2.09	0.02
0.20	0.90	1.89	-0.36	-1.02	1.89	0.01
0.25	1.02	1.56	-0.01	-1.02	1.55	-0.01
0.30	1.24	1.33	0.39	-1.00	1.33	0.00
0.35	1.51	1.15	0.75	-0.99	1.15	0.00
0.40	1.84	0.98	1.15	-0.97	0.98	0.00
0.45	2.15	0.81	1.50	-0.96	0.81	0.00
0.50	2.48	0.65	1.86	-0.95	0.65	0.00
0.55	2.78	0.51	2.19	-0.94	0.52	0.00
0.60	3.06	0.36	2.48	-0.94	0.36	0.00
0.65	3.32	0.26	2.77	-0.92	0.26	0.00
0.70	3.55	0.13	2.99	-0.93	0.12	0.00
0.75	3.75	0.06	3.22	-0.90	0.06	0.01
0.80	3.91	0.05	3.37	-0.92	-0.06	0.00
0.85	4.05	0.08	3.53	-0.89	-0.08	0.01
0.90	4.13	0.16	3.60	-0.91	-0.16	0.00
0.95	4.21	0.15	3.69	-0.89	-0.15	0.01
1.00	4.21	0.20	3.68	-0.91	-0.20	-0.01

Table 23: Case 8 Stress Results (Pa)

CP	IES	OES	ICS	IRS	OCS	ORS
0.00	27.43	36.40	7.91	-1.00	36.41	0.03
0.05	26.41	35.39	-26.90	-0.99	35.40	0.02
0.10	25.42	34.26	-25.89	-0.97	34.21	-0.10
0.15	22.23	31.05	-22.76	-1.12	31.02	-0.08
0.20	19.05	27.76	-19.68	-1.31	27.50	-0.52
0.25	15.61	21.51	-16.55	-2.09	20.94	-1.10
0.30	9.62	18.75	-10.15	-1.17	18.45	-0.60
0.35	5.87	13.33	-6.38	-1.21	13.03	-0.58
0.40	3.57	10.28	-2.69	1.35	10.02	-0.49
0.45	5.39	5.86	4.59	-1.35	5.79	-0.13
0.50	9.64	3.31	9.06	-1.08	2.85	-0.78
0.55	13.65	1.73	13.39	-0.59	0.75	-1.23
0.60	16.88	5.74	16.63	-0.49	-5.31	0.79
0.65	20.99	7.08	21.02	0.07	-6.59	0.89
0.70	22.97	11.16	22.93	-0.09	-10.73	0.81
0.75	26.79	11.60	27.04	0.50	-10.97	1.18
0.80	27.52	15.34	27.62	0.20	-14.85	0.93
0.85	30.80	14.74	31.19	0.79	-14.00	1.38
0.90	30.33	17.93	30.51	0.37	-17.41	1.01
0.95	32.84	16.34	33.30	0.94	-15.54	1.49
1.00	31.29	18.81	31.50	0.42	-18.22	1.04

Table 24: Case 9 Stress Results (Pa)

CP	IES	OES	ICS	IRS	OCS	ORS
0.00	92.15	107.80	-92.62	-0.95	107.86	-0.06
0.05	89.16	104.90	-89.66	-1.02	104.93	-0.06
0.10	85.43	101.03	-85.86	-0.95	100.86	-0.34
0.15	75.28	90.93	-75.96	-1.17	91.00	0.15
0.20	62.62	81.66	-63.17	-1.11	81.26	-0.81
0.25	53.61	60.16	-55.29	-3.53	58.90	-2.46
0.30	32.19	54.48	-32.74	-1.13	53.97	-1.02
0.35	22.54	34.81	-23.55	-2.18	34.22	-1.14
0.40	9.10	26.35	-8.26	1.51	25.71	-1.24
0.45	7.81	11.13	7.01	-1.41	11.06	-0.14
0.50	20.86	7.89	20.05	-1.56	-5.23	3.84
0.55	33.67	10.46	33.67	-0.01	-9.96	0.93
0.60	41.48	25.94	41.21	-0.53	-25.32	1.20
0.65	56.08	28.60	56.56	0.98	-27.71	1.71
0.70	58.79	44.57	58.82	0.06	-43.96	1.20
0.75	73.71	42.89	74.56	1.74	-41.67	2.34
0.80	71.72	58.63	71.95	0.46	-57.95	1.34
0.85	85.86	52.74	86.97	2.26	-51.30	2.78
0.90	79.69	67.32	80.04	0.70	-66.59	1.45
0.95	92.06	57.76	93.29	2.52	-56.20	3.00
1.00	82.44	70.26	82.81	0.74	-69.50	1.49

Table 25: Case 10 Stress Results (Pa)

CP	IES	OES	ICS	IRS	IAS	OCS	ORS	OAS
0.0000	79.24	79.36	-139.4	0.2170	-4.630	157.1	-1.605	80.11
0.0313	76.76	77.33	-135.4	-0.2678	-4.635	153.6	-1.008	78.00
0.0625	73.35	74.41	-129.9	0.1492	-6.105	147.3	-1.497	72.98
0.0938	64.95	67.35	-116.1	-0.3077	-7.316	133.8	-0.8455	63.83
0.1250	56.06	59.78	-101.1	0.7088	-9.528	117.4	-1.928	51.59
0.1563	40.94	46.76	-76.59	4.190	-11.24	92.49	-0.2513	35.71
0.1875	26.11	35.88	-47.80	2.630	-10.82	68.12	-1.358	17.86
0.2188	11.86	18.78	-25.83	-2.457	-10.67	27.67	-2.804	-6.562
0.2500	17.15	18.55	19.42	-2.705	-14.35	-7.038	12.62	-24.46
0.2813	31.16	21.31	48.01	1.349	-11.10	-15.75	2.021	-40.41
0.1250	56.06	59.78	-101.1	0.7088	-9.528	117.4	-1.928	51.59
0.1563	40.94	46.76	-76.59	-1.190	-11.24	92.49	-0.2513	35.71
0.1875	26.11	35.88	-47.80	2.630	-10.82	68.12	-1.358	17.86
0.2188	11.86	18.78	-25.83	-2.457	-10.67	27.67	-2.804	-6.562
0.2500	17.15	18.55	19.42	-2.705	-14.35	-7.038	12.62	-24.46
0.2813	31.16	21.31	48.01	1.349	-11.10	-15.75	2.021	-40.41
0.3125	39.87	36.16	61.58	-0.1387	-13.02	-52.61	2.539	-65.55
0.3438	60.68	38.02	107.2	5.476	-0.9607	-52.39	5.358	-66.35
0.3750	59.00	58.55	100.1	1.365	-5.219	-110.2	2.913	-79.86
0.4063	82.46	50.11	153.9	7.639	14.82	-86.42	7.691	-69.20
0.4375	71.85	76.12	130.5	2.427	10.81	-147.8	3.721	-84.65
0.4688	95.18	59.00	186.7	10.22	36.70	-107.6	9.429	-62.12
0.5000	74.96	83.19	143.7	2.290	20.80	-162.2	3.770	-68.20
0.5313	94.53	60.49	192.0	10.20	56.04	-110.6	9.525	-37.88
0.5625	88.71	82.92	137.0	2.151	46.49	-156.9	3.214	-39.62
0.5938	79.68	54.61	166.6	9.741	65.92	-91.44	9.198	-4.350
0.6250	53.48	70.34	108.4	1.517	52.90	-122.7	2.340	-4.446
0.6563	55.90	45.52	117.2	5.386	60.89	-62.60	5.544	23.76
0.6875	33.48	49.04	65.61	0.1617	45.12	-68.35	2.080	25.97
0.7188	25.39	31.16	49.89	1.368	39.60	-19.93	2.170	41.58
0.7500	13.76	27.94	16.36	-4.521	21.45	-10.31	11.15	45.10
0.7813	15.50	25.31	-28.40	-3.148	-0.2173	23.80	-2.930	47.66
0.8125	29.77	39.98	-56.02	2.688	-18.14	28.41	-1.543	37.75
0.8438	43.07	51.97	-97.22	-1.159	-41.11	102.5	0.0259	35.90
0.8750	56.67	66.98	112.5	0.7874	-58.96	128.1	-1.507	33.87
0.9063	64.42	75.83	-128.9	-0.3392	-72.97	143.8	-0.3815	30.24
0.9375	71.61	84.40	-142.3	0.2325	-93.42	158.0	-0.8619	29.11
0.9688	74.67	87.75	-148.7	-0.3278	-89.42	163.5	-0.4282	27.18
1.0000	76.67	90.34	-152.0	0.2891	-91.66	167.7	-0.7064	27.75

Table 26: Case 11 Stress Results (Pa)

CP	IES	OES	ICS	IRS	IAS	OCS	ORS	OAS
0.0000	128.1	115.9	-214.7	0.0455	14.75	229.2	-1.636	131.9
0.0313	125.1	113.7	-209.7	-0.6503	13.87	225.9	-0.8105	128.9
0.0625	120.2	109.9	-202.9	0.1149	9.983	217.6	-1.664	120.3
0.0938	108.9	101.4	-186.8	-0.9423	4.553	202.1	-0.5495	105.9
0.1250	96.15	91.63	-167.8	0.5082	-3.040	181.4	-1.791	85.84
0.1563	75.80	76.02	-137.3	-1.257	-11.35	150.8	-0.2066	60.07
0.1875	56.03	61.55	-103.7	2.590	-19.69	116.7	-2.205	29.52
0.2188	26.19	39.96	-54.70	-2.319	-28.04	64.71	-0.5955	-7.852
0.2500	25.86	36.29	16.24	-15.38	-35.03	25.17	-13.29	-47.36
0.2813	49.63	41.48	67.14	2.089	-30.29	-32.57	2.015	-80.57
0.3125	67.18	72.69	94.02	1.113	-33.81	-114.2	6.960	-123.2
0.3438	112.9	73.96	198.2	12.92	-6.208	-105.6	11.82	-124.7
0.3750	107.2	123.1	180.2	5.189	-14.53	-232.4	6.771	-163.4
0.4063	163.9	100.6	311.3	20.42	35.18	-172.5	17.60	-134.5
0.4375	132.0	164.0	243.4	7.623	22.55	-320.5	7.284	-166.9
0.4688	188.6	114.5	379.3	27.32	86.08	-204.6	23.89	-104.4
0.5000	137.0	185.9	269.6	8.162	67.70	-360.4	7.286	-128.3
0.5313	184.0	117.5	383.7	26.31	128.9	-206.3	23.05	-46.90
0.5625	120.5	178.1	246.9	7.642	102.9	-329.8	6.302	-59.32
0.5938	150.2	110.7	320.1	21.23	146.3	-173.4	19.30	17.63
0.6250	98.77	144.8	181.7	5.410	111.8	-241.9	5.449	12.27
0.6563	95.16	88.01	202.5	13.29	125.7	-105.9	12.71	66.08
0.6875	50.67	92.10	96.80	2.541	81.77	-118.4	6.780	61.28
0.7188	36.10	54.10	53.72	1.714	71.09	-26.15	3.095	78.69
0.7500	28.40	42.52	-25.07	12.50	30.61	28.65	-11.95	73.06
0.7813	36.08	41.39	-69.08	-1.638	-13.15	79.37	-0.4897	58.31
0.8125	60.98	64.80	-118.5	3.209	-50.73	127.7	-1.590	55.46
0.8438	76.71	82.28	-153.7	-0.5427	-84.24	159.9	0.2583	45.62
0.8750	93.10	101.4	-183.3	1.393	-111.5	190.4	-1.179	36.77
0.9063	103.3	113.3	-203.5	-0.1052	-133.1	208.8	-0.1956	28.30
0.9375	112.0	124.0	-219.1	1.067	-148.4	224.6	-0.9048	22.38
0.9688	116.0	128.5	-226.0	0.1646	-157.7	231.0	-0.2243	18.12
1.0000	118.2	131.5	-229.1	1.010	-160.8	235.4	-0.8530	17.11

Table 27: Case 12 Stress Results (Pa)

CP	IES	OES	ICS	IRS	IAS	OCS	ORS	OAS
0.0000	161.3	134.6	-251.1	-0.6747	50.20	262.2	-2.493	172.7
0.0208	158.7	133.2	-247.1	-0.4576	49.16	259.5	-2.590	169.5
0.0417	153.9	129.4	-242.3	-0.7302	43.61	253.1	-2.415	160.9
0.0625	143.6	122.4	-228.8	-0.5991	36.35	240.5	-2.340	145.8
0.0833	131.2	113.2	-213.9	-0.8678	25.08	223.8	-2.053	125.6
0.1042	112.4	100.3	-188.1	-0.8740	13.26	198.5	-2.028	98.43
0.1250	91.71	85.85	-159.9	-0.3564	-1.740	168.4	-2.262	66.41
0.1458	63.21	66.55	-117.0	-1.257	-15.06	126.0	-0.9017	27.83
0.1667	36.97	51.35	-71.37	2.259	-28.78	79.14	-2.809	-15.46
0.1875	22.06	41.76	-8.941	2.088	-40.42	13.64	-3.355	-65.68
0.2083	55.73	58.37	59.53	-2.317	-51.69	-53.38	1.020	-115.6
0.2292	82.53	70.46	118.3	4.688	-42.25	-84.73	2.036	-137.5
0.2500	99.77	87.58	146.5	-0.7681	-43.71	-136.4	-0.7957	-164.6
0.2708	127.0	95.58	205.0	4.807	-30.49	-146.4	3.576	-174.0
0.2917	136.1	116.0	217.9	-0.7415	-31.79	-208.5	-1.661	-195.4
0.3125	146.4	120.5	286.6	7.581	-9.911	-208.6	6.851	-194.3
0.3333	172.1	144.1	292.1	-1.942	-9.942	-276.3	-2.497	-217.2
0.3542	201.6	142.6	363.1	9.183	18.90	-263.1	7.767	-204.7
0.3750	201.1	170.0	355.8	-3.194	19.32	-337.7	-2.856	-221.8
0.3958	229.5	160.0	430.7	13.90	55.88	-304.6	18.38	-196.0
0.4167	221.7	193.2	406.7	-2.752	54.61	-390.5	-4.436	-211.2
0.4375	252.6	182.9	487.4	14.78	96.76	-355.1	10.68	-178.5
0.4583	232.9	207.7	441.3	-0.5487	22.65	-416.9	2.727	-181.4
0.4792	255.7	185.6	502.4	15.99	131.4	-358.3	10.75	-138.2
0.5000	233.0	214.9	451.7	-0.3490	127.7	-424.6	-3.366	-139.8
0.5208	251.6	191.5	504.7	13.59	164.3	-358.2	13.36	-91.89
0.5417	226.2	220.2	444.7	-1.638	158.1	-421.6	-3.429	-93.13
0.5625	240.6	197.1	493.0	16.98	193.9	-358.0	8.207	-48.31
0.5833	208.7	217.3	414.0	-2.248	179.1	-397.6	-2.746	-42.74
0.6042	211.8	182.7	437.6	14.77	204.1	-306.8	11.47	7.519
0.6250	179.9	200.2	359.0	-0.7013	187.0	-344.4	-3.571	7.757
0.6458	179.9	171.2	375.7	16.16	206.0	-266.1	8.911	48.02
0.6667	147.6	177.7	291.4	-0.6290	182.1	-281.5	-3.382	49.29
0.6875	142.9	149.7	291.1	8.806	188.2	-206.1	8.805	81.81
0.7083	112.6	148.4	216.8	0.7761	163.8	-206.8	-0.4047	81.08
0.7292	104.6	123.0	200.3	1.147	156.3	-144.2	4.630	99.77
0.7500	79.26	114.6	141.4	-1.562	129.0	-131.4	-0.2787	96.82
0.7708	63.13	91.49	107.4	5.449	121.0	-78.28	2.639	104.3
0.7917	44.13	75.09	48.74	0.1701	88.27	-45.84	1.765	103.3
0.8125	28.02	47.58	-20.00	2.446	35.58	24.13	-1.885	22.39
0.8333	48.03	51.64	-86.80	4.664	-15.95	95.13	-1.027	77.21
0.8542	68.78	70.34	-135.7	1.759	-61.78	140.5	-0.0160	64.19
0.8750	21.11	95.48	-179.0	2.475	-102.7	183.9	-0.2917	49.27
0.8958	707.5	114.3	-200.3	2.272	-136.6	212.4	-0.7903	36.65
0.9167	121.9	131.9	-234.4	2.336	-165.2	238.4	-0.5402	22.17
0.9375	131.7	143.5	-250.9	2.510	-186.8	253.1	-1.007	10.41
0.9583	139.2	153.3	-263.0	2.528	-202.7	266.4	-1.151	3.012
0.9792	143.4	157.7	-269.4	2.655	-211.9	271.0	-0.1701	-6.108
1.0000	144.9	160.3	-271.8	2.596	-215.2	275.0	-0.1524	-4.967

Table 2A: Case 13 Stress Results (Pa)

CP	IES	OES	ICS	IRS	IAS	OCS	ORS	OAS
0.0000	192.0	151.2	-282.2	3.230	83.04	294.9	-0.2955	204.5
0.0192	188.9	149.8	-277.8	3.631	81.13	292.4	-0.5956	200.1
0.0385	182.3	144.8	-272.1	3.175	72.62	284.3	-0.3361	188.1
0.0577	169.0	136.6	-256.3	3.475	60.86	270.0	-0.5215	167.3
0.0769	152.1	124.9	-237.9	3.001	43.41	249.0	-0.2307	138.9
0.0962	127.2	109.2	-205.7	3.258	24.62	217.5	-0.7277	101.2
0.1154	98.77	91.45	-168.0	4.204	1.872	177.4	-1.695	55.94
0.1346	60.33	69.17	-110.9	2.733	-1.888	120.9	-0.5114	2.294
0.1538	32.43	58.78	-49.24	11.49	-38.60	58.00	-6.984	-59.32
0.1731	52.70	66.03	50.37	-0.9044	-55.01	-43.11	0.3644	-129.4
0.1923	103.7	98.83	135.4	5.341	-69.55	-129.0	-3.012	-197.9
0.2115	139.8	114.2	216.4	13.53	-51.74	-163.1	1.587	-217.8
0.2308	156.0	131.5	235.9	-2.516	-57.65	-214.6	-2.454	-243.1
0.2500	183.2	139.7	295.7	4.415	-42.46	-222.7	4.884	-249.1
0.2692	190.9	159.1	307.5	0.2328	-42.54	-278.5	1.513	-269.3
0.2885	221.1	164.6	375.7	7.046	-20.07	-279.3	12.17	-266.1
0.3077	225.9	183.1	382.5	1.044	-17.73	-345.4	-3.305	-287.6
0.3269	254.2	185.0	453.1	13.85	11.90	-338.6	8.579	-275.8
0.3462	256.2	208.2	452.7	2.870	15.30	-403.1	1.107	-288.0
0.3654	283.0	203.9	521.3	15.42	48.60	-386.1	11.08	-267.7
0.3846	279.5	226.8	509.2	2.113	51.84	-448.9	0.8209	-274.9
0.4038	307.0	224.4	582.1	17.41	91.14	-436.2	10.47	-250.3
0.4231	294.9	244.4	556.7	7.852	95.12	-490.0	-1.309	-250.2
0.4423	319.9	234.9	619.7	17.05	132.4	-455.2	14.66	-213.3
0.4615	305.7	261.2	588.3	5.533	136.5	-518.0	1.735	-212.6
0.4808	326.4	252.5	648.7	21.93	177.3	-490.2	9.557	-177.3
0.5000	309.4	263.8	607.6	6.892	178.5	-512.8	3.589	-160.7
0.5192	322.2	258.5	647.5	18.29	212.1	-489.3	10.80	-126.0
0.5385	297.3	273.0	591.7	6.090	209.7	-518.1	1.146	-112.3
0.5577	305.0	251.1	620.7	17.62	239.4	-453.5	15.87	-64.07
0.5769	276.7	269.7	558.6	8.080	234.7	-495.1	-2.351	-58.52
0.5962	283.9	252.6	583.0	17.08	260.4	-440.7	9.556	-17.13
0.6154	255.3	259.2	512.8	2.310	248.9	-450.1	0.5979	-2.955
0.6346	254.2	237.4	526.0	17.60	269.7	-387.8	9.603	35.78
0.6538	221.2	246.8	443.9	2.672	251.1	-404.8	-0.3427	42.56
0.6731	220.7	221.4	454.0	14.19	265.4	-334.5	9.932	78.68
0.6923	190.8	224.7	377.0	0.6675	243.8	-342.4	-1.466	81.57
0.7115	179.8	200.8	366.5	13.02	247.6	-276.2	8.000	109.8
0.7308	154.5	197.2	302.8	5.324	226.8	-271.1	2.446	111.7
0.7500	143.4	172.7	283.1	9.216	220.0	-211.2	7.459	129.7
0.7692	121.1	165.8	225.9	2.625	195.4	-203.2	0.1574	125.2
0.7885	105.1	142.0	197.8	7.378	179.8	-150.4	2.116	133.4
0.8077	77.76	123.8	119.4	3.896	152.2	-116.1	-1.872	131.3
0.8269	42.26	76.56	24.89	-4.993	79.00	-29.32	2.721	117.1
0.8462	45.79	55.79	-72.08	4.438	0.746	73.30	-5.005	103.0
0.8654	68.42	68.65	-137.0	-1.033	-55.52	137.3	0.7756	81.49
0.8846	97.43	99.66	-193.6	0.5331	-111.4	193.8	-0.8653	59.53
0.9038	118.9	124.2	-233.0	-0.9707	-158.0	230.9	-0.2795	36.57
0.9231	136.9	146.7	-263.5	-0.2417	-196.0	262.4	0.3454	17.08
0.9423	150.0	162.0	-283.7	0.5163	-226.3	280.0	-1.959	0.8765
0.9615	159.2	174.2	-297.7	-0.0188	-247.8	295.0	0.3121	-13.40
0.9808	164.8	180.1	-304.8	0.2043	-260.3	300.3	-0.7183	-22.16

1.0000 166.4 183.1 -307.0 0.0370 -264.9 304.5 0.2962 -24.17

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MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

Table 29: Case 14 Stress Results (Pa)

CP	IES	OES	ICS	IRS	IAS	OCS	ORS	OAS
0.0000	112.2	94.63	-183.9	-0.2555	19.60	187.1	-1.522	105.9
0.0192	110.3	93.38	-180.7	-0.0685	19.36	184.7	-1.540	103.8
0.0385	107.3	91.12	-177.1	-0.2781	16.70	180.4	-1.470	99.11
0.0577	100.4	86.21	-166.7	-0.1394	13.74	170.9	-1.386	90.27
0.0769	92.52	80.42	-155.9	-0.3255	8.606	159.6	-1.263	79.95
0.0962	80.30	71.62	-137.3	-0.2726	3.720	141.9	-1.165	63.41
0.1154	67.29	67.07	-118.0	-0.2249	-2.812	122.3	-1.260	45.60
0.1346	49.25	49.42	-89.38	-0.4379	-8.227	94.61	-0.5371	23.83
0.1538	31.74	38.13	-60.14	0.7801	-14.25	65.49	-1.233	0.1437
0.1731	10.79	26.38	-21.00	-0.6271	-16.98	24.82	-0.8813	-2.792
0.1923	22.99	29.52	22.86	-1.539	-23.09	-16.06	2.253	-55.51
0.2115	37.40	35.12	53.47	1.259	-19.01	-34.95	1.660	-68.56
0.2308	49.29	43.16	74.54	-0.7763	-18.21	-61.44	0.1515	-83.03
0.2500	63.78	49.37	104.4	1.674	-12.49	-72.75	2.410	-90.62
0.2692	74.00	59.72	122.6	-0.1252	-10.46	-102.1	0.5246	-103.7
0.2885	90.30	65.27	158.1	3.866	-0.4800	-111.9	3.436	-107.2
0.3077	96.47	76.99	167.8	0.5599	0.8740	-144.3	0.1398	-118.3
0.3269	113.5	79.17	207.6	7.116	15.21	-144.8	4.496	-115.9
0.3462	117.2	92.90	209.8	-0.7274	15.24	-180.8	0.6264	-124.9
0.3654	134.3	94.47	252.2	7.674	33.49	-278.4	6.900	-117.8
0.3846	134.0	107.5	248.1	1.146	34.26	-214.7	-0.7380	-125.0
0.4038	149.7	104.7	287.2	8.632	53.20	-203.1	5.787	-112.1
0.4231	148.0	120.1	280.7	1.988	55.06	-241.1	-0.9243	-116.8
0.4423	159.5	114.7	312.5	10.09	73.67	-222.4	6.840	-99.12
0.4615	155.5	128.7	299.7	1.471	74.43	-255.8	-0.2781	-100.6
0.4808	164.5	121.6	325.9	8.502	92.00	-232.5	7.822	-79.87
0.5000	154.3	134.6	301.7	1.226	90.48	-263.9	-1.519	-80.38
0.5192	162.5	123.9	326.8	9.288	108.2	-231.3	8.041	-56.02
0.5385	151.8	136.0	299.0	0.7646	105.4	-258.1	0.1239	-54.81
0.5577	154.6	124.1	315.4	10.13	120.4	-225.8	5.536	-32.24
0.5769	141.0	132.1	280.2	-0.1118	113.8	-242.2	-0.9153	-28.56
0.5962	141.2	118.5	289.5	8.452	125.7	-204.2	6.954	-5.446
0.6154	125.3	125.4	250.9	0.5860	117.3	-219.5	-0.6929	-4.110
0.6346	123.3	110.7	253.7	7.228	124.6	-180.6	5.648	16.29
0.6538	105.8	111.7	211.6	0.2327	113.3	-184.6	-2.023	18.03
0.6731	101.7	97.13	208.8	5.708	116.4	-146.1	4.697	35.35
0.6923	84.80	96.31	166.1	0.1876	102.4	-146.0	-0.4049	36.07
0.7115	77.88	82.22	158.3	4.294	101.5	-111.6	3.406	47.72
0.7308	62.57	77.14	119.4	-1.423	84.54	-103.2	0.2020	47.73
0.7500	52.97	63.13	102.2	1.616	72.26	-70.10	4.364	55.43
0.7692	40.28	55.68	71.40	-2.059	63.33	-58.34	0.8627	52.25
0.7885	29.26	43.52	48.24	2.215	56.54	-31.24	0.4492	54.22
0.8077	20.32	34.24	17.85	-2.337	38.29	-12.66	3.576	53.08
0.8269	18.07	24.83	-25.21	-0.0710	8.901	31.59	-0.9647	47.78
0.8462	35.59	38.13	-67.76	1.727	-12.62	74.50	-1.322	43.67
0.8654	49.56	53.36	-28.37	0.5887	-44.75	104.7	-0.6145	36.00
0.8846	64.17	70.65	-127.3	0.2122	-67.29	134.1	-1.039	30.86
0.9038	74.61	83.16	-147.4	0.8724	-86.49	153.8	-0.9926	23.75
0.9231	84.25	95.23	-165.8	1.011	-102.8	172.2	-0.8581	18.40
0.9423	90.60	103.0	-177.5	1.084	-114.8	183.8	-1.069	12.83
0.9615	95.87	109.8	-187.2	1.126	-124.1	194.4	-0.9220	0.565
0.9808	98.35	112.7	-191.6	1.192	-132.0	197.2	-1.142	6.211

1.0000 99.67 114.7 -194.1 1.174 -131.2 201.2 -0.9300 6.410

Table 30: Case 15 Stress Results (Pa)

CP	IES	OES	ICS	IRS	IAS	OCS	ORS	OAS
0.0000	112.2	65.81	-89.81	2.850	133.5	94.97	2.037	123.5
0.0192	111.3	65.64	-91.09	2.939	130.6	97.80	-1.897	121.9
0.0385	109.3	64.54	-96.77	2.780	121.5	101.3	-1.931	116.8
0.0577	105.0	63.62	-103.0	2.890	107.0	112.4	-1.788	104.0
0.0769	99.39	62.51	-111.8	2.464	86.17	117.6	-1.486	91.00
0.0962	90.02	62.41	-115.5	2.836	61.23	122.7	-1.574	70.16
0.1154	78.63	62.84	-117.4	2.429	30.67	122.4	-1.467	42.07
0.1346	60.85	63.33	-105.0	2.358	-1.648	111.7	-0.6969	4.925
0.1538	43.87	66.91	-83.99	3.598	-35.58	88.86	-1.840	-41.69
0.1731	35.94	72.46	-33.98	2.307	-69.58	41.28	-1.649	-108.1
0.1923	65.56	90.69	23.14	0.2614	-100.1	-16.36	3.465	-162.6
0.2115	82.72	97.08	67.94	5.391	-95.98	-45.66	1.239	-185.4
0.2308	95.23	105.4	89.51	-4.127	-100.9	-74.71	6.406	-202.7
0.2500	112.7	110.3	132.7	3.716	-91.92	-91.87	4.251	-215.8
0.2692	121.7	117.3	149.8	0.8364	-91.52	-128.6	1.652	-232.5
0.2885	141.4	121.0	198.5	4.861	-76.87	-141.6	3.736	-236.6
0.3077	149.2	128.8	213.3	-1.749	-73.26	-183.7	0.2396	-248.2
0.3269	168.0	130.7	265.1	8.314	-50.95	-185.4	8.002	-241.0
0.3462	174.8	140.9	279.3	0.7242	-42.80	-238.5	-0.6195	-250.4
0.3654	193.0	140.7	326.5	4.816	-19.12	-236.7	7.863	-234.9
0.3846	198.4	153.2	342.7	1.664	-4.575	-283.1	2.955	-235.0
0.4038	214.6	150.8	389.8	10.91	25.84	-281.8	-6.347	-216.7
0.4231	209.6	162.5	383.1	4.733	37.54	-321.2	-0.7411	-207.6
0.4423	227.1	158.7	432.9	13.66	71.88	-304.4	11.12	-175.6
0.4615	219.4	176.3	418.2	5.466	82.81	-350.4	2.091	-166.5
0.4808	232.0	171.1	455.6	12.50	115.0	-333.8	6.643	-133.1
0.5000	219.3	181.6	429.1	3.688	124.1	-353.1	2.582	-111.8
0.5192	227.0	177.3	458.6	14.25	156.0	-332.3	8.039	-75.62
0.5385	207.8	190.2	413.1	1.471	158.2	-352.0	2.325	-55.21
0.5577	210.7	175.6	435.2	15.97	187.9	-304.3	9.775	-11.21
0.5769	189.8	188.9	386.3	6.849	188.7	-323.4	1.064	6.437
0.5962	190.0	178.9	389.6	9.700	207.0	-282.0	7.951	44.62
0.6154	168.8	185.5	342.1	6.477	206.4	-285.0	-1.614	62.53
0.6346	162.5	171.8	326.8	6.464	214.6	-234.2	5.782	98.78
0.6538	143.1	177.8	271.3	-3.055	204.5	-236.1	0.4685	112.0
0.6731	133.0	161.9	260.7	9.836	212.0	-179.8	7.693	142.6
0.6923	117.2	163.0	202.3	-2.541	190.2	-177.5	-0.7990	148.0
0.7115	106.9	149.1	203.7	8.009	180.4	-130.1	2.514	167.5
0.7308	93.21	144.4	131.1	0.9666	181.6	-116.7	-0.3480	170.3
0.7500	85.63	131.0	110.0	3.156	172.5	-75.21	4.476	180.7
0.7692	77.41	122.3	64.58	-4.445	150.1	-58.89	1.805	174.7
0.7885	68.87	108.9	39.20	1.040	134.7	-24.53	3.067	176.4
0.8077	66.05	96.42	-15.76	7.930	108.4	19.15	-5.822	172.3
0.8269	53.73	68.62	-65.19	-0.9256	41.52	68.13	0.6568	137.9
0.8462	60.77	64.38	-113.7	0.2044	-20.10	110.8	0.9062	103.2
0.8654	67.54	68.44	-136.4	-1.298	-68.50	138.4	1.594	67.08
0.8846	74.90	77.33	-145.5	-0.6416	-106.1	148.2	1.120	33.10
0.9038	80.25	81.72	-147.8	-0.5939	-129.6	142.5	-0.7592	2.670
0.9231	84.64	85.85	-129.2	-0.8020	-160.5	135.3	1.107	-24.70
0.9423	88.57	87.39	-121.0	-0.6648	-173.4	123.3	0.8546	-45.92
0.9615	91.48	88.92	-111.7	-0.5153	-192.0	114.3	0.8582	-60.26

0.9808	93.48	89.20	-106.2	-0.6481	-187.1	107.3	0.7402	-69.91
1.0000	94.09	89.50	-102.8	-0.4855	-188.4	105.3	0.7803	-72.83

Table 31: Case 16 Stress Results (Pa)

CP	IES	OES	ICS	IRS	IAS	OCS	ORS	OAS
0.0000	62.20	32.48	-7.863	2.202	104.5	8.697	0.4122	60.36
0.0192	62.26	31.82	-10.73	1.828	102.8	12.04	0.8118	60.67
0.0385	62.42	30.76	-18.62	2.159	98.37	19.19	0.3432	60.48
0.0577	62.56	29.60	-30.81	1.760	90.10	32.06	0.7438	59.90
0.0769	63.04	30.28	-47.47	1.747	77.66	47.25	0.4743	57.17
0.0962	62.30	33.84	-63.63	1.599	60.91	65.26	0.6816	50.48
0.1154	61.35	40.21	-81.58	1.201	38.24	81.06	0.6958	38.55
0.1346	55.90	48.34	-89.97	0.6714	12.03	92.00	0.9584	18.29
0.1538	49.16	57.21	-92.58	1.206	-20.17	92.58	0.5321	-12.30
0.1731	37.75	64.81	-73.00	0.9375	-49.23	73.71	1.127	-55.59
0.1923	46.91	78.94	-38.57	3.834	-89.85	45.49	-1.214	-108.5
0.2115	50.69	79.34	-11.90	1.537	-92.24	14.16	-2.623	-130.8
0.2308	58.23	84.04	13.77	0.1618	-93.20	-3.669	5.503	-144.4
0.2500	67.68	87.83	30.27	0.9791	-92.32	-20.88	2.857	-157.7
0.2692	76.67	91.14	63.28	3.408	-88.92	-43.10	2.670	-173.0
0.2885	87.25	93.57	90.91	2.328	-83.59	-59.68	2.147	-181.7
0.3011	97.62	96.90	117.3	1.036	-76.67	-90.63	2.212	-191.5
0.3269	113.0	99.07	159.1	5.974	-61.35	-102.0	6.616	-191.2
0.3462	117.2	102.9	170.7	2.855	-55.08	-137.5	3.273	-197.1
0.3654	134.0	102.8	213.9	4.771	-35.69	-145.4	6.231	-189.8
0.3846	140.3	110.0	232.6	3.382	-22.18	-179.2	6.395	-188.8
0.4038	154.1	107.6	271.3	8.045	0.9578	-188.7	4.046	-175.4
0.4231	155.5	116.6	280.4	6.566	15.92	-223.3	0.2387	-168.8
0.4423	169.4	112.9	318.1	10.04	41.92	-210.0	10.64	-141.4
0.4615	162.5	123.8	305.7	3.278	51.22	-245.5	1.894	-132.2
0.4808	172.9	120.8	340.9	13.32	81.14	-238.2	2.886	-105.5
0.5000	165.5	130.9	325.0	5.178	91.55	-255.0	2.190	-83.97
0.5192	170.0	128.2	338.7	5.654	113.2	-237.1	8.583	-50.54
0.5385	155.3	137.3	311.9	4.102	121.7	-249.4	3.146	-29.84
0.5577	153.0	126.8	320.2	15.47	144.5	-213.4	5.059	7.510
0.5769	135.3	136.0	273.0	2.456	142.6	-219.7	1.000	28.32
0.5962	133.9	130.0	274.8	7.634	158.1	-185.4	8.558	61.68
0.6154	115.8	133.5	229.5	2.277	154.8	-180.6	1.273	79.59
0.6346	107.0	126.3	213.0	7.334	161.1	-145.5	2.097	105.7
0.6538	93.20	127.9	172.5	2.403	153.5	-135.3	-1.726	120.4
0.6731	84.72	118.7	141.5	3.971	158.4	-91.59	4.918	144.5
0.6923	74.10	118.1	102.8	1.837	146.3	-81.56	0.5388	151.2
0.7115	67.48	108.7	77.87	4.541	139.3	-64.14	4.376	163.6
0.7308	62.71	103.7	38.33	-0.0880	122.5	-26.62	0.0127	164.8
0.7500	60.16	96.26	13.41	0.6595	110.6	0.7612	2.066	168.1
0.7692	59.81	89.73	-16.78	2.368	95.06	22.91	0.4176	165.9
0.7885	60.00	82.61	-41.79	1.589	76.80	40.05	-1.041	158.1
0.8077	63.37	78.00	-69.73	4.330	56.37	74.58	-2.351	153.6
0.8269	57.05	67.78	-98.78	2.155	-2.215	98.42	0.8325	118.0
0.8462	59.34	63.63	-116.3	1.990	-49.47	127.2	0.4332	73.15
0.8654	59.03	60.53	-113.0	1.680	-80.00	119.0	0.3328	38.72
0.8846	59.51	58.13	-91.55	2.400	-105.0	106.1	0.2336	11.47
0.9038	58.28	51.69	-75.33	2.223	-114.9	83.70	0.2313	-10.85
0.9231	59.03	46.48	-54.93	2.597	-115.4	64.25	0.0351	-26.06
0.9423	60.20	41.16	-37.99	2.163	-116.2	45.52	0.3349	-36.66
0.9615	61.65	37.61	-23.66	2.730	-114.8	31.67	0.0624	-43.25
0.9808	62.53	35.63	-16.06	2.126	-114.1	21.70	0.4561	-64.51

1.0000 62.95 34.80 -17.31 -2.945 -113.1 19.86 0.0944 -47.82

Table 32: Case 17 Stress Results (Pa)

CP	IES	OES	ICS	IRS	IAS	OCS	ORS	OAS
0.0000	22.48	24.35	-47.95	-1.036	-7.878	48.38	-0.1025	20.17
0.0277	21.64	23.56	-41.28	-0.9701	-7.477	46.78	-0.1243	19.53
0.0455	20.66	22.62	-39.54	-1.012	-7.319	44.96	-0.0717	18.60
0.0682	17.95	20.10	-34.42	-1.004	-6.350	39.89	-0.0988	16.38
0.0909	15.16	17.49	-29.24	-1.012	-5.562	34.67	-0.1006	13.90
0.1136	10.63	13.09	-20.73	-1.032	-3.957	25.99	0.0136	10.16
0.1364	6.283	8.697	-12.41	-0.3097	-2.434	17.00	-0.1787	6.046
0.1591	3.020	5.424	-6.407	-1.363	-1.175	10.11	-0.5120	2.887
0.1818	1.758	3.144	1.907	-1.606	0.2916	5.631	-0.1620	0.6153
0.2045	4.348	1.661	7.692	-0.8249	1.913	1.289	0.0563	-1.999
0.2273	7.564	2.877	13.92	-0.7514	3.395	-5.652	0.1009	-2.777
0.2500	10.81	4.535	20.67	-0.1538	5.199	-8.707	0.3320	-4.827
0.2727	13.48	7.055	25.56	-0.3997	6.248	-14.06	0.0337	-7.562
0.2955	16.69	8.501	32.43	0.3954	8.263	-16.40	0.5756	-9.728
0.3182	18.78	11.04	35.95	-0.1627	8.938	-22.02	0.0486	-11.10
0.3409	21.79	11.92	42.78	0.9935	11.21	-23.04	0.8023	-11.46
0.3636	23.20	14.32	44.65	-0.0242	11.47	-28.54	0.0918	-13.48
0.3864	25.78	14.74	50.79	1.227	13.68	-28.42	1.042	-13.10
0.4091	26.57	16.93	51.55	0.3060	13.77	-33.76	0.0221	-14.79
0.4318	28.61	16.74	56.73	1.578	15.91	-32.30	1.117	-13.63
0.4545	28.59	18.55	55.65	0.3200	15.49	-36.81	0.0568	-14.74
0.4773	30.00	17.86	59.74	1.726	17.49	-34.28	1.192	-12.94
0.5000	29.32	19.41	57.36	0.4146	16.76	-38.26	0.0361	-13.67
0.5227	30.06	18.10	60.03	1.658	18.36	-34.45	1.212	-11.25
0.5455	28.72	19.13	56.38	0.3885	17.32	-37.38	-0.0248	-11.52
0.5682	28.78	17.47	57.65	1.549	18.47	-32.85	1.135	-8.812
0.5909	26.76	18.01	52.68	0.2848	17.01	-34.60	0.0058	-8.666
0.6136	26.03	15.86	52.34	1.400	17.60	-29.24	0.9765	-5.792
0.6364	23.48	15.79	46.27	0.1159	15.70	-29.60	0.0051	-5.287
0.6591	21.94	13.33	44.06	0.9219	15.52	-23.66	0.9042	-2.406
0.6818	18.99	12.75	37.36	-0.1306	13.35	-22.90	0.0228	-1.755
0.7045	16.88	10.14	33.87	0.5322	12.56	-16.86	0.6270	0.7963
0.7273	13.61	8.945	26.56	-0.4361	10.04	-14.57	0.0137	1.694
0.7500	10.90	6.400	21.58	0.0606	8.510	-8.580	0.3412	3.828
0.7727	7.542	5.000	14.30	-0.7424	5.850	-5.154	-0.0104	4.845
0.7955	4.308	3.654	7.784	-0.8319	3.471	0.4072	0.0081	6.567
0.8182	1.764	4.185	1.645	-1.758	0.7445	4.663	-0.1835	8.149
0.8409	3.146	6.042	-7.135	-1.325	-2.137	11.39	-0.5036	7.276
0.8636	6.425	9.258	-13.34	-0.8240	-4.563	18.35	-0.1623	9.329
0.8864	10.67	13.89	-22.13	-1.051	-8.767	27.70	0.0719	11.28
0.9091	15.11	18.68	-31.00	-1.035	-12.72	36.87	0.0082	13.11
0.9318	17.84	21.53	-36.51	-1.032	-15.46	42.25	0.0072	13.93
0.9545	20.48	24.33	-41.77	-1.024	-17.95	47.64	0.0746	14.97
0.9773	21.44	25.34	-43.49	-1.004	-18.71	49.46	0.0019	15.12
1.0000	22.24	26.23	-45.33	-1.044	-19.59	51.21	0.0555	15.54

Appendix C:
Empirical Equations

General Form of Empirical Equation.

for stress:

$$\left(\frac{S}{P}\right) = K \left(\frac{R_0}{a}\right)^{X_1} \left(\frac{a}{t}\right)^{X_2} \left(\frac{a}{b}\right)^{X_3} \quad (1)$$

for: K, X_1, X_2, X_3 being constants..

Using logarithmic transformations, (1) becomes:

$$\log\left(\frac{S}{P}\right) = \log(K) + X_1 \log\left(\frac{R_0}{a}\right) + X_2 \log\left(\frac{a}{t}\right) + X_3 \log\left(\frac{a}{b}\right) \quad (2)$$

for change in radius of curvature:

$$\left(\frac{\Delta RE}{R_0 P}\right) = K \left(\frac{R_0}{a}\right)^{X_1} \left(\frac{a}{t}\right)^{X_2} \left(\frac{a}{b}\right)^{X_3} \quad (3)$$

and

$$\log\left(\frac{\Delta RE}{R_0 P}\right) = \log(K) + X_1 \log\left(\frac{R_0}{a}\right) + X_2 \log\left(\frac{a}{t}\right) + X_3 \log\left(\frac{a}{b}\right) \quad (4)$$

or

$$\left(\frac{\Delta R}{R_0}\right) = K \left(\frac{R_0}{a}\right)^{X_1} \left(\frac{a}{t}\right)^{X_2} \left(\frac{a}{b}\right)^{X_3} \quad (5)$$

and

$$\log\left(\frac{\Delta R}{R_0}\right) = \log(K) + X_1 \log\left(\frac{R_0}{a}\right) + X_2 \log\left(\frac{a}{t}\right) + X_3 \log\left(\frac{a}{b}\right) \quad (6)$$

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Effective Stress at the Major Axis:

for equation (2): $r^2 = 0.99404$

standard error of the logarithmic

response values = 0.02869

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	-	-	-	-
X_2	1.35258	0.04242	1.12548	1.45041
X_3	1.23862	0.18184	0.81929	1.65796
$\log(K)$	0.32467	0.05286	0.20277	0.44657
K	2.11190	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 2.1119 \left(\frac{a}{t}\right)^{1.35258} \left(\frac{a}{b}\right)^{1.23862}$$

and

$$\left(\frac{S}{P}\right) = 2.1119 \frac{a^{2.5912}}{t^{1.35258} b^{1.23862}}$$

Effective Stress at the Minor Axis:

for equation (2): $r^2 = 0.97385$

standard error of the logarithmic

response values = 0.02533

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	-	-	-	-
X_2	1.79045	0.15485	1.43338	2.14753
X_3	2.73102	0.25485	2.14334	3.31869
$\log(K)$	-0.21375	0.13164	-0.51731	0.08980
K	0.61129	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 0.61129 \left(\frac{a}{t}\right)^{1.79045} \left(\frac{a}{b}\right)^{2.73102}$$

and

$$\left(\frac{S}{P}\right) = 0.61129 \frac{a^{4.52147}}{t^{1.79045} b^{2.73102}}$$

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Circumferential Stress at the Major Axis:

for equation (2): $r^2 = 0.73666$

standard error of the logarithmic
response values = 0.34038

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	-	-	-	-
X_2	0.35527	0.01676	0.32223	0.38831
X_3	0.95450	0.07599	0.80473	1.10426
log(K)	0.85557	0.05501	0.74714	0.96399
K	7.17080	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 7.1708 \left(\frac{a}{t}\right)^{0.35527} \left(\frac{a}{b}\right)^{0.95450}$$

and

$$\left(\frac{S}{P}\right) = 7.1708 \frac{a^{1.30977}}{t^{0.35527} b^{0.95450}}$$

Annular Stress at the Major Axis:

for equation (2): $r^2 = 0.65054$

standard error of the logarithmic

response values = 0.66152

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	-	-	-	-
X_2	0.57965	0.03258	0.51544	0.64385
X_3	1.38015	0.14768	1.08909	1.67122
$\log(K)$	-0.09151	0.10692	-0.30223	0.11922
K	0.81001	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 0.81001 \left(\frac{a}{t}\right)^{0.57965} \left(\frac{a}{b}\right)^{1.38015}$$

and

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$$\left(\frac{S}{P}\right) = 0.81001 \frac{a^{1.9598}}{t^{0.57965} b^{1.38015}}$$

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Effective Stress at the Major Axis:

for equation (2): $r^2 = 0.69938$

standard error of the logarithmic
response values = 0.33587

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	-	-	-	-
X_2	0.31816	0.01654	0.28556	0.35076
X_3	0.89175	0.07498	0.74397	1.03953
$\log(K)$	0.88622	0.05428	0.77923	0.99321
K	7.69520	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 7.6952 \left(\frac{a}{t}\right)^{0.31816} \left(\frac{a}{b}\right)^{0.89175}$$

and

$$\left(\frac{S}{P}\right) = 7.6952 \frac{a^{1.20991}}{t^{0.31816} b^{0.89175}}$$

Cross-Sectional Model

Circumferential Stress at the Major Axis:

for equation (2): $r^2 = 0.95314$

standard error of the logarithmic

response values = 0.14703

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	-	-	-	-
X_2	1.14998	0.12151	0.85266	1.44729
X_3	4.10097	0.82285	2.08735	6.11422
$\log(K)$	0.04643	0.10336	-0.20649	0.29935
K	1.11283	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 1.11283 \left(\frac{a}{t}\right)^{1.14998} \left(\frac{a}{b}\right)^{4.10097}$$

and

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$$\left(\frac{S}{P}\right) = 1.11283 \frac{a^{5.25095}}{t^{1.14998} b^{4.10097}}$$

Effective Stress at the Major Axis:

for equation (2): $r^2 = 0.93734$

standard error of the logarithmic

response values = 0.15760

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	-	-	-	-
X_2	1.05418	0.13024	0.73550	1.37285
X_3	3.80277	0.88196	1.64471	5.96082
$\log(K)$	0.16462	0.11079	-0.10647	0.43570
K	1.46090	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 1.46090 \left(\frac{a}{t}\right)^{1.05418} \left(\frac{a}{b}\right)^{3.80277}$$

and

$$\left(\frac{S}{P}\right) = 1.46090 \frac{a^{4.85695}}{t^{1.05418} b^{3.80277}}$$

Circumferential Stress at the Minor Axis:

for equation (2): $r^2 = 0.96409$

standard error of the logarithmic
response values = 0.16761

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	-	-	-	-
X_2	1.52688	0.13851	1.18797	1.86580
X_3	5.12012	0.93798	2.82498	7.41526
$\log(K)$	-0.31215	0.11783	-0.60045	-0.02384
K	0.93065	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 0.93065 \left(\frac{a}{t}\right)^{1.52688} \left(\frac{a}{b}\right)^{5.12012}$$

and

$$\left(\frac{S}{P}\right) = 0.93065 \frac{a^{6.64700}}{t^{1.52688} b^{5.12012}}$$

Effective Stress at the Minor Axis:

for equation (2): $r^2 = 0.96409$

standard error of the logarithmic
response values = 0.16780

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	-	-	-	-
X_2	1.52912	0.13867	1.18981	1.86842
X_3	5.11861	0.93905	2.82085	7.41636
$\log(K)$	-0.31456	0.11796	-0.60320	-0.02592
K	0.48466	-	-	-

Equation (1) becomes;

$$\left(\frac{S}{P}\right) = 0.48466 \left(\frac{a}{t}\right)^{1.52912} \left(\frac{a}{b}\right)^{5.11861}$$

and

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$$\left(\frac{S}{P}\right) = 0.48466 \frac{a^{6.64773}}{t^{1.52912} b^{5.11861}}$$

Smith

Change in Radius of Curvature $\left(\frac{\Delta RE}{R_0 P}\right)$:

for equation (4): $r^2 = 0.98007$

standard error of the logarithmic
response values = 0.07567

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	0.28492	0.12550	0.03708	0.53276
X_2	2.37325	0.03278	2.30852	2.43797
X_3	0.59711	0.07197	0.45498	0.73925
$\log(K)$	0.07645	0.14223	-0.20443	0.35733
K	1.19248	-	-	-

Equation (3) becomes:

$$\left(\frac{\Delta RE}{R_0 P}\right) = 1.19248 \left(\frac{R_0}{a}\right)^{0.28492} \left(\frac{a}{t}\right)^{2.37325} \left(\frac{a}{b}\right)^{0.59711}$$

and

$$\left(\frac{\Delta RE}{R_0 P}\right) = 1.19248 \frac{R_0^{0.28492} a^{2.68544}}{t^{2.37325} b^{0.59711}}$$

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Kardos

Change in Radius of Curvature $\left(\frac{\Delta RE}{R_0 P}\right)$:

for equation (4): $r^2 = 0.93370$

standard error of the logarithmic
response values = 0.17940

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	1.63425	0.19047	1.25397	2.01453
X_2	2.71647	0.11971	2.47745	2.95549
X_3	0.78437	0.15102	0.48284	1.08589
$\log(K)$	-1.39824	0.25117	-1.89970	-0.89677
K	0.039972	-	-	-

Equation (3) becomes:

$$\left(\frac{\Delta RE}{R_0 P}\right) = 0.039972 \left(\frac{R_0}{a}\right)^{1.63425} \left(\frac{a}{t}\right)^{2.71647} \left(\frac{a}{b}\right)^{0.78437}$$

and

$$\left(\frac{\Delta RE}{R_0 P}\right) = 0.039972 \frac{R_0^{1.63425} a^{1.86659}}{t^{2.71647} b^{0.78437}}$$

Hamada

Observed Change of Radius of Curvature $\left(\frac{\Delta RE}{R_{OP}}\right)$:

for equation (4): $r^2 = 0.97627$

standard error of the logarithmic

response values = 0.10112

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	0.32421	0.41909	-0.54494	1.19335
X_2	2.11735	0.11918	1.87019	2.36450
X_3	0.87222	0.30246	0.24495	1.49950
$\log(K)$	0.12657	0.52962	-0.97180	1.22495
K	1.33840	-	-	-

Equation (3) becomes:

$$\left(\frac{\Delta RE}{R_{OP}}\right) = 1.3384 \left(\frac{R_0}{a}\right)^{0.32421} \left(\frac{a}{t}\right)^{2.11735} \left(\frac{a}{b}\right)^{0.87222}$$

and

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$$\left(\frac{\Delta RE}{R_{OP}}\right) = 1.3384 \frac{R_0^{0.32421} a^{2.66536}}{t^{2.11735} b^{0.87222}}$$

Predicted Change in Radius of Curvature $\left(\frac{\Delta RE}{R_0 P}\right)$:

for equation (4): $r^2 = 0.97669$

standard error of the logarithmic

response values = 0.09890

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	0.21894	0.40988	-0.63110	1.06898
X_2	2.15597	0.11656	1.91425	2.39770
X_3	0.48003	0.29582	-0.13346	1.09352
$\log(K)$	0.34646	0.51799	-0.72778	1.42069
K	2.22050	-	-	-

Equation (3) becomes:

$$\left(\frac{\Delta RE}{R_0 P}\right) = 2.2205 \left(\frac{R_0}{a}\right)^{0.21894} \left(\frac{a}{t}\right)^{2.15597} \left(\frac{a}{b}\right)^{0.48003}$$

and

$$\left(\frac{\Delta RE}{R_0 P}\right) = 2.2205 \frac{R_0^{0.21894} a^{2.41706}}{t^{2.15597} b^{0.48003}}$$

Andreeva

Circumferential Stress at the Major Axis:

for equation (2): $r^2 = 0.98656$

standard error of the logarithmic
response values = 0.06659

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	0.26923	0.01205	0.24559	0.29287
X_2	1.84320	0.00735	1.82878	1.85763
X_3	0.43496	0.00888	0.41754	0.45238
$\log(K)$	-0.27564	0.01220	-0.29958	-0.25170
K	0.53010	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 0.53010 \left(\frac{R_0}{a}\right)^{0.26923} \left(\frac{a}{t}\right)^{1.84320} \left(\frac{a}{b}\right)^{0.43496}$$

and

$$\left(\frac{S}{P}\right) = 0.53010 \frac{R_0^{0.26923} a^{2.00893}}{t^{1.84320} b^{0.43496}}$$

Annular Stress at the Major Axis:

for equation (2): $r^2 = 0.98656$

standard error of the logarithmic
response values = 0.06659

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	0.26919	0.01205	0.24554	0.29283
X_2	1.84319	0.00735	1.82876	1.85761
X_3	0.43496	0.00888	0.41754	0.45238
$\log(K)$	-0.79847	0.01220	-0.82241	-0.77453
K	0.15905	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 0.15905 \left(\frac{R_0}{a}\right)^{0.26919} \left(\frac{a}{t}\right)^{1.84319} \left(\frac{a}{b}\right)^{0.43496}$$

and

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$$\left(\frac{S}{P}\right) = 0.15905 \frac{R_0^{0.26919} a^{2.00896}}{t^{1.84319} b^{0.43496}}$$

Effective Stress at the Major Axis:

for equation (2): $r^2 = 0.98656$

standard error of the logarithmic
response values = 0.06659

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	0.26921	0.01205	0.24556	0.29285
X_2	1.84320	0.00735	1.82877	1.85762
X_3	0.43494	0.00888	0.41752	0.45237
$\log(K)$	-0.32679	0.01220	-0.35073	-0.30286
K	0.47121	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 0.47121 \left(\frac{R_0}{a}\right)^{0.26921} \left(\frac{a}{t}\right)^{1.84320} \left(\frac{a}{b}\right)^{0.43494}$$

and

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$$\left(\frac{S}{P}\right) = 0.47121 \frac{R_0^{0.26921} a^{2.00893}}{t^{1.84320} b^{0.43494}}$$

Circumferential Stress at the Minor Axis:

for equation (2): $r^2 = 0.99256$

standard error of the logarithmic

response values = 0.04959

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	0.20308	0.00897	0.18548	0.22069
X_2	1.89066	0.00547	1.87992	1.90140
X_3	0.05415	0.00661	0.04118	0.06713
$\log(K)$	-0.10911	0.00908	-0.12693	-0.09128
K	0.77784	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 0.77784 \left(\frac{R_0}{a}\right)^{0.20308} \left(\frac{a}{t}\right)^{1.89066} \left(\frac{a}{b}\right)^{0.05415}$$

and

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$$\left(\frac{S}{P}\right) = 0.77784 \frac{R_0^{0.20308} a^{2.22908}}{t^{1.89066} b^{0.05415}}$$

Annular Stress at the Minor Axis:

for equation (2): $r^2 = 0.99639$

standard error of the logarithmic
response values = 0.03957

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	-0.13413	0.00716	-0.14817	-0.12008
X_2	2.16759	0.00437	2.15902	2.17616
X_3	0.09034	0.00527	0.07999	0.10069
$\log(K)$	-0.37637	0.00725	-0.39059	-0.36214
K	0.42037	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 0.42037 \left(\frac{R_0}{a}\right)^{-0.13413} \left(\frac{a}{t}\right)^{2.16759} \left(\frac{a}{b}\right)^{0.09034}$$

and

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$$\left(\frac{S}{P}\right) = 0.42037 \frac{a^{2.39206}}{R_0^{0.13413} t^{2.16759} b^{0.09034}}$$

Effective Stress at the Minor Axis:

for equation (2): $r^2 = 0.99385$

standard error of the logarithmic
response values = 0.04508

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	0.18792	0.00815	0.17192	0.20392
X_2	1.89176	0.00498	1.88200	1.90153
X_3	0.05563	0.00601	0.04384	0.06742
$\log(K)$	-0.15697	0.00826	-0.17318	-0.14077
K	0.69668	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 0.69668 \left(\frac{R_0}{a}\right)^{0.18792} \left(\frac{a}{t}\right)^{1.89176} \left(\frac{a}{b}\right)^{0.05563}$$

and

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$$\left(\frac{S}{P}\right) = 0.69668 \frac{R_0^{0.18792} a^{1.75947}}{t^{1.89176} b^{0.05563}}$$

Change in Radius of Curvature $\left(\frac{\Delta R}{R_0}\right)$:

for equation (6): $r^2 = 0.99569$

standard error of the logarithmic
response values = 0.05890

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	0.26919	0.01065	0.24828	0.29010
X_2	2.84320	0.00650	2.83044	2.85596
X_3	0.96116	0.00785	0.94575	0.97656
$\log(K)$	-11.91205	0.01079	-11.93323	-11.89088
K	$1.224 \cdot 10^{-12}$	-	-	-

Equation (5) becomes:

$$\left(\frac{\Delta R}{R_0}\right) = 1.224 \cdot 10^{-12} \left(\frac{R_0}{a}\right)^{0.26919} \left(\frac{a}{t}\right)^{2.84320} \left(\frac{a}{b}\right)^{0.96116}$$

and

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$$\left(\frac{\Delta R}{R_0}\right) = 1.224 \cdot 10^{-12} \frac{R_0^{0.26919} a^{3.53517}}{t^{2.84320} b^{0.96116}}$$

Change in Radius of Curvature $\left(\frac{\Delta RE}{R_0 P}\right)$:

for equation (4): $r^2 = 0.99567$

standard error of the logarithmic
response values = 0.05908

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	0.26587	0.01044	0.24537	0.28636
X_2	2.84468	0.00652	2.83188	2.85747
X_3	0.96118	0.00787	0.94573	0.97664
$\log(K)$	-0.61024	0.01072	-0.63128	-0.58920
K	0.24534	-	-	-

Equation (3) becomes:

$$\left(\frac{\Delta RE}{R_0 P}\right) = 0.24534 \left(\frac{R_0}{a}\right)^{0.26587} \left(\frac{a}{t}\right)^{2.84468} \left(\frac{a}{b}\right)^{0.96118}$$

and

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$$\left(\frac{\Delta RE}{R_0 P}\right) = 0.24534 \frac{R_0^{0.26587} a^{3.53999}}{t^{2.84468} b^{0.96118}}$$

Critical Geometry:

Circumferential Stress:

$$0.53010 \left(\frac{R_0}{a}\right)^{0.26923} \left(\frac{a}{t}\right)^{1.84320} \left(\frac{a}{b}\right)^{0.43496}$$

$$= 0.77784 \left(\frac{R_0}{a}\right)^{0.20308} \left(\frac{a}{t}\right)^{1.89066} \left(\frac{a}{b}\right)^{0.05415}$$

$$1.46735 = \left(\frac{R_0}{a}\right)^{0.06615} \left(\frac{a}{t}\right)^{-0.04746} \left(\frac{a}{b}\right)^{0.3808}$$

and

$$1.46735 = \frac{R_0^{0.06615} t^{0.04746} a^{0.26719}}{b^{0.3808}}$$

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Annular Stress:

$$0.15905 \left(\frac{R_0}{a}\right)^{0.26919} \left(\frac{a}{t}\right)^{1.84319} \left(\frac{a}{b}\right)^{0.43496}$$

$$= 0.42037 \left(\frac{R_0}{a}\right)^{-0.13413} \left(\frac{a}{t}\right)^{2.16759} \left(\frac{a}{b}\right)^{0.09034}$$

$$2.64301 = \left(\frac{R_0}{a}\right)^{0.40332} \left(\frac{a}{t}\right)^{-0.3244} \left(\frac{a}{b}\right)^{0.34462}$$

and

$$2.64301 = \frac{R_0^{0.4033} t^{0.3244}}{a^{0.3831} b^{0.3446}}$$

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Effective Stress:

$$0.47121 \left(\frac{R_0}{a}\right)^{0.26921} \left(\frac{a}{t}\right)^{1.84320} \left(\frac{a}{b}\right)^{0.43494}$$

$$= 0.69667 \left(\frac{R_0}{a}\right)^{0.18792} \left(\frac{a}{t}\right)^{1.89176} \left(\frac{a}{b}\right)^{0.05563}$$

$$1.47849 = \left(\frac{R_0}{a}\right)^{0.08129} \left(\frac{a}{t}\right)^{-0.04856} \left(\frac{a}{b}\right)^{0.37931}$$

and

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$$1.47849 = \frac{R_0^{0.08129} t^{0.04856} a^{0.24946}}{b^{0.37931}}$$

Long Form

Change in Radius of Curvature $\left(\frac{\Delta RE}{R_{OP}}\right)$:

for equation (4): $r^2 = 0.99659$

standard error of the logarithmic

response values = 0.03953

For the constants:

Constant	Value	Standard Error of the Constant	95% Confidence Interval of the Constant	
			Lower Bound	Upper Bound
X_1	0.89157	0.09263	0.63440	1.14875
X_2	3.43370	0.14751	3.02417	3.84324
X_3	0.12449	0.16338	-0.32912	0.57810
$\log(K)$	-1.78806	0.19673	-2.33427	-1.24184
K	0.016291	-	-	-

Equation (3) becomes:

$$\left(\frac{\Delta RE}{R_{OP}}\right) = 0.016291 \left(\frac{R_0}{a}\right)^{0.89157} \left(\frac{a}{t}\right)^{3.43370} \left(\frac{a}{b}\right)^{0.12449}$$

and

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$$\left(\frac{\Delta RE}{R_{OP}}\right) = 0.016291 \frac{R_0^{0.89157} a^{2.66662}}{t^{3.43370} b^{0.12449}}$$

Effective Stress at the Minor Axis:

for equation (2): $r^2 = 0.99014$

standard error of the logarithmic

response values = 0.03813

for the constants:

Constant	Value	Standard Error of the Constant	95% Confidence Interval of the Constant	
			Lower Bound	Upper Bound
X_1	1.33845	0.08934	1.09041	1.58650
X_2	2.33847	0.14227	1.94347	2.73347
X_3	-0.42565	0.15758	-0.86316	0.01185
$\log(K)$	-1.87752	0.18975	-2.40434	-1.35069
K	0.013258	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 0.013258 \left(\frac{R_0}{a}\right)^{1.33845} \left(\frac{a}{t}\right)^{2.33847} \left(\frac{a}{b}\right)^{-0.42565}$$

and

$$\left(\frac{S}{P}\right) = 0.013258 \frac{R_0^{1.33845} a^{0.57437} b^{0.42565}}{t^{2.33847}}$$

Effective Stress at the Major Axis:

for equation (2): $r^2 = 0.99583$

standard error of the logarithmic
response values = 0.02775

for the constants:

Constant	Value	Standard Error of the Constant	95% Confidence Interval of the Constant	
			Lower Bound	Upper Bound
X_1	0.57339	0.06503	0.39284	0.75395
X_2	1.82211	0.10356	1.53458	2.10963
X_3	0.57891	0.11471	0.26043	0.89738
$\log(K)$	-0.82296	0.13812	-1.20645	-0.43947
K	0.15033	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 0.15033 \left(\frac{R_0}{a}\right)^{0.57339} \left(\frac{a}{t}\right)^{1.82211} \left(\frac{a}{b}\right)^{0.57891}$$

and

$$\left(\frac{S}{P}\right) = 0.15033 \frac{R_0^{0.57339} a^{1.82763}}{t^{1.82211} b^{0.57891}}$$

Circumferential Stress at the Minor Axis:

for equation (2): $r^2 = 0.93072$

standard error of the logarithmic
response values = 0.18897.

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	2.98531	0.44282	1.75586	4.21475
X_2	3.18287	0.70517	1.22505	5.14069
X_3	-0.60282	0.78106	-2.77134	1.56571
$\log(K)$	-4.13129	0.94050	-6.74251	-1.52007
K	$7.3911 \cdot 10^{-5}$	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 7.3911 \cdot 10^{-5} \left(\frac{R_0}{a}\right)^{2.98531} \left(\frac{a}{t}\right)^{3.18287} \left(\frac{a}{b}\right)^{-0.60282}$$

and

$$\left(\frac{S}{P}\right) = 7.3911 \cdot 10^{-5} \frac{R_0^{2.98531} b^{0.60282}}{a^{0.40526} t^{3.18287}}$$

Annular Stress at the Minor Axis:

for equation (2): $r^2 = 0.99194$

standard error of the logarithmic
response values = 0.03780

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	1.02829	0.08857	0.78237	1.27421
X_2	2.45286	0.14105	2.06125	2.84447
X_3	-0.13902	0.15623	-0.57278	0.29473
$\log(K)$	-1.78345	0.18812	-2.30576	-1.26114
K	0.016465	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 0.016465 \left(\frac{R_0}{a}\right)^{1.02829} \left(\frac{a}{t}\right)^{2.45286} \left(\frac{a}{b}\right)^{-0.13902}$$

and

$$\left(\frac{S}{P}\right) = 0.016465 \frac{R_0^{1.02829} b^{0.13902} a^{1.28555}}{t^{2.45286}}$$

Circumferential Stress at the Major Axis:

for equation (2): $r^2 = 0.99517$

standard error of the logarithmic
response values = 0.02972

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>.99% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	0.56779	0.06965	0.37442	0.76116
X_2	1.85274	0.11091	1.54481	2.16068
X_3	0.52005	0.12285	0.17898	0.86112
$\log(K)$	-0.52511	0.14793	-0.93581	-0.11440
K	0.29846	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 0.29846 \left(\frac{R_0}{a}\right)^{0.56779} \left(\frac{a}{t}\right)^{1.85274} \left(\frac{a}{b}\right)^{0.52005}$$

and

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$$\left(\frac{S}{P}\right) = 0.29846 \frac{R_0^{0.56779} a^{1.80500}}{t^{1.85274} b^{0.52005}}$$

Annular Stress at the Major Axis:

for equation (2): $r^2 = 0.98044$

standard error of the logarithmic
response values = 0.06313

for the constants:

<u>Constant</u>	<u>Value</u>	<u>Standard Error of the Constant</u>	<u>95% Confidence Interval of the Constant</u>	
			<u>Lower Bound</u>	<u>Upper Bound</u>
X_1	0.69949	0.14793	0.28879	1.11019
X_2	1.40420	0.23556	0.75018	2.05821
X_3	1.27883	0.26091	0.55442	2.00323
$\log(K)$	-1.11243	0.31418	-1.98471	-0.24014
K	0.077192	-	-	-

Equation (1) becomes:

$$\left(\frac{S}{P}\right) = 0.077192 \left(\frac{R_0}{a}\right)^{0.69949} \left(\frac{a}{t}\right)^{1.40420} \left(\frac{a}{b}\right)^{1.27883}$$

and

$$\left(\frac{S}{P}\right) = 0.077192 \frac{R_0^{0.69949} a^{1.98354}}{t^{1.40420} b^{1.27883}}$$

END / FIN

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