1

Micro

National Library
of Canada

Bibliothèque nationale
du Canada

Canadian Theses Service

Services des thèses canadiennes

Ottawa, Canada
K1A 0N4

CANADIAN THESES

NOTICE

The quality of this microfiche is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us an inferior photocopy.

Previously copyrighted materials (journal articles, published tests, etc.) are not filmed.

Reproduction in full or in part of this film is governed by the Canadian Copyright Act, R S C 1970, c C-30.

**THIS DISSERTATION
HAS BEEN MICROFILMED
EXACTLY AS RECEIVED**

THÈSES CANADIENNES

AVIS

La qualité de cette microfiche dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

*S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de qualité inférieure.

Les documents qui font déjà l'objet d'un droit d'auteur (articles de revue, examens publiés, etc.) ne sont pas microfilmés.

La reproduction, même partielle, de ce microfilm est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c C-30.

**LA THÈSE A ÉTÉ
MICROFILMÉE TELLE QUE
NOUS L'AVONS REÇUE**

Canada

OCEAN SCATTER PROPAGATION MODELS
FOR AERONAUTICAL AND MARITIME
SATELLITE COMMUNICATION
APPLICATIONS

by

KENNETH W. MORELAND, BSc HONOURS

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of
Master of Engineering

Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University

February, 1987

© copyright

1987, Kenneth W. Moreland

Permission has been granted to the National Library of Canada to microfilm this thesis and to lend or sell copies of the film.

The author (copyright owner) has reserved other publication rights, and neither the thesis nor extensive extracts from it may be printed or otherwise reproduced without his/her written permission.

L'autorisation a été accordée à la Bibliothèque nationale du Canada de microfilmer cette thèse et de prêter ou de vendre des exemplaires du film.

L'auteur (titulaire du droit d'auteur) se réserve les autres droits de publication; ni la thèse ni de longs extraits de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation écrite.

ISBN 0-315-35838-6

The undersigned hereby recommends to the Faculty of Graduate
Studies and Research acceptance of the thesis,
"OCEAN SCATTER PROPAGATION MODELS FOR AERONAUTICAL AND
MARITIME SATELLITE COMMUNICATION APPLICATIONS"
submitted by KENNETH W. MORELAND, in partial fulfilment of the
requirements for the degree of Master of Engineering.

Thesis Supervisor

Chairman,

Department of Systems and
Computer Engineering

Department of Systems and Computer Engineering
Faculty of Engineering
Carleton University

March, 1987

ABSTRACT

Ocean scatter propagation models for aircraft-to-satellite (aeronautical) and ship-to-satellite (maritime) communication applications are developed. These models incorporate the variability of the surface reflection coefficient and mobile antenna effects, the shadowing of primary scattering elements by the wave profile, and divergence and phase shift resulting from scattering from convex and concave surface elements. The propagation model is accurately validated against a known exact far-field theoretical solution for perfectly conducting sinusoidal surfaces. The simulated behaviour of aeronautical and maritime ocean scatter is characterized as a function of important physical variables such as surface roughness, antenna height, elevation angle to the satellite, and signal polarization. Simulation results are compared with available aeronautical and maritime measurement results. A detailed comparison of aeronautical and maritime channels for omni-directional antenna patterns is provided. An aeronautical stochastic analysis model, which reduces the computational complexity of establishing the first and second order statistics of the aeronautical channel by approximately two orders of magnitude, is also developed. A detailed validation of this stochastic model is presented.

ACKNOWLEDGEMENTS

The author would like to acknowledge the assistance provided throughout the course of this project by Dr. A.U.H. Sheikh and Dr. D. Falconer. I would also like to thank R. K. Tiedemann (formerly with Miller Communications Systems Ltd.) for his advice and many fruitful discussions. I would also like to thank Miller Communications Systems Ltd. for providing access to propriatory software and their assistance in preparing the manuscript. I would also like to express my sincere appreciation to S. Roy for her excellent typing job.

TABLE OF CONTENTS

ABSTRACT.....	(iii)
ACKNOWLEDGEMENTS.....	(iv)
TABLE OF CONTENTS.....	(v)
LIST OF FIGURES.....	(xiv)
1.0 INTRODUCTION.....	1
1.1 Summary of The Major Accomplishments and Results.....	8
2.0 OCEAN SCATTER PROPAGATION MODELS FOR THE AERONAUTICAL AND MARITIME APPLICATIONS.....	18
2.1 Identifying Contributing Scattering Facets On The Ocean Surface.....	24
2.1.1 Specular and Diffuse Reflection....	36
2.2 Derivation of Signal Attenuation Factors Which Account For Scattering From Curved Surfaces.....	38
2.2.1 Signal Attenuation Factors For Aeronautical Applications.....	42
2.2.1.1 Ratio of Fresnel Zone Areas Approach.....	43
2.2.1.2 Divergence Approach.....	45
2.2.1.3 Validation of the Simulation Solution Against The Theoretical	

Far-Field Solution For A Perfectly Conducting Sinusoidal Surface.....	47
2.2.1.3.1 Establishing An Appropriate Phase Shift Factor For The Sinusoidal Surface Model.....	54
2.2.1.3.2 Comparison of Simulation and Theoretical Reflection Coefficients	58
2.2.2 Divergence Factor For Maritime Applications.....	63
2.3 Incorporating Sea-Induced Vessel Motion And Antenna Stabilization For The Maritime Application.....	68
2.4 Direct Path Wave Blockage and Blockage of Reflected Paths By The Hull of The Vessel.....	77
3.0 AERONAUTICAL AND MARITIME SIMULATION RESULTS.....	86
3.1 Selection of Antenna Patterns For The Maritime and Aeronautical Applications....	87
3.1.1 Maritime Antenna Patterns.....	95
3.1.2 Aircraft Antenna Patterns.....	101
3.1.3 Amplitude Distribution of Ocean Scatter For Maritime and Aeronautical Applications With Rough Surface Conditions.....	103
3.2 Aeronautical Simulation Results.....	112

3.2.1	Baseline Simulation Results.....	113
3.2.1.1	Signal to Interference Ratio Results For Baseline Simulation Scenarios.....	120
3.2.1.2	Doppler Spectra of Ocean Scatter Multipath For The Aeronautical Application.....	123
3.2.2	Effect of Varying Sea State and Aircraft Altitude.....	129
3.2.3	Effect of Aircraft Manoeuvres.....	134
3.2.4	Simulation Results for a More Complex Switched Antenna.....	137
3.2.5	Comparison of Simulation Results And Measurement Data.....	141
3.3	Maritime Simulation Results.....	151
3.3.1	Summary of Maritime Measurement Results.....	152
3.3.2	Maritime Simulation Scenarios Considered.....	156
3.3.3	Signal to Interference Ratio Results For Shipborne Standard-C Antennas.....	161
3.3.3.1	S/I Ratios For The Antenna Stabilization Runs.....	161
3.3.3.2	Behaviour of S/I Ratio With Sea State.....	165

3.3.3.3	Behaviour of S/I Ratio With Antenna Height.....	167
3.3.3.4	Completing The S/I Ratio Characterization Versus Elevation Angle For Shipborne Standard-C Antennas.....	170
3.3.3.5	Summary of Maritime Signal to Interference Ratio Results.....	175
3.3.4	Second Order Statistics For Standard-C Antennas.....	180
3.3.5	Other Interesting Observations.....	193
3.4	Comparison of the Aeronautical and Maritime Multipath Channels.....	195
4.0	STOCHASTIC ANALYSIS MODEL FOR AERONAUTICAL APPLICATIONS.....	200
4.1	Aeronautical Stochastic Analysis Model Development.....	201
4.1.1	Formulating The Geometrical Search Process As A Random Process Curve Crossing Problem.....	202
4.1.2	Incorporating The Effects of Blockage And Shadowing By The Ocean Surface.....	211

4.1.3	Incorporating The Effect of Attenuation Introduced By Surface Curvature.....	226
4.1.4	Average Scatter Power Characterization.....	229
4.1.5	Stochastic Model Refinements For Alignment With The Simulation Solution.....	238
4.1.5.1	Comparing The Stochastic Model And Aeronautical Simulation Program Solution Density Results.....	246
4.1.5.2	Comparing The Behaviour of The Aproximate Radius of Curvature Obtained With The Stochastic Model And Aeronautical Propagation Simulation Program.....	250
4.1.5.3	Comparing The Stochastic Model And Aeronautical Propagation Simulation Program Clearance Probabilities....	258
4.2	Comparison of The First Order Statistics of The Stochastic Model And Aeronautical Simulation Program.....	262
4.3	Establishing Approximate Aeronautical Doppler Spectra With The Stochastic Model.	279

4.4	Effect of the Surface Model on the First Order Statistics of the Aeronautical Channel.....	283
4.5	Stochastic Model Enhancements and Conclusions.....	287
5.0	SUMMARY AND CONCLUSIONS.....	294
APPENDIX A	SEA SURFACE MODEL.....	299
1.0	Introduction.....	299
2.0	Surface Model Development.....	300
2.1	Establishing A Realistic Bandwidth Proportionality Constant.....	307
2.2	Establishing A Realistic Velocity Proportionality Constant.....	317
3.0	Important Conditional Distributions.....	321
4.0	Deviations From Gaussian Distributions....	328
APPENDIX B	REFLECTION COEFFICIENTS FOR THE SEA SURFACE.....	336
APPENDIX C	SIGNAL ATTENUATION FACTORS FOR SCATTERING FROM PARABOLIC CONVEX AND CONCAVE SURFACE FACETS.....	345
1.0	Introduction.....	345
2.0	Path Length Difference Derivations.....	345

3.0	Fresnel Diffraction Theory Approach.....	352
3.1	Using Fresnel Diffraction Integrals For Deriving Signal Attenuation Factors For The Aeronautical Application.....	359
3.1.1	Helmholtz Solution Procedure For The Aeronautical Application.....	362
3.1.2	Low Grazing Angle Attenuation Factor.....	366
4.0	Derivation of Appropriate Signal Attenuation Factors for Maritime Applications.....	370
4.1	Signal Attenuation Factor Evaluation For Concave Facets.....	370
4.1.1	Ratio of First Fresnel Zones Approach For Concave Facets.....	371
4.1.2	Modified Fresnel Zones Approach For Concave Facets.....	375
4.1.3	Comparison of the Qualitative Divergence Factors With The Traditional Ray-Optics Solution For Concave Facets.....	379
4.1.4	A More Exact Evaluation of the Signal Scale Factor For Concave Surface Facets In The Near-Field Region.....	385

4.2	Divergence Factor For Convex Facets For The Maritime Application.....	389
5.0	Approximate Analysis of The Effects of Surface Non-Uniformity in The Direction Perpendicular to Sea Surface Motion.....	394
APPENDIX D SIGNAL POLARIZATION AND ANTENNA ELLIPTICITY EFFECTS.....400		
1.0	Introduction.....	400
2.0	Signal Combination With Circular Polarization.....	400
3.0	Interrelationships Between The Polarization Ratio And The Axial Ratio Of A Polarization Ellipse.....	408
4.0	Effect of Satellite Antenna Polarization Purity.....	413
APPENDIX E AERONAUTICAL DOPPLER SPECTRA.....416		
1.0	Introduction.....	416
2.0	Specular Point Motion.....	416
3.0	Differential Doppler Shift From A Scattering Facet.....	426
4.0	Comparison with Doppler Expressions Derived In Reference [22].....	429
5.0	Justification Of The Doppler Spectrum Asymmetry.....	432

APPENDIX F	CHARACTERIZING SIGNAL TO INTERFERENCE RATIOS FOR AERONAUTICAL APPLICATIONS.....	438
ATTACHMENT I	ESTABLISHING CONFIDENCE LIMITS FOR THE ESTIMATED MEAN SCATTER POWER.....	445
APPENDIX G	STOCHASTIC MODEL FOR THE MARITIME APPLICATION.....	453
1.0	Introduction.....	453
2.0	Geometric Search Process.....	454
3.0	Incorporating Divergence.....	457
4.0	Incorporating Motion Coupling Effects.....	460
APPENDIX H	EFFECTS OF THE SEARCH PROCESS ON THE SURFACE SECOND DERIVATIVE AND WAVEHEIGHT DISTRIBUTIONS AT SPECULAR POINTS.....	463
REFERENCES.....		481
GLOSSARY OF NOTATION.....		485

LIST OF FIGURES

Figure 1.1	Channel model block diagram.....	9
Figure 1.2	Propagation model geometry.....	11
Figure 2.1	Detailed reflection geometry of the ocean surface scatter propagation model.....	25
Figure 2.2	Scatter signal contribution from i-th scattering facet.....	32
Figure 2.3	Path length difference for the i-th scattering facet.....	34
Figure 2.4	Divergence of rays scattered by convex and concave surface elements ...	41
Figure 2.5	Cosinusoidal surface model.....	48
Figure 2.6	Bessel function $J_0(x)$	48
Figure 2.7	Approximate location of specular reflection points on convex (crest) and concave (trough) surface elements when the receiver is in the far-field region.....	52
Figure 2.8	Comparison of the magnitude of the simulation and theoretical reflection coefficients for perfectly conducting sinusoidal surfaces.....	60
Figure 2.9	Comparison of scattering from convex and concave surface facets.....	65

Figure 2.10	Basic vessel motion definitions.....	69
Figure 2.11	Summary of fundamental processing for sea-induced vessel motion and gravity stabilized antenna platforms.....	72
Figure 2.12	Comparison of instantaneous waveheight (ZCT) and the waterline height (ZLT) of the vessel.....	76
Figure 2.13	Comparison of pitching angle (PIT) and stabilized antenna pointing angle error (PNT).....	76
Figure 2.14	Difference between X-coordinates of antenna and center of gravity of vessel.....	78
Figure 2.15	Difference between Z-coordinates of antenna and the nominal antenna height.	78
Figure 2.16	Direct path wave blockage by high sea waves.....	79
Figure 2.17	Effect of path clearance on radio transmission (from [17]).....	79
Figure 2.18	Geometry for evaluating path clearance on reflected paths.....	83
Figure 3.1	Relative orientation of horizontal and vertically polarized scatter signals...	90
Figure 3.2	Antenna pattern of a conical spiral Standard-C antenna, referred to by the mnemonic DFVLR3 (from [6]).....	97

Figure 3.3	Antenna pattern of a small backfire Standard-C antenna, referred to by the mnemonic DFVLRC5 (from [6]).....	97
Figure 3.4	Axial ratio characteristics of a short backfire antenna, designated by the mnemonic C14 in [6], at L-band.....	98
Figure 3.5	Hemispherical coverage pattern from a quadrifiler helix (minimal Standard- C) shipborne antenna (from [9]).....	98
Figure 3.6	Roll plane patterns of a low complexity crossed slot antenna (mounted on a Boeing 747) at L-band...	102
Figure 3.7	Roll plane patterns of a diversity mounted, switch selectable crossed slot antenna system at L-band.....	102
Figure 3.8	Observed and theoretical received voltage and scatter power density functions for a 10° elevation angle maritime case with sea state index SS4(ii).....	106
Figure 3.9	Observed and theoretical received voltage and scatter power density functions for a 15° elevation angle maritime case with sea state index SS4(ii).....	107

Figure 3.10	Observed and theoretical received voltage and scatter power density functions for a 20° elevation angle maritime case with sea state index SS4(ii).....	108
Figure 3.11	Observed and theoretical received voltage and scatter power density functions for a 31° elevation angle aeronautical run with sea state index SS4(ii).....	109
Figure 3.12	Observed and theoretical received voltage and scatter power density functions for a 10° elevation angle aeronautical run with SS3 surface conditions.....	110
Figure 3.13	Power density versus reflection angle curves for horizontal, vertical, and circular polarization with an omni antenna pattern for an aeronautical simulation run with an elevation angle of 10°.....	117
Figure 3.14	Power density versus reflection angle curves for horizontal, vertical, and circular polarization with an omni antenna pattern for an aeronautical simulation run with an elevation angle of 15°.....	118

Figure 3.15	Power density versus reflection angle curves for horizontal, vertical, and circular polarization with an omni antenna pattern for an aeronautical simulation run with an elevation angle of 31°	119
Figure 3.16	Aeronautical doppler spectrum for an elevation angle of 10° with a perfectly circular omni antenna pattern.....	125
Figure 3.17	Aeronautical doppler spectrum for an elevation angle of 15° with a perfectly circular omni antenna pattern.....	125
Figure 3.18	Aeronautical doppler spectra for an elevation angle of 31° with circularly polarized crossed slot and omni antennas.....	126
Figure 3.19	Aeronautical doppler spectrum for an elevation angle of 15° with a crossed slot antenna.....	126
Figure 3.20	Aeronautical doppler spectra for SS3 and SS5(i) surface conditions with an elevation angle of 10° and a perfectly circular omni antenna pattern.....	131

Figure 3.21	Comparison of measured and simulated horizontal and vertical scatter power levels for an omni reference antenna..	144
Figure 3.22	Radiation pattern (amplitude response to a rotating dipole source) of the aircraft slot-dipole antennas used during the measurement program reported in [5].....	150
Figure 3.23	Power density curves for a 5° elevation angle maritime scenario with an omni antenna pattern.....	181
Figure 3.24	Power density curves for a 10° elevation angle maritime scenario with an omni antenna pattern.....	182
Figure 3.25	Power density curves for a 15° elevation angle maritime scenario with an omni antenna pattern.....	183
Figure 3.26	Maritime doppler spectrum for an, elevation angle of 20° and for a vessel moving with the waves for SS4(ii) surface conditions.....	186
Figure 3.27	Maritime doppler spectrum for an elevation angle of 20° and for a vessel moving against the waves for SS4(ii) surface conditions.....	186

Figure 3.28	Maritime doppler spectrum for an elevation angle of 10° and for a vessel moving with the waves for SS4(ii) surface conditions.....	188
Figure 3.29	Maritime doppler spectrum for an elevation angle of 10° and for a vessel travelling against the waves for SS3 and SS4(ii) surface conditions	188
Figure 3.30	Maritime doppler spectrum for an elevation angle of 15° with a vessel travelling against the waves for SS4(ii) surface conditions.....	189
Figure 3.31	Maritime doppler spectrum for an elevation angle of 5° with a vessel travelling against the waves for SS4(ii) surface conditions.....	189
Figure 3.32	An example of a maritime scatter power versus time plot.....	194
Figure 4.1	Local reflection and elevation angles defined relative to the curvature of the earth.....	212
Figure 4.2	Geometry of the surface blockage (clearance) problem.....	216
Figure 4.3	Behaviour of the conditional wave- height density functions as a function of local surface slope.....	245

Figure 4.4	Comparison of solution density results for an elevation angle of 5° , SS4(ii) surface conditions, and an aircraft altitude of 9.1 km.....	247
Figure 4.5	Comparison of solution density results for an elevation angle of 15° , SS4(ii) surface conditions, and an aircraft altitude of 9.1 km.....	248
Figure 4.6	Comparison of solution density results for an elevation angle of 31° , SS4(ii) surface conditions, and an aircraft altitude of 9.1 km.....	249
Figure 4.7	Approximate radius of curvature curves for a 20° elevation angle scenario with SS4(ii) surface conditions.....	252
Figure 4.8	Approximate radius of curvature curves for a 31° elevation angle scenario with SS4(ii) surface conditions.....	252
Figure 4.9	Approximate radius of curvature for a 10° elevation angle scenario with SS3 surface conditions.....	253
Figure 4.10	Normalized radius of curvature curves for SS3 and SS5(i) surface conditions for an elevation angle of 10°	253
Figure 4.11	Behaviour of curvature scale factor for aircraft altitudes of 5 km and 9.1 km.....	257

Figure 4.12	Clearance probability versus local reflection angle curves for an elevation angle of 5°.....	257
Figure 4.13	Clearance probability versus local reflection angle curves for an elevation angle of 10°.....	260
Figure 4.14	Clearance probability versus local reflection angle curves for an elevation angle of 31°.....	260
Figure 4.15	Power density versus reflection angle curves for an elevation angle of 5°, SS4(ii) surface conditions, an aircraft altitude of 9.1 km, and an omni antenna pattern.....	273
Figure 4.16	Power density versus reflection angle curves for an elevation angle of 10°, SS4(ii) surface conditions, an aircraft altitude of 5.0 km, and an omni antenna pattern.....	274
Figure 4.17	Power density versus reflection angle curves for a horizontally polarized omni antenna pattern, an elevation angle of 10°, and SS3 and SS5(i) surface conditions.....	275
Figure 4.18	Power density curves for an elevation angle of 31°, SS4(ii) surface	

	conditions, an aircraft altitude of 9.1 km, and an omni antenna pattern... 276
Figure 4.19	Aeronautical doppler spectra for a horizontally polarized omni antenna pattern, an elevation angle of 10° , and SS4(ii) and SS5(i) surface conditions..... 281
Figure 4.20	Aeronautical doppler spectra for an elevation angle of 31° 282
Figure 4.21	Approximate radius of curvature curves for the Bandpass and Gaussian spectrum surface models..... 282
Figure 4.22	Conditional height density functions for the Bandpass and Gaussian spectrum surface models..... 285
Figure 4.23	Clearance probability versus local reflection angle curves for the Bandpass and Gaussian spectrum surface models..... 285
Figure 4.24	Integrated radius of curvature values, incorporating blockage and shadowing effects, for the Bandpass and Gaussian spectrum surface models..... 286
Figure 4.25	Clearance probability curves for the fixed and variable interval calculation approaches for an elevation angle of 10° 286

Figure A.1	A representative bandpass spatial spectrum for the wave process.....	306
Figure A.2	Correlation properties of a realistic dispersive wave along with Gaussian and cosinusoidal correlation functions (from [12]).....	309
Figure A.3	Normalized waveheight correlation function of the white Bandpass surface model.....	309
Figure A.4	Auto and cross-correlation properties of filtered random processes.....	311
Figure A.5	Equating the noise equivalent bandwidths of the Gaussian spectrum and white Bandpass surface models.....	316
Figure A.6	Normalized slope-height correlation function for the Bandpass surface model.....	329
Figure A.7	Comparison of height and slope occurrences with the SZERO surface profile with Gaussian confidence limits.....	333
Figure A.8	Comparison of height and slope occurrences with S5001 surface profile with Gaussian confidence limits.....	334
Figure B.1	Behaviour of the horizontal polarization reflection coefficient	

	for smooth sea water at L-band ($\lambda_c = 0.2$ m).....	338
Figure B.2	Behaviour of the vertical polarization reflection coefficient for smooth sea water at L-band ($\lambda_c = 0.2$ m).....	339
Figure B.3	Behaviour of the circular polarization reflection coefficient for smooth sea water at L-band ($\lambda_c = 0.2$ m).....	341
Figure B.4	Behaviour of the surface reflection ellipticity ratio for sea water at L-band ($\lambda_c = 0.2$ m).....	342
Figure B.5	Magnitude of reflection coefficients for first year sea ice (dielectric constant = 3.48, conductivity 0.02 mho/m) at L-band ($\lambda_c = 0.2$ m).....	344
Figure C.1	Geometry appropriate for evaluating Fresnel zone dimensions on scattering surfaces for satellite applications...	347
Figure C.2	Infinite tangent plane scattering slab	356
Figure C.3	Finite tangent plane scattering.....	358
Figure C.4	Geometry for the Helmholtz solution procedure (from [1]).....	363
Figure C.5	Illustration of illumination limit imposed by shadowing and blockage.....	363
Figure C.6	Magnitude and phase of the low grazing angle attenuation factor....,.....	368

Figure C.7	Phase shift introduced by a concave surface facet relative to that of a convex facet as a function of illumination.....	369
Figure C.8	Behaviour of the path length difference for a concave scattering facet.....	372
Figure C.9	Flowchart of divergence factor solution procedure in the near-field region of concave surface facets.....	376
Figure C.10	Comparison of the traditional (ray-optics) divergence factor and the qualitative ratio of Fresnel zones divergence factors in the near-field region of a concave scattering facet..	380
Figure C.11	Comparison of scattering from convex and concave surface facets.....	383
Figure C.12	Comparison of the traditional and qualitative divergence factors in the intermediate region just beyond the near-field region of a concave scattering facet.....	386
Figure C.13	Comparison of the magnitude of the more exact Fresnel diffraction near-field signal scale factor for concave scattering facets and the qualitative	

	divergence factors obtained with the ratio of Fresnel zones approach.....	388
Figure C.14	Coherent and non-coherent apertures...	396
Figure D.1	Polarization definitions.....	402
Figure D.2	The complex P plane with polarization ellipses (from [14]).....	402
Figure D.3	Signal combination for RHC polarization waves.....	404
Figure E.1	Illustration of specular point motion.	417
Figure E.2	Geometrical relationships used in doppler derivations.....	423
Figure E.3	Simulated aeronautical doppler spectra for climbing and diving manoeuvres for an elevation angle of 15°.....	437
Figure F.1	Received signal level, normalized with respect to the direct path, versus position for an elevation angle of 5°.	441
Figure H.1	Parabolic surface approximations as specular points become very close together.....	473
Figure H.2	Detection of zero crossings.....	473

LIST OF TABLES

Table 2.1	Zeros of the Bessel function $J_0(x)$ (from page 1039 of [15])	55
Table 2.2	Behaviour of height correction factor term $c(n)$	55
Table 2.3	Expressions for the ship motion functions, $z_t(t)$, $\theta_p(t)$, and $\theta_{ss}(t)$ for a superposition of sinusoids surface model	74
Table 3.1	Polarization ratio (amplitude and phase mismatch) parameters which provide an axial ratio of 5.5 dB.....	100
Table 3.2	Summary of baseline aeronautical simulation scenarios.....	114
Table 3.3	Summary of simulation scenarios for assessing the impact of varying sea state and aircraft altitude.....	114
Table 3.4	Summary of aircraft climbing and diving simulation runs.....	115
Table 3.5	Summary of aeronautical simulation runs conducted with the switched (diversity) antenna.....	115

Table 3.6	Signal to interference ratio results for the baseline aeronautical simulation runs with the crossed slot antenna and an omni antenna pattern... 122
Table 3.7	Extracted one-sided -10 dB bandwidths from the aeronautical doppler spectrum plots..... 127
Table 3.8	Signal to interference ratio results for the sea state and aircraft altitude variation simulation runs for an elevation angle of 10° and crossed slot and omni antenna patterns..... 133
Table 3.9	Signal to interference ratio results for aircraft climbing and diving manoeuvres..... 136
Table 3.10	Signal to interference ratio results for the aeronautical simulation runs conducted with the switched (diversity) antenna..... 138
Table 3.11	Performance comparison of switched (diversity) antenna with low complexity crossed slot antenna..... 140
Table 3.12	Effects of satellite antenna polarization ellipticity with the switched antenna system mounted on a Boeing 747 aircraft..... 142

Table 3.13	S/I measurement results for RHC polarization from [23].....	148
Table 3.14	Summary of maritime signal to interference ratio measurement results from [6].....	153
Table 3.15	Maritime simulation scenarios for Standard-C antenna stabilization runs.	158
Table 3.16	Maritime simulation scenarios examining the effect of sea state.....	158
Table 3.17	Maritime simulation scenarios examining the effect of varying the antenna height.....	160
Table 3.18	Maritime simulation scenarios that complete the multipath characterization versus elevation angle.....	160
Table 3.19	Maritime signal to interference ratio results for the antenna stabilization simulation runs with the DFVLR3 antenna pattern.....	162
Table 3.20	Maritime signal to interference ratio results for the antenna stabilization simulation runs for typical antenna polarization ratios consistent with an axial ratio of 5.5 dB for the DFVLR3 antenna.....	164

Table 3.21	-- Maritime signal to interference ratio results for the antenna stabilization simulation runs with an omni antenna pattern.....	164
Table 3.22	Maritime signal to interference ratio results for the sea state scenarios with the DFVLRC5 antenna pattern and an omni antenna pattern.....	166
Table 3.23	Maritime signal to interference ratio results for the sea state simulation runs for antenna polarization ratios consistent with an axial of 5.5 dB for the DFVLRC5 antenna.....	168
Table 3.24	Maritime signal to interference ratio results for the antenna height variation simulation runs with the QUAD antenna pattern and an omni antenna pattern.....	169
Table 3.25	Maritime signal to interference ratio results for the antenna height variation simulation runs for antenna polarization ratios consistent with an axial ratio of 5.5 dB for the QUAD antenna.....	171

Table 3.26	Maritime signal to interference ratio results for simulation runs conducted at 5° and 15° elevation angle with DFVLR3, DFVLR5, QUAD, and omni antenna patterns.....	173
Table 3.27	Maritime signal to interference ratio results for simulation runs conducted at 5° and 15° elevation angle for antenna polarization ratio consistent with an axial ratio of 5.5 dB for the DFVLR3, DFVLR5, and QUAD antennas...	174
Table 3.28	Summa y of the behaviour of S/I versus elevation angle for shipborne Standard-C antennas.....	176
Table 3.29	Fade margins and fade durations for simulated maritime channels.....	191
Table 3.30	Comparison of aeronautical and maritime signal to interference ratios versus elevation angle for an omni-reference antenna.....	198
Table 4.1	Interrelationship between the local and actual reflection angles for an aircraft altitude of 9.1 km.....	215
Table 4.2	Number of scattering facets and approximate radius of curvature versus height for a 15° elevation angle	

	aeronautical scenario with SS4(ii) surface conditions (SZERO wave profile).....	240
Table 4.3	Average scatter power levels obtained with the aeronautical simulation program and the stochastic analysis model for SS4(ii) surface conditions, an aircraft altitude of 9.1 km, level flight conditions, and an omni antenna pattern.....	264
Table 4.4	Average scatter power levels obtained with the aeronautical simulation program and the stochastic analysis model for the sea state and antenna height variation simulation scenarios with an omni antenna pattern.....	265
Table 4.5	Average scatter power levels obtained with the aeronautical simulation program and the stochastic analysis model for the baseline simulation scenarios with the crossed slot antenna pattern.....	267
Table 4.6	Average scatter power levels obtained with the aeronautical simulation program and the stochastic analysis model for the sea state and antenna	

	height variation simulation	
	scenarios with the crossed slot	
	antenna pattern.....	268
Table 4.7	Average scatter power levels for	
	aircraft climbing and diving	
	scenarios with the crossed slot	
	antenna pattern.....	270
Table 4.8	Average scatter power levels obtained	
	with the aeronautical simulation	
	program and the stochastic analysis	
	model for SS4(ii) surface conditions,	
	an aircraft altitude of 9.1 km, and	
	the switched (diversity) antenna	
	pattern.....	271
Table 4.9	Average scatter power behaviour	
	versus second derivative cutoff for	
	an elevation angle of 15°, SS4(ii)	
	surface conditions, and an omni	
	antenna pattern.....	278
Table 4.10	Average scatter power as a function	
	of the minimum distance between	
	specular points for an elevation	
	angle of 15°, SS4(ii) surface	
	conditions and an omni antenna	
	pattern.....	278

Table 4.11	Average scatter power levels obtained with the Bandpass and Gaussian spectrum surface models for an elevation angle of 10° , SS4(ii) surface conditions, and an omni-antenna pattern.....	288
Table 4.12	Average scatter power levels obtained with the stochastic analysis program for the variable and fixed interval clearance probability weighting factors for SS4(ii) surface conditions and an elevation angle of 10°	291
Table A.1	Sea State Table.....	302
Table A.2	Behaviour of rms surface slope versus sea state index.....	314
Table A.3	Summary of the relationships for the conditional second derivative density function for the white Bandpass and Gaussian spectrum surface models.....	325
Table A.4	Summary of the relationships for the conditional third derivative density function for the white Bandpass and Gaussian spectrum surface models.....	327
Table C.1	A tabulation of $Q D_0$ and the difference between the divergence factor obtained using the ratio of	

Fresnel zones and traditional ray-optics approaches versus local grazing angle for an antenna located one focal length from the specular point of a convex surface facet with $h = 10$ meters and $\lambda_c = 0.2$ meters.... 395

Table F.1

Signal to interference ratio results for level flight conditons, an aircraft altitude of 9.1 km, SS4(ii) surface conditions, and an omni antenna pattern..... 440

Table F.2

Signal to interference ratio results for SS3 surface conditions, an aircraft altitude of 9.1 km, an elevation angle of 10° , and an omni antenna pattern..... 444

CHAPTER 1.0

INTRODUCTION

For ship-to-satellite (maritime) and aircraft-to-satellite (aeronautical) communication applications over oceanic areas, the surface scatter (multipath) component is the most important component of the overall propagation model. This is especially true when relatively low gain, non-directive mobile antennas are employed. Aeronautical and maritime propagation data is extremely useful for developing realistic channel simulation models for characterizing the performance of narrowband analog and digital communication systems.

One of the major objectives of this thesis is to characterize the multipath process for these applications as a function of important physical variables such as (1) the elevation angle to the satellite, (2) antenna characteristics, (3) mobile antenna height, (4) mobile terminal velocity, (5) signal polarization and (6) sea state. The behaviour of ocean surface waveheight and wavelength parameters as a function of sea state index, which provides an indication of surface roughness, is summarized in the sea state table provided in Appendix A. To gain an appreciation for the difficult nature of this task, a few of the difficulties and limitations of theoretical approaches to the rough surface electromagnetic scattering problem are presented.

The evaluation of the electromagnetic field scattered by a rough surface is a difficult problem. The exact general solution is as yet unknown; making it necessary to resort to approximate solutions [1]. The complexity of the solution to the apparently simple problem of an electromagnetic wave reflecting from a plane interface between two media is considerable, and gives one an appreciation of the complexity of the more difficult rough surface scattering problem. The major difference between a rough surface and a smooth plane of sufficiently large dimensions is that the rough surface will scatter the incident wave into various directions, with some privileged directions receiving more energy than others, while the smooth surface reflects the incident wave specularly in a single direction [1]. In order, to make the rough surface scattering problem mathematically tractable, the surface is often assumed to be perfectly conducting, in which case the boundary conditions are simplified (i.e. the tangential component of the electric field on the surface is zero).

The majority of rough surface scatter theories employ the Helmholtz surface integral [1] to establish the electric field at the receiver. The surface boundary conditions required to solve the Helmholtz surface integral are based on the Kirchhoff approach, where the electric field on the surface is approximated by the field that would be present on a tangent plane at that point. This approximation is

reasonable when the radius of curvature of the surface irregularities is large compared to the carrier wavelength, but breaks down completely if the surface includes sharp edges. For the ocean surface, which seldom has sharp edges, the Kirchhoff approximation will be reasonable for L-band carrier frequency applications.

The Helmholtz surface integral reduces to a relatively simple formula when the surface is a perfect conductor and the observation point is in the far-field region of the scattering surface [1]. The Helmholtz solution simplifies further when the surface is periodic. However, for random surface profiles representative of the ocean surface, one will obtain complicated integrals. For tractability reasons, shadowing, blockage, and multiple scattering effects are not included in the Helmholtz integral solution approaches.

Many of the simplifying assumptions employed in typical theoretical treatments are violated in practice. The fact that the theoretical solution is valid only in the far-field region limits the usefulness of this approach. For example, in the maritime case, the antenna is typically in the near-field region. Aircraft are usually not flying at high enough altitudes to justify the assumption that the receiving antenna is in the far-field region of the entire active scattering region on the ocean surface.

Furthermore, for the ocean surface scattering problem, assuming a perfectly conducting surface is not very realistic.

When the surface has finite conductivity, the surface reflection coefficient is a function of the local grazing angle and the electromagnetic wave polarization. The behaviour of the reflection coefficients as a function of grazing angle for sea water and first year sea ice are presented in Appendix B. These reflection coefficient curves demonstrate that finite surface conductivity has a very significant impact on the reflection properties of the ocean surface for different signal polarizations. The most dramatic effect is on the magnitude of the vertical polarization reflection coefficient, which experiences a significant dip at the Brewster angle. At L-band, the Brewster angle is in the neighbourhood of 6° for sea water and around 28° for first year sea ice.

Incorporating the variation of the reflection coefficient over the surface complicates the Helmholtz solution procedure considerably (see pages 22-23 of [1]), forcing one to resort to numerical evaluation procedures. When the scattering surface is a perfect conductor, the reflection coefficient is constant over the entire surface, and the Helmholtz integral solution is dramatically simplified. If one ignores the finite conductivity of the surface, only

the effect of surface roughness on the scattered field strength is established. However, given the significant variation of the surface reflection coefficient with grazing angle and signal polarization, ignoring this would be a serious deficiency; especially for characterizing the scatter power for vertical and circular polarization. Consequently, for an accurate characterization of the ocean scatter process, the theoretical Helmholtz integral must be numerically evaluated. The complexity of this approach is prohibitive for both the aeronautical and maritime applications, mainly because of the size of the active scattering region.

While measurement results provide the most trustworthy characterizations of ocean surface scatter, one often finds that there is a lack of measurement data for many scenarios of interest. Furthermore, there are often situations where the only difference between a measurement scenario and the situation of interest to the investigator is the antenna used. Unfortunately, factoring out the effects of the antenna from the measurement data is usually extremely difficult, and in some cases, impossible.

In order to overcome the restrictive assumptions of theoretical solutions and to handle situations for which measurement data is unavailable, a physical propagation

simulation model for the scattering of electromagnetic waves from an ocean surface has been developed. This model is based on using a realistic and tractable surface profile and modelling the propagation effects introduced by this surface.

Although the complexity of directly evaluating Helmholtz surface integrals in a numerical fashion is prohibitive, a reasonable approximate solution can be obtained by employing the principle of stationary phase (pages 92-96 of [2] and page 99 of [3]). With this approach, attention is restricted to the surface facets in the neighbourhood of "specular reflection" points, where the phase of the electric field is essentially stationary (i.e. varies by less than 45° over the surface segment). These specular points are those places on the surface which reflect the incident ray to the receiving antenna position. In fact, the small stationary phase regions about the specular points are the regions on the surface which make the major contribution to the Helmholtz integral solution.

Establishing the individual scatter amplitudes introduced by these local surface facets is not particularly difficult, and more importantly, not very computationally demanding for each facet. A successful validation of the propagation simulation solution against the theoretical solution for an antenna in the far-fields of a perfectly

conducting sinusoidal surface demonstrates the accuracy of the individual scatter component solutions. The effect of finite surface conductivity on the surface reflection coefficient at the local surface facet is easily incorporated. The overall surface scatter signal is simply the superposition of the individual scatter components.

In spite of the simplifications employed in the local scattering solution approach, the computational complexity of the propagation simulation solution is still considerable because of the search for and large number of facets. Consequently, direct use of the physical propagation model as a channel model in the simulation of a narrowband communications system is unnecessarily complicated and wasteful of computing resources. Channel characterization is more efficiently performed in an offline mode, where the inphase and quadrature components of the multipath process can be characterized with a sampling rate commensurate with the fading bandwidth of the multipath process.

The complex baseband samples of both the horizontal and vertical polarization multipath processes are computed. The values appropriate for circular polarization can be established by a simple linear combination of the horizontal and vertical polarization components.

Nyquist interpolation (re-construction) can be used to characterize the multipath process for higher sampling rates commensurate with the spectrum occupancy of the communication signal. In this way, one can directly incorporate the propagation model output in the channel simulation model, as illustrated in Figure 1.1.

Alternatively, one can extract relevant parameters (average scatter power, magnitude of the coherent scatter component, multipath power spectrum (doppler) spectrum shape and bandwidth) for the general Rician channel model illustrated in Figure 1.1. This is the channel model that is often assumed for aeronautical and maritime applications [4], [5], and is based on sound underlying principles. With such a model, one can generate statistically similar fading characteristics in a very computationally efficient manner. Simulated propagation data has been analyzed to test the validity of this channel model. Simulation results generally confirm this model, and indicate that the magnitude of the coherent scatter component is negligible for rough surface conditions.

1.1 Summary of The Major Accomplishments and Results

Following review of existing models and measurements, it became apparent that an improved ocean scatter model was required to adequately characterize the sea-reflected multipath signal. The novel results obtained for the

Figure 1.1: Channel model block diagram.

aeronautical application ultimately justify this view. A detailed description of the physical ocean scatter propagation models developed for the aeronautical and maritime applications is provided in Chapter 2.

Figure 1.2 gives a simplified view of the physical model ultimately implemented as a computer simulation. To keep the model computationally tractable, a two-dimensional wave distribution in the azimuthal direction to the satellite is assumed. The wave profile is described by a sum of random phase sinusoidal components of sufficient number to give reasonable agreement with tabulated sea state data. To realistically limit the number of scattering facets, the curvature of the earth is incorporated. To model the dispersive nature of ocean surface motion, practical wave velocities, for each of the constituent wave profile components, are incorporated. Furthermore, the coupling between vessel motion and the motion of the ocean surface, as well as antenna stabilization by gravity stabilized antenna pedestals, is incorporated in the maritime propagation model.

The model illustrated in Figure 1.2 was implemented as a time domain software simulation in which the antenna position and wave profile are incremented, and sample records of channel data versus time are generated. The

Figure 1.2: Propagation model geometry

following effects are incorporated and applied to each scattering facet (of the type shown in Figure 1.2)

- (1) variability of the reflection coefficient with grazing angle for horizontal and vertical polarization,
- (2) attenuation (divergence) and phase shift resulting from scattering from convex and concave curved surface segments,
- (3) appropriate horizontal and vertical polarization antenna gains for the scatter signal, and
- (4) shadowing of primary scattering elements by the wave profile.

The major accomplishments were

- (1) the accurate validation of the simulation solution against a known exact far-field theoretical solution for a perfectly conducting sinusoidal surface, and
- (2) the development of a realistic divergence (attenuation) factor which also accounts for focussing effects of concave surface facets close to the receiving antenna.

Simulation results, which characterize ocean scatter for practical antenna patterns for the aeronautical and maritime applications, are presented in Chapter 3. The

effect of sea state, mobile antenna height, elevation angle to the satellite, and signal polarization on ocean scatter characteristics are established. The basic trends for the aeronautical and maritime models are essentially the same, and summarized below

- (1) for rough surface conditions, the scatter power level is relatively insensitive to sea state and mobile antenna height,
- (2) for circular polarization, there is a tendency for the interference level to increase as the elevation angle is lowered, and
- (3) the fading bandwidth decreases (fade durations increase) as the elevation angle is lowered.

Maritime measurement results [6], [7] exhibit similar behaviour. A comparison of maritime and aeronautical simulation results with measurement results for these applications generally validate the accuracy of the propagation simulation models.

Simulation results suggest that tapering the polarization characteristics of circularly polarized shipborne and aircraft antennas just below the horizon (low reflection angle region) towards vertical would be beneficial. At low elevation angles, the signal to interference ratio improvement is likely to be as great as 4 to 6 dB for the

maritime application. This behaviour is consistent with the surface discrimination against vertical polarization at low surface grazing angles.

The signal to interference ratios obtained with a low complexity crossed slot aircraft antenna are very good. For level flight conditions, signal to interference ratios exceed 20 dB for perfect circular polarization with this antenna. The worst case signal to interference ratio is around 14.4 dB (for an antenna phase mismatch of -90° between the vertical and horizontal polarization patterns and SS3 surface conditions). Even for a perfectly circular omni antenna pattern, the worst case signal to interference is not that severe (≈ 11.5 dB). Consequently, it is evident that ocean scatter multipath is not especially severe for aircraft-to-satellite communication applications.

For shipborne antennas with relatively low gain, hemispherical patterns, ocean scatter multipath is more severe than in the aeronautical application. Signal to interference ratios in the range of 8.4 dB - 10.8 dB are reasonable for low elevation angles ($5^\circ - 15^\circ$) with typical low gain shipborne antennas.

A detailed comparison of aeronautical and maritime channels, for omni-directional antenna patterns, is presented in Section 3.4. For circular polarization, the

interference levels obtained for the maritime application are roughly 2 to 3 dB worse than corresponding aeronautical values. This can be attributed to the much closer proximity of the shipborne antenna to the surface in the maritime application, which reduces the amount of attenuation suffered by the surface scatter signal. The difference between the velocities of the mobile terminals is responsible for the much greater direct path doppler shifts and fading rate of the aeronautical channel. The maritime channel experiences slow fading, with typical fading rates in the neighbourhood of 1 to 2 Hz. The fading experienced by the aeronautical channel is considerably faster, with typical values between 30 to 200 Hz.

One of the most interesting results was obtained with the aeronautical propagation simulation program, where it was found that the multipath interference (doppler) spectra exhibited noticeable asymmetry. It is claimed in some sources in the literature, in particular [8], that the theoretical doppler spectrum for aeronautical applications has a symmetric Gaussian shape. However, analysis performed in Appendix E reveals that this is only the case when some fairly restrictive assumptions, which are often violated in practice, are satisfied. A thorough justification for the doppler spectrum asymmetry is provided in Appendix E.

The computational requirements of the propagation simulation models are quite severe, especially for the aeronautical application. To obtain reasonable execution times for low elevation angle aeronautical simulation runs on general purpose mini-computer systems, some form of high speed arithmetic peripheral is essential. This significantly impairs the usability of the aeronautical propagation simulation program. To alleviate this deficiency, an analytic model suitable for the aeronautical application has been developed. The major components of this stochastic analysis model include

- (1) formulating the geometrical search process for potential scattering facets as a Gaussian random process curve crossing problem,
- (2) analytically estimating the effect of wave profile shadowing, and
- (3) analytically predicting the effect of divergence on the scatter power contributed by a scattering facet.

The development of this stochastic analysis model, which has the capability of estimating both first and second order statistics, is discussed in detail in Chapter 4. An impressive reduction in execution time (approximately two

orders of magnitude) is obtained with the stochastic analysis model. Of more importance, however, is the fact that this is accomplished without suffering any noticeable degradation in the accuracy of the first and second order statistics. With the stochastic analysis model, it is now practical to characterize the behaviour of ocean scatter for the aeronautical application on computer systems with relatively modest computational power, such as personal computers.

CHAPTER 2.0

OCEAN SCATTER PROPAGATION MODELS FOR THE AERONAUTICAL AND MARITIME APPLICATIONS

For L-band oceanic mobile satellite communications, with relatively low gain, non-directive antennas, the surface scatter (multipath) component is the most important component of the overall propagation model. A theoretical evaluation of the scattered field from a rough ocean surface involves computing a Helmholtz surface integral [1]. For anything but simple periodic and perfectly conducting surface characteristics, which are not very realistic representations of ocean surface characteristics, the theoretical Helmholtz integral must be numerically evaluated. The complexity of this theoretical approach is prohibitive for the aeronautical and maritime applications. While measurement results provide the most trustworthy characterizations, one finds that there is a lack of measurement data for many scenarios of interest.

In order to overcome the restrictive assumptions of theoretical solutions and to handle situations for which measurement data is unavailable, an ocean scatter propagation model has been developed. Some pioneering work on the development of an ocean scatter model for the maritime application was performed by COMSAT [9], [10].

Although certain aspects of this maritime model are reasonable, there are several glaring weaknesses which render the model useless for characterizing maritime multipath. The most serious weaknesses, and ways of alleviating them, are discussed in this chapter.

The objective of the propagation modelling effort is to characterize the behaviour of ocean scatter for the aeronautical and maritime applications as a function of important physical variables such as (1) the elevation angle to the satellite, (2) sea state, (3) mobile antenna characteristics, (4) antenna height, (5) velocity of the mobile terminal, and (6) signal polarization. Although there are several similarities between the propagation models for these applications, a few enhancements are specifically required for the maritime application. These include incorporating the coupling between ship motion and ocean surface motion, incorporating practical wave velocities in the ocean surface model to simulate dispersive wave motion, and developing a divergence factor which accounts for the potential focussing effects of concave scattering facets close to the shipborne antenna.

The propagation model is based on employing a realistic and tractable rough surface characteristic, and modelling the propagation effects introduced by this surface. The basic geometry of the physical model is shown in Figure 1.2. The

physical model developed is two dimensional in nature. That is, along the azimuthal direction to the satellite, a rough surface distribution is superimposed on the curved earth surface. The wave resulting surface profile is given by

$$z_s(x,t) = z(x,t) - \frac{x^2}{2a_e} \quad (2.1)$$

where

a_e is the radius of the earth (6.378 km) [11], and $z(x,t)$ represents the time varying rough surface wave distribution formed by a superposition of random phase sinusoidal components.

It should be noted that with a flat earth model, as was used in [9], the number of surface elements which can scatter energy to the receiver can become infinite at low elevation angles. Under such circumstances, one needs to restrict the extent of the scattering region. The major advantage of incorporating the curvature of the earth is that the number of potential scattering facets, especially at low elevation angles, is limited in a realistic fashion rather than arbitrarily.

The surface wave distribution is modelled by the superposition of N_T random phase sinusoids, where the number of components is under operator control. This surface model can be used to simulate a wide range of

possible surface characteristics, ranging from sinusoidal to Gaussian, depending on the selected value of N_T . The surface model, which is described in detail in Appendix A, incorporates wave motion to simulate dispersive wave motion and has a white bandpass spatial spectrum. The behaviour of the significant waveheight, rms waveheight, and average wavelength of the wave profile as a function of sea state is summarized in Table A.1 of Appendix A.

In [9], attention was restricted solely to a sinusoidal surface. It should be noted that this is not a very accurate representation of the ocean surface in many circumstances. In fact, in [12] it is claimed that a Gaussian process with a correlation distance of approximately $\lambda/4$, where λ is the average sea wavelength, is a fairly realistic surface model. The model developed in Appendix A satisfies these requirements. For the aeronautical and maritime simulation runs, 21 random phase sinusoidal components were used to approximate a Gaussian wave profile. Including the simple sinusoidal surface model is useful, however, for model calibration because theoretical solutions for perfectly conducting sinusoidal surfaces are available (see Chapter 4.3 of [1]).

Finite surface conductivity has a significant impact on the reflection properties of a surface for different signal

polarizations. Realistic di-electric and conductivity parameters for sea water and first year sea ice are incorporated in the propagation models. The behaviour of the surface reflection coefficients as a function of grazing angle at L-band ($\lambda_c = 0.2$ m) is presented in Appendix B.

The assumption that the waveheight does not vary perpendicular to the azimuthal direction to the satellite effectively makes the scattering problem two dimensional in nature and significantly reduces the computational complexity. For the maritime and aeronautical applications, a three dimensional surface model would be computationally prohibitive, especially at low elevation angles. This is simply because of the sheer size of the scattering region which must be examined for surface facets which scatter energy to the mobile antenna. Although the assumption that the waveheight does not vary perpendicular to the azimuthal direction is known to be false, simply from practical experience with water waves and also due to the curvature of the earth, a simple analysis presented in Section 5 of Appendix C suggests that, in terms of predicting scatter power, the assumption is not that unreasonable.

Another requirement of both propagation models is incorporating motion of the mobile terminal, where the

position is periodically updated with time. In the aeronautical application, the aircraft velocity is normalized. The aircraft position is updated as a function of distance (i.e. fraction of the average sea wavelength) and the ocean surface is assumed to be motionless. This is a fairly realistic assumption for the aeronautical application, because typical aircraft speeds greatly exceed realistic ocean wave velocities (typically by two orders of magnitude). The capability of simulating aircraft manoeuvres (climbing, diving, and banking) is also incorporated in the aeronautical propagation simulation program.

For the maritime application, incorporating ocean surface motion is important because of the relatively slow vessel speeds, as well as the intimate coupling between vessel and ocean surface motion. Consequently, one cannot normalize with respect to ship velocity and must explicitly supply an update sample interval as a program input parameter. This coupling of the motion of a ship and the ocean surface is addressed in Section 2.3.

The simulation solution, which restricts attention to the regions on the surface about the "specular reflection" points, can be viewed as a stationary phase approximation of the theoretical Helmholtz solution approach. Consequently, two of the major components of the

propagation simulation solution procedure involve locating the specular points on the surface and determining the attenuation resulting from divergence in scattering from curved surfaces. In the maritime model presented in [9], divergence is not incorporated (which is equivalent to assuming that the reflections occur from infinite tangent planes at the specular point locations). This is one of the most serious deficiencies of the maritime propagation model of [9]. The issue of locating contributing scattering facets is addressed in the next section.

2.1 Identifying Contributing Scattering Facets On The Ocean Surface

There is typically a large number of scattering facets which reflect energy to the mobile receiver, especially for the aeronautical application. One of the most computationally demanding portions of the propagation simulation involves locating these potential "specular reflection" points, which produce rays that intersect at the receiving antenna location. An expanded view of the geometry of the problem is provided in Figure 2.1. When determining the specular point locations on the surface, it is assumed that the satellite is so far away that the direct path ray and the incoming rays, which are ultimately reflected by the ocean surface, are essentially parallel near the receiver. With a smooth sea surface, the surface

Scattering Angle Definitions

Geometric Relations

α_i - local grazing angle

$$\alpha_i = E - \epsilon_i$$

θ_i - reflection angle relative to the horizontal

$$\theta_i = E - 2\epsilon_i$$

γ_i - surface slope angle

$$\epsilon_i = \tan^{-1} \left(\frac{dz_s}{dx} (x_i) \right)$$

Figure 2.1: Detailed reflection geometry of the ocean surface scatter propagation model.

grazing angle ψ , is essentially equal to the elevation angle to the satellite E.

The geometrical relationships that must be satisfied for rays to be reflected to the receiving antenna are provided in Figure 2.1. Consequently, the procedure for identifying the specular reflection points involves identifying the values of x where

$$\theta(x) = \beta(x)$$

$$\text{with } \theta(x) = \tan^{-1} \left(\frac{z_0 - z_s(x)}{x - x_0} \right) \quad (2.2)$$

$$\beta(x) = E - 2 \epsilon(x)$$

$$\epsilon(x) = \tan^{-1} \left\{ \frac{dz_s(x)}{dx} \right\}.$$

Here, $\epsilon(x)$ denotes the incline of the tangent plane to the surface, $\beta(x)$ denotes the angle of the reflected wave, and $\theta(x)$ denotes the angle between the receiving antenna and the point on the surface. Each of these angles are measured relative to the horizontal. The co-ordinates of the mobile antenna are represented by the ordered pair (x_0, z_0) . The local grazing angle is given by

$$\alpha(x) = E - \epsilon(x) = \frac{E + \beta(x)}{2} \quad (2.3)$$

If one has knowledge of the peak surface slope (z'_p), one can easily identify the region which must be examined for potential specular reflection points for the aeronautical application, as indicated below

$$x_{\text{start}} = x_0 + \frac{z_0}{\tan(E + 2 \tan^{-1}(z'_p))} \quad (2.4)$$

$$x_{\text{stop}} = x_0 + a_e \cos^{-1}\left(\frac{a_e}{a_e + z_0}\right) = x_0 + \sqrt{2a_e z_0} \quad (2.5)$$

where z_0 is the aircraft antenna height above the surface, and $h_R = \sqrt{2a_e z_0}$ represents the horizon range of the antenna.

For the maritime application, knowledge of the peak waveheight (z_p) is also required. To establish the start point of the search region, one must use ($z_0 - z_p$) instead of z_0 in (2.4), while using ($z_0 + z_p$) instead of z_0 when establishing the upper limit of the search region using (2.5).

Two of the major shadowing effects are incorporated in an elementary fashion in the propagation simulation programs. Whenever a specular point is invisible to incident rays (referred to as shadowing), or whenever a reflected wave is blocked from the antenna by the ocean surface (referred to as blockage), that specific specular point is not used in

the overall determination of the surface scatter signal. The extent of the region and number of discrete evaluation points considered when checking for surface blockage can be selected by the operator. The values used for the aeronautical and maritime simulation runs were an extent of 2.5 sea wavelengths and 50 evaluation points. It should be noted that if $\tan(E)$ exceeds the maximum surface slope, one need not worry about shadowing. Similarly, if $\tan(\theta)$ exceeds the maximum surface slope, one need not perform the blockage calculations. Theoretical treatments of scattering problems usually always ignore the above mentioned shadowing conditions to keep the analysis simpler. In the same spirit, multiple scattering effects are not incorporated in the propagation models.

The number of specular points (size of the active scattering region) is much greater for the aeronautical application than for the maritime application. This is a consequence of the much greater antenna heights in the aeronautical application. To get an appreciation for the complexity of the aeronautical propagation model, for an elevation angle of 10° , rough sea conditions and an aircraft altitude of 9.1 km, there are in excess of 10,000 contributing scattering facets. To ensure reasonable execution times, efficient specular point update procedures have been developed. These update algorithms use the

secant method (discrete approximation to Newton's method) for finding the zero crossings of the function $\theta(x) - \beta(x)$.

The number of potential specular reflection points obtained in the maritime application is considerably smaller than in the aeronautical application. For an elevation angle of 10° and SS4(ii) surface conditions, the number of potential scattering facets is in the neighbourhood of 1000, while the number of visible specular points is only around 150, for the maritime application. Consequently, one would expect the maritime application to be much more computationally efficient. Although it is more efficient, the processing requirements of the maritime application are not trivial. One of the major contributing reasons for this is the fact that the reflection angle at each scattering facet changes much more rapidly for the maritime application. This is because the ratio of the velocity of the terminal to the nominal antenna height above the ocean surface is much greater in maritime applications than for aeronautical applications. This combined with the time varying nature of the ocean surface results in the classification index of a specular point changing with time. The classification index refers to whether or not the specular point is shadowed, blocked, or visible. This classification reversal tendency makes the maritime ocean scatter model more complicated than originally anticipated.

The computational complexity of the specular point classification procedure⁴ is considerable. To reduce the computational load of this classification process for the maritime model, an algorithm for identifying "sensitive" specular points (i.e. those specular points whose classification indices are most likely to change) has been developed. This algorithm is invoked whenever a detailed (binary) search is used to locate specular points. A detailed search is essential, for example, on the first pass. Potential specular points are then updated using the fast converging secant method, using the previous estimate as a good initial guess. The major utility of the algorithm to identify sensitive points is that at each sampling (update) interval, only the restricted subset of sensitive reflection points need be classified, thereby resulting in a considerable computational savings.

During the development and testing of the maritime model, it was observed that the fast converging techniques for updating specular points would fail occasionally. This phenomenon could be attributed to the disappearance of specular point(s). It was originally proposed that whenever the fast converging update procedure failed or the vessel had traversed a sufficient distance, a detailed search would be performed again. Although undoubtedly the most accurate approach, it was found to be computationally unmanageable at low elevation angles, even when the

simulation program was executed on a high speed array processor. To reduce the number of computationally intensive detailed searches, a threshold was established for the number of lost specular points. Only when the number of lost specular points exceeded the threshold value or if the vessel had traversed a sufficient distance, would a detailed search be performed. This relatively simple modification improved the execution speed by more than a factor of four without compromising the accuracy of the results. It must be emphasized that it is important to periodically re-perform detailed searches to identify new potential scattering facets.

In the propagation simulation, unmodulated transmissions are considered and the horizontal and vertical polarization components are treated separately. Each polarization component is subjected to its corresponding surface reflection coefficient at the scattering facet, and at the antenna, each scatter component is subjected to the antenna gain specifically for that signal polarization. This is illustrated in Figure 2.2. To summarize, the horizontal and vertically polarized scatter components from the i -th scattering facet are given by

$$s_H(i) = r_H(\alpha_i) e^{-j\phi_i} G_H(g_a(i)) A_i \quad (2.6)$$

$$s_V(i) = r_V(\alpha_i) e^{-j\phi_i} G_V(g_a(i)) A_i \quad (2.7)$$

Contribution From i -th Scattering Facet

$$r_H(i) = \Gamma_H(\alpha_i) e^{-j\phi_i} G_H(g_a(i)) A_i$$

$$r_V(i) = \Gamma_V(\alpha_i) e^{-j\phi_i} G_V(g_a(i)) A_i$$

With

Γ_H, Γ_V - reflection coefficients

G_H, G_V - antenna gains

ϕ_i - phase difference (relative to direct path)

A_i - divergence and phase shift factor

α_i - local grazing angle

$g_a(i)$ - antenna gain angle

Figure 2.2: Scatter signal contribution from i -th scattering facet.

where

Γ_H, Γ_V denote the surface reflection coefficients for horizontal and vertical polarization,

G_H, G_V are the antenna (voltage) gains for horizontal and vertical polarization,

ϕ_i is the phase difference relative to the direct path (i.e. $\phi_i = \frac{-2\pi\delta_i}{\lambda_c}$),

δ_i is the path length difference of the reflected path and the direct path (see Figure 2.3),

$$g_a(i) = \begin{cases} B_A - \beta_i & \text{for aeronautical application} \\ -\theta_{ss} - \beta_i & \text{for maritime application} \end{cases}$$

B_A is the banking angle of the aircraft antenna, (positive direction toward the surface),

θ_{ss} is the shipborne antenna pointing angle (counter-clockwise direction assumed positive) supplied by the ship and ocean surface motion coupling routine, and

A_i is the signal attenuation and phase shift factor which accounts for scattering from curved surface facets.

Although not explicitly indicated, these parameters vary with time.

Determining the appropriate signal attenuation and phase shift factors, which account for the effects of surface

$$l_1 = \sqrt{(x_i - x_0)^2 + (z_i - z_0)^2}$$

$$l_2 = l_1 \cos (E + \beta_i)$$

δ_i = path length difference between i -th scatter path and the direct path

$$= l_1 - l_2 = l_1 (1 - \cos(E + \beta_i))$$

Figure 2.3: Path length difference for the i -th scattering facet.

curvature, is a critical component of the ocean surface propagation models. This subject is addressed in Section 2.2.

The horizontal and vertically polarized scatter signals are formed by adding the individual scatter signals from each contributing facet

$$s_H = \sum_{i=1}^{N_C} s_H(i) \quad (2.8)$$

$$s_V = \sum_{i=1}^{N_C} s_V(i) \quad (2.9)$$

where N_C represents the number of contributing facets.

The propagation simulation programs output sampled records of the inphase and quadrature components of the multipath process for both horizontal and vertical polarization. The signal combination appropriate for circular polarization, incorporating antenna ellipticity and phase mismatches, is derived in Appendix D. The multipath output for circular polarization is given by a simple linear combination of the individual horizontal and vertical polarization components. This has been incorporated in the propagation analysis program, which performs signal combination and analysis

according to operator inputs. The analysis program computes the signal to interference ratio, scatter power, and received signal voltage histograms, fade duration statistics, and multipath doppler (power) spectrum characteristics. One can easily examine the effects of different antenna polarization ellipticities with this analysis program.

2.1.1 Specular and Diffuse Reflection

In the literature, the terms specular and diffuse reflection (scattering), are sometimes used in a confusing manner. Specular reflection is a reflection of the same type as that caused by a smooth surface; that is, it is phase coherent. Furthermore, specular reflection from the surface is very directional (concentrated in the traditional specular direction). When the scattering is diffuse, energy is scattered in many different directions (very little directivity). Diffuse scattering is phase incoherent, and its fluctuations tend to be Rayleigh distributed.

Although the simulation solution is given by the superposition of the scatter components arising from those regions on the surface which reflect energy to the receiving antenna, it effectively incorporates both specular and diffuse components. In fact, for rough sea

conditions, simulation results obtained for both the maritime and aeronautical applications confirm that the specular component is negligible when compared to the diffuse component.

In [9], the output of the sinusoidal surface propagation model was incorrectly referred to as the specular component. An arbitrary complex Gaussian diffuse component was then combined with the output of the sinusoidal surface propagation model. The specular and diffuse components were weighted according to empirical results from measurement data, in which case the diffuse component swamped the so-called specular component (for rough sea conditions). If the surface model were accurate, one would not be justified in adding this random diffuse component. This random component could be thought of as compensating for the non-sinusoidal nature of the surface. It must be pointed out though, that the resulting field from a superposition of surfaces is not the superposition of their individual contributions.

The diffuse component in [9] was modelled by independent inphase and quadrature signals derived by filtering independent Gaussian processes by equivalent single pole filters. Furthermore, the horizontal and vertically polarized components of the diffuse scatter process were assigned equivalent power levels [9]. Given the surface

discrimination against vertical polarization at low elevation angles (Brewster angle effects), it is evident that the power scaling used in [9] is totally unrealistic.

2.2 Derivation of Signal Attenuation Factors Which Account For Scattering From Curved Surfaces

Qualitatively, one can use Fresnel zone dimensions on a plane surface to establish whether or not surface curvature has a significant impact on the amplitude of a scattered signal. Fresnel zones on a plane surface are ellipsoids of revolution where the phase difference of a path including that point on the surface, relative to the specular path, is " $n\pi$ ", where " n " denotes the Fresnel zone number. It is commonly assumed that it is the first Fresnel zone on a tangent plane which makes the most important contribution to the total field at a receiver location [1], [3], [12].

Fresnel zone derivations appropriate for satellite applications are provided in Appendix C. For satellite applications, the major and minor axis of the first Fresnel zone on a tangent plane at the specular point are given respectively by

$$x_{TP} = \frac{\sqrt{\lambda_c r}}{\sin \alpha} \quad (2.10)$$

$$Y_{TP} = \sqrt{\lambda_c r} \quad (2.11)$$

where

α is the surface grazing angle,

λ_c is the carrier wavelength, and

r is the distance between the scattering facet and the receiving antenna.

For an L-band satellite-to-aircraft communications link ($\lambda_c = 0.2$ m), with an aircraft altitude of 10 km, and a surface grazing angle of 20° , the major and semi-major axes of the first Fresnel zone on a tangent plane are

$$X_{TP} = 223.58 \text{ m}$$

$$Y_{TP} = 76.47 \text{ m.}$$

Because these first Fresnel zone dimensions are much greater than typical sea wavelengths, the first Fresnel zone will include many rough surface cycles. Consequently, the amplitude of the scattered signal will be much different than that obtained from an infinite tangent plane at the specular point.

For an L-band ship-to-satellite communications link ($\lambda_c = 0.2$ m), with an antenna height of 20 m, and a surface grazing angle of 20° , the major and minor axes of the first Fresnel zone on a tangent plane are

$$X_{TP} = 10.0 \text{ m}$$

$$Y_{TP} = 3.42 \text{ m.}$$

The Fresnel zone dimensions for maritime applications around the nominal specular point are comparable to typical sea wavelengths. However, for the scattering facets from the distant horizon region, the Fresnel zone dimensions will greatly exceed typical sea wavelengths. Consequently, one cannot justify simply selecting $A_1=1$ in equations (2.6) and (2.7) (which is equivalent to assuming reflection from an infinite tangent plane), as was done in [9].

It is evident that surface curvature (roughness) will have a significant impact on the amplitude of the scattered signal. This is especially true for the aeronautical application because the antenna is so far removed from the ocean surface. Figure 2.4 illustrates the fundamental divergence of rays from convex and concave surface facets. Because aircraft altitudes are much greater than typical surface radii of curvature, there will be little difference in the amplitude of signals scattered from convex and concave surface facets. This will definitely not be the case for the maritime application because of the much closer proximity of the shipborne antenna to the ocean surface. Concave surface facets close to the shipborne antenna may, in fact, focus energy. Establishing an appropriate attenuation factor for aeronautical applications is considered first.

Figure 2.4: Divergence of rays scattered by convex and concave surface elements.

2.2.1 Signal Attenuation Factors For Aeronautical Applications

Establishing an appropriate attenuation factor for scattering from curved facets for the aeronautical application is considered in this section. For this application, one need not make a distinction between convex and concave surface facets, when attention is restricted solely to the attenuation of the scatter signal. The surface radius of curvature at a specular point x_s is given by [13]

$$r_c = \left| \frac{(1 + z'_s(x_s)^2)^{3/2}}{z''_s(x_s)} \right|. \quad (2.12)$$

The focal length of a curved surface facet can be expressed as

$$f = \frac{r_c}{2} \sin \alpha \quad (2.13)$$

where α is the surface grazing angle.

Two heuristic approaches for establishing the signal attenuation factor for a convex surface facet are considered in the next two sub-sections. The results obtained are supported by a more rigorous Fresnel diffraction theory approach employing Huygens' principle (pages 488-509 of [14]).

2.2.1.1 Ratio of Fresnel Zone Areas Approach

With this approach, the signal attenuation factor is given by ratio of the areas of the first Fresnel zones on the curved surface facet and a tangent plane located at the specular point. For a parabolic convex surface facet, the path length difference relative to the specular point, for the aeronautical application, can be expressed as (see Appendix C)

$$\delta = \left[1 + \frac{r}{f} \right] \frac{\sin^2 \alpha}{2r} \Delta x^2 + \frac{\Delta y^2}{2r} \quad (2.14)$$

where

r is the distance between the specular point and the antenna location,

f is the focal length defined by (2.13),

α is the surface grazing angle, and

$\Delta x, \Delta y$ are the deviations from the specular point location along the x and y axis.

It should be noted that the x -axis being referred to here is actually a local x -axis which is aligned with the tangent plane to the surface at the specular point.

The path length difference expression of (2.14) can be used to establish the first Fresnel zone dimensions appropriate for the convex surface facet. These are given by

$$X_{\text{CONVEX}} = \sqrt{\frac{\lambda_c r}{1 + \frac{r}{f}}} / \sin \alpha \quad (2.15)$$

$$Y_{\text{CONVEX}} = \sqrt{\lambda_c r} = Y_{\text{TP}} \quad (2.16)$$

Because there is no variation in the waveheight along the y-axis (i.e. perpendicular to the azimuthal direction to the satellite), the minor axis of the Fresnel ellipse is identical to that on a tangent plane. As a consequence, the ratio of Fresnel zone areas is simply the ratio of the x-axis dimensions. Therefore, the magnitude of the signal attenuation factor is given by

$$\begin{aligned} |A| &= \frac{X_{\text{CONVEX}}}{X_{\text{TP}}} \\ &= \frac{1}{\sqrt{1 + \frac{r}{f}}} \\ &= \frac{1}{\sqrt{1 + \frac{2r}{r_c \sin \alpha}}} \end{aligned} \quad (2.17)$$

It is interesting to note that the above approach is not sensitive to the choice of the actual phase boundary. That is, it is not crucial that the phase boundary be 180°

($\delta = \lambda_c/2$), which is the appropriate value for establishing first Fresnel zone dimensions. From the path length difference expressions for a tangent plane and a convex surface facet, the generalized x-axis dimensions can be expressed as

$$x_{TP}(\delta) = \sqrt{2\delta r} / \sin \alpha \quad (2.18)$$

$$x_{CONVEX}(\delta) = \sqrt{\frac{2\delta r}{[1 + \frac{r}{f}]}} / \sin \alpha = \frac{x_{TP}(\delta)}{\sqrt{1 + \frac{r}{f}}} \quad (2.19)$$

It is evident that the ratio $\frac{x_{CONVEX}(\delta)}{x_{TP}(\delta)}$ is independent of the path length difference δ and equivalent to the first Fresnel zone ratio.

2.2.1.2 Divergence Approach

The divergence approach is based on standard ray-optics, which is exact in the optical (high frequency) limit. A general derivation of the divergence factor from a three dimensional convex surface facet is presented in pages 222 - 224 of [1]. The basic result is

$$D = \left[r + \frac{2 r_1 r_2}{a(r_1 + r_2) \sin \alpha} \right]^{-\frac{1}{2}} \left[1 + \frac{2 r_1 r_2}{b(r_1 + r_2)} \right]^{-\frac{1}{2}} \quad (2.20)$$

where

r_1 is the distance between the transmitter and the specular point,

r_2 is the distance between the specular point and the receiving antenna,

α is the surface grazing angle, and

a, b denote the surface radius of curvature along the x-axis and the y-axis, respectively.

This derivation is based on a simple ratio of flux densities approach.

For the aircraft-to-satellite application, with no surface curvature perpendicular to the azimuthal direction to the satellite, the divergence factor simplifies to

$$D = \frac{1}{\sqrt{1 + \frac{2r}{r_c \sin \alpha}}} \quad (2.21)$$

where

r simply denotes the distance between the aircraft and the specular point, and

r_c denotes the surface radius of curvature.

The divergence scale factor is in agreement with the attenuation factor derived using the qualitative ratio of Fresnel zone areas approach. This result is also in agreement with that obtained with more rigorous Fresnel diffraction integrals in Appendix C. The Fresnel diffraction approach is invaluable for determining the phase shift factors introduced by convex and concave surface facets, which have been ignored to this point. In the next section, which compares the simulation solution to the theoretical far-field solution for a perfectly conducting sinusoidal surface, this issue is addressed.

2.2.1.3 Validation of the Simulation Solution Against The Theoretical Far-Field Solution For A Perfectly Conducting Sinusoidal Surface

For perfectly conducting, cosinusoidal surfaces, as illustrated in Figure 2.5, the far-field Helmholtz surface integral solution is provided in Section 4.3 of [1]. To summarize, the extent of the surface is $2L$ and the angles in which energy is scattered are given by the solutions to the grating equation (page 35 of [1])

$$\sin \theta_{2m} = \sin \theta_1 + \frac{m \lambda_c}{\lambda_w}, \quad m=0, \pm 1, \pm 2, \dots \quad (2.22)$$

Figure 2.5: Cosinusoidal surface model.

Figure 2.6: Bessel function $J_0(x)$.

where

θ_1 is the angle of incidence of the incoming radiation, and

θ_{2m} denotes an angle, measured relative to the z-axis, in which energy is scattered.

The received electric field at a distance r from the origin and in the grating angle directions is given by

$$E_r = [\rho(\theta_1, \theta_{2m}) + \epsilon_f] E_{20}(r) \quad (2.23)$$

where

$E_{20}(r)$ denotes the reference field in the absence of surface roughness, at a distance r (far-field region) from the scattering surface,

ϵ_f is an edge effect term which becomes negligible for $L \gg \lambda_w$, and equal to zero when $\frac{L}{\lambda_w}$ is an integer,

$$\rho(\theta_1, \theta_{2m}) = - \left(\frac{\sec \theta_1 (1 + \cos(\theta_1 + \theta_{2m})) (-j)^m J_m(s)}{(\cos \theta_1 + \cos \theta_{2m})} \right) \quad (2.24)$$

$J_m(\cdot)$ is the Bessel function of the first kind of order m , and

$$s = \frac{2\pi h_w}{\lambda_c} (\cos \theta_1 + \cos \theta_{2m}) \quad (2.25)$$

Since it is assumed that the sea waves extend indefinitely in the y-direction, the reference field is that scattered

from an infinite tangent plane slab of width $2L$ in the far field region. This problem is solved in Appendix C, and the result for the specular reflection direction is

$$E_{20}(r) = e^{-j\theta_r} \frac{2L \sin \alpha}{\sqrt{\lambda_c} r} E_i \quad (2.26)$$

Where

α is the grazing angle (i.e. $\alpha = 90 - \theta_1$),

E_i is the incident field strength, and

$\theta_r = \frac{2\pi r}{\lambda_c}$ is the phase shift associated with the path between the antenna and the origin.

Attention has been restricted to the specular reflection direction for simulation model calibration. In this case, the effective reflection coefficient of equation (2.24) can be simplified to

$$\rho = -J_0\left(\frac{2\pi}{\lambda_c} 2 h_w \sin \alpha\right) \quad (2.27)$$

The theoretical field strength will have zero crossings as the argument of the Bessel function changes (see Figure 2.6).

The simulation solution is now considered for an omnidirectional antenna located in the specular direction and in the far-field region (i.e. $r > \frac{2(2L)^2}{\lambda_c}$). As shown in

Figure 2.7, the specular points will be near the peaks of the crests and the bottom of the troughs, when the receiver is in the far-field region. The path length difference between the concave and convex specular paths is given by

$$\delta = 2 (2h_w) \sin \alpha \quad (2.28)$$

and the phase difference ϕ_d is given by

$$\phi_d = 2 \left(\frac{2\pi}{\lambda_c} \right) 2 h_w \sin \alpha. \quad (2.29)$$

The signal attenuation factor appropriate for the convex and concave facets is accurately predicted by divergence

$$D = \frac{1}{\sqrt{1 + \frac{2r}{r_c \sin \alpha}}} = \sqrt{\frac{r_c \sin \alpha}{2r}}. \quad (2.30)$$

The number of crest reflection points, N , is given by

$$N = \frac{2L}{\lambda_w}. \quad (2.31)$$

The number of trough reflection points is given by the same relationship. The scattered field, relative to the direct path received signal, can be expressed as

Figure 2.7: Approximate location of specular reflection points on convex (crest) and concave (trough) surface elements when the receiver is in the far-field region.

$$E_r = - e^{j\theta_{rel}} N D (1 + e^{-j\phi_d} e^{j\phi_c}) \quad (2.32)$$

where

$$\theta_{rel} = \frac{2\pi}{\lambda_c} 2(z_o - h_w) \sin \alpha \quad \text{is the phase shift}$$

associated with the crest specular paths
relative to the direct path,

z_o is the aircraft altitude relative to the mean
surface height, and

ϕ_c represents an additional phase shift term
which accounts for the difference between
scattering from concave and convex surface
facets.

The importance of the ϕ_c phase shift factor is now
established. Note that the theoretical reflection
coefficient can be expressed as

$$\rho = - J_0\left(\frac{\phi_d}{2}\right) \quad (2.33)$$

If one ignores the ϕ_c phase shift factor, one obtains phase
cancellation with the simulation solution whenever
 $\phi_d = (2n-1)\pi$, which does not match the zero crossings of
the theoretical solution as $J_0\left(\frac{(2n-1)\pi}{2}\right) \neq 0$. A realistic
phase shift correction factor, ϕ_c , which aligns the
simulation and theoretical solutions, is now derived.

2.2.1.3.1 Establishing An Appropriate Phase Shift Factor For The Sinusoidal Surface Model

Aligning the theoretical solution with the simulation solution is the simplest way to derive the appropriate phase shift factor. The adjusted (increased) waveheight, $h_T(n)$, required to align the theoretical Bessel function zero crossings with the simulation solution zero crossings, assuming no phase correction, is established. In this case, the simulation solution zero crossings occur at waveheights

$$h_s(n) = (2n - 1) h_s(1) \quad (2.34)$$

where $h_s(1) = \frac{\lambda_c}{8 \sin \alpha}$, which is closely related to the Rayleigh roughness criterion.

The adjusted waveheights which align the theoretical and simulation solutions are given by the relationship

$$h_T(n) = \left(\frac{2 b(n)}{\pi} \right) h_s(1) , \quad (2.35)$$

where $b(n)$ represents the positions of the zero crossings of the Bessel function $J_0(x)$ (see Table 2.1).

This can be expressed in terms of the simulation solution waveheight as follows

Zero Crossing Number n	Zero Crossing Location b(n)
1	2.4048
2	5.5201
3	8.6537
4	11.7915
5	14.9309
6	18.0711
7	21.2116
8	24.3525

Table 2.1: Zeros of the Bessel function $J_0(x)$
(from page 1039 of [15]).

Zero Crossing Number n	Appropriate Height Correction Factor Term c(n)
1	0.531
2	0.514
3	0.509
4	0.507
5	0.505
6	0.504
7	0.504
8	0.503

Table 2.2: Behaviour of height correction factor
term c(n).

$$h_T(n) = h_s(n) + h_c(n) \quad (2.36)$$

where the height correction factor is given by

$$h_c(n) = c(n) \frac{\lambda_c}{8 \sin \alpha} \text{ and } c(n) \text{ is defined in Table 2.2.}$$

This alignment of the zero crossings establishes an appropriate asymptotic height correction factor for rough conditions, which is

$$h_c = \frac{\lambda_c}{16 \sin \alpha} \quad (2.37)$$

In this case, the correction factor is independent of waveheight and wavelength. However, as the surface becomes smooth, the height correction factor must change, because as $h_s \rightarrow 0$, one must also have $h_T \rightarrow 0$.

Since one really wants to align the simulation solution with the theoretical solution, the height correction factor must be applied to the simulation solution. For rough surface conditions, one obtains

$$h_s = h_T - \frac{\lambda_c}{16 \sin \alpha} \quad (2.38)$$

where h_T represents the actual waveheight, h_w . Using this corrected waveheight, the effective phase shift between the trough and crest scattering facets is

$$\phi_e = 2 \left(\frac{2\pi}{\lambda_c} \right) 2 h_s \sin \alpha = \phi_d - \pi/2, \quad (2.39)$$

where ϕ_d denotes the actual phase shift between the trough and crest specular paths.

The appropriate factor for combining the crest and trough scatter signals is

$$1 + e^{-j\phi_e} = 1 + e^{-j\phi_d} e^{j\frac{\pi}{2}},$$

which indicates that the appropriate phase correction factor (phase shift factor appropriate for a concave facet relative to a convex facet) is $\frac{\pi}{2}$ for rough surface conditions. The same result is obtained in Appendix C with Fresnel diffraction integrals for parabolic concave and convex surface facets.

The above waveheight correction factor is only reasonable when the actual waveheight h_w exceeds $\frac{\lambda_c}{16 \sin \alpha}$. An appropriate linear height correction factor for smooth surface conditions (i.e. $h_w < \frac{\lambda_c}{16 \sin \alpha}$), is $h_c = h_w$. This results in a linear phase shift correction factor

$$\phi_c = \pi \left(\frac{8 \sin \alpha}{\lambda_c} \right) h_w \quad \text{for } h_w < \frac{\lambda_c}{16 \sin \alpha}. \quad (2.40)$$

From an electromagnetic standpoint, the surface becomes smooth at very low grazing angles and the surface illumination can become truncated. This issue is addressed in Appendix C, where the low grazing angle attenuation factor is derived. As the illumination region becomes truncated, the relative phase shift between convex and concave facets approaches zero in a fairly linear fashion.

2.2.1.3.2 Comparison of Simulation And Theoretical Reflection Coefficients

The scattered field strength of the simulation solution, for rough surface conditions, can be established using (2.30), (2.31), and (2.32) with $\phi_c = \frac{\pi}{2}$,

$$|E_r| = \left(\frac{2L}{\lambda_w}\right) \sqrt{\frac{r_c \sin \alpha}{2r}} \left| 1 + e^{-j(\phi_d - \frac{\pi}{2})} \right| \quad (2.41)$$

In the crest and trough extremes of a sinusoidal surface, the radius of curvature is

$$r_c = \frac{\lambda_w^2}{(2\pi)^2 h_w} \quad (2.42)$$

Substituting this into (2.41) and simplifying yields

$$|E_r| = |\rho_{sim}| \frac{2L \sin \alpha}{\sqrt{\lambda_c} r} \quad (2.43)$$

where

$$|\rho_{sim}| = \frac{1}{\sqrt{\left(\frac{2\pi}{\lambda_c}\right) 2 h_w \sin \alpha}} \cdot \frac{1}{\sqrt{\pi}} \cdot \sqrt{1 + \cos \left(\phi_d - \frac{\pi}{2}\right)}$$

$$= \frac{1}{\sqrt{\frac{\phi_d}{\pi - \frac{\pi}{2}}}} \sqrt{1 + \cos \left(\phi_d - \frac{\pi}{2}\right)} \quad (2.44)$$

Similarly, the magnitude of the theoretical field strength is given by

$$|E_r| = |\rho| \frac{2L \sin \alpha}{\sqrt{\lambda_c} r} \quad (2.45)$$

where

$$|\rho| = |J_0 \left(\left(\frac{2\pi}{\lambda_c}\right) 2 h_w \sin \alpha \right)|$$

$$= |J_0 \left(\frac{\phi_d}{2} \right)| \quad (2.46)$$

The magnitude of the simulation and theoretical reflection coefficients are compared in Figure 2.8, as a function of the phase difference ϕ_d . For $\phi_d > \frac{\pi}{2}$, the simulation and theoretical coefficients are in close agreement, and this

2

MicroD

Figure 2.8: Comparison of the magnitude of the simulation and theoretical reflection coefficients for perfectly conducting sinusoidal surfaces.

improves as ϕ_d increases (rougher surfaces). This is not surprising because the magnitude of the simulation reflection coefficient can be simplified to yield

$$|\rho_{\text{sim}}| = \sqrt{\frac{2}{\pi \left(\frac{\phi_d}{2}\right)}} \left| \cos \left(\frac{\phi_d}{2} - \frac{\pi}{4} \right) \right|, \quad (2.47)$$

which is a standard approximation for $|J_0(\frac{\phi_d}{2})|$ for large $(\frac{\phi_d}{2})$ (page 869 of [15]).

When the phase difference ϕ_d is less than $\frac{\pi}{2}$, the waveheight is less than $\frac{\lambda_c}{16 \sin \alpha}$, indicating that the surface is becoming smooth. The accuracy of the theoretical solution, under these conditions, is suspect because it does not incorporate shadowing effects. For phase differences smaller than $\frac{\pi}{2}$ and small grazing angles,

$$h_w < \frac{\lambda_c}{16 \alpha}. \quad (2.48)$$

Shadowing sets in if

$$\alpha < \tan^{-1} \left(\frac{4h_w}{\lambda_w} \right) = \frac{4h_w}{\lambda_w}. \quad (2.49)$$

One can guarantee shadowing effects if

$$\alpha < \frac{\lambda_c}{16 h_w} < \frac{4 h_w}{\lambda_w}, \quad (2.50)$$

which can be rearranged as follows

$$\left(\frac{\lambda_c}{h_w} \right) \left(\frac{\lambda_w}{h_w} \right) < 64. \quad (2.51)$$

For a sinusoidal surface, the peak waveheight h_w and rms waveheight σ_z are related by

$$h_w = \sqrt{2} \sigma_z. \quad (2.52)$$

For L-band applications ($\lambda_c = 0.2$ m) and typical North Atlantic conditions (SS4(ii) sea/ state index where $\sigma_z = 0.61$ m and $\lambda_w = 34.26$ m),

$$\left(\frac{\lambda_c}{h_w} \right) \left(\frac{\lambda_w}{h_w} \right) = \frac{\lambda_c \lambda_w}{2 \sigma_z^2} = 9.21.$$

Since this is well below the threshold value of 64, shadowing will definitely be a problem for North Atlantic sea conditions when the surface becomes smooth according to the Rayleigh criterion. As a result $|\rho_{sim}|$ and $|\rho|$ are not compared for $\phi_d < \frac{\pi}{2}$.

The successful validation of the simulation solution against the theoretical far-field solution for perfectly conducting sinusoidal surfaces and rough surface conditions demonstrate that the attenuation (divergence) and phase shift correction factors introduced are realistic.

2.2.2 Divergence Factor For Maritime Applications

The signal attenuation after reflection can be significant for the aeronautical application, because of the appreciable distance between the scattering surface and the mobile antenna, and can be accurately predicted by ray divergence. For typical aircraft altitudes there is little difference between the attenuation (divergence) factor for convex and concave surface facets. This is definitely not the case for the maritime application, where the antenna is much closer to the scattering surface, and concave surface facets can, in fact, focus energy. Consequently, a major requirement for the maritime ocean scatter model is an accurate signal scale factor which accounts for scattering from concave surface facets.

For convex surface facets the appropriate signal scale factor is

$$A_{\text{convex}} = \sqrt{\frac{1}{1 + \frac{r}{f}}} \quad (2.53)$$

where r is the distance between the scattering facet and mobile antenna, and f is the focal length.

The magnitude of the signal attenuation factor for concave surface facets, when the antenna is not in the near-field region, can be established in a heuristic fashion. The scattering from individual convex and concave segments is illustrated in Figure 2.9. Once the distance between the mobile antenna and the specular point is greater than two focal lengths, the scattering from the concave facet is virtually equivalent to that which would be obtained from a convex surface segment (with equivalent radius of curvature) located a distance $2f$ closer to the receiver. Consequently, whenever the distance between the concave specular point is greater than two focal lengths, the signal attenuation factor can be approximated as follows

$$|A_{\text{concave}}| = D_{\text{concave}} = \frac{1}{\sqrt{1 + \frac{d}{f}}} \quad (\text{for } r > 2f) \quad (2.54)$$

where $d = r - 2f$.

(a) Scattering from a convex surface segment

(b) Scattering from a concave surface segment

Figure 2.9: Comparison of scattering from convex and concave surface facets.

This can be rearranged to give

$$D_{\text{concave}} = \frac{1}{\sqrt{\frac{r}{f} - 1}} \quad (\text{for } r > 2f) \quad (2.55)$$

which is the expression obtained using the traditional ratio of flux density (ray-optics) approach. Outside the near-field region, one obtains the same result using the qualitative ratio of Fresnel zone areas approach (see Section 4 of Appendix C).

The only difficulty with this approach is that the phase shift factor introduced by a concave facet is not evident. In Section 3 of Appendix C, it is shown that the appropriate signal scale factor is given by

$$A_{\text{concave}} = j \sqrt{\frac{1}{\frac{r}{f} - 1}} \quad (\text{for } r > 2f). \quad (2.56)$$

It is evident that, relative to a convex facet, an additional 90° phase shift is introduced by concave surface facets. The effect of truncated illumination, which occurs at low grazing angles, is addressed in Section 3.1.2 of Appendix C.

The impulsive nature of the ray-optics divergence factor of (2.55) around the focal point, makes this approach impractical in the near-field region (i.e. $r \in [0, 2f]$).

Note that there is a fundamental limit on the power that can be collected by a mobile antenna located at the focal point, which implies that the divergence factor at the focal point is finite (bounded).

The qualitative ratio of Fresnel zones approach has been used to establish a reasonable estimate of the divergence factor in the near-field region. The details are presented in Section 4.1 of Appendix C, and the resulting qualitative divergence factor is compared to the ray-optics solution in Figure C.10. Also presented in Section 4.1 of Appendix C, is a more exact signal scale factor for the near-field region. This involves Fresnel diffraction integrals, with a phase function which incorporates both quadratic and cubic factors. Although, the qualitative approach provides realistic results in some regions, there are differences in the transition region, where the phase function changes from being predominately cubic to quadratic (see Figure C.13). Consequently, the more exact Fresnel diffraction approach is incorporated in the maritime ocean scatter model for evaluating the signal scale factor introduced by a concave surface facet in the near-field region.

2.3 Incorporating Sea-Induced Vessel Motion and Antenna Stabilization For The Maritime Application

One of the major enhancements required for the maritime application is the coupling of the motion of the ship to wave motion. Since the model of the sea surface is two-dimensional, the model for vessel motion will remain two dimensional with only vertical translation of the waterline height of the ship and rotation about the pitch axis being considered. Figure 2.10 illustrates a ship on a large wave and defines the following terms

- z_o is the antenna height above the waterline of the ship,
- x_c is the horizontal position of the centre of gravity of the ship ($x_c = ut$, where u is the velocity of the ship),
- z_s is the instantaneous height of the sea surface above the datum at point x ,
- z_l is the instantaneous height of the waterline of the ship above the datum at point x ,
- θ_p is the pitch angle of the ship (counter-clockwise (CCW) direction assumed positive), and
- θ_{ss} is the antenna pointing angle error (CCW direction assumed positive).

(a) A ship on a large wave

(b) Antenna pointing angle definition

Figure 2.10: Basic vessel motion definitions.

It should be emphasized that it is assumed in the simulation program that the ship is moving in the positive x-direction (towards the satellite). Motion both with and against the waves can be simulated.

Using simple trigonometry, the co-ordinates of the antenna are given by

$$x_a(t) = x_c(t) + x_D \cos \theta_p(t) - z_o \sin \theta_p(t) \quad (2.57)$$

$$z_a(t) = z_l(t) + x_D \sin \theta_p(t) + z_o \cos \theta_p(t) \quad (2.58)$$

where x_D denotes the distance from the pitch axis where the antenna is fitted.

The antenna pointing angle error is important when applying antenna gains to incoming signals. If the incoming signal arrives at an angle χ relative to the horizontal, the appropriate angle to use when evaluating antenna gains is

$$g_a = \chi - \theta_{ss} \quad (2.59)$$

A realistic model for coupling ship motion with the motion of the sea surface has been developed by the author, with the underlying physics of sea induced vessel motion included in the model development. A detailed discussion of the development of this motion coupling model is

presented in Appendix D of [16], and only a brief summary is presented here. The parameters of a typical vessel that are relevant to the analysis of sea induced vessel motion are modelled in a simple, yet realistic fashion. These parameters are favourably compared to the parameters of an actual ocean going vessel in Appendix D of [16].

The fundamental processing required to produce the vessel's waterline height $z_t(t)$, pitch angle $\theta_p(t)$, and stabilized antenna pointing angle error $\theta_{ss}(t)$ is summarized in Figure 2.11. The frequency responses of the vertical translation, pitching motion, and antenna stabilization filters are provided below

$$H_v(f) = \frac{f_v^2 + j 1.4 f_v f}{(f_v^2 - f^2) + j 1.4 f_v f} \quad (2.60)$$

$$H_p(f) = \frac{f_p^2 + j 0.6 f_p f}{(f_p^2 - f^2) + j 0.6 f_p f} \quad (2.61)$$

$$H_s(f) = \frac{(f_o^2 - K f^2) + j f_o f}{(f_o^2 - f^2) + j f_o f} \quad (2.62)$$

where

$$f_v = f_p = \frac{1}{2\pi} \sqrt{\frac{10g}{L}} \text{ Hz} \quad (2.63)$$

$$f_o = \frac{f_p}{10} \text{ Hz} \quad (2.64)$$

Figure 2.11: Summary of fundamental processing for sea-induced vessel motion and gravity stabilized antenna platforms

$$K = \frac{h}{g} \omega_0^2, \quad (2.65)$$

h = height of the antenna gravity sensor (meters),

g = acceleration due to gravity (9.8 m/sec^2),

$\omega_0 = 2\pi f_0$, and

L = waterline length of the vessel in meters.

In the simulation program, the gravity sensor height is selected to be the antenna height, z_0 , or $0.5 L$, whichever is smaller. Consequently, the only ship parameters that are required are the velocity (m/sec) and length (m) of the vessel, as well as an antenna stabilization flag. If there is no stabilization, $\theta_{ss}(t) = \theta_p(t)$. It is also important to limit the velocity of the vessel to its maximum theoretical value given by the hull speed

$$u_h = 1.25 \sqrt{L} \text{ m/sec } [40], [41].$$

Although a discrete implementation of the processing indicated in Figure 2.11 is possible, one can take advantage of the sinusoidal nature of the surface model to directly evaluate expressions for $z_i(t)$, $\theta_p(t)$, and $\theta_{ss}(t)$. This approach is not only more computationally efficient, it also avoids the troublesome transient problems associated with discrete implementations as it directly provides the steady state solution. The expressions for $z_i(t)$, $\theta_p(t)$, and $\theta_{ss}(t)$ are summarized in Table 2.3 for a rough surface characteristic given by

$$\begin{aligned}
 z_t(t) &= \sum_{k=-N}^N A_k |H_v(f_k)| \cos(2\pi f_k t + \theta_k + \phi_v(f_k)) \\
 \theta_p(t) &= \sum_{k=-N}^N B_k |H_p(f_k)| \sin(2\pi f_k t + \theta_k + \phi_p(f_k)) \\
 \theta_{ss}(t) &= \sum_{k=-N}^N B_k |H_p(f_k)| |H_s(f_k)| \sin(2\pi f_k t + \theta_k + \phi_p(f_k) + \phi_s(f_k)) \\
 A_k &= c \frac{\sin(\frac{\pi L}{\lambda_k})}{(\frac{\pi L}{\lambda_k})} \\
 B_k &= \frac{12c}{L^3} \left(\frac{\lambda_k}{2\pi}\right)^2 \left[L \left(\frac{2\pi}{\lambda_k}\right) \cos\left(\frac{\pi L}{\lambda_k}\right) - 2 \sin\left(\frac{\pi L}{\lambda_k}\right) \right] \\
 f_k &= \frac{u - v_k}{\lambda_k} \\
 \phi_v(f_k) &= \arg \{H_v(f_k)\} \\
 \phi_p(f_k) &= \arg \{H_p(f_k)\} \\
 \phi_s(f_k) &= \arg \{H_s(f_k)\} \\
 u &= \text{ship velocity} \\
 v_k &= \text{velocity of the } k\text{-th surface component} \\
 c &= \sqrt{\frac{2}{2N+1}} \sigma_z \quad ; \quad \sigma_z = \text{rms waveheight}
 \end{aligned}$$

Table 2.3: Expressions for the ship motion functions, $z_t(t)$, $\theta_p(t)$, and $\theta_{ss}(t)$ for a superposition of sinusoids surface model.

$$z_g(x, t) = c \sum_{k=-N}^N \cos\left[\frac{2\pi}{\lambda_k} (x - v_k t) + \theta_k\right]. \quad (2.66)$$

Note that the curvature of the earth can be safely ignored for establishing vessel motion.

A ship motion monitoring routine has been developed as a separate stand alone program. This program produces a number of plottable output files which characterize vessel motion. Results will be shown for a vessel length of 69 m, an antenna height $z_0 = 20$ m, travelling against the sea waves with a speed of 5 m/sec. The sea state index selected was SS4(ii), which is representative of typical North Atlantic conditions. For this sea state, the rms waveheight σ_z is 0.61 m, the average sea wavelength λ is 34.26 m, and the average wavespeed $\bar{v} = 6$ m/sec. (see Appendix A).

Figure 2.12 compares the waterline height of the vessel to the instantaneous surface height that would occur at the centre of gravity (in the absence of the vessel) for this scenario. Significant filtering is evident, and the bobbing of the ship is less than ± 0.5 m. The pitch angle and stabilized antenna pointing angle error are shown in Figure 2.13. The pitching angle extremes are only $\pm 2.3^\circ$. The peak stabilized antenna pointing error is around 0.3° .

Figure 2.12 : Comparison of instantaneous waveheight (ZCT) and the waterline height (ZLT) of the vessel.

Figure 2.13 : Comparison of pitching angle (PIT) and stabilized antenna pointing angle error (PNT).

which demonstrates the effectiveness of the antenna stabilization network. To provide some indication of induced antenna motion, the difference between the x-antenna co-ordinate and the centre of gravity of the vessel is illustrated in Figure 2.14 and the difference between the z-antenna co-ordinate and the nominal antenna height z_0 is presented in Figure 2.15. For this vessel, the x-co-ordinate deviation is bounded by approximately ± 0.75 m, while the vertical bobbing of the antenna is less than 0.4 m (which is less than the rms surface waveheight for this scenario).

2.4 Direct Path Wave Blockage And Blockage of Reflected Paths By The Hull of The Vessel

In the maritime application, there is some possibility that the direct path may suffer from blockage by high waves, especially if the antenna is close to the surface. The situation is illustrated in Figure 2.16. The effect of path clearance (Fresnel zone blockage) on radio transmission is illustrated in Figure 2.17 [17]. The appropriate curve for this application is the one for knife edge diffraction. A functional approximation of this curve has been incorporated into the simulation program. It is apparent that when the ratio of the path clearance, h_c , to the first Fresnel zone radius, F_R , exceeds two, there is no significant direct path blockage.

Figure 2.14 : Difference between X-coordinate of antenna and center of gravity of vessel.

Figure 2.15 : Difference between Z-coordinate of antenna and the nominal antenna height.

Figure 2.16: Direct path wave blockage by high sea waves

Figure 2.17: Effect of path clearance on radio transmission (from [17]). Knife edge diffraction curve appropriate for wave blockage effects in the maritime application.

A simple pre-screening procedure, which approximates the surface by the peak waveheight h_p , which is determined in the initialization phase of the propagation simulation program, has been developed. This pre-screening procedure can identify those situations where direct path blockage will not have significant impact. Furthermore, when blockage may have a significant effect, the region which must be examined to establish the minimum clearance ratio (h_c/F_R) is provided. Using the knife edge diffraction functional approximation, one can then establish the appropriate voltage scale factor corresponding to the minimum clearance ratio discovered in the examination region.

The points which bound the region where the clearance ratio (for the constant surface with height equal to the peak waveheight) is less than or equal to 2 are given by the solutions to the quadratic equation

$$A(x-x_a)^2 + B(x-x_a) + C = 0 \quad (2.67)$$

where

$$A = \sin^2 E \quad (2.68)$$

$$B = 2(z_a - h_p) \cos E \sin E - 4 \lambda_c \cos E \quad (2.69)$$

$$C = (z_a - h_p)^2 \cos^2 E + 4 \lambda_c (z_a - h_p) \sin E \quad (2.70)$$

When $B^2 - 4AC < 0$, direct path wave blockage is guaranteed to be insignificant. Note that

$$B^2 - 4AC = 16 \lambda_c^2 \left(\cos^2 E - \frac{(z_a - h_p)}{\lambda_c} \sin E \right).$$

When $B^2 - 4AC > 0$, this quadratic has realistic solutions, and the interval that must be checked for the minimum clearance ratio is bounded by

$$x_L = \text{MAX} \left[x_a, x_a - \left(\frac{B + \sqrt{B^2 - 4AC}}{2A} \right) \right] \quad (2.71)$$

$$x_H = x_a - \left(\frac{B - \sqrt{B^2 - 4AC}}{2A} \right). \quad (2.72)$$

Before concluding the discussion on direct path wave blockage, a few numerical examples are considered to assess its impact under typical operating conditions. For an elevation angle of 5° and a carrier wavelength of 0.2 m, one finds that there is no significant blockage effects with $(z_a - h_p) = 5$ m. However, reducing $(z_a - h_p)$ to 1 m yields $A = 7.596 \times 10^{-3}$, $B = -0.623$, and $C = 1.062$, and the region one must search for potentially significant direct path blockage is defined by

$$(x - x_a) = [1.74 \text{ m}, 80.27 \text{ m}].$$

For typical shipborne antenna heights, it is apparent that direct path wave blockage will not be that significant. One requires very high waves, low antenna heights, and low elevation angles for direct path blockage to be significant. An application where this would be significant is when measurements are performed with antennas located on buoys floating on the ocean surface.

Incorporating the effects of the vessel's superstructure on the ocean multipath is not practical, especially at low elevation angles, because of the sheer number of reflected paths which must be considered. However, it is possible to account for blockage effects by the hull of the ship in a relatively easy fashion. Consequently, this has been included in the maritime propagation simulation program as an option.

The situation is illustrated in Figure 2.18, where θ denotes the reflection angle relative to the horizontal and θ_p denotes the pitch angle of the vessel. Letting θ_F denote the angle to the antenna from the front of the ship

$$\theta_F = \tan^{-1} \left(\frac{h_D}{x_F} \right) \quad (2.73)$$

where

$$x_F = L/2 - x_D,$$

Figure 2.18: Geometry for evaluating path clearance on reflected paths.

h_D = the height of the antenna above the deck, and
 x_D = the x-coordinate of the antenna relative to the
pitching axis,

and letting r_a denote the distance between the antenna and
the front of the ship

$$r_a = \sqrt{h_D^2 + x_F^2} , \quad (2.74)$$

the clearance angle θ_C and the path clearance h_C are given
by

$$\theta_C = \theta_F - \beta - \theta_P \quad (2.75)$$

$$h_C = r_a \sin \theta_C . \quad (2.76)$$

The first Fresnel zone radius is given by

$$F_R = \sqrt{\lambda_C r_a \cos \theta_C} . \quad (2.77)$$

The effects of blockage can be predicted by supplying the
ratio $R = (h_C/F_R)$ to the knife edge diffraction functional
approximation.

One can also establish a threshold reflection angle

β_{THRESH} , such that whenever $\beta < \beta_{THRESH}$, there will be no

significant blockage effects. This is based on the observation that $F_R < \sqrt{\lambda_c r_a}$, which implies that the actual clearance ratio, $h_c/F_R > h_c/\sqrt{\lambda_c r_a}$. The threshold angle that ensures that $h_c/\sqrt{\lambda_c r_a} > 2$ is given by

$$\theta_{\text{THRESH}} = \theta_F - \theta_P - \sin^{-1}\left(2\sqrt{\frac{\lambda_c}{r_a}}\right). \quad (2.78)$$

It is apparent that the blockage effects will be most severe for the close-in scattering facets, which have the largest reflection angles and also tend to be the strongest reflections. It is also evident that blockage on multipath components will be most severe when the antenna is not very high above the deck of the ship.

CHAPTER 3.0

AERONAUTICAL AND MARITIME SIMULATION RESULTS

In this chapter, simulation results for the aeronautical and maritime ocean scatter multipath channels are presented. Low gain (minimal Standard-C) antenna patterns and L-band carrier frequency applications are considered. The Standard-C earth station is the smallest ship earth station for two way communications to be offered by INMARSAT [18]. Standard-C embodies a wide range of earth station capabilities, and can be subdivided in various subclasses corresponding to different antenna realizations. For minimal Standard-C the transmission rates are 400 bps to the mobile and 800 bps on the return path. The associated antenna patterns provide low gain hemispherical coverage.

The objective of this chapter is to characterize the behaviour of ocean scatter for the aeronautical and maritime applications as a function of important physical variables such as the elevation angle to the satellite, signal polarization, velocity of the mobile terminal, antenna height and sea state (surface roughness). The aeronautical application is considered first. To assess the accuracy and validity of the aeronautical simulation

model, a brief comparison of aeronautical simulation and measurement results is presented in this chapter.

The availability of measurement data for the maritime application [6], makes this application a good candidate for validating the multipath propagation model.

Consequently simulation scenarios have been set up to closely resemble cases where measurement results are available in [6]. The maritime simulation results obtained and measurement data are compared in this chapter. This section concludes with a comparison of maritime and aeronautical ocean scatter multipath.

To accurately characterize the ocean scatter multipath obtained with right hand circular (RHC) polarization, one requires accurate antenna information, for both horizontal and vertical polarization. The selection of relevant antenna patterns for the maritime and aeronautical applications is considered before presenting simulation results.

3.1 Selection of Antenna Patterns For The Maritime And Aeronautical Applications

The selection of antenna patterns is an extremely important issue, because the multipath output for circular polarization can be quite sensitive to antenna

characteristics such as gain and phase mismatches.

Ideally, one wants to have accurate antenna patterns, for both horizontal and vertical polarization, specified in terms of amplitude and phase (or alternately as a complex number) as indicated below

$$G_H(\theta) = g_H(\theta) e^{j\phi_H(\theta)} \quad (3.1)$$

$$G_V(\theta) = g_V(\theta) e^{j\phi_V(\theta)}$$

When complete pattern information is available, the copolarized multipath output can be accurately characterized. However, the antenna pattern information one has access to is often far from complete. In many cases, only the amplitude response (no phase information) for RHC polarization is available. In some cases, cross-polarization (LHC) amplitude responses are also available. Unfortunately, these characterizations do not provide sufficient information to establish the individual horizontal and vertical polarization antenna patterns; which is a major deficiency.

One way of establishing horizontal and vertical gain patterns which give the prescribed pattern shape for RHC polarization, is to assume perfect circularity (matching gains and phases for each polarization pattern). This is

the approach used to establish the individual pattern shapes employed in the simulation program. However, it should be noted that the actual patterns could be significantly different due to the polarization ellipticity of the receiving antenna, and this could have a significant impact on the resulting multipath output. Consequently, a procedure for approximating the effects of antenna ellipticity has been incorporated.

To demonstrate the sensitivity of the copolarized multipath output to antenna gain and phase mismatches, consider the reflected component from a single scattering facet. The scattered signals prior to the application of antenna gains will be represented by the complex numbers r_H and r_V , where the subscript denotes the polarization type. The resultant copolarized output signal is given by

$$S_{\text{copol}} = \left\{ \frac{G_H r_H + G_V r_V}{2} \right\}$$

$$= \frac{e^{j\phi_H}}{2} [g_H r_H + g_V r_V e^{j(\phi_V - \phi_H)}]. \quad (3.2)$$

From the nature of the reflection coefficients for horizontal and vertical polarization, presented in Appendix B, one can establish typical orientations for the reflected r_H and r_V signals, as illustrated in Figure 3.1, for a few representative local grazing angles.

(a) Local grazing angle well above vertical polarization Brewster angle

(b) Local grazing angle in vicinity of Brewster angle

(c) Local grazing angle below Brewster angle

Figure 3.1: Relative orientation of horizontal and vertically polarized scatter signals

The first case illustrated in Figure 3.1 represents a situation where the local grazing angle is well above the vertical polarization Brewster angle, which is in the neighbourhood of 6° at L-band. Due to the different surface reflection properties for these polarizations, the horizontal and vertically polarized scatter components are virtually anti-phase with comparable magnitudes. For perfect circularly polarized antennas (both gain and phase matched), the copolarized (RHC) output is simply a scaled vector addition of these components, and one obtains almost complete cancellation, with only a small residue. Stated another way, the multipath component undergoes a polarization sense reversal upon reflection by the surface, and this is discriminated against by the antenna. In fact, this discrimination is the major reason why circular polarization is used in multipath environments. Practical mobile antennas will not be perfectly circular, however, especially below the horizon, which happens to be where the multipath arises. It is quite evident from (3.2) that gain and phase perturbation will tend to reduce the degree of cancellation, resulting in an increase in the amplitude level of the residual multipath output. The amount of increase is strongly dependent on the degree of imbalance, and can be substantial for significant phase mismatches.

The second situation illustrated in Figure 3.1 represents the case where the local grazing angle is in the vicinity of

the Brewster angle. In this case, the scatter power is predominantly horizontally polarized because of the surface discrimination against vertical polarization at low grazing angles. With a perfect circular receiving antenna, the amplitude of the copolarized multipath output will be virtually 6 dB below that obtained with an antenna receiving only horizontal polarization. Because of the considerably smaller magnitude of the vertical component, antenna phase mismatches will not have nearly as much impact as deviating the relative gains of the horizontal and vertical antenna patterns on the resultant multipath level. For example, if the magnitude of the polarization ratio, $|P| = \frac{g_V}{g_H}$, is less than unity, the resultant multipath level is increased relative to perfect circular polarization. To raise the resultant multipath level by 2 dB relative to perfect circular polarization, one requires a polarization ratio magnitude of approximately -6 dB. Even in the limit, as the polarization ratio approaches zero ($g_H = \sqrt{2}$ and $g_V = 0$), the multipath level is only increased by approximately 3 dB relative to perfect circular polarization. However, if the same ellipticity applies for the direct path signal direction, its level is also degraded by 3 dB, implying a signal to interference ratio degradation of 6 dB.

For very small local grazing angles, the orientation of the individual scatter components are as illustrated in Figure 3.1 (c). In this case, the horizontal and vertically polarized components are almost in phase and of comparable magnitude. From the vertical polarization reflection coefficient amplitude and phase responses provided in Appendix B, the local grazing angle would have to be less than 2.5° for this to occur. Note that this is the worst case for circular polarization, because the components are additive. It is evident that large antenna phase mismatches would tend to lower the copolarized multipath output. From equation (2.3), it is evident that the local grazing angle can be lower bounded by $E/2$, where E is the elevation angle to the satellite. Consequently, coherent combining of scatter components for RHC polarization will occur at very low elevation angles (i.e. elevation angles below 5°).

The above discussion demonstrates that the copolarized multipath output for circular polarization can be quite sensitive to antenna characteristics. A feature that has been incorporated in the simulation program, developed initially to investigate the effects of satellite antenna ellipticity, can be used to introduce realistic perturbations from ideal circular polarization in the

receiving antenna patterns. This approach is discussed below.

In this case, the polarization ratio of the satellite antenna, P_S , specified in terms of its amplitude and phase mismatch

$$P_S = E_S e^{j\theta_m} \quad (3.3)$$

where

$$E_S = \frac{\alpha_V}{\alpha_H} \quad \text{and} \quad \theta_m = \phi_V - \phi_H \quad (3.4)$$

The constant transmit and receive power constraint, discussed in detail in Appendix D, implies that

$$\alpha_H^2 + \alpha_V^2 = 2. \quad (3.5)$$

With knowledge of the magnitude of the polarization ratio, one obtains

$$\alpha_H = \sqrt{\frac{2}{1 + E_S^2}} \quad (3.6)$$

$$\alpha_V = \sqrt{\frac{2E_S^2}{1 + E_S^2}} \quad (3.7)$$

The copolarized output with this ellipticity is given by

$$\vec{S}_{\text{copol}}(n) = \frac{\alpha_H \vec{S}_H(n) + \alpha_V e^{j\theta_m} \vec{S}_V(n)}{2} \quad (3.8)$$

where

$\vec{S}_H(n)$, $\vec{S}_V(n)$ are complex vectors representing the horizontal and vertical polarization scatter processes at sample index n . As indicated in (2.6) and (2.7), the horizontal and vertically polarized scatter signals incorporate antenna effects on the reflected signals from each scattering facet.

If one has knowledge of the polarization ratio of an antenna, one can use this to define α_H , α_V , θ_m in (3.8), thereby introducing realistic perturbations in the horizontal and vertical polarization patterns derived assuming perfect circular polarization. The only deficiency inherent in this approach is that the selected ellipticity applies for all elevation angles. Although this may not be representative of actual conditions, it is the best one can do without detailed antenna pattern information.

3.1.1 Maritime Antenna Patterns

Maritime antenna patterns obtained from [6], designated by the mnemonics DFVLR3 and DFVLR5, are presented in

Figures 3.2 and 3.3, respectively. The DFVLR3 pattern only provides data for RHC polarization. In addition to the RHC pattern, the cross-polarized (LHC) pattern is also available for the DFVLR5 antenna. At an elevation angle of 20° , the relative RHC to LHC gain is roughly 13 dB, which allows one to establish a realistic axial ratio for the polarization ellipse using equation (D.34) of Appendix D

where

$$AR = \left| \frac{1 + |Q|}{1 - |Q|} \right|$$

(3.9)

$$|Q| = \left| \frac{E_{LHC}}{E_{RHC}} \right|$$

(3.10)

With the above gain ratio, one obtains an axial ratio of $A_R = 1.58$ (≈ 4 dB) for the polarization ellipse at 20° . It is not possible to establish the axial ratio below the horizon, but it is likely to be worse than that appropriate at 20° elevation angle.

A backfire antenna used in the measurements discussed in [6] (designated by the mnemonic C14) has an axial ratio pattern as shown in Figure 3.4. Here an axial ratio on the order of 5.5 dB is evident. Unfortunately, no other

Figure 3.2: Antenna pattern of a conical spiral Standard-C antenna, referred to by the mnemonic DFVLRCS (from [6]).

Figure 3.3: Antenna pattern of a small backfire Standard-C antenna, referred to by the mnemonic DFVLRCS (from [6]).

Figure 3.4: Axial ratio characteristics of a short backfire antenna, designated by the mnemonic C14 in [6], at L-band.

Figure 3.5: Hemispherical coverage pattern from a quadrifiler helix (minimal Standard-C) shipborne antenna (from [9]).

pattern information is provided, so this antenna could not be incorporated in the simulation.

Another Standard-C antenna pattern incorporated in the simulation program is that of a 5-turn quadrifilar helix antenna [9], with radiation patterns as shown in Figure 3.5. These radiation patterns are the copol and cross-polarized patterns. For reflection angles between approximately 10° to 30° below the horizon, the ratio of the magnitude of the cross-polarized to copolarized output is $Q = -10$ dB. Using equation (3.9), it is apparent that an axial ratio for the polarization ellipse of $A_R = 1.925$ (≈ 5.7 dB) is appropriate. When comparing measurement and simulation results for maritime applications with low gain Standard-C antennas, an axial ratio of 5.5 dB is assumed.

The relationship between the axial ratio and the magnitude of the polarization ratio, $|P| = E_R$, and the phase mismatch angle, θ_m , is derived in Appendix D. Using the basic relationship for RHC polarization,

$$\frac{A_R}{1 + A_R^2} = \frac{E_R \cos \theta_m}{1 + E_R^2} \quad (3.11)$$

it is easy to find values of E_R and θ_m which provide a specific axial ratio. Appropriate values for an axial ratio of 5.5 dB are presented in Table 3.1.

Magnitude of the Polarization Ratio [dB]	Phase Mismatch θ_m (deg)
± 5.5	0
± 5.0	± 14.2
± 4.5	± 19.6
± 4.0	± 23.4
± 3.5	± 26.3
± 3.0	± 28.6
± 2.5	± 30.4
± 2.0	± 31.75
± 1.5	± 32.8
± 1.0	± 33.5
± 0.5	± 33.9
0	± 34.1

Table 3.1: Polarization ratio (amplitude and phase mismatch) parameters which provide an axial ratio of 5.5 dB.

3.1.2 Aircraft Antenna Patterns

Aircraft antennas must have a low profile to prevent air drag and resistance. The restrictions on antenna placement and size are much more severe than for shipborne antennas. Aircraft antenna patterns for horizontal and vertical polarization have been determined by Racal-Decca [19] for a low complexity crossed slot antenna mounted on a Boeing 747 aircraft. The effects of the aircraft fuselage are incorporated in the geometrical theory of diffraction (GTD) solution approach. The roll plane patterns are illustrated in Figure 3.6. The peak gain of the crossed slot antenna is 5 dB_{1c} at zenith. The distinguishing feature of the crossed slot antenna pattern is the discrimination against horizontal polarization below the horizon.

While these patterns are a considerable improvement over those available for the maritime application, the lack of phase information for these patterns is a deficiency, necessitating a zero phase assumption. One can examine the sensitivity to antenna phase mismatches, however, using the relationship of (3.8) with $\alpha_H = \alpha_V = 1$ and realistic phase mismatches, θ_m . The simulated horizontally and vertically polarized multipath sequences are combined to provide multipath output records for circular polarization with $\theta_m = 0^\circ$, $\theta_m = -45^\circ$, and $\theta_m = -90^\circ$.

Figure 3.6: Roll plane patterns of a low complexity crossed slot antenna (mounted on a Boeing 747) at L-band.

Figure 3.7: Roll plane patterns of a diversity mounted, switch selectable crossed slot antenna system at L-band.

The other aircraft antenna pattern considered is the roll plane pattern which results when two crossed slot antennas are mounted on the sides of a Boeing 747 in such a fashion that they individually provide peak gain at 50° elevation angle on their respective sides of the aircraft. The antennas are switch-selectable, and on reception, the antenna providing the maximum signal output is selected. This switched antenna pattern is provided in Figure 3.7 [19]. Note that this antenna maintains much better circularity at low elevation angles and below the horizon than the crossed slot antenna, which results in an improved direct path signal to noise ratio (SNR). More complicated phased array antenna solutions are not considered because the cost and fuel penalties associated with their increased complexity and size make them less attractive [19].

Before presenting the aeronautical simulation results, the nature of the amplitude distribution of ocean scatter multipath for rough surface conditions is considered for the maritime and aeronautical applications.

3.1.3 Amplitude Distribution of Ocean Scatter For Maritime And Aeronautical Applications With Rough Surface Conditions

For rough seas, it is commonly assumed that the ocean scatter process has a Rayleigh amplitude distribution and

uniform phase (diffuse scattering). This behaviour has been confirmed by measurement results [5], [6], [7], [8], and [20]. This implies that the complex baseband scatter signal $s(n)$, can be represented by a complex Gaussian process

$$s(n) = x(n) + j y(n), \quad (3.12)$$

where $x(n)$ and $y(n)$ are independent, zero mean, Gaussian processes with equal variance σ^2 . In this case, the scatter power

$$P_{sc}(n) = x(n)^2 + y(n)^2, \quad (3.13)$$

has an exponential density function

$$f_{P_{sc}}(x) = \frac{1}{2\sigma^2} \exp\left(-\frac{x}{2\sigma^2}\right). \quad (3.14)$$

Addition of a constant direct path component D , yields a combined received signal

$$c(n) = (D + x(n)) + j y(n). \quad (3.15)$$

The envelope of this signal, $R(n)$, has a Rician density function

$$f_R(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + D^2}{2\sigma^2}\right) I_0\left(\frac{rD}{\sigma^2}\right) \quad (3.16)$$

where

$I_0(\cdot)$ is the modified Bessel function of the first kind of order zero.

The signal to interference ratio (S/I) is defined as

$$S/I = \frac{\bar{P}_D}{\bar{P}_{sc}} \quad (3.17)$$

where

$\bar{P}_D$ and $\bar{P}_{sc}$ represents the average direct path and scatter power levels, respectively.

The theoretical exponential scatter power and Rician received voltage density functions can be approximated using $\bar{P}_{sc}$ as an estimate of $2\sigma^2$ and $\bar{P}_D$ as a estimate of D^2 . Simulated and theoretical density functions for the maritime application with rough sea conditions (sea state index SS4(ii)) and elevation angles of 10° , 15° , and 20° are presented in Figure 3.8, 3.9, and 3.10, respectively. The agreement with theory is quite good, indicating that the maritime multipath is Rayleigh in nature.

Simulated and theoretical density functions for the aeronautical application are presented in Figure 3.11 and 3.12 for elevation angles of 31° (SS4(ii) surface conditions) and 10° (SS3 surface conditions). The scatter

Figure 3.8: Observed and theoretical received voltage and scatter power density functions for a 10° elevation angle maritime case with sea state index SS4(ii).

Figure 3.9: Observed and theoretical received voltage and scatter power density functions for a 15° elevation angle maritime case with sea state index SS4(ii).

Figure 3.10: Observed and theoretical received voltage and scatter power density functions for a 20° elevation angle maritime case with sea state index SS4(ii).

Figure 3.11: Observed and theoretical received voltage and scatter power density functions for a 31° elevation angle aeronautical run with sea state index SS4(ii).

Figure 3.12: Observed and theoretical received voltage and scatter power density functions for a 10° elevation angle aeronautical run with SS3 surface conditions.

power and received voltage histograms match theory fairly well. The received voltage histograms, in particular, exhibit considerably more fluctuation than those for the maritime case. This is because the signal records for the aeronautical case contain only 1024 samples, as opposed to the 4096 samples for the maritime application. It is evident that aeronautical ocean scatter is essentially Rayleigh in nature, as there is no evidence of a coherent component in the scatter power histogram. This behaviour is not that surprising, as the reflection coefficient for the specular (coherent) component for rough surface scattering is given by (see pages 244 and 316 of [1])

$$\rho_c = \rho_s D_e \rho_o \quad (3.18)$$

where

ρ_o is the smooth surface reflection coefficient,

D_e is the divergence factor of the earth, and

ρ_s is a random component that depends on the nature of the surface irregularities.

For a Gaussian surface model, the mean square value of ρ_s is given by (see pages 93, 246, and 316 of [1])

$$E\{|\rho_s|^2\} = \exp\left\{-\left(\frac{4\pi \sigma_z \sin \alpha}{\lambda_c}\right)^2\right\} \quad (3.19)$$

where σ_z is the rms waveheight, λ_c is the carrier wavelength, and α is the local grazing angle.

For North Atlantic conditions, a realistic sea state index is SS4(ii) where $\sigma_z = 0.61$ m. At L-band ($\lambda_c = 0.2$ m) and for a grazing angle of 5° , the rms rough surface reflection coefficient is 48.5 dB below the smooth surface value. For higher grazing angles, the rough surface reflection coefficient is even smaller. Consequently, the specular component of the scatter process for typical rough sea conditions at L-band will be extremely small, meaning that the scatter process is predominantly diffuse.

3.2 Aeronautical Simulation Results

Aeronautical simulation results are presented in this section. In all the simulation runs performed, the rough surface was generated using the general superposition of sinusoids surface model described in Appendix A, with 21 random phased sinusoidal components. The sequence of uniform random variables used to generate surface profiles are maintained in disk files. The filename is defined by a five character surface descriptor seed, which identifies the initial seed used to generate the uniform phase random variables.

A set of baseline simulation runs, designed to characterize ocean scatter as a function of elevation angle with the low complexity crossed slot antenna, are considered first. These baseline simulation scenarios are summarized in Table

3.2. These baseline runs employ level flight and rough surface conditions (sea state index SS4(ii)). Table 3.3 summarizes the simulation runs conducted to assess the impact of varying the sea state and aircraft altitude. The effect of climbing and diving manoeuvres on ocean scatter multipath, with a crossed slot antenna, is considered next. Table 3.4 summarizes the pertinent information characterizing these runs. For all the above mentioned simulation scenarios, the behaviour of ocean scatter multipath with an omni-directional antenna pattern is also established.

The behaviour of the multipath induced by the presence of the ocean surface versus elevation angle with a more complex switched antenna (diversity pattern derived using two crossed slot antennas) is then examined. The simulation runs performed are summarized in Table 3.5. The effect of polarization ellipticity of the satellite antenna on the first order statistics obtained with the switched antenna is also established.

This section concludes with a brief comparison of aeronautical measurement data and simulation results.

3.2.1 Baseline Simulation Results

This section characterizes the first order and second order statistics of ocean scatter multipath for the baseline

Elevation Angle	Surface Descriptor Seed	Number of Samples Per Average Sea Wavelength, N_w	Aircraft Altitude, $z_0 = 9.1$ km Carrier Wavelength $\lambda_c = 0.2$ m Sea State Index SS4(ii) Total Number of Samples $N = 1024$ Crossed Slot Antenna Level Flight Simulated
31°	S1111	256	
20°	S1111	256	
15°	S1111	200	
10°	SZERO	128	
5°	SZERO	128	

Table 3.2: Summary of baseline aeronautical simulation scenarios

Sea State Index	Surface Descriptor Seed	Aircraft Altitude	Carrier Wavelength $\lambda_c = 0.2$ m Number of Samples Per Average Sea Wavelength $N_w = 128$ Total Number of Samples $N = 1024$ Crossed Slot Antenna Elevation Angle $E = 10^\circ$ Level Flight Simulated
SS3	S1111	9.1 km	
SS3	SZERO	9.1 km	
SS5(i)	SZERO	9.1 km	
SS4(ii)	SZERO	5.0 km	

Table 3.3: Summary of simulation scenarios for assessing the impact of varying sea state and aircraft altitude.

Elevation Angle	Climb Angle	Surface Descriptor Seed
15°	-10°	S1111
15°	+10°	S1111

Aircraft Altitude

$$z_0 = 9.1 \text{ km}$$

Carrier Wavelength

$$\lambda_c = 0.2 \text{ m}$$

Sea State Index SS4(ii)

Number of Samples Per

Average Sea Wavelength

$$N_w = 256$$

Total Number of Samples

$$N = 512$$

Crossed Slot Antenna

Table 3.4: Summary of aircraft climbing and diving simulation runs.

Elevation Angle	Surface Descriptor Seed	Number of Samples Per Average Sea Wavelength N_w
31°	S1111	256
20°	S5001	256
15°	S1111	200
10°	SZERO	128
5°	S1111	128

Aircraft Altitude

$$z_0 = 9.1 \text{ km}$$

Carrier Wavelength

$$\lambda_c = 0.2 \text{ m}$$

Sea State Index SS4(ii)

Total Number of Samples

$$N = 1024$$

Switched (Diversity)

Antenna

Level Flight Simulated

Table 3.5: Summary of aeronautical simulation runs conducted with the switched (diversity) antenna

simulation scenarios. Two approaches for estimating the signal to interference ratio have been considered. The first of these is the traditional approach of averaging the instantaneous scatter power samples over the signal record. Under slow fading conditions, the average power estimated with this approach can exhibit considerable fluctuation, especially if the signal record does not include a sufficiently large number of fade cycles.

With the second approach, the average scatter power is determined by integrating the power density versus reflection angle curves. This power density characterization is based on the assumption that the individual contributions from the several underlying scattering facets add as power (i.e. the underlying phase shifts are independent and uniformly distributed).

Examples of these power density versus reflection angle curves for horizontal, vertical, and perfect circular polarization with an omni-directional antenna are provided in Figures 3.13, 3.14, and 3.15 for elevation angles of 10° , 15° , and 31° . These power density curves could be extremely useful for aircraft antenna design engineers. Figures 3.13 and 3.14 clearly illustrate the surface discrimination against vertical polarization at low reflection (grazing) angles. These curves also clearly illustrate that scatter power actually arises from a

Figure 3.11: Power density versus reflection angle curves for horizontal, vertical, and circular polarization with an omni antenna pattern for an aeronautical simulation - run with an elevation angle of 10° .

(a) Horizontal polarization.

(b) Vertical polarization.

(c) Circular polarization.

Figure 3.14: Power density versus reflection angle curves for horizontal, vertical, and circular polarization with an omni antenna pattern for an aeronautical simulation run with an elevation angle of 15° .

(a) Horizontal polarization.

(b) Vertical polarization.

(c) Circular polarization.

Figure 3.15: Power density versus reflection angle curves for horizontal, vertical, and circular polarization with an omni antenna pattern for an aeronautical simulation run with an elevation angle of 31° .

relatively wide reflection angle region. The significant beamwidth of the multipath scatter frustrates antenna null steering multipath rejection techniques. It is also evident that one cannot realistically define a single direction, such as $\theta = E$ or $\theta = E/2$, from which the majority of scatter power arises.

The superiority of the power density method for characterizing the signal to interference ratio is clearly demonstrated in Appendix F. Consequently, the signal to interference (S/I) ratios reported for horizontal, vertical, and perfect circular polarization are those obtained with the power density method. As indicated in Appendix F the results obtained with the power density method are consistent with confidence limits on the mean scatter power obtained by averaging instantaneous scatter power samples over the signal record. S/I values for phase mismatches of $\theta_m = -45^\circ$ and $\theta_m = -90^\circ$ with circular polarization can only be established by averaging instantaneous power samples. These values are provided to give some indication of worst case S/I values applicable for circular polarization.

3.2.1.1 Signal to Interference Ratio Results For Baseline Simulation Scenarios

The signal to interference (S/I) ratio results for the crossed slot and omni antenna patterns for the baseline

simulation scenarios are summarized in Table 3.6. These S/I results exhibit some interesting trends.

With a horizontally polarized crossed slot antenna, the S/I ratio tends to decrease as the elevation angle is lowered, levelling off to relatively constant value in the neighbourhood of 10-11 dB at low elevation angles. With a vertically polarized crossed slot antenna, the effect of the Brewster angle is clearly evident for elevation angles of 10° and 5° , where the S/I ratios tend to increase because the surface discriminates against vertical polarization at low grazing angles. For a perfectly circular crossed slot antenna, the S/I ratios remain relatively constant with elevation angle, and are excellent, exceeding 20 dB in each case. Even with a phase mismatch as extreme as -90° , the worst case S/I ratio is approximately 15 dB. With signal to interference ratios as high as this, ocean scatter will not have a severe impact on data communications integrity. The fact that S/I ratios are so high at low elevation angles with the crossed slot antenna is not that surprising; because the antenna pattern (see Figure 3:6) discriminates against horizontally polarized multipath, while the surface tends to discriminate against vertically polarized multipath.

For an omni-directional antenna, the S/I values tend to improve (increase) as the elevation angle is lowered for both horizontal and vertical signal polarizations (see

Signal to Interference Ratios [dB] For The Following Polarization Types					
Elevation Angle	Horizon al	Vertical	Copol $\theta_m = 0^\circ$	Copol $\theta_m = -45^\circ$	Copol $\theta_m = -90^\circ$
31°	24.6 (1.21)	16.86 (3.93)	21.58 (17.88)	19.91 (8.27)	17.0 (1.52)
20°	14.87 (1.02)	15.73 (5.86)	20.44 (13.6)	18.05 (7.62)	14.96 (1.73)
15°	11.88 (1.60)	16.46 (7.97)	21.27 (12.41)	18.57 (8.14)	15.25 (2.94)
10°	10.41 (2.74)	18.8 (11.27)	22.9 (11.72)	19.18 (8.41)	16.27 (4.0)
5°	11.18 (5.21)	21.0 (16.2)	22.87 (12.45)	20.1 (9.63)	18.04 (6.17)

Values in parentheses are for an OMNI antenna pattern

Table 3.6: Signal to interference ratio results for the baseline aeronautical simulation runs with the crossed slot antenna and an omni antenna pattern.

Table 3.6). Furthermore, the values obtained with vertical polarization are considerably better than those obtained with horizontal polarization. With a perfectly circular omni antenna pattern, the S/I ratio tends to get worse as the elevation angle is reduced. However, the S/I value at 5° elevation angle is 0.73 dB higher than that at 10° elevation angle. For elevation angles above 10°, the S/I values obtained with circular polarization are better than those obtained with either horizontal or vertical polarization. However, at 5° elevation angle, the S/I ratio with vertical polarization is 3.75 dB greater than the value obtained with perfect circular polarization.

3.2.1.2 Doppler Spectra of Ocean Scatter Multipath For The Aeronautical Application

The second order statistics of the multipath process can be established by examining doppler spectra, which are simply power spectra of the multipath process. The power spectra are estimated using the Welch method of averaging periodograms (computed using FFT processing and incorporating a Hanning window) [21]. The doppler spectra for the aeronautical application are constructed by averaging individual spectrum estimates based on 128 signal samples.

The doppler spectra for an omni-directional antenna and SS4(ii) surface conditions are presented in Figures 3.16,

3.17, and 3.18 for elevation angles of 10° , 15° , and 31° , respectively. The most striking feature of these doppler spectra is the asymmetry (spectra not centred about 0 Hz), with doppler shifts in the negative direction being larger in frequency shift but lower in power than positive doppler shifts. The frequency axis of these doppler plots is normalized, each with a different frequency normalization factor. This tends to obscure the increase in doppler spectrum width that occurs with increasing elevation angle. To clarify this, effective one-sided -10 dB bandwidth, B_{10} , values extracted from the doppler plots are provided in Table 3.7. For an aircraft speed of 600 mph under level flight conditions, B_{10} doppler bandwidths in the neighbourhood of 30 Hz are reasonable at low elevation angles ($E=10^\circ$) and around 150 Hz at intermediate elevation angles ($E=31^\circ$).

Figure 3.18 also shows the effect of the crossed slot antenna on the doppler spectrum for $E=31^\circ$. A slight narrowing of the spectrum is evident. Figure 3.19 shows the doppler spectrum obtained with the crossed slot antenna at 15° elevation angle. Comparing this with the doppler spectrum for an omni antenna pattern at 15° elevation angle (see Figure 3.17), it is evident that the doppler spectrum is broadened somewhat.

In [8], it is claimed that theory predicts that the average doppler spectrum for level flight conditions has a symmetric Gaussian shape given by

Figure 3.16: Aeronautical doppler spectrum for an elevation angle of 10° with a perfectly circular omni antenna pattern.

Figure 3.17: Aeronautical doppler spectrum for an elevation angle of 15° with a perfectly circular omni antenna pattern.

Figure 3.18: Aeronautical doppler spectra for an elevation angle of 31° with circularly polarized crossed slot and omni antennas.

Figure 3.19: Aeronautical doppler spectrum for an elevation angle of 15° with a crossed slot antenna.

Elevation Angle	Normalized B_{10}	Sampling Frequency (Hz)	B_{10} (Hz)
10°	0.0314	3.736 v_x	0.117 v_x
15°	0.0314	5.838 v_x	0.183 v_x
31°	0.077	7.472 v_x	0.575 v_x

v_x is the horizontal aircraft speed in the direction of the sub-satellite point

Table 3.7: Extracted one-sided -10 dB bandwidths from the aeronautical doppler spectrum plots.

$$S(f) = \frac{|D_e r|^2}{\sqrt{\pi} B} \exp\left\{-\left(\frac{f}{B}\right)^2\right\} \quad (3.20)$$

where D_e is the divergence factor of the earth,
 r is the smooth earth reflection coefficient,
 f is the frequency in Hz relative to the specular component,

$$B = \left(\frac{2}{\lambda_c}\right) \sqrt{2} \sigma_z \sin E v_p \text{ (one-sided e}^{-1} \text{ bandwidth),}$$

λ_c is the carrier wavelength,

E is the elevation angle to the satellite,

σ_z is the rms surface slope, and

v_p is the horizontal airspeed.

A derivation of this general doppler spectrum shape is contained in [22]. Upon careful review, it is apparent that the following conditions must be satisfied for a Gaussian doppler spectrum shape

- (i) the receiving antenna pattern must be such that it receives energy from only a small region in the vicinity of the nominal specular reflection point, and/or
- (ii) the rms sea slope σ_z must be small and $E \gg \sigma_z$.

The high gain, narrow beam antenna used to collect surface scatter in the measurements conducted in [8] satisfies

the first assumption above, and is likely the reason why experimental data in [8] generally confirms the Gaussian doppler spectrum shape.

For the doppler spectra considered in this section, none of the fundamental assumptions upon which the Gaussian spectrum is based are satisfied. Consequently it is not surprising that the simulated doppler spectra do not have symmetric Gaussian shapes. It is encouraging to see that at higher elevation angles ($E=31^\circ$), the spectrum assumes a more symmetric shape around the peak. A detailed doppler analysis, which justifies the observed doppler spectrum characteristics, is contained in Appendix E. It is important to note that one does in fact obtain a Gaussian spectrum shape with the detailed approach presented in Appendix E, when the above mentioned assumptions are made. The major consequence of the doppler asymmetry is that the inphase and quadrature components of the surface scatter process are correlated and not independent.

3.2.2 Effect of Varying Sea State and Aircraft Altitude

In this section, the effect of sea state and aircraft altitude on ocean scatter are examined. Runs have been performed with sea state indices SS3 and SS5(1) for an elevation angle of 10° , with an aircraft altitude of

9.1 km. To examine the effect of aircraft altitude, a run was conducted with the aircraft altitude reduced to 5 km, with an elevation angle of 10° and SS4(ii) surface conditions.

Figure 3.20 illustrates the doppler spectra for SS3 and SS5(i) surface conditions. The differences are not as great as they appear, as different normalization factors are applicable for each case (i.e. $f_s = 5.735 v_x$ for SS3 while $f_s = 3.06 v_x$ for SS5(i)). When these different normalization factors are taken into account, it is apparent that there is very little difference in doppler spectrum width with sea state. The extracted B_{10} value from Figure 3.20 for SS3 is $B_{10} = 0.1 v_x$ which compares favourably with the value of $0.117 v_x$ for SS4(ii). The bandwidth ratio

$$\frac{B_{10}(\text{SS4(ii)})}{B_{10}(\text{SS3})} = 1.17,$$

is in very good agreement with the corresponding ratio of rms sea slopes, which happens to be 1.14 for this case. It is evident that the fading is quite slow.

The S/I ratio results for the sea state and aircraft altitude variation simulation runs for a crossed slot and an omni-directional antenna are summarized in Table 3.8.

Figure 3.20: Aeronautical doppler spectra for SS3 and SS5(1) surface conditions with an elevation angle of 10° and a perfectly circular omni antenna pattern.

These results clearly demonstrate the superiority of the power density method for establishing the signal to interference ratio. The S/I ratios obtained by averaging the instantaneous power samples (for circular polarization with phase mismatches $\theta_m = -45^\circ$ and $\theta_m = -90^\circ$) for the two different SS3 surface profiles are noticeably different (by as much as 1.6 dB). However, the differences between the S/I values obtained with the power density approach for the two surface profiles are relatively minor (less than 0.2 dB). This stability makes the power density estimation approach superior, especially for making comparisons to assess the impact of varying the sea state and antenna height.

The signal to interference ratio results presented in Table 3.8 for a crossed slot antenna pattern clearly demonstrate that one obtains superior S/I values with perfect circular polarization at an elevation angle of 10° . The S/I values are excellent; exceeding 22.5 dB. Even with a phase mismatch as severe as -90° , S/I values still exceed 14.4 dB.

The effect of sea state, for an aircraft altitude of 9.1 km, can be established by comparing the S/I results obtained with the power density approach in Table 3.8 with

Signal to Interference Ratios [dB] For The Following Polarization Types							
Sea State Index	Sea Surface Seed	Aircraft Altitude	Hori- zontal	Vertical	Copol $\theta_m = 0^\circ$	Copol $\theta_m = -45^\circ$	Copol $\theta_m = -90^\circ$
SS3	S1111	9.1 km	9.87 (2.39)	18.39 (11.25)	22.69 (11.26)	19.07 (8.17)	16.0 (3.68)
SS3	SZERO	9.1 km	9.70 (2.31)	18.38 (11.16)	22.59 (11.12)	17.59 (6.71)	14.42 (2.52)
SS5(i)	SZERO	9.1 km	10.57 (2.82)	18.91 (11.22)	23.0 (11.86)	19.68 (8.71)	16.52 (4.27)
SS4(ii)	SZERO	5.0 km	9.98 (2.54)	18.74 (11.11)	22.5 (11.41)	18.2 (7.61)	15.4 (3.46)

Values in parentheses are for an OMNI antenna pattern

Table 3.8: Signal to interference ratio results for the sea state and aircraft altitude variation simulation runs for an elevation angle of 10° and crossed slot and omni antenna patterns.

the corresponding results for an elevation angle of 10° and SS4(ii) surface conditions presented in Table 3.6. It is evident that the effect of sea state is relatively minor (maximum difference between SS3 and SS5(i) results is only 0.74 dB for an omni antenna pattern and 0.87 dB for a crossed slot antenna). A slight trend of increasing scatter power (decreasing S/I) as the surface becomes smoother (decreasing sea state index) is apparent.

By comparing the S/I values obtained with the power density approach in Table 3.6 for an elevation angle of 10° and an aircraft altitude of 9.1 km with those for an elevation angle of 10° and an aircraft altitude of 5 km in Table 3.8, one can assess the impact of varying the antenna height. The impact is relatively minor, with the maximum difference being only 0.31 dB and 0.43 dB for an omni and crossed slot antenna, respectively. It should be noted that the scatter power is worse for the lower aircraft altitude, and this is consistently the case for each signal polarization considered.

3.2.3 Effect of Aircraft Manoeuvres

In this section, the effect of having an aircraft climb and dive at an angle of 10° with respect to level flight, for

an elevation angle of 15° , SS4(ii) surface conditions, an aircraft altitude of 9.1 km, and a crossed slot antenna, is considered. The diving manoeuvre, which rotates the antenna pattern towards the surface, will tend to increase the level of interference, but will also tend to improve the direct path signal gain. Conversely, the climbing manoeuvre, which rotates the antenna pattern away from the surface, will tend to reduce the level of interference received while degrading the direct path signal gain. The improvements in direct path signal to noise ratio (SNR) for the 10° dive and the SNR degradation for the 10° climb are summarized below

Polarization	SNR Improvement [dB]	SNR Degradation [dB]
	For 10° Dive	For 10° Climb
Horizontal	4.80	3.27
Vertical	1.24	1.86
Copol ($\theta_m = 0^\circ$)	2.05	2.12
Copol ($\theta_m = -45^\circ$)	1.93	2.08
Copol ($\theta_m = -90^\circ$)	1.53	1.93

The signal to interference ratio results presented in Table 3.9 confirm the above predictions. It is interesting to

Signal to Interference Ratios dB For The Following Polarization Types						
Elevation Angle	Climb Angle	Hori- zontal	Vert- ical	Copol $\theta_m = 0^\circ$	Copol $\theta_m = -45^\circ$	Copol $\theta_m = -90^\circ$
15°	-10°	12.44	14.5	21.1	17.0	12.8
15°	+10°	16.2	18.5	21.72	20.28	17.8

Table 3.9: Signal to interference ratio results for
aircraft climbing and diving manoeuvres.

note that while the S/I values are roughly 4 dB worse for horizontal and vertical polarization for the diving case, the difference is less than 1 dB for perfect circular polarization. The worse case S/I value for circular polarization ($\theta_m = -90^\circ$) is 12.8 dB and occurs for the diving scenario.

3.2.4 Simulation Results for a More Complex Switched Antenna

The signal to interference ratios obtained with the low complexity crossed slot antenna are excellent (a worse case value of 12.8 dB was observed for a diving manoeuvre, while typical values are in the neighbourhood of 20 dB). However, the signal to interference ratio is not the only relevant performance measure for an antenna. Often the direct path signal to noise ratio obtained is of more importance.

The major disadvantage of the crossed slot antenna is the low gain at low elevation angles. The switched (diversity) antenna, with patterns illustrated in Figure 3.7, improves this situation. The signal to interference ratio simulation results obtained with the switched antenna are presented in Table 3.10.

Signal to Interference Ratios [dB] For The Following Polarization Types					
Elevation Angle	Horizontal	Vertical	Copol $\theta_m = 0^\circ$	Copol $\theta_m = -45^\circ$	Copol $\theta_m = -90^\circ$
31°	13.25	15.1	31.5	20.1	13.3
20°	9.4	13.5	22.5	15.1	9.33
15°	8.2	14.22	18.98	14.5	9.4
10°	7.92	16.81	16.79	13.6	9.4
5°	9.08	20.82	16.08	13.26	10.2

Table 3.10: Signal to interference ratio results for the aeronautical simulation runs conducted with the switched (diversity) antenna.

The basic S/I trends for horizontal and vertical polarization with elevation angle are similar to those discussed in Section 3.2.1.1 for the crossed slot antenna. The only difference is the relative magnitudes, which are several decibels lower with the switched antenna. As the elevation angle is reduced, the S/I values obtained with perfect circular polarization become worse. The degree of degradation with decreasing elevation angle is much more severe at high elevation angles. The signal to interference ratios obtained with the switched antenna are very good, exceeding 16 dB with perfect circular polarization for all cases considered.

A performance comparison of the switched and crossed slot antennas for circular polarization is provided in Table 3.11. At the higher elevation angles, the improved circularity of the switched antenna pattern actually results in both improved S/I and SNR ratios. However, at the lower elevation angles (10° and 5°), the S/I degradation is around 6.1 to 6.8 dB while the improvement in E/N_0 is around 2.1 to 2.4 dB.

For the relatively low interference levels obtained with the crossed slot antenna, an examination of the modulation analysis results presented in Figure 3.2-2 of [19] suggests that the performance improvement associated with the increase in direct path SNR of 2 dB more than compensates

Elevation Angle	Polarization	Relative Decrease In S/I [dB]	Relative Increase In E/N_0 [dB]
31°	Copol ($\theta_m = 0^\circ$)	-9.92	1.47
	Copol ($\theta_m = -90^\circ$)	3.7	0.93
20°	Copol ($\theta_m = 0^\circ$)	-2.06	1.53
	Copol ($\theta_m = -90^\circ$)	5.63	0.36
15°	Copol ($\theta_m = 0^\circ$)	2.29	1.84
	Copol ($\theta_m = -90^\circ$)	5.85	0.45
10°	Copol ($\theta_m = 0^\circ$)	6.11	2.12
	Copol ($\theta_m = -90^\circ$)	6.87	0.6
5°	Copol ($\theta_m = 0^\circ$)	6.79	2.37
	Copol ($\theta_m = -90^\circ$)	7.84	0.8

Table 3.11: Performance comparison of switched (diversity) antenna with low complexity crossed slot antenna.

for the degradation associated with the reduction of roughly 6 dB in signal to interference ratio obtained with the switched antenna.

The effect of satellite antenna polarization purity on the first order statistics obtained with the switched antenna is summarized in Table 3.12. The satellite antenna axial ratios considered are -6 dB, -3 dB, 3 dB, and 6 dB. Worst case results are obtained with an axial ratio of -6 dB. At an elevation angle of 5°, the signal to interference ratio is approximately 12.3 dB, which represents a degradation of roughly 3.8 dB, and the SNR degradation is around 0.6 dB.

3.2.5 Comparison of Simulation Results and Measurement Data

Measurement scenarios and data which would allow direct comparisons with the simulation model could not be found. The difficulty is that information vital for setting up corresponding simulation runs (i.e. antenna data and surface characteristics) is often missing or incomplete. In light of the observed lack of sensitivity of scatter power with sea state, when the surface is rough, the most critical item is the lack of accurate antenna pattern information. Given the significant impact of antenna characteristics on ocean scatter multipath, absolute agreement between measurement data and simulation results is not a realistic objective.

Switched Antenna Reference Values For a Perfectly Circular Satellite Antenna			Signal to Interference Ratios [dB] For The Following Satellite Antenna Axial Ratios				Direct Path SRR Degradation [dB] For The Following Satellite Antenna Axial Ratios			
Elevation Angle	Direct Path Gain [dB]	S/I [dB]	-6 dB	-3 dB	+3 dB	+6 dB	-6 dB	-3 dB	+3 dB	+6 dB
31°	2.5	31.3	21.4	25.4	29.7	24.5	0.6	0.2	0.1	0.3
20°	1.0	21.0	14.8	17.3	25.3	25.1	0.5	0.15	0.1	0.4
15°	0.2	18.9	13.9	16.0	22.4	24.8	0.5	0.15	0.1	0.4
10°	-0.55	16.5	12.75	14.4	19.0	22.3	0.55	0.15	0.1	0.4
5°	-1.4	15.1	12.3	13.6	16.7	18.4	0.6	0.2	0.05	0.3

Table 3.12: Effects of satellite antenna polarization ellipticity with the switched antenna system mounted on a Boeing 747 aircraft.

Reference [8] presents results of a flight test program involving a KC-135 jet and the synchronous ATS-5 L-band satellite. In these experiments the direct path signal was received by a steerable 15-dB gain quad-helix antenna and the sea scatter signal by an 8 element 13 dB gain, cross-dipole array pointing roughly 15° below horizon. No indication of surface conditions or antenna pattern data is provided. The normalized mean of the vertical and horizontal reflected powers obtained during this measurement program are presented in Figure 3.21. The normalization factor was the antenna gain in the direction of the nominal specular point, which is clearly not an exact treatment. However, for a relatively narrow beam receive antenna, it is not likely to be too seriously in error. The solid curves in Figure 3.21, corresponds to the product of the divergence factor of the earth, D_e , and the smooth surface reflection coefficients, which the experimental results of [8] lie close to for elevation angles greater than 11° . This agreement is not a coincidence, and is discussed below.

A detailed summary of a technique for determining the ratio of scatter power to direct path power for a Gaussian random surface with Gaussian correlation properties is presented in [22]. It employs scattering cross-section results derived in [1]. The final result is the integral equation [22]

Figure 3.21 Comparison of measured and simulated horizontal and vertical scatter power levels for an omni reference antenna.

$$\frac{P_S}{P_D} = |\Gamma_0|^2 \iint_{-\infty}^{\infty} W(X,Y) G_C(X,Y) dX dY \quad (3.21)$$

where

Γ_0 is the surface reflection coefficient appropriate for the specular reflection direction,
X and Y are cartesian co-ordinates on a flat earth,
H is the aircraft altitude,

$$W(X,Y) = \frac{\sigma(X,Y)}{4\pi(X^2 + Y^2 + H^2)},$$

$G_C(X,Y)$ is the surface illumination of the combined transmitter and receiver antenna gains normalized by the direct path gains,

$$\sigma(X,Y) = \frac{\exp\left\{-\frac{\tan^2 \alpha(X,Y)}{2\sigma_z^2}\right\}}{2\sigma_z^2 \cos^4[\alpha(X,Y)]},$$

$\alpha(X,Y)$ is the local grazing angle, and
 σ_z is the rms surface slope.

If one were to examine a large scattering region, one would likely obtain better results if one made Γ_0 a function of local grazing angle and placed it inside the integral.

This integral is not actually solved in [22], but approximations due to Mallinckrodt and Staras are used to derive a simple final result

$$\frac{P_S}{P_D}(\text{flat earth}) = |\Gamma_0|^2 G_C(-E) \quad (3.22)$$

If one considers the curvature of the earth, the above equation must be modified to incorporate the effect of the divergence factor of the earth, D_e , giving

$$\frac{P_S}{P_D}(\text{curved earth}) = |r_0|^2 |D_e|^2 G_C(-E). \quad (3.23)$$

For this simple result to apply the following conditions must be satisfied:

- (i) σ_z , must be small,
- (ii) $E \gg \sigma_z$, , and/or
- (iii) G_C is such that the majority of the scatter power arises from the vicinity of the nominal specular point.

Also shown on Figure 3.21 are the horizontal and vertical polarization scatter powers obtained with the aeronautical propagation simulation program with an omni-directional antenna. The agreement with the experimental results of [8] is quite good. Given the wide angular range over which multipath arises, and the fact that it is not discriminated against by an omni antenna pattern, the level of agreement between the simulation results and the simple approximation of (3.23) for elevation angles above 10° is truly remarkable. The agreement is not as good at very low elevation angles. This is not unexpected because this is where approximations used in the derivation of (3.23)

break down. Furthermore, this is where blockage and shadowing effects, which are incorporated in the simulation solution but not in (3.23), will have a significant impact.

Reference [23] is a companion paper to [8], presenting results obtained with the same measurement system. Signal to interference ratio results obtained for right hand circular polarization are presented in Table 3.13. The major problem with this data is the lack of antenna pattern information, crucial for a proper comparison with aeronautical propagation simulation results. The difference between the peak gains of the direct and scatter receiving antennas (2 dB) was subtracted from the measured values as a crude adjustment for what would be obtained with an omni antenna pattern.

For elevation angles in the vicinity of 15° , the estimated S/I(omni) values are 6.3 dB and 10.6 dB. Simulated S/I values for this case can vary from 2.9 - 12.4 dB. For elevation angles in the vicinity of 20° , simulated S/I values can vary from 1.7 - 13.4 dB, a range which includes the observed value of 9.4 dB.

Some useful measurement data is presented in [5], where sea surface conditions were accurately monitored. An indication of the gain characteristics of the crossed-slot

Elevation Angle	Aircraft Altitude (feet)	Estimated Rms Waveheight (meters)	Approximate Sea State Index	Measured S/I [dB]	Estimated S/I [dB] For An Omni Reference
14.5°	27,000	0.39	SS3	12.6	10.6
15.3°	26,000	1.0	SS5(ii)	8.3	6.3
18.1°	33,000	0.55	SS4(i)	11.4	9.4
20.3°	27,000	0.11	SS2(i)	11.4	9.4
26.1°	29,000	N/A	N/A	9.75	7.75

Table 3.13: S/I measurement results for RHC polarization from [23].

dipole used in the measurements is provided in Figure 3.22, which shows the amplitude response to a rotating dipole source. The measurement scenario employed a carrier wavelength of 0.195 m, a nominal elevation angle of 31° , an aircraft altitude of 30,000 ft (≈ 9.1 km), and RHC polarization. Two basic measurement results for the crossed-slot dipole antenna are summarized below

Significant Waveheight (meters)	Approximate Rms Waveheight (meters)	Approximate Sea State Index	S/I [dB]
2.36	0.59	SS4(ii)	16.0
1.37	0.34	SS3	15.5

Simulation runs were not performed for these scenarios because of the lack of accurate antenna pattern information. Instead, the effects of the antenna were approximately factored out, assuming that the scatter power effectively arrives from a single direction. The direction considered in the analysis was -20° , because the power density curve in Figure 3.15 (c) for $E=31^\circ$ tends to peak for reflection angles in the vicinity of 20° . Also included in the analysis is the effect of surface

Peak Gain At 35° Elevation = 5 dB_1
Gain At Horizon = 1.5 dB_1

Figure 3.22: Radiation pattern (amplitude response to a rotating dipole source) of the aircraft slot-dipole antennas used during the measurement program reported in [5].

reflection coefficients and a realistic axial ratio of 3 dB is assumed. With this antenna factorization procedure, it can be shown that the measured S/I value of 16 dB for SS4(ii) surface conditions corresponds to effective horizontal and vertical polarization scatter power levels for an omni reference of $P_H = -0.75$ dB and $P_V = -4.07$ dB. These values are in reasonable agreement (within 0.5 dB) with those obtained with the simulation program for this scenario, where $P_H = -1.2$ dB and $P_V = -3.9$ dB.

For the most part, the measurement data examined is quite supportive of the aeronautical simulation model, especially the normalized horizontal and vertical polarization scatter power results versus elevation angle from [8].

3.3 Maritime Simulation Results

This section presents the simulation results obtained for the maritime application. Simulation scenarios which closely resemble conditions during the measurement program discussed in [6] have been considered. Additional simulation scenarios have been established to investigate the effect of sea state and antenna height on maritime multipath. The appropriate vessel parameters extracted from [6] are

length $L = 69$ m

maximum velocity $u_{\max} = 13.3 \text{ knots} = 6.85 \text{ m/sec}$,

antenna height above waterline $z_0 = 20 \text{ m}$, and

the main deck is approximately 6.3 m above the vessel's waterline.

Before discussing the simulation scenarios and results, a summary of the relevant measurement results presented in [6] is provided.

3.3.1 Summary of Maritime Measurement Results

The measurements performed by DFVLR [6] were under typical 'rough sea conditions' (approximately 2 m crest to trough waveheights were encountered). Measured signal to interference ratios (S/I), fading bandwidths, and fade duration characteristics are reported in [6].

The measured signal to interference ratios versus elevation angle presented in [6] are summarized in Table 3.14, with the specific antennas of interest identified. For the DFVLR3 and DFVLR5 antennas, it is evident that signal to interference ratios in the neighbourhood of 8 - 9 dB are typical around 5° elevation angle, while the typical range is 10 - 14 dB around 20° elevation angle. The trend

Elevation Angle Region	Signal to Interference Ratio in dB For The Following Antennas		
	DFVLR C3	DFVLR C5	*Other Standard-C Antennas
3.8° to 4°	8.6 to 9.1	7.9	9.7
7.1° to 7.9°	8.6	7.9	10.1
9.8° to 10.3°	9.7	**	10.8 to 12.1
16.3° to 16.8°	10.0	10.3	13.7 to 14.9
18.8° to 19.3°	10.2	12.2	11.5 to 13.8
22.1° to 22.4°	**	14.4	14.3
25.9° to 26.9°	10.7	15.0	13.8 to 15

*Results applicable for C11 and C14 antennas from [6]

**Special event destroyed results [6]

Table 3.14: Summary of maritime signal to interference ratio measurement results from [6].

is for the signal to interference ratio to increase as the elevation angle increases.

The measurement results presented in [6] indicate that the fading bandwidth increases with elevation angle, and 20 dB bandwidths are in the neighbourhood of 3 to 8 Hz. However, it must be noted that the spectra computed in [6] are actually the spectra of the envelopes of the received signal. Preferably, one should compute the spectrum of the complex baseband multipath signal, formed from the individual inphase and quadrature components. Two problems associated with using received signal envelopes for spectrum evaluation are

- (1) the process of forming the envelope is inherently non-linear, thereby introducing spectral spreading, and
- (2) it generates a real signal (which must have a symmetric spectrum) and consequently destroys any spectrum asymmetry or spectrum shifts from the nominal direct path doppler shift (which can also introduce an apparent bandwidth expansion).

The simulated maritime doppler spectra presented in this section were computed using the complex baseband multipath signal. The spectrum is estimated using the Welch method

of averaging periodograms (computed using FFT processing and incorporating a Hanning window [21]).

In [6], it is shown that fade durations decrease with increasing elevation angle. Given the nature of the spectral characteristics with elevation angle, this is not unexpected. Unfortunately, average fade durations are not presented. However, for a 5 dB fade margin, the probability that a fade duration exceeds a specified threshold value is considered. It is claimed that the threshold values of 0.45 sec and 0.65 sec are exceeded only 0.1% of the time for the DFVLR3 and DFVLR5 antennas, respectively. Characterizing fade duration statistics for exceedance percentages as low as 0.1%, with any degree of accuracy, requires a voluminous amount of propagation data; far more than is practical with the computationally intensive maritime propagation simulation program.

Some very valuable information was provided by D. R. Mullins of INMARSAT in a telex entitled "Characteristics of (Maritime) Multipath Propagation Effects As Deduced From the Results of Numerous Experimental Measurements" [7]. The major points are summarized below

- (i) 3 dB fading bandwidths typically range from 0.1 Hz to 1.0 Hz,

- (ii) mean fade durations typically range from 0.03 to 0.15 sec at low elevation angles for a 5 dB fade margin,
- (iii) signal to interference ratios do not vary significantly with sea state (when the surface is rough and not calm) or antenna height,
- (iv) the tendency is for the signal to interference ratio to increase with increasing elevation angle, and
- (v) under typical rough sea conditions, sea reflected multipath has a negligible specular component.

In Section 3.1.3, it has already been established that simulation results support this latter claim.

3.3.2 Maritime Simulation Scenarios Considered

Four basic simulation scenarios have been set up to investigate the behaviour of sea reflected multipath. Attention has been restricted to relatively low elevation angles ($E=5^\circ$, 10° , 15° , and 20°) because ocean surface scatter at high elevation angles is not expected to be particularly severe with RHC polarization. This is because the polarization sense reversal introduced by reflection from the ocean surface, which is discriminated against by the antenna, is much more pronounced at higher elevation angles. Attention was restricted to L-band applications, with a carrier wavelength of 0.2 m used in all the

3

MICRO

simulation runs. Given the relatively slow fading nature of the scatter process and the relatively slow nature of sea induced pitching and bobbing motion, it is important that one perform relatively long runs to encompass several fading cycles. Conducting long runs at low elevation angles is extremely computationally demanding, requiring execution times on the order of 2 hours at 20° elevation angle and around 6 hours at 5° elevation angle on a high speed FPS-164 array processor (which is over an order of magnitude faster than typical minicomputers such as an APOLLO DN-320 or a DEC VAX-11/750).

The major purposes of the first set of runs are to show that antenna stabilization is not essential for low gain Standard-C antennas and to gain insight into the behaviour of the S/I ratio obtained with the DFVLR3 antenna pattern. The distinguishing parameters for the antenna stabilization runs are provided in Table 3.15. The purpose of the last two runs is to quantify the effect of using the simple ratio of Fresnel zone near field divergence factor calculation procedure instead of the more exact approach presented in Appendix C. Ship speeds of 7 m/sec (with the waves), which is representative of the maximum velocity of the vessel, and 5 m/sec (against the waves) are considered. The corresponding relative velocities of the vessel with respect to the average wave speed are roughly 1 m/sec when the vessel moves with the waves and 11 m/sec with the

Scenario	Elevation Angle	Ship Speed (m/s)	Ship Direction	Antenna Stabilization	Near-Field Divergence Fac. Calc. Approach	Carrier Wavelength $\lambda_c = 0.2$ m Ship Length $L = 69$ m Nominal Antenna Height $z_o = 20$ m Number of Simulated Points $N = 4096$ Update Interval $\Delta t = 0.05$ sec SS4(ii) Surface Conditions DFVLRCS Antenna
ANTENNAS1	20°	7	with waves	No	Exact	
ANTENNAS2	20°	7	with waves	Yes	Exact	
ANTENNAS3	20°	5	against waves	No	Exact	
ANTENNAS4	20°	5	against waves	Yes	Exact	
DIVERG1	10°	7	with waves	Yes	Ratio of Fresnel Zones	
DIVERG2	10°	7	with waves	Yes	Exact	

Table 3.15: Maritime simulation scenarios for Standard-C antenna stabilization runs.

Scenario	Elevation Angle	Sea State	Carrier Wavelength $\lambda_c = 0.2$ m Ship Length $L = 69$ m Nominal Antenna Height $z_o = 20$ m Number of Simulated Points $N = 4096$ Update Interval $\Delta t = 0.05$ sec Ship Speed $v_s = 7$ m/sec (with waves) DFVLRCS Antenna
SEASTATE1	10°	SS4(ii)	
SEASTATE2	10°	SS3	
SEASTATE3	10°	SS5(i)	
SEASTATE4	20°	SS4(ii)	
SEASTATE5	20°	SS3	
SEASTATE6	20°	SS5(i)	

Table 3.16: Maritime simulation scenarios examining the effect of sea state.

vessel going against the waves (as the average wave speed for SS4(ii) is on the order of 6 m/sec [see Appendix A]).

The second set of runs investigate the influence of sea state on sea reflected multipath. In this case, the DFVLR5 antenna pattern is used, and the vessel is moving with the waves at a speed of 7 m/sec, with a gravity stabilized antenna platform. The distinguishing parameters for the sea state runs are summarized in Table 3.16.

The third set of runs investigate the impact of varying the nominal height of the antenna above the waterline on sea reflected multipath. The antenna heights considered were 10 m, 20 m, and 30 m with elevation angles of 10° and 20°. For these runs, the QUAD antenna pattern is used, and the ship moves against the waves with a velocity of 5 m/sec with a gravity stabilized antenna platform. The distinguishing parameters for these runs are summarized in Table 3.17.

The fourth simulation scenario completes the S/I characterization versus elevation angle for each of the Standard-C antennas with a nominal antenna height of 20 m. Table 3.18 contains a summary of the relevant parameters. Different random surface profiles have been considered for each run. This provides the opportunity to examine the S/I variation with different surface profiles typical of

Scenario	Nominal Antenna Height	Elevation Angle	Carrier Wavelength $\lambda_c = 0.2$ m Ship Length $L = 69$ m Number of Simulated Points $N = 4096$ Update Interval $\Delta t = 0.05$ sec SS4(ii) Surface Conditions Ship Speed $v_s = 5$ m/sec (against waves) QUAD Antenna
VHGT10E10	10 m	10°	
VHGT20E10	20 m	10°	
VHGT30E10	30 m	10°	
VHGT10E20	10 m	20°	
VHGT20E20	20 m	20°	
VHGT30E20	30 m	20°	

Table 3.17: Maritime simulation scenarios examining the effect of varying the antenna height.

Scenario	Elevation Angle	Surface Descriptor Seed	Antenna Pattern	Carrier Wavelength $\lambda_c = 0.2$ m Ship Length $L = 69$ m Ship Speed $v_s = 5$ m/sec (against waves) SS4(ii) Surface Conditions Nominal Antenna Height $z_o = 20$ m Number of Simulated Points $N = 4096$ Update Interval $\Delta t = 0.05$ sec
ELEV1	5°	S1111	DFVLR3	
ELEV2	15°	SZFR0	DFVLR3	
ELEV3	5°	S9199	DFVLR5	
ELEV4	15°	S3841	DFVLR5	
ELEV5	5°	S7777	QUAD	
ELEV6	15°	S4533	QUAD	

Table 3.18: Maritime simulation scenarios that complete the multipath characterization versus elevation angle.

SS4(ii) surface conditions when an omni-directional antenna pattern is used.

3.3.3 Signal to Interference Ratio Results For Shipborne Standard-C Antennas

To give a better appreciation for the behaviour of ocean scatter with signal polarization, signal to interference ratios are determined for horizontal and vertical polarization as well as with RHC polarization. Because of the sensitivity of the multipath output to antenna deviations from perfect RHC polarization, signal to interference ratios are also evaluated for antenna polarization ratios consistent with an axial ratio of 5.5 dB. The signal to interference ratio characteristics are also established for omni-directional antenna characteristics.

3.3.3.1 S/I Ratios For The Antenna Stabilization Runs

The signal to interference ratios for the antenna stabilization runs, with the DFVLR3 antenna pattern, are presented in Table 3.19. As anticipated, antenna stabilization has no significant influence on the signal to interference ratio with low gain Standard-C antenna patterns. At 20° elevation angle, the signal to interference ratio is slightly higher when the ship is

				Signal to Interference Ratios [dB] For The Following Polarization Types		
Scenario	Elevation Angle	Ship Velocity (m/sec)	Antenna Stabilization	Horizontal	Vertical	Copol $F_R=0$ dB $\theta_m=0^\circ$
ANTENNAS1	20°	+7	No	2.989	9.138	14.112
ANTENNAS2	20°	+7	Yes	2.991	9.139	14.115
ANTENNAS3	20°	-5	No	2.628	9.251	13.367
ANTENNAS4	20°	-5	Yes	2.661	9.284	13.401
DIVERG1 (Ratio of Fresnel Zones Near Field Calculation Approach)	10°	+7	Yes	4.09	14.51	11.76
DIVERG2 (More Exact Near Field Calculation Approach)	10°	+7	Yes	4.07	14.43	11.75

Table 3.19: Maritime signal to interference ratio results for the antenna stabilization simulation runs with the DFVLRCS antenna pattern.

travelling with the waves (slow fading case), but the difference is not that significant. The results for the DIVERG1 and DIVERG2 simulation runs indicate that there is little difference between the results obtained with the two near field divergence factor evaluation procedures.

From Table 3.14, the measured S/I values with the DFVLR3 antenna for $E=10^\circ$ and $E=20^\circ$ are in the neighbourhood of 9.7 dB and 10.2 dB, respectively. The simulated values with this antenna for perfect circular polarization are considerably higher than these measured values. Signal to interference ratios for polarization ratios consistent with an axial ratio of 5.5 dB with this antenna pattern are presented in Table 3.20. Note that the values obtained for $E_R = -1$ dB and $\theta_m = -33.5^\circ$ are in reasonable agreement with the measured values.

At 10° elevation angle, it is apparent that using vertical polarization is preferable to circular polarization (the S/I ratio is roughly 2.75 dB better than that obtained with perfect circular polarization, but is more likely to be around 4.8 dB better when typical axial ratios of Standard-C antennas are taken into account). Given the surface discrimination against vertical polarization, this is not that surprizing. This suggests that tapering the

			Signal to Interference Ratios [dB] For The Following Polarization Types		
Scenario	Elevation Angle	Ship Velocity (m/sec)	Copol $E_R = -5.5$ dB $\theta_m = 0^\circ$	Copol $E_R = -2$ dB $\theta_m = -31.75^\circ$	Copol $E_R = -1$ dB $\theta_m = -33.5^\circ$
ANTENNAS2	20°	+7	9.11	9.85	10.31
ANTENNAS4	20°	-5	8.62	9.37	9.84
DIVERG2	10°	+7	8.69	9.29	9.66

Table 3.20: Maritime signal to interference ratio results for the antenna stabilization simulation runs for typical antenna polarization ratios consistent with an axial ratio of 5.5 dB for the DFVLR3 antenna.

			Signal to Interference Ratios [dB] For The Following Polarization Types			
Scenario	Elevation Angle	Ship Velocity (m/sec)	Near Field Calc. Approach	Horizontal	Vertical	Copol $E_R = 0$ dB, $\theta_m = 0^\circ$
ANTENNAS2	20°	+7	Exact	0.07	5.67	11.73
ANTENNAS4	20°	-5	Exact	0.04	6.12	11.23
DIVERG1	10°	+7	Ratio of Fresnel Zones	1.67	11.18	9.76
DIVERG2	10°	+7	Exact	1.63	11.07	9.75

Table 3.21: Maritime signal to interference ratio results for the antenna stabilization simulation runs with an omni antenna pattern.

polarization of the antenna to vertical at low elevation angles would be beneficial. At 20° elevation angle, RHC polarization is superior to vertical polarization. It is clear that one does not want a horizontally polarized receive antenna at either elevation angle.

The signal to interference ratios for an omni antenna pattern for this case are presented in Table 3.21. The same polarization dependencies are evident here, and as one would expect, the signal to interference ratios are worse. One very interesting observation is the degradation in signal to interference ratio for horizontal polarization with increasing elevation angle.

3.3.3.2 Behaviour of S/I Ratio With Sea State

Table 3.22 presents the S/I results obtained for SS3, SS4(ii), and SS5(i) surface conditions with the DFVLRCS antenna pattern and an omni antenna pattern. An examination of these results indicate that at 20° elevation angle, the signal to interference ratio is not particularly sensitive to sea state. This behaviour is in agreement with that observed for measurement results [7]. However, at 10° elevation angle, there seems to be a clear trend of increasing scatter power (lower S/I ratios) as the surface becomes smoother (decreasing sea state index). As one goes from SS5(i) to SS4(ii), the S/I ratio is reduced by

J

Signal to Interference Ratios [dB] For The Following Polarization Types					
Scenario	Elevation Angle	Sea State	Horizontal	Vertical	Copol $E_R = 0 \text{ dB}$, $\theta_m = 0^\circ$
SEASTATE1	10°	SS4(ii)	5.40 (1.63)	15.87 (11.07)	12.99 (9.75)
SFASTATE2	10°	SS3	3.78 (0.01)	14.53 (9.78)	11.52 (8.2)
SFASTATE3	10°	SS5(i)	6.19 (2.35)	16.25 (11.39)	13.80 (10.55)
SEASTATE4	20°	SS4(ii)	6.30 (0.07)	12.44 (5.67)	17.41 (11.73)
SEASTATE5	20°	SS3	6.64 (0.39)	12.82 (6.06)	17.80 (12.05)
SFASTATE6	20°	SS5(i)	6.68 (0.49)	12.98 (6.15)	17.63 (12.05)

...Values in parentheses are for an OMNI antenna pattern

Table 3.22: Maritime signal to interference ratio results for the sea state scenarios with the DEVLRC5 antenna pattern and an omni antenna pattern.

approximately 0.8 dB, while the reduction is approximately 1.5 dB when the surface conditions change from SS4(ii) to SS3. It is evident that vertical polarization provides better signal to interference ratios than circular polarization at low elevation angles (i.e. $E=10^\circ$).

From Table 3.14, the measured S/I values with the DFVLRCS antenna for elevation angles around 10° are in the neighbourhood of 7.9 to 10.3 dB, while around 20° , values around 12.2 to 14.4 dB were obtained. The simulated values with this antenna for perfect circular polarization, presented in Table 3.22, are considerably higher than these measured values. Signal to interference ratios for polarization ratios consistent with an axial ratio of 5.5 dB for this antenna pattern are presented in Table 3.23. Values obtained with $E_R = -5.5$ dB and $\theta_m = 0^\circ$ as well as $E_R = -3$ dB and $\theta_m = -28.6^\circ$ are in reasonable agreement with the above measurement values.

3.3.3.3 Behaviour of S/I Ratio With Antenna Height

The signal to interference ratio results for nominal antenna heights of 10 m, 20 m, and 30 m with the QUAD antenna pattern and an omni antenna pattern are summarized in Table 3.24. An examination of these results reveal that the signal to interference ratio is not particularly

Signal to Interference Ratios [dB] For The Following Polarization Types					
Scenario	Elevation Angle	Sea State	Copol $E_R = -5.5$ dB $\theta_m = 0^\circ$	Copol $E_R = -3$ dB $\theta_m = -28.6^\circ$	Copol $E_R = -1$ dB $\theta_m = -33.5^\circ$
SEASTATE1	10°	SS4(ii)	9.98	10.27	10.94
SEASTATE2	10°	SS3	8.42	8.75	9.46
SEASTATE3	10°	SS5(i)	10.79	11.09	11.76
SEASTATE4	20°	SS4(ii)	12.42	12.78	13.61
SEASTATE5	20°	SS3	12.77	13.16	14.01
SEASTATE6	20°	SS5(i)	12.73	13.14	13.99

Table 3.23: Maritime signal to interference ratio results for the sea state simulation runs for antenna polarization ratios consistent with an axial ratio of 5.5 dB for the DEVLPCS antenna.

Signal to Interference Ratios [dB] For The Following Polarization Types					
Scenario	Elevation Angle	Antenna Height	Horizontal	Vertical	Oppol $E_R = 0$ dB, $\theta_m = 0^\circ$
VHGT10E10	10°	10 m	6.56 (0.32)	19.94 (8.76)	13.18 (8.94)
VHGT20E10	10°	20 m	6.38 (-0.02)	19.34 (8.39)	13.13 (8.66)
VHGT30E10	10°	30 m	6.54 (-0.36)	19.35 (7.62)	13.41 (8.61)
VHGT10E20	20°	10 m	4.40 (-0.55)	12.50 (4.8)	14.16 (11.18)
VHGT20E20	20°	20 m	4.61 (-0.57)	12.71 (4.78)	14.41 (11.19)
VHGT30E20	20°	30 m	5.09 (-0.40)	13.19 (4.74)	14.88 (11.57)

...Values in parentheses are for an OMNI antenna pattern

Table 3.24: Maritime signal to interference ratio results for the antenna height variation simulation runs with the QUAD antenna pattern and an omni antenna pattern.

sensitive to antenna height, a fact that was also observed from measurement results [7]. The differences for perfect RHC polarization with an omni antenna pattern are less than 0.4 dB.

The signal to interference ratios of Table 3.25 for a QUAD antenna are in reasonable agreement with the measurement results presented in Table 3.14 for other Standard-C antennas (when an axial ratio of 5.5 dB is applied to perturb the assumed perfect circularity of the QUAD pattern). At 10° elevation angle, the most appropriate polarization ratio parameters are $E_R = -2$ dB and $\theta_m = -31.75^\circ$, and at 20° elevation angle, $E_R = 0$ dB and $\theta_m = -34.1^\circ$ gives the best agreement with the range of measured values. For $E=20^\circ$, the omni antenna pattern results presented in Table 3.24, for vessel motion against the waves, are in reasonable agreement with those for SS4(ii) surface conditions presented in Table 3.22, for vessel motion with the waves (which results in a much slower fading process).

3.3.3.4 Completing The S/I Ratio Characterization Versus Elevation Angle For Shipborne Standard-C Antennas

The last set of simulation scenarios characterize maritime ocean scatter at elevation angles of 5° and 15° for SS4(i) surface conditions and a nominal antenna height of 20 m.

Signal to Interference Ratios [dB] For The Following Polarization Types					
Scenario	Elevation Angle	Antenna Height	Copol $E_R = -5.5 \text{ dB}$ $\theta_m = 0^\circ$	Copol $E_R = -2 \text{ dB}$ $\theta_m = -31.75^\circ$	Copol $E_R = 0 \text{ dB}$ $\theta_m = -34.1^\circ$
VHGT1OE10	10°	10 m	10.62	11.21	11.97
VHGT2OE10	10°	20 m	10.51	11.09	11.86
VHGT3OE10	10°	30 m	10.75	11.34	12.12
VHGT1OE20	20°	10 m	9.94	10.68	11.69
VHGT2OE20	20°	20 m	10.16	10.91	11.92
VHGT3OE20	20°	30 m	10.63	11.37	12.39

Table 3.25: Maritime signal to interference ratio results for the antenna height variation simulation runs for antenna polarization ratios consistent with an axial ratio of 5.5 dB for the QUAD antenna.

The S/I results with the DFVLR3, DFVLR5, QUAD, and omni antenna patterns are summarized in Table 3.26. The results for an omni antenna pattern indicate that the variation in signal to interference ratio for different representative SS4(ii) surface conditions is relatively minor. The observed fluctuations for circular polarization are less than 0.25 dB and 0.45 dB at 5° and 15° elevation angles, respectively. While the S/I values obtained with the DFVLR3, DFVLR5, and QUAD antennas for vertical and RHC polarization are comparable at 15° elevation angle, at low elevation angles ($E=5^\circ$), vertical polarization has a considerable advantage (between 4.5 - 6.7 dB) over circular polarization.

For elevation angles around 5°, measured values between 8.6 and 9.1 dB are appropriate for the DFVLR3 antenna pattern, and values for the DFVLR5 antenna pattern are around 8 dB. In the vicinity of 15° elevation angle, measured values of 10 to 10.3 dB are appropriate for both the DFVLR3 and DFVLR5 antenna patterns. Signal to interference ratios presented in Table 3.26 for perfect circular polarization are somewhat higher than these measured values. As is evident from Table 3.27, much more realistic values are obtained with $E_R = -1$ dB and $\theta_m = -33.5^\circ$ with the DFVLR3 antenna, and $E_R = -5.5$ dB and $\theta_m = 0^\circ$ with the DFVLR5 antenna, where the agreement is very good at $E=15^\circ$.

Signal to Interference Ratios dB For The Following Polarization Types					
Scenario	Sea Surface Descriptor Seed	Elev- tion Angle	Horizontal	Vertical	Copol $E_R = 0$ dB, $\theta_m = 0^\circ$
ELEV1	S1111 (DFVLRC3 antenna)	5°	4.27 (2.28)	15.28 (12.39)	10.71 (9.29)
ELEV2	SZERO (DFVLRC3 antenna)	15°	2.61 (-0.32)	10.73 (6.94)	11.97 (9.58)
ELEV3	S9199 (DFVLRC5 antenna)	5°	5.25 (2.33)	16.31 (12.67)	11.37 (9.04)
ELEV4	S3841 (DFVLRC5 antenna)	15°	5.11 (0.28)	13.5 (7.78)	14.32 (10.03)
ELEV5	S7777 (QUAD antenna)	5°	9.0 (2.2)	20.8 (12.61)	14.1 (9.13)
ELEV6	S4533 (QUAD antenna)	15°	5.48 (-0.14)	15.76 (6.99)	13.85 (9.90)

...Values in parentheses are for an OMNI antenna pattern

Table 3.26: Maritime signal to interference ratio results for simulation runs conducted at 5° and 15° elevation angle with DFVLRC3, DFVLRC5, QUAD, and omni antenna patterns.

		Signal to Interference Ratios [dB] For the Following Polarization Types		
Scenario	Elevation Angle	Copol $E_R = -5.5$ dB $\theta_m = 0^\circ$	Copol $E_R = -2$ dB $\theta_m = -31.75^\circ$	Copol $E_R = -1$ dB $\theta_m = -33.5^\circ$
ELEV1	5°	8.28	8.84	9.17
ELEV2	15°	8.01	8.72	9.15

(i) DFVLR3 antenna

		Signal to Interference Ratios [dB] For the Following Polarization Types		
Scenario	Elevation Angle	Copol $E_R = -5.5$ dB $\theta_m = 0^\circ$	Copol $E_R = -3$ dB $\theta_m = -28.6^\circ$	Copol $E_R = -1$ dB $\theta_m = -33.5^\circ$
ELEV3	5°	9.09	9.35	9.92
ELEV4	15°	10.43	10.8	11.58

(ii) DFVLR5 antenna

		Signal to Interference Ratios [dB] For the Following Polarization Types		
Scenario	Elevation Angle	Copol $E_R = -5.5$ dB $\theta_m = 0^\circ$	Copol $E_R = -2$ dB $\theta_m = -31.75^\circ$	Copol $E_R = 0$ dB $\theta_m = -34.1^\circ$
ELEV5	5°	12.25	12.72	13.32
ELEV6	15°	10.40	11.07	11.99

(iii) QUAD antenna

Table 3.27: Maritime signal to interference ratio results for simulation runs conducted at 5° and 15° elevation angle for antenna polarization ratios consistent with an axial ratio of 5.5 dB for the DFVLR3, DFVLR5, and QUAD antennas.

However, at $E=5^\circ$, the simulated S/I value with the DFVLRC5 antenna is roughly 1 dB higher than measured values.

3.3.3.5 Summary of Maritime Signal to Interference Ratio Results

The signal to interference ratio results versus elevation angle for the DFVLRC3, DFVLRC5, and QUAD antenna patterns are summarized in Table 3.28. The simulated circular polarization values for the DFVLRC3 and DFVLRC5 antenna patterns are in good agreement with the measurement values [6] presented in Table 3.14; the major exception being at 5° elevation angle, where the simulated S/I values are roughly 1 dB higher than measured values with the DFVLRC5 antenna. It could well be that the ellipticity of this antenna pattern below the horizon is actually worse than that used to analyze the simulated propagation data. With these antenna patterns, the basic trend is for the S/I ratio obtained with circular polarization to increase as the elevation angle increases. With the exception of the high S/I value at 5° elevation angle, the S/I results obtained with the QUAD antenna are in good agreement with the measurement results presented in Table 3.14 for other Standard-C antennas. For the most part, the simulation results obtained are in reasonable agreement with measurement results [6], [7], exhibiting the same

Elevation Angle	Polarization	Antenna		
		DFVLRC3	DFVLRC5	QUAD
5°	Horizontal	4.3	5.3	9.0
	Vertical	15.3	16.3	20.8
	Copol	9.2	9.1	12.3
10°	Horizontal	4.1	3.8-6.2	6.4-6.6
	Vertical	14.4	14.5-16.3	19.3-19.9
	Copol	9.7	8.4-10.8	11.1-11.3
15°	Horizontal	2.6	5.1	5.5
	Vertical	10.7	13.5	15.8
	Copol	9.2	10.4	12.0
20°	Horizontal	2.6-3.0	6.3-6.7	4.4-5.1
	Vertical	9.1-9.3	12.4-13.0	12.5-13.2
	Copol	9.8-10.3	12.4-12.8	11.7-12.4

Antenna Ellipticities Used to Derive Copol Values

DFVLRC3 + $E_R = -1$ dB, $\theta_m = -33.5^\circ$

DFVLRC5 + $E_R = -5.5$ dB, $\theta_m = 0^\circ$

QUAD + $E = 5^\circ$ $E = 10^\circ$ $E = 15^\circ$ and 20°
 $E_R = -5.5$ dB $E_R = -2$ dB $E_R = 0$ dB
 $\theta_m = 0^\circ$ $\theta_m = -31.75^\circ$ $\theta_m = -34.1^\circ$

Table 3.28: Summary of the behaviour of S/I versus elevation angle for shipborne Standard-C antennas.

fundamental trends with elevation angle, antenna height, and to some extent, sea state.

In an attempt to further quantify the effect of sea state, runs were conducted at $E=10^\circ$ and $E=20^\circ$ with SS3 surface conditions and a vessel going against the waves at 5 m/sec with a QUAD antenna pattern. The corresponding SS4(ii) and SS3 signal to interference ratios are summarized below.

Elevation Angle	Antenna Height	S/I [dB] For Perfect Circular Polarization With The QUAD Antenna And Sea State Index	
		SS4(ii)	SS3
10°	20 m	13.1	12.45
20°	20 m	14.4	14.5

The S/I ratio at 20° elevation angle is not very sensitive to sea state, which confirms earlier observations. However, at 10° elevation angle, the S/I ratio is indeed worse for smoother surface conditions.

The behaviour of the signal to interference ratio with vertical polarization suggests that tapering the polarization characteristics of the shipboard antenna just below the horizon (low reflection angle region) towards vertical would be quite beneficial. At low elevation angles, the S/I improvement can be as great as 4 to 6 dB. This behaviour with vertical polarization is consistent with the surface discrimination against vertical polarization that occurs at low grazing angles.

In [9], results of measurements conducted by Engels et. al. (reference [2-12] of [9]) are summarized. The major conclusions were that

- (1) the multipath signal power appears to come from the horizon rather than from the nominal specular reflection direction ($\theta = -E$, where E is the elevation angle to the satellite), and
- (2) the effective signal to interference ratio for an omni-directional reference antenna tends to be fairly constant at '8 dB' regardless of elevation angle.

The majority of measurements upon which the latter claim was based were performed at elevation angles around 8° and

17°. However, other results presented in [9] exhibit the trend of increasing S/I ratio with increasing elevation angle, which tends to contradict the above claim. For example, an examination of Figure 4-44 of [9] suggests that effective signal to interference ratios for an omni antenna pattern are in the neighbourhood of 6 - 8 dB for $E = 5^\circ$, 8 - 9.5 dB for $E = 10^\circ$, and 9 to 13 dB for elevation angles around 15° .

Simulation results obtained with a perfectly circular omni antenna are summarized below.

Elevation Angle	S/I (omni) [dB] For Perfect Circular Polarization
5°	9.0 - 9.3
10°	8.2 - 10.6
15°	9.6 - 10.0
20°	11.2 - 12.1

The agreement with the above mentioned values from [9] is very good for elevation angles of 10° and 15° . At 5° elevation angle, simulated values are somewhat high.

To further characterize the nature of maritime multipath, scatter power density versus reflection angle curves are

generated by the simulation program for horizontal and vertical polarization (with an omni antenna to eliminate antenna specific effects). Sample curves are presented in Figures 3.23, 3.24 and 3.25 for elevation angles of 5° , 10° , and 15° , respectively. The surface discrimination against vertical polarization at low reflection angles is clearly evident in these figures. It is also evident that the scatter power arises from a relatively wide reflection angle region. This behaviour casts doubt on the accuracy of simple multipath models which assume that the scatter power arises from a specific direction, such as half way between the horizon and the nominal specular reflection direction. The power density curves presented in Figures 3.24 and 3.25 for elevation angles of 10° and 15° , respectively, clearly demonstrate that a significant amount of multipath energy arises from very low reflection angles (horizon region). This tends to support, to some degree, the claim made by Engels et. al. [9] that the multipath signal power tends to come from the horizon rather than the nominal specular direction.

3.3.4 Second Order Statistics For Standard-C Antennas

Second order statistics can be specified in terms of fading bandwidth (spectral width of the doppler spectrum) or in terms of fade durations. Both characterizations are considered here.

(a) Horizontal polarization

(b) Vertical polarization

Figure 3.23: Power density curves for a 5° elevation angle maritime scenario with an omni antenna pattern.

(a) Horizontal polarization

(b) Vertical polarization

Figure 3.24: Power density curves for a 10° elevation angle maritime scenario with an omni antenna pattern.

(a) Horizontal polarization

(b) Vertical polarization

Figure 3.25: Power density curves for a 15° elevation angle maritime scenario with an omni antenna pattern.

The doppler spectral characteristics for the aeronautical application are carefully examined in Appendix E. These results do not directly apply to the maritime application because the antenna height (as well as the x-co-ordinate of the antenna) is modulated by the coupling of vessel and ocean surface motion. Furthermore, the specular point location is also influenced by the dispersive nature of the wave motion. However, one can crudely approximate the motion (velocity) of a specular point in terms of the average wave speed, which is on the order of 6 m/sec for SS4(ii) conditions and 4.8 m/sec for SS3 conditions. Even though the results of Appendix E do not apply exactly, a reasonable approximation for the differential doppler from a scattering facet is

$$f_d = \frac{v_r}{\lambda_c} (\cos \beta - \cos E) \quad (3.24)$$

where

v_r is the relative velocity of the ship with respect to the waves.

β is the reflection angle from the scattering facet,

E is the elevation angle to the satellite, and

λ_c is the carrier wavelength.

It is apparent that the spectrum width will be proportional to the relative velocity of the ship with respect to the waves. The power density curves presented in Figures 3.24

and 3.25 indicate that significant scatter power arises from very low reflection angles, significantly below the elevation angle. As can be seen from (3.24), these low reflection angles introduce positive doppler shifts, and one can approximately predict the location of the peak of the doppler spectrum by

$$f_p = \frac{v}{\lambda_c} (1 - \cos E). \quad (3.25)$$

Figures 3.26 and 3.27 clearly demonstrate the effect of relative velocity on the doppler spectrum for an elevation angle of 20° . When the relative velocity is 1 m/sec, the -10 dB spectrum width is on the order of 2 Hz. The peak is slightly offset in the positive direction (≈ 0.3 Hz), which is in excellent agreement with predicted value of 0.3 Hz obtained using (3.25). The doppler spectrum obtained when the vessel is going against the waves is noticeably different (see Figure 3.27), with a positive doppler shift in the neighbourhood of 3.2 Hz, which is in reasonable agreement with the predicted value of 3.3 Hz obtained using (3.25). The -10 dB spectrum width is in the neighbourhood of 4 Hz in this case.

Going with the waves ($v_r = 1$ m/sec) for an elevation angle of 10° and SS4(ii) surface conditions results in the very narrow, symmetric doppler spectrum presented in Figure

Figure 3.26: Maritime doppler spectrum for an elevation angle of 20° and for a vessel moving with the waves for SS4(ii) surface conditions.

Figure 3.27: Maritime doppler spectrum for an elevation angle of 20° and for a vessel moving against the waves for SS4(ii) surface conditions.

3.28. Figure 3.29 presents the doppler spectra obtained going against the waves at this elevation angle, for SS4(ii) ($v_r = 11$ m/sec) and SS3 ($v_r = 9.8$ m/sec) surface conditions. There is a slight shift of the doppler spectra in the positive direction, and the magnitude of this shift is accurately indicated by (3.25). The -10 dB spectrum width is on the order of 2 - 2.5 Hz.

The doppler spectrum obtained when the vessel is travelling against the waves ($v_r = 11$ m/sec), for an elevation angle of 15° , has a -10 dB spectrum width of approximately 3.3 Hz (see Figure 3.30). The doppler spectrum for an elevation angle of 5° and $v_r = 11$ m/sec has a -10 dB spectrum width of approximately 2.5 Hz (see Figure 3.31).

The -10 dB spectrum width, for a relative ship velocity of 11 m/sec and SS4(ii) surface conditions, versus elevation angle is tabulated below

Elevation Angle	Approximate -10 dB Spectrum Width	Approximate One-Sided Fading Bandwidth B_{10}
5°	2.5 Hz	1.25 Hz
10°	2.5 Hz	1.25 Hz
15°	3.3 Hz	1.65 Hz
20°	4.0 Hz	2.0 Hz

Figure 3.28: Maritime doppler spectrum for an elevation angle of 10° and for a vessel moving with the waves for SS4(ii) surface conditions.

Figure 3.29: Maritime doppler spectra for an elevation angle of 10° and for a vessel travelling against the waves for SS3 and SS4(ii) surface conditions.

Figure 3.30: Maritime doppler spectrum for an elevation angle of 15° with a vessel travelling against the waves for SS4(ii) surface conditions.

Figure 3.31: Maritime doppler spectrum for an elevation angle of 5° with a vessel travelling against the waves for SS4(ii) surface conditions.

The observed tendency of increasing fading bandwidth with increasing elevation angle is in agreement with theory and measurement results [6], [7].

The analysis program developed to process propagation data has the capability of characterizing fade margins and fade durations. The fade margins and average fade durations obtained from the simulated propagation data are summarized in Table 3.29. Here, the 99% availability (1% outage) fade margin and associated average fade duration are presented along with the system outage and average fade duration for a fade margin of 5 dB. It is evident that when the ship moves with the waves (low relative velocity) that the fading process is slow. The average fade durations are considerably longer than they are for the cases where the ship moves against the waves. It is also interesting to note that when the vessel is moving with the waves, the relative velocity is greatest for SS3, which accounts for the fact that the fade durations for the SEASTATE2 and SEASTATE5 are the smallest of the sea state runs.

It is evident that the average fade duration is reduced as the elevation angle is increased. This behaviour is consistent with theory and measurement results [6], [7]. It is evident that at low elevation angles (5° and 10°) and for vessel motion against the waves, the average fade durations obtained from the simulated propagation data

Scenario	Polarization Characteristics	Elevation Angle	S/I [dB]	1% Outage		5 dB Fade Margin	
				Fade Margin [dB]	Average Fade Dur. (sec)	Outage Level (%)	Average Fade Dur. (sec)
*ANTENNAS2	$E_R = -1$ dB $\theta_m = -33.5^\circ$	20°	10.3	5.3	0.23	1.3	0.19
ANTENNAS4	$E_R = -1$ dB $\theta_m = -33.5^\circ$	20°	9.8	6.2	0.064	1.7	0.0645
*DIVERG2	$E_R = -1$ dB $\theta_m = -33.5^\circ$	10°	9.7	6.1	0.29	2.25	0.46
*SEASTATE1 (SS4(ii))	$E_R = -5.5$ dB $\theta_m = 0^\circ$	10°	10.0	6.2	0.34	2.4	0.495
*SEASTATE2 (SS3)	$E_R = -5.5$ dB $\theta_m = 0^\circ$	10°	8.4	6.2	0.17	2.9	0.29
*SEASTATE3 (SS5(i))	$E_R = -5.5$ dB $\theta_m = 0^\circ$	10°	10.8	5.2	0.26	1.2	0.27
*SEASTATE4 (SS4(ii))	$E_R = -5.5$ dB $\theta_m = 0^\circ$	20°	12.4	4.2	0.16	0.4	0.14
*SEASTATE5 (SS3)	$E_R = -5.5$ dB $\theta_m = 0^\circ$	20°	12.8	3.4	0.16	0.3	0.12
*SEASTATE6 (SS5(i))	$E_R = -5.5$ dB $\theta_m = 0^\circ$	20°	12.7	4.0	0.23	0.2	0.15
VHGT10E10	$E_R = -2$ dB $\theta_m = -31.75^\circ$	10°	11.2	5.1	0.103	1.1	0.11

*Ship moving with waves at 7 m/sec

-- All other cases, vessel moving against the waves at 5 m/sec.

Table 3.29: Fade margins and fade durations for simulated maritime channels (cont'd)

Scenario	Polarization Characteristics	Elevation Angle	S/I [dB]	1% Outage		5 dB Fade Margin	
				Fade Margin [dB]	Average Fade Dur. (sec)	Outage Level (%)	Average Fade Dur. (sec)
VHGT20E10	$E_R = -2$ dB $\theta_m = -31.75^\circ$	10°	11.1	4.6	0.098	0.6	0.089
VHGT30E10	$E_R = -2$ dB $\theta_m = -31.75^\circ$	10°	11.3	4.8	0.12	0.8	0.11
VHGT10E20	$E_R = 0$ dB $\theta_m = -34.1^\circ$	20°	11.7	4.6	0.041	0.56	0.036
VHGT20E20	$E_R = 0$ dB $\theta_m = -34.1^\circ$	20°	11.9	4.3	0.043	0.66	0.036
VHGT30E20	$E_R = 0$ dB $\theta_m = -34.1^\circ$	20°	12.4	3.8	0.038	0.32	0.04
ELEV1	$E_R = -1$ dB $\theta_m = -33.5^\circ$	5°	9.2	5.85	0.093	1.66	0.106
ELEV2	$E_R = -1$ dB $\theta_m = -33.5^\circ$	15°	9.2	6.3	0.07	2.0	0.08
ELEV3	$E_R = -5.5$ dB $\theta_m = 0^\circ$	5°	9.1	7.2	0.073	2.6	0.098
ELEV4	$E_R = -5.5$ dB $\theta_m = 0^\circ$	15°	10.4	5.3	0.079	1.25	0.077
ELEV5	$E_R = -5.5$ dB $\theta_m = 0^\circ$	5°	12.2	4.3	0.228	0.3	0.15
ELEV6	$E_R = 0$ dB $\theta_m = -34.1^\circ$	15°	12.0	4.35	0.079	0.5	0.079

Table 3.29: Fade margins and fade durations for simulated maritime channels (cont'd)

for a 5 dB fade margin are between 0.09 sec and 0.15 sec. This is consistent with the claim in [7] that the average fade duration for a 5 dB fade depth at lower elevation angles is typically in the range of 0.03 to 0.15 sec.

It is also evident that one requires a significant fade margin (between 6 dB to 7 dB) to guarantee 99% system availability at low elevation angles with Standard-C shipborne antennas.

3.3.5 Other Interesting Observations

An examination of the detailed analysis outputs for the maritime simulation runs, too voluminous to be presented here, has yielded some interesting observations. These are summarized below.

First of all, it is evident that properly incorporating focussing effects of concave surface facets is important. A typical scatter power versus time plot is provided in Figure 3.32, and illustrates nicely the focussing of scatter power from concave facets. It is not uncommon to find between 50 to 100 near field occurrences over a 4096 sample run. It is also evident that the classification indices (blocked, shadowed, visible, or lost) change with time. It is not uncommon to observe several thousand of

Figure 3.32: An example of a maritime scatter power versus time plot.

these classification changes over a 4096 point run. As was anticipated, the number of reflection points increase as the elevation decreases, thereby increasing computational requirements. Computation requirements at 5° elevation angle were found to be comparable to those at 10° elevation angle, despite a significant increase in the number of reflection points. This occurred because more detailed searches for reflection points were required at 10° elevation angle, and these searches are very computationally demanding at low elevation angles. This behaviour is a consequence of the fact that specular points tend to deviate more at 10° elevation angle, thereby resulting in a greater likelihood of changing classification index or disappearing.

3.4 Comparison of the Aeronautical and Maritime Multipath Channels

The major differences between the aeronautical and maritime channels can be attributed to the very different link geometries (in particular, antenna heights) and mobile terminal velocities. Typical aircraft altitudes are between 5 to 20 km and shipborne antennas are usually between 3 to 30 m above the ocean surface. Typical aircraft speeds are in the neighbourhood of 500 to 600 mph, whereas typical ship speeds are in the neighbourhood of 15 mph, a factor of 40 lower.

The different antenna heights are directly responsible for the considerably different multipath delays, relative to the direct path, for the maritime and aeronautical applications. Typical multipath delays are between 10 - 100 nsec for the maritime application and between 10-100 μ sec for the aeronautical application. The delay spreads are also considerably greater for the aeronautical application, in which case, values around 5 - 8 μ sec are typical.

The differences in mobile terminal velocities are responsible for the significantly different direct path doppler shifts and fading rates of the aeronautical and maritime channels. For L-band carriers ($f = 1.5$ GHz), typical maximum doppler shifts for aeronautical and maritime applications are given below

Aeronautical Application		Maritime Application	
Velocity	Maximum Doppler Shift	Velocity	Maximum Doppler Shift
600 mph (268.2 m/sec)	1.34 kHz	15 mph (6.7 m/sec)	33.5 Hz

It is apparent that appreciable doppler shifts can be introduced by aircraft motion. Fortunately, commercial aircraft possess adequate navigation equipment to permit a mobile earth station to compute and compensate for most of the doppler effects caused by vehicle motion [24]. The maritime channel suffers from much slower fading than the aeronautical channel. Typical fading rates for the maritime channel are in the neighbourhood of 1 to 2 Hz, while typical values for the aeronautical channel are between 30 to 200 Hz.

Although minimal Standard-C antenna patterns for maritime and aeronautical applications are quite similar, when comparing aeronautical and maritime signal to interference ratios, an omni-reference antenna is considered. A summary of the S/I behaviour versus elevation angle is provided in Table 3.30. From an interference standpoint, it is evident that the maritime channel is worst case. For circular polarization, the maritime signal to interference ratios are roughly 2 to 3 dB lower than the corresponding aeronautical values.

At low elevation angles, the active scattering region effectively extends to the horizon limit of the antenna, $\sqrt{2a_e z_0}$, where a_e is the radius of the earth and z_0 is the antenna height. The $\sqrt{z_0}$ dependence indicates that the active scattering region for the aeronautical application

Signal to Interference Ratios [dB] For the Following Polarization Types						
Elevation Angle	Horizontal		Vertical		Circular	
	Aeronautical Model	Maritime Model	Aeronautical Model	Maritime Model	Aeronautical Model	Maritime Model
5°	5.2	2.2 to 2.3	16.2	12.4 to 12.7	12.5	9.0 to 9.3
10°	2.3 to 2.8	0.0 to 2.4	11.1 to 11.3	9.8 to 11.4	11.1 to 11.9	8.2 to 10.6
15°	1.6	-0.3 to 0.3	8.0	6.9 to 7.8	12.4	9.6 to 10.0
20°	1.0	-0.6 to 0.5	5.9	4.7 to 6.2	13.6	11.2 to 12.1

Table 3.30: Comparison of aeronautical and maritime signal to interference ratios versus elevation angle for an omni-reference antenna.

is significantly larger than that appropriate for the maritime case. Although the number of scattering facets on the ocean surface is considerably smaller in the maritime application, each has a stronger impact than in the aeronautical case because of the much closer proximity of the shipborne antenna to the surface. In the aeronautical application, the distance between the receiving antenna and the surface is considerable, and the reflected energy is attenuated significantly. The attenuation factor can be accurately predicted by the divergence factor

$$D = \sqrt{\frac{1}{1 + \frac{2d}{r_c \sin \alpha}}} \quad (3.26)$$

where d is the distance between the receiver and scattering facet, r_c is the surface radius of curvature, and α is the local grazing angle. In the aeronautical application, the divergence factor has a $\sqrt{1/d}$ dependence. However, the close proximity of the shipborne antenna to the ocean surface results in much lower attenuation than in the aeronautical application, especially for scattering facets close to the vessel. In fact, concave scattering facets close to the receiver can even focus energy. It is this lower divergence (attenuation) which is the most logical explanation for maritime multipath power levels being worse than aeronautical levels.

CHAPTER 4.0

STOCHASTIC ANALYSIS MODEL FOR AERONAUTICAL APPLICATIONS

When compared to the expensive alternative of performing field measurements, the propagation simulation approach is attractive. The appeal of the ocean scatter propagation programs is diminished somewhat by their extreme computational requirements. This is especially true for the aeronautical application, where a typical 10° elevation angle simulation run with SS4(ii) surface conditions has an execution time in the neighbourhood of 6.5 hours on a high speed FPS-164 array processor system. This array processor system has roughly an order of magnitude speed advantage over typical medium sized mini-computers, such as an APOLLO DN-320 and a DEC VAX-11/750. Consequently, it is not really practical to perform representative simulation runs at low elevation angles on mini-computer systems that do not have some form of high speed arithmetic peripheral. This significantly impairs the usability of the propagation simulation programs. In an attempt to rectify this situation, complexity reduction became a major goal of this research effort.

The key elements of a stochastic analysis model, designed primarily to reduce the computational complexity of

establishing the first order statistics of the aeronautical channel, are summarized in this chapter. The results obtained with this analysis model are compared with aeronautical propagation simulation program results.

4.1 Aeronautical Stochastic Analysis Model Development

The major components of the aeronautical stochastic analysis model include

- (i) formulating the geometrical search process for potential scattering facets as a Gaussian random process curve crossing problem,
- (ii) analytically approximating the effect of wave profile blockage, as a function of local waveheight and reflection angle, and
- (iii) analytically predicting the effect of the divergence factor on the scatter power contributed by a scattering facet.

The fundamental breakthrough in the development of the stochastic model was discovering that the search process could be modelled as a random process curve crossing

problem. This subject is addressed in the next subsection.

4.1.1 Formulating The Geometrical Search Process As A Random Process Curve Crossing Problem

As alluded to in Chapter 2.0, one of the most computationally demanding portions of the aeronautical and maritime propagation simulation programs involves locating the potential "specular reflection" points on the random surface. The geometrical relationships are summarized in Figure 2.1. The procedure involves identifying values x where a specular reflection point occurs, i.e.

$$\theta(x) = \beta(x) \quad (4.1)$$

with

$$\theta(x) = \tan^{-1} \left(\frac{z_0 - z_s(x)}{x} \right)$$

$$\beta(x) = E - 2 \epsilon(x)$$

$$\epsilon(x) = \tan^{-1} \left\{ \frac{dz_s(x)}{dx} \right\}$$

z_0 represents the aircraft altitude,

$z_s(x)$ represents the random surface superimposed on a curved earth surface, and

E is the elevation angle to the satellite.

It has been assumed that the initial x-co-ordinate of the aircraft is $x_0 = 0$.

The search region upper limit, at low elevation angles, is essentially the horizon range of the antenna

$$h_R = \sqrt{2a_e z_0}, \quad (4.2)$$

where a_e is the radius of the earth (6.378×10^6 m [11]).

The random surface profile and the first and second surface derivatives can be expressed as

$$z_s(x) = z(x) - \frac{x^2}{2a_e} \quad (4.3)$$

$$z'_s(x) = z'(x) - \frac{x}{a_e} \quad (4.4)$$

$$z''_s(x) = z''(x) - \frac{1}{a_e} = z''(x), \quad (4.5)$$

where $z(x)$, $z'(x)$, and $z''(x)$ represent the local waveheight, local surface slope, and local surface second derivative, respectively.

In this case,

$$\theta(x) = \tan^{-1}\left(\frac{h(x) - z(x)}{x}\right) \quad (4.6)$$

where

$$h(x) = z_0 + \frac{x^2}{2a_e} \quad (4.7)$$

Using the Taylor series expansion

$$\tan^{-1}(x+h) = \tan^{-1}(x) + \frac{h}{1+x^2} = \tan^{-1}(x) + \cos^2(\tan^{-1}(x))h \quad (4.8)$$

one obtains

$$\begin{aligned} \theta(x) &= \theta_0(x) - \cos^2(\theta_0(x)) \frac{z(x)}{x} \\ &= \theta_0(x) - \sin(2\theta_0(x)) \frac{z(x)}{2h(x)} \end{aligned} \quad (4.9)$$

where

$$\theta_0(x) = \tan^{-1}\left(\frac{h(x)}{x}\right)$$

Because the local waveheight is so much smaller than typical aircraft altitudes, the effect of the local waveheight can be effectively ignored, meaning that

$$\theta(x) = \theta_0(x) \quad (4.10)$$

This will not be the case in the maritime environment where the antenna is much closer to the surface. The effect of this on the development of a maritime stochastic analysis model is discussed in Appendix G.

Employing the approximation of (4.10) in the fundamental specular point constraint (4.1), one essentially obtains

$$E - 2 \tan^{-1}(z'_s(x)) = \theta_0(x) . \quad (4.11)$$

This can be simplified to give

$$z'(x) = m(x) \quad (4.12)$$

where

$$m(x) = \tan\left(\frac{E - \theta_0(x)}{2}\right) + \frac{x}{a} . \quad (4.13)$$

Consequently, specular points will be located at the points where the local slope process crosses the deterministic curve (function) represented by $m(x)$. If one considers a Gaussian random process for the local waveheight, $z'(x)$ is also a Gaussian random process. In this case, the specular point search process can be modelled as a Gaussian process curve crossing problem.

The slope of the curve is given by

$$m'(x) = \frac{(-\frac{1}{2})}{\cos^2[\frac{E-\theta_o(x)}{2}]} \frac{d\theta_o(x)}{dx} + \frac{1}{a_e} \quad (4.14)$$

The rate of change of the reflection angle from the surface can be expressed as

$$\frac{d\theta_o(x)}{dx} = \frac{-\sin^2(\theta_o(x))}{h(x)} \left[1 - \frac{h(x)}{a_e \tan^2(\theta_o(x))} \right] \quad (4.15)$$

Because typical aircraft altitudes are so high, the slope of the curve will usually be quite small. To demonstrate this,

$$|m'(x)| < \frac{\sin^2(\theta_o)}{2z_o \cos^2[\frac{E-\theta_o}{2}]} < \frac{1}{2z_o \cos^2[\frac{E-\theta_o}{2}]} \quad (4.16)$$

A reasonable extreme value for $(\frac{E-\theta_o}{2})$ is $5 \sigma_z$, where σ_z is the rms surface slope. For rough surface conditions, the rms surface slope is in the neighbourhood of 6° , and for an aircraft altitude of 10 km, one obtains

$$|m'(x)| < \frac{2/3}{z_o} = 6.67 \times 10^{-5} \quad (4.17)$$

The slope of the curve is so small that one can effectively assume that the curve is constant in the neighbourhood of a

specific point x . As a consequence, traditional treatments for the crossings of a fixed level by a stationary random process [25], [26] can be used to establish a reasonable approximate solution (crossing) density

$$\lambda(x) = \lim_{\Delta x \rightarrow 0} \left\{ \frac{\text{Number of Crossings in Interval } [x, x+\Delta x]}{\Delta x} \right\}$$

$$= \frac{1}{\pi} \left(\frac{\sigma_z''}{\sigma_z'} \right) \exp \left\{ -\frac{1}{2} \left(\frac{m(x)}{\sigma_z'} \right)^2 \right\} \quad (4.18)$$

A detailed treatment of the more general problem of curve crossings by Gaussian processes, which includes non-stationary processes, is presented in Chapter 13 of [27]. Equation 13.2.1 of [27] provides a general expression for the solution density for the curve crossings of a function $m(t)$ by a Gaussian process $\epsilon(t)$, which is

$$\lambda(t) = \frac{\gamma(t)}{\sigma(t)} (1 - \nu^2(t))^{\frac{1}{2}} \phi\left(\frac{m(t)}{\sigma(t)}\right) H(\eta(t)) \quad (4.19)$$

where

$$H(\eta(t)) = 2\phi(\eta(t)) + \eta(t)(2\phi(\eta(t)) - 1)$$

$$\sigma^2(t) = \text{Var} \{ \epsilon(t) \} ,$$

$$\gamma^2(t) = \text{Var} \{ \epsilon'(t) \} ,$$

$$\nu(t) = \frac{\text{Cov}[\epsilon(t), \epsilon'(t)]}{\sigma(t) \gamma(t)} ,$$

$$\eta(t) = \frac{m'(t) - \gamma(t) \mu(t) m(t)/\sigma(t)}{\gamma(t) (1 - \mu^2(t))^{1/2}}$$

$$\phi(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$$

$$\Phi(t) = \int_{-\infty}^t \phi(v) dv.$$

If $\epsilon(t)$ is stationary, $\sigma(t) = \sigma$ and $\gamma(t) = \gamma$ are independent of time, and more importantly, $\mu(t) = \mu = 0$. Using these simplifications to characterize the crossings of $m(x)$ by a general stationary Gaussian process $f(x)$, one obtains the following exact solution density expression

$$\lambda(x) = \frac{1}{\pi} \left(\frac{\sigma_{f'}}{\sigma_f} \right) \exp\left\{-\frac{1}{2} \left(\frac{m(x)}{\sigma_f} \right)^2\right\} G(\eta(x)) \quad (4.20)$$

where

$$G(\eta(x)) = \exp\left\{-\frac{\eta^2(x)}{2}\right\} + \frac{\sqrt{2\pi}}{2} \eta(x) (1 - 2Q(\eta(x))),$$

$$\eta(x) = \frac{m'(x)}{\sigma_{f'}},$$

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-v^2/2} dv,$$

$$\sigma_f^2 = \text{Var} \{f(x)\},$$

$$\sigma_{f'}^2 = \text{Var} \{f'(x)\}.$$

For the aeronautical application, $f(x) = z'(x)$, and $m'(x)$ is sufficiently small that $n(x) = 0$. This implies that $G(n(x)) = 1$, in which case the exact expression of (4.20) simplifies to the approximate expression presented in (4.18).

It should be noted that in the maritime application, the rate of change of the reflection angle from the surface $\frac{d\theta_o(x)}{dx}$ is much greater than in the aeronautical application. This is a simple consequence of the fact that the shipborne antenna is much closer to the ocean surface. Consequently, when establishing the maritime solution density one must use the exact solution approach (see Appendix G).

The mean number of crossings in a cell of width Δx centered about the point x is given by

$$E\{N_T\} = \lambda(x) \Delta x. \quad (4.21)$$

The overall average number of potential scattering facets can be established by integration

$$\bar{N}_O = \int_{x_{\text{start}}}^{h_R} \lambda(x) dx \quad (4.22)$$

where

$$x_{\text{start}} = z_0 \tan\left(\frac{\pi}{2} + \beta_{\text{max}}\right),$$

$$\beta_{\text{max}} = E + 2 \tan^{-1}(z'_p),$$

z'_p is the peak surface slope which is assigned a value of $4\sigma_z$, and

h_R is the horizon range given by (4.2).

It is interesting to note that the streams of upcrossings and downcrossings asymptotically approach a Poisson process as the level m tends to infinity [27], [28]. The variance of the number of crossings per cell under these circumstances is

$$\text{Var}(N_T) = \lambda(x) \Delta x. \quad (4.23)$$

This could be used as an approximation of the variance, providing that the ratio $\left(\frac{m}{\sigma_z}\right)$ is effectively in the tail region of a unit variance Gaussian process (i.e. $\left|\frac{m}{\sigma_z}\right| > 1$).

Determining the variance of the number of zero crossings (i.e. $m=0$) is a difficult problem. This is discussed in Section 10.7 of [27], and although the variance can be expressed in closed form, one obtains complicated integrals. It should be noted that when the reflection

angle from the surface is in the neighbourhood of the elevation angle to the satellite, the values of $\alpha(x)$ are in the neighbourhood of zero.

It should be noted that the above approach only provides solution densities, or mean numbers of potential scattering facets, and not exact specular point locations. Because of blockage by the wave profile, not all of the specular points which satisfy the geometric constraints will generate scatter signals which contribute to the overall scatter signal at the receiver. An approximate approach for incorporating the effects of surface blockage is described in the next subsection.

4.1.2 Incorporating The Effects of Blockage And Shadowing By The Ocean Surface

The effect of blockage on the reflected path from a scattering facet to the receiving antenna is strongly dependent on reflection angle. The most suitable reflection angle definition for analysis purposes is the local reflection angle θ_l , which is defined relative to the slope of the curved surface at the point of interest. The relationship between the local reflection angle and the actual reflection angle is illustrated in Figure 4.1. The actual reflection angle relative to the horizontal is essentially given by

Figure 4.1: Local reflection and elevation angles defined relative to the curvature of the earth.

$$\beta(x) = \tan^{-1} \left\{ \frac{z_0 + \frac{x^2}{2a_e}}{x} \right\} \quad (4.24)$$

and the local reflection angle is

$$\beta_l(x) = \beta(x) - \tan^{-1} \left\{ \frac{x}{a_e} \right\} \quad (4.25)$$

These reflection angles are explicitly defined in terms of the x-co-ordinate of a scattering facet.

It is important to establish interrelationships between these parameters to allow a characterization of scatter power as a function of reflection angle β or local reflection angle β_l . In the case where the actual reflection angle ($\beta(x) = \beta$) is known, the appropriate x-co-ordinate of the scattering facet is easily shown to be

$$x = a_e \tan \beta \left[1 - \sqrt{1 - \frac{2 z_0 a_e}{(a_e \tan \beta)^2}} \right] \quad (4.26)$$

and β_l is easily established using (4.25).

When the local reflection angle β_l is known,

$$\begin{aligned} \tan \beta_l &= \tan \left\{ \beta - \tan^{-1} \left(\frac{x}{a_e} \right) \right\} \\ &= \frac{\tan \beta - \left(\frac{x}{a_e} \right)}{1 + \left(\frac{x}{a_e} \right) \tan \beta} \end{aligned} \quad (4.27)$$

Using (4.24), this can be expressed as

$$\tan \beta_1 = \frac{z_0 - \frac{x^2}{2a_e}}{\left[1 + \frac{z_0}{a_e}\right]x + \frac{x^3}{2a_e^2}} \quad (4.28)$$

The cubic term is very small and can be effectively ignored, thereby resulting in a quadratic equation for x , which has a solution

$$x = (a_e + z_0) \tan \beta_1 \left[\sqrt{1 + \frac{2 z_0 a_e}{(a_e + z_0) \tan \beta_1}} - 1 \right] \quad (4.29)$$

and the actual reflection angle is established using (4.24).

The behaviour of the actual reflection angle as a function of local reflection angle, for an aircraft altitude of 9.1 km, is presented in Table 4.1.

Figure 4.2 illustrates the wave blockage problem. Here it is assumed that the scatter signal from a facet contributes if the ray to the receiver is not blocked by the surface, which is the approach used in the simulation program. This is a quantized approach which applies no blockage

Local Reflection Angle (degrees)	Corresponding Actual Reflection Angle (degrees)
0.125	3.060
0.375	3.081
0.625	3.121
0.875	3.180
1.125	3.258
1.375	3.352
1.625	3.462
1.875	3.586
2.125	3.723
2.375	3.871
2.625	4.029
2.875	4.196
3.125	4.371
3.375	4.553
3.625	4.741
3.875	4.935
4.125	5.133
4.375	5.336
4.625	5.543
4.875	5.753
5.125	5.966
5.375	6.182
5.625	6.400
5.875	6.621
6.125	6.843
6.375	7.068
6.625	7.294
6.875	7.521
7.125	7.750
7.375	7.981
7.625	8.212
7.875	8.444
8.125	8.678
8.375	8.912
8.625	9.147
8.875	9.383
9.125	9.620
9.375	9.857
9.625	10.095

Table 4.1: Interrelationship between the local and actual reflection angles for an aircraft altitude of 9.1 km.

Figure 4.2: Geometry of the surface blockage (clearance) problem.

attenuation when there is path clearance and complete blockage if there is no clearance. The absolute accuracy of this approach is debatable. A superior approach involves computing the minimum path clearance between the reflected ray and the surface, and applying a blockage attenuation factor based on Fresnel zone clearance (using the knife edge diffraction curve of Figure 2.17). This approach would be more computationally demanding than the current approach of simply determining whether or not there is path clearance, which is already very computationally demanding. Furthermore, from an analytic standpoint, this minimum path clearance problem is even more difficult than determining the probability of path clearance (which is itself a very difficult problem). With the current approach, the clearance probability can be used as a weighting factor to provide an estimate of the average scatter power from a specific region which accounts for the effects of surface blockage.

It is fundamental that one be able to establish the probability that the ray to the receiver clears the ocean surface over an interval length denoted by x_L . Expressed mathematically, the clearance probability P_c is

$$P_c(\beta_t, z_t) = \text{Prob}\{z(x_s - x_t) < z_t + \tan(\beta_t) x_t \mid \text{for all } x_t \in [0, x_L]\} \quad (4.30)$$

where

x_s represents the x-co-ordinate of the scattering facet, and

$z_s = z(x_s)$ is the local waveheight at the specular point.

As is evident from Figure 4.2, the clearance probability will depend strongly on the local waveheight z_s of the scattering facet as well as, the local reflection angle θ_s .

For the aeronautical simulation runs performed, the interval length x_L was arbitrarily selected to be 2.5λ , where λ is the average sea wavelength. The surface $z(x)$ is represented by a Gaussian random process. Ideally, one would like to be able to solve (4.30) for the clearance probability for general autocorrelation (spatial spectral) properties. For an ocean surface, [12] suggests that the surface becomes essentially de-correlated after a distance of $\lambda/4$. In fact, for the bandpass surface model, which was used in the aeronautical simulation program, the autocorrelation function (see Figure A.3 of Appendix A) has a zero crossing at a distance Δx from the correlation peak, where

$$\Delta x = \frac{\lambda}{4} \sqrt{1 + \frac{\gamma^2}{12}}, \quad (4.31)$$

and the bandwidth factor $\gamma = 1.344$.

For the stochastic model, a computationally efficient procedure for establishing the clearance probability $P_c(\beta_l, z_l)$ is required. Unfortunately, the author has not been able to locate a published solution to the clearance probability problem. It should be noted that this is a rather difficult problem, especially in light of the significant contribution from the low reflection angle (horizon) region evident from the power density curves presented in Chapter 3. Professor Lorne Campbell at Queen's University at Kingston, Ontario and M.R. Leadbetter, a contributing author for references [27] and [28], at the University of North Carolina, have been contacted concerning this problem, and they are not aware of any general solution to the clearance probability problem.

An appealing approximate approach for estimating the clearance probability is to select N sample points over the prescribed interval and determine the probability of the event that each of the associated surface random variables assume values below the ray to the receiver. The probability of this event is an estimate of the actual clearance probability. Because the above random variables are jointly Gaussian, one can easily establish a theoretical expression for this probability. This involves N -fold integrals over the joint Gaussian density function, which necessitates computing the inverse of an $N \times N$

covariance matrix. In addition to conditioning on the local waveheight z_l , one can also establish the conditional Gaussian distribution which takes into account the local slope z'_l at the scattering facet. The appeal of this sampling approach is that the clearance probability estimate asymptotically converges to the exact value (as $N \rightarrow \infty$). The difficulty of this approach is the computational complexity of performing the resulting N -fold integrals, especially when N is large.

From an intuitive standpoint, surface correlation introduces a practical upper limit on the number of sample points that need be considered. That is, after a certain point, reducing the sample spacing further will do little in terms of improving the accuracy of the clearance probability estimate that justifies the increased computational complexity associated with the smaller sample spacing.

The approach that has been adopted for estimating the clearance probability minimizes the computational requirements. With this approach, the sample width, Δx , is selected to be large enough that neighbouring surface samples have very low correlation. To simplify the clearance probability calculation, it is assumed that the resulting set of surface samples are all independent. The

clearance probability estimate that one obtains with this "independent sampling model" is

$$\hat{P}_C(\beta_l, z_l) = \prod_{i=1}^{N_I} \left[1 - Q \left(\frac{z_l + i \tan(\beta_l) \Delta x}{\sigma_z} \right) \right] \quad (4.32)$$

where

N_I is the effective number of independent samples over the interval x_L ,

σ_z is the rms waveheight, and

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt.$$

Matching the conditions used during the aeronautical simulation runs, where $x_L = 2.5 \lambda$, and using a sample interval of $\Delta x = \frac{\lambda}{4} \sqrt{1 + \frac{1.344^2}{12}}$ for the bandpass surface model, one obtains

$$\hat{P}_C(\beta_l, z_l) = \prod_{i=1}^{N_I} \left[1 - Q \left(\frac{z_l}{\sigma_z} + i \frac{K_{BP} \tan(\beta_l)}{\sigma_z} \right) \right] \quad (4.33)$$

where

σ_z is the rms surface slope,

$$K_{BP} = \frac{\pi}{2} \sqrt{1 + \frac{\gamma^2}{12}},$$

$\gamma = 1.344$, and

$N_I = 9$.

The clearance probability estimates obtained with this "independent sampling method" are compared with corresponding aeronautical simulation results in Section 4.1.5.3. It should be noted that the independent sampling approach does not incorporate any correlation effects that the local slope at the scattering facet may have on the set of waveheight samples.

Some study of slope related effects has been performed. To summarize, it was found that the geometrical constraints on specular points result in surface slopes that ensure that there is very little likelihood of blockage in the immediate neighbourhood of the specular point. A first order slope-waveheight correlation compensation approach, where slope correlation effects on the first surface sample are considered, has been developed for clearance probability calculations. The results obtained with this approach were not very encouraging, and indicate that one must reduce the sample spacing, which complicates the estimation of the clearance probability. Upper and lower clearance probability bounds, appropriate for the case where "correlated samples" half-way between the uncorrelated sampling instants are considered, have also been derived. The upper bound provides results that are not appreciably different from those obtained with the

"independent sampling method". It has been found that the lower bound developed is not very tight, and as a result, not particularly useful.

Some effort was devoted to deriving general solution approaches which exploit the continuous nature of the surface over the interval. It has been discovered that the approaches developed are not very accurate in the low reflection angle region, which makes them unsuitable for the stochastic analysis model. Consequently, only a brief summary of these approaches is warranted here.

It is interesting to note that the surface blockage problem can be formulated as a curve crossing problem, in that a scattering facet contributes if and only if the surface $z(x_s - x_t)$ does not cross the line " $z_t + \tan(\beta_t) x_t$ " over the prescribed interval. The local surface slope is such that there is no local blockage problem and one must have a minimum of one positive going curve crossing for blockage to occur. Using equation (13.3.2) of [27], one can determine the density of upcrossings $\lambda_u(x_t)$ and the mean number of upcrossings using

$$N_u = \int_0^{x_L = 2.5\lambda} \lambda_u(\rho) d\rho. \quad (4.34)$$

With this one can bound the clearance probability by

$$P_C > 1 - \bar{N}_u \quad (4.35)$$

The problem with this is that as the local reflection angle becomes relatively low (values representative of the horizon region), $\bar{N}_u$ exceeds unity, and (4.35) is a worthless bound.

Another interesting and related way of viewing the blockage problem is in terms of the extrema of the process

$$z(x_l) = z(x_s - x_l) - z(x_s) - \tan(\beta_l) x_l \quad (4.36)$$

Section 13.5 of [27] presents an approach which can be used to upper bound the clearance probability. For low reflection angles, it has been found that the clearance probability bounds can assume values which significantly exceed unity. Consequently, these clearance probability upper bounds are not very useful for the stochastic analysis model.

To summarize, the blockage weighting factor is defined to be

$$w_B(\theta_l, z_l) = \prod_{i=1}^9 \left[1 - Q\left(\frac{z_l}{\sigma_z} + i \frac{1.685 \tan \theta_l}{\sigma_z}\right) \right] \quad (4.37)$$

The local reflection angle θ_l is a function of the x-co-ordinate. To keep the notation simpler, this dependence is implicitly assumed and not explicitly indicated in (4.37).

Shadowing, which refers to the blockage of the path from the scattering facet to the satellite, can be handled in an analogous fashion, and will be strongly dependent on the elevation angle to the satellite. The local elevation angle, E_l , defined with respect to the slope of the earth, is

$$E_l(x) = E + \tan^{-1} \left(\frac{x}{a_e} \right) \quad (4.38)$$

The shadowing weighting factor is defined to be

$$w_S(E_l, z_l) = \prod_{i=1}^9 \left[1 - Q\left(\frac{z_l}{\sigma_z} + i \frac{1.685 \tan E_l}{\sigma_z}\right) \right] \quad (4.39)$$

These weighting factors are crucial for establishing the average scatter power as a function of position or reflection angle. In the next section, the important power weighting factor that accounts for the effects of surface curvature for the aeronautical application is derived.

4.1.3 Incorporating The Effect of Attenuation Introduced by Surface Curvature

For the aeronautical application, the voltage scale factor introduced by surface curvature can be accurately predicted by the divergence factor

$$D = \sqrt{\frac{1}{1 + \frac{2d}{r_c \sin \alpha}}} \quad (4.40)$$

where

d is the distance between the scattering facet and the receiving antenna,

α is the local grazing angle, and

r_c is the surface radius of curvature at the scattering facet.

The local grazing angle α can be expressed as

$$\alpha(x) = \frac{E + \beta(x)}{2} \quad (4.41)$$

and the distance to the receiving antenna, ignoring the small perturbations introduced by local waveheight variations, can be expressed as

$$d(x) = \frac{h(x)}{\sin \beta(x)} \quad (4.42)$$

where

$\beta(x)$ is the reflection angle relative to the horizontal, and

$$h(x) = z_0 + \frac{x^2}{2a_e}.$$

The appropriate weighting factor for the scatter power from the region around a point x is given by

$$\begin{aligned} D^2(x) &= \frac{1}{1 + \frac{2 d(x)}{r_c \sin \alpha(x)}} \\ &= \frac{r_c \sin \beta(x) \sin \alpha(x)}{2 h(x)} \left[\frac{1}{1 + \frac{r_c \sin \beta(x) \sin \alpha(x)}{2 h(x)}} \right]. \end{aligned} \quad (4.43)$$

For typical aircraft altitudes, $\frac{r_c \sin \beta(x) \sin \alpha(x)}{2 h(x)} \ll 1$, and the following approximation is reasonable,

$$D^2(x) = r_c \frac{\sin \beta(x) \sin \alpha(x)}{2 h(x)}. \quad (4.44)$$

The local radius of curvature of the surface is given by equation (2.12), and can be expressed as

$$r_c = \frac{[1 + z'_s(x)^2]^{3/2}}{|z''_l|} \quad (4.45)$$

where

$$z'_s(x) = \tan \left[\frac{E - \delta(x)}{2} \right], \text{ and}$$

z''_l is the second derivative of the local waveheight profile at the specular point.

Incorporating the radius of curvature expression in the divergence expression of (4.44) yields

$$D^2(x) = \sigma(\alpha, \beta, x) \hat{P}_c(z''_l) \quad (4.46)$$

where

$$q(\alpha, \beta, x) = \frac{\sin \alpha(x) \sin \beta(x)}{2 h(x)} (1 + \tan^2(\frac{E - \delta(x)}{2}))^{3/2} \quad (4.47)$$

$$\hat{P}_c(z''_l) = \frac{1}{|z''_l|} \text{ (approximate radius of curvature).} \quad (4.48)$$

As established in Appendix A, the surface second derivative, z'' , and the local waveheight z , are correlated as

$$F\{z'' | z\} = \sigma_z^2 \quad (4.49)$$

and the conditional second derivative density function is given by

$$f_{z''}(z''_l | z=z_l) = \frac{1}{\sqrt{2\pi} \sigma_{m_2}} \exp\left\{ \frac{-1}{2\sigma_{m_2}^2} (z''_l - \kappa_2 z_l)^2 \right\} \quad (4.50)$$

where

$$K_{2z} = \frac{E\{z'' z\}}{\sigma_z^2} = - \left(\frac{\sigma_{z'}}{\sigma_z} \right)^2,$$

$$r_2 = \frac{F\{z'' z\}}{\sigma_{z''} \sigma_z} = \frac{-\sigma_{z'}^2}{\sigma_{z''} \sigma_z},$$

$$\sigma_{m_2} = \sqrt{1 - r_2^2} \sigma_{z''}.$$

The appropriate parameters for the Bandpass and Gaussian spectrum surface models are summarized in Table A.3 of Appendix A.

The negative correlation between the second derivative and the waveheight is intuitively appealing, in that facets with large positive waveheights will tend to be convex (negative second derivative values) and conversely, facets with large negative waveheights will tend to be concave (positive second derivative values). Consequently, it is apparent that the approximate radius of curvature $\hat{R}_c$ is strongly dependent on local waveheight, as are the blockage and shadowing weighting factors w_B and w_S .

4.1.4 Average Scatter Power Characterization

In this section the behaviour of the average scatter power from a small cell of width Δx , centred about a point x , is examined. Here, it is assumed that the cell width is such

that the variations in the reflection angle β , the surface grazing angle α , and the distance between the antenna and scattering facet are very small. A brief examination of the actual simulation solution for this situation is informative. In this case, the scatter signal from the small cell is given by

$$r(x) = a(x) \sum_{i=1}^{N_C} D_i(x) e^{j\phi_i} \quad (4.51)$$

where

N_C is the number of contributing facets in the cell,

ϕ_i is the phase shift relative to the direct path,

$$a(x) = \begin{cases} G_H(g_a) \Gamma_H(\alpha) & \text{for horizontal polarization} \\ G_V(g_a) \Gamma_V(\alpha) & \text{for vertical polarization} \\ \frac{G_H(g_a) \Gamma_H(\alpha) + E_R e^{j\theta_m} G_V(g_a) \Gamma_V(\alpha)}{2} & \text{for circular polarization} \end{cases}$$

$g_a = B_A - \beta$ (angle appropriate for applying antenna gains),

B_A is the banking angle of the aircraft antenna pattern (rotations toward the surface represent positive angles),

$\Gamma_H(\cdot)$, $\Gamma_V(\cdot)$ are the surface reflection coefficients for horizontal and vertical polarization,

$G_H(\cdot)$, $G_V(\cdot)$ are the antenna (voltage) gain patterns for horizontal and vertical polarization,

E_R is the magnitude of the antenna polarization ratio, and

θ_m is the antenna phase mismatch.

The "polarization dependent" component of the "scatter signal is represented by the function $a(x)$.

The normalized scatter power is given by

$$|r(x)|^2 |a(x)|^2 \sum_{i=1}^{N_C} \sum_{k=1}^{N_C} D_i(x) D_k(x) e^{j[\theta_i - \theta_k]} \quad (4.52)$$

If one assumes that the phase shifts are independent and uniformly distributed,

$$f(\theta_i, \theta_k) = f(\theta_i) f(\theta_k) \quad (4.53)$$

$$f(\theta_i) = \begin{cases} \frac{1}{2\pi} & -\pi < \theta_i < \pi \\ 0 & \text{otherwise} \end{cases}$$

as well as being independent of the divergence terms, the incremental simulation scatter power is given by

$$\Delta P_{sim}(x) = E\{|r(x)|^2 / z, z''\}$$

$$= |a(x)|^2 \sum_{i=1}^{N_C} \sum_{k=1}^{N_C} D_i(x) D_k(x) \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} e^{j[\theta_i - \theta_k]} f(\theta_i) f(\theta_k) d\theta_i d\theta_k$$

$$= |a(x)|^2 \sum_{i=1}^{N_C} D_i^2(x). \quad (4.54)$$

Generalizing this for the stochastic model, it is apparent that an appropriate definition for the incremental scatter power is

$$\Delta P_S(x) = |a(x)|^2 \sum_{i=1}^{N_T} w_B(\beta_i, z_i) w_S(E_i, z_i) D_i^2(x) \quad (4.55)$$

where

N_T is the number of potential scattering facets in the cell, and

w_B and w_S are the blockage and shadowing weighting factors.

Using the expression for $D_i^2(x)$ presented in (4.46), in conjunction with the definition

$$R(\beta_i, E_i) = w_B(\beta_i, z_i) w_S(E_i, z_i) \hat{R}_C(z_i), \quad (4.56)$$

the conditional mean of the incremental scatter power can be expressed as

$$\begin{aligned} E\{\Delta P_S(x)/N_T\} &= N_T |a(x)|^2 E\{w_B(\beta_i, z_i) w_S(E_i, z_i) D_i^2(x)/N_T\} \\ &= N_T |a(x)|^2 q(\alpha, \beta, x) E\{R(\beta_i, E_i)\} \end{aligned} \quad (4.57)$$

where

$$E\{R(\beta_i, E_i)\} = \int \int w_B(\beta_i, z_i) w_S(E_i, z_i) \hat{R}_C(z_i) f_{zz''}(z_i, z_i) dz_i dz_i'' \quad (4.58)$$

$$= \int w_B(\beta_i, z_i) w_S(E_i, z_i) M_{RC}(z_i) f_z(z_i) dz_i \quad (4.59)$$

$$M_{RC}(z_l) = \int_{-\infty}^{\infty} \hat{R}_C(z_l'') f_{z''}(z_l''/z_l) dz_l'' \quad (4.60)$$

Here, the function $M_{RC}(z_l)$ represents the mean radius of curvature as function of local waveheight.

The average incremental scatter power from a cell is given by

$$\begin{aligned} E\{\Delta P_S(x)\} &= E\{E\{\Delta P_S(x)/N_T\}\} \\ &= E\{N_T\} |a(x)|^2 q(\alpha, \beta, x) E\{R(\beta_l, E_l)\} \\ &= \lambda(x) \Delta x |a(x)|^2 q(\alpha, \beta, x) E\{R(\beta_l, E_l)\}. \end{aligned} \quad (4.61)$$

Since $a(x)$ is the only component depending on signal polarization, one can define a polarization independent power density,

$$f_I(x) = \lambda(x) q(\alpha, \beta, x) E\{R(\beta_l, E_l)\} \quad (4.62)$$

and the overall average scatter power, accounting for signal polarization, can be determined by performing the integral

$$P_S = \int_{x_{start}}^{h_R} |a(x)|^2 f_I(x) dx. \quad (4.63)$$

One of the nice features of the stochastic model is that some indication of the variance of the scatter power can be provided. Note that the random variable $\Delta P_S^2(x)$ is given by

$$\Delta P_S^2(x) = |a(x)|^4 \sum_{i=1}^{N_T} \sum_{j=1}^{N_T} w_B^2(\beta_i, z_i) w_S^2(E_i, z_i) D_i^2(x) D_j^2(x). \quad (4.64)$$

The divergence factors appropriate at different scattering facets are independent and identically distributed, in which case the conditional mean is

$$\begin{aligned} E\{\Delta P_S^2(x)/N_T\} &= N_T |a(x)|^4 E\{w_B^2(\beta_i, z_i) w_S^2(E_i, z_i) D^2(x)\} \\ &\quad + N_T(N_T-1) |a(x)|^4 [E\{w_B(\beta_i, z_i) w_S(E_i, z_i) D^2(x)\}]^2 \\ &= |a(x)|^4 q^2(\alpha, \beta, x) (N_T E\{R^2(\beta_i, E_i)\} \\ &\quad + N_T(N_T-1) [E\{R(\beta_i, E_i)\}]^2). \end{aligned} \quad (4.65)$$

The mean squared incremental scatter power is given by

$$\begin{aligned} E\{\Delta P_S^2(x)\} &= E\{E\{\Delta P_S^2(x)/N_T\}\} \\ &= |a(x)|^4 q^2(\alpha, \beta, x) (E\{N_T\} (E\{R^2(\beta_i, E_i)\} - [E\{R(\beta_i, E_i)\}]^2) \\ &\quad + E\{N_T^2\} [E\{R(\beta_i, E_i)\}]^2) \end{aligned}$$

$$\begin{aligned}
 &= |a(x)|^4 q^2(\alpha, \beta, x) (E\{N_T\} \text{Var}\{R(\beta_l, E_l)\} \\
 &\quad + E\{N_T^2\} [E\{R(\beta_l, E_l)\}]^2) .
 \end{aligned}
 \tag{4.66}$$

The variance of the incremental scatter power is given by

$$\begin{aligned}
 \text{Var}\{\Delta P_S(x)\} &= E\{\Delta P_S^2(x)\} - [E\{\Delta P_S(x)\}]^2 \\
 &= |a(x)|^4 q^2(\alpha, \beta, x) (E\{N_T\} \text{Var}\{R(\beta_l, E_l)\} \\
 &\quad + \text{Var}(N_T) [E\{R(\beta_l, E_l)\}]^2) .
 \end{aligned}
 \tag{4.67}$$

Assuming that N_T is a Poisson random variable, which is exact in the asymptotic limit,

$$E(N_T) = \text{Var}(N_T) = \lambda(x) \Delta x , \tag{4.68}$$

and the variance of the incremental scatter power can be approximated by

$$\text{Var}\{\Delta P_S(x)\} = \lambda(x) \Delta x |a(x)|^4 q^2(\alpha, \beta, x) E\{R^2(\beta_l, E_l)\} .
 \tag{4.69}$$

In an analogous fashion to the mean power density, one can define a polarization independent variance density

$$v_I(x) = \lambda(x) q^2(\alpha, \beta, x) E\{R^2(\beta_l, E_l)\}, \quad (4.70)$$

where

$$E\{R^2(\beta_l, E_l)\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w_B^2(\beta_l, z_l) w_S^2(E_l, z_l) \hat{R}_C^2(z_l'') f_{zz''}(z_l, z_l'') dz_l dz_l'' \quad (4.71)$$

and the overall variance can be approximated by the integral

$$\text{Var}\{P_S\} = \int_{x_{\text{start}}}^{h_R} |a(x)|^4 v_I(x) dx. \quad (4.72)$$

As alluded to earlier, the power densities can be characterized as a function of reflection angle, in which case

$$f_I(\beta) = \lambda(\beta) q(\alpha, \beta) E\{R(\beta_l, E_l)\} \quad (4.73)$$

$$v_I(\beta) = \lambda(\beta) q^2(\alpha, \beta) E\{R^2(\beta_l, E_l)\} \quad (4.74)$$

where

$$\lambda(\beta) = \frac{h(x)}{\sin^2(\beta(x))} c(\beta, x) \frac{1}{\pi} \left(\frac{\sigma_{z''}}{\sigma_{z'}} \right) \exp\left\{-\frac{1}{2} \left(\frac{m(x)}{\sigma_{z'}} \right)^2\right\} \quad (4.75)$$

$$c(\beta, x) = \frac{1}{1 - \frac{h(x)}{a_e \tan^2(\beta(x))}} \quad (4.76)$$

and

$$m(x) = \tan\left(\frac{E - \beta(x)}{2}\right) + \frac{x}{a_e}$$

The integrals for the mean and variance of the scatter power are

$$E\{P_S\} = \int_{\beta_{\min}}^{\beta_{\max}} |a(\beta)|^2 E_I(\beta) d\beta \quad (4.77)$$

$$\text{Var}\{P_S\} = \int_{\beta_{\min}}^{\beta_{\max}} |a(\beta)|^4 v_I(\beta) d\beta, \quad (4.78)$$

where

$$a(\beta) = \begin{cases} G_H(B_A - \beta) r_H(\frac{E+\beta}{2}) & \text{for horizontal polarization} \\ G_V(B_A - \beta) r_V(\frac{E+\beta}{2}) & \text{for vertical polarization} \\ \frac{G_H(B_A - \beta) r_H(\frac{E+\beta}{2}) + E_R e^{j\theta_m} G_V(B_A - \beta) r_V(\frac{E+\beta}{2})}{2} & \text{for circular polarization} \end{cases} \quad (4.79)$$

B_A represents the aircraft banking angle,

$$\beta_{\min} = \tan^{-1} \left\{ \sqrt{\frac{2z_0}{a_e}} \right\}, \text{ and}$$

$$\beta_{\max} = E + 2 \tan^{-1} \{ 4 \sigma_z \}.$$

The above integrals are performed in a numerical fashion. Given the dependance of the blockage and shadowing weighting factors on local reflection angle, the integrals are effectively performed with respect to local reflection angle, with smaller increments used in the sensitive low local reflection angle regions.

4.1.5 Stochastic Model Refinements For Alignment With The Simulation Solution

In this section, stochastic model refinements required to align the stochastic and simulation solutions are discussed. The evaluation of $E\{R(\beta_i, E_i)\}$ is critical for establishing the mean scatter power from a specific region. The important relationships are provided in (4.58), (4.59), and (4.60). It is important to note that this expected value is a function of local reflection angle, which is a function of the slope level crossing value.

To obtain reasonable first and second moments for the approximate radius of curvature, one must modify the definition of $\hat{R}_c$. In the simulation program, a second derivative threshold is employed when establishing the surface radius of curvature at a scattering facet. That is, when the magnitude of the second derivative at a scattering facet is below the threshold value, the maximum magnitude of the second derivative in the neighbourhood of the specular point is used, if that value is above the threshold. Otherwise, the threshold (cutoff) second derivative is used. The following simple cutoff approach, is employed in the stochastic model

$$\hat{R}_c(z_i^*) = \begin{cases} \frac{1}{\text{cutoff}} & \text{if } |z_i^*| < \text{cutoff} \\ \frac{1}{|z_i^*|} & \text{if } |z_i^*| > \text{cutoff} \end{cases} \quad (4.80)$$

Although this approach does not match that used in the propagation simulation exactly, it is a reasonable analysis model. The cutoff threshold value was arbitrarily selected to be 0.0035 m^{-1} for the aeronautical simulation runs. It has been discovered that the sensitivity of the stochastic model results to the second derivative cutoff threshold is relatively minor (see Section 4.2).

It has been discovered that characterizing the first moment of the approximate radius of curvature versus height by direct application of (4.60) provides results which are noticeably different than simulation observations. The fundamental reason for this is that the search process alters both the waveheight and second derivative distributions at the specular points. That is, the constraint that a specular point be located at a level crossing of the surface slope process effectively modifies both the second derivative and waveheight density functions. A detailed discussion of this phenomenon is presented in Appendix H, where the waveheight and second derivative density functions conditioned on the slope level crossing event C are derived. Observation of aeronautical simulation results confirm that the search process does modify the associated density functions. For example, the results presented in Table 4.2, for a 15° elevation angle aeronautical scenario, clearly reveals that the waveheight

Normalized Height	Number of Scattering Facets	Approximate Radius of Curvature R_c (meters)
-4.00	0	--
-3.80	0	--
-3.60	0	--
-3.40	0	--
-3.20	0	--
-3.00	10	16.0
-2.80	59	17.7
-2.60	122	19.2
-2.40	106	18.9
-2.20	213	19.1
-2.00	270	19.9
-1.80	331	24.7
-1.60	524	30.8
-1.40	670	33.4
-1.20	787	39.0
-1.00	720	46.8
-0.80	677	54.1
-0.60	698	56.5
-0.40	642	60.3
-0.20	620	65.5
0.00	639	66.9
0.20	604	67.2
0.40	648	62.7
0.60	629	61.3
0.80	647	54.7
1.00	787	46.6
1.20	800	39.5
1.40	661	35.1
1.60	525	30.5
1.80	329	24.0
2.00	277	20.0
2.20	189	17.2
2.40	126	17.2
2.60	116	17.0
2.80	66	16.4
3.00	11	15.9
3.20	0	--
3.40	0	--
3.60	0	--
3.80	0	--
4.00	0	--

Table 4.2: Number of scattering facets and approximate radius of curvature versus height for a 15° elevation angle aeronautical scenario with SS4(ii) surface conditions (SZERO wave profile).

distribution at the potential scattering facets (before any blockage and shadowing classification) is not Gaussian. The number of scattering facets tends to peak for normalized waveheights in the neighbourhood of unity (actually 1.2).

A constraint on the distance between neighbouring specular points is also considered in Appendix H. To ensure that the distance is greater than or equal to $\Delta_{\min}$, the event

$$E_d = \{ |z'''| < \frac{2|z''|}{\Delta_{\min}} \} \quad (4.81)$$

must be satisfied. It is shown in Appendix H that one obtains a similar constraint for the detectability of a specular point. A sign-change detection approach for locating intervals with specular points, as used in the simulation program, was considered in this analysis. An appropriate choice for $\Delta_{\min}$ is

$$\Delta_{\min} = \frac{\Delta_s}{2} \quad (4.82)$$

where

Δ_s is the step sized used for the sign change detection procedure.

The step size used for all the aeronautical simulation runs was $\Delta_s = \frac{\lambda}{10}$, where λ is the average sea wavelength.

When evaluating the first and second moments of $R(\beta_l, E_l)$, one must replace $f_{zz''}(z_l, z_l'')$ in equations (4.58) and (4.71) with the conditional density function $f_{zz''}(z_l, z_l''/C, E_d)$. This conditional density function can be expressed in product form as

$$f_{zz''}(z_l, z_l''/C, E_d) = f_{z''}(z_l''/z=z_l, C, E_d) f_z(z_l/C, E_d) \quad (4.83)$$

The first and second moments of the random variable $R(\beta_l, E_l)$ can be expressed as

$$E\{R(\beta_l, E_l)\} = \int_{-\infty}^{\infty} w_B(\beta_l, h) w_S(E_l, h) M_{RC}(h) f_z(h/C, E_d) dh \quad (4.84)$$

$$E\{R^2(\beta_l, E_l)\} = \int_{-\infty}^{\infty} w_B^2(\beta_l, h) w_S^2(E_l, h) S_{RC}(h) f_z(h/C, E_d) dh \quad (4.85)$$

where

$$M_{RC}(h) = \int_{-\infty}^{\infty} \hat{R}_C(z_l'') f_{z''}(z_l''/z=h, C, E_d) dz_l'' \quad (4.86)$$

$$S_{RC}(h) = \int_{-\infty}^{\infty} \hat{R}_C^2(z_l'') f_{z''}(z_l''/z=h, C, E_d) dz_l'' \quad (4.87)$$

The conditional density functions required to evaluate these expressions are derived in Appendix H, and summarized below

$$f_{z''}(b/z=a, C, E_d) = g_{z, z', z''}(a, m_c, b) f_{z''}(b/z=a) \quad (4.88)$$

$$g_{z, z', z''}(a, m_c, b) = \frac{P(E_d/z'=m_c, z''=b) |b|}{\int_{-\infty}^{\infty} P(E_d/z'=m_c, z''=b) |b| f_{z''}(b/z=a) db} \quad (4.89)$$

$$f_z(a/C, E_d) = h_{z, z'}(a, m_c) f_z(a) \quad (4.90)$$

$$h_{z, z'}(a, m_c) = \frac{\int_{-\infty}^{\infty} P(E_d/z'=m_c, z''=b) |b| f_{z''}(b/z=a) db}{\int_{-\infty}^{\infty} P(E_d/z'=m_c, z''=b) |b| f_{z''}(b) db} \quad (4.91)$$

$$P(E_d/z'=m_c, z''=b) = Q\left(\frac{-2|b|}{\frac{\Delta_{\min}}{\sigma_{m_3}} - K_3 m_c}\right) - Q\left(\frac{2|b|}{\frac{\Delta_{\min}}{\sigma_{m_3}} - K_3 m_c}\right) \quad (4.92)$$

$$f_{z''}(b/z=a) = \frac{1}{\sqrt{2\pi} \sigma_{m_2}} \exp\left\{-\frac{1}{2\sigma_{m_2}^2} (b - K_2 a)^2\right\} \quad (4.93)$$

$$f_{z''}(b) = \frac{1}{\sqrt{2\pi} \sigma_{z''}} \exp\left\{-\frac{1}{2} \left(\frac{b}{\sigma_{z''}}\right)^2\right\} \quad (4.94)$$

$$f_z(a) = \frac{1}{\sqrt{2\pi} \sigma_z} \exp\left\{-\frac{1}{2} \left(\frac{a}{\sigma_z}\right)^2\right\} \quad (4.95)$$

The parameters K_2 , σ_{m_2} , K_3 , and σ_{m_3} are defined in Tables A.3 and A.4 of Appendix A.

It is evident that employing these conditional distributions introduces a slope (or equivalently a position or reflection angle) dependence in the first and second moments of the approximate radius of curvature (i.e. $M_{RC}(h, z')$ and $\sigma_{RC}(h, z')$). A stochastic model initialization routine has been developed to tabulate these functions for several normalized slope values. In the stochastic analysis program, values at intermediate slope values are approximated using linear interpolation. Having established the moments of the approximate radius of curvature as a function of slope, the moments of $R(\beta_l, E_l)$, which are clearly dependent on reflection angle and thus slope, are determined by numerical integration. The overall mean and variance of the scatter power are established by summing the individual contributions from each of the reflection angle regions.

Figure 4.3 illustrates the behaviour of the modified (conditional) height density as a function of slope. It is interesting to note that, with the bandpass surface model, the height density functions assume their maximum value for normalized heights in the neighbourhood of unity. This is consistent with the simulation results presented in Table 4.2.

Figure 4.3: Behaviour of the conditional waveheight density functions as a function of local surface slope.

4.1.5.1 Comparing The Stochastic Model And Aeronautical Simulation Program Solution Density Results

The usefulness of the curve crossing solution density, derived assuming Gaussian surface characteristics, for estimating the number of scattering facets is examined in this section. In the stochastic analysis program, an expected number of scattering facets versus local reflection angle curve is generated. The expected number of scattering facets is established for a cell width of 0.25° about the local reflection angle of interest. In the aeronautical simulation program, the number of observed facets versus local reflection angle is characterized in a similar fashion.

The agreement between the observed number of occurrences and the theoretical (stochastic model) mean curve, is for the most part, very good. The excellent agreement obtained at low elevation angles is illustrated in Figures 4.4 and 4.5. As the elevation angle is increased, the level of agreement in the sensitive low reflection angle (horizon) region tends to worsen. The results presented in Figure 4.6 for an elevation angle of 31° provide evidence of this. Deviations between the actual waveheight and surface slope distributions and the assumed Gaussian distributions used in the stochastic analysis model are responsible for this behaviour. These deviations tend to occur in the tail

Figure 4.4: Comparison of solution density results for an elevation angle of 5° , SS4 (11) surface conditions, and an aircraft altitude of 9.1 km.

Figure 4.5: Comparison of solution density results for an elevation angle of 15° , SS4(ii) surface conditions, and an aircraft altitude of 9.1 km.

Figure 4.6: Comparison of solution density results for an elevation angle of 31° , SS4(ii) surface conditions, and an aircraft altitude of 9.1 km.

region of the distributions, where the simulated density functions often abruptly stop. Consequently, as the elevation angle is increased, the curve

$$m(x) = \tan\left(\frac{E-\beta(x)}{2}\right) + \frac{x}{a_e}$$

will eventually assume values in the horizon region which exceed the maximum possible slope of the simulated wave profile. When this occurs, there can be no scattering facets in the horizon region with the superposition of sinusoids surface model. It should be noted that the agreement between the observed number of occurrences and the mean number of occurrences predicted by the stochastic model, for an elevation angle of 31°, is very good for local reflection angles above 10° (see Figure 4.6).

4.1.5.2 Comparing The Behaviour of The Approximate Radius of Curvature Obtained With The Stochastic Model And Aeronautical Propagation Simulation Program

In the aeronautical simulation program, the behaviour of the first moment of the approximate radius of curvature versus height is established for the complete set of potential scattering facets (prior to visibility classification). Since this incorporates scattering facets from the entire search region, this first moment curve is an effective average over all possible slope values in the

search region. For comparison purposes, a weighted average approximate radius of curvature curve, $\bar{M}_{RC}(h)$, is computed in the stochastic model program, as follows

$$\bar{M}_{RC}(h) = \frac{\sum_{i=1}^N w_i M_{RC}(h, z_i)}{\sum_{i=1}^N w_i} \quad (4.96)$$

where

$$w_i = \frac{N_T(i)}{\sum_{i=1}^N N_T(i)}$$

$$N_T(i) = \lambda(\beta_i) \Delta\beta_i$$

Simulated and stochastic model approximate radius of curvature curves versus height are presented in Figures 4.7 and 4.8, for elevation angles of 20° and 31° and SS4(ii) surface conditions. The agreement between these curves is fairly good. It is interesting to note that the height distributions for the superposition of random phase sinusoids surface model have well defined cutoff points, typically between $3\sigma_z$ and $4\sigma_z$. Consequently, the tail regions of the height distributions of the simulated bandpass surface model differ from the assumed Gaussian distribution used in the stochastic analysis model.

Figure 4.9 presents simulated and stochastic model approximate radius of curvature curves for SS3 surface

Figure 4.7: Approximate radius of curvature curves for a 20° elevation angle scenario with SS4(ii) surface conditions.

Figure 4.8: Approximate radius of curvature curves for a 31° elevation angle scenario with SS4(ii) surface conditions.

Figure 4.9: Approximate radius of curvature for a 10° elevation angle scenario with SS1 surface conditions.

Figure 4.10: Normalized radius of curvature curves for SS1 and SS5(1) surface conditions for an elevation angle of 10° .

4

Micro

conditions and an elevation angle of 10° . The stochastic model curve provides a reasonable estimate of the simulated radius of curvature behaviour versus height. Note that the radius of curvature values for SS3 surface conditions are lower than those obtained with SS4(ii) surface conditions.

The behaviour of the normalized first moment of the approximate radius of curvature,

$$M_{RC_{norm}}(z) = M_{RC}(z) \cdot \sigma_z'' \quad (4.97)$$

with sea state is illustrated in Figure 4.10. It is evident that the normalized first moment of the approximate radius of curvature is relatively insensitive to sea state. Consequently, the actual radius of curvature values are proportional to $1/\sigma_z''$. The variation of the rms second derivative with sea state is essentially given by $1/\sigma_z$ (because the rms surface slope varies little with sea state). As a result, the radius of curvature is directly proportional to the rms waveheight, which implies increasing radius of curvature values as the sea state index increases. The results presented here exhibit this behaviour.

The behaviour of the scatter power as a function of sea state is now approximated. The polarization independent power density is given by

$$f_I(\beta) = \lambda(\beta) q(\alpha, \beta) E\{R(\beta_l, E_l)\}$$

where

$q(\alpha, \beta)$, $\lambda(\beta)$, and $E\{R(\beta_l, E_l)\}$ are defined by (4.47), (4.75), and (4.84), respectively.

Because $M_{RC_{norm}}(z)$ is essentially independent of sea state,

$$E\{R(\beta_l, E_l)\} = \frac{K(\beta_l, E_l)}{\sigma_z''} \quad (4.98)$$

where

$K(\beta_l, E_l)$ simply denotes a proportionality factor that depends only on the reflection angle and the elevation angle.

Using this approximation in conjunction with (4.47) and (4.75), it is easily shown that

$$f_I(\beta) = c(\beta, x) \left(\frac{u(\beta, x) K(\beta_l, E_l) \sin \alpha}{2\pi \sigma_z' \sin \beta} \right) \quad (4.99)$$

where

$$u(\beta, x) = \left(1 + \tan^2 \left(\frac{E-\beta}{2} \right) \right)^{3/2} \exp \left\{ -\frac{1}{2} \left(\frac{m(x)}{\sigma_z'} \right)^2 \right\} \quad (4.100)$$

$$m(x) = \tan \left(\frac{E-\beta}{2} \right) + \frac{x}{a_e},$$

$$c(\beta, x) = \frac{1}{1 - \frac{h(x)}{a_e \tan^2 \beta}},$$

$$h(x) = z_0 + \frac{x^2}{2a_e}.$$

It is important to note that the polarization independent power density does not depend on σ_z (which is a parameter that is sensitive to sea state). The power density does depend on rms sea slope, which is fairly insensitive to sea state (see Appendix A). Therefore, for the aeronautical application, the scatter power should not vary dramatically with sea state. Aeronautical simulation results presented in Section 3.2.2 support this statement.

The effect of aircraft altitude on the average scatter power is now considered. The lower limit of the average scatter power integral, $\beta_{\min} = \tan^{-1}\left\{\sqrt{\frac{2z_0}{a_e}}\right\}$, depends on aircraft altitude. The only term in the power density expression of (4.99) which depends on aircraft altitude is the curvature induced scale factor $c(\beta, x)$, which only assumes significant values in the horizon (low reflection angle) region. Figure 4.11 illustrates the behaviour of this curvature scale factor, as a function of local reflection angle, for aircraft altitudes of 9100 m and 5000 m. It is evident that over the majority of the reflection angle region, the power density function does not depend on the aircraft altitude. In the very low reflection angle region, the curvature induced scale factor

Figure 4.11: Behaviour of curvature scale factor for aircraft altitudes of 5 km and 9.1 km.

Figure 4.12: Clearance probability versus local reflection angle curves for an elevation angle of 5° .

is marginally worse for the higher aircraft altitude. This tends to counterbalance the increase in scatter power, which accompanies a reduction in aircraft altitude through the reduction of $\beta_{\min}$ in the average scatter power integral. Therefore, some variation in average scatter power levels with aircraft altitude can be expected, but they should be relatively minor.

4.1.5.3 Comparing The Stochastic Model And Aeronautical Propagation Simulation Program Clearance Probabilities

Aeronautical simulation program clearance probability observations are useful for validating the stochastic model blockage weighting factor. Although the number of potential scattering facets is quite large for the aeronautical application, there are not enough to accurately characterize the clearance probability as a function of both waveheight and local reflection angle. As a result, the clearance probability is characterized as a function of local reflection only in the aeronautical simulation program. To obtain a comparable quantity with the stochastic model, one must de-condition the blockage weighting factor with respect to waveheight

$$P_c(\beta_l) = \int_{-\infty}^{\infty} w_B(\beta_l, h) f_z(h/C, E_d) dh \quad (4.101)$$

where the blockage weighting factor is defined in (4.37), and the conditional height density function is defined in (4.90).

The clearance probability curves obtained with the aeronautical simulation program and the stochastic model are presented in Figures 4.12 and 4.13, for elevation angles of 5° and 10° , respectively. The simulated clearance probability curves obtained with different random surface profiles exhibit considerable variation, and the analytically predicted clearance probability is in reasonable agreement with these observations. For characterizing the clearance probability behaviour in the low local reflection angle region, simulation runs with low elevation angles are preferred. This is because one obtains more specular points in the low reflection angle region, which improves the accuracy of the estimated clearance probability, when low elevation angle scenarios, as opposed to high elevation angle scenarios, are considered. The clearance probability results for an elevation angle of 31° , presented in Figure 4.14, confirm this. For example, for local reflection angles above 7.5° , no surface blockage effects were observed with the SZERO surface profile. The number of scattering facets observed in this reflection angle region is usually quite small, being in the neighbourhood of 20. When the number of

Figure 4.13: Clearance probability versus local reflection angle curves for an elevation angle of 10° .

Figure 4.14: Clearance probability versus local reflection angle curves for an elevation angle of 31° .

scattering facets is so small, the clearance probability estimate may not be very accurate. For example, reducing the number of contributing facets by only two would drop the clearance probability from 100% to 90%, which would result in better agreement with the analytic model prediction. It should be emphasized that for reflection angles below 7° (see Figure 4.14), the analytically computed clearance probability values provide a reasonable indication of typical simulated clearance probability values for an elevation angle of 31° .

Given the extremely difficult nature of the surface blockage problem, the level of agreement between the clearance probability values predicted by the computationally efficient analytic solution procedure and the observed simulation values is very encouraging, and suggests that the simple analytic solution procedure is accurate enough for this application.

Aeronautical simulation results and analytically predicted values for the three fundamental stochastic model components, i.e.

- (1) specular point solution density,
- (2) attenuation introduced by divergence, and
- (3) the clearance probability weighting factors,

are in reasonable agreement. Given this, one would expect that the mean scatter power predicted by the stochastic model should match aeronautical simulation values fairly well. The next section addresses this issue.

4.2- Comparison of The First Order Statistics of The Stochastic Model And Aeronautical Simulation Program

The ability of the computationally efficient stochastic analysis model to predict the first order statistics of the aeronautical channel is examined in this section.

Aeronautical simulation results have been obtained for three different random surface profiles for comparison purposes.

An impressive complexity reduction is obtained with the stochastic analysis model. With the aeronautical propagation simulation program, a typical 1024 point simulation run at 10° elevation angle with SS4(ii) surface conditions requires 6.5 hours to complete on a high speed FPS-164 array processor system. This array processor is approximately an order of magnitude faster than an Apollo DSP-90 processor. The execution time of the stochastic analysis program on an Apollo DSP-90 processor is less than 8 minutes, which corresponds to a speed up improvement factor of almost 500. Another advantage of the stochastic analysis model is that the execution time is independent of sea state and elevation angle. This is in direct contrast

to the execution time behaviour of the physical propagation simulation program, which increases dramatically as the elevation angle is lowered and the average sea wavelength is reduced.

The first order statistics for the baseline simulation scenarios (sea state index SS4(ii), an aircraft altitude of 9.1 km, and level flight conditions) with an omni antenna pattern are summarized in Table 4.3. The average scatter power levels predicted by the stochastic analysis model are typically within a few tenths of a decibel of the values obtained with the aeronautical propagation simulation program. The worst case deviation is 0.43 dB for circular polarization at an elevation angle of 5°. Differences of 0.1 - 0.2 dB between the analytic and simulation values are typical.

The average scatter power levels for the sea state and antenna height variation simulation scenarios, with an omni antenna pattern, are compared to the values obtained with the stochastic analysis model in Table 4.4. The agreement between the simulated and analytic values is excellent, especially for the S1111 and SZERO surface profiles.

Comparing the values in Table 4.4 for SS3 and SS5(i) surface conditions with those in Table 4.3 for an elevation angle of 10° and SS4(ii) conditions, reveals that the scatter power is relatively insensitive to sea state. The analysis presented in Section 4.1.5.2 supports this

Elevation Angle	Signal Polarization	Average Scatter Power [dB] Obtained With The Stochastic Model	Average Scatter Power [dB] Obtained With The Aeronautical Simulation Program For The Following Surface Profiles		
			S1111	SZERO	S7777
5°	Horizontal	-5.04	-5.20	-5.21	-4.76
	Vertical	-16.15	-16.27	-16.20	-16.17
	Circular	-12.02	-12.34	-12.45	-11.66
10°	Horizontal	-2.73	-2.79	-2.74	-2.50
	Vertical	-11.19	-11.23	-11.27	-11.31
	Circular	-11.76	-11.84	-11.72	-11.35
15°	Horizontal	-1.65	-1.60	-1.53	-1.62
	Vertical	-7.95	-7.97	-7.92	-7.96
	Circular	-12.53	-12.41	-12.31	-12.51
20°	Horizontal	-1.17	-1.02	-1.03	-1.30
	Vertical	-5.97	-5.86	-5.92	-6.06
	Circular	-13.84	-13.60	-13.58	-14.07
31°	Horizontal	-1.11	-1.21	-1.19	-1.34
	Vertical	-3.84	-3.93	-3.92	-4.05
	Circular	-17.77	-17.88	-17.86	-18.11

Table 4.3: Average scatter power levels obtained with the aeronautical simulation program and the stochastic analysis model for SS4(ii) surface conditions, an aircraft altitude of 9.1 km, level flight conditions, and an omni antenna pattern.

Elevation Angle	Sea State Index	Antenna Height	Signal Polarization	Average Scatter Power [dB] Obtained With The Stochastic Model	Average Scatter Power [dB] Obtained With The Aeronautical Simulation Program For The Following Surface Profiles		
					S1111	SZERO	S777
10°	SS3	9.1 km	Horizontal	-2.36	-2.39	-2.31	-2.21
			Vertical	-11.16	-11.25	-11.16	-11.29
			Circular	-11.25	-11.26	-11.12	-10.97
10°	SS5 (i)	9.1 km	Horizontal	-2.82	-2.90	-2.82	-2.48
			Vertical	-11.22	-11.28	-11.22	-11.08
			Circular	-11.88	-11.97	-11.86	-11.41
10°	SS4 (ii)	5 km	Horizontal	-2.59	-2.52	-2.54	-2.35
			Vertical	-11.24	-11.03	-11.11	-11.39
			Circular	-11.42	-11.44	-11.41	-11.0

Table 4.4: Average scatter power levels obtained with the aeronautical simulation program and the stochastic analysis model for the sea state and antenna height variation simulation scenarios with an omni antenna pattern.

behaviour. A tendency for the scatter power to increase somewhat as the surface becomes smoother is evident from both the stochastic model and simulated values. Comparing the values for an elevation angle of 10° and SS4(ii) surface conditions for aircraft altitudes of 9.1 km and 5 km presented in Tables 4.3 and 4.4, respectively, reveals that the average scatter power is not very sensitive to antenna height. The scatter power values for the lower aircraft altitude are marginally worse than those at the higher aircraft altitude.

The effects of aircraft antenna patterns, as well as the capability of handling aircraft manoeuvres (climbing, diving, and banking), are incorporated in the stochastic analysis model program. The average scatter power levels obtained with the crossed slot antenna pattern (see Figure 3.6), for the baseline simulation scenarios, are presented in Table 4.5. The agreement between the analytically predicted and simulation values for the crossed slot antenna is quite good. The deviations are usually less than 0.3 - 0.4 dB. However, deviations as great as 0.89 dB occur for horizontal polarization at an elevation angle of 31° .

The average scatter power levels for the sea state and antenna height variation simulation scenarios with the crossed slot antenna are summarized in Table 4.6. The

Elevation Angle	Signal Polarization	Average Scatter Power [dB] Obtained With The Stochastic Model	Average Scatter Power [dB] Obtained With The Aeronautical Simulation Program For The Following Surface Profiles		
			S1111	SZERO	S7777
5°	Horizontal	-23.77	-24.23	-24.39	-23.34
	Vertical	-21.55	-21.66	-21.67	-21.46
	Circular	-25.79	-26.38	-26.64	-25.67
10°	Horizontal	-22.18	-22.30	-22.13	-21.62
	Vertical	-17.09	-17.14	-17.12	-16.88
	Circular	-25.68	-25.67	-25.56	-25.59
15°	Horizontal	-22.05	-21.82	-21.66	-22.09
	Vertical	-13.99	-13.95	-13.88	-13.87
	Circular	-22.94	-22.92	-22.90	-22.83
20°	Horizontal	-22.83	-22.38	-22.35	-23.40
	Vertical	-12.58	-12.38	-12.35	-12.71
	Circular	-21.02	-20.93	-20.91	-21.02
31°	Horizontal	-27.14	-27.56	-27.52	-28.03
	Vertical	-13.01	-13.11	-13.11	-13.46
	Circular	-20.49	-20.55	-20.54	-20.81

Table 4.5: Average scatter power levels obtained with the aeronautical simulation program and the stochastic analysis model for the baseline simulation scenarios with the crossed slot antenna pattern.

Elevation Angle	Sea State Index	Antenna Height	Signal Polarization	Average Scatter Power [dB] Obtained With The Stochastic Model	Average Scatter Power [dB] Obtained With The Aeronautical Simulation Program For The Following Surface Profiles		
					S1111 ^P	SZERO	S7777
10°	SS3	9.1 km	Horizontal	-21.59	-21.59	-21.39	-21.21
			Vertical	-16.66	-16.71	-16.70	-16.52
			Circular	-25.38	-25.35	-25.25	-25.40
10°	SS5 (i)	9.1 km	Horizontal	-22.32	-22.44	-22.29	-21.73
			Vertical	-17.19	-17.27	-17.23	-16.90
			Circular	-25.76	-25.76	-25.66	-25.58
10°	SS4 (ii)	5 km	Horizontal	-21.67	-21.76	-21.70	-21.09
			Vertical	-17.07	-17.01	-17.06	-16.88
			Circular	-25.35	-25.23	-25.16	-25.29

Table 4.6: Average scatter power levels obtained with the aeronautical simulation program and the stochastic analysis model for the sea state and antenna height variation simulation scenarios with the crossed slot antenna pattern.

simulation results and analytically predicted values for these 10° elevation angle scenarios match very well.

The average scatter power levels obtained for the aircraft climbing and diving manoeuvres with a crossed slot antenna are summarized in Table 4.7. The analytically predicted values are in very good agreement with the simulation results.

The average scatter power levels versus elevation angle for SS4(ii) surface conditions, an aircraft altitude of 9.1 km, and the switched (diversity) antenna pattern (see Figure 3.7) are summarized in Table 4.8. The analytically predicted values match the simulation results obtained very well, especially for elevation angles of 10° , 15° , and 20° , where the agreement is excellent.

In light of the impressive complexity reduction of the analytic stochastic model relative to the detailed physical propagation model (approximately two orders of magnitude), the accuracy of the predicted first-order statistics for aeronautical channels is excellent. One can also estimate aeronautical doppler spectrum characteristics, which provides an indication of the second order statistics, with the stochastic model. Mean power density versus reflection angle curves are used in this doppler spectrum approximation procedure, which is the subject of Section 4.3.

Elevation Angle	Climb Angle	Signal Polarization	Average Scatter Power [dB] Obtained With The Stochastic Model	Average Scatter Power [dB] Obtained With The Aeronautical Simulation Program
15°	-10°	Horizontal	-17.71	-17.58
		Vertical	-10.77	-10.75
		Circular	-20.73	-20.71
15°	+10°	Horizontal	-29.85	-29.41
		Vertical	-17.91	-17.85
		Circular	-25.45	-25.49

Table 4.7: Average scatter power levels for aircraft climbing and diving scenarios with the crossed slot antenna pattern.

Elevation Angle	Signal Polarization	Average Scatter Power [dB] Obtained With The Stochastic Model	Average Scatter Power [dB] Obtained With The Aeronautical Simulation Program
5°	Horizontal	-10.62	-10.90
	Vertical	-21.75	-21.81
	Circular	-17.0	-17.48
10°	Horizontal	-8.73	-8.71
	Vertical	-17.07	-17.11
	Circular	-17.42	-17.33
15°	Horizontal	-8.21	-8.08
	Vertical	-13.97	-13.96
	Circular	-19.04	-18.79
20°	Horizontal	-8.48	-8.46
	Vertical	-12.41	-12.35
	Circular	-21.42	-21.51
31°	Horizontal	-10.97	-11.13
	Vertical	-12.11	-12.23
	Circular	-28.60	-29.03

Table 4.8: Average scatter power levels obtained with the aeronautical simulation program and the stochastic analysis model for SS4(ii) surface conditions, an aircraft altitude of 9.1 km, and the switched (diversity) antenna pattern.

In the stochastic model program, mean power density versus reflection angle curves are established. In addition, upper and lower 2- σ confidence limit curves are computed to provide some indication of reasonable deviations from the mean power density. The variance approximation of (4.69) is used to estimate the standard deviation required to compute the confidence limit curves.

The stochastic model power density curves for horizontal and vertical polarization, obtained with an omni antenna pattern and an elevation angle of 5° , are compared to corresponding simulation curves in Figure 4.15. The power density curves for an elevation angle of 10° , SS4(ii) surface conditions and an aircraft altitude of 5 km are presented in Figure 4.16. Figure 4.17 shows the power density curves for horizontal polarization and an omni antenna pattern for SS3 and SS5(i) surface conditions. The simulated power density curves are in reasonable agreement with the stochastic model curves for each of these cases. The agreement is also very good at higher elevation angles ($E=31^\circ$), as Figure 4.18 demonstrates.

Before concluding this section, the sensitivity of the average scatter power levels of the stochastic model to the second derivative cutoff, used when establishing the approximate radius of curvature, is examined. The

(a) Horizontal polarization

(b) Vertical Polarization

Figure 4.15: Power density versus reflection angle curves for an elevation angle of 5° , SS4(ii) surface conditions, an aircraft altitude of 9.1 km, and an omni antenna pattern.

(a) Horizontal polarization

(b) Vertical polarization

Figure 4.16: Power density versus reflection angle curves for an elevation angle of 10° , SS4(11) surface conditions, an aircraft altitude of 5.0 km, and an omni antenna pattern.

(a) SS3 surface conditions

(b) SS5(i) surface conditions

Figure 4.17: Power density versus reflection angle curves for a horizontally polarized omni antenna pattern, an elevation angle of 10° , and SS3 and SS5(i) surface conditions.

(a) Horizontal polarization

(b) Vertical polarization

Figure 4.18: Power density curves for an elevation angle of 31° , SS4(11) surface conditions, an aircraft altitude of 9.1 km, and an omni antenna pattern.

sensitivity of the average scatter power to the minimum distance, $\Delta_{\min}$, between specular points is also considered.

The sensitivity of the average scatter power to variations in the second derivative cutoff, about the nominal value of 0.0035 m^{-1} used in the aeronautical simulation runs, is summarized in Table 4.9. The low and high cutoff values considered are 0.001 m^{-1} and 0.005 m^{-1} , which correspond to limiting radius of curvature values of 1000 m and 200 m, respectively. It is evident that the scatter power is not very sensitive to the cutoff value, the maximum difference being only 0.11 dB.

Given the step size used in the search process of the physical simulation program, the most appropriate choice for the minimum distance between specular points is

$$\Delta_{\min} = \lambda/20,$$

where λ is the average sea wavelength. To examine the sensitivity to this parameter, average scatter power levels for $\Delta_{\min} = \lambda/30$ and $\Delta_{\min} = \lambda/15$ are established for an elevation angle of 15° and SS4(ii) surface conditions. The results are summarized in Table 4.10. The sensitivity to $\Delta_{\min}$ is greater than the second derivative cutoff sensitivity, with maximum deviations of 0.27 dB between the extreme $\Delta_{\min}$ values.

Signal Polarization	Average Scatter Power Levels [dB] For The Following Second Derivative Cutoff Values		
	0.001 m ⁻¹	0.0035 m ⁻¹	0.005 m ⁻¹
Horizontal	-1.60	-1.65	-1.71
Vertical	-7.91	-7.95	-8.01
Circular	-12.48	-12.53	-12.59

Table 4.9: Average scatter power behaviour versus second derivative cutoff for an elevation angle of 15°, SS4(ii) surface conditions, and an omni antenna pattern.

Signal Polarization	Average Scatter Power Levels [dB] For		
	$\Delta_{\min} = \lambda/30$	$\Delta_{\min} = \lambda/20$	$\Delta_{\min} = \lambda/15$
Horizontal	-1.53	-1.65	-1.78
Vertical	-7.82	-7.95	-8.09
Circular	-12.42	-12.53	-12.66

λ = average sea wavelength

Table 4.10: Average scatter power as a function of the minimum distance between specular points for an elevation angle of 15°, SS4(ii) surface conditions and an omni antenna pattern.

4.3 Establishing Approximate Aeronautical Doppler Spectra With The Stochastic Model

The differential doppler shift (relative to the direct path) introduced by a scattering facet is strongly dependent on reflection angle. The basic relationships are derived in Appendix E, and summarized by equation (E.35). The doppler (power) spectrum of the multipath process can be approximated by weighting the doppler shift at a specific reflection angle by the power weighting factor appropriate at that reflection angle. This power weighting factor is readily established from the mean power density versus reflection angle curves for the appropriate signal polarization. This sort of approach has been used in [29] and [30] for establishing the effect of mobile antenna patterns on mobile radio power spectral characteristics. In the stochastic model program, approximate doppler spectra are established using the mean power density versus reflection angle curves for the appropriate signal polarization.

The simulated aeronautical doppler spectra are presented as functions of normalized frequency, where the frequency normalization factor is

$$f_s = \frac{N_w v_x}{\lambda}$$

where N_w is the number of sample points per average sea wavelength λ , and v_x is the horizontal aircraft velocity.

Simulated aeronautical and stochastic model doppler spectra for a horizontally polarized omni-antenna pattern for SS4(ii) and SS5(i) surface conditions and an elevation angle of 10° are presented in Figure 4.19. The simulated doppler spectra are in fairly good agreement with the stochastic model doppler spectra. The doppler spectra for an elevation angle of 31° and SS4(ii) surface conditions obtained with a horizontally polarized omni-antenna pattern are presented in Figure 4.20. Over the frequency range containing the majority of the scatter power, the simulated and stochastic model doppler spectra are in fairly reasonable agreement.

In the extreme negative frequency region, which corresponds to the high reflection angle region, there is a tendency for the stochastic power spectrum values to exceed simulated values. Note that this high reflection angle region corresponds to large negative slope values. The slope distributions obtained with the superposition of sinusoids surface model tend to have sharp cutoff points, usually between $2.6 \sigma_z$ to $4.0 \sigma_z$. The magnitude of the slope process does not assume values beyond these cutoff points. As a result, there exists a maximum reflection angle, and thus a negative doppler shift, beyond which

(a) SS4(ii) surface conditions

(b) SS5(i) surface conditions

Figure 4.19: Aeronautical doppler spectra for a horizontally polarized omni antenna pattern, an elevation angle of 10° , and SS4(ii) and SS5(i) surface conditions.

Figure 4.20: Aeronautical doppler spectra for an elevation angle of 31° .

Figure 4.21: Approximate radius of curvature curves for the Bandpass and Gaussian spectrum surface models.

there will be no scatter power with the simulated wave profiles. However, there does not exist such a slope cutoff with the Gaussian distributions assumed in the analytical model, which explains the observed differences. One other minor difference is that the simulated doppler spectra near the doppler peak in the positive frequency region are somewhat broader than the stochastic doppler spectra. This is caused by the windowing employed in the power spectrum estimation procedure used to process simulated propagation data. These windowing effects are not included in the stochastic model doppler mapping process using the mean power density versus reflection angle curves.

4.4 Effect of the Surface Model on the First Order Statistics of the Aeronautical Channel

In this section, the first order statistics obtained with the Bandpass surface model are compared with results for a Gaussian spectrum surface model. These surface models are described in detail in Appendix A. It suffices to say that these models provide matching rms waveheights and rms surface slopes, but different rms second derivative values. As a consequence, the behaviour of the first moment of the approximate radius of curvature versus height will be significantly different with these two models. This is clearly illustrated in Figure 4.21. The conditional height

density functions for the two surface models, presented in Figure 4.22, are noticeably different. The behaviour of the clearance probability versus local reflection angle curves for the Bandpass and Gaussian surface models, shown in Figure 4.23, are not dramatically different.

In Section 4.1.5.2, the mean power density versus reflection angle is shown to be essentially independent of σ_z , which suggests that the average scatter power should not be especially sensitive to the detailed nature of the surface model spectrum shape. However, because of the significantly different behaviour of the radius of curvature curves, there will be some dependance on the surface model (i.e. the proportionality factor $K(\beta_l, E_l)$ of (4.98) and (4.99) will depend on the surface model).

Consequently, the normalized mean of the random variable $R(\beta_l, E_l)$,

$$M_{R_{\text{norm}}}(\beta_l, E) = E\{R(\beta_l, E_l)\} \cdot \sigma_z, \quad (4.102)$$

will depend on the surface model. The behaviour of this normalized mean for the Bandpass and Gaussian surface models, as a function of local reflection angle, for an elevation angle of 10° and SS4(ii) surface conditions, is presented in Figure 4.24. Note that the curve for the Bandpass model is consistently above the Gaussian model curve. At high local reflection angles, the normalized means are 0.81 and 0.72 for the Bandpass and Gaussian

Figure 4.22: Conditional height density functions for the Bandpass and Gaussian spectrum surface models.

Figure 4.23: Clearance probability versus local reflection angle curves for the Bandpass and Gaussian spectrum surface models.

Figure 4.24: Integrated radius of curvature values, incorporating blockage and shadowing effects, for the Bandpass and Gaussian spectrum surface models.

Figure 4.25: Clearance probability curves for the fixed and variable interval calculation approaches for an elevation angle of 10° .

surface models, respectively, which corresponds to a difference of 0.5 dB.

The average scatter power levels obtained with the Bandpass and Gaussian surface models for an elevation angle of 10° , SS4(ii) surface conditions, and an omni antenna pattern are presented in Table 4.11. The scatter power levels obtained with the Bandpass and Gaussian surface models are not that different (Bandpass values are only around 0.25 dB higher than Gaussian model values). It is evident that the average scatter power levels are not very sensitive to detailed surface model spectral characteristics. However, it is important that the rms waveheight and rms surface slope parameters match. This is an important observation, which suggests that the bandpass surface model developed in Appendix A and used in both the physical and analysis models, is sufficient for characterizing the behaviour of ocean scatter for the aeronautical application.

4.5 Stochastic Model Enhancements and Conclusions

To match the approach currently used in the physical propagation simulation program, the clearance probability weighting factor employed in the analytic model considers a fixed interval $X_L = 2.5 \lambda$, independent of the local reflection angle and local waveheight. Improving this, by making the interval length considered a function of both

Signal Polarization	Average Scatter Power Levels [dB] For The	
	Bandpass Surface Model	Gaussian Surface Model
Horizontal	-2.73	-2.98
Vertical	-11.19	-11.45
Circular	-11.76	-12.0

Table 4.11: Average scatter power levels obtained with the Bandpass and Gaussian spectrum surface models for an elevation angle of 10° , SS4(ii) surface conditions, and an omni-antenna pattern.

local reflection angle and local waveheight, is not very difficult. The incremental blockage effect, once the ray from the scattering facet emerges beyond a height of $4\sigma_z$, where σ_z is the rms waveheight, is minimal. With this in mind, an appropriate interval length to consider for clearance probability calculations is

$$X_L = \frac{4\sigma_z - z_l}{\tan \beta_l}, \quad (4.103)$$

where z_l and β_l represent the local waveheight and local reflection angle at the scattering facet. This approach has been incorporated in the analytic model, but time did not permit doing so with the computationally demanding physical propagation simulation program.

The impact of such an approach can be established in an approximate fashion. With the fixed interval approach considered in the stochastic model,

$$\frac{\Delta z}{\sigma_z} = \frac{z - z_l}{\sigma_z} = \frac{1.685 N_I \tan(\beta_l)}{\sigma_z}, \quad (4.104)$$

where $N_I = 9$. For SS4(ii) surface conditions and local reflection angles greater than 1° ,

$$\frac{\Delta z}{\sigma_z} > 2.37.$$

It is clearly evident that for low local waveheights, the clearance probabilities predicted by the fixed interval approach will be small. Making the interval a function waveheight and reflection angle will result in even lower probabilities. In fact, the deviation between the fixed and variable interval length approaches will be greatest in this low waveheight region. In the higher waveheight region, where the clearance probabilities are much more substantial, the deviations between the approaches will be relatively small. Since the average scatter power from a specific region involves an average over waveheight, the high local waveheight region will dominate and the impact of the new clearance probability calculation approach will be diminished. Evidence of this is provided in Figure 4.25, which shows the clearance probability versus local reflection angle curves for the two approaches. Differences are only apparent for very low reflection angles ($\beta_l < 1^\circ$).

Table 4.12 summarizes the effect of this clearance probability enhancement on the average scatter power for SS4(ii) surface conditions and an elevation angle of 10° . The impact is so minor that there is really no need upgrading the current approach incorporated in the physical propagation model.

Antenna Pattern	Signal Polar- ization	Average Scatter Power Levels [dB] For	
		Variable Interval Clearance Prob- ability Approach	Fixed Interval Clearance Prob- ability Approach
OMNI	Horizontal	-2.734	-2.728
	Vertical	-11.191	-11.190
	Circular	-11.769	-11.758
CROSSED SLOT	Horizontal	-22.201	-22.184
	Vertical	-17.088	-17.085
	Circular	-25.689	-25.680

Table 4.12: Average scatter power levels obtained with the stochastic analysis program for the variable and fixed interval clearance probability weighting factors for SS4(ii) surface conditions and an elevation angle of 10°.

A recommended enhancement for improving the blockage and shadowing weighting factors involves developing a blockage attenuation factor which depends on minimum path clearance between the reflected ray and the ocean surface.

Incorporating such an approach in the physical propagation simulation program would be much easier than incorporating it in the stochastic model. Before incorporating this approach in the stochastic model, one must solve, or at least come up with a reasonable approximate solution for this difficult analysis problem.

The complexity reduction of the aeronautical stochastic analysis model, relative to the physical propagation simulation model, is impressive (approximately two orders of magnitude), and the accuracy of the first and second order statistics is not compromised. With the stochastic analysis model, the behaviour of ocean scatter for aeronautical applications can be realistically characterized on computer systems with relatively modest computational power. The use of high speed arithmetic peripherals, such as array processors, is no longer a prerequisite. The software development environments and reliability of array processor systems are, generally speaking, vastly inferior to those of typical general purpose mini-computer systems. The reliability problems associated with array processor systems are especially disturbing; introducing delays which significantly diminish

the speed advantage of these processors. In light of these characteristic traits, eliminating the need for these high speed processors is an important achievement.

CHAPTER 5.0

SUMMARY AND CONCLUSIONS

Realistic ocean scatter propagation models, for characterizing the behaviour of multipath for aircraft-to-satellite and ship-to-satellite communication applications, have been developed.

Some major components of these propagation models include

- (1) a realistic ocean surface model, which incorporates practical wave velocities to simulate the dispersive nature of ocean surface motion,
- (2) incorporating the coupling between vessel motion and the motion of the ocean surface, as well as antenna stabilization,
- (3) incorporating the shadowing of primary scattering elements by the wave profile,
- (4) incorporating the variability of the reflection coefficient and mobile antenna effects on the scatter signals from each individual contributing scattering facet, and
- (5) incorporating the divergence and phase shift resulting from scattering from convex and concave surface segments.

One of the major accomplishments was the development of a realistic divergence factor which accounts for focussing effects of concave surface facets close to the mobile antenna. This is critical for a reasonable characterization of multipath for ship-to-satellite communication applications.

The resulting model was accurately validated against a known exact far-field theoretical solution for a perfectly conducting sinusoidal surface, clearly demonstrating that the divergence and phase correction factors introduced are realistic.

Comparisons between measurement data and simulation results generally validate the accuracy of the propagation simulation models. Simulation results for low complexity crossed slot aircraft antennas demonstrate that ocean scatter multipath is not very severe (worst case signal to interference ratio for level flight conditions is approximately 14.4 dB). Ocean scatter multipath is considerably worse for maritime applications. For representative low gain shipborne antennas, signal to interference ratios for elevation angles between 5° and 15° are typically in the neighbourhood of 8 to 11 dB. The fading experienced on maritime channels is considerably slower than that experienced on aeronautical channels.

Typical fading rates are 1 to 2 Hz and 30 to 200 Hz for the maritime and aeronautical applications, respectively.

Several fundamental trends in the behaviour of ocean generated multipath are the same for both propagation models. These include

- (1) at low elevation angles, the interference levels obtained with vertical polarization are appreciably lower than those obtained with circular polarization,
- (2) there is a tendency for the level of interference to increase as the elevation angle decreases with circular polarization,
- (3) the fading bandwidth decreases as the elevation angle is lowered, and
- (4) for rough surface conditions, the scatter power level is relatively insensitive to sea state and mobile antenna height.

Maritime measurement results [7] exhibit the same fundamental trends.

The most significant accomplishment was the development of a stochastic analysis model, which has the capability of estimating both the first and second order statistics of the aeronautical channel. Relative to the physical

propagation simulation program, an impressive (approximately two orders of magnitude) reduction in execution time is obtained with the stochastic analysis model. This computational reduction is accomplished without compromising the accuracy of the first and second order statistics. The same fundamental trends for the behaviour of ocean scatter are obtained with the stochastic analysis model. A much better appreciation and understanding of the fundamental reasons for this behaviour are obtained with the analytic model.

With the stochastic analysis model, one can characterize the behaviour of ocean scatter for aeronautical applications using computer systems with relatively modest computational power, such as an IBM-PC. This is in direct contrast to the physical propagation model, where simulation runs commonly require several hours to complete on high speed array processor computing systems.

The computational requirements of the maritime propagation model, although not as severe as the aeronautical propagation model, are not trivial. Given the significant computational improvements afforded by the aeronautical analytic model, and the accuracy of the results obtained, the development of an analysis model for the maritime environment would be a valuable contribution. Some

preliminary ground work for such a model has been reported in this thesis. The major recommendation for further work involves completing the development of the maritime model and validating each of the fundamental model components using the maritime propagation simulation program as a reference.

APPENDIX A

SEA SURFACE MODEL

1.0 INTRODUCTION

In this appendix, a general surface model for the ocean surface is presented. This model can be used to simulate a wide variety of surface characteristics, ranging from periodic sinusoidal surfaces to Gaussian random process profiles. The model also incorporates practical wave velocities for each of the fundamental components to simulate dispersive wave motion. This is critical for the maritime application, because of the coupling between the ocean surface and vessel motion.

The rough wave surface generated by this surface model is representative of a local waveheight profile, which is then superimposed on a curved earth surface. Consequently, the actual surface height function, $z_s(x,t)$, is given by

$$z_s(x,t) = z(x,t) - \frac{x^2}{2a_e}, \quad (A.1)$$

where

$z(x,t)$ is the local waveheight profile,

$a_e = 6378$ km (radius of the earth) [11], and

x is the x-coordinate measured relative to the starting position.

It is important to incorporate curvature because it limits the size of the active scattering region to the horizon range of the antenna, thereby limiting the number of potential scattering facets in a realistic fashion.

2.0 SURFACE MODEL DEVELOPMENT

The surface of the sea has been the subject of many theoretical and experimental investigations. Some of the results of these investigations are summarized in Section 3.4 and Section 4.6 of reference [12]. Pages 30-35 of [31] contain a detailed description of the sea surface. The consensus is that a periodic sinusoidal surface is usually not an accurate model of typical one-dimensional cross sections of the ocean surface. A more realistic model is a Gaussian random process surface profile, with a correlation distance of approximately $\lambda/4$, where λ is the average sea wavelength. It is often assumed that the surface autocorrelation function $R_{zz}(x)$, is also Gaussian [1], [12]

$$R_{zz}(x) = \sigma_z^2 \exp\left\{-\left(\frac{x}{\tau}\right)^2\right\} \quad (A.2)$$

where

τ denotes the e^{-1} correlation distance, and

σ_z represents the rms waveheight.

The interrelationship between surface parameters and the sea state characterization of the surface is addressed very well on pages 30-35 of [31]. This information was used to derive the sea state table presented in Table A.1. This gives the average sea wavelength λ , significant waveheight H_s , and rms waveheight σ_z versus sea state index. The rms waveheights are derived from the significant waveheights by the relationship [31]

$$\sigma_z = \frac{H_s}{4} \quad (A.3)$$

The reason for using the value 4 rather than 4.66 (98% confidence limit) in this relationship is because the significant waveheight does not represent extreme values, but rather the average of the 1/3 highest waves. The sea state table presented in Table A.1 is in reasonable agreement with that presented in Table 5.3 of [32].

Ignoring the time variation of the surface for a moment, one can easily generate a Gaussian wave process $z(x)$, which provides an arbitrary spatial spectral density, by appropriately filtering a white Gaussian noise process. Some of the difficulties associated with this approach include

Sea State Index	Significant Waveheight H_s (meters) (Crest to Trough)	RMS Waveheight σ_z (meters)	Average Sea Wavelength λ (meters)	$\frac{\lambda}{H_s}$
1	0.3048	0.0762	6.1	20.0
2(i)	0.67	0.1675	12.2	18.2
2(ii)	0.884	0.221	16.04	18.1
3	1.4	0.35	22.32	15.92
4(i)	2.1	0.525	30.75	14.64
4(ii)	2.44	0.61	34.26	14.05
5(i)	3.048	0.762	41.86	13.73
5(ii)	3.658	0.9145	48.77	13.33
6(i)	4.572	1.143	57.75	12.63
6(ii)	5.49	1.3725	65.82	12.0
7(i)	7.925	1.981	87.33	11.0
7(ii)	11.28	2.82	116.27	10.3

Table A.1: Sea State Table

- (1) it is difficult to evaluate derivatives of the surface process, which are required in the geometrical search process for specular points, and
- (2) it is difficult to incorporate the dispersive nature of the motion of the ocean surface with time.

The most suitable representation is one which involves the superposition of the standard basis functions which can be easily differentiated and for which motion can be easily incorporated.

A suitable candidate wave distribution that satisfies the above mentioned requirements, in general, is

$$z(x) = \sum_{k=-N}^N a_k \cos[2\pi u_k x + \theta_k] \quad (\text{A.4})$$

where

$u_k = u_0 + k \Delta u$ (k-th spatial frequency component with inverse wavelength dimensions),

θ_k is a uniformly distributed random variable associated with the k-th spatial frequency component,

$$\Delta u = \frac{B}{N_T},$$

$$N_T = 2N + 1, \text{ and}$$

B is the bandwidth of the bandpass spatial spectrum.

The spatial spectrum, $S_{zz}(u)$, consists of impulses located at $\pm u_k$ weighted by the factor $a_k^2/4$,

$$S_{zz}(u) = \sum_{k=-N}^N \frac{a_k^2}{4} [\delta(u-u_k) + \delta(u+u_k)]. \quad (A.5)$$

This is analogous to the standard approximation for representing the spectrum of a Gaussian noise process by a series of weighted impulses [33], [34]. The approximation becomes exact in the limit as $N \rightarrow \infty$ ($\Delta u \rightarrow 0$). From the treatments in [33] and [34], it is apparent that in order for the wave expression in (A.4) to be an accurate representation of the wave process with spatial spectrum $S_{zz}(u)$, one must choose a_k such that

$$a_k^2 = 4 \Delta u S_{zz}(u_k). \quad (A.6)$$

Although it is possible to handle any arbitrary spatial spectrum shape, a flat bandpass spectrum has been

selected as a representative case (see Figure A.1). With this surface model, each sinusoidal surface component has the same amplitude, c , and the bandwidth $B = \gamma u_0$. The spatial autocorrelation function, corresponding to the selected spectrum for this model is

$$R_{zz}(x) = \sigma_z^2 \text{sinc}(B x) \cos(2\pi u_0 x), \quad (\text{A.7})$$

which does not differ very much, for small values of x , from the Gaussian autocorrelation function of equation (A.2).

Another nice feature of the superposition of sinusoids surface model is that it allows one to incorporate wave motion relatively easily, as indicated below

$$z(x,t) = c \sum_{k=-N}^N \cos\left[\frac{2\pi}{\lambda_k} (x - v_k t) + \theta_k\right] \quad (\text{A.8})$$

where

λ_k = the wavelength of the k -th surface component
(i.e. $\lambda_k = \frac{1}{u_k}$),

v_k = the velocity of the k -th surface component in the x -direction,

Figure A.1: A representative bandpass spatial spectrum for the wave process.

$$c = \frac{\sqrt{2} \sigma_z}{\sqrt{N_T}},$$

$$N_T = 2N + 1, \text{ and}$$

σ_z is the rms waveheight.

It is well known that the velocities of each surface component are not identical, thereby resulting in the dispersive nature of the motion of the sea surface. From [31] and [35], it is apparent that

$$v_k = a \sqrt{\lambda_k} \tag{A.9}$$

where a is a proportionality constant. Consequently, to complete the surface model characterization, all one has to do is relate the spatial centre frequency to the average sea wavelength, and establish appropriate bandwidth and velocity proportionality constants.

2.1 Establishing A Realistic Bandwidth Proportionality Constant

For Gaussian random processes, it is relatively easy to establish an expression for the density of zero crossings λ (pages 485-491 of [26]). For the bandpass spatial spectrum of the surface model (page 489 of [26])

$$\lambda = 2 u_0 \sqrt{1 + \frac{\gamma^2}{12}} . \quad (\text{A.10})$$

A reasonable definition of the average wavelength is the distance between successive zero crossings in the same direction. That is

$$\lambda = \frac{2}{\frac{1}{u_0 \sqrt{1 + \frac{\gamma^2}{12}}}} = \frac{2 u_0}{\frac{1}{\sqrt{1 + \frac{\gamma^2}{12}}}} \quad (\text{A.11})$$

or

$$u_0 = \frac{1}{\lambda \sqrt{1 + \frac{\gamma^2}{12}}} . \quad (\text{A.12})$$

The normalized correlation function, $C_{zz}(x) = \frac{R_{zz}(x)}{\sigma_z^2}$, of a realistic example of a wave that ceases to be periodic [12] is illustrated in Figure A.2. Also shown in this figure are the correlation functions of a pure sinusoidal wave and the often assumed Gaussian correlation function [12].

The normalized correlation function of the white bandpass surface model is given by

$$C_{zz}(x) = \text{sinc}(\gamma u_0 x) \cos (2\pi u_0 x) . \quad (\text{A.13})$$

Figure A.2: Correlation properties of a realistic dispersive wave along with Gaussian and cosinusoidal correlation functions (from [12]).

Figure A.3: Normalized waveheight correlation function of the white Bandpass surface model.

This correlation function is shown in Figure A.3. It is evident that this correlation function is in reasonable agreement with the realistic ocean wave correlation function shown in Figure A.2.

A realistic approach for establishing the parameter τ of the Gaussian model is to select τ such that the rms surface slope matches that of the actual underlying wave process. The rms surface slope has a significant impact on ocean induced multipath, because it influences the number of potential scattering facets at low reflection angles and has a significant impact on the doppler spectrum characteristics (see Appendix E), (i.e. the doppler spectrum width is proportional to the rms surface slope). Consequently, when considering different surface models, it is important to match both the rms waveheights and rms surface slopes of the models.

At this point, it is important to establish the autocorrelation and cross-correlation properties of filtered random processes. These are derived on pages 344-347 of [26], and summarized in Figure A.4. Furthermore, if the input process is Gaussian in nature, the output of the linear filtering process is also Gaussian. These relationships can be used to establish the rms surface slope of the random surface profile. For the differentiation operation,

$$R_{zz}(\tau) = \mathcal{F}^{-1} \{ S_{zz}(u) \} = \int_{-\infty}^{\infty} S_{zz}(u) e^{+j2\pi u \tau} du$$

$$R_{yz}(\tau) = R_{zz}(\tau) \odot h(\tau)$$

$$S_{yz}(u) = S_{zz}(u) H(u)$$

$$R_{zy}(\tau) = R_{yz}^*(-\tau)$$

$$S_{zy}(u) = S_{zz}(u) H^*(u)$$

$$R_{yy}(\tau) = R_{zz}(\tau) \odot h^*(-\tau) \odot h(\tau)$$

$$S_{yy}(u) = S_{zz}(u) |H(u)|^2$$

Figure A.4: Auto and cross-correlation properties of filtered random processes.

$$H(u) = j 2\pi u \quad (A.14)$$

and

$$S_{z'z'}(u) = (2\pi u)^2 S_{zz}(u). \quad (A.15)$$

The autocorrelation function is given by

$$\begin{aligned} R_{z'z'}(x) &= \int_{-\infty}^{\infty} (2\pi u)^2 S_{zz}(u) e^{+j2\pi ux} du \\ &= - \frac{d^2}{dx^2} R_{zz}(x). \end{aligned} \quad (A.16)$$

The surface slope variance,

$$\sigma_{z'}^2 = E\{z'(x)^2\} = R_{z'z'}(0) = -R_{zz}''(0). \quad (A.17)$$

For the Gaussian correlation function of (A.2),

$$R_{zz}''(0) = - \frac{2\sigma_z^2}{r^2} \quad (A.18)$$

For the white bandpass surface model,

$$\begin{aligned} R_{zz}''(0) &= -(2\pi u_0)^2 \left(1 + \frac{\gamma^2}{12}\right) \sigma_z^2 \\ &= -\left(\frac{2\pi}{\lambda}\right)^2 \sigma_z^2, \end{aligned} \quad (A.19)$$

where this latter simplification follows directly from (A.11). Therefore, the rms surface slopes obtained with the Gaussian and Bandpass surface models are

$$\sigma_{z'}(G) = \frac{\sqrt{2} \sigma_z}{\tau}, \quad (A.20)$$

$$\sigma_{z'}(BP) = \frac{2\pi\sigma_z}{\lambda}. \quad (A.21)$$

Equating the rms surface slopes yields

$$\tau = \frac{\lambda}{\sqrt{2}} = \frac{\lambda}{4.44}. \quad (A.22)$$

This matches the result derived in [12] for the best match between a Gaussian and sinusoidal correlation function. However, it is evident that the above relationship applies in general for the white bandpass surface model, independent of the bandwidth parameter γ .

The behaviour of the rms surface slope $\sigma_{z'} = \frac{2\pi\sigma_z}{\lambda}$ versus sea state is summarized in Table A.2. Note that the surface slope increases as the sea state index increases. However, in the neighbourhood of sea state SS4(ii), which is typical of North Atlantic conditions, the surface slope is not very sensitive to sea state index.

The approach used to select an appropriate bandwidth parameter involves matching the spectral occupancy (in terms of noise equivalent bandwidth) of the white bandpass

Sea State Index	Rms Surface Slope
1	0.07849
2(i)	0.08627
2(ii)	0.08657
3	0.09853
4(i)	0.10727
4(ii)	0.11187
5(i)	0.11438
5(ii)	0.11782
6(i)	0.12436
6(ii)	0.13102
7(i)	0.14253
7(ii)	0.15239

Table A.2: Behaviour of rms surface slope versus sea state index.

surface model to that of the often assumed Gaussian model [12]. This is illustrated in Figure A.5.

For this Gaussian surface model, with $R_{zz}(x) = \sigma_z^2 e^{-x^2/\tau^2}$, the spatial spectrum can be expressed as

$$S_{zz}(u) = \frac{\sigma_z^2}{\sqrt{2\pi} \sigma_G} \exp \left\{ -\frac{1}{2} \left(\frac{u}{\sigma_G} \right)^2 \right\} \quad (\text{A.23})$$

where

$$\sigma_G = \frac{\sqrt{2}}{\sqrt{2\pi} \tau} \quad (\text{A.24})$$

Using (A.22), the noise equivalent bandwidth $B_E(G)$ of this spectrum can be expressed as

$$B_E(G) = \sqrt{2\pi} \sigma_G = \frac{\sqrt{2}}{\sqrt{2\pi} \tau} = \frac{\sqrt{2\pi}}{\Lambda} \quad (\text{A.25})$$

The noise equivalent bandwidth of the white bandpass surface model is easily seen to be

$$B_E(BP) = 2B = 2\gamma u_0 \quad (\text{A.26})$$

Equating the noise equivalent bandwidths of the two models and simplifying using (A.12) yields

$$\gamma^2 = \frac{\pi}{2} \left(1 + \frac{\gamma^2}{12} \right) \quad (\text{A.27})$$

Figure A.5: Equating the noise equivalent bandwidths of the Gaussian spectrum and white Bandpass surface models.

for which the solution is

$$\gamma = \sqrt{\frac{\frac{\pi}{2}}{(1 - \frac{\pi}{24})}} \quad (\text{A.28})$$

≈ 1.344

Substituting this into (A.12) gives

$$u_o = \frac{1}{1.0726 \lambda} \quad (\text{A.29})$$

It should be noted that this analysis does not apply for the special case when the operator selects a single random phased sinusoidal component. When $N_T = 1$,

$$\lambda_o = \frac{1}{u_o} = \lambda. \quad (\text{A.30})$$

2.2 Establishing a Realistic Velocity Proportionality Constant

The ocean surface does not move as a rigid body with each component travelling at the same velocity, but rather as a dispersive medium with the velocity of each fundamental component being different. In fact, the velocity of each component is proportional to the square root of its wavelength (A.9)

$$v_k = \omega \sqrt{\lambda_k}$$

From the relationships presented in [31], it can be shown that the average wave velocity $\bar{v}$ and average sea wavelength λ are related as follows

$$\bar{v} = 1.85 \sqrt{\lambda_{ft}} \quad \text{ft/sec} \quad (\text{A.31})$$

$$= 1.02 \sqrt{\lambda_m} \quad \text{m/sec} \quad (\text{A.32})$$

where λ_{ft} and λ_m denote the average sea wavelength in feet and meters respectively.

In [35], it is claimed that the relationship between wavespeed v and wavelength λ is given by

$$v = 2.26 \sqrt{\lambda_{ft}} \quad \text{ft/sec} \quad (\text{A.33})$$

$$= 1.25 \sqrt{\lambda_m} \quad \text{m/sec.} \quad (\text{A.34})$$

If one defines the average wave velocity as

$$\bar{v} = \frac{1}{N_T} \sum_{i=1}^{N_T} v_i \quad (\text{A.35})$$

where $v_1 = \alpha \sqrt{\lambda_1} = \frac{\alpha}{\sqrt{u_1}}$,

the average wavespeed can be approximated, for large N_T , by

$$\begin{aligned} \bar{v} &= \frac{\alpha}{B} \int_{(1-\frac{\gamma}{2})u_0}^{(1+\frac{\gamma}{2})u_0} \frac{dt}{\sqrt{t}} \\ &= \frac{2\alpha}{\gamma} \left(\sqrt{1 + \frac{\gamma}{2}} - \sqrt{1 - \frac{\gamma}{2}} \right) \frac{1}{\sqrt{u_0}}. \end{aligned} \quad (A.36)$$

Equating this expression for the average wave velocity to that in (A.32) and using $\gamma = 1.344$ and the fact that

$$u_0 = \frac{1}{\lambda \sqrt{1 + \frac{\gamma^2}{12}}}, \text{ one obtains a velocity proportionality}$$

constant $\alpha = 0.92$.

It is apparent that choosing a velocity proportionality constant anywhere between 0.92 and 1.25 can be justified. Consequently, the average velocity relationship has been selected for each individual component; that is

$$v_k = 1.02 \sqrt{\lambda_k} = \frac{1.02}{\sqrt{u_k}}. \quad (A.37)$$

At this point it is interesting to establish some typical average wavespeeds using (A.32) and average sea wavelength parameters from Table A.1. Appropriate values are

$$\bar{v} = \begin{cases} 4.8 \text{ m/sec} & \text{for SS3} \\ 6.0 \text{ m/sec} & \text{for SS4(ii)} \\ 6.6 \text{ m/sec} & \text{for SS5(i)} \end{cases} \quad (\text{A.38})$$

These wavespeeds are roughly two orders of magnitude lower than typical aircraft speeds of 268.2 m/sec (600 mph). Consequently, when one extracts relatively short records of propagation data, one can effectively ignore ocean surface motion, thereby considering a still surface (equation (A.4)).

This has significant implementation consequences for the aeronautical application, where one can effectively normalize with respect to aircraft velocity. This is done by taking ocean scatter multipath samples with respect to position rather than time. This is definitely not possible for the maritime application, where the wave speeds are of the same order as typical vessel velocities. Consequently, one must use the more general expression of (A.8) to describe the local waveheight for the maritime application. One can simulate wave motion in the same and opposite directions to vessel motion. The number of surface components N_T is under operator control. When attempting to simulate Gaussian surface profiles, one should choose a

relatively large value for N_T . However, one must avoid choosing excessively large values because of the computational impact on the evaluation of the local waveheight and surface derivatives. The total of sinusoidal components, used for the aeronautical and maritime simulation runs, was $N_T = 21$. To specify the actual surface conditions, the operator enters either one of the sea state indices contained in Table A.1, or specific values for the rms waveheight σ_z and the average sea wavelength λ directly.

3.0 IMPORTANT CONDITIONAL DISTRIBUTIONS

In this section, important conditional distributions required in the development of the stochastic analysis model are presented. First of all, the density function of the following vector of zero mean Gaussian random variables is considered

$$\vec{z} = \begin{bmatrix} z \\ z' \\ z'' \\ z''' \end{bmatrix} = \begin{bmatrix} z(x_s) \\ z'(x_s) \\ z''(x_s) \\ z'''(x_s) \end{bmatrix}$$

Using the expressions presented in Figure A.4, with $H(u) = (j2\pi u)^n$, the general expression for the moments

involving derivatives taken at the same sampling instant is given by

$$E\{z^{(m+n)}(x_s) z^{(m)}(x_s)\} = \int_{-\infty}^{\infty} (j2\pi u)^n S_{z^{(m)} z^{(m)}}(u) du$$

$$= \frac{d^n}{dx^n} R_{z^{(m)} z^{(m)}}(x) \Big|_{x=0} \quad (A.39)$$

where $z^{(m+n)}(x)$ and $z^{(m)}(x)$ denote the $(m+n)$ th and (m) th derivative of the local waveheight profile.

It is immediately evident that for n odd, the above correlation coefficients are zero, which implies that the associated random variables are independent (because they are Gaussian random variables). As a result,

$$f_{zz'z''z'''}(a,b,c,d) = f_{zz''}(a,c) f_{z'z'''}(b,d). \quad (A.40)$$

It is important to note that both the waveheight and surface second derivatives at a specular point are independent of the slope at that point.

The waveheight and surface second derivative density function can be expressed as (page 220 of [26])

$$f_{zz''}(a,c) = f_z(c/z=a) f_z(a) \quad (A.41)$$

where

$$f_{z''}(c/z=a) = \frac{1}{\sqrt{2\pi} \sigma_{m_2}} \exp\left\{-\frac{1}{2\sigma_{m_2}^2} (c - K_2 a)^2\right\} \quad (A.42)$$

and

$$K_2 = \frac{E[z'' z]}{\sigma_z^2}, \quad (A.43)$$

$$r_2 = \frac{E[z'' z]}{\sigma_{z''} \sigma_z}, \quad (A.44)$$

$$\sigma_{m_2} = \sqrt{1 - r_2^2} \sigma_{z''}. \quad (A.45)$$

Using (A.39), it is evident that

$$E[z'' z] = - \int_{-\infty}^{\infty} (2\pi u)^2 S_{zz}(u) du$$

$$= R''_{zz}(0)$$

$$= - \sigma_z^2. \quad (A.46)$$

where this later simplification follows directly from (A.17).

The variance of the second derivative can be established using the relationships of Figure A.4 with $H(u) = -(2\pi u)^2$,

$$\begin{aligned}\sigma_{z''}^2 &= E\{z''^2\} = \int_{-\infty}^{\infty} (2\pi u)^4 S_{zz}(u) du \\ &= R_{zz}^{(4)}(0).\end{aligned}\quad (A.47)$$

The appropriate parameter definitions for the conditional second derivative density function of (A.42) for the white Bandpass spectrum and Gaussian spectrum surface models are presented in Table A.3.

Similarly, the surface slope and surface third derivative density function can be expressed as

$$f_{z'z'''}(b,d) = f_{z'''}(d/z'=b) f_{z'}(b) \quad (A.48)$$

where

$$f_{z'''}(d/z'=b) = \frac{1}{\sqrt{2\pi} \sigma_{m_3}} \exp\left\{-\frac{1}{2\sigma_{m_3}^2} (d - K_3 b)^2\right\} \quad (A.49)$$

and

$$K_3 = \frac{E\{z''' z'\}}{\sigma_{z'}^2}, \quad (A.50)$$

$$r_3 = \frac{E\{z''' z''\}}{\sigma_{z'''} \sigma_{z''}}, \quad (A.51)$$

$$\sigma_{m_3} = \sqrt{1 - r_3^2} \sigma_{z''}. \quad (A.52)$$

Using (A.39), it is evident that

$$E\{z''' z'\} = \int_{-\infty}^{\infty} -(2\pi u)^2 S_{z'z'}(u) du$$

$$f_{z''}(c/z=a) = \frac{1}{\sqrt{2\pi} \sigma_{m_2}} \exp\left\{\frac{-1}{2\sigma_{m_2}^2} (c - K_2 a)^2\right\}$$

where

$$K_2 = - \left(\frac{\sigma_{z'}}{\sigma_z} \right)^2$$

$$r_2 = \frac{-\sigma_{z'}^2}{\sigma_{z''} \sigma_z} = - \frac{1}{\eta}$$

$$\sigma_{m_2} = \sqrt{1 - r_2^2} \sigma_{z''}$$

$$\sigma_{z'} = \frac{2\pi \sigma_z}{\lambda}$$

$$\sigma_{z''} = \eta \frac{\sigma_{z'}^2}{\sigma_z}$$

$$\eta = \begin{cases} \frac{\sqrt{1 + \frac{\gamma^2}{2} + \frac{\gamma^4}{80}}}{(1 + \frac{\gamma^2}{12})} & \text{for the Bandpass Model} \\ \sqrt{3} & \text{for the Gaussian Model} \end{cases}$$

$$\gamma = 1.344 \text{ (Bandwidth Factor for Bandpass Model)}$$

Table A.3: Summary of the relationships for the conditional second derivative density function for the white Bandpass and Gaussian spectrum surface models.

$$= - \int_{-\infty}^{\infty} (2\pi u)^4 S_{zz}(u) du$$

$$= - R_{zz}^{(4)}(0) = - \sigma_z^2 \quad (A.53)$$

The variance of the third derivative can be established using the relationships of Figure A.4 with $H(u) = (j2\pi u)^3$,

$$\sigma_{z'''}^2 = E\{z'''^2\} = \int_{-\infty}^{\infty} (2\pi u)^6 S_{zz}(u) du$$

$$= - R_{zz}^{(6)}(0) \quad (A.54)$$

The appropriate parameter definitions for the conditional third derivative density function of (A.49) for the white Bandpass spectrum and Gaussian spectrum surface models are presented in Table A.4.

The correlation function between the surface slope and the surface waveheight can be expressed as

$$R_{z',z}(x) = E\{z'(x_s) z(x_s + x)\}$$

$$= \int_{-\infty}^{\infty} (j2\pi u) S_{zz}(u) e^{j2\pi ux} du = \frac{d}{dx} R_{zz}(x).$$

$$(A.55)$$

$$f_{z'''}(d/z'=b) = \frac{1}{\sqrt{2\pi} \sigma_{m_3}} \exp\left\{-\frac{1}{2\sigma_{m_3}^2}(d - K_3 b)^2\right\}$$

where

$$K_3 = -\left(\frac{\sigma_{z''}}{\sigma_{z'}}\right)^2$$

$$r_3 = \frac{-\sigma_{z''}^2}{\sigma_{z'''} \sigma_{z'}}$$

$$\sigma_{m_3} = \sqrt{1 - r_3^2} \sigma_{z'''}$$

$$\sigma_{z'} = \frac{2\pi\sigma_z}{\lambda}$$

$$\sigma_{z''} = \eta \frac{\sigma_z^2}{\sigma_z}$$

$$\eta = \begin{cases} \frac{\sqrt{1 + \frac{\gamma^2}{2} + \frac{\gamma^4}{80}}}{(1 + \frac{\gamma^2}{12})} & \text{for the Bandpass Model} \\ \sqrt{3} & \text{for the Gaussian Model} \end{cases}$$

$$\sigma_{z'''} = \begin{cases} (\pi u_0)^3 \sigma_z \sqrt{64 + 80\gamma^2 + 12\gamma^4 + \frac{\gamma^6}{7}} & \text{for the Bandpass Model} \\ \frac{\sqrt{120} \sigma_z}{\tau^3} & \text{for the Gaussian Model} \end{cases}$$

$$u_0 = \frac{1}{\lambda \sqrt{1 + \frac{\gamma^2}{12}}}, \quad \tau = \frac{\lambda}{\sqrt{2\pi}}, \quad \gamma = 1.344$$

Table A.4: Summary of the relationships for the conditional third derivative density function for the white Bandpass and Gaussian spectrum surface models.

For the white bandpass model,

$$R_{z,z}(x) = \sigma_z^2 \left\{ \left[\cos(\gamma u_0 x) - \text{sinc}(\gamma u_0 x) \right] \frac{\cos(2\pi u_0 x)}{x} - (2\pi u_0) \text{sinc}(\gamma u_0 x) \sin(2\pi u_0 x) \right\}. \quad (\text{A.56})$$

The normalized correlation function

$$\rho(x) = \frac{R_{z,z}(x)}{\sigma_z \sigma_z} \quad (\text{A.57})$$

is plotted in Figure A.6 for SS4(ii) surface conditions with the bandpass surface model. This function is useful for incorporating slope-waveheight correlation effects in surface blockage (clearance probability) calculations.

4.0 DEVIATIONS FROM GAUSSIAN DISTRIBUTIONS

The superposition of random phase sinusoids bandpass surface model, with 21 surface components, provides rms waveheight, rms surface slope, and rms surface second derivatives that agree with theoretical values for a Gaussian random process model with a bandpass spatial spectrum. However, there are differences between the density functions obtained with the superposition of sinusoids surface model and the assumed Gaussian distributions used in the stochastic analysis model.

Figure A.6: Normalized slope-height correlation function for the Bandpass surface model.

One of the major differences is in the tail region, where there exists a cutoff point for waveheight and surface slopes with the superposition of sinusoids surface model, beyond which there are no occurrences. This cutoff point depends on the random seed used to generate the random phases. Specifically for the SZERO surface profile, no occurrences were observed in the regions

$$|z| > 3\sigma_z \quad (A.58)$$

$$|z'| > 2.8 \sigma_{z'} \quad (A.59)$$

It should be noted that rather than being the superposition of uniform random variables, the surface at a specific position is effectively given by the sum of the cosine of uniformly distributed and independent random phases. The random variable $Y_i = \cos \theta_i$, where θ_i is a uniformly distributed random variable on $[-\pi, \pi]$, has a density function

$$f_{Y_i}(y) = \begin{cases} \frac{1}{2\pi\sqrt{1-y^2}} & \text{for } y \in [-1, 1] \\ 0 & \text{otherwise.} \end{cases} \quad (A.60)$$

This density function has singularities at $y = \pm 1$.

Therefore, it is not that surprising that the density function of the sum of independent random variables of this type (given by convolutions of the above density function)

will have deviations in the tail region. The sensitivity to the specific phase values selected to define a wave profile is also not that surprising.

How well the actual normalized height and slope distributions agree with assumed Gaussian distributions has also been considered. To do so, the normalized height/slope axis is divided into a number of cells of width Δ , and the distribution of the number of occurrences, N_i , one obtains in the i -th cell,

$$C_i = \left[\left(i - \frac{1}{2}\right) \Delta, \left(i + \frac{1}{2}\right) \Delta \right], \quad (\text{A.61})$$

given N independent Gaussian observations, is derived. Since one has Bernoulli trials, the resulting distribution is Binomial

$$P_{N_i}(n) = \binom{N}{n} p_i^n q_i^{N-n} \quad (\text{A.62})$$

where

$$\binom{N}{n} = \frac{N!}{(N-n)!n!}$$

$$q_i = 1 - p_i \quad (\text{A.63})$$

$$p_i = Q\left(\left(i - \frac{1}{2}\right)\Delta\right) - Q\left(\left(i + \frac{1}{2}\right)\Delta\right) \quad (\text{A.64})$$

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

For a binomial random variable (page 92 of [36])

$$E\{N_i\} = N p_i \quad (A.65)$$

$$\text{Var}\{N_i\} = N p_i q_i \quad (A.66)$$

These relationships can be used to derive approximate 95% confidence limit curves on the number of occurrences, i.e.

$$u_{95}(i) = N p_i + 1.96 \sqrt{N p_i q_i} \quad (A.67)$$

$$l_{95}(i) = N p_i - 1.96 \sqrt{N p_i q_i} \quad (A.68)$$

These curves are compared with simulation observations for height and slope with the SZERO surface profile and parameters $N = 10000$ and $\Delta = 0.2$ in Figure A.7. It is evident that, in addition to tail distribution differences, significant differences around the origin also exist.

The curves for the S5001 surface profile are presented in Figure A.8. Once again, noticeable differences in the neighbourhood of the origin are evident. The tail distribution of the slope process for this surface profile differs significantly from that of a Gaussian distribution. This behaviour can have a major impact on the number of

Figure A.7: Comparison of height and slope occurrences with the SZERO surface profile with Gaussian confidence limits.

Figure A.8: Comparison of height and slope occurrences with the S5001 surface profile with Gaussian confidence limits.

potential scattering facets in the horizon region,
especially for higher elevation angles, which require the
slope process to cross relatively high levels.

APPENDIX B

REFLECTION COEFFICIENTS FOR THE SEA SURFACE

The plane surface reflection coefficients for horizontal and vertical polarization are given by (see Chapter 16 of [14])

$$\Gamma_H = \frac{\sin \psi - \sqrt{(\epsilon_r - j60\lambda\sigma) - \cos^2 \psi}}{\sin \psi + \sqrt{(\epsilon_r - j60\lambda\sigma) - \cos^2 \psi}} \quad (B.1)$$

$$\Gamma_V = \frac{(\epsilon_r - j60\lambda\sigma) \sin \psi - \sqrt{\epsilon_r - j60\lambda\sigma - \cos^2 \psi}}{(\epsilon_r - j60\lambda\sigma) \sin \psi + \sqrt{\epsilon_r - j60\lambda\sigma - \cos^2 \psi}} \quad (B.2)$$

where

ψ is the surface grazing angle,

ϵ_r is the relative di-electric constant,

σ is the conductivity (mho/m), and

λ is carrier wavelength in meters.

Table 5.1 of [3] provides approximate electromagnetic properties of soil and water. The relevant parameters for sea water at L-band ($\lambda_c = 0.2$ m) are $\sigma = 4.3$ mho/m and $\epsilon_r = 80$. Values presented in Table 4.1 and Figure 4.3 of [12] suggest that a realistic range for the relative di-electric constant is $73 < \epsilon_r < 78$ and the most appropriate conductivity value is $\sigma = 4.0$ mho/m, at L-band. The values that have been selected as being representative of sea surface conditions for this study are

$$\sigma = 4 \text{ mho/m} \quad \epsilon_r = 80. \quad (\text{B.3})$$

Figure B.1 shows the magnitude and phase of the horizontal polarization reflection coefficient as a function of grazing angle for a carrier wavelength of 0.2 m.

The magnitude and phase of the vertical polarization reflection coefficient is presented, as a function of grazing angle, in Figure B.2 for a carrier wavelength of 0.2 m. Note the pronounced dip in the magnitude of the vertical polarization reflection coefficient which occurs in the neighbourhood of 6° . The angle at which this dip occurs is referred to as the Brewster angle. It is also interesting to note the significant change which occurs in

(a) Magnitude of horizontal polarization reflection coefficient.

(b) Phase of horizontal polarization reflection coefficient.

Figure B.1 Behaviour of the horizontal polarization reflection coefficient for smooth sea water at L-band ($\lambda_c = 0.2$ m)

(a) Magnitude of vertical polarization reflection coefficient.

(b) Phase of vertical polarization reflection coefficient.

Figure B.2: Behaviour of the vertical polarization reflection coefficient for smooth sea water at L-band ($\lambda_c = 0.2$ m).

the phase angle of the vertical polarization reflection coefficient in the neighbourhood of the Brewster angle.

The reflection coefficient for circular polarization can be expressed as

$$\Gamma_{\text{copol}} = \frac{(\Gamma_H + \Gamma_V)}{2} \quad (B.4)$$

The magnitude and phase of the circular polarization reflection coefficient is shown in Figure B.3 for a carrier wavelength of 0.2 m.

When designing elliptically polarized antennas, it is useful to know the polarization properties of the multipath signal. Here, the polarization properties of smooth sea surface reflections are examined. The polarization ratio is defined as

$$P = \frac{\Gamma_V}{\Gamma_H} = \frac{\rho_V}{\rho_H} e^{j[\phi_V - \phi_H]} \quad (B.5)$$

The magnitude of the polarization ratio (in decibels) and relative phase is illustrated in Figure B.4.

Realistic di-electric and conductivity parameters for first year sea ice at L-band are [37]

(a) Magnitude of circular polarization reflection coefficient.

(b) Phase of circular polarization reflection coefficient.

Figure B.3: Behaviour of the circular polarization reflection coefficient for smooth sea water at L-band ($\lambda_c = 0.2$ m).

(a) Magnitude of the polarization ratio (V/H).

(b) Phase of the polarization ratio.

Figure B.4: Behaviour of the surface reflection ellipticity ratio for sea water at L-band ($\lambda_c = 0.2$ m).

$$\epsilon_r = 3.48$$

$$\sigma = 0.02 \text{ mho/m.}$$

(B.6)

These parameters are significantly different than those appropriate for sea water, and result in significantly different reflection coefficient properties. This is clearly illustrated in Figure B.5, which shows the magnitude of the horizontal and vertical polarization reflection coefficients for first year sea ice, as a function of grazing angle. Note that the Brewster angle for first year sea ice at L-band is in the neighbourhood of 28° , which is significantly higher than it is for sea water.

(a) Horizontal polarization

(b) Vertical polarization

Figure B.5: Magnitude of reflection coefficients for first year sea ice (dielectric constant = 3.48, conductivity = 0.02 mho/m) at L-band ($\lambda_c = 0.2$ m).

APPENDIX C

SIGNAL ATTENUATION FACTORS FOR SCATTERING FROM PARABOLIC CONVEX AND CONCAVE SURFACE FACETS

1.0 INTRODUCTION

In this appendix, signal attenuation factors appropriate for scattering from curved surface facets, for both the maritime and aeronautical applications, are derived. Fresnel diffraction theory solution approaches are presented, in addition to qualitative solution procedures. A simple analysis, which suggests that the computationally tractable two dimensional surface scattering model should provide realistic results, is also contained in this appendix. The derivation of path length differences relative to the nominal specular path, which are required to establish both Fresnel zone dimensions and parameters for the Fresnel diffraction integrals, are derived in the next section.

2.0 PATH LENGTH DIFFERENCE DERIVATIONS

To establish Fresnel zone dimensions appropriate for a tangent plane and parabolic surface facets, expressions for the path length difference, relative to the specular path, must be derived. For satellite communications

applications, the satellite transmitter is located a substantial distance from the specular point. In this case, the geometry of Figure C.1 is appropriate. A parabolic concave surface facet is provided as an example of a representative scattering element. Note that it is assumed that there is no variation in the scattering facet along the y-axis. The path length difference in Figure C.1 is given by

$$\delta = (RP - RS) + (PT - ST) . \quad (C.1)$$

It is not difficult to show that $(PT - ST) = -(\ell_x + \ell_z)$ where

$$\ell_x = (x - x_s) \cos \psi , \quad (C.2)$$

$$\ell_z = z(x,y) \sin \psi , \quad (C.3)$$

ψ is the surface grazing angle.

It is also evident that

$$(RP - RS) = \sqrt{x^2 + y^2 + (h - z(x,y))^2} - \frac{h}{\sin \psi} . \quad (C.4)$$

Combining these results and simplifying yields

$$\delta = \sqrt{x^2 + y^2 + (h - z(x,y))^2} - h \sin \psi - x \cos \psi - z(x,y) \sin \psi , \quad (C.5)$$

(a) Three dimensional geometry.

(b) Two dimensional geometry.

Figure C.1: Geometry appropriate for evaluating Fresnel zone dimensions on scattering surfaces for satellite applications.

where

$$z(x, y) = \begin{cases} 0 & \text{for a tangent plane} \\ -\frac{(x-x_s)^2}{2 r_c} & \text{for a convex surface facet} \\ \frac{(x-x_s)^2}{2 r_c} & \text{for a concave surface facet} \end{cases} \quad (C.6)$$

and r_c is the surface radius of curvature. The focal length, f , depends on the surface radius of curvature and the local grazing angle, as indicated below

$$f = \frac{r_c}{2} \sin \psi . \quad (C.7)$$

Defining $x = x_s + \Delta x$ and $y = \Delta y$, the path length difference $\delta(\Delta x, \Delta y)$ can be expressed as a function of the deviations from the specular point,

$$\delta(\Delta x, \Delta y) = \sqrt{r^2 + 2x_s \Delta x + \Delta x^2 + \Delta y^2 - 2 z(\Delta x) h + z(\Delta x)^2} - r - \Delta x \cos \psi - z(\Delta x) \sin \psi \quad (C.8)$$

where

$$z(\Delta x) = \begin{cases} 0 & \text{for a tangent plane} \\ -\frac{\Delta x^2}{2 r_c} & \text{for a convex surface facet} \\ \frac{\Delta x^2}{2 r_c} & \text{for a concave surface facet.} \end{cases} \quad (C.9)$$

If one lets $K = 2 x_s \Delta x + \Delta x^2 + \Delta y^2 - 2 z(\Delta x)h + z(\Delta x)^2$, the square root term in (C.8) can be expressed as

$$\sqrt{r^2 + K} = r \sqrt{1 + \frac{K}{r^2}}, \quad (C.10)$$

which can be approximated using the binominal theorem as follows

$$\begin{aligned} r \sqrt{1 + \frac{K}{r^2}} &= r \left(1 + \frac{1}{2} \left(\frac{K}{r^2} \right) - \frac{1}{8} \left(\frac{K}{r^2} \right)^2 + \frac{1}{16} \left(\frac{K}{r^2} \right)^3 \right) \\ &= r + \frac{1}{2} \left(\frac{K}{r} \right) - \frac{1}{8} \left(\frac{K^2}{r^3} \right) + \frac{1}{16} \left(\frac{K^3}{r^5} \right). \end{aligned} \quad (C.11)$$

Using this square root approximation, one obtains, after some tedious algebraic manipulation, the following expressions for the path length differences.

For a tangent plane,

$$\delta(\Delta x, \Delta y) = \frac{\Delta y^2}{2r} + \Delta x^2 \frac{\sin^2 \psi}{2r}. \quad (C.12)$$

For a parabolic convex surface facet,

$$\begin{aligned} \delta(\Delta x, \Delta y) &= \frac{\Delta y^2}{2r} + \Delta x^2 \frac{\sin^2 \psi}{2r} \left(1 + \frac{r}{f} \right) \\ &\quad - \Delta x^3 \frac{\sin^2 \psi \cos \psi}{2r} \left(\frac{1}{r} + \frac{1}{2f} \right). \end{aligned} \quad (C.13)$$

For a parabolic concave surface facet,

$$\delta(\Delta x, \Delta y) = \frac{\Delta y^2}{2r} + \Delta x^2 \frac{\sin^2 \psi}{2r} \left(1 + \frac{r}{f}\right) + \Delta x^3 \frac{\sin^2 \psi \cos \psi}{2r} \left(\frac{1}{2f} - \frac{1}{r}\right) \quad (C.14)$$

For the aeronautical application, the distance between the aircraft antenna and the ocean surface is appreciable, in which case the cubic terms in the path length difference expressions of (C.13) and (C.14) are negligible. As a result, the appropriate first Fresnel zone ($\delta = \pm \lambda_c/2$) dimensions for the aeronautical application are given by

$$X_{TP} = \frac{\sqrt{\lambda_c r}}{\sin \psi} \quad (C.15)$$

$$X_{CONVEX} = \frac{\sqrt{\lambda_c r}}{\sin \psi} \left(\frac{1}{\sqrt{1 + \frac{r}{f}}} \right) \quad (C.16)$$

$$X_{CONCAVE} = \frac{\sqrt{\lambda_c r}}{\sin \psi} \left(\frac{1}{\sqrt{\frac{r}{f} - 1}} \right) \quad (C.17)$$

$$Y_{CONVEX} = Y_{CONCAVE} = Y_{TP} = \sqrt{\lambda_c r} \quad (C.18)$$

A qualitative approach for establishing the magnitude of the signal attenuation (divergence) factor is to simply

5

Nidec

take the ratio of the area of the first Fresnel zone for the curved surface facet and the area of the first Fresnel zone on a tangent plane at the specular point. With this approach one obtains the following signal attenuation factors

$$|A| = D = \begin{cases} \frac{1}{\sqrt{1 + \frac{r}{F}}} & \text{for a convex surface facet} \\ \frac{1}{\sqrt{\frac{r}{F} - 1}} & \text{for a concave surface facet.} \end{cases} \quad (C.19)$$

The above result for a convex surface facet is in agreement with the traditional divergence factor, based on the ratio of flux densities, for a convex surface with no surface curvature along the y-axis (see equation (2.21) of Section 2.2.1.2). When using the ratio of Fresnel zones approach for estimating the divergence factor, the result is independent of the choice for the phase boundary on the curved surface and reference tangent plane when the phase function is parabolic in nature.

In the maritime environment, the antenna is much closer to the scattering surface, and as a consequence one cannot always ignore the cubic term in the path length difference expressions for convex and concave surface facets (see (C.13) and (C.14)). As a consequence, the qualitative ratio of Fresnel zones area approach for estimating the signal attenuation factor is somewhat more complex for the

maritime application. This is addressed in Sections 4.1 and 4.2 of this appendix. The next section discusses the Fresnel diffraction theory approach.

3.0 FRESNEL DIFFRACTION THEORY APPROACH

Huygens' principle is especially useful for solving electromagnetic diffraction and scattering problems (see Chapter 13 of [14]). The principle states that "each particle in any wave front acts as a new source of disturbance, sending out secondary waves, and these secondary waves combine to form a new wave front" (page 488 of [14]). This provides a method for computing radiated fields in terms of known fields on a scattering surface or aperture. Basically, the scattering surface can be subdivided into several Huygens' (or secondary) sources, and the resultant field is given by the superposition of the individual scattered fields from these secondary sources.

For satellite applications the incident field strength is constant ~~over the~~ scattering surface and is virtually the same as that of the direct path at the receiving antenna. The field present on the parabolic surface facet is assumed to be

$$E_s = \Gamma(\psi) E_i \quad (C.20)$$

where

E_i is the incident field strength,

ψ is the surface grazing angle at the specular point, and

r is the surface reflection coefficient.

The scattered field, relative to the direct path, can be easily estimated using a Fresnel diffraction theory approach (pages 498 - 503 of [14])

$$E_r = e^{-j\phi_s} E_i r(\psi) K_1 \int_{x_L}^{x_H} \int_{y_L}^{y_H} e^{-j\frac{2\pi}{\lambda} \delta(x,y)} dx dy \quad (C.21)$$

where

K_1 is a scale factor which is inversely proportional to the distance, r , between the specular point and the mobile antenna,

ϕ_s is the phase shift associated with the specular path measured relative to the direct path,

$\delta(x,y)$ is the path length difference relative to the specular path, where x and y denote the deviations from the specular point location,

E_i is the incident field strength, and

x_L, x_H, y_L, y_H represent the dimensions of the scattering facet.

To establish the form of the K_1 scale factor, consider an infinite tangent plane, in which case the scattered field is given by

$$E_r = e^{-j\phi_s} E_i \Gamma(\psi) \quad (C.22)$$

The path length difference and diffraction integral for a tangent plane are $\delta(x,y) = \frac{x^2 \sin^2 \psi}{2r} + \frac{y^2}{2r}$, and

$$E_r = e^{-j\phi_s} E_i \Gamma(\psi) K_1 \int_{-\infty}^{\infty} e^{-j\frac{\pi}{2} \left(\frac{2 \sin^2 \psi}{\lambda_c r} \right) x^2} dx \cdot \int_{-\infty}^{\infty} e^{-j\frac{\pi}{2} \left(\frac{2}{\lambda_c r} \right) y^2} dy. \quad (C.23)$$

Making the following changes of variables, $x_1 = \sqrt{\frac{2}{\lambda_c r}} \sin \psi x$ and $y_1 = \sqrt{\frac{2}{\lambda_c r}} y$, in the integrals of (C.23) yields

$$E_r = e^{-j\phi_s} E_i \Gamma(\psi) \left(\frac{\lambda_c r}{2 \sin \psi} \right) K_1 \int_{-\infty}^{\infty} e^{-j\frac{\pi}{2} x_1^2} dx_1 \cdot \int_{-\infty}^{\infty} e^{-j\frac{\pi}{2} y_1^2} dy_1. \quad (C.24)$$

The integrals in (C.24) are standard Fresnel integrals (page 500-501 of [14]), whose solution is

$$\int_{-\infty}^{\infty} e^{-j\frac{\pi}{2} x^2} dx = \sqrt{2} e^{-j\frac{\pi}{4}}. \quad (C.25)$$

Using this result to simplify (C.24) gives

$$E_r = -j K_1 \left(\frac{\lambda_c r}{\sin \psi} \right) e^{-j\phi_s} E_i r(\psi) . \quad (C.26)$$

Comparing the diffraction integral result of (C.26) to the known result for an infinite tangent plane (C.22), it is evident that the proportionality constant is

$$K_1 = j \frac{\sin \psi}{\lambda_c r} . \quad (C.27)$$

The 90° phase shift factor is expected, as it is present in the expression for the Huygens' source radiation fields for small rectangular elements given in equation (13-51) of [14].

Fresnel diffraction integrals can be used to characterize the scattered field from an infinite tangent plane slab, as shown in Figure C.2. The scattered field in this case is given by

$$E_r = e^{-j\phi_s} E_i r(\psi) \left(\frac{j \sin \psi}{\lambda_c r} \right) \int_{-\infty}^{\infty} e^{-j\frac{\pi}{2} \left(\frac{2}{\lambda_c r} \right) y^2} dy \cdot \int_{-\frac{A}{2}}^{\frac{A}{2}} e^{-j\frac{\pi}{2} \left(\frac{2 \sin^2 \psi}{\lambda_c r} \right) x^2} dx . \quad (C.28)$$

Employing the change of variables, $y_1 = \sqrt{\frac{2}{\lambda_c r}} y$, one obtains

(a) Infinite tangent plane slab.

(b) Projected tangent plane slab perpendicular to specular direction.

Figure C.2: Infinite tangent plane scattering slab.

$$\begin{aligned}
 E_r &= e^{-j\phi_s} E_i \Gamma(\psi) \left(\frac{j \sin \psi}{\sqrt{2} \sqrt{\lambda_c} r} \right) \int_{-\infty}^{\infty} e^{-j\frac{\pi}{2} y_1^2} dy_1 \int_{-\frac{A}{2}}^{\frac{A}{2}} e^{-j\frac{\pi}{2} \left(\frac{2 \sin^2 \psi}{\lambda_c r} \right) x^2} dx \\
 &= e^{-j\phi_s} E_i \Gamma(\psi) \frac{e^{+j\frac{\pi}{4}} \sin \psi}{\sqrt{\lambda_c} r} \int_{-\frac{A}{2}}^{\frac{A}{2}} e^{-j\frac{\pi}{2} \left(\frac{2 \sin^2 \psi}{\lambda_c r} \right) x^2} dx \\
 &= e^{-j\phi_s} E_i \Gamma(\psi) e^{+j\frac{\pi}{4}} \left(\frac{A \sin \psi}{\sqrt{\lambda_c} r} \right) \quad (C.29)
 \end{aligned}$$

This approximation is essentially valid when

$A < \frac{\sqrt{\lambda_c} r}{\sin \psi} = X_{TP}$. Note that the scattered field strength is proportional to the projected width of the tangent plane in the specular direction.

Fresnel diffraction integrals can also be used to characterize the scattered field from a small, finite tangent plane, as illustrated in Figure C.3. In this case,

$$\begin{aligned}
 E_r &= e^{-j\phi_s} E_i \Gamma(\psi) \left(\frac{j \sin \psi}{\lambda_c r} \right) \int_{-\frac{A}{2}}^{\frac{A}{2}} e^{-j\frac{\pi}{2} \left(\frac{2}{\lambda_c r} \right) \sin^2 \psi x^2} dx \int_{-\frac{B}{2}}^{\frac{B}{2}} e^{-j\frac{\pi}{2} \left(\frac{2}{\lambda_c r} \right) y^2} dy \\
 &= e^{-j\phi_s} E_i \Gamma(\psi) \frac{j(A \sin \psi) B}{\lambda_c r} \quad (C.30)
 \end{aligned}$$

(a) White tangent plane.

(b) Projected finite tangent plane perpendicular to specular direction.

Figure C.3: Finite tangent plane scattering.

This approximation is essentially valid when

$$A < \frac{\sqrt{\lambda_c r}}{\sin \psi} = X_{TP} \text{ and } B < \sqrt{\lambda_c r} = Y_{TP} .$$

This is the standard finite tangent plane scattering solution [1], [14]. This result is useful for defining the extent of the "stationary phase" region. Comparing the magnitude of the finite tangent plane scattered field of (C.30) with that for the infinite tangent plane slab (see (C.29)), one finds that they are equivalent if the width along the y-axis is

$$B = \sqrt{\lambda_c r} . \quad (C.31)$$

Note that this is equivalent to the standard first Fresnel zone minor axis dimension, Y_{TP} . The stationary phase limit is $\frac{B}{2} = \frac{\sqrt{\lambda_c r}}{2}$, which corresponds to a path length difference of $\delta = \frac{\lambda_c}{8}$. Consequently, an appropriate phase limit for the stationary phase region is 45° .

3.1 Using Fresnel Diffraction Integrals For Deriving Signal Attenuation Factors For The Aeronautical Application

The two-dimensional Fresnel diffraction integral, which can be used to characterize the scattered field is

$$E_r = e^{-j\phi_s} E_i r(\psi) \left(\frac{j \sin \psi}{\lambda_c r} \right) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-j \frac{2\pi}{\lambda_c} \delta(x,y)} dx dy. \quad (C.32)$$

No limit on x-axis illumination is considered at this point. For the aeronautical application, the appropriate path length difference expressions for parabolic surface facets are

$$\delta(x,y) = \begin{cases} \frac{\sin^2 \psi}{2r} \left(1 + \frac{r}{f} \right) x^2 + \frac{y^2}{2r} & \text{for convex facets} \\ - \frac{\sin^2 \psi}{2r} \left(\frac{r}{f} - 1 \right) x^2 + \frac{y^2}{2r} & \text{for concave facets.} \end{cases} \quad (C.33)$$

The quadratic phase shift along the y-axis is the same in both cases, and the above integral can be simplified to the following one-dimensional integral

$$E_r = e^{-j\phi_s} E_i r(\psi) \left(\frac{e^{j\frac{\pi}{4}} \sin \psi}{\sqrt{\lambda_c} r} \right) \int_{-\infty}^{\infty} e^{-j \frac{2\pi}{\lambda_c} \delta(x)} dx. \quad (C.34)$$

The quadratic phase integral in (C.34), has the following solutions for convex and concave facets

$$\int_{-\infty}^{\infty} e^{-j\frac{2\pi}{\lambda_c} x} \delta(x) dx = \begin{cases} e^{-j\frac{\pi}{4} \frac{\sqrt{\lambda_c} r}{\sin \psi}} \left(\frac{1}{\sqrt{1 + \frac{r}{f}}} \right) = e^{-j\frac{\pi}{4}} X_{\text{CONVEX}} & \text{for convex facets} \\ e^{+j\frac{\pi}{4} \frac{\sqrt{\lambda_c} r}{\sin \psi}} \left(\frac{1}{\sqrt{\frac{r}{f} - 1}} \right) = e^{j\frac{\pi}{4}} X_{\text{CONCAVE}} & \text{for concave facets.} \end{cases} \quad (\text{C.35})$$

Incorporating these results in (C.34), the scattered field can be expressed as

$$E_r = \Gamma(\psi) e^{-j\phi_s} A E_i \quad (\text{C.36})$$

where the signal scale factor A, which incorporates the effects of surface curvature, is given by

$$A = \begin{cases} \frac{1}{\sqrt{1 + \frac{r}{f}}} = \frac{X_{\text{CONVEX}}}{X_{\text{TP}}} & \text{for convex facets} \\ \frac{e^{j\frac{\pi}{2}}}{\sqrt{\frac{r}{f} - 1}} = j \frac{X_{\text{CONCAVE}}}{X_{\text{TP}}} & \text{for concave facets.} \end{cases} \quad (\text{C.37})$$

It is evident that the magnitude of the signal attenuation factor $|A|$ is the same as that obtained with the ratio of Fresnel zones approach. With a convex facet, the phase shift is the same as that introduced by a tangent plane. However, with a concave facet, an additional phase shift of

' $+90^\circ$ ' is introduced, and can be attributed to surface curvature.

3.1.1 Helmholtz Solution Procedure For The Aeronautical Application

The Helmholtz surface integral solution procedure can be used to characterize the scattered field from a local scattering facet, where there is no variation in waveheight along the y-axis. The geometry appropriate for this scattering problem and solution is presented in Figure C.4. A far-field solution to this problem is given in equations (26), (27), and (28) of Section 3.1 of [1]. Unfortunately, the scale factor in the integral equation (26) of [1] is not correct for a scattering element that is infinite in the y-direction, as acknowledged on pages 101 and 102 of [1]. In the solution procedure presented in [1], the phase factor used is $\exp(+j \frac{2\pi}{\lambda_c} \delta)$, where δ is the path length difference, instead of the more traditional $\exp(-j \frac{2\pi}{\lambda_c} \delta)$ definition. Reperforming the analysis presented in [1] for the traditional phase shift definition, and incorporating the phase shift along the y-axis, one obtains the following Helmholtz integral solution

Figure C.4: Geometry for the Helmholtz solution procedure (from [17])

Figure C.5: Illustration of illumination limit imposed by shadowing and blockage.

$$\begin{aligned}
 E_r &= \frac{1}{2\lambda_c R_0} \int_{-\frac{L}{2}}^{\frac{L}{2}} e^{-j\frac{\pi}{2}(\frac{2}{\lambda_c R_0})y^2} dy \int_{-\frac{L}{2}}^{\frac{L}{2}} [a z'(x) + b] e^{-j\frac{2\pi}{\lambda_c} [v_x x - v_z z(x)]} dx \\
 &= \frac{e^{+j\frac{\pi}{4}} e^{-j\frac{2\pi R_0}{\lambda_c}}}{2\sqrt{\lambda_c R_0}} \int_{-\frac{L}{2}}^{\frac{L}{2}} [a z'(x) + b] e^{-j\frac{2\pi}{\lambda_c} [v_x x - v_z z(x)]} dx
 \end{aligned}
 \tag{C.38}$$

where

R_0 is the distance between the specular point and the antenna position,

$$a = -(1-r) \sin \theta_1 - (1+r) \sin \theta_2, \tag{C.39}$$

$$b = (1+r) \cos \theta_2 - (1-r) \cos \theta_1, \tag{C.40}$$

r is the reflection coefficient,

$$v_x = \sin \theta_1 - \sin \theta_2, \tag{C.41}$$

$$v_z = \cos \theta_1 + \cos \theta_2, \tag{C.42}$$

θ_1 is the angle of incidence, and

θ_2 is the reflection angle relative to the vertical.

In the specular direction, $\theta_1 = \theta_2 = 90^\circ - \psi$, where ψ is the surface grazing angle. For a parabolic surface facet, $z(x) = \frac{-x^2}{2r_c}$, one obtains the following realistic approximation for the scattered field in the specular direction

$$E_r = e^{-j \frac{2\pi R_0}{\lambda_c}} \left(\frac{r \sin \psi}{\sqrt{\lambda_c R_0}} \right) e^{+j \frac{\pi}{4}} \int_{-\frac{L}{2}}^{\frac{L}{2}} e^{+j \frac{2\pi}{\lambda_c} (2 \sin \psi) z(x)} dx \quad (C.43)$$

Here, the effect of the slope of the surface facet is ignored, as well as the variation of the reflection coefficient with position. The reflection coefficient value at the specular point is assumed to be appropriate. Assuming that the extent of the facet is several Fresnel zones in length, the scattered field for a parabolic convex facet can be approximated by

$$\begin{aligned} E_r &= e^{-j \frac{2\pi R_0}{\lambda_c}} \left(\frac{r \sin \psi}{\sqrt{\lambda_c R_0}} \right) e^{+j \frac{\pi}{4}} \int_{-\infty}^{\infty} e^{-j \frac{\pi}{2} \left(\frac{4 \sin \psi}{\lambda_c r_c} \right) x^2} dx \\ &= e^{-j \frac{2\pi R_0}{\lambda_c}} \left(\frac{r \sin \psi}{\sqrt{\lambda_c R_0}} \right) \frac{\sqrt{\lambda_c r_c}}{2 \sqrt{\sin \psi}} e^{+j \frac{\pi}{4}} \int_{-\infty}^{\infty} e^{-j \frac{\pi}{2} v^2} dv \\ &= e^{-j \frac{2\pi R_0}{\lambda_c}} r \sqrt{\frac{r_c \sin \psi}{2 R_0}} \\ &= e^{-j \frac{2\pi R_0}{\lambda_c}} r \sqrt{\frac{f}{R_0}} \quad (C.44) \end{aligned}$$

The appropriate signal scale factor for the Helmholtz solution procedure is

$$A = \sqrt{\frac{F}{R_0}} \quad (C.45)$$

This result is in good agreement with the traditional divergence factor $D_{\text{trad}} = \sqrt{\frac{1}{1 + \frac{R_0}{F}}}$, especially for the large values of R_0 obtained in the aeronautical application.

3.1.2 Low Grazing Angle Attenuation Factor

At low grazing angles, there is a limit on the surface illumination imposed by shadowing and blockage, as illustrated in Figure C.5. The illumination limit is imposed when the rays are tangent to the surface. In this case, the limiting x-value is given by

$$x_l = r_c \tan \psi \quad (C.46)$$

The stationary phase limit x_{sp} , which corresponds to a path length difference of $\frac{\lambda_c}{8}$ (45° phase difference), is given by

$$x_{sp} = \frac{1}{2 \sin \psi} \sqrt{\frac{\lambda_c r}{1 + \frac{r}{F}}} \quad (C.47)$$

The signal attenuation factor is defined as

$$A = LGA \cdot D_{\text{trad}} \quad (\text{C.48})$$

where

$$LGA = \begin{cases} \frac{x_l}{\sqrt{2} x_{sp}} & \text{for convex} \\ \sqrt{2} e^{j\frac{\pi}{4}} \int_0^{\frac{x_l}{\sqrt{2} x_{sp}}} \exp(-j\frac{\pi}{2} v^2) dv & \text{facets} \\ 0 & \end{cases} \quad (\text{C.49})$$

$$\begin{cases} \frac{x_l}{\sqrt{2} x_{sp}} & \text{for concave} \\ \sqrt{2} e^{j\frac{\pi}{4}} \int_0^{\frac{x_l}{\sqrt{2} x_{sp}}} \exp(+j\frac{\pi}{2} v^2) dv & \text{facets} , \\ 0 & \end{cases}$$

$$D_{\text{trad}} = \begin{cases} \sqrt{\frac{1}{1 + \frac{r}{F}}} & \text{for convex facets} \\ \sqrt{\frac{1}{\frac{r}{F} - 1}} & \text{for concave facets.} \end{cases} \quad (\text{C.50})$$

The amplitude and phase of the low grazing angle attenuation factor as a function of $(\frac{x_l}{x_{sp}})$ is provided in Figure C.6. The magnitude of the low grazing angle attenuation factor oscillates about the value obtained with unrestricted illumination when $x_l > x_{sp}$. Note that for $x_l < x_{sp}$, the magnitude of the low grazing angle attenuation factor is essentially given by the linear approximation $|\hat{LGA}| = \frac{x_l}{x_{sp}}$, illustrated in Figure C.6(a).

The phase shift of a concave facet relative to a convex facet is illustrated in Figure C.7, as a function of the illumination ratio $(\frac{x_l}{x_{sp}})$. Note that as the illumination

Figure C.6: Magnitude and phase of the low grazing angle attenuation factor.

Figure C.7: Phase shift introduced by a concave surface facet relative to that of a convex facet as a function of illumination.

limit approaches zero, the relative phase approaches zero. However, when the grazing angle is lowered sufficiently to introduce significant illumination truncation, the rays to the receiver from concave surface facets are very likely to be blocked by the ocean surface profile.

4.0 DERIVATION OF APPROPRIATE SIGNAL ATTENUATION FACTORS FOR MARITIME APPLICATIONS

The evaluation of appropriate signal attenuation factors for the maritime environment is more difficult because the antenna is much closer to the ocean surface. Concave surface facets are especially interesting because they have the ability to focus energy. This phenomena will occur when the mobile antenna is within two focal lengths of the specular point.

4.1 Signal Attenuation Factor Evaluation For Concave Facets

The path length difference, as a function of the deviation Δx , can be suitably approximated by the cubic equation

$$\delta(\Delta x) = \frac{\Delta x^2}{2r} \sin^2 \psi \left(1 - \frac{r}{f}\right) + \frac{\Delta x^3}{2r} \cos \psi \sin^2 \psi \left[\frac{1}{2f} - \frac{1}{r}\right]. \quad (C.51)$$

For the critical near-field region (i.e. $r \in [0, 2f]$), the cubic term is always negative. Therefore, when $r > f$, the

path length difference will clearly be negative for positive values of Δx . However, for negative Δx , the second order term is negative, while the contribution from the third order term is positive. The effect of the third order term will tend to dominate for large Δx values. Consequently, the path length difference function tends to behave as indicated in Figure C.8(a).

When $r < f$, the quadratic term is positive, while the third order term is positive for negative Δx and is negative for positive Δx . Consequently, the path length difference function tends to behave as indicated in Figure C.8(b).

When $r = f$, one gets an antisymmetric cubic path length difference function as illustrated in Figure C.8(c).

4.1.1 Ratio of First Fresnel Zones Approach For Concave Facets

The stationary phase region for the concave scattering facet will be defined to be region where the magnitude of the phase difference with respect to the specular path is less than or equal to 180 degrees. The boundaries of this region will be defined by Δx^+ and Δx^- , which represent positive and negative deviations about the specular point. The signal attenuation (divergence) factor will be defined as the ratio of the length of this stationary phase region

(a) $r > f$

(a) $r < f$

(c) $r = f$

Figure C.8: Behaviour of the path length difference for a concave scattering facet.

to the length of the stationary phase region on a reference tangent plane at the specular point. Consequently, the divergence factor will be given by

$$D = \frac{\Delta x^+ + |\Delta x^-|}{2 R_N} = \frac{\Delta x^+ - \Delta x^-}{2 R_N} \quad (C.52)$$

where R_N is the first Fresnel zone length on a tangent plane, i.e.

$$R_N = \frac{\sqrt{\lambda_c r}}{\sin \psi} \quad (C.53)$$

When $r > f$, one must first find Δx^+ such that the path difference is $-\lambda_c/2$. One must then check to see if there is a value of Δx^- corresponding to a path length difference of $-\lambda_c/2$. As alluded to earlier, if r is too close to the focal length, such a value may not exist. It is not difficult to establish when this is the case. When there is no negative Δx^- value that gives a path length difference of $-\lambda_c/2$, one must find a value of Δx^- where the path length difference is $\frac{\lambda_c}{2}$. In this case $|\Delta x^-|$ will exceed the magnitude of Δx^+ .

The consequence of the above approach of assigning the divergence factor is that as one moves towards the focal

point, there will be a threshold distance where the divergence factor will jump in value. This will occur when the Δx^- solution corresponding to $\delta = -\frac{\lambda_c}{2}$ disappears. Stated another way, this occurs when the cubic nature of the phase difference function begins to dominate the quadratic term.

Similarly, when $r < f$, one first finds the value of Δx^- corresponding to a path length difference of $+\frac{\lambda_c}{2}$. One attempts to find a value of Δx^+ such that the path length difference is $+\lambda_c/2$. If this is not possible, one finds the value of Δx^+ that corresponds to $\delta = -\frac{\lambda_c}{2}$. The divergence factor is then evaluated using equation (C.52).

The path length difference equation can be rearranged to give

$$\Delta x^3 \cos \psi \left[\frac{1}{2f} - \frac{1}{r} \right] + \Delta x^2 \left[1 - \frac{r}{f} \right] = \frac{2\delta r}{\sin^2 \psi} \quad (C.54)$$

Normalizing this equation by R_N^2 , where R_N is the first

Fresnel zone radius, $R_N = \frac{\sqrt{\lambda_c r}}{\sin \psi}$, on a tangent plane and defining $\rho_x = \frac{\Delta x}{R_N}$ one obtains the following cubic equation

$$B \rho_x^3 + A \rho_x^2 = C \quad (C.55)$$

$$\text{where } B = \left[\frac{1}{r} - \frac{1}{2f} \right] \frac{\sqrt{\lambda_c r}}{\tan \psi} \quad (\text{C.56})$$

$$A = \left(\frac{r}{f} - 1 \right) \quad (\text{C.57})$$

$$C = \frac{-2 \delta}{\lambda_c} \quad (\text{C.58})$$

The divergence factor can be re-defined as

$$D = \frac{\rho_x^+ + |\rho_x^-|}{2} = \frac{\rho_x^+ - \rho_x^-}{2} \quad (\text{C.59})$$

where ρ_x^+ corresponds to a positive excursion in Δx and ρ_x^- corresponds to a negative excursion in Δx .

The procedure for assigning appropriate values to ρ_x^+ and ρ_x^- is summarized in the flow chart presented in Figure C.9.

An iterative Newtons method solution procedure has been developed for the above cubic equation.

4.1.2 Modified Fresnel Zones Approach For Concave Facets

When the phase function is parabolic in nature, one need not be concerned with the actual phase boundary value used

1

Figure C.9: Flowchart of divergence factor solution procedure in the near-field region of concave surface facets.

with the ratio of Fresnel zones approach. This is not the case when one has a significant cubic term, and suggests that there may be a more appropriate phase boundary. An alternative approach is to define a phase boundary which provides a qualitative divergence factor that matches the exact value at the focal point, determined using Fresnel diffraction integrals

$$D_{\text{exact}} = \frac{C_u}{(2B)^{1/3}} \quad (\text{C.60})$$

where

$$C_u = \int_{-\infty}^{\infty} e^{+j\frac{\pi}{2} v^3} dv = \int_{-\infty}^{\infty} \cos\left(\frac{\pi}{2} v^3\right) dv, \quad (\text{C.61})$$

and B is given by equation (C.56).

Numerical integration reveals that $C_u = 1.33$.

The path length difference equation of (C.54) can be expressed as

$$\Delta x^3 \cos\left[\frac{1}{r} - \frac{1}{2f}\right] + \Delta x^2 \left[\frac{r}{f} - 1\right] = -\left(\frac{2\delta}{\lambda_c}\right) R_N^2, \quad (\text{C.62})$$

where R_N is the first Fresnel zone radius.

If one defines $t_\delta = \left(\frac{2\delta}{\lambda_c}\right)$, $R_\delta = \sqrt{|t_\delta|} R_N$, and $\rho_x = \frac{\Delta x}{R_\delta}$, one obtains the following cubic equation.

$$B_{\delta} \rho_x^3 + A_{\delta} \rho_x^2 = C_{\delta} \quad (C.63)$$

where

$$B_{\delta} = \sqrt{|t_{\delta}|} B \quad (C.64)$$

$$A_{\delta} = A \quad (C.65)$$

$$C_{\delta} = \frac{-t_{\delta}}{|t_{\delta}|} \quad (C.66)$$

B and A are defined in (C.56) and (C.57), respectively.

The divergence factor definition of (C.59) is appropriate, and the solution procedure outlined in Figure C.9 can be used to establish appropriate values for ρ_x^+ and ρ_x^- .

At the focal point, $A_{\delta} = 0$, and the divergence factor is given by

$$D = \frac{1}{(B_{\delta})^{1/3}} = \left(\frac{1}{\sqrt{|t_{\delta}|} B} \right)^{1/3} \quad (C.67)$$

Equating this to the exact expression (C.60) at the focal point yields

$$|t_{\delta}| = \frac{4}{C_u^6} = 0.7227, \quad (C.68)$$

which corresponds to a phase boundary of 130.1° , as opposed to the 180° used with the first Fresnel zone ratio approach.

4.1.3 Comparison of the Qualitative Divergence Factors With The Traditional Ray-Optics Solution For Concave Facets

The traditional ratio of flux density (ray-optics) approach for evaluating the divergence factor of concave surface facets yields

$$D_{\text{ray}} = \sqrt{\frac{1}{\left| \frac{r}{f} - 1 \right|}} \quad (\text{C.69})$$

The impulsive nature of the flux density makes this approach impractical in the near-field region. The two ratio of Fresnel zones approaches developed here give finite values for the divergence factor at the focal point. A comparison of the divergence factors obtained with the qualitative ratio Fresnel zones approaches and the traditional flux density approach for a focal length of 10 meters, a local grazing angle of 30°, and a carrier wavelength of 0.2 meters is provided in Figure C.10. Around the focal point, the ratio of Fresnel zone approaches provide well behaved divergence factor values. Note that as the distance is increased beyond the focal length, the qualitative divergence factor values actually increase. The reason for this is that the phase difference function is still predominately cubic, and this introduces a skewing which results in $|\Delta x^-|$ exceeding the magnitude of

Figure C.10: Comparison of the traditional (ray-optics) divergence factor and the qualitative ratio of Fresnel zones divergence factors in the near-field region of a concave scattering facet.

Δx^+ . As the distance is increased further, the phase difference function becomes more and more quadratic until there are values of Δx^- corresponding to phase differences of 130.1° and 180° . When this point is reached, there is a significant drop in the magnitude of Δx^- , and consequently, a noticeable drop in the divergence factor is experienced. When this breakpoint is reached, the divergence values obtained with the ratio of Fresnel zones approaches are virtually identical to those obtained with the traditional ray-optics approach.

The absolute accuracy of the qualitative divergence factors in the near-field region is difficult to establish because there are no published results available in the literature. However it is informative to examine the behaviour of the solution in the far-field region ($r \gg f$).

When the antenna is not in the near-field region, one can develop an expression for the divergence factor of a concave surface facet solely on physical considerations. Once the receiving antenna is greater than two focal lengths from the specular point, the scattering from a concave facet is virtually equivalent to that which would be obtained from a convex surface segment (with equivalent radius of curvature) located a distance $2f$ closer to the

receiver (see Figure C.11). Noting that the divergence factor of a convex facet can be expressed as

$$D_{\text{convex}} = \frac{1}{\sqrt{1 + \frac{d}{f}}}$$

where d is the distance between the antenna and convex scattering facet, one would predict, based on physical considerations, that

$$\begin{aligned} D_{\text{concave}} &= \frac{1}{\sqrt{1 + \left(\frac{r - 2f}{f}\right)}} \\ &= \frac{1}{\sqrt{\frac{r}{f} - 1}} \quad \text{for } r > 2f \quad (C.70) \end{aligned}$$

where r is the distance between the antenna and concave scattering facet, and f is the focal length of the concave scattering element.

When $r \gg 2f$, the cubic terms in equations (C.55) and (C.63) are negligible, and the qualitative divergence factor is given by

$$D_{\text{concave}} = \frac{1}{\sqrt{\frac{r}{f} - 1}} \quad \text{for } r \gg 2f \quad (C.71)$$

(a) Scattering from a convex surface segment

(b) Scattering from a concave surface segment

Figure C.11: Comparison of scattering from convex and concave surface facets.

When $r \gg 2f$ the path length difference can be approximated by the quadratic expression

$$\delta = - \frac{\Delta x^2}{2r} \sin^2 \psi \left(\frac{r}{f} - 1 \right) .$$

Using the Fresnel diffraction approach, the appropriate signal scale factor is given by

$$A = \frac{\sin \psi}{\sqrt{\lambda_c r}} e^{j\frac{\pi}{4}} \int_{-\infty}^{\infty} e^{+j\frac{\pi}{2} \left(\frac{2 \sin^2 \psi}{\lambda_c r} \right) \left(\frac{r}{f} - 1 \right) x^2} dx$$

for $r \gg f$

$$= j \frac{1}{\sqrt{\frac{r}{f} - 1}} \quad (C.72)$$

Consequently, the magnitude of the exact divergence factor $D = |A|$ is equivalent to that obtained with the ratio of Fresnel zones approach and the heuristic approach based on physical equivalence of scattering from concave and convex facets, when the mobile antenna is not in the near-field region.

When the antenna is greater than two focal lengths from the specular point, the phase difference function about the specular point is predominately quadratic. In this case one should be able to use the expression in (C.72) for the signal scale factor without any appreciable error. It must

also be emphasized that there are computational benefits, as one does not have to solve any cubic equations.

To gain an appreciation for the difference between the divergence factors obtained with the ratio of Fresnel zones approach and the traditional flux density approach in the transition region between the near-field and far-field regions, these divergence factors are compared in Figure C.12. Here, the distance between the antenna and scattering facet is varied over the range $[2f, 7f]$, and the focal length is 10 meters, the local grazing angle is 30° , and the carrier wavelength is 0.2 meters. Given the minor difference in the divergence factor values, one need only employ the detailed divergence factor solution procedure in the near-field region (i.e. $r \in [0, 2f]$).

4.1.4 A More Exact Evaluation of the Signal Scale Factor For Concave Surface Facets In The Near-Field Region

Using the Fresnel diffraction theory approach, the signal scale factor in the near-field region of a concave surface facet can be expressed as

Figure C.12: Comparison of the traditional and qualitative divergence factors in the intermediate region just beyond the near-field region of a concave scattering facet.

$$A = \begin{cases} \left(\frac{1}{2B}\right)^{\frac{1}{3}} \int_{-\infty}^{\infty} \exp \{ j \frac{\pi}{2} [v^3 + K_q v^2] \} dv & \text{for } r \in [f, 2f] \\ \left(\frac{1}{2B}\right)^{\frac{1}{3}} \int_{-\infty}^{\infty} \exp \{ j \frac{\pi}{2} [v^3 - K_q v^2] \} dv & \text{for } r \in [0, f] \end{cases} \quad (C.73)$$

where

$$K_q^3 = \frac{2 \tan^2 \psi}{\lambda_c r} \frac{\left| \frac{r}{f} - 1 \right|^3}{\left(\frac{1}{r} - \frac{1}{2f} \right)^2} \quad (C.75)$$

and B is defined by equation (C.56).

The above integrals are evaluated numerically, with the lower integration limit set at -7.5 and the upper limit set at 7.5.

The qualitative ratio of Fresnel zones approach for establishing the magnitude of the signal scale (divergence) factor has a breakpoint where there is a considerable decrease in the divergence factor value. One of the major reasons for investigating this more exact solution was to determine if this behaviour is realistic.

The values obtained with the qualitative ratio of Fresnel zones approaches are compared with those obtained with the Fresnel diffraction integrals in Figure C.13, where $f = 10$ m and $\psi = 30^\circ$. The exact divergence factor is in excellent agreement with the qualitative divergence factor

Figure C.13: Comparison of the magnitude of the more exact Fresnel diffraction near-field signal scale factor, for concave scattering facets and the qualitative divergence factors obtained with the ratio of Fresnel zones approaches.

for $r > 1.6 f$, where the phase difference becomes predominately quadratic. It is also interesting to note that the exact divergence factor exhibits a similar sort of breakpoint behaviour to that of the qualitative approaches. However, in the transition region (cubic to quadratic phase difference), the Fresnel diffraction solution exhibits considerable and rapid oscillation. The magnitude of the divergence factor in this transition region is a volatile function of distance. The qualitative divergence factor curves represent a reasonable average for the oscillating Fresnel diffraction curve, however.

The differences between the qualitative Fresnel zones approaches and the more exact solution procedure are rather significant. As a consequence, the Fresnel diffraction approach is incorporated in the maritime propagation model to handle the near-field region of concave surface facets. Before concluding, the divergence factor for a convex facet, obtained using the qualitative ratio of Fresnel zones approach, is presented.

4.2 Divergence Factor For Convex Facets For the Maritime Application

In the far-field region, the path length difference is parabolic, and the magnitude of the signal scale factor is given by (C.19). The only region of concern is the near-

field region. One can express the path length difference as a function of Δx as follows

$$\Delta x^2 \left(1 + \frac{r}{f}\right) - \frac{\Delta x^3}{r} \cos \psi \left(1 + \frac{r}{2f}\right) = \frac{2\delta}{\lambda_c} R_N^2, \quad (C.76)$$

where R_N is the first Fresnel zone radius. Setting $\delta = \frac{\lambda_c}{2}$ and normalizing the above equation gives

$$\rho^2 \left(1 + \frac{r}{f}\right) - \rho^3 \left(\frac{\sqrt{\lambda_c/f}}{\tan \psi}\right) \left(1 + \frac{r}{2f}\right) = 1, \quad (C.77)$$

where

$$\rho = \frac{\Delta x}{R_N}.$$

In the far-field region ($r \gg f$) and in the high frequency

limit, one finds $\rho = \frac{1}{\sqrt{1 + \frac{r}{f}}}$, which is the expected

result. Using the ratio of Fresnel zones approach in the near-field region requires solving a cubic equation.

However, there is a dominant quadratic term with a convex facet, which is not necessarily the case in the near-field region of a concave scattering element.

A useful way of expressing the divergence factor cubic equation is as follows

$$\rho^2 [1 - Q\rho] = \frac{1}{1 + \frac{r}{f}} \quad (C.78)$$

where

$$Q = \frac{\sqrt{\lambda_c/r} (1 + r/2f)}{\tan \psi (1 + r/f)} \quad (C.79)$$

As is the case for a concave surface facet, the qualitative divergence factor can be defined as

$$D = \frac{\rho^+ + |\rho^-|}{2} = \frac{\rho^+ - \rho^-}{2} \quad (C.80)$$

where ρ^+ and ρ^- correspond to the positive and negative solutions of the cubic equation in (C.78) with smallest magnitude.

A reasonable iterative solution procedure for ρ^+ and ρ^- is outlined below

$$\rho_n^\pm = \frac{\pm 1}{\sqrt{1 - Q\rho_{n-1}^\pm}} D_0 \quad (C.81)$$

where

$$D_0 = \frac{1}{\sqrt{1 + \frac{r}{f}}}, \quad \rho_0^+ = D_0, \text{ and } \rho_0^- = -D_0. \quad (C.82)$$

The iteration is stopped when the difference between successive solutions is sufficiently small.

If Q is small enough, a first order correction should be sufficient. In this case,

$$\rho^+ = \frac{1}{\sqrt{1 - Q D_0}} D_0, \quad (C.83)$$

$$\rho^- = \frac{-1}{\sqrt{1 + Q D_0}} D_0. \quad (C.84)$$

For $Q D_0 \ll 1$, the divergence factor can be approximated by

$$D = \left(1 + \frac{3}{8}(Q D_0)^2\right) D_0. \quad (C.85)$$

This gives an expression for the percentage error relative to the divergence factor derived using the standard ray optics (ratio of flux densities) approach.

Note that the quantity $Q D_0$ can be expressed as follows

$$Q D_0 = \frac{\sqrt{\lambda_c/r}}{\tan \psi} \frac{(1 + r/2f)}{(1 + r/f)^{3/2}}. \quad (C.86)$$

At a first glance this expression seems troublesome because of the presence of the $(\frac{1}{\tan \psi})$ term, which seems to imply that $Q D_0$ may not be small at low grazing angles. However, there is a counter balancing force in that the focal length, $f = \frac{r_c}{2} \sin \psi$, decreases with lower grazing angles, thereby increasing the ratio r/f . Consequently, one needs large radii of curvature for the focal length to be comparable to the distance between the antenna and scattering facet at low grazing angles. For the maritime application, at low grazing angles one will typically be outside the near-field region ($r > 2f$). In this case, the product $Q D_0$ will be small, and one can use the standard ray-optics divergence factor without any significant error.

To get a feel for the degree of error there is relative to the ray-optics divergence factor in the near-field region, consider the case where the antenna is located one focal length from the specular point, and the height of the antenna above the specular point is represented by h , the carrier wavelength is represented by λ_c and the local grazing angle is given by ψ . In this case, $r = h/\sin \psi$, and

$$Q D_0 = \frac{3}{2} \sqrt{\frac{\lambda_c}{h}} \frac{\cos \psi}{\sqrt{8 \sin \psi}} \quad (C.87)$$

This ratio is tabulated versus local grazing angle for $h = 10$ m and $\lambda_c = 0.2$ m (L-band frequency) in Table C.1. Also provided in this table is the difference, in decibels, between the divergence factor obtained by solving the cubic equation provided by the ratio of Fresnel zones approach and the divergence factor given by ray-optics. It is apparent that the difference is only a few tenths of a decibel under worst case conditions. Consequently, one can use the standard ray-optics divergence factor without any appreciable error in the near-field region of a convex surface facet. This is in direct contrast to the situation for concave surface facets.

5.0 APPROXIMATE ANALYSIS OF THE EFFECTS OF SURFACE NON-UNIFORMITY IN THE DIRECTION PERPENDICULAR TO SEA SURFACE MOTION

For computational reasons, the surface model used in the propagation model assumes that there is no variation in waveheight in the direction perpendicular to sea surface motion. This can be thought of as a coherent aperture, as illustrated in Figure C.14(a), with an effective (45° stationary phase region) width

$$Y = \sqrt{\lambda_c r} , \quad (C.88)$$

where r is the distance between the antenna and the point of interest on the surface.

Surface Grazing Angle ψ	$Q D_0$	Difference, in dB, between divergence factor obtained with ratio of Fresnel zones and traditional ray-optics approaches
5°	0.253	0.206
10°	0.177	0.101
15°	0.142	0.065
20°	0.121	0.048
25°	0.105	0.036
30°	0.092	0.027

Table C.1: A tabulation of $Q D_0$ and the difference between the divergence factor obtained using the ratio of Fresnel zones and traditional ray-optics approaches versus local grazing angle for an antenna located one focal length from the specular point of a convex surface facet with $h = 10$ meters and $\lambda_c = 0.2$ meters.

(a) Coherent aperture

(b) Non-coherent aperture

Figure C.14: Coherent and non-coherent apertures.

It is evident that this surface model is not a very realistic model of actual surface conditions, as the surface is actually non-uniform in the direction perpendicular to wave motion. In fact, in [12], it is claimed that the ocean surface becomes effectively decorrelated after a distance

$$d = \frac{\lambda}{4}$$

(C.89)

where λ is the average sea wavelength.

A fairly realistic model for handling the surface non-uniformity in the y-direction, is to consider a number of adjacent but independent (non-coherent) apertures of width W . A reasonable choice for the aperture width is

$$W = d = \frac{\lambda}{4} .$$

(C.90)

For a non-coherent aperture of width W , the beamwidth

$$BW = \frac{\lambda}{W} .$$

(C.91)

The angular region θ over which the contributions from non-coherent apertures will have significant impact at the receiving antenna (see Figure C.14(b)), is

$$\theta = BW .$$

(C.92)

This angle can also be defined in terms of the number of non-coherent apertures n_{nch} as follows

$$\theta = 2 \tan^{-1} \left\{ \frac{n_{nch} W}{2r} \right\} \approx \frac{n_{nch} W}{r} . \quad (C.93)$$

Equating these expressions for θ yields

$$n_{nch} = \frac{\lambda_c r}{W^2} . \quad (C.94)$$

For the non-coherent aperture, the contributions from each aperture add as power. The resulting scatter power is

$$P_S(nch) = n_{nch} P_W , \quad (C.95)$$

where P_W is scatter power from each aperture of width W .

For the coherent aperture, the number of adjacent, coherent apertures n_{coh} of width W is given by

$$n_{coh} = \frac{Y}{W} = \frac{\sqrt{\lambda_c r}}{W} . \quad (C.96)$$

It is evident that the number of non-coherent apertures is given by

$$n_{\text{nch}} = n_{\text{coh}}^2 \quad (\text{C.97})$$

In the phase coherent (stationary phase) case, the contributions from each of the n_{coh} apertures of width W add as voltage, and as a result, the scatter power is $P_S(\text{coh}) = n_{\text{coh}}^2 P_W$. Because $n_{\text{coh}}^2 = n_{\text{nch}}$, it is evident that the scatter power obtained with the phase coherent (stationary phase) approach is equivalent to that obtained with the non-coherent aperture approach, which models surface non-uniformity along the y -axis.

This suggests that one should obtain realistic scattering results with the simple surface model which assumes no waveheight variation along the y -axis. With this surface model, the electromagnetic scattering problem becomes essentially two-dimensional in nature. Relative to the actual three-dimensional scattering problem, one obtains a dramatic reduction in computational complexity with this surface model. In fact, it is this simplification which makes the scattering problem mathematically and computationally tractable.

APPENDIX D

SIGNAL POLARIZATION AND ANTENNA ELLIPTICITY EFFECTS

1.0 INTRODUCTION

In this appendix, issues related to signal combination for circular polarization are addressed. First, the signal combination strategy for perfect circular polarization is addressed. The effect of transmit and receive antenna polarization purity is also examined.

2.0 SIGNAL COMBINATION WITH CIRCULAR POLARIZATION

The polarization of a wave radiated in the z-direction (see Figure D.1) can be specified by the polarization ratio P defined by

$$P = \frac{E_y}{E_x} = |P| e^{j\delta} \quad (D.1)$$

E_x and E_y denote the fields parallel to the x and y axes, where the x axis is the traditional horizontal polarization axis and the y axis is the traditional vertical polarization axis. With these definitions, $|P| = A_y/A_x$ and $\delta = \theta_y - \theta_x$.

The path traced out by the time varying vector

$$\vec{E}(t) = \text{Re}\left\{\begin{bmatrix} E_x \\ E_y \end{bmatrix} e^{j2\pi f_c t}\right\} \quad (\text{D.2})$$

is given by

$$a E_x^2 - b E_x E_y + c E_y^2 = 1 \quad (\text{D.3})$$

where

$$a = \frac{1}{A_x^2 \sin^2 \delta}, \quad b = \frac{2 \cos \delta}{A_x A_y \sin^2 \delta}, \quad c = \frac{1}{A_y^2 \sin^2 \delta}. \quad (\text{D.4})$$

This conic section is an ellipse. The polarization ellipses one obtains at various points on the complex plane P is provided in Figure D.2 [14]. When $P = -j$, one has right hand circular (RHC) polarization and when $P = j$, one has left hand circular (LHC) polarization.

To characterize the electric field vector $\vec{v}(t)$ at the transmitter or receiver, one only need establish the complex baseband field vector $\tilde{v}(t)$, as

$$\vec{v}(t) = \text{Re}\{\tilde{v}(t) e^{j2\pi f_c t}\} \quad (\text{D.5})$$

Furthermore, all power levels quoted are for the complex baseband equivalent signals.

With this notation, the complex baseband vector for RHC polarization can be represented as

Figure D.1: Polarization definitions

Figure D.2: The complex P plane with polarization ellipses (from [14]).

$$\tilde{E}_{RHC}(t) = \sqrt{\frac{P_T}{2}} \begin{bmatrix} 1 \\ e^{-j\frac{\pi}{2}} \end{bmatrix} \quad (D.6)$$

where P_T represents the complex baseband transmit power level.

In this case,

$$\tilde{E}_{RHC}(t) = \sqrt{\frac{P_T}{2}} \begin{bmatrix} \cos 2\pi f_c t \\ \sin 2\pi f_c t \end{bmatrix} \quad (D.7)$$

The complex baseband signal vector at the receiving antenna can, in general, be represented by

$$\tilde{E}_{REC}(t) = \begin{bmatrix} \tilde{s}_x(t) \\ \tilde{s}_y(t) \end{bmatrix} \quad (D.8)$$

where $\tilde{s}_x(t)$ and $\tilde{s}_y(t)$ represent the time varying complex phasors for horizontal and vertical polarization respectively.

The signal combination strategy at the receiving antenna site, for the same sense (co-polarized) reception of RHC polarization waves, is illustrated in Figure D.3. The complex baseband copolarized output is given by

Figure D.3: Signal combination for RHC polarization waves.

$$\tilde{s}_{\text{copol}}(t) = \frac{1}{\sqrt{2}} [\tilde{s}_x(t) + e^{j\frac{\pi}{2}} \tilde{s}_y(t)] \quad (\text{D.9})$$

Note that the $(\frac{\pi}{2})$ phase shift allows coherent combining of the individual horizontally and vertically polarized components of a RHC polarized wave. To demonstrate this, if one applies this signal strategy directly at the

transmitter output, where $\tilde{s}_x(t) = \sqrt{\frac{P_T}{2}}$ and $\tilde{s}_y(t) = \sqrt{\frac{P_T}{2}} e^{-j\frac{\pi}{2}}$, one obtains $\tilde{s}_{\text{copol}} = \sqrt{P_T}$.

The cross-polarized (opposite sense) output is obtained using a $(-\frac{\pi}{2})$ phase shift instead of the 90° phase shift in Figure D.3, and

$$\tilde{s}_{\text{xpol}}(t) = \frac{1}{\sqrt{2}} [\tilde{s}_x(t) + e^{-j\frac{\pi}{2}} \tilde{s}_y(t)] \quad (\text{D.10})$$

If the signal is transmitted with RHC polarization, the form of the direct path and surface scatter complex baseband signal vectors can be expressed as

$$\tilde{\mathbf{E}}_D(t) = \sqrt{\frac{P_D}{2}} e^{-j\theta(t)} \begin{bmatrix} G_H(D_a) \\ -jG_V(D_a) \end{bmatrix} \quad (\text{D.11})$$

$$\tilde{\mathbf{E}}_{sc}(t) = \sqrt{\frac{P_D}{2}} \begin{bmatrix} \tilde{s}_H(t) \\ -j\tilde{s}_V^*(t) \end{bmatrix} \quad (\text{D.12})$$

where

P_D denotes the power level of the complex baseband phasor that would be obtained with an omni-reference antenna, which incorporates the effects of space loss and the transmit power level,

$\theta(t)$ is the time varying phase shift introduced by the direct path,

$G_H(\cdot), G_V(\cdot)$ represents the antenna (voltage) gain patterns (relative to an omni-reference) for horizontal and vertical polarization,

$$D_a = \begin{cases} E + B_A & \text{for aeronautical applications} \\ E - \theta_{ss} & \text{for maritime applications,} \end{cases}$$

E is the elevation angle to the satellite,

B_A is the banking angle of the aircraft (where a rotation towards the surface is considered a positive angle),

θ_{ss} is the shipborne antenna pointing angle deviation (counter-clockwise direction assumed positive), and

$\tilde{s}_H(t), \tilde{s}_V(t)$ denote the complex baseband surface scatter signals that would be obtained for horizontal and vertical polarization.

The identical scale factor of $\sqrt{\frac{P_D}{2}}$ on the direct and scatter paths is simply a consequence of the fact that for

satellite applications, the incident field strength at the aircraft and the ocean surface are virtually identical. For characterizing the behaviour of ocean scatter relative to the direct path, the exact nature of the scale factor is irrelevant. One can assume that the power level P_D is normalized to unity (i.e. the received direct path power for an omni-reference is normalized to 0 dB).

The complex baseband surface scatter signals are given by

$$\tilde{s}_H(t) = \sum_{i=1}^N G_H(g_a(i)) r_H(\alpha_i) A_i e^{-j\phi_i} \quad (D.13)$$

$$\tilde{s}_V(t) = \sum_{i=1}^N G_V(g_a(i)) r_V(\alpha_i) A_i e^{-j\phi_i} \quad (D.14)$$

where

N is the number of contributing surface facets,

$$g_a(i) = \begin{cases} \beta_A - \beta_i & \text{for aeronautical applications} \\ -\theta_{ss} - \beta_i & \text{for maritime applications,} \end{cases}$$

β_i is the reflection angle, relative to the horizontal, at the i -th scattering facet,

A_i is the attenuation (divergence) factor for the i -th scattering facet,

ϕ_i is the differential phase shift relative to the direct path,

α_i is the local grazing angle, and

$\Gamma_H(\cdot), \Gamma_V(\cdot)$ represent the surface reflection coefficients, as a function of grazing angle, for horizontal and vertical polarization.

Although not explicitly indicated in the above expressions, $N, g_a(i), \beta_i, A_i, \phi_i$, and α_i are functions of time.

The complex baseband copolarized outputs for the direct path and the surface scatter process, obtained using (D.9), are given by

$$\tilde{s}_D(t) = \frac{\sqrt{P_D}}{2} e^{-j\theta(t)} (G_H(D_a) + G_V(D_a)) \quad (D.15)$$

$$\tilde{s}_{sc}(t) = \frac{\sqrt{P_D}}{2} (\tilde{s}_H(t) + \tilde{s}_V(t)) \quad (D.16)$$

It is evident that the copolarized output can be obtained by a simple linear combination of the individual horizontal and vertically polarized components. In the simulation program, the constant P_D was assigned a value of unity.

3.0 INTERRELATIONSHIPS BETWEEN THE POLARIZATION RATIO AND THE AXIAL RATIO OF A POLARIZATION ELLIPSE

In this section, expressions are derived which relate polarization ratio parameters and the axial ratio of an antenna.

The polarization ellipse is defined by equation (D.3).

Defining a new co-ordinate system by rotating the x-y axes by a tilt angle, ψ , one obtains

$$A E_{x'}^2 + C E_{x'} E_{y'} + B E_{y'}^2 = 1 \quad (D.17)$$

where

$$A = a \cos^2 \psi - b \cos \psi \sin \psi + c \sin^2 \psi$$

$$B = a \sin^2 \psi + b \cos \psi \sin \psi + c \cos^2 \psi$$

$$C = -2a \cos \psi \sin \psi + 2c \cos \psi \sin \psi - b \cos 2\psi.$$

The cross-term goes to zero when

$$\begin{aligned} \tan 2\psi &= \frac{b}{c - a} \\ &= \frac{2 A_x A_y \cos \delta}{A_x^2 - A_y^2} \\ &= \frac{2 (A_y/A_x) \cos \delta}{1 - (A_y/A_x)^2} = \frac{2 |P| \cos \delta}{1 - |P|^2} \end{aligned} \quad (D.18)$$

Defining $v = \tan^{-1} |P|$, it is apparent that

$$\tan 2\psi = \tan 2v \cos \delta. \quad (D.19)$$

The axes of the ellipse are defined by

$$F_{x'}^2 = \frac{1}{a \cos^2 \psi - b \cos \psi \sin \psi + c \sin^2 \psi} \quad (D.20)$$

$$F_y^2 = \frac{1}{a \sin^2 \psi + b \cos \psi \sin \psi + c \cos^2 \psi} \quad (D.21)$$

After some tedious algebraic manipulation, it can be shown the ratio of the semi-minor to semi-major axes, denoted by (M_b/M_a) is given by (see Section 12.06 of [14])

$$\left(\frac{M_b}{M_a}\right) = \pm \tan \epsilon \quad (D.22)$$

where ϵ is given by the relationship

$$\sin 2\epsilon = \sin 2\psi \sin \delta. \quad (D.23)$$

In the above, the minus sign indicates right-handed polarization and the plus sign indicates left-hand polarization.

The most common definition of the axial ratio is the magnitude of the ratio of the semi-major to semi-minor axes of the polarization ellipse

$$A_R = \left| \frac{M_a}{M_b} \right| = \frac{1}{|\tan \epsilon|} \quad (D.24)$$

In order to retain the indication of the sense of the polarization ellipse, an ellipticity ratio ϵ_R will be defined as follows

$$\epsilon_R = \frac{1}{\tan \epsilon} \quad (D.25)$$

In this case, the sign of ϵ_R will indicate the sense of the polarization.

In the signal analysis program, a phase mismatch θ_m and a gain mismatch can be introduced between the vertical and horizontal antenna gains. With a right hand circularly polarized antenna, a nominal phase shift of $-\pi/2$ is introduced between the vertically and horizontally polarized components.

Consequently, with gain and phase mismatches introduced, the polarization ratio is defined by

$$|P| = A_y/A_x, \quad (D.26)$$

$$\delta = \theta_m - \frac{\pi}{2}. \quad (D.27)$$

A way of establishing values of $|P|$ and θ_m which provide a specific axial ratio A_R is desired. It is evident (using (D.23), (D.25), and (D.27)) that

$$\epsilon_R = \frac{1}{\tan \epsilon}$$

where

$$\sin 2\epsilon = \frac{-2 |P| \cos \theta_m}{1 + |P|^2}. \quad (D.28)$$

Since we are concerned with phase mismatches

$\theta_m \in [-\pi/2, \pi/2]$, it is evident that the sign of the

ellipticity ratio is negative. This indicates right handed sense polarization. From (D.25), it is evident that

$$\sin \epsilon = \frac{1}{\sqrt{1 + \epsilon_R^2}} \quad (D.29)$$

$$\cos \epsilon = \frac{\epsilon_R}{\sqrt{1 + \epsilon_R^2}} \quad (D.30)$$

Since $\sin 2\epsilon = 2 \sin \epsilon \cos \epsilon$, one can express (D.28) as follows

$$\frac{2 \epsilon_R}{1 + \epsilon_R^2} = \frac{-2 |P| \cos \theta_m}{1 + |P|^2} \quad (D.31)$$

Consequently, the axial ratio, A_R , for which the sign is unimportant, is given by

$$\frac{A_R}{1 + A_R^2} = \frac{|P| \cos \theta_m}{1 + |P|^2} \quad (D.32)$$

Given an axial ratio, it is easy to establish magnitudes of the polarization ratio and phase mismatches which provide this axial ratio using (D.32).

Often the polarization characteristics of antennas are specified in terms of a polarization state

$$Q = \frac{E_{LHC}}{E_{RHC}} = |Q|e^{j\zeta}, \quad (D.33)$$

where E_{LHC} and E_{RHC} denote the LHC polarization and RHC polarization voltage gains. The resultant field is right handed for $|Q| < 1$ and left handed for $|Q| > 1$. Using the relationship presented on page 460 of [14], the axial ratio is given by

$$A_R = \left| \frac{1 + |Q|}{1 - |Q|} \right|. \quad (D.34)$$

This relationship is particularly useful for establishing typical axial ratios for the shipborne antenna patterns provided in [6] and [9].

4.0 EFFECT OF SATELLITE ANTENNA POLARIZATION PURITY

In this section, an expression for the combined multipath signal, which accounts for satellite antenna ellipticity, is derived. The satellite antenna polarization ratio is defined as follows.

$$P_S = E_S e^{j\delta} \quad (D.35)$$

where

$$E_S = |P_S| = \frac{a_V}{a_H}, \quad (D.36)$$

$$\delta = \theta_m - \frac{\pi}{2}, \text{ and}$$

θ_m denotes the phase mismatch from perfect RHC polarization.

In this case, the complex baseband transmitted electric field vector is

$$\tilde{E}_T(t) = \sqrt{\frac{P_T}{2}} \begin{bmatrix} a_H \\ -ja_V e^{j\theta_m} \end{bmatrix} \quad (D.37)$$

and the transmitted power is given by

$$P_t = \left[\frac{a_H^2 + a_V^2}{2} \right] P_T \quad (D.38)$$

The transmitted power level must not depend on the antenna ellipticity, which implies that

$$a_H^2 + a_V^2 = 2. \quad (D.39)$$

Using (D.36) and (D.39), the signal scale factors are given by

$$a_H = \sqrt{\frac{2}{1 + E_S^2}} \quad (D.40)$$

$$\alpha_V = \sqrt{\frac{2E_S^2}{1 + E_S^2}} \quad (D.41)$$

The appropriate direct path and multipath signals for circularly polarized receive antenna are given by

$$\tilde{s}_D(t) = \frac{\sqrt{P_D}}{2} e^{-j\theta(t)} [\alpha_H G_H(D_a) + \alpha_V e^{j\theta_m} G_V(D_a)] \quad (D.42)$$

$$\tilde{s}_{sc}(t) = \frac{\sqrt{P_D}}{2} [\alpha_H \tilde{s}_H(t) + \alpha_V e^{j\theta_m} \tilde{s}_V(t)] \quad (D.43)$$

In the simulation, the received power constant P_D is assumed to be 0 dB.

It should be noted that these expressions can also be used to establish the effects of receiving antenna phase mismatches and additional gain imbalance. The relationship between the axial ratio of an antenna and the polarization ratio parameters $|P|$ and θ_m , presented in (D.32), is invaluable for establishing realistic mismatch parameters.

APPENDIX E

AERONAUTICAL DOPPLER SPECTRA

1.0 INTRODUCTION

In this appendix, the doppler shift introduced by a scattering facet is derived. Justification for the observed doppler spectrum asymmetry obtained with the aeronautical propagation simulation is provided. First, the nature of specular point motion on the ocean surface, as an aircraft flies over the surface, is addressed.

2.0 SPECULAR POINT MOTION

The fundamental nature of specular point motion is most easily grasped through an informative example. The basic geometry of the situation is illustrated in Figure E.1, where the local grazing angle α is equal to the elevation angle E at the nominal specular point x_s (which is located at the crest of the wave).

Consider two specular points x_1 and x_2 located symmetrically about the crest,

$$x_1 = x_s - \Delta x_1 \quad (E.1)$$

Figure E.1 Illustration of specular point motion.

$$x_2 = x_s + \Delta x_t \quad (E.2)$$

where Δx_t is the specular point deviation, which is assumed to be small. In the neighbourhood of the parabolic scattering facet, the surface reflection angle is given by

$$\begin{aligned} \beta(x) &= E - 2 \tan^{-1} \{z'_s(x)\} \\ &= E - 2 \tan^{-1} \left\{ \frac{x_s - x}{r_c} \right\} \end{aligned} \quad (E.3)$$

Therefore, the reflection angles at the two specular points are given by

$$\beta_1 = E - 2\epsilon_0 \quad (E.4)$$

$$\beta_2 = E + 2\epsilon_0 \quad (E.5)$$

where

$$\epsilon_0 = \tan^{-1} \left(\frac{\Delta x_t}{r_c} \right) = \frac{\Delta x_t}{r_c} \quad (E.6)$$

It is interesting to establish the behaviour of the path length difference when the reflection angle is less than and greater than the elevation angle. For this scenario, the path length difference, δ , (see Figure 2.3) can be expressed as

$$\begin{aligned} \delta &= \frac{h_A}{\sin \theta} (1 - \cos (E + \theta)) \\ &= h_A \left(\frac{1}{\sin \theta} - \cos E \cot \theta + \sin E \right). \end{aligned} \quad (E.7)$$

If one considers small deviations, $(2c)$, in the reflection angle about the elevation angle (i.e. $\theta = E - 2c$), it can be shown that

$$\delta = 2h_A \sin E + \frac{(2c)^2 h_A}{2 \sin E}. \quad (E.8)$$

The Taylor series approximations

$$\frac{1}{\sin(x+h)} = \frac{1}{\sin x} - \left(\frac{\cos x}{\sin^2 x} \right) h + \left(\frac{\sin^3 x + 2 \sin x \cos^2 x}{\sin^4 x} \right) \frac{h^2}{2} \quad (E.9)$$

$$\cot(x+h) = \cot x - \left(\frac{1}{\sin^2 x} \right) h + \left(\frac{2 \cos x}{\sin^3 x} \right) \frac{h^2}{2} \quad (E.10)$$

were used in this derivation. It is evident that the path length difference is minimized at the nominal specular point, where $\theta = E$ ($c=0$). Therefore, as the specular point moves from x_1 to x_s , the relative path length difference decreases, which corresponds to positive doppler shifts. Conversely, as the specular point moves from x_s to x_2 , the relative path length difference increases, which corresponds to negative doppler shifts.

The distance between the aircraft position and the specular point locations can be approximated (ignoring the waveheight fluctuations) by

$$d_1 = \frac{h_A}{\tan(E - 2\epsilon_0)} \quad (E.11)$$

$$d_{\text{nom}} = \frac{h_A}{\tan E} \quad (E.12)$$

$$d_2 = \frac{h_A}{\tan(E + 2\epsilon_0)} \quad (E.13)$$

where h_A is the vertical distance between the aircraft and specular point positions.

The distance the aircraft must move for the specular point to move from x_1 to x_s is

$$\begin{aligned} \Delta x_1 &= d_1 - d_{\text{nom}} + \Delta x_\ell \\ &= h_A \cot(E - 2\epsilon_0) - h_A \cot E + \Delta x_\ell \end{aligned} \quad (E.14)$$

$$= \frac{h_A (2\epsilon_0)}{\sin^2 E} + \frac{h_A \cos E}{\sin^3 E} (2\epsilon_0)^2 + \Delta x_\ell$$

where this latter simplification follows after application of the Taylor series approximation of (E.10).

To get some indication for the amount of aircraft motion necessary to introduce small perturbations in the specular point location, the following realistic parameters are considered

$$\Delta x_l = 0.2 \text{ m}, r_c = 20 \text{ m}, \text{ and } h_A = 10 \text{ km}.$$

The resulting distances the aircraft must move, for elevation angles of 45° and 20° , established using (E.14), are

$$\Delta x_l = \begin{cases} 408.45 \text{ m} & E = 45^\circ \\ 1809.73 \text{ m} & E = 20^\circ \end{cases} \quad (\text{E.15})$$

It is evident that appreciable aircraft motion is required to introduce relatively small specular point deviations. Therefore, the specular point moves very slowly on the ocean surface.

The distance that the aircraft must move for the specular point to move from x_s to x_2 is

$$\begin{aligned} \Delta x_2 &= d_{\text{nom}} - d_2 + \Delta x_l \\ &= h_A \cot E - h_A \cot(E + 2\epsilon_0) + \Delta x_l \\ &= \frac{h_A (2\epsilon_0)}{\sin^2 E} - \frac{h_A \cos E}{\sin^3 E} (2\epsilon_0)^2 + \Delta x_l \end{aligned} \quad (\text{E.16})$$

where this latter simplification follows after application of the Taylor series approximation of (E.10). The difference in the distance travelled by the aircraft is

$$\Delta x_1 - \Delta x_2 = \frac{2 h_A \cos E}{\sin^3 E} (2c_0)^2 > 0. \quad (E.17)$$

Because this difference is positive, it is apparent that the specular point moves toward the crest (decreasing path length difference or positive doppler region) more slowly than the specular point moves away from the crest (increasing path length difference or negative doppler region). This is a very important property of specular point motion, which will have an influence on the doppler spectrum (second order statistics).

It is apparent that the specular point moves very slowly, and for characterizing the variation in the horizontal distance, $x_0(t)$, between the aircraft and specular point location, one can effectively ignore specular point motion. Because the aircraft altitude is so much greater than typical surface fluctuations, one can effectively ignore the variations in specular point height also. With these approximations, the parameters of Figure E.2 can be defined as functions of time as follows

$$x_0(t) = x_0 - v_x t \quad (E.18)$$

$$h_A(t) = h_A + v_z t \quad (E.19)$$

Figure E.2: Geometrical relationships used in doppler derivations.

$$\beta(t) = \tan^{-1} \left\{ \frac{h_A(t)}{x_O(t)} \right\} \quad (E.20)$$

where

$$\beta = \beta(0) = \tan^{-1} \left\{ \frac{h_A}{x_O} \right\},$$

$$h_A = h_A(0) = z_O + \frac{x_O^2}{2a_e},$$

a_e is the radius of the earth, and

v_x and v_z denote the horizontal and vertical velocity components of the aircraft.

These relationships can be used to derive general expressions for the amount of specular point motion. Evaluating the time variation of the reflection angle is critical. This can be expressed as

$$\begin{aligned} \frac{d\beta(t)}{dt} &= \frac{d}{dt} \tan^{-1} \left\{ \frac{h_A(t)}{x_O(t)} \right\} \\ &= \frac{1}{1 + \left(\frac{h_A(t)}{x_O(t)} \right)^2} \frac{d}{dt} \left\{ \frac{h_A(t)}{x_O(t)} \right\} \\ &= \frac{1}{x_O^2(t) + h_A^2(t)} (x_O(t) v_z + h_A(t) v_x) \\ &= \frac{h_A^2(t)}{x_O^2(t) + h_A^2(t)} \left[\frac{v_x}{h_A(t)} + \frac{v_z}{h_A(t)} \left(\frac{x_O(t)}{h_A(t)} \right) \right] \end{aligned}$$

$$= \frac{v_x \sin^2(\beta(t))}{h_A(t)} \left[1 + \frac{v_z}{v_x \tan(\beta(t))} \right] \quad (E.21)$$

The specular point constraint, $\beta = E - 2 \tan^{-1}\{z'_s(x)\}$, can be used to derive an alternative expression

$$\begin{aligned} \frac{d\beta(t)}{dt} &= \frac{-2 z''_s(x_s(t))}{(1 + z'_s(x_s(t))^2)} \frac{dx_s(t)}{dt} \\ &= \frac{-2 z''_s(x_s(t))}{(1 + \tan^2(\frac{E-\beta(t)}{2}))} \frac{dx_s(t)}{dt} \end{aligned} \quad (E.22)$$

where $\frac{dx_s(t)}{dt}$ denotes the variation of the specular point location with time.

Consequently, the specular point deviation is given by

$$\frac{dx_s(t)}{dt} = - \left[\frac{1 + \tan^2(\frac{E-\beta(t)}{2})}{2 z''_s(x_s)} \right] \frac{d\beta(t)}{dt} \quad (E.23)$$

where

$$\frac{d\beta(t)}{dt} \text{ is given by (E.21).}$$

Because $\frac{d\beta(t)}{dt}$ is so small, primarily because of high aircraft altitudes, specular point deviations occur very slowly for aeronautical applications. The same will not be true for the maritime application, where the rate of change of the reflection angle is significantly higher primarily because of the close proximity of the shipborne antenna to

the ocean surface. Furthermore, the motion of the surface, which is comparable to typical vessel speeds, also introduces specular point perturbations. For the maritime application, one can approximate the relative positions of the x-co-ordinate of the ship antenna and the specular point locations by

$$x_o(t) = x_o - (v_{ship} - \bar{v}) t \quad (E.24)$$

where v_{ship} and $\bar{v}$ represent the velocities of the vessel and the average wave speed, respectively.

The formula of (E.23), with the vertical velocity set to zero, is used to establish typical specular point deviations in the maritime simulation program. These are then used in the classification index (visible, blocked, or shadowed) sensitivity evaluation.

3.0 DIFFERENTIAL DOPPLER SHIFT FROM A SCATTERING FACET

The differential doppler shift (relative to the direct path) introduced by a scattering facet with a reflection angle, β , relative to the horizontal, is given by

$$f_d(t) = \frac{-1}{\lambda_c} \frac{d}{dt} \delta(t) \quad (E.25)$$

where

$$\delta(t) = \frac{h_A(t)}{\sin \beta(t)} [1 - \cos(E + \beta(t))] \quad (E.26)$$

and $h_A(t)$ and $\beta(t)$ are defined by (E.19) and (E.20), respectively.

The derivative of the path length difference can be expressed as

$$\frac{d\delta(t)}{dt} = h_A(t) \frac{da(t)}{dt} + a(t) \frac{dh_A(t)}{dt} \quad (E.27)$$

where

$$a(t) = \frac{1 - \cos(E + \beta(t))}{\sin \beta(t)} \quad (E.28)$$

$$\frac{dh_A(t)}{dt} = v_z \quad (E.29)$$

The only remaining unknown is $\frac{da(t)}{dt}$, which can be established using the chain rule for differentiation

$$\frac{da(t)}{dt} = \frac{da(\beta(t))}{d\beta} \frac{d\beta(t)}{dt} \quad (E.30)$$

Equation (E.21) defines $\frac{d\beta(t)}{dt}$, and it is easily shown that

$$\frac{da(\beta(t))}{d\beta} = \frac{\cos E - \cos \beta(t)}{\sin^2 \beta(t)} \quad (E.31)$$

Incorporating these expressions in (E.30) and simplifying yields

$$\frac{da(t)}{dt} = \frac{(\cos E - \cos \beta(t))}{h_A(t)} v_x \left(1 + \frac{v_z}{v_x \tan \beta(t)} \right) \quad (E.32)$$

Incorporating this expression in (E.27) gives, after some algebraic simplification,

$$\begin{aligned} \frac{d\delta(t)}{dt} = & v_x (\cos E - \cos \beta(t)) + \\ & v_z \left[\frac{(\cos E - \cos \beta(t))}{\tan \beta(t)} + \frac{(1 - \cos(E + \beta(t)))}{\sin \beta(t)} \right]. \end{aligned} \quad (E.33)$$

Using straightforward trigonometric identities and after some algebraic simplification, the derivative of the path length difference can be expressed as

$$\frac{d\delta(t)}{dt} = v_x [\cos E - \cos \beta(t)] + v_z [\sin E + \sin \beta(t)]. \quad (E.34)$$

Inserting this expression in (E.25) provides the general expression for the doppler shift as a function of time.

However, because $\frac{d\delta(t)}{dt}$ is so small for the aeronautical application, the doppler shift is essentially constant, and given by

$$f_d = \frac{v_x}{\lambda_c} (\cos \beta - \cos E) - \frac{v_z}{\lambda_c} (\sin \beta + \sin E). \quad (E.35)$$

4.0 COMPARISONS WITH DOPPLER EXPRESSIONS DERIVED IN REFERENCE [22]

In [22], a general expression for the doppler shift, f_D , relative to that appropriate at the nominal specular point, is derived for the aeronautical application. A flat earth surface is assumed. The derivation is specifically for scattering facets in the neighbourhood of the nominal specular point, which implies very small surface slopes. The basic result, for a two-dimensional surface, is

$$f_D(x) = \left[\frac{v}{\lambda_c} x \sin E + \frac{v}{\lambda_c} z \cos E \right] \sin^2 E \left(\frac{x}{h_A} - \frac{1}{\tan E} \right) \quad (\text{E.36})$$

or

$$f_D(\theta) = \left[\frac{v}{\lambda_c} x \sin E + \frac{v}{\lambda_c} z \cos E \right] \sin^2 E \left(\frac{1}{\tan \theta} - \frac{1}{\tan E} \right) \quad (\text{E.37})$$

For values of x in the neighbourhood of the nominal specular point, the reflection angle cannot deviate significantly from the elevation angle. This implies that surface slopes must be quite small, and the reflection angle can be accurately approximated by

$$\theta = E - 2 \tan^{-1}(z') = E - 2z', \quad (\text{E.38})$$

and

$$\left(\frac{1}{\tan \theta} - \frac{1}{\tan E} \right) = \frac{2z'}{\sin^2 E}, \quad (\text{E.39})$$

where z' is a Gaussian random variable representing the surface slope.

Using these approximations,

$$f_D = 2z' \left[\frac{v_x}{\lambda_c} \sin E + \frac{v_z}{\lambda_c} \cos E \right] \quad (E.40)$$

which demonstrates that the doppler shift in this case is a Gaussian random variable, with standard deviation

$$\sigma_{f_D} = 2\sigma_z \cdot \left[\frac{v_x}{\lambda_c} \sin E + \frac{v_z}{\lambda_c} \cos E \right]. \quad (E.41)$$

It must be emphasized that this doppler shift is relative to the doppler shift at the nominal specular point. The path length difference of the nominal specular path, relative to the direct path, is given by

$$\delta_{sp}(t) = 2 h_A(t) \sin E, \quad (E.42)$$

which introduces a corresponding doppler shift

$$\omega_{\mu} = - 2 \frac{v_z}{\lambda_c} \sin E. \quad (E.43)$$

The doppler shift relative to the direct path is given by

$$f_d = f_D + \mu$$

$$= 2z' \left[\frac{v_x}{\lambda_c} \sin E + \frac{v_z}{\lambda_c} \cos E \right] - \frac{2v_z \sin E}{\lambda_c}. \quad (E.44)$$

Because the power weighting factor in a small region about the nominal specular point is effectively constant, the doppler spectrum will essentially assume a Gaussian spectrum shape

$$S_{f_d}(f) = \frac{1}{\sqrt{\pi} B} \exp \left\{ - \left(\frac{f - \mu}{B} \right)^2 \right\} \quad (\text{E.45})$$

where

μ is defined by (E.43), and

$$\begin{aligned} B &= \sqrt{2} \sigma_{f_D} \\ &= 2\sqrt{2} \sigma_z \cdot \left[\frac{v_x}{\lambda_c} \sin E + \frac{v_z}{\lambda_c} \cos E \right]. \end{aligned} \quad (\text{E.46})$$

For level flight conditions, the e^{-1} bandwidth is given by

$$B_{1f} = \frac{2\sqrt{2} \sigma_z}{\lambda_c} \sin E \cdot v_x, \quad (\text{E.47})$$

which is the result quoted in [8].

The general doppler shift expression of (E.35) is simplified to see what is obtained for the small slope assumptions used in the derivation presented in [22]. Restricting attention to small surface slopes, the reflection angle can be expressed as in (E.38), and if the magnitude of the surface slope is also much less than $\tan E$, the following approximations are valid

$$\cos \beta - \cos E = 2z' \sin E \quad (E.48)$$

$$\sin \beta + \sin E = -(2z') \cos E + 2 \sin E. \quad (E.49)$$

Using these approximations in the general doppler shift expression of (E.35) yields

$$f_d = 2z' \left[\frac{v_x}{\lambda_c} \sin E + \frac{v_z}{\lambda_c} \cos E \right] - \frac{2v_z \sin E}{\lambda_c}, \quad (E.50)$$

which is identical to that obtained in (E.44) for the derivation approach of [22]. Consequently, under these restrictive assumptions on the slope (small z' values) and elevation angle ($\tan E \gg |z'|$), one will obtain a doppler spectrum with a Gaussian shape.

5.0 JUSTIFICATION OF THE DOPPLER SPECTRUM ASYMMETRY

The simulated doppler spectra for level flight conditions, presented in Figures 3.16, 3.17, 3.18, 3.19, and 3.20, are noticeably asymmetric. In this section, explanations for this doppler asymmetry are provided.

For level flight conditions, the doppler shift from a scattering facet with reflection angle β is given by

$$f_{dif} = \frac{v_x}{\lambda_c} (\cos \beta - \cos E). \quad (E.51)$$

It is apparent that the resulting doppler shift is positive when $\beta < E$, and negative when $\beta > E$. The behaviour of this doppler shift, as a function of $\Delta\beta = \beta - E = 2 \tan^{-1}(z')$, can be expressed as

$$f_{d1f}(\Delta\beta) = \frac{v}{\lambda_c} [\sin \Delta\beta \sin E - (1 - \cos \Delta\beta) \cos E]. \quad (E.52)$$

Only when the surface slopes are small and such that $|z'| \ll \tan E$, does one obtain a symmetric doppler spectrum. For typical rough sea conditions, the rms sea slopes are in excess of 6° , and, in the absence of antenna discrimination, the elevation angle must be quite large to obtain a Gaussian doppler spectrum shape. It should be pointed out that one can obtain a symmetric Gaussian doppler spectrum if the combination of the transmitter and receiver antenna gain patterns are such that only surface facets in the immediate neighbourhood of the nominal specular point contribute. This is because the effective slope values in this region are small, even though the rms slope is not. However, the antenna patterns considered in the aeronautical simulation program did not have this property.

The doppler shifts for reflection angles located symmetrically about the elevation angle are given by

$$f_{\text{dif}} = \begin{cases} -\frac{v_x}{\lambda c} [\sin w \sin E + (1 - \cos w) \cos E] & \Delta\beta = -w \\ +\frac{v_x}{\lambda c} [\sin w \sin E - (1 - \cos w) \cos E] & \Delta\beta = w \end{cases} \quad (\text{E.53})$$

and the difference in the magnitude of the negative doppler shift and the positive doppler shift is given by

$$\begin{aligned} \Delta f_{\text{dif}}(w) &= |f_{\text{dif}}(-w)| - f_{\text{dif}}(w) \\ &= 2 \left(\frac{v_x}{\lambda c} \right) (1 - \cos w) \cos E. \end{aligned} \quad (\text{E.54})$$

Since $-\frac{\pi}{2} < w < \frac{\pi}{2}$, it is evident that $\Delta f_{\text{dif}} > 0$.

Consequently, it is apparent that negative doppler shifts tend to be greater in magnitude than corresponding positive doppler shifts.

It is also evident from the power density versus reflection angle curves, presented in Figures 3.13, 3.14 and 3.15, as well as Figures 4.15, 4.16, 4.17, and 4.18, that the majority of scatter power arises from the region where $\beta < E$, which is the positive doppler region. The power weighting factor in the negative doppler region, where $\beta > E$, is considerably smaller than in the positive doppler region. Combining this with the fact that the magnitude of

the doppler shifts in the negative doppler region exceed those in the positive doppler region, the observed doppler spectrum characteristics are sensible.

Having access to scatter power density versus reflection angle curves can be particularly useful, as one can estimate the doppler (power) spectrum by a simple doppler mapping process. That is, associated with a specific doppler shift f_d is a unique reflection angle β_0 , and the power density value at this reflection angle is a reasonable estimate of the doppler spectrum value at this frequency. This power weighting versus reflection angle approach is also used in [29], [30] for establishing doppler spectral characteristics for land mobile radio applications. The usefulness of this doppler spectrum estimation approach is established in Section 4.3 of Chapter 4.

Before concluding this section, the effect of vertical aircraft motion on doppler spectrum characteristics is established. The doppler shift, as a function of the climb angle $C = \tan^{-1} \left\{ \frac{v_z}{v_x} \right\}$, is

$$f_d = \frac{v_x}{c} [(\cos \beta - \cos E) - \tan C (\sin \beta + \sin E)] \quad (E.55)$$

Note that at low reflection angles, the positive doppler shift that would have been obtained for level flight conditions is reduced for positive climb angles, while being increased for negative climb angles. Furthermore, for higher reflection angles, the negative doppler shift that would have been obtained for level flight conditions, is made more extreme for positive climb angles and less extreme for negative climb angles. Consequently, the positive doppler cutoff point will be larger for negative climb angles than positive climb angles, while more extreme negative doppler shifts will be obtained for positive climb angles. This behaviour is clearly illustrated in Figure E.3, which presents simulated aeronautical doppler spectra for climb angles of $\pm 10^\circ$ and an elevation angle of 15° .

Figure E.3: Simulated aeronautical doppler spectra for climbing and diving manoeuvres for an elevation angle of 15°.

APPENDIX F.

CHARACTERIZING SIGNAL TO INTERFERENCE RATIOS FOR AERONAUTICAL APPLICATIONS

In this appendix, two approaches for estimating the signal to interference ratio are compared. The first estimation procedure is the traditional approach of averaging the instantaneous scatter power samples over the signal record. The issue of assigning realistic confidence limits for the mean scatter power, appropriate for this estimation approach, is addressed in Attachment I of this appendix. Under slow fading conditions, the average power estimated with this approach can exhibit considerable fluctuation, especially if the signal record does not include a sufficiently large number of fade cycles.

With the second estimation approach, the average scatter power is determined by integrating the power density versus reflection angle curves. The power density characterization assumes that the individual components from different scattering facets have independent and uniformly distributed phase shifts. With these reasonable assumptions, the overall scatter power from a specific region is given by the sum of the scatter powers from the individual scattering facets in that region.

The signal to interference (S/I) ratio results for level flight conditions, an aircraft altitude of 9.1 km, SS4(ii) surface conditions, and an omni antenna pattern are summarized in Table F.1. The agreement between the two S/I ratio estimation approaches is quite good, especially for elevation angles above 10°. The agreement is not as good for $E=5^\circ$, but this is a very slow fading case, as is evident from the plot of the received signal voltage versus position provided in Figure F.1.

Using the approach obtained in Attachment I, and the extracted B_{10} fading bandwidth values of Table 3.7, one obtains the following representative 95% confidence limits for the actual average scatter power, $\bar{P}_{sc}$, with an omni antenna pattern

Elevation Angle	Lower Confidence Limit	Upper Confidence Limit	
31°	$\hat{P}_{dB} - 0.82 \text{ dB}$	$\hat{P}_{dB} + 0.69 \text{ dB}$	(F.1)
10°	$\hat{P}_{dB} - 1.36 \text{ dB}$	$\hat{P}_{dB} + 1.03 \text{ dB}$	

where $\hat{P}$ is the average over the record of instantaneous power samples.

Signal to Interference Ratios [dB] For The Following Polarization Types			
Elevation Angle	Horizontal	Vertical	Circular
31°	0.73 (1.21)	3.43 (3.93)	17.46 (17.88)
20°	0.55 (1.02)	5.23 (5.86)	13.36 (13.60)
15°	1.53 (1.60)	7.85 (7.97)	12.35 (12.41)
10°	2.41 (2.74)	10.6 (11.27)	11.47 (11.72)
5°	4.42 (5.21)	15.47 (16.20)	11.7 (12.45)

Values in parentheses obtained with power density method

Table F.1: Signal to interference ratio results for level flight conditions, an aircraft altitude of 9.1 km, SS4(ii) surface conditions, and an omni antenna pattern.

Figure F.1: Received signal level, normalized with respect to the direct path, versus position for an elevation angle of 50° .

Note that the confidence limit range at $E = 5^\circ$ will even be greater than that applicable at $E = 10^\circ$ because the fade rate decreases with lowering elevation angle.

An examination of Table F.1 reveals that the values of the average scatter power obtained with the power density approach are consistent with the above confidence limits.

For SS3 surface conditions, the fading bandwidth $B_{10} = 0.1 v_x$, where v_x is the horizontal airspeed of the aircraft. The appropriate 95% confidence limits on the mean scatter power for this scenario, using the approach of Attachment I, are

$$\bar{P}_{sc} \in [\hat{P}_{dB} - 1.97 \text{ dB}, \hat{P}_{dB} + 1.35 \text{ dB}]. \quad (F.2)$$

If one had to rely solely on the estimate of the mean scatter power obtained by averaging instantaneous power samples, under such conditions, one should increase the record length to reduce the amount of fluctuation. In light of the current computational requirements (a simulation run at 10° elevation angle with SS3 surface conditions, and a record length of 1024 samples has an execution time on the order of 9 hours on a high speed FPS-164 array processor system), this is not particularly attractive. However, it should be noted that one can obtain an estimate of the signal to interference ratio

using the power density method, which does not entail any increase in simulation complexity. Although time constraints prohibited this, it would be worthwhile comparing the two scatter power estimation techniques for a much longer record of simulated signal samples.

S/I ratio results for two different SS3 surface profiles are provided in Table F.2. The S/I ratios obtained by averaging the instantaneous power samples are noticeably different (by as much as 1.8 dB for circular polarization). However, the differences between the S/I values obtained with the two estimation procedures are consistent with the confidence limits of (F.2). Even of more importance, the differences between the S/I values obtained with the power density approach for the two surface profiles are very minor (maximum difference is only 0.14 dB, and this occurs for circular polarization). This stability makes the power density estimation procedure superior, especially for slow fading conditions. For assessing the impact of varying sea state and antenna height, the stability of the S/I ratios obtained with the power density method is extremely beneficial.

Signal to Interference Ratios [dB] For The Following Polarization Types			
Sea Surface Descriptor Seed	Horizontal	Vertical	Circular
S1111	2.09 (2.39)	10.31 (11.25)	11.31 (11.26)
SZERO	0.87 (2.31)	9.96 (11.16)	9.51 (11.12)

Values in parentheses obtained with power density method

Table F.2 Signal to interference ratio results for
SS3 surface conditions, an aircraft
altitude of 9.1 km, an elevation angle of
10°, and an omni antenna pattern.

ATTACHMENT I

ESTABLISHING CONFIDENCE LIMITS FOR THE ESTIMATED MEAN SCATTER POWER

In this attachment, the confidence limits on the estimated mean scatter power,

$$\hat{P} = \frac{1}{N} \sum_{i=1}^N R_i^2 \quad (1)$$

are established for multipath samples with a Rayleigh amplitude distribution

$$f_{R_i}(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} \quad (2)$$

The random variable, R_i , can be thought of as arising from two independent Gaussian random variables, X_i and Y_i , with equal variance σ^2 , via the transformation

$$R_i = \sqrt{X_i^2 + Y_i^2} \quad (3)$$

The actual underlying scatter power is denoted by $\bar{P}_{sc} = 2\sigma^2$.

The above estimator is unbiased as

$$E(\hat{P}) = 2\sigma^2 \quad (4)$$

The case where the individual amplitude samples are independent is considered first. By the central limit theorem, the random variable $\hat{P}$ approaches a Gaussian random variable as N becomes large. The variance of the random variable $\hat{P}$ in this case is given by:

$$\begin{aligned}
 \sigma_{\hat{P}}^2 &= E(\hat{P} - 2\sigma^2)^2 \\
 &= \frac{1}{N^2} E\left\{ \sum_{i=1}^N \sum_{j=1}^N (R_i^2 - 2\sigma^2)(R_j^2 - 2\sigma^2) \right\} \\
 &= \frac{1}{N} E\{(R_1^2 - 2\sigma^2)^2\} \\
 &= \frac{1}{N} E\{(X_1^2 - \sigma^2)^2 + (Y_1^2 - \sigma^2)^2\} \\
 &= \frac{4\sigma^4}{N} .
 \end{aligned} \tag{5}$$

The 95% confidence limits for $\bar{P}_{sc}$ in this case are:

$$\begin{aligned}
 l_c &= \hat{P} - 1.96 \sigma_{\hat{P}} \\
 u_c &= \hat{P} + 1.96 \sigma_{\hat{P}}
 \end{aligned} \tag{6}$$

where

$$\sigma_{\hat{P}} = \frac{2\sigma^2}{\sqrt{N}} .$$

Since the true value of $\bar{P}_{sc} = 2\sigma^2$ is unknown, $\sigma_{\hat{P}}$ cannot be determined exactly. However, it can be approximated using $\hat{P}$ as an estimate of $2\sigma^2$. With this approximation, the 95% confidence limits become:

$$\begin{aligned}
 l_c &= \hat{P} \left(1 - \frac{1.96}{\sqrt{N}}\right) \\
 u_c &= \hat{P} \left(1 + \frac{1.96}{\sqrt{N}}\right) .
 \end{aligned} \tag{7}$$

With 256 independent sample points, one finds that the following 95% confidence limits for $\bar{P}_{sc}$ dB are appropriate:

$$\begin{aligned} l_c &= \hat{P}_{dB} - 0.57 \text{ dB} \\ u_c &= \hat{P}_{dB} + 0.5 \text{ dB} \end{aligned} \quad (8)$$

Consequently, even with 256 independent samples, differences between $\bar{P}_{sc}$ dB and $\hat{P}$ dB on the order of ± 0.5 dB are not unreasonable.

Although the above approach is informative, one really needs an approach which takes into account the correlation between samples of the multipath signal record. One way of approximating the effects of correlation is compute an approximate number of independent samples $N_a = \frac{T}{\tau}$, where T is the overall record length and τ is the correlation period of the random process (which is related to the inverse bandwidth of the power spectrum). One then uses the approximate number of independent samples N_a instead of N in the confidence limit expressions. Establishing an appropriate proportionality constant between τ and the inverse bandwidth is important. For a Gaussian correlation function, the e^{-1} correlation period,

$$\tau = \frac{1}{\pi B} \quad (9)$$

6

of/de

6

MICRO

where B is the e^{-1} bandwidth, and the approximate number of independent samples is given by

$$N_a = \pi BT . \quad (10)$$

A more exact analysis of the effects of correlation is presented below for Gaussian doppler spectrum shapes. The complex baseband multipath signal is represented by:

$$S(t) = X(t) + j Y(t) \quad (11)$$

where $X(t)$ and $Y(t)$ are zero mean Gaussian processes, each with power σ^2 . The doppler spectrum of the complex baseband multipath signal

$$S(f) = 2\sigma^2 \frac{1}{\sqrt{2\pi}\sigma_f} \exp\left\{-\frac{1}{2} \left(\frac{f-f_0}{\sigma_f}\right)^2\right\} \quad (12)$$

where

$$\sigma_f = B/\sqrt{2} ,$$

B is the e^{-1} bandwidth, and

f_0 is a general frequency offset which disturbs the spectrum symmetry about the origin, thereby introducing correlation between the quadrature $X(t)$ and $Y(t)$ components.

The autocorrelation function of the complex baseband multipath signal,

$$\begin{aligned}
 R_{SS}(u) &= 2R_{XX}(u) - j 2R_{XY}(u) = \mathcal{F}^{-1} \{S(f)\} \\
 &= 2\sigma^2 \rho_n(u) e^{j2\pi f_o u}
 \end{aligned} \tag{13}$$

where

$$\begin{aligned}
 \rho_n(u) &= \exp\left\{-\frac{1}{2}\left(\frac{u}{\sigma_\tau}\right)^2\right\}, \text{ and} \\
 \sigma_\tau &= \frac{\tau}{\sqrt{2}} = \frac{1}{2\pi\sigma_f} = \frac{1}{\sqrt{2}\pi B}.
 \end{aligned}$$

For this doppler spectrum, the auto-correlation and cross-correlation functions are given by:

$$R_{YY}(u) = R_{XX}(u) = \sigma^2 \rho_n(u) \cos(2\pi f_o u) \tag{14}$$

and

$$R_{XY}(u) = -R_{YX}(u) = -\sigma^2 \rho_n(u) \sin(2\pi f_o u).$$

Using the result for independent samples (5), it is apparent that one can characterize the effects for correlated multipath environments by defining an effective number of independent samples, N_e , as follows:

$$\sigma_{\hat{p}}^2 = \frac{4\sigma^4}{N_e}. \tag{15}$$

The general expression for $\sigma_{\hat{p}}^2$ is derived below:

$$\begin{aligned}
 \sigma_{\hat{P}}^2 &= \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N E[(X_i^2 - \sigma^2) + (Y_i^2 - \sigma^2)][(X_j^2 - \sigma^2) + (Y_j^2 - \sigma^2)] \\
 &= \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N F(X_i^2 - \sigma^2)(X_j^2 - \sigma^2) + F(Y_i^2 - \sigma^2)(Y_j^2 - \sigma^2) + \\
 &\quad F(X_i^2 - \sigma^2)(Y_j^2 - \sigma^2) + F(Y_i^2 - \sigma^2)(X_j^2 - \sigma^2) \\
 &= \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N \{ 2 R_{XX}^2((i-j)\Delta t) + 2 R_{YY}^2((i-j)\Delta t) + \\
 &\quad 2 R_{XY}^2((i-j)\Delta t) + 2 R_{YX}^2((i-j)\Delta t) \}. \quad (16)
 \end{aligned}$$

This latter simplification is possible because for zero mean Gaussian random variables of equal variance (page 221 of [26]):

$$E(x^2 - \sigma^2)(y^2 - \sigma^2) = 2F^2(xy). \quad (17)$$

Substituting the expressions from (14) into (16) and simplifying yields:

$$\begin{aligned}
 \sigma_{\hat{P}}^2 &= \frac{4\sigma^4}{N^2} \sum_{i=1}^N \sum_{j=1}^N \rho_n^2((i-j)\Delta t) \\
 &= \frac{4\sigma^4}{N} \sum_{k=-(N-1)}^{N-1} \left(1 - \frac{|k|}{N}\right) \rho_n^2(k\Delta t). \quad (18)
 \end{aligned}$$

Since $\rho_n(u)$ is Gaussian in nature, $\rho_n^2(u)$ is also Gaussian.

Letting $\rho_s(u) = \rho_n^2(u)$,

$$\rho_s(u) = \exp\left\{-\frac{1}{2} \left(\frac{u}{\sigma_s}\right)^2\right\} \quad (19)$$

where

$$\sigma_s^2 = \sigma_r^2/2.$$

For a large observation interval (number of samples)

$$\begin{aligned} \sigma_{\hat{P}}^2 &= \frac{4\sigma^4}{N} \sum_{k=-\infty}^{\infty} \rho_s(k\Delta t) \\ &= \frac{4\sigma^4}{N \Delta t} \int_{-\infty}^{\infty} \rho_s(t) dt \\ &= \frac{4\sigma^4}{N \Delta t} (\sqrt{2\pi} \sigma_s) = \frac{4\sigma^4}{N \Delta t} \sqrt{\pi} \sigma_r \\ &= \frac{4\sigma^4}{N \Delta t} \sqrt{\frac{\pi}{2}} \tau = 4\sigma^4 \frac{\sqrt{\pi/2} \tau}{\pi} \end{aligned} \quad (20)$$

It is evident from (15) and (20) that the effective number independent samples is

$$N_e = \frac{T}{\sqrt{\pi/2} \tau} = \frac{\pi RT}{\sqrt{\pi/2}} = \left(\frac{\pi B \Delta t}{\sqrt{\pi/2}}\right) N. \quad (21)$$

†

For the aeronautical application, the sample period

$\Delta t = \frac{\lambda}{N_w v_x}$, where λ is the average sea wavelength, N_w is the number of sample points per sea wavelength, and v_x is the horizontal aircraft speed. Bandwidths extracted from doppler plots are usually normalized with respect to the sample frequency $f_s = \frac{1}{\Delta t}$, which yields

$$B_{\text{norm}} = B \Delta t. \quad (22)$$

In terms of normalized bandwidths,

$$N_e = \left(\frac{\pi B_{\text{norm}}}{\sqrt{\pi/2}} \right) N. \quad (23)$$

Often it is much easier to extract the equivalent one-sided -10 dB bandwidth, B_{10} , from doppler spectrum plots. For a Gaussian spectrum,

$$B = 0.659 B_{10}, \quad (24)$$

and the effective number of independent samples can be expressed as

$$N_e = 0.659 \left(\frac{\pi B_{10} \Delta t}{\sqrt{\pi/2}} \right) N \quad (25)$$

$$= 0.659 \left(\frac{\pi B_{10}(\text{norm})}{\sqrt{\pi/2}} \right) N. \quad (26)$$

To establish confidence limits on the average scatter power, the effective number of independent samples, N_e , is used instead of N in (7). Depending on the bandwidth parameter one has access to, the number of independent samples N_e can be established using one of either (21), (23), (25), or (26).

APPENDIX G

STOCHASTIC MODEL FOR THE MARITIME APPLICATION

1.0 INTRODUCTION

In the maritime application, the rate of change of the reflection angle versus time and position is much greater than that for the aeronautical application. One of the consequences of this is that the classification index, (which refers to whether the specular point is blocked, shadowed, or visible) varies with time. This behaviour complicates the implementation of the maritime propagation model. These same properties also complicate the development of a stochastic analysis model for the maritime application.

In the maritime application, the geometric search problem for potential scattering facets can also be modelled as a Gaussian random process curve crossing problem. The evaluation of the solution density is, however, more difficult than it is for the aeronautical application. Because the antenna is much closer to the ocean surface in the maritime environment, the application of the attenuation introduced by divergence must distinguish between concave and convex surface facets and is somewhat more difficult to apply. Coupling of ocean surface and

vessel motion introduces time varying antenna motions, which must be incorporated for estimating the doppler (power) spectrum of the multipath process. This is also important for characterizing the behaviour of the signal to interference ratio with directive shipborne antennas. The procedure developed for incorporating the effects of wave profile blockage for the aeronautical application is directly applicable to the maritime application.

2.0 GEOMETRIC SEARCH PROCESS

The reflection angle from the surface to the receiving antenna can be approximated by (see equation (4.9))

$$\theta(x) = \theta_0(x) - \frac{\sin(2\theta_0(x))}{2h(x)} z(x) \quad (G.1)$$

where

$$\theta_0(x) = \tan^{-1} \left(\frac{h(x)}{x} \right), \quad (G.2)$$

$$h(x) = z_0 + \frac{x^2}{2a_e}, \quad (G.3)$$

z_0 is the nominal antenna height above the waterline of the vessel,

a_e is the radius of the earth, and

$z(x)$ is the local waveheight, modelled by a Gaussian random process.

In the maritime environment, one cannot ignore the influence of the local waveheight because the antenna is relatively close to the ocean surface.

The geometric search process for potential scattering facets can be rearranged as follows

$$z'(x) = \tan \left(\frac{E - \theta(x)}{2} \right) + \frac{x}{a_e} \quad (G.4)$$

where $\theta(x)$ is a function of the random waveheight process. Using a Taylor series expansion for (G.4), it can be shown that the specular points are located at the curve crossings of the function $m(x)$ by the Gaussian random process $\varepsilon(x)$ where

$$\varepsilon(x) = z'(x) - a(x) z(x) \quad (G.5)$$

$$m(x) = \tan \left(\frac{E - \theta_0(x)}{2} \right) + \frac{x}{a_e} \quad (G.6)$$

$$a(x) = \frac{\sin(2 \theta_0(x))}{4 h(x) \cos^2 \left(\frac{E - \theta_0(x)}{2} \right)} \quad (G.7)$$

In this case, the Gaussian random process is non-stationary, and one must use the general expression of equation (4.19) for the solution density. The important parameters which are required to evaluate this solution density are

$$\sigma^2(x) = \sigma_z^2 + a(x)^2 \sigma_z^2 \quad (G.8)$$

$$\gamma^2(x) = \sigma_z^2 + [a(x)^2 + 2 a'(x)] \sigma_z^2 + a'(x)^2 \sigma_z^2, \quad (G.9)$$

$$\mu(x) = \frac{a(x) a'(x) \sigma_z^2}{\sigma(x) \gamma(x)}, \quad (G.10)$$

$$\eta(x) = \frac{m'(x) - \gamma(x) \mu(x) m(x)/\sigma(x)}{\gamma(x) \sqrt{1 - \mu^2(x)}}, \quad (G.11)$$

$$a'(x) = \frac{\{2 \cos(2\theta_o(x)) \frac{d\theta_o(x)}{dx} - \sin(2\theta_o(x)) [\frac{x}{a_e h(x)} + \tan(\frac{F-\theta_o(x)}{2}) \frac{d\theta_o(x)}{dx}]\}}{(4 h(x) \cos^2(\frac{E-\theta_o(x)}{2}))} \quad (G.12)$$

$$m'(x) = \frac{(-1/2)}{\cos^2(\frac{E-\theta_o(x)}{2})} \frac{d\theta_o(x)}{dx} + \frac{1}{a_e}, \quad (G.13)$$

$$\frac{d\theta_o(x)}{dx} = \frac{-\sin^2(\theta_o(x))}{h(x)} \left[1 - \frac{h(x)}{a_e \tan^2(\theta_o(x))}\right]. \quad (G.14)$$

Although this solution procedure is more involved than that for the aeronautical application, the increase in the computational complexity is relatively minor. A valuable validation exercise would involve comparing theoretical solution densities obtained using equation (4.19) with those obtained with the maritime propagation simulation program.

3.0 INCORPORATING DIVERGENCE

In the maritime application, one must distinguish between the scattering from concave and convex surface facets, because of the much closer proximity of the shipborne antenna to the scattering surface. The exact near-field solution procedure for concave surface facets is not amenable to simple analysis. Consequently, the traditional ray optics divergence factors are incorporated

$$D^2(x) = \begin{cases} \frac{1}{1 + \frac{2 d(x)}{r_c \sin \alpha(x)}} & \text{for } z'' < 0 \\ & \text{(convex facet)} \\ \frac{1}{\frac{2 d(x)}{r_c \sin \alpha(x)} - 1} & \text{for } z'' > 0 \\ & \text{(concave facet)} \end{cases} \quad (G.15)$$

where

$d(x)$ is the distance between the shipborne antenna and the surface element,

$\alpha(x)$ is the local grazing angle, and

r_c is the surface radius of curvature.

Following the same solution procedure as that used for the aeronautical application (see Section 4.1.3), one obtains

$$D^2(x) = \begin{cases} q(\alpha, \beta, x) \hat{R}_{\text{convex}} & \text{for } z'' < 0 \\ q(\alpha, \beta, x) \hat{R}_{\text{concave}} & \text{for } z'' > 0 \end{cases} \quad (G.16)$$

where

$$q(\alpha, \beta, x) = \frac{\sin \theta(x) \sin \beta(x)}{2 h(x)} \left[1 + \tan^2 \left(\frac{E - \beta(x)}{2} \right) \right]^{3/2}, \quad (G.17)$$

$\beta(x)$ is the local reflection angle,

$$\alpha(x) = \frac{E + \beta(x)}{2}, \quad (G.18)$$

$$\hat{R}_{\text{convex}} = \frac{1}{|z''| + q(\alpha, \beta, x)}, \quad (G.19)$$

$$\hat{R}_{\text{concave}} = \frac{1}{|z''| - q(\alpha, \beta, x)}. \quad (G.20)$$

It should be noted that the random variables, $\hat{R}_{\text{convex}}$ and $\hat{R}_{\text{concave}}$, are functions of the local slope z' and antenna height z_0 , which are deterministic, as well as the random surface second derivative z'' .

For evaluating the mean and variance of the scatter power, the basic procedures used for the aeronautical application (see Sections 4.1.4 and 4.1.5) are applicable with a few minor modifications. These include the expressions for the first and second moments of the random variable $R(\beta_i, E_i)$ in equations (4.84) and (4.85), where the first and second moment curves versus height are defined by

$$M_{RC}(h, z', z_0) = M_{\text{convex}}(h, z', z_0) + M_{\text{concave}}(h, z', z_0) \quad (G.21)$$

$$S_{RC}(h, z', z_0) = S_{\text{convex}}(h, z', z_0) + S_{\text{concave}}(h, z', z_0) \quad (G.22)$$

where

$$M_{\text{convex}}(h, z', z_0) = \int_{-\infty}^0 \hat{R}_{\text{convex}}(z'', z', z_0) f_c(z'') dz'' \quad (G.23)$$

$$S_{\text{convex}}(h, z', z_0) = \int_{-\infty}^0 \hat{R}_{\text{convex}}^2(z'', z', z_0) f_c(z'') dz'' \quad (G.24)$$

$$M_{\text{concave}}(h, z', z_0) = \int_0^{\infty} \hat{R}_{\text{concave}}(z'', z', z_0) f_c(z'') dz'' \quad (G.25)$$

$$S_{\text{concave}}(h, z', z_0) = \int_0^{\infty} \hat{R}_{\text{concave}}^2(z'', z', z_0) f_c(z'') dz'' \quad (G.26)$$

$f_c(z'') = f_{z''}(z''/z=h, C, E_d)$, which is defined in equation (4.88).

In the aeronautical stochastic analysis program, the first and second moments of the approximate radius of curvature curves are indexed according to both height and slope. For the maritime application, it may be important to index with respect to antenna height also. This would increase the complexity of the initialization programs and interpolation routines, but could prove especially useful for incorporating the effects of vertical antenna motion with time.

In the aeronautical application, it was found that the polarization independent scatter power density versus reflection angle is not a very strong function of antenna height. However, as the antenna height is reduced, the lower limit of the scatter power integral is reduced somewhat. This has the effect of increasing the scatter power as the antenna height is lowered. In fact, it is this behaviour that is responsible for the 2 to 3 dB increase in the interference levels of the maritime application relative to the aeronautical case. However, it should be noted that it takes dramatic changes in antenna height, like the antenna height differences of the maritime and aeronautical applications, to notice much difference.

To validate the maritime stochastic model one must compare analytically computed power density versus reflection angle curves to those obtained with the maritime propagation simulation. Generating realistic upper and lower confidence limit curves, which provide an indication of reasonable variations from the mean power density curve, is also essential.

4.0 INCORPORATING MOTION COUPLING EFFECTS

The stand alone motion coupling routine, discussed in Section 2.3, could be incorporated in the maritime stochastic model to characterize antenna motion and antenna

pointing angles. This is especially crucial for establishing maritime doppler spectral characteristics. For example, at a specific time instant, the instantaneous horizontal u_x and vertical u_z velocities of the shipborne antenna can be used, in conjunction with the average wavespeed $\bar{v}$, to predict the doppler shift as a function of reflection angle. Equation (E.35) of Appendix E would be used to establish the appropriate doppler shift. The v_x velocity component required in equation (E.35) can be approximated by

$$v_x = u_x - \text{DIR } \bar{v} \quad (\text{G.27})$$

where

$$\text{DIR} = \begin{cases} 1 & \text{for ship motion with the waves} \\ -1 & \text{for ship motion against the waves.} \end{cases} \quad (\text{G.28})$$

The doppler spectrum appropriate at this time instant can be established using the doppler mapping process discussed in Section 4.3 and Appendix E, which incorporates power weighting. Several of these individual doppler spectra can be averaged to obtain a representative long term power (doppler) spectrum.

With relatively low gain Standard-C shipborne antennas, antenna motion will not have a very significant impact on the direct path and ocean scatter power levels. However, with directive antennas, such as those appropriate for

Standard-A applications, this is not the case. One can incorporate the effect of time varying antenna motion simply by determining the direct path power levels and average scatter power levels at several time instants (i.e. different antenna positions and pointing angles).

Unfortunately, this will have a detrimental effect on the computational efficiency of the stochastic model. However, one only need select a sampling rate commensurate with the rate of change of antenna motion parameters.

APPENDIX H

EFFECTS OF THE SEARCH PROCESS ON THE SURFACE SECOND DERIVATIVE AND WAVEHEIGHT DISTRIBUTIONS AT SPECULAR POINTS

In this appendix, the effect of the geometrical search process for specular points on the resulting surface second derivative and waveheight distributions at these points are examined. In Section 4.1.1 of Chapter 4, it is shown that the geometrical search process for specular points can be modelled as a Gaussian process curve crossing problem of the form

$$z'(x) = m(x) \quad \text{--- (H.1)}$$

where

$z'(x)$ is the Gaussian surface slope process,

$$m(x) = \tan\left(\frac{E - \delta(x)}{2}\right) + \frac{x}{a_e},$$

$$\delta(x) = \tan^{-1}\left(\frac{z_0 + \frac{x^2}{2a_e}}{x}\right),$$

z_0 is the aircraft altitude,

a_e is the radius of the earth, and

E is the elevation angle to the satellite.

Here, attention is restricted to a small region in the neighbourhood of the point x , where $m(x)$ is essentially equal to the constant m_c .

The analytical approach considered here closely parallels the intuitive treatments presented in Rice [25], [38]. The intuitive approach is more enlightening than using mathematically precise, but tedious, formal proofs, as is the case in references [27] and [28].

To begin, the level crossing event C is defined as follows

$$\begin{aligned} C &= \left\{ \begin{array}{l} \text{Level Crossing of } z'=m_c \text{ By Slope} \\ \text{Process In The Neighbourhood of } x_1 \end{array} \right\} \\ &= \left\{ \begin{array}{l} \text{Level Crossing of } z'=m_c \text{ By Slope} \\ \text{Process In the Interval } [x_1, x_1+\Delta x] \end{array} \right\}. \quad (H.2) \end{aligned}$$

As in [25], it is assumed that Δx is chosen to be so small that one can effectively regard all the representative random slope curves as straight lines. The slope and second derivative random variables at the point x_1 are defined to be $z' = z'(x_1)$ and $z'' = z''(x_1)$, respectively. The conditions for a slope level crossing are summarized below.

If $z'' > 0$, any crossing in the interval $[x_1, x_1 + \Delta x]$ will be an up-crossing and for this to occur

$$m_c - z''\Delta x < z' < m_c. \quad (H.3)$$

Conversely, if $z'' < 0$, any crossing in the interval $[x_1, x_1 + \Delta x]$ will be a down-crossing and for this to occur

$$m_C < z' < m_C - z'' \Delta x. \quad (H.4)$$

The probability of a level crossing, conditioned on a specific second derivative value, will now be considered.

This conditional probability can be expressed as

$$P(C/z''=\eta) = \begin{cases} P\{m_C - \eta \Delta x < z' < m_C\} & \text{for } \eta > 0 \\ P\{m_C < z' < m_C - \eta \Delta x\} & \text{for } \eta < 0, \end{cases} \quad (H.5)$$

where

$$P\{m_C - \eta \Delta x < z' < m_C\} = \int_{m_C - \eta \Delta x}^{m_C} f_{z'}(v) dv = \eta \Delta x f_{z'}(m_C) \quad (H.6)$$

$$P\{m_C < z' < m_C - \eta \Delta x\} = \int_{m_C}^{m_C - \eta \Delta x} f_{z'}(v) dv = -\eta \Delta x f_{z'}(m_C). \quad (H.7)$$

The assumption that Δx is so small that the slope z' is effectively equal to the constant m_C has been utilized in (H.6) and (H.7). It is evident that

$$P(C/z''=\eta) = |\eta| \Delta x f_{z'}(m_C). \quad (H.8)$$

The probability of a slope level crossing in the interval $[x_1, x_1 + \Delta x]$ is given by

$$P(C) = \int_{-\infty}^{\infty} P(C/z''=\eta) f_{z''}(\eta) d\eta \quad (H.9)$$

$$= (\Delta x f_{z'}(m_c)) \int_{-\infty}^{\infty} |n| f_{z''}(n) dn.$$

For Gaussian processes

$$f_{z''}(n) = \frac{1}{\sqrt{2\pi} \sigma_{z''}} \exp \left\{ -\frac{1}{2} \left(\frac{n}{\sigma_{z''}} \right)^2 \right\} \quad (H.10)$$

$$f_{z'}(a) = \frac{1}{\sqrt{2\pi} \sigma_{z'}} \exp \left\{ -\frac{1}{2} \left(\frac{a}{\sigma_{z'}} \right)^2 \right\}, \quad (H.11)$$

and it is easily shown that

$$\int_{-\infty}^{\infty} |n| f_{z''}(n) dn = \sqrt{\frac{2}{\pi}} \sigma_{z''}, \quad (H.12)$$

and

$$P(C) = \frac{1}{\pi} \left(\frac{\sigma_{z''}}{\sigma_{z'}} \right) \exp \left\{ -\frac{1}{2} \left(\frac{m_c}{\sigma_{z'}} \right)^2 \right\}. \quad (H.13)$$

The major purpose of this section is to establish the behaviour of the second derivative and waveheight distributions at a specular point (slope level crossing). Note that the actual specular point location is in the neighbourhood of x_1 as

$$x_s \in [x_1, x_1 + \Delta x]. \quad (H.14)$$

For very small Δx values, the second derivative and waveheight distributions at the actual specular point will be essentially the same as those at the point x_1 . Using the "mixed form" of Bayes rule [39],

becomes a two-sided Rayleigh density function as opposed to the Gaussian density function in the unconstrained case. This behaviour is consistent with simulation observations at specular points.

Because of the correlation between z'' and z , it is evident that the slope level crossing constraint also modifies the waveheight distribution. In fact, the important conditional density function is $f_{zz''}(a, b/C)$ which can be expressed in product form as

$$f_{zz''}(a, b/C) = f_{z''}(b/z=a, C) f_z(a/C). \quad (H.19)$$

The conditional second derivative density function can be derived using mixed probability expressions and Bayes rule [39] as follows

$$\begin{aligned} f_{z''}(b/z=a, C) &= \frac{f_{zz''}(a, b, C)}{f_z(a, C)} \\ &= \frac{f_{zz''}(a, b, C)}{\int_{-\infty}^{\infty} f_{zz''}(a, b, C) db} \end{aligned} \quad (H.20)$$

Now

$$f_{zz''}(a, b, C) = P(C/z''=b, z=a) f_{zz''}(a, b) \quad (H.21)$$

where

$$P(C/z''=b, z=a) = |b| \Delta x f_{z''}(m_C), \quad (H.22)$$

as this conditional crossing probability is completely specified with knowledge of the second derivative value. Substituting the relationships of the (H.21) and (H.22) into equation (H.20) and simplifying yields

$$f_{z''}(b/z=a, C) = g_C(a, b) f_{z''}(b/z=a) \quad (H.23)$$

where

$$g_C(a, b) = \frac{|b|}{\int_{-\infty}^{\infty} |b| f_{z''}(b/z=a) db} \quad (H.24)$$

It is evident that the second derivative density function, conditioned on the level crossing event C and the surface waveheight, can be expressed as the product of a modification factor and the traditional conditional second derivative density with respect to waveheight.

The modified height density function can be expressed as

$$f_z(a/C) = \frac{f_z(a, C)}{P(C)} = \frac{\int_{-\infty}^{\infty} f_{zz''}(a, b, C) db}{\int_{-\infty}^{\infty} P(C/z''=b) f_{z''}(b) db} \quad (H.25)$$

Using (H.8), (H.21), and (H.22) in equation (H.25) and simplifying yields

$$f_z(a/C) = h_c(a) f_z(a) \quad (\text{H.26})$$

where

$$h_c(a) = \frac{\int_{-\infty}^{\infty} |b| f_{z''}(b/z=a) db}{\int_{-\infty}^{\infty} |b| f_{z''}(b) db} \quad (\text{H.27})$$

Note that this has the functional form of a modification factor multiplied by the unconstrained Gaussian waveheight density function.

There is one other extremely important constraint that is incorporated in the stochastic model. This is a constraint on the distance between neighbouring specular points.

Intuitively, as the level which the slope process must cross increases, the length of the interval where the slope process remains above the prescribed level (or in other words, the distance between adjacent level crossings) decreases. For small deviations, Δ , from the specular point location, the slope function can be accurately predicted by the Taylor series expansion

$$z'(x_s + \Delta) = z'(x_s) + z''(x_s) \Delta + \frac{z'''(x_s)}{2} \Delta^2. \quad (\text{H.28})$$

If there is a level crossing at $x_s + \Delta$,

$z'(x_s + \Delta) = z'(x_s) = \mu_c$, which implies that

$$\Delta = \frac{-2 z''(x_s)}{z'''(x_s)} \quad (H.29)$$

A Taylor series expansion of this form has also been used by Rice [38] to establish an asymptotic expression for the density function of the duration of the interval above a fixed level (as the level $\rightarrow \infty$) for Gaussian random processes.

It is evident from (H.29) that specular points will be close together for small second derivative values. In fact, as $z'' \rightarrow 0$, two unique specular points will ultimately merge into a single point. Under these circumstances, a few problems with the simulation solution arise. The first is that the instantaneous radius of curvature at the specular point is infinite. This is relatively easily handled by using neighbouring z'' values, in conjunction with a second derivative cutoff threshold, to establish a reasonable radius of curvature value.

The second problem is that the validity of the solution procedure used in the simulation model becomes suspect as the specular points become very close together (very small

z" values). The simulation approach, and thus the approach used in the stochastic model, effectively treats each specular point independently. However, when the distance between specular points becomes small, the assumed parabolic facets at the two specular points will have overlapping segments which tend to be contradictory. This situation is clarified in Figure H.1. For the convex facet, the parabolic approximation is no longer valid a short distance to one side of this specular point (because the surface becomes concave in nature about the nearby neighbouring specular point).

Furthermore, the relative path length difference through the adjacent specular points becomes small as the distance between the specular points becomes small. This destroys the otherwise reasonable assumption that the phase shifts introduced by neighbouring facets are independent and uniformly distributed. Under these circumstances there will be a form of coherent combining of the scatter signals. It should be noted, however, that the different phase shifts introduced by the convex and concave facets add to the phase shifts introduced by the path length differences. Since the differential phase introduced by curvature is 90° , the power of the resulting scatter signal is essentially the sum of the individual scatter powers from each facet (which is the same result one obtains assuming independent and uniformly distributed phase

Figure H.1: Parabolic surface approximations as specular points become very close together.

Figure H.2: Detection of zero crossings.

shifts). Even so, one really should use a "Huygen's source" solution approach (Chapter 13 of [14]), to establish the scattered field from such a surface segment, rather than treating the surface as two independent parabolic scattering facets under these circumstances.

To alleviate these problems, the distance between specular points is constrained. The distance must be greater than or equal to a specified minimum $\Delta_{\min}$, as indicated below

$$|\Delta| = \frac{2 |z''(x_g)|}{|z'''(x_g)|} > \Delta_{\min}. \quad (\text{H.30})$$

This restriction will be referred to by the event

$$E_d = \{ |z'''| < \frac{2}{\Delta_{\min}} |z''| \}. \quad (\text{H.31})$$

The issue of establishing an appropriate value for $\Delta_{\min}$ is now addressed. A practical search strategy for locating specular points is considered. The specular points occur at the zero crossings of the function

$$f(x) = z'(x) - m(x)$$

where

$$m(x) = \tan \left(\frac{E - \beta(x)}{2} \right) + \frac{x}{a_e}.$$

The deterministic function $m(x)$ varies slowly with x , effectively being constant over any relatively small region. In the propagation simulation program, a binary search process is used to establish the initial location of

specular points, where a sign change detection approach, using a step size Δ_s , is used to determine the intervals where these zero crossings occur. To establish the inter-relationships between the specular point distance constraint and the detectability of zero crossings, consider the situation illustrated in Figure H.2, where the zero crossing occurs in the middle of the step interval. In this case, the function values at the search step boundaries can be approximated using Taylor series expansions

$$\begin{aligned} f(x_s - \Delta_x) &= f(x_s) - f'(x_s) \Delta_x + \frac{f''(x_s)}{2} \Delta_x^2 \\ &= -z''(x_s) \Delta_x + \frac{z'''(x_s)}{2} \Delta_x^2 \end{aligned} \quad (\text{H.32})$$

$$\begin{aligned} f(x_s + \Delta_x) &= f(x_s) + f'(x_s) \Delta_x + \frac{f''(x_s)}{2} \Delta_x^2 \\ &= z''(x_s) \Delta_x + \frac{z'''(x_s)}{2} \Delta_x^2 \end{aligned} \quad (\text{H.33})$$

where

$$\Delta_x = \frac{\Delta_s}{2}. \quad (\text{H.34})$$

For the zero crossing to be detected,

$$f(x_s - \Delta_x) \cdot f(x_s + \Delta_x) < 0, \quad (\text{H.35})$$

which can be simplified to yield

$$\left(\frac{z'''(x_s)}{2} \Delta_x^2 \right)^2 < (z''(x_s) \Delta_x)^2$$

or

$$|z'''(x_s)| < \frac{2}{\Delta_x} |z''(x_s)|. \quad (H.36)$$

Note that the form of the detectability constraint for the binary search process used in the simulation program is identical to that of the event E_d defined in (H.31). As a result, the event E_d will commonly be referred to as the "detectability event". An appropriate choice for the minimum distance is

$$\Delta_{\min} = \Delta_x = \frac{\Delta_s}{2}. \quad (H.37)$$

The step size used for all the aeronautical simulation runs was $\Delta_s = \frac{\lambda}{10}$, where λ is the average sea wavelength.

The third and first derivatives of the surface are correlated, and the conditional density function (see Appendix A) is given by

$$f_{z'''}(c/z' = m_c) = \frac{1}{\sqrt{2\pi} \sigma_{m_3}} \exp\left\{ \frac{-1}{2\sigma_{m_3}^2} (c - K_3 m_c)^2 \right\} \quad (H.38)$$

where

$$K_3 = \frac{E\{z''' z'\}}{\sigma_{z'}^2} = -\left(\frac{\sigma_{z''}}{\sigma_{z'}}\right)^2$$

$$r_3 = \frac{E\{z''' z'\}}{\sigma_{z''} \sigma_{z'}} = \frac{-\sigma_{z''}^2}{\sigma_{z''} \sigma_{z'}}$$

$$\sigma_{m_3} = \sqrt{1 - r_3^2} \sigma_{z''}$$

The appropriate parameters for the Bandpass and Gaussian spectrum surface models are summarized in Table A.4 of Appendix A.

The probability of the detectability event, conditioned on specific slope and second derivative values is given by

$$\begin{aligned} P(E_d / z' = m_c, z'' = b) &= P(|z'''| < \frac{2|b|}{\Delta_{\min}} \mid z' = m_c) \\ &= Q\left(\frac{\frac{-2|b|}{\Delta_{\min}} - K_3 m_c}{\sigma_{m_3}}\right) - Q\left(\frac{\frac{2|b|}{\Delta_{\min}} - K_3 m_c}{\sigma_{m_3}}\right) \end{aligned}$$

(H.39)

To incorporate the effects of both the slope level crossing and the detectability constraint, one must use the conditional density function $f_{zz''}(z_f, z_f'' / C, E_d)$ instead of the unconditioned density $f_{zz''}(z_f, z_f'')$ in equations (4.58)

and (4.71) for establishing the mean and variance of the scatter power. This conditional density function can be expressed in product form as

$$f_{zz''}(a, b/C, E_d) = f_{z''}(b/z=a, C, E_d) f_z(a/C, E_d). \quad (H.40)$$

The modified second derivative density function can be expressed as

$$f_{z''}(b/z=a, C, E_d) = \frac{f_{zz''}(a, b, C, E_d)}{\int_{-\infty}^{\infty} f_{zz''}(a, b, C, E_d) db} \quad (H.41)$$

The mixed probability expression is given by

$$\begin{aligned} f_{zz''}(a, b, C, E_d) &= P(E_d/C, z''=b, z=a) f_{zz''}(a, b, C) \\ &= P(E_d/z'=m_C, z''=b, z=a) P(C/z''=b, z=a) f_{zz''}(a, b) \\ &= P(E_d/z'=m_C, z''=b) P(C/z''=b) f_{zz''}(a, b), \end{aligned} \quad (H.42)$$

where

$P(E_d/z'=m_C, z''=b)$ is defined by (H.39), and

$P(C/z''=b)$ is defined by (H.8).

Employing these relations in (H.41) and simplifying yields

$$f_{z''}(b/z=a, C, E_d) = g_{z, z', z''}(a, m_c, b) f_{z''}(b/z=a) \quad (H.43)$$

where

$$g_{z, z', z''}(a, m_c, b) = \frac{P(E_d/z'=m_c, z''=b) |b|}{\int_{-\infty}^{\infty} P(E_d/z'=m_c, z''=b) |b| f_{z''}(b/z=a) db} \quad (H.44)$$

The modified height density function in this case is given by

$$f_z(a/C, E_d) = \frac{f_z(a, C, E_d)}{P(C, E_d)} = \frac{\int_{-\infty}^{\infty} f_{zz''}(a, b, C, E_d) db}{\int_{-\infty}^{\infty} f_{z''}(b, C, E_d) db} \quad (H.45)$$

Using the mixed probability expression of (H.42), it is easily shown that the conditional height density can be expressed as

$$f_z(a/C, E_d) = h_{z, z'}(a, m_c) f_z(a) \quad (H.46)$$

where

$$h_{z,z'}(a, m_c) = \frac{\int_{-\infty}^{\infty} P(E_d/z' = m_c, z'' = b) |b| f_{z''}(b/z=a) db}{\int_{-\infty}^{\infty} P(E_d/z' = m_c, z'' = b) |b| f_{z''}(b) db} \quad (H.47)$$

It is evident that incorporating the effects of the detectability event does introduce some slope dependence, and thus position dependence, in the second derivative and waveheight distribution functions at the specular points.

These modified conditional density functions have been incorporated in the stochastic analysis model program for establishing the mean and variance of the scatter power for aeronautical applications.

REFERENCE LIST

- [1] P. Beckmann and A. Spizzichino, "The Scattering of Electromagnetic Waves From Rough Surfaces", The MacMillan Company, New York, 1963.
- [2] P. F. Panter, "Modulation, Noise, and Spectral Analysis", McGraw-Hill Book Company, New York, 1965.
- [3] D. E. Kerr, Editor, "Propagation of Short Radio Waves", McGraw-Hill Book Company, Inc., New York, 1951.
- [4] H. C. Salwen and C. B. Duncombe, "Performance Evaluation of Data Modems for the Aeronautical Satellite Channel", IEEE Trans. Comm., Vol. COM-23, No. 7, July 1975.
- [5] T. Reich, "Preliminary Results of Satellite-Aircraft Propagation Measurements", Department of Communications, Space Technology and Applications, July 1983.
- [6] "Multipath Fading Effects and Data Transmission For Small Ship Earth Stations (Standard-C)", DFVLR German Aerospace Research Establishment, ESA/ESTEC Contract No. 5323/82/NL/JS, November 1984.
- [7] Telex from D. R. Mullins of INMARSAT entitled, "Characteristics of Multipath Propagation Effects As Deduced From The Results of Numerous Experimental Measurements (for the Maritime Application)", 4 April 1985.
- [8] R. W. Sutton et. al., "Satellite-Aircraft Multipath and Ranging Experiments at L-Band", IEEE Trans. Comm., May 1973.
- [9] "Study of Various System Aspects Related to the Use of Standard-C Ship Earth Stations In The INMARSAT System: Chapters 2 and 4", prepared for International Maritime Satellite Organization under INMARSAT Contract No. 006, by Communications Satellite Corporation, COMSAT Laboratories, Clarksburg, Maryland, 1982.
- [10] L. Palmer, "Modeling and Simulating The L-Band Maritime Channel", IEEE Globecom '82, Conference Record Volume 1 of 3, pages 1038 - 1045, Nov. 29 - Dec. 2, 1982.
- [11] "Comsat Technical Review: Volume 9, Number 2A", Fall 1979, page 372.

- [12] P. Horn et. al., "Theoretical Study of Multipath Effects In An Aeronautical Satellite System", Messerschmitt - Bolkow - Blohm Gmbh, Contract 1064/70CG (RFQ/737) for European Space Research and Technology Centre, Noordwijk, Holland, 20 November 1970.
- [13] H. P. Greenspan and D. J. Benney, "Calculus: An Introduction to Applied Mathematics", McGraw-Hill Book Company, New York, 1973, pages 439 - 442.
- [14] E. C. Jordan and K. G. Balmain, "Electromagnetic Waves and Radiating Systems: Second Edition", Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1968.
- [15] G. A. Korn and T. M. Korn, "Mathematical Handbook For Scientists and Engineers: Second Edition", McGraw-Hill Book Company, New York, 1968.
- [16] K. W. Moreland, "Simulation of Maritime Multipath and Superstructure Effects", Miller Communications Systems Ltd. Technical Report, MCS File No. 8448, INMARSAT Contract No. INM/84-078, July 5, 1985.
- [17] "Reference Data For Radio Engineers, Fifth Edition", ITT, Howard W. Sams and Co., Inc., New York, 1972, page 28-16.
- [18] "INMARSAT Services to Small Terminals", INMARSAT Advisory Committee on Technical and Operational Matters, Fifteenth Session, ACTOM/15/29, 4 October 1983.
- [19] "Applicability of the INMARSAT System To The Aeronautical Mobile Satellite Service: Study Report", (INMARSAT Contract INM/83-054), Racal Satellite Systems Division and Miller Communications Systems Ltd., March 1, 1984.
- [20] F. S. Carr, "Aerosat Ground Environment and Test Plan", EASCON '73, pages 146-151.
- [21] A. V. Oppenheim and R. W. Schaffer, "Digital Signal Processing", Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1975, pages 553 - 556.
- [22] P. A. Bello, "Aeronautical Channel Characterization", IEEE Trans. Comm., Vol. COM-21, pages 548 - 563, May 1973.

- [23] J. H. Chinnick, "The Time Function Scatter Characteristics of The Ocean Surface at 1550 MHz As Encountered in a Satellite to Aircraft Communications System - Volume 1", Communications Research Centre, Ottawa, Technical Memorandum SA 5/74, March 1974.
- [24] "Some Features of Aeronautical Satellite Communications", INMARSAT Advisory Committee on Technical and Operational Matters, Fifteenth Session, ACTOM/15/34, 5 October 1983.
- [25] S. O. Rice, "Mathematical Analysis of Random Noise", Bell System Technical Journal 24 (1945), pages 46-156.
- [26] A. Papoulis, "Probability, Random Variables, and Stochastic Processes", McGraw-Hill Book Company, New York, 1965.
- [27] H. Cramer and M. R. Leadbetter, "Stationary and Related Stochastic Processes", John Wiley & Sons, Inc., New York, 1967.
- [28] M. R. Leadbetter, G. Lindgren, H. Rootzen, "Extremes and Related Properties of Random Sequences and Processes", Springer-Verlag, New York, 1983.
- [29] R. H. Clarke, "A Statistical Theory of Mobile-Radio Reception", Bell Systems Technical Journal, July - August, 1968, pages 957 - 1000.
- [30] M. J. Gans, "A Power-Spectral Theory of Propagation in the Mobile-Radio Environment", IEEE Trans. on Vehicular Technology, Vol. VT-21, No. 1, February 1972.
- [31] F. E. Nathanson, "Lecture IV-Radar Clutter", Course Notes From, "An Intensive Short Course on Radar Systems Engineering", by Technology Service Corporation, Silver Spring, Maryland, given in Ottawa, May 1982.
- [32] D. K. Barton and H. R. Ward, "Handbook of Radar Measurement", Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1969.
- [33] S. Haykin, "Communication Systems", John Wiley & Sons, New York, 1978.
- [34] H. Taub and D. L. Shilling, "Principles of Communication Systems", McGraw-Hill Book Company, New York, 1971.

- [35] Captain C. Stewart, "Appendix I from Heavy Weather Sailing", by K. A. Coles, New York Lithographing Corporation, 34 Oak Avenue, Tuckahoe, N.Y. 1968.
- [36] W. Mendenhall and R. L. Scheaffer, "Mathematical Statistics With Applications", Duxbury Press, North Scituate, Massachusetts, 1973.
- [37] M. R. Vant et. al., "The complex-dielectric constant of sea ice at frequencies in the range 0.1 - 40 GHz", Journal of Applied Physics, Vol. 49, No. 3, March 1978.
- [38] S. O. Rice, "Distribution of the Duration of Fades in Radio Transmission: Gaussian Noise Model", Bell System Technical Journal, Volume XXXVII, No. 3, May 1958, pages 581 - 635. /
- [39] J. M. Wozencraft and I. M. Jacobs, "Principles of Communication Engineering", John Wiley & Sons, New York, 1965, pages 73 - 77.
- [40] S. Donaldson, "Principles of Power - Displacement Hulls", Pacific Yachting, March 1985.
- [41] S. Colgate, "Manual of Basic Sailing Theory", Van Nostrand Reinhold Company, New York, 1973.

GLOSSARY OF NOTATION

Only the notation used often in this report is included below. In those cases where a symbol has more than one meaning, the context (or a specific statement) resolves the ambiguity.

<u>Notation</u>	<u>Definition</u>	<u>Typical Page Reference</u>
$a(x)$	Polarization dependent antenna gain and reflection coefficient scatter signal weighting factor	230
a_e	Radius of the earth	20
A_i	Curvature induced signal scale factor incorporating divergence and phase shift for the i-th scattering facet	33
AR	Axial ratio of an antenna	96
B	One-sided e^{-1} bandwidth	128
B_A	Aircraft banking angle	33
B_{10}	One-sided -10 dB bandwidth	127
C	Slope process level crossing event	464

$c(\beta, x)$	Curvature induced solution density scale factor	236
D	Divergence factor of a scattering facet	45
D_e	Divergence factor of the earth	111
E	Elevation angle to the satellite	26
E_d	Detectability event for specular point locations	241
E_i	Incident electric field	50
E_l	Local elevation angle	225
E_R	Magnitude of the antenna polarization ratio	99
E_r	Received electric field	49
f	Focal length of a scattering element	42
f_d	Differential doppler shift relative to the direct path	428
f_o	Corner frequency for the antenna stabilization network for shipborne antennas	71
f_p	Pitching motion resonant frequency of a ship	71
F_R	First Fresnel zone radius for circular Fresnel zones	77

f_s	Sampling frequency for the ocean scatter multipath signals	125
f_v	Vertical motion resonant frequency of a ship	71
q_a	Angle appropriate for applying antenna gains to scatter signals	33
G_H, G_V	Antenna (voltage) gains for horizontal and vertical polarization	33
$h(x)$	Vertical distance between antenna location and a point on the earth's surface	204
h_A	Vertical distance between scattering facet and antenna location	422
$H_p(f)$	Frequency response of ship pitching motion filter	71
h_R	Horizon range of an antenna	203
$H_s(f)$	Frequency response of shipborne antenna stabilization network	71
$H_v(f)$	Frequency response of vertical ship motion filter	71
h_w	Peak waveheight of sinusoidal surface model	48
$J_m(\cdot)$	Bessel function of the first kind of order m	49

L	Waterline length of an ocean going vessel	73
LHC	Left hand circular polarization	88
LGA	Low grazing angle attenuation factor	367
$M_{RC}(z)$	First moment of the approximate radius of curvature versus height z	242
N_C	Number of contributing (visible) surface facets	230
N_I	Effective number of independent surface samples for clearance probability estimation	221
N_T	Random variable representing total number of potential scattering facets in a cell	232
N_W	Number of surface scatter samples per average sea wavelength	114
P	Polarization ratio of an antenna	99
$\hat{P}$	Average over record of instantaneous power samples	439
P_C	Clearance probability	217
$\bar{P}_D$	Average direct path power level	105
$\bar{P}_S$	Average scatter power with stochastic analysis model	233
$\bar{P}_{sc}$	Average scatter power with propagation simulation model	105

Q	Ratio of LHC and RHC field strengths	96
$Q(x)$	Q-function $Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$	221
$q(\alpha, \beta, x)$	Weighting factor required for evaluating the divergence factor value squared	228
r	Distance between antenna location and a scattering facet	39
$R(\beta_t, E_t)$	Approximate radius of curvature incorporating effects of blockage and shadowing by ocean surface profile	232
r_c	Surface radius of curvature at a specular point	42
R_c	Approximate surface radius of curvature	228
RHC	Right hand circular polarization	87
$R_{zz}(x)$	Surface autocorrelation function	300
S/I	Signal to interference ratio	105
$s_H(i), s_V(i)$	Surface scatter signals for horizontal and vertical polarization for the i -th scattering facet	31
$S_{RC}(z)$	Second moment of the approximate radius of curvature versus height z	242

u	Ship velocity	72
u_h	Maximum ship velocity (hull speed)	73
u_o	Spatial centre frequency of ocean surface spectrum	308
$\bar{v}$	Average wavespeed	320
$v_I(x)$	Polarization independent variance density	236
v_k	Velocity of the k-th surface component of superposition of sinusoids surface model	307
v_r	Relative velocity of a ship with respect to the waves	184
v_x	Horizontal aircraft velocity in m/sec	127
w_B, w_S	Surface blockage and shadowing weighting factors	225
x_L	Interval length for clearance probability calculations	217
x_s	Specular point location	416
x_{sp}	Stationary phase limit	366
x_{start}, x_{stop}	Start and stop boundaries of the search region for specular points	27
$\left\{ \begin{array}{l} x_{CONCAVE} \\ x_{CONVEX} \\ x_{TP} \end{array} \right\}$	X-axis Fresnel dimensions for concave facets, convex facets, and a tangent plane	350
$\left\{ \begin{array}{l} y_{CONCAVE} \\ y_{CONVEX} \\ y_{TP} \end{array} \right\}$	Y-axis Fresnel dimensions for concave facets, convex facets, and a tangent plane	350

$\begin{Bmatrix} z(x) \\ z'(x) \\ z''(x) \end{Bmatrix}$	Local waveheight, surface slope, and surface second derivative functions	203
z_t, z_t''	Local waveheight and second derivative at a specular point	232
z_o	Antenna height	27
z_p, z_p'	Peak waveheight and peak surface slope	27
$\begin{Bmatrix} z_s(x) \\ z_s'(x) \\ z_s''(x) \end{Bmatrix}$	Surface waveheight, surface slope, and surface second derivative functions incorporating the curvature of the earth	203
α	Local grazing angle	26
β	Reflection angle relative to the horizontal	26
β_t	Local reflection angle relative to slope of the earth	213
ϵ	Incline of the tangent plane to the surface	26
ϵ_r	Relative di-electric constant of the ocean surface	336
θ	Reflection angle from surface to antenna location	26
θ_k	Random phase shift of k-th surface component	30

θ_m	Antenna phase mismatch between vertical and horizontal polarization components	99
θ_p	Pitching angle of ship	68
θ_{ss}	Shipborne antenna pointing angle	68
θ_1	Angle of incidence	49
θ_2	Scattering angle	363
λ	Average sea wavelength	218
λ_k	Wavelength of k-th surface component	305
λ_w	Wavelength of sinusoidal surface	48
σ	Conductivity parameter of surface	336
σ_z	Rms waveheight	62
σ_z'	Rms surface slope	128
σ_z''	Rms second derivative	207
ρ, ρ_{sim}	Theoretical and simulation reflection coefficients for scattering from perfectly conducting sinusoidal surfaces	59
Γ_H, Γ_V	Surface reflection coefficients for horizontal and vertical polarization	33
δ	Path length difference	52
ϕ_i	Phase difference (relative to direct path) for i-th scattering facet	34

ϕ_c	Phase shift correction factor for concave facets for a sinusoidal surface model	53
ϕ_d	Phase difference between trough and crest specular points for a sinusoidal surface	51
Δ_{mir}	Threshold distance between specular points for the stochastic model	241
ΔP_S	Incremental scatter power for the stochastic model	232
Δ_s	Step size for zero crossing detection algorithm	241
γ	Bandwidth factor of white Bandpass surface model	317
τ	e^{-1} correlation distance of Gaussian spectrum surface model	300
ψ	Surface grazing angle	336

END

23|·12·87

FIN