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**IONIZATION CHAMBER RESPONSE
FOR BRACHYTHERAPY SOURCES**

by

MARIA J. CORSTEN, B.Sc.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of

Master of Science

Department of Physics

Carleton University

Ottawa, Ontario

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a calibration factor and correction factors [9]:

$$D_{\text{gas}} = M N_{\text{gas}} K_{\text{ion}} \quad (\text{Gy}), \quad (1.1)$$

where D_{gas} is the dose to gas, M is the measured reading from the electrometer in the ionization chamber set-up, N_{gas} is the cavity-gas calibration factor and K_{ion} is a correction factor for the charge collection efficiency [16]. The cavity-gas calibration factor can be determined from the exposure calibration factor. The exposure calibration factor (N_x) is determined by a primary standards laboratory. The cavity-gas calibration factor does not depend on the energy of radiation but does depend on the ionization chamber being used. From the AAPM TG-21 protocol [9, 15], the equation for N_{gas} is as follows:

$$N_{\text{gas}} = N_x \frac{k(\frac{W}{e})_{\text{gas}} A_{\text{ion}} A_{\text{wall}}}{\alpha(\frac{L}{\rho})_{\text{gas}}^{\text{wall}} (\frac{\mu_{\text{en}}}{\rho})_{\text{wall}}^{\text{air}} + (1 - \alpha)(\frac{L}{\rho})_{\text{gas}}^{\text{cap}} (\frac{\mu_{\text{en}}}{\rho})_{\text{cap}}^{\text{air}}} \quad (\text{Gy/C}), \quad (1.2)$$

where k is charge per unit mass of air per unit exposure, $(\frac{W}{e})_{\text{gas}}$ is the mean energy expended per unit charge, A_{ion} is the ion-collection efficiency ($A_{\text{ion}} = (K_{\text{ion}})^{-1}$), A_{wall} is the wall correction factor, α is the fraction of ionization due to electrons from the chamber wall, $(\frac{L}{\rho})$'s are ratios of the mean restricted mass stopping powers and $(\frac{\mu_{\text{en}}}{\rho})$'s are energy absorption coefficient ratios. The subscripts and superscripts on the ratios denote the gas, wall and buildup cap for the particular ionization chamber being used.

The dose to gas can be calculated as above, but a more useful quantity in medical radiation physics is dose to water (or medium). Using Spencer-Attix [17] cavity

Abstract

The correction factor A_{pn} accounts for non-uniformity of an incident photon field in the vicinity of an ionization chamber cavity. The Kondo-Randolph theory for A_{pn} and its extension are reviewed. The effect of source orientation and distribution on A_{pn} theory is developed. The A_{wall} correction factor accounts for photon scatter and attenuation in the walls of the chamber. Bielajew's theory suggests a nonlinear extrapolation may be more appropriate in determining A_{wall} for spherical chambers than a linear one. Monte Carlo simulations were performed and the results confirm the theory. Simulations of cylindrical plane-parallel ionization chambers were performed to determine the effect of source orientation on A_{wall} . These show that small changes in the angle cause large changes in A_{wall} when the source orientation angle is near ninety degrees with respect to the symmetry axis. Experiments were performed by Tölli and Mattsson comparing the Kondo-Randolph theory to its extension. Simulations were performed to determine A_{wall} for the experimental conditions. The results suggest that the extension to the theory is more accurate. The improvement in accuracy is about three percent near the source and tenths of a percent for longer distances.

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Chapter 1

Introduction

Brachytherapy (from the Greek *brachy* meaning 'short') is literally short distance therapy. A radiation source is placed in or near the tumor site to give a high dose directly to the tumor. The radioactive sources that are commonly used are cesium-137, iridium-192, iodine-125 and gold-198. The size of these sources is typically on the order of millimeters. Ionization chambers have traditionally been used to calibrate these sources [1]. There are problems that occur when using an ionization chamber to calibrate a point source. This work focuses on two of these problems. 1) The ionization chamber will be irradiated non-uniformly by the point source because of the diverging field from the point source. 2) The walls of the ionization chamber will cause photon attenuation and scatter that will depend on the specifications of the ionization chamber being used. To begin, the fundamental concepts of dosimetry are reviewed.

1.1 Ionization chamber dosimetry

Three of the basic quantities in radiation physics are kerma, exposure and absorbed dose [2, 3]. When a photon interacts with an atom, the photon may give some energy to a charged particle and continue with a decreased amount of energy. The interaction can occur by way of the photoelectric process, pair production or Compton incoherent scattering. The photoelectric process occurs when a photon collides with an atom and ejects a bound electron from one of the shells. It is only possible if the photon has the same (or more) energy than the binding energy of the electron in the shell. In this process, most of the photon's energy is converted into the kinetic energy of the electron. Pair production causes the photon to become a positron-electron pair when the path of the photon passes through the strong Coulomb field of the nucleus of the atom. For pair production to occur, the photon must have at least twice the energy of one electronic mass (ie. the photon must have an energy of at least 1.022 MeV). The sum of the kinetic energies of the positron and electron is the energy of the photon less the 1.022 MeV to create the charged particles. Compton scattering occurs when a photon interacts with an electron and some of the photon's energy is transferred to kinetic energy of the electron. The photon is scattered at an angle determined by energy and momentum conservation; it continues to travel but with reduced energy. The energy of the incoming photon determines the probability of a particular process occurring. For example, photon interactions in water at low energies (10 keV) are

mostly photoelectric interactions, whereas at 1.0 MeV in water, the interactions are almost entirely due to Compton scattering [3].

Independent of the type of process occurring to yield the charged particles, the energy that is transferred to the charged particles from the photon is defined as kerma. As it travels, the electron (or positron) loses energy in a variety of ways. If the electron or positron interacts with a nucleus, a bremsstrahlung photon can be given off. If an electron encounters an atomic electron, a 'delta ray' may be set in motion. If a positron encounters an atomic electron, they can combine to form two photons. This process is positron annihilation. The electron or positron will also lose energy through small collisions (exciting and ionizing atoms) along its path. Johns and Cunningham [3] define the absorbed dose to be the energy transferred to the electron initially (kerma) minus the energy subsequently given off as bremsstrahlung photons.

Another important quantity to define is exposure. Exposure is measured in Coulombs per kilogram and is "a measure of the ability of radiation to ionize air" [3]. The ICRU [4] defines exposure to be "the quotient of dQ by dm where the value of dQ is the absolute value of the total charge of the ions of one sign produced in air when all the electrons liberated by photons in air of mass dm are completely stopped in air."

A primary instrument used in determining dose for cobalt-60 energies and higher

is the ionization chamber. Figure 1.1 is a schematic diagram of a spherical ionization chamber. The ionization chamber consists of a gas-filled cavity surrounded by the chamber wall. The gas is usually air and the chamber wall can be one of many materials including graphite, aluminium and lucite. There is a collecting electrode that extends from the stem of the chamber into the cavity. The electrode is connected to an electrometer that will measure the current from the ions collected in the cavity by the electrode. A high voltage is applied to the chamber wall while the electrode is at ground potential. The difference in the potentials causes either positive or negative ions to collect on the electrode and the current is measured with the electrometer. The ionization chamber shown in Figure 1.1 is spherical, but many ionization chambers are cylindrical in shape. When an ionization chamber is irradiated, the dose to the cavity gas is approximately proportional to the reading from the electrometer in the ionization chamber set-up. The point of measurement of an ionization chamber placed free-in-air in the field of a radiation source, is assumed to be the center of the chamber. Departures from this are accounted for using correction factors [5, 6, 7, 8, 9, 10, 11, 12, 13]. To measure dose to gas using an ionization chamber, the chamber is irradiated with a photon beam for a certain amount of time. The current from the electrode of the ionization chamber is measured using an electrometer. The time of irradiation is known, so the current can be converted to charge. The charge is corrected for temperature, pressure and humidity [14, 15]. The dose to the gas in an ionization chamber is calculated from the ionization chamber reading (in Coulombs),

Figure 1.1: Schematic diagram of a spherical ionization chamber.

a calibration factor and correction factors [9]:

$$D_{\text{gas}} = M N_{\text{gas}} K_{\text{ion}} \quad (\text{Gy}), \quad (1.1)$$

where D_{gas} is the dose to gas, M is the measured reading from the electrometer in the ionization chamber set-up, N_{gas} is the cavity-gas calibration factor and K_{ion} is a correction factor for the charge collection efficiency [16]. The cavity-gas calibration factor can be determined from the exposure calibration factor. The exposure calibration factor (N_x) is determined by a primary standards laboratory. The cavity-gas calibration factor does not depend on the energy of radiation but does depend on the ionization chamber being used. From the AAPM TG-21 protocol [9, 15], the equation for N_{gas} is as follows:

$$N_{\text{gas}} = N_x \frac{k(\frac{W}{e})_{\text{gas}} A_{\text{ion}} A_{\text{wall}}}{\alpha(\frac{L}{\rho})_{\text{gas}}^{\text{wall}} (\frac{\mu_{\text{en}}}{\rho})_{\text{wall}}^{\text{air}} + (1 - \alpha)(\frac{L}{\rho})_{\text{gas}}^{\text{cap}} (\frac{\mu_{\text{en}}}{\rho})_{\text{cap}}^{\text{air}}} \quad (\text{Gy/C}), \quad (1.2)$$

where k is charge per unit mass of air per unit exposure, $(\frac{W}{e})_{\text{gas}}$ is the mean energy expended per unit charge, A_{ion} is the ion-collection efficiency ($A_{\text{ion}} = (K_{\text{ion}})^{-1}$), A_{wall} is the wall correction factor, α is the fraction of ionization due to electrons from the chamber wall, $(\frac{L}{\rho})$'s are ratios of the mean restricted mass stopping powers and $(\frac{\mu_{\text{en}}}{\rho})$'s are energy absorption coefficient ratios. The subscripts and superscripts on the ratios denote the gas, wall and buildup cap for the particular ionization chamber being used.

The dose to gas can be calculated as above, but a more useful quantity in medical radiation physics is dose to water (or medium). Using Spencer-Attix [17] cavity

theory, the dose to water can be found from the dose to gas. The general equation for dose to a medium is [9]:

$$D_{\text{med}} = M N_{\text{gas}} \left(\frac{L}{\rho} \right)_{\text{gas}}^{\text{med}} P_{\text{ion}} P_{\text{repl}} P_{\text{wall}} \quad (\text{Gy}), \quad (1.3)$$

where $(\frac{L}{\rho})_{\text{gas}}^{\text{med}}$ is the stopping power ratio of the medium to that of the gas, P_{ion} is the correction factor for recombination losses [18], P_{repl} is the replacement correction factor (it depends on the energy of the beam, the radius of the chamber and the depth of measurement), and P_{wall} accounts for the differences in composition between the medium and the chamber wall [9, 19].

In the equations for dose, correction factors were introduced. There are some correction factors that are of particular importance when the radiation source is a point source. These correction factors account for the non-uniformity of the photon field across the ionization chamber. The photons from the point source diverge, resulting in a non-uniform irradiation of the ionization chamber. The axial non-uniformity correction factor, K_{an} , is a correction factor for the non-uniformity of the photon field along the line from the source through the center of the cavity. The radial non-uniformity correction factor, K_{rn} , is the correction factor for the non-uniformity in the transverse direction and the transverse field non-uniformity due to finite size source effects and scatter from the collimator and room. The deviation of the response of an ionization chamber from the inverse square law from such effects has been observed [5, 6, 7, 8, 11]. K_{pn} is the correction factor for the non-uniformity of the incident point source photon field across the chamber. This correction factor

includes K_{an} and some of K_{m} . A_{pn} is used in some recent literature [11, 12]; as is its reciprocal K_{pn} . The correction factor for the primary photon attenuation and scatter in the walls of the ionization chamber is A_{wall} . This correction factor also includes the correction for the electron drift (K_{cep}). In Figure 1.2 a graphite ionization chamber (with air cavity) is shown being irradiated by a point source. A primary photon (γ_{p}) from the point source travels into the ionization chamber and interacts. An electron is set in motion (e_{p}^-) and travels through the cavity. The photon beam is attenuated in the interaction and continues through the chamber with less energy. K_{att} is the correction factor that accounts for this attenuation of the primary photons. The electron drift correction is included in K_{att} ; it accounts for the electron being set in motion a short distance from the surface of the cavity. The scattered photon (γ_{s}) may interact again and another electron (e_{s}^-) may travel through the cavity. K_{scat} accounts for the contribution of the primary photon scatter. The correction factor K_{wall} is the wall attenuation and scatter correction factor, it is the product of K_{att} and K_{scat} . The diverging photon beam from the point source is accounted for using the point source non-uniformity correction factor, K_{pn} . Stopping power ratios are used to account for the fact that there is air in the cavity instead of carbon gas. Finally, Fano's theorem is used to replace the carbon gas in the cavity with carbon. Fano's theorem states that in an infinite medium of material exposed to radiation, the energy deposited is independent of density variations from point to point [2]. The second part of Figure 1.2 shows a graphite ionization chamber with a graphite filled

Figure 1.2: A diagram showing a ionization chamber being irradiated with a point source and an ionization chamber being irradiated with a parallel photon beam. The correction factors that account for the differences in the two cases (K_{att} , K_{scat} and K_{pn}) are labeled.

cavity being irradiated by a parallel beam of photons with no photon attenuation or scatter. The correction factors described above correct for the differences between this case and the point source irradiating a graphite ionization chamber with an air cavity.

1.2 Kondo-Randolph theory

Kondo and Randolph [8] developed a theory for the correction factor due to the non-uniformity of the photon field from a point source. The basis of their theory is that if an ionization chamber is placed in the field of a point source, the energy released in a differential volume element of the gas in the cavity by electrons entering the cavity from a surface element of the wall of the chamber is proportional to the volume of the element, the solid angle subtended by the surface element from volume element and the inverse square of the distance from the source to the surface element. The following equation shows the relationship described.

$$dE = c \frac{1}{r^2} d\Omega dV, \quad (1.4)$$

where c is a constant of proportionality, dV is the volume element of the gas in the cavity, $d\Omega$ is the solid angle subtended by the surface element (dA_1) from dV and r is the distance from the point source to dA_1 [8]. If one integrates over the surface of the cavity wall and the volume of the cavity, Equation 1.4 becomes:

$$E = c \int_{\text{wall}} \frac{d\Omega}{r^2} \int_{\text{cavity}} dV, \quad (1.5)$$

The solid angle $d\Omega$ can be represented as $\frac{\cos \psi_{21} dA_1}{u^2}$ where u is the distance between dV and dA_1 and ψ_{21} is the angle u makes with the inward normal from dA_1 . To reduce the integral over the cavity to another integral over the cavity surface, consider a second surface element dA_2 . The distance between dA_1 and dA_2 is h . The solid angle subtended by dA_2 from dA_1 is $\frac{\cos \psi_{12} dA_2}{h^2}$ where ψ_{12} is the angle h makes with the inward normal from dA_2 . As u varies from zero to h , $u^{-2}dV$ equals h times the solid angle between dA_1 and dA_2 (see Figure 1.3). The equation for the energy becomes:

$$E = c \int dA_1 \int dA_2 \frac{\cos \psi_{12} \cos \psi_{21}}{hr^2}. \quad (1.6)$$

If the distance between the source and the center of the chamber (r_0) is large enough, then r , the distance from the source to the surface element dA_1 , can be replaced by r_0 in Equation 1.5. Making this replacement and noting that $1/r_0^2$ is constant so it can be taken out of the integration yields a constant (c/r_0^2), the integral of $d\Omega$ and the integral of dV . The integral of dV over the cavity volume is simply V and the integral of the solid angle over the cavity surface is 4π . The result when the source-to-chamber-center distance is large is the inverse-square law [6],

$$E = \frac{4\pi cV}{r_0^2}. \quad (1.7)$$

Correction factors are defined such that the reading of the ionization chamber multiplied by the correction factor (in this case, A_{pn}) is proportional to the true dose rate. The correction factor, A_{pn} , accounts for the deviation from the inverse-square law due to the non-uniformity of the photon field from a point source; it is the ratio

Figure 1.3: A diagram showing the solid angle between surface elements dA_1 and dA_2 . dV is the volume element of gas in the cavity. h is the distance between dA_1 and dA_2 and u is an integration parameter that varies between zero and h .

of Equation 1.6 and Equation 1.7. The Kondo and Randolph [8] theory for the point source non-uniformity correction factor, A_{pn} is expressed as:

$$A_{pn} = \frac{r_0^2}{4\pi V_{cav}} \int \frac{dA_1}{r^2} \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h}, \quad (1.8)$$

where r_0 is the distance from the source to the center of the chamber, V_{cav} is the volume of the cavity, dA_1 and dA_2 are surface elements of the cavity, r is the distance from the source to dA_1 , h is the distance from dA_1 to dA_2 , ψ_{21} is the angle h makes with the inward normal from dA_1 , and ψ_{12} is the angle h makes with the inward normal from dA_2 .

The Kondo and Randolph [8] theory makes the following assumptions: 1) the radiation is emitted from a point source; 2) the primary photons suffer no appreciable attenuation while going from the source to the chamber nor while passing through the chamber; 3) the electrons entering the cavity are set in motion at the surface of the cavity; 4) the electrons travel in straight lines through the cavity; 5) electrons lose energy in proportion to their track lengths through the cavity; and 6) the angular distribution of the electrons entering the cavity is isotropic. Assumption 2 is not necessary if the wall attenuation and scatter correction (A_{wall}) is used. The third assumption is valid if the electron range within the walls of the ionization chamber is much smaller than the distance from the source to the chamber and smaller than the cavity dimensions. Assumption 4 is valid if the cavity is 'small' in comparison with the electron range in the cavity. Assumption 5 does not account for the increased

stopping power of low energy electrons in the cavity. The final assumption that the distribution of electrons is isotropic is only valid at extremely low energies. However, the Kondo and Randolph [8] theory accounts for the experimental deviations from the inverse square law with good accuracy. To achieve more accurate results, some of the assumptions made by Kondo and Randolph must be relaxed.

1.3 The extension to Kondo-Randolph theory

Bielajew [11, 12] has extended the theory of Kondo and Randolph [8] to include the anisotropic distribution of electrons entering the cavity. The equation for the point source non-uniformity correction factor then can be expressed as:

$$A_{\text{pn}} = \frac{r_0^2}{4\pi V_{\text{cav}}} \int \frac{dA_1}{r^2} \int dA_2 \frac{\cos \varphi_{21} \cos \varphi_{12}}{h} (1 + \omega \cos \Theta), \quad (1.9)$$

where Θ is the angle between the incident photon direction and the direction of the electron through the cavity and ω is the measure of the degree of anisotropy of the electron distribution [11]. The term ω is determined by the primary equilibrium electron angular distribution in the wall of the ionization chamber weighted by the energy deposition. The energy deposited is proportional to the stopping power multiplied by the track length of the electron. The inclusion of the $\omega \cos \Theta$ term relaxes some of the assumptions made by Kondo and Randolph [8]. The anisotropy of the electron distribution is accounted for by ω , thereby relaxing assumption 6. Furthermore, the

energy deposition is determined using stopping power effects, which relaxes the fourth assumption made by Kondo and Randolph [8]. The point source non-uniformity correction factor can be written as the sum of the Kondo and Randolph correction factor and a new correction factor. Equation 1.9 can be written as [11]:

$$A_{\text{pn}} = A_{\text{pn}}^{\text{KR}} + \omega A'_{\text{pn}}, \quad (1.10)$$

where $A_{\text{pn}}^{\text{KR}}$ is given in Kondo and Randolph's theory and A'_{pn} can be expressed as [11]:

$$A'_{\text{pn}} = \frac{r_0^2}{4\pi V_{\text{cav}}} \int \frac{dA_1}{r^2} \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12} \cos \Theta}{h}. \quad (1.11)$$

For spherical ionization chambers A'_{pn} is zero but for cylindrical chambers A'_{pn} is non-zero. The equations describing A'_{pn} for cylindrical chambers are complex and are shown in Bielajew's paper [11]. In order to simplify the equations, practical measurement distances are considered. The correction factor for cylindrical ionization chambers becomes [11]:

$$\lim_{L, a \ll r_0} \longrightarrow 1 + \omega \kappa'_{\text{S}} \left(\frac{a}{r_0} \right) + \kappa_{\text{E}}^{\text{KR}} \left(\frac{a}{r_0} \right)^2 + \dots \quad (1.12)$$

where L is the half-length of the chamber, a is the radius of the chamber, κ^{KR} is from Kondo and Randolph theory, κ' is the anisotropic term and the S and E denote whether the ionization chamber is irradiated from the side or from the end [11]. For a cylindrical ionization chamber irradiated from the side, the κ' term can be expressed as:

$$\begin{aligned}
\kappa'_S = & \frac{1}{2} \left[\int_{-1}^1 dx \left(\int_{-1}^1 dy [f_{1a}(x, y, \sigma) + f_{1b}(x, y, \sigma)] \right. \right. \\
& + \left. \int_0^1 dy [f_{2a}(x, y, \sigma) + f_{2b}(x, y, \sigma)] \right) \\
& + \int_0^1 dx \left(\int_{-1}^1 dy [f_{3a}(x, y, \sigma) + f_{3b}(x, y, \sigma)] \right. \\
& \left. \left. + \int_0^1 dy [f_{4a}(x, y, \sigma) + f_{4b}(x, y, \sigma)] \right) \right]. \quad (1.13)
\end{aligned}$$

where the f factors depend on the shape of the chamber only and $\sigma = a/L$ [11]. For the irradiation from the end of a cylindrical chamber the expression is [11]:

$$\kappa'_E = -2\kappa'_S. \quad (1.14)$$

The anisotropic portion of the correction factor in Equation 1.12 is first-order in a/r_0 (where a is the radius of the chamber and r_0 is the distance from the source to the center of the chamber) whereas unity and the second-order term arise from the isotropic portion of the theory. Figure 1.4 is a graph of κ factors versus σ for practical values of σ . Near unity, the κ' terms are close to zero and the Kondo-Randolph terms are close to one-third, the value for a sphere. A_{pn} can be determined from the graph given the dimensions of the chamber [11].

Bielajew's extension to the Kondo and Randolph theory is more accurate in accounting for the deviation of ionization chamber response from the inverse-square law. The $\omega \cos \Theta$ term is the first-order expansion in the Legendre polynomials. Using more terms in the Legendre series would be more accurate but the mathematics

Figure 1.4: First and second order κ values for A_{pn} when the source-to-chamber-center distance is large. (Taken from Bielajew [11] with permission.)

of the theory would be much more complicated and would yield a small increase in accuracy.

1.4 A_{wall} for spherical chambers

The correction factor for the attenuation of the incident beam by the wall of the ionization chamber and for the generation of scattered photons in the wall is A_{wall} . This correction factor is determined by making measurements with an ionization chamber in a photon beam. The thickness of the chamber wall is varied and a plot is made of chamber response as a function of wall thickness. For measurements with an ionization chamber, the walls of the chamber must be thick enough to insure that electrons originating outside of the walls do not reach the cavity. This full buildup thickness is estimated to be the CSDA (continuous slowing down approximation) range of an electron with the maximum energy from the photon that produced it [2]. The full buildup thickness of the walls of the chamber is maintained throughout the experiment to determine A_{wall} . The experimental results are fitted linearly and extrapolated back to zero wall thickness [20, 21]. The correction factor, A_{wall} , is calculated as the ratio of the linearly extrapolated value of the chamber response at zero wall thickness to the chamber response at the wall thickness of the chamber being used [21]. Bielajew [22] has challenged the linear extrapolation to zero wall thickness for spherical chambers. For spherical chambers, when the wall thickness is increased linearly, the path-length increases non-linearly. In Figure 1.5, the wall

thickness is increased by the same amount with each shell. The figure shows that although the wall thickness increases linearly, the path length of the photon increases non-linearly [22]. The closer the photon is to the inner surface the longer its path length through the shell with the maximum effect at the transverse edge.

Bielajew [22] has developed an analytic theory to explain the nonlinear extrapolation for spherical chambers. The theory is similar to that of the point source non-uniformity correction factor. The assumptions that Kondo and Randolph [8] made are also used in this theory. Using these assumptions the correction factor K_W can be expressed as [22]:

$$K_W^{-1} = 1 - \frac{\mu_{\text{eff}}}{4\pi V_{\text{cav}}} \int dA |R - r_s| \int dV \frac{\cos \eta}{u^2}, \quad (1.15)$$

where $A_{\text{wall}} = (K_W K_{\text{cep}})^{-1}$ and K_{cep} is a correction factor that accounts for electron drift. In equation 1.15, μ_{eff} is the effective attenuation coefficient that accounts for photon scatter and attenuation of the primary beam, V_{cav} is the cavity volume, dA is a surface element of the cavity, $|R - r_s|$ is the distance between the point where a photon hits the outside of the chamber to the point on the inside of the cavity, dV is a volume element of the cavity, u is the distance between dA and dV , and η is the angle between the line joining dA and dV and the inward normal line from dA [22].

For spherical chambers, equation 1.15 can be expressed as [22]:

$$K_W^{-1}(w) = 1 - \mu_{\text{eff}} w f_s(\alpha), \quad (1.16)$$

where μ_{eff} is the effective attenuation coefficient, w is the wall thickness, $f_s(\alpha)$ is the

Figure 1.5: A sphere with wall thickness increased linearly. The path-length of the photon through the layers increases non-linearly.

slope correction function for spheres and α is the radius of the cavity divided by the wall thickness [22]. The slope correction function is expressed as [22]:

$$f_s(\alpha) = \frac{[1 + (1 + \frac{1}{2\alpha}) \log(1 + 2\alpha)]}{2}. \quad (1.17)$$

Monte Carlo calculated wall correction factors and experimental data have been compared for a spherical chamber [22]. Both the linear fit and nonlinear fit match the data well in the measurement region but extrapolation to zero wall thickness differs by approximately 1%.

1.5 Monte Carlo and the EGS4 code

The EGS4 (Electron Gamma Shower Code Version 4) [23] simulates radiation transport. The code uses Monte Carlo techniques to simulate the random paths of photons and electrons. Using a random number generator, the probability distributions of occurrences are sampled. In simple terms, before a particle undergoes any process, dice are rolled to see if the process will occur. This procedure is repeated for all the processes that may happen with the particle. A large number of particles are simulated and from the histories, the average values or distributions of quantities can be evaluated. For example, the energy deposited by the particles can be obtained. To reduce the time for the simulations, condensed-history techniques are employed. These condensed-history techniques involve grouping together large numbers of similar interactions that cause small changes in a simulation [23, 24]. Without these

techniques, the simulations would become very time consuming. For example, small energy losses are accounted for by using a continuous slowing down approximation. Range rejection is another time-saving technique that can be used in EGS4. Electrons that do not have enough energy to reach the region of interest are discarded. This technique reduces the time of the simulation by not following particles that will not contribute to the quantities of interest. In the simulation of photon interactions, many photons can leave the area being simulated without interacting. Forcing photon interactions is a way to reduce the simulation time. If the photons are forced to interact, the results from the forced interaction must be weighted by the probability that the photon would have interacted before leaving the area being simulated. Following all the photons that would not interact in the region of the simulation would be a waste of time; forcing photon interactions significantly decreases the time involved. These techniques, as well as others, reduce the time to perform simulations using the EGS4 code.

To show how a Monte Carlo code works, consider the transport of photons. The photon has initial parameters of energy, position and direction. It is determined whether the energy of the photon is below the cutoff energy. If the energy is less than the cutoff energy the history of that photon is terminated and a new photon is selected to go through the process. If the energy of the photon is not less than the cutoff, the distance to the next interaction is determined by selecting a random number to sample the exponential distribution of the path length for photons. The

photon is transported to the new position. It is determined whether the photon has left the region of interest; if it has, the history is terminated. If it is still in the region of interest, the type of interaction is determined. Another random number is selected and is used to sample the cross sections of different interactions occurring (eg. Compton scattering, pair production, coherent scattering or the photoelectric effect). The particles that result have energy and direction distributions, these parameters are sampled using random numbers. Once these quantities are determined these particles follow the same procedure of transport. All the particle histories in the region of interest are followed until they leave the region or the energy falls below the cutoff energy [23].

The EGS4 code calculates many quantities from the particle histories. The calculation of the wall attenuation and scatter correction factor (A_{wall}) is important for this work. A_{wall} is calculated by multiplying the photon scatter factor, (A_{scat}), by the attenuation factor, (A_{att}). These corrections are calculated separately in the code. The scatter factor is determined from the total dose less the total primary dose. The remaining dose is attributed to photon scatter. The attenuation factor is determined from the total primary unattenuated dose less the total primary dose. The difference in these values is due to the attenuation [25]. A_{wall} is a combination of the photon scatter and the attenuation; these factors are multiplied to calculate A_{wall} .

Chapter 2

Development of general forms of A_{pn}

2.1 Source orientation

The correction factor A_{pn} has been determined for two source orientations: the source positioned beside the chamber, at the midpoint of the length, and the source centered at the end of the chamber. However, the source can be at any angle around the chamber and the point source non-uniformity correction factor should be known for all of these cases. The effect of the anisotropic distribution of primary electrons can be seen when the point source is moved around the chamber so it is not along one of the axes. For the source orientation effect, Equation 1.9 can be written as:

$$A_{pn} = \frac{r_0^2}{4\pi V_{cav}} \int \frac{dA_1}{r^2} \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \left(1 + \omega \frac{\vec{r} \cdot \vec{h}}{rh} \right), \quad (2.1)$$

where r_0 is the reference distance to the center of the cavity, $\vec{r}$ is the vector from the source to dA_1 (the incident photon direction) and $\vec{h}$ is the vector from dA_1 to dA_2 (the direction of the electron through the cavity). The position of the point source

is:

$$\vec{r}_0 = r_0[\sin \Theta \cos \Phi \hat{x} + \sin \Theta \sin \Phi \hat{y} + \cos \Theta \hat{z}]. \quad (2.2)$$

Figure 2.1 shows a schematic diagram of a cylindrical ionization chamber with the quantities defined above labeled.

2.2 Source orientation at large distance

For large source-to-chamber-center distances, the equation for A_{pn} can be simplified. By rearranging Equation 2.1, the expression for the correction factor A_{pn} can be written as:

$$A_{\text{pn}} = \frac{r_0^2}{4\pi V_{\text{cav}}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \left(\frac{1}{r^2} + \frac{\omega}{r^2} \frac{\vec{r} \cdot \vec{h}}{rh} \right). \quad (2.3)$$

Define $\vec{s}_1$ as the coordinates of the surface element dA_1 (ie. the vector from the center of the chamber to dA_1). The vector $\vec{s}_1$ can be expressed as:

$$\vec{s}_1 = r_1 \cos \theta_1 \hat{x} + r_1 \sin \theta_1 \hat{y} + z_1 \hat{z}. \quad (2.4)$$

The vector from the source to dA_1 ($\vec{r}$) can be expressed as:

$$\vec{r} = \vec{s}_1 - \vec{r}_0. \quad (2.5)$$

Squaring both sides of the previous equation yields:

$$r^2 = r_0^2 - 2\vec{s}_1 \cdot \vec{r}_0 + s_1^2. \quad (2.6)$$

$$\vec{r}_0 = r_0 (\sin\theta\cos\Phi, \sin\theta\sin\Phi, \cos\theta)$$

$$\vec{s}_1 = (r_1 \cos\theta_1, r_1 \sin\theta_1, z_1)$$

Figure 2.1: A schematic diagram of a cylindrical ionization chamber being irradiated with a point source from the side.

Since the source is a large distance away from the chamber, s_1^2 is small when compared to r_0 so it can be ignored. Therefore Equation 2.6 becomes:

$$r^2 \approx r_0^2 - 2\vec{s}_1 \cdot \vec{r}_0. \quad (2.7)$$

Rearranging the above equation gives:

$$r^2 \approx r_0^2 \left[1 - \frac{2\vec{s}_1 \cdot \vec{r}_0}{r_0^2} \right]. \quad (2.8)$$

The terms to be evaluated in Equation 2.3 are $1/r^2$ and $1/r^3$.

$$\frac{1}{r^2} \approx \frac{1}{r_0^2 \left[1 - \frac{2\vec{s}_1 \cdot \vec{r}_0}{r_0^2} \right]}. \quad (2.9)$$

Multiplying top and bottom by $(1 + \frac{2\vec{s}_1 \cdot \vec{r}_0}{r_0^2})$ and noting that $4(\frac{\vec{s}_1 \cdot \vec{r}_0}{r_0^2})^2$ is small when compared to unity the result is:

$$\frac{1}{r^2} \approx \frac{1}{r_0^2} \left[1 + \frac{2\vec{s}_1 \cdot \vec{r}_0}{r_0^2} \right]. \quad (2.10)$$

Similarly, the equation for $1/r^3$ is:

$$\frac{1}{r^3} \approx \frac{1}{r_0^3} \left[1 + \frac{3\vec{s}_1 \cdot \vec{r}_0}{r_0^2} \right]. \quad (2.11)$$

Replacing $1/r^2$ and $1/r^3$ with the above expressions and replacing $\vec{r}$ with Equation 2.5 in Equation 2.3 gives:

$$\begin{aligned} A_{pn} = & \frac{r_0^2}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \left[\frac{1}{r_0^2} \left(1 + \frac{2\vec{s}_1 \cdot \vec{r}_0}{r_0^2} \right) \right. \\ & \left. + \frac{\omega}{hr_0^3} \left(1 + \frac{3\vec{s}_1 \cdot \vec{r}_0}{r_0^2} \right) ((\vec{s}_1 - \vec{r}_0) \cdot \vec{h}) \right]. \end{aligned} \quad (2.12)$$

Expanding the above equation yields:

$$\begin{aligned}
 A_{pn} &= \frac{1}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \\
 &+ \frac{1}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \frac{2\vec{s}_1 \cdot \vec{r}_0}{r_0^2} \\
 &+ \frac{1}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \\
 &\quad \left[\frac{\omega}{hr_0} \left(1 + \frac{3\vec{s}_1 \cdot \vec{r}_0}{r_0^2} \right) (\vec{s}_1 \cdot \vec{h} - \vec{r}_0 \cdot \vec{h}) \right].
 \end{aligned} \tag{2.13}$$

Simplifying the third term only:

$$\begin{aligned}
 &\frac{\omega}{hr_0} \left(1 + \frac{3\vec{s}_1 \cdot \vec{r}_0}{r_0^2} \right) (\vec{s}_1 \cdot \vec{h} - \vec{r}_0 \cdot \vec{h}) \\
 &= \frac{\omega}{hr_0} \left(\vec{s}_1 \cdot \vec{h} - \vec{r}_0 \cdot \vec{h} + \frac{3\vec{s}_1 \cdot \vec{r}_0 \vec{s}_1 \cdot \vec{h}}{r_0^2} - \frac{3\vec{s}_1 \cdot \vec{r}_0 \vec{r}_0 \cdot \vec{h}}{r_0^2} \right).
 \end{aligned} \tag{2.14}$$

The s_1^2 term is small compared to the others and can be ignored. The above equation becomes:

$$\frac{\omega}{hr_0} \left(1 + \frac{3\vec{s}_1 \cdot \vec{r}_0}{r_0^2} \right) (\vec{s}_1 \cdot \vec{h} - \vec{r}_0 \cdot \vec{h}) = \frac{\omega}{h} \left(\frac{\vec{s}_1 \cdot \vec{h}}{r_0} - \frac{\vec{r}_0 \cdot \vec{h}}{r_0} - \frac{3\vec{s}_1 \cdot \vec{r}_0 \vec{r}_0 \cdot \vec{h}}{r_0^3} \right). \tag{2.15}$$

Substituting Equation 2.15 into Equation 2.13 yields:

$$\begin{aligned}
 A_{pn} &= \frac{1}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \\
 &+ \frac{1}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \frac{2\vec{s}_1 \cdot \vec{r}_0}{r_0^2} \\
 &+ \frac{1}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \\
 &\quad \left[\frac{\omega}{h} \left(\frac{\vec{s}_1 \cdot \vec{h}}{r_0} - \frac{\vec{r}_0 \cdot \vec{h}}{r_0} - \frac{3\vec{s}_1 \cdot \vec{r}_0 \vec{r}_0 \cdot \vec{h}}{r_0^3} \right) \right].
 \end{aligned} \tag{2.16}$$

Simplifying the above equation gives:

$$\begin{aligned}
 A_{pn} = & \frac{1}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \\
 & + \frac{1}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \frac{2\vec{s}_1 \cdot \vec{r}_0}{r_0^2} \\
 & + \frac{\omega}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \\
 & \left[\frac{\vec{s}_1 \cdot \vec{h}}{hr_0} - \frac{\vec{r}_0 \cdot \vec{h}}{hr_0} - \frac{3\vec{s}_1 \cdot \vec{r}_0 \vec{r}_0 \cdot \vec{h}}{hr_0^3} \right]. \quad (2.17)
 \end{aligned}$$

Rearranging the previous equation yields:

$$\begin{aligned}
 A_{pn} = & \frac{1}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \\
 & + \frac{1}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \frac{2\vec{s}_1 \cdot \vec{r}_0}{r_0^2} \\
 & - \frac{\omega}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \\
 & \left[\frac{\vec{r}_0 \cdot \vec{h}}{hr_0} - \frac{\vec{s}_1 \cdot \vec{h}}{hr_0} + \frac{3\vec{s}_1 \cdot \vec{r}_0 \vec{r}_0 \cdot \vec{h}}{hr_0^3} \right]. \quad (2.18)
 \end{aligned}$$

As r_0 goes to infinity, the correction factor goes to unity. Equation 2.18 becomes:

$$\begin{aligned}
 A_{pn} = & 1 + \frac{1}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \frac{2\vec{s}_1 \cdot \vec{r}_0}{r_0^2} \\
 & - \frac{\omega}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \\
 & \left[\frac{\vec{r}_0 \cdot \vec{h}}{hr_0} - \frac{\vec{s}_1 \cdot \vec{h}}{hr_0} + \frac{3\vec{s}_1 \cdot \vec{r}_0 \vec{r}_0 \cdot \vec{h}}{hr_0^3} \right]. \quad (2.19)
 \end{aligned}$$

The integrals over dA_1 and dA_2 reduce to four integrals over $d\theta_1$ (from 0 to 2π), $d\theta_2$ (from 0 to 2π), z_1 (from -L to L) and z_2 (from -L to L). s_1 is linear in $\cos \theta$, $\sin \theta$ and z_1 so by the following rules the second term vanishes.

$$\int_0^{2\pi} \cos \theta d\theta = 0 \quad (2.20)$$

$$\int_0^{2\pi} \sin \theta d\theta = 0 \quad (2.21)$$

$$\int_{-L}^L z dz = 0. \quad (2.22)$$

The third term of Equation 2.19 can be broken up into three terms.

$$\frac{\omega}{4\pi V_{\text{cav}}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \left[\frac{\vec{r}_0 \cdot \vec{h}}{hr_0} - \frac{\vec{s}_1 \cdot \vec{h}}{hr_0} + \frac{3\vec{s}_1 \cdot \vec{r}_0 \vec{r}_0 \cdot \vec{h}}{hr_0^3} \right]. \quad (2.23)$$

Consider the first term of the above expression.

$$\frac{\omega}{4\pi V_{\text{cav}}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \left(\frac{\vec{r}_0 \cdot \vec{h}}{hr_0} \right). \quad (2.24)$$

Changing variables $1 = 2$ and given that $h_{12} = -h_{21}$ the term becomes:

$$\frac{\omega}{4\pi V_{\text{cav}}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \left(\frac{-\vec{r}_0 \cdot \vec{h}}{hr_0} \right). \quad (2.25)$$

Incorporating this change in sign the term can be written as:

$$\begin{aligned} & \frac{\omega}{4\pi V_{\text{cav}}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \left(\frac{\vec{r}_0 \cdot \vec{h}}{hr_0} \right) \\ &= \frac{1}{2} \left[\frac{\omega}{4\pi V_{\text{cav}}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \left(\frac{\vec{r}_0 \cdot \vec{h}}{hr_0} \right) \right. \\ &+ \left. \frac{\omega}{4\pi V_{\text{cav}}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \left(\frac{-\vec{r}_0 \cdot \vec{h}}{hr_0} \right) \right] \end{aligned} \quad (2.26)$$

$$= 0. \quad (2.27)$$

Taking the second and third terms of Equation 2.23 together and making the change in variables $1 = 2$, as above, yields:

$$\begin{aligned}
& \frac{\omega}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \left[-\frac{\vec{s}_1 \cdot \vec{h}}{hr_0} + \frac{3\vec{s}_1 \cdot \vec{r}_0 \vec{r}_0 \cdot \vec{h}}{hr_0^3} \right] \\
&= \frac{1}{2} \left[\frac{\omega}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \frac{1}{h} \left(-\frac{\vec{s}_1 \cdot \vec{h}}{r_0} + \frac{3\vec{r}_0 \cdot \vec{h} \vec{s}_1 \cdot \vec{r}_0}{r_0^3} \right) \right. \\
&+ \left. \frac{\omega}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \frac{1}{h} \left(\frac{\vec{s}_2 \cdot \vec{h}}{r_0} - \frac{3\vec{r}_0 \cdot \vec{h} \vec{s}_2 \cdot \vec{r}_0}{r_0^3} \right) \right] \quad (2.28)
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2} \left[\frac{\omega}{4\pi V_{cav}} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \frac{1}{h} \right. \\
&\quad \left. \left(-\frac{\vec{s}_1 \cdot \vec{h}}{r_0} + \frac{3\vec{r}_0 \cdot \vec{h} \vec{s}_1 \cdot \vec{r}_0}{r_0^3} + \frac{\vec{s}_2 \cdot \vec{h}}{r_0} - \frac{3\vec{r}_0 \cdot \vec{h} \vec{s}_2 \cdot \vec{r}_0}{r_0^3} \right) \right] \quad (2.29)
\end{aligned}$$

$$\begin{aligned}
&= \frac{\omega}{4\pi V_{cav} r_0} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{2h} \frac{1}{h} \\
&\quad \left[(\vec{s}_2 - \vec{s}_1) \cdot \vec{h} - \frac{3\vec{r}_0 \cdot \vec{h} (\vec{s}_2 - \vec{s}_1) \cdot \vec{r}_0}{r_0^2} \right] \quad (2.30)
\end{aligned}$$

$$= \frac{\omega}{4\pi V_{cav} r_0} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{2h} \frac{1}{h} \left(h^2 - \frac{3(\vec{r}_0 \cdot \vec{h})^2}{r_0^2} \right), \quad (2.31)$$

where $\vec{s}_2$ is the vector from the origin (the center of the chamber) to the surface element dA_2 ($\vec{s}_2 = r_2 \cos \theta_2 \hat{x} + r_2 \sin \theta_2 \hat{y} + z_2 \hat{z}$) and the vector between dA_1 and dA_2 is $\vec{h}$. By definition, $\vec{h} = \vec{s}_2 - \vec{s}_1$. Consider only:

$$h^2 - \frac{3(\vec{r}_0 \cdot \vec{h})^2}{r_0^2}, \quad (2.32)$$

let $\vec{h} = h_x \hat{x} + h_y \hat{y} + h_z \hat{z}$ and Equation 2.32 becomes:

$$\begin{aligned}
& h_x^2 + h_y^2 + h_z^2 - \frac{3r_0^2}{r_0^2} (\sin^2 \Theta \cos^2 \Phi h_x^2 + \sin^2 \Theta \sin^2 \Phi h_y^2 + \cos^2 \Theta h_z^2 \\
&+ 2 \sin^2 \Theta \cos \Phi \sin \Phi h_x h_y + 2 \sin \Theta \cos \Theta \cos \Phi h_x h_z \\
&+ 2 \sin \Theta \cos \Theta \sin \Phi h_y h_z). \quad (2.33)
\end{aligned}$$

As defined above:

$$\vec{h} = \vec{s}_2 - \vec{s}_1 \quad (2.34)$$

$$= (r_2 \cos \theta_2 \hat{x} + r_2 \sin \theta_2 \hat{y} + z_2 \hat{z}) - (r_1 \cos \theta_1 \hat{x} + r_1 \sin \theta_1 \hat{y} + z_1 \hat{z}) \quad (2.35)$$

$$= (r_2 \cos \theta_2 - r_1 \cos \theta_1) \hat{x} + (r_2 \sin \theta_2 - r_1 \sin \theta_1) \hat{y} + (z_2 - z_1) \hat{z} \quad (2.36)$$

$$= h_x \hat{x} + h_y \hat{y} + h_z \hat{z}. \quad (2.37)$$

Consider the term $h_x h_y$

$$h_x h_y = (r_2 \cos \theta_2 - r_1 \cos \theta_1)(r_2 \sin \theta_2 - r_1 \sin \theta_1). \quad (2.38)$$

Let $\theta_1 = -\theta_1$ and $\theta_2 = -\theta_2$, then $h_x h_y$ becomes:

$$h_x h_y = (r_2 \cos(-\theta_2) - r_1 \cos(-\theta_1))(r_2 \sin(-\theta_2) - r_1 \sin(-\theta_1)) \quad (2.39)$$

$$= (r_2 \cos \theta_2 - r_1 \cos \theta_1)(-r_2 \sin \theta_2 + r_1 \sin \theta_1) \quad (2.40)$$

$$= -(r_2 \cos \theta_2 - r_1 \cos \theta_1)(r_2 \sin \theta_2 - r_1 \sin \theta_1). \quad (2.41)$$

The $h_x h_y$ term vanishes because of the change in sign resulting from the change of variables. Now consider the $h_x h_z$ term, it can be expressed as:

$$h_x h_z = (r_2 \cos \theta_2 - r_1 \cos \theta_1)(z_2 - z_1). \quad (2.42)$$

Let $z_1 = -z_1$ and $z_2 = -z_2$ and the $h_x h_z$ term changes sign and therefore vanishes.

Similarly the $h_y h_z$ term

$$h_y h_z = (r_2 \sin \theta_2 - r_1 \sin \theta_1)(z_2 - z_1), \quad (2.43)$$

vanishes when the change of variables is made. All the cross-terms in Equation 2.33 go to zero and as a result the equation can be written as:

$$h_x^2 + h_y^2 + h_z^2 - 3(\sin^2 \Theta \cos^2 \Phi h_x^2 + \sin^2 \Theta \sin^2 \Phi h_y^2 + \cos^2 \Theta h_z^2). \quad (2.44)$$

Rotating so $h_x^2 = h_y^2$ simplifies the above equation to:

$$\begin{aligned} & 2h_x^2 + h_z^2 - 3(h_x^2 \sin^2 \Theta (\cos^2 \Phi + \sin^2 \Phi) + h_z^2 \cos^2 \Theta) \\ = & 2h_x^2 + h_z^2 - 3(h_x^2 \sin^2 \Theta + h_z^2 \cos^2 \Theta) \end{aligned} \quad (2.45)$$

$$= h_z^2(1 - 3\cos^2 \Theta) + h_x^2(2 - 3\sin^2 \Theta) \quad (2.46)$$

$$= h_z^2(\sin^2 \Theta + \cos^2 \Theta - 3\cos^2 \Theta) + h_x^2(2\cos^2 \Theta + 2\sin^2 \Theta - 3\sin^2 \Theta) \quad (2.47)$$

$$= h_z^2(\sin^2 \Theta - 2\cos^2 \Theta) + h_x^2(2\cos^2 \Theta - \sin^2 \Theta) \quad (2.48)$$

$$= \sin^2 \Theta (h_z^2 - h_x^2) + 2\cos^2 \Theta (h_x^2 - h_z^2) \quad (2.49)$$

$$= 2\cos^2 \Theta (h_x^2 - h_z^2) - \sin^2 \Theta (h_x^2 - h_z^2). \quad (2.50)$$

Substituting Equation 2.50 into Equation 2.31 yields:

$$\begin{aligned} A_{pn} = & 1 - \frac{\omega}{4\pi V_{cav} r_0} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{2h} \\ & \frac{1}{h} \left(2\cos^2 \Theta (h_x^2 - h_z^2) - \sin^2 \Theta (h_x^2 - h_z^2) \right). \end{aligned} \quad (2.51)$$

For irradiation of the chamber from the side, the variable κ'_S can be defined as:

$$\kappa'_S = \frac{\omega}{4\pi V_{cav} r_0} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h^2} \frac{(h_x^2 - h_z^2)}{2}. \quad (2.52)$$

The factor for end-on irradiation is:

$$\kappa'_E = -2\kappa'_S \quad (2.53)$$

$$= \frac{-2\omega}{4\pi V_{cav} r_0} \int dA_1 \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h^2} \frac{(h_x^2 - h_z^2)}{2}. \quad (2.54)$$

Equation 2.51 can be written as:

$$A_{pn} = 1 + \kappa'_E \cos^2 \Theta + \kappa'_S \sin^2 \Theta, \quad (2.55)$$

where κ'_E is the point source non-uniformity correction factor for the end-on irradiation of the chamber, κ'_S is the correction factor for the side-on irradiation of the chamber and Θ is the source orientation angle. This result is for a cylindrical chamber at a large distance from the source.

2.3 Source distribution

The source distribution effect is an extension of the source orientation effect. The line source of length l centered with respect to the ionization chamber is in one of two orientations, either along the axis to the chamber or perpendicular to it. Consider that the line source is beside the chamber, parallel to the axis of the chamber. The schematic diagram is shown in Figure 2.2.

The vector from the source to the first surface element, $\vec{r}$, can be expressed as:

$$\vec{r} = \vec{r}_0 + \vec{s}_1 - s\vec{n}, \quad (2.56)$$

Figure 2.2: A schematic diagram of a cylindrical ionization chamber being irradiated by a line source from the side.

where $\vec{r}_0$ is the vector from the source to the chamber center, $\vec{s}_1$ is the vector from dA_1 to the chamber center, s is the distance along the line source and $\vec{n}$ is the direction of the line source (orthogonal to $\vec{r}_0$).

Generalizing Equation 1.9 to include the line source yields:

$$A_{pn} = \frac{r_0^2}{4\pi l V_{cav}} \int dA_1 \int_{-l/2}^{l/2} \frac{ds}{r^2} \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \left(1 + \omega \frac{\vec{r} \cdot \vec{h}}{rh} \right) \quad (2.57)$$

$$= \frac{r_0^2}{4\pi l V_{cav}} \int dA_1 \int_{-l/2}^{l/2} \frac{ds}{r_0^2 + s_1^2 + s^2} \int dA_2 \frac{\cos \psi_{21} \cos \psi_{12}}{h} \left(1 + \omega \frac{(\vec{r}_0 + \vec{s}_1 - s\vec{n}) \cdot \vec{h}}{h\sqrt{r_0^2 + s_1^2 + s^2}} \right), \quad (2.58)$$

where l is the length of the line source and s is the position along the line source. The above equation is the general form for the point source non-uniformity correction factor (including anisotropic electron distribution) with a line source. The line source increases the complexity of the equation by adding the integral over ds .

Chapter 3

A_{wall} for varying source orientation

3.1 MC calculations for a pancake chamber

A calculation was performed to investigate the response of an ionization chamber when the position of the point source is varied from zero to ninety degrees around the chamber. A_{wall} was calculated for each source position and the response of the chamber was corrected using these factors. The residual response was compared with the theory for A_{pn} at large distances. A Monte Carlo simulation of a pancake ionization chamber in the field of a point source was performed using the EGS4 code. In the simulation the chamber had a minimal full buildup wall thickness. The source was placed at various points around the chamber to see the effect that source orientation has on A_{wall} , the photon attenuation and scatter correction factor. Monte Carlo simulations of ionization chambers have been performed to determine A_{wall} in the past [22, 26, 27, 28, 29, for example].

The ionization chamber consisted of a cavity that was 1 cm by 5 cm with 0.3

cm graphite wall. A chamber of this shape was chosen because the position of the source around a pancake chamber would have a large effect on the response of the chamber. Figure 3.1 shows a schematic diagram of the ionization chamber with the point source. The source was a cobalt-60 point source, 100 cm and 50 cm from the center of the ionization chamber. The EGS4 code CAVRZ was used for the simulation. The result was that the largest variation in the correction factor occurred when the source orientation changed by ninety degrees. For angles near ninety degrees, small changes in orientation caused large changes in A_{wall} . Figure 3.2 shows a graph of A_{wall} versus the source orientation angle for a source-to-chamber-center distance of 100 cm, as well as the raw response of the chamber and the raw response divided by A_{wall} for the same set-up. The solid line is the anisotropic theory for the point source non-uniformity correction factor, A_{pn} . When the chamber response was corrected for A_{wall} , the residual response agreed well with the curve from the anisotropic theory of A_{pn} . Figure 3.3 shows the same quantities as the previous graph but with a source-to-chamber-center distance of 50 cm. The results are similar; the largest change in A_{wall} is near ninety degrees and when the raw response is corrected for A_{wall} the response agrees with the theory for A_{pn} . The wall attenuation and scatter correction factor changes dramatically when the source orientation angle changes from zero to ninety degrees. The reason for the large change stems from the shape of the chamber. When the chamber is irradiated from the end (zero degrees), there is a large homogeneous area of 0.3 cm of graphite and then 1 cm of cavity. Relatively few photons will go

Figure 3.1: A schematic diagram of a pancake ionization chamber being irradiated from the end (0°) and from the side (90°) with point sources. The ionization chamber has 0.3 cm graphite walls and a 1 cm x 5 cm air cavity.

Point Source Anisotropy - MC and Theory

Figure 3.2: A graph of the response of a pancake ionization chamber in the field of a cobalt-60 point source versus the source orientation angle with a source-to-chamber-center distance of 100 cm. The Monte Carlo results for the photon attenuation and scatter correction factor (A_{pn}) and the theory for the point source non-uniformity correction factor (A_{pn}) are plotted. The Monte Carlo results for the raw response of the chamber and the raw response corrected for photon attenuation and scatter are also plotted. The response of the chamber varies with the source orientation angle. When the response is corrected for photon attenuation and scatter, the residual response agrees well with the theory for A_{pn} (extension to Kondo and Randolph theory).

Point Source Anisotropy - MC and Theory

Figure 3.3: A graph of the response of a pancake ionization chamber in the field of a cobalt-60 point source versus the source orientation angle with a source-to-chamber-center distance of 50 cm. The Monte Carlo results for the photon attenuation and scatter correction factor (A_{wall}) and the theory for the point source non-uniformity correction factor (A_{pn}) are plotted. The Monte Carlo results for the raw response of the chamber and the raw response corrected for photon attenuation and scatter are also plotted. The response of the chamber varies with the source orientation angle. When the response is corrected for photon attenuation and scatter, the residual response agrees well with the theory for A_{pn} (extension to Kondo and Randolph theory).

through the side walls of the chamber (through 1.6 cm graphite). When the chamber is irradiated from the side (ninety degrees), the homogeneous area is much smaller and more photons are going to travel through the walls on the top and bottom of the cavity (through 5.6 cm graphite). The increase in the number of photons travelling through more wall material causes the wall attenuation correction to be larger. It is important to note that the wall attenuation and scatter correction factor for a pancake ionization chamber changes with the source orientation angle.

3.2 MC calculations for a thick chamber

To reduce the difference in A_{wall} , the thickness of the walls of the pancake ionization chamber was increased. Figure 3.4 shows the ionization chamber with the top and bottom wall thickness of 1.22 cm and the side wall retaining the 0.3 cm thickness of the previous chamber. The walls are graphite and the point source is cobalt-60 .

These dimensions of the pancake chamber were chosen so that at 100 cm the values of A_{wall} at zero and ninety degrees were within 1% of each other. A Monte Carlo study using the CAVRZ/EGS4 code was performed for this chamber. The source-to-chamber-center distances were 100 cm and 50 cm. Figure 3.5 is a graph of A_{wall} versus the source orientation angle for a source-to-chamber-center distance of 100 cm. The result is that the response of the correction factor was almost flat for this chamber except near ninety degrees where a small change in angle caused a large change in the correction factor. Figure 3.6 is a plot of A_{wall} versus the source

Figure 3.4: A schematic diagram of a modified pancake ionization chamber being irradiated from the side and from the end. The chamber has a 1 x 5 cm air cavity surrounded by graphite; 1.22 cm graphite on the top and bottom and 0.3 cm graphite on the sides.

Figure 3.5: A graph of A_{wall} as a function of source orientation angle the modified pancake ionization chamber 100 cm from a cobalt-60 point source. The response is basically flat except near ninety degrees where a small change in angle causes a large change in A_{wall} .

Figure 3.6: A graph of A_{wall} as a function of source orientation angle the modified pancake ionization chamber 50 cm from a cobalt-60 point source. The response is basically flat except near ninety degrees where a small change in angle causes a large change in A_{wall} .

orientation angle when the source is 50 cm away from the center of the chamber. Again the response of the correction factor is nearly flat except near ninety degrees where it changes rapidly. Figure 3.7 is a graph of the ratio of A_{wall} at 100 cm to A_{wall} at 50 cm versus angle. The ratio is near unity except at ninety degrees. This result is important because it suggests that the response of the wall attenuation correction factor for different source orientations does not change with the distance from the source to the center of the chamber.

Figure 3.7: A graph of the ratio of A_{wall} with the source 100 cm from the center of the chamber to A_{wall} with the source 50 cm from the chamber versus the source orientation angle for the modified pancake ionization chamber. The ratio is close to unity except at ninety degrees.

Chapter 4

Non-linear extrapolation for A_{wall}

4.1 Determination

As explained in the introduction, the determination of A_{wall} involves the extrapolation back to zero wall thickness. For spherical ionization chambers, the extrapolation should be nonlinear. The equation for K_W is given in Equation 1.16 as

$$K_W^{-1}(w) = 1 - \mu_{\text{eff}} w f_s(\alpha). \quad (4.1)$$

$A_{\text{wall}} = (K_W K_{\text{cep}})^{-1}$, where K_{cep} is a correction factor that accounts for electron drift and μ_{eff} is the effective attenuation coefficient. To show the non-linearity, measurements should be taken where the curvature of the curve joining the A_{wall} data points is greatest. A Monte Carlo simulation was performed using EGS4/CAVSPH code. A cesium-137 point source was used because a large $\mu_{\text{eff}}\Delta$ was desired. Graphite was chosen for the wall material because it is widely used in ionization chambers and is readily available. The radius of the chamber should be as large as possible to show the non-linearity. Three radii were chosen (5 cm, 1.3 cm and 1 cm), the radius of five

centimeters was chosen because the non-linearity would be obvious with such a large chamber. The other radii were chosen for practicality; ionization chambers of these sizes are more widely used and are easier to construct. In the simulation, the wall thickness was gradually increased and the values of the wall attenuation and scatter correction factor were recorded for each of the ionization chambers.

4.2 Non-linearity for three spherical chambers

The simulation of the five centimeter radius spherical graphite chamber shows most clearly the nonlinearity of the extrapolation back to zero wall thickness. Graphite shells were added to the base chamber in steps of one-quarter millimeter for the first millimeter, one-half millimeter up to ten millimeters, and steps of five millimeters up to seventy millimeters. The wall attenuation and scatter correction factor was plotted versus the thickness of the graphite wall of the ionization chamber. The data were fitted by linear regression. The slope correction function was defined in the introduction as [22]:

$$f_s(\alpha) = \frac{[1 + (1 + \frac{1}{2\alpha}) \log(1 + 2\alpha)]}{2}, \quad (4.2)$$

where α is the radius of the cavity divided by the wall thickness. This slope correction function was applied to the wall thickness data. For the five centimeter (50 mm) base radius chamber the slope correction factor used was:

$$f_s(\frac{50\text{mm}}{x}) = \frac{[1 + (1 + \frac{x}{100}) \log(1 + \frac{100}{x})]}{2}, \quad (4.3)$$

where x is the wall thickness in millimeters. The scaled data were also linearly fitted using a regression technique. Figure 4.1 shows A_{wall} versus the wall thickness up to twenty millimeters. The χ^2_ν value is χ^2 divided by the number of degrees of freedom. A χ^2_ν value of approximately equal to unity suggests a good fit [30]. The χ^2_ν for the original data is 32.14 suggesting the data do not agree well with the linear fit. For the slope corrected data, the χ^2_ν value is 1.89; the linear fit is much better with the corrected data. The graph shows that the original data should not be linearly extrapolated back to zero wall thickness.

Figure 4.2 shows the same graph but with the wall thickness up to seventy millimeters. These points were done to show the limit of the slope correction technique. The assumptions made to simplify the correction factor to Equation 1.16 include assuming the ionization chamber is 'small'. As the wall thickness gets larger, this assumption is no longer valid and the graph shows the break-down of the theory. The figure shows that once the chamber gets large, the slope corrected data are nonlinear, and it is clear that the original data are nonlinear as well. The χ^2_ν value of the original data is 516.70 and the value for the slope corrected data is 63.13. These results confirm the prediction that the nonlinear theory will break down for large chambers. The data in this region are not practical values, (a 5 cm cavity enclosed in 7 cm of graphite on each side). The slope correction factor is useful in the practical range for ionization chambers.

A simulation of a smaller spherical ionization chamber was done using a one

Figure 4.1: A graph of A_{wall} versus the thickness of graphite on a 5 cm base radius spherical graphite ionization chamber. The slope correction was applied to the data and each set of points was fitted linearly. The slope corrected data are more linear than the original data suggesting the extrapolation back to zero wall thickness should be nonlinear.

Figure 4.2: A graph of A_{wall} versus the thickness of graphite on a 5 cm base radius spherical graphite ionization chamber. The slope correction was applied to the data and each set of points was fitted linearly. This graph shows thicknesses of graphite up to 70 mm. The slope corrected data are also becoming nonlinear as a result of the theory breaking down when the walls are thick.

centimeter radius cavity. Again, the graphite shells were added in small steps at first and then with increasing step size. Figure 4.3 shows A_{wall} as a function of wall thickness up to nineteen millimeters of graphite. The data were fitted linearly, but the graph shows that the data does not lie in a straight line. The χ^2_ν value is 46.14 for this data. A slope correction factor was applied to the data. For the one centimeter (10 mm) radius cavity, the correction factor was:

$$f_s\left(\frac{10\text{mm}}{x}\right) = \frac{[1 + (1 + \frac{x}{20})\log(1 + \frac{20}{x})]}{2}, \quad (4.4)$$

where x is the thickness of the wall in millimeters. The slope corrected data were also plotted and fitted linearly. The χ^2_ν value is 4.89 for the slope corrected data. The graph shows that the slope corrected data are more linear than the original data. Figure 4.4 shows the same parameters but with the wall thickness going to ninety millimeters. For a chamber this large the slope correction theory should break down. The χ^2_ν values for the original data and the slope corrected data are 68.72 and 20.99, respectively. Neither set of data agree well with the linear fit. This result was predicted in the slope correction theory in the case when the chamber is large. These two graphs suggest that the extrapolation should be nonlinear for a one centimeter radius ionization chamber.

A third spherical ionization chamber was investigated in the same manner. This chamber was chosen to simulate a realistic experiment. An experiment could be performed with a chamber of this size and thin graphite shells that could be added to the chamber. Measurements could be taken with the ionization chamber free-in-air in a

Figure 4.3: A graph of A_{wall} versus the thickness of graphite on a 1 cm base radius spherical graphite ionization chamber. The slope correction was applied to the data and each set of points was fitted linearly. The slope corrected data are more linear than the original data suggesting the extrapolation back to zero wall thickness should be nonlinear.

Figure 4.4: A graph of A_{wall} versus the thickness of graphite on a 1 cm base radius spherical graphite ionization chamber. The slope correction was applied to the data and each set of points was fitted linearly. The graph shows thicknesses of graphite up to 70 mm.

Cs-137 beam to verify the Monte Carlo calculations. In the Monte Carlo study, a base radius of 1.3 centimeter chamber was used to show that the extrapolation to zero wall thickness should be nonlinear. Again, the simulation was done using EGS4/CAVSPH code. A_{wall} was plotted versus the wall thickness (as shown in Figure 4.5). The wall thickness was increased in one-half millimeter steps up to nineteen millimeters. The data were fitted linearly using a regression technique; it is clear that the points do not follow a straight line. The χ^2_ν value is 12.21 for this data, suggesting that the points do not agree well with the linear fit. The data were scaled with the slope correction function. For the 1.3 centimeter (13 mm) radius chamber, the correction function used was:

$$f_s\left(\frac{13mm}{x}\right) = \frac{[1 + (1 + \frac{x}{26})\log(1 + \frac{26}{x})]}{2}. \quad (4.5)$$

The slope corrected data were fitted using a regression and the points fit the line much better than the original data (χ^2_ν value is 1.87).

The simulations of three spherical ionization chambers show that the extrapolation back to zero wall thickness in the determination of A_{wall} should be nonlinear. The slope correction function can be utilized to make the points fit a line. In the derivation of the slope correction function, the ionization chamber was assumed to be 'small'. For large wall thicknesses, this assumption is no longer valid. The slope correction is not accurate for ionization chambers with very thick walls but is accurate for chambers used in practical situations.

A spherical graphite ionization chamber with a 1.3 cm radius air cavity was manu-

Figure 4.5: A graph of A_{wall} versus the thickness of graphite on a 1.3 cm base radius spherical graphite ionization chamber. The slope correction was applied to the data and each set of points was fitted linearly. The slope corrected data are more linear than the original data suggesting the extrapolation back to zero wall thickness should be nonlinear.

factured at the National Research Council of Canada. The chamber has three graphite shells of different thicknesses that can be added over the cavity. Measurements could be made of the ionization chamber response as a function of the thickness of the graphite walls. If cesium-137 was used as the energy source, the experiment would correspond to the simulation performed for the 1.3 cm radius chamber. The experimental results could be used to test the theory for the nonlinear extrapolation of A_{wall} .

Chapter 5

Calculations for the Göteborg experiment

5.1 Comparison of Kondo-Randolph theory and the extension

An experiment was performed in Sweden by Heikki Tölli and Olle Mattsson [31] to compare the theory of Kondo-Randolph [8] and the extension to the theory [11]. My contribution to this work was in the Monte Carlo calculations of A_{wall} for the chambers used. Measurements using three ionization chambers were made with source-to-chamber-center distances between 10 mm and 140 mm. The source used was a line source iridium-192 (3.6 mm active diameter and 3.5 mm full length). The chambers consisted of a PTW chamber (cylindrical with flat ends), an NE2571 chamber (cylindrical with one conical end and one flat end) and an NE2581 which is a modified NE2571 chamber (also cylindrical with one flat end and one conical end). The measurement values used are averages of the data measured on five separate occa-

sions [31]. Figure 5.1 shows the shape of the NE2571 (or NE2581) ionization chamber. The one conical end is shown with one end of the cavity conical as well. The electrode is shown as a cylinder in the center of the chamber.

The following graphs show the theory for A_{pn} divided by the measured results. Ideally, the ratio of these values should be unity. The first graph (Figure 5.2) is a plot, for the PTW chamber, of the theory divided by the experiment as a function of the distance from the center of the chamber to the source. The error bars are those of the experiment only. A χ^2_ν value of 1.07 was obtained for data from the extension to the theory. The extension to Kondo-Randolph theory is close to unity (within error bars) with the exception of the case where the source-to-chamber-center distance is the smallest (10 mm). The cause of deviation may be the effect of the finite size of the source or the effect of the electrode. These possibilities are discussed in some detail in the next two sections. The Kondo-Randolph theory does not yield values that are as close to unity as the extension does; the χ^2_ν value is 37.58 for the Kondo-Randolph points. The data deviate from unity by up to 4.5%.

Figure 5.3 is a graph of the theories divided by the experimental results for the NE2571 chamber. The chamber has one conical end and the length of the cavity is 24.1 mm to the tip of the cone, 22.2 mm to the base of the cone and 22.83 mm using a volume averaging technique. The volume averaging length is the length of a cylinder with the same volume as the chamber with one conical end. The length used to determine A_{pn} is 22.8 mm based on the angle of the cone being 118° from

Figure 5.1: A diagram of the NE2571 (or NE2581) chamber showing the conical end and the flat end of both the chamber and the cavity. The electrode is the cylinder in the center of the chamber.

Figure 5.2: A graph comparing the Kondo and Randolph theory for A_{pn} to the extension to the theory. The theory divided by the results from the experiment (using the PTW chamber) performed by Tölli and Mattsson [31] is plotted as a function of the distance from the source to the center of the chamber. The extension to the yields results closer to unity than the original theory. The results of the extension are near unity except for the closest source-to-chamber-center distance.

Figure 5.3: A graph comparing the Kondo and Randolph theory for A_{pn} to the extension to the theory. The theory divided by the results from the experiment (using the NE2571 chamber) performed by Tölli and Mattsson [31] is plotted as a function of the distance from the source to the center of the chamber. The extension to the yields results closer to unity than the original theory.

measurements taken by Tölle and Mattsson [31]. The graph shows that the Kondo-Randolph theory divided by the measurements is not as close to unity as the extension to the theory. The χ^2_ν values for the Kondo-Randolph theory and its extension are 24.19 and 2.54, respectively. The Kondo-Randolph data differs from unity by up to 5%. The extension to the theory yields results that are close to unity with the exception of the very short source-to-chamber-center distances.

Figure 5.4 is a similar graph to the previous one. It is theory divided by the experiment versus the source-to-chamber-center distance for the NE2581 chamber. The NE2581 chamber is the same as a NE2571 chamber with the exception that the walls and electrode are A-150 whereas in the NE2571 chamber the walls are graphite with a graphite electrode. The graph shows the extension to the Kondo-Randolph theory giving results that are closer to unity than the original theory. The χ^2_ν value for data from the original theory is 21.18, whereas a value of 1.15 was obtained for the extension. Again, the deviation from unity occurs at the short source-to-chamber-center distances.

The results of the experiment involving the three cylindrical ionization chambers imply that the extension to the Kondo-Randolph theory for the point source non-uniformity correction factor is more accurate. The largest deviations in both theories occur when the source is very close to the chamber. The wall attenuation and scatter correction factor had not been applied to the experimental results. The next section discusses the calculation of A_{wall} for each of the three chambers and how the correction

Figure 5.4: A graph comparing the Kondo and Randolph theory for A_{pn} to the extension to the theory. The theory divided by the results from the experiment (using the NE2581 chamber) performed by Tölli and Mattsson [31] is plotted as a function of the distance from the source to the center of the chamber. The extension to the yields results closer to unity than the original theory.

factor affects the comparison of the theories. The effect of the electrode and the effect of using a line source as opposed to a point source has not been taken into account as yet and will be examined in a later section.

5.2 A_{wall} calculations for three ion chambers

The three ionization chambers used in the experiment were a PTW chamber, a NE2571 chamber and a NE2581 chamber. The experimental results in the previous section did not account for the wall attenuation and scatter correction factor. The A_{wall} correction factor was calculated for each chamber by Monte Carlo simulation using the EGS4/CAVRZ code. Each chamber was modelled using the dimensions, shape and composition of that chamber. The PTW chamber (Model 23331) is a cylindrical chamber with flat ends. It is 22.0 mm in length, 8.0 mm in diameter. The walls are lucite and the electrode is aluminium (0.75 mm radius). The chamber was simulated, ignoring the electrode. The wall attenuation and scatter correction factors were calculated for source-to-chamber-center distances of 10 mm to 140 mm to coincide with the measurements taken. The results from this study are shown in Figure 5.5. The closer the source is to the ionization chamber, the larger the correction factor. A large number of particle histories was used to ensure the statistics on A_{wall} were good. The error bars reflect the statistical uncertainty on the wall attenuation and scatter correction factor.

A similar study was performed for the NE2571 chamber. This chamber is cylin-

Figure 5.5: A graph of A_{wall} from Monte Carlo simulations versus the source-to-chamber-center distance for the PTW chamber. The the A_{wall} correction is large when the source is close to the chamber.

drical with one flat and one conical end. The chamber is graphite with a graphite electrode. The conical end of the ionization chamber was modelled in a stepwise manner. The cavity has a full length (to the tip of the cone) of 24.1 mm and a diameter of 6.3 mm. The chamber was modelled using these dimensions but ignoring the electrode. Figure 5.6 shows A_{wall} versus the source-to-chamber-center distance for the NE2571 chamber. The correction factor is largest when the source is close to the chamber. It is relatively flat as the source-to-chamber-center distance gets large.

A simulation of the NE2581 chamber was done in the same way. The NE2581 chamber is identical in size and shape to the NE2571 but the composition is different. The chamber was simulated without the electrode. The values of A_{wall} versus the source-to-chamber-center distance are plotted in Figure 5.7. The results are constant with slightly larger corrections when the source is close to the chamber.

Figure 5.8 shows a graph similar to Figure 5.5 except that the wall attenuation and scatter correction factor has been applied to the experimental values. The error bars reflect the the uncertainty from the measurements and the statistical uncertainty in A_{wall} . The graph shows the comparison of the two theories for A_{pn} divided by the experimental results. After A_{wall} is applied to the experiment and the values normalized at the distance of 140 mm, the extension to the Kondo-Randolph theory yields results that are close to unity. The χ^2_ν value for the extension to the theory is 0.77; the value for the original theory is 23.22. The results of the Kondo-Randolph theory show deviations from unity by up to 3.5%.

Figure 5.6: A graph of A_{wall} from Monte Carlo simulations versus the source-to-chamber-center distance for the NE2571 chamber. The A_{wall} correction is large when the source is close to the chamber.

Figure 5.7: A graph of A_{wall} from Monte Carlo simulations versus the source-to-chamber-center distance for the NE2581 chamber. The A_{wall} correction does not vary by a significant amount for all the source-to-chamber-center distances.

Figure 5.8: A graph comparing the two theories for A_{pn} . The experimental results have been corrected for A_{wall} using the results of the Monte Carlo simulations. The graph shows the theory divided by the corrected experimental values for the PTW chamber versus the distance from the source to the chamber center. The extension to the theory gives results closer to unity than the original theory. Again, at the short distances the results deviate from unity.

Figure 5.9 shows the comparison of the theories divided by the experiment including the A_{wall} correction factor for the NE2571 chamber. The results are similar to those for the PTW chamber. The extension to the theory gives results that are close to unity with a deviation at 10 mm source-to-chamber-center distance. The χ^2_ν value for the extension to the theory is 1.86 and for the original theory is 12.65. The Kondo-Randolph theory gives results that are close to unity for distance of 30 mm or more but for shorter distances deviates from unity.

Figure 5.10 shows a similar graph to the previous one but the data are for the NE2581 chamber. The response of the extension to Kondo-Randolph theory is close to unity except at very short distances. The Kondo-Randolph theory deviates from unity by up to 5%. The χ^2_ν values for the Kondo-Randolph theory and its extension are 13.00 and 0.91, respectively.

The simulations of the three ionization chambers yield results for A_{wall} showing the wall attenuation and scatter correction being largest at short source-to-chamber-center distances. When the values of A_{wall} are incorporated into the experimental measurements performed by Tölle and Mattsson [31], the comparison between Kondo-Randolph theory for the point source non-uniformity correction factor and the extension to the theory show that the extension to the theory is more accurate.

Figure 5.9: A graph comparing the two theories for A_{pu} . The experimental results have been corrected for A_{wall} using the results of the Monte Carlo simulations. The graph shows the theory divided by the corrected experimental values for the NE2571 chamber versus the distance from the source to the chamber center. The extension to the theory gives results closer to unity than the original theory. Again, at the short distances the results deviate from unity.

Figure 5.10: A graph comparing the two theories for A_{pn} . The experimental results have been corrected for A_{wall} using the results of the Monte Carlo simulations. The graph shows the theory divided by the corrected experimental values for the NE2581 chamber versus the distance from the source to the chamber center. The extension to the theory gives results closer to unity than the original theory. Again, at the short distances the results deviate from unity.

5.3 Line source distribution and electrode effects

In the experiment, the radiation source was a line source of iridium-192, not a point source as was assumed in the simulations to obtain values for A_{wall} . The length of the source used in the experiment was 3.5 mm. A Monte Carlo simulation was performed using the PTW chamber with the position of the source varying from 0 mm to 1.75 mm away from the middle of the line source. The simulation modelled the position along one-half of the source, by symmetry, the other half would be the same. The results are shown in Figure 5.11 which shows A_{wall} as a function of the distance along the line source. The error bars reflect the statistical uncertainty on A_{wall} . The source-to-chamber-center distance used was 10 mm; it was chosen because the closer the source is to the chamber, the larger the effect the walls of the chamber will have. The open circles show that the response is flat (within error bars) along the line source so the difference between the line source and point source is negligible. The graph also shows another set of points (closed squares) from a simulation of the PTW chamber with the electrode modelled as well. The electrode is aluminium, with a radius of 0.75 mm and a length of 20 mm. The source-to-chamber-center distance is again 10 mm. The electrode appears to have little effect on the wall attenuation and scatter correction factor.

Figure 5.11: A graph of A_{wall} from Monte Carlo simulations versus the distance along the line source for the PTW chamber. The chamber was modelled with the electrode and without. The position of the source along the line source does not seem to have an effect on A_{wall} . The inclusion of the electrode does not have a consistent effect on the values of A_{wall} .

Chapter 6

Conclusions

The work presented is meant to provide insight into some of the factors influencing the response of ionization chambers to brachytherapy sources. The Kondo-Randolph theory and its extension for the point source non-uniformity correction factor, A_{pn} , were reviewed. The effects that source orientation and source distribution have on A_{pn} were developed and shown in equation form. The source orientation effect was simplified to the following equation for large source to chamber distances:

$$A_{\text{pn}} = 1 + \kappa'_E \cos^2 \Theta + \kappa'_S \sin^2 \Theta, \quad (6.1)$$

where κ'_E is for end-on irradiation, κ'_S is for side-on irradiation and Θ is the source orientation angle. The analytic expressions give a quick way to determine the point source non-uniformity correction factor when the source is not positioned along one of the axes.

Monte Carlo calculations were performed on a pancake ionization chamber in a cobalt-60 beam. The wall attenuation and scatter correction factor, A_{wall} , was

calculated for different source orientation angles. The correction factor was found to vary significantly with source orientation angle, with the largest changes near ninety degrees. The response of the pancake ionization chamber was also plotted and once A_{wall} was factored out of the response, it agreed well with the anisotropic theory for A_{pn} .

A chamber was designed so that A_{wall} was the same at zero and ninety degrees. The effect source orientation has on A_{wall} was observed again using a Monte Carlo simulation. The response of the correction factor was nearly flat except close to ninety degrees where small changes in angle cause large changes in A_{wall} for the modified pancake chamber.

The method used to determine the correction factor, A_{wall} , for spherical chambers was reviewed. A case was made for using a nonlinear extrapolation to zero wall thickness instead of the conventional linear extrapolation. Monte Carlo simulations were performed for three spherical graphite ionization chambers with different radii. The data from the simulations shows that the extrapolation of A_{wall} should be nonlinear. This result is most obvious in the data from the largest chamber and is evident in the two smaller chambers.

An experiment performed by Tölli and Mattson [31] compared the Kondo and Randolph theory with its extension. The correction factor, A_{wall} , was calculated using a Monte Carlo simulation for the chambers involved in the experiment. The

experimental data fit the extension to the theory significantly better than the original theory. When the wall attenuation and scatter correction factor is taken into account, the results show that the extension to the Kondo-Randolph theory is more accurate than the original theory. The results for the very short source-to-chamber-center distances show the largest deviation from unity, perhaps due to the effect the line source has on A_{pn} which was not taken into account in this work. The effect on A_{wall} of the line source (used in experiment) compared to the point source (used in simulation) was investigated. Simulations showed that A_{wall} did not change significantly when the point source was placed along the position of the line source. The electrode was not modelled for the A_{wall} calculations. The electrode was modelled in one case to see if the effect would be significant. The case with the electrode showed no consistent deviation in A_{wall} from the cases without the electrode.

Some suggestions for future work include practical experiments to test theory and Monte Carlo results. As well, more Monte Carlo simulations could be performed to further investigate ionization chamber response to point sources.

The extension to the Kondo and Randolph theory for A_{pn} could be extended further. The $\omega \cos \Theta$ term is only one in the Legendre series. More terms of the series could be used to make the theory for A_{pn} more accurate but the increase in accuracy is small when weighed against to the mathematical complications that will result.

The Monte Carlo studies of A_{wall} show that for spherical ionization chambers, the

extrapolation back to zero wall thickness should be nonlinear. Experiments could be performed to confirm the results of the Monte Carlo simulations. The theory for the slope correction function could be extended for cylindrical chambers and Monte Carlo simulations and experiments could be performed to confirm the results. If it is further established that the extrapolation in the determination of A_{wall} should be nonlinear, then this method should be used to determine the values of A_{wall} for the ionization chambers used in primary standards laboratories.

Appendix A

Derivation of the Kondo-Randolph Correction Factor

In their paper, Kondo and Randolph [8] skip steps in the derivation of the point source non-uniformity correction factor. This appendix provides a full derivation of one of the cases in the derivation. The other cases follow similar steps. The purpose of including this appendix is to provide a way to work through Kondo and Randolph's derivations with relative ease. The derivation of the Kondo and Randolph expression for the point source non-uniformity correction factor, A_{pn} , can be split into four cases and taken separately. The four cases are 1) both surface elements, dA_1 and dA_2 , are on the side of the cylindrical chamber, 2) dA_1 is on the side and dA_2 is on the end of the chamber, 3) dA_1 is on the end of the chamber and dA_2 is on the side and 4) both dA_1 and dA_2 are on the end of the chamber. This appendix contains a derivation of the second case where the first surface element is on the side and the second surface element is on the end of the chamber; the chamber is irradiated from the side (ie.

the coordinates of the source are $(0, r_0, 0)$). The position of the first surface element, dA_1 , is given by the following expression:

$$\vec{s}_1 = a \cos \theta_1 \hat{x} + a \sin \theta_1 \hat{y} + z_1 \hat{z}, \quad (\text{A.1})$$

where a is the radius of the chamber, θ_1 is the angle in the xy -plane and z_1 is the distance to dA_1 in the z -direction. The position of the second surface element, dA_2 , is given by:

$$\vec{s}_2 = \rho_2 \cos \theta_2 \hat{x} + \rho_2 \sin \theta_2 \hat{y} \pm L \hat{z}, \quad (\text{A.2})$$

where ρ_2 is the distance in the xy -plane, θ_2 is the angle in the xy -plane and L is the half-length of the chamber (the $\pm$ denotes whether dA_2 is on the top (+) or bottom (-) of the cylindrical chamber. The vector from dA_1 to dA_2 is $\vec{h}_{12}$ which can be expressed as:

$$\vec{h}_{12} = (\rho_2 \cos \theta_2 - a \cos \theta_1) \hat{x} + (\rho_2 \sin \theta_2 - a \sin \theta_1) \hat{y} + (\pm L - z_1) \hat{z}. \quad (\text{A.3})$$

The vector from dA_2 to dA_1 is $\vec{h}_{21}$ where:

$$\vec{h}_{21} = (a \cos \theta_1 - \rho_2 \cos \theta_2) \hat{x} + (a \sin \theta_1 - \rho_2 \sin \theta_2) \hat{y} + (z_1 \mp L) \hat{z}. \quad (\text{A.4})$$

The angle ψ_{21} is the angle between $\vec{h}$ and the inward normal from dA_1 . The $\cos \psi_{21}$ can be written as:

$$\cos \psi_{21} = -\frac{(a \cos \theta_1 \hat{x} + a \sin \theta_1 \hat{y})}{\sqrt{(a \cos \theta_1)^2 + (a \sin \theta_1)^2}} \frac{\vec{h}_{12}}{h} \quad (\text{A.5})$$

$$\begin{aligned} &= -\frac{(a \cos \theta_1 \hat{x} + a \sin \theta_1 \hat{y})(\rho_2 \cos \theta_2 - a \cos \theta_1) \hat{x}}{a \sqrt{(\rho_2 \cos \theta_2 - a \cos \theta_1)^2 + (\rho_2 \sin \theta_2 - a \sin \theta_1)^2 + (\pm L - z_1)^2}} \\ &\quad - \frac{(a \cos \theta_1 \hat{x} + a \sin \theta_1 \hat{y})(\rho_2 \sin \theta_2 - a \sin \theta_1) \hat{y}}{a \sqrt{(\rho_2 \cos \theta_2 - a \cos \theta_1)^2 + (\rho_2 \sin \theta_2 - a \sin \theta_1)^2 + (\pm L - z_1)^2}} \\ &\quad - \frac{(a \cos \theta_1 \hat{x} + a \sin \theta_1 \hat{y})(\pm L - z_1) \hat{z}}{a \sqrt{(\rho_2 \cos \theta_2 - a \cos \theta_1)^2 + (\rho_2 \sin \theta_2 - a \sin \theta_1)^2 + (\pm L - z_1)^2}} \quad (\text{A.6}) \\ &= -(\cos \theta_1 \hat{x} + \sin \theta_1 \hat{y}) \end{aligned}$$

$$\frac{(\rho_2 \cos \theta_2 - a \cos \theta_1) \hat{x} + (\rho_2 \sin \theta_2 - a \sin \theta_1) \hat{y} + (\pm L - z_1) \hat{z}}{\sqrt{\rho_2^2 + a^2 - 2\rho_2 a \cos(\theta_2 - \theta_1) + (L \mp z_1)^2}} \quad (\text{A.7})$$

$$= \frac{-\rho_2 \cos \theta_2 \cos \theta_1 + a \cos^2 \theta_1 - \rho_2 \sin \theta_2 \sin \theta_1 + a \sin^2 \theta_1}{\sqrt{\rho_2^2 + a^2 - 2\rho_2 a \cos(\theta_2 - \theta_1) + (L \mp z_1)^2}} \quad (\text{A.8})$$

$$= \frac{a - \rho_2 \cos(\theta_2 - \theta_1)}{h}, \quad (\text{A.9})$$

where h is given by:

$$h = \sqrt{\rho_2^2 + a^2 - 2\rho_2 a \cos(\theta_2 - \theta_1) + (L \mp z_1)^2}. \quad (\text{A.10})$$

The angle ψ_{12} is the angle between $\vec{h}$ and the inward normal from dA_2 . The cosine of the angle can be expressed as:

$$\cos \psi_{12} = \frac{\mp L \hat{z}}{\sqrt{L^2}} \frac{\vec{h}_{21}}{h} \quad (\text{A.11})$$

$$= \mp \hat{z} \frac{(a \cos \theta_1 - \rho_2 \cos \theta_2) \hat{x} + (a \sin \theta_1 - \rho_2 \sin \theta_2) \hat{y} + (z_1 \mp L) \hat{z}}{h} \quad (\text{A.12})$$

$$= \frac{\mp(z_1 \pm L)}{h} \quad (\text{A.13})$$

$$= \frac{(L \mp z_1)}{h}. \quad (\text{A.14})$$

The expression for r^2 , the square of the distance from the source to dA_1 can be expressed as:

$$r^2 = r_0^2 - 2\vec{s}_1 \cdot \vec{r}_0 + s_1^2 \quad (\text{A.15})$$

$$= r_0^2 - 2(a \cos \theta_1 \hat{x} + a \sin \theta_1 \hat{y} + z_1 \hat{z}) \cdot (r_0 \hat{y})$$

$$+ (a \cos \theta_1 \hat{x} + a \sin \theta_1 \hat{y} + z_1 \hat{z})^2 \quad (\text{A.16})$$

$$= r_0^2 - 2ar_0 \sin \theta_1 + a^2 + z_1^2. \quad (\text{A.17})$$

The correction factor A_{pn} is expressed as [8]:

$$A_{\text{pn}} = \frac{r_0^2}{4\pi V_{\text{cav}}} \int \frac{dA_1}{r^2} \int dA_2 \frac{\cos \psi_{12} \cos \psi_{21}}{h}. \quad (\text{A.18})$$

Substituting the expressions for r^2 , $\cos \psi_{21}$, $\cos \psi_{12}$ and h in the above equation yields:

$$A_{\text{pn}} = \frac{r_0^2}{4\pi V_{\text{cav}}} \int \frac{dA_1}{r_0^2 - 2ar_0 \sin \theta_1 + a^2 + z_1^2} \int dA_2 \frac{\frac{a - \rho_2 \cos(\theta_2 - \theta_1)}{h} \frac{(L \mp z_1)}{h}}{h} \quad (\text{A.19})$$

$$= \frac{r_0^2}{4\pi V_{\text{cav}}} \int_0^{2\pi} d\theta_1 a \int_{-L}^L dz_1 \frac{1}{r_0^2 - 2ar_0 \sin \theta_1 + a^2 + z_1^2}$$

$$\int_0^a \rho_2 d\rho_2 \int_0^{2\pi} d\theta_2 \frac{(a - \rho_2 \cos(\theta_2 - \theta_1))(L \mp z_1)}{(\rho_2^2 + a^2 - 2\rho_2 a \cos(\theta_2 - \theta_1) + (L \mp z_1)^2)^{\frac{3}{2}}}. \quad (\text{A.20})$$

Let $z_1 = L\xi_1$, $dz_1 = Ld\xi_1$, $\sigma = \frac{a}{L}$, $\lambda = \frac{L}{r_0}$, $\alpha = \frac{a}{r_0}$ and $\theta_{21} = \theta_2 - \theta_1$. The equation for A_{pn} become :

$$A_{\text{pn}} = \frac{r_0^2}{4\pi V_{\text{cav}}} \int_0^{2\pi} d\theta_1 a \int_{-1}^1 Ld\xi_1 \frac{1}{r_0^2(1 - 2\alpha \sin \theta_1 + \alpha^2 + \lambda^2 \xi_1^2)} \quad (\text{A.21})$$

$$= \int_0^a \rho_2 d\rho_2 \int_0^{2\pi} d\theta_2 \frac{(\sigma L - \rho_2 \cos \theta_{21})(L \mp L\xi_1)}{(\rho_2^2 + \sigma^2 L^2 - 2\rho_2 \sigma L \cos \theta_{21} + (L \mp L\xi_1)^2)^{\frac{3}{2}}}. \quad (\text{A.22})$$

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When the substitutions $\rho_2 \rightarrow a\rho_2$ and $d\rho_2 \rightarrow ad\rho_2$ are made the equation becomes:

$$A_{\text{pn}} = \frac{r_0^2}{4\pi V_{\text{cav}}} \int_0^{2\pi} d\theta_1 a \int_{-1}^1 L d\xi_1 \frac{1}{r_0^2(1 - 2\alpha \sin \theta_1 + \alpha^2 + \lambda^2 \xi_1^2)} \int_0^1 a^2 \rho_2 d\rho_2 \int_0^{2\pi} d\theta_2 \frac{\sigma L^2 (1 - \rho_2 \cos \theta_{21})(1 \mp \xi_1)}{(\sigma^2 L^2 (\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + L^2 (1 \mp \xi_1)^2)^{\frac{3}{2}}} \quad (\text{A.23})$$

$$= \frac{r_0^2}{4\pi V_{\text{cav}}} \int_0^{2\pi} d\theta_1 a \int_{-1}^1 L d\xi_1 \frac{1}{r_0^2(1 - 2\alpha \sin \theta_1 + \alpha^2 + \lambda^2 \xi_1^2)} \int_0^1 a^2 \rho_2 d\rho_2 \int_0^{2\pi} d\theta_2 \frac{\sigma}{L} \frac{(1 - \rho_2 \cos \theta_{21})(1 \mp \xi_1)}{(\sigma^2 (\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{3}{2}}}. \quad (\text{A.24})$$

Replace V_{cav} with $(2\pi a^2 L)$ and the above equation becomes:

$$A_{\text{pn}} = \frac{r_0^2 a L a^2 \sigma}{4\pi (2\pi a^2 L) L} \int_0^{2\pi} d\theta_1 \int_{-1}^1 d\xi_1 \frac{1}{r_0^2(1 - 2\alpha \sin \theta_1 + \alpha^2 + \lambda^2 \xi_1^2)} \int_0^1 \rho_2 d\rho_2 \int_0^{2\pi} d\theta_2 \frac{(1 - \rho_2 \cos \theta_{21})(1 \mp \xi_1)}{(\sigma^2 (\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{3}{2}}}. \quad (\text{A.25})$$

The above equation simplifies to :

$$A_{\text{pn}} = \frac{\sigma^2}{8\pi^2} \int_0^{2\pi} d\theta_1 \int_{-1}^1 d\xi_1 \frac{1}{r_0^2(1 - 2\alpha \sin \theta_1 + \alpha^2 + \lambda^2 \xi_1^2)} \int_0^1 \rho_2 d\rho_2 \int_0^{2\pi} d\theta_2 \frac{(1 - \rho_2 \cos \theta_{21})(1 \mp \xi_1)}{(\sigma^2 (\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{3}{2}}}. \quad (\text{A.26})$$

Consider the following term alone:

$$\int_0^1 d\rho_2 \int_0^{2\pi} d\theta_2 \frac{\rho_2 (1 - \rho_2 \cos \theta_{21})}{(\sigma^2 (\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{3}{2}}}. \quad (\text{A.27})$$

This can be simplified as follows:

$$\begin{aligned}
& \int_0^1 d\rho_2 \int_0^{2\pi} d\theta_2 \frac{\rho_2(1 - \rho_2 \cos \theta_{21})}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{3}{2}}} \\
&= \int_0^{2\pi} d\theta_2 \int_0^1 d\rho_2 \frac{\rho_2}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{3}{2}}} \\
&\quad - \frac{\rho_2^2 \cos \theta_{21}}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{3}{2}}}
\end{aligned} \tag{A.28}$$

$$\begin{aligned}
&= \int_0^{2\pi} d\theta_2 \int_0^1 d\rho_2 \frac{\rho_2}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{3}{2}}} \\
&\quad - \frac{\frac{\cos \theta_{21}}{\sigma^2} [\sigma^2(1 + \rho_2^2 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2 - \sigma^2(1 - 2\rho_2 \cos \theta_{21}) - (1 \mp \xi_1)^2]}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{3}{2}}}
\end{aligned} \tag{A.29}$$

$$\begin{aligned}
&= \int_0^{2\pi} d\theta_2 \int_0^1 d\rho_2 \frac{\rho_2}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{3}{2}}} \\
&\quad + \frac{\frac{\cos \theta_{21}}{\sigma^2} [\sigma^2(1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2]}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{3}{2}}} \\
&\quad - \frac{\frac{\cos \theta_{21}}{\sigma^2} [\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2]}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{3}{2}}}
\end{aligned} \tag{A.30}$$

$$\begin{aligned}
&= \int_0^{2\pi} d\theta_2 \int_0^1 d\rho_2 \frac{\rho_2(1 - 2 \cos^2 \theta_{21})}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{3}{2}}} \\
&\quad + \frac{\frac{\cos \theta_{21}}{\sigma^2} (\sigma^2 + (1 \mp \xi_1)^2)}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{3}{2}}} \\
&\quad - \frac{\cos \theta_{21}}{\sigma^2} \frac{1}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{1}{2}}}.
\end{aligned} \tag{A.31}$$

Consider integrating the final term of Equation A.31 with respect to $d\theta_2$. Note that $\cos \theta_{21} = \cos \theta_2$ from the periodicity of the cosine function. The integration to be performed is then:

$$\int_0^{2\pi} d\theta_2 \frac{\cos \theta_2}{\sigma^2} \frac{1}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_2) + (1 \mp \xi_1)^2)^{\frac{1}{2}}}. \tag{A.32}$$

Integrating by parts with $v = \sin \theta_2$, $dv = \cos \theta_2 d\theta_2$, $u = (\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_2) + (1 \mp \xi_1)^2)^{-\frac{1}{2}}$ and $du = \frac{\sigma^2 \rho_2 \sin \theta_2 d\theta_2}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_2) + (1 \mp \xi_1)^2)^{\frac{3}{2}}}$ gives:

$$\begin{aligned} & \int_0^{2\pi} d\theta_2 \frac{\cos \theta_2}{\sigma^2} \frac{1}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_2) + (1 \mp \xi_1)^2)^{\frac{1}{2}}} \\ &= \left[\frac{\sin \theta_2}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_2) + (1 \mp \xi_1)^2)^{\frac{1}{2}}} \right]_0^{2\pi} \\ &+ \int_0^{2\pi} \frac{\sin^2 \theta_2 \rho_2 d\theta_2}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_2) + (1 \mp \xi_1)^2)^{\frac{3}{2}}} \end{aligned} \quad (\text{A.33})$$

$$= 0 + \rho_2 \int_0^{2\pi} \frac{\sin^2 \theta_2 d\theta_2}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_2) + (1 \mp \xi_1)^2)^{\frac{3}{2}}}. \quad (\text{A.34})$$

Substituting Equation A.34 back into Equation A.31 yields:

$$\begin{aligned} & \int_0^{2\pi} d\theta_2 \int_0^1 d\rho_2 \frac{\rho_2(1 - 2\cos^2 \theta_{21})}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{3}{2}}} \\ &+ \frac{\frac{\cos \theta_{21}}{\sigma^2}(\sigma^2 + (1 \mp \xi_1)^2)}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{3}{2}}} \\ &- \frac{\cos \theta_{21}}{\sigma^2} \frac{1}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_{21}) + (1 \mp \xi_1)^2)^{\frac{1}{2}}} \\ &= \int_0^1 d\rho_2 \int_0^{2\pi} d\theta_2 \frac{\rho_2(1 - 2\cos^2 \theta_2)}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_2) + (1 \mp \xi_1)^2)^{\frac{3}{2}}} \\ &+ \frac{\cos \theta_2}{\sigma^2} \frac{\sigma^2 + (1 \mp \xi_1)^2}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_2) + (1 \mp \xi_1)^2)^{\frac{3}{2}}} \\ &- \frac{\rho_2 \sin^2 \theta_2}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_2) + (1 \mp \xi_1)^2)^{\frac{3}{2}}} \end{aligned} \quad (\text{A.35})$$

$$\begin{aligned} &= \int_0^1 d\rho_2 \int_0^{2\pi} d\theta_2 \frac{\rho_2 - 2\rho_2 \cos^2 \theta_2 - \rho_2 \sin^2 \theta_2}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_2) + (1 \mp \xi_1)^2)^{\frac{3}{2}}} \\ &+ \frac{\cos \theta_2}{\sigma^2} \frac{\sigma^2 + (1 \mp \xi_1)^2}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_2) + (1 \mp \xi_1)^2)^{\frac{3}{2}}} \end{aligned} \quad (\text{A.36})$$

$$\begin{aligned} &= \int_0^1 d\rho_2 \int_0^{2\pi} d\theta_2 \frac{-\rho_2 \cos^2 \theta_2}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_2) + (1 \mp \xi_1)^2)^{\frac{3}{2}}} \\ &+ \frac{\cos \theta_2}{\sigma^2} \frac{\sigma^2 + (1 \mp \xi_1)^2}{(\sigma^2(\rho_2^2 + 1 - 2\rho_2 \cos \theta_2) + (1 \mp \xi_1)^2)^{\frac{3}{2}}}. \end{aligned} \quad (\text{A.37})$$

Let $x = \sigma(\rho_2 - \cos \theta_2)$ so $x^2 = \sigma^2(\rho_2^2 - 2\rho_2 \cos \theta_2 + 1) - \sigma^2(1 - \cos^2 \theta_2)$ and let $a^2 = \sigma^2(1 - \cos^2 \theta) + (1 \mp \xi_1)^2$, $\rho_2 = \frac{x}{\sigma} + \cos \theta_2$ and $d\rho_2 = \frac{dx}{\sigma}$. Equation A.37 becomes:

$$\begin{aligned} & \int_0^{2\pi} d\theta_2 \int_{\sigma \cos \theta_2}^{\sigma(1 - \cos \theta_2)} \frac{dx}{\sigma} \left[\frac{\cos \theta_2 (\sigma^2 + (1 \mp \xi_1)^2)}{\sigma^2 (x^2 + a^2)^{\frac{3}{2}}} - \frac{(\frac{x}{\sigma} + \cos \theta_2) \cos^2 \theta_2}{(x^2 + a^2)^{\frac{3}{2}}} \right] \\ &= \frac{1}{\sigma} \int_0^{2\pi} d\theta_2 \int_{\sigma \cos \theta_2}^{\sigma(1 - \cos \theta_2)} dx \left[\frac{\cos \theta_2 (\sigma^2(1 - \cos^2 \theta_2) + (1 \mp \xi_1)^2)}{\sigma^2 (x^2 + a^2)^{\frac{3}{2}}} - \frac{x \cos^2 \theta_2}{\sigma (x^2 + a^2)^{\frac{3}{2}}} \right] \end{aligned} \quad (\text{A.38})$$

$$= \frac{1}{\sigma} \int_0^{2\pi} d\theta_2 \int_{\sigma \cos \theta_2}^{\sigma(1 - \cos \theta_2)} dx \left[\frac{\cos \theta_2}{\sigma^2} \frac{a^2}{(x^2 + a^2)^{\frac{3}{2}}} - \frac{x \cos^2 \theta_2}{\sigma (x^2 + a^2)^{\frac{3}{2}}} \right]. \quad (\text{A.39})$$

Since $\int \frac{a^2 dx}{[x^2 + a^2]^{\frac{3}{2}}} = \frac{x}{[x^2 + a^2]^{\frac{1}{2}}}$ and $\int \frac{x dx}{[x^2 + a^2]^{\frac{3}{2}}} = \frac{1}{[x^2 + a^2]^{\frac{1}{2}}}$, Equation A.39 becomes:

$$\frac{1}{\sigma} \int_0^{2\pi} d\theta_2 \left[\frac{\cos \theta_2}{\sigma^2} \frac{x}{[x^2 + a^2]^{\frac{1}{2}}} \right]_{-\sigma \cos \theta_2}^{\sigma(1 - \cos \theta_2)} + \left[\frac{\cos^2 \theta_2}{\sigma} \frac{1}{[x^2 + a^2]^{\frac{1}{2}}} \right]_{-\sigma \cos \theta_2}^{\sigma(1 - \cos \theta_2)}. \quad (\text{A.40})$$

When $x = \sigma(1 - \cos \theta_2)$ then:

$$x^2 + a^2 = (\sigma(1 - \cos \theta_2))^2 + \sigma^2(1 - \cos^2 \theta_2) + (1 \mp \xi_1)^2 \quad (\text{A.41})$$

$$= \sigma^2(1 - 2\cos \theta_2 + \cos^2 \theta_2) + \sigma^2(1 - \cos^2 \theta_2) + (1 \mp \xi_1)^2 \quad (\text{A.42})$$

$$= 2\sigma(1 - \cos \theta_2) + (1 \mp \xi_1)^2. \quad (\text{A.43})$$

When $x = -\sigma \cos \theta_2$ then:

$$x^2 + a^2 = \sigma^2 \cos^2 \theta_2 + \sigma^2(1 - \cos^2 \theta_2) + (1 \mp \xi_1)^2 \quad (\text{A.44})$$

$$= \sigma^2 + (1 \mp \xi_1)^2. \quad (\text{A.45})$$

Using Equation A.43 and Equation A.45, Equation A.40 becomes:

$$\begin{aligned}
& \frac{1}{\sigma} \int_0^{2\pi} d\theta_2 \left[\frac{\cos \theta_2 \sigma (1 - \cos \theta_2)}{\sigma^2 [2\sigma^2 (1 - \cos \theta_2) + (1 \mp \xi_1)^2]^{\frac{1}{2}}} + \frac{\cos \theta_2}{\sigma^2} \frac{\sigma \cos \theta_2}{[\sigma^2 + (1 \mp \xi_1)^2]^{\frac{1}{2}}} \right. \\
& + \left. \frac{\cos^2 \theta_2}{\sigma} \frac{1}{[2\sigma^2 (1 - \cos \theta_2) + (1 \mp \xi_1)^2]^{\frac{1}{2}}} - \frac{\cos^2 \theta_2}{\sigma} \frac{1}{[\sigma^2 + (1 \mp \xi_1)^2]^{\frac{1}{2}}} \right] \\
& = \frac{1}{\sigma} \int_0^{2\pi} d\theta_2 \left[\frac{\cos \theta_2}{\sigma^2 [2\sigma^2 (1 - \cos \theta_2) + (1 \mp \xi_1)^2]^{\frac{1}{2}}} \right]. \tag{A.46}
\end{aligned}$$

Substitute the above equation into Equation A.26, to obtain:

$$\begin{aligned}
A_{\text{pn}} &= \frac{\sigma^2}{8\pi^2} \int_0^{2\pi} d\theta_1 \int_{-1}^1 d\xi_1 \frac{(1 \mp \xi_1)}{1 + \alpha^2 + \frac{\alpha^2}{\sigma^2} \xi_1^2 - 2\alpha \sin \theta_1} \\
& \quad \int_0^{2\pi} d\theta_2 \frac{\cos \theta_2}{\sigma^2 [2\sigma^2 (1 - \cos \theta_2) + (1 \mp \xi_1)^2]^{\frac{1}{2}}}. \tag{A.47}
\end{aligned}$$

Consider the integral over θ_1 .

$$\int_0^{2\pi} d\theta_1 \frac{1}{1 + \alpha^2 + \frac{\alpha^2}{\sigma^2} \xi_1^2 - 2\alpha \sin \theta_1}. \tag{A.48}$$

Let $a = 1 + \alpha^2 + \frac{\alpha^2}{\sigma^2} \xi_1^2$ and $b = 2\alpha$, and using $\int_0^{2\pi} d\theta \frac{1}{a + b \sin \theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}$ the integral over θ_1 becomes:

$$\int_0^{2\pi} d\theta_1 \frac{1}{1 + \alpha^2 + \frac{\alpha^2}{\sigma^2} \xi_1^2 - 2\alpha \sin \theta_1} = \frac{2\pi}{\sqrt{(1 + \alpha^2 + \frac{\alpha^2}{\sigma^2} \xi_1^2)^2 - 4\alpha^2}}. \tag{A.49}$$

The equation for A_{pn} becomes:

$$\begin{aligned}
A_{\text{pn}} &= \frac{\sigma^2}{8\pi^2} \int_{-1}^1 d\xi_1 \frac{2\pi(1 \mp \xi_1)}{[(1 + \alpha^2 + \frac{\alpha^2}{\sigma^2} \xi_1^2)^2 - 4\alpha^2]^{\frac{1}{2}}} \\
& \quad \int_0^{2\pi} d\theta_2 \frac{\cos \theta_2}{\sigma^2 [2\sigma^2 (1 - \cos \theta_2) + (1 \mp \xi_1)^2]^{\frac{1}{2}}} \tag{A.50}
\end{aligned}$$

$$\begin{aligned}
A_{\text{pn}} = & \frac{1}{4\pi} \int_{-1}^1 d\xi_1 \frac{1}{[(1 + \alpha^2 + \frac{\alpha^2}{\sigma^2} \xi_1^2)^2 - 4\alpha^2]^{\frac{1}{2}}} \\
& \int_0^{2\pi} d\theta_2 \frac{\cos \theta_2 (1 - \xi_1)}{[2\sigma^2(1 - \cos \theta_2) + (1 - \xi_1)^2]^{\frac{1}{2}}} \\
& + \frac{1}{4\pi} \int_{-1}^1 d\xi_1 \frac{1}{[(1 + \alpha^2 + \frac{\alpha^2}{\sigma^2} \xi_1^2)^2 - 4\alpha^2]^{\frac{1}{2}}} \\
& \int_0^{2\pi} d\theta_2 \frac{\cos \theta_2 (1 + \xi_1)}{[2\sigma^2(1 - \cos \theta_2) + (1 + \xi_1)^2]^{\frac{1}{2}}}. \quad (\text{A.51})
\end{aligned}$$

In the first term of Equation A.51, change variables so that $\xi_1 = -\xi_1$ and $d\xi_1 = -d\xi_1$.

Using the fact that $\int_{-1}^1 = -\int_1^{-1}$ the equation for A_{pn} can be written as:

$$\begin{aligned}
A_{\text{pn}} = & \frac{-1}{4\pi} \int_1^{-1} d\xi_1 \frac{1}{[1 + \alpha^2 + \frac{\alpha^2}{\sigma^2} \xi_1^2)^2 - 4\alpha^2]^{\frac{1}{2}}} \\
& \int_0^{2\pi} d\theta_2 \frac{\cos \theta_2 (1 + \xi_1)}{[2\sigma^2(1 - \cos \theta_2) + (1 + \xi_1)^2]^{\frac{1}{2}}} \\
& + \frac{1}{4\pi} \int_{-1}^1 d\xi_1 \frac{1}{[(1 + \alpha^2 + \frac{\alpha^2}{\sigma^2} \xi_1^2)^2 - 4\alpha^2]^{\frac{1}{2}}} \\
& \int_0^{2\pi} d\theta_2 \frac{\cos \theta_2 (1 + \xi_1)}{[2\sigma^2(1 - \cos \theta_2) + (1 + \xi_1)^2]^{\frac{1}{2}}} \quad (\text{A.52})
\end{aligned}$$

$$\begin{aligned}
= & \frac{2}{4\pi} \int_{-1}^1 d\xi_1 \frac{1}{[(1 + \alpha^2 + \frac{\alpha^2}{\sigma^2} \xi_1^2)^2 - 4\alpha^2]^{\frac{1}{2}}} \\
& \int_0^{2\pi} d\theta_2 \frac{\cos \theta_2 (1 + \xi_1)}{[2\sigma^2(1 - \cos \theta_2) + (1 + \xi_1)^2]^{\frac{1}{2}}} \quad (\text{A.53})
\end{aligned}$$

$$= \frac{1}{2\pi} \int_{-1}^1 d\xi_1 G_1(\xi_1, \alpha, \sigma) \int_0^{2\pi} d\theta_2 B_2(\xi_1, \theta_2, \sigma), \quad (\text{A.54})$$

where G_1 and B_2 are defined as follows:

$$G_1(\xi_1, \alpha, \sigma) = \frac{1}{[(1 + \alpha^2 + \frac{\alpha^2}{\sigma^2} \xi_1^2)^2 - 4\alpha^2]^{\frac{1}{2}}}, \quad (\text{A.55})$$

$$B_2(\xi_1, \theta_2, \sigma) = \frac{\cos \theta_2 (1 + \xi_1)}{[2\sigma^2(1 - \cos \theta_2) + (1 + \xi_1)^2]^{\frac{1}{2}}}. \quad (\text{A.56})$$

Equation A.54 is the Kondo and Randolph [8] equation for the point source non-uniformity correction factor in the case where the source is to the side of the ionization chamber and the first surface element is on the side of the chamber and the second surface element is on the end of the chamber. The derivations of the other cases are similar.

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