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**ACOUSTOOPTIC FOURIER TRANSFORMING CONFIGURATION
FOR SPECTRUM ANALYSIS AND DETERMINATION OF DIRECTION-OF-ARRIVAL**

by

© Jim Pak-Yee Lee, M.A.Sc.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Doctor of Philosophy

Department of Electronics

Carleton University

Ottawa, Ontario

29 August, 1986

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ABSTRACT

The Fourier transforming configuration employing Bragg cells as spatial light modulators, is investigated for wideband radar receiver applications, in which the primary signal parameters such as frequency spectrum, pulse width and direction-of-arrival are of interest. The one-dimensional acoustooptic spectrum analyzer using a single-channel Bragg cell can easily be extended to a two-dimensional multi-channel configuration to provide simultaneous frequency spectrum analysis and direction-of-arrival determination.

The temporal and spatial characteristics of the acoustooptic Fourier transforming configuration are analyzed for pulsed signals. In frequency spectrum analysis, two operating modes of the photodetector array are investigated. In the real-time mode of operation, two simple characteristics, namely signal duration stretching and the existence of a maximum flat region, are identified from the outputs of the photodetector array and can be utilized in the determination of the pulse width. In the time-integrating mode of operation, a good estimate of the frequency spectrum can be obtained. However the spectrum is distorted due to truncation and amplitude weighting by the Bragg cell aperture and the optical illumination profile. The distortion is characterized by the effect of broadening the main lobe and smoothing out its sidelobes.

A two-dimensional Fourier transforming configuration employing a multi-channel Bragg cell, optimized in a linear aperiodic array configuration is given to provide simultaneous frequency spectrum analysis and determination of direction-of-arrival. The effects of the amplitude window weighting function, scaling factor, error sources and deviations of the Bragg cell diffraction pattern from the ideal traveling wave modulator model, on the processor are investigated. A simple synthesis procedure is also given on designing a 4-element aperiodic array with equal weightings.

A modified multi-channel Bragg cell with a phased array structure is proposed to provide an improvement of about five times on the output intensity of the processor. Finally a novel simple method of illuminating two adjacent channels at a time and then comparing the resultant diffraction fringe patterns, is developed for the phase-tracking measurement of a multi-channel Bragg cell.

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TABLE OF CONTENTS

	<u>PAGE</u>
ACCEPTANCE SHEET	ii
ABSTRACT	iii
ACKNOWLEDGEMENTS	v
TABLE OF CONTENTS	vi
LIST OF TABLES	x
LIST OF FIGURES	xi
LIST OF PRINCIPAL SYMBOLS AND ABBREVIATIONS	xv
 CHAPTER 1. INTRODUCTION	 1
1.1 INTRODUCTION	1
1.2 BACKGROUND AND OBJECTIVE	6
1.3 THESIS ORGANIZATION	12
 CHAPTER 2. ACOUSTOOPTIC DIFFRACTION	 14
2.1 INTRODUCTION	14
2.2 THEORY OF ACOUSTOOPTIC DIFFRACTION	15
2.2.1 COUPLED MODE FORMULATION	17
2.2.2 ISOTROPIC DIFFRACTION	21
2.2.2.1 Raman-Nath Region ($Q \ll 1$)	24
2.2.2.2 Bragg Region ($Q \gg 1$)	24
2.2.3 ANISOTROPIC DIFFRACTION	34
2.2.3.1 Bragg Region ($Q \gg 1$)	35
2.3 SUMMARY AND CONCLUSIONS	39

	<u>PAGE</u>
CHAPTER 3. ACOUSTOOPTIC FOURIER TRANSFORMING CONFIGURATION	40
3.1 INTRODUCTION	40
3.2 FOURIER TRANSFORMING CONFIGURATION	41
3.3 USE OF BRAGG CELLS AS SPATIAL LIGHT MODULATORS	43
3.4 BRAGG CELL DIFFRACTION PATTERNS	49
3.5 SUMMARY AND CONCLUSIONS	60
CHAPTER 4. ACOUSTOOPTIC SPECTRUM ANALYZER: TEMPORAL RESPONSE AND DETECTION OF PULSED SIGNALS	62
4.1 INTRODUCTION	62
4.2 SPATIAL AND TEMPORAL DISTRIBUTIONS	63
4.3 DETECTION SCHEME	77
4.4 1/D BANDWIDTH DETECTION	80
4.5 PULSE WIDTH DETERMINATION	88
4.6 SUMMARY AND CONCLUSIONS	92
CHAPTER 5. ACOUSTOOPTIC SPECTRUM ANALYZER: DETECTION USING A TIME-INTEGRATING PHOTODETECTOR ARRAY	95
5.1 INTRODUCTION	95
5.2 PULSE-MODULATED cw AND LINEAR FM SIGNALS	98
5.3 EXPERIMENTAL ARRANGEMENT	101
5.4 COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS FOR PULSE-MODULATED cw SIGNALS	103
5.5 COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS FOR A 5- μ SEC PULSE-MODULATED LINEAR FM SIGNAL	112

	<u>PAGE</u>
5.6 EFFECTS OF THE AMPLITUDE WINDOW WEIGHTING FUNCTION ON THE TIME-INTEGRATED WAVEFORM OF A SIGNAL	112
5.7 SUMMARY AND CONCLUSIONS	120
CHAPTER 6. INTERFEROMETRIC ACOUSTOOPTIC SIGNAL PROCESSOR FOR SIMULTANEOUS DIRECTION FINDING AND SPECTRUM ANALYSIS.	122
6.1 INTRODUCTION	122
6.2 DIRECTION FINDING AND SPECTRUM ANALYSIS EMPLOYING A LINEAR ARRAY	123
6.3 EFFECTS OF AMPLITUDE WINDOW WEIGHTING FUNCTION $w(x_0, y_0)$ ON OUTPUT WAVEFORM DISTRIBUTIONS	128
6.4 SCALING FACTOR (S.F.)	130
6.5 ERROR TOLERANCE ANALYSIS	132
6.6 LINEAR PERIODIC ARRAY VERSUS APERIODIC ARRAY	136
6.7 COMPARISON BETWEEN MICROWAVE AND OPTICAL ANTENNA PHASED ARRAY FAR-FIELD PATTERNS	138
6.8 A SPECIFIC EXAMPLE	139
6.9 SUMMARY AND CONCLUSIONS	144
CHAPTER 7. EFFECTS OF BRAGG CELL DIFFRACTION CURVATURES ON 2-D ACOUSTOOPTIC SIGNAL PROCESSOR	147
7.1 INTRODUCTION	147
7.2 MULTI-CHANNEL BRAGG CELL DIFFRACTION PATTERNS	148
7.3 SAMPLING ERROR VERSUS DEVIATION IN THE ANGULAR SPATIAL FREQUENCY γ	150

	<u>PAGE</u>
7.4 RELATIONSHIP AMONG THE ANISOTROPIC FACTOR (α), SCALING FACTOR (S.F.) AND THE ANGULAR SPATIAL FREQUENCY (γ)	159
7.5 SUMMARY AND CONCLUSIONS	163
CHAPTER 8. IMPROVEMENT ³ ON THE INTENSITY OF THE DIFFRACTION PATTERN USING A MODIFIED MULTI-CHANNEL BRAGG CELL ...	165
8.1 INTRODUCTION	165
8.2 MODIFIED MULTI-CHANNEL BRAGG CELL CONFIGURATION	167
8.3 GAUSSIAN LASER ILLUMINATION PROFILE	177
8.4 DOA MAIN LOBE INTENSITY VERSUS GAUSSIAN PROFILE PARAMETER T_y	181
8.5 SUMMARY AND CONCLUSIONS	189
CHAPTER 9. PHASE-TRACKING MEASUREMENT OF A MULTI-CHANNEL BRAGG CELL	190
9.1 INTRODUCTION	190
9.2 DIFFRACTION PATTERN OF TWO CHANNELS	191
9.3 RELATIVE PHASE MEASUREMENT OF A MULTI-CHANNEL BRAGG CELL	194
9.4 EXPERIMENTAL EVALUATION OF THE RELATIVE PHASE DISTRIBUTION OF A 4-CHANNEL BRAGG CELL	198
9.5 PHASE-TRACKING MEASUREMENT	208
9.6 SUMMARY AND CONCLUSIONS	212
CHAPTER 10. CONCLUSION AND RECOMMENDATIONS	214
10.1 SUMMARY	214
10.2 FUTURE RESEARCH WORK	222
REFERENCES	226

LIST OF TABLES

<u>TABLE</u>	<u>DESCRIPTION</u>	<u>PAGE</u>
I	Comparison between theoretical and experimental integrated intensity distributions for a cw signal, and a pulsed signal with the width equal to the aperture transit time	108
II	Comparison between theoretical and experimental integrated intensity distributions for a 1- μ sec and 2- μ sec pulse-modulated cw signals	113
III	Comparison between theoretical and experimental integrated intensity distribution for the linear FM signal	116
IV	An optimum two-dimensional plot showing the peak sidelobe to main lobe values for an array of 10.5 wavelengths long	142

LIST OF FIGURES

<u>FIGURE</u>	<u>DESCRIPTION</u>	<u>PAGE</u>
1	Interferometric acoustooptic signal processor	3
2	Geometry of acoustooptic interaction	18
3	Wave vector construction for isotropic diffraction at the Bragg incident angle	27
4	Wave vector construction for birefringent diffractions (a) General case of birefringent diffraction (b) 90° birefringent phase matched diffraction	36
5	Fourier transforming configuration	42
6	Phase vector $k_g(\phi)$ and Poynting vector $k_e(\chi)$ plotted for $s \rightarrow 0$	51
7	Fourier transforming configuration for displaying diffraction patterns	55
8	Singularity curves in the Fourier plane for anisotropic Bragg cells	58
9	Normalized plot of $ \text{sinc}[(uD - j\alpha\tau/(2\pi)] S) ^2$ versus uD	71
10	Plot of $[1 - \exp(-\alpha\tau S)]/(\alpha\tau)^2$ versus $\alpha\tau$	72
11	Relative light intensity distributions at five different time intervals	79
12	1/D bandwidth detection of a pulsed signal vs photodetector location $[n/(2D)]$ from the signal centroid	83
13	Experimental 1/D bandwidth detection of a pulsed signal vs photodetector location $[n/(2D)]$ from the signal centroid: (a) 1 and 2 (b) 3, 4 and 5	84

14	1/D bandwidth detection of equipower pulsed signals as a function of the weighting parameters: ($PW/\tau = 1.0$ and 0.5)	87
15	Schematic diagram of acoustooptic spectrum analyzer ..	99
16	Orientation of Bragg cell diffraction singularity curves with respect to linear photodetector array	102
17	Light intensity distribution in the frequency plane for a 5- μ sec pulse-modulated cw at two different instances of time	104
18	Integrated light intensity distribution in the frequency plane for both cw and a 5- μ sec pulse-modulated cw	106
19	Integrated light intensity distribution in the frequency plane detected by photodetector array	107
20	Integrated light intensity distribution in the frequency plane for both cw and a 1- μ sec pulse-modulated cw	109
21	Integrated light intensity distribution in the frequency plane for both cw and a 2- μ sec pulse-modulated cw	110
22	Integrated light intensity distribution in the frequency plane detected by photodetector array (a) cw and a 1- μ sec pulse-modulated cw (b) cw and a 2- μ sec pulse-modulated cw	111
23	Integrated light intensity distribution in the frequency plane for a 5- μ sec pulse-modulated linear FM	114
24	Integrated light intensity distribution in the frequency plane of a 5- μ sec pulse-modulated linear FM detected by the photodetector array	115
25	Main-lobe broadening of a time-integrated pulse vs the normalized pulse width as a function of amplitude weighting	118
26	Configuration of a linear aperiodic array	124
27	Multi-channel Bragg cell configuration for a linear aperiodic antenna array	126

	<u>PAGE</u>
28	Normalized intensity patterns of the optimum array ... 143
29	Simulated intensity pattern of the optimum array 145
30	Orientation of Bragg cell diffraction singularity curves with respect to photodetector area array 151
31	Photodetector location with respect to the sinc^2 function distribution to give maximum and minimum outputs 153
32	Maximum sampling error versus number of photodetectors per resolution width 155
33	Percent change in output level versus shift in horizontal position (u) with no dead area 157
34	Percent change in output level versus shift in horizontal position (u) with $A_2 = 0.4$ 158
35	Diffraction patterns for a fully illuminated lithium niobate Bragg cell driven at 2.3, 2.5 and 2.7 GHz 161-
36	Square of the optimum total weighting function (w_1) and $\text{sinc} [0.25 \sin(\theta_0)]$ 173
37	Square of the total weighting function (w_1) versus $\sin(\theta_0)$ for $P_2 = -.36, -.38$ and $-.40$ 175
38	Schematic diagram of the modified 4-channel Bragg cell configuration showing the transducer locations .. 176
39	Total interacting optical power and its maximum DOA pattern intensity versus Gaussian profile parameter T_y (a) Total optical power interacting with acoustic beams (b) Relative maximum intensity of DOA pattern 183
40	Normalized DOA patterns of the 4-channel Bragg cell for a signal received on boresight and illuminated with (a) a rectangular uniform weighting (b) the optimum Gaussian profile 186

41	Relative DOA patterns for a signal received on boresight and illuminated with the optimum Gaussian profile (a) The 4-channel Bragg cell (b) The modified 4-channel Bragg cell using a phased array structure	187
42	Normalized DOA patterns for a signal received on boresight and illuminated with the optimum Gaussian profile (a) The 4-channel Bragg cell (b) The modified 4-channel Bragg cell using a phased array structure	188
43	Diffraction patterns along DOA axis by illuminating two adjacent slits at a time of a 4-slit photomask ...	200
44	Diffraction pattern along DOA axis by illuminating a 4-slit photomask	201
45	Diffraction pattern of an ideal 4-channel Bragg cell along the DOA axis	202
46	Diffraction patterns along DOA axis by illuminating two adjacent channels at a time of a 4-channel Bragg cell driven at 2.5 GHz	203
47	Diffraction pattern along DOA axis of a 4-channel Bragg cell	204
48	Diffraction pattern of a 4-channel Bragg cell along DOA axis at 2.5 GHz, using measured amplitude and phase values in each channel	206
49	Diffraction pattern of a 4-channel Bragg cell along DOA axis with equal phase values and with normalized amplitudes of ($C_1/C_4 = .81$, $C_2/C_4 = .86$, $C_3/C_4 = .92$ and $C_4/C_4 = 1.0$)	207
50	Diffraction patterns along DOA axis by illuminating two adjacent channels at a time of a 4-channel Bragg cell driven at 2.45 GHz	211

LIST OF PRINCIPAL SYMBOLS AND ABBREVIATIONS

a_n	Amplitude of channel n
AOSA	Acoustooptic spectrum analyzer
c	Velocity of light
cw	Continuous wave
D	Aperture width of Bragg cell
d_n	Location of antenna element n
DOA	Direction-of-arrival
E	Electric field
erf	Error function
F	Focal length
f_{RF}	RF frequency
f_s	Acoustic frequency
H	Height of acoustic transducer
I	Light intensity
IF	Intermediate frequency
k	Optical phase constant
k_B^2	Acoustooptic diffraction efficiency of Bragg cell
k_s	Acoustic phase constant
L	Length of acoustic transducer

PW	Pulse width
$\text{rect}(x)$	$\begin{cases} 1, & x \leq 1/2 \\ 0, & \text{otherwise} \end{cases}$
RF	Radio frequency
s	Anisotropy parameter of crystal
S	Normalized effective pulse width
S.F.	Scaling factor
T_x, T_y	Gaussian laser beam profile parameters in the x_0 and y_0 coordinates respectively
(u, v)	Spatial frequency components corresponding to x_1 and y_1 axes respectively
$U_1(x_1, y_1)$	Amplitude distribution in Fourier plane
v_s	Acoustic phase velocity
w	Height of acoustic beam
$w(x_0, y_0)$	Amplitude window weighting function
(x_0, y_0)	Coordinates of Bragg cell
(x_1, y_1)	Fourier plane coordinates

α	Acoustic loss coefficient
β_n	Phase delay of channel n
(γ, β)	Angular spatial frequency components corresponding to the x_1 and y_1 axes respectively
θ_a	Angle of arrival
θ_B	Bragg angle
θ_d	Diffracted angle
λ	Optical wavelength
Λ	Acoustic wavelength
λ_R	RF wavelength
τ	Transit time across Bragg cell aperture
τ'	Time delay
ω	Optical angular frequency
ω_s	Acoustic angular frequency
l_n	Location of acoustic transducer n

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Optical signal processing is a fast-growing technology which is capable of handling large amounts of information in parallel and in two dimensions. Operations such as Fourier transformation, correlation or convolution are performed at a rate limited only by the characteristics of the input and output devices. Acoustooptic technology has rapidly emerged as a major real-time optical signal processing technique with the advent of new components. Significant improvements in the performance of processing systems were gained in the last decade when devices such as acoustooptic cells and photodetector arrays were developed to the level necessary to implement practical systems.

Many architectural optical processors have been developed and they can in general be classified into two major divisions; time and space integrating systems [1]. Among the space integrating systems, the Fourier transforming configuration is the most important and well known [2]. In this thesis, the characteristics of the Fourier transforming configuration are investigated for processing radar signals over a wide frequency bandwidth.

One of the key potential applications of acoustooptic technology is in the development of wideband receiving systems where it is desirable to have real-time determination of such signal parameters as frequency spectrum and direction-of-arrival (DOA).

The most basic and well-known one-dimensional acoustooptic Fourier transforming configuration is the spectrum analyzer. This one-dimensional configuration can be extended to a two-dimensional one where the spectrum of the signal is given on one dimension and its direction-of-arrival is on the other [3]. A block diagram of this interferometric acoustooptic configuration is shown in Fig. 1., where an interferometer is used to determine the direction-of-arrival of a received signal. The signal received by each antenna element is down-converted to an intermediate frequency (IF) signal. The signals are then applied to a multi-channel Bragg cell where the ratios of the transducer spacings are made the same as those of the interferometer. Through the interaction of acoustooptic diffraction, the signals are coherently processed in parallel to produce a two-dimensional Fourier transform in the back focal plane of the lens. The intensity distribution of this Fourier transform gives the power spectrum of the signal on one axis and its direction-of-arrival on the other. This is indeed a very powerful signal sorting technique in which the incoming signals are spatially separated in frequency and direction-of-arrival. The direction-of-arrival and frequency resolutions are obtained in this configuration, by combining the inherent high angular resolution of an interferometer with the frequency spectral analysis capability of a Bragg cell receiver.

Fig. 1. Interferometric acoustooptic signal processor.

The aspect of frequency spectrum analysis is presented in the first part of the thesis. This can either be the operation of the Fourier transforming configuration employing a single-channel Bragg cell as an acoustooptic spectrum analyzer (AOSA), or in the case of the multi-channel Bragg cell configuration, it is the operation along the frequency dimension. The AOSA is a real-time processor in which the instantaneous Fourier transform of the weighted input signal in the Bragg cell aperture is obtained. The weighting function is composed of a number of factors including the aperture width, acoustic attenuation and optical amplitude profile. In the case of a continuous wave (cw) signal, the spatial frequency distribution is completely determined by the Fourier transform of the weighting function [4]. However in the case of a non-cw signal such as a pulse-modulated cw signal, the spatial distribution is more complex and is a function of where the signal is located in the aperture. Furthermore, the output data from the AOSA is also a function of the characteristics of the photodetector array used for detecting the spatial distribution. The purpose of the first part of this thesis is to extend the analysis on the AOSA to pulse-modulated cw signals. A pulse-modulated cw signal is often referred to as a pulsed signal and the two are used interchangeably throughout this thesis. Pulsed signals are of great interest because the great majority of radar signals are of this type. A good understanding of the spatial and temporal responses of the AOSA is of great importance. It is needed to interpret intelligently the complicated output data generated by such a processor for the purpose of estimating signal parameters such as

frequency spectrum, pulse width, and pulse repetition frequency. Moreover, once the characteristics of the AOSA are understood, and depending on the requirement, an optimum configuration can be designed in terms of its component parameters.

The basic 2-D configuration using a multi-channel Bragg cell was first proposed by Lambert et al in processing received signals from a phased array radar [3]. The configuration considered was a large linear periodic array with less than half a wavelength in element-to-element spacings. The bandwidth considered, was wide enough to accommodate the pulsed signal and the anticipated spread in Doppler frequency shifts.

For wideband operation, a microwave receiver is usually required to cover about an octave and more in frequency, and over a wide angular range with good DOA accuracy. As a result, the configuration using a linear periodic array is impracticable due to antenna size constraint, problems associated with grating lobes, the complexity and cost of having a large number of channels. In this thesis, a linear aperiodic design with four to five elements is proposed.

The second part of this thesis deals with the DOA determination of the 2-D Fourier transforming configuration employing a multi-channel Bragg cell. Deviations of the Bragg cell diffraction pattern from the ideal traveling wave modulator model and its effects on the DOA and frequency outputs of the processor are also investigated. A novel multi-channel Bragg cell configuration is proposed to improve the diffracted intensity of the DOA pattern and finally a simple novel

technique is developed for measuring the phase-tracking characteristics of a multi-channel Bragg cell.

In Section 1.2, the background and objective of the thesis are presented in greater details, while the thesis organization is presented in Section 1.3.

1.2 BACKGROUND AND OBJECTIVE

With the advent of new components such as Bragg cells and photodetector arrays, acoustooptic technology has rapidly emerged as a major optical signal processing technique for wideband receiver applications. The Fourier transforming configuration is the most important and practically the easiest to implement. The Fourier transforming configuration using a single-channel Bragg cell as a spatial light modulator is well known for its inherent capability of wideband spectrum analysis on a real-time basis with many simultaneous signals present.

Direction finding employing the phase interferometer technique can provide very high accuracy in determining the direction-of-arrival of signals. The single-channel Bragg cell configuration can easily be extended to a 2-D Fourier transforming configuration by using a multi-channel Bragg cell with the same number and relative channel spacings as the antenna linear array [3].

The operation of spectrum analysis is addressed in the first part of this thesis. A unique feature of the Fourier transforming configuration is that the Fourier transform is computed over a sliding window of the input signal. At different instances in time, the Fourier transform of a different portion of the weighted signal is performed. The amplitude weighting function takes into account the acoustic attenuation, and the interaction profile between the laser beam and the acoustic beam in the aperture. For cw signals, the shape of the spectrum is constant and completely determined by the Fourier transform of the amplitude weighting function of the aperture. This subject has been addressed and an excellent treatment can be found in the open literature [4]. When the signal is of finite duration such as a pulsed signal, the spectrum is a function of where the signal is located in the aperture and is more complex. In general, there are two general effects on the spectrum, due to the truncation and weighting on the signal, namely spectral spreading and pulse width extension. The purpose of the first part of the thesis is to extend the analysis of the AOSA to input pulsed signals. We are interested in pulsed signals because the great majority of radar signals are of this type. The temporal and spatial responses of the AOSA to pulse-modulated cw signals are investigated in this thesis. This work is carried out for the purpose of estimating the parameters of the signal and it can also be used for the optimization of the system configuration in terms of its component parameters.

A photodetector array is usually used for detecting the spectrum. The output from the AOSA also depends on a number of other factors such

as the number of photodetectors used to cover a given frequency band, the response of the photodetector element, and whether it is operating in the time-integrating or the real-time mode.

In Chapter 4, the spatial and temporal characteristics of the AOSA are investigated for an input pulsed signal as it propagates through the Bragg cell aperture. The detected signal waveform from a photodetector array operating in the real-time mode is analyzed as a function of pulse width, weighting function parameters, and the relative position of the photodetector to the spectral centroid. In this photodetector array configuration, the AOSA is used mainly as a frequency channelizer in which multiple simultaneous signals of different frequencies can be separated spatially. In this case, a coarse frequency resolution is obtained while the time resolution is very good. Methods of determining the pulse width from the detected outputs are also discussed.

Another configuration of the AOSA is to use a time-integrating photodetector array for detection. In this case, one is mainly interested in the total power spectral density of the signal than its instantaneous spectra. The use of a time-integrated photodetector array also helps to simplify and reduce the amount of data generated from such a processor. Another advantage is the increased sensitivity which may be achieved through the non-coherent integration produced by the time-integrating photodetector array. The effect on the spectrum of the signal, due to truncation and weighting by the Bragg cell aperture, is given in Chapter 5. Pulse-modulated cw signals with various pulse

widths are investigated experimentally and then compared with theory.

The second part of the thesis deals with the DOA determination of the Fourier transforming configuration using a multi-channel Bragg cell. The basic configuration as proposed by Lambert et al [3] is modified to be used for wideband applications where the frequency coverage of the order of an octave and more is required. Due to antenna size constraint, the need to keep the number of channels to a small number, and at the same time to cover a quadrant or more in angular range with very good angular accuracy and no grating lobes, a linear aperiodic array design with four to five elements is proposed in this thesis.

The excitation of the elements of a linear antenna array and the resulting far-field pattern form Fourier transform pairs. In addition the Fourier plane and object plane light amplitude distributions for the optical configuration also form Fourier transform pairs. In the Fourier transforming configuration considered in this thesis, the received signals from the antenna elements of the linear array are processed in parallel. From the theorem of reciprocity, the resultant diffraction pattern in the Fourier plane should reproduce closely the far-field pattern of the linear array at an angle which is related to the incident angle.

In Chapter 6, we investigate the 2-D acoustooptic configuration using a multi-channel Bragg cell for DOA determination in detail. The multi-channel Bragg cell is modeled by an ideal traveling wave modulator. Specific topics to be addressed, include the effects of the

amplitude weighting function and the scaling factor on the diffracted waveform, as well as error tolerance analysis on the shape of the diffracted DOA pattern and on the beam-pointing accuracy of the processor. Trade-offs between the periodic and aperiodic configurations in implementing a practical processor along with comparisons between the optical and microwave far-field patterns are discussed. Finally a typical example is given to illustrate a synthesis technique of designing an aperiodic array configuration using four elements of equal weightings.

A 2-D diffraction pattern giving the DOA pattern and frequency spectral distribution of the signal, is produced by the 2-D Fourier transforming configuration. The DOA pattern and the spectral distribution are orthogonal if the Bragg cell can be represented by the model of an ideal traveling wave modulator, in which a self-collimating acoustic beam is produced. In this case, the DOA pattern and frequency spectral distribution are oriented in parallel and match exactly with the columns and rows of a photodetector array respectively. However in a practical case, the DOA pattern and frequency spectral distribution are not orthogonal and thus will result in a mismatch with the columns and rows. The effect of this mismatch on the 2-D signal processor and the minimization of it by a choice of the system and component parameters, are investigated in Chapter 7.

The multi-channel Bragg cell is normally illuminated fully with a collimated laser beam and as a result, a great portion of the incident

light is not interacting with the acoustic beams. In order to utilize the incident light more efficiently, a novel scheme is proposed in which the multi-channel Bragg cell is modified by using a phased array structure. An optimum configuration which can easily be implemented and can provide a substantial improvement on the intensity of the DOA pattern, is given in Chapter 8.

The 2-D acoustooptic signal processor, employing a multi-channel Bragg cell for the determination of DOA of an incoming signal is a phase DF system. Phase tracking among the channels, is the most important specification of the system, which determines the accuracy as well as the shape of the DOA pattern.

The multi-channel Bragg cell is the key component in the 2-D acoustooptic signal processor and is also the most difficult to evaluate. Only a very few multi-channel Bragg cells with a large bandwidth have recently been developed. The inputs to a multi-channel Bragg cell are the IF signals applying to the acoustic transducers, and the outputs are the diffracted optical beams. All the channels have to be phase tracked in the microwave, acoustic and optical domains.

A novel simple method of illuminating two adjacent channels at a time is used, in which the relative phase information is extracted from its interference diffraction pattern. This novel scheme is developed in Chapter 9 in which the phase-tracking characteristics of a multi-channel Bragg cell can be obtained. This technique is verified experimentally using a 4-channel Bragg cell.

1.3 THESIS ORGANIZATION

This thesis is mainly divided into two parts. The first part deals with the frequency spectral analysis of the Fourier transforming configuration. The second part is mainly on the DOA determination of the 2-D Fourier transforming configuration using a multi-channel Bragg cell.

The most important component in an acoustooptic processor is the Bragg cell and thus a good understanding of its characteristics is essential. The basic theory of acoustooptic diffraction is reviewed in Chapter 2, where a coupled mode formulation is slightly modified to analyze both the cases of isotropic and anisotropic diffractions.

The acoustooptic Fourier transforming configuration is the basic architecture investigated in this thesis. The optical Fourier transformation and the use of Bragg cells as spatial light modulators are presented in Chapter 3.

The frequency spectrum analysis capability of the acoustooptic Fourier transforming configuration for a pulse-modulated cw signal is developed in Chapters 4 and 5. Chapter 4 is written on the configuration where a photodetector array operating in the real-time mode is used. In Chapter 5, spectrum analysis of radar signals using a time-integrating photodetector array is given along with some experimental results.

Chapters 6 to 9 are written mainly on the DOA determination of the

acoustooptic Fourier transforming configuration. The direction finding capability of the processor employing a linear aperiodic array design is developed in Chapter 6. The non-ideal traveling wave characteristics of the multi-channel Bragg cell are analyzed and discussed in Chapter 7. The novel multi-channel Bragg cell configuration used to improve the overall efficiency of the processor is presented in Chapter 8. A simple novel technique for measuring the phase-tracking characteristics of a multi-channel Bragg cell is given in Chapter 9.

Final conclusion and recommendations for future research are given in Chapter 10.

CHAPTER 2

ACOUSTOOPTIC DIFFRACTION

2.1 INTRODUCTION

The word "acoustooptic" describes the field of light-sound interaction phenomena. When an acoustic wave propagates in a transparent medium, it induces localized refractive index variations via the elastooptical effect. The acoustic wave acts like a moving phase grating which may diffract portions of an incident light beam into one or more orders. The angle between the various orders is a function of the acoustic frequency (f_s). In addition, the light frequency is Doppler shifted by nf_s , where n denotes the diffraction order. This phenomenon, known as acoustooptic diffraction, has led to various interesting optical devices such as deflectors, modulators, filters, etc.

The basic theory of acoustooptic diffraction is well understood. Comprehensive and informative reviews of the principles and history of acoustooptic diffraction can be found in texts [6],[7] and review papers [8]-[13].

Starting with Maxwell's equations, the theory of acoustooptic diffraction is derived by using the coupled mode formulation. The coupled mode formulation as given by Klein and Cook [14] on isotropic interaction is slightly modified to include also the anisotropic case. Special emphasis is placed on the Bragg region for both isotropic and anisotropic interactions. Performance characteristics of Bragg cells such as bandwidth, incident and diffracted angles, and diffraction efficiency are also discussed.

2.2 THEORY OF ACOUSTOOPTIC DIFFRACTION

Let us consider a general case where an optical field is incident on an acoustooptic medium in which an acoustic wave is propagating. The perturbed medium can be characterized by a time-varying refractive index (n). The medium is assumed to be source free with a scalar permeability (μ) and a tensor permittivity ($\hat{\epsilon}$). The total field inside the medium must satisfy Maxwell's equations

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t} \quad (2.1)$$

$$\nabla \times \bar{E} = - \frac{\partial \bar{B}}{\partial t} \quad (2.2)$$

$$\nabla \cdot \bar{D} = 0 \quad (2.3)$$

$$\nabla \cdot \bar{B} = 0 \quad (2.4)$$

The constitutive relations are

$$\bar{D} = \hat{\epsilon} \bar{E} \quad (2.5)$$

$$\bar{B} = \mu \bar{H} \quad (2.6)$$

The field quantities, E , H , B , D , and ϵ , μ are in general a function of space ($\bar{r}$) and time (t).

Taking the curl of Eq. (2.2) and making use of Eq. (2.6), one obtains

$$\begin{aligned} \nabla \times \nabla \times \bar{E} &= \nabla \times \left(-\frac{\partial \bar{B}}{\partial t} \right) = -\frac{\partial}{\partial t} [\nabla \times (\mu \bar{H})] \\ &= \nabla \times \left[-\bar{H} \frac{\partial \mu}{\partial t} - \mu \frac{\partial \bar{H}}{\partial t} \right] \\ &= \nabla \times \left(-\mu \frac{\partial \bar{H}}{\partial t} \right) \end{aligned} \quad (2.7)$$

where

$$\frac{\partial \bar{H}}{\partial t} \gg \frac{\partial \mu}{\partial t}$$

Using a vector identity and substitution of Eqs. (2.2) and (2.5), gives

$$\nabla (\nabla \cdot \bar{E}) - \nabla^2 \bar{E} = -\mu \frac{\partial^2}{\partial t^2} (\hat{\epsilon} \bar{E})$$

$$\begin{aligned}
\therefore \nabla^2 \bar{E} &= \mu \left[\hat{\epsilon} \frac{\partial^2 \bar{E}}{\partial t^2} + 2 \frac{\partial \bar{E}}{\partial t} \frac{\partial \hat{\epsilon}}{\partial t} + \bar{E} \frac{\partial^2 \hat{\epsilon}}{\partial t^2} \right] \\
&= \mu \hat{\epsilon} \frac{\partial^2 \bar{E}}{\partial t^2} \\
&= \frac{\hat{n}^2}{c^2} \frac{\partial^2 \bar{E}}{\partial t^2}
\end{aligned} \tag{2.8}$$

where

$$\frac{\partial \bar{E}}{\partial t} \gg \frac{\partial \hat{\epsilon}}{\partial t}$$

$\hat{n}$ is a tensor refractive index and c is the velocity of light in free space.

Equation (2.8) is the wave equation which is the starting point for the analysis of the acoustooptic interaction.

2.2.1 COUPLED MODE FORMULATION

We will follow an analysis similar to that given by Klein and Cook [14] on isotropic interaction, but with a slight modification to include also the anisotropic case.

The basic interaction geometry is shown in Fig. 2. The acoustic wave is propagating approximately along the x axis, and the light beam is in the x - z plane with an incident angle θ . It is assumed that both

Fig. 2. Geometry of acoustooptic interaction.

the acoustic and electric fields have no variations in the y direction. The subscript s is used to denote the acoustic wave.

The refractive index in the region of the acoustic wave can be expanded in plane waves as

$$n(x,t) = n_{m0} + \sum_{i=1}^{\infty} n_i \sin[i(\omega_s t - k_s x) + \delta_i] \quad (2.9)$$

ω_s and k_s are the angular frequency and the phase constant respectively of the acoustic wave, n_i is the i th Fourier component of the perturbed refractive index distribution, and δ_i is its relative phase, n_{m0} is the refractive index of the m th optical wave.

The electric intensity E is periodic in both space and time with the acoustic field. It can also be expanded in plane waves

$$E(x,z,t) = \exp(j \omega t + \phi_0) \sum_{m=-\infty}^{\infty} \psi_m(z) \exp[j(m \omega_s t - \bar{k}_m \cdot \bar{r})] \quad (2.10)$$

where $\bar{k}_m \cdot \bar{r} = n_0 k [z \cos(\theta) + x \sin(\theta)] + m k_s x$

n_0 is the refractive index of the incident optical wave, ω and k are the angular frequency and phase constant of the incident light respectively, ψ_m is the amplitude of the m th diffracted optical wave, and ϕ_0 is the initial phase of the light at $x = t = 0$.

By substituting Eqs. (2.9) and (2.10) into the wave equation (2.8), neglecting second order terms and equating the coefficient of each exponential to zero, one obtains a set of coupled, difference - differential equations which relate the amplitudes in the plane-wave expansion:

$$\begin{aligned} \frac{d\psi_m}{dz} + \frac{k_m^2 n_{m0}}{2n_0 k \cos(\theta)} \sum_{i=1}^{\infty} n_i [\psi_{m-i} \exp(j\delta_i) - \psi_{m+i} \exp(-j\delta_i)] \\ = \frac{jn_0 k}{2 \cos(\theta)} \left\{ \left[1 - \left(\frac{n_{m0}}{n_0} \right)^2 \right] + \left(\frac{mk}{n_0} \right)^2 + \frac{2mk \sin(\theta)}{n_0 k} \right\} \psi_m \end{aligned} \quad (2.11)$$

$$\text{where } k_m = \frac{\omega + m\omega_s}{c} \quad (2.12)$$

In the usual experimental conditions, the acoustic frequency is very small compared with the light frequency and consequently Eq. (2.11) can be approximated by

$$\begin{aligned} \frac{d\psi_m}{dz} + \frac{kn_{m0}}{2n_0 \cos(\theta)} \sum_{i=1}^{\infty} n_i [\psi_{m-i} \exp(j\delta_i) - \psi_{m+i} \exp(-j\delta_i)] \\ = \frac{jn_0 k}{2 \cos(\theta)} \left\{ \left[1 - \left(\frac{n_{m0}}{n_0} \right)^2 \right] + \left(\frac{mk}{n_0} \right)^2 + \frac{2mk \sin(\theta)}{n_0 k} \right\} \psi_m \end{aligned} \quad (2.13)$$

Let us define the coefficient of ψ_m by

$$\Delta k_m = \frac{n k}{2 \cos(\theta)} \left\{ 1 - \left(\frac{n_{m0}}{n_0} \right)^2 \right\} + \left(\frac{mk}{n_0 k} \right)^2 + \frac{2mk \sin(\theta)}{n_0 k} \quad (2.14)$$

Physically Δk_m is the magnitude of the momentum mismatch between the polarization wave and the free waves of the medium, i.e.,

$$\Delta \bar{k}_m = \bar{k} + m \bar{k}_s - \bar{k}_m \quad (2.15)$$

The momentum mismatch will be explained and illustrated further in this chapter.

2.2.2. ISOTROPIC DIFFRACTION

The acoustooptic diffraction is isotropic when either the medium is isotropic or the process does not change the polarization of the light beam. In this case, the refractive indices of the incident and diffracted light are about equal. Letting $n_{m0} = n_0$ in Eq. (2.13), one obtains

$$\begin{aligned} \frac{d\psi}{dz} + \frac{1}{2L} \sum_{i=1}^{\infty} v_i [\psi_{m-i} \exp(j\delta_i) - \psi_{m+i} \exp(-j\delta_i)] \\ = j \frac{mQ}{2L} (m - 2\alpha) \psi_m \end{aligned} \quad (2.16)$$

$$\text{where } v_1 = \frac{kn_1 L}{\cos(\theta)} \quad (2.17)$$

$$Q = \frac{k_s^2 L}{n_0 k \cos(\theta)} \quad (2.18)$$

$$\alpha = -\frac{n_0 k}{k_s} \sin(\theta) \quad (2.19)$$

and the momentum mismatch is now

$$\Delta k_m = \frac{mQ}{2L} (m - 2\alpha) \quad (2.20)$$

where L is the width of the acoustic column in the z direction, v_1 is the normalized index of refraction modulation amplitude corresponding to n_1 , α is a measure of the angle of incidence of the light on the acoustic field, normalized to the angle between diffracted orders, Q is a measure of the angle between diffraction orders, normalized to the diffraction spread angle of the acoustic field.

Equation (2.16) is written in the same form as defined by Klein and Cook. This equation was first given by Raman and Nath in 1936. It is an infinite set of coupled mode equations. The normal modes are the plane waves given in Eq. (2.10), which give rise to the diffraction orders. Modes whose order numbers differ by 1 are coupled by the coupling coefficients v_1 .

The amount of energy transferred between modes depends not only upon the coupling coefficients v_1 , but also upon the degree of synchronization of the modes. The degree of synchronization of two modes is determined by the relative value of the momentum mismatch defined by Eq. (2.20). Modes with the same value are synchronized in space, and energy can be transferred directly between them provided that the appropriate v_1 does not vanish.

For a sinusoidal acoustic wave, Eq. (2.16) can be simplified to

$$\frac{d\psi_m}{dz} + \frac{v_1}{2L} [\psi_{m-1} \exp(j\delta_1) - \psi_{m+1} \exp(-j\delta_1)] = j \frac{mQ}{2L} (m - 2\alpha) \psi_m \quad (2.21)$$

It can be solved by using the boundary conditions at $z = 0$

where

$$\begin{aligned} \psi_0 &= 1, \text{ and} \\ \psi_m &= 0 \quad (m \neq 0) \end{aligned} \quad (2.22)$$

The momentum mismatch for the zeroth order is always zero. An appreciable amount of light can be transferred out of the zeroth order only if either or both of the momentum mismatches are small for $m = \pm 1$. This can be accomplished in two ways:

1. By working at or near normal incidence and having Q small or
2. By working at angles of incidence corresponding to $\alpha = \pm \frac{1}{2}$ with large Q .

2.2.2.1 Raman-Nath Region ($Q \ll 1$)

The first condition of operation is known as the Raman-Nath region in which $Q \ll 1$. The solution can be shown to be [14]

$$\psi_m = \exp\left[-j \frac{mQ\alpha z}{2L}\right] J_m\left[\frac{2v_1}{Q\alpha} \sin\left(\frac{Q\alpha z}{2L}\right)\right] \quad (2.23)$$

where J_m is the ordinary Bessel function. At $z = L$, the intensity of the m th diffraction order is

$$\begin{aligned} I_m &= |\psi_m|^2 \\ &= J_m^2\left[v_1 \frac{\sin(Q\alpha/2)}{Q\alpha/2}\right] \\ &= J_m^2(v_1), \end{aligned} \quad (2.24)$$

at $\alpha = 0$ for normal incidence. In this region of operation, light is not only transferred from the zeroth to the first orders, but also from the first to the second, the second to the third, etc.

2.2.2.2 Bragg Region ($Q \gg 1$)

The second condition of operation is in the Bragg region in which Q is large and the light is transferred only to one order corresponding to the appropriate choice of α . Physically, this is analogous to the

Bragg diffraction of X-rays in which only one strong diffraction order is obtained.

For Bragg region diffraction, only two orders need to be considered. The momentum mismatches for the other orders are appreciable and thus can be neglected. To obtain exact momentum matching between the zeroth order and the first order requires that $\Delta k_x = 0$, i.e., from Eq. (2.20)

$$\frac{2n_o k}{k_s} \sin(\theta) = -1$$

$$\therefore \sin(\theta)_B = -\sin(\theta) = -\frac{k_s}{2n_o k} = -\frac{\lambda}{2n_o \Lambda} \quad (2.25)$$

where θ_B is the Bragg angle and Λ is the acoustic wavelength. Exact momentum matching can occur only at the Bragg incident angle as defined by the above equation.

The wave vector of the optical plane waves defined by Eq. (2.10) is given by

$$\vec{k}_m = n_o k \cos(\theta) \hat{z} + n_o k \sin(\theta) \hat{x} + m k_s \hat{x} \quad (2.26)$$

The direction of the wave vector for the mth optical wave is given by

$$\phi = \tan^{-1} \left[\frac{n_o k \sin(\theta) + m k_s}{n_o k \cos(\theta)} \right] \quad (2.27)$$

At the exact Bragg angle of incidence, we have

$$\theta_i = -\theta_B \quad (2.28)$$

The direction of the first diffraction order ($m = 1$) wave vector is now

$$\theta_d = \phi = \tan^{-1} \left[\frac{-n_o k \sin(\theta_B) + k_s}{n_o k \cos(\theta_B)} \right] = \theta_B \quad (2.29)$$

The angular relationship between the incident light and diffracted light is best illustrated by using a wave vector interaction diagram as shown in Fig. 3. The triangle is isosceles because the angle of incidence is equal to the angle of diffraction.

The angular frequency of the first diffraction order is the sum of the incident light frequency and the acoustic frequency, as given in Eq. (2.10)

$$\omega_1 = \omega + \omega_s \quad (2.30)$$

The interaction of plane waves of light and sound can also be

Fig. 3. Wave vector construction for isotropic diffraction at the Bragg incident angle.

depicted as a collision process between photons and phonons in which the total momentum must be conserved,

Two kinds of collision processes are possible corresponding to the quantum mechanical model of absorption and stimulated emission respectively. In the first one, that of absorption, one phonon is absorbed for every scattered photon generated. The condition for the conservation of momentum is

$$\frac{h}{2\pi} \bar{k}_0 + \frac{h}{2\pi} \bar{k}_s = \frac{h}{2\pi} \bar{k}_1 \quad (2.31)$$

where h is Planck's constant. This corresponds to the Bragg angle diffraction in which $m = 1$.

Since $\omega \gg \omega_s$, $k_0 \approx k_1$ and the conservation of momentum diagram is identical to the wave vector interaction diagram.

In addition to the momentum, energy must also be conserved in the process. The law of conservation of energy is

$$\frac{h}{2\pi} \omega_1 = \frac{h}{2\pi} \omega_s + \frac{h}{2\pi} \omega \quad (2.32)$$

This equation is identical to Eq. (2.30)

In the second process, the incident phonon stimulates the emission of a photon plus another phonon. This emitted phonon has the same

momentum as the incident one. The conditions for the conservation of momentum and energy are

$$\frac{h}{2\pi} \bar{k}_0 + \frac{h}{2\pi} \bar{k}_s = \frac{h}{2\pi} \bar{k}_{-1} + \frac{h}{2\pi} \bar{k}_s + \frac{h}{2\pi} \bar{k}_s$$

$$\bar{k}_0 - \bar{k}_s = \bar{k}_{-1} \quad (2.33)$$

$$\text{and} \quad \omega_{-1} = \omega - \omega_s \quad (2.34)$$

The above two equations specify the other case where the diffraction order is $m = -1$.

Near the Bragg angle of incidence, the momentum mismatch between the zeroth order and the first order is small, while the momentum mismatches for the other orders are appreciable. For $\delta_1 = \pi/2$, the coupled mode equations for the zeroth and first orders are approximately defined by

$$\frac{d\psi_0}{dz} + \frac{j}{2L} v_1 \psi_1 = 0 \quad (2.35)$$

$$\frac{d\psi_1}{dz} + \frac{j}{2L} v_1 \psi_0 = \frac{jQ}{2L} (1 - 2\alpha) \psi_1 \quad (2.36)$$

The two differential equations can be solved by using the boundary conditions given by Eq. (2.22). The solutions at $z = L$, are

$$\psi_1 = \frac{-v_1}{2\alpha} \sin(\alpha) \exp[j(1 - 2\alpha) Q/4] \quad (2.37)$$

$$\psi_0 = \frac{-j}{8\vartheta} [2jQ(1-2\alpha)\sin(\vartheta) - 8\vartheta\cos(\vartheta)] \exp[j(1-2\alpha)Q/4] \quad (2.38)$$

$$\text{where } \vartheta = \{[(1-2\alpha)Q/2]^2 + v_j^2\}^{1/2} \quad (2.39)$$

The light intensity of the diffracted first order is expressed by

$$I_1 = |\psi_1|^2 = \frac{\sin^2[\eta + (\Delta k_1 L/2)^2]}{\eta^2 + (\Delta k_1 L/2)^2} \quad (2.40)$$

$$\text{where } \eta = \frac{v_1^2}{4} = \frac{k^2 n_1^2 L^2}{4 \cos^2(\theta)} \quad (2.41)$$

n_1 is the amplitude of the refractive index perturbed by the acoustic wave. It is related to the acoustic strain field (S) by [9]

$$n_1 = 1/2 n_0^3 p |S| \quad (2.42)$$

where p is the effective elastooptic coefficient for the particular mode of acoustooptic interaction. Substituting Eq. (2.42) into Eq. (2.41),

one obtains

$$\eta = \frac{\pi^2 n_0^6 p^2 |S|^2 L^2}{4 \lambda^2 \cos^2(\theta)} = \frac{\pi^2}{2 \lambda^2} M_2 \frac{P_s L}{H \cos^2(\theta)} \quad (2.43)$$

where $P_s = 1/2 \rho v_s^3 |S|^2 L H$ (2.44)

$$M_2 = \frac{n_0^6 p^2}{\rho v_s^3} \quad (2.45)$$

P_s is the acoustic power, M_2 is the acoustooptic figure of merit, ρ is the mass density, v_s is the acoustic wave velocity, and H is the height of the acoustic beam.

In Eq.(2.40), η is usually small compared with $(\Delta k_1 L/2)^2$, insignificant error is introduced if the equation is approximated by

$$I_1 = \sin^2(\eta) \text{sinc}^2\left(\frac{\Delta k_1 L}{2\pi}\right) \quad (2.46)$$

If we normalize the acoustic frequency by a center frequency f_{so} of wavelength Λ_0 , where the Bragg condition is satisfied, then $\Delta k_1 L/(2\pi)$ can be expressed in terms of $f = f_s/f_{so}$ as

$$\begin{aligned} \frac{\Delta k_1 L}{2\pi} &= \frac{L}{2\pi} \frac{Q}{2L} (1 - 2\alpha) \\ &= \frac{\lambda L}{2n_0 \Lambda_0^2 \cos(\theta_B)} \left\{ f - \frac{2n_0 \Lambda_0}{\lambda} \sin(\theta_B) \right\} f \end{aligned}$$

$$= \frac{L}{2L_0} (f - 1) f \quad (2.47)$$

where L_0 is the characteristic length and is defined by

$$L_0 = n_0 \Lambda_0^2 \cos(\theta_B) / \lambda \quad (2.48)$$

Equation (2.46) is the expression for the band shape of a Bragg cell by assuming the acoustic wave is a plane sinusoidal wave with a width L . It has been proven by Gordon [9] that the diffracted light intensity (I) as a function of the angle of incidence is proportional to the angular distribution of the acoustic energy. This distribution, except for phase factors, cannot change as a function of x . As a result, it is convenient to evaluate the angular spectra at a location where the amplitude distribution of the acoustic wave is known. The acoustic amplitude distribution on the surface of a simple uniform transducer is approximately given by a plane wavefront with width L corresponding to the transducer width. In other words, the analysis presented so far using a sinusoidal plane acoustic wave is a good approximation to the case where a uniform transducer is used for launching the acoustic wave.

At the exact Bragg angle of incidence, the amplitudes for the zeroth and first orders at $z = L$, are given by Eq. (2.38) and (2.37) respectively, and

$$\psi_0 = j \cos(v_1/2) \quad (2.49)$$

$$\psi_1 = -\sin(v_1/2) \quad (2.50)$$

The diffracted order is 90° out of phase with respect to the zeroth order. This phase difference is a property of all phase modulators [15].

The total light distribution from the acoustooptic cell operating in the Bragg region is obtained by substituting Eqs. (2.49) and (2.50) into Eq. (2.10), and at $z = L$,

$$E(x,t) = \exp\{j[\omega t - n_0 k(L \cos \theta_B - x \sin \theta_B) + \pi/2 + \phi_0]\} \\ \{ \cos(v_1/2) + j \sin(v_1/2) \exp[j(\omega_{s_0} t - k_{s_0} x)] \} \quad (2.51)$$

where ω_{s_0} and k_{s_0} are the angular frequency and the phase constant of the acoustic wave respectively, where the Bragg condition is satisfied. As a check, the total light energy should be conserved with

$$I = I_0 + I_1 = |\psi_0|^2 + |\psi_1|^2 = 1 \quad (2.52)$$

2.2.3 ANISOTROPIC DIFFRACTION

Dixon [16] has pointed out that acoustooptic diffraction becomes significantly different from normal (isotropic) diffraction when it takes place between an ordinary wave and an extraordinary wave in an optically anisotropic medium. We shall refer to this type of diffraction as birefringent since the refractive indices for the incident and diffracted waves are different.

The general coupled mode equation from Eq. (2.13) can be written in terms of the normalized parameters as

$$\begin{aligned} \frac{d\psi_m}{dz} + \frac{1}{2L} \sum_{i=1}^{\infty} v_{im} [\psi_{m-i} \exp(j\delta_i) - \psi_{m+i} \exp(-j\delta_i)] \\ = \frac{j\alpha_m Q}{2L} (m - 2\alpha_m) \psi_m \end{aligned} \quad (2.53)$$

$$\text{where } v_{im} = \frac{kn_{mo} n_i L}{n_o \cos(\theta)} \quad (2.54)$$

$$Q = \frac{k_s^2 L}{n_o k \cos(\theta)} \quad (2.55)$$

$$\alpha_m = - \frac{n_o k}{k_s} \sin(\theta) - \left[1 - \left(\frac{n_{mo}}{n_o} \right)^2 \right] \frac{n_o^2 k^2}{2mk_s^2} \quad (2.56)$$

The expression for the momentum mismatch is significantly modified by the birefringence as

$$\Delta k_m = \frac{mk_s^2}{2n_0 k \cos(\theta)} \left\{ m + \frac{2n_0 k}{k_s} \sin(\theta) + \left[1 - \left(\frac{n_{mo}}{n_0} \right)^2 \right] \frac{n_0^2 k^2}{mk_s^2} \right\} \quad (2.57)$$

2.2.3.1 BRAGG REGION ($Q \gg 1$)

The momentum mismatch for the zeroth order is always zero with $n_{mo}^* = n_0$. In the Bragg region of operation, significant energy can be transferred to the first order only if $\Delta k_1 = 0$. Equating Eq. (2.57) to zero for $m = 1$, one obtains the incident angle for Bragg angle diffraction as

$$\sin(\theta_B) = -\sin(\theta) = \frac{\lambda}{2n_0 \Lambda} \left[1 + \frac{\Lambda^2}{\lambda^2} (n_0^2 - n_{10}^2) \right] \quad (2.58)$$

Substituting Eq.(2.58) into Eq.(2.29), the angle for the first diffraction order is

$$\sin(\theta_d) = \frac{\lambda}{2n_{10} \Lambda} \left[1 - \frac{\Lambda^2}{\lambda^2} (n_0^2 - n_{10}^2) \right] \quad (2.59)$$

The wave vector construction of a general birefringent diffraction for $n_0 > n_{10}$ is shown in Fig. 4(a). It shows the birefringent diffraction in a uniaxial crystal when the optical wave vectors are

Fig. 4. Wave vector construction for birefringent diffractions.
 (a) General case of birefringent diffraction.
 (b) 90° birefringent phase matched diffraction.

normal to the optic axis. For a given acoustic wave direction, there exists two distinct acoustic frequencies that satisfy exact momentum matching conditions. One interesting case, which is usually used in designing broadband Bragg cells, is shown in Fig. 4(b). In this limiting case, the two solutions of momentum matching become degenerate. The diffracted light wave vector is perpendicular to the acoustic wave, and the momentum matching condition can be approximately satisfied for a given incident light direction over a broad range of acoustic frequencies. The band shape characteristic can be determined from the momentum mismatch function

$$\frac{\Delta k_1 L}{2\pi} = \frac{L}{2n_0 \lambda \cos(\theta)} \left[\frac{\lambda^2}{\Lambda^2} + 2n_0 \frac{\lambda}{\Lambda} \sin(\theta) + (n_0^2 - n_{10}^2) \right] \quad (2.60)$$

By choosing the center acoustic frequency to satisfy the condition as given in Fig. 4(b), that is,

$$f_{s0} = \frac{v_s}{\lambda} (n_0^2 - n_{10}^2)^{\frac{1}{2}} \quad (2.61)$$

and normalize the acoustic frequency by $f = f_s / f_{s0}$, Eq. (2.60) can be rewritten as

$$\frac{\Delta k_1 L}{2\pi} = \frac{L}{2L_0} \left\{ f^2 - 2 \left[\frac{n_0 \Lambda_0}{\lambda} \sin(\theta_0) \right] f + 1 \right\} \quad (2.62)$$

where the characteristic length L is

$$L_0 = \frac{n \Lambda_0^2 \cos(\theta_0)}{\lambda} \quad (2.63)$$

At the incident angle θ_0 defined by

$$\sin(\theta_0) = \frac{\lambda}{n_0 \Lambda_0} \quad (2.64)$$

the phase mismatch function is

$$\frac{\Delta k_1 L}{2\pi} = \frac{L}{2L_0} (f - 1)^2 \quad (2.65)$$

Comparing Eq. (2.65) to the isotropic case, Eq. (2.47), and for the same characteristic length (L_0) and center frequency, the birefringent case gives a much broader bandwidth.

The analysis presented so far is for the case where the Poynting vector directions, for both the optical and acoustic waves, are in the same directions as the wave vector directions. However, in a more general case, the Poynting vector directions may no longer be in parallel with the wave vector directions, and the anisotropy of the interaction medium has to be taken into account [17]-[20]. The acoustically anisotropic case will be treated later in Chapter 3.

2.3 SUMMARY AND CONCLUSIONS

The basic theory of acoustooptic diffraction has been presented in this chapter. A coupled mode formulation as given by Klein and Cook has been slightly modified to analyze both the isotropic and anisotropic interactions in the Bragg regime. Performance characteristics of Bragg cells such as bandwidth, incident and diffracted angles, and diffraction efficiency are discussed in terms of simple phase mismatch considerations. A much broader bandwidth is obtained for a Bragg cell utilizing the anisotropic interaction.

CHAPTER 3

ACOUSTOOPTIC FOURIER TRANSFORMING CONFIGURATION

3.1 INTRODUCTION

A general theory on acoustooptic diffraction has been presented in Chapter 2. The acoustooptic cell operating in the Bragg region is of particular interest as an input device in an optical signal processor. Only one diffraction order is generated, the frequency bandwidth is broad and the diffraction efficiency is high.

In this chapter, the optical Fourier transforming configuration using Bragg cells as spatial light modulators is analyzed in detail. A mathematical model of the acoustooptic Fourier transforming configuration is developed.

3.2 FOURIER TRANSFORMING CONFIGURATION

A Fourier transforming configuration is shown in Fig. 5, where a monochromatic, collimated light wave is impinging on the Bragg cell located in the object plane. We will use a different sign convention in which the phase of the wave is decreasing with time. This is the opposite of the sign convention used in Chapter 2. By using this new sign convention, the light amplitude distribution in the back focal plane is the Fourier transform of the object plane with a negative sign in the exponent [21].

The incident light wave impinging on the object plane is given by

$$\begin{aligned} E(x, y, z, t) &= E_0(x_0, y_0) \cos(2\pi\nu t - kz + \phi_0) \\ &= \text{Re}\{ E_0(x_0, y_0) \exp[-j(2\pi\nu t - kz + \phi_0)] \} \end{aligned} \quad (3.1)$$

where ν is the optical frequency with phase constant k ,
 ϕ_0 is some initial phase of the wave at $t=z=0$, and
 $E_0(x_0, y_0)$ is the amplitude distribution, which is normally Gaussian, of the collimated optical wave.

Assuming the Fourier transform lens is ideal and using phasor quantities, the field distribution in the back focal plane is related to the object plane by the Fourier transform [21].

Fig. 5. Fourier transforming configuration.

$$\begin{aligned}
 U_1(x_1, y_1, t) &= \iint_{-\infty}^{\infty} \frac{E_0(x_0, y_0)}{j \lambda F} T(x_0, y_0, t) \exp[-j \frac{2 \pi}{\lambda F} (x_0 x_1 + y_0 y_1)] dx_0 dy_0 \\
 &= K_1 \iint_{-\infty}^{\infty} E_0(x_0, y_0) T(x_0, y_0, t) \exp[-j 2 \pi (u x_0 + v y_0)] dx_0 dy_0
 \end{aligned}
 \tag{3.2}$$

where a constant phase factor has been dropped,

$$K_1 = \frac{1}{j \lambda F}$$

$$u = \frac{x_1}{\lambda F} \quad \text{and} \quad v = \frac{y_1}{\lambda F}$$

are expressed in normalized coordinates, F is the focal length of the lens, $T(x_0, y_0, t)$ is the complex amplitude transmission function of the object, and λ is the optical wavelength.

3.3 USE OF BRAGG CELLS AS SPATIAL LIGHT MODULATORS

The total light amplitude distribution inside the acoustooptic interaction region has been analyzed in the Bragg region and the material has been presented in Chapter 2. At the Bragg angle of incidence, the light amplitude distribution at $z=L$ is obtained from Eq. (2.51)

$$E(x_0, L, t) = \exp\{j[\omega t - n_0 k(L \cos \theta_B - x_0 \sin \theta_B) + \pi/2 + \phi_0]\}$$

$$\left\{ \cos\left(\frac{v_1}{2}\right) + j \sin\left(\frac{v_1}{2}\right) \exp[j(\omega_{so} t - k_{so} x_0)] \right\}$$

(3.3)

The incident light is assumed to have a uniform intensity profile and the acoustic signal is a sinusoidal wave. The acoustic beam profile where the optical beam transverses is assumed to have no variations in the y_0 direction. This is true when the height (H) of the acoustic transducer is many orders larger than the acoustic wavelength in which the diffraction effects are negligible.

For signal processing applications, it is desirable to operate in the linear region where the amount of light diffracted is linearly proportional to the acoustic power. This requires that $v_1 \ll 1$, and Eq. (3.3) can be approximated by

$$E(x_0, L, t) = \exp\{j[\omega t - n_0 k(L \cos \theta_B - x_0 \sin \theta_B) + \pi/2 + \phi_0]\}$$

$$\left\{ 1 + j \frac{v_1}{2} \exp[j(\omega_{so} t - k_{so} x_0)] \right\} \quad (3.4)$$

The above operation is written for the case where the diffracted

order has the same polarization as the undiffracted (zeroth) order. If the acoustic wave is a shear wave, then a 90° change in polarization also occurs [16].

The sinusoidal acoustic wave can be replaced by a more general input signal of the form

$$V_{in}(t) = A(t) \cos(2\pi f_s t + \phi_0) \quad (3.5)$$

provided the frequency bandwidth of $V_{in}(t)$ is within the Bragg cell bandwidth. Eq. (3.4) can also be modified directly, with insignificant errors introduced [22], by incorporating the acoustic attenuation and the Gaussian incident light profile. Using the new sign convention, Eq. (3.4) can be generalized to

$$E(x_0, L, t) = E_0(x_0, y_0) \exp\{-j[\omega t - n_0 k(L \cos \theta_B - x_0 \sin \theta_B)]\} \\ \cdot \left\{1 + jK_B A\left(t - \frac{x_0}{v_s}\right) \exp\left[-\alpha\tau\left(\frac{x_0}{D} + \frac{1}{2}\right)\right]\right. \\ \cdot \left.\exp[-j(\omega_s t - k_s x_0)]\right\} \quad (3.6)$$

The acoustic signal is attenuated as it propagates across the aperture in the x direction with velocity v_s , α is the acoustic loss coefficient in Np/sec and is a function of frequency; τ is the

acoustic transit time across the aperture D , $A(t - x_0/v_s)$ is the normalized complex amplitude function of the input signal, K_B is a constant which is proportional to the square root of the acoustooptic diffraction efficiency of the Bragg cell given by Eq.(2.40), K_B also takes into account the conversion efficiency of the piezoelectric transducer, and a constant phase term has been dropped.

In the optical transforming configuration, the incident optical axis is chosen to be parallel to the z axis. In the acoustooptic diffraction analysis, the incident optical beam is chosen to be at an angle to the z axis. The two coordinate systems are related by a simple rotation. In addition, the coordinate origin of the Bragg cell in the optical transforming configuration, has been shifted in the x_0 direction by $D/2$. The Bragg angle θ_B refers to the incident angle inside the medium and is usually much less than unity. Its value external to the Bragg cell is obtained by using Snell's law of refraction and is given by $n/n_0 \sin(\theta)$, n is the refractive index of the external medium and it is air in this case. θ is referred to either the Bragg or diffraction angle inside the acoustooptic medium. By transforming Eq.(3.6) in the acoustooptic diffraction coordinates into the optical transformation configuration coordinates and assuming $\theta_B \ll 1$, one obtains

$$E(x_0, L, t) = E_0(x_0, y_0) \exp[-j(\omega t - n_0 k L)]$$

$$\left\{ 1 + jK_B A\left(t - \frac{x_0}{v_s}\right) \exp\left[-j\left(\frac{\omega}{v_s} \frac{x_0}{D} + \frac{1}{2}\right)\right] \exp[-j(\omega_s t - k_s x_0)] \right\} \quad (3.7)$$

$$= \exp[-j(\omega t - n_0 kL)] \exp\left[-(2Tx \frac{x_0^2}{D} - (2Ty \frac{y_0^2}{H})\right] \\ \cdot T(x_0, y_0, t)$$

(3.8)

where T_x and T_y specify the Gaussian profile in the x_0 and y_0 dimensions respectively, and

$$T(x_0, y_0, t) = \text{rect}\left(\frac{x_0}{D}\right) \text{rect}\left(\frac{y_0}{H}\right) \left\{ 1 + jK_B A(t - \frac{x_0}{v_s}) \right. \\ \left. \cdot \exp\left[-\alpha\tau\left(\frac{x_0}{D} + \frac{1}{2}\right)\right] \exp[-j2\pi f_s(t - \frac{x_0}{v_s})] \right\} \quad (3.9)$$

is the amplitude transmission function of the Bragg cell with $m=1$. There are two terms in the function; one is the zeroth order with a value of unity and the other term contains the signal information.

Substituting Eq. (3.9) into Eq.(3.2), the amplitude distribution of the waveform which contains the signal information is

$$U_1(u, v, t) = \frac{K}{\lambda F} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(t - \frac{x_0}{v_s}) w(x_0, y_0) \\ \cdot \exp[-j2\pi (ux_0 + vy_0)] dx_0 dy_0 \quad (3.10)$$

$$\text{where } g(t - \frac{x}{v_s}) = A(t - \frac{x}{v_s}) \exp[-j2\pi f_s (t - \frac{x}{v_s})] \quad (3.11)$$

and

$$w(x_0, y_0) = \text{rect}(\frac{x_0}{D}) \text{rect}(\frac{y_0}{H}) \exp[-(2Tx \frac{x_0^2}{D}) - (2Ty \frac{y_0^2}{H})] \\ \exp[-\alpha T (\frac{x_0}{D} + \frac{1}{2})] \quad (3.12)$$

is the normalized amplitude weighting function and a constant phase term has been dropped.

Equation (3.10) simply states that the diffracted instantaneous amplitude distribution in the focal plane is the one-sided Fourier transform of the weighted input signal. A unique feature of this configuration is that the Fourier transform is computed over a sliding window of the input signal. At different instances in time, the Fourier transform of a different portion of the weighted signal is performed. With the acoustic diffraction effects in the y direction neglected, the acoustooptic Fourier transformation is completely characterized by this equation, and it will be used extensively in the next few chapters. In the case of a continuous wave (cw) signal and for a rectangular weighting function, the amplitude distribution in the focal plane can be expressed in closed forms. At a certain time t_0 , when the phase of

the signal is zero at $x_0 = -D/2$, the amplitude distribution is

$$U_1(u,v) = \frac{K_B}{\lambda F} DH \exp(j \frac{\pi D}{\lambda F} v) \text{sinc}(vH) \text{sinc}[D(u - \frac{f_s}{v_s})]$$

(3.13)

The amplitude distribution, parallel to the normalized u axis, is the one-sided Fourier transform of the truncated input signal and the distance from $u=0$ to the centroid is linearly proportional to the acoustic frequency.

3.4. BRAGG CELL DIFFRACTION PATTERNS

The analysis presented so far, is for the case where the acoustic wave in the y direction is assumed to be uniform in amplitude and plane in its phase front. In other words, the acoustic diffraction effects, due to the finite height (H) of the rectangular acoustic transducer, have been neglected. However, as given in Eq.(2.43), the diffraction efficiency of a Bragg cell is directly proportional to the ratio of L/H , where L is the width of the transducer. For broadband operation, L must be kept small to give an appreciable spread in the acoustic angular spectra to satisfy the momentum conservation criterion. In order to maintain a large diffraction efficiency which is directly proportional to L/H , H must also be kept to a small value. Moreover, as will be

discussed in Chapter 6, the height H must be kept small in a multi-channel Bragg cell configuration to preserve the diffraction pattern for direction-of-arrival determination. Under these circumstances, the acoustic diffraction effects on the characteristics of the diffraction pattern can be appreciable and must be considered.

Let us consider the case where the Bragg cell is operating in the longitudinal mode and is acoustically anisotropic. In an anisotropic crystal, the phase velocity of the acoustic wave varies as a function of the direction of propagation and the orientation of the crystal relative to the transducer.

For propagating directions close to a three, four, or sixfold pure-mode axis, the acoustic longitudinal phase velocity is given by [17],[23]

$$v_s(\phi) = v_s (1 - s \phi^2) \quad (3.14)$$

where s is the anisotropy parameter of the crystal. Since the acoustic wavelength is $\Lambda = 2\pi v_s / \omega_s$, the wavelength dependence on ϕ has the same form as Eq.(3.14). Let the normal of the acoustic transducer be parallel to the crystal's pure-mode axis as shown in Fig. 6.

For small values of ϕ , the acoustic k_s vector associated with the phase velocity of a propagating wave is

Fig. 6. Phase vector $k_g(\phi)$ and Poynting vector $k_e(x)$ plotted

for $s < 0$.

$$k_s(\phi) = \frac{\omega}{v_s(\phi)} = \frac{2\pi}{\Lambda(1 - s\phi^2)} \quad (3.15)$$

The coupling of the optical and acoustic wave fronts requires that we first find the acoustic pressure field within the cell. The pressure at each point within the cell is defined by the scalar product of the Poynting vector $\bar{k}_e$ and a distance vector $\bar{r}$ [17],[23]. In an anisotropic material, the acoustic energy propagates at an angle χ which is related to the acoustic k_s vector angle ϕ by

$$\chi = \phi + \psi \quad (3.16)$$

where ψ is the angle between the phase vector and the energy (Poynting) vector. The magnitudes of the phase velocity and the energy velocity are related by [24]

$$v_e(\chi) = v_s(\phi) / \cos(\psi) \quad (3.17)$$

Furthermore, the tangent plane of the ray (energy) surface is always perpendicular to the corresponding wave normal [6]. By making use of this relationship and Eq.(3.14), it can be shown that [23] $\psi = -2s\phi$, so that

$$v_e(\chi) = v_s \left[1 - \frac{s\chi^2}{(1 - 2s)} \right] \quad (3.18)$$

which is valid to terms of the order χ^2 . The magnitude of the acoustic k vector associated with the energy flow is then

$$k_e(\chi) = \frac{\omega s}{v_e(\chi)} = \frac{2\pi}{\Lambda [1 - s\chi^2/(1-2s)]} \quad (3.19)$$

The scalar product of the Poynting vector and a distance vector is

$$\begin{aligned} \bar{k}_e \cdot \bar{r} &= \frac{2\pi}{\Lambda [1 - s\chi^2/(1-2s)]} [(x_0 + D/2)^2 + y_0^2]^{\frac{1}{2}} \\ &= \frac{2\pi}{\Lambda} \left[(x_0 + D/2) + \frac{y_0^2}{2(x_0 + D/2)(1-2s)} \right] \end{aligned} \quad (3.20)$$

where we have used the relationships that for small angle,

$$\chi = \arctan \left(\frac{y_0}{x_0 + D/2} \right) \approx \frac{y_0}{x_0 + D/2} \quad (3.21)$$

$$[(x_0 + D/2)^2 + y_0^2]^{\frac{1}{2}} \approx (x_0 + D/2) + \frac{y_0^2}{2(x_0 + D/2)} \quad (3.22)$$

Now we have established the distribution of the acoustic energy in the x_0 and y_0 directions as a function of the anisotropy factor s . Let us consider the Fourier transforming configuration again as given in Fig. 7 and take into account the acoustic field variation in the y_0 direction. We will use the same assumptions as outlined in Sections 3.2-3.3, by assuming the input Bragg and diffracted angles are small. We will first consider the acoustic field produced by a transducer of infinitesimal height and then account for its finite height after we find the Fourier transform of the coupled optical waveform. Suppressing the time dependence of the acoustic wave by assuming a cw signal with zero phase at $x_0 = -D/2$ and at a certain time t , the acoustic wave function for the first higher-order diffracted component for a line acoustic source is

$$\begin{aligned}
 g(x_0, y_0) &= \frac{A}{\sqrt{j \Lambda r (1-2s)}} \exp[j \bar{k}_e(\chi) \cdot \bar{r}] \\
 &\approx \frac{A}{\sqrt{j \Lambda (x_0 + D/2) (1-2s)}} \\
 &\quad \cdot \exp \left\{ j \frac{2\pi}{\Lambda} \left[(x_0 + D/2) + \frac{y_0^2}{2(x_0 + D/2)(1-2s)} \right] \right\} \quad (3.23)
 \end{aligned}$$

where we have replaced r by $(x_0 + D/2)$ and have also introduced the $(1-2s)$ factor into the denominator. The above diffraction formula in the Fresnel region with the $(1-2s)$ factor has been established by Cohen [25]

Fig. 7. Fourier transforming configuration for displaying diffraction patterns.

as a more rigorously correct formula for anisotropic media.

For a fully illuminated Bragg cell, the amplitude of the diffracted signal is

$$U_1(\gamma, \beta) = \frac{K}{\lambda F} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x_0, y_0) w(x_0, y_0) \cdot \exp\left[-j \frac{2\pi}{\lambda} (\gamma x_0 + \beta y_0)\right] dx_0 dy_0 \cdot H \operatorname{sinc}\left(\frac{H\beta}{\lambda}\right) \quad (3.24)$$

where the weighting function is in general given by

$$w(x_0, y_0) = \operatorname{rect}\left(\frac{x_0}{D}\right) \exp\left[-(2Tx \frac{x_0}{D})^2 - (2Ty \frac{y_0}{H})^2\right] \cdot \exp\left[-\alpha\tau\left(\frac{x_0}{D} + \frac{1}{2}\right)\right] \quad (3.25)$$

and the finite height of the transducer has been accounted for by the multiplication of the integral by the $\operatorname{sinc}(H\beta/\lambda)$ function, r in the attenuation factor has been replaced by $x_0 + D/2$ and

$$\begin{aligned} \gamma &= \arctan(x_1/F) \approx \lambda u \\ \beta &= \arctan(y_1/F) \approx \lambda v \end{aligned} \quad (3.26)$$

In the case where the illuminating laser profile is uniform and the attenuation factor is negligible, Eq. (3.24) can be expressed in a closed form solution by following a similar procedure given by Vander Lugt [17], hence

$$U_1(\gamma, \beta) = \frac{K_B}{\lambda F} DH \operatorname{sinc}\left(-\frac{H\beta}{\lambda}\right) \exp\left\{j \frac{\pi D}{\lambda} \left[\frac{\lambda}{\Lambda} - \frac{\Lambda (1/2 - s) \beta^2}{\lambda} \right]\right\} \\ \cdot \operatorname{sinc}\left\{\frac{D}{\lambda} \left[\gamma - \frac{\lambda}{\Lambda} + \frac{\Lambda (1/2 - s) \beta^2}{\lambda} \right]\right\}$$

(3.27)

The maximum value of the amplitude distribution in the $\gamma\beta$ plane, lies along a curvature defined by the argument of the last sinc function, which is

$$\gamma - \frac{\lambda}{\Lambda} + \frac{\Lambda (1/2 - s) \beta^2}{\lambda} = 0$$

or

$$\beta^2 = \frac{\lambda (\lambda / \Lambda - \gamma)}{\Lambda (1/2 - s)}$$

(3.28)

The above equation is plotted in Fig. 8 for different values of s , the anisotropy parameter and at $\lambda / \Lambda = 1$. This set of curves are known as the

Fig. 8. Singularity curves in the Fourier plane for anisotropic Bragg cells.

singularity curves.

For isotropic materials, $s = 0$ and the singularity function curve is a sector of a circle and the derivation has been obtained by Vander Lugt [17]. For $s < 0$, the singularity function curves more rapidly toward the optical axis, whereas, for $s > 0$, the singularity function curves less rapidly toward the source. Papadakis [26] gives some limits on the value of s in terms of the elastic constants and argues that s must be less than $1/2$. For $s = 1/2$ results in a singularity function that is a straight vertical line passing through $\gamma = \lambda/\Lambda$, and this is a completely self-collimating medium. For $s = 1/2$, the diffraction pattern of the signal in the Fourier plane is given by

$$U_1(u,v) = \frac{K_B DH}{\lambda F} \exp(j \frac{\pi D}{\lambda}) \text{sinc}(vH) \text{sinc}[D(u - 1/\Lambda)] \quad (3.29)$$

Equation (3.29) is identical to Eq. (3.13) in Section 3.3. In other words, the signal function derived in Section 3.3 by neglecting the acoustic diffraction effects in the y direction, is the exact expression for a self-collimating material.

As shown in Fig. 8, the horizontal difference between the spatial frequency at an angle ϕ and that at $\phi = 0$ is given by

$$\Delta\gamma = \gamma_0 - \gamma(\phi) = -\frac{\Lambda}{\lambda} (1/2-s) \beta^2 \quad (3.30)$$

From Eq. (3.27), we also know that most of the optical energy is contained within the first nulls of the sinc $(H\beta/\lambda)$ function and the first null is located at $\beta = \lambda/H$. Substituting β into Eq.(3.30) and normalizing $\Delta\gamma$ by γ_0 , we obtain

$$\frac{\Delta\gamma}{\gamma_0} = (1/2-s) \left(\frac{\Lambda}{H} \right)^2 \quad (3.31)$$

From the above equation, for an anisotropic material with a certain value s , the normalized spatial frequency difference at the first null point, is inversely proportional to $(H/\Lambda)^2$. If the acoustic transducer height (H) is much larger than the acoustic wavelength, the curvature effect due to acoustic diffraction can be neglected, and thus the signal function can be approximated by assuming the medium to be a self-collimating one.

3.5 SUMMARY AND CONCLUSIONS

The optical Fourier transforming configuration using Bragg cells as spatial light modulators has been analyzed in this chapter. A unique feature of this configuration is that the Fourier transform is computed over a sliding window of the input signal. At different instances in

time, the Fourier transform of a different portion of the weighted signal is performed.

The Bragg cell was first modeled as an ideal traveling wave modulator in which the acoustic diffraction effects along the height dimension are neglected. The diffraction pattern is the familiar 2-D sinc^2 function. This simplified model is very useful for general system analysis such as in the development of processing architectures. The degradation or deviation from this ideal model is then analyzed for both isotropic and anisotropic interaction materials. The diffraction pattern is still characterized by the expected 2-D sinc^2 function, except that in one dimension the function is curved toward the optical axis. The degree of curvature depends on the anisotropic parameter of the interaction medium.

CHAPTER 4

ACOUSTOOPTIC SPECTRUM ANALYZER: TEMPORAL RESPONSE AND DETECTION OF PULSED SIGNALS

4.1 INTRODUCTION

The optical configuration analyzed in the last chapter, where a single-channel Bragg cell is employed, is an acoustooptic spectrum analyzer. This configuration is one of the most basic and important functions performed by optics.

The acoustooptic spectrum analyzer is a real-time processor in which the output is the instantaneous Fourier transform of the weighted input signal in the aperture. The amplitude weighting function takes into account the acoustic attenuation and the interaction profile between the laser beam and acoustic beam in the aperture. For cw signals, the spatial and temporal characteristics of the spectrum are constant and are determined by the Fourier transform of the product of the amplitude weighting function and the signal [4],[17]. On the other hand, in the case where the signal is of finite duration, such as with pulse-modulated cw signals, the instantaneous spectrum is a function of where the signal is located in the aperture and is a more complex problem [27]-[29].

A photodetector array is usually used for detecting the spectrum. The output from such a processor also depends on a number of other factors such as the number of photodetectors used to cover a given frequency band, the response of the photodetector, and whether it is operating in the time-integrating or the real-time mode.

In this chapter we investigate both the spatial and temporal characteristics of the acoustooptic spectrum analyzer to pulse-modulated cw signals. The formation of the temporal and spatial distributions in the Fourier plane as a pulsed signal propagates through the Bragg cell aperture is analyzed in detail. The detected signal waveform from a photodetector array operating in the real-time mode is then analyzed as a function of the pulse width, weighting function parameters, and the relative position of the photodetector to the spectral centroid. Two simple methods of determining the pulse width from the detected signal waveforms are also discussed.

4.2 SPATIAL AND TEMPORAL DISTRIBUTIONS

A Fourier transforming configuration is shown in Fig. 5, where the Bragg cell is illuminated with a collimated source of monochromatic light at the Bragg input angle. An electrical signal is applied to the transducer of the Bragg cell and a portion of the light is diffracted into one of the first-order components. The interaction between sound and light is assumed to be ideal and the effect of the acoustic spreading in the y direction is neglected. The light amplitude distribution of the diffracted first-order component, in the back focal plane of the lens, is given by Eq.(3.10), that is,

$$U_1(x_1, y_1, t) = \frac{K_B}{\lambda F} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(t - x_0/v_s) w(x_0, y_0) \cdot \exp[-j2\pi(ux_0 + vy_0)] dx_0 dy_0, \quad (4.1)$$

where

$$u = \frac{x_1}{\lambda F}, \quad v = \frac{y_1}{\lambda F}$$

are expressed in normalized coordinates, λ is the optical wavelength, F is the focal length of the lens, K_B is a constant which is proportional to the square root of the acoustooptic diffraction efficiency of the Bragg cell and the illuminating laser power, $g(t - x_0/v_s)$ is the acoustic signal function, v_s is the acoustic velocity, and $w(x_0, y_0)$ is the amplitude window weighting function.

In the implementation of spectrum analyzers, the Bragg cell is often

illuminated with a line source. The optics is configured to perform a 1-D Fourier transform in the x_1 direction and the line source is imaged in the y_1 direction. This operation is generated by inserting a cylindrical lens in front of the Bragg cell and another in between the Bragg cell and the Fourier transform lens. In this arrangement, the effects of the acoustic curvature in the y direction, as described in Section 3.4, on the intensity distribution have been analyzed [17] and found to be small. As a result, the analysis presented in this chapter can be simplified by assuming the acoustooptic medium to be self-focusing.

In this arrangement, the weighting function can in general be approximated by

$$\begin{aligned}
 w(x_0, y_0) &= \text{rect}\left(\frac{x_0}{D}\right) \delta(y_0) \exp\left[-\alpha_T\left(\frac{x_0}{D} + \frac{1}{2}\right)\right] \\
 &\quad \cdot \exp\left[-\left(2Tx \frac{x_0^2}{D}\right)\right] \\
 &= \text{rect}\left(\frac{x_0}{D}\right) \cdot \delta(y_0) \exp\left(\alpha_T/2\left[\alpha_T/(8Tx^2) - 1\right]\right) \\
 &\quad \cdot \exp\left(-\left(2Tx\left[x_0/D + \alpha_T/(8Tx^2)\right]\right)^2\right)
 \end{aligned}
 \tag{4.2}$$

where δ is the Dirac delta function, rect is the rectangular function, the input laser profile is of the Gaussian form with Tx specifying the

intensity truncation ratio at the $1/e^2$ points, α is the acoustic attenuation coefficient in Np/s and increases with the input frequency, and τ is the acoustic transit time across the effective aperture D . In the latter expression of Eq. (4.2), both the acoustic exponential attenuation and the Gaussian illumination profile have been combined into a displaced Gaussian function.

In the case of a pulse-modulated cw signal of the form

$$V_{in}(t) = C \text{ rect}[(t - PW/2)/PW] \cos(2\pi f_s t + \phi_0), \quad (4.3)$$

the corresponding acoustic wave function with only the first higher-order component generated is given by

$$g(t - x_0/v_s) = C \text{ rect}[(t - PW/2 - x_0/v_s - \tau/2)/PW] \cdot \exp[-j2\pi f_s(t - x_0/v_s - \tau/2) - j\phi_0], \quad (4.4)$$

where f_s is the carrier frequency, C is the pulse amplitude, and PW is the pulse width. The leading edge of the pulse arrives at $x_0 = -D/2$ at $t = 0$ with phase ϕ_0 .

With this modified configuration for spectrum analysis, the amplitude distribution of the signal in the frequency plane is obtained by substituting Eqs. (4.2) and (4.4) into Eq. (4.1) and solving the

integral in terms of error functions [36], we have

$$\begin{aligned}
 U_1(x_1, y_1, t) &= K \delta(y_1) \int_{-\infty}^{\infty} g(t - x_0/v_s) w(x_0) \exp(-j2\pi u x_0) dx_0 \\
 &= K C \delta(y_1)/2 \exp[-j2\pi f_s (t - \tau/2) - \alpha\tau/2] \\
 &\quad \cdot (\pi/a)^{1/2} \exp(b^2/a) [\operatorname{erf}(a^{1/2} B + b/a^{1/2}) \\
 &\quad - \operatorname{erf}(a^{1/2} A + b/a^{1/2})]
 \end{aligned} \tag{4.5}$$

where

$$\begin{aligned}
 a &= (2T\pi/D)^2, \\
 b &= j\pi [u - f_s/v_s - j\alpha\tau/(2\pi D)], \\
 K &= K_B / (\lambda F)^{1/2},
 \end{aligned}$$

B and A are the upper and lower limits, respectively, of the integral and they are a function of the pulse position in the aperture. For example, when only the leading edge of the pulse is in the aperture, the limits are defined by $A = -D/2$ and $B = -D/2 + v_s t$.

The optical illumination profile can be easily modified while the acoustic loss is inherent as it depends upon the material of the Bragg cell. Let us analyze the case where the laser illumination profile is uniform in the x dimension, to obtain insight on the characteristics of the field distribution. If the input signal is pulse-modulated with an amplitude C, then, the amplitude distribution along the x_1 axis at $y_1 = 0$

can be expressed as

$$\begin{aligned} \dot{U}_1(u, t) = & K C \exp[-j2\pi f_s(t - \tau/2)] \exp(-\alpha\tau/2) \\ & \int_{-\infty}^{\infty} \text{rect}[(t - PW/2 - x_0/v_s - \tau/2)/PW] \text{rect}(x_0/D) \\ & \cdot \exp\{-j2\pi[u - f_s/v_s - j\alpha\tau/(2\pi D)]\} dx_0 \end{aligned} \quad (4.6)$$

where the initial phase ϕ_0 has been dropped from the above equation. There are four possible cases when considering a pulse-modulated cw signal and they are

- (A) When the pulse fills up the aperture.
- (B) Only the leading edge of the pulse in the aperture.
- (C) Only the trailing edge of the pulse in the aperture and
- (D) both the leading and trailing edges in the aperture.

These four cases will be considered in some details in this section.

(A) The Pulse Completely Fills the Aperture

This is the only case for an input cw signal, but can also occur when using a pulsed signal which has a pulse width longer than the aperture transit time. In both cases, the limits of integration are now defined by the aperture, namely, $A = -D/2$ and $B = D/2$ and the amplitude distribution is

$$U_1(u, t) = C K D \operatorname{sinc}[uD - f_s \tau - j \alpha \tau / (2 \pi)] \cdot \exp(-\alpha \tau / 2) \exp[-j 2 \pi f_s (t - \tau / 2)] \quad (4.7)$$

The last phase term shows that the light frequency is upshifted by the input signal carrier frequency f_s . The magnitude is directly proportional to the pulse amplitude C , and the aperture width D . It is also attenuated by an average exponential factor given at one half of the aperture transit time. The spatial amplitude distribution along the u axis is a complex sinc function which is defined by

$$\operatorname{sinc}[(uD - f_s \tau - j \alpha \tau / (2 \pi))] = \frac{\sin[\pi(uD - f_s \tau)] \cosh(-\alpha \tau / 2) + j \cos[\pi(uD - f_s \tau)] \sinh(-\alpha \tau / 2)}{\pi [uD - f_s \tau - j \alpha \tau / (2 \pi)]} \quad (4.8)$$

It is to be noted that the sinc function with a complex argument is uncommon in the literature. However since it appears very often in this chapter and is in a much more compact form than the expression given on the right hand side, it will therefore be used.

The intensity of the distribution is proportional to the square of

the absolute value of the sinc distribution. It can be shown that the maximum value of the sinc square function with a complex argument occurs at the normalized spatial coordinate of $u = f_s / y_s$.

The square of the normalized absolute sinc function is plotted versus the normalized variable uD for various acoustic loss values of $\alpha\tau$ and is shown in Fig.9. The location of the centroid is shifted by f_s and its magnitude is normalized to unity. S is the normalized effective pulse width which is that portion of the pulse inside the aperture normalized by the total aperture width D , and is unity in this case. As can be seen from the plot, the effect of acoustic attenuation on the intensity distribution is to smooth out the waveform by slightly broadening its main lobe, increasing its sidelobe levels and filling up its nulls.

The peak amplitude of the distribution is given by

$$U_{lp} = C K D \left[\frac{1 - \exp(-\alpha\tau)}{\alpha\tau} \right] \quad (4.9)$$

The square of the last term is plotted in Fig. 10 as a function of $\alpha\tau$. The normalized intensity distribution is plotted in Fig. 9, with its peak value normalized to unity, while the absolute value of the peak is plotted in Fig. 10. By using the plots of both Figs. 9 and 10, a semi-graphical technique can be used to obtain the absolute light

Fig. 9. Normalized plot of $|\text{sinc} \{ [uD - j\alpha r/(2\pi)] S \}|^2$ versus uD .

Fig. 10. Plot of $\left\{ \left[1 - \exp(-atS) \right] / (at) \right\}^2$ versus at .

intensity distribution as a function of $\alpha\tau$ and S .

In the limit when the loss is negligible, the amplitude distribution is simply given by the familiar sinc function

$$U_1(u, t) = C K D \operatorname{sinc}(uD - f_s \tau) \exp[-j2\pi f_s (t - \tau/2)]$$

(4.10)

(B) Only the Leading Edge of the Pulse in the Aperture

In this case, the lower limit of integration is defined by the aperture boundary at $A = -D/2$ and the upper limit by the leading edge of the pulse, which is moving in time, that is,

$$B = -D/2 + v_s t, \quad t < PW, \quad \tau \quad (4.11)$$

The amplitude distribution in the back focal plane is now

$$U_1(u, t) = K C D S \exp(-\alpha\tau S/2) \operatorname{sinc}([uD - f_s \tau - j\alpha\tau/(2\pi)]S) \\ \cdot \exp(-j\pi[(uv_s + f_s)t - uD])$$

(4.12)

where the normalized effective pulse width $S = t/\tau$. The temporal frequency is given in the last exponential term as

$$f = (uv_s + f_s)/2 \quad (4.13)$$

The spatial amplitude distribution is again a complex sinc function as in the case of a cw signal except that it is a function of the effective pulse width. The temporal frequency at the centroid of the distribution ($u=f_s/\tau$) is also upshifted by the signal carrier frequency. However as we move away from the centroid, the temporal frequency varies continuously as u varies. At some incremental spatial frequency of Δu from the centroid, the temporal frequency deviation is $\Delta f/2$. The explanation for this phenomenon is that in this case, one edge moves, and the other one is stationary. The absolute square of the normalized sinc function is plotted in Fig. 9.

As t increases, the effective pulse width increases proportionally and the magnitude of the centroid increases while its spatial width decreases. The peak amplitude of the centroid is given by

$$U_{1D}(t) = C K D \left[\frac{1 - \exp(-\alpha TS)}{\alpha \tau} \right] \quad (4.14)$$

and the square of the bracketed term is plotted in Fig. 10.

(C) Only the Trailing Edge of the Pulse in the Aperture

The upper limit is now defined by the aperture at $B = D/2$. The trailing edge of the pulse is moving in time and defined by

$$A = -D/2 + v_g (t - PW) \quad , \quad 0 < t - PW < \tau \quad (4.15)$$

The amplitude distribution in the back focal plane is

$$\begin{aligned} U_1(u, t) = & K C D S \exp[-\alpha\tau(1-S)] \exp(-\alpha\tau S/2) \\ & \cdot \text{sinc}([uD - f_g \tau - j\alpha\tau/(2\pi)]S) \\ & \cdot \exp\{-j\pi[(uv_g + f_g)t - f_g \tau - (uv_g - f_g)PW]\} \end{aligned} \quad (4.16)$$

$$\text{and} \quad S = 1 - (t - PW)/\tau \quad (4.17)$$

As expected, the distribution is of the same form as for the leading edge case. The only difference is that the amplitude is attenuated more for the same effective pulse width since it is now located at the exit end of the aperture. The peak amplitude of the centroid is

$$U_{1p}(u, t) = K C D \exp[-\alpha(t - PW)] \{ [1 - \exp(-\alpha\tau S)] / (\alpha\tau) \} \quad (4.18)$$

(D) Both the Leading and Trailing Edges in the Aperture

This case happens only when the pulse width PW is shorter than the aperture transit time τ . Both the leading and trailing edges are moving in time, that is,

$$\begin{aligned} A &= -D/2 + v_s(t - PW) \\ B &= -D/2 + v_s t \end{aligned} \quad (4.19)$$

for $0 < t - PW < \tau$

The amplitude distribution in the back focal plane is

$$\begin{aligned} U_1(u, t) &= K C D S \exp[-\alpha(t - PW)] \exp(-\alpha\tau S/2) \\ &\quad \text{sinc}\{ [uD - f_s\tau - j\alpha\tau/(2\pi)]S \} \\ &\quad \exp\{-j\pi[2uv_s t + (f_s - uv_s)PW - uD]\} \end{aligned} \quad (4.20)$$

and $S = PW/\tau \quad (4.21)$

The temporal frequency is now

$$f = uv_s \quad (4.22)$$

It varies continuously as a function of the normalized spatial frequency u . At the centroid, the temporal frequency is again equal to f_s . At

some incremental frequency Δu away from the centroid, we find that the temporal frequency of the sidelobes differs from the centroid by Δf . The explanation for this phenomenon is that now both the leading and trailing edges move.

The spatial distribution is a complex sinc function with its magnitude decreasing with time. The square of the absolute value of the sinc function is given in Fig. 9. The amplitude of the centroid is

$$U_1(t) = K C D \exp[-\alpha(t-PW)] \{ [1 - \exp(\alpha T S)] / (\alpha T) \},$$

(4.23)

The square of the last term is plotted in Fig. 10 as a function of $\alpha T S$.

4.3 DETECTION SCHEME

In power spectrum analyzers, the intensity distribution along the x_1 axis as a function of time is of interest. For a given pulse width, this distribution is completely characterized by Eq.(4.5) and is a function of only two parameters, T_x and αT .

We have seen in the last section for $T_x = 0$, the waveform distribution is rather complicated as it is a function of where the pulse is located in the aperture. With the additional parameter T_x ,

the power pattern is expected to be much more complex.

A typical case is used to illustrate the formation of the power spectral distribution of a pulsed signal as it propagates across the aperture. The weighting function parameters are $T_x = 0.8$ and $\alpha T = 0.4$.

The plots shown in Fig. 11 are for the case where the normalized pulse width (PW/τ) is unity. The normalized instantaneous intensity distribution is plotted at five different positions as the pulse propagates through the aperture. Due to symmetry in the waveform distribution, only the right half of the waveform is plotted for each location and the spectral centroid is chosen to be located at the origin. As can be seen from the plots, the instantaneous intensity distribution changes with the pulse position in the aperture and is complicated in shape.

In the implementation of a spectrum analyzer, solid-state linear photodetector arrays are usually used for detection. They operate either in the real-time mode with parallel individual outputs or in a time-integrating mode where the output is from detectors arrayed in serial/parallel configurations. In the time-integrating mode, there is a trade-off between the frequency resolution and the integration time for a given data rate. If a large number of photodetectors are used to obtain a fine resolution of the spectrum, the integration time is usually increased due to finite clock rates. In the real-time mode of operation, the number of photodetectors used must be limited to keep the number of parallel channels to a manageable level.

2

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Fig. 11. Relative light intensity distributions at five different time intervals: ($PW/\tau = 1.0$, $T_x = 0.8$, and $\alpha\tau = 0.4$).

The time-integrated waveform distributions for pulse-modulated cw signals will be addressed in Chapter 5 and therefore will not be considered further in this chapter.

In the real-time mode of operation, one is more interested in the instantaneous spectrum of the signal detected by the photodetector array. Using this accurate measurement of the output voltage from the photodetector array as a function of time, not only the pulse repetition frequency (PRF) but also the pulse width (PW) of the signal can be determined.

In general, the output voltage from a given photodetector is a function of the frequency response of the photodetector and the portion of the intensity distribution sampled by it. To keep the number of possible cases to a minimum, the most commonly employed 1/D bandwidth detection scheme is investigated. In this photodetector array configuration, the spectrum analyzer is used mainly as a frequency channelizer in which multiple simultaneous signals of different frequencies can be spatially separated. In this case, a coarse frequency resolution is obtained while the time resolution is very good.

4.4 1/D BANDWIDTH DETECTION

In this scheme of detection, the spectrum analyzer is configured in such a way that the photodetector width has a normalized spatial bandwidth equal to the reciprocal of the aperture width (D). If we

designate the photodetector where the spatial distribution of the signal is approximately centered at $K = 0$, the output voltage from the K th photodetector is

$$V_K(t) = \iint_{-\infty}^{\infty} |U_1(u, v, t)|^2 H\left[\frac{u - (u_s + K \Delta u + u_0)}{\Delta u}, \frac{v}{\Delta v}\right] du dv * h(t) \quad (4.24)$$

where H is the spatial weighting function of the photodetector array; $\Delta u = 1/D$ is the center-to-center spacing, Δv is the height of the photodetector array, $u_s = f/v_s$ is the location of the signal spectral centroid, u_0 is the mismatch between the spectral centroid and the centered photodetector, $h(t)$ is the impulse response of the detection circuit which normally consists of a photodetector followed by an amplifier, and $*$ denotes convolution.

If the bandwidth of the detection circuit is much larger than the electrical bandwidth of the sampled waveform by the photodetector, the impulse response can be approximated by the delta function $\delta(t)$.

A simple case is used to illustrate the detected waveform characteristics with the weighting parameters $T_x = \alpha\tau = 0$ and a normalized pulse width (PW/τ) of unity. It was carried out experimentally using a slit with a normalized spatial bandwidth of $1/D$ placed in front of a large photodetector. The slit with the

photodetector was initially centered on the centroid of the signal spectrum and was then moved in steps of $1/(2D)$ away from it. The reason for the $1/(2D)$ step (half of the photodetector width) is to simulate the two extreme relative positions of the photodetector array with respect to the centroid of the signal spectrum. One case is when the spectral centroid is centered with respect to one of the photodetectors and the photodetectors on that array are designated by $n = 2K = \text{even integers}$. The other case is when the spectral centroid is located in between two adjacent photodetectors with $n = 2K = \text{odd integers}$.

The theoretical and experimental waveforms at the various photodetector locations are plotted in Figs. 12 and 13, respectively. The bandwidth of the detection circuit is much larger than the detected signal bandwidth and $h(t)$ is approximated by the delta function $\delta(t)$. The spatial weighting function of the photodetector in this arrangement is of rectangular form. The photodetector located at the centroid of the spectrum is designated by $n = 0$, and all the curves are normalized by the maximum output value at $n = 0$ and $t/\tau = 1$. In Fig. 12, only the first half of the response is plotted because the output waveform is symmetrical for the rectangular weighting function. In Fig. 13(b), the four experimental curves are displaced vertically by one division from each other to show a clearer picture.

For the rectangular weighting function, with $T_x = \alpha\tau = 0$, the power spectral distribution is always of the sinc squared function. Its magnitude increases with the square of the effective pulse width in the

Fig. 12. 1/D bandwidth detection of a pulsed signal vs photodetector location [$n/(2D)$] from the signal centroid: ($PW/\tau = 1.0$, $T_x = \alpha\tau = 0$), (a) $|n| = 0, 1$, and 2, (b) $|n| = 2, 3, 4$, and 5.

NORMALIZED AMPLITUDE

(a) $n = 0, 1$ and 2 , $0.2 \mu\text{sec/division}$

NORMALIZED AMPLITUDE

(b) $n = 2, 3, 4$ and 5 , $0.2 \mu\text{sec/division}$

Fig. 13. Experimental $1/D$ bandwidth detection of a pulsed signal vs photodetector location $[n/(2D)]$ from the signal centroid: ($PW/\tau = 1.0$, $PW = 0.625 \mu\text{s}$, $T_x = \alpha\tau = 0$)
 (a) $n = 0, 1$, and 2 , $0.2 \mu\text{s/div}$,
 (b) $n = 2, 3, 4$ and 5 , $0.2 \mu\text{s/div}$.

aperture while its main lobe width decreases proportionally to it. The minimum main lobe width is reached when the pulse completely fills the aperture, with its first nulls located at $u = \pm 1/D$ from the centroid and the other nulls spaced at $1/D$ apart.

In the case where $|n| = 0, 1$, the photodetector is always situated inside the main lobe of the spectrum and, as a result, the detected output increases monotonically as the pulse enters the aperture. It reaches a maximum level when the pulse completely fills the aperture and then decreases to zero in the same fashion as the pulse leaves the aperture.

On the other hand, for a photodetector situated in other locations, it is initially located inside the main lobe of the spectrum as the pulse enters the aperture and the detected output increases with time. As the pulse moves in further, the nulls start moving in closer toward the centroid; every time a null crosses the photodetector, a dip in the output magnitude occurs and thus gives rise to the appearance of a hump in the detected waveform.

The number of humps appearing on the detected waveform from the n th photodetector is $|n|$ for $n \neq 0$. For example, if $|n| = 4$, the output registers four humps as the pulse propagates through the aperture, and with a maximum level of only 14.8 dB below that of the photodetector located at the spectral centroid.

So far we have only considered the cases where the spectral centroid

is located either at the center or in between two photodetectors. However, in a practical situation, the spectral centroid of a given signal may be located anywhere along the photodetector array. When this happens, the magnitude and the number of humps appearing on the detected output will be slightly different from the above two cases, and they will fall somewhere in between.

As can be seen from Fig. 12 , the maximum output from the photodetector array has only one peak (without the multiple-hump structure) appearing on the waveform. In the case where the spectral centroid is located in between two adjacent photodetectors, there are two identical maximum outputs. In all the other cases, there is only one such maximum output. If the weighting function is not a rectangular one, its effect on the instantaneous spectrum is to broaden the main lobe and to alter the sidelobe levels and spacings. This effect increases in direct proportion to the amount of overlapping between the pulse signal and the aperture. The broadening effect will move the first nulls further away from the centroid thus ensuring that there is at least one maximum output with only one peak appearing on the waveform. The change in the sidelobe levels and spacings will affect the shape and the number of humps detected by those photodetectors located off the signal spectral centroid.

The effect of the weighting parameters on the detected waveform is illustrated in Fig. 14. The detected waveform is taken from the photodetector which is centered with respect to the spectral centroid.

Fig. 14. 1/D bandwidth detection of equipower pulsed signals as a function of the weighting parameters: ($PW/r = 1.0$ and 0.5).

For each set of weighting parameters, the response is plotted for two normalized pulse widths of $PW/\tau = 0.5$ and 1. The maximum response is normalized to a value of unity. As can be seen from the graph, the shape of the detected waveform is a strong function of the weighting function parameters.

The response for other pulse widths can also be derived from Fig. 12 when the weighting function is of the same rectangular form. If the normalized pulse width is greater than unity by $\Delta t/\tau$, the waveform at $t/\tau = 1.0$ will remain at that level for the same time duration. On the other hand, if the normalized pulse width is less than unity by $\Delta t/\tau$, the waveform will only reach the level at $1 - \Delta t/\tau$ and remain at that level until t/τ equals unity. It will then retrace the waveform from $t/\tau = (1 - \Delta t/\tau)$ back to $t/\tau = 0$.

In all the discussions presented so far, the impulse response of the detection circuit is approximated by the delta function $\delta(t)$. If the bandwidth of the detection circuit is not much larger than the detected signal bandwidth, the exact output waveform will depend on the bandpass nature of the detection circuit. It has the general effect of smoothing out the hump structure and stretching the output in time.

4.5 PULSE WIDTH DETERMINATION

As given in Eq. (4.24), the detected output waveform from a

photodetector is a function of many parameters. It is in general a function of the pulse width, relative photodetector to spectral centroid location, spatial weighting function of the photodetector array, amplitude weighting function parameters, and the bandwidth of the detection circuit. The first two parameters are the unknowns to be determined by the acoustooptic spectrum analyzer. However, there are two characteristics which are independent of the above parameters, namely, (i) the input pulse width of the signal is stretched by an amount equal to the aperture transit time, and (ii) if the pulse width exceeds the aperture transit time by ΔT , the detected output reaches a maximum level for the same duration ΔT .

The above two characteristics can be utilized to determine the pulse width of an input signal. Let us take a signal scenario where there may be simultaneous signals present but nonoverlapping in the frequency plane. The first function performed by the post-processor is to identify the photodetector output where the signal spectral centroid is located. This is obtained simply by finding the maximum photodetector output free of the multiple-hump structure. The second function is to determine the pulse width from this maximum output.

One approach is to make use of characteristic (i) where the pulse width is stretched by the aperture transit time,

$$T = PW + \tau$$

(4.25)

The pulse width can be estimated by measuring the duration of the detected signal and then subtracting from it the aperture transit time. This can be done by setting a certain threshold level above the noise floor to determine both the leading and trailing edges of the signal. It is apparent that this scheme works well for output signals with a large signal-to-noise ratio where both the leading and trailing edges are clearly defined. The measured pulse width becomes shorter as the signal-to-noise ratio decreases. If the parameters of the weighting function are large, the effective aperture transit time is reduced and an adjusting factor has to be used. Furthermore, if the impulse response of the detection circuit cannot be approximated by the delta function, the output waveform will be stretched further and an additional adjusting factor must be used.

Another approach to determine the pulse width is to make use of the second characteristic. If the pulse width exceeds the aperture transit time by ΔT , the detected output increases initially until a maximum level is reached when the pulse fills the aperture. The output stays at this maximum level for the same time duration ΔT and then decreases to zero as the trailing edge of the pulse leaves the aperture. The pulse width of the input signal is related to ΔT by

$$PW = \Delta T + \tau \quad (4.26)$$

To ensure that there is always a flat maximum region in the output of the detected waveform, the aperture transit time must be chosen shorter than the narrowest pulse width to be detected. This is not a serious constraint and it actually turns out to be what can be achieved in practice for broadband receiver applications. A Bragg cell with a bandwidth of 1 GHz has a limited useful aperture width due to high acoustic loss arising from operating in the higher frequency range. The input laser beam is confined to the region closest to the transducer to achieve a high diffraction efficiency. In addition, the number of parallel outputs must be limited in this mode of operation and thus fine frequency resolution is not required. A coarser frequency resolution means a shorter aperture transit time.

For example, if the aperture transit time is $0.25 \mu s$, any pulse width exceeding that value will have a flat maximum level in the detected output. This aperture will give a spot size equivalent to a frequency resolution of 4 MHz.

In a practical situation, both approaches to pulse width determination can be employed to take advantage of the merits each one has to offer.

In a real environment, the amplitude of the received pulse may be distorted. Both the rise and fall times are of finite duration. Moreover, the pulse envelope is usually corrugated with ripples. These perturbations will generally distort the shape of the detected waveform and increase both the rise and fall times.

The ripples are usually small in amplitude and have a higher spatial frequency content than the pulse envelope. The power intercepted by the photodetector depends on the frequency components of the ripples. In general, there is an averaging effect applied to those spatial components falling within the photodetector's spatial bandwidth. This will in turn contribute some distortions to the detected output and affect the shape of the maximum flat region. However, these distortions are small in amplitude in comparison with the rising and falling edges of the detected output and can be taken into account by the postprocessor. If the spatial components fall outside the photodetector of interest, they will not affect the output but will contribute to the outputs of the adjacent photodetectors.

4.6 SUMMARY AND CONCLUSIONS

The temporal and spatial characteristics of the acoustooptic spectrum analyzer for pulse-modulated cw signals, have been investigated. The instantaneous spectral distribution generated by the spectrum analyzer as the pulse propagates through the Bragg cell aperture, is detected by using a photodetector array operating in the real-time mode. The center-to-center element spacing used in the analysis and measurements, has a normalized spatial bandwidth equal to the reciprocal of the aperture. This configuration is mainly used as a frequency channelizer in which a coarse frequency of the signal is

obtained while the time resolution is very good. The detected signal waveform has been studied as a function of pulse width, amplitude weighting function parameters, and relative spectral centroid to photodetector location.

It was observed that the input pulse envelope is modified by the acoustooptic spectrum analyzer in a number of ways. It is always stretched in duration by an amount equal to the aperture transit time. Moreover, in the case where the pulse width is longer than the aperture time, the difference in duration was found to appear as a flat top on the detected output. The shape of the detected waveform is strongly affected by the amplitude weighting parameters. Furthermore, if the photodetector is located off the main lobe of the signal spectrum and detecting its sidelobes, multiple humps are detected.

However, despite the appearance of complicated waveform distributions, the first two characteristics, namely, the signal duration stretching and the duration of the maximum flat region were found to be simply related to the aperture transit time. This is especially true for the photodetector located at the spectral centroid where the output level is the highest and free of the multiple-hump structure. These two characteristics can then be utilized effectively in the determination of the pulse width.

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CHAPTER 5

ACOUSTOOPTIC SPECTRUM ANALYZER: DETECTION USING

A TIME-INTEGRATING PHOTODETECTOR ARRAY

5.1 INTRODUCTION

In the last chapter, both the spatial and temporal characteristics of the acoustooptic spectrum analyzer to pulse-modulated cw signals have been analyzed. The detected signal waveform from a photodetector array operating in the real-time mode has been analyzed as a function of the pulse width, weighting function parameters, and the relative position of the photodetector to the spectral centroid. In this chapter, we investigate the signal waveform distributions of pulse-modulated cw signals and a linear FM signal using a time-integrating photodetector array.

In the Fourier transforming configuration, the Fourier transform is computed over a sliding window of the weighted input signal. The transformed signal appears as a light distribution in the back focal plane of the Fourier transform lens. This light distribution, which is the magnitude squared of the Fourier transform of the signal, is detected by a time-integrating photodetector array. At different

instances in time, the Fourier transform of a different portion of the weighted signal is performed. As the signal passes through the Bragg cell aperture, the instantaneous power spectra generated, are continuously integrated by the photodetector array. The time-integrated charge packets at discrete photodetector elements are then multiplexed to produce a sampled data waveform.

The key function of the acoustooptic spectrum analyzer is to provide power spectral density estimates of signals. In this operation, one is more interested in the total power spectral density rather than the instantaneous spectra. The use of a time-integrated photodetector array for detection, also helps to simplify and reduce the amount of data generated from such a processor. Moreover, silicon MOSFET scanned and CCD scanned solid-state linear photodetector arrays with up to 2048 elements are commercially available [30]-[33]. Another advantage is the increased sensitivity which may be achieved through the non-coherent integration produced by the time-integrating array [34],[35].

The time-integrated spectrum is obtained by integrating the instantaneous power spectrum in time and a voltage is produced in each photodetector as

$$V_{\text{out},K} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H(x_1 - x_{1K}, y_1 - y_{1K}) \int_{t_0}^{t_0 + T} |U_1(x_1, y_1, t)|^2 dt dx_1 dy_1 \quad (5.1)$$

where x_{1K} and y_{1K} are the spatial coordinates of the k th photodetector element, H is the spatial weighting function of the photodetector element, T is the integration period and t_0 is some initial time, $|U_1(x_1, y_1, t)|^2$ is the instantaneous spectral distribution of the signal and the integral with respect to time is the time-integrated spectrum.

In the case of a cw signal, the time variable is no longer a parameter and the shape of the spectrum is just a Fourier transform of the weighting function. The effects of the weighting function on the power spectrum of a cw signal have been analyzed [4]. A general statistical analysis on the noncoherent processing gain for various types of noise statistics as a function of the integration period T , has been given for the case of a cw signal [35]. In addition, the degradation in signal sensitivity due to noise arising from the photodetection process, and the trade-off between noncoherent processing gain versus dynamic range have also been investigated [34].

In the case of a non-cw signal, the instantaneous spectrum is more complex and it changes as the signal propagates across the Bragg cell aperture. The time-integrated spectrum is in general different from the true spectrum of the signal. The effects on the time-integrated spectral distribution, due to the truncation and weighting by the Bragg cell aperture, are analyzed in this chapter. Pulse-modulated cw signals with various pulse widths and a linear FM signal are investigated

experimentally and then compared with theory.

5.2 PULSE-MODULATED CW AND LINEAR FM SIGNALS

The block diagram of the experimental set-up is shown in Fig. 15, where a 2-D Fourier transform is performed by the spherical lens. In the frequency plane, the amplitude distribution for a given input signal $g(t)$, is obtained by using Eq. (3.10), which is

$$U_1(x_1, y_1, t) = K_y \int_{-\infty}^{\infty} g\left(t - \frac{x_0}{v_s}\right) w(x_0) \exp\left(-j \frac{2\pi}{\lambda F} x_1 x_0\right) dx_0 \quad (5.2)$$

where

$$K_y = \frac{K}{\lambda F} \int_{-H/2}^{H/2} \exp\left[-(2Ty - \frac{y_0}{H})^2\right] \exp(-j2\pi v y_0) dy_0 \quad (5.3)$$

A pulse-modulated cw signal is characterized by a fixed carrier frequency (f_s) and the acoustic signal function is given by Eq.(3.11), that is,

$$g(t - x_0/v_s) = A(t - x_0/v_s) \exp[-j2\pi f_s (t - x_0/v_s)] \quad (5.4)$$

The acoustic signal function has been approximated by assuming the acoustooptic crystal produces a self-collimating beam. The justification of this will be given in Section 5.3. The instantaneous

Fig. 15. Schematic diagram of acoustooptic spectrum analyzer.

spectrum for a pulse of amplitude C is obtained by substituting Eq. (5.4) into Eq. (5.2). After solving the integral and expressing the solution in terms of the error function [36], we have

$$U_1(x_1, y_1, t) = C K y / 2 \exp(-j 2 \pi f_s t - \alpha \tau / 2) (\pi / a)^{1/2} \cdot \exp(b^2 / a) [\operatorname{erf}(a^{1/2} B + b / a^{1/2}) - \operatorname{erf}(a^{1/2} A + b / a^{1/2})] \quad (5.5)$$

$$\text{where } a = (2 T x / D)^2, \text{ and } b = j \pi [u - f / v_s - j \alpha \tau / (2 \pi D)] \quad (5.6)$$

B and A are the upper and lower limits respectively of the integral and they are functions of time.

In the case of a linear FM signal, the signal function is characterized by

$$g(t - x_0 / v_s) = C(t - x_0 / v_s) \exp\{-j 2 \pi [f_0(t - x_0 / v_s) + k/2 (t - x_0 / v_s)^2]\} \quad (5.7)$$

where f_0 is the frequency referenced to the center of the aperture and k is the rate of change of frequency in (Hz/sec).

5.3 EXPERIMENTAL ARRANGEMENT

Referring to Fig. 15, the optical source is a Spectra Physics Model 124B helium neon laser which delivers 15 mW of coherent optical power at $0.6328 \mu\text{m}$. The laser beam is expanded in one dimension by the anamorphic beam expander to a width of 20.5 mm with a Gaussian intensity profile truncated at $1/e^2$ points. The isotropic bulk Bragg cell used is a FJW D-150 acousto-optic deflector with the Zenith phased array transducer giving a bandwidth of about 100 MHz at a center frequency of 150 MHz. The aperture dimensions used in the experimental measurements are 2 mm by 20.5 mm with a corresponding transit time of 5 μsec . The acoustic loss coefficient is measured to be 0.1 nepers/ μsec at the center frequency. A thin circular Achromat lens of diameter 50.8 mm with a focal length of 0.4 m is used to Fourier transform the weighted signal as it forms the far-field intensity distribution in its back focal plane. The output intensity distribution is detected and integrated on a Fairchild CCD 110/110F linear image sensor with 256 elements. The cell size is 13 μm by 17 μm on 13 μm centers with a channel stop width of 5 μm . The information stored in the elements is clocked out serially by CCD shift registers and displayed on an oscilloscope.

For the isotropic Bragg cell, the singularity curve appears as a circle in the Fourier plane as described in Section 3.4. The photodetector array used in the experimental set-up is located along the frequency axis and centered at $y_1 = 0$, as shown in Fig. 16. The sector of the circle subtended by the photodetector array is slightly deviated

Fig. 16. Orientation of Bragg cell diffraction singularity curves with respect to linear photodetector array.

from that of the vertical line. The maximum deviation which appears at $\Delta y_1 = 8.5 \mu\text{m}$ (half the photodetector height) and at $f_s = 150 \text{ MHz}$, is obtained by using Eqs. (3.26) and (3.30), therefore

$$\begin{aligned}\Delta x_1 &= \Lambda / (2 \lambda F) (\Delta y_1)^2 \\ &= 4 \times 10^{-9} \text{ m}\end{aligned}\tag{5.8}$$

Normalizing this deviation by the element center-to-center spacing of $13 \mu\text{m}$, we obtain a deviation of 3×10^{-4} element.— This is indeed very small and thus the acoustic signal function can be approximated by assuming the crystal produces a self-collimating acoustic beam.

5.4 COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS FOR PULSE-MODULATED cw SIGNALS

Using Eq. (5.5) the instantaneous power spectra for a 5- μsec pulse-modulated cw at 150 MHz are plotted in Fig. 17. They are plotted at different instants of time as the pulse propagates across the Bragg cell. The acoustic signal is attenuated and illuminated by different portions of the Gaussian profile on its course through the aperture. The effect of the amplitude weighting function on the instantaneous power spectrum is to broaden the main lobe and suppress the sidelobe levels. Shown in Fig. 17 are the two theoretical plots of the instantaneous power spectra, one with the signal completely coincident

Fig. 17. Light intensity distribution in the frequency plane for a 5 μ sec pulse-modulated cw at two different instances of time ($\alpha = 0.5 N_p/5 \mu$ sec, $T_x = 1$, $D = 20.5$ mm, $cw = 150$ MHz, and $F = 0.4$ m).

with the aperture and the other with one half of the pulse interacting. As can be seen from the graph, the truncation and amplitude weighting on the signal cause the frequency components to spread out with a corresponding drop in power.

Summing up all the spectra at different instants of time, an integrated power spectrum is obtained and shown in Fig. 18. The total integration time is 10 μ sec which is the time interval for the pulse to propagate across the aperture. Figure 18 also shows the spectrum of a cw signal integrated for the same interval of time, or equivalently, it is the spectrum of a stationary 5 μ sec-pulse filling up the aperture and time integrated for 10 μ sec. By comparing the two plots, the effect due to the truncation and amplitude weighting on the signal by the finite aperture is to broaden the main lobe and to smooth out the sidelobes. This effect is measured experimentally using a narrower aperture width of 0.5125 cm and is shown in Fig. 19. Identical photocells are used in recording the two waveforms in order to minimize the effect of cell response variations. Note the definite spread in the main lobe and the disappearance of the sidelobe structure in the figures. The theoretical light intensity distribution is graphically integrated with a cell width of 13 μ m and the results are tabulated in Table I along with the experimental values.

Theoretical results are plotted in Figs. 20 and 21 for a 1- μ sec and 2- μ sec pulse-modulated carriers along with the corresponding experimental measurements shown in Figs. 22. The theoretical and

Fig. 18. Integrated light intensity distribution in the frequency plane for both cw and a 5- μ sec pulse-modulated cw. (Integration time = 10 μ sec, α = 0.5 Np/5 μ sec, T_x = 1, ω = 150 MHz, D = 20.5 mm, and F = 0.4 m).

RELATIVE POWER

(a) - CW

RELATIVE POWER

(b) - 1.25 μ SEC PULSE

Fig. 19, Integrated light intensity distribution in the frequency plane detected by photodetector array (cell-to-cell center spacing = $13\ \mu\text{m}$, integration time = $50\ \mu\text{sec}$, $\alpha = 0.5\ \text{Np}/5\ \mu\text{sec}$, $T_x = 0.25$, $D = 5.125\ \text{mm}$, $F = 0.4\ \text{mm}$, and $\text{cw} = 150\ \text{MHz}$).

TABLE I

COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL INTEGRATED INTENSITY DISTRIBUTIONS FOR A cw SIGNAL, AND A PULSED SIGNAL WITH THE WIDTH EQUAL TO THE APERTURE TRANSIT TIME

(D = 0.5125 cm, $T_X = 0.25$, $\alpha = 0.125/\tau$, F = 0.4 m)

PHOTOCELL LOCATION	cw SIGNAL		PULSED SIGNAL PW = 1.25 μ SEC	
	THEORETICAL INTENSITY	EXPERIMENTAL INTENSITY	THEORETICAL INTENSITY	EXPERIMENTAL INTENSITY
Center Cell (normalized to unity)	1.0	1.0	1.0	1.0
1st Cell from center	0.81	0.82	0.88	0.90
2nd	0.38	0.37	0.58	0.55
3rd	0.06	0.07	0.03	0.29
4th	0.01	0.01	0.13	0.12
5th	0.03	0.03	0.08	0.08
6th	0.03	0.03	0.06	0.05

Fig. 20. Integrated light intensity distribution in the frequency plane for both cw and a 1- μ sec pulse-modulated cw. (Integration time = 6 μ sec, cw = 150 MHz, α = 0.5 Np/5 μ sec, T_x = 1, D = 20.5 mm, and F = 0.4 m).

Fig. 21. Integrated light intensity distribution in the frequency plane for both cw and a 2- μ sec pulse-modulated cw.

(Integration time = 7 μ sec, cw = 150 MHz, = 0.5 Np/5 μ sec, Tx = 1, D = 20.5 mm, and F = 0.4 m).

(a) cw and 1 μ sec pulse-modulated cw with an offset in frequency.

(b) cw and a 2 μ sec pulse-modulated cw with an offset in frequency.

Fig. 22. Integrated light intensity distribution in the frequency plane detected by photodetector array (cell-to-cell center spacing = 13 μ m, integration time = 50 μ sec, α = 0.5 Np/5 μ sec, T_x = 1, D = 20.5 mm, and F = 0.4 m).

experimental values are tabulated in Table II.

The experimental results presented agree very well with the theoretical calculations. The major errors are due to the cell-to-cell boundary structure and the photoresponse nonuniformity of the cells.

5.5 COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESULTS FOR A 5- μ SEC PULSE-MODULATED LINEAR FM SIGNAL

The theoretical time-integrated power spectrum of a linear FM signal is plotted in Fig. 23. and the power spectrum of the same signal, but stationary in the aperture and time integrated for the same interval of time, is also plotted for comparison. The duration of the signal is 5 μ sec with a center frequency of 150 MHz and a frequency excursion of 1 MHz. As expected, there is a spread in frequency and smoothing of the sidelobes for the time-integrated output. The experimental output power spectrum is also measured and shown in Fig. 24. Some comparisons between the experimental and theoretical values are tabulated in Table III. The theoretical power spectrum is graphically integrated with a cell width of 13 μ m, and they agree reasonably well.

5.6 EFFECTS OF THE AMPLITUDE WINDOW WEIGHTING FUNCTION ON THE TIME-INTEGRATED WAVEFORM OF A SIGNAL

A few specific examples have been presented, showing the

TABLE II

COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL INTEGRATED INTENSITY
DISTRIBUTIONS FOR A 1 μ SEC AND 2 μ SEC PULSE-MODULATED cw SIGNALS

PHOTOCELL LOCATION	PW = 1 μ SEC		PW = 2 μ SEC	
	THEORETICAL INTENSITY	EXPERIMENTAL INTENSITY	THEORETICAL INTENSITY	EXPERIMENTAL INTENSITY
Center Cell (normalized to unity)	1.0	1.0	1.0	1.0
1st Cell from center	0.87	0.88	0.59	0.58
2nd	0.55	0.53	0.11	0.13
3rd	0.24	0.23	0.04	0.05
4th	0.06	0.06	0.03	0.03
5th	0.02	0.02		
6th	0.03	0.04		
7th	0.05	0.05

Fig. 23. Integrated light intensity distribution in the frequency for a 5- μ sec pulse-modulated linear FM (integration time = 10 μ sec, $f_0 = 150$ MHz, $\Delta f = 1$ MHz, $\alpha = 0.5$ Np/5 μ sec, $T_x = 1$, $D = 20.5$ mm, and $F = 0.4$ m):

Fig. 24. Integrated light intensity distribution in the frequency plane of a 5- μ sec pulse-modulated linear FM detected by photodetector array (cell-to-cell center spacing = 13 μ m, integration time = 50 μ sec, f_0 = 150 MHz, Δf = 1 MHz, α = 0.5 Np/5 μ sec, T_x = 1, D = 20.5 mm, F = 0.4 m).

TABLE III

COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL INTEGRATED
INTENSITY DISTRIBUTION FOR THE LINEAR FM SIGNAL

LINEAR FM PULSE WIDTH = 5 μ SEC f_0 = 150 MHz Δf = 1 MHz	THEORETICAL INTENSITY	EXPERIMENTAL INTENSITY
Center cell (at 150 MHz, Normalized to unity)	1.0	1.0
1st cell on right of center cell	0.79	0.84
2nd	0.62	0.36
3rd	0.34	0.10
1st cell on left of center cell	0.95	0.96
2nd	0.92	0.88
3rd	0.57	0.30
4th	0.27	0.12

relationship between the time-integrated output waveform distribution and the input signal.

The time-integrated output waveform is distorted from the spatial Fourier spectrum of the signal by two contributing factors of the amplitude window weighting function. One is due to the truncation of the signal by the finite aperture. The other is due to the amplitude weighting on the signal when it is in the aperture. The time-integrated output is just a summation of the various waveforms generated as the pulse propagates across the aperture.

As exemplified by the few specific examples, the window amplitude weighting function modifies the signal, and thus the time integrated output waveform differs from the true spatial Fourier transform of the signal. There are in general two effects: (1) a broadening effect on the main lobe width; (2) the smoothing effect on the sidelobe structure.

A. MAIN LOBE WIDTH BROADENING

A plot showing the normalized half-power main-lobe width of the time-integrated spectrum versus the normalized pulse width as a function of the amplitude weighting function is shown in Fig. 25. The main-lobe width (Δ) of the time-integrated spectrum is normalized by the main-lobe width (Δ_0) of the Fourier transform of the same pulse signal but with an infinitely long rectangular weighting function. In other words, Δ_0 is the main-lobe width of the spectrum of the pulsed signal and which is

Fig. 25. Main-lobe broadening of a time-integrated pulse vs the normalized pulse width as a function of amplitude weighting.

characterized by the familiar sinc function. The pulse width is normalized by the aperture transit time of the Bragg cell. The broadening effect due to truncation on the main-lobe width of the spectrum is given by the curve $T_x \rho = 0$. The distortion increases with increasing normalized pulse width, T_x and α . As the normalized pulse width becomes smaller, the distortion introduced will be small. The reason is that the pulse will appear to be more rectangular in shape at all instants of time, and the truncation effect is negligible. As the normalized pulse width becomes much greater than unity, the output waveform will resemble more and more of the output of a cw signal. The main-lobe width will approach a constant value while Δ_0 is inversely proportional to the normalized pulse width. As a result, each curve in the graph will approach a constant slope as PW/τ approaches infinity. The constant slope can be found by taking the ratio of the main-lobe width (Δ) to the main-lobe width (Δ_0) of τ seconds in duration. The slopes for the various curves are also given in Fig. 25.

B. SIDELOBE STRUCTURE

It is difficult to describe the sidelobe levels quantitatively as a function of the window amplitude weighting. The general effect on the time-integrated output is to smooth out the sidelobe structure. The greatest effect occurs when the pulse width matches the Bragg cell transit time. With PW/τ around unity, the output waveform distribution becomes very smooth, and the sidelobe structure disappears completely.

The smoothing effect increases with increasing values of both T_x and α .

5.7 SUMMARY AND CONCLUSIONS

The spatial frequency characteristics of the acoustooptic spectrum analyzer for pulse-modulated cw and a linear FM signals have been analyzed using a time-integrating photodetector array. The time-integrated power spectra of three pulsed signals with different pulse widths and a linear FM signal were measured and shown to confirm with the analysis given.

The time-integrated spectrum is an estimate of the spectrum of the input signal. The distortion on the spectrum due to truncation of the signal by the finite Bragg cell aperture and the amplitude weighting on it, was shown to have an effect of broadening the main lobe and smoothing its sidelobes. This effect increases with increasing values of the amplitude weighting parameters T_x and α . The broadening effect on the main lobe of the spectrum for pulsed signals has also been quantitatively characterized in a form of a normalized plot as a function of the amplitude weighting parameters. Furthermore, When the pulse width is about the same duration as the Bragg cell aperture transit time, the shape of the time-integrated spectrum was found to be very smooth with no sidelobe structures.

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CHAPTER 6

INTERFEROMETRIC ACOUSTOOPTIC SIGNAL PROCESSOR FOR SIMULTANEOUS DIRECTION FINDING AND SPECTRUM ANALYSIS

6.1 INTRODUCTION

The basic two-dimensional acoustooptic configuration was first proposed by Lambert et al [3] to process radar echo signals received by the elements of a large phased array. The array structure considered, was periodic with spacings less than half of the radar signal carrier wavelength. It was used for radar applications where the signal was known and the bandwidth of operation was narrow.

In wideband receiver applications, the requirements are quite different, as mentioned in Chapter 1, and a different set of design criteria has to be used. The key differences are that the frequency bandwidth is much broader and the number of antenna elements used must, for economic reasons, be kept small. Some simulated and experimental results on a multi-channel Bragg cell configuration have also been reported [37].

In this chapter, a general mathematical formulation is given on this two-dimensional optical configuration. Analyses are presented for a linear array configuration for both direction finding and spectrum analysis. The effects of the amplitude window weighting function, scaling factor and error sources on the output waveform distribution are analyzed. Trade-offs between periodic and aperiodic configurations in implementing a practical processor along with comparisons on both the optical and microwave far-field patterns are discussed. Finally a typical example is given of the synthesis of an aperiodic array design using four nonuniformly spaced elements of equal weightings.

6.2 DIRECTION FINDING AND SPECTRUM ANALYSIS EMPLOYING A LINEAR ARRAY

In this section, analysis is presented of spectrum determination and direction finding, employing a multi-channel Bragg cell using signals derived from a linear antenna array as shown in Fig. 26.

For a cw or pulse-modulated carrier signal received from an emitter at an angle θ_0 , the relative signal strength at the n th antenna is given by

$$V_n(t) = E_n(f_R, \theta_0) A(t - \tau_n') \cos[2\pi f_R(t - \tau_n')] \quad (6.1)$$

Fig. 26. Configuration of a linear aperiodic array.

$$\text{where } \tau'_n = \frac{d_n}{c} \sin(\theta_0) = \frac{d_n}{\lambda_R f_R} \sin(\theta_0), \quad n=1,2,3,\dots,N \quad (6.2)$$

$E(f_R, \theta_0)$ is the normalized antenna element far-field pattern, f_R is the RF frequency, d_n is the distance measured from the array center to the n th antenna element. It is positive when located to the right of the center and negative when on the left.

If the maximum delay $\tau'_n \ll$ the duration of the signal, we can neglect the envelope delay, and the heterodyned n th IF signal is

$$v_{IF,n} = A(t) \cos[2\pi(f_1 t - \beta_n)] E(f_R, \theta_0) \quad (6.3)$$

$$\text{where } \beta_n = \tau'_n f_R = \frac{d_n}{\lambda_R} \sin(\theta_0) \quad (6.4)$$

The IF signal derived from each antenna element of frequency (f_1) is fed into a multi-channel Bragg cell as shown in Fig. 27. The Bragg cell consists of separate and parallel acoustic columns. The ratios of the acoustic channel separation spacings are the same as those of the microwave antenna spacings.

In this chapter, we will assume that the acoustooptic material produces a self-collimating beam in the y direction (an ideal traveling wave modulator) and thus the acoustic beam can simply be represented by a rectangular function. We will investigate the general case in the

INPUT SIGNALS FROM ANTENNA ELEMENTS

Fig. 27. Multi-channel Bragg cell configuration for a linear aperiodic antenna array.

next chapter when the acoustic beam is not self-collimating. Substituting Eq. (6.3) into Eq.(3.9), the complex amplitude transmission function of the Bragg cell is

$$\begin{aligned}
 T(x_0, y_0, t) = & \sum_{n=1}^N \text{rect}\left(\frac{x_0}{D}\right) \text{rect}\left(\frac{y_0 - \ell}{w}\right) \\
 & \cdot \{1 + jK_B A(t - x_0/v_s) \exp[-\alpha\tau(\frac{x_0}{D} + \frac{1}{2})] E_n(f_R, 0_0) \\
 & \cdot \exp[-j[2\pi f_i(t - x_0/v_s) - 2\pi\beta_n + \phi_0]]\} \quad (6.5)
 \end{aligned}$$

The multi-channel Bragg cell is used in the same Fourier transforming configuration as described in Chapter 3. A collimated laser source is used to illuminate the multi-channel Bragg cell at the Bragg input angle. A 2-D Fourier transform is carried out using a spherical lens with a focal length F . Substituting Eq.(6.5) into Eq.(3.2) and making use of the Shift theorem of the Fourier transform in the y_0 dimension, the diffracted order which contains the signal information is given by

$$\begin{aligned}
 U_1(x_1, y_1, t) = & K_2 f(v) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(t - x_0/v_s) w(x_0, y_0) \\
 & \cdot \exp[-j \frac{2\pi}{\lambda F} (x_0 x_1 + y_0 y_1)] dx_0 dy_0 \quad (6.6)
 \end{aligned}$$

where

$$K_2 = \frac{NK E(f, \theta_0)}{\lambda F} \exp(-j\phi_0)$$

(6.7)

$$f(v) = \frac{1}{N} \sum_{n=1}^N a_n(f_R, \theta_0) \exp[-j2\pi(v\ell_n - \beta_n)]$$

$f(v)$ is the normalized complex far-field pattern of the optical antenna array, $v = y_1/(\lambda F)$, ℓ_n is the distance measured from the center of the acoustic transducer array to the n th transducer element, a_n is the normalized amplitude weighting of the diffracted light beam appearing in the n th channel, and

$$w(x_0, y_0) = \text{rect}\left(\frac{x_0}{D}\right) \text{rect}\left(\frac{y_0}{w}\right) \exp\left[-\alpha_T \left(\frac{x_0}{D} + \frac{1}{2}\right)\right] \exp\left[-(2Tx \frac{x_0^2}{D} - (2Ty \frac{y_0^2}{w})^2)\right] \quad (6.8)$$

is the window weighting function for Gaussian interaction profile. T_x and T_y specify the Gaussian profile in the x_0 and y_0 dimensions respectively. w is the height of the acoustic beam.

6.3. EFFECTS OF AMPLITUDE WINDOW WEIGHTING FUNCTION

$w(x_0, y_0)$ ON OUTPUT WAVEFORM DISTRIBUTIONS

For the simple case of a continuous wave or pulse-modulated cw filling up the Bragg cell aperture, and with

$$w(x_0, y_0) = \text{rect}\left(\frac{x_0}{D}\right) \text{rect}\left(\frac{y_0}{w}\right) \quad (6.9)$$

The instantaneous diffracted light distribution in Eq.(6.6) can be simplified to

$$U_1(u,v,t) = \frac{K_B E(f, \theta_0) D w}{\lambda F} \exp[-j2\pi f_i(t + \phi_0)] \\ \cdot \text{sinc}[D(u - \frac{1}{\Lambda})] \text{sinc}(vw) \sum_{n=1}^N \exp[-j2\pi (v \ell_n - f_R \tau_n')] \quad (6.10)$$

where Λ is the acoustic wavelength.

The angle of arrival information is completely contained in the optical array pattern term. Its shape must be preserved in order to determine the main beam location. This can be achieved if the weighting function $[\text{sinc}(vw)]$ is kept relatively constant and varies smoothly over all values of v of interest.

If the requirement is made that the $\text{sinc}(vw)$ function is not going to vary by more than 10% from its maximum value of unity within the scan angle (θ_0) of $\pm 90^\circ$. That is at

$$v = \frac{d}{\lambda_R} \frac{n}{\ell_n} \sin(\theta_0) \quad , \theta_0 = \pm 90^\circ \quad (6.11)$$

From Eqs. (6.10) and (6.2) the following condition must be met.

$$\text{sinc}(vw) = \frac{\sin\left[\pi\left(\frac{d_n w}{\lambda_R \ell_n}\right)\right]}{\pi \frac{d_n w}{\lambda_R \ell_n}} \geq 0.9 \quad (6.12)$$

$$\text{Therefore } w \leq \frac{\ell_n \lambda_R}{4 d_n} \quad (6.13)$$

In practical systems, the amplitude window weighting function is given by Eq. (6.8). D and w specify the regions where the collimated laser beam and acoustic beam interact. The function is assumed to be of truncated Gaussian distributions in both the x and y dimensions. The general effect of the truncated Gaussian laser profile and acoustic attenuation on both the instantaneous and time-integrated waveform distributions are well known [4],[27]. The general effect of the weighting function is to broaden the main lobe of the sinc function while the sidelobe level are suppressed. The condition stated by Eq. (6.13) will be relaxed slightly due to this broadening effect.

6.4 SCALING FACTOR (S.F.)

The main beam of the pattern is located at

$$v \ell_n = f_R \tau_n', \quad \text{for } n = 1, 2, \dots, N$$

$$y_1 = \frac{\lambda d_n}{\lambda_R \ell_n} F \sin(\theta_0) \quad (6.14)$$

where the ratio d_n / ℓ_n for this configuration, is a constant for all n , the location of the main lobe (y_1), is proportional to $\sin(\theta_0)$ and θ_0 is the angle of arrival of the incoming signal. Let us define the ratio of the normalized optical antenna array length in wavelengths by the scaling factor (S.F.), that is

$$\text{S.F.} = (d_n / \lambda_R) / (\ell_n / \lambda) = y_1 / [F \sin(\theta_0)] \quad (6.15)$$

The scaling factor is normally much less than unity and therefore the optical deflection angle (ϕ) of the main lobe is given by

$$\begin{aligned} \phi &= \tan^{-1}(y_1 / F) \approx y_1 / F, \quad \text{for } y_1 / F \ll 1 \\ &= \text{S.F.} \sin(\theta_0) \\ &\approx \text{S.F.} \theta_0, \quad \text{for } \theta_0 \ll 1 \end{aligned} \quad (6.16)$$

The above equation simply states that the optical antenna array steering angle is scaled down from the microwave antenna array steering angle by the scaling factor (S.F.). Once the location of the main lobe is located in the back focal plane, the angle of arrival of the emitter can be calculated as

$$\theta_0 = \sin^{-1} \left(\frac{y_1}{F \cdot \text{S.F.}} \right) \quad (6.17)$$

6.5 ERROR TOLERANCE ANALYSIS

An incoming electromagnetic wave front is sampled spatially by the linear antenna array. The signal received by each antenna element is then separately down-converted and amplified by various active and passive components. It is then finally converted into a traveling acoustic wave by the piezoelectric transducer. The resulting acoustic columns interact with the collimated laser beam, producing the desired diffracted wavefront.

A large number of occurrences can cause the actual optical array pattern to be far from the ideal predicted by the assumed deterministic models. Since most components are amplitude and phase sensitive to variations in frequency, then the signal passing through each channel experiences phase and amplitude errors. Phase errors are also introduced by errors in the placement of antenna elements and acoustic transducers. Mutual coupling can also be a major phenomenon affecting both amplitude and phase in both arrays. The optical system also introduces aberrations which affect both the phase and amplitude of the optical wavefront.

Irrespective of which phenomenon is responsible for an error, the only effect it may have is on the amplitudes and the phases of the signals. Hence any error sources may be typified by error δa_n and $\delta \phi_n$ to

the amplitude and phase of the n th channel, respectively.

A. Optical Array Pattern

Introducing the amplitude and phase errors into Eq. (6.7), the normalized optical array pattern is given by

$$f(v) = \frac{1}{N} \sum_{n=1}^N (a_n + \delta a_n) \exp(j \delta \phi_n) \cdot \exp[-j2 \pi (v \ell_n - f_R \tau_n')] \quad (6.18)$$

where the dependent variables have been dropped for simplification of notations. The expected value of the intensity pattern is

$$E[f(v) f^*(v)] = \frac{1}{N^2} E \sum_{n=1}^N \sum_{m=1}^N (a_n + \delta a_n)(a_m + \delta a_m) \cdot \exp[j(\delta \phi_n - \delta \phi_m)] \exp\{-j2 \pi [v(\ell_n - \ell_m) - f_R(\tau_n' - \tau_m')]\} \quad (6.19)$$

where * denotes complex conjugate.

In the following analysis, the means of the amplitude and the phase errors are assumed to be zero. All errors are independent of each other and the n th amplitude error is independent of the n th amplitude.

Following a similar analysis as given by Steinberg [38], the

expected intensity pattern for the array with equal amplitudes (a) can be shown to be

$$E[f(v)f^*(v)] = |\overline{\exp(j\delta\phi)}|^2 f_0(v) f_0^*(v) + \frac{a^2[1 - |\overline{\exp(j\delta\phi)}|^2] + \sigma_a^2}{N} \quad (6.20)$$

where σ_a^2 is the variance of the amplitude error. $f_0(v)$ is the designed pattern with no errors.

If the phase error is a normal power spectral distribution function with variance σ_ϕ^2 , then

$$\begin{aligned} |\overline{\exp(j\delta\phi)}|^2 &= |E[\exp(j\delta\phi)]|^2 \\ &= \left| \int_{-\infty}^{\infty} \exp(j\delta\phi) \frac{\exp[-\delta\phi^2/(2\sigma_\phi^2)]}{\sqrt{2\pi} \sigma_\phi} d\delta\phi \right|^2 \\ &= \exp(-\sigma_\phi^2) \end{aligned} \quad (6.21)$$

All the direction properties are in the first term of Eq. (6.20) and are unchanged from the intensity pattern of the error-free array. The remainder is an angle-independent contribution to the expected pattern, and its magnitude is inversely proportional to the number of channels. The dominant effect is a reduction in gain and a change in the main lobe to sidelobe ratios.

B. RMS Beam-Pointing Error

The amplitude errors alone produce no pointing errors. Hence only the effect of phase errors will be analyzed.

The assumed statistics are

$$\begin{aligned} E(\delta\phi_n) &= E(\delta\phi_m) = 0, \text{ all } n, m \\ E(\delta\phi_n \delta\phi_m) &= 0, \quad n \neq m, \\ \sigma_{\phi}^2 &= E(\delta\phi^2) \ll 1. \end{aligned} \quad (6.22)$$

Let $\delta\phi_n - \delta\phi_m = \delta_{nm}$ and $\ell_n - \ell_m = \ell_{nm}$. By arbitrarily steering the array to $v = 0$, the intensity pattern is given by

$$f(v)f^*(v) = \frac{1}{N^2} \sum_{n=1}^N \sum_{m=1}^N a_n a_m \exp[-j(2\pi v \ell_{nm} - \delta_{nm})] \quad (6.23)$$

The pointing direction is found by setting the derivative of the above equation with respect to the variable v to zero. Assuming the pointing error is small and follow a similar procedure as given by Steinberg [38], the expected second moment can be shown to be

$$E(2\pi v)^2 = \frac{\sigma_{\phi}^2 \left(\sum_{n=1}^N a_n \right)^2 \sum_{n=1}^N a_n^2 \ell_n^2}{\left[\sum_{n=1}^N a_n \sum_{n=1}^N a_n \ell_n^2 - \left(\sum_{n=1}^N a_n \ell_n \right)^2 \right]^2} \quad (6.24)$$

The normalized optical RMS beam-pointing error is then given by

$$\sigma_v = \frac{\sigma_\phi \sum_{n=1}^N a_n \left[\sum_{n=1}^N a_n^2 \ell_n^2 \right]^{\frac{1}{2}}}{2\pi \left[\sum_{n=1}^N a_n \sum_{n=1}^N a_n \ell_n^2 - \left(\sum_{n=1}^N a_n \ell_n \right)^2 \right]} \quad (6.25)$$

Expressing in Radians

$$\frac{\sigma_{y_1}}{F} = \frac{\Delta\phi}{2\pi} \sigma_\phi \frac{\sum_{n=1}^N a_n \left[\sum_{n=1}^N a_n^2 \ell_n'^2 \right]^{\frac{1}{2}}}{\left[\sum_{n=1}^N a_n \sum_{n=1}^N a_n \ell_n'^2 - \left(\sum_{n=1}^N a_n \ell_n' \right)^2 \right]} \quad (6.26)$$

where $\Delta\phi = \lambda/2\ell_N$ is the optical beam width, $\ell_n' = \ell_n/2\ell_N$ is the normalized acoustic column separation measured from the array center and σ_ϕ is the RMS phase error. The beam-pointing error is inversely proportional to the array length ($2\ell_N$) and for N large, it can be shown that it is approximately inversely proportional to the square root of N .

6.6 LINEAR PERIODIC ARRAY VERSUS APERIODIC ARRAY

The condition to avoid grating lobes under all conditions of beam-steering for a linear periodic array is

$$d \leq \lambda_R / 2 \quad (6.27)$$

where d is the element spacing and λ_R is the wavelength of the received signal.

Two important implications follow, the first pertaining to cost and the second to mutual coupling. The cost of a linear array with its separate RF channels is approximately proportional to the length $2\ell_N$. The resolving power of the array or its beamwidth is always proportional to the length $2\ell_N$. The angular resolution is just a function of the length of the array and not the number or distribution of elements. The effect of mutual coupling on closely spaced elements is another factor. The effect of mutual coupling among antenna elements can be reduced by overspacing the elements in the array, a process known as thinning. This process will introduce grating lobes unless the array is made nonperiodic. The problem of a thin array is the reduction of the designer's control of the radiation pattern in the sidelobe region. This arises because of its reduced number of elements, which offers the designer fewer degrees of design freedom than a filled array. There exists no single theory which governs the design of an aperiodic array [38]. With the exception of a random array, aperiodic arrays are designed ad hoc. Various design techniques for the aperiodic arrays have appeared in the literature to reduce the large sidelobes [39],[40], but the success has been quite limited.

Equal gain channels are highly practical in terms of design,

manufacture and optimum use of components. For broadband receiver designs, the physical size of the antenna may be a constraint in which the antenna diameter may not satisfy the requirement given by Eq. (6.27). In order to keep the number of channels to a reasonable number while still having some control over the main lobe width and sidelobe levels, four to five antenna elements with unequal spacings seem to be the optimum number.

6.7 COMPARISON BETWEEN MICROWAVE AND OPTICAL ANTENNA PHASED ARRAY FAR-FIELD PATTERNS

The far-field pattern of the optical configuration can also be split into a product of two functions; the element pattern and the array pattern.

The element pattern is determined by the interacting profile of the acoustic column and the laser beam. The effect of the element pattern on the far-field pattern has been presented above.

The far-field pattern distribution of the optical antenna array factor is a continuous function of the spatial coordinate with no bounds on the range of y_1 . In other words, there are no "invisible" regions as in the case of a microwave antenna array pattern. The microwave antenna array factor pattern, on the other hand, is a function of $\sin(\theta_0)$ which is periodic and within the range of ± 1 .

The shape of the optical array far-field pattern is unchanged by the phase tilt introduced when the signal arrives off boresight. Only the center or beam axis is altered. However, the radiation pattern of the microwave antenna array is distorted due to the trigonometric transformation relating the variables y_1 and θ_0 .

6.8 A SPECIFIC EXAMPLE

An example is given of the synthesis of an array using four nonuniformly spaced elements of equal weightings.

Consider a design where it is desired to cover the frequency range of 6.0 to 12.5 GHz. The bearing accuracy is to be in the order of 1° for the worst case of 30° RMS phase and 3 dB power imbalances. The total angular coverage is chosen as $\pm 45^\circ$ in order to provide a system to cover one quadrant in azimuth.

The main design criteria used are:

- (i) To minimize the mainbeam width while the peak sidelobe-to-main-lobe ratio is kept to a reasonable minimum.
- (ii) To optimize the channel spacings so that the ratio of the peak sidelobe-to-main-lobe level is least sensitive to perturbations of channel locations.

The beam pointing error is inversely proportional to the length of the baseline as given by Eq. (6.26) while the angular resolution is

directly proportional to it. It is also well known that the peak sidelobe to main lobe ratio is less susceptible to channel spacing variations for a longer baseline. On the other hand, the difficulty in controlling the peak sidelobe-to-main lobe ratio for a given number of channels is directly proportional to the length of the baseline. As a result, a compromise has to be made.

In order to realize the array, antenna elements and acoustic columns are chosen to be separated as far apart from each other as possible for ease of fabrication and reduction of mutual coupling effects.

For a baseline length of about 10.5 wavelengths at 12.5 GHz, there are up to 15 sidelobes on either side of the main lobe within the scanning angular range of $\pm 45^\circ$. Since there are only two degrees of freedom corresponding to the inner two element locations, at least two sidelobes out of the possible 15 can be optimized. However, no matter which algorithmic design is used, it is going to have little control of the radiation pattern.

On the other hand, since there are only four channels, the optimum locations of the inner two channels can be determined by using a computer. This is done by moving the two inner channels, one at a time, in discrete steps within the baseline and recording the main lobe to peak sidelobe ratios within the scanning angular range of $\pm 45^\circ$. A two-dimensional grid plot, showing the peak sidelobe-to-main-lobe ratios as a function of the two inner channel spacings, is used to determine the optimum channel spacings. By the conservation of radiation energy,

keeping the peak sidelobes to a minimum value has more or less the same effect of trying to equalize the sidelobe levels. It is found that there are various regions of minima of different sizes. The purpose is to identify some of the largest regions of about the same lengths and widths so that the two-dimensional plot can be reduced to a number of smaller and more manageable elements. These two-dimensional plots are then extended to three-dimensional plots by plotting perturbations of the baseline length on the other axis. The purpose is to find the largest pocket in a volume where the peak sidelobe to main lobe ratio is least sensitive to channel location perturbations and thus to phase errors as well.

An optimum two-dimensional plot is shown in Table IV for this specific example of a 10.5 wavelength baseline at 12.5 GHz. The optimum element spacings are 0, 3.71, 6.23 and 10.5 wavelengths, measured from element number 1. The normalized optical intensity patterns of the array are plotted in Fig. 28 for both a signal arriving on boresight with no errors and for one with 30° RMS phase error and 3 dB power error. Since the patterns are symmetrical, only one side is shown. The interaction profile of the acoustic column and laser is assumed to be of uniform width ($\lambda_N/30$). From Eqs. (6.16) and (6.26) the RMS pointing errors, for a 30° RMS phase error are 1.27° and 0.61° at 6.0 and 12.5 GHz respectively. The worst case peak sidelobe to main lobe ratio is 0.69. The 3 dB beamwidths of the array factor are 11.5° and 5.5° at 6 and 12.5 GHz respectively. A photodetector array is usually used for detection and interpolation is employed to obtain about 1° angular resolution.

TABLE IV

AN OPTIMUM TWO-DIMENSIONAL PLOT SHOWING THE PEAK SIDELobe TO
MAIN LOBE VALUES FOR AN ARRAY OF 10.5 WAVELENGTHS LONG

$$(D_u = 10.5/2 \lambda_R)$$

	$D_2 (\lambda_R)$						
$D_1 (\lambda_R)$	-1.3625	-1.4250	-1.4875	-1.5500	-1.6125	-1.6750	-1.7375
1.2500	.98	.95	.89	.81	.83	.85	.86
1.1875	.94	.92	.87	.80	.80	.82	.83
1.1250	.87	.86	.82	.76	.76	.89	.93
1.0625	.85	.79	.75	.70	.77	.93	1.00
1.0000	.86	.81	.74	.68	.79	.86	.96
0.9375	.86	.81	.75	.71	.85	.94	.97
0.8750	.86	.82	.82	.75	.84	.95	1.00
0.7500	.84	.92	.95	.90	.76	.89	.96

Fig. 28. Normalized intensity patterns of the optimum array (baseline length = $10.5 \lambda_R$ at 12.5 GHz, $w = \lambda_R/30$, element locations (λ_R) : -5.25, -1.54, 0.98 and 5.25).

The experimental simulation using a photomask to represent the 4-channel Bragg cell is shown in Fig. 29. Comparing the theoretical and simulated patterns, they indeed agree very well. Some experimental optical array patterns using a multi-channel Bragg cell, will be given in Chapter 9.

6.9 SUMMARY AND CONCLUSIONS

The characteristics of the 2-D acoustooptic signal processor for simultaneous direction finding and spectrum analysis have been investigated. It produces optically the weighted spectrum of an incoming signal on one axis and the optical far-field pattern on the other. The deflection angle of the mainbeam of the pattern has been shown to be related to the angle of arrival θ_0 by the product of the scaling factor (S.F.) and $\sin(\theta_0)$. In addition, there are no "invisible" regions in the optical far-field pattern.

An error tolerance analysis on the processor has also been given in which the amplitude and phase errors in all the channels are assumed to be of zero mean, independent Gaussian statistics. The amplitude error is found to affect only the shape of the pattern, and the dominant effect of both the amplitude and phase errors is a reduction in gain and a change in the main lobe to sidelobe ratios. The beam pointing error is directly proportional to the phase errors. It has been shown to be inversely proportional to the array length and approximately inversely proportional to the square root of the number of channels.

RELATIVE INTENSITY

SPATIAL COORDINATE (y_1)

(a) Right half

RELATIVE INTENSITY

SPATIAL COORDINATE (y_1)

(b) Left half

Fig. 29. Simulated intensity pattern of the optimum array
 (photodetector array cell-to-cell center spacing
 $= 13 \mu\text{m}$, $F = 25 \text{ cm}$, $\text{S.F.} = 4.43 \times 10^{-3}$, $w = \lambda_0/30$).

For applications requiring a broad frequency and angular coverage with high angular accuracy, an aperiodic array configuration with four or five elements, seems to be optimum. This conclusion is based on parameters of cost, fabrication and required main lobe beamwidth and sidelobe levels.

For this aperiodic array configuration with a limited number of elements (channels), a simple synthesis procedure of designing an aperiodic array has been shown to be useful. The main design criteria used are (i) The mainbeam width is minimized while the peak sidelobe-to-main lobe ratio is kept to a reasonable minimum and (ii) the channel spacings are optimized so that the ratio of the peak sidelobe-to-main lobe level is least sensitive to perturbations of channel locations.

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CHAPTER 7

EFFECTS OF BRAGG CELL DIFFRACTION CURVATURES ON

2-D ACOUSTOOPTIC SIGNAL PROCESSOR

7.1 INTRODUCTION

As discussed in Section 3.4, the principal effect of the acoustic diffraction along the height (y) direction of a Bragg cell, is a curvature of the Fourier spectral components. The locus of this curve is a parabola whose vertex is at the position corresponding to the IF frequency and whose focus is at the optical axis. The degree of curvature is dependent on the anisotropic nature of the interaction medium.

A 2-D acoustooptic signal processor which employs a multi-channel Bragg cell for simultaneous direction-of-arrival (DOA) determination and spectrum analysis, was presented in the last chapter. The acoustooptic medium was assumed to produce a self-collimating beam in the y direction, and in the case of a uniform amplitude weighting, the diffraction pattern is an exact orthogonal sinc function. This diffraction pattern maps exactly to the rectangular coordinates of a 2-D photodetector array with its elements configured in the form of rows and columns.

In this chapter, we investigate the general case, where the acoustooptic medium may not produce a self-collimating acoustic beam (the ideal traveling wave modulator model) and thus the diffraction pattern no longer exactly matches the area array. The effects of the diffraction curvature on the DOA and frequency information of the 2-D acoustooptic signal processor will be analyzed.

7.2 MULTI-CHANNEL BRAGG CELL DIFFRACTION PATTERNS

A block diagram of a multi-channel Bragg cell is shown in Fig. 27. In the following analysis, the time variable is suppressed by assuming a cw signal. For a fully illuminated Bragg cell, the amplitude distribution of the diffracted signal is obtained by combining Eqs. (3.24) and (6.6), to give

$$\begin{aligned}
 U_1(\gamma, \beta) = & \frac{K_B}{\lambda F} E(f_R, \theta_0) \iint_{-\infty}^{\infty} \sum_{n=1}^N g_n(x_0, y_0 - \ell_n) \\
 & \cdot w(x_0, y_0 - \ell_n) \exp(j2\pi x_0 / \Lambda) \exp(j2\pi \beta_n) \\
 & \cdot \exp[-j2\pi / \lambda (\gamma x_0 + \beta y_0)] dx_0 dy_0 \cdot H \operatorname{sinc}(H\beta / \lambda)
 \end{aligned}
 \tag{7.1}$$

where β without a subscript is the vertical angular spatial frequency, which is related to the DOA information by $\sin(\theta_0)$, β_n with subscript n is the phase delay in channel n , Λ is the acoustic wavelength of the cw signal and a constant phase term has been dropped. The finite height of

the acoustic transducer has been accounted for by the multiplication of the integral by the $\text{sinc}(H\beta/\lambda)$ function.

If the acoustic phase velocity can be approximated by a parabolic function as given in Eq. (3.14) and for a uniform window weighting function of the form

$$w(x_0, y_0 - \ell_n) = \text{rect}(x_0/D) \quad (7.2)$$

in which the acoustic loss has been neglected, then Eq. (7.1) can be solved in closed forms. Making use of Eqs. (3.27) and (6.7), we have

$$U_1(\gamma, \beta) = \frac{NK}{\lambda F} DH E(f_R, \theta_0) \exp\left\{j \frac{\pi D}{\lambda} \left[\frac{\lambda}{\Lambda} - \frac{\Lambda (1/2 - s) \beta^2}{\lambda} \right]\right\}$$

$$\text{sinc}\left\{ \frac{D}{\lambda} \left[\gamma - \frac{\lambda}{\Lambda} + \frac{\Lambda (1/2 - s) \beta^2}{\lambda} \right] \right\} \text{sinc}(H\beta/\lambda)$$

$$\frac{1}{N} \sum_{n=1}^N a_n(f_R, \theta_0) \exp[-j2\pi(\beta \ell_n/\lambda - \beta_n)] \quad (7.3)$$

The above equation is similar to Eq. (6.10), the only difference is that the two sinc functions are no longer exactly orthogonal to each other for $s \neq 1/2$. The horizontal difference between the angular spatial frequency at an angle β and that at $\beta = 0$ is obtained from Eq. (3.30) in which

$$\Delta\gamma = \gamma_0 - \gamma(\beta)$$

$$= \Lambda/\lambda (1/2 - s) \beta^2 \quad (7.4)$$

7.3 SAMPLING ERROR VERSUS DEVIATION IN THE ANGULAR SPATIAL FREQUENCY γ

For the 2-D acoustooptic signal processor, a solid-state photodetector area array is used for the detection of the Fourier spectral distribution in the (x_1, y_1) plane. The (x_1, y_1) and (γ, β) planes are simply related by

$$\begin{aligned} x_1 &= \gamma F_1 \\ y_1 &= \beta F_2 \end{aligned} \quad (7.5)$$

where F_1 and F_2 are the focal lengths of the Fourier transforming lens in the x and y dimensions respectively. The photodetectors of the area array are configured in rows and columns, which are orthogonal to each other. As discussed in Section 3.4, for a self-collimating acoustic medium in which $s = 1/2$, the diffraction pattern is a 2-D orthogonal sinc function for a uniform amplitude weighting function. The frequency and DOA data map exactly into the rows and columns of the area array respectively and is illustrated in Fig. 30.

In the case when $s < 1/2$, the columns are no longer parallel to the

Fig. 30. Orientation of Bragg cell diffraction singularity curves with respect to photodetector area array.

singularity curve other than at $y_1 = 0$. The frequency spectral information along the row direction is no longer independent of the DOA information. Only a small error is introduced to the frequency data if the two are assumed to be independent. However and more significantly, this small deviation in γ may have a great impact on the DOA pattern read out from each column, because the main lobe level is only a few dB above the highest sidelobe levels. The DOA information stored in the columns may be distorted to a significant level as a result of this mismatch. The degree of distortion is a function of the anisotropy factor s , angular spatial frequency γ , the spatial weighting function of the photodetector array, the relative size and position of the column to the spectral width of the distribution. From Eq. (7.4), the maximum distortion is between $\beta = 0$ and $\beta = \beta_{\max}$.

The angles $\Delta\beta$ and $\Delta\gamma$ subtended by each photodetector element are very small and the singularity curve across each element can be approximated by assuming it to be vertical. For a uniform amplitude weighting of width D , the magnitude of the spectral distribution is a $\text{sinc}(uD)$ function. The main lobe to the first null width is $\Delta u = 1/D$ and is equivalent to the resolution width of the acoustooptic spectrum analyzer. The output from each photodetector is a voltage which is proportional to the total amount of light integrated by the photodetector.

Fig. 31 shows the case where the number of photodetectors (N) per resolution width is one. A maximum output is obtained when the

Fig. 31. Photodetector location with respect to the sinc^2 function distribution to give maximum and minimum outputs. (Number of photodetectors per resolution width $N = 1$)

photodetector is centered with respect to the centroid of the sinc function. The minimum occurs when the centroid is located in between two photodetectors in which there are two equal outputs. The maximum possible sampling error due to the mismatch of the centroid with respect to the photodetector is given by

Maximum sampling error

$$= \frac{\text{Maximum output} - \text{Minimum output}}{\text{Maximum output}} \quad (7.6)$$

The maximum sampling error for the sinc spectral distribution is a function of the number of photodetectors used per resolution width, and the spatial weighting function of the photodetector array. The maximum sampling errors versus the number of photodetectors per resolution width and A_2 are shown in Fig. 32. A_2 is the ratio of the total dead area to the total photodetector area. The dead area is the light insensitive region in between two columns. As can be seen from the graph, for $N \geq 3$, the maximum sampling error becomes less sensitive to variations in A_2 and decreases with increasing values of N . The tradeoffs of reducing the maximum sampling error by increasing the number of photodetectors per resolution width are the loss in receiver sensitivity due to the spreading in light power, and the requirement of a longer photodetector array.

As given in Chapters 4 and 5, if the weighting function is other than the rectangular one, then the effect of the weighting will broaden

Fig. 32 Maximum sampling error versus number of photodetectors per resolution width.

(A_2 = Total dead area / Total photodetector area)

the main lobe and thus decrease the maximum sampling error. In addition, if the signal is a pulse-modulated cw signal, the effective aperture width D will be reduced accordingly and the truncation effect by the finite aperture width, will broaden the main lobe further.

The change in the output level as a function of the relative shift in the position of the photodetector to the centroid of the sinc function distribution is plotted in Figs. 33 and 34, for A_2 equals to zero and 0.4 respectively. A maximum output is obtained when the shift in position is zero. The output level decreases with an increase in the shift in position (Δu) to at least $0.5/D$ as plotted.

In the case of $N = 1$, the maximum change in the output level occurs at $\Delta u = 0.5/D$ in which the centroid of the sinc function is centered between two photodetectors. We have plotted this change for Δu up to $0.5/D$ because we are only interested in taking information from the two adjacent columns which have the two highest outputs.

For $N = 4$, the maximum change in the output level is now shifted to $\Delta u = 0.125/D$ from the centroid, at which there are again two equal outputs. The change in the output level is much less than the case for $N = 1$ at $\Delta u = 0.5/D$. For example, from the plot, a maximum 5% drop in the output level occurs when there is shift in position of $\Delta u = 0.125/D$. This is equivalent to $1/8$ the resolution width and it occurs when the centroid is initially centered on one of the columns.

Fig. 33. Percent change in output level versus shift in horizontal position (u) with no dead area.
 (Number of photodetectors per resolution width
 $N = 1, 2, 3$ and 20)

Fig. 34. Percent change in output level versus shift in horizontal position (u) with $A_2^2 = 0.4$.
 (Number of photodetectors per resolution width $N = 1, 2, 3$ and 20)

7.4 RELATIONSHIP AMONG THE ANISOTROPY FACTOR(s),

SCALING FACTOR (S.F.) AND THE ANGULAR SPATIAL FREQUENCY (γ)

After finding the maximum allowable deviation $\Delta\gamma$, the minimum value of s can be found by using Eq. (7.4). That is

$$s \geq \frac{1}{2} - \frac{\Delta\gamma \lambda}{\Lambda \beta_{\max}^2} \quad (7.7)$$

where $\beta_{\max}$ is related to the scaling factor, from Eq. (6.16)

$$\beta_{\max} = [\text{S.F.} \sin(\theta_0)]_{\max} \quad (7.8)$$

Substituting Eq.(7.8) into Eq.(7.7), with $\gamma_0 = \lambda / \Lambda$, we have

$$s \geq \frac{1}{2} - \frac{\gamma_0 \Delta\gamma}{\{[\text{S.F.} \sin(\theta_0)]_{\max}\}^2} \quad (7.9)$$

The above condition is met in all cases for $s = 1/2$, which is the upper limit for all materials in which a self-collimating acoustic beam is produced. The requirement on the anisotropy factor s from the limiting self-collimating case can be relaxed by a certain choice of the parameters as given in the above equation. The degree of relaxation is directly proportional to the product of the horizontal angular spatial frequency γ_0 and its maximum tolerable value $\Delta\gamma$, and inversely proportional to the square of the product of the scaling factor (S.F.) and the sine of the maximum steering angle θ_0 .

A typical example is given to illustrate the minimum requirement for the anisotropy factor s , by using an anisotropic Bragg cell on Lithium niobate. The phase velocity is calculated by using the elastic constants and it can be accurately approximated by the parabolic form [41]

$$v_s(\phi) = v_s(1 - 1.12\phi^2) \quad (7.10)$$

where $v_s = 3.45$ km/sec. Using the set-up as outlined in Fig. 7, the Bragg cell is fully illuminated. The resulting singularity curves are given in Fig. 35 where the acoustic frequencies are spaced at 200 MHz apart. By measuring the locus for each singularity curve and by using Eq. (3.14), we find that $s = -1.02$. It agrees reasonably well with the theoretical value of -1.12 .

We are going to use the typical system requirements as outlined in Section 6.8 for this example, and we have

$$\begin{aligned} [S.F. \sin(\theta_0)]_{\max} &= 4.43 \times 10^{-3} \sin(45^\circ) \\ &= 3.13 \times 10^{-3} \end{aligned} \quad (7.11)$$

A HeNe laser with a $0.6328 \mu\text{m}$ wavelength is assumed. The minimum frequency of operation of the Bragg cell is assumed to be 2 GHz with an aperture transit time (τ) of $0.5 \mu\text{sec}$. If the criterion of using $N = 4$ photodetectors per resolution width is used to allow a maximum drop in

Fig. 35. Diffraction patterns for a fully illuminated Lithium niobate Bragg cell driven at 2.3 , 2.5 and 2.7 GHz.
(Multiple exposure, focal length of lens = .565 m)

output level of 5% as given in Section 7.3, then

$$\gamma_0 \Delta\gamma = \frac{\lambda \lambda}{\Lambda \delta D} = 1.68 \times 10^{-5} \quad (7.12)$$

Substituting Eqs. (7.11) and (7.12) into Eq.(7.9), the minimum requirement on the anisotropy factor s is calculated to be -1.22. This value is smaller than the anisotropy factor s of -1.12 used in this example and thus will meet the design criterion.

As illustrated in the above example for a practical receiver, there are a number of degrees of freedom on the choice of parameters, which the designer can choose to satisfy the requirement given by Eq. (7.9). However, if the requirement cannot be met even by a change in parameters, there are two other ways in which the effect of the acoustic curvature can be compensated. One way is to remap all the DOA and frequency data in the postprocessing. This technique can be difficult and complicated because the diffraction curvature is frequency dependent and the sampling error is signal dependent. Another technique to compensate for the spreading of the acoustic wave in the height direction is by using a holographic element [42]. The action of the holographic element is to correct, or conjugate, the cylindrical phase factor introduced by the spread of the acoustic beam. However, in addition to the complexity of dealing with holograms, other degradations such as the reduction in the modulation transfer function and the loss in efficiency of the overall system, have to be considered.

7.5 SUMMARY AND CONCLUSIONS

The principal effect of the acoustic diffraction along the height direction of a Bragg cell is a curvature of the Fourier spectral components. In the 2-D acoustooptic signal processor employing a multi-channel Bragg cell, the spectral information along the β axis is related to the DOA by $\sin(\theta_0)$. A solid-state photodetector area array is usually used for detection and in order to simplify the detected data, the columns and rows are chosen to be parallel to the DOA and γ axes respectively. For a self-collimating material in which the anisotropy factor $s = 1/2$, the frequency and DOA spectral components are orthogonal. The data read out from each column will register the exact DOA pattern at a constant frequency. For $s \neq 1/2$, the Bragg cell diffraction distribution is a curvature which no longer matches exactly with the orthogonal configuration of the photodetector array, and the frequency content from a column is thus no longer constant. As a result, a small error is introduced if the data from a column is taken to be constant throughout. However, the amplitude distribution of the DOA pattern detected in each column may be distorted to a significant level as a result of the mismatch because the sidelobes are only a few dB below the main lobe in amplitude. The degree of distortion has been shown to be a function of the anisotropy factor s , angular spatial frequency γ , the spatial weighting function of the photodetector

array, the relative size and position of the column to the frequency spectral width of the signal distribution.

In order to keep the amplitude sampling error of an input signal to a minimum, and insensitive to the spatial weighting function of the photodetector array, it was found desirable to choose the minimum number of photodetectors per frequency resolution width to be three. In addition, the distortion on the DOA pattern is minimized if the column located at the frequency centroid of the signal is chosen, where the change in amplitude as a function of shift in angular spatial frequency $\Delta\gamma$ is the smallest.

Once the maximum allowable deviation in the angular spatial frequency ($\Delta\gamma$) is chosen to satisfy a certain maximum distortion level, there are a number of degrees of freedom on the choice of system and component parameters. If it is expressed in terms of the requirement on the anisotropy factor s from the limiting self-collimating case, then the requirement can be relaxed by a certain choice of the parameters. The degree of relaxation was shown to be directly proportional to the product of the angular spatial frequency γ_0 and its maximum tolerable deviation $\Delta\gamma$, and inversely proportional to the square of the product of the scaling factor (S.F.) and the sine of the maximum steering angle θ_0 .

CHAPTER 8

IMPROVEMENT ON THE INTENSITY OF THE DIFFRACTION PATTERN USING A MODIFIED MULTI-CHANNEL BRAGG CELL

8.1 INTRODUCTION

The 2-D acoustooptic signal processor was analyzed in detail and presented in Chapter 6. It was concluded that an aperiodic array with four or five elements was the optimum configuration for broadband receiver designs. This was based on factors such as complexity, cost, required main lobe beamwidth, sidelobe levels and antenna size constraint. It was also pointed out in Section 6.3 that for a rectangular acoustic transducer of height H , the optical DQA array pattern $f(v)$ was modified by a sinc function which is related to the scanning angle θ_0 by

$$\text{sinc}(vH) = \frac{\sin\left[\pi \left(\frac{d_N H}{\lambda_R \ell_N} \right) \sin(\theta_0)\right]}{\pi \left[\frac{d_N H}{\lambda_R \ell_N} \sin(\theta_0) \right]} \quad (8.1)$$

For a scanning angular range of $\theta_0 = \pm 45^\circ$, and with a maximum drop in intensity of 10%, the transducer height (H) must satisfy

$$H \leq \frac{\ell_N \lambda_R}{4 d_N} \quad (8.2)$$

For the typical 4-element interferometer design given in Section 6.8 with a baseline length of 10.5 wavelengths, we have

$$H \leq 2 \ell_N / 42 \quad (8.3)$$

where $2 \ell_N$ is the total baseline length of the transducer array in the multi-channel Bragg cell. For the 4-element transducer array, the inter-transducer spacings are much larger than the space occupied by the four acoustic transducers. If the multi-channel Bragg cell is uniformly illuminated, a great portion of the incident light will not be interacting with the acoustic beams even though there may be an appreciable spread in the acoustic beam profile. This will reduce the amount of light diffracted with a corresponding drop in the sensitivity of such a receiver.

In order to utilize the incident light more efficiently, a novel scheme is proposed to modify the multi-channel Bragg cell by putting on

more acoustic transducers along the array. Instead of feeding N acoustic transducers from N antenna elements, N sets of acoustic transducers arranged in a phased array are used.

An alternate technique is to use one or more holograms to split and reshape the incident beam into multiple coherent beams, matching the acoustic beams in profile. However this technique has some disadvantages such as the complexity associated with the generation and handling of holograms, phase distortions and loss introduced by the additional elements. Furthermore a hologram can not be dynamically varied and thus it can be optimized for one acoustic frequency only. As a result this technique will not be considered in this thesis.

In this chapter, the modified Bragg cell configuration using a phased array structure is investigated. An optimum configuration which can easily be implemented and provides a substantial improvement on the intensity level of the DOA pattern is given. This output intensity level is also a function of the Gaussian laser illumination profile parameter T_y and its optimum value is also discussed.

8.2 MODIFIED MULTI-CHANNEL BRAGG CELL CONFIGURATION

With the N -channel Bragg cell in the 2-D acoustooptic signal processor illuminated fully, the amplitude distribution in the Fourier plane is given by Eq. (7.3). That is

$$\begin{aligned}
 U_1(\gamma, \beta) = & \frac{NK_B}{\lambda F} DH E(f_R, \theta_0) \exp\left\{j \frac{\pi D}{\lambda} \left[\frac{\lambda}{\Lambda} - \frac{\Lambda (1/2 - s) \beta^2}{\lambda} \right]\right\} \\
 & \cdot \text{sinc}\left\{ \frac{D}{\lambda} \left[\gamma - \frac{\lambda}{\Lambda} + \frac{\Lambda (1/2 - s) \beta^2}{\lambda} \right] \right\} \text{sinc}(H\beta / \lambda) \\
 & \cdot \frac{1}{N} \sum_{n=1}^N a_n \exp[-j2\pi (\beta l_n / \lambda - \beta_n)] \quad (8.4)
 \end{aligned}$$

where the time variable has been suppressed by assuming a cw input signal with an acoustic wavelength Λ , and the acoustic loss has been neglected. In this case, we are more interested in the DOA pattern, and let us define the normalized DOA pattern by

$$f_1(\beta) = \text{sinc}\left(\frac{H\beta}{\lambda}\right) \frac{1}{N} \sum_{n=1}^N a_n \exp\left[-j2\pi \left(\frac{\beta l_n}{\lambda} - \beta_n\right)\right] \quad (8.5)$$

where a_n is the amplitude and β_n is the phase delay of channel n respectively, l_n is the position of acoustic transducer n on the acoustic transducer array. The number of antenna elements is equal to the number of acoustic transducers. The ratios of the acoustic transducer separation spacings are the same as those of the microwave antenna spacings.

In the modified Bragg cell configuration, instead of using a single acoustic transducer for each antenna element, a phased array of acoustic

transducers is used. For an N-element interferometer, there are N phased arrays.

There are many ways in which N phased arrays can be designed, such as in terms of the transducer to transducer spacing, transducer cross-sectional area, number of transducers used in the array and the excitation of each transducer. For ease of fabrication and excitation, N identical phased arrays are considered. The transducer elements of the phased array are of the same cross-sectional area, with uniform spacing of inter-transducer spacing ℓ and the phase excitation can either be 0 or π radians. Now the normalized DOA pattern of the modified multi-channel Bragg cell is

$$f_2(\beta) = \text{sinc}\left(\frac{H\beta}{\lambda}\right) \cdot \frac{1}{N} \sum_{n=1}^N \sum_{m=-M/2}^{M/2} a_{nm} \exp\left\{-j2\pi \left[\frac{\beta(\ell_n - m\ell)}{\lambda} - \beta_n \right]\right\}$$

(8.6)

where the number of transducer elements of the phased array is $(M+1)$, M is an even integer, and the total number of acoustic transducers on the modified multi-channel Bragg cell is $N(M+1)$.

A synthesis of a typical aperiodic array has been given in Section 6.8. For a given baseline length, the antenna elements are chosen to be separated from each other as far as possible for ease of fabrication

of both the antenna and acoustic elements and reduction of mutual coupling effects. In order to avoid overlapping of transducer elements from adjacent phased arrays, the baseline length and thus the number of transducer elements of the phased array, must be kept small. On the other hand, the degree of control over the modified DOA pattern is directly proportional to the number of transducer elements in the phased array. For a typical broadband receiver design, a phased array with five acoustic transducer elements is a good compromise. For a symmetrically excited phased array with five acoustic transducer elements, we have

$$a_{n,m} = a_{n,-m} \quad (8.7)$$

and Eq. (8.6) can be simplified by factoring out the common phased array factor

$$f_2(\beta) = [1 + 2P_1 \cos(2\pi\beta l / \lambda) + 2P_2 \cos(4\pi\beta l / \lambda)] \text{sinc}(H\beta / \lambda)$$

$$\frac{1}{N} \sum_{n=1}^N a_n \exp[-j2\pi (\frac{\beta l_n}{\lambda} - \beta_n)] \quad (8.8)$$

$$\text{where } P_1 = \frac{a_{n,1}}{a_n} = \frac{a_{n,-1}}{a_n} \quad (8.9)$$

$$P_2 = \frac{a_{n,2}}{a_n} = \frac{a_{n,-2}}{a_n} \quad (8.10)$$

Comparing Eq. (8.8) to Eq. (8.5), we see that for the modified Bragg cell configuration, the normalized DOA pattern is modified by the common phased array factor. The total weighting function on the DOA array pattern $f(\beta)$ is

$$w_1(\beta) = \frac{[1 + 2P_1 \cos(2\pi\beta l / \lambda) + 2P_2 \cos(4\pi\beta l / \lambda)]}{\text{sinc}(H\beta / \lambda)}$$

(8.11)

There are three parameters, namely P_1 , P_2 and l , which the designer can choose to optimize the weighting function. The two most important optimization criteria are

1. The modified Bragg cell must provide a substantial improvement on the intensity of the DOA pattern.
2. In order to preserve the DOA array pattern, the amplitude of the weighting function must be kept relatively constant and varies smoothly over all values of β of interest.

In addition, l must be small enough to avoid overlappings of transducers from adjacent phased arrays. A small l will also increase the periods of the two cosine terms to give a smooth weighting function over all β of interest. On the other hand, l must be wide enough to allow two transducers to be placed close together without touching. As a result, $l = 1.1H$ is chosen with H being the height of the acoustic transducer. Moreover, for ease of fabrication, transducers of equal

weightings with 0 or π radians are used.

Expressing the total weighting function w_1 in terms of the scanning angle θ_0 by making use of Eq. (8.1) and meeting the minimum criterion given by Eq. (8.2), we have

$$w_1(\theta_0) = \{ 1 + 2P_1 \cos[1.1/2\pi \sin(\theta_0)] + 2P_2 \cos[1.1\pi \sin(\theta_0)] \} \cdot \text{sinc} [\sin(\theta_0)/4] \quad (8.12)$$

using the above guidelines, an optimum design is obtained with

$$\begin{aligned} P_1 &= 1 \\ P_2 &= -0.38 \end{aligned} \quad (8.13)$$

In this case, the center and its adjacent transducers of the phased array are excited with equal amplitude and phase. The two outer transducers are excited with a relative amplitude of 0.38 and π radians in phase. The square of the total weighting function (w_1) of the modified Bragg cell and the sinc function of the unmodified Bragg cell are plotted in Fig. 36 as a function of $\sin(\theta_0)$. Comparing the two plots, there is an improvement on the intensity of the DOA pattern of about five times as a result of the modification. There are two peaks located symmetrically at $\sin(\theta_0) = \pm .445$ and a dip at the center. The

Fig. 36. Square of the optimum total weighting function (w_1) and $\text{sinc} [0.25 \sin(\theta_0)]$.

peaks have a relative intensity of 5.59 times that of the peak of the sinc function, which is the total weighting function for the case of the unmodified Bragg cell. The magnitude of the dip is 90% of the peak value, and at the maximum steering angles of $\pm 45^\circ$, the intensity is reduced to 83% of the peak value. For the sinc function, it is maximum at the center with a value of unity and at $\theta_0 = \pm 45^\circ$, it is down to 0.9. The total weighting function (w_1) is plotted in Fig. 37 as a function of P_2 , showing the effect of varying the value of P_2 around the optimum.

The modified multi-channel Bragg cell configuration is illustrated by using the specific design example of a 4-element interferometer given in Section 6.8. A schematic diagram of the modified 4-channel Bragg cell configuration showing the transducer locations is shown in Fig. 38. This is a typical transducer structure used in the fabrication of a multi-channel Bragg cell [43],[44]. The top electrodes of the transducer array are deposited on the piezoelectric layer using a "lift off" technique [44]. The pattern of the top electrode determines the acoustic beam profile. Whereas the thickness of the piezoelectric layer determines its frequency response.

For the input feeding network, a microwave transmission line either in the form of a miniature 50 Ω coaxial cable or microstrip line is used, in conjunction with an impedance matching network. The impedance matching network is used to match the 50 Ω transmission line to the low-impedance transducer. The output is then stitch bonded to the top electrode of the transducer by using a fine wire. To implement a

Fig. 37. Square of the total weighting function (w_1) versus $\sin(\theta)$ for $P_2 = -0.36, -0.38$ and -0.40

3

OUTSIDE

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Fig. 38. Schematic diagram of the modified 4-channel Bragg cell configuration showing the transducer locations.

reversal in polarity (with π radians in phase), the simplest technique is by interchanging the wire bondings to the top and bottom electrodes, in which the ground electrode must be isolated from the ground plane.

To implement the excitation of the optimum phased arrays on the modified multi-channel Bragg cell, there are several methods one can use. One method is to use one transmission line with an impedance matching network feeding all the five transducers of each phased array in parallel. The inner three transducers are excited with an equal amplitude weighting and phase. The outer two electrodes are excited with π radians out of phase and this is achieved with a reversal in the wire bondings. The amplitude weighting of 0.38 may be implemented by metal loading the top electrode [45]. This technique has to be investigated further to find out how effective and what other effects it may have on the overall impedance characteristics of the transducer.

Another method is to use two transmission lines with their own impedance matching networks. One is used to feed the inner three transducers and the other for the outer two transducers. The reversal in sign and amplitude weighting for the two outer transducers are implemented externally using microwave components.

8.3 GAUSSIAN LASER ILLUMINATION PROFILE

Laser beam profile of Gaussian distribution is important in optical

signal processing because the intensity profile of most gas lasers such as HeNe is characterized by the Gaussian distribution [46]. In addition, the intensity profile of the lowest order beam from a double heterostructure laser is also of Gaussian shape. The Gaussian profile can be modified (expanded or reduced) easily in one or two dimensions by using a beam-expander such as an anamorphic prism beam-expander [47], [48]. In this section, the effect of the Gaussian illumination profile on the DOA pattern will be investigated.

In the case of a multi-channel Bragg cell illuminated fully with a Gaussian laser beam, the amplitude weighting function with the acoustic attenuation included, is

$$\begin{aligned}
 w(x_0, y_0) &= w(x_0) w(y_0) \\
 &= \text{rect}\left(\frac{x_0}{D}\right) \exp\left[-(2Tx\frac{x_0}{D})^2\right] \\
 &\quad \cdot \exp\left[-\alpha\tau\left(\frac{x}{D} + \frac{1}{2}\right)\right] \exp\left[-(2Ty\frac{y_0}{2\ell_N})^2\right] \quad (8.14)
 \end{aligned}$$

We will follow a similar procedure as given in Section 3.4, by calculating the amplitude distribution of the case where the amplitude weighting function is of a rectangular distribution. The Gaussian and acoustic exponential functions are then accounted for by convolving the amplitude distribution with the Fourier transform of $w(x_0, y_0)$. The Fourier transform of the weighting function $w(x_0, y_0)$ can be separated into two functions of γ and β respectively. That is

$$W(\gamma, \beta) = W(\gamma) W(\beta) \quad (8.15)$$

$$\text{where } W(\gamma) = \int_{-\infty}^{\infty} w(x_0) \exp(-j \frac{2\pi}{\lambda} \gamma x_0) dx_0$$

$$\text{and } W(\beta) = \int_{-\infty}^{\infty} w(y_0) \exp(-j \frac{2\pi}{\lambda} \beta y_0) dy_0$$

$$= \sqrt{\pi} \frac{\ell_N}{T_y} \exp\left[-\left(\frac{\pi \beta \ell_N}{\lambda T_y}\right)^2\right] \quad (8.16)$$

is again of Gaussian distribution. The amplitude distribution in the Fourier plane is obtained by convolving Eq. (7.3) with Eq.(8.15) to give

$$U_1(\gamma, \beta) = \frac{NK_B}{\lambda F} DH E(f_R, \theta_0) \exp\left\{j \frac{\pi D}{\lambda} \left[\frac{\lambda}{\Lambda} - \frac{\Lambda (1/2 - s)\beta^2}{\lambda} \right]\right\}$$

$$\cdot \text{sinc}\left\{\frac{D}{\lambda} \left[\gamma - \frac{\lambda}{\Lambda} + \frac{\Lambda (1/2 - s)\beta^2}{\lambda} \right]\right\} \text{sinc}\left(\frac{H \beta}{\lambda}\right)$$

$$\cdot \frac{1}{N} \sum_{n=1}^N a_n \exp\left[-j 2\pi \left(\frac{\beta \ell_n}{\lambda} - \beta_n\right)\right]$$

$$\cdot \sqrt{\pi} \frac{\ell_N}{T_y} \exp\left[-\left(\frac{\pi \beta \ell_N}{\lambda T_y}\right)^2\right] W(\gamma) \quad (8.17)$$

We are mainly interested in determining the effect of the Gaussian input laser profile $w(y_0)$ on the DOA pattern and from the above

equation, only the $W(\beta)$ component is of interest. It is expected that if the acoustic beam width including the acoustic spreading in the y_0 dimension, is much smaller than the Gaussian illumination beam width, the Gaussian weighting on each acoustic beam can be assumed to be approximately constant. In this case, the convolution given in Eq. (8.17) in the β dimension can be simplified by performing it in the (x_0, y_0) domain. This is done by just multiplying the amplitude of each acoustic channel by the Gaussian weighting $w(y_0)$ located at the channel center. This simplification can also be obtained by using Eq. (8.17) expressed in the Fourier (γ, β) domain. The shape of the distribution is unaffected by the convolution with $W(\beta)$, if the width of the Gaussian distribution $W(\beta)$ is much smaller than the widths of the two sinc distributions. If the widths of the sinc and Gaussian functions are defined by the $\text{sinc}(\pm 0.5)$ and $\exp(-1)$ points respectively, we have

$$H \ll \frac{\pi}{2} \frac{\ell_N}{T_y} \quad (8.18)$$

and

$$D\left(\frac{\Lambda}{H}\right)(1/2 - s) \ll \frac{2\pi^2}{H} \left(\frac{\ell_N}{T_y}\right)^2 \quad (8.19)$$

where ℓ_N/T_y is the Gaussian amplitude half-width in y_0 . From Eq. (8.18), the transducer height H must be much smaller than the Gaussian half-width and Eq. (8.19) states that the acoustic spread must be much smaller than the square of the Gaussian half-width divided by the height

of the acoustic transducer. If the above two conditions are met, then the acoustic beam can be approximated by a self-collimating beam with height H in the analysis of the effect of the Gaussian laser illumination profile on the DOA pattern.

8.4 DOA MAIN LOBE INTENSITY VERSUS GAUSSIAN PROFILE

PARAMETER T_y

In Section 8.3, the input Gaussian illumination profile has been chosen to be located symmetrically with respect to the acoustic array center. This is due to the reason that a displaced Gaussian function can be expressed in terms of a product of a centrally located Gaussian function of the same distribution and an exponential function as given in Section 4.2. The exponential function has the undesirable effect of increasing the sidelobe levels in the Fourier plane as illustrated in Fig. 9.

We are going to investigate the intensity distribution of the DOA pattern as a function of the input Gaussian illumination profile parameter T_y . For a fixed total laser power interacting with the acoustic beams, it can be shown that the intensity of the main lobe DOA pattern will reach a maximum when all the acoustic channels are of equal amplitude. In the case of the Gaussian illumination profile, the output intensity is a function of the Gaussian profile parameter T_y . A maximum

in the total optical power interacting with the acoustic beams usually may not result in a maximum main lobe intensity in the DOA pattern. This will be illustrated by using the optimum 4-element array as given in Section 6.8 as an example. In this case, the optimum relative acoustic array spacings are as follows:

$$\begin{aligned} \ell_1 &= -\ell_4 \\ \ell_2 &= -0.2933 \ell_4 \\ \ell_3 &= 0.1867 \ell_4 \end{aligned} \quad (8.20)$$

In this case, a laser with a total power of unity is used to illuminate the 4-channel Bragg cell fully. The illumination profile is of Gaussian distribution characterized by the parameter T_y . The total power interacting with the four acoustic beams is plotted in Fig. 39, curve (a), as a function of the Gaussian profile T_y . For T_y close to zero, the Gaussian beam is very large in comparison with the acoustic array and thus a very small portion of the input laser power is utilized in the interaction. As the Gaussian beamwidth is decreased, more and more of the input laser power is utilized, and depending on the acoustic channel locations, a number of maxima may occur. For this specific 4-element array, the first maximum occurs at around $T_y = 2.14$. The resultant relative maximum intensity of the main lobe level of the DOA pattern is also plotted in Fig. 39, curve b, as a function of T_y . However, by comparing the two plots, the maximum main lobe intensity occurs at $T_y = 0.8$, and the two curves are different in shape. We are

Fig. 39. Total interacting optical power and its maximum DOA pattern intensity versus Gaussian profile parameter T_y .
 (a) Total optical power interacting with acoustic beams.
 (b) Relative maximum intensity of DOA pattern.

only interested in small T_y because the main lobe width of the DOA pattern increases with T_y . This will result in a reduction of the angular resolution of the processor. At $T_y = 0.8$, where the maximum main lobe intensity occurs, we have the following relative amplitude distribution in the four acoustic channels,

$$\begin{aligned} a_1 &= a_4 = 0.092 \\ a_2 &= 0.165 \\ a_3 &= 0.170 \end{aligned} \quad (8.21)$$

The relative maximum intensity of the main lobe is

$$(a_1 + a_2 + a_3 + a_4)^2 = 0.27 \quad (8.22)$$

while the maximum total laser power interacting with the four acoustic beams is

$$a_1^2 + a_2^2 + a_3^2 + a_4^2 = 0.073 \quad (8.23)$$

out of a total maximum of unity.

For comparison, a rectangular laser profile with a width of $(2L + H)$ in the y_0 dimension, and having the same total power is used. The transformation of the Gaussian beam into a rectangular beam could be performed using a holographic filter [49]. If we assume that the transformation were 100% efficient, and the acoustic beams were self-collimating, the maximum total laser power interacting with the acoustic

beams would have a value of $0.0232 \times 4 = 0.093$. The relative maximum intensity of the main lobe would have a value of 0.37 which is 1.38 times that of the case where the optimum Gaussian illumination of $T_y = 0.8$ is used.

The DOA patterns for the two cases, rectangular uniform weighting and optimum Gaussian weighting, are plotted in Fig. 40 for a signal coming in on boresight. The maximum values of the two cases have been normalized to a value of unity. As expected, the Gaussian weighting has the effect of broadening the main lobe and suppressing the sidelobe levels.

Using the optimum Gaussian profile of $T_y = 0.8$, the DOA patterns of the 4-channel Bragg cell and its modified version using a phased array structure are plotted in Fig. 41. They are plotted for the case where the signal is received on boresight. By comparing the two plots, the modified Bragg cell gives an improvement on the output intensity of at least five times in the angular range of interest. Outside the angular range of interest in which $|y_2| > 1$, the sidelobes are suppressed further by the phased array weighting. Fig. 42 shows the two plots again but with their maximum values normalized to unity.

Fig. 40. Normalized DOA patterns of the 4-channel Bragg cell for a signal received on boresight and illuminated with (a) a rectangular uniform profile, (b) the optimum Gaussian profile.

Fig. 41. Relative DOA patterns for a signal received on boresight and illuminated with the optimum Gaussian profile.

(a) The 4-channel Bragg cell.

(b) The modified 4-channel Bragg cell using a phased array structure.

Fig. Normalized DOA patterns for a signal received on
 presight and illuminated with the optimum
 Gaussian profile.

- (a) The 4-channel Bragg cell.
- (b) The modified 4-channel Bragg cell using a phased array structure.

8.5 SUMMARY AND CONCLUSIONS

The novel modified multi-channel Bragg cell using a phased array structure, has been shown to give an improvement of at least five times on the intensity of the DOA pattern. The phased array is composed of five acoustic transducers with a uniform inter-transducer spacing of $1.1H$ and H is the height of the acoustic transducer element. The inner three transducers are excited with the same amplitude and phase, while the outer two transducers are excited with a phase difference of π radians and a smaller amplitude of 0.38 times that of the inner three transducers.

For a certain multi-channel Bragg cell, illuminated fully with a Gaussian laser beam, the main lobe intensity of the DOA pattern is a function of the profile parameter T_y . It was shown that an optimum T_y can be chosen to maximize the main lobe intensity which is the signal output from the processor. However this optimum value of T_y usually does not result in a maximum total laser power interacting with the acoustic beams.

CHAPTER 9

PHASE-TRACKING MEASUREMENT OF A MULTI-CHANNEL BRAGG CELL

9.1 INTRODUCTION

The 2-D acoustooptic signal processor, employing a multi-channel Bragg cell for the determination of DOA of an incoming signal is a phase DF system. Phase tracking among the channels is the most important specification of the system, which determines the accuracy as well as the shape of the DOA pattern.

The multi-channel Bragg cell is the key component in such a processor, and is usually the most difficult to evaluate. The inputs to the multi-channel Bragg cell are the IF signals applied to the acoustic transducers, and the outputs are the diffracted optical beams. Phase tracking has to perform well in all the three different domains, namely the microwave, acoustic and optical.

In this chapter, we investigate the phase-tracking characteristics of the multi-channel Bragg cell by making use of the inherent interferometric nature of this processor. A simple novel method of illuminating two adjacent channels at the same time is used, in which the relative phase information is extracted from the interference diffraction pattern. A scheme is developed to calculate the relative phase distribution and phase-tracking characteristics in all the channels. Finally this scheme is illustrated using a 4-channel Bragg cell.

9.2 DIFFRACTION PATTERN OF TWO CHANNELS

In this section, the diffraction pattern of only two channels of a multi-channel Bragg cell is investigated. The two acoustic channels are excited equally in both amplitude and phase by using a cw signal. The acoustic beams are illuminated fully at the Bragg input angle with a laser source of a uniform profile. The input optical wavefront and the Fourier transform lens are assumed to be ideal and the acoustic loss to be negligible. The diffracted light distributions from the acoustic channels are assumed to be of the same form and may differ from each other in both phase and amplitude. Making use of Eq.(7.3), the diffraction pattern is given by

$$\begin{aligned}
 U_1(\gamma, \beta) = & \frac{K_B}{\lambda F} DH \exp\left\{j \frac{\pi D}{\lambda} \left[\frac{\gamma}{\Lambda} - \frac{\Lambda (1/2 - s) \beta^2}{\lambda} \right]\right\} \\
 & \cdot \operatorname{sinc}\left\{\frac{D}{\lambda} \left[\gamma - \frac{\lambda}{\Lambda} + \frac{\Lambda (1/2 - s) \beta^2}{\lambda} \right]\right\} \operatorname{sinc}\left(\frac{H \beta}{\lambda}\right) \\
 & \cdot \{ C_1 \exp[-j(\frac{2\pi \beta \ell_1}{\lambda} + \phi_1)] + C_2 \exp[-j(\frac{2\pi \beta \ell_2}{\lambda} + \phi_2)] \}
 \end{aligned}
 \tag{9.1}$$

where $C_n = a_n + \delta a_n$, a_n is the designed amplitude of channel n and in this case it is (a) for all n , δa_n is its amplitude error, ϕ_n is the phase of channel n .

The differences in amplitude and phase among the channels may come from the microwave network feeding the acoustic transducers, from the acoustic transducers in which there may be alignment errors and thickness variations, and from the optical path difference through the Bragg cell medium.

The intensity distribution in the Fourier plane (x_1, y_1) is

$$\begin{aligned}
 I_1(x_1, y_1) = & U_1(x_1, y_1) U_1^*(x_1, y_1) \\
 = & \left(\frac{K C_1 DH}{\lambda F} \right)^2 \operatorname{sinc}^2 \left\{ D \left[\frac{x_1}{\lambda F} - \frac{1}{\Lambda} + \frac{\Lambda (1/2 - s) y_1^2}{\lambda^2 F^2} \right] \right\} \\
 & \operatorname{sinc}^2 \left(\frac{H y_1}{\lambda F} \right) \left\{ 1 + \left(\frac{C_2}{C_1} \right)^2 + \frac{C_2}{C_1} \left[2 \cos \left(\frac{2\pi y_1 \ell_1}{\lambda F} + \phi_{21} \right) \right] \right\}
 \end{aligned}
 \tag{9.2}$$

where

$$y_1 = \beta F$$

$$x_1 = \gamma F$$

$$\ell_{21} = \ell_2 - \ell_1$$

$$\phi_{21} = \phi_2 - \phi_1$$

From the above equation, we see that the normal single channel diffraction pattern is multiplied by a fringe pattern in the y_1 coordinate with period $= \lambda F / \ell_{21}$. Along $x_1 = \lambda F / \Lambda$, the period of the fringe pattern is always much smaller than the widths of the two sinc envelopes. For other window weighting functions such as the Gaussian illumination profile and acoustic loss, only the sinc functions are modified and the fringe pattern is not affected appreciably. The maximum value of the fringe pattern

$$I_{fp}(y_1) = 1 + \left(\frac{C_2}{C_1}\right)^2 + \frac{C_2}{C_1} \left[2 \cos\left(\frac{2\pi y_1 \ell_{21}}{\lambda F} + \phi_{21}\right) \right] \quad (9.3)$$

occurs when

$$\frac{2\pi y_1 \ell_{21}}{\lambda F} + \phi_{21} = \pm n\pi, \quad n = 0, 2, 4, \dots \infty \quad (9.4)$$

and the maximum value is

$$I_{fp,max} = \left(1 + \frac{C_2}{C_1}\right)^2 \quad (9.5)$$

The minimum value occurs when

$$\frac{2\pi y_1 \ell_{21}}{\lambda F} + \phi_{21} = \pm m\pi, \quad m = 1, 3, 5, \dots \infty \quad (9.6)$$

$$\text{and } I_{fp, \min} = \left(1 - \frac{C_2}{C_1}\right)^2 \quad (9.7)$$

In practice, ϕ_{21} should be small and much less than $\pm\pi$ radians. The maximum intensity distribution I_1 , occurs at $y_1 = 0$, when $\phi_1 = \phi_2$. If $\phi_1 \neq \phi_2$, then the peak of the fringe pattern will shift in position with a slight drop in the total intensity. A change in ϕ_{21} , of 2π radians corresponds to one period of the fringe pattern.

9.3 RELATIVE PHASE MEASUREMENT OF A MULTI-CHANNEL BRAGG CELL

Using the information contained in the diffraction fringe pattern of two channels, the relative phase between the two channels can be measured. We are going to make use of this technique and present a simple novel method in which the relative phase distribution of a multi-channel Bragg cell can be evaluated.

The procedure is to start with the first two adjacent acoustic channels denoted here as channels #1 and #2. Since the origin of y_1 is not known exactly, the peak location of the fringe pattern (y_{11}) is

measured relative to some arbitrary reference (y_{10}). From Eq. (9.2), we have

$$(y_{21} - y_{10}) = \frac{\phi_1 - \phi_2}{2\pi\ell_{21}} \lambda F \quad (9.8)$$

The second step is to compare the next two adjacent acoustic channels, #2 and #3, and we have

$$(y_{32} - y_{10}) = \frac{\phi_2 - \phi_3}{2\pi\ell_{32}} \lambda F \quad (9.9)$$

This step is repeated until all the channels are used. For N channels, $(N-1)$ peak locations and equations are obtained. The unknown y_{10} can be eliminated by taking the difference in peak locations between two consecutive channel pairs. For channel pairs (#1, #2) and (#2, #3), we have

$$(y_{32} - y_{21}) = \left(\frac{\phi_2 - \phi_3}{\ell_{32}} - \frac{\phi_1 - \phi_2}{\ell_{21}} \right) \frac{\lambda F}{2\pi} \quad (9.10)$$

Expressing the difference in peak locations in terms of fractions of the fringe period of channels #2 and #3, we get

$$(y_{32} - y_{21}) \frac{\ell_{32}}{\lambda F} = \frac{\phi_2 - \phi_3}{2\pi} - \frac{\phi_1 - \phi_2}{2\pi} \frac{\ell_{32}}{\ell_{21}} = Q_1 \quad (9.11)$$

The next two pairs of acoustic channels (#2, #3) and (#3, #4) are compared to obtain

$$Q_2 = (y_{43} - y_{32}) \frac{\ell_{43}}{\lambda F} = \frac{\phi_3 - \phi_4}{2\pi} - \frac{\phi_2 - \phi_3}{2\pi} \frac{\ell_{43}}{\ell_{32}} \quad (9.12)$$

For the last two pairs of acoustic channels, we have

$$Q_{N-2} = (y_{N,N-1} - y_{N-1,N-2}) \frac{\ell_{N,N-1}}{\lambda F} \\ = \frac{\phi_{N-1} - \phi_N}{2\pi} - \frac{\phi_{N-2} - \phi_{N-1}}{2\pi} \frac{\ell_{N,N-1}}{\ell_{N-1,N-2}} \quad (9.13)$$

Now there are (N-2) independent equations with N unknown phase values. However, we are only interested in the relative phase differences among the various channels. Using channel #1 as the reference, the above (N-2) equations and Eq. (9.8) can be solved to give

$$\phi_{11} = 0$$

$$\phi_{21} = \frac{-(y_{21} - y_{10})}{\lambda F} 2\pi \ell_{21}$$

$$\begin{aligned}
\phi_{31} &= \phi_{21} \left(\frac{\ell_{21} + \ell_{32}}{\ell_{21}} \right) - 2\pi Q_1 \\
\phi_{41} &= \phi_{21} \left(\frac{\ell_{21} + \ell_{32} + \ell_{43}}{\ell_{21}} \right) - 2\pi Q_1 \left(\frac{\ell_{32} + \ell_{43}}{\ell_{32}} \right) - 2\pi Q_2 \\
\phi_{N1} &= \phi_{21} \left(\frac{\ell_{21} + \ell_{32} + \ell_{43} \dots + \ell_{N,N-1}}{\ell_{21}} \right) \\
&\quad - 2\pi Q_1 \left(\frac{\ell_{32} + \ell_{43} \dots + \ell_{N,N-1}}{\ell_{32}} \right) - 2\pi Q_2 \left(\frac{\ell_{43} + \ell_{54} \dots + \ell_{N,N-1}}{\ell_{43}} \right) \\
&\quad \dots - 2\pi Q_{N-2}
\end{aligned} \tag{9.14}$$

The only unknown is the value y_{10} which is the assumed value for the origin ($y_1 = 0$). Unless there are other means of locating $y_1 = 0$, in the Fourier plane, there will be an error in estimating ϕ_{21} . From the above set of equations, an error in estimating ϕ_{21} , will introduce a linear phase shift across the whole array. However, this linear phase shift will only shift the DOA pattern and will not affect its shape. As will be addressed in Section 9.5, this doesn't pose a problem for the determination of DOA in this architecture. As a result, the relative phase distributions in the multi-channel Bragg cell using this simple method can be measured within an uncertainty of a linear phase shift across the array.

In a practical situation, the relative phase differences of all the channels are expected to be small in order for this configuration to

work as an interferometer. When all the channels are illuminated, the main lobe of the DOA pattern should be located very close to $y_1=0$. The deviation is the beam-pointing error discussed in Section 6.5. The beam-pointing error is inversely proportional to the array length ($2 \ell_N$) and for N large, it is inversely proportional to the square root of N . As a result, by illuminating all the channels simultaneously, the location of $y_1=0$ can be estimated. The smaller the phase errors in the various channels, the better the estimation.

2.4 EXPERIMENTAL EVALUATION OF THE RELATIVE PHASE DISTRIBUTION OF A 4-CHANNEL BRAGG CELL

In the experimental set-up, a HeNe laser is collimated and expanded to illuminate a 4-channel Bragg cell fully with a uniform profile. A good quality spherical lens is used to form a 2-D diffraction pattern in the Fourier plane. The spherical lens is evaluated separately as a Fourier transform lens using the two-aperture optical path difference technique [50]. It is found to have a maximum optical path difference of about $1/50 \lambda$ for the input spatial frequency range of interest. A Reticon S-series linear photodetector array with a $25 \mu\text{m}$ cell-to-cell center spacing and a height of 2.5 mm is used to detect the diffraction pattern along the DOA axis. A cw signal is divided equally in both amplitude and phase and applied to two adjacent channels of the multi-channel Bragg cell at a time.

Initially, a 4-slit array photomask with the same array dimensions as the multi-channel Bragg cell is used to evaluate the quality of the input illumination profile and the spherical lens. The height (H) of each slit is $67 \mu\text{m}$ and the array spacings are

$$\begin{aligned} \ell_1 &= -1.267 \text{ mm} \\ \ell_2 &= -0.320 \text{ mm} \\ \ell_3 &= 0.107 \text{ mm} \\ \ell_4 &= 1.267 \text{ mm} \end{aligned} \tag{9.15}$$

The fringe diffraction patterns of the three pairs of channels using the photomask are given in Fig. 43. As can be seen from the fringe patterns, the phase differences among the four channels are very small. The DOA pattern with all four channels illuminated, is shown in Fig. 44. The theoretical plot of the ideal DOA pattern with a baseline length of $2\ell_N = 9.5\lambda_R$ is given in Fig. 45. By comparing the theoretical and experimental plots, they agree very well.

Next the 4-channel Bragg cell is used and energized with a cw signal at 2.5 GHz. The fringe patterns of the three pairs of adjacent channels are given in Fig. 46 and the DOA pattern with all four channels illuminated is shown in Fig. 47. The centroid of the main lobe is assumed to be located at $y_1=0$ because the phase differences of all the four channels are very small. Using the fringe patterns and the procedure outlined in the last section, the relative phase values of the

Fig. 43. Diffraction patterns along DOA axis by illuminating two adjacent slits at a time of a 4-slit photomask. (photodetector array cell-to-cell center spacing = $25\text{ }\mu\text{m}$, $F = .565\text{ m}$, slit width = $67\text{ }\mu\text{m}$, 4-slit array length = 4 mm)

RELATIVE
INTENSITY

DOA AXIS

Fig. 44. Diffraction pattern along DOA axis by illuminating a 4-slit photomask. (Photodetector array cell-to-cell center spacing = $25\text{ }\mu\text{m}$, $F = .565\text{ m}$, slit width = $67\text{ }\mu\text{m}$, 4-slit array length = 4 mm)

Fig. 45. Diffraction pattern of an ideal 4-channel Bragg cell along the DOA axis.

Fig. 46. Diffraction patterns along DOA axis by illuminating two adjacent channels at a time of a 4-channel Bragg cell driven at 2.5 GHz. (Photodetector array cell-to-cell center spacing = 25 μm , $F = .565 \text{ m}$)

Fig. 47. Diffraction pattern along DOA axis of a 4-channel Bragg cell. (Photodetector array cell-to-cell center spacing = $25\text{ }\mu\text{m}$, $F = .565\text{ m}$, input frequency = 2.5 GHz)

four channels are calculated and given below in terms of cycles,

$$\phi_{11} = 0.$$

$$\phi_{21} = 0.$$

$$\phi_{31} = 0.03$$

(9.16)

$$\phi_{41} = -0.03$$

The amplitudes of the diffracted optical beams for the four channels are obtained by energizing one channel at a time and the diffracted power is measured using a photometer. The relative amplitude values of the four channels are

$$C_1/C_4 = 0.81$$

$$C_2/C_4 = 0.86$$

$$C_3/C_4 = 0.92$$

$$C_4/C_4 = 1.0$$

(9.17)

With the measured amplitude and phase values, the theoretical DOA pattern is calculated and plotted in Fig. 48. By comparing this calculated pattern with the experimental pattern shown in Fig. 47, they do agree quite well.

As discussed earlier in Section 6.5, the phase errors have a much greater effect than the amplitude errors on the DOA pattern. Fig. 49 is

Fig. 48. Diffraction pattern of 4-channel Bragg cell along DOA axis at 2.5 GHz, using measured amplitude and phase values in each channel.

(Normalized amplitudes: $C_1/C_4 = .81$

$C_2/C_4 = .86$

$C_3/C_4 = .92$

$C_4/C_4 = 1.0$

Relative phases (cycles): $\phi_{11} = 0.0$

$\phi_{21} = 0.0$

$\phi_{31} = .03$

$\phi_{41} = -.03$)

Fig. 49. Diffraction pattern of a 4-channel Bragg cell along DOA axis with equal phase values⁷ and with normalized amplitudes of

$$C_1/C_4 = .81$$

$$C_2/C_4 = .86$$

$$C_3/C_4 = .92$$

$$C_4/C_4 = 1.0$$

a plot where the same amplitude distribution as given in Eq. (9.17) is used but with zero phase errors. By comparing Fig. 49 to the ideal DOA pattern given in Fig. 45, the effect of amplitude error on the DOA pattern is indeed very small.

9.5 PHASE-TRACKING MEASUREMENTS

In the last two sections, the technique of comparing two adjacent channels at a time was presented to measure the relative phase distribution of a multi-channel Bragg cell. The relative phase distribution can be measured within an uncertainty of a small linear phase shift across the array. A linear phase shift across the array doesn't affect the shape of the DOA pattern and thus is not important as long as it doesn't vary appreciably as a function of frequency. In other words, we are only interested in the phase-tracking characteristics of the multi-channel Bragg cell within its operating bandwidth.

In this section, the measurement technique for evaluating the phase-tracking characteristics of a multi-channel Bragg cell is investigated. A slightly different procedure is used for the phase-tracking analysis. The first step is to illuminate all the channels simultaneously at a certain frequency f_1 . The centroid of the main lobe of the DOA pattern (y_{1R}) is used as the reference through out all the measurements. The same procedure of using two adjacent channels at

a time is used and the only difference is that the peak location of the fringe pattern (y_{nm}) is measured with respect to y_{1R} . The relative phase differences at frequency f_1 can be calculated and given by

$$\phi_{11} = 0$$

$$\phi_{21} = \frac{-(y_{21} \ell_{21})}{\lambda F} 2\pi$$

$$\phi_{31} = -(y_{21} \ell_{21} + y_{32} \ell_{32}) \frac{2\pi}{\lambda F} \quad (9.18)$$

$$\phi_{N1} = -(y_{21} \ell_{21} + y_{32} \ell_{32} \dots + y_{N,N-1} \ell_{N,N-1}) \frac{2\pi}{\lambda F}$$

Now if the input frequency to the Bragg cell is changed to f_2 , then a new set of ϕ values are generated. Using the same reference point y_{1R} for measuring the location of the peak of the fringe pattern, a new set of ϕ_{N1} values can be calculated. Since we are using the same reference y_{1R} for both frequencies f_1 and f_2 , the differences in the two sets of ϕ_{N1} values will give the phase-tracking errors of the Bragg cell for those two frequencies. Denoting the 2nd set of ϕ_{N1} values using primed notations, the phase-tracking errors are

$$\Delta\phi_{11} = \phi_{11} - \phi'_{11} = -\phi'_{11}$$

$$\Delta\phi_{21} = \phi_{21} - \phi'_{21} = -(y_{21} - y'_{21}) \frac{2\pi \ell_{21}}{\lambda F}$$

$$\Delta\phi_{31} = \phi_{31} - \phi'_{31} = -[(y_{21} - y'_{21}) \ell_{21} + (y_{32} - y'_{32}) \ell_{32}] \frac{2\pi}{\lambda F}$$

$$\begin{aligned} \Delta\phi_{N1} = \phi_{N1} - \phi'_{N1} = & -[(y_{21} - y'_{21})\ell_{21} + (y_{32} - y'_{32})\ell_{32} \\ & + \dots + (y_{N,N-1} - y'_{N,N-1})\ell_{N,N-1}] \frac{2\pi}{\lambda F} \end{aligned} \quad (9.19)$$

If the above procedure is repeated at other frequencies, then a plot of the phase-tracking errors for all the channels as a function of frequency can be obtained.

Fig. 50 shows the fringe patterns obtained, when the input frequency is changed to $f_2 = 2.45$ GHz. This new set of patterns is obtained by physically moving the linear photodetector array to the new x_1 location corresponding to the new location of the diffraction pattern at f_2 . By comparing Figs. 46 and 50, the phase tracking errors between this two frequencies can be calculated.

There are two practical set-ups in which the phase-tracking measurement can be carried out. One set-up is to use the same arrangement as outlined in Section 9.4 by using a single spherical lens. The only change is to replace the linear photodetector array with an area array to cover a fractional bandwidth (Δf) of the Bragg cell. As the frequency is varied by more than (Δf) , the lens and the area array are moved to follow the diffracted beam. The movement of the lens will allow the diffracted light beam to go through the central region of the lens where distortions are minimal. By using this step and repeat process, the phase-tracking characteristics of the Bragg cell over its full bandwidth can be measured.

Fig. 50. Diffraction patterns along DOA axis by illuminating two adjacent channels at a time of a 4-channel Bragg cell driven at 2.45 GHz. (photodetector array cell-to-cell center spacing = 25 μ m, $F = .565$ m)

Another set-up is to introduce a cylindrical lens placed in between the Bragg cell and the spherical lens. In this arrangement, the image of the multi-channel Bragg cell is formed on the x dimension and the Fourier transform is performed on the y dimension. The DOA pattern is now stationary independent of the input frequency and a linear photodetector array can be used. However, in this arrangement, the combination of the cylindrical and spherical lenses must be able to maintain a low optical path difference (OPD) over a relatively wide range of input spatial frequencies.

In all the measurements discussed so far in this chapter, the electrical crosstalks or mutual couplings among the various channels are assumed to be negligible.

9.6 SUMMARY AND CONCLUSIONS

DOA determination using a multi-channel Bragg cell operating in the Fourier transforming configuration, is a phase DF system. Phase tracking among the channels is of paramount importance because it determines the angular accuracy as well as the shape of the DOA pattern. The multi-channel Bragg cell is the most important and at the same time, the most difficult component to be evaluated in the receiver.

A simple technique, of illuminating two channels at a time and then

comparing the resultant diffraction fringe patterns in the Fourier plane, has been developed to determine the phase distribution of a multi-channel Bragg cell. It has been shown both theoretically and experimentally with a 4-channel Bragg cell, that this simple measurement scheme can be used to measure the relative phase values of the channels with an uncertainty of a small linear phase shift across the array. However this linear phase shift will only shift the DOA pattern by a small offset but not its shape. In this 2-D acoustooptic signal processor, this offset is not important as long as it doesn't vary appreciably over the frequency band of operation. In other words, we are only interested in the phase-tracking characteristics of the multi-channel Bragg cell within its operating bandwidth. As a result, this simple novel technique is ideal for the phase-tracking measurement of a multi-channel Bragg cell.

CHAPTER 10

CONCLUSION AND RECOMMENDATIONS

10.1 SUMMARY

The acoustooptic Fourier transforming configuration for spectrum analysis and DOA determination has been investigated in this thesis. The objective of the study was two-fold.

- (a) To study the response of the acoustooptic spectrum analyzer for pulse-modulated cw signals, using a photodetector array operating either in the real-time mode or the time-integrating mode, for the purpose of estimating the frequency spectrum, pulse width and pulse repetition frequency of an input signal.
- (b) To investigate the aspect of DOA determination of the 2-D acoustooptic Fourier transforming configuration employing a multi-channel Bragg cell, optimized in a linear aperiodic configuration for wideband radar receiver applications.

The thesis is divided into two parts. The first part concentrates on the operation of spectrum analysis. This can either be the operation of the Fourier transforming configuration employing a single-channel Bragg cell or, in the case of the multi-channel Bragg cell configuration, it is the operation along the frequency dimension. The second part of the thesis deals with the DOA determination of the 2-D Fourier transforming configuration using a multi-channel Bragg cell.

The key component in the acoustooptic, Fourier transforming configuration is the Bragg cell which is used as a spatial light modulator. The theory of acoustooptic diffraction has been analyzed in Chapter 2, where the coupled mode formulation as given by Klein and Cook [14] on isotropic interaction is modified to include also the anisotropic case.

The optical Fourier transforming configuration using a Bragg cell as a spatial light modulator has been investigated in Chapter 3. The Bragg cell was first modeled as an ideal traveling wave modulator in which the material is assumed to produce a self-collimating acoustic beam. The diffraction pattern is the familiar 2-D sinc² function. This simplified model is very useful for general system analysis such as in the development of signal processing architectures. The deviation from this ideal model was then analyzed for both the isotropic and anisotropic materials. The diffraction pattern is still characterized by the expected 2-D sinc² function, except that in one dimension the function is curved toward the optical axis. The degree of curvature depends on the anisotropic parameter of the interaction medium.

Once the basic acoustooptic Fourier transforming configuration had been characterized, attention was focused on using it as a spectrum analyzer for detecting pulsed signals. For cw signals, the shape of the spectrum is constant and completely determined by the Fourier transform of the amplitude weighting function of the aperture. This subject has been addressed and can be found in the open literature [4]. When the

signal is of finite duration such as a pulsed signal, the spectrum is a function of where the signal is located in the aperture and is more complex. In addition, the output from the AOSA is also a function of the mode of operation of the photodetector array, used for detecting the spectrum of the signal. The purpose of the first part of the thesis is to extend the analysis of the AOSA to input pulsed signals. Pulsed signals are of great importance because the great majority of radar signals are of this type. The spatial and temporal characteristics of the AOSA using a photodetector array operating in the real-time and time-integrating modes are given in Chapters 4 and 5, respectively.

In Chapter 4, the spatial and temporal responses of the AOSA for pulsed signals are given. The formation of the instantaneous spatial frequency distribution has been fully characterized as a pulse propagates through the Bragg cell aperture. In the case where the window weighting function contains only the acoustic attenuation exponential function, the spatial distributions have been shown to be characterized by a combination of exponential and complex sinc functions.

The instantaneous spatial distribution generated by the spectrum analyzer as the pulse propagates through the Bragg cell aperture, is detected using a photodetector array operating in the real-time mode. The center-to-center element spacing used in the analysis and measurements, has a normalized spatial bandwidth equal to the reciprocal of the aperture. This spectrum analyzer configuration is used mainly as

a frequency channelizer in which multiple simultaneous signals of different frequencies can be spatially separated. In this case, a coarse frequency resolution is obtained while the time resolution is fine. The detected signal waveform from the photodetector has been studied and measured as a function of the pulse width, amplitude weighting function parameters, and the relative location of the spectral centroid with respect to the photodetector.

It was observed that the input pulse envelope is modified by the AOSA in which the duration of the signal is always stretched by an amount equal to the aperture transit time. In the case where the pulse width is longer than the aperture transit time, the difference in duration was found to appear as a flat top on the detected output. The shape of the detected waveform is strongly affected by the amplitude weighting parameters. Furthermore, if the photodetector is located off the main lobe of the signal spectrum and detecting its sidelobes, multiple humps are detected.

However, despite the appearance of complicated waveform distributions, the first two characteristics, namely, the signal duration stretching and the duration of the maximum flat region, were found to be simply related to the aperture transit time. This is especially true for the photodetector located at the spectral centroid, where the output level is the highest and free of the multiple-hump structure. These two characteristics can then be used in the determination of the pulse width.

The other mode of operation of the AOSA is the use of a time-integrating photodetector array, which is presented in Chapter 5. In this case, one is more interested in the total power spectral density of the signal than its instantaneous spectra. The use of a time-integrating photodetector array also helps to simplify and reduce large amount of data generated by such a processor. The time-integrated spectrum is distorted from the spectrum of the signal by two contributing factors of the window weighting function. One is due to the truncation of the signal by the finite aperture and the other is due to the amplitude weighting on the signal when it is in the aperture.

The distortion on the spectrum due to truncation and amplitude weighting, was shown to have an effect of broadening the main lobe and smoothing out its sidelobes. This effect increases with increasing values of the amplitude weighting parameters T_x and α , and has also been quantitatively characterized in a form of a normalized plot. Furthermore, when the pulse width is about the same duration as the Bragg cell aperture transit time, the shape of the time-integrated spectrum was found to be very smooth with no sidelobe structures.

The second part of this thesis is mainly on the DOA determination aspect of the 2-D Fourier transforming configuration using a multi-channel Bragg cell. The basic configuration as proposed by Lambert et al [3] has been modified to be used for wideband radar receiver applications where the frequency coverage of the order of an octave and more is required. A general system analysis in which the multi-channel

Bragg cell is modeled by an ideal traveling wave modulator is given in Chapter 6. This 2-D configuration produces optically the weighted spectrum of an incoming signal on one axis and the optical far-field pattern on the other. The optical far-field pattern was shown to be similar in shape to the microwave antenna array far-field pattern, except that the element pattern is determined by the interacting profile of the acoustic and laser beams. Furthermore, there are no "invisible" regions in the optical far-field pattern and what is "invisible" in the microwave domain becomes "visible" in the optical domain. The deflection angle of the main beam of the optical pattern has also been shown to be related to the angle of arrival (θ_0) by the product of the scaling factor (S.F.) and $\sin \theta_0$.

An error tolerance analysis is given in which the amplitude and phase error distributions in all the channels are assumed to be of zero mean, with independent Gaussian statistics. The amplitude error was found to affect only the shape of the pattern, and the dominant effect of both the amplitude and phase errors is a reduction in gain and a change in the main-lobe-to-sidelobe ratios. The beam-pointing error of the processor is directly proportional to the phase errors. It has also been shown to be inversely proportional to the array length and approximately inversely proportional to the square root of the number of channels.

It was concluded that an aperiodic array configuration with four or five elements is the optimum design, for applications requiring a broad

frequency band and angular coverage with good angular accuracy. This conclusion was based on the criteria of cost, ease of fabrication, required main lobe beamwidth and sidelobe levels. A simple synthesis procedure of designing an aperiodic array configuration with four elements, has been shown to be useful. The main optimization parameters used, are minimum peak sidelobe-to main lobe ratio, and maximum tolerance to perturbations of channel locations.

For a non-ideal Bragg cell, the DOA pattern and frequency distribution are no longer orthogonal. This results in a mismatch with the rectangular coordinates of a photodetector area array configured in terms of rows and columns. The effect of this mismatch has a far greater impact on the DOA pattern than the frequency information because the main lobe level is only a few dB above its sidelobes. It was shown, in Chapter 7 that in order to keep the amplitude-sampling error of an input signal to a minimum and insensitive to the spatial weighting function of the photodetector array, it is desirable to choose the minimum number of photodetectors per frequency resolution width to be three. Furthermore the distortion on the DOA pattern is minimized if the column located at the frequency centroid of the signal is chosen. No distortions are introduced in the DOA pattern when the Bragg cell is ideal in which the material produces a self-collimating acoustic beam with the anisotropy factor $s = 1/2$. The requirement on the value s from the ideal case can be relaxed by a certain choice of system and component parameters. The degree of relaxation was shown to be directly proportional to the product of the horizontal angular spatial frequency

(γ_0) and its maximum tolerable value $(\Delta\gamma)$, and inversely proportional to the square of the product of the scaling factor (S.F.) and the sine of the maximum steering angle (θ_0) .

In Chapter 8, a novel modified multi-channel Bragg cell using a phased array structure has been shown to give an improvement of at least five times in the intensity of the DOA pattern. The phased array is composed of five acoustic transducers with a uniform inter-transducer spacing of $1.1H$ and H is the height of the transducer. The inner three transducers are excited with the same amplitude and phase, while the outer two transducers are excited with a phase difference of π radians and a smaller amplitude of 0.38 times that of the inner three transducers. It was also shown that depending on the channel spacings, an optimum choice of the laser Gaussian profile parameter T_y can be chosen to maximize the output DOA pattern.

DOA determination using a multi-channel Bragg cell configuration, is a phase DF system. Phase tracking among the channels is of paramount importance because it determines the angular accuracy as well as the shape of the DOA pattern. The multi-channel Bragg cell is the most important and at the same time, the most difficult component to be evaluated in the receiver. A simple novel technique, of illuminating two adjacent channels at a time and then comparing the resultant diffraction fringe patterns in the Fourier plane, has been developed in Chapter 9 to determine the phase distributions of a multi-channel Bragg cell. It has been shown both theoretically and experimentally with a

4-channel Bragg cell, that this simple technique is ideal for the phase-tracking measurement of a multi-channel Bragg cell.

10.2 FUTURE RESEARCH WORK

The acoustooptic Fourier transforming configuration has been investigated in this thesis for wideband radar receiver applications. The major attraction of this type of processor is its inherent parallelism and its tremendous speed. This is especially true for the elegant 2-D Fourier transforming configuration using a multi-channel Bragg cell, in which the incoming simultaneous signals are spatially separated in terms of frequency and direction-of-arrival on a real-time basis.

The 2-D configuration, using a 4-channel Bragg cell has been successfully demonstrated in the laboratory at the Defence Research Establishment Ottawa. However, practical wideband radar receivers must exhibit good sensitivity and large dynamic range. At present, acoustooptic receiver technology using the Fourier transforming configuration is still limited in its dynamic range and its sensitivity is poor for pulsed signals with a low duty cycle and narrow pulse width.

Acoustooptic receiver technology has been advancing rapidly over the last few years. However there are still two major technological

areas where more research is needed to make the acoustooptic technology more attractive and be able to compete with other receiver technologies. The two key component technological areas are the photodetector array and the multi-channel Bragg cell.

The overall performance, in particular the dynamic range, of acoustooptic signal processors tends to be limited by the performance of the photodetector array. Although one and two-dimensional solid-state photodetector arrays are now incorporated into operational systems, the driving force behind the development of these self-scanned arrays relates to imaging applications, that do not require the wide dynamic range, linearity or read-out flexibility desired for signal processing applications. Recently there have been some developments in addressing some of these limitations [32].

A novel solid-state photodetector structure which can be integrated on the same silicon chip with MOSFET circuits or be integrated on a chip which has CCD shift registers, has been conceived and developed [33]. This novel photodetector device has the advantageous properties of a silicon photodiode plus an additional important property. Its output does not saturate since its voltage varies logarithmically as a function of the incident light intensity. The logarithmic response and the wide dynamic range associated with it, make this device attractive as a photodetector element.

This novel photodetector structure is being incorporated into linear and area arrays. In making use of this type of photodetector

arrays for detecting pulsed signals, more investigation is needed on the interpretation of the photodetector output due to its logarithmic response and its relatively slow response. The improvement on the speed of response of this photodetector structure is another area where more research is needed.

Another photodetector structure of interest, which can be used for acoustooptic signal processing, is the avalanche photodiode. It has the advantage of having a multiplication gain which makes it more sensitive. It is also very fast. The avalanche photodiode has been recently incorporated into small linear arrays [51]. However more research work is needed to reduce the element-to-element center spacing before a large linear array and area arrays can be fabricated.

The other component which needs to be developed further is the multi-channel Bragg cell. Single-channel Bragg cell technology has reached a relatively mature state of development in the last few years [52]. More research is being carried out on the development of the multi-channel Bragg cell. Only a very few multi-channel Bragg cells with a large bandwidth have been developed recently. The state-of-the-art multi-channel Bragg cells are still limited in performance in particular the electrical isolation among the various channels. No data is available on the phase-tracking characteristics of these multi-channel Bragg cells due to the difficulty in carrying out the evaluation. The simple novel technique developed in this thesis should be very useful for this type of measurement. The sensitivity of the 2-D configuration

can be improved by making use of the novel multi-channel Bragg cell structure proposed in this thesis. The feasibility of practical implementation of the modified multi-channel Bragg cell needs to be investigated further.

Another area which has not been addressed in this thesis is on the post-processing of the data generated by the photodetector array. The data rate coming out from the photodetector array and, in particular the one operating in the real-time mode, is very high and special preprocessors are needed to reduce this data rate before it can be passed onto the digital computer for further processing.

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