A Numerical Investigation of Dual-Rotor Horizontal Axis Wind Turbines using an In-House Vortex Filament Code (DR_HAWT)

by

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Abstract

A numerical study was carried out using an in-house code named DR_HAWT (Dual-Rotor Horizontal-Axis Wind Turbine code), to identify non-dimensional parameters for dual-rotor wind turbines (DRWTs). DR_HAWT, which was implemented by the current author to predict the performance of single and dual-rotor wind turbines, uses a free vortex wake method with vortex filaments to represent the rotor and its wake. This vortex code was verified and validated using various blade-vortex interaction case studies and two well-known wind tunnel experiments (NREL Phase VI and MEXICO wind turbines).

Based on some important DRWT parameters such as the rotor speeds, rotor diameters and the separation distance between the rotors, three dimensionless parameters were derived from the Buckingham Pi theorem. These three parameters define the ratio of diameters (diameter ratio, DR), the ratio of the separation distance and the downwind diameter (gap ratio, GR), and the ratio of the product of the upwind angular velocity and the downwind rotor radius and the oncoming wind velocity (combined tip-speed ratio, CTSR). In addition, the power output of each DRWT was normalized with the total power generated by equivalent single-rotor turbines.

Hypothetical DRWT models were created using geometrically-scaled NREL Phase VI rotor geometry and operating conditions in order to confirm the validity of these

parameters. The variation in normalized output power as a function of the dimensionless parameters suggests that an improved performance of DRWTs can be obtained at lower diameter and gap ratios. The study also revealed that the effect of having an auxiliary upwind rotor reduces the angle of attack along the inboard section of the downwind blade. Finally, the NREL Phase VI rotor equipped with a 5 m geometrically-scaled upwind rotor was found to generate more than the combined power output of two equivalent single-rotors at 7.5 CTSR. It was hypothesized that this extra power results from a shift in the operating range of the blade due to the variation in angle of attack.

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Nomenclature

Latin Letters

A	Swept area of wind turbine rotor	$[m^2]$
$\mathcal{R}$	Aspect ratio	$[-]$
BN	Blade number	$[-]$
c	Chord length	$[m]$
C_D	Coefficient of drag	$[-]$
C_L	Coefficient of lift	$[-]$
C_n	Coefficient of normal force	$[N]$
C_P	Coefficient of power	$[-]$
C_Q	Coefficient of torque	$[-]$
C_T	Coefficient of thrust	$[-]$
C_t	Coefficient of tangential force	$[N]$
D	Drag force	$[N]$
D, D_1, D_2	Rotor diameter	$[m]$

F_n	Normal force	[N]
F_t	Tangential force	[N]
L	Lift force	[N]
L	Separation distance between rotors	[m]
NB	Number of blades	[-]
NT	Timestep	[-]
P_{dual}	Power generated by dual-rotor wind turbine	[W]
P_{max}	Maximum available power	[W]
P_{norm}	Normalized power	[-]
$P_{singles}$	Sum of power generated by two single-rotor wind turbines	[W]
Q	Rotor torque	[Nm]
R	Radius of wind turbine rotor	[m]
r	Radial distance from the hub center	[m]
r_c	Vortex core radius	[m]
Re	Reynolds number	[-]
r_o	Vortex core radius at $t = 0$	[m]
$\mathbf{s}, \mathbf{n}, \mathbf{c}$	Unit vectors defining the blade element	[-]

T	Axial thrust	$[N]$
t	Vortex age	$[s]$
U_t, V_t, W_t	Magnitude of induced velocity in x , y , and z directions at time t	$[m/s]$
$\mathbf{V}_{ind}$	Induced velocity vector	$[m/s]$
V_o	Magnitude of the oncoming wind velocity	$[m/s]$
$\mathbf{V}_o$	Wind velocity vector	$[m/s]$
$\mathbf{V}_{rel}$	Relative velocity vector	$[m/s]$
$\mathbf{V}_T$	Blade velocity vector	$[m/s]$
x, y, z	Global coordinates of the aerodynamic centre	$[m]$

Greek Letters

α	Angle of attack	$[\circ]$
β	Sum of twist and pitch angles	$[\circ]$
Γ_B	Bound vortex strength	$[m^2/s]$
Γ_S	Spanwise vortex strength	$[m^2/s]$
Γ_T	Trailing vortex strength	$[m^2/s]$
δ	Eddy viscosity coefficient	$[-]$
δ_c	Cut-off radius parameter	$[\%]$
Δt	Timestep increment	$[s]$
ζ	Vorticity	$[s^{-1}]$
θ	Angle Rotated by Blade 1	$[\circ]$
θ_B	Blade Azimuth Angle	$[\circ]$
λ	Tip-speed ratio	$[-]$
μ	Fluid Viscosity	$[kg/ms]$
ν	Fluid Kinematic Viscosity	$[m^2/s]$
ρ	Fluid density	$[kg/m^3]$
ϕ	Flow angle ($\phi = \alpha + \beta$)	$[\circ]$
$\omega, \omega_1, \omega_2$	Rotor angular velocity	$[rad/s]$

Abbreviations

AC	Aerodynamic center
AOA	Angle of attack
BEM	Blade element momentum theory
BET	Blade element theory
CFD	Computational fluid dynamics
CTSR	Combined tip-speed ratio
DES	Detached eddy simulation
DR	Diameter ratio
DR_HAWT	Dual-rotor horizontal axis wind turbine code
DRWT	Dual-rotor wind turbine
FWM	Free-wake method
GCS	Global coordinate system
GR	Gap ratio
HAWT	Horizontal axis wind turbine
LCS	Local coordinate system
LES	Large eddy simulation
LFA	Local flow angle

LLT	Lifting-line theory
MEXICO	Model experiments in controlled conditions
NREL	National renewable energy laboratory
PWM	Prescribed-wake method
RANS	Reynolds averaged Navier-Stokes
SRWT	Single-rotor wind turbine
TSR	Tip-speed ratio
UAE	Unsteady aerodynamic experiments
VAWT	Vertical axis wind turbine

Chapter 1

Introduction

With the depletion of petroleum resources as well as the increasing environmental issues, viz. climate change, renewable energy such as hydro, wind, and solar, are potential replacements for fossil-fuel-based energy as a source of electricity generation. In 2014, Ontario became the first North American jurisdiction to replace coal with wind and solar energies as a source of power generation [1]. Moreover, in 2008, the Canadian Wind Energy Association (CanWEA) proposed a long term goal of produce 20% of Canada's electricity from wind by 2025 [2]. Since then, wind energy has rapidly expanded, as can be seen in Figure 1.1, with over 11,000 MW of electricity generated by wind turbines at the end of 2015, which is equivalent to approximately 5% of Canada's demand in electrical power [3].

Over the past decades, greater emphases have been directed to improve the efficiency of single-rotor wind turbines (SRWTs) by increasing their swept area (i.e., the area of the circle produced by rotating blades) which allows for more power to be extracted since the power output scales with the rotor diameter squared [4]. Other examples to increase wind turbines' efficiency are to optimize the layout of wind farms, and install vortex generators on turbine blades. In more recent years, a growing interest in dual-rotor systems which have been extensively employed in the

Figure 1.1: Wind power generation in Canada over the past 16 years [3]

aerospace (e.g., the Soviet Ka-32 helicopter) and marine (e.g., the Mark 46 torpedo with contra rotating propellers) applications—to increase the aerodynamic efficiency of the systems while eliminating the asymmetrical torque experienced by conventional single-rotors—have resurfaced. Although the idea of increasing the amount of rotors per tower is not well accepted within the wind industry, university researchers from the University of Ottawa [5] and the Iowa State University (ISU) [6], to name a few, are investigating the potential of Dual-Rotor Wind Turbine (DRWT) systems due to its ability to extract more energy per tower. In addition, dual-rotor systems have the potential to lower the overall cost by reducing the number of towers which accounts for around 14%-17% of the total cost of the turbine including operation and management, grid connection, and foundation costs [7–9], albeit the need for stronger tower and more complex gearbox.

This chapter describes the fundamental of conventional wind turbines (horizontal and vertical axis wind turbines) and provides a literature survey on recent DRWT studies. Finally, the motivation behind this work and the structure of the thesis will also be outlined.

1.1 Principles of Wind Turbines

Wind turbines are rotary machines that extract the kinetic energy of the wind and convert it to electricity via generators. There are two main types of wind turbines, namely Vertical Axis Wind Turbine (VAWT) and Horizontal Axis Wind Turbine (HAWT), as shown in Figure 1.2. As its name implies, VAWT which can either be lift-based or drag-based turbine, rotates vertically with its rotational axis perpendicular

Figure 1.2: Schematic of a typical (a) Vertical Axis Wind Turbine (VAWT) and (b) Horizontal Axis Wind Turbine (HAWT)

to the incoming wind to generate electricity. HAWT, on the other hand, is a lift-based turbine which uses a propeller-type rotor with its rotational axis parallel to the incoming wind. Since the current work focuses on HAWTs, the term wind turbine will be used to refer to the horizontal axis configuration throughout this thesis.

The blade of a wind turbine is shaped, from a thick cylindrical section at the root to a thin airfoil section at the tip, to achieve the best equilibrium between its aerodynamic and structural properties. In addition to having various cross-sectional shapes, the blade is twisted to take advantage of a more ideal angle of attack (α)—the angle of the blade relative to the apparent wind—along the blade span. As the blade rotates, aerodynamic loads are generated and converted to mechanical power through the turbine shaft. These loads comprise of the lift (L) and drag (D) forces and can be resolved into normal (F_n) and tangential (F_t) forces, acting perpendicular and parallel to the rotor plane, respectively. Figure 1.3 illustrates the aerodynamic loads experienced on an airfoil section of the blade.

Figure 1.3: Aerodynamic loads acting on a section of a wind turbine blade, twisted at an angle β

The thrust (T) and torque (Q) produced by the normal and tangential forces, respectively, determine the performance of a wind turbine. In order to compare various designs, the thrust and torque of wind turbines can be non-dimensionalized. The following equations define the coefficients of thrust (C_T) and torque (C_Q), respectively.

$$C_T = \frac{T}{1/2\rho AV_o^2} \quad (1.1)$$

$$C_Q = \frac{Q}{1/2\rho RAV_o^2} \quad (1.2)$$

where V_o is the oncoming wind speed, ρ is the fluid density, A is the swept area of the rotor (i.e., $A = \pi R^2$) and R is the radius of the rotor.

Similarly, the coefficient of power (C_P), another non-dimensional performance parameter, is defined as the ratio between the actual output power obtained from a wind turbine and the theoretical maximum available power (P_{max}). Equation 1.3 defines the maximum available power that can be obtained if, in theory, the wind velocity could be reduced to zero.

$$P_{max} = 1/2\rho AV_o^3 \quad (1.3)$$

This maximum coefficient of power is about 16/27 (59.3%) according to the Betz law. In practice, however, conventional SRWTs can generate up to 75% to 80% of the Betz limit due to various losses such as viscous loss, three dimensional loss, and transmission loss [9–12].

1.2 Dual-Rotor Wind Turbines

Dual-rotor wind turbines (DRWTs) are equipped with two co-axial rotors in a back-to-back configuration or with an upwind and a downwind rotor separated by the nacelle, as illustrated in Figure 1.4. Since the shape of the inboard sections of SRWT blades extract little energy from the wind, it is hypothesized that by adding a smaller rotor upstream of a SRWT, more energy can be harvested.

Moreover, since the wake behind a rotor rotates in opposite direction to the rotor itself, more energy can be extracted by having the downwind rotor taking advantage of the wake rotation of the upwind rotor. In other words, counter-rotating DRWT can harvest more energy as compared to co-rotating DRWT [13,14]. The reason why counter-rotating configuration can extract more energy than co-rotating configuration

Figure 1.4: Schematic of the two main type of dual-rotor wind turbines

can be explained by comparing the velocity triangles of the downwind rotor. Figure 1.5 illustrates the velocity triangles for both configurations where V_{up} represents the effect of the upwind wake as the rotor rotates either in a clockwise or counter-clockwise direction. As can be observed, the swirl velocity (V_{up}) from a co-rotating upwind rotor induces a smaller angle of attack (α_{co}) thereby decreasing the lift force component on the blade while increasing the relative velocity (V_{co}) acting on the blade. On the other hand, in counter-rotating configuration, a larger angle of attack, $\alpha_{counter}$ (i.e., larger lift coefficient) and lower relative velocity ($V_{counter}$) act on the downwind blade due to the induced swirl from the upwind wake. In short, higher lift coefficient due to a larger angle of attack ($\alpha_{counter} > \alpha_{co}$) results in a greater torque from the counter-rotating configuration [14].

Figure 1.5: Schematic of the velocity triangles for co- (left) and counter-rotating (right) wind turbines

During the study performed by Jung *et al.* [15] using a 30 kW counter-rotating DRWT consisting of an 11 m diameter main rotor with a 5.5 m auxiliary upwind rotor, an increase in power output of about 21% was achieved over a conventional 11 m diameter rotor turbine at a rated wind speed of 10.6 m/s. Shen *et al.* [16] investigated the performance of a counter-rotating DRWT equipped with two Nordtank 500 kW turbines using the Navier-Stokes solver EllipSys3D. The analysis showed that the turbine can produce 43.5% more annual energy than a similar SRWT. Habash *et al.* [5] carried out a wind tunnel study with a model DRWT system equipped with a pair of three-bladed, 23 cm diameter counter-rotating rotors and a distance between the rotors that varied from 7-54 cm, which resulted in up to 60% more energy than a SRWT system.

More recently, ISU researchers started investigating DRWT concepts both numerically and experimentally in an attempt to reduce losses near the root of the main rotor (downwind rotor). Utilizing a secondary, smaller, co-axial rotor upstream of the main rotor, Rosenberg *et al.* [17] studied the effect of varying the secondary rotor diameter, the spacing between the rotors and the secondary rotor tip-speed. The DRWT model used consisted of the NREL 5 MW turbine as the main rotor and an optimized secondary rotor using an inversed BEM method. The analysis was performed using the Reynolds Averaged Navier-Stokes (RANS) solver OpenFOAM coupled with an actuator disk method to obtain an optimized DRWT model. The RANS results showed a net increase of 7% in power coefficient can be obtained when the swept area of the secondary rotor is 25% that of the main rotor with a distance separating the rotors of 20% of the main rotor radius. Furthermore, Ozbay *et al.* [6] conducted a wind tunnel investigation of DRWTs in either co-rotating or counter-rotating configuration using a model DRWT equipped with two 24 cm diameter rotors. It was found that the counter-rotating model which harvested approximately 7.8% more energy than the

co-rotating model, and produced up to 60% more wind energy as compared with the SRWT model.

1.3 Motivation

The work performed in this thesis is a continuation of the collaboration with ZEC Wind Power Corp. [18], a company partner in Ottawa, who envisioned Dual-Rotor Wind Turbines (DRWTs) as a promising alternative to solar panels and diesel-fueled generators which are generally used in remote locations and areas without access to the electricity grid. As previously mentioned, to capture more energy from the wind, the swept area of conventional HAWTs is growing in size. However, larger turbines come with a number of drawbacks such as increasing weight (therefore, larger stresses on the blades), transportation difficulties, increasing tower height, as well as bigger footprint (more land or sea). A potential alternative to increasing the size of conventional HAWTs, is multi-rotor technology which had fallen out of consideration over the years due to its complexity [19]. In the early 2016, Vestas installed a multi-rotor prototype consisting of four refurbished V29-225 kW turbines each having a 29-m-diameter rotor, attached to a single support structure, near Roskilde in Denmark. The support structure is composed of one central steel tower and two rotor arm assemblies arranged vertically at two levels which can independently rotate around the tower [20].

This thesis aims at providing a preliminary work to establish an optimal configuration for DRWT, in an attempt to generate more power per tower while keeping the blade span within a adequate range. With the low fidelity BEM theory not capable of simulating DRWT and the more sophisticated Navier-Stokes solver having

high computational cost, a vortex code that has the ability to compute the performance of dual-rotor wind turbine will be implemented. In addition, to compare the performance of various DRWT models, dimensionless parameters will be derived and tested.

1.4 Structure of Thesis

This thesis consists of six chapters. Chapter 2 provides a limited literature survey of the various performance prediction methods such as wind tunnel experiment, blade element momentum theory, vortex theory, and Navier-Stokes solvers. Chapter 3 describes DR_HAWT, the vortex filament code implemented in this thesis, including the mathematical formulations as well as the structure of the code. Chapter 4 details various test cases performed to verify if the code was well implemented and the validation of DR_HAWT against two well-known experiments, namely the NREL Phase VI and MEXICO wind turbines. Chapter 5 describes the dimensionless parameters derived in this work and the parametric study performed to investigate their validity as well as any potential effect of varying each parameter. Finally, Chapter 6 outlines the results drawn from the parametric study and provides some suggestions for future work.

Chapter 2

Review of Performance Prediction

Methods

During the design process, the aerodynamic performance of wind turbines are typically analyzed using numerical models ranging from models based on the Blade Element Momentum (BEM) theory to more sophisticated unsteady aerodynamic models using either a Prescribed-Wake Method (PWM) or a Free-Wake Method (FWM) to Computational Fluid Dynamics (CFD) models [21–27]. Despite being widely used nowadays, these models were built on empirical assumptions and validated based on specific wind turbines. Due to this drawback, numerical modeling does not provide enough flexibility to accurately design better performing wind turbines, however, with the increasing research and development in numerical models, vortex wake theories and CFD models based on Euler and Navier-Stokes solutions are promising prediction methods for the future of wind turbine design [12].

The BEM approach is generally employed by wind turbine manufacturers due to its fast computational runtime which makes it well suited for assessing new design performance. A summary of relevant performance prediction softwares for wind turbines available in the literature is compiled in Table 2.1. It should be noted that for

Table 2.1: Summary of performance prediction software for wind turbines

Author	Code Name	Affiliation	Method ¹	Reference
Marten	QBlade	TU Berlin	BEM, FWM	[28, 29]
Jonkman	AeroDyn	NREL	BEM	[30]
Hassan	Bladed	DNV_GL	BEM	[31]
van Garrel	AWSM	ECN	FWM	[32, 33]
Sant	HAWT-FWC	TU Delft	FWM	[34]
Kloosterman	–	TU Delft	FWM	[35]
Sebastian	WInDs	UMass	FWM	[36]
Abedi	–	Chalmers	PWM, FWM	[37]
Rosenberg	–	Iowa	PWM	[38]
Voutsinas	GENUVP	NTUA	FWM	[39]
Coton	HAWTDAWG	UGlasgow	PWM	[40]
Strickland	VDART3	Sandia Lab	FWM	[41]
Michelsen	EllipSys3D	DTU/Risø	RANS	[42–44]

1. Blade Element Momentum theory (BEM), Free-Wake Method (FWM), Prescribed-Wake Method (PWM), Reynolds Averaged Navier-Stokes (RANS)

each code, only the name of the first author is shown in Table 2.1 for compactness. A review of other commercial softwares is presented by Molenaar in Ref. [45]. The following sections describe various types of codes (BEM, vortex models, CFD models) as well as the commonly used validation experiments.

2.1 Wind Tunnel Experiments

A significant number of experiments which were performed to characterize the performance of wind turbines, are available in the literature (see Refs. [23, 46]). Two well-known wind turbine experiments which were carried out in a controlled environment, are commonly used to validate performance codes—the National Renewable

Energy Laboratory (NREL) Unsteady Aerodynamics Experiment (UAE) [47–49] and the Model Experiments in Controlled Conditions (MEXICO) project [50, 51]. Figure 2.1 shows the NREL Phase VI and MEXICO wind turbines mounted in their respective wind tunnel.

Figure 2.1: NREL Phase VI turbine mounted in NASA Ames $24.4 \text{ m} \times 36.6 \text{ m}$ wind tunnel (left) [49] and MEXICO turbine mounted in DNW $9.5 \text{ m} \times 9.5 \text{ m}$ wind tunnel (right) [51]

2.1.1 NREL Phase VI Wind Turbine

The Unsteady Aerodynamics Experiment (UAE) was carried out throughout 1999 and 2000 in the NASA Ames $24.4 \text{ m} \times 36.6 \text{ m}$ wind tunnel using the 10 m diameter NREL Phase VI wind turbine. This twisted, two-bladed, stall-controlled rotor consisting of mainly S809 airfoil sections as shown in Figure 2.2, was tested under 14 different turbine configurations. More details about the geometry of this turbine

will be provided in Section 4.3. The objective of this experiment was to provide accurate and reliable quantitative information on full-scale, 3D aerodynamic behaviour of HAWTs. These measurement data have contributed to the development and validation of engineering models for the design of better performing wind turbines. The reader is referred to Refs. [47, 48] for a complete description of the test plan, model geometry, instrumentation, and calibration procedures as well as a summary of the experimental results.

Figure 2.2: NREL Phase VI blade geometry. Adapted from Ref. [47]

Following the experiment, a blind comparison was performed with 30 academic and industrial wind turbine experts using prediction codes ranging from blade element models, prescribed-wake models, free-wake models, and full Navier-Stokes codes. A summary of the participants who took part of the blind comparison and a description of their code can be found in Ref. [48]. Figure 2.3 shows the Sequence S measurements and some of the best performance predictions resulting from the blind comparison. As can be seen, the results demonstrated large deviations in the prediction accuracy. In 0° yaw, steady-state, and pre-stall conditions, the power predictions ranged from 25% to 175% of the measured data. On the other hand, in the stall regimes, where dynamic stall and unsteady blade loading are present, the power prediction ranged from 30% to 275% of the measured data [48].

Figure 2.3: NREL Phase VI blind comparison results extracted from Ref. [48]. Experimental data provided by Dr. Schreck [52]

2.1.2 MEXICO Project

Similar to the UAE, the MEXICO Project was performed to build an experimental database as a means of validating engineering models. The experiment consisting of blade pressure distributions, loads and flow field measurements, was conducted in 2006. The 4.5 m diameter model rotor and operating conditions were designed to complement the UAE database. This three-bladed rotor employing three different aerodynamic profiles (DU91-W2-250, RISOE-A1-21 and NACA 64-418), as shown in Figure 2.4, was placed in the 9.5 m $\times$ 9.5 m LLF (large scale low speed facility) of the German Dutch Wind Tunnel (DNW) in the Netherlands. To trigger the laminar to turbulent flow transition, a 0.25 mm thick zig-zag tape placed at 5% of the chord was applied on both pressure and suction sides of the turbine blades. The measurements were carried out under several flow conditions, including 2 different rotational speeds (324 rpm and 424 rpm), wind speeds ranging from 10 m/s to 30 m/s, pitch angles ranging from -5.3° to 1.7° , and yaw-inflow angles ranging from 0° to 45° . The reader

Figure 2.4: MEXICO blade geometry

is referred to Refs. [50, 51, 53, 54] for more information regarding the setup, rotor configuration, and measurements performed.

Both yawed and non-yawed measurement cases were compared with different types of codes including BEM, vortex model, and CFD. To reduce errors, the blade geometry was provided as IGES and STEP files to CFD participants. Moreover, 2D airfoil wind tunnel experiments were performed at Delft University of Technology and Risø National Laboratory thereby creating a polar database to be used as an input to codes using engineering approximations (e.g., BEM). The same type of zig-zag tapes were applied for the 2D airfoil experiments although the thickness of the strips might vary between experiments. Figures 2.5 and 2.6 show the torque and thrust results, respectively, from various prediction codes compared to the experimental data of the non-yawed case. A description of each code can be found in Ref. [51].

2.2 Blade Element Momentum (BEM) Theory

The Blade Element Momentum (BEM) theory, developed by Glauert in 1935 [56], is the simplest and most commonly used prediction tool for wind turbines. This method uses a combination of two mathematical methods—blade element theory

Figure 2.5: MEXICO torque measurements compared with the prediction from various codes (extracted from Ref. [51]). Experimental data provided by Dr. Schepers [55]

Figure 2.6: MEXICO axial thrust measurements compared with the prediction from various codes (extracted from Ref. [51]). Experimental data provided by Dr. Schepers [55]

and momentum theory—along with various empirical approximations to evaluate the loads acting on discretized radial elements of a wind turbine blade. Typical corrections that are applied to BEM methods are: i) blade tip and root effects, ii) turbulent wake state, iii) dynamic inflow, iv) dynamic stall, and v) 3D corrections [45].

The momentum theory is a one-dimensional (1D) model in which the flow is assumed to be inviscid, incompressible, and axisymmetric. It basically computes the conservation of mass, momentum, and energy over control volumes fixed by stream tubes to solve for the loads acting on the blade elements. In other words, the induced velocities from the momentum lost in the flow can be calculated using the momentum theory and can be modeled using both the axial and radial inflow [9–11] or solely based on the axial inflow [57]. These induced velocities which are usually specified as induction factors, affect the inflow in the rotor plane and therefore also affect the loads acting on the blades. On the other hand, the blade element theory assumes that blades can be divided into small independent elements thereby operating as 2D airfoils. This allows the lift and drag forces on each blade elements to be evaluated based on the local 2D airfoil characteristics, angle of attack and relative velocity. A detailed derivation of the BEM theory can be found in Refs. [9–11].

The BEM method provides a simple and fast way to compute the performance of wind turbines, however, in situations such as yawed flow and unsteady aerodynamics (particularly dynamic inflow effects), this method lacks accuracy [24, 58]. Although there are corrections and models that can be integrated into the BEM algorithm to account for these effects, the use of induced velocities compared to induction factors provide a more accurate performance prediction.

2.3 Vortex Methods

As previously mentioned, although computationally more expensive compared to BEM methods, vortex models provide more accurate performance predictions due to better spatial and temporal representation of the wake. In this method, the wind turbine blades and wake (spanwise and trailing vortices) are modeled by either vortex particles (also known as vortex blobs) [39], vortex lines (straight or curved filaments) [32, 36, 41], vortex panels [37], or a hybrid method [29, 59]. Based on the Kutta-Joukowski Theorem, the vortex strength of an element on the blade (commonly known as bound circulation) is determined from the lift generated by the local inflow field [23, 24]. This flow field is related to the oncoming wind, blade, and induced velocities which are determined using the Biot-Savart law (see Section 3.6). As the blades pass through the flow field, vorticity is generated in the wake due to spatial and temporal variations in bound circulation. In other words, the resulting wake consists of spanwise vortices generated by spatial variations of the bound circulation along the blade and trailing vortices generated by the temporal variations as the blades rotate. This method is considered to be an efficient and flexible way to represent complex wake structures [35, 57, 60].

The convection of the wake can either be prescribed or free. The former method involves specifying the wake geometry based on analytical or empirical solutions. On the other hand, a free-wake geometry is obtained from first principle thereby allowing the vortices in the wake to interact with themselves (self-induced effect), with other vortices (mutually induced effect) or with the blades [57]. The free-wake method, which is the most computationally expensive vortex method, is able to predict the wake geometry and loads more accurately than the prescribed-wake because of less restrictive assumptions. After obtaining the positions and strengths of the vortices,

the loads acting on the blades and therefore the performance of the wind turbine can be evaluated. A review of vortex methods and their applications is presented by Stock in Ref. [61].

2.4 CFD Models

Computational Fluid Dynamics (CFD) models, which rely on the first principles of conservation of mass, momentum, and energy, are based on numerical solutions to the Euler and Navier-Stokes equations. CFD is, in theory, the most comprehensive way to analyze the entire flow field around any body in steady or unsteady flow such as wind turbines. In practice, however, these codes are computationally very expensive and require large amount of memory. In addition, CFD models have various numerical issues such as turbulence modeling and wake diffusion [22].

There are many commercially available CFD softwares that have demonstrated a clear ability to predict the forces acting on wind turbine blades. The Reynolds Averaged Navier-Stokes equations (RANS) are commonly solved for wind turbine applications with various turbulence models such as the k - ω SST (Shear Stress Transport) model of Menter [62], the Spalart-Allmaras model [63], and the Baldwin-Barth model [64]. All RANS models, however, cannot model large separations accurately. An alternative to RANS simulations is Large Eddy Simulation (LES) which provides better physical representation of the eddy dynamics in separate flows, yet is limited to moderate Reynolds numbers. Detached Eddy Simulation (DES), on the other hand, is a hybrid technique which provides a more accurate and affordable option to RANS and LES. DES combines the capability of RANS to model boundary layers using “attached” eddies, and LES to model separated regions using “detached” eddies [65].

2.5 Comparison of Prediction Tools

As previously discussed, the blade element momentum theory (BEM) provides a simple and fast method to predict the performance of a single-rotor wind turbine (SRWT). However, this method cannot accurately capture all the flow phenomena which occurs in the wake of a HAWT and therefore, cannot be used to simulate a dual-rotor wind turbine. CFD, which has the ability to accurately simulate the entire flow field around a dual-rotor wind turbine, is too computationally expensive and therefore, impractical for parametric studies. Free-wake method (FWM), on the other hand, has superior capability to predict aerodynamic performance of wind turbine compared with the BEM method [32, 66]. In addition, this method, which requires much less computational time than CFD simulations, can be used for parametric studies.

To highlight the accuracy and computational cost of each method, the QBlade, DR_HAWT and EllipSys3D codes (BEM, FWM, and CFD, respectively) were used to compute the performance of the NREL Phase VI. Figure 2.7 compares the torque measurements to the computed results from the three codes. The data for QBlade and Ellipsys3D were extracted from Ref. [29] and Ref. [67], respectively, whereas the results computed in DR_HAWT will be described in Section 4.3. As can be seen, all three methods show good agreement with a maximum of 30% difference as compared to the measured data in the pre-stall and stall regimes (7 m/s to 15 m/s).

Marten *et al.* computed the performance of the two-bladed Phase VI rotor using 18 discretized elements. Although the computational time was not provided in Ref. [29], the computational cost of BEM simulations are known to be in the order of seconds. As part of the UAE blind comparison, Sørensen *et al.* used EllipSys3D which is a RANS solver, to perform steady and unsteady computations. In these simulations,

Figure 2.7: Power prediction of the NREL Phase VI wind turbine using the QBlade, DR_HAWT, and Ellipsys3D codes

the tower and nacelle were neglected, and a single rotor blade was modeled. With over 3 million cells used to model the rotor and computational domain, the steady state simulation required about 30 hours on average using 4 CPUs [67]. DR_HAWT, on the other hand, required 15-30 mins to compute the two-bladed Phase VI rotor discretized with 26 elements using 8 CPUs (more details are provided in Section 4.3).

To summarize, a free-wake method was determined to be better suited for a dual-rotor wind turbine parametric study. It provides a relatively fast computational cost and possesses the ability to accurately model rotor wakes as compared to BEM and CFD methods.

Chapter 3

DR_HAWT: Vortex Filament Code

This chapter describes the aerodynamic model used in DR_HAWT (Dual-Rotor Horizontal-Axis Wind Turbine code) for the aerodynamic performance prediction of a single or dual-rotor horizontal axis wind turbine. The general procedure used is based on the code devised by Strickland *et al.* for the Darrieus Vertical Axis Wind Turbine [41, 68]. In this three-dimensional (3D) free vortex model, the rotor blades are divided into a number of elements along their span and each element is replaced by a bound vortex filament based on the Lifting Line Theory (LLT). The vortex filaments are then shed at each time step in terms of spanwise and trailing vortices. These shed vortices are free to convect in the fluid domain thereby providing a suitable spatial and temporal representation of the wake. The aerodynamic loads acting on the blades can then be determined following the Blade Element Theory (BET).

The material presented in the following sections will allow the reader to understand the mathematical formulations implemented in DR_HAWT including any correction factors employed.

3.1 Governing Equations

An incompressible, inviscid flow with no body forces, is governed by the conservation of mass and momentum, given by:

$$\nabla \cdot \mathbf{v} = 0 \quad (3.1)$$

$$\frac{D\mathbf{v}}{Dt} = \frac{\delta\mathbf{v}}{\delta t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{\nabla p}{\rho} \quad (3.2)$$

where $\mathbf{v}$, t , p , and ρ denote the velocity vector, time, pressure, and fluid density, respectively and ∇ is the gradient operator. The inviscid vorticity transport can be obtained by taking the curl of Equation 3.2, giving

$$\frac{D\zeta}{Dt} = (\zeta \cdot \nabla) \mathbf{v} \quad (3.3)$$

where the vorticity ζ is a measure of the local rotation or spin of a fluid element about an axis through the element and is defined as the curl of the velocity ($\zeta = \nabla \times \mathbf{v}$). The term on the right-hand side of Equation 3.3 denotes the tilting and stretching of vorticity. This equation is the basis of the Helmholtz's theorem, where the strength of a vortex tube remains constant along its length and does not vary with time. Vortex lines, which are defined as the line fields that are parallel to the local vorticity vector, passing through a closed curve in space form a vortex tube. A vortex filament is then defined as a vortex tube with infinitesimal cross-sectional area [69, 70].

Circulation (also referred to as vortex strength) is defined as the negative of the line integral of velocity around a closed curve in the flow, given by:

$$\Gamma = - \oint_C \mathbf{v} \cdot d\mathbf{s} \quad (3.4)$$

Figure 3.1: Definition of circulation

where $\mathbf{v}$ and $d\mathbf{s}$ are the velocity vector and infinitesimal line segment, respectively, at a point on the closed curve C , as illustrated in Figure 3.1. Therefore, the time rate of change of the circulation of this fluid curve C gives

$$\frac{D\Gamma}{Dt} = - \oint_C \frac{D\mathbf{v}}{Dt} \cdot d\mathbf{s} \quad (3.5)$$

When substituting Equation 3.2 into Equation 3.5, the Kelvin's theorem is obtained where the circulation about any closed path moving with the fluid is a constant.

$$\frac{D\Gamma}{Dt} = 0 \quad (3.6)$$

3.2 Coordinate System

As previously mentioned, each blade is discretized into elements, defined by unit vectors in the spanwise ($\mathbf{s}$), chordwise ($\mathbf{c}$), and normal ($\mathbf{n}$) directions, located at their aerodynamic center (AC), as shown in Figure 3.2. It should be noted that the chordwise vector lies on the rotor plane and therefore, does not take into account

Figure 3.2: 3D model representing wind turbine in DR_HAWT

blade twist and pitch. In this thesis, these unit vectors are being identified as local coordinate system (LCS) which allows for the calculation of the local flow conditions (Re, AOA) and the aerodynamic coefficients (C_L , C_D). Equations 3.7 - 3.9 define the LCS in a generalized form.

$$\mathbf{s} = \frac{(x_{i+1} - x_i)\mathbf{i} + (y_{i+1} - y_i)\mathbf{j} + (z_{i+1} - z_i)\mathbf{k}}{[(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 + (z_{i+1} - z_i)^2]^{1/2}} \quad (3.7)$$

$$\mathbf{c} = \cos \theta_B \mathbf{j} + \sin \theta_B \mathbf{k} \quad (3.8)$$

$$\mathbf{n} = \mathbf{c} \times \mathbf{s} \quad (3.9)$$

where the subscript i identifies the element number along the blade and θ_B is the blade azimuth angle. x , y and z are the coordinates of the AC of any section of the blades defined in the global coordinate system (GCS). A schematic of the wind turbine model used in DR_HAWT is illustrated in Figure 3.2. As can be seen, the origin of GCS is located at the center of the upwind hub with the $\mathbf{j}$ - and $\mathbf{k}$ -axes in the vertical and horizontal directions, respectively. The $\mathbf{i}$ -axis is oriented parallel to the axis of rotation with the positive axis in the direction of the wind during zero yaw condition. Note that either clockwise or counter-clockwise rotation can be modeled where a '-1' factor is introduced in Equations 3.8 and 3.9 to account for counter-clockwise rotation. Equations 3.10 - 3.13 define the coordinates and azimuth angle of an element in a generalized form.

$$\theta_B = \frac{2\pi}{NB}(BN - 1) + \theta \quad (3.10)$$

$$x = 0 \text{ or } L \quad (3.11)$$

$$y = -r \sin \theta_B \quad (3.12)$$

$$z = r \cos \theta_B \quad (3.13)$$

where r is the radial distance from the center of the hub to the AC of the blade element, NB is the number of blades for each rotor, BN is the blade number and θ is the angle rotated by blade 1 from the positive $\mathbf{k}$ -axis in the clockwise direction. Since it is assumed that the blades are rigid, the aerodynamic centers of the sections of the blade follow the perimeter of a circle with radius, r in one revolution. In other words, all the ACs of the same rotor lie along the rotor plane. Therefore, for all the blade elements of the same rotor, a fixed value of $x = 0$ for the upwind rotor, and $x = L$ where L is the distance between the rotors in the positive $\mathbf{i}$ -direction, for the downwind rotor is considered.

3.3 Element Bound Vorticity

Based on the global coordinate system described in Section 3.2 and the velocity vectors acting on the blade element (see Figure 3.3), the relative velocity, $\mathbf{V}_{rel}$ of each blade element can be evaluated by summing the blade rotational velocity, $\mathbf{V}_T$, free stream velocity, $\mathbf{V}_o$, and the induced velocity caused by the vortex filaments, $\mathbf{V}_{ind}$. The approach used for calculating the velocity induced by the vortex filaments

Figure 3.3: Velocity components acting on blade element

is described in Section 3.6. The vector sum of the relative velocity is given below:

$$\mathbf{V}_{rel} = (\mathbf{V}_{ind,x} + \mathbf{V}_o)\mathbf{i} + (\mathbf{V}_{ind,y} + \mathbf{V}_T \cos \theta_B)\mathbf{j} + (\mathbf{V}_{ind,z} + \mathbf{V}_T \sin \theta_B)\mathbf{k} \quad (3.14)$$

Equations 3.15 and 3.16 define the Reynolds number (Re) and the angle of attack (α) at each element section using vector manipulation of the relative velocity.

$$Re = \frac{|\mathbf{V}_{rel}|c}{\nu} \quad (3.15)$$

$$\alpha = \tan^{-1} \left(\frac{\mathbf{V}_{rel} \cdot \mathbf{n}}{\mathbf{V}_{rel} \cdot \mathbf{c}} \right) - \beta \quad (3.16)$$

where c is the local chord length, β is the sum of the local twist and pitch angle, and ν is the kinematic viscosity of the fluid. Knowing the local Reynolds number and angle of attack, the lift and drag coefficients are linearly interpolated from look-up tables consisting of experimental or numerical (e.g., XFOIL) aerodynamic polars.

XFOIL [71] which employs a viscid/inviscid panel method, is used to compute the lift and drag coefficients. Although the latter method over-predicts lift and under-predicts drag, especially under post-stall conditions [72–74], its low computational runtime, allows new designs to be tested quickly.

Based on Prandtl’s lifting line theory, each blade element is associated with a bound vortex filament of strength, Γ_B . This strength can be expressed using the Kutta-Joukowski theorem as follows:

$$\Gamma_B = \frac{1}{2} C_{Lc} |\mathbf{V}_{rel}| \quad (3.17)$$

Since the bound vortex strength is a function of the relative velocity, which takes into account the induced velocity, a predictor corrector method is employed for which the bound vortex strength is predicted based on the induced velocities from the previous timestep. The predicted strength is then used to correct the induced velocities which in turn corrects the bound vortex strength. This process is repeated until the differences between the predicted and corrected values are below 1×10^{-3} . Figure 3.4 illustrates the procedure for the predictor corrector method employed in this code.

3.4 Vortex Shedding

Figure 3.5 illustrates the representation of blade elements with the wake which is made up of vortex filaments. Each vortex strength, shown in Figure 3.5 is associated with a superscript and a subscript notation where the superscript denotes the blade element end section and the subscript denotes the time step. In other words, for a timestep NT , the bound vortex strength of an element i is written as Γ_{NT}^i . As the blade rotates, vortices are shed in the form of spanwise and trailing vortex filaments

Figure 3.4: Bound vortex calculation using the predictor corrector method

Figure 3.5: Schematic of the bound (Green), spanwise (Red) and trailing (Blue) vortices representing the wake behind a blade consisting of two elements after several timesteps. Adapted from Strickland *et al.* [41].

which are parallel to the span and chord of the blade, respectively. It should be noted that there are n spanwise vortices and $n + 1$ trailing vortices, where n is the number of discretized sections along the blade. The strength of these vortex filaments are calculated based on the Kelvin's theorem for which any change in the bound vortex strength over time must be accompanied by an equal and opposite vortex strength in the wake [70], given by:

$$\frac{D\Gamma_B}{Dt} = 0 \quad (3.18)$$

In order to satisfy the Kelvin's theorem, the spanwise vortex strength of the element i for timestep $(NT - 1)$ is given by

$$\Gamma_{S(NT-1)}^i = \Gamma_{B(NT-1)}^i - \Gamma_{B(NT)}^i \quad (3.19)$$

On the other hand, the strength of the trailing vortex filaments, at both end sections of the element is dictated by the Helmholtz Theorem and is equal to the strength of the bound vortex of that element. Therefore, the difference between the strengths of the bound vortices associated to two consecutive blade elements, results in the trailing vortex. The strength of the trailing vortex of element end j for timestep $(NT - 1)$ is given as

$$\Gamma_{T(NT-1)}^j = \Gamma_{B(NT-1)}^i - \Gamma_{B(NT-1)}^{i-1} \quad (3.20)$$

In the absence of a subsequent section, namely at the blade tip, the strength of the tip vortex is assumed to have the same magnitude and opposite direction as the bound vortex closest to the tip, given by:

$$\Gamma_{T(NT-1)}^j = -\Gamma_{B(NT-1)}^i \quad (3.21)$$

3.5 Wake Convection

Each filament vortex contains two points, one at the head and another at the tail, which are known as Lagrangian markers. Each marker (or node) in the wake convects with a local velocity. As the filaments first leaves the blade, its initial velocity is assumed to be equal to the sum of the induced velocities at the blade element end and the oncoming freestream velocity. Therefore the initial distance travelled by each node of the filament vortex is given by:

$$\Delta x = (U_t + V_o) \Delta t \quad (3.22a)$$

$$\Delta y = V_t \Delta t \quad (3.22b)$$

$$\Delta z = W_t \Delta t \quad (3.22c)$$

where Δt is the time increment and U_t , V_t , and W_t are the induced velocities in the x , y , and z directions at time t . In order to calculate the distance travelled by any node in the wake for a given time step, a second order Adams-Bashforth explicit integration formula is used. The predicted distance travelled is given as follows:

$$\Delta x = (1.5U_t - 0.5U_{t-\Delta t} + V_o) \Delta t \quad (3.23a)$$

$$\Delta y = (1.5V_t - 0.5V_{t-\Delta t}) \Delta t \quad (3.23b)$$

$$\Delta z = (1.5W_t - 0.5W_{t-\Delta t}) \Delta t \quad (3.23c)$$

It should be noted that the two markers, which constitute a vortex filament, move independently of each other. Therefore, each vortex filament is allowed to stretch and rotate while being convected in the flow field.

3.6 Induced Velocity

The induced velocity $\mathbf{V}_{ind}$ at a point P , located at position $\mathbf{r}$ relative to the filament vortex of infinitesimal length $d\mathbf{l}$ of strength Γ_v , can be evaluated using the well-known Biot-Savart law, written as:

$$\begin{aligned} \mathbf{V}_{ind} &= \frac{\Gamma_v}{4\pi} \int \frac{\mathbf{r} \times d\mathbf{l}}{|\mathbf{r}|^3} \\ &= \frac{\Gamma_v}{4\pi h} (\cos \beta_1 - \cos \beta_2) \mathbf{e} \end{aligned} \quad (3.24)$$

where the angles β_1 and β_2 are called the view angles, the unit vector $\mathbf{e}$ is the direction vector of the induced velocity and h is the perpendicular distance between the filament vortex and evaluation point P . Figure 3.6 shows a representation of the vortex filament for a point of interest. The reader can consult Refs. [69, 70, 75] for the derivation of the Biot-Savart law.

Figure 3.6: Velocity induced by a straight filament vortex of strength Γ_v with vortex core representation. Adapted from Ref. [76]

For a straight filament vortex illustrated in Figure 3.6, $\mathbf{r}_0$, $\mathbf{r}_1$, and $\mathbf{r}_2$ represent the direction vectors AB, AP, and BP, respectively. A simplified form of Equation 3.24 can be derived by applying trigonometric relations, giving:

$$\mathbf{V}_{ind} = \frac{\Gamma_v}{4\pi} \frac{\mathbf{r}_1 \times \mathbf{r}_2}{|\mathbf{r}_1 \times \mathbf{r}_2|^2} \left(\frac{\mathbf{r}_0 \cdot \mathbf{r}_1}{|\mathbf{r}_1|} - \frac{\mathbf{r}_0 \cdot \mathbf{r}_2}{|\mathbf{r}_2|} \right) \quad (3.25)$$

Although this equation is commonly found in textbooks [70,75], a singular solution is obtained whenever $\mathbf{r}_1$ and $\mathbf{r}_2$ are collinear [77]. The latter can be eliminated by removing the $(\mathbf{r}_1 \times \mathbf{r}_2)$ term from the denominator, giving:

$$\mathbf{V}_{ind} = \frac{\Gamma_v}{4\pi} \frac{(|\mathbf{r}_1| + |\mathbf{r}_2|)(\mathbf{r}_1 \times \mathbf{r}_2)}{|\mathbf{r}_1||\mathbf{r}_2|(|\mathbf{r}_1||\mathbf{r}_2| + \mathbf{r}_1 \cdot \mathbf{r}_2)} \quad (3.26)$$

Moreover, as $(\mathbf{r}_1 \cdot \mathbf{r}_2)$ approaches zero (i.e., the angle between $\mathbf{r}_1$ and $\mathbf{r}_2$ is $\pm\pi$), Equation 3.26 shows a singular behaviour. In other words, as the evaluation point is approaching the vortex core which is represented by a cylinder and two spherical

Figure 3.7: Induced velocity of a filament vortex evaluated using (i) Biot-Savart Law, (ii) viscous core model, and (iii) cut-off radius method, as a function of dimensionless distance from the vortex core r/r_c

caps with a core radius r_c in Figure 3.6, the induced velocity experienced increases exponentially. This nonphysical behaviour can be removed with the introduction of a cut-off radius [32, 69] or viscous core model [78, 79] that mimic the viscous behaviour of a real vortex. Figure 3.7 illustrates the induced velocity distribution as a function of the distance r from the point of interest to the vortex core.

The Biot-Savart law which is desingularized using the cut-off radius can be written as

$$\mathbf{V}_{ind} = \frac{\Gamma_v}{4\pi} \frac{(|\mathbf{r}_1| + |\mathbf{r}_2|)(\mathbf{r}_1 \times \mathbf{r}_2)}{|\mathbf{r}_1||\mathbf{r}_2|(|\mathbf{r}_1||\mathbf{r}_2| + \mathbf{r}_1 \cdot \mathbf{r}_2) + (\delta_c|\mathbf{r}_0|)^2} \quad (3.27)$$

where the cut-off radius parameter δ_c defines the percentage of the vortex length $\mathbf{r}_0$ which represents vortex core. Within this region, the induced velocity smoothly decreases to zero as it approaches the vortex line. On the other hand, the viscous core method, defined in Equation 3.28, uses a factor K which desingularizes the Biot-Savart equation as r tends to zero and prevents a high induced velocity in the vicinity region of the vortex core radius [37].

$$\mathbf{V}_{ind} = K \frac{\Gamma_v}{4\pi} \frac{(|\mathbf{r}_1| + |\mathbf{r}_2|) (\mathbf{r}_1 \times \mathbf{r}_2)}{|\mathbf{r}_1||\mathbf{r}_2| (|\mathbf{r}_1||\mathbf{r}_2| + \mathbf{r}_1 \cdot \mathbf{r}_2)} \quad (3.28)$$

where

$$K = \frac{r^2}{(r_c^{2n} + r^{2n})^{1/n}} \quad (3.29)$$

The traditional core regularization (i.e., $r = h$) was preferred to the recent improved correction proposed by van Hoydonck *et al.* (2014) [76]. Although the latter method showed better agreement with the analytical approximation of the induced velocity produced by a thin-cored vortex ring (formulation not shown here), van Hoydonck *et al.*'s approach works only for very small discretizations, i.e., for $\Delta\theta < r_c/R \approx 10^{-5}$ where $\Delta\theta$ is the azimuthal discretization for the vortex ring. Figure 3.8 illustrates the induced velocity of a vortex ring discretized into small straight-line vortex elements as a function of the core radius. The reader is referred to Saffman (1992) [69] for a detailed description of the analytical approximation of an inviscid vortex ring.

Furthermore, the accuracy of the simulation is highly dependent on the viscous core model used since the induced velocity field has a significant effect on the wake geometry and aerodynamic loads. The induced velocity can be represented using either a constant vortex model based on a desingularized algebraic profile such as Rankine ($n \rightarrow \infty$) [81], Scully ($n = 1$) [82] and Vatisstas ($n = 2$) [83] models or

Figure 3.8: Axial induced velocity vortex ring comparison between (i) analytical approximation, (ii) traditional core regularization, and (iii) modified core regularization by van Hoydonck *et al.* [76], as a function of relative core size r_c/R . Adapted from Ref. [80]

a diffusive vortex model where the core radius grows with time according to the Lamb-Oseen’s model [79]. The latter model, which is employed in this work, can be approximated using the Vatistas vortex model ($n = 2$).

The viscous core radius r_c defined as the radial position around the filament vortex where the swirl velocity is maximum, was found to grow with time as

$$r_c(t) = \sqrt{4\alpha\nu t} \quad (3.30)$$

where $\alpha = 1.25643$, ν is the fluid viscosity and t is the age of the vortex. However, at $t = 0$, the swirl velocity given by Equation 3.30 is singular. To remove this singularity,

the filament vortex can be assumed to have a finite core size (r_o) at its origin ($t = 0$). The vortex core growth which is a modified Lamb-Oseen model proposed by Squire in 1954 and employed in this work, is given by Equation 3.31. This model includes an average apparent or "eddy" viscosity ($\delta\nu$) to account for the effects of turbulence generation on the enhanced diffusion of vorticity [79, 84].

$$r_c(t) = \sqrt{r_o^2 + 4\alpha\delta\nu t} \quad (3.31)$$

Typical initial core radius of wing tip vortices was measured to be 5-10% chord length by Bhagwat and Leishman (2002) [79]. In this current work, r_o is assumed to be a percentage of the chord length and blade radius when evaluating the bound vortex strength and shed vortices, respectively. The eddy viscosity coefficient δ was found to be dependent on the vortex Reynolds number ($Re_v = \Gamma_v/\nu$), given by

$$\delta = 1 + a_1 \left(\frac{\Gamma_v}{\nu} \right) \quad (3.32)$$

where a_1 is in the order of 10^{-4} based on the experiments performed by Bhagwat and Leishman [79, 85].

3.7 Stall Delay Model

The effect of rotation affects the boundary layer on rotor blades at the onset of stall. These rotational effects are induced by a phenomenon referred to as rotational augmentation which delay the separation point at higher angle of attack as compared to two-dimensional (2D) case. In the 1940's, Himmelskamp [86] was the first to quantitatively analyze the three-dimensional (3D) effect on propeller blades and confirmed that rotation does delay the laminar separation point. Since then, various

rotational effects or ‘stall delay’ models have been formulated by Snel *et al.* [87], Du & Selig [88], Chaviaropoulos & Hansen [89], Lindenburg [90], and Bak *et al.* [91], to name a few. These models correct the non-rotating lift ($C_{L,2D}$) coefficients with an increment (ΔC_L). The rotational effect on 2D drag coefficient ($C_{D,2D}$), however, is not clearly understood [92]. Lindenburg [93] predicts an increase in drag since energy is required for ‘centrifugal pumping’ due to the radial flows which exist in rotating blades under stall conditions. Most 3D correction models predict an increment in drag [89, 91, 93] which correspond to the measurements on the NREL Phase VI experiment, showing a drastic increase in the aerodynamic drag coefficient [67, 92, 94]. However, some models describe a reduction [88] or no influence at all [87] on the drag coefficient.

Equations 3.33 and 3.34 describe the methodology used in most stall delay models

$$C_{L,3D} = C_{L,2D} + f_L \Delta C_L \quad (3.33)$$

$$C_{D,3D} = C_{D,2D} + f_D \Delta C_D \quad (3.34)$$

where the subscripts 2D and 3D refer to non-rotating and rotating characteristics, respectively. ΔC_L and ΔC_D are the difference between the $C_{L,2D}$ and $C_{D,2D}$ wind tunnel measurements and the 2D C_L and C_D if hypothetically the flow would remained attached at all angle of attacks [91]. In other words, ΔC_L is the difference between $C_{L,2D}$ and the inviscid C_L ($C_{L,inv} = 2\pi(\alpha - \alpha_0)$ where α_0 is the zero-lift angle) and ΔC_D is the difference between the $C_{D,2D}$ and the drag coefficient at zero angle of attack, $C_{D,0}$ [88–90, 95]. Sometimes, the extended linear lift ($C_{L,lin}$) is used in place of the $C_{L,inv}$ [96]. This method was employed in this code as it is believed to provide a better representation (extrapolation) of the airfoil characteristic. f_L and f_D are

Table 3.1: Summary of functions used in stall delay models

Model	Functions
Snel <i>et al.</i> [87]	$f_L = 3 \left(\frac{c}{r} \right)^2$
Du & Selig [88]	$f_L = \frac{1}{2\pi} \left[\frac{1.6(c/r)}{0.1267} \frac{a-(c/r) \frac{d}{\lambda} \frac{R}{r}}{b+(c/r) \frac{d}{\lambda} \frac{R}{r}} - 1 \right]$ $f_D = -\frac{1}{2\pi} \left[\frac{1.6(c/r)}{0.1267} \frac{a-(c/r) \frac{d}{2\lambda} \frac{R}{r}}{b+(c/r) \frac{d}{2\lambda} \frac{R}{r}} - 1 \right]$ <p>where $a = b = d = 1$,</p> <p>and $\lambda = \frac{\omega R}{\sqrt{V_o^2 + (\omega R)^2}}$</p>
Chaviaropoulos & Hansen [89]	$f_L = f_D = 2.2 \left(\frac{c}{r} \right) \cos^4 \beta$
Lindenburg [90]	$f_L = 3.1 \left(\frac{\omega r}{V_{rel}} \right)^2 \left(\frac{c}{r} \right)^2$

functions used in the specific stall delay models and are briefly summarized in Table 3.1. It should be noted that the lift and drag corrections were applied up to 30° angle of attack, after which the correction was linearly decreased to zero at an angle of attack of 50° [89, 90, 97].

The stall delay models previously described do not in fact correct for blade-specific and solution dependent effects since sectional airfoil data are corrected prior to the actual simulation. Other models proposed by Bak *et al.* [91] and Dowler & Schmitz [95] provide promising attempt to include these effects (not shown here). Figure 3.9 shows various stall delay models used to correct the 2D wind tunnel lift and drag coefficients for an S809 airfoil. Since modeling stall delay is not the focus of the current study, only the contemporary stall delay models such as Snel *et al.* and Du & Selig to correct for rotational effects, are employed.

Figure 3.9: Effect of various 3D corrections applied to the wind tunnel (2D) data from OSU [47] for the S809 airfoil located at 46.6% R along a blade rotating at 7.5 rad/s and $Re = 6.5 \times 10^5$

3.8 Blade loading and Rotor Performance

Once the effect of the wake (i.e., the total induced velocity) is known, the loads acting in the tangential (F_t) and normal (F_n) direction on the each blade element can be determined as:

$$F_t = \frac{1}{2} \rho |\mathbf{V}_{rel}|^2 c C_t \quad (3.35)$$

$$F_n = \frac{1}{2} \rho |\mathbf{V}_{rel}|^2 c C_n \quad (3.36)$$

Figure 3.10: Load coefficients acting on blade element

where the normal (C_n) and tangential (C_t) coefficients are calculated using Equations 3.37 and 3.38 based on the airfoil coefficients for a specific angle of attack, as illustrated in Figure 3.10.

$$C_n = C_L \cos(\phi) + C_D \sin(\phi) \quad (3.37)$$

$$C_t = C_L \sin(\phi) - C_D \cos(\phi) \quad (3.38)$$

where the flow angle, ϕ is the sum of the local angle of attack (α), twist and pitch (β) acting on the blade element.

Once the normal and tangential force for each section have been determined, the torque (Q) and axial thrust (T) can be calculated by assuming linear variation along each element [11], giving the following.

$$\begin{aligned}
Q = \sum_{i=2}^{NB(n-1)} & \left[\frac{1}{3} \frac{(F_{t,i} - F_{t,i-1})}{(r_i - r_{i-1})} (r_i^3 - r_{i-1}^3) \right. \\
& \left. + \frac{1}{2} \frac{(F_{t,i-1}r_i - F_{t,i}r_{i-1})}{(r_i - r_{i-1})} (r_i^2 - r_{i-1}^2) \right]
\end{aligned} \tag{3.39}$$

$$\begin{aligned}
T = \sum_{i=2}^{NB(n-1)} & \left[\frac{1}{2} \frac{(F_{n,i} - F_{n,i-1})}{(r_i - r_{i-1})} (r_i^2 - r_{i-1}^2) \right. \\
& \left. + \frac{(F_{n,i-1}r_i - F_{n,i}r_{i-1})}{(r_i - r_{i-1})} (r_i - r_{i-1}) \right]
\end{aligned} \tag{3.40}$$

where subscript i denotes the element section and n denotes the number of sections that discretized the blade. It should be noted that each element is bounded by two sections.

The aerodynamic power produced by the wind turbine can then be calculated as the product of the rotational velocity and the torque of the rotor. Furthermore, for comparison purposes, the coefficients of thrust (C_T), torque (C_Q), and power (C_P) are calculated as described in Section 1.1. For completeness, the equations are repeated below. Also, note that for the purpose of this work, the rotor performance is averaged over 1 revolution.

$$C_T = \frac{T}{1/2\rho AV_o^2} \tag{3.41}$$

$$C_Q = \frac{Q}{1/2\rho RAV_o^2} \tag{3.42}$$

$$C_P = \frac{Q\omega}{1/2\rho AV_o^3} \tag{3.43}$$

3.9 Parallel Implementation

Although the vortex method employed in DR_HAWT is considerably less computationally expensive than a full CFD simulation, when parametric studies are conducted to investigate the effect of varying the geometry and flow conditions, the computational cost becomes more pronounced. Using similar implementation to Fereidooni's FEM-Vort [68], parallel capability was implemented into DR_HAWT.

Parallel computing takes advantage of dividing the computational workload amongst several processors thereby reducing the number of tasks each processor must accomplish. These distributed tasks are then solved simultaneously (in parallel) by each processor and the results are recombined in the shared memory. However, it should be noted that the current vortex model is not suited for distributed-memory machines. DR_HAWT implements parallel computing using OpenMP, a shared memory Fortran parallelization Application Programming Interface (API). By integrating “!\$OMP” directive into the code, repeated calculations such as the induced velocity calculations, can be solved in parallel thereby decreasing the computational runtime.

To illustrate the capability of DR_HAWT to run simulations in parallel configuration, the MEXICO three-bladed wind turbine, where each blade was discretized into 21 sections, was simulated for 200 steps (4D wake lengths) at a wind velocity of 24 m/s and a rotational velocity of 424.5 rpm. Figure 3.11 shows the speed-up ratio S , which is defined as T_1/T_N where T is the computational runtime and the subscript refers to the number of cores. All simulations investigated in this work, unless otherwise stated, were performed on the departmental fortran computer running Centos 6.5. This computer consists of 2 quad cores Intel (R) Xeon (R) CPU L5410 @ 2.33 GHz with 16 GB ECC memory. The computational runtime for the aforementioned example was 3.5 hours in serial and 0.5 hour using 8 cores. As can be

Figure 3.11: Parallel capability of DR_HAWT **Figure 3.12:** Verification of parallel implementation in DR_HAWT

seen in Figure 3.11, DR_HAWT was about 90% efficient when the maximum available number of cores (i.e., 8 cores) were used. In addition, to ensure that the parallel computing is implemented correctly, various outputs from the parallel simulations were compared to the serial results. Figure 3.12 illustrates the tangential force obtained from the study mentioned previously. The exact match between the results of the parallelized code for various number of cores and the serial code confirms that the parallel computing was correctly implemented in DR_HAWT.

3.10 Program Structure

To sum-up, the Dual-Rotor Horizontal-Axis Wind Turbine (DR_HAWT) code, implemented using FORTRAN 95, is capable of simulating a horizontal axis wind turbine in single or dual-rotor configuration. The general procedure used in DR_HAWT, as illustrated in Figure 3.13, is based on the code developed by Strickland *et al.* in 1980.

The flow characteristics are input in terms of oncoming wind velocity, as well as the density and viscosity of the fluid. Due to the transient nature of the vortex filament method, the timestep increment in terms of seconds or degrees per step and the number of steps are also input. In addition, the user chooses the wind turbine configuration and the rotor geometry and specifications are extracted from the rotor input files accordingly. Based on the user input rotor configuration, the required blade matrices are created. The user can also choose the desired corrections such as rotational augmentation and vortex model, as well as the output parameters.

The bound vorticity at each blade section is assigned a zero value for the first iteration (or step). The coordinates of each blade element are determined and the induced velocity based on the bound vortex strength is computed for each element. For the first iteration, since the initial vorticity has been set to zero and no vortices

Figure 3.13: Program structure

have previously been shed, the induced velocities have a zero value. To calculate the bound vorticities, a predictor corrector method is employed where the vortex strength is calculated based on the previous calculated induced velocities (see Section 3.3). This process is repeated until the differences between the predicted and corrected values are below 1×10^{-3} .

Once the bound vorticities and the induced velocities have been corrected, the tangential and normal forces at each blade element are computed. Applying the blade element theory (BET), the torque, thrust and power coefficients (C_Q , C_T , and C_P , respectively) can be calculated for the current iteration. Next, a second order Adams-Bashforth explicit integration formula is used to calculate the new location of each lattice point. The timestep is then increased and the bound vortices are shed. For each iteration, the results files are updated and a new set of video files are created, if required. These video files, which are .VTK format, can then be opened in ParaView [98], an open-source data visualization and analysis tool which allows the user to animate the simulation. To illustrate DR_HAWT's ability to produce wake visualization, the NREL Phase VI two-bladed wind turbine was simulated. Figure 3.14 shows the wake structure for the Phase VI turbine.

This process is repeated for the number of desired steps or until the average C_P value over a full rotation is below 1×10^{-4} between two consecutive rotations. Furthermore, various error checking procedures are implemented in DR_HAWT to alert the user of any possible issues with the current configuration.

Figure 3.14: NREL Phase VI Wake visualization using ParaView where the wake from each blade is illustrated with a different color

Chapter 4

Verification and Validation Methodology

To ensure that DR_HAWT was implemented properly, several verification and validation test cases were performed. As a first verification test, the accuracy of the computational solution is measured against the analytical solution for an elliptical planform wing with high aspect ratio. Subsequently, the effect on the downwind rotor from various oncoming vortices was analyzed to verify that the code responds correctly to a change in vortex strength and direction. Finally, to validate the single-rotor portion of the code, two well-known wind tunnel experiments—the National Renewable Energy Laboratory (NREL) Unsteady Aerodynamics Experiment (UAE) [47] and the Model Experiments in Controlled conditions (MEXICO) project [50]—were computed in DR_HAWT and compared to their measurements.

4.1 Elliptic Wing

The lift distribution on an elliptic planform wing with no twist causes the downwash to be a constant along the span. This constant induced downwash velocity $w(y)$ can be analytically evaluated by Equation 4.1. For a wing with non-elliptic planform, the local twist or camber along the span has to be adjusted to produce an elliptic loading

Figure 4.1: Chord and spanwise circulation distributions for a typical elliptical wing with low aspect ratio. Adapted from Bertin and Smith (1998) [75]

and therefore, constant downwash. Figure 4.1 illustrates a typical elliptic wing with span b , root chord c_o ($c_o = c(y = 0)$) and its corresponding circulation $\Gamma(y)$ and chord $c(y)$ distributions as a function of distance y along the span, defined by Equations 4.2 and 4.3. The reader is referred to Refs. [75] and [70] for a detailed description of the elliptical lift distribution.

$$w(y) = -\frac{\Gamma_o}{2b} \quad (4.1)$$

$$\Gamma(y) = \Gamma_o \left[1 - \left(\frac{y}{b/2} \right)^2 \right]^{1/2} \quad (4.2)$$

$$c(y) = c_o \left[1 - \left(\frac{y}{b/2} \right)^2 \right]^{1/2} \quad (4.3)$$

where Γ_o is maximum circulation ($\Gamma_o = \Gamma(y = 0)$) which is a function of the oncoming

wind velocity V_o , the lift coefficient C_L , and the aspect ratio $\mathcal{R}$ (commonly defined as $\mathcal{R} = b^2/S$ where $S = \pi b c_o/4$), given by the following equation.

$$\Gamma_o = \frac{2bV_oC_L}{\pi\mathcal{R}} \quad (4.4)$$

Figure 4.2 shows the analytical and computed downwash of an untwisted elliptic wing having a wing span of 10 m, a root chord of 0.1 m, and a constant 5° angle of attack. The wing consists of constant airfoil sections with lift coefficients based on thin airfoil theory (i.e., $C_L = 2\pi(\alpha - \alpha_o)$ where α_o is the angle of attack at $C_L = 0$). To mimic a fixed-wing case in DR.HAWT, a single-bladed rotor with the elliptic geometry defined by Equation 4.3 is placed in a flow field where the oncoming wind velocity is 10 m/s and a zero angular velocity. In addition, the blade was pitched 85° from the rotor plane to produce an angle of attack of 5° .

The analytical downwash for the elliptic wing is compared with three computed results in which the discretized elements were varied from 25 elements equally-spaced to 50 elements equally-spaced to 25 elements distributed using a full-cosine discretization scheme (see Appendix A) over the span of the blade. As can be seen in Figure 4.2, the downwash computed using DR.HAWT shows good agreement as compared to the analytical solution with the exception of the wing tips. However, the computed results tend to provide better agreement as the number of elements increased near the tip region. The discrepancy at the tips of the blade is also due to the fact that the analytical solution assumes a rigidly flat wake [32, 34, 70, 75] whereas the wake in DR.HAWT is free to evolve in three-dimensional (3D) space which results in a significant effect from the tip vortices.

Figure 4.2: Comparison between computed and analytical downwash distribution over an elliptic wing with high aspect ratio. The three simulated wings consist of 25 elements equally-spaced, 50 elements equally-spaced, and 25 elements distributed using a full-cosine discretization scheme.

4.2 Blade-Vortex Interaction

Blade-Vortex Interaction (BVI) is the phenomenon where a vortex with prescribed strength, either fixed at a location or convecting with the free-stream interacts with the flow field of a blade. The axis of the vortex can either be parallel or perpendicular to the chord of the downwind blade. Figure 4.3 illustrates the span (starting vortex) and trailing (tip vortex) vortices approaching a blade element. As a span vortex approaches a blade, a downwash is induced on the blade which decreases the angle of attack, thereby decreasing lift. This induced velocity changes direction as the vortex passes the leading edge of the blade. On the other hand, an approaching

Figure 4.3: Schematic of a blade-vortex interaction phenomenon

trailing vortex tends to induce a downwash on the blade element. Furthermore, this downwash is expected to peak as the vortex passes the leading edge of the blade element. Although this work does not focus on BVI, it is important to verify that DR_HAWT accounts for these interactions correctly.

To simulate the effect of an approaching vortex in DR_HAWT, two stationary single-bladed rotors pitched at 90° which induces a zero angle of attack on the blade, were modeled with the influence of an oncoming wind velocity of 10 m/s. The downwind blade is placed 5 m downstream of the upwind blade which is equivalent to approximately 20 timesteps. Furthermore, both blade elements were prescribed zero lift to prevent their respective wake from affecting the simulation. A starting (span) vortex with a prescribed strength ($\Gamma_S = +\Gamma$) was then released from the upwind blade element and allowed to freely convect in the flow field. As expected, the downwind blade element experienced an initial downwash followed by an upwash as shown in Figure 4.4(a). In addition, to verify that the vortex direction is also accounted for, an approaching vortex with “negative” strength ($\Gamma_S = -\Gamma$) was analyzed, which resulted in the opposite trend.

Figure 4.4: Induced velocity variation of a stationary blade element as a vortex with prescribed strength approaches. The approximate location of the stationary blade element is also shown

Using the same technique, a tip (trailing) vortex with a prescribed strength ($\Gamma_S = \Gamma$) was released from the upwind blade element. As can be seen in Figure 4.4(b), a downwash velocity is induced on the blade element. As previously mentioned, the induced velocity is expected to peak when passing the leading edge of the blade element. However, since the blade elements are replaced by lifting lines (vortex filaments) in this vortex code, the peak occurs at the aerodynamic center of the element. The analysis was also performed with a “negative” strength ($\Gamma_S = -\Gamma$) and as expected, the opposite trend was obtained. The interaction observed between the convecting vortex and the blade is comparable to that found in the literature [99–101].

4.3 NREL Phase VI Wind Turbine

The NREL Sequence S experiment which was briefly described in Section 2.1.1, was used as the first validation test case. As previously mentioned, NREL performed extensive controlled experiments using the two-bladed, 10 m diameter Phase VI turbine. The blade geometry at various spanwise sections, which can be obtained from Ref. [47], is summarized in Table 4.1. For the purpose of this work, the geometry provided by NREL will be referred to as NREL geometry. The blade is tapered and twisted with a blade tip pitch angle of 3° . In other words, the blade tip is pitched 3° from the rotor plane. During this validation test case, various operating conditions were computed with the oncoming wind velocities ranging from 5 m/s to 25 m/s at a rotational velocity of 72 rpm and 0° yaw angle.

After a thorough calibration, the overall computed and measured performance of the Phase VI turbine were compared. Further comparison were performed to correlate the distribution of local angle of attack along the blade with the measured data. The following sections describe the calibration and outcome of this test case.

4.3.1 Convergence Study

Prior to comparing with the experimental data, a comprehensive parameter calibration was performed to determine if the solution is independent of various parameters such as the azimuthal angle increment, vortex core radius, number of discretized section, and the wake length (i.e., the total number of steps). Although a parameter calibration should be performed for all operating conditions over a wide range of samples, the calibration was only performed at a TSR of 3.75 ($V_o = 10$ m/s) and 5 samples for each parameter. The latter was assumed sufficient to calibrate the NREL Phase VI model since 140 simulations would be required in place of 20 simulations,

Table 4.1: Description of the NREL Phase VI wind turbine geometry provided in Ref. [47]

Spanwise Distance [m]	Chord Length [m]	Twist ¹ [°]	Thickness [m]	Airfoil Section
0.000	0.218	0.000	0.218	Cylinder
0.152	0.218	0.000	0.218	Cylinder
0.375	0.183	0.000	0.183	Cylinder
0.500	0.349	6.700	0.163	Transition
0.559	0.441	9.900	0.154	Transition
0.625	0.544	13.400	0.154	Transition
0.749	0.737	20.040	0.154	S809
0.835	0.728	18.074	0.153	S809
1.002	0.711	14.292	0.149	S809
1.140	0.697	11.909	0.146	S809
1.444	0.666	7.979	0.140	S809
1.749	0.636	5.308	0.133	S809
1.835	0.627	4.712	0.131	S809
2.054	0.605	3.425	0.127	S809
2.359	0.574	2.083	0.120	S809
2.664	0.543	1.150	0.114	S809
2.677	0.542	1.115	0.114	S809
2.968	0.512	0.494	0.107	S809
3.273	0.482	-0.015	0.101	S809
3.515	0.457	-0.381	0.096	S809
3.578	0.451	-0.475	0.094	S809
3.883	0.420	-0.920	0.088	S809
4.188	0.389	-1.352	0.081	S809
4.272	0.381	-1.469	0.080	S809
4.492	0.358	-1.775	0.075	S809
4.521	0.355	-1.814	0.074	S809

1. Twist convention is positive towards feather

if a calibration was performed for each operating conditions.

The effect of each parameter was analyzed by varying one parameter only in each case as can be seen in Figure 4.5. It should be noted that the baseline simulation was performed using the NREL geometry with 10° azimuthal angle and 10% core radius for a wake length of $4D$ (where D is the rotor diameter).

Azimuthal Angle Increment

In this vortex code, time increment can be in the form of seconds or degrees. The latter was preferred to maintain the number of steps per revolution between simulations. To accurately capture the flow around the wind turbine rotor, small time increment is required, however, accuracy comes with high computational cost and therefore, a sensitivity study was performed to determine an adequate time increment for the NREL Phase VI computation. Figure 4.5(a) shows the variation in power coefficient as the time increment in degrees increases. Based on the results, an angle increment of 10° was found to provide short computational runtime without affecting the accuracy of the prediction with less than 1% difference between the different cases.

Vortex Core Radius

To calibrate the vortex core radii, a sensitivity analysis with the core radius ranging from 1% to 20% was studied as shown in Figure 4.5(b). Although the result seems to be converging at about 15-20%, larger core radii tend to reduce the effect of neighboring vortices thereby negating the effect of having small timesteps as well as large number of discretized elements. At the other end of the range investigated, inconclusive results were obtained for a vortex core of 1% due to large fluctuations in power coefficient at every timestep. Therefore, core radii between 5% and 10%

Figure 4.5: Coefficient of power as a function of the variation in (a) azimuth angle increment, (b) vortex core radius, (c) wake length, and (d) number of discretized sections

were assumed to be adequate. This range also agrees with typical values obtained from the literature (see Section 3.6). Since the calibration was performed at only one operating condition (i.e, $TSR = 3.75$), a conservative vortex core radius of 10% chord length was preferred.

Number of Steps

In this work, the number of steps required to obtain a converged solution is assumed to be dependent on the wake length (i.e., the distance travelled by the wake downstream of the turbine) which is usually measured in terms of rotor diameter. The NREL Phase VI was simulated until its wake reached 10D. As can be seen in Figure 4.5(c), the performance of the turbine decreases as the wake length increases. It was found that performance varied by 0.03% from 4D to 10D and since the wake produced by a wind turbine is generally fully-developed after reaching 4D, a wake length of 4D was used for all simulation, unless otherwise stated. In addition, since computational cost increases drastically with the number of steps, the 4D wake length assumption allows for shorter computational runtime. For example, it took 14 times longer to simulate the NREL Phase VI from a wake length of 4D to 10D.

Number of Discretized Sections

To analyzed the effect of the number of discretized section and the discretization scheme used to model the wind turbine blades, three test cases with 13, 26, and 52 sections discretized using an equally-spaced or a full-cosine distribution (see Appendix A) were investigated. Since the inboard sections typically generate little to no torque, the discretization scheme was applied along the blade region consisting of S809 airfoil sections only (see Table 4.1). As can be seen in Figure 4.5(d), both discretization schemes showed similar trend with about 0.20% difference as the number of section

increases from 26 to 52. In addition, with more sections at the tip region using the full-cosine distribution, a 0.35%-0.45% higher performance is predicted.

The NREL geometry was also used to compute the performance of the Phase VI turbine. This resulted in a 0.15% difference with the 26 full-cosine distribution and therefore, it was assumed to provide the necessary details in the performance prediction of the NREL Phase VI turbine. Furthermore, it is believed that the use of the NREL geometry provides a better comparison with other performance codes since most codes, available in the literature, employed the NREL geometry without any discretization scheme.

4.3.2 Comparison with Experimental Data

The performance of the Phase VI turbine was computed using DR_HAWT based on the parameters calibrated previously. Furthermore, as described in Section 3.7, the aerodynamic polars which are obtained either experimentally or numerically (e.g., XFOIL) are two-dimensional and therefore, must be corrected for rotational effects. For this validation case, four polar databases were employed namely, the corrected (using Snel *et al.* and Du & Selig 3D corrections) and uncorrected (2D) experimental data obtained from Ref. [47], as well as the uncorrected XFOIL data. Figure 4.6 shows the measured and computed torque and axial thrust of the NREL Phase VI wind turbine as a function of wind velocity. The NREL Phase VI measurements were provided by Dr. Scott Schreck from NREL [52]. The measured torques were obtained from pressure and strain gauge measurements whereas the axial thrust measurements were obtained from the pressure data only.

As can be seen in the torque comparison, the computed torque for all four polars showed good agreements in the pre-stall region (5 m/s and 7 m/s) with a maximum

Figure 4.6: Performance prediction comparison with NREL experimental data: (a) Torque (Top) and (b) Thrust (Bottom)

deviation of 13% with the measurements. The reason for this consistent results is because in this region, the flow over the blade is attached and therefore, the aerodynamic polars are more accurate between 2D wind tunnel experiments and numerical methods. In addition, there is no rotational effects acting along the span of the blade. However, in the stall region (between 10 m/s and 15 m/s) where flow separation and rotational effects are present, greater deviations ranging from 5% to 52% were obtained between DR_HAWT and the NREL measurements. At high wind speeds (20 m/s and 25 m/s) where most of the blade operate in deep-stall, the accuracy of the prediction is very low, with deviations ranging from 74% to 271%. The reason for this large inaccuracy is that the 2D modeling and wind tunnel measurement of lift and drag coefficients in these flow regimes are difficult to obtain [23].

Similarly, comparing the measured and computed axial thrust, consistent results were obtained in the pre-stall region with a maximum of 15% deviation. As expected, in the stall and deep-stall region (10 m/s to 25 m/s), better agreements were obtained when using the corrected polars for rotational effects. Overall, the computed torque and thrust using corrected experimental polars (Du & Selig and Snel *et al.*) showed a maximum of 20% (excluding deep-stall region) and 9% deviations with the NREL Phase VI measurements, respectively.

Figure 4.7 compares the computed and measured average angle of attack (AOA) distribution along the blade for several wind velocities (5 m/s to 15 m/s). During the NREL Phase VI experiment, the local flow angles (LFA) at several span locations along the blade (0.34R, 0.51R, 0.67R, 0.84R and 0.91R) were measured using five-hole pressure probes installed ahead of the leading edge of the blade. Moreover, an inverse BEM technique (see Appendix B) was employed to determine the AOA based on the C_t and C_n values from surface pressure measurements along the blade (0.30R, 0.47R,

Figure 4.7: Comparison between computed and measured angle of attack distribution over the span of the NREL Phase VI blade

0.63R, 0.80R and 0.95R). The large differences observed between LFA measurements and inverse BEM data (see Figure 4.7), correspond to the study performed by Sant *et al.* [102] using the free wake vortex code, developed at Delft University of Technology. These differences can be explained from the drawbacks of the LFA technique where 2D wind tunnel calibration of the probes may not always be accurate enough to correct the complex flow around wind turbine blades. Furthermore, the probes may alter the flow around the blade, thereby interfering with the pressure measurements [102,103].

For the purpose of this thesis, the computed AOA from DR_HAWT is compared to the inverse BEM since the LFA measurement technique used by NREL may have a considerable amount of error associated with it [102,104]. As can be seen in Figure 4.7, the computed AOA displays good agreement when compared to the inverse BEM data with the exception of the inboard region at higher oncoming wind velocities. As expected, the computed AOA with 2D polars exhibit large differences due to the effect of rotation and flow separation and when the 3D corrections are applied, the flow field in the inboard region is captured more accurately.

In summary, DR_HAWT accurately predicted the performance of the NREL Phase VI wind turbine with the exception of the deep-stall region. Additionally, it was observed that the accuracy of the prediction is highly dependent on the aerodynamic polars input in DR_HAWT.

4.4 MEXICO Wind Turbine

To further validate the vortex code, the three-bladed wind turbine which was performed within the framework of the EU FP5 project MEXICO, was chosen for its extensive coverage of the entire operational range, from turbulent wake state to stalled conditions. This 4.5 m diameter rotor has a standard tip-speed of 100 m/s, which

Figure 4.8: MEXICO blade chord length and twist distribution

maximizes typical tip Reynolds number while maintaining the Mach number below 0.3. The blades are twisted and tapered with chord and twist distributions as illustrated in Figure 4.8. As described in Section 2.1.2, the geometry of the blade consists of three different airfoil sections (DU91-W2-250, RISO-A1-21, and NACA 64-418). The DU91-W2-250 airfoil is applied from 20% to 45.6% span, the RISO-A1-21 airfoil from 54.4% to 65.6% span and the NACA 64-418 airfoil outboard of 74.4% span.

The analysis was carried out using the non-yawed configuration with a pitch angle of -2.3° at various operating conditions: wind velocity ranging from 10 m/s to 24 m/s and rotational speed of 424 rpm. Similar to the NREL Phase VI validation, a parameter calibration was performed prior to computing the performance of the MEXICO turbine. Based on the results (not shown here), the blades were discretized with 21 sections following the data provided in Ref. [53]. The turbine was modeled with a vortex core radius of 5% and azimuthal increment of 10° . In addition, the computation was assumed to have reached a converged value after traveling 4D downstream of the turbine.

Figure 4.9 displays the computed and measured torque and axial thrust of the MEXICO wind turbine. The aerodynamic polar database which was based on the experimental 2D polars provided by Dr. Schepers [55], was either uncorrected (2D) or corrected using either the Snel *et al.* or the Du & Selig 3D corrections. As can be seen in both figures, a large over-prediction was obtained for the pre-stall regime (10 m/s and 15 m/s) whereas better accuracy was obtained for the stall regime (24 m/s) with the corrected polars. Although a large discrepancy can be observed, the DR_HAWT's prediction falls within that of other codes from simple BEM to more sophisticated CFD (see Figures 2.5 and 2.6).

Furthermore, the local angle of attack over the span of the blade was compared with the measured data. Using the inversed BEM method described in Appendix B, the angle of attack was computed from the tangential and normal forces at 5 spanwise locations on the blade. The comparison shown in Figure 4.10, demonstrates good agreement over the range of operating conditions investigated. Furthermore, for the stalled regime ($V_o = 24$ m/s), the rotational effect was well captured when the 3D corrections were used. Despite the overestimation for the torque and axial thrust, the good agreement shown with the measured angle of attack outlines the ability of DR_HAWT to accurately predict the performance of HAWT, provided that a comprehensive calibration is carried out.

4.5 Dual-Rotor Validation

In the current work, several attempts in obtaining dual-rotor wind turbine (DRWT) wind tunnel measurements were pursued, however, with the limited resources and time available during this thesis, strict dual-rotor validation case was not performed.

Figure 4.9: Performance prediction comparison with MEXICO experimental data: (a) Torque (Top) and (b) axial thrust (Bottom)

Figure 4.10: Comparison between computed and measured angle of attack distribution over the span of the MEXICO blade

A wind tunnel experiment, using similar rotor design to the MAAE rotor used by McTavish during his PhD thesis (see Ref. [105]), was performed in the atmospheric boundary layer wind tunnel at Carleton University. The tapered and twisted blade consisted of Eppler 387 airfoil. Since the aerodynamic polars were not available at such low Reynolds numbers, the in-house measurements could not be used to validate DR_HAWT. Furthermore, an approach was made to initiate a collaboration with Dr. Hui Hu from Iowa State University and is yet to be finalized. With the extent of the study, it is believed that the verification and validation test cases previously described, are sufficient to provide an insight on the DRWT performance.

Chapter 5

Study Approach and Results

This chapter describes the numerical study to identify potential dimensionless parameters for dual-rotor wind turbine (DRWT). Based on some important DRWT parameters such as the rotor speeds, rotor diameters and the distance between the rotors, three dimensionless parameters were derived using the Buckingham Pi theorem. Hypothetical DRWT models were created using geometrically-scaled National Renewable Energy Laboratory (NREL) Phase VI rotor geometry and operating conditions in order to confirm the validity of these parameters. Each DRWT was modeled and computed in DR_HAWT using aerodynamic polars from XFOIL. To compare the performance of each turbine, their output power is normalized with respect to the output power of equivalent single-rotor wind turbines (SRWTs).

5.1 Dimensional Analysis

The overall performance of a wind turbine can be defined as the sum of its properties which, at a given operating conditions, determine its annual energy production (AEP), lifetime, production cost and maintenance cost, etc.. However, from a purely aerodynamic point of view, the performance of a wind turbine can be defined based

on the rotor swept area (rotor diameter) and its operating condition (oncoming wind speed and rotational speed). Similarly, as illustrated in Figure 5.1, the performance of dual-rotor wind turbines can be characterized as a function of the oncoming wind velocity (V_o), the rotor angular velocities (ω_1 and ω_2), the rotor diameters (D_1 and D_2), and the distance separating the upwind and downwind rotor (L). Other flow conditions considered are the fluid density (ρ) and viscosity (μ). The dependent and independent variables considered in the dimensional analysis for dual-rotor wind turbines, are summarized in Table 5.1. Note that subscripts 1 and 2 which are associated to the rotor diameter and angular velocity, represent the upwind and downwind rotors, respectively.

Figure 5.1: Schematic of the dual-rotor horizontal axis wind turbine's performance parameters used in the present study

Table 5.1: Summary of dependent and independent variables for DRWT

Variable	Description	Dimensions
D_1, D_2	Rotor diameters	L
ω_1, ω_2	Rotor angular velocity	T^{-1}
L	Separation distance	L
V_o	Oncoming wind velocity	LT^{-1}
ρ	Fluid density	ML^{-3}
μ	Fluid viscosity	$ML^{-1}T^{-1}$
P_{out}	Power generated	ML^2T^{-3}

By using the Buckingham Pi Theorem, six dimensionless numbers were derived from the 9 variables listed in Table 5.1. The analysis was performed with reference to the downwind rotor (main rotor) since the upwind rotor is considered as an auxiliary rotor. These dimensionless numbers are given below:

$$\Pi_1 = \frac{\rho V_o D_2}{\mu}; \quad \Pi_2 = \frac{\omega_2 D_2}{V_o}; \quad \Pi_3 = \frac{P_{out}}{\rho D_2^2 V_o^3}; \quad \Pi_4 = \frac{D_1}{D_2}; \quad \Pi_5 = \frac{L}{D_2}; \quad \Pi_6 = \frac{\omega_1 D_2}{V_o}$$

where Π_1 , Π_2 , and Π_3 are similar to the Reynolds number ($Re = (\rho V_o D_2)/\mu$), tip-speed ratio ($\lambda = (\omega_2 R_2)/V_o$), and power coefficient ($C_P = P_{out}/(0.5\rho\pi R_2^2 V_o^3)$) with reference to the downwind rotor, respectively. The aforementioned numbers are commonly used to non-dimensionalize SRWT. On the other hand, Π_4 , Π_5 , and Π_6 constitute geometric and operational characteristics from both the upwind and downwind rotors and therefore, since the goal of this work is to identify parameters for DRWT, these three parameters will be considered in the parametric study.

For the purpose of this thesis, the first parameter which is the ratio of the upwind rotor diameter (D_1) and the downwind rotor diameter (D_2) is called the diameter

ratio (DR). The gap ratio (GR) describes the ratio of the distance between the rotors and the diameter of the downwind rotor. Finally, the combined tip-speed ratio (CTSR) is defined as the ratio of the product of the upwind angular velocity and the downwind rotor radius and the oncoming wind velocity. It should be noted that the downwind rotor radius, R_2 was used as the length scale for CTSR to be consistent with the tip-speed ratio ($\lambda = \omega R/V_o$) of a SRWT. For completeness, the three potential dimensionless parameters are repeated below:

$$DR = \frac{D_1}{D_2} \quad (5.1)$$

$$GR = \frac{L}{D_2} \quad (5.2)$$

$$CTSR = \frac{\omega_1 R_2}{V_o} \quad (5.3)$$

Furthermore, unlike other dual-rotor studies found in the literature [5, 6, 15, 16], where the power output produced by DRWTs are compared with the main rotor (usually the downwind rotor) in single-rotor configuration, the power output of a DRWT is compared with the power generated by individual SRWTs equipped with the equivalent rotors. In other words, the normalized power (P_{norm}) is given by Equation 5.4 as the ratio of the power output generated by a DRWT (P_{dual}) to the sum of the power output generated by the upwind and downwind rotors in single-rotor configurations ($P_{singles}$). This method is believed to give a better value to the potential that dual-rotor systems can provide.

$$P_{norm} = \frac{P_{dual}}{P_{singles}} \quad (5.4)$$

5.2 Parametric Sweep

Based on the 3 dimensionless parameters derived previously, a parametric study which was divided into 3 test cases (D01, D02, and D03), was performed to verify their applicability to dual-rotor wind turbines. Furthermore, the scalability (D04) of these dimensionless parameters were verified where the baseline model was scaled by a factor of 0.5 and 2. Table 5.2 summarizes the specification for each case. In each test case, two parameters were varied one at a time with the third parameter kept constant. A total of 16 hypothetical wind turbine models were created ranging from 0.25 – 1.00 diameter ratio and 0.25 – 1.00 gap ratio. Moreover, since counter-rotating rotors were more popular in the past and proved to be more efficient as compared to co-rotating rotors, all the simulations performed during this study employed counter-rotating configuration.

Table 5.2: Summary of DRWT parametric study

Case	Diameter Ratio	Gap Ratio	CTSR	Scale Factor
D01	0.5	0.25 – 1.00	6.0 – 10.0	–
D02	0.25 – 1.00	0.5	6.0 – 10.0	–
D03	0.25 – 1.00	0.25 – 1.00	7.5	–
D04	0.5	0.5	6.0 – 10.0	0.5, 1, 2

These models were based on the geometry and operating condition of the NREL Phase VI wind turbine provided in Ref. [47] and summarized in Section 4.3. With reference to the wind turbine design of ZEC Wind Power Copr. (i.e, 0.5 diameter and gap ratios) [18], the baseline model is equipped with a 5 m upwind and 10 m downwind rotors and a separation distance of 5 m between the rotors. Due to aero-acoustic noise generation, land-based HAWTs are typically constraint to operate at

tip-speeds ranging from 75 m/s to 80 m/s [106]. Although the tip-speed of the NREL Phase VI rotor is 37.5 m/s, it was believed adequate to geometrically scale the Phase VI rotor with the assumption that the blade tip-speed remains constant.

For all simulations, unless otherwise stated, the blade was modeled in DR_HAWT with 26 discretized sections, as provided in Table 4.1 with the appropriate scaling factor. Similar to the NREL Phase VI validation case, a 10° azimuth angle increment with 10% vortex core radius was used with the assumption that the converged solution is obtained when the wake traveled $4D_2$ (4 times the diameter of the downwind rotor) downstream of the downwind rotor. In addition, the aerodynamic polars which were computed in XFOIL for Reynolds numbers ranging from 1×10^4 to 2×10^6 , were employed without applying any corrections. It should also be noted that the tower and nacelle are not modeled in this study. Finally, since the performance of the DRWTs are normalized with equivalent single-rotor turbines, the latter are also modeled with similar operating and flow conditions as their respective DRWT models.

5.3 Results and Discussion

This section details the results the four case studies previously described (i.e., Cases D01, D02, D03 and D04). Potential trends to determine an optimal design for dual-rotor wind turbines are also explained.

5.3.1 CASE D01: Effect of Gap Ratio vs CTSR

The effect of varying the distance separating the upwind and downwind rotors is analyzed in this case study. The baseline turbine which is equipped with a 5.0 m and 10.0 m diameter upwind and downwind rotors, respectively, was utilized with various gap ratio (0.25 - 1.00) as illustrated in Figure 5.2. The performance of these four

DRWT models were simulated at combined TSR of 6.0, 7.5, and 10.0. As can be seen in Figure 5.3, comparable performances were obtained between the different models with almost 100% normalized power obtained at 6.0 and 7.5 CTSR. These similar performances can be explained by analyzing the wake expansion of the upwind rotor. Figure 5.4 shows the wake visualization which was obtained from DR_HAWT, of a half-scale NREL Phase VI rotor (i.e, the upwind rotor in a single-rotor configuration) operating at a TSR of 3.75 ($V_o = 10$ m/s and $\omega = 144$ rpm). Similar to the NREL Phase VI smoke visualization, there is hardly any wake expansion due to its very low thrust values. That being said, at higher CTSR, the turbine models which were expected to perform as efficiently as the other two cases, produced only about 90% normalized power.

Figure 5.2: Schematic of the four DRWT models investigated in Case D01

To understand why this deficit was obtained, the power output of the upwind and downwind rotors, which are displayed in Figure 5.5, were compared to their respective equivalent single-rotor wind turbine. It can be observed that the performance of the upwind rotor for both the 6.0 and 7.5 CTSR were slightly affected at 0.25 gap ratio whereas at 10.0 CTSR, a maximum of 20% deficit in upwind power was obtained,

Figure 5.3: Power output comparison between four DRWT having diameter ratio of 0.50 at various gap ratios ($GR = 0.25, 0.50, 0.75$, and 1.00)

Figure 5.4: Wake visualization of a half-scaled NREL Phase VI rotor generated from DR.HAWT

Figure 5.5: Performance of the upwind (solid lines) and downwind (dashed lines) rotors compared to their respective SRWT as a function of gap ratio

which decreases as the downwind rotor was moved further downstream. Further investigation revealed that, due to the interaction with the downwind rotor, the upwind rotor experienced similar relative velocity than its equivalent SRWT while the angle of attack (AOA) distribution for each operating condition was lower. As a result, at 10.0 CTSR, the flow around the upwind blade re-attached causing a drop in lift and therefore, lowering its performance. However, as the distance between the rotors increases, the effect becomes less pronounced thereby generating single-rotor-like performances. On the other hand, the downwind rotor performance has a reversed trend where the output power decreases with an increasing gap ratio. Similar to the upwind blade, this decreasing trend is caused by variation in AOA experienced by the downwind blade.

Moreover, as can be observed in Figure 5.5, the downwind rotor extracted more than its equivalent SRWT when operating at 7.5 CTSR ($V_o = 10$ m/s). At this operating condition, the inner region of the blade in single-rotor configuration is

Figure 5.6: S809 lift curve at $Re = 360,000$

considered to be in deep-stall. However, in a dual-rotor configuration, since the angle of attack of the downwind rotor is lowered as a result of the upwind wake, it is hypothesized that at 10 m/s, the operating range of the downwind blade's inboard sections was shifted from deep-stalled to stall condition. For instance, at 40% along the span of the blade, the AOA dropped from about 18° to 14° . Figure 5.6 shows the lift curve for the S809 airfoil at a Reynolds number of 360,000. As can be seen, this drop in AOA (circled in red) increases the lift coefficient by about 20% thereby increasing the power generation.

5.3.2 CASE D02: Effect of Diameter Ratio vs CTSR

The second case study investigates the effect of varying the swept area of the upwind rotor with respect to the downwind rotor. The four turbine models studied in this case, are equipped with 2.5 – 10.0 m diameter upwind rotor and a 10.0 m diameter downwind rotor separated by a 5.0 m long nacelle. A schematic of each turbine

is shown in Figure 5.7. The overall trend, as seen in Figure 5.8, shows that more power can be extracted at lower diameter ratio with the smallest ratio generating 100% normalized power at all CTSRs. This trend can be explained by comparing the power generated by the downwind rotor of each turbine. Figure 5.9 shows the performance of the upwind and downwind rotors as compared to their respective equivalent SRWT. Since the outer region of a rotor is typically designed to extract the most energy, the performance of the downwind rotor would be affected if the swept area of the upwind rotor overshadows the outer region of the former. Further investigation showed that the downwind rotor of the 1.0 diameter ratio appeared not to be producing any power at 10.0 CTSR (see Figure 5.9). The reason for this lack of power generation is due to a combination of the momentum loss from the upwind rotor and the fact that the freestream velocity was below the turbine's cut-in speed.

Figure 5.7: Schematic of the four DRWT models investigated in Case D02

The general effect of diameter ratio on the turbine's performance seems to be well represented in Figure 5.8, however, using combined TSR to compare the significance of this dimensionless parameter is believed to be an inadequate method. Contrary to the previous case (D01), in order to investigate similar CTSRs, large variation in the oncoming wind velocity was utilized between the turbine models since CTSR

Figure 5.8: Power output comparison between four DRWT having various diameter ratios ($DR = 0.25, 0.50, 0.75$, and 1.00) and a gap ratio of 0.50 as a function of combined tip-speed ratio

Figure 5.9: Performance of the upwind (solid lines) and downwind (dashed lines) rotors compared to their respective SRWT as a function of combined tip-speed ratio

is a function of the upwind rotational speed, downwind rotor radius and freestream velocity (i.e., $\text{CTSR} = f(\omega_1, R_2, V_o)$). The turbines are therefore operating in various conditions from attached flows to deep-stall regions. That being said, comparing against constant wind velocity is considered to provide better representation since four turbines with different diameter ratio experiencing the same wind velocity provide a better contrast than turbines experiencing constant CTSR (i.e., different wind velocities).

To confirm the trend displayed in Figure 5.8, similar simulations were performed with oncoming wind velocities of 7.5 m/s, 10.0 m/s, and 12.5 m/s. Figure 5.10 displays the normalized power of four turbine models as a function of the freestream velocity. As previously explained, the overall performance of the turbine can be observed to decrease with increasing diameter ratio at all wind speeds. As expected, the effect of the upwind wake becomes more pronounced with increasing upwind rotor diameter. In addition, similar to Case D01, the upwind turbines generated the same amount of power compared to their respective SRWT, with the exception of the models operating at 7.5 m/s. The reason for this exception is again due to the variation in AOA as described in the previous case.

The effect of diameter ratio can further be explain by analyzing the angle of attack distribution of each downwind rotor. Figure 5.11 displays the angle of attack distribution along the blade span for the downwind rotor of four different DR models and for their equivalent SRWT. It can be observed that at 0.5 DR, the reduction in AOA is more pronounced in the inboard section between 25% to 50% span. Although not clearly shown in this figure, the effect is not as apparent near the root since this region consists mostly of cylindrical sections where vortex filaments of lower strength are shed. In addition, the wake of the upwind rotor still affects the downwind rotor

Figure 5.10: Power output comparison between four DRWT having various diameter ratios ($DR = 0.25, 0.50, 0.75$, and 1.00) and a gap ratio of 0.50 as a function of freestream velocity

Figure 5.11: Angle of attack comparison between the downwind rotor of DRWT with diameter ratios ranging from 0.25 to 1.00 and its equivalent SRWT at a freestream velocity of 10.0 m/s

outer 50% span due to wake expansion which gradually dissipates nearing the blade tip. A similar trend can be observed for all models with the difference in AOA increasing as the upwind rotor grows in size.

5.3.3 CASE D03: Effect of Diameter Ratio vs Gap Ratio

To complement the first two cases, the effect of varying the diameter ratio as a function of the gap ratio was studied. In this case study, two set of computations were performed to verify the reasoning previously discussed, regarding using CTSR as a means of comparison. Both set of computations investigated 16 turbine models with diameter and gap ratio ranging from 0.25 to 1.00, as illustrated in Figure 5.12.

The first set of computations was performed at a constant CTSR of 7.5 to avoid simulating the downwind rotor outside its design operating range. Figure 5.13 displays the results for the turbine operating at constant CTSR. As expected, the dual-rotor systems with diameter ratios of 0.25 and 0.5 produced the best overall performance, as seen in Figure 5.13. The normalized power for each model with similar diameter ratio was also found to be fairly constant across the range of gap ratio investigated. Furthermore, similar overall trend to that obtained from the constant CTSR simulations was reported in the second set of computations (see Figure 5.14) which was performed at constant oncoming wind velocity of 10 m/s. Although different normalized powers were obtained between the two sets of computations, due to different operating conditions, this comparable trend suggests that the combined tip-speed ratio can be used to compare the general performance trend of various DRWT models.

Figure 5.15 gives the variation in power output of the upwind and downwind rotors separately as a function of gap ratio. At diameter ratio greater than 0.5, the results showed that the power generated by the downwind rotor decreases with increasing gap

Figure 5.12: Schematic of the 16 DRWT models investigated in Case D03

ratio whereas the upwind power output increases with increasing gap ratio. Due to the interaction between the downwind rotor and the upwind wake, the power loss of the downwind rotor was found to be as high as 25% for the turbine with equivalent rotors, in comparison to the power generation of similar rotor in single-rotor configuration. Moreover, at the lowest diameter ratio investigated (i.e., $L/D_2 = 0.25$), the power

Figure 5.13: Power output comparison between four DRWT having various diameter ratios ($DR = 0.25, 0.50, 0.75$, and 1.00) at a CTSR of 7.5

Figure 5.14: Power output comparison between four DRWT having various diameter ratios ($DR = 0.25, 0.50, 0.75$, and 1.00) at 10 m/s freestream velocity

Figure 5.15: Performance of the upwind (solid lines) and downwind (dashed lines) rotors compared to their respective SRWT as a function of gap ratio at 10 m/s freestream velocity

deficit of 21% obtained corresponds to the lower end of the range (25% - 40% power loss from the downwind rotor) found in the literature [5,6,16]. It is believed that this lower deficit is due to the lack of tower effect in this study.

5.3.4 CASE D04: Scaling Effect

To verify the scalability of the three dimensionless parameters derived in this thesis, the performance of the baseline model was compared to two geometrically similar models scaled by a factor of 0.5 and 2. Figure 5.16 illustrates the baseline model along with the two scaled models. In other words, the three models investigated are equipped with upwind rotors having diameters of 2.5 m, 5.0 m, and 10.0 m and downwind rotor diameters of 5.0 m, 10.0 m, and 20.0 m, respectively. Figure 5.17 shows the performance of the three geometrically-scaled dual-rotor models having

Figure 5.16: Schematic of the three DRWT models investigated in Case D04

diameter and gap ratios of 0.50. The large discrepancy which observed at 7.5 CTSR, can be justified by the fact that Reynolds number was not constant between the three models. Therefore, for the purpose of this study, Case D04 was repeated with the Reynolds number artificially kept constant by varying the fluid viscosity between the models. The results which are displayed using dashed lines in Figure 5.17, confirmed that the differences in power output between the models are solely due to Reynolds number effect.

Figure 5.17: Power output comparison between three geometrically-scaled DRWT with diameter and gap ratios of 0.5 (Dashed lines represent the constant Reynolds number case)

Chapter 6

Conclusions and Recommendations for Future Work

With dual-rotor wind turbines (DRWTs) attracting more attention in recent years, the present work aimed at developing a prediction tool and identifying potential dimensionless parameters for DRWTs as well as establishing an optimal DRWT configuration. This section is divided into two sub-sections summarizing (a) the work performed on the prediction tool including potential improvements and (b) the dimensionless parameters involved in the dual-rotor study.

6.1 Prediction Tool

An in-house vortex filament code, named DR_HAWT (Dual-Rotor Horizontal-Axis Wind Turbine code) was implemented in FORTRAN 95 for the aerodynamic performance prediction of a single or dual-rotor horizontal-axis wind turbine (HAWT). This three-dimensional (3D) free vortex model which is based on the lifting line theory, represents a wind turbine blade and its wake using straight vortex filaments. The performance prediction of DR_HAWT was shown to agree well with the single rotor results from two well-known wind tunnel experiments, namely the NREL Phase VI

and the MEXICO wind turbines. However, in deep-stall condition, the accuracy of the prediction, after correcting for rotational effects, is poor which corresponds to the results found in the literature. Due to a lack of available DRWT experimental data, the vortex code was assumed validated after further investigations using various blade-vortex interaction scenarios.

From the current study, several additions can be included in DR_HAWT to improve its accuracy and computational cost. These improvements are listed below:

- Although parallel computing capabilities have already been implemented in DR_HAWT, the author believes that further parallelization of the vortex code is possible which will further reduce the computational cost. Moreover, in the current code, the effect of every vortex filaments is evaluated. This tedious task can be trimmed by either eliminating weaker vortices, or combining the nearby vortices into a stronger one, thereby reducing computational cost.
- Most single rotor wind turbine simulations are modeled without their tower, however, for dual-rotor configurations, the latter will significantly influence the performance of the downwind rotor. The effect of the tower may be aerodynamically modeled using semi-empirical tower shadow wake models such as Powles', Blevins', Schlichtings' and the JET wake models. These steady wake models describe the velocity deficit behind a body as a decaying Gaussian or cosine square shaped profile [107].
- Since wind turbines do not always experience head on winds, an additional input parameter such as yaw angle, can be included in the present version of DR_HAWT. This parameter, which describes the direction of the wind, can therefore allow for yaw flow conditions to be investigated. In addition, the yaw flow measurements from the NREL Phase VI and MEXICO experiments can

be used to validate the single-rotor portion of the vortex code.

- Although the vortex code was compared to single-rotor experiments and showed good performance prediction, it is important to validate dual-rotor cases with in-house wind tunnel experiments employing either a dual-rotor setup or two single-rotors co-axially positioned. Based on the author's experience, XFOIL does not generate good aerodynamic polars at very low Reynolds number and therefore, 2D wind tunnel experiments may be required to obtain the necessary polar database to input in DR_HAWT.
- In this vortex code, a viscous core model incorporating the Biot-Savart law, is employed to simulate viscous diffusion of the vortex filaments based on the vortex age. However, vortex filaments also experience positive or negative strain (stretching or squeezing, respectively) in unsteady flow fields. This effect will cause a change in the local induced velocity field, which will influence the motion of the vortex filaments relative to one other [84]. Although viscous diffusion may be sufficient for single-rotor simulations, better DRWT performance prediction may be obtained if the downwind rotor sees a more accurate representation of the upwind wake.

6.2 Dual-Rotor Study

Three non-dimensional parameters, namely diameter ratio (D_1/D_2), gap ratio (L/D_2), and combined tip-speed ratio ($\omega_1 R_2/V_o$), were derived for the characterization of DRWTs. With the assumption that the blade tip-speed is conserved, 16 hypothetical DRWT models were created based on blade geometry and operating conditions of the NREL Phase VI wind turbine. For the purpose of the present work, uncorrected 2D aerodynamic polars obtained from XFOIL were employed to remove

additional unknowns from the study since stall delay models are based on empirical work. Through three case studies, the effect of each parameter was analyzed using DR_HAWT. The following are observations and conclusions gathered from this study:

- Effect of Diameter Ratio: Diameter ratio plays a significant role in the power extraction of the downwind rotor. The addition of a smaller auxiliary rotor placed upstream of the main rotor allows for further kinetic energy extraction from the wind. However, as the swept area of the upwind rotor increases, the performance of the downwind rotor is affected as the latter is overshadowed by the former. It was also found that for a particular situation where a 0.5 diameter ratio turbine experienced an oncoming wind velocity of 10 m/s, more energy than that of its equivalent SRWTs was extracted since the operating range of the turbine shifted from deep-stall to stall condition.
- Effect of Gap Ratio: Due to a lack of wake expansion from the NREL Phase VI rotor, hardly any differences were obtained when analyzing the effect of gap ratio. However, if the effect of the tower and nacelle was also modeled, it is expected that performance of the downwind rotor will decrease for the case where the tower is located in-between the rotors.
- Effect of Combined Tip-Speed Ratio: The combined tip-speed ratio (CTSR) uses the characteristic of both the upwind and downwind rotors as well as the oncoming wind velocity. The present work showed that this dimensionless parameter can be used to compare the power generation of various dual-rotor systems with different diameter and gap ratios. Moreover, it is believed that the combined TSR will provide an insight on the performance of various DRWTs during the initial design stages when large number of configurations are being studied.

- The results from the current work suggest that having a counter-rotating 5.0 m diameter auxiliary rotor located 5.0 m upstream of the 10 m diameter NREL Phase VI rotor could provide an optimal DRWT configuration. The overall trend suggests that the performance of DRWTs is enhanced at low diameter and gap ratios, keeping in mind that the minimum gap ratio allowed by a specific turbine is dependent on the mechanical system used. From this study, such optimal DRWT model could produce more than 90% of the power generated by equivalent SRWTs at an oncoming wind speed of 7.5 m/s.
- The addition of either a half-sized or full-sized rotor upstream of the NREL Phase VI rotor showed an increase in power extraction of up to about 25% and 76%, respectively, compared to the power output of the downwind rotor only. This correlates well with the improved performance found in the literature (see Section 4.5).

From the current study, several paths can be followed to improve both the dimensionless parameters and in establishing an optimal DRWT configuration. The recommendations for future work are listed below:

- Although experimental work is associated to high cost, the effects of each dimensionless parameter can be analyzed through in-house wind tunnel experiments since the latter generally provide a more realistic representation. As previously mentioned, these measurements can also be used to further validate DR.HAWT.
- To better understand the effect of each dimensionless parameter on the performance of DRWTs, other wind turbine operating conditions and blade geometries should be investigated such as the three-bladed 4.5 m diameter MEXICO wind turbine. Moreover, since the blade geometry employed in the present work was

designed to operate in a single-rotor configuration, a blade optimization can be performed for both the upwind and downwind rotors.

- The current study neglects the effect of the tower, however, since the latter will definitely influence the turbine's performance, it would be interesting to investigate whether the tower and its location with respect to the downwind rotor, affects the ability of the turbine to extract energy.
- Since dual-rotor system may be a potential solution to the increasing size of SRWT, it would be interesting to see how much larger a SRWT should be to extract similar power to a DRWT. In addition, further analyses should be performed under yaw conditions and flow misalignment to investigate if DRWT performs as well as SRWT under such conditions.
- In addition to the three dimensionless numbers found in this thesis, other numbers such as L/D_1 and $\omega_2 R_1/V_o$, can be derived which could allow for better comparison between DRWT models operating under different Reynolds number. Hence, based on the results found in this study, further investigation into DRWT dimensionless parameters should be performed.

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Appendix A

Full-Cosine Discretization

In order to enhance the accuracy at the root and tip regions of a blade where large gradient in the bound circulation are obtained, the blade is divided into elements using a full-cosine discretization method [32, 34, 108]. For a given number of discretized elements n , the blade span is divided into $n + 1$ equi-angle increments as shown in Figure A.1. Equation A.1 defines the radial position r for a given section i .

$$r_i = \frac{R}{2} (1 - \cos \gamma_i) \quad (\text{A.1})$$

where R is the blade radius and γ_i is incremented angle for section i as defined in Figure A.1.

Figure A.1: Full-cosine discretization method applied on the NREL Phase VI

Appendix B

Inverse BEM

The inverse BEM methodology which is adapted from the classical blade element momentum (BEM) theory, utilized the normal and tangential force coefficients obtained from experiments to evaluate the angle of attack, and the 3D lift and drag coefficients. The following describe the formulation used based on Refs. [93, 109]:

1. Initialize the axial (a) and tangential (a') induction factors, typically 0 or 1/3.
2. Compute the local flow angle ϕ :

$$\phi = \arctan \left[\frac{(1 - a)V_o}{(1 + a')\omega r} \right] \quad (\text{B.1})$$

3. Calculate the new induction factors using the experimental C_t and C_n :

$$a = \frac{1}{\frac{4 \sin^2 \phi}{\sigma C_n} + 1} \quad (\text{B.2})$$

$$a' = \frac{1}{\frac{4 \sin \phi \cos \phi}{\sigma C_t} - 1} \quad (\text{B.3})$$

4. If a and a' has changed more than a certain tolerance, go to step 3 or else finish.

5. Determine the local angle of attack α :

$$\alpha = \phi - \beta - \theta_p \quad (\text{B.4})$$

6. Calculate the 3D lift and drag coefficients:

$$C_{L,3D} = C_n \cos \phi + C_t \sin \phi \quad (\text{B.5})$$

$$C_{L,3D} = C_n \sin \phi - C_t \cos \phi \quad (\text{B.6})$$

It should be noted that Prandtl's tip and root losses as well as Glauert's correction for high induction factors can also be included.