An Experimental and Numerical Investigation of Capillary Limit in an
Arterial Heat Pipe

by

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A Thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Master of Applied Science

Ottawa-Carleton Institute for
Mechanical and Aerospace Engineering

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Carleton University

Ottawa, Ontario, Canada

August 2009

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ISBN: 978-0-494-60234-8
Our file Notre référence
ISBN: 978-0-494-60234-8

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Abstract

The operating characteristics and performance limits of a novel space qualified arterial heat pipe design were experimentally investigated. The arterial heat pipe was found to be capillary limited to a maximum dry-out power of 350 W at horizontal elevation. Performance was repeatable and stable throughout all testing. A numerical model that accounted for excess charging of working fluid was proposed and found to compare well with the results of experimental testing.

Acknowledgments

I would like to thank my supervisor, Professor Tarik Kaya, for providing me with the opportunity to further my education and for being there to answer my questions. I would also like to thank Michel Garcia for his insights and suggestions that have helped me so much. Finally I would like to thank my generous family and friends for their support and encouragement.

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Nomenclature

Roman

a	Polynomial coefficient	—
A_c	Exponential factor in Arrhenius equation	—
C	Chord	m
C_c	Nusselt number coefficient	—
c_p	Specific heat at constant pressure	$kJkg^{-1}K^{-1}$
d	Diameter	m
d_{hw}	Screen mesh wire diameter	m
E_a	Activation energy	—
f	Frictional drag coefficient	—
F	Area	m^2
g	Gravitational acceleration	ms^{-2}
Gr	Grashof number	—
H	Height	m
$\bar{h}$	Mean heat transfer coefficient	$Wm^{-2}K^{-1}$
h_{fg}	Latent heat of vapourization	Jkg^{-1}
I_p	Polar moment of inertia	m^4
k	Thermal conductivity	$Wm^{-1}K^{-1}$
k_c	Rate constant	—
L	Length	m
L_w	Length from evaporator to condenser (width)	m

m	Mass	kg
M	Molar mass	$gmol^{-1}$
m_c	Nusselt number exponential coefficient	—
$\dot{m}$	Mass flow rate	$kg s^{-1}$
n	Number of moles	—
$\overline{Nu}$	Mean Nusselt number	—
P	Pressure	Pa
P_c	Capillary pressure	Pa
Po	Poiseuille number	—
Pr	Prandtl number	—
$\dot{Q}$	Heat transfer rate	W
R	Radius	m
r	Radius of thread bottom	m
Ra	Rayleigh number	—
R_c	Universal gas constant	—
Re	Reynolds number	—
r_{har}	Hydraulic radius of artery space	m
r_{hl}	Hydraulic radius of liquid space	m
r_{hv}	Hydraulic radius of vapour space	m
S	Segment length	m
S_p	Circumferential groove thread pitch	m
S_t	Circumferential groove thread width	m

T	Temperature	$^{\circ}\text{C or } K$
t_s	Time	s
U	Uncertainty	—
V	Volume	m^3
w	Velocity	ms^{-1}
x, y, z	Cartesian coordinates	m
ΔP_{fla}	Flange pressure drop	Pa
ΔP_{t1}	Total pressure drop (vent hole criteria)	Pa
ΔP_{t2}	Total pressure drop (threaded groove criteria)	Pa
ΔP_{thr}	Thread pressure drop	Pa

Greek

α	Groove half angle	$rad \text{ or } ^{\circ}$
α_d	Thermal diffusivity	m^2s^{-1}
β	Complimentary groove angle	$rad \text{ or } ^{\circ}$
β_f	Volume coefficient of expansion	K^{-1}
δ_w	Screen mesh thickness	m
ϵ_s	Emissivity	—
ϵ_w	Porosity	—
θ	Contact angle	$rad \text{ or } ^{\circ}$
μ	Dynamic viscosity	$kgm^{-1}s^{-1}$
ν	Kinematic viscosity	m^2s^{-1}

ξ	Segment half-angle	$rad\ or\ ^\circ$
Π	Perimeter	m
ρ	Density	kgm^{-3}
σ	Surface tension	Nm^{-1}
σ_c	Stefan-Boltzmann constant	$Wm^{-2}K^{-4}$
τ	Shear stress	Pa
ϕ	Puddle angle	$rad\ or\ ^\circ$
χ	Artery angle	$rad\ or\ ^\circ$
ψ	Tilt angle	$rad\ or\ ^\circ$

Script

$\mathcal{I}$	Current	A
$\mathcal{P}$	Power	W
$\mathcal{R}$	Thermal Resistance	KW^{-1}
$\mathcal{V}$	Voltage	V

Subscript

0	Zero	—
$\perp$	Perpendicular	—
∞	Infinity	—
<i>ad</i>	Adiabatic	—
<i>ar</i>	Artery	—

<i>co</i>	Condenser	—
<i>eff</i>	Effective	—
<i>ev</i>	Evaporator	—
<i>f</i>	Film	—
<i>i</i>	Inside	—
<i>in</i>	Input	—
<i>l</i>	Liquid	—
<i>m</i>	Meniscus	—
<i>max</i>	Maximum	—
<i>min</i>	Minimum	—
<i>o</i>	Outside	—
<i>out</i>	Output	—
<i>s</i>	Surface	—
<i>sat</i>	Saturation	—
<i>t</i>	Thread	—
<i>v</i>	Vapor	—

Acronyms		—
-----------------	--	---

ArHP	Arterial heat pipe	—
FEM	Finite element method	—
LPM	Liters per minute	—
UMF	Uncertainty magnification factor	—

UPC	Uncertainty percentage contribution	—
NCG	Non-condensable gas	—
TC	Thermocouple	—

Chapter 1 Introduction

1.1 Introduction and Literature Review

Heat pipes are heat transport devices employing two phase flow that are capable of extremely high equivalent thermal conductivity. The heat pipe was initially patented by Grover in 1963 while working for the United States Atomic Energy Commission [1]. It was then described in a paper entitled “Structures of very high thermal conductance” by the same author the following year which defined a heat pipe in [2] “as a synergistic engineering structure which is equivalent to a material having a thermal conductivity greatly exceeding any known metal”. After that, the field expanded quickly and by 1967 a heat pipe was tested in space [3]. Heat pipes have been used extensively for spacecraft

thermal control since the launch of the American GEOS-B satellite in 1968 [4]. Heat pipe technology takes advantage of two phase flows to achieve much greater heat transfer rates than via conduction or convection alone. A typical heat pipe, shown in Fig. 1.1, has three main sections, the evaporator where heat is input, an adiabatic section that transports the flow at constant temperature and a condenser where heat is output. The main sealed tube has a wick structure on the inner wall and includes a working fluid.

Fig. 1.1 – Typical heat pipe operation

Heat vapourizes liquid in the evaporator. This vapour is driven to the condenser through a pressure difference. The vapour condenses and releases its latent heat of vapourization. The liquid then moves back to the evaporator through the capillary wick. The liquid is forced through the wick structure by a pressure difference caused by the differing radii of curvature of the meniscus on the evaporator and condenser sides. Heat pipes of various designs have been used for many applications including those in spacecraft, nuclear power plants, permafrost layer stabilization and in electronic cooling [5]. The

microelectronics industry, in particular, requires ever increasing cooling capabilities to cope with the increasing power densities of new microprocessors. Heat pipes are already used as integral cooling system components in laptop computers and in some desktop air coolers. Their extremely high effective thermal conductivity and ability to work passively will continue to play a critical role in electronic cooling [6].

Arterial heat pipes are similar to standard heat pipes but differ in that they have a dedicated artery for liquid flow as shown in Fig. 1.2. This results in lower friction along the liquid path and in turn allows much higher performance for a given cross sectional diameter. This style of heat pipe is also very sensitive to any adverse tilt while in a gravitational field. A small tilt angle can be enough to cause total dry-out of the artery. With this high sensitivity to gravitationally induced adverse pressure gradients and excellent performance per mass ratios compared with typical heat pipes, space applications have clear potential.

The term dry-out refers to the point at which the capability of the heat pipe to return liquid to the evaporator is insufficient to replenish the quantity of vapour that is leaving. This effectively leaves a section at the beginning of the evaporator void of fluid, hence the term dry-out. This is undesirable as it would lead to a local spike in temperature which could potentially damage the equipment that is relying on the heat pipe for cooling.

Fig. 1.2 – Typical arterial heat pipe operation

A somewhat similar concept to the arterial heat pipe presented in Fig. 1.2 was the monogroove heat pipe introduced in [7]. This design and its subsequent design iterations flew on several space shuttle flights for zero gravity testing. The initial Space Station Heat Pipe Advanced Radiator Element (SHARE) design flew on STS-29 in March 1989. During testing vapour bubbles formed in the liquid artery which caused significant performance degradation [8]. This design was modified and in August 1991 on STS-43 the SHARE-II experiment was launched. This redesign changed the configuration of the monogroove artery to promote venting of non-condensable gas. The improved monogroove heat pipe design functioned as expected [9]. Analytical models for predicting de-prime and rewetting of the artery were developed and agreed well with the experimental values [10]. A radiator composed of multiple monogroove style heat pipes

in parallel is used on the International Space Station. This is advantageous because if one of the pipes is damaged from a micro meteoroid or orbital debris impact the remaining pipes will continue to operate [11].

There are very few other instances of arterial style heat pipes in western literature, however there have been many Russian spacecraft, some of which are presented in the following table adapted from [12], that have successfully used arterial heat pipes.

Table 1 – Various Soviet and Russian satellites using arterial heat pipes

Spacecraft Name	Launch Year	Number of arterial heat pipes	Working Fluid
OKO-1-0	1991	14	Freon-22
OKO-1-1	1992	14	Freon-22
OKO-1-2	1994	14	Freon-22
OBZOR	1994	1	Ammonia
MARS-96	1996	2	Ammonia
ARCON-1	1997	32	Ammonia
COUPON	1997	14	Ammonia
OKO-1-3	1998	14	Freon-22
OKO-1-4	2001	14	Freon-22
ARCON-2	2002	32	Ammonia
OKO-1-6	2003	10	Propylene

Over three hundred arterial heat pipes with segmented artery have been manufactured by Lavochkin Association and TAIS in Russia and more than one hundred sixty have been launched onboard various spacecraft. They have exhibited reliable performance, in some cases operating for more than 6 years, as noted in [13].

The test article is presented in Fig. 1.3 and a cross section of the internal arterial structure in Fig. 1.4. To prevent the accumulation of vapour pockets in the artery, vapour vent holes are arranged along the length of the screen which separates the liquid and vapour sections. These holes ensure any excess vapour in the artery will be vented to the vapour space. The vapour vent holes are sized by the following equation, given in [14],

$$d_{vent} \geq 0.9 \sqrt{\frac{H_{ar} \delta_w}{\cos(\theta)} \left(2 - \frac{\delta_w \cos(\theta)}{H_{ar}} \right)} \quad (1.1)$$

which is a function of the height of the artery H_{ar} , the mesh thickness δ_w and the meniscus contact angle θ .

Fig. 1.3 – Arterial heat pipe (all dimensions in mm)

Fig. 1.4 – Arterial heat pipe cross section (all dimensions in mm)

The arterial heat pipe examined in this thesis has circumferential grooves along its entire length as shown in Fig. 1.5. These internal threads serve to pump liquid from the liquid artery up to the top of the vapour space through capillary action. Each small groove along the inside of the pipe wall acts as a very small heat pipe. This feature is not unique to this design as it was also employed in the monogroove heat pipe [7]. Internal threaded grooves have been analyzed extensively with analytical methods for determining dry-out and flooding limits [15]. Experimental and analytical work has also been done on the rewetting characteristics of circular channels with internal grooves [16].

Fig. 1.5 – Circumferential threads (all dimensions in mm)

Several Russian papers, translated into English, were found to be relevant to arterial heat pipes of the same configuration as the test article. A numerical method was developed to predict the maximum heat transfer limit and was used as the basis for the development of the numerical model presented in Chapter 4 of this thesis [17]. Another paper of note is [18], which provides a means to estimate the thermal resistance of the vapourization zone along internal circumferential threaded grooves.

The most relevant paper that examined this style of heat pipe was that of Garcia and Kaya in 2007 [19]. This work presented experimental test results for the same test article as used in this thesis and a numerical model based on the work in the standard text on heat pipes by Chi [20]. Michel Garcia's thesis provides further details on the experimental work performed, including detailed information about the test apparatus [21].

This thesis sets out to define the operating characteristics and performance of a novel space qualified arterial heat pipe design. Performance limits and dry-out characteristics are thoroughly tested experimentally. A numerical model that can account for excess charging of fluid is proposed and compared with the results of experimental testing.

1.2 Organization

This thesis is organized into the following five chapters:

Chapter 1 – Introduction: Introduces background on the history of heat pipes. A literature review concerning arterial heat pipes is presented with the aim of summarizing the existing state of research on this type of heat pipe. This section also presents the problem statement of this thesis.

Chapter 2 – Experimental Setup: Presents the necessary background information on the experimental apparatus and methods used for testing the arterial heat pipe. An uncertainty analysis is presented and all relevant uncertainty values are given.

Chapter 3 – Experimental Results: Presents the results of experimental work. Figures summarizing the tests performed and the overall performance of the heat pipe are presented and discussed.

Chapter 4 – Numerical Model: Presents the theory and results for a typical numerical model as found in [20], as well as the development of an improved model that takes into account excess fluid in the heat pipe. The results of the numerical model are compared with the experimental results.

Chapter 5 – Conclusion: Presents conclusions, list of personal contributions and recommendations for future work.

Chapter 2 Experimental Setup

2.1 Introduction

The following chapter summarizes the experimental setup including all equipment used in testing, introduces an uncertainty analysis and details how major sources of experimental error were identified and then accounted for. An arterial heat pipe, manufactured by TAIS Thermal Systems Design and Manufacturing, was tested in order to understand and characterize its performance limits. Tests were performed to determine operating limits as well as the pipes tolerance to adverse pressure gradients through extensive tilt testing. All experimental work was performed in the Heat Pipe Laboratory, in room 2233 in the Mackenzie Engineering building at Carleton University. Ambient

conditions were not actively controlled as temperature and pressure fluctuated with local conditions.

2.2 Apparatus

A similar test apparatus was used in [19] and [21] to compare the performance of an axially grooved heat pipe to that of an arterial design. That test apparatus was modified to allow the heat exchanger to be mounted on the underside of the condenser where the liquid in the artery impedes the heat exchange. If the heat exchanger were mounted on the top, then better performance would be expected. Bottom cooling is the more penalizing case since the liquid in the artery at the bottom of the condenser represents an additional resistance. The heater must be mounted on the vapour side of the evaporator. If the heater were mounted on the artery side, local boiling could occur in the arterial fluid.

2.2.1 Overall Layout

The test apparatus has the ArHP held by three polypropylene blocks affixed with two-sided mounting tape to an aluminum table with four independently adjustable machine screw legs. A larger threaded rod assembly, used for inclination testing, was mounted to one side of the metal table.

Fig. 2.1 – Test assembly

2.2.2 Cooling System

A heat exchanger was attached to the condenser side saddle. The heat exchanger consisted of a tube welded to a flat saddle which was attached to the underside of the condenser saddle with hex screws. The screws were tightened to a torque of 1.7 Nm as per the heat pipe instruction manual [22]. Wakefield Engineering 120 series silicone oil-based thermal joint compound with a thermal conductivity of $0.735 \frac{W}{mK}$ was used to minimize the thermal contact resistance between the mating surfaces.

A Lytron recirculation chiller, part number RC011, filled with a 70% ethylene glycol to 30% water mixture supplied chilled fluid to the heat exchanger. Much of the testing was at 5°C and this value was chosen to avoid formation of ice crystals as noted in [21]. The cooling capacity of this chiller is rated at 500 W and it used R-134a refrigerant. The mean flow rate of coolant was 9.0 LPM as measured using an Omega FL-2068 flow

meter which has an uncertainty of $\pm 3\%$ across the full scale range. A schematic representation of the cooling liquid flow is in Fig. 2.2.

Fig. 2.2 – Cooling system

2.2.3 Heating System

On the evaporator side saddle, two different heaters were used during testing. A larger 1000 W cylindrical cartridge heater, coated with thermal grease and enclosed in an aluminum block, was used for high power testing. The smaller heater, rated up to 200 W, was supplied by the manufacturer of the heat pipe and was used for all low power and tilt testing. It consists of a resistive wire glued to the back of a 0.5 cm thick aluminum block. In both cases, the heaters were attached to the evaporator saddle using hex screws with a torque of 1.7 Nm.

2.2.4 Test Stand Base and Insulation

An aluminum table with four machine screw legs was used as a base onto which the heat pipe was attached. This table had a two holes passing through it on either side allowing both the heat exchanger and the tilt screw mechanism to pass through. The test article was fixed to the test table with three polyethylene blocks that were machined to have longitudinal holes. The pipe passed through these holes and was clamped in place by the matching tops of the polypropylene blocks. The blocks were attached to the aluminum base plate securely with two sided adhesive mounting tape. Great care was taken to ensure the heat pipe was aligned with the base plate so that an inclination measurement at the center of the plate would be equal that of the heat pipe. This was necessary because the heat pipe was then fully wrapped in two layers of Polyolefin closed cell foam type pipe insulation with an approximate thermal conductivity of $0.036 \frac{W}{mK}$.

2.2.5 Inclinator

The inclinometer used to determine the tilt angle of the heat pipe was manufactured by M-D Building Products. It had a display precision of 0.1° . The uncertainty was taken to be half of the smallest scale division. Therefore, the uncertainty was assumed to be $\pm 0.05^\circ$. Multiplying the distance between the evaporator and condenser by the sine of 0.05° , it can be shown that the maximum height difference is ± 0.31 mm. In the perpendicular axis this increases to ± 0.42 mm. In any case, the height difference is within the ± 0.5 mm maximum specified in [22].

2.2.6 Data Acquisition System

Data was collected by a Keithley Model 2700 data acquisition system with 6 ½ digit (22 bit) resolution. Thirty-six copper/constantan (type T) thermocouples were positioned along the heat pipe at various positions. The Keithley used an internally regulated reference in all temperature acquisition. This data was passed to a Pentium IV, 2.4 GHz PC where it was recorded to the hard drive. The analog to digital conversion error is estimated to be $\pm 0.8\%$.

2.2.7 Power System

Omega multi-meters were used to measure the applied current and voltage, the product of which is power. Model HHM32 was used to measure AC current with an uncertainty of 1.2% of reading, and model number HHM31 was used to measure AC voltage with an uncertainty of 1.0% of reading.

Power was regulated by a 240 V variable AC transformer while using the larger heater, and by two 120 V variable AC transformers in series for the low power heater. The larger 240 V power supply was necessary for the high power horizontal dry-out testing that required in excess of 120 V. Two 120 V variable AC transformers were put in series to allow greater control of the much smaller applied power required for inclination testing. A schematic representation of both the large and small heater configurations is provided in Fig. 2.3

Fig. 2.3 – Power system

2.3 Uncertainty Analysis

Experimental Uncertainty Analysis provides a method to quantify the expected uncertainty of a result from the errors that propagated through a set of measurements.

Before commencing any test plan or assembling a test apparatus, it is vital to perform a general uncertainty analysis. This general analysis will provide the information required to select appropriate sensors and equipment and can determine the validity of an experimental approach well before the test apparatus is ever built [23].

The approach to uncertainty analysis that follows is based on the ANSI/ASME Standard on Measurement Uncertainty [24].

2.3.1 General Uncertainty Analysis

General Uncertainty Analysis is used during the initial concept phase of an experiment. It is an inexpensive way of creating and testing several possible experimental methods to determine which one has the least error. Another valuable product of the analysis is that it can pinpoint the main sources of error so that the experimenter can focus time, energy and money on addressing the largest sources. This general uncertainty analysis procedure that follows is from [25].

To begin a general uncertainty analysis, one must start with the data reduction equation, r , which is a function of several measured variables, X_i .

$$r = r(X_1, X_2, \dots, X_J) \quad (2.1)$$

The uncertainty of the calculated results is given by,

$$U_r^2 = \left(\frac{\partial r}{\partial X_1}\right)^2 U_{X_1}^2 + \left(\frac{\partial r}{\partial X_2}\right)^2 U_{X_2}^2 + \dots + \left(\frac{\partial r}{\partial X_J}\right)^2 U_{X_J}^2 \quad (2.2)$$

where U_r is the absolute uncertainty of the result and U_{X_i} is the absolute uncertainty of a given variable. Dividing through by r^2 , the following non-dimensionalized form is obtained,

$$\frac{U_r^2}{r^2} = \left(\frac{X_1}{r} \frac{\partial r}{\partial X_1}\right)^2 \left(\frac{U_{X_1}}{X_1}\right)^2 + \left(\frac{X_2}{r} \frac{\partial r}{\partial X_2}\right)^2 \left(\frac{U_{X_2}}{X_2}\right)^2 + \dots + \left(\frac{X_J}{r} \frac{\partial r}{\partial X_J}\right)^2 \left(\frac{U_{X_J}}{X_J}\right)^2 \quad (2.3)$$

where $\frac{U_r}{r}$ and $\frac{U_{X_i}}{X_i}$ are the relative uncertainties of the absolute result and of the various variables.

The partial derivative term, is also referred to as the uncertainty magnification factor (UMF). This value, when larger than unity will result in the uncertainty from that variable increasing as it propagates through the data reduction equation. If the UMF is less than unity, then the uncertainty will decrease as it propagates through the data reduction equation. A specific UMF for the X_i variable is given by,

$$UMF_i = \frac{X_i}{r} \frac{\partial r}{\partial X_i} \quad (2.4)$$

Dividing equation (2.2) by U_r^2 , permits one to determine the uncertainty percentage contribution (UPC) of each of the variables in the overall uncertainty of the result from,

$$1 = \frac{\left(\frac{dr}{dX_1}\right)^2 U_{X_1}^2 + \left(\frac{dr}{dX_2}\right)^2 U_{X_2}^2 + \dots + \left(\frac{dr}{dX_J}\right)^2 U_{X_J}^2}{U_r^2} \quad (2.5)$$

as

$$UPC_i = \frac{\left(\frac{dr}{dX_1}\right)^2 U_{X_1}^2}{U_r^2} \times 100 = \frac{\left(\frac{X_1}{r} \frac{dr}{dX_1}\right)^2 \left(\frac{U_{X_1}}{X_1}\right)^2}{\left(\frac{U_r}{r}\right)^2} \times 100 \quad (2.6)$$

Following this method it is possible to quickly isolate the largest contributors to the overall error. This can save a great deal of time and money as it allows an experimenter to focus on the sources of error that will make the most impact on the overall uncertainty. This general method can also quickly show that certain experimental configurations may never give the required accuracy [26].

2.3.2 Input Power Uncertainty

To further clarify the derivation of the general uncertainty analysis a simple example follows. The example is the calculation of the uncertainty of the applied power from the product of a measured voltage, $\mathcal{V}$, and current, $\mathcal{I}$.

$$\mathcal{P} = \mathcal{V}\mathcal{I} \quad (2.7)$$

Thus, this equation is a function of two variables.

$$\mathcal{P} = \mathcal{P}(\mathcal{V}, \mathcal{I}) \quad (2.8)$$

The form of the uncertainty equation, equation (2.2), is

$$U_{\mathcal{P}}^2 = \left(\frac{\partial \mathcal{P}}{\partial \mathcal{V}}\right)^2 U_{\mathcal{V}}^2 + \left(\frac{\partial \mathcal{P}}{\partial \mathcal{I}}\right)^2 U_{\mathcal{I}}^2 \quad (2.9)$$

Dividing through by the power squared yields,

$$\frac{U_P^2}{P^2} = \left(\frac{V}{P} \frac{\partial P}{\partial V} \right)^2 \left(\frac{U_V}{V} \right)^2 + \left(\frac{I}{P} \frac{\partial P}{\partial I} \right)^2 \left(\frac{U_I}{I} \right)^2 \quad (2.10)$$

Performing the derivatives, the uncertainty magnification factor for both voltage and current is found to be equal to 1.

$$UMF_V = \frac{V}{P} \frac{\partial P}{\partial V} = \frac{V}{(VI)} \frac{\partial (VI)}{\partial V} = \frac{VI}{VI} = 1 \quad (2.11)$$

$$UMF_I = \frac{I}{P} \frac{\partial P}{\partial I} = \frac{I}{(VI)} \frac{\partial (VI)}{\partial I} = \frac{VI}{VI} = 1 \quad (2.12)$$

Therefore, equation (2.10) reduces to,

$$\frac{U_P^2}{P^2} = (1)^2 \left(\frac{U_V}{V} \right)^2 + (1)^2 \left(\frac{U_I}{I} \right)^2 \quad (2.13)$$

By taking the square root of both sides of the equation, the relative uncertainty in terms of a percentage of applied power can be expressed as the following.

$$\frac{U_P}{VI} = \sqrt{\left(\frac{U_V}{V} \right)^2 + \left(\frac{U_I}{I} \right)^2} \quad (2.14)$$

The result of this percent error calculation is presented in Fig. 2.4 and is it clear that the smaller heater has much lower percent uncertainty in its operational range up to 200 W. The large heater does not drop below 1% uncertainty until approximately 80 W of applied power.

Fig. 2.4 – Percent uncertainty

Multiplying equation (2.14) by the applied power, $\mathcal{P} = \mathcal{V}\mathcal{I}$, gives the absolute uncertainty as,

$$U_{\mathcal{P}} = (\mathcal{V}\mathcal{I}) \sqrt{\left(\frac{U_{\mathcal{V}}}{\mathcal{V}}\right)^2 + \left(\frac{U_{\mathcal{I}}}{\mathcal{I}}\right)^2} \quad (2.15)$$

This absolute error is presented in Fig. 2.5 and clearly shows that the large heater has more error than the small heater, but that the maximum error is still well under 2 W at a highest power levels examined.

Fig. 2.5 – Absolute uncertainty

Thus, based on these results it was concluded that the large heater was to be used only when the small heater's maximum capacity of approximately 200 W was insufficient to reach dry-out power.

2.3.3 Thermocouple Uncertainty

Temperature data was collected for a period of several days without any applied power at ambient conditions. This data was then reduced by dividing it into one minute segments. The standard deviation was calculated for each of these one minute segments. This was done to isolate the random variation of the thermocouples from the fluctuations in the ambient temperature. It was assumed for this analysis that the ambient temperature

would essentially remain constant over each one minute segment. The following figure presents the relative frequency of the standard deviation of the one minute segments.

Fig. 2.6 – Thermocouple standard deviation

The maximum standard deviation in the thermocouple data is less than 0.025°C.

2.3.4 Chiller Uncertainty

Temperature data was collected for one hour with no applied load and the chiller running at a stable 5°C. The data was then reduced into one minute blocks. The standard deviation of each block was determined and plotted in the following figure. The spread of standard deviation is slightly smaller for the chiller than for the thermocouples. This

indicates that the chiller had a temperature fluctuation smaller than that of the thermocouples. Also, it demonstrates that the chiller is capable of holding a constant temperature that fluctuates less than the random precision error of the thermocouples.

Fig. 2.7 – Compare chiller vs. thermocouple standard deviation

2.3.5 Ambient Temperature Uncertainty

The following figure presents the relative frequency of ambient temperature throughout all testing. Ambient temperature was always larger than 21°C and smaller than 26°C and is characterized by a random distribution such as that shown in Fig. 2.8.

Fig. 2.8 – Ambient temperature readings during all testing

2.4 Non-Condensable Gas

It has been less than 5 years since the ammonia in the heat pipe was degassed. In this time it is possible that the ammonia working fluid could have reacted with the stainless steel structure of the heat pipe and generated non-condensable gasses. The approach taken to determine whether non-condensable gasses were present was two-fold. Firstly, a theoretical analysis was done to determine a best estimate of the quantity of non-condensable gas that had formed in the pipe. The second method used for determining whether any non-condensable gasses were formed, was to place as many thermocouples as possible in the end of the condenser. This is where non-condensable gasses would build up forming a bubble. If a large enough bubble existed, it could be detected by a drop in temperature at the tip of the condenser.

Knowing that a buildup of non-condensable gas would have a significant impact on performance it was of interest to determine how a vapour bubble would diffuse back into the fluid. It was also planned to perform a theoretical analysis of the non-condensable gas supported bubble life in the arteries which would have required the solubility and diffusivity of nitrogen and hydrogen in ammonia. This was later deemed to be unnecessary since the following model indicated that the amount of non-condensable gasses generated is negligible and no significant amount of non-condensable gas was detected experimentally. A literature review of the methods for determining the diffusivity of interest is presented in Appendix A for future reference.

2.4.1 Non-Condensable Gas Model

As shown in [27], the quantity of non-condensable gas increases with time and it is possible to predict the amount of non-condensable gas formation [28]. Arrhenius' equation solves for the rate constant k_c as a function of activation energy E_a , the universal gas constant R_c , absolute temperature T and a pre-exponential factor, A_c . The value of A_c is 0.023 and the value of E_a is 70000, as presented in [29], for the combination of ammonia and stainless steel.

$$k_c = A_c e^{-E_a/R_c T} \quad (2.16)$$

Assuming a worst case scenario of 50°C ambient storage temperature for the last 5 years, the rate k_c can be solved for and multiplied by the elapsed time in seconds, t_s . Taking this value and multiplying by the mass of ammonia m_{NH3} , gives the mass of non-condensable gas as,

$$m_{NCG} = k_c t_s m_{NH3} \quad (2.17)$$

As both N_2 and H_2 are formed in this reaction, they both makeup a portion of the calculated mass of non-condensable gas. When ammonia dissociates it results in one nitrogen and three hydrogen atoms for a total of four atoms. Therefore, the molar mass, M , of the mixture of N_2 and H_2 is given as $\frac{1}{4}$ the molar mass of hydrogen plus $\frac{3}{4}$ the molar mass of nitrogen, or

$$M = \frac{1}{4} (M_{Hydrogen}) + \frac{3}{4} (M_{Nitrogen}) \quad (2.18)$$

This yields a calculated molar mass for the non-condensable gasses as 8.5 since the molar mass of hydrogen is 2 and that of nitrogen is 28. Dividing the mass, m_{NCG} , by the molar mass, M , gives the total number of moles of non-condensable gas, n . The ideal gas law can be used to calculate the volume of gas using the number of moles of gas n , the saturation pressure of ammonia P_{sat} at the absolute temperature T and the universal gas constant R_c as,

$$V_{gas} = \frac{nR_cT}{P_{sat}} \quad (2.19)$$

As given in [30], the saturation pressure of ammonia at 50°C is 2.0331 megapascals so the volume from equation (2.19) is 6.33 mm³ which is equivalent to 0.00633 mL. The length of a cylindrical bubble of non-condensable gas, L_{bubble} , that would form at the tip of the condenser with this given volume can be found using the volume of a cylinder expression,

$$L_{bubble} = \frac{V_{gas}}{\pi r_i^2} \quad (2.20)$$

which yields 0.0666 mm. Such a small quantity of gas would likely be undetectable by thermocouples placed on the exterior of the pipe.

2.5 Thermocouple Placement

A total of 36 thermocouples were used throughout testing. Of these, 33 were attached directly to the heat pipe.

The thermocouples attached to the evaporator were mounted along the central axis of the bottom saddle as seen in Fig. 2.9. This was done to allow proper mating of the heater with the top saddle. The exact position of these thermocouples is presented in the following table. Thermocouple TC_8 and TC_9 are mounted on each side of the pipe halfway between the top and bottom saddles.

Table 2 – Evaporator section thermocouple distances

Thermocouple placement	Linear distance in meters from	
	evaporator end	condenser end
TC_1	0.000	1.295
TC_2	0.010	1.285
TC_3	0.020	1.275
TC_4	0.030	1.265
TC_5	0.040	1.255
TC_6	0.072	1.223
TC_7	0.117	1.178
TC_8	0.140	1.155
TC_9	0.140	1.155
TC_{10}	0.162	1.133
TC_{11}	0.207	1.088
TC_{12}	0.252	1.043

Fig. 2.9 – Thermocouple placement (bottom view)

The thermocouples in the adiabatic section are all mounted on the top side of the heat pipe with the exception of TC_{13} and TC_{18} which are mounted directly below TC_{14} and TC_{19} , respectively. Thermocouple TC_{16} is mounted off center to allow space for a support bracket which attaches at the straight section in the middle of the adiabatic section. The following table presents the linear distances of the thermocouples from the tip of the evaporator and condenser.

Table 3 – Adiabatic section thermocouple distances

Thermocouple placement	Linear distance in meters from	
	evaporator end	condenser end
TC_{13}	0.290	1.005
TC_{14}	0.290	1.005
TC_{15}	0.498	0.797
TC_{16}	0.616	0.679
TC_{17}	0.797	0.498
TC_{18}	1.005	0.290
TC_{19}	1.005	0.290

The thermocouples mounted to the condenser were mounted along the central axis of the top saddle as seen in Fig. 2.10. This was done to allow proper mating of the heat exchanger to the bottom saddle. The exact position of these thermocouples is presented in the following table. Thermocouple TC_{23} and TC_{24} are mounted on each side of the pipe halfway between the top and bottom saddles and are visible in Fig. 2.11.

Table 4 – Condenser section thermocouple distances

Thermocouple placement	Linear distance in meters from	
	evaporator end	condenser end
TC_{20}	1.043	0.252
TC_{21}	1.088	0.207
TC_{22}	1.133	0.162
TC_{23}	1.155	0.140
TC_{24}	1.155	0.140
TC_{25}	1.178	0.117
TC_{26}	1.200	0.095
TC_{27}	1.235	0.060
TC_{28}	1.245	0.050
TC_{29}	1.255	0.040
TC_{30}	1.265	0.030
TC_{31}	1.275	0.020
TC_{32}	1.285	0.010
TC_{33}	1.295	0.000

Fig. 2.10 – Thermocouple placement (top view)

Fig. 2.11 – Thermocouple placement (side view)

2.6 Heat Leak

If the test assembly was perfectly insulated, then it would follow that all the input heat would be removed via the cooling system. In reality, some of the applied heat escapes into the environment. This phenomenon is referred to as heat leak and it can be characterized through,

$$\dot{Q}_{in} = \dot{Q}_{leak} + \dot{Q}_{out} \quad (2.21)$$

Knowing the input heat from the input power, if it were possible to calculate the heat removed from the system, the heat leak could be established. In initial testing an approach was examined that would measure the temperature of the fluid entering and leaving the heat exchanger directly. Using the following relation it is theoretically possible to determine the heat that the fluid removed from the heat pipe from the mass flow rate $\dot{m}$, the coolant's specific heat c_p , and the temperature difference of the warmer fluid exiting to the cold fluid entering the heat exchanger. Thus,

$$\dot{Q}_{out} = \dot{m}c_p(T_{out} - T_{in}) \quad (2.22)$$

It was observed during initial testing that the temperature differences were very small which lead to uncertainty in excess of 100%. This method was shown to be insufficient for quantifying the amount of heat escaping to the environment. Another approach was selected. Knowing that a small fraction of the heat input to the evaporator would leak through the insulation and warm the surrounding air, it was decided to model the natural convection occurring around the evaporator. The evaporator covered with insulation was

modeled as a horizontal cylinder. Two thermocouples, TC_{34} and TC_{35} , were placed on top of each end of the evaporator outside of the insulation. The recorded temperatures from these two thermocouples were used in the following model to determine the approximate amount of parasitic heat loss.

2.6.1 Free Convection from Horizontal Cylinders

The convective heat transfer coefficient is a value that is generally determined experimentally. There exist several empirical correlations that can be used to solve for this value. Five of these correlations were analyzed to determine which one was the most applicable to the tests that were conducted:

For natural convection, the convention is to use a film temperature. This is the average between the surface T_s and ambient temperature T_∞ as given by,

$$T_f = \frac{T_s - T_\infty}{2} \quad (2.23)$$

Another parameter used in this analysis is the volume coefficient of expansion β_f . For an ideal gas this is simply the inverse of temperature given as,

$$\beta_f = \frac{1}{T_f} \quad (2.24)$$

The Grashof number is a dimensionless parameter that is also used in natural convection analysis. It physically represents a ratio of the buoyancy to the viscous forces [31] as,

$$Gr = \frac{g\beta_f(T_s - T_\infty)D^3}{\nu^2} \quad (2.25)$$

where the kinematic viscosity ν is a function of temperature and is the ratio of the dynamic viscosity μ divided by the density ρ .

$$\nu = \frac{\mu}{\rho} \quad (2.26)$$

Gravitational acceleration g and the outer diameter of the cylinder D are also needed to find the Grashof number. The Prandtl number is a dimensionless parameter that physically represents the ratio between kinematic viscosity ν and thermal diffusivity α_d . This can be further simplified to the product of the specific heat c_p and dynamic viscosity μ divided by the thermal conductivity k , or

$$Pr = \frac{\nu}{\alpha_d} = \frac{c_p \mu}{k} \quad (2.27)$$

For the purposes of this analysis, second order polynomial fits were found based on tabulated property data in [31].

Table 5 – Properties of air at atmospheric pressure

Temperature	Kinematic Viscosity	Prandtl Number	Thermal Conductivity
T_f	$\nu \times 10^6$	Pr	k
K	m^2/s		W/mK
100	1.923	0.77	0.009246
150	4.343	0.753	0.013735
200	7.49	0.739	0.01809
250	11.31	0.722	0.02227
300	15.69	0.708	0.02624
350	20.76	0.697	0.03003
400	25.9	0.689	0.03365

This yielded the following equations:

$$\nu = (a_0T^2 + a_1T + a_2)10^6 \quad (2.28)$$

$$Pr = a_0T^2 + a_1T + a_2 \quad (2.29)$$

$$k = a_0T^2 + a_1T + a_2 \quad (2.30)$$

where the constants a_0 , a_1 and a_2 are given below in Table 6.

Table 6 – Parameters used in property equations

	a_0	a_1	a_2	Maximum Error (in temperature range used)
ν	1.1588095E-04	2.2748810E-02	-1.6005714E+00	0.978%
Pr	3.1429000E-07	-4.3285714E-04	8.1085714E-01	0.182%
k	-3.6140000E-08	9.9465710E-05	-3.5164286E-04	0.017%

The Nusselt number is a dimensionless value denoting the ratio of convection to conduction heat transfer [32]. It can be determined using empirical correlations based on the product of the Grashof and Prandtl numbers, which is also known as the Rayleigh number, or

$$Ra = GrPr \quad (2.31)$$

For the range of Raleigh numbers considered here, there exist five applicable empirical formulas for horizontal cylinders that could be used to calculate the average Nusselt number. The first three were in the following form,

$$\overline{Nu} = C_c(GrPr)^{m_c} \quad (2.32)$$

with C_c and m_c as constants given in Table 7.

Table 7 – Coefficients used to calculate the average Nusselt number [31]

					Applicable range of Rayleigh number
Formula 1	C_{c1}	0.53	m_{c1}	0.25	$10^4 - 10^9$
Formula 2	C_{c2}	0.13	m_{c2}	0.33	$10^9 - 10^{12}$
Formula 3	C_{c3}	0.48	m_{c3}	0.25	$10^4 - 10^7$

A more complex correlation is given by Churchill and Chu[33] as,

$$\overline{Nu}^{1/2} = 0.60 + 0.387 \left\{ \frac{GrPr}{[1 + (0.559/Pr)^{9/16}]^{16/9}} \right\}^{1/6} \quad (2.33)$$

This expression is valid for a range of Rayleigh numbers from 10^{-5} to 10^{12} . Finally, a simpler correlation from [33] that is only valid for Rayleigh numbers from 10^{-6} to 10^9 is,

$$\overline{Nu} = 0.36 + \frac{0.518(GrPr)^{1/4}}{[1 + (0.559/Pr)^{9/16}]^{4/9}} \quad (2.34)$$

Once a Nusselt number is obtained, it is possible to determine the average heat transfer coefficient $\bar{h}$ as,

$$\bar{h} = \overline{Nu} \left(\frac{k}{D} \right) \quad (2.35)$$

We have evaluated an average heat transfer coefficient for each of the five correlations considered. Thus combining equation (2.32), the coefficients in Table 7 and equation (2.35) it is possible to solve for h_1 , h_2 and h_3 . Combining equation (2.33) and (2.35) yields h_4 . Similarly combining equation (2.34) with (2.35) yields h_5 .

The following figure gives the calculated heat transfer coefficients at the maximum applied power seen in testing. This represents the highest possible heat transfer coefficient seen during testing.

Fig. 2.12 – Heat transfer coefficient

The fourth relation, given by equation (2.33), was chosen as it yielded a heat transfer coefficient that was approximately the mean of the other values, and was applicable across the largest spectrum of Rayleigh numbers.

2.6.2 Total Heat Leak

The total heat leak is the sum of the heat lost due to natural convection and the heat lost via radiation, or

$$\dot{Q}_{total} = \dot{Q}_{convection} + \dot{Q}_{radiation} \quad (2.36)$$

With the average natural convection heat transfer coefficient, $\bar{h}$, it is possible to determine the heat lost with Newton's law of cooling, or

$$\dot{Q}_{convection} = \bar{h}F_s(T_s - T_\infty) \quad (2.37)$$

The radiation emitted from the evaporator section insulation was investigated to determine if it was a significant part of the total parasitic heat loss. The equation for radiation heat transfer from a convex surface to a large enclosure is given by,

$$\dot{Q}_{radiation} = \sigma_c \epsilon_s F_s (T_s^4 - T_\infty^4) \quad (2.38)$$

This equation includes σ_c , called the Stefan-Boltzmann constant with a value of $5.669 \times 10^{-8} \frac{W}{m^2 K^4}$. The emissivity, ϵ_s , is physically the ratio of emissive power to that of an ideal black body and was estimated to be 0.8. In this case, the convex surface, F_s , is modeled as the outer surface of the insulation wrapping the evaporator section of the heat pipe at temperature T_s . The large enclosure is the room in which the tests were performed at the temperature T_∞ and for the purposes of this analysis was assumed to be an ideal black

body. The value of F_s was set to be $\frac{2}{3}$ of the cylindrical insulation surrounding the evaporator section. This was to take into account that the heater and TC₃₄ and TC₃₅ were located on the top side of the pipe. Parasitic heat loss was calculated to never exceed 1% of the applied heat load. This is shown in the following figure where the applied power was brought up to the maximum power used in testing via steps but the heat loss did not exceed 0.9%.

Fig. 2.13 – Percent heat leak

It is interesting to note that some heat is absorbed by the insulation from the environment while the pipe is colder than ambient temperature. This occurred with applied powers lower than 50 W. Fig. 2.13 only shows the percent of heat lost; as such it does not

display the heat that is added to the pipe via convection when the temperature is below ambient. It was assumed that this very small heat addition could be neglected.

2.6.3 Insulation Critical Thickness Calculation

It is possible that, by adding insulation, the area for convective heat transfer could increase proportionally more than the reduced conductive heat transfer. It is possible to find the critical thickness of the insulation where if the outer radius of the insulation is smaller than the critical thickness, adding insulation would serve to increase rather than decrease the heat transfer from the pipe. Using the resistance network method [31], it is possible to define an equation for the heat transfer Q , as a function of the internal and external radii and temperatures, length, thermal conductivity of the insulation and the convective heat transfer coefficient as,

$$Q = \frac{2\pi L(T_i - T_\infty)}{\frac{\ln(r_o/r_i)}{k} + \frac{1}{r_o h}} \quad (2.39)$$

Taking the derivative with respect to the outer radius and setting it to zero it is possible to find the maximum as,

$$\frac{dQ}{dr_o} = 0 = \frac{-2\pi L(T_i - T_\infty) \left(\frac{1}{kr_o} + \frac{1}{hr_o^2} \right)}{\left[\frac{\ln(r_o/r_i)}{k} + \frac{1}{r_o h} \right]^2} \quad (2.40)$$

which simplifies to,

$$r_0 = \frac{k}{h} \quad (2.41)$$

The thermal conductivity of the insulation is a property of the material. For the type of Armaflex pipe insulation used, the thermal conductivity is approximately $0.036 \frac{W}{mK}$.

Taking the maximum heat transfer coefficient seen in Fig. 2.12 as approximately 4, the outer radius is calculated to be 9 mm. Therefore, as long as the insulation outer radius is larger than 9 mm adding more insulation will reduce the overall heat transfer to the pipe.

The pipe outer radius is 6 mm before addition of any insulation. This analysis shows that the first 3 mm of insulation actually enhance the convective heat transfer to the environment. This problem was overcome by adding a minimum of two layers of insulation with each layer being 10 mm thick. This gives a minimum of 26 mm outer radius for the insulation which is much more than the critical value.

Chapter 3 Experimental Results

3.1 Introduction to Experimental Results

Very little performance data is available in the open literature about the arterial heat pipe we tested. Existing work that is directly related to our research is limited to [17] ,[19] and [21].

This chapter presents the extensive tests that were performed to better characterize the performance of our arterial heat pipe. The operating envelope was probed with the aim of determining the test articles performance limits including, maximum dry-out power at horizontal as well as adverse elevations. Further, tests consisting of a step change in applied power were explored with the aim of assessing whether application of a large

fraction of dry-out power would initiate nucleate boiling. Nucleate boiling is undesirable because it can cause a vapour blockage in the artery and result in degradation of performance. The final set of tests examines the heat pipes response to a step change decrease in condenser temperature with a constant applied power.

3.2 Horizontal Dry-out Testing

Fig. 3.1 – Horizontal dry-out test

In Fig. 3.1 the results of a typical horizontal dry-out test are given. The temperatures presented are the average section temperatures of the evaporator (T_{ev}), adiabatic section (T_{ad}) and condenser (T_{co}) as given by the following expressions

$$T_{ev} = \frac{1}{10} (TC_1 + TC_2 + \dots + TC_6 + TC_7 + TC_{10} + TC_{11} + TC_{12}) \quad (3.1)$$

$$T_{ad} = \frac{1}{3} (TC_{15} + TC_{16} + TC_{17}) \quad (3.2)$$

and

$$T_{co} = \frac{1}{12}(TC_{20} + TC_{21} + TC_{22} + TC_{25} + TC_{26} + \dots + TC_{32} + TC_{33}) \quad (3.3)$$

Temperatures are plotted as a function of time in seconds. The sink temperature was maintained at 5°C for the duration of the test. Ambient temperature (T_{∞}) was measured by thermocouple TC_{36} attached to the base of the test stand. The base acted as a thermal mass to reduce fluctuations in the ambient temperature measurement. Before each increase in power, temperatures were allowed to stabilize until no temperature varied by more than 0.1°C for 600 seconds. Power was then applied in 50 W increments until dry-out was observed. At 350 W a sudden increase in the evaporator temperature is a sign of dry-out. Power is plotted as the input power before correction for parasitic heat loss. Since parasitic heat loss is smaller than 1% in all cases, its inclusion in the figures would only serve to make them less clear. There are several criteria that are widely used to denote dry-out. Method 1 uses the temperature difference across the evaporator to define dry-out, as is proposed in [34], as

$$Method_1 = TC_1 - TC_{12} \quad (3.4)$$

Method 2 uses the difference between the mean evaporator temperature and the mean adiabatic section temperature as the definition of dry-out and is used by the European Space Agency [35]. Method 2 yields the expression

$$Method_2 = T_{ev} - T_{ad} \quad (3.5)$$

Method 2 has a secondary criterion, useful especially for low power testing, where dry-out is defined to be one of two limits, whichever occurs first. Either when the temperature difference across the evaporator exceeds 5°C or when evaporator equivalent resistance calculated by the following equation

$$\mathcal{R}_{ev} = \frac{TC_1 - TC_{12}}{Q_{applied}} \quad (3.6)$$

starts to increase. Corrected power is used for all calculations of resistance. As power is increased, the heat pipe evaporator resistance should decrease until it reaches an optimum operating condition. After reaching this optimum operating condition with minimum resistance, when the applied power is increased the resistance will go up. When this begins to occur it indicates that the end of the evaporator is no longer being replenished with liquid and is effectively experiencing dry-out.

Fig. 3.2 – Method 1 for determining dry-out

Fig. 3.3 – Method 2 for determining dry-out

Fig. 3.4 – Evaporator resistance method

In all three plots it is clear that dry-out occurs once power is increased to 350 W.

In addition to this initial test, two more dry-out tests were performed at zero elevation.

This was done to show repeatability in the results and to better locate the exact point of dry-out. Subsequent tests followed the same pattern as the initial dry-out test using 50 W increments up to the 300 W power level achieved before dry-out. The power was then added in steps of 10 W until dry-out was observed.

Fig. 3.5 – Follow-up dry-out test at horizontal

Dry-out was determined to occur at 350 W in both follow-up tests. This was confirmed using all available criteria. The evaporator resistance is given in the following figure.

Fig. 3.6 – Evaporator resistance method (follow-up test)

It is clear that the evaporator resistance increases rapidly after the power is increased to 350 W. This clearly demonstrates that dry-out occurs at 350 W.

This dry-out power compares well with that found in [19] where the maximum dry-out power at horizontal was found to be between 360 W and 370 W. The discrepancy in power can be attributed to the added thermal resistance of having the heat exchanger mounted below the condenser in this series of tests.

3.3 Small Elevation Dry-out Testing

In an arterial heat pipe it is possible that at horizontal and at very small elevations, the liquid in the artery could penetrate into the vapour space. This phenomenon can cause a puddle to form [17] and it can lead to a change in operational capability. It is therefore desirable to determine the performance of the heat pipe when it does not have a puddle present and when liquid is still being provided to the evaporator.

Equating the maximum capillary pressure to the gravitational pressure drop gives the following equation

$$\frac{2\sigma}{r_{vent}} = L_w \rho_l g \sin(\psi) \quad (3.7)$$

Solving for the tilt angle (ψ) we get

$$\psi = \sin^{-1} \left(\frac{2\sigma}{r_{vent} L_w \rho_l g} \right) \quad (3.8)$$

At dry-out in Fig. 3.1 and Fig. 3.5 T_{ad} is approximately 50°C. Evaluating the surface tension (σ) and liquid density (ρ_l) using T_{ad} as the heat pipe saturation temperature, from the relations given in [36], we can solve for a tilt angle of 0.9°. This means that the maximum capillary pressure of the heat pipe is exactly balanced by the adverse hydrostatic pressure created when the pipe is tilted by just 0.9°. This analysis is simplified since, in reality, the liquid would be moving during operation and there would be non-trivial contributions to the pressure drop from the vapour flow and bends in the

pipe. It was therefore decided that an angle of 0.5° would be a good compromise between the situation of driving the liquid out of the artery at too high of an angle and of allowing a puddle to form at too low of an angle. Typical tilt orientation is given in Fig. 3.7 with the evaporator side higher than the condenser side.

Fig. 3.7 – Tilt

Dry-out testing was performed at a tilt angle of 0.5° to the horizontal. This resulted in an elevation of 3.14 mm so that the heat pipe evaporator was 3.14 mm higher than the condenser. At this adverse elevation it was expected that the heat pipe performance would be reduced. This is in fact seen in the following figure, where the maximum dry-out power observed during testing was 180 W or approximately one-half the power obtained when the heat pipe was horizontal.

Fig. 3.8 – 3.14 mm elevation dry-out (0.5° adverse tilt)

This is nearly a 50% reduction in maximum dry-out power from the tests performed at horizontal. It is clear that the arterial heat pipe's performance degrades rapidly when subjected to adverse pressure gradients. The following figure clearly shows that the pipe experiences a large increase in evaporator resistance at 180 W of applied power.

Fig. 3.9 – Evaporator resistance (0.5° adverse tilt)

3.4 Comparison Between Large and Small Heaters

As was shown in the last chapter the uncertainty of the large 1000 W heater shown in Fig. 2.4 and Fig. 2.5 is very high at low power settings. It is for this reason that a smaller 200 W heater was used for all tests at 6.28 mm elevation and above where dry-out occurs at powers of 50 W or less. At the 6.28 mm test elevation, both heaters were used to evaluate how well the collected data correlated between the two heaters.

Fig. 3.10 – Large vs. small heater results

Two tests using the large heater were compared with three tests using the small heater from the beginning of the test to the power level before dry-out. The largest root mean square difference in temperature was found to be 0.85°C in TC₃₆. This thermocouple measured the ambient temperature. Ignoring the ambient temperature fluctuations, the

largest root mean square difference in temperature in Fig. 3.10 was found in TC₇ and was found to be 0.54°C. These results indicate that there was excellent consistency and uniformity in the data generated while using both the large and small heaters.

It is clear from Fig. 3.11 that dry-out occurs at 50 W. This figure presents the resistance calculated from the small heater test up to 30 W and it then plots the values calculated from the large heater test from 30 W to dry-out. The overlapping values for resistance at 30 W were within 3.3% difference on average.

Fig. 3.11 – 6.28 mm elevation resistance (1.0° adverse tilt)

3.5 Larger Elevation Testing

Using the small heater, dry-out testing was performed at the elevations summarized in Table 8.

Table 8 – Elevation testing

Angle	Elevation
°	mm
1.5	9.42
2.0	12.56
2.5	15.70
3.0	18.84
3.5	21.98
4.0	25.11

Evaporator resistance was used to judge when dry-out occurred for these tests. It was clear in all of the following tests that the evaporator resistance decreases with increasing applied power that it reached a minimum value and then increased rapidly with the addition of more power. The power at which the evaporator reached a minimum resistance was the maximum operating power for a given adverse elevation and the power at which the resistance increased was the dry-out power. A summary of all elevation tests is provided in the following figures.

In all tests the heat pipe temperature was allowed to come to equilibrium for a minimum of 600 seconds before any power was applied. Power was applied in even increments and the pipe was allowed to come to equilibrium before advancing to the next power level. Ambient temperatures fluctuated throughout all tests and from day to day from 21°C to 26°C.

Dry-out behaviour in Fig. 3.12, Fig. 3.14 and Fig. 3.16 exhibit the same pattern as previous tests. The general shape of the graphs is similar, with smaller temperature corresponding to smaller applied powers.

In Fig. 3.13, Fig. 3.15 and Fig. 3.17 the evaporator resistance clearly decreases with each increase in applied power until it reaches a minimum, then increases with each increase in power.

Even though the applied power is quite small, the dry-out behaviour in Fig. 3.18, Fig. 3.20 and Fig. 3.22 remains consistent with previous tests. The evaporator, adiabatic and condenser temperatures all increase and then stabilize in the same manner as the higher power tests.

The evaporator resistances plotted in Fig. 3.19, Fig. 3.21 and Fig. 3.23 confirm that even at the highest elevations examined, the heat pipe is operational up to a point. The evaporator resistance being stable, then decreasing and stabilizing with an increase in the application of power clearly shows that the pipe is still pumping fluid into the evaporator. The heat pipe was found to dry-out beyond 1 W of applied power as given in Fig. 3.23. Testing was not continued beyond 4.0 degree tilt corresponding to 25.11 mm.

Fig. 3.12 – 9.42 mm elevation dry-out test (1.5° adverse tilt)

Fig. 3.13 – 9.42 mm elevation resistance (1.5° adverse tilt)

Fig. 3.14 – 12.56 mm elevation dry-out test (2.0° adverse tilt)

Fig. 3.15 – 12.56 mm elevation resistance (2.0° adverse tilt)

Fig. 3.16 – 15.70 mm elevation dry-out test (2.5° adverse tilt)

Fig. 3.17 – 15.70 mm elevation resistance (2.5° adverse tilt)

Fig. 3.18 – 18.84 mm elevation dry-out test (3.0° adverse tilt)

Fig. 3.19 – 18.84 mm elevation resistance (3.0° adverse tilt)

Fig. 3.20 – 21.98 mm elevation dry-out test (3.5° adverse tilt)

Fig. 3.21 – 21.98 mm elevation resistance (3.5° adverse tilt)

Fig. 3.22 – 25.11 mm elevation dry-out test (4.0° adverse tilt)

Fig. 3.23 – 25.11 mm elevation resistance (4.0° adverse tilt)

3.6 Overall Dry-out Performance

After performing a complete set of dry-out tests at various adverse elevations and power levels, it is useful to combine all the test data together to summarize the overall dry-out performance of the heat pipe. The following table summarizes the dry-out power at all the adverse elevations tested.

Table 9 – Dry-out summary

Angle	Elevation	Max Operating Power	Dry-out Power
deg	mm	W	W
0.0	0.00	340	350
0.5	3.14	170	180
1.0	6.28	40	50
1.5	9.42	35	40
2.0	12.56	25	30
2.5	15.70	20	25
3.0	18.84	4	6
3.5	21.98	2	3
4.0	25.11	1	1.5

Dry-out power vs. elevation is presented in Fig. 3.24 and indicates that there is an exponential decrease in maximum achievable dry-out power with an increase in adverse elevation.

Fig. 3.24 – Dry-out power vs. elevation

It is also possible to plot the minimum operating resistance against the dry-out power as shown in Fig. 3.25. The clear trend in this figure is linear when plotted using a logarithmic scale on both axes. It would appear from this figure that the minimum operating evaporator resistance is achieved at applied power levels just below maximum dry-out power.

Fig. 3.25 – Evaporator minimum operating resistance vs. applied power

3.7 Step Change Tests

Tests were performed to determine if the heat pipe was susceptible to vapour blockages in the artery when a large percentage of dry-out power was applied instantaneously.

These were done at both horizontal as well as 3.14 mm elevations at power levels from 80% to 90% of dry-out power. As can be seen in the following figures, instantaneous application of 80% to 90% of dry-out power did not result in any adverse performance effects.

3.7.1 Horizontal Step Change in Power

Fig. 3.26 – Step change in applied power

To ensure the temperature in the heat pipe is stable a delay of 600 seconds was used before 300 W of power was applied as a step change. The heat pipe was allowed to stabilize in temperature before shutting off the power and allowing the temperatures to fall back to their original values. With a dry-out power in the range of 350 W, this test clearly demonstrates that it is possible to instantaneously apply over 80% of the pipe's dry-out power without encountering any reduction in performance.

3.7.2 Adverse Tilt Step Change in Power

This test was repeated at a 3.14 mm elevation to determine if the pipe would encounter any difficulty managing over 80% of dry-out power at this reduced performance orientation.

Fig. 3.27 – Step change in applied power with adverse tilt

The test showed that instantaneously applying a power of 150 W, representing over 80% of dry-out power, at this reduced performance regime was still possible. This clearly demonstrates that the ArHP is capable of sustaining sudden power application in excess of 80% of dry-out power at a given adverse elevation without experiencing any performance degradation.

3.8 Drop Chiller Temperature Tests

Tests were performed to determine what would occur if, while at a load of 80% of dry-out power, the condenser temperature was reduced. To perform these tests, the chiller was set to 20°C and the heat pipe was allowed to stabilize for 600 seconds. Power, amounting to 80% of dry-out, was then applied and the pipe was again allowed to stabilize. Once stable for more than 600 seconds the chiller was set to 5°C while the applied power remained constant. A plot of temperature vs. time as shown in Fig. 3.28 does not appear to show any unusual results but if the first six thermocouples are plotted, as provided in Fig. 3.29 rather than the average of all evaporator measurements, something more interesting can be seen. The temperature at the tip of the evaporator increases just after the condenser temperature is decreased.

The reason the temperature increases in the tip of the evaporator is that as the condenser side cools the saturation temperature of the pipe drops. Simultaneously, the evaporator side still has walls that are hot. It takes some time to cool the walls given that they have some thermal mass. Thus, the difference between the wall temperature and the saturation temperature of the ammonia in the pipe increases. This leads to some superheat. The amount of superheat causing nucleation is stochastic. Therefore, in some cases, a sudden boiling was observed, but in other cases, there was no boiling, and no temperature overshoot as plotted in Fig. 3.29. The behaviour shown in this figure was only observed three times in fifteen total tests.

Fig. 3.28 – Chiller drop temperature test overall

Fig. 3.29 – Chiller drop temperature (first six thermocouples)

3.9 Summary of Results

The arterial heat pipe has been tested under different configurations. Dry-out powers at various adverse elevations were found. It was shown (Fig. 3.24) that the dry-out power vs. elevation relationship is exponential in nature. It was also shown (Fig. 3.25) that the relationship between the minimum operating evaporator resistance and applied power is apparently linear when plotted on a log-log graph.

The pipes response to a step change in power larger than 80% of dry-out was shown to be stable and did not demonstrate any degradation in performance. The heat pipe did not de-prime even when this test was done with adverse tilt.

It was shown in the last series of tests, that if the sink temperature was decreased too quickly, while a significant load is being applied to the evaporator, nucleate boiling may occur in the tip of the evaporator. In all cases observed, this nucleation and subsequent increase in temperature was a transient phenomenon that subsided within about 10 minutes when all temperatures converged back to the evaporator average values.

Overall, this arterial heat pipe exhibits reliable, repeatable performance across all tests with a maximum stable operating power at horizontal orientation of 340 W.

Chapter 4 Numerical Model

4.1 Introduction

It is desirable to create a numerical model to predict the performance of the arterial heat pipe. With a robust numerical model, it is possible to run parametric studies to determine the optimum layout for a heat pipe for any given application. Previous models mostly relied on the methods introduced in [20]. In this chapter this approach is first presented and then an improved model is presented. The details of this model are explained, the most important feature of which is that it can take into account excess fluid charge.

The key assumptions of both models are that the heat pipe is limited by the available capillary pumping pressure, the flow of liquid and vapour are laminar and that the liquid and vapour regions do not interact.

A function to calculate property data was derived from polynomial fit equations for ammonia provided in the appendix of [36].

4.2 Chi Model

The book by Chi [20], is a standard reference in the heat pipe industry. The basic theory behind the Chi model relies on the fact that most heat pipes are limited in their heat transport capacity by the available capillary pressure. This is known as the capillary limit. To find the maximum applied power that will result in the capillary limit being reached, all pressure drops along the heat pipe are equated to the capillary limit.

4.2.1 Maximum Capillary Pressure

The maximum capillary pressure P_c is given by the Laplace and Young equation where R_1 and R_2 are the mutually perpendicular radii of curvature of the meniscus and σ is the surface tension of the fluid,

$$P_c = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (4.1)$$

At a circular section, such as a vapour vent hole, the radii will be equal and thus the maximum capillary pressure can be written as,

$$P_c = \frac{2\sigma}{r_{vent}} \quad (4.2)$$

where the value r_{vent} is the radius of the circular vapour vent hole.

Operation in a gravitational field will reduce the performance of the heat pipe by an amount equal to the perpendicular pressure drop $P_\perp$,

$$P_\perp = \rho_l g 2r_{hv} \cos(\psi) \quad (4.3)$$

where ρ_l is the liquid density, r_{hv} is the hydraulic radius of the vapour space and ψ is the tilt angle.

4.2.2 Liquid Pressure Drop

For laminar flow the longitudinal liquid pressure gradient $\frac{dP_l}{dz}$ can be shown to be,

$$\frac{dP_l}{dz} = -\frac{2\tau_l}{r_{hl}} \pm \rho_l g \sin(\psi) \quad (4.4)$$

where τ_l is the liquid shear stress and r_{hl} is the liquid artery hydraulic radius.

Critical dimensionless parameters are the Reynolds number, Re_l , and the friction factor, f_l , given as

$$Re_l = \frac{2r_{hl}\rho_l\bar{u}_l}{\mu_l} \quad (4.5)$$

$$f_l = \frac{2\tau_l}{\rho_l\bar{u}_l^2} \quad (4.6)$$

When these two values are combined, they form another dimensionless parameter known as the Poiseuille number Po_l ,

$$Po_l = Re_l f_l \quad (4.7)$$

The mean liquid velocity $\bar{w}_l$ can be shown to be,

$$\bar{w}_l = \frac{Q}{F_l \rho_l h_{fg}} \quad (4.8)$$

where Q is the applied heat load, F_l is the area of the liquid artery and h_{fg} is the latent heat of vapourization of the working fluid. Re-arranging equations (4.4), (4.5), (4.6), (4.7) and (4.8) one can obtain,

$$\frac{dP_l}{dz} = -\frac{Po_l \mu_l}{2F_l \rho_l h_{fg} r_{hl}^2} Q \pm \rho_l g \sin(\psi) \quad (4.9)$$

4.2.3 Vapor Pressure Drop

For laminar flow the vapour pressure gradient $\frac{dP_v}{dz}$ can be shown to be a combination of the first viscous term and the second inertial term as follows,

$$\frac{dP_v}{dz} = -\frac{(f_v Re_v) \mu_v \dot{m}_v}{2F_v \rho_v r_{hv}^2} - \frac{8}{3F_v^2 \rho_v} \frac{d\dot{m}_v^2}{dz} \quad (4.10)$$

Mass flow rate of vapour can be written as follows,

$$\dot{m}_v = \frac{Q}{h_{fg}} \quad (4.11)$$

Noting that the relationships noted in (4.5), (4.6), (4.7) and (4.8) hold true for vapour and combining with (4.10) and (4.11), the following equation can be derived,

$$\frac{dP_v}{dz} = -\frac{Po_v\mu_v}{2F_v\rho_v h_{fg} r_{hv}^2} Q - \frac{4}{3F_v^2\rho_v h_{fg}^2} \frac{dQ^2}{dz} \quad (4.12)$$

4.2.4 Maximum Heat Transfer

From [20], it can be shown that when the effective maximum capillary pressure is set to equal the liquid and vapour pressure drops along the heat pipe it is possible to solve for the maximum dry-out power.

$$P_c - P_\perp = \int_0^{L_t} \left(\frac{dP_v}{dz} - \frac{dP_l}{dz} \right) dz \quad (4.13)$$

Substituting equations (4.2), (4.3), (4.9) and (4.12) one can obtain,

$$\begin{aligned} & \frac{2\sigma}{r_{vent}} - \rho_l g 2r_{hv} \cos(\psi) \\ &= \int_0^{L_t} \left(\frac{Po_v\mu_v}{2F_v\rho_v h_{fg} r_{hv}^2} Q - \frac{4}{3F_v^2\rho_v h_{fg}^2} \frac{dQ^2}{dz} \right. \\ & \quad \left. + \frac{Po_l\mu_l}{2F_l\rho_l h_{fg} r_{hl}^2} Q + \rho_l g \sin(\psi) \right) dz \end{aligned} \quad (4.14)$$

Knowing that the pipe is fully insulated, and the ends of the pipe are set to have no heat transfer it is possible to eliminate the following term from equation (4.14).

$$\begin{aligned} & \int_0^{L_t} \frac{4}{3F_v^2\rho_v h_{fg}^2} \frac{dQ^2}{dz} \\ &= \frac{8}{3F_v^2\rho_v h_{fg}^2} \int_{Q(0)}^{Q(L_t)} Q dQ \\ &= \frac{4}{3F_v^2\rho_v h_{fg}^2} (Q^2(L_t) - Q^2(0)) \\ &= 0 \end{aligned} \quad (4.15)$$

Therefore the maximum heat transport factor can be found to be,

$$(QL_{eff})_{max} = \frac{\frac{2\sigma}{r_{vent}} - \rho_l g 2r_{hv} \cos(\psi) - \rho_l g \sin(\psi)}{\frac{Po_v \mu_v}{2F_v \rho_v h_{fg} r_{hv}^2} + \frac{Po_l \mu_l}{2F_l \rho_l h_{fg} r_{hl}^2}} \quad (4.16)$$

which can also be written as,

$$(QL_{eff})_{max} = \int_0^{L_t} Q dz = Q_{max} (0.5L_{ev} + L_{ad} + 0.5L_{co}) \quad (4.17)$$

Therefore the capillary limited maximum heat transfer is,

$$Q_{max} = \frac{\frac{2\sigma}{r_{vent}} - \rho_l g 2r_{hv} \cos(\psi) - \rho_l g \sin(\psi)}{\left(\frac{Po_v \mu_v}{2F_v \rho_v h_{fg} r_{hv}^2} + \frac{Po_l \mu_l}{2F_l \rho_l h_{fg} r_{hl}^2} \right)} (0.5L_{ev} + L_{ad} + 0.5L_{co}) \quad (4.18)$$

4.2.5 Poiseuille Number

It is necessary to solve for the Poiseuille number of both the liquid and vapour flows.

This was done using two techniques as presented below. Both techniques yielded values that are somewhat close to the theoretical value of 16 for laminar flow in a circular cross section.

Solution using finite element method

The governing equation for both the liquid and vapour regions was taken to be the Poisson equation.

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{1}{\mu} \frac{dP}{dz} \quad (4.19)$$

The area and perimeter of the vapour and artery spaces can be calculated from geometry as follows,

Fig. 4.1 – Heat pipe geometry

$$\chi = 2 \cos^{-1} \left(\frac{r_i - H_{ar}}{r_i} \right) \quad (4.20)$$

$$C = 2 r_i \sin\left(\frac{\chi}{2}\right) \quad (4.21)$$

$$S = r_i \chi \quad (4.22)$$

$$F_v = (\pi r_i^2) - \frac{r_i^2}{2} (\chi - \sin(\chi)) \quad (4.23)$$

$$\Pi_v = 2\pi r_i - S + C \quad (4.24)$$

$$F_{ar} = \frac{r_i^2}{2} (\chi - \sin(\chi)) \quad (4.25)$$

$$\Pi_{ar} = S + C \quad (4.26)$$

where F refers to area and Π refers to the perimeter.

Non-dimensionalizing the x and y coordinates and the velocity w with the following

equations which are equally applicable to both the vapour and liquid region,

$$x^* = \frac{x}{r_i} \quad (4.27)$$

$$y^* = \frac{y}{r_i} \quad (4.28)$$

$$w^* = \frac{w}{\left(\frac{1}{\mu}\right) \left(-\frac{dP}{dz}\right) r_i^2} \quad (4.29)$$

$$\Pi^* = \frac{\Pi}{r_i} \quad (4.30)$$

$$F^* = \frac{F}{r_i^2} \quad (4.31)$$

Equation (4.19) can now be written non-dimensionally as,

$$\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}} = -1 \quad (4.32)$$

The boundary conditions for both the liquid and vapour are set to the no-slip condition.

$$w^* = 0 \quad (4.33)$$

The Poiseuille number can then be found from the mean non-dimensional velocity as given by,

$$Po = \frac{8}{\bar{w}^*} \left(\frac{F^*}{\Pi^*} \right)^2 \quad (4.34)$$

Matlab PDE Toolbox was used to solve the velocity field. Once the velocity field is obtained, the Poiseuille number was computed from equation (4.34). The following

tables are a summary of the grid convergence analysis that was performed. Refinements were halted after the relative change was below one hundredth of a percent.

Table 10 – Grid convergence summary for Po_v

Vapour Space			
# Triangles	$\bar{w}^*$	Po_v	% change
254	1.096E-01	16.09435125	
1016	1.107E-01	15.92714364	1.050%
4064	1.110E-01	15.88390307	0.272%
16256	1.111E-01	15.87287648	0.069%
65024	1.111E-01	15.87008707	0.018%
260096	1.111E-01	15.86938782	0.004%

Table 11 – Grid convergence summary for Po_l

Liquid Artery Space			
# Triangles	$\bar{w}^*$	Po_l	% change
40	2.917E-03	21.80239962	
160	3.771E-03	16.86773852	29.255%
640	3.992E-03	15.93067260	5.882%
2560	4.049E-03	15.70888352	1.412%
10240	4.063E-03	15.65459370	0.347%
40960	4.067E-03	15.64017469	0.092%
163840	4.067E-03	15.63684245	0.021%
655360	4.068E-03	15.63598113	0.006%

From this analysis it was found that the Poiseuille number for the vapour space is 15.87, and that of the liquid artery is 15.64.

Approximate method based on geometry

The method described in [37] is solely based on geometry and does not require the flow field to be solved. The technique relies on finding the polar moment of inertia given for a circular segment channel as follows,

$$I_p = r_i^4 \left[\frac{\xi}{2} - \frac{\sin(2\xi) (1 + \cos^2(\xi))}{12} - \frac{(2 \sin(\xi) - \cos(\xi) \sin(2\xi))^2}{9(\xi - 0.5 \sin(2\xi))} \right] \quad (4.35)$$

Fig. 4.2 – Circular segment geometry

where the following angles for the liquid artery and the vapour space come from geometry and the angle given in equation (4.20),

$$\xi_l = \frac{\chi}{2} \quad (4.36)$$

$$\xi_v = \pi - \frac{\chi}{2} \quad (4.37)$$

Using the specific polar moment of inertia defined in equation (4.38), it is possible to define a Poiseuille like value that is normalized with $\sqrt{F}$ rather than the hydraulic diameter as given in equation (4.39).

$$I_p^* = \frac{I_p}{F^2} \quad (4.38)$$

$$fRe_{\sqrt{F}} = 32\pi^2 I_p^* \frac{\sqrt{F}}{\Pi} \quad (4.39)$$

It is then possible to convert this value into the standard form of the Poiseuille number as follows,

$$Po = fRe_{\sqrt{F}} \frac{4\sqrt{F}}{\Pi} \quad (4.40)$$

The results of this analysis added further confidence to the values obtained through the finite element solution. This approximate method yielded values that are summarized in the following table and compared very well with the more precise finite element solutions.

Table 12 – Comparison between FEM and approximate Poiseuille solutions

	FEM	Approximate	Percent Error
Po_v	15.86939	15.64203	-1.43267%
Po_l	15.63598	15.59852	-0.23959%

4.3 Improved Model

The Chi model, while useful for initial prediction of the performance of a given style of heat pipe, does not take into account important aspects of the physics of the arterial heat pipe. Specifically the Chi model does not take into account the excess liquid that will invariably be in the heat pipe at operating temperature. This excess fluid comes from the initial fill level of the heat pipe when assembled. Normally, the pipe is slightly overfilled to ensure that there is always enough fluid to keep the liquid artery filled. In a gravitational field this excess fluid will result in a puddle forming in the heat pipe. The model specifically examines the evaporator as puddle formation there would significantly change the dry-out limit. Since the main capillary structure of the arterial pipe is the vapour vent holes, a puddle will have the effect of disabling a portion of the heat pipe. There are two criteria that define effective dry-out in the arterial heat pipe. The first is if the capillary pumping limit of the vapour vent holes is reached, and the second is if the fluid can no longer be pumped to the top of the heat pipe through circumferential grooves in the evaporator. These are the same criteria defined in [19] but are approached differently based primarily on the work in [17].

Fig. 4.3 – Groove geometry

It is first necessary to define several meniscus radius values starting at the minimum and increasing. The general meniscus radius of the groove can be derived from geometry in Fig. 4.3 as follows,

From triangle A,

$$h = \frac{r}{\sin(\alpha)} \quad (4.41)$$

From triangle B,

$$w = (H_t + h - r) \tan(\alpha) \quad (4.42)$$

At a meniscus height of zero and a minimum meniscus radius it can be shown in triangle C,

$$w_{min} = r \cos(\alpha) \quad (4.43)$$

Therefore generally from triangle D it can be shown that the general meniscus radius is,

$$R_g = \frac{w}{\cos(\alpha + \theta)} \quad (4.44)$$

With the incorporation of equations (4.41) and (4.42),

$$R_g = \frac{\left(H_t + \frac{r}{\sin(\alpha)} - r\right) \tan(\alpha)}{\cos(\alpha + \theta)} \quad (4.45)$$

The minimum radius collapses to the following based on equations (4.43) and (4.44),

$$R_{min} = \frac{r \cos(\alpha)}{\cos(\alpha + \theta)} \quad (4.46)$$

In a gravitational field the maximum meniscus radius is infinite as it approaches a flat surface.

$$R_{max} = \infty \quad (4.47)$$

Since it is impossible to define a list of meniscus radius values that goes all the way to R_{max} , the values were set up to a maximum of approximately three times the value of R_g . Combining equations (4.41) and (4.42) it is possible to get an expression for half the width of the groove. Since the value of w is given per Fig. 1.5, it is possible to solve for the angle α . The Matlab built in 'fzero' function is used to solve for the angle that satisfies the following equation,

$$0 = \left(H_t + \frac{r}{\sin(\alpha)} - r \right) \tan(\alpha) - w \quad (4.48)$$

The 'fzero' function is derived from a FORTRAN program given in [38] and is a blend of the bisection, secant, and inverse quadratic interpolation methods. The maximum area and perimeter of fluid in the threaded groove can be written as follows,

$$F_{tmax} = \left(H_t + \frac{r}{\sin(\alpha)} - r \right)^2 \tan(\alpha) - r^2 \left(\tan^{-1}(\alpha) - \frac{\pi}{2} + \alpha \right) \quad (4.49)$$

$$\Pi_{tmax} = 2 \left[\frac{H_t(1 + \sin(\alpha))}{\cos(\alpha)} + r \left(\frac{\pi}{2} - \alpha \right) \right] \quad (4.50)$$

The volume taken up by the threads can be determined by establishing the thread pitch S_p . Therefore,

$$V_t = \frac{F_{tmax} 2\pi r_i L_{total} \rho_l}{S_p} \quad (4.51)$$

The total mass of ammonia in the pipe is the sum of the volume of the vapour space subtracted by the volume of the puddle multiplied by the vapour density, and the volume

of the artery added to the volume of the threads and the puddle then multiplied by the liquid density as given in the following equation,

$$m_{pr} = (V_v - V_{exc})\rho_v + (V_{ar} + V_t + V_{exc})\rho_l \quad (4.52)$$

Re-arranging this equation to solve for the excess volume of the liquid puddle yields,

$$V_{exc} = \frac{m_{pr} - V_v\rho_v - (V_{ar} + V_t)\rho_l}{\rho_l - \rho_v} \quad (4.53)$$

It is next necessary to solve for the length z_0 of the puddle caused by the excess of fluid in the evaporator for each of the defined meniscus radius values. The puddle geometry is given in Fig. 4.4.

Fig. 4.4 – Puddle geometry (adapted from [17])

The slope of the puddle is given by the following equation where ϕ_p is the angle the puddle form at a given distance along the puddle length as shown in cross-section A-A in Fig. 4.4.

$$\frac{dy}{dz} = \frac{d_i}{2} \sin(\phi_p) \frac{d\phi_p}{dz} \quad (4.54)$$

The pressure distribution along the puddle can be shown to be,

$$\frac{dP}{dz} = \frac{\nu_l Q_{max} P_{O_z}}{2F_z R_z^2 h_{fg}} = (\rho_l - \rho_v) g \left(\frac{dy}{dz} \cos(\psi) - \sin(\psi) \right) \quad (4.55)$$

The differential volume of the liquid in the puddle can be shown to be given by,

$$\frac{dV}{dz} = F_z - F_{ar} \quad (4.56)$$

where the area and perimeter based upon the angle ϕ_p of the puddle at a given location is given in the following two equations,

$$F_z = \frac{d_i^2}{8} (2\phi_p - \sin(2\phi_p)) \quad (4.57)$$

$$\Pi_z = d_i^2 (\phi_p + 2\sin(\phi_p)) \quad (4.58)$$

For a given R_m , a numerical method is used to find the length of the puddle from the following equations,

$$\Delta P_{t1} = \sigma \left(\frac{1}{R_m} - \frac{2}{R_{max}} \right) - g(\rho_l - \rho_v) L_w \sin(\psi) \quad (4.59)$$

$$\Delta P_{ar} = \frac{\nu_l P_{O_l} Q_{max1} L_{eff}}{2F_{ar} r_{har}^2 h_{fg}} \quad (4.60)$$

$$\Delta P_v = \frac{\nu_v P_{o_v} Q_{max1} L_{eff}}{2 F_v r_{hv}^2 h_{fg}} \quad (4.61)$$

$$\Delta P_{t1} = \Delta P_{ar} + \Delta P_v \quad (4.62)$$

$$L_{eff} = L_{total} - 0.5[(L_{ev} - z_0) + L_{co}] \quad (4.63)$$

Replacing the terms in equation (4.62) with the expressions (4.59) to (4.61) and solving for the maximum power, the following equation can be written,

$$Q_{max1} = \frac{2 \Delta P_{t1} h_{fg}}{L_{eff} \left[\frac{P_{o_l} \nu_l}{F_{ar} r_{har}^2} + \frac{P_{o_v} \nu_v}{F_v r_{hv}^2} \right]} \quad (4.64)$$

By re-arranging equations (4.54) and (4.55) it is possible to get the following equation,

$$\frac{d\phi}{dz} = \frac{\nu_l Q_{max1} P_{o_z}}{d_i F_z R_z^2 h_{fg} (\rho_l - \rho_v) g \cos(\psi) \sin(\phi)} + \frac{2 \tan(\psi)}{d_i \sin(\phi)} \quad (4.65)$$

where P_{o_z} is found using the approximate geometric method from [37] using equations (4.35) to (4.40).

Equation (4.65) is then numerically integrated from 0 to z_0 to solve for the profile of the angle of the puddle. The boundary conditions for this numerical integration are given as,

At $z = 0$

$$\phi = \phi_a \quad (4.66)$$

and at $z = z_0$

$$V = V_{exc} \quad (4.67)$$

Matlab built in Runge-Kutta ode45 solver was used to perform this integration.

With the distribution of ϕ , the integral of equation (4.56) is found by using the trapezoid rule. It is then possible to compare the calculated excess volume with the theoretical excess volume given in (4.53). If they are equal then the value of z_0 is stored to be used to calculate Q_{max1} using equations (4.59) and (4.64). If they are not equal, the value of z_0 must be incremented to a new value and the whole solution procedure repeated. The Matlab built in function 'fzero' was used rather than implementing a slower bisection search.

These steps are repeated until a z_0 and Q_{max1} are found for each radius meniscus value. R_m is then incremented, and the process repeats.

Fig. 4.5 – Groove cross-section (adapted from [17])

The value of Q_{max2} is solved for by looking at the secondary dry-out criterion. The second criterion differs from the first in that it is only concerned with the circumferential grooves. For a given meniscus radius in the vapour vent hole there exists a value z_0 which represents the length of the puddle. The groove at the edge of the puddle is then treated as a separate very small heat pipe.

There will be some pressure drop associated with the fluid passing under the flange from point A to point C per Fig. 4.5. The pressure drop can be calculated as follows per the analysis in [17],

$$\Delta P_{fla} = \frac{\nu_l d_i (\phi_b - \phi_a) Q_{max2}}{2 h_{fg} L_{ev} \left[\frac{F_{tmax}^3}{(S_p \Pi_{tmax}^2)} + \frac{\delta_w \epsilon_w d_{hw}^2}{16} \right]} \quad (4.68)$$

After pumping the fluid from the vapour vent hole, through and then to the edge of the flange, it is required to determine the pumping pressure from the edge of the flange to the

top of the pipe. To go from point C to D per Fig. 4.5, a new meniscus radius must be defined as follows,

$$R'_m = \left(\frac{\Delta P_{fla}}{\sigma} + \frac{1}{R_m} \right)^{-1} \quad (4.69)$$

The total effective capillary pumping pressure is therefore given by,

$$\Delta P_{t2} = \sigma \left(\frac{1}{R_{min}} - \frac{1}{R'_m} \right) \quad (4.70)$$

As the radius of the meniscus at a groove changes so too does the area and perimeter of the fluid in the threaded groove. The area and perimeter of the fluid in the threaded groove as a function of the height and angle of the meniscus can be written as follows,

$$F_t = \frac{r^2}{2} (\pi - 2\alpha + \sin(2\alpha)) + 2r(H_m - r) \cos(\alpha) + [(H_m - r) + r \sin(\alpha)]^2 \tan(\alpha) - R_m^2 (\beta - \sin(\beta) \cos(\beta)) \quad (4.71)$$

$$\Pi_t = 2 \left[\frac{H_m - r(1 - \sin(\alpha))}{\sin(\alpha)} + r \left(\frac{\pi}{2} - \alpha \right) \right] \quad (4.72)$$

Angle β and the height of the meniscus H_m are found with the following equations,

If $R_{min} \leq R_m < R_g$

$$\beta = \frac{\pi}{2} - (\alpha + \theta) \quad (4.73)$$

$$H_m = \frac{R_m \sin(\beta)}{\tan(\alpha)} - \frac{r}{\sin(\alpha)} + r \quad (4.74)$$

and if $R_{max} \geq R_m \geq R_g$

$$\beta = \sin^{-1} \left(\frac{\left(H_t + \frac{r}{\sin(\alpha)} - r \right) \tan(\alpha)}{R_m} \right) \quad (4.75)$$

$$H_m = H_t \quad (4.76)$$

The pressure distribution in the threaded groove from the edge of the flange up to the top of the pipe (from point C to D per Fig. 4.5) is given by,

$$\frac{dP}{d\phi} = \frac{v_l \Pi_t^2 d_i Q_{max2} S_p}{2 F_t^3 L_{ev} h_{fg}} \frac{\pi - \phi}{\pi - \phi_b} + (\rho_l - \rho_v) \frac{d_i}{2} g \frac{d(\cos(\phi))}{d\phi} \quad (4.77)$$

Integrating from the edge of the flange at an angle of ϕ_b up to the top of the pipe at an angle of π it is possible to find the pressure drop through the groove channel as follows,

$$\Delta P_{thr} = \int_{\phi_b}^{\pi} \frac{dP}{d\phi} \quad (4.78)$$

$$\Delta P_{thr} = \frac{v_l \Pi_t^2 d_i Q_{max2} S_p}{2 F_t^3 L_{ev} h_{fg}} \left(\frac{\pi - \phi_b}{2} \right) + (\rho_l - \rho_v) \frac{d_i}{2} g (-\cos(\phi_b) - 1) \quad (4.79)$$

Equating equation (4.70) with the sum of equations (4.68) and (4.79) one gets,

$$\Delta P_{t2} = \Delta P_{fla} + \Delta P_{thr} \quad (4.80)$$

By rearranging the resulting equation and isolating for Q_{max2} it is possible to write,

$$Q_{max2} = \frac{2h_{fg} \left[\Delta P_{t2} - (\rho_l - \rho_v) \frac{d_i}{2} g(-\cos(\phi_b) - 1) \right]}{v_l d_i \left[\frac{(\phi_b - \phi_a)}{L_{ev} \left[\frac{F_t^3}{(S_p \Pi_t^2)} + \frac{\delta_w \epsilon_w d_{hw}^2}{16} \right]} + \frac{\Pi_t^2 S_p}{F_t^3 L_{ev}} \left(\frac{\pi - \phi_b}{2} \right) \right]} \quad (4.81)$$

The value of Q_{max2} is needed to determine R'_m so this set of equations must be solved iteratively.

Both Q_{max1} and Q_{max2} solved for each meniscus radius are then plotted. The two resulting lines will cross in a manner similar to either the first or second method as seen in Fig. 4.6.

Fig. 4.6 – Capillary pumping limits

If the intersection of the two curves is at a point larger than R_{limit} as shown in Fig. 4.6 then the value of Q_{max} is at that intersection. If they intersect below R_{limit} the value of Q_{max} is the calculated value of Q_{max1} at R_{limit} .

From [36] it is known that for small radiuses,

$$R_1 = R_2 = R = \frac{r_{\text{vent}}}{\cos(\theta)} \quad (4.82)$$

By equating equations (4.1) and (4.2) and taking into account equation (4.82) the following relation can be derived,

$$\sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{2\sigma \cos(\theta)}{r_{\text{vent}}} \quad (4.83)$$

The value R_{limit} is the smallest possible meniscus radius. By replacing $r_{\text{vh}} = \frac{d_{\text{vent}}}{2}$ and noting that $R_2 = \infty$ in equation (4.83), R_{limit} can be derived to be,

$$R_{\text{limit}} = \frac{d_{\text{vent}}}{4 \cos(\theta)} \quad (4.84)$$

Having solved for Q_{max} with a guessed operating temperature, the guess is updated via the experimentally derived linear function based on all data from TC₁₆, and the solution procedure is repeated until the difference between the guess and the saturation temperature solved for in equation (4.85) is less than one 0.01°C. The estimated T_{sat} value is added to half of the difference between the previous guess and the newly calculated T_{sat} . This ratio of adding half the difference was found to quickly converge as long as the initial guess was larger than the actual T_{sat} .

$$T_{sat} = (0.12841(Q_{max}) + 6.4858) + 273.15 \quad (4.85)$$

The overall solution procedure was found to converge in approximately 20 seconds on an AMD Athlon 64 processor running at 2.20 GHz.

The solution algorithm is shown in Fig. 4.7 where the values with the ‘*’ are calculated numerically, V_{exc} is calculated from equation (4.53), the initial T_{sat} guess is 330 K and the initial Q_{max2} guess is 300 W.

Fig. 4.7 – Solution algorithm

4.4 Discussion of Results

Both the Chi model and the improved model that took into account puddle formation were solved. The following section discusses the results of these two models.

4.4.1 Chi Model Results

Using the Chi model and taking into account the T_{sat} temperature using equation (4.85) the maximum dry-out power was found to be 322.6 W. This represents approximately 8.5% error when compared to the 350 W dry-out power found during testing.

4.4.2 Improved Model Results

The improved model yielded somewhat better results. A grid convergence analysis was performed where the model was solved with various numbers of meniscus radius values from R_{min} of approximately $5.0\text{e-}5$ m up to $3.0\text{e-}4$ m with a contact angle of 45° , a very small tilt angle of -0.01° to ensure the puddle would stay in the evaporator, and a fill mass of 10 g. Since the value of Q_{max} is the point at which the Q_{max1} and Q_{max2} lines cross, the precision of the calculation depends on how closely spaced the meniscus radius values are. As shown in the following figure the Q_{max} value stabilizes at 16 points with less than 0.03% error vs. a calculation with 29 points. This result means that using points separated by approximately $1.0909\text{e-}5$ m or smaller should be used in further study. The grid convergence analysis is shown graphically in Fig. 4.8 and typical results using the same parameters are given in Fig. 4.9 giving a maximum predicted dry-out power of 346.05 W. Matlab has several Runge-Kutta solvers that are available for use. Both

ode45 which is a 4th and 5th order solver and ode23 which is a 2nd and 3rd order solver could be used. It was found that by using the ode23 solver the solution could be found 30% faster and had a negligible difference in results as compared with the ode45 results.

Fig. 4.8 – Model grid convergence

Fig. 4.9 – Typical results using 45° contact angle and $m_{pr} = 10$ g

A contact angle of 45 degrees at dry-out is possible, especially if the internal surfaces of the heat pipe have been poorly cleaned [6]. In Fig. 4.10 it is notable that at a contact angle of 0° and a fill mass of 10 g, the Q_{max1} and Q_{max2} lines cross at a meniscus radius value small than R_{limit} . This means that the dry-out value can be found where the Q_{max1} curve intercepts R_{limit} .

Fig. 4.10 – Typical results using 0° contact angle and $m_{pr} = 10$ g

The model is a function of the contact angle and the fill mass. It was necessary to conduct a parametric analysis to determine how the performance changed with changes in these parameters. The contact angle in this model was assumed to be the value just as dry-out is reached. As such, this dynamic contact angle could be any value. Certain authors have given a range of 0 to 40 degrees [39]. Other authors have noted that the dynamic contact angle is an unknown that, with internal surface contamination, could be in excess of 45 degrees [6]. Values from 0 to 50 degrees were explored in the presented analysis.

The range for the mass of ammonia was set from 9.7 g up to 10.0 g. The chosen values represent the lowest possible fill mass that would still result in a puddle forming at the operating temperature of 326 K, up to a maximum value that would keep the resulting

puddle length less than the length of the evaporator. The length of the puddle, z_0 , has to be smaller than the length of the evaporator because the vapour vent holes are only within this section. If the puddle was longer than the evaporator, the vapour vent holes would no longer be able to provide a capillary pumping action and the pipe would be limited by the centrifugal groove criteria alone.

Fig. 4.11 – Parametric analysis

The region that comes closest to the experimental dry-out value of 350 W is at the larger contact angles and is given in more detail in Fig. 4.12.

Fig. 4.12 – Parametric analysis (zoomed)

For a fill mass of 10.00 g, the numerical model predicts a maximum dry-out power within 5% of the experimental 350 W when the contact angle is between 43 degrees and 47 degrees. These are reasonable values since it would be nearly impossible to get the theoretical maximum of a zero degree contact angle due to imperfections in the material or contamination on the surface.

The improved numerical model yields values for maximum dry-out power that are quite close to the values found during experimental testing. Since the values of fill mass and contact angle are not known exactly, it is difficult to provide an exact value from the numerical model to compare with experiment. Within a range of valid fill masses and from approximately 43 to 47 degrees contact angle, the maximum dry-out value predicted

by the numerical model fall between 330 W and 370 W. This falls perfectly within the expected range for dry-out power based on the experimental observations. The improved numerical model has therefore been shown to give results that are reasonable and within the same range as found in experimental analysis.

Chapter 5 Conclusions and Recommendations

5.1 Conclusion

The arterial heat pipe tested has been found to have repeatable performance during extensive dry-out testing. The heat pipe has been found not to be prone to accumulation of vapour in the artery that would lead to premature dry-out. The test article shows no signs of non-condensable gas formation. A maximum dry-out power of 350 W was achieved at horizontal. Tilt testing demonstrated that the heat pipe exhibits an exponential decrease in performance with adverse evaporator elevation. Similarly, the minimum operating evaporator resistance follows a linear relationship when plotted on a log-log graph for the applied power from zero up to the maximum dry-out power.

Instantaneous application of up to 80% of dry-out power was handled smoothly by the heat pipe with no indication of dry-out or local boiling. Dropping the chiller temperature while at 80% of maximum load was found to cause local boiling in the tip of the evaporator in 20% of cases. This local spike in temperature was found to return to average conditions within ten minutes. A numerical model was developed that took into account excess charging and the resulting puddle that would form. This model was found to yield reasonable results. The arterial heat pipe exhibited predictable stable performance but was very sensitive to tilt.

5.2 List of Personal Contributions

Detailed testing of arterial heat pipe was accomplished. A thorough uncertainty analysis was performed and based on the results improvements to the experimental apparatus were made, including procurement of more accurate multi-meters. The test stand was modified to allow cooling the condenser from the bottom rather than the top.

Thermocouples were clustered at the tip of the evaporator to detect any potential local boiling and at the condenser to detect any potential non-condensable gas formation. The experimental testing involved approximately a dozen different tests each being repeated from a minimum of three to a maximum of fifteen times, representing hundreds of hours of detailed work. These detailed tests probed the operating envelope of an arterial heat pipe with minimal performance data available in the open literature. The obtained results were self consistent and fit well with previous experimental results.

A novel numerical model that takes into account excess charge of fluid was also developed. The previous work [17] on which much of the numerical model is based did not account for the effect the skewed shape of the liquid and vapour cross sections. Effectively the model in [17] assumed a Poiseuille number of 16 for both flows. The model presented in this work solved for the exact value of the Poiseuille number in the artery and in the vapour space and made use of geometric correlations to solve for the values of Poiseuille number in the puddle region. Therefore, the improved model presented in this work is more accurate. A good fit was obtained between experimental results and the numerical model.

5.3 Suggestions for Future Work

It is clear from the extensive experimental testing performed in this thesis that this configuration of arterial heat pipe performs extremely well. Further experimental and numerical research work on this configuration of heat pipe can lead to a better understanding of these heat pipes and more confidence in using them for future space missions.

5.3.1 Experimental Recommendations

Extensive testing has been done that has yielded a quality data set probing the capillary limited dry-out behaviour of the arterial heat pipe. To improve upon these results any further testing should consider the use of a power supply that could be controlled by software. Many of the tests took upwards of seven to eight hours to complete. This involved changing setting manually approximately every half an hour. Automation of power cycling would enable more repeatable results. It is also suggested that space flight tests should be done to collect data on the performance of the arterial heat pipe in microgravity conditions since we do not have access to any of the data from the previously flown pipes on Russian spacecraft.

5.3.2 Numerical Recommendations

The presented numerical model has been shown to produce accurate estimates of maximum dry-out power for the arterial heat pipe. It would be interesting to take the presented model and incorporate an optimization technique to probe the limits of the

operational capacity of this heat pipe. A technique such as a genetic algorithm could quickly vary design parameters such as groove geometry, fill mass, wick properties and vent hole sizing to optimize for maximum dry-out power and minimum mass. It is recommended that this model should be extended to simulate the performance of the arterial heat pipe in microgravity conditions.

With ever increasing capabilities, it is possible that a three dimensional computational fluid dynamics and heat transfer simulation could be undertaken for this arterial heat pipe configuration. Any such model would require solving the full three-dimensional Navier-Stokes, energy and mass transfer equations. A major challenge in such a simulation would be to resolve the very small scale phenomena occurring at the screen mesh and circumferential grooves while keeping the number of nodes of the domain within a manageable range.

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Appendix A

Diffusivity Estimation—Liquids

The following analysis provides a means to estimate the diffusivity of gaseous nitrogen and hydrogen in liquid ammonia. There exist many correlations for liquid phase diffusivity that can be applied to this problem [40]. All of the available correlations are empirical and vary in accuracy on the order of 20%. This is acceptable as this analysis is only used as an initial estimate of the potential quantity of non-condensable gas.

Many correlations exist for estimating liquid phase diffusion. The most basic equation, based on only on fluid mechanics and thermodynamics is called the Stokes-Einstein equation. This equation is also the starting point for most other correlations.

	$D_{AB} = \frac{k_B T}{6\pi r_A \mu_B}$	(1)
	Returns value in m ² /s k_B = Boltzmann's constant T = Temperature r_A = radius of molecule μ_B = solvent viscosity	

Since this theory is derived based on the movement of a large solid sphere moving through a continuous liquid, the Stokes-Einstein equation is limited to cases where the solute is much larger than the solvent. In cases where they are comparable in size, other correlations have been developed. Sutherland developed a parallel to the Stokes-Einstein equation that replaces the 6 in the denominator with a 4 this making it “no stick” at the surface of the sphere. Glasstone et al. modelled diffusion as a rate process replacing the 6π with 2.

In these equations the radius can be assumed to be half the collision diameter.

Table 13 – Collision diameters of selected substances [41]

Substance	Collision diameter [angstroms]
H ₂	2.827
O ₂	3.467
N ₂	3.798
NH ₃	2.900
H ₂ O	2.641

The most well known liquid diffusion correlation is given by Wilke and Chang.

[42]

Wilke and Chang	$D_{AB} = \frac{1.173 \times 10^{-16} \phi_B^{\frac{1}{2}} M_B^{\frac{1}{2}} T}{\mu V_A^{0.6}}$	(2)
	Returns value in m ² /s ϕ_B = association factor M_B = Molecular Weight (g/mol) T = Temperature (K) μ = Viscosity of solution (Ns/m ²) V_A = Molecular volume of the solute (m ³ /kmol)	

Table 14 – Solvent association factors [40]

Solvent	ϕ_B
Water	2.26
Methanol	1.9
Ethanol	1.5
Propanol	1.2
Non-hydrogen bonded	1.0

Table 15 – Molecular volumes of simple molecules for V_A and V_B (m³/kmol) [42]

H ₂	0.0143	CO ₂	0.0340	NH ₃	0.0258
O ₂	0.0256	H ₂ O	0.0189	H ₂ S	0.0329
N ₂	0.0312	SO ₂	0.0448	Cl ₂	0.0484
Air	0.0299	NO	0.0236	Br ₂	0.0532
CO	0.0307	N ₂ O	0.0364	I ₂	0.0715

Another empirical equation for diffusion in liquids is given by Scheibel (1954). From [41]

Scheibel	$D_{AB} = \frac{AT}{(\mu \times 10^{-3})(V_A \times 10^{-3})^{\frac{1}{3}}} \left[1 + \left(\frac{3V_B}{V_A} \right)^{\frac{2}{3}} \right]$	(3)
	Returns value in cm ² /s A = Constant (see table below) T = Temperature (K) μ = Viscosity of solution (Ns/m ²) V_A = Molecular volume of the solute (m ³ /kmol) V_B = Molecular volume of the solvent (m ³ /kmol)	

Table 16 – Constant A [41]

Constant A equals	8.2×10^{-8}
Except as follows,	---
Water when $V_A < V_B$	25.2×10^{-8}
Benzene when $V_A < 2V_B$	18.9×10^{-8}
Others when $V_A < 2.5V_B$	17.5×10^{-8}

Another empirical equation for diffusion in liquids is given by Siddiqi-Lucas.

[40]

Siddiqi-Lucas	$D_{AB} = \frac{9.89 \times 10^{-8} (V_B \times 10^3)^{0.265} T}{(V_A \times 10^3)^{0.45} (\mu_B \times 10^3)^{0.907}}$	(4)
	Returns value in cm ² /s T = Temperature (K) μ_B = Viscosity of solution (Ns/m ²) V_A = Molecular volume of the solute (m ³ /kmol) V_B = Molecular volume of the solvent (m ³ /kmol)	

Another empirical equation for diffusion in liquids is given by Umes-Danner.

[40]

	$D_{AB} = \frac{2.75 \times 10^{-8} \left(\frac{R_B}{R_A^{2/3}} \right) T}{\mu_B}$	(5)
	Returns value in cm ² /s R_A = Radius of gyration in ångströms of the solute R_B = Radius of gyration in ångströms of the solvent T = Temperature (K) μ_B = Viscosity of solution (Ns/m ²)	

Analysis – Example (Oxygen in Water)

All of these equations were briefly analyzed to gain a better understanding of how the relations work and to confirm that they are indeed valid.

The value of the diffusion parameter for Oxygen in Water is known experimentally to be 1.8×10^{-5} cm²/s. The values calculated are:

Calculated Diffusivity Values for O₂ in H₂O

	Diffusivity	Error	% Error
Stokes-Einstein	1.26E-05	-5.41E-06	-30%
Sutherland	1.89E-05	8.90E-07	5%
Glasstone	1.19E-04	1.01E-04	559%
Wilke and Chang	2.01E-05	2.10E-06	12%
Scheibel	2.24E-05	4.38E-06	24%
Siddiqi – Lucas	1.49E-05	-3.07E-06	-17%
Umes-Danner	7.50E-06	-1.05E-05	-58%
Average	3.07E-05	1.27E-05	71%

Removing the outliers, the following relations give:

Values (Outliers Removed)

	Diffusivity	Error	% Error
Stokes-Einstein	1.26E-05	-5.41E-06	-30%
Sutherland	1.89E-05	8.90E-07	5%
Wilke and Chang	2.01E-05	2.10E-06	12%
Scheibel	2.24E-05	4.38E-06	24%
Siddiqi – Lucas	1.49E-05	-3.07E-06	-17%
Average	1.78E-05	-2.22E-07	-1%

Analysis – Estimation of the Diffusivity of H₂ and N₂ in NH₃ (Ammonia)

Calculated Diffusivity Values of H₂ and N₂ in NH₃

	H2	N2
Stokes-Einstein	1.10E-04	8.17E-05
Sutherland	1.65E-04	1.23E-04
Glasstone	1.04E-03	7.70E-04
Wilke and Chang (1)	1.31E-04	8.21E-05
Wilke and Chang (2)	1.86E-04	1.16E-04
Scheibel	6.24E-04	3.34E-04
Siddiqi – Lucas	1.25E-04	8.79E-05
Umesu – Danner	6.71E-05	5.51E-05
Average	3.05E-04	2.06E-04

(using $\phi=1$ and $\phi=2$ for the Wilke and Chang Equation)