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Spatial Processing and Arithmetic Performance

by

Alison G. Kulak

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of

Doctor of Philosophy

Department of Psychology

Carleton University

9 July 1993

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ABSTRACT

There are strong reasons for believing that visuo-spatial skills are important to arithmetic performance. For example, correlations between spatial skills and arithmetic performance have been observed among children at all levels of arithmetic ability (i.e., arithmetic learning disabled to mathematically gifted). In the present experiments, Baddeley's model of working memory was used as a framework to assess the hypothesis that codes associated with the visuo-spatial sketchpad mediate performance on a complex addition task. In the first experiment, adults solved double- plus single-digit arithmetic problems (e.g., $35 + 7$) while also retaining either a verbal or a spatial working-memory load. The most intriguing finding was that less-skilled subjects showed improved performance in the experimental condition that would be expected to be most demanding of a visuo-spatial code. It was hypothesized that subjects' improved performance could be attributed to a shift in the types of solution strategies used to solve the addition problems: From relatively demanding procedural strategies (e.g., counting) to less-demanding retrieval strategies (e.g., direct retrieval of the sum). In the second experiment the strategy shift hypothesis was tested using a similar experimental procedure, in which subjects also reported the strategy used to solve the addition problem. The analyses of the strategy data supported the strategy-shift hypothesis: Less-skilled subjects reported relying more on retrieval strategies in the experimental condition most demanding of visuo-spatial codes than in the less demanding conditions. It was also hypothesized that

children's arithmetic performance may be more dependent on visuo-spatial codes than that of adults. In the third experiment, sixth grade children completed the same set of problems completed by adults in the first experiment. Overall the problems were more difficult for children than for adults. Again, a subgroup of subjects showed improved performance in the condition most demanding of visuo-spatial codes; in this experiment, however, it was skilled subjects who showed improved performance. These results support the hypothesis that visuo-spatial codes are involved in arithmetic performance, and suggest that particular sub-groups of subjects are especially reliant on spatially-mediated solution strategies.

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INTRODUCTION

Visuo-spatial skills have long been considered to be important to mathematical skill. According to one definition (Hamley, 1935, cited in McGee, 1979), mathematical ability comprises a constellation of abilities, including general intelligence, visual imagery, and the "ability to perceive number and space configurations and to retain such configurations as mental patterns" (p. 897). This definition clearly suggests that visuo-spatial processes contribute to mathematical ability, but does not define what role visuo-spatial processes may play.

Spatial ability has been defined as the mental manipulation of shapes (e.g., mental rotation; Kelley, 1928, cited in McGee, 1979), or as the ability to see relations among visually presented objects (Sattler, 1974). The results of several factor analytic studies suggest that spatial ability is comprised of at least two factors; spatial visualization and spatial orientation (McGee, 1979). Spatial visualization refers to the ability to mentally manipulate stimuli in space, whereas spatial orientation refers to the ability to detect spatial patterns and to relate spatial positions to the body orientation of the observer. In this paper the term "visuo-spatial" is used to refer to those abilities associated with spatial visualization; that is, the mental manipulation of visually presented or self-generated stimuli in an internal visual space.

There are strong reasons for believing that visuo-spatial skills would play a role in the development of early arithmetic skill. Research has shown that children use their fingers extensively in

early counting procedures and that counting is an important precursor to the development of arithmetic skill (Fuson, 1988, 1993; Siegler, 1988). As children learn to manipulate large numbers they are often taught to arrange objects in groups of five or ten; thus, words and written symbols continue to be associated with concrete visual representations of quantity (Hartje, 1987). Other early mathematical skills and concepts such as absolute and relative magnitude, manipulation of quantity, and seriation, also require visuo-spatial skills. Moreover, certain mathematics specialties such as geometry, trigonometry, and calculus appear likely to be subserved by visuo-spatial abilities (Halpern, 1992).

Greenes (1979) has asserted that the perceptual skill most closely associated with the learning of mathematics is visuo-spatial perception and that spatial abilities are requisite to success in arithmetic. She has pointed out that sequencing (the ability to order) is essential to the development of counting skills; counting skills are, in turn, "perhaps the major vehicle for finding answers to simple addition, subtraction, and multiplication problems in the primary school classes" (p. 7). According to her argument, children who lack the visuo-spatial skills requisite to efficient sequencing will encounter difficulty in learning to count and in using counting methods to obtain answers to simple arithmetic problems. Inefficient counting strategies may, in turn, compromise the development of fast and accurate retrieval of arithmetic facts (Siegler, 1988). Thus, although there may be no obvious use of visuo-spatial strategies among individuals who are skilled at basic computations, the acquisition of

computational skill may derive, in part, from the early mediation of component skills by visuo-spatial processes.

The ability to maintain spatial alignment also appears to be important to success in the solution of more advanced arithmetic problems. Webster (1984) has asserted that the spatial component of arithmetic disability is most evident in problems requiring numbers to be carried or borrowed from one column to another. In the carrying or borrowing process "numbers may be misplaced relative to each other, rows and columns confused or entire lines of numbers omitted" (Webster, 1984, p. 46). It appears that successful arithmetic performance requires that an individual's spatial ability be sufficient to enable the execution of the complex series of cognitive operations that comprise arithmetic.

Hayes (1973) assessed the use of spatial imagery in mathematics problem solving by asking adults to report any imagery that they used to solve a variety of mathematics problems (e.g., simple addition, addition with carrying, adding fractions). He found that subjects commonly reported using spatial imagery in order to (a) set up the problem for solution, (b) store partial results, and (c) execute solution processes. In general, he concluded that subjects make use of self-generated spatial images to solve arithmetic problems, and that these images include a variety of information that is important to mathematical problem solving. Hitch (1978) reported a similar use of visual imagery among adult subjects solving multidigit addition problems: The more flexible problem solvers reported quickly visualizing the carrying structure of an addition problem without

adding.

Hayes (1973) has also discussed the importance of spatial symbolism in our number and notation systems, and the relation of "notation-related imagery" to mathematics performance. He suggested, for example, that spatial position is critical to the development of concepts of place value (e.g., in the number 261, the 2 represents 200 and the 6 represents 60). In a recent study, Johnson and Artuso (1993) found evidence for a relation between spatial ability and children's understanding of place value concepts: Among children in grades 2, 4, and 6, performance on a spatial task was found to be a strong predictor of performance on a task assessing place-value concepts.

In summary, there are both logical and empirical reasons for postulating a link between spatial processes and arithmetic performance. From the results of existing studies, however, it is not possible to determine how spatial skills contribute to mathematics ability. To date, little research has been designed to directly test the hypothesized role of spatial processes in arithmetic skill, and the majority of this research has not been theoretically motivated. Because the concept of working memory provides a useful theoretical framework for generating hypothesis about the relation between spatial processes and arithmetic performance, the model of working memory proposed by Baddeley and his colleagues (Baddeley & Hitch, 1974; Baddeley, 1985, 1986) is discussed below.

Working Memory

The concept of working memory is similar to that of short-term memory, but adds a dimension of active processing that is missing in the traditional view of short-term memory (Hulme & Mackenzie, 1992). Short-term memory is viewed solely as an information storage system, whereas working memory comprises both storage and processing components. Because the concept of working memory encompasses both the storage and processing aspects inherent in most real life tasks, the concept of working memory serves as a useful framework for exploring the relations between memory performance and task performance in a variety of domains.

An influential model of the structure of working memory has been developed by Baddeley (Baddeley, 1979, 1981, 1983, 1985, 1986; Baddeley & Hitch, 1974). He has proposed that working memory includes three structural components: A central executive system that integrates incoming information with prior knowledge, and two 'slave' systems, (a) an articulatory loop and (b) a visuo-spatial scratchpad. These hypothesized components of working memory, and supporting research, are reviewed below.

Components of working memory

The central executive. The central executive component of working memory can be described as the control centre of the working memory system. The central executive is assumed to select, operate, and monitor processes that are used to operate on internal memory representations and to integrate incoming information (Baddeley, 1981, 1986; Baddeley & Hitch, 1974). Baddeley (1981) has suggested that the

central executive may be a resource limited, modality-free attentional component that directs the work of the slave systems.

The articulatory loop. The articulatory loop is a speech-based system that is used for the subvocal, verbal rehearsal of material. Thus, a memory trace is repeatedly refreshed and is not lost to the system (Baddeley, 1979). Considerable evidence has now accumulated in support of the notion that subvocal verbal rehearsal is mediated by the articulatory loop. For example, people make more errors on memory tasks that involve phonologically similar words (e.g., bun, fun, pun) or letters (e.g., b, p, v) than on immediate memory tasks than involve phonologically non-confusable items (e.g., bun, fat, pill; see Baddeley, 1976 for a review of this literature). The phonological similarity effect is believed to reflect confusability, in the articulatory loop, of similar codes that are refreshed by articulation.

Because of the relation between memory performance and phonological similarity, this phonological similarity effect is believed to reflect the sub-vocal rehearsal of information in the articulatory loop. Another variable believed to reflect the operation of the articulatory loop is word length. In general, strings of short words (e.g., run, swim, cat) are recalled more accurately than are longer words of equal frequency (e.g., opportunity, university; Baddeley, Thompson, & Buchanan, 1975). This effect is believed to reflect limits in articulatory rehearsal; longer words take longer to articulate and, hence, are more susceptible to decay than are short words (Baddeley, 1986; Ellis & Henneley, 1980). Finally, data showing

that performance deteriorates when verbal rehearsal is suppressed indicate the importance of phonological coding in a variety of reading tasks (Baddeley, 1986).

The visuo-spatial scratchpad. The visuo-spatial scratchpad (VSSP) is a system believed to be involved in generating and manipulating visuo-spatial images (Baddeley, 1983). This system has been the subject of substantially less research than the articulatory loop, but there is some evidence supporting the notion of a short-term store for both simple visual stimuli and for relatively complex patterns (Christie & Phillips, 1979; Baddeley, 1986; Farmer, Berman, & Fletcher, 1986; Logie, 1986; Morris, 1987; Phillips, 1983, 1989). This system is hypothesized to be capable of holding and manipulating images, and to be vulnerable to disruption by concurrent spatial processing tasks (Baddeley, 1986, p. 143). The results of several studies support this hypothesis (Brooks, 1968, 1970; Morris, 1987, 1989; Murray & Newman, 1973).

Working memory is a construct that has been used successfully to generate a number of different experimental tasks designed to explore both individual differences in basic processing abilities, and to uncover the cognitive processes underlying task performance in a variety of domains (e.g., Daneman & Carpenter, 1980; Dark & Benbow, 1990, 1991; DineI, Glover, & Ronning, 1984; LaPointe & Engle, 1990). The majority of studies designed to assess working memory make use of a variant of the dual-task paradigm. Before reviewing these studies some examples of the dual-task paradigm will be presented, in order to provide a context for evaluating the research that follows.

Measuring Working Memory: The Dual-Task Paradigm

Measurement of working memory is based largely on the assumption that people are limited in the amount of information that they can process at one time (Johnston, Greenberg, Fischer, & Martin, 1970; Siegel & Ryan, 1989). If this assumption is correct, it should be possible to assess the limits of working memory by devising tasks to exceed the system's capacity. Furthermore, it should be possible to assess the components of working memory using tasks designed to selectively affect performance believed to be mediated primarily by a single component of working memory. Most often selective measurement of the components of working memory is achieved using some form of the dual-task paradigm.

Use of the dual-task paradigm is based on the assumption that if two tasks require the same processing codes, decrements in task performance will be greater than if two tasks require different processing codes. In the dual-task, subjects are asked to perform two tasks simultaneously: Typically, a preload in the form of a memory task is presented concurrently with, or immediately prior to, the presentation of an intervening activity. For example, Baddeley and Hitch (1974) used a concurrent dual task to investigate the role of working memory in reasoning. Subjects were auditorally presented with a series of digits and a series of sentences were presented visually for true/false verification. Subjects were to maintain the digits for subsequent recall while performing the verification task. The digit memory task was found to impair verification performance, particularly for linguistically complex sentences.

A second example of the dual-task paradigm is the reading-span task developed by Daneman & Carpenter (1980), who were interested in assessing the limits of memory in a context that involved the processing of text. These investigators developed the reading-span task in order to determine the relation between reading skill and memory span with a reading-specific task. Daneman and Carpenter had subjects read a series of sentences and then asked them to recall the final word of each sentence. The number of words correctly recalled defined an individual's reading span; this span measure was highly correlated with independent measures of reading comprehension. Note that the reading span task differs from traditional simple span tasks (e.g., digit span) in that a dimension of active processing is an integral part of the task.

A study conducted by Brooks (1968) provides a third example of the dual-task method. Brooks was interested in whether two tasks presented in the same modality would interfere with one another more than two tasks that were presented in different modalities. As in Baddeley and Hitch (1974), subjects were required to perform two tasks concurrently. There were two primary tasks: In the verbal primary task subjects read sentences silently while performing one of two secondary tasks, either (a) verbally identifying the nouns in the sentence by saying "yes" or "no" as they read, or (b) pointing to a written "Yes" or "No" as they read the sentence. In the spatial primary task subjects were presented with a capital letter (F or E), which was then removed. Subjects were instructed to visualize the presented letter while they performed one of two secondary tasks (a)

verbally identifying the internal and external corners of the letter, or (b) pointing to the internal and external corners. Brooks found that performance on the verbal primary task was poorer when the secondary task was verbal rather than spatial. Similarly, performance on the spatial primary task was poorer when the secondary task was spatial. These results provided evidence in support of the hypothesis that material is processed in a modality specific manner, and that resource limitations may also be modality specific.

In the model of working memory proposed by Baddeley (Baddeley, 1981, 1983, 1986; Baddeley & Hitch, 1974) a distinction is made between the central executive and the subsystems of working memory. In this model, all measures of working memory reflect, to some extent, the functioning of the central executive as well as the contributions of the subsystems. This poses a problem for the measurement of working memory: It can be difficult to determine the relative contributions of the central executive and the subsystems to task performance. The research of Engle and his colleagues (Engle, Nations, & Cantor, 1990; LaPointe & Engle, 1990; Turner & Engle, 1989) illustrates this difficulty.

Engle has suggested that working memory is task independent; that people with larger working memory capacities will perform well across a range of tasks (such as reading and mathematics) because their general working memory capacity is large, not because of task-specific processing requirements (Turner & Engle, 1989; Engle et al., 1990). This argument is based largely on the finding that measures of working memory capacity for college students correlate well with scores on

both the quantitative and verbal sections of the Scholastic Aptitude Test (e.g., Turner & Engle, 1989). This correlation holds regardless of whether the measure of working memory involves sentences or arithmetic operations. There are two problems with this research that weaken the argument made by Turner and Engle: First, both the quantitative and verbal subtests of the SAT assess general reasoning and analytic skills, in addition to domain-specific skills. Reasoning and logic presumably reflect substantial involvement of the central executive in addition to contributions from the subsystems. Second, their measures of working memory span included complex tasks that required processing of one type of material (e.g., arithmetic) and recall of another (e.g., words). Tasks requiring recall of words correlated most highly with verbal SAT (VSAT) scores whereas tasks requiring recall of digits correlated more highly with quantitative (QSAT) than with verbal SAT scores. Hence, it is possible that the overall correlations between working memory and both the QSAT and VSAT scores reflects central executive processing. The difference in magnitude of the correlations may reflect the operation of domain-specific processes controlled by the different subsystems of working memory.

It is difficult to evaluate the relative contributions of the components of working memory to measures of performance, particularly when the performance measures include a broad range of unspecified abilities. Thus, it is important that (a) the working memory task be dependent primarily on the processing of one of the components of working memory, and (b) the performance measure(s) must be chosen to

minimize the likelihood that performance is primarily dependent on general working memory capacity.

Current research suggests that working memory may be related to individual differences in reading skill (e.g., Baddeley, Logie, Nimmo-Smith, & Brereton, 1985; Daneman & Carpenter, 1980; Dixon, LeFevre, & Twilley, 1988; Siegel & Linder, 1984). For example, researchers have demonstrated that good and poor readers differ in their ability to recall the last word of each sentence in a series (Daneman & Carpenter, 1980). In contrast, little research has been designed to clarify the relation between working memory performance and arithmetic skill. There is, however, a considerable body of literature documenting a relation between visuo-spatial abilities and arithmetic skill (e.g., Battista, 1980; Greenes, 1979; McLeod & Crump, 1978). The relation between visuo-spatial ability and arithmetic skill suggests a possible relation between the scratchpad component of working memory and arithmetic performance that has yet to be explored.

The research outlined above provides support for the hypothesis that working memory comprises subsystems that are specialized for specific types of materials (i.e., verbal and spatial; Daneman & Carpenter, 1980; Brooks, 1967, 1968). This literature also suggests that the dual-task paradigm is useful for assessing the relation between the slave systems of working memory and performance on tasks believed to be mediated primarily by a specific slave system.

Working Memory and Cognitive Abilities

There is now strong evidence that the component of working memory that is of particular relevance to the study of reading is the articulatory loop (e.g., Baddeley et al., 1985; Daneman & Carpenter, 1980; Dixon et al., 1988; Siegel & Feldman, 1983; Siegel & Linder, 1984). In contrast to the area of reading, little research has been designed to explore the relation between working memory and arithmetic performance. Moreover, the majority of this research has been based on the assumption that the articulatory loop and central executive components of working memory are most directly involved in mediating arithmetic performance. However, the results of several of these studies are also consistent with the hypothesis that the visuo-spatial scratchpad component is involved in arithmetic performance. These studies are reviewed below.

Working memory performance and arithmetic skill. Researchers concerned with individual differences in arithmetic skill have recently attempted to use the model of working memory outlined above (Baddeley, 1979, 1986) to explore possible relations between working memory performance and arithmetic skill. This majority of this research has been conducted with subjects who represent the extremes of the continuum of arithmetic skill. That is, the arithmetic disabled and students who are mathematically gifted. The relatively restricted range of the population sampled must therefore be considered when evaluating the results of these studies.

Siegel and her colleagues (Siegel & Feldman, 1983; Siegel & Linder, 1984) have used the concept of working memory to design a

series of studies conducted with academically impaired children. These researchers have suggested that the articulatory loop may play a similar role in both reading and arithmetic. Siegel has suggested that the arithmetic sign and numbers of a problem are processed and stored in the articulatory loop, and that the executive system is used to retrieve the algorithm required to solve the problem (no role was posited for the visuo-spatial scratchpad).

To test the hypothesis that the articulatory loop is involved in arithmetic performance, Siegel and Linder (1984) presented a series of letters that were either phonologically similar (e.g., g, d, p) or dissimilar (e.g., a, l, n) to children with either a reading or an arithmetic disability¹. The letter strings were presented in both auditory and visual modes and the children were asked to recall as many letters as they could. Normally achieving children showed an effect of phonological similarity in both presentation conditions (i.e., they recalled fewer letters from the confusable than from the non-confusable sequence). In contrast, reading-disabled children were not sensitive to phonological similarity, regardless of whether the letters were presented visually or auditorily, suggesting some impairment of articulatory loop processes. Children with an arithmetic disability, however, were sensitive to phonological similarity in the auditory, but not the visual presentation condition.

The finding that phonological confusability differentiated the performance of arithmetic-disabled and reading-disabled children only in the visual presentation condition (Siegel & Linder, 1984) suggests that the visual aspects of the task may underly the differential

performance of learners with arithmetic and reading disabilities.

Siegel and Linder (1984) have suggested that the learning difficulties experienced by arithmetic-disabled children may be related to recoding from visual symbols to a verbal representation; presumably, recoding from a visual to a verbal format involves some communication between the visuo-spatial scratchpad and the articulatory loop.

Alternatively, the ability to recode may be intact, but some other aspect of the visuo-spatial scratchpad, such as holding spatial information in memory, may be impaired. The visuo-spatial scratchpad may play a role in mediating arithmetic performance, and children with an arithmetic disability may have particular difficulty with one or more aspects of visual encoding, storage, and manipulation.

Research conducted by Webster (1979, 1980) provides tentative support for this hypothesis. Webster (1980) found that, as compared to mathematically proficient students, children with arithmetic disability demonstrated short-term memory deficits under conditions of visual, but not auditory stimulus presentation. Taken together, the Siegel and Linder (1984) and Webster (1979, 1980) studies suggest that arithmetic-disabled children have particular difficulty in the processing of visual, but not linguistic (auditory) stimuli. This supports the hypothesis that the ability to process and/or store visual information is one aspect of arithmetic skill.

In a more recent study, Siegel and Ryan (1989) used two different working memory tasks in an effort to discriminate between children with reading and arithmetic learning disabilities. In the verbal memory task children were asked to generate the final word in a series

of sentences. On completion of the sentences the children were asked to recall the final word in each sentence; reading-span was defined as the number of words correctly recalled. The second memory task was the counting-span task developed by Case and Kurland (1978; cited in Case, 1985), in which children are asked to count the number of dots in a series of arrays. On completion of the series, the children are asked to recall the number of dots in each array of the series; counting-span is defined as the number of counts correctly recalled. Siegel and Ryan found that, in comparison with a normally achieving control group, children with a reading disability showed a memory deficit on both of the working memory tasks. In contrast, children with an arithmetic disability showed a deficit only on the dot counting task.

The finding of deficient working memory performance among reading-disabled children, regardless of the type of stimuli or the mode of presentation, suggests that reading-disabled children may have sub-optimal central executive processing (Siegel & Linder, 1984; Siegel & Ryan, 1989). Thus, it appears plausible that a general working-memory deficit is characteristic of children with a reading disability. In contrast, arithmetic-disabled children demonstrated relatively specific deficits, limited to visual stimuli (Siegel & Linder, 1984; Webster, 1979) and to tasks closely related to arithmetic skills (Siegel & Ryan, 1989). These results suggest that, in contrast to reading-disabled children, children with arithmetic difficulties may have relatively specific working memory deficits; deficits that may be related to the functioning of the visuo-spatial

scratchpad.

Although not specifically designed to do so, a study by Fletcher (1985) provides a test of the hypothesis that the visuo-spatial scratchpad is particularly involved in arithmetic performance. In this study, children with specific reading and arithmetic disabilities (defined on the basis of subtest scores on the WRAT²) completed a short-term memory task in which the stimuli were either lists of animal names, or random dot patterns. Fletcher found that the children with an arithmetic disability performed significantly more poorly than either controls or children with reading disability on the visual (dot pattern) task. In contrast, children with a reading disability performed more poorly on the verbal task. Because the dot pattern task is clearly visuo-spatial in nature, these results support the theoretical differentiation of the articulatory and visuo-spatial slave systems proposed by Baddeley (1979, 1986). Moreover, the specific memory performance deficit of the arithmetic-disabled subjects suggests that the visuo-spatial scratchpad may contribute to arithmetic performance. Involvement of the visuo-spatial scratchpad in arithmetic performance would be theoretically consistent with findings that children with arithmetic learning disabilities are often characterized by deficits on a variety of visuo-spatial tasks (e.g., Ozols & Rourke, 1988; Rourke, 1978, 1983; Rourke & Finlayson, 1978; Share, Moffit, & Silva, 1988; Van der Vlugt & Satz, 1985).

To date, no research has been conducted that is explicitly designed to test possible relations between performance in a particular domain (e.g., arithmetic) and the visuo-spatial scratchpad.

However, a study conducted by Phillips and Christie (1977) provides indirect evidence of a relation between arithmetic performance and the visuo-spatial scratchpad. These investigators found that performance on a visual tracking task deteriorated when visual tracking was combined with concurrent mental addition. The interference of mental addition with visual tracking suggests that both tasks share limited processing resources. Thus, from these results one could infer that visuo-spatial processing resources were required for both the visual tracking and the addition task, and that the dual-task exceeded the available resources of the visuo-spatial scratchpad.

In summary, although some studies have been designed to explore the relation between working memory and arithmetic performance (e.g., Hitch, 1978; Siegel & Feldman, 1983; Siegel & Linder, 1984) these studies have focussed on the central executive and articulatory loop components of working memory, rather than the visuo-spatial scratchpad. However, the results of studies in which both verbal and visuo-spatial working memory tasks were used have yielded results that are consistent with the hypothesis that visuo-spatial processes may mediate arithmetic performance. The plausibility of a relation between visuo-spatial processes and arithmetic performance is further supported by the results of studies demonstrating a relation between measures of spatial ability and arithmetic skill. These studies are reviewed below.

Spatial Ability and Arithmetic Performance

A recent series of studies conducted by Dark and Benbow (1990, 1991) have documented a relation between spatial ability and arithmetic performance among academically gifted students. Using the model of working memory proposed by Baddeley (1979, 1986; Baddeley & Hitch, 1974) as a conceptual framework, Dark and Benbow (1990, Experiment 2) compared the performance of mathematically gifted, verbally gifted, (defined on the bases of scores on the Scholastic Aptitude Test [SAT]³), and average seventh- and eighth-grade students, and average college students on a variety of memory tasks. Dark and Benbow included two traditional short-term memory tasks (a) a digit-span task, and (b) a spatial-span task (recalling a series of asterisks presented in a grid). They also included two continuous paired-associate tasks intended to assess working memory that involved the pairing of (a) letters and digits, and (b) letters and a location in a grid. On the spatial-span task the mathematically gifted children performed better than both their average ability and verbally gifted peers, and at a level similar to that of the college students. A similar pattern was obtained on the spatial continuous paired-associate task, although the college students consistently outperformed the mathematically gifted students.

Based on the results of the simple span and continuous paired-associate tasks, Dark and Benbow (1990) concluded that the mathematically gifted children were superior to their average and verbally gifted peers in their ability to maintain spatial information in memory (simple spatial span). Mathematically gifted students were

also superior to all groups except the college students in their ability to process spatial information, as demonstrated by their performance on the spatial continuous paired-associate task.

In a similar study involving mathematically and verbally gifted students, Dark and Benbow (1991) used the simple span and continuous paired-associate tasks described above, as well as a semantic priming task. They found that the mathematically gifted students outperformed the verbally gifted students with digit and spatial location stimuli, whereas the verbally gifted outperformed the mathematically gifted when verbal stimuli were used. Thus, even among a group of highly talented students, facility with spatial stimuli appears to be an important component of mathematical skill.

In summary, research with arithmetic-disabled and mathematically gifted children suggests that, among these exceptional learners, there is a relation between spatial processes and arithmetic performance. Children with arithmetic-specific learning disabilities demonstrate particular difficulty in manipulating visuo-spatial information, whereas mathematically gifted students show an above-average facility with these types of stimuli.

There is also evidence that spatial ability and arithmetic skill may be linked at all levels of academic skill, not just at the extremes of the continuum. Within the last two decades a body of research has emerged documenting a positive relation between visuo-spatial skills and arithmetic performance. Researchers working in the 1970's (e.g., Burnett, Lane, & Dratt, 1979; Eisenberg & McGinty, 1977; Fennema & Sherman, 1977a, 1977b; Harris, 1978; Sherman, 1979)

consistently reported a significant relation between measures of spatial ability and mathematics performance, with males typically performing better than females on both tasks. These findings were supported by the results of the Studies of Mathematically Precocious Youth (SMPY) conducted in the United States by Benbow and Stanley (1980). In a series of mathematics talent searches, these researchers observed a significant gender difference in mathematical ability in favor of boys. They concluded that the observed gender differences in mathematics achievement reflected a male superiority in mathematical ability. They further hypothesized that male superiority in mathematics was related to greater male ability in spatial processing tasks (Benbow & Stanley, 1980). This conclusion generated considerable debate (e.g., Kimball, 1989; Linn & Peterson, 1986) and led to further research concerning the hypothesized role of visual-spatial factors in mathematics performance and putative gender differences in performance.

Subsequent research has shown that evidence for gender differences in spatial visualization depends, in part, on the particular tasks used to measure this ability (e.g., Armstrong, 1981; Linn & Petersen, 1986; see Halpern, 1992 for an extensive evaluation of this literature). Consequently, conclusions concerning the relations among gender, spatial skills, and arithmetic ability must be evaluated with caution (Kimball, 1989).

Although findings concerning gender differences in spatial ability are equivocal, there does appear to be a relation between spatial ability and arithmetic performance that is independent of

gender. Spatial skills are consistently related to mathematics performance for both males and females. For example, Fennema and Sherman (1978) obtained measures of spatial ability and mathematics performance from boys and girls in grades 6 to 8. Although spatial ability did not differentiate between males and females it did correlate highly with mathematics ability (correlations ranged from .51 to .60). Bennett et al. (1974) found that the spatial subtest of the Differential Aptitude Test was significantly correlated with grades in school geometry ($r=.57$) and with a measure of quantitative thinking ($r=.69$). Similarly, Eisenberg and McGinty (1977) found that scores on a test of spatial visualization were higher among students in calculus courses than among students in other courses. Similar results have been obtained in other studies, indicating that spatial ability may be a significant predictor of mathematics performance (Fennema & Sherman, 1977; Sherman, 1980).

In sum, the research cited above provides strong evidence for the involvement of a spatial component in arithmetic processing. This research does not, however, provide strong support for the theory that observed gender differences in mathematics performance are necessarily a secondary consequence of gender differences in spatial ability (Halpern, 1992).

Skill Acquisition and Models of Arithmetic Performance

The relation between spatial processing and performance may depend on the level of arithmetic skill that has been achieved. The sequence of skills that children go through in achieving proficiency in arithmetic may vary in terms of the demands on spatial processes or

spatial memory: For example, counting both addends to solve an addition problem may place greater demands on spatial processes than counting only one of the addends. Below I will present a brief review of literature concerning children's acquisition of arithmetic skill, followed by a summary of two influential models of arithmetic performance.

Acquisition of Arithmetic Skill

The primary task facing the young learner of mathematics involves the acquisition of basic arithmetic facts (Ashcraft & Fierman, 1982; Kaye, Post, Hall, & Dineen, 1986; Siegler & Shrager, 1984). The acquisition process can be broadly characterized as a transition from procedural, counting-based, strategies to increasingly automatic retrieval of arithmetic facts. Considerable research has now been conducted concerning the sequence of children's acquisition of strategies for solving simple addition problems. This research suggests that young learners rely primarily on counting-based procedures that become increasingly sophisticated with age and experience (Groen & Parkman, 1972; Goldman, Pellegrino, & Mertz, 1988; Nesher, 1986; Siegler & Jenkins, 1989). Briefly, early in the developmental sequence a child might arrive at a solution using the sum strategy; counting-out each addend, and then re-counting to achieve the total. The shortcut-sum strategy is a variation of the sum strategy that involves counting-out each addend contiguously, so that it is not necessary to re-count to achieve the total. A more sophisticated and efficient counting-on procedure is the min strategy: Execution of this procedure requires that the child choose the larger

of the two addends and count on the number of times indicated by the smaller addend (Groen & Parkman, 1972; Siegler & Jenkins, 1989; see Nesher, 1986 for a review of these procedural models).

In the transition from counting-based strategies to automatic retrieval of arithmetic facts a relatively effortful procedural strategy (counting) gives way to an automatic process (fact retrieval) that presumably requires fewer cognitive resources. Achieving automatic access to arithmetic facts is believed to be important because when the resources required to solve an addition problem are reduced, resources are then available for tasks that require these facts, such as solving word problems (e.g., Kaye et al., 1986; Kaye, deWinstanley, Chen, & Bonnefil, 1989). In contrast to older children, young children may devote a large proportion of their processing resources to executing procedural strategies when solving addition problems. Thus, after solving a simple addition problem, young children may have few resources available for higher level problem-solving, such as regrouping for multicolumn addition problems. The spatial processing demands of arithmetic tasks may, accordingly, vary as a function of the stage of skill acquisition that has been reached (e.g., relying primarily on counting versus fact retrieval for problem solution).

In summary, the process of skill acquisition in arithmetic is characterized by a transition from reconstructive procedural strategies, such as counting, to increasingly automatic retrieval of arithmetic facts. Young learners rely primarily on procedural strategies to solve arithmetic problems (Goldman et al., 1988; Groen &

Parkman, 1972; Nesher, 1986). With age, and increasing skill, children increasingly rely on retrieval strategies to achieve problem solution (Cooney, Swanson, & Ladd, 1988; Siegler, 1988).

Theories of arithmetic performance are based on a body of research that documents several general effects that reflect the structural aspects of arithmetic problems (e.g., small problems are solved more quickly than large problems; Ashcraft, 1982; Groen & Parkman, 1972), and a number of models of arithmetic performance have now been developed (e.g., Ashcraft, 1982, 1987; Campbell, 1987; Campbell & Graham, 1985; Groen & Parkman, 1972; Siegler, 1988; Siegler & Shrager, 1984). These models will be reviewed briefly, with particular attention to Ashcraft's (1982, 1987) network retrieval model, and Siegler's (1988, Siegler & Shrager, 1984) distribution of associations model.

Models of Arithmetic Performance

Ashcraft (1982, 1987) has proposed a model of arithmetic performance in which basic number facts (e.g., $3 + 4 = 7$ and $3 \times 4 = 12$) are represented in a network of associative connections among numbers. Access to this network of associations is assumed to be automatic, and to be relatively undemanding of processing resources. It is also assumed that the strength of associative connections between problems and their sums is variable. The hypothesized variability in the strength of associative connections is necessary to account for a phenomenon known as the problem-difficulty effect: Subjects' arithmetic performance typically becomes slower and more error prone as the size of the problem increases (Ashcraft, 1992). In

his original model, Ashcraft (1982) proposed that adults achieve most solutions to arithmetic problems via direct retrieval. However, he also included a role for procedural strategies which, in his view, were activated in parallel with retrieval. Because retrieval is faster than procedural strategies, problems will generally be solved via retrieval. In this view, procedural strategies will be used primarily for (a) solving "exception" problems (e.g., $N + 0 = N$ or $N \times 0 = 0$), or (b) in the event of retrieval failure.

In his original formulations of the network retrieval model (1982, 1987), the role posited by Ashcraft for procedural knowledge was primarily that of a backup strategy. In contrast, perhaps because he was working from a developmental perspective, Siegler (1988, Siegler & Shrager, 1984; Siegler & Jenkins, 1989) posited a more central role for procedural strategies in his model of children's arithmetic performance. In Siegler's view, a memorial representation of arithmetic facts develops via the solutions that children achieve using various procedural solution strategies (e.g., the sum and min strategies). In this model any solution that a child achieves has some effect on the memory representation; that is, a connection is established between the problem and the achieved sum (whether correct or not). Ideally, with practice the association between a problem and its sum becomes "peaked", and is stronger than the associations between the same problem and other possible answers. However, because of factors such as individual learning histories and frequency of problem presentation, not all distributions will achieve the same level of "peakedness"; that is, problems will differ in terms of the

strength of their connections with their sums. Even in the case of a relatively peaked (i.e., leptokurtic) distribution there is a role for procedural strategies. The probability that a sum will be retrieved and stated (versus solved via a procedural strategy) is assumed to be a function of two factors; (a) the peakedness of the association, and (b) whether the associative strength exceeds a response threshold, or confidence criterion (Siegler, 1987, 1988; Siegler & Jenkins, 1989).

For the purposes of this paper, a central aspect of Siegler's work concerns the ways that strategies are chosen to solve particular problems. In the initial formulation of his model, procedural strategies were invoked only when retrieval strategies failed to generate answers that exceeded the confidence criterion (Siegler, 1988). More recently, Siegler and Jenkins (1989) have posited that strategies, as well as sums, are connected via associative links to specific problems, to particular classes of problems, and to arithmetic problems as a whole. The strength of connections between a strategy and various problems is believed to be dependent on the speed and accuracy with which it has been used to solve problems in the past. Moreover, even young children are remarkably adept at matching problem characteristics with effective solution strategies (Siegler & Jenkins, 1989). In this theoretical extension, procedural strategies have equal status with retrieval strategies, rather than being relegated to a back-up role.

On the whole, investigators working in the area of mathematical cognition have been remarkably reluctant to extend the logic of Siegler's work on strategy choice from children to adults. The

dominant assumption is that children use procedural strategies whereas adults retrieve; this assumption is a feature of most models of adults' arithmetic performance (e.g., Ashcraft, 1982, 1987; Campbell, 1987; Campbell & Graham, 1985; Widaman, Geary, Cormier, & Little, 1989). If one extends Siegler's hypotheses regarding a distribution of associations among problems and strategies to adults, a logical possibility is that adults may also solve some proportion of problems using a procedural, rather than a retrieval, strategy. One of the methodological issues that Siegler's work has highlighted is that misleading conclusions may arise when solution latencies are averaged over different solution strategies. It was this technique of averaging over strategies that had obscured the variability in children's strategy choices in addition (and other domains, such as reading; Siegler, 1987, 1988), and average solution latencies are used almost exclusively to study arithmetic performance among adults. Although there is very little research concerning the strategies used by adults to solve addition problems, the research that has been done suggests that adults do occasionally use procedural strategies to solve addition problems (Geary & Wiley, 1991, 1993; LeFevre, Sadesky, & Bisanz, 1993; Svenson, 1975). Thus, adults may not necessarily retrieve the sums to all problems, particularly if they are asked to solve problems more complex than the single-digit problems typically used to study arithmetic processing.

Most models of arithmetic performance have been developed to account for the relation between performance on simple (single-digit) addition and multiplication problems, and various structural

properties of these problems (e.g., the magnitude of the sum or minimum addend). Relatively little research has been directed toward understanding the processes used to solve more complex problems. However, Widaman and his colleagues (Widaman et al., 1989; Widaman, Little, Geary, & Cormier, 1992) have proposed a model for addition that includes parameters to account for the solution of multidigit problems (e.g., parameters to represent encoding, retrieval, and the presence or absence of a carry operation). More specifically, their model includes a parameter to account for the additional time needed to execute a carry operation, as well as parameters to estimate the time to retrieve (or count) the sum in the ones' and tens' columns (in addition to parameters specific to the verification task that they used). For the present purposes, there are two important results of their research. First, the results of Widaman et al. (1989) support the logical assumption that problems involving a carry operation are solved more slowly than no-carry problems: Both children and adults solved carry problems more slowly than no-carry problems. Second, in testing the fit of alternate models to the RT data, the model that included a parameter consistent with retrieval (the column product) fit the obtained data only modestly better than did the model that included a parameter consistent with a counting strategy (the minimum addend; R^2 s of .742 and .741 respectively). Thus, from these results one can conclude that double-plus-single digit problems with a carry operation are more difficult than problems without a carry operation; however, it is less clear whether subjects consistently use retrieval strategies to achieve problem solution.

Additional Considerations: The Role of the Visuo-spatial Scratchpad in Skill Acquisition

It is generally accepted that there is an age-related increase in short-term memory span. This increase has been interpreted as reflecting either increased storage capacity, or increased efficiency of control processes (Chi, 1976). Case, Kurland, and Goldberg (1982) suggested that the age-related increases in counting span that they observed reflect increases in the efficiency of counting operations, rather than increases in total processing capacity. As operational efficiency increases, the amount of space available for storage also increases. Hitch and Halliday (1983) designed a series of experiments designed to examine developmental changes in the components of working memory (i.e., the articulatory loop and visuo-spatial scratchpad). Using a picture naming task, Hitch and Halliday demonstrated that older children were sensitive to phonemic similarity, whereas younger children were more sensitive to visual similarities. On the basis of these results they suggested that younger children may be more reliant on processes associated with the visuo-spatial scratchpad, perhaps because they are generally less adept with linguistic material.

Hitch and Halliday (1983) suggested that a better understanding of developmental changes in the subsystems of working memory might contribute to an understanding of more general cognitive abilities, such as reading and arithmetic. Clearly, working memory capacity places constraints on skill acquisition in development, and identifying these constraints may improve our understanding of the processes involved in skilled performance.

The role of working memory in mediating skill acquisition may also vary developmentally because children at different stages of skill acquisition also vary in the strategies that they use to solve problems. As outlined above, skill acquisition involves a transition from the primary use of procedural solution strategies to increasing reliance on direct retrieval of arithmetic facts (Ashcraft & Fierman, 1982; Goldman et al., 1988; Kaye et al., 1986; Siegler & Shrager, 1984; Siegler & Jenkins, 1989). Procedural strategies are assumed to be more demanding of cognitive resources than are retrieval strategies (although this assumption is debatable; Ashcraft, Donley, Halas, & Vakali, 1992). Moreover, even within the class of procedural strategies, procedures may vary in terms of processing demands. For example, use of the sum strategy (counting-out each addend, and re-counting to achieve the total) may require more cognitive effort than does use of the min strategy (counting-on from the larger addend the number of times indicated by the smaller). Thus, working memory demands may depend on the sophistication of the procedural strategies that a child uses. The spatial processing demands of arithmetic tasks may, therefore, vary as a function of the type of procedural solution strategies that children use (e.g., the sum strategy may draw more from the resources of the VSSP than the min strategy). Older and more skilled children generally rely on relatively sophisticated procedural strategies (e.g., the min strategy). Therefore, the spatial processing demands of arithmetic may be less for skilled and older children than for younger or less-skilled children.

Summary of the Literature on Skill Acquisition and Arithmetic
Performance

What is perhaps most clear in the review of the literature on mathematical cognition is the richness of the domain. There are a number of variables that contribute to the characteristics of individuals' performance on arithmetic problems. Generally, large problems take longer to solve than small problems (Ashcraft, 1982, 1987), and multidigit problems are more complex than single digit problems (Widaman et al., 1989, 1992). Moreover, research with children indicates that they are flexible problem solvers, who adjust their strategy choices to accomodate problem characteristics (Siegler, 1988, Siegler & Shrager, 1984, Siegler & Jenkins, 1989). Although very little research has explored adults' strategy choices (but see LeFevre et al. 1993; Geary & Wiley, 1991, 1993; Svenson, 1975), it seems plausible that adults may be similarly adaptive problem solvers. Variability in the solution strategies used by adults may be more evident when they are asked to solve multidigit, rather than single-digit addition problems. Moreover, solving multidigit problems may be more demanding of visuo-spatial codes than are single-digit problems. Thus, in the present research, there were three broad questions; (a) does a visuo-spatial code mediate adults' arithmetic performance?, (b) does strategy choice vary among adults as a function of subject and problem characteristics?, and (c) is visuo-spatial mediation central to children's arithmetic performance?

Overview of the present research

The empirical findings of a relation between spatial skill and arithmetic performance, as well as theoretical considerations, suggest that the visuo-spatial scratchpad may be involved in arithmetic performance. One method that can be used to assess the relation between spatial processing and arithmetic performance involves the use of a concurrent processing load or dual-task that would, theoretically, be expected to interfere specifically with arithmetic processing. Specifically, if spatial processing is involved in arithmetic performance, then a spatial-load task would be expected to interfere with arithmetic performance. If adults use visual imagery to solve complex problems (Hayes, 1973; Hitch, 1978), then solution of carry problems may depend more on a visuo-spatial code than does solution of no-carry problems. Moreover, less-skilled adults may rely relatively heavily on procedural solution strategies; they may, therefore, find the combination of carry problems in the context of a heavy spatial working memory load more difficult than do skilled adults.

In the present studies a dual-task methodology was used to assess the relation between the visuo-spatial component of working memory and arithmetic skill. Subjects were asked to complete arithmetic problems concurrently with either a verbal or a spatial memory preload task. Thus, it was possible to evaluate the hypothesis that spatial processes mediate arithmetic problem solving. Complex addition problems were used as the intervening task. For the present purposes, complex addition was defined as a double- plus a single-digit number

(e.g., $23 + 5$). The complex addition problems also varied in terms of whether a carry operation was required. This manipulation allowed a comparison of the processing demands of these two types of complex problems. Because the spatial processing requirements of arithmetic may also vary with age and skill, the first two experiments were conducted with adults, whereas sixth grade children participated in the final experiment.

The experiments presented below were designed to: (a) determine whether a visuo-spatial memory preload interferes more with arithmetic performance than does a verbal memory preload, (b) evaluate whether a visuo-spatial memory preload differentially affects arithmetic procedures varying in complexity (i.e., complex addition with or without a carry operation), and (c) determine whether the effects of a visuo-spatial preload differ with age and skill. It was assumed that performing the visuo-spatial memory task requires processing codes associated with the visuo-spatial scratchpad component of working memory. Consequently, any interaction of the spatial memory task with arithmetic performance was assumed to reflect an underlying relation between the visuo-spatial scratchpad and performance of the arithmetic task.

INTRODUCTION TO THE EXPERIMENTS

Pilot Research

In a series of pilot studies, hypotheses concerning the relation between the VSSP and arithmetic performance were assessed by having subjects complete arithmetic problems concurrently with either a verbal or a spatial memory task. Because the results of one of these studies generated further predictions, those results will be briefly reviewed below.

In this pilot study adult subjects solved complex addition problems while maintaining either (a) a verbal working memory load, or (b) a spatial working memory load. Within each type of memory task (verbal or spatial) there were two levels of memory load difficulty (low or high). In addition, the arithmetic problems were presented at two levels of difficulty; carry (e.g., $36 + 7$) and no-carry (e.g., $36 + 2$). This methodology is described in detail in the methods section of Experiment 1. It was hypothesized that (a) the spatial task would interfere more with arithmetic performance than would the verbal memory task, (b) interference would be greater for carry than for no-carry problems, (c) a high memory load would interfere more than a low memory load with arithmetic performance, and (d) interference would be greater for less-skilled than for skilled subjects, especially in the spatial memory condition.

In general, these hypotheses were confirmed. There was however, a striking and unexpected finding; females showed a decrease in latencies to the carry problems as the difficulty of the spatial memory load was increased. This is the condition where resource

demands on the VSSP would be expected to be greatest. Paradoxically, it appeared that taxing the spatial processing resources of these subjects actually enhanced their performance in the most demanding condition. Because the females in this sample were also less skilled than the males, this finding may reflect differences due either to gender or to skill. If this finding is related to skill, the unexpected reduction in latencies may be due to skill differences in the strategies used to solve addition problems: That is, strategy use may interact with memory task and load in terms of the resource demands placed on the VSSP. This hypothesis is discussed in more detail below.

General Predictions

Current research and theory suggests two alternative assumptions concerning the interaction of visuo-spatial memory resources and arithmetic problem solving. In the first instance, arithmetic problem solving is assumed to require general, but not code-specific working memory resources. If this assumption is correct, then it follows that there should be undifferentiated interference from both the verbal and spatial working memory tasks. Moreover, in this scenario there is no reason to hypothesize that interference for skilled and less-skilled subjects would interact with memory task: That is, interference may be greater for less-skilled than for skilled subjects, but there should not be a skill by memory task interaction.

The alternative assumption is that arithmetic problem solving makes specific demands on codes associated with the VSSP. If this hypothesis is correct, the following predictions can be made. First,

the spatial memory task should produce greater interference than the verbal memory task. Second, if basic facts are largely automatized, complex addition that does not involve a carry operation should be less affected than carry problems by the additional demands placed upon the VSSP. Third, if less-skilled subjects are more reliant on spatially mediated solution strategies than are skilled subjects, interference should be especially pronounced for less-skilled subjects in the spatial memory task condition.

The results of the pilot study suggest some additional predictions based on hypotheses concerning how skilled and less-skilled subjects solve arithmetic problems. It is possible that less-skilled subjects rely relatively heavily on procedural strategies to solve arithmetic problems, whereas skilled subjects rely primarily on automatic retrieval of arithmetic facts. Automatic fact retrieval is believed to place few demands on processing resources (e.g., Ashcraft, 1982; LeFevre, Bisanz, & Mrkonjic, 1988; Logan, 1978; Logan & Klapp, 1991; Kaye, deWinstanley, Chen, & Bonnefil, 1989). In contrast, procedural strategies are, by definition, effortful and are believed to require working memory resources (LeFevre & Kulak, 1993), perhaps a visuo-spatial code associated with the VSSP. Moreover, the strength of associative connections between arithmetic problems and their sums appears to vary among adults as a function of their level of arithmetic skill, with more-skilled subjects showing greater evidence of automatic fact retrieval (LeFevre & Kulak, 1990; LeFevre, Kulak, & Bisanz, 1991).

In the pilot study, female subjects (who were also less skilled) showed improved performance (i.e., faster latencies) in precisely that condition where the demands on a visuo-spatial code would be expected to be greatest: That is, the combination of carry problems with a high spatial memory load. One plausible explanation for the improved performance in this condition is that less-skilled subjects switch strategies when the task demands overwhelm the resources of the VSSP. Specifically, one way to reduce demands on a visuo-spatial code would be to switch from a procedural solution strategy (which may rely on a visuo-spatial code) to direct fact retrieval. Thus, in the experimental condition that places the greatest demands on VSSP resources, subjects who typically rely on procedural solution strategies may, because of resource depletion, adopt a retrieval-based solution strategy. That is, to minimize the degree of overlap in processing requirements of the two tasks, subjects may switch from a spatially mediated procedural strategy to automatic fact retrieval (see Navon & Gopher, 1979 for a discussion of flexible resource use). Based on these hypotheses regarding the resource demands of retrieval and procedural solution strategies the following prediction can be made: Less-skilled subjects will show improved (faster) arithmetic performance in the spatial memory, high load, carry problem condition. Further, this improvement will be accompanied by a relative shift in strategy use; from procedural to retrieval strategies. Because skilled subjects presumably rely on retrieval strategies, it is not expected that their arithmetic performance will be affected differentially as a function of memory task, load, and problem type.

Overview of the Experiments

The first experiment was designed to evaluate the relative contributions of codes associated with the articulatory loop and the visuo-spatial sketchpad to arithmetic performance among adults. Specifically, the goal was to determine whether visuo-spatial codes play a greater role in mediating performance on complex arithmetic problems than do verbal codes. The second experiment was designed to explore the types of strategies that adults use to solve complex problems, and to evaluate the possibility that adults' strategy use may vary as a function of task demands. Finally, the third experiment was designed to evaluate the relation between visuo-spatial processing and performance on complex arithmetic problems among sixth grade children.

EXPERIMENT 1

Purpose

The purpose of this experiment was to explore the relations between the components of working memory and performance on complex addition problems (i.e., multidigit problems with and without a carry operation). Researchers have hypothesized that automatic access to basic facts frees working memory capacity to deal with the more involved procedural aspects of complex arithmetic problems (Geary, Widaman, Cormier, & Little, 1987; Goldman, Pellegrino, & Mertz, 1988; Hitch, 1974). Hayes (1973; see also Hitch, 1978; Webster, 1974) also found that adults report greater use of spatial symbolism with complex as compared to simple addition, suggesting that visuo-spatial representations may be used as memory aids (e.g., recording carries) in the execution of complex addition problems. Both these sources of evidence suggest that visuo-spatial codes may be heavily taxed in the solution of complex addition problems. Consequently, this experiment is designed to evaluate the relative contributions of verbal and visuo-spatial codes associated with working memory to performance on a complex addition task.

Method

Pretest selection

Participation was solicited from introductory psychology students who had previously completed a pretest package that included a speeded test of arithmetic fluency (the French Kit; French, Ekstrom, & Price, 1963). This test comprises four pages of problems: Two pages of addition (120 problems) and two pages of subtraction and

multiplication problems (60 problems of each type). A more detailed description of this test, and representative test items, are presented in Appendix A.1. The total number of items correct was used as a measure of arithmetic fluency, and students were rank-ordered according to their obtained score. Because the goal was to obtain a subject sample with two distinct levels of fluency (skilled and less-skilled), subjects' participation was solicited based on fluency scores. To obtain a skilled group, participation was solicited by starting with the highest scoring student and working down the list. For the less-skilled group, participation was solicited from students who were near the bottom of the list, and working up the list. Students with scores less than 35 were not contacted, on the assumption that these extremely low scores may be invalid for reasons unrelated to skill (e.g., motivation, distraction, misunderstanding the test instructions). Fluency test scores for the full sample, for the skilled and less-skilled groups, and for males and females in each skill group are presented in Appendix A.2.

Subjects

Forty-eight subjects (24 males, 24 females) participated in this experiment. Using the fluency scores as outlined above, the participation of 12 males and 12 females in each of the two fluency groups was obtained. Subjects received either course credit or a five-dollar payment for their participation.

Experimental tasks

In conjunction with a multidigit arithmetic task, subjects completed (a) a spatial memory task that involved remembering the positions of

asterisks in a random pattern, and (b) a verbal memory task that involved remembering a list of pronounceable non-words. These memory tasks were each presented at two levels of load difficulty; low (4 asterisks and 2 non-words respectively), and high (8 asterisks and 4 non-words). Following presentation of the memory preload subjects solved a double- plus single-digit addition problem. The addition problems were presented at two levels of difficulty; with or without a carry operation. In pilot testing using a "resource neutral" tone identification task, it was determined that latencies to the tone identification task increased as a function of these load levels. With this neutral (tone) primary task latencies were longer in the verbal than in the spatial memory task condition, suggesting that the spatial was somewhat easier than the verbal memory task. This finding is not problematic for the present experiments because the initial difference in task difficulty works against the hypothesis that the spatial task will result in more interference than the verbal memory task.

All tasks were presented on a computer. A graphic representation of the task sequence is shown in Figures 1(a) and (b). For the spatial memory preload subjects were presented with a pattern of asterisks. After a short time interval (2,000 ms) the pattern disappeared from the screen and an arithmetic problem was presented. After the subject answered the arithmetic problem aloud, a second pattern of asterisks (the probe) appeared and subjects decided whether the probe pattern exactly matched the preload pattern. For half the problems the probe pattern matched the preload pattern and "yes" was the correct response. Subjects indicated their response to the memory task by pressing the

appropriate button on a three key response panel. Thus, subjects had to remember the spatial pattern formed by the asterisks while they completed the arithmetic task. The patterns of asterisks were generated randomly within a five-by-five matrix, with the two following constraints (a) the pattern was not easily identified using a verbal label (e.g., a star, a letter shape), and (b) the non-matching preload and probe patterns differed only in the position of one asterisk.

For the verbal memory task a memory preload of pronounceable non-words (e.g., cav, boj, hap, nar) was presented, followed by an arithmetic problem. After solving the arithmetic problem subjects were presented with a single non-word probe and were asked to decide whether it had been in the initial list. The verbal memory items were simple consonant-vowel-consonant (cvc) combinations. The cvc's were constructed so that they did not spell a word, proper name, or common abbreviation. The cvc's appeared in sets of two or four as the memory preload items. Each cvc combination was used only once, and cvc's that rhymed were not included in the same set. For the memory preload the cvc's were presented in uppercase (e.g., CAV, NAR, ROG, ZAM), whereas the cvc probe was presented in lowercase letters (e.g., cav). This was to minimize the possibility that subjects would use a visual matching strategy to perform the verbal memory task.

In the instructions, subjects were asked to place equal emphasis on the memory and arithmetic tasks. Further, subjects were explicitly told to rehearse the verbal memory items subvocally and to retain a visual image of the pattern for the spatial memory task.

The arithmetic problems were double- plus single-digit problems; 12 of the problems involved carrying (e.g., $23 + 8$) and 12 had no carry operation (e.g., $23 + 5$). All sums were under 60 (range; 24 - 54 for no-carry and 27 - 59 for carry problems), and the carry and no-carry problem sets had an equal number of sums in the same decade. Each problem was presented twice; once with the larger digit first (e.g., $27 + 6$) and once with the smaller digit first (e.g., $6 + 27$). No problems involving the addition of one (e.g., $26 + 1$) or with ties as part of the problem (e.g., $26 + 6$) were included. Two equivalent lists of the same 48 addition problems and memory items were generated randomly within the following constraints; (a) there was at least one problem intervening between the two orders of the same problem (e.g., $23 + 8$ and $8 + 23$), (b) the two orders of the same problem did not have the same type of memory probe (i.e., true or false) (c) the same numeral did not appear more than twice in succession, (d) the same problem type (carry or no-carry) did not appear more than twice in succession, and (e) the same probe response (true or false) was not correct more than three times in succession. The complete set of arithmetic problems is presented in Appendix B. In a session lasting approximately 45 minutes, each subject completed one list of arithmetic problems in each of the two memory conditions. List order was counterbalanced across memory task, skill, and gender. The experimental trials were preceded by ten practice trials, which were repeated if the subject achieved fewer than 7 of 10 problems correct or felt uncomfortable with the response procedure.

Results

Subjects were preselected based on their total score on the arithmetic fluency test. Based on this score subjects were classified as either skilled or less-skilled in arithmetic. These skill groups were balanced with respect to gender (12 males and 12 females per group). The mean, standard deviation, and range of scores on this test, for each of Experiments 1 through 3, is presented in Appendix A.2.

It was hypothesized that (a) the spatial memory task would interfere more with arithmetic performance than would the verbal memory task, (b) interference would be greater for carry than for no-carry problems, (c) a high memory load would interfere more with arithmetic performance than a low memory load, (d) interference would be greater for less-skilled than for skilled subjects, especially in the spatial memory condition, except that (e) interference would be reduced for less-skilled subjects in the spatial memory, carry problem, high load condition relative to the low load condition. To test these hypotheses, median response latencies (correct responses only) and percentage of errors to the arithmetic problems were analyzed in separate ANOVAs with two between-subjects factors; 2 (arithmetic skill; high or low) $\times$ 2 (gender; male or female), and three repeated measures, 2 (problem type; carry or no-carry) $\times$ 2 (memory task; verbal or spatial) $\times$ 2 (memory load; low or high). These results are presented below.

Analysis of latencies: medians

Skilled subjects responded more rapidly than less-skilled subjects (1547 versus 2240 ms; $F(1,44) = 25.19$, $p < .001$, $MSe = 46,004,166$). This finding is consistent with the assumption that skilled subjects are relying relatively more on retrieval (a fast strategy) than are less-skilled subjects. However, it is also possible that skilled subjects are simply more efficient than less-skilled subjects in executing the same strategy. Carry problems were responded to more slowly than were no-carry problems (2261 versus 1526 ms; $F(1,44) = 150.49$, $p < .001$, $MSe = 51,698,558$). This finding indicates that, as expected, carry problems required more processing than no-carry problems. Surprisingly, problems were solved somewhat more quickly in the high than in the low load condition (1876 versus 1911 ms), $F(1,44) = 4.69$, $p < .05$, $MSe = 115,787$. However, this effect was subsumed under a significant interaction of memory task and load ($F(1,44) = 15.73$, $p < .001$, $MSe = 319,358$). As shown in Table 1, latencies in the verbal condition showed a moderate increase (23 ms) from the low- to high-load conditions. However, in the spatial memory condition latencies decreased significantly (94 ms; $F(1,44) = 5.15$, $p < .05$) across load. Thus, it appears that performance is facilitated in the spatial - high load condition relative to the low-load condition. This finding suggests that taxing subjects' spatial processing resources beyond some critical level may actually enhance their performance on a relatively complex arithmetic task.

There was also a significant interaction of skill and problem type, $F(1,44) = 9.16$, $p < .01$, $MSe = 425,334$. As shown in Table 2,

skilled subjects took longer to solve carry than no-carry problems (554 ms), but this effect was larger for less-skilled subjects (915 ms; both simple main effects were significant, $F(1,44) = 10.72$ and 29.24 respectively, $p < .01$). In addition, there was an interaction of memory task and problem type, $F(1,44) = 5.01$, $p < .05$, $MSe = 138,472$. As shown in Table 3, latencies to no-carry problems were somewhat faster (24 ms) in the spatial than in the verbal condition, whereas latencies to carry problems were slower in the spatial than in the verbal condition (a difference of 54 ms; neither simple main effect was significant). This result provides some support for the hypothesis that interference resulting from the spatial task would be especially pronounced for carry problems. In addition, there was an interaction of gender and load, $F(1,44) = 11.17$, $p < .01$, $MSe = 275,525$. As shown in Table 4, latencies for males increased slightly across load (19 ms), whereas latencies for females decreased as a function of load (90 ms; this decrease in latencies approached significance, $F(1,44) = 3.51$, $p < .10$).

The two-way interactions were, however, qualified by two three-way interactions. There was a significant interaction of gender, memory task, and problem type, $F(1,44) = 14.85$, $p < .001$, $MSe = 410,424$. As shown in Table 5, males showed a similar pattern of increased latencies across problem type in the two memory conditions, whereas females showed a greater increase in latencies as a function of problem type in the spatial than in the verbal memory condition. This suggests that interference produced by the spatial memory task was somewhat greater for females than for males.

The interaction of gender, problem type, and load was also significant, $F(1,44) = 10.80$, $p < .01$, $MSe = 227,176$. As shown in Table 6, males showed a modest decrease in latencies to no-carry problems (14 ms) and a modest increase in latencies to carry problems (52 ms) across load. In contrast, females showed a pattern of decreased latencies to both no-carry and carry problems as a function of load, with a particularly pronounced decrease in latencies to carry problems (-152 ms). Thus, females' performance was actually enhanced in the carry problem - high load condition.

There were also two significant four-way interactions. First, there was an interaction of skill, gender, memory task, and problem type, $F(1,44) = 4.44$, $p < .05$, $MSe = 122,765$. As shown in Figure 2, latencies increased as a function of problem type for male and female subjects, regardless of skill, in both the memory task conditions. However, the magnitude of the increase in latencies was greatest for less-skilled females in the spatial task condition: This interaction is due to the greater slowing of latencies for less-skilled females in the spatial task - carry problem condition. Second, there was an interaction of skill, gender, memory task, and load, $F(1,44) = 4.34$, $p < .05$, $MSe = 89,426$. As shown in Figure 3, in the verbal task condition the pattern of latencies for skilled and less-skilled males and females was essentially the same across load. However, in the spatial task condition the pattern for males and females was somewhat different. Less-skilled males showed essentially no change in latencies as a function of load, and skilled males showed a moderate (nonsignificant) decrease in latencies. In contrast, less-skilled

females showed a significant decrease in latencies as a function of load, $F(1,44) = 34.1$, $p < .001$. This interaction suggests that less-skilled females benefit from a high spatial memory load.

Planned Comparisons. On the basis of the results of the pilot study it was hypothesized that less-skilled subjects might show a decrease in latencies in the spatial - carry - high load condition. Therefore, planned comparisons were conducted to compare median latencies across load for skilled and less-skilled males and females in each of the memory task and problem type conditions (e.g., the difference in latencies across load in the spatial - no-carry condition). Presented in Figures 4a and 4b are median latencies to carry and no-carry problems across load, at each combination of memory task and subject skill. In the spatial task - carry problem condition less-skilled females showed a significant decrease in latencies across load (354 ms), $F(1,44) = 17.31$, $p < .001$; none of the other simple main effects of load were significant. Thus, it appears that the reduction in latencies observed for less-skilled females is primarily due to a decrease in the spatial task - high load - carry problem condition.

The results of the planned comparisons indicate that the facilitation in the spatial - high load condition observed for females reflects a benefit to less-skilled females solving carry problems. Why would less-skilled subjects benefit in precisely that condition that would intuitively be most difficult for them? One possibility is that their enhanced performance in this condition reflects a change in the strategies that they use to complete the task. If less-skilled

subjects switch from strategies that are demanding of visuo-spatial codes to less demanding strategies, the resulting decrease in resource demands may be reflected in faster latencies. This hypothesis will be discussed in more detail in the summary of results.

Analysis of errors

More errors were made to carry (14.5%) than to no-carry problems (3.9%), $F(1,44) = 45.01$, $p < .001$, $MSe = 10,819$, and more errors were made in the high load (10.2%) than in the low load (8.2%) condition ($F(1,44) = 7.53$, $p < .01$, $MSe = 380$). Less-skilled subjects made more errors (10.4%) than skilled subjects (7.6%); this effect approached significance, $F(1,44) = 3.55$, $p = .06$, $MSe = 948$.

There was also a significant interaction of skill, memory task, and load, $F(1,44) = 4.98$, $p < .05$, $MSe = 288$. As shown in Table 7, in the verbal task condition errors increased as a function of load for both skilled and less-skilled subjects (this increase in errors was significant for less-skilled subjects, $F(1,44) = 6.27$, $p < .05$). In contrast, in the spatial task condition errors were stable across load for both skilled and less-skilled subjects. The stability of errors across load for less-skilled subjects in the spatial condition is interesting in light of the decrease in latencies across load in this condition. It appears that the observed decrease in latencies among less-skilled subjects cannot be attributed to a speed-accuracy trade-off on the arithmetic task in the spatial-high load condition.

It is possible that the decrease in latencies and the stability of errors across load in the spatial condition reflect a trade-off in performance between the arithmetic and the memory tasks: Performance

may improve because subjects devote fewer resources to completing the memory task. To evaluate this hypothesis percent errors to the memory probe items were analyzed in a 2 (gender) x 2 (skill) x 2 (memory task) x 2 (problem type) x 2 (load) repeated measures ANOVA. In this analysis there was no interaction of skill, gender, memory task, and load or of skill, gender, memory task, and problem type. The percentage of errors to the memory probe items for skilled and less-skilled females in the spatial task condition are presented across the load and problem type variables in Table 8. As compared to skilled females, less-skilled females in the spatial task condition made more errors to the memory probes in both the no-carry problem conditions, and in the carry problem - low load condition. In contrast, in the carry problem - high load condition the error rate for skilled and less-skilled subjects was essentially the same (31.2 and 30.5% errors respectively). Less-skilled female's arithmetic performance improved in the most demanding spatial memory condition, but these subjects did not make more errors on the memory probe items. Thus, this analysis does not support the hypothesis that the improved arithmetic performance of less-skilled females was due to a speed-accuracy trade-off on the spatial memory task.

Another way to evaluate the possibility of a speed-accuracy trade-off is to examine the correlations between latencies to the arithmetic problems and errors to the memory probe items. A negative correlation between problem latencies and probe errors in the same condition would suggest the presence of a speed-accuracy trade-off between the arithmetic and memory tasks. In the sample as a whole

there were no significant correlations between latencies to the arithmetic problems and errors to the probe items in any condition (correlations ranged from .06 to .12 in the spatial task condition, and between .09 and .16 in the verbal task condition). Correlations between latencies and errors for each of the four combinations of gender and skill were also non-significant.

Summary of results: Experiment 1

The results of this experiment support several of the main hypotheses. As expected, latencies were longer for carry than for no-carry problems (suggesting that carry problems require more processing), and this effect was larger for less-skilled than for skilled subjects. One of the most intriguing findings was that in the spatial, but not the verbal condition, latencies decreased as a function of load: This is attributable to a decrease in latencies among less-skilled female subjects. Moreover, the results of the planned comparisons show that the benefit to less-skilled females is due to faster latencies to carry problems in the spatial - high load condition; precisely that condition where the demands on the VSSP would be expected to be greatest. In combination, these results suggest that taxing the resources of the VSSP beyond some critical level results in enhanced performance, particularly for less-skilled subjects. This improvement in performance cannot be explained on the basis of a speed-accuracy performance trade-off on either the arithmetic or the memory task.

One explanation for these findings involves the strategies that subjects may use to solve these complex addition problems. Procedural solution procedures are assumed to be resource demanding (Geary et al., 1987; Goldman et al., 1988; Hitch, 1974), and may involve visuo-spatial codes (Hayes, 1973; Hitch, 1978; Webster, 1974). In contrast, retrieval strategies are believed to make few demands on processing resources. The demands on a visuo-spatial code resulting from the combination of procedural strategies, a high spatial memory load, and a carry problem may exceed the codes available in the VSSP. Thus, subjects who rely heavily on procedural strategies (less-skilled subjects) are most likely to exceed the available codes of the VSSP. If these subjects switch strategies, from procedural to retrieval, they may reduce the demands on a visuo-spatial code. A reduction in code-specific demands may be reflected in the observed decrease in solution latencies. The hypothesis that less-skilled subjects may show a shift in the relative proportion of retrieval and procedural strategies used was directly tested in Experiment 2.

EXPERIMENT 2

Purpose

The second experiment was designed to explore the solution strategies used by adults to solve complex addition problems. Despite a growing literature concerning adults' and childrens' solution of simple arithmetic problems (e.g., Ashcraft, 1982, 1987; Ashcraft & Battaglia, 1978; Campbell, 1987; Goldman et al., 1988; Siegler, 1988), relatively little is known about the strategies that adults use to solve more complex problems. This experiment was intended to clarify the relations between code-specific working memory resources and solution strategies in adults' solution of complex addition problems. More specifically, this experiment was designed to assess the hypothesis that in the spatial memory - high load - carry problem condition less-skilled subjects may show a relative shift in the proportion of procedural versus retrieval strategies used. This experiment represents a first step toward clarifying the relations between resource use and solution strategies in adults' solution of complex arithmetic problems. The goal was to determine whether subject protocols could be used to help interpret the results of the first experiment; in particular, to determine whether subjects vary their strategy use according to task demands.

Method

Subjects

Subjects were not preselected for this experiment, but completed the arithmetic fluency test (French Kit; French et al., 1963) prior to completing the experimental tasks. On the basis of subjects scores on

the fluency test, an effort was made to retain only those subjects who were judged to be clearly skilled or less-skilled. A median split on the total fluency score was then used to divide subjects into skilled and less-skilled groups, which were balanced with respect to gender. Sixteen undergraduate students (4 males and 4 females in each of the two skill groups) participated in this study. Subjects received either experimental credit or payment for their participation.

Experimental tasks

Subjects completed the spatial memory task used in the first experiment, following the procedure described for Experiment 1 (except that only one memory task was used), and a test of arithmetic fluency (the French Kit; French et al., 1963). Based on a review of the literature, and logical considerations, five methods of problem solution were derived and described to the subjects. Immediately on completion of a problem set (a memory item and an addition problem), subjects reported the strategy that they had used to solve the problem. Subjects who had not used one of the five methods of problem solution were asked to describe the procedure that they had used. Thus, problem solution strategies were coded 1 to 6; these strategies are presented below.

The first strategy corresponds to direct fact retrieval. Subjects were instructed to indicate that they had used this strategy when they felt that they "just knew" the answer without having to carry out any computations. The second strategy was similar, in that retrieval was the primary mode of problem solution. However, there was a procedural component whereby subjects incremented or actively

maintained the digit in the tens column. Thus, subjects using the "retrieve-plus-increment" strategy retrieved the sum for the one's column, and either incremented the ten's column, or consciously maintained the digit in this position. The count-on strategy involved selecting the larger of the two digits and incrementing the number of times indicated by the smaller digit. Subjects using a recombination strategy manipulated the problem in some way that involved recombining the elements of the problem. For example, the problem $35 + 3$ could be solved by recasting the problem as $30 + 8$. Subjects using the count-with-carry strategy obtained the sum for the one's column using a counting strategy and, if the result was greater than 9, incremented the digit in the ten's column. Finally, if subjects' could not characterize their performance using one of these category descriptors, they were to report that they had used some other strategy, and were asked to describe that strategy.

Results

The results of the 2 (skill: high or low) $\times$ 2 (gender: male or female) $\times$ 2 (memory task; spatial or verbal) $\times$ 2 (problem type; carry or no-carry) $\times$ 2 (load; high or low) ANOVAs of median latencies (correct responses only) and errors to the arithmetic problems are presented below.

It was hypothesized that (a) interference would be greater for the carry than for the no-carry problems, (b) a high memory load would interfere more with arithmetic performance than a low memory load, and (c) interference would be greater for less-skilled than for skilled subjects. Of particular interest in this study were any differences

in the pattern of strategy use between skilled and less-skilled subjects. If less-skilled subjects' reduced latencies in the high load - carry problem - spatial task condition in Experiment 1 do reflect changes in strategy use, then it is expected that these subjects will report a relatively greater reliance on retrieval strategies in the high load - carry problem condition as compared to the low load - carry problem condition.

Analyses of latency and error data

Analysis of latencies: medians

Skilled subjects responded more rapidly than less-skilled subjects (1303 and 2226 ms respectively; $F(1,12) = 12.96$, $MSe = 13,634,556$), and latencies to no-carry problems were shorter than those to carry problems (1393 and 2136 ms; $F(1,12) = 32.16$, $p < .001$, $MSe = 9,561,006$). There was also an interaction of problem type and load ($F(1,12) = 6.13$, $p < .05$, $MSe = 255,278$). As shown in Table 9, there was a small increase in latencies to no-carry problems across load whereas latencies to carry problems decreased substantially from the low to high load conditions. There was also an interaction of skill and problem type, $F(1,12) = 9.59$, $p < .01$, $MSe = 263,552$. As shown in Table 10, the latencies of both skilled and less-skilled subjects increased from the no-carry to carry problem conditions, but the magnitude of this increase was greater for less-skilled (1,150 ms) than for skilled subjects (340 ms).

Planned comparisons. Analyses of simple main effects were again conducted to evaluate latencies across load for the different combinations of skill, gender, and problem type. As shown in Figure 5,

less-skilled male subjects showed the same pattern of latencies observed among less-skilled female subjects in Experiment 1: Latencies to carry problems decreased significantly across load (-606 ms; $F(1,12) = 17.62, p < .01$). No other differences in latency across load were significant. Thus, as in the first experiment, less-skilled subjects show improved performance in the condition that places the greatest demands on a visuo-spatial code. In the first experiment this effect was shown by less-skilled females whereas it was shown in this second experiment by less-skilled males. The source of this gender difference is not clear. One possible explanation is that although the average fluency scores of the less-skilled males and females did not differ significantly in either experiment, in the first experiment the lowest scores were obtained by females, whereas in the second experiment the lowest scores were obtained by males. It is therefore possible that the observed decrease in latencies is primarily due to the least skilled of these subjects; females in the first experiment and males in the second experiment (see Appendix A.2).

Analysis of errors

Two main effects were observed in this analysis. Subjects made more errors when responding to carry than to no-carry problems (8.9% versus 2.1% errors), $F(1,14) = 26.41, p < .001, MSe = 1152$. This finding is consistent with those of the first experiment and suggests that, as expected, the carry problems were more difficult than the no-carry problems. There was also a significant effect of gender, $F(1,12) = 4.96, p < .05, MSe = 219$; females made significantly more errors (8.2%) than did males (4.5%). This suggests that, overall, females

found the task somewhat more difficult than did males.

Analyses of strategy data

Descriptive analyses

The number of times that each strategy was used was calculated for each combination of problem type and load. This strategy use score was then converted into a percentage use score; based on the number of times that each strategy was used in relation to the total number of arithmetic problems at each level of problem type and load (i.e., 12). Presented in Table 11 are the percentage use scores for each strategy at each level of problem type and load. In Tables 12 and 13 the percentage use scores are presented for skilled and less-skilled subjects respectively. Overall, 69% of subjects reported using more than one strategy, and 31% reported using only one strategy (retrieval in all cases). Of the 31% of subjects using only retrieval, all but one were classified as skilled. As shown in Tables 12 and 13, retrieval was used equally often by skilled and less-skilled subjects to solve no-carry problems, whereas skilled subjects reported using retrieval more often than less-skilled subjects to solve carry problems. Perhaps the most striking skill difference was in the use of the count-on procedural strategy; skilled subjects almost never used this strategy, whereas less-skilled subjects relied quite heavily on this procedural strategy.

Because the percentage use scores were extremely small in several cells (less than 10%), strategies were combined to create two broad categories of strategy use: Retrieval and non-retrieval. The retrieval score was the average of the retrieval and the retrieve-

plus-increment strategy use scores expressed as a percentage. The procedural score was the average of the count-on, recombination, and count-with-carry strategy use scores, also expressed as a percentage. The retrieval - procedural distinction was the most logically and theoretically plausible way to recombine the strategy use data. The retrieval strategy clearly falls into the retrieval category, and the retrieve-plus-increment strategy was judged to be primarily a retrieval strategy. In contrast, the count-on, count-with-carry, and recombination strategies were judged to (a) largely involve procedures, such as counting, and (b) be more demanding of spatial codes than the retrieval strategies. These combined percentage use scores are presented in Table 14.

Analyses of Variance

The carry and no-carry problems were analyzed separately. For each strategy type (retrieval or nonretrieval) the percentage use scores were analyzed using an ANOVA with two between subjects factors; gender (2; male or female) and skill (2; skilled or less-skilled), and one repeated measures factor; memory load (2; low or high).

Analysis of no-carry problems. In these analyses there were no significant effects for either retrieval or nonretrieval trials. However, retrieval strategies were used more often than non-retrieval strategies (on an average of 94.3 and 4.7% of trials respectively). Overall, subjects are more reliant on retrieval than non-retrieval strategies to solve simple columnar addition problems.

Analysis of carry problems. As with no-carry problems subjects used retrieval strategies more often than nonretrieval strategies (an

average of 73.4 and 22.7% respectively). In the analysis of retrieval trials there was an interaction of skill and load, $F(1,12) = 5.12$, $p < .05$, $MSe = 109$. As shown in Table 15, the pattern of use of retrieval strategies as a function of load was strikingly different for skilled and less-skilled subjects. Among skilled subjects the percentage of trials on which a retrieval strategy was used decreased slightly (by 3%) across load, whereas there was an increase in the use of retrieval strategies among less-skilled subjects (by almost 14%). In the analysis of nonretrieval trials the interaction of skill and load approached significance, $F(1, 12) = 3.16$, $p = .10$, $MSe = 263$. As shown in Table 15, the increase in the use of retrieval strategies by less-skilled subjects in the high load condition was accompanied by a corresponding decrease in the use of nonretrieval strategies. skilled subjects. These results are consistent with the hypothesis that placing greater demands on the VSSP results in a tendency to shift away from using the more demanding procedural strategies. Moreover, this shift appears to occur among less-skilled subjects, who report relying more heavily than skilled processors on procedural solution strategies - strategies that presumably place greater demands on the resources of the VSSP. There were no effects involving gender in the analyses of either retrieval or nonretrieval trials. However, for comparison with the latency data, the percentage strategy use for males and females in each of the skill groups is also presented in Table 15.

Summary of results: Experiment 2

Two important conclusions can be derived from these data. First, the effect whereby increased demands on the VSSP results in decreased latencies is clearly replicable. Moreover, it is clear that this effect is not gender specific, but occurs among less-skilled subjects. To obtain this effect in such a small sample lends clear support to the hypothesis that this effect reflects some real processing differences or changes as a function of task demands and subject characteristics.

Second, these data clearly support the hypothesis that a large visuo-spatial working memory load, in combination with resource demanding arithmetic problems, results in a shift in the types of strategies used to achieve problem solution. This strategy shift is such that as the resource requirements of the task increase, subjects become relatively more reliant on less demanding solution procedures (i.e., retrieval). Moreover, the strategy shift appears to occur primarily among less-skilled subjects.

EXPERIMENT 3

Purpose

The third experiment was designed to evaluate the relation between visuo-spatial processing and arithmetic performance among children who are in the latter stages of skill acquisition, but who may not yet have achieved full automaticity of arithmetic facts. Hitch (1974) has suggested that, in general, children are more reliant than adults on spatially based strategies (such as counting) for solving arithmetic problems (see also Nesher, 1986). If these assumptions are correct, children should be more affected than adults by the spatial memory task. It is also possible that, like less-skilled adults, children will show improved performance as the demands placed on VSSP resources increase. However, because less-skilled children presumably have fewer solution strategies available to them it may be only the more skilled children who show a pattern of improvement in the condition that places most demands on the resources of the VSSP.

Method

Subjects

Forty-eight sixth grade students (24 males, 24 females) participated in this study. Subjects were from two classrooms in the same suburban elementary school.

Experimental tasks

Subjects completed the spatial and verbal memory tasks in combination with the complex problems used in the first experiment.

The only change in procedure was that subjects completed the experiment in two sessions; with the spatial condition presented in one session and the verbal condition in the other session. Session order was counterbalanced across gender, and each session took approximately 15 minutes. Subjects also completed the arithmetic fluency test (the French Kit; French et al., 1963) in group testing.

Results

A median split on the total fluency score was used to divide subjects into two skill groups. The fluency score was not available for one subject, whose data were excluded from analyses involving this variable. Skill and gender were not perfectly counterbalanced; in the skilled group there were 13 males and 11 females, and in the less-skilled group there were 10 males and 13 females. Median response latencies (correct responses only) and percentage of errors on the arithmetic problems were analyzed in a set of ANOVAs that included two between subjects variables; 2 (gender; male or female) $\times$ 2 (skill; skilled or less-skilled), and three repeated measures factors; 2 (problem difficulty; carry or no-carry) $\times$ 2 (memory task; verbal or spatial) $\times$ 2 (memory load; high or low). The results of these analyses are presented below.

Analysis of latencies: medians

In addition to a main effect of skill ($F(1,43) = 9.06, p < .01, \text{MSe} = 23,935,059$; latencies of 2323 and 2825 ms for skilled and less-skilled subjects respectively), there was an effect of problem type, $F(1,43) = 114.77, p < .001, \text{MSe} = 86,884,546$: Latencies to no-carry problems were shorter than those to carry problems (2087 and 3055 ms

respectively). There was also an interaction of skill and problem type, $F(1,46) = 20.66$, $MSe = 12,437,280$. As with adults in Experiments 1 and 2, less-skilled subjects showed a greater relative difference in latencies as a function of problem type than did skilled subjects (a difference of 1231 and 704 ms respectively; see Table 16). This effect was, however, subsumed under a significant five-way interaction of skill, gender, memory task, problem type, and load, $F(1,43) = 5.33$, $p < .05$, $MSe = 245,926$. This interaction is shown in Figures 6 (a) and (b). In the verbal task condition (Figure 6[a]) changes in latencies as a function of load were modest in all conditions. In the spatial task condition (Figure 6[b]) the latencies for males tended to decrease as a function of load, with the exception of less-skilled males in the carry problem condition who showed a significant increase in latencies across load ($F(1,10) = 10.65$, $p < .05$). Females also showed a tendency toward decreasing latencies as a function of load; however none of these simple main effects were significant. In the analyses of these subjects' data the analyses of errors were more informative than those of latencies. The analyses of errors are presented below.

Analysis of errors

Generally, the error rate was quite low (range 1.9 to 16.4 across all conditions). More errors were made to carry than to no-carry problems (11.2 and 4.6% respectively; $F(1,43) = 27.57$, $p < .001$, $MSe = 3,770$). There was an interaction of skill, memory task, and load, $F(1,43) = 4.86$, $p < .05$, $MSe = 169$. As shown in Table 17, for less-skilled subjects there were essentially no changes in error rates

across load in either memory condition. In contrast, skilled subjects showed an increase in errors in the verbal task condition and a decrease in errors in the spatial task condition (none of the simple main effects were significant). This suggests that skilled subjects may derive some benefit, in terms of arithmetic performance, in the spatial task condition.

There were also two significant four-way interactions. First, there was an interaction of skill, gender, problem type, and load, $F(1,43) = 6.31, p < .05, \text{MSe} = 266$. As shown in Figure 7, there was a general trend toward an increase in error rates for all skill and gender combinations, for both no-carry and carry problems, as a function of load. There were, however, two exceptions to this trend: Less-skilled males showed a significant decrease in errors to no-carry problems from the low to high load conditions (6.9 and 3.9% errors respectively; $F(1,43) = 7.99, p < .01$), and skilled females showed a small (nonsignificant) decrease in errors to no-carry problems across load (5.7 and 4.7% respectively). Thus, less-skilled males showed enhanced performance on no-carry problems in the high load condition. There was also an interaction of skill, memory task, problem type, and load, $F(1,43) = 4.40, p < .05, \text{MSe} = 140$. The picture that emerges in this interaction is somewhat different: As shown in Figure 8, in the spatial task condition the error rate for both skilled and less-skilled subjects to no-carry problems decreased slightly as a function of load, and there was a significant decrease in errors for skilled subjects in the carry problem condition ($F(1,43) = 5.73, p < .05$).

Thus, in both of the significant four-way interactions the pattern is such that there are unexpected decreases in the percentage of errors to the arithmetic problems. However, the locus of this effect is not clear. To disambiguate these results, tests of simple main effects were conducted for the interaction of gender, skill, memory task, problem type, and load. As shown in Figure 9 (a), in the verbal task condition error rates to both problem types either increased or remained stable from the low to high load conditions. However, in the spatial task condition (Figure 9 [b]) the pattern was such that, with the exception of less-skilled males in the high-load - carry problem condition, errors generally decreased or remained stable across load. The reduction in errors was significant for skilled males in the carry problem condition ($F(1,43) = 4.07, p < .05$), and the increase in errors for less-skilled males in this condition was also significant ($F(1,43) = 6.1, p < .05$). Thus, it appears that, in this sample of sixth grade students, the performance of skilled male subjects improved in the most demanding spatial memory task condition. It is also interesting that less-skilled males showed a trend toward reduced errors across load to no-carry problems, and skilled females showed this trend in both problem type conditions. Among children, skilled males clearly show improved performance in the same condition as do less-skilled adults.

It is possible that skill and gender differences in errors to the arithmetic problems reflect differences in performance on the memory task. To evaluate this possibility, percent errors to the memory probe items were analyzed in an repeated measures ANOVA. In this

analysis there was an interaction of memory task, problem type, and load ($F(1,46) = 4.35$, $p = .07$, $MSe = 626$). As shown in Table 18, errors to the memory items increased for both no-carry and carry problems across load in both memory task conditions. However, the increase in probe errors across load in the no-carry problem condition was somewhat greater in the spatial than in the verbal memory condition (a difference of 3.1%). The reverse was the case in the carry condition; errors increased more in the verbal than in the spatial memory condition (a difference of 7.2%). Thus, if there is a speed accuracy trade-off, it may be in the spatial - no-carry condition, the verbal - carry condition, or both.

To disambiguate these results, correlations between latencies and probe errors were calculated. There were no significant correlation between latencies and probe errors in the spatial task (correlations ranged from $-.07$ to $.24$). In the verbal task condition there were significant positive correlations between probe errors and problem latencies in the no-carry - low load and carry - high load conditions (r s of $.34$ and $.43$ respectively). However, a positive correlation indicates that as latencies increase (or decrease), errors also increase (or decrease). Thus, there is no evidence in this analysis that a speed-accuracy trade-off accounts for the decreased latencies in the spatial task - no-carry - high load condition.

Summary of results: Experiment 3

As in the second experiment several of the results supported the main hypotheses, but there were also several unexpected and intriguing results. As expected, carry problems were more difficult than were

no-carry problems. This finding is particularly important given the dearth of literature on the processes involved in complex addition (but see Ashcraft et al., 1992; Widaman et al., 1989, 1992). Moreover, less-skilled subjects showed greater increases in latency to carry problems than did skilled subjects.

What was unexpected was the finding that in the spatial memory task condition errors decreased significantly as a function of load for skilled male subjects. As with adults in Experiment 1, it appears that taxing a visuo-spatial processing code can enhance performance for certain groups of subjects. These results can, perhaps, be addressed using the same hypothesis that was used to explain the similar findings in Experiment 2: That is, that taxing the VSSP beyond some critical level results in a strategy shift from more- to less- resource demanding solution strategies. The locus of this effect may be more variable for children than for adults, perhaps because of greater variability in the efficiency and number of solution strategies used.

GENERAL DISCUSSION

The question that motivated these studies concerned the nature of the cognitive processes that underly arithmetic performance. There is very little research concerning the relation between working memory and arithmetic performance (Ashcraft et al., 1992; Hitch, 1978; Hitch & McAuley, 1991; Siegel, & Linder, 1984; Siegel, & Ryan, 1989), and virtually no research has directly addressed the possibility that arithmetic performance may be mediated by a visuo-spatial code. The present research was designed to test the hypothesis that procedural aspects of arithmetic performance are mediated primarily by a visuo-spatial code rather than a verbal code. Of particular interest are the results of analyses that show a differential effect of the spatial and verbal memory tasks on arithmetic performance. These results are of interest for three reasons: First, because it is important to understand the cognitive processes that support, or enable, efficient processing of arithmetic information. Second, these results may improve our understanding of skill acquisition, and of strategy choice. Finally, these data also bear on the more general question of whether working memory is best thought of as a general resource, or whether it comprises two or more code-specific subsystems (e.g., verbal and visuo-spatial in Baddeley's model).

Arithmetic Performance and Visuo-Spatial Processing

The concept of working memory has been particularly useful in providing a framework for the study of individual differences in reading skill (e.g., Daneman & Carpenter, 1980; Dixon et al., 1988; Herdman & LeFevre, 1992). Researchers concerned with individual

differences in arithmetic skill have also sought to understand skill differences in terms of the efficiency of working memory processes (Ashcraft et al., 1992; Siegel & Linder, 1984; Siegel & Ryan, 1989). However, these efforts have been focussed on the central executive and articulatory loop components of working memory as the locus of individual differences in arithmetic skill, and the results are inconclusive with respect to the nature of the relation between working memory and arithmetic performance. Moreover, the results of studies that have included both verbal and nonverbal working memory tasks are consistent with the hypothesis that the component of working memory most directly associated with arithmetic performance is the visuo-spatial sketchpad (Fletcher, 1985; Siegel & Linder, 1984; Siegel & Ryan, 1989; Webster, 1979, 1980)

The present results are consistent with the hypothesis that procedural arithmetic solution strategies are mediated by a visuo-spatial code. However, these data cannot be adequately understood without reference to the interaction of task and subject characteristics; that is, the difficulty of the spatial memory and arithmetic tasks, and the subjects' level of arithmetic skill. In the series of experiments presented in this paper, there was no evidence that solving arithmetic problems concurrently with a spatial memory task was generally more difficult than solving these problems concurrently with a verbal memory task; that is, there were no main effects involving memory task. However, in each experiment there was evidence that the spatial was more difficult than the verbal memory task for subgroups of subjects in particular task conditions. In

Experiments 1 and 2 (adults), latencies were slower for less-skilled subjects (females in Experiment 1; males in Experiment 2) in the spatial task - low load - carry problem condition than for subjects in any other condition. In Experiment 3 (children) percent errors to the arithmetic problems were higher for less-skilled males in the spatial task - low load - carry problem condition than for subjects in any other condition. Thus, it appears that for at least these groups of subjects, arithmetic performance may be mediated by a visuo-spatial code.

Patterns of enhanced performance were also observed for subjects in the spatial-task condition. Consider the results of Experiments 1 and 2. In these experiments the pattern of performance was such that less-skilled subjects showed improved performance (faster latencies) in the condition that was expected to place the greatest demands on a visuo-spatial code; the spatial task - high load - carry problem condition (Figures 4[a] and 5). A comparable pattern of performance was observed in the analysis of children's errors to the arithmetic problems in Experiment 3. In this experiment, however, it was skilled (male) subjects who showed a clear pattern of improved performance (i.e., reduced errors) in the spatial task - high load - carry problem condition (Figure 9[b]).

A plausible interpretation of these results must take into consideration the relation between demands on a visuo-spatial code and the strategies that subjects use to solve the arithmetic problems. It was suggested earlier that retrieval strategies would make minimal demands on a processing code, whereas procedural strategies would be

relatively more demanding of visuo-spatial codes. There is evidence that among adults, skilled subjects are more likely than less-skilled subjects to have automatic access to basic arithmetic facts (i.e., retrieval; LeFevre & Kulak, 1990, 1993; LeFevre et al., 1993). A logical conclusion, then, is that less-skilled subjects are more reliant than skilled subjects on procedural strategies. The results of Experiment 2 support this assertion (see Table 16): Less-skilled subjects reported using procedural strategies more often than did skilled subjects. In a recent study, LeFevre and her colleagues (1993) also found that adults report using a variety of procedural strategies to solve simple addition problems, in addition to direct fact retrieval.

Among less-skilled adults, improved arithmetic performance in the spatial task condition was noted in the high load - carry problem combination. Presumably, a high memory load is more demanding of a visuo-spatial code than is a low memory load, and carry problems are more demanding than are no-carry problems. In the most demanding spatial task condition (carry problems with a high memory load), subjects who use a procedural strategy may exceed the capacity of the visuo-spatial sketchpad. To make this hypothesis more concrete, consider the following hypothetical situation. Assume that the visuo-spatial sketchpad has 10 units of a spatial code available for processing. The visuo-spatial code requirements of the spatial task - high load - carry problem condition can be considered to require a certain number of these units. For example, assume that a low memory load requires 3 units and a high memory load 6 units of this code,

that a retrieval strategy requires 1 unit and a procedural strategy 6 units to solve a carry problem. Combining these (arbitrary) units of measurement yields the following estimate of the code requirements in the spatial task - carry problem condition: Retrieval plus low load (3 units); retrieval plus high load (7 units); procedural plus low load (8 units); procedural plus high load (11 units). Because less-skilled subjects use procedural more frequently than retrieval solution strategies, the available visuo-spatial coding units will be exceeded in the spatial task - high load - carry problem condition for these subjects. In order to accomplish the experimental task, subjects must reduce the demands on the visuo-spatial code. This could be accomplished in one of two ways; by reallocating resources between the two tasks (i.e., by trading-off speed and accuracy), or by shifting from a procedural to a retrieval-based solution strategy. Analyses of errors to the arithmetic problems and memory probe items do not support the first possibility; there was no evidence of a speed-accuracy trade-off on either the arithmetic or memory tasks. However, subjects' self-reported strategy use in the second experiment support the strategy shift hypothesis; less-skilled subjects report a shift from the use of procedural strategies to relatively greater use of the retrieve-plus-increment strategy.

A question that arises from the interpretation offered above is the following: If less-skilled subjects can retrieve, why would they typically use procedural strategies? The answer may lie in the Siegler and Jenkins (1989) conceptualization of the distribution of associations between arithmetic problems and solution strategies, and

their notion of a variable confidence criterion. As outlined previously, Siegler and Jenkins hypothesized that strategies are selected on the basis of the strength of the associative link between a particular strategy and a problem, or group of problems (i.e., the strategy strength). The strength of an associative link is believed to reflect the history of the association between a strategy and a problem (or class of problems) with respect to accuracy and speed of problem solution. Another feature of Siegler's model (1988; Siegler & Shrager, 1984; Siegler & Jenkins, 1989) that is important here concerns the feature of the distribution-of-associations model that determines whether an obtained answer will be stated, or whether an alternate solution strategy will be invoked: That is, the confidence criterion. In this model, the associative strength of an obtained answer must exceed a certain confidence criterion before it will be advanced. The confidence criterion is a threshold value that varies both between and within individuals. Siegler (1988) has used the concept of individual differences in the stringency of the confidence criterion to explain strategy differences among competent first grade students. In his terms, "perfectionists" are students who have a peaked distribution of associations, and a very stringent confidence criterion; thus, these students use retrieval less often than would be predicted on the basis of their overall competence. "Good" students, have a similarly peaked distribution of associations, but a somewhat less stringent confidence criterion. Thus, "perfectionists" rely more on procedural strategies than do their "good" peers.

The results of the present experiments are interpretable within a framework that combines the concept of the confidence criterion with that of strategy choice (Siegler & Jenkins, 1989). Strategy choice for a particular problem is believed to reflect the history of speed and accuracy for that strategy - problem combination. It is not unreasonable to assume that this history would result in a strategy-to-problem (or problem-type) distribution of associations that would vary in terms of strength, or "peakedness", in the same way that the distribution of associations between problems and potential answers is assumed to vary (Siegler, 1988; Siegler & Shrager, 1984; Siegler & Jenkins, 1989). Similarly, just as an answer will be stated only if the "peakedness" of the association between that answer and the stated problem exceeds a variable threshold, or confidence criterion, the "peakedness" of the association between a strategy and a particular problem (or class of problems) may need to exceed a threshold, or confidence criterion, before that strategy will be executed.

It is reasonable to hypothesize that there would be individual differences in the 'peakedness' of associations between strategy types and problem types. Thus, for some individuals the association between procedural strategies and addition problems may be stronger than the association between retrieval strategies and addition problems. If the distribution of associations between procedural strategies and addition problems is more 'peaked' than the distribution of associations between retrieval strategies and addition problems it may be more likely to exceed a stringent (high) strategy-related confidence criterion. In the present studies, less-skilled adults

generally used procedural strategies, but were able to use retrieval strategies to reduce the demands on a visuo-spatial code. It is possible that for these adults, task demands that exceeded the capacity of the VSSP resulted in a lowering of the strategy-related confidence criterion. Lowering of the strategy-related confidence criterion would increase the probability that the less-peaked distribution of retrieval strategy associations would exceed the confidence criterion; hence retrieval strategies would be used more often.

If one accepts the plausibility of the above argument, the performance of less-skilled adults in the most demanding experimental condition can be interpreted in the following way. Because less-skilled subjects rely largely on procedural strategies to solve carry problems, the availability of visuo-spatial codes is exceeded in the spatial task - high load - carry problem condition. In order to reduce spatial coding demands to a level that allows the task to be completed without sacrificing performance, subjects lower the strategy-related confidence criterion. When this confidence criterion is lowered, the less 'peaked' distribution of associations between retrieval strategies and arithmetic problems is more likely to exceed the confidence criterion than it was previously; hence, the probability of a retrieval strategy being executed increases.

In terms of skill acquisition, the above interpretation suggests that a child's learning history influences the development of a distribution-of-associations between strategies and classes of arithmetic problems, in addition to the distribution-of-associations

between arithmetic problems and their sums. Variables that may influence strategy-related confidence might include the accuracy of different solution procedures (Siegler & Jenkins, 1989), and the degree of emphasis on speeded performance in the classroom. It is important to note that in none of the three experiments did skilled and less-skilled subjects differ, overall, in terms of the accuracy of their performance on the arithmetic problems; these groups differed only in the speed of their performance. This suggests that less-skilled subjects (as defined for the present studies) were less capable with respect to speeded arithmetic performance than were skilled subjects. Thus, it seems plausible that one variable that may influence the use of retrieval versus procedural strategies may be the relative emphasis placed on the speed of arithmetic solution in skill acquisition.

The pattern of results for sixth grade children in the third experiment was similar to the pattern of results for Experiments 1 and 2, but with two important differences: In this case it was skilled children (males) who showed a clear pattern of improved performance in the spatial task - high load - carry problem condition, and this pattern was observed in the analyses of errors, not latencies. The observed decrement in errors in the spatial task condition was significant only for skilled males in the high load - carry problem condition. However, a similar pattern of decreasing errors across load was shown by less-skilled males in the no-carry problem condition, and by skilled females in both the no-carry and carry problem condition. The pattern of improved performance for the

different skill groupings suggests that children's ability to adjust their strategy use may be somewhat dependent on their level of skill. Regardless of gender, in the spatial-task condition skilled children were able to reduce the task demands in the most demanding condition (see Figure 8), and to improve their accuracy on the arithmetic problems. In contrast, less-skilled children seemed unable to adopt a compensatory solution strategy in this most demanding condition, and their accuracy deteriorated.

As with adults, a particular sub-group of children demonstrated improved performance in the most demanding experimental condition. In contrast to adults, it was skilled children who showed this improved performance. This suggests that skilled sixth grade children are able to compensate for excessive demands on a visuo-spatial code in a manner that is similar to that of less-skilled adults.

Recall the hypothesis that less-skilled adults showed improved performance in this condition because they were able to shift from use of procedural to retrieval strategies in order to reduce visuo-spatial processing demands. To apply this logic to children is to assume that skilled children are able to compensate in a similar way. The question then concerns how skilled and less-skilled children differ in their approaches to solving these complex arithmetic problems. Among adults, less-skilled subjects relied primarily on procedural strategies (Experiment 2). If skilled children are like less-skilled adults they, too, may rely primarily on procedural strategies. Similarly, they may be able to reduce task demands by either adopting a retrieval strategy or a relatively less-demanding procedural

strategy. Thus, skilled and less-skilled children may not differ in the class of preferred strategy (i.e., the class of procedural strategies), but instead may differ in terms of the efficiency of the procedural strategy used to solve these complex problems.

The applicability of the strategy shift hypothesis is less clear with these younger subjects. Clearly, some change in strategy or allocation of resources is going on. It seems unlikely that a change in the allocation of resources can account for skilled male subjects' improved performance; there was no evidence of a speed-accuracy trade-off in the analyses of errors to either the arithmetic problems or the memory probe items. Thus, it seems probable that, like adults, skilled children are able to adjust their strategies to meet variable task demands (even children as young as 4 and 5 seem able to do this; Siegler & Jenkins, 1989). Because no decrease in latencies was observed, it appears that if strategies are being changed, the change is occurring in a way that improves accuracy but has little effect on speed of responding. It is possible that a strategy shift may occur entirely within the class of procedural strategies. For example, in combination with the visuo-spatial coding requirements of the spatial memory task, the min counting strategy may result in fair number of errors for carry problems. In contrast, a recombination strategy might result in higher accuracy, without having a significant effect on latencies. To illustrate, the majority of the carry problems had minimum addends ranging in value between 5 and 8; to use the min strategy in this instance may make substantial demands of a limited visuo-spatial code. If children adopted a procedural strategy that

was less demanding of visuo-spatial codes, possibly recombination, the demands on the visuo-spatial code may have been reduced without substantially affecting latencies. For example, the problem $7 + 38$ could be recast as $7 + 40 = 47$ (retrieved), and $47 - 2 = 45$ (procedural). Evaluating the tens structure of the problem may be a time consuming process relative to retrieval of the sum of $7 + 8$, but is also likely less prone to error than would be counting to achieve either the sum of 7 and 8 or the sum of 7 and 38. It is also possible that these skilled children are directly retrieving the sums to these problems. Two findings cast doubt on this possibility, (a) no decrease in latencies was observed for skilled children (male) in this condition (i.e., spatial - high load -carry problem), and (b) their latencies were still substantially slower than those of adults in the same condition (an average of 378 ms slower). It is, however, possible that a change in strategy use would not be accompanied by an immediate change in latencies, particularly if the strategy were not highly practiced (Siegler & Jenkins, 1989).

Thus, with the children there are two hypotheses that may account for the reduced errors in the most demanding condition. These subjects may be shifting either from a procedural to retrieval strategy, or from a relatively demanding to a less demanding procedural strategy. From the results of the present study it is not possible to differentiate between these hypotheses.

Spatial Processing and the Development of Skilled Arithmetic Performance

As noted in the introduction to this paper, the hypothesis that visuo-spatial skills are important to mathematical skill is not new (Hamley, 1935, cited in McGee, 1979; Halpern, 1992). The contribution of this research is that the studies provide a direct test of this hypothesis. The results of the experiments conducted with adults suggest that visuo-spatial processes (or codes) mediate procedural, but not retrieval, solution strategies. In the experiment conducted with children, the visuo-spatial coding demands of the most demanding task condition essentially overwhelmed the capacity of the less-skilled processors; in contrast, skilled subjects appear to have adjusted their strategy use to reduce demands on a visuo-spatial code.

One of the developmental hypotheses that arises from these results is that visuo-spatial codes may play a larger role in the acquisition of arithmetic skills than they do in skilled performance: Children are more reliant than adults on procedural solution strategies (Ashcraft, 1992; Groen & Parkman, 1972; Nesher, 1986), and these strategies seem to be more sensitive to manipulation of visuo-spatial codes. Automatization and retrieval are believed to be achieved via procedural solution strategies: That is, a distribution, or network, of automatically accessed arithmetic facts is built-up over time as children repeatedly achieve the same answers to arithmetic problems using procedural strategies (Ashcraft, 1992; Goldman, Pellegrino, & Mertz, 1988; Goldman, Mertz, & Pellegrino, 1989; Hamann & Ashcraft, 1985; Siegler & Jenkins, 1989). Thus, the

role of spatial processing in arithmetic performance may be relatively larger for young learners than for relative experts in the domain of arithmetic. If spatial processes are centrally involved in early arithmetic performance, the facility that children have in executing spatially mediated processes may influence the ease with which they acquire a 'mature' representation of arithmetic knowledge.

The results of the third experiment suggest that it was the more skilled children who were able to reduce visuo-spatial coding demands by adopting a solution strategy that was less demanding of visuo-spatial codes. Less-skilled children did not appear able to make this adjustment to task demands. Thus, it appears that less-skilled children remain reliant on strategies that are mediated by a visuo-spatial code longer than do more skilled children. The apparent tautology is that skilled children may be skilled precisely because they have good visuo-spatial skills. Good visuo-spatial skills may allow them to execute procedural strategies efficiently and accurately, resulting in the development of a 'peaked' distribution of associations between arithmetic problems and their sums.

An early advantage in visuo-spatial processing may result in the development of fluent automatic access to an accurate representation of arithmetic facts. In contrast, less-skilled children may be less skilled partially because of relatively poor visuo-spatial processes. Relatively poor visuo-spatial processing may impede efficient and accurate early use of procedural strategies, which in turn may delay the development of a 'peaked' distribution of associations between problems and sums. A situation similar to that described by Stanovich

A resolution test chart featuring various patterns of horizontal and vertical lines of decreasing size. Each pattern is accompanied by a numerical value indicating its resolution. The values include 1.0, 1.1, 1.25, 1.4, 1.6, 1.8, 2.0, 2.2, 2.5, 2.8, 3.2, 3.6, 4.0, 4.5, 5.0, 5.6, 6.3, 7.1, 8.0, 9.0, 10, 11.2, 12.5, 14, 16, 18, 20, 22.4, 25, 28, 31.5, 36, 40, 45, 50, 56, 63, 71, 80, 90, 100, 112, 125, 140, 160, 180, 200, 224, 250, 280, 315, 360, 400, 450, 500, 560, 630, 710, 800, 900, 1000, 1120, 1250, 1400, 1600, 1800, 2000, 2240, 2500, 2800, 3150, 3600, 4000, 4500, 5000, 5600, 6300, 7100, 8000, 9000, 10000.

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

(1986, 1988) as "Matthew Effects" in reading may also arise in arithmetic. That is, children who have more facility in the processes underlying a skill (phonemic skills in reading; visuo-spatial processes in arithmetic) acquire the skill earlier in development, and achieve a higher level of fluency. This is the developmental equivalent of the "rich get richer and the poor get poorer" maxim. For an example of how visuo-spatial processes may mediate early procedural solution strategies consider the role of finger recognition (i.e., the quantity represented by fingers) in early solution procedures. Among their 4- and 5-year-old subjects, Siegler and Jenkins (1989) reported that children who were proficient in their use of the sum strategy were able to put up the correct number of fingers to represent each addend without counting them. They also reported that children became increasingly proficient at recognizing the sum of a problem by recognizing the configuration of the fingers, also without counting (pp. 120-121). Thus, visuo-spatial processes appear to be involved in the solution procedures used by young children. These children are clearly able to visually recognize a spatial configuration that represents quantity.

Both Goldman (Goldman et al., 1988; Goldman et al., 1989) and Geary (Geary & Brown, 1991; Geary, Brown, & Samaranayake, 1991) have demonstrated that, overall, children with a math disability appear to show a delay in the shift from less to more efficient counting strategies and direct retrieval. That is, children with a math disability are characterized by a less mature mix of strategies than is shown by normally achieving learners of the same age. Goldman and

her colleagues (Goldman et al., 1988) have also demonstrated that it is possible, via intervention, to facilitate the movement of most children with arithmetic learning disabilities through the sequence of less- to more - efficient procedural strategies, and retrieval. It is possible that interventions designed to improve visuo-spatial skills might have a similarly beneficial effect for very young learners who are experiencing difficulty with numerical concepts and procedures.

Visuo-spatial Processing and Arithmetic Performance: Evidence for Code-Specific Processing in Working Memory.

One assumption that I have made in the design of these studies is that the efficiency of working memory is domain-dependent; that is, that the contribution of basic working memory processes may vary from task to task (from reading to arithmetic, for example). This assumption arises from the hypothesis that, in addition to the central executive, working memory is comprised of subsystems (e.g., the articulatory loop and visuo-spatial sketchpad; Baddeley, 1985, 1986) that process information using distinct codes or resources. In contrast to this view, Engle and his colleagues (Engle et al., 1990; Turner & Engle, 1989; Engle, Cantor, & Carullo, 1992) have suggested that working memory is task independent; people with larger working memory capacities will perform well across a range of tasks (such as reading and mathematics) because the general processes associated with working memory capacity are efficient, not because of task-specific variability in working memory efficiency. With respect to the present studies, the prediction that arises from this view is that the specific nature of the working memory task (verbal or spatial) should

not affect performance on the primary task (arithmetic); that is, arithmetic performance should not be differentially affected by the verbal and spatial working memory tasks.

The results of these studies with respect to the task-specific versus task-independent views of working memory are quite clear. There was a uniform effect of the verbal working memory task across the three experiments, such that increases in the verbal working memory load were generally unrelated to changes in performance on the arithmetic task. In contrast, performance on the arithmetic task was clearly related to manipulations of the spatial working memory load for less-skilled subjects. Generally, less-skilled subjects showed decrements in performance to carry problems in the low spatial load condition, and improved performance in the high spatial load condition.

No doubt, central executive processing contributes to performance on a task such as addition. However, these results are based on both (a) working memory tasks that are clearly differentiated in terms of the codes required for successful task execution, and (b) a primary task (addition) that is relatively circumscribed and well-defined. Thus, the results of these studies provide clear evidence for the operation of domain-specific processes that are controlled by the different subsystems of working memory. These domain-specific processes appear to mediate performance in ways that go beyond the contribution of general domain-independent processes.

Future Directions

In the present studies there were no baseline data to which the latencies and errors in the memory task conditions could be compared; that is, there was not a no-memory load condition to compare to the verbal and spatial working memory load conditions. This limits the generality of the conclusions in two ways. First, it is difficult to interpret the effect of the verbal task manipulations beyond noting that the verbal memory task did not affect arithmetic performance differentially as a function of task difficulty and subject characteristics. If performance in the verbal task condition was slower or more error-prone than performance in a no-memory task condition, this would suggest that the verbal memory task required general working memory resources that were also required to accomplish the arithmetic task. Second, a no-memory task experiment (or condition) could provide information about the processing demands of complex addition problems under less 'artificial' conditions. In comparison with single-digit addition, very little is known about the processing demands of more complex addition (but see Ashcraft, 1992; Widaman et al., 1989, 1992). In future research a comparable no-memory task condition, in conjunction with subjects' self-reported strategy use, would provide useful insights into the processes involved in complex addition.

A second productive avenue for future research would be to replicate the third experiment (children), but also ask subjects to describe their strategy use. In the third experiment subjects were asked to describe, in a general way, the types of strategies that they

had used; however, this information was not collected on a trial-by-trial basis. Consequently, the hypotheses offered above concerning children's strategy use, and task-related changes in strategy use, remain speculative. When designing this experiment, the concern was that sixth grade subjects would not be able to monitor and accurately describe their performance in the context of these relatively demanding experimental tasks. This does not appear to be an issue; very few subjects had difficulty with either the memory or arithmetic tasks, and with the exception of two children the experimental procedure did not appear to be unduly anxiety provoking.

Finally, our understanding of mathematical cognition could be enhanced by directing future research toward exploring the conditions under which adults (and children) use particular strategies, and the conditions that result in a "strategy shift". The present data suggest that less-skilled adults do not rely heavily on retrieval-based solution strategies, but that they are capable of doing so when it will benefit their performance. The strategy shift hypothesis could be explored further by manipulating task difficulty in ways other than was done here; for example, the time available for problem solution could be varied. If such a manipulation results in a strategy shift, the hypothesis that for some adults retrieval may be a backup strategy, rather than a dominant strategy, is strengthened. A further intriguing question concerns why adults who are capable of direct retrieval would, instead, rely on procedural solution strategies. Above, I offered one hypothesis; less-skilled subjects may be more confident in the accuracy of procedural than retrieval solution

strategies; the "confidence criterion" hypothesis. One way to evaluate this hypothesis would be to have subjects rate their confidence in their solutions on a trial-by-trial basis. In the context of an experimental design where the task demands may result in a strategy shift, it would be possible to determine whether subjects who change their strategies are more confident in solutions achieved via non-retrieval than retrieval-based strategies.

It was suggested above that visuo-spatial processes may play a role in the development of arithmetic fluency. Specifically, I have suggested that visuo-spatial processes may mediate early counting strategies; strategies that play a central role in childrens' solution of simple arithmetic problems. In the context of the model of skill acquisition proposed by Siegler (1988, Siegler & Jenkins, 1989), the absence of counting strategies that reliably yield the correct answer to a problem will compromise the development of "peaked" distributions of associations between problems and their sums. It is, therefore, important to directly assess the relation among young childrens' visuo-spatial abilities, the types of strategies they use to solve simple arithmetic problems, and the accuracy of those strategies. In assessing this hypothesized relationship it may be best to begin by determining whether the correlation between arithmetic and spatial skills observed in older children and adults (Bennett et al., 1974; Dark & Benbow, 1990, 1991; Eisenberg & McGinty; Fennema & Sherman, 1977, 1978; Sherman, 1980) is also evident among children in the early stages of arithmetic skill acquisition.

General Conclusions

In conclusion, the results of the present series of experiments show that, under some conditions and for certain subjects, arithmetic performance is mediated by processes associated with the visuo-spatial sketchpad component of working memory. The most intriguing results were those related to individual differences in the demands placed on visuo-spatial processing codes. Individual differences in coding demands appear to be related to the types of solution strategies that are used to solve arithmetic problems. These results suggest that adults do not uniformly solve arithmetic problems using retrieval-based strategies. Childrens' performance also varied as a function of task demands and individual differences in arithmetic skill, with skilled children exhibiting a pattern of performance similar to that of less-skilled adults. These results suggest that skilled children may use solution processes that are similar to those used by less-skilled adults. Finally, the pattern of performance observed among children suggests that visuo-spatial processes may be especially involved in mediating the performance of less-skilled children; less-skilled children may rely quite heavily on solution strategies mediated by the VSSP. Further exploration of individual differences in the solution strategies used by adults, and the conditions under which they may change strategies may be a fruitful avenue for future research. It may also be useful to devise a dual-task similar to those used in these experiments for use with younger children. This would be necessary to allow a test of the hypothesis that early visuo-spatial skills contribute to efficient execution of procedural

solution strategies. Understanding the cognitive "underpinnings" of early arithmetic solution strategies may, in turn, contribute to our growing understanding of mathematical cognition.

Notes

¹ The reading disabled group comprised children whose Reading score on the WRAT was below the 21st percentile. Arithmetic disabled children achieved WRAT Arithmetic scores below the 21st percentile and WRAT Reading scores above the 34th percentile. All subjects achieved a score of 80 or better on the Peabody Picture Vocabulary Test.

² Reading and Spelling disabled children achieved WRAT Reading and Spelling subtest scores below the 31st percentile, and Arithmetic scores above the 30th percentile and at least 1/2 standard deviation above their reading scores. The arithmetic disabled subgroup achieved arithmetic scores below the 31st percentile, and at least 1 standard deviation below the Reading score. All children had WISC-R full scale IQ scores between 80 and 115.

³ Subjects in this study were seventh and eighth grade students with scores on the mathematics portion of the SAT (SAT-M) greater than or equal to 500, and scores on the verbal portion of the SAT (SAT-V) greater than or equal to 430 (levels equal to the scores of college bound seniors). Extreme mathematical precocity was defined as a SAT-M score greater than or equal to 580 and a SAT-V score greater than or equal to 450. Extreme verbal precocity was defined as a SAT-M score greater than or equal to 550 and a SAT-V score greater than or equal to 490.

Table 1

Experiment 1: Median Latencies (ms) to the Arithmetic Problems as a
Function of Memory Task and Load

<u>Load</u>	<u>Memory Task</u>	
	<u>Spatial</u>	<u>Verbal</u>
<u>Low</u>	1948	1874
<u>High</u>	1854	1897

Table 2

Experiment 1: Median Latencies (ms) to the Arithmetic Problems as a
Function of Problem Type and Skill

<u>Skill</u>	<u>Problem Type</u>	
	<u>No-carry</u>	<u>Carry</u>
<u>Less-skilled</u>	1782	2697
<u>Skilled</u>	1270	1824

Table 3

Experiment 1: Median Latencies (ms) to the Arithmetic Problems as a
Function of Memory Task and Problem Type

<u>Problem Type</u>	<u>Memory Task</u>	
	<u>Spatial</u>	<u>Verbal</u>
No-carry	1514	1538
Carry	2288	2234

Table 4

Experiment 1: Median Latencies (ms) to the Arithmetic Problems as a
Function of Gender and Load

<u>Load</u>	<u>Gender</u>	
	<u>Male</u>	<u>Female</u>
Low	1779	2043
High	1798	1953

Table 5

Experiment 1: Median Latencies (ms) to the Arithmetic Problems as a
Function of Gender, Memory Task, and Problem Type

	<u>Gender</u>	
	<u>Male</u>	<u>Female</u>
Verbal Task		
No-carry	1447	1628
Carry	2142	2325
Spatial Task		
No-carry	1462	1567
Carry	2102	2473

Table 6

Experiment 1: Median Latencies (ms) to the Arithmetic Problems as a
Function of Gender, Problem Type, and Load

	<u>Gender</u>	
	<u>Male</u>	<u>Female</u>
No-Carry Problems		
Low Load	1452	1610
High Load	1448	1548
Carry Problems		
Low Load	2096	2475
High Load	2148	2323

Table 7

Experiment 1: Percent Errors to the Arithmetic Problems as a Function of Skill, Memory Task, and Load

		Skill	
		Less-skilled	Skilled
Verbal Task			
Low Load	8.4		6.8
High Load	13.9		8.1
Spatial Task			
Low Load	10.8		6.8
High Load	10.0		8.9

Table 8

Experiment 1: Percent Errors to the Memory Probe Items in the Spatial Task Condition at Each Level of Problem Type and Load (Females; n=24)

		Skill	
		Less-skilled	Skilled
No-carry Problems			
Low Load	10.4		6.2
High Load	27.7		20.1
Carry Problems			
Low Load	19.6		15.3
High Load	30.5		31.2

Table 9

Experiment 2: Median Latencies (ms) to the Arithmetic Problems as a
Function of Problem Type and Load

	<u>Problem Type</u>	
<u>Load</u>	<u>No-carry</u>	<u>Carry</u>
Low	1369	2239
High	1416	2033

Table 10

Experiment 2: Median Latencies (ms) to the Arithmetic Problems as a
Function of Skill and Problem Type

	<u>Problem Type</u>	
<u>Skill</u>	<u>No-carry</u>	<u>Carry</u>
Less-skilled	1651	2800
Skilled	1134	1471

Table 11

Experiment 2: Percentage Strategy Use as a Function of Problem Type and Load (Full Sample; n=16)

Problem		Retrieve +			Count +		
Type	Load	Retrieve	Increment	Count-on	Recomb.	Carry	Other
NC	Low	74.2	20.0	4.2	1.1	0.0	0.5
NC	High	75.8	19.2	3.2	0.5	0.5	0.5
C	Low	33.7	38.5	9.8	6.8	6.2	3.1
C	High	32.5	44.2	6.8	7.3	7.3	1.1

NC = No-carry problems; C = Carry problems

Recomb. = Recombination strategy

Table 12

Experiment 2: Percentage Strategy Use as a Function of Problem Type and Load (Skilled Subjects; n=8)

Problem		Retrieve +			Count +		
Type	Load	Retrieve	Increment	Count-on	Recomb.	Carry	Other
NC	Low	74.2	23.3	1.1	0.0	0.0	1.1
NC	High	75.0	21.7	0.0	1.1	1.1	0.0
C	Low	41.7	40.0	0.0	7.3	10.8	0.0
C	High	36.7	40.0	0.0	10.8	10.8	1.1

NC = No-carry problems; C = Carry problems

Recomb. = Recombination strategy

Table 13

Experiment 2: Percentage Strategy Use as a Function of Problem Type and Load (Less-skilled Subjects; n=8)

Problem		Retrieve +				Count +	
Type	Load	Retrieve	Increment	Count-on	Recomb.	Carry	Other
NC	Low	74.2	15.0	7.3	2.1	0.0	0.0
NC	High	75.8	16.7	6.3	0.0	0.0	1.1
C	Low	25.0	35.8	17.5	11.7	1.1	4.2
C	High	28.3	48.3	13.3	5.3	4.2	1.1

NC = No-carry problems; C = Carry problems

Recomb. = Recombination strategy

Table 14

Experiment 2: Combined Percentage Strategy Use Scores (Full Sample; n=16)

Problem			
Type	Load	Retrieval	Non-Retrieval
NC	Low	93.7	5.2
NC	High	94.8	4.2
C	Low	70.8	24.0
C	High	76.1	21.4

NC = No-carry problems; C = Carry problems

Table 15

Experiment 2: Percentage Strategy Use for Carry and No-carry problems
as a Function of Skill and Load

		Skill	
		Less-skilled (M/F)	Skilled(M/F)
No-Carry Problems			
Retrieval			
Low Load	90.6 (96/85)		96.9 (98/96)
High Load	92.7 (96/90)		96.8 (96/98)
Non-retrieval			
Low Load	9.4 (4/15)		1.0 (0/2)
High Load	6.3 (2/10)		2.1 (4/0)
Carry Problems			
Retrieval			
Low Load	55.2 (54/56)		86.5 (75/97)
High Load	68.7 (69/68)		83.3 (73/94)
Non-retrieval			
Low Load	38.5 (42/35)		9.3 (19/0)
High Load	30.3 (29/31)		12.5 (25/0)

M = Male; F = Female

Table 16

Experiment 3: Median Latencies (ms) to the Arithmetic Problems as a Function of Problem Type and Skill

Skill	Problem Type	
	No-carry	Carry
Less-skilled	2209	3440
Skilled	1965	2669

Table 17

Experiment 3: Percent Errors to the Arithmetic Problems as a Function of Skill, Memory Task, and Load

	Skill	
	Less-skilled	Skilled
Verbal Task		
Low Load	8.1	5.4
High Load	8.2	8.1
Spatial Task		
Low Load	8.2	8.7
High Load	8.6	6.3

Table 18

Experiment 3: Percent Errors to the Memory Probe Items as a Function of Memory Task, Problem Type, and Load

	<u>Problem Type</u>	
	<u>No-carry</u>	<u>Carry</u>
<u>Verbal Task</u>		
Low Load	16.3	13.3
High Load	22.6	31.9
<u>Spatial Task</u>		
Low Load	9.7	29.1
High Load	27.3	34.8

Figure 1(a)

Dual-task Procedure: Spatial Memory Task

Figure 1(b)

Dual-task Procedure: Verbal Memory Task

Figure 2

Experiment 1: Median Latencies (ms) as a Function of Skill, Gender, Memory Task and Problem Type

Figure 3

Experiment 1: Median Latencies (ms) as a Function of Skill, Gender, Memory Task and Load

Figure 4 (a)

Experiment 1: Median Latencies (ms) in the Spatial Task
Condition as a Function of Skill, Gender, Problem Type,
and Load

Figure 4 (b)

Experiment 1: Median Latencies (ms) in the Verbal Task
Condition as a Function of Skill, Gender, Problem Type,
and Load

Figure 5

Experiment 2: Median Latencies (ms) as a Function of Skill, Gender, Problem Type, and Load

Figure 6 (a)

Experiment 3: Median Latencies (ms) in the Verbal Task
Condition as a Function of Skill, Gender, Problem Type,
and Load

Figure 6 (b)

Experiment 3: Median Latencies (ms) in the Spatial Task
Condition as a Function of Skill, Gender, Problem Type,
and Load

Figure 7

Experiment 3: Percent Errors to the Arithmetic Problems as a Function of Skill, Gender, Problem Type, and Load

No-carry Problems - Males

No-carry Problems - Females

Carry Problems - Males

Carry Problems - Females

Figure 8

Experiment 3: Percent Errors to the Arithmetic Problems as a Function of Skill, Memory Task, Problem Type, and Load

Carry Problems - Verbal Task

Carry Problems - Spatial Task

No-carry Problems - Verbal Task

No-carry Problems - Spatial Task

Figure 9 (a)

Experiment 3: Percent Errors to the Arithmetic Problems in the Verbal Task Condition as a Function of Skill, Gender, Problem Type, and Load

No-carry Problems - Males

No-carry Problems - Females

Carry Problems - Males

Carry Problems - Females

Figure 9 (b)

Experiment 3: Percent Errors to the Arithmetic Problems in the Spatial Task Condition as a Function of Skill, Gender, Problem Type, and Load

No-carry Problems - Males **No-carry Problems - Females** **Carry Problems - Males** **Carry Problems - Females**

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Appendix A.1
Descriptive Information and Sample Items From the Arithmetic
Fluency Test

Descriptive Information and Sample Items From the French

Kit

The arithmetic subtest of the French Kit (French et al., 1963) comprises four pages of arithmetic problems; two pages of addition problems (120 in total), and two pages of combined subtraction and multiplication problems (60 problems of each type). On each page subjects complete as many consecutive problems as possible in two minutes. Prior to beginning the timed administration of the addition problems, subjects are asked to solve 10 practice problems. Similarly, prior to the timed administration of the subtraction and multiplication problems subjects have the opportunity to solve 10 practice problems of each type. Note that in group administration it is not possible to ensure that subjects complete all of the practice problems. Within each section, the arithmetic problems become progressively more difficult. For example, the addition section begins with addition of three single-digit numbers, and progresses to addition of three double-digit numbers.

Successful performance on this arithmetic test involves at least three components. First, because it is a timed test direct retrieval of basic arithmetic facts should enhance performance; other things being equal, individuals who retrieve most basic facts should benefit in terms of speed. Second, the test is designed so that people who can identify numeric relations will benefit. For example, in many of the addition problems two of the three numbers in the ones' column sum to 10. Thus, someone who can readily identify a unit of 10 will be at an advantage because they can reduce the three-digit problem to a

simple two-digit (10-plus) problem. Third, the test requires proficiency in the procedural operations of carrying and borrowing. Thus, individuals who are practiced at these procedural operations should perform better than individuals who are less-skilled in executing these procedures. Individuals who perform well on this test likely retrieve the sums, or partial sums, for a relatively large number of the problems, are skilled at executing the requisite procedural operations, and are able to see time-saving relations among numbers. Individuals who are less successful on this test likely solve a greater proportion of problems using procedural strategies to obtain basic number facts (i.e., partial sums), are less likely to recognize important relations among numbers, and are less skilled at executing procedures such as carrying.

Examples of the addition, subtraction, and multiplication problems are given on the following page.

1. Addition Problems

8	23	99	54	48	77
3	74	88	34	29	84
<u>7</u>	<u>8</u>	<u>77</u>	<u>78</u>	<u>77</u>	<u>59</u>

2. Subtraction Problems

68	89	39	48	93	31
<u>-47</u>	<u>-60</u>	<u>-23</u>	<u>-19</u>	<u>-45</u>	<u>-27</u>

3. Multiplication Problems

14	50	72	67	80	94
<u>x 3</u>	<u>x 2</u>	<u>x 7</u>	<u>x 8</u>	<u>x 9</u>	<u>x 7</u>

Appendix A.2

Descriptive Statistics for the Arithmetic Fluency Test

All Experiments: Scores on the Arithmetic Fluency Test for the Full Sample, Skilled and Less-skilled Subjects, and Each Combination of Sex and Skill

<u>Full Sample</u>					
Standard					
<u>Experiment</u>	<u>Mean</u>	<u>Deviation</u>	<u>Minimum</u>	<u>Maximum</u>	<u>N</u>
One	87.1	33.7	36	140	48
Two	85.7	25.4	36	125	16
Three	63.0	17.5	2	97	47
<u>Skilled Subjects</u>					
Standard					
<u>Experiment</u>	<u>Mean</u>	<u>Deviation</u>	<u>Minimum</u>	<u>Maximum</u>	<u>N</u>
One	118.7	11.1	104	140	24
Two	104.5	14.7	87	125	8
Three	76.3	9.8	64	97	24
<u>Less-skilled Subjects</u>					
Standard					
<u>Experiment</u>	<u>Mean</u>	<u>Deviation</u>	<u>Minimum</u>	<u>Maximum</u>	<u>N</u>
One	55.3	9.87	36	68	24
Two	67.0	19.0	36	76	8
Three	49.2	12.3	2	60	23

 Skilled Males

Standard					
<u>Experiment</u>	<u>Mean</u>	<u>Deviation</u>	<u>Minimum</u>	<u>Maximum</u>	<u>N</u>
One	118.5	10.6	104	134	12
Two	107.3	14.6	93	125	4
Three	76.3	9.1	64	93	13

 Skilled Females

Standard					
<u>Experiment</u>	<u>Mean</u>	<u>Deviation</u>	<u>Minimum</u>	<u>Maximum</u>	<u>N</u>
One	119.0	12.0	105	140	12
Two	101.8	16.5	87	124	4
Three	76.3	11.0	64	97	11

 Less-skilled Males

Standard					
<u>Experiment</u>	<u>Mean</u>	<u>Deviation</u>	<u>Minimum</u>	<u>Maximum</u>	<u>N</u>
One	57.5	8.5	44	68	12
Two	62.0	18.3	36	76	4
Three	49.5	9.5	30	60	10

 Less-skilled Females

Standard					
<u>Experiment</u>	<u>Mean</u>	<u>Deviation</u>	<u>Minimum</u>	<u>Maximum</u>	<u>N</u>
One	53.3	11.0	36	67	12
Two	72.0	13.7	52	82	4
Three	49.0	14.5	2	58	13

Appendix B

Addition Problems used in Experiments One Through Three

Carry and No-carry Problems (and correct sums) Used in The Three Experiments

<u>Carry</u> (sum)	<u>No-carry</u> (sum)
5 + 17 (22)	3 + 24 (27)
6 + 17 (23)	6 + 22 (28)
8 + 16 (24)	6 + 23 (29)
8 + 23 (31)	3 + 32 (35)
5 + 28 (33)	3 + 36 (39)
6 + 28 (34)	4 + 35 (39)
4 + 37 (41)	4 + 42 (46)
7 + 36 (43)	5 + 42 (47)
7 + 38 (45)	6 + 43 (49)
3 + 48 (51)	5 + 52 (57)
5 + 47 (52)	5 + 53 (58)
8 + 46 (54)	7 + 52 (59)

END

1 8 - 0 7 - 9 4

FIN