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"(Anti)-Neutrino-Deuteron Scattering and the
Structure of the Weak Hadronic Neutral Currents"

by

Mahfuzur Rahman

A thesis submitted to the Faculty of Graduate Studies
in partial fulfillment of the requirements for the Degree of
Doctor of Philosophy.

Carleton University

Ottawa, Ontario

1978

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The undersigned hereby recommend to the Faculty of Graduate Studies acceptance of the thesis, "(Anti)-Neutrino-Deuteron Scattering and the Structure of the Weak Hadronic Neutral Currents" submitted by Mahfuzur Rahman, in partial fulfillment of the requirements of the Degree of Doctor of Philosophy.

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ABSTRACT

Neutrino (antineutrino) deuteron elastic and neutral current breakup processes are studied to provide information about the structure of the weak hadronic neutral currents. The scattering cross-sections for these processes have been calculated with the help of the impulse approximation, an assessment of the validity of which is also given. It gives an additional input in determining the weak hadronic neutral current couplings. Some of the popular models can be put to test as they give distinguishable predictions for these processes. Also, since the deuterium filled bubble chambers would be exposed to neutrino beams, our results may serve as a guide as to what to expect theoretically.

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INTRODUCTION

The interest to find a single universal theory of weak interactions dates back to the discovery of the nuclear beta-decay. In recent years, an unified theory of the weak and the electromagnetic interactions is being sought.

The early search for the form of weak interactions started with the problem of the apparent missing energy and angular momentum in the nuclear beta-decay. Pauli's neutrino hypothesis in 1933 made it possible for Fermi to give a theory of weak interactions in 1934. According to it, the nuclear beta-decay is a manifestation of the fundamental processes in the nucleus,

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

$$p \rightarrow n + e^{+} + \nu_e$$

the first of which is pictorially represented by Figure 1.

Figure 1

Fermi assumed that the weak interaction Hamiltonian would depend on the wave functions of the particles involved and that the Hamiltonian was Lorentz invariant. Thus, the four-fermion interaction gets the general form

$$H_{\text{beta-decay}} = \sum_{i=1}^5 G_i \{ (\bar{\psi}_p O_i \psi_n) (\bar{\psi}_e O_i \psi_{\nu e}) + \text{h.c.} \}$$

where G_i 's are the weak coupling constants and O_i 's are suitable combinations of the Dirac operators so that $(\bar{\psi}_p O_i \psi_n)$ or $(\bar{\psi}_e O_i \psi_{\nu e})$ are Lorentz bilinear covariants, given by

Scalar, abbreviated as S, $\bar{\psi}_p \psi_n$

Pseudoscalar, abbreviated as P, $\bar{\psi}_p \gamma_5 \psi_n$

Vector, abbreviated as V, $\bar{\psi}_p \gamma_\mu \psi_n$

Axial vector, abbreviated as A, $\bar{\psi}_p \gamma_\mu \gamma_5 \psi_n$

and

Tensor, abbreviated as T, $\bar{\psi}_p \sigma_{\alpha\beta} \psi_n$

Fermi's ideas were formulated in close analogy to the electromagnetic interaction and that is why he believed that only vector bilinear covariants contributed. But soon, it was found to be wrong.

Now the question was, which Dirac variants contributed. For allowed beta-decays, one finds that for Fermi transition where one finds emission of e^- and $\bar{\nu}_e$ with spins anti-parallel, S and V contribute. On the other hand, for Gamow-Teller transition, where one finds emission of e^- and $\bar{\nu}_e$ with parallel spins, T and A contribute. A large number of experiments were done to see which combination was favoured. After some of the experiments were redone which were giving conflicting results, it was established that the (V,A) combination was the one contributing to weak interactions. The direct measurement of the neutrino helicity established the left-handedness of neutrino and confirmed the V-A structure.

Once the V-A current-current structure was accepted, the concept of the universal weak interaction acquired a precise meaning. One could obtain a unified description of the weak decays of nucleon, muon and pion by means of the single current-current interaction.

$$H_w = \frac{G}{\sqrt{2}} J_\lambda J^{\lambda\dagger}$$

where

$$J_\lambda = J_\lambda^V - J_\lambda^A = \bar{\psi}_e \gamma_\lambda (1 - \gamma_5) \psi_\nu + \bar{\psi}_\nu \gamma_\lambda (1 - \gamma_5) \psi_e + \bar{\psi}_p \gamma_\lambda (1 - \gamma_5) \psi_n$$

and G is Fermi's dimensional weak coupling constant given by $G_F^2 \approx 10^{-5}$. Cabibbo connected the strangeness conserving and

the strangeness changing charged weak currents saying that they belong to the same octet representation. Using his hypothesis, one can write the weak charged current as

$$J_{\lambda} = G \cos \theta_c (V_{\lambda}^{1-i2} + A_{\lambda}^{1-i2}) + G \sin \theta_c (V_{\lambda}^{4-i5} + A_{\lambda}^{4-i5})$$

We use $\hbar = c = 1$ throughout.

This current-current theory was very successful in explaining the then observed characteristic features of weak processes. But such a local current-current theory violates unitarity at energies of about 900 GeV^1 in $\nu_e e$ scattering. In an effort to handle this problem, the intermediate vector boson hypothesis was proposed. According to this hypothesis, weak processes are mediated by vector bosons just as electromagnetic processes are mediated by photons and it was expected to give an acceptable high energy behaviour for the theory. The nuclear beta-decay would then be pictured by means of the Feynman diagram shown in Figure 2.

Figure 2

Even this intermediate vector boson hypothesis does not rectify the unitarity problems. For example, in the process indicated by Figure 3, we get unitarity breakdown at about 2700 GeV.

Figure 3

Theorists have been working for a long time to solve this problem and at the same time obtain a unified theory of weak and electromagnetic interactions. Weinberg²⁾ and Salam³⁾ gave the first practical model to unify weak and electromagnetic interactions. Their model, known as the W-S, $SU(2) \otimes U(1)$ gauge model, is quite elegant and simple. The model gained in importance as a result of the work performed by G. t'Hooft⁴⁾ in 1971, who showed the renormalizability of a class of theories of which W-S model was one. A good reference work reviewing such theories and ideas of renormalizability etc. is the paper by Abers and Lee.⁵⁾ However, the Weinberg-Salam model needed the presence of neutral currents which couples with zero net charge and with a coupling strength comparable to the conventional charged current coupling. As an example, to cancel the divergent contribution of the previous figure, one needs the contribution of

a process expressed in Figure 4.

Figure 4

This prompted experimentalists to look for the signal of the neutral current processes and by 1973, Hasert et al.⁶⁾ had the first positive indications of neutral currents in the neutrino-electron⁶⁾ and the neutrino nucleon deep inelastic scattering.⁷⁾

Once the neutral-current process was detected, a large number of experiments have been performed to learn more about the neutral currents. A good review on the development of experiments regarding neutral currents can be found in Sciuli's report.⁸⁾

From the theorists point of view, the question of general space-time and isospin structure has been the most important one. A large number of papers have been written in the last few years on this subject.

Although neutral currents were first looked for as a consequence of gauge theories, the neutral current phenomena are interesting in their own right, regardless of whether they help one in constructing gauge theories or not.⁹⁾

To illustrate what we want to do, we see that the charged

weak current, for the process $\nu_\mu + n \rightarrow \mu^- + p$ is isovector and can be written as

$$J_\mu^{1+i2} = V_\mu^{1+i2} - A_\mu^{1+i2}$$

We can write for the electromagnetic current in the same notation, for the process $e p \rightarrow e p$ as

$$J_\mu^{e,m} = V_\mu^3 + V_\mu^0$$

Here V_μ^3 term is related to V_μ^{1+i2} . In fact, V_μ^3 is the isospin partner of V_μ^1 and V_μ^2 . This connection between V_μ^{1+i2} with V_μ^3 by isospin rotation resulted in the proposition of the conserved vector current hypothesis. V_μ^0 is the isoscalar vector piece in the electromagnetic current.

Since the discovery of the neutral current process, the question of its space-time structure and isospin structure have been the focal point of many investigations. There seems to be good evidence that the space-time or the Lorentz structure of the neutral currents is a mixture of vector (V) and axial vector (A).

For the purely leptonic neutral current, that the current is of (V,A) form follows from Faissner's¹⁰⁾ analysis of neutrino-electron scattering data. The experimental results¹¹⁾ indicate, as shown in Figure 5 in the following page, that

$$\sigma(\nu_\mu e) \neq \sigma(\bar{\nu}_\mu e)$$

Fig. 5: $\nu_e e$ and $\bar{\nu}_e e$ total cross sections, as measured by AD-PD and GARGAMELLE. Straight lines delineate simple V, A interactions, the curve is for the minimal gauge model.

and average value of the quantity y given by $\frac{E_e}{E_\nu}$,

$$\langle y \rangle = 0.30 \pm 0.07 \quad \text{for } \nu_\mu e$$

$$= 0.54 \pm 0.09 \quad \text{for } \bar{\nu}_\mu e$$

On the other hand, an arbitrary mixture of scalar (S) and pseudoscalar (P) would imply equality of $\sigma(\nu_\mu e)$ and $\sigma(\bar{\nu}_\mu e)$ and $\langle y \rangle = 3/4$. Thus, the data clearly do not support any hint on the occurrence of Dirac variants other than (V,A). A point to keep in mind is the "confusion theorem" which states that any (V,A) combination can be reproduced by a combination of S,P and tensor (T).

For the structure of the hadronic neutral current, the information about the space-time structure is extracted from the neutrino-nucleon inclusive reactions. The implications of the anomalous Lorentz structure in the neutral weak processes have been discussed by many authors.

To discuss the deep-inelastic scattering, one uses parton model and the cross-section $\frac{d\sigma}{dy}$ is given in terms of

$$y = \frac{E_\nu - E'_\nu}{E_\nu}$$

with

E_ν = incident lab. neutrino energy

E'_ν = final lab. neutrino energy.

Following Kingsley, Wilczek and Zee¹²⁾

$$\frac{d\sigma^V}{dy} = \frac{S}{4\pi} \int_0^1 dx \times \sum_i f_i(x) \{ 2[(g_V^i + g_A^i)^2 + (g_V^i - g_A^i)^2(1-y)^2] + (|g_S^i|^2 + |g_P^i|^2)y^2 + 32|g_T^i|(1-\frac{1}{2}y)^2 - 8\text{Re}[g_T^i(g_S^{*i} + g_P^{*i})]y(1-\frac{1}{2}y) \}$$

where

$$S = (\text{c.m. energy})^2$$

$f_i(x)$ denotes the longitudinal momentum distribution function of quark partons of type i in the nucleon.

For the antineutrino process, one has to change the sign of g_A^i and g_T^i terms.

Now, by looking at the y -distribution, one can predict about the space-time structure. As an example, any combination of S and P would give

$$\frac{1}{E_V} \frac{d\sigma^V}{dy} = \text{constant } xy^2,$$

for a pure T , one would get

$$\frac{1}{E_V} \frac{d\sigma^V}{dy} = \text{constant} \times (1 - \frac{1}{2}y)^2.$$

For a combination of V and A ,

$$\frac{1}{E_V} \frac{d\sigma^V}{dy} = \text{constant} [a + b(1-y)^2]$$

From the observed distribution of $\frac{d\sigma}{dy}$ vs y , any y^2 dependence is ruled out at the level of five standard deviations.¹³⁾

Benvenuti et al.¹⁴⁾ used the hadronic energy distributions and total cross-sections and showed that the hadronic neutral currents favour (V,A) structure.

Quoting their results,

Form of Weak Neutral Current	$\sigma_{\bar{\nu}N}/\sigma_{\nu N}$	
	Corrected (for cut at $E_{\text{hadrons}} = 4 \text{ GeV}$) experimental value	Theoretically expected value
V-A	0.61 ± 0.25	0.38
V or A	0.40 ± 0.17	1.00
V + A	0.37 ± 0.16	2.65
S, P or S and P	0.26 ± 0.09	1.00
S, P interference with T	0.31 ± 0.11	1.00
T	0.45 ± 0.15	1.00

H.P. Paar¹⁵⁾ used the y distribution for neutral current to charged current ratio, given by $(\frac{d\sigma}{dy})_{\text{NC}} / (\frac{d\sigma}{dy})_{\text{CC}}$ or

$$\frac{NC}{CC} = \frac{g_L + g_R(1-y)^2 + g_{SP}y^2}{f_L + f_R(1-y)^2}$$

$$= \frac{g_L(1-y)^2 + g_R + g_{SP}y^2}{\bar{f}_L(1-y)^2 + \bar{f}_R}$$

for ν for $\bar{\nu}$

with $f_L=1.05$, $f_R=0.07$, $\bar{f}_L=0.98$ and $\bar{f}_R=0.10$ as suggested by quark-parton model as input parameters for charged current y -distributions. Most results are insensitive to the specific value of these parameters. They found

$$\frac{g_{SP}}{g_L + g_R} = 0.02 \pm 0.07$$

These results imply that the neutral current is of the form (V,A).

Fischbach et al.¹⁶⁾ noted that in elastic neutrino-proton scattering, a pure S, P combination is ruled out since $\sigma(\nu_\mu p) \neq \sigma(\bar{\nu}_\mu p)$.

Thus, from all this information, one believes that the space-time structure of the weak neutral current is a combination of vector and axial vector terms.

Next comes the question of the isospin structure of the neutral currents. Since currents are generally formed out of bilinear combination of quarks with isospin restricted to $I=\frac{1}{2}$ and 0, the general structure of neutral currents is

$$J_\mu^{NC} = v_\mu^3 + a_\mu^3 + v_\mu^0 + a_\mu^0$$

In comparison to charged currents and electromagnetic currents, in neutral currents, the isoscalar axial vector part a_μ^0 appears. Knowledge of its presence would be interesting and a good place to investigate its presence would be in $\nu(\bar{\nu})$ deuteron-elastic scattering.¹⁷⁾ Any difference in ν and $\bar{\nu}$ cross-section would signal its presence.

Now we can write the effective Lagrangian for neutral current interactions as¹⁸⁾ (with Bjorken and Drell conventions)

$$L = \frac{G}{\sqrt{2}} \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu \left\{ \frac{1}{2} [\bar{u} \gamma^\mu (\alpha - \beta \gamma_5) u - \bar{d} \gamma^\mu (\alpha - \beta \gamma_5) d] \right. \\ \left. + \frac{1}{2} [\bar{u} \gamma^\mu (\frac{1}{3} \gamma - \delta \gamma_5) u + \bar{d} \gamma^\mu (\frac{1}{3} \gamma - \delta \gamma_5) d] + \text{possible s, c, t, b, terms} \right\}$$

With a different notation¹⁹⁾,

$$L = \frac{G}{\sqrt{2}} \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu \{ \bar{u} \gamma_\mu [u_L (1 + \gamma_5) + u_R (1 - \gamma_5)] + \bar{d} \gamma_\mu [d_L (1 + \gamma_5) + d_R (1 - \gamma_5)] \\ + \text{possible s, c, t, b, terms} \}$$

where the Lagrangian is written with right and left-handed couplings separated out and the relation between $(\alpha, \beta, \gamma, \delta)$ and (u_L, u_R, d_L, d_R) is given by

$$\alpha = u_L + u_R - d_L - d_R$$

$$\beta = u_L - u_R - d_L + d_R$$

$$\gamma = 3(u_L + u_R + d_L + d_R)$$

$$\delta = u_L - u_R + d_L - d_R$$

We use the values of α , β , γ and δ in Albright²⁰⁾ et al., in which γ is three times that of Hung and Sakurai¹⁸⁾, other constants remaining the same.

Once the Lagrangian is expressed in terms of the coupling parameters α , β , γ and δ , it remains to determine the values of them. To determine them, the processes considered are:

- i) Inclusive neutrino scattering²¹⁾ ($\nu N \rightarrow \nu X$)
- ii) Pion semi-inclusive scattering²²⁾ ($\nu N \rightarrow \nu \pi X$)
- iii) Elastic neutrino-proton scattering²³⁾ ($\nu p \rightarrow \nu p$)
- iv) Exclusive pion scattering²⁴⁾ ($\nu N \rightarrow \nu \pi N$)

Different combinations of the couplings appear in these processes and many people have utilised the neutral current data to determine the coupling parameters.

The information from inclusive neutrino scattering is obtained and compared with the corresponding charged current processes. Using quark-parton techniques²⁵⁾

$$\frac{[(\frac{d\sigma^{\nu p}}{dy} + \frac{d\sigma^{\nu n}}{dy}) + (\frac{d\sigma^{\bar{\nu} p}}{dy} + \frac{d\sigma^{\bar{\nu} n}}{dy})]_{\text{neutral current}}}{[(\frac{d\sigma^{\nu p}}{dy} + \frac{d\sigma^{\nu n}}{dy}) + (\frac{d\sigma^{\bar{\nu} p}}{dy} + \frac{d\sigma^{\bar{\nu} n}}{dy})]_{\text{charged current}}} = \frac{1}{4}(\alpha^2 + \beta^2 + \frac{1}{9}\gamma^2 + \delta^2)$$

and

$$\frac{[(\frac{d\sigma}{dy}^{vp} + \frac{d\sigma}{dy}^{vn}) - (\frac{d\sigma}{dy}^{\bar{v}p} + \frac{d\sigma}{dy}^{\bar{v}n})]_{\text{neutral current}}}{[(\frac{d\sigma}{dy}^{vp} + \frac{d\sigma}{dy}^{vn}) - (\frac{d\sigma}{dy}^{\bar{v}p} + \frac{d\sigma}{dy}^{\bar{v}n})]_{\text{charged current}}} = \frac{1}{2} (\alpha\beta + \frac{1}{3}\gamma\delta)$$

Here

$$\left. \frac{d\sigma}{dy}^{v(\bar{v})N} \right|_{\text{neutral current}} = \frac{d\sigma}{dy} (v(\bar{v})N \rightarrow v(\bar{v})+x)$$

and

$$\left. \frac{d\sigma}{dy}^{v(\bar{v})N} \right|_{\text{charged current}} = \frac{d\sigma}{dy} (v(\bar{v})N \rightarrow \mu^-(\mu^+)+x)$$

Neutrino-induced semi-inclusive pion reaction gives us information about $(\frac{1}{3}\gamma\alpha + \beta\delta)$ and $(\alpha\delta + \frac{1}{3}\gamma\beta)$. Hung²⁶⁾ used quark fragmentation model to get

$$\left(\frac{\pi^-}{\pi^0} \right)_{\text{NC}}^{vN} = 1 - \left(\frac{1-\omega(z)}{1+\omega(z)} \right) \left(\frac{2\alpha \frac{1}{3}\gamma + 2\beta\delta + \alpha\delta + \frac{1}{3}\beta\gamma}{\alpha^2 + \beta^2 + \frac{1}{9}\gamma^2 + \delta^2 + \alpha\beta + \frac{1}{3}\gamma\delta} \right)$$

and

$$\left(\frac{\pi^-}{\pi^0} \right)_{\text{CC}}^{\bar{v}N} = 1 - \left(\frac{1-\omega(z)}{1+\omega(z)} \right) \left(\frac{2\alpha \frac{1}{3}\gamma + 2\beta\delta - \alpha\delta - \frac{1}{3}\beta\gamma}{\alpha^2 + \beta^2 + \frac{1}{9}\gamma^2 + \delta^2 - \alpha\beta - \frac{1}{3}\gamma\delta} \right)$$

where

$$\left(\frac{\pi^-}{\pi^0}\right)_{NC}^{v(\bar{\nu})N} (x, z) = \frac{\frac{d\sigma}{dx dz} [v(\bar{\nu}) + N \rightarrow v(\bar{\nu}) + \pi + x]}{\left(\frac{d\sigma}{dy}\right)_{NC} [v(\bar{\nu}) + N \rightarrow v(\bar{\nu}) + x]}$$

$$z = \frac{E_{\text{hadron } x}}{E_{\nu}}$$

and

$$\omega(z) = \frac{D_u^{\pi^-}(z)}{D_u^{\pi^+}(z)}$$

with $D_q^h(z)$ as the probability for a parton q to fragment into a hadron h plus anything.

But for quark fragmentation model to work, one must have

$$\left(\frac{\pi^+ + \pi^-}{\pi^0}\right)_{\nu \rightarrow \nu} = \left(\frac{\pi^+ + \pi^-}{\pi^0}\right)_{\bar{\nu} \rightarrow \bar{\nu}} = 2$$

The data of Kluttig et al.²²⁾ seem to support it.

Numerically,

$$\left(\frac{\pi^+ + \pi^-}{\pi^0}\right)_{\nu \rightarrow \nu} = 1.90 \pm 0.30 \quad \text{and} \quad \left(\frac{\pi^+ + \pi^-}{\pi^0}\right)_{\bar{\nu} \rightarrow \bar{\nu}} = 2.03 \pm 0.37$$

Next, from the elastic neutrino-proton scattering, one can reduce the number of acceptable set of $\alpha, \beta, \gamma, \delta$ to within two.²⁷⁾

These two sets of solutions are known as Hung and Sakurai solutions.

A and B and hereafter referred to as H-S(A) and H-S(B) respectively.

Abbott and Barnett²⁸⁾ used exclusive pion production processes to determine a unique set of solutions for α , β , γ , δ . The neutrino induced exclusive pion production process was analyzed first by Adler.²⁹⁾ Adler used current algebra techniques to discuss the process. He started out with the soft-pion formula for pion emission and obtained a relation between the threshold pion production cross-section and the νp elastic cross-section. The model is only valid for small invariant pion-nucleon mass, namely, it is required to be less than 1.4 GeV. Also, the calculation has $O(q)$ corrections. There are some uncertainties as to how these corrections can be handled. However, Abbott and Barnett note that even with these limitations, one can determine a unique set of α , β , γ , δ . The value of the ratio of the isoscalar axial vector form factor to isovector axial vector form factor at zero momentum transfer is taken as 3/5 from the quark phenomenological relation³⁰⁾ or the SU(6) quark representation of protons and neutrons.

The aim of this thesis is to calculate $\nu(\bar{\nu})$ deuteron scattering. It will give information on the existence of isoscalar axial vector currents. In view of the previous discussions, it would also give extra information pertaining to the determination of the coupling parameters. Apart from the interest in the determination of the coupling parameters, the calculations would be of some intrinsic interest since deuterium filled bubble chambers will soon be exposed to neutrino beams and one would like to know what to expect.

It will be found that the $\nu(\bar{\nu})d$ elastic differential cross-section goes down very fast with the increase of momentum transfer. Typically, it is found that at zero momentum transfer νd elastic cross-section is large compared to νp but goes down much faster. For the νp elastic process, using the numbers obtained from the predictions of H-S(A) and H-S(B), both of which give results which satisfy the data, we find that at a square of momentum transfer 0.07 (GeV/c)^2 , H-S(A) gives $3.03 \times 10^{-39} \text{ cm}^2/(\text{GeV/c})^2$ and $1.38 \times 10^{-39} \text{ cm}^2/(\text{GeV/c})^2$ for νp and νd elastic cross-section respectively but at 0.4 (GeV/c)^2 , we get $0.84 \times 10^{-39} \text{ cm}^2/(\text{GeV/c})^2$ and $0.013 \times 10^{-39} \text{ cm}^2/(\text{GeV/c})^2$ respectively for them. According to H-S(B), we get at 0.07 (GeV/c)^2 , $2.86 \times 10^{-39} \text{ cm}^2/(\text{GeV/c})^2$ for νp and $0.495 \times 10^{-39} \text{ cm}^2/(\text{GeV/c})^2$ for νd , whereas at 0.4 (GeV/c)^2 , $1.14 \times 10^{-39} \text{ cm}^2/(\text{GeV/c})^2$ for νp and $0.015 \times 10^{-39} \text{ cm}^2/(\text{GeV/c})^2$ for νd . Thus, it may be that νd elastic process may be really hard to detect experimentally, specially at larger momentum transfer. Also, the observation of any difference between the νd and the $\bar{\nu} d$ elastic cross-sections may be almost impossible since according to H-S(A) one would need to measure a difference of $0.01 \times 10^{-39} \text{ cm}^2/(\text{GeV/c})^2$ at 0.07 (GeV/c)^2 . It may be mentioned here that in νp elastic scattering the quoted experimental error is about $10^{-39} \text{ cm}^2/(\text{GeV/c})^2$ at a momentum transfer-squared of 0.3 (GeV/c)^2 . For this reason, we calculate the $\nu(\bar{\nu})d$ neutral current breakup cross-section. A typical result of the calculation is that, for H-S(A) the νd breakup differential cross-section at 0.07 (GeV/c)^2 is $4.689 \times 10^{-39} \text{ cm}^2/(\text{GeV/c})^2$ and $0.893 \times 10^{-39} \text{ cm}^2/(\text{GeV/c})^2$ at 0.4 (GeV/c)^2 . We can compare

these numbers to the charged current process where one gets³¹⁾ $16.8 \times 10^{-39} \text{ cm}^2/(\text{GeV}/c)^2$ at $0.07 (\text{GeV}/c)^2$ and $10 \times 10^{-39} \text{ cm}^2/(\text{GeV}/c)^2$ at $0.4 (\text{GeV}/c)^2$. The calculation of $\nu(\bar{\nu})d$ elastic process closely parallels the calculation of ed elastic scattering and the calculation of the breakup process is similar to the calculation of quasi-elastic $\nu d \rightarrow \mu pp$ process with the help of impulse approximation.

The results of our calculation gives distinguishable results for the different models considered. As an example, our results show that at a momentum transfer-squared of $0.07 (\text{GeV}/c)^2$, the difference in the differential cross-section for νd elastic process, for models H-S(A) and H-S(B) is $0.886 \times 10^{-39} \text{ cm}^2/(\text{GeV}/c)^2$ which should be possible to detect experimentally. Also, in the $\nu(\bar{\nu})d$ breakup process, it may be even possible to look at νd and $\bar{\nu} d$ difference in some models. The detailed results are given later.

The thesis is arranged in the following way. In Chapter I, we review the elastic electron-deuteron scattering as an input for the calculation of the elastic (anti) neutrino-deuteron cross-section. In Chapter II, we calculate the differential cross-section for the $\nu(\bar{\nu})d$ elastic scattering in terms of the incident neutrino energy in the laboratory frame, the momentum transfer squared and the deuteron charge, the quadrupole, magnetic and the axial form factors. In Chapter III, we calculate the differential cross-section for the $\nu(\bar{\nu})d$ neutral breakup process. In Chapter IV, we give the different sets of neutral current couplings which we use to make numerical predictions. Here we also mention what choices of the form factors are used. In Chapter V, we give all of the

results of our numerical calculations based on the different choices of models, couplings, etc. Chapter VI discusses our findings and possible improvements.

CHAPTER I

Elastic Electron-Deuteron Scattering

Before we attempt to calculate the cross-section for νd elastic scattering, we review the calculation of ed elastic scattering to get information about the deuteron form factors. ed scattering gives us information about the deuteron electromagnetic form factors. We can use this information to calculate the deuteron weak form factors.

Electron-deuteron scattering was first investigated by Jankus¹⁾ and afterwards by many others. The simplest graph for ed elastic scattering is shown in Figure 6. The scattering amplitude for this simple 'tree' graph, under the one photon exchange approximation is

$$T = \langle D_2 | J^\mu | D \rangle \frac{1}{q^2 - i\epsilon} \langle k_2 | j_\mu | k_1 \rangle \dots \quad (I-1)$$

Here $q = D_2 - D_1 =$ the momentum transfer. The energy-momentum four vectors are indicated in Figure 6.

Figure 6

The differential cross-section is proportional to

$$\sum_{\substack{\lambda_1, \lambda_2 \\ \sigma_1, \sigma_2}} |\langle D_2, \xi_2 | J^\mu | D_1, \xi_1 \rangle \bar{u}_{\sigma_2}(k_2) \gamma_\mu u_{\sigma_1}(k_1) |^2 \quad (I-2)$$

where $u_\sigma(k)$ is a Dirac spinor associated with the electron with four momentum k and spin σ . ξ_1, ξ_2 are the polarization vectors associated with the deuteron and can be written as $\xi_2^\mu = \xi_2^\mu(D_2, \lambda_2)$ and $\xi_1^\mu = \xi_1^\mu(D_1, \lambda_1)$.

We can write

$$\sum_{\substack{\lambda_1, \lambda_2 \\ \sigma_1, \sigma_2}} |\langle D_2, \xi_2 | J^\mu | D_1, \xi_1 \rangle \bar{u}_{\sigma_2}(k_2) \gamma_\mu u_{\sigma_1}(k_1) |^2 = H^{\mu\nu} l_{\mu\nu} \quad (I-3)$$

with

$$H^{\mu\nu} = \sum_{\lambda_1, \lambda_2} \langle D_2, \xi_2 | J^\mu | D_1, \xi_1 \rangle \langle D_1, \xi_1 | J^{\nu*} | D_2, \xi_2 \rangle$$

and

$$l_{\mu\nu} = \sum_{\sigma_1, \sigma_2} [\bar{u}_{\sigma_2}(k_2) \gamma_\mu u_{\sigma_1}(k_1) \bar{u}_{\sigma_1} \gamma_\nu u_{\sigma_2}]$$

The leptonic tensor $l_{\mu\nu}$ is easy to calculate and is given

by

$$l_{\mu\nu} = 4(k_{1\mu} k_{2\nu} + k_{1\nu} k_{2\mu} + \frac{q^2}{2} g_{\mu\nu}) \quad (I-4)$$

The hadronic tensor $H^{\mu\nu}$ is not so trivial since it involves looking into the deuteron structure. The deuteron electromagnetic form factor has been calculated by many people, some of whom are mentioned in Reference 2.

Following Glaser and Jaksic³⁾, the deuteron current $G_D^\mu = \langle D_2 | J^\mu | D_1 \rangle$ can be written as a function of the four vectors D_1 , D_2 and ξ_1 , ξ_2 consistent with relativity and the continuity equation. Assuming time reversal invariance, we get

$$\begin{aligned}
 G_D^\mu &= \langle D_2 | J^\mu | D_1 \rangle \\
 &= G_1(q^2) (\xi_2^* \cdot \xi_1) d^\mu + G_2(q^2) [\xi_1^\mu (\xi_2^* \cdot q) - \xi_2^{*\mu} (\xi_1 \cdot q)] \\
 &\quad - G_3(q^2) \frac{(\xi_1 \cdot q) (\xi_2^* \cdot q)}{2M_d^2} d^\mu
 \end{aligned} \tag{I-5}$$

as obtained by Gross.²⁾

Where $d^\mu = D_2^\mu + D_1^\mu$ and G_1 , G_2 , G_3 are the form factors depending only on q^2 . These form factors carry all the information regarding the deuteron structure and are all real. Thus,

$$\begin{aligned}
 H^{\mu\nu} &= \sum_{\lambda_1, \lambda_2} \left\{ [G_1(q^2) (\xi_2^* \cdot \xi_1) d^\mu + G_2(q^2) \{ \xi_1^\mu (\xi_2^* \cdot q) - \xi_2^{*\mu} (\xi_1 \cdot q) \}] \right. \\
 &\quad - \frac{G_3(q^2)}{2M_d^2} (\xi_1 \cdot q) (\xi_2^* \cdot q) d^\mu] \\
 &\quad \times [G_1(q^2) (\xi_2 \cdot \xi_1^*) d^\nu + G_2(q^2) \{ \xi_1^\nu (\xi_2 \cdot q) - \xi_2^\nu (\xi_1^* \cdot q) \}] \\
 &\quad \left. - \frac{G_3(q^2)}{2M_d^2} (\xi_1^* \cdot q) (\xi_2 \cdot q) d^\nu \right\}
 \end{aligned} \tag{I-6}$$

Now we do the polarization sum, using

$$\sum_{\lambda} \xi^{\mu}(D, \lambda) \xi^{*\nu}(D, \lambda) = -g^{\mu\nu} + \frac{D^{\mu} D^{\nu}}{M_d^2} \quad (I-7)$$

After completing the polarization sum,

$$H^{\mu\nu} = 3[G_c^2(q^2) + \frac{q^4}{18M_d^4} G_Q^2] d^{\mu} d^{\nu} + 2G_2^2(q^2) [q^2(1 - \frac{q^2}{4M_d^2}) g^{\mu\nu} - q^{\mu} q^{\nu} - \frac{q^2}{4M_d^2} d^{\mu} d^{\nu}]$$

where

$$G_c(q^2) = G_1(q^2) - \frac{q^2}{6M_d^2} [G_1(q^2) - G_2(q^2) + G_3(q^2) \{1 - \frac{q^2}{4M_d^2}\}] \quad (I-8)$$

$$G_Q(q^2) = G_1(q^2) - G_2(q^2) + G_3(q^2) \{1 - \frac{q^2}{4M_d^2}\}$$

Thus,

$$\begin{aligned} H^{\mu\nu} \epsilon_{\mu\nu} = & 12 \left[G_c^2(q^2) + \frac{q^4}{18M_d^4} G_Q^2(q^2) - \frac{q^2}{6M_d^2} G_2^2(q^2) \right] \left\{ 2d \cdot k_1 d \cdot k_2 \right. \\ & \left. + \frac{q^2}{2} d^2 \right\} \\ & + 8G_2^2(q^2) \left[q^2(1 - \frac{q^2}{4M_d^2})(2k_1 \cdot k_2 + 2q^2) \right] \\ & - 8G_2^2(q^2) \left[2(q \cdot k_1)(q \cdot k_2) + \frac{q^4}{4} \right] \quad (I-9) \end{aligned}$$

with $S^{\mu} = (D_1 + k_1)^{\mu} = (D_2 + k_2)^{\mu}$,

$$H^{\mu\nu} \ell_{\mu\nu} = 12 \left[G_c^2(q^2) + \frac{q^4}{18M_d^4} G_Q^2(q^2) - \frac{q^2}{6M_d^2} G_2^2(q^2) \right] \times 2[(S^2 - M_d^2)^2 + q^2 S^2] + 8G_2^2(q^2) q^4 \left(1 - \frac{q^2}{4M_d^2}\right) \quad (I-10)$$

The cross-section is now given by

$$\sigma = \frac{1}{2\lambda^{1/2}(S^2, m^2, M_d^2)} \int d\text{lips}(s; k_2, D_2) H^{\mu\nu} \ell_{\mu\nu} \times \frac{1}{3 \cdot 2 \cdot q} e^4 \quad (I-11)$$

where $\lambda(a, b, c) = (a - b - c)^2 - 4bc$ and where the factors 3 and 2 arise from averaging over the initial spins. After performing the phase space calculations⁴, we get

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dq^2} (ed \rightarrow ed) = \frac{1}{16\pi\lambda(S^2, m^2, M_d^2)} \cdot \frac{1}{6} \frac{e^4}{q} H^{\mu\nu} \ell_{\mu\nu} \quad (I-12)$$

$$\begin{aligned} &= \frac{1}{16\pi\lambda(S^2, m^2, M_d^2)} \frac{1}{6q^4} (16\pi^2 \alpha^2) H^{\mu\nu} \ell_{\mu\nu} \\ &= \frac{2\pi\alpha^2}{q^4} \left[G_c^2(q^2) + \frac{q^4}{18M_d^4} G_Q^2(q^2) - \frac{q^2}{6M_d^2} G_2^2(q^2) + (G_2^2(q^2)) \frac{q^4 \left(1 - \frac{q^2}{4M_d^2}\right)}{3\{(S^2 - M_d^2)^2 + q^2 S^2\}} \right] \\ &\quad \times \frac{(S^2 - M_d^2)^2 + q^2 S^2}{\lambda(S^2, m^2, M_d^2)} \quad (I-13) \end{aligned}$$

In the laboratory frame, we get

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha^2}{4E^2}\right) \left(\frac{\cos^2 \theta/2}{\sin^4 \theta/2}\right) \frac{1}{1 + \frac{2E}{M} \sin^2 \theta/2} \\ \times G_c^2(q^2) + \frac{q^4}{18M_d^4} G_Q^2(q^2) - \frac{q^2}{6M_d^2} \left\{1 + 2\left(1 - \frac{q^2}{4M_d^2}\right) \tan^2 \theta/2\right\} G_2^2(q^2) \quad (I-14)$$

$$= \left(\frac{d\sigma}{d\Omega}\right)_{\text{mott}} \times \left[G_c^2(q^2) + \frac{q^4}{18M_d^4} G_Q^2(q^2) - \frac{q^2}{6M_d^2} \left\{1 + 2\left(1 - \frac{q^2}{4M_d^2}\right) \tan^2 \theta/2\right\} G_2^2(q^2)\right] \quad (I-15)$$

where E is the lab incident electron energy and θ is the lab scattering angle,

G_c , G_Q and G_2 are generally used instead of G_1 , G_2 and G_3 since in the non-relativistic limit, they reduce to the form factors used by Jankus¹⁾ and their static values correspond to the charge, quadrupole moment and the magnetic moment of the deuteron respectively.

So the calculation of ed elastic cross-section really involves the calculation of the form factors G_c , G_Q and G_2 . In view of the fact that the ed elastic scattering has been measured for very high momentum transfer, many schemes for calculating the relativistic form factors G_c , G_Q and G_2 have been worked out. Some of the more important ones has been done by F. Gross and his collaborators⁵⁾. Many others also contributed in this field, incorporating in them the meson exchange contributions.

We intend to use the knowledge gained from ed elastic scattering to compute vd scattering. Thus, we defer our discussion

of the deuteron form factors G_C , G_Q and G_2 until then. We do this more so because the calculation of νd scattering involves the form factor corresponding to the axial vector deuteron form factor in addition to those arising from the deuteron electromagnetic form factors.

CHAPTER II

Elastic $\nu(\bar{\nu})$ Deuteron Scattering

Now we can calculate the νd elastic cross-section in a way similar to the ed elastic scattering. In the present case, the deuteron current would involve the axial vector current in addition to the vector current. Also, in the case of ed scattering, we had one photon exchange; now we have one neutral vector boson exchange as shown in Figure 7.

Figure 7

The amplitude for the process is given by

$$g_1 g_2 \langle D_2, \epsilon_2 | J^\mu | D_1, \epsilon_1 \rangle \frac{-g_{\mu\alpha} + \frac{q_\mu q_\alpha}{M_Z^2}}{q^2 - M_Z^2 + i\epsilon} \langle k_2 | \gamma_\alpha (1 - \gamma_5) | k_1 \rangle$$

So, the differential cross-section is proportional to

$$\sum_{\text{spins}} \frac{g_1^2 g_2^2}{(q^2 - M_Z^2)^2} \left\{ \langle D_2, \epsilon_2 | J^\mu | D_1, \epsilon_1 \rangle \langle D_1, \epsilon_1 | J^{\nu*} | D_2, \epsilon_2 \rangle \right. \\ \left. \times \langle k_2 | \left(\gamma_\mu - \frac{q_\mu}{M_Z^2} \right) (1 - \gamma_5) | k_1 \rangle \langle k_1 | \left(\gamma_\nu - \frac{q_\nu}{M_Z^2} \right) (1 - \gamma_5) | k_2 \rangle \right\}$$

$$\begin{aligned}
&= \frac{g_1^2 g_2^2}{(q^2 - M_Z^2)^2} \sum_{\text{spins}} \langle D_2, \xi_2 | J^\mu | D_1, \xi_1 \rangle \langle D_1, \xi_1 | J^{\nu*} | D_2, \xi_2 \rangle \\
&\quad \times \text{Tr} \left\{ \left(\gamma_\mu - \frac{\not{q}_\mu}{M_Z} \right) (1 - \gamma_5) \not{K}_1 \left(\gamma_\nu - \frac{\not{q}_\nu}{M_Z} \right) (1 - \gamma_5) \not{K}_2 \right\} \\
&= \frac{g_1^2 g_2^2}{(q^2 - M_Z^2)^2} H^{\mu\nu} l_{\mu\nu} \quad (\text{say}) \quad (\text{II-1})
\end{aligned}$$

Trace calculation gives

$$\begin{aligned}
l_{\mu\nu} = 8 & \left[(k_{1\mu} k_{2\nu} + k_{1\nu} k_{2\mu} - g_{\mu\nu} k_1 \cdot k_2 + i \epsilon_{\mu\alpha\nu\beta} k_1^\alpha k_2^\beta) \right. \\
& - \frac{q \cdot k_2}{M_Z^2} (q_\mu k_{1\nu} + q_\nu k_{1\mu}) - \frac{q \cdot k_1}{M_Z^2} (q_\mu k_{2\nu} + q_\nu k_{2\mu}) \\
& \left. + \frac{q_\mu q_\nu}{M_Z^2} \left\{ \left(2 - \frac{q^2}{M_Z^2} \right) k_1 \cdot k_2 + \frac{2 q \cdot k_1 q \cdot k_2}{M_Z^2} \right\} \right] \quad (\text{II-2})
\end{aligned}$$

To calculate $H^{\mu\nu}$ we need to know about the weak deuteron current. The only new part to find is due to the axial vector. Going through arguments similar to those in ed scattering, i.e. time reversal invariance, absence of second class currents,

$$\langle D_2, \xi_2 | A^\mu | D_1, \xi_1 \rangle = -G_A \epsilon^{\mu\nu\sigma\lambda} \xi_{1\nu} \xi_{2\sigma} d_\lambda \quad \text{with} \quad d_\lambda = (D_1 + D_2)_\lambda \quad (\text{II-3})$$

All other terms do not contribute because they are proportional to q^μ and upon contraction with leptonic tensor gives

terms with m_V^2 and hence vanish.

$$\begin{aligned}
 \langle D_2, \xi_2 | J^\mu | D_1, \xi_1 \rangle &= G_0^\mu \\
 &= G_1(q^2)(\xi_2^* \cdot \xi_1) d^\mu + G_2(q^2) \left\{ \xi_1^\mu(\xi_2^* \cdot q) - \xi_2^{*\mu}(\xi_1 \cdot q) \right\} \\
 &\quad - \frac{G_3(q^2)}{2M_d^2} (\xi_1 \cdot q)(\xi_2^* \cdot q) d^\mu - i G_4(q^2) \epsilon^{\mu\rho\delta\sigma} \xi_{2\rho}^* d_\delta \xi_{1\sigma}
 \end{aligned} \tag{II-4}$$

where G_1, G_2, G_3, G_4 are all real.

Now,

$$\begin{aligned}
 H^{\mu\nu} &= \sum_{\text{spins}} \langle D_2, \xi_2 | J^\mu | D_1, \xi_1 \rangle \langle D_1, \xi_1 | J^{\nu*} | D_2, \xi_2 \rangle \\
 &= \sum_{\text{spins}} \left[G_1(q^2)(\xi_2^* \cdot \xi_1) d^\mu + G_2(q^2) \{ \xi_1^\mu(\xi_2^* \cdot q) - \xi_2^{*\mu}(\xi_1 \cdot q) \} \right. \\
 &\quad \left. - \frac{G_3(q^2)}{2M_d^2} (\xi_1 \cdot q)(\xi_2^* \cdot q) d^\mu - i G_4(q^2) \epsilon^{\mu\rho\delta\sigma} \xi_{2\rho}^* d_\delta \xi_{1\sigma} \right] \\
 &\quad \times \left[G_1(q^2)(\xi_1^* \cdot \xi_2) d^\nu + G_2(q^2) \{ \xi_1^{*\nu}(\xi_2 \cdot q) - \xi_2^\nu(\xi_1^* \cdot q) \} \right. \\
 &\quad \left. - \frac{G_3(q^2)}{2M_d^2} (\xi_1^* \cdot q)(\xi_2 \cdot q) d^\nu + i G_4(q^2) \epsilon^{\nu\rho'\delta'\sigma'} \xi_{2\rho'}^* d_{\delta'} \xi_{1\sigma'} \right]
 \end{aligned}$$

Doing the polarization sum, we get

$$\begin{aligned}
H^{\mu\nu} = & D_1^\mu D_1^\nu \left[12 \{ G_c^2(q^2) + \frac{q^4}{18M_d^4} G_Q^2(q^2) - \frac{q^2}{6M_d^2} G_2^2(q^2) \} + 8G_4^2(q^2) \left(1 + \frac{q^2}{4M_d^2} \right) \right] \\
& + (d_1^\mu q^\nu + d_1^\nu q^\mu) \left[6 \{ G_c^2(q^2) + \frac{q^4}{18M_d^4} G_Q^2(q^2) - \frac{q^2}{6M_d^2} G_2^2(q^2) \} + 4G_4^2(q^2) \left(1 + \frac{q^2}{4M_d^2} \right) \right] \\
& + q^\mu q^\nu \left[3 \{ G_c^2(q^2) + \frac{q^4}{18M_d^4} G_Q^2(q^2) \} + 2G_2^2(q^2) \left(1 + \frac{q^2}{4M_d^2} \right) \right] \\
& + g^{\mu\nu} \left[2q^2 \left(1 - \frac{q^2}{4M_d^2} \right) G_2^2(q^2) + G_4^2 \left(\frac{q^4}{2M_d^2} - 8M_d^2 \right) \right] \\
& - 4i G_2(q^2) G_4(q^2) \epsilon^{\mu\nu\rho\delta} q_\rho D_{1\delta} \left(1 - \frac{q^2}{4M_d^2} \right)
\end{aligned} \tag{II-5}$$

where G_c, G_2 are defined as before, or

$$\begin{aligned}
\frac{H^{\mu\nu}}{12M_d^2} = & -g^{\mu\nu} W_1 + \frac{D_1^\mu D_1^\nu}{M_d^2} W_2 - \frac{i}{2M_d^2} \epsilon^{\mu\nu\rho\delta} q_\rho D_{1\delta} W_3 \\
& + \frac{q^\mu q^\nu}{M_d^2} W_4 + \frac{D_1^\mu q^\nu + D_1^\nu q^\mu}{2M_d^2} W_5
\end{aligned} \tag{II-6}$$

where all the W 's are functions of q^2 , and

$$W_1 = \frac{2}{3} G_4^2(q^2) \left(1 - \frac{q^4}{16M_d^4} \right) - \frac{q^2}{6M_d^2} \left(1 - \frac{q^2}{4M_d^2} \right) G_2^2(q^2),$$

$$W_2 = G_c^2(q^2) + \frac{q^4}{18M_d^4} G_Q^2(q^2) - \frac{q^2}{6M_d^2} G_2^2(q^2) + \frac{2}{3} G_4^2(q^2) \left(1 + \frac{q^2}{4M_d^2}\right),$$

$$W_3 = \frac{2G_2(q^2)G_4(q^2)}{3} \left(1 - \frac{q^2}{4M_d^2}\right), \quad (II-7)$$

$$W_4 = G_c^2(q^2) + \frac{q^4}{18M_d^4} G_Q^2(q^2) + \frac{2}{3} G_2^2(q^2) \left(1 - \frac{q^2}{4M_d^2}\right),$$

$$W_5 = W_2$$

We shall disregard terms with W_4 and W_5 , since upon contracting with the leptonic tensor, we get vanishing contribution from terms involving these. Thus,

$$H^{\mu\nu} l_{\mu\nu} = 96M_d^2 \left[-q^2 W_1 + \frac{W_2}{M_d^2} \left\{ \frac{(S^2 - M_d^2)^2}{2} + \frac{q^2 S^2}{2} - \frac{q^4 (S^2 - M^2)}{4M_z^2} \right\} - \frac{q^2 W_3}{2M_d^2} (S^2 - M_d^2 + \frac{q^2}{2}) \right] \quad (II-8)$$

Now, performing the phase space calculation, with $S^2 = (\text{c.m. energy})^2$

$$\frac{d\sigma}{dq^2} (\nu d \rightarrow \nu d) = \frac{(2S_\nu + 1)^{-1} (2S_d + 1)^{-1}}{16\pi \lambda (S^2, m_\nu^2, M_d^2)} \frac{g_1^2 g_2^2}{(q^2 - M_z^2)^2} H^{\mu\nu} l_{\mu\nu} \quad (II-9)$$

Following Abers and Lee¹, $(\frac{g_1^2 + g_2^2}{4M_z^2})$ is given by

$$\frac{(g_1^2 + g_2^2)}{8M_z^2} = \frac{G^2}{2} \quad (\text{II-10})$$

* We will be interested mainly with low values of q^2 . So, taking M_z^2 large, we get

$$\frac{d\sigma}{dq^2} (\nu d + \bar{\nu} d) = \frac{G^2 (2S_\nu + 1)^{-1} (2S_d + 1)^{-1}}{32\pi\lambda (S^2, m_\nu^2, M_d^2)} H^{\mu\nu} \epsilon_{\mu\nu} \quad (\text{II-11})$$

where $\lambda(S^2, m_\nu^2, M_d^2) = (S^2 - M_d^2)^2$ since $m_\nu^2 \approx 0$. With m_z^2 large,

$$H^{\mu\nu} \epsilon_{\mu\nu} = 96M_d^2 [-q^2 W_1 + \frac{W_2}{M_d^2} \{ \frac{(S^2 - M_d^2)^2}{2} + \frac{q^2 S^2}{2} \} - \frac{q^2 W_3}{2M_d^2} (S^2 - M_d^2)] \quad (\text{II-12})$$

Since the incoming neutrino is of fixed helicity with the deuteron spin being one, we get

$$\frac{d\sigma}{dq^2} (\nu d + \bar{\nu} d) = \frac{G^2 M_d^2}{\pi} \left[\frac{-q^2 W_1}{(S^2 - M_d^2)^2} + \frac{W_2}{M_d^2} \left\{ \frac{1}{2} + \frac{q^2 S^2}{2(S^2 - M_d^2)^2} \right\} - \frac{q^2 W_3}{2M_d^2 (S^2 - M_d^2)} \right] \quad (\text{II-13})$$

In the lab. frame,

$$\begin{aligned} S^2 &= (D_1 + k_1)^2 \\ &= (M_d + E_\nu)^2 - (\vec{0} + \vec{k}_1)^2 = M_d^2 + 2M_d E_\nu \end{aligned}$$

or

$$s^2 - M_d^2 = 2M_d E_\nu$$

where E_ν is the incident neutrino energy. So, we get, with $q^2 = -Q^2$ with some rearrangements,

$$\frac{d\sigma}{dQ^2}(\nu d \rightarrow \nu d) = \frac{G^2}{2\pi} \left[W_2 \left(1 - \frac{Q^2}{2M_d E_\nu} \right) + (2W_1 - W_2) \frac{Q^2}{4E_\nu^2} + W_3 \frac{Q^2}{2M_d E_\nu} \left(1 - \frac{Q^2}{4M_d E_\nu} \right) \right]$$

(II-14)

which agrees with the expression given by Pais and Treiman²⁾ with the definition of W 's as given by us above.

For $\bar{\nu}d$ elastic scattering, we have to change the sign of the W_3 term.

Now our task is to calculate W_1 , W_2 and W_3 or to calculate G_C , G_Q , G_2 and G_4 . So we have to calculate the deuteron current in terms of the deuteron radial functions. We can see the graphs contributing to neutrino (antineutrino) deuteron elastic process are given by Figure 8.

Figure 8

We will use impulse approximation³⁾ to calculate the deuteron current. The basis of this assumption is that the interaction is with the individual nucleons in the deuteron. Other contributions would include interactions with some particles exchanged between the nucleons. These exchange current contributions⁴⁾ are rather small in the low momentum transfer region. As an example, calculation of Chemtob, Moniz and Rho⁵⁾ shows that the exchange current contributions to deuteron electric, magnetic and quadrupole form factors become important only for momentum-transfer squared larger than about 10 fm^{-2} or $0.4 (\text{GeV}/c)^2$. Since, we hope to look at a region lower than $0.4 (\text{GeV}/c)^2$, our results with the impulse approximation should be reliable.

We also assume that one of the nucleons in the deuteron is on the mass-shell. This seems to be a good approximation as shown by Gross.⁶⁾ He showed that the fractional error introduced in this way in the Breit frame, explained in Pages 36 and 37, is given by

$$\Delta = \frac{1}{2} \frac{[\alpha^2 + (\vec{p} + \frac{1}{4} \vec{q})^2][\alpha^2 + (\vec{p} - \frac{1}{4} \vec{q})^2]}{[M_\mu + (\vec{p} + \frac{1}{4} \vec{q})^2][M_\mu + (\vec{p} - \frac{1}{4} \vec{q})^2]}$$

where

α = deuteron binding energy

$\vec{p}$ = spectator nucleon 3-momentum

$\vec{q}$ = 3-momentum transfer

and

μ = pion mass.

He shows that the contribution comes mainly from the region $\vec{p}^2/\mu M \ll 1$ and the error introduced near zero momentum transfer is about 0.35%.

Another assumption we make is that the nucleon form factors are taken for the nucleons on the mass-shell. Very little is known about the form factors with both the nucleons off their mass-shell. We hope that this is a reasonable assumption. A supporting evidence for this is that the charge nucleon form factor is unity at zero momentum transfer independent of nucleon masses. If one expands it, namely F_1 , in a power series in q^2 , the four momentum transfer, $y = t - M_N^2$, $y' = t' - M_N^2$ where t and t' are the masses of the off-shell nucleons, we get⁷⁾

$$F_1(q^2, t, t') = F_1(0, M_N^2, M_N^2) + C_1 \frac{q^2}{M_N^2} + C_2 \frac{q^2 y}{M_N^4} + C_3 \frac{q^2 y'}{M_N^4} + \dots$$

showing that neglecting the off-mass shell contributions amount to neglecting terms of order M_N^{-4} . Also the assumption is hoped to be reasonable for small values of q^2 as the nucleons in the deuterons are quasi-real due to the small binding energy.³⁾

We now proceed with our calculation. For convenience, we shall calculate in a frame where the deuteron is in the Breit system. In this system, the sum of the incoming and the outgoing deuteron three momentum is zero. This makes sure that the fourth component of the momentum transfer is zero. The kinematics is explained in Figure 9.⁸⁾ The three-momentum of the nucleon

spectator is taken to be $-\vec{p}/2$. Also, by symmetry since the fourth component of the momentum transfer is zero in the Breit system, we have $\vec{p} \cdot \vec{q} = 0$.

Figure 9

Let $\frac{\vec{p}-\vec{q}}{2} = \vec{p}_1$; $\frac{\vec{p}+\vec{q}}{2} = \vec{p}_2$

So, $E_1^2 = E_2^2 = \frac{p^2}{4} + \frac{q^2}{4} + M_N^2 = M_N^2 + \frac{p^2}{4} + \frac{q^2}{4} = E^2$ (say)

The nucleonic current is given by

$$j^\lambda = \bar{u}(p_2) [g_V \gamma^\lambda - g_A \gamma^\lambda \gamma_5 - i f_2 \sigma^{\lambda\nu} q_\nu - q^\lambda \gamma_5 h_A] u(p_1) \quad (\text{II-15})$$

The Pauli reduction of the current gives

$$j^0 = \chi_f^\dagger \left[F_C + \frac{i(\vec{\sigma} \times \vec{q}) \cdot \vec{p}}{4M_N^2} \left(F_M - \frac{F_C}{2} \right) - g_A \frac{\vec{\sigma} \cdot \vec{p}}{2M_N} \right] \chi_i$$

and

$$j^k = \chi_f^\dagger \left[\frac{F_M}{2M_N} i(\sigma \times q)^k + \frac{p^k}{2M_N} \left(F_C + \frac{Q^2}{8M_N^2} F_M \right) - g_A \left\{ \alpha^k \left(1 + \frac{Q^2}{8M_N^2} - \frac{p^2}{8M_N^2} \right) + \frac{p^k \sigma \cdot p - q^k \sigma \cdot q - i(\vec{q} \times \vec{p})^k}{8M_N^2} \right\} \right] \chi_i$$

where χ_f, χ_i are two-component Pauli Spinors, and

$$F_C = g_V + \frac{Q^2}{4M_N^2} 2M_N f_2,$$

$$F_M = g_V - 2M_N f_2$$

(II-16)

M_N is the nucleon mass.

We note here that this result is different from the one obtained by Singh⁹⁾ where

$$j_0 = \chi_f^\dagger \left[F_C \left(1 + \frac{p^2}{8M_N^2} \right) + \dots \right] \chi_i$$

which would need a renormalization of the wave function.

Now, if we write $\vec{p}$ in the coordinate representation, $\frac{\vec{p}}{2} \equiv -i\vec{\nabla}$, where $\vec{\nabla}$ operates in both directions, we get

$$j^0 = \chi_f^\dagger \left[F_C + \frac{F_M}{2M_N} \frac{Q^2}{2} (\vec{\sigma} \times \vec{q}) \cdot \vec{\nabla} + i g_A \frac{\vec{\sigma} \cdot \vec{\nabla}}{M_N} \right] \chi_i$$

(II-17)

$$j^k = \chi_f^+ \left[i \frac{F_M}{2M_N} (\sigma \times \vec{q})^k - \frac{i}{M_N} \left(F_C + \frac{Q^2 F_M}{8M_N^2} \right) \vec{v}^k - g_A \left\{ \sigma^k \left(1 + \frac{Q^2}{8M_N^2} + \frac{\vec{v}^2}{2M_N^2} \right) - \frac{1}{2M_N^2} (\vec{v}^k \sigma \cdot \vec{v} + (\frac{\vec{q}}{2} \times \vec{v})^k) \right\} \right] \chi_i \quad (II-17)$$

We omit any term with q^k since it gives vanishing contribution upon contracting with the leptonic tensor.

Now, we write the deuteron wave function in terms of the nucleon spinors. In view of the fact that we shall be working with small Q^2 , we may use the non-relativistic deuteron wave function.

Thus, the deuteron wave function is given by

$$\phi_d(\vec{r}) = \frac{\chi_n^+}{\sqrt{8\pi}} \frac{u(r)}{r} \vec{\sigma} \cdot \vec{\xi} - \frac{1}{\sqrt{2}} \frac{\omega(r)}{r} \left\{ \frac{3(\vec{\sigma} \cdot \vec{r})(\vec{r} \cdot \vec{\xi})}{r^2} - \sigma \cdot \vec{\xi} \right\} i \sigma_2 \chi_p \quad (II-18)$$

where u and ω are the radial S and D state wave functions, respectively.

In impulse approximation, the deuteron current is given by

$$G_d^\mu(q^2) = \langle \phi_{d2} | j^\mu | \phi_{d1} \rangle \quad (II-19)$$

$$= \langle \xi_2 | j^{\mu,p} + j^{\mu,n} | \xi_1 \rangle$$

Here the integration is over the relative co-ordinates of the nucleon in the deuteron. The dependence with respect to $\vec{q}$ in the integral is given by $e^{i\vec{q} \cdot \frac{\vec{r}}{2}}$ for the proton part and $e^{-i\vec{q} \cdot \frac{\vec{r}}{2}}$ for the neutron

part since $\frac{1}{2} \vec{r}$ and $-\frac{1}{2} \vec{r}$ are the positions of the proton and the neutron respectively with respect to the mass centre of the deuteron. Thus, we can write, from (II-17),

$$j^0 = \chi_f^+ \left[F_C^p e^{i\vec{q} \cdot \vec{r}/2} + F_C^n e^{-i\vec{q} \cdot \vec{r}/2} + \left(\frac{F_M^p}{2M_N} - \frac{F_C^p}{2} \right) e^{i\vec{q} \cdot \vec{r}/2} (\vec{\sigma} \times \vec{q}) \cdot \vec{\nabla} - \left(\frac{F_M^n}{2M_N} - \frac{F_C^n}{2} \right) e^{-i\vec{q} \cdot \vec{r}/2} (\vec{\sigma} \times \vec{q}) \cdot \vec{\nabla} + i \frac{g_A^p}{M_N} e^{i\vec{q} \cdot \vec{r}/2} \vec{\sigma} \cdot \vec{\nabla} - i \frac{g_A^n}{M_N} e^{-i\vec{q} \cdot \vec{r}/2} \vec{\sigma} \cdot \vec{\nabla} \right] \chi_i = \chi_f^+ \Lambda^0 \chi_i \text{ (say)}$$

(II-20)

$$j^k = \chi_f^+ \left[\frac{i}{2M_N} F_M^p e^{i\vec{q} \cdot \vec{r}/2} (\vec{\sigma} \times \vec{q})^k + \frac{i}{2M_N} F_M^n e^{-i\vec{q} \cdot \vec{r}/2} (\vec{\sigma} \times \vec{q})^k - \frac{i}{M_N} \left(F_C^p + \frac{Q^2}{8M_N^2} F_M^p \right) e^{i\vec{q} \cdot \vec{r}/2} \nabla^k + \frac{i}{M_N} \left(F_C^n + \frac{Q^2}{8M_N^2} F_M^n \right) e^{-i\vec{q} \cdot \vec{r}/2} \nabla^k - (g_A^p e^{i\vec{q} \cdot \vec{r}/2} + g_A^n e^{-i\vec{q} \cdot \vec{r}/2}) \left\{ \sigma^k \left(1 + \frac{Q^2}{8M_N^2} + \frac{\nabla^2}{2M_N^2} \right) - \frac{1}{2M_N^2} \nabla^k \sigma \cdot \vec{\nabla} \right\} + (g_A^p e^{i\vec{q} \cdot \vec{r}/2} - g_A^n e^{-i\vec{q} \cdot \vec{r}/2}) \frac{1}{2M_N^2} \left(\frac{\vec{q} \times \vec{\nabla}}{2} \right)^k \right] \chi_i = \chi_f^+ \Lambda^k \chi_i \text{ (say)}$$

with $Q^2 = \vec{q}^2$.

To calculate νd or $\bar{\nu} d$ elastic cross-section, we know that we are dealing only with the isoscalar currents since deuteron is isoscalar and thus it is legitimate to calculate only the proton part and then replace it by $F^p + F^n$.³⁾

Now we can calculate the deuteron current, defined by

$$G_d^\mu(q^2) = \langle \phi_{d2} | j^\mu | \phi_{d1} \rangle$$

$$= \int \frac{d^3r}{8\pi r^2} \text{Tr} \left\{ u(r) \sigma \cdot \vec{\xi}_2 - \frac{\omega(r)}{\sqrt{2}} \left(\frac{3 \sigma \cdot \vec{r} \vec{r} \cdot \vec{\xi}_2}{r^2} - \sigma \cdot \vec{\xi}_2 \right) \right\} \Lambda^u -$$

$$\times \left\{ u(r) \sigma \cdot \vec{\xi}_1 - \frac{\omega(r)}{\sqrt{2}} \left(\frac{3 \sigma \cdot \vec{r} \vec{r} \cdot \vec{\xi}_1}{r^2} - \sigma \cdot \vec{\xi}_1 \right) \right\}$$
(II-21)

We shall use $\sigma \cdot \vec{\nabla} f(r) = \frac{\partial f}{\partial r} \frac{\sigma \cdot \vec{r}}{r}$. Also,

$$(\psi_a, \nabla \psi_b) = \frac{1}{2} [\int \psi_a^* \nabla \psi_b d^3r - \int (\nabla \psi_a)^* \psi_b d^3r]$$
(II-22)

Some of the integrals over the angles like $\int d\Omega e^{i\vec{r} \cdot \vec{k}}$ are evaluated in Appendix A. Also, as mentioned before, we shall neglect the terms with q^k . Once we have $G_D^\mu(q^2)$, we can compare it with the non-relativistic reduction obtained in the case of ed scattering given by ¹⁰⁾

$$G_D^0(q^2) = (\vec{\xi}_2 \cdot \vec{\xi}_1) G_C(q^2) + [(\vec{\xi}_2 \cdot \vec{q}) - \frac{1}{3} q^2 (\vec{\xi}_2 \cdot \vec{\xi}_1)] \frac{G_Q(q^2)}{2M_N^2}$$

and

(II-23)

$$G_D^k(q^2) = \frac{1}{4M_N} [\xi_1^k (\vec{\xi}_2 \cdot \vec{q}) - \xi_2^k (\vec{\xi}_1 \cdot \vec{q})] G_2(q^2)$$

The term with G_4 will have $-iG_4(\vec{\xi}_1 \times \vec{\xi}_2)^k$ in $G_D^k(Q^2)$.

We now expand G 's in powers of Q^2/M_N^2 , keeping to terms of order (Q^2/M_N^2) . Under the integral, however, we keep $j_0(Qr/2)$ since the length scale in the integral will depend on the binding energy of the deuteron. Then,

$$G_C(Q^2) = G_{C0}(Q^2) + G_{C1}(Q^2)$$

where

$$G_{C0}(Q^2) = (F_C^p + F_C^n) \int_0^\infty dr (u^2 + \omega^2) j_0\left(\frac{Qr}{2}\right) \quad (\text{II-24})$$

and

$$G_{C1}(Q^2) = \frac{3Q^2}{4M_N^2} (F_M^p + F_M^n - \frac{F_C^p + F_C^n}{2}) \int_0^\infty dr \omega^2 [j_0\left(\frac{Qr}{2}\right) + j_2\left(\frac{Qr}{2}\right)]$$

$$G_Q(Q^2) = G_{Q0}(Q^2) + G_{Q1}(Q^2)$$

with

$$G_{Q0}(Q^2) = (F_C^p + F_C^n) \frac{6\sqrt{2} M_d^2}{Q^2} \int_0^\infty dr (u\omega - \frac{\omega^2}{2\sqrt{2}}) j_2\left(\frac{Qr}{2}\right)$$

and

$$G_{Q1}(Q^2) = \frac{3}{2} \frac{M_d^2}{M_N^2} (F_M^p + F_M^n - \frac{F_C^p + F_C^n}{2})$$

$$\times \int_0^\infty dr \left\{ \frac{\omega^2}{2} \left[\frac{1}{5} j_0\left(\frac{Qr}{2}\right) - \frac{1}{7} j_2\left(\frac{Qr}{2}\right) - \frac{12}{35} j_4\left(\frac{Qr}{2}\right) \right] - \frac{3u\omega}{\sqrt{2}} \left[\frac{1}{5} j_0\left(\frac{Qr}{2}\right) + \frac{1}{7} j_2\left(\frac{Qr}{2}\right) - \frac{2}{35} j_4\left(\frac{Qr}{2}\right) \right] \right. \\ \left. - \frac{r^2}{\sqrt{2}} \left(\frac{u}{r} \frac{\partial \omega}{\partial r} - \frac{\omega}{r} \frac{\partial \xi}{\partial r} \right) \left[\frac{1}{5} j_0\left(\frac{Qr}{2}\right) + \frac{2}{7} j_2\left(\frac{Qr}{2}\right) + \frac{3}{35} j_4\left(\frac{Qr}{2}\right) \right] \right\}$$

(II-25)

Similarly,

$$G_2(Q^2) = G_{20}(Q^2) + G_{21}(Q^2)$$

with

$$G_{20}(Q^2) = (F_c^p + F_c^n) \frac{3}{2} \int_0^\infty dr \omega^2 [j_0(\frac{Qr}{2}) + j_2(\frac{Qr}{2})] \\ + (F_M^p + F_M^n) [2 \int_0^\infty dr (u^2 - \frac{\omega^2}{2}) j_0(\frac{Qr}{2}) + \sqrt{2} \int_0^\infty dr (\frac{u\omega}{\sqrt{2}} + \frac{\omega^2}{2}) j_2(\frac{Qr}{2})]$$

and

$$G_{21}(Q^2) = \frac{Q^2}{8M_N^2} (F_M^p + F_M^n) \frac{3}{2} \int_0^\infty dr \omega^2 [j_0(\frac{Qr}{2}) + j_2(\frac{Qr}{2})] \quad (II-26)$$

Also

$$G_4(Q^2) = G_{40}(Q^2) + G_{41}(Q^2)$$

with

$$G_{40}(Q^2) = (g_A^p + g_A^n) \int_0^\infty dr [(u^2 - \frac{\omega^2}{2}) j_0(\frac{Qr}{2}) + (\frac{u\omega}{\sqrt{2}} + \frac{\omega^2}{2}) j_2(\frac{Qr}{2})]$$

and

$$G_{41}(Q^2) = \frac{Q^2}{8M_N^2} G_{40}(Q^2) \\ + \frac{g_A^p + g_A^n}{2M_N^2} \int_0^\infty dr \left\{ j_0(\frac{Qr}{2}) \left[\frac{3Q^2}{16} \omega^2 - (uu_1 - \frac{\omega\omega_1}{2}) \right] \right. \\ \left. + j_2(\frac{Qr}{2}) \left[\frac{3Q^2}{16} \omega^2 - \frac{1}{2}(\omega\omega_1 + \frac{u\omega_1}{\sqrt{2}} + \frac{\omega u_1}{\sqrt{2}}) \right] \right\} \quad (II-27)$$

where

$$u_1 = \frac{\partial^2 u}{\partial r^2} - \frac{1}{8} \frac{\partial u}{\partial r} + \frac{u}{r^2} - \frac{2\sqrt{2}\omega}{r^2} - \frac{1}{\sqrt{2}r} \frac{\partial \omega}{\partial r}$$

and

$$\omega_1 = \frac{\partial^2 \omega}{\partial r^2} - \frac{3\omega}{r^2}$$

Again $Q^2 = -q^2 = -(\text{four momentum transfer})^2$ and so the result can be expressed in terms of invariant quantities and hence in the lab. frame later. We wrote the form factors as sum of two terms since in the lowest order, they reduce to only the first term.

Thus, to calculate νd or $\bar{\nu} d$ elastic cross-section, the only thing remaining to be known are the values of F_C , F_M , g_A and the deuteron radial wavefunctions. We shall come back to these after we have calculated the expression for differential cross-section for the break-up reaction, i.e., $\nu(\bar{\nu})d \rightarrow \nu(\bar{\nu})pn$ since they involve a knowledge of the same quantities.

Before ending this Chapter, as a comparison, we give the expression for the neutrino-nucleon elastic differential cross-section, we have the same result as (II-14) with the differences that the W 's are now given in terms of F_C , F_M and g_A as follows¹¹⁾.

$$W_1^N = \left(1 + \frac{Q^2}{4M_N^2}\right) g_A^N{}^2 + \frac{Q^2}{4M_N^2} F_M^N{}^2 \quad (\text{II-28})$$

$$W_2^{N2} = g_A^{N2} + \frac{F_C^{N2} - F_M^{N2}}{1 + \frac{Q^2}{4M_N^2}}$$

and

$$W_3^N = 2g_A^N f_M^N$$

(II-28)

CHAPTER III

(Anti)-Neutrino-Deuteron Neutral Current Breakup Processes

The calculation of the $\nu(\bar{\nu})d$ neutral current breakup reaction is more involved. Here the deuteron current may have a combination of isovector and isoscalar components since the final np state can be in both $I=1$ and $I=0$ state. For the calculation we follow a procedure similar to the one adopted to calculate νd charged current breakup process. But there is one fundamental difference between these. In the case of the charged current process we have only Figure 10 contributing but in the case of the neutral current process, we have in addition, Figures 11, 12 and Figure 13, where the kinematics is also shown.

Figure 10

Figure 11

Figure 12

Figure 13

Many people have done calculations on the breakup of the deuteron by the charged current process. We shall follow a procedure similar to the one adopted by Singh.¹ We wish to calculate the deuteron current in the impulse approximation given by

$$G_D^\mu(Q^2) = \langle (np) | \Lambda^\mu | \psi_D \rangle \quad (\text{III-1})$$

where ψ_D is the deuteron wave function given by

$$\psi_D(\vec{r}) = \left(\frac{u}{r}\right) Y_{00}(\hat{r}) + \frac{1}{\sqrt{10}} \frac{w}{r} Y_{20}(\hat{r}) Y_1^1 + \sqrt{\frac{3}{5}} \frac{w}{r} Y_{22}(\hat{r}) Y_1^{-1} - \sqrt{\frac{3}{10}} Y_{21}(\hat{r}) Y_1^0, \quad (\text{III-2})$$

Λ^μ is given, as in (II-20), by

$$\begin{aligned} \Lambda^0 = & \{F_C^p + (\vec{\sigma} \times \vec{q}) \cdot \vec{\nabla} \frac{F_M^p - \frac{F_C^p}{2}}{2M_N^2} + \frac{i}{M_N} \vec{\sigma} \cdot \vec{\nabla} \vec{q} \cdot \vec{p}\} e^{i\vec{q} \cdot \vec{r}/2} \\ & + \{F_C^n + (\vec{\sigma} \times \vec{q}) \cdot \vec{\nabla} \frac{F_M^n - \frac{F_C^n}{2}}{2M_N^2} + \frac{i}{M_N} \vec{\sigma} \cdot \vec{\nabla} g_A^n\} e^{-i\vec{q} \cdot \vec{r}/2} \end{aligned} \quad (\text{III-3})$$

$$\begin{aligned} \Lambda^k = & \left\{ \frac{i}{2M_N} (\vec{q} \times \vec{\nabla})^k F_M^p - \frac{i}{M_N} \nabla^k (F_C^p + \frac{Q^2}{8M_N^2} F_M^p) + \frac{1}{2M_N^2} \nabla^k \vec{\sigma} \cdot \vec{\nabla} g_A^p \right. \\ & \left. + \frac{1}{4M_N^2} (\frac{\vec{q}}{2} \times \vec{\nabla})^k g_A^p - \sigma^k (1 + \frac{Q^2}{8M_N^2} + \frac{\nabla^2}{2M_N^2}) g_A^p \right\} e^{i\vec{q} \cdot \vec{r}/2} \end{aligned} \quad (\text{III-4})$$

$$+ \left\{ \frac{i}{2M_N} (\sigma \times \mathbf{q})^k F_M^n - \frac{i}{M_N} \nabla^k (F_C^n + \frac{Q^2}{8M_N^2} F_M^n) + \frac{1}{2M_N^2} \nabla^k \sigma \cdot \nabla g_A^n \right. \\ \left. + \frac{1}{4M_N^2} (\frac{\mathbf{q}}{2} \times \nabla)^k g_A^n - \sigma^k (1 + \frac{Q^2}{8M_N^2} + \frac{\nabla^2}{2M_N^2}) g_A^n \right\} e^{-i\mathbf{q} \cdot \mathbf{r}/2} \quad \text{(III-4)}$$

and χ_1^m 's are the usual triplet spinors.

The final np state can be in both the spin triplet and the spin singlet states. Also, it can occur in isospin triplet and singlet states. So, the spin states can be written as

$$\psi_{\text{triplet}} = \left\{ \frac{1}{\sqrt{2}} (a+b) e^{i\vec{p} \cdot \vec{r}} - \frac{1}{\sqrt{2}} (a-b) e^{-i\vec{p} \cdot \vec{r}} \right\} \chi_1^m$$

$$\psi_{\text{singlet}} = \left\{ \frac{1}{\sqrt{2}} (a+b) e^{i\vec{p} \cdot \vec{r}} + \frac{1}{\sqrt{2}} (a-b) e^{-i\vec{p} \cdot \vec{r}} \right\} \chi_0^0$$

where a and b represent the strengths of $I=1$ and $I=0$ respectively

and $\vec{p}$ is the relative momentum of the outgoing nucleons. Thus,

$$\text{writing } t_1 = \frac{1}{\sqrt{2}} (a+b) \text{ and } t_2 = \frac{1}{\sqrt{2}} (a-b)$$

$$\psi_{\text{triplet}} = (t_1 e^{i\vec{p} \cdot \vec{r}} - t_2 e^{-i\vec{p} \cdot \vec{r}}) \chi_1^m = \chi_t \chi_1^m$$

and

(III-5)

$$\psi_{\text{singlet}} = (t_1 e^{i\vec{p} \cdot \vec{r}} + t_2 e^{-i\vec{p} \cdot \vec{r}}) \chi_0^0$$

$$\text{where } t_1^2 + t_2^2 = 1.$$

With this information, we can now calculate the expressions for

G_D^0 singlet, G_D^0 triplet, G_D^k singlet and G_D^k triplet.

They are given by

$$G_D^0 \text{ singlet } (q^2) = \int d^3r \left[\frac{i}{M_N} \left\{ I_{3,s}^3 + \frac{1}{\sqrt{2}} (I_{2,s}^- - I_{1,s}^+) \right\} \{ g_A^p e^{\frac{i\vec{q} \cdot \vec{r}}{2}} - g_A^n e^{\frac{i\vec{q} \cdot \vec{r}}{2}} \} \right. \\ \left. + \{ (\vec{q} \times \vec{I}_{3,s})^3 + \frac{1}{\sqrt{2}} (\vec{q} \times \vec{I}_{1,s})^+ + \frac{1}{\sqrt{2}} (\vec{q} \times \vec{I}_{2,s})^- \} \right. \\ \left. \times \frac{1}{2M_N^2} \{ (F_M^p - \frac{1}{2} F_C^p) e^{\frac{i\vec{q} \cdot \vec{r}}{2}} - (F_M^n - \frac{1}{2} F_C^n) e^{-\frac{i\vec{q} \cdot \vec{r}}{2}} \} \right] \quad (\text{III-6})$$

$$G_D^0 \text{ triplet } (m=+1) = \int d^3r \left[I_{1,t}^0 (F_C^p e^{\frac{i\vec{q} \cdot \vec{r}}{2}} + F_C^n e^{-\frac{i\vec{q} \cdot \vec{r}}{2}}) + \frac{i}{M_N} \right. \\ \left. (I_{1,t}^3 + \frac{1}{\sqrt{2}} I_{3,t}^-) (g_A^p e^{\frac{i\vec{q} \cdot \vec{r}}{2}} + g_A^n e^{-\frac{i\vec{q} \cdot \vec{r}}{2}}) \right. \\ \left. + \{ (\vec{q} \times \vec{I}_{1,t})^3 + \frac{1}{\sqrt{2}} (\vec{q} \times \vec{I}_{3,t})^- \} \frac{1}{2M_N^2} \right. \\ \left. (F_M^p - \frac{1}{2} F_C^p) e^{\frac{i\vec{q} \cdot \vec{r}}{2}} + (F_M^n - \frac{1}{2} F_C^n) e^{-\frac{i\vec{q} \cdot \vec{r}}{2}} \} \right] \quad (\text{III-7})$$

$$\begin{aligned}
G_D^0 \text{ triplet } (m=0) = & \int d^3r \left[I_{3,t}^0 (F_c^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} + F_c^n e^{\frac{-i\vec{q}\cdot\vec{r}}{2}}) + \frac{i}{\sqrt{2} M_N} \right. \\
& (I_{1,t}^+ + I_{2,t}^-) (g_A^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} + g_A^n e^{\frac{-i\vec{q}\cdot\vec{r}}{2}}) \\
& + \frac{1}{2\sqrt{2} M_N^2} \{ (\vec{q} \times \vec{I}_{1,t})^+ + (\vec{q} \times \vec{I}_{2,t})^- \} \\
& \left. \{ (F_M^p - \frac{1}{2} F_c^p) e^{\frac{i\vec{q}\cdot\vec{r}}{2}} + (F_M^n - \frac{1}{2} F_c^n) e^{\frac{-i\vec{q}\cdot\vec{r}}{2}} \} \right] \\
& \text{(III-8)}
\end{aligned}$$

$$\begin{aligned}
G_D^0 \text{ triplet } (m=-1) = & \int d^3r \left[I_{2,t}^0 (F_c^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} + F_c^n e^{\frac{-i\vec{q}\cdot\vec{r}}{2}}) - \frac{i}{M_N} \right. \\
& (I_{2,t}^3 + \frac{1}{\sqrt{2}} I_{3,t}^+) (g_A^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} + g_A^n e^{\frac{-i\vec{q}\cdot\vec{r}}{2}}) \\
& - \frac{1}{2M_N^2} \{ (\vec{q} \times \vec{I}_{2,t})^3 - \frac{1}{\sqrt{2}} (\vec{q} \times \vec{I}_{3,t})^+ \} \\
& \left. \{ (F_M^k - \frac{1}{2} F_c^p) e^{\frac{i\vec{q}\cdot\vec{r}}{2}} + (F_M^n - \frac{1}{2} F_c^n) e^{\frac{-i\vec{q}\cdot\vec{r}}{2}} \} \right] \\
& \text{(III-9)}
\end{aligned}$$

$$G_D^{k=1} \text{ singlet} = \int d^3r \left[-\frac{i}{2M_N} \{q_2 I_{3,s}^0 + \frac{i}{\sqrt{2}} q_3 (I_{1,s}^0 + I_{2,s}^0)\} (F_M^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} - F_M^n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}}) \right. \\ \left. + \frac{1}{2M_N^2} \{I_{3,s}^{1,3} + \frac{1}{\sqrt{2}} (I_{2,s}^{1,-} - I_{1,s}^{1,+})\} (g_A^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} - g_A^n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}}) \right. \\ \left. - \frac{1}{\sqrt{2}} (I_{2,s}^F - I_{1,s}^F) (g_A^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} - g_A^n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}}) \right]$$

(III-10)

$$G_D^{k=2} \text{ singlet} = \int d^3r \left[-\frac{i}{2M_N} \{q_1 I_{3,s}^0 - \frac{1}{\sqrt{2}} q_3 (I_{1,s}^0 + I_{2,s}^0)\} (F_M^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} - F_M^n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}}) \right. \\ \left. + \frac{1}{2M_N^2} \{I_{3,s}^{2,3} + \frac{1}{\sqrt{2}} (I_{2,s}^{2,-} - I_{1,s}^{2,+})\} (g_A^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} - g_A^n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}}) \right. \\ \left. + \frac{i}{\sqrt{2}} (I_{2,s}^F + I_{1,s}^F) (g_A^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} - g_A^n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}}) \right]$$

(III-11)

$$G_D^{k=3} \text{ singlet} = \int d^3r \left[-\frac{1}{2\sqrt{2} M_N} (q^+ I_{1,s}^0 + q^- I_{2,s}^0) (F_M^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} - F_M^n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}}) \right. \\ \left. + \frac{1}{2M_N^2} \{I_{3,s}^{3,3} + \frac{1}{\sqrt{2}} (I_{2,s}^{3,-} - I_{1,s}^{3,+})\} (g_A^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} - g_A^n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}}) \right. \\ \left. - I_{3,s}^F (g_A^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} - g_A^n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}}) \right]$$

(III-12)

$$\begin{aligned}
G_D^k \text{ triplet } (m=1) = \int d^3r & \left[-\frac{i}{M_N} I_{1,t}^k \left\{ (F_c^p + \frac{Q^2}{8M_N^2} F_M^p) e^{\frac{i\vec{q}\cdot\vec{r}}{2}} + (F_c^n + \frac{Q^2}{8M_N^2} F_M^n) e^{-\frac{i\vec{q}\cdot\vec{r}}{2}} \right\} \right. \\
& + \frac{i}{2M_N} \left\{ -\delta_{k,1} (q_2 I_{1,t}^0 + \frac{i}{\sqrt{2}} q_3 I_{3,t}^0) + \delta_{k,2} (q_1 I_{1,t}^0 - \frac{1}{\sqrt{2}} q_3 I_{3,t}^0) \right. \\
& \quad \left. \left. + \delta_{k,3} \frac{i}{\sqrt{2}} q^- I_{3,t}^0 \right\} (F_M^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} + F_M^n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}}) \right. \\
& + \frac{1}{4M_N^2} (\vec{q} \times \vec{I}_{1,t})^k (g_A^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} + g_A^n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}}) \\
& + \frac{1}{2M_N^2} \left\{ I_{1,t}^{k,3} + \frac{1}{\sqrt{2}} I_{3,t}^{k,-} \right\} (g_A^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} + g_A^n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}}) \\
& \left. - \left\{ \delta_{k,1} \frac{1}{\sqrt{2}} I_{3,t}^F - i\delta_{k,2} \frac{1}{\sqrt{2}} I_{3,t}^F + \delta_{k,3} I_{1,t}^F \right\} \right. \\
& \quad \left. (g_A^p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} + g_A^n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}}) \right]
\end{aligned}$$

(III-13)

$$\begin{aligned}
G_D^k \text{ triplet } (m=0) = \int d^3r & \left[\frac{i}{M_N} I_{3,t}^k \left\{ (F_c^p + \frac{Q^2}{8M_N^2} F_M^p) e^{\frac{i\vec{q} \cdot \vec{r}}{2}} + (F_c^n + \frac{Q^2}{8M_N^2} F_M^n) e^{-\frac{i\vec{q} \cdot \vec{r}}{2}} \right\} \right. \\
& + \frac{i}{2M_N} \left\{ \delta_{k,1} \frac{1}{\sqrt{2}} q_3 (I_{1,t}^0 - I_{2,t}^0) - \delta_{k,2} \frac{1}{\sqrt{2}} q_3 (I_{1,t}^0 + I_{2,t}^0) \right. \\
& \left. \left. + \delta_{k,3} \frac{1}{\sqrt{2}} (-I_{1,t}^0 q^+ + I_{2,t}^0 q^-) \right\} \right. \\
& \left. (F_M^p e^{\frac{i\vec{q} \cdot \vec{r}}{2}} + F_M^n e^{-\frac{i\vec{q} \cdot \vec{r}}{2}}) \right. \\
& + \frac{1}{4M_N^2} (\vec{q} \times \vec{I}_{3,t})^k (g_A^p e^{\frac{i\vec{q} \cdot \vec{r}}{2}} + g_A^n e^{-\frac{i\vec{q} \cdot \vec{r}}{2}}) \\
& + \frac{1}{2\sqrt{2}M_N^2} (I_{1,t}^{k,+} + I_{2,t}^{k,-}) (g_A^p e^{\frac{i\vec{q} \cdot \vec{r}}{2}} + g_A^n e^{-\frac{i\vec{q} \cdot \vec{r}}{2}}) \\
& - \left\{ \delta_{k,1} \frac{1}{\sqrt{2}} (I_{1,t}^F + I_{2,t}^F) + \frac{i}{\sqrt{2}} \delta_{k,2} (I_{1,t}^F - I_{2,t}^F) \right\} \\
& \left. (g_A^p e^{\frac{i\vec{q} \cdot \vec{r}}{2}} + g_A^n e^{-\frac{i\vec{q} \cdot \vec{r}}{2}}) \right]
\end{aligned}$$

(III-14)

$$\begin{aligned}
G_D^k \text{ triplet } (m=-1) = & \int d^3r \left[\frac{i}{M_N} I_{2,t}^k \left\{ (F_c^p + \frac{Q^2}{8M_N^2} F_M^p) e^{\frac{i\vec{q} \cdot \vec{r}}{2}} + (F_c^n + \frac{Q^2}{8M_N^2} F_M^n) e^{\frac{-i\vec{q} \cdot \vec{r}}{2}} \right\} \right. \\
& + \frac{i}{2M_N} \{ \delta_{k,1} (q_2 I_{2,t}^0 + \frac{i}{\sqrt{2}} q_3 I_{3,t}^0) \\
& - \delta_{k,2} (q_1 I_{2,t}^0 + \frac{1}{\sqrt{2}} q_3 I_{3,t}^0) \\
& + \frac{1}{\sqrt{2}} \delta_{k,3} q^{-1} I_{3,t}^0 \} (F_M^p e^{\frac{i\vec{q} \cdot \vec{r}}{2}} + F_M^n e^{\frac{-i\vec{q} \cdot \vec{r}}{2}}) \\
& + \frac{1}{4M_N^2} (\vec{q} \times \vec{I}_{2,t})^k (g_A^p e^{\frac{i\vec{q} \cdot \vec{r}}{2}} + g_A^n e^{\frac{-i\vec{q} \cdot \vec{r}}{2}}) \\
& + \frac{1}{2M_N^2} (\frac{1}{\sqrt{2}} I_{3,t}^{k,+} - I_{2,t}^{k,3}) (g_A^p e^{\frac{i\vec{q} \cdot \vec{r}}{2}} + g_A^n e^{\frac{-i\vec{q} \cdot \vec{r}}{2}}) \\
& - (\delta_{k,1} \frac{1}{\sqrt{2}} I_{3,t}^F + \frac{i}{\sqrt{2}} \delta_{k,2} I_{3,t}^F - \delta_{k,3} I_{2,t}^F) \\
& \left. (g_A^p e^{\frac{i\vec{q} \cdot \vec{r}}{2}} + g_A^n e^{\frac{-i\vec{q} \cdot \vec{r}}{2}}) \right]
\end{aligned}$$

In the relations (III-(6)-(15)), the operators I 's are defined as follows:

$$I_{1,s}^0 = (t_1 e^{-i\vec{p}\cdot\vec{r}} \pm t_2 e^{i\vec{p}\cdot\vec{r}}) \left(\frac{u}{r} \gamma_{00}(\hat{r}) + \frac{1}{\sqrt{10}} \frac{\omega}{r} \gamma_{20}(\hat{r}) \right)$$

$$I_{2,s}^0 = (t_1 e^{-i\vec{p}\cdot\vec{r}} \pm t_2 e^{i\vec{p}\cdot\vec{r}}) \sqrt{\frac{3}{5}} \frac{\omega}{r} \gamma_{22}(\hat{r}) \quad (\text{III-16})$$

$$I_{3,s}^0 = (t_1 e^{-i\vec{p}\cdot\vec{r}} \pm t_2 e^{i\vec{p}\cdot\vec{r}}) \left(-\sqrt{\frac{3}{10}} \frac{\omega}{r} \gamma_{21}(\hat{r}) \right)$$

$$I_{1,s}^j = (t_1 e^{-i\vec{p}\cdot\vec{r}} \pm t_2 e^{i\vec{p}\cdot\vec{r}}) \nabla^j \left(\frac{u}{r} \gamma_{00}(\hat{r}) + \frac{1}{\sqrt{10}} \frac{\omega}{r} \gamma_{20}(\hat{r}) \right)$$

$$I_{2,s}^j = (t_1 e^{-i\vec{p}\cdot\vec{r}} \pm t_2 e^{i\vec{p}\cdot\vec{r}}) \nabla^j \sqrt{\frac{3}{5}} \frac{\omega}{r} \gamma_{22}(\hat{r}) \quad (\text{III-17})$$

$$I_{3,s}^j = (t_1 e^{-i\vec{p}\cdot\vec{r}} \pm t_2 e^{i\vec{p}\cdot\vec{r}}) \nabla^j \left(-\sqrt{\frac{3}{10}} \frac{\omega}{r} \gamma_{21}(\hat{r}) \right)$$

$$I_{1,s}^{j,k} = (t_1 e^{-i\vec{p}\cdot\vec{r}} \pm t_2 e^{i\vec{p}\cdot\vec{r}}) \nabla^j \nabla^k \left(\frac{u}{r} \gamma_{00}(\hat{r}) + \frac{1}{\sqrt{10}} \frac{\omega}{r} \gamma_{20}(\hat{r}) \right)$$

$$I_{2,s}^{j,k} = (t_1 e^{-i\vec{p}\cdot\vec{r}} \pm t_2 e^{i\vec{p}\cdot\vec{r}}) \nabla^j \nabla^k \sqrt{\frac{3}{5}} \frac{\omega}{r} \gamma_{22}(\hat{r}) \quad (\text{III-18})$$

$$I_{3,s,t}^{j,k} = (t_1 e^{-i\vec{p}\cdot\vec{r}} \pm t_2 e^{i\vec{p}\cdot\vec{r}}) \nabla^j \nabla^k \left(-\sqrt{\frac{3}{10}} \frac{\omega}{r} Y_{21}(\hat{r}) \right)$$

$$I_{1,s,t}^F = (t_1 e^{-i\vec{p}\cdot\vec{r}} \pm t_2 e^{i\vec{p}\cdot\vec{r}}) \left(1 + \frac{\nabla^2}{2M_N^2} + \frac{Q^2}{8M_N^2} \right) \left(\frac{\omega}{r} Y_{00}(\hat{r}) + \frac{1}{\sqrt{10}} \frac{\omega}{r} Y_{20}(\hat{r}) \right)$$

$$I_{2,s,t}^F = (t_1 e^{-i\vec{p}\cdot\vec{r}} \pm t_2 e^{i\vec{p}\cdot\vec{r}}) \left(1 + \frac{\nabla^2}{2M_N^2} + \frac{Q^2}{8M_N^2} \right) \sqrt{\frac{3}{5}} \frac{\omega}{r} Y_{22}(\hat{r}) \quad (\text{III-19})$$

$$I_{3,s,t}^F = (t_1 e^{-i\vec{p}\cdot\vec{r}} \pm t_2 e^{i\vec{p}\cdot\vec{r}}) \left(1 + \frac{\nabla^2}{2M_N^2} + \frac{Q^2}{8M_N^2} \right) \left(-\sqrt{\frac{3}{10}} \frac{\omega}{r} Y_{21}(\hat{r}) \right)$$

Also, we have any quantity

$$Q^\pm = Q^1 \pm iQ^2,$$

for example,

$$I_{1,s}^{3,\pm} = I_{1,s}^{3,1} \pm iI_{1,s}^{3,2}$$

We have to remember that the operators act on both the right and the left side.

Thus, we have relations like,

$$\int I_{1,s}^{j,k} F_p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} d^3r = \int d^3r (t_1 e^{-i\vec{p}\cdot\vec{r}} \pm t_2 e^{i\vec{p}\cdot\vec{r}}) F_p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} (\nabla^j + i\frac{q^j}{2}) (\frac{u}{r} \gamma_{00}(\hat{r}) + \frac{1}{\sqrt{10}} \frac{\omega}{r} \gamma_{20}(\hat{r}))$$

$$\int I_{1,s}^{j,k} F_n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}} d^3r = \int d^3r (t_1 e^{-i\vec{p}\cdot\vec{r}} \pm t_2 e^{i\vec{p}\cdot\vec{r}}) F_n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}} (\nabla^j - i\frac{q^j}{2}) (\frac{u}{r} \gamma_{00}(\hat{r}) + \frac{1}{\sqrt{10}} \frac{\omega}{r} \gamma_{20}(\hat{r}))$$

$$\int I_{1,s}^{j,k} F_p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} d^3r = \int d^3r (t_1 e^{-i\vec{p}\cdot\vec{r}} \pm t_2 e^{i\vec{p}\cdot\vec{r}}) F_p e^{\frac{i\vec{q}\cdot\vec{r}}{2}} \{ \nabla^j \nabla^k - \frac{q^j q^k}{4} + \frac{i}{2} (q^j \nabla^k + q^k \nabla^j) \} (\frac{u}{r} \gamma_{00}(\hat{r}) + \frac{1}{\sqrt{10}} \frac{\omega}{r} \gamma_{20}(\hat{r}))$$

$$\int I_{1,s}^{j,k} F_n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}} d^3r = \int d^3r (t_1 e^{-i\vec{p}\cdot\vec{r}} \pm t_2 e^{i\vec{p}\cdot\vec{r}}) F_n e^{-\frac{i\vec{q}\cdot\vec{r}}{2}} \{ \nabla^j \nabla^k - \frac{q^j q^k}{4} - \frac{i}{2} (q^j \nabla^k + q^k \nabla^j) \} (\frac{u}{r} \gamma_{00}(\hat{r}) + \frac{1}{\sqrt{10}} \frac{\omega}{r} \gamma_{20}(\hat{r}))$$

Using these relations, we get

$$G_{D \text{ singlet}}^u(Q^2) = \int d^3r (G_p^u \text{ singlet} e^{\frac{i\vec{q}\cdot\vec{r}}{2}} - G_n^u \text{ singlet} e^{\frac{-i\vec{q}\cdot\vec{r}}{2}}) (t_1 e^{-i\vec{p}\cdot\vec{r}} + t_2 e^{i\vec{p}\cdot\vec{r}})$$

$$G_{D \text{ triplet}}^u(Q^2) = \int d^3r (G_p^u \text{ triplet} e^{\frac{i\vec{q}\cdot\vec{r}}{2}} + G_n^u \text{ triplet} e^{\frac{-i\vec{q}\cdot\vec{r}}{2}}) (t_1 e^{-i\vec{p}\cdot\vec{r}} - t_2 e^{i\vec{p}\cdot\vec{r}})$$

(III-20)

where $G_p^u \text{ singlet}(\vec{q}) = G_n^u \text{ singlet}(-\vec{q})$ and these are functions of the triplet

deuteron radial wave functions, F_c , F_M and g_A and of course q .

We are interested in terms like

$$\int G^u G^{*v} d^3p = \int H^{uv} d^3p$$

since these are the ones that come in the calculation of cross-section.

We first evaluate these in the frame we have chosen given by Figure 13,

where the three momentum of the initial deuteron and the final np

system is zero. The result is first given in terms of Q^2 ,

the deuteron mass, and the cm-energy and so can be written in terms of

the invariant quantities. Later, we express our results in the

laboratory frame.

The phase space calculation for $\nu d + \mu^- pp$ has been done by many authors.²⁾ To get the differential cross-section for the process $\nu(\bar{\nu})d \rightarrow \nu(\bar{\nu})pn$. We have a similar relation, and proceeding as in the case of the elastic process, we get,

($\frac{q^2}{M_Z^2}$ being very small),

$$\frac{d\sigma}{dQ^2} = \frac{G^2}{4\pi E_\nu^2} \left[\sum_{\text{spins}} \int L_{\mu\nu} H^{\mu\nu} d^3p \right] = \frac{G^2}{4\pi E_\nu^2} I \quad (\text{III-21})$$

Where I stands for the expression in $[\]$ and where E_ν is the incident (anti) neutrino energy in the lab frame and $L_{\mu\nu}$ is given by

$$L_{\mu\nu} = (k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k \cdot k' + i\epsilon_{\mu\nu\rho\lambda} k_\rho k'_\lambda)$$

(III-22)

At first we evaluate $\sum_{\text{spins}} \int H^{\mu\nu} d^3p$, since we can take the $L_{\mu\nu}$ outside the integration by using free nucleon kinematics and the leptonic tensor is replaced by its expectation value.

The calculation of the integrated hadronic tensor is done in the deuteron Breit frame. The result is very long and

messy and is given in Appendix B where we give expressions for

$$\int H^{\mu\nu} d^3p$$

for $\mu, \nu = 0, 1, 2, 3$ and for different spin states. Originally, it was our intention to look at the polarized deuteron cross-section as well. But we abstained from doing that, mainly because, even with unpolarized target, the experiment is very difficult and large volume polarized deuteron targets would be really difficult to get. Another reason for postponing it for the present is that the calculation would be very messy and extremely long. In principle, given enough time, we can use the expressions given in Appendix B and calculate the (anti) neutrino-deuteron polarized neutral current breakup cross-section. We hope to do it at a later time, when hopefully it will also be possible to do a $\nu(\bar{\nu})$ -polarized-deuteron experiment.

However, we think that the unpolarized cross-section could be experimentally measured at present and so we calculated the unpolarized differential cross-section for these processes. To facilitate these calculations, we go to a frame where

$$q = (0, 0, 0, q)$$

so that the (four momentum)², $q^2 = -\vec{q}^2 = -Q^2$.

Then we express $L_{\mu\nu} \Sigma \int H^{\mu\nu} d^3x$ in terms of invariant quantities given by, $I = \int I_r r^2 dr$, with

$$I_r = (F_c^{p2} + F_c^{n2}) \left\{ \frac{2}{r^2} (u^2 + \omega^2) (Q_0^2 - \frac{Q^2}{4}) (t_1^2 + t_2^2 - 2t_1 t_2 j_0(Qr)) \right. \\ \left. + \frac{1}{2M_N^2} \left[(Q_0^2 + \frac{Q^2}{4}) (\frac{4}{3} A^2 + \frac{47}{75} B^2 + \frac{21}{25} C^2 - \frac{4\sqrt{2}}{15} AB) \right. \right. \\ \left. \left. + Q_0 Q \cdot \frac{3}{5} (B^2 - C^2) \right] \right. \\ \left. \left[\times (t_1^2 + t_2^2 + 2t_1 t_2 j_0(Qr)) \right] \right\}$$

$$- \frac{Q^2}{8M_N^4} (F_c^p F_M^p + F_c^n F_M^n) \left\{ (Q_0^2 + \frac{Q^2}{4}) (\frac{4}{3} A^2 + \frac{47}{75} B^2 + \frac{21}{25} C^2 - \frac{4\sqrt{2}}{15} AB) \right. \\ \left. + \frac{3Q_0 Q}{5} (B^2 - C^2) \right\} (t_1^2 + t_2^2 + 2t_1 t_2 j_0(Qr))$$

$$+ \frac{Q^2}{4M_N^2} (F_M^{p2} + F_M^{n2}) \left\{ (\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2}) (Q_0^2 + \frac{Q^2}{4} + Q_0 Q) (t_1^2 + t_2^2 + 2t_1 t_2 j_0(Qr)) \right. \\ \left[+ (\frac{u^2}{r^2} + \frac{13}{10} \frac{\omega^2}{r^2}) (Q_0^2 + \frac{Q^2}{4}) + Q_0 Q (\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2}) \right] \\ \left. \left[\times t_1^2 + t_2^2 - 2t_1 t_2 j_0(Qr) \right] \right\}$$

$$\begin{aligned}
 & + (g_A^{p2} + g_A^{n2}) \left\{ \left[\left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right) (Q_0^2 + \frac{Q^2}{4}) + Q_0 Q \left(\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2} \right) \right] \right. \\
 & \quad \left. + \frac{1}{2M_N^2} \left[(Q_0^2 - \frac{Q^2}{4}) \left(\frac{48}{35} C^2 + \frac{3}{10} Q^2 \frac{\omega^2}{r^2} + \frac{(B + \sqrt{2} A)^2}{12} \right) \right. \right. \\
 & \quad \left. \left. + (Q_0^2 + \frac{Q^2}{4} + Q_0 Q) \frac{u}{r} \sqrt{2} \left(\frac{7}{15} E + \frac{2\sqrt{2}}{3} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} \right) \right. \right. \\
 & \quad \left. \left. + (Q_0^2 + \frac{Q^2}{4} - \frac{10}{7} Q_0 Q) \frac{7}{15} \frac{\omega}{r} \left(D + \sqrt{2} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} - \frac{3\sqrt{2} A}{r} \right) \right] \right\} \\
 & \quad (t_1^2 + t_2^2 + 2t_1 t_2 j_0(Qr))
 \end{aligned}$$

$$\begin{aligned}
 & + (g_A^{p2} + g_A^{n2}) \left\{ \left[\left(\frac{u^2}{r^2} + \frac{13}{10} \frac{\omega^2}{r^2} \right) (Q_0^2 + \frac{Q^2}{4}) - Q_0 Q \left(\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2} \right) \right] \right. \\
 & \quad \left. + \frac{1}{2M_N^2} \left[Q^2 (Q_0^2 - \frac{Q^2}{4}) \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right) + \frac{4}{3} (Q_0^2 - \frac{Q^2}{4}) (A - \sqrt{2} B)^2 \right. \right. \\
 & \quad \left. \left. + (Q_0^2 + \frac{Q^2}{4}) \left\{ \frac{2u}{r} \left(\frac{2}{3} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} - \frac{E}{3\sqrt{2}} \right) + \frac{1}{15} \frac{\omega}{r} \right. \right. \right. \\
 & \quad \left. \left. \left. \left(31D - \frac{8\sqrt{2}}{r} \frac{\partial^2 u}{\partial r^2} + \frac{24\sqrt{2}}{r} A \right) \right\} \right. \right. \\
 & \quad \left. \left. + \frac{1}{5} Q_0 Q \frac{\omega}{r} \left(D + \sqrt{2} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} - \frac{3\sqrt{2} A}{r} \right) \right] \right\} \\
 & \quad \times ((t_1^2 + t_2^2) - 2t_1 t_2 j_0(Qr))
 \end{aligned}$$

$$- \frac{2Q}{M_N} (F_C^P g_A^P - F_C^N g_A^N) (Q_0^2 - \frac{Q^2}{4}) (\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2}) (t_1^2 + t_2^2 - 2t_1 t_2 j_0(Qr))$$

$$+ \frac{Q}{M_N} (F_M^P g_A^P - F_M^N g_A^N) \{ (Q_0^2 + \frac{Q^2}{4}) (\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2}) + \frac{3}{5} Q_0 Q \frac{\omega^2}{r^2} \}$$

$$\{ t_1^2 + t_2^2 - 2t_1 t_2 j_0(Qr) \}$$

$$+ 2F_C^P F_C^N \left\{ \begin{aligned} & \frac{2}{r^2} (u^2 + \omega^2) (Q_0^2 - \frac{Q^2}{4}) [(t_1^2 + t_2^2) j_0(Qr) - 2t_1 t_2] \\ & - \frac{1}{2M_N^2} [(Q_0^2 + \frac{Q^2}{4}) (\frac{4}{3} A^2 + \frac{47}{75} B^2 + \frac{21}{25} C^2 - \frac{4\sqrt{2} AB}{15}) + \frac{3}{5} Q_0 Q (B^2 - C^2)] \\ & \times [(t_1^2 + t_2^2) j_0(Qr) + 2t_1 t_2] \end{aligned} \right\}$$

$$+ \frac{Q^2}{8M_N^4} (F_C^P F_M^N + F_C^N F_M^P) \left[\begin{aligned} & (Q_0^2 + \frac{Q^2}{4}) (\frac{4}{3} A^2 + \frac{47}{75} B^2 + \frac{21}{25} C^2 - \frac{4\sqrt{2} AB}{15}) \\ & + \frac{3}{5} Q_0 Q (B^2 - C^2) [(t_1^2 + t_2^2) j_0(Qr) + 2t_1 t_2] \end{aligned} \right]$$

$$+ \frac{Q^2}{4M_N^2} 2F_M^P F_M^N \left[\begin{aligned} & (\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2}) (Q_0^2 + \frac{Q^2}{4} + Q_0 Q) [(t_1^2 + t_2^2) j_0(Qr) + 2t_1 t_2] \\ & - \{ (\frac{u^2}{r^2} + \frac{13}{10} \frac{\omega^2}{r^2}) (Q_0^2 + \frac{Q^2}{4}) + Q_0 Q (\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2}) \} \\ & \times [(t_1^2 + t_2^2) j_0(Qr) - 2t_1 t_2] \end{aligned} \right]$$

$$\begin{aligned}
 & - 2g_A^p g_A^n \left[\left\{ \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right) (Q_0^2 + \frac{Q^2}{4}) + Q_0 Q \left(\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2} \right) \right\} \right. \\
 & \quad + \frac{1}{2M_N^2} \left\{ (Q_0^2 - \frac{Q^2}{4}) \left(\frac{48}{35} C^2 + \frac{3}{10} Q^2 \frac{\omega^2}{r^2} + \frac{(B + \sqrt{2} A)^2}{12} \right) \right. \\
 & \quad \quad + (Q_0^2 + \frac{Q^2}{4} + Q_0 Q) \frac{u}{r} \sqrt{2} \left(\frac{7}{15} E + \frac{2\sqrt{2}}{3} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} \right) \\
 & \quad \quad \left. \left. + (Q_0^2 + \frac{Q^2}{4} - \frac{10}{7} Q_0 Q) \frac{7}{15} \frac{\omega}{r} \left(D + \sqrt{2} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} - \frac{3\sqrt{2} A}{r} \right) \right\} \right] \\
 & \quad \quad \quad [(t_1^2 + t_2^2) j_0(Qr) + 2t_1 t_2]
 \end{aligned}$$

$$\begin{aligned}
 & + 2g_A^p g_A^n \left[\left\{ \left(\frac{u^2}{r^2} + \frac{13}{10} \frac{\omega^2}{r^2} \right) (Q_0^2 + \frac{Q^2}{4}) - Q_0 Q \left(\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2} \right) \right\} \right. \\
 & \quad + \frac{1}{2M_N^2} \left\{ Q^2 (Q_0^2 - \frac{Q^2}{4}) \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right) + \frac{4}{3} (Q_0^2 - \frac{Q^2}{4}) (A - \sqrt{2} B)^2 \right. \\
 & \quad \quad + (Q_0^2 + \frac{Q^2}{4}) \left[\frac{2u}{r} \left(\frac{2}{3} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} - \frac{E}{3\sqrt{2}} \right) + \frac{1}{15} \frac{\omega}{r} \right. \\
 & \quad \quad \quad \left. \left. \left(31D - \frac{8\sqrt{2}}{r} \frac{\partial^2 u}{\partial r^2} + \frac{24\sqrt{2} A}{r} \right) \right] \right. \\
 & \quad \quad \quad \left. \left. + \frac{1}{5} Q_0 Q \frac{\omega}{r} \left(D + \sqrt{2} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} - \frac{3\sqrt{2} A}{r} \right) \right\} \right] [(t_1^2 + t_2^2) j_0(Qr) - 2t_1 t_2]
 \end{aligned}$$

$$\begin{aligned}
& + \frac{2Q}{M_N} (F_C^p g_A^n - F_C^n g_A^p) (Q_0^2 - \frac{Q^2}{4}) (\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2}) \{ (t_1^2 + t_2^2) j_0(Qr) - 2t_1 t_2 \} \\
& + \frac{Q}{M_N} (F_M^p g_A^n - F_M^n g_A^p) \{ (Q_0^2 + \frac{Q^2}{4}) (\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2}) + \frac{3}{5} \frac{\omega^2}{r^2} Q_0 Q \} \\
& \{ (t_1^2 + t_2^2) j_0(Qr) - 2t_1 t_2 \} \\
& + (Q_0^2 - \frac{Q^2}{4}) (\frac{\omega^2}{r^2} - 2\sqrt{2} \frac{u\omega}{r^2}) j_2(Qr) \{ (t_1^2 + t_2^2) 2F_C^p F_C^n - 2t_1 t_2 (F_C^{p2} + F_C^{n2}) \} \\
& + \{ 2Q_0 Q (\sqrt{2} \frac{u\omega}{r^2} + \frac{\omega^2}{r^2}) - (Q_0^2 + \frac{Q^2}{4}) \frac{3}{14} \frac{\omega^2}{r^2} \} j_2(Qr) \\
& \times \{ (t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 (g_A^{p2} + g_A^{n2}) \}
\end{aligned}$$

(III-23)

where

$$Q^2 = - (\text{four momentum transfer})^2 = -q^2$$

$$Q_0 = \frac{s - M_d^2 + q^2/2}{2M_d (1 - \frac{q^2}{4M_d^2})^{1/2}} = \frac{s - M_d^2 - Q^2/2}{2M_d (1 + \frac{Q^2}{4M_d^2})^{1/2}}$$

with

$$S = (\text{sum of the initial (final) four momentum})^2 = M_d^2 + 2M_d E_\nu$$

in the lab frame.

M_d = mass of deuteron

j_0, j_2 are spherical Bessel functions,

F 's and g 's are functions of Q^2 , and

$$A = \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2}$$

$$B = \frac{1}{r} \frac{\partial \omega}{\partial r} + \frac{2\omega}{r^2}$$

$$C = \frac{1}{r} \frac{\partial \omega}{\partial r} - \frac{3\omega}{r^2}$$

$$D = \frac{1}{r} \frac{\partial^2 \omega}{\partial r^2} - \frac{6\omega}{r^3}$$

$$E = \frac{1}{r} \frac{\partial^2 \omega}{\partial r^2} + \frac{3}{r^2} \frac{\partial \omega}{\partial r}$$

The expression (III-23) is for neutrino-deuteron scattering;

for anti neutrino-deuteron scattering, we need to change all g_A 's to

$-g_A$'s.

In the next chapter we discuss the different models and choices used to calculate the differential cross-sections.

CHAPTER FOUR

Selected Models

In this Chapter we discuss the various models we use to calculate the differential cross-sections for $\nu(\bar{\nu})d$ processes. We remember that we need to know about the form factors F_C , F_M and g_A . We also know that various gauge model¹⁾ predictions, most important of which is the Weinberg-Salam model, fit the experimental results rather well. We want to investigate the predictions of these models. To see how they are introduced into our calculations, we start by looking at the general structure of the charged and neutral currents.²⁾ The charged current has the form for $SU(2) \otimes U(1)$ gauge models,

$$J_\mu^{(\pm)} = \sum_L \bar{\psi}_L \tau_\pm \gamma_\mu (1 - \gamma_5) \psi_L + \sum_R \bar{\psi}_R \tau_\pm \gamma_\mu (1 + \gamma_5) \psi_R = V_\mu^{1 \pm i 2} - A_\mu^{1 \pm i 2} \text{ (say)} \quad (IV-1)$$

where the first sum runs over the left-handed doublets and the second runs over right-handed doublets, if any. In ref. 2, they have the opposite sign of γ_5 .

The weak neutral current, believed to be of (V,A) structure is given by,

$$J_\mu^{(0)} = \frac{1}{2} \sum_L \bar{\psi}_L \tau_3 \gamma_\mu (1 - \gamma_5) \psi_L + \frac{1}{2} \sum_R \bar{\psi}_R \tau_3 \gamma_\mu (1 + \gamma_5) \psi_R - 2 \sin^2 \theta_w J_\mu^{\text{em}} \quad (IV-2)$$

where J_μ^{em} is the electromagnetic current. τ 's are isospin matrices and θ_w is the Weinberg angle.

The neutral current can be decomposed into isoscalar and isovector components in (u,d) space according to

$$J_\mu^{(0)} = \alpha V_\mu^3 - \beta A_\mu^3 + \frac{1}{3} \gamma V_\mu^0 - \delta A_\mu^0 \quad (\text{IV-3})$$

$$J_\mu^{\text{em}} = V_\mu^3 + \frac{1}{3} V_\mu^0$$

In equation (IV-3), V^3 and A^3 are related to $V^{1\pm i2}$ and $A^{1\pm i2}$ appearing in the equation (IV-1) for the charged current by the conserved vector current hypothesis which states that they are members of the same isotriplet current, being isospin partners of one another. V^0 appears both in the electromagnetic and the neutral current and is the isoscalar vector piece. However, in the neutral current A^0 is the isoscalar axial vector piece. The weak hadronic neutral current can thus be written as

$$\frac{1}{2} [\bar{u} \gamma_\lambda (\alpha - \beta \gamma_5) u - \bar{d} \gamma_\lambda (\alpha - \beta \gamma_5) d] + \frac{1}{2} [\bar{u} \gamma_\lambda (\frac{1}{3} \gamma - \delta \gamma_5) u + \bar{d} \gamma_\lambda (\frac{1}{3} \gamma - \delta \gamma_5) d] + \dots$$

(IV-4)

where ... stands for $s\bar{s}$, $c\bar{c}$... terms to which the processes under consideration are sensitive. The theoretical predictions of the coupling parameters α , β , γ and δ have been the focal point of many authors. We mention some of them in ref. 3. The processes considered are:

- i) Inclusive reactions $\nu(\bar{\nu})N \rightarrow \nu(\bar{\nu}) + \text{hadrons}$
- ii) Pion semi-inclusive reactions $\nu(\bar{\nu})N \rightarrow \nu(\bar{\nu}) + \pi + \text{hadrons}$
- iii) Elastic neutrino-proton reactions $\nu(\bar{\nu})p \rightarrow \nu(\bar{\nu})p$,
and
- iv) Exclusive pion production $\nu(\bar{\nu})N \rightarrow \nu(\bar{\nu}) \pi N$

Different combinations of the parameters appear for these processes and are utilized to determine the structure of the neutral currents. Some authors have tried to determine the values of α , β , γ and δ from the experimental data by using these relations.⁴⁾ Hung and Sakurai used the first three types of processes and their analysis from the experimental data gave two sets of solutions, A and B given by

$$\alpha = 0.45 \pm 0.14; \beta = 0.92 \pm 0.14; \gamma = -1.05 \pm 0.45; \delta = 0.12 \pm 0.15$$

Hung and Sakurai, solution A or H-S(A), and

$$\alpha = 0.92 \pm 0.14; \beta = 0.45 \pm 0.14; \gamma = 0.36 \pm 0.45; \delta = -0.35 \pm 0.15$$

Hung and Sakurai solution B or H-S(B).

Abbott and Barnett used the exclusive pion production data to obtain a set of solution, in short (A-B), given by

$$U_L = 0.33 \pm 0.07; U_R = -0.18 \pm 0.06; d_L = -0.40 \pm 0.07; d_R = 0.0 \pm 0.11$$

where

$$\alpha = U_L + U_R - d_L - d_R$$

$$\beta = U_L - U_R - d_L + d_R$$

$$\gamma = 3(U_L + U_R + d_L + d_R)$$

$$\delta = U_L - U_R + d_L - d_R$$

(IV-6)

As we can see, there is a large uncertainty in the values of α , β , γ and δ . To calculate the $\nu(\bar{\nu})d$ processes we also use some theoretical models, especially the Weinberg-Salam model with two values of the Weinberg angle. We also use models of Achiman, Koller and Walsh⁵⁾ and that of Gursey and Sikivie.⁶⁾ These models with more than four quarks were proposed to explain the so-called "y-anomaly". However, even though now "y-anomaly" is believed not to be present, it might be of interest to see what these models predict for $\nu(\bar{\nu})d$

processes. Sets of $(\alpha, \beta, \gamma, \delta)$ for these models were chosen so that they were sufficiently close to the limits given by H-S(A) or H-S(B) or (A-B) solutions. We also use another important model, known as $SU(2)_L \otimes SU(2)_R \otimes U(1)$ model. Such a model is left-right symmetric⁷⁾ and is constructed to accommodate the results of the experiments that failed to detect any "parity violation in atomic transition experiments". A variety of ingenious experiments have been devised to search for and measure weak parity violating effects in transitions between atomic levels. The Oxford⁸⁾ and Washington⁹⁾ experiments measured the rotation R due to parity non conserving effects in atomic bismuth to be

$$R_{876\text{nm}} = (-0.7 \pm 3.2) \times 10^{-8} \quad (\text{Washington})$$

$$R_{648\text{nm}} = (2.7 \pm 4.7) \times 10^{-8} \quad (\text{Oxford})$$

where

$$R = \text{Im} E_1 / M_1$$

the ratio of electric and magnetic states of the same parity.

Thus, it is found that within systematic uncertainty, the optical rotation R is zero.

The theoretical calculation for R is not very easy. This is mainly because the atomic calculations are very difficult. The estimation of the electron wave-functions for complex systems like bismuth with more than one valence electron obscures the interpretation of the experimental results. Radiative corrections are involved as well.

However, with naive atomic physics calculations, the standard $SU(2) \otimes U(1)$ Weinberg-Salam model predicts a value of R to be of order 10^{-7} , the exact value of which varies with the Weinberg angle.

Left-right symmetric $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models were introduced to explain the null results, assuming the atomic calculations to be correct. By construction, parity is conserved in the first order. For this reason, we included an $SU(2)_L \otimes SU(2)_R \otimes U(1)$ model in our calculation.

However, recently there have been new developments in this direction. An experiment by L. Barkov and M. Zolotarev at Novobirsk found an optical rotation in bismuth equal to 1.1 ± 0.3 times the Weinberg-Salam prediction with $\sin^2 \theta_W = 0.25$.

Thus, the contradiction between these experimental findings needs to be resolved. The recent experiment of Prescott et al.¹⁰⁾ seem to have resolved this question. Their experiment measured deep inelastic polarized-electron-deuteron asymmetry at SLAC. To see how it can resolve the contradiction, it is known that the optical rotation R is directly proportional

to the weak charge given by

$$Q_w(Z,A) = 2[C_{1p} Z + C_{1n} (A-Z)]$$

for an atom (Z,A) . Here C_{1p} and C_{1n} are constants depending on the gauge model. The same parameters are used to calculate the deep inelastic asymmetry parameters.¹¹⁾ This parametrization emphasizes an important distinction between the deep inelastic asymmetries and atomic parity violation due to the respective incoherence and coherence of these processes.

The experiment of Prescott et al.,

$$e^{\pm} + D \rightarrow e^{\pm} + X$$

concludes that parity-violation does exist. They found that the result of their difference experiment gave,

$$A/Q_0^2 = (-9.6 \pm 0.8 \pm 0.9) \times 10^{-5} \times Q^2$$

where

$$A = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$$

The experimental result seem to favour the standard Weinberg-Salam model with $\sin^2 \theta_w = 0.20 \pm 0.03$. At the same

time, it seems to rule out the left-right symmetric models.

Furthermore, the experiments support the result of the Barkov experiment.

In $SU(2) \times SU(2) \times U(1)$ models, the coupling parameters are given by,¹²⁾

$$\alpha = (1 + 2x') \cdot (1 - \sin^2 \theta) R_V$$

$$\beta = (1 - 2x') R_A$$

$$\gamma = (1 + 2x') (2x - \sin^2 \theta) R_V$$

$$\delta = (1 - 2x') (-2x) R_A$$

(IV-7)

where $R_V = R_A$ for left-right symmetric model. It was shown

that the model with $x = x' = 0$ is acceptable to the solutions

H-S(A) and H-S(B).¹³⁾ When $R_V = R_A = 1$ and $\sin^2 \theta = 2 \sin^2 \theta_W$,

we get predictions identical to the Weinberg-Salam model for

α , β , γ and δ . We use $\sin^2 \theta = 2 \sin^2 \theta_W$ and $R_V = R_A = 0.9$ and

look at the prediction for $\nu(\bar{\nu})d$ processes. Thus, we use the

following sets of $(\alpha, \beta, \gamma, \delta)$ to calculate differential

cross-sections for $\nu(\bar{\nu})d$ processes:

i) $\alpha = 0.45; \beta = 0.92; \gamma = -1.05; \delta = 0.12$ H-S(A)

ii) $\alpha = 0.92; \beta = 0.45; \gamma = 0.36; \delta = -0.35$ H-S(B)

• iii) $\alpha = 0.55; \beta = 0.91; \gamma = -0.75; \delta = 0.11$ A-B

iv) $\alpha = 0.48; \beta = 1.0; \gamma = -0.52; \delta = 0$ W-S; $\sin^2 \theta_W = 0.26$

$$v) \alpha = 0.4; \beta = 1.0; \gamma = -0.6; \delta = 0$$

$$W-S; \sin^2 \theta_W = 0.3$$

$$vi) \alpha = 0.53; \beta = 0.95; \gamma = -0.37; \delta = -0.03$$

$$AKW; \sin^2 \theta_W = 0.26;$$

$$\cos^2 \phi = 0.1$$

$$vii) \alpha = 0.45; \beta = 0.95; \gamma = -0.45; \delta = -0.03$$

$$AKW; \sin^2 \theta_W = 0.3;$$

$$\cos^2 \phi = 0.1$$

$$viii) \alpha = 0.98; \beta = 0.5; \gamma = 0.98; \delta = -0.3$$

$$G-S(B); \sin^2 \theta_W = 0.26$$

$$ix) \alpha = 0.90; \beta = 0.5; \gamma = 0.9; \delta = -0.3$$

$$G-S(B); \sin^2 \theta_W = 0.3$$

$$x) \alpha = 0.93; \beta = 0.45; \gamma = 1.13; \delta = -0.27$$

$$G-S(C); \sin^2 \theta_W = 0.26;$$

$$\cos^2 \alpha/2 = 0.9$$

$$xi) \alpha = 0.85; \beta = 0.45; \gamma = 1.05; \delta = -0.27$$

$$G-S(C); \sin^2 \theta_W = 0.3;$$

$$\cos^2 \alpha/2 = 0.9$$

$$xii) \alpha = 0.36; \beta = 0.9; \gamma = -0.54; \delta = 0$$

$$SU(2)_L \times SU(2)_R \times U(1)$$

model, or parity-non-violating model or PNV.

Next we assume that the neutral current form factors are proportional to the charged current form factors which in turn are conventionally described by dipole forms. Thus, we get, 2)

$$F_C = \frac{1}{2} (\pm\alpha + \gamma) \left(1 + \frac{q^2}{m_V^2}\right)^{-2}$$

$$F_M = \frac{1}{2} (\pm 4.7\alpha + 0.88\gamma) \left(1 + \frac{q^2}{m_V^2}\right)^{-2}$$

(IV-8)

$$g_A = \frac{1}{2} (\pm\beta + \delta) \times 1.24 \left(1 + \frac{q^2}{m_A^2}\right)^{-2}$$

where in $\pm$, '+' refers to proton and '-' refers to neutron:

m_V^2 is conventionally taken as 0.71 GeV^2 . The axial mass is less precisely fixed, so we took two values of it, $m_A^2 = 0.71 \text{ GeV}^2$ and 1.32 GeV^2 .

For the purpose of calculating $\nu(\bar{\nu})d$ processes, we use the non-relativistic deuteron wave-function of Hamada and Johnston.¹⁴⁾ The modified form of Humberston and Wallace¹⁵⁾ was used which gives the correct binding energy (with $L = 7$, $N=5$). The normalization used is,

$$\int_0^\infty (u^2 + w^2) dr = 1 \quad (\text{IV-9})$$

The wave-function is given in Appendix C. In the calculation of the $\nu(\bar{\nu})d$ breakup processes, the final np state was taken as (i) pure $I = 1$ state and (ii) a mixture of 70% $I = 1$ and 30% $I = 0$ states.

We now have all the information to calculate the differential cross-section for the $\nu(\bar{\nu})d$ processes being considered. The integration over ' r ' is done numerically by using Gaussian quadrature. The results are given in the next chapter.

CHAPTER V

Results

The numerical values of the differential cross-sections, obtained for different sets of neutral current couplings α , β , γ , δ as given in the last chapter will be discussed in this chapter. Let us recapitulate the different choices;

- (i) Hung and Sakurai, solution A, abbreviated as H-S(A)
- (ii) Hung and Sakurai, solution B, abbreviated as H-S(B)
- (iii) Abbott and Barnett solution, abbreviated as A-B
- (iv), (v) Weinberg and Salam model abbreviated as W-S ($\sin^2\theta_W = 0.26$) and W-S ($\sin^2\theta_W = 0.3$)
- (vi), (vii) Ackiman, Koller and Walsh model with $\cos^2\phi = 0.1$, abbreviated as AKW ($\sin^2\theta_W = 0.26$) and AKW ($\sin^2\theta_W = 0.3$)
- (viii), (iv) Gürsey - Sikivie model B, abbreviated as G-S (B) ($\sin^2\theta_W = 0.26$) and G-S (B) ($\sin^2\theta_W = 0.3$)
- (x), (xi) Gürsey - Sikivie model C with $\cos^2\alpha/2 = 0.9$ abbreviated as G-S (C) ($\sin^2\theta_W = 0.26$) and G-S (C) ($\sin^2\theta_W = 0.3$)
- (xii) An $SU(2)_L \times SU(2)_R \times (1)$ model, a parity non-

violating one, supposed to satisfy "atomic parity violation" experiments, abbreviated as PNV.

Since the axial mass is not precisely known, we used $m_A^2 = 0.71 \text{ GeV}^2$ and 1.22 GeV^2 . Also, the neutrino and the antineutrino processes are calculated.

Let us at first discuss the elastic processes. Graphs 1 - 6 show the differential cross-section for $\nu(\bar{\nu})d$ elastic scattering. The differential cross-sections is expressed in terms of G , the fermi coupling constant. We intend to look at low momentum transfer since the differential cross-section goes down very fast with increasing momentum transfer. Also, we kept terms only up to first order in q^2/M^2 . Again we are not interested in very low momentum transfer since that process would be difficult to detect and the binding energy effect would become important. Mathematically speaking, when $q^2/2M_N$ becomes comparable to the binding energy of deuteron in magnitude, our results would not be valid. Thus, with the above reasons in mind, we think a good region to look at would be the momentum transfer-squared range of $0.07 (\text{GeV}/c)^2$ to $0.4 (\text{GeV}/c)^2$. We keep in mind that these numbers are not very strict, in any case. The differential cross-sections are given in units of $G^2/2\pi$ which is equal to $8.425 \times 10^{-39} \text{ cm}^2/(\text{GeV}/c)^2$.

The graphs are all drawn for an incident (anti) neutrino lab energy of 1 GeV. In these first six graphs, we see that the differential cross-section goes down really fast with increasing momentum transfer. The difference in the νd elastic and the $\bar{\nu} d$ elastic cross-section is very, very small. In terms of numbers, at 0.07 (GeV/c)^2 for H-S(A), the differential cross-section is $0.1640 \times (8.425 \times 10^{-39}) \text{ cm}^2/(\text{GeV/c})^2$ for the νd and $0.1655 \times (8.425 \times 10^{-39}) \text{ cm}^2/(\text{GeV/c})^2$ for the $\bar{\nu} d$ elastic scattering. Thus, we can see that the difference in them will be almost impossible to detect at present. The change in the axial mass does not alter the situation. However, it should be possible to distinguish between the different conventional models for example, as explained in Table 1, with $m_A^2 = 0.71 \text{ GeV}^2$.

For the sake of comparison, we also calculated the differential cross-section for $\nu(\bar{\nu})$ proton and $\nu(\bar{\nu})$ neutron elastic scattering. Here also, the variation in axial mass did not matter much. So, for the sake of clarity, we show graphs for two values of m_A^2 only for H-S(A) and H-S(B). To compare νd elastic process with νp elastic process, we present the following table, Table 2, with the values of the νp differential cross-section, much like that which appears in Table 1. for νd differential cross-section, for $m_A^2 = 0.71 \text{ GeV}^2$.

Table 1

 $E_{\nu} = 1 \text{ GeV}$

Elastic ν d Differential Cross-Section in Units
 of $8.425 \times 10^{-39} \text{ cm}^2/(\text{GeV}/c)^2$

Momentum Transfer Squared	H-S(A)	H-S(B)	A-B	W-S(0.26)	W-S(0.3)	PNV
$0.07 (\text{GeV}/c)^2$	0.1640	0.0588	0.0851	0.0392	0.0522	0.0423
$0.4 (\text{GeV}/c)^2$	0.0016	0.0018	0.0009	0.0004	0.0005	0.0004

Table 2

 $E_{\nu} = 1 \text{ GeV}$ vp Differential Cross-Section in Units $8.425 \times 10^{-39} \text{ cm}^2/(\text{GeV}/c)^2$

Momentum Transfer Squared	H-S(A)	H-S(B)	W-S(0.26)	W-S(0.3)	PNV
0.07 $(\text{GeV}/c)^2$	0.3612	0.3411	0.2978	0.2923	0.3843
0.4 $(\text{GeV}/c)^2$	0.1026	0.1357	0.1067	0.0934	0.1251

One thing is really clear from Tables I and II, that is the $\nu(\bar{\nu})d$ elastic differential cross-section goes down very fast with increase in momentum transfer although at low momentum transfer the νd elastic differential cross-section is of the same order as that of the νp elastic process. Graphs 7 to 18 give the differential cross-section for $\nu(\bar{\nu})p$ and $\nu(\bar{\nu})n$ elastic processes.

Before leaving the topic of the $\nu(\bar{\nu})d$ elastic processes, we calculated what would be the differential cross-section of $\nu(\bar{\nu})d$ elastic process without inclusion of the D-state in deuteron. So, we put the part of the deuteron radial wave function associated with the $3D_1$ state to be zero, namely $\omega(r) = 0$, and calculated the differential cross-section for $\nu(\bar{\nu})d$ elastic cross-section. Graph 19 to 24 show the differential cross-section with D state effects absent. The contribution depends on the particular model chosen, but it is about 10 to 15 percent at momentum transfer-squared $0.07 (\text{GeV}/c)^2$ where we have to remember the normalization effect is about 7 percent.

Since the differential cross-section for the elastic $\nu(\bar{\nu})d$ falls off very rapidly with increasing momentum transfer, we also calculated the differential cross-section for the breakup processes $\nu(\bar{\nu})d \rightarrow \nu(\bar{\nu})pn$. It was of interest to see if we could get a significant difference in the differential cross-sections of $\nu d \rightarrow \nu pn$ and $\bar{\nu} d \rightarrow \bar{\nu} pn$. Also, to look at how the differential cross-section changed with the structure of the final np state, we considered the final state as (i) Isovector and (ii) Mixed

with 30% isoscalar and 70% isovector mixing. The variation with axial mass appears to be quite significant. The differential cross-section goes down much slower with momentum transfer for $m_A^2 = 1.32 \text{ GeV}^2$ compared to the case with $m_A^2 = 0.71 \text{ GeV}^2$. We show it by some typical numbers in Table 3.

However, since the pattern is similar for all the models, for the sake of clarity we show the variation with the axial mass only for the models H-S(A) and H-S(B) in the graphs 25 and 31. For all the other models, we show results only for $m_A^2 = 0.71 \text{ GeV}^2$.

As expected, the differential cross-section goes down much slower for the breakup processes compared to the elastic νd process. As in the case of the $\nu(\bar{\nu})d$ elastic process, it should be possible to distinguish between different models with the help of these breakup processes. The neutral $\nu(\bar{\nu})d$ breakup processes should also be able to give some information about the isoscalar axial current piece since the difference in the νd and the $\bar{\nu} d$ process is significant. This information is tabulated in Table 4 for a few models and for $m_A^2 = 0.71 \text{ GeV}^2$.

From Table 4, we can see that it should be possible, for example to distinguish the predictions for H-S(A) and H-S(B) models for the values of the differential cross-sections of νd breakup and also to see the difference between νd and $\bar{\nu} d$ differential cross-sections in each model. Thus, it should give us more knowledge about the validity of these models and

Table 3

$$E_v = 1 \text{ GeV}$$

vd Breakup

Differential Cross-Section in Units of $8.425 \times 10^{-39} \text{ cm}^2/(\text{GeV}/c)^2$

Momentum Transfer Squared	H-S(A)		H-S(A)	
	$M_A^2 = 0.71 \text{ GeV}^2$		$M_A^2 = 1.32 \text{ GeV}^2$	
	Isovector	Mixed	Isovector	Mixed
$0.07 (\text{GeV}/c)^2$	0.5565	0.6953	0.6386	0.7524
$0.4 (\text{GeV}/c)^2$	0.1059	0.1758	0.2093	0.2527

Table 4

$$E_\nu = 1 \text{ GeV}$$

$\nu(\bar{\nu})d$ Breakup Differential Cross-Section
in Units of $8.425 \times 10^{-39} \text{ cm}^2/(\text{GeV}/c)^2$

Momentum Transfer Squared		H-S(A)	H-S(B)	A-B	W-S(0.26)	W-S(0.3)	PNV
$0.07 (\text{GeV}/c)^2$	Isovector ν	0.5565	0.7535	0.6173	0.6848	0.6356	0.5148
	Mixed ν	0.6953	0.6040	0.6104	0.5520	0.5352	0.4335
	Isovector $\bar{\nu}$	0.7404	0.6072	0.7555	0.7731	0.7374	0.5973
	Mixed $\bar{\nu}$	0.6732	0.6330	9.5931	9.5425	0.5243	0.4247
$0.4 (\text{GeV}/c)^2$	Isovector ν	0.1059	0.1357	0.1102	0.1361	0.1293	0.1048
	Mixed ν	0.1758	0.2176	0.1669	0.1484	0.1375	0.1114
	Isovector $\bar{\nu}$	0.2253	0.0539	0.1989	0.1946	0.1968	0.1594
	Mixed $\bar{\nu}$	0.1950	0.2253	0.1796	0.1598	0.1506	0.1220

might also give more insight into this isoscalar axial vector piece in the weak hadronic neutral current.

In the next chapter, we conclude by saying what our findings would mean and what improvements can be made.

$\nu(\bar{\nu})d$ -ELASTIC PROCESS H-S(A)

$\nu(\bar{\nu})d$ -ELASTIC PROCESS H-S(B)

LEGEND :

$\square$ $M_h^2 = 0.71 GeV^2 (\nu)$
 Δ $M_h^2 = 1.32 GeV^2 (\nu)$

$\circ$ $M_h^2 = 0.71 GeV^2 (\bar{\nu})$
 $\times$ $M_h^2 = 1.32 GeV^2 (\bar{\nu})$

LEGEND :

□ $M_A^2 = 0.71 \text{Gev}^2 (\nu)$

△ $M_A^2 = 1.32 \text{Gev}^2 (\bar{\nu})$

○ $M_A^2 = 0.71 \text{Gev}^2 (\bar{\nu})$

* $M_A^2 = 1.32 \text{Gev}^2 (\nu)$

$\nu(\bar{\nu})d$ -ELASTIC PROCESS

W-S (0.3)

$\nu(\bar{\nu})d$ -ELASTIC PROCESS

AKW (0.26)

LEGEND :

□ $M_A^2 = 0.71 \text{ GeV}^2(\nu)$
 Δ $M_A^2 = 1.32 \text{ GeV}^2(\nu)$

○ $M_A^2 = 0.71 \text{ GeV}^2(\bar{\nu})$
 * $M_A^2 = 1.32 \text{ GeV}^2(\bar{\nu})$

$\nu(\bar{\nu})d$ -ELASTIC PROCESS

AKW (0.3)

$\nu(\bar{\nu})d$ -ELASTIC PROCESS

G-S(B) (0.26)

LEGEND :

$\square$ $M_A^2 = 0.71 \text{Gev}^2 (\nu)$
 Δ $M_A^2 = 1.32 \text{Gev}^2 (\bar{\nu})$

$\circ$ $M_A^2 = 0.71 \text{Gev}^2 (\bar{\nu})$
 $*$ $M_A^2 = 1.32 \text{Gev}^2 (\nu)$

$\nu(\bar{\nu})d$ -ELASTIC PROCESS $G-S(B)(0.3)$ $\nu(\bar{\nu})d$ -ELASTIC PROCESS $G-S(C)(0.26)$

LEGEND :

□ $M_A^2 = 0.71 \text{ Gev}^2(\nu)$
 Δ $M_A^2 = 1.32 \text{ Gev}^2(\nu)$

○ $M_A^2 = 0.71 \text{ Gev}^2(\bar{\nu})$
 * $M_A^2 = 1.32 \text{ Gev}^2(\bar{\nu})$

$\nu(\bar{\nu})d$ -ELASTIC PROCESS

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G-S(c) (0.3)

$\nu(\bar{\nu})d$ -ELASTIC PROCESS

PNV

LEGEND :

$\square$ $M_A^2 = 0.71 \text{Gev}^2(\nu)$
 Δ $M_A^2 = 1.32 \text{Gev}^2(\nu)$

$\circ$ $M_A^2 = 0.71 \text{Gev}^2(\bar{\nu})$
 $*$ $M_A^2 = 1.32 \text{Gev}^2(\bar{\nu})$

$\nu(\bar{\nu})p$ -ELASTIC PROCESS H-S (A)

$\nu(\bar{\nu})p$ -ELASTIC PROCESS H-S (B)

LEGEND :

$\square$ $M_A^2 = 0.71 \text{ GeV}^2 (\nu)$
 Δ $M_A^2 = 1.32 \text{ GeV}^2 (\nu)$

O $M_A^2 = 0.71 \text{ GeV}^2 (\bar{\nu})$
 * $M_A^2 = 1.32 \text{ GeV}^2 (\bar{\nu})$

LEGEND :

$\square$ $M_A^2 = 0.71 \text{ GeV}^2 (\nu)$

$\circ$ $M_A^2 = 0.71 \text{ GeV}^2 (\bar{\nu})$

$\nu(\bar{\nu})p$ -ELASTIC PROCESS
W-S (0.3)

$\nu(\bar{\nu})p$ -ELASTIC PROCESS
AKW (0.26)

LEGEND :

$\square$ $M_A^2 = 0.71 \text{Gev}^2(\nu)$

$\circ$ $M_A^2 = 0.71 \text{Gev}^2(\bar{\nu})$

$\nu(\bar{\nu})p$ -ELASTIC PROCESS AKW (0.3)

$\nu(\bar{\nu})p$ -ELASTIC PROCESS G-S (B) (0.26)

LEGEND :

$\square$ $M_A^2 = 0.71 \text{ GeV}^2 (\nu)$

$\circ$ $M_A^2 = 0.71 \text{ GeV}^2 (\bar{\nu})$

LEGEND :

□ $M_A^2 = 0.71 \text{Gev}^2 (\nu)$

○ $M_A^2 = 0.71 \text{Gev}^2 (\bar{\nu})$

$\nu(\bar{\nu})p$ -ELASTIC PROCESS
G-S(c) (0.3)

$\nu(\bar{\nu})p$ -ELASTIC PROCESS
PNV

LEGEND :

$\square$ $M_A^2 = 0.71 \text{ Gev}^2 (\nu)$

$\circ$ $M_A^2 = 0.71 \text{ Gev}^2 (\bar{\nu})$

$\nu(\bar{\nu})n$ -ELASTIC PROCESS

H-S (A)

$\nu(\bar{\nu})n$ -ELASTIC PROCESS

H-S (B)

LEGEND :

 $\square$ $M_A^2 = 0.71 \text{Gev}^2 (\nu)$ $\triangle$ $M_A^2 = 1.32 \text{Gev}^2 (\nu)$ $\circ$ $M_A^2 = 0.71 \text{Gev}^2 (\bar{\nu})$ $*$ $M_A^2 = 1.32 \text{Gev}^2 (\bar{\nu})$

$\nu(\bar{\nu})n$ -ELASTIC PROCESS

A-B

$\nu(\bar{\nu})n$ -ELASTIC PROCESS

W-S (0.26)

LEGEND :

 $\square \quad M_A^2 = 0.71 \text{ Gev}^2 (\nu)$ $\circ \quad M_A^2 = 0.71 \text{ Gev}^2 (\bar{\nu})$

$\nu(\bar{\nu})n$ -ELASTIC PROCESS W-S (0.3)

$\nu(\bar{\nu})n$ -ELASTIC PROCESS AKW (0.26)

LEGEND :

□ $M_A^2 = 0.71 \text{ GeV}^2(\nu)$

○ $M_A^2 = 0.71 \text{ GeV}^2(\bar{\nu})$

LEGEND :

□ $M_A^2 = 0.71 \text{ GeV}^2 (\nu)$

○ $M_A^2 = 0.71 \text{ GeV}^2 (\bar{\nu})$

$\nu(\bar{\nu})n$ -ELASTIC PROCESS $G_1 - S(8) (0.3)$ $\nu(\bar{\nu})n$ -ELASTIC PROCESS $G_1 - S(8) (0.26)$

LEGEND :

 $\square M_A^2 = 0.71 \text{ GeV}^2 (\nu)$ $\circ M_A^2 = 0.71 \text{ GeV}^2 (\bar{\nu})$

$\nu(\bar{\nu})n$ -ELASTIC PROCESS $G = 5(c)(0.3)$

LEGEND :

$\square$ $M_A^2 = 0.71 \text{ GeV}^2 (\nu)$

$\circ$ $M_A^2 = 0.71 \text{ GeV}^2 (\bar{\nu})$

$\nu(\bar{\nu})d$ -ELASTIC PROCESS H-S(A)

$\nu(\bar{\nu})d$ -ELASTIC PROCESS H-S(B)

LEGEND :

$\square$ $M_A^2 = 0.71 \text{Gev}^2 (\nu)$
 Δ $M_A^2 = 1.32 \text{Gev}^2 (\nu)$

$\circ$ $M_A^2 = 0.71 \text{Gev}^2 (\bar{\nu})$
 $*$ $M_A^2 = 1.32 \text{Gev}^2 (\bar{\nu})$

$\nu(\bar{\nu})d$ -ELASTIC PROCESS

A-B

$\nu(\bar{\nu})d$ -ELASTIC PROCESS

W-S (0.26)

LEGEND :

□ $M_A^2 = 0.71 \text{Gev}^2 (\nu)$
 Δ $M_A^2 = 1.32 \text{Gev}^2 (\nu)$

○ $M_A^2 = 0.71 \text{Gev}^2 (\bar{\nu})$
 * $M_A^2 = 1.32 \text{Gev}^2 (\bar{\nu})$

$\nu(\bar{\nu})d$ -ELASTIC PROCESS W-S (0.3)

$\nu(\bar{\nu})d$ -ELASTIC PROCESS AKW (0.26)

LEGEND :

□ $M_A^2 = 0.71 \text{Gev}^2(\nu)$
 Δ $M_A^2 = 1.32 \text{Gev}^2(\nu)$

○ $M_A^2 = 0.71 \text{Gev}^2(\bar{\nu})$
 * $M_A^2 = 1.32 \text{Gev}^2(\bar{\nu})$

$\nu(\bar{\nu})d$ -ELASTIC PROCESS

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AKW (0.3)

$\nu(\bar{\nu})d$ -ELASTIC PROCESS

G-S (B) (0.26)

LEGEND :

$\square$: $M_A^2 = 0.71 GeV^2 (\nu)$
 Δ : $M_A^2 = 1.32 GeV^2 (\nu)$

$\circ$: $M_A^2 = 0.71 GeV^2 (\bar{\nu})$
 $*$: $M_A^2 = 1.32 GeV^2 (\bar{\nu})$

$\nu(\bar{\nu})d$ -ELASTIC PROCESS
G-S(B)(0.3)

$\nu(\bar{\nu})d$ -ELASTIC PROCESS
G-S(C)(0.26)

LEGEND :

□ $M_A^2 = 0.71 \text{ GeV}^2 (\nu)$
 △ $M_A^2 = 1.32 \text{ GeV}^2 (\nu)$

○ $M_A^2 = 0.71 \text{ GeV}^2 (\bar{\nu})$
 * $M_A^2 = 1.32 \text{ GeV}^2 (\bar{\nu})$

$\nu(\bar{\nu})d$ -ELASTIC PROCESS
G-S(c) (0.3)

$\nu(\bar{\nu})d$ -ELASTIC PROCESS
PNV

LEGEND :

□ $M_A^2 = 0.71 \text{ GeV}^2 (\nu)$
 Δ $M_A^2 = 1.32 \text{ GeV}^2 (\nu)$

○ $M_A^2 = 0.71 \text{ GeV}^2 (\bar{\nu})$
 * $M_A^2 = 1.32 \text{ GeV}^2 (\bar{\nu})$

ν d-BREAKUP PROCESS H-S (A)

112

ν d-BREAKUP PROCESS H-S (B)

LEGEND :

□ 0.71 (Isovector)
Δ 1.32 (Isovector)

○ 0.71 (Mixed)
* 1.32 (Mixed)

ν d-BREAKUP PROCESS

A-B

ν d-BREAKUP PROCESS

W-S (0.26)

LEGEND :

0.71 (Isovector)

0.71 (Mixed)

ν d-BREAKUP PROCESS W-S (0.3)

114

ν d-BREAKUP PROCESS AKW (0.26)

LEGEND :

□ 0.71 (Isovector)

○ 0.71 (Mixed)

ν d-BREAKUP PROCESS AKW (0.3)

ν d-BREAKUP PROCESS G-S(B) (0.26)

LEGEND :

□ 0.71 (Isovector)

○ 0.71 (Mixed)

νd -BREAKUP PROCESS
 $G-S(0)(0.3)$

νd -BREAKUP PROCESS
 $G-S(c)(0.26)$

LEGEND :

□ 0.71 (Isoscalar)

○ 0.71 (Mixed)

ν d-BREAKUP PROCESS G-S(c) (0.3)

ν d-BREAKUP PROCESS PNU

LEGEND :

- $\square$ 0.71 (Isoscalar)
- $\circ$ 0.71 (Mixed)

$\bar{\nu}$ d-BREAKUP PROCESS H-S(A)

$\bar{\nu}$ d-BREAKUP PROCESS H-S(B)

LEGEND :

□ 0.71 (Isovector)
 △ 1.32 (Isovector)

○ 0.71 (Mixed)
 * 1.32 (Mixed)

$\bar{\nu}$ d-BREAKUP PROCESS

A-B

$\bar{\nu}$ d-BREAKUP PROCESS

W-5 (0.26)

LEGEND :

□ 0.71 (Isovector)

○ 0.71 (Mixed)

$\bar{\nu}$ d-BREAKUP PROCESS W-S (0.3)

$\bar{\nu}$ d-BREAKUP PROCESS AKW (0.26)

LEGEND :

□ 0.71 (Isovector)

○ 0.71 (Mixed)

$\bar{\nu}d$ -BREAKUP PROCESS AKW (0.3)

$\bar{\nu}d$ -BREAKUP PROCESS G-S(B) (0.26)

LEGEND :

□ 0.71 (Isovector)

○ 0.71 (Mixed)

$\bar{\nu}$ d-BREAKUP PROCESS G-S(B)(0.3)

$\bar{\nu}$ d-BREAKUP PROCESS G-S(c)(0.26)

LEGEND :

 $\square$ 0.7N(Isovector)

O 0.7I(Mixed)

$\bar{\nu}d$ -BREAKUP PROCESS

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G-5(c) (0.3)

$\bar{\nu}d$ -BREAKUP PROCESS

PNV

LEGEND :

□ 0.71(Isovector)

○ 0.71(Mixed)

CHAPTER VI

Conclusion

The (anti) neutrino-deuteron neutral processes should give us further information about the structure of the weak hadronic neutral current. The present theoretical calculations should also be interesting from the point of view that soon such experiments may be performed.

The $\nu(\bar{\nu})d$ elastic and neutral breakup processes should be able to test the validity of the existing models. Elaborating this point a little more, we see that the models we considered fall roughly in two different classes. Hung and Sakurai solutions, type A and type B. As explained in Chapter V, we get predictions for differential cross-sections for the $\nu(\bar{\nu})d$ process with fairly significant differences for the two types of models. Thus, experiments should be able to rule out one of these types. Also, in the $\nu(\bar{\nu})d$ neutral breakup processes, we have seen that we can get some information about the axial mass and also a significant difference between the νd and the $\bar{\nu} d$ processes, giving information about the isoscalar axial vector piece.

Some discussion about the assumptions that were taken to calculate the differential cross-sections for the $\nu(\bar{\nu})d$ elastic and the $\nu(\bar{\nu})d$ neutral breakup processes are now in order. Going back to Chapter II, we have taken the nucleonic

currents which are associated with the nucleons on the mass shell. This seems to be reasonable from the point of view that the charge form factor is independent of nucleon masses.¹ We have calculated the results in the impulse approximation which is believed to be valid at low momentum transfer. We have used the non-relativistic deuteron wave function since we want to look at low momentum transfer and the deuteron is a loosely bound system. We have chosen Hamada-Johnston² wave functions for the deuteron as it is the one which gives satisfactory results in the elastic electron-deuteron scattering. Another reason for choosing it is that it is easy to use for numerical work. We used a modified version³ of Hamada-Johnston wave function so as to give the correct deuteron binding energy.

The choice of the different sets of hadronic neutral current couplings were based on reasonable models; some of them based on experimental data and some of them based on theoretical interest.

In the case of (anti) neutrino-deuteron neutral breakup processes, plane wave states were taken for the final np state. To see how results vary with the isospin structure of the final np state, we considered two cases. Once with the final state as a pure isovector and then with thirty percent mixture of an isoscalar contribution.

We considered the range of the momentum transfer

squared valid for our calculation to be about 0.07 (GeV/c)^2 to 0.4 (GeV/c)^2 . At a lower momentum transfer, the processes would be almost impossible to detect. As a comparison, we can say that in $\nu(\bar{\nu})p$ elastic scattering, a cut at 0.3 (GeV/c)^2 is used. Also, at lower momentum transfer, even the deuteron binding energy would be important. At higher momentum transfer, one would get much smaller differential cross-section.

However, in anticipation that one can measure much smaller differential cross-sections in the future, one can see what would happen at the larger momentum transfers.

At larger momentum transfers, the impulse approximation would not be valid and the exchange current contributions would be important. One would need to use the relativistic deuteron wave functions. F. Gross⁴ has done a lot of work in this direction for the electron-deuteron scattering. He showed that for a relativistic treatment, all four deuteron form-factors in the $d-np$ vertex should be used. One should really use the fully relativistic deuteron wave function. F. Gross⁵ used two-body Green's function for the propagation of one free particle (on the mass shell) and one virtual particle nucleon (off the mass shell) to eliminate the integration over the relative energy to get a relativistic deuteron wave function. That would also mean the inclusion of the P-states of the deuteron. The calculation of W. Buck and F. Gross⁶ showed the probability of P-states to be about 1 percent.

At large momentum transfer, the off-mass shell effects on the form factors could be important.

When considering the (anti) neutrino-deuteron neutral breakup process, the final state interaction for the final np state should also be considered.

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APPENDIX A

To calculate the deuteron form factors, we need to calculate the integrals like

$$\int e^{i\vec{q} \cdot \vec{r}/2} \times (\text{function of angles}) d\Omega(\vec{r})$$

Here, the results are given.

$$\int e^{i\vec{q} \cdot \vec{r}/2} d\Omega = 4\pi j_0\left(\frac{|\vec{q}||\vec{r}|}{2}\right)$$

$$\int e^{i\vec{q} \cdot \vec{r}/2} \vec{r} \cdot \vec{a} d\Omega = 4\pi i \frac{|\vec{r}|}{|\vec{q}|} j_1\left(\frac{|\vec{q}||\vec{r}|}{2}\right) \vec{q} \cdot \vec{a}$$

$$\int e^{i\vec{q} \cdot \vec{r}/2} \vec{r} \cdot \vec{a} \vec{r} \cdot \vec{b} d\Omega = 4\pi |\vec{r}|^2 \frac{\vec{a} \cdot \vec{b}}{3} j_0\left(\frac{|\vec{q}||\vec{r}|}{2}\right) - \frac{(\vec{q} \cdot \vec{a})(\vec{q} \cdot \vec{b})}{|\vec{q}|^2} - \frac{\vec{a} \cdot \vec{b}}{3} j_2\left(\frac{|\vec{q}||\vec{r}|}{2}\right)$$

$$\begin{aligned} & \int e^{i\vec{q} \cdot \vec{r}/2} \vec{r} \cdot \vec{a} \vec{r} \cdot \vec{b} \vec{r} \cdot \vec{c} d\Omega \\ &= 4\pi i \frac{|\vec{r}|^3}{|\vec{q}|^3} \frac{\vec{q} \cdot \vec{a} \vec{b} \cdot \vec{c} + \vec{q} \cdot \vec{b} \vec{c} \cdot \vec{a} + \vec{q} \cdot \vec{c} \vec{a} \cdot \vec{b}}{5} |\vec{q}|^2 \{j_1\left(\frac{|\vec{q}||\vec{r}|}{2}\right) + j_3\left(\frac{|\vec{q}||\vec{r}|}{2}\right)\} \\ & \quad - j_3\left(\frac{|\vec{q}||\vec{r}|}{2}\right) (\vec{q} \cdot \vec{a})(\vec{q} \cdot \vec{b})(\vec{q} \cdot \vec{c}) \end{aligned}$$

where j 's are spherical Bessel functions.

APPENDIX B

Here are the integrated hadronic tensor part associated with the $\nu(\bar{\nu})d$ breakup processes, given for the singlet and the three triplet states of the np final state. Thus,

$$H^{\mu\nu} = \int H^{\mu\nu} d^3p$$

for $\mu, \nu = 0, 1, 2, 3$ and for different spin states of the final np state is given in what follows.

$$H^{00}(\text{np singlet}) = \int r^2 dr \times$$

$$\{(t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)\}(g_A^{p2} + g_A^{n2})H1 + \{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\}$$

$$(g_A^{p2} + g_A^{n2})H2$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2\} 2g_A^p g_A^n H1 - \{(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2\}$$

$$2g_A^p g_A^n H2$$

with

$$H1 = \frac{1}{M_N^2} \left\{ \frac{q^+ q^-}{8} \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right) + \frac{3}{40} q_3^2 \frac{\omega^2}{r^2} \right\} \text{ and } H2 = \frac{(A - \sqrt{2} B)^2}{3M_N^2}$$

where

$$q^{\pm} = q_1 \pm iq_2.$$

$$H^{00}(\text{np triplet, } m=-1) = \int r^2 dr x$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \{(F_C^{p2} + F_C^{n2}) \frac{3}{5} \frac{\omega^2}{r^2} + (g_A^{p2} + g_A^{n2}) H_3$$

$$+ (F_C^p g_A^p - F_C^n g_A^n) \frac{3}{5} \frac{q_3}{M_N} \frac{\omega^2}{r^2}\}$$

$$+ \{(t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)\} (g_A^{p2} + g_A^{n2}) \frac{12}{35} \frac{c^2}{M_N^2} - \{(t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2\}$$

$$2g_A^p g_A^n \frac{12}{35} \frac{c^2}{M_N^2}$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2\} \{2F_C^p F_C^n \frac{3}{5} \frac{\omega^2}{r^2} - 2g_A^p g_A^n H_3 + (F_C^n g_A^p - F_C^p g_A^n)$$

$$\frac{3}{5} \frac{q_3}{M_N} \frac{\omega^2}{r^2}\}$$

$$+ j_2(qr) Y_{20}(\hat{q}) \frac{6}{35} \sqrt{\frac{5}{4\pi}} \frac{\omega^2}{r^2} \{(t_1^2 + t_2^2) 2F_C^p F_C^n - 2t_1 t_2 (F_C^{p2} + F_C^{n2})\}$$

with

$$H_3 = \frac{3}{80M_N^2} (q^2 + 3q_3^2) \frac{\omega^2}{r^2}$$

$$\underline{H^{00} \text{ (np triplet, } m=0)} = \int r^2 dr \times$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \{F_c^{p2} + F_c^{n2}\} \frac{3}{10} \frac{\omega^2}{r^2} + \frac{g_A^{p2} + g_A^{n2}}{2M_N^2} \frac{q^+ q^-}{4}$$

$$\left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right) \}$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2\} \{2F_c^p F_c^n \frac{3}{10} \frac{\omega^2}{r^2} - \frac{2g_A^p g_A^n}{2M_N^2} \frac{q^+ q^-}{4} \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right) \}$$

$$+ \{(t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)\} \frac{(g_A^{p2} + g_A^{n2})}{6M_N^2} (B + \sqrt{2} A)^2$$

$$- \{(t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2\} \frac{2g_A^p g_A^n}{6M_N^2} (B + \sqrt{2} A)^2$$

$$- 4\pi j_2(qr) Y_{20}(\hat{q}) \frac{3}{70} \sqrt{\frac{5}{4\pi}} \frac{\omega^2}{r^2} \{(t_1^2 + t_2^2) 2F_c^p F_c^n - 2t_1 t_2 (F_c^{p2} + F_c^{n2})\}$$

$$\underline{H^{00} \text{ (np triplet, } m=+1)} = \int r^2 dr \times$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \{F_c^{p2} + F_c^{n2}\} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) + (g_A^{p2} + g_A^{n2}) H_4$$

$$- (F_c^p g_A^p - F_c^n g_A^n) H_5 \}$$

$$+[(t_1^2 + t_2^2)j_0(qr) - 2t_1 t_2] \{ 2F_C^p F_C^n \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) - 2g_A^p g_A^n H4$$

$$- (F_C^n g_A^p - F_C^p g_A^n) H5 \}$$

$$+[(t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)] \left\{ \frac{g_A^{p2} + g_A^{n2}}{6M_N^2} (B + \sqrt{2} A)^2 \right\} - [(t_1^2 + t_2^2)j_0(qr) + 2$$

$$\frac{2g_A^p g_A^n}{6M_N^2} (B + \sqrt{2} A)^2$$

$$-4\pi j_2(qr) y_{20}(\hat{q}) \left\{ \frac{u\omega}{r^2} \sqrt{\frac{1}{10\pi}} + \frac{1}{35} \sqrt{\frac{5}{4\pi}} \frac{\omega^2}{r^2} \right\}$$

with

$$H4 = \frac{1}{4M_N^2} (q_3^2 \frac{u^2}{r^2} + \frac{q^2}{10} \frac{\omega^2}{r^2} + \frac{q^+ q^-}{20} \frac{\omega^2}{r^2}) \text{ and}$$

$$H5 = \frac{1}{M_N} q_3 \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right)$$

$$\underline{H^{11} \text{ (np singlet)}} = \int r^2 dr x$$

$$[(t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)] \{ (F_M^{p2} + F_M^{n2}) H6 + (g_A^{p2} + g_A^{n2}) H7$$

$$+ \frac{F_M^p g_A^p - F_M^n g_A^n}{2M_n} q_3 \left(\frac{u^2}{r^2} + \frac{1}{2} \frac{\omega^2}{r^2} \right) \}$$

$$+ \{ (t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2 \} \{ 2F_M^p F_M^n H6 - 2g_A^p g_A^n H7 - \frac{F_M^p g_A^n - F_M^n g_A^p}{2M_N} \}$$

$$q_3 \left(\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2} \right)$$

$$+ 2\pi j_2(qr) \left\{ \frac{2}{7} \sqrt{\frac{5}{4\pi}} \frac{\omega^2}{r^2} \left[\sqrt{\frac{3}{50}} (\gamma_{22}(\hat{q}) + \gamma_{2-2}(\hat{q})) - \frac{1}{2} \gamma_{20}(\hat{q}) \right] \right.$$

$$\left. - \frac{1}{\sqrt{10\pi}} \frac{u\omega}{r^2} \left[\sqrt{\frac{3}{2}} (\gamma_{22}(\hat{q}) + \gamma_{2-2}(\hat{q})) - \gamma_{20}(\hat{q}) \right] \right\}$$

$$\{ (t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 (g_A^{p2} + g_A^{n2}) \}$$

with

$$H6 = \frac{1}{8M_N^2} \left\{ q_3^2 \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right) + \frac{3}{5} q_2^2 \frac{\omega^2}{r^2} \right\}$$

and

$$H7 = \frac{1}{2} \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right) + \frac{1}{2\sqrt{2} M_N^2} \left\{ \frac{7}{15\sqrt{2}} \frac{\omega}{r} \left(D + \sqrt{2} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} - \frac{3\sqrt{2}}{r} A \right) \right.$$

$$\left. + \frac{u}{r} \left(\frac{7}{15} E + \frac{2\sqrt{2}}{3} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} \right) + \left(\frac{q^+ + q^-}{2} \right)^2 \frac{1}{2\sqrt{2}} \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right) \right\}$$

$$H^{11}(\text{np triplet, } m=-1) = \int r^2 dr x$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \{(F_M^{p2} + F_M^{n2})H_8 + \frac{3}{20} (g_A^{p2} + g_A^{n2}) \frac{\omega^2}{r^2}$$

$$+ (g_A^{p2} + g_A^{n2})H_9$$

$$+ \frac{[(F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2 + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2]}{4M_N^2} + \frac{3}{5} q_1^2 \frac{\omega^2}{r^2} + \frac{F_M^p g_A^p - F_M^n g_A^n}{2M_N} \frac{3}{10} q_3 \frac{\omega^2}{r^2}$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2\} \{\frac{3}{20} 2g_A^p g_A^n \frac{\omega^2}{r^2} - 2F_M^p F_M^n H_8 + 2g_A^p g_A^n H_9$$

$$- \frac{3q_1^2}{20M_N^2} \frac{\omega^2}{r^2} 2(F_C^p + \frac{q^2}{8M_N^2} F_M^p)(F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$+ \frac{F_M^p g_A^n - F_M^n g_A^p}{2M_N} \frac{3}{10} q_3 \frac{\omega^2}{r^2}$$

$$+ \{(t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)\} (\frac{96}{175} C^2 + \frac{12}{25} B^2) \frac{1}{4M_N^2} \{(F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2$$

$$+ (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2\}$$

$$-\{(t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2 \left(\frac{96}{175} c^2 + \frac{12}{25} B^2\right) \frac{1}{4M_N^2} (F_C^p + \frac{q^2}{8M_N^2} F_M^p)\}$$

$$(F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$-4\pi j_2(qr) Y_{20}(\hat{q}) \frac{3}{140} \sqrt{\frac{5}{4\pi}} \frac{\omega^2}{r^2} \{(t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 (g_A^{p2} + g_A^{n2})\}$$

with

$$H8 = \frac{3}{20M_N^2} \frac{\omega^2}{r^2} (q_2^2 + \frac{q_3^2}{4}) \text{ and}$$

$$H9 = \frac{3}{40M_N^2} \frac{\omega}{r} (2D + \frac{q_1^2}{2} \frac{\omega}{r})$$

$$\underline{H^{11} (np \text{ triplet, } m=0)} = \int r^2 dr \times$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \left\{ \frac{q_3^2}{8M_N^2} (F_M^{p2} + F_M^{n2}) \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right) + \frac{1}{2} (g_A^{p2} + g_A^{n2}) \right\}$$

$$\left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right)$$

$$- \frac{1}{4\sqrt{2} M_N^2} (g_A^{p2} + g_A^{n2}) H10 + \frac{3q_1^2}{40M_N^2} \frac{\omega^2}{r^2} \left[(F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2 + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2 \right]$$

$$+ \frac{q_3}{2M_N} (F_M^p g_A^p - F_M^n g_A^n) \left(\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2} \right)$$

$$+ \left\{ (t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2 \right\} \left\{ g_A^p g_A^n \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right) - \frac{q_3^2}{8M_N^2} 2F_M^p F_M^n \right.$$

$$\left. \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right) + \frac{2g_A^p g_A^n}{4\sqrt{2} M_N^2} H10 \right.$$

$$- \frac{3q_1^2}{40M_N^2} \frac{\omega^2}{r^2} 2(F_C^p + \frac{q^2}{8M_N^2} F_M^p) (F_C^n + \frac{q^2}{8M_N^2} F_M^n) + \frac{q_3}{2M_N} (F_M^p g_A^n - F_M^n g_A^p)$$

$$\left. \left(\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2} \right) \right\}$$

$$+ \left\{ (t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr) \right\} \frac{1}{4M_N^2} \left(\frac{3}{25} B^2 + \frac{39}{175} C^2 \right) \left\{ (F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2 \right.$$

$$+ (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2 \left. \right\}$$

$$- \left\{ (t_1^2 - t_2^2) j_0(qr) + 2t_1 t_2 \right\} \frac{1}{4M_N^2} \left(\frac{3}{25} B^2 + \frac{39}{175} C^2 \right) 2(F_C^p + \frac{q^2}{8M_N^2} F_M^p)$$

$$(F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$+ 2\pi j_2(qr) \left[\frac{2}{7} \sqrt{\frac{5}{4\pi}} \frac{\omega^2}{r^2} \left\{ \sqrt{\frac{3}{50}} (\gamma_{22}(\hat{q}) + \gamma_{2-2}(\hat{q})) + \frac{1}{2} \gamma_{20}(\hat{q}) \right\} - \sqrt{\frac{1}{10\pi}} \frac{\omega}{r^2} \right.$$

$$\left. \left\{ \sqrt{\frac{3}{2}} (\gamma_{22}(\hat{q}) + \gamma_{2-2}(\hat{q})) + \gamma_{20}(\hat{q}) \right\} \right]$$

$$\times [2g_A^p g_A^n (t_1^2 + t_2^2) - 2t_1 t_2 (g_A^{p2} + g_A^{n2})]$$

with

$$H_{10} = \frac{u}{r} \left(\frac{2}{3} E - \frac{8\sqrt{2}}{3} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} - \frac{q_1^2}{\sqrt{2}} \frac{u}{r} \right) - \frac{1}{15} \frac{\omega}{r} \left(\frac{2}{r} \frac{\partial^2 u}{\partial r^2} - \frac{6A}{r} + 4\sqrt{2} D + \frac{3q_1^2}{2\sqrt{2}} \frac{\omega}{r} \right)$$

$$+ \frac{2}{5} \frac{\omega}{r} \left(\frac{2}{r} \frac{\partial^2 u}{\partial r^2} - \frac{6A}{r} - 2\sqrt{2} D - \frac{3q_1^2}{2\sqrt{2}} \frac{\omega}{r} \right)$$

$$H_{11} \text{ (np triplet, } m=+1) = \int r^2 dr \times$$

$$\{ (t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr) \} \{ (F_M^{p2} + F_M^{n2}) H_{11} + \frac{3}{20} \frac{\omega^2}{r^2} (g_A^{p2} + g_A^{n2}) + (g_A^{p2} + g_A^{n2}) \}$$

$$- H_{12} + \frac{q_1^2}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \left[(F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2 + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2 \right]$$

$$- \frac{3q_3}{20M_N^2} \frac{\omega^2}{r^2} (F_M^p g_A^p - F_M^n g_A^n)$$

$$+ \{ (t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2 \} \left\{ \frac{3}{10} \frac{\omega^2}{r^2} g_A^p g_A^n - 2F_M^p F_M^n H_{11} + 2g_A^p g_A^n H_{12} \right\}$$

$$- \frac{q_1^2}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right)^2 (F_C^p + \frac{q^2}{8M_N^2} F_M^p) (F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$- \frac{3q_3}{20M_N^2} \frac{\omega^2}{r^2} (F_M^p g_A^n - F_M^n g_A^p)$$

$$+ \{ (t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr) \} \frac{1}{4M_N^2} \{ (F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2 + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2 \}$$

$$\left(\frac{4}{3} A^2 + \frac{2B^2}{75} - \frac{4\sqrt{2}}{15} AB + \frac{12C^2}{175} \right)$$

$$- \{ (t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2 \} \frac{1}{2M_N^2} (F_C^p + \frac{q^2}{8M_N^2} F_M^p) (F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$\left(\frac{4}{3} A^2 + \frac{2}{75} B^2 - \frac{4\sqrt{2}}{15} AB + \frac{12}{175} C^2 \right)$$

$$- 4\pi j_2(qr) \gamma_{20}(\hat{q}) \frac{3}{140} \sqrt{\frac{5}{4\pi}} \frac{\omega^2}{r^2} \{ (t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 (g_A^{p2} + g_A^{n2}) \}$$

with

$$H11 = \frac{1}{4M_N^2} \{ q_2^2 \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) + \frac{3}{20} q_3^2 \frac{\omega^2}{r^2} \} \text{ and}$$

$$H12 = \frac{1}{8M_N^2} \frac{\omega}{r} \left\{ \frac{2}{5} \left(2D - \frac{\sqrt{2}}{r} \frac{\partial^2 u}{\partial r^2} + \frac{3\sqrt{2}}{r} A \right) + \frac{3q_1^2}{10} \frac{\omega}{r} \right\}$$

$$H^{22} (\text{np singlet}) = \int r^2 dr x$$

$$\{(t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)\} \{(F_M^{p2} + F_M^{n2}) H13 + \frac{1}{2} (g_A^{p2} + g_A^{n2}) \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2}\right)\}$$

$$+ (g_A^{p2} + g_A^{n2}) H14$$

$$+ \frac{1}{2M_N} q_3 (F_M^p g_A^p - F_M^n g_A^n) \left(\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2}\right)$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2\} \{2F_M^p F_M^n H13 - g_A^p g_A^n \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2}\right)\}$$

$$- 2g_A^p g_A^n H14 - \frac{q_3}{2M_N} (F_M^p g_A^n - F_M^n g_A^p) \left(\frac{u^2}{r^2} - \frac{\omega^2}{2r^2}\right)$$

$$- 2\pi j_2(qr) \left\{ \frac{2}{7} \sqrt{\frac{5}{4\pi}} \frac{\omega^2}{r^2} \left[\sqrt{\frac{3}{50}} (\gamma_{22}(\hat{q}) + \gamma_{2-2}(\hat{q})) + \frac{1}{2} \gamma_{20}(\hat{q}) \right] - \frac{1}{\sqrt{10\pi}} \frac{u\omega}{r^2} \right.$$

$$\left. \left[\sqrt{\frac{3}{2}} (\gamma_{22}(\hat{q}) + \gamma_{2-2}(\hat{q})) + \gamma_{20}(\hat{q}) \right] \right\}$$

$$x \{(t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 (g_A^{p2} + g_A^{n2})\}$$

with

$$H13 = \frac{1}{8M_N^2} \{q_3^2 \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2}\right) + \frac{3}{5} q_1^2 \frac{\omega^2}{r^2}\} \text{ and}$$

$$H14 = \frac{1}{2\sqrt{2} M_N^2} \left\{ \frac{u}{r} \left(\frac{7}{15} E + \frac{2\sqrt{2}}{3} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} \right) + \frac{7}{15\sqrt{2}} \frac{\omega}{r} \left(D + \sqrt{2} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} - \frac{3\sqrt{2}}{r} A \right) \right. \\ \left. + \frac{q_2^2}{2\sqrt{2}} \left(\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2} \right) \right\}$$

$$H^{22} (\text{np triplet, } m=-1) = \int r^2 dr \times$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \left\{ (F_M^p)^2 + F_M^{n2} \right\} \frac{3}{20M_N^2} \frac{\omega^2}{r^2} (q_1^2 + \frac{q_3^2}{4})$$

$$+ \frac{3}{20} (g_A^p)^2 + g_A^{n2} \frac{\omega^2}{r^2} + \frac{3}{20} \frac{q_3}{M} \frac{\omega^2}{r^2} (F_M^p g_A^p - F_M^n g_A^n)$$

$$+ \frac{3}{20M_N^2} (g_A^p)^2 + g_A^{n2} \frac{\omega}{r} \left(2D - \frac{q_2^2}{2} \frac{\omega}{r} \right) + \frac{3q_2^2}{20M_N^2} \frac{\omega^2}{r^2}$$

$$\left[F_C^p + \frac{q^2}{8M_N^2} F_M^p \right]^2 + \left(F_C^n + \frac{q^2}{8M_N^2} F_M^n \right)^2 \}$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2\} \left\{ \frac{3}{10} g_A^p g_A^n \frac{\omega^2}{r^2} - \frac{3}{10M_N^2} F_M^p F_M^n \frac{\omega^2}{r^2} (q_1^2 + \frac{q_3^2}{4}) \right.$$

$$+ \frac{3q_3}{20M_N^2} \frac{\omega^2}{r^2} (F_M^p g_A^n - F_M^n g_A^p) \}$$

$$+ \frac{3}{20M_N^2} g_A^p g_A^n \frac{\omega}{r} \left(2D - \frac{q_2^2}{2} \frac{\omega}{r} \right) - \frac{3q_2^2}{10M_N^2} \frac{\omega^2}{r^2} \left(F_C^p + \frac{q^2}{8M_N^2} F_M^p \right)$$

$$(F_c^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$+ \{(t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)\} \frac{1}{4M_N^2} (\frac{12}{25} B^2 + \frac{96}{175} C^2) \{(F_c^p + \frac{q^2}{8M_N^2} F_M^p)^2$$

$$+ (F_c^n + \frac{q^2}{8M_N^2} F_M^n)^2$$

$$- \{(t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2\} \frac{1}{2M_N^2} (\frac{12}{25} B^2 + \frac{96}{175} C^2) (F_c^p + \frac{q^2}{8M_N^2} F_M^p)$$

$$(F_c^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$-4\pi j_2(qr) \frac{3}{140} \sqrt{\frac{5}{4\pi}} \frac{\omega^2}{r^2} Y_{20}(\hat{q}) [(t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 (g_A^{p2} + g_A^{n2})]$$

$$\underline{H^{22} (np \text{ triplet, } m=0) = \int r^2 dr \times}$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \{\frac{1}{8M_N^2} q_3^2 (F_M^{p2} + F_M^{n2}) (\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2})$$

$$+ (g_A^{p2} + g_A^{n2}) \frac{1}{2} (\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2})$$

$$+ \frac{q_3}{2M_N} (F_M^p g_A^p - F_M^n g_A^n) (\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2}) - \frac{1}{4\sqrt{2} M_N^2} (g_A^{p2} + g_A^{n2}) H15$$

$$+ \frac{3}{40M_N^2} q_2^2 \frac{\omega^2}{r^2} \left[(F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2 + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2 \right]$$

$$+ \{ (t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2 \} \{ g_A^p g_A^n (\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2}) - \frac{1}{4M_N^2} q_3^2 F_M^p F_M^n$$

$$(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2})$$

$$+ \frac{q_3}{2M_N} (F_M^p g_A^n - F_M^n g_A^p) (\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2}) - \frac{1}{2\sqrt{2} M_N^2} g_A^p g_A^n \} H15$$

$$- \frac{3}{20M_N^2} q_2^2 \frac{\omega^2}{r^2} (F_C^p + \frac{q^2}{8M_N^2} F_M^p) (F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$+ \{ (t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr) \} \frac{1}{4M_N^2} (\frac{3}{25} B^2 + \frac{39}{175} C^2) \{ (F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2$$

$$+ (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2 \}$$

$$- \{ (t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2 \} \frac{1}{2M_N^2} (\frac{3}{25} B^2 + \frac{39}{175} C^2) (F_C^p + \frac{q^2}{8M_N^2} F_M^p)$$

$$(F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$-2\pi j_2 (qr) \{2g_A^p g_A^n (t_1^2 + t_2^2) - 2t_1 t_2 (g_A^{p2} + g_A^{n2})\} \left\{ \frac{2}{7} \sqrt{\frac{5}{4\pi}} \frac{\omega^2}{r^2} \right.$$

$$\left[\sqrt{\frac{3}{50}} (Y_{22}(\hat{q}) + Y_{2-2}(\hat{q})) - \frac{1}{2} Y_{20}(\hat{q}) \right]$$

$$- \frac{1}{\sqrt{10\pi}} \frac{u\omega}{r^2} \left[\sqrt{\frac{3}{2}} (Y_{22}(\hat{q}) + Y_{2-2}(\hat{q})) - Y_{20}(\hat{q}) \right]$$

with

$$H15 = \frac{u}{r} \left(\frac{2}{3} E - \frac{4\sqrt{2}}{3} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} \right) - \frac{q_2^2}{\sqrt{2}} \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right)$$

$$- \frac{2}{15} \frac{\omega}{r} \left(\frac{1}{r} \frac{\partial^2 u}{\partial r^2} - \frac{3A}{r} + 2\sqrt{2} D \right) + \frac{4}{5} \frac{\omega}{r} \left(\frac{1}{r} \frac{\partial^2 u}{\partial r^2} - \frac{3A}{r} - \sqrt{2} D \right)$$

$$\underline{H^{22} (np \text{ triplet, } m=+1)} = -\int r^2 dr x$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \{ (F_M^{p2} + F_M^{n2}) H16 + \frac{3}{20} \frac{\omega^2}{r^2} (g_A^{p2} + g_A^{n2})$$

$$- \frac{3}{20} \frac{q_3}{M_N} \frac{\omega^2}{r^2} (F_M^p g_A^p - F_M^n g_A^n)$$

$$+ (g_A^{p2} + g_A^{n2}) H17 + \frac{q_2^2}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \left[(F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2 \right.$$

$$\left. + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2 \right]$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2\} \left\{ \frac{3}{10} \frac{\omega^2}{r^2} g_A^p g_A^n - 2F_M^p F_M^n H16 - \frac{3}{20} \frac{q_3^2 \omega^2}{M_N r^2} \right.$$

$$\left. (F_M^p g_A^n - F_M^n g_A^p) + 2g_A^p g_A^n H17 \right.$$

$$\left. - \frac{1}{2M_N^2} q_2^2 \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \left(F_C^p + \frac{q^2}{8M_N^2} F_M^p \right) \left(F_C^n + \frac{q^2}{8M_N^2} F_M^n \right) \right\}$$

$$+ \{(t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)\} \frac{1}{4M_N^2} \left(\frac{4}{3} A^2 + \frac{2B^2}{75} - \frac{4\sqrt{2}}{15} AB + \frac{12}{175} C^2 \right)$$

$$\left\{ \left(F_C^p + \frac{q^2}{8M_N^2} F_M^p \right)^2 + \left(F_C^n + \frac{q^2}{8M_N^2} F_M^n \right)^2 \right\}$$

$$- \{(t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2\} \frac{1}{2M_N^2} \left(\frac{4}{3} A^2 - \frac{2}{75} B^2 - \frac{4\sqrt{2}}{15} AB + \frac{12}{175} C^2 \right)$$

$$\left(F_C^p + \frac{q^2}{8M_N^2} F_M^p \right) \left(F_C^n + \frac{q^2}{8M_N^2} F_M^n \right)$$

$$- 4\pi j_2(qr) Y_{20}(\hat{q}) \frac{3}{140} \sqrt{\frac{5}{4\pi}} \frac{\omega^2}{r^2} \{(t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 (g_A^{p2} + g_A^{n2})\}$$

with

$$H16 = \frac{1}{4M_N^2} \left\{ q_1^2 \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) + \frac{3}{20} q_3^2 \frac{\omega^2}{r^2} \right\} \text{ and}$$

$$H17 = \frac{1}{8M_N^2} \left\{ \frac{2}{5} \frac{\omega}{r} (2D - \sqrt{2} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} + \frac{3\sqrt{2} A}{r}) + \frac{3}{10} q_2^2 \frac{\omega^2}{r^2} \right\}$$

$$H^{33} (\text{np singlet}) = \int r^2 dr x$$

$$\{ (t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr) \} \{ (F_M^{p2} + F_M^{n2}) \frac{q^+ q^-}{8M_N^2} (\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2})$$

$$+ \frac{3}{10} (g_A^{p2} + g_A^{n2}) \frac{\omega^2}{r^2} + \frac{1}{2M_N^2} (g_A^{p2} + g_A^{n2}) H18 \}$$

$$+ \{ (t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2 \} \{ 2F_M^p F_M^n \frac{q^+ q^-}{8M_N^2} (\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2}) - \frac{3}{5} g_A^p g_A^n \frac{\omega^2}{r^2}$$

$$- \frac{1}{2M_N^2} 2g_A^p g_A^n H18 \}$$

$$+ 4\pi j_2(qr) Y_{20}(\hat{q}) \frac{3}{140} \sqrt{\frac{5}{4\pi}} \frac{\omega^2}{r^2} \{ (t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 (g_A^{p2} + g_A^{n2}) \}$$

with

$$H18 = \frac{3}{20} q_3^2 \frac{\omega^2}{r^2} + \frac{1}{5} \frac{\omega}{r} (D + \sqrt{2} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} - \frac{3\sqrt{2} A}{r})$$

$$H^{33}(\text{np triplet, } m=-1) = \int r^2 dr x$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \left\{ \frac{3}{80} \frac{q^+ q^-}{M_N^2} \frac{\omega^2}{r^2} (F_M^p{}^2 + F_M^n{}^2) + \frac{3}{5} (g_A^p{}^2 + g_A^n{}^2) H_{19} \right.$$

$$\left. + \frac{3}{20} \frac{\omega^2}{r^2} \frac{q_3^2}{M_N^2} \left[(F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2 + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2 \right] \right.$$

$$\left. + \frac{3}{5} \frac{q_3}{M_N} \frac{\omega^2}{r^2} \left[g_A^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - g_A^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \right] \right\}$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2\} \left\{ \frac{3}{5} 2g_A^p g_A^n H_{19} - \frac{3}{40} \frac{q^+ q^-}{M_N^2} \frac{\omega^2}{r^2} F_M^p F_M^n \right.$$

$$\left. - \frac{3}{10} \frac{\omega^2}{r^2} \frac{q_3^2}{M_N^2} (F_C^p + \frac{q^2}{8M_N^2} F_M^p) (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \right.$$

$$\left. - \frac{3}{5} \frac{q_3}{M_N} \frac{\omega^2}{r^2} \left[g_A^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n) - g_A^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p) \right] \right\}$$

$$+ \{(t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)\} \frac{3}{35} \frac{C^2}{M_N^2} \left[(F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2 + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2 \right]$$

$$- \{(t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2\} \frac{3}{35} \frac{C^2}{M_N^2} 2 (F_C^p + \frac{q^2}{8M_N^2} F_M^p) (F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$-4\pi j_2(qr) Y_{20}(\hat{q}) \frac{6}{35} \sqrt{\frac{5}{4\pi}} \frac{\omega^2}{r^2} \{ (t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 (g_A^{p2} + g_A^{n2}) \}$$

with

$$H_{19} = \frac{\omega^2}{r^2} + \frac{1}{M_N} \frac{\omega}{r} (D + \frac{q_3^2}{4})$$

$$H^{33}(\text{np triplet, } m=0) = \int r^2 dr \times$$

$$\{ (t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr) \} \{ \frac{q^+ q^-}{8M_N^2} (F_M^{p2} + F_M^{n2}) (\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2})$$

$$+ \frac{3}{40} \frac{q_3^2}{M_N^2} \frac{\omega^2}{r^2} [(F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2 + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2] \}$$

$$- \{ (t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2 \} \{ \frac{q^+ q^-}{8M_N^2} 2F_M^p F_M^n (\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2})$$

$$+ \frac{3}{20} \frac{q_3^2}{M_N^2} \frac{\omega^2}{r^2} (F_C^p + \frac{q^2}{8M_N^2} F_M^p) (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \}$$

$$+ \{ (t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr) \} \frac{1}{M_N^2} (\frac{3}{50} B^2 + \frac{12}{175} C^2) \{ (F_C^p + \frac{q^2}{8M_N^2})^2$$

$$+ (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2 \}$$

$$-\{(t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2\} \frac{1}{M_N^2} \left(\frac{3}{50} B^2 + \frac{12}{175} C^2 \right) 2 (F_C^p + \frac{q^2}{8M_N^2} F_M^p)$$

$$(F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$H^{33} (\text{np triplet, } m=+1) = \int r^2 dr \times$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \left\{ \frac{3q^+ q^-}{80M_N^2} (F_M^{p2} + F_M^{n2}) \frac{\omega^2}{r^2} + (g_A^{p2} + g_A^{n2}) \right.$$

$$\left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) + \frac{1}{2M_N^2} (g_A^{p2} + g_A^{n2}) H_{20}$$

$$+ \frac{q_3}{M_N} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) [g_A^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - g_A^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n)]$$

$$+ \frac{q_3^2}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) [(F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2 + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2]$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2\} \{2g_A^p g_A^n \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right)$$

$$- \frac{3}{40} \frac{q^+ q^-}{M_N^2} F_M^p F_M^n \frac{\omega^2}{r^2} + \frac{1}{2M_N^2} 2g_A^p g_A^n H_{20}$$

$$- \frac{q_3}{M_N} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \left[g_A^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n) - g_A^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p) \right]$$

$$- \frac{q_3^2}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) 2 (F_C^p + \frac{q^2}{8M_N^2} F_M^p) (F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$+ \{ (t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr) \} \frac{1}{M_N^2} \left\{ \frac{9C^2}{350} + \frac{1}{75} (5A + \sqrt{2} B)^2 \right\}$$

$$\{ (F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2 + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2 \}$$

$$- \{ (t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2 \} \frac{1}{M_N^2} \left\{ \frac{9C^2}{350} + \frac{1}{75} (5A + \sqrt{2} B)^2 \right\}$$

$$2 (F_C^p + \frac{q^2}{8M_N^2} F_M^p) (F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$- 4\pi j_2(qr) Y_{20}(\hat{q}) \left\{ \frac{1}{35} \sqrt{\frac{5}{4\pi}} \frac{\omega^2}{r^2} + \frac{1}{\sqrt{10\pi}} \frac{\omega\omega}{r^2} \right\} \{ (t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 (g_A^{p2} +$$

with

$$H_{20} = \frac{u}{r} \left(\frac{q_3^2}{4} + \frac{4}{3} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} + \frac{\sqrt{2} E}{5} \right) + \frac{q_3^2}{40} \frac{\omega^2}{r^2} - \frac{1}{15\sqrt{2}} \frac{\omega}{r} (\sqrt{2} D - \frac{4}{r} \frac{\partial^2 u}{\partial r^2} + \frac{12A}{r})$$

$$\underline{H^{01} \text{ (np singlet)}} = \int r^2 dr x$$

$$\{(t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)\} \{(g_A^p F_M^p + g_A^n F_M^n) \frac{1}{2\sqrt{2} M_N^2} q_3$$

$$[\frac{q^+}{2\sqrt{2}} (\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2}) - \frac{3}{20\sqrt{2}} (3q^- - q^+) \frac{\omega^2}{r^2}]$$

$$+ \frac{1}{\sqrt{2} M_N} (g_A^{p2} - g_A^{n2}) [\frac{3}{10\sqrt{2}} q^- \frac{\omega^2}{r^2} + \frac{q^+}{\sqrt{8}} (\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2})]\}$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2\} \{(g_A^p F_M^n + g_A^n F_M^p) \frac{1}{2\sqrt{2} M_N^2} q_3$$

$$[\frac{q^+}{2\sqrt{2}} (\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2}) - \frac{3}{20\sqrt{2}} (3q^- - q^+) \frac{\omega^2}{r^2}]\}$$

$$- 4\pi r j_0(qr) \frac{A - \sqrt{2} B}{3\sqrt{2} M_N} (2t_1 t_2) (g_A^{p2} - g_A^{n2}) [\frac{3}{20\pi} q^- \frac{\omega}{r} - \frac{1}{4\sqrt{2}} q^+ \frac{u}{r} + \frac{1}{40\pi} q^+ \frac{\omega}{r}]$$

$$\underline{H^{01} \text{ (np triplet, } m=-1)}} = \int r^2 dr x$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \{\frac{3}{10} \frac{q_1}{M_N} \frac{\omega^2}{r^2} [F_c^p (F_c^p + \frac{q^2}{8M_N^2} F_M^p)$$

$$- F_c^n (F_c^n + \frac{q^2}{8M_N^2} F_M^n)] - \frac{3}{20} \frac{(q^+ - q^-)}{M_N} \frac{\omega^2}{r^2} (F_c^p F_n^p - F_c^n F_M^n)$$

$$\begin{aligned}
& + \frac{3}{80} \frac{q_3}{M_N^2} (q^+ + q^-) \frac{\omega^2}{r^2} (F_C^P g_A^P + F_C^N g_A^N) - \frac{3}{40} \frac{q^+}{M_N^2} \frac{\omega^2}{r^2} (g_A^{P2} - g_A^{N2}) \\
& + \frac{3}{40} \frac{q_3(q^+ + q^-)}{M_N^2} \frac{\omega^2}{r^2} [(F_C^P + \frac{q^2}{8M_N^2} F_M^P) g_A^P + (F_C^N + \frac{q^2}{8M_N^2} F_M^P) g_A^N] \\
& - \frac{3}{80M_N^2} q_3 (3q^+ - 2q^-) \frac{\omega^2}{r^2} (F_M^P g_A^P + F_M^N g_A^N) \\
& + ((t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2) \{ \frac{3}{10} \frac{q_1}{M_N^2} \frac{\omega^2}{r^2} [F_C^N (F_C^P + \frac{q^2}{8M_N^2} F_M^P) \\
& - F_C^P (F_C^N + \frac{q^2}{8M_N^2} F_M^N)] - \frac{3}{20} \frac{(q^+ - q^-)}{M_N^2} \frac{\omega^2}{r^2} (F_C^N F_M^P - F_C^P F_M^N) \\
& + \frac{3}{80} \frac{q_3}{M_N^2} (q^+ + q^-) \frac{\omega^2}{r^2} (F_C^P g_A^N + F_C^N g_A^P) + \frac{3}{80} \frac{q_3(3q^+ - 2q^-)}{M_N^2} \\
& (F_M^P g_A^N + F_M^N g_A^P) \\
& - \frac{3}{40} \frac{q_3(q^+ + q^-)}{M_N^2} \frac{\omega^2}{r^2} [g_A^P (F_C^P + \frac{q^2}{8M_N^2} F_M^P) + g_A^N (F_C^P + \frac{q^2}{8M_N^2} F_M^P)] \} \\
& - 4\pi j_2(qr) Y_{21}(\hat{q}) \frac{3}{70} \sqrt{\frac{15}{2\pi}} \frac{\omega^2}{r^2} [(t_1^2 + t_2^2) (F_C^P g_A^N + F_C^N g_A^P) \\
& - 2t_1 t_2 (F_C^P g_A^P + F_C^N g_A^N)]
\end{aligned}$$

$$-4\pi j_0(qr) \frac{3}{200\pi M_N} \frac{\omega}{r} \left[\frac{c}{7} (q^+ + 15q^-) + 2q^+ B \right] [(t_1^2 + t_2^2) \{ F_C^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n) - F_C^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p) \} - 2t_1 t_2 \{ F_C^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - F_C^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \}]$$

$$- F_C^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p) \}$$

$$- 2t_1 t_2 \{ F_C^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - F_C^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \}]$$

$$+4\pi j_0(qr) 2t_1 t_2 \frac{3cq^+}{28\pi M_N} \frac{\omega}{r} (g_A^{p2} - g_A^{n2})$$

$$H^{01}(\text{np triplet } m=0) = \int r^2 dr \times$$

$$\{ (t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr) \} \{ (F_M^p g_A^{p2} + F_M^n g_A^{n2}) \frac{q_3}{4M_N^2} \left[\frac{q^+}{2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) - \frac{3}{10} q^- \frac{\omega^2}{r^2} \right] \right.$$

$$+ \frac{3(q^+ + q^-)}{40M_N} \frac{\omega^2}{r^2} [F_C^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - F_C^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n)]$$

$$+ \frac{g_A^{p2} - g_A^{n2}}{2M_N} \left[\frac{q^+}{2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) + \frac{3}{10} q^- \frac{\omega^2}{r^2} \right] \}$$

$$+ \{ (t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2 \} \{ \frac{3}{40M_N} (q^+ + q^-) \frac{\omega^2}{r^2} [(F_C^p + \frac{q^2}{8M_N^2} F_M^p) F_C^n - F_C^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n)]$$

$$- F_C^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \}$$

$$\begin{aligned}
& - (F_M^p g_A^n + F_M^n g_A^p) \frac{q_3}{4M_N^2} \left[\frac{q^+}{2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) - \frac{3}{10} q^- \frac{\omega^2}{r^2} \right] \\
& + 4\pi j_2(qr) \sqrt{\frac{3}{20}} \frac{\omega}{r} \left[\frac{3}{7\sqrt{2}\pi} \frac{\omega}{r} Y_{2-1}(\hat{q}) - \frac{1}{\sqrt{4\pi}} \frac{u}{r} Y_{21}(\hat{q}) - \frac{1}{14\sqrt{2}\pi} \frac{\omega}{r} Y_{21}(\hat{q}) \right] \\
& \times [(t_1^2 + t_2^2) (F_C^p g_A^n + F_C^n g_A^p) - 2t_1 t_2 (F_C^p g_A^p + F_C^n g_A^n)] \\
& + 4\pi j_0(qr) \frac{3}{40\pi M_N} \frac{\omega}{r} \left[(q^- + \frac{3q^+}{10}) \frac{c}{7} + \frac{q^+ B}{10} \right] [(t_1^2 + t_2^2) \{ F_C^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \\
& - F_C^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p) \} \\
& - 2t_1 t_2 \{ F_C^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - F_C^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \}] \\
& - 2\pi j_0(qr) 2t_1 t_2 (g_A^{p2} - g_A^{n2}) \frac{B+\sqrt{2}A}{12\pi M_N} \left[\frac{u}{r} \frac{q^+}{\sqrt{2}} - \frac{q^+}{10} \frac{\omega}{r} + \frac{3}{5} \frac{\omega}{r} q^- \right]
\end{aligned}$$

$$H^{01}(\text{np triplet, } m=+1) = \int r^2 dr \times$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \left\{ \frac{q^+ - q^-}{4M_N} (F_C^p F_M^p - F_C^n F_M^n) \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \right\}$$

$$- \frac{q_3(q^+ + q^-)}{16M_N^2} (F_C^p g_A^p + F_C^n g_A^n) \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right)$$

$$+ \frac{3q^-}{40M_N} (g_A^{p2} - g_A^{n2}) + \frac{q^+ + q^-}{4M_N} [F_C^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p)$$

$$- F_C^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n)] \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right)$$

$$- \frac{q_3 q_1}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) [g_A^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) + g_A^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n)]$$

$$- \frac{q_3}{2M_N^2} (F_M^p g_A^p + F_M^n g_A^n) \left[\frac{3}{40} q^- \frac{\omega^2}{r^2} + \frac{q^+ - q^-}{4} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \right]$$

$$+ \{ (t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2 \} \left(\frac{q^+ - q^-}{4M_N} (F_M^p F_C^n - F_M^n F_C^p) \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \right.$$

$$- \frac{q_3 (q^+ + q^-)}{16M_N^2} (F_C^p g_A^n + F_C^n g_A^p) \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right)$$

$$+ \frac{q^+ + q^-}{4M_N} [F_C^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - F_C^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n)] \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right)$$

$$+ \frac{q_3 q_1}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) [g_A^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n) + g_A^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p)]$$

$$+ \frac{q_3}{2M_N^2} (F_M^p g_A^n + F_M^n g_A^p) \left[\frac{3}{40} q^- \frac{\omega^2}{r^2} + \frac{q^+ - q^-}{4} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \right]$$

$$+ 2\pi j_2(qr) \sqrt{\frac{3}{20\pi}} Y_{2-1}(\hat{q}) \frac{\omega}{r} \left[\frac{u}{r} + \frac{1}{7\sqrt{2}} \frac{\omega}{r} \right] [(t_1^2 + t_2^2) (F_C^p g_A^n + F_C^n g_A^p)$$

$$- 2t_1 t_2 (F_C^p g_A^p + F_C^n g_A^n)].$$

$$+ 4\pi j_0(qr) \frac{q_1}{12\pi M_N} \left[\frac{u}{r} \left(A - \frac{B}{5\sqrt{2}} \right) + \frac{3c}{175} \frac{\omega}{r} \right] [(t_1^2 + t_2^2) \{ F_C^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$- F_C^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p) \}$$

$$- 2t_1 t_2 \{ F_C^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - F_C^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \}]$$

$$- 2\pi j_0(qr) 2t_1 t_2 (g_A^{p2} - g_A^{n2}) \frac{q^-}{40\pi M_N} \frac{\omega}{r} (B + \sqrt{2} A)$$

$$\underline{H^{02}(\text{np singlet})} = \int r^2 dr x$$

$$\{(t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)\} \left\{ - \frac{iq_3}{8M_N^2} (g_A^p F_M^p + g_A^n F_M^n) \left[q^+ \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \right. \right.$$

$$- \frac{3}{10} (q^+ - q^-) \frac{\omega^2}{r^2}]$$

$$- \frac{i}{4M_N} (g_A^{p2} - g_A^{n2}) \left[\frac{q^+}{2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) - \frac{3}{5} q^- \frac{\omega^2}{r^2} \right]$$

$$+ \{ (t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2 \} \left\{ - \frac{i q_3}{8M_N^2} (g_A^p F_M^n + g_A^n F_M^p) \left[q^+ \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \right. \right.$$

$$\left. - \frac{3}{10} (q^+ - q^-) \frac{\omega^2}{r^2} \right\}$$

$$- 4\pi r j_0(qr) 2t_1 t_2 (g_A^{p2} - g_A^{n2}) \frac{A - \sqrt{2} B}{12\pi M_N} \left(\frac{3}{5\sqrt{2}} q^- \frac{\omega}{r} + \frac{q^+}{2} \frac{u}{r} - \frac{q^+}{10\sqrt{2}} \frac{\omega}{r} \right)$$

$$H^{02}(\text{np triplet, } m=-1) = \int r^2 dr x$$

$$\{ (t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr) \} \left\{ \frac{3iq_1}{10M_N^2} \frac{\omega^2}{r^2} (F_C^p F_M^p - F_C^n F_M^n) + \frac{3q_2}{10M_N^2} \frac{\omega^2}{r^2} \right.$$

$$\left[F_C^p \left(F_C^p + \frac{q^2}{8M_N^2} F_M^p \right) - F_C^n \left(F_C^n + \frac{q^2}{8M_N^2} F_M^n \right) \right]$$

$$+ \frac{3iq^+}{40M_N^2} \frac{\omega^2}{r^2} (g_A^{p2} - g_A^{n2}) + \frac{3q_2 q_3}{40M_N^2} \frac{\omega^2}{r^2} (F_C^p g_A^p + F_C^n g_A^n)$$

$$+ \frac{3iq_3}{80M_N^2} (3q^+ + 2q^-) \frac{\omega^2}{r^2} (g_A^p F_M^p + g_A^n F_M^n)$$

$$+ \frac{3q_3 q_2}{20M_N} \frac{\omega^2}{r^2} \left[g_A^p (F_C^p + \frac{q^2}{8M_N} F_M^p) + g_A^n (F_C^n + \frac{q^2}{8M_N} F_M^n) \right]$$

$$+ \{ (t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2 \} \left\{ \frac{3iq_1}{10M_N} \frac{\omega^2}{r^2} (F_C^n F_M^p - F_C^p F_M^n) \right.$$

$$+ \frac{3q_2}{10M_N} \frac{\omega^2}{r^2} [F_C^n (F_C^p + \frac{q^2}{8M_N} F_M^p) - F_C^p (F_C^n + \frac{q^2}{8M_N} F_M^n)]$$

$$+ \frac{3q_2 q_3}{40M_N} \frac{\omega^2}{r^2} (F_C^p g_A^n + F_C^n g_A^p) - \frac{3iq_3}{80M_N} (3q^+ + 2q^-) \frac{\omega^2}{r^2}$$

$$(g_A^p F_M^n + g_A^n F_M^p)$$

$$- \frac{3q_2 q_3}{20M_N} \frac{\omega^2}{r^2} \left[g_A^p (F_C^n + \frac{q^2}{8M_N} F_M^n) + g_A^n (F_C^p + \frac{q^2}{8M_N} F_M^p) \right]$$

$$+ 2\pi i j_2(qr) Y_{21}(\hat{q}) \frac{3}{7} \sqrt{\frac{3}{10\pi}} \frac{\omega^2}{r^2} [(t_1^2 + t_2^2) (F_C^p g_A^n + F_C^n g_A^p)$$

$$- 2t_1 t_2 (F_C^p g_A^p + F_C^n g_A^n)]$$

$$+ 2\pi i r j_0(qr) \frac{3}{100\pi M_N} \frac{\omega}{r} [28q^+ + \frac{c}{7} (q^+ - 15q^-)] [(t_1^2 + t_2^2)$$

$$(F_C^p (F_C^n + \frac{q^2}{8M_N} F_M^n) - F_C^n (F_C^p + \frac{q^2}{8M_N} F_M^p))$$

$$- 2t_1 t_2 \left\{ F_C^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - F_C^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \right\}$$

$$+ 4\pi i r j_0(qr) 2t_1 t_2 \frac{3q^+ C}{140\pi M_N} \frac{\omega}{r} (g_A^{p2} - g_A^{n2})$$

$$H^{02}(\text{np triplet, } m=0) = \int r^2 dr \times$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \left\{ \frac{3q_2}{20M_N} \frac{\omega^2}{r^2} [F_C^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - F_C^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n)] \right.$$

$$\left. - \frac{i}{4M_N} (g_A^{p2} - g_A^{n2}) [q^+ (\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2}) - \frac{3}{5} q^- \frac{\omega^2}{r^2}] \right.$$

$$\left. + \frac{i q_3}{8M_N^2} (F_M^p g_A^p + F_M^n g_A^n) [q^+ (\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2}) + \frac{3}{5} q^- \frac{\omega^2}{r^2}] \right\}$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2\} \left\{ \frac{3q_2}{20M_N} \frac{\omega^2}{r^2} [F_C^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p) \right.$$

$$\left. - F_C^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \right]$$

$$+ \frac{i q_3}{8M_N^2} (F_M^p g_A^n + F_M^n g_A^p) [q^+ (\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2}) + \frac{3}{5} q^- \frac{\omega^2}{r^2}] \}$$

$$+2\pi i j_2(qr) \sqrt{\frac{3}{10\pi}} \frac{\omega}{r} \left[\frac{u}{r\sqrt{2}} Y_{21}(\hat{q}) + \frac{1}{7} \frac{\omega}{r} Y_{21}(\hat{q}) + \frac{3}{7} \frac{\omega}{r} Y_{2-1}(\hat{q}) \right]$$

$$\times [(t_1^2 + t_2^2) (F_C^p g_A^n + F_C^n g_A^p) - 2t_1 t_2 (F_C^p g_A^p + F_C^n g_A^n)]$$

$$-4\pi i r j_0(qr) \frac{3}{40\pi M_N} \frac{\omega}{r} \left[\frac{c}{7} (q^- - \frac{3}{10} q^+) - \frac{B}{10} q^+ \right]$$

$$\times [(t_1^2 + t_2^2) \{F_C^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n) - F_C^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p)\}$$

$$- 2t_1 t_2 \{F_C^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - F_C^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n)\}]$$

$$+4\pi i r j_0(qr) 2t_1 t_2 \frac{B+\sqrt{2}A}{24\pi M_N} \left[\frac{1}{\sqrt{2}} \frac{u}{r} q^+ - \frac{1}{10} \frac{\omega}{r} q^+ - \frac{3}{5} q^- \frac{\omega}{r} \right] (g_A^{p2} - g_A^{n2})$$

$$H^{02}(\text{np triplet } m=+1) = \int r^2 dr \times$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \left\{ \frac{-iq_1}{2M_N} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) (F_C^p F_M^p - F_C^n F_M^n) \right.$$

$$\left. - \frac{q_2 q_3}{8M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) (F_C^p g_A^p + F_C^n g_A^n) \right\}$$

$$\begin{aligned}
 & + \frac{q_2}{2M_N} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) [F_c^p (F_c^p + \frac{q^2}{8M_N^2} F_M^p) - F_c^n (F_c^n + \frac{q^2}{8M_N^2} F_M^n)] \\
 & + \frac{3iq^-}{40M_N} \frac{\omega^2}{r^2} (g_A^{p2} - g_A^{n2}) \\
 & + \frac{iq_3}{4M_N^2} [q_1 \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) - \frac{3}{20} q^- \frac{\omega^2}{r^2}] (g_A^p F_M^p + g_A^n F_M^n) \\
 & - \frac{q_3 q_2}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) [g_A^p (F_c^p + \frac{q^2}{8M_N^2} F_M^p) + g_A^n (F_c^n + \frac{q^2}{8M_N^2} F_M^n)] \\
 & + [(t_1^2 + t_2^2) (j_0(qr) - qt_1 t_2) \{ \frac{-iq_1}{2M_N} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) (F_c^n F_M^p - F_c^p F_M^n) \\
 & - \frac{q_2 q_3}{8M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) (F_c^p g_A^n + F_c^n g_A^p) \\
 & + \frac{q_2}{2M_N} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) [F_c^n (F_c^p + \frac{q^2}{8M_N^2} F_M^p) - F_c^p (F_c^n + \frac{q^2}{8M_N^2} F_M^n)] \\
 & - \frac{iq_3}{4M_N^2} [q_1 \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) - \frac{3}{20} q^- \frac{\omega^2}{r^2}] (g_A^p F_M^n + g_A^n F_M^p) \\
 & + \frac{q_3 q_2}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) [g_A^p (F_c^n + \frac{q^2}{8M_N^2} F_M^n) + g_A^n (F_c^p + \frac{q^2}{8M_N^2} F_M^p)] \}
 \end{aligned}$$

$$+2\pi i j_2(qr) \gamma_{2-1}(\hat{q}) \sqrt{\frac{3}{10\pi}} \frac{\omega}{r} \left(\frac{u}{r\sqrt{2}} + \frac{\omega}{7r} \right) [(t_1^2 + t_2^2) (F_c^p g_A^n + F_c^n g_A^p)$$

$$- 2t_1 t_2 (F_c^p g_A^p + F_c^n g_A^n)]$$

$$+4\pi r j_0(qr) \frac{q_2}{12\pi M_N} \left[\left(A - \frac{B}{5\sqrt{2}} \right) \left(\frac{u}{r} + \frac{\sqrt{2}\omega}{10r} \right) + \frac{9c}{175} \frac{\omega}{r} \right] [(t_1^2 + t_2^2) \{ F_c^p (F_c^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$- F_c^n (F_c^p + \frac{q^2}{8M_N^2} F_M^p) \}$$

$$- 2t_1 t_2 \{ F_c^p (F_c^p + \frac{q^2}{8M_N^2} F_M^p) - F_c^n (F_c^n + \frac{q^2}{8M_N^2} F_M^n) \}]$$

$$-2\pi r j_0(qr) 2t_1 t_2 \frac{q}{40\pi M_N} \frac{\omega}{r} (B + \sqrt{2} A) (g_A^{p2} - g_A^{n2})$$

$$H^{03}(\text{np singlet}) = \int r^2 dr x$$

$$\{ (t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr) \} \{ -\frac{q^+ q^-}{8M_N^2} \left(\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2} \right) (g_A^p F_M^p + g_A^n F_M^n) \}$$

$$+ \{ (t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2 \} \{ \frac{q^+ q^-}{8M_N^2} \left(\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2} \right) (g_A^p F_M^n + g_A^n F_M^p) \}$$

$$-4\pi r j_0(qr) 2t_1 t_2 \frac{q_3}{M_N} \frac{\omega}{r} \frac{A - \sqrt{2} B}{20\pi\sqrt{2}} (g_A^{p2} - g_A^{n2})$$

$$H^{03}(\text{np triplet, } m=-1) = \int r^2 dr \times$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \left\{ \frac{3}{20} \frac{q_3}{M_N} \frac{\omega^2}{r^2} [F_C^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - F_C^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \right.$$

$$+ \frac{3}{5} \frac{\omega^2}{r^2} (F_C^p g_A^p + F_C^n g_A^n) \cdot$$

$$+ \frac{3}{10M_N^2} (F_C^p g_A^p + F_C^n g_A^n) \frac{\omega}{r} (D + \frac{q_3^2}{4} \frac{\omega}{r}) - \frac{3q^+ q^-}{90M_N^2} \frac{\omega^2}{r^2} (F_M^p g_A^p + F_M^n g_A^n)$$

$$+ \frac{3q_3^2}{20M_N^2} [g_A^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) + g_A^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n)]$$

$$+ \frac{q_3}{10M_N} \frac{\omega^2}{r^2} (g_A^{p2} - g_A^{n2}) \}$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2\} \left\{ \frac{3}{20} \frac{q_3}{M_N} \frac{\omega^2}{r^2} [F_C^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p) \right.$$

$$- F_C^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n)] + \frac{3}{5} \frac{\omega^2}{r^2} (F_C^p g_A^n + F_C^n g_A^p)$$

$$+ \frac{3}{10M_N^2} (F_C^p g_A^n + F_C^n g_A^p) \frac{\omega}{r} (D + \frac{q_3^2}{4} \frac{\omega}{r}) + \frac{3q^+ q^-}{80M_N^2} \frac{\omega^2}{r^2} (F_M^p g_A^n + F_M^n g_A^p)$$

$$- \frac{3}{20} \frac{q_3^2}{M_N^2} [g_A^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n) - g_A^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p)] \}$$

$$+((t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)) \frac{6}{35} \frac{c^2}{M_N^2} [g_A^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) + g_A^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n)]$$

$$-((t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2) \frac{6}{35} \frac{c^2}{M_N^2} [g_A^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n) + g_A^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p)]$$

$$+4\pi j_2(qr) Y_{20}(\hat{q}) \frac{3}{7} \sqrt{\frac{1}{5\pi}} \frac{\omega^2}{r^2} [(t_1^2 + t_2^2) (F_C^p g_A^n + F_C^n g_A^p)$$

$$- 2t_1 t_2 (F_C^p g_A^p + F_C^n g_A^n)]$$

$$-4\pi j_0(qr) \frac{3cq_3}{70\pi M_N} \frac{\omega}{r} 2t_1 t_2 (g_A^{p2} - g_A^{n2})$$

$$\underline{H^{03} \text{ (np triplet, } m=0)} = \int r^2 dr \times$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \left\{ \frac{3}{20} \frac{q_3}{M_N} \frac{\omega^2}{r^2} [F_C^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - F_C^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n)] \right.$$

$$\left. + \frac{1}{20M_N} \frac{\omega}{r} (D + \sqrt{2} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} - \frac{3\sqrt{2} A}{r}) (F_C^p g_A^p + F_C^n g_A^n) \right\}$$

$$+((t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2) \left\{ \frac{3}{20} \frac{q_3}{M_N} \frac{\omega^2}{r^2} [F_C^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p) \right.$$

$$\left. - F_C^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n)] \right\}$$

$$+ \frac{1}{20M_N^2} \frac{\omega}{r} (D + \sqrt{2} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} - \frac{3\sqrt{2} A}{r}) (F_C^p g_A^n + F_C^n g_A^p)$$

$$- 4\pi r j_0(qr) \frac{3}{25\pi} \frac{q_3}{M_N} \frac{\omega}{r} (\frac{B}{8} + \frac{C}{7}) [(t_1^2 + t_2^2) \{F_C^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$- F_C^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p)\}$$

$$- 2t_1 t_2 \{F_C^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - F_C^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n)\}]$$

$$H^{03} (np \text{ triplet}, m=+1) = \int r^2 dr x$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \{ \frac{q_3}{2M_N} (\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2}) [F_C^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p)$$

$$- F_C^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n)] - (F_C^p g_A^p + F_C^n g_A^n) (\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2})$$

$$- \frac{1}{4M_N^2} (F_C^p g_A^p + F_C^n g_A^n) H_{21} + \frac{3q^+ q^-}{80M_N^2} \frac{\omega^2}{r^2} (F_M^p g_A^p + F_M^n g_A^n)$$

$$+ \frac{q_3}{2M_N} (g_A^{p2} - g_A^{n2}) (\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2})$$

$$\begin{aligned}
& - \frac{q_3^2}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \left[g_A^P (F_C^P + \frac{q^2}{8M_N^2} F_M^P) + g_A^N (F_C^N + \frac{q^2}{8M_N^2} F_M^N) \right] \\
& + \{ (t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2 \} \left\{ \frac{q_3}{2M_N} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) [F_C^N (F_C^P + \frac{q^2}{8M_N^2} F_M^P) \right. \\
& \quad \left. - F_C^P (F_C^N + \frac{q^2}{8M_N^2} F_M^N)] - (F_C^P g_A^N + F_C^N g_A^P) \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \right. \\
& \quad \left. - \frac{1}{4M_N^2} (F_C^P g_A^N + F_C^N g_A^P) H_2 \right\} - \frac{3q^+ q^-}{80M_N^2} \frac{\omega^2}{r^2} (F_M^P g_A^N + F_M^N g_A^P) \\
& \quad + \frac{q_3^2}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \left[g_A^P (F_C^N + \frac{q^2}{8M_N^2} F_M^N) + g_A^N (F_C^P + \frac{q^2}{8M_N^2} F_M^P) \right] \\
& - \{ (t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr) \} \frac{(B + \sqrt{2} A)(5A + \sqrt{2} B)}{15\sqrt{2} M_N^2} \left[g_A^P (F_C^P + \frac{q^2}{8M_N^2} F_M^P) \right. \\
& \quad \left. + g_A^N (F_C^N + \frac{q^2}{8M_N^2} F_M^N) \right] \\
& + \{ (t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2 \} \frac{(B + \sqrt{2} A)(5A + \sqrt{2} B)}{15\sqrt{2} M_N^2} \left[g_A^P (F_C^N + \frac{q^2}{8M_N^2} F_M^N) \right. \\
& \quad \left. + g_A^N (F_C^P + \frac{q^2}{8M_N^2} F_M^P) \right]
\end{aligned}$$

$$+4\pi j_2 (qr) \gamma_{20} (q) \frac{1}{\sqrt{10}\pi} \frac{\omega}{r} \left(\frac{u}{r} + \frac{1}{7\sqrt{2}} \frac{\omega}{r} \right) [(t_1^2 + t_2^2) (F_C^p g_A^n + F_C^n g_A^p)$$

$$- 2t_1 t_2 (F_C^p g_A^p + F_C^n g_A^n)]$$

$$-4\pi r j_0 (qr) \frac{q_3}{60\pi M_N} [(5A + \sqrt{2} B) \left(\frac{u}{r} + \frac{\sqrt{2}}{5} \frac{\omega}{r} \right) + \frac{27}{70} C \frac{\omega}{r}] [(t_1^2 + t_2^2) \{F_C^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$- F_C^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p)\}]$$

$$- 2t_1 t_2 \{F_C^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - F_C^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n)\}]$$

$$-4\pi r j_0 (qr) \frac{q_3}{M_N} \frac{B + \sqrt{2} A}{12\pi} \left(\frac{u}{r\sqrt{2}} + \frac{1}{5} \frac{\omega}{r} \right) 2t_1 t_2 (g_A^{p2} - g_A^{n2})$$

with

$$H_{21} = \frac{q_3^2}{4} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) + \frac{u}{r} \left(\frac{4}{3} \frac{1}{r} \frac{\partial^2 r}{\partial r^2} + \frac{\sqrt{2} E}{5} \right)$$

$$+ \frac{1}{15\sqrt{2}} \frac{\omega}{r} (\sqrt{2} D - \frac{4}{r} \frac{\partial^2 u}{\partial r^2} + \frac{12A}{r})$$

$$\underline{H^{12} (np \text{ singlet})} = \int r^2 dr x$$

$$\{(t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)\} \left\{ - \frac{F_M^{p2} + F_M^{n2}}{8M_N^2} H_{22} - \frac{1}{2} (g_A^{p2} + g_A^{n2}) \left(\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2} \right) \right\}$$

$$- \frac{iq_3}{2M_N} \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right) (F_M^p g_A^p - F_M^n g_A^n)$$

$$- \frac{1}{8M_N^2} (g_A^{p2} + g_A^{n2}) H_{23} \}$$

$$+ \{ (t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2 \} \left\{ \frac{2F_M^p F_M^n}{8M_N^2} H_{22} + i g_A^p g_A^n \left(\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2} \right) \right.$$

$$- \frac{iq_3}{2M_N} \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right) (F_M^n g_A^p - F_M^p g_A^n)$$

$$+ \frac{1}{4M_N^2} g_A^p g_A^n H_{23} \}$$

$$- 2\pi i j_2(qr) \left[\left(\frac{u\omega}{r^2} + \frac{1}{\sqrt{2}} \frac{\omega^2}{r^2} \right) \frac{1}{\sqrt{10\pi}} Y_{20}(\hat{q}) - \sqrt{\frac{3}{20\pi}} \left(\frac{\sqrt{2}}{7} \frac{\omega^2}{r^2} - \frac{u\omega}{r^2} \right) (Y_{22}(\hat{q}) + Y_{2-2}(\hat{q})) \right]$$

$$\times [(t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 (g_A^{p2} + g_A^{n2})]$$

with

$$H_{22} = \frac{3}{5} \frac{\omega^2}{r^2} q_1 q_2 + i q_3^2 \left(\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2} \right) \text{ and}$$

$$H_{23} = 2\sqrt{2} \frac{u}{r} \left(\frac{7}{15} E + \frac{2\sqrt{2}}{3} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} \right) - \frac{2}{3} \frac{\omega}{r} \left(D + \frac{\sqrt{2}}{r} \frac{\partial^2 u}{\partial r^2} - \frac{3\sqrt{2}}{r} A \right)$$

$$+ \frac{1}{4} (q^+ q^+ - q^- q^-) \left(\frac{u^2}{r^2} + \frac{7}{10} \frac{\omega^2}{r^2} \right)$$

$$+ \frac{1}{2} q^+ q^- \left(\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2} \right)$$

$$\underline{H^{12}_{np}} (np \text{ triplet, } m=-1) = \int r^2 dr \times$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \left\{ -\frac{3}{20M_N^2} (F_M^p{}^2 + F_M^n{}^2) \frac{\omega^2}{r^2} (q_1 q_2 + i \frac{q_3^2}{4}) \right.$$

$$- \frac{3i}{20} (g_A^p{}^2 + g_A^n{}^2) \frac{\omega^2}{r^2} - \frac{3iq_3}{20M_N^2} \frac{\omega^2}{r^2} (F_M^p g_A^p - F_M^n g_A^n)$$

$$+ \frac{3iq^+ q^-}{20M_N^2} \frac{\omega^2}{r^2} [F_M^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) + F_M^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n)]$$

$$- \frac{3i}{40M_N^2} (g_A^p{}^2 + g_A^n{}^2) H_{24}$$

$$+ \frac{3q_1 q_2}{20M_N^2} \frac{\omega^2}{r^2} [(F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2 + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2]$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2\} \left\{ \frac{3}{10M_N^2} F_M^p F_M^n \frac{\omega^2}{r^2} (q_1 q_2 + i \frac{q_3^2}{4}) \right.$$

$$- \frac{3i}{10} g_A^p g_A^n \frac{\omega^2}{r^2} - \frac{3iq_3}{20M_N^2} \frac{\omega^2}{r^2} (F_M^p g_A^n - F_M^n g_A^p)$$

$$- \frac{3iq^+ q^-}{20M_N^2} \frac{\omega^2}{r^2} [F_M^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n) + F_M^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p)]$$

$$- \frac{3i}{20M_N^2} g_A^p g_A^n H_{24}$$

$$- \frac{3iq_1 q_2}{10M_N^2} (F_C^p + \frac{q^2}{8M_N^2} F_M^p) (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \frac{\omega^2}{r^2}$$

$$+ [(t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)] \frac{3i}{25} (C^2 - B^2) [(F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2$$

$$+ (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2]$$

$$- [(t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2] \frac{6i}{25} (C^2 - B^2) (F_C^p + \frac{q^2}{8M_N^2} F_M^p) (F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$+ 4\pi i j_2(qr) \frac{3}{140} \sqrt{\frac{5}{4\pi}} Y_{20}(\hat{q}) \frac{\omega^2}{r^2} [(t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 (g_A^{p2} + g_A^{n2})]$$

with

$$H_{24} = \frac{\omega}{r} [2D + \frac{1}{8} \frac{\omega}{r} (+q^+ + q^- q^-)]$$

$$H^{12}(\text{np. triplet, } m=0) = \int r^2 dr x$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \{\frac{1}{2} (g_A^{p2} + g_A^{n2}) (\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2})$$

$$- \frac{1q_3^2}{8M_N^2} (F_M^{p2} + F_M^{n2}) (\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2})$$

$$\begin{aligned}
& + \frac{3q_1 q_2}{40M_N^2} \frac{\omega^2}{r^2} \left[(F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2 + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2 \right] \\
& - \frac{i}{32M_N^2} (g_A^{p2} + g_A^{n2}) H_{25} \} \\
& + \{ (t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2 \} \{ i g_A^p g_A^n (\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2}) + \frac{i q_3^2}{4M_N^2} F_M^p F_M^n (\frac{u^2}{r^2} - \frac{1}{2} \frac{\omega^2}{r^2}) \\
& - \frac{3q_1 q_2}{20M_N^2} \frac{\omega^2}{r^2} (F_C^p + \frac{q^2}{8M_N^2} F_M^p) (F_C^n + \frac{q^2}{8M_N^2} F_M^n) - \frac{i}{16M_N^2} g_A^p g_A^n H_{25} \} \\
& + \{ (t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr) \} \frac{3i}{100M_N^2} (C^2 - B^2) [F_C^p + \frac{q^2}{8M_N^2} F_M^p]^2 \\
& + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2 \} \\
& - \{ (t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2 \} \frac{3i}{50M_N^2} (C^2 - B^2) (F_C^p + \frac{q^2}{8M_N^2} F_M^p) (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \\
& - 2\pi i j_2(qr) [\frac{1}{\sqrt{10\pi}} \gamma_{20}(\hat{q}) \{ \frac{u\omega}{r^2} + \frac{1}{\sqrt{2}} \frac{\omega^2}{r^2} \} - \sqrt{\frac{3}{20\pi}} \{ \frac{u\omega}{r^2} - \frac{\sqrt{2}}{7} \frac{\omega^2}{r^2} \}] \\
& [(t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 (g_A^{p2} + g_A^{n2})]
\end{aligned}$$

with

$$H_{25} = \frac{3}{5} \frac{\omega^2}{r^2} (q^+ q^+ - q^- q^-) - \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) (q^+ q^+ - q^- q^-)$$

$$H_{12}^{12} (\text{np triplet, } m=+1) = \int r^2 dr \times$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \left\{ \frac{3i}{20} \frac{\omega^2}{r^2} (g_A^{p2} + g_A^{n2}) - \frac{1}{4M_N^2} (F_M^{p2} + F_M^{n2}) \right\} H_{26}$$

$$- \frac{3iq_3}{20M_N} \frac{\omega^2}{r^2} (F_M^p g_A^p - F_M^n g_A^n)$$

$$- \frac{iq^+ q^-}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) [F_M^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) + F_M^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n)]$$

$$+ \frac{1}{20M_N^2} (g_A^{p2} + g_A^{n2}) H_{27}$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2\} \left\{ \frac{3i}{10} \frac{\omega^2}{r^2} g_A^p g_A^n - \frac{1}{2M_N^2} F_M^p F_M^n \right\} H_{26}$$

$$- \frac{3iq_3}{20M_N} \frac{\omega^2}{r^2} (F_M^p g_A^n - F_M^n g_A^p)$$

$$+ \frac{iq^+ q^-}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) [F_M^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n) + F_M^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p)]$$

$$+ \frac{1}{10M_N^2} g_A^p g_A^n H_{27}$$

$$-4\pi j_2(qr) \frac{3}{140} \sqrt{\frac{5}{4\pi}} Y_{20}(\hat{q}) \frac{\omega^2}{r^2} [(t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 (g_A^{p2} + g_A^{n2})]$$

with

$$H_{26} = q_1 q_2 \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) - \frac{3i}{20} q_3^2 \frac{\omega^2}{r^2} \quad \text{and}$$

$$H_{27} = \frac{\omega}{r} \left[(2D - \sqrt{2} \frac{1}{r} \frac{\partial^2 u}{\partial r^2} + \frac{3\sqrt{2}}{r} A) - \frac{3}{2} \frac{\omega}{r} (q^+ q^+ - 2q^+ q^- - q^- q^-) \right]$$

$$H^{13}(\text{np singlet}) = \int r^2 dr x$$

$$\{(t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)\} \left\{ \frac{1}{4M_N} (F_M^p g_A^p - F_M^n g_A^n) H_{28} - \frac{1}{8M_N^2} (F_M^{p2} + F_M^{n2}) \right.$$

$$H_{29} - \frac{1}{16M_N^2} (g_A^{p2} + g_A^{n2}) H_{29} \}$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2\} \left\{ \frac{1}{4M_N} (F_M^n g_A^p - F_M^p g_A^n) H_{28} - \frac{1}{4M_N^2} F_M^p F_M^n H_{29} + \right.$$

$$\left. \frac{1}{8M_N^2} g_A^p g_A^n H_{29} \right\}$$

$$+ 2\pi j_2(qr) \sqrt{\frac{3}{20\pi}} \frac{\omega}{r} \left[\frac{3\sqrt{2}}{7} \frac{\omega}{r} Y_{21}(\hat{q}) + \left(\frac{u}{r} + \frac{1}{2\sqrt{2}} \frac{\omega}{r} \right) Y_{2-1}(\hat{q}) \right] [2t_1 t_2 (g_A^{p2} + g_A^{n2})$$

$$- (t_1^2 + t_2^2) 2g_A^p g_A^n]$$

with

$$H_{28} = \frac{9}{10} q^+ \frac{\omega^2}{r^2} - q^- \left(\frac{u^2}{r^2} + \frac{2}{5} \frac{\omega^2}{r^2} \right) \text{ and}$$

$$H_{29} = q_3 \left\{ q^- \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) + \frac{3}{5} q^+ \frac{\omega^2}{r^2} \right\}$$

$$H^{13} (\text{np triplet, } m=-1) = \int r^2 dr x$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \left\{ \frac{3q^+ q_3}{80M_N^2} \frac{\omega^2}{r^2} (F_M^{p2} + F_M^{n2}) + \frac{3q_1 q_3}{20M_N^2} \frac{\omega^2}{r^2} \right.$$

$$\left. \left[(F_C^p + \frac{q}{8M_N^2} F_M^p)^2 + (F_C^n + \frac{q}{8M_N^2} F_M^n)^2 \right] + \frac{3q_3}{160M_N^2} \frac{\omega^2}{r^2} (2q^+ + 3q^-) \right.$$

$$(g_A^{p2} + g_A^{n2})$$

$$+ \frac{3i}{20M_N^2} q_3 q_2 \frac{\omega^2}{r^2} [F_M^p (F_C^p + \frac{q}{8M_N^2} F_M^p) + F_M^n (F_C^n + \frac{q}{8M_N^2} F_M^n)]$$

$$+ \frac{3}{40M_N^2} \frac{\omega^2}{r^2} (3q^+ - 2q^-) (F_M^p g_A^p - F_M^n g_A^n)$$

$$+ \frac{3}{10} \frac{q_1}{M_N^2} \frac{\omega^2}{r^2} [g_A^p (F_C^p + \frac{q}{8M_N^2} F_M^p) - g_A^n (F_C^n + \frac{q}{8M_N^2} F_M^n)] \}$$

$$+[(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2] \left\{ -\frac{3q^+ q_3}{40M_N^2} \frac{\omega^2}{r^2} F_M^p F_M^n - \frac{3q_1 q_3}{10M_N^2} \frac{\omega^2}{r^2} \right.$$

$$\left. (F_C^p + \frac{q^2}{8M_N^2} F_M^p) (F_C^n + \frac{q^2}{8M_N^2} F_M^n) + \frac{3q_3}{80M_N^2} \frac{\omega^2}{r^2} (2q^+ + 3q^-) \cdot g_A^p g_A^n \right.$$

$$\left. - \frac{3i}{20M_N^2} q_3 q_2 \frac{\omega^2}{r^2} [F_M^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n) + F_M^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p)] \right.$$

$$\left. + \frac{3}{40M_N^2} \frac{\omega^2}{r^2} (3q^+ - 2q^-) (F_M^p g_A^n - F_M^n g_A^p) \right.$$

$$\left. + \frac{3q_1}{10M_N^2} \frac{\omega^2}{r^2} [g_A^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - g_A^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n)] \right\}$$

$$+ 4\pi j_2(qr) Y_{2-1}(\hat{q}) \frac{3}{70} \sqrt{\frac{15}{2\pi}} \frac{\omega^2}{r^2} [(t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 (g_A^{p2} + g_A^{n2})]$$

$$+ 4\pi r j_0(qr) \frac{3}{1400\pi M_N^2} \frac{\omega}{r} [(6q^+ + 15q^-) \cdot C + 14q^+ B] [(t_1^2 + t_2^2) (g_A^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$- g_A^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p))]$$

$$- 2t_1 t_2 \{ g_A^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - g_A^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \}]$$

$$H^{13}(\text{np triplet, } m=0) = \int r^2 dr x$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \left\{ \frac{3q_3 q_1}{40M_N^2} \frac{\omega^2}{r^2} \left[(F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2 + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2 \right] \right.$$

$$- \frac{1}{8M_N^2} (F_M^{p2} + F_M^{n2}) H_{29}$$

$$+ \frac{1}{16M_N^2} (g_A^{p2} + g_A^{n2}) H_{29} - \frac{1}{4M_N} (F_M^p g_A^p - F_M^n g_A^n) \left[q^- \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \right.$$

$$\left. - \frac{3}{5} q^+ \frac{\omega^2}{r^2} \right\}$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2\} \left\{ \frac{3q_3 q_1}{20M_N^2} \frac{\omega^2}{r^2} (F_C^p + \frac{q^2}{8M_N^2} F_M^p) (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \right.$$

$$+ \frac{1}{4M_N^2} F_M^p F_M^n H_{29}$$

$$+ \frac{1}{8M_N^2} g_A^p g_A^n H_{29} + \frac{1}{4M_N} (F_M^n g_A^p - F_M^p g_A^n) \left[q^- \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \right.$$

$$\left. - \frac{3}{5} q^+ \frac{\omega^2}{r^2} \right\}$$

$$+ 4\pi r j_0(qr) \frac{1}{20\pi M_N} \left[B \left(\frac{u}{r} \frac{q^-}{2\sqrt{2}} - \frac{q^-}{20} \frac{\omega}{r} + \frac{3}{10} q^+ \frac{\omega}{r} \right) - \frac{3C}{35} (q^+ - q^-) \frac{\omega}{r} \right]$$

$$x, [(t_1^2 + t_2^2) \{g_A^p (F_c^n + \frac{q^2}{8M_N^2} F_M^n) - g_A^n (F_c^p + \frac{q^2}{8M_N^2} F_M^p)\} \\ - 2t_1 t_2 \{g_A^p (F_c^p + \frac{q^2}{8M_N^2} F_M^p) - g_A^n (F_c^n + \frac{q^2}{8M_N^2} F_M^n)\}]$$

with

$$H_{29} = q_3 \{q^- (\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2}) + \frac{3}{5} q^+ \frac{\omega^2}{r^2}\}$$

$$H^{13} (\text{np triplet, } m=+1) = \int r^2 dr x$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \{ \frac{q_3}{4M_N^2} (g_A^{p2} + g_A^{n2}) [\frac{3}{40} q^+ \frac{\omega^2}{r^2} + \frac{q_1}{2} (\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2})]$$

$$- \frac{3q_3 q^+}{80M_N^2} (F_M^{p2} + F_M^{n2}) \frac{\omega^2}{r^2}$$

$$+ \frac{q_3 q_1}{4M_N^2} (\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2}) [(F_c^p + \frac{q^2}{8M_N^2} F_M^p)^2 + (F_c^n + \frac{q^2}{8M_N^2} F_M^n)^2]$$

$$+ \frac{(F_M^p g_A^p - F_M^n g_A^n)}{2M_N} [iq_2 (\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2}) + \frac{3q^+ \omega^2}{20r^2}]$$

$$- \frac{iq_2 q_3}{4M_N^2} (\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2}) [F_M^p (F_c^p + \frac{q^2}{8M_N^2} F_M^p) + F_M^n (F_c^n + \frac{q^2}{8M_N^2} F_M^n)]$$

$$\begin{aligned}
& - \frac{q_1}{2M_N} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \left[g_A^P (F_C^P + \frac{q^2}{8M_N^2} F_M^P) - g_A^N (F_C^N + \frac{q^2}{8M_N^2} F_M^N) \right] \\
& + [(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2] \left\{ \frac{q_3}{2M_N^2} g_A^P g_A^N \left[\frac{3}{40} q^2 + \frac{\omega^2}{r^2} + \frac{q_1}{2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \right] \right. \\
& \quad \left. + \frac{3q_3 q^+}{40M_N^2} F_M^P F_M^N \frac{\omega^2}{r^2} \right. \\
& \quad - \frac{q_3 q_1}{2M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) (F_C^P + \frac{q^2}{8M_N^2} F_M^P) (F_C^N + \frac{q^2}{8M_N^2} F_M^N) \\
& \quad + \frac{F_M^P g_A^N - F_M^N g_A^P}{2M_N} \left[i q_2 \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) + \frac{3q^+}{20} \frac{\omega^2}{r^2} \right] \\
& \quad \left. + \frac{i q_2 q_3}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \left[F_M^P (F_C^N + \frac{q^2}{8M_N^2} F_M^N) + F_M^N (F_C^P + \frac{q^2}{8M_N^2} F_M^P) \right] \right. \\
& \quad \left. - \frac{q_1}{2M_N} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \left[g_A^N (F_C^P + \frac{q^2}{8M_N^2} F_M^P) - g_A^P (F_C^N + \frac{q^2}{8M_N^2} F_M^N) \right] \right\} \\
& + 2\pi r j_0(qr) \frac{1}{12\pi M_N} \left[\frac{2u}{r} q_1 \left(A - \frac{B}{5\sqrt{2}} \right) + \frac{\omega}{r} \frac{1}{5\sqrt{2}} \left\{ A (4q^+ + q^-) + \frac{B}{5\sqrt{2}} (2q^+ - q^-) \right\} \right. \\
& \quad \left. - \frac{9c}{350} \frac{\omega}{r} (5q^+ + 2q^-) \right]
\end{aligned}$$

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$$\begin{aligned}
& \times [(t_1^2 + t_2^2) \{g_A^p (F_C^n + \frac{q^2}{8M_N^2} F_M^n) - g_A^n (F_C^p + \frac{q^2}{8M_N^2} F_M^p)\} \\
& - 2t_1 t_2 \{g_A^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) - g_A^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n)\}] \\
& + 2\pi j_2(qr) \sqrt{\frac{3}{20\pi}} \gamma_{21}(\hat{q}) \frac{\omega}{r} (\frac{u}{r} + \frac{1}{7\sqrt{2}} \frac{\omega}{r}) [(t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 \\
& (g_A^{p2} + g_A^{n2})]
\end{aligned}$$

$$H^{23}(\text{np singlet}) = \int r^2 dr \times$$

$$\begin{aligned}
& \{[(t_1^2 + t_2^2) + 2t_1 t_2 j_0(qr)] \{ \frac{iq_3}{32M_N^2} (g_A^{p2} + g_A^{n2}) H_{30} - \frac{iq_3}{8M_N^2} (F_M^{p2} + F_M^{n2}) H_{31} \\
& - \frac{i}{4M_N} (F_M^p g_A^p - F_M^n g_A^n) H_{32} \} \\
& + [(t_1^2 + t_2^2) j_0(qr) + 2t_1 t_2] \{ - \frac{iq_3}{16M_N^2} g_A^p g_A^n H_{30} + \frac{iq_3}{4M_N^2} F_M^p F_M^n H_{31} \\
& + \frac{i}{4M_N} (F_M^p g_A^n - F_M^n g_A^p) H_{32} \}
\end{aligned}$$

$$+ 4\pi i j_2(qr) \sqrt{\frac{3}{80\pi}} \frac{\omega}{r} [(\frac{u}{r} + \frac{1}{7\sqrt{2}} \frac{\omega}{r}) \gamma_{2-1}(\hat{q}) - \frac{3\sqrt{2}}{7} \frac{\omega}{r} \gamma_{21}(\hat{q})]$$

$$\times [2t_1 t_2 (g_A^{p2} + g_A^{n2}) - (t_1^2 + t_2^2) 2g_A^p g_A^n]$$

with

$$H30 = q^- \left(\frac{u^2}{r^2} + \frac{2}{5} \frac{\omega^2}{r^2} \right) - \frac{9}{10} q^+ \frac{\omega^2}{r^2} ;$$

$$H31 = q^- \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) - \frac{3}{5} q^+ \frac{\omega^2}{r^2} \text{ and}$$

$$H32 = q^- \left(\frac{u^2}{r^2} + \frac{2}{5} \frac{\omega^2}{r^2} \right) + \frac{9}{10} q^+ \frac{\omega^2}{r^2}$$

$$H^{23}(\text{np triplet, } m=-1) = \int r^2 dr x$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \left\{ \frac{3iq_3 q^+}{80M_N^2} \frac{\omega^2}{r^2} (F_M^{p2} + F_M^{n2}) + \frac{3q_3 q_2}{20M_N^2} \frac{\omega^2}{r^2} [F_C^p + \frac{q^3}{8M_N^2} F_M^p]^2 \right.$$

$$\left. + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2 \right\}$$

$$- \frac{3i}{40M_N^2} (2q^- + q^+) \frac{\omega^2}{r^2} (F_M^p g_A^p - F_M^n g_A^n) - \frac{3iq_3 q_1}{20M_N^2} \frac{\omega^2}{r^2}$$

$$[F_M^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) + F_M^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n)]$$

$$+ \frac{3iq_3}{160M_N^2} (3q^- - 2q^+) \frac{\omega^2}{r^2} (g_A^{p2} + g_A^{n2}) + \frac{3q_2}{10M_N^2} \frac{\omega^2}{r^2} [g_A^p (F_c^p + \frac{q^2}{8M_N^2} F_M^p) - g_A^n (F_c^n + \frac{q^2}{8M_N^2} F_M^n)]$$

$$+ [(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2] \{ \frac{3iq_3 q^+}{40M_N^2} \frac{\omega^2}{r^2} F_M^p F_M^n - \frac{3q_3 q_2}{10M_N^2} \frac{\omega^2}{r^2} (F_c^p + \frac{q^2}{8M_N^2} F_M^p) (F_c^n + \frac{q^2}{8M_N^2} F_M^n) \}$$

$$+ \frac{3i}{40M_N^2} (2q^- + q^+) \frac{\omega^2}{r^2} (F_M^p g_A^n - F_M^n g_A^p) + \frac{3iq_3 q_1}{20M_N^2} \frac{\omega^2}{r^2}$$

$$[F_M^p (F_c^n + \frac{q^2}{8M_N^2} F_M^n) + F_M^n (F_c^p + \frac{q^2}{8M_N^2} F_M^p)]$$

$$+ \frac{3iq_3}{80M_N^2} (3q^- - 2q^+) \frac{\omega^2}{r^2} g_A^p g_A^n + \frac{3q_2}{10M_N^2} \frac{\omega^2}{r^2} [g_A^n (F_c^p + \frac{q^2}{8M_N^2} F_M^p) - g_A^p (F_c^n + \frac{q^2}{8M_N^2} F_M^n)]$$

$$+2\pi i j_0(qr) \frac{3}{\pi M_N} \frac{\omega}{r} \left(\frac{3c}{350} q^- + \frac{Bq^-}{50} - \frac{3c}{140} q^+ \right) [(t_1^2 + t_2^2) \{g_A^p (F_c^n + \frac{q^2}{8M_N^2} F_M^n) - g_A^n (F_c^p + \frac{q^2}{8M_N^2} F_M^p)\}]$$

$$= 2t_1 t_2 \{g_A^p (F_c^p + \frac{q^2}{8M_N^2} F_M^p) - g_A^n (F_c^n + \frac{q^2}{8M_N^2} F_M^n)\}$$

$$+4i j_2(qr) Y_{2-1}(\hat{q}) \frac{3}{70} \sqrt{\frac{15}{2\pi}} \frac{\omega^2}{r^2} [(t_1^2 + t_2^2) 2g_A^p g_A^n] 2t_1 t_2 (g_A^{p2} + g_A^{n2})$$

$$H^{23}(\text{np triplet}, m=0) = \int r^2 dr \times$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \left\{ \frac{i q_3}{16M_N^2} (g_A^{p2} + g_A^{n2}) H_{31} + \frac{3q_3 q_2}{40M_N^2} \frac{\omega^2}{r^2} [(F_c^p + \frac{q^2}{8M_N^2} F_M^p)^2 \right.$$

$$+ (F_c^n + \frac{q^2}{8M_N^2} F_M^n)^2]$$

$$- \frac{i q_3}{8M_N^2} (F_M^{p2} + F_M^{n2}) H_{31} - \frac{i}{4M_N} (F_M^p g_A^p - F_M^n g_A^n) [q^- (\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2})$$

$$+ \frac{3}{5} q^+ \frac{\omega^2}{r^2}]\}$$

$$+((t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2) \left\{ \frac{i q_3}{8M_N^2} g_A^p g_A^n H_{31} - \frac{3q_3 q_2}{20M_N^2} (F_C^p + \frac{q^2}{8M_N^2} F_M^p) \right\}$$

$$(F_C^n + \frac{q^2}{8M_N^2} F_M^n)$$

$$- \frac{i q_3}{4M_N^2} F_M^p F_M^n H_{31} + \frac{i}{4M_N} (g_A^p F_M^n - g_A^n F_M^p) \left[q^- \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) + \frac{3}{5} q^+ \frac{\omega^2}{r^2} \right]$$

$$+ 4\pi i r j_0(qr) \frac{1}{40\pi M_N} \left[B \left(\frac{u}{r\sqrt{2}} q^- - \frac{1}{5} \frac{\omega}{r} (3q^+ + \frac{q^-}{2}) \right) + \frac{3c}{35} q_1 \frac{\omega}{r} \right]$$

$$\times \left[(t_1^2 + t_2^2) \left\{ g_A^p \left(F_C^n + \frac{q^2}{8M_N^2} F_M^n \right) - g_A^n + \frac{q^2}{8M_N^2} F_M^p \right\} \right.$$

$$\left. - 2t_1 t_2 \left\{ g_A^p \left(F_C^p + \frac{q^2}{8M_N^2} F_M^p \right) - g_A^n \left(F_C^n + \frac{q^2}{8M_N^2} F_M^n \right) \right\} \right]$$

with

$$H_{31} = q^- \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) - \frac{3}{5} q^+ \frac{\omega^2}{r^2}$$

$$H^{23} (\text{np triplet, } m=+1) = \int r^2 dr \times$$

$$\{(t_1^2 + t_2^2) - 2t_1 t_2 j_0(qr)\} \left\{ \frac{3iq_3 q^+}{80M_N^2} \frac{\omega^2}{r^2} (F_M^{p2} + F_M^{n2}) + \frac{q_3 q_2}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \right.$$

$$\left[(F_C^p + \frac{q^2}{8M_N^2} F_M^p)^2 + (F_C^n + \frac{q^2}{8M_N^2} F_M^n)^2 \right]$$

$$- \frac{i}{2M_N} (F_M^p g_A^p - F_M^n g_A^n) H_{33} - \frac{q_2}{2M_N} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \left[g_A^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) \right.$$

$$\left. - g_A^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \right]$$

$$+ \frac{iq_3 q_1}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \left[F_M^p (F_C^p + \frac{q^2}{8M_N^2} F_M^p) + F_M^n (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \right]$$

$$- \frac{1q_3}{8M_N^2} (g_A^{p2} + g_A^{n2}) H_{34} \}$$

$$+ \{(t_1^2 + t_2^2) j_0(qr) - 2t_1 t_2\} \left\{ - \frac{3i}{40M_N^2} q_3 q^+ \frac{\omega^2}{r^2} F_M^p F_M^n + \frac{q_3 q_2}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \right.$$

$$\left. (F_C^p + \frac{q^2}{8M_N^2} F_M^p) (F_C^n + \frac{q^2}{8M_N^2} F_M^n) \right\}$$

$$- \frac{i}{2M_N} (F_M^p g_A^n - F_M^n g_A^p) H_{33} + \frac{q_2}{2M_N} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \left[g_A^n \left(F_C^p + \frac{q^2}{8M_N^2} F_M^p \right) \right.$$

$$\left. - g_A^p \left(F_C^n + \frac{q^2}{8M_N^2} F_M^n \right) \right]$$

$$- \frac{i q_3 q_1}{4M_N^2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) \left[F_M^p \left(F_C^n + \frac{q^2}{8M_N^2} F_M^n \right) + F_M^n \left(F_C^p + \frac{q^2}{8M_N^2} F_M^p \right) \right]$$

$$- \frac{i q_3}{4M_N^2} g_A^p g_A^n H_{34} \}$$

$$- 2\pi i j_0(qr) \frac{1}{12\pi M_N} \left[(q^+ - q^-) \frac{u}{r} \left(A - \frac{B}{5\sqrt{2}} \right) + \frac{9c}{350} \frac{\omega}{r} (q^+ + 2q^-) + \frac{1}{50} \frac{\omega}{r} \right.$$

$$\left. \{ 5\sqrt{2} A (4q^+ + q^-) + B(5q^+ + q^-) \} \right]$$

$$- \times \left[(t_1^2 + t_2^2) \left\{ g_A^p \left(F_C^n + \frac{q^2}{8M_N^2} F_M^n \right) - g_A^n \left(F_C^p + \frac{q^2}{8M_N^2} F_M^p \right) \right\} \right.$$

$$\left. - 2t_1 t_2 \left\{ g_A^p \left(F_C^p + \frac{q^2}{8M_N^2} F_M^p \right) - g_A^n \left(F_C^n + \frac{q^2}{8M_N^2} F_M^n \right) \right\} \right]$$

$$- 2\pi i j_2(qr) \sqrt{\frac{3}{20\pi}} \gamma_{21}(\hat{q}) \frac{\omega}{r} \left(\frac{u}{r} + \frac{1}{7\sqrt{2}} \frac{\omega}{r} \right) \left[(t_1^2 + t_2^2) 2g_A^p g_A^n - 2t_1 t_2 \right.$$

$$\left. (g_A^{p2} + g_A^{n2}) \right]$$

with

$$H_{33} = q_1 \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) + \frac{3}{20} q^+ \frac{\omega^2}{r^2} \quad \text{and}$$

$$H_{34} = \frac{q^+ - q^-}{2} \left(\frac{u^2}{r^2} + \frac{1}{10} \frac{\omega^2}{r^2} \right) + \frac{3}{20} q^+ \frac{\omega^2}{r^2}$$

APPENDIX C

Hamada-Johnston deuteron radial wave-functions, the modified form by Humberston and Wallace.

$$u(r) = e^{-\alpha r} (1 - e^{-\delta(r-r_0)}) \sum_{i=1}^L c_i e^{-(i-1)\mu r}$$

and

$$w(r) = e^{-\alpha r} (1 - e^{-\rho(r-r_0)}) \left(1 + \frac{3}{\alpha r} + \frac{3}{\alpha^2 r^2}\right) \sum_{i=1}^N d_i e^{-(i-1)\mu r}$$

where

$$\mu = \text{pion mass} = 0.70671 \text{ fm}^{-1}$$

$$\alpha = 0.23195 \text{ fm}^{-1}$$

$$\delta = 4.6 \text{ fm}^{-1}$$

$$\rho = 3.4 \text{ fm}^{-1}$$

$$x_0 = \mu r_0 = 0.343$$

We use

$$L = 7$$

$$N = S \text{ which gives deuteron binding energy} = 2.22599 \text{ MeV}$$

$$c_1 = 0.884598$$

$$c_2 = -0.0408152$$

$$c_3 = -1.333233$$

$$c_4 = 3.557742$$

$$c_5 = -15.21926$$

$$c_6 = 23.95358$$

$$c_7 = -12.55528$$

$$d_1 = 0.0232954$$

$$d_2 = -0.0530118$$

$$d_3 = 0.00833263$$

$$d_4 = 0.0625671$$

$$d_5 = -0.0446299$$

The normalization was done so that $\int_{r_0}^{\infty} (u^2 + \omega^2) dr = 1$.

END

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