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The One Loop Radiative corrections to W pair production in electron positron annihilation in the Supersymmetric extension of the Salam-Weinberg model of the Electroweak Interactions

by

S. Alam, B.Sc. (Hons.), ARCS, M.Sc.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Doctor of Philosophy

Department of Physics
Ottawa-Carleton Institute for Physics
Carleton University
Ottawa, Ontario
May, 1992

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the Supersymmetric extension of the Salam-Weinberg
model of the Electroweak Interactions"**

submitted by

Sher Alam, B.Sc., M.Sc.

in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

Chair, Department of Physics

Thesis Supervisor

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Abstract

The one loop radiative corrections to W pair production in e^+e^- annihilation in the supersymmetric extension of the Salam-Weinberg (SW) model of the electroweak interactions are calculated. Since our model contains the SW theory, and several calculations have been reported on the latter we compare these results with ours. In general agreement is found, a detailed comparison is not possible since the explicit details have not been published. However we have cross checked many of SW model results with Sundaresan and Kalyniak who have performed the calculation using the same renormalization scheme as ours.

The virtual corrections are determined in the on-mass-shell renormalization scheme (OMRS) of Sakakibara. The OMRS scheme has several advantages, one being that it is a transparent (i.e. in terms of the physics) renormalization procedure. Moreover the fundamental set of input parameters of OMRS is well determined. By this we mean the accurate determination of the Z -boson mass at LEP I and the expected precise measurement of the W mass at LEP II, and the already well determined α value constitute a good set of the fundamental input parameters. Of course the Higgs boson mass and the fermion masses have also to be put in. So far the top quark and the Higgs boson have eluded detection and consequently their masses have to be put in as free parameters.

One very important feature in determining how good the standard model (SM) is involves the measurement of the tri-boson coupling. Such a coupling occurs in $e^+e^- \rightarrow W^+W^-$ in the SM, at the tree level. For a precision check, one must calculate one loop radiative corrections in SM, using the tri-boson coupling. We have also evaluated W pair production with non standard couplings to get an intuitive feel for deviations away from the SM. Supersymmetry has the effect of

reducing the virtual corrections. The effect of supersymmetry is examined on the differential cross section of $e^+e^- \rightarrow W^+W^-$, the 'A' term, the magnetic moment of W-boson and the quadrupole moment of the W-boson.

Acknowledgements

First and foremost I must thank God¹ Almighty Who created me and everything around me (things we know, don't know and will never know). As a human being (and theoretician)² it was very difficult for me to accept 'the never know' part. Simply speaking our knowledge by definition and inherently suffers from *incompleteness*. I have found Newton's discussion of God in Principia very useful and interesting. And I am tempted to quote some of this discussion here, so I will. "This most beautiful system of the sun, planets, and comets, could only proceed from the counsel and dominion of an intelligent and powerful Being. And....He is not eternity or infinity, but eternal and infinite; he is not duration or space, but he endures and is present. He endures forever, and is everywhere present;... All that diversity of natural things which we find suited to different times and places could arise from nothing but the ideas and a will of a Being necessarily existing....." It should be noted that Newton and his colleague Cotes considered the possibility of the universe arising out of pure chance. They also considered pantheism polytheism etc and rejected these notions.

Next I must thank my parents who cultivated somehow the love of knowledge in my stubborn and stupid head. I wonder how they did this? But their patience and long-suffering leaves me thinking about the goodness which God has programmed in the very nature of the human soul. My parents taught me discipline too but I lost it along the way, I hope to gain it back.

Next I thank all my teachers, this includes anyone from whom I learned anything (even the atheists). In particular I thank my M.Sc. supervisor Prof. R.N

¹I have used the word God rather than a more appropriate name for the All-Wise creator for the benefit of the English speaking people.

²The question arises if the concept of a human being and a theoretician are really compatible (i.e do they commute?), but that is a separate issue.

Henriksen, Prof. W.R Conkie and Prof. B.C Robertson (all as far as I know are at Queen's University). I also extend my gratitude to Prof. C.J. Isham at Imperial College. I really thank my Ph.D. thesis supervisor Prof. M.K. Sundaresan for being an excellent teacher, very helpful and providing financial support. In 'real' life providing financial support is no academic issue. It indeed is appreciated. I also thank Prof. P. Kalyniak for her thoroughness in checking of the results. Special thanks go to Prof. P.J.S. Watson for help he provided which cannot easily be described , maybe it has something to do with his name? I also thank Prof. W.J Romo and Prof. J.E Hardy for their compassion , good physics insights and being a joy to discuss physics with. Mention must be made of Prof. R.J Hemingway for his delightful, cheerful and colorful character not easily found among the physicist species. I also thank Prof. Armitage for a good course on experimental particle physics and his willingness to help other people. I also take this opportunity to thank the computer staff Bill Jack, Alan Barney, Markus Sperling and Wade Hong. In particular Alan Barney for his excellent disposition, hell can't get him mad. Bill Jack must be thanked for his willingness to increase my computer quota. I thank J-G. Boutin for helping me organize my research papers. I thank Roselyn Tighe for being a mother figure and always trying in vain to suppress the wild Afghan in me. I thank the CRPP secretary Marilyn McGregor for being compassionate and helpful. I also thank L. Resnik, J. Guy, A. Peterson, H. Konig, S. Godfrey, P. Estabrooks, R.K Carnegie ,B.J Jarosz, M. Ogg and the rest of faculty and other staff.

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This thesis is dedicated to
My parents,
Prof. M.K. Sundaresan,
Prof. R.N. Henriksen,
and
My wife Leslie and son Yusaf

**"There are therefore Agents in Nature
able to make the Particles of Bodies
stick together by very strong attractions.
And it is the Business of experimental
Philosphy to find them out."**

Newton, Opticks p394.

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Chapter 1

Introduction

Symmetry, as wide or as narrow you may define it, is one idea by which man through ages has tried to create order, beauty and perfection.

Herman Weyl

in Symmetry

It is amazing yet humbling experience to realize that our current knowledge of elementary particle physics can be summarized on a single sheet of paper.

Anonymous !

1.1 General Remarks

The main purpose of this section is to motivate the reader as to why it is useful to study supersymmetric gauge theories. To this end we first state certain well known facts regarding our current knowledge of Elementary Particle Physics [abbreviated hereafter as EPP] (EPP is the study of the fundamental particles and their interactions), then give a brief historical review as to how we arrived at our present day knowledge of fundamental physics and finally discuss briefly gauge theories and

supersymmetric gauge theories.

In the present framework of EPP all matter is described in terms of elementary fermions, which are classified as either quarks or leptons, while interactions between them are described in terms of exchange of elementary bosons, see Table 1.1 through 1.5 for some physical characteristics of these particles. Experimental research in this field of science is mainly carried out with giant particle accelerators and their associated detection equipment. However neutrino physics and proton decay experiments use huge underground detectors. High energies are necessary mainly for two reasons. First, in order to localize the investigations to the very small scales of distance associated with the elementary constituents, one requires radiation of the smallest possible wavelength and highest possible energy; second, many of the fundamental constituents have large masses and require correspondingly high energy for their creation and study.

The electromagnetic [EM] interactions are mediated by the photon between any two particles carrying Electric Charge. Note that with the exception of the neutrinos all fermions and the charged vector bosons ($W^\pm$) experience electromagnetic interactions, (Tables 1.1, 1.2 and 1.3). From a field theoretic view point photons are the quanta of the electromagnetic field. It is well known from classical electromagnetic theory that the Maxwell's Equations are the best description known to date for the electromagnetic field. We also know from classical physics that the electromagnetic interactions have infinite range. When this fact is combined with uncertainty principle of quantum physics one immediately deduces that the quanta responsible for mediating the EM interaction must be massless¹. Indeed the photon is found to be massless [3]. The masslessness of the photon is

¹According to the Heisenberg uncertainty principle, the range of an interaction is inversely proportional to the exchanged particle's mass, a concept exploited by Yukawa [1] in 1935.

related to the gauge invariance of the Maxwell Lagrangian. The spin half fermions are described by the Dirac equation. One of the astounding predictions of the Dirac equation, which was confirmed, is that of antiparticles [2]. For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol. According to the Dirac theory particle and antiparticle have identical mass and spin but opposite charge. The last three specific predictions about the charge, mass and spin have been confirmed experimentally. Some but not all electrically neutral bosons are their own antiparticles (for e.g. Z^0 and γ are their own antiparticles) but $K^0 = d\bar{s}$ is not.

The Weak interactions [hereafter referred to as WI] are mediated by the $W^\pm$ and the Z^0 gauge bosons between particles carrying the weak charge. The WI are short ranged (10^{-18} m). The range implies that the quanta responsible for this interaction must be very massive (100 GeV). The weak interaction is responsible for reactions such as the familiar beta decay and is experienced by all known fermions. The beta decay in which a neutron decays into a proton, an electron and an antineutrino is more accurately viewed as the decay of d quark into u quark, electron and antineutrino via a virtual W^- boson. The relative strength of the weak to electromagnetic forces is on the order of 10^{-3} for distance scales of approximately 10^{-17} m. The weak and the electromagnetic fields have been successfully combined [Unified] into the electroweak field, a development parallel to the unification of electricity, magnetism and light by Maxwell.

The strong forces are mediated by the gluons between particles carrying color charges. The quarks are the only fermions known to possess this color charge. Each quark carries 3 color charges. There are eight possible types of color charges which the mediating quanta of the strong force may carry. Just as the electromagnetic

forces are responsible for the rich atomic structures enshrined in the Mendeleev Periodic Table, the strong force is thought to be responsible for the internal structure of hadrons [a hadron refers collectively to mesons and baryons]. Such 'Particle Periodic Tables' may be found in the Particle Data Book [3]. For the convenience of the reader the quark content of some of the common hadrons has been listed in Tables 1.6 and 1.7. The relative strength of the strong to electromagnetic forces is on the order of 10 for distance scales of approximately 10^{-17} m. Unlike the photon the gluon can interact with itself, thus allowing non linear interactions. It should be noted that as color charged particles (quarks and gluons) are separated the color force between them approaches a constant value and the energy in color force field increases. At some distance creation of additional quark-antiquark pairs becomes preferable energetically. The objects which appear in the final state are colorless hadrons. It is instructive to compare this situation with the electromagnetic field where one can separate the contents of the atom. One cannot isolate quarks and gluons; they are confined into color-neutral hadrons. Physicists conjecture that this confinement is the result of multiple exchanges of gluons. It is natural to attempt the unification of the electroweak field with the strong field. Many such attempts have been made, these efforts are referred to as Grand Unified Theories [GUTS] [5] in the jargon of EPP. But it is fair to say that none of the grand unification schemes possess the elegance or the exact predictions of the electroweak unification.

The gravitational forces are mediated by the gravitons [Table 1.5] between any two objects having mass or energy or both! In other words everything we know in the material universe participates in gravitational interactions making gravity a universal interaction. Like the EM interactions the gravitational interactions have

an infinite range. In other words the graviton must be massless. The graviton is a natural consequence of the Einstein field equations [6] just as the photon is of Maxwell equations. The relative strength of the gravitational to electromagnetic forces is on the order of 10^{-41} for distance scales of approximately 10^{-17} m. In spite of many heroic efforts no gravity waves have been detected to date. The gravitational field is the most subtle of the interactions. A glimpse into this subtlety is got by comparing it with the gluonic field. The gluonic field can be looked at as a non linear field propagating on a four dimensional Minkowskian flat space time since we can neglect the gluons gravitational field. Hence there is a clear separation between space time and the field. For the case of the graviton no such separation exist. This partly explains why gravity has resisted unification with the other interactions and refused quantization in the context of point field theory.

At the turn of this century, the observation of processes which are basically electromagnetic in nature (the constant speed of light, the spectrum of the black body radiation and the photoelectric effect) brought about a conceptual revolution and with it the breakdown of classical physics. At the same time, the discovery of radioactive decay brought with it the discovery of the short range weak and strong interactions; the establishment of quantum mechanics had opened the way for understanding them as well. So that by 1930, relativity and quantum mechanics were established, and it was clear that matter is made of nuclear atoms. But the excitement of the new physics was far from over. Indeed the next half century was characterized by startling experimental and theoretical discoveries and an assortment of puzzles wherever one looked. Before we proceed to outline some of the history of EPP and Quantum Field Theory [QFT], we give a superficial outline of the developments which took place in our conception of space and time.

One can view the history of 'relativity' as the history of generalizations of the group of transformation of space time. By this one simply means the following: if one looks back at the history of relativity, one notices how our notions of space time gradually matured, starting from Aristotle. The concept of 'force' and 'acceleration' were foreign to Aristotle. The state of rest was given a preferred or an invariant meaning. One could talk about spatial separation of two events even if they possessed non-zero time difference, so the state of rest of a body for which the spatial separation vanished for all times, acquired a special meaning. But for Galileo and Newton, this had to be generalized so that "a body at rest or in uniform motion continues to do so until compelled by external forces to do otherwise" so that the state of rest and uniform motion could not be distinguished. This introduces the concept of inertial observers, which are a set of observers in constant relative velocity to each other. It is reasonable to assume that such a motion occurs with respect to the distant "fixed stars". The departure from the Aristotelian view is that it makes sense only to talk of spatial separation between two events as long as their time difference vanishes. As is well known the Galileo group was generalized to Minkowski or the Lorentz group where the "invariant" is not the time nor the spatial separation but a certain combination of these [i.e., proper time] which is defined to ensure that a physical signal cannot travel faster than light. However the notion of the preferred or inertial observer is retained from the Galileo group. The concept of the inertial observer is dispensed with globally in General Relativity [GR] where its validity is kept only 'locally' i.e., space time is locally flat but not globally. The curvature of space time is then connected to gravity. The curvature of space time required by Einstein's Theory can be discussed in terms of the "parallel displacement" of a vector, through which the transport of a

vector around a closed loop by parallel displacement results in the final direction of the vector differing from its initial direction. As the space time is not globally flat, then the partial derivative with which one is familiar for calculating “rates of change” is not adequate and must be replaced by a generalized derivative

$$\frac{\partial V^i}{\partial x^j} \longrightarrow \frac{\partial V^i}{\partial x^j} + \Gamma_{jk}^i V^k \quad (1.1)$$

called the covariant derivative. Γ_{jk}^i is called the Riemann-Christoffel symbol. The curvature of space time is represented by the Riemann tensor. As can be guessed the Riemann tensor is written directly in terms of the Christoffel symbol viz

$$R_{lpn}^m = \frac{\partial \Gamma_{ln}^m}{\partial x^p} - \frac{\partial \Gamma_{lp}^m}{\partial x^n} + \Gamma_{rp}^m \Gamma_{ln}^r - \Gamma_{rn}^m \Gamma_{lp}^r \quad (1.2)$$

In summary the Aristotelian relativity was replaced by Galilean which was replaced by Minkowskian which in turn was replaced by Einsteinian Relativity. In terms of group theory² the space time group of Aristotle is much more restricted than that of Galileo which is more restricted than of Minkowski which is more restrictive than that of Einstein.

We now turn to a brief outline of the history of EPP and QFT, for details of the historical development see [2]. Most of the experimental work in the early part of this century depended on observation of particles in cosmic rays. Indeed this is how the positron³ was discovered [2]. Cosmic rays were discovered as a result

²for example the group of GR is GL(4,R)

³The positron, discovered by Carl David Anderson in the cosmic rays in 1932, was used to complete the picture of nuclear β decay when Irene Curie and Frederic Joliot produced artificially radioactive light elements that decayed by positron emission. The neutron was discovered by James Chadwick in 1932, and this resolved the nucleus problem since before the discovery of the neutron one had to understand the nucleus in terms of the only two particles known at that time i.e., the proton and the electron. The neutrino was proposed by Wolfgang Pauli in 1930 in order to maintain conservation of energy in β decays. The neutron and the neutrino along with electron and the proton were neatly incorporated by Enrico Fermi in 1934 in a theory of β decay. It is a credit to Fermi's genius how well this theory worked as a model of weak interactions until it ran into problems at higher energies.

of investigations, after 1900, of fine-weather “atmospheric electricity”(i.e., in the absence of an electrical thunderstorm). At about this time it was realized that the nonzero electrical conductivity of the atmosphere implied the presence of an ionizing radiation. Indeed, minute amounts of radioactive substances, first discovered by Henri Becquerel, were found to be present in the earth, air, water, (and in the measuring instruments themselves), and these did provide ionizing rays. However, after all known sources of ionization had been accounted for, there remained a “residual” conductivity, even in closed vessels that were heavily shielded. This phenomenon implied the existence of a “penetrating radiation of unknown origin”. The altitude dependence of the atmospheric conductivity was investigated by means of balloon flights. The manned balloons carried sealed electrometers whose rates of discharge first decreased with altitude, but then (above 2 km) began a marked increase. This pattern of ionization suggested, if it did not prove, the existence of an extraterrestrial source for the penetrating radiation, so that by the late 1920’s one spoke of the *cosmic rays*. The next major particle discovered in cosmic ray studies is now known as the muon. During its earlier history it was falsely identified as Yukawa’s nuclear meson, posing the following problem: if the cosmic-ray meson were Yukawa’s strongly interacting particle, why did it not seem to interact at all? The puzzle was called $\pi - \mu$ paradox and was resolved by the observation of the complete decay chain (pion $\rightarrow$ muon $\rightarrow$ electron), together with the long muon lifetime, strongly suggested that the muon was a heavy version of the electron, in modern terms a second generation lepton. Another puzzle the $\theta - \tau$ puzzle, involving the decay of the strange K meson caused the first questioning of parity conservation. This culminated in the famous experiment by Wu [2] which established that parity is violated in weak interactions and conserved in the

strong and electromagnetic interactions. The discovery of parity violation in weak interactions was crucial in formulating a successful model of these interactions, i.e., the SW model. The so called strange particles (K-meson, Λ ,.....) were discovered in 1950-1960. As the number of these strongly interacting particles grew, physicists looking for simplicity and symmetry began to think of theories in which they could explain all these strongly interacting particles arising out of some fundamental constituents. These efforts led to the proposal of the three quark model by Gell-Mann [2]. Gell-Mann proposed the "eight-fold way" [2] in which all baryons and mesons are constructed from three quarks up, down, and strange, see Tables 1.6 and 1.7.

Relativistic electron theory and the quantum theory of fields were both on the agenda of theoretical physics after the invention of quantum mechanics by Heisenberg [2] and Schrodinger [2] in 1925 and 1926. In a series of papers in 1927⁴ Dirac quantized the electromagnetic field in interaction with electrons (the electron field was not quantized so it was a semiclassical approach). He essentially treated the electromagnetic field as a Bose-Einstein gas of light quanta and successfully derived Einstein's coefficients. The analogous treatment for a Fermi-Dirac gas, applicable to electrons, was given the following year by Pascual Jordan [2] and Eugene Wigner [2]. The Jordon-Wigner type of quantization, designed to forbid more than one electron to occupy a given state, was just what Dirac needed to formulate his theory of holes in 1929 [2]. But in the context of old QED and field theories proposed for strong interactions problems with infinities arose. These problems, at least for QED, were resolved successfully by the procedure of renormalization pioneered by Schwinger [2], Feynman [2], and Tomonaga [2] around 1948.

⁴This paper is regarded as the first paper on QED

We now turn to the concept of gauge invariance. This concept of grew out of the observation that if a 'charge' (e.g. electric charge, total energy, isospin, etc.) is conserved in a dynamical system, then the Lagrangian for the system is invariant under 'global gauge transformations' of the fields.⁵ For example, the electric charge is related to invariance under phase transformations $\psi \rightarrow e^{iQ\theta}\psi$ for all fields ψ which describe particles with charge Q [Just as the energy is related to time translations, 3-momenta to space translations and angular momentum to rotational invariance]. The converse is also true: if the Lagrangian is invariant under some infinitesimal transformations $\psi \rightarrow \psi + \delta\psi$ then there is a conserved current and a conserved charge associated with this gauge invariance. One calls the transformations 'global' if their parameters θ are space time independent. This relationship between conserved quantum numbers and global symmetries of the Lagrangian led in the 1960's, to a search for globally gauge invariant field theories capable of describing and classifying all elementary particles. If one allows the parameters θ to be space time dependent the Lagrangian can still be maintained as gauge invariant provided one introduces additional fields, the gauge potential, whose quanta will interact with all the charged particles. This interaction, which results in the exchange of field quanta, will generate forces between the particles. Hence the concept of symmetry and the attempt to generalize it leads in an elegant manner to the introduction of interactions between fields. By "gauging" the phase transformations (i.e. making the gauge parameters space time dependent) associated with electric charge forces us to introduce the electromagnetic four vector potential and, as its quanta, the photons. Similarly gauging the weak and the color charges lead respectively to theories of weak and strong interactions. We

⁵global gauge transformations are also referred to as gauge invariance of the first kind

now briefly describe the two gauge theories for electromagnetism and strong interactions. The remaining gauge theory of electroweak interactions is the subject of this thesis.

By the fifties, Quantum Electrodynamics [QED] [2] was well established both theoretically and experimentally. As is well known QED is the simplest example of a local gauge theory describing the interaction of the electron field with the photon field. QED is describable by a Lagrangian which is a function of ψ (fermion field, it could be an electron or a muon field for example), A_μ (electromagnetic field) and their first derivatives. The restriction of the Lagrangian to contain only first derivatives is made so that we can have second order field equations, for as yet we do not have a consistent way of dealing with differential equations of order greater than two. Now QED is renormalizable⁶, unitary,⁷ and Lorentz invariant. One crucial fact that the physicists learned while quantizing QED is that the gauge invariance [via the Ward identities] was essential in establishing the renormalizability and the unitarity of the theory [2]. Obviously a physical theory must have predictive power. By its very definition a renormalizable theory meets this criterion, since, given a few input parameters one can calculate all the physical processes allowed by the fundamental Lagrangian. In a non renormalizable theory one must put in new parameters at each order of quantum corrections in order to absorb the infinities and the predictive power is lost. The experimental and theoretical successes of QED are well known and we refer the interested reader to [5].

The gauge theory for the strong interactions is dubbed Quantum Chromodynamics [QCD]. Let us briefly discuss the features of strong interactions that suggest

⁶Roughly speaking by a renormalisable theory one means that the counterterms induced by the procedure of quantisation are of the same form as present in the original bare Lagrangian

⁷Unitarity is a direct generalisation to quantum field theory of the quantum mechanical rule that states that the sum of probabilities for any physical process is one

its description in terms of a non-abelian gauge theory. The first important step in the understanding of nuclear forces was the success of $SU(3)$ symmetry for hadrons, which led Gell-Mann and Zweig to introduce a quark picture of hadrons. According to this picture, the quarks are assumed to have spin $1/2$ and baryon number $1/3$ and transform as $SU(3)$ triplets denoted by (u, d, s) . (u, d) carry isospin and s carries strangeness. This picture provides a clear understanding of the spectroscopy of low-lying mesons and baryons in terms of simple S- and P-wave bound states of quarks (u, d, s) or quark and antiquarks. The baryon spectroscopy is understood in terms of three-quark bound states and meson spectroscopy in terms of quark anti-quark bound states. Accepting quarks as the constituents of hadrons and mesons, we have to search for a field theory which provides a binding force between quarks. In trying to understand the Fermi statistics for baryons (such as Ω^-), it became clear that if they are S-wave bound states, the space part of their wave function is totally symmetric; since a particle such as Ω^- consists of three strange quarks, and has spin $3/2$, the spin part of its wave function is also symmetric. If there are no other degrees of freedom, this would lead to disagreement with the required antisymmetry of wave function in Fermi statistics. A simple way to resolve this problem is to introduce [5] a threefold degree of freedom for quarks, called color (quarks being color triplets) and assume that all known baryons are singlets under this new $SU(3)$. Since an $SU(3)_c$ singlet constructed out of three triplets is antisymmetric in the interchange of indices (quarks), the total baryon wave function is antisymmetric in the interchange of any two constituents as demanded by Fermi statistics. It is now tempting to introduce strong forces by making $SU(3)_c$ into a local symmetry. In fact, if this is done, we can show that the exchange of the associated gauge bosons [gluons] provide a force for which the $SU(3)_c$ color singlet

is the lowest lying state; and the triplet, sextet, and octet states all have higher mass. By choosing this mass gap large, we can understand why excited states corresponding to the color degree of freedom have not been found. While this argument in favor of an $SU(3)_c$ gauge theory of strong interactions was attractive it was not conclusive. The most convincing argument in favor of $SU(3)_c$ gauge theory came from the experimental studies of deep inelastic neutrino and electron scattering off nucleons and the observation of three jet events. The result of the first experiment implies among other things that at high momentum transfer the strong coupling constant gets weaker. This property called asymptotic freedom [40] can be understood as a consequence of self interacting gluons provided there are no more than 16 flavors of fermions in $SU(3)_c$. Hence, unlike QED, in QCD, the coupling constant decreases at high momentum transfer. The three jet events confirms the existence of gluons. Note that the gluons form an octet under $SU(3)_c$. So in the QCD picture, all observed nuclear forces are residual effects of gluons exchanges between quarks which are bound inside the nucleon not unlike the Van der Waal's force which are the residual forces arising from photon exchanges. An immediate problem was to understand why long range forces resulting from single gluon exchange have not been seen. Similarly, since gluons are massless they should be copiously produced in all strong interaction processes. Furthermore, since at high energies quarks become free, they should have been seen since their masses are expected to be less than the nucleon mass. To resolve this conundrum a new principle, known as "confinement," was introduced. According to this principle, all unbroken non-abelian gauge theories with the property of asymptotic freedom should confine all the gauge nonsinglet states, and only observable sectors of Hilbert space correspond to states which are singlets under this local symmetry.

This would then explain why quarks and gluons, and indeed any other nonsinglet bound hadron states have not been seen. Many plausible arguments and techniques have been developed to prove confinement, however it would be fair to say that a nice rigorous proof is still lacking.

Now we briefly turn to the electroweak group. Here we shall focus on the historical development of this gauge group since the technical details will be mentioned throughout the thesis. The vector and the axial-vector nature of the weak force was established by the work of Fermi, Feynman and Gell-Mann, and Marshak and Sudarshan [2]. However this work was purely phenomenological. Two separate ideas were proposed which culminated in a successful theory of electroweak interactions. One of these was work of Yang and Mills [7] and Shaw [8]. These authors extended the idea of a local gauge symmetry from an Abelian $U(1)$ group of phase rotations to a non-Abelian $SU(2)$ symmetry. This led to the prediction of new massless vector bosons in addition to the photon. This was in contradiction with experiment. However an $SU(2)$ local gauge theory is renormalizable. In such a gauge theory one cannot put in masses by hand for then the renormalizability of the theory would be compromised. An important new idea was introduced, namely, that of spontaneous symmetry breaking due to Anderson [9], Higgs [10], Kibble [11], Guralnik [12] and Englert [13]. These authors showed that the massless gauge bosons of the Yang-Mills theory and massless Nambu-Goldstone⁸ bosons could both be circumvented if one absorbs the latter as the longitudinal degree of freedom of the former giving the gauge bosons a mass. After a few attempts Salam [16], Weinberg [18] and Glashow [17] were able to combine successfully the ideas of spontaneous symmetry breaking into a theory for electroweak interactions. The

⁸Nambu [14] and Goldstone [15] had shown that for global symmetries a spontaneously broken symmetry gives rise to massless spin 0 bosons

low energy limit of the EW theory is Fermi's four-point-fermion theory of weak interactions.

If one combines the electroweak group with the color group one obtains what is known as the *Standard Model* [SM]. So the SM group is $SU(3)_c \times SU(2)_L \times U(1)_Y$. The SM is compatible with all known facts of elementary particle physics and is believed to be free of mathematical inconsistencies. Moreover, the basic interactions are all described by gauge theories, which implies among other things that the forms of the couplings of the vector bosons that mediate the interactions are determined by the underlying gauge symmetry. Despite these attractive features, few physicists believe that the SM is the ultimate theory of elementary particle interactions. The problem is not that there is anything fundamentally wrong with the SM, but rather that it needs many parameters for making predictions. A few reasons as to why the standard model must be replaced:

1. :In the SM the strong, weak, and electromagnetic interactions are largely independent of each other, as is illustrated by the fact that the gauge group $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$ is a direct product of three factors with different gauge coupling constants.
2. :The pattern of fermion representation is arbitrary and complicated. There is therefore no fundamental explanation for the repetition of fermion families or for the violation of parity in the weak interactions but not in the strong interactions.
3. : Electric charge is not quantized. That is, there is no a priori reason for the quark and lepton charges to be related by a simple factor of three.
4. : Even given the groups, the representations, and the electric charge assign-

ments, the standard model has too many free parameters (19 or 26, depending on whether the neutrinos are assumed to be massless). This means the observable quantities, such as the fermion masses, mixing angles, and CP violating phase are completely arbitrary.

5. : Finally, the Standard model does not incorporate gravity, nor does it offer an explanation for the empirical absence of a large cosmological term.

One way to constrain or determine some of the features that are arbitrary in the standard model is to consider models with more symmetry. Several grand unified theories [GUTS] have been proposed in which the strong, weak, and the electromagnetic interactions are embedded in a larger underlying gauge theory with a single gauge coupling constant g . There are several features of GUTS which we now mention

1. : The basic idea of combining the strong and electroweak interactions into a unified theory is very attractive and it makes the observed pattern of low energy interactions somewhat less arbitrary.
2. : Most GUTS theories explain the quantization of charge.
3. : Many GUTS models predict $\sin^2 \theta_W = .20 - .21$ which is reasonably close to the experimental value $(\sin^2 \theta_W)_{\text{expt.}} = .229 \pm .014$
4. : Most models (with the exception of the simplest SU(5) model) predict massive neutrinos, with masses typically in the range $10^{-5} - 10^{+2}$ eV. The associated neutrino oscillations could account for the missing solar neutrinos.
5. : Some of the complicated models predict flavor changing neutral currents [FCNC] at some level.

6. : Finally, one of the most exciting implications of GUTS is that they may explain the excess of matter over antimatter in the universe (baryon asymmetry).

GUTS have several shortcomings, some of the more serious draw backs are as follows

1. : GUTS shed little light on the pattern of fermion masses and mixings, or on the repetition of families.
2. : The appearance of two very different mass scales (M_W and M_X) is not a natural feature. It appears necessary to put this in by hand by fine tuning parameters, and when radiative corrections are included this leads to a serious problem known as the hierarchy problem (to be explained below)
3. : The unification of GUTS is the unification of the forces, and a weak unification of fermions. There is no unification of the matter (fermions) and forces (bosons). Hence the unification proposed by GUTS is at best incomplete.
4. : Gravity is not included in most accepted GUTS

It has been mentioned above the hierarchy problem relates to the appearance of two very different mass scales, M_X and M_W , in a typical GUTS. The physical Higgs scalar particle associated with electroweak symmetry breaking must have a mass of order M_W in order that the cancellation of diagrams in perturbation theory occurs at sufficiently low energy to satisfy unitarity. The Higgs bosons associated with GUT symmetry breaking on the other hand must have masses of the order of M_X , in order to give masses of this order to the GUTS bosons which mediate proton decay. Hence the scalar potential must give a hierarchy of vacuum expectation

values. One could imagine adjusting the parameters of the scalar potential to give this hierarchy. However, the precision to which the scalar self-couplings can be specified is limited by radiative corrections. To make these remarks about the hierarchy problem precise consider the one loop radiative corrections contributing to the mass renormalization of a Higgs boson (Fig. 1.1). It can be shown that these contributions diverge quadratically and in the solution of the renormalization group equations [RGE] the running mass square acquires a quadratic dependence on the scale μ of the form

$$M^2(\mu_2) = M^2(\mu_1) + Cg^2(\mu_2^2 - \mu_1^2) + g^2R + O(g^4) \quad (1.3)$$

where g is a coupling, C is dimensionless and R grows at most logarithmically as $\mu_1 - \mu_2 \rightarrow \infty$. Applied to the electroweak-breaking Higgs H , this equation gives

$$M_H^2(M_W) \simeq M_H^2(M_X) - Cg^2M_X^2 \quad (1.4)$$

In order that the physical mass $M_H(M_W)$ be of the order M_W , there would have to be a cancellation between the terms on the right-hand side. This would require a fine-tuning of the parameters to an accuracy of order 10^{-26} in each order of perturbation theory, which is unnatural. One way of bringing the radiative corrections under control is to postulate a cut-off $\Lambda \sim 1$ TeV in the loop integrals, by making the Higgs scalars effectively condensates of pair of new fermions. This is the *Technicolor* approach [5]. With elementary Higgs scalars, there is an elegant way of solving the hierarchy problem by introducing a boson-fermion symmetry called *Supersymmetry* (SUSY) [5]. If the boson-fermion pairs have identical couplings, their contributions to the loop diagrams like Fig. 1.1 are of opposite sign and cancel against each other. Naturalness in the hierarchy problem requires that

their masses satisfy

$$|m_B^2 - m_F^2| \leq O(1\text{TeV}^2) \quad (1.5)$$

The existence of an approximate supersymmetry would entail a complete doubling of the presently known particle spectrum. Let us now see what are the other motivations and advantages of introducing SUSY. One of the criticism we gave for GUTS was that it does not relate or unify matter and force fields. One of the main ambitions of fundamental physics has been to achieve such a unification. For sometime it was thought that the symmetries which would naturally relate forces and fermionic matter would be in conflict with field theory. The progress in understanding elementary particles through the SU(3) classification of the "eight-fold way"(global symmetry) had led to attempts to a unifying symmetry which would directly relate to each other several of the SU(3) multiplets (baryon octet, decuplet, etc.), even if these had different spins. The failure of attempts to make those "spin symmetries" relativistically covariant led to the formulation of a series of no-go theorems, culminating in an interesting paper by Coleman and Mandula [19] which was widely understood to show that it is impossible, within the theoretical framework of relativistic field theory, to unify space-time symmetry⁹ with internal symmetries. More precisely, the theorem says that the charge operators whose eigenvalues represent the "internal" quantum numbers such as electric charge, isospin, hypercharge, etc., must be translationally and rotationally invariant. This means that these operators commute with the energy, the momentum and angular momentum operators. Indeed, the only symmetry generators which transform at all under both translations and rotations are those of the Lorentz

⁹Space-time symmetries are also called external-symmetries. Ignoring gravity the generators of external-symmetry are P^μ of translation and $M^{\mu\nu}$ of rotations. These generators form the Poincare algebra [20].

transformations themselves (rotations and transformations to coordinate systems which move with constant velocity). The generators of internal symmetries cannot relate eigenstates with different eigenvalues of m^2 and $l(l+1)\hbar^2$ of the mass and the spin operators. This means that irreducible multiplets of symmetry groups cannot contain particles of different mass or of different spin. Hence the no-go theorem of Coleman and Mandula would seem to rule out exactly the sort of unity which was sought. Different spins in the same multiplet are allowed if one includes symmetry operations whose generators obey anticommutation relations of the form $AB + BA \equiv \{A, B\} = C$. This was first proposed in 1971 by Gol'fand and Likhtman [21], and followed by Volkov and Akulov [22] who arrived at what we now call a non-linear realization of supersymmetry. Their model was not renormalizable. In 1973, Wess and Zumino [23] presented a renormalizable field theoretical model of a spin $1/2$ particle in interaction with two spin 0 particles where the particles are related by symmetry transformations, and "sit" in the same multiplet. The limitations of the Coleman-Mandula no-go theorem had been circumvented by the introduction of a fermionic symmetry operator which carried spin $1/2$, and thus when acting on a state of spin j resulted in a linear combinations of states with spins $j + \frac{1}{2}$ and $j - \frac{1}{2}$. Such operators must and do observe anticommutator relations with each other. They do not generate Lie groups and therefore are not ruled out by the Coleman-Mandula no-go theorem. In the light of this discovery, Haag, Łopuszanski, and Sohnius [24] extended the results of Coleman and Mandula to include symmetry operations which obey Fermi statistics. They showed that in the context of relativistic field theory the only models which can lead to a solution of the unification problems are the supersymmetric theories. The class of such theories is, however, rather restricted: for it was shown in [24] that space-time

and internal symmetries can only be related to each other by fermionic symmetry operators Q of spin $1/2$ (not $3/2$ or higher) whose properties are either exactly those of the Wess-Zumino model or are at least closely related to them. Only in the presence of supersymmetry can multiplets contain particles of different spin, such as the graviton and the photon. A large set of supersymmetric algebras that could play a role in field theory have been classified in [24]. Supersymmetry transformations are generated by quantum operator Q which changes fermionic states into bosonic ones and vice versa,

$$Q|\text{fermion}\rangle = |\text{boson}\rangle \quad (1.6)$$

$$Q|\text{boson}\rangle = |\text{fermion}\rangle \quad (1.7)$$

Which particular bosons and fermions are related to each other by the operations of some such Q , how many Q 's there are, and which other properties other than statistics are changed by the operations depends on the supersymmetric model under study. There are, however, a number of properties which are common to the Q 's in all supersymmetric models. By definition, the Q 's change the statistics and hence the spin of the states. Spin is related to behaviour under spatial rotations, and thus supersymmetry is in some sense a space-time symmetry. Normally, and particularly so in models of "extended supersymmetry" (which is defined in next section), the Q 's also affect some of the internal quantum numbers of the states. It is this property of combining internal with space-time behaviour that makes supersymmetric field theories interesting in the attempt to unify all interactions. After the original work in supersymmetry there was an intense period of activity. Supersymmetric gauge theories were immediately constructed, so were supersymmetric grand unified theories [SGUTS] [25]. Interesting phenomenology was worked out

for these models. As is clear from our discussion of the hierarchy problem SGUTS do not suffer from this problem. In fact the hierarchy problem continues to be a big motivation for serious consideration of supersymmetric theories. Generalizing the Minkowski flat space-time metric to a curved metric in the presence of supersymmetric field theories led to supergravity models [33], hence incorporating gravity in non-trivial fashion with the rest of the interactions. However supergravity shares the problem of having a dimensional coupling constant with the Einstein theory of gravity and the Fermi theory of weak interactions. From simple power counting [27] any theory with a dimensional constant is non-renormalizable. Recent attempts to overcome this problem [i.e. the non-renormalizability of supergravity among others has , led to the constructions of superstrings theories. Very simply speaking the fundamental objects in superstrings are strings rather than the point particles of ordinary field theory. The fundamental particles arise in string theory from the ground state vibrations of the string. The string carries a tension which is the string equivalent of a coupling constant of ordinary field theory. In the limit of infinite tension the string reduces to a point particle. In the infinite tension limit superstring theory reduces to supergravity. Very crudely speaking the relationship of supergravity to superstrings is the same as of Fermi theory of weak interactions to SW model of weak force. This has led some physicists to suggest that superstrings may be renormalizable or even finite! The phenomenology of superstrings is very complicated and there is no satisfactory single model which has been able to reduce superstring predictions into being testable. The supersymmetric particles along with the SW model Higgs and the top quark have yet to be detected. But that is no reason to get discouraged; after all it was quite some time after the neutrino was predicted that it was actually observed experimentally. Supersymmetry

seems such an elegant solution for the hierarchy problem that it is theoretically very attractive. However, for SUSY to be relevant for the solution of the hierarchy problem, SUSY particles must be seen experimentally around a few TeV at most. The relevant information on supersymmetry such as recent experimental limits on masses of SUSY particles and a discussion of the Supersymmetric SW lagrangian will be given in Chapter3.

1.2 Supersymmetric Algebra

We now translate our comments about Lie algebra, Poincare group, graded Lie algebra into equations. The generators T_a of a non-abelian internal symmetry obey the following commutation relation;

$$[T_a, T_b] = if_{abc}T_c \quad (1.8)$$

in other words they form a Lie algebra. Moreover T_a commute with the Hamiltonian and the Poincare generators,

$$[T_a, H] = 0 \quad (1.9)$$

$$[T_a, P^\mu] = 0 \quad (1.10)$$

$$[T_a, M^{\mu\nu}] = 0 \quad (1.11)$$

The 10 Poincare generators themselves obey the commutation relations

$$[P^\mu, P^\nu] = 0 \quad (1.12)$$

$$[M^{\mu\nu}, P^\rho] = -ig^{\mu\rho}P^\nu + ig^{\nu\rho}P^\mu \quad (1.13)$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = i(g^{\mu\sigma}M^{\nu\rho} + g^{\nu\rho}M^{\mu\sigma} - g^{\mu\rho}M^{\nu\sigma} - g^{\nu\sigma}M^{\mu\rho}) \quad (1.14)$$

In its simplest form [$N = 1$], the SUSY algebra has a self-conjugate spin half Majorana generator Q_α ($\alpha = 1, 2, 3, 4$) with the interesting and important property that it changes the total angular momentum by half a unit. An immediate consequence of this crucial property is that it changes a fermion into a boson and vice versa. In other words it changes matter fields into force fields. Thus unifying matter and

Figure 1.1: Radiative corrections to a Higgs boson mass from gauge boson, scalar boson and fermion loops.

forces. The graded lie algebra is obtained from the Lie algebra by supplementing the above relations with the following commutation and anticommutation relations

$$[Q_\alpha, M^{\mu\nu}] = i(\sigma^{\mu\nu} Q)_\alpha, \quad (1.15)$$

$$[Q_\alpha, P^\mu] = 0, \quad (1.16)$$

$$\{Q_\alpha, \bar{Q}_\beta\} = -2(\gamma_\mu)_{\alpha\beta} P^\mu, \quad (1.17)$$

$$\{Q_\alpha, Q_\beta\} = 0, \quad (1.18)$$

$$\{\bar{Q}_\alpha, \bar{Q}_\beta\} = 0 \quad (1.19)$$

where $\sigma^{\mu\nu} = \frac{1}{4}[\gamma^\mu, \gamma^\nu]$ and $\bar{Q}_\alpha = Q_\alpha^T \gamma^0$, (i.e. real). The commutation relation (1.15) expresses that Q_α transforms as a spinor under the Lorentz group, while (1.16) states that spinor charges are conserved. The anticommutation relation (1.17) is the essence or the main ingredient of SUSY and it indicates that two successive SUSY transformations involve a space time translation. Hence this expression (1.17) combines or relates “internal symmetries” with external symmetry. When the relation (1.17) is generalized in the presence of gravity one obtains Supergravity [33]. There could exist N different operators $Q_\alpha^i (i = 1, 2, \dots, N)$ of this kind, but only $N = 1$ SUSY allows fermions in chiral representations, as observed. When $N \neq 1$, relation (1.17) generalizes to

$$\{Q_\alpha^i, \bar{Q}_\beta^j\} = -2\delta_{ij}(\gamma_\mu)_{\alpha\beta} P^\mu + C_{\alpha\beta} Z_{ij} + (\gamma_5 C)_{\alpha\beta} Z'_{ij} \quad (1.20)$$

The generators Z_{ij} and Z'_{ij} , known as the central charges, are antisymmetric in i and j and they commute with all the other generators of the SUSY algebra. Since the central charges are antisymmetric in i and j they exist only when $N > 1$.

1.3 Outline

The main purpose of this thesis is to examine the one loop radiative corrections to the process $e^+e^- \rightarrow W^+W^-$ in the context of both the SW model and its supersymmetric extension. Although the radiative corrections up to one loop for this process have been calculated by several authors [28, 29, 30, 31, 44] in the context of the SW model, no general consensus exist. The arrangement of this thesis is as follows. In Chapter 2 we examine the weak Lagrangian in the context of this calculation and establish our notation, convention, and Feynman rules. We discuss the regularization and renormalization schemes we have adopted and establish their consistency, i.e., show that all infinities (that are not infrared) cancel as they should. This also provides a check on the calculation. In Chapter 3 and Chapter 4 we essentially repeat the same steps as in Chapter 2 when the SUSY particles are included. We also include all the relevant information about supersymmetry that we need in this thesis, such as experimental limits on the masses of the SUSY particles. A discussion of the cancellation of quadratic and linear divergences is also included. Chapter 5 deals with the tree-level and bremsstrahlung cross section for the process $e^+e^- \rightarrow W^+W^-$. The bremsstrahlung is included in this chapter for the reason of its simple relation with the tree-level cross-section. Since as is well-known [28] that the bremsstrahlung cross section factors out into a product of the tree-level cross-section and a finite piece(this piece is essentially composed of Spence and logarithmic functions). Chapters 6, 7, and 8 deal respectively with the contributions of self-energy, triangle and box diagrams to the one loop corrections. These Chapters and their associated appendices contain results of this long and tedious calculation. Chapter 9 contains a simple discussion of the magnetic and the quadrupole moments of the W-boson in the framework of SUSY.

Finally chapter 10 contains concluding remarks. Several appendices are included for completeness and for the convenience of the reader.

Table 1.1: Matter constituents, FERMIONS, Leptons, spin = $\frac{1}{2}$.

Flavor	Mass GeV/c ²	Electric charge
ν_e , electron neutrino	$< 2 \times 10^{-8}$	0
e , electron	5.1×10^{-4}	-1
ν_μ , muon neutrino	$< 3 \times 10^{-4}$	0
μ , muon	.106	-1
ν_τ , tau neutrino	$< 4 \times 10^{-2}$	0
τ , tau	1.784	-1

Table 1.2: Matter constituents, FERMIONS, Quarks, spin = $\frac{1}{2}$.

Flavor	Approx. Mass GeV/c ²	Electric charge
u, up	4×10^{-3}	$\frac{2}{3}$
d, down	7×10^{-3}	$-\frac{1}{3}$
c, charm	1.5	$\frac{2}{3}$
s, down	.15	$-\frac{1}{3}$
t, top (not yet observed)	> 89	$\frac{2}{3}$
b, bottom	4.7	$-\frac{1}{3}$

Table 1.3: Force carriers, BCSONS, Unified Electroweak Force, spin = 1.

Particle (force carrier)	Mass GeV/c^2	Electric charge
γ , photon	0	0
W^- , W-boson	80.6	-1
W^+ , W-boson	80.6	+1
Z^0 , Z-boson	91.16	0

Table 1.4: Force carriers, BOSONS, Strong or Color Force, spin = 1.

Particle (force carrier)	Mass GeV/c^2	Electric charge
g , gluon	0	0

Table 1.5: Force carriers, BOSONS, Gravitational Force, spin = 2.

Particle (force carrier)	Mass GeV/c^2	Electric charge
G , graviton (not yet observed)	0	0

Table 1.6: Sample Fermionic Hadrons (Baryons and Antibaryons).

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	+1	0.938	$\frac{1}{2}$
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	-1	0.938	$\frac{1}{2}$
n	neutron	udd	0	0.940	$\frac{1}{2}$
Λ	lambda	uds	0	1.116	$\frac{1}{2}$
Ω^-	omega	sss	-1	1.672	$\frac{3}{2}$

Table 1.7: Sample Bosonic Hadrons (Mesons).

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	-1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
D^+	D^+	$c\bar{d}$	+1	1.869	0
η_c	eta-c	$c\bar{c}$	0	2.980	0

Chapter 2

The Weak Lagrangian

“Now if we adopt the view that this arbitrary convention should be independently chosen at every space-time point, then we are naturally led to the concept of gauge fields.” Yang and Mills [7]

2.1 Introduction

The purpose of this Chapter along with Chapter 3 is to gather all the machinery necessary to calculate the one loop corrections to the cross section of the process $e^+e^- \rightarrow W^+W^-$ in the context of the Salam-Weinberg (hereafter referred to as SW) model¹ and its supersymmetric extension (hereafter referred to as SQFD). In this chapter we concentrate on the SW model. To this end

1. We first briefly review some of the technical aspects of the QFD Lagrangian, its particle content and the various couplings which arise in this model. The confrontation of this model with experiment is also summarized.
2. We then recall some well known aspects of renormalization and regularization which are to be used in our calculation.

¹also called Quantum flavor dynamics (QFD) and Electroweak model (EW)

3. In the last section we specialize these remarks to the specific renormalization scheme we use in this thesis, i.e. the OMRS procedure and apply them to the SW model.

Radiative corrections to $e^+e^- \rightarrow W^+W^-$ in QFD were first calculated by Lemoine and Veltman [28] and Phillipe [29]. More recently they were repeated by Bohm et al. [30], Fleischer et al. [31] and us² [44]. It should be pointed out that despite these efforts there is no consensus on the actual magnitude of the radiative corrections in QFD. The importance of the reaction $e^+e^- \rightarrow W^+W^-$ as a test of the standard model cannot be overemphasized. Despite the recent results at LEP I pointing to the success of the standard model, the Higgs boson and the top quark have eluded detection. Moreover the crucial feature of SW model i.e., a test of the triple boson vertices has still to be made. One place where the triple boson vertex will be tested is in the electron positron annihilation into pairs of charged vector bosons at LEP II. Another quantity to be measured (with great precision) at LEP II is of course the W-boson mass, expected to be measured with an accuracy of 100 MeV. This coupled with the now well determined Z-boson mass [3] will make it possible, to detect deviations from the standard model. This necessitates all the more that the standard model radiative corrections (at least up to 1-loop!) be accurately estimated. Thus many independent evaluations are necessary.

As was pointed out by Kalyniak and Sundaresan [44] there are several reasons as to why we must take another look at the calculations of radiative corrections in QFD to the $e^+e^- \rightarrow W^+W^-$ reaction

1. The results of two calculations by Lemoine and Veltman [28] and Phillipe [29] though very similar, disagree with one another as to the actual magnitude

²By us we mean S. Alam, M.K. Sundaresan and P. Kalyniak.

of the corrections.

2. Also, the previous calculations were carried out before the advent of UA1 and UA2 data on W and Z bosons.
3. The results of the previous calculations were given in graphical form for selected energies, and for selected values of certain input parameters. They are not available in a form which experimentalists can use readily to obtain the radiative corrections under various experimental conditions.
4. In references [28] and [29] $\sin^2 \theta_W$ was used as one of the free parameters which is not convenient for the analysis of the data.
5. It is moreover desirable to have analytical expressions for the corrections available so that renormalization scheme dependence of the corrections can be studied.
6. Furthermore it is desirable to incorporate the finite width effects of the W's since W's were taken to be on mass shell in [28, 29, 30].

It was with all these aims in mind that the present calculation was undertaken in QFD and SQFD [44].

2.2 Electroweak Lagrangian

The construction of a Lagrangian for the weak interaction started with the proposal of Fermi to model the weak force by a four-point fermion interaction. To this end he suggested the following Lagrangian to model beta decay [2, 40]

$$\mathcal{L}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}} \bar{\Psi}_p \gamma^\mu \Psi_n \bar{\Psi}_e \gamma_\mu \Psi_{\nu_e}. \quad (2.1)$$

Observation of parity violation in weak decays led Feynman and Gell-Mann [46], and independently Sudarshan and Marshak [47] to a modification of the Fermi Lagrangian which incorporated this violation:

$$\mathcal{L} = \frac{G_F}{2\sqrt{2}} \bar{\Psi}_p [\gamma^\mu (1 - \gamma_5)] \Psi_n \bar{\Psi}_e [\gamma_\mu (1 - \gamma_5)] \Psi_{\nu_e} \quad (2.2)$$

Both the Lagrangians, Eq. 2.1 and Eq. 2.2 are non-renormalizable. It is an experimental fact that there is maximal parity violation in weak decays involving the neutrinos. Another way of saying this is that there are no right handed neutrinos³. As already mentioned in Chapter 1, the notion that all interactions between fermions are transmitted by intermediate bosons, has its origin in QED and in Yukawa's meson theory of nuclear forces [1]. So that once the V-A law, i.e. Eq. 2.2, was established, it seemed reasonable, indeed compelling, that the charged weak interactions also proceed by exchange of intermediate vector bosons (hereafter referred to as IVB) $W^\pm$, which would necessarily be quite massive, since the weak interaction has a very short range. Incorporating the IVB hypothesis⁴ modifies Eq. 2.2 to

$$\mathcal{L} = g^2 \bar{\Psi}_p [\gamma^\mu (1 - \gamma_5)] \Psi_n \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}}{k^2 - M_W^2} \bar{\Psi}_e [\gamma_\nu (1 - \gamma_5)] \Psi_{\nu_e} \quad (2.3)$$

Unfortunately as it turns out [40] the IVB Lagrangian is not only nonrenormalizable like its predecessors, it is also violates unitarity at high energies. The above facts indicate the challenge to incorporate all the observed facts about the weak interactions into a consistent renormalizable theory. Moreover it would seem that the weak and electromagnetic interactions could be unified since they both are mediated by vector particles. As already mentioned in the introductory chapter

³To date the left handedness of the neutrino is put in 'by hand'. It would be nice to construct a theory in which the handedness of neutrinos arose out of some underlying symmetry

⁴The IVB idea is largely credited to Schwinger [48]

this difficult task was accomplished by Salam [16], Weinberg [18] and Glashow [17] who used the idea of gauge invariance and successfully merged it with the concept of spontaneous symmetry breaking (SSB) to obtain massive vector bosons yet retaining renormalizability and unitarity of the model. In a set of brilliant papers 't Hooft [40] proved the renormalizability and unitarity of SW model.

It was mentioned in the introductory Chapter that by making a global symmetry local, one is automatically led to the idea of a gauge field (i.e. interaction). The idea of gauge invariance is well established in physics today and is extensively applied to modern field theories. One may illustrate the idea of gauge invariance and gauge field very simply by considering the electromagnetic interactions. Consider the Lagrangian for a free Dirac fermion field

$$\mathcal{L}_f = \bar{\Psi}_f(i\partial - m)\Psi_f \quad (2.4)$$

It is clear that the above Lagrangian is invariant under the global transformations

$$\begin{aligned} \Psi_f &\longrightarrow e^{ic\theta}\Psi_f \\ \bar{\Psi}_f &\longrightarrow \bar{\Psi}_f e^{-ic\theta} \end{aligned} \quad (2.5)$$

where e is the charge or coupling and θ is an arbitrary number. Now comes a simple but important observation—what if θ is made a function of space-time coordinates. Do we still have form invariance for the Lagrangian under the generalized U(1) phase transformations ?

$$\begin{aligned} \Psi_f &\longrightarrow e^{ic\theta(x)}\Psi_f \\ \bar{\Psi}_f &\longrightarrow \bar{\Psi}_f e^{-ic\theta(x)} \end{aligned} \quad (2.6)$$

The answer is yes provided we are willing to replace ∂_μ by the generalized or the 'covariant' derivative D_μ , where

$$D_\mu = \partial_\mu - ieB_\mu \quad (2.7)$$

and provided B_μ is chosen to cancel the extra term, $\partial_\mu\theta$, which arises through θ being space-time dependent. To accomplish this B_μ must satisfy the transformation

$$B_\mu \longrightarrow B_\mu + \partial_\mu\theta \quad (2.8)$$

which is taken to be the definition of the gauge field B_μ for the U(1) group. Eq. 2.6, and Eq. 2.7 constitute the local U(1) gauge transformations and these leave the Lagrangian

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \bar{\Psi}_f(i\gamma^\mu\partial_\mu - m)\Psi_f + e\bar{\Psi}_f\gamma^\mu B_\mu\Psi_f \quad (2.9)$$

invariant. $G_{\mu\nu}$ is the electromagnetic field tensor

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad (2.10)$$

and the first term in Eq. 2.9 is the Lagrangian⁵ for the B_μ field just as the second term is the Lagrangian for the Fermi field. The final term is an interaction Lagrangian. Notice that there are no self interaction terms for the B_μ field. In short Eq. 2.9 represents the Lagrangian for a massless electromagnetic field of spin 1 interacting with a spin $\frac{1}{2}$ field Ψ_f of bare mass m .

The above arguments about abelian gauge group can readily be generalized to the case of a non abelian gauge group such as SU(2) of the internal symmetry isospin [7]. SU(2) is chosen for the obvious reason that it can serve as the weak isospin group of SW model. Consider the case of the internal symmetry group SU(2) under which the spin $\frac{1}{2}$ fields transform as doublets. Their free field Lagrangian is

$$\mathcal{L}_f = \bar{\Psi}_f(i\partial - m)\Psi_f \quad (2.11)$$

⁵The Lagrangian in field theory is a simple generalisation of the Lagrangian in classical mechanics, where [i.e in classical mechanics] one writes $\mathcal{L} = \text{KE} - \text{PE}$. KE stands for kinetic energy and PE for potential energy. In field theory the KE terms are called propagators and PE terms represent the interaction vertices.

where now $\bar{\Psi}_f$ is a 1×2 row vector and Ψ_f is a 2×1 column vector. In analogy with QED, we require invariance under the local gauge transformation

$$\Psi_f \longrightarrow e^{ig\mathbf{T}\cdot\theta(x)}\Psi_f \quad (2.12)$$

where $\theta(x)$ is an arbitrary vector in isospin space and $\mathbf{T} = (T_1, T_2, T_3)$ is the isospin operator whose components T_i are the generators of SU(2) symmetry transformations. The T_i do not commute,

$$[T_i, T_j] = i\epsilon_{ijk}T_k \quad (2.13)$$

and the gauge group is said to be nonabelian. Operating on isospin doublets, the matrix representation is $T_i = \frac{1}{2}\tau_i$ where τ_i are the Pauli matrices. The Ψ_f field part of the Lagrangian can be made gauge invariant by introducing the appropriate covariant derivative D_μ

$$\mathcal{L}_f = \bar{\Psi}_f(i\gamma^\mu D_\mu - m)\Psi_f, \quad (2.14)$$

$$D_\mu = \partial_\mu - igW_{i\mu}T_i, \quad (2.15)$$

provided that an isospin triplet of Yang-Mills gauge field exist and transforming simultaneously as

$$W_{i\mu}(x) \longrightarrow W_{i\mu} + \partial_\mu\theta_i(x) - g\epsilon_{ijk}\theta_j(x)W_{k\mu}(x) \quad (2.16)$$

This transformation (i.e., Eq. 2.16) is more complicated than in the corresponding QED case, because of the non abelian property. The third term in Eq. 2.16 which was absent in the QED case is needed because the isospin operators in the infinitesimal transformation on $\bar{\Psi}_f$ and Ψ_f do not commute with the isospin

operators in D_μ . The gauge-invariant Lagrangian for the SU(2) vector fields is again a simple generalization of the QED case

$$\mathcal{L}_W = -\frac{1}{4}F_{\mu\nu}^i F_i^{\mu\nu} \quad (2.17)$$

where

$$F_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g\epsilon^{ijk}W_{j\mu}W_{k\nu} \quad (2.18)$$

In addition to the usual KE terms, this introduces triple and quartic self-couplings of the $W_{i\mu}$ fields. An SU(2) gauge model could be a candidate for weak interaction theory, since the isospin triplet could consist of $W^{0,\pm}$ bosons to transmit the weak force. The unification with electromagnetism could be achieved by combining the abelian and the non-abelian groups given above into a group product like $SU(2) \times U(1)$. However several considerations stand in the way

1. As mentioned before the effective low energy form of weak interactions implies, Eq. 2.1 -Eq. 2.3, that the charged vector bosons must be very massive. Adding a mass term $M^2 W^{i\mu} W_{i\mu}$ for the vector particles in Eq. 2.17 would not do since such a term is not gauge invariant. Breaking the gauge invariance by an explicit mass term would again lead us to non-renormalizability and unitarity problems.
2. The fermions cannot be given a mass either. This is because the effective low energy model implies a left handed structure for charged current couplings and such a mass term would destroy the invariance under SU(2) gauge symmetry.

Let us ignore the masses for the moment and write a Lagrangian from the above pieces incorporating the vector nature of both weak and electromagnetic interac-

tions and the handedness of the fermions. The partial SW Lagrangian based on the gauge group $SU(2)_L \times U(1)_Y$ can be written as

$$\mathcal{L}_{SW} = \mathcal{L}_1 + \mathcal{L}_2 \quad (2.19)$$

where $\mathcal{L}_1$ and $\mathcal{L}_2$ are respectively given by

$$\mathcal{L}_1 = -\frac{1}{4}F_{\mu\nu}^i F_i^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} \quad (2.20)$$

and

$$\mathcal{L}_2 = \sum_{f,c} \bar{L}_f \gamma^\mu (\partial_\mu - \frac{ig}{2}\tau_i A_\mu^i - \frac{ig'}{2}Y_f B_\mu)L_f + \sum_{f,c} \bar{R}_f \gamma^\mu (\partial_\mu - \frac{ig'}{2}Y_f B_\mu)R_f \quad (2.21)$$

In the above Lagrangian the left handed fields are doublets and the right handed fields are singlets under the weak isospin group $SU(2)$. Both these fields carry the hypercharge quantum number Y of the $U(1)$ group. The constraint between Y and the third quantum number T_{3f} of the $SU(2)$ group satisfies a Gell-Mann like relation

$$Q = T_3 + \frac{Y}{2} \quad (2.22)$$

Note that this relation generates the correct electrical charge quantum numbers for fermions [Table 2.1], and indicates the unification of the electromagnetic and weak interactions.

We now turn to the question of giving the gauge bosons and the fermions masses. To this end we go back to the SSB or the Higgs mechanism. The SW Higgs field is a spin 0 field that transforms as complex doublet under $SU(2)$ and carries hypercharge quantum number $Y = 1$

$$\Phi = \begin{pmatrix} G^+ \\ \frac{(H_1^0 + G^0)}{\sqrt{2}} \end{pmatrix}. \quad (2.23)$$

A gauge-invariant Lagrangian for the Higgs field can be written as

$$\mathcal{L}_3 = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \quad (2.24)$$

where

$$D_\mu \Phi = (\partial_\mu - \frac{ig}{2} \tau_i A_\mu^i - \frac{ig'}{2} B_\mu) \Phi \quad (2.25)$$

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \quad (2.26)$$

The above Lagrangian by definition and choice is renormalizable. The self interaction term in Eq. 2.26 provides the mechanism for SSB. For the PE term in Eq. 2.24 has a minimum at

$$v = \frac{\mu^2}{\lambda} \quad (2.27)$$

where $\frac{v}{\sqrt{2}}$ is the vacuum expectation value (VEV) of Higgs field. The VEV defines the ground state about which we must expand the field in perturbation theory. If one expands the Higgs field about the ground state value and substitutes it in the Lagrangian given in Eq. 2.24 one finds that 3 members of this doublet field give the gauge bosons their masses without breaking the gauge invariance and consequently maintaining the renormalizability of the SW model. The remaining member is a new neutral scalar particle, the Higgs boson. After the SSB the following combinations are chosen to get a massless photon and QED intact. The mass eigenstates are

$$W^\pm = \frac{A^1 \mp iA^2}{\sqrt{2}}, \quad (2.28)$$

$$M_W = \frac{g}{2} v, \quad (2.29)$$

$$Z = \frac{gA^3 - g'B}{\sqrt{g^2 + g'^2}}, \quad (2.30)$$

$$M_Z = \frac{\sqrt{g^2 + g'^2}}{2} v, \quad (2.31)$$

$$A = \frac{g' A^3 + g B}{\sqrt{g^2 + g'^2}}, \quad (2.32)$$

$$M_\gamma = 0. \quad (2.33)$$

It is convenient to define a weak interaction mixing angle θ_W ⁶ by

$$\cos \theta_W = c_W = \frac{g}{\sqrt{g^2 + g'^2}} \quad (2.34)$$

$$\sin \theta_W = s_W = \frac{g'}{\sqrt{g^2 + g'^2}} \quad (2.35)$$

The standard electric charge is given in terms of g, g' , etc by

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}}, \quad (2.36)$$

$$e = g s_W, \quad (2.36)$$

$$e = g' c_W. \quad (2.37)$$

The fermions get their masses from Yukawa-like interactions with the doublet Higgs field.

$$\mathcal{L}_A = f^e \bar{l}_L \Phi e_R + f^d \bar{q}_L \Phi d_R + f^u \bar{q}_L \tilde{\Phi} u_R + h.c \quad (2.38)$$

$$\tilde{\Phi} = i\tau_2 \Phi^* \quad (2.39)$$

$$Y(\tilde{\Phi}) = -1. \quad (2.40)$$

We note that to give masses to both the u and the d quarks, we need in addition to the Higgs doublet its conjugate with hypercharge $Y = -1$. The Higgs mechanism for generating masses introduces an arbitrary coupling parameter for each fermion mass, and hence provides no fundamental understanding of the masses of the fermions. The neutrinos cannot acquire masses or couplings to the H_1^0 field in an analogous way since by fiat there are no ν_R fields in SW model. The fermion structure of the SW model, among other things, is chosen to comply with the following restrictions:

⁶One may regard θ_W as the angle between the weak and electromagnetic interactions.

1. : It is experimentally observed that flavor changing neutral currents are highly suppressed [40],
2. : In a renormalizable gauge theory there should be anomaly [40, 41] cancellation, otherwise it would lead to the break down of renormalization and consequently the theory would lose its predictive power.

In order to cope with the first restriction one introduces the Glashow, Iliopoulos and Maiani mechanism (GIM) [36] for quark mixing. The GIM mechanism was validated when the charm or the c quark was observed (with properties predicted for it by GIM) in 1974. The discovery of the τ lepton (Perl et al. [38], 1975) and the bottom or b quark of charge $Q = -1/3$ (Herb et al. [39], 1977) suggests a third generation of quarks and leptons. Assigning the b quark to the $T_{3f} = -1/2$ state of a weak left handed doublet thus predicts the existence of another quark t or top quark with charge $Q = +2/3$, see Fig. 2.1. Hence having three families one generalizes the GIM mechanism (which works for 2 families) to the Cabibbo-Kobayashi-Maskawa (hereafter referred to as CKM) (for 3 families) in order to cope with the weak decays of the quarks of the second and the third generations. The discussion of CKM mixings does not directly have a bearing on this thesis so we relegate some relevant information on this subject to Appendix D. The anomalies cancel due to the quantum numbers chosen for the fermions [see Table 2.1]. The observed universe around us consists largely of the first family. However, it is believed that the two more exotic families must have flourished in the early universe. The SW model does not predict the number of families. So there could be more families than three as long they follow the pattern indicated for the fermions. Recent experiments have (assuming that neutrinos continue to have small masses relative to the other fermions) shown that there is no room for the number of

$$\begin{array}{l} \text{quarks } \left(\begin{array}{c} u \\ d' \end{array} \right)_L, \left(\begin{array}{c} c \\ s' \end{array} \right)_L, \left(\begin{array}{c} t \\ b' \end{array} \right)_L \\ \text{leptons } \left(\begin{array}{c} \nu_e \\ e \end{array} \right)_L, \left(\begin{array}{c} \nu_\mu \\ \mu \end{array} \right)_L, \left(\begin{array}{c} \nu_\tau \\ \tau \end{array} \right)_L \end{array}$$

Figure 2.1: The standard model fermion generation structure. The mass of each fermion generation increases from left to right. Also in each quark-doublet the mass decreases from top to bottom, and in the lepton-doublets this is reversed. d', s', b' are the CKM mixed eigenstates of d, s, b

families to exceed three. All the particles in Fig. 2.1 have been observed with the exception of the top quark. This implies that if the SW model is correct then the top must be very massive [3], $m_t \geq 75 \text{ GeV}$ ⁷

In terms of the physical fields the SW Lagrangian can be written as, using the definitions (2.28 through 2.37)

$$\mathcal{L}_{SW} = \sum_{f,c} \mathcal{L}_{f1} + \sum_{d,c} \mathcal{L}_{f2} + \mathcal{L}_{II} + \mathcal{L}_{III} + \mathcal{L}_{IV} \quad (2.41)$$

The explicit form of each term in eq. 2.41 is displayed in Appendix B. Quantization requires gauge fixing terms [40] which we choose as, (using 't Hooft-Feynman gauge)

$$\mathcal{L}_{gf} = -\frac{1}{2}(\partial^\mu A_\mu)^2 - |\partial_\mu W_\mu^+ - iM_W^2 G^+|^2 - \frac{1}{2}(\partial_\mu Z_\mu - M_Z G^0)^2 \quad (2.42)$$

The corresponding Faddeev-Popov (FP) ghost [40] Lagrangian is given in Appendix B. Adding the gauge fixing and FP Lagrangians to the SW Lagrangian one obtains the effective Lagrangian for the EW model

$$\mathcal{L}_{eff} = \mathcal{L}_{SW} + \mathcal{L}_{gf} + \mathcal{L}_{FP}. \quad (2.43)$$

The Feynman rules can now be read off from this Lagrangian.

⁷Recent experiments [4] indicate that $m_t = 150 \pm 22 \pm 18 \text{ GeV}$.

Let us now review some specific details of the predictions of the SW model and their confrontation with experimental measurements. The particle content of the SW model consists of 4 spin one particles i.e. the gauge bosons which mediate the electroweak interactions between the fermions (see Fig 2.1). The gauge bosons of the SW model are the charged W-bosons, the Z-boson, and the photon, which are denoted respectively by $[W^+, W^-, Z_\mu, A_\mu]$. The W-boson is responsible for the so called charged electroweak current whereas the Z-boson is responsible for the weak neutral current.⁸ Last but not least, the photon arises from the electromagnetic current. Since QED is so successful the SW model must incorporate this field in a manner which leaves it unchanged. This is indeed achieved in the EW model.

The actual explicit observation of the vector bosons W and Z indeed constitutes an amazing vindication of the SW model. Besides this triumph the SW model also correctly predicts the couplings between the known fermions and the vector bosons, up to energies of about 100 GeV, modulo radiative corrections. At the lowest order there are three free parameters g, g' and v in the SW model. g describes the self-coupling of the Non-Abelian $SU(2)$ gauge fields and the coupling of these fields to the fermions and the Higgs doublet before SSB. Similarly g' couples the Abelian $U(1)_Y$ to the fermions and the Higgs before SSB. Finally v is the ground state or vacuum expectation value of the Higgs. This value gives masses to the gauge bosons and the fermions. The formulae [2.29, 2.31, 2.36, and 2.37] above imply the important relation

$$\frac{M_W^2}{M_Z^2 c_W^2} = \rho, \quad (2.44)$$

⁸Weak neutral currents were discovered by observation of the reaction $\nu_\mu e \rightarrow \nu_\mu e$ in a heavy liquid bubble chamber (Hasert et al. [32], 1973). This discovery inspired the search for the actual observation of the gauge bosons. These heavy gauge bosons were observed in pp reactions (Arnison et al. [33], 1983; Banner et al. [34], 1983). Bagnaia et al. [35], 1983).

$$\rho = 1 \quad (2.45)$$

Experimentally [3], this relation holds to within 1% accuracy, and so it is important to understand its origin. In this analysis, (2.45) would seem to be a special consequence of the assumption that the Higgs field is the agent which breaks $SU(2) \times U(1)$. But it may be shown that (2.45) holds at zeroth order in any model in which the field, which acquires the vacuum expectation value, is an isodoublet. The SW model predicts the following relations at the lowest order or the tree level

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu s_W^2} \quad (2.46)$$

$$\frac{M_W^2}{M_Z^2} = 1 - s_W^2 \quad (2.47)$$

In Eq. 2.46 we have exploited the following relation⁹ between the Fermi constant, G , and g

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2}, \quad (2.48)$$

and then used Eq 2.46 to express G in terms of α , the fine structure constant. From the above relations (2.46, 2.47, and 2.48) we can predict the gauge boson masses provided we input the values for the 3 parameters. Two observables are known with very high precision at low momentum transfer. These are the value of the basic electric charge

$$\alpha = [137.03604(11)]^{-1} \quad (2.49)$$

obtained from Thomson scattering [3] or better still from more recent measurement of the Josephson effect [3], and the value of the Fermi constant

$$G_\mu = 1.166344(11) \times 10^{-5} (GeV)^{-2} \quad (2.50)$$

⁹Eq 2.48 follows since the Fermi theory of 4 fermion point interaction must result if one takes the low momentum transfer limit of the SW model [40].

obtained from the muon lifetime [3]. Using these two values and the value of s_W^2 obtained from neutrino-quark scattering [3]

$$s_W^2 = .231 \pm .006, \quad (2.51)$$

in Eqs. 2.46 and 2.47 one arrives at

$$M_W = 75.9 \pm 0.9 \text{ GeV}, \quad (2.52)$$

$$M_Z = 87.1 \pm 0.7 \text{ GeV}, \quad (2.53)$$

Experimentally, [3], combining all the measurements from UA2, UA1, MARK II, ALEPH, DELPHI, L3, OPAL and CDF, one has

$$(M_W)_{\text{expt}} = 80.6 \pm 0.4 \text{ GeV}, \quad (2.54)$$

$$(M_Z)_{\text{expt}} = 91.161 \pm 0.031 \text{ GeV}. \quad (2.55)$$

The difference between Eqs. 2.52, 2.53 and Eqs. 2.54, 2.55 clearly indicates the need for inclusion of radiative corrections (rc). Radiative corrections completed [3] up to one loop give values for the masses of the W and Z bosons, which depend on the assumed mass value for the top quark. If the mass of the top quark is taken to be 100 (200)¹⁰ GeV, one gets

$$(M_W)_{\text{rc}} = 79.6(78.6) \pm 0.9 \quad (2.56)$$

$$(M_Z)_{\text{rc}} = 90.8(89.6) \pm 0.7 \quad (2.57)$$

These numbers are mentioned here to give a feeling for the effect of radiative corrections and their size and dependence on top quark mass.

¹⁰The first (second) values for the W and Z boson masses are for $m_t = 100(200)$ GeV.

Table 2.1: The Weak Quantum Numbers for the first generation of quarks and leptons.

	T	T_3	Y	Q
ν_{eL}	$\frac{1}{2}$	$\frac{1}{2}$	-1	0
e_L	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
u_L	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$
d_L	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$
e_R	0	0	-2	-1
u_R	0	0	$\frac{4}{3}$	$\frac{2}{3}$
d_R	0	0	$-\frac{2}{3}$	$-\frac{1}{3}$

2.3 Some remarks on Dimensional Regularization and Renormalization

Since there are many references on the subject of renormalization and regularization [40, 41, 42] we confine ourselves to a few simple remarks and some straightforward illustrations.

One encounters divergent integrals in perturbative field theory calculations. One must regulate these integrals or isolate the infinities in these integrals before extracting the physical content via renormalization. Crudely speaking a field theory is said to be renormalizable if the loop effects do not generate terms that are absent at the tree level¹¹

$$\tilde{\mathcal{L}}_R(\phi_R, g_R, m_R) = \tilde{\mathcal{L}}_B(\phi_B, g_B, m_B) \quad (2.58)$$

This has several consequences

1. The theory has predictive power since once we specify a set of input independent variables at the tree level. This suffices at any loop level i.e. we don't have to introduce any more input variables.
2. The infinities can be got rid of in a systematic manner. There are a countable number of infinities.
3. If the field theory possesses gauge invariance, the infinities are further constrained by gauge invariance. These constraints are called Ward identities which arise due to the conservation laws which are themselves a consequence of the underlying symmetry (gauge invariance) of the field theory.

¹¹In general one can write for any Lagrangian after loop effects have been included $\tilde{\mathcal{L}}_R(\phi_R, g_R, m_R) + C.T = \tilde{\mathcal{L}}_B(\phi_B, g_B, m_B)$ for a renormalisable one this equation reduces to Eq. 2.58

In short one can calculate quantum corrections to any degree of accuracy for any physical quantity.

To give a concrete realization of the above remarks let us consider the case of QED. The Lagrangian for QED is

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}_f\gamma^\mu(i\partial_\mu + eA_\mu)\Psi_f - m_f\bar{\Psi}_f\Psi_f \quad (2.59)$$

In view of the above remarks we introduce four renormalization constants Z_m , Z_1 , Z_2 and Z_3 for the two parameters m and e , and fermion field and photon field. For renormalizability alone, it is sufficient that all divergences be absorbed into these four constants. From gauge symmetry $Z_1 = Z_2$ hence we can write the QED Lagrangian plus its counterterms as

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}Z_3F_{\mu\nu}F^{\mu\nu} + Z_2\bar{\Psi}_f\gamma^\mu(i\partial_\mu + eA_\mu)\Psi_f - Z_m m_f\bar{\Psi}_f\Psi_f \quad (2.60)$$

$$Z = 1 + \delta Z \quad (2.61)$$

Now renormalizability is established by showing that all divergences may be absorbed into the three constants. This is indeed the case, as is well known [42]. We note that 2.59 has same form as 2.60. Coming back to regularization schemes, there are many, such as dimensional reduction, Pauli-Villars regularization, dimensional regularization (DR) and more recently operator regularization. In DR the spacetime dimensionality is analytically continued from the real physical value, four, to a complex generic value, n . With real n sufficiently small the ultraviolet behaviour is convergent; eventually the limit $n \rightarrow 4$ is taken with the poles in $(n - 4)^{-1} \rightarrow (\varepsilon)^{-1}$ defining counterterms. More specifically the infinities are isolated as poles of the Γ function. Let us consider an example to illustrate DR; to this end, let us consider the integral

$$I_1(m) = \int \frac{d^4q}{2\pi^4} \frac{1}{(q^2 - m^2)^2}. \quad (2.62)$$

Clearly 2.62 is divergent in 4D (four dimensions), whereas it is convergent in 3D.

In general in $n = 4 + \varepsilon$ dimensions

$$I_n(m) = \int \frac{d^n q}{2\pi^n} \frac{1}{(q^2 - m^2)^2} \quad (2.63)$$

$$I_n(m) = \frac{i}{16\pi^2} \Gamma(-\frac{\varepsilon}{2}) \left(\frac{4\pi}{m^2}\right)^{-\frac{\varepsilon}{2}} \quad (2.64)$$

which has a pole at $\varepsilon = 0$

$$I_{4+\varepsilon}(m) = \frac{i}{16\pi^2} \left[-\frac{2}{\varepsilon} - \gamma_E + \ln\left(\frac{4\pi\mu^2}{m^2}\right) + O(\varepsilon) \right] \quad (2.65)$$

where

$$\gamma_E = -\Gamma'(1) \simeq .577251 \quad (2.66)$$

is the Euler-Mascheroni constant. The divergent part of 2.63 is now uniquely identified by DR, but the finite part needs some care. In fact the separation of the finite part still involves ambiguities which are discussed in detail by Sakakibara [43]. In our calculation of the one loop corrections to the process $e^+e^- \rightarrow W^+W^-$ we encounter ultraviolet (UV), linear and infrared (IR) divergences. To cope with UV and linear divergences we use DR together with the notation (Appendix C)

$$\Delta = -\frac{2}{\varepsilon} - \gamma_E + \ln\left(\frac{4\pi\mu^2}{M_W^2}\right) + O(\varepsilon) \quad (2.67)$$

To deal with the IR divergences the photon mass λ is introduced. This can be achieved, without destroying gauge invariance, by a SSB, the effects of which vanish as $\lambda \rightarrow 0$ except for normal $\ln \lambda$ in the infrared divergent terms. The UV and linear divergences are dealt with in the remainder of this Chapter and Chapter 4. The IR divergences are dealt with in Chapter 5.

2.4 On Mass Shell Renormalization Scheme

In this section we introduce and give some details of the on mass shell renormalization scheme (OMRS) as worked out by Sakakibara [43]. In chapter 4 we apply this subtraction procedure to the one loop corrections of the $e^+e^- \rightarrow W^+W^-$ reaction in SW model and its supersymmetric extension. In both cases we find that infinities cancel, thus demonstrating the consistency of the adopted renormalization scheme and the correctness of the Feynman rules. Cancellation of these infinities is not an academic exercise for it acts as an excellent check on the correctness of the finite pieces.

In the SW model the set of independent parameters we choose in the OMRS are $(\alpha, M_W, M_Z, M_{H_1^0}, m_f)$. Here m_f represents collectively the fermion masses. We note that the number of independent input parameters has not changed from the tree level, as should be the case in a renormalizable theory. In Appendix E we discuss the changes when G_μ is chosen instead of α as an input parameter. The renormalization conditions in the OMRS are

1. The charge e is defined in the Thomson limit, and
2. All one particle irreducible two-point functions (including the vanishing of γZ mixing at $k^2 = 0$) have their zeroes at their physical masses.

The gauge-invariant counterterms are generated by the following rescalings of the bare fields

$$A_\mu^a \rightarrow (Z_W)^{1/2} A_\mu^a \quad (2.68)$$

$$B_\mu \rightarrow (Z_B)^{1/2} B_\mu \quad (2.69)$$

$$L_f \rightarrow (Z_L)^{1/2} L_f \quad (2.70)$$

$$R_f \longrightarrow (Z_R)^{1/2} R_f \quad (2.71)$$

$$\Phi \longrightarrow (Z_\phi)^{1/2} \Phi \quad (2.72)$$

$$g \longrightarrow (Z_W)^{-1/2} (g + \delta g) \quad (2.73)$$

$$g' \longrightarrow (Z_B)^{-1/2} (g' + \delta g') \quad (2.74)$$

in the Lagrangian $\mathcal{L}$. Here the fields and coupling constants on the right-hand side are the renormalized ones. We then express the Lagrangian in terms of the physical fields. Alternatively, we can rewrite the expressions 2.68 through 2.74 in terms of the physical fields and plug these into the physical Lagrangian. Note that the counterterm Lagrangian follows, when each of the Z is written as $Z = 1 + \delta Z$. The explicit form of the counterterm Lagrangian is given in Appendix B. Similarly the renormalization constants for the ghosts are generated by the above rescalings and

$$c^a \longrightarrow \bar{Z}_W^{-1/2} c^a \quad (2.75)$$

$$c^0 \longrightarrow \bar{Z}_B^{-1/2} c^0 \quad (2.76)$$

We start implementing the OMRS by first determining the counterterms δg and $\delta g'$. This is done by requiring that the photon vertex function of the lepton defines the physical (renormalized) charge. This means that δg and $\delta g'$ must be independent of the lepton mass m ; otherwise the renormalized charge would be different for the electron and the muon, destroying universality of EM coupling. For this consistency check, all powers of m^2/M_W^2 are kept in the section and wherever the question of universality arises. In order to determine δg , $\delta g'$, δZ_L and $\delta Z'_R$, we must consider the lepton self-energies and the photon-lepton vertex. To determine δZ_L we choose to make a subtraction on the neutrino mass shell. Let $-i\nu(k^2)\not{k}^{\frac{1-\gamma_5}{2}} = -i\nu(k^2)\not{k}L$ be the contribution from one loop diagrams

[Chapter 6] to the ν_e self energy. The counterterm contribution reads in this case, Appendix B, is $i\delta Z_L \not{L}$. Then renormalized contribution $\tilde{\nu}$ may be written as

$$-i\tilde{\nu}(k^2)\not{L} = -i[\nu(k^2) - \delta Z_L]\not{L} \quad (2.77)$$

Hence one has

$$\tilde{\nu}(k^2) = \nu(k^2) - \nu(0) \quad (2.78)$$

To subtract on mass shell we must have $\tilde{\nu}(0) = 0$, on using this condition in 2.77 one immediately has

$$\delta Z_L = \nu(0), \quad (2.79)$$

so that now δZ_L is determined. Similarly let $-i\Sigma$ denote the self-energy contribution of the charged lepton [Chapter 6]. This has the structure

$$-i\Sigma(k) = -i(\sigma_L(k^2)\not{L} + \sigma_R(k^2)\not{R} + \rho(k^2)m) \quad (2.80)$$

Adding the counterterm contribution to this we have

$$-i\tilde{\Sigma}(k) = -i(\Sigma(k) - \delta Z_L \not{L} - \delta Z_R \not{R} + \delta m) \quad (2.81)$$

To determine δZ_R , since δZ_L has already been determined, we must impose one condition which we take to be

$$\frac{\tilde{\Sigma}(k)}{k - m} \longrightarrow f_L L \quad (2.82)$$

in the limit as $k \rightarrow m$. This means that the right handed lepton (l_R) is subtracted on mass shell. Clearly we cannot subtract l_L on mass shell since it is the partner of the neutrino which has been subtracted on mass shell. Therefore f_L must be a non zero finite number. The condition 2.82 fixes δZ_R and f_L as

$$\delta Z_R = \sigma_{R0} + 2\sigma_{R1} + 2\rho_1 \quad (2.83)$$

$$f_L + \delta Z_L = \sigma_{L0} + 2\sigma_{L1} + 2\rho_1 \quad (2.84)$$

where σ_i, ρ_i are coefficients of the expansion of $\sigma(k^2), \rho(k^2)$ about $k^2 = m^2$. We now give the results [Chapter 6] obtained after calculation of the self energy diagrams (considering only the SM particles circulating in the loops since at the moment we are dealing with the QFD case) and then use these in conjunction with the above definitions to obtain the renormalization constants. δZ_L follows immediately from the neutrino self energy diagrams (see diagrams 1, 2, and 3 in Fig. 6.3)

$$\delta Z_L = \delta Z_L^W + \delta Z_L^Z + \delta Z_L^G \quad (2.85)$$

where (the symbol $\doteq$ means that the right-hand side must be multiplied by a common factor of $\frac{g^2}{16\pi^2}$)

$$\delta Z_L^W \doteq -\frac{1}{2}[\Delta - \frac{1}{2}], \quad (2.86)$$

$$\delta Z_L^Z \doteq -\frac{1}{4c_W^2}[\Delta + \ln(\frac{M_W^2}{M_Z^2}) - \frac{1}{2}], \quad (2.87)$$

$$\delta Z_L^G \doteq -\frac{m^2}{4M_W^2}[\Delta + \frac{1}{2}]. \quad (2.88)$$

Similarly the electron self energy diagram yield the following results for δZ_R and f_L

$$\delta Z_R^\gamma \doteq -s_W^2[\Delta + \ln(\frac{M_W^2}{m^2}) + 4 + 2\ln(\frac{\lambda^2}{m^2})] \quad (2.89)$$

$$\begin{aligned} \delta Z_R^Z \doteq & \frac{2s_W^4}{c_W^2}[-\frac{1}{2}\Delta + \frac{1}{2} + I_1(M_Z, m, m) - 2\frac{m^2}{M_Z^2}I_{12}(M_Z, m, m)] - \\ & \frac{4s_W^2 \cos 2\theta_W}{c_W^2} \frac{m^2}{M_Z^2} I_{11}(M_Z, m, m) \end{aligned} \quad (2.90)$$

where the integrals I_1, I_{11}, I_{12} are defined in Appendix C. Since we expect f_L to be finite we use the following groupings. This allows us to see the cancellation of infinities immediately within each group.

$$f_L^W + \delta Z_L^W \doteq -\frac{1}{2}\Delta + \frac{1}{2} + I_1(M_W, 0, m) - 2\frac{m^2}{M_W^2}I_{12}(M_W, 0, m) \quad (2.91)$$

$$\begin{aligned}
f_L^Z + f_L^\gamma + \delta Z_L^Z &\doteq \frac{\cos^2 2\theta_W}{4c_W^2} \left[-\Delta - \ln\left(\frac{M_W^2}{M_Z^2}\right) + 1 + 2I_1(M_Z, m, m) \right. \\
&\quad \left. - 4\frac{m^2}{M_Z^2} I_{12}(M_Z, m, m) \right] - \frac{4s_W^2 \cos 2\theta_W}{c_W^2} \frac{m^2}{M_Z^2} I_{11}(M_Z, m, m) \\
&\quad - s_W^2 \left[\Delta + \ln\left(\frac{M_W^2}{m^2}\right) + 4 + 2\ln\left(\frac{\lambda^2}{m^2}\right) \right] \quad (2.92)
\end{aligned}$$

$$\begin{aligned}
f_L^{G^0} + f_L^{H_1^0} + \delta Z_L^{G^0} &\doteq -\frac{m^2}{4M_W^2} \left[\Delta - I_1(M_Z, m, m) + 2\frac{m^2}{M_W^2} I_{12}(M_Z, m, m) - \right. \\
&\quad \left. 2\frac{m^2}{M_W^2} I_{11}(M_Z, m, m) - I_1(M_{H_1^0}, m, m) \right] + (2.93) \\
&\quad \left. 2\frac{m^2}{M_W^2} I_{12}(M_{H_1^0}, m, m) - 2\frac{m^2}{M_W^2} I_{11}(M_{H_1^0}, m, m) \right]
\end{aligned}$$

Plugging the values obtained for δZ_L in the above expressions. We see that divergences cancel in each group and f_L becomes finite,

$$f_L \doteq -s_W^2 \left[+\ln\left(\frac{M_Z^2}{m^2}\right) + \frac{9}{2} + 2\ln\left(\frac{\lambda^2}{m^2}\right) \right] \quad (2.94)$$

Now we turn to the determination of the counterterms δg and $\delta g'$. To this end we consider the lepton-lepton-photon vertex. We may write these contributions as

$$-ie\Lambda_L\gamma^\mu L - ie\Lambda_R\gamma^\mu R, \quad (2.95)$$

we find

$$\delta Z_R^A + \Lambda_R^A = 0, \quad (2.96)$$

where

$$A = Z, \gamma, G, G^0, H_1^0. \quad (2.97)$$

The contribution Λ_R due to the Z-boson in Eq. 2.96 is (we do not repeat here the contributions from the other particles [44]),

$$\begin{aligned}
\Lambda_R^Z &\doteq \frac{2s_W^4}{c_W^2} \left[+\frac{1}{2} \Delta - 1 - I_0(M_Z, m, m) + \frac{m^2}{M_Z^2} (I_{12}(M_Z, m, m) + \right. \\
&\quad \left. I_{10}(M_Z, m, m)) \right] + \frac{4s_W^2 \cos 2\theta_W}{c_W^2} \frac{m^2}{M_Z^2} I_{11}(M_Z, m, m). \quad (2.98)
\end{aligned}$$

Summing all these contributions (see Eq. 2.96) we find

$$\delta Z_R + \Lambda_R = 0. \quad (2.99)$$

The relevant counterterm contribution for this case is [Appendix B]

$$\Lambda_R^c = \delta Z_R + \frac{\delta g'}{g} \cot \theta_W. \quad (2.100)$$

The renormalization condition implies

$$\Lambda_R^c + \Lambda_R = 0. \quad (2.101)$$

Hence we have 2.100, 2.101

$$\Lambda_R + \delta Z_R + \frac{\delta g'}{g} \cot \theta_W = 0 \quad (2.102)$$

This i.e. 2.102 together with 2.99 fixes $\delta g'$,

$$\delta g' = 0. \quad (2.103)$$

The argument for the left handed component is similar

$$\Lambda_L^W \doteq \frac{3}{2} \Delta - \frac{1}{2} - I_1(M_W, 0, m) + 2 \frac{m^2}{M_W^2} I_{12}(M_W, 0, m) \quad (2.104)$$

$$f_L^W + \delta Z_L^W + \Lambda_L^W \doteq \Delta, \quad (2.105)$$

$$f_L^Z + f_L^\gamma + \delta Z_L^Z + \Lambda_L^Z + \Lambda_L^\gamma = 0, \quad (2.106)$$

$$f_L^{G^0} + f_L^{H_1^0} + \delta Z_L^G + \Lambda_L^{G^0} + \Lambda_L^{H_1^0} = 0. \quad (2.107)$$

The sum of these terms is

$$f_L + \delta Z_L + \Lambda_L \doteq \Delta. \quad (2.108)$$

The relevant counterterm contribution for this case is

$$\Lambda'_L = \delta Z_L + \frac{\delta g'}{2g} \cot \theta_W + \frac{\delta g}{2g}. \quad (2.109)$$

The renormalization condition implies

$$\Lambda'_L + \Lambda_L = 0. \quad (2.110)$$

Hence we have 2.109, 2.110

$$f_L + \Lambda_L + \delta Z_L + \frac{\delta g'}{2g} \cot \theta_W + \frac{\delta g}{2g} = 0 \quad (2.111)$$

Eq. 2.111 together with 2.108 fixes δg and we find

$$\delta g \doteq -2g \Delta. \quad (2.112)$$

We now turn to the gauge boson sector and determine the renormalization constants relevant to this sector. These are δM_W^2 , δM_Z^2 , Z_W and Z_B . At first one may think that we can simply subtract on mass shell of W and Z and determine these four constants. However the photon does not carry an independent wave function renormalization constant(WFRC), hence we cannot subtract on shell of the photon. This will not only complicate the determination of δg but will also destroy the Ward identities. To satisfy the the two conditions

1. : subtract photon on shell
2. : make subtraction which maintain the Ward identities

we must calculate ghost self-energies. There are four FP ghost fields in SW theory [43]. If $ie^W(k^2)$ and $ie^Z(k^2)$ denote the sum of all one loop contributions to the c^+

and c^z self-energy respectively. Then the corresponding renormalized self-energies read after account is taken of the counter terms, section 2.2 and Appendix B

$$\bar{e}^W(k^2) = e^W(k^2) + \delta\bar{Z}_W(k^2 - M_W^2) - M_W\delta M_W \quad (2.113)$$

$$\bar{e}^Z(k^2) = e^Z(k^2) + \delta\bar{Z}_Z(k^2 - M_Z^2) - M_Z\delta M_Z \quad (2.114)$$

Gauge invariance of the theory requires that the ghost self-energy must be related to the self-energy of the corresponding gauge- boson via a Ward identity. This relation takes a particularly simple form in the 't Hooft-Feynman gauge. In this gauge the FP ghosts as well as the Goldstone fields are nothing but longitudinal degrees of freedom of the corresponding gauge bosons. This is easily seen by looking at the propagators. c^+ has a pole at M_W^2 in its propagator, likewise c^z has a pole at M_Z^2 . If these poles are to be maintained at the same place after renormalization, then we must require

$$\bar{e}^W(M_W^2) = 0 \quad (2.115)$$

$$\bar{e}^Z(M_Z^2) = 0 \quad (2.116)$$

These conditions in conjunction with relations 2.113 and 2.114 give

$$\delta M_W^2 = 2e^W(M_W^2) \quad (2.117)$$

$$\delta M_Z^2 = 2e^Z(M_Z^2) \quad (2.118)$$

Carrying out the actual calculation of the FP ghosts self-energies we find

$$\delta Z_\phi + \frac{\delta M_W^2}{M_W^2} \doteq (\frac{1}{2}t_W^2 - 2\frac{1}{2})\Delta - 4s_W^2 - (3 - \frac{1}{2}t_W^2 + 2s_W^2)[B_0(M_Z, M_W, M_W^2) - \Delta] + \frac{1}{2}[B_0(M_{H_0^1}, M_W, M_W^2) - \Delta], \quad (2.119)$$

$$\delta Z_\phi + \frac{\delta M_Z^2}{M_Z^2} \doteq (4s_W^2 + \frac{1}{2}t_W^2 - 2\frac{1}{2})\Delta - (3 - 4s_W^2)[B_0(M_W, M_W, M_Z^2) - \Delta] + \frac{1}{2}\sec^2_W[B_0(M_{H_0^1}, M_Z, M_Z^2) - \Delta]. \quad (2.120)$$

$$\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} \doteq -4s_W^2 \Delta + \delta\chi, \quad (2.121)$$

$\delta\chi$ is the finite piece

$$\begin{aligned} \delta\chi = & -\left(3 - \frac{1}{2}t_W^2\right)[B_0(M_Z, M_W, M_W^2) - \Delta] + \frac{1}{2}[B_0(M_{H_0^1}, M_W, M_W^2) - \Delta] \\ & + \left(3 - 4s_W^2\right)[B_0(M_W, M_W, M_Z^2) - \Delta] - \frac{1}{2}\sec^2_W[B_0(M_{H_0^1}, M_Z, M_Z^2) - \Delta] - 4s_W^2. \end{aligned} \quad (2.122)$$

Translating 2.122 into explicit integral form, Appendix C, and defining $R_1 = \frac{M_Z^2}{M_W^2}$, $R_2 = \frac{m_{H_0^1}^2}{M_W^2}$, we get

$$\begin{aligned} \delta\chi = & -[(3 - 4s_W^2) \int_0^1 dx \ln(1 - x(1 - x)R_1) \\ & - \frac{1}{2c_W^2} \int_0^1 dx \ln(x^2 R_1 + (1 - x)R_2) + 4s_W^2 \\ & - (3 - \frac{1}{2}t_W^2 + 2s_W^2) \int_0^1 dx \ln(x^2 + (1 - x)R_1) \\ & + (\frac{1}{2}) \int_0^1 dx \ln(x^2 + (1 - x)R_2)]. \end{aligned} \quad (2.123)$$

Now we turn to determine the WFR constants Z_W and Z_B . To this end consider the photon self-energy contributions chapter 6. After taking into account the counterterm, the renormalized photon self-energy may be written as

$$i\tilde{\pi}_{\mu\nu}^\gamma(k^2) = i\tilde{\pi}^\gamma(k^2)[k^2 g_{\mu\nu} - k_\mu k_\nu] \quad (2.124)$$

$$\tilde{\pi}^\gamma(k^2) = \pi^\gamma(k^2) - [\delta Z_W s_W^2 + \delta Z_B c_W^2] \quad (2.125)$$

Making the subtraction on shell requires the condition

$$\tilde{\pi}(0) = 0 \quad (2.126)$$

Putting this condition in 2.125 one has,

$$\pi^\gamma(0) = \delta Z_A = \delta Z_W s_W^2 + \delta Z_B c_W^2 \quad (2.127)$$

We proceed similarly for the W and Z gauge bosons. We first write their self energies [Chapter 6].

$$i\pi_{\mu\nu}^{Z,W}(k^2) = ia^{Z,W}(k^2)[g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}] + ib^{Z,W}(k^2)\frac{k_\mu k_\nu}{k^2} \quad (2.128)$$

In our gauge we need only consider the coefficient of $g_{\mu\nu}$ i.e a. The renormalized a's read after the appropriate counter terms have been included, Appendix B

$$\tilde{a}^{Z,W}(k^2) = a^{Z,W}(k^2) - \delta Z_{Z,W} k^2 + \delta Z_\phi M_{Z,W}^2 + \delta M_{Z,W}^2, \quad (2.129)$$

with the notation

$$\delta Z_Z = \delta Z_W c_W^2 + \delta Z_B s_W^2. \quad (2.130)$$

Once again we note that in our gauge the relevant Ward identity reduces to the statement that the renormalized propagators must maintain the poles at $k^2 = M_{Z,W}^2$ so that it follows that

$$\tilde{a}(M_{Z,W}^2) = 0, \quad (2.131)$$

and consequently

$$\frac{a^{Z,W}(M_{Z,W}^2)}{M_{Z,W}^2} - \delta Z_{Z,W} + \delta Z_\phi + \frac{\delta M_{Z,W}^2}{M_{Z,W}^2} = 0. \quad (2.132)$$

By eliminating δZ_ϕ from 2.132, we obtain

$$Y_{WZ} = \delta Z_W - \delta Z_Z \quad (2.133)$$

$$Y_{WZ} = [\delta Z_W - \delta Z_B] s_W^2 \quad (2.134)$$

where we have defined

$$Y_{WZ} = \frac{a^W(M_W^2)}{M_W^2} - \frac{a^Z(M_Z^2)}{M_Z^2} + \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}, \quad (2.135)$$

Finally, from 2.127, 2.130, 2.133 and 2.134 we find

$$\delta Z_B = \pi^\gamma(0) - Y_{WZ}, \quad (2.136)$$

$$\delta Z_W = \pi^\gamma(0) + Y_{WZ} \cot^2 \theta_W, \quad (2.137)$$

$$\delta Z_Z = \pi^\gamma(0) + Y_{WZ}(\cot^2 \theta_W - 1). \quad (2.138)$$

Having found these constants, the renormalized W and Z self-energies are given by, on using the definitions given in 2.129

$$\bar{a}^{W,Z}(k^2) = a^{W,Z}(k^2) - a^{W,Z}(M_{W,Z}^2) - \delta Z_{W,Z}(k^2 - M_{W,Z}^2) \quad (2.139)$$

Chapter 3

The Supersymmetric Weak Lagrangian

3.1 Introduction

When studying the phenomenology of various processes involving supersymmetric particles, it is convenient to choose some model. However it is desirable to proceed as much as possible in a model independent way. With this in mind we have chosen to work with a softly broken supersymmetric extension of QFD (from now on referred to as SQFD) of Haber and Kane [25] The complete Lagrangian is given in [25], [44] (PhD Thesis Notes) and will not be repeated here, but the motivation, general structure and the relevant parts will be elucidated in the following sections.

The objective of this chapter is three fold

1. To describe and summarize the essential features of Supersymmetric Weak Lagrangian when supersymmetry is not broken but the gauge symmetry is broken.
2. To give a discussion of the cancellation of quadratic and linear divergences first in context of a toy model (i.e Wess-Zumino). We have demonstrated these cancellations in the context of SQFD, we include some details of this

in the Appendix F.

3. To briefly discuss the breaking of the supersymmetry by the soft-terms.

We now summarize some of the salient features of Supersymmetry mentioned in the introductory chapter. Supersymmetry is one of the most elegant extensions of the standard model. It solves the hierarchy problem, one of the major drawbacks of grand unified theories, by introducing a Fermion-Boson symmetry. It is precisely this beautiful property of supersymmetry which provides a hope of unifying all forces of nature ¹, and also allowing forces and matter to be treated on the same footing. As a consequence of the Fermion-Boson symmetry, many new degrees of freedom corresponding to the supersymmetric partners (S-P) of the ordinary particles are predicted by the theory. However aesthetically appealing a theory might be, it must stand the test of experiment. Therefore it is of crucial interest to explore all the phenomenological implications of supersymmetric theories in order to eventually confront experiment. A lot of work has been done on supersymmetric phenomenology [49, 25, 50, 5]. The effect of supersymmetric particles on the physical properties such as $g - 2$ of leptons [51], the two-photon decay widths [52], the magnetic and the quadrupole [72] moments of the W-boson has received some attention. Radiative corrections in N=1 supersymmetry in the context of neutral current scattering $e\nu_\mu \rightarrow e\nu_\mu$ have been given Grifols et al. [54] and Schwarzer [55], for $e^+e^- \rightarrow \mu^+\mu^-$ by Lynn [56]. In reference [54] no mention is made of the renormalization scheme adopted. Schwarzer [55] adopts the MS scheme and finds the electroweak parameters dependence on the supersymmetric effects to be small; for example, a 0.002 increase is found in the value of $\sin^2 \theta_W(\mu^2)$ at scale $\mu^2 = M_W^2$ and a small decrease of the order of a few MeV in the predicted gauge boson

¹We mean generalisations of global supersymmetry like supergravity and superstring theories.

masses. A good review of phenomenology and work on the radiative corrections both in and beyond the standard model may be found in [49]. One of the main purposes of this thesis is to give the effect of introducing S-P on the one loop radiative corrections to the process $e^+e^- \rightarrow W^+W^-$.

3.2 The Supersymmetric Electroweak Lagrangian

In field theory if one ignores gravity there are only three types of fields that one encounters. Scalar fields (spin 0) fields, Fermi fields (spin $\frac{1}{2}$) and vector fields (spin 1) fields.² A very convenient and common approach used in field theory to formulate problems is via the method of Lagrangians and the method of least Action. The Action is related to the Lagrangian.³

$$S = \int d^4x \mathcal{L} \quad (3.1)$$

In classical theory the Action has units of angular momentum ML^2T^{-1} or equivalently the units of $\hbar$. In a unit system where $\hbar = 1$, S is dimensionless. Then in four dimensions the Lagrangian has natural dimensions of L^{-4} or simply -4 . Keeping this information in mind we can deduce the dimensions of any field. For example, if we consider the Lagrangian of scalar field Eq. 2.24 in Chapter 2, we can see from the KE term that Φ must have dimensions of -1 . For a spin $\frac{1}{2}$ fermion by the same token we deduce that it must have dimensions of $-\frac{3}{2}$ and a vector field has dimensions of -1 . One can use dimension or weight argument to write out the various Lorentz scalars for the fields. The constraint of Lorentz invariance is too broad. We can further restrict the choice by some other physical principle

²Spin $\frac{3}{2}$ or Rarita-Schwinger field in ordinary field theory in 4D leads to problems. Same applies to fields with spin $> \frac{5}{2}$.

³Correctly speaking what we call Lagrangian is strictly Lagrangian density, but it has become customary in field theory to refer to the Lagrangian density simply as the Lagrangian.

like gauge invariance (Chapter 2). The beauty of gauge invariance is that it highly constrains the forms of interactions between various fields; see Eqs. 2.20, 2.21, and 2.24.

We now recall our discussion about supersymmetry up to the end of section 1.3. Since Poincare invariance seems to be the symmetry of the world around us, the known particles and their interactions must obey this invariance. In other words the Lagrangian representing these particles must obey this invariance. Indeed it is well known that all fields transform as irreducible unitary representations of the Poincare group. A good discussion of this and how to build Lagrangians may be found in Chapter 1 of [27]. From the algebra of the Poincare group, see Eqs. 1.12 through 1.14, we can write a representation for the generators of this group. The most general representation of the ten Poincare group generators is [27]

$$\begin{aligned}
 P_\mu &= -i\partial_\mu, \\
 M_{\mu\nu} &= L_{\mu\nu} + S_{\mu\nu}, \\
 \text{with,} \\
 L_{\mu\nu} &= i[x_\mu\partial_\nu - x_\nu\partial_\mu].
 \end{aligned} \tag{3.2}$$

Now for example a scalar, spinor and a vector field transforms respectively as⁴

$$\begin{aligned}
 \delta_0\phi &= -\frac{i}{2}\epsilon^{\mu\nu}M_{\mu\nu}\phi \\
 \Psi'_L(x') &= e^{\frac{i}{2}\vec{\tau}\cdot(\vec{\omega}-i\vec{\nu})}\Psi_L(x) \\
 \Psi'_R(x') &= e^{\frac{i}{2}\vec{\tau}\cdot(\vec{\omega}+i\vec{\nu})}\Psi_R(x) \\
 \delta_0 A_\rho &= -\frac{1}{2}\epsilon^{\mu\nu}L_{\mu\nu}A_\rho - \frac{i}{2}(\epsilon^{\mu\nu}S_{\mu\nu})^\lambda_\rho A_\lambda
 \end{aligned} \tag{3.3}$$

⁴ δ_0 is the functional change at the same x , all the fields transform under a translation in space time, there is no change in a local field, that is $\delta_0 = -\epsilon^\mu\partial_\mu f$.

under Poincare group.⁵ The question now arises as to how one introduces supersymmetry into the Lagrangian. One way is to take the Lagrangian from ordinary field theory and supersymmetrize it by introducing supersymmetric partners of each of the fields present in the original Lagrangian. We then note that the new Lagrangian is invariant under any of the symmetries that were present in the original Lagrangian and under supersymmetric transformations. This approach will be referred to as the component approach.⁶ To illustrate these remarks let us consider a concrete but simple toy model, the Wess-Zumino Lagrangian. We start with the KE part of this Lagrangian

$$\mathcal{L}_0^{WZ} = \frac{1}{2} \partial_\mu A \partial^\mu A + \frac{1}{2} \partial_\mu B \partial^\mu B + i \frac{1}{4} \bar{\chi} \gamma^\mu \overleftrightarrow{\partial}_\mu \chi \quad (3.4)$$

Note the several differences between the simple KE term of spinor fields and its counterpart for scalar field. The former involves one derivative, the latter involves two; while the Fermi field is a Grassmann field the scalar field is a normal field, and finally the Fermi KE term has a global phase invariance; a single scalar field's KE has no such symmetry. The only similarity they share is that they are both conformally invariant [27]. With these points in mind we come back to the Lagrangian given in Eq. 3.4 and note that besides being conformally invariant it has two independent global phase invariances

$$\begin{aligned} \chi &\longrightarrow e^{i\alpha\gamma_5} \chi \\ (A + iB) &\longrightarrow e^{i\beta} (A + iB) \end{aligned} \quad (3.5)$$

Now we want to find the transformation which will turn the spinless field A and B into spinor field χ . The transformation must have the following characteristics

⁵Recall that Poincare group is Lorents group plus translations.

⁶This was the procedure adopted by Wess and Zumino in their original paper [23].

1. Its parameter must itself be a Grassmann spinor field, α , a global infinitesimal Majorana spinor parameter. This is for the simple reason that we are trying to turn a spinless field into one with spin.
2. In its simplest form, the transformation must involve no derivatives because the Fermi fields have one derivative less than Bose fields in their KE term.

One finds that the Lagrangian

$$\mathcal{L}_0^{WZ} = \frac{1}{2}\partial_\mu A\partial^\mu A + \frac{1}{2}\partial_\mu B\partial^\mu B + i\frac{1}{4}\bar{\chi}\gamma^\mu \overleftrightarrow{\partial}_\mu \chi - \frac{1}{2a^2}(F^2 + G^2) \quad (3.6)$$

is invariant under the supersymmetry transformations

$$\begin{aligned} \delta A &= ia\bar{\alpha}\chi \\ \delta B &= a\bar{\alpha}\gamma_5\chi \\ \delta F &= -a^2\bar{\alpha}\gamma^\mu\partial_\mu\chi \\ \delta G &= -ia^2\bar{\alpha}\gamma_5\gamma^\mu\partial_\mu\chi \\ \delta\chi &= ia\gamma^\mu\alpha\partial_\mu A + a\gamma^\mu\gamma_5\alpha\partial_\mu B + (F + i\gamma_5 G)\alpha \end{aligned} \quad (3.7)$$

One can introduce mass and interaction (or Yukawa terms) without disturbing the supersymmetry of the above Lagrangian

$$\mathcal{L}_{Yuk}^{WZ} = -h\left(\bar{\chi}\chi A - i\bar{\chi}\gamma_5\chi B - \frac{F}{a}(B^2 - A^2) - \frac{2}{a}GAB\right) \quad (3.8)$$

$$\mathcal{L}_{mass}^{WZ} = -\frac{m}{2}\left(\bar{\chi}\chi + \frac{2}{a}AF - \frac{2}{a}BG\right) \quad (3.9)$$

A few comments are in order

1. In Eqs. 3.6 through 3.9 we have introduced auxiliary fields (F, G) ⁷ to close the supersymmetric algebra off mass shell. When particles are on shell or satisfy their equations of motion the algebra closes without the auxiliary fields.
2. With the introduction of the auxiliary fields we have the same number of spinless (A, B, F , and G) and spinor (the four real components of χ) fields irrespective of the equations of motion. The balance between the spinor and spinless degrees of freedom remains true even on shell when F and G are no longer needed. This balance between boson (even spin) and fermion (odd spin) degrees of freedom is a general feature of relativistic supersymmetric theories.

Defining

$$\mathcal{L}^{WZ} = \mathcal{L}_0^{WZ} + \mathcal{L}_{Yuk}^{WZ} + \mathcal{L}_{mass}^{WZ} \quad (3.10)$$

we can eliminate F and G by using the equation of motion for these fields

$$\begin{aligned} F &= ha(B^2 - A^2) - maA \\ G &= 2haAB + maB \end{aligned} \quad (3.11)$$

and by substituting them back into Eq. 3.10 we can obtain the familar form, Eq. 3.47, of the Lagrangian for three interacting fields. We set $a = 1$ and $h = g$.

The second method of constructing supersymmetric Lagrangians is via the route of superfields. Superfields are fields that live in superspace. Superspace is Minkowski space-time extended to include the Grassmann variable θ . So a point in

⁷An auxiliary field is simply a non propagating field, in other words it does not have a K. term.

2

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superspace is labelled as $(x, \theta, \bar{\theta})$. The question arises as to why we want to use the formalism of superspace. The concept of superspace and superfields arises directly out of a need to construct representations of the GLA of supersymmetry given in Chapter 1 section 1.2. In the formalism of superfield the supersymmetry of the Lagrangian is manifest. This is not surprising since the Lagrangian by definition is a scalar in the space it lives in.

Let us briefly go over the construction of the representations of the algebra given in Eqs. 1.15 through 1.19. To take into account the anticommutator in these Eqs. we introduce the anticommuting parameters θ^α ($\alpha = 1, 2$) and $\bar{\theta}^{\dot{\beta}}$ ($\dot{\beta} = 1, 2$) which are elements of a Grassmann algebra

$$\{\theta^\alpha, \theta^\beta\} = \{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = \{\theta^\alpha, \bar{\theta}^{\dot{\beta}}\} = 0 \quad (3.12)$$

The supersymmetry algebra can then be rewritten as

$$[\theta Q, \bar{Q} \bar{\theta}] = 2\theta \sigma_\mu \bar{\theta} P^\mu \quad (3.13)$$

$$[\theta Q, \theta Q] = [\bar{Q} \bar{\theta}, \bar{Q} \bar{\theta}] = 0 \quad (3.14)$$

where we have dropped the indices ($\theta Q = \theta^\alpha Q_\alpha = \theta^\alpha Q^\beta \epsilon_{\alpha\beta}$ where $\epsilon_{\alpha\beta}$ is antisymmetric in α & β , and $\epsilon_{12} = +1$). A finite supersymmetry transformation can now be defined as

$$S(x, \theta, \bar{\theta}) = e^{i(-x_\mu P^\mu + \theta Q + \bar{Q} \bar{\theta})} \quad (3.15)$$

In the above Q is the generator of θ translations in superspace just as P is a generator of space-time translation in Minkowski space, see Eq. 3.1. In a similar fashion to Eq. 3.1 we can write a representation for the supercharges (i.e. Q 's)

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - i\sigma_{\alpha\dot{\beta}}^\mu \bar{\theta}^{\dot{\beta}} \partial_\mu \quad (3.16)$$

$$\bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + i\theta^{\beta}\sigma_{\beta\dot{\alpha}}^{\mu}\partial_{\mu} \quad (3.17)$$

which along with Eq. 3.1 is a representation of superalgebra in terms of differential operators. One can define covariant derivatives. These are derivatives that anticommute with the variation δ_S

$$\delta_S\phi = \left[\eta\frac{\partial}{\partial\theta} + \bar{\eta}\frac{\partial}{\partial\bar{\theta}} - i(\eta\sigma_{\mu}\bar{\theta} - \theta\sigma_{\mu}\bar{\eta})\partial^{\mu} \right] \quad (3.18)$$

$$D_{\alpha}(\delta_S\phi) = -\delta_S(D_{\alpha}\phi) \quad (3.19)$$

these Eqs. 3.18 and 3.19 give

$$D_{\alpha}\phi = \left(\frac{\partial}{\partial\theta^{\alpha}} + i\sigma_{\alpha\dot{\beta}}^{\mu}\bar{\theta}^{\dot{\beta}}\partial_{\mu} \right)\phi \quad (3.20)$$

$$\bar{D}_{\dot{\alpha}} = \left(-\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^{\beta}\sigma_{\beta\dot{\alpha}}^{\mu}\partial_{\mu} \right)\phi \quad (3.21)$$

The multiplication of two successive transformations (Eq. 3.15) can be computed with help of Hausdorf's formula

$$S(x, \theta, \bar{\theta})S(y, \eta, \bar{\eta}) = S(x + y - i\eta\sigma\bar{\theta} + i\theta\sigma\bar{\eta}, \theta + \eta, \bar{\theta} + \bar{\eta}). \quad (3.22)$$

A superfield is now defined as a function of the parameters $\theta, \bar{\theta}$ in addition to x_{μ} such that it transforms under a supersymmetry transformation as follows:

$$S(y, \eta, \bar{\eta})\phi(x, \theta, \bar{\theta}) = \phi(x + y - i\eta\sigma\bar{\theta} + i\theta\sigma\bar{\eta}, \theta + \eta, \bar{\theta} + \bar{\eta}). \quad (3.23)$$

A general superfield, $\hat{S}$, can be understood in terms of its power series expansions in terms of θ and $\bar{\theta}$

$$\hat{S}(x, \theta, \bar{\theta}) = f(x) + \theta\phi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta m(x) + \bar{\theta}\bar{\theta}n(x) + \theta\sigma_{\mu}\bar{\theta}v^{\mu}(x) + \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\psi(x) + \theta\theta\bar{\theta}\bar{\theta}d(x) \quad (3.24)$$

Because of the anticommutation property of θ and $\bar{\theta}$, Eq. 3.12, higher powers of θ and $\bar{\theta}$ than indicated in Eq. 3.24 vanish. $(f, \phi, \chi, m, n, v, \psi, d)$ are called

component fields. The transformation law of superfields Eq. 3.15 may now be applied to 3.24

$$\delta_\eta \hat{S}(x, \theta, \bar{\theta}) \equiv (\eta Q + \bar{\eta} \bar{Q}) \hat{S}, \quad (3.25)$$

$$\delta_\eta \hat{S}(x, \theta, \bar{\theta}) = \delta_\eta f(x) + \theta \delta_\eta \phi(x) + \bar{\theta} \delta_\eta \bar{\chi}(x) + \theta \theta \delta_\eta m(x) + \bar{\theta} \bar{\theta} \delta_\eta n(x) + \theta \sigma_\mu \bar{\theta} \delta_\eta v^\mu(x) + \theta \theta \bar{\theta} \delta_\eta \bar{\lambda}(x) + \bar{\theta} \bar{\theta} \theta \delta_\eta \psi(x) + \theta \theta \bar{\theta} \bar{\theta} \delta_\eta d(x). \quad (3.26)$$

The transformation laws for the components fields may be found from Eq. 3.26 by matching appropriate powers of θ and $\bar{\theta}$. By imposing various constraints on 3.24, we can find superfields of physical interest. For example the constraint

$$\bar{D}_\alpha \hat{\Phi} = 0 \quad (3.27)$$

gives a scalar or chiral superfield, where as the reality condition

$$\hat{V} = \hat{V}^\dagger \quad (3.28)$$

yields a vector superfield. It is possible to construct all supersymmetric renormalizable Lagrangians in terms of scalar and vector superfields. If one solves the constraint 3.27 one obtains

$$\hat{\Phi} = \phi(x) - i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\square\phi(x) + \sqrt{2}\theta\psi(x) + \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta\theta Z(x). \quad (3.29)$$

The components fields of the scalar superfield are a complex scalar, a spinor and auxiliary field. These fields were discussed when we were discussing the component approach. Indeed one can show that the transformation laws of ϕ , ψ , and Z derived through 3.29 are the same as those given in 3.7^a. From 3.29 we note that the highest component of $\hat{\Phi}$ is Z . Similarly we can show that $\hat{\Phi}^*$ is a superfield and its highest component is Z^* . All higher powers in θ , $\bar{\theta}$ are space time derivatives.

^aNote that $\phi = A + iB$ etc since 3.10 was given in terms of real fields

Thus the Z or Z^* components of a scalar superfield always transforms into space-time derivatives. Since products in θ terminate because of the anticommutative property, we expect products of superfields to be again be a superfield. This is indeed found to be usually the case; however note that the product $\hat{\Phi}^\dagger \hat{\Phi}$ is not a superfield. However its highest component i.e. the coefficient of $\theta\theta\bar{\theta}\bar{\theta}$ transforms into a total space-time derivative, so that it can represent the KE terms for the component fields. At this moment it is worth recalling the property of Grassmannian coordinates under integration

$$\begin{aligned}\int d\theta &= 0 \\ \int d\theta \cdot \theta &= 1\end{aligned}\tag{3.30}$$

These properties under integration allows us to project out the highest components of superfields or their products. This last observation is crucial in constructing physical Lagrangians starting from superfields. For example the most general supersymmetric renormalizable Lagrangian involving only scalar superfields is

$$\mathcal{L} = \hat{\Phi}_i^\dagger \hat{\Phi}_i |_{\theta\theta\bar{\theta}\bar{\theta}\text{component}} + [(\frac{1}{2}m_{ij}\hat{\Phi}_i\hat{\Phi}_j + \frac{1}{3}g_{ijk}\hat{\Phi}_i\hat{\Phi}_j\hat{\Phi}_k + \lambda_i\hat{\Phi}_i)|_{\theta\theta\text{component}} + H.C.] \tag{3.31}$$

The above is cubic in the scalar superfield since we want no more than quartic terms for the component scalar field. This is because it is well known that this is required by renormalizability in ordinary field theory. The first term in 3.31 is the KE term, the second being the mass, the third and fourth being the interaction terms. Once the superfields are expressed in terms of their components Eq. 3.29, 3.31 reduces to the Wess-Zumino model after the auxiliary fields are eliminated in their usual manner via their equations of motion. In a similar fashion one finds that the *physical* components of a vector superfield are a spin 1/2 and spin 1 field after the auxiliary fields are eliminated and the gauge is fixed. The method of

constructing Lagrangians for supersymmetric gauge fields is now clear; we know that

$$\mathcal{L} = \mathcal{L}_{KE} + \mathcal{L}_{\text{interaction}} \quad (3.32)$$

so we hunt for various combinations of scalar and vector superfields which will reproduce the Lagrangian for the gauge theory we are trying to supersymmetrize in addition to the Lagrangian which involves the supersymmetric partners after we express them in terms of components and eliminate the auxiliary fields. In the jargon of EPP the highest components of scalar and vector superfield are called F and D terms, respectively.

With these points in mind the minimal globally supersymmetric renormalizable $SU(2)_L \times U(1)_Y$ -gauge invariant Lagrangian [25] is constructed out of the following set of chiral matter superfields:

$\hat{Q}_L(\frac{1}{2}, \frac{1}{3})$	$\hat{U}_R(0, -\frac{4}{3})$	$\hat{D}_R(0, \frac{2}{3})$
$\hat{L}(\frac{1}{2}, -1)$	$\hat{E}_R(0, 2)$	
$\hat{H}_1(\frac{1}{2}, -1)$	$\hat{H}_2(\frac{1}{2}, +1)$	$\hat{N}(0, 0)$

In the above $SU(2)_L \times U(1)_Y$ representations are given in the parentheses. The vector superfields describing the gauge fields of $SU(2)_L$ and $U(1)_Y$ are denoted by $\hat{V}^i$ and $\hat{V}'$ where i is an $SU(2)_L$ index. The gauge invariant chiral field strengths are $W_\alpha = -\frac{1}{4}\bar{D}\bar{D}e^{-g\tau\cdot\hat{V}}D_\alpha e^{g\tau\cdot\hat{V}}$ and $W'_\alpha = -\frac{1}{4}\bar{D}\bar{D}D_\alpha\hat{V}'$ where D_α are covariant derivatives defined before in Eqs. 3.20, and 3.21. Putting together all the pieces of

information indicated above we can write the Lagrangian for the supersymmetric extension of SW model based on $SU(2)_L \times U(1)_Y$ as

$$\begin{aligned} \mathcal{L}_{SQFD} = & \int D^2\theta \left[\frac{1}{4} (W^\alpha W_\alpha + W^{\dot{\alpha}} W_{\dot{\alpha}} + W'^\alpha W'_\alpha + W'^{\dot{\alpha}} W'_{\dot{\alpha}}) + \right. \\ & h_{\epsilon_{ij}} \hat{H}_1^i \hat{H}_2^j \hat{N} + s \hat{N} + f_{\epsilon_{ij}} \hat{H}_1^i \hat{L}^j \hat{E}_R + h_d \epsilon_{ij} \hat{H}_1^i \hat{Q}_L^j \hat{D}_R + \\ & h_u \epsilon_{ij} \hat{Q}_L^i \hat{H}_2^j \hat{U}_R \left. \right] + \int D^4\theta \left[\hat{N}^\dagger \hat{N} + \hat{L}^\dagger e^{g\tau_i V^i - g'V'} \hat{L} + \hat{E}^\dagger e^{2g'V'} \hat{E} + \right. \\ & \hat{Q}_L^\dagger e^{g\tau_i V^i + Y_Q g'V'} \hat{Q}_L + \hat{U}_R^\dagger e^{Y_U g'V'} \hat{U}_R + \hat{D}_R^\dagger e^{Y_D g'V'} \hat{D}_R + \\ & \left. \hat{H}_1^\dagger e^{g\tau_i V^i - g'V'} \hat{H}_1 + \hat{H}_2^\dagger e^{g\tau_i V^i + g'V'} \hat{H}_2 \right] \end{aligned} \quad (3.33)$$

We now briefly summarize the particle content of the above Lagrangian, the actual explicit reduction of 3.33 in terms of component fields and the associated Feynman rules may be found in [[25],[44] (Phd Thesis notes), [52]]. As already mentioned we work in t'Hooft-Feynman gauge for $SU(2)_L \times U(1)_Y$ and write the component fields in the Wess-Zumino gauge. Note that in [25] the authors work in the unitary gauge. The superfields and their associated component fields with their quantum numbers are summarized in Table 3.1. The gauge vector superfield $\hat{V}^i$ yields the usual $SU(2)_L$ triplet of vector bosons, A_μ^i ($i = 1, 2, 3$), along with their corresponding fermionic partners represented by the two component spinors λ^i and the auxiliary fields, D^i . The $U(1)_Y$ vector superfield denoted by $\hat{V}'$ reduces to a single vector boson, its fermionic partner λ' and the D term D' . This completes the gauge sector. The matter sector contains a left-handed $SU(2)_L$ quark doublet of two component fermions Ψ_Q^i $i = (1, 2)$ along with a pair of two component right handed $SU(2)_L$ singlets Ψ_u and Ψ_d . The SUSY scalar partners of the quark doublets and singlets are respectively denoted by $\tilde{Q}^i$, $\tilde{u}$, and $\tilde{d}$. Similarly the lepton sector contains a doublet Ψ_L^i and one singlet Ψ_e . The scalar-lepton $SU(2)_L$ doublet and singlet are $\tilde{L}^i$ and $\tilde{R}$, respectively. The remaining members of the matter sector are the Higgs multiplets and their SUSY partners. At least two scalar $SU(2)_L$ doublets are necessary to give mass to both the up- and down- type quarks. The reason is that in the SW model, see section 2.2, in order to generate quark mass terms

for both u-and d-type quarks we need not only the doublet Φ with $Y = 1$ but also the conjugate multiplet $\bar{\Phi}$ with $Y = -1$. Now supersymmetry does not allow the latter to couple to the quark fields. Hence we must introduce an extra Higgs doublet with $Y = -1$. The Higgs doublets are written as

$$H_1 = \begin{bmatrix} \frac{1}{2}[H_1^0 - H_2^0 + i(H_3^0 - G^0)] \\ \frac{1}{\sqrt{2}}[H^- - G^-] \end{bmatrix}$$

$$H_2 = \begin{bmatrix} \frac{1}{\sqrt{2}}[H^+ + G^+] \\ \frac{1}{2}[H_1^0 + H_2^0 + i(H_3^0 + G^0)] \end{bmatrix}$$

In order to construct the desired fermion mass terms H_1 must have weak hypercharge $Y = -1$ while H_2 has $Y = +1$. The various Higgs field H_i^0 etc are extracted from the above, for example H^+ is defined as,

$$H^+ = \frac{1}{\sqrt{2}}(H_2^1 + H_1^{2*}).$$

So we have eight Higgs fields in all. Of these $G^\pm$ and G^0 are the would be Goldstone bosons, as in SW model, see section 2.2. In our gauge these are the longitudinal degrees of freedom of $W^\pm$ and Z^0 respectively. There are two physical charged Higgs scalars $H^\pm$, two scalars H_1^0, H_2^0 (H_1^0 can be identified with the SW Higgs scalar) and a pseudoscalar H_3^0 . Finally a singlet scalar N is needed in addition to the two Higgs doublets to break the $SU(2)_L \times U(1)_Y$ to $U(1)_{EM}$ in the SUSY limit. These Higgs fields are partnered by their fermionic counterparts $\Psi_{H_1^\pm}$, $\Psi_{H_3^0}$ and Ψ_N , and auxiliary fields in the off shell case. We make the assumption that the two Higgs doublets acquire equal VEV's⁹

$$\langle H_1 \rangle_0 = \begin{bmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{bmatrix}$$

$$\langle H_2 \rangle_0 = \begin{bmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{bmatrix}$$

⁹In order to stay close to the SW model, i.e. there Ψ and its conjugate had the same VEV!

while the singlet does not acquire a VEV. The more general case will be commented on later in section 3.4. The scalar potential which results from the Lagrangian is [25]

$$V = F_a^* F_a + \frac{1}{2} [D^i D^i + D'^2] \quad (3.34)$$

Here F_a is the auxiliary field for a matter field A denoted by the label a , and given by

$$F_a = \frac{\partial W}{\partial A_a} \quad (3.35)$$

Similarly D^i are the auxiliary fields for the $SU(2)$ gauge fields

$$D^k = -g A_i^* T_{ij}^k A_j \quad (3.36)$$

W is the superpotential but with the superfields replaced by their scalar components. The following expressions follow for the gauge and matter auxiliary fields from the lagrangian of Eq. 3.33,

$$D^a = \frac{g}{2} [H_2^{i*} \tau_{ij}^a H_2^j + H_1^{i*} \tau_{ij}^a H_1^j + \tilde{Q}^{i*} \tau_{ij}^a \tilde{Q}^j + \tilde{L}^{i*} \tau_{ij}^a \tilde{L}^j], \quad (3.37)$$

$$D' = \frac{g'}{2} [H_2^{i*} H_2^i - H_1^{i*} H_1^i + Y_Q \tilde{Q}^{i*} \tilde{Q}^i + Y_U \tilde{U}^* \tilde{U} + Y_d \tilde{D}^* \tilde{D} + Y_L \tilde{L}^{i*} \tilde{L}^i + Y_R \tilde{R}^* \tilde{R}], \quad (3.38)$$

$$F_N = h \epsilon_{ij} H_1^i H_2^j + s \quad (3.39)$$

$$F_{H_1}^i = h \epsilon_{ij} H_2^j N + f \epsilon_{ij} \tilde{L}^j \tilde{E}_R + h_d \epsilon_{ij} \tilde{Q}^j \tilde{d}_R, \quad (3.40)$$

$$F_{H_2}^i = h \epsilon_{ij} H_1^j N + h_u \epsilon_{ij} \tilde{Q}^j \tilde{u}_R, \quad (3.41)$$

$$F_{\tilde{Q}}^i = h_d \epsilon_{ij} H_1^j \tilde{d}_R^* + h_u \epsilon_{ij} H_2^j \tilde{u}_R^*, \quad (3.42)$$

$$F_{\tilde{L}}^i = f \epsilon_{ij} H_1^j \tilde{R}, \quad (3.43)$$

$$F_R = f\epsilon_{ij}H_1^j\tilde{L}^i, \quad (3.44)$$

$$F_u = h_u\epsilon_{ij}\tilde{Q}^iH_2^j, \quad (3.45)$$

$$F_d = h_d\epsilon_{ij}H_1^i\tilde{Q}^j. \quad (3.46)$$

Once these expressions are plugged into Eq. 3.34 we obtain the scalar potential in terms of component fields.

We now briefly describe the particle content of this model [25]. It consists of the following 5 physical multiplets $(\gamma, \tilde{\gamma})$, $(W^-, \tilde{\omega}_1^-, H^-)$ $(W^+, \tilde{\omega}_2^+, H^+)$, $(Z, \tilde{z}, H_2^0)$ and $(H_1^0, H_3^0, \sqrt{2}\text{Re}N, \sqrt{2}\text{Im}N, \tilde{h})$ In the supersymmetric(SUSY) limit they have masses 0, M_W , M_W , M_Z and $m_{H_1^0}$ respectively. The above multiplets consist of an equal number of Bose and Fermi degrees of freedom. In the above multiplets $\tilde{\omega}_1^-$, $\tilde{\omega}_2^+$, $\tilde{z}$, $\tilde{h}$, are given by

$$\tilde{\omega}_1^- = \begin{bmatrix} \Psi_{H_1^2} \\ i\tilde{\lambda}^+ \end{bmatrix}$$

$$\tilde{\omega}_2^+ = \begin{bmatrix} \Psi_{H_1^1} \\ i\tilde{\lambda}^- \end{bmatrix}$$

$$\tilde{z} = \begin{bmatrix} \frac{\psi_{H_1^1} - \psi_{H_2^2}}{\sqrt{2}} \\ i\tilde{\lambda}_z \end{bmatrix}$$

$$\tilde{h} = \begin{bmatrix} \frac{\psi_{H_1^1} + \psi_{H_2^2}}{\sqrt{2}} \\ i\psi_N \end{bmatrix}$$

$$H^+ = \frac{1}{\sqrt{2}}(H_2^1 + H_1^{2*})$$

$$H_1^0 = \text{Re}(H_1^1 + H_2^2)$$

$$H_2^0 = \text{Re}(H_2^2 - H_1^1)$$

$$H_3^0 = \text{Im}(H_1^1 + H_2^2)$$

$$m_{H_1^0} = M_{H_1^0} = \hbar v$$

$$m_{H_2^0} = M_Z = \frac{gv}{\sqrt{2}c_W}$$

$$m_{H^\pm} = M_W = \frac{gv}{\sqrt{2}}$$

$$\frac{1}{2}\hbar v^2 + s = 0$$

The ghost sector reduces to that of the SW model [58, 52]. That is, the Faddeev-Popov ghost couples to H_0^1 , the scalar resembling the SW Higgs boson, and to photon, W , Z etc via the usual rules. The other bosons $H^\pm$, H_2^0 , H_3^0 as well as the N fields do not couple to these ghosts.

3.3 The Cancellation of the linear and quadratic divergences in Supersymmetry

As already mentioned one of the amazing properties of supersymmetric theories is the cancellation of linear and quadratic divergences to all orders of perturbation theory. This happens because fermions appear in loops with a negative sign relative to the bosons. And there are equal Bose and Fermi degrees of freedom in supersymmetric theories. The cancellation holds even when the supersymmetry is spontaneously broken or broken by 'soft' terms. In ordinary renormalizable theories the worst divergences one encounters are quadratic. As we have seen (Chapter 2) one introduces various renormalization constants to get rid of these infinities. If we supersymmetrize such a theory the quadratic divergences will go away. Hence in supersymmetry the task of renormalization is made very easy since there are at most logarithmic divergences. So one needs only a single renormalization constant.

To illustrate these cancellations let us consider the simplest supersymmetric model, the Wess-Zumino model. As already mentioned this consists of single chiral

superfield. Or in terms of component fields there are two real scalar fields(A and B) and a four component Majorana spinor(ψ) in this model. The Lagrangian is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2}(\partial_\mu B)^2 - \frac{1}{2}m^2(A^2 + B^2) + i\frac{1}{2}\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{1}{2}m\bar{\psi}\psi - gmA(A^2 + B^2) - \frac{1}{2}g^2(A^2 + B^2)^2 - g\bar{\psi}(A - iB\gamma_5)\psi \quad (3.47)$$

The Feynman rules follow immediately from the Lagrangian. Quadratic divergences only occur in this model in the scalar self energies. It is sufficient to consider the self energy of one scalar(say A, since the same arguments apply to B). The quadratically divergent graphs contributing to the self energy of A are shown are shown in Fig. 3.1. Note that we should multiply each of these graphs by a symmetry factor of 1/2. The boson contribution is found to be (c and d Fig. 3.1)

$$B = 8g^2 \int d^4q \frac{1}{(q^2 - m^2)} \quad (3.48)$$

The fermion loop contributes(diagram c Fig. 3.1)

$$F = -2g^2 \int d^4q \frac{N}{(q^2 - m^2)((q + p)^2 - m^2)} \quad (3.49)$$

$$N = \text{Tr}(q + m)(q + p + m) = 4(q^2 + q \cdot p + m^2) \quad (3.50)$$

N can be rewritten as

$$N = 2[(q^2 - m^2) + ((q + p)^2 - m^2) - p^2 + 4m^2] \quad (3.51)$$

Substituting this value of N in 3.49

$$F = -4g^2 \int d^4q \frac{1}{(q^2 - m^2)} - 4g^2 \int d^4q \frac{1}{((q + p)^2 - m^2)} + L(p, m) \quad (3.52)$$

Replacing q by q+p in the second integral in 3.52 we get

$$F = -8g^2 \int d^4q \frac{1}{(q^2 - m^2)} + L(p, m). \quad (3.53)$$

L(p,m) is a logarithmically divergent integral. On adding 3.48 and 3.53 we immediately see that quadratic divergences have indeed cancelled out.

We have shown that quadratic and linear divergences go away in SQFD [44].

Some details of this are given in the Appendix F.

3.4 Supersymmetry Breaking

As is clear from section 3.2 a phenomenologically acceptable supersymmetric gauge models must contain the standard particles quarks, leptons and gauge bosons and their superpartners scalar quarks, scalar leptons and gauginos. In addition, two Higgs doublets and a singlet are needed, to break the $SU(2) \times U(1)$ and give the up and down quarks different masses. However we know that supersymmetry must be broken since there is no evidence for such symmetry in the low energy particle spectrum. But at the same time we don't want to break supersymmetry in a way which will reintroduce quadratic and linear divergences, otherwise we will be back to the hierarchy problem and the introduction of supersymmetry would have become an academic exercise. One can achieve this by introducing soft-breaking¹⁰ terms as was first done by Girardello and Grisaru [57]. These authors categorized all possible soft-supersymmetry-breaking (from now on referred to as SSSB) terms as

$$\begin{aligned} \mathcal{L}_{SSSB} = & \tilde{M}_1 \text{Re} A^2 + \tilde{M}_2 \text{Im} A^2 + c(A^3 + \text{h.c.}) \\ & + \tilde{M}_3(\lambda^a \lambda^a + \bar{\lambda}^a \bar{\lambda}^a) + \tilde{M}_4(\lambda' \lambda' + \bar{\lambda}' \bar{\lambda}') \end{aligned} \quad (3.54)$$

The parameter $\tilde{M}_1$ splits the mass of the complex scalar A_i from its fermionic partner Ψ_i . If we express A_i in terms of two real scalars, then $\tilde{M}_2$ splits the masses of these two fields. The coupling constant c introduces a new non-supersymmetric scalar interaction term. Finally $\tilde{M}_3$ and $\tilde{M}_4$ are Majorana mass terms for the gauginos respectively of $SU(2)$ and $U(1)$ gauge groups. All terms in Eq. 3.54 are gauge invariant. One would like constrain the above parameters by some physical

¹⁰By soft-breaking terms in this context one means terms that will not introduce unwanted divergences like quadratic

argument. Amazingly one can show that the low energy limit of spontaneously broken supergravity is a globally supersymmetric theory modified by SSB terms¹¹

$$\mathcal{L}(N=1\text{Supergravity}) \longrightarrow \mathcal{L}_{\text{SQFD in the supersymmetric limit}} + \mathcal{L}_{\text{SSSB}} \quad (3.55)$$

We choose to work with this low energy effective Lagrangian, in this case we can write [52]

$$\mathcal{L}_{\text{SSSB}} = -\sum_A m_A^2 A A^* - m_{\frac{3}{2}} [h(Z) + h.c.] - \frac{1}{2} \bar{m} (\lambda^a \lambda^a + \bar{\lambda}^a \bar{\lambda}^a) - \frac{1}{2} \bar{m}' (\lambda' \lambda' + \bar{\lambda}' \bar{\lambda}') \quad (3.56)$$

$$h(Z) = (A-3)w(Z) + \sum_a \frac{\partial w}{\partial Z_a} Z_a \quad (3.57)$$

$$w(Z) = +h\epsilon_{ij} H_1^i H_2^j N + sN + f\epsilon_{ij} H_1^i \bar{L}^j \bar{e}_R + h_d \epsilon_{ij} H^i \bar{Q}_L^j \bar{d}_R + h_u \epsilon_{ij} \bar{Q}_L^i \bar{H}_2^j \bar{u}_R + h.c. \quad (3.58)$$

In the above, the gaugino masses, $\bar{m}$, $\bar{m}'$ as well as the gravitino mass $m_{\frac{3}{2}}$ are free parameters. In Eq. 3.57 the parameter A determines the Goldstone-fermion-matter coupling and is usually set equal to 3 [52]. We follow this choice. We shall also assume that all scalar masses m_A are different in general and not all equal to the gravitino mass as is sometimes assumed [52].

Since we follow the method given in [52] for SSSB, we only summarize some relevant features here. Let us first start with the Higgs sector. The gauge symmetry breaking is achieved by letting the three Higgs fields H_1 , H_2 and N develop VEV's. We relax our previous conditions, see section 3.2, and allow the following different VEV for the Higgs fields

$$\begin{aligned} \langle H_1 \rangle_0 &= \begin{bmatrix} \frac{v_1}{\sqrt{2}} \\ 0 \end{bmatrix} \\ \langle H_2 \rangle_0 &= \begin{bmatrix} 0 \\ \frac{v_2}{\sqrt{2}} \end{bmatrix} \end{aligned}$$

¹¹One also finds that the superpotential given in section 3.2 is modified somewhat but in a way consistent with supersymmetry. This adds new supersymmetric interaction terms

$$\langle N \rangle_0 = \frac{v_3}{\sqrt{2}}$$

These choices for the VEV allows us to generalize the previous constraint equation, the last Eq. of section 3.2, to a set of three equations

$$\begin{aligned} m_{\frac{3}{2}} \left[\frac{3}{2} h v_1 v_2 + s \right] + \frac{v_3}{\sqrt{2}} \left[m_N^2 + \frac{1}{2} h^2 v^2 \right] &= 0 \\ \frac{3}{\sqrt{2}} m_{\frac{3}{2}} h v_2 v_3 + m_{H_1}^2 v_1 \\ + \frac{h}{2} [h v_1 (v_2^2 + v_3^2) + 2 s v_2] + \frac{1}{8} \bar{g}^2 v_1 (v_1^2 - v_2^2) &= 0 \\ \frac{3}{\sqrt{2}} m_{\frac{3}{2}} h v_1 v_3 + m_{H_2}^2 v_2 \\ + \frac{h}{2} [h v_2 (v_1^2 + v_3^2) + 2 s v_1] - \frac{1}{8} \bar{g}^2 v_2 (v_1^2 - v_2^2) &= 0 \end{aligned} \quad (3.59)$$

where $v^2 = v_1^2 + v_2^2$, $\bar{g}^2 = g^2 + g'^2$ and m_{H_1} , m_{H_2} , m_N are bare mass terms of H_1 , H_2 and N . The three Higgs fields H_1 , H_2 and N are not the physical mass eigenstates and a diagonalization has to be performed. The relations to extract various Higgs given before in the SUSY limit, see the Eqs. at the end of section 3.2, and for equal VEV and $v_3 = 0$, now generalize to

$$\begin{aligned} H^+ &= \frac{1}{v} (v_1 H_2^1 + v_2 H_1^{2*}), \\ G^+ &= \frac{1}{v} (v_2 H_2^1 - v_1 H_1^{2*}), \\ G^0 &= \frac{1}{v} \text{Im}(v_2 H_2^2 - v_1 H_1^1), \\ H_3^0 &= \frac{1}{v} \text{Im}(v_1 H_2^2 + v_2 H_1^1), \\ H_1^0 &= \frac{1}{v} \text{Re}(v_1 H_2^2 + v_2 H_1^1), \\ H_2^0 &= \frac{1}{v} \text{Re}(v_2 H_2^2 - v_1 H_1^1). \end{aligned} \quad (3.60)$$

H^- and G^- are given by the conjugates of their positively charged counterparts.

We also have $h_0^1 = \text{Re}N$ and $h_0^3 = \text{Im}N$. Hence we have six neutral spin-0

bosons, which further divide into three scalars (H_0^1, H_0^2, h_0^1) and three psuedoscalars (H_0^3, G_0, h_0^3). $G^\pm$ and G^0 are eaten up in the usual manner by $W^\pm$ and Z . The mass matrix of the remaining two psuedoscalars is nondiagonal and can be diagonalized in the usual way by defining a mixing angle ξ , the resulting two physical states can be written as

$$\begin{aligned} H_4^0 &= H_3^0 \cos \xi - h_3^0 \sin \xi, \\ H_5^0 &= H_3^0 \sin \xi + h_3^0 \cos \xi. \end{aligned} \quad (3.61)$$

The mixing angle is determined by the parameters of the scalar potential

$$\tan 2\xi = \frac{6hm_{\frac{3}{2}}v}{\sqrt{2}[h^2v_3^2 + (m_{H_1}^2 + m_{H_2}^2 - m_N^2)]}. \quad (3.62)$$

The following sum rule holds for the two physical psuedoscalars

$$m_{H_4^0}^2 + m_{H_5^0}^2 = h^2[v^2 + v_3^2] + m_{H_1}^2 + m_{H_2}^2 + m_N^2 \quad (3.63)$$

The mass matrix of the three neutral scalars is also nondiagonal. Three new physical fields can be defined which are related to the old scalars by a unitary transformation [52]. The elements of this matrix are rather involved expressions in terms of $h, v_1, v_2, v_3, m_{H_1}, m_{H_2}, m_N$ and will not be given here[24 (Higgs Hunter's Guide)]. Suffice is to note the following sum rule

$$m_{H_6^0}^2 + m_{H_7^0}^2 + m_{H_8^0}^2 = h^2[v^2 + v_3^2] + m_{H_1}^2 + m_{H_2}^2 + m_N^2 \quad (3.64)$$

implies

$$m_{H_6^0}^2 + m_{H_7^0}^2 + m_{H_8^0}^2 = m_{H_4^0}^2 + m_{H_5^0}^2, \quad (3.65)$$

where we have denoted the new scalars by H_6^0, H_7^0 and H_8^0 . The mass of the physical charged Higgs scalars is given by

$$m_{H^\pm}^2 = h^2v_3^2 + m_{H_1}^2 + m_{H_2}^2 + M_W^2 \quad (3.66)$$

It can be shown [52] that if $v_1 = v_2$ and $m_{H_1} = m_{H_2} = m_{\frac{3}{2}}$ then Eq. 3.66 takes a particularly simple form

$$m_{H\pm}^2 = 4m_{\frac{3}{2}}^2 + M_W^2 \quad (3.67)$$

So the simple solution indicates that the charged Higgs boson is heavier than the W boson quite independent of the coupling parameters in the scalar potential. It is important to emphasize that this need not be true in general i.e. when $v_1 \neq v_2$ and $m_{H_1} \neq m_{H_2}$. To summarize, in the Higgs sector one expects from the above discussion, for reasonable values of the parameter space, the spin-0 bosons would typically have masses of the order of M_W or $m_{\frac{3}{2}}$. The analysis of the mass spectrum in terms of the parameter space is an interesting problem in itself. However, since it is not the focus of our thesis, we are interested in it in so far as an indicator to input sensible masses for the SUSY particles. The other indicator is of course the recent experimental limits on the SUSY particles [3], see Tables 3.2 through 3.4.

Let us now turn to the generalization of the winos, see section 3.2. In the presence of SSSB, Eq. 3.56, the physical mass eigenstates are the Dirac spinors $\tilde{\chi}_1$ and $\tilde{\chi}_2$

$$\begin{aligned} \tilde{\chi}_1 &= \begin{bmatrix} -i\lambda^+ \cos \phi_+ + \Psi_{H_2}^1 \sin \phi_+ \\ i\bar{\lambda}^- \cos \phi_- + \bar{\Psi}_{H_1}^2 \sin \phi_- \end{bmatrix} \\ \tilde{\chi}_2 &= \begin{bmatrix} -i\lambda^+ \sin \phi_+ - \Psi_{H_2}^1 \cos \phi_+ \\ i\bar{\lambda}^- \sin \phi_- + \bar{\Psi}_{H_1}^2 \cos \phi_- \end{bmatrix} \end{aligned}$$

The mixing angles are given by

$$\sin 2\phi_{\pm} = \frac{\bar{m}}{\sqrt{2}M_W} \frac{(1 + \sin 2\alpha)^{\frac{1}{2}} \pm (1 - \sin 2\alpha)^{\frac{1}{2}}}{[1 + \frac{\bar{m}^2}{2M_W^2}]^2 - \sin^2 \alpha]^{\frac{1}{2}}}, \quad (3.68)$$

where

$$\tan \alpha \equiv \frac{v_1}{v_2}. \quad (3.69)$$

The masses of the physical states are

$$\begin{aligned}\tilde{M}_{1,2} = & \frac{M_W}{\sqrt{2}} \left[\left[1 + \sin 2\alpha \right) + \frac{\tilde{m}^2}{2M_W^2} \right]^{\frac{1}{2}} \\ & \pm \left[1 - \sin 2\alpha \right) + \frac{\tilde{m}^2}{2M_W^2} \right]^{\frac{1}{2}} \}. \end{aligned} \quad (3.70)$$

As a check we can make connection with the SUSY limit by setting first $\phi_+ = \phi_-$

$$\sin 2\phi = \frac{1}{\left[1 + \frac{\tilde{m}^2}{4M_W^2} \right]}. \quad (3.71)$$

Setting $\tilde{m} = 0$ in the above equation gives $\phi = 45^\circ$. $v_1 = v_2$ implies that $\sin 2\alpha = 1$. Hence in this limit we find from Eq. 3.70 that $\tilde{M}_{1,2} = M_W$. In this limit the connection with the winos is

$$\begin{aligned}\tilde{\omega}_1^- &= \frac{1}{\sqrt{2}} [\tilde{\chi}_2^c + \tilde{\chi}_1^c] \\ \tilde{\omega}_2^+ &= \frac{1}{\sqrt{2}} [\tilde{\chi}_1 - \tilde{\chi}_2] \end{aligned} \quad (3.72)$$

(c denotes charge conjugation).

The discussion of the neutral mass eigenstates is similar to the charged mass eigenstates. The main complication is that here one has five neutral states mixing: λ_γ , λ_z , $\Psi_{H_1^0}$, $\Psi_{H_2^0}$, and Ψ_N [25]. Under certain restrictions [25] one can obtain a light photino, two massive Majorana fermions χ_2^0 , χ_3^0 (which combine to give the zino in the SUSY limit), another two massive Majorana fermions, χ_4^0 , χ_5^0 which combine to give $\tilde{h}$, defined in section 3.2, in the SUSY limit. We choose the simplest SSSB scenario for their masses [25]

$$\begin{aligned}\tilde{M}_{2,3}^0 &= [M_Z^2 + \frac{1}{4}\tilde{m}^2]^{\frac{1}{2}} \pm \frac{1}{2}\tilde{m} \\ \tilde{M}_{4,5}^0 &= [m_{H_1^0}^2 + \frac{1}{4}\tilde{m}_x^2]^{\frac{1}{2}} \pm \frac{1}{2}\tilde{m}_x \end{aligned} \quad (3.73)$$

In other words we have assumed that their masses are distributed about their values in the SUSY limit, i.e. about M_Z and $m_{H_1^0}$.

Turning now to the slepton and squark sectors. Once again there is mixing, and one must diagonalize mass matrix to obtain mass eigenstates [25]. However in this sector we take advantage of the stringent experimental constraints imposed by limits from FCNC and anomalous parity violation effects in nuclei. These FCNC contributions are suppressed if the $\tilde{\gamma}$ and $\tilde{g}^{12}$ couplings are flavor diagonal, the SUSY CKM angles are identical to the conventional CKM angles, and the squark masses of five flavors are nearly degenerate

$$m_{\tilde{u}} \simeq m_{\tilde{d}} \simeq m_{\tilde{c}} \simeq m_{\tilde{s}} \simeq m_{\tilde{b}}. \quad (3.74)$$

Parity violation in nuclear transitions can occur through penguin diagrams. The absence of anomalous parity violating effects requires

$$\begin{aligned} m_{\tilde{u}_L} &\simeq m_{\tilde{u}_R}, \\ m_{\tilde{d}_L} &\simeq m_{\tilde{d}_R}. \end{aligned} \quad (3.75)$$

We assume this left right squark mass degeneracy for all squark flavors. In the SSSB scenario we are working with, we may assume that the squarks and sleptons have a common SUSY-breaking mass, $m_{\frac{3}{2}}$. In addition there are two other contributions to the masses of the S-fermions one of which may be expressed in terms of the gluino mass. The remaining contribution arises from the D-term. The resulting masses at the M_W scale, assuming three generations are

$$\begin{aligned} \tilde{m}_{\tilde{u}}^0 &= m_{\frac{3}{2}}^2 + 0.805m_g^2 - 0.250rM_Z^2 \\ \tilde{m}_{\tilde{d}}^0 &= m_{\frac{3}{2}}^2 + 0.805m_g^2 + 0.250rM_Z^2 \\ \tilde{m}_{\tilde{\nu}_e}^0 &= m_{\frac{3}{2}}^2 + 0.060m_g^2 + 0.250rM_Z^2 \\ \tilde{m}_{\tilde{\nu}_\tau}^0 &= m_{\frac{3}{2}}^2 + 0.035m_g^2 - 0.500rM_Z^2 \end{aligned} \quad (3.76)$$

¹² $\tilde{g}$ refers to the gluino, the SUSY partner of the gluon

where $r = \frac{v_2^2 - v_1^2}{v_2^2 + v_1^2}$. We note that $m_{\text{squark}} > m_{\text{septon}}$. We have implicitly assumed that the photino is the lightest supersymmetric particle (hereafter referred to as LSP). The R parity defined by

$$R = (-1)^{3B+L+2J} \quad (3.77)$$

is taken to be conserved. B is the baryon number, L is the lepton number, and J is the spin. R works out to be +1 for the usual particles and -1 for their SUSY partners. An immediate of R conservation is that SUSY particles are always pair produced and decay to the LSP which is stable. We close this chapter with a cautionary remark that while SUSY is a beautiful theory, to date there is no experimental evidence to support the existence of the SUSY particles.

Figure 3.1: One loop radiative corrections to the mass of scalar A .

Table 3.1: The Gauge and Matter Multiplets of Supersymmetric $SU(2)_L \times U(1)_1$ model.

Superfield	Auxiliary fields	Component fields	Spin	$SU(2)_L$	(T, Y)
Gauge Multiplets					
$\hat{V}^i$	D^i	A_μ^i, λ^i	$(1, \frac{1}{2})$	triplet	(1, 0)
$\hat{V}'$	D'	B_μ, λ'	$(1, \frac{1}{2})$	singlet	(0, 0)
Matter Multiplets					
$\hat{Q}_L$	F_Q	$\Psi_Q^i, \tilde{Q}^i$	$(\frac{1}{2}, 0)$	doublet	$(\frac{1}{2}, \frac{1}{3})$
$\hat{U}_R$	F_u	$\Psi_u, \bar{u}$	$(\frac{1}{2}, 0)$	singlet	$(0, -\frac{2}{3})$
$\hat{D}_R$	F_d	$\Psi_d, \bar{d}$	$(\frac{1}{2}, 0)$	singlet	$(0, \frac{2}{3})$
$\hat{L}$	F_L	$\Psi_L^i, \tilde{L}^i$	$(\frac{1}{2}, 0)$	doublet	$(\frac{1}{2}, -1)$
$\hat{E}_R$	F_R	$\Psi_e, \tilde{R}$	$(\frac{1}{2}, 0)$	singlet	(0, 2)
$\hat{H}_1$	F_{H_1}	$H_1^i, \Psi_{H_1^i}$	$(0, \frac{1}{2})$	doublet	$(\frac{1}{2}, -1)$
$\hat{H}_2$	F_{H_2}	$H_2^i, \Psi_{H_2^i}$	$(0, \frac{1}{2})$	doublet	$(\frac{1}{2}, +1)$
$\hat{N}$	F_N	$N, \Psi N$	$(0, \frac{1}{2})$	singlet	(0, 0)

Table 3.2: Experimental Mass Limits of Supersymmetric Particles, S-leptons.

S-Particle	Symbol	Mass in GeV
Photino	$\tilde{\gamma}$	0 – 30
s-electron	$\tilde{e}$	> 43.4 (95%CL)
s-muon	$\tilde{\mu}$	> 43.0 (95%CL)
s-tau	$\tilde{\tau}$	> 43.0 (95%CL)
electron s-neutrino	$\tilde{\nu}_e$	≥ 36 (90%CL)
muon s-neutrino	$\tilde{\nu}_\mu$	≥ 36 (90%CL)
tau s-neutrino	$\tilde{\nu}_\tau$	≥ 36 (90%CL)

Table 3.3: Experimental Mass Limits of Supersymmetric Particles, S-quarks.

S-Particle	Symbol	Mass in GeV
Photino	$\tilde{\gamma}$	0 – 30
s-down	$\tilde{d}$	≥ 39 (95%CL)
s-strange	$\tilde{s}$	≥ 39 (95%CL)
s-bottom	$\tilde{b}$	≥ 39 (95%CL)
s-up	$\tilde{u}$	≥ 32.9 (90%CL)
s-charm	$\tilde{c}$	≥ 32.9 (90%CL)
s-top	$\tilde{t}$	$74 \leq m_i \leq 1000$

Table 3.4: Experimental Mass Limits of Supersymmetric Particles, Supersymmetric Higgs, Charginos, Neutralinos

S-Particle	Symbol	Mass in GeV
Photino	$\tilde{\gamma}$	0 – 30
Charged Higgs	$H^{\pm}$	≥ 35 (95%CL)
CP-even SW Higgs	H_1^0	≥ 37.1 (95%CL)
CP-even Higgs	H_2^0	$\geq 37.5, 45$ (95%CL)
CP-odd Higgs	H_3^0	$\geq 37.5, 45$ (95%CL)
Charginos	$\tilde{\chi}_j^{\pm}$	> 45 (95%CL)
Neutralinos	$\tilde{\chi}_j^0$	> 31 (95%CL)

Chapter 4

On Mass Shell Renormalization of the one loop Radiative Corrections in $e^+e^- \rightarrow W^+W^-$ in QFD and SQFD

4.1 Introduction

In Chapter 2 and 3 we gave

1. The Weak Lagrangian
2. The Lagrangian for the minimally broken supersymmetric extension of the EW model
3. An introduction to the OMRS scheme and applied it when the standard particles were present

Having the complete Feynman rules for both QFD and SQFD and the basic structure of the OMRS scheme at hand the purpose of this Chapter is to

1. Work out the details of the OMRS procedure parallel to that of section 2.4 when the supersymmetric particles are present

2. Then show the consistency of the OMRS procedure in the context of both QFD and SQFD by demonstrating that all infinities cancel when the appropriate counterterms are put in, as should in any renormalizable theory.

In this chapter, we will present only details of the OMRS regarding the cancellation of infinities in SQFD when the supersymmetry is unbroken. The model we use has been fully described in section 3.2. Recall that each multiplet contains equal number of Bose and Fermi degrees of freedom. As already mentioned we have intentionally chosen this multiplet structure to render the cancellation of quadratic and linear divergences explicit. The cancellation of quadratic and linear divergences leads to the non-renormalization theorems [25].¹ Indeed these cancellations are the prime reason why supersymmetry disturbs very little the radiative corrections of the standard model, indeed lowering them.

The outline of this chapter is as follows; in section 2 we derive the self energy counterterms in the gauge sector concentrating on the infinite parts. The calculation of the self energy counterterms is rendered transparent by the cancellation of quadratic and linear divergences. Section 3 shows that the definition of Thomson charge as developed in section 2.4 is left unchanged due to the addition of s -particles, section 4 deals with the counterterm Lagrangian and the cancellation of infinities as required in a renormalizable theory. Finally in section 5 we give some remarks on the general structure of the quantum corrections.

¹Some subtleties connected with the non-renormalization theorem in supersymmetric gauge theories are pointed out in reference [59]. Essentially they demonstrate the dependence of counterterms on the gauge chosen for supersymmetry i.e. if one chooses the *Wess-Zumino* gauge then there is an infinite mass counterterm, however this does not imply a violation of non-renormalisation theorem as shown in reference [59].

4.2 Self-energy counterterms in the gauge-sector

In the SQFD model the set of independent parameters we choose in the OMRS are $(\alpha, M_W, M_Z, M_{H_1^0}, m_f, m_s)$. Here m_s refers collectively to the masses of the supersymmetric particles. All the other parameters are the same as used in QFD, see section 2.4, and consequently have already been defined. Note that the number of independent input parameters has not changed from the tree level, as required in a renormalizable theory. The masses of supersymmetric particles don't enter as independent input parameters in the SUSY limit. And this is the limit which is relevant for the purpose of this Chapter. Now we start where we had left off in section 2.4. We had defined all the counterterms. Now we give the actual infinite contributions to these counterterms coming from the various self energy diagrams. To this end first consider the photon self energy diagrams. The self energy contributions to the photon are given in Fig. 6.5. The diagrams contributing to photon self energy as given in Fig. 6.5 may be conveniently broken down into the (gauge-boson + wino + charged Higgs scalar) and (Fermion + S Fermion) contributions. The total contribution in the gauge-boson, wino and charged Higgs to the photon self energy works out to be Fig. 6.5

$$i\pi_{\mu\nu}^{\gamma} = g^2 s_W^2 \int \frac{d^n k}{2\pi^n} \frac{(n-4)k_{\mu}k_{\nu} + (4-n)k^2}{(q^2 - M_W^2)((q+k)^2 - M_W^2)} \quad (4.1)$$

This reduces on using the integrals defined in Appendix C

$$i\pi_{\mu\nu}^{\gamma} = \frac{ig^2 s_W^2}{16\pi^2} \epsilon B_0(k_{\mu}k_{\nu} - k^2) \quad (4.2)$$

Using the definition given in Eq. 2.124 of section 2.4 one immediately obtains from Eq. 4.2

$$\pi^{\gamma}(k^2) = \frac{\alpha}{4\pi} \quad (4.3)$$

Several comments are in order. In arriving at Eq. 4.2 all the quadratic and linear divergences have cancelled, only a logarithmic divergence superficially remained which disappeared because of the factor $\epsilon = n - 4$, moreover the expression 4.2 is gauge invariant as it should. Turning now to the (fermion + S fermion) contribution Fig. 6.5 one obtains

$$i\pi_{\mu\nu}^{\gamma} = \frac{ig^2 s_W^2}{16\pi^2} \left[\sum_{f,c} Q_f^2 (k_{\mu} k_{\nu} (-2\epsilon B_{21} - 2\epsilon B_1 + 2B_0) + g_{\mu\nu} (-2\epsilon B_{22} - \epsilon B_0 m_f^2)) \right] \quad (4.4)$$

Once again the linear and quadratic divergences have disappeared since all the B integrals appearing in Eq. 4.4 which are linearly and quadratically divergent are being multiplied by the ϵ term hence giving only a finite contribution. On using the definitions of B integrals Eq. 4.4 reduces to the gauge invariant form

$$i\pi_{\mu\nu}^{\gamma} = \frac{ig^2 s_W^2}{16\pi^2} \left[\sum_{f,c} Q_f^2 (-2(k^2 g_{\mu\nu} - k_{\mu} k_{\nu}) [\Delta - \int_0^1 dx \ln(\frac{m_f^2}{M_W^2} - \frac{k^2}{M_W^2} x(1-x))] + \frac{2}{3}(k^2 g_{\mu\nu} - k_{\mu} k_{\nu})) \right]. \quad (4.5)$$

From Eq. 4.5 $\pi^{\gamma}(k^2)$ reads

$$\pi^{\gamma}(k^2) = -2 \frac{g^2 s_W^2}{16\pi^2} \sum_{f,c} Q_f^2 \Delta + \text{finite}, \quad (4.6)$$

where we have only written out explicitly the infinite part. Note that the contributions to the infinite part in Eq. 4.6 coming from fermions is

$$\pi^{\gamma}(k^2) = -\frac{1}{3} \frac{g^2 s_W^2}{16\pi^2} \sum_{f,c} Q_f^2 \Delta, \quad (4.7)$$

and from s-fermions is

$$\pi^{\gamma}(k^2) = -\frac{2}{3} \frac{g^2 s_W^2}{16\pi^2} \sum_{f,c} Q_f^2 \Delta. \quad (4.8)$$

Clearly Eq. 4.7 is in agreement with the previous calculation [43] in the OMRS scheme. Similarly to calculate a^W and a^Z the self energy diagrams for W and Z bosons were considered (Fig. 6.6 and Fig. 6.7). Following the hint that quadratic

and linear divergences must cancel between particles and their S-partners the expressions for self energies were grouped as displayed by the diagrams of Fig. 6.6 and Fig. 6.7. Following the same procedure as for the photon (Appendix F) the B integrals containing the quadratic and linear divergences gave no contribution from diagrams of Fig. 6.6, excluding the fermion and their S-partners in SUSY limit (section 6.6) $\frac{a^W(M_W^2)}{M_W^2}$ is

$$\frac{a^W(M_W^2)}{M_W^2} = \frac{g^2}{16\pi^2} \left[4 - \sec^2 \theta_W + \frac{1}{2} \frac{M_{H_1^0}^2}{M_W^2} + \frac{1}{2c_W^2} \right] \Delta + \text{finite}. \quad (4.9)$$

We don't display the finite part here since we are not concerned with it in the present Chapter. Similarly for $\frac{a^Z(M_Z^2)}{M_Z^2}$ excluding the fermions and S-fermions one finds

$$\frac{a^Z(M_Z^2)}{M_Z^2} = \frac{g^2}{16\pi^2} \left[4c_W^2 - \sec^2 \theta_W + \frac{1}{2} \frac{M_{H_1^0}^2}{M_W^2} + 1 - \frac{1}{2c_W^2} \right] \Delta + \text{finite} \quad (4.10)$$

Now plugging Eq. 4.9 and Eq. 4.10 in the expression for Y_{WZ} (see section 2.4) one obtains

$$Y_{WZ} = \frac{g^2}{16\pi^2} [\tan^2 \theta_W] \Delta + \text{finite} \quad (4.11)$$

excluding the contribution from fermions and S-fermions. Note that when only the gauge-sector and Higgs sector of QFD is considered one gets

$$\pi^{\gamma}(0) = \frac{g^2 s_W^2}{16\pi^2} [3\Delta] \quad (4.12)$$

$$\frac{a^W(M_W^2)}{M_W^2} = \frac{g^2}{16\pi^2} \left[2 - \sec^2 \theta_W + \frac{19}{6} \right] \Delta \quad (4.13)$$

$$\frac{a^Z(M_Z^2)}{M_Z^2} = \frac{g^2}{16\pi^2} \left[4c_W^2 - 2 - \sec^2 \theta_W + 3c_W^2 + \frac{1}{3} - \frac{1}{6c_W^2} \right] \Delta \quad (4.14)$$

$$Y_{WZ} = \frac{g^2}{16\pi^2} \left[3s_W^2 + \frac{1}{6} \tan^2 \theta_W \right] \Delta \quad (4.15)$$

$$\delta Z_W = \frac{g^2}{16\pi^2} \frac{19}{6} \Delta \quad (4.16)$$

Correspondingly we note that, when only the gaugino-sector and Higgs sector of SQFD is considered one gets

$$\pi^\gamma(0) = \frac{g^2 s_W^2}{16\pi^2} [-3\Delta], \quad (4.17)$$

$$\frac{a^W(M_W^2)}{M_W^2} = \frac{g^2}{16\pi^2} \left[-\frac{7}{6} + \frac{1}{2} \frac{M_{H_1^0}^2}{M_W^2} + \frac{1}{2} \sec^2 \theta_W \right] \Delta, \quad (4.18)$$

$$\frac{a^Z(M_Z^2)}{M_Z^2} = \frac{g^2}{16\pi^2} \left[+3s_W^2 - \frac{1}{3} + \frac{1}{2} \frac{M_{H_1^0}^2}{M_W^2} - \frac{1}{3} \sec^2 \theta_W \right] \Delta, \quad (4.19)$$

$$Y_{WZ} = \frac{g^2}{16\pi^2} \left[-3s_W^2 + \frac{5}{6} \tan^2 \theta_W \right] \Delta, \quad (4.20)$$

$$\delta Z_W = -\frac{g^2}{16\pi^2} \frac{13}{6} \Delta. \quad (4.21)$$

For quarks and leptons one gets

$$Y_{WZ} = \frac{g^2}{16\pi^2} \sum_f \left[-\frac{1}{6} n_c + \frac{1}{6} n_c - \frac{1}{6} n_c \tan^2 \theta_W + \frac{4}{3} n_c Q_f^2 s_W^2 \tan^2 \theta_W \right] \Delta + \text{finite}. \quad (4.22)$$

Similarly for S-quarks and S-leptons one gets

$$Y_{WZ} = \frac{g^2}{16\pi^2} \sum_f \left[-\frac{1}{12} n_c + \frac{1}{12} n_c - \frac{1}{12} n_c \tan^2 \theta_W + \frac{2}{3} n_c Q_f^2 s_W^2 \tan^2 \theta_W \right] \Delta + \text{finite}. \quad (4.23)$$

On using 4.3, 4.7, 4.8, 4.11, 4.22, and 4.23, in the definition of δZ_W excluding the fermions and S-fermions, one gets

$$\delta Z_W = \frac{g^2}{16\pi^2} \Delta, \quad (4.24)$$

for one generation of quarks and leptons

$$\delta Z_W = \frac{g^2}{16\pi^2} \sum_f \left[-\frac{1}{6} n_c \right] \Delta = \frac{g^2}{16\pi^2} \left[-\frac{4}{3} \right] \Delta, \quad (4.25)$$

and finally for one generation of S-quarks and S-leptons.

$$\delta Z_W = \frac{g^2}{16\pi^2} \sum_f \left[-\frac{1}{12} n_c \right] \Delta = \frac{g^2}{16\pi^2} \left[-\frac{2}{3} \right] \Delta. \quad (4.26)$$

The expressions for δZ_Z follow immediately and will not be given here. The counter terms δg , $\delta g'$, δZ_L , and δZ_R will be determined in the next section when we deal with the definition of Thomson limit.

4.3 Self energies of leptons and Thomson charge

Consider first the neutrino self energy diagrams Fig. 6.3 and recall here $\Psi_{H^\pm_1}$ is the SUSY partner of charged Higgs scalar; this part of $\tilde{\omega}$ will couple to fermions-S fermions via a coupling proportional to m_f/M_W since the Higgs fermion-fermion coupling goes as m_f/M_W , and $\lambda^\pm$ are the fermionic partners of the W gauge bosons. The diagrams 2, 2' and 2a' of Fig. 6.3 all contribute to the same order [i.e. to the order m^2/M^2] and hence are grouped together. The self energy diagrams of Fig. 6.3 are calculated and the linear divergences are shown to cancel between a particle and its S-partner's contribution (Appendix F). Having the self energy expression (section 6.2) of the neutrino at hand one finds in the OMRS δZ_L by subtracting the neutrino on the mass shell, section 2.4

$$\delta Z_L = \delta Z_L^{\tilde{\omega}} + \delta Z_L^i + \delta Z_L^H \quad (4.27)$$

where

$$\delta Z_L^{\tilde{\omega}} \doteq -\frac{1}{2} [\Delta + \text{finite}] \quad (4.28)$$

$$\delta Z_L^{\tilde{\omega}} \doteq -\frac{m^2}{2M_W^2} [\Delta + \text{finite}] \quad (4.29)$$

$$\delta Z_L^i \doteq -\frac{1}{4c_W^2} [\Delta + \text{finite}] \quad (4.30)$$

$$\delta Z_L^H \doteq -\frac{m^2}{4M_W^2} [\Delta + \text{finite}] \quad (4.31)$$

Using the definitions (2.83) through (2.84) and calculating the self energy diagrams for the electron (Fig. 6.4) we get

$$\begin{aligned}
f_L^W + \delta Z_L^W &\doteq -\frac{1}{2}[\Delta] + \text{finite}, \\
f_L^Z + f_L^\gamma + \delta Z_L^Z &\doteq -\frac{\cos^2 2\theta_W}{4c_W^2}[-\Delta] - s_W^2[\Delta] + \text{finite}, \\
f_L^{G^0} + f_L^{H_1^0} + \delta Z_L^G &\doteq -\frac{m^2}{4M_W^2}[\Delta] + \text{finite}, \\
f_L^{\tilde{W}} + \delta Z_L^{\tilde{W}} &\doteq -\frac{1}{2}[\Delta] + \text{finite}, \\
f_L^{\tilde{Z}} + f_L^{\tilde{h}} + \delta Z_L^{\tilde{W}} &\doteq -\frac{m^2}{4M_W^2}[\Delta] - \frac{m^2}{4M_W^2}[\Delta] + \text{finite}, \\
f_L^{\tilde{Z}} + f_L^{\tilde{\gamma}} + \delta Z_L^{\tilde{Z}} &\doteq -\frac{\cos^2 2\theta_W}{4c_W^2}[-\Delta] - s_W^2[\Delta] + \text{finite}, \\
f_L^{H_2^0} + f_L^{H_3^0} + \delta Z_L^H &\doteq -\frac{m^2}{4M_W^2}[\Delta] + \text{finite}.
\end{aligned} \tag{4.32}$$

Using the calculated values of δZ_L , from Eqs. (2.86, 2.87, 2.88, 4.28, 4.29, 4.30, and 4.31) in the above relationships Eq. 4.32, it immediately follows that the divergences cancel within each group, hence each f_L is finite as claimed before. Now turning to the lepton-lepton-photon vertex corrections. We define $\Lambda_{L,R}$ via [43]

$$-ie\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)\Lambda_L - ie\gamma^\mu\left(\frac{1+\gamma_5}{2}\right)\Lambda_R \tag{4.33}$$

and find

$$\begin{aligned}
\Lambda_L^W &\doteq +\frac{3}{2}[\Delta] + \text{finite}, \\
\Lambda_L^Z &\doteq +\frac{\cos^2 2\theta_W}{4c_W^2}[\Delta] + \text{finite}, \\
\Lambda_L^\gamma &\doteq +s_W^2[\Delta] + \text{finite}, \\
\Lambda_L^{G^0} &\doteq +\frac{m^2}{8M_W^2}[\Delta] + \text{finite}, \\
\Lambda_L^{H_1^0} &\doteq +\frac{m^2}{8M_W^2}[\Delta] + \text{finite},
\end{aligned}$$

$$\begin{aligned}
\Lambda_L^{\tilde{\omega}} &\doteq +\frac{1}{2}[\Delta] + \text{finite}, \\
\Lambda_L^{\tilde{z}} &\doteq +\frac{\cos^2 2\theta_W}{4c_W^2}[\Delta] + \text{finite}, \\
\Lambda_L^{\tilde{\gamma}} &\doteq +s_W^2[\Delta] + \text{finite}, \\
\Lambda_L^{\tilde{z}} &\doteq +\frac{m^2}{4M_W^2}[\Delta] + \text{finite}, \\
\Lambda_L^{\tilde{h}} &\doteq +\frac{m^2}{4M_W^2}[\Delta] + \text{finite}, \\
\Lambda_L^{H_2^0} &\doteq +\frac{m^2}{8M_W^2}[\Delta] + \text{finite}, \\
\Lambda_L^{H_3^0} &\doteq +\frac{m^2}{8M_W^2}[\Delta] + \text{finite}.
\end{aligned} \tag{4.34}$$

The above equations given in Eq. 4.34 are added up with those given in Eq. 4.32 the following results are found

$$\begin{aligned}
f_L^W + \delta Z_L^W + \Lambda_L^W &= \frac{g^2}{16\pi^2}\Delta, \\
f_L^Z + f_L^\gamma + \delta Z_L^Z + \Lambda_L^Z + \Lambda_L^\gamma &= 0, \\
f_L^{G^0} + f_L^{H_1^0} + \delta Z_L^G + \Lambda_L^{G^0} + \Lambda_L^{H_1^0} &= 0, \\
f_L^{H_1^0} + f_L^{H_3^0} + \delta Z_L^H + \Lambda_L^{H_2^0} + \Lambda_L^{H_3^0} &= 0, \\
f_L^{\tilde{\omega}} + \delta Z_L^{\tilde{\omega}} + \Lambda_L^{\tilde{\omega}} &= 0, \\
f_L^{\tilde{z}} + f_L^{\tilde{\gamma}} + \delta Z_L^{\tilde{z}} + \Lambda_L^{\tilde{z}} + \Lambda_L^{\tilde{\gamma}} &= 0, \\
f_L^{\tilde{z}} + f_L^{\tilde{h}} + \delta Z_L^{\tilde{\omega}} + \Lambda_L^{\tilde{z}} + \Lambda_L^{\tilde{h}} &= 0.
\end{aligned} \tag{4.35}$$

Summing over all contributions, in Eq. 4.35 the total may be written as

$$f_L + \delta Z_L + \Lambda_L \doteq \Delta \tag{4.36}$$

In a similar manner one finds

$$\delta Z_R^C + \Lambda_R^C = 0, \quad C = (Z, \gamma, G^0, G, H, \tilde{z}, \tilde{\gamma}, H_2^0, H_3^0, \tilde{h}, \tilde{\omega}) \tag{4.37}$$

The total of which may be written as

$$\delta Z_R + \Lambda_R = 0. \quad (4.38)$$

The counterterm diagram 5e in Fig 4.1. gives, see also section 2.4

$$\begin{aligned} \Lambda_R^c &= \delta Z_R + \frac{\delta g'}{e} \cos \theta_W, \\ \Lambda_L^c &= \delta Z_L + \frac{\delta g}{2g} + \frac{\delta g'}{2g} \cot \theta_W. \end{aligned} \quad (4.39)$$

Our renormalization condition requires

$$\begin{aligned} \Lambda_R + \delta Z_R + \frac{\delta g'}{e} \cos \theta_W &= 0, \\ f_L + \Lambda_L + \delta Z_L + \frac{\delta g}{2g} + \frac{\delta g'}{2g} \cot \theta_W &= 0. \end{aligned} \quad (4.40)$$

Together with Eq. 4.36 and 4.38 this implies

$$\begin{aligned} \delta g' &= 0, \\ \frac{\delta g}{2g} &= -\frac{g^2}{16\pi^2} \Delta, \end{aligned} \quad (4.41)$$

which are valid to all order in m^2/M^2 , as it should be since we require universality as mentioned before. The relations in Eq. 4.41 are the Thomson conditions. In the next section we will go on to use the Thomson condition to show the consistency of our renormalization procedure within our model.

4.4 The Counterterm Lagrangian and the cancellation of infinities:

In this section we apply the considerations of the previous sections 2 and 3 [see also Chapter 2, section 2.4] to the $e^+e^-\gamma$, e^+e^-Z , γW^+W^- , ZW^+W^- , $e^+\bar{\nu}_e W^+$, and $e^-\nu_e W^-$ vertices, since all these vertices appear in the one loop corrections to the $e^+e^- \rightarrow W^+W^-$ reaction. The vertex corrections to the tree level diagram can

be classified into three types: $e^+e^-\gamma$, e^+e^-Z ; γW^+W^- , ZW^+W^- ; and $e^+\bar{\nu}_e W^+$, $e^-\nu_e W^-$. To this end we consider the counterterm Lagrangian, and we recall that this can be generated from the original QFD Lagrangian by the rescalings listed in Chapter 2². The parts of the counterterm Lagrangian relevant to these vertices are given by

$$\begin{aligned}
 \mathcal{L}_{counter} = & \sum_{f,c} \bar{f} \not{Z} \{ \{ \frac{g}{c_W} (T_{3f} - Q_f s_W^2) \delta Z_L + c_W \delta g T_{3f} - s_W \delta g' (Q_f - T_{3f}) \} L \\
 & - \{ g Q_f \frac{s_W}{c_W} \delta Z_R + s_W \delta g' Q_f \} R \} f + \sum_{f,c} \bar{f} A [\{ g s_W Q_f \delta Z_L + s_W \delta g T_{3f} + c_W \delta g' (Q_f - T_{3f}) \} L \\
 & + Q_f \{ g s_W \delta Z_R + c_W \delta g' \} R] f + \sum_{d,c} \frac{g}{\sqrt{2}} \left(\delta Z_L + \frac{\delta g}{g} \right) \bar{L}_f \gamma^\mu (\tau_+ W_\mu^+ + \tau_- W_\mu^-) L_f \\
 & - \frac{ig}{2} \left(\delta Z_W + \frac{\delta g}{g} \right) \{ F_{\mu\nu}^+ [(c_W Z^\mu + s_W A^\mu) W^{-\nu} - (c_W Z^\nu + s_W A^\nu) W^{-\mu}] + F_{\mu\nu}^- [(c_W Z^\nu + s_W A^\nu) W^{+\mu} \\
 & - (c_W Z^\mu + s_W A^\mu) W^{+\nu}] + [c_W Z_{\mu\nu} + s_W A_{\mu\nu}] [W^{-\mu} W^{+\nu} - W^{+\mu} W^{-\nu}] \}
 \end{aligned} \quad (4.42)$$

here

$$\begin{aligned}
 \tau_\pm &= \frac{\tau_1 \pm i\tau_2}{2}, \\
 A_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \\
 Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu, \\
 W_{\mu\nu}^\pm &= \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm, \\
 \begin{pmatrix} L \\ R \end{pmatrix} &::= \frac{1 \mp \gamma_5}{2}.
 \end{aligned} \quad (4.43)$$

It will be shown below that divergences in all the diagrams contributing to the six relevant vertices are cancelled by the allowed counterterms given in the above Lagrangian, Eq. 4.42, as is required in a renormalizable theory. Since a lot of diagrams are involved in these cancellations, this provides an excellent check on the calculation. The contribution of the finite parts of the counterterms to the

²We need only consider the QFD part of the full SQFD Lagrangian since the external particles involved in the reaction $e^+e^- \rightarrow W^+W^-$ are all standard particles

one loop corrected cross section will be given in Chapter 7. For the moment we focus our attention on the infinite parts. Consider first the corrections to the $e^+e^-\gamma$ vertex. Note that this has already been considered implicitly in the previous section. Tables 4.1 and 4.2 list the infinite contributions to this vertex³. It is clear from the counterterm Lagrangian Eq. 4.42 and Tables 4.1, 4.2 that

$$\Lambda_L + \Lambda_L^c = \text{finite}, \quad (4.44)$$

$$\Lambda_L^c = \delta Z_L + \frac{\delta g}{2g} + \frac{\delta g'}{2g} \cot \theta_W, \quad (4.45)$$

$$\Lambda_R + \Lambda_R^c = \text{finite}, \quad (4.46)$$

$$\Lambda_R^c = \delta Z_R + \frac{\delta g'}{g} \cot \theta_W. \quad (4.47)$$

The values of δg and $\delta g'$ have already been determined in the previous section Eqs. 4.41. Recalling the values δZ_L and δZ_R for the Standard particles (see Eqs. 2.86 through 2.90 Chapter 2) we have

$$\Lambda_L^c \doteq -\frac{1}{2} \Delta - \frac{1}{4c_W^2} \Delta - \Delta, \quad (4.48)$$

$$\Lambda_R^c \doteq -s_W^2 \Delta - \frac{s_W^4}{c_W^2} \Delta. \quad (4.49)$$

On adding these Eqs. [4.48 and 4.49] to the values given in Table 4.1 we immediately see that the infinities cancel. The contribution from the S-P to the counterterms is

$$\Lambda_L^c \doteq -\frac{1}{2} \Delta - \frac{1}{4c_W^2} \Delta, \quad (4.50)$$

$$\Lambda_R^c \doteq -s_W^2 \Delta - \frac{s_W^4}{c_W^2} \Delta. \quad (4.51)$$

The contributions from these Eqs. [4.50 and 4.51] cancel the infinities arising from the triangle diagrams given in Table 4.2.

³from now on we ignore terms proportional to the electron mass

Now consider the e^+e^-Z vertex. This vertex is similar to the $e^+e^-\gamma$ vertex except for the couplings. We merely state the results. Renormalization demands

$$\Lambda_L + \Lambda_L^{Z;c} = \text{finite}, \quad (4.52)$$

$$\Lambda_L^{Z;c} = 2(1 - 2s_W^2)\delta Z_L + 2\cos\theta_W^2\frac{\delta g}{g} - 2\frac{\delta g'}{g}\cos\theta_W\sin\theta_W, \quad (4.53)$$

$$\Lambda_R + \Lambda_R^{Z;c} = \text{finite}, \quad (4.54)$$

$$\Lambda_R^{Z;c} = -4\sin\theta_W^2\delta Z_R - \frac{\delta g'}{g}\cos\theta_W\sin\theta_W. \quad (4.55)$$

Substituting the contributions from the standard particles into 4.53 and 4.55 we get

$$\Lambda_L^{Z;c} \doteq -2(1 - 2s_W^2)\left[\frac{1}{2}\Delta + \frac{1}{4c_W^2}\Delta\right] - 4c_W^2\Delta, \quad (4.56)$$

$$\Lambda_R^{Z;c} \doteq 4s_W^4\Delta + 4\frac{s_W^6}{c_W^2}\Delta. \quad (4.57)$$

These infinities cancel against those given in Table 4.3 Substituting the contributions from the S-P into 4.53 and 4.55 we get

$$\Lambda_L^{Z;c} \doteq -2(1 - 2s_W^2)\left[\frac{1}{2}\Delta + \frac{1}{4c_W^2}\Delta\right], \quad (4.58)$$

$$\Lambda_R^{Z;c} \doteq 4s_W^4\Delta + 4\frac{s_W^6}{c_W^2}\Delta. \quad (4.59)$$

These infinities cancel against those given in Table 4.4

Let us now consider the corrections to the γW^+W^- Among the many vertex correction diagrams, most give infinite parts, Figs. 7.7, 7.8, 7.9 and 7.10. The diagrams in Figs. 7.7, 7.8, 7.9, and 7.10 that only give finite contributions will be ignored for the moment since as already mentioned in this Chapter we only concentrate on the infinities. We note that our notation for the 3 vector vertex is defined in Fig. 5.1. There are lengthy and tedious algebraic calculations associated with the some of the diagrams contributing to this vertex (see Figs. 7.7, 7.8, 7.9

and 7.10). These have been derived, checked with Macsyma, and checked against the calculations of Sundaresan and Kalyniak [44]. The various contributions are listed in Tables 4.5 and 4.6. The relevant counterterm for the $\gamma W^+ W^-$ vertex from the Lagrangian, Eq. 4.42 is $(\delta Z_W + \frac{\delta g}{g})$. Renormalizability demands that this counterterm cancel all the infinities arising from all the vertex contributions to the $\gamma W^+ W^-$ vertex listed in Tables 4.5 and 4.6. We have already calculated the various contributions to δZ_W in section 4.2. For the standard particles we have first consider the fermionic contribution from one generation of quarks and leptons on collecting δZ_W (Eq. 4.25), and total contribution from Table 4.5 we have

$$(\delta Z_W)^{\text{fermions}} + \left(\frac{\delta g}{g}\right)_{\gamma W^+ W^-, \text{fermions}} \doteq -\frac{4}{3} \Delta + \frac{4}{3} \Delta = 0 \quad (4.60)$$

The gauge and the Higgs sectors of the SW, Eq. 4.16, along with the contribution from $\frac{\delta g}{g}$, Eq. 4.41, and total $\frac{\delta g}{g}$ in Table 4.6 yield

$$\left(\delta Z_W + \frac{\delta g}{g}\right)^{\text{gauge+Higgs+Thomson}} + \left(\frac{\delta g}{g}\right)^{\gamma W^+ W^-, \text{gauge+Higgs}} \doteq +\frac{19}{6} \Delta - 2\Delta - \frac{7}{6} \Delta = 0 \quad (4.61)$$

For one generation of S-leptons and S-quarks, see Eq. 4.26 and Table 4.5

$$(\delta Z_W)^{\text{S-fermions}} + \left(\frac{\delta g}{g}\right)^{\gamma W^+ W^-, \text{S-fermions}} \doteq -\frac{2}{3} \Delta + \frac{2}{3} \Delta = 0 \quad (4.62)$$

For gauginos and the extra Higgs introduced by SUSY, see Eq. 4.21 and Table 4.6 we have

$$(\delta Z_W)^{\text{gauginos+Higgs}} + \left(\frac{\delta g}{g}\right)^{\gamma W^+ W^-, \text{gauginos+Higgs}} \doteq -\frac{13}{6} \Delta + \frac{13}{6} \Delta = 0 \quad (4.63)$$

This completes our discussion for the $\gamma W^+ W^-$ vertex. The argument for the $Z W^+ W^-$ vertex follows very closely that of $\gamma W^+ W^-$ vertex, so we will not discuss it in full but note the following points. Most of the $Z W^+ W^-$ vertex diagrams can be obtained from the $\gamma W^+ W^-$ vertex by changing the external ' γ ' line to a ' Z '

line and some simple changes in coupling. There are few other extra diagrams for the ZW^+W^- vertex. These extra diagrams and the changed couplings conspire together so that the total contribution to $\frac{\delta g}{g}$ still remain the same as that of the γW^+W^- vertex. The required cancellations follow immediately since for ZW^+W^- vertex the required counterterm is also $(\delta Z_W + \frac{\delta g}{g})$. Finally we turn to $e^+\bar{\nu}_e W^+$ vertex. In this case the counterterm is $(\delta Z_L + \frac{\delta g}{g})$, (see Eq. 4.42) For the contribution of the standard particles we have, including the Thomson contribution, Eq. 4.41, see section 2.4 for δZ_L and Table 4.9 ⁴

$$\left(\delta Z_L + \frac{\delta g}{g}\right)^{\text{infinite}} + \left(\frac{\delta g}{g}\right)^{e^+\bar{\nu}_e W^+ \text{ vertex}} = -\frac{1}{2}\Delta - \frac{\sec^2 \theta_W}{4}\Delta - 2\Delta + \frac{5}{2}\Delta + \frac{\sec^2 \theta_W}{4}\Delta = 0(4.64)$$

for the contribution of the S-P we have, see Eqs. 4.28, 4.30 and Table 4.9

$$\left(\delta Z_L + \frac{\delta g}{g}\right)^{\text{infinite}} + \left(\frac{\delta g}{g}\right)^{e^+\bar{\nu}_e W^+ \text{ vertex}} = -\frac{1}{2}\Delta - \frac{\sec^2 \theta_W}{4}\Delta + \frac{1}{2}\Delta + \frac{\sec^2 \theta_W}{4}\Delta = 0(4.65)$$

In conclusion we have provided the details of the cancellation of the divergences in the process $e^+e^- \rightarrow W^+W^-$ using the on mass-shell renormalization procedure. There are many remaining diagrams which lead to finite contributions all of which must be included in finally calculating the numerical magnitude of these corrections. The finite contributions from the counterterms must also be included in these calculations. The remainder of this thesis will deal with these finite effects and the cancellation of the infrared infinities.

⁴The superscript 'infinite' in Eqs. 4.64 and 4.65 emphasises that we are dealing here only with the infinite contributions.

4.5 General Structure of the Quantum Corrections

In calculating the one loop corrections to $e^+e^- \rightarrow W^+W^-$ in SQFD one encounters a large number of Feynman diagrams. A systematic procedure was pioneered by Passarino and Veltman [45] to deal with this plethora of integrals. Using this approach we must evaluate in the dimensional regularization scheme integrals of the following type: [Appendices C, and H]

$$A(m) = -\frac{1}{i\pi^2} \int d^n q \frac{1}{(q^2 - m^2)} \quad (4.66)$$

$$B_0; B_\mu; B_{\mu\nu} = \frac{1}{i\pi^2} \int d^n q \frac{1; q_\mu; q_\mu q_\nu}{(q^2 - m_1^2)((q+k)^2 - m_2^2)} \quad (4.67)$$

$$C_0; C_\mu; C_{\mu\nu}; C_{\mu\nu\alpha} = -\frac{1}{i\pi^2} \int d^n q \frac{1; q_\mu; q_\mu q_\nu; q_\mu q_\nu q_\alpha}{(q^2 - m_1^2)((q+k)^2 - m_2^2)((q+p)^2 - m_3^2)} \quad (4.68)$$

where

$$p_5 = k + p$$

$$D_0; D_\mu; D_{\mu\nu}; D_{\mu\nu\alpha} = \frac{1}{i\pi^2} \int d^n q \frac{1; q_\mu; q_\mu q_\nu; q_\mu q_\nu q_\alpha}{(q^2 - m_1^2)((q+k)^2 - m_2^2)((q+k+p)^2 - m_3^2)((q+k+p+l)^2 - m_4^2)} \quad (4.69)$$

Every such one loop integral can be decomposed in terms of form factors[44]. For example consider C_μ this can be written as

$$C_\mu = C_{11}k_\mu + C_{12}p_\mu \quad (4.70)$$

After some algebra one can express C_{11} and C_{12} in terms of the scalar integrals C_0 and 3 different B_0 's [44, 45]. In this manner one can iteratively express all the invariant functions

$$B_1, \dots, B_{22}, C_{11}, \dots, C_{36}, D_{11}, \dots, D_{313}$$

in terms of the scalar integrals A, B_0, C_0 and D_0 . General expressions for these

Figure 4.1: Diagrams which are needed to arrive at the definition of the Thomson charge i.e. to determine the counterterms δg and $\delta g'$. These are the one particle irreducible diagrams contributing to the lepton-lepton-photon vertex.

scalar integrals were given in terms of logarithms and Spence functions by 't Hooft and Veltman [73]. The explicit details of this procedure may be found in [44](Ph.d Thesis Notes) and the Appendices C, H, and K. We [44](Ph.d Thesis Notes) have also given complete expressions for the infrared [IR] singular integrals and other relevant integrals encountered. One great advantage of the above procedure is that it allows a consistency check on the computer program, since some integrals like C_{23}, C_{34} are computed in two different ways [45]. In this manner we obtain numerical consistency up to at least eight places of decimal.

Table 4.1: Infinite contributions of the Standard particles triangle loops to the $e^+e^-\gamma$ vertex. A common factor of $-ie\frac{g^2}{16\pi^2}(\frac{1\mp\gamma_5}{2})$ has been taken out.

Lefthanded	Contribution	Righthanded	Contribution
7.3, 1	$s_W^2 \Delta$	7.3, 1	$s_W^2 \Delta$
7.3, 2	$\frac{(1-2s_W^2)^2}{4c_W^2} \Delta$	7.3, 2	$\frac{s_W^4}{c_W^2} \Delta$
7.3, 3	$\frac{3}{2} \Delta$	7.3, 3	—
Total	$(\frac{1}{4c_W^2} + \frac{3}{2})\Delta$	Total	$(s_W^2 + \frac{s_W^4}{c_W^2})\Delta$

Table 4.2: Infinite contributions of the s particles triangle loops to the $e^+e^-\gamma$ vertex. A common factor of $-ie\frac{g^2}{16\pi^2}(\frac{1\mp\gamma_5}{2})$ has been taken out.

Lefthanded	Contribution	Righthanded	Contribution
7.3, 1'	$s_W^2 \Delta$	7.3, 1'	$s_W^2 \Delta$
7.3, 2'	$\frac{(1-2s_W^2)^2}{4c_W^2} \Delta$	7.3, 2'	$\frac{s_W^4}{c_W^2} \Delta$
7.3, 3'	$\frac{1}{2} \Delta$	7.3, 3'	—
Total	$(\frac{1}{4c_W^2} + \frac{1}{2})\Delta$	Total	$(s_W^2 + \frac{s_W^4}{c_W^2})\Delta$

Table 4.3: Infinite contributions of the Standard particles triangle loops to the e^+e^-Z vertex. A common factor of $-i\frac{g}{4c_W}\frac{g^2}{16\pi^2}(\frac{1\mp\gamma_5}{2})$ has been taken out.

Lefthanded	Contribution	Righthanded	Contribution
7.4, 1	$2s_W^2(1 - 2s_W^2)\Delta$	7.4, 1	$-4s_W^4\Delta$
7.4, 2	$\frac{(1-2s_W^2)^3}{2c_W^2}\Delta$	7.4, 2	$\frac{-4s_W^6}{c_W^2}\Delta$
7.4, 3	$6c_W^2\Delta$	7.4, 3	---
7.4, 4	$-\Delta$	7.4, 4	---
Total	$[2(1 - 2s_W^2)(\frac{1}{4c_W^2} + \frac{1}{2}) + 4c_W^2]\Delta$	Total	$(-4s_W^4 - 4\frac{s_W^6}{c_W^2})\Delta$

Table 4.4: Infinite contributions of the s-particles triangle loops to the e^+e^-Z vertex. A common factor of $-i\frac{g}{4c_W}\frac{g^2}{16\pi^2}(\frac{1\mp\gamma_5}{2})$ has been taken out.

Lefthanded	Contribution	Righthanded	Contribution
7.4, 1'	$2s_W^2(1 - 2s_W^2)\Delta$	7.4, 1'	$-4s_W^4\Delta$
7.4, 2'	$\frac{(1-2s_W^2)^3}{2c_W^2}\Delta$	7.4, 2'	$\frac{-4s_W^6}{c_W^2}\Delta$
7.4, 3'	$2c_W^2\Delta$	7.4, 3'	---
7.4, 4'	$-\Delta$	7.4, 4'	---
Total	$2(1 - 2s_W^2)(\frac{1}{4c_W^2} + \frac{1}{2})\Delta$	Total	$(-4s_W^4 - 4\frac{s_W^6}{c_W^2})\Delta$

Table 4.5: Infinite contributions of the fermion and the s-fermion triangle loops to the $\gamma W^+ W^-$ vertex, s-graph is the contribution of the s-fermion. A common factor of $-ie\frac{g^2}{16\pi^2}\Gamma^{\mu\nu\lambda}$ has been taken out.

Graph	Contribution	S-graph	Contribution
7.10, 1	$\frac{1}{3}\Delta$	7.10 1'	$\frac{1}{6}\Delta$
7.10, 2	$\frac{1}{3}\Delta$	7.10, 2'	$\frac{1}{6}\Delta$
7.10, 3	$\frac{2}{3}\Delta$	7.10, 3'	$\frac{1}{3}\Delta$
Total	$\frac{4}{3}\Delta$	Total	$\frac{2}{3}\Delta$

Table 4.6: Infinite contributions of the Higgs-Higgsino and the gauge-gaugino triangle loops to the $\gamma W^+ W^-$ vertex. A common factor of $-ie\frac{g^2}{16\pi^2}\Gamma^{\mu\nu\lambda}$ has been taken out. Note that y.7 is the sum of five diagrams, 7, and 8 in Fig. 7.8, and 7, 8, and 9 in Fig. 7.9.

Graph	Contribution	S-graph	Contribution
7.7, 1	$\frac{1}{12}\Delta$	7.7, 4'	$\frac{1}{12}\Delta$
7.8, 1	$\frac{13}{4}s_W^2\Delta$	7.8, 1'	$\frac{1}{3}\Delta$
7.8, 5,6	$-\frac{1}{12}s_W^2\Delta$	7.8, 1'	$\frac{4}{3}s_W^2\Delta$
7.9, 1	$\frac{13}{4}c_W^2\Delta$	7.9,5'	$(\frac{4}{3}c_W^2 + \frac{1}{3})\Delta$
7.9,5,6	$-\frac{1}{12}c_W^2\Delta$	7.9,4'	$\frac{1}{12}\Delta$
7.9,13	$\frac{1}{12}\Delta$	-	-
y.7	$-\frac{9}{2}\Delta$	-	-
Total	$-\frac{7}{6}\Delta$	Total	$\frac{13}{6}\Delta$

Table 4.7: Infinite contributions of the fermion and the s-fermion triangle loops to the ZW^+W^- vertex, s-graph is the contribution of the s-fermion. A common factor of $-igc_W \frac{g^2}{16\pi^2} \Gamma^{\mu\nu\lambda}$ has been taken out.

Graph	Contribution	S-graph	Contribution
7.10, 1	$\frac{1-2s_W^2}{6c_W^2} \Delta$	7.10, 1'	$\frac{1-2s_W^2}{12c_W^2} \Delta$
7.10, 2	$\frac{1}{6c_W^2} \Delta$	7.10, 2'	$\frac{1}{12c_W^2} \Delta$
7.10, 3	$\frac{1-\frac{4}{3}s_W^2}{2c_W^2} \Delta$	7.10, 3'	$\frac{1-\frac{4}{3}s_W^2}{4c_W^2} \Delta$
7.10, 4	$\frac{1-\frac{4}{3}s_W^2}{2c_W^2} \Delta$	7.10, 4'	$\frac{1-\frac{4}{3}s_W^2}{4c_W^2} \Delta$
Total	$\frac{4}{3} \Delta$	Total	$\frac{2}{3} \Delta$

Table 4.8: Infinite contributions of the Higgs-Higgsino and the gauge-gaugino triangle loops to the ZW^+W^- vertex. A common factor of $-igc_W \frac{g^2}{16\pi^2} \Gamma^{\mu\nu\lambda}$ has been taken out. Note that y.9 is the sum of five diagrams, 7, and 8 in Fig. 7.8, and 7, 8, and 9 in Fig. 7.9.

Graph	Contribution	S-graph	Contribution
7.7, 1	$\frac{1-2s_W^2}{24c_W^2} \Delta$	7.7,4'	$\frac{1-2s_W^2}{24c_W^2} \Delta$
7.8,1	$\frac{13}{4}s_W^2 \Delta$	7.7,1'	$\frac{1-2s_W^2}{6c_W^2} \Delta$
7.8,5,6	$-\frac{1}{12}s_W^2 \Delta$	7.8,1'	$\frac{4}{3}s_W^2 \Delta$
7.9,1	$\frac{13}{4}c_W^2 \Delta$	7.9,5'	$(\frac{4}{3}c_W^2 + \frac{1-2s_W^2}{6c_W^2})\Delta$
7.9,5,6	$-\frac{1}{12}c_W^2 \Delta$	7.7,2'	$\frac{1}{6c_W^2} \Delta$
7.9,13	$\frac{1-2s_W^2}{24c_W^2} \Delta$	7.9,4'	$\frac{1-2s_W^2}{24c_W^2} \Delta$
7.7,5	$\frac{1}{24c_W^2} \Delta$	7.7,5'	$\frac{1}{24c_W^2} \Delta$
7.7,6	$\frac{1}{24c_W^2} \Delta$	7.7,6'	$\frac{1}{24c_W^2} \Delta$
y.9	$-\frac{9}{2} \Delta$	7.7,3'	$\frac{1}{6c_W^2} \Delta$
Total	$-\frac{7}{6} \Delta$	Total	$\frac{13}{6} \Delta$

Table 4.9: Infinite contributions to the $e^+\bar{\nu}_e W^+$ vertex. A common factor $(ig^3)/(2(2)^{\frac{1}{2}}16\pi^2)\gamma_\nu(1 - \gamma_5)$ has been taken out.

Graph	Contribution	S-graph	Contribution
7.5,4	$-\frac{(2-4s_W^2)}{8c_W^2}\Delta$	7.5,4'	$-\frac{(2-4s_W^2)}{8c_W^2}\Delta$
7.5,2	$\frac{3(2-4s_W^2)}{4}\Delta$	7.5,2'	$\frac{2-4s_W^2}{4}\Delta$
7.5,1	$3s_W^2\Delta$	7.5,1'	$s_W^2\Delta$
7.5,3	$\frac{3}{2}\Delta$	7.5,3'	$\frac{1}{2}\Delta$
Total	$(\frac{5}{2} + \frac{\sec^2\theta_W}{4})\Delta$	Total	$(\frac{1}{2} + \frac{\sec^2\theta_W}{4})\Delta$

Chapter 5

Tree level and Bremsstrahlung cross section.

5.1 Introduction

The tree level cross section for W pair production was calculated by Alles et al. [60], and since then has been extensively quoted in the literature [50]. As we need the general form of the cross section for the discussion of radiative corrections, we have calculated the tree level cross section [44] for W pair production with W's off mass shell and extracted

1. The differential cross section at the tree level with on shell W's, and,
2. The total cross section.

One can write for the total differential cross section,

$$d\sigma^T = d\sigma^0 + d\sigma^{self} + d\sigma^{vertex} + d\sigma^{box} + d\sigma^{brem} + d\sigma^{h.l.}, \quad (5.1)$$

Here $d\sigma^0$ is the lowest order (or tree level) cross section, $d\sigma^{self}$ contains self energy and wave function renormalization terms, $d\sigma^{vertex}$ is the total contribution of corrections to the vertices in the lowest order diagrams, $d\sigma^{box}$ contains corrections arising from 4-point irreducible one loop diagrams, $d\sigma^{brem}$ is the bremsstrahlung

contribution, $d\sigma^{ren}$ contains renormalization counterterms and finally $d\sigma^{h,l}$ contains higher loops correction [beyond one loop] not calculated in this thesis. It is expected to be small in a perturbatively renormalizable theory.

Although not directly related to the main topic of the thesis, the calculation of radiative corrections, a question arises what if the three vector coupling is modified with the W being a composite object with an anomalous magnetic and quadrupole moment different from the SM values. Then the gauge cancellations seen in the SM would not occur. To this end we have recalculated the tree level cross section when the W possesses phenomenological magnetic and quadrupole moments different from the SM values.

The outline of this chapter is as follows. In the next section, we give the tree level differential cross section for both on and off shell W 's and the total cross section. In the following section the tree level differential cross section is given when the 3-vector vertices are allowed to take a non standard form. The last section in this chapter deals with differential cross section due to bremsstrahlung when the W 's are assumed to be on the mass shell. As is well known infrared infinities arise in this cross section. These are in turn expected to be cancelled by virtual loop corrections. After assembling all the loop corrections this is indeed shown to be the case.

5.2 The Standard Model differential and the total cross section for on and off shell W 's

As already mentioned the tree level cross section for the process $e^+e^- \rightarrow W^+W^-$ in QFD has been extensively quoted in literature since its first calculation by Alles et al. [60]. Here we establish notation, give definitions, and briefly discuss the unpolarized differential cross section for this process. As is well known, to get more realistic results, the decay of W 's into fermion pairs must be considered. To achieve this W 's must be considered off shell [61]. The expression for the cross section for the W 's off shell was given in [61]. Incidentally, no such expression to our knowledge has been given for the differential cross section. Here we give the result for the differential cross section with W 's off shell. In the limit that W 's are on shell our expression reduces to the well known result [60, 28, 29]. A number of misprints in [28] have been pointed out by Phillippe [29]. A discussion when the electron and positron beams are polarized may be found in [31]. Our results for the total cross section agree with those given in [61] and we will discuss them at the end of this section. The momenta of the incoming positron and electron and the outgoing W^+ and W^- are denoted respectively by p_1 , p_2 , q_1 and q_2 . The invariants of the process are given by (when the W 's are off shell)

$$p_1^2 = m_e^2, \tag{5.2}$$

$$p_2^2 = m_e^2, \tag{5.3}$$

$$q_1^2 = s_3, \tag{5.4}$$

$$q_2^2 = s_4. \tag{5.5}$$

For the on mass W 's, $s_3 = s_4 = M_W^2$. The Mandelstam variables are defined in the usual manner

$$s = (p_1 + p_2)^2 = (q_1 + q_2)^2, \quad (5.6)$$

$$t = (q_1 - p_1)^2 = (p_2 - q_2)^2, \quad (5.7)$$

$$u = (q_2 - p_1)^2 = (p_2 - q_1)^2, \quad (5.8)$$

$$s + t + u = 2m_e^2 + s_3 + s_4. \quad (5.9)$$

In terms of the center of mass (CM) scattering angle, θ ,¹ the beam energy, E_b , and the CM energy, E_{CM} the above expressions (5.6 through 5.9) may be rewritten as

$$s = E_{CM}^2 = 4E_b^2, \quad (5.10)$$

$$t = m_e^2 + s_4 - s(1 - \beta_e \beta_W \cos \theta), \quad (5.11)$$

$$u = s_3 + s_4 + 2m_e^2 - s - t, \quad (5.12)$$

where

$$\beta_i = \sqrt{1 - \frac{4p_i^2}{s}} \quad (5.13)$$

is the velocity of the i^{th} particle, with p_i being its four momenta. The contributions to the lowest order Born cross section comes from the three diagrams of Fig. 5.1, b, c, d. Since the Higgs exchange contributes $\frac{m_f^2}{M_W^2}$ relative to the remaining three it can be neglected. The contribution to lowest order amplitude can be succinctly written as

$$\begin{aligned} \mathcal{M}_0 = & ig^2 [\bar{v}(p_1) \gamma^\mu (a_s + b_s \gamma_5) u(p_2) \Gamma_{\mu\nu\lambda} \epsilon_1^\nu \epsilon_2^\lambda] \\ & - ig^2 [\bar{v}(p_1) \not{\epsilon}_1 (\not{q}_1 - \not{p}_1) \not{\epsilon}_2 (a_t + b_t \gamma_5) u(p_2)] \end{aligned} \quad (5.14)$$

¹where we take θ to be the angle between the electron and the W^- direction.

Here

$$a_s = \frac{s_W^2}{s} + \frac{(1 - 4s_W^2)}{(4(s - M_Z^2))}, \quad (5.15)$$

$$b_s = \frac{-1}{(4(s - M_Z^2))}, \quad (5.16)$$

$$a_t = \frac{1}{(4t)}, \quad (5.17)$$

$$b_t = -a_t, \quad (5.18)$$

and

$$\Gamma_{\mu\nu\lambda} = g_{\mu\lambda}(p_5 + q_2)_\nu + g_{\nu\lambda}(q_1 - q_2)_\mu + g_{\mu\nu}(-p_5 - q_1)_\lambda, \quad (5.19)$$

In Eq. 5.14 we have reduced the s-channel contributions from the photon (γ) and the Z-boson into one term. From the amplitude Eq. 5.14 one readily obtains the differential cross section for the off shell case

$$\left(\frac{d\sigma(s, t, s_3, s_4)}{d \cos \theta} \right)_0 = \Phi \mathcal{A}_0 \quad (5.20)$$

where

$$\Phi = \left(\frac{4g^4(.6238959)^2 \beta_W \beta_c}{(32\pi s)} \right), \quad (5.21)$$

$$\mathcal{A}_0 = ((a_s^2 + b_s^2)\mathcal{A}_{ss} + 2(a_t^2)\mathcal{A}_{tt} - 2(a_s a_t + b_s b_t)\mathcal{A}_{st}), \quad (5.22)$$

and $\mathcal{A}_{ss}$ through $\mathcal{A}_{st}$ are given by

$$\begin{aligned} \mathcal{A}_{ss} = & \frac{1}{s_3 s_4} (q_1 \cdot q_2)^2 (2p_1 \cdot p_2 q_1 \cdot q_2 - 2p_1 \cdot q_1 p_2 \cdot q_2 - 2p_2 \cdot q_1 p_1 \cdot q_2 + \\ & 2p_1 \cdot q_1 p_2 \cdot q_1 + 2p_2 \cdot q_2 p_1 \cdot q_2) + \frac{1}{s_4} q_1 \cdot q_2 (3q_1 \cdot q_2 p_1 \cdot p_2 + \\ & 8p_1 \cdot q_2 p_2 \cdot q_2 + 4p_1 \cdot q_2 p_2 \cdot q_1 + 4p_1 \cdot q_1 p_2 \cdot q_2) + \frac{1}{s_3} q_1 \cdot q_2 (3q_1 \cdot \\ & q_2 p_1 \cdot p_2 + 8p_1 \cdot q_1 p_2 \cdot q_1 + 4p_1 \cdot q_2 p_2 \cdot q_1 + 4p_1 \cdot q_1 p_2 \cdot q_2) + \\ & 4p_1 \cdot p_2 q_1 \cdot q_2 - 4p_1 \cdot q_2 p_2 \cdot q_2 - 12p_1 \cdot q_2 p_2 \cdot q_1 - 12p_1 \cdot q_1 p_2 \cdot \\ & q_2 - 6s_3 p_1 \cdot p_2 - 4p_1 \cdot q_1 p_2 \cdot q_1 - 6s_4 p_1 \cdot p_2, \end{aligned} \quad (5.23)$$

$$\begin{aligned}
\mathcal{A}_{tt} = & \frac{2}{s_2 s_4} (q_1 \cdot q_2)(p_2 \cdot q_2)(-p_1 \cdot p_2 q_1 \cdot q_2 - 2p_1 \cdot q_1 p_2 \cdot q_1 + p_1 \cdot q_2 p_2 \cdot \\
& q_1 + p_2 \cdot q_2 p_1 \cdot q_1) + \frac{1}{s_4} p_2 \cdot q_2 (-4q_1 \cdot q_2 p_1 \cdot p_2 - 2p_2 \cdot q_2 (p_1 \cdot \\
& q_1 + 2p_1 \cdot q_2) - 2p_2 \cdot q_1 p_1 \cdot q_2) + \frac{1}{s_2} (2p_2 \cdot q_1 ((p_2 \cdot q_1)[2p_1 \cdot q_1 + \\
& p_1 \cdot q_2] + p_1 \cdot q_1 p_2 \cdot q_2) + q_1 \cdot q_2 (p_1 \cdot q_2 p_2 \cdot q_1 - 2p_1 \cdot q_1 p_2 \cdot q_1 - \\
& 7p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 q_1 \cdot q_2 - 2p_1 \cdot p_2 p_2 \cdot q_1)) - 3p_1 \cdot p_2 q_1 \cdot \\
& q_2 + 8p_1 \cdot q_1 p_2 \cdot q_2 + 4s_4 p_1 \cdot p_2 + 2p_1 \cdot p_2 p_2 \cdot q_2 + 6p_1 \cdot p_2 p_2 \cdot q_1,
\end{aligned} \tag{5.24}$$

$$\begin{aligned}
\mathcal{A}_{tt} = & [-s_4 - 4p_2 \cdot q_2 + \frac{4}{s_4} ((p_2 \cdot q_2)^2)](p_1 \cdot p_2 + \frac{2}{s_2} p_1 \cdot q_1 p_2 \cdot q_1) + \\
& 4(p_2 \cdot q_2)[p_1 \cdot q_2 + \frac{2}{s_2} p_1 \cdot q_1 q_1 \cdot q_2].
\end{aligned} \tag{5.25}$$

Note that the differential cross section is in the units of millibarns (mb) since the masses are measured in GeV. Numerical results are given in Tables 5.1 and 5.2 for the tree level differential cross section. The values in Table 5.1 are in agreement with Phillipe[28], taking into account that he uses different values for the masses of the W and Z bosons. Also he uses an angle θ which is the angle between the direction of the electron and the W^+ . In Table 5.2 we compare the effect of using two different schemes (i.e. α and the G_μ schemes) on the tree level differential cross section. For a definition of the two schemes see Appendix E and Chapter 2. The change $(\delta\sigma)^2$ of approximately 11 percent is quite substantial. This difference drops down significantly, as expected, after the inclusion of the one loop corrections. However there remains a non negligible difference which signals higher order effects. It should be pointed out that we have not averaged over the initial polarization states, to do so one simply divides by a factor of 4. The reason we have not divided by four is to keep the numerical values large.

If one one wants [61] to consider the reaction $e^+e^- \rightarrow W^+W^-$ followed by the subsequent decay $W \rightarrow f_i \bar{f}_j$, where $f_i, \bar{f}_j$ are leptons or quarks, then the total

2

$$\frac{\left(\frac{d\sigma}{d\cos\theta}\right)_0(\alpha) - \left(\frac{d\sigma}{d\cos\theta}\right)_0(G_\mu)}{\left(\frac{d\sigma}{d\cos\theta}\right)_0(G_\mu)}$$

cross section for this process is given in terms of the corresponding total cross section for the process $e^+e^- \rightarrow W^+W^-$ with two virtual W's $\sigma(s, s_3, s_4)$ by

$$\sigma(s) = B_{f_1 f_2} B_{f_3 f_4} \int_0^s ds_3 \rho(s_-) \int_0^{(\sqrt{s} - \sqrt{s_3})^2} ds_4 \rho(s_4) \sigma(s, t, s_3, s_4) \quad (5.26)$$

where $B_{f_i f_j}$ is the branching ratio of the W-boson decaying into the i th and j th fermion, s_3 and s_4 are the invariant masses squared of the two virtual W-bosons and $\rho(s)$ is the weight factor coming from the W-boson propagator,

$$\rho(s) = \frac{1}{\pi} \frac{\sqrt{s} \Gamma(s)}{[s - M_W^2]^2 + M_W^2 \Gamma^2(s)} \quad (5.27)$$

Here $\Gamma(s)$ is the full width at half maximum of the W-boson decay. We note that $\sigma(s)$ in Eq. 5.26 reduces to the on shell total cross section for $e^+e^- \rightarrow W^+W^-$ in the limit of vanishing W-boson width. One can obtain the total unpolarized tree level cross section $\sigma(s, s_3, s_4)$ for $e^+e^- \rightarrow W^+W^-$ from the differential cross section given in Eq. 5.20, by simply directly integrating³ the latter over all CM angles or simply t . We find (note here we have averaged over the initial polarization states) the following expression for the total cross section $\sigma(s, s_3, s_4)$ for the e^+e^- annihilations into two virtual W's with masses squared s_3 , and s_4

$$\sigma_T = \sigma_{\gamma\gamma} + \sigma_{\gamma Z} + \sigma_{ZZ} + \sigma_{\nu\gamma} + \sigma_{\nu Z} + \sigma_{\nu\nu}, \quad (5.28)$$

$$\begin{aligned} \sigma_{\gamma\gamma} &= -\Upsilon \frac{s_W^4}{s^2} \frac{1}{s_3 s_4} \mathcal{G}_1, \\ \sigma_{ZZ} &= -\Upsilon \frac{1}{16} \frac{(1 - 4s_W^4)^2 + 1}{(s - M_Z^2)^2} \frac{1}{s_3 s_4} \mathcal{G}_1, \\ \sigma_{\gamma Z} &= -\Upsilon \frac{1}{2} \frac{s_W^2}{s} \frac{1 - 4s_W^4}{s - M_Z^2} \frac{1}{s_3 s_4} \mathcal{G}_1, \\ \sigma_{\nu\nu} &= -\Upsilon \frac{1}{8} \frac{1}{s_3 s_4} \mathcal{G}_2, \end{aligned}$$

³In general when the differential cross section expressions are more involved one usually resorts to numerical techniques and writes $\sigma = \int_{\delta}^{\pi-\delta} d\theta \sin \theta \frac{d\sigma}{d\cos \theta}$ where δ is the appropriate cut.

$$\begin{aligned}
\sigma_{\nu Z} &= \Upsilon \frac{1}{8} \frac{1 - 4s_W^2}{s - M_Z^2} \frac{1}{s_3 s_4} \mathcal{G}_3, \\
\sigma_{\nu\gamma} &= \Upsilon \frac{s_W^2}{s} \frac{1}{s_3 s_4} \mathcal{G}_3,
\end{aligned} \tag{5.29}$$

where,

$$\Upsilon = \frac{g^4 (.6238959)^2 \beta_c}{32\pi s^2}, \tag{5.30}$$

$$\mathcal{G}_1 = -\Lambda^{\frac{3}{2}}(s, s_3, s_4) \left[\frac{1}{6} \Lambda(s, s_3, s_4) + 2[s(s_3 + s_4) + s_3 s_4] \right], \tag{5.31}$$

$$\mathcal{G}_2 = -\Lambda^{\frac{1}{2}}(s, s_3, s_4) \left[\frac{1}{6} \Lambda(s, s_3, s_4) + 2[s(s_3 + s_4) - 4s_3 s_4] + 4s_3 s_4 (s - s_3 - s_4) \ln[f(s, s_3, s_4)] \right], \tag{5.32}$$

$$\mathcal{G}_3 = -\Lambda^{\frac{1}{2}}(s, s_3, s_4) \left[\frac{1}{6} (s + 11s_3 + 11s_4) \Lambda(s, s_3, s_4) + 2[s(s_3^2 + s_4^2 + 3s_3 s_4)] - 2(s_3^3 + s_4^3) - 4s_3 s_4 [s(s_3 + s_4) + s_3 s_4] \ln[f(s, s_3, s_4)] \right], \tag{5.33}$$

$$\Lambda(s, s_3, s_4) = s^2 + s_3^2 + s_4^2 - 2[s s_3 + s s_4 + s_3 s_4], \tag{5.34}$$

$$f(s, s_3, s_4) = \frac{[s - s_3 - s_4 - \Lambda^{\frac{1}{2}}]}{[s - s_3 - s_4 + \Lambda^{\frac{1}{2}}]}. \tag{5.35}$$

$\sigma_{\gamma\gamma}$, σ_{ZZ} , and $\sigma_{\nu\nu}$ denote the contributions of the photon-, Z-, and ν -exchange diagrams, respectively and $\sigma_{\gamma Z}$, $\sigma_{\nu\gamma}$, and $\sigma_{\nu Z}$ denote interference terms. Our expressions for the cross section agree with the ones obtained by the various authors[61]. The above expression for the cross section are complete so that anyone can use them to perform numerical evaluations. A simple Fortran program was written to calculate the total cross section. The numerical results for the on shell total cross section for various CM energies are given in Table 5.3 for three cases A, B, and C

1. : Case A, $M_W = 81.0$, $M_Z = 92.0$
2. : Case B, $M_W = 80.2$, $M_Z = 91.1$
3. : Case C, $M_W = 82.5$, $M_Z = 92.9$

The sensitivity of the cross section on the gauge boson masses is evident from Table 5.3. In order to determine the W mass precisely one must also consider the radiative corrections to the tree level cross section for the process $e^+e^- \rightarrow W^+W^-$. What we see here is that there exists another source of the corrections due to finite W width. The W width corrections are of the same order as the one loop radiative corrections i.e. of the order α . If the finite W width [61] is taken into account, the total cross section (see next section for the finite width effect on differential cross section) is reduced by 5 – 9% compared with the zero width cross section in the region of the maximum value of the cross section and is smeared around the threshold of the zero width process resulting in a long tail below this threshold. This effect of the finite W width disappears gradually at higher energies beyond CM energies of 300 GeV. This dependence of the cross section on the width may be considered a problem in so far as the accurate measurement of W mass from the threshold behavior of the cross section. Fortunately, however, the cross section has a strong dependence on the W mass as can be seen from Table 5.3, and hence the value of mass can be determined by fitting the cross section in its maximum region rather than in the threshold region. We see that the W mass obtained by fitting the data by the zero width cross section should be lower than the one obtained in terms of the finite width cross section. As already mentioned, the one loop radiative corrections are of the same order as the finite width effect, this fact must be kept in mind when analysing the effect of the higher order effects on the cross section or differential cross section.

5.3 The non standard tree level differential cross section for on and off shell W's

In this section we consider the W boson to have a magnetic moment and quadrupole moment different from the SM so as to get a feeling for the magnitude of the radiative corrections arising due to dressed ⁴ $\gamma W^+ W^-$ and $Z W^+ W^-$ vertices. We also take the liberty to vary the parameters to enter the nonperturbative region.⁵ The obvious motivation of doing this to see the effect of having W's which are composite and not fundamental. For in some composite models the W's have rather large magnetic and quadrupole moments [62].

We follow Kim et al. [63] and parametrize the $\gamma W^+ W^-$ vertex as follows,

$$V_{\mu\nu\lambda}^{\gamma W^+ W^-} = -ie[\Gamma_{\mu\nu\lambda} - \lambda_\gamma \frac{s}{M_W^2} q_{2\mu} g_{\nu\lambda} + [(\kappa_\gamma - 1) + \lambda_\gamma \frac{s_3}{M_W^2}] q_{2\nu} g_{\mu\lambda} - [(\kappa_\gamma - 1) + \lambda_\gamma \frac{s_4}{M_W^2}] q_{1\lambda} g_{\mu\nu} + 2[\frac{\lambda_\gamma}{M_W^2}] q_{2\mu} q_{2\nu} q_{1\lambda}], \quad (5.36)$$

$$= -ie[\Gamma_{\mu\nu\lambda}^{\text{SM}} + \Gamma_{\mu\nu\lambda}^{\gamma W^+ W^-, \text{NSM}}], \quad (5.37)$$

where we have written

$$\Gamma_{\mu\nu\lambda}^{\gamma W^+ W^-, \text{NSM}} = [A_{1\gamma} q_{2\mu} g_{\nu\lambda} + A_{2\gamma} q_{2\nu} g_{\mu\lambda} + A_{3\gamma} q_{1\lambda} g_{\mu\nu} + B_{1\gamma} q_{2\mu} q_{2\nu} q_{1\lambda}] \quad (5.38)$$

and where

$$\begin{aligned} A_{1\gamma} &= -s_1 \lambda_\gamma, \\ A_{2\gamma} &= \kappa_\gamma - 1 + \lambda_\gamma \frac{s_3}{M_W^2}, \\ A_{3\gamma} &= -[\kappa_\gamma - 1 + \lambda_\gamma \frac{s_4}{M_W^2}], \\ B_{1\gamma} &= 2 \frac{\lambda_\gamma}{M_W^2}. \end{aligned} \quad (5.39)$$

⁴By dressed we simply mean modified by radiative corrections

⁵In other words we choose large values for the parameters such that these cannot be accounted for by radiative corrections by using perturbation theory.

We note that s_1 is a dimensionless variable given by $s_1 = s/M_W^2$. The ZW^+W^- vertex can be parameterized in a similar manner

$$V_{\mu\nu\lambda}^{ZW^+W^-} = -igc_W W [\Gamma_{\mu\nu\lambda} - \lambda_Z \frac{s_1}{M_W^2} q_{2\mu} g_{\nu\lambda} + [(\kappa_Z - 1) + \lambda_Z \frac{s_3}{M_W^2}] q_{2\nu} g_{\mu\lambda} - [(\kappa_Z - 1) + \lambda_Z \frac{s_4}{M_W^2}] q_{1\lambda} g_{\mu\nu} + 2[\frac{\lambda_Z}{M_W^2}] q_{2\mu} q_{2\nu} q_{1\lambda}], \quad (5.40)$$

$$= -igc_W W [\Gamma_{\mu\nu\lambda}^{\text{SM}} + \Gamma_{\mu\nu\lambda}^{ZW^+W^-, \text{NSM}}], \quad (5.41)$$

where we have written

$$\Gamma_{\mu\nu\lambda}^{ZW^+W^-, \text{NSM}} = [A_{1Z} q_{2\mu} g_{\nu\lambda} + A_{2Z} q_{2\nu} g_{\mu\lambda} + A_{3Z} q_{1\lambda} g_{\mu\nu} + B_{1Z} q_{2\mu} q_{2\nu} q_{1\lambda}], \quad (5.42)$$

and where

$$\begin{aligned} A_{1Z} &= -s_1 \lambda_Z, \\ A_{2Z} &= \kappa_Z - 1 + \lambda_Z \frac{s_3}{M_W^2}, \\ A_{3Z} &= -[\kappa_Z - 1 + \lambda_Z \frac{s_4}{M_W^2}], \\ B_{1Z} &= 2 \frac{\lambda_Z}{M_W^2}. \end{aligned} \quad (5.43)$$

We have denoted the five non standard parameters by W , κ_γ , λ_γ , κ_Z and λ_Z . W measures the relative strength of the γWW and ZWW couplings, κ_γ and λ_γ are related to the magnetic moment and the quadrupole moment of the W -boson, κ_Z and λ_Z are the corresponding parameters to κ_γ and λ_γ respectively for the ZWW vertex. Our motivation for this particular parameterization for the triple boson vertices is governed by the fact that radiative corrections (Chapter 7) induce terms similar to the ones we have introduced besides those present at the tree level in the SW model. The SW case is recovered from the above Eqs. 5.36 and 5.40 by setting $W = 1$, the κ 's equal to 1 and the λ 's equal to zero. In terms of the parameters κ_γ and λ_γ the magnetic dipole moment and the electric quadrupole moment of the

W can be expressed respectively as

$$\begin{aligned}\mu_\gamma &= -e \frac{1 + \kappa_\gamma + \lambda_\gamma}{2M_W}, \\ Q_\gamma &= e \frac{\kappa_\gamma - \lambda_\gamma}{M_W^2}.\end{aligned}\quad (5.44)$$

Similarly in terms of the parameters κ_Z and λ_Z the “magnetic dipole moment” and the “electric quadrupole moment” of the W arising due to its coupling with the Z boson can be expressed respectively as

$$\begin{aligned}\mu_Z &= -gc_W W \frac{1 + \kappa_Z + \lambda_Z}{2M_W}, \\ Q_Z &= gc_W W \frac{\kappa_Z - \lambda_Z}{M_W^2}.\end{aligned}\quad (5.45)$$

Assuming the above non standard form for the triple boson couplings when the W's are off the mass shell, the non standard differential cross section is found to be (after a straightforward but tedious calculation),

$$\left(\frac{d\sigma(s, t, s_3, s_4)}{d\cos\theta} \right)_{\text{NS}} = \Phi[\mathcal{A}_{\text{NS1}} + \mathcal{A}_{\text{rmNS2}}] \quad (5.46)$$

$$\mathcal{A}_{\text{NS1}} = [(a_0^2 + b_0^2)\mathcal{A}_{ss} + (a_t^2 + b_t^2)\mathcal{A}_{tt} - 2(a_0 a_t + b_0 b_t)\mathcal{A}_{st}] \quad (5.47)$$

$$\mathcal{A}_{\text{NS2}} = \mathcal{A}_{\gamma\gamma} + \mathcal{A}_{ZZ} + \mathcal{A}_{\gamma Z} + \mathcal{A}_{\gamma W} + \mathcal{A}_{ZW} \quad (5.48)$$

$$\begin{aligned}\mathcal{A}_{\gamma\gamma} = \frac{s_W^4}{s} [&A_{1\gamma}[A_{1\gamma}\mathcal{W}_0 + B_{1\gamma}\mathcal{W}_1 + A_{2\gamma}\mathcal{W}_2 + A_{3\gamma}\mathcal{W}_3] + A_{2\gamma}[A_{2\gamma}\mathcal{W}_4 + \\ &A_{3\gamma}\mathcal{W}_5 + B_{1\gamma}\mathcal{W}_6 + A_{1\gamma}\mathcal{W}_2] + A_{3\gamma}[A_{3\gamma}\mathcal{W}_7 + A_{1\gamma}\mathcal{W}_5 + B_{1\gamma}\mathcal{W}_8 + \\ &A_{1\gamma}\mathcal{W}_3] + B_{1\gamma}[B_{1\gamma}\mathcal{W}_9 + A_{3\gamma}\mathcal{W}_8 + A_{2\gamma}\mathcal{W}_6 + A_{1\gamma}\mathcal{W}_1]]\end{aligned}\quad (5.49)$$

$$\begin{aligned}\mathcal{A}_{ZZ} = \frac{[1 - 4s_W^2]^2 + 1}{[s - M_Z^2]^2} \frac{W^2}{16} [&A_{1Z}[A_{1Z}\mathcal{W}_0 + B_{1Z}\mathcal{W}_1 + A_{2Z}\mathcal{W}_2 + A_{3Z}\mathcal{W}_3] + \\ &A_{2Z}[A_{2Z}\mathcal{W}_4 + A_{3Z}\mathcal{W}_5 + B_{1Z}\mathcal{W}_6 + A_{1Z}\mathcal{W}_2] + A_{3Z}[A_{3Z}\mathcal{W}_7 + \\ &A_{2Z}\mathcal{W}_5 + B_{1Z}\mathcal{W}_8 + A_{1Z}\mathcal{W}_3] + B_{1Z}[B_{1Z}\mathcal{W}_9 + A_{3Z}\mathcal{W}_8 + \\ &A_{2Z}\mathcal{W}_6 + A_{1Z}\mathcal{W}_1]]\end{aligned}\quad (5.50)$$

$$\begin{aligned}\mathcal{A}_{\gamma Z} = \frac{1}{2} \frac{[1 - 4s_W^2]}{[s - M_Z^2]} \frac{s_W^2}{s} W [&A_{1\gamma}[A_{1Z}\mathcal{W}_0 + B_{1Z}\mathcal{W}_1 + A_{2Z}\mathcal{W}_2 + A_{3Z}\mathcal{W}_3] + \\ &A_{2\gamma}[A_{2Z}\mathcal{W}_4 + A_{3Z}\mathcal{W}_5 + B_{1Z}\mathcal{W}_6 + A_{1Z}\mathcal{W}_2] + A_{3\gamma}[A_{3Z}\mathcal{W}_7 + \\ &A_{2Z}\mathcal{W}_5 + B_{1Z}\mathcal{W}_8 + A_{1Z}\mathcal{W}_3] + B_{1\gamma}[B_{1Z}\mathcal{W}_9 + A_{3Z}\mathcal{W}_8 + \\ &A_{2Z}\mathcal{W}_6 + A_{1Z}\mathcal{W}_1]]\end{aligned}\quad (5.51)$$

$$\begin{aligned} \mathcal{A}_{\gamma W} = & 2 \frac{s_W^2}{s} a_0 [A_{2\gamma} \mathcal{W}\mathcal{W}_{1ss} + A_{3\gamma} \mathcal{W}\mathcal{W}_{2ss} + A_{1\gamma} \mathcal{W}\mathcal{W}_{4ss} + \\ & B_{1\gamma} \mathcal{W}\mathcal{W}_{6ss}] - 2 \frac{s_W^2}{s} a_t [A_{2\gamma} \mathcal{W}\mathcal{W}_{1st} + A_{3\gamma} \mathcal{W}\mathcal{W}_{2st} + \\ & A_{1\gamma} \mathcal{W}\mathcal{W}_{4st} + B_{1\gamma} \mathcal{W}\mathcal{W}_{6st}] \end{aligned} \quad (5.52)$$

$$\begin{aligned} \mathcal{A}_{ZW} = & \frac{1}{2} W \frac{1-4s_W^2}{s-M_Z^2} a_0 [A_{2Z} \mathcal{W}\mathcal{W}_{1ss} + A_{3Z} \mathcal{W}\mathcal{W}_{2ss} + A_{1Z} \mathcal{W}\mathcal{W}_{4ss} + \\ & B_{1Z} \mathcal{W}\mathcal{W}_{6ss}] - \frac{1}{2} W \frac{1-4s_W^2}{s-M_Z^2} a_t [A_{2Z} \mathcal{W}\mathcal{W}_{1st} + A_{3Z} \mathcal{W}\mathcal{W}_{2st} + \\ & A_{1Z} \mathcal{W}\mathcal{W}_{4st} + B_{1Z} \mathcal{W}\mathcal{W}_{6st}] \end{aligned} \quad (5.53)$$

$$\begin{aligned} a_0 &= \frac{s_W^2}{s} + W \frac{1-4s_W^2}{[4[s-M_Z^2]]} \\ b_0 &= W \frac{-1}{[4[s-M_Z^2]]} \\ &= W b_s \end{aligned} \quad (5.54)$$

The expressions $\mathcal{W}_0, \dots, \mathcal{W}_9, \mathcal{W}\mathcal{W}_{1ss}, \dots, \mathcal{W}\mathcal{W}_{6st}$ involving various dot products of momenta may be found in Appendix G. In Eq. 5.46 we have intentionally split the differential cross section into two parts, the first piece represents the standard electroweak differential cross section with the W's off shell except for the multiplicative factor W . The second portion comprises of several parts which allow deviations away from the standard model depending on all the five NSM parameters. This split makes it straightforward, if one wishes to, to incorporate radiative corrections to the tri-boson vertices. In the limit when the vector bosons are on shell and the standard model values $W = 1, \kappa_\gamma = \kappa_Z = 1, \lambda_\gamma = \lambda_Z = 0$ are chosen we obtain agreement with the known result [29]. The second term in Eq. 5.46, appears deceptively lengthy due to the manner in which we have written it. This is done to exhibit separately the contribution of each term which gives rise to the non standard contribution. It allows us to examine the effect of varying each of the parameters independently. For example if one wants to change only the $\gamma W^+ W^-$ vertex from its SM form while leaving the $Z W^+ W^-$ vertex intact,

this way of writing allows us to do that. It is straightforward to see that in the standard model limit all these terms vanish. As can be seen from Eq. 5.48 $\mathcal{A}_{NS2}$ is written as the sum of five terms the first term is purely due to the photon, the second arises due to the Z-boson, the third being due to the photon and Z-boson interference. The remaining terms arise due to the interference of the non standard amplitude with that of the "standard model" like amplitude (i.e. an amplitude like the SM except where the Z vertex has an extra multiplicative factor W). It is straightforward to program the above into the computer. This is the reason why we have not given the reduced form of the momenta dot products in terms of the Mandelstam variables. The above results have been checked many times by us. We now turn to numerical results. A program was written to calculate the non standard differential cross section for off and on shell W's. For the on shell W's the differential cross section was evaluated for two different CM energies of 200 GeV and 260 GeV for the following 3 choices of the NS parameters

1. : Case I, $\kappa_\gamma = 1.01$, $\lambda_\gamma = 0.01$, $\kappa_Z = 1.02$, $\lambda_Z = 0.02$, $W = 1.01$,
2. : Case II, $\kappa_\gamma = 1.1$, $\lambda_\gamma = 0.1$, $\kappa_Z = 1.2$, $\lambda_Z = 0.2$, $W = 1.1$,
3. : Case III, $\kappa_\gamma = 2.$, $\lambda_\gamma = 1.$, $\kappa_Z = 2.5$, $\lambda_Z = 1.5$, $W = 1.1$.

The parameters in Case I were chosen to mimic the radiative corrections to the triple boson vertices in the framework of the SW model. So that the deviations of the parameters away from their SM values was taken to be of the order α ⁶. The values chosen for the parameters in Case III was governed by the fact that the W may be composite. Case II allows for the values of the parameters between these two extremes, i.e. Cases I and III. We also evaluated the cross section at

⁶Note until otherwise stated α refers to the fine structure constant

260 GeV since the deviations away from the SM become more significant at higher CM energies. The numerical results for non standard differential cross section are exhibited in Tables 5.5 and 5.6

Now we briefly turn to the case of allowing the W 's to be off shell in the non standard case. The discussion of the calculation of the differential cross section of the process $e^+e^- \rightarrow W^+W^-$ with two virtual W 's followed by the process when the W 's decay into fermions proceeds much like the discussion of the cross section for this process, see section 5.2. Eq. 5.26, gets replaced by

$$\frac{d\sigma(s, t)}{d\cos\theta} = B_{f_1 f_2} B_{f_3 f_4} \int_0^s ds_3 \rho(s_3) \int_0^{(\sqrt{s}-\sqrt{s_3})^2} ds_4 \rho(s_4) \frac{d\sigma(s, t, s_3, s_4)}{d\cos\theta} \quad (5.55)$$

As an illustration, [see Table 5.7 of the effect of non zero width] we evaluate the differential cross section for various values of the angle θ for a center of mass energy of 200 GeV. In our numerical calculation we take $M_W = 81$ GeV, $M_Z = 92$ GeV. As is clear from Table 5.7 the non zero width causes a 5 – 14% reduction compared to the zero width differential cross section of the non standard W pair production in the electron-positron collisions. It should be pointed out that we have not averaged over the initial polarization states.

5.4 Bremsstrahlung

As is well known, in order to obtain finite cross sections, in the limit of vanishing photon mass, we have to include emission of real photons by external charged particles. Since the photon interacts both with the electron and the W -boson the relevant Bremsstrahlung diagrams are displayed in Fig. 5.2. The charged would-be-Goldstone boson, which is a longitudinal mode of the W -boson in the t'Hooft-Feynman gauge, must also be included as in Fig. 5.2. In the soft photon

approximation (i.e. the emission of photons with small energy) we obtain with the help of Ward identity [28, 44] and the assumption that the W 's be on mass shell the usual factorization of the Bremsstrahlung cross section [28].

$$\frac{d\sigma_H}{d\cos\theta} = \frac{d\sigma_0}{d\cos\theta} \delta_B \quad (5.56)$$

Here

$$\delta_H = \delta_e + \delta_W + \delta_t + \delta_u \quad (5.57)$$

$$\delta_e = -\left(\frac{\alpha}{\pi}\right) \left(+\log\left(\frac{s}{\lambda^2}\right) - \log\left(\frac{s}{\lambda^2}\right) \log\left(\frac{s}{m_e^2}\right) + \log(a^2) - \log(a^2) \log\left(\frac{s}{m_e^2}\right) + .5 \log^2\left(\frac{m_e^2}{s}\right) + \log\left(\frac{m_e^2}{s}\right) + \frac{\pi^2}{3} \right) \quad (5.58)$$

$$\begin{aligned} \delta_W = & -\left(\frac{\alpha}{\pi}\right) \left(+\log\left(\frac{s}{\lambda^2}\right) + \frac{s-2M_W^2}{s\beta_W} \log\left(\frac{s}{\lambda^2}\right) \log\left(\frac{1-\beta_W}{1+\beta_W}\right) + \log(a^2) + \right. \\ & \left. \frac{s-2M_W^2}{s\beta_W} \log(a^2) \log\left(\frac{1-\beta_W}{1+\beta_W}\right) + \right. \\ & \left. \frac{1}{\beta} \log\left(\frac{1-\beta_W}{1+\beta_W}\right) - \frac{s-2M_W^2}{s\beta_W} [2Sp\left(\frac{1-\beta_W}{1+\beta_W}\right) - .5 \log^2\left(\frac{1-\beta_W}{1+\beta_W}\right) - \frac{\pi^2}{3} + \right. \\ & \left. 2 \log\left(\frac{1-\beta_W}{1+\beta_W}\right) \log\left(\frac{2\beta_W}{1+\beta_W}\right)] \right) \end{aligned} \quad (5.59)$$

$$\delta_t = -\left(\frac{\alpha}{\pi}\right) \left(-2 \log\left(\frac{t}{\lambda^2}\right) \log\left(\frac{M_W^2-t}{m_e M_W}\right) - 2 \log(a^2) \log\left(\frac{M_W^2-t}{m_e M_W}\right) + \right. \\ \left. 2[Sp(1 - .5 \frac{s(1-\beta_W)}{M_W^2-t}) + Sp(1 - .5 \frac{s(1+\beta_W)}{M_W^2-t})] \right) \quad (5.60)$$

$$\delta_u = -\left(\frac{\alpha}{\pi}\right) \left(+2 \log\left(\frac{u}{\lambda^2}\right) \log\left(\frac{M_W^2-u}{m_e M_W}\right) + 2 \log(a^2) \log\left(\frac{M_W^2-u}{m_e M_W}\right) - \right. \\ \left. 2[Sp(1 - .5 \frac{s(1-\beta_W)}{M_W^2-u}) + Sp(1 - .5 \frac{s(1+\beta_W)}{M_W^2-u})] \right) \quad (5.61)$$

$$\begin{aligned} a &= \frac{2 \Delta E_b}{\sqrt{s}} \\ &= \frac{\Delta E_b}{E_b} \end{aligned} \quad (5.62)$$

To the best of our knowledge the only other places the complete expression for the bremsstrahlung (infrared infinite) cross section of the process $e^+e^- \rightarrow W^+W^-$ has been given is in references [30, 31]. The above expression 5.56 agrees with [30]. We note however the misprint in [30] (2nd term of line 3 of eq. 5.18). Although

it is well expected that the infrared infinite bremsstrahlung (IRBI) contributions cancel against infrared virtual infinities (IRVI), details of the precise cancellation of the infrared infinities for the process $e^+e^- \rightarrow W^+W^-$ are lacking. Moreover, for the sake of completeness one would like the complete infrared finite expression. To this end we summarize the steps needed to arrive at the infrared finite expression. Consider the two single logarithmic infinite terms in the electron and the W expressions (Eqs. 5.58 and 5.59). These are cancelled by the WFNR contributions of the electron and the W to the $e^+e^- \rightarrow W^+W^-$ cross section shown in Fig. 6.1, f, g. The W self-energy(SE) contribution works out to be (considering now only the infinite part, Chapter 6)

$$\frac{d\sigma_{W(SE)}^{(IRVI)}}{d\cos\theta} = -\frac{\alpha}{\pi} \left[\log\left(\frac{\lambda^2}{M_W^2}\right) \right] \frac{d\sigma_0}{d\cos\theta} \quad (5.63)$$

On adding this to the infinite single logarithm term in 5.59 we get

$$\frac{d\sigma_{W(WFNR)+W(Brem)}}{d\cos\theta} = -\frac{\alpha}{\pi} \left[\log\left(\frac{s}{M_W^2}\right) \right] \frac{d\sigma_0}{d\cos\theta} \quad (5.64)$$

which is finite. As can be inferred, the electron produces similar contribution to the IRVI (Chapter 6)

$$\frac{d\sigma_{e(WFNR)}^{(IRVI)}}{d\cos\theta} = -\frac{\alpha}{\pi} \left[\log\left(\frac{\lambda^2}{m_e^2}\right) \right] \frac{d\sigma_0}{d\cos\theta} \quad (5.65)$$

which when considered along with the remaining single logarithm term in Eq. 5.58 gives

$$\frac{d\sigma_{e(WFNR)+e(Brem)}}{d\cos\theta} = -\frac{\alpha}{\pi} \left[\log\left(\frac{s}{m_e^2}\right) \right] \frac{d\sigma_0}{d\cos\theta} \quad (5.66)$$

which is again finite.

The double logarithm terms in Eq. 5.58 through Eq. 5.61 need much more work for their cancellation. Essentially, certain combinations of triangle (Δ) and

box ($\square$) diagrams conspire to produce an effective IRVI term which cancel the corresponding IRBI.

We start with the simplest case that of the triangle diagrams with $e^+e^-\gamma$ as internal particles, and the box diagram with $\gamma e^+e^-\nu_e$ as the internal particles, Fig. 5.3,a . First consider the triangle contribution, which works out to be

$$V^{e^+e^-\gamma} + V^{e^+e^-Z} = -2x\left(\frac{\alpha}{4\pi}\right)\Phi[(a_s^2 + b_s^2)\mathcal{A}_{ss} - (a_s a_t + b_s b_t)\mathcal{A}_{st}] \quad (5.67)$$

where

$$x = 2 - 4C_{24}(-p_1, -p_2, m_e, \lambda, m_e) + 2s(C_{23} + C_{11}) \quad (5.68)$$

We recall that all the C integrals can be expressed in terms of C_0 and B integrals [45, 44]. It can be shown [44] (Ph.d Notes) that the C integrals which are infrared divergent (IRD) are C_0 , C_{11} , C_{21} and C_{31} .

$$C_{11} \simeq -C_0, \quad C_{21} \simeq C_0, \quad C_{31} \simeq -C_0$$

This allows us to write the IRVI contribution of Eq. 5.67 as

$$(V^{e^+e^-\gamma} + V^{e^+e^-Z})^{IRVI} = -2\left(\frac{\alpha}{4\pi}\right)\Phi(2\log(\frac{s}{\lambda^2}))\log(\frac{s}{m_e^2}) \\ [(a_s^2 + b_s^2)\mathcal{A}_{ss} - (a_s a_t + b_s b_t)\mathcal{A}_{st}] \quad (5.69)$$

Turning now to the box contribution Fig. 5.3,a. We know that each D_0 integral can be decomposed into a D-like term and two C_0 integrals [45], that is,

$$D_0 = \frac{1}{i\pi^2} \int d^n q \frac{1}{\prod_i Q_i},$$

$$\frac{1}{\prod_i Q_i} = -\frac{2}{y - m_2^2 - m_3^2} \left[\frac{(q + p_1) \cdot (q + p_1 + p_2)}{\prod_i Q_i} - \frac{1}{2 \prod_{i \neq 2} Q_i} - \frac{1}{2 \prod_{i \neq 3} Q_i} \right],$$

$$Q_1 = q^2 - m_1^2,$$

$$Q_2 = (q + p_1)^2 - m_2^2,$$

$$Q_3 = (q + p_1 + p_2)^2 - m_3^2,$$

$$\begin{aligned}
Q_4 &= (q + p_1 + p_2 + p_3)^2 - m_4^2, \\
y &= (p_1 + p_2)^2 + p_1^2 - 2p_1 \cdot (p_1 + p_2).
\end{aligned}
\tag{5.70}$$

Applying Eq. 5.70 to the case of Fig. 5.3,a (see the box diagram there) and writing only the IRD part of D_0 we have

$$D_0^{IRD} \simeq -\frac{2}{t} C_0^{IRD}(p_2, p_1, m_e, \lambda, m_e) \tag{5.71}$$

The IRVI box contribution of Fig. 5.3,a to the one loop cross section works out to be

$$B_{\gamma e e \nu_e}^{IRVI} = \left(\frac{\alpha}{\pi}\right) \Phi\left(-\log\left(\frac{s}{\lambda^2}\right) \log\left(\frac{s}{m_e^2}\right)\right) [(a_t^2 + b_t^2) \mathcal{A}_{it} - (a_s a_t + b_s b_t) \mathcal{A}_{st}] \tag{5.72}$$

Adding both these contributions, i.e. Eqs. 5.69 and 5.72, the result is

$$\frac{d\sigma_{electron}^{IRVI}}{d\cos\theta} = -\frac{\alpha}{\pi} \left[\log\left(\frac{s}{\lambda^2}\right) \log\left(\frac{s}{m_e^2}\right) \right] \frac{d\sigma_0}{d\cos\theta} \tag{5.73}$$

Using the electron part of IRBI i.e. Eq. 5.58 we immediately see that the double logarithm infinity cancels between Eqs. 5.58 and 5.73.

Next we consider the contribution of diagrams shown in Fig. 5.3, b. The amplitude from the triangle graphs of Fig. 5.53, b is reduced after a tedious calculation and some rearrangement to

$$\begin{aligned}
\mathcal{M}^\Delta &= \frac{ig^4}{16\pi^2} (4q_1 \cdot q_2) (s_W^2) [\bar{v}(p_1) \gamma^\mu (a_s + b_s \gamma_5) u(p_2)] \\
&\quad \Gamma_{\mu\nu\lambda} \epsilon_1^\nu \epsilon_2^\lambda C_0^{IRD}(q_1, q_2, M_W, \lambda, M_W) \\
&\quad - \frac{ig^4}{16\pi^2} \left(\frac{s_W^2}{4}\right) [\bar{v}(p_1) \gamma^\mu (1 - \gamma_5) u(p_2)] \\
&\quad \Gamma_{\mu\nu\lambda}^x \epsilon_1^\nu \epsilon_2^\lambda C_0^{IRD}(q_1, q_2, M_W, \lambda, M_W)
\end{aligned}
\tag{5.74}$$

$$\Gamma_{\mu\nu\lambda}^x = \Gamma_{\mu\nu\lambda} - q_{1\mu} g_{\nu\lambda} \tag{5.75}$$

Amazingly the box contribution of Fig. 5.3,b generates a term which exactly cancels the unwanted term in Eq. 5.75 i.e. the second term. For

$$\begin{aligned}\mathcal{M}^{\square} = & -\frac{ig^4}{16\pi^2}(4q_1 \cdot q_2)(s_W^2) \\ & [\bar{v}(p_1)\epsilon_1(q_1 - p_1)\epsilon_2(a_t + b_t\gamma_5)u(p_2)]C_0^{IRD}(q_1, q_2, M_W, \lambda, M_W) \\ & + \frac{ig^4}{16\pi^2}\left(\frac{s_W^2}{4}\right)[\bar{v}(p_1)\gamma^\mu(1 - \gamma_5)u(p_2) \\ & \Gamma_{\mu\nu\lambda}^x \epsilon_1^\nu \epsilon_2^\lambda]C_0^{IRD}(q_1, q_2, M_W, \lambda, M_W)\end{aligned}\quad (5.76)$$

Clearly the second terms in Eqs. 5.74 and 5.76 cancel each other. The remainder is precisely what we need to cancel the W boson double logarithm infinity in Eq. 5.59. Since on adding Eqs. 5.74 and 5.75 we are able to factor out the tree level amplitude ($\mathcal{M}^T$)

$$\mathcal{M}^{\Delta+\square} = \frac{\alpha}{4\pi}(4q_1 \cdot q_2)\mathcal{M}^T C_0^{IRD}(q_1, q_2, M_W, \lambda, M_W) \quad (5.77)$$

This yields the following cross section contribution, after substituting the infinite part of C_0

$$\frac{d\sigma_{W-boson}^{IRVI}}{d\cos\theta} = +\frac{\alpha}{\pi}\frac{s - 2M_W^2}{s\beta_W}\log\left(\frac{s}{\lambda^2}\right)\log\left(\frac{1 - \beta_W}{1 + \beta_W}\right)\frac{d\sigma_0}{d\cos\theta} \quad (5.78)$$

Once again this cancels the infinity in IRIB Eq. 5.59.

Now we turn to the diagrams of Fig. 5.53 ,c i.e. the T channel IRVI diagrams.

The amplitude for the triangle diagrams is [infrared parts only]

$$\mathcal{M}_{\Delta 1} = -ig^2\left(\frac{\alpha}{4\pi}\right)\bar{v}(p_1)[V_{11}^x\epsilon_1(q_1 - p_1)\epsilon_2 + V_{13}^x(p_1 \cdot \epsilon_1)\epsilon_2](a_t + b_t\gamma_5)u(p_2) \quad (5.79)$$

$$\mathcal{M}_{\Delta 2} = -ig^2\left(\frac{\alpha}{4\pi}\right)\bar{v}(p_1)[V_{21}^x\epsilon_1(q_1 - p_1)\epsilon_2 - V_{23}^x(p_2 \cdot \epsilon_2)\epsilon_1](a_t + b_t\gamma_5)u(p_2) \quad (5.80)$$

After some manipulation the combined amplitude from the box diagrams in Fig. 5.3, c is reduced to

$$\mathcal{M}_{\square 1} + \mathcal{M}_{\square 2} = -8ig^2(p_1 \cdot q_1)\left(\frac{\alpha}{4\pi}\right)[\bar{v}(p_1)\gamma^\mu(a_s + b_s\gamma_5)u(p_2)]$$

$$\begin{aligned}
& \Gamma_{\mu\nu\lambda} \epsilon_1^\nu \epsilon_2^\lambda] C_0^{IRD}(p_1, -q_1, m_e, \lambda, M_W) \\
& -2ig^2 \left(\frac{\alpha}{4\pi}\right) \bar{v}(p_1) [(p_2 \cdot \epsilon_2) \epsilon_1 - (p_1 \cdot \epsilon_1) \epsilon_2] \\
& (a_t + b_t \gamma_5) u(p_2) C_0^{IRD}(p_1, -q_1, m_e, \lambda, M_W)
\end{aligned} \tag{5.81}$$

Now since

$$V_{13}^\pi = V_{23}^\pi = 2C_0^{IRD}(p_1, -q_1, m_e, \lambda, M_W) \tag{5.82}$$

$$V_{11}^\pi = V_{21}^\pi = -4(p_1 \cdot q_1) C_0^{IRD}(p_1, -q_1, m_e, \lambda, M_W) \tag{5.83}$$

Adding all the contributions from diagrams in Fig 5.3, c

$$\mathcal{M}_{\Delta 1} + \mathcal{M}_{\Delta 2} + \mathcal{M}_{\Box 1} + \mathcal{M}_{Box 2} = -8ig^2(p_1 \cdot q_1) \left(\frac{\alpha}{4\pi}\right) C_0^{IRD}(p_1, -q_1, m_e, \lambda, M_W) \mathcal{M}^T \tag{5.84}$$

Once again we have obtained the factorization of the amplitude. The cross section arising from the above amplitude is

$$\frac{d\sigma_{t-channel}}{d\cos\theta}^{IRVI} = -\frac{\alpha}{\pi} (4p_1 \cdot q_1) C_0^{IRD}(p_1, -q_1, m_e, \lambda, M_W) \frac{d\sigma_0}{d\cos\theta} \tag{5.85}$$

Using

$$C_0^{IRD}(p_1, -q_1, m_e, \lambda, M_W) = \left(\frac{1}{2p_1 \cdot q_1}\right) \log\left(\frac{t}{\lambda^2}\right) \log\left(\frac{M_W^2 - t}{m_e M_W}\right) \tag{5.86}$$

$$\frac{d\sigma_{t-channel}}{d\cos\theta}^{IRVI} = -2\frac{\alpha}{\pi} \frac{d\sigma_0}{d\cos\theta} \log\left(\frac{t}{\lambda^2}\right) \log\left(\frac{M_W^2 - t}{m_e M_W}\right) \tag{5.87}$$

The corresponding IRD bremsstrahlung contribution from Eq. 5.60

$$\frac{d\sigma_{t-channel}}{d\cos\theta}^{IRIB} = 2\frac{\alpha}{\pi} \frac{d\sigma_0}{d\cos\theta} \log\left(\frac{t}{\lambda^2}\right) \log\left(\frac{M_W^2 - t}{m_e M_W}\right) \tag{5.88}$$

Again the divergences cancel as is explicit from the above two Eqs. The remaining u-channel IRIB Eq. 61 is cancelled by a similar argument as the above t-channel calculation. We do not repeat the steps here. However, we note that here

1. The two fermion line crossed box diagrams of Fig. 5.3, d replace the triangle diagrams of Fig. 5.3, c, since there are no u-channel virtual triangle corrections. These precisely generate a contribution similar to the one given in Eqs. 5.79 and 5.80.
2. The crossed box diagrams of Fig. 5.3, d take the place of the direct boxes in Fig. 5.3, c. The expression generated by diagrams of Fig. 5.3, d can be got by replacing $t \rightarrow u$ in Eq. 5.88 and adding an overall sign change.

The total finite contribution is now written as

$$C_{01} = C_0^{IR,finite}(q_2, q_1, M_W, \lambda, M_W) \quad (5.89)$$

$$C_{02} = C_0^{IR,finite}(p_1, -q_1, m_e, \lambda, M_W) \quad (5.90)$$

$$C_{03} = C_0^{IR,finite}(p_1, -q_2, m_e, \lambda, M_W) \quad (5.91)$$

$$\delta_{B,finite}^{Total} = \delta_{B,finite}^1 + \delta_{B,finite}^2 + \delta_{B,finite}^3 + \delta_{B,finite}^4 + \delta_{B,finite}^5 + \delta_{B,finite}^6 + \delta_{B,finite}^7 \quad (5.92)$$

$$\begin{aligned} \delta_{B,finite}^1 = & -\left(\frac{\alpha}{\pi}\right)((\log(a^2) + \log(a^2) + \log(\frac{s}{M_W^2})) \\ & + (\frac{1}{\beta_W}) \log(\frac{1 - \beta_W}{1 + \beta_W})) \end{aligned} \quad (5.93)$$

$$\delta_{B,finite}^2 = -\left(\frac{\alpha}{\pi}\right)((\frac{s - 2M_W^2}{s\beta_W}) \log(\frac{1 - \beta_W}{1 + \beta_W}) \log(a^2) - \log(\frac{s}{m_e^2}) \log(a^2)) \quad (5.94)$$

$$\begin{aligned} \delta_{B,finite}^3 = & -\left(\frac{\alpha}{\pi}\right)(2 \log(\frac{M_W^2 - u}{M_W^2 - t}) \log(a^2) + 2 \log(\frac{M_W^2 - u}{M_W m_e}) \log(\frac{s}{-u}) \\ & - 2 \log(\frac{M_W^2 - t}{M_W m_e}) \log(\frac{s}{-t})) \end{aligned} \quad (5.95)$$

$$\begin{aligned} \delta_{B,finite}^4 = & -\left(\frac{\alpha}{\pi}\right)(-\frac{s - 2M_W^2}{s\beta_W})[2Sp(\frac{1 - \beta_W}{1 + \beta_W}) \\ & - .5 \log^2(\frac{1 - \beta_W}{1 + \beta_W}) - \frac{\pi^2}{2} \end{aligned}$$

$$+2 \log\left(\frac{1-\beta_W}{1+\beta_W}\right) \log\left(\frac{2\beta_W}{1+\beta_W}\right)) \quad (5.96)$$

$$\begin{aligned} \delta_{B,\text{finite}}^5 = & -\left(\frac{\alpha}{\pi}\right) \left(2 \left[Sp \left(1 - .5 \frac{s(1-\beta_W)}{M_W^2 - t} \right) \right. \right. \\ & \left. \left. + Sp \left(1 - .5 \frac{s(1+\beta_W)}{M_W^2 - t} \right) \right] \right) \end{aligned} \quad (5.97)$$

$$\begin{aligned} \delta_{B,\text{finite}}^6 = & -\left(\frac{\alpha}{\pi}\right) \left(-2 \left[Sp \left(1 - .5 \frac{s(1-\beta_W)}{M_W^2 - u} \right) \right. \right. \\ & \left. \left. + Sp \left(1 - .5 \frac{s(1+\beta_W)}{M_W^2 - u} \right) \right] \right) \end{aligned} \quad (5.98)$$

$$\delta_{B,\text{finite}}^7 = -\left(\frac{\alpha}{\pi}\right) (-4(q_1 \cdot q_2)C_{01} + 4(p_1 \cdot q_1)C_{02} - 4(p_1 \cdot q_2)C_{03}) \quad (5.99)$$

The finite Bremsstrahlung is calculated for $a = 0.1$ and $a = 0.05$ and the results are listed in Tables 5.8 and 5.9 respectively.

Figure 5.1: The lowest order Feynman diagrams for the process $e^+e^- \rightarrow W^+W^-$ and the three vertex momentum convention.

Figure 5.2: Diagrams for the process $e^+e^- \rightarrow W^+W^-\gamma$

Table 5.1: Standard model differential cross section in pb.

$\theta(deg)$	$(\frac{d\sigma}{d\cos\theta})_0(200GeV)$	$(\frac{d\sigma}{d\cos\theta})_0(500GeV)$	$(\frac{d\sigma}{d\cos\theta})_0(1000GeV)$
10	155.6123	291.2080	83.0610
30	106.9359	33.1306	8.0082
50	59.3622	9.4963	2.2021
70	33.5360	3.7453	0.8379
90	20.5578	1.9066	0.4209
110	13.7370	1.1660	0.2582
130	9.8762	0.7161	0.1542
150	7.6402	0.3712	0.0684
170	6.5744	0.1665	0.0157

Figure 5.3: Combination of diagrams which cancel the infrared infinities in the Bremsstrahlung.

Table 5.2: Standard model differential cross section in pb for the α and the G_μ -scheme.

$\theta(deg)$	$(\frac{d\sigma}{d\cos\theta})_0(200GeV)$	$(\frac{d\sigma}{d\cos\theta})_0(200GeV)$	Difference
-	α -scheme	G_μ -scheme	$\delta\sigma$
10	155.6123	175.3002	-0.1123
30	106.9359	120.4653	-0.1123
50	59.3622	66.8726	-0.1123
70	33.5360	37.7789	-0.1123
90	20.5578	23.1588	-0.1123
110	13.7370	15.4749	-0.1123
130	9.8762	11.1258	-0.1123
150	7.6402	8.6069	-0.1123
170	6.5744	7.4062	-0.1123
180	6.4424	7.2575	-0.1123

Table 5.3: Standard Model cross section in pb for various choices of gauge boson masses

CM Energy	Case A	Case B	Case C
166	10.0511	11.7830	5.7746
170	13.1461	14.2781	11.9213
200	17.5402	17.8298	19.1232
230	16.1727	16.3373	17.8632
260	14.3966	14.5164	15.9624
290	12.7982	12.8940	14.2160
320	11.4355	11.5152	12.7171
350	10.2813	10.3491	11.4436
380	9.2991	9.3576	10.3579
410	8.4570	8.5081	9.4258
500	6.5416	6.5770	7.3017

Table 5.4: Effects of the W-boson width on the Standard Model cross section (pb)

CM Energy	σ_0	σ_W	$\frac{\sigma_0 - \sigma_W}{\sigma_0} \%$
177	15.8287	14.3173	9.5485
180	16.4579	15.1868	7.7235
190	17.4460	16.6289	4.6838
200	17.5402	16.9452	3.3924
230	16.1727	15.8391	2.0627
260	14.3966	14.1540	1.6853
290	12.7982	12.6031	1.5244
320	11.4355	11.2715	1.4341
350	10.2813	10.1403	1.3714
380	9.2991	9.1762	1.3213
410	8.4570	8.3489	1.2995
500	6.5416	6.4647	1.1776

Table 5.5: Non Standard differential cross section (in pb) for CM energy of 200 GeV

$\theta(deg)$	Case I	Case II	Case III
10	156.8818	169.4364	272.2052
30	107.4659	113.4268	195.7971
50	59.4522	61.6074	138.4342
70	33.4267	33.9383	112.7116
90	20.3565	20.0981	98.8269
110	13.4929	12.7778	85.5568
130	9.6133	8.5567	70.4095
150	7.3701	6.0540	56.1788
170	6.3021	4.8400	47.4438
180	6.1699	4.6887	46.2659

Table 5.6: Non Standard differential cross section (in pb) for CM energy of 260 GeV

$\theta(deg)$	Case I	Case II	Case III
10	279.4467	296.6217	476.6278
30	107.1888	111.8087	267.9966
50	40.1415	42.6891	237.3995
70	18.2345	20.6048	256.2358
90	9.8262	12.1697	263.8282
110	6.0399	8.0396	241.0617
130	3.9466	5.2855	192.8530
150	2.6323	3.2455	139.4072
170	1.9486	2.0914	104.7538
180	1.8606	1.9390	100.0085

$\theta(deg)$	on-shell dcs (pb)	off-shell dcs (pb)	% change compared to the on-shell value
10	242.7952	209.4243	-13.7445%
30	164.7610	147.9847	-10.1822%
50	103.0870	95.4175	-7.4398%
70	73.0903	67.6607	-7.4286%
90	57.5752	52.7164	-8.4391%
110	46.3598	42.1887	-8.9972%
130	36.1223	33.0436	-8.5231%
150	27.4254	25.5174	-6.9569%
170	22.2929	21.1403	-5.1701%

Table 5.7: The On-shell and off-shell differential cross-section (dcs) for the non-standard values $W = 1.1$, $\kappa_\gamma = \kappa_Z = 2$, $\lambda_\gamma = \lambda_Z = 1$ when the center of mass energy is 200 GeV for various angles. θ is the angle of the W^- momentum to the electron direction.

Table 5.8: Bremsstrahlung differential cross section in pb for various center of mass energies and $a = 0.1$

$\theta(deg)$	$(\frac{d\sigma}{d\cos\theta})_b(200GeV), \delta_b^{Total}$	$(\frac{d\sigma}{d\cos\theta})_b(500GeV), \delta_b^{Total}$	$(\frac{d\sigma}{d\cos\theta})_b(1000GeV), \delta_b^{Total}$
10	(-34.2446, -0.2201)	(-65.3503, -0.2244)	(-19.5905, -0.2359)
30	(-24.0332, -0.2247)	(-8.1994, -0.2475)	(-2.1896, -0.2734)
50	(-13.8036, -0.2325)	(-2.5477, -0.2683)	(-0.6537, -0.2976)
70	(-8.1109, -0.2419)	(-1.0713, -0.2860)	(-0.2653, -0.3167)
90	(-5.1752, -0.2517)	(-0.5767, -0.3025)	(-0.1405, -0.3339)
110	(-3.5939, -0.2616)	(-0.3718, -0.3189)	(-0.0906, -0.3511)
130	(-2.6759, -0.2709)	(-0.2411, -0.3367)	(-0.0571, -0.3701)
150	(-2.1297, -0.2787)	(-0.1327, -0.3574)	(-0.0270, -0.3943)
170	(-1.8633, -0.2834)	(-0.0634, -0.3805)	(-0.0068, -0.4319)

Table 5.9: Bremsstrahlung differential cross section
in pb for $\alpha = 0.05$

$\theta(deg)$	$(\frac{d\sigma}{d\cos\theta})_b(200GeV)$	$(\frac{d\sigma}{d\cos\theta})_b(500GeV)$	$(\frac{d\sigma}{d\cos\theta})_b(1000GeV)$
—	δ_b^{Total}	δ_b^{Total}	δ_b^{Total}
10	(-45.5995, -0.2930)	(-86.4441, -0.2968)	(-25.8741, -0.3115)
30	(-31.9715, -0.2990)	(-10.8196, -0.3266)	(-2.8831, -0.3600)
50	(-18.3357, -0.3089)	(-3.3536, -0.3531)	(-0.8610, -0.3910)
70	(-10.7547, -0.3207)	(-1.4070, -0.3757)	(-0.3479, -0.4151)
90	(-6.8497, -0.3332)	(-0.7558, -0.3964)	(-0.1839, -0.4369)
110	(-4.7488, -0.3457)	(-0.4864, -0.4172)	(-0.1184, -0.4586)
130	(-3.5308, -0.3575)	(-0.3149, -0.4397)	(-0.0744, -0.4828)
150	(-2.8071, -0.3674)	(-0.1731, -0.4663)	(-0.0352, -0.5137)
170	(-2.4546, -0.3733)	(-0.0826, -0.4960)	(-0.0088, -0.5622)

Chapter 6

One Loop Self-Energy Contributions

"I remember that when someone had started to teach me about creation and annihilation operators, that this operator creates an electron, I said, 'How do you create an electron? It disagrees with conservation of charge.'"

R.P. Feynman, Nobel Lecture

6.1 Introduction

The purpose of this chapter is to calculate the self energy diagrams for the neutrino, electron, photon, Z-boson, W-boson and photon-Z boson mixing. From these diagrams one extracts not only the self energy insertions (Fig. 6.1) needed to compute the one loop corrected cross section of $e^+e^- \rightarrow W^+W^-$ reaction, but also gets all the renormalizations constants needed for the consistency of the underlying one loop renormalizable Lagrangian [Chapter 2 and Chapter 4]. In general the self-energy diagrams contain terms which are infrared, linear¹ and ultraviolet divergent. Clearly the wave-function renormalizations of the electron and W-boson

¹A linear divergence may be transformed into a logarithmic divergence by a shift of the momentum variable, this transformation is not always allowed, for example when anomalies are present [40].

are extracted from their respective self-energies. As was pointed out in Chapter 2, when calculating the lepton self energies we must retain terms of the order of m^2/M_W^2 to check the consistency of our renormalization scheme since the counter terms δg and $\delta g'$ are determined by requiring that the photon vertex function of the lepton defines the physical (renormalized) charge. The charge thus defined must be the same for the electron and muon. This means that δg and $\delta g'$ must be independent of the lepton mass m ; otherwise the renormalized charge would be different for the electron and muon and our renormalization scheme would fail. The consistency of this remark has already been verified in Chapter 2 and Chapter 4. It is clear that when dealing with finite contributions to the one loop cross section that terms of order m may be ignored at high energies, (the complete expressions for all the self energies, i.e. including terms of order m will be given, since these are needed as explained in Chapters 2 and 4 for the purpose of renormalization). We will now concentrate mainly on the finite one loop corrections to the cross section arising out of the self energy insertions and WFNR as shown in Fig 6.1. We may subdivide $d\sigma^{\text{self}}$, given in Eq. 5.1, in accord with Fig. 6.1 as

$$d\sigma^{\text{self}} = d\sigma^{\nu_e} + d\sigma^{\gamma\gamma} + d\sigma^{\gamma Z} + d\sigma^{Z\gamma} + d\sigma^{ZZ} + d\sigma^W + d\sigma^e \quad (6.1)$$

The first term in Eq. 6.1 represents the contribution of the neutrino self energy as shown in Fig 6.1 a. This contribution is purely of the t channel type. The next four contributions are purely s channel, they are respectively the photon self energy, photon Z mixing and Z self energy contributions. As is well known the photon contribution is the most dominant since it contains the huge QED contribution. In other words it represents the correction to α , which already at LEP I energies i.e. around the Z mass (91 GeV) changes α from approximately $\frac{1}{137}$ to $\frac{1}{128}$. The sixth and seventh terms are the WFNR of the W and the electron. Both these

contributions are made up of both s and t channels terms.

The number of diagrams to be calculated in QFD and SQFD is given in Table 6.1. For the self energy diagrams there are two topologies t_1 and t_2 possible within the framework of these theories, see Fig. 6.2.

In order to make a surveyable computation and description we define a convenient decomposition of the first order amplitude for every set of self energy, triangle or box diagrams, which give contributions to the one loop corrections to the process $e^+e^- \rightarrow W^+W^-$. In defining such a decomposition we follow closely the method of Lemoine and Veltman [28]. To illustrate the procedure consider the general form of any first order amplitude²

$$\mathcal{M}_{11} = ig^4 f(s_W) [\bar{v}(p_1) \gamma^\mu (a_i + b_i \gamma_5) u(p_2) X G_{\mu\nu\lambda} \epsilon_1^\nu \epsilon_2^\lambda] \quad (6.2)$$

Each diagram will have its own set of $f(s_W)$, a_i , b_i and X . So the contribution of each diagram can be given by specifying the various X , the axial and vector coefficients a_i and b_i , and an overall factor f , that depends in general on the weak mixing angle. X in general will contain the A, B, C and the D integrals. The contribution of a term of the form 6.2 to the differential cross section is given by the interference with the lowest order amplitude Eq. 5.14, averaged over the spins of the incoming particles³ and summed over the required polarizations of the outgoing vector bosons. The result is⁴

$$\left(\frac{d\sigma}{d\cos\theta} \right)_1 = 2g^2 \Phi f(s_W) \text{Re}[(a_s a_i + b_s b_i) X Y^s - (a_t a_i + b_t b_i) X Y^t] \quad (6.3)$$

²The subscript 1 that appears on the amplitude is an indicator of the fact that we are working to one loop order.

³Until otherwise stated we have not averaged over the initial polarisations, one may simply divide our values by a factor of 4 in order to average over the initial polarisations.

The subscript 1 which appears on the differential cross section indicates we are working to one loop order.

with

$$Y^* = \sum_{\text{spin}} [\bar{v}(p_1) \gamma_\lambda u(p_2) \Gamma_{\mu\nu\lambda} \epsilon_1^\mu \epsilon_2^\nu]^* \times [\bar{v}(p_1) \gamma_\lambda u(p_2) G_{\lambda\rho\sigma} \epsilon_1^\rho \epsilon_2^\sigma], \quad (6.4)$$

For the purposes of numerical evaluation we assume the following values for our set of independent input parameters [3], (all masses are given in GeV), $\alpha = 1/137.035985$, $M_W = 81.0$, $M_Z = 92.0$, $m_e = .000511$, $m_\mu = .105658$, $m_\tau = 1.78410$, $m_{\nu_e} = 0$, $m_{\nu_\mu} = 0$, $m_{\nu_\tau} = 0$, $m_u = 0.032$, $m_d = 0.032$, $m_c = 1.5$, $m_s = 0.15$, $m_t = 100.0$, $m_b = 4.5$, $m_{\bar{c}} = 45.0$, $m_{\bar{u}} = 45.0 + .105658$, $m_{\bar{\tau}} = 45.0 + 1.78410$, $m_{\bar{\nu}_e} = 30.0$, $m_{\bar{\nu}_\mu} = 30.0$, $m_{\bar{\nu}_\tau} = 30.0$, $m_{\bar{u}} = 45.0 + 0.032$, $m_{\bar{d}} = 45.0 + 0.032$, $m_{\bar{c}} = 45.0 + 1.5$, $m_{\bar{s}} = 45.0 + 0.15$, $m_{\bar{b}} = 45.0 + 4.5$, $m_{\bar{t}} = 50.0$, $m_{H_1^0} = 50.0$, 100.0 , $m_{H_2^0} = 100.0$, $m_{H_3^0} = 75.0$, $m_H = 91.0$, $m_{\tilde{\gamma}} = 15.0$, $m_{\tilde{W}} = 50.0$, $m_{\tilde{Z}} = 35.0$, and $m_{\tilde{h}} = 40.0$. The following remarks are in order as to why we have chosen the above values for the set of the input parameters

1. : In order to make a sensible comparison with the results given by other authors [28, 29, 30, 31]. For the purposes of exact comparison with the work of [30] we take $M_W = 82.0$, $M_Z = 93.0$ and $m_{H_1^0} = 100.0$.
2. : The input masses for the supersymmetric particles given above were inspired by the work of Baer et al. [64], and by the experimental limits [3]. The masses given above for the supersymmetric Higgs sector were arrived at by keeping in mind the recent experimental limits [3] and the standard wisdom (that at the tree-level) $m_H > M_W$, $H_2^0 > M_Z$, $m_{H_3^0} > m_{H_1^0}$ and $m_{H_1^0} < M_Z$. In addition to the case given above for the SUSY particles masses we have also examined the case when the masses of most SUSY particles are taken above the W-boson mass.⁵ For this case we as-

⁵The case, i.e. when the SUSY particles masses are above W-mass was suggested to us by a Physical Review D referee.

sumed the following values for the masses of the SUSY particles, $m_{\tilde{e}} = 90.0$
 $m_{\tilde{\mu}} = 90.0 + .105658$, $m_{\tilde{\tau}} = 90.0 + 1.78410$, $m_{\tilde{\nu}_e} = 87.0$, $m_{\tilde{\nu}_\mu} = 87.0$,
 $m_{\tilde{\nu}_\tau} = 87.0$, $m_{\tilde{u}} = 120.0 + 0.032$, $m_{\tilde{d}} = 120.0 + 0.032$, $m_{\tilde{c}} = 125.0 + 1.5$,
 $m_{\tilde{s}} = 125.0 + 0.15$, $m_{\tilde{b}} = 130.0 + 4.5$, $m_{\tilde{t}} = 135.0$, $m_{H_1^0} = 50.0$, $m_{H_2^0} = 100.0$,
 $m_{H_3^0} = 75.0$, $m_H = 91.0$, $m_{\tilde{\gamma}} = 30.0$, $m_{\tilde{\omega}} = 85.0$, $m_{\tilde{z}} = 95.0$, and $m_{\tilde{h}} = 70.0$.

6.2 Self-energy of the electron neutrino

There are three diagrams, see Fig. 6.3 contributing to the neutrino self energy in QFD. It is straightforward to write out the contributions of these, we will illustrate the calculation by working out the W contribution to neutrino self energy and then give the result for the remaining two diagrams. Defining Σ^i where $\Sigma_{\nu_e}^i = \nu_e'(k^2) \not{\epsilon} \frac{1-\gamma_5}{2}$ to be the self-energy of the neutrino due to the i th particle one has Fig. 6.3

$$-i\Sigma_{\nu_e}^W = -\frac{g^2(2-n)}{4} \int \frac{d^n q}{(2\pi)^n} \frac{(\not{q} + \not{k})(1-\gamma_5)}{(q^2 - M_W^2)((q+k)^2 - m_e^2)} \quad (6.5)$$

Using the definitions of the A and B-integrals given in Appendix C, this reduces to

$$\Sigma_{\nu_e}^W = \frac{(2-n)}{2} [B_0(M_W, m_e, k^2) + B_1] \not{\epsilon} \frac{(1-\gamma_5)}{2} \quad (6.6)$$

The expression for ν_e^W immediately follows from Eq. 6.6

$$\nu_e^W = \frac{(2-n)}{2} [B_0(M_W, m_e, k^2) + B_1] \quad (6.7)$$

Plugging in the explicit forms for the A and B-integrals one can show that this result agrees with Sakakibara [43],

$$\begin{aligned} \nu_e^W(k^2) &= \left[-\frac{1}{2} \Delta + \frac{1}{2} + \int_0^1 (1-x) \ln(M_1^2) \right], \\ M_1^2 &= \frac{xm_e^2 + (1-x)M_W^2 - x(1-x)k^2}{M_W^2}. \end{aligned} \quad (6.8)$$

Similarly for diagrams 2 and 3 one obtains Fig. 6.3

$$\nu^{G^+}(k^2) \doteq -(m_e^2/2M_W^2)[B_0(m_{H^+}, m_e, k^2) + B_1], \quad (6.9)$$

and

$$\nu^Z(k^2) \doteq \frac{(2-n)}{2}(1/2c_W^2)[B_0(M_Z, m_{\nu_e}, k^2) + B_1]. \quad (6.10)$$

These results agree with Sakakibara [43], thus providing a check on our calculations. Replacing each particle in the self energy loop (Fig. 6.3) by its s partners one obtains the SUSY contribution to neutrino self energy Fig.6.3, 1', 2', 2a', and 3'. These are written as

$$\nu^{\tilde{\omega}}(k^2) \doteq [B_1(m_{\tilde{\omega}}, m_{\tilde{e}}, k^2)], \quad (6.11)$$

$$\nu^{H^+}(k^2) \doteq -(m_e^2/2M_W^2)[B_0(m_{H^+}, m_{\tilde{e}}, k^2) + B_1], \quad (6.12)$$

$$\nu^{\tilde{\omega}}(k^2) \doteq (m_e^2/M_W^2)[B_1(m_{\tilde{\omega}}, m_{\tilde{e}}, k^2)], \quad (6.13)$$

$$\nu^{\tilde{z}}(k^2) \doteq (1/2c_W^2)[B_1(m_{\tilde{z}}, m_{\tilde{\nu}_e}, k^2)]. \quad (6.14)$$

We follow the notation that in each bracket all the B's have the same argument as the first B in the bracket. Notice that there are two contributions due to the wino. One to the order (m_e^2/M_W^2) . This arises because the wino in our model [25, 44] (see also section 3.3) is made of two parts, one being the fermionic partner of the charged Higgs scalar (H) and the other being of course the s partner of the W boson.

6.3 Self-energy of the electron

The calculation of the self-energy diagrams for the electron follows much like the neutrino case except that there are a few additional diagrams Fig. 6.4. Defining

$$L = \frac{(1 - \gamma_5)}{2}, R = \frac{(1 + \gamma_5)}{2}, a_1 = (1/4)(2 - 4s_W^2), b_1 = (1/4)(-4s_W^2) \quad (6.15)$$

the contributions for the diagrams of Fig. 6.4 may be promptly written as

$$\Sigma^\gamma \doteq s_W^2 [(2 - n)(B_0(\lambda, m_e, k^2) + B_1)\not{\epsilon} + nm_e B_0](L + R) \quad (6.16)$$

where a mass λ has been given to the photon. We also find

$$\Sigma_W \doteq \frac{(2 - n)}{2} [B_0(M_W, m_{\nu_e}, k^2) + B_1]\not{\epsilon} L, \quad (6.17)$$

$$\Sigma_Z \doteq \frac{1}{c_W^2} [(2 - n)(B_0(m_Z, m_e, k^2) + B_1)\not{\epsilon}(a_1^2 L + b_1^2 R) + nm_e a_1 b_1 B_0(L + R)], \quad (6.18)$$

$$\Sigma^{G^+} \doteq -(m_e^2/2M_W^2)[B_0(M_W, m_{\nu_e}, k^2) + B_1]\not{\epsilon} R, \quad (6.19)$$

$$\Sigma^{H^0} \doteq -(m_e^2/4M_W^2)[(B_0(m_{H^0}, m_e, k^2) + B_1)\not{\epsilon} + m_e B_0](L + R) \quad (6.20)$$

$$\Sigma^{G^0} \doteq -(m_e^2/4M_W^2)[(B_0(m_{G^0}, m_e, k^2) + B_1)\not{\epsilon} + m_e B_0](R + L), \quad (6.21)$$

$$\Sigma_{\tilde{\gamma}} \doteq 2s_W^2 [B_1(m_{\tilde{\gamma}}, m_{\tilde{e}}, k^2)]\not{\epsilon} L,$$

$$\Sigma_{\tilde{\gamma}} \doteq 2s_W^2 [B_1(m_{\tilde{\gamma}}, m_{\tilde{e}}, k^2)]\not{\epsilon} R, \quad (6.22)$$

$$\Sigma_{\tilde{\omega}} \doteq [B_1(m_{\tilde{\omega}}, m_{\tilde{\nu}_e}, k^2)]\not{\epsilon} L,$$

$$\Sigma_{\tilde{\omega}} \doteq \frac{m_e^2}{M_W^2} [B_1(m_{\tilde{\omega}}, m_{\tilde{\nu}_e}, k^2)]\not{\epsilon} R, \quad (6.23)$$

$$\Sigma_H \doteq -\frac{m_e^2}{2M_W^2} [B_0(m_H, m_{\tilde{\nu}_e}, k^2) + B_1]\not{\epsilon} R, \quad (6.24)$$

$$\begin{aligned}
\Sigma_{\tilde{e}} &\doteq \frac{2a_1^2}{c_W^2} [B_1(m_{\tilde{e}}, m_{\tilde{e}}, k^2)] \not{L}, \\
\Sigma_{\tilde{e}} &\doteq \frac{2b_1^2}{c_W^2} [B_1(m_{\tilde{e}}, m_{\tilde{e}}, k^2)] \not{R}, \\
\Sigma_{\tilde{e}} &\doteq \frac{m_e^2}{2M_W^2} [B_1(m_{\tilde{e}}, m_{\tilde{e}}, k^2)] \not{L}, \\
\Sigma_{\tilde{e}} &\doteq \frac{m_e^2}{2M_W^2} [B_1(m_{\tilde{e}}, m_{\tilde{e}}, k^2)] \not{R},
\end{aligned} \tag{6.25}$$

$$\Sigma_{H_2^0} \doteq -(m_e^2/4M_W^2)[(B_0(m_{H_2^0}, m_e, k^2) + B_1)\not{L} + m_e B_0], \tag{6.26}$$

$$\begin{aligned}
\Sigma_{\tilde{h}} &\doteq \frac{m_e^2}{2M_W^2} [B_1(m_{\tilde{h}}, m_{\tilde{e}}, k^2)] \not{L}, \\
\Sigma_{\tilde{h}} &\doteq \frac{m_e^2}{2M_W^2} [B_1(m_{\tilde{h}}, m_{\tilde{e}}, k^2)] \not{R},
\end{aligned} \tag{6.27}$$

$$\Sigma_{H_3^0} \doteq -(m_e^2/4M_W^2)[(B_0(m_{H_3^0}, m_e, k^2) + B_1)\not{L} + m_e B_0](L + R). \tag{6.28}$$

From the photon contribution to the electron self-energy we extract the infrared infinite parts

$$\begin{aligned}
f_L &\doteq -s_W^2 [2 \log(\frac{\lambda^2}{m_e^2})], \\
\delta Z_R &\doteq -s_W^2 [2 \log(\frac{\lambda^2}{m_e^2})].
\end{aligned} \tag{6.29}$$

Following the same method as in the neutrino self energy (Appendix F) we have checked [[44] (Ph.d Thesis Notes)] the explicit cancellation of all linear divergences occurring in the electron self energy in the SUSY limit.

6.4 Self-energy of the photon

In this section and the three subsequent ones we concern ourselves with computation of self energies of gauge mesons. We make a few some simple remarks. First of all, as is clear from Fig. 6.5, γ self energy can be categorized into two sets. One

entirely due to fermions and other arises out of γ interaction with W-boson [G in our gauge is simply the longitudinal made of W and $c^\pm$ are the Faddeev-Popov ghosts corresponding to each of the W^+ and W^-]. The photon self energy will be denoted by $i\pi_f^\gamma(k^2)[k^2 g_{\mu\nu} - k_\mu k_\nu]$ One immediately writes down the fermion contribution to photon's self energy

$$i\pi_f^\gamma(k^2)[k^2 g_{\mu\nu} - k_\mu k_\nu] = - \sum_{f,c} (g^2 s_W^2 Q_f^2) \int \frac{d^n q}{(2\pi)^n} \frac{\text{Tr}(\gamma_\mu(\not{q} + m_f)\gamma_\nu(\not{q} + \not{k} + m_f))}{(q^2 - m_f^2)((q+k)^2 - m_f^2)} \quad (6.30)$$

After dimensional regularization, Appendix C, one obtains

$$\pi_f^\gamma(k^2) \doteq \sum_{f,c} n s_W^2 Q_f^2 [2B_{21}(m_f, m_f, k^2) + 2B_1] \quad (6.31)$$

On using the explicit form of the B integrals in Eq. 6.31 one obtains with the well known result [43]. The Gauge-boson (g.b) contribution may be written down as

$$\pi_{g.b}^\gamma(k^2) \doteq s_W^2 [(B_0(M_W, M_W, k^2) - 12B_1 - 12B_{21}) + \frac{2}{3}] \quad (6.32)$$

Eqs. 6.31 and 6.32 allow us to write renormalized $\bar{\pi}$

$$\bar{\pi}_{f+g.b}^\gamma(k^2) \doteq \pi_{f+g.b}^\gamma(k^2) - \pi_{f+g.b}^\gamma(0) \quad (6.33)$$

Here

$$\pi_{f+g.b}^\gamma(0) \doteq [3\Delta + \frac{2}{3} + \frac{4}{3}Q_f^2 \sum_{f,c} (-\Delta + \ln(\frac{m_f^2}{M_W^2}) + \frac{1}{2})] \quad (6.34)$$

The SUSY-contribution to the photon self energy is displayed in Fig. 6.5, 1', 2', 3', 4'.

The contribution of the wino works out to be

$$\pi_{\tilde{W}}^\gamma(k^2) \doteq 2n s_W^2 [2B_{21}(m_{\tilde{W}}, m_{\tilde{W}}, k^2) + 2B_1] \quad (6.35)$$

The charged Higgs (H) gives

$$\pi_H^\gamma(k^2) \doteq s_W^2 [-(B_0(M_H, M_H, k^2) + 4B_1 + 4B_{21})] \quad (6.36)$$

The calculation for Fig. 6.5, 4' follows a similar pattern since it is also a vector-scalar-scalar interaction and in a renormalizable theory it has a universal form, summing over all colors and flavors

$$\pi_f^\gamma(k^2) \doteq \sum_{f,c} 2s_W^2 Q_f^2 [-(B_0(m_f, m_f, k^2) + 4B_1 + 4B_{21})] \quad (6.37)$$

We have checked that quadratic and linear divergences cancel as expected in SUSY for the case of photon self-energy.

6.5 $\gamma - Z$ mixing

The diagrams which contribute to the $\gamma - Z$ mixing are not displayed since they are identical to the diagrams which contribute to the photon self energy (see Fig. 6.5). The fermion contribution in QFD is

$$a_f^{\gamma-Z}(k^2) \doteq - \sum_{f,c} n t_W Q_f (\frac{1}{2} T_{3f}^a - Q_f^a s_W^2) [(n-2)B_{22}(m_f, m_f, k^2) - nB_{21}k^2 - nB_1k^2 + nm_f^2 B_0], \quad (6.38)$$

The Gauge-boson contribution may be written down as

$$a_{g,b}^{\gamma-Z}(k^2) \doteq t_W [+ c_W^2 k^2 (B_0(M_W, M_W, k^2) - 12B_1 - 12B_{21}) + \frac{2}{3} + \frac{1}{2} k^2 (+B_0(M_W, M_W, k^2) + 4B_1 + 4B_{21}) + 2M_W^2 B_0]. \quad (6.39)$$

The SUSY-contribution to $\gamma - Z$ are now given. The contribution of the wino works out to be

$$\pi_{\tilde{W}}^{\gamma-Z}(k^2) \doteq 2ns_W^2 [2B_{21}(m_{\tilde{W}}, m_{\tilde{W}}, k^2) + 2B_1]. \quad (6.40)$$

The charged Higgs (H) gives

$$a_H^{\gamma-Z}(k^2) \doteq \frac{1}{2} t_W k^2 (1 - 2s_W^2) [-(B_0(M_H, M_H, k^2) + 4B_1 + 4B_{21})], \quad (6.41)$$

$$a_f^{\gamma-Z}(k^2) \doteq k^2 \sum_{i,f,c} t_W [(T_{3f} - Q_f s_W^2) Q_f (-B_0(m_f, m_f, k^2) - 4B_1 - 4B_{21})]. \quad (6.42)$$

We have checked that quadratic and linear divergences cancel as expected in SUSY for the case of photon-Z mixing diagrams.

6.6 W Self-Energy

The W-self energy diagrams can be divided into 4 categories, the contribution from fermions, from Higgs, from the photon and finally from the Z-boson (Fig. 6.6).

The contribution from the fermions may be written as

$$a_f^W \doteq \sum_{\text{doublets}, c} \left[\frac{n}{4} [(2-n)B_{22}(m_1, m_2, k^2) + k^2(B_1 + B_{21})] \right], \quad (6.43)$$

from Higgs as

$$a_{H_1^0}^W \doteq \frac{1}{4} [-4B_{22}(M_{H_1^0}, M_W, k^2) - 4M_W^2 B_0 + A(M_{H_1^0})], \quad (6.44)$$

and from vector bosons as

$$\begin{aligned} a_{v,b}^W \doteq & s_W^2 \left(-\frac{k^2}{3} + M_W^2 \right) + c_W^2 \left(+\frac{2k^2}{3} - 2M_W^2 + 2M_Z^2 \right) - \frac{1}{2} M_W^2 + \\ & \frac{1}{4} M_Z^2 + \left(+\frac{M_Z^2}{4} + 3M_W^2 \right) \ln \frac{M_W^2}{M_Z^2} - s_W^2 [5k^2(-B_0(\lambda, M_W, k^2) - \\ & 2B_1 - 2B_{21}) + M_W^2(B_0 + 8B_1)] + \\ & c_W^2 [k^2(-3B_0(M_Z, M_W, k^2) - 20B_1 - 12B_{21}) - 6(M_W^2 - \\ & M_Z^2)(-B_0 - B_1)] + \left(\frac{1}{4} - c_W^2 \right) [k^2(-2B_0(M_Z, M_W, k^2) - \\ & 6B_1 - 4B_{21}) - 2(M_W^2 - M_Z^2)(-B_0 - B_1)] + (c_W^2 + 2 - \\ & \sec^2 \theta_W^2) M_W^2 B_0(M_Z, M_W, k^2). \end{aligned} \quad (6.45)$$

Now the contribution from the s particles are found to be

$$a_h^W \doteq \frac{n}{4} [(2-n)B_{22}(m_h, m_\omega, k^2) + k^2(B_1 + B_{21})], \quad (6.46)$$

$$a_{\tilde{\gamma}}^W \doteq n s_W^2 [(2-n)B_{22}(m_{\tilde{\gamma}}, m_\omega, k^2) + k^2(B_1 + B_{21})], \quad (6.47)$$

$$a_{\tilde{z}}^W \doteq n \left(c_W^2 + \frac{1}{4} \right) [(2-n)B_{22}(m_{\tilde{z}}, m_\omega, k^2) + k^2(B_1 + B_{21})] - n m_\omega m_{\tilde{z}} c_W B_0, \quad (6.48)$$

$$a_{H_2^0}^W \doteq \frac{1}{4} [-4B_{22}(M_{H_2^0}, M_H, k^2) + A(M_{H_2^0}) + A(M_H)], \quad (6.49)$$

$$a_{H_3^0}^W \doteq \frac{1}{4} [-4B_{22}(M_{H_3^0}, M_H, k^2) + A(M_{H_3^0}) + A(M_H)], \quad (6.50)$$

$$a_f^W \doteq \sum_{\text{doublets}, c} \left[\frac{1}{2} [-4B_{22}(m_1, m_2, k^2) + A(m_1) + A(m_2)] \right] \quad (6.51)$$

m_1 and m_2 being respectively the masses of the down and the up s-fermions.

The derivative of the renormalized a_W which enters as the relevant quantity is easily extracted from the above evaluations of the various a 's and the definition (section 2.4) of δZ_W .

$$\frac{\partial \bar{a}^W(s)}{\partial s} \Big|_{s=M_W^2} = \frac{\partial a^W(s)}{\partial s} \Big|_{s=M_W^2} - \delta Z_W. \quad (6.52)$$

From the photon contribution to the W self-energy we extract the infrared infinite part

$$\bar{a}^W \doteq -s_W^2 [2 \log(\frac{\lambda^2}{M_W^2})]. \quad (6.53)$$

6.7 Z Self Energy

For the Z self energy (Fig. 6.7), we get contributions similar to the above

$$a_f^Z \doteq \sum_{f,c} \left[\frac{n}{4} [(2-n)B_{22}(m_1, m_2, k^2) + k^2(B_1 + B_{21})] \right], \quad (6.54)$$

$$a_{H_1^0}^Z \doteq \frac{1}{4c_W^2} [-4B_{22}(M_{H_1^0}, M_Z, k^2) - 4M_Z^2 B_0 + A(M_{H_1^0})], \quad (6.55)$$

$$\begin{aligned} a_{v,b}^Z \doteq & \frac{2k^2 c_W^2}{3} - \frac{M_Z^2}{4c_W^2} (1 + \ln \frac{M_W^2}{M_Z^2}) - c_W^2 k^2 (-B_0(M_W, M_W, k^2) + \\ & 12B_1 + 12B_{21}) + (\frac{1}{4c_W^2} - 1)k^2 (-B_0(M_W, M_W, k^2) - 4B_1 - \\ & 4B_{21}) + (4 - \frac{2}{c_W^2})M_W^2 B_0(M_W, M_W, k^2), \end{aligned} \quad (6.56)$$

and the contribution from the s-particles is

$$a_h^Z \doteq \frac{n}{4c_W^2} [(2-n)B_{22}(m_h, m_h, k^2) + k^2(B_1 + B_{21})], \quad (6.57)$$

$$\begin{aligned} a_{\tilde{\omega}}^Z \doteq & n(c_W^2 + \frac{(1-2s_W^2)^2}{4c_W^2}) [(2-n)B_{22}(m_{\tilde{\omega}}, m_{\tilde{\omega}}, k^2) + k^2(B_1 + B_{21})] - \\ & n(1 - 2s_W^2)m_{\tilde{\omega}}m_{\tilde{\omega}}B_0, \end{aligned} \quad (6.58)$$

$$a_{H_2^0, H_3^0}^Z \doteq \frac{1}{4c_W^2} [-4B_{22}(M_{H_2^0}, M_{H_2^0}, k^2) + A(M_{H_2^0}) + A(M_{H_3^0})], \quad (6.59)$$

$$a_H'' \doteq \frac{(1 - 2s_W^2)^2}{4c_W^2} [-4B_{22}(M_H, M_H, k^2) + 2A(M_H)] \quad (6.60)$$

$$a_j'' \doteq \sum_{i,f,c} \left[\frac{1}{c_W^2} (T_{3f}^i - Q_f^i s_W^2)^2 [-4B_{22}(m_j, m_j, k^2) + 2A(m_j)] \right], \quad (6.61)$$

where m_j the mass of the s -fermion.

6.8 The Self-Energy and the Wave Function Renormalization contribution to the one loop differential cross section

In this section we give the contribution of the self-energy insertions and the Wave Function Renormalization [WFNR] to the one loop differential cross section. Consider first the neutrino contribution Fig 6.1. a. Contracting this with the tree-level amplitude (as described in the introductory section 6.1) one has

$$\left(\frac{d\sigma}{d \cos \theta} \right)_1^\nu = 2\bar{\nu}(t)\Phi[-(a_s a_t + b_s b_t)\mathcal{A}_{st} + (a_t^2 + b_t^2)\mathcal{A}_{tt}]. \quad (6.62)$$

The renormalized neutrino self-energy $\bar{\nu}$ entering in the above expression is defined as

$$\bar{\nu}(k^2) = \nu(k^2) - \nu(0). \quad (6.63)$$

Ignoring the terms proportional to the electron mass, for QFD one has, on using results in section 6.2,

$$\begin{aligned} \bar{\nu}(k^2) \doteq & -[B_0(M_W, m_e, k^2) - B_0(M_W, m_e, 0) + B_1(M_W, m_e, k^2) - \\ & B_1(M_W, m_e, 0)] - \frac{(1/2c_W^2)[B_0(M_Z, m_{\nu_e}, k^2) - B_0(M_Z, m_{\nu_e}, 0) + B_1(M_Z, m_{\nu_e}, k^2) - B_1(M_Z, m_{\nu_e}, 0)]}{1} \end{aligned} \quad (6.64)$$

and for SQFD in addition to the above contribution from QFD

$$\begin{aligned} \bar{\nu}(k^2) \doteq & [B_1(m_{\tilde{W}}, m_{\tilde{e}}, k^2) - B_1(m_{\tilde{W}}, m_{\tilde{e}}, 0)] \\ & + \frac{(1/2c_W^2)[B_1(m_{\tilde{Z}}, m_{\tilde{\nu}_e}, k^2) - B_1(m_{\tilde{Z}}, m_{\tilde{\nu}_e}, 0)]}{1}. \end{aligned} \quad (6.65)$$

The numerical values for $\left(\frac{d\sigma}{d\cos\theta}\right)_1^\nu$ are listed in Table 6.3 for CM energy of 200 GeV and various CM angles. The W-boson WFNR, Fig. 6.1. f, contribution is

$$\left(\frac{d\sigma}{d\cos\theta}\right)_1^W = \Phi \mathcal{A}_0 \left[\left(\frac{\partial \bar{a}^W(s_3)}{\partial s_3} \right)_{s_3=M_W^2} + \left(\frac{\partial \bar{a}^W(s_4)}{\partial s_4} \right)_{s_4=M_W^2} \right]. \quad (6.66)$$

The renormalized derivative of $\bar{a}^W$ is easily got from the definition in Chapter 2 (section 2.4) and the results for the self-energy contributions listed in section 6.6. The numerical values for Eq. 6.66 are given Table 6.8. The self-energy insertions from Fig. 6.1. b-e, are successively written as

$$\left(\frac{d\sigma}{d\cos\theta}\right)_1^{\gamma\gamma} = 2\Phi \frac{s_W^2}{s^2} [a_s \mathcal{A}_{ss} - a_t \mathcal{A}_{st}] \bar{\pi}^\gamma(s) \quad (6.67)$$

$$\left(\frac{d\sigma}{d\cos\theta}\right)_1^{\gamma Z} = 2\Phi \frac{s_W c_W}{(s)(s-M_Z^2)} [a_s \mathcal{A}_{ss} - a_t \mathcal{A}_{st}] \bar{a}^{\gamma Z}(s) \quad (6.68)$$

$$\left(\frac{d\sigma}{d\cos\theta}\right)_1^{Z\gamma} = 2\Phi \frac{s_W}{(4c_W)(s)(s-M_Z^2)} [(a_s(1-4s_W^2) - b_s) \mathcal{A}_{ss} - (a_t(1-4s_W^2) - b_t) \mathcal{A}_{st}] \bar{a}^{Z\gamma}(s) \quad (6.69)$$

$$\left(\frac{d\sigma}{d\cos\theta}\right)_1^{ZZ} = 2\Phi \frac{1}{4(s-M_Z^2)^2} [(a_s(1-4s_W^2) - b_s) \mathcal{A}_{ss} - (a_t(1-4s_W^2) - b_t) \mathcal{A}_{st}] \bar{a}^{ZZ}(s) \quad (6.70)$$

The renormalized a (i.e. $\bar{a}^{\gamma-Z}$) of the $\gamma - Z$ mixing is given by

$$\bar{a}^{\gamma-Z}(s) = a^{\gamma-Z}(s) - Y_{WZ} \cot \theta_W + t_W M_W^2 \frac{\delta g}{g} \quad (6.71)$$

The numerical values for Eqs. 6.67 through 6.70 are listed respectively in Tables 6.4, 6.5, 6.6 and 6.7. We note that in Tables 6.5, 6.6, 6.7, and 6.8 the first entry in the QFD column is for Higgs boson mass of 100 GeV, the second entry in the same column corresponds to a Higgs mass of 50 GeV. The first and second entry in the S-P column corresponds to with and without a mass splitting term respectively. To get the total contribution in context of SQFD (i.e. third column) we have added the second entry in the first column (i.e. Higgs mass of 50 GeV) to both the

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entries in the second column. The reason being that we have assumed a 50 GeV Higgs mass in SQFD. Now we turn to the contribution by the electron WFNR to the one loop corrected cross section Fig. 1g. This is given by

$$\left(\frac{d\sigma}{d\cos\theta}\right)_1^e = \left(\frac{d\sigma}{d\cos\theta}\right)_1^{e_L} + \left(\frac{d\sigma}{d\cos\theta}\right)_{1fct}^{e^+e^-\gamma+e^+e^-Z,e_R} \quad (6.72)$$

where

$$\left(\frac{d\sigma}{d\cos\theta}\right)_1^{e_L} = \Phi f_L [(a_s - b_s)^2 \mathcal{A}_{ss} - 2(a_t - b_t)(a_s - b_s) \mathcal{A}_{st} + (a_t - b_t)^2 \mathcal{A}_{tt}], \quad (6.73)$$

$$\left(\frac{d\sigma}{d\cos\theta}\right)_{1fct}^{e^+e^-\gamma+e^+e^-Z,e_R} = \Phi \delta \bar{Z}_R [(a_s + b_s)^2 \mathcal{A}_{ss} - 2(a_t + b_t)(a_s + b_s) \mathcal{A}_{st} + (a_t + b_t)^2 \mathcal{A}_{tt}]. \quad (6.74)$$

A few remarks are in order to clarify the appearance of the right-handed electron term in the electron WFNR contribution (see Eq. 6.72). As mentioned before, (see Chapters 2 and 4) the right-handed electron is subtracted on shell and does not contribute. The right-handed term in Eq. 6.72 comes from the counterterm contribution considered in Chapter 7. This is why we have labelled it with the names of the relevant vertices from which it arises. We have included it here in the electron WFNR term because we need it for the infrared cancellations considered in the previous Chapter. δZ_R is written with a bar since the quadratic divergence in it has been cancelled in Chapter 7. The superscript γ appearing on δZ_R indicates that only the photon's contribution to δZ_R has been considered here. The quantity f_L can be easily extracted by using the definition of f_L given in Chapter 2, (see section 2.4) and the expressions for the electron self-energy given in section 6.3. For the case of QFD one finds ⁶

$$f_L \doteq -s_W^2 \left[\frac{9}{2} + \ln \frac{M_Z^2}{m_e^2} \right], \quad (6.75)$$

⁶We note that the infrared divergences in f_L and δZ_R have been cancelled against the bremsstrahlung IR infinities in the previous Chapter.

$$\delta\widetilde{Z}_R \doteq -s_W^2[4.0]. \quad (6.76)$$

For the case of SQFD in addition to the above one has

$$f_L^S \doteq +\frac{g_1^2}{c_W^2}[B_1(m_z, m_{\tilde{e}}, m_e^2)] + 2s_W^2[F_1(m_{\tilde{\gamma}}, m_{\tilde{e}}, m_e^2)] + [B_1(m_{\tilde{\omega}}, m_{\tilde{\nu}_e}, m_e^2)] - [B_1(m_{\tilde{\omega}}, m_{\tilde{e}}, 0)] - \frac{1}{2c_W^2}[\mathcal{J}_1(m_z, m_{\tilde{\nu}_e}, 0)]. \quad (6.77)$$

This reduces in the SUSY limit to

$$f_L \doteq -s_W^2[-\frac{1}{2} + \ln \frac{M_Z^2}{m_e^2}]. \quad (6.78)$$

The results for the electron WFNR are listed in Table 6.2 Note that all the momentum dot products which entered in the one loop corrections due to self energy insertions and WFNR are exactly the momentum dot products we encountered at the tree level Eqs. 5.23, 5.24, and 5.25.

Figure 6.1: Feynman diagrams for self-energy and wave-function corrections.

Figure 6.2: Topologies of self energy corrections

Figure 6.3: One loop Feynman diagrams contributing to the neutrino self-energy in SQFD.

Figure 6.4: One loop Feynman diagrams contributing to the electron self-energy in SQFD.

Figure 6.5: One loop Feynman diagrams contributing to the photon self-energy in SQFD.

Figure 6.6: One loop Feynman diagrams contributing to the W-boson self-energy in SQFD.

Table 6.1: Self Energy diagram count

self energy of	# of diagrams in QFD	# of diagrams due to S-P	Total # in SQFD
ν_e	(2,0)	(2,0)	(4,0)
e	(3,0)	(3,0)	(6,0)
γ	$(7n_g + 6, 2)$	$(7n_g + 3, 7n_g + 1)$	$(14n_g + 9, 7n_g + 3)$
$\gamma - Z$	$(7n_g + 6, 2)$	$(7n_g + 3, 7n_g + 1)$	$(14n_g + 9, 7n_g + 3)$
W	$(4n_g + 11, 6)$	$(4n_g + 8, 4n_g + 3)$	$(8n_g + 19, 4n_g + 9)$
Z	$(8n_g + 8, 4)$	$(8n_g + 6, 8n_g + 3)$	$(16n_g + 14, 8n_g + 7)$

Figure 6.7: One loop Feynman diagrams contributing to the Z-boson self-energy in SQFD.

Table 6.2: Electron WFNR contribution to one loop radiative corrections in pb for center of mass energy of 200 GeV.

$\theta(deg)$	$(\frac{d\sigma}{d\cos\theta})_1^{QFD}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-I'}$	$(\frac{d\sigma}{d\cos\theta})_1^{SQFD}$
10	-5.2997	+0.4867	-4.8130
30	-3.6883	+0.3341	-3.3542
50	-2.1070	+0.1850	-1.9220
70	-1.2545	+0.1040	-1.1505
90	-0.8252	+0.0633	-0.7619
110	-0.5945	+0.0419	-0.5526
130	-0.4574	+0.0299	-0.4275
150	-0.3733	+0.0230	-0.3503
170	-0.3315	+0.0197	-0.3118
180	-0.3263	+0.0193	-0.3070

Table 6.3: Neutrino Self-energy contribution to one loop radiative corrections in pb for center of mass energy of 200 GeV.

$\theta(deg)$	$(\frac{d\sigma}{d\cos\theta})_1^{QFD}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$	$(\frac{d\sigma}{d\cos\theta})_1^{SQFD}$
10	+0.0346	+0.4416	+0.4762
30	+0.0491	+0.4162	+0.4653
50	+0.0552	+0.3215	+0.3767
70	+0.0565	+0.2550	+0.3115
90	+0.0568	+0.2170	+0.2738
110	+0.0569	+0.1951	+0.2520
130	+0.0568	+0.1812	+0.2380
150	+0.0563	+0.1722	+0.2285
170	+0.0559	+0.1676	+0.2235
180	+0.0559	+0.1670	+0.2229

Table 6.4: γ Self-energy contribution to one loop radiative corrections in pb for center of mass energy of 200 GeV.

$\theta(deg)$	$(\frac{d\sigma}{d\cos\theta})_1^{QFD}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$	$(\frac{d\sigma}{d\cos\theta})_1^{SQFD}$
10	+2.8626	-0.0032	+2.8594
30	+1.0384	-0.0011	+1.0373
50	-0.0320	+0.00004	-0.0320
70	-0.5000	+0.0006	-0.4994
90	-0.7012	+0.0008	-0.7004
110	-0.7844	+0.0009	-0.7835
130	-0.8143	+0.0009	-0.8134
150	-0.8218	+0.0009	-0.8209
170	-0.8224	+0.0004	-0.8220
180	-0.8223	+0.0009	-0.8214

Table 7.5: γ -Z Self-energy contribution to one loop radiative corrections in pb for center of mass energy of 200 GeV.

$\theta(deg)$	$(\frac{d\sigma}{d\cos\theta})_1^{QFD}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$	$(\frac{d\sigma}{d\cos\theta})_1^{SQFD}$
10	+0.0152,+0.0181	+0.0966,+0.3222	+0.1147,+0.3403
30	+0.0055,+0.0066	+0.0350,+0.1169	+0.0416,+0.1235
50	-0.0002,-0.0002	-0.0011,-0.0036	-0.0012,-0.0038
70	-0.0027,-0.0032	-0.0169,-0.0563	-0.0201,-0.0595
90	-0.0037,-0.0044	-0.0237,-0.0789	-0.0281,-0.0833
110	-0.0042,-0.0050	-0.0265,-0.0883	-0.0315,-0.0933
130	-0.0043,-0.0051	-0.0265,-0.0916	-0.0316,-0.0967
150	-0.0044,-0.0052	-0.0277,-0.0925	-0.0329,-0.0977
170	-0.0044,-0.0052	-0.0277,-0.0925	-0.0329,-0.0978
180	-0.0044,-0.0052	-0.0277,-0.0926	-0.0329,-0.0978

Table 6.6: Z- γ Self-energy contribution to one loop radiative corrections in pb for center of mass energy of 200 GeV.

$\theta(deg)$	$(\frac{d\sigma}{d\cos\theta})_1^{QFD}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$	$(\frac{d\sigma}{d\cos\theta})_1^{SQFD}$
10	+0.0059,+0.0070	+0.0375,+0.1251	+0.0445,+0.1321
30	+0.0025,+0.0030	+0.0158,+0.0528	+0.0188,+0.0558
50	+0.0005,+0.0006	+0.0033,+0.0109	+0.0039,+0.0115
70	-0.0003,-0.0003	-0.0021,-0.0067	-0.0024,-0.0070
90	-0.0007,-0.0008	-0.0044,-0.0147	-0.0052,-0.0155
110	-0.0009,-0.0010	-0.0055,-0.0183	-0.0065,-0.0193
130	-0.0010,-0.0011	-0.0061,-0.0203	-0.0072,-0.0214
150	-0.0010,-0.0012	-0.0065,-0.0215	-0.0077,-0.0227
170	-0.0010,-0.0012	-0.0066,-0.0222	-0.0078,-0.0234
180	-0.0011,-0.0013	-0.0067,-0.0223	-0.0080,-0.0236

Table 6.7: Z-Boson Self-energy contribution to one loop radiative corrections in pb for center of mass energy of 200 GeV.

$\theta(deg)$	$(\frac{d\sigma}{d\cos\theta})_1^{QFD}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$	$(\frac{d\sigma}{d\cos\theta})_1^{SQFD}$
10	+1.8726,+1.9374	-0.1438,+0.0706	+1.7936,+2.0080
30	+0.7903,+0.8177	-0.0607,+0.0298	+0.7570,+0.8475
50	+0.1636,+0.1693	-0.0126,+0.0061	+0.1567,+0.1754
70	-0.1043,-0.1079	+0.0080,-0.0039	-0.0999,-0.1118
90	-0.2200,-0.2276	+0.0169,-0.0083	-0.2107,-0.2359
110	-0.2745,-0.2840	+0.0211,-0.0103	-0.2629,-0.2943
130	-0.3043,-0.3148	+0.0234,-0.0115	-0.2914,-0.3263
150	-0.3224,-0.3336	+0.0248,-0.0121	-0.3088,-0.3457
170	-0.3317,-0.3431	+0.0255,-0.0125	-0.3176,-0.3556
180	-0.3328,-0.3444	+0.0256,-0.0125	-0.3188,-0.3569

Table 6.8: W-Boson WFNR contribution to one loop radiative corrections in pb for center of mass energy of 200 GeV.

$\theta(deg)$	$(\frac{d\sigma}{d\cos\theta})_1^{QFD}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$	$(\frac{d\sigma}{d\cos\theta})_1^{SQFD}$
10	+3.2802,+3.0756	-10.1466,-8.8322	-7.0735,-5.7566
30	+2.2542,+2.1135	-6.9744,-6.0694	-4.8609,-3.9559
50	+1.2513,1.1733	-3.8716,-3.3693	-2.6983,-2.1960
70	+0.7069,+0.6628,	-2.1872,-1.9034	-1.5244,-1.2406
90	+0.4333,+0.4063	-1.3408,-1.1668	-0.9345,-0.7605
110	+0.2896,+0.2715	-0.8959,-0.7797	-0.6244,-0.5082
130	+0.2082,+0.1952	-0.6441,-0.5606	-0.4489,-0.3654
150	+0.1611,+0.1510	-0.4983,-0.4336	-0.3473,-0.2826
170	+0.1386,+0.1299	-0.4288,-0.3731	-0.2989,-0.2432
180	+0.1358,+0.1273	-0.4202,-0.3657	-0.2929,-0.2384

Chapter 7

One Loop Triangle Contributions

7.1 Introduction

The triangle diagrams give us the vertex corrections. From the physics point of view they are the most interesting among the one loop corrections since one can extract from them the magnetic moment of the electron, the magnetic and the quadrupole moment of the W-bosons. We shall discuss those physical applications in Chapter 9. The purpose of this Chapter is the straightforward but tedious computation of some 300 triangle diagrams, Table 6.1, relevant to the total contribution of vertex correction to the cross section. As is apparent from Table 6.1 there are some 116 diagrams in QFD and 277 in SQFD (i.e. 161 due to the addition of the s-particles [S-P]) to be considered, for 3 generations of quark and leptons and their s partners¹. We note that for the case of $\gamma W^+ W^-$ and $Z W^+ W^-$ vertices there are two topologies T_1 and T_2 of vertex corrections present, see Fig. 6.1. We note from Fig. 6.1 that one can obtain T_2 from T_1 by shrinking one of the internal lines and then seeing if the new vertex is allowed by the Lagrangian. Since both $\gamma\gamma WW$ and $WWWW$ vertices are allowed by QFD one gets in this case (i.e. the

¹Our results for 2 generations of quarks and leptons for the $\gamma W^+ W^-$ and the $Z W^+ W^-$ vertices do not agree with those given by Phillippe [29] his Table III.

case of the three vector vertex corrections) 3 diagrams of topology T_2 from every topology T_1 .

Once all the triangle diagrams are evaluated we can write the expressions for the contribution to the differential cross section of $d\sigma^{\text{vertex}}$ and $d\sigma^{\text{ren}}$ in Eq. 5.1. We may subdivide $d\sigma^{\text{vertex}}$, given in Eq. 5.1, in accord with Fig. 6.2 as

$$d\sigma^{\text{vertex}} = d\sigma^{e^+e^-\gamma} + d\sigma^{e^+e^-Z} + d\sigma^{e^+\bar{\nu}_e W^+} + d\sigma^{e^-\nu_e W^-} + d\sigma^{\gamma W^+W^-} + d\sigma^{ZW^+W^-} \quad (7.1)$$

$d\sigma^{\text{ren}}$ is subdivided in the same way, see next section. This Chapter is arranged as follows. The next section contains the contribution of the finite parts of the counterterms (fct) to the one loop corrected differential cross section (i.e. $d\sigma^{\text{ren}}$). The next five sections contain the analytic and numerical results of all the vertex corrections. Finally the Appendices (G, I, and J) contains the necessary collection of the formulas (for the virtual corrections), that are not included in the main body of the Chapter.

7.2 Counterterm Lagrangian

In section 2.4 and Chapter 4 we gave details of our renormalization scheme. Now we apply these considerations to the finite parts of the counterterms that arise out of the vertex corrections and self energy corrections to the process $e^+e^- \rightarrow W^+W^-$. We summarize some of our previous findings (Chapter 4).

1. The vertex corrections to the tree level diagram can be classified into three types: $e^+e^-\gamma$, e^+e^-Z ; γW^+W^- , ZW^+W^- ; and $e^+\bar{\nu}_e W^+$, $e^-\nu_e W^-$.
2. The counterterm Lagrangian can be generated from the original QFD Lagrangian by the rescalings², $A_\mu^a \rightarrow Z_W^{\frac{1}{2}} A_\mu^a$, $B_\mu \rightarrow Z_B^{\frac{1}{2}} B_\mu$, $L \rightarrow Z_L^{\frac{1}{2}} L$,

²We need only consider the QFD part of the full SQFD Lagrangian since the external particles involved in the reaction $e^+e^- \rightarrow W^+W^-$ are all standard particles.

$R \longrightarrow Z_R^{\frac{1}{2}} R$, $g \longrightarrow Z_W^{-\frac{1}{2}}(g + \delta g)$ and $g' \longrightarrow Z_B^{-\frac{1}{2}}(g' + \delta g')$ The parts of the counterterm Lagrangian relevant to these vertices is given in Chapter 4 (see Eq. 4.42).

3. It was shown in Chapter 4 that divergences in all the diagrams contributing to the six relevant vertices are cancelled by the allowed counterterms given in the Lagrangian of Eq. 4.42, as it should be in a renormalizable theory.

Since a lot of diagrams are involved in these cancellations, this provides an excellent check on the calculation. The contribution of the finite parts of the counterterms to the one loop corrected differential cross section can now be written as

$$d\sigma^{\text{ren}} = d\sigma_{1fct}^{e^+e^-\gamma} + d\sigma_{1fct}^{e^+e^-Z} + d\sigma_{1fct}^{e^+\bar{\nu}_e W^+} + d\sigma_{1fct}^{e^-\nu_e W^-} + d\sigma_{1fct}^{\gamma W^+ W^-} + d\sigma_{1fct}^{Z W^+ W^-} \quad (7.2)$$

The various contributions appearing in Eq. 7.2 are given by

$$\left(\frac{d\sigma}{d\cos\theta}\right)_{1fct}^{e^+e^-\gamma} = \Phi \frac{s_W^2}{s} ([a_s(A_{1\gamma} + B_{1\gamma}) - b_s(A_{1\gamma} - B_{1\gamma})][\mathcal{A}_{ss}] - [a_t(A_{1\gamma} + B_{1\gamma}) - b_t(A_{1\gamma} - B_{1\gamma})][\mathcal{A}_{st}]) \quad (7.3)$$

The subscript 1 has been used in Eqs. 7.2, 7.3 and elsewhere to remind ourselves that we are working to the first order in the coupling constant.

$$\left(\frac{d\sigma}{d\cos\theta}\right)_{1fct}^{e^+e^-Z} = \frac{\Phi}{4(s-M_Z^2)} ([a_s(A_{1Z} + B_{1Z}) - b_s(A_{1Z} - B_{1Z})][\mathcal{A}_{ss}] - [a_t(A_{1Z} + B_{1Z}) - b_t(A_{1Z} - B_{1Z})][\mathcal{A}_{st}]), \quad (7.4)$$

here

$$A_{1\gamma} = A_{\gamma}^{\text{infinite}} + \delta Z_L + \frac{\delta g}{2g} + \frac{\delta g'}{2g} \cot \theta_W, \quad (7.5)$$

$$B_{1\gamma} = B_{\gamma}^{\text{infinite}} + \delta Z_R + \frac{\delta g'}{e} \cos \theta_W, \quad (7.6)$$

$$A_{1Z} = A_Z^{\text{infinite}} + 2(1 - 2s_W^2)\delta Z_L + 2\cos\theta_W^2 \frac{\delta g}{g} - 2\frac{\delta g'}{g} \cos\theta_W \sin\theta_W, \quad (7.7)$$

$$B_{1Z} = B_Z^{\text{infinite}} - 4\sin\theta_W^2 \delta Z_R - \frac{\delta g'}{g} \cos\theta_W \sin\theta_W. \quad (7.8)$$

$$\left(\frac{d\sigma}{d\cos\theta}\right)_{1fct}^{e^+\bar{\nu}_e W^+} = 2\Phi(\delta Z_L + \frac{\delta g}{g})[(a_t^2 + b_t^2)\mathcal{A}_{tt} - (a_s a_t + b_s b_t)\mathcal{A}_{st}] \quad (7.9)$$

$$\left(\frac{d\sigma}{d\cos\theta}\right)_{1fct}^{\gamma W^+ W^-} = 2\Phi(\delta Z_W + \frac{\delta g}{g})[(a_s a_\gamma)\mathcal{A}_{ss} - (a_t a_\gamma)\mathcal{A}_{st}] \quad (7.10)$$

$$\left(\frac{d\sigma}{d\cos\theta}\right)_{1fct}^{ZW^+ W^-} = 2\Phi(\delta Z_W + \frac{\delta g}{g})[(a_s a_s + b_s b_s)\mathcal{A}_{ss} - (a_t a_s + b_t b_s)\mathcal{A}_{st}] \quad (7.11)$$

Here

$$a_\gamma = \frac{s_W^2}{s}, \quad (7.12)$$

$$b_\gamma = 0, \quad (7.13)$$

$$a_s = \frac{(1 - 4s_W^2)}{(4(s - M_Z^2))}, \quad (7.14)$$

$$b_s = \frac{-1}{(4(s - M_Z^2))}. \quad (7.15)$$

We recall that

$$\Phi = \left(\frac{4g^4(.6238959)^2\beta_W\beta_e}{(32\pi s)}\right) \quad (7.16)$$

and a_s , b_s , a_t , and b_t are defined in section 5.1. We note that the contribution of the $e^-\nu_e W^-$ vertex is the same as that of the $e^+\bar{\nu}_e W^+$ vertex. The numerical values of these contributions are listed in Tables 6.2 through 6.10. Listing the counterterm contributions separately has the advantage that one can compare the various renormalization schemes. Moreover we have given the complete analytical expressions so that the renormalization scheme dependence of the corrections may be studied.

7.3 $e^+e^-\gamma$ Vertex

The most general fermion-fermion-vector can be reduced to a combination of six form factors [45]

$$T^\mu = (2\pi)^4 ig[F_V\gamma^\mu + G_A\gamma^\mu\gamma^5 + F_M\sigma^{\mu\nu}p_{5\nu} + F_S p_5^\mu + G_P\gamma_5 p_5^\mu + G_E\gamma^5(p_1 - p_2)^\mu] \quad (7.17)$$

$$p_5 = p_1 + p_2$$

In the limit of small lepton mass only F_V and G_A contribute whereas F_M gives the magnetic moment of the lepton. Using the superfield technique it was shown in [65] that in the SUSY -limit F_M vanishes. We [[44] (Ph.d Thesis Notes)] have shown this result to be true using component fields, thus providing a check on our calculations. In this section and from now we work in the limit of small lepton mass since we are interested in contributions of the various vertices to the total cross section. In this context, terms of the order of the electron mass are negligible. As noted before in section 2.4 and Chapter 4 these terms cannot be neglected when showing the consistency of the renormalization procedure. In QFD we must consider 3 diagrams Fig. 7.3 for the $e^+e^-\gamma$ vertex. We recall that in QED only diagram 1 is present. One promptly writes the expression for (diag.1 Fig. 7.3) as

$$\Gamma^\mu = -(i)^3(-ie)^3 \int \frac{d^n q}{(2\pi)^n} \frac{\bar{v}(p_1)\gamma^\alpha(\not{q})\gamma^\mu(\not{q} - \not{p}_5)\gamma_\alpha u(p_2)}{(q^2 - m_e^2)((q - p_1)^2 - \lambda^2)((q - p_5)^2 - m_e^2)} \quad (7.18)$$

Using the definitions of the C integrals [Appendix H] the above reduces to

$$\Gamma^\mu \doteq ies_W^2 \bar{v}(p_1)[(2-4C_{24})+4p_1 \cdot p_2(C_{11}+C_{23})\gamma^\mu(\frac{1}{2}(1-\gamma_5)+\frac{1}{2}(1+\gamma_5))u(p_2) \quad (7.19)$$

Here the functional dependence of C on the momenta and the masses is given by

$$C \equiv C(-p_1, -p_2, m_e, \lambda, m_e)$$

Calling the coefficient of $(1 \mp \gamma_5)/2$ as A_γ and B_γ respectively, and working out expressions similar to Eq. 7.19 for diagrams 2 and 3 in Fig. 7.3 one has [after factoring a common factor of -ie]

$$A_\gamma \doteq -s_W^2 \cdot [(2-4C_{24}(-p_1, -p_2, m_e, \lambda, m_e))+4p_1 \cdot p_2(C_{11}+C_{23})] - ((1 - 2s_W^2)^2/4c_W^2)[(2 - 4C_{24}(-p_1, -p_2, m_e, M_Z, m_e)) + 4p_1 \cdot p_2(C_{11}+C_{23})] - \frac{1}{2}[(2-12C_{24}(-p_1, -p_2, M_W, 0, M_W))+4p_1 \cdot p_2(C_{11} + C_{23} + C_0)] \quad (7.20)$$

$$B_\gamma \doteq -s_W^2 \cdot [(2-4C_{24}(-p_1, -p_2, m_e, \lambda, m_e))+4p_1 \cdot p_2(C_{11}+C_{23})] - (s_W^4/c_W^2)[(2-4C_{24}(-p_1, -p_2, m_e, M_Z, m_e))+4p_1 \cdot p_2(C_{11}+C_{23})] \quad (7.21)$$

In the above we have followed the notation that in each square bracket each C function has the same arguments as the first C functions. To obtain the total contribution of radiatively corrected $e^+e^-\gamma$ vertex to one loop corrected differential cross section, one must calculate the interference of the dressed diagram, Fig. 7.2, with the tree level S and T channels diagrams. On doing this one obtains

$$\left(\frac{d\sigma}{d\cos\theta}\right)_1^{e^+e^-\gamma} = \Phi \frac{s_W^2}{s} ([a_s(A_\gamma + B_\gamma) - b_s(A_\gamma - B_\gamma)][\mathcal{A}_{ss}] - [a_t(A_\gamma + B_\gamma) - b_t(A_\gamma - B_\gamma)][\mathcal{A}_{st}]) \quad (7.22)$$

where $\mathcal{A}_{ss}$ and $\mathcal{A}_{st}$ have already been given in Chapter 5 (see Eqs. 5.23 through 5.25). Now we consider the SUSY contribution to $e^+e^-\gamma$ vertex to the same order as the QFD diagrams. To this end consider the diagrams of due to the s-particles (Fig. 7.3). The contribution of the left handed s-electron [diag.1' of Fig. 7.3] is immediately written as

$$\Gamma_L^\mu = \frac{1}{2}(i)^3(-ie)^3 \int \frac{d^n q}{(2\pi)^n} \frac{\bar{v}(p_1)(1 + \gamma_5)(\not{q} - \not{p}_1 + m_{\tilde{\gamma}})(p_5 - 2q)^\mu(1 - \gamma_5)u(p_2)}{(q^2 - m_{\tilde{e}}^2)((q - p_1)^2 - m_{\tilde{\gamma}}^2)((q - p_5)^2 - m_{\tilde{e}}^2)} \quad (7.23)$$

Using the same convention as used in QFD it follows from Eq. 7.25

$$A_\gamma^S \doteq -s_W^2(-4C_{24}(-p_1, -p_2, m_{\tilde{e}}, m_{\tilde{\gamma}}, m_{\tilde{e}})) \quad (7.24)$$

Similarly working out the other diagrams in Fig. 7.3, the total SUSY contribution to $e^+e^-\gamma$ vertex is written in terms of A_γ^S and B_γ^S

$$A_\gamma^S \doteq -s_W^2(-4C_{24}(-p_1, -p_2, m_{\tilde{e}}, m_{\tilde{\gamma}}, m_{\tilde{e}})) - \frac{1}{2}(1 - ((1 - 2s_W^2)^2/4c_W^2)(-4C_{24}(-p_1, -p_2, m_{\tilde{e}}, m_{\tilde{\gamma}}, m_{\tilde{e}})) - 4C_{24}(-p_1, -p_2, m_{\tilde{\omega}}, m_{\tilde{\nu}_e}, m_{\tilde{\omega}}) + 4p_1 \cdot p_2(C_{12} + C_{23}) - 2m_{\tilde{\omega}}^2 C_0) \quad (7.25)$$

and

$$B_\gamma^S \doteq -s_W^2(-4C_{24}(-p_1, -p_2, m_{\tilde{e}}, m_{\tilde{\gamma}}, m_{\tilde{e}})) - (s_W^4/c_W^2)(-4C_{24}(-p_1, -p_2, m_{\tilde{e}}, m_{\tilde{\gamma}}, m_{\tilde{e}})) \quad (7.26)$$

Having obtained A_γ and B_γ we can immediately write out the SUSY contribution to the cross section

$$\left(\frac{d\sigma}{d\cos\theta}\right)_{1S}^{e^+e^-\gamma} = \Phi \frac{s_W^2}{s} ([a_s(A_\gamma^S + B_\gamma^S) - b_s(A_\gamma^S - B_\gamma^S)][\mathcal{A}_{ss}] - [a_t(A_\gamma^S + B_\gamma^S) - b_t(A_\gamma^S - B_\gamma^S)][\mathcal{A}_{st}]) \quad (7.27)$$

The numerical values for the corrections to the $e^+e^-\gamma$ vertex are given in Table 2. From column one ³ (i.e. the QED case) of our Table 2 we see that the corrections decrease in absolute magnitude as the angle is increased, they also change sign from positive to negative as the angle increases. The radiative corrections due to the SUSY particles first decrease with increasing CM angle in absolute magnitude and then gradually increase in absolute magnitude as the CM angle is increased. From column two of Table 2 we also note a sign change from positive to negative as the CM angle is increased.

7.4 e^+e^-Z Vertex

The evaluations for this vertex follow very closely the $e^+e^-\gamma$ vertex in QED and also when the s-particles are included, so we merely state the end results here following the same notations as before. First consider the case of QED, here we have four diagrams , Fig. 7.4, and we have,

$$\begin{aligned}
 A_Z \doteq & -2s_W^2(1 - \frac{2s_W^2}{c_W^2})[(2 - 4C_{24}(-p_1, -p_2, m_e, \lambda, m_e)) + 4p_1 \cdot p_2(C_{11} + C_{23})] - \\
 & ((1 - \frac{2s_W^2}{c_W^2})^3 / 2c_W^2)[(2 - 4C_{24}(-p_1, -p_2, m_e, M_Z, m_e)) + \\
 & 4p_1 \cdot p_2(C_{11} + C_{23})] - \frac{2c_W^2}{c_W^2}[(2 - 12C_{24}(-p_1, -p_2, M_W, m_{\nu_e}, M_W)) + 4p_1 \cdot p_2(C_{11} + C_{23} + \\
 & C_0)] + [(2 - 4C_{24}(-p_1, -p_2, m_{\nu_e}, M_W, m_{\nu_e})) + 4p_1 \cdot p_2(C_{11} + C_{23})], \tag{7.28}
 \end{aligned}$$

$$\begin{aligned}
 B_Z \doteq & 4s_W^4[(2 - 4C_{24}(-p_1, -p_2, m_e, \lambda, m_e)) + 4p_1 \cdot p_2(C_{11} + C_{23})] + \\
 & 4(s_W^6/c_W^2)[(2 - 4C_{24}(-p_1, -p_2, m_e, M_Z, m_e)) + 4p_1 \cdot p_2(C_{11} + C_{23})]. \tag{7.29}
 \end{aligned}$$

³We note that in this and subsequent sections we are referring to the first entry in each column until otherwise stated, since the second entry in each column, (as indicated in the tables), refers to the contributions due to the counterterms.

Having obtained A^Z and B^Z we can immediately write the total contribution of QFD correction to the total differential cross section.

$$\left(\frac{d\sigma}{d\cos\theta}\right)_1^{e^+e^-Z} \doteq \frac{\Phi}{4(s-M_Z^2)} ([a_s(A_Z + B_Z) - b_s(A_Z - B_Z)][\mathcal{A}_{ss}] - [a_t(A_Z + B_Z) - b_t(A_Z - B_Z)][\mathcal{A}_{st}]) \quad (7.30)$$

Finally we write out the radiative correction due to SUSY-Particles to the e^+e^-Z vertex. There are four diagrams due to the s-particles corresponding to the four diagrams in QFD. The total contribution in terms of A_Z^S and B_Z^S for SUSY diagrams in Fig. 7.4 work out to be

$$\begin{aligned} A_Z^S \doteq & -2s_W^2(1 - 2s_W^2)(-4C_{24}(-p_1, -p_2, m_{\tilde{e}}, m_{\tilde{\gamma}}, m_{\tilde{e}})) - ((1 - \\ & 2s_W^2)^3/2c_W^2)(-4C_{24}(-p_1, -p_2, m_{\tilde{e}}, m_{\tilde{z}}, m_{\tilde{e}})) - 2c_W^2(1 - \\ & 4C_{24}(-p_1, -p_2, m_{\tilde{\omega}}, m_{\tilde{\nu}_e}, m_{\tilde{\omega}}) + 4p_1 \cdot p_2(C_{12} + C_{23}) - \\ & 2m_{\tilde{\omega}}^2 C_0) + (-4C_{24}(-p_1, -p_2, m_{\tilde{\nu}_e}, m_{\tilde{\omega}}, m_{\tilde{\nu}_e})), \end{aligned} \quad (7.31)$$

and

$$\begin{aligned} B_Z^S \doteq & 4s_W^4(-4C_{24}(-p_1, -p_2, m_{\tilde{e}}, m_{\tilde{\gamma}}, m_{\tilde{e}})) \\ & 4(s_W^6/c_W^2)(-4C_{24}(-p_1, -p_2, m_{\tilde{e}}, m_{\tilde{z}}, m_{\tilde{e}})). \end{aligned} \quad (7.32)$$

The cross section contribution arising out of Eqs. 7.31 and 7.32 is given by

$$\left(\frac{d\sigma}{d\cos\theta}\right)_{1S}^{e^+e^-Z} \doteq \frac{\Phi}{4(s-M_Z^2)} ([a_s(A_Z^S + B_Z^S) - b_s(A_Z^S - B_Z^S)][\mathcal{A}_{ss}] - [a_t(A_Z^S + B_Z^S) - b_t(A_Z^S - B_Z^S)][\mathcal{A}_{st}]) \quad (7.33)$$

The numerical values for the one loop corrections to the e^+e^-Z vertex are listed in Table 3. The one loop radiative corrections of this vertex in QFD decrease in magnitude as the CM angle increases, these corrections also change sign as the CM angle is increased, changing from positive to negative values. The one loop corrections due to SUSY particles first decrease and then increase slowly in absolute magnitude with increasing CM angle. A sign change from negative to positive values is also noted.

7.5 $e^+ \bar{\nu}_e W^+$ Vertex

Now we move on to the T-channel vertex correction Fig. 7.2. The amplitude in this case may be written as

$$\mathcal{M}_i^{e^+ \bar{\nu}_e W^+} = ig^2 \bar{v}(p_1) [V_{11} \gamma_\nu + V_{12} \not{q}_1 q_{1\nu} + V_{13} \not{q}_1 p_{1\nu}] (\not{q}_1 - \not{p}_1) \gamma_\lambda (a_i + b_i \gamma_5) u(p_2) e^{1\nu} e^{2\lambda} \quad (7.34)$$

The condition $q_1 \cdot e_1 = 0$ simplifies Eq. 7.34 to

$$\mathcal{M}_i^{e^+ \bar{\nu}_e W^+} = ig^2 \bar{v}(p_1) [V_{11} \gamma_\nu + V_{13} \not{q}_1 p_{1\nu}] (\not{q}_1 - \not{p}_1) \gamma_\lambda (a_i + b_i \gamma_5) u(p_2) e^{1\nu} e^{2\lambda} \quad (7.35)$$

First we must explicitly find V_{11} and V_{13} in terms of the C integrals and finally calculate the cross term of 7.34 with the tree level S and T channels to get the total radiative contribution to the cross section of $e^+ \bar{\nu}_e W^+$ vertex correction. Let us first compute V_{11} and V_{13} to this end consider 4 diagrams Fig. 7.5 in QFD. To illustrate the calculation let us consider diagram 1 of Fig. 7.5 one may write this as

$$A^\nu = -\frac{g^3}{2\sqrt{2}} s_W^2 \int \frac{d^n c}{(2\pi)^n} \frac{\bar{v}(p_1) \gamma_\alpha (\not{q}) \gamma_\beta (1 - \gamma_5) u(p_2) \Gamma^{\alpha\beta\nu}}{(q^2 - m_e^2)((q + p_1)^2 - \lambda^2)((q + p_1 - q_1)^2 - M_W^2)} \quad (7.36)$$

where $\Gamma^{\alpha\beta\nu}$ is the three vector vertex. Taking out a common factor of $\frac{ig(1-\gamma_5)}{2\sqrt{2}}$, V_{11} and V_{13} read for this diagram as

$$\begin{aligned} V_{11} &\doteq s_W^2 [4(n-1)C_{24}(p_1, -q_1, m_e, \lambda, M_W) \\ &\quad - s_3(C_{12} + 2C_{22}) + 2p_1 \cdot q_1 (+C_{12} + 2C_{11} + 2C_{23})], \end{aligned} \quad (7.37)$$

$$V_{13} \doteq -2s_W^2 [-2C_{23}(p_1, -q_1, m_e, \lambda, M_W) - C_{12} + C_{11}]. \quad (7.38)$$

Defining $a = 1 - 4s_W^2$, and working out the contribution of all the remaining QFD diagrams in Fig. 7.5 one has,

$$\begin{aligned}
 V_{11} \doteq & s_W^2 [4(n-1)C_{24}(p_1, -q_1, m_e, \lambda, M_W) - s_3(C_{12} + 2C_{22}) + \\
 & 2p_1 \cdot q_1 (+C_{12} + 2C_{11} + 2C_{23})] + \\
 & [(a+1)/4][4(n-1)C_{24}(p_1, -q_1, m_e, M_Z, M_W) - s_3(C_{12} + \\
 & 2C_{22}) + 2p_1 \cdot q_1 (+C_{12} + 2C_{11} + 2C_{23})] + [(a+1)/8c_W^2][-(2-n)^2 C_{24}(q_1, p_1 - q_1, m_e, m_{\nu_e}, M_Z) + s_3(+2C_{21} + 2C_{22} - \\
 & 4C_{23} + 2C_{11} - 2C_{12}) + 4p_1 \cdot q_1 (+C_{12} - C_{22} + C_{23})] + \\
 & .5[4(n-1)C_{24}(p_1, -q_1, m_{\nu_e}, M_W, M_Z) - s_3(C_{12} + 2C_{22}) + \\
 & 2p_1 \cdot q_1 (+C_{12} + 2C_{11} + 2C_{23})], \tag{7.39}
 \end{aligned}$$

$$\begin{aligned}
 V_{13} \doteq & -2s_W^2 [-2C_{23}(p_1, -q_1, m_e, \lambda, M_W) - C_{12} + C_{11}] + [(a+1)/4][4C_{23}(p_1, -q_1, m_e, M_Z, M_W) + 2C_{12} - 2C_{11}] + [(a+1)/8c_W^2][-4C_{23}(q_1, p_1 - q_1, m_e, m_{\nu_e}, M_Z) + 4C_{22} - 4C_{12}] + \\
 & .5[4C_{23}(p_1, -q_1, m_{\nu_e}, M_W, M_Z) + 2C_{12} - 2C_{11}]. \tag{7.40}
 \end{aligned}$$

Now we must consider S-P contribution to V_{11} and V_{13} Fig. 7.5. To illustrate the calculation consider diagram 1' of Fig. 7.5

$$\begin{aligned}
 & A_S^\nu \left[\frac{(i)^3 (ie)^2 (ig)}{4\sqrt{2}} \right] \\
 & \int \frac{d^n q}{(2\pi)^n} \frac{\bar{u}(p_1)(1+\gamma_5)(\not{q} - \not{p}_1 + m_{\tilde{\tau}})\gamma^\nu(1+\gamma_5)(\not{q} + \not{p}_1 - \not{q}_1 + m_{\tilde{\omega}})(1-\gamma_5)u(p_1)}{(q^2 - m_{\tilde{\tau}}^2)((q+p_1)^2 - m_{\tilde{\tau}}^2)((q+p_1-q_1)^2 - m_{\tilde{\omega}}^2)} \tag{7.41}
 \end{aligned}$$

Taking out a common factor of $\frac{ig(1-\gamma_5)}{2\sqrt{2}}$ one gets for V_{11} and V_{13} in this case

$$\begin{aligned}
 V_{11}^S \doteq & 2s_W^2 [(n-2)C_{24}(p_1, -q_1, m_{\tilde{\tau}}, m_{\tilde{\tau}}, m_{\tilde{\omega}}) \\
 & + 2(p_1 \cdot q_1)(C_{12} + C_{23}) - s_3(C_{12} + C_{22})], \tag{7.42}
 \end{aligned}$$

$$V_{13}^S \doteq 2s_W^2 [-2C_{12}(p_1, -q_1, m_{\tilde{\tau}}, m_{\tilde{\tau}}, m_{\tilde{\omega}}) - 2C_{23}]. \tag{7.43}$$

After a similar calculation for the other diagrams, the total contribution to V_{11}^S and V_{13}^S may be written as

$$\begin{aligned}
 V_{11}^S \doteq & 2s_W^2 [(n-2)C_{24}(p_1, -q_1, m_{\tilde{\tau}}, m_{\tilde{\tau}}, m_{\tilde{\omega}}) + \\
 & 2(p_1 \cdot q_1)(C_{12} + C_{23}) - s_3(C_{12} + C_{22})] + [(a+1)/2][(n-2)C_{24}(p_1, -q_1, m_{\tilde{\tau}}, m_{\tilde{\tau}}, m_{\tilde{\omega}}) + 2(p_1 \cdot q_1)(C_{12} + C_{23}) - s_3(C_{12} + C_{22})] + \\
 & [(a+1)/2c_W^2][-C_{24}(q_1, p_1 - q_1, m_{\tilde{\tau}}, m_{\nu_e}, m_{\tilde{\tau}}) + \\
 & [(n-2)C_{24}(p_1, -q_1, m_{\nu_e}, m_{\tilde{\omega}}, m_{\tilde{\tau}}) + 2(p_1 \cdot q_1)(C_{12} + C_{23}) - s_3(C_{12} + C_{22})], \tag{7.44}
 \end{aligned}$$

$$\begin{aligned}
V_{13}^S = & 2s_W^2 \{ -2C_{12}(p_1, -q_1, m_{\tilde{e}}, m_{\tilde{\gamma}}, m_{\tilde{\omega}}) - \\
& 2C_{23} \} + [(a+1)/2] \{ -2C_{12}(p_1, -q_1, m_{\tilde{e}}, m_{\tilde{z}}, m_{\tilde{\omega}}) - 2C_{23} \} + \\
& [(a+1)/2c_W^2] \{ +C_{23}(q_1, p_1 - q_1, m_{\tilde{e}}, m_{\tilde{\nu}_e}, m_{\tilde{z}}) - C_{22} \} + \\
& [-2C_{12}(p_1, -q_1, m_{\tilde{\nu}_e}, m_{\tilde{\omega}}, m_{\tilde{z}}) - 2C_{23} \}.
\end{aligned} \tag{7.45}$$

Finally calculating the interference of Eq. 7.35 with the tree level amplitude we can write the contribution of the $e^+ \bar{\nu}_e W^+$ vertex correction to the cross section as

$$\left(\frac{d\sigma}{d\cos\theta} \right)_1^{e^+ \nu_e W^+} = -2\Phi[(a_s a_t + b_s b_t)[V_{11}\mathcal{A}_{11st} + V_{13}\mathcal{A}_{13st}] - (a_t a_t + b_t b_t)[V_{11}\mathcal{A}_{11tt} + V_{13}\mathcal{A}_{13tt}]], \tag{7.46}$$

and

$$\left(\frac{d\sigma}{d\cos\theta} \right)_{1S}^{e^+ \nu_e W^+} = -2\Phi[(a_s a_t + b_s b_t)[V_{11}^S\mathcal{A}_{11st} + V_{13}^S\mathcal{A}_{13st}] - (a_t a_t + b_t b_t)[V_{11}^S\mathcal{A}_{11tt} + V_{13}^S\mathcal{A}_{13tt}]], \tag{7.47}$$

where

$$\mathcal{A}_{11tt} = \mathcal{A}_{tt} \tag{7.48}$$

$$\mathcal{A}_{11st} = \mathcal{A}_{st} \tag{7.49}$$

$$\begin{aligned}
\mathcal{A}_{13st} = & [2(q_1 \cdot q_2)(p_1 \cdot q_1)(p_2 \cdot q_2)/(s_3 s_4)][(p_1 \cdot q_1)((p_2 \cdot q_2) - 2(p_2 \cdot q_1)) + \\
& (p_2 \cdot q_1)(p_1 \cdot q_2) - (p_1 \cdot p_2)(q_1 \cdot q_2)] + \\
& [2(p_2 \cdot q_2)/s_4][(p_1 \cdot q_2)((s_3(p_1 \cdot p_2) + (q_1 \cdot q_2)(p_1 \cdot p_2) - \\
& 2(p_1 \cdot q_1)(p_2 \cdot q_1) - \\
& (p_1 \cdot q_1)(p_2 \cdot q_2) - (p_2 \cdot q_1)(p_1 \cdot q_2)) + (q_1 \cdot q_2)(p_1 \cdot q_1)(p_1 \cdot p_2)] + \\
& [(p_1 \cdot q_1)/s_3][4(p_2 \cdot q_1)^2(p_1 \cdot q_1) + 2(p_1 \cdot q_1)(p_2 \cdot q_1)(p_2 \cdot q_2) + \\
& 2(p_2 \cdot q_1)^2(p_1 \cdot q_2) + (q_1 \cdot q_2)(-2(p_1 \cdot p_2)(p_2 \cdot q_1) - \\
& 2(p_1 \cdot q_1)(p_2 \cdot q_1) - 7(p_2 \cdot q_2)(p_1 \cdot q_1) - (q_1 \cdot q_2)(p_1 \cdot p_2) + \\
& (p_2 \cdot q_1)(p_1 \cdot q_2))] + s_3[+(p_1 \cdot q_2)(p_1 \cdot p_2) - 2(p_1 \cdot p_2)^2] + \\
& [p_1 \cdot q_1][+s_4(p_1 \cdot p_2) + 5(p_1 \cdot q_2)(p_2 \cdot q_2) + 2(p_1 \cdot p_2)(p_2 \cdot q_1) - \\
& 2(p_1 \cdot p_2)(p_2 \cdot q_2) + (p_1 \cdot q_2)(p_2 \cdot q_1) - 2(p_1 \cdot p_2)(q_1 \cdot q_2) + \\
& 3(p_1 \cdot q_1)(p_2 \cdot q_2)] + [(p_1 \cdot p_2)(q_1 \cdot q_2) - (p_1 \cdot q_2)(p_2 \cdot q_1)][(p_1 \cdot q_2) + \\
& 2(p_1 \cdot p_2)]
\end{aligned} \tag{7.50}$$

$$\begin{aligned}
\mathcal{A}_{13tt} = & [-s_4 - 4(p_2 \cdot q_2) + 4(p_2 \cdot q_2)^2/s_4][2(p_1 \cdot q_1)^2(p_2 \cdot q_1)/s_3 - \\
& (p_1 \cdot p_2)(p_1 \cdot q_1)] + 4(p_2 \cdot q_2)[2(p_1 \cdot q_1)^2(q_1 \cdot q_2)/s_3 - \\
& (p_1 \cdot q_2)(p_1 \cdot q_1)]
\end{aligned} \tag{7.51}$$

The numerical values for the one loop corrections to this vertex are displayed in Table 4. For the QFD case the absolute magnitude of these corrections decreases

with increasing CM angle. For this vertex there is no change in sign of the corrections as the CM angle is varied. The one loop corrections due to SUSY particles decrease slowly in absolute magnitude with increasing CM angle. The corrections due to the SUSY particles are opposite in sign to the QFD corrections and hence negative in sign.

7.6 $e^- \nu_e W^-$ Vertex

Since the calculation of this vertex follows almost parallel steps to the $e^+ \nu_e W^+$ vertex, except for some straightforward changes ($p_1 \rightarrow p_2, q_1 \rightarrow q_2, s_3 \rightarrow s_4$) we will merely state the results here. For the sake of completeness diagrams contributing to the $e^- \nu_e W^-$ vertex in SQFD are given in Fig. 7.6. The amplitude in this case may be written as

$$\mathcal{M}_i^{e^- \nu_e W^-} = \bar{v}(p_1) \gamma_\nu (\not{p}_2 - \not{q}_2) [V_{21} \gamma_\lambda + V_{22} \not{q}_2 q_{2\lambda} + V_{23} \not{q}_2 p_{2\lambda}] (a_i + b_i \gamma_5) u(p_2) e^{1\nu} e^{2\lambda} \quad (7.52)$$

On using the condition $q_2 \cdot e_2 = 0$ Eq. 7.52 simplifies to

$$\mathcal{M}_i^{e^- \nu_e W^-} = \bar{v}(p_1) \gamma_\nu (\not{p}_2 - \not{q}_2) [V_{21} \gamma_\lambda + V_{23} \not{q}_2 p_{2\lambda}] (a_i + b_i \gamma_5) u(p_2) e^{1\nu} e^{2\lambda} \quad (7.53)$$

The interference of Eq. 7.53 with the lowest order tree S and T channels yields

$$\left(\frac{d\sigma}{d\cos\theta} \right)_1^{e^- \nu_e W^-} = -2\Phi[(a_s a_t + b_s b_t) [V_{21} \mathcal{A}_{21st} + V_{23} \mathcal{A}_{23st}] - (a_t a_t + b_t b_t) [V_{21} \mathcal{A}_{21tt} + V_{23} \mathcal{A}_{23tt}]] \quad (7.54)$$

and

$$\left(\frac{d\sigma}{d\cos\theta} \right)_{1S}^{e^- \nu_e W^-} = -2\Phi[(a_s a_t + b_s b_t) [V_{21}^S(\mathcal{A}_{21st}) + V_{23}^S(\mathcal{A}_{23st})] - (a_t a_t + b_t b_t) [V_{21}^S(\mathcal{A}_{21tt}) + V_{23}^S(\mathcal{A}_{23tt})]] \quad (7.55)$$

where

$$\begin{aligned}
 V_{21} \doteq & s_W^2 [4(n-1)C_{24}(-p_2, q_2, m_e, \lambda, M_W) - s_4(C_{12} + 2C_{22}) + \\
 & 2q_2 \cdot p_2 (+C_{12} + 2C_{11} + 2C_{23})] + \\
 & [(a+1)/4][4(n-1)C_{24}(-p_2, q_2, m_e, M_Z, M_W) - s_4(C_{12} + \\
 & 2C_{22}) + 2q_2 \cdot p_2 (+C_{12} + 2C_{11} + 2C_{23})] + [a+1]/8c_W^2 [-(2- \\
 & n)^2 C_{24}(-q_2, q_2 - p_2, m_e, 0, M_Z) + s_4(+2C_{21} + 2C_{22} - \\
 & 4C_{23} + 2C_{11} - 2C_{12}) + 4q_2 \cdot p_2 (+C_{12} - C_{22} + C_{23})] + \\
 & .5[4(n-1)C_{24}(-p_2, q_2, 0, M_W, M_Z) - s_4(C_{12} + 2C_{22}) + \\
 & 2q_2 \cdot p_2 (+C_{12} + 2C_{11} + 2C_{23})]
 \end{aligned} \tag{7.56}$$

$$\begin{aligned}
 V_{23} \doteq & -2s_W^2 [-2C_{23}(-p_2, q_2, m_e, \lambda, M_W) - C_{12} + C_{11}] + [(a+ \\
 & 1)/4][4C_{23}(-p_2, q_2, m_e, M_Z, M_W) + 2C_{12} - 2C_{11}] + [(a+ \\
 & 1)/8c_W^2] [-4C_{23}(-q_2, q_2 - p_2, m_e, 0, M_Z) + 4C_{22} - 4C_{12}] + \\
 & .5[4C_{23}(-p_2, q_2, 0, M_W, M_Z) + 2C_{12} - 2C_{11}]
 \end{aligned} \tag{7.57}$$

and

$$\begin{aligned}
 V_{21}^S \doteq & 2s_W^2 [(n-2)C_{24}(-p_2, q_2, m_{\tilde{e}}, m_{\tilde{\gamma}}, m_{\tilde{\omega}}) + \\
 & 2(p_2 \cdot q_2)(C_{12} + C_{23}) - s_4(C_{12} + C_{22})] + [(a+1)/2][(n- \\
 & 2)C_{24}(-m, q_2, m_{\tilde{e}}, m_{\tilde{z}}, m_{\tilde{\omega}}) + 2(p_2 \cdot q_2)(C_{12} + C_{23}) - s_4(C_{12} + \\
 & C_{22})] + [(a+1)/2c_W^2] [-C_{24}(-q_2, q_2 - p_2, m_{\tilde{e}}, m_{\tilde{\nu}_e}, m_{\tilde{z}})] + \\
 & [(n-2)C_{24}(-p_2, q_2, m_{\tilde{\nu}_e}, m_{\tilde{\omega}}, m_{\tilde{z}}) + 2(p_2 \cdot q_2)(C_{12} + C_{23}) - \\
 & s_4(C_{12} + C_{22})]
 \end{aligned} \tag{7.58}$$

$$\begin{aligned}
 V_{23}^S \doteq & 2s_W^2 [-2C_{12}(-p_2, q_2, m_{\tilde{e}}, m_{\tilde{\gamma}}, m_{\tilde{\omega}}) - \\
 & 2C_{23}] + [(a+1)/2] [-2C_{12}(-p_2, q_2, m_{\tilde{e}}, m_{\tilde{z}}, m_{\tilde{\omega}}) - 2C_{23}] + \\
 & [(a+1)/2c_W^2] [+C_{23}(-q_2, q_2 - p_2, m_{\tilde{e}}, m_{\tilde{\nu}_e}, m_{\tilde{z}}) - C_{22}] + \\
 & [-2C_{12}(-p_2, q_2, m_{\tilde{\nu}_e}, m_{\tilde{\omega}}, m_{\tilde{z}}) - 2C_{23}]
 \end{aligned} \tag{7.59}$$

In the approximation that the W's are on mass-shell the product of momenta factors are

$$\mathcal{A}_{21st} = \mathcal{A}_{11st} \tag{7.60}$$

$$\mathcal{A}_{21tt} = \mathcal{A}_{11tt} \tag{7.61}$$

$$\mathcal{A}_{23st} = \mathcal{A}_{13st} \tag{7.62}$$

$$\mathcal{A}_{23tt} = \mathcal{A}_{13tt} \tag{7.63}$$

For W's not on shell the expressions for $\mathcal{A}_{21st}$ through $\mathcal{A}_{23tt}$ are given in the Appendix G. The numerical values for the correction to this vertex are same as those

for the $e^+ \bar{\nu}_e W^+$ (therefore we do not list them) when W 's are on shell, which is the limit we have used in the numerical evaluations.

7.7 $\gamma W^+ W^-$ Vertex

The general form of the dressed $\gamma W^+ W^-$ vertex may be written in the form [27], Fig. 7.2,

$$\mathcal{M}^{\gamma W^+ W^-} = (ig^2) \bar{v}(p_1) \gamma^\mu (a_\gamma + b_\gamma \gamma_5) u(p_2) [V_2^\gamma q_{2\nu} g_{\mu\lambda} + V_3^\gamma q_{1\lambda} g_{\mu\nu} + V_5^\gamma q_{1\mu} g_{\nu\lambda} + V_6^\gamma q_{2\mu} g_{\nu\lambda} + V_8^\gamma q_{1\mu} q_{2\nu} q_{1\lambda} + V_{12}^\gamma q_{1\lambda} q_{2\mu} q_{2\nu}] e^{1\nu} e^{2\lambda} \quad (7.64)$$

When the interference (see section 6.1) of Eq. 7.64 with the lowest order tree S and T channels is performed one obtains the contribution of $\gamma W^+ W^-$ vertex to the total cross section to the order α [fine structure constant] and this is given by

$$\left(\frac{d\sigma}{d\cos\theta} \right)_1^{\gamma W^+ W^-} = 2\Phi[(a_s a_\gamma + b_s b_\gamma) [V_2^\gamma \mathcal{W}\mathcal{W}_{1ss} + V_3^\gamma \mathcal{W}\mathcal{W}_{2ss} + V_5^\gamma \mathcal{W}\mathcal{W}_{3ss} + V_6^\gamma \mathcal{W}\mathcal{W}_{4ss} + V_8^\gamma \mathcal{W}\mathcal{W}_{5ss} + V_{12}^\gamma \mathcal{W}\mathcal{W}_{6ss}] - (a_t a_\gamma + b_t b_\gamma) [V_2^\gamma \mathcal{W}\mathcal{W}_{1st} + V_3^\gamma \mathcal{W}\mathcal{W}_{2st} + V_5^\gamma \mathcal{W}\mathcal{W}_{3st} + V_6^\gamma \mathcal{W}\mathcal{W}_{4st} + V_8^\gamma \mathcal{W}\mathcal{W}_{5st} + V_{12}^\gamma \mathcal{W}\mathcal{W}_{6st}]] \quad (7.65)$$

In the above $\mathcal{W}\mathcal{W}_{1ss} \cdots \mathcal{W}\mathcal{W}_{6st}$ are the various products of the dot products of momenta. These are listed in the Appendix G. Now we proceed to give expressions for V_i^γ . These can be written in terms of B and C integrals [Appendices C, and H]. We introduce the following compact notation for the C's $1 \equiv k, 2 \equiv p, W \equiv M_W, Z \equiv M_Z, H \equiv m_{H_1^0}$

$$C_{ij}(1, 2, W, \lambda, W) \equiv C_{ij1} \quad (7.66)$$

$$C_{ij}(1, 2, W, Z, W) \equiv C_{ij1Z} \quad (7.67)$$

$$C_{ij}(1, 2, W, H_1^0, W) \equiv C_{ij1H} \quad (7.68)$$

$$C_{ij}(2, 1, W, \lambda, W) \equiv C_{ij2} \quad (7.69)$$

$$C_{ij}(2, 1, W, Z, W) \equiv C_{ij2Z} \quad (7.70)$$

$$C_{ij}(2, 1, W, H_1^0, W) \equiv C_{ij2H} \quad (7.71)$$

The $\gamma W^+ W^-$ triangle diagrams can be split into 4 categories, Category I (Fig. 7.7) where a Higgs particle is exchanged, category II (Fig. 7.8) where a photon or its related ghost particle is exchanged, category III (Fig. 7.9) where a Z-boson, its related Higgs-Kibble ghost scalar and its ghost particle is exchanged and finally category IV (Fig. 7.10) involves fermions. The corresponding diagrams with the S-P are also enumerated in Fig. 7.7 through Fig. 7.10. It should be noted that within each category the masses of all internal particles are the same. Now we shall proceed to give results for V_{2j}^γ through V_{12j}^γ ; where j denotes the exchanged particle. A factor of $-ig$ has been taken out to comply with the notation of Eq. 7.64. We have for category I

$$V_{2H}^\gamma \doteq 2(C_{352H} + C_{242H} + M_W^2 C_{02H}), \quad (7.72)$$

$$V_{3H}^\gamma \doteq 2(C_{362H} - M_W^2 C_{02H}), \quad (7.73)$$

$$V_{5H}^\gamma \doteq (2C_{382H} + C_{242H} + M_W^2(2C_{122H} + C_{02H})), \quad (7.74)$$

$$V_{6H}^\gamma \doteq (2C_{352H} + C_{242H} + M_W^2(2C_{112H} + C_{02H})), \quad (7.75)$$

$$V_{8H}^\gamma \doteq -(2C_{342H} + 2C_{222H} + C_{223H} + C_{122H}), \quad (7.76)$$

$$V_{12H}^\gamma \doteq -(2C_{332H} + 2C_{232H} + C_{232H} + C_{122H}). \quad (7.77)$$

We have written out the contributions of each category separately to facilitate a discussion of the Magnetic and the Quadrupole moments of the W's[5, Chapter 9].

Similarly for the ' γ ' -exchange terms one obtains

$$\begin{aligned} V_{2\gamma}^\gamma \doteq & -s_W^2 [(1-n)(10C_{352} + 14C_{242}) - 4C_{242} + s_3(2C_{342} + 2C_{232} + \\ & 6C_{222} + 2C_{122} - C_{112} + C_{02}) + s_4(2C_{312} + 10C_{212} + 16C_{112} + \\ & 8C_{02}) + (q_1 \cdot q_2)(4C_{332} + 16C_{232} + 2C_{212} + 4C_{122} + 14C_{112} + \\ & 8C_{02})] - s_W^2 M_W^2 [C_{112} + 3C_{02}] - s_W^2 [C_{352} + C_{242} - C_{361}] - \\ & s_W^2 [5B_{02} + B_{12}], \end{aligned} \quad (7.78)$$

$$V_{3\gamma}^\gamma \doteq -s_W^2 [(1-n)(10C_{362}) + 4nC_{242} + s_3(2C_{322} + 2C_{122} - 4C_{222}) + s_4(2C_{332} + 2C_{232} - 4C_{212} - C_{122} - 8C_{112} - 6C_{02}) + (q_1 \cdot q_2)(4C_{342} - 8C_{232} + 2C_{222} + 2C_{122} - 8C_{112} - 4C_{02})] - s_W^2 M_W^2 [C_{122} - 2C_{02}] + s_W^2 [C_{351} + C_{241} - C_{362}] + s_W^2 [5B_{01} + B_{11}], \quad (7.79)$$

$$V_{5\gamma}^\gamma \doteq -s_W^2 [(1-n)(10C_{362}) + (5-7n)C_{242} + s_3(2C_{322} + 2C_{122} + 5C_{222}) + s_4(2C_{332} + 2C_{232} + 3C_{212} - C_{122} + 7C_{112} + 5C_{02}) + (q_1 \cdot q_2)(4C_{342} + 8C_{232} + 2C_{222} + 3C_{112} - 5C_{122})] + s_W^2 M_W^2 [4C_{122}] + s_W^2 [C_{351} + C_{241} - C_{362} - C_{242}] - s_W^2 [4B_{01} - B_{11}], \quad (7.80)$$

$$V_{6\gamma}^\gamma \doteq -s_W^2 [(1-n)(10C_{352}) + (5-3n)C_{242} + s_3(2C_{342} + 2C_{232} - C_{222} - C_{112} + 3C_{122} - 2C_{02}) + s_4(2C_{312} + C_{212} - 2C_{112} - C_{02}) + (q_1 \cdot q_2)(4C_{332} + 2C_{212} - 9C_{112} - C_{122} - 8C_{02})] + s_W^2 M_W^2 [4C_{112} + 4C_{02}] - s_W^2 [C_{352} + C_{242} - C_{361} - C_{241}] + s_W^2 [4B_{02} - B_{12}], \quad (7.81)$$

$$V_{8\gamma}^\gamma \doteq -s_W^2 [18C_{342} + 9C_{232} + 18C_{222} + 12C_{122} + 3C_{112} + 3C_{02}] - s_W^2 [-C_{342} - C_{122} - C_{232} - C_{222}] - s_W^2 [C_{331} + C_{121} + C_{231} + C_{231}], \quad (7.82)$$

$$V_{12\gamma}^\gamma \doteq -s_W^2 [18C_{332} + 9C_{232} + 18C_{232} + 12C_{122} + 3C_{112} + 3C_{02}] - s_W^2 [-C_{332} - C_{122} - C_{232} - C_{232}] - s_W^2 [C_{341} + C_{121} + C_{231} + C_{221}], \quad (7.83)$$

The 'Z' exchange diagrams (Fig. 7.9) are similar to the γ exchange diagrams except for change in couplings and the photon mass is replaced by the Z mass [i.e. compare diagrams given in Fig. 7.9 with those in Fig. 7.8]. However, we note that there are additional diagrams [Fig. 7.9] present. We now write down V_2^γ through V_{12Z}^γ for the Z exchange diagrams

$$V_{2Z}^\gamma \doteq -c_W^2 [(1-n)(10C_{352Z} + 14C_{242Z}) - 4C_{242Z} + s_3(2C_{342Z} + 2C_{232Z} + 6C_{222Z} + 2C_{122Z} - C_{112Z} + C_{02Z}) + s_4(2C_{312Z} + 10C_{212Z} + 16C_{112Z} + 8C_{02Z}) + (q_1 \cdot q_2)(4C_{332Z} + 16C_{232Z} + 2C_{212Z} + 4C_{122Z} + 14C_{112Z} + 8C_{02Z})] + s_W^2 M_W^2 [C_{112Z} + 3C_{02Z}] - c_W^2 [C_{352Z} + C_{242Z} - C_{361Z}] - c_W^2 [5B_{02Z} + B_{12Z}] + 2[C_{352Z} + C_{242Z}] - [5B_{05W} + B_{15W}], \quad (7.84)$$

$$V_{3Z}^\gamma \doteq c_W^2 [(1-n)(10C_{362Z}) + 4nC_{242Z} + s_3(2C_{322Z} + 2C_{122Z} - 4C_{222Z}) + s_4(2C_{332Z} + 2C_{232Z} - 4C_{212Z} - C_{122Z} - 8C_{112Z} - 6C_{02Z}) + (q_1 \cdot q_2)(4C_{342Z} - 8C_{232Z} + 2C_{222Z} + 2C_{122Z} - 8C_{112Z} - 4C_{02Z})] + s_W^2 M_W^2 [C_{122Z} - 2C_{02Z}] + c_W^2 [C_{351Z} + C_{241Z} - C_{362Z}] + c_W^2 [5B_{01Z} + B_{11Z}] + 2[C_{362Z}] + [4B_{05W} - B_{15W}], \quad (7.85)$$

$$V_{5Z}^\gamma \doteq -c_W^2 [(1-n)(10C_{362Z}) + (5-7n)C_{242Z} + s_3(2C_{322Z} + 2C_{122Z} + 5C_{222Z}) + s_4(2C_{332Z} + 2C_{232Z} + 3C_{212Z} - C_{122Z} + 7C_{112Z} + 5C_{02Z}) + (q_1 \cdot q_2)(4C_{342Z} + 8C_{232Z} + 2C_{222Z} + 3C_{112Z} - 5C_{122Z})] + s_W^2 M_W^2 t_W^2 [2C_{122Z} + C_{02Z}] - s_W^2 M_W^2 [2C_{122Z} - 2C_{02Z}] - c_W^2 [-C_{351Z} - C_{241Z} + C_{362Z} + C_{242Z}] - c_W^2 [4B_{01Z} - B_{11Z}] + [2C_{362Z} + C_{242Z}] - [.5B_{05W} + B_{15W}] + (2-n)[B_{05W} + 2B_{15W}], \quad (7.86)$$

$$V_{6Z}^\gamma \doteq -c_W^2 [(1-n)(10C_{352Z}) + (5-3n)C_{242Z} + s_3(2C_{342Z} + 2C_{232Z} - C_{222Z} - C_{112Z} + 3C_{122Z} - 2C_{02Z}) + s_4(2C_{312Z} + C_{212Z} - 2C_{112Z} - C_{02Z}) + (q_1 \cdot q_2)(4C_{332Z} + 2C_{212Z} - 9C_{112Z} - C_{122Z} - 8C_{02Z})] + s_W^2 t_W^2 M_W^2 [2C_{112Z} + C_{02Z}] - s_W^2 M_W^2 [2C_{112Z} + 2C_{02Z}] - c_W^2 [C_{352Z} + C_{242Z} - C_{361Z} - C_{241Z}] - c_W^2 [-4B_{02Z} + B_{12Z}] + [2C_{352Z} + C_{242Z}] - [.5B_{05W} + B_{15W}] + (2-n)[B_{05W} + 2B_{15W}], \quad (7.87)$$

$$V_{8Z}^\gamma \doteq -c_W^2 [18C_{342Z} + 9C_{232Z} + 18C_{222Z} + 12C_{122Z} + 3C_{112Z} + 3C_{02Z}] - c_W^2 [-C_{342Z} - C_{122Z} - C_{232Z} - C_{222Z}] - c_W^2 [C_{331Z} + C_{121Z} + C_{231Z} + C_{231Z}] - [2C_{342Z} + 2C_{222Z} + C_{232Z} + C_{122Z}], \quad (7.88)$$

$$V_{12Z}^\gamma \doteq -c_W^2 [18C_{332Z} + 9C_{232Z} + 18C_{232Z} + 12C_{122Z} + 3C_{112Z} + 3C_{02Z}] - c_W^2 [-C_{332Z} - C_{122Z} - C_{232Z} - C_{232Z}] - c_W^2 [C_{341Z} + C_{121Z} + C_{231Z} + C_{221Z}] - [2C_{332Z} + 2C_{223Z} + C_{232Z} + C_{122Z}]. \quad (7.89)$$

Now we turn to category IV, i.e. the fermion contribution to the $\gamma W^+ W^-$ vertex

(Fig. 7.10). For these the following results were found

$$V_{2F}^\gamma \doteq -(n/4)s_W^2 [(2-n)(C_{241e} - C_{361e}) - s_3(C_{331e} + 2C_{231e} - C_{211e} + C_{121e} - C_{111e}) - s_4(C_{321e} + C_{221e}) - 2(q_1 \cdot q_2)(C_{341e} + C_{221e}) + m_e^2 C_{121e}] - (n/4)s_W^2 [(2-n)(C_{241d} - C_{361d}) - s_3(C_{331d} + 2C_{231d} - C_{211d} + C_{121d} - C_{111d}) - s_4(C_{321d} + C_{221d}) - 2(q_1 \cdot q_2)(C_{341d} + C_{221d}) + m_d^2 C_{121d}] + (n/2)s_W^2 [(n-2)(2C_{242u} + C_{352u}) - s_3(C_{342u} + 2C_{222u} + C_{122u}) - s_4(C_{312u} + 2C_{212u} + C_{112u}) - 2(q_1 \cdot q_2)(C_{332u} + 2C_{232u} + C_{122u}) + m_u^2 (C_{112u} + C_{02u})], \quad (7.90)$$

$$\begin{aligned}
V_{3F}^\gamma \doteq & -(n/4)s_W^2[(n-2)(2C_{241e} + C_{351e}) - s_3(C_{311e} + 2C_{211e} + \\
& C_{111e}) - s_4(C_{341e} + 2C_{221e} + C_{121e}) - 2(q_1 \cdot q_2)(C_{331e} + \\
& 2C_{231e} + C_{121e}) + m_e^2(C_{111e} + C_{01e})] - (n/4)s_W^2[(n-2)(2C_{241d} + C_{351d}) - s_3(C_{311d} + 2C_{211d} + C_{111d}) - s_4(C_{341d} + \\
& 2C_{221d} + C_{121d}) - 2(q_1 \cdot q_2)(C_{331d} + 2C_{231d} + C_{121d}) + \\
& m_d^2(C_{111d} + C_{01d})] + (n/2)s_W^2[(2-n)(C_{242u} - C_{362u}) - \\
& s_4(C_{332u} + 2C_{232u} - C_{212u} + C_{122u} - C_{112u}) - s_3(C_{322u} + \\
& C_{222u}) - 2(q_1 \cdot q_2)(C_{342u} + C_{222u}) + m_u^2 C_{122u}],
\end{aligned} \tag{7.91}$$

$$\begin{aligned}
V_{5F}^\gamma \doteq & -(n/4)s_W^2[(n-2)C_{351e} - s_3(C_{311e} + 2C_{211e} + C_{111e}) - \\
& s_4(C_{341e} - C_{121e}) - 2(q_1 \cdot q_2)(C_{331e} + C_{231e}) - m_e^2(C_{111e} + \\
& C_{01e})] - (n/4)s_W^2[(n-2)C_{351d} - s_3(C_{311d} + 2C_{211d} + \\
& C_{111d}) - s_4(C_{341d} - C_{121d}) - 2(q_1 \cdot q_2)(C_{331d} + C_{231d}) - \\
& m_d^2(C_{111d} + C_{01d})] + (n/2)s_W^2[(n-2)(C_{362u} + C_{242u}) - \\
& s_4(C_{332u} + 2C_{232u} + C_{212u} + C_{122u} + C_{112u}) - s_3(C_{322u} + \\
& C_{222u}) - 2(q_1 \cdot q_2)(C_{342u} + C_{222u} + C_{232u} + C_{122u}) - m_u^2 C_{122u}],
\end{aligned} \tag{7.92}$$

$$\begin{aligned}
V_{6F}^\gamma \doteq & -(n/4)s_W^2[(n-2)(C_{361e} + C_{241e}) - s_3(C_{331e} + 2C_{231e} + \\
& C_{211e} + C_{121e} + C_{111e}) - s_4(C_{321e} + C_{221e}) - 2(q_1 \cdot q_2)(C_{341e} + \\
& C_{221e} + C_{231e} + C_{121e}) - m_e^2 C_{121e}] - (n/4)s_W^2[(n-2)(C_{361d} + \\
& C_{241d}) - s_3(C_{331d} + 2C_{231d} + C_{211d} + C_{121d} + C_{111d}) - \\
& s_4(C_{321d} + C_{221d}) - 2(q_1 \cdot q_2)(C_{341d} + C_{221d} + C_{231d} + C_{121d}) - \\
& m_d^2 C_{121d}] + (n/2)s_W^2[(n-2)C_{352u} - s_4(C_{312u} + 2C_{212u} + \\
& C_{112u}) - s_3(C_{342u} - C_{122u}) - 2(q_1 \cdot q_2)(C_{332u} + C_{232u}) - \\
& m_u^2(C_{112u} + C_{02u})],
\end{aligned} \tag{7.93}$$

$$\begin{aligned}
V_{8F}^\gamma \doteq & -(n/4)s_W^2[+4C_{331e} + 6C_{231e} + 2C_{121e}] - (n/4)s_W^2[+4C_{331d} + \\
& 6C_{231d} + 2C_{121d}] + (n/2)s_W^2[+4C_{342u} + 4C_{222u} + 2C_{232u} + \\
& 2C_{122u}],
\end{aligned} \tag{7.94}$$

$$\begin{aligned}
V_{12F}^\gamma \doteq & -(n/4)s_W^2[+4C_{341e} + 4C_{221e} + 2C_{231e} + \\
& 2C_{121e}] - (n/4)s_W^2[+4C_{341d} + 4C_{221d} + 2C_{231d} + 2C_{121d}] + \\
& (n/2)s_W^2[+4C_{332u} + 6C_{232u} + 2C_{122u}].
\end{aligned} \tag{7.95}$$

The numerical values for the cross section contribution of the $\gamma W^+ W^-$ vertex are given in Tables 7.5, 7.7 and 7.9. Table 7.5 and 7.7 respectively give values when the top mass is greater and less than the W mass. This gives a general indication of the sensitivity of the radiative corrections to the top quark mass. The absolute magnitude of the corrections decreases with increasing center of mass angle. We note that the sign changes from negative to positive as the center of mass angle is increased.

7.8 ZW^+W^- Vertex

The corrections to the ZW^+W^- vertex proceed in much the same way as those of the γW^+W^- vertex. So we mainly give the end results. All the diagrams that contribute to the corrections of this vertex are listed in Fig. 7.7 through 7.10. Most of the diagrams are the same as those of the γW^+W^- except for a few additional ones. The general form of the dressed ZW^+W^- vertex, Fig. 7.2 may be written in the form

$$\mathcal{M}^{ZW^+W^-} = (ig^2) \bar{v}(p_1) \gamma^\mu (a_s + b_s \gamma_5) u(p_2) [V_2^Z q_{2\nu} g_{\mu\lambda} + V_3^Z q_{1\lambda} g_{\mu\nu} + V_5^Z q_{1\mu} g_{\nu\lambda} + V_6^Z q_{2\mu} g_{\nu\lambda} + V_8^Z q_{1\mu} q_{2\nu} q_{1\lambda} + V_{12}^Z q_{1\lambda} \xi_{\nu\mu} q_{2\nu}] e^{1\nu} e^{2\lambda} \quad (7.96)$$

The contribution to the one loop corrected cross section of this vertex reads

$$\left(\frac{d\sigma}{d\cos\theta} \right)_1^{ZW^+W^-} \doteq 2\Phi[(a_s a_z + b_s b_z) [V_2^Z \mathcal{W}\mathcal{W}_{1ss} + V_3^Z \mathcal{W}\mathcal{W}_{2ss} + V_5^Z \mathcal{W}\mathcal{W}_{3ss} + V_6^Z \mathcal{W}\mathcal{W}_{4ss} + V_8^Z \mathcal{W}\mathcal{W}_{5ss} + V_{12}^Z \mathcal{W}\mathcal{W}_{6ss}] - (a_t a_z + b_t b_z) [V_2^Z \mathcal{W}\mathcal{W}_{1st} + V_3^Z \mathcal{W}\mathcal{W}_{2st} + V_5^Z \mathcal{W}\mathcal{W}_{3st} + V_6^Z \mathcal{W}\mathcal{W}_{4st} + V_8^Z \mathcal{W}\mathcal{W}_{5st} + V_{12}^Z \mathcal{W}\mathcal{W}_{6st}]] \quad (7.97)$$

Complete expressions for V_i^Z through V_{12}^Z are listed in the Appendix J. The numerical values for the cross section contribution of the ZW^+W^- vertex are given in Tables 7.6, 7.8 and 7.10. Remarks similar to the γW^+W^- vertex apply to the numerics of the corrections to this vertex.

Figure 7.1: Two different topologies for the vertex corrections

Figure 7.2: Feynman diagrams for the vertex corrections.

Figure 7.3: Feynman diagrams for the $e^+e^-\gamma$ vertex corrections in SQFD

Figure 7.4: Feynman diagrams for the e^+e^-Z vertex corrections in SQFD

Figure 7.5: Feynman diagrams for the $e^+ \bar{\nu}_e W^+$ vertex corrections in SQFD

Figure 7.6: Feynman diagrams for the $e^- \nu_e W^-$ vertex corrections in SQFD

Figure 7.7: The Higgs-Higgsino exchange diagrams for the $\gamma W^+ W^-$ and the $Z W^+ W^-$ vertex corrections in SQFD

Figure 7.8: The photon-photino exchange diagrams for the $\gamma W^+ W^-$ and the $Z W^+ W^-$ vertex corrections in SQED

Figure 7.9: The Z-zino exchange diagrams for the $\gamma W^+ W^-$ and the $Z W^+ W^-$ vertex corrections in SQFD

Table 7.1: Triangle diagram count

Vertex —	# of diagrams in QFD	# of diagrams due to S-P	Topology —
$e^+e^-\gamma$	3	3	T1
e^+e^-Z	4	4	T1
$e^+\bar{\nu}_e W^+$	4	4	T1
$e^-\nu_e W^-$	4	4	T1
γW^+W^-	$(17+7n_g, 10)$	$(5+7n_g, 13n_g)$	(T1, T2)
ZW^+W^-	$(19+8n_g, 10)$	$(9+8n_g, 16n_g)$	(T1, T2)

Figure 7.10: The fermion-s fermion exchange diagrams for the $\gamma IV^+ W^-$ and the $Z W^+ W^-$ vertex corrections in SQFD

Table 7.2: $e^+e^-\gamma$ vertex contribution to the one loop radiative corrections in pb for ECM=200 GeV.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{QFD}$ $, (\frac{d\sigma}{d\cos\theta})_1^{QFD}$ $, fct$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $, (\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $, fct$	$(\frac{d\sigma}{d\cos\theta})_1^{SQFD}$ $, (\frac{d\sigma}{d\cos\theta})_1^{SQFD}$ $, fct$
10	+2.3042,+0.0561	+0.0472,-0.2114	+2.3514,-0.1553
30	+0.8301,+0.0221	+0.0169,-0.0825	+0.8470,-0.0604
50	-0.0353,+0.0022	-0.0009,-0.0073	-0.0362,-0.0051
70	-0.4140,-0.0063	-0.0087,+0.0252	-0.4227,+0.0189
90	-0.5768,-0.0100	-0.0120,+0.0393	-0.5888,+0.0293
110	-0.6438,-0.0117	-0.0134,+0.0454	-0.6572,+0.0337
130	-0.6673,-0.0124	-0.0139,+0.0481	-0.6812,+0.0357
150	-0.6727,-0.0128	-0.0140,+0.0494	-0.6867,+0.0366
170	-0.6727,-0.0129	-0.0140,+0.0449	-0.6867,+0.0320
180	-0.6726,-0.0129	-0.0140,+0.0449	-0.6866,+0.0320

Table 7.3: e^+e^-Z vertex contribution to the one loop radiative corrections in pb for ECM=200 GeV.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{QFD}$ $(\frac{d\sigma}{d\cos\theta})_{1fct}^{QFD}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $(\frac{d\sigma}{d\cos\theta})_{1fct}^{S-P}$	$(\frac{d\sigma}{d\cos\theta})_1^{SQFD}$ $(\frac{d\sigma}{d\cos\theta})_{1fct}^{SQFD}$
10	+3.3646,+0.0878	-0.1793,-0.3339	+3.1853,-0.2461
30	+1.4412,+0.0351	-0.0688,-0.1342	+1.3724,-0.0991
50	+0.3292,+0.0044	-0.0043,-0.0179	+0.3249,-0.0135
70	-0.1450,-0.0089	+0.0237,+0.0322	-0.1213,+0.0233
90	-0.3498,-0.0146	+0.0358,+0.0538	-0.3140,+0.0392
110	-0.4476,-0.0172	+0.0410,+0.0635	-0.4066,+0.0463
130	-0.5032,-0.0184	+0.0432,+0.0682	-0.4600,+0.0498
150	-0.5383,-0.0190	+0.0441,+0.0706	-0.4942,+0.0516
170	-0.5566,-0.0193	+0.0445,+0.0717	-0.5121,+0.0524
180	-0.5589,-0.0193	+0.0445,+0.0718	-0.4144,+0.0525

Table 7.4: $e^+\nu_e W^+$ vertex contribution to the one loop radiative corrections in pb for ECM=200 GeV.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{QFD}$ $(\frac{d\sigma}{d\cos\theta})_{1fct}^{QFD}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $(\frac{d\sigma}{d\cos\theta})_{1fct}^{S-P}$	$(\frac{d\sigma}{d\cos\theta})_1^{SQFD}$ $(\frac{d\sigma}{d\cos\theta})_{1fct}^{SQFD}$
10	+4.0734,+0.2521	+0.9088,-0.9552	+4.9822,-0.7031
30	+3.4161,+0.2147	+0.6784,-0.8137	+4.0945,-0.5990
50	+2.1259,+0.1439	+0.3774,-0.5454	+2.5033,-0.4015
70	+1.3692,+0.0999	+0.2070,-0.3785	+1.5762,-0.2786
90	+0.9923,+0.0762	+0.1185,-0.2887	+1.1108,-0.2125
110	+0.7996,+0.0630	+0.0691,-0.2388	+0.8687,-0.1708
130	+0.6950,+0.0552	+0.0395,-0.2091	+0.7345,-0.1539
150	+0.6374,+0.0505	+0.0222,-0.1914	+0.6596,-0.1409
170	+0.6111,+0.0483	+0.0141,-0.1828	+0.6252,-0.1345
180	+0.6079,+0.0480	+0.0131,-0.1818	+0.6210,-0.1338

Table 7.5: $\gamma W^+ W^-$ vertex contribution to the one loop radiative corrections in pb for ECM=200 GeV in QFD. The Higgs mass is 100 GeV and top mass is 100 GeV.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^B$ $, (\frac{d\sigma}{d\cos\theta})_{1\,fct}^B$	$(\frac{d\sigma}{d\cos\theta})_1^F$ $, (\frac{d\sigma}{d\cos\theta})_{1\,fct}^F$	$(\frac{d\sigma}{d\cos\theta})_1^{total}$ $, (\frac{d\sigma}{d\cos\theta})_{1\,fct}^{total}$
10	-0.0551,+0.3017	-0.1466,-0.5343	-0.2017,-0.2324
30	-0.0249,+0.1095	-0.0602,-0.1939	-0.0851,-0.0844
50	-0.0014,-0.0034	-0.0010,+0.0060	-0.0024,+0.0026
70	+0.0142,-0.0527	+0.0321,+0.0933	+0.0463,+0.0406
90	+0.0230,-0.0739	+0.0493,+0.1309	+0.0723,+0.0570
110	+0.0255,-0.0827	+0.0549,+0.1464	+0.0804,+0.0637
130	+0.0232,-0.0858	+0.0523,+0.1520	+0.0755,+0.0662
150	+0.0189,-0.0866	+0.0466,+0.1534	+0.0655,+0.0668
170	+0.0158,-0.0867	+0.0420,+0.1535	+0.0578,+0.0668
180	+0.0154,-0.0867	+0.0414,+0.1535	+0.0568,+0.0668

Table 7.6: ZW^+W^- vertex contribution to the one loop radiative corrections in pb for $ECM=200$ GeV in QFD. The Higgs mass is 100 GeV and top mass is 100 GeV.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^B$ $, (\frac{d\sigma}{d\cos\theta})_{1fct}^B$	$(\frac{d\sigma}{d\cos\theta})_1^F$ $, (\frac{d\sigma}{d\cos\theta})_{1fct}^F$	$(\frac{d\sigma}{d\cos\theta})_1^{total}$ $, (\frac{d\sigma}{d\cos\theta})_{1fct}^{total}$
10	-0.1702,+0.5122	-0.0765,-0.9070	-0.2476,-0.3948
30	-0.0873,+0.2162	-0.0308,-0.3828	-0.1181,-0.1666
50	-0.0266,+0.0448	-0.0057,-0.0792	-0.0323,-0.0344
70	+0.0111,-0.0285	+0.0041,+0.0505	+0.0152,+0.0220
90	+0.0329,-0.0602	+0.0070,+0.1066	+0.0399,+0.0464
110	+0.0416,-0.0751	+0.0091,+0.1329	+0.0507,+0.0578
130	+0.0405,-0.0832	+0.0106,+0.1474	+0.0511,+0.0642
150	+0.0348,-0.0882	+0.0127,+0.1562	+0.0475,+0.0680
170	+0.0301,-0.0907	+0.0137,+0.1606	+0.0438,+0.0699
180	+0.0295,-0.0910	+0.0135,+0.1612	+0.0430,+0.0702

Table 7.7: $\gamma W^+ W^-$ vertex contribution to the one loop radiative corrections in pb for ECM=200 GeV in QFD with $m_{H_1^0} = 50$ GeV, and $m_t = 75$ GeV.

$\theta(deg)$ —	$\left(\frac{d\sigma}{d\cos\theta}\right)_1^B$ $\left(\frac{d\sigma}{d\cos\theta}\right)_{1\,fct}^B$	$\left(\frac{d\sigma}{d\cos\theta}\right)_1^F$ $\left(\frac{d\sigma}{d\cos\theta}\right)_{1\,fct}^F$	$\left(\frac{d\sigma}{d\cos\theta}\right)_1^{total}$ $\left(\frac{d\sigma}{d\cos\theta}\right)_{1\,fct}^{total}$
10	-0.0348,+0.3000	-0.5434,-1.0256	-0.5782,-0.7256
30	-0.0180,+0.1087	-0.1965,-0.3720	-0.2145,-0.2633
50	-0.0017,-0.0033	+0.0053,+0.0115	+0.0036,+0.0082
70	+0.0111,-0.0523	+0.0916,+0.1791	+0.1027,+0.1268
90	+0.0190,-0.0734	+0.1283,+0.2512	+0.1473,+0.1778
110	+0.0209,-0.0821	+0.1442,+0.2810	+0.1651,+0.1989
130	+0.0182,-0.0852	+0.1512,+0.2917	+0.1694,+0.2065
150	+0.0134,-0.0860	+0.1546,+0.2944	+0.1680,+0.2084
170	+0.0100,-0.0860	+0.1560,+0.2946	+0.1660,+0.2086
180	+0.0096,-0.0860	+0.1561,+0.2946	+0.1657,+0.2086

Table 7.8: ZW^+W^- vertex contribution to the one loop radiative corrections in pb for $ECM=200$ GeV in QFD with $m_{H_1^0} = 50$ GeV, and $m_t = 75$ GeV.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^B$ $(\frac{d\sigma}{d\cos\theta})_{1fct}^B$	$(\frac{d\sigma}{d\cos\theta})_1^F$ $(\frac{d\sigma}{d\cos\theta})_{1fct}^F$	$(\frac{d\sigma}{d\cos\theta})_1^{total}$ $(\frac{d\sigma}{d\cos\theta})_{1fct}^{total}$
10	-0.1234,+0.5085	-0.5387,-1.7410	-0.6621,-1.2325
30	-0.0677,+0.2146	-0.2160,-0.7348	-0.2837,-0.5202
50	-0.0227,+0.0444	-0.0392,-0.1521	-0.0619,-0.1077
70	+0.0086,-0.0283	+0.0274,+0.0970	+0.0360,+0.087
90	+0.0278,-0.0602	+0.0514,+0.2045	+0.0792,+0.1443
110	+0.0351,-0.0745	+0.0647,+0.2552	+0.0998,+0.1807
130	+0.0331,-0.0826	+0.0765,+0.2829	+0.1096,+0.2003
150	+0.0268,-0.0876	+0.0883,+0.2998	+0.1151,+0.2122
170	+0.0218,-0.0901	+0.0935,+0.3084	+0.1153,+0.2183
180	+0.0212,-0.0905	+0.0962,+0.3095	+0.1174,+0.2190

Table 7.9: $\gamma W^+ W^-$ vertex contribution to the one loop radiative corrections in pb for ECM=200 GeV in QFD. The Higgs mass is 50 GeV and top mass is 100 GeV.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^B$ $, (\frac{d\sigma}{d\cos\theta})_{1fc}^B$	$(\frac{d\sigma}{d\cos\theta})_1^F$ $, (\frac{d\sigma}{d\cos\theta})_{1fc}^F$	$(\frac{d\sigma}{d\cos\theta})_1^{total}$ $, (\frac{d\sigma}{d\cos\theta})_{1fc}^{total}$
10	-0.0348,+0.3000	-0.1466,-0.5343	-0.1814,-0.2343
30	-0.0180,+0.1087	-0.0602,-0.1939	-0.0782,-0.0852
50	-0.0017,-0.0033	-0.0010,+0.0060	-0.0027,+0.0027
70	+0.0111,-0.0523	+0.0321,+0.0933	+0.0433,+0.0410
90	+0.0190,-0.0734	+0.0493,+0.1309	+0.0683,+0.0575
110	+0.0209,-0.0821	+0.0549,+0.1464	+0.0758,+0.0643
130	+0.0182,-0.0852	+0.0523,+0.1520	+0.0705,+0.0668
150	+0.0134,-0.0860	+0.0466,+0.1534	+0.0600,+0.0674
170	+0.0100,-0.0860	+0.0420,+0.1535	+0.0520,+0.0675
180	+0.0096,-0.0860	+0.0414,+0.1535	+0.0510,+0.0675

Table 7.10: ZW^+W^- vertex contribution to the one loop radiative corrections in pb for $ECM=200$ GeV in QFD. The Higgs mass is 50 GeV and top mass is 100 GeV.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^B$ $, (\frac{d\sigma}{d\cos\theta})_{1fct}^B$	$(\frac{d\sigma}{d\cos\theta})_1^F$ $, (\frac{d\sigma}{d\cos\theta})_{1fct}^F$	$(\frac{d\sigma}{d\cos\theta})_1^{total}$ $, (\frac{d\sigma}{d\cos\theta})_{1fct}^{total}$
10	-0.1234,+0.5085	-0.0765,-0.9070	-0.1999,-0.3985
30	-0.0677,+0.2146	-0.0308,-0.3828	-0.0985,-0.1682
50	-0.0227,+0.0444	-0.0057,-0.0792	-0.0284,-0.0348
70	+0.0086,-0.0283	+0.0041,+0.0505	+0.0127,+0.0222
90	+0.0278,-0.0597	+0.0070,+0.1066	+0.0348,+0.0469
110	+0.0351,-0.0745	+0.0091,+0.1329	+0.0442,+0.0584
130	+0.0331,-0.0826	+0.0106,+0.1474	+0.0437,+0.0648
150	+0.0268,-0.0876	+0.0127,+0.1562	+0.0395,+0.0686
170	+0.0218,-0.0901	+0.0137,+0.1606	+0.0355,+0.0705
180	+0.0212,-0.0904	+0.0135,+0.1612	+0.0347,+0.0708

Table 7.11: $\gamma W^+ W^-$ vertex contribution to the one loop radiative corrections in pb for ECM=200 GeV of the S-P.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{S-B}$ $, (\frac{d\sigma}{d\cos\theta})_{1\,fct}^{S-B}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-F}$ $, (\frac{d\sigma}{d\cos\theta})_{1\,fct}^{S-F}$	$(\frac{d\sigma}{d\cos\theta})_1^{total}$ $, (\frac{d\sigma}{d\cos\theta})_{1\,fct}^{total}$
10	-0.1906,-0.5029	+0.2549,+0.0301	+0.0643,-0.4728
30	-0.0686,-0.1824	+0.0841,+0.0109	+0.0155,-0.1715
50	+0.0013,+0.0056	-0.0076,-0.0003	-0.0063,+0.0053
70	+0.0305,+0.0878	-0.0401,-0.0053	-0.0096,+0.0825
90	+0.0426,+0.1232	-0.0507,-0.0074	-0.0081,+0.1158
110	+0.0481,+0.1378	-0.0564,-0.0082	-0.0083,+0.1296
130	+0.0514,+0.1430	-0.0626,-0.0086	-0.0112,+0.1344
150	+0.0535,+0.1444	-0.0691,-0.0086	-0.0156,+0.1358
170	+0.0547,+0.1445	-0.0734,-0.0086	-0.0187,+0.1359
180	+0.0548,+0.1445	-0.0740,-0.0086	-0.0192,+0.1359

Table 7.12: ZW^+W^- vertex contribution to the one loop radiative corrections in pb for ECM=200 GeV due to the S-P.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{S-B}$ $(\frac{d\sigma}{d\cos\theta})_{1fct}^{S-B}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-F}$ $(\frac{d\sigma}{d\cos\theta})_{1fct}^{S-F}$	$(\frac{d\sigma}{d\cos\theta})_1^{total}$ $(\frac{d\sigma}{d\cos\theta})_{1fct}^{total}$
10	-0.3225,-0.8537	+0.4201,+0.0510	+0.0976,-0.8027
30	-0.1347,-0.3603	+0.1610,+0.0215	+0.0263,-0.3388
50	-0.0277,-0.0746	+0.0233,+0.0045	-0.0044,-0.0701
70	+0.0164,+0.0476	-0.0238,-0.0028	-0.0074,+0.0448
90	+0.0348,+0.1003	-0.0385,-0.0060	-0.0037,+0.0943
110	+0.0439,+0.1251	-0.0466,-0.0075	-0.0027,+0.1176
130	+0.0500,+0.1387	-0.0565,-0.0083	-0.0065,+0.1304
150	+0.0545,+0.1470	-0.0673,-0.0088	-0.0128,+0.1382
170	+0.0571,+0.1512	-0.0745,-0.0090	-0.0174,+0.1422
180	+0.0574,+0.1517	-0.0755,-0.0091	-0.0181,+0.1426

Table 7.13: $\gamma W^+ W^-$ vertex contribution to the one loop radiative corrections in pb for ECM=200 GeV of the S-P. The mass splitting subtraction has been included in the counterterm.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{S-B}$ $, (\frac{d\sigma}{d\cos\theta})_{1\,fct}^{S-B}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-F}$ $, (\frac{d\sigma}{d\cos\theta})_{1\,fct}^{S-F}$	$(\frac{d\sigma}{d\cos\theta})_1^{total}$ $, (\frac{d\sigma}{d\cos\theta})_{1\,fct}^{total}$
10	-0.1906,-0.6808	+0.2549,+0.0301	+0.0643,-0.6507
30	-0.0686,-0.2470	+0.0841,+0.0109	+0.0155,-0.2361
50	+0.0013,+0.0076	-0.0076,-0.0003	-0.0063,+0.0073
70	+0.0305,+0.1189	-0.0401,-0.0053	-0.0096,+0.1136
90	+0.0426,+0.1668	-0.0507,-0.0074	-0.0081,+0.1594
110	+0.0481,+0.1865	-0.0564,-0.0082	-0.0083,+0.1783
130	+0.0514,+0.1936	-0.0626,-0.0086	-0.0112,+0.1850
150	+0.0535,+0.1954	-0.0691,-0.0086	-0.0156,+0.1862
170	+0.0547,+0.1956	-0.0734,-0.0086	-0.0187,+0.1870
180	+0.0548,+0.1956	-0.0740,-0.0086	-0.0192,+0.1870

Table 7.14: ZW^+W^- vertex contribution to the one loop radiative corrections in pb for $ECM=200$ GeV due to the S-P. The mass splitting subtraction has been included in the counterterm.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{S-B}$ $, (\frac{d\sigma}{d\cos\theta})_{1\,fct}^{S-B}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-F}$ $, (\frac{d\sigma}{d\cos\theta})_{1\,fct}^{S-F}$	$(\frac{d\sigma}{d\cos\theta})_1^{total}$ $, (\frac{d\sigma}{d\cos\theta})_{1\,fct}^{total}$
10	-0.3225,-1.1557	+0.4201,+0.0510	+0.0976,-1.1047
30	-0.1347,-0.4878	+0.1610,+0.0215	+0.0263,-0.4663
50	-0.0277,-0.1010	+0.0233,+0.0045	-0.0044,-0.0965
70	+0.0164,+0.0644	-0.0238,-0.0028	-0.0074,+0.0616
90	+0.0348,+0.1358	-0.0385,-0.0060	-0.0037,+0.1298
110	+0.0439,+0.1694	-0.0466,-0.0075	-0.0027,+0.1619
130	+0.0500,+0.1878	-0.0565,-0.0083	-0.0065,+0.1795
150	+0.0545,+0.1990	-0.0673,-0.0088	-0.0128,+0.1902
170	+0.0571,+0.2046	-0.0745,-0.0090	-0.0174,+0.1956
180	+0.0574,+0.2051	-0.0755,-0.0091	-0.0181,+0.1960

Chapter 8

One Loop Box Contributions

“I fear those big words, Stephen said, which make me so unhappy.” From *Ulysses*, J. Joyce

8.1 General Remarks

The purpose of this Chapter is to give the contribution of the box diagrams to the one loop differential cross section. Like the self energy and the triangle diagrams their contribution $d\sigma^{\text{box}}$ to the total cross section is given by their interference with the three lowest order tree diagrams.

There are three different topologies of the box diagrams allowed by both QFD and SQFD. The three different topologies are, (see Fig. 8.1), triangular or the collapsed box, direct or the square box, and the crossed box. We note that the three different topologies may be seen to arise as follows. We may first write all the direct box diagrams allowed by the tree level Lagrangian, then cross the internal lines 1 and 2 in Fig. 8.2 of the direct box and see if these boxes are allowed by basic Lagrangian and charge conservation to obtain the crossed topology. The triangular topology is obtained by collapsing line 4 in Fig. 8.2 to a point and seeing if the new vertex is allowed by the fundamental Lagrangian. The only box diagrams which

cannot be generated in this manner are diagrams of crossed box topology with two internal fermion lines. As mentioned in chapter 5 these diagrams play the role of providing “u” channel type of terms which are needed to cancel the u-channel infrared infinities. We recall that there are no u-channel vertex corrections.

The box diagrams are enumerated in Table 8.1, where we have classified them under triangular topology (TP), three categories of the direct boxes (DB) and their corresponding crossed box (CB) counterparts and finally the crossed boxes with two internal fermion lines¹. In Table 8.1 (and everywhere else) F, V, S respectively refer to an fermion, vector and scalar internal line in the box loop. We may divide the contribution of the box diagrams to the one loop differential cross section as follows,

$$d\sigma^{\text{box}} = d\sigma^{\text{TP}} + d\sigma^{\text{DB}} + d\sigma^{\text{CB}}. \quad (8.1)$$

The DB (similar remarks apply to CB) can be further subdivided as

$$d\sigma^{\text{DB}} = d\sigma^{\text{category I}} + d\sigma^{\text{category II}} + d\sigma^{\text{category III}}. \quad (8.2)$$

Category I is a collection of DB diagrams with three fermions and one vector boson circulating in the loop. Category II is made up of diagrams with three vector boson lines and one fermion line. Category III consists of diagrams with one fermion, two vector boson and one Higgs (or unphysical Higgs) circulating in the box loop. We have not considered diagrams of the type given in Fig. 8.2 b, since they are of the order of $\sim (m_e^2/M_W^2)$ as can be immediately seen from tree level Lagrangian for any fermion-fermion couples to the Higgs sector via a $\sim (m_f/M_W)$ factor. A discussion of the additional diagrams introduced by SUSY is given in section 8.6.

¹We note the misprint or error in [29] where the number of DB diagrams is given to be 13, this number should be 14.

We note that all the loop integrals arising out of Fig. 8.1 are ultraviolet finite, for at the most there are 3 factors of q (where q is internal momentum) in the numerator in any of the diagrams in Fig. 8.1 i.e.

$$D_{\mu\nu\lambda} \sim \int \frac{d^n q}{2\pi^n} \frac{q_\mu q_\nu q_\lambda}{(q^2 - m_1^2)((q+k)^2 - m_2^2)((q+k+p)^2 - m_3^2)((q+k+p+l)^2 - m_4^2)}$$

So when $n=4$, from the dimensional counting degree of divergence = number of q 's in numerator - number of q 's in the denominator = $7 - 8 = -1$. Since this is negative it follows that all loop integrals are finite. However a subtlety arises in the infrared case for then one or both the masses in the above integral may be zero coupled with the fact that in the infrared region one has $q = p_i$ [45] where p_i is one of the momenta factor appearing in the above integral i.e. it is one of k , $k + p$, $k + p + l$. These conditions naturally lead to infrared infinities as discussed in Chapter 5.

This chapter is organized as follows; in the next section we discuss the cross section contributions of the box diagrams. In the remaining sections we list the explicit contributions of each of the individual set of box diagrams that are allowed by SQFD. We list the numerical results for all the box diagrams in Table 8.2 through 8.12.

8.2 Box contribution to the one loop differential cross section

The contribution from box diagrams may be written as

$$B^X \doteq i(g^2)(f(s_W))\bar{v}(p_1)B_{\nu\lambda}^X(a_k + b_k\gamma_5)u(p_2)e^{1\nu}e^{2\lambda} \quad (8.3)$$

where $f(s_W)$, a_k and b_k are factors which depend on s_W and may be different for different diagrams. Here

$$B_{\nu\lambda}^X = \gamma^\mu B_{\mu\nu\lambda}^X + \gamma_\nu (\not{q}_1 - \not{p}_1) \gamma_\lambda B_{21}^X \quad (8.4)$$

where

$$\begin{aligned} B_{\mu\nu\lambda}^X = & (B_3^X p_{1\nu} + B_4^X p_{2\nu}) g_{\mu\lambda} + (B_5^X q_{1\lambda} + B_7^X p_{1\lambda} + B_8^X p_{2\lambda}) g_{\mu\nu} + \\ & B_{11}^X q_{1\mu} q_{1\lambda} p_{1\nu} + B_{12}^X q_{1\lambda} q_{1\mu} p_{2\nu} + B_{15}^X q_{1\mu} p_{1\lambda} p_{1\nu} + \\ & B_{16}^X q_{1\mu} p_{1\lambda} p_{2\nu} + B_{17}^X q_{1\mu} p_{2\lambda} p_{1\nu} + B_{18}^X q_{1\mu} p_{2\nu} p_{2\lambda} + B_{19}^X q_{1\mu} g_{\nu\lambda} \end{aligned} \quad (8.5)$$

The same notation of Passarino and Veltman [45] is followed except that we have dropped terms which don't count i.e. when $q_{1\nu}$ is contracted with $e_{1\nu}$ etc. The coefficient functions B_3^X — B_{21}^X above are expressible in terms of the scalar D integrals introduced in reference [45] and discussed in Appendix K. The contribution of the box diagrams to the one loop cross section is given by the interference of their amplitude with the amplitude of the three lowest order tree diagrams. Therefore we must calculate the interference of equation 8.3 with the S and T channels tree level amplitude given in Chapter 5.

$$\left(\frac{d\sigma}{d\cos\theta} \right)_{1,\text{box}}^{e^+e^- \rightarrow W^+W^-} \doteq 2\Phi[f(s_W)(a_s a_k + b_s b_k) \sum B_i^X \mathcal{X}_i - f(s_W)(a_t a_k + b_t b_k) \sum B_i^X \mathcal{X}_{ji}] \quad (8.6)$$

Here $i = j = 3, 4, 5, \dots, 21$

and $\mathcal{X}_1$ through $\mathcal{X}_{21}$ are the various momentum dot products. Though we have not indicated explicitly in Eqs. 8.3, and 8.6, each diagram or set of diagrams has its own set of a_k , b_k and $f(s_W)$. After a straightforward but lengthy calculation $\mathcal{X}_1$ through $\mathcal{X}_{21}$ are found and are listed in the Appendix G.

8.3 Triangular Topology

There are only 5 independent diagrams allowed in this topology, (Fig. 8.3). These arise because four vector interactions exist due to the non-abelian nature of the

$SU(2) \times U(1)$ gauge group. The TP diagrams are the only set of diagrams in our calculation which involve the four vector vertex corrections and are interesting from this point of view. It is clear that these diagrams are finite and will involve the C-integrals. The momentum convention for TP is illustrated in Fig. 8.3. The loop integral for diagram a in Fig. 8.3 is readily written down and is given by

$$B_{\nu\lambda}^{X(TP)} = (i)^3 (-ie)^2 (-ie^2) \int \frac{d^m q}{2\pi^n} \frac{\bar{u}(p_1) \gamma^{\mu_1} (\not{q}) \gamma^{\mu_2} u(p_2)}{(q^2 - m_e^2)((q - p_2)^2 - \lambda^2)((q - p_2 + p_3)^2 - \lambda^2)} \quad (8.7)$$

using the definition of C integrals, Appendix H this immediately reduces to

$$B_{\nu\lambda}^X = -\frac{i}{16\pi^2} g^4 s_W^4 \bar{u}(p_1) [-\gamma_\nu (\not{p}_1 C_{12} + \not{p}_2 (C_{12} - C_{11})) \gamma_\lambda - [\gamma_\lambda (\not{p}_1 C_{12} + \not{p}_2 (C_{12} - C_{11})) \gamma_\nu] u(p_2)] \quad (8.8)$$

where the definition of $S_{\mu_1 \mu_2, \nu\lambda}$

$$S_{\mu_1 \mu_2, \nu\lambda} = 2g_{\mu_1 \mu_2} g_{\nu\lambda} - g_{\mu_1 \nu} g_{\mu_2 \lambda} - g_{\mu_2 \nu} g_{\mu_1 \lambda} \quad (8.9)$$

has been used. The coefficients of B tensor read in this case

$$B_3^{X(TP)} = 2C_{12} \quad (8.10)$$

$$B_7^{X(TP)} = 2C_{12} \quad (8.11)$$

$$B_4^{X(TP)} = 2(C_{12} - C_{11}) \quad (8.12)$$

$$B_8^{X(TP)} = 2(C_{12} - C_{11}) \quad (8.13)$$

These Eqs. imply

$$B_3^{X(TP)} = B_7^{X(TP)} \quad (8.14)$$

$$B_4^{X(TP)} = B_8^{X(TP)} \quad (8.15)$$

Relations 8.14 and 8.15 holds for diagrams a through d in Fig. 8.3. Diagram e in Fig. 8.3 requires a little special attention and we have for this case

$$B_3^{X(TP)} = -2C_{12}(-p_2, p_5, m_{\nu_e}, M_W, M_W) \quad (8.16)$$

$$B_7^{X(TP)} = 4C_{12}(-p_2, p_5, m_{\nu_e}, M_W, M_W) \quad (8.17)$$

$$B_8^{X(TP)} = -2[C_{12}(-p_2, p_5, m_{\nu_e}, M_W, M_W) - C_{11}] \quad (8.18)$$

$$B_4^{X(TP)} = 4[C_{12}(-p_2, p_5, m_{\nu_e}, M_W, M_W) - C_{11}] \quad (8.19)$$

Leading to the relations

$$B_7^{X(TP)} = -2B_3^{X(TP)} \quad (8.20)$$

$$B_4^{X(TP)} = -2B_8^{X(TP)} \quad (8.21)$$

which are the equivalent of relation 8.14 and 8.15. The total contribution for this category can thus be written as

$$\begin{aligned} B^{X(TP)} \doteq & ig^2 \bar{v}(p_1) [+ s_W^4 B_{\nu\lambda}^{X(TP)}(-p_2, p_5, m_e, \lambda, \lambda) + (s_W^2/4)(a - \\ & b\gamma_5)[B_{\nu\lambda}^{X(TP)}(-p_2, p_5, m_e, \lambda, M_Z) + \\ & B_{\nu\lambda}^{X(TP)}(-p_2, p_5, m_e, M_Z, \lambda)] + (s_W^4/16)(a^2 + \\ & b^2 - 2ab\gamma_5)[B_{\nu\lambda}^{X(TP)}(-p_2, p_5, m_e, M_Z, M_Z)] + (1/4)(1 - \\ & \gamma_5)[B_{\nu\lambda}^{X(TP)}(-p_2, p_5, m_{\nu_e}, M_W, M_W)]] u(p_2) e^{1\nu} e^{2\lambda}. \end{aligned} \quad (8.22)$$

We recall that

$$a = 1 - 4s_W^2, \quad (8.23)$$

$$b = 1. \quad (8.24)$$

The numerical results for the triangular box diagrams are listed in Table 8.2.

8.4 Direct Box Topology

The 14 DB diagrams are given in Fig. 8.4 where they are divided into three categories for computational simplicity. It suffices to calculate only one diagram

in each category for the rest follow from these by a simple change of couplings and internal masses. We start by calculating the least difficult of the categories to compute [i.e. category III or HVV box diagrams]. It suffices to calculate one of these, we choose diagram a in Fig. 8.4. The contribution of this diagram reads,

$$B_{\nu\lambda}^X = (i)^4(-ie)^2(ieM_W)^2 \int \frac{d^n q}{2\pi^n} \frac{\bar{v}(p_1)N_{\nu\lambda}u(p_2)}{Q_1 Q_2 Q_3 Q_4} \quad (8.25)$$

We have used the following compact notation in the above loop integral,

$$\begin{aligned} N_{\nu\lambda} &= \gamma_\nu(\not{q} + m_e)\gamma_\lambda \\ Q_1 &= (q^2 - m_e^2) \\ Q_2 &= ((q - p_2)^2 - \lambda^2) \\ Q_3 &= ((q - p_2 + p_5)^2 - \lambda^2) \\ Q_4 &= ((q - p_2 + p_5 - q_1)^2 - M_W^2) \end{aligned}$$

henceforth we shall use this notation in every loop integral, so that each such integral will have its own set of $N_{\nu\lambda}$ and Q 's. On using the definition of the D integrals Eq. 8.25 reduces to

$$B_{\nu\lambda}^X = \frac{i}{16\pi^2} (e)^4 M_W^2 (\bar{v}(p_1)\gamma_\nu \not{D}\gamma_\lambda u(p_2)) \quad (8.26)$$

where

$$\not{D} = \not{p}_2(D_{12} - D_{11}) + \not{p}_1(D_{12} - D_{13}) - (\not{q}_1 - \not{p}_1)D_{13} \quad (8.27)$$

Using the fact that terms like $\not{p}_2 u(p_2) = m_e u(p_2)$ can be ignored we have

$$B_{\nu\lambda}^X = \frac{i}{16\pi^2} g^4 s_W^4 M_W^2 \bar{v}(p_1) [2p_{2\lambda}\gamma_\nu(D_{12} - D_{11}) + 2p_{1\nu}\gamma_\lambda(D_{12} - D_{13}) - \gamma_\nu(q_1 - p_1)D_{13}\gamma_\lambda] u(p_2) \quad (8.28)$$

On using the definitions (8.4,8.5)

$$B_3^{X(H)} = 2(D_{12} - D_{13})M_W^2 \quad (8.29)$$

$$B_8^{X(H)} = 2(D_{12} - D_{11})M_W^2 \quad (8.30)$$

$$B_{21}^{X(H)} = -(D_{13})M_W^2 \quad (8.31)$$

Proceeding similarly for the remaining diagrams in Category III we have the total contribution of this category

$$\begin{aligned} B^{X(H)} = & ig^2 \bar{u}(p_1) [s_W^4 B_{\nu\lambda}^H(-p_2, p_5, -q_1, m_e, \lambda, \lambda, M_W) - \\ & (s_W^4/4c_W^2)(a - b\gamma_5)[B_{\nu\lambda}^H(-p_2, p_5, -q_1, m_e, \lambda, M_Z, M_W) + \\ & B_{\nu\lambda}^H(-p_2, p_5, -q_1, m_e, M_Z, \lambda, M_W)] + (s_W^4/16c_W^4)(a^2 + b^2 - \\ & 2ab\gamma_5)[B_{\nu\lambda}^H(-p_2, p_5, -q_1, m_e, M_Z, M_Z, M_W)] + (1/4)(1 - \\ & \gamma_5)[B_{\nu\lambda}^H(-p_2, p_5, -q_1, m_{\nu_e}, M_W, M_W, M_{H_1^0})]] u(p_2) e^{1\nu} e^{2\lambda} \end{aligned} \quad (8.32)$$

The numerical results for the above direct box diagrams involving one scalar (i.e. corresponding to Eq. 8.32) are listed in Tables 8.4, and 8.6. We note that the first entry in the column 3 assumes the physical Higgs mass to be 100 GeV, whereas the second entry corresponds to the Higgs mass being 50 GeV. Now we proceed to calculate category I which has one vector boson and three fermions as internal particles, (see Fig. 8.4). The loop integral for this case reads

$$B_{\nu\lambda}^{X(F)} = -(i)^4 (-ie)^2 \left(\frac{ig}{2\sqrt{2}} \right)^2 \int \frac{d^n q}{2\pi^n} \frac{\bar{u}(p_1) N_{\nu\lambda} u(p_2)}{Q_1 Q_2 Q_3 Q_4} \quad (8.33)$$

Here

$$N_{\nu\lambda} = \gamma^\alpha (-\not{q} - \not{p}_1) \gamma_\nu (-\not{q} + \not{q}_1 - \not{p}_1)$$

$$\gamma_\lambda (-\not{q} + \not{p}_2) \gamma_\alpha 2(1 - \gamma_5)$$

$$Q_1 = (q^2 - \lambda^2)$$

$$Q_2 = ((q - p_2)^2 - m_e^2)$$

$$Q_3 = ((q - p_2 + p_5)^2 - m_e^2)$$

$$Q_4 = ((q - p_2 + p_5 - q_1)^2 - m_{\nu_e}^2)$$

Several comments are in order. The above loop integral is simplified by the repeated use of the formulae

$$\gamma^\alpha \not{q} \not{p} \not{q} \gamma_\alpha = -2 \not{q} \not{p} \not{q} + (4 - n) \not{q} \not{p} \not{q} \quad (8.34)$$

Since the integral is finite as far as quadratic or linear divergences are concerned the $(4-n)$ term can safely be dropped. Using this simplification Eq. 8.34 is easily generalized to the case when 5 momenta are sandwiched between two γ matrices. We obtain after some algebra

$$\gamma^\alpha \not{q} \not{b} \not{c} \not{d} \not{e} \gamma_\alpha = -2 \not{q} \not{b} \not{c} \not{d} \not{e} + 2 \not{q} \not{b} \not{c} \not{d} - 2 \not{q} \not{b} \not{c} \not{e} \quad (8.35)$$

It is Eq. 8.35 we use to simplify the loop integral given in 8.33. As before we repeatedly drop terms like $\not{p}_2 u(p_2)$ which are of the order of $m_e u(p_2)$. From the definition of D integrals given in Appendix H we immediately see that Eq. 8.33 will involve all the D-integrals from D_0 through to D_{313} since we have 3q, 2q, 1q and no q terms. After a tedious but a straightforward calculation we obtain from Eq. 8.33 following expressions for the B's.

$$\begin{aligned} B_3^{X(F)} = & 8(p_2 \cdot q_1 - p_1 \cdot q_1) D_{13} - 8(p_1 \cdot p_2) D_{12} + 4s_3 D_{13} + 2(+2(s_3 - \\ & 2(p_1 \cdot q_1)) D_{26} - 4(p_1 \cdot p_2 - p_2 \cdot q_1)(D_{25} - D_{26}) + 4(p_1 \cdot p_2)(D_{22} - \\ & D_{24}) + 4(p_2 \cdot q_1)(D_{25} - D_{26}) + 4(p_1 \cdot p_2 - p_2 \cdot q_1)(D_{22} - D_{24}) - \\ & 4(p_2 \cdot q_1 - p_1 \cdot p_2)(D_{25} - D_{26}) - 4(p_1 \cdot p_2) D_{24} + 4(p_1 \cdot q_1) D_{23} + \\ & 4(p_1 \cdot p_2) D_{25} - 4(p_1 \cdot q_1) D_{26} + 4(p_2 \cdot q_1 - p_1 \cdot p_2) D_{26} - 4 D_{27} + \\ & 2(4(p_1 \cdot p_2)(D_{32} - D_{36} - D_{38} + D_{310}) + 4(p_2 \cdot q_1)(-D_{37} + \\ & D_{39} - D_{38} + D_{310}) + 4(p_1 \cdot q_1)(D_{39} - D_{38}) + 2s_3(D_{39} - \\ & D_{33}) - 8 D_{312} + 12 D_{313}) \end{aligned} \quad (8.36)$$

$$\begin{aligned} B_4^{X(F)} = & 4(s_3 - 2(p_1 \cdot q_1)) D_{13} + 2[+4(p_1 \cdot q_1)(D_{22} - D_{24}) + 2s_3(D_{25} - \\ & D_{26}) + 2s_3 D_{23} - 4(p_1 \cdot q_1) D_{26} - 4 D_{27}] + 8(D_{312} - D_{311}) \end{aligned} \quad (8.37)$$

$$B_5^{X(F)} = 8(p_1 \cdot q_1) D_{13} + 8[(p_1 \cdot q_1) D_{26} - (p_1 \cdot p_2)(D_{22} - D_{24})] - 8 D_{313} \quad (8.38)$$

$$B_7^{X(F)} = -4s_3 D_{13} + 2[+4(p_2 \cdot q_1)(D_{22} - D_{24}) - 2s_3 D_{26} + 4 D_{27}] + 8 D_{312} \quad (8.39)$$

$$\begin{aligned}
B_8^{X(F)} = & -4(s_3 - \\
& 2(p_1 \cdot q_1))D_{13} + 8(p_1 \cdot p_2)(D_{11} - D_{12}) + 2[-4(p_1 \cdot q_1)(D_{22} - \\
& D_{24}) - (2s_3 + 4(p_2 \cdot q_1))(D_{25} - D_{26}) + 4(p_1 \cdot p_2)(D_{21} - D_{24}) + \\
& 4(p_1 \cdot q_1)D_{26} - 2s_3D_{23} + 4D_{27}] + 4[+2(p_1 \cdot p_2)(D_{32} + D_{34}) - \\
& 2(p_2 \cdot q_1)D_{35} - 4(p_1 \cdot p_2)D_{36} - s_3D_{37} - 2(p_1 \cdot q_1 + p_2 \cdot q_1)D_{38} + \\
& s_3D_{39} + (2(p_1 \cdot q_1) + 4(p_2 \cdot q_1))D_{310} + 4(D_{311} - D_{312})]
\end{aligned} \tag{8.40}$$

$$B_{11}^{X(F)} = -8[D_{13} + D_{23} + D_{26} + D_{39}] \tag{8.41}$$

$$B_{12}^{X(F)} = 8[D_{37} - D_{39}] \tag{8.42}$$

$$B_{15}^{X(F)} = 8[D_{13} + 2D_{26} + D_{38}] \tag{8.43}$$

$$B_{16}^{X(F)} = -8[D_{22} - D_{24} + D_{25} - D_{26} + D_{310} - D_{38}] \tag{8.44}$$

$$B_{17}^{X(F)} = 8[-D_{13} - D_{25} + D_{22} - D_{24} + D_{38} - D_{310}] \tag{8.45}$$

$$B_{18}^{X(F)} = 8[+D_{25} - D_{26} + D_{35} + D_{38} - 2D_{310}] \tag{8.46}$$

$$\begin{aligned}
B_{19}^{X(F)} = & 2[+4(p_1 \cdot p_2)(D_{22} - \\
& D_{24}) - 2s_3D_{23} + 4(p_1 \cdot q_1)D_{23} + 4(p_1 \cdot p_2)(D_{25} - D_{26})] + \\
& 2[+8D_{313} + 2(-s_3D_{33} - 2(p_2 \cdot q_1)D_{37} - 2(p_1 \cdot p_2)D_{38} + \\
& 2(p_1 \cdot q_1 + p_2 \cdot q_1)D_{39} + 2(p_1 \cdot p_2)D_{310})]
\end{aligned} \tag{8.47}$$

$$\begin{aligned}
B_{21}^{X(F)} = & +4(p_1 \cdot p_2)D_0 + 4(p_1 \cdot p_2)D_{11} + 2[+2(p_1 \cdot p_2)(D_{22} - D_{24}) - \\
& s_3D_{23} + 2(p_1 \cdot q_1)D_{23} + 2(p_1 \cdot p_2)(D_{25} - D_{26})] + 2[+6D_{313} - \\
& s_3D_{33} - 2(p_2 \cdot q_1)D_{37} - 2(p_1 \cdot p_2)D_{38} + 2(p_1 \cdot q_1 + p_2 \cdot q_1)D_{39} + \\
& 2(p_1 \cdot p_2)D_{310}]
\end{aligned} \tag{8.48}$$

Letting $\gamma \longrightarrow Z$ and noting the change in coupling we similarly calculate diagram

2 in Category I. The sum of the first two diagrams in category I reads

$$\begin{aligned}
B^{X(F)} \doteq & ig^2 \bar{v}(p_1)[(s_W^2/4)B_{\nu\lambda}^F(-p_2, p_5, -q_1, \lambda, m_e, m_e, m_{\nu_e})(1 - \\
& \gamma_5) + (1/64c_W^2)B_{\nu\lambda}^F(-p_2, p_5, -q_1, m_e, \lambda, M_Z, M_W)(a + \\
& 1)^2(1 - \gamma_5)]u(p_2)e^{1\nu}e^{2\lambda}
\end{aligned} \tag{8.49}$$

Similarly diagram 3 of category I yields

$$\begin{aligned}
B^{X(F)} \doteq & ig^2 \bar{v}(p_1)[+(1/16)A_{\nu\lambda}^F(-p_2, p_5, -q_1, M_W, m_{\nu_e}, m_{\nu_e}, m_e)2(1 - \\
& \gamma_5)]u(p_2)e^{1\nu}e^{2\lambda}
\end{aligned} \tag{8.50}$$

Thence the total contribution of category I amounts to

$$\begin{aligned}
 B^{X(F)} \doteq & ig^2 \bar{v}(p_1) [(s_W^2/4) B_{\nu\lambda}^F(-p_2, p_5, -q_1, \lambda, m_c, m_c, m_{\nu_c})(1 - \gamma_5) \\
 & + (1/64 c_W^2) B_{\nu\lambda}^F(-p_2, p_5, -q_1, M_Z, m_c, m_c, m_{\nu_c})(a+1)^2(1 - \gamma_5) \\
 & + (1/8) A_{\nu\lambda}^F(-p_2, p_5, -q_1, M_W, m_{\nu_c}, m_{\nu_c}, m_c)(1 - \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda}
 \end{aligned} \quad (8.51)$$

The numerical results corresponding to the differential cross section arising from Eq. 8.51 are listed in Table 8.3. We recall that there are more equations than unknowns in the Passarino and Veltman scheme [45], (see Appendices H and K and our discussion in section 4.5) of determining the D integrals in terms of D_0 and various C integrals. As already mentioned this provides an excellent check on the internal consistency of the numerical scheme. We have fully exploited this method of cross checking our results and obtained agreement up to 10 decimal places. Now we turn to Category II and choose to evaluate $e\gamma\gamma W$, we shall denote the form factors of this category by $B_i^{X(V)}$ for the obvious reason that there are three internal vector boson, Fig. 8.4b. The loop integral involved in this diagram is

$$B_{\nu\lambda}^{X(V)} = -(i)^4 (-ie)^4 \int \frac{d^n q}{2\pi^n} \frac{\bar{v}(p_1) N_{\nu\lambda} u(p_2)}{Q_1 Q_2 Q_3 Q_4} \quad (8.52)$$

$$N_{\nu\lambda} = \gamma^{\mu_1} (\not{q}) \gamma^{\mu_2} \Gamma_{\mu_1 \alpha \nu} \Gamma_{\mu_2 \lambda \alpha}$$

$$Q_1 = (q^2 - m_c^2)$$

$$Q_2 = ((q - p_2)^2 - \lambda^2)$$

$$Q_3 = ((q - p_2 + p_5)^2 - \lambda^2)$$

$$Q_4 = ((q - p_2 + p_5 - q_1)^2 - M_W^2)$$

where the Γ 's are three vector boson vertices. Following our adopted form of the form factors we eliminate q_2 in the above calculation of the loop-integral in favour

of q_1 , p_1 and p_2 , dropping terms like $\not{p}_2 u(p_2)$ etc and using the definition of D integrals, Eq. 8.52 reduces to the following set of B's after a straightforward but a tedious calculation.

$$\begin{aligned}
 B_3^{X(V)} = & 4(2(p_1 \cdot p_2) - p_1 \cdot q_1)D_{12} + 2(s_3 - 4(p_2 \cdot q_1))D_{13} + 8(+p_2 \cdot q_1 - \\
 & p_1 \cdot p_2)(D_{12} - D_{11}) - 4(s_3 - 2(p_1 \cdot q_1))D_{13} - 4(p_1 \cdot q_1)D_{12} - \\
 & 2s_3 D_{13} - 2(D_{12} - D_{13})(2(p_1 \cdot p_2) + 2(p_2 \cdot q_1) - s_3) - \\
 & 10(p_1 \cdot p_2)(D_{22} - D_{24}) - 5s_3 D_{23} - 10(p_2 \cdot q_1)(D_{25} - D_{26}) + \\
 & 10(p_1 \cdot q_1)D_{26} - 4(p_2 \cdot q_1)(D_{24} - D_{25}) - 4((p_2 \cdot q_1) - s_3)(D_{23} - \\
 & D_{26}) - 8(p_1 \cdot p_2 - p_2 \cdot q_1)(D_{22} - D_{24}) + 8(p_1 \cdot p_2 - p_1 \cdot q_1)D_{22} - \\
 & 4(+2(p_2 \cdot q_1) - s_3)D_{26} - 4(p_1 \cdot p_2 - p_1 \cdot q_1 - p_2 \cdot q_1)(D_{22} - D_{24}) - \\
 & 4(+s_3 - 2(p_1 \cdot q_1))D_{26} - 2(p_1 \cdot p_2)(D_{32} - D_{36}) + 2(p_1 \cdot q_1) + \\
 & p_2 \cdot q_1 - 2p_1 \cdot p_2)D_{38} + (-s_3 + 4(p_1 \cdot q_1) + 4(p_2 \cdot q_1))D_{39} + \\
 & 2(2(p_1 \cdot p_2) - p_2 \cdot q_1)D_{310} - 4(p_2 \cdot q_1)D_{37} - 2s_3 D_{33} + 14D_{312} + \\
 & 12D_{313}
 \end{aligned} \tag{8.53}$$

$$\begin{aligned}
 B_4^{X(V)} = & 4(2(p_1 \cdot q_1)D_{12} - s_3 D_{13}) + 8(p_1 \cdot p_2 - p_1 \cdot q_1)(D_{22} - D_{24}) + \\
 & 4(2(p_2 \cdot q_1) - s_3)(D_{25} - D_{26}) - 4(p_1 \cdot p_2)(D_{22} - D_{24}) - \\
 & 4(p_2 \cdot q_1)(D_{25} - D_{26}) + 4(p_1 \cdot q_1)D_{26} - 2s_3 D_{23} + 8(p_2 \cdot q_1 - \\
 & p_1 \cdot p_2)(+D_{21} + D_{22} - 2D_{24}) + 4(+s_3 - 2(p_1 \cdot q_1))(D_{25} - \\
 & D_{26}) + 26(D_{312} - D_{311}) - 6(p_1 \cdot p_2)(D_{32} + D_{34} - 2D_{36}) + \\
 & 6(p_2 \cdot q_1)D_{35} + 3s_3(+D_{37} - D_{39}) + 6(p_1 \cdot q_1 + p_2 \cdot q_1)D_{38} - \\
 & 6(p_1 \cdot q_1 + 2(p_2 \cdot q_1))D_{310}
 \end{aligned} \tag{8.54}$$

$$\begin{aligned}
 B_5^{X(V)} = & 4(p_1 \cdot p_2 - p_2 \cdot q_1)(D_{12} - D_{11}) + 2(s_3 - 2(p_1 \cdot q_1))D_{13} - \\
 & 2(2(p_1 \cdot q_1)D_{12} - s_3 D_{13}) - (s_3 + 2(p_1 \cdot q_1))D_{13} + 2(p_2 \cdot q_1 + \\
 & p_1 \cdot p_2)(D_{12} - D_{11}) + 6(p_1 \cdot p_2)(D_{22} - D_{24}) - 2(p_2 \cdot q_1)(D_{25} - \\
 & D_{26}) - 14(p_1 \cdot q_1)D_{26} + 3s_3 D_{23} + 3s_3 D_{33} + 6(p_2 \cdot q_1)D_{37} + \\
 & 6(p_1 \cdot p_2)(D_{38} - D_{310}) - 6(p_1 \cdot q_1 + p_2 \cdot q_1)D_{39} - 26D_{313}
 \end{aligned} \tag{8.55}$$

$$\begin{aligned}
 B_7^{X(V)} = & +(2(p_1 \cdot p_2) - 6(p_2 \cdot q_1))(D_{12} - D_{11}) + s_3 D_{13} + 2(p_1 \cdot q_1)(2D_{12} - \\
 & D_{13}) - 2(p_1 \cdot p_2)(D_{22} - D_{24}) - 2(p_2 \cdot q_1)(D_{25} - D_{26}) + \\
 & 2(p_1 \cdot q_1)D_{26} - 8(p_2 \cdot q_1)(D_{22} - D_{24}) - s_3 D_{23} + 8(p_1 \cdot q_1)D_{22} - \\
 & 6(p_1 \cdot p_2)(D_{32} - D_{36}) + 6(p_2 \cdot q_1)(D_{38} - D_{310}) - 3s_3 D_{39} + \\
 & 6(p_1 \cdot q_1)D_{37} + 26D_{312}
 \end{aligned} \tag{8.56}$$

$$\begin{aligned}
 B_8^{X(V)} = & -2(p_1 \cdot p_2 + p_2 \cdot q_1 - s_3)(D_{12} - D_{11}) + s_3 D_{13} - 2(p_1 \cdot q_1)(2D_{12} + \\
 & D_{13}) + 4(p_1 \cdot q_1)(3D_{22} - 3D_{24} - D_{26}) + 2s_3(+D_{23} + 2D_{25} - \\
 & 2D_{26}) - 4(p_2 \cdot q_1)(+D_{21} + D_{22} - 2D_{24}) + 14(D_{312} - D_{311}) + \\
 & s_3(D_{37} - D_{39}) + 2(p_2 \cdot q_1)(D_{35} + D_{38} - 2D_{310}) + 2(p_1 \cdot q_1)(D_{38} - \\
 & D_{310}) - 2(p_1 \cdot p_2)(D_{32} + D_{34} - 2D_{36})
 \end{aligned} \tag{8.57}$$

$$B_{11}^{X(V)} = -2(D_{12} - D_{13}) - 8D_{23} + 4D_{26} - 8D_{39} \quad (8.58)$$

$$B_{12}^{X(V)} = -4(D_{12} - D_{11}) + 12(D_{25} - D_{26}) + 8(D_{37} - D_{39}) \quad (8.59)$$

$$B_{15}^{X(V)} = +2(D_{12} - D_{13}) + 4D_{26} + 8D_{38} \quad (8.60)$$

$$B_{16}^{X(V)} = +4(D_{12} - D_{11}) + 16(D_{22} - D_{24}) + 4(D_{25} - D_{26}) + 8(D_{38} - D_{310}) \quad (8.61)$$

$$B_{17}^{X(V)} = +2D_{12} + 6D_{13} - 16(D_{22} - D_{24}) - 8(D_{25} - D_{26}) + 4D_{26} + 8(D_{38} - D_{310}) \quad (8.62)$$

$$B_{18}^{X(V)} = +4(D_{12} - D_{11}) - 4(D_{25} - D_{26}) + 8(D_{35} + D_{38} - 2D_{310}) \quad (8.63)$$

$$B_{19}^{X(V)} = +8(p_1 \cdot q_1)D_{12} + 8(p_2 \cdot q_1 - p_1 \cdot p_2)(D_{12} - D_{11}) - 4s_3 D_{13} - 8(p_1 \cdot p_2)(D_{22} - D_{24}) - 8(p_2 \cdot q_1)(D_{25} - D_{26}) + 8(p_1 \cdot q_1)D_{26} - 4s_3 D_{23} - s_3 D_{33} - 2(p_2 \cdot q_1)(D_{37} - D_{39}) - 2(p_1 \cdot p_2)(D_{38} - D_{310}) + 2(p_1 \cdot q_1)D_{39} - 2D_{313} \quad (8.64)$$

$$B_{21}^{X(V)} = +(-s_3 + 2(p_1 \cdot p_2) + 2(p_2 \cdot q_1))D_{13} + 2(s_3 - p_2 \cdot q_1)D_{23} - 2(p_1 \cdot q_1 - p_1 \cdot p_2)D_{26} + 2(p_2 \cdot q_1)(D_{25} - D_{26}) - 18D_{27} - s_3 D_{33} - 2(p_2 \cdot q_1)(D_{37} - D_{39}) - 2(p_1 \cdot p_2)(D_{38} - D_{310}) + 2(p_1 \cdot q_1)D_{39} + 6D_{313} \quad (8.65)$$

Similarly evaluating all the diagrams of Category II, Fig. 8.4, one obtains the following result for the total contribution of this category

$$B^{X(V)} \doteq -ig^2 \bar{v}(p_1) [s_W^4 B_{\nu\lambda}^V(-p_2, p_5, -q_1, m_e, \lambda, \lambda, M_W) + (s_W^2/4)(a - b\gamma_5)[B_{\nu\lambda}^V(-p_2, p_5, -q_1, m_e, \lambda, M_Z, M_W) + B_{\nu\lambda}^V(-p_2, p_5, -q_1, m_e, M_Z, \lambda, M_W)] + (1/16)(a^2 + b^2 - 2ab\gamma_5)[B_{\nu\lambda}^V(-p_2, p_5, -q_1, m_e, M_Z, M_Z, M_W)] + (s_W^2/4)(1 - \gamma_5)[B_{\nu\lambda}^V(-p_2, p_5, -q_1, m_{\nu_e}, M_W, M_W, \lambda)] + (c_W^2/4)(1 - \gamma_5)[B_{\nu\lambda}^V(-p_2, p_5, -q_1, m_{\nu_e}, M_W, M_W, M_Z)]u(p_2)e^{1\nu}e^{2\lambda} \quad (8.66)$$

The numerical results for the direct box diagrams with 3 vector bosons are listed in Tables 8.5, and 8.6.

8.5 Crossed Box Topology

As was pointed out before the CB is obtained from DB by crossing lines 2 and 3 in Fig. 8.2, essentially momentum p_1 will feed into the vertex of $W_\lambda^-(q_2)$ and p_2 will go into the vertex of $W_\nu^+(q_1)$ i.e. the reverse of the DB. Thence we expect that $p_2 \rightarrow p_1$ or $q_1 \rightarrow q_2$. All the diagrams and the momentum convention of CB are given in Fig. 8.5. Just as for the DB we start calculating Category III, by evaluating the first diagram in this category, Fig. 8.5. The loop integral for this case reads

$$B_{\nu\lambda}^{CX(H)} = (i)^4 (-ie)^2 (ieM_W)^2 \int \frac{d^n q}{2\pi^n} \frac{\bar{v}(p_1) N_{\lambda\nu} u(p_2)}{Q_1 Q_2 Q_3 Q_4} \quad (8.67)$$

$$N_{\lambda\nu} = \gamma_\lambda(\not{q})\gamma_\nu$$

$$Q_1 = (q^2 - m_c^2)$$

$$Q_2 = ((q + p_1)^2 - \lambda^2)$$

$$Q_3 = ((q + p_1 - p_5)^2 - \lambda^2)$$

$$Q_4 = ((q + p_1 - p_5 + q_1)^2 - M_W^2)$$

using the definition of the D integrals, Eq. 8.67 reduces to

$$B_{\nu\lambda}^{CX(H)} = (i/16\pi^2)(e)^4 M_W^2 \bar{v}(p_1) \gamma_\lambda [\not{q}_1 (D_{11} - D_{12}) - \not{q}_2 D_{12} + \not{q}_1 D_{13}] \gamma_\nu u(p_2) \quad (8.68)$$

Interchanging $(\nu \leftrightarrow \lambda)$ and $(p_1 \leftrightarrow -p_2)$ in the above, reproduces the DB case given in Eq. 8.26. Evaluating Eq 8.68 yields

$$B_3^{CX(H)} = -2D_{13}M_W^2 \quad (8.69)$$

$$B_4^{CX(H)} = -2D_{12}M_W^2 \quad (8.70)$$

$$B_5^{CX(H)} = +2D_{13}M_W^2 \quad (8.71)$$

$$B_7^{CX(H)} = -2(D_{12} - D_{11})M_W^2 \quad (8.72)$$

$$B_{19}^{CX(H)} = -2D_{13}M_W^2 \quad (8.73)$$

$$B_{21}^{CX(H)} = -D_{13}M_W^2 \quad (8.74)$$

From Eqs. (8.69...8.74) and (8.29...8.31) we observe the following relationships

$$B_{21}^{CX(H)} = -D_{13}M_W^2 \quad (8.75)$$

$$B_3^{CX(H)} = -B_4^{X(H)} + 2B_{21}^{CX(H)} \quad (8.76)$$

$$B_4^{CX(H)} = -B_3^{X(H)} + 2B_{21}^{CX(H)} \quad (8.77)$$

$$B_5^{CX(H)} = -B_5^{X(H)} - 2B_{21}^{CX(H)} \quad (8.78)$$

$$B_7^{CX(H)} = -B_8^{X(H)} \quad (8.79)$$

$$B_{19}^{CX(H)} = -B_{19}^{X(H)} + 2B_{21}^{CX(H)} \quad (8.80)$$

We have indicated the relationships (8.75...8.80) to display the crossing symmetry ($p_2 \rightarrow p_1$). Moreover it provides a cross check on our calculations, especially in the case of lengthy evaluations involved in categories II' and I'. After evaluating the rest of the diagrams in Category III' one has the following result for this category

$$\begin{aligned} B^{CX(H)} = & ig^2 \bar{v}(p_1) [s_W^4 B_{\nu\lambda}^{CH}(p_1, -p_5, q_1, m_e, \lambda, \lambda, M_W) - \\ & (s_W^4/4c_W^2)(a - b\gamma_5) [B_{\nu\lambda}^{CH}(p_1, -p_5, q_1, m_e, \lambda, M_Z, M_W) + \\ & B_{\nu\lambda}^{CH}(p_1, -p_5, q_1, m_e, M_Z, \lambda, M_W)] + (s_W^4/16c_W^4)(a^2 + b^2 - \\ & 2ab\gamma_5) [B_{\nu\lambda}^{CH}(p_1, -p_5, q_1, m_e, M_Z, M_Z, M_W)]] u(p_2) e^{1\nu} e^{2\lambda} \end{aligned} \quad (8.81)$$

The numerical results for the above crossed box diagrams are listed in Tables 8.4, and 8.6.

8.5.1 Crossed Box Topology of category I'

The loop integral for the CB three fermion diagram given in Fig. 8.5a

$$B_{\nu\lambda}^{CX(F)} = (i)^4 (ig/2\sqrt{2})^4 \int \frac{d^n q}{2\pi^n} \frac{\bar{v}(p_1) N_{\lambda\nu} u(p_2)}{Q_1 Q_2 Q_3 Q_4} \quad (8.82)$$

$$N_{\lambda\nu} = \gamma^\alpha(\not{q} + \not{p}_1)\gamma_\lambda(\not{q} + \not{p}_1 - \not{p}_5 + \not{q}_1)\gamma_\nu(\not{q} - \not{p}_5 + \not{p}_1)\gamma_\alpha 8(1 - \gamma_5)$$

$$Q_1 = (q^2 - M_W^2)$$

$$Q_2 = ((q + p_1)^2 - m_{\nu_e}^2)$$

$$Q_3 = ((q + p_1 - p_5)^2 - m_{\nu_e}^2)$$

$$Q_4 = ((q + p_1 - p_5 + q_1)^2 - m_e^2)$$

Applying the same arguments as in Category III' (i.e. of crossing symmetry we deduce the following relationships for the form factors involved here

$$\begin{aligned} B_{21}^{CX(F)} = & 4(p_1 \cdot p_2)D_0 + 4(p_1 \cdot p_2)D_{11} + 4(p_1 \cdot p_2)D_{22} - 2(s_3 - \\ & 2(p_2 \cdot q_1))D_{23} - 4(p_1 \cdot p_2)(D_{24} - D_{25}) - 4(p_1 \cdot p_2)D_{26} + 4D_{27} + \\ & 2s_3D_{33} + 4(p_2 \cdot q_1)D_{37} + 4(p_1 \cdot p_2)D_{38} - 4(p_1 \cdot q_1 + p_2 \cdot q_1)D_{39} - \\ & 4(p_1 \cdot p_2)D_{310} - 12D_{313} \end{aligned} \quad (8.83)$$

$$B_3^{CX(F)} = -B_4^{X(F)} + 2B_{21}^{CX(F)} \quad (8.84)$$

$$B_4^{CX(F)} = -B_3^{X(F)} + 2B_{21}^{CX(F)} \quad (8.85)$$

$$B_5^{CX(F)} = -B_5^{X(F)} - 2B_{21}^{CX(F)} \quad (8.86)$$

$$B_7^{CX(F)} = -B_8^{X(F)} \quad (8.87)$$

$$B_8^{CX(F)} = -B_7^{X(F)} \quad (8.88)$$

$$B_{11}^{CX(F)} = -B_{12}^{X(F)} \quad (8.89)$$

$$B_{12}^{CX(F)} = -B_{11}^{X(F)} \quad (8.90)$$

$$B_{15}^{CX(F)} = -B_{18}^{X(F)} \quad (8.91)$$

$$B_{16}^{CX(F)} = -B_{17}^{X(F)} \quad (8.92)$$

$$B_{17}^{CX(F)} = -B_{16}^{X(F)} \quad (8.93)$$

$$B_{18}^{CX(F)} = -B_{15}^{X(F)} \quad (8.94)$$

$$B_{19}^{CX(F)} = -B_{19}^{X(F)} + 2B_{21}^{CX(F)} \quad (8.95)$$

Having relations (8.83...8.95) one can immediately write the total contribution of this category as

$$B_{\nu\lambda}^{CX(F)} \doteq i g^2 \bar{v}(p_1) [+ (1/8) A_{\nu\lambda}^{CF}(p_1, -p_5, q_1, M_W, m_{\nu_e}, m_{\nu_e}, m_e)(1 - \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.96)$$

The numerical results for the above crossed box diagram are listed in Table 8.3.

8.5.2 Crossed Box Topology of category II'

We turn to CB Category II' and choose to evaluate $e\gamma\gamma W$ diagram Fig. 8.5b The loop integral for this case is given by

$$B_{\nu\lambda}^{CX(V)} = -(i)^4 (-ie)^4 \int \frac{d^n q}{2\pi^n} \frac{\bar{v}(p_1) N_{\lambda\nu} u(p_2)}{Q_1 Q_2 Q_3 Q_4} \quad (8.97)$$

$$N_{\lambda\nu} = \gamma^{\mu_1} (q) \gamma^{\mu_2} \Gamma_{\mu_1 \lambda \alpha} \Gamma_{\mu_2 \alpha \nu}$$

$$Q_1 = (q^2 - m_e^2)$$

$$Q_2 = ((q + p_1)^2 - \lambda^2)$$

$$Q_3 = ((q + p_1 - p_5)^2 - \lambda^2)$$

$$Q_4 = ((q + p_1 - p_5 + q_1)^2 - M_W^2)$$

Here the Γ 's are the usual three vector vertices. We expect the same type of relations between form factors here as in X, it turns out indeed to be the case. We will not repeat these. We merely replace F in (8.84...8.95) by V. The $B_{21}^{CX(V)}$ is given by

$$B_{21}^{CX(V)} = (-s_3 + 2(p_1 \cdot p_2) + 2(p_1 \cdot q_1)) D_{13} + 2(-s_3 + p_1 \cdot q_1) D_{23} - 2(p_1 \cdot p_2 - p_2 \cdot q_1) D_{26} - 2(p_1 \cdot q_1)(D_{25} - D_{26}) - s_3 D_{33} - 2(p_1 \cdot q_1)(D_{37} - D_{39}) - 2(p_1 \cdot p_2)(D_{38} - D_{310}) + 2(p_2 \cdot q_1) D_{39} + 6 D_{313} \quad (8.98)$$

Bearing in mind the contributions from other 3 diagrams in category II' we have the total

$$B^{CX(V)} \doteq -ig^2 \bar{v}(p_1) [+s_W^4 B_{\nu\lambda}^{CV}(p_1, -p_5, q_1, m_e, \lambda, \lambda, M_W) + (s_W^2/4)(a - b\gamma_5)[B_{\nu\lambda}^{CV}(p_1, -p_5, q_1, m_e, \lambda, M_Z, M_W) + B_{\nu\lambda}^{CV}(p_1, -p_5, q_1, m_e, M_Z, \lambda, M_W)] + (1/16)(a^2 + b^2 - 2ab\gamma_5)[B_{\nu\lambda}^{CV}(p_1, -p_5, q_1, m_e, M_Z, M_Z, M_W)]] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.99)$$

The numerical results for the above crossed box diagrams with 3 vector bosons are listed in Tables 8.5, and 8.6.

8.5.3 Crossed Box Topology of category IV with two internal fermion lines

The following expressions were obtained for the first set of diagrams of category IV

$$B_3^{CX(2F)} = -8(p_2 \cdot q_2)D_{11} + 8(p_2 \cdot q_1 - p_1 \cdot p_2)D_{12} + 8(p_2 \cdot q_2 - p_2 \cdot q_1 + p_1 \cdot p_2)D_{13} + 4s_4 D_{12} - 2s_4 D_{13} + 6X + 16D_{27} + 4(q_1 \cdot q_2 - p_2 \cdot q_2 - p_2 \cdot q_1 + s_4)D_{22} - 4(p_1 \cdot p_2 + p_1 \cdot q_2)D_{23} - 8(p_2 \cdot q_2)D_{24} + 4(2(p_2 \cdot q_2) - p_2 \cdot q_1)D_{25} + 4(p_1 \cdot p_2 + p_2 \cdot q_1 + p_1 \cdot q_2 - s_4)D_{26} + 2s_4 D_{32} - 4(p_2 \cdot q_2)D_{36} - 4(p_1 \cdot p_2)D_{37} - 2(s_4 + 2(p_1 \cdot q_2))D_{38} + 4(p_1 \cdot q_2)D_{39} + 4(p_2 \cdot q_2 + p_2 \cdot q_1)D_{310} + 20(D_{313} - D_{312}) \quad (8.100)$$

where X is given by

$$X = s_4 D_{22} - 2(p_2 \cdot q_2)D_{24} + 2(p_1 \cdot p_2)D_{25} - 2(p_1 \cdot q_2)D_{26} - 4D_{27} \quad (8.101)$$

$$B_4^{CX(2F)} = -8(p_1 \cdot q_1)D_{12} + 8(p_1 \cdot q_1)D_{13} + 4s_3 D_{12} + 4X + 8D_{27} + 8(p_2 \cdot q_2)D_{21} + 4(q_1 \cdot q_2 - p_2 \cdot q_2 - p_2 \cdot q_1 + s_4)D_{22} + 4(p_2 \cdot q_1 - q_1 \cdot q_2 - p_2 \cdot q_2 - s_4)D_{24} + 4(p_1 \cdot q_1 - p_1 \cdot q_2 - p_1 \cdot p_2)D_{25} + 4(p_1 \cdot p_2 + p_1 \cdot q_2 - p_1 \cdot q_1)D_{26} + 2s_4 D_{32} + 4(p_2 \cdot q_2)D_{34} - 4(p_1 \cdot p_2)D_{35} - 2(s_4 + 2(p_2 \cdot q_2))D_{36} - 4(p_1 \cdot q_2)D_{38} + 4(p_1 \cdot q_2 + p_1 \cdot p_2)D_{310} + 20(D_{311} - D_{312}) \quad (8.102)$$

$$B_5^{CX(2F)} = 8(p_2 \cdot q_2)(D_{11} - D_{13}) + 4s_4(D_{13} - D_{12}) - 6X - 8D_{27} \quad (8.103)$$

$$B_7^{CX(2F)} = 4X + 8D_{27} - 4s_3 D_{22} + 8(p_2 \cdot q_1)D_{25} + 4(p_1 \cdot q_1 - p_2 \cdot q_1)D_{26} + 2s_4 D_{38} + 4(p_1 \cdot p_2)D_{37} - 4(p_1 \cdot q_2)D_{39} - 4(p_2 \cdot q_2)D_{310} - 4D_{313} \quad (8.104)$$

$$B_8^{CX(2F)} = 8(p_1 \cdot q_1)(D_{12} - D_{13}) + 2s_3(D_{11} - D_{12} - 2D_{13}) + 4(p_2 \cdot q_1)D_{21} - 4(q_1 \cdot q_2)D_{22} + 4(-q_1 \cdot q_2 + p_2 \cdot q_1 + 2(p_1 \cdot q_1) - s_3)D_{24} + 4(p_1 \cdot q_1)(D_{26} - D_{25}) + 2s_4D_{36} + 4(p_1 \cdot p_2)D_{35} - 4(p_2 \cdot q_2)D_{34} - 4(p_1 \cdot q_2)D_{310} - 4D_{311} \quad (8.105)$$

$$B_{15}^{CX(2F)} = 8(D_{26} - D_{23}) + 8(D_{38} - D_{39}) \quad (8.106)$$

$$B_{16}^{CX(2F)} = 16(D_{26} - D_{25}) + 8(D_{38} - D_{310}) \quad (8.107)$$

$$B_{17}^{CX(2F)} = 8(D_{13} - D_{12}) + 8(D_{25} - D_{24} - D_{26} + D_{22}) + 8(D_{36} - D_{310}) \quad (8.108)$$

$$B_{18}^{CX(2F)} = 8(D_{22} - D_{24}) + 8(D_{36} - D_{34}) \quad (8.109)$$

$$B_{19}^{CX(2F)} = 4s_4D_{12} - 8(p_2 \cdot q_2)(D_{11} - D_{13} + 6X + 16D_{27} + 2(s_4D_{32} - 2(p_2 \cdot q_2)D_{36} - 2(p_1 \cdot q_2)D_{38} + 2(p_1 \cdot p_2)D_{310}) - 4D_{312} \quad (8.110)$$

$$B_{21}^{CX(2F)} = 3s_3D_{12} - 4(p_2 \cdot q_2)(D_{11} - D_{13}) - 4(p_1 \cdot p_2)(D_{12} - D_{13}) + 5X + 14D_{27} + 2(+ (q_1 \cdot q_2)D_{22} - (p_2 \cdot q_1)D_{24} - (p_1 \cdot q_1)D_{26}) \quad (8.111)$$

We now give the expressions for the second set of diagrams in category IV

$$B_3^{CX(2F)} = -8(p_1 \cdot q_1)(D_{11} + D_{12} - 2D_{13}) - 2s_3(D_{11} - D_{12}) + 4s_3D_{12} + 6X + 8D_{27} - 4(p_1 \cdot q_2)D_{21} + 4(q_1 \cdot q_2)D_{22} - 4(p_2 \cdot q_2)(D_{24} - D_{25} - D_{26}) - 2s_3D_{36} + 4(p_1 \cdot q_1)D_{34} - 4(p_1 \cdot p_2)D_{35} + 4(p_2 \cdot q_1)D_{310} + 4D_{311} \quad (8.112)$$

$$B_4^{CX(2F)} = -8(p_1 \cdot q_1)(D_{11} - D_{13}) + 4s_3(D_{12} - D_{13}) + 2X + 8D_{27} - 8(p_1 \cdot q_2)(D_{25} - D_{26}) - 4(s_4)D_{26} + 2s_3D_{38} - 4(p_1 \cdot p_2)D_{37} + 4(p_2 \cdot q_1)D_{39} + 4(p_1 \cdot q_1)D_{310} + 4D_{313} \quad (8.113)$$

$$B_5^{CX(2F)} = 8(p_1 \cdot q_1)D_{12} - 8(p_1 \cdot p_2 - p_2 \cdot q_1)D_{13} - 4s_3D_{12} - 2s_3D_{32} + 4(p_1 \cdot q_1)D_{36} + 4(p_2 \cdot q_1)D_{38} - 4(p_1 \cdot p_2)D_{310} + 20D_{312} + 8(p_1 \cdot q_1)D_{24} - 8(p_2 \cdot q_1)D_{26} - 4D_{27} \quad (8.114)$$

$$B_7^{CX(2F)} = 2X + 4D_{27} + 8(p_1 \cdot q_1)D_{21} + 4(p_2 \cdot q_2 - p_1 \cdot p_2 - p_2 \cdot q_1)D_{25} + 2s_3D_{36} - 4(p_1 \cdot q_1)D_{34} + 4(p_1 \cdot p_2)D_{35} - 4(p_2 \cdot q_1)D_{310} - 20D_{311} \quad (8.115)$$

$$B_8^{CX(2F)} = 8(p_1 \cdot q_1)(D_{11} - D_{13} + 2s_3D_{13} + 16D_{27} + 4X + 4(p_1 \cdot q_2 - 2(p_1 \cdot q_1))D_{25} + 4(p_2 \cdot q_1 + p_1 \cdot p_2)D_{23} + 4(s_3 - p_2 \cdot q_1 - p_1 \cdot q_1)D_{26} + 2s_4D_{38} - 4(p_1 \cdot p_2)D_{37} - 4(p_2 \cdot q_1)D_{39} - 4(p_1 \cdot q_1)D_{310} - 20D_{313} \quad (8.116)$$

$$B_{15}^{CX(2F)} = 8(D_{22} - D_{24}) + 8(D_{36} - D_{34}) \quad (8.117)$$

$$B_{16}^{CX(2F)} = 16(D_{26} - D_{25}) + 8(D_{38} - D_{310}) \quad (8.118)$$

$$B_{17}^{CX(2F)} = 8(D_{25} - D_{26}) + 8(D_{22} - D_{24}) + 8(D_{36} - D_{310}) + 8(D_{13} - D_{12}) \quad (8.119)$$

$$B_{18}^{CX(2F)} = 8(D_{26} - D_{23}) + 8(D_{38} - D_{39}) \quad (8.120)$$

$$B_{19}^{CX(2F)} = 4s_3 D_{12} - 8(p_1 \cdot q_1)(D_{11} - D_{13}) + 6X + 16D_{27} + 2(s_4 D_{32} - 2(p_2 \cdot q_2)D_{36} - 2(p_1 \cdot q_2)D_{38} + 2(p_1 \cdot p_2)D_{310}) - 4D_{312} \quad (8.121)$$

$$B_{21}^{CX(2F)} = 3s_3 D_{12} - 4(p_1 \cdot q_1)(D_{11} - D_{13}) - 4(p_1 \cdot p_2)(D_{12} - D_{13}) + 5X + 14D_{27} + 2((q_1 \cdot q_2)D_{22} - (p_2 \cdot q_1)D_{24} - (p_1 \cdot q_1)D_{26}) \quad (8.122)$$

Bearing in mind the contributions from all diagrams in Category IV we have the total

$$B^{CX(2F)} \doteq i g^2 \bar{v}(p_1) [+ (s_W^2/4) B_{\nu\lambda}^{CX(2F)}(-p_2, q_2, -p_1, m_{\nu e}, M_W, \lambda, m_e)(1 - \gamma_5) + (1 - 2s_W^2)/4 B_{\nu\lambda}^{CX(2F)}(-p_2, q_2, -p_1, m_{\nu e}, M_W, M_Z, m_e)(1 - \gamma_5) + (s_W^2/4) B_{\nu\lambda}^{CX(2F)}(p_1, -q_1, p_2, m_{\nu e}, M_W, \lambda, m_e)(1 - \gamma_5) + (1 - 2s_W^2)/4 B_{\nu\lambda}^{CX(2F)}(p_1, -q_1, p_2, m_{\nu e}, M_W, M_Z, m_e)(1 - \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.123)$$

The numerical results for the above crossed box diagrams are listed in Table 8.7. This concludes the calculation of the 32 box diagrams we encounter in the Weinberg-Salam model of electroweak interactions. Next we proceed to give the extra diagrams introduced as a result of supersymmetrizing the $SU(2) \times U(1)$ Lagrangian.

8.6 The Supersymmetry Box Diagrams

To enumerate all the SUSY box diagrams one may proceed as follows, by replacing all the standard model internal particles by their S -partners. This is not enough,

for supersymmetry also introduces some new vertices, for example there is no four point interaction between a pair of fermions and bosons but SUSY introduces a four point interaction between a pair of scalar leptons and vector bosons, see Fig. 8.6. However this diagram gives a contribution $\sim m_e$ and therefore can be neglected. Also note that when we replace the SM particles by their S -partners some of the new vertices may not be allowed by SUSY. A prime example is the SM triangular topology Fig. 8.3. Their SUSY equivalent is clearly not allowed for there is no two photino -two vector boson vertex.

8.6.1 The Supersymmetric Direct Box

All the SUSY direct box diagrams are given in two categories Fig. 8.7. Note that Category IA is the SUSY equivalent of Category I of Fig. 8.4 whereas Category IIA is generated from Category II and III of Fig. 8.4. We now turn to evaluate category IA and write the loop integral for diagram 1,

$$B_{\nu\lambda}^{X(SS)} = (i)^4 (-e^2/2)(-g^2/2) \int \frac{d^n q}{2\pi^n} \frac{\bar{u}(p_1) N_{\nu\lambda} u(p_2)}{Q_1 Q_2 Q_3 Q_4} \quad (8.124)$$

$$N_{\nu\lambda} = (\not{q})(-2q + \not{f})_\nu (-2q + \not{n})_\lambda 2(1 - \gamma_5)$$

$$Q_1 = (q^2 - m_{\tilde{\gamma}}^2)$$

$$Q_2 = ((q - p_2)^2 - m_{\tilde{e}}^2)$$

$$Q_3 = ((q - p_2 + p_5)^2 - m_{\tilde{e}}^2)$$

$$Q_4 = ((q - p_2 + p_5 - q_1)^2 - m_{\tilde{\nu}_e}^2)$$

$$\text{Here } f = 2p_2 - 2q_2 - q_1, n = 2p_2 - q_2$$

We recall the notation for the masses of the supersymmetric particles

mass of photino= $m_{\tilde{\gamma}}$, mass of zino= $m_{\tilde{z}}$, mass of wino= $m_{\tilde{w}}$

mass of higgsino= $m_{\tilde{h}}$, mass of selectron= $m_{\tilde{e}}$, mass of sneutrino= $m_{\tilde{\nu}_e}$.

We have followed the same momentum convention as for the SM. The form factors reduce for the above case to

$$B_3^{X(SS)} = -4(D_{312} + D_{27}) \quad (8.125)$$

$$B_4^{X(SS)} = -4(D_{312} - D_{311}) \quad (8.126)$$

$$B_5^{X(SS)} = 4D_{313} \quad (8.127)$$

$$B_7^{X(SS)} = -4D_{312} \quad (8.128)$$

$$B_6^{X(SS)} = 4(D_{311} - D_{312}) + 4D_{27} \quad (8.129)$$

$$B_{11}^{X(SS)} = 4(D_{39} + D_{23}) \quad (8.130)$$

$$B_{12}^{X(SS)} = 4(D_{39} - D_{37}) \quad (8.131)$$

$$B_{15}^{X(SS)} = -4(D_{38} + D_{26}) \quad (8.132)$$

$$B_{16}^{X(SS)} = 4(D_{310} - D_{38}) \quad (8.133)$$

$$B_{17}^{X(SS)} = 4(D_{310} - D_{38}) + 4(D_{13} + D_{25}) \quad (8.134)$$

$$B_{18}^{X(SS)} = -4(D_{38} + D_{35} - 2D_{310}) + 4(D_{26} - D_{25}) \quad (8.135)$$

$$B_{19}^{X(SS)} = 4D_{313} \quad (8.136)$$

The total contribution of category IA may thus be written as

$$B^{X(SS)} = i g^2 \bar{v}(p_1) [(s_W^2/2) B_{\nu\lambda}^{SS}(-p_2, p_5, -q_1, m_{\tilde{\gamma}}, m_{\tilde{e}}, m_{\tilde{e}}, m_{\tilde{\nu}_e})(1 - \gamma_5) + (1/32c_W^2) B_{\nu\lambda}^{SS}(-p_2, p_5, -q_1, m_{\tilde{z}}, m_{\tilde{e}}, m_{\tilde{e}}, m_{\tilde{\nu}_e})(a + 1)^2(1 - \gamma_5) + (1/4) B_{\nu\lambda}^{SS}(-p_2, p_5, -q_1, m_{\tilde{\omega}}, m_{\tilde{\nu}_e}, m_{\tilde{\nu}_e}, m_{\tilde{e}})(1 - \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.137)$$

The numerical results for the above direct box diagrams with 3 scalars in supersymmetry are listed in Table 8.8. In the SUSY -limit this (i.e. Eq. 8.137) reduces

to

$$\begin{aligned}
 B^{X(SS)} \doteq & ig^2 \bar{v}(p_1) [(s_W^2/2) B_{\nu\lambda}^{SS}(-p_2, p_5, -q_1, \lambda, m_e, m_e, m_{\nu_e})(1 - \gamma_5) \\
 & + (1/32c_W^2) B_{\nu\lambda}^{SS}(-p_2, p_5, -q_1, M_Z, m_e, m_e, m_{\nu_e})(a+1)^2(1 - \gamma_5) \\
 & + (1/4) B_{\nu\lambda}^{SS}(-p_2, p_5, -q_1, M_W, m_{\nu_e}, m_{\nu_e}, m_e)(1 - \gamma_5)] u(p_2) e^{i\nu} e^{2\lambda}
 \end{aligned} \quad (8.138)$$

We now turn to category IIA and choose to evaluate the $\bar{e}\tilde{\gamma}\tilde{\gamma}\tilde{\omega}$ diagram in Fig 8.7.

The loop integral for this diagram reduces after some simple manipulations to

$$B_{\nu\lambda}^{X(SF)} = -g^4 s_W^4 \int \frac{d^n q}{2\pi^n} \frac{\bar{v}(p_1) N_{\nu\lambda} u(p_2)}{Q_1 Q_2 Q_3 Q_4} \quad (8.139)$$

$$N_{\nu\lambda} = (\not{q} + \not{p}_1 - m_{\tilde{\gamma}}) \gamma_\nu (\not{q} - \not{q}_1 + \not{p}_1 - m_{\tilde{\omega}}) \gamma_\lambda (\not{q} - \not{p}_2 - m_{\tilde{\gamma}}) (1 - \gamma_5)$$

$$Q_1 = (q^2 - m_{\tilde{e}}^2)$$

$$Q_2 = ((q - p_2)^2 - m_{\tilde{\gamma}}^2)$$

$$Q_3 = ((q - p_2 + p_5)^2 - m_{\tilde{\gamma}}^2)$$

$$Q_4 = ((q - p_2 + p_5 - q_1)^2 - m_{\tilde{\omega}}^2)$$

After a straightforward calculation, we obtain for the form factors the following expressions

$$\begin{aligned}
 B_3^{X(SF1)} = & 4D_{27} + 4(p_1 \cdot p_2 - p_2 \cdot q_1)(D_{25} - D_{26}) + 4(p_1 \cdot q_1 - s_3)D_{23} + \\
 & 2s_3 D_{26} + 4(p_1 \cdot p_2 - p_2 \cdot q_1)(D_{22} - D_{24}) - 2s_3 D_{33} - \\
 & 4(p_2 \cdot q_1)(D_{37} - D_{39}) - 4(p_1 \cdot p_2)(D_{38} - D_{310}) + 4(p_1 \cdot q_1)D_{39} - \\
 & 4D_{312} + 12D_{313}
 \end{aligned} \quad (8.140)$$

$$\begin{aligned}
 B_4^{X(SF1)} = & +2s_3(D_{25} - D_{26}) + 4(p_1 \cdot q_1)(D_{22} - D_{24}) - 4(p_1 \cdot p_2)(D_{32} + \\
 & D_{34} - 2D_{36}) + 4(p_2 \cdot q_1)(D_{35} + D_{38} - 2D_{310}) + 4(p_1 \cdot q_1)(D_{38} - \\
 & D_{310}) + 2s_3(D_{37} - D_{39}) - 8(D_{311} - D_{312})
 \end{aligned} \quad (8.141)$$

$$\begin{aligned}
 B_5^{X(SF1)} = & -4D_{27} - 4(p_1 \cdot p_2 - p_2 \cdot q_1)(D_{25} - D_{26}) - 2(2(p_1 \cdot q_1) - s_3)D_{23} + \\
 & 2s_3 D_{33} + 4(p_2 \cdot q_1)(D_{37} - D_{39}) + 4(p_1 \cdot p_2)(D_{38} - D_{310}) - \\
 & 4(p_1 \cdot q_1)D_{39} - 8D_{313}
 \end{aligned} \quad (8.142)$$

$$B_7^{X(SF1)} = +4D_{27} + 4(p_1 \cdot p_2 - p_2 \cdot q_1)(D_{24} - D_{22}) + 2(2(p_1 \cdot q_1) - s_3)D_{26} - 2s_3D_{39} + 4(p_1 \cdot p_2)(D_{36} - D_{32}) + 4(p_1 \cdot q_1 + p_2 \cdot q_1)D_{38} - 4(p_2 \cdot q_1)D_{310} + 8D_{312} \quad (8.143)$$

$$B_8^{X(SF1)} = +4(p_1 \cdot q_1)(D_{24} - D_{22}) - 2s_3(D_{25} - D_{26}) + 4(D_{311} - D_{312}) \quad (8.144)$$

$$B_{11}^{X(SF1)} = +4(D_{23} + D_{39}) \quad (8.145)$$

$$B_{12}^{X(SF1)} = +4(D_{26} - D_{25}) + 4(D_{39} - D_{37}) \quad (8.146)$$

$$B_{15}^{X(SF1)} = -4(D_{26} + D_{38}) \quad (8.147)$$

$$B_{16}^{X(SF1)} = +4(D_{24} - D_{22}) + 4(D_{310} - D_{38}) \quad (8.148)$$

$$B_{17}^{X(SF1)} = +4(D_{25} - D_{26}) + 4(D_{22} - D_{24}) + 4(D_{310} - D_{38}) \quad (8.149)$$

$$B_{18}^{X(SF1)} = +4(2D_{310} - D_{35} - D_{38}) \quad (8.150)$$

$$B_{19}^{X(SF1)} = +4(D_{313} + D_{27}) \quad (8.151)$$

$$B_{21}^{X(SF1)} = 2D_{27} + 2(p_1 \cdot p_2)(D_{25} - D_{26}) + (2(p_1 \cdot q_1) - s_3)D_{23} + 2(p_1 \cdot p_2)(D_{22} - D_{24}) - s_3D_{33} - 2(p_2 \cdot q_1)(D_{37} - D_{39}) + 2(p_1 \cdot q_1)D_{39} - 2(p_1 \cdot p_2)(D_{38} - D_{310}) + 6D_{313} \quad (8.152)$$

Thus the photino contribution can be written as

$$B^{X(SF)} = -ig^2 \bar{v}(p_1) [+s_W^4 B_{\nu\lambda}^{SF1}(-p_2, p_5, -q_1, m_{\tilde{e}}, m_{\tilde{\gamma}}, m_{\tilde{\gamma}}, m_{\tilde{\omega}})(1 - \gamma_5) + s_W^4 (m_{\tilde{\gamma}}^2) B_{\nu\lambda}^{SF2}(1 + \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.153)$$

where $B^{X(SF1)}$ is made up of the collection of the form factors given in (8.140...8.152)

and $B^{X(SF2)}$ is generated from the following B's

$$B_{21}^{X(SF2)} = -(D_0 + D_{13}) \quad (8.154)$$

$$B_3^{X(SF2)} = 2(D_{12} - D_{13}) \quad (8.155)$$

$$B_8^{X(SF2)} = 2(D_{12} - D_{11}) \quad (8.156)$$

Clearly in the SUSY -limit 8.153 reduces to

$$B^{X(SF)} \doteq -ig^2 \bar{v}(p_1) [+ s_W^4 B_{\nu\lambda}^{SF1}(-p_2, p_5, -q_1, m_e, \lambda, \lambda, M_H)(1 - \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.157)$$

Next we move on to diagram 2 of category IIA the total contribution of which is given by

$$B^{X(SF)} \doteq -ig^2 \bar{v}(p_1) [+ (s_W^2/2)(1 - 2s_W^2) B_{\nu\lambda}^{SF1}(-p_2, p_5, -q_1, m_e, m_z, m_{\tilde{\gamma}}, m_{\tilde{\omega}})(1 - \gamma_5) + (s_W^2/4c_W)(1 - 2s_W^2)(m_{\tilde{z}} m_{\tilde{\omega}}) B_{\nu\lambda}^{SF4}(1 - \gamma_5) + s_W^4(m_{\tilde{\gamma}} m_{\tilde{z}}) B_{\nu\lambda}^{SF2}(1 + \gamma_5) - (s_W^4/2c_W)(m_{\tilde{\gamma}} m_{\tilde{\omega}}) B_{\nu\lambda}^{SF4}(1 + \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.158)$$

Here $B^{X(SF1)}$, $B^{X(SF2)}$ are defined by form factors given in (8.140...8.152) and (8.154...8.156), respectively whereas $B^{X(SF3)}$ is defined by the following B's

$$B_{21}^{X(SF3)} = +D_{13} \quad (8.159)$$

$$B_3^{X(SF3)} = 2D_{12} - 2(D_{12} - D_{13}) \quad (8.160)$$

$$B_4^{X(SF3)} = 2(D_{12} - D_{11}) \quad (8.161)$$

$$B_8^{X(SF3)} = -2(D_{12} - D_{11}) \quad (8.162)$$

and $B^{X(SF4)}$ by

$$B_{21}^{X(SF4)} = +D_{13} \quad (8.163)$$

$$B_3^{X(SF4)} = -2(D_{12} - D_{13}) \quad (8.164)$$

$$B_5^{X(SF4)} = -2D_{13} \quad (8.165)$$

$$B_7^{X(SF4)} = 2D_{12} \quad (8.166)$$

Similarly the total contribution of diagram 3 is given by (which is only dissimilar from diagram 2 in that positions of the photino and zino are interchanged),

$$B^{X(SF)} \doteq -ig^2 \bar{v}(p_1) [+ (s_W^2/2)(1 - 2s_W^2) B_{\nu\lambda}^{SF1}(-p_2, p_5, -q_1, m_{\tilde{e}}, m_{\tilde{\gamma}}, m_{\tilde{z}}, m_{\tilde{\omega}})(1 - \gamma_5) + (s_W^2/4c_W)(1 - 2s_W^2)(m_{\tilde{z}} m_{\tilde{\omega}}) B_{\nu\lambda}^{SF4}(1 - \gamma_5) + s_W^4(m_{\tilde{\gamma}} m_{\tilde{z}}) B_{\nu\lambda}^{SF2}(1 + \gamma_5) - (s_W^4/2c_W)(m_{\tilde{\gamma}} m_{\tilde{\omega}}) B_{\nu\lambda}^{SF4}(1 + \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.167)$$

Note that the interchange of the photino and zino has led to change in the positions of $B^{X(SF4)}$ and $B^{X(SF3)}$. The total zino contribution is a little bit more involved than the previous diagrams in category IIA primarily due to its more elaborate coupling with the wino. After some algebraic manipulations the total contribution from the zino may be written as

$$\begin{aligned}
 B^{X(SF)} \doteq & -ig^2 \bar{v}(p_1) [+ (1 - \frac{2s_W^2}{4}) [B_{\nu\lambda}^{SF1}(-p_2, p_5, -q_1, m_{\tilde{e}}, m_{\tilde{z}}, m_{\tilde{z}}, m_{\tilde{\omega}})(1 - \gamma_5) + \\
 & (m_{\tilde{z}}^2/4c_W^2) B_{\nu\lambda}^{SF2}(1 - \gamma_5) + (m_{\tilde{\omega}} m_{\tilde{z}}/2c_W) B_{\nu\lambda}^{SF3}(1 - \gamma_5) + (m_{\tilde{\omega}} m_{\tilde{z}}/2c_W) B_{\nu\lambda}^{SF4}(1 - \gamma_5)] \\
 & + (s_W^4) [B_{\nu\lambda}^{SF1}(-p_2, p_5, -q_1, m_{\tilde{e}}, m_{\tilde{z}}, m_{\tilde{z}}, m_{\tilde{\omega}})(1 + \gamma_5) + \\
 & ((m_{\tilde{z}}^2/4c_W^2) B_{\nu\lambda}^{SF2}(1 + \gamma_5) + (m_{\tilde{\omega}} m_{\tilde{z}}/2c_W) B_{\nu\lambda}^{SF3}(1 + \gamma_5) + \\
 & (m_{\tilde{\omega}} m_{\tilde{z}}/2c_W) B_{\nu\lambda}^{SF4}(1 + \gamma_5)]] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.168)
 \end{aligned}$$

Finally we turn to diagrams 5, 6 and 7 of category IIA. These are given respectively by

$$B^{X(SF)} \doteq -ig^2 \bar{v}(p_1) [+ (s_W^2/2) B_{\nu\lambda}^{SF1}(-p_2, p_5, -q_1, m_{\tilde{\nu}_e}, m_{\tilde{\omega}}, m_{\tilde{\omega}}, m_{\tilde{z}})(1 - \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.169)$$

$$\begin{aligned}
 B^{X(SF)} \doteq & -ig^2 \bar{v}(p_1) [+ (c_W^2/2) B_{\nu\lambda}^{SF1}(-p_2, p_5, -q_1, m_{\tilde{\nu}_e}, m_{\tilde{\omega}}, m_{\tilde{\omega}}, m_{\tilde{z}})(1 - \gamma_5) \\
 & + (c_W m_{\tilde{z}} m_{\tilde{\omega}}/4) B_{\nu\lambda}^{SF3}(1 - \gamma_5) + (m_{\tilde{\omega}} m_{\tilde{z}} c_W/4) B_{\nu\lambda}^{SF4}(1 - \gamma_5) \\
 & + (m_{\tilde{\omega}}^2/8) B_{\nu\lambda}^{SF2}(1 - \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.170)
 \end{aligned}$$

and

$$B^{X(SF)} \doteq -ig^2 \bar{v}(p_1) [+ (m_{\tilde{\omega}}^2/8) B_{\nu\lambda}^{SF2}(-p_2, p_5, -q_1, m_{\tilde{\nu}_e}, m_{\tilde{\omega}}, m_{\tilde{\omega}}, m_{\tilde{z}})(1 - \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.171)$$

The numerical results for the direct box diagrams of category IIA with 3 fermions in supersymmetry are listed in Tables 8.10 and 8.11.

8.6.2 The Supersymmetric Crossed Box Diagrams

Following the same arguments as in the Weinberg-Salam model one can obtain the SUSY CB from the SUSY DB; the only diagrams allowed are given in Fig. 8.8. Let us consider category IA' (see Fig. 8.8) for momentum convention. For obvious

reasons we have followed the same momentum convention as the SM crossed box.

The loop integral for the CB $\bar{\omega}\bar{\nu}_e\bar{\nu}_e\bar{e}$ diagram of category IA is given by

$$B_{\nu\lambda}^{CX(SS)} = (i)^4(-g^2/4)(-g^2/2) \int \frac{d^n q}{2\pi^n} \frac{\bar{v}(p_1)N_{\lambda\nu}u(p_2)}{Q_1 Q_2 Q_3 Q_4} \quad (8.172)$$

$$N_{\lambda\nu} = (\not{q})(-2q + f')_\nu(-2q + n')_\lambda 2(1 - \gamma_5)$$

$$Q_1 = (q^2 - m_\omega^2)$$

$$Q_2 = ((q + p_1)^2 - m_{\bar{\nu}_e}^2)$$

$$Q_3 = ((q + p_1 - p_5)^2 - m_{\bar{\nu}_e}^2)$$

$$Q_4 = ((q + p_1 - p_5 + q_1)^2 - m_e^2)$$

where $f' = p_5 - q_1 - p_1 + p_2$

and $n' = p_5 - 2p_1 - q_1$

The form factors reduce in the above case to

$$B_4^{CX(SS)} = 4(D_{312} + D_{27}) \quad (8.173)$$

$$B_4^{CX(SS)} = -B_3^{X(SS)} \quad (8.174)$$

$$B_3^{CX(SS)} = 4(D_{312} - D_{311}) = -B_4^{X(SS)} \quad (8.175)$$

$$B_3^{CX(SS)} = -B_4^{X(SS)} \quad (8.176)$$

$$B_5^{CX(SS)} = -4D_{313} = -B_5^{X(SS)} \quad (8.177)$$

$$B_5^{CX(SS)} = -B_5^{X(SS)} \quad (8.178)$$

$$B_8^{CX(SS)} = 4D_{312} = -B_7^{X(SS)} \quad (8.179)$$

$$B_8^{CX(SS)} = -B_7^{X(SS)} \quad (8.180)$$

$$B_7^{CX(SS)} = -4(D_{311} - D_{312}) - 4D_{27} = -B_8^{X(SS)} \quad (8.181)$$

$$B_7^{CX(SS)} = -B_8^{X(SS)} \quad (8.182)$$

$$B_{12}^{CX(SS)} = -4(D_{39} + D_{23}) = -B_{11}^{X(SS)} \quad (8.183)$$

$$B_{12}^{CX(SS)} = -B_{11}^{X(SS)} \quad (8.184)$$

$$B_{11}^{CX(SS)} = -4(D_{39} - D_{37}) = -B_{12}^{X(SS)} \quad (8.185)$$

$$B_{11}^{CX(SS)} = -B_{12}^{X(SS)} \quad (8.186)$$

$$B_{18}^{CX(SS)} = 4(D_{38} + D_{26}) = -B_{15}^{X(SS)} \quad (8.187)$$

$$B_{18}^{CX(SS)} = -B_{15}^{X(SS)} \quad (8.188)$$

$$B_{17}^{CX(SS)} = -4(D_{310} - D_{38}) = -B_{16}^{X(SS)} \quad (8.189)$$

$$B_{17}^{CX(SS)} = -B_{16}^{X(SS)} \quad (8.190)$$

$$B_{16}^{CX(SS)} = -4(D_{310} - D_{38}) - 4(D_{13} + D_{25}) \quad (8.191)$$

$$B_{16}^{CX(SS)} = -B_{17}^{X(SS)} \quad (8.192)$$

$$B_{15}^{CX(SS)} = 4(D_{38} + D_{35} - 2D_{310}) - 4(D_{26} - D_{25}) \quad (8.193)$$

$$B_{15}^{CX(SS)} = -B_{18}^{X(SS)} \quad (8.194)$$

$$B_{19}^{CX(SS)} = -4D_{313} = -B_{19}^{X(SS)} \quad (8.195)$$

$$B_{19}^{CX(SS)} = -B_{19}^{X(SS)} \quad (8.196)$$

Note that we calculated all the B's directly in the above and then used crossing symmetry (as displayed in equations (8.173...8.196)) to verify our results. The final result with all the factors reads in this case

$$B^{CX(SS)} = ig^2 \bar{v}(p_1) [+1/4 B_{\nu\lambda}^{CSS}(p_1, -p_5, q_1, m_{\tilde{\omega}}, m_{\tilde{\nu}_e}, m_{\tilde{\nu}_e}, m_{\tilde{e}})(1 - \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.197)$$

The numerical results for the above crossed box diagram with 3 scalar bosons are listed in Table 8.8. Finally we consider CB category IIA' Fig. 8.8, in this subsection, to this end we calculate diagram $\tilde{e}\tilde{\gamma}\tilde{\gamma}\tilde{\omega}$ of Fig. 8.8 the loop integral can

be written as

$$B_{\nu\lambda}^{CX(SF)} = -g^4 s_W^4 \int \frac{d^n q}{2\pi^n} \frac{\bar{v}(p_1) N_{\lambda\nu} u(p_2)}{Q_1 Q_2 Q_3 Q_4} \quad (8.198)$$

$$N_{\lambda\nu} = (\not{q} + \not{p}_1 - m_{\tilde{\gamma}}) \gamma_\lambda (\not{q} + \not{p}_1 - \not{p}_2 - m_{\tilde{\omega}}) \gamma_\nu (\not{q} - \not{p}_2 - m_{\tilde{\gamma}}) (1 - \gamma_5)$$

$$Q_1 = (q^2 - m_{\tilde{e}}^2)$$

$$Q_2 = ((q + p_1)^2 - m_{\tilde{\gamma}}^2)$$

$$Q_3 = ((q + p_1 - p_5)^2 - m_{\tilde{\gamma}}^2)$$

$$Q_4 = ((q + p_1 - p_5 + q_1)^2 - m_{\tilde{\omega}}^2)$$

Since this is the cross symmetric version of equation 8.139 this is readily evaluated

$$B_{\nu\lambda}^{CX(SF)} \doteq -ig^2 \bar{v}(p_1) [+s_W^4 B_{\nu\lambda}^{CSF1}(-p_2, p_5, -q_1, m_{\tilde{e}}, m_{\tilde{\gamma}}, m_{\tilde{\gamma}}, m_{\tilde{\omega}})(1 - \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.199)$$

Similarly evaluating the right handed part and on adding this to 8.199 one obtains the total contribution of the first diagram of Fig. 8.8b

$$B^{CX(SF)} \doteq -ig^2 \bar{v}(p_1) [+s_W^4 B_{\nu\lambda}^{CSF1}(-p_2, p_5, -q_1, m_{\tilde{e}}, m_{\tilde{\gamma}}, m_{\tilde{\gamma}}, m_{\tilde{\omega}})(1 - \gamma_5) + s_W^4 (m_{\tilde{\gamma}}^2) B_{\nu\lambda}^{CSF2}(1 + \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.200)$$

$B^{CX(SF1)}$ is consists of the following B's

$$B_{21}^{CX(SF1)} = 2D_{27} + 2(p_1 \cdot p_2)(D_{25} - D_{26}) + (2(p_2 \cdot q_1) - s_3)D_{23} + 2(p_1 \cdot p_2)(D_{22} - D_{24}) - s_3 D_{33} - 2(p_1 \cdot q_1)(D_{37} - D_{39}) + 2(p_2 \cdot q_1)D_{39} - 2(p_1 \cdot p_2)(D_{38} - D_{310}) + 6D_{313} \quad (8.201)$$

$$B_3^{CX(SF1)} = -2s_3(D_{25} - D_{26}) - 4(p_2 \cdot q_1)(D_{22} - D_{24}) + 4(p_1 \cdot p_2)(D_{32} + D_{34} - 2D_{36}) - 4(p_1 \cdot q_1)(D_{35} + D_{38} - 2D_{310}) - 4(p_2 \cdot q_1)(D_{38} - D_{310}) - 2s_3(D_{37} - D_{39}) + 8(D_{311} - D_{312}) + 2B_{21}^{CX(SF1)} \quad (8.202)$$

$$B_3^{CX(SF1)} = -B_4^{X(SF1)} + 2B_{21}^{CX(SF1)} \quad (8.203)$$

$$B_4^{CX(SF1)} = -4D_{27} - 4(p_1 \cdot p_2 - p_1 \cdot p_1)(D_{25} - D_{26}) - 4(p_2 \cdot q_1 - s_3)D_{23} - 2s_3 D_{26} - 4(p_1 \cdot p_2 - p_1 \cdot q_1)(D_{22} - D_{24}) + 2s_3 D_{33} + 4(p_1 \cdot q_1)(D_{37} - D_{39}) + 4(p_1 \cdot p_2)(D_{38} - D_{310}) - 4(p_2 \cdot q_1)D_{39} + 4D_{312} - 12D_{313} + 2B_{21}^{CX(SF1)} \quad (8.204)$$

$$B_4^{CX(SF1)} = -B_3^{X(SF1)} + 2B_{21}^{CX(SF1)} \quad (8.205)$$

$$B_5^{CX(SF1)} = 4D_{27} + 4(p_1 \cdot p_2 - p_1 \cdot q_1)(D_{25} - D_{26}) + 2(2(p_2 \cdot q_1) - s_3)D_{23} - \\ 2s_3D_{33} - 4(p_1 \cdot q_1)(D_{37} - D_{39}) - 4(p_1 \cdot p_2)(D_{38} - D_{310}) + \\ 4(p_2 \cdot q_1)D_{39} - 8D_{313} - 2B_{21}^{CX(SF1)} \quad (8.206)$$

$$B_5^{CX(SF1)} = -B_5^{X(SF1)} - 2B_{21}^{CX(SF1)} \quad (8.207)$$

$$B_7^{CX(SF1)} = -4(p_2 \cdot q_1)(D_{24} - D_{22}) + 2s_3(D_{25} - D_{26}) - 4(D_{311} - D_{312}) \quad (8.208)$$

$$B_7^{CX(SF1)} = -B_8^{X(SF1)} \quad (8.209)$$

$$B_8^{CX(SF1)} = -4D_{27} - 4(p_1 \cdot p_2 - p_1 \cdot q_1)(D_{24} - D_{22}) - 2(2(p_2 \cdot q_1) - s_3)D_{26} + \\ 2s_3D_{39} - 4(p_1 \cdot p_2)(D_{36} - D_{32}) - 4(p_2 \cdot q_1 + p_1 \cdot q_1)D_{38} + \\ 4(p_1 \cdot q_1)D_{310} - 8D_{312} \quad (8.210)$$

$$B_8^{CX(SF1)} = -B_7^{X(SF1)} \quad (8.211)$$

$$B_{11}^{CX(SF1)} = -4(D_{26} - D_{25}) - 4(D_{39} - D_{37}) \quad (8.212)$$

$$B_{11}^{CX(SF1)} = -B_{12}^{X(SF1)} \quad (8.213)$$

$$B_{12}^{CX(SF1)} = -4(D_{23} - D_{39}) \quad (8.214)$$

$$B_{12}^{CX(SF1)} = -B_{11}^{X(SF1)} \quad (8.215)$$

$$B_{15}^{CX(SF1)} = -4(2D_{310} - D_{35} - D_{38}) \quad (8.216)$$

$$B_{15}^{CX(SF1)} = -B_{18}^{X(SF1)} \quad (8.217)$$

$$B_{16}^{CX(SF1)} = -4(D_{25} - D_{26}) - 4(D_{22} - D_{24}) - 4(D_{310} - D_{38}) \quad (8.218)$$

$$B_{16}^{CX(SF1)} = -B_{17}^{X(SF1)} \quad (8.219)$$

$$B_{17}^{CX(SF1)} = -4(D_{24} - D_{22}) - 4(D_{310} - D_{38}) \quad (8.220)$$

$$B_{17}^{CX(SF1)} = -B_{16}^{X(SF1)} \quad (8.221)$$

$$B_{18}^{CX(SF1)} = +4(D_{26} + D_{38}) \quad (8.222)$$

$$B_{18}^{CX(SF1)} = -B_{15}^{X(SF1)} \quad (8.223)$$

$$B_{19}^{CX(SF1)} = -4(D_{313} + D_{27}) + 2B_{21}^{CX(SF1)} \quad (8.224)$$

$$B_{19}^{CX(SF1)} = -B_{19}^{X(SF1)} + 2B_{21}^{CX(SF1)} \quad (8.225)$$

Note that in the above we have exhibited the crossing symmetry. Similarly $B^{CX(SF2)}$ is the cross-symmetric version of $B^{X(SF2)}$ which is given in equations (8.154...8.156). For $B^{CX(SF2)}$ one finds

$$B_{21}^{CX(SF2)} = -(D_0 + D_{13}) \quad (8.226)$$

$$B_3^{CX(SF2)} = -2(D_0 + D_{13}) \quad (8.227)$$

$$B_3^{CX(SF2)} = -B_4^{X(SF2)} + 2B_{21}^{CX(SF2)} \quad (8.228)$$

$$B_4^{CX(SF2)} = -2(D_0 + D_{12}) \quad (8.229)$$

$$B_4^{CX(SF2)} = -B_3^{X(SF2)} + 2B_{21}^{CX(SF2)} \quad (8.230)$$

$$B_5^{CX(SF2)} = 2(D_0 + D_{13}) \quad (8.231)$$

$$B_5^{CX(SF2)} = -B_5^{X(SF2)} - 2B_{21}^{CX(SF2)} \quad (8.232)$$

$$B_7^{CX(SF2)} = 2(D_{11} - D_{12}) \quad (8.233)$$

$$B_7^{CX(SF2)} = -B_8^{X(SF2)} \quad (8.234)$$

$$B_8^{CX(SF2)} = 0 \quad (8.235)$$

$$B_8^{CX(SF2)} = -B_7^{X(SF2)} \quad (8.236)$$

$$B_{19}^{CX(SF2)} = -2(D_0 + D_{13}) \quad (8.237)$$

$$B_{19}^{CX(SF2)} = -B_{19}^{X(SF2)} + 2B_{21}^{CX(SF2)} \quad (8.238)$$

The contribution from diagram 2 of category IIA' is

$$\begin{aligned} B^{CX(SF)} \doteq & -ig^2 \bar{v}(p_1) [+ (s_W^2/2)(1 - 2s_W^2) B_{\nu\lambda}^{CSF1}(p_1, -p_5, q_1, m_{\tilde{e}}, m_{\tilde{z}}, m_{\tilde{\gamma}}, m_{\tilde{\omega}})(1 - \gamma_5) \\ & + (s_W^2/4c_W)(1 - 2s_W^2)(m_{\tilde{z}}m_{\tilde{\omega}})A_{\nu\lambda}^{CSF4}(1 - \gamma_5) - s_W^4(m_{\tilde{\gamma}}m_{\tilde{z}})B_{\nu\lambda}^{CSF2}(1 + \gamma_5) \\ & - (s_W^2/2c_W)(m_{\tilde{\gamma}}m_{\tilde{\omega}})A_{\nu\lambda}^{CSF3}(1 + \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda} \end{aligned} \quad (8.239)$$

Here $B^{CX(SF1)}$ and $B^{CX(SF2)}$ obey the relations (8.201...8.225) and (8.226...8.238) respectively, whereas $B^{CX(SF4)}$ and $B^{CX(SF3)}$ obey (recall the relations (8.159...8.162) and (8.163...8.166)), respectively,

$$B_{21}^{CX(SF4)} = D_{13} \quad (8.240)$$

$$B_3^{CX(SF4)} = 2D_{13} \quad (8.241)$$

$$B_3^{CX(SF4)} = -B_4^{X(SF4)} + 2B_{21}^{CX(SF4)} \quad (8.242)$$

$$B_4^{CX(SF4)} = 2(D_{12} - D_{13}) + 2D_{13} \quad (8.243)$$

$$B_4^{CX(SF4)} = -B_3^{X(SF4)} + 2B_{21}^{CX(SF4)} \quad (8.244)$$

$$B_5^{CX(SF4)} = 0 \quad (8.245)$$

$$B_5^{CX(SF4)} = -B_5^{X(SF4)} - 2B_{21}^{CX(SF4)} \quad (8.246)$$

$$B_7^{CX(SF4)} = 0 \quad (8.247)$$

$$B_7^{CX(SF4)} = -B_8^{X(SF4)} \quad (8.248)$$

$$B_8^{CX(SF4)} = -2D_{12} \quad (8.249)$$

$$B_8^{CX(SF4)} = -B_7^{X(SF4)} \quad (8.250)$$

$$B_{19}^{CX(SF4)} = 2D_{13} \quad (8.251)$$

$$B_{19}^{CX(SF4)} = -B_{19}^{X(SF4)} + 2B_{21}^{CX(SF4)} \quad (8.252)$$

$$B_{21}^{CX(SF3)} = D_{13} \quad (8.253)$$

$$B_3^{CX(SF3)} = 2(D_{11} - D_{12}) + 2D_{13} \quad (8.254)$$

$$B_3^{CX(SF3)} = -B_4^{X(SF3)} + 2B_{21}^{CX(SF3)} \quad (8.255)$$

$$B_4^{CX(SF3)} = 0 \quad (8.256)$$

$$B_4^{CX(SF3)} = -B_3^{X(SF3)} + 2B_{21}^{CX(SF3)} \quad (8.257)$$

$$B_5^{CX(SF3)} = -2D_{13} \quad (8.258)$$

$$B_5^{CX(SF3)} = -B_5^{X(SF3)} - 2B_{21}^{CX(SF3)} \quad (8.259)$$

$$B_7^{CX(SF3)} = 2(D_{12} - D_{11}) \quad (8.260)$$

$$B_7^{CX(SF3)} = -B_8^{X(SF3)} \quad (8.261)$$

$$B_8^{CX(SF3)} = 0 \quad (8.262)$$

$$B_8^{CX(SF3)} = -B_7^{X(SF3)} \quad (8.263)$$

$$B_{19}^{CX(SF3)} = 2D_{13} \quad (8.264)$$

$$B_{19}^{CX(SF3)} = -B_{19}^{X(SF3)} + 2B_{21}^{CX(SF3)} \quad (8.265)$$

Similarly diagram 3 of this category yields

$$B^{CX(SF)} \doteq -ig^2 \bar{v}(p_1) [+ (s_W^2/2)(1 - 2s_W^2) B_{\nu\lambda}^{CSF1}(p_1, -p_5, q_1, m_{\tilde{e}}, m_{\tilde{\tau}}, m_{\tilde{z}}, m_{\tilde{\omega}})(1 - \gamma_5) + (s_W^2/4c_W)(1 - 2s_W^2)(m_{\tilde{z}} m_{\tilde{\omega}}) B_{\nu\lambda}^{CSF3}(1 - \gamma_5) - s_W^4(m_{\tilde{\tau}} m_{\tilde{z}}) B_{\nu\lambda}^{CSF2}(1 + \gamma_5) - (s_W^2/2c_W)(m_{\tilde{\tau}} m_{\tilde{\omega}}) B_{\nu\lambda}^{CSF4}(1 + \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.266)$$

And the zino contribution is

$$B^{CX(SF)} \doteq -ig^2 \bar{v}(p_1) [+ ((1 - 2s_W^2)^2/4) [B_{\nu\lambda}^{CSF1}(p_1, -p_5, q_1, m_{\tilde{e}}, m_{\tilde{\tau}}, m_{\tilde{z}}, m_{\tilde{\omega}})(1 - \gamma_5) + ((m_{\tilde{z}}^2/4c_W^2) B_{\nu\lambda}^{CSF2}(1 - \gamma_5) + (m_{\tilde{\omega}} m_{\tilde{z}}/2c_W) B_{\nu\lambda}^{CSF4}(1 - \gamma_5) + (m_{\tilde{\omega}} m_{\tilde{z}}/2c_W) B_{\nu\lambda}^{CSF3}(1 - \gamma_5)] + (s_W^4) [B_{\nu\lambda}^{CSF1}(p_1, -p_5, q_1, m_{\tilde{e}}, m_{\tilde{\tau}}, m_{\tilde{z}}, m_{\tilde{\omega}})(1 + \gamma_5) + ((m_{\tilde{z}}^2/4c_W^2) B_{\nu\lambda}^{CSF2}(1 + \gamma_5) + (m_{\tilde{\omega}} m_{\tilde{z}}/2c_W) B_{\nu\lambda}^{CSF3}(1 + \gamma_5) + (m_{\tilde{\omega}} m_{\tilde{z}}/2c_W) B_{\nu\lambda}^{CSF4}(1 + \gamma_5)]] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.267)$$

The numerical results for the above crossed box diagrams of category IIA' with 3 supersymmetric fermions are listed in Table 8.12.

8.6.3 Supersymmetric Crossed Box Topology of Category IVA with two internal fermion lines

Finally we write down the results for the SUSY diagrams, see Fig. 8.8, which are the equivalent of the CB diagrams of category IV of the SM with two internal fermion lines. For the first set of diagrams in category IVA, Fig. 8.8

$$B_3^{CX(2SF)} = 4D_{27} + 2(2p_2 \cdot q_2 - s_4)D_{22} + 4(p_1 \cdot p_2 - p_1 \cdot q_2)D_{23} - 2(2p_1 \cdot p_2 + 2p_2 \cdot q_2 - 2p_1 \cdot q_2 - s_4)D_{26} - 2s_4 D_{32} + 4p_2 \cdot q_2 D_{36} + 4p_1 \cdot p_2 D_{37} + 2(s_4 + 2p_1 \cdot q_2)D_{38} - 4p_1 \cdot q_2 D_{39} - 4(p_2 \cdot q_2 + p_1 \cdot p_2)D_{310} - 8(D_{313} - D_{312}) \quad (8.268)$$

$$B_4^{CX(2SF)} = 2(2p_2 \cdot q_2 - s_4)D_{22} - 2(2p_2 \cdot q_2 - s_4)D_{24} - 4(p_1 \cdot q_2 - p_1 \cdot p_2)D_{25} - 4(p_1 \cdot p_2 - p_1 \cdot q_2)D_{26} - 2s_4 D_{32} - 4p_2 \cdot q_2 D_{34} + 4p_1 \cdot p_2 D_{35} + 2(s_4 + 2p_2 \cdot q_2)D_{36} + 4p_1 \cdot q_2 D_{38} - 4(p_1 \cdot q_2 + p_1 \cdot p_2)D_{310} - 8(D_{311} - D_{312}) \quad (8.269)$$

$$B_{15}^{CX(2SF)} = 4(D_{26} - D_{23}) + 4(D_{38} - D_{39}) \quad (8.270)$$

$$B_{16}^{CX(2SF)} = 4(D_{26} - D_{25}) + 4(D_{38} - D_{310}) \quad (8.271)$$

$$B_{17}^{CX(2SF)} = 4(D_{22} - D_{26}) + 4(D_{36} - D_{310}) \quad (8.272)$$

$$B_{18}^{CX(2SF)} = 4(D_{22} - D_{24}) + 4(D_{36} - D_{34}) \quad (8.273)$$

$$B_{19}^{CX(2SF)} = 4D_{27} + 4D_{312} \quad (8.274)$$

$$B_{21}^{CX(2SF)} = 2D_{27} \quad (8.275)$$

We now give the expressions for the second set of diagrams in category IVA

$$B_7^{CX(2SF)} = -2(2p_1 \cdot q_1 - s_3)D_{22} + 2(2p_1 \cdot q_1 - s_3)D_{24} + 4(p_2 \cdot q_1 - p_1 \cdot p_2)D_{25} + 4(p_1 \cdot p_2 - p_2 \cdot q_1)D_{26} + 2s_3D_{32} + 4p_1 \cdot q_1D_{34} - 4p_1 \cdot p_2D_{35} - 2(s_3 + 2p_1 \cdot q_1)D_{36} - 4p_2 \cdot q_1D_{38} + 4(p_2 \cdot q_1 + p_1 \cdot p_2)D_{310} - 8(D_{312} - D_{311}) \quad (8.276)$$

$$B_8^{CX(2SF)} = -4D_{27} - 2(2p_1 \cdot q_1 - s_3)D_{22} - 4p_1 \cdot p_2D_{23} + 4p_2 \cdot q_1D_{25} + 2(2p_1 \cdot p_2 + 2p_1 \cdot q_1 - 2p_2 \cdot q_1 - s_3)D_{26} + 2s_3D_{32} - 4p_1 \cdot q_1D_{36} - 4p_1 \cdot p_2D_{37} - 2(s_3 + 2p_2 \cdot q_1)D_{38} + 4p_2 \cdot q_1D_{39} + 4(p_1 \cdot q_1 + p_1 \cdot p_2)D_{310} - 8(D_{312} - D_{313}) \quad (8.277)$$

$$B_{15}^{CX(2SF)} = 4(D_{22} - D_{24}) + 4(D_{36} - D_{34}) \quad (8.278)$$

$$B_{16}^{CX(2SF)} = 4(D_{26} - D_{25}) + 4(D_{38} - D_{310}) \quad (8.279)$$

$$B_{17}^{CX(2SF)} = 4(D_{22} - D_{26}) + 4(D_{36} - D_{310}) \quad (8.280)$$

$$B_{18}^{CX(2SF)} = 4(D_{26} - D_{23}) + 4(D_{38} - D_{39}) \quad (8.281)$$

$$B_{19}^{CX(2SF)} = 4D_{27} + 4D_{312} \quad (8.282)$$

$$B_{21}^{CX(2SF)} = 2D_{27} \quad (8.283)$$

Bearing in mind the contributions from all diagrams in Category IV we have the total

$$B^{CX(2F)} = ig^2 \bar{u}(p_1) [- (s_W^2/2) B_{\nu\lambda}^{CX(2SF)}(-p_2, q_2, -p_1, m_{\nu_e}, m_{\bar{\nu}}, m_{\tilde{\nu}}, m_{\tilde{\nu}})(1 - \gamma_5) - (1 - 2s_W^2)/4 B_{\nu\lambda}^{CX(2SF)}(-p_2, q_2, -p_1, m_{\nu_e}, m_{\bar{\nu}}, m_{\tilde{\nu}}, m_{\tilde{\nu}})(1 - \gamma_5) - (s_W^2/2) B_{\nu\lambda}^{CX(2SF)}(p_1, -q_1, p_2, m_{\nu_e}, m_{\bar{\nu}}, m_{\tilde{\nu}}, m_{\tilde{\nu}})(1 - \gamma_5) - (1 - 2s_W^2)/2 B_{\nu\lambda}^{CX(2SF)}(p_1, -q_1, p_2, m_{\nu_e}, m_{\bar{\nu}}, m_{\tilde{\nu}}, m_{\tilde{\nu}})(1 - \gamma_5)] u(p_2) e^{1\nu} e^{2\lambda} \quad (8.284)$$

The numerical results for the above crossed box diagrams of category IVA with 2 supersymmetric fermions are listed in Table 8.9.

Figure 8.1: Three different topologies of the box diagrams

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(a)

(b)

Figure 8.2: (a) DB and its momentum convention, (b) Diagrams Ignored.

Figure 8.3: 5 diagrams of the triangular topology

Figure 8.4: (a) 3 diagrams of Category I, (b) 6 diagrams of Category II, (c) 5 diagrams of category III.

Figure 8.5: All the CB diagrams of categories I' (a), II' (b), III' (c), and IV' (d).

Figure 8.6: Triangular topology diagrams introduced by SUSY.

Figure 8.7: All the SUSY DB diagrams of categories IA (a), and IIA (b).

Table 8.1: Box diagram number count

Box	# of diagrams in QFD	# of diagrams due to S-P	Topology
FVV	5	5	Triangular
VFFF,FSSS	3	3	Direct
VFFF,FSSS	1	1	Crossed
FVVF,SFFS	4	4	Crossed
FVVV,SFFF	6	6	Direct
FVVV,SFFF	4	4	Crossed
FVVS,SFFF	5	1	Direct
FVVS,SFFF	4	-	Crossed

Figure 8.8: All the SUSY CB diagrams of categories IA' (a), IIA' (b), and IVA (c).

Table 8.2: Triangular box contribution to the one loop radiative corrections in pb for ECM = 200 GeV.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{QFD}$ CC	$(\frac{d\sigma}{d\cos\theta})_1^{QFD}$ NN	$(\frac{d\sigma}{d\cos\theta})_1^{Total}$ —
10	+0.9429	-2.7820	-1.8391
30	+0.4886	-1.8882	-1.3994
50	+0.2035	-1.3218	-1.1183
70	+0.0615	-1.0318	-0.9703
90	-0.0098	-0.8702	-0.8800
110	-0.0429	-0.7644	-0.8073
130	-0.0550	-0.6807	-0.7357
150	-0.0573	-0.6218	-0.6701
170	-0.0568	-0.5723	-0.6291

Table 8.3: Contribution of category I and I' to the one loop radiative corrections in pb for ECM = 200 GeV.

$\theta(deg)$ -	$(\frac{d\sigma}{d\cos\theta})_1$ $\gamma e e \nu_e$	$(\frac{d\sigma}{d\cos\theta})_1$ $Z e e \nu_e$	$(\frac{d\sigma}{d\cos\theta})_1$ $W \nu_e \nu_e e$	$(\frac{d\sigma}{d\cos\theta})_1$ $W \nu_e \nu_e e$
10	+1.7750	+0.0257	+0.2164	-2.0956
30	+1.5119	+0.0063	+0.0533	-0.6730
50	+1.0135	-0.0014	-0.0138	-0.2160
70	+0.7034	-0.0022	-0.0243	-0.0441
90	+0.5365	-0.0018	-0.0226	+0.0127
110	+0.4437	-0.0018	-0.0228	+0.0266
130	+0.3886	-0.0024	-0.0262	+0.0295
150	+0.3557	-0.0032	-0.0307	+0.0332
170	+0.3398	-0.0037	-0.0338	+0.0394

Table 8.4: Contribution of category III and III' to the one loop radiative corrections in pb for ECM = 200 GeV.

$\theta(deg)$ - -	$(\frac{d\sigma}{d\cos\theta})_1$ $e Z Z G$ DB	$(\frac{d\sigma}{d\cos\theta})_1$ $\nu_e W W H_1^0$ DB	$(\frac{d\sigma}{d\cos\theta})_1$ $e Z Z G$ CB
10	+0.0014	+0.0257,+0.0192	+0.0042
30	+0.0015	+0.0304,+0.0319	+0.0023
50	+0.0015	+0.0301,+0.0258	+0.0011
70	+0.0014	+0.0272,+0.0154	+0.0006
90	+0.0014	+0.0242,+0.0085	+0.0004
110	+0.0013	+0.0219,+0.0060	+0.0003
130	+0.0012	+0.0203,+0.0058	+0.0003
150	+0.0011	+0.0194,+0.0071	+0.0003
170	+0.0011	+0.0189,+0.0082	+0.0003

Table 8.5: Contribution of category II and II' to the one loop radiative corrections in pb for ECM = 200 GeV.

$\theta(deg)$ — —	$(\frac{d\sigma}{d\cos\theta})_1$ $e Z Z W$ DB	$(\frac{d\sigma}{d\cos\theta})_1$ $e Z Z W$ CB	$(\frac{d\sigma}{d\cos\theta})_1$ $\nu_e W W Z$ DB	$(\frac{d\sigma}{d\cos\theta})_1$ $\nu_e W W \gamma$ DB
10	+0.0067	-1.0759	-1.0269	+1.2088
30	+0.0443	-0.5157	-0.3321	+0.4923
50	+0.0767	-0.1974	+0.0482	+0.0444
70	+0.0828	-0.0704	+0.2039	-0.0910
90	+0.0810	-0.0294	+0.2795	-0.1178
110	+0.0794	-0.0239	+0.3291	-0.1163
130	+0.0782	-0.0296	+0.3673	-0.1101
150	+0.0773	-0.0357	+0.3954	-0.1050
170	+0.1184	-0.0389	+0.4107	-0.1024

Table 8.6: The contribution of the diagrams with at least one photon in the box loop of categories II, II', III, and III' to the one loop radiative corrections in pb for ECM = 200 GeV.

$\theta(deg)$ — —	$\left(\frac{d\sigma}{d\cos\theta}\right)_1$ $e \sum(ij) W$ DB	$\left(\frac{d\sigma}{d\cos\theta}\right)_1$ $e \sum(ij) W$ CB	$\left(\frac{d\sigma}{d\cos\theta}\right)_1$ $e \sum(ij) G$ DB	$\left(\frac{d\sigma}{d\cos\theta}\right)_1$ $e \sum(ij) G$ CB
10	+0.3654	+14.3634	+2.1733	-1.0908
30	+4.1447	+5.5624	+1.8909	-0.6437
50	+3.1280	+2.2503	+1.5433	-0.3666
70	+0.6150	+2.0311	+1.2612	-0.2357
90	-1.6826	+2.4071	+1.0530	-0.1821
110	-3.4636	+3.3423	+0.8921	-0.1755
130	-4.7826	+2.9459	+0.7623	-0.1972
150	-5.6876	+2.6078	+0.6652	-0.2283
170	-6.1588	+2.1392	+0.6119	-0.2488

Table 8.7: Contribution of category IV and to the one loop radiative corrections in pb for ECM = 200 GeV.

$\theta(deg)$	$(\frac{d\sigma}{d\cos\theta})_1$ $\nu_e W Z e$ CB, Diag. 1	$(\frac{d\sigma}{d\cos\theta})_1$ $\nu_e W Z e$ CB, Diag. 3	$(\frac{d\sigma}{d\cos\theta})_1$ $\nu_e W \gamma e$ CB, Diag. 2	$(\frac{d\sigma}{d\cos\theta})_1$ $\nu_e W \gamma e$ CB, Diag. 4
10	-0.2544	+0.3798	-0.2644	-0.2644
30	-0.1062	+0.0983	-0.1562	-0.1562
50	-0.0075	+0.0458	+0.1795	+0.1795
70	+0.0239	-0.0015	+0.5844	+0.5844
90	+0.0286	-0.0481	+0.9935	+0.9935
110	+0.0225	-0.0897	+1.1419	+1.1419
130	+0.0132	-0.1241	+1.8913	+1.8913
150	+0.0052	-0.1471	+2.4293	+2.4293
170	+0.0034	-0.1619	+2.9229	+2.9229

Table 8.8: Contribution of category IA and IA' to the one loop radiative corrections in pb for $ECM = 200$ GeV.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $\tilde{\gamma}\tilde{e}\tilde{e}\tilde{\nu}_e$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $\tilde{z}\tilde{e}\tilde{e}\tilde{\nu}_e$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $\tilde{\omega}\tilde{\nu}_e\tilde{\nu}_e\tilde{e}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $\tilde{\omega}\tilde{\nu}_e\tilde{\nu}_e\tilde{e}$
10	+0.0981	+0.0330	+0.1379	+0.1148
30	+0.0570	+0.0205	+0.0846	+0.0748
50	+0.0373	+0.0143	+0.0559	+0.0500
70	+0.0299	+0.0120	+0.0444	+0.0380
90	+0.0260	+0.0107	+0.0386	+0.0324
110	+0.0228	+0.0095	+0.0342	+0.0301
130	+0.0198	+0.0084	+0.0301	+0.0295
150	+0.0172	+0.0074	+0.0266	+0.0295
170	+0.0158	+0.0068	+0.0246	+0.0296

Table 8.9: Contribution of category IVA to the one loop radiative corrections in pb for ECM = 200 GeV.

$\theta(deg)$ –	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $\bar{\nu}_e\tilde{\omega}\tilde{\gamma}\bar{e}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $\bar{\nu}_e\tilde{\omega}\tilde{z}\bar{e}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $\bar{\nu}_e\tilde{\omega}\tilde{\gamma}\bar{e}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $\bar{\nu}_e\tilde{\omega}\tilde{z}\bar{e}$
10	+0.6968	+0.4132	-0.9479	-0.8216
30	+0.5188	+0.3372	-0.8306	-0.7349
50	+0.2873	+0.2000	-0.7150	-0.6591
70	+0.0849	+0.0529	-0.6406	-0.6132
90	-0.0812	-0.0844	-0.6032	-0.5939
110	-0.2167	-0.2072	-0.5885	-0.5901
130	-0.3203	-0.3084	-0.5848	-0.5926
150	-0.3881	-0.3794	-0.5848	-0.5957
170	-0.4208	-0.4153	-0.5860	-0.5971

Table 8.10: Contribution of category IIA to the one loop radiative corrections in pb for ECM = 200 GeV.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $e\bar{\gamma}\gamma\bar{\omega}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $e\bar{z}\gamma\bar{\omega}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $e\bar{\gamma}z\bar{\omega}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P'}$ $e\bar{z}z\bar{\omega}$
10	+0.0441	+0.0675	+0.0706	+0.1102
30	+0.0098	+0.0189	+0.0218	+0.0347
50	-0.0081	-0.0066	-0.0040	-0.0056
70	-0.0137	-0.0148	-0.0124	-0.0196
90	-0.0141	-0.0159	-0.0137	-0.0230
110	-0.0126	-0.0143	-0.0124	-0.0224
130	-0.0107	-0.0120	-0.0106	-0.0207
150	-0.0090	-0.0098	-0.0091	-0.0192
170	-0.0075	+0.0162	-0.0339	-0.0200

Table 8.11: Contribution of category IIA to the one loop radiative corrections in pb for ECM = 200 GeV.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $\bar{\nu}_e \bar{\omega} \bar{\omega} \tilde{\gamma}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $\bar{\nu}_e \bar{\omega} \bar{\omega} \bar{z}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $\bar{\nu}_e \bar{\omega} \bar{\omega} \bar{h}$
10	+0.1105	+0.7382	+0.0308
30	+0.0149	+0.2347	+0.0135
50	-0.0282	-0.0146	+0.0040
70	-0.0364	-0.0889	-0.0006
90	-0.0329	-0.1001	-0.0040
110	-0.0267	-0.0921	-0.0072
130	-0.0211	-0.0809	-0.0102
150	-0.0172	-0.0724	-0.0125
170	-0.0320	-0.1278	-0.0137

Table 8.12: Contribution of category IIA' to the one loop radiative corrections in pb for ECM = 200 GeV.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $\bar{e}\tilde{\gamma}\tilde{\gamma}\tilde{\omega}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $\bar{e}\tilde{z}\tilde{\gamma}\tilde{\omega}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $\bar{e}\tilde{\gamma}\tilde{z}\tilde{\omega}$	$(\frac{d\sigma}{d\cos\theta})_1^{S-P}$ $\bar{e}\tilde{z}\tilde{z}\tilde{\omega}$
10	-0.0332	+0.0698	-0.1617	-0.0795
30	-0.0200	-0.0278	-0.0187	-0.0309
50	-0.0090	-0.0102	-0.0083	-0.0050
70	-0.0027	-0.0017	-0.00003	+0.0085
90	+0.0018	+0.0042	+0.0052	+0.0165
110	+0.0051	+0.0086	+0.0094	+0.0219
130	+0.0076	+0.0119	+0.0126	+0.0255
150	+0.0093	+0.0143	+0.0149	+0.0280
170	+0.0102	+0.0156	+0.0162	+0.0293

Chapter 9

The Magnetic and Quadrupole moment of the W boson in SQFD

9.1 Introduction

The purpose of this chapter is to give a straightforward discussion of the radiative corrections contributions to the quadrupole moment, the magnetic moment and the 'A' terms in the context of SQFD.

Lautrup, Bardeen and Gastmans [66] were the first to calculate the loop corrections to the quadrupole and magnetic moments of the W boson in QFD. It is well known that both these moments are finite since we are dealing with a renormalizable theory. It is easy to see from the form of the 'quadrupole' like term, (see Eq. 5.36), that it must be finite in the SW model since there is no counterterm to subtract out the divergent term. For the magnetic and 'A' terms there are counterterms. We note that both magnetic and quadrupole moment terms are renormalization scheme independent where as the 'A' term is renormalization scheme dependent. In the context of supersymmetry where there are equal numbers of Bose and Fermi degrees of freedom in each multiplet, one would like

¹see Eq. 9.1 for definition of the 'A' term

to examine the constraints this would place on the magnetic and quadrupole moments of the particles. Using superfield methods it was shown long ago by Ferrara and Remiddi [65] that $g-2$ vanishes identically in supersymmetric Abelian gauge theories, the latter result can be easily generalized to SQFD. Using component fields we have shown [[44] (PhD Thesis Notes)] that $g-2$ vanishes in SQFD. The quadrupole and magnetic moments of the W boson have been extensively discussed in the literature [66, 67, 68, 69, 70, 72].

If ϵ_μ , ϵ_ν and ϵ_λ are the polarization vectors of the photon and outgoing W^+ and W^- and p_5^μ , k^ν , and p^λ , are, respectively their four-momenta then the most general CP-invariant $\gamma W^+ W^-$ vertex is²

$$V_{\mu\nu\lambda}^{\gamma W^+ W^-} = -ie[A(-2p_\mu g_{\nu\lambda} + 2[p_\nu g_{\mu\lambda} - k_\lambda g_{\mu\nu}]) + [K - 1][p_\nu g_{\mu\lambda} - k_\lambda g_{\mu\nu}] - \frac{\Delta Q}{M_W^2} p_\mu p_\nu k_\lambda] \quad (9.1)$$

for all particles on the shell. Eq. 9.1 can be obtained from Eq. 5.36 of Chapter 5 by putting all the particles on shell and some further redefinitions. To reduce Eq. 9.1 to the SW tree level $\gamma W^+ W^-$ vertex we set $\Delta Q = 0$, $K = 1$ and $A = 1$. As pointed out in Chapter 5, combinations of $\Delta Q = 0$ and K are equivalent to the magnetic and the quadrupole moments of the W , [see Eq. 5.39]. The ‘A’ in Eq. 9.1 term arises as a result of ‘charge’ renormalization of the three vertex and wave function renormalization of the W -boson.

9.2 The quadrupole moment of the W boson

We find in QFD the following contributions to ΔQ ,

$$\Delta Q^{H_1^0} = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} \frac{1}{3} \int_0^1 dx \frac{x^3(1-x)}{x^2 + \mu^2(1-x)}, \quad (9.2)$$

²Note the misprint in [72] where $2A$ must be replaced by A and p_μ must be multiplied by $g_{\nu\lambda}$

here $\mu^2 = \frac{m_{H^\pm}^2}{M_W^2}$

$$\Delta Q^\gamma = \frac{\alpha}{9\pi}, \quad (9.3)$$

$$\Delta Q^Z = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} \frac{1}{3R} \int_0^1 dx \frac{x^3(1-x)(8+R)}{x^2 + R(1-x)}, \quad (9.4)$$

here $R = \frac{M_i^2}{M_W^2}$, from Higgs, photon, and Z-boson respectively. The above expressions are in agreement with the work of [66], thus providing a check on our calculations. We note that the contributions to ΔQ given in the above equations can be roughly estimated from the combinations of constants appearing in front of the integrals. These approximately amount to $\sim 8.6 \cdot 10^{-4}$, $\sim 2.6 \cdot 10^{-4}$, and $\sim 6.7 \cdot 10^{-4}$ for the Higgs, the photon and the Z-boson respectively. In order to give the contributions to ΔQ in SQFD we recall the multiplet structure given in Chapter 4. We start with the photon-photino multiplet. Since the photon contribution is already given in Eq. 9.3, we turn to the photino contribution, this works out to be,

$$\Delta Q^{\tilde{\gamma}} = -\frac{e^2}{6\pi^2} \int_0^1 dx \frac{x^3(1-x)}{x \frac{m_{\tilde{\gamma}}^2}{M_W^2} + (1-x) \frac{m_{\tilde{Z}}^2}{M_W^2} - x(1-x)} \quad (9.5)$$

which in the supersymmetric limit ($m_{\tilde{\gamma}} = M_W$, and $m_{\tilde{Z}} = 0$) reduces to

$$\Delta Q^{\tilde{\gamma}} = -\frac{\alpha}{9\pi}. \quad (9.6)$$

This clearly cancels the photon contribution given in 9.3. We now consider the Z - zino - H_2^0 multiplet. The total zino contribution may be written as

$$\Delta Q^i = \Delta Q_1^i + \Delta Q_2^i \quad (9.7)$$

where ΔQ_1^i and ΔQ_2^i are found to be respectively,

$$\Delta Q_1^i = -\frac{g^2}{6\pi^2} c_W^2 \int_0^1 dx \frac{x^3(1-x)}{x \frac{m_{\tilde{Z}}^2}{M_W^2} + (1-x) \frac{m_{\tilde{H}_2^0}^2}{M_W^2} - x(1-x)} \quad (9.8)$$

and

$$\Delta Q_2^i = -\frac{g^2}{24\pi^2} \int_0^1 dx \frac{x^3(1-x)}{x \frac{m_i^2}{M_W^2} + (1-x) \frac{m_1^2}{M_W^2} - x(1-x)} \quad (9.9)$$

In the SUSY limit ($m_{\tilde{\omega}} = M_W$, and $m_i = M_Z$) these reduce to

$$\Delta Q_1^i = -\frac{G_F M_W^2}{\sqrt{2}\pi^2} \frac{4}{3R} \int_0^1 dx \frac{x^3(1-x)}{x^2 + R(1-x)}, \quad (9.10)$$

and

$$\Delta Q_2^i = -\frac{G_F M_W^2}{\sqrt{2}\pi^2} \frac{1}{3} \int_0^1 dx \frac{x^3(1-x)}{x^2 + R(1-x)}. \quad (9.11)$$

We note that Eq. 9.11 arises from the Higgs-fermion part of the massive zino. We must also consider the contribution from H_2^0 . This can be promptly written down as

$$\Delta Q^{H_2^0} = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} \frac{1}{3} \int_0^1 dx \frac{x^3(1-x)}{x \frac{m_H^2}{M_W^2} + (1-x) \frac{m_{H_2^0}^2}{M_W^2} - x(1-x)}, \quad (9.12)$$

Taking the SUSY limit ($m_H = M_W$, and $m_{H_2^0} = M_Z$) this reduces to

$$\Delta Q^{H_2^0} = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} \frac{1}{3} \int_0^1 dx \frac{x^3(1-x)}{x^2 + (1-x)R}, \quad (9.13)$$

Thus from Eqs. 9.10, 9.11, 9.12, and 9.13 we have

$$\Delta Q^Z + \Delta Q^i + \Delta Q^{H_2^0} = 0. \quad (9.14)$$

Finally we consider the Higgs-Higgsino multiplet. The contributions from the members of this multiplet can be written as

$$\Delta Q^{\tilde{h}} = -\frac{g^2}{24\pi^2} \int_0^1 dx \frac{x^3(1-x)}{x \frac{m_{\tilde{h}}^2}{M_W^2} + (1-x) \frac{m_{\tilde{h}^0}^2}{M_W^2} - x(1-x)} \quad (9.15)$$

$$\Delta Q^{H_1^0} = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} \frac{1}{3} \int_0^1 dx \frac{x^3(1-x)}{x \frac{m_{\tilde{h}}^2}{M_W^2} + (1-x) \frac{m_{H_1^0}^2}{M_W^2} - x(1-x)}, \quad (9.16)$$

and $\Delta Q^{H_1^0}$ is already given in. In the SUSY limit ($m_H = M_W$, and $m_{H_2^0} = m_{H_1^0}$) the contribution of this multiplet to $\Delta Q^{H_1^0}$ again reduces to zero

$$\Delta Q^{H_1^0} + \Delta Q^{\tilde{h}} + \Delta Q^{H_2^0} = 0. \quad (9.17)$$

We note that the Ψ_N part of the $\tilde{h}$ fermion does not interact with the gauge fields since it is a singlet under $SU(2) \times U(1)$. It is only the Higgs-fermion part of $\tilde{h}$ which interacts with the W-bosons and winos.

Now we turn to consider some numerical results. We expect from the above discussion that a non-zero value of the quadrupole parameter must be generated by supersymmetry breaking. Moreover we expect small values for ΔQ provided the mass splitting among the SUSY particles circulating in the loops does not exceed $\sim 50\text{GeV}$. Indeed we find this to be the case after explicit numerical evaluations.

We may parameterize a non-zero value of ΔQ in the broken supersymmetry, for photino-photon as

$$\Delta Q^{\gamma-\tilde{\gamma}} = \frac{\alpha}{9\pi} \left[1 - 6 \int_0^1 dx \frac{x^3(1-x)}{x \frac{m_{\tilde{g}}^2}{M_W^2} + (1-x) \frac{m_{\tilde{t}}^2}{M_W^2} - x(1-x)} \right] \quad (9.18)$$

We expect numerically to obtain the cancellation we found by algebraic computation. This indeed turns out to be the case, [see Table 9.1]. We note that we have factored out $\frac{\alpha}{9\pi}$ in Table 9.1. For the choices of the various values of masses indicated in Table 9.1, the variation of ΔQ from Table 9.1 is,

$$-3.3144 \leq \Delta Q^{\gamma-\tilde{\gamma}} \leq 0.8556. \quad (9.19)$$

In a similar manner the variation of ΔQ for the Z - zino - H_2^0 and H_1^0 - higgsino - H_3^0 sectors is found to be, [see Tables 9.2, 9.3],

$$-7.0377 \leq \Delta Q^{Z-\tilde{z}-H_2^0} \leq -1.9704 \quad (9.20)$$

and

$$-0.5442 \leq \Delta Q^{H_1^0-\tilde{h}-H_2^0} \leq -1.3191 \quad (9.21)$$

respectively. We note that for the numerical results given in Tables 9.2 and 9.3 we took $m_H = 90\text{GeV}$ and $m_{\tilde{\omega}} = 50\text{GeV}$. The numerical results corresponding to the contributions to ΔQ in the SW model Eqs. 9.2, 9.3, and 9.4 are as follows (after taking out the factors indicated in Tables (9.1, 9.2 and 9.3),

$$\begin{aligned} \Delta Q^\gamma &= 1.0000 \\ \Delta Q^Z &= 0.5134 \\ \Delta Q^{H_1^0} &= 0.0891 \end{aligned} \quad (9.22)$$

In general the addition of SUSY particles tends to decrease the radiative corrections, especially in the SUSY -limit, as shown above. However in the individual cases indicated above (for our choice of the parameter space) there has been an increase in the radiative corrections by the inclusion of the SUSY particles. We note that we have not given the contribution due to the fermions and s-fermions. A more detailed investigation will be reported else where [71], since the examination of the magnetic and quadrupole moments of the W's is not in the mainline of this thesis.

9.3 The “A” term in QFD

When the one loop radiative corrections are calculated to the $\gamma W^+ W^-$ vertex, [see Chapter 7], a counterterm must be included in the Lagrangian. The part of the counterterm Lagrangian relevant to our discussion here is:

$$\mathcal{L}_{\text{counter}} = -\frac{ie}{2} \left(\delta Z_W + \frac{\delta g}{g} \right) \{ F_{\mu\nu}^+ [A^\mu W^{-\nu} - A^\nu W^{-\mu}]$$

$$\begin{aligned}
& + F_{\mu\nu}^- [A^\nu W^{+\mu} - A^\mu W^{+\nu}] \} \\
\mathcal{L}_{\text{counter}} &= -ie \left(\delta Z_W + \frac{\delta g}{g} \right) \\
& [-2p_\mu g_{\nu\lambda} + 2[p_\nu g_{\mu\lambda} - k_\lambda g_{\mu\nu}]] \epsilon^\mu \epsilon^\nu \epsilon^\lambda \\
& \mathcal{L}_{\text{counter}} + \mathcal{L}_{\text{radiative corrections}} \\
&= -ie \left[\left(\delta Z_W + \left(\frac{\delta g}{g} \right)_1 \right) [-2p_\mu g_{\nu\lambda}] \right. \\
& \quad \left. \left(\delta Z_W + \left(\frac{\delta g}{g} \right)_2 \right) 2[p_\nu g_{\mu\lambda} - k_\lambda g_{\mu\nu}] \right] \epsilon^\mu \epsilon^\nu \epsilon^\lambda \tag{9.23}
\end{aligned}$$

As already mentioned δZ_W comes from the wave function renormalization of the W boson and $\frac{\delta g}{g}$ is from the vertex correction. We have already demonstrated the finiteness of the factor $\delta Z_W + \frac{\delta g}{g}$ (Chapter 4) and given the finite contribution of this factor in Chapter 7 to the one loop differential cross section of the process $e^+e^- \longrightarrow W^+W^-$, where the photon is off shell. Now we turn to consider the contribution of this factor in the case when all the particles are on shell. To this end consider Eqs. 9.1 and 9.23, it follows from these two equations that

$$A = \delta Z_W + \left(\frac{\delta g}{g} \right)_1, \tag{9.24}$$

$$\frac{K-1}{2} + A = \left(\frac{\delta g}{g} \right)_2. \tag{9.25}$$

From Eqs. 9.24 and 9.25 we can obtain an expression for $\frac{K-1}{2}$ by eliminating A ,

$$\frac{K-1}{2} = \left(\frac{\delta g}{g} \right)_2 - \left(\frac{\delta g}{g} \right)_1. \tag{9.26}$$

We note that the anomalous magnetic moment parameter (AMM) parameter K is renormalization scheme independent (as should) whereas A is very much renormalization scheme dependent. Of course physical results are renormalization scheme independent but that occurs only after all loops have been taken into account. From now on we shall denote $K-1$ by ΔK (i.e. deviation away from the standard

model). One finds the following contributions to ΔK in QFD

$$\Delta K^{H_1^0} = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} \int_0^1 dx \frac{x^2[2 + x(x-1) + \frac{1}{2}\mu^2(1-x)]}{x^2 + \mu^2(1-x)}, \quad (9.27)$$

$$\Delta K^\gamma = \frac{5}{3} \frac{\alpha}{\pi}, \quad (9.28)$$

$$\Delta K^Z = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} \int_0^1 dx \frac{Z(x)}{x^2 + R(1-x)}, \quad (9.29)$$

here $Z(x)$ is given by

$$\begin{aligned} Z(x) = & x[(8x^3 - 8x^2 + 8x) + R(x^3 - 5x^2 - 2x) \\ & + \frac{1}{2}R^2(-x^2 + 5x - 4)] \end{aligned}$$

from the Higgs, photon, and Z-boson respectively. For leptons and down-quarks

$$\Delta K^{l,d} = -\frac{G_F M_W^2}{2\sqrt{2}\pi^2} Q_{l,d} n_c I_F, \quad (9.30)$$

and for up-quarks

$$\Delta K^u = -\frac{G_F M_W^2}{2\sqrt{2}\pi^2} Q_u n_c I_F. \quad (9.31)$$

Here

$$\begin{aligned} I_F = & \int_0^1 dx \frac{F(x)}{x \frac{m_1^2}{M_W^2} + (1-x) \frac{m_2^2}{M_W^2} - x(1-x)}, \\ F(x) = & x^2[x - \frac{1}{2}x^2 - \frac{1}{2} \\ & - \frac{1}{2} \frac{m_1^2}{M_W^2} + \frac{1}{2}(1-x)(\frac{m_2^2}{M_W^2} + 1 - \frac{m_1^2}{M_W^2})] \end{aligned} \quad (9.32)$$

where in 9.32 we have allowed for the general case when quarks and leptons are included with arbitrary masses. For example, the top quark mass cannot certainly be neglected. In the massless limit 9.32 reduces to, (for the case of lepton)

$$\Delta K^l = -\frac{G_F M_W^2}{2\sqrt{2}\pi^2} \frac{1}{6} \quad (9.33)$$

The above are in complete agreement with Lautrup et al. [66]³. We note however that their leptonic contribution is twice ours since they have added the contribution from the muon and the electron. We find from Eq. 9.30 and Eq. 9.31 that in the massless limit the lepton contributions cancels that of the quarks. We note that in 9.30 and 9.31 we have used the notation that $n_c = 1$ for the leptons and $n_c = 3$ for the quarks. m_1 and m_2 are defined such that when the lepton⁴ and the down-quark contributions are considered m_1 represents the mass of the lepton or the down-quark and m_2 is the mass of the neutrino or the up-quark. Similarly if the up-quark contribution is considered m_1 (m_2) represents the mass of the up-quark or the neutrino (lepton or the down-quark).

For the "A" term we have divided the contributions into fermionic and non-fermionic parts. From the definition 9.24 in order to find the "A" term we must first evaluate δZ_W and $(\frac{\delta g}{g})_1$. Since δZ_W has already been evaluated, see Chapter 2, 4 and 6, we now give expressions for $(\frac{\delta g}{g})_1$. To this end we consider first the non-fermionic contributions. These read

$$\begin{aligned}
 \left(\frac{\delta g}{g}\right)_1^{H^0} &= \frac{G_F M_W^2}{2\sqrt{2}\pi^2} \int_0^1 dx \frac{[x(1-x) + \frac{1}{2}x^2(\frac{1}{2} - \frac{\pi}{3})(2x - \mu^2)]}{x^2 + \mu^2(1-x)}, \\
 \left(\frac{\delta g}{g}\right)_1^\gamma &= \frac{G_F M_W^2}{2\sqrt{2}\pi^2} s_W^2 \left[-\frac{11}{6} \right. \\
 &\quad \left. + \int_0^1 dx \frac{[-\frac{11}{6} + 7x^3 - 4x^2 - 4x + \frac{\lambda^2}{M_W^2}(\frac{7}{3}x^3 - 6x^2 + 5x)]}{x^2 + \frac{\lambda^2}{M_W^2}(1-x)} \right], \\
 \left(\frac{\delta g}{g}\right)_1^Z &= \frac{G_F M_W^2}{2\sqrt{2}\pi^2} [c_W^2 \left[-\frac{11}{6} \right. \\
 &\quad \left. + \int_0^1 dx \frac{[-\frac{11}{6} + 7x^3 - 4x^2 - 4x + R(\frac{7}{3}x^3 - 6x^2 + 5x)]}{x^2 + R(1-x)} \right] \\
 &\quad \left. + \int_0^1 dx \frac{[\frac{1}{2}x^2(\frac{1}{2} - \frac{\pi}{3})(2x - R)]}{x^2 + R(1-x)} \right]
 \end{aligned}$$

³We note that there should not be a factor of 2 in their [66] value of κ^{ZW} .

⁴lepton in the context of this discussion refers only to the electron, muon and tau

$$\begin{aligned}
& + R \int_0^1 dx \frac{[x(1-x)]}{x^2 + R(1-x)} \\
& + \int_0^1 dx \frac{[3x^2 - x]}{x^2 + R(1-x)}].
\end{aligned} \tag{9.34}$$

Similarly, the fermionic contribution to $(\frac{\delta g}{g})_1$ term may be evaluated and one finds for one generation of quarks and leptons,

$$\begin{aligned}
\left(\frac{\delta g}{g}\right)_1^e &= \frac{G_F M_W^2}{2\sqrt{2}\pi^2} \left[-\frac{1}{2} \right. \\
&\quad \left. - \frac{1}{2} \int_0^1 dx \frac{[x^2(-\frac{1}{3}x^2 + \frac{1}{3}x(\frac{m_e^2}{M_W^2} - \frac{m_\nu^2}{M_W^2} - 1) + \frac{m_\nu^2}{M_W^2} - 2\frac{m_e^2}{M_W^2})]}{x\frac{m_e^2}{M_W^2} + (1-x)\frac{m_\nu^2}{M_W^2} - x(1-x)} \right], \\
\left(\frac{\delta g}{g}\right)_1^d &= \frac{G_F M_W^2}{2\sqrt{2}\pi^2} \left[-\frac{1}{2} \right. \\
&\quad \left. - \frac{1}{2} \int_0^1 dx \frac{[x^2(-\frac{1}{3}x^2 + \frac{1}{3}x(\frac{m_d^2}{M_W^2} - \frac{m_u^2}{M_W^2} - 1) + \frac{m_u^2}{M_W^2} - 2\frac{m_d^2}{M_W^2})]}{x\frac{m_d^2}{M_W^2} + (1-x)\frac{m_u^2}{M_W^2} - x(1-x)} \right], \\
\left(\frac{\delta g}{g}\right)_1^u &= \frac{G_F M_W^2}{2\sqrt{2}\pi^2} \left[-1 - \right. \\
&\quad \left. \int_0^1 dx \frac{[x^2(-\frac{1}{3}x^2 + \frac{1}{3}x(\frac{m_u^2}{M_W^2} - \frac{m_d^2}{M_W^2} - 1) + \frac{m_d^2}{M_W^2} - 2\frac{m_u^2}{M_W^2})]}{x\frac{m_u^2}{M_W^2} + (1-x)\frac{m_d^2}{M_W^2} - x(1-x)} \right].
\end{aligned} \tag{9.35}$$

It is straightforward and immediate to generalize the above to more generations, especially where massive quarks like top quark is involved whose mass is expected to be greater than 130 GeV.

9.4 The magnetic moment and “A” term in SQFD

The purpose of this section is to discuss the AMM parameter and the “A” term within the context of SQFD. We first consider the AMM parameter. To this end consider the photino-photon multiplet, the contribution from the photon has already been given, Eq. 9.28. For the photino one finds

$$\Delta K^{\tilde{\gamma}} = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} [4s_W^2$$

$$\int_0^1 dx \frac{x^2[-\frac{1}{2}x^2 + \frac{1}{2}x - \frac{1}{2}\frac{m_{\tilde{\nu}}^2}{M_W^2}(2-x) + \frac{1}{2}\frac{m_{\tilde{\nu}}^2}{M_W^2}(1-x)]}{x\frac{m_{\tilde{\nu}}^2}{M_W^2} + (1-x)\frac{m_{\tilde{\nu}}^2}{M_W^2} - x(1-x)}. \quad (9.36)$$

On taking the SUSY -limit ($m_{\tilde{\nu}} = M_W$, and $m_{\tilde{\nu}} = 0$), Eq. 9.36 reduces to

$$\Delta K^{\tilde{\gamma}} = -\frac{2}{3} \frac{\alpha}{\pi}. \quad (9.37)$$

On using equations 9.28 and 9.37 the net contribution of photon-photino multiplet (in the SUSY -limit) reduces to

$$\Delta K^{\tilde{\gamma}-\gamma} = \frac{\alpha}{\pi}. \quad (9.38)$$

Hence there is a change of 40% from the contribution of the photon in QFD. We note that the values given by Aliyev [67] not only differ from our result by a factor of 2, both for magnetic and quadrupole moment parameters but more crucially by sign (this applies to all multiplets).

Next let us consider the Z - zino - H_2^0 multiplet. The contribution by the Z -boson to the AMM parameter has already been given in Eq. 9.29. The remaining members of this multiplet give the following contributions

$$\Delta K^i = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} [(4c_W^2 + 1) \int_0^1 dx \frac{x^2[-\frac{1}{2}x^2 + \frac{1}{2}x - \frac{1}{2}\frac{m_{\tilde{\nu}}^2}{M_W^2}(2-x) + \frac{1}{2}\frac{m_{\tilde{\nu}}^2}{M_W^2}(1-x)]}{x\frac{m_{\tilde{\nu}}^2}{M_W^2} + (1-x)\frac{m_{\tilde{\nu}}^2}{M_W^2} - x(1-x)}], \quad (9.39)$$

and

$$\Delta K^{H_2^0} = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} \int_0^1 dx \frac{x^2[x(x-1) + \frac{1}{2}(\frac{m_{H_2^0}^2}{M_W^2} + 1 - \frac{m_{\tilde{\nu}}^2}{M_W^2})(1-x)]}{x\frac{m_{\tilde{\nu}}^2}{M_W^2} + (1-x)\frac{m_{H_2^0}^2}{M_W^2} - x(1-x)}. \quad (9.40)$$

Similarly for the Higgs-Higgsino multiplet, the Higgs contribution is given by Eq. 9.27, while the remaining members yield

$$\Delta K^h = \frac{G_F M_W^2}{2\sqrt{2}\pi^2}$$

$$\left[\int_0^1 dx \frac{x^2 \left[-\frac{1}{2}x^2 + \frac{1}{2}x - \frac{1}{2} \frac{m_A^2}{M_W^2} (2-x) + \frac{1}{2} \frac{m_A^2}{M_W^2} (1-x) \right]}{x \frac{m_A^2}{M_W^2} + (1-x) \frac{m_A^2}{M_W^2} - x(1-x)} \right], \quad (9.41)$$

and

$$\Delta K^{H_3^0} = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} \int_0^1 dx \frac{x^2 \left[x(x-1) + \frac{1}{2} \left(\frac{m_{H_2^0}^2}{M_W^2} + 1 - \frac{m_A^2}{M_W^2} \right) (1-x) \right]}{x \frac{m_A^2}{M_W^2} + (1-x) \frac{m_{H_2^0}^2}{M_W^2} - x(1-x)}. \quad (9.42)$$

The numerical results corresponding to the contributions to ΔK in the SW model Eqs. 9.27, 9.28, and 9.29 are as follows (after taking out a factor of $\frac{g^2}{16\pi^2} = 2.5828 \times 10^{-3}$ and assuming $m_{H_1^0} = 50\text{GeV}$),

$$\begin{aligned} \Delta K^\gamma &= 1.4989 \\ \Delta K^Z &= -0.3594 \\ \Delta K^{H_1^0} &= 0.9890 \end{aligned} \quad (9.43)$$

For the choices of the various values of masses indicated in Table 9.4 (same choices as in Table 9.1), the variation of ΔK from Table 9.4 is,

$$1.0015 \leq \Delta K^{\gamma-\tilde{\gamma}} \leq 1.4522. \quad (9.44)$$

In a similar manner the variation of ΔK for the Z -zino- H_2^0 and H_1^0 -higgsino- H_3^0 sectors is found to be, see Tables 9.5, 9.6,

$$-2.2071 \leq \Delta K^{Z-\tilde{z}-H_2^0} \leq -0.9064 \quad (9.45)$$

and

$$0.3444 \leq \Delta K^{H_1^0-\tilde{h}-H_3^0} \leq 0.7577 \quad (9.46)$$

respectively. We note that in the above three sectors in the SUSY -limit the expressions given above take on simpler forms[71]. Taking advantage of this simplification we have cross checked our numerical computations against these exact

results and obtain agreement up to 16 places of decimals. Finally, we consider the contribution of the fermion and s-fermion multiplet to the AMM parameter. We have already given the contribution of one generation of fermions. For one generation of sleptons and squarks we find

$$\Delta K^e = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} [(-1) \int_0^1 dx \frac{x^2 [2x(1-x) + (\frac{m_{\tilde{L}}^2}{M_W^2} - \frac{m_{\tilde{e}}^2}{M_W^2} - 1)(1-x)]}{x \frac{m_{\tilde{L}}^2}{M_W^2} + (1-x) \frac{m_{\tilde{e}}^2}{M_W^2} - x(1-x)}], \quad (9.47)$$

$$\Delta K^d = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} [(-\frac{1}{3} \times 3) \int_0^1 dx \frac{x^2 [2x(1-x) + (\frac{m_{\tilde{L}}^2}{M_W^2} - \frac{m_{\tilde{d}}^2}{M_W^2} - 1)(1-x)]}{x \frac{m_{\tilde{L}}^2}{M_W^2} + (1-x) \frac{m_{\tilde{d}}^2}{M_W^2} - x(1-x)}], \quad (9.48)$$

$$\Delta K^i = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} [(\frac{2}{3} \times 3) \int_0^1 dx \frac{x^2 [2x(1-x) + (\frac{m_{\tilde{L}}^2}{M_W^2} - \frac{m_{\tilde{u}}^2}{M_W^2} - 1)(1-x)]}{x \frac{m_{\tilde{L}}^2}{M_W^2} + (1-x) \frac{m_{\tilde{u}}^2}{M_W^2} - x(1-x)}]. \quad (9.49)$$

We now give expressions for $(\frac{\delta g}{g})_1$ which are needed for the "A" term in SQFD. We note that $(\frac{\delta g}{g})_1$'s have already been given for the photon, the Z-boson, Higgs and one generation of leptons and quarks. The others are

$$(\frac{\delta g}{g})_1^{H^0} = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} \int_0^1 dx \frac{x^2 (\frac{1}{2} - \frac{x}{3}) [x - \frac{1}{2} (\frac{m_{\tilde{A}}^2}{M_W^2} + 1 - \frac{m_{\tilde{H}}^2}{M_W^2})]}{x \frac{m_{\tilde{A}}^2}{M_W^2} + (1-x) \frac{m_{\tilde{H}}^2}{M_W^2} - x(1-x)}, \quad (9.50)$$

$$(\frac{\delta g}{g})_1^h = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} [\frac{1}{2} + \frac{1}{2} \int_0^1 dx \frac{x^2 [-\frac{1}{3}x^2 + \frac{1}{3}x (\frac{m_{\tilde{A}}^2}{M_W^2} - \frac{m_{\tilde{H}}^2}{M_W^2} - 1) + \frac{m_{\tilde{A}}^2}{M_W^2} - 2\frac{m_{\tilde{H}}^2}{M_W^2}]}{x \frac{m_{\tilde{A}}^2}{M_W^2} + (1-x) \frac{m_{\tilde{H}}^2}{M_W^2} - x(1-x)}], \quad (9.51)$$

$$\left(\frac{\delta g}{g}\right)_1^{\tilde{\gamma}} = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} [2s_W^2 + 2s_W^2 \int_0^1 dx \frac{x^2[-\frac{1}{3}x^2 + \frac{1}{3}x(\frac{m_{\tilde{\gamma}}^2}{M_W^2} - \frac{m_{\tilde{\gamma}}^2}{M_W^2} - 1) + \frac{m_{\tilde{\gamma}}^2}{M_W^2} - 2\frac{m_{\tilde{\gamma}}^2}{M_W^2}]}{x\frac{m_{\tilde{\gamma}}^2}{M_W^2} + (1-x)\frac{m_{\tilde{\gamma}}^2}{M_W^2} - x(1-x)}], \quad (9.52)$$

$$\left(\frac{\delta g}{g}\right)_1^{H^0} = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} \int_0^1 dx \frac{x^2(\frac{1}{2} - \frac{x}{3})[x - \frac{1}{2}(\frac{m_{H^0}^2}{M_W^2} + 1 - \frac{m_{H^0}^2}{M_W^2})]}{x\frac{m_{H^0}^2}{M_W^2} + (1-x)\frac{m_{H^0}^2}{M_W^2} - x(1-x)}, \quad (9.53)$$

$$\left(\frac{\delta g}{g}\right)_1^{\tilde{z}} = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} [2c_W^2 + \frac{1}{2} + (2c_W^2 + \frac{1}{2}) \int_0^1 dx \frac{x^2[-\frac{1}{3}x^2 + \frac{1}{3}x(\frac{m_{\tilde{z}}^2}{M_W^2} - \frac{m_{\tilde{z}}^2}{M_W^2} - 1) + \frac{m_{\tilde{z}}^2}{M_W^2} - 2\frac{m_{\tilde{z}}^2}{M_W^2}]}{x\frac{m_{\tilde{z}}^2}{M_W^2} + (1-x)\frac{m_{\tilde{z}}^2}{M_W^2} - x(1-x)}], \quad (9.54)$$

$$\left(\frac{\delta g}{g}\right)_1^{\tilde{e}} = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} [(-1) \int_0^1 dx \frac{x^2(\frac{1}{2} - \frac{x}{3})[-2x - (\frac{m_{\tilde{e}}^2}{M_W^2} - \frac{m_{\tilde{e}}^2}{M_W^2} - 1)]}{x\frac{m_{\tilde{e}}^2}{M_W^2} + (1-x)\frac{m_{\tilde{e}}^2}{M_W^2} - x(1-x)}], \quad (9.55)$$

$$\left(\frac{\delta g}{g}\right)_1^{\tilde{d}} = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} [(-\frac{1}{3} \times 3) \int_0^1 dx \frac{x^2(\frac{1}{2} - \frac{x}{3})[-2x - (\frac{m_{\tilde{d}}^2}{M_W^2} - \frac{m_{\tilde{d}}^2}{M_W^2} - 1)]}{x\frac{m_{\tilde{d}}^2}{M_W^2} + (1-x)\frac{m_{\tilde{d}}^2}{M_W^2} - x(1-x)}], \quad (9.56)$$

$$\left(\frac{\delta g}{g}\right)_1^{\tilde{u}} = \frac{G_F M_W^2}{2\sqrt{2}\pi^2} [(\frac{2}{3} \times 3) \int_0^1 dx \frac{x^2(\frac{1}{2} - \frac{x}{3})[-2x - (\frac{m_{\tilde{u}}^2}{M_W^2} - \frac{m_{\tilde{u}}^2}{M_W^2} - 1)(1-x)]}{x\frac{m_{\tilde{u}}^2}{M_W^2} + (1-x)\frac{m_{\tilde{u}}^2}{M_W^2} - x(1-x)}]. \quad (9.57)$$

9.5 Conclusion

In conclusion we have given the AQM, and AMM parameters and the “A” term generated by the one loop corrections to the $\gamma W^+ W^-$ vertex in the context of SQFD. For both “A” term and magnetic moment partial cancellation occur which lower the one loop QFD values by about 50% in the SUSY -limit. We have also given numerical results for the case when SUSY is broken.

Table 9.1: The photon-photino contribution to the quadrupole parameter for various photino and wino masses. 2.5809×10^{-4} has been taken out.

photino mass ($m_{\tilde{\gamma}}$)	wino mass ($m_{\tilde{\omega}}$)	$\Delta Q^{\gamma-\tilde{\gamma}}$
0	81	0
5	50	+0.8556
15	50	+0.7698
20	50	+0.7042
25	50	+0.6404
30	50	+0.6031
35	50	-3.3144
35	55	-1.3785
35	85	0.4269
5	85	0.2160
15	85	0.2802
15	75	-0.1753
15	65	-0.9258
15	55	0.0754

Table 9.2: Z - zino - H_2^0 contribution to the quadrupole parameter for various H_2^0 and zino masses 6.6737×10^{-4} has been taken out.

$m_{H_2^0}$	zino mass $m_{\tilde{z}}$	$\Delta Q^{Z-\tilde{z}-H_2^0}$
92	92	-6.9389×10^{-17}
100	35	-7.0377
100	40	-4.2734
100	45	-3.1194
100	50	-2.4363
100	55	-1.9704
95	35	-7.0351
105	35	-7.0401
110	35	-7.0424
115	35	-7.0446
120	35	-7.0466

Table 9.3: The Higgs-Higgsino contribution to the quadrupole parameter for various H_1^0 , H_3^0 and higgsino masses. 8.6094×10^{-4} has been taken out.

$m_{H_1^0}$	$m_{H_3^0}$	$m_{\tilde{h}}$	$\Delta Q^{H_1^0-\tilde{h}-H_3^0}$
50	50	50	0
50	55	40	-0.7590
50	60	45	-0.5442
50	70	45	-0.5503
50	80	45	-0.5556
60	80	40	-0.7840
65	80	40	-0.7885
70	80	40	-0.7927
75	80	40	-0.7966
75	80	35	-1.3191

Table 9.4: The photon-photino contribution to the quadrupole parameter for various photino and wino masses. 2.5828×10^{-3} has been taken out.

photino mass ($m_{\tilde{\gamma}}$)	wino mass ($m_{\tilde{\omega}}$)	$\Delta K^{\gamma-\tilde{\gamma}}$
0	81	0.8993
5	50	1.4292
15	50	1.4341
20	50	1.4397
25	50	1.4465
30	50	1.4522
35	50	1.0942
35	55	1.1550
35	85	1.1953
5	85	1.0340
15	85	1.0930
15	75	1.0015
15	65	1.1857
15	55	1.3894

Table 9.5: Z – zino – H_2^0 contribution to the AMM parameter for various H_2^0 and zino masses 2.5828×10^{-3} has been taken out.

$m_{H_2^0}$	zino mass m_i	$\Delta Q^{Z-i-H_2^0}$
92	92	-0.9308
100	35	-2.2020
100	40	-1.5234
100	45	-1.2257
100	50	-1.0400
100	55	-0.9064
95	35	-2.2071
105	35	-2.1971
110	35	-2.1923
115	35	-2.1878
120	35	-2.1835

Table 9.6: The Higgs-Higgsino contribution to the quadrupole parameter for various H_1^0 , H_3^0 and higgsino masses. 2.5828×10^{-3} has been taken out.

$m_{H_1^0}$	$m_{H_3^0}$	$m_{\tilde{h}}$	$\Delta K^{H_1^0-\tilde{h}-H_3^0}$
50	50	50	0.6633
50	55	40	0.6509
50	60	45	0.7309
50	70	45	0.7450
50	80	45	0.7577
60	80	40	0.6062
65	80	40	0.5714
70	80	40	0.5395
75	80	40	0.5099
75	80	35	0.3444

Chapter 10

Results, Summary and Conclusions

10.1 Introduction

The purpose of this final chapter is to first give the results then summarize the main ingredients of the thesis and finally give some concluding remarks. This thesis has been divided into essentially three parts. The first part has been devoted to the motivation for considering supersymmetry. To some extent it was indicated that supersymmetry seems to be the next logical theory in the quest for unification of particles and forces. Supersymmetry if it exists can be detected in two different ways. One way is to directly observe the SUSY particles. The other ‘indication’ of the existence of supersymmetry could come from radiative corrections. This thesis has been devoted to the latter possibility. We further restricted ourself to studying the radiative corrections introduced by supersymmetry at the one loop level in the reaction $e^+e^- \rightarrow W^+W^-$. The tree level cross section of the reaction $e^+e^- \rightarrow W^+W^-$ in the context of the SW is well known and has been verified by several independent authors. To the best of our knowledge , there are five different calculations [28, 29, 30, 31, 44] on the cross section for this reaction up to one loop

level in the context of QFD. It is fair to say that there is no exact agreement on the actual numerical magnitude of the corrections.¹ We shall discuss this issue a little further in the next section.

The second part of this thesis dealt with the formalism necessary to calculate the cross section up one loop, involving, the Lagrangian for both QFD and SQFD, the Feynman rules resulting from these Lagrangians, the adopted regularization and the renormalization schemes, and the application of these schemes in showing the cancellation of infinities (non infrared) in the context of both QFD and SQFD. As emphasized before, once the infinities are shown to cancel this provides an excellent check on the correctness of the relative sign of the finite terms and also to some extent provides a marker on the magnitude of the radiative corrections.

The third part of the thesis has been devoted to giving the analytic expressions and the numerical magnitude of the finite corrections. The infrared infinities have been shown to cancel, providing yet another check on the analytic results. We have given all the final results in their complete details so that these can be checked by anyone. In order to make the calculation surveyable we divided and subdivided it into many parts. This also facilitates the task of isolating and locating errors.

10.2 Results

Let us first consider the results given in Chapter 5 for the tree level differential as well as total cross section both in the SM and the NSM. As can be seen from Table 5.3 the total cross section in the SM is of the order of 10 pb, it rises sharply from the energy to pair produce W's i.e. around $ECM=164$ GeV and attains not far from the threshold its maximum (approximately 17.5 pb at $ECM=200$ GeV) ,

¹The authors of [30] and [31] claim to agree with each other.

from where it decreases slowly with energy. From Eq. 5.28 we see that the lowest order total cross section can be written as arising from six different contributions. Each of these individual contributions is quite large, but there are very strong cancellations between these contributions which makes the total cross section well behaved at all energies. The same remarks [i.e. about strong cancellations among the various parts] apply to the differential cross section and the one loop differential cross section. The lowest order differential cross section is very asymmetric with respect to scattering angle (θ), (see Fig. 10.1 and Table 5.1). It peaks very strongly near small angles. This peaking gets more pronounced as the center of mass energy is increased, as can be seen from Fig. 10.1. This forward peaking comes entirely from the T-channel neutrino exchange diagram, (see Fig. 5.1).

Next we consider the bremsstrahlung contributions to the one loop corrected differential cross section. We have given the complete finite expression for this contribution in Chapter 5, (see Eq. 5.92). The numerical results are listed in Tables 5.8 and 5.9 and plotted in Figs. 10.2 and 10.3 respectively. We recall the definition of the parameter a , which is the cutoff parameter and essentially represents a measure of the W energy hidden in the soft photons. We notice from Tables 5.8 and 5.9 and Figs. 10.2 and 10.3 that although there is asymmetric peaking at small CM angles, it is less pronounced than at the tree level. The reason for this is that the bremsstrahlung differential cross section is related to the tree level differential cross section via δ , (see Eq. 5.56), which is a non trivial function of center of mass energy and center of mass angle. In fact the absolute magnitude of δ increases with increasing center of mass angle and energy. The bremsstrahlung contribution by itself is not infrared finite and the one loop virtual radiative corrections must be added to it to make it finite. After summing up all the contributions to the virtual

corrections from Chapters 6, 7, and 8, the numerical values found are listed in Table 10.1, for Higgs mass of 100 GeV. As expected these come with opposite sign to the real bremsstrahlung contributions, thus lowering the total one loop radiative corrections in SM relative to the bremsstrahlung values. The one loop radiative corrections in the SM in terms of percentages for two values of the Higgs mass, and for two different values of the photon cutoff are plotted in Fig. 10.4 and Fig 10.5. For comparison with previous work we have plotted the results of Phillippe [29] in Fig. 10.6. It can be seen from our results given in Fig. 10.4, that there is a fair agreement between our results and those of Phillippe [29], bearing in mind the several “missed” diagrams in [29]. However, it is clear from Figs. 10.4 and 10.6 that we find a much weaker dependence of the one loop radiative corrections on the Higgs mass than Phillippe [29]. One might think that this disagreement might be traced to the Higgs box diagram missed by Phillippe [29]. However, the contribution from this Higgs box diagram is very small and cannot by itself account for the different dependence on the Higgs mass displayed in Figs. 10.4 and 10.6. Our results regarding the dependence of the one loop differential cross section on the Higgs mass seem to agree more with Bohm et al. [30] (see their Table 1). We find [44] a change of approximately 3% – 4% in the differential cross section if the Higgs mass is varied from 10 GeV to 500 GeV. The differential cross section varies approximately 8% if the top mass is varied between 40 GeV and 150 GeV. Our results for the dependence of the differential cross section on the Higgs and top quark are in agreement with Bohm et al. [30] (see Table 2 of [30]). When the total one loop radiative corrections are considered our results are in agreement with the work reported by Bohm et al. [30] is not as good, it is at the order of 0.6% – 10% of δ . Bohm et al. [30] have reported disagreement with both Phillippe [29] and

Lemoine and Veltman [28], but claimed agreement with Fleisher et al. [31]. Since Bohm et al. [30] have reported agreement with Lemoine and Veltman [28] at the order of 10% of δ . Our results are hence in very close agreement with Lemoine and Veltman [28], on the order of $\frac{1}{2}\% - \frac{3}{2}\%$ of δ . It has not been possible to pinpoint the exact sources of disagreement.

We now turn to the main objective of the thesis the contribution of the SUSY particles to the one loop differential cross section. To this end we sum up all the supersymmetric virtual corrections from Chapter 6, 7, and 8. The numerical results are listed in Table 10.6. We may now extract the percentage one loop radiative corrections from the results in Table 10.6, by adding the latter to the standard model virtual and real corrections, as is done in Tables 10.7 and 10.8, for two values of the photon cutoff. The SUSY particles tend to increase the absolute magnitude of the percentage loop corrections (see Figs. 10.7, and 10.8). The reason for this is straightforward. As already mentioned the contribution from SUSY virtual loop corrections comes in opposite sign to the virtual loop corrections generated by the standard particles. Bearing in mind that the bremsstrahlung comes with an opposite sign to the SM virtual corrections, there is an overall increase in the magnitude of the percentage corrections in the context of SQFD. In Figs. 10.8 and 10.9 we have compared our results for QFD and SQFD. As is easily noticed there is almost a constant increase or enhancement of approximately 6.7% relative to the SM result. This enhancement gradually increases with the CM angle so that at $\theta = 170$ deg it rises approximately to around 11.7%. The introduction of a mass subtraction “counterterm” slightly decreases this enhancement by about 1%, see Figs. 10.9 and 10.10. The question arises as to what happens if the masses of all the supersymmetric partners of the standard particles are taken to be heavier than

the W mass ? (see Chapter 6) For this case of S-P masses we choose the values given in Chapter 6. The results we obtained are listed in Table 12 and plotted in Figure 11. As can be seen from Figure 11 δ is reduced in the forward direction and increased in the backward direction resulting in an overall greater variation of δ compared to the other case of S-P masses (shown in Figure 11 for the sake of comparison).

10.3 Summary

We now summarize the main points of the thesis.

1. : The main theme of Chapter 1 was how the idea of symmetry (in particular gauge symmetry) has been used in physics to explain the fundamental interactions of nature, and the more ambitious task of unification of these interactions. Supersymmetry, as the name implies is a possible attempt in this quest to get a deeper understanding of the workings of matter at the fundamental level. We emphasize, once again that this chapter is not aimed at experts in the field.
2. : Chapter 2 dealt with the electroweak Lagrangian, dimensional regularization, and introduced the on mass shell renormalization scheme. This machinery was needed to compute the one loop corrections in the context of QFD, which was one of the tasks of the thesis.
3. : Chapter 3 was devoted mainly to the details of the supersymmetric model that has been employed to calculate the one loop radiative corrections introduced by S-P.
4. : In Chapter 4 the OMRS was applied to QFD and SQFD.

So by the end of the first part of the thesis we had accomplished the task of setting up the Lagrangian for both QFD and SQFD, and showing the internal consistency of our adopted renormalization scheme.

The second part of thesis was devoted to detailing mainly the analytic and numerical results of all the finite one loop radiative corrections in QFD and SQFD.

1. : In chapter 5, the Born cross section and differential cross section were discussed in the SM. We also tried to study deviations away from the SM by parameterizing the three vector vertex by NS values. By varying the parameters slightly [on the order of α] away from their SM values we got rough estimate of the one loop radiative corrections in the SM.
2. : Chapters 6, 7, and 8 dealt with the one loop radiative corrections from the self energy, triangle and box diagrams respectively.

In this part of the thesis all the results were given in their full detail, so that these may be checked easily. The corresponding numerical results were listed in several tables. One of the advantages of a full listing of the numerical results is that it allows a detailed comparison with other works.

In the third and final part of the thesis, we have given a brief discussion of the magnetic and quadrupole moments of the W boson, and summed up all the one loop radiative corrections in the context of QFD and SQFD.

10.4 Conclusions

In conclusion we find that our results for the QFD one loop radiative corrections to the process $e^+e^- \rightarrow W^+W^-$ agree with those given by Lemoine and Veltman [28] at the order of $\frac{1}{2}\% - \frac{3}{2}\%$ of δ , and on the order of $0.6\% - 10\%$ of δ with Bohm et al.

[30]. For the benefit of the reader we list below the explicit numerical comparison of the percentage one loop corrections between our work and that reported by Bohm et al. [30], (using the same parameters as Bohm et al. [30] , i.e. Higgs mass of 100 GeV, top quark mass of 100 GeV, W-boson mass of 82 GeV, Z-boson mass of 93 GeV, ECM=200 GeV. All the other masses and parameters are given in the beginning of Chapter 6.)

θ	Bohm et al. [30]	Us [44]
10°	-4.29%	-3.87%
90°	-7.24%	-7.29%
170°	-10.04%	-10.68%

The major aim of the thesis was to calculate the SQFD corrections, the addition of the SUSY particles tends to increase the percentage loop corrections on the order of 8%, with an accurate measurement of the cross section at LEP II, one can in principle detect such a deviation away from the SM.

Figure 10.1: The tree level differential cross section for three different center of mass energies.

Figure 10.2: Angular dependence of the bremsstrahlung corrections relative to the tree level differential cross section with $a=0.1$.

Figure 10.3: Angular dependence of the bremsstrahlung corrections relative to the tree level differential cross section with $a=0.05$.

Figure 10.4: The percentage one loop radiative corrections to the lowest order cross section in QFD compared for two values of Higgs mass. Top quark mass is 100 GeV, and the photon cutoff is here taken as $a = 0.1$.

Figure 10.5: The percentage one loop radiative corrections to the lowest order cross section in QFD compared for two values of Higgs mass. Top quark mass is 100 GeV, and the photon cutoff is here taken as $a = 0.05$.

Figure 10.6: The percentage one loop radiative corrections to the lowest order cross section in QFD given by Phillippe [29].

Figure 10.7: The percentage one loop radiative corrections to the lowest order cross section compared for QFD and SQFD, the Higgs mass and top masses are 50 GeV and 100 GeV respectively. The photon cutoff is here taken as $a = 0.1$.

Figure 10.8: The percentage one loop radiative corrections to the lowest order cross section compared for QFD and SQFD, the Higgs mass and top masses are 50 GeV and 100 GeV respectively. The photon cutoff is here taken as $a = 0.05$.

Figure 10.9: The percentage one loop radiative corrections to the lowest order cross section compared for QFD and two cases of SQFD, the Higgs mass and top masses are 50 GeV and 100 GeV respectively. The photon cutoff is here taken as $a = 0.1$.

Figure 10.10: The percentage one loop radiative corrections to the lowest order cross section compared for QFD and two cases of SQFD, the Higgs mass and top masses are 50 GeV and 100 GeV respectively. The photon cutoff is here taken as $a = 0.05$.

Table 10.1: The virtual one loop radiative corrections in pb for $ECM = 200\text{GeV}$ for the Higgs and the top quark mass of 100 GeV in the SM.

$\theta(deg)$	$(\frac{d\sigma}{d\cos\theta})_1^{\bigcirc}$	$(\frac{d\sigma}{d\cos\theta})_1^{\bigtriangleup}$	$(\frac{d\sigma}{d\cos\theta})_1^{\bigsquare+\gamma}$
10	+8.0671	+8.0873	+12.5996
30	+4.1400	+5.4477	+9.8596
50	+1.4384	+2.6666	+6.6243
70	+0.1561	+1.2336	+4.6825
90	-0.4418	+0.5762	+3.4500
110	-0.7175	+0.2630	+2.7449
130	-0.8589	+0.0987	+2.3840
150	-0.9322	+0.0075	+2.1086
170	-0.9640	-0.1325	+2.1117

Figure 10.11: The percentage one loop radiative corrections to the lowest order cross section compared for the two cases of SUSY particles masses. SQFD* is the case when most of the S-P masses are above the W boson mass. The photon cutoff is here taken as $a = 0.1$.

Table 10.2: The percentage one loop radiative corrections to the lowest order differential cross section for the Higgs mass and the top quark mass of 100 GeV in the SM. The photon cutoff is taken to be $a = 0.1$.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{virtual}$ pb	$(\frac{d\sigma}{d\cos\theta})_1^{real}$ pb	δ^{total} %
10	+28.7540	-34.2446	-3.5284
30	+19.4473	-24.0332	-4.2885
50	+10.7293	-13.8036	-5.1789
70	+6.0722	-8.1109	-6.0791
90	+3.5844	-5.1752	-7.7382
110	+2.2904	-3.5939	-9.4890
130	+1.6238	-2.6759	-10.6529
150	+1.1839	-2.1297	-12.3793
170	+1.0152	-1.8633	-12.9000

Table 10.3: The percentage one loop radiative corrections to the lowest order differential cross section for the Higgs mass and the top quark mass of 100 GeV in the SM. The photon cutoff is taken to be $a = 0.05$.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{virtual}$ pb	$(\frac{d\sigma}{d\cos\theta})_1^{real}$ pb	δ^{total} %
10	+28.7540	-45.5995	-10.8253
30	+19.4473	-31.9715	-11.7119
50	+10.7293	-18.3357	-12.8135
70	+6.0722	-10.7547	-13.9626
90	+3.5844	-6.8497	-15.8835
110	+2.2904	-4.7488	-17.8962
130	+1.6238	-3.5308	-19.3090
150	+1.1839	-2.8071	-21.2455
170	+1.0152	-2.4546	-21.8940

Table 10.4: The percentage one loop radiative corrections to the lowest order differential cross section for the Higgs mass of 50 GeV and the top quark mass of 100 GeV in the SM. The photon cutoff is taken to be $a = 0.1$.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{virtual}$ pb	$(\frac{d\sigma}{d\cos\theta})_1^{real}$ pb	δ^{total} %
10	+28.5798	-34.2446	-3.6403
30	+19.3220	-24.0332	-4.4056
50	+10.6483	-13.8036	-5.3153
70	+6.0072	-8.1109	-6.2730
90	+3.5252	-5.1752	-8.0262
110	+2.2361	-3.5939	-9.8843
130	+1.5737	-2.6759	-11.1602
150	+1.1370	-2.1297	-12.9931
170	+0.9706	-1.8633	-13.5784

Table 10.5: The percentage one loop radiative corrections to the lowest order differential cross section for the Higgs mass of 50 GeV and the top quark mass of 100 GeV in the SM. The photon cutoff is taken to be $a = 0.05$.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{virtual}$ pb	$(\frac{d\sigma}{d\cos\theta})_1^{real}$ pb	δ^{total} %
10	+28.5798	-45.5995	-10.9372
30	+19.3220	-31.9715	-10.8290
50	+10.6483	-18.3357	-12.9500
70	+6.0072	-10.7547	-14.1564
90	+3.5252	-6.8497	-16.1715
110	+2.2361	-4.7488	-18.2915
130	+1.5737	-3.5308	-19.8163
150	+1.1370	-2.8071	-21.8594
170	+0.9706	-2.4546	-22.5724

Table 10.6: The additional virtual one loop radiative corrections in pb for $ECM = 200$ GeV due to the addition of the supersymmetric particles.

$\theta(deg)$	$(\frac{d\sigma}{d\cos\theta})_1^{\bigcirc}$	$(\frac{d\sigma}{d\cos\theta})_1^{\triangle}$	$(\frac{d\sigma}{d\cos\theta})_1^{\square}$	$(\frac{d\sigma}{d\cos\theta})_1^{Total}$
10	-9.2337	-1.8838	+0.2842	-10.8333
30	-6.2351	-1.0077	-0.3435	-7.5863
50	-3.3755	-0.4449	-0.5927	-4.4131
70	-1.8386	-0.1602	-0.6985	-2.6973
90	-1.0709	-0.0252	-0.5744	-1.6705
110	-0.6689	+0.0333	-0.5215	-1.1571
130	-0.4413	-0.1277	-0.4723	-1.0413
150	-0.3121	-0.0349	-0.4330	-0.7800
170	-0.2499	+0.0063	-0.5213	-0.7649

Table 10.7: The percentage one loop radiative corrections to the lowest order differential cross section of SQFD. The photon cutoff is taken to be $a = 0.1$.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{virtual}$ pb	$(\frac{d\sigma}{d\cos\theta})_1^{real}$ pb	δ^{total} %
10	+17.7465	-34.2446	-10.6021
30	+11.7357	-24.0332	-11.4999
50	+6.2352	-13.8036	-12.7495
70	+3.3099	-8.1109	-14.3160
90	+1.8547	-5.1752	-16.1520
110	+1.0790	-3.5939	-18.3075
130	+0.5324	-2.6759	-21.7037
150	+0.3570	-2.1297	-23.2023
170	+0.2057	-1.8633	-25.2129

Table 10.8: The percentage one loop radiative corrections to the lowest order differential cross section of SQFD. The photon cutoff is taken to be $a = 0.05$.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{virtual}$ pb	$(\frac{d\sigma}{d\cos\theta})_1^{real}$ pb	δ^{total} %
10	+17.7465	-45.5995	-17.8990
30	+12.0792	-31.9715	-18.6021
50	+6.2352	-18.3357	-20.3842
70	+3.3099	-10.7547	-22.1994
90	+1.8547	-6.8497	-24.2973
110	+1.0790	-4.7488	-26.7147
130	+0.5324	-3.5308	-30.3599
150	+0.3570	-2.8071	-32.0685
170	+0.2057	-2.4546	-34.2069

Table 10.9: The additional virtual one loop radiative corrections in pb for $ECM = 200$ GeV due to the addition of the supersymmetric particles when the mass splitting subtraction term is included.

$\theta(deg)$	$(\frac{d\sigma}{d\cos\theta})_1^{\bigcirc}$	$(\frac{d\sigma}{d\cos\theta})_1^{\Delta}$	$(\frac{d\sigma}{d\cos\theta})_1^{\square}$	$(\frac{d\sigma}{d\cos\theta})_1^{Total}$
10	-7.3892	-2.3637	+0.2842	-9.4687
30	-5.1185	-1.1997	-0.3435	-6.6617
50	-2.8494	-0.4693	-0.5927	-3.9114
70	-1.6109	-0.1124	-0.6985	-2.4218
90	-0.9876	+0.0539	-0.5744	-1.5081
110	-0.6587	+0.1263	-0.5215	-1.0539
130	-0.4721	-0.0279	-0.4723	-0.9723
150	-0.3636	+0.0681	-0.4330	-0.7285
170	-0.3126	+0.1056	-0.5213	-0.7283

Table 10.10: The percentage one loop radiative corrections to the lowest order differential cross section of SQFD with mass splitting subtraction. The photon cutoff is taken to be $a = 0.1$.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{virtual}$ pb	$(\frac{d\sigma}{d\cos\theta})_1^{real}$ pb	δ^{total} %
10	+19.1111	-34.2446	-9.7251
30	+12.6603	-24.0332	-10.6353
50	+6.7369	-13.8036	-11.9044
70	+3.5854	-8.1109	-13.4945
90	+2.0171	-5.1752	-15.3621
110	+1.1822	-3.5939	-17.5562
130	+0.6014	-2.6759	-21.0050
150	+0.4085	-2.1297	-22.5282
170	+0.2423	-1.8633	-24.6562

Table 10.11: The percentage one loop radiative corrections to the lowest order differential cross section of SQFD with mass splitting subtraction. The photon cutoff is taken to be $a = 0.05$.

$\theta(deg)$ —	$(\frac{d\sigma}{d\cos\theta})_1^{virtual}$ pb	$(\frac{d\sigma}{d\cos\theta})_1^{real}$ pb	δ^{total} %
10	+19.1111	-45.5995	-17.0220
30	+12.6603	-31.9715	-18.0587
50	+6.7369	-18.3357	-19.5390
70	+3.5854	-10.7547	-21.3779
90	+2.0171	-6.8497	-23.5074
110	+1.1822	-4.7488	-25.9635
130	+0.6014	-3.5308	-29.6612
150	+0.4085	-2.8071	-31.3945
170	+0.2423	-2.4546	-33.6502

Table 10.12: The percentage one loop radiative corrections to the lowest order differential cross section of SQFD with masses of most SUSY particles above the W mass. The photon cutoff is taken to be $a = 0.1$.

$\theta(deg)$ —	δ^{QFD} %	δ^{S-P} %	δ^{SQFD} %
10	-3.6403	-0.7706	-4.4109
30	-4.4056	-1.9865	-6.3921
50	-5.3153	-3.1505	-8.4658
70	-6.2730	-4.6759	-10.9489
90	-8.0262	-6.6909	-14.7171
110	-9.8843	-9.1408	-19.0251
130	-11.1600	-11.9398	-23.0998
150	-12.9930	-14.8138	-27.8068
170	-13.5500	-16.4882	-30.0382

Appendix A

List of Acronyms and Symbols

α	Fine structure constant
CKM	Cabibbo-Kobayashi-Maskawa
CM	Center of mass
θ_W	Weinberg angle of Weak interactions
c_W	$\cos \theta_W$
DR	Dimensional Regularization
$\doteq$	$= \frac{g^2}{16\pi^2}$
EPP	Elementary Particle Physics
EM	Electromagnetism
EW	Electroweak Model
FP	Faddeev-Popov
FCNC	Flavor changing neutral currents
GIM	Glashow-Iliopoulos-Maiani mechanism
GLA	Graded Lie Algebras
GR	General Relativity
G_μ	Fermi constant
GUTS	Grand Unified Theories

SGUTS	Supersymmetric Grand Unified Theories
IR	Infrared
IRD	Infrared divergent
IVB	Intermediate vector boson
LSP	Lightest Supersymmetric Particle
$\mathcal{L}$	Lagrangian, strictly speaking Lagrangian Density
M_W	mass of the W vector boson
M_Z	mass of the Z vector boson
NSM	Non Standard Model
OMRS	On the mass shell renormalization scheme
QED	Quantum Electrodynamics
QCD	Quantum Chromodynamics
QFD	Quantum Flavor Dynamics
QFT	Quantum field theory
RGE	Renormalization group equation
SQFD	Supersymmetric Quantum Flavor Dynamics
SW	Salam-Weinberg Model
SM	Standard Model
s_W	$\sin \theta_W$
SSB	Spontaneous Symmetry Breaking
SSSB	Soft Supersymmetry Breaking
SUSY	Supersymmetry
S-P	S-Particles, the partners of ordinary particles introduced by Supersymmetry
SUGRA	Supergravity

t_W	$\tan \theta_W$
u, d, c, s, t, b	quark flavors
$\nu_e, e, \nu_\mu, \mu, \nu_\tau, \tau$	lepton flavors
$V - A$	Vector-Axial vector
VEV	Vacuum expectation value
W	W vector boson
WI	Weak interactions
WFNR	Wave function renormalization
WFRC	Wave function renormalization constant
Z	Z vector boson
γ	Photon

Appendix B

Tree-level EW Lagrangian; Counterterm Lagrangian

The explicit form of the QFD Lagrangian is now given in terms of the physical fields. For the fermions,

$$\mathcal{L}_{f1} = \bar{f}(i\partial - m_f + eQ_f A)f + \frac{g}{c_w} \bar{f} \gamma^\mu (T_{3f} \frac{1-\gamma_5}{2} - Q_f s_w^2) f Z_\mu \quad (\text{B.1})$$

$$\mathcal{L}_{f2} = \frac{g}{\sqrt{2}} \bar{L}_f \gamma^\mu (\tau_+ W_\mu^+ + \tau_- W_\mu^-) L_f - \frac{g}{\sqrt{2}} \frac{m_f}{M_w} (\bar{L}_f \Phi' R_{df} + h.c) - \frac{g}{\sqrt{2}} \frac{m_f}{M_w} (\bar{L}_f \bar{\Phi}' R_{uf} + h.c) \quad (\text{B.2})$$

For the gauge bosons,

$$\mathcal{L}_{II} = -\frac{1}{2} |\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ie(W_\mu^+ A_\nu - W_\nu^+ A_\mu) + igc_w(W_\mu^+ Z_\nu - W_\nu^+ Z_\mu)|^2 - \frac{1}{4} |\partial_\mu Z_\nu - \partial_\nu Z_\mu + igc_w(W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+)|^2 - \frac{1}{4} |\partial_\mu A_\nu - \partial_\nu A_\mu + ie(W_\mu^- W_\nu^+ - W_\nu^- W_\mu^+)|^2 \quad (\text{B.3})$$

For the KE terms and gauge couplings of the scalars,

$$\mathcal{L}_{III} = |\partial_\mu G^+ - ig \frac{s_w}{2c_w} Z_\mu G^+ - ie A_\mu G^+ - \frac{1}{2} g W_\mu^+ G^+ (v + H_1^0 + iG^0)|^2 + \frac{1}{2} |\partial_\mu (H_1^0 + iG^0) - ig W_\mu^- G^+ + \frac{ig}{2c_w} Z_\mu (v + H_1^0 + iG^0)|^2 \quad (\text{B.4})$$

For the mass and self coupling of the scalars

$$\mathcal{L}_{IV} = -\frac{1}{2} M_{H_1^0}^2 (H_1^0)^2 - \frac{g^2}{2} \frac{M_{H_1^0}^2}{M_w^2} G^0 (G^+ G^- + \frac{1}{2} |H_1^0 + iG^0|^2) - \frac{g^2}{8} \frac{M_{H_1^0}^2}{M_w^2} (G^+ G^- + \frac{1}{2} |H_1^0 + iG^0|^2)^2 \quad (\text{B.5})$$

The Faddeev-Popov ghost Lagrangian is

$$\begin{aligned} \mathcal{L}_{F.P.} = & -\bar{c}^+(\partial^2 + M_W^2)c^+ + igc_W\bar{c}^+\partial^\mu(Z_\mu c^+) + iec^+\partial^\mu(A_\mu c^+) - \\ & igc^+\partial^\mu[W_\mu^+(c_s c_W + c_A s_W)] - M_W c^+[\frac{g s_W}{2c_W}c_s G^+ + ec_A G^+ + \\ & \frac{g}{2}c^+(H_1^0 + iG^0)] + [c^+ \rightarrow c^-, W^+ \rightarrow W^-, i \rightarrow -i] - \bar{c}_s(\partial^2 + \\ & M_Z^2)c_s - igc_W\bar{c}_s\partial^\mu(W_\mu^- c^+ - W_\mu^+ c^-) - M_Z\bar{c}_s[-\frac{g}{2}c^- G^+ - \\ & \frac{g}{2}c^+ G^- + \frac{g}{2c_W}c^s G^0] - \bar{c}_A\partial^2 c_A - ig s_W\bar{c}_A\partial^\mu(W_\mu^- c^+ - W_\mu^+ c^-) \end{aligned} \quad (B.6)$$

Now let us display the counterterm Lagrangian.

$$\begin{aligned} \mathcal{L}_{SW}^c = & \sum_{flavor,color} \bar{f}[i\partial(\delta Z_{Lf}L + \delta Z_{Rf}R) - \delta m_f]f + \\ & \sum_{flavor,color} \bar{f}\gamma^\mu[\frac{g}{c_W}(T_{3f} - Q_f s_W^2)\delta Z_{Lf}L - \\ & \frac{g}{c_W}Q_f s_W^2\delta Z_{Rf}R]fZ_\mu + \sum_{flavor,color} \bar{f}\gamma^\mu[-(-eQ_f\delta Z_{Lf} + \\ & \delta g T_{3f} s_W + \delta g'(Q_f - T_{3f} c_W))L + Q_f(e\delta Z_{Rf} + \\ & \delta g' c_W)R]fA_\mu + \sum_{doublet,color} \frac{g}{\sqrt{2}}(\delta Z_{Lf} + \frac{\delta g}{g})\bar{L}_f\gamma^\mu(\tau_+ W_\mu^+ + \\ & \tau_- W_\mu^-)L_f - \frac{1}{2}\delta Z_W|W_{\mu\nu}^+|^2 + (\delta Z_\phi M_W^2 + \delta M_W^2)W_\mu^+ W^{-\mu} - \\ & \frac{1}{4}\delta Z_Z|Z_{\mu\nu}|^2 + \frac{1}{2}(\delta Z_\phi M_Z^2 + \delta M_Z^2)Z_\mu Z^\mu - \frac{1}{4}\delta Z_A|A_{\mu\nu}|^2 - \\ & \frac{1}{2}(\delta Z_B - \delta Z_W)c_W s_W A_{\mu\nu} Z^{\mu\nu} - \frac{\delta g}{g}M_W^2 t_W Z_\mu A^\mu \end{aligned} \quad (B.7)$$

The counterterms for the ghosts

$$\begin{aligned} \mathcal{L}_{F.P.}^c = & -\delta\bar{Z}_W\bar{c}^+(\partial^2 + M_W^2)c^+ - \frac{1}{2}(\delta Z_\phi M_W^2 + \delta M_W^2)\bar{c}^+c^+ - \\ & \delta\bar{Z}_Z\bar{c}_s(\partial^2 + M_Z^2)c_s - \frac{1}{2}(\delta Z_\phi M_Z^2 + \delta M_Z^2)\bar{c}_s c_s \end{aligned} \quad (B.8)$$

Appendix C

Dimensional Regularization (DR) formulae; A and B integrals

In this appendix we give an outline of the derivation of the integrals involved in the self-energy one and two point functions i.e the A and B integrals of reference [45]. We also give of these integrals in our notation. Note that unlike reference [45] we use the metric convention $(+, -, -, -)$. Let us consider the following useful n -dimensional loop integrals.

$$\int \frac{d^n q}{(2\pi)^n} \frac{1}{(q^2 - M^2 + i\delta)^N} = \frac{i(-1)^N}{(4\pi)^{n/2}} \frac{\Gamma(N - n/2)}{\Gamma(N)} \frac{1}{(M^2 - i\delta)^{N - n/2}} \quad (C.1)$$

$$\int \frac{d^n q}{(2\pi)^n} \frac{q^2}{(q^2 - M^2 + i\delta)^N} = \frac{i(-1)^{N-1}}{2(4\pi)^{n/2}} \frac{\Gamma(N - 1 - n/2)}{\Gamma(N)} \frac{n}{(M^2 - i\delta)^{N - 1 - n/2}} \quad (C.2)$$

$$\int \frac{d^n q}{(2\pi)^n} \frac{q_\mu q_\nu f(q^2)}{(q^2 - M^2 + i\delta)^N} = \frac{1}{n} g_{\mu\nu} \int \frac{d^n q}{(2\pi)^n} \frac{q^2 f(q^2)}{(q^2 - M^2 + i\delta)^N} \quad (C.3)$$

$$\int \frac{d^n q}{(2\pi)^n} \frac{q_\mu q_\nu}{(q^2 - M^2 + i\delta)^N} = \frac{i(-1)^{N-1}}{2(4\pi)^{n/2}} \frac{\Gamma(N - 1 - n/2)}{\Gamma(N)} \frac{g_{\mu\nu}}{(M^2 - i\delta)^{N - 1 - n/2}} \quad (C.4)$$

Now consider the explicit derivation of the A integrals. To this end consider the definition [reference [45]]

$$A(m) = -\frac{1}{i\pi^2} \int d^n q \frac{1}{(q^2 - m^2)} \quad (C.5)$$

Using the formula C.1 in C.5 one obtains after some algebra

$$A(m) = m^2[-\Delta - 1 + \ln(\frac{m^2}{M_W^2})] \quad (\text{C.6})$$

Here and everywhere else

$$\Delta = -\frac{2}{\epsilon} - \gamma_E + \ln(4\pi\mu^2) - \ln(M_W^2) + O(\epsilon) \quad (\text{C.7})$$

$$n = 4 + \epsilon \quad (\text{C.8})$$

Next we must evaluate B_0 , B_1 , B_{21} and B_{22} where these arise in the definitions

$$B_0; B_\mu; B_{\mu\nu} = \frac{1}{i\pi^2} \int d^n q \frac{1; q_\mu; q_\mu q_\nu}{(q^2 - m_1^2)((q+k)^2 - m_2^2)} \quad (\text{C.9})$$

The function B_μ must be proportional to k_μ ; this defines B_1 . Similarly B_{21} and B_{22} are defined or extracted from $B_{\mu\nu}$.

$$B_\mu = B_1 k_\mu \quad (\text{C.10})$$

$$B_{\mu\nu} = B_{21} k_\mu k_\nu - B_{22} g_{\mu\nu} \quad (\text{C.11})$$

Let us start with the evaluation of B_0

$$B_0 = \frac{1}{i\pi^2} \int d^n q \int_0^1 dx \frac{1}{[+(1-x)(q^2 - m_1^2) + x((q+k)^2 - m_2^2)]^2} \quad (\text{C.12})$$

Where we have used Feynman-parametric technique. Rearranging the above i.e and using the shift of variable $q \longrightarrow q' - xp$ one may rewrite C.12

$$B_0 = \frac{1}{i\pi^2} \int d^n q \int_0^1 dx \frac{1}{[q^2 - M^2]^2} \quad (\text{C.13})$$

where

$$M^2 = xm_2^2 + (1-x)m_1^2 - x(1-x)k^2 \quad (\text{C.14})$$

Using dimensional regularization on C.13 one gets

$$B_0 = \Delta - \int_0^1 dx \ln\left(\frac{M^2}{M_W^2}\right) \quad (\text{C.15})$$

Following the same procedure as for B_0 one obtains for B_1 , B_{21} and B_{22}

$$B_1 = -(1/2) \Delta + \int_0^1 x dx \ln\left(\frac{M^2}{M_W^2}\right) \quad (\text{C.16})$$

$$B_{21} = (1/3) \Delta - \int_0^1 x^2 dx \ln\left(\frac{M^2}{M_W^2}\right) \quad (\text{C.17})$$

$$B_{22} = -(1/4)(\Delta + 1)(m_1^2 + m_2^2 - (1/3)k^2) + (1/2) \int_0^1 dx M^2 \ln\left(\frac{M^2}{M_W^2}\right) \quad (\text{C.18})$$

Alternatively one knows that the functions B_1 , B_{21} and B_{22} are algebraically related to A and B_0 and one may find these relations by using the definitions C.5 and C.9. Once these relations are found and having explicit forms for A and B_0 on hand one may use these algebraic relations to express B_1 , B_{21} and B_{22} in terms of A and B_0 . Let us demonstrate these remarks for B_1 . To this end consider the definitions C.5 and C.9 then contracting both sides with k_μ of C.9 one obtains

$$k^2 B_1 = \frac{1}{i\pi^2} \int d^n q \frac{q \cdot p}{(q^2 - m_1^2)((q + k)^2 - m_2^2)} \quad (\text{C.19})$$

using the identity

$$q \cdot p = (1/2)[((q + k)^2 - m_2^2) - (q^2 - m_1^2) - k^2 + m_2^2 - m_1^2] \quad (\text{C.20})$$

in C.19 one may rewrite it as

$$p^2 B_1 = (1/2)[-A(m_1) + A(m_2) + (m_2^2 - m_1^2 - k^2)B_0] \quad (\text{C.21})$$

Similarly we obtain

$$p^2 B_{21} - B_{22} = (1/2)[-A(m_2) + (m_2^2 - m_1^2 - k^2)B_1] \quad (\text{C.22})$$

Figure C.1: Higgs contribution to the self energy of the W-boson.

and

$$p^2 B_{21} - 4B_{22} = m_1^2 B_0 - A(m_2) + \epsilon B_{22} \quad (\text{C.23})$$

C.23 may be reduced to

$$p^2 B_{21} - 4B_{22} = m_1^2 B_0 - A(m_2) + (1/2)(m_1^2 + m_2^2 - (1/3)k^2) \quad (\text{C.24})$$

C.22 and C.24 can be solved to obtain B_{21} and B_{22} in terms of A and B_0 .

As an illustration as to how A and B integrals arise in self-energy contributions. Let us consider the case of the Higgs contributions to W self-energy, Fig. C.1 Diagram(a) yields

$$T_{\mu\nu}^a = -\frac{g^2}{4} g_{\mu\nu} \int \frac{d^n q}{(2\pi)^n} \frac{1}{(q^2 - M_H^2)} \quad (\text{C.25})$$

or

$$T_{\mu\nu}^a \doteq \frac{i}{4} g_{\mu\nu} A(M_H) \quad (\text{C.26})$$

Similarly diag(b) gives

$$T_{\mu\nu}^b \doteq -iM_W^2 g_{\mu\nu} B_0 \quad (\text{C.27})$$

Diagram(c) requires a little more work

$$T_{\mu\nu}^c = \frac{g^2}{4} \int \frac{d^n q}{(2\pi)^n} \frac{(2q+k)_\mu (2q+k)_\nu}{(q^2 - M_H^2)((q+k)^2 - M_W^2)} \quad (\text{C.28})$$

$$T_{\mu\nu}^c \doteq \frac{i}{4} [-4B_{22}g_{\mu\nu} + (B_0 + 4B_1 + 4B_{21})k_\mu k_\nu] \quad (\text{C.29})$$

So that a_W^H (which is the coefficient of $g_{\mu\nu}$) may be written from the above three expressions C.26, C.27 and C.29 as

$$a_W^H(k^2) \doteq \frac{1}{4} [A(M_H) - 4B_0 M_W^2 - 4B_{22}] \quad (\text{C.30})$$

Using the explicit forms for integrals in C.30

$$a_W^H(k^2) \doteq -(\frac{1}{12}k^2 + \frac{3}{4}M_W^2) \triangle + \frac{1}{4}M_H^2 \ln \frac{M_H^2}{M_W^2} + \frac{1}{4}M_W^2 - \frac{1}{12}k^2 - \frac{1}{2} \int_0^1 dx (M'^2 - 2M_W^2) \ln(\frac{M'^2}{M_W^2}) \quad (\text{C.31})$$

Here

$$M'^2 = xM_W^2 + (1-x)M_H^2 - x(1-x)k^2 \quad (\text{C.32})$$

In agreement with reference [43] Now we define the integrals which were used in section 2.4

$$I_0(m_1, m_2, k) = \int_0^1 dx x \ln(x \frac{m_2^2}{m_1^2} + (1-x) - x(1-x) \frac{k^2}{m_1^2}) \quad (\text{C.33})$$

$$I_1(m_1, m_2, k) = \int_0^1 dx (1-x) \ln(x \frac{m_2^2}{m_1^2} + (1-x) - x(1-x) \frac{k^2}{m_1^2}) \quad (\text{C.34})$$

$$I_{ij}(m_1, m_2, k) = \int_0^1 dx \frac{(x)^i (1-x)^j}{(x \frac{m_2^2}{m_1^2} + (1-x) - x(1-x) \frac{k^2}{m_1^2})} \quad (\text{C.35})$$

Appendix D

Quark Mixing

In the standard model with $SU(2) \times U(1)$ as the gauge group for electroweak interactions, both the quarks and leptons are assigned to be left-handed doublets and right-handed singlets. The quark mass eigenstates are not the same as the weak eigenstates. Hence a matrix must be given to relate these bases. This was done for six quarks and given an explicit parameterization by Kobayashi and Maskawa in 1973. It generalizes the four quark case, where the matrix is parameterized by a single angle, the Cabibbo angle. Intuitively one suspects that the CKM angles represent "angles" between the strong and the weak forces, not unlike the θ_W which is a link between the electromagnetic and the weak forces. Hence these CKM angles may contain hints of strong and weak unification.

By convention, the three charge 2/3 quarks are unmixed, and all the mixing is expressed in terms of 3×3 unitary matrix M operating on the charge $-1/3$ quarks viz:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = M \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (D.1)$$

$$M = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad (D.2)$$

There are several parameterization of the CKM matrix, the one due to Kobayashi and Maskawa reads

$$M = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 c_3 + s_2 s_3 e^{i\delta} \\ s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & -c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix} \quad (\text{D.3})$$

In the above matrix c_i, s_i denote $\cos \theta_i, \sin \theta_i$, with θ_1 equal to the Cabibbo angle θ_C . The angles $\theta_{1,2}$, turn out be small experimentally. δ is a measure of CP violation in weak decays. All these parameters have been experimentally determined by various decays [3], and we refer the reader for a detailed discussion of CKM matrix to [3]. We now list the experimentally determined values of the CKM matrix [3], for the convenience of the reader

$$M = \begin{pmatrix} 0.974 - 0.976 & 0.218 - 0.222 & 0.000 - 0.012 \\ 0.183 - 0.213 & 0.810 - 1.000 & 0.035 - 0.049 \\ 0.000 - 0.022 & 0.032 - 0.050 & 0.998 - 0.999 \end{pmatrix} \quad (\text{D.4})$$

We notice that in the above matrix the diagonal elements have values close to 1, whereas the off-diagonal elements are small, with values close to 0. The values also decrease as one goes further from the diagonal.

Appendix E

The α and the G_μ schemes

As can be inferred from the name of the two schemes the difference between them lies in whether one uses α as one of the fundamental input parameters or the Fermi constant G_μ . In both schemes the physical gauge boson masses, M_W and M_Z appear as input parameters, and consequently the definition

$$\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2} \quad (\text{E.1})$$

holds for the weak mixing angle. Then, $\sin^2 \theta_W$ is renormalized by the relation

$$\frac{\delta \sin^2 \theta_W}{\sin^2 \theta_W} = \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} \quad (\text{E.2})$$

The vacuum parameter v is given either by

$$v^{-2} = \sqrt{2} G_\mu \quad (\text{E.3})$$

or

$$v^{-2} = \frac{\pi \alpha}{M_W^2 \sin^2 \theta_W} \quad (\text{E.4})$$

depending on whether the Fermi constant G_μ or the fine structure constant $\alpha = \frac{e^2}{4\pi}$ is chosen as an input parameter in addition to the masses. We now list the relationships which arise in the two schemes. For the G_μ procedure one has

$$\frac{\delta v^{-1}}{v^{-1}} = \frac{1}{2} \frac{\delta G_\mu}{G_\mu} \quad (\text{E.5})$$

$$\frac{\delta e}{e} = \frac{1}{2} \left[\frac{\delta G_\mu}{G_\mu} + \frac{\left\{ (M_Z^2 - 2M_W^2) \frac{\delta M_W^2}{M_W^2} + M_W^2 \frac{\delta M_Z^2}{M_Z^2} \right\}}{M_Z^2 - M_W^2} \right] \quad (\text{E.6})$$

In the alpha scheme

$$\frac{\delta e}{e} = \frac{1}{2} \frac{\delta \alpha}{\alpha} \quad (\text{E.7})$$

Now the $\frac{\delta v^{-1}}{v^{-1}}$ is more complicated

$$\frac{\delta v^{-1}}{v^{-1}} = \frac{1}{2} \left[\frac{\delta \alpha}{\alpha} - \frac{\left\{ (M_Z^2 - 2M_W^2) \frac{\delta M_W^2}{M_W^2} + M_W^2 \frac{\delta M_Z^2}{M_Z^2} \right\}}{M_Z^2 - M_W^2} \right] \quad (\text{E.8})$$

Appendix F

Cancellation of quadratic and linear divergences

To illustrate the cancellation of quadratic and linear divergences in our SUSY $SU(2)_L \times U(1)_Y$ model consider the W self energy. As is clear from Fig. 6.6 we have suitably divided the various contributions to W self energy to make such cancellations apparent, for example we would expect the fermion contribution (Diagram 4 in Fig. 6.6) to receive such cancellations from the S-fermions (Diagram 4' in Fig. 6.6). Considering only one doublet of fermions for simplicity we have (taking the SUSY limit i.e. $m_{\bar{f}} = m_f$)

$$T_{\mu\nu}^f + T_{\mu\nu}^{sf} = g^2 \int \frac{d^n q}{(2\pi)^n} \frac{N_{\mu\nu}}{(q^2 - m_1^2)((q+k)^2 - m_2^2)} \quad (F.1)$$

Here

$$N_{\mu\nu} = (2 - \frac{n}{2})q_\mu q_\nu + (1 - \frac{n}{4})(q_\mu k_\nu + k_\mu q_\nu) + (\frac{1}{2})k_\mu k_\nu + [(\frac{n}{4} - 1)q^2 + (\frac{n}{4} - 1)k \cdot q + \frac{1}{2}(m_1^2 + m_2^2 - k^2)]g_{\mu\nu} \quad (F.2)$$

Note that in F.1 m_1 and m_2 are the mass of the up and down fermion (S-fermion). Setting $n = 4 + \epsilon$, it is clear that the quadratic and linearly divergent terms are rendered harmless. Using the definitions of B's in Appendix C and definition of $a^{sf}(q^2)$ after some algebra one gets

$$a_{W^{sf}}^{f+sf} \doteq [\epsilon(-\frac{1}{2}B_{22} + \frac{k^2}{4}B_{21} + \frac{k^2}{4}B_1 + \frac{1}{2}(m_1^2 + m_2^2 - k^2)B_0] \quad (F.3)$$

Hence the only divergence left is an logarithmic one which is enshrined in B_0 . The Higgs-Higgsino sector gives (see Diag. 3 and 3' in Fig. 6.6) in the SUSY limit

$$m_{\tilde{h}} = m_{H_1^0} = m_{H_1^{\pm}}, \quad m_{\tilde{\omega}} = m_H = M_W$$

$$T_{\mu\nu}^{\tilde{h}} + T_{\mu\nu}^{H_1^0} + T_{\mu\nu}^{H_1^{\pm}} + T_{\mu\nu}^H = g^2 \int \frac{d^n q}{(2\pi)^n} \frac{N_{\mu\nu}}{(q^2 - M_{H_1^0}^2)((q+k)^2 - M_W^2)} \quad (\text{F.4})$$

Here

$$N_{\mu\nu} = (2 - \frac{n}{2})q_\mu q_\nu + (1 - \frac{n}{4})(q_\mu k_\nu + k_\mu q_\nu) + (\frac{1}{2})k_\mu k_\nu + [(\frac{n}{4} - 1)q^2 + (\frac{n}{4} - 1)k \cdot q + \frac{1}{2}(M_{H_1^0}^2 + M_W^2 - k^2) - M_W^2]g_{\mu\nu} \quad (\text{F.5})$$

Setting $n = 4 + \epsilon$ in F.4 it is clear again that quadratic and linear divergences cancel. Finally $a_{\text{Higgs-Higgsino}}$ in extracted from F.4 this reads

$$a_W^{Higgs+Higgsino} \doteq [\epsilon(-\frac{1}{2}B_{22} + \frac{k^2}{4}B_{21} + \frac{k^2}{4}B_1) + \frac{1}{2}(M_H^2 + M_W^2 - k^2)B_0 - M_W^2 B_0] \quad (\text{F.6})$$

Similar results were found in the $(\gamma, \tilde{\gamma})$ and $(Z, H_2^0, \tilde{z})$ sectors and will not be repeated here. One similarly observes the cancellation of linear and quadratic divergences for the Z self energy Fig. 6.7, neutrino self-energy Fig. 6.3 and electron self energy Fig. 6.4. For illustration consider the contribution of $(\tilde{\omega}, H, W)$ multiplet to the neutrino self energy Fig. 6.3, one obtains from Eqs. 6.7, 6.9, 6.10 for the bose degrees of freedom [note we only retain B integrals which are linearly or quadratically divergent i.e. B_1 here]

$$\sum_{\text{boson}}^{\nu_e} = \sum_{W, H^+, G^+}^{\nu_e} \quad (\text{F.7})$$

$$\sum_{W, H^+, G^+}^{\nu_e} \doteq [-1 - (m_e^2/2M_W^2) - (m_e^2/2M_W^2)]B_1 \quad (\text{F.8})$$

whereas for the fermi degrees of freedom, (see Eqs. 6.11, 6.12, 6.13, and 6.14)

$$\sum_{\text{fermi}}^{\nu_e} = \sum_{\tilde{\omega}}^{\nu_e} \quad (\text{F.9})$$

$$\sum_{\tilde{\omega}}^{\nu_e} \doteq [1 + (m_e^2/M_W^2)]B_1 \quad (\text{F.10})$$

It is clear from F.8 and F.10 that the linear divergences cancel.

Appendix G

Momentum Dot Products

In this Appendix we give the results of evaluating traces encountered in the evaluation of the cross section. The momentum dot products found for the $e^- \nu_e W^-$, and $\gamma W^+ W^-$ ($Z W^+ W^-$ vertex has the same momentum dot products as the $\gamma W^+ W^-$ vertex), vertices respectively are given by

$$\begin{aligned} \mathcal{A}_{21st} = & \frac{2}{s_3 s_4} (q_1 \cdot q_2) (p_1 \cdot q_1) [-p_1 \cdot p_2 q_1 \cdot q_2 - 2p_2 \cdot q_2 p_1 \cdot q_2 + p_1 \cdot q_2 p_2 \cdot q_1 + \\ & p_2 \cdot q_2 p_1 \cdot q_1] + \frac{1}{s_3} p_1 \cdot q_1 (-4q_1 \cdot q_2 p_1 \cdot p_2 - 2p_1 \cdot q_1 (p_2 \cdot q_2 + 2p_2 \cdot q_1) - \\ & 2p_2 \cdot q_1 p_1 \cdot q_2) + \frac{1}{s_4} [2p_1 \cdot q_2 (p_1 \cdot q_2 [2p_2 \cdot q_2 + p_2 \cdot q_1] + p_1 \cdot q_1 p_2 \cdot q_2) + \\ & q_1 \cdot q_2 (p_2 \cdot q_1 p_1 \cdot q_2 - 2p_2 \cdot q_2 p_1 \cdot q_2 - 7p_1 \cdot q_1 p_2 \cdot q_2 - p_1 \cdot p_2 q_1 \cdot q_2 - \\ & 2p_1 \cdot p_2 p_1 \cdot q_2)] - 3p_1 \cdot p_2 q_1 \cdot q_2 + 8p_1 \cdot q_1 p_2 \cdot q_2 + 4s_3 p_1 \cdot p_2 + \\ & 2p_1 \cdot p_2 p_1 \cdot q_1 + 6p_1 \cdot p_2 p_1 \cdot q_2 \end{aligned} \quad (G.1)$$

$$\begin{aligned} \mathcal{A}_{21tt} = & [-s_3 - 4p_1 \cdot q_1 + 4\frac{1}{s_3} ((p_1 \cdot q_1)^2)] (p_1 \cdot p_2 + \frac{2}{s_4} p_2 \cdot q_2 p_1 \cdot q_2) + \\ & 4(p_1 \cdot q_1) [p_2 \cdot q_1 + \frac{2}{s_4} p_2 \cdot q_2 q_1 \cdot q_2] \end{aligned} \quad (G.2)$$

$$\begin{aligned}
\mathcal{A}_{23st} = & [2(q_1 \cdot q_2)(p_1 \cdot q_1)(p_2 \cdot q_2)/(s_3 s_4)] [(p_2 \cdot q_2)((p_1 \cdot q_1) - 2(p_1 \cdot q_2)) + \\
& (p_2 \cdot q_1)(p_1 \cdot q_2) - (p_1 \cdot p_2)(q_1 \cdot q_2)] + \\
& [2(p_1 \cdot q_1)/s_3] [(p_2 \cdot q_1)((s_4(p_1 \cdot p_2) + (q_1 \cdot q_2)(p_1 \cdot p_2) - \\
& 2(p_1 \cdot q_2)(p_2 \cdot q_2) - \\
& (p_1 \cdot q_2)(p_2 \cdot q_1) - (p_2 \cdot q_2)(p_1 \cdot q_1)) + (q_1 \cdot q_2)(p_2 \cdot q_2)(p_1 \cdot p_2)] + \\
& [(p_2 \cdot q_2)/s_4] [4(p_1 \cdot q_2)^2(p_2 \cdot q_2) + 2(p_1 \cdot q_1)(p_1 \cdot q_2)(p_2 \cdot q_2) + \\
& 2(p_1 \cdot q_2)^2(p_2 \cdot q_1) + (q_1 \cdot q_2)(-2(p_1 \cdot p_2)(p_1 \cdot q_2) - \\
& 2(p_2 \cdot q_2)(p_1 \cdot q_2) - 7(p_2 \cdot q_2)(p_1 \cdot q_1) - (q_1 \cdot q_2)(p_1 \cdot p_2) + \\
& (p_2 \cdot q_1)(p_1 \cdot q_2))] + s_4 [(p_2 \cdot q_1)(p_1 \cdot p_2) - 2(p_1 \cdot p_2)^2] + \\
& [p_2 \cdot q_2][s_3(p_1 \cdot p_2) + 5(p_1 \cdot q_1)(p_2 \cdot q_1) + 2(p_1 \cdot p_2)(p_1 \cdot q_2) - \\
& 2(p_1 \cdot p_2)(p_1 \cdot q_1) + (p_1 \cdot q_2)(p_2 \cdot q_1) - 2(p_1 \cdot p_2)(q_1 \cdot q_2) + \\
& 3(p_1 \cdot q_1)(p_2 \cdot q_2)] + [(p_1 \cdot p_2)(q_1 \cdot q_2) - (p_1 \cdot q_2)(p_2 \cdot q_1)] [(p_2 \cdot q_1) + \\
& 2(p_1 \cdot p_2)]
\end{aligned} \tag{G.3}$$

$$\begin{aligned}
\mathcal{A}_{23st} = & [-s_3 - 4(p_1 \cdot q_1) + 4(p_1 \cdot q_1)^2/s_3] [2(p_2 \cdot q_2)^2(p_1 \cdot q_2)/s_4 - \\
& (p_1 \cdot p_2)(p_2 \cdot q_2)] + 4(p_1 \cdot q_1)[2(p_2 \cdot q_2)^2(q_1 \cdot q_2)/s_4 - \\
& (p_2 \cdot q_1)(p_2 \cdot q_2)]
\end{aligned} \tag{G.4}$$

$$\begin{aligned}
\mathcal{W}\mathcal{W}_{1ss} = & [p_1 \cdot p_2] [-2s_4 - (q_1 \cdot q_2) + 2\frac{(q_1 \cdot q_2)^2}{s_3} + \frac{(q_1 \cdot q_2)^3}{s_3 s_4}] + \\
& 2\frac{(q_1 \cdot p_1)(q_1 \cdot p_2)(q_1 \cdot q_2)}{s_3} + 2(p_1 \cdot q_2)(p_2 \cdot q_2) [-2 + 2\frac{(q_1 \cdot q_2)}{s_4} + \frac{(q_1 \cdot q_2)^2}{s_3 s_4}] + \\
& [(q_1 \cdot p_1)(q_2 \cdot p_2) + (q_1 \cdot p_2)(q_2 \cdot p_1)] [-2 + \frac{(q_1 \cdot q_2)}{s_3} - \frac{(q_1 \cdot q_2)^2}{s_3 s_4}]
\end{aligned} \tag{G.5}$$

$$\begin{aligned}
\mathcal{W}\mathcal{W}_{2ss} = & [p_1 \cdot p_2] [+2s_3 + (q_1 \cdot q_2) - 2\frac{(q_1 \cdot q_2)^2}{s_4} - \frac{(q_1 \cdot q_2)^3}{s_3 s_4}] - \\
& 2\frac{(q_2 \cdot p_2)(q_2 \cdot p_1)(q_1 \cdot q_2)}{s_4} + 2(p_2 \cdot q_1)(p_1 \cdot q_1) [+2 - 2\frac{(q_1 \cdot q_2)}{s_3} - \frac{(q_1 \cdot q_2)^2}{s_3 s_4}] + \\
& [(q_1 \cdot p_1)(q_2 \cdot p_2) + (q_1 \cdot p_2)(q_2 \cdot p_1)] [+2 - \frac{(q_1 \cdot q_2)}{s_4} + \frac{(q_1 \cdot q_2)^2}{s_3 s_4}]
\end{aligned} \tag{G.6}$$

$$\begin{aligned}
\mathcal{W}\mathcal{W}_{3ss} = & [+2(q_1 \cdot p_1)(q_1 \cdot p_2) - s_3(p_1 \cdot p_2)] [+2 - 2\frac{(q_1 \cdot q_2)}{s_3} - \frac{(q_1 \cdot q_2)^2}{s_3 s_4}] - \\
& [(q_1 \cdot p_1)(q_2 \cdot p_2) + (p_2 \cdot q_1)(p_1 \cdot q_2) - (q_1 \cdot q_2)(p_1 \cdot p_2)] [+2 - \\
& 2\frac{(q_1 \cdot q_2)}{s_4} - \frac{(q_1 \cdot q_2)^2}{s_3 s_4}]
\end{aligned} \tag{G.7}$$

$$\begin{aligned}
\mathcal{W}\mathcal{W}_{4ss} = & -[+2(q_2 \cdot p_1)(q_2 \cdot p_2) - s_4(p_1 \cdot p_2)] [+2 - 2\frac{(q_1 \cdot q_2)}{s_4} - \frac{(q_1 \cdot q_2)^2}{s_3 s_4}] + \\
& [(q_1 \cdot p_1)(q_2 \cdot p_2) + (p_2 \cdot q_1)(p_1 \cdot q_2) - (q_1 \cdot q_2)(p_1 \cdot p_2)] [+2 - \\
& 2\frac{(q_1 \cdot q_2)}{s_3} - \frac{(q_1 \cdot q_2)^2}{s_3 s_4}]
\end{aligned} \tag{G.8}$$

$$\begin{aligned}
\mathcal{W}\mathcal{W}_{5ss} = & -[(q_1 \cdot p_1)(q_2 \cdot p_2) + (q_1 \cdot p_2)(q_2 \cdot p_1) - (q_1 \cdot q_2)(p_1 \cdot p_2)] [(s_3 + \\
& (q_1 \cdot q_2))(2 - \frac{(q_1 \cdot q_2)^2}{s_3 s_4}) - \frac{(q_1 \cdot q_2)}{s_4} (s_4 + (q_1 \cdot q_2))] + [2(q_1 \cdot p_1)(q_1 \cdot p_2) - \\
& s_3(p_1 \cdot p_2)] [(s_4 + (q_1 \cdot q_2))(2 - \frac{(q_1 \cdot q_2)^2}{s_3 s_4}) - \frac{(q_1 \cdot q_2)}{s_3} (s_3 + (q_1 \cdot q_2))]
\end{aligned} \tag{G.9}$$

$$\begin{aligned}
\mathcal{W}\mathcal{W}_{6ss} = & [(q_1 \cdot p_1)(q_2 \cdot p_2) + (q_1 \cdot p_2)(q_2 \cdot p_1) - (q_1 \cdot q_2)(p_1 \cdot p_2)] [(s_4 + \\
& (q_1 \cdot q_2))(2 - \frac{(q_1 \cdot q_2)^2}{s_3 s_4}) - \frac{(q_1 \cdot q_2)}{s_3} (s_3 + (q_1 \cdot q_2))] - [2(q_2 \cdot p_1)(q_2 \cdot p_2) - \\
& s_4(p_1 \cdot p_2)] [(s_3 + (q_1 \cdot q_2))(2 - \frac{(q_1 \cdot q_2)^2}{s_3 s_4}) - \frac{(q_1 \cdot q_2)}{s_4} (s_4 + (q_1 \cdot q_2))]
\end{aligned} \tag{G.10}$$

$$\begin{aligned} \mathcal{WW}_{1st} = & 3(p_1 \cdot p_4)(q_2 \cdot p_2) - \frac{(p_2 \cdot p_4)(q_2 \cdot p_1)}{4 \frac{(q_2 \cdot p_1)(q_2 \cdot p_2)(q_2 \cdot p_4)}{s_4}} - \frac{(p_1 \cdot p_2)(q_2 \cdot p_4)}{\frac{(q_1 \cdot q_2)}{s_3}} - \\ & \frac{(q_1 \cdot p_4)(p_1 \cdot p_2)}{s_4} + 2 \frac{(q_1 \cdot q_2)(q_2 \cdot p_2)}{s_3 s_4} [-(q_1 \cdot q_2)(p_1 \cdot p_4) + \\ & (q_2 \cdot p_1)(q_1 \cdot p_4) + (q_2 \cdot p_4)(q_1 \cdot p_1)] \end{aligned} \quad (G.11)$$

$$\begin{aligned} \mathcal{WW}_{2st} = & 3(p_2 \cdot p_4)(q_1 \cdot p_1) - \frac{(p_1 \cdot p_4)(q_1 \cdot p_2)}{4 \frac{(q_1 \cdot p_2)(q_1 \cdot p_1)(q_1 \cdot p_4)}{s_3}} - \frac{(p_1 \cdot p_2)(q_1 \cdot p_4)}{\frac{(q_1 \cdot q_2)}{s_4}} - \\ & \frac{(q_2 \cdot p_4)(p_1 \cdot p_2)}{s_3} + 2 \frac{(q_1 \cdot q_2)(q_1 \cdot p_1)}{s_3 s_4} [-(q_1 \cdot q_2)(p_2 \cdot p_4) + \\ & (q_1 \cdot p_2)(q_2 \cdot p_4) + (q_1 \cdot p_4)(q_2 \cdot p_2)] \end{aligned} \quad (G.12)$$

$$\begin{aligned} \mathcal{WW}_{3st} = & 2(q_1 \cdot p_4)(p_1 \cdot p_2) - \frac{4 \frac{(q_1 \cdot p_1)(q_1 \cdot p_2)(q_1 \cdot p_4)}{s_3}}{\frac{1}{s_4}} + \\ & \frac{1}{s_4} [-2(q_2 \cdot p_4)(q_2 \cdot p_2)(q_1 \cdot p_1) - 2(q_2 \cdot p_4)(q_2 \cdot p_1)(q_1 \cdot p_2) + \\ & (q_2 \cdot p_4)(p_1 \cdot p_2)(q_1 \cdot q_2) + (q_1 \cdot q_2)(-(q_2 \cdot p_2)(p_1 \cdot p_4) + \\ & (q_2 \cdot p_1)(p_2 \cdot p_4))] + 2 \frac{(q_1 \cdot q_2)(q_1 \cdot p_1)}{s_3 s_4} [-(q_1 \cdot q_2)(p_2 \cdot p_4) + \\ & (q_2 \cdot p_2)(q_1 \cdot p_4) + (q_2 \cdot p_4)(q_1 \cdot p_2)] \end{aligned} \quad (G.13)$$

$$\begin{aligned} \mathcal{WW}_{4st} = & 2(q_2 \cdot p_4)(p_1 \cdot p_2) - \frac{4 \frac{(q_2 \cdot p_1)(q_2 \cdot p_2)(q_2 \cdot p_4)}{s_4}}{\frac{1}{s_3}} + \\ & \frac{1}{s_3} [-2(q_1 \cdot p_4)(q_1 \cdot p_2)(q_1 \cdot p_1) - 2(q_1 \cdot p_4)(q_1 \cdot p_1)(q_2 \cdot p_2) + \\ & (q_1 \cdot q_2)[(q_1 \cdot p_2)(p_1 \cdot p_4) - (q_1 \cdot p_1)(p_2 \cdot p_4) + (q_1 \cdot p_4)(p_1 \cdot p_2)] + \\ & 2 \frac{(q_1 \cdot q_2)(q_2 \cdot p_2)}{s_3 s_4} [-(q_1 \cdot q_2)(p_1 \cdot p_4) + (q_2 \cdot p_1)(q_1 \cdot p_4) + \\ & (q_2 \cdot p_4)(q_1 \cdot p_1)] \end{aligned} \quad (G.14)$$

$$\begin{aligned} \mathcal{WW}_{5st} = & 2(p_2 \cdot q_1)[-(q_1 \cdot q_2)(p_1 \cdot p_4) + (p_1 \cdot q_1)(q_2 \cdot p_4) + (p_1 \cdot q_2)(q_1 \cdot p_4)] + \\ & \frac{1}{s_3} [-(p_1 \cdot q_2)(p_2 \cdot p_4) + (p_2 \cdot q_2)(p_1 \cdot p_4) - \\ & (p_1 \cdot p_2)(q_2 \cdot p_4)] + 2(q_1 \cdot q_2)[(p_2 \cdot q_1)(p_1 \cdot p_4) + (p_1 \cdot q_1)(p_2 \cdot p_4)] - \\ & \frac{4 \frac{(q_1 \cdot q_2)(p_2 \cdot q_1)(p_1 \cdot q_1)(q_1 \cdot p_4)}{s_3}}{\frac{1}{s_4}} - 2 \frac{(q_1 \cdot q_2)(q_2 \cdot p_4)}{s_3} [-(q_1 \cdot q_2)(p_1 \cdot p_2) + \\ & (p_2 \cdot q_2)(q_1 \cdot p_1) + (p_1 \cdot q_2)(q_1 \cdot p_2)] + \\ & 2 \frac{(q_1 \cdot q_2)^2(q_1 \cdot p_1)}{s_3 s_4} [-(q_1 \cdot q_2)(p_2 \cdot p_4) + (q_1 \cdot p_2)(q_2 \cdot p_4) + \\ & (p_2 \cdot q_2)(q_1 \cdot p_4)] + \frac{(q_1 \cdot q_2)^2}{s_4} [-(q_2 \cdot p_4)(p_1 \cdot p_2) - (p_1 \cdot p_4)(q_2 \cdot p_2) + \\ & (p_2 \cdot p_4)(q_2 \cdot p_1)] \end{aligned} \quad (G.15)$$

$$\begin{aligned} \mathcal{WW}_{6st} = & 2(p_1 \cdot q_2)[-(q_1 \cdot q_2)(p_2 \cdot p_4) + (p_2 \cdot q_2)(q_1 \cdot p_4) + (p_2 \cdot q_1)(q_2 \cdot p_4)] + \\ & \frac{1}{s_4} [-(p_2 \cdot q_1)(p_1 \cdot p_4) + (p_1 \cdot q_1)(p_2 \cdot p_4) - \\ & (p_1 \cdot p_2)(q_1 \cdot p_4)] + 2(q_1 \cdot q_2)[(p_1 \cdot q_2)(p_2 \cdot p_4) + (p_2 \cdot q_2)(p_1 \cdot p_4)] - \\ & \frac{4 \frac{(q_1 \cdot q_2)(p_1 \cdot q_2)(p_2 \cdot q_2)(q_2 \cdot p_4)}{s_4}}{\frac{1}{s_3}} - 2 \frac{(q_1 \cdot q_2)(q_1 \cdot p_4)}{s_3} [-(q_1 \cdot q_2)(p_1 \cdot p_2) + \\ & (p_2 \cdot q_2)(q_1 \cdot p_1) + (p_2 \cdot q_1)(q_2 \cdot p_1)] + \\ & 2 \frac{(q_1 \cdot q_2)^2(q_2 \cdot p_2)}{s_3 s_4} [-(q_1 \cdot q_2)(p_1 \cdot p_4) + (q_2 \cdot p_1)(q_1 \cdot p_4) + \\ & (p_1 \cdot q_1)(q_2 \cdot p_4)] + \frac{(q_1 \cdot q_2)^2}{s_3} [-(q_1 \cdot p_4)(p_1 \cdot p_2) - (p_2 \cdot p_4)(q_1 \cdot p_1) + \\ & (p_1 \cdot p_4)(q_1 \cdot p_2)] \end{aligned} \quad (G.16)$$

where p_4 is given by

$$p_4 = p_2 - q_2 \quad (\text{G.17})$$

$$p_4 = q_1 - p_1 \quad (\text{G.18})$$

since $p_1^2 = p_2^2 = m_c^2$ one can neglect p_1^2 and p_2^2 and write

$$p_1 \cdot p_4 = p_1 \cdot q_1 \quad (\text{G.19})$$

$$p_2 \cdot p_4 = -p_2 \cdot q_2 \quad (\text{G.20})$$

$$q_1 \cdot p_4 = q_1 \cdot p_2 - q_1 \cdot q_2 \quad (\text{G.21})$$

$$q_2 \cdot p_4 = q_2 \cdot q_1 - q_2 \cdot p_1 \quad (\text{G.22})$$

We now give the extra dot products involved in the NSM differential cross section (Chapter 5).

$$\mathcal{W}_0 = -\frac{1}{s_3 s_4} (+p_1 \cdot p_2 (q_1 \cdot q_2)^2 s_4 + 2p_1 \cdot p_2 s_3 (s_4)^2 - 2p_1 \cdot q_2 p_2 \cdot q_2 (q_1 \cdot q_2)^2 - 4p_1 \cdot q_2 p_2 \cdot q_2 s_3 s_4) \quad (\text{G.23})$$

$$\mathcal{W}_1 = -\frac{1}{s_3 s_4} (+q_1 \cdot q_2 (+p_1 \cdot p_2 (q_1 \cdot q_2)^2 s_4 - p_1 \cdot p_2 s_3 (s_4)^2 - 2p_1 \cdot q_2 p_2 \cdot q_2 (q_1 \cdot q_2)^2 + 2p_1 \cdot q_2 p_2 \cdot q_2 s_3 s_4)) \quad (\text{G.24})$$

$$\mathcal{W}_2 = -\frac{1}{s_3 s_4} (+q_1 \cdot q_2 (p_1 \cdot q_1 p_2 \cdot q_2 s_4 + p_1 \cdot q_2 p_2 \cdot q_1 s_4 - 2p_1 \cdot q_2 p_2 \cdot q_2 q_1 \cdot q_2)) \quad (\text{G.25})$$

$$\mathcal{W}_3 = -\frac{1}{s_3 s_4} (+q_1 \cdot q_2 (p_1 \cdot p_2 (q_1 \cdot q_2)^2 - p_1 \cdot p_2 s_3 s_4 - p_1 \cdot q_1 p_2 \cdot q_2 q_1 \cdot q_2 - p_1 \cdot q_2 p_2 \cdot q_1 q_1 \cdot q_2 + 2p_1 \cdot q_2 p_2 \cdot q_2 s_3)) \quad (\text{G.26})$$

$$\mathcal{W}_4 = \frac{1}{s_3 s_4} (p_1 \cdot p_2 s_4 (q_1 \cdot q_2)^2 - p_1 \cdot p_2 s_3 s_4 s_4 + 2p_1 \cdot q_2 p_2 \cdot q_2 (q_1 \cdot q_2)^2 - 2p_1 \cdot q_2 p_2 \cdot q_2 s_3 s_4) \quad (\text{G.27})$$

$$\mathcal{W}_5 = -\frac{1}{s_3 s_4} (p_1 \cdot p_2 (q_1 \cdot q_2)^3 - p_1 \cdot p_2 q_1 \cdot q_2 s_3 s_4 + 2p_1 \cdot q_1 p_2 \cdot q_1 q_1 \cdot q_2 s_3 - p_1 \cdot q_1 p_2 \cdot q_2 (q_1 \cdot q_2)^2 - p_1 \cdot q_1 p_2 \cdot q_2 s_3 s_4 - p_1 \cdot q_2 p_2 \cdot q_1 (q_1 \cdot q_2)^2 - p_1 \cdot q_2 p_2 \cdot q_1 s_3 s_4 + 2p_1 \cdot q_2 p_2 \cdot q_2 q_1 \cdot q_2 s_3) \quad (\text{G.28})$$

$$\mathcal{W}_6 = -\frac{1}{s_3 s_4} (p_1 \cdot q_1 p_2 \cdot q_2 s_4 (q_1 \cdot q_2)^2 - p_1 \cdot q_1 p_2 \cdot q_2 s_3 s_4 s_4 + p_1 \cdot q_2 p_2 \cdot q_1 (q_1 \cdot q_2)^2 s_4 - p_1 \cdot q_2 p_2 \cdot q_1 s_3 s_4 s_4 - 2p_1 \cdot q_2 p_2 \cdot q_1 (q_1 \cdot q_2)^3 + 2p_1 \cdot q_2 p_2 \cdot q_2 q_1 \cdot q_2 s_3 s_4) \quad (\text{G.29})$$

$$\mathcal{W}_7 = \frac{1}{s_3 s_4} (p_1 \cdot p_2 (q_1 \cdot q_2)^2 s_3 - p_1 \cdot p_2 s_3 s_3 s_4 + 2p_1 \cdot q_1 p_2 \cdot q_1 (q_1 \cdot q_2)^2 - 2p_1 \cdot q_1 p_2 \cdot q_1 s_3 s_4) \quad (\text{G.30})$$

$$\mathcal{W}_8 = -\frac{1}{s_3 s_4} (p_1 \cdot p_2 (q_1 \cdot q_2)^4 - 2p_1 \cdot p_2 (q_1 \cdot q_2)^2 s_3 s_4 + p_1 \cdot p_2 s_3 s_3 s_4 s_4 - p_1 \cdot q_1 p_2 \cdot q_2 (q_1 \cdot q_2)^3 - p_1 \cdot q_1 p_2 \cdot q_2 s_3 s_4 + p_1 \cdot q_1 p_2 \cdot q_2 q_1 \cdot q_2 s_3 s_4 - p_1 \cdot q_2 p_2 \cdot q_1 (q_1 \cdot q_2)^3 + p_1 \cdot q_2 p_2 \cdot q_1 q_1 \cdot q_2 s_3 s_4 + 2p_1 \cdot q_2 p_2 \cdot q_2 (q_1 \cdot q_2)^2 s_3 - 2p_1 \cdot q_2 p_2 \cdot q_2 s_3 s_3 s_4) \quad (\text{G.31})$$

$$\mathcal{W}_9 = -\frac{1}{s_3 s_4} (p_1 \cdot p_2 (q_1 \cdot q_2)^4 s_4 - 2p_1 \cdot p_2 (q_1 \cdot q_2)^2 s_3 s_4 s_4 + p_1 \cdot p_2 s_3 s_3 s_4 s_4 s_4 - 2p_1 \cdot q_2 p_2 \cdot q_2 (q_1 \cdot q_2)^4 + 4p_1 \cdot q_2 p_2 \cdot q_2 (q_1 \cdot q_2)^2 s_3 s_4 - 2p_1 \cdot q_2 p_2 \cdot q_2 s_3 s_3 s_4 s_4) \quad (\text{G.32})$$

The following momentum dot products arise in evaluating the one loop differential cross section due to the box diagrams of Chapter 8. $\mathcal{X}_3$, through $\mathcal{X}_{21}$, are given by the following expressions

$$\mathcal{X}_3 = -(p_1 \cdot p_2) [q_1 \cdot (3p_2 + p_1) + p_1 \cdot q_2] - \frac{p_1 \cdot q_1}{s_3} [-2p_1 \cdot p_2 q_1 \cdot q_2 - 2p_2 \cdot q_1 p_1 \cdot q_1 - p_2 \cdot q_2 p_1 \cdot q_1 - p_2 \cdot q_1 p_1 \cdot q_2] + \frac{p_1 \cdot q_1 q_1 \cdot q_2}{s_3 s_4} [p_2 \cdot q_2 p_1 \cdot (2q_2 - q_1) - p_2 \cdot q_1 p_1 \cdot q_2 + q_1 \cdot q_2 p_1 \cdot p_2] \quad (\text{G.33})$$

$$\mathcal{X}_4 = -(p_1 \cdot p_2) [p_2 \cdot (4q_2 + q_1)] + \frac{p_1 \cdot q_1}{s_3} [2p_1 \cdot p_2 q_1 \cdot q_2 + 2p_2 \cdot q_1 p_1 \cdot q_1 + p_2 \cdot q_1 p_1 \cdot q_2 + p_2 \cdot q_2 p_1 \cdot q_1] + \frac{p_1 \cdot q_1 q_1 \cdot q_2}{s_3 s_4} [p_1 \cdot q_2 p_2 \cdot (2q_2 - q_1) - p_1 \cdot q_1 p_2 \cdot q_2 + q_1 \cdot q_2 p_1 \cdot p_2] \quad (\text{G.34})$$

$$\mathcal{X}_{5s} = \mathcal{W} \mathcal{W}_{2ss} \quad (\text{G.35})$$

$$\mathcal{X}_{7s} = p_1 \cdot p_2 [p_1 \cdot (3q_1 + q_2)] - \frac{p_1 \cdot q_2 p_1 \cdot p_2}{s_4} [p_2 \cdot q_2 + p_1 \cdot q_2 + 2q_1 \cdot q_2] - \frac{p_1 \cdot q_1}{s_3} [-p_1 \cdot q_1 p_2 \cdot (2q_1 + q_2) + q_1 \cdot q_2 p_1 \cdot p_2 - p_2 \cdot q_1 p_1 \cdot q_2] + \frac{p_1 \cdot q_1 q_1 \cdot q_2}{s_3 s_4} [p_2 \cdot q_1 p_1 \cdot (q_2 - 2q_1) + p_1 \cdot q_1 p_2 \cdot q_2 - q_1 \cdot q_2 p_1 \cdot p_2] \quad (\text{G.36})$$

$$\mathcal{X}_{8s} = p_1 \cdot p_2 [p_2 \cdot (4q_1 + q_2)] - \frac{p_1 \cdot q_2 p_1 \cdot p_2}{s_4} [p_2 \cdot q_2 + p_1 \cdot q_2 + 2q_1 \cdot q_2] + \frac{p_2 \cdot q_2 q_1 \cdot q_2}{s_3 s_4} [p_2 \cdot q_1 p_1 \cdot (q_2 - 2q_1) + p_1 \cdot q_1 p_2 \cdot q_2 - q_1 \cdot q_2 p_1 \cdot p_2] \quad (\text{G.37})$$

$$\mathcal{X}_{11s} = 2[2p_1 \cdot q_2 p_2 \cdot q_1 p_1 \cdot q_1 - s_3 (p_1 \cdot p_2)^2 + 2p_2 \cdot q_1 (p_1 \cdot q_1)^2 + p_1 \cdot q_1 p_1 \cdot p_2 q_1 \cdot q_2] - \frac{q_1 \cdot q_2}{s_4} [-q_1 \cdot q_2 (p_1 \cdot p_2)^2 + p_1 \cdot q_2 (2p_1 \cdot q_1 p_2 \cdot q_1 - s_3 p_1 \cdot p_2 + p_2 \cdot q_2 p_1 \cdot q_1 + p_2 \cdot q_1 p_1 \cdot q_2)] - \frac{4q_1 \cdot q_2 p_2 \cdot q_1 (p_1 \cdot q_1)^2}{s_3} + \frac{p_1 \cdot q_1 (q_1 \cdot q_2)^2}{s_3 s_4} [p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 q_1 \cdot q_2 + p_2 \cdot q_1 p_1 \cdot (q_2 - 2q_1)] \quad (\text{G.38})$$

$$\mathcal{X}_{12s} = 2p_1 \cdot p_2 [p_2 \cdot q_1 (2p_1 \cdot q_1 + q_1 \cdot q_2) - s_3 (p_1 \cdot p_2)] - \frac{q_1 \cdot q_2}{s_4} [p_2 \cdot q_2 (p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 q_1 \cdot q_2 + p_2 \cdot q_1 p_1 \cdot (q_2 + 2q_1) - s_3 p_1 \cdot p_2) - p_2 \cdot q_1 p_1 \cdot p_2 q_1 \cdot q_2] - \frac{4q_1 \cdot q_2 p_1 \cdot q_1 (p_2 \cdot q_1)^2}{s_3} + \frac{p_2 \cdot q_1 (q_1 \cdot q_2)^2}{s_3 s_4} [p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 q_1 \cdot q_2 + p_2 \cdot q_1 p_1 \cdot (q_2 - 2q_1)] \quad (\text{G.39})$$

$$\begin{aligned} \mathcal{X}_{15s} = & -p_1 \cdot p_2 (p_1 \cdot q_1)^2 - \frac{p_1 \cdot q_2}{s_4} [p_1 \cdot q_2 (p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 (q_1 \cdot q_2 + s_3) + \\ & p_2 \cdot q_1 p_1 \cdot (q_2 + 2q_1)) - p_1 \cdot p_2 q_1 \cdot q_2 p_1 \cdot q_1] - \frac{(p_1 \cdot q_1)^2}{s_3} [p_1 \cdot p_2 q_1 \cdot (q_2 - \\ & 2p_2) - p_2 \cdot q_2 p_1 \cdot q_1 + p_2 \cdot q_1 p_1 \cdot q_2] + \frac{p_1 \cdot q_1 p_1 \cdot q_2 q_1 \cdot q_2}{s_3 s_4} [p_2 \cdot q_2 p_1 \cdot q_1 - \\ & p_1 \cdot p_2 q_1 \cdot q_2 + p_2 \cdot q_1 p_1 \cdot (q_2 - 2q_1)] \end{aligned} \quad (\text{G.40})$$

$$\begin{aligned} \mathcal{X}_{16s} = & p_1 \cdot p_2 [p_1 \cdot q_1 p_2 \cdot (2q_1 - q_2) + p_1 \cdot p_2 (q_1 \cdot q_2 - \\ & s_3 - p_2 \cdot q_1)] - \frac{p_1 \cdot q_2}{s_4} [p_2 \cdot q_2 (p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 (q_1 \cdot q_2 + s_3) + \\ & p_2 \cdot q_1 p_1 \cdot (q_2 + 2q_1)) - p_1 \cdot p_2 q_1 \cdot q_2 p_2 \cdot q_1] - \frac{p_2 \cdot q_1 p_1 \cdot q_1}{s_3} [p_2 \cdot q_1 p_1 \cdot (q_2 - \\ & 2p_2) - p_2 \cdot q_2 p_1 \cdot q_1 + p_1 \cdot p_2 q_1 \cdot q_2] + \frac{p_1 \cdot q_1 p_2 \cdot q_2 q_1 \cdot q_2}{s_3 s_4} [p_2 \cdot q_2 p_1 \cdot q_1 - \\ & p_1 \cdot p_2 q_1 \cdot q_2 + p_2 \cdot q_1 p_1 \cdot (q_2 - 2q_1)] \end{aligned} \quad (\text{G.41})$$

$$\begin{aligned} \mathcal{X}_{17s} = & p_1 \cdot p_2 [p_2 \cdot q_1 p_1 \cdot (2q_1 - q_2) + p_1 \cdot p_2 (q_1 \cdot q_2 - \\ & s_3 - p_1 \cdot q_1)] - \frac{p_2 \cdot q_2}{s_4} [p_1 \cdot q_2 (p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 (q_1 \cdot q_2 + s_3) + \\ & p_2 \cdot q_1 p_1 \cdot (q_2 + 2q_1)) - p_1 \cdot p_2 q_1 \cdot q_2 p_1 \cdot q_1] - \frac{p_1 \cdot q_1 p_2 \cdot q_1}{s_3} [p_1 \cdot p_2 q_1 \cdot (q_2 - \\ & 2p_1) - p_2 \cdot q_1 p_1 \cdot q_2 + p_2 \cdot q_2 p_1 \cdot q_1] + \frac{p_1 \cdot q_1 p_2 \cdot q_2 q_1 \cdot q_2}{s_3 s_4} [p_2 \cdot q_2 p_1 \cdot q_1 - \\ & p_1 \cdot p_2 q_1 \cdot q_2 + p_2 \cdot q_1 p_1 \cdot (q_2 - 2q_1)] \end{aligned} \quad (\text{G.42})$$

$$\begin{aligned} \mathcal{X}_{18s} = & -p_1 \cdot p_2 (p_2 \cdot q_1)^2 - \frac{p_2 \cdot q_2}{s_4} [p_2 \cdot q_2 (p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 (q_1 \cdot q_2 + s_3) + \\ & p_2 \cdot q_1 p_1 \cdot (q_2 + 2q_1)) - p_1 \cdot p_2 q_1 \cdot q_2 p_2 \cdot q_1] - \frac{(p_2 \cdot q_1)^2}{s_3} [p_1 \cdot p_2 q_1 \cdot (q_2 - \\ & 2p_1) - p_2 \cdot q_1 p_1 \cdot q_2 + p_2 \cdot q_2 p_1 \cdot q_1] + \frac{p_2 \cdot q_1 p_2 \cdot q_2 q_1 \cdot q_2}{s_3 s_4} [p_2 \cdot q_2 p_1 \cdot q_1 - \\ & p_1 \cdot p_2 q_1 \cdot q_2 + p_2 \cdot q_1 p_1 \cdot (q_2 - 2q_1)] \end{aligned} \quad (\text{G.43})$$

$$\mathcal{X}_{19s} = \mathcal{W}\mathcal{W}_{3ss} \quad (\text{G.44})$$

$$\mathcal{X}_{21s} = \mathcal{A}_{11st} \quad (\text{G.45})$$

$$\mathcal{X}_{3t} = p_1 \cdot q_1 [1 - \frac{2p_1 \cdot q_1}{s_3}] [p_1 \cdot p_2 + \frac{2p_2 \cdot q_2 p_1 \cdot q_2}{s_4}] \quad (\text{G.46})$$

$$\begin{aligned} \mathcal{X}_{4t} = & p_1 \cdot p_2 [p_2 \cdot (2p_1 - q_1)] - 2 \frac{p_1 \cdot p_2 p_2 \cdot q_1 p_1 \cdot q_1}{s_3} - \frac{2p_2 \cdot q_2}{s_4} [p_1 \cdot p_2 q_2 \cdot (q_1 - \\ & 2p_2) - p_1 \cdot q_1 p_2 \cdot q_2] - 4 \frac{p_1 \cdot q_1 p_2 \cdot q_1 p_2 \cdot q_2 p_1 \cdot q_2}{s_3 s_4} \end{aligned} \quad (\text{G.47})$$

$$\mathcal{X}_{5t} = \mathcal{W}\mathcal{W}_{2tt} \quad (\text{G.48})$$

$$\begin{aligned} \mathcal{X}_{7t} = & -4p_1 \cdot q_1 p_1 \cdot p_2 - \frac{p_1 \cdot q_2}{s_4} [p_2 \cdot q_1 p_1 \cdot q_2 - 3p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 q_1 \cdot q_2] - \\ & 2 \frac{p_1 \cdot q_1 p_1 \cdot q_2}{s_3 s_4} [p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 q_1 \cdot q_2 + p_2 \cdot q_1 p_1 \cdot q_2] \end{aligned} \quad (\text{G.49})$$

$$\begin{aligned} \mathcal{X}_{8t} = & \frac{p_2 \cdot q_2}{s_4} [-p_2 \cdot q_1 p_1 \cdot q_2 + 3p_2 \cdot q_2 p_1 \cdot q_1 + p_1 \cdot p_2 q_1 \cdot q_2] - \\ & 2 \frac{p_1 \cdot q_1 p_2 \cdot q_2}{s_3 s_4} [p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 q_1 \cdot q_2 + p_2 \cdot q_1 p_1 \cdot q_2] \end{aligned} \quad (\text{G.50})$$

$$\begin{aligned} \mathcal{X}_{11t} = & -p_1 \cdot q_1 [p_1 \cdot p_2 (2p_1 \cdot q_1 - s_3) + 2p_2 \cdot q_1 p_1 \cdot q_1] + \\ & \frac{p_1 \cdot q_1 q_1 \cdot q_2}{s_3} [p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 q_1 \cdot q_2 - p_2 \cdot q_1 p_1 \cdot q_2] + \frac{4p_2 \cdot q_1 (p_1 \cdot q_1)^3}{s_3} - \\ & \frac{2q_1 \cdot q_2 (p_1 \cdot q_1)^2}{s_3 s_4} [p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 q_1 \cdot q_2 + p_2 \cdot q_1 p_1 \cdot q_2] \end{aligned} \quad (\text{G.51})$$

$$\begin{aligned} \mathcal{X}_{12t} = & [s_3 p_1 \cdot p_2 (2p_1 \cdot p_2 + p_2 \cdot q_1) - \\ & 2p_2 \cdot q_1 p_1 \cdot q_1 p_1 \cdot (q_2 + 3p_2)] - \frac{q_1 \cdot q_2}{s_4} [2p_1 \cdot p_2 (s_3 p_2 \cdot q_2 - p_1 \cdot q_1 p_2 \cdot q_2 + \\ & p_1 \cdot p_2 q_1 \cdot q_2) - p_2 \cdot q_1 (p_1 \cdot p_2 q_2 \cdot (2p_1 + q_1) + 3p_2 \cdot q_2 p_1 \cdot q_1 - \\ & p_2 \cdot q_1 p_1 \cdot q_2)] + \frac{4(p_1 \cdot q_1)^2 (p_2 \cdot q_1)^2}{s_3} - \frac{2p_1 \cdot q_1 p_2 \cdot q_1 q_1 \cdot q_2}{s_3 s_4} [p_2 \cdot q_2 p_1 \cdot q_1 - \\ & p_1 \cdot p_2 q_1 \cdot q_2 + p_2 \cdot q_1 p_1 \cdot q_2] \end{aligned} \quad (\text{G.52})$$

$$\mathcal{X}_{15t} = \frac{[p_1 \cdot q_1 p_1 \cdot q_2]}{s_3 s_4} [s_3 - 2p_1 \cdot q_1] [p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 q_1 \cdot q_2 + p_2 \cdot q_1 p_1 \cdot q_2] \quad (\text{G.53})$$

$$\begin{aligned} \mathcal{X}_{16t} = & 2p_1 \cdot p_2 [s_3 p_1 \cdot p_2 - \\ & 2p_1 \cdot q_1 p_2 \cdot q_1] - \frac{p_1 \cdot q_2}{s_4} [p_1 \cdot p_2 (2p_2 \cdot q_2 (s_3 - p_1 \cdot q_1) + 2p_1 \cdot p_2 q_1 \cdot q_2 - \\ & p_2 \cdot q_1 q_2 \cdot (2p_1 + q_1)) - p_2 \cdot q_1 (3p_2 \cdot q_2 p_1 \cdot q_1 - p_2 \cdot q_1 p_1 \cdot q_2)] - \\ & \frac{2p_1 \cdot q_1 p_1 \cdot q_2 p_2 \cdot q_1}{s_3 s_4} [p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 q_1 \cdot q_2 + p_2 \cdot q_1 p_1 \cdot q_2] \end{aligned} \quad (\text{G.54})$$

$$\mathcal{X}_{17t} = \frac{[p_1 \cdot q_1 p_2 \cdot q_2]}{s_3 s_4} [s_3 - 2p_1 \cdot q_1] [p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 q_1 \cdot q_2 + p_2 \cdot q_1 p_1 \cdot q_2] \quad (\text{G.55})$$

$$\begin{aligned} \mathcal{X}_{18t} = & -\frac{p_2 \cdot q_2}{s_4} [p_1 \cdot p_2 (2p_2 \cdot q_2 (s_3 - p_1 \cdot q_1) + 2p_1 \cdot p_2 q_1 \cdot q_2 - p_2 \cdot q_1 q_2 \cdot (2p_1 + \\ & q_1)) + p_2 \cdot q_1 (p_2 \cdot q_1 p_1 \cdot q_2 - 3p_2 \cdot q_2 p_1 \cdot q_1)] - \\ & \frac{2p_1 \cdot q_1 p_2 \cdot q_1 p_2 \cdot q_2}{s_3 s_4} [p_2 \cdot q_2 p_1 \cdot q_1 - p_1 \cdot p_2 q_1 \cdot q_2 + p_2 \cdot q_1 p_1 \cdot q_2] \end{aligned} \quad (\text{G.56})$$

$$\mathcal{X}_{19t} = \mathcal{W}\mathcal{W}_{3tt} \quad (\text{G.57})$$

$$\mathcal{X}_{21t} = \mathcal{A}_{11tt} \quad (\text{G.58})$$

Appendix H

C Integrals

The C integrals are defined, derived and discussed in this Appendix. Let us first derive explicit expressions for the C integrals. This is done in much the same way as the A and B integrals. Define

$$C_0; C_\mu; C_{\mu\nu}; C_{\mu\nu\alpha} = -\frac{1}{i\pi^2} \int d^n q \frac{1; q_\mu; q_\mu q_\nu; q_\mu q_\nu q_\alpha}{(q^2 - m_1^2)((q+k)^2 - m_2^2)((q+p_5)^2 - m_3^2)} \quad (\text{H.1})$$

where

$$p_5 = k + p \quad (\text{H.2})$$

Using the Feynman parametric technique

$$\frac{1}{abc} = 2 \int_0^1 dx \int_0^{1-x} dy \frac{1}{[a(1-x-y) + bx + cy]^3} \quad (\text{H.3})$$

C_0 in Eq. H.1 may be reduced to

$$C_0 = -\frac{2}{i\pi^2} \int d^n q \int_0^1 dx \int_0^{1-x} dy \frac{1}{[q^2 - M^2]^3} \quad (\text{H.4})$$

Here

$$M^2 = (1-x-y)m_1^2 + xm_2^2 + ym_3^2 - x(1-x)k^2 - y(1-y)p_5^2 + 2xy(k.p_5) \quad (\text{H.5})$$

Performing dimensional regularization Eq. H.4 reduces to

$$C_0 = \int_0^1 dx \int_0^{1-x} dy \frac{1}{M^2} \quad (\text{H.6})$$

Now we must find expressions for C_{11} and C_{12} to this end define

$$C_\mu = k_\mu C_{11} + p_\mu C_{12} \quad (\text{H.7})$$

Together with Eq. H.1, Eq. H.7 gives us

$$k_\mu C_{11} + p_\mu C_{12} = -\frac{1}{i\pi^2} \int d^n q \frac{q_\mu}{(q^2 - m_1^2)((q+k)^2 - m_2^2)((q+p_5)^2 - m_3^2)} \quad (\text{H.8})$$

Following steps similar to the ones used in obtaining C_0 , Eq. H.8 may be reduced to

$$k_\mu C_{11} + p_\mu C_{12} = -\int_0^1 dx \int_0^{1-x} dy \frac{xk_\mu + yp_\mu}{M^2} \quad (\text{H.9})$$

Since $p_5 = k + p$, on equating coefficients in Eq. H.9 one obtains

$$C_{11} = -\int_0^1 dx \int_0^{1-x} dy \frac{x+y}{M^2} \quad (\text{H.10})$$

and

$$C_{12} = -\int_0^1 dx \int_0^{1-x} dy \frac{y}{M^2} \quad (\text{H.11})$$

Similarly expanding $C_{\mu\nu}$ as

$$C_{\mu\nu} = k_\mu k_\nu C_{21} + p_\mu p_\nu C_{22} + [k, p]_{\mu\nu} C_{23} - g_{\mu\nu} C_{24} \quad (\text{H.12})$$

where

$$[k, p]_{\mu\nu} = k_\mu p_\nu - k_\nu p_\mu \quad (\text{H.13})$$

and following procedure indicated for C_μ one obtains C_{21} through C_{24}

$$C_{21} = \int_0^1 dx \int_0^{1-x} dy \frac{x^2 + 2xy + y^2}{M^2} \quad (\text{H.14})$$

$$C_{22} = \int_0^1 dx \int_0^{1-x} dy \frac{y^2}{M^2} \quad (\text{H.15})$$

$$C_{23} = \int_0^1 dx \int_0^{1-x} dy \frac{y^2 + xy}{M^2} \quad (\text{H.16})$$

and

$$C_{24} = (1/2) \int_0^1 dx \int_0^{1-x} dy (\Delta - \ln(M^2)) \quad (\text{H.17})$$

It is only a matter of tedious algebra to find C_{31} through C_{36} one expands $C_{\mu\nu\alpha}$ as

$$C_{\mu\nu\alpha} = k_\mu k_\nu k_\alpha C_{31} + p_\mu p_\nu p_\alpha C_{32} + [pkk]_{\mu\nu\alpha} C_{33} + [kpp]_{\mu\nu\alpha} C_{34} + [k\delta]_{\mu\nu\alpha} C_{35} - [p\delta]_{\mu\nu\alpha} C_{36} \quad (\text{H.18})$$

where

$$[pkk]_{\mu\nu\alpha} = p_\mu k_\nu k_\alpha + p_\nu k_\mu k_\alpha + p_\alpha k_\mu k_\nu \quad (\text{H.19})$$

and

$$[k\delta]_{\mu\nu\alpha} = k_\mu \delta_{\nu\alpha} + k_\nu \delta_{\mu\alpha} + k_\alpha \delta_{\mu\nu} \quad (\text{H.20})$$

and follows the same procedure on $C_{\mu\nu\alpha}$ as on C_μ , thus obtaining the following Feynman-parametric representations for C_{31} through C_{36} . We merely give here the final results

$$\begin{aligned} C_{31} &= - \int_0^1 dx \int_0^{1-x} dy \frac{(x+y)^3}{M^2}, \\ C_{32} &= - \int_0^1 dx \int_0^{1-x} dy \frac{y^3}{M^2}, \\ C_{33} &= - \int_0^1 dx \int_0^{1-x} dy \frac{y(x+y)^2}{M^2}, \\ C_{34} &= - \int_0^1 dx \int_0^{1-x} dy \frac{y^2(x+y)}{M^2}, \\ C_{35} &= -(1/6) \Delta + (1/2) \int_0^1 dx \int_0^{1-x} dy (y+x) \ln(M^2), \\ C_{36} &= -(1/12) \Delta + (1/2) \int_0^1 dx \int_0^{1-x} dy y \ln(M^2) \end{aligned} \quad (\text{H.21})$$

As is clear from Power-counting and definitions of C_0 , C_μ , $C_{\mu\nu}$ and $C_{\mu\nu\alpha}$ that only C_{24} , C_{35} and C_{36} must contain infinities. This is indeed the case as is given in Eq. H.21.

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Alternatively one can find expressions for C_1 through C_{36} in terms of C, B and A integrals [45]. To this end consider the definition of the C integrals as given Eq. H.1 and let us demonstrate this method by considering the evaluation of C_{11} and C_{12} in terms of C_0 and two and one point integrals B's and A's. Recalling Eq. H.8 and contracting it with k and p one obtains respectively

$$k^2 C_{11} + p \cdot k C_{12} = -\frac{1}{i\pi^2} \int d^n q \frac{q \cdot k}{(q^2 - m_1^2)((q+k)^2 - m_2^2)((q+p_5)^2 - m_3^2)} \quad (\text{H.22})$$

$$k \cdot p C_{11} + p^2 C_{12} = -\frac{1}{i\pi^2} \int d^n q \frac{q \cdot p}{(q^2 - m_1^2)((q+k)^2 - m_2^2)((q+p_5)^2 - m_3^2)} \quad (\text{H.23})$$

one may now expand $q \cdot k$ and $q \cdot p$ as

$$q \cdot k = (1/2)[(q+k)^2 - m_2^2 - (q^2 - m_1^2) - m_1^2 + m_2^2 - k^2] \quad (\text{H.24})$$

$$q \cdot p = (1/2)[(q+k+p)^2 - m_3^2 - ((q+k)^2 - m_2^2) - m_2^2 + m_3^2 + k^2 - p_5^2] \quad (\text{H.25})$$

Plugging in H.24 and H.25 into H.22 and H.23 respectively, one obtains,

$$k^2 C_{11} + p \cdot k C_{12} = (1/2) f_1 C_0 - (1/2) B_0(1, 3) + (1/2) B_0(2, 3) \quad (\text{H.26})$$

$$k \cdot p C_{11} + p^2 C_{12} = (1/2) f_2 C_0 - (1/2) B_0(1, 2) + (1/2) B_0(1, 3) \quad (\text{H.27})$$

where

$$f_1 = m_2^2 - m_1^2 - k^2 \quad (\text{H.28})$$

and

$$f_2 = m_3^2 - m_2^2 + k^2 - p_5^2 \quad (\text{H.29})$$

The solution of H.26 and H.27 may be immediately written in the matrix form

$$\begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix} = X^{-1} \begin{pmatrix} R_1 \\ R_2 \end{pmatrix} \quad (\text{H.30})$$

where

$$X = \begin{pmatrix} k^2 & k.p \\ k.p & p^2 \end{pmatrix} \quad (\text{H.31})$$

and R_1 and R_2 are the right hand sides of the equations H.26 and H.27 respectively.

The above method is easily generalized to $C_{\mu\nu}$ one thus obtains C_{21} through C_{24} .

We merely state results here

$$\begin{pmatrix} C_{21} \\ C_{23} \end{pmatrix} = X^{-1} \begin{pmatrix} R_3 \\ R_5 \end{pmatrix} \quad (\text{H.32})$$

where

$$R_3 = (1/2)(f_1 C_{11} - B_1(1, 3) - B_0(2, 3)) + C_{24} \quad (\text{H.33})$$

and

$$R_5 = (1/2)(f_2 C_{11} - B_1(1, 2) + B_1(1, 3)) \quad (\text{H.34})$$

Similarly

$$\begin{pmatrix} C_{23} \\ C_{22} \end{pmatrix} = X^{-1} \begin{pmatrix} R_4 \\ R_6 \end{pmatrix} \quad (\text{H.35})$$

where

$$R_4 = (1/2)(f_1 C_{12} - B_1(1, 3) + B_1(2, 3)) \quad (\text{H.36})$$

$$R_6 = (1/2)(f_2 C_{12} + B_1(1, 3)) + C_{24} \quad (\text{H.37})$$

and

$$C_{24} = (1/4) - (1/2)m_1^2 C_0 + (1/4)(B_0(2, 3) + f_1 C_{11} + f_2 C_{12}) \quad (\text{H.38})$$

Similarly proceeding to obtain C_{31} through C_{36} one gets where

$$\begin{pmatrix} C_{35} \\ C_{36} \end{pmatrix} = X^{-1} \begin{pmatrix} R_{11} \\ R_{22} \end{pmatrix} \quad (\text{H.39})$$

$$R_{11} = (1/2)(f_1 C_{24} + B_{22}(2, 3) - B_{22}(1, 3)) \quad (\text{H.40})$$

$$R_{22} = (1/2)(f_2 C_{24} - B_{22}(1, 2) + B_{22}(1, 3)) \quad (\text{H.41})$$

where

$$\begin{pmatrix} C_{34} \\ C_{32} \end{pmatrix} = X^{-1} \begin{pmatrix} R_9 \\ R_{13} \end{pmatrix} \quad (\text{H.42})$$

$$R_9 = (1/2)(f_1 C_{22} + B_{21}(2, 3) - B_{21}(2, 3)) \quad (\text{H.43})$$

$$R_{13} = (1/2)(f_2 C_{22} + B_{21}(1, 3)) + 2C_{36} \quad (\text{H.44})$$

$$\begin{pmatrix} C_{31} \\ C_{33} \end{pmatrix} = X^{-1} \begin{pmatrix} R_8 \\ R_{12} \end{pmatrix} \quad (\text{H.45})$$

$$R_8 = (1/2)(f_1 C_{21} - B_{21}(1, 3) + B_0(2, 3)) + 2C_{35} \quad (\text{H.46})$$

$$R_{12} = (1/2)(f_2 C_{21} - B_{21}(1, 2) + B_{21}(1, 3)) \quad (\text{H.47})$$

and

$$\begin{pmatrix} C_{33} \\ C_{34} \end{pmatrix} = X^{-1} \begin{pmatrix} R_{10} \\ R_{14} \end{pmatrix} \quad (\text{H.48})$$

$$R_{10} = (1/2)(f_1 C_{23} - B_{21}(1, 3) - B_1(2, 3)) + C_{36} \quad (\text{H.49})$$

$$R_{14} = (1/2)(f_2 C_{23} + B_{21}(1, 3)) + C_{35} \quad (\text{H.50})$$

Appendix I

Results for the $\gamma W^+ W^-$ vertex

The contribution of the S-P is now given

$$V_{2H_2^0}^\gamma \doteq 2[C_{35}(q_2, q_1, m_H, m_{H_2^0}, m_H) + C_{24}] \quad (\text{I.1})$$

$$V_{3H_2^0}^\gamma \doteq 2[C_{36}(q_2, q_1, m_H, m_{H_2^0}, m_H)] \quad (\text{I.2})$$

$$V_{5H_2^0}^\gamma \doteq 2[C_{36}(q_2, q_1, m_H, m_{H_2^0}, m_H) + .5C_{24}] \quad (\text{I.3})$$

$$V_{6H_2^0}^\gamma \doteq 2[C_{35}(q_2, q_1, m_H, m_{H_2^0}, m_H) + .5C_{24}] \quad (\text{I.4})$$

$$V_{8H_2^0}^\gamma \doteq 2[C_{34}(q_2, q_1, m_H, m_{H_2^0}, m_H) + C_{22} + .5C_{23} + .5C_{12}] \quad (\text{I.5})$$

$$V_{12H_2^0}^\gamma \doteq 2[C_{33}(q_2, q_1, m_H, m_{H_2^0}, m_H) + C_{23} + .5C_{23} + .5C_{12}] \quad (\text{I.6})$$

For the contribution of H_3^0 simply substitute H_3^0 for H_2^0 .

$$V_{2\tilde{\gamma}}^\gamma \doteq -ns_W^2 [+ (2 - n)(C_{24}(q_1, q_2, m_{\tilde{\omega}}, m_{\tilde{\gamma}}, m_{\tilde{\omega}}) - C_{36}) - s_3(C_{33} + 2C_{23} - C_{21} + C_{12} - C_{11}) - s_4(C_{32} + C_{22}) - 2q_1 \cdot q_2(C_{34} + C_{22}) + m_{\tilde{\omega}}^2 C_{12}] \quad (\text{I.7})$$

$$V_{3\tilde{\gamma}}^\gamma \doteq -ns_W^2 [+ (n - 2)(2C_{24}(q_1, q_2, m_{\tilde{\omega}}, m_{\tilde{\gamma}}, m_{\tilde{\omega}}) + C_{35}) - s_3(C_{31} + 2C_{21} + C_{11}) - s_4(C_{34} + 2C_{22} + C_{12}) - 2q_1 \cdot q_2(C_{33} + 2C_{23} + C_{12}) + m_{\tilde{\omega}}^2(C_{11} + C_0)] \quad (\text{I.8})$$

$$V_{5\tilde{\gamma}}^\gamma \doteq -ns_W^2 [+ (n - 2)(C_{35}(q_1, q_2, m_{\tilde{\omega}}, m_{\tilde{\gamma}}, m_{\tilde{\omega}})) - s_3(C_{31} + 2C_{21} + C_{11}) - s_4(C_{34} - C_{12}) - 2q_1 \cdot q_2(C_{33} + C_{23}) - m_{\tilde{\omega}}^2(C_{11} + C_0)] \quad (\text{I.9})$$

$$V_{6\tilde{\gamma}}^\gamma \doteq -ns_W^2 [+(n-2)(C_{36}(q_1, q_2, m_{\tilde{\omega}}, m_{\tilde{\gamma}}, m_{\tilde{\omega}}) + C_{24}) - s_3(C_{33} + 2C_{23} + C_{21} + C_{12} + C_{11}) - s_4(C_{32} + C_{22}) - 2q_1 \cdot q_2(C_{34} + C_{23} + C_{22} + C_{12}) - m_{\tilde{\omega}}^2 C_{12}] \quad (I.10)$$

$$V_{8\tilde{\gamma}}^\gamma \doteq -ns_W^2 [C_{33}(q_1, q_2, m_{\tilde{\omega}}, m_{\tilde{\gamma}}, m_{\tilde{\omega}}) + 1.5C_{23} + .5C_{12}] \quad (I.11)$$

$$V_{12\tilde{\gamma}}^\gamma \doteq -ns_W^2 [C_{34}(q_1, q_2, m_{\tilde{\omega}}, m_{\tilde{\gamma}}, m_{\tilde{\omega}}) + C_{22} + .5C_{23} + .5C_{12}] \quad (I.12)$$

The zino contribution is obtained by replacing $m_{\tilde{\gamma}}$ by $m_{\tilde{z}}$ and s_W^2 by $c_W^2 + .25$. Similarly the higgsino contribution is obtained by replacing $m_{\tilde{\gamma}}$ by $m_{\tilde{h}}$ and s_W^2 by .25

$$V_{2\tilde{F}}^\gamma \doteq -4[C_{36}(q_1, q_2, m_{\tilde{e}}, m_{\tilde{\nu}_e}, m_{\tilde{e}})] - 4[C_{36}(q_1, q_2, m_{\tilde{d}}, m_{\tilde{u}}, m_{\tilde{d}})] + (4)(2)[C_{35}(q_2, q_1, m_{\tilde{u}}, m_{\tilde{d}}, m_{\tilde{u}}) + C_{24}] \quad (I.13)$$

$$V_{3\tilde{F}}^\gamma \doteq -4[C_{36}(q_1, q_2, m_{\tilde{e}}, m_{\tilde{\nu}_e}, m_{\tilde{e}})] + \frac{C_{24}}{C_{24}} - 4[C_{36}(q_1, q_2, m_{\tilde{d}}, m_{\tilde{u}}, m_{\tilde{d}})] + \frac{C_{24}}{C_{24}} + (4)(2)[C_{36}(q_2, q_1, m_{\tilde{u}}, m_{\tilde{d}}, m_{\tilde{u}})] \quad (I.14)$$

$$V_{5\tilde{F}}^\gamma \doteq -4[C_{35}(q_1, q_2, m_{\tilde{e}}, m_{\tilde{\nu}_e}, m_{\tilde{e}})] - 4[C_{35}(q_1, q_2, m_{\tilde{d}}, m_{\tilde{u}}, m_{\tilde{d}})] + (4)(2)[C_{36}(q_2, q_1, m_{\tilde{u}}, m_{\tilde{d}}, m_{\tilde{u}})] \quad (I.15)$$

$$V_{6\tilde{F}}^\gamma \doteq -4[C_{36}(q_1, q_2, m_{\tilde{e}}, m_{\tilde{\nu}_e}, m_{\tilde{e}})] - 4[C_{36}(q_1, q_2, m_{\tilde{d}}, m_{\tilde{u}}, m_{\tilde{d}})] + (4)(2)[C_{35}(q_2, q_1, m_{\tilde{u}}, m_{\tilde{d}}, m_{\tilde{u}})] \quad (I.16)$$

$$V_{8\tilde{F}}^\gamma \doteq -4[C_{33}(q_1, q_2, m_{\tilde{e}}, m_{\tilde{\nu}_e}, m_{\tilde{e}})] + \frac{C_{23}}{C_{23}} - 4[C_{33}(q_1, q_2, m_{\tilde{d}}, m_{\tilde{u}}, m_{\tilde{d}})] + \frac{C_{23}}{C_{23}} + (4)(2)[C_{34}(q_2, q_1, m_{\tilde{u}}, m_{\tilde{d}}, m_{\tilde{u}}) + C_{22}] \quad (I.17)$$

$$V_{12\tilde{F}}^\gamma \doteq -4[C_{34}(q_1, q_2, m_{\tilde{e}}, m_{\tilde{\nu}_e}, m_{\tilde{e}})] + \frac{C_{24}}{C_{24}} - 4[C_{34}(q_1, q_2, m_{\tilde{d}}, m_{\tilde{u}}, m_{\tilde{d}})] + \frac{C_{24}}{C_{24}} + (4)(2)[C_{33}(q_2, q_1, m_{\tilde{u}}, m_{\tilde{d}}, m_{\tilde{u}}) + C_{23}] \quad (I.18)$$

Appendix J

Results for the ZW^+W^- vertex

For the Higgs exchange diagrams we have

$$C_{i,j}(2, 1, Z, W, H_1^0) \equiv C_{i,j3H} \quad (\text{J.1})$$

$$C_{i,j}(2, 1, H_1^0, W, Z) \equiv C_{i,j4H} \quad (\text{J.2})$$

$$V_{2H}^Z \doteq \sec^2 \theta_W [(1 - 2s_W^2)(C_{352H} + C_{242H}) + (C_{353H} + C_{243H}) + (C_{354H} + C_{244H}) - s_W^2 M_W^2 (C_{112H} + C_{02H}) - c_W^2 M_W^2 (C_{112H} - C_{02H})] \quad (\text{J.3})$$

$$V_{3H}^Z \doteq \sec^2 \theta_W [(1 - 2s_W^2)(C_{362H}) + (C_{363H}) + (C_{364H}) - s_W^2 M_W^2 (C_{122H}) - c_W^2 M_W^2 (C_{122H} + 2C_{02H})] \quad (\text{J.4})$$

$$V_{5H}^Z \doteq \sec^2 \theta_W [(1 - 2s_W^2)(C_{362H} + .5C_{242H}) + (C_{363H} + .5C_{243H}) + (C_{364H} + .5C_{244H}) + c_W^2 M_W^2 (2C_{122H} + C_{02H})] \quad (\text{J.5})$$

$$V_{6H}^Z \doteq \sec^2 \theta_W [(1 - 2s_W^2)(C_{352H} + .5C_{242H}) + (C_{353H} + .5C_{243H}) + (C_{354H} + .5C_{244H}) + c_W^2 M_W^2 (2C_{112H} + C_{02H})] \quad (\text{J.6})$$

$$V_{8H}^Z \doteq \sec^2 \theta_W [-(1 - 2s_W^2)(C_{342H} + C_{222H} + .5C_{232H} + .5C_{122H}) - (C_{343H} + C_{223H} + .5C_{233H} + .5C_{123H}) - (C_{344H} + C_{224H} + .5C_{234H} + .5C_{124H})] \quad (\text{J.7})$$

$$V_{12H}^Z \doteq \sec^2 \theta_W [-(1 - 2s_W^2)(C_{332H} + C_{232H} + .5C_{232H} + .5C_{122H}) - (C_{333H} + C_{233H} + .5C_{233H} + .5C_{123H}) - (C_{334H} + C_{234H} + .5C_{234H} + .5C_{124H})] \quad (\text{J.8})$$

Now the γ exchange diagrams

$$V_{2\gamma}^Z \doteq -s_W^2 [(1-n)(10C_{352} + 14C_{242}) - 4C_{242} + s_3(2C_{342} + 2C_{232} + 6C_{222} + 2C_{122} - C_{112} + C_{02}) + s_4(2C_{312} + 10C_{212} + 16C_{112} + 8C_{02}) + (q_1 \cdot q_2)(4C_{332} + 16C_{232} + 2C_{212} + 4C_{122} + 14C_{112} + 8C_{02})] + s_W^2 t_W^2 M_W^2 [C_{112} + 3C_{02}] - s_W^2 [C_{352} + C_{242} - C_{361}] - s_W^2 [5B_{02} + B_{12}] \quad (\text{J.9})$$

$$V_{3\gamma}^Z \doteq -s_W^2 [(1-n)(10C_{362}) + 4nC_{242} + s_3(2C_{322} + 2C_{122} - 4C_{222}) + s_4(2C_{332} + 2C_{232} - 4C_{212} - C_{122} - 8C_{112} - 6C_{02}) + (q_1 \cdot q_2)(4C_{342} - 8C_{232} + 2C_{222} + 2C_{122} - 8C_{112} - 4C_{02})] + s_W^2 t_W^2 M_W^2 [C_{122} - 2C_{02}] - s_W^2 [-C_{351} - C_{241} + C_{362}] - s_W^2 [-5B_{01} - B_{11}] \quad (\text{J.10})$$

$$V_{5\gamma}^Z \doteq -s_W^2 [(1-n)(10C_{362}) + (5-7n)C_{242} + s_3(2C_{322} + 2C_{122} + 5C_{222}) + s_4(2C_{332} + 2C_{232} + 3C_{212} - C_{122} + 7C_{112} + 5C_{02}) + (q_1 \cdot q_2)(4C_{342} + 8C_{232} + 2C_{222} + 3C_{112} - 5C_{122})] + .5(1 - 2s_W^2) M_W^2 t_W^2 [2C_{122}] - 2s_W^2 M_W^2 t_W^2 [C_{122} - C_{02}] - s_W^2 [-C_{351} - C_{241} + C_{362} + C_{242}] - s_W^2 [4B_{01} - B_{11}] \quad (\text{J.11})$$

$$V_{6\gamma}^Z \doteq -s_W^2 [(1-n)(10C_{352}) + (5-3n)C_{242} + s_3(2C_{342} + 2C_{232} - C_{222} - C_{112} + 3C_{122} - 2C_{02}) + s_4(2C_{312} + C_{212} - 2C_{112} - C_{02}) + (q_1 \cdot q_2)(4C_{332} + 2C_{212} - 9C_{112} - C_{122} - 8C_{02})] + .5(1 - 2s_W^2) M_W^2 t_W^2 [2C_{112}] - 2s_W^2 M_W^2 t_W^2 [C_{112} + C_{02}] - s_W^2 [C_{352} + C_{242} - C_{361} - C_{241}] - s_W^2 [-4B_{02} + B_{12}] \quad (\text{J.12})$$

$$V_{8\gamma}^Z \doteq -s_W^2 [18C_{342} + 9C_{232} + 18C_{222} + 12C_{122} + 3C_{112} + 3C_{02}] - s_W^2 [-C_{342} - C_{122} - C_{232} - C_{222}] - s_W^2 [C_{331} + C_{121} + C_{231} + C_{231}] \quad (\text{J.13})$$

$$V_{12\gamma}^Z \doteq -s_W^2 [18C_{332} + 9C_{232} + 18C_{232} + 12C_{122} + 3C_{112} + 3C_{02}] - s_W^2 [-C_{332} - C_{122} - C_{232} - C_{232}] - s_W^2 [C_{341} + C_{121} + C_{231} + C_{221}] \quad (\text{J.14})$$

The 'Z' exchange diagrams

$$V_{2Z}^Z \doteq -c_W^2 [(1-n)(10C_{352Z} + 14C_{242Z}) - 4C_{242Z} + s_3(2C_{342Z} + 2C_{232Z} + 6C_{222Z} + 2C_{122Z} - C_{112Z} + C_{02Z}) + s_4(2C_{312Z} + 10C_{212Z} + 16C_{112Z} + 8C_{02Z}) + (q_1 \cdot q_2)(4C_{332Z} + 16C_{232Z} + 2C_{212Z} + 4C_{122Z} + 14C_{112Z} + 8C_{02Z})] - c_W^2 [C_{352Z} + C_{242Z} - C_{361Z}] - s_W^4 M_Z^2 [C_{112Z} + 3C_{02Z}] - c_W^2 [5B_{02Z} + B_{12Z}] + (1 - 2s_W^2) \sec^2 \theta_W [C_{352Z} + C_{242Z}] - [5B_{05W} + B_{15W}] \quad (\text{J.15})$$

$$V_{1Z}^Z \doteq -c_W^2 [(1-n)(10C_{362Z}) + 4nC_{242Z} + s_3(2C_{322Z} + 2C_{122Z} - 4C_{222Z}) + s_4(2C_{332Z} + 2C_{232Z} - 4C_{212Z} - C_{122Z} - 8C_{112Z} - 6C_{02Z}) + (q_1 \cdot q_2)(4C_{342Z} - 8C_{232Z} + 2C_{222Z} + 2C_{122Z} - 8C_{112Z} - 4C_{02Z})] + c_W^2 [C_{351Z} + C_{241Z} - C_{362Z}] - s_W^4 M_Z^2 [C_{122Z} - 2C_{02Z}] + c_W^2 [5B_{01Z} + B_{11Z}] + (1 - 2s_W^2) \sec^2 \theta_W [C_{362Z}] + [4B_{05W} - B_{15W}] \quad (\text{J.16})$$

$$V_{5Z}^Z \doteq -c_W^2 [(1-n)(10C_{362Z}) + (5-7n)C_{242Z} + s_3(2C_{322Z} + 2C_{122Z} + 5C_{222Z}) + s_4(2C_{332Z} + 2C_{232Z} + 3C_{212Z} - C_{122Z} + 7C_{112Z} + 5C_{02Z}) + (q_1 \cdot q_2)(4C_{342Z} + 8C_{232Z} + 2C_{222Z} + 3C_{112Z} - 5C_{122Z})] - c_W^2 [-C_{351Z} - C_{241Z} + C_{362Z} + C_{242Z}] + .5(1 - 2s_W^2) t_W^4 M_W^2 [2C_{122Z} + C_{02Z}] + s_W^4 M_Z^2 [2C_{122Z} - 2C_{02Z}] - c_W^2 [4B_{01Z} - B_{11Z}] + (1 - 2s_W^2) \sec^2 \theta_W [C_{362Z}] - [.5B_{05W} + B_{15W}] + (2-n)[B_{05W} + 2B_{15W}] \quad (\text{J.17})$$

$$V_{6Z}^Z \doteq -c_W^2 [(1-n)(10C_{352Z}) + (5-3n)C_{242Z} + s_3(2C_{342Z} + 2C_{232Z} - C_{222Z} - C_{112Z} + 3C_{122Z} - 2C_{02Z}) + s_4(2C_{312Z} + C_{212Z} - 2C_{112Z} - C_{02Z}) + (q_1 \cdot q_2)(4C_{332Z} + 2C_{212Z} - 9C_{112Z} - C_{122Z} - 8C_{02Z})] - c_W^2 [C_{352Z} + C_{242Z} - C_{361Z} - C_{241Z}] + .5(1 - 2s_W^2) t_W^4 M_W^2 [2C_{112Z} + C_{02Z}] + s_W^4 M_Z^2 [2C_{112Z} + 2C_{02Z}] - c_W^2 [-4B_{02Z} + B_{12Z}] + (1 - 2s_W^2) \sec^2 \theta_W [C_{352Z}] - [.5B_{05W} + B_{15W}] + (2-n)s_W^2 [B_{05W} + 2B_{15W}] \quad (\text{J.18})$$

$$V_{8Z}^Z \doteq -c_W^2 [18C_{342Z} + 9C_{232Z} + 18C_{222Z} + 12C_{122Z} + 3C_{112Z} + 3C_{02Z}] - c_W^2 [-C_{342Z} - C_{122Z} - C_{232Z} - C_{222Z}] - c_W^2 [C_{331Z} + C_{121Z} + C_{231Z} + C_{221Z}] - .5 \sec^2 \theta_W (1 - 2s_W^2) [2C_{342Z} + 2C_{222Z} + C_{232Z} + C_{122Z}] \quad (\text{J.19})$$

$$V_{12Z}^Z \doteq -c_W^2 [18C_{332Z} + 9C_{232Z} + 18C_{232Z} + 12C_{122Z} + 3C_{112Z} + 3C_{02Z}] - c_W^2 [-C_{332Z} - C_{122Z} - C_{232Z} - C_{232Z}] - c_W^2 [C_{341Z} + C_{121Z} + C_{231Z} + C_{221Z}] - .5 \sec^2 \theta_W (1 - 2s_W^2) [2C_{332Z} + 2C_{232Z} + C_{232Z} + C_{122Z}] \quad (\text{J.20})$$

The fermion contribution

$$V_{2F}^Z \doteq -(n/8)(1 - 2s_W^2) \sec^2 \theta_W [(2-n)(C_{241e} - C_{361e}) - s_3(C_{331e} + 2C_{231e} - C_{211e} + C_{121e} - C_{111e}) - s_4(C_{321e} + C_{221e}) - 2(q_1 \cdot q_2)(C_{341e} + C_{221e}) + m_e^2 C_{121e}] + (n/8) \sec^2 \theta_W [(n-2)(2C_{242n} + C_{352n}) - s_3(C_{342n} + 2C_{222n} + C_{122n}) - s_4(C_{312n} + 2C_{212n} + C_{112n}) - 2(q_1 \cdot q_2)(C_{332n} + 2C_{232n} + C_{122n})] - (3n/8)(1 - \frac{2s_W^2}{3}) \sec^2 \theta_W [(2-n)(C_{241d} - C_{361d}) - s_3(C_{331d} + 2C_{231d} - C_{211d} + C_{121d} - C_{111d}) - s_4(C_{321d} + C_{221d}) - 2(q_1 \cdot q_2)(C_{341d} + C_{221d}) + m_d^2 C_{121d}] + (3n/8)(1 - \frac{4s_W^2}{3}) \sec^2 \theta_W [(n-2)(2C_{242u} + C_{352u}) - s_3(C_{342u} + 2C_{222u} + C_{122u}) - s_4(C_{312u} + 2C_{212u} + C_{112u}) - 2(q_1 \cdot q_2)(C_{332u} + 2C_{232u} + C_{122u}) + m_u^2 (C_{112u} + C_{02u})] \quad (\text{J.21})$$

$$\begin{aligned}
V_{5F}^Z \doteq & -(n/8)(1 - 2s_W^2) \sec^2 \theta_W [(n-2)(2C_{241e} + C_{351e}) - \\
& s_3(C_{311e} + 2C_{211e} + C_{111e}) - s_4(C_{341e} + 2C_{221e} + C_{121e}) - \\
& 2(q_1 \cdot q_2)(C_{331e} + 2C_{231e} + C_{121e}) + m_e^2(C_{111e} + C_{01e})] + \\
& (n/8) \sec^2 \theta_W [(2-n)(C_{242n} - C_{362n}) - s_4(C_{332n} + 2C_{232n} - \\
& C_{212n} + C_{122n} - C_{112n}) - s_3(C_{322n} + C_{222n}) - 2(q_1 \cdot q_2)(C_{342n} + \\
& C_{222n})] - (3n/8)(1 - \frac{2s_W^2}{3}) \sec^2 \theta_W [(n-2)(2C_{241d} + \\
& C_{351d}) - s_3(C_{311d} + 2C_{211d} + C_{111d}) - s_4(C_{341d} + 2C_{221d} + \\
& C_{121d}) - 2(q_1 \cdot q_2)(C_{331d} + 2C_{231d} + C_{121d}) + m_d^2(C_{111d} + \\
& C_{01d})] + (3n/8)(1 - \frac{4s_W^2}{3}) \sec^2 \theta_W [(2-n)(C_{242u} - C_{362u}) - \\
& s_4(C_{332u} + 2C_{232u} - C_{212u} + C_{122u} - C_{112u}) - s_3(C_{322u} + \\
& C_{222u}) - 2(q_1 \cdot q_2)(C_{342u} + C_{222u}) + m_u^2 C_{122u}]
\end{aligned} \tag{J.22}$$

$$\begin{aligned}
V_{5F}^Z \doteq & -(n/8)(1 - 2s_W^2) \sec^2 \theta_W [(n-2)C_{351e} - s_3(C_{311e} + \\
& 2C_{211e} + C_{111e}) - s_4(C_{341e} - C_{121e}) - 2(q_1 \cdot q_2)(C_{331e} + C_{231e}) - \\
& m_e^2(C_{111e} + C_{01e})] + (n/8) \sec^2 \theta_W [(n-2)(C_{362n} + C_{242n}) - \\
& s_4(C_{332n} + 2C_{232n} + C_{212u} + C_{122n} + C_{112n}) - s_3(C_{322n} + \\
& C_{222n}) - 2(q_1 \cdot q_2)(C_{342n} + C_{222n} + C_{232n} + C_{122n})] - (3n/8)(1 - \\
& \frac{2s_W^2}{3}) \sec^2 \theta_W [(n-2)C_{351d} - s_3(C_{311d} + 2C_{211d} + C_{111d}) - \\
& s_4(C_{341d} - C_{121d}) - 2(q_1 \cdot q_2)(C_{331d} + C_{231d}) - m_d^2(C_{111d} + \\
& C_{01d})] + (3n/8)(1 - \frac{4s_W^2}{3}) \sec^2 \theta_W [(n-2)(C_{362u} + C_{242u}) - \\
& s_4(C_{332u} + 2C_{232u} + C_{212u} + C_{122u} + C_{112u}) - s_3(C_{322u} + \\
& C_{222u}) - 2(q_1 \cdot q_2)(C_{342u} + C_{222u} + C_{232u} + C_{122u}) - m_u^2 C_{122u}]
\end{aligned} \tag{J.23}$$

$$\begin{aligned}
V_{6F}^Z \doteq & -(n/8)(1 - 2s_W^2) \sec^2 \theta_W [(n-2)(C_{361e} + C_{241e}) - \\
& s_4(C_{331e} + 2C_{231e} + C_{211e} + C_{121e} + C_{111e}) - s_3(C_{321e} + \\
& C_{221e}) - 2(q_1 \cdot q_2)(C_{341e} + C_{221e} + C_{231e} + C_{121e}) - \\
& m_e^2 C_{121e}] + (n/8) \sec^2 \theta_W [(n-2)C_{352n} - s_4(C_{312n} + \\
& 2C_{212n} + C_{112n}) - s_3(C_{342n} - C_{122n}) - 2(q_1 \cdot q_2)(C_{332n} + \\
& C_{232n})] - (3n/8)(1 - \frac{2s_W^2}{3}) \sec^2 \theta_W [(n-2)(C_{361d} + C_{241d}) - \\
& s_4(C_{332d} + 2C_{231d} + C_{211d} + C_{121d} + C_{111d}) - s_3(C_{321d} + \\
& C_{221d}) - 2(q_1 \cdot q_2)(C_{341d} + C_{221d} + C_{231d} + C_{121d}) - m_d^2 C_{121d}] + \\
& (3n/8)(1 - \frac{4s_W^2}{3}) \sec^2 \theta_W [(n-2)C_{352u} - s_4(C_{312u} + 2C_{212u} + \\
& C_{112u}) - s_3(C_{342u} - C_{122u}) - 2(q_1 \cdot q_2)(C_{332u} + C_{232u}) - \\
& m_u^2(C_{112u} + C_{02u})]
\end{aligned} \tag{J.24}$$

$$\begin{aligned}
V_{8F}^Z \doteq & -(n/8)(1 - 2s_W^2) \sec^2 \theta_W [4C_{331e} + 6C_{231e} + 2C_{121e}] + \\
& (n/8) \sec^2 \theta_W [4C_{342n} + 4C_{222n} + 2C_{232n} + 2C_{122n} - \\
& (3n/8)(1 - \frac{2s_W^2}{3}) \sec^2 \theta_W [4C_{331d} + 6C_{231d} + 2C_{121d}] + \\
& (3n/8)(1 - \frac{4s_W^2}{3}) \sec^2 \theta_W [4C_{342u} + 4C_{222u} + 2C_{232u} + 2C_{122u}]
\end{aligned} \tag{J.25}$$

$$\begin{aligned}
V_{12F}^Z \doteq & -(n/8)(1 - 2s_W^2) \sec^2 \theta_W [+4C_{341e} + 4C_{221e} + 2C_{231e} + \\
& 2C_{121e}] + (n/8) \sec^2 \theta_W [+4C_{332n} + 6C_{232n} + 2C_{122n}] - \\
& (3n/8)(1 - \frac{2s_W^2}{3}) \sec^2 \theta_W [+4C_{341d} + 4C_{221d} + 2C_{231d} + \\
& 2C_{121d}] + (3n/8)(1 - \frac{4s_W^2}{3}) \sec^2 \theta_W [+4C_{332u} + 6C_{232u} + \\
& 2C_{122u}]
\end{aligned} \tag{J.26}$$

Finally the contribution from the SUSY particles

$$C_{ij}(2, 1, M_H, M_{H_2^0}, M_H) \equiv C_{ij6} \tag{J.27}$$

$$C_{ij}(2, 1, M_H, M_{H_3^0}, M_H) \equiv C_{ij7} \tag{J.28}$$

$$C_{ij}(2, 1, M_{H_2^0}, M_H, M_{H_3^0}) \equiv C_{ij6H} \tag{J.29}$$

$$C_{ij}(2, 1, M_{H_3^0}, M_H, M_{H_2^0}) \equiv C_{ij6H} \tag{J.30}$$

$$V_{2[H, H_2^0, H_3^0]}^Z \doteq \sec^2 \theta_W [+ (1 - 2s_W^2)(C_{356} + C_{246} + C_{357} + C_{247}) + C_{356H} + C_{246H} + C_{357H} + C_{247H}] \tag{J.31}$$

$$V_{3[H, H_2^0, H_3^0]}^Z \doteq \sec^2 \theta_W [+ (1 - 2s_W^2)(C_{366} + C_{357}) + C_{366H} + C_{367H}] \tag{J.32}$$

$$V_{5[H, H_2^0, H_3^0]}^Z \doteq \sec^2 \theta_W [(1 - 2s_W^2)(C_{366} + .5C_{246} + C_{367} + .5C_{247}) + C_{366H} + .5C_{246H} + C_{367H} + .5C_{247}] \tag{J.33}$$

$$V_{6[H, H_2^0, H_3^0]}^Z \doteq \sec^2 \theta_W [(1 - 2s_W^2)(C_{356} + .5C_{246} + C_{357} + .5C_{247}) + C_{356H} + .5C_{246H} + C_{357H} + .5C_{247}] \tag{J.34}$$

$$\begin{aligned}
V_{8[H, H_2^0, H_3^0]}^Z \doteq & -\sec^2 \theta_W [+ (1 - 2s_W^2)(C_{346} + C_{226} + .5C_{236} + .5C_{126} + \\
& C_{347} + C_{227} + .5C_{237} + .5C_{127}) + (C_{346H} + C_{226H} + .5C_{236H} + \\
& .5C_{126H}) + (C_{347H} + C_{227H} + .5C_{237H} + .5C_{127H})]
\end{aligned} \tag{J.35}$$

$$\begin{aligned}
V_{12[H, H_2^0, H_3^0]}^Z \doteq & -\sec^2 \theta_W [+ (1 - 2s_W^2)(C_{336} + C_{236} + .5C_{236} + .5C_{126} + \\
& C_{337} + C_{237} + .5C_{237} + .5C_{127}) + (C_{336H} + C_{236H} + .5C_{236H} + \\
& .5C_{126H}) + (C_{337H} + C_{237H} + .5C_{237H} + .5C_{127H})]
\end{aligned} \tag{J.36}$$

$$V_{k\bar{i}}^Z = [\frac{4c_W^2 + .5 \sec^2 \theta_W (1 - 2s_W^2)}{4c_W^2 + 1}] V_{k\bar{i}}^\gamma \tag{J.37}$$

$$V_{k\bar{\gamma}}^Z = V_{k\bar{\gamma}}^\gamma \tag{J.38}$$

$$C_{ij}(2, 1, m_{\tilde{t}}, m_{\tilde{\omega}}, m_{\tilde{h}}) \equiv C_{ij5} \tag{J.39}$$

$$C_{ij}(2, 1, m_{\tilde{h}}, m_{\tilde{\omega}}, m_{\tilde{t}}) \equiv C_{ij8} \tag{J.40}$$

$$V_{k\bar{h}}^Z = [.5 \sec^2 \theta_W (1 - 2s_W^2)] V_{k\bar{h}}^\gamma + V_{k[\bar{\omega}, \bar{h}, \bar{i}]}^Z \quad (\text{J.41})$$

$$V_{2[\bar{\omega}, \bar{h}, \bar{i}]}^Z \doteq -(.5 \sec^2 \theta_W) [+ .75 + .25 - 2(C_{245} - C_{365}) - s_3(C_{335} + 2C_{235} - C_{215} + C_{125} - C_{115}) - s_4(C_{325} + C_{225}) - 2(q_1 \cdot q_2)(C_{345} + C_{225}) + m_{\bar{i}} m_{\bar{h}} C_{125}] - (.5 \sec^2 \theta_W) [+ .75 + .25 - 2(C_{248} - C_{368}) - s_3(C_{338} + 2C_{238} - C_{218} + C_{128} - C_{118}) - s_4(C_{328} + C_{228}) - 2(q_1 \cdot q_2)(C_{348} + C_{228}) + m_{\bar{i}} m_{\bar{h}} C_{128}] \quad (\text{J.42})$$

$$V_{3[\bar{\omega}, \bar{h}, \bar{i}]}^Z \doteq -(.5 \sec^2 \theta_W) [+ 0.50 - 1.50 + 2(C_{355} + 2C_{245}) - s_3(C_{315} + 2C_{215} + C_{115}) - s_4(C_{345} + 2C_{225} + C_{125}) - 2(q_1 \cdot q_2)(C_{335} + 2C_{235} + C_{125}) + m_{\bar{i}} m_{\bar{h}} (C_{115} + C_{05})] - (.5 \sec^2 \theta_W) [+ 0.50 - 1.50 + 2(C_{358} + 2C_{248}) - s_3(C_{318} + 2C_{218} + C_{118}) - s_4(C_{348} + 2C_{228} + C_{128}) - 2(q_1 \cdot q_2)(C_{338} + 2C_{238} + C_{128}) + m_{\bar{i}} m_{\bar{h}} (C_{118} + C_{08})] \quad (\text{J.43})$$

$$V_{5[\bar{\omega}, \bar{h}, \bar{i}]}^Z \doteq -(.5 \sec^2 \theta_W) [+ .50 + 2C_{355} - s_3(C_{315} + 2C_{215} + C_{115}) - s_4(C_{345} - C_{125}) - 2(q_1 \cdot q_2)(C_{335} + C_{235}) - m_{\bar{i}} m_{\bar{h}} (C_{115} + C_{05})] - (.5 \sec^2 \theta_W) [+ .50 + 2C_{358} - s_3(C_{318} + 2C_{218} + C_{118}) - s_4(C_{348} - C_{128}) - 2(q_1 \cdot q_2)(C_{338} + C_{238}) - m_{\bar{i}} m_{\bar{h}} (C_{118} + C_{08})] \quad (\text{J.44})$$

$$V_{6[\bar{\omega}, \bar{h}, \bar{i}]}^Z \doteq -(.5 \sec^2 \theta_W) [+ .25 - .75 + 2(C_{245} + C_{365}) - s_3(C_{335} + 2C_{235} + C_{215} + C_{125} + C_{115}) - s_4(C_{325} + C_{225}) - 2(q_1 \cdot q_2)(C_{345} + C_{225} + C_{235} + C_{125}) - m_{\bar{i}} m_{\bar{h}} C_{125}] - (.5 \sec^2 \theta_W) [+ .25 - .75 + 2(C_{248} + C_{368}) - s_3(C_{338} + 2C_{238} + C_{218} + C_{128} + C_{118}) - s_4(C_{328} + C_{228}) - 2(q_1 \cdot q_2)(C_{348} + C_{228} + C_{238} + C_{128}) - m_{\bar{i}} m_{\bar{h}} C_{128}] \quad (\text{J.45})$$

$$V_{8[\bar{\omega}, \bar{h}, \bar{i}]}^Z \doteq -(.5 \sec^2 \theta_W) [4C_{335} + 6C_{235} + 2C_{125}] - (.5 \sec^2 \theta_W) [4C_{338} + 6C_{238} + 2C_{128}] \quad (\text{J.46})$$

$$V_{12[\bar{\omega}, \bar{h}, \bar{i}]}^Z \doteq -(.5 \sec^2 \theta_W) [4C_{345} + 4C_{225} + 2C_{235} + 2C_{125}] - (.5 \sec^2 \theta_W) [4C_{348} + 4C_{228} + 2C_{238} + 2C_{128}] \quad (\text{J.47})$$

$$V_{2\bar{F}}^Z \doteq -(4)(.5)(1 - 2s_W^2)(\sec^2 \theta_W) [C_{36}(q_1, q_2, m_{\bar{e}}, m_{\bar{\nu}_e}, m_{\bar{\tau}})] - (4)(1.5)(1 - (2/3)s_W^2)(\sec^2 \theta_W) [C_{36}(q_1, q_2, m_{\bar{d}}, m_{\bar{u}}, m_{\bar{j}})] + (4)(.5)(\sec^2 \theta_W) [C_{35}(q_2, q_1, m_{\bar{\nu}_e}, m_{\bar{e}}, m_{\bar{\nu}_e}) + C_{24}] + (4)(1.5)(1 - (4/3)s_W^2)(\sec^2 \theta_W) [C_{35}(q_2, q_1, m_{\bar{u}}, m_{\bar{d}}, m_{\bar{u}}) + C_{24}] \quad (\text{J.48})$$

$$V_{3\bar{F}}^Z \doteq -(4)(.5)(1 - 2s_W^2)(\sec^2 \theta_W) [C_{35}(q_1, q_2, m_{\bar{e}}, m_{\bar{\nu}_e}, m_{\bar{e}}) + C_{24}] - (4)(1.5)(1 - (2/3)s_W^2)(\sec^2 \theta_W) [C_{35}(q_1, q_2, m_{\bar{d}}, m_{\bar{u}}, m_{\bar{d}}) + C_{24}] + (4)(.5)(\sec^2 \theta_W) [C_{36}(q_2, q_1, m_{\bar{\nu}_e}, m_{\bar{e}}, m_{\bar{\nu}_e})] + (4)(1.5)(1 - (4/3)s_W^2)(\sec^2 \theta_W) [C_{36}(q_2, q_1, m_{\bar{u}}, m_{\bar{d}}, m_{\bar{u}})] \quad (\text{J.49})$$

$$\begin{aligned}
V_{5\hat{F}}^Z \doteq & -(4)(.5)(1 - 2s_W^2)(\sec^2 \theta_W)[C_{35}(q_1, q_2, m_{\tilde{e}}, m_{\tilde{\nu}_e}, m_{\tilde{e}})] - \\
& (4)(1.5)(1 - (2/3)s_W^2)(\sec^2 \theta_W)[C_{35}(q_1, q_2, m_{\tilde{d}}, m_{\tilde{u}}, m_{\tilde{d}})] + \\
& (4)(.5)(\sec^2 \theta_W)[C_{36}(q_2, q_1, m_{\tilde{\nu}_e}, m_{\tilde{e}}, m_{\tilde{\nu}_e})] + (4)(1.5)(1 - \\
& (4/3)s_W^2)(\sec^2 \theta_W)[C_{36}(q_2, q_1, m_{\tilde{u}}, m_{\tilde{d}}, m_{\tilde{u}})]
\end{aligned} \tag{J.50}$$

$$\begin{aligned}
V_{6\hat{F}}^Z \doteq & -(4)(.5)(1 - 2s_W^2)(\sec^2 \theta_W)[C_{36}(q_1, q_2, m_{\tilde{e}}, m_{\tilde{\nu}_e}, m_{\tilde{e}})] - \\
& (4)(1.5)(1 - (2/3)s_W^2)(\sec^2 \theta_W)[C_{36}(q_1, q_2, m_{\tilde{d}}, m_{\tilde{u}}, m_{\tilde{d}})] + \\
& (4)(.5)(\sec^2 \theta_W)[C_{35}(q_2, q_1, m_{\tilde{\nu}_e}, m_{\tilde{e}}, m_{\tilde{\nu}_e})] + (4)(1.5)(1 - \\
& (4/3)s_W^2)(\sec^2 \theta_W)[C_{35}(q_2, q_1, m_{\tilde{u}}, m_{\tilde{d}}, m_{\tilde{u}})]
\end{aligned} \tag{J.51}$$

$$\begin{aligned}
V_{8\hat{F}}^Z \doteq & -(4)(.5)(1 - 2s_W^2)(\sec^2 \theta_W)[C_{33}(q_1, q_2, m_{\tilde{e}}, m_{\tilde{\nu}_e}, m_{\tilde{e}}) + \\
& C_{23}] - \\
& (4)(1.5)(1 - (2/3)s_W^2)(\sec^2 \theta_W)[C_{33}(q_1, q_2, m_{\tilde{d}}, m_{\tilde{u}}, m_{\tilde{d}}) + \\
& C_{23}] + (4)(.5)(\sec^2 \theta_W)[C_{34}(q_2, q_1, m_{\tilde{\nu}_e}, m_{\tilde{e}}, m_{\tilde{\nu}_e}) + C_{22}] + \\
& (4)(1.5)(1 - (4/3)s_W^2)(\sec^2 \theta_W)[C_{34}(q_2, q_1, m_{\tilde{u}}, m_{\tilde{d}}, m_{\tilde{u}}) + \\
& C_{22}]
\end{aligned} \tag{J.52}$$

$$\begin{aligned}
V_{12\hat{F}}^Z \doteq & -(4)(.5)(1 - 2s_W^2)(\sec^2 \theta_W)[C_{34}(q_1, q_2, m_{\tilde{e}}, m_{\tilde{\nu}_e}, m_{\tilde{e}}) + \\
& C_{22}] - \\
& (4)(1.5)(1 - (2/3)s_W^2)(\sec^2 \theta_W)[C_{34}(q_1, q_2, m_{\tilde{d}}, m_{\tilde{u}}, m_{\tilde{d}}) + \\
& C_{22}] + (4)(.5)(\sec^2 \theta_W)[C_{33}(q_2, q_1, m_{\tilde{\nu}_e}, m_{\tilde{e}}, m_{\tilde{\nu}_e}) + C_{23}] + \\
& (4)(1.5)(1 - (4/3)s_W^2)(\sec^2 \theta_W)[C_{33}(q_2, q_1, m_{\tilde{u}}, m_{\tilde{d}}, m_{\tilde{u}}) + \\
& C_{23}]
\end{aligned} \tag{J.53}$$

Appendix K

D Integrals

This appendix deals with definitions and explicit expressions for D integrals as was done for A, B and C integrals. Let us first derive explicit expressions for these integrals using Feynman parametric approach. Starting from the Feynman's identity (relevant to four factors in the denominator)

$$\frac{1}{abcd} = (n-1)! \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{1}{[a(1-x-y-z) + bx + cy + dz]^n} \quad (\text{K.1})$$

and definitions [45] for D integrals

$$D_0; D_\mu; D_{\mu\nu}; D_{\mu\nu\alpha} = \frac{1}{i\pi^2} \int d^n q \frac{1; q_\mu; q_\mu q_\nu; q_\mu q_\nu q_\alpha}{(q^2 - m_1^2)((q+k)^2 - m_2^2)((q+k+p)^2 - m_3^2)((q+k+p+l)^2 - m_4^2)} \quad (\text{K.2})$$

D_0 in Eq. K.1 may be reduced to

$$D_0 = \frac{\Gamma(4)}{i\pi^2} \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{1}{[q^2 - M^2]^4} \quad (\text{K.3})$$

where

$$M^2 = (1-x-y-z)m_1^2 + xm_2^2 + ym_3^2 + zm_4^2 - xk^2 - yp_5^2 - z(1-z)(k^2 + p_6^2 + 2k.p_6) + x^2k^2 + y^2p_5^2 + 2xy(k.p_5) + 2xzk.(k + p_6) + 2yzp_5.(k + p_6) \quad (\text{K.4})$$

Here

$$\begin{aligned} p_5 &= k + p, \\ p_6 &= p + l \end{aligned} \quad (\text{K.5})$$

Performing dimensional regularization Eq. K.3 reduces to

$$D_0 = \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{1}{(M^2 - i\delta)^2} \quad (\text{K.6})$$

Now consider the evaluation of D_{11}, D_{12} and D_{13} to this end consider the definition

$$k_\mu D_{11} + p_\mu D_{12} + l_\mu D_{13} = \frac{1}{i\pi^2} \int d^n q \frac{q_\mu}{(q^2 - m_1^2)((q+k)^2 - m_2^2)((q+p_5)^2 - m_3^2)((q+k+p_6)^2 - m_4^2)} \quad (\text{K.7})$$

Following steps similar to the ones used in obtaining D_0 , Eq. K.6 may be reduced to

$$k_\mu D_{11} + p_\mu D_{12} + l_\mu D_{13} = - \int dx \int dy \int dz \frac{(xk + yp_5 + z(k + p_6))_\mu}{(M^2 - i\delta)^2} \quad (\text{K.8})$$

on equating the coefficients of the momenta's we obtain

$$\begin{aligned} D_{11} &= - \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{x + y + z}{(M^2 - i\delta)^2}, \\ D_{12} &= - \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{y + z}{(M^2 - i\delta)^2}, \\ D_{13} &= - \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{z}{(M^2 - i\delta)^2} \end{aligned} \quad (\text{K.9})$$

The above procedure is easily extended to calculate D_{21} through D_{313} , we merely give the final forms. For the case of $D_{\mu\nu}$ these read

$$\begin{aligned} D_{21} &= \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{(x + y + z)^2}{(M^2 - i\delta)^2}, \\ D_{22} &= \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{(y + z)^2}{(M^2 - i\delta)^2}, \\ D_{23} &= \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{z^2}{(M^2 - i\delta)^2}, \\ D_{24} &= \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{(x + y + z)(y + z)}{(M^2 - i\delta)^2}, \\ D_{25} &= \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{(x + y + z)z}{(M^2 - i\delta)^2}, \\ D_{26} &= \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{(y + z)z}{(M^2 - i\delta)^2}, \\ D_{27} &= (1/2) \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{1}{(M^2 - i\delta)} \end{aligned} \quad (\text{K.10})$$

and for $D_{\mu\nu\alpha}$

$$\begin{aligned}
D_{311} &= -(1/2) \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{x+y+z}{(M^2-i\delta)}, \\
D_{312} &= -(1/2) \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{y+z}{(M^2-i\delta)}, \\
D_{313} &= -(1/2) \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{z}{(M^2-i\delta)}, \\
D_{31} &= - \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{(x+y+z)^3}{(M^2-i\delta)^2}, \\
D_{32} &= - \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{(y+z)^3}{(M^2-i\delta)^2}, \\
D_{33} &= - \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{z^3}{(M^2-i\delta)^2}, \\
D_{34} &= - \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{(x+y+z)^2(y+z)}{(M^2-i\delta)^2}, \\
D_{35} &= - \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{z(x+y+z)^2}{(M^2-i\delta)^2}, \\
D_{36} &= - \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{(y+z)^2(x+y+z)}{(M^2-i\delta)^2}, \\
D_{37} &= - \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{z^2(x+y+z)}{(M^2-i\delta)^2}, \\
D_{38} &= - \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{z(y+z)^2}{(M^2-i\delta)^2}, \\
D_{39} &= - \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{z^2(y+z)}{(M^2-i\delta)^2}, \\
D_{310} &= - \int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} dz \frac{z(y+z)(x+y+z)}{(M^2-i\delta)^2} \quad (K.11)
\end{aligned}$$

An alternative to evaluate the D integrals is to express them in terms of the C integrals and D_0 . The results can be found in reference [45], we have rechecked these and they are found to be correct except for some minor misprints. Note however that in [45] the convention for metric is not the same as ours. One can illustrate this procedure by considering the case of D_μ

$$k_\mu D_{11} + p_\mu D_{12} + l_\mu D_{13} = \frac{1}{i\pi^3} \int d^m q \frac{q_\mu}{(q^2-m_1^2)((q+k)^2-m_2^2)((q+p)^2-m_3^2)((q+k+p)^2-m_4^2)} \quad (K.12)$$

Let us abbreviate the above by denoting $q^2 - m_1^2$ by 1 $(q + k)^2 - m_2^2$ by 2 etc

$$k_\mu D_{11} + p_\mu D_{12} + l_\mu D_{13} = \frac{1}{i\pi^2} \int d^n q \frac{q_\mu}{(1)(2)(3)(4)} \quad (\text{K.13})$$

by contracting Eq. 12 with k^μ , p^μ , and l^μ , one obtains

$$\begin{aligned} k^2 D_{11} + k \cdot p D_{12} + k \cdot l D_{13} &= \frac{1}{i\pi^2} \int d^n q \frac{k \cdot q}{(1)(2)(3)(4)}, \\ k \cdot p D_{11} + p^2 D_{12} + p \cdot l D_{13} &= \frac{1}{i\pi^2} \int d^n q \frac{p \cdot q}{(1)(2)(3)(4)}, \\ k \cdot l D_{11} + l \cdot p D_{12} + l^2 D_{13} &= \frac{1}{i\pi^2} \int d^n q \frac{l \cdot q}{(1)(2)(3)(4)} \end{aligned} \quad (\text{K.14})$$

Now using identities

$$\begin{aligned} q \cdot k &= (1/2)[(q + k)^2 - m_2^2 - (q^2 - m_1^2) - m_1^2 + m_2^2 - k^2], \\ q \cdot k &= (1/2)[(2) - (1) + f_1], \\ q \cdot p &= (1/2)[(q + p)^2 - m_3^2 - ((q + k)^2 - m_2^2) - m_2^2 + m_3^2 + k^2 - p^2], \\ q \cdot p &= (1/2)[(3) - (2) + f_2], \\ q \cdot l &= (1/2)[(q + k + p)^2 - m_4^2 - ((q + p)^2 - m_3^2) - m_3^2 + m_4^2 - p^2 + p^2], \\ q \cdot l &= (1/2)[(4) - (3) + f_3] \end{aligned} \quad (\text{K.15})$$

(note that $p_4 = k + p + l$), Eq. K.13 be may be rewritten as

$$\begin{aligned} k^2 D_{11} + k \cdot p D_{12} + k \cdot l D_{13} &= (1/2)(f_1 D_0 - C_0(1, 3, 4) + C_0(2, 3, 4)), \\ &= R_{20}, \\ k \cdot p D_{11} + p^2 D_{12} + p \cdot l D_{13} &= (1/2)(f_2 D_0 - C_0(1, 2, 4) + C_0(1, 3, 4)), \\ &= R_{21}, \\ k \cdot l D_{11} + l \cdot p D_{12} + l^2 D_{13} &= (1/2)(f_3 D_0 - C_0(1, 2, 3) + C_0(1, 2, 4)), \\ &= R_{22} \end{aligned} \quad (\text{K.16})$$

Now clearly K.16 represents a set of algebraic Eqs. which are solved by matrix methods i.e. defining

$$X = \begin{pmatrix} k^2 & k.p & k.l \\ k.p & p^2 & p.l \\ k.l & p.l & l^2 \end{pmatrix} \quad (\text{K.17})$$

we may write K.16 as

$$\begin{pmatrix} D_{11} \\ D_{12} \\ D_{13} \end{pmatrix} = X^{-1} \begin{pmatrix} R_{20} \\ R_{21} \\ R_{22} \end{pmatrix} \quad (\text{K.18})$$

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