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ANALYTICAL AND EXPERIMENTAL MULTI-ROW,
FINNED-TUBE HEAT EXCHANGER PERFORMANCE
DURING COOLING AND DEHUMIDIFICATION PROCESSES

by

ABDELHAKIM H. ELMAHDY

A thesis submitted to the Faculty of
Graduate Studies and Research in
partial fulfillment of the requirements
for the degree of Doctor of Philosophy.

Department of Mechanical and
Aeronautical Engineering
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Carleton University

Ottawa, Canada

December 1975

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The undersigned hereby recommend to the Faculty of Graduate Studies and Research, Carleton University, Ottawa, acceptance of the thesis "Analytical and Experimental Multi-Row, Finned-Tube Heat Exchanger Performance during Cooling and Dehumidification Processes", submitted by Mr. Abdelhakim H. Elmahdy, B.Sc., M.Sc., in partial fulfillment of the requirements of the degree of Doctor of Philosophy in Mechanical Engineering.

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ANALYTICAL AND EXPERIMENTAL MULTI-ROW,
FINNED-TUBE HEAT EXCHANGER PERFORMANCE
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ABSTRACT

Experimental and theoretical analysis of a multi-row finned-tube heat exchanger has been made for the purpose of simulating the coil performance. The analysis is done in cases of both dry and wet surface conditions.

Regression analysis is used to determine the most influential coil dimensions on the overall heat exchanger performance. This resulted in an expression of the average heat transfer J-factor ($J = St \cdot Pr^{2/3}$) in terms of the air flow Reynolds number Re , and some of the coil dimensions.

A mathematical model to describe the multi-row heat exchanger performance is developed for both dry and wet surface conditions. The results obtained from the regression analysis are used in conjunction with the mathematical model to simulate row-to-row cooling coil performance.

A complete design and construction of a test facility was made to conduct tests on multi-row heat exchangers.

An 8-row 2 ft. x 2 ft. face area, continuous fins staggered tube heat exchanger was tested, where the air flow Reynolds Number was varied between 400 and 1500 approximately. The analytical and previously published data are compared with the present experimental results.

Dry and wet surface test results show good agreement between the experimental and analytical coil performance. It is concluded that the theoretical model of cooling coil performance presented can be used as an algorithm for coil performance simulation. The latter is necessary for heating, ventilation and air-conditioning system analysis and energy requirements.

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CHAPTER I

INTRODUCTION

Energy and energy crisis are today the most popular issues in the entire world. The impending energy shortage is beginning to seriously threaten the world including the Western World and North America, due to the high rate of energy consumption. For example: W. Fleming (1)* pointed out that the United States, where the population is only 6 percent of the whole world, consumes about 33 percent of the world's energy. This is more than five times as much, on a per capita basis, as the world's average.

The rapidly increasing rate of energy consumption on one side, and the limited and unrenewable natural energy resources on the other side, makes world-wide energy crisis inevitable. This drama was put to Canadians by Canada's Prime Minister in his New Year's message of 1974 (2): "Recent events did not create the current situation; they merely hastened its coming. What we are beginning to experience now was inevitable because man has been consuming the resources of this planet with a voracious appetite that no wisdom or insight - be it religious, philosophical, scientific or political - has so far been able to curb." And he went on to say: "All the resources we now take from the earth come to an enormous price. This is in effect a distress signal, a warning that thoughtless exploitation can in the end lead only to tragedy."

* Numbers in brackets indicate references listed beginning on page (144).

In Canada, the expected energy demand in 1990 is about three times as much as the actual energy consumption during the year 1966. See Table (1) (taken from Reference 3) and Fig.(1). This prediction indicates that at such a high rate of energy demand, an energy crisis is unavoidable, unless considerable effort is made to decelerate this rate.

In 1971, space heating and air-conditioning consumed about 25 percent of the total energy consumed in Canada (2). This percentage was about 30 percent in 1973, as estimated by D. Brousson (4). This considerable portion of the total energy consumption directed the attention of many research workers in this field toward detailed studies of the heating, ventilation and air-conditioning (HVAC) systems for the purpose of saving energy. Most of the current studies on energy utilization in space heating and air-conditioning take advantage of the available powerful electronic computers to simulate the buildings and the associated HVAC systems.

Several computer programs are now available for energy analysis and requirements of residential and commercial buildings. In these programs, the performances of the different components are simulated. Eventually, the accuracy of such analyses is directly related to how well the system components are described, as well as the accuracy of the program.

This thesis presents a detailed study of cooling coils. Particular attention is given to those conditions where sensible and latent heat transfer occur simultaneously. Analytical and experimental verification are presented for better understanding of the coil performance at full-load and off-design conditions.

Table (1): Energy Demand by Sectors
(Trillions of Btu's) (3).

Sector	Actual	Estimated	
	1966	1975	1990
Residential and Commercial	1,392	1,795	2,879
Industrial	1,240	1,861	3,516
Transportation	982	1,463	2,706
Total	3,614	5,119	9,101

FIG. (1) CANADA'S ENERGY CONSUMPTION UNDER
ALTERNATIVE FUTURES (2).

The literature relevant to this study is reviewed and summarized and is followed by a theoretical analysis of coil performance with dry and wet surface. Tables and graphs present the analytical and experimental results of this work from which conclusions and recommendations are outlined.

Following the lists of nomenclature and references, are appendices describing in detail, some of the topics covered in the text.

CHAPTER II

LITERATURE SURVEY

This chapter sheds some light on buildings and their associated air-conditioning systems. Also included is a brief description of cooling coils and the role they play in the entire system. Finally, the available literature dealing with the performance of cooling and dehumidifying coils is reviewed.

1. Buildings and Air-Conditioning System Simulation:

In order to perform an energy analysis for buildings, detailed information is required on: (a) the imposed cooling and heating loads and, (b) the performance of each of the components in the HVAC system, such as: boilers, fans, cooling coils, chillers, etc.

Available computer programs include those developed by the U.S. Postal Services (5), Edison Electric Institute, American Gas Ass., Westinghouse Electric Corp., Ross F. Meriwether and Ass. (6) and many others. In general, such programs can be used in the pre-design selection of the basic system, during the design stage, during the construction of the system, or after completion of the building. The use of this type of analysis can result in considerable saving in owning and operating cost. This is true since the mechanical and electrical systems of a modern building represents about 50 percent of the

total owning and operating cost (7). This saving is achieved by using energy systems analysis to evaluate the performance of various design alternatives.

Since air-conditioning systems seldom operate at full load (8), the characteristic performance of the different components at part load is particularly important for accurate determination of energy consumption. The components, which will be given consideration in this study are the cooling and dehumidifying coils, since they seem to be the least well understood elements, and because of their importance in the energy analysis and requirements.

In the next section, the impact of the cooling and dehumidifying coils on the HVAC system will be discussed.

2. Cooling and Dehumidifying Coils: Simulation and Energy Analysis:

Cooling and dehumidifying coils are among the most influential components in a HVAC process involving sensible and latent heat removal, i.e. cooling, heat recovery, and humidity control.

The analysis given in the Post Office Program for cooling and dehumidifying coils simulation is not complete as indicated in Reference 5. The authors stated that in the fan system analysis no provision is made to simulate coil performance, and that the dehumidification effect of the coil is approximated as a linear relation with the outside air dry bulb temperature.

Another method of simulating the cooling coil performance is by using the manufacturer's catalogue data, in tabular or

equation form. As indicated by the manufacturers, these coils are generally certified by the Air-Conditioning and Refrigeration Institute (ARI) (9). To satisfy the ARI standards, the capacity determined by a standard test must not be less than 95% of the catalogue rating. The catalogue data are not necessarily experimental data, but include extrapolated or interpolated points. For example, the data published by the Trane Co. (10) are in the form of tables giving outlet air dry and wet bulb temperatures and cooling capacity as a function of coil geometry, air inlet dry and wet bulb temperatures, air face velocity, water inlet and outlet temperatures and water velocity. Together with these tables, some characteristic curves are plotted on a psychrometric chart (prepared by the same company) to describe the shape of the characteristics air-water curve. These curves (which correspond to the catalogue data) show that the cooling and dehumidifying coil characteristics depend only on the dry and wet bulb temperature of the moist air at inlet. However, the technique suggested by Threlkeld (11) to draw the air-water characteristic curve on the psychrometric chart shows the dependence of the shape of the curve on the film heat transfer coefficient. This is confirmed when an air-water characteristics curve is drawn on a psychrometric chart, where a substantial difference between the former curve and that given by the manufacturer, is noted (the effect on the energy estimate is discussed below).

Recently, Stoecker et al (12) studied the problem of accuracy and model complexity in system simulation for energy

calculation. Their study showed that whereas the routines which determine the energy required by chillers can be simplified, those associated with the thermal quantities in the air systems require a refined calculation. As an example, the authors considered the error in cooling load which results from the assumption that the outlet air temperature from a "wild" cooling coil (constant inlet air and water temperatures and water flow rate) in a dual duct system is independent of air flow rate. At an air flow rate of 50% of the full flow this assumption results in the cooling energy being underestimated by about 26%. This underestimate of cooling energy results in an identical underestimate of heating energy. Although the "wild coil" is not a realistic assumption, this example gives some idea of the influence of cooling coil performance representation on the predicted energy consumption.

The above examples show that improper representation of the cooling coil performance can lead to inaccurate, and sometimes misleading, conclusions in energy requirements and analysis. At this stage, it may be beneficial to present a general description of some of the coils generally used in the HVAC systems.

3. Cooling Coils: General Features and Description:

According to the ASHRAE Standard 33-64 (13), a forced-circulation air-cooling coil is one for use in an air stream whose circulation is caused by a difference in pressure produced by a fan or blower. Also, it is a heat exchanger,

with or without extended surfaces, through which either chilled water or volatile refrigerant is circulated, for the purpose of sensible cooling, or sensible cooling and dehumidification, of a forced-circulation air stream. Fig. (2) shows heating and cooling coils typical of those used in air-conditioning systems. Circular or flat tubes are arranged in banks in either in-line or staggered arrangements. These tubes are usually provided with a secondary surface in the form of circular or continuous fins. To explain the idea behind the provision of secondary surfaces, it is known that heat transfer between water flowing in tubes and air flowing across the tube banks depends on the overall thermal resistance between these two fluids, and the driving force between them, which is an appropriate temperature difference in this case. To increase the heat transferred between the two fluids, for a specified temperature difference, the heat transfer coefficients on both sides and the heat transfer area should be increased for a given coil-core volume. Because the heat transfer coefficient on the air side is usually poor, the area on this side is increased by providing a secondary surface in the form of circular or continuous fins, or spines. In other words, the fins are generally provided on the surface of greater resistance to heat flow (greater thermal resistance).

It is worthwhile to explain briefly the heat transfer mechanisms encountered in both dry surface coils (sensible heating or cooling) and wet surface coils (cooling and

TUBULAR SURFACES

FINNED TUBE SURFACES

FIG. (2) TYPICAL KINDS OF HEAT EXCHANGERS (19).

dehumidifying). In the former case, there is only the sensible heat transfer, where heat is transferred by convection between coil surface and the flowing air. In the latter case, simultaneous sensible and latent heat transfer takes place, i.e. both heat and mass transfer mechanisms act in the wet region of the coil. The sensible heat part of the total heat transfer is dependent on the temperature differential between the fluid and the wall, and the convective heat transfer coefficients whereas the latent heat transfer is dependent on the partial vapor pressure differential between the fluid and the wall, and the mass transfer coefficient (14).

Because of the complexity of these transfer mechanisms, and the unpredictable flow patterns over the different types of tube banks, it is virtually impossible to present an analytical solution for such flows (15). This is the main reason responsible for the lack of understanding of the cooling and dehumidifying coil characteristics. As Sabersky says (16): "Despite the wide field of utilization of heat exchangers, it may be surprising to some that there is a lack of understanding of many details of the flow and transfer mechanisms encountered. The details on which more information is needed include flow of single phase and multi-phase fluids over various arrangements of tubes. In fact, in some fairly common exchangers the lack of understanding may lead to an over-design of by as much as 100 percent, and if deterioration of the surface is involved, this figure could go considerably higher." This strongly emphasizes and confirms the pressing need for an accurate simulation of cooling and dehumidifying coil behaviour.

A summary of the available literature related to the cooling coil performance is outlined briefly.

4. Heat Exchanger Performance:

The literature survey is an outline of the available work which is considered to be related to the simulation of heat exchangers. Some of the relationships given in this section are as they appeared in the original publication; others have been reformulated to be in a similar format for comparison purposes.

This survey covers the following areas, which are believed to have considerable influence on the overall heat exchanger simulation:

- a) dry surface heat exchanger performance, where sensible heat transfer is the only mechanism involved,
- b) the combined effect of sensible and latent heat transfer mechanisms on the overall heat exchanger performance,
- c) fin efficiency calculation, when sensible, or combined sensible and latent heat transfer exists; and its effect on the overall heat exchanger efficiency.

Since the overall resistance on the air-side of the heat exchanger has a great influence on the overall performance, special consideration has been given in this review, to the air-side parameters.

Information about the dry surface heat exchangers is relatively easy to obtain. The available information is usually in the form of graphs or equations representing the relationship between the average heat transfer parameters of

the coil, i.e. the average J-factor, and the average Reynolds number of the flow over the tube banks. The J-factor is defined as:

$$J = St \cdot Pr^{2/3} \quad (1)$$

where the Stanton number St , is evaluated at the air-flow Reynolds number Re . The latter is based on the maximum air mass flux, as will be discussed later.

Colburn (17) carried out experiments on bare staggered tube banks and recommended the following relation for $2000 < Re < 32,000$:

$$J = .33 \cdot Re^{-.4} \quad (2)$$

In the case of in-line tube banks, the author suggested that the constant in equation (2) be reduced from 0.33 to 0.26. Similar results were obtained by Grimison (18) for bare tube banks used in steam boilers.

Extensive work has been done by Kays and London (19) on the performance of finned tube banks of different types. Their results are reported in the form of tables and graphs of J-factor and friction factor f , versus Reynolds number based on the air-side hydraulic diameter, D_h ,

where

$$D_h = 4 \cdot \frac{A_{m,f}}{A_o} \cdot L$$

London et al (20), Kays (21) and Kays and London (22) did not report their results in the form of a mathematical expression. Hence the quantitative effects of fin and tube spacing, and other characteristic dimensions are not easily obtained. The authors do, however, indicate that their results may be dependent on these factors by pointing out that the results are only applicable to cases in which complete geometric similarity exists.

Some effort has been made to derive empirical relations between the characteristic heat transfer J-factor and the coil physical dimensions. As an example, Gram et al (23) considered the coil configuration shown in Fig.(3), and with the aid of dimensional analysis defined the following functional relationships for the Stanton number and the pressure drop factor, f:

$$St = \phi_1 [Re, Pr, S_t/S_d, S_l/S_d, C_w/S_d, C_l/S_d, N_r] \quad (3)$$

and

$$f = \phi_2 [Re, S_t/S_d, S_l/S_d, C_w/S_d, C_l/S_d, N_r] \quad (4)$$

Jameson (24) correlated his data by using a modified hydraulic diameter which he defined as:

$$D_{h,m} = D_h / \left[\left(\frac{F_h}{2 \cdot S_{t,r}} \right)^{.4} * \left(\frac{1}{2 \cdot \sqrt{S-1}} + \frac{1}{2 \cdot \sqrt{r_1-1}} \right) \right]^{0.4} \quad (5)$$

where

$$S_{t,r} \text{ is defined as } \frac{S_t/S_d - 1}{S_t/S_d}$$

See Fig.(4) for the definition of the different parameters in equation (5).

FIG.(3) DIFFERENT DIMENSIONS OF FINNED TUBE HEAT EXCHANGER(23).

This attempt was, to some extent, successful in correlating the friction coefficient, but not the heat transfer J-factor.

Briggs and Young (25) extended the work carried out by Ward and Young (26) to correlate the published heat transfer data. The authors presented the following formula for the average heat transfer J-factor, in terms of the air-flow Reynolds number and some physical dimensions of the coil (see Fig.(5)):

$$J = 0.134 \cdot Re^{-.39} \cdot \left(\frac{F_s}{F_h} \right)^{0.2} \cdot \left(\frac{F_s}{F_t} \right)^{0.1134} \quad (6)$$

Little information is available on the effect of row-to-row variation in heat transfer coefficient in heat exchangers. McAdams (27), summarized the data of Pierson and Kays and Lo, presenting a table showing the row-to-row variation in heat transfer coefficient for staggered banks of bare tubes. These data show that the heat transfer coefficient increases from row-to-row, being a minimum for the first row and asymptotically approaching a maximum value. This seems to contradict the test results reported by Jones and Monroe (28). They concluded that the average heat transfer coefficient, in case of bare-tube deep heat exchangers, decreases along the air-flow path, as shown in Fig.(6), and approaches an almost constant average value. Unfortunately, details on the test procedures and data reduction techniques are not available for comparison.

t_{de} is the equivalent diameter of finned tube

FIG.(4) TUBE DIMENSIONS DEFINED BY JAMESON(24).

FIG.(5) FINNED TUBE HEAT TRANSFER DATA WITH THE CORRELATION OBTAINED BY REGRESSION ANALYSIS (25).

FIG.(6) VARIATION OF INDIVIDUAL ROW HEAT TRANSFER COEFFICIENT(28).

Recently, Rich (29) investigated the effect of the number of tube rows on the heat transfer performance of flat-plate finned tube banks. The author concluded that the average heat transfer coefficient for a deep row coil can be higher or lower than that for a shallow coil, depending upon the Reynolds number. Likewise, heat transfer coefficients for the downstream rows of a multi-row coil may be higher or lower than for the upstream rows. The author suggested that the effect of stable vortex patterns on the local air temperature distribution within the flow passage is responsible for this Reynolds number effect. This is, to some extent, in agreement with test results of 1, 2, 4 and 6 row, deep helical fin tube banks tested by Tuve (30), who found that curves of overall average heat transfer coefficient, when plotted versus flowing air face velocity, had different slopes for the different depths of coils.*

Lapin and Schuring (31) used the number of rows, N_r , in correlating the average heat transfer J-factor of multi-row circular finned tube banks. The expression for the J-factor is given by:

$$J = \frac{0.13 \cdot \text{Pr}^{2/3}}{G \cdot C_p} \cdot (0.63 - 0.01 \cdot N_r) \cdot N_r^{-1} \cdot \text{Re}^{(0.39 + 0.07 \cdot N_r)} \quad (7)$$

As stated by the authors, equation (7) is only applicable to the type of finned tubes investigated and is valid only within the tested range of Reynolds number between 1600 and 11,000, based

* Rich (29), showed that at higher Re , the heat transfer coefficient for the downstream rows is higher than for the upstream rows. Similarly the average coefficient for deep coils is higher than for shallow coils.

on an equivalent diameter D_e , defined as:

$$D_e = \frac{2 * \text{outside heat transfer area, ft}^2}{\pi * \text{projected perimeter, ft}^2/\text{linear ft}}$$

However, the authors suggested that similar results would be obtained with fins of other types and sizes, so long as the ratio of fin-to-tube diameter is of the order of 2 with 8 to 10 fins per inch. In a study conducted by Rich (32), the author investigated the effect of the number of fins per inch on the average heat transfer J-factor. For the tested range of fin spacing (3 to 21 fins per inch), the author concluded that average heat transfer J-factor is essentially independent of the fin spacing.

LaHaye et al (33) tried to correlate the dry surface heat transfer and friction data reported by Kays and London (19). A modified "dimensionless transfer performance factor" - JJ was plotted versus "dimensionless pumping factor" - PF for several heat exchangers, where:

$$JJ = J * Re \quad (8)$$

and

$$PF = f * (Re)^3 \quad (*) \quad (9)$$

The above correlation is not useful in predicting the performance of the dry surface heat exchanger since the friction factor f has to be known first.

(*) The complete definition of the friction factor f can be found in Reference (19).

In summary, it is obvious from the available literature that the performance of a dry-surface heat exchanger is dependent on its physical dimensions, particularly whether or not a secondary surface is provided, and is strongly dependent on Re .

Literature on the combined effect of sensible and latent heat transfer mechanisms on the overall heat exchanger performance is reviewed in the following paragraphs. Unfortunately, a very limited amount of information is available on the performance of finned tube banks where both sensible and latent heat transfer mechanisms occur. Ashley (34) presented an analytical method to describe the performance of cooling coils using chilled water or brine as coolant, without considering any latent heat removal. It is not clear whether the coil surface was dry during the test period or the author neglected the dehumidification effect on the overall coil performance.

Kusuda (35) extended the effectiveness concept, used in determining the dry surface performance, and applied it in evaluating the coil performance during the dehumidification processes. He suggested the modified effectiveness method as a replacement for the log-mean relation reported in the ASRE-ASHAE Joint Standard (36).

Experimental results reported by Bryan (37, 38) showed that the average heat transfer J -factor of the tested bare tube bank is increased when condensation occurs on the primary surface. On the other hand, he found that for a finned tube heat exchanger

the presence of latent heat removal decreased the average J-factor. This result is in contradiction with that reported by Myers (39), which showed that the dehumidification process increased the average J-factor.

Besides the experimental studies, analytical models to study the performance of heat exchangers are found in the literature, e.g. Tamm (40). Tamm's model was modified by McCullagh et al (41), but the authors did not consider the effect of the moist air conditions on the fin efficiency and hence on overall heat exchanger efficiency. Guillory et al (42) used the enthalpy potential method described by Threlkeld (11) to calculate the performance of parallel plate heat exchanger. The authors concluded that in the range of Reynolds number tested ($800 < Re < 6000$) the use of dry heat transfer data in the design of wet heat exchanger can result in an over-estimate of fin area and an underestimate of pressure drop of the order of 30%.

In summary, information on wet-surface heat exchanger performance is limited, and a better understanding of such components requires further analytical and experimental work.

The rest of this chapter contains a review of the research work done on dry-and wet-surface fin efficiency, which strongly affects the overall heat exchanger efficiency and performance.

By definition, fin efficiency is the ratio of the total heat transferred to or from the fin surface, to the total heat that would be transferred if the entire fin surface were at a

uniform temperature equal to that of the fin base. Extensive work has been done by Gardner (43), Stewart (44) and others to find closed-form solutions of the fin efficiency for different fin geometries, when sensible heat transfer occurs. When the fin efficiency $\eta_{f,d}$ as defined above is combined with the assumption of a constant heat transfer coefficient over the fin surface and tube surface, an expression for the overall external surface efficiency η_{ov} is obtained (45):

$$\eta_{ov} = 1 - \frac{A_s}{A_o} * (1 - \eta_{f,d}) \quad (10)$$

where A_s is the secondary heat transfer surface area (fin area), and A_o is the total heat transfer area on the fin side (primary surface plus secondary surface areas).

The assumption of an equal film heat transfer coefficient on fin surface and tube surface is a common postulation for the type of heat exchangers under consideration. However, Kern and Kraus (46) indicated that such an assumption is unrealistic in certain cases. The variation of the film heat transfer coefficient over the fin surface is not well known. Different assumptions have been made to find the temperature distribution over the fin surface. An exponential variation of film heat transfer coefficient over a straight fin was assumed by Chen and Zyskowski (47). Their results appeared to be more realistic than the power relationship assumed by Han and Lefkowitz (48) for the same fin geometry, in which the value of zero is assigned for the film heat transfer coefficient at the fin root. The only experimental work on the

variation of film heat transfer coefficient over the fin surface, that is known to the author, is that of Abdelmessih and Alkhirdaji (49). They reported the radial and circumferential variation of film heat transfer coefficient on both circular and square fin surfaces. As expected, there were marked differences in the values of the local coefficient of heat transfer between different points for both types of fins. These differences appeared to the authors to be due primarily to the flow pattern and the air temperature. Despite the local variation of the film coefficient of heat transfer over the fin surface, it is agreed that the assumption of an average value is adequate for the purpose of describing heat exchanger performance without an appreciable sacrifice in the accuracy of calculation. This is confirmed by experiment.

The general definition of fin efficiency is the same whether the fin is wet or dry. However, the value of fin efficiency is not the same for both cases. This is evident by comparing the different values of fin efficiency for the wet and dry cases of a straight fin, as reported by O'Brien and Turner (50), (see Fig.(7)).

The Air-Conditioning and Refrigeration Institute (ARI) (9) has adopted the work done by Ware and Hacha (51) for design and rating of cooling and heating coils, despite the inaccurate method used to evaluate the fin efficiency in the wet-surface case. Recently, McQuiston (52) presented a simplified solution of the fin efficiency during dehumidifying processes for a

* Details can be found in References 9 and 51.

FIG.(7) FIN EFFICIENCY WITH AND WITHOUT MOISTURE CONDENSATION FOR A STRAIGHT FIN GEOMETRY(50).

simple fin geometry (longitudinal fin with a rectangular cross-section).

The available dry surface heat transfer data show that the average heat transfer J-factor of a heat exchanger is dependent on the physical dimensions of the coil. A general correlation of dry surface heat transfer data is not available, for the purpose of cooling coil simulation.

Limited information is known about the characteristic performance of heat exchangers under wet surface conditions. This resulted in an incomplete simulation program of the coil performance under such conditions.

CHAPTER III

THEORETICAL ANALYSIS

In this chapter, a mathematical model to simulate the cooling and dehumidifying coil is developed. The solution of the suggested model is performed on a digital computer; and the procedures are outlined in Section 2 of this chapter. Section 3 summarizes the regression analysis method used to develop a relationship between the average heat transfer J-factor and the physical dimensions of the coil. The effect of water film condensation on the overall heat exchanger performance is explained in Section 4. Finally, the evaluation of fin efficiency under dry and wet surface operating conditions is given in Section 5.

1. Cooling and Dehumidifying Coil: Theoretical Analysis:

The analysis presented in this section is used to predict the psychrometric conditions of the moist air leaving the cooling coil, and the refrigerant condition (chilled water in this case) when the inlet conditions for both fluids are known. In this way, the cooling capacity, the process line, and the sensible and latent portions of the total heat transferred will be evaluated. This analysis will be applied at the coil design point as well as at off-design conditions.

Due to the variability in design of heating and cooling coils, the analysis presented here is restricted to that for

circular tube banks with circular or continuous fins. The tube banks can be either staggered or in-line. These types of heat exchangers are believed to be those most commonly used in commercial HVAC systems.

For completeness, a dynamic model of cooling and dehumidifying coil is given, which can be used for future studies. However, the steady state model is extracted and is used to satisfy the requirements of the present work.

Fig.(8) shows a schematic diagram of a cooling and dehumidifying coil arrangement. Some of the basic assumptions are described in Fig.(9), which can be summarized as follows:

- 1 - Uniform conditions of moist air and chilled water at the entry to the cooling coil.
- 2 - All the heat is transferred from the air to both the primary and secondary surface areas, then through the tube wall, and finally to the chilled water.
- 3 - The condensed water film capacitance (on the outside surface area) is neglected, since the time constant introduced by the very thin water film is negligible (41).
- 4 - Because of the cross-counter flow arrangement considered, the air is assumed to be completely mixed between rows.
- 5 - The temperatures of air and chilled water are the mixed mean temperatures.
- 6 - Constant mass flux of both fluids.

Any further assumptions will be stated where needed in the text.

FIG.(8) COOLING COIL ARRANGEMENT.

FIG.(9) INCREMENT OF AN AIR-COIL AREA.

The energy balance on an incremental volume of dimension $dx * dy * \text{coil width}$, is (see Fig.(9)):

$$\dot{m}_a * di_a = m_a * \frac{dE}{dt} * dy + dq \quad (11)$$

where i_a is the moist air enthalpy, E is the stored energy, and q is the net heat transferred. Note that $\dot{m}_a$ is the mass rate of flow of air in lb_a/hr , whereas m_a is the mass of air per unit length in the direction of flow of air in lb_a/ft , and:

$$\frac{\dot{m}_a}{m_a} = V_a \quad \text{the air velocity ft/hr.}$$

The heat transferred, q , is the sum of the sensible and latent heat. The sensible heat part is given by the usual convective heat transfer equation. The latent heat transfer part is the product of the convective mass transfer coefficient h_m based on humidity ratio potential, and the humidity ratio difference times the area of mass transfer, as follows:

$$\begin{aligned}
 q = & \underbrace{\int_{A'_f} h * (t_a - t_f) * dA'_f}_{\text{sensible heat (fin)}} * dy + \underbrace{\int_{A'_f} h_m * \lambda * (W_a - W_{f,st}) * dA'_f}_{\text{latent heat (fin)}} * dy \\
 & + \underbrace{h * A'_p * (t_a - t_t) * dy}_{\text{sensible heat (tube)}} + \underbrace{h_m * A'_p * (W_a - W_{t,st}) * \lambda * dy}_{\text{latent heat (tube)}}
 \end{aligned} \quad (12)$$

In the above equation A'_f and A'_p are the fin and tube heat transfer area per unit length in the direction of flow of air, respectively (ft^2/ft). It is assumed also, that the convective film heat transfer and convective mass transfer coefficients on the tubes are equal to those on the fins, and are a function of the air side Reynolds number only (11).

The following fin parameters are introduced, which can be considered as measures of the efficiency of the fin in transferring sensible and latent heat separately:

$$\eta_s = \frac{\int_{A'_f} h^*(t_f - t_a) * dA'_f}{h^*(t_{t,o} - t_a) * A'_f} \quad (13)$$

and,

$$\eta_c = \frac{\int_{A'_f} h_m \lambda (W_{f,st} - W_a) dA'_f}{h_m \lambda (W_{t,o,st} - W_a) A'_f} \quad (14)$$

The total heat transfer area per unit length in the direction of flow of air is the sum of the fin and bare tube heat transfer areas per unit length, i.e.:

$$A'_o = A'_p + A'_f \quad (15)$$

From equations (13) and (14) we get:

$$\int_{A'_f} h^*(t_f - t_a) * dA'_f = \eta_s * h^*(t_{t,o} - t_a) * A'_f \quad (16)$$

and :

$$\int_{A'_f} h_m \lambda (W_{f,st} - W_a) dA'_f = \eta_c h_m \lambda (W_{t,o,st} - W_a) A'_f \quad (17)$$

Substituting equations (16), (17) and (15) into equation (12), then rearranging, yields:

$$q = (t_a - t_{t,o}) h_m A'_O \left[1 - \frac{A'_f}{A'_O} (1 - \eta_s) \right] dy + (W_a - W_{t,o,st}) h_m \lambda A'_O \left[1 - \frac{A'_f}{A'_O} (1 - \eta_c) \right] dy \quad (18)$$

From equations (10) and (18), we can write:

$$q = h A'_O \eta_{s,ov} (t_a - t_{t,o}) dy + h_m \lambda A'_O \eta_{c,ov} (W_a - W_{t,o,st}) dy \quad (19)$$

where (see equation (10)):

$$\eta_{s,ov} = 1 - \frac{A'_S}{A'_O} (1 - \eta_s) \quad (10-A)$$

and

$$\eta_{c,ov} = 1 - \frac{A'_S}{A'_O} (1 - \eta_c) \quad (10-B)$$

The enthalpy change $\dot{m}_a di_a$ can be written as:

$$-\dot{m}_a di_a = \dot{m}_a i_a - (\dot{m}_a i_a + \dot{m}_a \frac{\partial i_a}{\partial y} dy)$$

or:

$$\dot{m}_a di_a = \dot{m}_a \frac{\partial i_a}{\partial y} dy \quad (20)$$

But, the enthalpy of moist air is given by:

$$i_a = c_p * (t_a - t_{ref}) + W_a * i_g \quad (21)$$

where: t_{ref} is the reference temperature (0°F).

It is found that the enthalpy of saturated water vapor (i_g), taken from the steam tables, over the temperature range encountered in air conditioning applications, can be written as (11):

$$i_g = 1060.6 + .444 * t_{st} \quad (22)$$

t_{st} is the saturation temperature in degrees F.

Over the air temperature range covered in this analysis, the quantity i_g can be assumed to be constant. This may produce an error less than one percent in the final analysis (41).

The partial derivative of the enthalpy of moist air can be written as:

$$\frac{\partial i_a}{\partial t} = c_p * \frac{\partial t_a}{\partial t} + i_g * \frac{\partial W_a}{\partial t} \quad (23)$$

$$\frac{\partial i_a}{\partial y} = c_p * \frac{\partial t_a}{\partial y} + i_g * \frac{\partial W_a}{\partial y} \quad (24)$$

From a mass balance of moisture in the air:

$$\dot{m}_a * \frac{\partial W_a}{\partial t} + \dot{m}_a * \frac{\partial W_a}{\partial y} = - \int_{A'_f} h_m * (W_a - W_{f,st}) * dA'_f -$$

$$- h_m * (W_a - W_{t,o,st}) * A'_p$$

or

$$\begin{aligned}
\dot{m}_a * \frac{\partial W_a}{\partial t} + \dot{m}_a * \frac{\partial W_a}{\partial y} &= \\
&= -\eta_c * h_m * A'_f * (W_a - W_{t,o,st}) - h_m * A'_p * (W_a - W_{t,o,st}) \\
&= -h_m * (W_a - W_{t,o,st}) * [A'_p + \eta_c * A'_f] \quad (25)
\end{aligned}$$

Using equation (10B), then equation (25) becomes:

$$\dot{m}_a * \frac{\partial W_a}{\partial t} + \dot{m}_a * \frac{\partial W_a}{\partial y} = -h_m * \eta_{c,ov} * A'_o * (W_a - W_{t,o,st}) \quad (26)$$

The change in the stored energy (E) is given by:

$$\dot{m}_a * \frac{\partial E}{\partial t} * dy = \dot{m}_a * \frac{\partial u_a}{\partial t} * dy \quad (27)$$

Substituting equations (19), (20) and (27) into (11) we can write:

$$\begin{aligned}
-\dot{m}_a * \frac{\partial i_a}{\partial y} * dy &= \dot{m}_a * \frac{\partial u_a}{\partial t} * dy + h * A'_o * \eta_{s,ov} * (t_a - t_{t,o}) * dy \\
&\quad + h_m * \lambda * A'_o * \eta_{c,ov} * (W_a - W_{t,o,st}) * dy \quad (28)
\end{aligned}$$

Substituting for $\frac{\partial i_a}{\partial y}$ and $\frac{\partial u_a}{\partial t}$ from equations (23) and (24)

into equation (28), and rearrange:

$$\begin{aligned}
-h * A'_o * \eta_{s,ov} * (t_a - t_{t,o}) - h_m * \lambda * A'_o * \eta_{c,ov} * (W_a - W_{t,o,st}) &= \\
= \dot{m}_a * (c_p * \frac{\partial t_a}{\partial y} + i_g * \frac{\partial W_a}{\partial y}) + \dot{m}_a * (c_v * \frac{\partial t_a}{\partial t} + u_g * \frac{\partial W_a}{\partial t}) \quad (29)
\end{aligned}$$

The difference between λ and i_g (when evaluated at the same temperature) is the enthalpy of saturated water. When multiplying the latter by the quantity $(W_a - W_{t,o,st})$ in equation (29), the product is negligible (11). So, the assumption that: $i_g = \lambda$ will not result in serious error.

Rewriting equation (29) as follows:

$$\begin{aligned} & -h^*A'_O \eta_{s,ov} (t_a - t_{t,o}) - h^*i_g A'_O \eta_{c,ov} (W_a - W_{t,o,st}) = \\ & = (\dot{m}_a c_p \frac{\partial t_a}{\partial y} + m_a c_v \frac{\partial t_a}{\partial t}) + (\dot{m}_a i_g \frac{\partial W_a}{\partial y} + m_a u_g \frac{\partial W_a}{\partial t}) \end{aligned} \quad (30)$$

Rearrangement of equations (26) and (30) will result in the following two equations:

$$m_a \frac{\partial W_a}{\partial t} + \dot{m}_a \frac{\partial W_a}{\partial y} = -h^*A'_O \eta_{c,ov} (W_a - W_{t,o,st}) \quad (26)$$

and:

$$\begin{aligned} m_a c_v \frac{\partial t_a}{\partial t} + \dot{m}_a c_p \frac{\partial t_a}{\partial y} = & -h^*A'_O \eta_{s,ov} (t_a - t_{t,o}) \\ & + m_a (i_g - u_g) \frac{\partial W_a}{\partial t} \end{aligned} \quad (31)$$

Define the following parameters:

$$M_a = m_a c_p \quad (32)$$

and

$$R_a = \frac{1}{h^*A'_O} \quad (33)$$

Note that $(i_g - u_g) \frac{\partial W_a}{\partial t} = \bar{R} t_a \frac{\partial W_a}{\partial t}$

where $\bar{R}$ is the gas constant for moist air in BTU/lb.°F.

Lewis number can be written as:

$$Le = \frac{h}{h_m * c_p}$$

Also, noting that $\frac{\dot{m}_a}{\dot{m}_a} = v_a$, and $\frac{c_{p_a}}{c_{v_a}} = \gamma_a$, then equation (31)

becomes:

$$\frac{\partial t_a}{\partial t} + \gamma_a * v_a * \frac{\partial t_a}{\partial y} = - \frac{\gamma_a * \eta_{s,ov}}{M_a * R_a} * (t_a - t_{t,o}) + (\gamma_a - 1) * t_a * \frac{\partial W_a}{\partial t} \quad (35)$$

and equation (26) becomes:

$$\frac{\partial W_a}{\partial t} + v_a * \frac{\partial W_a}{\partial y} = - \frac{\eta_{c,ov}}{M_a * Le * R_a} * (W_a - W_{t,o,st}) \quad (36)$$

Part of the total heat transferred through the water film on the fin and bare tube surfaces is stored in the fin and tube materials, and the rest is transferred to the chilled water flowing inside the tubes. This can be expressed mathematically as:

$$\begin{aligned} m_f * c_f * \frac{1}{A_f} * \frac{\partial}{\partial t} \left(\int_{A_f} t_f * dA_f \right) + m_t * c_t * \frac{\partial t_{t,o}}{\partial t} = \\ = h * A'_o * \eta_{s,ov} * (t_a - t_{t,o}) + \eta_{c,ov} * h_m * \lambda * A'_o * (W_a - W_{t,o,st}) - h_i * A'_i * (t_{t,o} - t_w) \end{aligned} \quad (37)$$

where the temperature drop across the thin tube wall is neglected and m_f , m_t , c_f and c_t are masses per unit length and specific heats of fins and tubes respectively, h_i and t_w are the convective

film heat transfer coefficient and temperature on the water side,* respectively. $A'_{t,i}$ is the heat transfer area on the water side per unit length in the direction of flow of air.

Equation (13) can be rearranged into the form:

$$\frac{\partial}{\partial t} \left(\int_{A'_f} t_f * dA'_f \right) = \left\{ \frac{\partial}{\partial t} (t_a - \eta_s * (t_a - t_{t,o})) \right\} * A'_f \quad (38)$$

Substituting equation (38) into equation (37); we get:

$$\begin{aligned} m_f * c_f * \frac{\partial}{\partial t} [t_a - \eta_s * (t_a - t_{t,o})] + m_t * c_t * \frac{\partial t_{t,o}}{\partial t} = \\ = h * A'_o * \eta_{s,ov} * (t_a - t_{t,o}) + h_m * \lambda * A'_o * \eta_{c,ov} * (W_a - W_{t,o,st}) \\ - h_i * A'_{t,i} * (t_{t,o} - t_w) \end{aligned} \quad (39)$$

or, after rearranging:

$$\begin{aligned} \frac{\partial t_a}{\partial t} + \frac{\eta_s + \frac{m_t * c_t}{m_f * c_f}}{1 - \eta_s} * \frac{\partial t_{t,o}}{\partial t} = \frac{\eta_{s,ov} * (t_a - t_{t,o})}{R_a * (1 - \eta_s) * m_f * c_f} \\ + \frac{\eta_{c,ov} * \lambda}{Le * R_a * c_p} * \frac{W_a - W_{t,o,st}}{1 - \eta_s} - \frac{t_{t,o} - t_w}{R_w * c_f * m_f * (1 - \eta_s)} \end{aligned} \quad (40)$$

$$\text{where } R_w = 1 / (h_i * A'_{t,i}) \quad (41)$$

* h_1 is calculated using the fully developed flow relationship by McAdams (27). Also, see footnote on page 89.

Now, making heat balance on the chilled water only:

$$\begin{aligned} \dot{m}_w * i_w - (\dot{m}_w * i_w + \frac{\partial(\dot{m}_w * i_w)}{\partial x} * dx) + \\ + h_i * A'_{t,i} * (t_{t,o} - t_w) * dx = \frac{\partial(\dot{m}_w * u_w)}{\partial t} * dx \end{aligned} \quad (42)$$

where: i_w and u_w are the water enthalpy, and internal energy respectively, dx is the length of the control volume in the direction of flow of air. m_w and $\dot{m}_w$ are the mass of water enclosed in the control volume (lb_w/ft), and the mass rate of flow of water (lb_w/hr) respectively. Notice that $\frac{\dot{m}_w}{m_w} = V_w$ the velocity of flow of water (ft/hr), and $i_w = u_w$.

Since the water specific heat c_w is almost constant over the temperature range considered and equals unity, then:

$$\frac{\partial i_w}{\partial t} = c_w * \frac{\partial t_w}{\partial t} \quad (43)$$

and

$$\frac{\partial i_w}{\partial x} = c_w * \frac{\partial t_w}{\partial x} \quad (44)$$

From equations (43), (44), and (42), we may write:

$$\frac{\partial t_w}{\partial t} + V_w * \frac{\partial t_w}{\partial x} = \frac{1}{R_w * m_w * c_w} * (t_{t,o} - t_w) \quad (45)$$

Finally, an empirical relationship to calculate the specific humidity of saturated air at a given temperature is needed. This is given by (41):

$$W_{t,o,st} = - .00793 + .00031 * t_{t,o} + .0000075 * (t_{t,o} - 53)^2 \quad (46)$$

This relationship is correct within less than 0.1 percent over the temperature range 43°F to 95°F.

In summary, we have five unknowns:

$$t_a, t_w, t_{t,o}, W_a, W_{t,o,st}$$

and five equations to be solved simultaneously, equations (35), (36), (40), (45) and (46).

The above equations describe the dynamic model of the cooling coil. The solution gives:

$$\begin{aligned} t_a &= t_a(y,t) \\ t_w &= t_w(x,t) \\ t_{t,o} &= t_{t,o}(y,t) \\ W_a &= W_a(y,t) \quad , \text{ and } \\ W_{t,o,st} &= W_{t,o,st}(y,t) \end{aligned}$$

This model can be used in determining the time constant of the heat exchanger under both full and partial load conditions. This has its application in designing the control systems coupled to the cooling and dehumidifying coils and to the HVAC systems in general.

However, the steady state model, which is the purpose of this study, is obtained from the dynamic model by eliminating the $(\frac{\partial}{\partial t})$ terms. In this case:

t_a , t_w , $t_{t,o}$, W_a and $W_{t,o,st}$ will be functions of space dimension only, i.e. either x or y . Putting $\frac{\partial}{\partial t} = 0$ in equations (35), (36), (40) and (45) results in the following steady state model:

$$\frac{dt_a}{dy} = - \frac{\eta_{s,ov}}{R_a \cdot \dot{m}_a \cdot c_p} * (t_a - t_{t,o}) \quad (47)$$

$$\frac{dW_a}{dy} = - \frac{\eta_{c,ov}}{\dot{m}_a \cdot Le \cdot R_a} * (W_a - W_{t,o,st}) \quad (48)$$

$$\frac{dt_w}{dx} = - \frac{1}{\dot{m}_w \cdot R_w \cdot c_w} * (t_w - t_{t,o}) \quad (49)$$

$$t_{t,o} = \frac{R_w \cdot \eta_{s,ov}}{R_a + R_w \cdot \eta_{s,ov}} \{ t_a + t_w \cdot \frac{R_a}{R_w \cdot \eta_{s,ov}} + \frac{\eta_{c,ov} \cdot i_g \cdot (W_a - W_{t,o,st})}{Le \cdot c_p \cdot \eta_{s,ov}} \} \quad (50)$$

$$W_{t,o,st} = - .00793 + .00031 * t_{t,o} + .0000075(t_{t,o} - 53)^2 \quad (51)$$

Five equations to be solved simultaneously for the unknowns:

$$t_a, t_w, t_{t,o}, W_a \text{ and } W_{t,o,st}$$

The different parameters included in equations (47) through (51) are, primarily, the convective film heat transfer coefficients on both air and water sides, and the fin sensible and latent efficiencies defined by equations (13), and (14). The evaluation of these parameters will be discussed in Sections 3 and 5 of this chapter.

2. Numerical Solution of Steady State Model of the Cooling and Dehumidifying Coil:

Equations (47) through (51), which represent the steady state model of a cooling coil, are numerically solved for the psychrometric conditions of air leaving the cooling coil, and also the chilled water condition. Fig.(10) shows the flow chart of the computer program used to solve the above equations.

In solving the steady state model, it is assumed that the air-side convective heat transfer coefficient is the mean average coefficient, and is constant for a given Reynolds number. It is also assumed that Lewis number is assumed to be a constant and equals 0.92 (11).

Equations (47) through (51) are integrated over the cross-section of one row assuming that all parameters are constant throughout the depth of one row.*

Because of the cross-counter flow fashion, a value of the chilled water temperature is assumed in the first row. Going backward, the chilled water temperature at the last row (which is the temperature of the chilled water entering the coil) is

* Note that $dx = -dy$.

FIG. (10) FLOW CHART OF THE SIMULATION PROGRAM OF COOLING COILS IN CASES OF DRY AND WET SURFACE CONDITIONS.

calculated. Steady state criterion is achieved when the calculated chilled water temperature is equal to (or within $\pm 0.005^\circ\text{F}$) the actual temperature. At this stage, calculation is terminated and the required results are printed.

Next, the method used to predict the average film heat transfer coefficient on the air-side is outlined.

3. Average Film Heat Transfer Coefficient:

The relationship between the average film heat transfer coefficient on the air-side, and the air Reynolds number is required in order to solve the coil model. This relationship can be assumed by comparing the present coil with a "similar" one, of known characteristics. Eventually, the final results of such analysis is strongly affected by the assumed h-Re relationship.

In this section, the available dry surface test data, that have been previously published, are used to predict a general relationship between the heat transfer J-factor

$(J = (\frac{h}{G \cdot c_p}) * Pr^{2/3})$ and Re. Such a relationship can be

expressed as:

$$J = C_1 * Re^{C_2} \quad (52)$$

where C_1 and C_2 are constants for a particular coil. This has been shown to be true over the Reynolds number range $200 < Re < 1500$, which is the range of operation of most cooling and dehumidifying coils in HVAC systems.

C_1 and C_2 vary from one coil to another, and it is believed that they are strongly dependent on some of the coil physical dimensions. Since the flow characteristics over finned tube banks are not clearly defined (15), it becomes very difficult to identify the influential coil dimensions. However, based on previously published work of many investigators, a non-dimensional relation can be assumed for C_1 (or C_2) as follows:

$$C_1 = \phi(F_s/F_h, F_d/S_t, F_t/D_h, F_d/S_l, F_t/F_s, S_d/F_d, F_t/F_h, Ar) \quad (53)$$

In equation (53), the longitudinal and transverse tube spacing, S_l and S_t respectively, are seen to affect the characteristic performance of the coil, as concluded by Gram et al (23). This is in agreement with similar work by Jones (28). Briggs et al (25) suggested that the performance of finned tube banks, tested by the authors, can be described using the fin spacing F_s , fin thickness F_t , and fin height F_h , in non-dimensional forms. The ratio (F_t/F_s) is proportional to the heat transmitted through the fin base to that transmitted through the primary surface approximately. The spacing of the tubes is described by the ratios F_d/S_t and F_d/S_l . Finally, Ar , the area ratio (minimum flow area to coil face area ratio) is included together with the other coil dimensions which are used in non-dimensional forms.

Equation (53) is reformulated in a workable fashion, to be used in the regression analysis, as follows:

$$C_1 = a_0 * (Ar)^{a_1} * \left(\frac{F_s}{F_h}\right)^{a_2} * \left(\frac{F_d}{S_t}\right)^{a_3} * \left(\frac{F_t}{D_h}\right)^{a_4} * \left(\frac{F_d}{S_l}\right)^{a_5} * \left(\frac{F_t}{F_s}\right)^{a_6} * \left(\frac{S_d}{F_d}\right)^{a_7} * \left(\frac{F_t}{F_h}\right)^{a_8} \quad (54)$$

Step-by-step multiple linear regression routines of the Sigma 9 computer scientific library are used in conjunction with equation (54) to eliminate the insignificant groups of the above equation. Details of the step-by-step regression analysis can be found in References (53) and (54).

Previously published test data of twenty different heat exchangers are used in the regression analysis. The result is summarized in the following two equations expressing C_1 and C_2 :

$$C_1 = 0.159 * \left(\frac{F_t}{D_h}\right)^{-0.065} * \left(\frac{F_t}{F_h}\right)^{0.141} \quad (55)$$

and

$$C_2 = -0.323 * \left(\frac{F_s}{F_h}\right)^{0.049} * \left(\frac{F_d}{S_l}\right)^{0.549} * \left(\frac{F_t}{F_s}\right)^{-0.028} \quad (56)$$

Table (2) shows the percentage deviations from the observed C_1 and C_2 as calculated using equations (55) and (56); the standard deviation of residuals in each case is also given, and details of the different coils included in the regression analyses are given in Appendix J.

Table (2)

Standard deviation and percentage deviation of C_1 and C_2 based on the observed values of each.

	Max. deviation %	Min. deviation %	Standard deviation as a percentage of the mean %
C_1	4.0	-4.3	2.2
C_2	4.0	-3.0	3.1

It should be noted that the above relationships are restricted to the range of each dimensionless group tested in this analysis. Table (3) gives the respective range of each group.

Table (3)

Dimensionless groups used in the regression analysis

	$\frac{F_t}{D_h}$	$\frac{F_t}{F_h}$	$\frac{F_s}{F_h}$	$\frac{F_d}{S_l}$	$\frac{F_t}{F_s}$	$\frac{F_d}{S_t}$	$\frac{S_d}{F_d}$	Ar
Range	0.03	0.01	0.15	0.87	0.04	0.76	0.37	0.35
	0.33	0.45	0.91	1.27	0.50	1.4	0.85	0.62

The only reason for the previous restriction is that the tested coils fall in this range, and no other coils are available for inclusion in the regression analysis.

In summary, using equations (55) and (56), one can calculate C_1 and C_2 , given the coil physical dimensions. Then, equation (52) can be used to describe the coil performance over the Reynolds number range between 200 and 1500 approximately.

4. Effect of Water Film Condensate on Overall Heat Exchanger Performance:

Condensation of moisture from humid air on the outside surface of cooling coils occurs wherever the surface temperature is less than the dew point temperature of the air in contact with the surface. Usually, it is a combination of drop-wise and film-wise condensation, which makes it very difficult to identify the water film thickness. However, previous investigators, (11) and (39), have concluded that the water film thickness is very small compared with coil dimensions. In this analysis, a uniform water film thickness of 0.004 inches will be assumed.*

Condensation of moisture on the coil surface decreases the minimum flow area $A_{m,f}$, and the coil hydraulic diameter D_h . The decrease in $A_{m,f}$ results in an equivalent increase in the maximum air velocity through the coil, V_a . The net result of moisture condensation has no effect on the air Reynolds number, as explained by equation (57):

$$\frac{Re_w}{Re_d} = \frac{(D_h * V_a)_{wet}}{(D_h * V_a)_{dry}} = \frac{A_{m,f,w}}{A_{m,f,d}} * \frac{A_{m,f,d}}{A_{m,f,w}} = 1.0 \quad (57)$$

* As assumed in similar tests by Myers (39).

where $A_{m,f,w}$ and $A_{m,f,d}$ are the coil minimum flow areas of wet and dry surfaces, respectively. However, a noticeable increase in the overall heat transfer coefficient is reported by some investigators (39, 55) due to the presence of water film condensate. This has been confirmed by the results of the present experimental work, (see Chapter VI).

Present and previously published experimental work showed that the average heat transfer J-factor increased (15 to 25 percent depending on Re) as a result of moisture condensation on the heat transfer surface. Therefore:

$$\frac{J_{\text{wet}}}{J_{\text{dry}}} > 1.0$$

but we have

$$\frac{J_{\text{wet}}}{J_{\text{dry}}} = \frac{h_w A_{m,f,w}}{h A_{m,f,d}} > 1.0$$

Despite the deficiency in the minimum flow area (about 10 percent for $z_w = 0.004$ in) due to the presence of water film, the J-factor of a wet surface is greater than the corresponding J-factor of a dry surface. This is due to the increase in average h . This increase in h may be caused by the increased turbulence level in the flow field due to the increased surface roughness by the water film (39), and also by the slight increase in V_a for a given mass velocity.

In summary, despite the negative effect of the water film on the coil hydraulic diameter, the average heat transfer coefficient h is increased and this results in an increase in the overall J-factor.

5. Fin Efficiency Calculation:

5.1 Dry Surface Fin Efficiency:

Fins of various geometries and thermal conductivities respond differently to identical and uniform heat sources and sinks. Similarly, there are numerous ways in which the temperatures and heat transfer coefficients of sources and sinks may vary (46). The analysis of the radial fin with a rectangular cross-section (or a continuous fin with rectangular cross-section) utilized the assumptions of Murray (56) and Gardner (43). These assumptions can be summarized as:

1. The heat flow in the fin and its temperatures remain constant with time.
2. The fin material is homogeneous and its thermal conductivity is the same in all directions and remains constant.
3. The convective film heat transfer coefficient on the fin surface is constant and uniform over the entire fin surface.
4. The temperature of the medium surrounding the fin is uniform.
5. The fin thickness is small compared to its height, so that temperature gradients across the fin thickness may be neglected.
6. The temperature at the base of the fin is uniform.
7. There is no contact resistance where the base of the fin joins the prime surface.
8. There is no heat source or sink in the fin itself.
9. The fin diameter of a continuous fin, is an equivalent diameter, $f_{d,e}$ based on an equal circular fin area, See Fig. (11).

FIG.(11) RADIAL FIN SECTION SHOWING THE EQUIVALENT
FIN DIAMETER OF A-CONTINUOUS SURFACE FIN.

It should be indicated here that the complete analysis of the fin efficiency calculations can be found in References (43, 56). The summary given in this thesis is to help in deriving the wet surface fin efficiency. The latter will be explained in the next section.

Applying the foregoing assumptions, and the heat transfer equation for the radial fin shown in Fig.(11), one can write:

$$r^2 * \frac{d^2 T_r}{dr^2} + r * \frac{dT_r}{dr} - m^2 * r^2 * T_r = 0 \quad (58)$$

where T_r is the temperature excess $t_r - t_a$, and m is defined as:

$$m = \left(\frac{2 * h}{k_f * F_t} \right)^{\frac{1}{2}} \quad (59)$$

Using the appropriate boundary conditions, equation (58) can be solved for the temperature distribution over the fin surface.

Using the definition of fin efficiency (see Chapter II), one may write:

$$\eta_{f,d} = \frac{2 * r_o}{m * (r_e^2 - r_o^2)} * \left\{ \frac{I_1(m * r_e) * K_1(m * r_o) - K_1(m * r_e) * I_1(m * r_o)}{I_0(m * r_o) * K_1(m * r_e) + I_1(m * r_e) * K_0(m * r_o)} \right\} \quad (60)$$

The modified Bessel functions of zero and first orders can be found in many mathematics texts. Fig.(12) shows the fin efficiency of a radial fin of a rectangular cross section.

FIG.(12) EFFICIENCY OF A RADIAL FIN OF A RECTANGULAR CROSS-SECTION (46).

Next, the fin efficiency, when simultaneous heat and mass transfer exist, is discussed in more detail.

5.2 Wet Surface Fin Efficiency:

When condensation occurs on the heat transfer surface both temperature potential and specific humidity potential exist.

The difference between the air dry-bulb temperature and the heat transfer surface temperature is the potential for the sensible heat transfer.

The difference between the surrounding air specific humidity and the specific humidity of saturated air at the wet surface temperature is the potential for the transfer of latent heat (moisture transfer in the present case).

The present analysis of wet surface efficiency concentrates on the types of fin geometry which are commonly used in commercial heat exchangers. These are the circular fin (or continuous fins) of a rectangular cross-section.

Consider the basic assumptions stated in Section 5.1, and assume a circular fin of rectangular cross-section as shown in Fig.(13). For each infinitesimal circular fin area dA_f of the fin surface ($dA_f = 2\pi r dr$), the total heat exchanged between the area dA_f and the surrounding air is given by:

$$dq = h T_r dA_f + h_m \lambda W_r dA_f \quad (61)$$

where T_r is the excess temperature $t_r - t_a$, and W_r is the excess specific humidity $W_{r,st} - W_a$.

FIG.(13) SCHEMATIC DIAGRAM OF A FIN UNDER WET SURFACE CONDITION.

Rearranging equation (61) and substituting for dA_f ;

$$\frac{dq}{dr} = 4\pi(hT_r + h_m\lambda W_r)r \quad (62)$$

The integration of equation (62) over the entire fin surface gives the total heat transfer through both sides of the fin surface. Therefore:

$$q = 4\pi \int_{r_o}^{r_e} (hT_r + h_m\lambda W_r)r \cdot dr \quad (63)$$

The total heat that would be transferred if the fin surface is at a constant temperature equal to the temperature at the base of the fin, and constant specific humidity equal to the specific humidity of saturated air at the fin base temperature, is given by:

$$q_{id} = 2\pi(r_e^2 - r_o^2) \cdot (hT_{r,o} + h_m \lambda W_o) \quad (64)$$

$T_{r,o}$ is the excess temperature $t_{r,o} - t_a$ and W_o is the excess air specific humidity $W_{r,o,st} - W_a$. The ratio between the actual total heat transferred through the fin surface and that which would be transferred under the ideal conditions described above, is defined as the fin overall efficiency.

$$\eta_{f,w} = \frac{q}{q_{id}} \quad (65)$$

Substituting equations (63) and (64) into (65):

$$\eta_{f,w} = \frac{2 \left(h \int_{r_o}^{r_e} T_r r dr + h_m \lambda \int_{r_o}^{r_e} W_r r dr \right)}{(h T_{r,o} + h_m \lambda W_o) (r_e^2 - r_o^2)} \quad (66)$$

This can be written in terms of the Lewis number, Le , where:

$$Le = \frac{h}{h_m c_{pa}} \quad (67)$$

as:

$$\eta_{f,w} = \frac{2 \left(\int_{r_o}^{r_e} T_r r dr + \frac{\lambda}{Le c_{pa}} \int_{r_o}^{r_e} W_r r dr \right)}{(T_{r,o} + \frac{\lambda}{Le c_{pa}} W_o) (r_e^2 - r_o^2)} \quad (68)$$

Integration of the numerator requires a knowledge of the temperature and specific humidity distribution over the entire fin surface.

Consider the energy equation applied on a control volume taken from the fin as shown in Fig.(14):

Fig.(14) SCHEMATIC DIAGRAM OF A FIN SURFACE

The heat transferred by conduction into the cylindrical surface at radius (r) is given by:

$$q_{\text{cond}} = -k_f * (2 * \pi * r * F_t) * \frac{dT_r}{dr} \quad (69)$$

The amount of heat leaving the cylindrical surface at radius (r + dr) is given by:

$$q_{\text{cond}} + dq_{\text{cond}} = -k_f * (2 * \pi * r * F_t) * \frac{dT_r}{dr} + \frac{d}{dr} (-k * 2 * \pi * r * F_t \frac{dT_r}{dr}) * dr \quad (70)$$

The total heat (sensible and latent) transferred from the fin surfaces is written as:

$$dq_{\text{out}} = 2 * (2 * \pi * h * T_r * r * dr + 2 * \pi * h_m * \lambda * W_r * r * dr) \quad (71)$$

Applying the energy equation on the control volume gives:

$$dq_{\text{out}} = q_{\text{cond}} - (q_{\text{cond}} + dq_{\text{cond}})$$

or

$$k_f * F_t * (r * \frac{d^2 T_r}{dr^2} + \frac{dT_r}{dr}) = 2 * (h * T_r + h_m * \lambda * W_r) * r \quad (72)$$

Rearranging and substituting for h_m from equation (67), gives:

$$r \frac{d^2 T_r}{dr^2} + \frac{dT_r}{dr} - r \frac{2h}{k_f F_t} * [T_r + \frac{\lambda}{Le c_{pa}} * W_r] = 0 \quad (73)$$

The excess specific humidity W_r is a function of the local fin surface temperature, and is given by:

$$W_r = W_{r,st} - W_a \quad (74)$$

where $W_{r,st}$ is the specific humidity of saturated air at the local fin surface temperature.

A linear relationship between the saturation temperature and the corresponding W_{st} , is assumed. So, W_{st} can be written as (11, 39):

$$W_{st} = a + b * t_{st} \quad (75)$$

where a and b are constants to be determined from the psychrometric data. The above approximation is true for a small interval of temperature difference. To overcome this restriction, the constants a and b are calculated at each point assuming a linear approximation over the temperature range, $t_{st} + 1.0$ and $t_{st} - 1.0$ degrees F. In such cases, W_{st} is calculated within $\pm 0.5\%$ of the correct value.

Substituting $W_{r,st}$ from equation (75) into equation (73):

$$r \frac{d^2 T_r}{dr^2} + \frac{dT_r}{dr} - r \frac{2h}{k_f F_t} * (T_r + \frac{\lambda}{Le c_{pa}} * (a + b * (T_r + t_a) - W_a)) = 0 \quad (76)$$

Equation (76) is solved numerically for the temperature distribution over the fin surface. By substituting the temperature distribution into equation (68), the overall fin efficiency can be determined for the case of simultaneous heat and mass transfer.

CHAPTER IV

APPARATUS DESCRIPTION AND INSTRUMENTATION

1. Apparatus Description:

Since a new approach to simulate cooling and dehumidifying coils has been presented in Chapter III, the results of such simulation would be incomplete without some experimental validation. The purpose of this chapter is to outline the means used to perform this validation.

The experimental work was conducted at the Division of Building Research, National Research Council of Canada. The design of the testing facility is such that multi-rows coils, having a face area of 24 in x 24 in, can be installed in the working section. The flowing air-face velocity can be varied from about 100 fpm to 800 fpm. This face velocity range is the practical range of typical cooling coils associated with commercial air-conditioning systems. Fig.(15) shows a partial view of the test set-up showing the closed-loop air duct and the coil assembly.

The following description of the apparatus is divided into three sub-sections, as follows:

1 - Air circuit

2 - Chilled water (or coolant) circuit

3 - Finned - tube coil and assembly.

FIG. (15) PARTIAL VIEW OF THE COOLING AND DEHUMIDIFYING TEST FACILITY.

1.1 Air Circuit:

A schematic diagram of the overall air circuit assembly is shown in Fig.(16). This consists of a closed loop in which air is circulated by a constant speed fan (3450 rpm, 7.9 HP). The air duct is made of galvanized steel sheet and has a 24 * 24 inches cross-section. Two transition sections are installed upstream and downstream of the fan between the square section of the duct and the circular section of the fan intake and discharge sections. These transition duct pieces are designed with a diffuser angle sufficiently gentle so as to prevent separation of air. Also, all elbows in the air circuit are provided with guide vanes to minimize the total losses in the air circuit.

The air velocity flowing through the coil is varied by by-passing portions of the total circulated air through a by-pass section as shown in Fig.(16). Two sets of mechanical dampers are used to control the air flow; these dampers are operated by a minimum position switch, see Fig.(17).

The return air from the by-pass section is mixed with the main air stream in a mixing chamber (see Fig.(18)), to improve the flow quality upstream of the fan. This process also eliminates the chance of flow discontinuity, which has an adverse effect on the fan performance.

The main air duct houses a 24 * 24 inch electric duct heater. The power needed for the 50 kilowatt heater, is supplied by a separate motor-generator (M-G) set to provide

32 ft.

- 1-Cooling coil
- 2-Air circulating fan
- 3-Electric duct heater
- 4-Condensate collecting panel
- 5-By-pass duct
- 6-Location of the steam humidifier
- 7-Mechanical dampers

FIG. (16) CLOSED LOOP AIR DUCT.

FIG.(17)AIR BY-PASS DAMPER CONTROL.

Details of the air slot.

FIG.(18)RETURN AIR MIXING CHAMBER.

complete control on the power supplied. Fig.(19) shows the wiring circuit used to vary the excitation field of the M-G set.

Following the mixing chamber, a steam humidifier is installed in the main air duct between the mixing chamber and the intake to the air fan. The 10-gallons per hour (at 15 psig) steam humidifier is supplied by steam from a 30 kilowatt electric steam boiler. The discharge pressure of the steam boiler is reduced to about 3 to 5 psig to maintain a constant rate of flow of steam into the air stream which is essential for the steady state measurements.

The section of the duct immediately downstream of the cooling coil is designed to collect the condensate from the cooling coil; and to equalize the pressure in this section. The bottom of the duct is made in two levels; the upper level is flat and slotted to collect all of the condensate; and the lower level is tapered to form a funnel which leads to a collecting jar for condensate measurements.

1.2 Chilled Water Circuit:

Chilled water is supplied, to the cooling coil from the main chilled water line in the laboratory. The chilled water temperature entering the cooling coil is controlled by a three-way diverter valve as shown in Fig.(20). The valve position is changed in response to an electric controller acting on signal from a resistance thermometer located in the water entering the cooling coil.

FIG. (19) AIR DUCT HEATER: POWER AND CONTROL CIRCUIT.

FIG. (20) CHILLED WATER: LAYOUT AND CONTROL CIRCUIT.

Two immersion heaters are installed in the chilled water circuit on the discharge side of the cooling coil (8 kilowatts and 4 kilowatts rated power). The immersion heaters are used to measure the chilled water flow rate. See Appendix E.

1.3 Finned Tube Coil and Assembly:

The finned coil tested in the present work is a circular tube with flat plate fins made by KEEPRITE PRODUCTS LTD. with the following specifications:

Tubes:

No. of rows	8
Tube arrangement	staggered
Tube outside diameter, in.	.625
Tube inside diameter, in.	.575
Tube longitudinal spacing, in	1.299
Tube transverse spacing, in	1.5
Tube material	copper

Fins:*

Fin material	aluminum
Fin thickness, in	.0065
No. of fins per inch	11.8

Coil:

Coil face area, in ²	24 * 24
Coil depth, in	13.5
Coil primary surface area, ft ²	38.9
Coil secondary surface area, ft ²	827
Minimum flow area/frontal area	.54
No. of water circuits	16

All the chilled water circuits are fed from a common header and discharge into another header, on the same side of the coil.

The straight duct section upstream of the cooling coil contains a 16-point thermocouple grid and a grid of 16-total pressure probes. The two grids are separated by about 9 inches.

* The fins have collars and the contact resistance between the fins and tubes is neglected.

Each thermocouple and pressure probe is located at the geometric center of a square area of 6 in * 6 in.

A similar grid of thermocouples and pressure probes are installed in the section of the air duct immediately downstream of the cooling coil.

At the two cross-sections where the total pressure grids are located, 4-static pressure tapings are provided at each section to measure the average static pressure. The pressure probes and their relative position are made and located according to the ASME standards (57), the ARI 410-72 (9), and the ASHRAE 33-64 standards (13). Fig.(21) is a schematic diagram of the air duct showing the location of the different measuring elements.

Two more thermocouple grids, each consisting of 16 thermocouples, are installed on each side of the electric duct heater. All the thermocouples (except the grid which is located downstream of the air duct heater) are shielded to minimize radiation effect, see Fig.(22). Specially designed wooden wedges are used to protect the thermocouple grid downstream of the duct heater, because it faces the glowing heating elements. Each thermocouple junction is hidden behind a wedge which is covered by reflecting aluminum tape. All the wooden wedges are mounted on a $\frac{1}{2}$ -inch diameter wooden bar to prevent conduction heat transfer to or from the sheet metal duct, see Fig.(22).

The cooling coil assembly (i.e. finned tube coil and the upstream and downstream duct sections which contain most of the

FIG. (21) LOCATION OF THERMOCOUPLE AND TOTAL PRESSURE PROBE GRIDS INSTALLED IN THE MAIN AIR DUCT.

Thermocouple radiation
shield (Copper)

Wooden thermocouple
radiation shield

FIG. (22) DIFFERENT TYPES OF THERMOCOUPLE RADIATION
SHIELDS.

measuring elements) is housed in a plywood box which is filled with polystyrene beads to provide 5 to 6 inches thickness of insulation around the entire assembly.

2. Instrumentation:

Because of the large size of the cooling coil test facility it was not practical to house all the measuring instruments in one location, see Fig.(23). Most of the outputs from the measuring instruments are in the form of electrical signals which are fed through a 64-channel automatic scanner, and are recorded on punched paper tape.

The description of the instrumentation will be divided into the following sub-sections:

- 1 - Air system measuring instruments
- 2 - Chilled water (coolant) measuring instruments.

2.1 Air System Measuring Instruments:

2.1.1 Air dry-bulb temperature:

The dry bulb temperature of the air stream is measured at the following positions:

- a - upstream of the cooling coil, 16 measurement points
- b - downstream of the cooling coil, 16 measurement points
- c - an average of 16 points to measure the air dry-bulb temperature before and after the electric duct heater and in the branched by-pass circuit.

(A)

(B)

FIG. (23) A - PARTIAL VIEW OF THE DATA RECORDING ARRANGEMENT

B - THE CONTROL PANEL

The air dry-bulb temperatures are sensed by 24 gauge copper-constantan thermocouples. The output of all the thermocouples is measured by a high impedance D.V.M. which is connected to an automatic scanner and paper tape punch to facilitate data processing on the computer. Fig.(24) shows a sketch of the thermocouple and recorder arrangement. The output from the thermocouples is measured with reference to a constant-temperature common junction. The latter is a solid aluminum rod 3 inches in diameter inside a 3 1/16 inches diameter pipe. The temperature of the reference block is measured with reference to an ice bath, as shown in Fig.(24). Before installation, the thermocouples were calibrated in a silicon oil bath against a standard platinum-resistance thermometer. Details of the calibration are given in Appendix A.

2.1.2 Air dew-point temperature:

Dew-point temperature of the flowing air is measured at two points: upstream of the cooling coil and downstream of it. Two DEWCEL elements are used for these measurements. The output is measured in the same way as the air dry-bulb temperatures discussed in the previous section. A description of the DEWCEL element and their calibration is given in Appendix B.

2.1.3 Total and static air-pressure:

Fig.(25) shows the arrangement for velocity head measurements. Calibration of the pressure probes is described in Appendix I.

FIG. (24) SCHEMATIC DIAGRAM SHOWING ARRANGEMENT FOR TEMPERATURE MEASUREMENT AND RECORDING.

FIG. (25) SCHEMATIC DIAGRAM SHOWING VELOCITY PRESSURE MEASUREMENT AND RECORDING.

2.1.4 Condensate measurements:

The condensate collected (every 15 minutes) from the condensate panel is weighed on a balance. The condensate temperature is measured by a mercury-in-glass thermometer.

2.1.5 Air flow rate:

The air flow rate through the cooling coil is calculated using the velocity profile and temperatures measured upstream of the cooling coil. Details are given in Appendix D.

2.2 Chilled Water Measurements:

2.2.1 Chilled water temperature:

Chilled water temperature is measured at three locations: before and after the cooling coil, and after the immersion heaters. Three platinum resistance thermometers are used for these measurements, and their output signal is recorded on punched tape through the D.V.M.-Scanner arrangement. Upstream of each chilled water temperature measuring point, there is a mechanical mixer to ensure proper mixing of water and to minimize the chance of temperature gradients at that section.

2.2.2 Chilled water flow rate:

The flow rate of chilled water circulated through the cooling coil is determined by measuring the power input to the immersion heaters and the water temperature rise across them. Description of the chilled water flow rate calculation is described in Appendix E.

CHAPTER V

DRY SURFACE PHASE

In this chapter, the experimental work conducted to test the previously described heat exchanger, under dry surface condition, is discussed. Comparison will be made between previously reported data for a similar coil and the present experimental data. Finally, the analytical simulation program presented in Chapter III will be used to examine the agreement between the measured and predicted coil performance..

1. Analysis: Dry Surface Phase:

When the minimum surface temperature of a heat exchanger is higher than the dew-point temperature of the surrounding air, no condensation occurs and the entire external heat exchanger surface is dry. This is called the "sensible cooling" process.

1.1 Heat Transfer Areas:

The following are definitions of the different heat transfer areas, previously defined in this thesis. The primary heat transfer area A_p , is the total area of the bare tubes on the air side. It is calculated as:

$$A_p = \pi * S_d * C_l * (1 - 12 * N_f * F_t) * N_r * N_{pr} \quad (77)$$

The secondary heat transfer area on the air side is the area of the extended surfaces, which is the flat plate fins in the present case. It is calculated as:

$$A_s = 24 * C_w * C_d * C_l * N_f - 0.5 * \pi * S_d^2 * N_{pr} * N_r \quad (78)$$

The total external heat transfer area is the sum of equations (77) and (78):

$$A_o = A_p + A_s \quad (79)$$

The minimum flow area is defined as the net cross-sectional area through which the air flows:

$$A_{m,f} = (C_l - 12 * N_f * C_l * F_t) * (C_w - S_d * N_{pr}) \quad (80)$$

The internal heat transfer area is the area of heat transfer on the chilled water side:

$$A_{t,i} = \pi * S_{d,i} * C_l * N_r * N_{pr} \quad (81)$$

The ratio between the heat transfer area on the air side to that on the water side B, is frequently used in the analysis; it is defined as:

$$B = \frac{A_o}{A_{t,i}} \quad (82)$$

Fig.(26) shows the different physical dimensions of the heat exchanger tested in this thesis.

FIG.(26) DIFFERENT DIMENSIONS OF THE TESTED FINNED TUBES HEAT EXCHANGER.

1.2 Dry Surface Fin Efficiency:

Equation (60) in Chapter III is used to evaluate the fin efficiency, where the fin diameter is substituted by an equivalent fin diameter; see Fig.(11):

$$F_{d,e} = \sqrt{\frac{4 * C_d * C_w}{\pi * N_{pr} * N_r}} \quad (83)$$

$F_{d,e}$ is the equivalent fin diameter of a continuous fin based on an equal circular fin area (46, 58).

1.3 Mean Temperature and Log-Mean Temperature Difference:

Fluid properties are usually calculated at the mean fluid temperature. For a finned tube heat exchanger, in a cross-counter flow arrangement, the air mean temperature is given by:

$$t_{a,m} = t_{a,1} - \frac{(t_{a,1} - t_{a,2}) * (t_{a,1} - t_{w,2})}{(t_{a,1} - t_{a,2}) - (t_{w,2} - t_{w,1})} + \frac{(t_{a,1} - t_{a,2})}{\ln\left(\frac{t_{a,1} - t_{w,2}}{t_{a,2} - t_{w,1}}\right)} \quad (84)$$

and that of the chilled water is:

$$t_{w,m} = t_{w,2} - \frac{(t_{w,2} - t_{w,1}) * (t_{a,1} - t_{w,2})}{(t_{a,1} - t_{a,2}) - (t_{w,2} - t_{w,1})} + \frac{(t_{w,2} - t_{w,1})}{\ln\left(\frac{t_{a,1} - t_{w,2}}{t_{a,2} - t_{w,1}}\right)} \quad (85)$$

Fig.(27) shows a diagram illustrating the meaning of the different temperatures in equations (84) and (85), and the

FIG. (27) SCHEMATIC DIAGRAM SHOWING THE DIFFERENT QUANTITIES IN EQUATION (85).

derivation of these equations can be found in Reference (59).

The heat transfer rate in case of a finned-tube heat exchanger is calculated in terms of an appropriate temperature difference between air and chilled water as will be explained later in this section. This temperature difference is "the logarithmic temperature difference" and is given by:

$$\Delta t_{l,m} = \frac{(t_{a,1} - t_{w,2}) - (t_{a,2} - t_{w,1})}{\ln\left(\frac{t_{a,1} - t_{w,2}}{t_{a,2} - t_{w,1}}\right)} * P \quad (86)$$

This form is acceptable for a cross-counter flow with more than two tube passes (39). P is a correction factor.*

1.4 Overall Dry Surface Heat Transfer Coefficient:

The basic equation for calculating the heat transfer rate of a finned tube coil is given by:

$$q = U_o * A_o * \Delta t_{l,m} \quad (87)$$

where U_o is the overall heat transfer coefficient based on the outside total heat transfer area, A_o . Generally in experimental work on finned tube coils, A_o is calculated from the physical dimensions of the heat exchanger, $\Delta t_{l,m}$ is calculated using equation (86) and the heat extraction is one of the outcomes of the experimental work. So, the only parameter which is to be evaluated from equation (87) is U_o .

* For the temperature range considered and the cross-counter flow fashion, P is approximately unity. This is true in case of multi-row tube banks.

The aim of the analysis given in this section is to express the overall heat transfer coefficient U_o in terms of the outside average film heat transfer coefficient and some other coefficients that will be defined later. In this analysis it is assumed that:

- 1 - the system is in steady state conditions
- 2 - no contact resistance between the fin and the outside tube surface (in other words, the fin base temperature t_b is equal to the outside bare tube surface $t_{t,o}$).
- 3 - The sum of the heat transferred through the fin surface and the bare tube surface is transferred to the chilled water (in case of dry cooling) and no radiation exchange occurs.

Referring to Fig.(28), one may write the expression of heat transfer from the scale deposit on the inner tube surface to the chilled water as:

$$q = h_i \cdot A_{t,i} \cdot (t_d - t_w) \quad (88)$$

where h_i is the average film heat transfer coefficient for coolant inside the tubes, t_d is the temperature of the deposit layer on the inside of the tube, and t_w is the chilled water temperature. If the fouling factor is FF, then the apparent film heat transfer coefficient of the deposit layer is (11):

$$h_d = \frac{1}{FF} \quad (89)$$

then the heat transferred through the deposit layer is written as:

FIG.(28) SCHEMATIC DIAGRAM SHOWING HEAT TRANSFER PATHS DURING COOLING OF AIR WITH A FINNED TUBE SURFACE.

$$q_d = h_d * A_{t,i} * (t_{t,i} - t_d) \quad (90)$$

where $t_{t,i}$ is the temperature of the inner surface of the tube wall.

The heat transfer from the thin wall tube can be written as (39):

$$q_t = \frac{k_t * A_{t,m}}{Z} * (t_{t,o} - t_{t,i}) \quad (91)$$

where:

$$A_{t,m} = \frac{A_{t,o} - A_{t,i}}{\ln\left(\frac{A_{t,o}}{A_{t,i}}\right)} \quad (92)$$

$A_{t,o}$ and $A_{t,i}$ are the outside and inside tube surface area respectively. Z is the tube wall thickness.

Referring to Fig.(28), heat transfer through the outside surface from the surrounding air flows through two parallel paths; the primary bare tube surface and the fin surface to the tube wall. The heat transfer equation which describes the total heat transferred to the tube wall can be written as:

$$q_o = h_t * A_p * (t_a - t_{t,o}) + h_f * A_s * \left(\int_{A_s} (t_a - t_f) * dA \right) \frac{1}{A_s} \quad (93)$$

Recalling the definition of the dry surface fin efficiency as:

$$\eta_{f,d} = \frac{\int_{A_s} (t_a - t_f) dA_s}{A_s * (t_a - t_{t,o})} \quad (94)$$

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then, from equations (93) and (94), one can write:

$$q_o = (t_a - t_{t,o}) * (h_t * A_p + h_f * A_s * \eta_{f,d}) \quad (95)$$

For steady state conditions, assuming no heat sources or sinks

$$q_o = q_t = q_d = q_i = q \quad (96)$$

Investigating equations (88), (90), (91), (95) and (96), one can write:

$$(t_a - t_w) = q * \left(\frac{1}{h_i * A_{t,i}} + \frac{1}{h_d * A_{t,i}} + \frac{z}{k_t * A_{t,m}} + \frac{1}{h_t * A_p + h_f * A_s * \eta_{f,d}} \right) \quad (97)$$

For a cross-counter flow, as in the case of multi-tube bank, the appropriate temperature difference is the logarithmic mean temperature difference. Then from equations (87) and (97), one may write:

$$\frac{1}{U_o} = \frac{A_o}{A_{t,i} * h_i} + \frac{A_o}{h_d * A_{t,i}} + \frac{A_o * z}{A_{t,m} * k_t} + \frac{A_o}{h_t * A_p + h_f * A_s * \eta_{f,d}} \quad (98)$$

In the case of the present finned tube heat exchanger, A_s represents 95 percent approximately of the total outside heat transfer area. On the other hand, a 10 percent increase in h_t results in less than one percent change in the quantity $(h_t * A_p + h_f * A_s * \eta_{f,d})$. So, the assumption $h_t = h_f = h$ will not

seriously affect the overall accuracy of results (39).

Therefore, equation (98) can be written as:

$$\frac{1}{U_o} = \frac{B}{h_i} + FF * B + B * \frac{Z}{k_t} + \frac{1}{h * \eta_{ov}} \quad (99)$$

where FF is the fouling factor given by equation (89) and η_{ov} is the overall heat exchanger efficiency defined by equation (10). It is assumed that $A_{t,m} = A_{t,i}$ in the derivation of equation (99) with an error of about 0.1 percent, since the tube wall resistance represents less than 0.1 percent of the total resistance to heat flow.

2. Test Procedure: Dry Surface Phase:

This section includes a general description of how the test was conducted. A brief listing of the data recorded is also given together with how to process these data in order to present the performance of the dry surface heat exchanger.

2.1 General Test Procedure:

The chilled water flow rate is adjusted such that the water temperature rise across the cooling coil is about 4 to 10 degrees F. This is typical of the temperature rise experienced by cooling coils.

The air flow rate is modulated as described in Chapter IV, and the power input to the duct heater is varied to provide the desired inlet air temperature to the cooling coil.

The whole system achieves steady state conditions in about 30 to 45 minutes. Once the steady state conditions are achieved the necessary measurements are recorded. The measured air temperatures, the power transducer output and chilled water temperatures are recorded automatically every 90 seconds. In addition, the following quantities are recorded manually:

- velocity pressure of the air flow upstream of the cooling coil,
- static pressure of the air stream before and after the cooling coil,
- the barometric pressure.

2.2 Data Analysis:

The heat extraction on the water side is calculated as explained in Appendix F. Equation (86) is used to evaluate $\Delta t_{l,m}$, then, the overall heat transfer coefficient U_o is calculated using equation (87) (for a complete uncertainty analysis see Appendix H).

The average film heat transfer coefficient on the chilled water side is calculated using the turbulent flow relation (17):

$$Nu_w = 0.023 * Re_w^{0.8} * Pr_w^{0.4} ** \quad (100)$$

FF is estimated using previously published data in Reference (11). Therefore, the only unknown in equation (99) is the combined resistance to heat flow on the air side, i.e. $1/(h * \eta_{ov})$.

** Equation (100) may underestimate h_1 for the first row by about 7% since the flow is not fully developed. However, this does not result in a serious error since the water side resistance varied between 16 and 25% of the total resistance.

Since η_{ov} is dependent on h , iteration is necessary to calculate h .

A constant value of h is assumed on the fin surface, when evaluating η_{ov} , (the actual distribution of h over the fin surface is very difficult to predict and is beyond the scope of this work).

3. Test Results: Dry Surface Phase:

In this section, experimental results obtained under dry surface conditions are presented. The Cooling Coil simulation program discussed in Chapter III, is used for comparison between the experimental and analytical results.

3.1 Present Experimental Results:

The Reynolds number on the air side is evaluated, based on the coil hydraulic diameter which is defined as (19, 22):

$$D_h = 4 * \frac{A_{m,f}}{A_o} * L \quad (101)$$

For the tested range of air side Reynolds number ($300 < Re < 1500$), the different properties of air and chilled water are calculated using the corresponding fluid mean temperature given by equations (84 and 85). The cooling coil is tested at different air inlet temperature, see Fig.(29).

The condition line of the tested coil is a horizontal straight line going from right to left on the psychrometric chart. Figs.(30, 31 and 32) show some test points for three different loads, details of which are given in Table (4).

TABLE 4

DRY SURFACE TEST RESULTS
RUNS D-A, D-B, AND D-C

Description	Run D-A	Run D-B	Run D-C
Average air temperature:			
- entering the cooling coil °F	63.9	80.6	93.9
- leaving the cooling coil °F	50.3	53.4	50.7
Average specific humidity:			
- entering the cooling coil lb _w /lb _a	0.0061	.0065	.0075
- leaving the cooling coil lb _w /lb _a	0.0061	.0065	.0075
Chilled water temperature:			
- entering the cooling coil °F	46.8	46.7	46.7
- leaving the cooling coil °F	49.6	52.2	51.5
Heat extraction:			
Air side Btu/hr * 10 ⁻³	43.6	85.3	74.0
Water side Btu/hr * 10 ⁻³	43.3	85.5	74.1
Air flow Re	1250	1210	640
Average J-factor	.0072	.0072	.0094

FIG. (29) SUMMARY OF DRY-TEST AIR INLET CONDITIONS.

FIG.(30) SENSIBLE COOLING OF AIR, RUN D-A.

FIG. (31) SENSIBLE COOLING OF AIR, RUN D-B.

FIG. (32) SENSIBLE COOLING OF AIR, RUN D-C.

The J-Re relationship is shown in Fig.(33), from which the following relation is obtained,

$$J = 0.101 * Re^{-0.369} \quad (102)$$

This means that the constants C_1 and C_2 of the tested coil are 0.101 and -0.369 respectively. See equation (52).

Characteristic performance of a finned tube heat exchanger, similar to the one tested in this work, was reported by Rich (32). A comparison between the different coil dimensions is given in Table (5), and the J-Re relationships for the two coils are compared in Fig.(34). The present experimental heat transfer data is 1 to 4 percent higher than those reported by Rich (32). This difference is related to the slight physical difference between the two coils. See Table (5). This is confirmed by the analytical results shown in Fig.(34).

3.2 Analytical Simulation Program:

Given the coil physical dimensions and by using equations (55 and 56), C_1 and C_2 of the coil can be predicted analytically. Knowing the air flow Re , then by using equation (52), J , and hence h , can be evaluated. Given the air and water inlet conditions together with the calculated h , the simulation program can be used to calculate the air and water outlet conditions as well as the cooling coil capacity.

The quantities C_1 and C_2 are calculated, as described above, for the tested coil and found to be 0.104 and -0.366, respectively.

TABLE 5
COMPARISON BETWEEN THE TESTED
COIL DIMENSIONS AND THOSE TESTED
BY DON RICH (32)

Description	Present Coil	Literature Coil
Ar	0.539	0.543
F_s/F_h	0.17	0.215
F_d/S_t	1.067	1.055
F_t/D_h	0.0593	0.0545
F_d/S_d	1.248	1.218
F_t/F_s	0.077	0.070
S_d/F_d	0.391	0.398
F_t/F_n	0.013	0.015
No. of fins per inch,	11.8	11.7
Tube diameter, in,	0.625 (Copper)	0.525 (Copper)
Fin Thickness, in,	0.0065 (Alum.)	0.006 (Copper)
Tube row spacing, in,	1.3	1.085
Tube face spacing, in,	1.5	1.25
Hydraulic diameter, ft,	0.0913	0.0912

FIG. (33) CORRELATION OF DRY SURFACE HEAT TRANSFER J-FACTOR.

FIG(34) COMPARISON BETWEEN EXPERIMENTAL AND ANALYTICAL DRY SURFACE HEAT TRANSFER CORRELATIONS FOR BOTH THE PRESENT TESTED COIL AND THAT PREVIOUSLY REPORTED IN THE LITERATURE(39).

The conditions of air and water obtained from the analytical calculation (Chapter III) versus those obtained experimentally for the three tests given in Figs. (30, 31, 32), are shown in Tables (6, 7 and 8) respectively.

The average value of h , calculated from the regression analysis is assumed to be constant over the coil surface. As indicated before, h may vary from one row to another. However, for a multi-row heat exchanger (more than four rows), it is expected that the average h is slightly less than the average value for the first two rows, and is slightly greater than that of the successive rows (see Fig.16)). This restricts the application of the analytical method to coils having four or more rows.

4. Summary of the Dry Surface Phase:

Analytical and experimental methods were used to predict the performance of an 8-row finned tube heat exchanger operating under dry surface conditions.

Experimental tests showed that the average J -factor is 3 to 5 percent lower than that predicted by the analytical simulation program.

Present experimental results showed that the average J -factor is 1 to 4 percent higher than that previously reported for tests on a 4-row coil.

The dry surface coefficients obtained from experimental data can be correlated using the following relation:

$$J = 0.101 \cdot Re^{-0.369} \quad (102)$$

TABLE 6
COMPARISON BETWEEN EXPERIMENTAL
AND ANALYTICAL RESULTS
DRY TEST D-A

Description	Experimental	Analytical
Average air temperature: °F		
- entering the cooling coil:	63.9	63.9
- leaving the cooling coil:	50.3	50.1
Average chilled water temperature: °F		
- entering the cooling coil:	46.8	46.8
- leaving the cooling coil:	49.6	49.7
Air specific humidity: lb_w/lb_a		
- entering the cooling coil:	0.0061	0.0061
- leaving the cooling coil:	0.0061	0.0061
Cooling load: Btu/hr * 10^{-3}		
- air side:	43.6	44.0
- water side:	43.3	43.5
Average air side heat transfer coefficient: Btu/ft ² -hr.*F	12.9	12.9
Average J-factor:	0.0072	0.0072

TABLE 7
COMPARISON BETWEEN EXPERIMENTAL
AND ANALYTICAL RESULTS
DRY TEST D-B

Description	Experimental	Analytical
Average air temperature: °F		
- entering the cooling coil:	80.6	80.6
- leaving the cooling coil:	53.4	52.9
Average chilled water temperature: °F		
- entering the cooling coil:	46.7	46.7
- leaving the cooling coil:	52.2	52.2
Air specific humidity: lb _w /lb _a		
- entering the cooling coil:	0.0065	0.0065
- leaving the cooling coil:	0.0065	0.0065
Cooling load: Btu/hr * 10 ⁻³		
- air side:	85.3	86.4
- water side:	85.5	85.4
Average air side heat transfer coefficient: Btu/ft ² -hr.°F	12.6	12.9
Average J-factor:	0.0072	0.0073

TABLE 8
COMPARISON BETWEEN EXPERIMENTAL
AND ANALYTICAL RESULTS
DRY TEST D-C

Description	Experimental	Analytical
Average air temperature: °F		
- entering the cooling coil:	93.6	93.6
- leaving the cooling coil:	50.7	49.9
Average chilled water temperature: °F		
- entering the cooling coil:	46.7	46.7
- leaving the cooling coil:	51.5	51.5
Air specific humidity: lb_w/lb_a		
- entering the cooling coil:	0.0075	0.0075
- leaving the cooling coil:	0.0075	0.0075
Cooling load: $\text{Btu/hr} \times 10^{-3}$		
- air side:	74.1	73.9
- water side:	74.1	74.9
Average air side heat transfer coefficient: $\text{Btu/ft}^2\text{-hr.}^\circ\text{F}$	8.7	8.6
Average J-factor:	0.0094	0.0092

CHAPTER VI

WET SURFACE PHASE

A finned tube heat exchanger was tested under wet surface conditions. Analysis of the heat exchanger performance is presented under such conditions; and a comparison is made between the experimental results and results obtained from the simulation program.

1. Analysis: Wet Surface Phase:

1.1 Water Film Thickness:

Condensate on heat exchange surfaces may result in a slight change in the area of heat transfer, but since the water film thickness is very small compared with the major physical dimensions of the heat exchanger, the change in the heat transfer area will be ignored. However, the condensate film thickness is usually of the same order of magnitude as the fin thickness (for commonly used heat exchangers). This may result in a change in the minimum air flow area, and hence the coil hydraulic diameter. Fig.(35) shows the effect of water film thickness on the free flow area of the tested coil.

It is difficult to measure the water film thickness on the outside surface of the heat exchanger during dehumidification process. However, based on visual inspection of the trailing section of the coil and results of previously published work (11, 39), it is assumed* that the water film is about

* The assumption is used to evaluate the water film resistance and to correct the minimum area of flow.

FIG.(35) THE EFFECT OF WATER FILM THICKNESS ON THE MINIMUM FLOW AREA OF A FINNED TUBE HEAT EXCHANGER.

0.004-inch thick. It is also assumed that the condensate film is uniform and of a constant thickness over the entire surface.

1.2 The Enthalpy Potential Method:

In this section, the enthalpy potential method, used to describe the performance of wet-surface heat exchangers, is explained. This technique has proven to be applicable when simultaneous heat and mass transfer mechanisms occur.

The chilled water stream is in contact with an infinitesimal surface area, as shown in Fig. (36), while the air stream is in contact on the other side of the surface. The air stream is considered to be composed of two distinct regions; the boundary layer region immediately adjacent to the water film surface, and the main air stream. Similarly, on the chilled water side, the boundary layer region after the metal surface is followed by the main water stream.

In the steady state condition, it is assumed that, for the small area dA_0 , the change in the main air-stream properties (W_a and t_a) is negligible compared to that across the boundary layer. Across the boundary layer, the air dry bulb temperature, t_a , and specific humidity W_a , decreases towards the water film. At the water film surface, $t_a = t_c$ and $W_a = W_{c,at}$. Applying an energy balance on the small area, dA_0 :

$$- \dot{m}_a di_a = dq + d\dot{m}_c \cdot i_c \quad (103)$$

FIG.(36) COOLING AND DEHUMIDIFICATION OF MOIST AIR BY AN INFINITESIMAL COLD SURFACE AREA dA_o .

where dq is the heat exchanged between air and water film.

A mass balance on the air flow gives:

$$d\dot{m}_c = -\dot{m}_a * dW_a \quad (104)$$

From equations (103) and (104),

$$-\dot{m}_a * di_a = dq - \dot{m}_a * i_c * dW_a \quad (105)$$

Neglecting the radiation effect, the net heat transferred dq can be expressed as:

$$dq = h_w * dA_o * (t_a - t_c) + h_m * dA_o * (W_a - W_{c,st}) * (i_{g,t_a} - i_c) \quad (106)$$

where

h_w is the average film heat transfer coefficient of a wet surface, and h_m is the average mass transfer coefficient.

A moisture mass balance gives:

$$d\dot{m}_c = h_m * (W_a - W_{c,st}) * dA_o \quad (107)$$

Introducing the Lewis number relation:

$$Le = \frac{h_w}{h_m * c_{p,a}} \quad (108)$$

Then, from equations (107), (108) and (106) one can write:

$$dq = \frac{h_w}{c_{p,a}} * \{c_{p,a} * (t_a - t_c) + \frac{(W_a - W_{c,st}) * (i_{g,t_a} - i_c)}{Le}\} * dA_o \quad (109)$$

The enthalpy of moist air can be written as (11):

$$i_a = c_{p,a} * t_a + 1061 * W_a \quad (110)$$

From equations (110) and (109) one can write:

$$dq = \frac{h_w * dA_o}{c_{p,a}} * \left\{ (i_a - i_{c,st}) + \frac{(W_a - W_{c,st}) * (i_{g,t_a} - i_c - 1061 * Le)}{Le} \right\} \quad (111)$$

The second group of terms of the R.H.S. of equation (111) is very small compared to the enthalpy difference $(i_a - i_{c,st})$. For typical moist air conditions, the second group of terms is less than 2 percent of the enthalpy difference (11)* Therefore, equation (111) can be simplified to:

$$dq = \frac{h_w}{c_{p,a}} * (i_a - i_{c,st}) * dA \quad (112)$$

Equation (112) is the basic heat transfer equation of moist air in contact with a wet surface.

1.3 Mean Enthalpy and Log-Mean Enthalpy Difference:

In a counter-cross flow heat exchanger, the mean enthalpy of air $i_{a,m}$ and saturated air enthalpy at chilled water temperature $i_{w,m,st}$ are (39):

* This is primarily due to the fact that Le is close to unity.

$$i_{a,m} = i_{a,1} + \frac{i_{a,2} - i_{a,1}}{\ln\left(\frac{i_{a,1} - i_{w,2,st}}{i_{a,2} - i_{w,1,st}}\right)} - \frac{(i_{a,2} - i_{a,1}) * (i_{a,1} - i_{w,2,st})}{(i_{a,1} - i_{w,2,st}) - (i_{a,2} - i_{w,1,st})} \quad (113)$$

and

$$i_{w,m,st} = i_{w,2,st} + \frac{i_{w,2,st} - i_{w,1,st}}{\ln\left(\frac{i_{a,1} - i_{w,2,st}}{i_{a,2} - i_{w,1,st}}\right)} - \frac{(i_{w,2,st} - i_{w,1,st}) * (i_{a,1} - i_{w,2,st})}{(i_{a,1} - i_{w,2,st}) - (i_{a,2} - i_{w,1,st})} \quad (114)$$

Equations (113) and (114) are analogous to the corresponding mean temperature equations given in Section 1.2 of Chapter V.

The log-mean enthalpy difference is:*

$$\Delta i_{l,m} = \frac{(i_{a,1} - i_{w,2,st}) - (i_{a,2} - i_{w,1,st})}{\ln\left(\frac{i_{a,1} - i_{w,2,st}}{i_{a,2} - i_{w,1,st}}\right)} \quad (115)$$

The complete derivation of equation (115) is given in Appendix G.

1.4 Overall Heat Transfer Coefficient:

For a multi-row heat exchanger, when both heat and mass transfer occur, the basic heat transfer equation becomes:

$$q = U_{o,w} * A_o * \Delta i_{l,m} \quad (116)$$

where $U_{o,w}$ is the overall heat transfer coefficient of a wet surface based on the total heat transfer area on the air side, A_o and $\Delta i_{l,m}$ as the driving force.

* In the case of cross-counter flow, correction factor is assumed to be unity (see footnote on page 83).

In a steady state condition, consider the schematic diagram in Fig.(37), where all the temperatures and enthalpies are assumed to be constant.

The enthalpy potential is used to develop an expression of $U_{o,w}$. One basic assumption is made in the following derivation. That is the saturation curve on the psychrometric chart, over a small interval of temperature, is a straight line. For example, the slope of the saturation curve between two temperatures, t_1 and t_2 , and the corresponding saturated enthalpies, $i_{1,st}$ and $i_{2,st}$ is (11):

$$b = \frac{i_{1,st} - i_{2,st}}{t_1 - t_2} \quad (117)$$

Referring to Fig.(37), and applying the enthalpy potential method, the overall heat transfer coefficient can be written as:

$$\frac{1}{U_{o,w}} = \frac{b_{w,a}}{h_{x,w} \eta_{o,w}} + \frac{z \cdot b_t \cdot B}{k_t} + FF \cdot b_d \cdot B + \frac{b_w \cdot B}{h_i} \quad (118)$$

The derivation of this relation is the same as described in Section 1.4 of Chapter V. The terms on the R.H.S. of equation (118) represent the outside air resistance, the tube wall resistance, the deposit layer resistance, and the inside resistance on the water side, respectively. $b_{w,a}$, b_t , b_d and b_w are the slopes of the saturation curves at the particular locations. By considering the ratio between the appropriate resistance to heat flow and the corresponding enthalpy potential, the different b 's can be calculated.

FIG.(37) SCHEMATIC DIAGRAM SHOWING HEAT TRANSFER PATHS DURING COOLING AND DEHUMIDIFICATION OF MOIST AIR WITH A FINNED TUBE SURFACE.

The quantity $h_{x,w}$ is the modified film heat transfer coefficient (enthalpy basis), taking the effect of the water film resistance into consideration (11), and is defined as:

$$h_{x,w} = \frac{1}{\frac{c_{p,a}}{b_{w,m} h_w} + \frac{z_w}{k_w}} \quad (119)$$

where h_w is the wet surface film heat transfer coefficient.

2. Test Procedure: Wet Surface Phase:

This section contains a general description of the wet surface tests, together with a summary of the experimental data.

2.1 General Test Procedure:

For the wet surface tests, the system is started in the same manner as described in Section 2.1 of Chapter V. In addition, a steam humidifier is switched on to supply the required humidification. When steady state conditions are achieved, the condensate is collected and weighed over two 15 minute periods to ensure that the condensate flow rate is steady.

2.2 Data Analysis:

Measured quantities are recorded on punched tape as explained in Chapter V. Fig.(38) shows a block diagram of the general data reduction program used for both dry and wet surface

FIG.(38) BLOCK DIAGRAM SHOWING THE INTERACTION BETWEEN THE EXPERIMENTAL AND ANALYTICAL PROGRAMS.

experimental data analysis. The heat picked up by the chilled water is calculated, see Appendix F. The log-mean enthalpy difference $\Delta i_{l,m}$ is evaluated using equation (115), then the overall heat transfer coefficient $U_{o,w}$ is calculated. Equation (100) is used to calculate the average heat transfer coefficient on the water side h_i . The different b's in equation (118) are evaluated as described in Section 1.4 of this chapter.

Since $\eta_{o,w}$ is dependent on h_w , therefore iteration is used to solve equation (118).

3. Test Results: Wet Surface Phase:

In this section, results of the present experimental work are presented. Comparison between the experimental and analytical coil performance is made.

3.1 Present Experimental Results:

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Inlet air conditions are selected to correspond to those of practical applications. Fig.(34) shows the entering air condition of the tested cases.

Fig.(40) shows the J_w -Re relationship of the tested coil, where J_w is the average heat transfer J-factor based on wet surface parameters. The above relationship can be expressed in the form:

$$J_w = C_{11} * Re^{C_{21}} \quad (120)$$

FIG.(39) SUMMARY OF WET-TEST AIR INLET CONDITIONS.

FIG. (40) CORRELATION OF WET SURFACE HEAT TRANSFER J-FACTOR.

The coil constants C_{11} and C_{22} are equivalent to C_1 and C_2 of the dry surface coil. From Fig.(40) one finds that C_{11} and C_{22} are 0.22 and -0.46, respectively. Unfortunately, no published data is available for a similar coil tested under wet-surface conditions for comparison.

In Fig.(41), the J-factor of the dry surface and wet surface of the test coil are plotted. There is an increase of about 25 percent in the J-factor at a Re of about 400. This increase dropped to about 12 percent at a Re of about 1200.

In general, the average heat transfer J-factor of a wet surface coil is higher than that of a dry surface heat exchanger at the same air flow Re, for the range of Re tested. This is a result of the combined heat and mass transfer mechanism occurring during the-cooling and dehumidification process. Certainly the surface of the water film would not be perfectly smooth, even if complete film-wise condensation occurred on the surface, and since a combination of drop-wise and film-wise condensation was present, the surface of the water film would be much rougher than the dry coil surface. This rough water film surface would probably promote additional turbulence in the air stream. The latter resulted in an increase in h_w and hence the average J-factor. The net result shows that the increase in h_w has compensated for the reduction in the J-factor due to the decrease in the $A_{m,f}$ (the J_w is proportional to the product $(h_w * A_{m,f,w})$).

FIG. (41) COMPARISON BETWEEN DRY AND WET SURFACES HEAT TRANSFER J-FACTOR CORRELATIONS.

The experimental data can be demonstrated on the psychrometric chart by plotting the entering and leaving conditions of the moist air. The actual shape of the process line cannot be determined from the present experimental work, since there were no measurements taken from row to row to determine intermediate conditions of the fluids. However, the analytical simulation program given in Chapter III can be used to predict the process line as well as the condition of air and chilled water, as explained in the next section.

Figs.(42 through 44) show some experimental results as plotted on the psychrometric chart, using the entering and leaving air condition. Details of these tests are given in Table (9).

3.2 Analytical Simulation Program:

The simulation program outlined in Chapter III, and the regression analysis are used to predict the J_w -Re relationship for a wet surface coil in the same manner as described in Chapter V.

Equations (55) and (56) are used to calculate C_1 and C_2 of a dry surface coil. Then equation (52) is used to evaluate J at a given Re. As the execution proceeds, the surface condition is checked for moisture condensation. Whenever the average fin surface temperature equals to or is lower than the air dew point temperature, condensation occurs and the

TABLE 9

WET SURFACE TEST RESULTS
RUNS W-A, W-B, and W-C

Description	Run W-A	Run W-B	Run W-C
Average air temperature:			
- entering the cooling coil °F	70.4	80.3	93.1
- leaving the cooling coil °F	54.1	51.5	52.0
Average specific humidity:			
- entering the cooling coil lb_w/lb_a	0.0125	0.0093	0.0168
- leaving the cooling coil lb_w/lb_a	0.0086	0.0077	0.0081
Chilled water temperature:			
- entering the cooling coil °F	46.8	47.0	46.8
- leaving the cooling coil °F	52.6	51.5	53.4
Heat extraction:			
Air side: Btu/hr * 10^{-3}	90.7	70.3	102.7
Water side: Btu/hr * 10^{-3}	91.0	70.3	102.3
Air flow Re	1030	730	480
Average J-factor	0.0096	0.0109	0.0115
Sensible heat Btu/hr * 10^{-3}	41.9	53.1	49.5

FIG.(42) COOLING AND DEHUMIDIFICATION OF MOIST AIR
DATA OF EXPERIMENT W-A.

Dry-bulb temperature °F
FIG.(43) COOLING AND DEHUMIDIFICATION OF MOIST AIR
DATA OF EXPERIMENT W-B.

FIG.(44) COOLING AND DEHUMIDIFICATION OF MOIST AIR
DATA OF EXPERIMENT W-C.

J-factor of the dry surface is modified to calculate J_w as follows:

$$J_w = J * C_J \quad (121)$$

where C_J is given by:

$$C_J = 1.425 - 0.51 * \left(\frac{Re}{1000}\right) + 0.263 * \left(\frac{Re}{1000}\right)^2 \quad (122)$$

The above equation is extracted from the present experimental work. Equation (122) shows good agreement with a previously reported work by Myers (39) for a different heat exchanger. It should be noted that equation (122) is restricted to coils having four or more rows and $400 < Re < 1500$. More experimental data is necessary to extend the range of applicability of the relationships.

The analytical simulation program is checked against the present experimental results. The row-by-row analysis method proved to be in good agreement with the experimental ~~results~~, as illustrated in Figs.(45), (46) and (47). The process line is plotted as a dotted line, since some approximations were made regarding the performance of the first two rows of the heat exchanger. Meanwhile, the final results show that the heat extraction rate and the outlet conditions of the two fluids are within 1 to 2 percent of the measured quantities for the tested cases. Further details regarding uncertainty are given in Appendix H. Tables (10), (11) and (12) give the details of the particular runs shown in Figs.(45), (46) and (47). The boundary between the wet, and dry portions of the coil can be predicted readily with the simulation program. This boundary, however,

TABLE 10
COMPARISON BETWEEN EXPERIMENTAL
AND ANALYTICAL RESULTS
WET TEST W-A

Description	Experimental	Analytical
Average air temperature: °F		
- entering the cooling coil:	70.4	70.4
- leaving the cooling coil:	54.1	54.4
Average chilled water temperature: °F		
- entering the cooling coil:	46.8	46.8
- leaving the cooling coil:	52.6	52.6
Air specific humidity: lb _w /lb _a		
- entering the cooling coil:	0.0125	0.0125
- leaving the cooling coil:	0.0086	0.0089
Cooling load: Btu/hr * 10 ⁻³		
- air side	90.7	86.7
- water side	91.0	88.3
Average air side heat transfer coefficient: Btu/ft ² -hr.°F	14.3	13.6
Average J-factor:	0.0096	0.0091
Sensible heat load: Btu/hr * 10 ⁻³	41.9	42.0

TABLE 11
COMPARISON BETWEEN EXPERIMENTAL
AND ANALYTICAL RESULTS
WET TEST W-B

Description	Experimental	Analytical
Average air temperature: °F		
- entering the cooling coil:	80.3	80.3
- leaving the cooling coil:	51.5	51.3
Average chilled water temperature: °F		
- entering the cooling coil:	47.0	47.0
- leaving the cooling coil:	51.5	51.5
Air specific humidity, lb _w /lb _a		
- entering the cooling coil:	0.0093	0.0093
- leaving the cooling coil:	0.0077	0.0079
Cooling load: Btu/hr * 10 ⁻³		
- air side	70.3	68.3
- water side	70.3	69.3
Average air side heat transfer coefficient: Btu/ft ² -hr.*F	11.6	11.1
Average J-factor:	0.0109	0.0104
Sensible heat load Btu/hr * 10 ⁻³	53.7	53.5

TABLE 12
COMPARISON BETWEEN EXPERIMENTAL
AND ANALYTICAL RESULTS
WET TEST W-C

Description	Experimental	Analytical
Average air temperature: °F		
- entering the cooling coil:	93.1	93.1
- leaving the cooling coil:	52.0	52.4
Average chilled water temperature: °F		
- entering the cooling coil:	46.8	46.8
- leaving the cooling coil:	53.4	53.3
Air specific humidity: lb _w /lb _a		
- entering the cooling coil:	0.0168	0.0168
- leaving the cooling coil:	0.0081	0.0082
Cooling load: Btu/hr * 10 ⁻³		
- air side	102.7	103.8
- water side	102.3	103.0
Average air side heat transfer coefficient: Btu/ft ² -hr.*F	8.0	8.6
Average J-factor:	0.0115	0.0125
Sensible heat load Btu/hr * 10 ⁻³	49.5	49.7

FIG.(45) COMPARISON BETWEEN EXPERIMENTAL AND ANALYTICAL COOLING COIL PERFORMANCE DURING DEHUMIDIFICATION PROCESS, RUN W-A.

FIG.(46) COMPARISON BETWEEN EXPERIMENTAL AND ANALYTICAL COOLING COIL PERFORMANCE DURING DEHUMIDIFICATION PROCESS, RUN W-B.

FIG.(47) COMPARISON BETWEEN EXPERIMENTAL AND ANALYTICAL COOLING COIL PERFORMANCE DURING DEHUMIDIFICATION PROCESS, RUN W-C.

cannot be easily verified, since it requires a rather complicated experimental set-up.

As the moist air flows through the cooling coil core the air temperature (and also in some cases, specific humidity) changes. The simulation program gives the air temperature and specific humidity at the boundary between each two successive rows. Fig.(47) shows the condition line of the tested cooling coil when cooling and dehumidification occur. In the first two rows sensible cooling is the only mechanism of heat transfer existing, therefore, no change in the air specific humidity is detected. As the air flows through the rest of the coil core both sensible and latent heat are removed from the air stream and a drop in the air temperature and specific humidity are noticed, as shown in Fig.(47). This example shows that the boundary between the dry and wet surface can be determined by using the simulation program.

Dry and wet surface heat transfer data reported by Myers (39) were used to test the simulation program. The results of the simulation program (air and water temperatures, heat transfer rate and air specific humidity) were ± 1 to ± 2 percent of the corresponding measured quantities. For example, the appropriate input information was fed into the program and the results were compared with the published data. Fig.(48) shows the process line as predicted by the analytical solution, together with the experimentally measured air conditions, as plotted on a psychrometric chart. Table (13) gives more detail about these experimental and analytical results. It should be noted here that the average heat transfer coefficient of the dry surface was calculated using the results of the regression analysis

TABLE 13
COMPARISON BETWEEN EXPERIMENTAL
AND ANALYTICAL RESULTS OF
PREVIOUSLY REPORTED TEST (39)

Description	Experimental	Analytical
Average air temperature: °F		
- entering the cooling coil:	80.5	80.5
- leaving the cooling coil:	52.0	52.2
Average chilled water temperature: °F		
- entering the cooling coil:	40.0	40.0
- leaving the cooling coil:	43.5	43.7
Air specific humidity: lb _w /lb _a		
- entering the cooling coil:	0.0109	0.0109
- leaving the cooling coil:	0.0073	0.0074
Cooling load: Btu/hr *10 ⁻³		
- air side	30.0	29.8
- water side	30.0	30.2
Average air side heat transfer coefficient: Btu/ft ² -hr.°F	12.3	12.6
Average J-factor:	0.0105	0.0106
Sensible heat load Btu/hr *10 ⁻³	19.3	19.2

FIG.(48) COOLING AND DEHUMIDIFICATION OF MOIST AIR: COMPARISON
BETWEEN PRESENT ANALYTICAL RESULTS WITH PREVIOUSLY -
REPORTED EXPERIMENTAL DATA (39).

discussed in Chapter III, and then the correction factor given by equation (122) was introduced to calculate the wet surface average heat transfer coefficient.

4. Summary of the Wet Surface Phase:

The coil under consideration was tested under wet surface conditions. Observations of the fin surface indicated that a combination of film-wise and drop-wise condensation occurred. It was therefore very difficult to make an accurate determination of the water film thickness.

Experimental results showed that the wet surface coefficients can be formulated as follows:

$$J_w = 0.22 * Re^{-0.46} \quad (123)$$

The analytical simulation program was used to predict the cooling coil performance under wet surface conditions. The analytical results were found to be 3 to 5 percent higher than the corresponding experimental figures.

The wet surface J-factor was increased by 16 to 25 percent over the dry surface J-factor, for air flow Re of about 1500 and 400, respectively.

The simulation program was successfully used to determine the boundary between dry and wet regions of the cooling coil surface during cooling and dehumidifying process.

CHAPTER VII

CONCLUSIONS AND RECOMMENDATIONS

1. Conclusions:

Performance of a finned tube, multi-row heat exchanger is presented both analytically and experimentally. The coil was tested under both dry and wet surface conditions.

Regression analysis was successfully used to predict the heat transfer J-factor of a multi-row heat exchanger operating under dry surface conditions. The J-factor predicted, by the above method, was 3 to 5% higher than those values reported previously.

The dry surface tests showed that the average heat transfer J-factor was about 1 to 4 percent higher than that of a similar coil previously reported in the literature. The difference is believed to be due to the uncertainties in both sets of test results.

The analytical simulation program was used to predict the cooling coil performance with both wet and dry surface conditions. The predicted coil performance parameters were found to be 3 to 5 percent higher than the corresponding experimental parameters.

The present work is a new source of information to be added to the few already available regarding the performance of multi-row heat exchangers in the presence of sensible and latent heat transfer. It provides also a test facility for furthering understanding of such components.

2. Recommendations:

The regression analysis presented in this thesis would be more useful if all the varieties of coils in HVAC systems were included. The suggestion is therefore made that more coils be tested to extend the range of coil parameters given in Chapter III to cover all the coils in common use in practice, if possible.

Certainly the limited amount of information about wet surface heat transfer data can be overcome by performing more experimental work under such surface conditions. In this case a similar regression analysis of wet surface heat transfer data can be done to present a simple J_w -Re relationship that would be applicable for most of the cooling and dehumidifying coils.

Perhaps the model presented in this thesis for cooling and dehumidifying coil performance can be simplified without significant sacrifice of accuracy of the results. This goal is planned in the future as an extension of the present work.

Finally, for comparison reasons, standard methods of testing coils (both dry and wet surface tests) reducing the heat transfer data should be established. It is, therefore, suggested that a technical association such as the ASHRAE, ASME, CSME or a similar engineering group, undertake preparation of such standard method.

NOMENCLATURE

a	Constant	lb_w/lb_a
a_o through a_s	Constants in equation (54)	-
A	Area	ft^2
A_r	Coil minimum flow area/face area	-
A'	Area per unit length	ft^2/ft
$A_{m,f}$	Minimum flow area	ft^2
b	Constant in equation (75)	$\text{lb}_w/\text{lb}_a \cdot ^\circ\text{F}$
b_d	Slope of the saturation line between temperatures $t_{t,i}$ and t_d	$\text{Btu}/\text{lb}_a \cdot ^\circ\text{F}$
b_t	Slope of the saturation line between temperatures $t_{t,o}$ and $t_{t,i}$	$\text{Btu}/\text{lb}_a \cdot ^\circ\text{F}$
b_w	Slope of the saturation line between temperatures t_d and t_w	$\text{Btu}/\text{lb}_a \cdot ^\circ\text{F}$
$b_{w,m}$	Slope of the saturation line at the mean water film temperature	$\text{Btu}/\text{lb}_a \cdot ^\circ\text{F}$
B	External to internal heat transfer area ratio	-
c	Specific heat	$\text{Btu}/\text{lb} \cdot ^\circ\text{F}$
$c_{p,a}$	Air specific heat	$\text{Btu}/\text{lb} \cdot ^\circ\text{F}$
C	Coil dimension	ft
C_1 and C_2	Constants in equation (52)	-
C_{11} and C_{22}	Constants in equation (120)	-

D_e	Equivalent diameter as referred to in Reference 31	ft
D_h	Hydraulic diameter as defined by equation (101)	ft
e	Dimensionless emf as defined by equation (125)	-
$e_1, e_{\min},$ and $e_{\max}$	Thermocouple outputs	m.v.
E	Stored energy	Btu/lb
f	Pressure drop factor as referred to in Reference 19	-
F	Fin dimension	ft
$F_{d,e}$	Equivalent fin diameter as defined in Reference 24	ft
FF	Fouling factor	-
G	Mass flux	lb/hr·ft ²
h	Average film heat transfer coefficient	Btu/hr·ft ² ·°F
h_m	Mass transfer coefficient based on the air specific humidity difference	lb _w /hr·ft ² · $\frac{lb_w}{lb_a}$
$h_{x,w}$	Combined heat transfer coefficient of a wet surface as defined by equation (119)	Btu/hr·ft ² ·°F
i	Enthalpy	Btu/lb
I_0, I_1	Modified Bessel's functions of the first kind and of zero and first order respectively	-

J	Heat transfer factor defined by equation (1)	-
JJ	Dimensionless heat transfer factor defined by equation (8)	-
k	Thermal conductivity	Btu/hr·ft ² ·°F/ft
K ₀ , K ₁	Modified Bessel's function of the second kind and of zero and first order respectively	-
K _m , K _D , K _w	Calibration constants defined in Appendices D and E	-
L	Flow pass length	ft
Le	Lewis number	-
m	Fin parameter defined by equation (59)	-
m	Mass per unit length	lb/ft
$\dot{m}$	Mass rate of flow	lb/hr
M _a	Quantity defined by equation (32)	Btu/ft·°F
N _f	Number of fins per inch	in ⁻¹
N _{pr}	Number of tubes per row in the tube bank	-
N _r	Number of rows of a tube bank	-
Nu	Nusselt number	-
PF	Dimensionless pumping factor as defined by equation (9)	-
P	Pressure	lb/ft ²
Pr	Prandtl number	-

q	Heat transfer rate	Btu/hr
Q	Heat transfer rate	Btu/hr
r	Radius	ft
r_e, r_o	Outer and base fin radius respectively	ft
r_1	Parameter defined on Fig.(4)	-
R_a	Resistance to heat flow per unit length	hr·ft·°F/Btu
Re	Reynolds number	-
s	Parameter defined on Fig.(4)	-
S	Tube dimension	ft
St	Stanton number	-
t	Time	hr
t	Temperature	°F
T_r	Temperature excess ($t_r - t_a$)	°F
$T_{r,o}$	Temperature excess ($t_{r,o} - t_a$)	°F
u	Internal energy	Btu/lb.
U_o	Overall heat transfer coefficient (temperature basis)	Btu/hr·ft ² ·°F
$U_{o,w}$	Overall heat transfer coefficient (enthalpy basis)	lb _a /hr·ft ²
V	Velocity	ft/hr
W	Specific humidity	lb _w /lb _a
W_o	Specific humidity excess ($W_{r,o,st} - W_a$)	lb _w /lb _a
W_r	Specific humidity excess ($W_{r,st} - W_a$)	lb _w /lb _a

x	Variable	-
X	Distance	ft
Y	Distance	ft
Y	Coefficient in equation (124)	-
Z	Thickness	-
Δ	Difference	
$\Delta i_{l,m}$	Log-mean enthalpy difference defined by equation (115)	Btu/lb _a
Δp	Pressure difference	lb/ft ²
$\Delta t_{l,m}$	Log-mean temperature difference defined by equation (86)	°F
η	Fin efficiency	-
η_c, η_s	Fin parameters defined by equations (14) and (13) respectively	-
η_{ov}	Overall heat exchanger efficiency	-
λ	Latent heat of evaporation	Btu/lb _w
μ	Dynamic viscosity	lb/ft·hr
ϕ	Function	-
σ	Standard deviation defined by equation (147)	-

Subscripts

a	Dry air
b	Fin base
c	Saturated water
d	Deposit
d	Depth
d	Diameter
d	Dry
f	Fin
g	Saturated water vapor
h	Height
h	Hydraulic
i	Inside tube condition
l	Longitudinal
m	Mean
max	Maximum
min	Minimum
o or ov	Overall (or total)
p	Primary
r	Condition at a radius
s	Fin spacing
s	Secondary
st	Saturation
w	Water
w	Wet
t_a	Refers to a condition at t_a
t	Thickness

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APPENDIX A

THERMOCOUPLES CALIBRATION

All the thermocouples used in the present work are 24-gauge copper-constantan. Fig.(49) shows a schematic diagram of the apparatus used for calibration. The thermocouples and their shields are totally immersed in a silicone oil bath, the latter is connected to a cooler-heater oil supply. The oil bath temperature is maintained constant at a predetermined temperature by means of an auxiliary heater and stirrer. An ice bath is used as a cold junction for all thermocouples.

The thermocouples were calibrated against a standard platinum resistance thermometer between temperatures 60 and 115°F. Fig.(50) shows the thermocouple output in μv , versus the reference temperature as measured by the Pt-resistance thermometer.

Comparing the present temperature correlations with those reported by Shirtliffe (60) for the same type of thermocouple, it was found that the present results were within ± 0.02 °F of the reported temperature correlations over the tested temperature range. The following polynomial relationship is used to determine the measured temperature in terms of the measured thermocouple output in μv (60):

$$\frac{t_i - t_{\min}}{t_{\max} - t_{\min}} = \sum_{i=0}^3 y_i \cdot e^i \quad (124)$$

FIG.(49) SET-UP FOR THERMOCOUPLE CALIBRATION.

FIG. (50) CALIBRATION CURVE FOR A COPPER-CONSTANTAN THERMOCOUPLE.

where: t_1 is the measured temperature, and $t_{\min}$ and $t_{\max}$ are the lower and upper temperatures of the considered temperature range; 0°F and 200°F respectively.

The dimensionless thermocouple output e is defined as:

$$e = \frac{e_1 - e_{\min}}{e_{\max} - e_{\min}} \quad (125)$$

where e_1 , $e_{\min}$ and $e_{\max}$ are the corresponding thermocouple readings at t_1 , $t_{\min}$ and $t_{\max}$ respectively. The coefficients in equation (124) can be found in Reference (60).

APPENDIX B

DEW-POINT TEMPERATURE MEASUREMENT

The dew-point temperature of the air stream is measured at two locations in the air duct: upstream and downstream of the cooling coil. The dew-point sensing element (DEWCEL ELEMENT) is covered with lithium chloride solution and the heating element is connected to a 25-volt A.C. power supply. Details and theory of operation of the DEWCEL element can be found in Reference (62).

A 30-gauge copper-constantan thermocouple is attached to the humidity sensing element to measure the equilibrium temperature. The latter is plotted against the dew-point temperature of the surrounding air.

Calibration of the DEWCEL elements was done at the Division of Building Research of the National Research Council. The relationship between the DEWCEL thermocouple reading and the air dew-point temperature is shown in Fig.(51).

As recommended by the manufacturer, the DEWCEL element functions perfectly in still air and does not require a high velocity for accurate measurement. In cases where air velocity exceeds 50 feet per minute, a special draft shield has to be used to house the DEWCEL element. Fig.(52) shows a sketch of the draft shield used in this work, as suggested by the manufacturer, which is suitable for flow velocities up to 2000 feet per minute.

FIG(51) CALIBRATION CURVE FOR THE DEWCEL-ELEMENT INSTALLED
UPSTREAM OF THE COOLING COIL.

Draft shield

Air duct

Dewcel element

Air flow

Installation of the Dewcel element

FIG(52) DRAFT SHIELD FOR THE INSTALLATION OF THE DEWCEL ELEMENT.

APPENDIX C

CALIBRATION OF THE ELECTRIC POWER TRANSDUCERS
AND THE RESISTANCE THERMOMETERS

1. Electric Power Transducers:

Electric power transducers are installed to measure the electric power supplied to the immersion heaters, air duct heaters, and the calibration heaters. Calibration of these transducers is done at the Division of Building Research of the National Research Council of Canada. It is found that the uncertainty in measuring the electric power in terms of standard deviation is about + 0.5 percent.

2. Pt-Resistance Thermometer:

Three Pt-resistance thermometers were used to measure the chilled water temperature before and after the cooling coil and after the immersion heaters ($t_{w,1}$, $t_{w,2}$ and $t_{w,3}$ respectively), see the chilled water circuit in Fig.(20).

The Pt-resistance thermometers were calibrated indirectly in terms of the temperature differences across the cooling coil and the immersion heaters, since the latter are of primary importance in the data reduction procedure.

For the calibration purposes, the cooling coil is replaced by a 24-kilowatt electric heater (substitute heater).

If P_1 and P_2 are the electric power input in kilowatts to the substitute heater and immersion heaters respectively, then

referring to Fig.(20) we may write:

$$\frac{P_1}{P_2} = \frac{t_{w,2} - t_{w,1}}{t_{w,3} - t_{w,2}} = \frac{\Delta t_{2-1}}{\Delta t_{3-2}}$$

The calibration procedure was as follows:

The power input P_1 and P_2 are measured using the electric power transducers. The quantities Δt_{2-1} and Δt_{3-2} are calculated using the Pt-resistance thermometers under consideration. This was repeated for P_1 from 0 to about 20 kilowatts, and for P_2 from 0 to about 12 kilowatts.

Experimental results showed that the uncertainty in measuring the temperature difference ratio $\frac{\Delta t_{2-1}}{\Delta t_{3-2}}$ in terms of standard deviation was about 0.5 to 2 percent of the measured value.

Maintaining P_2 constant and changing P_1 , the same margin of uncertainty was noticed in the measured Δt_{2-1} . This procedure is repeated while keeping P_1 constant and changing P_2 .

APPENDIX D

AIR FLOW RATE MEASUREMENT

Two different methods were used to calculate the air flow rate circulating in the air duct:

- 1 - velocity profile method
- 2 - electric duct heater method.

The theoretical background of both methods is covered in this appendix.

1. Velocity Profile Method:

The mass rate of flow of air $\dot{m}_a$, passing through an area A, can be written as:

$$\dot{m}_a = \rho_a * A * V_a \quad (126)$$

where ρ_a is the average air density. The average air velocity V_a can be expressed in terms of the velocity head as follows:

$$V_a = K_v * \sqrt{\frac{\Delta p}{p}} * T \quad (127)$$

where:

Δp is the velocity head in inches of water, p is the static air pressure, and T is the average air temperature in degrees Rankine.

Substituting equation (127) into equation (126), and summing over the entire duct area (16 points), then:

$$\dot{m}_a = K_m \sum_{i=1}^{16} A \sqrt{\frac{\Delta P_i \cdot P_i}{T_i}} \quad (128)$$

A 16-point total pressure-probe grid is installed upstream of the cooling coil to measure the total pressure of the air stream. An average value of the static pressure (average of four readings at 4 different points) is taken at the same section of the duct. The dynamic head is measured by a pressure transducer. The output of the pressure transducer is recorded on a strip-chart and a digital voltmeter. The calibration of the pressure transducer is done at the National Research Council (61), and that of the static pressure probes is given in Appendix I.

A 16-point thermocouple grid is used to measure the average air dry-bulb temperature. Fig.(25) shows a schematic diagram of the pressure and temperature measuring circuit. Figs.(53) and (54) show schematic diagrams of the velocity and temperature profiles measured upstream of the cooling coil, respectively.

2. Electric Duct Heater Method:

At relatively low air velocities (below 250 feet per minute), the velocity head becomes very small and the accuracy of measurement becomes uncertain. Therefore the power input to the duct heater is used to calculate the average air flow rate as follows:

Assume that the average air dry-bulb temperature before and after the duct heater are $t_{a,3}$ and $t_{a,4}$, respectively, then the air flow rate is given by:

FIG.(53) VELOCITY PROFILES OVER THE DUCT SECTION UPSTREAM OF THE COOLING COIL FOR THREE DIFFERENT TESTS.

FIG. (54) TEMPERATURE PROFILES OVER THE DUCT SECTION UPSTREAM OF THE COOLING COIL
FOR TWO DIFFERENT TESTS.

$$\dot{m}_a = K_D * \frac{P_D}{c_{p,a} * (t_{a,4} - t_{a,3})} \quad (129)$$

where K_D is a constant to convert the power transducer output P_D (power input to the duct heater) in (mv) into Btu/hr.

Two 16-points thermocouple grids are used to measure the average air temperatures $t_{a,3}$ and $t_{a,4}$. Specially designed thermocouple shields are used to prevent radiation between the heating elements of the duct heaters and the thermocouples, (see Chapter IV).

Cross-checking tests on the two methods of calculating $\dot{m}_a$ showed that for air velocity higher than 250 ft/min, the difference between the calculated $\dot{m}_a$ using the two methods was up to about 2.0 percent of the average value. In cases where V_a varied between 150 and 250 ft/min, the difference in $\dot{m}_a$ calculated by the two methods was about 4.0 to 5.0 percent of the average. No attempt was made to check the two methods when V_a was less than 150 ft/min.

APPENDIX E

CHILLED WATER FLOW RATE MEASUREMENT

The chilled water flow rate is measured, indirectly, by measuring the power input to two electric immersion heaters placed in the chilled water circuit, and the corresponding water temperature rise. Referring to Fig.(20), the average water temperatures are measured at two points; before and after the immersion heaters, by two platinum-resistance thermometers. A mechanical mixer is placed ahead of each temperature measuring element to ensure proper mixing and to avoid temperature gradients in the pipe cross-section. Immersion heaters with about 12 kilowatt capacity are used to heat the circulated water, and the power input is measured by an electric power transducer. The output of the power transducer is recorded on the punched paper tape for data reduction purposes.

If $t_{w,2}$ and $t_{w,3}$ are the average chilled water temperatures before and after the immersion heaters, respectively, then the water flow rate $\dot{m}_w$, can be written as:

$$\dot{m}_w = K_w * \frac{P_w}{(t_{w,3} - t_{w,2}) * c_p} \quad (130)$$

where K_w is a conversion factor to convert the power transducer reading P_w (in m.v.) into Btu/hr.

APPENDIX F

HEAT EXTRACTION CALCULATION

The heat extraction is calculated on both the air and the water sides, as a means of checking the results. Comparison between the results of both methods is discussed in this appendix.

1. Air Side Calculation

The dry bulb and dew-point temperatures are measured upstream and downstream of the cooling coil. The mass rate of flow of air is calculated as explained in Appendix D, therefore, the heat extraction on the air side is given by,

$$Q_a = \dot{m}_a * (i_{a,1} - i_{a,2}) - \dot{m}_c * i_c \quad (131)$$

The average air enthalpies $i_{a,1}$ and $i_{a,2}$ are calculated, knowing the corresponding dry-bulb and dew-point temperatures. The condensate rate $\dot{m}_c$, and its temperature are measured directly by collecting the condensate, as described in Chapter V.

2. Chilled Water Side Calculation

Average chilled water temperatures are measured upstream and downstream of the cooling coil. The mass rate of flow of the chilled water is calculated as described in Appendix E. Referring to Fig.(20), the heat extraction on the water side can be written as:

$$Q_w = \dot{m}_w * c_p * (t_{w,2} - t_{w,1}) \quad (132)$$

Substituting the quantity $\dot{m}_w$ from equation (130) of Appendix E into equation (132), then,

$$Q_w = K_w * P_w * \frac{t_{w,2} - t_{w,1}}{t_{w,3} - t_{w,2}} \quad (133)$$

Equation (133) gives the heat extraction on the water side.

The calibration of the power transducers and the resistance thermometers is given in Appendix C.

Cross-checking tests on the results of the given two methods of calculating the heat extraction is done in the data reduction program. In cases of air velocity $V_a > 250$ ft/min., the difference $(Q_a - Q_w)$ was about ± 2.0 percent of the average value $(Q_a + Q_w)/2.0$. In some cases where $V_a < 250$ ft/min that difference increased up to about 5.0 percent. This is explained as a result of the higher uncertainty in calculating the air flow rate $\dot{m}_a$ at the lower range of air velocities, see Appendix H for complete uncertainty analysis.

APPENDIX G

LOG-MEAN ENTHALPY DIFFERENCE $\Delta i_{l,m}$

Consider the section of the coil shown on Fig.(55).
Neglecting the energy of condensate over an infinitesimal
area dA_o , then we may write:

$$U_{o,w} * (i_a - i_{w,st}) * dA_o = -\dot{m}_a * di_a \quad (134)$$

From the energy balance, we have:

$$q = \dot{m}_a * (i_{a,1} - i_a) = \dot{m}_w * c_r * (t_{w,2} - t_w) \quad (135)$$

Introducing the slope of the saturation line between the two
temperatures $t_{w,2}$ and t_w , and the two fictitious enthalpies
 $i_{w,2,st}$ and $i_{w,st}$, then we may write:

$$\dot{m}_a * (i_{a,1} - i_a) = \frac{\dot{m}_w * c_r}{b_f} * (i_{w,2,st} - i_{w,st}) \quad (136)$$

where b_f is defined as:

$$b_f = \frac{i_{w,2,st} - i_{w,st}}{t_{w,2} - t_w} \quad (137)$$

Rearranging equation (136), we may write:

$$\frac{\dot{m}_a * b_f}{\dot{m}_w * c_r} = \frac{i_{w,2,st} - i_{w,st}}{i_{a,1} - i_a} \quad (138)$$

FIG.(55) ENTHALPY PROFILES IN A COUNTER-FLOW
HEAT EXCHANGER.

From equations (134) and (138), and by eliminating $i_{w,st}$, we get:

$$\frac{U_{o,w} * dA_o}{\dot{m}_a} = \frac{di_a}{i_{w,2,st} - \frac{\dot{m}_a * b_f}{\dot{m}_w * c_r} * (i_{a,1} - i_a) - i_a} \quad (139)$$

Equation (139) can be rearranged in the form:

$$\frac{\dot{m}_a}{h_{a,1}} \frac{di_a}{L_1 - L_2 * i_a} = \int_0^{A_o} \frac{U_{o,w} * dA}{\dot{m}_a} \quad (140)$$

where:

$$L_1 = i_{w,2,st} - \frac{\dot{m}_a * b_f}{\dot{m}_w * c_r} * i_{a,1} \quad (141)$$

and

$$L_2 = 1 - \frac{\dot{m}_a * b_f}{\dot{m}_w * c_r} \quad (142)$$

Taking advantage of the previous assumptions that both $U_{o,w}$ and $\dot{m}_a$ are constants, then integration of equation (140) gives:

$$\ln \left(\frac{L_1 - L_2 * i_{a,1}}{L_1 - L_2 * i_{a,2}} \right) = L_2 * \frac{U_{o,w} * A_o}{\dot{m}_a} \quad (143)$$

Combining equations (138) and (143), and using the fact that

$q = \dot{m}_a * (i_{a,1} - i_{a,2})$, then we get:

$$\ln\left(\frac{i_{a,1} - i_{a,2}}{i_{w,2,st} - i_{w,1,st}}\right) = \frac{U_{o,w} * A_o}{q} * ((i_{a,1} - i_{w,2,st}) - (i_{a,2} - i_{w,1,st})) \quad (144)$$

Now, by definition, the log-mean enthalpy difference is expressed as:

$$\Delta i_{l,m} = \frac{q}{U_{o,w} * A_o} \quad (145)$$

Therefore, from equations (144) and (145), we obtain:

$$\Delta i_{l,m} = \frac{(i_{a,1} - i_{w,2,st}) - (i_{a,2} - i_{w,1,st})}{\ln\left(\frac{i_{a,1} - i_{w,2,st}}{i_{a,2} - i_{w,1,st}}\right)} \quad (146)$$

Equation (146) is used for counterflow coils, or multi-pass (three or more) counter-crossflow coils.

APPENDIX H
UNCERTAINTY ANALYSIS

In this appendix, the uncertainties in evaluating the different parameters are discussed.

The uncertainties associated with the quantities of interest in this study are determined by the well-known methods of statistics described in References 63 and 64. The uncertainty is expressed in terms of the standard deviation σ , which is defined as:

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - x_m)^2} \quad (147)$$

where x_m is the mean value of the variable x_i taken over an N number of measurements.

Assuming a normal distribution of error, then the 95 percent confidence limit is equal to two standard deviations.

In this analysis, it is assumed that for uncertainties which are known only in terms of absolute limits, the 2σ or 95 percent confidence limit corresponds to the absolute limit of the uncertainties.

For example, the thermocouples were calibrated against a standard accurate to $\pm 0.2^\circ\text{F}$; in this case $2\sigma = 0.2^\circ\text{F}$, or $\sigma = 0.1^\circ\text{F}$.

To express the standard deviation as a percentage of the mean temperature difference, let us assume that the mean air temperature is about 70°F , (with respect to 0°F as a reference).

Then:

$$\sigma_t = \frac{0.1}{70} \times 100 = 0.14\%$$

For a quantity, Q , calculated from several other quantities, $a, b, c \dots$, when $Q = f(a, b, c \dots)$ then the combined uncertainty associated with Q is given by:

$$\sigma_Q^2 = \left(\frac{\partial Q}{\partial a}\right)^2 \sigma_a^2 + \left(\frac{\partial Q}{\partial b}\right)^2 \sigma_b^2 + \left(\frac{\partial Q}{\partial c}\right)^2 \sigma_c^2 + \dots \quad (148)$$

where a, b and c are independent quantities.

Air Flow Rate:

The uncertainty associated with $\dot{m}_a$ depends on the method used to evaluate $\dot{m}_a$, and also on the associated uncertainties of the different quantities encountered.

When using the velocity method (see Appendix D) in evaluating $\dot{m}_a$, then:

$$\dot{m}_a = A * \left(\frac{\Delta p * \rho}{T} \right)^{1/2} \quad (149)$$

The associated uncertainties with the different quantities in equation (149) are as follows:

$$\sigma_{\Delta p} = 1.0\%$$

$$\sigma_p = 0.50\%$$

$$\sigma_T = 0.1\%$$

$$\sigma_A = 0.1\%$$

Then

$$\sigma_{\dot{m}_a} = 0.62\%$$

On the other hand, at low air flow rate ($V_a < 250$ ft/min), the electric duct heater is used for calculating the air flow, the associated equation is given in Appendix D:

$$\dot{m}_a = \frac{P_D}{c_p \Delta t_a} \quad (150)$$

and

$$\sigma_{P_D} = 0.5\%$$

$$\sigma_{\Delta t_a} = 1.0\%$$

$$\sigma_{c_p} = 0.3\%$$

Therefore:

$$\sigma_{\dot{m}_a} = (0.5^2 + 1.0^2 + 0.3^2)^{1/2} = 1.34\%$$

The uncertainty associated with the remaining quantities are calculated from equation (148) in a similar manner. These results are summarized in the remainder of this appendix.

Water Flow Rate:

$$\dot{m}_w \propto \frac{P_w}{\Delta t_w \cdot C_c}$$

$$\sigma_{P_w} = 0.5\%$$

$$\sigma_{\Delta t_w} = 1.0\%$$

$$\sigma_{C_c} = 0.3\%$$

$$\sigma_{\dot{m}_w} = 1.34\%$$

Heat Extraction Q:

At small loading conditions ($V_a < 250$ ft/min), the uncertainty in calculating Q is about 2.0%, and reaches about 1.0% at higher loads ($V_a > 250$ ft/min). Therefore, we may say:

$$\sigma_{Q_{\min}} = 1.0\%$$

$$\sigma_{Q_{\max}} = 2.0\%$$

Reynolds Number:

$$Re = \frac{\dot{m}_a}{\mu_a} \quad \text{and}$$

$$\mu_a \propto T_a^{0.7}$$

$$\sigma_{\mu_a} = 0.43\%$$

Because there are two different methods used in evaluating $\dot{m}_a$, then at low values of Reynolds number ($V_a < 250$ ft/min):

$$\sigma_{Re} = 1.19\%$$

and at high Reynolds number ($V_a > 250$ ft/min)

$$\sigma_{Re} = 0.76\%$$

Film Heat Transfer Coefficient:

$$h = \frac{Q}{A_o \Delta t_a}$$

$$\sigma_{\Delta t_a} = 1.0\%$$

$$\sigma_{A_o} = 0.1\%$$

$$\sigma_{Q_{\min}} = 1.0\%$$

$$\sigma_{h_{\min}} = 1.4\%$$

$$\sigma_{Q_{\max}} = 2.0\%$$

$$\sigma_{h_{\max}} = 2.3\%$$

J-Factor:

$$J \propto \frac{A_o \cdot h}{\dot{m}_a} \cdot \frac{\mu_a^{2/3}}{k_a^{2/3}} \cdot c_p^{2/3}$$

$$\sigma_{\mu_a} = 0.43\%$$

$$\sigma_{A_o} = 0.1\%$$

$$\sigma_{k_a} = 0.47\%$$

$$\sigma_{c_p} = 0.3\%$$

Then at the higher range of Reynolds number,

$$\sigma_{\dot{m}_a \min} = 0.62\%$$

$$\sigma_{J \min} = 2.37\%$$

and at the lower range of Reynolds number,

$$\sigma_{\dot{m}_a \max} = 1.34\%$$

$$\sigma_{J \max} = 2.9\%$$

The following table summarizes the uncertainties associated with the quantities of interest in this study.

TABLE 14

SUMMARY OF UNCERTAINTIES

Quantity	σ % min	σ % (max) max
Re	0.8	1.2
h	1.4	2.3
J	2.4	2.9

APPENDIX I

STATIC PRESSURE PROBES CALIBRATION

The static pressure probes were used to measure the air flow static pressure. Details of the probes can be found in Reference 57, page 208.

Calibration of these probes is done by comparing the measured static pressure with the corresponding reading of a calibrated Betz manometer. Fig.(56) is a plot of the present probe and Betz static pressure readings from 0 to about 6 inches of water. The associated uncertainty with the static pressure reading in terms of the standard deviation is found to be ± 0.5 percent. Note that the velocity is measured by a pressure transducer as described in Appendix D. Calibration of the pressure transducer is given in Reference 61.

FIG.(56) CALIBRATION CURVE OF THE STATIC PRESSURE PROBES.

APPENDIX J

COILS DATA FOR REGRESSION ANALYSIS

Coil parameters used in the regression analysis discussed in Chapter III are given in Table 15. The number of rows is six for coils 1 through 12 and 20, and is four rows for coils 13 through 17. No information is available about the number of rows of coils 18 and 19.

The fins of coils 1 through 12 are circular and those of coils 13 through 20 are flat plate fins.

Information about coils 1 through 12 is taken from Reference 25, for coils 13 through 17 is taken from Reference 32, for coils 18 and 19 is taken from Reference 19, and that for coil 20 is taken from Reference 39.

For all the given coils, the tube and fin materials are the same. Coils 4 and 7 through 12 are made of aluminum and the rest are made of copper.

TABLE 15

COILS DATA FOR REGRESSION ANALYSIS

Coil No.	S _d in	F _d in	N _f fin/in	D _h in	F _t in	Ar -	S _t in	S _l in
1	.625	.737	19.5	.187	.016	.352	.964	.838
2	.546	.762	11.3	.209	.017	.434	.964	.835
3	.554	.894	11.2	.186	.018	.487	1.08	.935
4	.438	.895	7.2	.293	.022	.59	1.08	.935
5	.549	.89	8.0	.22	.042	.49	1.08	.935
6	.535	.89	6.25	.24	.08	.50	1.08	.935
7	1.15	2.3	8.0	.24	.017	.53	2.45	2.13
8	.93	1.97	9.82	.203	.021	.59	2.25	1.95
9	.95	2.23	9.65	.181	.022	.613	2.45	2.13
10	1.16	2.75	7.9	.188	.018	.59	2.8	2.44
11	1.1	2.02	9.02	.196	.017	.5	2.19	1.9
12	1.03	2.0	10.1	.184	.013	.53	2.19	1.9
13	.53	1.32	6.67	.183	.006	.56	1.25	1.08
14	.53	1.32	7.67	.157	.006	.56	1.25	1.08
15	.53	1.32	9.17	.133	.006	.56	1.25	1.08
16	.53	1.32	11.7	.11	.006	.54	1.25	1.08
17	.53	1.32	14.5	.08	.006	.54	1.25	1.08
18	.4	1.05	8.0	.14	.013	.53	1.0	.87
19	.68	1.83	7.75	.14	.016	.48	1.5	1.75
20	.39	1.04	8.1	.15	.006	.58	1.0	.87

END

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