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Trailing Edge Flap Control of Dynamic Stall on Helicopter Rotor Blades

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Abstract

The use of an actively controlled trailing edge flap for the reduction of dynamic stall induced vibration is demonstrated via CFD. Two separate flap actuations are considered. In the first instance, a moderate advance ratio dynamic stall case is examined by using a 3D aeroelastic code, GAST, which couples a vortex particle method, and dynamics equations for a beam model. This case employs a downward flap deflection to reduce the portion of the rotor disk area over which dynamic stall is encountered. The second approach targets dynamic stall occurring in high advance ratio forward flight configurations. 2D RANS simulations were performed. In this case, an upward flap deflection is employed in order to control the strength of the trailing edge vortex. In both cases the flap served to reduce the amplitudes of vertical blade reaction forces, blade torsional moment and negative aerodynamic damping, indicating a net reduction in the vibration experienced by the rotors.

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List of Symbols

Chapter 1

R	Rotor radius, or Rotor blade span
$\bar{r}$	Radial location of the blade section
T	Thrust of main rotor
T_{TAIL}	Thrust of tail rotor
V_{∞}	Freestream velocity
ψ	Azimuth angle
Ω	Angular velocity

Chapter 2

C_d	Sectional drag coefficient
C_l	Sectional lift coefficient
C_m	Sectional moment coefficient
C_W	Work Coefficient
D.F	Torsional Damping Factor
V_{∞}	Freestream velocity
α	Angle of attack
$\dot{\alpha}$	Time rate of change of angle of attack

ψ	Azimuth angle
Ω	Angular velocity

Chapter 3

A	Cross section
a^D	Semi-empirical constant developed for the ONERA model
A_k	Complete rotational transformation matrix
a^M	Semi-empirical constant developed for the ONERA model
C	Influence coefficients
c	Chord length
c	Damping
c	Speed of sound
C_{Dlin}	Drag coefficient of the potential flow reference angle
C_l	Sectional lift coefficient
C_m	Sectional moment coefficient
C_{Mlin}	Moment coefficient of the potential flow reference angle
C_p	Pressure coefficient, at a point
C_W	Work Coefficient
D	Drag
dF	Net elastic internal force

$D.F$	Torsional Damping Factor
d^L	Semi-empirical constant developed for the ONERA model
d^M	Semi-empirical constant developed for the ONERA model
dy	Section width
D_ω	The flow domain
E	Young's Modulus
E^D	Semi-empirical constant developed for the ONERA model
E^L	Semi-empirical constant developed for the ONERA model
E^M	Semi-empirical constant developed for the ONERA model
F	Local normal velocity
G	Shear modulus
g	Acceleration due to gravity
g_G	Gravitational acceleration in the global coordinate system
$H(f)$	Heaviside function
k	Body number
k	Stiffness
k^L	Semi-empirical constant developed for the ONERA model
L	Lift
$l_{c/4}$	Distance from elastic axis to quarter chord point

l_i	Local pressure on the fluid
M	Moment
m	Mass
N	Number of blades
N	Number of independent bodies
$O_{x_G y_G z_G}$	Global coordinate system
O_{xyz}	Local coordinate system
$\bar{p}$	Arbitrary point in space
p'	Acoustic pressure measured at the observer position
p'_L	Loading acoustic pressure
p'_T	Thickness acoustic pressure
q	Degrees of freedom
q_0	Arbitrary degree of freedom reference state
$\tilde{q}$	Varying degree of freedom term
R	Rotor radius
$\mathbf{r}$	Deformed position of any radial point
$\mathbf{r}_0$	Initial undeformed position of any radial point
$\mathbf{R}_1^{(q)}$	Elementary rotation matrix
$\mathbf{R}_2^{(q)}$	Elementary rotation matrix

$\mathbf{R}_3^{(q)}$	Elementary rotation matrix
r^D	Semi-empirical constant developed for the ONERA model
ret	Denotes an evaluation with respect to retarded time
r^L	Semi-empirical constant developed for the ONERA model
r^M	Semi-empirical constant developed for the ONERA model
S	Surface
s^L	Semi-empirical constant developed for the ONERA model
s^M	Semi-empirical constant developed for the ONERA model
t	Time
t	Observer time frame, acoustic model
T_{ij}	Lighthill's stress tensor
U	Elastic flap displacement
u	Velocity vector, in terms of time and position
$\mathbf{U}_{el}$	Elastic displacement
$\mathbf{u}_k$	Degrees of freedom
V_∞	Freestream velocity
W	Lead-lag displacement
W_0	Velocity normal to the local chord
W_1	Angular velocity

x	Position vector
x_0	Chordwise local position of the aerodynamic centre
z_0	Thicknesswise local position of the aerodynamic centre
$\bar{Z}_j$	Position vector defining cut-off length for a vortex particle
α	Angle of attack
$\dot{\alpha}$	Time rate of change of angle of attack
α^L	Semi-empirical constant developed for the ONERA model
Γ	Circulation
δ	Flap deflection angle
$\delta\alpha$	Elastic movement of the blade
$\delta(f)$	Dirac delta function
$\delta\mathcal{P}$	External loads acting on section
Θ_{el}	Rotation
θ_w	Local twist angle
Θ_y	Torsion angle of the blade section
λ^L	Semi-empirical constant developed for the ONERA model
μ	Dipole intensity, panel method
μ	Advance ratio
v_n	Local normal velocity on the surface of the body

ξ	Elementary reference angle
ρ	Freestream Density
ρ	Local density of the working fluid
ρ	Mass of the beam structure, structural model
ρ	Undisturbed density of the fluid, acoustic model
ρ_k	Position of the local coordinate system origin for each body
σ	Local source intensity distribution on the body
σ^D	Semi-empirical constant developed for the ONERA model
σ^L	Semi-empirical constant developed for the ONERA model
σ^M	Semi-empirical constant developed for the ONERA model
$\bar{\sigma}^M$	Semi-empirical constant developed for the ONERA model
τ	Time parameter
τ	Retarded time frame
ϕ	Scalar velocity potential
ϕ_p	Local inflow velocity and lead-lag direction
ψ	Azimuth angle
Ω	Angular velocity
ω	Vorticity scalar
$\vec{\Omega}_j$	Vorticity vector

Chapter 4

C_d	Sectional drag coefficient
C_l	Sectional lift coefficient
C_{lmax}	Maximum attainable lift coefficient
C_m	Sectional moment coefficient
F_z	Vertical reaction force at the blade root
$M^2 C_l$	Non-dimensional parameter for dynamic stall model verification
$M^2 C_m$	Non-dimensional parameter for dynamic stall model verification
M_y	Torsional moment at the blade root
t	Non-dimensional time
α	Angle of attack

Chapter 5

b	Blade chord
c	Blade semi chord
$CD_{k\omega}$	Cross-diffusion term for the k - ω turbulence model
E	Total energy per unit mass
e	Internal energy
e^*	Dimensional internal energy

F_1	SST coefficient blending function
F_2	SST closure forcing function
$\mathbf{F}^i$	Inviscid flux vector in Cartesian x direction
$\mathbf{F}^v$	Diffusive viscous flux vector in Cartesian x direction
$\hat{\mathbf{F}}^i$	Inviscid flux vector in curvilinear ζ direction
$\hat{\mathbf{F}}^v$	Diffusive viscous flux vector in curvilinear ζ direction
$\mathbf{G}^i$	Inviscid flux vector in Cartesian y direction
$\mathbf{G}^v$	Diffusive viscous flux vector in Cartesian y direction
$\hat{\mathbf{G}}^i$	Inviscid flux vector in curvilinear η direction
$\hat{\mathbf{G}}^v$	Diffusive viscous flux vector in curvilinear η direction
H	Enthalpy
$\mathbf{H}^i$	Inviscid flux vector in Cartesian z direction
$\mathbf{H}^v$	Diffusive viscous flux vector in Cartesian z direction
$\hat{\mathbf{H}}^i$	Inviscid flux vector in curvilinear ς direction
$\hat{\mathbf{H}}^v$	Diffusive viscous flux vector in curvilinear ς direction
J	Jacobian determinant
k	Reduced frequency
k	Turbulent kinetic energy, Navier-Stokes equations
L^*	Dimensional length
M	Mach number

M_{∞}	Freestream Mach number
p	Pressure
Pr	Prandtl number
Pr_T	Turbulent Prandtl number
q	Heat flow
Re	Reynolds Number
Q	Heat flow vector
R	Residual vector
T	Temperature
t	Non-dimensional time
T_0	Reference temperature
t^*	Dimensional quantity time
T^*	Dimensional quantity temperature
T_{∞}^*	Dimensional quantity temperature of the freestream
u	Velocity in Cartesian x direction
V	Reference velocity
V_{∞}^*	Dimensional freestream velocity
V	Velocity in Cartesian y direction
W	Solution vector

w	Velocity in Cartesian z direction
x	Cartesian x direction
x^*	Dimensional quantity in Cartesian x direction
y	Cartesian y direction
y^*	Dimensional quantity in Cartesian y direction
z	Cartesian z direction
z^*	Dimensional quantity in Cartesian z direction
α	Angle of attack
α_0	Mean angle of attack
γ	Specific heat ratio
δ	Flap deflection angle
η	Curvilinear coordinate
M	Viscosity
μ_0	Reference viscosity
μ^*	Dimensional quantity viscosity
μ_∞^*	Dimensional quantity viscosity of the freestream
μ_T	Eddy viscosity
ζ	Curvilinear coordinate
ρ	Density

ρ^*	Dimensional quantity density
ρ_∞^*	Dimensional quantity density of the freestream
ς	Curvilinear coordinate
ψ	Azimuth angle
ω	Angular frequency

Chapter 6

C_d	Sectional drag coefficient
C_l	Sectional lift coefficient
C_m	Sectional moment coefficient
B	Logarithmic overlap layer coefficient
u	Freestream velocity
u^*	Friction velocity
y^+	Function of wall proximity
y	Distance from wall to first computational layer
α	Angle of attack
δ	Flap deflection angle
κ	Logarithmic overlap layer coefficient
ψ	Azimuth angle

Contents

Abstract.....	iii
Acknowledgements.....	iv
List of Symbols.....	v
Contents.....	xvii
List of Figures.....	xxi
List of Tables.....	xxvii
Chapter 1 Introduction.....	1
1.1 Rotorcraft Aerodynamics.....	1
1.2 Aerodynamic Effects.....	5
1.3 Dynamic Stall Alleviation.....	6
1.4 The SHARCS Project.....	7
1.5 Aims and Objectives.....	9
Chapter 2 Dynamic Stall.....	10
2.1 Static and Dynamic Stall.....	10
2.2 Stall Flutter.....	14
2.3 Factors affecting Dynamic Stall Flow Separation.....	17
2.4 Mechanism of Dynamic Stall.....	18

2.5	Parameters Affecting Dynamic Stall.....	25
Chapter 3	3D Test Case and Numerical Method.....	27
3.1	Test Case.....	27
3.1.1	Rotor System.....	28
3.1.2	Flap Details.....	30
3.2	Numerical Method.....	33
3.2.1	Aerodynamic Component.....	36
3.2.2	Structural Component.....	49
3.2.3	Acoustic Component.....	60
3.2.4	Modification to the GAST Code.....	62
Chapter 4	3D Simulation.....	67
4.1	Validation.....	67
4.1.1	HELINOISE Validation.....	68
4.1.2	ONERA Validation.....	71
4.2	Computational Method.....	72
4.2.1	Trim Conditions.....	73
4.2.2	Outputs.....	75
4.3	Results.....	77
4.3.1	Lift Examination.....	78
4.3.2	Angle of Attack Examination.....	80

4.3.3	Force Examination.....	82
4.3.4	Moment Examination.....	83
Chapter 5	2D Test Case and Numerical Method.....	86
5.1	Test Case.....	88
5.1.1	Airfoil Details.....	89
5.1.2	Flap Details.....	90
5.2	Numerical Method.....	92
5.2.1	Non-Dimensional Reynolds Averaged Navier-Stokes Equations in Curvilinear Form.....	93
5.2.2	Steady Solver.....	103
5.2.3	Unsteady Solver.....	107
5.2.4	Turbulence Model.....	110
5.2.5	Modifications to the Solver.....	114
Chapter 6	2D Simulation.....	117
6.1	Mesh Generation.....	117
6.2	Verification of the Numerical Method.....	121
6.2.1	Mesh Dependency Test.....	122
6.2.2	Flap Size Effect on Grid Topology.....	123
6.3	Validation.....	127
6.4	Parametric Studies.....	130

6.4.1	Effect of Flap Deflection.....	131
6.4.2	Effect of Flap Size.....	133
6.4.3	Effect of Timing.....	135
6.4.4	Optimum Flap Strategy.....	138
6.5	Flow Mechanism.....	138
Chapter 7	Conclusion.....	150
	References.....	153
	Appendix A.....	159

List of Figures

<u>Figure 1.1: Azimuth angle description for a counter-clockwise rotation..</u>	2
<u>Figure 1.2: Velocity variation (Red) at $\psi = 90^\circ$ and 270° [1].....</u>	4
<u>Figure 1.3: Flow structure and areas of aerodynamic interest [2].....</u>	5
<u>Figure 1.4: The SHARCS rotor blade concept.....</u>	8
<u>Figure 2.1: Shear layer forms with flow separation.....</u>	11
<u>Figure 2.2: Dynamic stall vortex formation.....</u>	12
<u>Figure 2.3: $C_l-\alpha$, $C_m-\alpha$ and $C_d-\alpha$ comparing unsteady and static pitch cases [13].....</u>	13
<u>Figure 2.4: $C_m-\alpha$ curve for a typical stall flutter case, demonstrating shape of loops.....</u>	15
<u>Figure 2.5: $C_l-\alpha$ indicating the characteristic points for dynamic stall.....</u>	19
<u>Figure 2.6: $C_m-\alpha$ indicating the characteristic points for dynamic stall....</u>	19
<u>Figure 2.7: $C_d-\alpha$ indicating the characteristic points for dynamic stall....</u>	20
<u>Figure 2.8: Pressure (left) and velocity (right) contours for a NACA 0012 airfoil at $\alpha=15^\circ$ (Point “1”, Figs. 2.5 – 2.7). $\alpha=15^\circ$.....</u>	21
<u>Figure 2.9: Pressure (left) and velocity (right) contour for a NACA 0012 airfoil at $\alpha=23^\circ$ (Point “2”, Figs. 2.5 – 2.7).....</u>	21

<u>Figure 2.10: Pressure (left) and velocity (right) contour for a NACA 0012 airfoil at $\alpha=25^\circ$ (Point 3 in Figs. 2.5 – 2.7).</u>	22
<u>Figure 2.11: Pressure (left) and velocity (right) contour for a NACA 0012 airfoil at $\alpha\approx 24^\circ$. Notice the trailing edge vortex overtaking the dynamic stall vortex as the principal source of aerodynamic pitching moment (Arbitrary location between Points “3” and “3b”. Figs. 2.5 – 2.7).</u>	23
<u>Figure 2.12: Pressure (left) and velocity (right) contour for a NACA 0012 airfoil at $\alpha=23^\circ$ (Point “3b”, Figs. 2.5 – 2.7).</u>	23
<u>Figure 2.13: Pressure (left) and velocity (right) contour for a NACA 0012 airfoil at $\alpha=20^\circ$ (Point “4” in Figs. 2.5 – 2.7).</u>	24
<u>Figure 2.14: Pressure (left) and velocity (right) contour for a NACA 0012 airfoil at $\alpha=8^\circ$ (Point “5” in Figs. 2.5 – 2.7).</u>	25
<u>Figure 3.1: Location and size of the trailing edge flap.</u>	32
<u>Figure 3.2: Flap deflection with respect to azimuth angle.</u>	33
<u>Figure 3.3: Panel distribution for GAST simulation.</u>	35
<u>Figure 3.4: Panel distribution with flap deflected.</u>	36
<u>Figure 3.5: Geometry of a blade element [24].</u>	55
<u>Figure 3.6: Forces and moments on a blade element [24].</u>	56
<u>Figure 3.7: Communication of interactions [25].</u>	58

<u>Figure 3.8: Unmodified GAST code structure.....</u>	64
<u>Figure 3.9: GAST code including modification for flap.....</u>	65
<u>Figure 3.10: Chordwise distribution of nodes superimposed over a typical NACA 23012 cross section.....</u>	66
<u>Figure 3.11: Nodes at an angle of attack with a reference line through the first and final nodes on the airfoil.....</u>	66
<u>Figure 3.12: Flap deflection, $\delta = 15^\circ$, measured from the reference line..</u>	66
<u>Figure 4.1: C_N-ψ for HELINOISE case 1333 [34].....</u>	70
<u>Figure 4.2: C_N-ψ for GAST representation of HELINOISE case 1333 [34].....</u>	70
<u>Figure 4.3: Prediction of comparative C_l for dynamic stall models [35]..</u>	71
<u>Figure 4.4: Prediction of comparative C_m for dynamic stall models [35].</u>	72
<u>Figure 4.5: C_l-t comparison controlled and uncontrolled cases.....</u>	79
<u>Figure 4.6: α-t comparison for controlled and uncontrolled cases.....</u>	82
<u>Figure 4.7: F_z-t comparison for controlled and uncontrolled cases.....</u>	83
<u>Figure 4.8: M_y-t comparison for controlled and uncontrolled cases.....</u>	85
<u>Figure 5.1: Effect of flap on lift curve slope.....</u>	87
<u>Figure 5.2: Flap size in percent of relative chord.....</u>	91
<u>Figure 5.3: Flap deflection with respect to azimuth angle for the 2D simulations.....</u>	92

<u>Figure 6.1: Computational mesh with 45000 nodes in 10 Blocks for the 20% chord flap configuration.....</u>	118
<u>Figure 6.2: Boundary layer showing y^+ and that the law of the wall region ends at 10.....</u>	120
<u>Figure 6.3: C_l history for the mesh dependency test.....</u>	122
<u>Figure 6.4: C_m history for the mesh dependency test.....</u>	123
<u>Figure 6.5: Lift coefficient time history comparing flap sizes.....</u>	124
<u>Figure 6.6: Pitching moment coefficient time history comparing flap sizes.....</u>	125
<u>Figure 6.7: Lift Coefficient angle of attack history comparing flap sizes.....</u>	126
<u>Figure 6.8: Pitching moment coefficient angle of attack history comparing flap sizes.....</u>	126
<u>Figure 6.9: C_l history of CFD validation.....</u>	127
<u>Figure 6.10: C_m history of CFD validation.....</u>	128
<u>Figure 6.11: C_l-α comparing uncontrolled CFD case to experimental data.....</u>	129
<u>Figure 6.12: C_m-α comparing uncontrolled CFD case to experimental data.....</u>	130
<u>Figure 6.13: Effect of flap deflection on the Cl-α curve. All cases with 15% chord flap.....</u>	132

<u>Figure 6.14: Effect of flap deflection on the C_m-α curve. All cases with 15% chord flap.....</u>	132
<u>Figure 6.15: Effect of flap size on the C_l -α curve. All cases with $\delta = 15^\circ$.....</u>	134
<u>Figure 6.16: Effect of flap size on the C_m-α curve. All cases with $\delta = 15^\circ$.....</u>	134
<u>Figure 6.17: C_l -α comparing effect of flap timing.....</u>	137
<u>Figure 6.18: C_m -α curve comparing effect of timing.....</u>	137
<u>Figure 6.19: C_l-α indicating the characteristic points for dynamic stall...</u>	140
<u>Figure 6.20: C_m-α indicating the characteristic points for dynamic stall..</u>	140
<u>Figure 6.21: C_d-α indicating the characteristic points for dynamic stall..</u>	141
<u>Figure 6.22: C_d-α indicating the characteristic points for dynamic stall..</u>	141
<u>Figure 6.23: Surface pressure corresponding to numbered points on Figs. 6.19 - 6.22.....</u>	142
<u>Figure 6.24 a – b. (a) Pressure (b) Velocity, corresponding to Point “2.” Notice the flow is unaltered by the flap. Uncontrolled (Top), Controlled (Bottom)</u>	143
<u>Figure 6.25 a – b. (a) Pressure (b) Velocity, corresponding to Point “3.” Reduction in the core size and magnitude of the dynamic stall vortex is visible.....</u>	145

<u>Figure 6.26 a – b: (a) Pressure (b) Velocity, corresponding to Point “3a.” Maximum strength of the controlled trailing edge vortex is less than the still developing uncontrolled case.....</u>	146
<u>Figure 6.27 a – b: (a) Pressure (b) Velocity, corresponding to Point “3b.”</u>	147
<u>Figure 6.28 a – b: (a) Pressure (b) Velocity, corresponding to Point “4.”</u>	148
<u>Figure 6.29 a – b: (a) Pressure (b) Velocity, corresponding to Point “5.”</u>	
<u>Once again, the controlled and uncontrolled cases have become synchronized.....</u>	149
<u>Figure A1: Frame 8222 C_l, C_m, and C_d with respect to non-dimensional time and α.....</u>	161
<u>Figure A2: Frame 9218 C_l, C_m, and C_d with respect to non-dimensional time and α.....</u>	162
<u>Figure A3: Frame 8222 C_l, C_m, and C_d with respect to non-dimensional time and α.....</u>	164
<u>Figure A4: Frame 9218 C_l, C_m, and C_d with respect to non-dimensional time and α.....</u>	165

List of Tables

<u>Table 3.1: Test Case for the 3D simulations.....</u>	30
<u>Table 4.1: The HELINOISE BO105 scaled rotor test case [30].....</u>	69
<u>Table 4.2: Initial and converged trim results.....</u>	74
<u>Table 5.1: 2D dynamic stall test case.....</u>	90
<u>Table 5.2: Code number and description of designated boundaries.....</u>	115
<u>Table 6.1: y^+ and related y values.....</u>	120
<u>Table 6.2: Effect of start and end azimuth angle on pitching moment coefficient and negative aerodynamic damping.....</u>	135
<u>Table A1: Description of McCroskey pitching airfoil cases.....</u>	160
<u>Table A2: Characteristic results from McCroskey pitching airfoil cases.....</u>	163
<u>Table A3: Characteristic results from computational simulations.....</u>	163

Chapter 1 - Introduction

The use of a helicopter has become widespread for a range of civil, military and search and rescue operations because of their unique capacity for vertical take-off and landing (VTOL) and hover. This capability comes at the cost of relatively large fuel consumption, limited forward flight speed, and large noise and vibration levels when compared to a conventional fixed-wing aircraft. It is the complex aerodynamic environment in which the rotor blades operate that is responsible for all these effects.

1.1 Rotorcraft Aerodynamics

The rotor of a helicopter provides the thrust necessary for flight, and also contributes to the generation of asymmetric moments for the purpose of flight control. Torque produced by a conventional helicopter is counteracted by a tail rotor,

Chapter 1 - Introduction

although it is not always the tail rotor that counteracts rotor torque. Co-axial, no tail rotor (NOTAR) or tandem helicopters use the main rotor(s) for both thrust and control.

The lift produced by a section of the blade is related to the effective angle of attack and the dynamic pressure at that section. The position of a rotor blade is described by the azimuth angle ψ , which is set to zero at the aft most position of the blade, Fig. 1.1.

Figure 1.1: Definition of azimuth angle on helicopter rotors.

Chapter 1 - Introduction

Since a rotor blade is set to operate at a continuous rotational speed, the tip speed of the rotor blade is a constant $V_{tip} = \Omega R$, where Ω is the rotational speed and R is the radius, or span, of the blade. The freestream velocity, V_∞ , is equal to the flight speed of the helicopter. For a given V_∞ , assuming forward flight, there will be a variation in the effective velocity over the rotor blades, as they travel through the azimuth angles. The velocity acting on a blade section becomes $V_{blade\ section} = \Omega r + V_\infty \sin\psi$, where r is the radial location along the blade section [1].

This means that the effective velocity acting on the advancing blade will be larger than that on the retreating one, with a region of reversed flow appearing at the root of the retreating blade, Fig. 1.2.

Since the velocity varies with azimuth angle, the angle of attack of the rotor blades needs to be adjusted in order to maintain equilibrium forward flight, free of asymmetric pitching, or rolling moments. Due to the higher velocity at the advancing side the blades will operate at relatively low angles of attack. At high forward flight speeds, this can lead to transonic effects around the blade tip. To counteract the lift on the advancing side, the retreating blades must operate at relatively high angles of attack. However, the reversed flow portion does not contribute effectively to lift and the remainder of the retreating rotor blade is conducive to dynamic stall, since the blade must pitch past the static stall angle.

Figure 1.2: Velocity variation (Red) at $\psi = 90^\circ$ and 270° [2].

The lift producing circulation is collected at the tip of the rotor blades, in the form of tip vortices. These vortices are shed from a given rotor blade in a helical pattern, and will impact or otherwise interact with the following rotor blades in what is known as Blade Vortex Interaction (BVI). In addition to the rotor system aerodynamic interactions, there are interactions between the rotor wake and the fuselage, and the tail rotor, Fig. 1.3.

Figure 1.3: Flow structure and areas of aerodynamic interest [3].

1.2 Aerodynamic Effects

Transonic effects and dynamic stall occurring on the advancing and retreating blades result in characteristic noise and vibration. BVI also creates a disturbance of the flow resulting in vibration, and a distinct and pronounced noise, known as blade slap. Blade slap is especially prominent during low speed forward descending flight, during which the rotor blades are inappositely positioned and receive maximum contact with the shed helical tip vortices [4].

In particular, the vibration induced by dynamic stall is of principal interest in this thesis. Apart from the vibratory loads transferred through the rotor hub to the fuselage, dynamic stall is associated with large nose down torsional pitching

Chapter 1 - Introduction

moments that translate directly into the control system via the pitch links. Furthermore, aerodynamically induced loads interact and combine with the inertial and structural forces and moments to further exacerbate the vibration problem.

The operational impact of dynamic stall effects include increased maintenance costs due to limit life cycles; reduced passenger comfort at high speeds; a less stable targeting platform; reduced maximum forward flight speed; and possible catastrophic failure of the rotor blades.

1.3 Dynamic Stall Alleviation

There are many clear benefits to the reduction of dynamic stall effects. Perhaps the most tantalizing is the ability to increase the forward flight speed of the helicopter. The reduction in control loads would result in higher safety factors for existing control systems, or a reduction in control system weight. The reduction in harmonic loading would provide a smoother ride, providing better service to passengers and a smoother operating or targeting platform for the crew, and the reduction in vibration would reduce maintenance costs. An ancillary benefit is the possible reduction in acoustic pressure caused by dynamic stall which would reduce noise, decreasing noise pollution and noise signature for combat roles.

Chapter 1 - Introduction

There exist several potential methods for noise and vibration mitigation. Passive methods include dampers [5], anhedral rotor blade tip shape [6], and integrated tail-rotors [7]. Dampers are a structural method intended to reduce the transmission of vibration from the rotor blades to the fuselage. The anhedral blade tip is intended to displace the tip vortices in order to increase the miss distance between a vortex and the adjacent rotor blade. The integrated tail rotor is designed to avoid interaction between the tail rotor and the tip vortices. Passive methods do not require a control input.

Active flow control methods include the Active Twist Rotor (ATR) [8] and the Actively Controlled Flap (ACF) [9], which are forms of Individual Blade Control (IBC), and Higher Harmonic Control (HHC) [10]. Active structural control methods include the Active Impedance Control Device (AIC) and the Smart-Spring [11].

1.4 The SHARCS Project

To date, it has proven difficult to eliminate vibration and noise concurrently. Typically, a reduction in vibration has resulted in an increase in noise, and *vice-versa*. The SHARCS (Smart Hybrid Active Rotor Control System) [12] project, conducted at Carleton University, aims to reduce both vibration and noise simultaneously, by combining four control devices: An Actively Controlled Flap

Chapter 1 - Introduction

located at the trailing edge of the rotor blades; an Active Shape Control device in the form of a variable tip anhedral; a Smart-Spring active altering the structural response of the blades; and Active Impedance Control device “filtering” the loads transferred to the pitch links, Fig. 1.4.

Figure 1.4: The SHARCS rotor blade concept.

The SHARCS project will include the construction of a complete scaled rotor-system for wind tunnel testing. Computational Fluid Dynamics (CFD) is seen as an appropriate tool for the evaluation of the Actively Controlled Flap concept for the mitigation of dynamic stall induced vibrations. Prior to physical experimentation, it is necessary to create the control laws for the ACF and CFD simulations are able to provide the basis for these laws.

1.5 Aims and Objectives

This thesis aims to demonstrate through the use of Computational Fluid Dynamics that an actively controlled flap can be used to mitigate the negative effects of dynamic stall. The principal aim is to show that aerodynamic changes brought on by the flap will result in a reduction of the loads and vibrations typically associated with dynamic stall. This is relevant to the SHARCS project, which incorporates the flap into its design.

It is desired that the validation of the actively controlled flap concept should include all aspects of flight in the dynamic stall regime. As such, the flap is tested in flow conditions representative of typical forward flight, and also for more extreme flow conditions representative of high speed forward flight. The typical flight case is to show the practicality of the flap for normal operations. The high speed flight case is to show that the flap may be used to increase the flight envelope of a helicopter by allowing higher forward flight speeds for a conventional helicopter design.

In order to perform the computational examination of the ACF it is necessary to determine the flap actuation required to appropriately modify the flow field. It is desired that the necessary actuation in terms of the deflection angle and timing (in terms of azimuth angle) will be determined. From this, a suitable control law for the SHARCS model will be made possible.

Chapter 2 - Dynamic Stall

It was described in Chapter 1 that dynamic stall is a principal source of vibration on helicopter rotors and that the objective of this thesis will be to alleviate the effects of dynamic stall through the use of trailing edge flap flow control. In order to identify an effective control strategy, the phenomenon of dynamic stall must be understood. The aim of this chapter is to give a thorough overview of this topic.

2.1 Static and Dynamic Stall

First, the difference between static and dynamic stall will be established. The angle of attack is referred to as static when the flow affecting the airfoil is held steady. Static stall over an airfoil occurs when the static angle of attack exceeds the so-called critical angle of attack, at which the flow becomes separated from the

Chapter 2 - Dynamic Stall

surface of the airfoil and stall occurs, as characterized by a sudden loss of lift. The stall angle is dependant on the Reynolds number and factors such as turbulence.

However, when the angle of attack for an airfoil is changed at a high pitch rate, lift can be maintained even beyond the critical angle of attack of the airfoil. Dynamic stall then occurs at a higher angle of attack, when the airflow separates from the leading edge of the airfoil. The shear layer created by the flow separation accumulates into a dynamic stall vortex, which gains strength as the airfoil pitches until the vortex detaches from the leading edge and convects downstream, across the upper surface of the airfoil, Figs. 2.1 & 2.2. The dynamic stall vortex changes the pressure distribution over the airfoil, causing the centre of pressure to track towards the vortex as it convects.

Figure 2.1: Initial phase of dynamic stall.

Figure 2.2: Convection of the dynamic stall vortex.

The motion of the centre of pressure results in an increase in nose down pitching moment, until the vortex passes free of the airfoil which then returns from the stall regime. The presence of the dynamic stall vortex also contributes to drag, requiring additional driving power in order to maintain forward speed.

The physics of the flow separation and the characteristics of the stall development differ greatly between dynamic and static stall. Dynamic stall will occur at an angle of attack higher than the static stall angle. This has both advantageous and disadvantageous effects. The benefit of the dynamic stall vortex is that it decreases the pressure over the upper surface of the airfoil causing an increase in lift known as dynamic lift. The disadvantage is that as the dynamic stall vortex convects downstream a large nose down pitching moment is created as it approaches the trailing edge. The static and dynamic stall are compared in Fig. 2.3.

Figure 2.3: C_l - α , C_m - α and C_d - α comparing unsteady and static pitch cases [13].

2.2 Stall Flutter

Stall flutter is the result of the highly nonlinear aerodynamic loading on the rotor blades, caused by dynamic stall. Its primary effect is the reduction in aerodynamic damping which causes a stable aeroelastic blade mode to diverge or go into high amplitude limit cycle oscillations, which may result in either rapid or eventual failure of the rotor system. Even in moderate amplitude stall flutter cases, the limit cycle oscillations are realizable. For a rotor operating at 200 RPM this will mean 24,000,000 cycles in two thousand hours of flying time at a frequency of 1 per rev. As such, it is evident that stall flutter is of concern when considering the flight envelope of a helicopter.

Leishman [14] introduces the Torsional Damping factor, D.F., as a means of measuring the effects of stall flutter. In fact, the torsional damping factor is defined identically to the work coefficient, C_w , in that it is the area with the C_m - α loop(s) for a pitching aerodynamic body. The equation below takes the circular integral of C_m with the change in the angle of attack, α .

$$D.F. = C_w = \oint C_m(\alpha) d\alpha$$

Chapter 2 - Dynamic Stall

Since the aerodynamic loads are unsteady and nonlinear, the resulting hysteresis creates two prominent loops in the C_m - α plot, which in general resembles an irregular figure eight, Fig. 2.4.

Figure 2.4: C_m - α curve for a typical stall flutter case, demonstrating shape of loops.

By definition positive aerodynamic damping, in this case corresponding to positive torsional damping, acts to reduce the total reaction force acting on the dynamic system. On the contrary, negative aerodynamic damping is analogous to amplification and will serve to increase the reaction forces.

Chapter 2 - Dynamic Stall

The physical mechanism of the aerodynamic damping effects, either positive or negative, is intuitive, as is the effect that the degree of unsteadiness has on the system. Positive aerodynamic damping corresponds to the area within a counter clockwise C_m - α loop, as indicated by the direction arrows in Fig. 2.4. A clockwise loop, on the other hand, indicates negative damping.

If the path is counter clockwise the moment coefficient becomes more negative at the angle of attack is increasing. A negative C_m value corresponds to a nose down pitching moment. If the C_m - α loop were entirely counter clockwise the physical phenomenon would be a nose up motion applied to the rotor blade through the control system in opposition to an ever-increasing aerodynamic moment, resulting in an increase in the stability of the system.

If the C_m - α loop were entirely clockwise the exact opposite effect would be seen. The pitching moment would be increasingly nose up as the airfoil was pitching up, and profoundly nose down as the airfoil was pitching down, resulting in a decrease of the stability of the system to the point where the system may become truly unstable.

Recall that there are two main loops in the C_m - α plot, corresponding to dynamic stall. The first loop is invariably counter clockwise owing to the predominantly linear flow in the lower angle of attack region. As such, even if a clockwise second loop is present, the total damping may remain positive. The

Chapter 2 - Dynamic Stall

difference between the two cases is that one will be wholly divergent, the negative C_w case, whereas the positive C_w case is more likely to result in the limit life cycle failure.

2.3 Factors affecting Dynamic Stall Flow Separation

The onset of dynamic stall flow separation is delayed in comparison with static stall flow separation based on three principal factors [15].

The first factor applies to the time varying increase in angle of attack. With a pitching or plunging motion unsteadiness is introduced into the flow by the circulation that is shed from the airfoil at the trailing edge by altering the effective angle of attack. The result is a decrease in the lift and a reduction in the adverse pressure gradients with respect to the steady stall case, at the same effective angle of attack. The difference between the steady and dynamic effects is highlighted mathematically by the presence of an $\dot{\alpha}$ term, which indicates the dependence on the time rate of change of angle of attack.

The second factor is the induced camber effect that, when combined with a positive pitch rate, will result in a reduction in the pressure at the leading edge and the pressure gradient over the airfoil when comparing the static and dynamic cases, at a given angle of attack. This is a viscous effect that results from the shape of the boundary layer.

The third factor is the unsteady boundary layer effects that result from the external pressure gradients. This is typified by the existence of flow reversal in the boundary layer prior to the onset of significant flow separation.

2.4 Mechanism of Dynamic Stall

Leishman [16] divides the dynamic stall process into five stages. However, the dynamic stall case characterized by Leishman does not include the formation of a trailing edge vortex, indicating that the case is moderately unsteady, since the formation of a trailing edge vortex is characteristic for highly unsteady cases. As such, the case presented below is at a higher level of unsteadiness, and includes all of the characteristic effects encountered in dynamic stall. Leishman's [17] five stage description is presented below, with the formation trailing edge vortex being appended to the third stage. To facilitate comparison the C_l - α , C_m - α , and C_d - α curves are marked at each of the characteristic points, Figs. 2.5, 2.6 and 2.7, respectively.

The first stage is the delay in the onset of separation due to the reduction in the adverse pressure gradients created by the rate of change of angle of attack, including the influence of the unsteady wake, and the unsteady response of the boundary layer, described in section 2.3 as the third factor affecting flow separation. This stage is denoted by "1" on Figs. 2.5 – 2.7. The pressure and velocity plots in Fig. 2.8 illustrate the flow conditions.

Figure 2.5: C_L - α indicating the characteristic points for dynamic stall.

Figure 2.6: C_m - α indicating the characteristic points for dynamic stall.

Figure 2.7: C_d - α indicating the characteristic points for dynamic stall.

The second stage is denoted by separation of the boundary layer at the leading edge and the resulting formation of the dynamic stall vortex from the shear layer. The C_l - α curve shows elevated values of lift for the period of time while the dynamic stall vortex is in contact with the lifting surface. Also present in the second stage is a pronounced increase in the nose down pitching moment resulting from the downstream convection of the dynamic stall vortex. Experimental research has documented the convection rate of the vortex to be between 1/3 and 1/2 of the freestream velocity. This stage is denoted by “2” on Figs. 2.5 – 2.7. The pressure and velocity plots in Fig. 2.9 illustrate the flow conditions.

Chapter 2 - Dynamic Stall

Figure 2.8: Pressure (left) and velocity (right) contours for a NACA 0012 airfoil at $\alpha=15^\circ$ (Point “1”, Figs. 2.5 – 2.7).

Figure 2.9: Pressure (left) and velocity (right) contour for a NACA 0012 airfoil at $\alpha=23^\circ$ (Point “2”, Figs. 2.5 – 2.7).

The third stage coincides with the peak angle of attack. Lift increases steadily with angle of attack until the peak angle of attack is reached. The third stage also coincides with the peak negative pitching moment due to the location of the dynamic stall vortex, as denoted by “3” on Figs. 2.5 – 2.7 with the requisite pressure and velocity plots in Fig. 2.10.

Chapter 2 - Dynamic Stall

Figure 2.10: Pressure (left) and velocity (right) contour for a NACA 0012 airfoil at $\alpha=25^\circ$ (Point 3 in Figs. 2.5 – 2.7).

As the dynamic stall vortex moves towards the trailing edge of the airfoil, the pressure difference between the upper and lower surfaces may result in the formation of a trailing edge vortex. If the flow is only moderately unsteady, the pressure difference may be such that the disturbed shear layer from the trailing edge can convect away in a similar manner as it would in a static stall case. If the flow is highly unsteady, the pressure difference will be too great for the shear layer to convect freely, and it will be drawn up and around the trailing edge. The highly rotational shear layer will coalesce into the trailing edge vortex. For the moderately unsteady case, the characteristic second peak of the negative pitching moment coefficient will be absent. The transition from the dynamic stall vortex moment peak to the trailing edge moment peak is captured in Fig. 2.11. The point of secondary negative aerodynamic pitching moment peak is denoted by “3b” in Figs. 2.5 – 2.7, and is shown in Fig. 2.12.

Chapter 2 - Dynamic Stall

Figure 2.11: Pressure (left) and velocity (right) contour for a NACA 0012 airfoil at $\alpha \approx 24^\circ$. Notice the trailing edge vortex overtaking the dynamic stall vortex as the principal source of aerodynamic pitching moment (Arbitrary location between Points “3” and “3b”, Figs. 2.5 – 2.7).

Figure 2.12: Pressure (left) and velocity (right) contour for a NACA 0012 airfoil at $\alpha = 23^\circ$ (Point “3b”, Figs. 2.5 – 2.7).

The fourth stage in dynamic stall occurs when the dynamic stall vortex convects past the lifting surface. At this point, the fully separated flow on the lifting surface is analogous to an airfoil in static stall. As the airfoil continues to pitch down, the flow will reattach, but the same mechanism responsible for the delay in the onset

Chapter 2 - Dynamic Stall

of separation results in a delay in the reattachment. The point denoted by “4” in Figs. 2.5 – 2.7 is shown in Fig. 2.13.

Figure 2.13: Pressure (left) and velocity (right) contour for a NACA 0012 airfoil at $\alpha=20^\circ$ (Point “4” in Figs. 2.5 – 2.7).

The fifth and final stage sees the flow reattaching to the airfoil as the angle of attack is sufficiently reduced. In a quasi-static stall, occurring when the unsteady effects are small which coincides with a low reduced frequency, the reattachment will occur near the critical angle of attack. In the dynamic stall case, the reattachment may not occur until the angle of attack is well below the critical angle, resulting in hysteresis. The reattached flow is shown in Fig. 2.14, showing an airfoil pitching towards point “5” in Figs. 2.5 – 2.7.

Figure 2.14: Pressure (left) and velocity (right) contour for a NACA 0012 airfoil at $\alpha=8^\circ$ (Point “5” in Figs. 2.5 – 2.7).

2.5 Parameters Affecting Dynamic Stall

In the study of dynamic stall the relationship between the angle of attack and pitching moment coefficient is of key interest. In particular, the size and shape of the C_m - α plots provide insight into the effects of dynamic stall. Experimental examination of dynamic stall was performed by McCroskey *et. al.* [18] in the early 1980s, in which several airfoil types were cyclically pitched through the dynamic stall regime. Effects of airfoil geometry, Reynolds number, Mach number, and reduced frequency were considered. Reduced frequency is an indicator of the rate of pitching motion relative to the freestream velocity.

The experiments showed that Reynolds number had an effect on the amplitude of the pitching moment coefficient and the lift coefficient. This suggested a link between the energy in the flow and the intensity of the dynamic stall vortex.

Chapter 2 - Dynamic Stall

Changes in the reduced frequency, a value that relates the pitching rate of the airfoil to the freestream velocity, altered the size of the second loop in the C_m - α plot, which is shown to have a major effect on aerodynamic damping and stall flutter. The change in size of second loop demonstrated a relationship between the convection rate of the vortex, and the pitching rate of the airfoil.

The study also showed that the shape and size of the pitching moment coefficient curves were dependent on the Mach number. Higher Mach numbers were observed to lead to a counterclockwise second loop, corresponding to positive aerodynamic damping of the system. Lower Mach numbers resulted in a clockwise second loop, corresponding to negative aerodynamic damping. Negative aerodynamic damping coincides with stall flutter.

Chapter 3 - 3D Test Case and Numerical Method

3D simulations will be performed for the SHARCS rotor system, which is a research project in which the Rotorcraft Research Group at Carleton University is currently involved [19]. The flow conditions used in this model are representative of the wind tunnel environment in which the SHARCS rotor will be tested. The aim of the 3D simulations is to validate the trailing edge flap concept for controlling dynamic stall induced vibrations, and to define the necessary actuation strategy in terms of the flap deflection time history.

3.1 Test Case

The basis for the test case in the 3D simulations is the SHARCS (Smart Hybrid Active Rotor Control System) rotor. Recall from Chapter 1 that the SHARCS concept aims to demonstrate that vibrations and noise can be reduced

Chapter 3 - 3D Test Case and Numerical Method

simultaneously by incorporating both structural and flow control devices into the blade. It is anticipated that an Actively Controlled Flap will be implemented for the suppression of vibration and the 3D simulations conducted here are intended to validate this concept.

The intent is to actuate the trailing edge flap near the peak angle of attack on the retreating side of the rotor disk. The deflected flap increases the effective camber of the airfoil so the required blade lift is expected to be achieved with minimal intrusion into the dynamic stall regime. Thus, the large vibratory loads associated with dynamic stall can be reduced. It was determined that there is a void in the recorded flap actuation history in the existing literature. It is desired that the 3D simulation section describe adequately the logic behind the actuation of the flap, including the timing and deflection, and also the size and position of the flap.

3.1.1 Rotor System

The key element of the SHARCS project is to demonstrate the feasibility of the hybrid active control concept on a Mach scaled aeroelastically similar rotor in wind tunnel tests. Mach scaling has been chosen to maintain aeroacoustic similarity and it means that the tip speeds of the scaled and full size rotor are the same. For the SHARCS project, a BO-105 helicopter rotor has been chosen to be scaled, with a tip speed of Mach 0.6. Aeroelastic similarity means identical dynamic response of the

Chapter 3 - 3D Test Case and Numerical Method

scaled and full size blades, *i.e.* the first four significant modes have to be maintained. There are several constraints on the design of the blade resulting from the wind tunnel requirements, and the need to achieve Mach scaling and aeroelastic similarity.

The design requirements specify that each rotor blade have a span of 1 m, chord of 0.075 m, and zero taper. The blades are to be of composite construction, with the centre of mass located at the quarter chord point for the entire span. The Lock number γ , which is the ratio of the aerodynamic and inertial forces acting on the blade, has to be maintained between 5 and 6. The tip speed of the blade is to be Mach 0.6 for the Mach scaling. Finally, the rotor hub to be used should be fully articulated, accommodating 4 blades. The hub radius or root cut-out is 0.126 m.

The goal of the 3D numerical simulations is to provide qualitative evidence that a trailing edge flap will provide some reduction in the vertical reaction force and the torsional moment at the blade root, resulting from dynamic stall. As such, it was not necessary that the rotor system used in the simulation exactly resemble the rotor assembly used in the SHARCS project. However, the model used in the numerical simulations was based on the SHARCS requirements. It was necessary to model the rotor system as hingeless, with a cut-out instead of representing the hub.

The values for the rotor system used in the numerical simulations are given in Tab. 3.1. The flow characteristics are derived from the SHARCS requirements. The differences between the physical setup of the numerical simulation and those

Chapter 3 - 3D Test Case and Numerical Method

requirements mandated for the wind tunnel experiments are small enough that the effect of the trailing edge flap on the force and moments at the root of the blade will provide a realistic depiction of the real life system.

Description	Value
Rotor Radius (R)	1m
Chord Length (c)	0.075 m
Number of Blades (N)	4
Type	Hingeless
Airfoil Section	NACA 23012 (modified)
Twist	-4°
RPM	1340 min-1
Advance Ratio (μ)	0.35
Freestream Density (ρ)	1.225 kg/m ³

Table 3.1: Test Case for the 3D simulations.

3.1.2 Flap Details

The parameters for the size and actuation of the flap were determined based on the physical limitations associated with the SHARCS project. The small size of the blades limits both the relative chord length of the flap and the size of the flap actuators. Since the actuators are small, constraints on response time and the force they are able to provide must be considered. This will provide a limit to the size and deflection of the flap.

It is proposed that the flap will be actuated by piezoelectric actuators and to accommodate these a maximum relative chord length of 15% was established. Given the 0.075 m chord length, the flap thus is 0.01125 m long.

Chapter 3 - 3D Test Case and Numerical Method

Two factors constrained the spanwise sizing of the flap in the numerical model. The first was the location of the inside edge of the flap, determined based on the expected flow regime. With a forward flight speed of 50 m/s, and an advance ratio of 0.35, the retreating portion of the rotor disk near azimuth angle $\psi=270^\circ$ is reversed between the root and 35% span. The relationship between advance ratio and the region of reversed flow on the retreating rotor blade is a convenient gauge of the unsteadiness of the flow. The region between 35% and 65% span has measurable 3D effects. To avoid these, the inside flap edge was located at 70% span.

The second constraint was the implementation of a variable tip anhedral. Among the features included in the SHARCS rotor is a variable droop blade tip that occupies the outside 10% of the span. This required that the trailing edge flap not be located beyond 90% Span, which was the chosen value for a flap size of 20% relative span. This location has additional merit because of the influence of the tip effects that are present in the outermost 5% of the span. In fact, the tip effects are the target of the variable tip anhedral. It is worth noting that the location of the flap would remain acceptable even without the spatial constraint imposed by the tip anhedral. The resulting flap has a relative chord length of 15%, and occupies 20% of the span, as shown in Fig. 3.1.

Figure 3.1: Location and size of the trailing edge flap.

The flap actuation was determined from the angle of attack profile of the rotor blade, from which the timing for the maximum deflection was derived, taking into account the physical limitations of the SHARCS rotor blade system. Examination of the angle of attack profile of the rotor blade indicates that for the moderate advance ratio equilibrium forward flight case, the peak angle of attack is achieved very near to the azimuth angle of $\psi=270^\circ$. To reduce the peak angle of attack, the actuation sequence would see the flap begin to deflect prior to the peak angle of attack, achieve its maximum deflection coincident with the peak angle of attack, and then return to neutral as the rotor blade pitches down from its maximum angle of attack. This motion is accomplished through a sinusoidal actuation with an equal proportion of flap deflection to the change in relative velocity. To facilitate a conservative actuation frequency it was decided that the motion last for 90° azimuth angle (one quarter of the rotor disk), for the simulations.

The maximum deflection achievable in the time available through one quarter of a revolution is on the order of 10° flap deflection, so this was the value used in the numerical model. Given a rotational speed of 1340 min^{-1} for the SHARCS rotor

Chapter 3 - 3D Test Case and Numerical Method

system, the flap will be required to actuate at the equivalent rate of 178.6 deg/s. The full flap deflection history is shown in Fig. 3.2.

Figure 3.2: Flap deflection with respect to azimuth angle.

3.2 Numerical Method

The GAST code was used to model the 3D rotor system in equilibrium forward flight. Developed by the National Technical University of Athens (NTUA), GAST is capable of aeroelastic and aeroacoustic computations. The programme couples the aerodynamic code GENUVP with a linearized beam model that was

Chapter 3 - 3D Test Case and Numerical Method

developed from first principles at the NTUA. The aerodynamic data is such that a discretization of the Ffowcs Williams-Hawkings equation is able to model acoustic effects too [20].

The GAST code is ideally suited to a preliminary study such as this of the effect of the trailing edge flap on the internal structural response of the rotor blades and the specific effects that the flap application will have at the root of the blades.

The coupling of the aerodynamic and structural components of GAST resolves each step in series, with the aerodynamic information being passed to the structural code in order to calculate the aeroelastic deformation, the introduction of the trailing edge flap was made in the structural component. The shape of the trailing edge was modified prior to the structural information being passed back into the aerodynamic component. The base assumption was that the rotor blades are essentially rigid in the x (chord wise) direction, when compared to the deflection in the y (span wise) direction. As such, it was possible to reference the flap deflection to the mean chord line, in order to ensure parity between each of the rotor blades without the concern that wake interaction between blades will lead to asymmetric flap deflection.

The aerodynamic component uses the ONERA model for dynamic stall and compressibility correction. Since the ONERA model is specific to a given airfoil, some error is introduced in the calculation of the dynamic stall effects. However,

Chapter 3 - 3D Test Case and Numerical Method

given that the flap deflections are small enough to be of the same scale as aeroelastic deformation, the difference will not have an effect on the qualitative analysis of the flap deflection. Since there is no flap deflection on the advancing blades, the compressibility correction will not be adversely effected. Specific to the numerical model is the existence of webbing between the flap and the rotor blade. This means that although the flap will be deflected as desired, the panels at the edge of the flap will remain joined to their adjacent panels on the rotor blade, in the numerical model. The effect of the webbing is that there will be no vortex shedding from the edge of the flap, or from the adjacent edge on the rotor blade. The panel distribution is shown in Fig. 3.3, with webbing evident in the flap deflection shown in Fig. 3.4.

Figure 3.3: Panel distribution for GAST simulation.

Figure 3.4: Panel distribution with flap deflected.

3.2.1 Aerodynamic Component

A major advantage of GENUVP is that it does not require the generation of a computational mesh for the solution of the flow field. It generates the geometry through the creation of a set of panels, typical to any panel method, and the strength and position of the vortex particles are determined by integrating the vorticity of each of the dipole elements in the near-wake panels.

The GENUVP code is based on the Helmholtz decomposition, which permits the flowfield around an aerodynamic body to be divided into irrotational (inviscid) and rotational (viscous) components. As such, GENUVP is a hybrid code using the panel method to solve the irrotational flow created by the surfaces of the rotor blades and a vortex particle method to calculate the effects the wake induced rotational flow

Chapter 3 - 3D Test Case and Numerical Method

has on the surroundings of the aerodynamic bodies. To determine the velocity $\vec{u}(\vec{x}, t)$ as a function of position $\vec{x} \in D$ and time $t \geq 0$, the Helmholtz decomposition yields:

$$\vec{u}(\vec{x}, t) = \vec{u}_{ext}(\vec{x}, t) + \vec{u}_{solid}(\vec{x}, t) + \vec{u}_{near-wake}(\vec{x}, t) + \vec{u}_{far-wake}(\vec{x}, t) \quad (3.0)$$

In this case, $\vec{u}_{ext}$ is the freestream velocity, $\vec{u}_{solid}$ and $\vec{u}_{near-wake}$ are irrotational terms that are determinable from the panel method portion of the code, and $\vec{u}_{far-wake}$ is the rotational term that will be resolved through the vortex particle method ‘UVP’ portion of the GENUVP code.

Panel Method

The panel method solution for calculating the induced velocities $\vec{u}_{solid}$ and $\vec{u}_{near-wake}$ is based on the formulation developed by Hess. The irrotational flow is described according to the Laplace equation:

$$\nabla^2 \phi = 0 \quad (3.1)$$

With ϕ representing the scalar velocity potential. The Laplace equation is solvable only if the solid boundaries are impermeable, and the condition of regularity is

Chapter 3 - 3D Test Case and Numerical Method

satisfied. If the aerodynamic body is described by the surface S , then these two conditions are:

$$\left. \text{grad} \phi \cdot \bar{n} \right|_S = \left. \frac{\partial \phi}{\partial n} \right|_S = (\bar{u}_{ext} \cdot \bar{n} - F)_S \quad (3.2)$$

$$\left| \text{grad} \phi \right|_\infty \rightarrow 0 \quad (3.3)$$

In eq. 3.2, $\bar{n}$ is the local unit outward normal vector on the surface S such that freestream velocity can be employed as a scalar quantity, and F is the local normal velocity resulting from the motion of the body and the influence of the far wake.

Green's identity is used to express the Laplace equation in terms of source and dipole distributions over the aerodynamic bodies. If the aerodynamic body does not contribute lift, the potential is described by a continuous source distribution over its surface, with no dipoles. Where σ is the local source intensity distribution on the body, then the potential induced at a point $\bar{P}$, due to the presence of the body is given by:

$$\phi(\bar{P}) = \int_S \sigma \left(\frac{1}{r} \right) dS \quad (3.4)$$

Chapter 3 - 3D Test Case and Numerical Method

For a lifting body, the potential is described according to dipole distribution if the body is thin, or a mix of sources and dipoles if the body is thick. In GENUVP, only the thin lifting body can be modeled. The thin body is formed by establishing a dipole distribution around the lifting body in order to model the circulation produced to create a lift force. The dipoles around the body will be shown to contribute the $\vec{u}_{solid}$ term to the Helmholtz decomposition. Additional dipole sheets are trailed downstream of the body, in order to establish the near-wake. These dipole sheets are necessary for conservation of circulation according to Kelvin's theorem, and are used to compute $\vec{u}_{near-wake}$. If μ is the dipole intensity distribution on the body or trailed wake, then the potential induced at a point $\vec{P}$, due to the dipoles is given by:

$$\phi(\vec{P}) = \int_S \mu \vec{n} \cdot \nabla \left(\frac{1}{r} \right) dS \quad (3.5)$$

Having incorporated this into the model, it is now possible to write out the equation combining the presence of all solid bodies at a point $\vec{P}$ in the flowfield:

$$\phi(\vec{P}) = -\frac{1}{4\pi} \int_{Lifting} \mu \vec{n} \cdot \nabla \left(\frac{1}{r} \right) dS - \frac{1}{4\pi} \int_{Non\ Lifting} \sigma \left(\frac{1}{r} \right) dS - \frac{1}{4\pi} \int_{Wake} \mu \vec{n} \cdot \nabla \left(\frac{1}{r} \right) dS \quad (3.6)$$

Chapter 3 - 3D Test Case and Numerical Method

Special treatment of eq. 3.6 is required if point $\bar{P}$ is on one of the surfaces. Since eq. 3.4 and eq. 3.5 each individually satisfy the regularity condition, it remains that the impermeability condition must be satisfied. Thus, to satisfy all conditions and establish the governing equation for the panel method portion of GENUVP, the combined potential equation eq. 3.6 is substituted into eq. 3.2 giving:

$$\left(-\frac{1}{4\pi} \int_{Lifting} \mu \nabla \left[\frac{\partial}{\partial n} \left(\frac{1}{r} \right) \right] dS - \frac{1}{4\pi} \int_{Non\ Lifting} \sigma \left(\frac{1}{r} \right) dS - \frac{1}{4\pi} \int_{Wake} \mu \nabla \left[\frac{\partial}{\partial n} \left(\frac{1}{r} \right) \right] dS \right) \cdot \bar{n} = \bar{u}_{ext} \cdot \bar{n} - F \quad (3.7)$$

In order to permit the numerical solution, the aerodynamic bodies must be separated into a set of discrete panel elements, which will establish a system of linear equations. Only one strip of wake panels is retained as the near wake in each time step. Prior to each subsequent time step, the panel is converted to a vortex particle and becomes part of the far-wake.

Each panel is assigned a value of source σ intensity for a non-lifting surface, or dipole μ intensity for a lifting surface, that is held constant within its boundaries. Since they are invariant for a given panel, the source and dipole intensities can be moved out of the integrals in eq. 3.7. Thus, the remaining integrals are dependant only on the geometry and the discretization, and are evaluated at control points. This

Chapter 3 - 3D Test Case and Numerical Method

results in constant matrices of influence coefficient, which can be written as a linear system approximation of eq. 3.7:

$$\left[C^{Lifting}_{ij} \right] \{ \mu_j \} + \left[C^{Near-wake}_{ik} \right] \{ \mu_k \} + \left[C^{Non-lifting}_{il} \right] \{ \sigma_l \} = \{ \bar{u}_{ext} \cdot \bar{n}_i - F_i \} \quad (3.8)$$

$$i = 1, (N_{Lifting} + N_{Non-lifting}) \quad j = 1, N_{Lifting} \quad k = 1, N_{Near-wake} \quad l = 1, N_{Non-lifting}$$

where $N_{Lifting}$, $N_{Non-lifting}$, and $N_{Near-wake}$ are the total number of panels on their respective surfaces.

In order to obtain a unique solution to this system the Kutta condition must be satisfied. This is accomplished by setting the dipole intensity of the wake strip elements equal to the value of the adjacent emitting elements, at the tip and the trailing edge of the lifting body. From this, the geometry of the near wake is determined.

After obtaining the source and dipole distributions from the solution of eq. 3.8 it is possible to solve for the scalar velocity potential at any point in the flowfield using a discretized version of the potential equation eq. 3.6. The Helmholtz decomposition can then be used to find the velocity field from the potential according to:

Chapter 3 - 3D Test Case and Numerical Method

$$\vec{u}_{panel} = \vec{u}_{solid} + \vec{u}_{near-wake} = grad(\phi) \quad (3.9)$$

The non-dimensional pressure distribution on the solid bodies can be determined from the unsteady Bernoulli equation:

$$C_p = 1 - \frac{u^2}{u_{ref}^2} - \frac{2}{u_{ref}^2} \frac{\partial \phi}{\partial t} \quad (3.10)$$

from which the potential loads on the lifting bodies can be calculated. Additionally, in GENUVP, the potential load distribution on thin lifting bodies is corrected to include leading edge suction force.

Vortex Particle Method

The vortex particle method in GENUVP is a free-wake model, meaning that the vortex particles are free to move and are not bound to a pre-specified location or density. The Biot-Savart law that defines the velocity in rotational flow field in terms of vorticity and position (distance), and is used as the basis for modeling the far-wake is as follows:

$$\vec{u}_{far-wake}(\vec{x}, t) = \int_{D_w(t)} \frac{\vec{\omega}(\vec{x}_o, t) \times (\vec{x} - \vec{x}_o)}{4\pi |\vec{x} - \vec{x}_o|^3} dD \quad (3.11)$$

Chapter 3 - 3D Test Case and Numerical Method

The flow domain $D_\omega(t)$ in the original Biot-Savart formulation above, is redefined in terms of the volume elements, $D_{\omega,j}(t), j \in J(t)$, with each individual element representing a vortex particle. If every vortex particle has the two properties of vorticity $\bar{\Omega}_j(t)$, and a position vector $\bar{Z}_j(t)$, where the vorticity is defined as:

$$\bar{\Omega}_j(t) = \int_{D_{\omega,j}} \bar{\omega}(\bar{x}, t) dD \quad (3.12)$$

such that,

$$\bar{\omega}(\bar{x}, t) = \sum_j \bar{\Omega}_j(t) \delta(\bar{x} - \bar{Z}_j(t)) \quad (3.13)$$

and,

$$\bar{\Omega}_j(t) \times \bar{Z}_j(t) = \int_{D_{\omega,j}} \bar{\omega}(\bar{x}, t) \times \bar{x} dD \quad (3.14)$$

This allows the Biot-Savart law to be re-written to define the velocity induced for a vortex particle as:

Chapter 3 - 3D Test Case and Numerical Method

$$\bar{u}_{far-wake}(\bar{x}, t) = \sum_j \frac{\bar{\Omega}_j(t) \times (\bar{x} - \bar{Z}_j(t))}{4\pi |\bar{x} - \bar{Z}_j(t)|^3} \quad (3.15)$$

For GENUVP, the smoothed approximation of Beal and Majda [21] allows the Biot-Savart law to be expressed as:

$$\bar{u}_{far-wake}(\bar{x}, t) = \sum_j \frac{\bar{\Omega}_j(t) \times \bar{R}_j}{4\pi \bar{R}_j^3} f_\varepsilon(R_j) \quad (3.16)$$

$$\bar{R}_j = \bar{x} - \bar{Z}_j(t) \quad (3.17)$$

$$f_\varepsilon(R_j) = 1 - e^{-(R_j/\varepsilon)^3} \quad (3.18)$$

where ε is a defined cut-off length for the influence of each vortex particle, which infers that each vortex particle will have a finite sphere of influence. Finally, the vortex particles are convected in the flow field according to the Lagrangian equations:

$$\frac{d\bar{Z}_j}{dt} = \bar{u}(\bar{Z}_j, t) \quad (3.19)$$

Chapter 3 - 3D Test Case and Numerical Method

$$\frac{d\bar{\Omega}_j}{dt} = (\bar{\Omega}_j \nabla) \bar{u}(\bar{Z}_j, t) = \bar{D} \cdot \bar{\Omega}_j \quad (3.20)$$

Where $\bar{D}$ is the deformation tensor.

ONERA Dynamic Stall Model

The remaining component of the aerodynamic solver is the ONERA (Office National d'Études et de Recherches Aérospatiales) dynamic stall model. Developed initially in 1981 by Tran & Pitot [22], the model has been revised throughout the 1980s and 1990s. The ONERA model describes the aerodynamic behaviour of an airfoil in unsteady flow conditions for both attached and separated flow conditions, using a set of non-linear differential equations, based on the Hopf Bifurcation. The model is semi-empirical, deriving the necessary coefficients through parameter identification from sets of data taken from oscillating airfoil experiments.

In the ONERA model, the aerodynamic loading is expressed as a sum of the linear attached component of flow, and a second component representing the deviation from the linear flow caused by flow separation during stall. The ONERA model employed in GENUVP is as created by Truong in 1996 [23], requires 18 coefficients and employs the Kirchhoff/Helmholtz trailing-edge separation scheme from the Leishman-Beddoes model [24]. Advantageous to the solution of aeroelastic problems is that equations for lift, drag, and pitching moment are expressed as

Chapter 3 - 3D Test Case and Numerical Method

differential equations. This model was left unmodified in GENUVP, and is described below.

The ONERA model consists of a set of differential equations describing circulation. There are two circulation terms for lift, Γ_{1L} and Γ_{2L} and two terms Γ_{2D} and Γ_{2M} for drag, and moment, respectively. The subscript 1 denotes that the term is described by a first order differential equation, and the subscript 2 denotes that those terms are described by second order differential equations. The equations are all non-linear and are, therefore linearized with respect to a reference state, which is determined by the far-wake. The linearized ONERA model equations are:

$$\begin{aligned}
 \dot{\Gamma}_{1L} + \frac{\lambda^L}{\tau} \Gamma_{1L} &= \frac{1}{2} \frac{\lambda^L}{\tau} \left(\frac{dC_L}{d\alpha} \right)_{ref} \sin[2(\alpha_{ref} - \alpha_0)] + \frac{\lambda^L}{\tau} V_{ref} \left(\frac{dC_L}{d\alpha} \right)_{ref} \cdot \delta\alpha \cdot \cos[2(\alpha_{ref} - \alpha_0)] \\
 &+ \frac{\lambda^L}{\tau} \sigma^L W_1 + \left(a^L \left(\frac{dC_L}{d\alpha} \right)_{ref} + d^L \right) \dot{W}_0 + a^L \sigma^L \dot{W}_1 \\
 \ddot{\Gamma}_{2L} + \frac{a^L}{\tau} \cdot \dot{\Gamma}_{2L} + r^L \cdot \frac{\Gamma_{2L}}{\tau^2} &= - \left[\frac{r^L}{\tau^2} \cdot V_{ref} \cdot (\Delta C_L)_{ref} + \frac{E^L}{\tau} \cdot \dot{W}_0 \right] \\
 \ddot{\Gamma}_{2D} + \frac{a^D}{\tau} \cdot \dot{\Gamma}_{2D} + r^D \cdot \frac{\Gamma_{2D}}{\tau^2} &= - \left[\frac{r^D}{\tau^2} \cdot V_{ref} \cdot (\Delta C_D)_{ref} + \frac{E^D}{\tau} \cdot \dot{W}_0 \right] \\
 \ddot{\Gamma}_{2M} + \frac{a^M}{\tau} \cdot \dot{\Gamma}_{2M} + r^M \cdot \frac{\Gamma_{2M}}{\tau^2} &= - \left[\frac{r^M}{\tau^2} \cdot V_{ref} \cdot (\Delta C_M)_{ref} + \frac{E^M}{\tau} \cdot \dot{W}_0 \right]
 \end{aligned} \tag{3.21}$$

Chapter 3 - 3D Test Case and Numerical Method

where the reference velocity V_{ref} and reference angle of attack α_{ref} denote the effective values corresponding to the wake and the linear (inviscid) portion of the lift curve slope represented by $\left(\frac{dC_L}{d\alpha}\right)_{ref}$. Viscous effects based on the reference angle of attack are introduced by the terms $(\Delta C_L)_{ref}$, $(\Delta C_D)_{ref}$ and $(\Delta C_M)_{ref}$ for the lift, drag, and moment coefficients, respectively, and are corrected for local Mach number. The time parameter is $\tau = \frac{c}{2V_{ref}}$ where c is the local chord length of the rotor blade. The terms λ^L , σ^L , α^L , d^L , r^L , E^L , a^D , r^D , E^D , a^M , r^M , and E^M are semi-empirical constants developed for the ONERA model.

Variation in the local angle of attack resulting from elastic movement of the blade is denoted by $\delta\alpha$, W_0 is the velocity normal to the local chord, and W_1 is the angular velocity of the section. The terms $\delta\alpha$, W_0 and W_1 are described as:

$$\begin{aligned}\delta\alpha &= \frac{1}{V_{ref}}(\dot{U} \cdot \sin \phi_p - \dot{W} \cdot \cos \phi_p) + \dot{\Theta}_y \\ W_0 &= V_{ref} \cdot \cos \alpha_{ref} - \sin \theta_w \cdot \dot{U} - \cos \theta_w \cdot \dot{W} - l_{c/4} \cdot \dot{\Theta}_y \\ W_1 &= \frac{c}{2} \dot{\Theta}_y\end{aligned}\tag{3.22}$$

Chapter 3 - 3D Test Case and Numerical Method

where the distance offsetting the quarter chord point from the elastic axis is given by $l_{c/4}$, the local twist angle is θ_w , and the angle formed between the local inflow velocity and the lead-lag direction is ϕ_p . U is the elastic flap displacement, W is the lead-lag displacement, and Θ_y is the torsion angle of the blade section.

The aerodynamic loads that are used in the aeroelastic calculations are given by the following expressions:

$$\begin{aligned}\delta P_x &= -L \sin \phi_p + D \cos \phi_p \\ \delta P_z &= L \cos \phi_p + D \sin \phi_p \\ \delta M_y &= M\end{aligned}\tag{3.23}$$

where the lift L , drag D , and moment M terms are defined as:

$$\begin{aligned}L &= \frac{\rho c}{2} \cdot \left[V_{ref} \cdot (\Gamma_{1L} + \Gamma_{2L}) + \frac{s^L c}{2} \dot{W}_0 + \frac{k^L \cdot c}{2} \dot{W}_1 \right] \\ D &= \frac{\rho c}{2} \cdot \left[V_{ref}^2 \cdot C_{Dlin} + \frac{\sigma^D c}{2} \dot{W}_0 + V_{ref} \cdot \Gamma_{2D} \right] \\ M &= \frac{\rho c^2}{2} \cdot \left[V_{ref}^2 \cdot C_{Mlin} + \frac{(\bar{\sigma}^M + d^M) \cdot c}{2} \dot{W}_0 + \sigma^M V_{ref} \cdot W_1 + \frac{s^M c}{2} \cdot \dot{W}_1 + V_{ref} \cdot \Gamma_{2M} \right]\end{aligned}\tag{3.24}$$

Chapter 3 - 3D Test Case and Numerical Method

and where the local density of the working fluid is given by ρ , and the terms C_{Dlin} and C_{Mlin} are the drag and moment coefficients of the potential flow reference angle. The terms s^L , κ^L , σ^D , $\bar{\sigma}^M$, d^M , σ^M , and s^M are semi-empirical constants developed for the ONERA model.

3.2.2 Structural Component

The structural component of GAST is a linearized beam model that was developed from first principles at NTUA. The beam model permits the resolution of lead-lag bending, flapping, and the torsional moment about the feathering axis, and any additional coupling caused by changes in the inertial conditions of the blade. The model allows the specification of material and sectional properties throughout the span of the rotor blades, and is capable of computing the loads for a varying elastic axis.

The undeformed elastic axis of a beam is set to occupy the y axis of an O_{xyz} Cartesian coordinate system. The beam is subject to combined flap and lead-lag bending in the respective x and z directions, and radial flexion and torsion in the y direction. Let $\mathbf{r}_0 = (x_0, y_0, z_0)^T$ and $\mathbf{r} = (x, y, z)^T$ denote the position of any radial point on the structure, with $\mathbf{r}_0$ indicating the initial undeformed position and $\mathbf{r}$ the deformed position. If the elastic displacement is represented by

Chapter 3 - 3D Test Case and Numerical Method

$\mathbf{U}_{el} = (U_{el}, V_{el}, W_{el})^T$, and the rotation is denoted by $\Theta_{el} = (\Theta_{x,el}, \Theta_{y,el}, \Theta_{z,el})^T$, then

the deformed position $\mathbf{r}$ can be expressed as:

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{E} \begin{Bmatrix} \mathbf{U}_{el} \\ \Theta_{el} \end{Bmatrix} = \mathbf{r}_0 + \begin{bmatrix} 1 & 0 & 0 & 0 & z_0 & 0 \\ 0 & 1 & 0 & -z_0 & 0 & x_0 \\ 0 & 0 & 1 & 0 & -x_0 & 0 \end{bmatrix} \begin{Bmatrix} \mathbf{U}_{el} \\ \Theta_{el} \end{Bmatrix} \quad (3.25)$$

and according to beam theory:

$$\Theta_{x,el} = \frac{\partial W_{el}}{\partial y} \quad \Theta_{el,z} = -\frac{\partial U_{el}}{\partial y} \quad (3.26)$$

These relationships are approximations of the real system, since they do not account for changes to the radial length resulting from bending deflections, and applies to small displacements. If a beam element is expressed with cross section A and width dy , the linear density of the external loads acting on it can be expressed as

$\delta \mathbf{P} = \{\delta P_x, \delta P_y, \delta P_z\}^T$. The balance of the forces becomes:

$$\int_A \rho dA \ddot{\mathbf{r}} dy = \mathbf{dF} + \int_A \rho dA \mathbf{g} dy + \delta \mathbf{P} dy \quad (3.27)$$

Chapter 3 - 3D Test Case and Numerical Method

where dot denotes a differentiation with respect to time, ρ is the mass of the beam structure, $\mathbf{g} = \{g_x, g_y, g_z\}^T$ is the acceleration due to gravity and $\mathbf{dF} = \{dF_x, dF_y, dF_z\}^T$ is the net elastic internal force. The balance of moments at a point '1' is then expressed as:

$$\int_A \rho dA \mathbf{r}_1 \times \ddot{\mathbf{r}} dy = \mathbf{dM} + \mathbf{dr} \times (\mathbf{F} + \mathbf{dF}) + \int_A \rho dA \mathbf{r}_1 \times \mathbf{g} dy + \mathbf{r}_1 \times \delta \mathbf{P} dy \quad (3.28)$$

Here, the total elastic force at some point '2' is denoted by $\mathbf{F} + \mathbf{dF}$, and $\mathbf{dM}$ is the net elastic moment. Assuming small displacements, the position of point '1' is expressed as:

$$\begin{aligned} \mathbf{r}_1 &= \{dU_{el} + x + z\Theta_{el}, dV_{el} + dy, dW_{el} + z - x\Theta_{el}\}^T \\ &\cong \{x + z\Theta_{el}, dy, z - x\Theta_{el}\}^T \end{aligned} \quad (3.29)$$

The stresses on the cross section A can be expressed as:

$$\begin{aligned} \sigma_{yy} &= E\epsilon_{yy} = E \frac{\partial V_{el}}{\partial y} - E x \frac{\partial^2 U_{el}}{\partial y^2} - E z \frac{\partial^2 W_{el}}{\partial y^2} = EV_{el}' - ExU_{el}'' - EzW_{el}'' \\ \tau_{xy} &= G\gamma_{xy} = Gz \frac{\partial \Theta_{el}}{\partial y} = Gz\Theta_{el}' \end{aligned}$$

Chapter 3 - 3D Test Case and Numerical Method

$$\tau_{yz} = G\gamma_{yz} = -Gx \frac{\partial \Theta_{el}}{\partial y} = -Gx \Theta'_{el} \quad (3.30)$$

where prime denotes a differentiation with respect to y , and E and G are Young's Modulus, and the shear modulus, which are the material constants. The force $\mathbf{F}$ and moment $\mathbf{M}$ are determined by integrating the stresses over the cross section A according to:

$$\begin{aligned} F_y &= \int_A \sigma_{yy} dA = \int_A E V'_{el} dA = (EA) V'_{el}, \\ M_y &= \int_A (\tau_{xy} z - \tau_{yz} x) dA = \int_A G(x^2 + z^2) \Theta'_{el} dA = (GI_t) \Theta'_{el} \\ M_x &= - \int_A \sigma_{yy} z dA = - \int_A (-E x z U''_{el} - E z^2 W''_{el}) dA = (EI_{xz}) U''_{el} + (EI_{xx}) W''_{el} \\ M_z &= \int_A \sigma_{yy} x dA = \int_A (-E x^2 U''_{el} - E x z W''_{el}) dA = -(EI_{zz}) U''_{el} - (EI_{xz}) W''_{el} \end{aligned} \quad (3.31)$$

where:

$$\begin{aligned} (EA) &= \int_A E dA, \quad (GI_t) = \int_A G(x^2 + z^2) dA \\ (EI_{xx}) &= \int_A E z^2 dA, \quad (EI_{zz}) = \int_A E x^2 dA, \quad (EI_{xz}) = \int_A E x z dA \end{aligned}$$

Chapter 3 - 3D Test Case and Numerical Method

The relations for the solution of the force and moments assumes that there is not net torsion and that $\int_A EzdA = 0$ and $\int_A ExdA = 0$. The balance of moments in x and z are used to determine dF_x and dF_z in their respective force balances. As such, the set of equations written with respect to $\mathbf{u} = \{U_{el}, V_{el}, W_{el}, \Theta_{el}\}^T$ can be expressed in the compact matrix form:

$$\int_A (\rho dA) \cdot \mathbf{II} \cdot \ddot{\mathbf{r}} = [\mathbf{T} \cdot \mathbf{u}']' + [\mathbf{K} \cdot \mathbf{u}']'' + \int_A (\rho dA) \cdot \mathbf{II} \cdot \mathbf{g} + \mathbf{II}_0 \cdot \delta \mathbf{P} \quad (3.32)$$

where,

$$\mathbf{II} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ z & 0 & -x \end{bmatrix}, \quad \mathbf{II}_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ z_0 & 0 & -x_0 \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} T & 0 & 0 & 0 \\ 0 & (EA) & 0 & 0 \\ 0 & 0 & T & 0 \\ 0 & 0 & 0 & (GI_t) \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} -(EI_{zz}) & 0 & -(EI_{xz}) & 0 \\ 0 & 0 & 0 & 0 \\ -(EI_{xz}) & 0 & -(EI_{xx}) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

And in bending theory, the cross-section remains undeformed, so x_0 and z_0 will denote the local position of the aerodynamic centre.

In addition to the resolution of structural reactions, the complete aeroelastic solution requires the calculation of inertial reactions based on the revolution of the rotor system, and the system motion with respect to an inertial frame of reference,

Chapter 3 - 3D Test Case and Numerical Method

typically denoted by the gravitational acceleration. GAST uses the typical rigid degree of freedom equations of multi-body dynamics.

The inertial, or global, coordinate system is first defined according to $O_{G x_G y_G z_G}$. It is then assumed that all deformable components behave as beam structures. For each component, a local coordinate system O_{xyz} is introduced. The y axis is defined as the elastic axis of the specific beam structure, while the x axis corresponds to flapping, and z with lead-lag bending.

The position of the origin of the local coordinate system for each body k is defined by $\rho_k = \rho_k(t)$, in respect to the global coordinate system. The complete rotational transformation matrix is $A_k = A_k(t)$ and is sufficient to go from the local to the global coordinate system. As such, the global position vector of any point on body k is given as

$$\mathbf{r}_{Gk} = \rho_k + \mathbf{A}_k \cdot \mathbf{r}_k = \rho_k + \mathbf{A}_k \cdot (\mathbf{r}_0 + \mathbf{E} \cdot \mathbf{u}_k), \quad k = 1, N \quad (3.33)$$

where N is the total number of independent bodies.

The equations describing ρ_k and A_k are joint specific and will vary based on the type of connection, in order to properly describe the dynamics of the complete configuration. Each joint is defined by three translations and three rotations, the set

Chapter 3 - 3D Test Case and Numerical Method

of which is defined in terms of degrees of freedom q . Thus, the position of the local coordinate system is more appropriately define as $\rho_k = \rho_k(q, t)$. Likewise, the transformation matrix will be defined as $A_k = A_k(q, t)$. The choice of q will depend on the system in consideration, with the translations defined according to ρ_k and the rotations by A_k .

Figs. 3.5 and 3.6 illustrate the kinematics and dynamics of a section of rotor blade, according to the deformable beam concept.

Figure 3.5: Geometry of a blade element [25].

Figure 3.6: Forces and moments on a blade element [25].

If the elementary rotation matrices are defined as $\mathbf{R}_1^{(q)}, \mathbf{R}_2^{(q)}, \mathbf{R}_3^{(q)}$, in terms of angle ξ , with respect to the x, y, and z axis respectively, then:

$$\mathbf{R}_1^{(\xi)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \xi & -\sin \xi \\ 0 & \sin \xi & \cos \xi \end{bmatrix}, \mathbf{R}_2^{(\xi)} = \begin{bmatrix} \cos \xi & 0 & \sin \xi \\ 0 & 1 & 0 \\ -\sin \xi & 0 & \cos \xi \end{bmatrix}, \mathbf{R}_3^{(\xi)} = \begin{bmatrix} \cos \xi & -\sin \xi & 0 \\ \sin \xi & \cos \xi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.34)$$

Returning to the complex matrix form of the structural model and introducing the equation for the global position vector results in the equation:

$$\begin{aligned} \int_A \rho dA \cdot \mathbf{II} \cdot \mathbf{A}_k^T \left(\ddot{\mathbf{p}}_k + \ddot{\mathbf{A}}_k \cdot (\mathbf{r}_0 + \mathbf{E}\mathbf{u}_k) + 2\dot{\mathbf{A}}_k \cdot \mathbf{E} \cdot \dot{\mathbf{u}}_k + \mathbf{A}_k \cdot \mathbf{E} \cdot \ddot{\mathbf{u}}_k \right) = \\ [\mathbf{T} \cdot \mathbf{u}'_k]' + [\mathbf{K} \cdot \mathbf{u}''_k]'' + \int_A (\rho dA \cdot \mathbf{II} \cdot \mathbf{A}_k^T \cdot \mathbf{g}_G) + \mathbf{II}_0 \cdot \delta \mathbf{P}_k, \quad k = 1, N \end{aligned} \quad (3.35)$$

where $\mathbf{g}_G$ is the gravitational acceleration in the global coordinate system. In this equation, the time derivative of the rotation term $\mathbf{A}_k$ introduces the centrifugal and Coriolis accelerations. Since the equations are non-linear due to the unknown degrees of freedom $\mathbf{u}_k$ and $\mathbf{q}$, the equation is supplemented with the appropriate equations defining the properties of all degrees of freedom, typically from the existing degree of freedom equations. In the specific cases of cyclic and collective inputs, special control laws are established governing the effect on blade pitch and teeter angle.

The boundary conditions for the solution of the above equations rely on the dynamic constraints of the system. In the case of the interaction of two bodies, that are attached to each other, the first body will communicate to the second body the translation and rotations of its endpoint, and the second body will communicate the internal loads at its endpoint, with the designation of the two bodies being arbitrary, as shown in Fig. 3.7.

Figure 3.7: Communication of interactions [26].

In order to permit the numerical solution of the combined structural and dynamic matrix equation, it must be linearized. The non-linearity is due to the dependence of $\mathbf{A}^T(\mathbf{q}, t)$, $\dot{\mathbf{A}}(\mathbf{q}, t)$, $\ddot{\mathbf{A}}(\mathbf{q}, t)$, and $\ddot{\mathbf{p}}(\mathbf{q}, t)$ on $\mathbf{q}$. Linearization is done by expressing $\mathbf{q}$ in terms of a reference state $\mathbf{q}_0$ and a varying term $\tilde{\mathbf{q}}$. This is done simply as:

$$\mathbf{q} = \mathbf{q}_0 + \tilde{\mathbf{q}}$$

Chapter 3 - 3D Test Case and Numerical Method

and the rotation matrices can be re-expressed as:

$$\begin{aligned}
 \mathbf{R}(\mathbf{q}) &= \mathbf{R}(\mathbf{q}_0) + \mathbf{R}'(\mathbf{q}_0) \cdot \tilde{\mathbf{q}} \\
 \dot{\mathbf{R}}(\mathbf{q}) &= \mathbf{R}'(\mathbf{q}_0) \cdot \dot{\mathbf{q}}_0 + \mathbf{R}'(\mathbf{q}_0) \cdot \ddot{\tilde{\mathbf{q}}} + \mathbf{R}''(\mathbf{q}_0) \cdot \dot{\mathbf{q}}_0 \cdot \tilde{\mathbf{q}} \\
 \ddot{\mathbf{R}}(\mathbf{q}) &= \mathbf{R}''(\mathbf{q}_0) \cdot \dot{\mathbf{q}}_0^2 + 2 \cdot \mathbf{R}''(\mathbf{q}_0) \cdot \dot{\mathbf{q}}_0 \cdot \ddot{\tilde{\mathbf{q}}} + \mathbf{R}'''(\mathbf{q}_0) \cdot \dot{\mathbf{q}}_0^2 \cdot \tilde{\mathbf{q}} + \dots \\
 &\quad \mathbf{R}'(\mathbf{q}_0) \cdot \ddot{\tilde{\mathbf{q}}} + \mathbf{R}'(\mathbf{q}_0) \cdot \ddot{\tilde{\mathbf{q}}} + \mathbf{R}''(\mathbf{q}_0) \cdot \ddot{\tilde{\mathbf{q}}} \cdot \tilde{\mathbf{q}}
 \end{aligned} \tag{3.36}$$

where prime denotes differentiation with respect to $\mathbf{q}_0$. Having so expressed the rotation matrices allows the linearized form of $\mathbf{A}^T(\mathbf{q}, t)$, $\dot{\mathbf{A}}(\mathbf{q}, t)$, $\ddot{\mathbf{A}}(\mathbf{q}, t)$, and $\ddot{\mathbf{p}}(\mathbf{q}, t)$ to be determined, and reintroduced into the governing equation (eq. 3.35), resulting in:

$$\begin{aligned}
 \int_A \rho dA \cdot \mathbf{\Pi} \cdot \left\{ \mathbf{E} \cdot \ddot{\mathbf{u}}_k + 2 \cdot \left(\mathbf{A}_k^T \dot{\mathbf{A}}_k \right)_0 \cdot \mathbf{E} \cdot \dot{\mathbf{u}}_k + \left(\mathbf{A}_k^T \ddot{\mathbf{A}}_k \right)_0 \cdot \mathbf{E} \cdot \mathbf{u}_k + \right. \\
 \left. + \left(\mathbf{A}_k^T \ddot{\mathbf{p}}_k + \mathbf{A}_k^T \ddot{\mathbf{A}}_k \mathbf{r}_0 \right)_m \cdot \ddot{\tilde{\mathbf{q}}} + \left(\mathbf{A}_k^T \ddot{\mathbf{p}}_k + \mathbf{A}_k^T \ddot{\mathbf{A}}_k \mathbf{r}_0 \right)_c \cdot \ddot{\tilde{\mathbf{q}}} \right. \\
 \left. + \left(\mathbf{A}_k^T \ddot{\mathbf{p}}_k + \mathbf{A}_k^T \ddot{\mathbf{A}}_k \mathbf{r}_0 \right)_k \cdot \tilde{\mathbf{q}} + \left(\mathbf{A}_k^T \ddot{\mathbf{p}}_k + \mathbf{A}_k^T \ddot{\mathbf{A}}_k \mathbf{r}_0 \right)_0 \right\} = \\
 [\mathbf{T} \cdot \mathbf{u}'_k]' + [\mathbf{K} \cdot \mathbf{u}''_k]'' + \int_A \left(\rho dA \cdot \mathbf{\Pi} \cdot \mathbf{A}_k^T \cdot \mathbf{g}_G \right) + \mathbf{\Pi}_0 \cdot \delta \mathbf{P}_k, \quad k=1, N
 \end{aligned} \tag{3.37}$$

where the naught subscript represents the reference state, and m , c , and k denote mass, damping, and stiffness respectively, as resulting from the linearization of the corresponding matrices.

3.2.3 Acoustic Component

The acoustic component of GAST is a modified Farassat 1A [27] model of the Ffowcs Williams and Hawkins (FW-H) equation. The FW-H equation, based on Lighthill's acoustic analogy, is the most widely used method for the prediction of rotorcraft aeroacoustics. In the FW-H equation sound generated by a body is treated as an arbitrary motion in a fluid according to the principles of mass and momentum conservation. The body is expressed mathematically as a discontinuity. The flow outside the body is similar to the physical flow field, and the flow within the body is arbitrarily assumed to be at rest. The mass and momentum sources that represent the body act as the source of the sound. The acoustic analogy is applied and the fluid mass and momentum sources are combined to form a wave equation.

If the position of an observer and a source are represented as $\bar{x}$ and $\bar{y}$ respectively, and the motion of the surface of a body according to $f(\bar{y}, t) = 0$, where $f > 0$ is outside the body, then the FW-H equation can be written as:

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) p' = \frac{\partial}{\partial t} [\rho_0 v_n |\nabla f| \delta(f)] - \frac{\partial}{\partial x_i} [l_i |\nabla f| \delta(f)] + \frac{\partial^2}{\partial x_i \partial x_j} [T_{ij} H(f)] \quad (3.38)$$

This equation gives the sound generated by a body moving through a fluid, where p' is the acoustic pressure measured at the observer position $\vec{x}$, c is the speed of sound and ρ is the undisturbed density of the fluid, $v_n = v_i n_i$ is the local normal velocity on the surface of the body, l_i is the local pressure on the fluid, and T_{ij} is Lighthill's stress tensor. $H(f)$ is the Heaviside function, and $\delta(f)$ is the Dirac delta function.

The three terms on the right hand side of the FW-H equation are thus the thickness noise, loading noise, and quadrupole noise terms, respectively. The thickness noise is the result of fluid displaced by the volume of the body, the loading noise is caused by the change in pressure distribution around the body, and the quadrupole noise results from the change in the density of the fluid caused by compressibility. Thickness and loading noise are known as surface sources, and the quadrupole noise is labeled a volume source.

The acoustic formulation in GAST is based on Farassat's 1A solution of the FW-H equation. Farassat's formulation solves the FW-H equation for thickness and loading noise by integrating the source and dipole information from the panel method portion of the aerodynamic component. The three equations belonging to the 1A model are written:

$$\begin{aligned}
 4\pi p'_L(\vec{x}, t) &= \frac{1}{c} \int_{f=0} \left[\frac{\dot{l}_i \hat{r}_i}{r(1-M_r)^2} \right]_{ret} dS + \int_{f=0} \left[\frac{l_r - l_i M_i}{r^2(1-M_r)^2} \right]_{ret} dS + \frac{1}{c} \int_{f=0} \left[\frac{l_r (r \dot{M}_i \hat{r}_i + c M_r - c M^2)}{r^2(1-M_r)^3} \right] dS \\
 4\pi p'_T(\vec{x}, t) &= \int_{f=0} \left[\frac{\rho_o v_n (r \dot{M}_i \hat{r}_i + c M_r - c M^2)}{r^2(1-M_r)^3} \right]_{ret} dS \\
 p'(\vec{x}, t) &= p'_L(\vec{x}, t) + p'_T(\vec{x}, t)
 \end{aligned} \tag{3.39}$$

where $\vec{r} = \vec{x} - \vec{y}$, $M_i = v_i / c$, $Mr = M_i r_i / r$, and $l_r = l_i r_i / r$. The Farassat 1A model gives the loading p'_L , and thickness p'_T acoustic pressure, while negating the effect of quadrupole noise. Since there is a distance between the observer and the source of the pressure, the subscript ret is used to denote an evaluation of certain terms with respect to a retarded time frame τ , which is the time at the source. The acoustic signal history is received by the observer in the observer time frame t .

3.2.4 Modification to the GAST Code

In order to actuate the flap in the existing GAST code, it would be necessary to establish a system of governing equations for the relative motion of the flap as located on the rotor blades, with respect to the origin. GAST is capable of determining an aeroelastic solution for a rotor system, as described in the previous sections. However, implementation of the flap as a new body, with a full set of

Chapter 3 - 3D Test Case and Numerical Method

parameters would require the introduction of a new model for the effects of dynamic stall that is capable of integrating the aerodynamic profile of the blade section with the inclusion of the additional body representing the flap.

Given that the development of a new relationship for dynamic stall is a lengthy undertaking, it was decided that an alternative solution for the implementation of the flap should be determined. In examining the requirements of the SHARCS project, the required size and deflection of the flap were determined to be on the order of an aeroelastic disturbance, meaning that the unaltered ONERA model should be suitable for the modeling of a flap. As such, implementation of the flap was done from within the structural model, such that the code perceives the flap as a deformation.

The typical GAST code functions according to the flow chart depicted in Fig. 3.8. The initial conditions are as determined by the structural properties of the rotor blades, the rotational speed, and all the other parameters as set out in section 3.1.1, and include the geometry of the panel elements. The panel method component defines the aerodynamic influence of the body and the near-wake, the vortex particle method component determines the aerodynamic effects of the far-wake, and the ONERA model component computes the effects of dynamic stall and compressibility, as outlined in section 3.2.1. The structural component computes the inertial and dynamic effects of the rotor blades, as outlined in section 3.2.2.

Figure 3.8: Unmodified GAST code structure.

Also included in the structural model is an algorithm that recomputes the nodes for the panel method, according to the resolved aeroelastic deformations at the end of each time step. This information is then fed back into the panel method component and serves as the new basis for the aerodynamic computations. This pattern repeats for each and every time step until the solution has reached the desired convergence criteria.

The flap is implemented after the structural model, prior to the introduction of the new nodal information for the panels. The nodes were intercepted and modified for the appropriate flap deflection for the specific time step. The numerical array contains the nodes for each panel and maintains a constant distribution, such

that the nodes remain indexed to the same location with it, despite the modification of their geometry. As such, it was possible to select the nodes specifically located on the flap for modification. The revised GAST flow chart is illustrated in Fig. 3.9.

Figure 3.9: GAST code including modification for flap.

Actuation of the flap is controlled in terms of timing and deflection. The timing is defined with respect to the azimuth angle of the blade root, which is linked to the rotational speed of the rotor hub and is not subject to lead-lag effects. The deflection of the flap is updated with respect to the angle of attack of the rotor blade, as outlined in Figs. 3.10 – 3.12. Fig. 3.10 shows the chordwise distribution of nodes, with the cross section of a NACA 23012 airfoil superimposed for clarity. The reference line through the first and second to last node provides a basis of orientation

Chapter 3 - 3D Test Case and Numerical Method

for the flap deflection. Since, the GAST code uses a trimming algorithm to calculate the cyclic and collective inputs needed to maintain equilibrium forward flight, there is no predicted angle of attack distribution prior to each time step. As such, the numerical description of the flap deflection is based on an instantaneous assessment of the location of the leading edge and flap hinge, from which the required displacement of the trailing edge of the flap panel is determined for each deflection. Fig. 3.11 shows the reference line at some angle of attack, and Fig. 3.12 demonstrates how the flap is deflected, based on the reference line.

Figure 3.10: Chordwise distribution of nodes superimposed over a typical NACA 23012 cross section.

Figure 3.11: Nodes at an angle of attack with a reference line through the first and final nodes on the airfoil.

Figure 3.12: Flap deflection, $\delta = 15^\circ$, measured from the reference line.

Chapter 4 – 3D Simulation

Presented in this chapter is the validation of the numerical method. The purpose of the verification and validation is to ensure that the computational model is capable of accurately modeling the test cases. Also presented here are the results from the simulations.

4.1 Validation

The aerodynamic component of GAST has been evaluated in terms of GENUVP and the ONERA model. The GENUVP validation presented here was undertaken by Opoku at Carleton University [28] in order to confirm the suitability of the code for rotorcraft modeling in forward flight. Key to Opoku's work was correct acoustic results, for which aerodynamic fidelity is a necessity. The test case used by Opoku was parametrically similar to the SHARCS model used in the 3D

modeling outlined in Chapter 3 of this thesis. Additional validation of GAST is incorporated into the comparison of the acoustic results obtained by Opoku and the HELINOISE experiment [29].

The ONERA model is popular in rotorcraft research, including the modeling of the effects of trailing edge flap on the rotor system [30] [31]. Validation of the ONERA model is presented by Leishman in comparison to a number of other unsteady aerodynamic models, and experimental measurements.

4.1.1 HELINOISE Validation

Validation of the Aerodynamic component of GAST was done in comparison to the HELINOISE experiments of the 1990's, by Opoku [32]. The test case for these validations was a hingeless BO105 four bladed rotor system, with similar parameters to the SHARCS case explored in this research. The properties of the BO105 rotor are outlined in Tab. 4.1.

The code used for the validation is called SMARTROTOR [33], and has the same aerodynamic and aeroacoustic models as GAST, but with a different structural solver. Structural approximations of the BO105 scale rotor were made by Opoku. Only the lifting surfaces of the blades were modeled with both the structural and aerodynamic components, with a cantilever boundary condition at the root for the flapping and lead-lag motion of the blade, which resulted in some additional stiffness

Chapter 4 - 3D Simulation

at the blade root. Additional approximations of the structural properties of the blades themselves were made from available data of other similar blade sections.

Description	Value
Scaling Factor	2.455
Number of Blades	4
Rotor Type	Hingeless
Airfoil Type	NACA 23012 modified
Rotor Radius	2 m
Blade Chord	0.121 m
Pretwist	-4° /m
Nominal Rotor Speed	1050 RPM (110 rad/s)

Table 4.1: The HELINOISE BO105 scaled rotor test case [34].

The test case presented here is for low-speed flight, with an advance ratio of 0.15. As such, the degree of unsteadiness does not make the helicopter encounter dynamic stall. The ONERA model is, therefore, not tested in this case, and requires separate validation.

Examination of the thrust coefficient measured by the numerical simulation and the experimentation revealed a typical difference of about 15% for 4 different flight regimes, including hover, steady forward flight, ascending flight, and descending flight [35]. Additional comparison was made between the measure normal force coefficient for the numerical model and the HELINOISE experiment

Chapter 4 - 3D Simulation

[36], Fig. 4.1 and 4.2. These two charts show good agreement, and are typical of all four cases.

Figure 4.1: C_N - ψ for HELINOISE case 1333 [37].

Figure 4.2: C_N - ψ for GAST representation of HELINOISE case 1333 [37].

4.1.2 ONERA Validation

A typical dynamic stall case in which the ONERA model, and other semi-empirical dynamic stall models are compared to experimental data for a UH-60 is presented by Leishman [38]. The results are presented in terms of $M^2 C_l$ and $M^2 C_m$ with respect to blade azimuth angle. The ONERA model displays adequate prediction of lift and moment effects, though there is considerable over-prediction of negative $M^2 C_m$ on the advance portion of the rotor disk. However, the ONERA model displays the highest fidelity in comparison to the experimental case, in the dynamic stall region of the rotor disk, which is the region of interest. The results from Leishman are presented with other dynamic stall models in Figs. 4.3 and 4.4.

Figure 4.3: Prediction of comparative C_l for dynamic stall models [38].

Figure 4.4: Prediction of comparative C_m for dynamic stall models [38].

4.2 Computational Method

This section details the input and output variables required and provided by GAST for each computational run. This includes a description of numerical representation of the test case, and a listing of the obtainable output parameters, from which the results of the simulations were determined.

4.2.1 Trim Conditions

The GAST code is capable of determining the required cyclic and collective angles for the maintenance of equilibrium forward flight. This is accomplished through the numerical integration of the thrust differential of the blade as it completes a revolution. Corrections are then made to the trim using a proportional-integrator controller until such time as the variation ceases. This serves as the principle error indication of the simulation, and is the basis for convergence.

Since the total thrust of the rotor is user specified, it is the determining factor for the angle of attack of the rotor blades, as controlled by the cyclic and the collective. This provides the basis for the initial guess of the cyclic and collective values. There were two cases tested, the uncontrolled case, with no flap deflection, and the controlled case, where the flap was deflected downward, Tab. 4.2.

An initial guess was made for the uncontrolled case, from which the simulation proceeded to calculate the real cyclic and collective angles. The uncontrolled case required 103 complete revolutions prior to obtaining a converged value. The converged cyclic and collective angles were extracted from the uncontrolled case, and used as the starting values for the controlled case which converged after only 23 revolutions.

Variable	Uncontrolled Case	Controlled Case
Thrust (N)	1550	1550
Starting Collective (°)	6.426	6.691
Starting Lateral Cyclic (°)	1.047	1.119
Starting Longitudinal Cyclic (°)	-1.202	-1.509
Final Collective (°)	6.691	6.502
Final Lateral Cyclic (°)	1.119	1.064
Final Longitudinal Cyclic (°)	-1.509	-1.271
Error (N)	57.40	52.62
Steps to Convergence	103	23

Table 4.2: Initial and converged trim results.

Since the error term is reported dimensionally in Newtons, the acceptable value for a converged solution will vary with different test cases. Furthermore, the calculation of acceptable error will also depend on the number of steps in a revolution. In the SHARCS test case, there are 72 steps per rev, each corresponding to a 5° change in the azimuth angle. If the total thrust is at all times equal to 1500 N, and the accepted error is 100 N then the equivalent non-dimensional convergence criteria per step would be less than 10^{-3} , which is an acceptable value for unsteady simulation. Since the thrust and convergence for both the uncontrolled and controlled

cases were above 1500 and below 100 N, respectively, both cases are deemed to be suitably converged.

Since the trim values and error are reported once for each complete rotation, they apply equally to the 72 steps in that rotation. As such, the comparison of the results for each rotation can be normalized to some time at which the convergence criteria has been satisfied.

4.2.2 Outputs

The output files are created initially at the end of the first time step and then modified with new output information being appended at the end of each subsequent time step. The output files are composed of aerodynamic and structural data pertaining to the rotor blades. The aerodynamic data includes the lift, drag, and moment coefficients, and the angle of attack, as determined prior to and after the ONERA model calculation, and it is outlined at each station on the rotor blade. The structural files contain the integrated internal reaction force at each node from the tip to the root.

In order to judge the effects of the trailing edge flap it was important to determine the following parameters. Aerodynamically, the lift was determined to be most important, since it provides the key to the aerodynamic load distribution in terms of the azimuth angle, and shows whether or not the flap adversely effects the

Chapter 4 - 3D Simulation

controllability or stability of the helicopter, since equilibrium must be maintained. The angle of attack information was also important since it provides a glimpse into the physics of the flow; because the reduced frequency is unchanged for the two cases, and the unsteady aerodynamic effects should be the same. This means that changes to the angle of attack combined with the same lift profile indicate that the flap is working, and the panel method is solving it properly.

The two most important structural properties to monitor are the vertical hub shear and the torsional moment at the root. The vertical load is transferred to the helicopter fuselage through the rotor hub, causing vibrations and often structural failure as a result. Hence, it is desired that the vibration levels and root vertical loads be reduced. The torsional moment at the root is transferred to the swashplate via the pitch link. In fact, the ratio of the pitching moment and the pitch horn arm length (the distance from the feathering axis to the pitch link) yields the exact pitch link load. Even though the blade used in the GAST simulations is hingeless, this value is still very indicative of the effects on the pitch link loads. Dynamic stall causes a large increase in the nose down aerodynamic pitching moment, and thus the pitch link loads. Excessive vibration levels in the pitch link loads limit the maximum forward flight speed of the helicopter and the principle function of the flap is to reduce this moment. A reduction in the torsional moment will be the key measure of the success of the flap.

Chapter 4 - 3D Simulation

To recapitulate, there are four factors that will be measured in the results section. The first is lift, to determine whether or not the collective and cyclic calculations are working with the flap actuation. The second is the angle of attack, which should show a reduction near the region where the flap is deployed, corresponding to a reduction in the unsteady loading and thus in dynamic stall. The third is the vertical reaction force at the root, which indicates vibration force transferred to the fuselage of the helicopter. Finally, the torsional moment at the blade root is representative of the pitch link loads.

4.3 Results

The results were compared between the uncontrolled and controlled case for the 3D simulations.

Recall that the duration of the calculations was dependant on the trim conditions of the rotor. The uncontrolled case converged after 103 revolutions. The final trim values from that case were then used as a basis for the controlled case, which converged after 23 revolutions. The non-dimensional time displayed in Figs. 4.5 – 4.8 are normalized to the first converged step for both cases, with the case of least computational error centered at $t = 0.29$.

The analysis of the aerodynamic results, to be presented in the following sections, was conducted at 75% span, since this region of flow is least altered by 3D

effects, as stated in section 3.1.2, and this is also where dynamic stall is most prominent.

4.3.1 Lift Examination

In order to accurately assess the effects of the trailing edge flap in the 3D simulations, it was necessary to ensure that the lift provided by the modified system was identical to the uncontrolled case. This was done as a form of validation that the lift values produced by GAST remain correct, even with the introduction of the moving trailing edge flap, a new feature in the code.

First, the uncontrolled test case was run, and allowed to trim, resulting in a stable set of trimmed revolutions, forming a basis for comparison. The controlled case was run until trim and a set of converged revolutions were extracted from the data.

Both the uncontrolled and controlled data sets were normalized to the same start time, based on the beginning of the converged revolutions. A comparison was then made of the lift coefficients for two revolutions, Fig. 4.5. This was achieved by integrating the total lift beneath the C_l -t curves for the controlled and uncontrolled cases over two revolutions. An error of 1.44% was discovered for an average error of 0.72% per revolution. This value was deemed to be acceptable proof that the lift conditions for the specified collective angle and cyclic angle has been met.

Figure 4.5: C_l - t comparison controlled and uncontrolled cases.

The maximum value of C_l for the controlled case was higher than that of the uncontrolled case. This is attributed to the deflection of the trailing edge flap, which is expected to increase $C_{l_{max}}$. This is compensated for through a higher drop off after the peak value. Symmetry is maintained through the appearance of a steeper slope leading up to the peak value as well.

4.3.2 Angle of Attack Examination

Since GAST has not previously been used for simulations with an embedded trailing edge flap, it was necessary to ensure that the angle of attack history properly represented the effects of the flap. For an airfoil in a static flow field, the lift curve slope will shift vertically for a downward flap deflection. This means that the lift coefficient will be higher at a given airfoil angle of attack, or conversely that the same lift coefficient can be achieved at a lower angle of attack. Since the examination of the lift coefficient history in the previous section revealed a preservation of the necessary values, it is anticipated that an analysis of the angle of attack history reveal a decrease in angle of attack in the region where the flap is deflected.

Much of the success of the trailing edge flap in this regime is dependent on the maximum angle of attack to which the rotor blade is exposed. The effects of reducing the angle of attack are twofold. First, a decrease in the peak angle of attack of a dynamically pitching airfoil section results in a reduction of the dynamic stall effects. The strength of the dynamic stall vortex that is formed at the leading edge will decrease, as will the strength of the induced trailing edge vortex, and the overall pressure difference between the lower and upper surfaces. Second, a reduction in the peak angle of attack is also a reduction in the degree of unsteadiness of the system, since the airfoil has a smaller amplitude of oscillation over a given time period. This

Chapter 4 - 3D Simulation

reduction in unsteadiness will provide the favourable stability condition of having the pitching motion opposed by the aerodynamic pitching moment, allowing for a preservation of aerodynamic damping, in comparison to a more unsteady case.

Analysis of the angle of attack history is conducted using the same two revolutions as the lift coefficient history and is provided in Fig. 4.6. As can be seen in Fig. 4.6, there is a measured drop in the maximum angle of attack, from 16.2° to 15° . The critical angle of attack for the flow conditions in the test case is 14.6° . This, however, is the static value and it has been detailed in section 1.3 that among the effects of flow unsteadiness on a dynamically pitching airfoil is the retardation of stall. As such, the effective stall angle for this test case is somewhat higher than 14.6° , indicating that it is likely that the angle of attack distribution for the controlled case represents a reduction in the area of the rotor disk to which the blades are exposed to dynamic stall.

These angle of attack results, combined with the lift history, are as desired. They indicate that the GAST code is capable of resolving the aerodynamic effects of the trailing edge flap, and should provide reasonable results for the structural reactions to these effects. The satisfactory reduction in the peak angle of attack, and the corresponding change to the unsteadiness of the system are good indications that the structural reactions should be reduced as desired.

Figure 4.6: α -t comparison for controlled and uncontrolled cases.

4.3.3 Force Examination

Dynamic stall effects are greatest in the vertical direction due to the influence of stall on lift and thrust. Furthermore, the rotor blade is markedly stiffer with respect to chord than with respect to thickness, based on the sectional properties of the blade. It is desired that the application of the trailing edge flap will reduce the peak loads experienced by the rotor blade.

Fig. 4.7 shows the comparison of the internal structural reactions, F_z . The average decrease in the amplitude of vibration over the analyzed time period was

23% due to the flap actuation, when comparing the peak amplitudes for the converged results.

Figure 4.7: F_z - t comparison for controlled and uncontrolled cases.

4.3.4 Moment Examination

The moments at the root of the blade are directly related to the pitch link loads. Since the pitch link controls the angle of attack of the rotor blade, according to the collective and cyclic inputs through the swash plate, the moment at the blade root must be countered by the product of the force at the pitch link the distance from the

Chapter 4 - 3D Simulation

feathering axis of the blade to the pitch link. The pitch link is weak in comparison to the rest of the hub, and is of concern in terms of overloading. It is desired that the application of the trailing edge flap serve to reduce the pitch link loads through more favourable aerodynamic damping, and an overall reduction in the magnitude of the negative pitching moment.

A comparison of the integrated torsional moments at the blade root is shown in Fig. 4.8. It can be seen that the reduction of these loads is much more dramatic than that of the vertical loads, with the peak moments reduced by as much as 44%. Furthermore, the average variation from the mean torsional moment is reduced from 3.21 N·m to 1.46 N·m, a reduction of 54.4%. Since the torsional moments are transferred to the rotor hub via the pitch link rods, it can be concluded that a major benefit of the ACF concept in the moderate advance ratio case is the reduction of the vibratory levels in the pitch link loads.

Figure 4.8: M_y - t comparison for controlled and uncontrolled cases.

Chapter 5 - 2D Test Case and Numerical Method

In Chapters 3 and 4, it has been shown that dynamic stall induced vibrations can be greatly reduced by employing a downward deflected trailing edge flap on the retreating side of rotor disk. By this, the lift required on the retreating rotor blade is achieved by increasing the effective camber of the airfoil, instead of using the extra ‘dynamic’ lift produced as a consequence of dynamic stall, Fig. 5.1.

However, this strategy can only work up to a certain, moderate, forward flight speed. As the helicopter flies faster, the amplitude of the cyclic pitch oscillations and the maximum pitch angle on the retreating rotor blades increase. Close to the upper limit of the forward flight speed, this will mean the occurrence of dynamic stall again, even if the flap is deflected down.

Thus, the next objective of this work was to examine whether or not the same trailing edge flap can be used for mitigating the negative effects of dynamic stall,

Chapter 5 - 2D Test Case and Numerical Method

particularly those leading to vibrations. Such investigation, however, would necessitate the use of a computational tool capable of simulating dynamic stall, as opposed to only modeling its effects, as was done in the GAST code through the use of GENUVP and the ONERA dynamic stall model.

Figure 5.1: Effect of flap on lift curve slope.

Since a fully viscous 3D aeroelastic rotorcraft code is currently beyond the state-of-the-art of CFD, it has been decided to explore this problem in a test case representative of 2D dynamic stall. Such a case can be regarded as representative of the blade section at 75% of the rotor radius.

5.1 Test Case

Two-dimensional dynamic stall has been traditionally investigated on oscillatory pitching or plunging airfoils. Pitch and plunge are equivalent in that they both lead to the oscillatory change in the effective angle of attack at the airfoil, according to:

$$\alpha(t) = \alpha_0 + \alpha_1 \sin(2kt)$$

where α_0 is the mean angle of attack, and k is the reduced frequency of the oscillation defined as:

$$k = \frac{\omega b}{V} = \frac{\omega c}{2V}$$

where b is the chord, c is semi-chord, V is the reference velocity of the flow, and ω is the angular frequency of the oscillation.

There is a wide range of dynamic stall experiments available in the open literature, of which the most famous is probably that of McCroskey *et. al.* from 1981 [39]. Their work into categorization of the effects of Reynolds number, Mach number, reduced frequency, and airfoil section is exhaustive. Experimental study of

Chapter 5 - 2D Test Case and Numerical Method

dynamic stall was also conducted at the University of Glasgow [40], based on the method of McCroskey. The availability of highly detailed data from the Glasgow experiment made it ideally suited to form the basis of the 2D numerical simulation, since validation of the uncontrolled test case is a necessity.

5.1.1 Airfoil Details

The University of Glasgow experiment was performed using a NACA 0012 airfoil which is well suited to represent a typical rotor blade section based on its thickness. The NACA 0012 airfoil is symmetric about its chord, and has no static pitching moment, meaning that any change in pitching moment is caused by a change in flow conditions.

The simulation of dynamic stall is accomplished via the sinusoidal pitching of the airfoil through a range of 5° to 25° angle of attack, as indicated in Tab. 5.1. The Reynolds and Mach number are representative of the flow conditions that would be experienced by a real rotor blade at the retreating portion of the azimuth. Since the freestream velocity is time invariant, the Reynolds and Mach numbers are likewise invariant. The test case details are located in Tab. 5.1.

Description	Value
Mach Number (M)	0.117
Reynolds Number (Re)	1,463,296
Reduced Frequency (k)	0.173
Airfoil Section	NACA 0012
Airfoil Motion	$\alpha(t) = 15^\circ + 10^\circ \sin(2kt)$

Table 5.1: 2D dynamic stall test case.

5.1.2 Flap Details

In order to determine the effects the trailing edge flap has on dynamic stall, it is necessary to evaluate the impact of flap size, flap deflection, and flap timing. Three flap sizes, and three flap deflections are used. The flap sizes measured in percent relative chord, were selected based on a reasonable assumption of manufacturability in respect to the SHARCS rotor blades, Fig. 5.2. The same consideration was given to the deflection and timing, since this research is intended to benefit the SHARCS project in addition to providing fundamental results for the effectiveness of the trailing edge flap.

The effects of the flap are determined through two sets of parametric studies. In the first set, each of the three flap deflections will be tested while the flap size is held constant. This permits the effect of flap deflection to be determined. In the

Chapter 5 - 2D Test Case and Numerical Method

second set of parametric studies, each of the three flap sizes will be tested while holding the peak flap deflection constant, permitting the effect of flap size to be determined.

Figure 5.2: Flap size in percent of relative chord.

The flap is to be deflected over an area of roughly $\frac{1}{4}$ the rotor disk, similar to the 3D test case as described in section 3.1.2. However, a major difference is that in this case the flap deflects upwards instead of downward, as in the 3D case. It is hoped that such a flap deflection strategy can alter the negative effects of dynamic stall if the avoidance of dynamic stall is no longer feasible. Preliminary examination of the upward flap deflection showed promising results [41]. Therefore, in the 2D case the peak deflection of flap is timed to maximize its effect on the dynamic stall vortex, as opposed to coinciding with the peak angle of attack as in the 3D case.

Chapter 5 - 2D Test Case and Numerical Method

Represented in Fig. 5.3 is timing typical of the 2D test cases. Note that negative δ coincides with upward flap deflection.

Figure 5.3: Flap deflection with respect to azimuth angle for the 2D simulations.

5.2 Numerical Method

The numerical method used for the 2D simulations is CMB (Carleton Multi Block), which is a derivative of PMB (Parallel Multi Block), a code developed at the University of Glasgow [42]. The CMB code is capable of 3D RANS (Reynolds Averaged Navier Stokes) simulations, in addition to Euler simulations. The effectiveness of RANS to model complex flows is dependent on turbulence

modeling. This is particularly true in the case of dynamic stall where the accurate prediction of flow separation and reattachment are important.

5.2.1 Non-Dimensional RANS Equations in Curvilinear Form

The governing equations are expressed in conservative form. The conservative form is well suited to use in Computational Fluid Dynamics since the continuity, energy and momentum equations can all be transformed to the same generic equations, simplifying the discretization process.

The Reynolds Averaged Navier-Stokes equations in conservative form for a Cartesian Coordinate System can be written as:

$$\frac{\partial \mathbf{W}}{\partial t} + \frac{\partial (\mathbf{F}^i - \mathbf{F}^v)}{\partial x} + \frac{\partial (\mathbf{G}^i - \mathbf{G}^v)}{\partial y} + \frac{\partial (\mathbf{H}^i - \mathbf{H}^v)}{\partial z} = 0 \quad (5.1)$$

where $\mathbf{W}$ is the solution vector, which contains the flow variables and is defined for 3D flows as:

Chapter 5 - 2D Test Case and Numerical Method

$$\mathbf{W} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho E \end{pmatrix} \quad (5.2)$$

The density of the fluid is represented by ρ , and the components of velocity in the Cartesian plains of x , y , and z are represented at u , v , and w respectively. E is the total energy per unit mass.

The conservative form of the Navier-Stokes equations are obtained through a derivation based upon a spatially fixed control volume. This imposes the necessary consideration of the flux of energy, mass and momentum through the control volume. It is the net conservation of these properties that leads to the term conservative form. The flux vectors $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{H}$, as appearing above, are composed of inviscid terms as described with the superscript i and diffusive viscous terms as described by the superscript v . The inviscid flux terms are described below:

$$\mathbf{F}^i = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uw \\ \rho uH \end{pmatrix} \quad \mathbf{G}^i = \begin{pmatrix} \rho v \\ \rho vu \\ \rho v^2 + p \\ \rho vw \\ \rho vH \end{pmatrix} \quad \mathbf{H}^i = \begin{pmatrix} \rho w \\ \rho wu \\ \rho wv \\ \rho w^2 + p \\ \rho wH \end{pmatrix} \quad (5.3)$$

Chapter 5 - 2D Test Case and Numerical Method

where pressure is denoted by p and the enthalpy by H . The viscous terms are:

$$\mathbf{F}^v = \frac{1}{\text{Re}} \begin{pmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{xz} \\ u\tau_{xx} + u\tau_{xy} + u\tau_{xz} + q_x \end{pmatrix}$$

$$\mathbf{G}^v = \frac{1}{\text{Re}} \begin{pmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ \tau_{yz} \\ u\tau_{xy} + u\tau_{yy} + u\tau_{yz} + q_y \end{pmatrix}$$

$$\mathbf{H}^v = \frac{1}{\text{Re}} \begin{pmatrix} 0 \\ \tau_{xz} \\ \tau_{yz} \\ \tau_{zz} \\ u\tau_{xz} + u\tau_{yz} + u\tau_{zz} + q_z \end{pmatrix}$$

(5.4)

The stress tensors are defined as:

$$\tau_{xx} = -\mu \left(2 \frac{\partial u}{\partial x} - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right)$$

Chapter 5 - 2D Test Case and Numerical Method

$$\begin{aligned}
 \tau_{yy} &= -\mu \left(2 \frac{\partial v}{\partial y} - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right) \\
 \tau_{zz} &= -\mu \left(2 \frac{\partial w}{\partial z} - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right) \\
 \tau_{xy} &= -\mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad \tau_{xz} = -\mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \quad \tau_{yz} = -\mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)
 \end{aligned}
 \tag{5.5}$$

and the heat flux vectors are written as:

$$\begin{aligned}
 q_x &= -\frac{1}{(\gamma-1)M_\infty^2} \frac{\mu}{Pr} \frac{\partial T}{\partial x} \\
 q_y &= -\frac{1}{(\gamma-1)M_\infty^2} \frac{\mu}{Pr} \frac{\partial T}{\partial y} \\
 q_z &= -\frac{1}{(\gamma-1)M_\infty^2} \frac{\mu}{Pr} \frac{\partial T}{\partial z}
 \end{aligned}
 \tag{5.6}$$

The variables in the heat flux terms are the specific heat ratio γ , the Prandtl number Pr , the static temperature T , and the freestream Mach number M_∞ . The ideal gas law provides the relationship between many of the variables:

Chapter 5 - 2D Test Case and Numerical Method

$$H = E + \frac{p}{\rho}$$

$$E = e + \frac{1}{2}(u^2 + v^2)$$

$$p = (\gamma - 1)\rho e$$

$$\frac{p}{\rho} = \frac{T}{\gamma M_\infty^2}$$

(5.7)

The coefficient of bulk viscosity, or laminar viscosity, term μ is determined according to Sutherland's law:

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0} \right)^{3/2} \frac{T_0 + 110}{T + 110} \quad (5.8)$$

where $\mu_0 = 1.7894 \times 10^{-5} \frac{kg}{m \cdot s}$ is a reference viscosity at a reference temperature of

$T_0 = 288.16K$. In each of the above cases, the variables represent the non-dimensionalized quantities according to:

$$x = \frac{x^*}{L^*}, y = \frac{y^*}{L^*}, z = \frac{z^*}{L^*}, t = \frac{t^*}{L^* / V_\infty^*},$$

Chapter 5 - 2D Test Case and Numerical Method

$$\begin{aligned}
 u &= \frac{u^*}{V_\infty^*}, v = \frac{v^*}{V_\infty^*}, w = \frac{w^*}{V_\infty^*}, \mu = \frac{\mu^*}{\mu_\infty^*}, \\
 \rho &= \frac{\rho^*}{\rho_\infty^*}, p = \frac{p^*}{\rho_\infty^* V_\infty^{*2}}, T = \frac{T^*}{T_\infty^*}, e = \frac{e^*}{V_\infty^{*2}}
 \end{aligned}
 \tag{5.9}$$

where terms with superscript * represent dimensional terms.

The Navier-Stokes equations permit the complete and accurate solution of a physical flow. In order to allow this solution to occur in a timely manner, turbulence modeling is introduced as a means of predicting turbulent transition, flow separation, and flow reattachment. The Reynolds-averaged form of the Navier-Stokes equation decomposes the flow variables into a mean, or time averaged component, and a turbulent fluctuation describing the instantaneous variation from the mean value. For example, the velocity in x would be decomposed as $u = \bar{u} + u'$.

The decomposition of the flow variables leads to the creation of new stress terms called Reynolds stresses. These terms are inherently fictitious in that they are the result of the decomposition process and would not exist in a non Reynolds-averaged equation. To permit the modeling of these terms, the Boussinesq assumption is used to create additional variables, namely the turbulent kinetic energy k , the turbulent or eddy viscosity μ_T , and the turbulent Prandtl number Pr_T . The

Chapter 5 - 2D Test Case and Numerical Method

stress tensors and heat flux vector components of the Navier-Stokes equations then become:

$$\begin{aligned}
 \tau_{xx} &= -(\mu + \mu_T) \left(2 \frac{\partial u}{\partial x} - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right) + \frac{2}{3} \rho k \\
 \tau_{yy} &= -(\mu + \mu_T) \left(2 \frac{\partial v}{\partial y} - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right) + \frac{2}{3} \rho k \\
 \tau_{zz} &= -(\mu + \mu_T) \left(2 \frac{\partial w}{\partial z} - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right) + \frac{2}{3} \rho k \\
 \tau_{xy} &= -(\mu + \mu_T) \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \\
 \tau_{xz} &= -(\mu + \mu_T) \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\
 \tau_{yz} &= -(\mu + \mu_T) \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)
 \end{aligned}
 \tag{5.10}$$

and,

$$q_x = -\frac{1}{(\gamma - 1)M_\infty^2} \left(\frac{\mu}{\text{Pr}} + \frac{\mu_T}{\text{Pr}_T} \right) \frac{\partial T}{\partial x}$$

Chapter 5 - 2D Test Case and Numerical Method

$$\begin{aligned} q_y &= -\frac{1}{(\gamma-1)M_\infty^2} \left(\frac{\mu}{\text{Pr}} + \frac{\mu_T}{\text{Pr}_T} \right) \frac{\partial T}{\partial y} \\ q_z &= -\frac{1}{(\gamma-1)M_\infty^2} \left(\frac{\mu}{\text{Pr}} + \frac{\mu_T}{\text{Pr}_T} \right) \frac{\partial T}{\partial z} \end{aligned} \tag{5.11}$$

where the variables represent the mean flow values, with the superscripts and identifiers discarded in favour of clarity. The turbulence properties of the flow are modeled by the turbulent viscosity μ_T , and the turbulent kinetic energy k , and an additional set of closure equations. The closure equations present a mathematical resolution for the effects of turbulence, and are known as turbulence models.

The governing equations are re-expressed in curvilinear form, in order to permit their adaptation to arbitrary grid orientations and densities. A spatial transformation from the Cartesian x,y,z coordinate system to the local curvilinear ξ,η,ζ coordinate system is introduced as:

$$\xi = \xi(x, y, z)$$

$$\eta = \eta(x, y, z)$$

$$\zeta = \zeta(x, y, z)$$

$$t = t.$$

and the Jacobian determinant of the transformation is given by:

$$J = \frac{\partial(\xi, \eta, \varsigma)}{\partial(x, y, z)}$$

The governing equation can then be expressed in curvilinear form as:

$$\frac{\partial \hat{\mathbf{W}}}{\partial t} + \frac{\partial(\hat{\mathbf{F}}^i - \hat{\mathbf{F}}^v)}{\partial \xi} + \frac{\partial(\hat{\mathbf{G}}^i - \hat{\mathbf{G}}^v)}{\partial \eta} + \frac{\partial(\hat{\mathbf{H}}^i - \hat{\mathbf{H}}^v)}{\partial \varsigma} = 0 \quad (5.12)$$

where the variables are represented by:

$$\hat{\mathbf{W}} = \frac{\mathbf{W}}{J}$$

$$\hat{\mathbf{F}}^i = \frac{1}{J}(\xi_x \mathbf{F}^i + \xi_y \mathbf{G}^i + \xi_z \mathbf{H}^i)$$

$$\hat{\mathbf{G}}^i = \frac{1}{J}(\eta_x \mathbf{F}^i + \eta_y \mathbf{G}^i + \eta_z \mathbf{H}^i)$$

$$\hat{\mathbf{H}}^i = \frac{1}{J}(\varsigma_x \mathbf{F}^i + \varsigma_y \mathbf{G}^i + \varsigma_z \mathbf{H}^i)$$

$$\hat{\mathbf{F}}^v = \frac{1}{J}(\xi_x \mathbf{F}^v + \xi_y \mathbf{G}^v + \xi_z \mathbf{H}^v)$$

Chapter 5 - 2D Test Case and Numerical Method

$$\hat{\mathbf{G}}^v = \frac{1}{J} (\eta_x \mathbf{F}^v + \eta_y \mathbf{G}^v + \eta_z \mathbf{H}^v)$$

$$\hat{\mathbf{H}}^v = \frac{1}{J} (\varsigma_x \mathbf{F}^v + \varsigma_y \mathbf{G}^v + \varsigma_z \mathbf{H}^v)$$

(5.13)

The expressions for the inviscid fluxes become complicated. In order to make them more manageable, U , V , and W are defined as:

$$U = \xi_x u + \xi_y v + \xi_z w$$

$$V = \eta_x u + \eta_y v + \eta_z w$$

$$W = \varsigma_x u + \varsigma_y v + \varsigma_z w$$

(5.14)

This permits the inviscid flux terms to be expressed more reasonably as:

$$\hat{\mathbf{F}}^i = \begin{pmatrix} \rho U \\ \rho u U + \xi_x p \\ \rho v U + \xi_y p \\ \rho w U + \xi_z p \\ \rho U H \end{pmatrix}$$

$$\begin{aligned}\hat{\mathbf{G}}^i &= \begin{pmatrix} \rho V \\ \rho u V + \eta_x p \\ \rho v V + \eta_y p \\ \rho w V + \eta_z p \\ \rho V H \end{pmatrix} \\ \hat{\mathbf{H}}^i &= \begin{pmatrix} \rho W \\ \rho u W + \varsigma_x p \\ \rho v W + \varsigma_y p \\ \rho w W + \varsigma_z p \\ \rho W H \end{pmatrix}\end{aligned}\tag{5.15}$$

The Cartesian stress tensors contain derivative terms that must also be converted into the curvilinear form. This is accomplished via the chain rule. An example is the decomposition:

$$\frac{\partial u}{\partial x} = \xi_x \frac{\partial u}{\partial \xi} + \eta_x \frac{\partial u}{\partial \eta} + \varsigma_x \frac{\partial u}{\partial \varsigma}$$

5.2.2 Steady State Solver

The steady state solver of CMB employs a cell-centred finite volume method for the discretization of the curvilinear Navier-Stokes equations, Eq. 5.12. The finite

Chapter 5 - 2D Test Case and Numerical Method

volume method decomposes the computational domain into a finite number of non-overlapping control-volumes, to which the governing equations are applied. From the previous section, the governing equation is:

$$\frac{\partial \hat{\mathbf{W}}}{\partial t} + \frac{\partial (\hat{\mathbf{F}}^i - \hat{\mathbf{F}}^v)}{\partial \xi} + \frac{\partial (\hat{\mathbf{G}}^i - \hat{\mathbf{G}}^v)}{\partial \eta} + \frac{\partial (\hat{\mathbf{H}}^i - \hat{\mathbf{H}}^v)}{\partial \zeta} = 0 \quad (5.12)$$

The discretization of this equation with the finite volume method leads to a set of temporal ordinary differential equations of the form:

$$\frac{d}{dt}(\mathbf{W}_{i,j,k} V_{i,j,k}) = -\mathbf{R}_{i,j,k}(\mathbf{W}) \quad (5.16)$$

where vectors $\mathbf{W}$ and $\mathbf{R}$ are the vectors of the cell conserved variables and the cell residuals, respectively. $V_{i,j,k}$ is the volume of the cell. Osher's approximate Riemann solver is used for the discretization of the convective terms, due to its robustness, accuracy and stability properties. The MUSCL variable interpolation is used in conjunction with the Van Albada limiter in order to provide second order accuracy without erroneous fluctuations around shocks. Two layers of Ghost cells are generated outside of the computational domain providing boundary conditions. For the far-field, the ghost cells are set universally to the freestream conditions. For

Chapter 5 - 2D Test Case and Numerical Method

viscous flows, the no-slip boundary condition is imposed at the walls, while the normal component of velocity is set to zero for inviscid ones. Ghost cell values are then extrapolated from the interior.

A numerical integration allows the solution of the set of differential equations created by the finite volume method. The steady state solution is determined from an implicit time-marching scheme according to:

$$\frac{\mathbf{W}_{i,j,k}^{n+1} - \mathbf{W}_{i,j,k}^n}{\Delta t} = -\frac{1}{V_{i,j,k}} \mathbf{R}_{i,j,k}(\mathbf{W}_{i,j,k}^{n+1}) \quad (5.17)$$

where the superscripts n and $n+1$ refer to time $n\Delta t$ and $(n+1)\Delta t$ respectively. The numerical integration expressed above represents a system of non-linear equations. In order to simplify the solution procedure, the residual term $\mathbf{R}_{i,j,k}$ is linearized:

$$\begin{aligned} \mathbf{R}_{i,j,k}(\mathbf{W}^{n+1}) &= \mathbf{R}_{i,j,k}(\mathbf{W}^n) + \frac{\partial \mathbf{R}_{i,j,k}}{\partial t} \Delta t + O(\Delta t^2) \\ &\approx \mathbf{R}_{i,j,k}^n(\mathbf{W}^n) + \frac{\partial \mathbf{R}_{i,j,k}}{\partial \mathbf{W}_{i,j,k}} \frac{\partial \mathbf{W}_{i,j,k}}{\partial t} \Delta t \\ &\approx \mathbf{R}_{i,j,k}^n(\mathbf{W}^n) + \frac{\partial \mathbf{R}_{i,j,k}}{\partial \mathbf{W}_{i,j,k}} \Delta \mathbf{W}_{i,j,k} \end{aligned} \quad (5.18)$$

where $\Delta \mathbf{W}_{i,j,k} = \mathbf{W}_{i,j,k}^{n+1} - \mathbf{W}_{i,j,k}^n$, and the integration becomes linear according to:

$$\left[\frac{V_{i,j,k}}{\Delta t} \mathbf{I} + \frac{\partial \mathbf{R}_{i,j,k}}{\partial \mathbf{W}_{i,j,k}} \right] \Delta \mathbf{W}_{i,j,k} = -\mathbf{R}_{i,j,k}^n(\mathbf{W}^n) \quad (5.19)$$

The above linear system is solved through a Generalized Conjugate Gradient (GCG) method. The CGC method provides an approximate solution of a linear system by minimizing an error function in a finite-dimensional space of potential solution vectors. The preconditioning strategy for the linear system is based on a block incomplete lower-upper (BILU) factorization. This was selected because the sparsity pattern of the BILU method is the same as that of the Jacobian matrix. The BILU factorization is decoupled between blocks, and thus does not restrict the efficiency of parallel computation.

Since implicit schemes require special treatment prior to the full fledged resolution of a computational domain, the flow is initially solved explicitly. This allows for some smoothing of the flow field, before the solver can be switched to the less robust but much faster implicit scheme. The resulting Jacobian matrix has more than one non-zero entries per row.

Chapter 5 - 2D Test Case and Numerical Method

The solution for the steady state turbulent case is identical to the steady state mean flow case described above. The eddy viscosity is determined from the turbulent kinetic energy, and other pertinent values, and then used to calculate the mean flow values, which are in turn used to update the turbulent terms. An approximate Jacobian matrix is used as the source term by disregarding the production terms and accounting only for the dissipation terms.

5.2.3 Unsteady Solver

The unsteady solver enables time-accurate simulations of unsteady flows on meshes. The case described here is for turbulent flow, and accounts for mesh deformation. The laminar and inviscid cases are calculable through the elimination of terms, and constant mesh size is solvable without adjustment.

The basis of the unsteady solver is the multigrid false-time formulation developed by Jameson [43]. The Jameson formulation for the updated steady state problem is given by:

$$\mathbf{R}_{i,j,k}^* = \frac{3\mathbf{w}_{i,j,k}^{n+1}V_{i,j,k}^{n+1} - 4\mathbf{w}_{i,j,k}^nV_{i,j,k}^n + \mathbf{w}_{i,j,k}^{n-1}V_{i,j,k}^{n-1}}{2\Delta t} + \mathbf{R}_{i,j,k}(\tilde{\mathbf{w}}_{i,j,k}^{k_m}, \tilde{\mathbf{q}}_{i,j,k}^{k_l}) = 0 \quad (5.20)$$

Chapter 5 - 2D Test Case and Numerical Method

$$\mathbf{Q}_{i,j,k}^* = \frac{3\mathbf{q}_{i,j,k}^{n+1}V_{i,j,k}^{n+1} - 4\mathbf{q}_{i,j,k}^nV_{i,j,k}^n + \mathbf{q}_{i,j,k}^{n-1}V_{i,j,k}^{n-1}}{2\Delta t} + \mathbf{Q}_{i,j,k}(\tilde{\mathbf{w}}_{i,j,k}^{l_m}, \tilde{\mathbf{q}}_{i,j,k}^{l_t}) = 0 \quad (5.21)$$

Here, the superscripts k_m, k_t, l_m , and l_t represent the time level of the spatially discretized variables. These superscripts are used rather than of simply tying them in with $n+1, n$, or $n-1$ so that the unsteady solution method may be varied. For instance, if $k_m = k_t = l_m = l_t = n+1$, then the mean and turbulent quantities are all advanced in real time in a fully coupled manner. However, if $k_m = l_m = l_t = n+1$ and $k_t = n$, then the equations are advanced in sequence in real time, since the turbulent value is determined and frozen and then used to establish the new mean values, allowing new turbulence values to be found recursively. In this formulation, the only modification to the laminar case is the addition of the eddy viscosity from the previous time step. Since the turbulence model is tied only to the eddy viscosity, it allows a two equation turbulence model to be implemented without modification to the mean flow solver, and is the method of choice for the computational code. However, since the sequenced solution decouples the system, it is important to recognize that there is an upper limit to the size of the time step with which the system will remain stable.

In order to solve Jameson's equations for the updated steady state problem, a false or pseudo time τ is introduced, giving the system of equations:

Chapter 5 - 2D Test Case and Numerical Method

$$\frac{\mathbf{w}_{i,j,k}^{n+1,m+1}V_{i,j,k}^{n+1,m+1} - \mathbf{w}_{i,j,k}^{n+1,m}V_{i,j,k}^{n+1,m}}{\Delta\tau} + \frac{3\mathbf{w}_{i,j,k}^{k_m}V_{i,j,k}^{k_m} - 4\mathbf{w}_{i,j,k}^nV_{i,j,k}^n + \mathbf{w}_{i,j,k}^{n-1}V_{i,j,k}^{n-1}}{2\Delta t} + R_{i,j,k}(\tilde{\mathbf{w}}_{i,j,k}^{k_m}, \mathbf{q}_{i,j,k}^{k_t}) = 0$$

(5.22)

$$\frac{\mathbf{q}_{i,j,k}^{n+1,m+1}V_{i,j,k}^{n+1,m+1} - \mathbf{q}_{i,j,k}^{n+1,m}V_{i,j,k}^{n+1,m}}{\Delta\tau} + \frac{3\mathbf{q}_{i,j,k}^{l_t}V_{i,j,k}^{l_t} - 4\mathbf{q}_{i,j,k}^nV_{i,j,k}^n + \mathbf{q}_{i,j,k}^{n-1}V_{i,j,k}^{n-1}}{2\Delta t} + Q_{i,j,k}(\tilde{\mathbf{w}}_{i,j,k}^{l_m}, \mathbf{q}_{i,j,k}^{l_t}) = 0$$

(5.23)

where the superscript n continues to describe the real time step, and the superscript m is introduced in order to describe the pseudo time step. The iteration scheme effects only the efficiency of the simulation and not the result, allowing the solution to be sequenced in pseudo time without compromising accuracy. An implicit time stepping method for pseudo time is employed in CMB, with $k_m = l_m = l_t = n+1, m+1$ and $k_t = n+1, m$. Here, the solution of the equations is decoupled by freezing values, but the real time stepping proceeds without sequential error. The above provides second order accuracy in time.

5.2.4 Turbulence Model

In order to have a closed solution to the Reynolds Averaged Navier Stokes equations, closure equations must be implemented. The closure equation comes in the form of a turbulence model. For the oscillating airfoil test cases, it was decided that the Shear Stress Transport (SST) turbulence model would provide the best solution. The SST model is known to be well suited to cases involving complex surface interaction, and it is also capable of resolving the free stream. Since dynamic stall is characterized by the formation and convection of a dynamic stall vortex, the ability to resolve both the complex surface interaction and the free stream is recognized as being important.

Menter's SST model [44] blends the $k-\epsilon$ model which is typically accepted for turbulence modeling outside of the boundary layer, and the $k-\omega$ model which is accepted for turbulence modeling within the boundary layer. The principle was to extract the best qualities of the two models by applying them only in their respective zones, by converting the shear stresses from one form to the other at the interface, hence the name Shear Stress Transport. As stated in section 5.2.3, a two equation turbulence model can be introduced through the eddy viscosity term, meaning the mean flow solver is not directly dependant on the closure equation. The eddy viscosity term is:

Chapter 5 - 2D Test Case and Numerical Method

$$\mu_T = \frac{\rho k / \omega}{\max[1 : \Omega F_2 / (a_1 \omega)]}, \quad a_1 = 0.31 \quad (5.24)$$

which describes that in a turbulent boundary layer the maximum value of eddy viscosity is limited by forcing the shear stress to be bounded by the turbulent kinetic energy ω time a constant $a_1 = 0.31$. Additionally, the absolute vorticity is multiplied by a forcing function F_2 which is defined as a function of the wall distance y according to:

$$F_2 = \tanh \left[\left(\max \left[2 \frac{\sqrt{k}}{0.09 \omega y}; \frac{500 \mu}{\rho y^2 \omega} \right] \right)^2 \right] \quad (5.25)$$

The two equations of the SST model are defined in terms of the blending function F_1 , which established the appropriate mix of the ε and ω model equations. The transport equation is given by:

$$\rho \frac{\partial k}{\partial t} + \rho \mathbf{V} \nabla k - \frac{1}{\text{Re}} \nabla \cdot [(\mu + \sigma^* \mu_T) \nabla k] = P_k - \beta^* \rho k \omega \quad (5.26)$$

and the specific dissipation rate by:

Chapter 5 - 2D Test Case and Numerical Method

$$\rho \frac{\partial \omega}{\partial t} + \rho \mathbf{V} \nabla \omega - \frac{1}{\text{Re}} \nabla [(\mu + \sigma_{\omega} \mu_T) \nabla \omega] = P_{\omega} - \beta \rho \omega^2 + 2(1 - F_1) \frac{\rho \sigma_{\omega 2}}{\omega} \nabla k \nabla \omega \quad (5.27)$$

The blending function F_1 is set to unity for a no-slip boundary and goes to zero towards the outer edge of the boundary layer. The function is defined as:

$$F_1 = \tanh \left[\left[\min \left(\max \left[\frac{\sqrt{k}}{0.09 \omega y}, \frac{500 \mu}{\rho y^2 \omega} \right]; \frac{4 \rho \sigma_{\omega 2} k}{CD_{k\omega} y^2} \right) \right]^4 \right] \quad (5.28)$$

where $CD_{k\omega}$ is the cross-diffusion term for the k - ω turbulence model, and is defined as:

$$CD_{k\omega} = \max \left[\frac{2 \rho \sigma_{\omega 2}}{\omega} \nabla k \nabla \omega; 10^{-20} \right] \quad (5.29)$$

The appropriate constants must also be determined from the k - ε and k - ω models. Three coefficients remain constant for all cases:

$$a_1 = 0.31 \quad \beta^* = 0.09 \quad \kappa = 0.41$$

Chapter 5 - 2D Test Case and Numerical Method

The remainder of the coefficients, β , γ , σ_k , and σ_ω are found by blending the constants from the other two turbulence models. Allowing ϕ to represent a characteristic coefficient (such that $\phi = [\sigma_k, \sigma_\omega, \beta, \gamma]$), with ϕ_1 representing the value of the coefficients in the k - ω model, and ϕ_2 so doing for the k - ε model, the pattern of coefficient blending according to F_1 is:

$$\phi = F_1 \phi_1 + (1 - F_1) \phi_2 \quad (5.30)$$

The coefficients of the k - ω model are:

$$\sigma_{k1} = 0.85 \quad \sigma_{\omega1} = 0.5 \quad \beta_1 = 0.075 \quad \gamma_1 = \beta_1 / \beta^* - \sigma_{\omega1} \kappa^2 / \sqrt{\beta^*} = 0.553$$

and for the k - ε model are:

$$\sigma_{k2} = 1.0 \quad \sigma_{\omega2} = 0.856 \quad \beta_2 = 0.0828 \quad \gamma_2 = \beta_2 / \beta^* - \sigma_{\omega2} \kappa^2 / \sqrt{\beta^*} = 0.440$$

5.2.5 Modifications to the Solver

In order to facilitate the actuation of the trailing edge flap in CMB it was necessary to make changes to the code. CMB was originally designed to support only one solid body motion. Even though CMB can tolerate mesh deformation based on aeroelastic inputs, there was no capacity to solve the relative motion of solid bodies for a non-aeroelastic case.

The existing mesh deformation algorithm, known as the trans-finite interpolation (TFI) algorithm maintains the number of nodes between the solid surface and the farfield. The algorithm determines a new nodal distribution based on the relative motion between the surface and the far field in order to preserve the characteristics of the mesh. The new nodes are then resolved into volumes in order to fill in the volume term V in Eqs. 5.22 and 5.23, Each of the boundaries, solid or otherwise is given a code number that is understood by the solver, Tab. 5.2.

The TFI algorithm applies the motion specified through a parameter file to the solid surface, while rigidly maintaining the outer boundaries. The parameter file specifies the desired rotation and translation for the solid surface. However, since it is desired that there be more than one motion, in this case the relative motion between the airfoil and the flap, it is necessary to have a second set of parameters that act on a second surface.

Chapter 5 - 2D Test Case and Numerical Method

Code Number	Type of Boundary
9998	Secondary solid wall (flap)
9999	Primary solid wall (airfoil)
10000	Farfield (characteristic)
10002	y-symmetry
10003	z-symmetry
10004	Extrapolation (linear)
10005	Poiseille
10006	Mirror of first order extrapolation
11000	2D Condition

Table 5.2: Code number and description of designated boundaries.

The second surface and the second set of parameters were implemented as follows. A new code number was assigned to represent a secondary surface (9998). The solver was set to recognize this in the same way it have previously recognized the first surface. The solver was then set to recognize the code number assigned to the primary surface and the secondary surface simultaneously. This means that the secondary surface can be moved in isolation, but movement of the primary surface will include movement of the secondary surface as though they were rigidly attached. This was done so that the TFI algorithm would only have to be run once per time step, in order to reduce computational cost.

The input parameter file was modified to contain the motion of both bodies. The flap was defined as a motion of the secondary solid body in terms of the primary

Chapter 5 - 2D Test Case and Numerical Method

solid body, and then the entire system was told to rotate to the desired angle of attack. Since the motion of the flap was done outside of the flow solver, it remains the domain of the RANS equations, and does not require additional modification to the code. However, since the flap changes the shape, and therefore the pressure distribution of the airfoil, considerations have to be made in terms of the size and number of time steps needed to achieve a converged solution.

Chapter 6 - 2D Simulation

In order to ensure that the results obtained from the 2D RANS simulations are valid, the computational method was tested with respect to experimental data, and to account for variation in the setup of each computational run. The creation of the numerical mesh, the verification and validation of the numerical method, and results for the test cases are presented in this chapter.

6.1 Mesh Generation

A NACA0012 computational mesh of 95,000 nodes with a y^+ value of 8 was generated using ICEM CFD. This mesh will be referred to as ‘fine’ in the following chapter and was selected in order to surpass the grid density assessed as necessary to resolve an accurate solution of the flow field. Starting with a fine mesh simplifies the mesh coarsening process required later for verification, since the removal of nodes

does not impact the shape of the splines connecting them. The mesh topology is located in Fig. 6.1 for the medium mesh.

Figure 6.1: Computational mesh with 41,500 nodes in 10 Blocks for the 20% chord flap configuration.

The y^+ value determines the distance between the first gridline and the surface of the airfoil. For a viscous simulation it is necessary to ensure that the first node is located in the law of the wall area of the boundary layer. This area is governed by the equations:

$$y^+ = \frac{1}{\kappa} \ln \left(\frac{yu^*}{\nu} \right) + B \quad (6.1)$$

and,

$$y^+ = \frac{u}{u^*}. \quad (6.2)$$

where κ and B are constants with values of 0.41 and 5 respectively, y is the dimensional distance from the surface to the first node, ν is the kinematic viscosity, u is the freestream velocity, and u^* is the friction velocity [45]. Figure 6.2 shows that the law of the wall is bounded by $y^+ = 1$ and 10. Thus the value of y must correspond to y^+ less than 10, in order to capture the viscous sublayer. In fact, it is preferable for there to be several nodes within the law of the wall region, for completeness.

For the flow conditions of this test case, the table below shows the y^+ value with the corresponding dimensional distance y . In selecting a y^+ value, it is important to be conscious of the computational significance of having substantial variation in the size of the cells, since such parameters as the Courant-Friedrichs-Levy (CFL) number are based partly on cell size.

Figure 6.2: Boundary layer showing y^+ and that the law of the wall region ends at 10.

y^+	y (m)
1	7.38E-8
2	2.22E-7
5	1.90E-6
8	1.04E-5
10	2.95E-5

Table 6.1: y^+ and related y values.

It can be seen that there is considerable variation in magnitude of y between $y^+ = 1, 2, 5$, and 8 . In fact $y^+ = 8$ is 141 times larger than $y^+ = 1$. In contrast, $y^+ = 10$ is only 2.84 times the size of $y^+ = 8$. Selecting $y^+ = 8$ is preferable to $y^+ = 1$ since the variation in cell size between the surface of the airfoil and the far field is two orders of magnitude less. The extra consistency in the size of the cells with $y^+ = 8$ will improve the stability of the system, and aid in convergence. This y^+ value is used for all meshes, regardless of density or blocking.

6.2 Verification of the Numerical Method

Verification of the numerical method is necessary to ensure that the computational mesh used does not affect the results of the simulation. The verification process has two stages: a mesh dependency test to assess the impact of the grid point density on the results; and generation of the mesh for all test cases.

The blocking of the computational mesh was based on the location of the flap. The mesh dependency test was done on a mesh with blocking for a 15% chord flap. The mesh was made fine and then refined to a reasonable density. The other two flap sizes were then generated with a reasonable density, and a final comparison was made to ensure that the blocking did not alter the performance of the mesh.

6.2.1 Mesh Dependency Test

A mesh dependency test was performed by creating a medium and a coarse mesh by halving the number of nodes in the fine mesh from 95,000 to 41,500 and 21,500, respectively. Figs. 6.3 and 6.4 show the comparison of C_l and C_m histories for the three grid levels. While the coarse mesh shows some agreement with the medium mesh, it lacks the resolution necessary to predict accurately the fine details in the lift and pitching moment coefficient histories. The fine mesh and the medium mesh, on the other hand, give similar results, demonstrating little deviation in lift coefficient, and only minor deviation between the pitching moments. Thus, the medium mesh of 41,500 nodes was used for the remainder of the simulations.

Figure 6.3: C_l history for the mesh dependency test.

Figure 6.4: C_m history for the mesh dependency test.

6.2.2 Flap Size Effect on Grid Topology

Blocking was arranged in accordance with the location of the trailing edge flap. The blocks joining the freestream to the leading and trailing edges of the airfoil are kept invariant. The four blocks on the upper and lower surfaces of the airfoil were modified for the specific flap sizes by creating a single block over the flap, while maintaining the overall nodal distribution over the airfoil, as shown in Fig. 6.1. This resulted in a change in the number of nodes per block, but a nearly constant ratio of block volume to the number of nodes. The initial generation process was

Chapter 6 - 2D Simulations

done for the 15% chord flap case. The 10% and 20% chord flap meshes were generated after the mesh dependency test, in order to ensure that they had the appropriate mesh density.

The variation of C_l and C_m with respect to the time and angle of attack histories is plotted in Figs 6.5-6.8. Inspection of the time histories reveals only slight variation in the moment coefficient at the primary and secondary peak negative values near non-dimensional time of 5 and 7 respectively, with no perceptible variation in lift or moment, Figs 6.5 & 6.6.

Figure 6.5: Lift coefficient time history comparing flap sizes.

Figure 6.6: Pitching moment coefficient time history comparing flap sizes.

The C_l - α and C_m - α loops demonstrate good agreement between the three meshes. This indicates that the flap size has negligible effect on the mesh topology.

Figure 6.7: Lift Coefficient angle of attack history comparing flap sizes.

Figure 6.8: Pitching moment coefficient angle of attack history comparing flap sizes.

6.3 Validation

A comparison of the medium mesh results to the experimental data [46] showed good agreement in terms of time histories, Figs. 6.9 and 6.10. The magnitude of peak C_l was underpredicted, although the general trend of the curves showed good agreement. The C_m history is in better agreement with the measurements both in terms of the magnitude of minimum C_m and the general trends of the curves. The deviations could likely be due to experimental errors since more extensive validation shows good agreement with other experimental results over a range of Mach and Reynolds numbers and reduced frequencies.

Figure 6.9: C_l history of CFD validation.

Figure 6.10: C_m history of CFD validation.

The C_l - α and C_m - α comparisons between the experimental and CFD results differ more than the time histories, Figs. 6.11 and 6.12. Since the rate of change of angle of attack is not constant with time, the areas in the loops nearest to the maximum and minimum angle of attack show some distortion. This is perceived to be a flaw in the collected experimental data, since it is expected that the C_l - α curve would follow a straight line on the upstroke of the airfoil as is captured in CFD. Also, examination of the C_l - α plot reveals that there is very little hysteresis in the upper region of angle of attack. This is not as expected and indicates the presence of lag in the experimental results. Lag would also result in the large size of the second loop of the C_m - α curve, representing negative aerodynamic damping, in the

experimental data. Nevertheless, even with the skewed results in the angle of attack plots, the major trends are all captured in both CFD and the experiment.

Additional validation was conducted with respect to cases taken from the original McCroskey work that was done at NASA [47]. The characteristic values are compared, and the results are located in Appendix A. In general the CFD results compared much better with these experiments than with University of Glasgow data, further increasing the confidence in the CFD simulations.

Figure 6.11: C_l - α comparing uncontrolled CFD case to experimental data.

Figure 6.12: C_m - α comparing uncontrolled CFD case to experimental data.

6.4 Parametric Studies

In the following sections, parametric studies of varying flap size, magnitude of flap deflection and timing will be presented. The purpose of these tests is to determine the sensitivity of the results on the flap actuation parameters as well as to be able to determine an optimum flap actuation strategy. The baseline case for this parametric study was that of a 15% chord flap, with 15° upward deflection. The effect of the flap size, flap deflection and timing will be examined.

6.4.1 Effect of Flap Deflection

The first parametric study is of the effects of varying the maximum deflection of the trailing edge flap while maintaining a constant flap size and timing. This allows the effects of the flap deflection angle to be isolated from the other parameters.

The C_l - α and C_m - α loops for the constant flap size study are shown in Figs. 6.13 and 6.14. The C_l - α loop indicates a slight decrease in the dynamic lift when the flap is deflected upward. The decrease is most pronounced with the largest flap deflection, although the shape of the loop is inherently similar for all three cases. The C_m - α loop indicates that there is a decrease in the peak negative pitching moment, and a corresponding decrease in the size of the second loop which represents negative aerodynamic damping. Again, the largest decrease occurs with the maximum flap deflection. It can be concluded therefore, that the larger the flap deflection, the smaller the peak dynamic lift and the peak negative pitching moment will be, within the range of flap deflections tested. Increased flap size has marginal effects on the preservation of dynamic lift with a greater drop near the peak deflection, but earlier lift recovery.

Figure 6.13: Effect of flap deflection on the C_l - α curve. with 15% chord flap.

Figure 6.14: Effect of flap deflection on the C_m - α curve with 15% chord flap.

6.4.2 Effect of Flap Size

The results of varying flap size are shown in Figs. 6.15 and 6.16. Similar to above, the results show a slight reduction in dynamic lift and a pronounced reduction in the negative pitching moment and negative aerodynamic damping.

The decrease in negative pitching moment corresponds to an increase in flap size. The magnitude of change in negative pitching moment is very small for the constant deflection case, when compared to the changes yielded by the different flap deflections for a constant flap size. However, even small changes in the negative pitching moment result in a measurable reduction of the negative damping loop. Therefore, it appears that the flap size has a less pronounced effect on dynamic stall, when compared to the flap deflection. This is of interest since manufacturability plays a great role in determining the applicability of the trailing edge flap. Since the flap size is not of great concern, a flap may be selected based on ease of implementation.

Figure 6.15: Effect of flap size on the C_l - α curve. All cases with $\delta = 15^\circ$.

Figure 6.16: Effect of flap size on the C_m - α curve. All cases with $\delta = 15^\circ$.

6.4.3 Effect of Timing

For timing, the start and end of the actuation are expressed in terms of azimuth angle. Even though the simulations are 2D, the azimuth angle concept eases the comparison since the angle of attack changes sinusoidally, introducing multiple values in a single test case, *e.g.* the flap deflects at 23.8° degrees while pitching up, and stops at 20.9° while pitching down.

In the baseline case, the flap actuation commenced at 23.8° angle of attack ($\psi=243^\circ$) and lasted for 81° azimuth angle. For all timing tests, $\Delta\psi=81^\circ$ was therefore maintained. Several additional actuations were tested and are presented in Tab. 6.2, with the C_m - α loops presented graphically in Fig. 6.17.

Start Azimuth ($^\circ$)	End Azimuth ($^\circ$)	Peak Negative C_m	Area of Negative Damping Loop
Uncontrolled	Uncontrolled	-0.5	-0.674
207	288	-0.424	-0.516
220.5	301.5	-0.386	-0.470
243	324	-0.378	-0.349
261	342	-0.427	-0.321

Table 6.2: Effect of start and end azimuth angle on pitching moment coefficient and negative aerodynamic damping.

Chapter 6 - 2D Simulations

The most effective flap actuation for the reduction of negative pitching moment was the base case. This coincides with the trailing edge flap reaching its maximum deflection when the dynamic stall vortex is right above the trailing edge of the airfoil. The greatest reduction in negative aerodynamic damping (second loop) is associated with a slightly later flap deflection, although the negative pitching moment is again increased for this case. This indicates that there may be an optimum timing for the flow conditions located between these values. However, further examination reveals that the decrease in negative aerodynamic damping is accompanied by a strong decrease in the lift coefficient as the airfoil is pitching down. This would reduce dynamic lift, which is undesired.

Based on this, it can be concluded that the best timing is associated with the appearance of certain flow features during dynamic stall. The flap should reach its maximum deflection when the trailing edge vortex is at its closest proximity.

Figure 6.17: C_l - α comparing effect of flap timing.

Figure 6.18: C_m - α curve comparing effect of timing.

6.4.4 Optimum Flap Strategy

Based on the above parametric studies, the most important factors for a successful flap actuation strategy are the flap deflection and timing. The flap deflection should be as large as possible and the timing should be established so that the maximum deflection is attained when the dynamic stall vortex is passing over the trailing edge of the airfoil. Variation of the flap size, within the 10 to 20% chord range, has only a modest effect and can be selected based on manufacturability requirements. The optimum flap actuation was achieved by $\delta = -20^\circ$, $c = 20\%$, $\psi_{\text{START}} = 243^\circ$ ($\Delta\psi = 81^\circ$).

6.5 Flow Mechanism

Now that the optimum flap actuation strategy has been identified, it is possible to use the CFD results to understand the vibration reduction mechanism introduced by the flap. The uncontrolled flow field for a dynamically stalled airfoil is discussed in Chapter 2, and serves as the basis of comparison for the flap effects. There are several features highlighted in section 2.3, relating physical phenomena in the flow to the measured properties as represented by the C_l - α and C_m - α plots. These include the onset of dynamic stall, which is characterized by the formation of the dynamic stall vortex from the shear layer at the leading edge of the airfoil, the

Chapter 6 - 2D Simulations

movement of the dynamic stall vortex, the formation of the trailing edge vortex, and the reattachment of flow leading to the end of stall.

The examination of the flow field for the controlled case has been divided into three sections. The first section deals with the formation and influence of the dynamic stall vortex, the second section with the trailing edge vortex, and the third with the effects of the flap on the pressure field. The examination will be conducted using the optimum flap actuation strategy of 20° deflection, 20% chord flap, and starting at $\psi=243^\circ$.

The effects of the trailing edge flap on the negative pitching moment and negative aerodynamic damping have been discussed in section 6.4, in terms of the C_m - α and C_l - α curves. The flow field comparison conducted below will assign the flap effects to physical differences in the flow field.

The flow field for the controlled and uncontrolled cases are presented in terms of velocity contour plots, pressure contour plots and C_p distributions for each frame with characteristic locations marked on the C_l - α , C_m - α and C_d - α curves, Figs. 6.19 to 6.21, and α & δ with respect to non-dimensional time, Fig. 6.22. This enables the physical differences to be identified in terms of the associated differences in lift and moment coefficients. The C_p distributions are located in Fig. 6.23, velocity and pressure contours are named below. The goal of the analysis is to identify the cause of the reduction in negative aerodynamic damping and negative pitching moment.

Figure 6.19: C_l - α indicating the characteristic points for dynamic stall.

Figure 6.20: C_m - α indicating the characteristic points for dynamic stall.

Figure 6.21: C_d - α indicating the characteristic points for dynamic stall.

Figure 6.22: C_d - α indicating the characteristic points for dynamic stall.

Chapter 6 - 2D Simulations

Figure 6.23: Surface pressure corresponding to numbered points on Figs. 6.19 - 6.22.

Chapter 6 - 2D Simulations

The flow field for both the controlled and uncontrolled cases does not change until the flap begins its deployment. This is as expected, since the flow has an opportunity to reattach in both cases, and the angle of attack remains in the unstalled range for approximately half of the actuation. Initiation of the leading edge stall, characterized by boundary layer separation on the upper surface due to the adverse pressure gradient, is shown in Fig. 6.24, which is denoted by Point “2” on Figs. 6.19 to 6.22. Presented on the left of Fig. 6.24 are the pressure and velocity contours for the uncontrolled case, with those for the controlled case, on the right side.

Figure 6.24 a – b. (a) Pressure (b) Velocity, corresponding to Point “2.” Notice the flow is unaltered by the flap. Uncontrolled (Top), Controlled (Bottom).

Chapter 6 - 2D Simulations

The dynamic stall vortex initially forms at the same rate for both the uncontrolled and controlled cases. However, as the peak angle of attack is approached, the upward flap deflection alters the pressure difference between the lower and upper surfaces of the airfoil (clearly visible from the C_p plots for Point “3”, reducing circulation and resulting in a reduction of the peak strength of the dynamic stall vortex, and a change in its trajectory. Fig. 6.25 corresponds to Stage 3 of the uncontrolled case, and is indicated by Point “3” on the C_m - α and C_l - α plots. The pressure and velocity contour plots clearly show the core of the vortex being smaller for the controlled case, and the pressure plot confirms that the core is weaker. Notice that the trailing edge vortex has already started to form in the controlled case. This is likely due in part to the reduced distance from the dynamic stall vortex to the trailing edge of the flap, in the controlled case. Furthermore, the trailing edge vortex normally formed as a result of the strong pressure difference between the upper and lower surfaces of the airfoil, characteristically when the dynamic stall vortex passed over the trailing edge.

A reduction in the time between the dynamic stall vortex and the trailing edge vortex results in a reduction in the duration of time in which the force created by the vortices will cause negative aerodynamic damping.

Figure 6.25 a – b. (a) Pressure (b) Velocity, corresponding to Point “3.” Reduction in the core size and magnitude of the dynamic stall vortex is visible.

The trailing edge vortex remains attached to the uncontrolled airfoil substantially longer than for the controlled case (Point “3a” controlled case, Point “3b” uncontrolled case). The size of the uncontrolled trailing edge vortex is greater than the controlled vortex, due to the reduced pressure difference in the flapped case. The uncontrolled trailing edge vortex is firmly attached to the airfoil, whereas the controlled trailing edge vortex has convected free of the airfoil in the controlled case, as the flap returns to the undeflected position. As such, even though the trailing edge

Chapter 6 - 2D Simulations

vortex remains intact in the controlled case, the detachment results in a sharp reduction in negative pitching moment, and will allow for earlier flow reattachment as the airfoil pitches back towards the mean angle of attack. Fig 6.26 shows the plots for the maximum trailing edge vortex strength for the controlled case, denoted by “3a”, and Fig. 6.27 shows the plots for the maximum trailing edge vortex strength for the uncontrolled case, “3b”. Fig. 6.28 shows the vortex as it convects free of the airfoil Point “4.”

Figure 6.26 a – b: (a) Pressure (b) Velocity, corresponding to Point “3a.” Maximum strength of the controlled trailing edge vortex is less than the still developing uncontrolled case.

Figure 6.27 a – b: (a) Pressure (b) Velocity, corresponding to Point “3b.”

Chapter 6 - 2D Simulations

Figure 6.28 a – b: (a) Pressure (b) Velocity, corresponding to Point “4.”

Finally, as the airfoil reaches its minimum angle of attack, the two cases are once again identical, with no variation remaining from any lingering flap effects, Fig 6.29. This corresponds to “5” on the C_m - α and C_l - α plots.

Chapter 6 - 2D Simulations

Figure 6.29 a – b: (a) Pressure (b) Velocity, corresponding to Point “5.” Once again, the controlled and uncontrolled cases have become synchronized.

To sum up, the reduction in negative pitching moment and associated negative aerodynamic damping is caused by the flap altering the pressure distribution at the trailing edge of the airfoil. The upward deflection of the flap allows the flow on the bottom surface to expand, which reduced the pressure gradient between the upper and lower surfaces of the airfoil at the trailing edge. This directly reduces the moment about the feathering axis of the airfoil, and also reduces the strength of the trailing edge vortex.

Chapter 7 –Conclusion

A flap actuation strategy was proposed and shown to have a beneficial effect for the moderately unsteady 3D test case. The timing of the flap was, in this case, governed by the projected activation frequency deliverable by the smart material actuators that are to be used in the SHARCS project. The effect of timing variation for this case was not studied, but it is anticipated that the suggested flap strategy will yield benefits similar to the 23% reduction in vertical force, 44% reduction in peak moment, and 54% reduction in the amplitude of torsional vibration at the root of the blades, obtained in the simulation. The key feature for the flap actuation strategy is to set the flap to achieve peak deflection at the peak angle of attack, for the moderate advance ratio case. It may be necessary to modify the deflection slightly, in order to account for lead or lag.

For the highly unsteady (high advance ratio) case, the timing of the flap deflection had considerable effect on the reduction of negative pitching moment, and

Chapter 7 - Conclusion

negative aerodynamic damping. As such, it is necessary to ensure that the flap deploys in a specific time frame in order to achieve the desirable mitigation of dynamic stall effects. It is the interaction between the flap and the dynamic and trailing edge vortices that makes the timing more important in this case, than in the 3D test case. While it may be possible to base a control strategy on azimuth angle alone, since the convection of the dynamic stall vortex is known to occur within a specific range of speeds with respect to the freestream velocity (tip speed), in order to implement the flap in a real world situation will be necessary to determine a measurable parameter on which to base the flap deflection.

In order to ensure that the effects of the flap are correctly modeled, in both the 3D and 2D cases, a more detailed examination into the paneling and meshing of the computational bodies should be done. The verification of the 3D method did not account for the effect of webbing, as opposed to vorticity (wake) shedding on the inboard and outboard edges of the trailing edge flap. Likewise, the distribution of nodes around the flap, and junction between the flap and the airfoil for the 2D simulation was such that a sharp edge was produced at the top and bottom surfaces when the flap was deflected. The effects of these omissions should be measured with respect to experimental flap data, when it becomes available.

The numerical code for the 3D simulation, GAST, represents the state of the art for CFD vortex particle method codes. However, it is still not capable of

Chapter 7 - Conclusion

simulating the effects of dynamic stall, rather it relies on the ONERA dynamic stall model for those effects and the effects of other flow characteristics, such as compressibility.

The 2D computational code, CMB, is also representative of the current state of the art in CFD. Indeed, the code is itself capable of 3D simulation, but lacks the structural and dynamic models needed to compute a full simulation of a rotorcraft, or rotor system. In order to obtain more complete dynamic stall results, for a rotor blade, it is recommended that a 3D simulation be run when the technology becomes available.

References

- [1] Johnson W., Helicopter Theory, *Princeton University Press*, Princeton, New Jersey, 1980.
- [2] Nitzsche F. et al. The SHARCS Project: Smart Hybrid Active Rotor Control System for Noise and Vibration Attenuation of Helicopter Blades, *Paper no. 052, 31st European Rotorcraft Forum*, Florence, Italy, 2005.
- [3] Leishman J.G., Principles of Helicopter Aerodynamics, *Cambridge University Press*, 2002. pp. 97.
- [4] Lowson M.V., Focusing on Helicopter BVI Noise, *Journal of Sound and Vibration*. Vol. 190, No. 3, 1996. pp. 477-494.
- [5] Nitzsche, F., Zimcik, D., Wickramasinghe, V. and Yong, C., “Control Laws for an Active Tunable Vibration Absorber Designed for Aeroelastic Damping Augmentation,” *The Aeronautical Journal*, Vol. 108, No. 1079, 2004. pp. 35-42.
- [6] Aoyama T. et al., Calculation of Rotor Blade-Vortex Interaction Noise using Parallel Super Computer, *Paper 81, Proceedings: 22nd European Rotorcraft Forum, The Royal Aeronautical Society*, Vol. 2, 1996.
- [7] Mouille R., The Fenestron Shrouded Tail of the SA.341 Gazelle, *Journal of the*

References

American Helicopter Society, Vol. 15, No. 4, 1970.

- [8] Sekula M., Wilbur M., Yeager W., Aerodynamic Design Study of an Advanced Active Twist Rotor. *AHS International 4th Decennial Specialist's Conference on Aeromechanics, San Fransisco*. 2004. pp. 49-60.
- [9] Kloeppel V., Enenkl B. Rotor Blade Control by Active Helicopter Servo Flaps. *Paper no. IF-158 IFASD Conference, Munich, Germany*, 2005.
- [10] Nixon M.W., AHS Technical Committee Highlights: Dynamics 2003-2004. *Vertiflite, Winter 2004*, pp. 52-56
- [11] Nitzsche, F., Harold, T., Wickramasinghe, V. K., Young, C. and Zimcik, D. G., Development of a Maximum Energy Extraction Control for the Smart Spring, *Journal of Intelligent Material Systems and Structures (accepted for publication, 2005)*.
- [12] Nitzsche F. et al. The SHARCS Project: Smart Hybrid Active Rotor Control System for Noise and Vibration Attenuation of Helicopter Blades, *Paper no. 052, 31st European Rotorcraft Forum, Florence, Italy*, 2005.
- [13] Nitzsche F. et al. The SHARCS Project: Smart Hybrid Active Rotor Control System for Noise and Vibration Attenuation of Helicopter Blades, *Paper no. 052, 31st European Rotorcraft Forum, Florence, Italy*, 2005.
- [14] Leishman J.G., Principles of Helicopter Aerodynamics, Cambridge University Press, 2002. pp. 382.
- [15] Leishman J.G., Principles of Helicopter Aerodynamics, Cambridge University Press, 2002. pp. 397.
- [16] Leishman J.G., Principles of Helicopter Aerodynamics, Cambridge University Press, 2002. pp. 380.

References

- [17] Leishman J.G., Principles of Helicopter Aerodynamics, *Cambridge University Press*, 2002. pp. 381.
- [18] McCroskey W.J., McAlister K.W., Carr L.W., and Pucci S.L., An Experimental Study of Dynamic Stall on Advanced Airfoil Sections Volume 1. Summary of Experiments, *NASA TM 84245*, July 1982.
- [19] Nitzsche F. et al. The SHARCS Project: Smart Hybrid Active Rotor Control System for Noise and Vibration Attenuation of Helicopter Blades, *Paper no. 052, 31st European Rotorcraft Forum*, Florence, Italy, 2005.
- [20] “NTUA Internal Report on the GAST code”, Aerodynamics Laboratory, Department of Mechanical Engineering, National Technical University of Athens, Greece, 2005.
- [21] Beale J.T., and Majda A., Vortex Methods I: Convergence in Three Dimensions, *Mathematics of Computation*, Vol. 39, No. 159, July 1982, pp. 1-27.
- [22] Tran C.T., and Pitot D., Semi-Empirical Model for the Dynamic Stall of Airfoils in View of the Application to the Calculation of the Responses of a Helicopter Blade in Forward Flight. *Vertica*, Vol. 5, No. 1, 1981. pp. 35-53.
- [23] Truong V.K., A 2-D Dynamic Stall Model Based on a Hopf Bifurcation. *19th European Rotorcraft Forum*, Cernobbio, Italy, September 14-16, 1993.
- [24] Leishmann J.G., and Beddoes T.S., A Semi-Empirical Model for Dynamic Stall. *Journal of the American Helicopter Society*, Vol. 34, No. 3, 1989. pp. 3-17.
- [25] “NTUA Internal Report on the GAST code”, *Aerodynamics Laboratory, Department of Mechanical Engineering, National Technical University of Athens*, Greece, 2005.
- [26] “NTUA Internal Report on the GAST code”, *Aerodynamics Laboratory Department of Mechanical Engineering, National Technical University of Athens*, Greece, 2005.

References

- [27] Brentner K.S., Prediction of Helicopter Rotor Discrete Frequency Noise: A Computer Program Incorporating Realistic Blade Motions and Advanced Acoustic Formulation, *TM-87721, NASA Langley Research Centre*, Hampton, VA, 1986.
- [28] Opoku D., Aeroelastic and Aeroacoustic Modelling of Rotorcraft , M.ASc Thesis, Dept. of Mechanical and Aerospace Engineering, Carleton University, Ottawa, Canada, 2002.
- [29] Splettstoesser W. R. et. al., The HELINOISE Aeroacoustic Rotor Test in the DNW – Test Documentation and Representative Results, *DLR-Mitt. 93-09, DLR, Braunschweig*, Germany, 1993.
- [30] Friedmann P.P., Vibration and Noise Reduction Using Actively Controlled Flaps- Their Evolution and Potential for Improving Rotorcraft Technology, *paper no. IF-079, IFASD Conference*, Munich, Germany, 2005.
- [31] Sekula M., Wilbur M., Yeager W., Aerodynamic Design Study of an Advanced Active Twist Rotor. AHS International 4th Decennial Specialist's Conference on Aeromechanics, San Fransisco, 2004. *pp. 49-60*.
- [32] Opoku D. and Nitzsche, F., Acoustic Validation of a New Code Using Particle Wake Aerodynamics and Geometrically Exact Beam Structural Dynamics, *Paper No. 11, Proceedings: 29th European Rotorcraft Forum, 16-18 September 2003*, Friedrichshafen, Germany.
- [33] Opoku D., Aeroelastic and Aeroacoustic Modelling of Rotorcraft , M.ASc Thesis, *Dept. of Mechanical and Aerospace Engineering, Carleton University*, Ottawa, Canada, 2002, *pp. 88*.
- [34] Opoku D.G., Triantos D.G., Nitzsche F., Voustinas S.G., Rotorcraft Aerodynamic

References

and Aeroacoustic Modelling Using Vortex Particle Methods. *23rd ICAS Congress*, Toronto, 2002.

[35] Opoku D., Aeroelastic and Aeroacoustic Modelling of Rotorcraft , M.ASc Thesis, Dept. of Mechanical and Aerospace Engineering, Carleton University, Ottawa, Canada, 2002, *pp.* 88.

[36] Splettstoesser W. R. et. al., The HELINOISE Aeroacoustic Rotor Test in the DNW – Test Documentation and Representative Results, *DLR-Mitt. 93-09*, DLR, Braunschweig, Germany, 1993.

[37] Opoku D., Aeroelastic and Aeroacoustic Modelling of Rotorcraft , M.ASc Thesis, *Dept. of Mechanical and Aerospace Engineering, Carleton University*, Ottawa, Canada, 2002, *pp.* 91-92.

[38] Leishman J.G., Principles of Helicopter Aerodynamics, *Cambridge University Press*, 2002.

[39] McAlister K.W., Pucci S.L., McCroskey W.J., and Carr L.W., An Experimental Study of Dynamic Stall on Advanced Airfoil Sections Volume 2. Pressure and Force Data, *NASA TM 84245*, September 1982.

[40] Galbraith R., Gracey M., Gilmour R., Collected Data for Tests on a NACA 0012 Aerofoil. Volume II: Pressure Data from Oscillatory Tests, *University of Glasgow, Aero Report 9208*, Glasgow, Scotland, UK, February 1992.

[41] Feszty D., Gillies E.A., Vezza M., Alleviation of Airfoil Dynamic Stall Moments via Trailing Edge Flap Flow Control. *AIAA Journal*, Vol. 42, No.1, January 2004. *pp* 17-25.

[42] Badcock K.J., Richards B.E. and Woodgate M.A., Elements of Computational

References

Fluid Dynamics on Block Structured Grids Using Implicit Solvers.
Progress in Aerospace Sciences, Vol 36, 2000. pp. 351-392.

[43] Jameson A., Time Dependent Calculations Using Multigrid, with Applications to Unsteady Flows Past Airfoils and Wings. Paper No. 11 91-1596, AIAA 10th Computational Fluid Dynamics Conference, Honolulu, United States, 24-26 June 1991.

[44] “Modelling of turbulence in EROS-UK”, *DOC.N NO. TN02-012, CFD Research Group, University of Glasgow*, United Kingdom, November 2002.

[45] White F.M., Fluid Mechanics, 3rd Edition, *McGraw-Hill*, New York, 1994. pp. 304 – 306.

[46] Galbraith R., Gracey M., Gilmour R., Collected Data for Tests on a NACA 0012 Aerofoil. Volume II: Pressure Data from Oscillatory Tests, University of Glasgow, Aero Report 9208, Glasgow, Scotland, UK, February 1992.

[47] Leishman J.G., Principles of Helicopter Aerodynamics, Cambridge University Press, 2002.

[48] Galbraith R., Gracey M., Gilmour R., Collected Data for Tests on a NACA 0012 Aerofoil. Volume II: Pressure Data from Oscillatory Tests, University of Glasgow, Aero Report 9208, Glasgow, Scotland, UK, February 1992.

[49] McAlister K.W., Pucci S.L., McCroskey W.J., and Carr L.W., An Experimental Study of Dynamic Stall on Advanced Airfoil Sections Volume 2. Pressure and Force Data, NASA TM 84245, September 1982.

Appendix A – Additional Validation

The validation presented in Chapter 6 was done using the University of Glasgow experimental data for a pitching airfoil in dynamic stall [48]. There is some question as to the correctness of this data, since there are certain characteristic flaws in the shape of the C_l - t and C_l - α curves. It appears in the case of the Glasgow data that $C_{l_{max}}$ is larger than is to be expected. Additionally, the upward portion of the C_l - α curve is expected to be represented by a straight line for much of the upstroke as is typically shown in a C_l - α curve. In the experimental data from the University of Glasgow, the line deviates from the desired path.

Despite the shortcomings of the abovementioned experimental data, the characteristic points, including the maximum negative pitching moment and the secondary negative pitching moment, and the underlying shape of the C_m - t and C_m - α curves compare favourably between the experimental and computational data. Since the accuracy of the computational data is of chief concern, it was decided that additional validation should be made in order to ensure that the similarity between

Appendix A – Additional Validation

the experimental and computational data was not mere coincidence. Most importantly, it is desired that the shape of the lift curve slope be confirmed.

The McCroskey experiments of the 1981 [49] included several test cases that were similar to the baseline uncontrolled case presented in Chapter 6. Two of these cases were selected for simulation using CMB. Tab. A1 presents the McCroskey cases. The input parameters used in the computational simulation are as shown in this table.

Frame	Airfoil	A_0	A_1	k	Re	M
8222	NACA0012	14.84°	9.90°	0.149	2.42 E6	0.184
9218	NACA0012	14.91°	9.88°	0.151	3.45 E6	0.263

Table A1: Description of McCroskey pitching airfoil cases.

Here the oscillation occurs according to the formula $\alpha = A_0 + A_1 \sin(kt)$. Note that the A_0 and A_1 values are near the 15° and 10°, respectively.

The C_l , C_m , and C_d values for the two McCroskey cases are plotted versus time and angle of attack in Figs. A1-A2. The poor quality of these figures is a result of the microfiche scanner.

Appendix A – Additional Validation

Figure A1: Frame 8222 C_l , C_m , and C_d with respect to non-dimensional time and α .

Appendix A – Additional Validation

Figure A2: Frame 9218 C_L , C_M , and C_D with respect to non-dimensional time and α .

The characteristic results for the two McCroskey cases, in terms of $C_{L_{\max}}$, $C_{M_{\max}}$, and $C_{D_{\max}}$ are shown in Tab. A2.

Appendix A – Additional Validation

Frame	C_{lmax}	$-C_{mmax}$	C_{dmax}
8222	2.38	-0.47	1.08
9218	2.17	-0.42	0.89

Table A2: Characteristic results from McCroskey pitching airfoil cases.

The comparative simulations for the two McCroskey test cases are presented in Figs. A3 – A4. The figures will once again display the C_l , C_m , and C_d values with respect to non-dimensional time and angle of attack. The characteristic values of C_{lmax} , C_{mmax} , and C_{dmax} are presented in Tab. A3.

Frame	C_{lmax}	$-C_{mmax}$	C_{dmax}
8222	2.39	-0.52	1.37
9218	2.35	-0.53	1.4

Table A3: Characteristic results from computational simulations.

It is evident that there are some discrepancies between the two cases, particularly in terms of the drag coefficient. However, the overprediction of the results appears to be consistent for both frames. A brief examination of the data files revealed that there is a lack of convergence post onset of dynamic stall, which may account for some of the spurious flow features. This is likely a contributing factor. Nevertheless, the characteristic shapes of the curves remain preserved.

Appendix A – Additional Validation

Figure A3: Frame 8222 C_l , C_m , and C_d with respect to non-dimensional time and α .

Appendix A – Additional Validation

Figure A4: Frame 9218 C_l , C_m , and C_d with respect to non-dimensional time and α .