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Adaptive Equalization for a Multipath Fading Channel in the Presence of Interference

by

Norm W.K. Lo, B.A.Sc., M.Eng.

**A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy**

**Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University
April, 1994
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**“Adaptive Equalization for a Multipath Fading Channel
in the Presence of Interference”**

submitted by **Norm W.K. Lo, B.A.Sc., M.Eng.**
in partial fulfillment of the requirements for
the degree of Doctor of Philosophy

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Abstract

The current demand for wireless communications is motivating research into highly utilized cellular radio systems which are subjected to the performance-limiting impairment of interference due to synchronous data from other adjacent (ACI) and/or co-channel users (CCI). This thesis investigates the performance of a time-division multiple access (TDMA) indoor wireless cellular communications system employing adaptive equalization to combat such inter-user interference in a quasi-static multipath fading environment.

It is shown that a conventional adaptive fractionally spaced decision feedback equalizer (DFE) can achieve performance gains in the presence of strong interference and negligible ambient noise relative to the case in an equivalent amount of stationary noise. Unfortunately, such performance gains diminish considerably as the ambient noise becomes appreciable. However, it is observed that the use of a wide-bandwidth DFE can result in either an increase in radio capacity through a reduction in channel spacing or the capability to operate in a noisier environment while maintaining a desired system performance.

It is demonstrated that the recursive least squares (RLS) DFE out-performs the computed MMSE DFE which requires very accurate channel and interference plus noise autocorrelation estimates and is highly sensitive to the eigenvalue spread of its input autocorrelation matrix. It is also observed that this eigenvalue spread is much larger in the presence of inter-user interference than in stationary noise. Moreover, it is suggested that this larger eigenvalue spread may necessitate a greater precision digital implementation of an adaptive fractionally spaced equalizer operating in the presence of inter-user interference.

It is shown that the least mean square (LMS) DFE exhibits prohibitively slow MSE convergence in the presence of inter-user interference due to its sensitivity to the received signal eigenstructure and the existence of a very low MMSE. Furthermore, it is revealed that this extremely slow convergence is caused by one or two very slow-decaying significant eigenmodes. However, it is observed that the RLS DFE is essentially insensitive to the received signal eigenstructure and thus exhibits much faster MSE convergence than the LMS DFE in the presence of inter-user interference. Consequently, it is concluded that the RLS algorithm should be employed to train the DFE in an interference-limited quasi-static multipath fading environment.

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Table of Contents

Abstract.....	iii
Acknowledgments.....	iv
Table of Contents.....	v
List of Tables.....	viii
List of Figures.....	ix
List of Abbreviations and Acronyms.....	xv
List of Mathematical Symbols and Notation.....	xviii
Chapter 1 Introduction.....	1
1.1 Thesis Motivation.....	1
1.2 Thesis Contributions.....	3
1.3 Thesis Organization.....	5
Chapter 2 Cellular Communications: Background and Related Research.....	6
2.1 Introduction.....	6
2.2 Cellular Radio Communications.....	6
2.2.1 System Capacity.....	7
2.3 Channel Properties.....	10
2.3.1 Multipath-Induced Time Dispersion and Frequency Selectivity.....	10
2.3.2 Time-Variant Fading and Doppler Frequency Dispersion.....	11
2.3.3 Channel Characterization.....	13
2.3.4 Channel Model.....	16
2.4 Receiver Signal Processing to Mitigate Channel Impairments.....	17
2.4.1 Fading and ISI Mitigation.....	17
2.4.2 Interference Mitigation.....	18
2.5 Summary.....	25
Chapter 3 System Description and Ideal MMSE DFE Performance.....	26
3.1 Introduction.....	26
3.2 System Model.....	28
3.2.1 Signal to Interference plus Noise Ratios.....	31

3.2.2	System Capacity.....	39
3.3	Receiver Design.....	43
3.3.1	Stationary Inputs: Wiener Filter Approach.....	43
3.3.2	Non-stationary Inputs: Direct vs. Indirect Wiener Filter Update.....	44
3.4	Finite-Complexity MMSE DFE Formulation	47
3.4.1	Optimum Signal to Distortion Ratio at the DFE Output.....	52
3.5	MMSE DFE SDR Performance.....	53
3.5.1	In the Presence of a Dominant Interferer and Negligible Noise	55
3.5.2	In the Presence of Equal-Strength Interferers and Noise	61
3.5.3	In the Presence of Interferers with SIR= 10 dB and Noise.....	70
3.5.4	In the Presence of Interferers with SCCIR= 10 dB and Noise.....	75
3.6	SDR Probability Distribution	81
3.7	Summary.....	90
Chapter 4	Adaptive DFE Strategies and Convergence Performance	92
4.1	Introduction	92
4.2	Computed MMSE DFE Employing CIR and Interference plus Noise Autocorrelation Estimates	94
4.2.1	CIR Estimation.....	94
4.2.1.1	BLS CIR Estimation in Interference plus Noise.....	96
4.2.1.2	Simulated MNTE Performance	99
4.2.2	Interference plus Noise Autocorrelation Matrix	104
4.2.2.1	Interference plus Noise Sample Autocorrelation Estimation	108
4.2.3	Excess MSE Analysis	111
4.2.3.1	Verification of the Excess MSE Upper Bound.....	115
4.2.4	MSE Perturbation Analysis.....	126
4.3	DFE Directly Adapted with the LMS Algorithm	128
4.3.1	MSE Convergence Analysis	131
4.3.2	Calculated and Simulated MSE Convergence	132
4.3.2.1	Investigation of the LMS Equalizer MSE Convergence	138
4.4	DFE Directly Adapted with the RLS Algorithm.....	149
4.4.1	MSE Convergence Analysis	153
4.4.2	Calculated and Simulated MSE Convergence	154

4.5	Comparative MSE Performance of the Computed MMSE DFE and the Directly Adapted RLS DFE.....	157
4.6	Finite-Precision Considerations.....	159
4.7	Summary.....	163
Chapter 5 Adaptive DFE and System Performance		165
5.1	Introduction	165
5.2	Adaptive DFE SDR Performance.....	167
5.2.1	DFE Output MSE and Average SDR.....	167
5.2.2	Computed MMSE DFE and Directly Adapted RLS DFE	167
5.3	System Performance	175
5.3.1	Average Bit Error Rate and Outage Probability	175
5.3.2	In the Presence of a Dominant Interferer and Negligible Noise	175
5.3.3	In the Presence of a Dominant Interferer and Noise.....	182
5.3.4	In the Presence of Interferers with SIR = 10 dB and Noise.....	190
5.3.5	In the Presence of Interferers with SCCIR = 10 dB and Noise.....	194
5.4	Summary.....	204
Chapter 6 Conclusions.....		205
6.1	Recommendations for Further Research	207
Appendix A Variance of a BLS CIR Estimator		209
Appendix B Excess MSE Analysis of a Computed MMSE DFE		213
B.1	First Term of the Excess MSE.....	213
B.2	Second Term of the Excess MSE	217
B.3	Third Term of the Excess MSE	219
Appendix C Variance of an Interference plus Noise Sample Variance Estimator		220
Appendix D Autocorrelation Error Matrix and Cross-Correlation Error Vector of a Computed MMSE DFE.....		227
Appendix E MSE Perturbation Analysis of a Wiener Filter.....		231
Appendix F MSE Convergence Analysis of the LMS Algorithm.....		234
References.....		237

List of Tables

Table 3.1	Receiver input SINR for various additive noise and interference powers	63
Table 3.2	Receiver input SCCIR, SACIR and normalized channel spacing for various cases of equal strength interference and normalized receiver bandwidths, assuming a worst-case ACI reduction factor	64
Table 4.1	Calculated and simulated MSE and MMSE equalizer coefficients and eigenstructure in the presence of additive noise or interference due to one CCI user for an example with static channel and co-channel responses. RLS CIR estimation MNTTE (averaged over 100 trials) = - 24.23 dB and - 24.93 dB for the noise and interference cases, respectively, with a training length of 100 symbol periods.	125
Table 4.2	Calculated and simulated LMS equalizer MSE convergence and steady state MSE in the presence of additive noise or interference due to one CCI user for an example with static channel and co-channel responses. Frame and training lengths are 20,000 and 19,900 symbol periods, respectively.	143
Table 4.3	Weights, eigenvalues and time constants of the exponentially decaying LMS equalizer MSE eigenmodes in the presence of additive noise or interference due to one CCI user for an example with static channel and co-channel responses.	144

List of Figures

Figure 3.1	Complex baseband DCR communications system employing a computed MMSE DFE receiver.....	32
Figure 3.2	Complex baseband DCR communications system employing a directly adapted DFE receiver.....	33
Figure 3.3	SACIR as a function of the normalized channel spacing, assuming a worst case ACI reduction factor.	40
Figure 3.4	Fractionally spaced DFE receiver as a tapped delay line structure.....	49
Figure 3.5	DFE(Nf,2) SDR as a function of receiver input SIR, filter excess bandwidth and DFE forward filter length, with MPDP-1 channel multipath profile.....	57
Figure 3.6	DFE(5,Nb) SDR as a function of receiver input SIR, filter excess bandwidth and DFE feedback filter length, with MPDP-1 channel multipath profile	59
Figure 3.7	DFE(Nf,2) SDR as a function of receiver input SIR, multipath channel profile and DFE forward filter length for 100% pulse filter excess bandwidth.	60
Figure 3.8	DFE(5,2) SDR as a function of receiver input SNR and SIR with a 100% excess bandwidth receive filter	65
Figure 3.9	DFE(5,2) SDR as a function of receiver input SNR and SIR with a 300% excess bandwidth receive filter	66
Figure 3.10	DFE(5,2) SDR as a function of receiver input SNR and normalized receiver bandwidth with SIR = 0 dB.....	68
Figure 3.11	DFE(5,2) SDR as a function of receiver input SNR and normalized receiver bandwidth with SIR = 10 dB.....	69
Figure 3.12	DFE(5,2) SDR as a function of normalized channel spacing and receiver input SNR, for a normalized receiver bandwidth of 1 and SIR = 10 dB	72
Figure 3.13	DFE(5,2) SDR as a function of receiver input SNR and normalized channel spacing, for a normalized receiver bandwidth of 1 and SIR = 10 dB	73
Figure 3.14	Normalized radio capacity as a function of normalized channel spacing with SIR = 10 dB.....	74

Figure 3.15	DFE(5,2) SDR as a function of normalized channel spacing and receiver input SNR, for normalized receiver bandwidth of 1, and SCCIR = 10 dB	77
Figure 3.16	DFE(5,2) SDR as a function of normalized channel spacing and receiver input SNR, for normalized receiver bandwidth of 2, and SCCIR = 10 dB	78
Figure 3.17	DFE(5,2) SDR as a function of receiver input SNR, normalized channel spacing and normalized receiver bandwidth, with SCCIR = 10 dB	79
Figure 3.18	Normalized radio capacity as a function of normalized channel spacing with SCCIR = 10 dB	80
Figure 3.19	Cumulative distribution function for the SDR of an ideally computed MMSE DFE with SIR = 0 dB and SNR = 60 dB. The probability distribution is over an ensemble of 1000 random channel realizations. Transmit filter is SRRC with 100% excess bandwidth.	84
Figure 3.20	Cumulative distribution function for the SDR of an ideally computed MMSE DFE with SIR = 0 dB and SNR = 20 dB. The probability distribution is over an ensemble of 1000 random channel realizations. Transmit filter is SRRC with 100% excess bandwidth.	85
Figure 3.21	Cumulative distribution function for the SDR of an ideally computed MMSE DFE with SIR = 10 dB and SNR = 60 dB. The probability distribution is over an ensemble of 1000 random channel realizations. Transmit filter is SRRC with 100% excess bandwidth.	86
Figure 3.22	Cumulative distribution function for the SDR of an ideally computed MMSE DFE with SIR = 10 dB and SNR = 20 dB. The probability distribution is over an ensemble of 1000 random channel realizations. Transmit filter is SRRC with 100% excess bandwidth.	87
Figure 3.23	SDR histogram for case (2): "2 ACI & 0 CCI" with SIR = 10 dB and SNR = 60 dB. Receive filter has 100% excess bandwidth.	88
Figure 3.24	SDR histogram for case (2): "2 ACI & 0 CCI" with SIR = 10 dB and SNR = 60 dB. Receive filter has 300% excess bandwidth.	89
Figure 4.1	BLS CIR estimation MNTF (averaged over a set of 1000 independently fading channels and randomly selected training sequences) as a function of training length and channel Doppler fade rate in the presence of additive noise (SNR = 10 dB) or one CCI user (SIR = 10 dB)	102

Figure 4.2	BLS CIR estimation MNTE (averaged over a set of 1000 independently fading channels) as a function of channel SNR or SIR (due to one CCI user). With 25% excess bandwidth filtering, the estimated channel and training sequence lengths are 6 and 15, respectively. With 100% excess bandwidth, the estimated channel and training sequence lengths are 3 and 7, respectively.	103
Figure 4.3	RLS CIR estimation MNTE, averaged over 100 independent realizations of the noise and data, as a function of the training sequence length in the presence of additive noise (SNR = 0, 10 or 20 dB) or one CCI user (SIR = 0, 10 or 20 dB).	117
Figure 4.4	Comparison of the calculated and simulated SDR of a computed LE(5,0) in the presence of additive noise as a function of the training sequence length.	118
Figure 4.5	Comparison of the calculated and simulated SDR of a computed DFE(5,2) in the presence of additive noise as a function of the training sequence length.	119
Figure 4.6	Comparison of the calculated and simulated SDR of a computed LE(5,0) in the presence of one CCI user as a function of the training sequence length.	121
Figure 4.7	Comparison of the calculated and simulated SDR of a computed DFE(5,2) in the presence of one CCI user as a function of the training sequence length.	122
Figure 4.8	MSE perturbation upper bound of the Wiener equalizer as a function of small inaccuracies in its autocorrelation matrix and cross-correlation vector.	127
Figure 4.9	Gear-shifting and stochastic approximation LMS step size adjustment during training period of 19,000 symbols and fixed at 0.003 during tracking period of 1000 symbols. For SIR = 0, 5 dB, GSL-1,2,3 are used. For SIR = 10 to 30 dB, GSH-1,2,3 are used. For all SNR, GSNR-1 is used.	130
Figure 4.10	Simulated gear-shifting or stochastic approximation LMS DFE(5,2) MSE convergence in the presence of additive noise (SNR = 10 dB) or in one CCI user (SIR = 10 dB).	134

Figure 4.11	Simulated and calculated constant step size LMS DFE(5,2) MSE convergence in the presence of noise (SNR = 0 dB) or in the presence of one CCI user (SIR = 0 dB)	135
Figure 4.12	Simulated and calculated constant step size LMS DFE(5,2) MSE convergence in the presence of noise (SNR = 10 dB) or in the presence of one CCI user (SIR = 10 dB)	136
Figure 4.13	Simulated and calculated constant step size LMS LE(5,0) MSE convergence in the presence of noise (SNR = 10 dB) or in the presence of one CCI user (SIR = 10 dB) for an example with static channel and co-channel responses	141
Figure 4.14	Simulated and calculated constant step size LMS DFE(5,2) MSE convergence in the presence of noise (SNR = 10 dB) or in the presence of one CCI user (SIR = 10 dB) for an example with static channel and co-channel responses	142
Figure 4.15	Calculated convergence of the MSE and its constituent eigenmodes of a constant step size LMS LE(5,0) in the presence of noise (SNR = 10 dB) for an example with static channel and co-channel responses	145
Figure 4.16	Calculated convergence of the MSE and its constituent eigenmodes of a constant step size LMS DFE(5,2) in the presence of noise (SNR = 10 dB) for an example with static channel and co-channel responses	146
Figure 4.17	Calculated convergence of the MSE and its constituent eigenmodes of a constant step size LMS LE(5,0) in the presence of one CCI user (SIR = 10 dB) for an example with static channel and co-channel responses	147
Figure 4.18	Calculated convergence of the MSE and its constituent eigenmodes of a constant step size LMS DFE(5,2) in the presence of one CCI user (SIR = 10 dB) for an example with static channel and co-channel responses	148
Figure 4.19	Simulated and calculated RLS DFE(5,2) MSE convergence in the presence of noise (SNR = 0 dB) or in the presence of one CCI user (SIR = 0 dB) ..	155
Figure 4.20	Simulated and calculated RLS DFE(5,2) MSE convergence in the presence of noise (SNR = 10 dB) or in the presence of one CCI user (SIR = 10 dB)	156

Figure 5.1	Simulated SDR performance for an RLS and a computed MMSE DFE(5.2) in the presence of noise or in the presence of one CCI user for a training overhead of 15 symbols.....	170
Figure 5.2	Simulated SDR performance for an RLS and a computed MMSE DFE(5.2) in the presence of noise or in the presence of one CCI user for a training overhead of 60 symbols.....	171
Figure 5.3	Simulated SDR performance for an RLS and a computed MMSE DFE(5.2) in the presence of noise or in the presence of one CCI user (SIR = 0 dB) as a function of training overhead	173
Figure 5.4	Simulated SDR performance for an RLS and a computed MMSE DFE(5.2) in the presence of noise or in the presence of one CCI user (SIR = 10 dB) as a function of training overhead	174
Figure 5.5	Average BER for an RLS DFE and a computed MMSE DFE as a function of SIR due to a dominant CCI user with negligible additive noise for a training overhead of 100 symbols	177
Figure 5.6	Outage probability for an RLS DFE and a computed MMSE DFE with SIR = 0 dB due to a dominant CCI user with negligible additive noise and a training overhead of 100 symbols	179
Figure 5.7	Outage probability for an RLS DFE and a computed MMSE DFE with SIR = 10 dB due to a dominant CCI user with negligible additive noise and a training overhead of 100 symbols	180
Figure 5.8	Outage probability for an RLS DFE and a computed MMSE DFE as a function of SIR due to a dominant CCI user with negligible additive noise and a training overhead of 100 symbols.....	181
Figure 5.9	Average BER for an RLS DFE and a computed MMSE DFE as a function of SNR with SIR = 0 dB due to a dominant CCI user for a training overhead of 100 symbols.....	183
Figure 5.10	Average BER for an RLS DFE and a computed MMSE DFE as a function of SNR with SIR = 10 dB due to a dominant CCI user for a training overhead of 100 symbols.....	184
Figure 5.11	Average BER for an RLS DFE and a computed MMSE DFE as a function of SNR with SIR = 0 or 10 dB due to a dominant CCI user for a training overhead of 100 symbols.....	186

Figure 5.12	Outage probability for an RLS DFE and a computed MMSE DFE as a function of SNR with SIR = 0 dB due to a dominant CCI user for a training overhead of 100 symbols.....	187
Figure 5.13	Outage probability for an RLS DFE and a computed MMSE DFE as a function of SNR with SIR = 10 dB due to a dominant CCI user for a training overhead of 100 symbols	188
Figure 5.14	Outage probability for an RLS DFE and a computed MMSE DFE as a function of SNR and SIR due to a dominant CCI user for a training overhead of 100 symbols.....	189
Figure 5.15	RLS DFE average BER as a function of normalized channel spacing and SNR, for a normalized receiver bandwidth of 1 and SIR = 10 dB.....	191
Figure 5.16	RLS DFE average BER as a function of SNR and normalized channel spacing, for a normalized receiver bandwidth of 1 and SIR = 10 dB	192
Figure 5.17	RLS DFE outage probability, for an outage criterion of 0.01, as a function of SNR and normalized channel spacing, for a normalized receiver bandwidth of 1 and SIR = 10 dB.....	193
Figure 5.18	SINR as a function of the normalized channel spacing, receiver bandwidth and SNR. The CCI power is fixed at $\gamma_{CCIR} = 10$ dB while a nominal ACI reduction factor is assumed.	195
Figure 5.19	RLS DFE average BER as a function of SNR, normalized channel spacing and normalized receiver bandwidth, with $\gamma_{CCIR} = 10$ dB.....	196
Figure 5.20	RLS DFE average BER as a function of SNR, normalized channel spacing and normalized receiver bandwidth, with $\gamma_{CCIR} = 10$ dB.....	199
Figure 5.21	RLS DFE outage probability, for an outage criterion of 0.01, as a function of SNR, normalized channel spacing and normalized receiver bandwidth, with $\gamma_{CCIR} = 10$ dB	200
Figure 5.22	Outage probability for an RLS DFE as a function of normalized receiver bandwidth and SNR for a normalized channel spacing of 1.0	202
Figure 5.23	Outage probability for an RLS DFE as a function of normalized receiver bandwidth and SNR for a normalized channel spacing of 1.75.....	203

List of Abbreviations and Acronyms

ACI	- adjacent channel interference
AM	- amplitude modulation
AMPS	- advanced mobile phone system
ASK	- amplitude shift keying
AWGN	- additive white Gaussian noise
BER	- bit error rate (probability of bit error)
BLS	- block least squares
BPSK	- binary phase shift keying
BS	- base station
CCI	- co-channel interference
CDMA	- code division multiple access
CTDMA	- code time division multiple access
CT2	- second generation cordless telephony
CIR	- channel impulse response
co-CIR	- co-channel impulse response
dB	- decibel
DCR	- digital cellular radio
DECT	- digital European cordless telecommunications
DFE	- decision feedback equalizer
FDD	- frequency division duplex
FDMA	- frequency division multiple access
FEC	- forward error correction
FIR	- finite impulse response
FM	- frequency modulation
FSK	- frequency shift keying

GS	- gear shifting
GSM	- groupe spécial mobile or global system for mobile communications
Hz	- Hertz (cycles per second)
iid	- independent, identically distributed
INR	- average (total) interference to noise ratio at equalizer input
IS-54	- interim standard for North American DCR
ISDN	- integrated services digital network
ISI	- intersymbol interference
kbits/s	- kilo (10^3) bits per second
kHz	- kilo (10^3) Hertz (cycles per second)
km/hr	- kilo (10^3) meters per hour
ksymbols/s	- kilo (10^3) symbols per second
LE	- linear equalizer
LMS	- least mean square
LS	- least squares
LTI	- linear time invariant
LTV	- linear time variant
MAP	- maximum <i>a posteriori</i>
MHz	- mega (10^6) Hertz (cycles per second)
μs	- micro (10^{-6}) seconds
ML	- maximum likelihood
MLSE	- maximum likelihood sequence estimation
MMSE	- minimum mean squared error
MNTE	- mean norm tap error of CIR estimate
MPDP	- multipath power delay profile
MS	- mobile station
<i>ms</i>	- milli (10^{-3}) seconds

MSE	- mean squared error
MSK	- minimum shift keying
ns	- nano (10^{-9}) seconds
NTE	- norm tap error of CIR estimate
OC	- outage criterion
QAM	- quadrature amplitude modulation
QPSK	- quadrature phase shift keying
RC	- raised cosine
rf	-radio frequency
RLS	- recursive least squares
RMS	- root mean squared
SACIR	- average signal to ACI ratio at equalizer input
SCCIR	- average signal to CCI ratio at equalizer input
SDR	- average signal to distortion/disturbance ratio at equalizer output
SER	- average signal to CIR estimation error ratio
SIR	- average signal to (total) interference ratio at equalizer input
SINR	- average signal to (total) interference plus noise ratio at equalizer input
SNR	- average signal to noise ratio at equalizer input
SRRC	- square root raised cosine
TACS	- total access communications system
TDD	- time division duplex
TDL	- tapped delay line
TDMA	- time division multiple access
WSS	- wide sense stationary
WSSUS	- wide sense stationary, uncorrelated scattering
ZF	- zero forcing

List of Mathematical Symbols and Notation

$A^{(l)}$	- desired training ($l = 0$) and interferer ($l > 0$) data Toeplitz matrices (see Equation (4.8) on page 98)
A	- desired training data Toeplitz matrix (see Equation (4.10) on page 98)
$a^{(l)}(kT)$	- desired ($l = 0$) and interferer ($l > 0$) transmitted data symbols (see Equation (3.1) on page 30)
$a_k^{(l)}$	- portion of the desired training ($l = 0$) and interferer ($l = 1, 2, \dots, n_l$) sequences spanning the overall T/s -spaced CIRs at the k th observation time index of the CIR estimation (see Equation (4.3) on page 97)
a_k	- desired training sequence (see Equation (4.11) on page 98)
$\hat{a}(kT)$	- signal vector spanning the DFE feedback TDL filter (see Equation (3.43) on page 50)
B	- real, symmetric, positive definite matrix driving the time evolution of the LMS MSE (see Equation (F.7) on page 235)
$\bar{B}$	- normalized (by $1/T$) contributing ACI bandwidth (see Equation (3.28) on page 38)
$\bar{B}_r$	- normalized (by $1/T$) receiver bandwidth (near Equation (3.25) on page 38)
$\bar{B}_t$	- normalized (by $1/T$) transmitter bandwidth (near Equation (3.25) on page 38)
B_t	- total available system bandwidth (see Equation (3.33) on page 41)
BER	- average BER obtained from simulation (see Equation (5.3) on page 175)
BER_{frame}	- BER of a data frame for each independent trial in an ensemble obtained from simulation (see Equation (5.4) on page 175)
$C_{e_{h_r}}$	- BLS CIR estimation error covariance matrix (see Equation (A.3) on page 209)
C_e	- fourth-order moment terms involving the CIR estimation error (see Equation (C.23) on page 224)
$C_{e,i}$	- i th ($i = 1$ to 6) fourth-order moment term involving the CIR estimation error (see Equation (C.24) on page 224 and Equation (C.26) on page 225)
C_i	- fourth-order moment interference term (see Equation (C.11) on page 222)
C_{1234}	- fourth-order moment interference term with four equal data subscripts (see Equation (C.13) on page 222)

C_{1i}	- fourth-order moment interference terms with two equal data subscripts ($i = 2, 3$ or 4) (see Equation (C.16), Equation (C.18) and Equation (C.21) on page 223)
C_r	- radio capacity of a cellular system, #rf channels/cell, (see Equation (3.33) on page 41)
C_u	- user capacity of a TDMA cellular system, #users/cell, (see Equation (3.34) on page 41)
$\bar{C}_r$	- normalized radio capacity of a cellular system, #rf channels/bandwidth (in units of the symbol rate, $R = 1/T$)/cell, (see Equation (3.36) on page 42)
$\delta\bar{C}_r$	- change in normalized radio capacity of a cellular system (see Equation (3.39) on page 43)
$C(kT)$	- inverse of the sample autocorrelation matrix for the RLS DFE (see Equation (4.85) on page 152)
C	- diagonal matrix corresponding to the unitary similarity transformation matrix G in the LMS MSE analysis (see Equation (F.12) on page 235)
c_i	- i th eigenvalue of the LMS MSE time evolution matrix B (see Equation (F.12) on page 235)
c	- electromagnetic (e.g., light) propagation speed (see Equation (2.8) on page 15)
D	- equalizer delay in symbol periods which minimizes the output MSE (see Equation (3.45) on page 50)
d	- integer greater than unity which specifies the number of samples per symbol period of forward filter of DFE at the receiver (see Equation (3.42) on page 48 and Figure 3.4 on page 49)
d_a	- minimum distance between adjacent and desired channel cell sites (see Equation (3.16) on page 36)
d_c	- minimum distance between co-channel cell sites, i.e., base stations, (see Equation (3.8) on page 35)
E_b	- energy per bit (see Equation (3.6) on page 34)
E_h	- desired ($l = 0$) channel error matrix (see Equation (D.5) on page 228)
e	- natural logarithm constant = 2.718281...
$e(kT)$	- adaptation error signal for either the LMS algorithm (see Equation (4.59) on page 128) or the RLS algorithm (see Equation (4.80) on page 151)
ϵ_{h_p}	- CIR estimation error vector for the p th T -spaced phase (see Equation (A.4) on page 209)

$e_{h,i}$	- BLS estimation error for the i th CIR tap coefficient (see Equation (D.15) on page 230)
$\mathbf{e}_r$	- T/d -spaced CIR estimator output error vector (see Equation (4.21) on page 105)
$e_{r,k}$	- T/d -spaced CIR estimator output error at k th symbol instant (see Equation (4.29) on page 109)
$\mathbf{e}_{r,p}$	- p th phase of T -spaced error vector between estimated and actual received signal vectors (see Equation (4.18) on page 105)
$\mathbf{e}_w$	- equalizer tap coefficient error vector (see Equation (4.38) on page 111)
$F(f, \epsilon)$	- SRRC frequency response (see Equation (3.18) on page 37)
$F_t^{(l)}(f, \epsilon)$	- SRRC transfer function of the transmit filter of the l th user (see Equation (3.17) on page 36)
$F_r^{(0)}(f, \epsilon)$	- SRRC transfer function of the receive filter (see Equation (3.17) on page 36)
$f_t(kT/s)$	- T/s -spaced impulse response of the transmit filter (see Section 3.2 on page 28)
$f_r(kT/s)$	- T/s -spaced impulse response of the receive filter (see Section 3.2 on page 28)
f_c	- signal carrier frequency (see Equation (2.1) on page 13)
f_d	- Doppler frequency (see Equation (2.8) on page 15)
f_{dm}	- maximum Doppler frequency (see Equation (2.8) on page 15)
f_s	- channel frequency spacing (see Equation (3.30) on page 39)
$\tilde{f}_s$	- normalized (by $1/T$) channel frequency spacing (near Equation (3.17) on page 36)
$\delta\tilde{f}_s$	- change in normalized channel frequency spacing (see Equation (3.39) on page 43)
$G^{(l)}(f)$	- transfer function of the actual desired ($l = 0$) and adjacent ($l > n_c$) multipath fading channels (see Equation (3.17) on page 36)
G	- unitary similarity transformation matrix in LMS MSE convergence analysis (see Equation (F.12) on page 235)
$\mathbf{g}_i$	- i th column vector of the unitary matrix G and an eigenvector of the autocorrelation matrix B (see Equation (F.12) on page 235)
$\mathbf{g}(kT)$	- directly adapted RLS DFE Kalman gain vector (see Equation (4.83) on page 152)
$g(\tau; t)$	- complex baseband time-varying actual multipath fading CIR (see Equation (2.1) on page 13)

$g_m(kT/s)$	- m th T/s -spaced actual CIR coefficient (see Equation (2.9) on page 16)
H	- desired ($l = 0$) channel matrix (see Equation (D.4) on page 228)
$\hat{H}$	- estimated desired ($l = 0$) channel matrix (see Equation (D.6) on page 228)
$h^{(l)}(t)$	- overall desired ($l = 0$) and interfering ($l > 0$) CIRs (see Equation (3.1) on page 30)
h	- desired overall CIR coefficient vector (see Equation (4.1) on page 96)
h_i	- i th component of the desired overall CIR coefficient vector (see Equation (D.15) on page 230)
$\hat{h}_p^{(l)}$	- p th phase of overall desired ($l = 0$) and interfering ($l > 0$) CIRs (see Equation (4.4) on page 97)
$\hat{h}_p$	- p th phase of overall desired CIR (see Equation (4.15) on page 99)
$\hat{b}_p$	- p th phase of overall desired BLS CIR estimate (see Equation (4.13) on page 99)
$\hat{h}_{MAP}$	- MAP estimator of the desired overall CIR (see Equation (4.1) on page 96)
$\hat{h}_{ML}$	- ML estimator of the desired overall CIR (see Equation (4.2) on page 96)
$h_e(k)$	- k th cursor, including the overall CIR and DFE forward filter (see Equation (3.57) and Equation (3.58) on page 53)
h_r	- time-reversed T/d -spaced D -lagged CIR spanning the DFE forward filter (see Equation (3.59) on page 53)
I	- identity matrix (see Equation (4.43) on page 112)
I_N	- $N \times N$ identity matrix (see Equation (4.27) on page 107)
INR	- average (total) interference to noise ratio at the overall channel output (i.e., at the equalizer input) (see Equation (3.4) on page 34)
i	- integer or discrete-time index
i_m	- interference at m th sample instant (see Equation (C.4) on page 220)
J	- computed equalizer output MSE (see Equation (E.2) on page 231)
δJ	- perturbation in computed equalizer output MSE (see Equation (E.1) on page 231)
j	- constant $= \sqrt{-1}$; or integer or discrete-time index
K	- frequency reuse factor or cluster size (see Equation (3.11) on page 35)
$K(kT)$	- autocorrelation matrix of the LMS DFE tap coefficient error vector (see Equation (F.3) on page 234)
k	- discrete-time index

l	- index for summation over users (see Equation (3.1) on page 30) or the T/d -spaced lag index for the BLS CIR estimator output error (see Equation (4.29) on page 109)
M	- modulation alphabet size (see Equation (3.30) on page 39)
m	- discrete-time index
MSE	- MSE at DFE output (see Equation (3.45) on page 50)
$MSE(kT)$	- directly adapted LMS DFE MSE as a function of training time (see Appendix F on page 234)
$\langle MSE \rangle$	- time-averaged MSE at DFE output obtained from simulation (see Equation (5.2) on page 167)
$MMSE$	- minimum MSE at DFE output (see Equation (3.49) on page 50)
N	- number of independent channel realizations in simulation ensemble (see Equation (4.15) on page 99, Equation (5.2) on page 167 and Equation (5.3) on page 175)
N_b	- number of T -spaced DFE feedback taps (see Section 3.4 on page 47)
N_C	- total number of available rf channels (see Equation (3.33) on page 41)
N_c	- number of T -spaced taps in overall CIR (see Equation (4.4) on page 97)
N_d	- QPSK data symbol sequence length (see Section 3.2 on page 28)
N_e	- observation length, in symbols, of the CIR and interference plus noise estimators (see Equation (4.9) on page 98 and Equation (4.30) on page 109)
N_f	- number of T/d -spaced DFE forward taps (see Section 3.4 on page 47)
N_{fT}	- number of T -spaced DFE forward taps (see below Equation (4.55) on page 123)
N_t	- $\pi/4$ -rotated BPSK training data symbol sequence length (see Section 3.2 on page 28)
N_0	- power spectral density of complex-valued AWGN (see Equation (3.6) on page 34)
$N_a(\hat{f}_s)$	- number of contributing (upper or lower) 100% excess bandwidth ACI users (see Equation (3.24) on page 38)
NTE	- norm tap error for a BLS CIR estimate (see Equation (4.14) on page 99)
n	- index for summation over trials in simulation ensemble (e.g., see Equation (4.15) on page 99)
n_a	- number of ACI users (see Equation (3.1) on page 30)
n_c	- number of CCI users (see Equation (3.1) on page 30)

n_t	- total number of ACI and CCI users (see Equation (3.1) on page 30)
$n_r(t)$	- complex-valued receive-filtered noise (see Equation (3.1) on page 30)
$n_{r,p}$	- p th phase of the receive-filtered noise vector spanning the observation interval of length N_p symbols (see Equation (4.6) on page 97)
$\hat{n}_{r,p}$	- p th phase error vector of the receive-filtered noise (see Equation (4.25) on page 107)
P	- perturbation matrix (see Equation (4.45) on page 112)
P_Δ	- perturbation in matrix inverse norm (see Equation (E.8) on page 232)
P_{aci}	- power of either upper or lower ACI in desired bandwidth (see Equation (3.17) on page 36)
$P_{desired}$	- power of desired signal in desired bandwidth (see Equation (3.17) on page 36)
P_{in}	- average input signal power to the LMS filter (see Equation (4.67) on page 131)
P_{oc}	- abscissa value representing the outage criterion for a data frame obtained from simulation (see Equation (5.4) on page 175)
p	- integer index for phase of T -spaced CIR or signal vector (see Equation (4.4) and Equation (4.5) on page 97)
$\underline{p}$	- MMSE DFE cross-correlation vector (see Equation (3.48) on page 50)
$\delta \underline{p}$	- perturbation in the MMSE DFE cross-correlation vector (see Equation (E.5) on page 231)
$\hat{\underline{p}}$	- MMSE DFE estimated cross-correlation vector (see Equation (4.41) on page 112)
$\tilde{\underline{p}}$	- MMSE DFE cross-correlation error vector (see Equation (D.14) on page 230)
$\hat{\underline{p}}(kT)$	- directly adapted RLS DFE sample cross-correlation vector (see Equation (4.73) on page 150)
$\underline{p}_{ra}$	- MMSE DFE received signal and feedback signal cross-correlation vector (see Equation (3.54) on page 52)
$\underline{p}_{ua}$	- MMSE DFE feedback signal D -lagged zero-valued correlation vector (see Equation (3.51) on page 51)
q_a	- ACI reduction factor (see Equation (3.16) on page 36)
q_c	- CCI reduction (or co-channel reuse) factor (see Equation (3.8) on page 35)
R	- MMSE DFE input autocorrelation matrix (see Equation (3.47) on page 50)

$\hat{R}$	- MMSE DFE estimated input autocorrelation matrix (see Equation (4.40) on page 112)
$\hat{R}(kT)$	- directly adapted RLS DFE sample autocorrelation matrix (see Equation (4.72) on page 150)
$\tilde{R}$	- MMSE DFE input autocorrelation error matrix (see Equation (D.1) on page 227)
R_{e_h}	- T/d -spaced CIR estimation error correlation matrix (see Equation (4.21) on page 105)
$R_{e_r}(l)$	- T/d -spaced CIR estimator output error correlation matrix with lag l (see Equation (4.21) on page 105)
$\tilde{R}_{e_r}(l)$	- unbiased autocorrelation estimator of $R_{e_r}(l)$ (see Equation (4.29) on page 109)
$\hat{R}_{e_r}(l)$	- asymptotically unbiased autocorrelation estimator of $R_{e_r}(l)$ (see Equation (4.30) on page 109)
$\hat{R}_{i+n}(i, j)$	- asymptotically unbiased estimator of the interference plus noise autocorrelation (see Equation (4.36) on page 111)
$R_{e_r, p}$	- p th phase of the CIR estimator output error correlation matrix (see Equation (4.19) on page 105)
R_{rr}	- MMSE DFE received signal autocorrelation matrix (see Equation (3.52) on page 51 and Equation (D.2) on page 228)
$\tilde{R}_{rr}$	- MMSE DFE received signal autocorrelation error matrix (see Equation (D.1) on page 227)
R_{rrp}	- p th phase T -spaced received signal autocorrelation matrix (see Equation (3.52) on page 51)
R_h	- MMSE DFE desired CIR autocorrelation matrix (see Equation (3.52) on page 51 and Equation (D.2) on page 228)
$\hat{R}_h$	- MMSE DFE estimated desired CIR autocorrelation matrix (see Equation (D.6) on page 228)
$\tilde{R}_h$	- MMSE DFE desired CIR autocorrelation error matrix (see Equation (D.7) on page 228)
$R_{h, pp}$	- p th phase of the desired received signal autocorrelation matrix (see Equation (4.49) on page 114)
R_i	- MMSE DFE interference autocorrelation matrix (see Equation (3.52) on page 51)
R_{i+n}	- MMSE DFE interference plus noise autocorrelation matrix (see Equation (D.2) on page 228)

$\hat{R}_{i+n}$	- MMSE DFE (asymptotically unbiased) estimated interference plus noise autocorrelation matrix (see Equation (D.12) on page 229)
$\tilde{R}_{i+n}$	- MMSE DFE zero-mean interference plus noise autocorrelation error matrix (see Equation (D.13) on page 229)
$R_{i+n,pp}$	- p th phase of the interference plus noise autocorrelation matrix (see Equation (4.49) on page 114)
$R_{i+n,pl}$	- p th phase of the interference plus noise autocorrelation matrix at lag l (see Equation (4.48) on page 113)
R_a	- MMSE DFE ACI autocorrelation matrix (see Equation (3.52) on page 51)
R_c	- MMSE DFE CCI autocorrelation matrix (see Equation (3.52) on page 51)
R_n	- MMSE DFE receive-filtered AWGN autocorrelation matrix (see Equation (3.52) on page 51)
$R_n(mT/d)$	- T/d -spaced receive filter autocorrelation function (see Equation (3.52) on page 51)
$R_{rr,pp}$	- p th phase of the received signal autocorrelation matrix (see Equation (4.49) on page 114)
R_{ra}	- MMSE DFE received signal and feedback data correlation matrix (see Equation (3.53) on page 51)
$\tilde{R}_{ra}$	- MMSE DFE received signal and feedback data correlation error matrix (see Equation (D.1) on page 227)
R_{aa}	- MMSE DFE feedback data autocorrelation (identity) matrix (see Equation (3.50) on page 50)
$\tilde{R}_{aa}$	- MMSE DFE feedback data autocorrelation (zero-valued) error matrix (see Equation (D.1) on page 227)
R_b	- transmission bit rate (see Equation (3.30) on page 39)
R_s	- transmission symbol rate, $1/T$, (see Equation (3.30) on page 39)
r	- (hexagonal) cell radius (see Equation (3.8) on page 35)
$r(t)$	- continuous-time received signal at output of overall CIR (see Equation (3.1) on page 30)
r_k	- discrete-time (at k th symbol instant) received signal at output of overall CIR (see Equation (4.12) on page 98)
$r(kT)$	- received signal vector spanning the T/d -spaced forward TDL filter of the DFE (see Equation (3.42) on page 48)
r_p	- p th phase of the received signal vector spanning the observation interval of length N_e symbols (see Equation (4.5) on page 97 and Equation (4.7) on page 98)

$\hat{r}_p$	- p th phase of the estimated received signal vector spanning the observation interval of length N_c symbols (see Equation (4.9) on page 98)
$SACIR$	- average signal to ACI ratio at the overall channel output (i.e., at the equalizer input) (see Equation (3.22) on page 37 or Equation (3.23) on page 38)
$SCCIR$	- average signal to CCI ratio at the overall channel output (i.e., at the equalizer input) (see Equation (3.10) or Equation (3.13) on page 35)
SDR	- average SDR at DFE output obtained from simulation (see Equation (5.1) on page 167)
SDR_o	- optimum SDR at DFE output (see Equation (3.60) on page 53)
SDR_{frame}	- random variable distributed over the fading channel statistics (see Equation (3.63) on page 81)
SDR_{oc}	- abscissa value representing the outage criterion for a data frame (see Equation (3.63) on page 81)
SER	- average signal to BLS CIR estimation error ratio (see Equation (A.10) on page 211)
$SINR$	- average signal to (total) interference plus noise ratio at the overall channel output (i.e., at the equalizer input) (see Equation (3.5) and Equation (3.7) on page 34)
$SINR_{in}$	- $SINR$ at the input to a T/d -spaced equalizer (see Equation (4.49) on page 114)
SIR	- average signal to (total) interference ratio at the overall channel output (i.e., at the equalizer input) (see Equation (3.3) on page 34)
SNR	- average signal to noise ratio at the overall channel output (i.e., at the equalizer input) (see Equation (3.2) on page 31)
$S(\tau; f_d)$	- scattering function of a multipath fading CIR (see Equation (2.6) on page 15)
$S(kT)$	- matrix corresponding to the unitary similarity transformation of the matrix $K(kT)$ in the LMS MSE convergence analysis (see Equation (F.3) on page 234)
$s(kT)$	- column vector of the diagonal elements of the matrix $S(kT)$ (see Equation (F.5) on page 234)
s	- integer greater than unity which specifies the number of samples per symbol period of overall system (see Equation (2.9) on page 16)
T	- symbol period (see Equation (2.9) on page 16 or Section 3.2 on page 28)
t	- continuous-time variable

U	- unitary matrix for a linear rotational transformation in LMS MSE convergence analysis (see Equation (F.2) on page 234)
$\underline{u}_i$	- i th column vector of the unitary matrix U and an eigenvector of the auto-correlation matrix R (see Equation (F.2) on page 234)
$u(kT/s)$	- discrete-time input signal to multipath fading CIR (see Equation (2.9) on page 16)
v_m	- mobile speed (see Equation (2.8) on page 15)
W	- complex baseband signal bandwidth (e.g., see Equation (2.9) on page 16)
$\underline{w}$	- DFE tap coefficient vector (see Equation (3.44) on page 50)
$\delta \underline{w}$	- perturbation in the DFE tap coefficient vector (see Equation (E.6) on page 231)
$\hat{\underline{w}}$	- estimated Wiener DFE tap coefficient vector computed with estimates of the CIR and interference plus noise autocorrelation (see Equation (4.39) on page 111)
$\underline{w}_b$	- DFE feedback filter tap coefficient vector (see Equation (3.44) on page 50)
$\underline{w}_f$	- DFE forward filter tap coefficient vector (see Equation (3.44) on page 50)
$\underline{w}_o$	- optimum MMSE DFE tap coefficient vector (see Equation (3.46) on page 50)
$\underline{w}_{of}$	- optimum MMSE DFE forward filter tap coefficient vector (see Equation (4.48) on page 113)
$XSMSE$	- computed DFE excess MSE due to CIR and interference plus noise autocorrelation estimation errors (see Equation (4.37) on page 111)
$XSMSE_{EST}$	- computed DFE excess MSE (see Equation (4.93) on page 158)
$XSMSE_{RLS}$	- directly adapted RLS ($\lambda = 1$) DFE excess MSE (see Equation (4.92) on page 158)
$XSMSE(kT)$	- directly adapted LMS DFE excess MSE as a function of training time (see Appendix F on page 234)
$XSMSE_{tr}(kT)$	- directly adapted LMS DFE transient excess MSE (see Equation (F.9) on page 235)
$XSMSE(\infty)$	- directly adapted LMS DFE steady state excess MSE (see Equation (F.10) on page 235 and Equation (F.15) on page 236)
$\underline{x}(kT)$	- input signal vector spanning the DFE forward and feedback TDL filters (see Equation (3.41) on page 48)
x_i	- sample of equalizer input signal (see Equation (B.3) on page 214)

x_p	- p th phase T -spaced sample of equalizer input signal (see Equation (B.4) on page 214)
$y(kT)$	- $1/T$ -decimated DFE output signal (see Equation (3.40) on page 48)
$z(kT)$	- directly adapted RLS DFE dummy vector (see Equation (4.84) on page 152)
$\tilde{z}_N$	- $N \times 1$ column vector of zeroes (see Equation (D.4) on page 228)
$\alpha(\tau; t)$	- attenuation of time-varying CIR at delay τ (see Equation (2.1) on page 13)
$\alpha_m(t)$	- attenuation of m th multipath of time-varying CIR (see Equation (2.2) on page 13)
β	- LMS fixed step size (see Equation (4.60) on page 128)
$\beta(kT)$	- LMS time-varying (gear-shifting) step size (see Equation (4.61) and Equation (4.63) on page 129)
β_i	- LMS initial step size (see Equation (4.63) on page 129)
β_f	- LMS final step size (see Equation (4.63) on page 129)
β_e	- estimation bias factor (see Equation (D.11) on page 229)
γ	- transmission path loss exponent (see Equation (3.9) on page 35)
γ_i	- weighting factor for the i th transient LMS MSE eigenmode (see Equation (F.14) on page 236)
Δ	- incremental difference parameter (e.g., see Equation (4.95) on page 158)
δ	- lag difference dummy index (see Equation (C.14) on page 222)
$\delta(\tau)$	- impulse signal (i.e., delta function) with time (delay) variable τ (e.g., see Equation (2.2) on page 13)
δ_{ij}	- Kronecker delta function (see Equation (B.8) on page 215 and Equation (D.10) on page 229)
ϵ	- SRRC frequency response roll-off factor, $0 \leq \epsilon \leq 1$ (see Equation (3.18) on page 37)
η_c	- overall spectral efficiency of a cellular system, bit/s/Hz/cell, (see Equation (3.32) on page 41)
η_f	- spectral efficiency of an FDMA system, bit/s/Hz, (see Equation (3.30) on page 39)
η_s	- spatial efficiency of a cellular system, cluster/#cells, (see Equation (3.31) on page 41)
θ	- incident angle of the received electromagnetic signal

$\kappa(R)$	- condition number (eigenvalue spread) of the input autocorrelation matrix (see Equation (E.14) on page 232 and Equation (E.23) on page 233)
Λ	- real-valued diagonal matrix of the corresponding eigenvalues, λ_i , of the autocorrelation matrix R (see Equation (F.2) on page 234)
λ_i	- i th eigenvalue of the autocorrelation matrix R (see Equation (F.2) on page 234)
$\underline{\lambda}$	- column vector of the eigenvalues, λ_i , of the autocorrelation matrix R (see Equation (F.4) on page 234)
λ_{max}	- maximum eigenvalue of the autocorrelation matrix R (see Equation (E.21) on page 233)
λ_{min}	- minimum eigenvalue of the autocorrelation matrix R (see Equation (E.22) on page 233)
λ	- RLS prewindow parameter which exponentially weights the data (see Equation (4.70) on page 149)
μ	- RLS inverse matrix initialization (small positive) constant (see Equation (4.87) on page 152)
π	- circumference-to-diameter ratio constant = 3.141592...
ρ	- LMS gear-shifting "stochastic approximation" parameter (see Equation (4.61) on page 129)
$\rho_{\alpha}^{(l)}$	- l th user CIR autocorrelation with lag α (see Equation (C.7) on page 221)
σ_B^2	- average data bias component in equalizer output MMSE (see Equation (3.56) on page 52)
σ_D^2	- average residual distortion component in equalizer output MMSE (see Equation (3.55) on page 52 and Equation (3.60) on page 53)
σ_S^2	- average desired main cursor signal power at equalizer output (see Equation (3.60) on page 53)
σ_a^2	- ACI variance at equalizer input (see Equation (3.14) on page 36)
σ_{af}^2	- ACI filter attenuation factor (see Equation (3.17) on page 36)
σ_{ap}^2	- ACI propagation attenuation factor (see Equation (3.15) on page 36)
σ_c^2	- CCI variance at equalizer input (see Equation (3.9) on page 35)
σ_d^2	- (assumed unit) variance of transmitted QPSK data symbol (see Section 3.2 on page 28)
$\sigma_{ d ^4}^2$	- fourth-order moment of the user data symbol (see Equation (C.6) on page 220)
σ_i^2	- total inter-user interference variance at equalizer input (see Equation (3.3) on page 34)

σ_n^2	- receive-filtered AWGN variance (see Equation (3.2) on page 31)
$\hat{\sigma}_n^2$	- unbiased estimator of the AWGN variance (see Equation (4.28) on page 107)
σ_s^2	- variance of signal at output of multipath fading channel (see Equation (3.2) on page 31)
τ	- continuous-time propagation delay of CIR (see Equation (2.1) on page 13)
τ_l	- random continuous-time (assumed zero) propagation delay of the l th interfering CIR relative to the desired ($l = 0$) CIR (Section 3.2 on page 28)
$\tau_m(t)$	- discrete-time propagation delay of m th multipath of time-varying CIR (see Equation (2.2) on page 13)
τ_i	- time constant of the i th transient excess MSE eigenmode of an LMS equalizer (see Equation (4.69) on page 132)
$\Phi(\tau; t)$	- autocorrelation function of a multipath fading CIR (see Equation (2.3) on page 14)
$\oplus$	- exclusive-OR logical operator (see Equation (5.3) on page 175)
$ \cdot ^2$	- magnitude squared operator (see Equation (4.53) on page 123)
$\ \cdot\ $	- norm of a matrix or vector (see Equation (4.56) on page 126)
$\ \cdot\ _2$	- spectral norm of a matrix (see Equation (E.19) on page 233)
$\lceil \cdot \rceil$	- round up (to next integer) operator (see Equation (4.29) on page 109)
$B(\cdot)$	- estimation bias operator (see Equation (4.34) on page 110)
$E[\cdot]$	- expectation operator with respect to random process(es) or signal(s)
$O(\cdot)$	- order of magnitude (see Equation (4.46) on page 112)
$Re\{\cdot\}$	- real part of complex quantity, e.g., number, vector or matrix, (see Equation (4.50) on page 114)
$(\cdot)^*$	- complex conjugate (transpose) operator for a complex signal (matrix or vector)
$(\cdot)^{-1}$	- matrix inverse operator (see Equation (3.46) on page 50)
$tr(\cdot)$	- trace (of a matrix) operator (see Equation (A.9) on page 211)
$Var(\cdot)$	- estimation variance operator (see Equation (4.32) on page 110)
$\max \lambda(\cdot)$	- maximum eigenvalue operator (see Equation (E.20) on page 233)

Chapter 1 Introduction

1.1 Thesis Motivation

Cellular radio communications is currently experiencing rapid growth in many parts of the industrialized world. Hitherto, this unprecedented increase in user demand has been spurred on primarily by the business sector. However, with the promise of personal convenience and freedom through tetherless communications at relatively low cost, cellular radio communications is becoming rather attractive to the general public [17],[45]. It is predicted that user demand for such communications will likely increase ten-fold within the next few years [115]. In fact, mobile cellular telephone systems in some large cities are currently congested to near capacity. Thus, it is imperative that the capacity of present systems be increased while improving their quality, reliability and potential for new user services. This applies to both terrestrial mobile radio and indoor wireless communications.

Present mobile cellular radio systems primarily employ analogue narrowband frequency modulation (FM) technology. Examples are the North American Advanced Mobile Phone System (AMPS) and the British total access communication system (TACS) [45].

However, the trend is towards mobile cellular systems based on digital communications which can utilize powerful and relatively inexpensive digital signal processing integrated circuit technology. Furthermore, digital technology has beneficial properties such as superior speech quality, improved privacy or immunity to eavesdropping, simpler control and supervisory signalling and a natural extension to data transmission and the integrated services digital network (ISDN). Most importantly, digital cellular radio systems have the potential to provide significantly higher capacity than existing analogue cellular radio systems while utilizing the same available bandwidth [17],[38],[45],[64],[102].

Proposed digital cellular systems for Europe (i.e., groupe spécial mobile (GSM)), North America (i.e., interim standard IS-54) and Japan (similar to IS-54) are based on a form of time division multiple access (TDMA) where a sub-group of users share a common frequency band with each user using disjoint time slots in regular intervals [45],[102]. This multiple access scheme also incorporates frequency division multiple access (FDMA) where the total available spectrum is divided into disjoint frequency carrier bands. Pro-

posed indoor wireless systems include the second generation cordless telephony (CT2) and digital European cordless telecommunications (DECT) systems [91],[118].

The narrowband TDMA digital cellular systems promise to improve capacity by accommodating several time-multiplexed users within essentially the same bandwidth that presently supports just one analogue user. Further capacity gains can also be realized by these digital systems through low bit-rate speech coding, higher trunking efficiency and more effective frequency re-use due to increased robustness to co-channel interference (CCI). However, under conditions of time-dispersive transmission due to frequency selective fading resulting from high signalling rates and stringent bandwidth constraints, TDMA digital cellular systems are susceptible to multipath-induced intersymbol interference (ISI) which must be mitigated with relatively complex adaptive equalization and/or diversity combining techniques [8],[16],[21],[31],[58],[69],[84],[85],[99],[119],[135].

For the case of a narrowband TDMA mobile cellular system, a relatively low signalling rate makes it difficult for the receiver to estimate and track the rapidly fading channel [31],[69],[99]. Furthermore, due to cell splitting and cell size reduction to increase frequency re-use and thus the capacity of a cellular system, the attendant increase in CCI becomes a limiting factor in the performance of the communications system. In the case of indoor wireless communications, with its lower mobile speeds and higher data rates, the adaptation problem is inherently less severe than for mobile cellular communications. However, the performance limitation due to inter-user interference can be a worse problem for indoor wireless communications due to its typically denser traffic [17].

Recently, there has been an intense campaign to promote an alternative wideband code division multiple access (CDMA) digital cellular system based on spread spectrum techniques [38],[64],[97],[110]. Claims that a CDMA cellular system offers even higher capacity than the proposed TDMA cellular systems have yet to be substantiated by field trials and there is continued controversy over whether TDMA or CDMA is the superior approach. CDMA can also be used in an indoor wireless system [110]. A related narrow bandwidth direct sequence spread spectrum system employing a fractionally spaced DFE, which does not necessarily require knowledge of the user spreading codes, has been shown to yield promising performance in an interference-limited indoor wireless communications system [3]. Alternatively, a code time division multiple access (CTDMA) approach has also been considered for indoor cellular communications [105].

1.2 Thesis Contributions

An objective of this thesis is to study the performance of a TDMA-based digital cellular system for indoor wireless communications in a quasi-static multipath fading channel environment which exhibits the impairment of inter-user interference, i.e., ACI and CCI. The purpose is to evaluate this performance in relation to system design choices such as frequency and spatial channel spacing and transmission and receiver bandwidth which determine the interference conditions and affect the receiver's ability to mitigate such disturbances. Thus, a tantamount aim of this thesis is to investigate the initial convergence and steady state performance of adaptive equalization schemes required at the receiver. The following is a summary of the thesis contributions towards these goals.

A computer simulation study has demonstrated that average SDR, BER and outage performance gains in the presence of inter-user interference and negligible ambient noise relative to an equivalent amount of stationary noise are achievable with an ideally computed or a conventional adaptive fractionally spaced DFE in a quasi-static multipath fading environment. However, it has been observed that the performance gain in interference degrades considerably as the ambient stationary noise becomes appreciable. In fact, such a performance gain can be achieved only if the interference is of greater strength than the ambient thermal noise. It has also been shown that the use of a wide receiver bandwidth can result in improved interference mitigation as long as there is sufficient spectral overlap between the ACI and the desired signal. It is noted that this improvement can result in either an increase in radio capacity through a reduction in channel spacing or the ability to operate in a noisier environment while maintaining a desired equalizer performance.

A computed MMSE DFE which employs estimates of the CIR and interference plus noise autocorrelation has been proposed and demonstrated to function in the presence of inter-user interference and noise. An expression for the upper bound of the computed DFE excess MSE due to CIR and interference plus noise autocorrelation estimation errors has been derived and used to show that the directly adapted RLS DFE performs better than the computed MMSE DFE. Computer simulation has confirmed this result by indicating that the average SDR, BER and outage performance in an interference-limited quasi-static fading channel environment of a DFE which is directly adapted with an RLS algorithm is superior to that of a computed MMSE DFE which requires explicit estimates of the CIR and interference plus noise autocorrelation.

A perturbation analysis is employed to illustrate the sensitivity of the computed MMSE equalizer performance to small inaccuracies in its input autocorrelation matrix and cross-correlation vector. Simulation has corroborated this result by showing that the computed MMSE equalizer performance requires very accurate estimates of the CIR and interference plus noise autocorrelation. This perturbation analysis also demonstrated that its input autocorrelation matrix eigenvalue spread can have a strong effect on the performance sensitivity of an adaptive equalizer to errors due to algorithmic and finite-precision effects.

It has been observed that the eigenvalue spread of the received signal autocorrelation matrix is typically orders of magnitude larger in the presence of inter-user interference than in stationary noise. Thus, it is suggested that this larger eigenvalue spread of the input signal autocorrelation matrix of an adaptive fractionally spaced equalizer operating in the presence of inter-user interference may necessitate a higher precision digital implementation in order to achieve sufficient numerical accuracy and stability.

Simulation and calculation results have shown that the directly adapted LMS DFE exhibits prohibitively slow MSE convergence in the presence of inter-user interference due to: (1) its sensitivity to the eigenvalue spread of its input autocorrelation matrix; and (2) the existence of a significantly lower MMSE in inter-user interference than in an equivalent amount of stationary noise. Furthermore, the application of an LMS MSE convergence analysis has revealed that the extremely slow convergence of the LMS DFE in the presence of inter-user interference is caused by one or two very slow-decaying eigenmodes which have weights that are significant relative to the MMSE. It is concluded that the LMS algorithm for training the DFE in the presence of inter-user interference does not achieve the full potential reduction in MMSE within a reasonable convergence time.

Alternatively, simulation and calculation results have demonstrated that the directly adapted RLS DFE is essentially insensitive to the eigenstructure of its input signal and thus exhibits much faster MSE convergence in the presence of inter-user interference than the directly adapted LMS DFE. In fact, it has been observed that the MSE convergence of the RLS DFE in the presence of inter-user interference is practically as fast as that in the presence of stationary noise. Thus, it is concluded that the RLS algorithm should be used to train the DFE in an interference-limited quasi-static multipath fading environment. It is suggested that in order to reduce the computational effort under slowly fading conditions, the DFE adaptation algorithm could switch to normalized LMS after RLS convergence.

1.3 Thesis Organization

Following this introduction, Chapter 2 presents an overview of the elements of cellular radio communications and discusses the pertinent issues of system capacity and performance improvement strategies. It also describes the salient properties and characteristics of the mobile radio channel and introduces the channel model used in this thesis. Moreover, it reviews the state-of-the-art on various receiver signal processing methods for dealing with channel impairments, with particular emphasis on interference mitigation.

Chapter 3 describes the communications system model of interest and the relevant signal to disturbance ratios and radio capacity of such a cellular system. In addition, it discusses the Wiener filter receiver approach employed in this thesis and formulates the optimum coefficients and the corresponding output MMSE and SDR of a finite-complexity DFE. In order to establish an upper bound on the DFE performance, this chapter investigates the ideal static SDR performance of an MMSE DFE computed with exact knowledge of the channel and disturbance parameters.

Chapter 4 considers three adaptive equalization schemes. The first scheme entails a computed MMSE DFE which utilizes estimates of the desired CIR and interference plus noise autocorrelation. The second and third schemes employ the LMS and RLS algorithms, respectively, to adapt the DFE. This chapter also analyzes the excess MSE of the computed DFE due to CIR and interference plus noise autocorrelation estimation errors. Moreover, it investigates the convergence performance of the three adaptive equalizers and compares the steady state performance of the computed DFE and the directly adapted RLS DFE. It also discusses finite-precision effects of a digitally implemented fractionally spaced equalizer operating in the presence of inter-user interference.

Chapter 5 presents computer simulation results of the steady state performance of a system employing an adaptive fractionally spaced DFE. Specifically, it compares the average SDR performance of the computed MMSE DFE and the directly adapted RLS DFE. Moreover, it studies the average bit error rate and outage performance of the adaptive DFE in the presence of inter-user interference and ambient thermal noise. It also investigates the effects of appreciable ambient noise and a wide receiver bandwidth on the system performance and their implications on user capacity.

Chapter 6 offers conclusions of this thesis and suggestions for further research.

Chapter 2 Cellular Communications: Background and Related Research

2.1 Introduction

This chapter provides a context in which to view the work undertaken in this thesis. It offers a succinct description of cellular radio communications and discusses the pertinent issues of system capacity and performance improvement methods. In particular, it reviews the literature of related research on receiver signal processing strategies to combat impairments present in cellular communications. Section 2.2 introduces the approach of cellular radio for high-capacity mobile communications. It also discusses available strategies to improve the system capacity which can be hampered by impairments such as adjacent and co-channel interference (i.e., ACI and CCI). Section 2.3 describes the salient properties of a multipath fading channel and presents the tapped delay line (TDL) channel model used in this thesis. Section 2.4 surveys the published literature of related research on receiver signal processing techniques to combat channel impairments such as multipath-induced intersymbol interference (ISI) and inter-user interference (ACI and CCI). This review of the state-of-the-art provides a frame of reference for the contributions of this thesis.

2.2 Cellular Radio Communications

The use of cellular technology is motivated by the need to alleviate spectral congestion by increasing capacity without requiring additional spectrum resources [17],[45],[76]. A cellular system achieves this capacity gain through the re-use of frequency bands in non-adjacent cells which are sufficiently spaced to ensure negligible CCI contention. Frequency re-use relies on the phenomenon that the path loss of the mean power of a radio signal is proportional to an exponent of the propagation distance.

In a narrowband TDMA/FDMA cellular system, regularly shaped cells are grouped into cell clusters which tessellate the geographic service area. For example, a cellular system can be organized into cell cluster sizes of four, seven, nine, twelve or more [76]. The total available radio spectrum is divided into disjoint frequency carrier channels, each of which is either assigned to one user (i.e., FDMA) or shared in time by more than one user each

occupying a distinct regularly sequenced time slot (i.e. TDMA). The total number of available channels is then divided amongst the cells in each cluster and they are re-used in every cell cluster. Each cell in a given cluster is assigned a different set of channel frequencies to minimize ACI, while the cell size and cluster spacing are chosen to minimize CCI. Examples of digital cellular radio include the North American IS-54, Pan-European GSM, DECT and CT2 systems [24],[25],[45],[91],[118]. The first two are TDMA-FDD (frequency division duplex) terrestrial mobile systems while the latter two are TDMA- and FDMA-TDD (time division duplex) indoor cordless systems, respectively.

In the case of a wideband CDMA cellular system, a geographic area is divided into contiguous identical cells, without the need for cell clustering or frequency channel assignment [110]. In a CDMA system, the total available spectrum is shared simultaneously by all users in a given cell to achieve communication without co-ordinated access. Thus, a CDMA system is inherently CCI-limited since all users communicate at the same time on the same channel. To achieve reliable communications, CDMA users typically are distinguished from each other by their unique spreading codes. Alternatively, a wideband fractionally spaced equalizer with a moderate processing gain of eight has been shown to function without requiring knowledge of user spreading codes [3].

With the use of uncorrelated spreading sequences, all other CDMA users appear as random noise after de-spreading at a given user's receiver. As the system capacity increases through the addition of more users in the same wideband channel, the overall interference level increases and the voice quality degrades gracefully. However, because of the simultaneous presence of all co-channel users, there are situations when the desired signal of a mobile located far from a base station will be swamped by the interfering signal of another mobile in proximity to the base station (i.e., the "near-far" problem). Thus, it is crucial that a CDMA cellular system employ an adaptive power control strategy to ensure that all the mobile signals arrive at the base station receiver with approximately the same strength regardless of their location within a cell [38],[64],[97],[110].

2.2.1 System Capacity

In general, one can increase the capacity of a cellular system by either increasing the available spectrum or improving the system's spectral efficiency, defined as the number of users per unit bandwidth per coverage area [17],[63]. The former approach requires some form of spectrum re-allocation which is unlikely to happen within the near future because

of political, legal and economic issues [110]. Thus, one must resort to the latter approach of increasing spectral efficiency.

In highly utilized cellular systems where frequency bands are re-used to maximize user capacity, CCI is the main impairment which limits system performance and capacity. To ensure an acceptably low level of CCI, cells using co-channels must be spaced sufficiently far apart. As an alternative or additional countermeasure, some form of signal processing can be employed at the receiver to suppress the CCI. Furthermore, in situations where a transmitter and a receiver operating on adjacent channels are physically close together, the receive filtering must be sufficiently tight so as to reject most of the ACI. Again, some form of signal processing at the receiver can be used to reduce the ACI. If the receiver can sufficiently mitigate the ACI and CCI, then there exists the potential for increased system capacity through further reduction in the spacing between co-channel and adjacent channel users in distance and frequency, respectively. The study of such adaptive receiver strategies and structures which are capable of mitigating interference in a multipath fading environment is the primary objective of this thesis.

Capacity can also be increased through the use of digital technology employing either TDMA or CDMA techniques. Moreover, the use of lower bit rate speech coders which provide toll quality speech can increase capacity at the expense of transceiver complexity, power requirements and cost [61]. The TDMA system capacity can also be increased by improving the trunking efficiency¹ or by employing dynamic channel allocation which assigns channels to each call based on the current traffic and interference conditions [4],[102]. In addition, the capacity of both TDMA and CDMA systems can be increased through the use of speech interpolation which exploits the silent periods of speech to accommodate more users [64],[110].

In a mature highly utilized cellular system where the main impairment is CCI, capacity can also be increased through the use of cell sectorization and directional antennas at the base station to reduce the received CCI power which in turn allows for smaller cell sectors. For example, cells can be divided into three 120 degree sectors or into six 60 degree sectors. Smaller sized cells with a larger number of sectors result in higher capacity

1. In a narrowband TDMA system which allocates a fixed number of TDMA channels per cell, a user will occasionally be blocked from obtaining a free channel. Thus, with a trunking efficiency of typically 75%, one reduces this blocking probability by ensuring that the available spectrum is not fully utilized on average.

while maintaining an acceptable CCI level. Since each sector requires a different set of frequency channels, cell sectorization essentially increases the cell cluster size by a factor equal to the number of sectors per cell. This results in a reduction in the trunking efficiency since there are fewer available frequency channels per sector. However, the capacity gain due to cell sectorization far outweighs this loss in trunking efficiency.

CCI can also be reduced by adaptive power control at the mobile or through an increase in the distance between co-channel cells. The latter approach can be achieved, at the expense of reduced system capacity, by either increasing the cell size, for a given cluster size (i.e., number of cells using disjoint channel sets per cell cluster), or increasing the cluster size, for a given cell size. Conversely, system capacity can be increased through cell splitting by reducing cell sizes, with an attendant reduction in transmit power levels to maintain an acceptable level of CCI. However, this strategy incurs the expense of increased cost and maintenance of the additional number of required base stations and increased complexity in the system co-ordination of the additional handoffs between cells [17].

A cellular system design typically entails a trade-off between spectral efficiency (i.e., capacity) and signal quality (i.e., performance). Note that the increase in the frequency of handoffs due to a reduction in cell sizes requires more complex control, forward error correction (FEC) and diversity techniques to maintain proper signal connectivity during a handoff procedure. This microcellular approach of cell size reduction in a cell splitting strategy also incurs the expense of requiring an additional base station for each newly created cell. In principle an arbitrarily large system capacity can be achieved with sufficient cell splitting. However, in practice, there is a lower limit on cell size due to the variability in the radio propagation characteristics of a small-cell channel and increased difficulties in base station location and handoff co-ordination.

The aforementioned strategies to increase the capacity of narrowband TDMA systems can result in comparable capacity performance to that claimed for CDMA systems [102]. However, while the performance of CDMA systems hinges on the ability to perform accurate and rapid power control to minimize CCI, the performance of high-capacity TDMA systems hinges on the ability of its receiver to deal effectively with the attendant increase in CCI due to a reduction in cell geometries. Thus, if a receiver structure and strategy can mitigate CCI, then narrowband TDMA systems can achieve the capacity and performance claimed for CDMA systems.

2.3 Channel Properties

This section describes the salient features and impairments which characterize a multipath fading channel environment such as for mobile radio or indoor wireless communications. Furthermore, an appropriate channel model suitable for indoor wireless under quasi-static slow fading conditions is presented and the performance issues arising from the nature of the TDMA block data format and channel impairments are discussed.

2.3.1 Multipath-Induced Time Dispersion and Frequency Selectivity

Cellular radio communications typically occur over a multipath fading channel. Due to reflections, refractions and scattering from various structures, a transmitted signal arrives at the receiver via multiple paths with random delays and attenuations. This multipath propagation can be characterized in the propagation time delay domain by the multipath power delay profile (MPDP) which is essentially the average received power (i.e., signal autocorrelation with zero time lag) as a function of the propagation delay. Multipath propagation results in a received signal that is dispersed in time [17],[54],[62],[98],[114].

For the case of digital signalling, this time dispersion leads to ISI whereby a given received data sample is corrupted by the responses of neighbouring data symbols. The severity of the ISI depends on the degree of multipath-induced time dispersion (e.g., can be quantified by the multipath root-mean-square (RMS) delay spread of the multipath fading channel) relative to the data symbol period. It is known that if the ratio of the RMS delay spread to the symbol period is greater than about 0.3, then the multipath-induced ISI must be corrected by adaptive equalization techniques in order to maintain acceptable performance [14]. It should be noted that the ISI experienced at the receiver is due to the overall channel impulse response (CIR) which includes the physical multipath fading channel response and the combined responses of the transmit and receive filters. Thus, the severity of the ISI problem is determined not only by the RMS multipath delay spread of the channel but also by the excess bandwidth constraints of the system.

In the 900 MHz frequency band for mobile radio communications, wideband multipath propagation measurements have shown that worst-case RMS delay spreads are usually less than 12 μs . Specifically, urban areas tend to have RMS delay spreads of about 2 to 3 μs , with significant echo power up to about 5 μs , while rural and hilly areas have RMS delay spreads of about 5 to 7 μs . In exceptional circumstances where there are specular

and diffuse reflections from distant landmarks such as mountains and city skylines, RMS delay spreads can be over 20 μs .

For indoor environments, the RMS delay spread can range from about 20 ns for small office buildings to about 200 ns for factories [47]. Thus, it is observed that the RMS delay spreads for indoor wireless channels are considerably shorter in the 100 ns range than those for terrestrial mobile channels. However, because of the typically higher signalling rates involved, the indoor wireless ISI problem can be quite significant.

The degree of multipath-induced time dispersion can also be quantified equivalently in the frequency domain (relating to the propagation delay via the Fourier transform) by the coherence bandwidth, which is roughly the inverse of the RMS delay spread. The coherence bandwidth gives a measure of the spectral flatness of the transfer function of a multipath fading channel. Again, the degree of ISI impairment due to the time-dispersive channel depends on the channel coherence bandwidth relative to the signal bandwidth.

If two signal components are separated by greater than the coherence bandwidth of a channel, then they fade independently and the overall signal is said to experience frequency-selective fading. In this case, multipaths are resolvable and an implicit diversity gain can be realized. However, the linear phase distortion must be corrected by ISI-mitigation techniques such as channel equalization. On the other hand, if a signal bandwidth is less than the coherence bandwidth of the channel, then the channel exhibits spectrally flat fading in which all the signal components fade simultaneously. In this situation, channel equalization is not necessary but no implicit diversity gain can be achieved.

2.3.2 Time-Variant Fading and Doppler Frequency Dispersion

Besides time dispersion of the received signal, another primary consequence of multipath propagation is signal fading. Multipath signals arriving at the receiver interfere incoherently with each other such that at some locations their phases may add constructively while at other locations destructively. Thus, the channel transfer function (or equivalently the CIR) is not only frequency-selective but also a function of mobile location, with a spatial standing wave pattern of the signal strength exhibiting nulls spaced about every half wavelength. As a mobile moves through this approximately stationary spatial pattern, the channel impulse response it encounters is, in effect, randomly time-varying with the received signal power experiencing fades every half wavelength. Such fades can occasion-

ally be over 30 decibels. For the situation when the scatterers are in random motion and there does not exist a direct propagation path between the transmitter and receiver, this random fading results in a zero mean CIR with Rayleigh and uniformly distributed signal envelope and phase, respectively [17],[54],[62],[98]. This small-scale Rayleigh fading is assumed for the indoor wireless channel model used in this thesis.

Signal fading due to multipath propagation can be characterized by the Doppler power spectrum which represents the fading signal power distribution over the range of Doppler frequencies. The severity of the time variations in the channel due to multipath fading can be quantified by the RMS Doppler spread. Furthermore, the degree of signal fading depends on the ratio of the RMS Doppler spread and the signal bandwidth. Equivalently, the extent of the time variation in a signal is related to the ratio of the channel coherence time (approximately the inverse of the Doppler spread) and the data symbol period.

In the case of mobile cellular communications, the channel Doppler spectrum (for vertically polarized dipole antenna reception) has peaks at its band edges. Consequently, this fading channel is better characterized by its maximum Doppler frequency or fade rate than by its RMS Doppler spread. Specifically, for mobiles travelling at highway speeds (e.g., 120 km/hr) and communicating in the 900 MHz band, the maximum Doppler frequency can be up to 100 Hz. Furthermore, for a symbol rate of 24 ksymols/s as that for the IS-54 standard, the CIR can vary by up to about 2.6% of its peak amplitude in one symbol period [68]. Over a typical signal transmission period corresponding to a TDMA burst of about 100 symbol periods, the time variation in the fading channel can be quite significant and thus some form of adaptive receiver strategy is required to track the changing channel conditions. On the other hand, due to its much slower mobile speeds and higher symbol rates, indoor wireless communications does not suffer from significant time variation in its fading channel. This quasi-static fading is assumed for the indoor wireless channel model used in this thesis.

Doppler fading resulting from multipath propagation occurs over short distances on the order of signal wavelength. In addition, a mobile radio signal also experiences relatively slow fading of its mean amplitude due to shadowing caused by signal blockage by large obstacles such as hills and buildings. This slow shadow fading of the mean signal amplitude is spectrally flat and is characterized by log-normal statistics. The overall mobile radio signal typically experiences relatively fast Rayleigh fading superimposed onto this

slow log-normal fading. Mitigation of this large-scale slow fading can be accomplished with macroscopic diversity implemented at the base stations [17],[54],[62],[98].

The large scale mean signal power is subjected to a propagation path loss which is proportional to an exponent of the propagation distance. Depending on the propagation environment, this exponent can range from about one to six. This path loss exponent is typically much larger for indoor channels as compared with terrestrial mobile channels [47].

2.3.3 Channel Characterization

A multipath fading channel can be modelled as a time-varying filter with random attenuation and phase. In the case of modulated communications, the channel can be represented as a complex baseband time-varying CIR, $g(\tau; t)$, where τ is the propagation time delay and t is the time dependence of the channel. A general multipath fading channel can assume either a continuous or discrete multipath model. The continuous multipath channel model assumes a continuum of propagation paths and its CIR can be written as [98],[114]:

$$g(\tau; t) = \alpha(\tau; t) e^{-j2\pi f_c \tau} \quad (2.1)$$

where α is the attenuation factor

and f_c is the signal carrier frequency.

The discrete multipath channel model assumes a finite number of statistically independent propagation paths, with the m th multipath exhibiting an attenuation, $\alpha_m(t)$, and propagation delay, $\tau_m(t)$. Such a discrete multipath CIR can be written as [98],[114]:

$$g(\tau; t) = \sum_m \alpha_m(t) e^{-j2\pi f_c \tau_m(t)} \delta(\tau - \tau_m(t)) \quad (2.2)$$

where $\delta(\tau)$ is the input impulse signal to the channel.

The cellular radio channel can be represented by a discrete multipath CIR. Assuming a sufficiently small signal bandwidth such that it does not resolve the individual propagation paths, then each component of the CIR, $g(\tau; t)$, is composed of a sum of a large number of multipath signals. Therefore, from the central limit theorem, each component of the CIR, $g(\tau; t)$, has real and imaginary parts which are independent identically distributed (iid) jointly Gaussian random processes. Furthermore, it is assumed that these Gaussian channel processes have zero mean. This implies that their envelope, phase and instanta-

neous power are described by statistics whose distributions are Rayleigh, uniform and exponential, respectively [54],[62],[98],[114].

Since the real and imaginary parts of the complex baseband CIR, $g(\tau; t)$, of a multipath fading channel are iid jointly Gaussian random processes, the channel can be completely characterized by its mean and covariance functions. Under the assumption of Rayleigh fading, the CIR has zero mean and can be completely characterized by its second order statistics, viz., correlation functions or frequency responses which are related by the Fourier transform (assuming wide sense stationarity).

A multipath fading channel can thus be characterized by the autocorrelation function of its complex baseband CIR:

$$\Phi(\tau_1, \tau_2; t_1, t_2) = E[g(\tau_1; t_1) g^*(\tau_2; t_2)] \quad (2.3)$$

where $E[\]$ is the expectation operator with respect to the random channel process and $*$ is the complex conjugate operator.

This autocorrelation function can be further simplified by assuming that $g(\tau; t)$ is wide sense stationary (WSS) in the time variable, t , over short time intervals on the order of a TDMA burst and that $g(\tau; t)$ is uncorrelated at different propagation delays, τ . Consequently, with this wide-sense stationary uncorrelated scattering (WSSUS) assumption, the channel autocorrelation function can be expressed as:

$$E[g(\tau; t) g^*(\tau + \Delta\tau; (t + \Delta t))] = \Phi(\tau; \Delta t) \delta(\Delta\tau) \quad (2.4)$$

where $\Phi(\tau; \Delta t)$ is a function of the time difference, Δt (WSS) and $\delta(\Delta\tau)$ is the Dirac delta function of the propagation delay difference, $\Delta\tau$ (US).

For the case of zero time lag (i.e., $\Delta t = 0$), the MPDP is obtained as:

$$\Phi(\tau; 0) = \Phi(\tau) = E[|g(\tau; t)|^2] \quad (2.5)$$

which represents the variance of the CIR (i.e., averaged received power) due to an impulse input as a function of the propagation delay, τ . The MPDP can be obtained from wideband channel measurements and is a useful characterization of the time-dispersive nature of a multipath fading channel. A concise measure of the time dispersion is the RMS delay spread of the MPDP. Furthermore, a multipath fading channel has an approximately finite impulse response (FIR) since its MPDP has a finite range of propagation delays with significant multipath power.

The channel time dispersion can be characterized equivalently in the frequency domain (i.e., Fourier transform with respect to the propagation delay domain). In this representation, time-dispersive propagation manifests as frequency-selective fading for signal bandwidths greater than the channel coherence bandwidth which is defined as roughly the inverse of the RMS multipath delay spread. On the other hand, if the signal bandwidth is less than the channel coherence bandwidth, then the signal experiences frequency-nonselective or flat fading.

The MPDP characterizes only the multipath-induced time dispersion of the channel and does not contain any information about the time variation of the fading channel. Both the time-dispersive and fading characterizations are encapsulated in the scattering function which is related to the channel autocorrelation function, $\Phi(\tau; \Delta t)$, via the Fourier transform with respect to the time difference variable, Δt , [98],[114]:

$$S(\tau; f_d) = \int_{-\infty}^{\infty} \Phi(\tau; \Delta t) e^{-j2\pi f_d \Delta t} d(\Delta t) \quad (2.6)$$

The channel time variation due to fading can be explicitly characterized by the Doppler power spectrum which can be obtained by integrating out the time-dispersive effect from the scattering function, $S(\tau; f_d)$:

$$S(f_d) = \int_{-\infty}^{\infty} S(\tau; f_d) d\tau \quad (2.7)$$

The Doppler power spectrum can be obtained experimentally from narrowband (i.e., sinusoidal) channel measurements and yields a characterization of the Doppler frequency dispersion of the fading channel. Analogous to the RMS delay spread of the MPDP, the RMS Doppler spread of the Doppler power spectrum can give a concise measure of the degree of fading in the channel. For the case of terrestrial mobile radio whose Doppler power spectrum has its peaks at its band edges, a more appropriate measure is the maximum Doppler frequency [54]:

$$f_{dm} = f_d|_{\theta=0} = f_c \left(\frac{v_m}{c} \right) \cos\theta|_{\theta=0} = f_c \left(\frac{v_m}{c} \right) \quad (2.8)$$

where f_c is the signal carrier frequency.

v_m is the mobile speed,

c is the electromagnetic (e.g., light) propagation speed
and θ is the incident angle of the received electromagnetic signal.

The channel frequency dispersion as represented by the Doppler power spectrum can be characterized equivalently in the time domain (i.e., Fourier transform of the Doppler frequency domain). In this time correlation domain, the time variation due to fading can be represented concisely by the channel coherence time, which is defined as roughly the inverse of the RMS Doppler spread.

2.3.4 Channel Model

In general, a time-dispersive and time-varying propagation radio channel is described by a linear time-varying FIR filter. The cellular radio transmission medium considered in this thesis can be modelled as a discrete multipath fading channel with Rayleigh statistics. In particular, the frequency-selective fading channel is modelled as a sampled FIR filter implemented as a tapped delay line (TDL).

The TDL channel model is composed of evenly spaced tap coefficients which represent the resolvable multipaths. Moreover, the TDL should be sufficiently long to span the significant portion of the channel MPDP while the TDL tap spacing (i.e., multipath resolution) should be fine enough to represent the signal adequately. In order to avoid aliasing, the channel frequency resolution should in general be at least twice the system bandwidth which can include adjacent and co-channel signals. Specifically, the complex baseband input-output relationship of the frequency-selective channel can be written as a convolution (assuming no additive noise and interference for the moment) [98]:

$$r\left(\frac{kT}{s}\right) = \sum_{m=1}^M g_m\left(\frac{kT}{s}\right) u\left(\frac{kT}{s} - \frac{m}{2W}\right) \quad (2.9)$$

where $r(kT/s)$ is the channel output signal,

$g_m(kT/s)$ is the m th T/s -spaced CIR coefficient,

$u(kT/s - m/2W)$ is the channel input signal spanning the CIR,

$W = s/2T$ is the complex baseband signal bandwidth,

and T is the symbol period.

This TDL channel model distinguishes multipaths to a time resolution of $1/2W = T/s$. For example, in order to represent signals with up to 100% excess bandwidth (i.e.,

$W = 1/T$), a $T/2$ -spaced TDL channel model is required. In general for a system bandwidth of $W = s/2T$, a T/s -spaced TDL system model is required, with signal and channel filter samples that are T/s -spaced and indexed by m , while the time variation in the channel is also in T/s -spaced increments, as indexed by k . To model the Rayleigh fading in the multipath channel, each TDL tap coefficient, representing the m th resolvable multipath component of the discrete channel model, has a complex-valued gain with zero mean and Gaussian statistics. By the uncorrelated scattering assumption, the TDL tap coefficients also fade independently of each other.

2.4 Receiver Signal Processing to Mitigate Channel Impairments

This thesis is concerned with adaptive signal processing at the receiver in the presence of inter-user interference in a quasi-static multipath fading environment typical of indoor wireless wideband TDMA communications. The candidate adaptive receiver strategies considered in this thesis are: (1) computing the MMSE equalizer from updated estimates of the CIR and autocorrelation of the interference plus noise; and (2) directly adapting the equalizer coefficients in an iterative manner, e.g., using the LMS or RLS algorithm, without explicit knowledge of the channel and interference correlation properties.

In order to provide some background for this thesis' contributions regarding the study of these adaptive receiver approaches, this section reviews the literature of relevant research on receiver structures and strategies used to mitigate the multipath fading channel impairments of ISI, time-varying CIR, ACI, CCI and AWGN. Hitherto, there has been a vast amount of work done in these research areas. To be concise, the literature survey will concentrate on previous research which is most closely related to the problem of communications over an interference-limited multipath fading (e.g., the mobile cellular or indoor wireless) channel, with particular emphasis on the interference mitigation problem.

2.4.1 Fading and ISI Mitigation

The system design choices of the data symbol rate, signal bandwidth and TDMA block length determine the relative severity of multipath-induced ISI and random time variation in the received signal due to the multipath fading channel. For instance, communicating at sufficiently low signalling rates results in negligible ISI but significant time variation due

to Doppler fading. On the other hand, the use of high symbol rates can lead to appreciable ISI which must be mitigated with channel equalization techniques. However, channel adaptation is a much easier task at high signalling rates since the rate of channel fading relative to the symbol rate is lower. Thus, the selection of the data signalling rate, bandwidth and TDMA burst length involves a performance trade-off between adaptation to the CIR time variations and equalization of the multipath-induced ISI.

In general, bandwidth-efficient signalling over a frequency-selective fading channel typically results in multipath-induced ISI which can be minimized with adaptive equalization or data sequence detection, e.g., MLSE, techniques [98],[100]. With the tremendous recent interest in digital wireless communication, there has been a plethora of research in the area of adaptive equalization for multipath fading channels such as those for land mobile and indoor wireless communications [31],[99]. In order to deal effectively with the ISI problem in a rapidly fading environment, such equalization techniques must address the problem of tracking the time variation in the fading channel. Typical solutions can employ channel coding and/or diversity combining in addition to equalization [7],[8],[21],[68]. Furthermore, adaptive strategies may include equalizer coefficient update or CIR estimation with the RLS or BLS algorithm and CIR tracking with the LMS algorithm or an interpolation approach [30],[59],[68],[84],[85],[135]. However, since the scope of this thesis is limited to communications over relatively slow fading, i.e., quasi-static, channels typical of indoor wireless communications, this brief review of the problem of adaptive equalization for rapidly fading channels will suffice.

2.4.2 Interference Mitigation

Equalization techniques can mitigate user data interference such as ISI, ACI and CCI [112],[122]. Moreover, the use of relatively large bandwidths in the presence of inter-user interference (e.g., ACI and/or CCI) can enhance the cyclostationarity of the interference and thus improve the performance of a fractionally spaced equalizer which can resolve and exploit the periodic variation (over a baud interval) of the interference second order statistics [1],[51],[94]. However, for a spectrally efficient TDMA/FDMA system with fixed frequency spacing between adjacent channels, an increase in signal bandwidth leads to an increase in ACI power at the receiver which must be mitigated by the equalizer. Also, the issues of data throughput and system complexity can be important due to the

overhead required for receiver convergence with training sequences and the practical implementation of the receiver structures and algorithms.

A receiver's ability to mitigate a given level of interference depends on important factors such as the number of interferers present, the transmission bandwidth and the differences between the desired signal and interferer responses. A receiver can achieve performance gains in the presence of interference relative to the equivalent case of stationary noise by exploiting the cyclostationary property of the interference which is distinct from that of stationary noise [1],[3],[34],[36],[49],[51],[56],[95]. That is, interference arising from synchronous data transmission is known to be cyclostationary with non-Gaussian statistics that vary with a period equal to the baud rate [12],[35]. In contrast, additive Gaussian noise has statistics which are stationary.

The above-mentioned factors essentially determine the degree to which the interference is distinct from additive Gaussian stationary noise. For instance, many random interferers produce an overall interference which is essentially Gaussian stationary noise, resulting from the Central Limit Theorem, while a small number of interferers tends to produce non-Gaussian cyclostationary interference [56]. Furthermore, interference occupying a large bandwidth relative to the minimum Nyquist bandwidth possesses more cyclostationarity while interference confined to the Nyquist bandwidth manifests merely as stationary noise [51],[95]. For instance, CCI resulting from a small number of, or a large number of phase-aligned, co-channel data streams occupying a relatively large bandwidth will tend to possess a large degree of cyclostationarity and distinction from the case of additive Gaussian stationary noise [94].

In a mature highly utilized cellular radio system with a large number of users distributed over relatively small cells, the dominant source of interference is CCI while the additive thermal noise power can be relatively insignificant on the order of tens of decibels below the interference power. Furthermore, the CCI tends to originate from one dominant co-channel interferer [9],[134]. Consequently, improved receiver performance can be achieved by accounting for this distinctly cyclostationary CCI rather than assuming the worst case that it is additive Gaussian stationary noise. Thus, the receiver performance is not necessarily limited by CCI but by stationary noise.

In fact, it has been shown that under certain conditions and requirements, (viz., static channel, negligible ambient noise, infinite length equalizer and at least 100% excess trans-

mission bandwidth), an MMSE fractionally spaced equalizer can completely suppress all ISI and CCI from one interferer [36],[51],[95]. A more general result states that for an increase in the available bandwidth by the Nyquist bandwidth (i.e., symbol rate), a linear equalizer with fractional tap spacing commensurate with the signal bandwidth can suppress an additional cyclostationary (at the baud rate) interferer. This result has been found in the context of complex baseband QAM linear equalization for digital subscriber line and frequency-shift filtering for PSK or QAM signals with equal symbol rates and arbitrary carrier frequencies [36],[51],[95].

It has also been shown that an asynchronous direct-sequence spread spectrum multiple access system with an optimum, i.e., maximum likelihood sequence detector, multiuser receiver consisting of a bank of filters matched to the user waveforms followed by a Viterbi decoder is not interference-limited [126]. For the best distribution of the random relative delays amongst the users, any level of interference can be completely suppressed. Even for the case with worst-case relative delays, the system is not interference-limited and, indeed, sufficiently powerful interference, e.g. equal energy users, can still be completely suppressed. Similarly, the case that sufficiently strong interference does not limit system performance in terms of capacity has been shown for a Gaussian interference channel with two users [13].

The relative phase alignment of the CCI and desired data is generally random and not known. However, it may be acceptable to assume the worst case, in terms of differentiating between the desired and interfering data, that there is no misalignment in the transmitted data streams. Because the cellular radio channel is randomly fading, the relative alignment of the desired and interfering data at the receiver is random and the desired and interfering responses will in general be quite different. As for the use of relatively large transmission bandwidths to improve the CCI suppression capability of an equalizer with commensurately spaced taps, there is an attendant increase in ACI, for a fixed channel frequency spacing, which also has to be mitigated.

In the case when the interference possesses significant cyclostationarity, a reduced state sequence estimation receiver with appropriately chosen sampling phase has been shown for a digital subscriber loop application to perform better in such CCI than in an equivalent stationary coloured noise [56]. Furthermore, a receiver consisting of a synchronously spaced equalizer with optimally selected sampling phase has also been shown to exhibit

similar performance improvements [1]. In addition, the performance of a fractionally spaced equalizer has been shown to be relatively insensitive to the receiver sampling phase and can yield superior performance in CCI relative to the equivalent stationary noise case [1],[51],[95]. This use of a fractionally spaced equalizer as a Wiener filter to mitigate the inter-user interference exploits the temporal correlation of the cyclostationary interference.

For a Wiener filter such as a fractionally spaced MMSE DFE at the receiver, the difference between cyclostationary inter-user interference and stationary noise can be viewed in terms of their second order statistics (see Section 3.4 on page 47). The autocorrelation function of receive-filtered stationary noise is completely determined by the receive filter response and is fixed in time. This stationarity implies that the noise autocorrelation matrix has Toeplitz symmetry. On the other hand, the autocorrelation function of cyclostationary inter-user interference (or desired signal) also depends on the interferer (or desired) CIR. This cyclostationarity implies that the overall fractionally spaced interference (or desired signal) autocorrelation matrix has only Hermitian symmetry since the synchronous user data statistics can vary over a baud period. Because the interference (or desired signal) cyclostationary statistics are periodic at the baud rate, the autocorrelation sub-matrix of each phase of the oversampled interference (or desired signal) is stationary and therefore possess Toeplitz symmetry.

In complex baseband representation, the autocorrelation of the receive-filtered noise is real symmetric while the interference autocorrelation is conjugate symmetric. Depending on the interferer CIR and the degree of interference cyclostationarity, the second order statistics of interference can be quite different from that of stationary noise. In one sense, the MMSE DFE can perform better in the presence of inter-user interference than in stationary noise because the DFE can achieve a smaller prediction error at the output of its feed-forward filter by exploiting the correlation of the interference. Furthermore, the inter-user interference characteristics vary randomly with time in a fading environment. Thus, in order for the DFE to perform better in the presence of inter-user interference than in stationary noise, it requires updated knowledge of the interference statistics.

Another, perhaps more general, way in which to view the cyclostationarity in a signal is in terms of its spectral correlation [33]. That is, a cyclostationary signal exhibits spectral coherence at integer multiples of its fundamental periodicities such as the baud rate, chip

rate or carrier frequency. This spectral correlation can be exploited by frequency shift filtering to reduce the effect of the cyclostationary interference [33],[34],[36],[49],[104]. This frequency-shift filtering performs a linear sum of the weighted frequency-shifted received signal components and can be implemented as a parallel bank of filters, each consisting of a frequency-shifted product modulator followed by an FIR filter, whose outputs are summed. The number of cyclic frequency translates employed in the filtering is typically determined from experimentation.

For digital signals such as QPSK, QAM or non-coherent FSK which do not exhibit spectral correlation between their positive and negative frequency images, the weighted frequencies which result in non-zero spectral correlation occur only at integer multiples of the baud rate [33],[104]. On the other hand, digital signals such as BPSK, MSK and ASK do possess spectral correlation between their positive and negative frequency images and thus their corresponding receivers can exploit this additional correlation by also having weighted frequencies at twice the carrier frequency plus integer multiples of the baud rate [33],[104]. However, baseband signals do not exhibit such spectral correlation resulting from the carrier periodicity.

In general, a frequency-shift filter estimates all parameters of the desired waveform [34],[36]. However, only the baud rate sampled data sequence is of interest in a typical digital communications system. Furthermore, if the spectral correlation associated with the carrier frequency is non-existent or is not exploited then the receiver is simplified as a linear processor. Thus, when the output of the frequency-shift filter is sampled at the baud rate, this simplified structure is equivalent to, and can be implemented as, a fractionally spaced FIR equalizer [34],[36]. For a system employing 100% excess bandwidth raised-cosine filtered BPSK with zero carrier offset, such a fractionally spaced linear equalizer has been shown to out-perform a synchronously spaced linear equalizer in the presence of partial-band AM interference or multipath-induced ISI [36]. Moreover, when the interference is at a different baud rate and carrier offset from the desired signal, an LMS-trained frequency-shift filter (called a time-dependent adaptive filter) has been shown to achieve superior performance relative to a standard time-independent adaptive filter which does not exploit the signal cyclostationarity [104]. The ability of a sufficiently complex equalizer to suppress cyclostationary interference with a different baud periodicity from the desired signal has also been shown [94].

The effects of interference can also be mitigated with an optimum (in the MMSE sense) combiner employing space diversity which exploits the spatial correlation of the interference in each diversity branch [16],[21],[46],[82],[129],[130]. The BER performance of such an optimum combiner has been studied for a flat fading indoor wireless or mobile radio environment in the presence of CCI [46],[129],[130]. An investigation into the performance of an optimum combiner was also performed for a frequency-selective mobile cellular environment, although the CCI was modelled as worst-case AWGN [16]. An alternative method to deal with both multipath-induced ISI and inter-user interference (ACI or CCI) in an indoor wireless or microwave digital radio environment incorporates both optimum combining and equalization, with fractionally spaced TDL filtering in each diversity branch and synchronous decision feedback TDL filtering, into the receiver structure [21],[82]. The BER performance is computed via the method of moments for each quasi-static fading channel realization and averaged over the fading channel statistics via a Monte-Carlo simulation [21].

It is known that, for a given receiver complexity and bandwidth constraint, interference cancellation results in a diversity loss which in turn reduces the receiver's capability to handle channel fading [82]. If protection against fading is of concern, then channel coding, explicit diversity combining and/or implicit diversity via adaptive equalization techniques can be employed [7],[8],[16],[17],[21],[30],[54],[57],[62],[69]. Furthermore, it has been suggested that a linear fractionally spaced equalizer in conjunction with an antenna diversity combiner can suppress a number of interferers equal to the product of the available bandwidth, in integer multiples of the Nyquist bandwidth, and the number of antennas [94]. Thus, there exists a flexibility to employ additional excess bandwidth to suppress the interference while maintaining an acceptable level of fading protection.

In the case of multi-source systems with more than one independent data stream on the same physical channel, receiver decisions from adjacent channels can be used to aid in the ACI post-cursor reduction [82]. Similarly, a high-complexity multi-dimensional DFE also uses receiver decisions from the two adjacent channels to reduce some of the intersymbol and crosstalk interference from these adjacent channels which are present in the oversampled received signal [128]. This method requires a multi-dimensional DFE with essentially three times the number of coefficients as a conventional DFE since it has to process the desired channel and the two adjacent channels. This increased number of DFE coefficients would tend to aggravate the adaptation problem.

Other studies have shown that a conventional fractionally spaced decision feedback equalizer is capable of mitigating not only ISI but also ACI [20],[44],[58],[86],[94],[119]. For non-fading channel conditions with no CCI, MMSE calculations indicate that a fractionally spaced DFE can mitigate ACI even under significant spectral overlap [94]. This has been similarly observed in the context of frequency-shift filtering of partial-band AM interference [36]. In addition, BER simulations under similar system conditions have also confirmed such performance of a fractionally spaced DFE in ACI [44]. It was also found that for a given ACI level, a larger channel spacing yields better equalizer MMSE performance. The use of a frequency-shift filter in the form of a fractionally spaced DFE has been shown to mitigate ACI [50].

In terms of the performance criterion of the equalizer output SDR averaged over an ensemble of quasi-static fading channel realizations, it was found that ACI in a digital microwave radio channel is suppressible with a fractionally spaced linear equalizer (LE) but not with a synchronously spaced LE [20]. Also for a digital microwave radio environment, it was found that a fractionally spaced LE or DFE can increase the ACI immunity for a flat or frequency-selective fading channel [86]. In addition, the BER in the presence of noise, ISI and ACI was calculated using a moment method.

A study of the fractionally spaced LE static performance in a mobile radio environment with one ACI in the upper band has shown that a wideband fractionally spaced equalizer is effective in minimizing ISI and in rejecting ACI, especially when the channel spacing is small [58]. If the channel spacing is relatively large, it is suggested to avoid the ACI by employing tight transmit and receive filters with the resulting ISI removed by the equalizer. In this study, the adaptive equalizer was trained with the RLS algorithm.

An investigation of the performance of a GSM-based system employing a fractionally spaced DFE has been reported [119]. This simulation study considered the BER performance under typical frequency-selective fading conditions in the presence of ACI and CCI. A CIR estimate is obtained with a correlation method and is used to specify the feedback filter taps of the DFE. The synchronously spaced feedforward filter taps are found via the Wiener solution and are interpolated to determine the fractionally spaced taps. The DFE feedforward and feedback taps are tracked separately with the LMS algorithm. Under flat fading conditions with a maximum Doppler frequency of 40 Hz and in the presence of ACI, it was found that the performance was significantly better with a fractionally

spaced DFE. Alternatively, an approach for mobile radio communications which does not employ an equalizer estimates the ACI with the use of bandpass filtering and coherent detection and subtracts the interference from the received bandpass signal [109].

Research on interference mitigation often studies the optimum achievable performance by assuming that the receiver has complete *a priori* knowledge of the desired and interferer channels. The practical means by which an adaptive receiver converges to such an optimum setting has received relatively little attention. Moreover, the quasi-static fading channel conditions are usually assumed to be time-invariant or very slowly time-varying. Typically, the LMS algorithm is employed to train an equalizer during the initial convergence period [21],[36],[46][94],[104],[119]. The RLS algorithm has been used to train a linear equalizer in the presence of ACI, although subject to only transmit/receive filtering but no realistic channel response [58]. Furthermore, the receiver decision-directed tracking problem in the presence of interference and under time-varying channel conditions has not been addressed.

2.5 Summary

Cellular radio communications and relevant issues of capacity and receiver performance in the presence of inter-user interference in a quasi-static multipath fading environment have been discussed. Moreover, the pertinent characteristics of a multipath fading channel are reviewed and a tapped delay line channel model is presented. In order to provide a perspective on the work of this thesis, the literature on the state-of-the-art of related research on receiver signal processing to deal with multipath-induced ISI and inter-user interference (ACI and CCI) has been surveyed.

Chapter 3 System Description and Ideal MMSE DFE Performance

3.1 Introduction

This chapter investigates the ideal performance of an MMSE DFE which is computed with exact knowledge of the desired and interferer channel impulse responses and disturbance parameters. The performance measure is the optimum signal to distortion ratio (SDR) at the output of the equalizer, or equivalently, at the input to the decision device. Digital communication in a quasi-static multipath fading channel environment typical of a high-capacity indoor wireless cellular system is subjected to the impairments of multipath-induced ISI, inter-user interference (i.e., ACI and CCI) and ambient thermal noise. Moreover, the ideal performance is conditioned on the channel fading statistics. In particular, the calculated performance is averaged over an ensemble of 1000 random channels. Furthermore, the probability distribution of this performance is also considered.

Section 3.2 presents the communications system model of interest in this thesis and the relevant signal to disturbance ratios and radio capacity of such a cellular system. Section 3.3 discusses the Wiener filter receiver design approach employed in this thesis, while Section 3.4 formulates the optimum filter coefficients and the corresponding output MMSE and SDR of the finite-complexity MMSE DFE receiver. Section 3.5 investigates the average SDR performance of the ideally computed MMSE DFE and the associated radio capacity under various disturbance situations with inter-user interference and ambient noise. Furthermore, Section 3.6 considers the DFE SDR probability distribution for the disturbance conditions of Section 3.5.2.

Specifically, Section 3.5.1 investigates the optimum SDR performance of a DFE in the presence a dominant CCI user and negligible ambient noise. The main result of this section is the confirmation that a system employing a relatively wide 100% excess bandwidth operating in a multipath fading environment can achieve significant DFE performance gains in the presence of interference relative to the case in an equivalent amount of stationary noise. Related results have been obtained previously for an infinite-complexity receiver and/or without considering multipath fading statistics [1],[36],[51],[94].

Section 3.5.2 studies the optimum SDR performance of a DFE in the presence of appreciable ambient noise and equal-strength ACI and CCI users. This section investigates this performance as a function of the ambient noise level. Specifically, it is confirmed that the DFE performance in a multipath fading environment is ultimately limited by the stationary noise. However, a DFE performance advantage in the presence of interference with negligible ambient noise over that in an equivalent amount of stationary noise can be achieved. Moreover, the DFE performance in the presence of interference improves with the use of wider receiver bandwidths. These last two results have been obtained previously for a non-fading channel environment [51],[94].

Section 3.5.3 considers the optimum SDR performance of a DFE in the presence of up to two ACI users, one CCI user and ambient noise with a hypothetical constraint of $SIR = 10$ dB. This example serves to illustrate the inherent tradeoff between the DFE performance and the radio capacity of a cellular system. It is shown that to achieve a desired DFE performance criterion with a fixed overall interference level, it is not recommended to reduce the CCI power by increasing the frequency reuse factor since the radio capacity decreases to an unacceptable level. Rather, it is more beneficial to avoid a significant portion of the ACI by increasing the channel spacing, at the expense of a slight decrease in the radio capacity, and to employ the DFE to mitigate the dominant CCI and remaining ACI.

Section 3.5.4 investigates the optimum SDR performance of a DFE in the presence of up to two ACI users, one CCI user and ambient noise with a fixed CCI level at $SCCIR = 10$ dB. It is observed that the DFE performance improves through better interference suppression with the use of a wider receiver bandwidth as long as the channel spacing is small enough to allow sufficient spectral overlap of the ACI with the desired signal. The use of a wider receiver bandwidth can maintain a desired equalizer performance while permitting either an increase in radio capacity through a reduction in channel spacing or operation in a noisier environment. This benefit of using a wide-bandwidth DFE to mitigate ACI has been observed previously in non-fading channel environments [44],[51],[58],[94].

Section 3.6 considers the DFE SDR probability distribution for the disturbance conditions of Section 3.5.2. This section demonstrates that the SDR probability distribution results are consistent with the average SDR performance results obtained earlier in Section 3.5. Moreover, the SDR probability distribution can be used directly to calculate the outage performance.

3.2 System Model

The DCR communications system studied in this thesis is a TDMA system which can be applied generally to digital mobile radio or, under quasi-static or slow fading conditions, to indoor wireless communications. For the example of narrowband digital mobile radio, the communications system transmits an overall bit rate of 48 kbits/s employing QPSK modulation with a corresponding signalling rate, $1/T$, and minimum transmission bandwidth of 24 ksymbols/s and 24 kHz, respectively. The data format has a typical burst length of $N_t + N_d$ symbols consisting of an initial N_t $\pi/4$ -rotated BPSK training symbols followed by N_d QPSK data symbols. The uncorrelated complex baseband QPSK symbols have unit variance, $\sigma_d^2 = 1$, and can take on values of $(\pm 1 \pm j) / \sqrt{2}$.

The DCR communications system is modelled as a T/s -spaced, complex baseband system, where s is an integer greater than unity which specifies the number of samples per symbol period. In order to avoid aliasing, the frequency resolution as determined by s should be at least twice the system bandwidth, spanning both adjacent and co-channels, in integer units of the reciprocal of the symbol period, $1/T$. The T -spaced data symbols are transmitted in a T/s -spaced data sequence with $s - 1$ T/s -phase offset, zero-valued samples following each data symbol. The received signal, formed by convolving this data sequence with the T/s -spaced overall CIR, is corrupted by additive noise and inter-user interference (e.g., ACI and CCI) samples before entering the receive filter. At the receiver, coherent demodulation with ideal carrier and timing recovery is assumed.

In general, the frequency-selective TDL channel model, $g_m(kT/s)$, consists of T/s -spaced tap coefficients. For the specific case of a $T/2$ -spaced system which is representative of land mobile radio, one multipath channel model, denoted as MPDP-2, has RMS values, α_m , of 0.836, 0.484 and 0.153, corresponding to three resolvable $T/2$ -spaced paths with relative average powers of 0, -5 and -15 dB spanning approximately one symbol period [68]. For a symbol period, T , of 41.7 μs , this MPDP-2 model has RMS and average delay spreads of 10.4 μs and 5.9 μs , respectively. Another "worst-case" channel model, denoted as MPDP-1, has three equal strength $T/2$ -spaced paths each with an RMS value, α_m , of 0.577 corresponding to RMS and average delay spreads 17.0 μs and 20.8 μs , respectively. While such multipath delay spreads are typical of outdoor mobile cellular environments, this system can also model indoor wireless communications with delay spreads on the order of 100 ns by appropriately scaling the data rate to about 10 Mbits/s.

The MPDP-1 channel causes more multipath-induced ISI than the MPDP-2 channel. However, in a fading situation the MPDP-1 channel also provides more implicit diversity which an equalizer can exploit. Under fading conditions, the MPDP-1 channel can exhibit non-minimum phase behaviour. Despite such severe and perhaps unrealistic multipath fading conditions, this MPDP-1 channel serves as a worst case for receiver performance. In general for a T/s -spaced system, the MPDP-1 channel model consists of $s + 1$ equal strength T/s -spaced paths each with an RMS value, $\alpha_m = (s + 1)^{-1/2}$, and spanning approximately one symbol period. Unless otherwise stated, this “worst-case” MPDP-1 multipath channel model is used in this thesis.

Rayleigh fading is simulated for the mutually independent TDL channel tap coefficients, $g_m(kT/s)$, each of which has real and imaginary components with Gaussian statistics. Moreover, the time variation in each tap coefficient of the multipath channel can be modelled as a sum of weighted sinusoids with random phases and discrete frequencies spanning the Doppler spectrum [54],[68]. However, a time-varying channel (i.e., with non-zero Doppler frequency, $f_{dm} > 0$ Hz) is only considered in the context of CIR estimation to illustrate the deleterious effects of a time-varying environment (see Section 4.2.1.2 on page 99). Otherwise, this thesis deals mainly with quasi-static channel conditions where the channel tap coefficients are randomly initialized for each trial of an ensemble of channel realizations.

The T/s -spaced transmit and receive filters, $f_t(kT/s)$ and $f_r(kT/s)$, can have square root raised cosine (SRRC) frequency responses, $F(f, \epsilon)$, with a $100\epsilon\%$ excess bandwidth ranging from 25% to 100% (i.e., $0.25 \leq \epsilon \leq 1$) (see Equation (3.18)). In this case, the combined response of the transmit and receive filters has a raised cosine frequency response which satisfies Nyquist’s criterion for zero ISI at the symbol instants. However, the overall channel response, $h(kT/s)$, resulting from the convolution of the transmit and receive filters with the actual multipath fading channel does result in ISI. Even though the actual channel has a multipath span of only one symbol period, the overall channel impulse response can have an ISI span of up to about five symbol periods for the case of 25% excess bandwidth filtering. In addition, some subsequent simulation experiments consider the effect of greater than 100% excess bandwidth receive filtering with a normalized energy 100% excess bandwidth SRRC filter which is stretched linearly in frequency to the appropriate wider bandwidth [94].

The overall T/s -spaced channel impulse response (CIR), $h(kT/s)$, is represented by s T -spaced CIRs, $h_m(kT)$, with $m = 0, 1, \dots, s-1$, which are offset by T/s in phase. For the case when channel estimation is performed explicitly (see Section 4.2.1 on page 94), each of the T/s -phase offset CIRs is estimated as an $N_c = 6$ and $N_a = 3$ T -spaced coefficient response for 25% and 100% excess bandwidth filtering, respectively. For the case of 100% excess bandwidth filtering, a shorter estimated CIR can result in an attendant improvement in the adaptive equalization performance.

In addition to the multipath-induced ISI which is aggravated by the potentially stringent filtering requirements, the multipath fading channel output is also corrupted by complex-valued additive white Gaussian noise (AWGN), with power spectral density N_0 and per sample variance σ_n^2 , co-channel interference (CCI) from $n_c = 1$ dominant interferer with per sample variance σ_c^2 , and adjacent channel interference (ACI) from $n_a = 2$ interferers, one from the immediate upper and lower adjacent frequency channels, each with per sample variance, σ_a^2 . The desired and interfering data symbols and noise samples are assumed to be mutually uncorrelated. Moreover, all channels have identical MPDP structures and fading characteristics. However, it is assumed that the tap coefficients of the desired and interfering channel impulse responses fade independently of each other.

Receive filtering, adaptive channel estimation, interference plus noise correlation estimation and decision-feedback equalization (DFE) are employed to mitigate the impairments of ISI, ACI, CCI and AWGN. This chapter deals exclusively with the ideal MMSE DFE while Chapter 4 considers the adaptive DFE. The forward and feedback filters of the DFE are both implemented as a TDL with $N_f T/d$ -spaced (d is an integer greater than unity) and $N_b T$ -spaced tap coefficients, respectively. A block diagram of the overall complex baseband DCR communications system is shown in Figure 3.1 for the case of a computed DFE or in Figure 3.2 for the case of a directly adapted DFE. The receive-filtered signal at the input to the adaptive channel estimator and DFE can be written as:¹

$$r(t) = \sum_{l=0}^{n_i} \sum_k a^{(l)}(kT) h^{(l)*}(t-kT) + n_r(t) \quad (3.1)$$

where $n_i = n_c + n_a$ is the total number of CCI and ACI users,

$a^{(l)}(kT)$ are the desired ($l = 0$), co-channel ($l = 1, \dots, n_c$) and adjacent

1. For the sake of notational brevity, the superscript (0) for the desired ($l = 0$) data and CIR is usually implicit, i.e., $a(kT) = a^{(0)}(kT)$ and $h(t) = h^{(0)}(t)$.

channel ($l = n_d + 1, \dots, n_i$) data symbols,

$h^{(l)}(t)$ are the overall desired ($l = 0$) and interfering ($l > 0$) CIRs,

$n_r(t)$ is the complex-valued receive-filtered noise

and $*$ is the complex conjugate transpose operator.

In general, each of the interfering CIR, $h^{(l)}(t - \tau_l)$, $l > 0$, has a random delay, τ_l , relative to the desired ($l = 0$) CIR, which is assumed to be zero here. This is the worst case in terms of the receiver's ability to differentiate between the desired and interfering data. In fact, if the CIR and interfering CIRs are identical (e.g., same MPDP and no fading), then the receiver cannot distinguish between the desired and interfering data because these signals have the same Karhunen-Loève expansion and are indistinguishable in signal space [133]. However, the above assumption is acceptable in a typical DCR system since the CIR and interfering CIRs are generally different (even though they have the same MPDP) because they experience mutually random fading which essentially results in a random relative delay between the main cursors of the desired and interfering data streams. In case there is an additional random or fixed relative delay between the desired and interfering channel responses, the receiver should be able to differentiate more effectively the data streams and thereby achieve better interference suppression [126].

3.2.1 Signal to Interference plus Noise Ratios

In the T/s -spaced system, the transmit and receive filters are scaled so that the overall desired and interfering CIRs and the signal, $s(t)$, at the output of the actual multipath fading channel have unit variance, $\sigma_s^2 = 1$. Thus, at the output of the multipath fading overall channel (i.e., at the input to the equalizer), the average (over the ensemble of fading channels) signal to noise ratio (SNR), signal to total interference ratio (SIR), total interference to noise ratio (INR) and signal to total interference plus noise ratio (SINR) can be written respectively as:²

$$SNR = \frac{\sigma_d^2 \sum_m \left| f_t\left(\frac{mT}{s}\right) \right|^2 E \left[\sum_m \left| g_m\left(\frac{kT}{s}\right) \right|^2 \right]}{\sigma_n^2} = \frac{\sigma_s^2}{\sigma_n^2} = \frac{1}{\sigma_n^2} \quad (3.2)$$

2. The additive noise variance is stationary while the user data variances are cyclostationary with a period of T . Thus, the instantaneous user data powers vary periodically within a baud interval [65],[98].

Figure 3.1 Complex baseband DCR communications system employing a computed MMSE DFE receiver

Figure 3.2 Complex baseband DCR communications system employing a directly adapted DFE receiver

$$SIR = \frac{\sigma_d^2 \sum_m \left| f_t \left(\frac{mT}{s} \right) \right|^2 E \left[\sum_m \left| g_m \left(\frac{kT}{s} \right) \right|^2 \right]}{(n_c \sigma_c^2 + n_a \sigma_a^2) \sigma_d^2 \sum_m \left| f_t \left(\frac{mT}{s} \right) \right|^2 E \left[\sum_m \left| g_m \left(\frac{kT}{s} \right) \right|^2 \right]} = \frac{1}{n_c \sigma_c^2 + n_a \sigma_a^2} = \frac{1}{\sigma_i^2} \quad (3.3)$$

$$INR = \frac{(n_c \sigma_c^2 + n_a \sigma_a^2) \sigma_d^2 \sum_m \left| f_t \left(\frac{mT}{s} \right) \right|^2 E \left[\sum_m \left| g_m \left(\frac{kT}{s} \right) \right|^2 \right]}{\sigma_n^2} = \frac{\sigma_i^2}{\sigma_n^2} \quad (3.4)$$

and

$$SINR = \frac{\sigma_d^2 \sum_m \left| f_t \left(\frac{mT}{s} \right) \right|^2 E \left[\sum_m \left| g_m \left(\frac{kT}{s} \right) \right|^2 \right]}{(n_c \sigma_c^2 + n_a \sigma_a^2) \sigma_d^2 \sum_m \left| f_t \left(\frac{mT}{s} \right) \right|^2 E \left[\sum_m \left| g_m \left(\frac{kT}{s} \right) \right|^2 \right] + \sigma_n^2} = \frac{1}{\sigma_i^2 + \sigma_n^2} \quad (3.5)$$

where $E[\]$ is the expectation operator with respect to the random channel processes and σ_n^2 , σ_c^2 , σ_a^2 , σ_i^2 and $\sigma_d^2 = 1$ are the AWGN, CCI, ACI, total interference and normalized QPSK data symbol variances, respectively.

Furthermore, the ratio of the energy per bit (which is half the energy per QPSK symbol) to the noise power spectral density is:

$$\frac{E_b}{N_0} = \frac{\sigma_d^2 T \sum_m \left| f_t \left(\frac{mT}{s} \right) \right|^2 E \left[\sum_m \left| g_m \left(\frac{kT}{s} \right) \right|^2 \right]}{2 \sigma_n^2 T} = \frac{\sigma_i^2}{2 \sigma_n^2} = \frac{SNR}{2} \quad (3.6)$$

For a cellular system, the average (over the ensemble of fading channels) SINR can also be written as:

$$SINR = \frac{1}{SCCIR^{-1} + SACIR^{-1} + SNR^{-1}} \quad (3.7)$$

where the average signal to CCI ratio (SCCIR) and the signal to ACI ratio (SACIR) can be expressed as $1/(n_c \sigma_c^2)$ and $1/(n_a \sigma_a^2)$, respectively.

The SCCIR is a function of the number of co-channel interferers, n_c , and the CCI variance, σ_c^2 , as seen by the receiver. In a cellular environment, the CCI variance is a function of the transmission path loss exponent, γ (typically valued from 2 to 4), and the CCI reduction (or co-channel reuse) factor, which can be written as [64]:

$$q_c = \frac{d_c}{r} \quad (3.8)$$

where d_c is the minimum distance between co-channel cell sites (i.e., base stations) and r is the (hexagonal) cell radius.

In general, the average received signal power as a function of the propagation distance is governed by an inverse power law:

$$\sigma_c^2 = q_c^{-\gamma} \quad (3.9)$$

Thus, the nominal SCCIR can be written as:

$$SCCIR = \frac{1}{n_c \sigma_c^2} = \frac{1}{n_c} \left(\frac{r}{d_c} \right)^{-\gamma} = \frac{q_c^\gamma}{n_c} \quad (3.10)$$

This SCCIR definition is considered nominal in the sense that it is not averaged over the spatial distribution of the mobiles (MS) in their respective cells. Instead, the desired MS is assumed (as a worst case) to be located at its cell boundary at a distance r from the desired base station (BS). Furthermore, the dominant co-channel interferer MS (e.g., uplink) or BS (e.g., downlink) is assumed (as a nominal case) to be at an average distance d_c from the receiver at the BS (e.g., uplink) or MS (e.g., downlink), respectively.

It is known that the frequency reuse factor or cluster size (i.e., number of (hexagonal) cells per cluster), K , can be written in terms of the co-channel reuse factor, q_c , [64],[76]:

$$K = \frac{q_c^2}{3} \quad (3.11)$$

where the cluster size satisfies the following constraint for positive integers i and j :

$$K = i^2 + ij + j^2 \quad (3.12)$$

Thus, the SCCIR can also be related to the cell cluster size:

$$SCCIR = \frac{(3K)^{\gamma/2}}{n_c} \quad (3.13)$$

On the other hand, the SACIR is a function of the number of adjacent channel interferers, n_a , the ACI propagation attenuation variance, σ_{ap}^2 , and the ACI filter attenuation factor, σ_{af}^2 , with the overall ACI variance being:

$$\sigma_a^2 = \sigma_{ap}^2 \sigma_{at}^2 \quad (3.14)$$

Similar to the CCI case, the ACI propagation attenuation variance can be written as:

$$\sigma_{ap}^2 = q_a^{-\gamma} \quad (3.15)$$

where the ACI reduction factor is defined as:

$$q_a = \frac{d_a}{r} \quad (3.16)$$

with d_a being the minimum distance between adjacent and desired channel cell sites.

If the ACI is assumed to originate from the immediately adjacent (hexagonal) cell, then nominally $d_a = \sqrt{3}r$ and $q_a = \sqrt{3}$. However, a worst case is when the ACI and desired signals are both at the desired cell edge, resulting in $q_a = 1$ and implying that the SACIR does not depend on the propagation distance, with $\sigma_{ap}^2 = 1$.

Unlike the CCI case, the SACIR also depends on a filter attenuation component as represented by the factor, σ_{af}^2 , which is a function of the adjacent channel spacing, f_s (or the normalized (by $1/T$) channel spacing, $\tilde{f}_s = f_s T$), and the transmit and receive filter responses and excess bandwidth. An expression, as formulated in the frequency domain, for the filter attenuation factor of the ACI due to either the upper or lower immediately adjacent channel spilling into the desired channel bandwidth is:

$$\sigma_{af}^2 = \frac{P_{aci}}{P_{desired}} = \frac{\frac{1}{T} E \left[\int_{f_s - (1+\epsilon)/2T}^{(1+\epsilon)/2T} |F_t^{(l)}(f, \epsilon)|^2 |G^{(l)}(f)|^2 |F_r^{(0)}(f, \epsilon)|^2 df \right]}{\frac{2}{T} E \left[\int_0^{(1+\epsilon)/2T} |F_t^{(0)}(f, \epsilon)|^2 |G^{(0)}(f)|^2 |F_r^{(0)}(f, \epsilon)|^2 df \right]} \quad (3.17)$$

where $G^{(l)}(f)$ is the transfer function of the actual desired ($l = 0$) and adjacent ($l > n_c$) multipath fading channels

and $F_t^{(l)}(f, \epsilon)$ and $F_r^{(0)}(f, \epsilon)$ are the transfer functions of the transmit and receive filters, respectively.

Specifically, the SRRC filter frequency response, $F(f, \epsilon)$, with $100\epsilon\%$ excess bandwidth, is expressed as:

$$F(f, \epsilon) = \begin{cases} T & 0 \leq f < \frac{(1-\epsilon)}{2T} \\ \frac{T}{2} \left[1 - \sin\left(\frac{\pi}{2\epsilon}(2Tf - 1)\right) \right] & \frac{(1-\epsilon)}{2T} \leq f < \frac{(1+\epsilon)}{2T} \\ 0 & f \geq \frac{(1+\epsilon)}{2T} \end{cases} \quad (3.18)$$

Furthermore, the transfer functions of the transmit SRRC filters can be expressed as:

$$F_t^{(l)}(f, \epsilon) = F_t^{(0)}(f - nf_s, \epsilon) = F(f - nf_s, \epsilon) \quad (3.19)$$

with integer $n = \pm 1$ for immediately adjacent channels.³

Without loss of generality, the actual multipath fading channels are assumed to have unit average energy, i.e., $E[|g^{(l)}|^2] = 1$. Thus, the expression for σ_{af}^2 due to either the upper or lower immediately adjacent channel depends only on the filter frequency response, $F(f, \epsilon)$, and is a function of the filter excess bandwidth, ϵ , and the channel spacing, f_s :

$$\sigma_{af}^2 = \frac{\int_{f_s - (1+\epsilon)/2T}^{(1+\epsilon)/2T} |F(f - f_s, \epsilon)|^2 |F(f, \epsilon)|^2 df}{2 \int_0^{(1+\epsilon)/2T} |F(f, \epsilon)|^4 df} \quad (3.20)$$

For the particular case with 100% excess bandwidth SRRC transmit and receive filtering, the ACI filter attenuation factor can be obtained (from Equation (3.18) and Equation (3.20)) as:

$$\sigma_{af}^2(f_s) = \frac{1}{6} (2 - \tilde{f}_s) (2 + \cos(\pi \tilde{f}_s)) + \frac{1}{2\pi} \sin(\pi \tilde{f}_s) \quad (3.21)$$

For the case of $n_a = 2$ (due to the upper and lower immediate adjacent channels), the SACIR is expressed as:

$$SACIR = \frac{1}{n_a \sigma_a^2} = \frac{1}{2 \sigma_{ap}^2 \sigma_{af}^2} = \frac{q_a^\gamma}{2 \sigma_{af}^2} \quad (3.22)$$

However, in the more general case ($n_a > 2$) where more than just the immediately adjacent channels contribute to the ACI, the overall ACI has to be summed up appropriately:

3. There is an implicit mapping between l and n for the adjacent channel responses.

$$SACIR = \frac{1}{N_a(f_s)} = \frac{q_a^2}{2 \sum_{n=1}^{N_a(f_s)} \sigma_{af}^2(nf_s)} \quad (3.23)$$

where the number of contributing (upper or lower) 100% excess bandwidth adjacent channel interferers is [94]:

$$N_a(\tilde{f}_s) = \begin{cases} \text{int}(\frac{2}{\tilde{f}_s}) & \tilde{f}_s \leq 2 \\ 0 & \tilde{f}_s > 2 \end{cases} \quad (3.24)$$

where $\text{int}(x)$ is the integer part of x .

In addition, when considering greater than 100% excess bandwidth (i.e., $B_r > 1/T$ or $\bar{B}_r = B_r T > 1$ when normalized by $1/T$) receive filtering with 100% excess bandwidth (i.e., $B_t = 1/T$ or $\bar{B}_t = B_t T = 1$) SRRC transmit filtering, the formula for σ_{af}^2 will be different from that in Equation (3.21). In this case, the transfer function of the transmit filter is:

$$F_t^{(t)}(f, \epsilon) = F_t^{(0)}(f - f_s, \epsilon = 1) = F(f - f_s, \epsilon = 1) \quad (3.25)$$

while the transfer function of the receive filter is:

$$F_r^{(0)}(f, \epsilon) = F(\frac{f}{B_r}, \epsilon = 1) \quad (3.26)$$

Therefore, σ_{af}^2 is a function of both the receive filter bandwidth, B_r , and the channel spacing, f_s :

$$\sigma_{af}^2 = \frac{\int_{f_s - 1/T}^{\bar{B}/T} |F(f - f_s, \epsilon = 1)|^2 |F(\frac{f}{B_r}, \epsilon = 1)|^2 df}{2 \int_0^{1/T} |F(f, \epsilon = 1)|^2 |F(\frac{f}{B_r}, \epsilon = 1)|^2 df} \quad (3.27)$$

where the contributing ACI normalized (by $1/T$) bandwidth is defined as:

$$\bar{B} = \min(\bar{B}_t = 1, \bar{B}_r) \quad (3.28)$$

The receiver actually encounters the ACI over a normalized bandwidth of $\bar{B} = \bar{B}_r$. However, it is assumed that the effective bandwidth spans only the desired signal spectrum. Thus, the receiver should have sufficient capability to deal with the additional ACI present beyond the desired signal bandwidth.

For the particular case with 300% excess bandwidth (i.e., $\bar{B}_r = 2$) receive filtering, the ACI filtering attenuation factor can be expressed (from Equation (3.27)) as:

$$\sigma_{af}^2(\hat{f}_s) = \frac{1}{2(8+3\pi)} \{ 6 + 3\pi(2 - \hat{f}_s) + 8\cos(\frac{\pi\hat{f}_s}{2}) + 2\cos(\pi\hat{f}_s) + \sin(\pi\hat{f}_s) \} \quad (3.29)$$

Figure 3.3 displays the SACIR, assuming $q_a = 1$, as a function of the normalized channel spacing, $\hat{f}_s = 1$ to 2, for the two cases of $\bar{B}_r = 1$ (see Equation (3.21)) and $\bar{B}_r = 2$ (see Equation (3.29)). Curve (1) corresponds to $T/4$ -spaced 100% excess bandwidth SRRC transmit and receive filtering with $\bar{B}_t = \bar{B}_r = \bar{B} = 1$ while curve (2) corresponds to $T/8$ -spaced 100% excess bandwidth SRRC transmit and 300% excess bandwidth receive filtering with $\bar{B}_t = 1$, $\bar{B}_r = 2$ and $\bar{B} = \min(\bar{B}_t, \bar{B}_r) = 1$.

3.2.2 System Capacity

For an FDMA digital communications system, the spectral efficiency, η_f , in units of bit/s/Hz, can be expressed as [65],[98]:

$$\eta_f = \frac{R_s \log_2 M}{f_s} = \frac{R_b}{f_s} \quad (3.30)$$

where $R_s = 1/T$ is the symbol rate,

$R_b = 1/T_b$ is the bit rate ($R_b = 2R_s$ for QPSK),

M is the alphabet size ($M = 4$ for QPSK)

and f_s is the channel spacing (i.e., bandwidth per radio frequency (rf) channel).

Under the constraint of zero tolerance of ACI, the spectral efficiency ranges from a value of 2 to 1 for QPSK modulation with 0% (Nyquist bandwidth) and 100% excess bandwidth, respectively. In general, the spectral efficiency depends on the SACIR which is a function of the channel spacing, filter response and excess bandwidth. However, if some of the ACI can be cancelled by the receiver, e.g., a fractionally spaced equalizer, then the spectral efficiency can improve by allowing a reduction in the channel spacing while maintaining a desired performance [29],[44],[50],[95].

Figure 3.3 SACIR as a function of the normalized channel spacing, assuming a worst case ACI reduction factor.

For a cellular system, the spatial efficiency, η_s , in units of cluster/#cells, can be expressed as the inverse of the frequency reuse factor, K :

$$\eta_s = \frac{1}{K} \quad (3.31)$$

Consequently, the spatial efficiency depends on the SCCIR which is a function of the frequency reuse factor. Note that the maximum spatial efficiency is unity when $K = 1$, i.e., when all available rf channels are reused in every cell. Furthermore, the overall spectral efficiency of a cellular system, η_c , in units of bit/s/Hz/cell can be expressed as a product of η_f and η_s :

$$\eta_c = \eta_f \eta_s = \frac{R_b}{f_s K} \quad (3.32)$$

In addition, this spectral efficiency of a cellular system can be transformed into a radio capacity by considering the total available bandwidth of the system, B_T , (or equivalently, the total bit rate the system can support). Thus, the radio capacity of the cellular system, C_r , in units of #rf channels/cell, can be expressed as [63],[64]:

$$C_r = \frac{B_T}{f_s K} = \frac{N_c}{K} = \frac{3N_c}{q_c^2} = \frac{3N_c}{SCCIR^{2/\gamma}} \quad (3.33)$$

where $N_c = B_T/f_s$ is the total number of available rf channels.

For a TDMA digital cellular system which can support n_s TDMA user time slots per rf channel, the overall user capacity, in units of #users/cell, can be written as:

$$C_s = n_s C_r \quad (3.34)$$

Furthermore, this cellular capacity or efficiency can be converted to an absolute user capacity by expressing a cell in terms of its geographical area, e.g., in units of km^2 . Thus, the overall efficiency of a cellular system can be stated in units of bits/s/Hz/ km^2 or #users/Hz/ km^2 . The physical size of a cell also has ramifications on the communication quality of the system. For instance, as the cell size decreases, the overall cellular system capacity increases at the expense of increased system complexity such as more complicated handoff procedures. Moreover, the radio propagation characteristics will vary with cell size [17],[47].

Of relevance in this thesis, a reduction in the cell size (without a corresponding reduction in the transmitted signal power) can increase user capacity not only through the ostensibly larger number of available cells, but also through better receiver performance since it will be observed later that equalization of inter-user interference is more effective under lower SIR and higher SNR values representative of the smaller cell geometries.

Following from Equation (3.33), a normalized (by the total available bandwidth or equivalently, the total supportable bit rate) radio capacity, in units of #rf channels/Hz/cell (if bandwidth frequencies are in units of Hz), can be expressed as:

$$\frac{C_r}{B_T} = \frac{1}{f_s K} \quad (3.35)$$

The results presented in this chapter are in terms of the normalized (by the symbol rate, $1/T$) channel spacing, $\bar{f}_s$. Thus, the normalized radio capacity, $\bar{C}_r$, in units of #rf channels/bandwidth (in units of the symbol rate, $1/T$)/cell, can be defined as:

$$\bar{C}_r = \frac{C_r}{B_T T} = \frac{1}{\bar{f}_s K} \quad (3.36)$$

Multiplying the normalized radio capacity, $\bar{C}_r$, by the total available normalized (by the symbol rate, $1/T$) system bandwidth results in the radio capacity, C_r , (#rf channels/cell).

For a given level of total interference, the radio capacity is affected by ACI via the channel spacing, f_s , and by the CCI via the frequency reuse factor, K . Thus, there exists a trade-off between the degree of ACI and CCI in determining the optimum radio capacity and the resulting system performance in terms of average BER or outage probability. Moreover, the influence of changes in the channel spacing and frequency reuse factor on the radio capacity can be expressed by the total differential of the (normalized) radio capacity:

$$d\bar{C}_r = \frac{\partial \bar{C}_r}{\partial \bar{f}_s} d\bar{f}_s + \frac{\partial \bar{C}_r}{\partial K} dK \quad (3.37)$$

Substituting Equation (3.36) and the partial derivatives of the normalized radio capacity with respect to $\bar{f}_s$ and K into Equation (3.37) yields the following expression for the relative change in the normalized radio capacity:

$$\frac{d\bar{C}_r}{\bar{C}_r} = - \left(\frac{d\bar{f}_s}{\bar{f}_s} + \frac{dK}{K} \right) \quad (3.38)$$

The negative sign in the first-order derivatives of Equation (3.38) implies that a reduction in f_s (at the expense of incurring more ACI) and/or K (at the expense of incurring more CCI) results in an increase in the radio capacity. Furthermore, the use of a fractionally spaced equalizer with a sufficiently wide bandwidth can potentially mitigate the additional interference introduced to the receiver [29],[44],[51],[58],[95],[95]. Thus, an increase in radio capacity may be realized while maintaining a desired receiver performance.

Since the second-order derivatives of $\bar{C}_r$ with respect to f_s and K are positive, the relative change in radio capacity for a given change in f_s and/or K is greater at lower channel spacings and/or frequency reuse factors. Subsequent simulation results will demonstrate that the ability of the fractionally spaced equalizer to suppress interference is best when the interference power is relatively strong. Fortunately, this situation typically occurs when the channel spacings and/or frequency reuse factors are small.

As an example with a fixed average amount of CCI due to a fixed frequency reuse factor (i.e., $dK = 0$), the change in the normalized radio capacity, $\delta\bar{C}_r$, as a function of the change in normalized channel spacing, $\delta\tilde{f}_s$, can be expressed as (using Equation (3.38)):

$$\frac{\delta\bar{C}_r}{\bar{C}_r} = 1 - \frac{(\tilde{f}_s + \delta\tilde{f}_s)}{\tilde{f}_s} \quad (3.39)$$

The MMSE DFE performance for this scenario is considered in Section 3.5.4 on page 75. In addition, the average BER and outage performance of a system employing a directly adapted RLS DFE for this scenario is considered in Section 5.3.5 on page 194.

3.3 Receiver Design

3.3.1 Stationary Inputs: Wiener Filter Approach

As discussed earlier, the impairments to reliable DCR communications include ISI due to the time-dispersive channel and transmit/receive filtering, time-varying CIR due to the fading channel, ACI and CCI from other cellular users and AWGN from the thermal noise at the receiver front-end. The focus of this thesis is the design of an adaptive filter (e.g., equalizer) at the receiver to overcome these impairments introduced along the transmission path. It is assumed that the adaptive filter is linear in the sense that its output is a linear combination of the samples of its input signal, although some of these samples may

include detected symbols from a non-linear decision device. One such example is the decision feedback equalizer (DFE) with an FIR/TDL structure. In this case, the DFE is linear in its filtering of its input signal but is non-linear in its structure which includes feedback filtering of decision symbols [48],[52],[65],[98],[100].

The objective of the adaptive structure is to filter the corrupted received signal so as to minimize the effects of the channel impairments. The filter attempts to extract information about an earlier transmitted data symbol, $a((k-D)T)$, (delayed by DT time units by the equalizer structure and the overall channel and filter response), from its input data vector, $r(kT)$, consisting of present and previous receive-filtered samples of the channel output signal and possibly previously detected data symbols (in the case of the DFE). The adaptive filter minimizes the channel impairments in order to find the “best” estimate of the transmitted data, $\hat{a}((k-D)T)$. The approach is to choose the statistically optimum filter which yields the minimum mean squared error (MMSE) at its output. For the case of a stationary input signal, this corresponds to the Wiener filter. On the other hand, if the input signal is non-stationary (e.g., due to a fading channel) then the Wiener solution varies with time and the filter needs to adapt to the changing statistics of its input signal.

In order to find the Wiener solution to the filtering problem, *a priori* information about the statistics of the received signal at the input to the filter is required. Specifically for the DCR system considered in this thesis, this *a priori* knowledge consists of the statistics of the CIR, interference and noise processes. For the Wiener filter, the first and second order statistics, i.e., mean and correlation functions or matrices, are sufficient. Typically for DCR communications, no *a priori* knowledge of the above random processes is available at the beginning of a TDMA burst. Thus, an initial training period is required for the filter to learn this statistical information and to converge to the Wiener (i.e., MMSE) solution. Furthermore, in the multipath fading DCR environment the channel properties (e.g., desired and interfering CIRs) are time-varying. Consequently, the optimum Wiener filter is also time-varying. Therefore, an adaptive algorithm is required for the filter to track the time-variations of its input signal in order to maintain its MMSE operating point.

3.3.2 Non-stationary Inputs: Direct vs. Indirect Wiener Filter Update

In the above discussion on the adaptive Wiener filter, convergence to the MMSE filter during training-aided learning mode and maintenance of the MMSE filter during decision-directed tracking mode can be accomplished by two distinct methods. Specifically, the

filter coefficients can be adapted (i.e., estimated and tracked) directly without having to acquire explicit estimates of the random CIR and interference plus noise processes. An alternative approach is to update the filter coefficients indirectly by first estimating and tracking the time-varying CIR and interference plus noise and using this explicit knowledge (viz., first and second order statistics) of these random processes to compute the Wiener filter coefficients [30],[69],[135].⁴ For the particular system investigated in this thesis, the adaptive filter is a DFE implemented as an FIR TDL.

Both of the above-mentioned adaptation approaches can be implemented with similar adaptive algorithms. For instance, if a typical TDMA burst length is only about 100 symbols, then to ensure an adequately high data throughput, the initial training sequence is restricted to about 15 symbols [24]. This short training period precludes the use of simple stochastic gradient training algorithms like the least mean square (LMS) algorithm for either of the above approaches. In the presence of noise, LMS equalizers or channel estimators of typical size take on the order of 100 symbol periods to converge during training, while the convergence time of a deterministic least squares (LS) equalizer or channel estimator is an order of magnitude less, within about twice the length of the adaptive filter [48],[52],[65],[98],[100]. Thus, for either adaptation approach, an LS-type algorithm (e.g., recursive least squares (RLS) or block least squares (BLS)) should be used for initial convergence during the training period. Assuming the channel time variations are slow during the training period, the two LS adaptation approaches have similar performance.

During the data mode under sufficiently slow time-varying channel conditions, the direct approach of equalizer tracking can be accomplished by either the RLS or LMS algorithms. While the RLS equalizer has better tracking performance than the LMS equalizer, even an RLS equalizer can fail to track properly under fast fading conditions (e.g., when the ratio of the maximum Doppler fade rate to symbol rate, $f_{dm}T > 10^{-4}$) [27]. Thus, for DCR fading conditions that can be more than an order of magnitude worse than this example, the approach of directly tracking the equalizer coefficients is potentially inadequate.

Alternatively, for the indirect approach employing channel tracking, the LMS algorithm can perform just as well as the more complex RLS algorithm [26]. This stems from the

4. Note that this so-called (in this thesis) indirect approach of CIR and interference plus noise estimation followed by Wiener filter computation is referred by some as a direct approach, e.g., direct matrix inversion (DMI) [83].

fact that the stationary input (transmitted data) samples to the CIR estimator are uncorrelated, unlike the equalizer whose non-stationary input (received data) samples are correlated. While the RLS algorithm has the advantage of insensitivity to the eigenvalue spread of its input data correlation matrix, the LMS algorithm does not possess this quality, thus explaining the poorer performance of the LMS equalizer [48],[52],[63],[98],[100]. However, the LMS CIR estimator does not have to contend with correlated input signals and consequently, its tracking performance rivals that of the RLS CIR estimator.

The channel tracking problem is intrinsically less difficult than the equalizer tracking problem. The main reason for this is the fact that the forward equalizer coefficients, $\underline{w}_f(kT)$, can vary significantly faster than channel coefficients, $\underline{h}(kT)$. That is, a small change in the channel coefficients can lead to huge changes to the forward equalizer coefficients. This is especially apparent when the channel coefficients are small (i.e., in a deep fade) since the function of the forward (i.e., linear) part of the DFE is essentially to invert the channel so as to produce minimal precursor ISI (i.e., satisfy Nyquist's criterion of zero ISI in the precursor) [19].

The magnified sensitivity of the forward DFE coefficients to the CIR can also be seen in the Wiener solution where the part of the autocorrelation matrix inverse which is proportional to the forward DFE coefficients contains channel autocorrelation terms. Thus, small channel coefficients lead to large forward DFE coefficients, while small changes (or rate of change) in channel coefficients may lead to large changes (or rate of change) in forward DFE coefficients. Since steady state tracking (i.e., lag) error is proportional to the time variation in the filter coefficients (also proportional to the number of filter coefficients and inverse of LMS step size), the CIR tracking lag error can be significantly less than the DFE tracking lag error.

For LMS tracking, the steady state error has another fluctuation ("self") noise component which is proportional to the channel noise variance, number of filter coefficients and the LMS step size. Thus the trade-off between self noise and tracking noise leads to an optimum value of the LMS step size [48],[52],[65],[98],[100]. Moreover, overall adaptation performance improves with decreasing number of filter coefficients, channel noise variance or channel time variance. The channel noise and time variances are determined by the nominal conditions of the physical DCR medium. However, the number of filter coefficients can be chosen as a design parameter.

For the example of CIR estimation, the choice of the number of T -spaced CIR tap coefficients, N_c , requires a trade-off between CIR modelling accuracy and adaptation speed. In addition, the transmission filter bandwidth also determines the ISI span of the overall CIR. Specifically, 25% SRRC filtering requires a CIR model of $N_c = 6$ while a relatively wider transmission bandwidth of 100% SRRC requires only $N_c = 3$. For IS-54 DCR which specifies 35% SRRC filtering, some have suggested improving the CIR adaptation performance by reducing N_c to a value of 2 at the expense of decreased CIR modelling accuracy [59]. In order to exploit the cyclostationary nature of the CCI, a relatively large transmission bandwidth (e.g., at least 100% excess bandwidth) is required. In this case, $N_c = 3$ is sufficient to model the CIR accurately. While the use of relatively large transmission bandwidths enhances the CCI nature and reduces the ISI span, it does suffer the cost of increased ACI which has to be mitigated so as not to compromise the performance gains achieved by accounting for the CCI nature and shorter ISI span. Assuming a fixed channel spacing so as to maintain spectral efficiency, then the ACI must be reduced via filtering, e.g., with a fractionally spaced equalizer.

Thus, for the case of narrowband TDMA communications in a DCR environment where rapid and deep fades are likely to be encountered, the indirect approach of computing the MMSE DFE from LMS-tracked CIR and interference plus noise estimates is a possible receiver adaptation method. While the above discussion has suggested several candidate adaptation approaches, it has not shown whether such approaches can yield adequate adaptive equalization performance under severe time-varying ISI and interference conditions. In an indoor wireless environment where the time variation in the channel is significantly slower and the interference levels can be quite severe, it will be shown later in Chapter 4 that the direct approach of adapting the DFE coefficients yields better performance. The advantage of adapting the DFE coefficients directly stems from the fact that this approach does not require explicit estimates of the CIR and interference correlation.

3.4 Finite-Complexity MMSE DFE Formulation

The computation of the optimum (in the MMSE sense) discrete-time, finite-length DFE coefficients in the presence of CCI and noise has been formulated previously [1],[57],[80]. The equivalent result for a continuous-time, infinite-length DFE in interference is presented in [94],[95]. This result in interference can be seen as an extension of the case in noise by incorporating the interference autocorrelation into the overall received signal

autocorrelation matrix [19],[28],[100],[107]. A related formulation of the MMSF filter coefficients is the case of a discrete-time, finite-length diversity combiner and DFE in noise [68],[69]. The equivalent continuous-time, infinite-length diversity combiner and DFE is derived in [8]. In this section, the discrete-time, finite-length DFE in the presence of inter-user interference (i.e., ACI and CCI) and ambient noise is of interest.

The DFE consists of a fractionally T/d -spaced TDL forward filter, $\underline{w}_f$, with N_f tap coefficients and a T -spaced TDL feedback filter, $\underline{w}_b$, with N_b tap coefficients (see Figure 3.4).⁵ For example, a $T/2$ -spaced forward filter has $d = 2$ tap coefficients per symbol period. Furthermore, the forward filter tap spacing needs to be finer for the case with relatively wide receiver excess bandwidths greater than 100%. In general, the input signal to the forward filter is the d/T -sampled received signal, $r(kT - mT/d + \theta)$, while the input signal to the feedback filter is the synchronously detected data symbol, $\hat{a}((k - D)T)$ with an equalizer delay of D symbol periods. The receiver sampling phase, $0 \leq \theta < T$, is assumed to be zero since it is known that a fractionally spaced equalizer's performance is fairly insensitive to this sampling phase in the presence of ISI [100]. Furthermore, it has also been shown that this same insensitivity to receiver sampling phase is also true in the presence of cyclostationary CCI [1].

The $1/T$ -decimated signal at the DFE output (i.e., input to the decision device), $y(kT)$, can be expressed compactly in vector notation as:

$$y(kT) = \underline{w}^* \underline{x}(kT) \quad (3.40)$$

where $*$ denotes the conjugate transpose operator, while the input signal vector spanning the DFE forward and feedback TDL filters is:

$$\underline{x}(kT) = \left[\underline{r}^*(kT) \ \hat{\underline{a}}^*(kT) \right]^* \quad (3.41)$$

with the received signal vector spanning the forward TDL:

$$\underline{r}(kT) = \left[r(kT) \ r(kT - \frac{T}{d}) \ \dots \ r(kT - (N_f - 1) \frac{T}{d}) \right]^T \quad (3.42)$$

5. In the general case of a non-stationary environment, the optimum (MMSE) DFE tap coefficients are time-varying. This time dependence of the DFE tap coefficients is implicit in the notation.

Figure 3.4 Fractionally spaced DFE receiver as a tapped delay line structure

and the feedback signal vector spanning the feedback TDL:

$$\hat{\underline{d}}(kT) = [\hat{d}((k-D-1)T) \ \hat{d}((k-D-2)T) \ \dots \ \hat{d}((k-D-N_b)T)]^T \quad (3.43)$$

while the DFE tap coefficient vector is:

$$\underline{w} = [\underline{w}_f^* \ \underline{w}_b^*]^* = \left[[w_0 \ w_1 \ \dots \ w_{N_f-1}]^* \ [w_{N_f} \ w_{N_f+1} \ \dots \ w_{N_f+N_b-1}]^* \right]^T \quad (3.44)$$

Assuming correct decisions (i.e., $\hat{d}(kT) = d(kT)$), the Wiener solution is obtained by minimizing the DFE output MSE at each k th symbol instant with respect to the DFE tap coefficients [48],[52],[65],[98],[100]:

$$MSE = E[|y(kT) - d((k-D)T)|^2] \quad (3.45)$$

For a given CIR and second order statistics of the interference plus noise, the optimum (in the MMSE sense) DFE tap coefficients are:

$$\underline{w}_o = R^{-1} \underline{p} \quad (3.46)$$

where the $(N_f + N_b) \times (N_f + N_b)$ Hermitian autocorrelation matrix is:

$$R = E[\underline{x}(kT) \underline{x}^*(kT)] \quad (3.47)$$

and the $(N_f + N_b) \times 1$ cross-correlation vector is:

$$\underline{p} = E[\underline{x}(kT) d^*((k-D)T)] \quad (3.48)$$

with $E[\]$ denoting expectation with respect to the noise and user (i.e., desired and interfering) data while the desired and interfering CIRs are assumed fixed.

Assuming unit variance desired data symbols, $\sigma_d^2 = 1$, the corresponding MMSE is:

$$MMSE = 1 - \underline{p}^* R^{-1} \underline{p} = 1 - \underline{p}^* \underline{w}_o \quad (3.49)$$

Under the assumption that the noise, desired and interfering data samples are mutually uncorrelated, the autocorrelation matrix and cross-correlation vector can be written as:

$$R = \begin{bmatrix} R_{rr} & R_{ra} \\ R_{ra}^* & R_{aa} \end{bmatrix} = \begin{bmatrix} R_{rr} & R_{ra} \\ R_{ra}^* & I \end{bmatrix} \quad (3.50)$$

and

$$\underline{p} = \begin{bmatrix} p_{ra} \\ p_{aa} \end{bmatrix} = \begin{bmatrix} p_{ra}^* \\ p_{aa}^* \end{bmatrix} \quad (3.51)$$

where the (i, j) th $(i, j = 0, 1, \dots, N_f - 1)$ element of the $(N_f \times N_f)$ Hermitian sub-matrix for received signal correlation is:

$$\begin{aligned} R_{rr}(i, j) &= R_h(i, j) + R_c(i, j) + R_n(i, j) \\ &= R_h(i, j) + R_c(i, j) + R_a(i, j) + R_n(i, j) \\ &= \sum_k h(kT - i\frac{T}{d}) h^*(kT - j\frac{T}{d}) + \sigma_c^2 \sum_{l=1}^{n_c} \sum_k h^{(l)}(kT - i\frac{T}{d}) h^{(l)*}(kT - j\frac{T}{d}) \\ &\quad + \sigma_a^2 \sum_{l=n_c+1}^{n_c+n_a} \sum_k h^{(l)}(kT - i\frac{T}{d}) h^{(l)*}(kT - j\frac{T}{d}) + \sigma_n^2 R_n((i-j)\frac{T}{d}) \end{aligned} \quad (3.52)$$

where the first term, $R_h(i, j)$, is the desired CIR autocorrelation,

the second and third terms, $R_c(i, j) = R_c(i, j) + R_a(i, j)$, are the CCI and ACI autocorrelation, respectively,

$R_n((i-j)T/d)$ is the T/d -spaced SRRC receive filter autocorrelation function and σ_n^2 , σ_c^2 and σ_a^2 are the additive noise, CCI and ACI variances, respectively.

The k -indexed summation is over all (up to N_c) non-zero T -spaced desired or interfering CIR correlation products. In particular, each product term of the (i, j) th element consists of a cross-correlation between the $(i(s/d)) \text{MOD}(s)$ and $(j(s/d)) \text{MOD}(s)$ indexed T -spaced phases of the T/s -spaced CIRs, where the value of $(x) \text{MOD}(y)$ is the remainder from the division of x by y . Note that the T/d -spaced DFE forward filter resolves d out of s T -spaced channel phases. Furthermore, each p th $(p = 0, \dots, d-1)$ phase T -spaced sub-matrix, R_{rrp} , is Toeplitz while the overall autocorrelation matrix, R_{rr} , comprising the d interleaved sub-matrices, R_{rrp} , is only Hermitian.

Furthermore, the (i, j) th $(i = 0, 1, \dots, N_f - 1$ and $j = 0, 1, \dots, N_b - 1)$ element of the $(N_f \times N_b)$ sub-matrix for received signal and feedback data correlation is:

$$R_{ra}(i, j) = h((D+1)T - i\frac{T}{d} + jT) \quad (3.53)$$

and the i th $(i = 0, 1, \dots, N_f - 1)$ element of the $(N_f \times 1)$ sub-vector for received signal and feedback data cross-correlation is:

$$p(i) = p_{eq}(i) = h(DF + 1_d)F_d \quad (3.54)$$

Moreover, $R_{dd} = I$ is an $(N_b \times N_b)$ identity matrix and $p_{dd} = 0$ is an $(N_b \times 1)$ zero-valued vector.

3.4.1 Optimum Signal to Distortion Ratio at the DFE Output

The MMSE at the DFE output gives a measure of the optimum equalizer performance in terms of its residual output error corresponding to an implicit signal level which is usually normalized to unit variance. A more meaningful performance measure which explicitly incorporates the average power of both the signal and its error is the optimum signal to distortion ratio (SDR) at the DFE output.

Under the MMSE criterion which jointly minimizes all distortion present, e.g., ISI, inter-user interference (i.e., ACI and CCI) and/or noise, the MMSE equalizer generally produces a biased output signal since not all of the ISI is removed. That is, a unit variance input signal to the equalizer results in a reduced variance output signal. This is noticeable under low SINR conditions when the MMSE equalizer has to compensate more (relative to the ISI) for the high level of interference and/or noise. This is in contrast to an equalizer designed with the zero-forcing (ZF) criterion which achieves zero output data bias by disregarding the interference and/or noise while concentrating on removing all the ISI. This ZF criterion is satisfied when the main cursor is unity while the precursor and post-cursor are zero. That is, in this zero ISI case, an equalized overall CIR has unit amplitude at the desired sampling instant and zero crossings at all other symbol instants.

The MMSE expression in Equation (3.49) consists of average power components at the equalizer output due to the above-mentioned data bias, σ_B^2 , and residual distortion σ_D^2 , which is composed of residual additive noise, inter-user interference and precursor ISI:⁶

$$MMSE = \sigma_B^2 + \sigma_D^2 \quad (3.55)$$

where, assuming unit variance data samples, $\sigma_d^2 = 1$, the bias variance is:

$$\sigma_B^2 = |1 - h_p(0)|^2 = MMSE^2 \quad (3.56)$$

6. It is assumed that the DFE feedback filter is of sufficient length to remove fully the post-cursor ISI without interference/noise enhancement if correct decisions are fed back.

with the main ($k = 0$) cursor at the desired data sampling instant written as:

$$h_e(0) = \underline{w}_f^* h_r(DT) = \underline{w}_n^* p = 1 - MMSE \quad (3.57)$$

where the k th cursor, including the overall CIR and DFE forward filter, is:

$$h_e(k) = \underline{w}_f^* h_r((k+D)T) \quad (3.58)$$

and the part of the time-reversed T/d -spaced CIR spanning the DFE forward filter is:

$$h_r((k+D)T) = \left[h((k+D)T) \dots h((k+D)T - (N_f - 1)\frac{T}{d}) \right]^T \quad (3.59)$$

Therefore the optimum SDR at the equalizer output can be expressed as a ratio of the average desired signal power in the main cursor, σ_S^2 , to the average distortion power due to residual precursor ISI, ACI, CCI and/or noise, σ_D^2 :

$$SDR_o = \frac{\sigma_S^2}{\sigma_D^2} = \frac{\sigma_S^2}{MMSE - \sigma_B^2} = \frac{|h_e(0)|^2}{MMSE - |1 - h_e(0)|^2} \quad (3.60)$$

By using Equation (3.57), the optimum SDR can be expressed as [98]:

$$SDR_o = \frac{1 - MMSE}{MMSE} = \frac{1}{MMSE} - 1 \quad (3.61)$$

Note that if the residual ISI and noise enhancement are negligible, then the SDR_o is essentially equal to the average SINR at the input to the equalizer. Under high SINR conditions, the SDR_o is effectively equal to the inverse of the MMSE. Under low SINR (or SDR_o) conditions, the MMSE is close to unity (i.e., 0 dB). In particular, the SDR_o is approximately 0 dB when the MMSE is about -3 dB.

3.5 MMSE DFE SDR Performance

In general, the DFE performance gains in the presence of inter-user interference (e.g., ACI and CCI) relative to the equivalent stationary noise case can be significant at low SIR and high SNR values where interference is the dominant disturbance. In this case, a properly designed DFE is able to filter not only the ISI but also the inter-user interference. Consequently, the DFE SDR can be significantly greater than the SIR. In fact, if the DFE filters out all of the ISI, ACI and CCI without noise enhancement, then the SDR is equal to the SNR. That is, the performance is fundamentally limited by the stationary noise rather than

by the cyclostationary interference. Thus, if the dominant disturbance is indeed interference, then it is better to model it as such rather than as stationary noise since the DFE has an opportunity to improve its performance in such interference

Using Equation (3.49) and Equation (3.61), the optimum SDR at the output of the DFE is calculated under ideal conditions where the quasi-static desired and interfering CIRs are known. The desired and interfering channels are static for each trial but are random realizations over an ensemble. The ideal MMSE performance is averaged over 1000 such random channel realizations and the resulting optimum average SDR is computed to yield an upper bound on DFE performance in a multipath fading environment subjected to interuser interference and/or additive noise. Note that the equivalent additive noise is modeled as stationary Gaussian receive-filtered noise whose spectral properties are determined by the receive filter response, e.g., 100% excess bandwidth raised cosine power spectrum.

The average SDR could be expressed as the average of the SDR for each channel realization in the ensemble, where the averaging is over the desired and interfering CIRs and noise statistics. Because of the random fading experienced by both the desired and interfering CIRs, there will be situations in the presence of negligible noise when the interfering CIRs, but not the desired CIR, will be in a deep fade (more than 30 dB below its nominal power level) resulting in a very high SDR which would tend to bias the average SDR towards this higher value. By averaging the MSE (essentially the inverse of the SDR) instead, the problem with the skewing of the average SDR to high values is avoided since the MSE is upper bounded by unity (since the MMSE DFE is designed so that its output signal is as close as possible, in the MSE sense, to the unit variance desired signal). Thus, the large fluctuations in the interference power level is averaged out by the MSE. Moreover, if the DFE can filter out some or all of the interference and the effects of the multipath fading, then the SDR is not upper bounded by the SIR but rather by the SNR.

The DFE is denoted as $DFE(N_f, N_b)$ where N_f is the number of T/d -spaced forward filter taps and N_b is the number of T -spaced feedback filter taps. Furthermore, the equalizer delay in the forward filter is assumed to be optimally chosen. For example, in the presence of interference and multipath fading represented by the MPDP-1 profile, the reference tap coefficient is chosen to be at the middle of the DFE forward filter. In addition, the transmit and receive filters are specified by their normalized (by $1/T$) bandwidths B_t and $\bar{B}_r$, respectively. For instance, the transmit and receive filter excess bandwidths are

either SRRC 25% (i.e., $\hat{B}_t = \hat{B}_r = 0.625$) or 100% (i.e., $\bar{B}_t = \bar{B}_r = 1$) while the receive filter excess bandwidth can also be 300% (i.e., $\bar{B}_t = 1$ and $\bar{B}_r = 2$).

3.5.1 In the Presence of a Dominant Interferer and Negligible Noise

This section investigates the optimum SDR performance of a DFE in the presence of one dominant CCI user and negligible noise. For the sake of brevity, the DFE performance curves for the equivalent stationary noise case are not included in this section (see reference [71]). Instead, a short discussion is presented here. In the presence of additive noise and zero interference, the DFE SDR is generally a couple of dB below that of the SNR due to residual multipath-induced ISI and noise enhancement. Moreover, this SDR performance in additive noise is typically a couple of dB worse than the equivalent SRRC 25% one dominant CCI user case at low SIR and only marginally worse than the high SIR case. In fact, at high disturbance levels, the 25% excess bandwidth DFE performance in CCI is better than the 100% excess bandwidth DFE performance in stationary noise, despite the reduced ISI span of the latter case. However, at low disturbance levels where ISI is dominant, the 25% excess bandwidth DFE performance in CCI is a little worse than the 100% excess bandwidth DFE performance in stationary noise. More importantly, as the pulse filter excess bandwidth widens to 100%, the cyclostationarity in the CCI increases and the DFE performance improves substantially at all SIR values.

The transmitted pulse bandwidth's effect on DFE performance is more significant in the presence of interference than in stationary noise [71]. In the noise case, an increase in the filter excess bandwidth improves the DFE performance by reducing the ISI span, which is only evident at high SNR where the noise is negligible. For the case of an optimally delayed DFE(5,2) with a MPDP-1 channel, an increase in filter excess bandwidth from 25% to 100% at low SNR values of less than 10 dB yields practically the same DFE performance while at high SNR values of over 20 dB the DFE performance gains are a couple of dB in SNR. On the other hand, an increase in the filter bandwidth in the CCI case results in DFE performance gains at all SIR values. This is because the DFE performance improves with an increase in filter bandwidth due to not only the reduced ISI span but also the increase in cyclostationarity in the CCI which allows the DFE to suppress the interference more effectively. Specifically, for the case of an optimally delayed DFE(5,2) with a MPDP-1 channel, an increase in filter excess bandwidth from 25% to 100% yields a DFE performance gain of at least 10 dB in SIR at all levels of CCI [71].

As in the choice of the pulse filter bandwidth, the choice of the equalizer delay also affects the DFE performance more in the presence of interference than in noise [71]. In general, the optimum equalizer delay will be different in interference than in noise since the DFE forward filter in the former case has the task of filtering not only ISI but also inter-user interference. For instance, without any knowledge of the co-channel data stream, the DFE feedback filter cannot process any of the CCI and can only suppress the post-cursor ISI. Depending on the channel MPDP and transmit and receive filter responses (which in turn determine the overall CIR and co-CIR), the optimum equalizer delay minimizes the MSE by accounting for the CCI and precursor ISI. In the case of the MPDP-1 channel which corresponds to a symmetric CIR and co-CIR, the optimum equalizer delay in CCI is such that the main reference tap is at around the middle of the DFE forward filter. As another example, the MPDP-2 channel which has significant post-cursor but minimal precursor energy results in an optimum equalizer delay of zero which allows the DFE forward filter to process more effectively the co-CIR post-cursor.

The results detailed in the next three figures in this section represent the scenario when only 25% and 100% filter excess bandwidths are considered, requiring a $T/2$ -spaced system response ($s = 2$) and a $T/2$ -spaced DFE forward filter ($d = 2$). The disturbance is due to one dominant CCI user, with its average power level being specified by the SIR, while the noise is negligible at $\text{SNR} = 100$ dB. The two multipath channels considered are MPDP-1 and MPDP-2, corresponding to $T/2$ -spaced average relative tap powers of 0, 0, 0 dB and 0, -5, -15 dB, respectively.

Figure 3.5 displays the DFE SDR performance in CCI as a function of the receiver input SIR, pulse filter excess bandwidth and number of $T/2$ -spaced taps in the optimally delayed DFE forward filter. At all SIR values, the performance of the DFE is much better with 100% excess bandwidth filtering than with 25% excess bandwidth filtering because of the improved suppression of the CCI which exhibits more cyclostationarity at wider bandwidths. This performance difference is more pronounced at low SIR values and for a longer DFE forward filter which can mitigate more effectively the CCI with enhanced cyclostationarity. At low SIR values where CCI is the dominant impairment, the DFE(3,2) performance improvement at 100% relative to 25% excess bandwidth filtering is about 5 dB in SIR while the DFE(9,2) performance improvement is almost 20 dB in SIR.

Figure 3.5 DFE($N_f, 2$) SDR as a function of receiver input SIR, filter excess bandwidth and DFE forward filter length, with MPDP-1 channel multipath profile

Although the DFE performance gain is monotonic with its forward filter length, there is a diminishing rate of improvement with increasing filter length at any pulse filter bandwidth. Furthermore, the DFE performance gain in CCI relative to the equivalent case in noise is not nearly as significant at 25% filter excess bandwidth as at 100% [71]. This is because at small filter excess bandwidths the CCI possesses little cyclostationarity for the fractionally spaced DFE to exploit. In fact, at zero percent filter excess bandwidth the CCI behaves just like stationary noise [12],[35],[95]. This is somewhat apparent even at 25% excess bandwidth as the DFE(3,2) performance at high SIR values exhibits a performance ceiling due to residual precursor ISI, similar to the noise case. On the other hand, the DFE performance gains with 100% filter excess bandwidth in CCI relative to the equivalent noise case can be almost 20 dB in SIR under high disturbance levels.

Figure 3.6 shows the DFE SDR performance in CCI as a function of the receiver input SIR, pulse filter excess bandwidth and number of T -spaced taps in the DFE feedback filter. As is evident from both Figure 3.5 and Figure 3.6, the DFE(5,2) performance gains with 100% filter excess bandwidth over that with 25% are about 13 and 11 dB in SIR under high and low disturbance levels, respectively. On the other hand, the DFE performance gains with 25% filter excess bandwidth in CCI with respect to the equivalent noise case are just a couple of dB in SIR and marginal under high and low disturbance levels, respectively [71]. Furthermore, Figure 3.6 indicates that with 25% and, to a lesser extent, 100% excess bandwidth filtering in negligible CCI (i.e., high SIR), the linear equalizer (viz., DFE(5,0)) suffers a performance ceiling due to residual post-cursor ISI just as in the noise case. Thus in CCI as in noise, an optimally delayed DFE requires a feedback filter to remove the post-cursor ISI caused by previously detected symbols. The optimum number of DFE feedback taps required is about two in this case.

Figure 3.7 displays the 100% excess bandwidth DFE SDR performance in CCI as a function of the receiver input SIR and DFE forward filter size for two different multipath channels, MPDP-1 and MPDP-2 (see Section 3.2 on page 28). The MPDP-1 and MPDP-2 multipath profiles correspond to optimum DFE delays of $(N_f - 1)/2$ and 0, respectively. The general observations of the DFE performance for each multipath channel are similar to those for the case of the MPDP-1 channel with 100% excess bandwidth filtering as shown in Figure 3.5. Since the MPDP-2 channel has a less severe multipath profile than the MPDP-1 channel, the DFE performs better in the former channel as it can filter effectively not only the reduced amount of ISI but also the CCI.

Figure 3.6 DFE(5,Nb) SDR as a function of receiver input SIR, filter excess bandwidth and DFE feedback filter length, with MPDP-1 channel multipath profile

Figure 3.7 DFE(Nf,2) SDR as a function of receiver input SIR, multipath channel profile and DFE forward filter length for 100% pulse filter excess bandwidth.

This performance difference between the two channels is marginal for short equalizers, e.g., DFE(3,2), whose ability to suppress CCI is limited, especially under low SIR conditions. However, a longer equalizer yields greater performance improvements for the less severe MPDP-2 channel. Specifically, the DFE(9,2) performance gains with the MPDP-2 channel over that of the MPDP-1 channel are about 13 and 8 dB in SIR under high and low CCI levels, respectively. In fact, a shorter DFE(7,2) with the MPDP-2 channel performs better than a longer DFE(9,2) with the MPDP-1 channel by about 7 and 3 dB in SIR under high and low CCI levels, respectively. Furthermore, a DFE(5,2) with the MPDP-2 channel and a DFE(7,2) with the MPDP-1 channel have similar performance. Note that these results are for the case of known static channels in CCI. In practice when the fading channels are time-varying, the potential for greater implicit diversity with the MPDP-1 channel will change the relative performance results. In particular for the case in noise, the MPDP-1 channel might yield better performance due to its additional implicit diversity.

3.5.2 In the Presence of Equal-Strength Interferers and Noise

This section considers the optimum SDR performance of a DFE with different receive filter bandwidths in the presence of interferers of equal strength and additive stationary noise. The MPDP-1 channel is used. At the receiver input, the SNR ranges from 0 dB to 80 dB while the SIR is specified at 0 or 10 dB and the corresponding SINR values are listed in Table 3.1. The results in this section reflect DFE performance under four disturbance cases comprising different types of interference and whose total interference power level is specified by the SIR: case (1) - one dominant CCI user (denoted as "0 ACI & 1 CCI"); case (2) - 2 equal strength ACI users, one from each of the upper and lower adjacent channels (denoted as "2 ACI & 0 CCI"); case (3) - 3 equal strength interferers including those in both cases (1) and (2) (denoted as "2 ACI & 1 CCI") and case (4) - no interference but rather an equivalent amount of additive stationary noise. The receiver is subjected, in addition to the above-mentioned disturbances for the different cases, to additive stationary ambient noise whose level is specified by the receiver input SNR.

The normalized transmit bandwidth is $\bar{B}_t = 1$, while the normalized receive bandwidth is $\bar{B}_r = 1$ or 2. The time resolutions (e.g., ranging from $T/2$ to $T/8$ spacing) of the fractionally spaced system response and DFE forward filter are specified appropriately for the interference and bandwidth conditions (i.e., the number of ACI users and the receiver bandwidth). Furthermore, a worst-case ACI reduction factor, $q_u = 1$, is assumed so that

the SACIR is independent of the propagation conditions and is solely determined by filtering parameters such as the normalized channel spacing and the receive filter bandwidth. For specified interference power levels (i.e., SIR = 0 or 10 dB) and a particular disturbance case and receiver bandwidth, the corresponding values of SCCIR, SACIR and normalized channel spacing, $\tilde{f}_c$, are listed in Table 3.2.

The DFE performance of each of the four disturbance cases are compared at the same overall SIR in the presence of ambient noise specified by the receiver input SNR. While each case has the same amount of interference plus noise, the DFE performance can vary significantly among the cases due to the different statistical and spectral properties of the disturbance. Figure 3.8 and Figure 3.9 display the optimum DFE SDR performance as a function of the receiver input ambient noise and interference powers under the various disturbance scenarios for a normalized receiver bandwidth of $B_r = 1$ and 2, respectively. The $\tilde{B}_r = 1$ case employs a $T/2$ -spaced DFE(5,2) while the $B_r = 2$ case employs a $T/4$ -spaced DFE(9,2). The DFE forward filter has a span of two symbol periods.

These results indicate that under negligible noise conditions (i.e., high SNR), there is a significant performance advantage in interference (cases 1 to 3) over that in noise (case 4). Note that this same conclusion was made in the previous section for the case when the disturbance is due to one dominant CCI user. Here, it is further observed that for a given overall level of interference, an increase in the number of interferers tends to weaken the effectiveness of the DFE to combat the disturbance and to reduce the performance advantage in interference over that in noise. This has been previously observed for an infinite-complexity non-adaptive equalizer [29],[94],[95]. In fact, the largest performance gains in interference over that in noise occur under the scenario with one high power interferer. Specifically, comparing curves (5) and (8) in Figure 3.8 shows that this SDR performance gain can be up to 12 dB for a disturbance as strong as the desired signal (i.e., SIR = 0 dB). Moreover, comparing curves (5) and (6) shows that the SDR performance gain with just one CCI relative to the case with 2 ACI is over 8 dB.

When the overall interference power is reduced relative to the desired signal power, this performance advantage is also reduced. For instance, comparing curves (1) and (4) in Figure 3.8 shows that the SDR performance gain in one dominant interferer over that in noise is only about 9 dB for the case of SIR = 10 dB. Furthermore, comparing curves (1) and (2) shows that the performance gain in one CCI over that in 2 ACI is marginal.

SNR [dB]	SINR [dB]	
	SIR = 0 dB	SIR = 10 dB
0	- 3.01	- 0.41
10	- 0.41	6.99
20	- 0.04	9.59
30	- 0.01	9.96
≥ 40	0.00	10.0

Table 3.1 Receiver input SINR for various additive noise and interference powers

Equal Strength Interferer Case	SIR = 0 dB				SIR = 10 dB			
	SCCIR [dB]	SACIR [dB]	f_N		SCCIR [dB]	SACIR [dB]	f_N	
			$\bar{B}_r = 1$	$B_r = 2$			$\bar{B}_r = 1$	$B_r = 2$
(1) 0 ACI & 1 CCI	0	∞	n/a	n/a	10	∞	n/a	n/a
(2) 2 ACI & 0 CCI	∞	0	0.641	0.848	∞	10	1.247	1.741
(3) 2 ACI & 1 CCI	4.77	1.76	0.798	1.074	14.77	11.76	1.312	1.821

Table 3.2 Receiver input SCCIR, SACIR and normalized channel spacing for various cases of equal strength interference and normalized receiver bandwidths, assuming a worst-case ACI reduction factor

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Figure 3.8 DFE(5,2) SDR as a function of receiver input SNR and SIR with a 100% excess bandwidth receive filter

Figure 3.9 DFE(5,2) SDR as a function of receiver input SNR and SIR with a 300% excess bandwidth receive filter

An increase in the receiver bandwidth can allow the DFE more freedom with which to mitigate the interference present at the receiver [94],[95]. This is evident from the high SNR results in Figure 3.9 which demonstrate that an increase in receiver bandwidth, while hardly influencing the DFE performance in noise, does significantly improve DFE performance in interference. Specifically with $\bar{B}_r = 2$ in Figure 3.9, comparing curves (5) and (8), for $\text{SIR} = 0$ dB, reveals an SDR performance gain of 21 dB, almost 10 dB greater than the same case with $\bar{B}_r = 1$. In fact, for the case of $\text{SIR} = 10$ dB, the gain is even greater at about 26 dB while being about 17 dB greater than the same case with $B_r = 1$.

The improvement in the DFE performance in interference with wider receiver bandwidths can also be observed in Figure 3.10 and Figure 3.11, each of which explicitly compares such performance improvements with receiver bandwidth for the case of $\text{SIR} = 0$ dB and 10 dB, respectively. Interestingly, when increasing from $\bar{B}_r = 1$ to $B_r = 2$, the disturbance case (2) with 2 ACI achieves the largest SDR performance gains of 13 and 19 dB for $\text{SIR} = 0$ and 10 dB, respectively. Moreover, by comparing curves (6) with (8) and (2) with (4) with $\bar{B}_r = 2$ in Figure 3.9, this case with 2 ACI achieves DFE SDR performance gains of 16 and 27 dB for $\text{SIR} = 0$ and 10 dB, respectively. In fact, for the case of $\text{SIR} = 10$ dB and $\bar{B}_r = 2$, this disturbance case (2) actually performs about 9 dB better than disturbance case (1) which has one CCI. An increase in receiver bandwidth in this particular situation is more beneficial for the case where there are 2 ACI and no CCI. This is not necessarily the case in general as the relative performance depends on the receiver bandwidth and the power level, statistical and spectral properties of the interference.

From the results for any of the disturbance cases, it is clear that as the SNR decreases such that the additive stationary noise becomes significant, the DFE performance tends to converge to a relatively poor operating point which is dominated by the noise. That is, the DFE performance is indeed noise-limited. Specifically, a performance advantage in interference over that in additive noise is not possible, regardless of the SIR value, if $\text{SNR}/\text{SIR} = \text{INR} < 0$ dB. This performance threshold can be observed in Figure 3.8 to Figure 3.11 as the point where the four disturbance case curves for a given SIR value converge. Putting it another way, as long as there is sufficient interference (i.e., $\text{SIR} < 30$ dB), a performance gain in interference over that in additive noise can be achieved only if the interference is of greater strength than the noise, i.e., $\text{INR} > 0$ dB. The maximum performance gain in interference over that in additive noise is achievable if $\text{INR} > 20$ or 30 dB and $\text{INR} > 50$ or 60 dB for $\bar{B}_r = 1$ and $\bar{B}_r = 2$, respectively.

Figure 3.10 DFE(5,2) SDR as a function of receiver input SNR and normalized receiver bandwidth with $SIR = 0$ dB

Figure 3.11 DFE(5,2) SDR as a function of receiver input SNR and normalized receiver bandwidth with $SIR = 10$ dB

3.5.3 In the Presence of Interferers with SIR= 10 dB and Noise

This section deals with the optimum DFE SDR performance with a normalized receiver bandwidth, $\bar{B}_r = 1$, in the presence of ACI, CCI and additive stationary noise, subject to the constraint of a fixed total interference level at SIR = 10 dB. Figure 3.12 and Figure 3.13 present the DFE SDR performance as a function of receiver input SNR and normalized channel spacing, $\bar{f}_s$, assuming a worst-case ACI reduction factor, $q_u = 1$. The results in Figure 3.12 demonstrate again that under low SNR conditions when the additive noise is the limiting impairment, the DFE SDR performance is quite poor with the normalized channel spacing and, thus, the composition of the interference, having little or no effect on the performance. However, the DFE SDR performance improves significantly under low noise conditions as the DFE is able to filter out some of the interference.

Moreover, in the case when interference is the dominant disturbance, its statistical and spectral properties, as influenced by the relative contribution of the ACI and CCI, does affect the DFE performance. For instance, the DFE SDR performance degrades and then improves as the normalized channel spacing is reduced from 2.0 (when there is no ACI with just one CCI user) to a minimum of 1.247 in this case (when there is no CCI and two ACI users which provide the required maximum interference to meet the SIR = 10 dB constraint). The best performance is typically at either the minimum (1.247) or maximum (2.0) normalized channel spacing where the overall interference is the least noise-like and the DFE is best able to filter it. On the other hand, the worst performance occurs at $\bar{f}_s \approx 1.4$ where the overall interference consisting of 2 ACI and 1 CCI users is the most noise-like.

By comparing curve (4) with those in Figure 3.13, it is again evident that under high noise conditions, there is negligible DFE performance improvement in interference over that in additive noise. However, such a DFE performance improvement does exist for SNR > 10 dB. Furthermore, this performance gain increases monotonically with the SNR, with the maximum gains achievable for SNR > 40 dB when the effect of the noise on the DFE performance becomes negligible. This result is consistent with the conclusion made in the previous section which states that the maximum DFE performance gain in interference over that in additive noise is achievable if INR > 20 or 30 dB for the $\bar{B}_r = 1$ case.

Also, note that the DFE SDR performance gain is the least (less than 3 dB) for the worst case with 2 ACI and 1 CCI (curve (3)) where the sum of the three interferers is somewhat noise-like. On the other hand, the DFE SDR performance gains are greatest (about 9 dB)

for the best cases of one CCI (curve (1)) or 2 ACI (curve (2)). Moreover, the former case is marginally better when the additive noise is negligible (i.e., $\text{SNR} > 30$ dB) while the opposite is true when there is significant additive noise (i.e., $10 \text{ dB} < \text{SNR} < 30 \text{ dB}$).

Figure 3.13 shows that the DFE SDR performance in negligible noise can vary by over 5 dB with the normalized channel spacing. Moreover, the normalized channel spacing also determines the achievable cellular radio capacity. Figure 3.14 displays the normalized radio capacity, in units of #rf channels/bandwidth (in units of the symbol rate, $1/T$)/cell (see Equation (3.36)), as a function of the normalized channel spacing. The worst-case ACI normalized radio capacity is represented by curve (2) in Figure 3.14. The transmit and receive filters are 100% excess bandwidth SRRC with normalized bandwidths of 1. Curve (1) corresponds to a nominal ACI reduction factor, $q_a = d_a/r = \sqrt{3}$ (i.e., ACI from adjacent cells) while curve (2) corresponds to a worst-case ACI reduction factor, $q_a = d_a/r = 1$ (i.e., ACI at cell edge). The ACI distance attenuation factor is q_a^γ , with a transmission path loss exponent $\gamma = 4$. The normalized channel spacing ranges from a minimum (with no CCI) of 0.702 (for case of nominal ACI reduction) or 1.247 (for case of worst-case ACI reduction) to a maximum (with no ACI) of 2.0.

Under the constraint of $\text{SIR} = 10$ dB, the radio capacity approaches zero at the minimum $\tilde{f}_s = 1.247$ as the CCI is removed by unrealistically requiring a frequency reuse factor of infinity. On the other hand, at $\tilde{f}_s = 2.0$ where the ACI is removed, the normalized radio capacity has a value of almost one half, while the DFE SDR performance is near optimum, similar to that at the minimum $\tilde{f}_s = 1.247$. Thus, it is inferred that to achieve a given DFE performance criterion, it is not recommended to reduce the CCI by increasing the frequency reuse factor (since the radio capacity decreases to an unacceptable level). Rather, it is more beneficial to avoid a significant portion of the ACI by increasing the channel spacing (at the expense of a relatively slight decrease in the radio capacity) and to employ the DFE to mitigate the remaining ACI and the significant amount of CCI.

Moreover, the maximum normalized radio capacity, with a value of almost 0.6, occurs at $\tilde{f}_s \approx 1.5$. However, this is close to the operating point ($\tilde{f}_s \approx 1.4$) at which the DFE SDR performance is worst. Thus, in order to achieve an optimum performance while trading off between the DFE SDR performance and radio capacity, the normalized channel spacing should be selected somewhere in the range of $1.5 < \tilde{f}_s < 2.0$.

Figure 3.12 DFE(5,2) SDR as a function of normalized channel spacing and receiver input SNR, for a normalized receiver bandwidth of 1 and SIR = 10 dB

Figure 3.13 DFE(5,2) SDR as a function of receiver input SNR and normalized channel spacing, for a normalized receiver bandwidth of 1 and SIR = 10 dB

Figure 3.14 Normalized radio capacity as a function of normalized channel spacing with SIR = 10 dB.

3.5.4 In the Presence of Interferers with SCCIR= 10 dB and Noise

This section considers the optimum SDR performance of a DFE in the presence of ACI, CCI and additive stationary noise, with a fixed CCI level at SCCIR = 10 dB. Contrary to the scenario in the previous section which specified that SIR = 10 dB, the scenario in this section does not require an explicit tradeoff between the ACI and CCI powers as the channel spacing is varied. The specification of SCCIR = 10 dB fixes the frequency reuse factor, or equivalently the cell cluster size, which is therefore independent of the ACI and channel spacing. This scenario with SCCIR = 10 dB illustrates the degree of DFE SDR performance degradation due to an increase in ACI resulting from a reduction in the channel spacing and the possible benefits of using a wider receiver bandwidth.

Figure 3.15 and Figure 3.16 present the DFE SDR performance as a function of the receiver input SNR and the normalized channel spacing, $\hat{f}_s$, with an assumption of a worst-case ACI reduction factor, $q_a = 1$, for a normalized receiver bandwidth, $\hat{B}_r = 1$, and 2, respectively. These results again indicate that under low SNR conditions where the additive noise is the dominant impairment the DFE SDR performance is quite poor and is independent of the ACI and CCI. In the presence of lower noise levels, the DFE SDR performance degrades monotonically with a decreasing channel spacing as more ACI is introduced into the receiver. This degradation in SDR performance by going from the scenario with no ACI to that with significant ACI (e.g., $\hat{f}_s = 1.0$) ranges from about a couple of dB (for SNR = 10 dB) to about 15 dB (for SNR = 50 dB where the noise is negligible).

This variation in the DFE performance is greater when employing a wider receiver bandwidth as in this case the DFE has more flexibility to process both the ACI and CCI. This has been observed previously for an infinite-length equalizer in a non-fading channel environment [94]. Comparing the corresponding SDR performance curves for $\hat{B}_r = 1$, and 2, when the additive noise is not the dominant impairment, shows that the use of the wider receiver bandwidth can result in better performance as long as there is sufficient spectral overlap between the ACI and desired signal, which is a function of the channel spacing. For instance, negligible performance gains are available in the range of $1.5 < \hat{f}_s < 2.0$. However, gains are achieved at smaller channel spacings due to significant spectral overlap such that the DFE can better distinguish and filter the ACI from the desired signal.

Figure 3.17 shows the DFE SDR performance for selected values of normalized channel spacing and receiver bandwidth. Under high noise conditions, all cases tend to converge to

a relatively poor performance as the dominant noise severely limits the ability of the DFE to filter the interference. However, under negligible noise conditions, the DFE performance can be quite good. In particular, the use of a wider receiver bandwidth can significantly improve the DFE performance as long as the channel spacing is small enough to ensure sufficient spectral overlap of the ACI with the desired signal. For the case when only CCI is present, comparing curves (1) and (6) shows that the DFE SDR performance gain with $\bar{B}_r = 2$ relative to that with $\bar{B}_r = 1$ is about 8 dB. Note that this performance gain comes at the expense of a reduction in the radio capacity since the channel spacing requirements for the former and latter cases are $\tilde{f}_s = 3.0$ and $\tilde{f}_s = 2.0$, respectively.

Comparing curves (3) and (6) shows that by using a wider receiver bandwidth $\bar{B}_r = 2$ while reducing the channel spacing to $\tilde{f}_s = 2.0$, i.e., going from curve (1) to (3), in order to achieve the same radio capacity performance as that of curve (6), results in an SDR performance which is about one dB worse than the case of curve (6) with $\bar{B}_r = 1$. That is, for this case when the ACI spills into the wide receiver bandwidth but does not overlap the desired signal in frequency, the use of a wider receiver bandwidth can lead to marginally worse DFE performance as the gain in the DFE's freedom to mitigate interference by using a wider receiver bandwidth does not offset the attendant increase in the ACI power.

However, further reduction in the channel spacing improves the wide-bandwidth DFE performance. Comparing curves (4) and (7) indicates that the $\bar{B}_r = 1$ and $\bar{B}_r = 2$ DFE SDR performances are the same when $\tilde{f}_s = 1.5$. Moreover, comparing curves (5) and (8) shows that the use of a wider receiver bandwidth leads to superior DFE SDR performance (e.g., a 3 dB gain) as the channel spacing is further reduced to $\tilde{f}_s = 1.0$. In this case, there is sufficient spectral overlap of the ACI with the desired signal to allow the DFE to improve its interference suppression, despite the fact that the ACI power has increased. This can result in either an increase in radio capacity through a reduction in channel spacing or the ability to operate in a noisier environment while maintaining a desired equalizer performance.

Figure 3.18 displays the normalized radio capacity as a function of the normalized channel spacing. Curve (1), for a nominal ACI reduction factor, $q_a = d_a/r = \sqrt{3}$ (i.e., ACI from adjacent cells), and curve (2), for a worst-case ACI reduction factor, $q_a = d_a/r = 1$ (i.e., ACI at cell edge), coincide since the radio capacity is independent of the ACI reduction factor. With the specification of SCCIR = 10 dB, the frequency reuse factor is fixed at $K = 1.054$. Thus, the radio capacity increases monotonically with a decreasing channel spacing (see Equation (3.36) and Equation (3.39)).

Figure 3.15 DFE(5,2) SDR as a function of normalized channel spacing and receiver input SNR, for normalized receiver bandwidth of 1 and $SCCIR = 10$ dB

Figure 3.16 DFE(5,2) SDR as a function of normalized channel spacing and receiver input SNR, for normalized receiver bandwidth of 2, and SCCIR = 10 dB

Figure 3.17 DFE(5,2) SDR as a function of receiver input SNR, normalized channel spacing and normalized receiver bandwidth, with SCCIR = 10 dB

Figure 3.18 Normalized radio capacity as a function of normalized channel spacing with SCCIR = 10 dB

3.6 SDR Probability Distribution

In Section 3.5, the MMSE DFE is computed from exact knowledge of the desired and interfering static channels and its performance is averaged over an ensemble of such random channel realizations. For TDMA communications in a quasi-static multipath fading environment where symbol errors tend to occur in bursts, it is of interest to quantify the probability that a given TDMA transmission is of unacceptable quality due to a severe channel environment. A commonly used performance measure is the outage probability which can be defined as the probability that the BER during a given TDMA transmission exceeds an outage criterion which specifies the acceptable BER. This performance measure regards the BER of a TDMA frame as a random variable whose probability distribution is conditioned on the multipath channel fading statistics [5],[8],[32],[79].

In the presence of user interference, e.g., ISI, ACI and CCI, and additive noise, the BER expression can be complicated since it has to be averaged over the probability distributions of the disturbances. However, it is clear that the BER is a monotonically decreasing or increasing function of the equalizer output (i.e., detection) SDR or MSE, respectively. As an example, a common form of the Saltzberg upper bound on the BER, which tends to be tightest for interference with Gaussian statistics, can be expressed as [106]:

$$BER < e^{-SDR} \quad (3.62)$$

While the BER is the preferred measure of system performance, the SDR can nevertheless be used to determine the outage probability. Thus, the outage probability can also be defined by the cumulative distribution function (cdf) of the SDR of a TDMA frame:

$$P_{out} = Prob(SDR_{frame} \leq SDR_{oc}) \quad (3.63)$$

where SDR_{frame} is a random variable distributed over the fading channel statistics and SDR_{oc} is the abscissa value representing the outage criterion for a data frame.

In this section, the SDR distribution over the fading channel statistics is investigated for the different disturbance scenarios as discussed in Section 3.5.2. The SDR probability distribution (cdf) over an ensemble of 1000 random channel realizations is obtained for each of the disturbance cases with interferers of equal strength, i.e., SIR = 0 and 10 dB, of Figure 3.10 and Figure 3.11, respectively. These eight cases include 100% and 300% excess bandwidth receive filtering for each of the four disturbance cases as discussed at

the beginning of Section 3.5.2. Moreover, both negligible (i.e., SNR = 60 dB) and appreciable (i.e., SNR = 20 dB) additive ambient noise cases are considered.

Figure 3.19 and Figure 3.20 depict the DFE SDR probability distributions with SIR = 0 dB under negligible (i.e., SNR = 60 dB) and significant (i.e., SNR = 20 dB) additive noise conditions, respectively. The probability distributions range from SDR = 0 to 35 dB. When a cdf curve is not plotted below a particular SDR value, this means that there is zero probability for the SDR to be in this lower range. For a given SDR value, the performance improves with a decreasing cumulative probability. While for a given cumulative probability, the performance improves with an increasing SDR abscissa value. Thus, an improvement in the SDR performance corresponds with a translation of its cdf curve downwards and to the right.

As shown in Figure 3.19 under negligible noise conditions, the performance is significantly better in interference than in an equivalent amount of additive noise, as evidenced by curves (1) and (5), corresponding to the noise case for $\bar{B}_r = 1$ and $\bar{B}_r = 2$, respectively, being at the extreme upper left. For both receiver bandwidth cases, a decrease in the number of interferers results in a performance improvement. For the $\bar{B}_r = 2$ case, comparing curves (6) to (8) for a cumulative probability of 0.1 shows that the corresponding SDR is about 1, 11 and 16 dB for the cases of "2 ACI & 1 CCI", "2 ACI & 0 CCI" and "0 ACI & 1 CCI", respectively. In other words, there is a 90% probability that the SDR is greater than 1, 11 and 16 dB for the respective disturbance cases. Comparing again curves (6) to (8) for an SDR abscissa value of 10 dB results in a cumulative probability of 0.7, 0.07, 0.01 for the respective disturbance cases. That is, the probability that the SDR is greater than 10 dB is about 30%, 93% and 99% for the cases of "2 ACI & 1 CCI", "2 ACI & 0 CCI" and "0 ACI & 1 CCI", respectively.

Since a larger receiver bandwidth can allow the DFE more freedom with which to mitigate the interference, the relative performance gains among the different disturbance cases are greater for $\bar{B}_r = 2$ [29],[94],[95]. In fact, comparing curves (4) and (7) of Figure 3.19 for an SDR abscissa value of 10 dB indicates that the "2 ACI & 0 CCI" case with $\bar{B}_r = 2$ (curve (7)) results in superior performance to the "0 ACI & 1 CCI" case with $\bar{B}_r = 1$ (curve (4)) as the cumulative probability is 0.07 for the former case as compared with 0.3 for the latter case. In this particular example, an increase in receiver bandwidth is most

beneficial for the "2 ACI & 0 CCI" disturbance case. This is evident by comparing the relative positions of the $\bar{B}_r = 1$ and $\bar{B}_r = 2$ curves for the different disturbance cases.

As also observed in Section 3.5, when the additive stationary noise becomes appreciable, e.g., with $\text{SNR} = 20$ dB, the receiver performance degrades and is limited by the noise. The outage probability performance degradation with decreasing SNR is illustrated as the cdf curves in Figure 3.20 under significant noise conditions with $\text{SNR} = 20$ dB are all shifted to the upper left from their corresponding curves in Figure 3.19 with $\text{SNR} = 60$ dB. This is also evident by comparing the curves of Figure 3.21 and Figure 3.22 which depict the DFE SDR probability distributions with $\text{SIR} = 10$ dB under negligible (i.e., $\text{SNR} = 60$ dB) and significant (i.e., $\text{SNR} = 20$ dB) additive noise conditions, respectively. Moreover, the cdf curves of Figure 3.19 and Figure 3.20 are shifted to the upper left relative to their corresponding curves of Figure 3.21 and Figure 3.22. This is because the outage performance of the former cases with $\text{SIR} = 0$ dB is inferior to that of the latter cases with $\text{SIR} = 10$ dB. The probability distributions presented in Figure 3.21 and Figure 3.22 for the $\text{SIR} = 10$ dB case have similar relative performance amongst the different disturbance cases as those presented in Figure 3.19 and Figure 3.20 for the $\text{SIR} = 0$ dB case.

Figure 3.23 and Figure 3.24 display the histograms of the DFE SDR for the "2 ACI & 0 CCI" disturbance case with $\text{SIR} = 10$ dB and $\text{SNR} = 60$ dB. Figure 3.23 corresponds to curve (3) of Figure 3.21 with 100% excess bandwidth receive filtering while Figure 3.24 corresponds to curve (7) of Figure 3.21 with 300% excess bandwidth receive filtering. These histograms are analogous to probability density functions (pdf) and are related to the respective cumulative distribution functions (cdf) as depicted in Figure 3.21.

The mean SDR values of 15.48 dB and 33.60 dB, marked as vertical lines in the histograms and specified at the legend bottoms in Figure 3.23 and Figure 3.24, respectively, are not averaged over the SDR distributions but are calculated with Equation (3.61) using the MMSE which is averaged over the ensemble of channel realizations. As discussed at the beginning of Section 3.5, this approach of calculating the average SDR from the average MMSE is employed to avoid the skewing of the mean value by large SDR values in a fading channel ensemble.

Figure 3.19 Cumulative distribution function for the SDR of an ideally computed MMSE DFE with $SIR = 0\text{ dB}$ and $SNR = 60\text{ dB}$. The probability distribution is over an ensemble of 1000 random channel realizations. Transmit filter is SRRC with 100% excess bandwidth.

Figure 3.20 Cumulative distribution function for the SDR of an ideally computed MMSE DFE with SIR = 0 dB and SNR = 20 dB. The probability distribution is over an ensemble of 1000 random channel realizations. Transmit filter is SRRC with 100% excess bandwidth.

Figure 3.21 Cumulative distribution function for the SDR of an ideally computed MMSE DFE with SIR = 10 dB and SNR = 60 dB. The probability distribution is over an ensemble of 1000 random channel realizations. Transmit filter is SRRC with 100% excess bandwidth.

Figure 3.22 Cumulative distribution function for the SDR of an ideally computed MMSE DFE with $SIR = 10\text{ dB}$ and $SNR = 20\text{ dB}$. The probability distribution is over an ensemble of 1000 random channel realizations. Transmit filter is SRRC with 100% excess bandwidth.

Figure 3.23 SDR histogram for case (2): “2 ACI & 0 CCI” with SIR = 10 dB and SNR = 60 dB. Receive filter has 100% excess bandwidth.

Figure 3.24 SDR histogram for case (2): “2 ACI & 0 CCT” with SIR = 10 dB and SNR = 60 dB. Receive filter has 300% excess bandwidth.

3.7 Summary

The optimum coefficients and corresponding output MMSE and SDR of a finite-complexity MMSE DFE have been formulated to study equalization performance in a quasi-static multipath fading environment corrupted by inter-user interference and ambient noise. Simulation results, taken over an ensemble of fading channels, demonstrate that *average SDR and outage probability performance gains in the presence of inter-user interference, e.g., ACI and CCI, relative to the equivalent stationary noise case are achievable for an ideally computed DFE in a quasi-static multipath fading environment*. As the pulse filter excess bandwidth widens, the interference cyclostationarity increases and the DFE performance can improve substantially. Such performance gains are greatest at high SNR and low SIR values where the additive thermal noise is negligible and the interference is the dominant disturbance.

Simulation results also confirmed that the DFE performance in a multipath fading environment is fundamentally limited by the stationary noise rather than by the cyclostationary interference. In fact, *the DFE performance gain in the presence of interference degrades considerably as the SNR decreases such that the additive stationary noise becomes appreciable*. That is, the DFE response which minimizes the effect of interference in negligible ambient noise is not as effective in the presence of appreciable noise since the DFE can introduce noise enhancement. Thus, the DFE in the presence of significant interference and noise tends to compromise its interference suppression capability in its attempt to minimize both noise enhancement and interference.

If the $\text{INR} < 0$ dB for the system considered in this thesis, then a performance advantage in interference over that in an equivalent amount of additive stationary noise is not possible, regardless of the SIR value. Putting it another way, *as long as there is sufficient interference (i.e., $\text{SIR} < 30$ dB), such a performance gain can be achieved only if the interference is of greater strength than the ambient thermal noise, i.e., $\text{INR} > 0$ dB*. This performance advantage improves with increasing INR and is maximum when $\text{INR} > 20$ dB and $\text{INR} > 50$ dB for $\bar{B}_r = 1$ and $\bar{B}_r = 2$, respectively.

On the other hand, there can be a significant performance advantage in interference with negligible noise, i.e., high SNR, over that in an equivalent amount of stationary noise. It is further observed that for a given overall level of interference, an increase in the number of interferers tends to weaken the effectiveness of the DFE. However, an increase in the

receiver bandwidth can allow the DFE more freedom with which to filter the interferers present at the receiver. This benefit of using a wide receiver bandwidth can apply to ACI suppression.

In the presence of ACI and negligible noise, the DFE SDR performance degrades monotonically with a decreasing channel spacing as more ACI is introduced into the receiver. However, it has been confirmed that *the use of a wider receiver bandwidth can result in improved performance as long as there is sufficient spectral overlap between the ACI and the desired signal*. That is, the degree of performance improvement with an increase in receiver bandwidth depends on the channel spacing.

As the channel spacing decreases and the ACI spills into the wide receiver bandwidth but does not overlap the desired signal in frequency, the use of a wider receiver bandwidth can lead to marginally worse DFE performance. This is because the DFE's increased freedom to deal with interference by using a wider receiver bandwidth does not offset the attendant increase in the ACI power. However, *the wide-bandwidth DFE performance improves, relative to that with a narrower receiver bandwidth, when the channel spacing is reduced such that there is sufficient spectral overlap of the ACI with the desired signal to allow the DFE to distinguish and thus suppress the ACI more effectively, despite the fact that the ACI power has increased*.

Under low SNR conditions when the additive noise is the limiting impairment, the DFE SDR performance in interference is quite poor. In this case, the performance is relatively insensitive to the channel spacing and, thus, the composition of the interference. When there is negligible additive noise, the channel spacing can significantly influence both the DFE performance and the achievable radio capacity of a cellular system. *The ability of a wide-bandwidth DFE to mitigate interference can result in either an increase in radio capacity through a reduction in channel spacing or the ability to operate in a noisier environment while maintaining a desired equalizer performance*.

Chapter 4 Adaptive DFE Strategies and Convergence Performance

4.1 Introduction

This chapter considers three adaptive equalization schemes which require training at the beginning of a TDMA burst to learn the quasi-static multipath fading channel statistics. The first scheme entails a computed MMSE DFE which utilizes estimates of the desired CIR and interference plus noise autocorrelation. In contrast to this indirect approach, i.e., channel parameter estimation followed by equalizer computation, is the direct approach where the equalizer coefficients are adapted recursively without explicit estimation of the channel parameters. This latter approach includes the second and third schemes in which a DFE is directly adapted with the LMS or RLS algorithms, respectively.

Section 4.2 presents the computed MMSE DFE and details the LS CIR and interference plus noise autocorrelation estimators which are employed in this first scheme. A similarly computed DFE employing CIR estimation has been considered previously for multipath fading channels [30],[68],[85],[135]. A related approach is the computed linear equalizer using estimates of the autocorrelation matrix and cross-correlation vector [11]. Note that these previous studies considered a computed equalizer in the presence of only noise.

Section 4.2.3 derives an expression for the upper bound of the excess MSE of the computed fractionally spaced MMSE DFE. The excess MSE of a computed DFE has been obtained by assuming that the CIR estimation error is additional noise to the receiver [85]. Moreover, the excess MSE due to CIR estimation error for a short-block detector has been analyzed [18]. The excess MSE of a filter directly adapted with the RLS algorithm has also been formulated [26]. All these related results did not consider inter-user interference.

Section 4.2.3.1 uses the excess MSE analysis to illustrate some inherent differences in equalizer performance in the presence of interference and in noise. Furthermore, Section 4.2.4 employs an MSE perturbation analysis to demonstrate the sensitivity of the computed MMSE equalizer performance to small inaccuracies in the autocorrelation matrix and cross-correlation vector of the Wiener solution. The eigenvalue spread of the autocorrelation matrix is shown to play a central role in the equalizer performance.

Section 4.3 discusses the second scheme of a directly adapted LMS DFE. A well-known MSE convergence analysis for an LMS filter in the presence of additive Gaussian noise has been shown to apply also to an LMS equalizer in the presence of interference. As corroboration of simulation results, this analytical result is used to calculate the LMS MSE convergence in the presence of interference and/or noise. These simulation and calculation results, averaged over an ensemble of quasi-static fading channels, show that the LMS DFE MSE convergence in interference is slower by orders of magnitude than that in an equivalent amount of stationary noise. This slower MSE convergence in interference is due in part to a much lower DFE MMSE in interference than in stationary noise. The slow convergence of the LMS equalizer in interference has been previously observed in a non-fading subscriber loop channel [94]. Also, it has been observed that a normalized LMS DFE in a multipath fading CDMA environment converges faster than an LMS DFE [3].

Section 4.3.2.1 demonstrates that the slow MSE convergence of the LMS DFE in the presence of interference is also due to the existence of one or two very slow-decaying significant eigenmodes. It is observed that the eigenvalue spread of the input autocorrelation matrix is much larger in the presence of interference than that in noise. Thus, the sensitivity of the LMS algorithm to the eigenstructure of the input signal has a much greater negative effect on the MSE convergence in the presence of interference than in noise.

Section 4.4 discusses the third scheme of a directly adapted RLS DFE. Simulation and calculation results, averaged over an ensemble of quasi-static fading channels, show that the MSE convergence in the presence of interference of the RLS DFE is several orders of magnitude faster than that of the LMS DFE. Moreover, it has been confirmed that the RLS DFE's insensitivity to the eigenstructure of its input signal applies not only in the presence of noise but also in interference. In particular, its MSE convergence in the presence of interference is practically as fast as that in the presence of stationary noise.

Section 4.5 uses the computed DFE excess MSE result of Section 4.2.3 to demonstrate that the directly adapted RLS DFE performs better than the computed MMSE DFE. Thus, the directly adapted RLS DFE is the preferred adaptive equalizer for a typical interference-limited quasi-static fading channel environment.

Section 4.6 discusses finite-precision effects of a digitally implemented adaptive fractionally spaced equalizer in the presence of inter-user interference.

4.2 Computed MMSE DFE Employing CIR and Interference plus Noise Autocorrelation Estimates

4.2.1 CIR Estimation

Due to the channel variations caused by movement relative to base stations, other mobile users and obstacles, a DCR user can encounter a considerably different channel environment at each TDMA burst. Consequently, a different optimum MMSE DFE has to be estimated at the start of each time slot. Furthermore, in the case of a time-varying fading channel, the optimum DFE solution also has to be tracked during the time slot. As discussed in Section 3.4 on page 47, the computation of the MMSE DFE tap coefficients requires updated estimates of the overall desired CIR, h , and the autocorrelation function/matrix of the interferer CIRs, $h^{(l)}$, ($l > 0$), plus additive receive-filtered noise, $n_r(t)$. These estimates can be obtained with the assistance of a known training sequence which is typically positioned at the beginning of the TDMA burst before the data sequence.

As a basis for performance comparison and to gain insight into the CIR and interference plus noise autocorrelation estimation schemes, it is assumed that the multipath fading channel is quasi-static and is not time-varying during a TDMA burst. If such time-invariant estimates are sufficiently accurate, then the ideal MMSE performance can be obtained. However, the required training overhead to achieve this optimum MMSE performance may be impractical. Furthermore, in the DCR scenario with a time-varying channel, inaccurate time-invariant estimates may not yield acceptable performance. This estimation and tracking problem in a time-varying environment will not be addressed in this thesis.

As discussed earlier, in DCR communications the dominant and performance-limiting impairment is interference rather than additive noise. When the relative transmission bandwidth is sufficiently large (e.g., 100% excess bandwidth) to allow the cyclostationary characteristics of interference to manifest, a fractionally spaced equalizer can filter the interference to achieve superior performance to the equivalent case of stationary noise. Chapter 3 has shown how the interference and noise can influence the MMSE DFE performance. In the case of the Wiener solution, the DFE autocorrelation matrix has components that are explicitly due to the interference and noise (see Equation (3.52) on page 51).

Furthermore, the interference also affects the CIR estimation process. It is known that an efficient (i.e., unbiased and minimum variance) ML CIR estimator exists in the presence

of additive white Gaussian noise (AWGN) [18],[48],[66],[135]. However, it is not immediately obvious that an ML CIR estimate can be accurately obtained in the presence of interference. If the CIR estimate is somehow biased or degraded (i.e., larger variance) in interference, then the MMSE DFE performance gain in interference relative to noise may be compromised. On the other hand, if the CIR estimator is the same (in the mean sense) in interference as in noise, with the possible difference in variance (in the mean square error sense), then the computed MMSE DFE is likely to achieve improved performance in interference than in an equivalent amount of stationary noise by explicitly accounting for the cyclostationary autocorrelation of the interference.

An unbiased CIR estimate has no knowledge of the interference. The only explicit information about the interference is contained in the interference autocorrelation matrix involved in the computation of the MMSE DFE. In order to achieve the theoretical DFE performance gain in interference relative to that in stationary noise, an accurate estimate of the interference autocorrelation matrix is required. As will be shown later, the accuracy of the interference autocorrelation matrix estimator depends on the accuracy of the CIR estimator. Consequently, the purpose of this section is not only to demonstrate the feasibility of CIR estimation in the presence of interference, but also to understand the requirements for an accurate CIR estimate. Note that even if the same unbiased CIR estimator is obtained in interference as in noise, their variances may not necessarily be the same.

The many standard adaptive methods for CIR estimation include statistical approaches such as the LMS algorithm and deterministic data fitting strategies such as RLS and BLS algorithms [48],[52],[66],[67],[98]. CIR estimation algorithms can also be classified as recursive, e.g. LMS and RLS, or block-based, e.g., BLS. The choice of a CIR estimation algorithm involves trade-offs amongst the requirements of estimation accuracy, computational complexity, convergence speed and training data overhead. As a rule of thumb, in the presence of additive noise, LS algorithms can converge to within 3 dB of the MMSE in about $2N_c$ symbol periods, where N_c is the number of filter, e.g., CIR, tap coefficients. While the LMS algorithm is computationally simpler, its convergence speed is typically an order of magnitude slower than that of an LS algorithm. This slow convergence may not be objectional for some telephone modem applications where the start-up time is short compared to the data transmission duration. However, the LMS algorithm is not suitable for DCR TDMA applications with short data bursts on the order of a hundred symbols.

4.2.1.1 BLS CIR Estimation in Interference plus Noise

A general non-linear parametric CIR estimation scheme requires *a priori* knowledge of the probability distribution, e.g., probability density function (pdf), of the received data vector, r , and the CIR parameters to be estimated, h . Specifically, such a Bayesian estimator requires knowledge of the *a posteriori* conditional pdf, $p(h|r)$. A common example is the maximum *a posteriori* (MAP) estimator, $\hat{h}_{MAP}$, which satisfies the MAP equation [48],[66],[124]:

$$\left. \frac{\partial}{\partial h} p(h|r) \right|_{h=\hat{h}_{MAP}} = 0 \quad (4.1)$$

If the parameters to be estimated are not random variables but rather deterministic unknown parameters with no *a priori* knowledge of its pdf, $p(h)$, then the maximum likelihood (ML) estimator, $\hat{h}_{ML}$, is appropriate. A necessary condition for an ML estimator is the ML equation:

$$\left. \frac{\partial}{\partial h} p(r|h) \right|_{h=\hat{h}_{ML}} = 0 \quad (4.2)$$

This is also a sufficient condition if $p(r|h)$ is uni-modal, e.g., Gaussian pdf. Note also that if $p(h)$ is constant, i.e., uniformly distributed, over the range of values of h , then the MAP and ML estimators are equivalent.

The linear BLS CIR estimator is based on such an ML estimator. For the case when the received samples in r are jointly Gaussian and uncorrelated (and therefore independent), as is the situation when the interference is AWGN, the deterministic BLS CIR estimator is the ML estimator [48],[66]. Furthermore, it is an unbiased and efficient estimator.

It is assumed that the noise, interferer and desired data samples are zero-mean, mutually uncorrelated and have variances of σ_n^2 , σ_i^2 and $\sigma_d^2 = 1$, respectively. Moreover, the desired and interferer data samples are assumed to be equally likely and uncorrelated while the SRRC receive-filtered noise samples are uncorrelated at the symbol instants. In general, the interference samples are correlated since they result from the convolution of uncorrelated data sequences with finite memory interferer CIRs. The nature of this correlation depends on the interferer CIRs. It is essentially this interference autocorrelation which is accounted for in the MMSE DFE solution (see Equation (3.52) on page 51).

While the training symbols take on only two possible QPSK symbols (i.e., $\pi/4$ -rotated BPSK), which is sufficient for CIR estimation purposes, the subsequent data symbols can assume any of the four possible QPSK symbols. The training sequence consists of N_t known symbols, of which the initial $N_c - 1$ are used to fill up the CIR memory since no assumption is made about the data symbols immediately prior to the training sequence. The portion of the desired training ($l = 0$) and interferer ($l = 1, 2, \dots, n_i$) sequences spanning the overall T/s -spaced CIRs, $h^{(l)}$, at the k th ($k = N_c - 1, N_c, \dots, N_t - 1$) observation time index can be represented by the following data vectors:²

$$\mathbf{a}_k^{(l)} = \left[a_k^{(l)} \ a_{k-1}^{(l)} \ \dots \ a_{k-N_c+1}^{(l)} \right]^T \quad (4.3)$$

BLS estimation can be used to compute each of the s interleaved T -spaced CIR estimates, h_p , with T/s phase offset, which forms the overall T/s -spaced desired CIR, h . It follows that the time-invariant p th phase ($p = 0, 1, \dots, s-1$ with T/s offset for T/s -spaced representation) T -spaced overall desired ($l = 0$) CIR, h_p , and interferer ($l > 0$) CIRs, $h_p^{(l)}$, are assumed to have N_c tap coefficients and can be represented as follows:

$$\mathbf{h}_p^{(l)} = \left[h_{0+p/s}^{(l)} \ h_{1+p/s}^{(l)} \ \dots \ h_{N_c-1+p/s}^{(l)} \right]^T \quad (4.4)$$

Furthermore, the p th T -spaced phase of the received signal vector and receive-filtered noise vector spanning the observation interval of length $N_e = N_t - N_c + 1$ can be written respectively as:

$$\mathbf{r}_p = \left[r_{N_c-1+p/s} \ r_{N_c+p/s} \ \dots \ r_{N_t-1+p/s} \right]^* \quad (4.5)$$

and

$$\mathbf{u}_{r,p} = \left[n_{r,N_c-1+p/s} \ n_{r,N_c+p/s} \ \dots \ n_{r,N_t-1+p/s} \right]^* \quad (4.6)$$

With the above definitions, the p th T -spaced phase of the conjugated received signal vector spanning the observation period can be written compactly in matrix form as:

1. $n_i = n_c + n_a$ is the total number of CCI and ACI users.

2. For the sake of notational brevity, the symbol period, T , and the desired user superscript ($l = 0$) are omitted.

$$r_p = \sum_{l=0}^n A^{(l)} h_p^{(l)} + n_{r,p} \quad (4.7)$$

where the $N_c \times N_c$ desired training ($l = 0$) and interferer ($l > 0$) data Toeplitz matrices are defined to be:

$$A^{(l)} = \begin{bmatrix} a_{N_c-1}^{(l)*} \\ a_{N_c}^{(l)*} \\ \vdots \\ a_{N_c-1}^{(l)*} \end{bmatrix} \quad (4.8)$$

The BLS estimate for the p th T -spaced desired CIR phase is that CIR vector, $\hat{h}_p$, which minimizes the following sum of squared errors during the observation interval of N_c symbols (cf. covariance window method) [18],[48],[66],[135]:

$$|\hat{r}_p - r_p|^2 = |A\hat{h}_p - r_p|^2 = \sum_{k=N_c-1}^{N_t-1} \left| \sum_{m=0}^{N_c-1} a_{k-m}^* \hat{h}_{m+p/s} - r_{k+p/s}^* \right|^2 \quad (4.9)$$

with the resulting deterministic BLS estimate of the p th T -spaced CIR phase satisfying the following set of normal equations:

$$(A^* A) \hat{h}_p = A^* r_p \quad (4.10)$$

where the sample autocorrelation matrix of the training sequence can be written as:

$$A^* A = \sum_{k=N_c-1}^{N_t-1} a_k a_k^* \quad (4.11)$$

and the sample cross-correlation vector between the training and received sequences is:

$$A^* r_p = \sum_{k=N_c-1}^{N_t-1} a_k r_{k+p/2}^* \quad (4.12)$$

In order to solve for the N_c unknown BLS CIR coefficients for the p th T -spaced phase, there has to be at least the same number of normal equations such that $\text{rank}(A) \geq N_c$; otherwise the system of equations is under-determined with an infinite number of solutions. That is, the observation period has to be at least as long as the CIR length. As long

as $N_t \geq 2N_c - 1$, the Hermitian, but not necessarily Toeplitz, matrix A^*A is invertible and a BLS estimate for the p th T -spaced CIR phase can be obtained as [18],[48],[66],[135]:

$$\hat{h}_p = (A^*A)^{-1}A^*r_p \quad (4.13)$$

where the matrix $(A^*A)^{-1}A^*$ is the Moore-Penrose pseudo-inverse of A .

Moreover, it can be shown that an efficient (i.e., minimum NTE) BLS CIR estimate can be obtained with a training sequence whose autocorrelation matrix is diagonal [48],[66]. In this case, the CIR estimation errors for each CIR tap are uncorrelated and the NTE for a T -spaced CIR estimate can be expressed as (see Appendix A on page 209):

$$NTE = \frac{(\sigma_t^2 + \sigma_n^2)N_c}{N_t - N_c + 1} \quad (4.14)$$

The BLS CIR estimator is consistent since its NTE is asymptotically zero for an increasing training sequence length. In general, better CIR estimates result from less correlated training sequences with autocorrelation matrices that are more diagonal. Intuitively, longer training sequences tend to be less correlated and thus produce better CIR estimates, assuming the CIR is time-invariant. If the CIR is fading, then a time-invariant CIR estimate can worsen with increasing training sequence length beyond a certain optimum length. Furthermore, the use of long sequences also means higher training overhead.

4.2.1.2 Simulated MNTTE Performance

The performance measure of the BLS CIR estimate, $\hat{h}$, as computed using Equation (4.13), is the mean norm tap error (MNTTE) (for the s T -spaced phases of the T/s -spaced CIR):

$$MNTTE = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{p=0}^{s-1} |\hat{h}_{n,p} - h_{n,p}|^2 \quad (4.15)$$

which is averaged over the disturbance and $N = 1000$ random channel realizations.

Figure 4.1 displays the BLS CIR estimation MNTTE performance as a function of the length of the randomly specified training sequences for both the desired and single CCI signal. The 25% excess bandwidth filtered MPDP-2 channel is estimated as an $N_c = 6$ T -spaced vector in the presence of noise or one CCI user with SNR or SIR equal to 10 dB,

respectively. The training sequences, along with the channel realizations, are randomly specified from trial to trial in an ensemble. On average, such a resulting CIR estimate would not be as accurate as that obtained with the use of an optimum training sequence, i.e., a training sequence which minimizes the NTE as defined in Equation (A.12) on page 212. Nevertheless, these results demonstrate the relative accuracy of the BLS CIR estimation procedure in noise or in interference subjected to various degrees of channel Doppler fading, e.g., $f_d = 0, 50$ or 100 Hz. The time variation in each tap coefficient of the multipath channel is modelled as a sum of weighted sinusoids with random phases and discrete frequencies spanning the Doppler spectrum [54],[68]. As a benchmark, the minimum NTE (corresponding to a training sequence with a diagonal correlation matrix) of a time-invariant BLS CIR estimate as computed from Equation (4.14) is also shown in the plot.

Figure 4.1 shows that under all channel fading conditions, BLS CIR estimation produces an unbiased CIR estimate with essentially the same accuracy in either noise or interference. This is plausible since with a relatively small 25% pulse filter excess bandwidth, the interference behaves almost like stationary noise and the effect of interferer CIR correlation is negligible. In addition, under the condition of a time-invariant channel, i.e., $f_d = 0$ Hz, the BLS CIR estimate is consistent as its MNTE asymptotically approaches zero with increasing training sequence length. The BLS CIR estimation accuracy is quite poor with a short training sequence close to the minimum length of $N_T = 11$, but improves considerably as the training sequence length increases. In fact, for a training sequence length of over 20 symbols, the MNTE of a BLS CIR estimate obtained with a randomly specified training sequence is less than a couple of dB above the minimum achievable BLS CIR NTE. This is expected since a longer training sequence not only provides more training data observation energy, but also tends to be more uncorrelated and thereby achieves a more accurate BLS CIR estimate.

When the training sequence length is close to the minimum value of $N_T = 11$, the BLS CIR estimation accuracy is uniformly poor at all channel fading rates. As the training sequence length increases, the BLS CIR estimation accuracy improves monotonically only for a time-invariant or sufficiently slow fading channel. Otherwise under rapid time-varying channel conditions, there exists an optimum training sequence length beyond which a further increase results in monotonically decreasing BLS CIR estimation accuracy due to the time variation in the CIR over the actual training sequence. In this case, the optimum training sequence length appears to be around 20 symbols. The choice of the

optimum training sequence length will depend on factors such as the severity of the channel fading and the required data throughput efficiency. These results as depicted in Figure 4.1 are the only cases where the effects of channel time variations are investigated in this thesis. The rest of the thesis considers exclusively a quasi-static channel environment indicative of indoor wireless communications.

Figure 4.2 displays the BLS CIR estimation MNTE performance as a function of the channel output SNR or SIR value for different static, i.e., $f_d = 0$ Hz, multipath channel and filtering conditions. For 25% excess bandwidth filtering, the overall channel is estimated as an $N_c = 6$ T -spaced vector with a 15-symbol training sequence which is either randomly specified or optimally chosen according to Equation (A.12) on page 212. On the other hand for 100% excess bandwidth filtering, the overall channel is estimated as an $N_c = 3$ T -spaced vector with an optimally chosen 7-symbol training sequence. The minimum NTE, as expressed in Equation (4.14), is the same for both of these cases.

For the cases with 25% filter excess bandwidth, BLS CIR estimation with either randomly specified or optimally chosen training sequences produces the same unbiased and equally accurate CIR estimate in either noise or interference. Again, the use of an optimally chosen training sequence yields a considerably more accurate BLS CIR estimate than that resulting from randomly specified training sequences. This BLS CIR estimation accuracy improvement is less, about a couple of dB in MNTE, under low SINR conditions where the disturbance is dominant. However, under high SINR conditions, this BLS CIR estimation accuracy improvement is over 10 dB in MNTE. Furthermore, under such high SINR conditions where the disturbance is negligible, the MNTE improvement tends to flatten out due to the sub-optimum training sequence.

For the cases with 100% filter excess bandwidth, BLS CIR estimation with an optimally chosen training sequence results in an unbiased CIR estimate. However, the use of this relatively wide pulse bandwidth enhances the difference between the cyclostationary interference and the stationary noise. In this case, the MNTE of the BLS CIR estimate is about 1 to 2 dB less in interference than in noise. This difference in BLS CIR estimation accuracy in interference and in noise depends on the interferer CIR correlation according to Equation (A.11) on page 211. Although there can be a slight difference in BLS CIR estimation accuracy in noise and in interference, depending on the interferer CIR correlation, this difference is less important than the absolute accuracy in the BLS CIR estimate.

Figure 4.1 BLS CIR estimation MNTE (averaged over a set of 1000 independently fading channels and randomly selected training sequences) as a function of training length and channel Doppler fade rate in the presence of additive noise (SNR = 10 dB) or one CCI user (SIR = 10 dB)

Figure 4.2 BLS CIR estimation MNTE (averaged over a set of 1000 independently fading channels) as a function of channel SNR or SIR (due to one CCI user). With 25% excess bandwidth filtering, the estimated channel and training sequence lengths are 6 and 15, respectively. With 100% excess bandwidth, the estimated channel and training sequence lengths are 3 and 7, respectively.

4.2.2 Interference plus Noise Autocorrelation Matrix

The computation of the MMSE DFE tap coefficients requires not only an updated CIR estimate but also updated estimates of the interference and noise autocorrelation matrices. Section 4.2.1 has shown that as LS CIR estimate can be obtained via Equation (4.13). This section demonstrates that a sample estimate of the interference plus noise autocorrelation matrix can be acquired with Equation (4.36).

The receive-filtered noise autocorrelation matrix is completely determined by the channel SNR, or equivalently the noise variance, and the receive filter response. For a T/d -spaced receiver, this stationary noise autocorrelation Toeplitz matrix is completely specified by its autocorrelation function (see Equation (3.52) on page 51):

$$R_n(i, j) = \sigma_n^2 R_n\left((i - j) \frac{T}{d}\right) \quad (4.16)$$

In general, it is the additive noise which limits the performance of a communications system. However, in DCR communications, it is the interference and not the additive noise which is the dominant disturbance. Thus, an appropriately designed receiver, e.g., fractionally spaced DFE, which accounts for the cyclostationary interference should perform better than if it assumes the interference to be stationary noise. When occupying close to the minimum Nyquist bandwidth, the interference is noise-like and there is no advantage in designing a receiver to account for this interference as it will perform no better than in the equivalent stationary noise case. On the other hand, when relatively large pulse bandwidths (e.g., 100% excess bandwidth) are employed, the cyclostationary interference is distinctly different from stationary noise and a fractionally spaced equalizer can account for this characteristic to mitigate the interference [51],[95].

The T/d -spaced MMSE DFE explicitly accounts for this cyclostationary interference characteristic in terms of its autocorrelation matrix, R_i , whose (i, j) *th* element is (see Equation (3.52) on page 51):

$$R_i(i, j) = \sigma_i^2 \sum_{l=1}^{n_i} \sum_k h^{(l)}\left(kT - i \frac{T}{d}\right) h^{(l)*}\left(kT - j \frac{T}{d}\right) \quad (4.17)$$

If the channel CIR, or equivalently the interference variance, and the interferer CIRs, $h^{(l)}$, are known, then the interference autocorrelation matrix, R_i , can be readily computed. However, the interferer CIRs are generally not known at the receiver and the interference

autocorrelation matrix must be estimated from a finite record of the received signal. The approach is to form the autocorrelation matrix, R_{e_r} , from the T/d -spaced conjugated error vector, $e_{r,k}$, between the d/T -sampled received signal at the output of the CIR estimator, $\hat{r}_k$ and the d/T -sampled received signal at the output of the actual CIR, r_k (see Figure 3.1 on page 32). Consistent with the BLS CIR estimation procedure described in Section 4.2.1.1, each p th ($p = 0, 1, \dots, d-1$) T -spaced phase of this error vector spans a correlation length of the $N_e = N_t - N_c + 1$ observed training symbols and can be written as:

$$e_{r,p} = \hat{r}_p - r_p = A e_{h_r} - \sum_{l=1}^{n_i} A^{(l)} h_p^{(l)} - n_{r,p} \quad (4.18)$$

where the error vector for the p th T -spaced phase of the CIR estimate, e_{h_r} , is defined in Equation (A.4) on page 209. Moreover, the autocorrelation matrix of the p th T -spaced CIR estimator output error vector can be expressed as:

$$\begin{aligned} R_{e_r} &= E[e_{r,p} e_{r,p}^*] \\ &= E \left[\left(A e_{h_r} - \sum_{l=1}^{n_i} A^{(l)} h_p^{(l)} - n_{r,p} \right) \left(e_{h_r}^* A^* - \sum_{l=1}^{n_i} h_p^{(l)*} A^{(l)*} - n_{r,p}^* \right) \right] \end{aligned} \quad (4.19)$$

where the expectation is with respect to the noise and interference.

Furthermore, from the assumption of mutually uncorrelated data and noise samples, the cross-correlation terms disappear and the simplified expression is:

$$\begin{aligned} R_{e_r} &= E[A e_{h_r} e_{h_r}^* A^*] + \sum_{l=1}^{n_i} E[A^{(l)} h_p^{(l)} h_p^{(l)*} A^{(l)*}] + E[n_{r,p} n_{r,p}^*] \\ &= A C_{e_{h_r}} A^* + R_{i,pp} + R_{n,pp} \end{aligned} \quad (4.20)$$

where the correlation matrix of the error vector for the p th T -spaced phase of the CIR estimate, $C_{e_{h_r}}$, is expressed in Equation (A.7) on page 210.

In order to obtain the T/d -spaced interference and noise autocorrelation matrices required to compute the T/d -spaced MMSE DFE coefficients, the T/d -spaced CIR estimator output error vector, e_r , is formed by interleaving its d T -spaced vector phases, $e_{r,p}$, as defined in Equation (4.18). Correlating this combined T/d -spaced error vector yields a matrix whose (i, j) th ($i, j = 0, 1, \dots, dN_e - 1$) element is:

$$R_{e_r}(i, j) = \{E[e_r e_r^*]\}_{ij} = R_{e_h}(i, j) + R_i(i, j) + R_n(i, j) \quad (4.21)$$

This is the T/d -spaced analogue of the T -spaced autocorrelation matrix as defined in Equation (4.20). According to the definition of the data matrix, A , of Equation (4.8), the first training symbol of the observation period is indexed as $k = N_t - 1$.

In general, imperfect CIR estimation will bias the estimation of the interference and noise autocorrelation matrices. This bias term due to imperfect CIR estimation is represented by $R_{e_k}(i, j)$ in Equation (4.21). The norm, e.g., NTE, of the consistent BLS time-invariant CIR estimator error matrix decreases as the training observation interval increases. That is, for a well chosen training data matrix, A , whose size (i.e., number of rows or training observation length) is sufficiently large, the CIR estimate can be obtained with sufficient accuracy. In this case, the CIR estimation error correlation matrix, C_{e_k} , is small in some norm sense relative to the interference and noise autocorrelation matrices. Thus, the autocorrelation matrix of the CIR estimator output error can be a good estimator of the sum of the interference and noise autocorrelation matrices. In addition to imperfect CIR estimation, another source of error in the estimation of the interference and noise autocorrelation matrices is in the sample autocorrelation estimation itself (see Section 4.2.2.1).

Assuming sufficiently accurate CIR estimation, the first CIR estimation error term is negligible compared to the interference and noise autocorrelation terms:

$$R_{e_k}(i, j) = E[e_{r, N_t - 1 + i/d}^* e_{r, N_t - 1 + j/d}] = R_i(i, j) + R_n(i, j) \quad (4.22)$$

where the interference autocorrelation term can be expressed as:

$$R_i(i, j) = \sigma_i^2 \sum_{l=1}^{n_i} \sum_{k=0}^{N_t-1} h_{k-i/d}^{(l)} h_{k-j/d}^{(l)*} \quad (4.23)$$

and the receive-filtered noise autocorrelation term as:

$$R_n(i, j) = \sigma_n^2 R_n((i - j)/d) \quad (4.24)$$

For the case when cyclostationary interference is present, an estimate of its autocorrelation matrix should be used to compute the fractionally spaced MMSE DFE. This estimation of the autocorrelation of each phase of a cyclostationary process provides additional information of the interference autocorrelation within a baud period to the MMSE DFE so that it can filter the cyclostationary interference more effectively. Consequently, this performance is better than that if the DFE assumes the interference to be just stationary noise.

For the case when only stationary noise is present, if the noise variance is known, then the stationary noise autocorrelation is completely determined by the receive filter response which is known at the receiver. Thus, in this case, there is no advantage to estimating the noise autocorrelation explicitly from the CIR estimator output error. In fact, because of the errors due to imperfect CIR and noise autocorrelation estimation, the resulting performance can be worse than that when the noise autocorrelation function is pre-computed with knowledge of the receive filter response. Consequently, an explicit estimation of the disturbance autocorrelation is performed only in the likely and advantageous scenario where a significant portion of the disturbance is due to cyclostationary interference.

For the case of stationary noise with unknown variance, one can obtain an unbiased ML estimate of the noise variance, $\hat{\sigma}_n^2$, as follows [66]. The p th phase error vector spanning the N_e observation samples at the output of the CIR estimator is due to the noise and, using Equation (4.7) and Equation (4.13), can be written as:

$$\begin{aligned}\hat{\mathbf{u}}_{r,p} = \mathbf{e}_{r,p} &= \mathbf{A}\hat{\mathbf{h}}_p - \mathbf{r}_p = (\mathbf{A}(\mathbf{A}^*\mathbf{A})^{-1}\mathbf{A}^* - \mathbf{I})(\mathbf{A}\mathbf{h}_p + \mathbf{u}_{r,p}) \\ &= (\mathbf{A}(\mathbf{A}^*\mathbf{A})^{-1}\mathbf{A}^* - \mathbf{I})\mathbf{u}_{r,p}\end{aligned}\quad (4.25)$$

Furthermore, the MSE at the CIR estimator output can now be written as:

$$E[|\hat{\mathbf{u}}_{r,p}|^2] = \text{tr}((\mathbf{A}(\mathbf{A}^*\mathbf{A})^{-1}\mathbf{A}^* - \mathbf{I})E[\mathbf{u}_{r,p}\mathbf{u}_{r,p}^*](\mathbf{A}(\mathbf{A}^*\mathbf{A})^{-1}\mathbf{A}^* - \mathbf{I})) \quad (4.26)$$

and from the fact that $\text{tr}(\mathbf{ABC}) = \text{tr}(\mathbf{BCA})$ and the assumption of uncorrelated noise samples, this MSE can be simplified as:

$$\begin{aligned}E[|\hat{\mathbf{u}}_{r,p}|^2] &= \sigma_n^2(\text{tr}(\mathbf{I}_{N_t - N_c + 1}) - \text{tr}(\mathbf{A}(\mathbf{A}^*\mathbf{A})^{-1}\mathbf{A}^*)) \\ &= \sigma_n^2(\text{tr}(\mathbf{I}_{N_t - N_c + 1}) - \text{tr}(\mathbf{I}_{N_c}))\end{aligned}\quad (4.27)$$

Therefore, by approximating the ensemble average with a sample time average, an unbiased estimate of the noise variance can be obtained:

$$\hat{\sigma}_n^2 = \frac{\sum_{p=0}^{d-1} |\hat{\mathbf{u}}_{r,p}|^2}{N_t - 2N_c + 1} = \frac{\sum_{p=0}^{d-1} \sum_{k=N_c-1}^{N_t-1} \left| r_{k+p/d} - \sum_{l=0}^{N_c-1} \mathbf{a}_{k-l}^* \hat{\mathbf{h}}_{l+p/d} \right|^2}{N_t - 2N_c + 1} \quad (4.28)$$

Recall the formulation of the interference plus noise autocorrelation estimation in Equation (4.22). Consistent with the BLS CIR estimation procedure, it also does not make any

assumption about the data prior to the training sequence. However, if there are zero-valued data in the guard period immediately prior to the training sequence, then the initial $N_t - 1$ training symbols need not be omitted in the interference plus noise autocorrelation estimation. Thus, the observed autocorrelation length can be increased by the initial $N_t - 1$ training symbols. In this case, the data matrix, A , defined in Equation (4.8) is enlarged to size $N_t \times N_t$ to reflect the longer autocorrelation length spanning all the N_t training symbols. Furthermore, the first N_t rows of the data matrix, A , are now lower triangular with a zero-valued upper right triangle to reflect the assumed pre-windowed zero-valued data.

While it may be acceptable to assume front zero-valued data padding prior to the training sequence, e.g., during the guard time between TDMA bursts, no assumptions can be made about the unknown data symbols subsequent to the training sequence. For a minimum phase overall CIR with zero delay, this unknown trailing data is unimportant since they do not affect the response due to the N_t training symbols, i.e., the unknown trailing data do not appear in the data matrix, A . However, for a non-minimum phase overall CIR with non-zero delay, the last few observations, i.e., rows of A , spanning the delay in the overall CIR will be corrupted by the corresponding few unknown trailing data symbols. To avoid corrupted observations, the autocorrelation length can be reduced from N_t by the span of the CIR delay. Depending on the channel filtering, this delay may not be that significant. Moreover, because of the time averaging involved in the autocorrelation procedure, as long as the autocorrelation length is much greater than the channel delay, the effect of these few unknown trailing symbols on the last few observations of the autocorrelation estimate may be negligible. Another alternative is to window the observation data, viz., CIR estimator output error sequence, in order to de-emphasize the leading and trailing samples before performing the autocorrelation [48],[52],[77],[90],[111].

4.2.2.1 Interference plus Noise Sample Autocorrelation Estimation

Up to now, the interference and noise autocorrelation matrices have been formulated as second order ensemble statistics averaged over random data and noise realizations (see Equation (4.20)). In practice, such ensemble statistics are not known and must be estimated from a given sample data sequence of finite length. That is, an ensemble average of a random process is estimated by a time average of a finite length sample realization. This is possible for ergodic processes where, in the limit of an infinite length sample sequence, the time average equals the ensemble average with probability of one. While the non-

Gaussian cyclostationary error autocorrelation sequence may not necessarily be autocorrelation-ergodic, enough modes of the random process can manifest over a sufficiently long sequence. Therefore it can be assumed that, for a sufficiently long sample sequence, the sample time averaged autocorrelation is an acceptable approximation of the ensemble averaged autocorrelation [77],[90],[92],[93],[111].

From the cyclostationarity property of the overall T/d -spaced CIR estimator output error sequence, e_r , its autocorrelation matrix is Hermitian while its interleaved sub-matrices, corresponding to each of the d T -spaced phases of the error sequence, are Toeplitz. Thus, the overall autocorrelation matrix is completely specified by the non-negative T/d -spaced lags of the autocorrelation and cross-correlation sequences of the d phases. A non-parametric estimator is required since no assumption is made about the particular analytical form for the autocorrelation sequence. Two candidates for the autocorrelation estimator of the T/d -spaced error sequence, e_r , spanning N_e T -spaced known data samples, are [55],[77],[90],[92],[111]:

$$\hat{R}_{e_r}(l) = \frac{1}{N_e - \lceil l/d \rceil} \sum_{k=0}^{N_e - 1 - \lceil l/d \rceil} e_{r,k} e_{r,k+l/d}^* \quad (4.29)$$

and

$$\hat{R}_{e_r}(l) = \frac{1}{N_e} \sum_{k=0}^{N_e - 1 - \lceil l/d \rceil} e_{r,k} e_{r,k+l/d}^* \quad (4.30)$$

where l is the T/d -spaced lag index,

k is the summation index over all $N_e - 1 - \lceil l/d \rceil$ non-zero error products

and $\lceil l/d \rceil < N_e$ is rounded up to the next positive integer.

The error sequence, e_r , is defined in Equation (4.18) for the case when $N_e = N_t - N_r + 1$. As discussed at the end of Section 4.2.2, assuming the availability of a suitable data window or a zero padded guard period prior to the training sequence and a minimum phase overall CIR with zero delay, the autocorrelation length can be as long as the training sequence, i.e., $N_e = N_t$. Furthermore, if the sequence of the error product, $e_{r,k} e_{r,k+l/d}^*$, is Gaussian, then $\hat{R}_{e_r}(l)$ is the ML estimate of $R_{e_r}(l)$. Of course, the error product sequence does not in general have Gaussian statistics and it is difficult to determine whether a particular estimator is optimum in any sense, e.g., in the ML sense. Instead, relatively simple statistical performance measures such as estimation bias, variance and mean square error (which equals the sum of the variance and squared bias) are employed.

The first estimator, $\vec{R}_{e_r}(l)$, is unbiased since:

$$E[\vec{R}_{e_r}(l)] = \frac{1}{N_e - \lceil l/d \rceil} \sum_{k=0}^{N_e - 1 - \lceil l/d \rceil} E[e_{e_r, k} e_{e_r, k + l/d}^*] = R_{e_r}(l) \quad (4.31)$$

while its variance, or MSE, has been shown approximately to be [55]:

$$Var(\vec{R}_{e_r}(l)) = E[|\vec{R}_{e_r}(l) - R_{e_r}(l)|^2] \propto \frac{N_e}{(N_e - \lceil l/d \rceil)^2} \sum_l |R_{e_r}(l)|^2 \quad (4.32)$$

This implies that $\vec{R}_{e_r}(l)$ is a consistent estimator since its variance decreases to zero as the observation length, N_e , tends to infinity. However, for a fixed observation length, N_e , its variance grows as the lag index increases towards the observation length. That is, for a fixed data sequence length, as the lag index increases, there are fewer samples to average over and consequently the estimate becomes worse.

The second estimator, $\hat{R}_{e_r}(l)$, differs from the first unbiased estimator, $\vec{R}_{e_r}(l)$, by a bias factor, β_e :

$$E[\hat{R}_{e_r}(l)] = \frac{N_e - \lceil l/d \rceil}{N_e} \cdot E[\vec{R}_{e_r}(l)] = (1 - \frac{\lceil l/d \rceil}{N_e}) R_{e_r}(l) = \beta_e R_{e_r}(l) \quad (4.33)$$

resulting in a bias of:

$$B(\hat{R}_{e_r}(l)) = E[\hat{R}_{e_r}(l)] - R_{e_r}(l) = (\beta_e - 1) R_{e_r}(l) \quad (4.34)$$

However, as the observation length, N_e , tends to infinity, the scale factor, β_e , tends to unity and thus the second estimator, $\hat{R}_{e_r}(l)$, is asymptotically unbiased. Furthermore, its variance is related to that of the first unbiased estimator:

$$\begin{aligned} Var(\hat{R}_{e_r}(l)) &= E[|\hat{R}_{e_r}(l) - E[\hat{R}_{e_r}(l)]|^2] \\ &= \left(\frac{N_e - \lceil l/d \rceil}{N_e} \right)^2 Var(\vec{R}_{e_r}(l)) \propto \frac{1}{N_e} \sum_l |R_{e_r}(l)|^2 \end{aligned} \quad (4.35)$$

Therefore, unlike the first unbiased estimator, the second biased estimator, $\hat{R}_{e_r}(l)$, has a variance which doesn't grow as the lag index is increased towards a fixed observation length. For many applications, it is suggested that the estimation MSE is higher for the unbiased estimator than for the biased estimator [55]. Moreover, the unbiased estimator can in some cases result in a negative definite autocorrelation matrix while the biased esti-

mator always produces a positive semi-definite autocorrelation matrix [77]. The performance of the two estimators are identical at zero lag, i.e., as variance estimators, and are very similar for lags much less than the observation length, i.e., $|l - d| \ll N_e$. However, from the reasoning of the above discussion, the biased estimator, $\hat{R}_{e_n}(l)$, is chosen to estimate the interference plus noise autocorrelation matrix as defined in Equation (4.23) and Equation (4.24). Specifically, the (i, j) th $(i, j = 0, 1, \dots, N_f - 1)$ element of the interference plus noise autocorrelation matrix is estimated as:

$$\hat{R}_{i+n}(i, j) = \frac{1}{N_e} \sum_{k=0}^{N_e-1-\lceil (i-j)/d \rceil} e_{r,k-i-d} e_{r,k-j-d}^* \quad (4.36)$$

where $\lceil (i-j)/d \rceil$ is rounded up to the next positive integer.

4.2.3 Excess MSE Analysis

The objective of this section is to develop an expression for the excess MSE of a computed fractionally spaced MMSE DFE which employs an LS CIR estimate and a sample interference plus noise autocorrelation estimate. An upper bound of this excess MSE is expressed by Equation (4.52). The two main assumptions used in the development of this upper bound excess MSE expression are that the CIR estimation error is small and that only diagonal matrix elements contribute to the excess MSE.

The excess MSE at the output of a Wiener filter such as the adaptive DFE considered in this thesis can be expressed as the following quadratic form [48],[52],[100]:

$$XSME = E[\epsilon_w^* R \epsilon_w] \quad (4.37)$$

where the expectation, $E[\]$, is with respect to the interference data and noise samples while the desired and interferer CIRs are assumed fixed.

The equalizer tap coefficient error vector is defined as:

$$\epsilon_w = \hat{w} - w_o \quad (4.38)$$

with the estimated equalizer tap coefficient vector computed from the Wiener solution employing estimates of the CIR and interference plus noise autocorrelation (compare with the ideal case of Equation (3.46) on page 50):

$$\hat{w} = \hat{R}^{-1} \hat{p} \quad (4.39)$$

Furthermore, the estimated input signal autocorrelation matrix, $\hat{R}$, consists of the sum of the actual autocorrelation matrix, R , and an error matrix, $\tilde{R}$:

$$\hat{R} = R + \tilde{R} \quad (4.40)$$

while the estimated cross-correlation vector, $\hat{p}$, can be similarly expressed as the actual vector, p , perturbed by an additive error vector, $\tilde{p}$:

$$\hat{p} = p + \tilde{p} \quad (4.41)$$

Substituting Equation (4.38) and Equation (4.39) into Equation (4.37) yields:

$$XSMSE = E [\hat{p}^* \hat{R}^{-1} R \hat{R}^{-1} \hat{p} - \hat{p}^* \hat{R}^{-1} R \underline{w}_o - \underline{w}_o^* R \hat{R}^{-1} \hat{p} + \underline{w}_o^* R \underline{w}_o] \quad (4.42)$$

From Equation (4.40), the inverse of the estimated autocorrelation matrix is expressed as:

$$\hat{R}^{-1} = (R (I + R^{-1} \tilde{R}))^{-1} = (I + R^{-1} \tilde{R})^{-1} R^{-1} \quad (4.43)$$

A subsequent application of an approximation of the binomial expansion results in:

$$\hat{R}^{-1} R = (I + P)^{-1} \approx I - P \quad (4.44)$$

where the perturbation matrix is defined as:

$$P = R^{-1} \tilde{R} \quad (4.45)$$

and is assumed to be small in some norm sense (i.e., $\|P\| \ll 1$).

This assumption is valid as long as the CIR estimation error, e_h , is small. It is shown in Appendix A on page 209 that under this assumption, the error matrix, $\tilde{R}$, and thus the perturbation matrix, P , is of linear order in terms of the CIR estimation error, e_h , (i.e., $O(e_h)$) and is thus approximately zero-mean since the CIR estimation error is zero-mean.

Substituting Equation (4.44) into Equation (4.42) and after some algebra yields:

$$\begin{aligned} XSMSE = E [& p^* P^2 \underline{w}_o - p^* P R^{-1} \tilde{p} - \tilde{p}^* P \underline{w}_o + \tilde{p}^* R^{-1} \tilde{p} \\ & + p^* P^2 R^{-1} \tilde{p} + \tilde{p}^* P^2 \underline{w}_o - 2 \tilde{p}^* P R^{-1} \tilde{p} + \tilde{p}^* P^2 R^{-1} \tilde{p}] \end{aligned} \quad (4.46)$$

However, the last four terms are either $O(e_h^3)$ or $O(e_h^4)$ and are thus negligible under the assumption of small CIR estimation error. Hence, substituting Equation (4.45) into Equation (4.46) results in the following simplified expression for the excess MSE:

$$\begin{aligned}
 XSMSE &= \underline{w}_o^* E[\tilde{R}R^{-1}\tilde{R}] \underline{w}_o - \underline{w}_o^* E[\tilde{R}R^{-1}\tilde{p}] - E[\tilde{p}^* R^{-1}\tilde{R}] \underline{w}_o + E[\tilde{p}^* R^{-1}\tilde{p}] \quad (4.47) \\
 &= \underline{w}_o^* E[\tilde{R}R^{-1}\tilde{R}] \underline{w}_o - 2\text{Re}\{\underline{w}_o^* E[\tilde{R}R^{-1}\tilde{p}]\} + E[\tilde{p}^* R^{-1}\tilde{p}]
 \end{aligned}$$

For the situation of a quasi-static channel environment where there are no lag errors associated with the tracking of time variations in the channel parameters, only CIR and interference plus noise autocorrelation estimation errors contribute to this excess MSE.

Specifically, it will be shown that the CIR estimation error affects all the terms in Equation (4.47) while the interference plus noise autocorrelation estimation error manifests only in the first term. In order to proceed with the derivation of the excess MSE, the autocorrelation error matrix, $\tilde{R}$, and cross-correlation error vector, $\tilde{p}$, need to be expressed in terms of these estimation errors.

Section 3.4 on page 47 details the computation of a finite-complexity T/d -spaced MMSE DFE. Without loss of generality, the following analysis considers a T -spaced DFE. In order to generalize to a fractionally spaced DFE, the result for a T -spaced DFE is taken to represent one T -spaced phase of the overall fractionally spaced DFE and the excess MSE is obtained by summing over all d phases within a baud interval of the equalizer. A summary of the resulting upper bound on the excess MSE is presented here while the details are found in Appendix A on page 209.

In order to obtain a mathematically tractable solution which reasonably approximates the dependence of the computed DFE excess MSE on the CIR and interference plus noise autocorrelation estimation errors, it is assumed that R and $\tilde{R}$ are diagonal matrices. The implication of this assumption is discussed in Appendix A on page 209. Essentially, this assumption can result in an upper bound on the excess MSE whose accuracy is dependent on the channel multipath and disturbance conditions. Moreover, the accuracy or tightness of the upper bound is inversely proportional to the energy of the equalizer forward filter, $|\underline{w}_f|^2$, (see Section 4.2.3.1). In any case, with the above-mentioned diagonal matrix assumption, it can be shown (see Equation (B.18) on page 217) that the first term of the excess MSE expression of Equation (4.47) can be upper bounded as:

$$\underline{w}_o^* E[\tilde{R}R^{-1}\tilde{R}] \underline{w}_o \leq \frac{4|\underline{w}_o|^2}{N_e(1 + SINR_{in})} \left(1 + \frac{\sum_{p=0}^{d-1} \sum_{l=0}^{N_f-1} |R_{l+n,pl}|^2}{\sum_{p=0}^{d-1} R_{l+n,pp}} \right) \quad (4.48)$$

where $\underline{w}_{of}$ is the optimum DFE forward filter tap coefficient vector

$N_e = N_f - N_c + 1$ is the number of training symbols used for CIR and interference plus noise autocorrelation estimation

and $SINR_{in}$ is the SINR at the input to the T/d -spaced equalizer and is expressed as:

$$SINR_{in} = \frac{\sum_{p=0}^{d-1} R_{h,pp}}{\sum_{p=0}^{d-1} R_{i+n,pp}} = \frac{\sum_{p=0}^{d-1} R_{rr,pp}}{\sum_{p=0}^{d-1} R_{i+n,pp}} - 1 \quad (4.49)$$

with $R_{rr,ij} = R_{rr}(i,j)$, $R_{h,ij} = R_h(i,j)$ and $R_{i+n,ij} = R_{i+n}(i,j)$ being the (i,j) th elements of the DFE input autocorrelation matrix of the received signal, desired received signal and interference plus noise signal, respectively (see Equation (D.2) on page 228). Note that the numerator and denominator of Equation (4.49) are the variances (sum of diagonal (i.e., $i = j$) elements over the d phases) at the equalizer input of the desired received data and interference plus noise, respectively. Furthermore, the additive unit term is negligible for appreciably high channel SINR conditions.

The first term of the excess MSE expression of Equation (4.47) has two additive components (see Equation (4.48)). The first is a contribution from the CIR estimation error while the second is due to the interference plus noise sample autocorrelation estimation error, with $R_{i+n,pl}$ being the p th phase of the interference plus noise autocorrelation at lag l spanning the $N_f T/d$ -spaced DFE forward filter taps.

The second term of the excess MSE expression of Equation (4.47) is due to only the CIR estimation error, assuming that this error is small. Moreover, this excess MSE component is lower bounded as (see Equation (B.24) on page 218):

$$-2Re\{\underline{w}_o^* E[\tilde{R}R^{-1}\tilde{p}]\} \geq \frac{-4Re\{\underline{w}_o\} Re\{\underline{p}\}}{N_e(1 + SINR_{in})} \quad (4.50)$$

Finally, the third term of the excess MSE expression of Equation (4.47) is also due to only the CIR estimation error manifesting only in the cross-correlation vector, $\underline{p}$. This excess MSE component is expressed as (see Equation (B.27) on page 219):

$$E\{\tilde{p}^* R^{-1}\tilde{p}\} = \frac{N_f}{N_e(1 + SINR_{in})} \quad (4.51)$$

Thus, the overall excess MSE at the fractionally $T/2$ -spaced DFE output can be approximately upper bounded by:

$$XSMSE \leq \frac{1}{N_c (1 + SINR_{in})} \quad (4.52)$$

$$\times \left\{ 4|\underline{w}_f|^2 \left(1 + \frac{\sum_{p=0}^{d-1} \sum_{l=0}^{N_c-1} |R_{l+n,pl}|^2}{\sum_{p=0}^{d-1} R_{l+n,pp}} \right) - 4Re\{\underline{w}_f\} Re\{p\} + N_f \right\}$$

Note that the excess MSE asymptotically approaches zero as the training estimation length increases, i.e., $N_c \rightarrow \infty$. This is in agreement with the fact that the MMSE DFE computation uses CIR and interference plus noise autocorrelation estimators which are consistent. Moreover, the excess MSE is inversely proportional to the SINR at the input to the equalizer. This is intuitive since the excess MSE is proportional to the disturbance to the input signal of the equalizer. Furthermore, the excess MSE is proportional to the energy of the equalizer forward filter, $|\underline{w}_f|^2$. This has ramifications on the accuracy of this upper bound on the excess MSE for different interference and/or noise conditions (see Section 4.2.3.1).

4.2.3.1 Verification of the Excess MSE Upper Bound

The expression of Equation (4.52) for the upper bound of the computed fractionally spaced equalizer excess MSE is verified for a typical realization of a quasi-static multipath fading channel environment. The disturbance is due to interference from a CCI user or additive stationary noise while the excess MSE is caused by the resulting CIR and interference plus noise estimation errors. In the interference case, the equalizer is computed with knowledge of RLS CIR and interference plus noise autocorrelation estimates, while in the noise case, the computed equalizer requires only an RLS CIR estimate. Static real-valued $T/2$ -spaced channels and $T/2$ -spaced equalizers are considered.

With 100% excess bandwidth SRRC transmit and receive filters, the CIR estimate length is $N_c = 3$ symbols. The multipath channel and co-channel span one symbol period with $T/2$ -spaced channel tap coefficients being (1.0, 0.6, 0.2) and (0.2, 1.0, 0.8), respectively. The unit energy (i.e., $|h|^2 = 1$) $T/2$ -spaced CIR and co-CIR tap coefficients are (0.0, 0.262, 0.681, 0.629, 0.262, 0.052) and (0.0, 0.047, 0.326, 0.699, 0.606, 0.187), respectively. These CIRs are non-causal due to the non-causal SRRC filter responses.

Thus, even though the desired channel MPDP is minimum phase, the desired signal path requires a delay of one symbol period for realizability.

Two of the main assumptions used in the derivation of the upper bound on the excess MSE of the computed equalizer (see Equation (4.52)) are that the CIR estimation error is small and that the autocorrelation and corresponding error matrices are diagonal. In order to explore the effect of the first assumption, the RLS CIR estimation MNTTE, averaged over 100 trials, is found under various channel disturbance levels and training sequence lengths. These results as shown in Figure 4.3 indicate that for sufficiently long training periods, the MNTTE can be reduced to validate the assumption of small CIR estimation error. Specifically, for training lengths of 15, 100 and 200 symbol periods, the respective MNTTE is about 5, 15 and 18 dB below the disturbance level. Moreover, the MNTTE performance in CCI is typically about 1 dB better than that in noise for this particular case.

The equalizer output MSE is inversely proportional to the SDR as specified by Equation (3.61) on page 53. Thus, an upper bound on the MSE is equivalent to a lower bound on the SDR. Figure 4.4 and Figure 4.5 display the respective SDR performance of a $T/2$ -spaced linear equalizer, denoted LE(5,0), and a $T/2$ -spaced decision feedback equalizer, denoted DFE(5,2), as a function of the training length and stationary noise level. These results demonstrate that using Equation (4.52) for calculating the upper bound on the excess MSE results in a very tight lower bound on the SDR of a computed linear or decision feedback equalizer in the presence of noise. This performance bound is very tight for training lengths of at least 50 symbol periods. For shorter training lengths down to 15 symbol periods, the performance bound becomes a little looser since the assumption of small CIR estimation error is not as valid in this case. However, the bound is still within 1 dB in SDR of the simulated performance.

As the SNR increases from 0 dB to 20 dB, the eigenvalue spread of the equalizer input autocorrelation matrix increases from approximately 15 dB to 30 dB, while the forward filter energy, $|\underline{w}_f|^2$, increases from about 0.3 to 2. These values are consistent with the fact that at high noise levels where the noise is dominant with respect to the ISI, the equalizer attempts to minimize the effect of the noise by reducing its forward filter energy. Furthermore, the dominant impairment of stationary noise tends to result in a relatively smooth power spectrum which corresponds asymptotically with a smaller eigenvalue spread [48]. This in turn affects the energy of the equalizer taps according to Equation (4.55).

Figure 4.3 RLS CIR estimation MTE, averaged over 100 independent realizations of the noise and data, as a function of the training sequence length in the presence of additive noise (SNR = 0, 10 or 20 dB) or one CCI user (SIR = 0, 10 or 20 dB)

Figure 4.4 Comparison of the calculated and simulated SDR of a computed LE(5,0) in the presence of additive noise as a function of the training sequence length

Figure 4.5 Comparison of the calculated and simulated SDR of a computed DFE(5,2) in the presence of additive noise as a function of the training sequence length

It is also insightful to observe the relative contributions to the excess MSE of the four terms in the upper bound expression of Equation (4.52). In the presence of only noise, the second term is zero since noise autocorrelation estimation is not performed, assuming that the receive filter response and noise variance is known at the receiver. Thus, only the first term is dependent on the equalizer filter energy. At high SNR levels, the first term is the largest contributor (about 90%) while terms three and four contribute roughly -40% and 50%, respectively. As the equalizer energy decreases with increasing noise levels (say to $\text{SNR} = 0$ dB), the importance of term one diminishes (down to 30% contribution) while terms three and four increases to -50% and 120% contribution to the excess MSE, respectively. From these relative contributions to the excess MSE, it is concluded that all three terms are significant in the upper bound expression for the excess MSE. These results and conclusion are applicable to both the linear and decision feedback equalizers.

Figure 4.4 and Figure 4.5 also display the simulated SDR performance of the directly adapted RLS equalizer. For sufficiently long training of at least 50 symbol periods and sufficiently high SNR of no less than 10 dB, the directly adapted RLS and computed equalizers have the same SDR performance. However, under high noise conditions, e.g., $\text{SNR} = 0$ dB, and shorter training lengths of less than about 50 symbol periods, the directly adapted RLS equalizer exhibits slower convergence and a slightly worse SDR performance.

Figure 4.6 and Figure 4.7 show the respective SDR performance of the LE(5,0) and the DFE(5,2) as a function of the training length and interference level. Comparing the simulated and calculated SDR performance of the computed equalizers indicates that the excess MSE expression of Equation (4.52) can yield very loose lower bounds on the SDR performance in interference. For the linear equalizer under low interference conditions, e.g., $\text{SIR} = 20$ dB, and long training lengths greater than 100 symbol periods, the SDR lower bound is fairly tight, being within about 1 dB of the simulated performance. But as the training length decreases down to 15 symbol periods, the bound loosens to only within 5 dB of the simulated performance. For increasing levels of interference, the bound worsens. In fact, when $\text{SIR} = 0$ dB, the performance bound fails (as the upper bound on the excess MSE exceeds unity) for training lengths less than 100 symbol periods.

Figure 4.7 demonstrates that the accuracy of the performance bound is further degraded in the case of the DFE(5,2). In fact, the bound of Equation (4.52) cannot be applied, i.e., excess MSE exceeds unity, when the interference is very strong, e.g., $\text{SIR} = 0$ dB.

Figure 4.6 Comparison of the calculated and simulated SDR of a computed LE(5,0) in the presence of one CCI user as a function of the training sequence length

Figure 4.7 Comparison of the calculated and simulated SDR of a computed DFE(5,2) in the presence of one CCI user as a function of the training sequence length

However, Figure 4.3 shows that the CIR estimation error is quite small for the cases when the performance bound is very loose. This indicates that the loose bound in the presence of interference results from the inaccurate assumption of diagonal matrices.

Since the received signal spectrum in the presence of interference can vary appreciably over the signal frequency band, the eigenvalue spreads of its equalizer input autocorrelation matrix are significantly higher than those in the noise case. Specifically for the linear equalizer in the presence of interference, the eigenvalue spread ranges from about 25 dB to about 27 dB with increasing SIR. For the DFE, the eigenvalue spreads are higher in the range of 30 dB to 45 dB. Moreover, a larger equalizer filter energy is associated with a higher eigenvalue spread. For instance, linear equalizer energies are around 10 to 20 while DFE energies (most of which is in the forward filter) are greater at around 60 to 80.

The eigenvalue spread of the equalizer input autocorrelation matrix can be related to the equalizer energy as follows. From the Wiener solution for the equalizer coefficients (see Equation (3.46) on page 50), the energy of the MMSE equalizer can be written as:

$$|\underline{w}_o|^2 = \underline{p}^* R^{-2} \underline{p} \quad (4.53)$$

Since R and thus R^{-2} are Hermitian matrices, the following diagonalization can be performed (see Equation (F.2) on page 234):

$$R^{-2} = U \Lambda^{-2} U^* \quad (4.54)$$

where U is a unitary matrix whose column vectors are the eigenvectors, $\underline{u}_l$, of R and Λ is a real-valued diagonal matrix of the corresponding eigenvalues, λ_l , of R

Substituting Equation (4.54) into Equation (4.53) yields:

$$|\underline{w}_o|^2 = \underline{p}^* U \Lambda^{-2} U^* \underline{p} = \sum_{l=0}^{N_f + N_b - 1} \lambda_l^{-2} |\underline{p}^* \underline{u}_l|^2 \quad (4.55)$$

The equalizer energy is inversely proportional to the squared eigenvalues of its input autocorrelation matrix, thus confirming the above observations. Table 4.1 also demonstrates this relationship between the equalizer energy and the eigenvalue spread. Moreover, note that each of the four $T/2$ -spaced equalizers have two very small eigenvalues. This is because a T/d -spaced equalizer with $N_f = d(N_{ff} - 1) + 1$ taps, with the same span as an N_{ff} -tap T -spaced equalizer, has $N_f - N_{ff} = (d - 1)(N_{ff} - 1)$ approximately zero-valued eigenvalues [40],[42],[75],[101],[121]. Of particular significance is that the small-

est eigenvalue for each equalizer is much smaller for the interference case, especially with DFE(5,2) which has a small MMSE. The commensurate increase in the eigenvalue spread also has ramifications on the LMS MSE convergence (see Section 4.3.2.1 on page 138).

Because the equalizer energy can be significantly larger in the presence of interference than in noise, the assumption of diagonal matrices, leading to the equalizer energy being a scale factor in the excess MSE upper bound expression of Equation (4.52), can result in a very loose upper bound on the excess MSE in the case with interference. This strong influence of a large equalizer energy on the excess MSE bound is evident from the observation that terms one and two, which have the equalizer energy as a scale factor, in the excess MSE upper bound expression of Equation (4.52) typically account for well over 90% of the excess MSE in the presence of interference. Of these two terms, the first term is the dominant contributor to the excess MSE under conditions of low interference while the two terms are of similar magnitude under high interference conditions, e.g., SIR = 0 dB. These results are applicable to both the linear and decision feedback equalizers.

The eigenvalue spread of the input autocorrelation matrix corresponds roughly to the variation in the power spectrum of the received signal at the input to the equalizer. The minimum and maximum eigenvalues are the minimum and maximum asymptotic values of the input signal power spectrum [48]. It is essentially this variation in the input signal spectrum that has to be equalized. In order for an equalizer to achieve superior MSE performance in the presence of interference than in noise, its forward filter has to form an overall equalized response which minimizes both the ISI and inter-user interference. Thus, the forward filter tap coefficients can have some large amplitudes in trying to equalize a received signal with a highly variable eigenstructure as is the case in interference (see Table 4.1). In forming this optimum response, the forward filter tap coefficients typically have large amplitudes so that the main cursor is as close as possible to the desired signal level (e.g., unity for a normalized data signal), consequently minimizing the output MSE.

Figure 4.6 and Figure 4.7 also display the simulated SDR performance of the RLS equalizer as a comparison with that of the computed equalizer. These results show that, for both the linear and decision feedback equalizers, under low interference conditions, e.g., SIR = 20 dB, and with sufficient training so that the CIR estimation error is small, the computed equalizer performs slightly better than the RLS equalizer. However, when the interference level increases, e.g., SIR = 0 dB, the RLS linear equalizer performs a bit better while the RLS DFE performs significantly better, i.e., up to 5 dB in SDR, than the computed DFE.

Disturbance:		Noise: SNR = 10 dB		Interference: SIR = 10 dB	
Equalizer:		LE(5,0)	DFE(5,2)	LE(5,0)	DFE(5,2)
Calculated Computed Equalizer MSE [dB]	MMSE	- 9.714	- 10.136	- 13.456	- 20.513
	XSMSE	- 21.471	- 22.307	- 12.953	- 5.258
	MSE	- 9.433	- 9.881	- 10.187	- 5.130
Simulated Equalizer MSE [dB]	Computed	- 9.512	- 9.920	- 13.038	- 15.339
	RLS	- 9.452	- 9.764	- 12.974	- 15.762
Optimum Equalizer Energy	overall filter	1.351	1.128	12.533	75.585
	forward filter	1.351	1.062	12.533	75.082
Optimum Equalizer Coefficients	w_0	- 0.192	0.051	0.347	1.263
	w_1	0.618	- 0.161	- 0.974	- 3.362
	w_2	0.921	- 0.031	2.843	5.969
	w_3	- 0.276	0.510	- 1.757	- 4.932
	w_4	- 0.096	0.879	0.541	1.492
	w_5		- 0.257		0.697
	w_6		0.000001		- 0.133
Eigen- Structure of the Auto- correlation Matrix of the Input Signal to Equalizer	eigenvalue spread [dB]	21.974	22.824	29.294	34.590
	λ_0	1.546	1.574	1.580	2.192
	λ_1	0.858	1.120	0.847	1.179
	λ_2	0.364	1.000	0.325	1.001
	λ_3	0.0462	0.743	0.0261	0.358
	λ_4	0.00981	0.346	0.00186	0.0367
	λ_5		0.0330		0.0129
	λ_6		0.00822		0.000762

Table 4.1 Calculated and simulated MSE and MMSE equalizer coefficients and eigenstructure in the presence of additive noise or interference due to one CCI user for an example with static channel and co-channel responses. RLS CIR estimation MNTE (averaged over 100 trials) = - 24.23 dB and - 24.93 dB for the noise and interference cases, respectively, with a training length of 100 symbol periods.

4.2.4 MSE Perturbation Analysis

In order to gain further insight into how small variations in the input signal autocorrelation matrix, R , and cross-correlation vector, p , affect the Wiener equalizer tap coefficients, $\underline{w}$, (see Equation (3.46) on page 50) and thus the equalizer output MSE (see Equation (3.49) on page 50), an upper bound on the increase in the equalizer output MSE is considered. As was detailed in Section 4.2.3, this excess MSE is due to CIR and interference plus noise correlation estimation errors, assuming a quasi-static channel environment such that there are no lag errors associated with the tracking of time-varying channel parameters. The main result of the upper bound on the perturbation in the MMSE equalizer is presented here while the details of the derivation are found in Appendix E on page 231.

Specifically, for small average perturbations, δR , in the matrix, R , the perturbation in the equalizer MSE, J , is upper bounded as follows:

$$\delta J \leq E \left[\|R\| \|\underline{w}\|^2 \kappa^2(R) \left(\frac{\|\delta p\|}{\|p\|} + \frac{\|\delta R\|}{\|R\|} \right)^2 \right] \quad (4.56)$$

and the condition number, $\kappa(R)$, is the eigenvalue spread of the autocorrelation matrix:

$$\kappa(R) = \frac{\lambda_{max}}{\lambda_{min}} \quad (4.57)$$

Thus, from Equation (4.56) it is evident how, for a given perturbation in R and p , the eigenvalue spread of the autocorrelation matrix and equalizer energy can magnify the perturbation in the MSE. To illustrate how the equalizer excess MSE is affected by small inaccuracies in R and p , as expressed by $(\|\delta p\|/\|p\| + \|\delta R\|/\|R\|)^2$, this perturbation analysis is applied to the example in Section 4.2.3.1. Figure 4.8 shows the upper bound of the perturbation in the equalizer MSE as a function of the aggregate inaccuracy in R and p . The magnification factor of the four cases is $\|R\| \|\underline{w}\|^2 \kappa^2(R)$, with values taken from Table 4.1. Because these magnification factors are very large, ranging from about 45 to 90 dB for an LE(5,0) in SNR = 10 dB of additive noise (case (4)) to a DFE(5,2) in SIR = 10 dB of interference (case (1)), respectively, the upper bound on the equalizer MSE perturbation is highly sensitive to small inaccuracies in R and p , which are on the order of the LS CIR estimation NTE . In particular, the MSE of the DFE in the presence of interference is most sensitive to perturbations, being about 20 dB and 40 dB more sensitive than that of the LE in interference and the LE or DFE in noise, respectively.

Figure 4.8 MSE perturbation upper bound of the Wiener equalizer as a function of small inaccuracies in its autocorrelation matrix and cross-correlation vector

4.3 DFE Directly Adapted with the LMS Algorithm

As a comparison with the strategy where the DFE coefficients are computed with estimates of the CIR and interference plus noise autocorrelation, an alternative approach is investigated where the DFE coefficients, $\underline{w}(kT)$, are directly adapted with the LMS algorithm. This LMS algorithm is also called the stochastic gradient method because it is designed to minimize the MSE by employing an approximation of the true statistical MSE gradient based on a sample squared error gradient. Specifically, the complex-valued LMS DFE coefficient update equation can be written as [48],[52],[65],[98],[100],[127]:

$$\underline{w}((k+1)T) = \underline{w}(kT) + \beta(kT) \underline{x}(kT) e^*(kT) \quad (4.58)$$

where the adaptation error signal, $e(kT)$, is defined as the difference between the transmitted data (during training or ideal-reference mode) or the decision data (during decision-directed data mode) and the DFE output signal:

$$e(kT) = a((k-D)T) - y(kT) = a((k-D)T) - \underline{w}^*(kT) \underline{x}(kT) \quad (4.59)$$

and the relevant vectors are defined in Equation (3.40) to Equation (3.44) on page 50.

The derivation of the LMS algorithm is based on the objective of minimizing a statistical MSE measure. However, because it explicitly uses actual data samples the LMS algorithm can only approximate the statistical optimum performance. In general, the LMS algorithm exhibits relatively slow convergence since it performs only a one-dimensional adjustment of the filter coefficients at each iteration. Moreover, its convergence speed is sensitive to the spectral properties (i.e., eigenstructure) of the filter input data. Specifically, a less spectrally flat input signal (i.e., with an autocorrelation matrix possessing a greater eigenvalue spread) tends to result in a slower MSE convergence [48],[52],[65],[98],[100],[127].

In order to achieve stability of the LMS algorithm, the step size has to satisfy the following condition, assuming unit average signal power at the input to the equalizer:

$$0 < \beta < \frac{2}{(N_f + N_b)} \quad (4.60)$$

The LMS adaptation step size, $\beta(kT)$, may be fixed or variable during adaptation. For instance, in order to speed up convergence while maintaining a sufficiently small steady state MSE, a “gear-shifting” (GS-1) strategy could be used to adjust the LMS step size. In this algorithm, the LMS step size is fixed at a relatively high initial value, β_i , during training and is reduced, in one step, to a much smaller final value, β_f , during the data mode.

Alternatively, the LMS step size can be gradually reduced from a high initial value to a suitably small value which is then fixed for the duration of the data sequence. Typically this gradual change in the step size can be implemented during the convergence phase in the training period or throughout the training and data sequence. A non-linear version of this "stochastic approximation" algorithm, GS-2, can be expressed as:

$$\beta(kT) = \frac{\rho\beta_i}{\rho + k} \quad (4.61)$$

where the decrease in step size is non-linear in time with the parameter, ρ , determining the variable rate of change in step size:

$$\frac{\Delta\beta(kT)}{\Delta k} = -\frac{\beta(kT)}{\rho + k} \quad (4.62)$$

A linear version, GS-3, of this "stochastic approximation" algorithm can be written as:

$$\beta(kT) = \beta_i - (\beta_i - \beta_f) \left(\frac{k}{N_t} \right) \quad (4.63)$$

where an initial LMS step size, β_i , is reduced linearly to a final value, β_f , in time intervals of N_β symbol periods over a training duration of N_t symbols. This GS-3 algorithm has a constant rate of change in step size which decreases linearly with increasing time:

$$\frac{\Delta\beta(kT)}{\Delta k} = -\frac{\beta_i - \beta_f}{N_t} \quad (4.64)$$

The time evolution of the LMS step size for these three algorithms are shown in Figure 4.9 on page 130. For the GS-2 algorithm, the parameter, ρ , has a value of 10,000. Furthermore, all the LMS adaptation parameters have been chosen from preliminary trial-and-error experimentation.

It will be shown in Section 4.3.2 on page 132 that these three GS LMS step size adjustment strategies produce essentially the same slow MSE convergence in the presence of interference. While it was not pursued in this study, a normalized LMS algorithm where the step size is dependent on the input signal level can lead to improved MSE convergence in a quasi-static fading environment [3],[48]. As an alternative to the LMS algorithm, the more computationally intensive RLS algorithm is considered for training an equalizer in the presence of interference (see Section 4.4 on page 149).

Figure 4.9 Gear-shifting and stochastic approximation LMS step size adjustment during training period of 19,000 symbols and fixed at 0.003 during tracking period of 1000 symbols. For SIR = 0, 5 dB, GSL-1,2,3 are used. For SIR = 10 to 30 dB, GSH-1,2,3 are used. For all SNR, GSNR-1 is used.

4.3.1 MSE Convergence Analysis

The LMS DFE MSE is defined as the expected value of the squared magnitude of the adaptation error (see Equation (4.59)). The time evolution of the MSE at the output of an LMS DFE in the presence of interference can be calculated analytically. Such an MSE convergence analysis for the constant step size LMS algorithm as applied to filtering in the presence of noise is well known under reasonable approximating assumptions (e.g., "independence theory") [48],[52],[53],[120]. This analysis has also been applied to the MSE convergence of a wide-bandwidth DFE in the presence of CDMA interferers [3]. The derivation of this analytical result is presented in Appendix F on page 234.

To summarize, the MSE as a function of the k th symbol time instant can be expressed as:

$$MSE(kT) = \sum_{i=0}^{N_f+N_b-1} \gamma_i c_i^k + \frac{MMSE}{1 - \sum_{i=0}^{N_f+N_b-1} \beta \lambda_i / (2 - \beta \lambda_i)} \quad (4.65)$$

Equation (4.65) shows that the MSE convergence and steady state performance depends on the LMS constant step size, β , the number of filter (e.g., DFE) tap coefficients, $N_f + N_b$, and the eigenvalues, c_i , of the matrix B which are related to the eigenvalues, λ_i , of the input signal autocorrelation matrix R (see Appendix F on page 234). For instance, MSE convergence stability requires that the LMS step size, β , satisfy the following inequality [48]:

$$\sum_{i=0}^{N_f+N_b-1} \beta \lambda_i / (2 - \beta \lambda_i) < 1 \quad (4.66)$$

Under the approximating condition that $\beta \ll 2/\lambda_{max}$, this condition for MSE convergence stability becomes:

$$\beta < \frac{2}{(N_f + N_b) P_{in}} \quad (4.67)$$

where P_{in} is the average input signal power to the LMS filter.

Furthermore, the LMS MSE convergence is stable as long as all the real positive eigenvalues, c_i , of the matrix B are less than unity. In this case, the transient component of the excess MSE consists of a sum of weighted geometric sequences each of which decay with increasing time. Moreover, since the geometric sequences of the transient excess MSE

eigenmodes have exponential envelopes, their exponential decay with time can be characterized with time constants, τ_i [48],[127]. That is, the exponential envelope for the i th transient excess MSE eigenmode can be written as:

$$c_i = e^{-1/\tau_i} \quad (4.68)$$

where τ_i is the time for the i th transient excess MSE eigenmode to decay by a factor of $1/e$ from its initial value. Note that under relatively slow convergence with c_i close to but less than unity such that $\tau_i \gg 1$, the time constant can be approximated as:

$$\tau_i = \frac{1}{1 - c_i} \quad (4.69)$$

Equation (4.65) indicates that the MSE convergence performance can also be viewed as dependent on two components of its constituent eigenmodes: 1) the exponential decay of the geometric sequences as governed by the eigenvalues, c_i , or equivalently the time constants, τ_i ; and 2) the relative magnitude of the MSE eigenmodes as dictated by the weighting factors, γ_i . The smaller the eigenvalue, c_i , the faster the decay for the i th component of the excess MSE. Conversely, the MSE convergence speed tends to be limited by the largest eigenvalue, c_i , with an appreciable weighting factor, γ_i , which exhibits the slowest MSE decay (e.g., see Section 4.3.2.1 on page 138).

Moreover, the MSE convergence speed is influenced significantly by the eigenvalue spread, $\lambda_{max}/\lambda_{min}$, of the autocorrelation matrix R . In general, a larger eigenvalue spread tends to result in slower MSE convergence. Section 4.3.2 on page 132, which studies the LMS DFE MSE convergence in noise or in interference, shows that the significantly slower convergence of the LMS DFE in the presence of interference relative to that in noise is partially due to the larger eigenvalue spread of the input signal autocorrelation matrix in the interference case. This extremely slow MSE decay of the LMS DFE in interference is also due to the very low MMSE level to which it has to converge.

4.3.2 Calculated and Simulated MSE Convergence

The simulated MSE convergence performance of the LMS DFE employing the three GS step size adjustment algorithms is investigated. Figure 4.10 displays the time evolution of the MSE at the output of an optimally delayed DFE(5,2), representing 5 $T/2$ -spaced forward taps and 2 T -spaced feedback taps, whose weights are adapted with one of the three GS LMS algorithms (see Figure 4.9). The disturbance is either additive noise with

SNR = 10 dB or one CCI user with SIR = 10 dB. The ambient thermal noise is negligible at 100 dB below the desired signal power. A $T/2$ -spaced MPDP-1 multipath fading channel with three equal average strength paths, 100% excess bandwidth filtering and an ensemble of 100 trials with independent channel realizations are employed.

The training-assisted convergence period spans the initial 19,000 known data symbols while the decision-directed tracking data period spans the subsequent 1,000 unknown data symbols. For the case in noise with SNR = 10 dB, the GS-1 LMS algorithm is employed with $\beta_i = 0.03$ during training (i.e., convergence) and $\beta_f = 0.003$ during tracking (i.e., data) modes. For the case in the presence of one CCI user with SIR = 10 dB, all 3 GS LMS algorithms are employed during training while the LMS step size, $\beta_f = 0.003$, during the tracking data mode is the same as in the noise case for all 3 methods. For training in interference, the GS-1 LMS method uses a value of $\beta_i = 0.1$ while the GS-2 and GS-3 LMS algorithms use a value of $\beta_i = 0.14$.

Figure 4.10 shows that in the presence of noise the DFE MSE converges in less than a hundred symbol periods to its MMSE level close to the noise floor. However in the interference case, the DFE MSE converges much more slowly, requiring about 8,000 symbol periods to converge to within 2 dB of its MMSE level which is significantly lower than that for the noise case. Moreover, all three GS LMS algorithms exhibit similarly slow MSE convergence in the presence of interference. Thus, only the constant step size GS-1 algorithm is used in the subsequent study of the LMS equalizer.

The calculated and simulated MSE convergence of the constant step size GS-1 LMS DFE in the presence of noise or one CCI user is now investigated. Figure 4.11 and Figure 4.12, corresponding to SNR or SIR = 0 dB and 10 dB, respectively, depict the time evolution of the MSE at the output of a DFE(5,2) whose weights are adapted with a one-step GS-1 gear-shifting LMS algorithm. The $T/2$ -spaced MPDP-1 channel, 100% excess bandwidth filtering and an ensemble of 1000 trials with independent channel realizations are employed while the ambient thermal noise is assumed negligible at 100 dB below the desired signal power. The initial training period spans 19,900 known data symbols while the decision-directed tracking data period spans the subsequent 100 data symbols. In the noise case, the LMS algorithm is employed with $\beta_i = 0.03$ during training (i.e., convergence) and $\beta_f = 0.003$ during tracking (i.e., data) modes. For training in interference, the LMS algorithm uses values of $\beta_i = 0.05$ and 0.1 for SIR = 0 dB and 10 dB, respectively, while the LMS step size is again reduced to $\beta_f = 0.003$ during the tracking data mode.

Figure 4.10 Simulated gear-shifting or stochastic approximation LMS DFE(5,2) MSE convergence in the presence of additive noise (SNR = 10 dB) or in one CCI user (SIR = 10 dB).

Figure 4.11 Simulated and calculated constant step size LMS DFE(5,2) MSE convergence in the presence of noise (SNR = 0 dB) or in the presence of one CCI user (SIR = 0 dB)

Figure 4.12 Simulated and calculated constant step size LMS DFE(5,2) MSE convergence in the presence of noise (SNR = 10 dB) or in the presence of one CCI user (SIR = 10 dB)

The MSE convergence analysis for the constant step size LMS algorithm as applied to filtering in the presence of noise is well known under reasonable approximating assumptions [48],[52],[53],[120],[127]. Applying this same result, the time evolution of the MSE at the output of an LMS DFE in the presence of interference can also be computed analytically (see Section 4.3.1 on page 131). The calculated LMS DFE MSE convergence is shown as a heavily dotted curve for each of the cases corresponding to SNR or SIR = 0 dB and 10 dB in Figure 4.11 and Figure 4.12, respectively. These calculated MSE convergence curves are averaged over the same ensemble of channel realizations as for the simulated case. Furthermore, the DFE is deemed to have converged when its output MSE has decreased to within three decibels of its MMSE. Both the MMSE and MMSE+3dB levels are marked with solid horizontal lines on the DFE MSE convergence plots.

Figure 4.11 shows the DFE MSE convergence in the presence of noise or interference with SNR or SIR equal to 0 dB, respectively. In the noise case, the DFE MSE converges within 100 symbol periods, to its MMSE level close to the noise floor. However in the interference case, the DFE MSE converges much more slowly, requiring almost 3,000 symbol periods. Furthermore, the DFE MSE convergence in interference is particularly slow after it has dropped below the MMSE level of the noise case. Because the MMSE level in interference is almost 10 dB lower than that in noise, the LMS DFE takes appreciably longer to converge to this much lower MMSE level in interference. Thus, while the steady state DFE MMSE performance in interference is significantly better than that in noise, the LMS convergence time required to reach this improved operating point is prohibitive as it is well over an order of magnitude longer than that in noise.

A closer examination of the DFE MSE time evolution during the initial few hundred symbol periods shows that the convergence rate of the DFE MSE in interference is actually faster than that in noise. This is due to the higher initial value of the LMS step size used in the interference case. In fact, the LMS DFE in interference actually reaches the MMSE level corresponding to the noise case slightly faster than the LMS DFE in noise.

Figure 4.12 displays the LMS DFE MSE convergence in the presence of noise or interference with SNR or SIR equal to 10 dB, respectively. For this operating point, the MSE convergence time in noise is again less than 100 symbol periods. However, the MSE convergence in interference requires about 8,000 symbol periods, roughly two orders of magnitude longer than for the equivalent noise case and more than twice as long as for the SIR = 0 dB case. The MSE convergence time increases significantly for cases with a lower

MMSE level, e.g., in the presence of interference rather than noise, or with an increasing SIR. While a high interference (e.g., $\text{SIR} < 10$ dB) environment is of interest in order to maximize system capacity, it appears that the extremely slow MSE convergence of the LMS algorithm precludes its use for training a DFE in the presence of interference.

Nevertheless, there exists a potential trade-off between DFE MSE convergence time and steady state DFE MSE operating point. That is, one does not have to wait until the LMS DFE has fully converged before operating the equalizer in a decision-directed mode. For instance, Figure 4.12 shows that for $\text{SIR} = 10$ dB, the LMS DFE in interference converges to the MMSE level corresponding to an equivalent amount of noise in less than 100 symbol periods. However, for the LMS DFE in interference to yield 3 and 6 dB MSE improvements in interference over that in noise requires training periods on the order of 1000 and 10,000 symbol periods, respectively. Thus, it is evident that there is a diminishing rate of return on MSE reduction at the expense of training overhead. While it may be feasible to compromise between DFE MSE convergence time and steady state DFE MSE operating point, for a given desired or acceptable steady state MSE level, the long training time required for the LMS DFE is still prohibitive.

4.3.2.1 Investigation of the LMS Equalizer MSE Convergence

This section studies the influence of the individual eigenmodes on the MSE convergence of an LMS equalizer. In particular, it investigates the causes of the extremely slow MSE convergence of the LMS DFE in the presence of interference. MSE convergence results are obtained with the static real-valued $T/2$ -spaced multipath channels and $T/2$ -spaced equalizers of Section 4.2.3.1 on page 115. Thus, the optimum equalizer coefficients, MMSE performance and input signal eigenstructure as summarized in Table 4.1 also apply to this investigation. The particular LMS step sizes and training period are the same as those used for the constant step size GS-1 LMS algorithm of Section 4.3.2 on page 132. Moreover, the calculated and simulated MSE convergence curves are averaged over 100 independent trials while the ambient thermal noise is assumed to be negligible.

Figure 4.13 depicts the simulated and calculated time evolution, for the first 2000 training symbol periods, of the LMS LE(5,0) output MSE in the presence of $\text{SNR} = 10$ dB stationary noise or in $\text{SIR} = 10$ dB CCI due to one interferer. Figure 4.14 displays the same MSE convergence curves, for the complete training interval of 19,900 symbol periods, of an LMS DFE(5,2). The calculated LMS equalizer MSE convergence is shown as a heavy

dotted curve while both the MMSE and MMSE+3dB levels are marked with solid horizontal lines for each of the disturbance cases. As stated before, the equalizer is deemed to have converged when its output MSE has decreased to within three decibels of its MMSE.

The calculated and simulated MSE and convergence time for each of the four cases are listed in Table 4.2. In particular, the MSE convergence times of both the LE(5,0) and the DFE(5,2) in the presence of noise are similar at approximately 50 symbol periods. This is consistent with the fact that these two equalizers have about the same number of tap coefficients and MMSE performance. As for the MSE convergence in the presence of interference, the LE(5,0) converges within a slightly longer time of about 100 symbol periods while the DFE(5,2) convergence requires a drastically longer time of roughly 17,000 symbol periods. These convergence times are also consistent with their respective equalizer MMSE levels since the MMSE of the LE(5,0) and DFE(5,2) in interference is more than 3 dB and 10 dB, respectively, lower than that in noise. That is, convergence to a lower MMSE level generally requires a longer training period. However, this relationship is highly non-linear since the MSE convergence *rate* decreases as the MSE decreases.

This decay of both the MSE and its convergence rate with time can be observed in the MSE convergence curves of Figure 4.13 and Figure 4.14. In particular, the MSE convergence *rate* falls off dramatically after the initial, e.g., roughly 200, symbol periods. The reason for this is because the MSE convergence is a sum of weighted eigenmodes which have different exponential decay rates, or equivalently time constants (see Section 4.3.1 on page 131). The eigenmodes with higher decay rates, or smaller time constants, diminish quickly at the beginning of the training period. As training proceeds, the remaining significant contributors to the MSE are those eigenmodes with lower decay rates, or larger time constants. Thus, the MSE convergence curve begins to flatten out after the fast eigenmodes have essentially decayed, leaving the slow eigenmodes which limit the rate of convergence to the low MMSE level associated with operation in the presence of interference.

Slow eigenmodes stem from the small eigenvalues of the equalizer input signal autocorrelation matrix which result in a very large overall eigenvalue spread. Specifically, it has been observed (e.g., see Table 4.1) that there exist smaller eigenvalues, and thus larger eigenvalue spreads, of the DFE input autocorrelation matrix for the interference case which results in the much slower LMS DFE MSE convergence in the presence of interference than in noise. Note that these slow eigenmodes only dominate the MSE convergence if their weights are significant relative to the MMSE. Since the DFE typically performs

much better than the linear equalizer, the DFE in the presence of interference is most susceptible to one or more of these very slow eigenmodes dominating its MSE convergence.

The influence of the eigenmodes on the MSE convergence is investigated by observing their individual convergence behaviour. Table 4.3 lists the weighting factors, γ_i (see Equation (F.14) on page 236), geometric ratios, c_i , which are eigenvalues of the matrix B (see Equation (F.7) and Equation (F.12) on page 235), and the corresponding exponential decay time constants, τ_i (see Equation (4.68)), of each eigenmode for the four equalizer and disturbance cases. Furthermore, the time evolution of the MSE and its constituent eigenmodes are displayed in Figure 4.15 to Figure 4.18.

Figure 4.15 shows that the LE(5,0) MSE in SNR = 10 dB noise converges within 50 symbol periods. In particular, the three eigenmodes with appreciable weights that are larger than the MMSE, γ_0 to γ_2 , decay rapidly since they also have small time constants, τ_0 to τ_2 (see Table 4.3). Moreover, the two slower eigenmodes with longer time constants, τ_3 and τ_4 , are insignificant since they have weights, γ_3 and γ_4 , which are much smaller than the MMSE. This aggregate behaviour of the eigenmodes result in a relatively rapid MSE convergence for the LE(5,0) in the presence of noise. Similarly, Figure 4.16 shows that the DFE(5,2) MSE in SNR = 10 dB noise also converges within 50 symbol periods. In this case, there are five rapidly decaying eigenmodes while the two much slower eigenmodes have insignificant weights, γ_5 and γ_6 . This has been observed previously [120].

Figure 4.17 depicts the LE(5,0) MSE convergence in SIR = 10 dB CCI. Similar to the LE(5,0) in noise, the three significantly weighted eigenmodes exhibit rapid convergence with small time constants, τ_0 to τ_2 . However, the two slower eigenmodes possess larger weights, γ_3 and γ_4 , than those of the noise case. Consequently, the LE(5,0) convergence time in interference is somewhat longer at about 100 symbol periods. However, the DFE(5,2) convergence in interference as displayed in Figure 4.18 is markedly slower, requiring about 17,000 symbol periods for convergence. The MMSE is much smaller than that of the other three cases while the eigenmode weights and time constants are not that different. However, the DFE MSE convergence is dominated by the slowest eigenmode with a very large time constant, τ_6 , which also has a weight, γ_6 , that is significantly larger than the small MMSE. This eigenmode is depicted in Figure 4.18 as a nearly horizontal (i.e., very slow decay) heavy-dotted line that is appreciably higher than the MMSE level. This LMS MSE eigenmode behaviour in interference has not been observed previously.

Figure 4.13 Simulated and calculated constant step size LMS LE(5,0) MSE convergence in the presence of noise (SNR = 10 dB) or in the presence of one CCI user (SIR = 10 dB) for an example with static channel and co-channel responses

Figure 4.14 Simulated and calculated constant step size LMS DFE(5,2) MSE convergence in the presence of noise (SNR = 10 dB) or in the presence of one CCI user (SIR = 10 dB) for an example with static channel and co-channel responses

Disturbance:		Noise: SNR = 10 dB		Interference: SIR = 10 dB	
Equalizer:		LE(5,0)	DFE(5,2)	LE(5,0)	DFE(5,2)
Calculated MMSE [dB]		- 9.714	- 10.136	- 13.456	- 20.513
Calculated LMS Equalizer MSE [dB]	Steady State XSMSE	- 23.177	- 21.135	- 21.060	- 25.074
	Steady State MSE	- 9.522	- 9.804	- 12.761	- 19.210
	MSE at end of training	- 9.522	- 9.804	- 12.760	- 18.056
Simulated LMS Equalizer MSE [dB]		- 9.554	- 9.804	- 13.104	- 18.359
LMS Equalizer MSE Convergence Time [symbol periods]		50	50	100	17,000

Table 4.2 Calculated and simulated LMS equalizer MSE convergence and steady state MSE in the presence of additive noise or interference due to one CCI user for an example with static channel and co-channel responses. Frame and training lengths are 20,000 and 19,900 symbol periods, respectively.

Disturbance:		Noise: SNR = 10 dB		Interference: SIR = 10 dB	
Equalizer:		LE(5,0)	DFE(5,2)	LE(5,0)	DFE(5,2)
Weighting Factors	γ_0	0.483	0.289	0.395	- 0.072
	γ_1	0.152	0.022	0.165	0.285
	γ_2	0.246	- 0.026	0.340	0.307
	γ_3	0.0069	0.499	0.025	0.363
	γ_4	0.00074	0.110	0.023	0.022
	γ_5		0.00061		0.0074
	γ_6		0.000044		0.076
Geometric Ratios: Eigenvalues of the Matrix, B	c_0	0.912	0.910	0.732	0.651
	c_1	0.950	0.935	0.846	0.789
	c_2	0.978	0.942	0.937	0.829
	c_3	0.997	0.957	0.995	0.932
	c_4	0.9994	0.980	0.9996	0.993
	c_5		0.998		0.997
	c_6		0.9995		0.9998
Exponential Decay Time Constants [symbol periods]	τ_0	11	11	3	2
	τ_1	19	15	6	4
	τ_2	46	17	16	5
	τ_3	357	23	192	14
	τ_4	1666	48	2500	137
	τ_5		500		384
	τ_6		2000		5000

Table 4.3 Weights, eigenvalues and time constants of the exponentially decaying LMS equalizer MSE eigenmodes in the presence of additive noise or interference due to one CCI user for an example with static channel and co-channel responses.

Figure 4.15 Calculated convergence of the MSE and its constituent eigenmodes of a constant step size LMS LE(5,0) in the presence of noise (SNR = 10 dB) for an example with static channel and co-channel responses

Figure 4.16 Calculated convergence of the MSE and its constituent eigenmodes of a constant step size LMS DFE(5,2) in the presence of noise (SNR = 10 dB) for an example with static channel and co-channel responses

Figure 4.17 Calculated convergence of the MSE and its constituent eigenmodes of a constant step size LMS LE(5,0) in the presence of one CCI user (SIR = 10 dB) for an example with static channel and co-channel responses

Figure 4.18 Calculated convergence of the MSE and its constituent eigenmodes of a constant step size LMS DFE(5,2) in the presence of one CCI user (SIR = 10 dB) for an example with static channel and co-channel responses

4.4 DFE Directly Adapted with the RLS Algorithm

Least squares (LS) algorithms are designed subject to the criterion of minimizing a deterministic sum of squared error measure based on actual data samples. Consequently, in this problem of fitting a set of filter coefficients to a set of data samples, the LS solution is obtained with knowledge of all data up to the current time instant. With this greater use of the available data, an LS algorithm has more degrees of freedom to adjust the filter coefficients and thus exhibits much faster convergence than the LMS algorithm. Moreover, unlike the LMS algorithm, LS algorithms are relatively insensitive to the spectral properties of the input data to the adaptive filter [48],[52],[65],[98],[100]. The deterministic BLS algorithm used for CIR estimation as discussed in Section 4.2.1.1 is an example of an LS adaptive method. Furthermore, the recursive least squares (RLS) algorithm is an iterative LS adaptive method which is equivalent to the BLS algorithm with essentially the same performance, depending on the channel time variation, the RLS data window function and the BLS length over the data sequence.

As observed in Section 4.3.2 on page 132, the convergence of a DFE adapted with the LMS algorithm is extremely slow in the presence of interference. An alternative is to train the equalizer with the RLS algorithm. While the RLS algorithm is much more computationally intensive than the LMS algorithm³, under high SNR conditions the former algorithm exhibits much faster convergence in noise, with convergence times on the order of twice the number of filter tap coefficients [48],[52],[65],[98],[100]. Thus, this rapid convergence of the RLS algorithm in noise might also be mirrored in the case of interference.

The deterministic LS criterion attempts to minimize with respect to the adaptive filter (e.g., equalizer) coefficients, $\underline{w}(kT)$, the following sum of prewindowed squared errors (i.e., convolution of the squared error with an exponential window function) over an observation interval spanning all $k + 1$ data samples (indexed by $i = 0, 1, \dots, k$) up to the current time instant, $k = 0, 1, \dots, N_t - 1$, indexed up to the end of the training interval:

$$\sum_{i=0}^k \lambda^{k-i} |a((i-D)T) - y(iT)|^2 = \sum_{i=0}^k \lambda^{k-i} |a((i-D)T) - \underline{w}^*(kT) \underline{x}(iT)|^2 \quad (4.70)$$

3. The fast RLS (i.e., FRLS) family of algorithms, while achieving the same adaptive performance as an ordinary RLS algorithm, have a lower computational complexity on the same order as the LMS algorithm. However, FRLS algorithms suffer from numerical stability problems [48],[52],[65],[98],[100].

where the various data and coefficient vectors are defined earlier from Equation (3.40) to Equation (3.44) on page 50 with i and k being the cumulative and current time indices, respectively. Furthermore, D is the equalizer delay in symbol periods and $0 < \lambda \leq 1$ is a prewindow parameter which exponentially weights the data. Specifically, $\lambda = 1$ under stationary conditions weights all the prewindowed data equally. On the other hand, $\lambda < 1$ under non-stationary conditions is a forgetting factor which reduces the effective data window and weights older data samples progressively less. In the RLS formulation, the data samples are assumed to be zero-valued prior to time zero (i.e., $k = 0$).

The resulting deterministic LS solution for the optimum equalizer coefficient vector at the current time instant, k , is:

$$\underline{w}(kT) = \hat{R}^{-1}(kT) \hat{p}(kT) \quad (4.71)$$

where the sample autocorrelation matrix is:

$$\hat{R}(kT) = \sum_{i=0}^k \lambda^{k-i} \underline{x}(iT) \underline{x}^*(iT) \quad (4.72)$$

and the sample cross-correlation vector is:

$$\hat{p}(kT) = \sum_{i=0}^k \lambda^{k-i} \underline{x}(iT) a^*((i-D)T) \quad (4.73)$$

The performance advantage of the LS algorithm is due to the fact that a new optimum solution for a particular set of data is obtained at every time instant, k , which incorporates the new data information. This is in contrast to the simpler LMS solution which minimizes a single statistical MSE measure and is valid for a class of statistical data [48].

Note that for assumed jointly stationary data, $\underline{a}(kT)$ and $\underline{x}(kT)$, the LS time-averaged sample autocorrelation matrix and cross-correlation vector as defined in Equation (4.72) and Equation (4.73), respectively, are consistent estimators, for the infinite memory ($\lambda = 1$) case, and are asymptotically (as $k \rightarrow \infty$) related to their respective MMSE ensemble-averaged counterparts of Equation (3.47) and Equation (3.48) on page 50 as:

$$R = \begin{cases} (1/(1+k)) E[\hat{R}(kT)] & \lambda = 1 \\ (1-\lambda) E[\hat{R}(kT)] & \lambda < 1 \end{cases} \quad (4.74)$$

and

$$\underline{p} = \begin{cases} (1/(1+k)) E[\hat{\underline{p}}(kT)] & \lambda = 1 \\ (1-\lambda) E[\hat{\underline{p}}(kT)] & \lambda < 1 \end{cases} \quad (4.75)$$

In general, the LS solution of Equation (4.71) requires a matrix inversion which can be computationally prohibitive if it is performed at every time instant, k . One approach is the BLS solution (see Section 4.2.1.1) which is computed just once at the end of a given training interval. Furthermore, given a known training data sequence, the BLS sample autocorrelation matrix can be precomputed and stored at the receiver. An alternative iterative approach is the RLS algorithm which computes the LS solution at every time instant, k , without requiring an inversion of the sample autocorrelation matrix.

For the purposes of a time-recursive formulation, the LS solution of Equation (4.71) is written for time instant $k-1$:

$$\underline{w}((k-1)T) = \hat{R}^{-1}((k-1)T) \hat{\underline{p}}((k-1)T) \quad (4.76)$$

while Equation (4.72) and Equation (4.73) are rewritten respectively as:

$$\hat{R}(kT) = \lambda \hat{R}((k-1)T) + \underline{x}(kT) \underline{x}^*(kT) \quad (4.77)$$

and

$$\hat{\underline{p}}(kT) = \lambda \hat{\underline{p}}((k-1)T) + \underline{x}(kT) \underline{a}^*((k-D)T) \quad (4.78)$$

After appropriate substitutions involving these three equations, the RLS DFE coefficient update equation can be expressed as:

$$\underline{w}(kT) = \underline{w}((k-1)T) + \hat{R}^{-1}(kT) \underline{x}(kT) e^*(kT) \quad (4.79)$$

where the *a priori* adaptation error signal is defined as the difference between the transmitted data (during training or ideal-reference mode) or the decision data (during decision-directed mode) and the DFE output signal based on the LS DFE coefficient vector estimate of the previous time instant:

$$e(kT) = \underline{a}((k-D)T) - \underline{w}^*((k-1)T) \underline{x}(kT) \quad (4.80)$$

and the inverse of the sample autocorrelation matrix is obtained recursively, without explicit matrix inversion, by using the matrix inversion lemma [48],[52],[66]:

$$\hat{R}^{-1}(kT) = \frac{1}{\lambda} \left[\hat{R}^{-1}((k-1)T) - \frac{\hat{R}^{-1}((k-1)T) \underline{x}(kT) \underline{x}^*(kT) \hat{R}^{-1}((k-1)T)}{\lambda + \underline{x}^*(kT) \hat{R}^{-1}((k-1)T) \underline{x}(kT)} \right] \quad (4.81)$$

The LMS and RLS adaptive algorithms shown in Equation (4.58) and Equation (4.79), respectively, are quite similar with the exception of their adaptive gain parameter, $\beta(kT)$ and $\hat{R}^{-1}(kT)$, respectively. Specifically, the LMS update gain is simply a number which scales the input data vector while the RLS update gain consists of a more complicated matrix inverse which premultiplies the input data vector. In fact, this matrix inverse update gain factor, by orthogonalizing or decorrelating the autocorrelation matrix, essentially produces the RLS algorithm's insensitivity to the eigenstructure of the input data and therefore its superior convergence performance over that of the LMS algorithm.

The RLS algorithm can be realized more efficiently by writing the coefficient update equation, for time index $k = 1, 2, \dots, N_f - 1$, as [48]:

$$\underline{w}(kT) = \underline{w}((k-1)T) + \underline{g}(kT) e^*(kT) \quad (4.82)$$

where the Kalman gain vector is defined as:

$$\underline{g}(kT) = \hat{R}^{-1}(kT) \underline{x}(kT) = [1 + \underline{x}^*(kT) \underline{z}(kT)]^{-1} \underline{z}(kT) \quad (4.83)$$

with a dummy vector written as:

$$\underline{z}(kT) = \lambda^{-1} C((k-1)T) \underline{x}(kT) \quad (4.84)$$

and the inverse of the sample autocorrelation matrix defined as:

$$C(kT) = \hat{R}^{-1}(kT) \quad (4.85)$$

with the corresponding update equation of:

$$C(kT) = \lambda^{-1} C((k-1)T) - \underline{g}(kT) \underline{z}^*(kT) \quad (4.86)$$

The RLS algorithm updates the DFE coefficient vector with knowledge of the input and desired output data at the current time instant, k , and an assumed initial zero-valued DFE coefficient vector and an initial value for the sample autocorrelation matrix inverse:

$$C(0) = \hat{R}^{-1}(0) = \mu^{-1} I \quad (4.87)$$

where μ is a small positive constant

and I is the identity matrix.

With this specification of the initial conditions, the sample autocorrelation matrix in Equation (4.72) has an additional term, $\lambda^k \mu I$. As time evolves, this initial condition term is a small additive constant or tends to zero for $\lambda = 1$ and $\lambda < 1$, respectively. Furthermore,

the initial condition specified in Equation (4.87) results in a small bias in the RLS coefficient estimate in Equation (4.71). However, this estimate can be shown to be asymptotically unbiased so that the RLS estimate is convergent in the mean [48].

4.4.1 MSE Convergence Analysis

The RLS MSE at the k th symbol time instant is defined to be the expected value of the squared magnitude of the *a priori* adaptation error signal for the RLS algorithm (see Equation (4.80)). Furthermore, it can be shown that the time dependence of the RLS MSE can be expressed as [48],[52]:

$$MSE(kT) = (\sigma_i^2 + \sigma_n^2) + tr[RK((k-1)T)] \quad (4.88)$$

For large values of symbol time instant k , the following approximation is valid:

$$K(kT) \approx \frac{(\sigma_i^2 + \sigma_n^2)}{k} R^{-1} \quad (4.89)$$

Therefore, for the case when $\lambda = 1$, the RLS MSE can be approximately expressed as a function of time as:

$$MSE(kT) = (\sigma_i^2 + \sigma_n^2) \left(1 + \frac{(N_f + N_b)}{k}\right) \quad (4.90)$$

As a rule of thumb, the MSE of a filter trained with the RLS algorithm with infinite memory converges, in terms of the number of iterations, within about twice the number of filter tap coefficients. Moreover, in contrast to the stochastic gradient algorithm which has a finite steady state excess MSE, the deterministic RLS algorithm with infinite memory has zero steady state excess MSE.

On the other hand, for the exponentially windowed case when $\lambda < 1$, the RLS MSE is essentially independent of time:

$$MSE(kT) \approx (\sigma_i^2 + \sigma_n^2) \left(1 + (N_f + N_b) \frac{(1 - \lambda)}{(1 + \lambda)}\right) \quad (4.91)$$

Thus, the RLS algorithm with finite memory exhibits an asymptotic excess MSE which is due to the exponential windowing of the data.

4.4.2 Calculated and Simulated MSE Convergence

As an alternative to the LMS algorithm, the DFE can be trained with the more computationally complex RLS algorithm to achieve faster MSE convergence in the presence of additive noise and/or interference. In this case, there exists a trade-off between computational complexity and performance in terms of MSE convergence time and training overhead. Figure 4.19 and Figure 4.20, corresponding to SNR or SIR = 0 dB and 10 dB, respectively, display the time evolution of the MSE at the output of a DFE(5,2) whose weights are adapted with the RLS algorithm. In both the noise case and the interference case, the RLS initialization matrix constant, μ , is 0.01 while the RLS adaptation parameter, λ , takes on values ranging from 0.95 to 1.0.

The $T/2$ -spaced MPDP-1 channel and 100% excess bandwidth filtering as for the LMS DFE case are used here. In addition, the MSE results are averaged over an ensemble of 1000 trials with independent channel realizations. The training-assisted convergence period spans the initial 900 known data symbols while the decision-directed tracking data period spans the subsequent 100 unknown data symbols. The calculated RLS DFE MSE convergence is shown as a heavy dotted curve for each of the cases corresponding to SNR or SIR = 0 dB and 10 dB in Figure 4.19 and Figure 4.20, respectively. These calculated MSE convergence curves are averaged over the same ensemble of channel realizations as for the simulated case. The DFE is deemed to have converged when its output MSE has decreased to within three decibels of its MMSE. Both the MMSE and MMSE+3dB levels are marked with solid horizontal lines on the DFE MSE convergence plots.

Figure 4.19 displays the RLS DFE MSE convergence behaviour for either SNR or SIR equal to 0 dB. Specifically, for the case in noise, the RLS DFE converges within about 20 symbol periods while for the equivalent case in the presence of one CCI user, convergence is achieved within about 30 symbol periods. Furthermore, Figure 4.20 shows similar RLS DFE MSE convergence behaviour for the cases of either SNR or SIR equal to 10 dB. At the higher SIR condition, the MSE convergence time in interference is slightly longer at about 50 symbol periods. These convergence times for a seven tap filter (i.e., DFE(5,2)), is roughly consistent with the rule of thumb that RLS MSE convergence is attained within the number of symbol periods equalling about twice the number of filter coefficients.

Figure 4.19 Simulated and calculated RLS DFE(5,2) MSE convergence in the presence of noise (SNR = 0 dB) or in the presence of one CCI user (SIR = 0 dB)

Figure 4.20 Simulated and calculated RLS DFE(5,2) MSE convergence in the presence of noise (SNR = 10 dB) or in the presence of one CCI user (SIR = 10 dB)

More importantly, the convergence speed of the RLS DFE is clearly much faster than that of the LMS DFE. This is particularly significant for the case in interference where the RLS algorithm is shown to be essentially insensitive to the nature of the interference. This is in contrast to the LMS algorithm whose convergence performance depends strongly on the interference power and the eigenvalue spread of the input signal autocorrelation matrix. For instance when $SIR = 10$ dB, the MSE convergence in interference is over two orders of magnitude faster with RLS than with LMS.

Under the quasi-static fading channel conditions, the MSE convergence speed does not really depend on the value of the RLS adaptation parameter, λ , as long as it is close to unity. This is particularly true for the case in noise. However, for the case in interference, a value of λ slightly less than unity results in marginally faster convergence, especially for higher SIR conditions. As inferred from the RLS DFE MSE convergence curves, values of λ below 0.98 result in numerical instability in the RLS algorithm. Such MSE divergences are evident in both Figure 4.19 and Figure 4.20. Note also that the RLS algorithm is also susceptible to decision-directed error propagation. For a given channel disturbance level, the DFE tends to be more susceptible to decision errors during tracking in interference than in noise. This stems from the fact that the DFE MSE operating level in interference is much lower than that in an equivalent amount of noise. Furthermore, for the case in interference this problem is more noticeable under high interference levels when decisions are more likely to be in error. For instance when $SIR = 0$ dB, adaptation subject to possible decision errors can degrade the MSE operating level by about two decibels.

4.5 Comparative MSE Performance of the Computed MMSE DFE and the Directly Adapted RLS DFE

It is of interest to compare the MSE performance of the computed DFE and the RLS DFE under typical interference-limited quasi-static channel conditions. The approach taken here is to compare the excess MSE expressions for these two adaptive DFE schemes and to arrive at an approximate analytical comparison of their relative performance.

From the asymptotic MSE expression of the RLS DFE with infinite memory, i.e., $\lambda = 1$, (see Equation (4.90)), the corresponding asymptotic excess MSE, denoted here by $XSMSE_{RLS}$, as a function of the estimation overhead in terms of the training sequence length, N_e , can be expressed as:

$$XSMSE_{RLS} = \frac{(N_f + N_b)}{N_e} \sum_{p=0}^{d-1} R_{i+n,pp} = \frac{(N_f + N_b)}{N_e} \left(\frac{\sum_{p=0}^{d-1} R_{rr,pp}}{1 + SINR_{in}} \right) \quad (4.92)$$

where the last equality used the $SINR_{in}$ expression of Equation (4.49).

An upper bound for the excess MSE for the computed DFE employing CIR and interference plus noise autocorrelation estimators is given in Equation (4.52) and is denoted here as $XSMSE_{EST}$. Thus, substituting Equation (4.92) into Equation (4.52) yields:

$$XSMSE_{EST} \leq \frac{XSMSE_{RLS}}{\frac{d-1}{(N_f + N_b) \sum_{p=0}^{d-1} R_{rr,pp}}} \times \left\{ 4|\underline{w}_{of}|^2 \left(1 + \frac{\sum_{p=0}^{d-1} \sum_{l=0}^{N_f-1} |R_{i+n,pl}|^2}{\sum_{p=0}^{d-1} R_{i+n,pp}} \right) - 4Re\{\underline{w}_o\} Re\{\underline{p}\} + N_f \right\} \quad (4.93)$$

Without loss of generality, it is assumed that the desired overall CIR has unit energy, $|h|^2 = 1$, hence implying that:

$$\sum_{p=0}^{d-1} R_{rr,pp} = 1 \quad (4.94)$$

If it is further assumed that the number of DFE feedback filter taps, N_b , is small compared to the number of DFE forward filter taps, N_f , (or in the limit, a linear equalizer is assumed), then Equation (4.93) can be written as:

$$XSMSE_{EST} \leq XSMSE_{RLS} (1 + \Delta) \quad (4.95)$$

where the difference parameter, Δ , is expressed as:

$$\Delta = \frac{4}{(N_f + N_b)} \left\{ |\underline{w}_{of}|^2 \left(1 + \frac{\sum_{p=0}^{d-1} \sum_{l=0}^{N_f-1} |R_{i+n,pl}|^2}{\sum_{p=0}^{d-1} R_{i+n,pp}} \right) - Re\{\underline{w}_o\} Re\{\underline{p}\} \right\} \quad (4.96)$$

For typical channel statistics, the energy in the DFE forward filter is greater than unity (i.e., $\|\underline{w}_{df}\|^2 > 1$). From the MMSE expression of Equation (3.49) on page 50 which assumes that the desired data has unit variance, it is evident that:

$$\text{Re}\{\underline{w}_{df}\} \text{Re}\{p\} \leq \underline{w}_{df}^* p < 1 \quad (4.97)$$

Thus, it can be inferred from Equation (4.96) that the difference parameter, Δ , is a positive quantity. Furthermore, comparing Equation (4.52) and Equation (4.96) shows that for typically large N_e and $SINR_{in}$, the upper bound on $XSMSE_{EST}$ is tighter than Δ . Thus, it follows from Equation (4.95) that the RLS DFE performs better than the computed DFE.

$$XSMSE_{EST} > XSMSE_{RLS} \quad (4.98)$$

Note that when the communications system is real-valued, then the excess MSE of the computed equalizer can almost equal the upper bound in Equation (4.52). However, as discussed and demonstrated in Appendix B on page 213 and in Section 4.2.3.1, the accuracy of the upper bound on the excess MSE (and the difference parameter, Δ) is also dependent on the channel multipath and disturbance conditions. This upper bound can be fairly tight in the presence of noise while it can be quite loose in the presence of interference. This excess MSE analysis demonstrates that the RLS DFE can perform better than the computed MMSE DFE. Moreover, Section 5.2.2 on page 167 and Section 5.3.2 on page 175 present average SDR, BER and outage probability simulation results which also confirm this result in a quasi-static multipath fading channel environment.

4.6 Finite-Precision Considerations

This section discusses the finite-precision effects of a digitally implemented adaptive fractionally spaced equalizer operating in the presence of inter-user interference. Specifically, finite-precision effects for the three adaptive equalizers considered in this chapter are described. It is suggested that interference and receiver excess bandwidth can potentially degrade the ability of a digitally implemented adaptive equalizer to realize a significantly lower MSE than in the case of stationary noise with no interference, especially with fixed-point arithmetic. In addition, the nature of the disturbance and the receiver structure can impact the cost, design and implementation of a practical adaptive receiver whose performance can either degrade unacceptably from the ideal infinite-precision design or be improved at the expense of implementing higher-precision or floating-point digital signal processing and/or an alternative adaptive algorithm and structure.

As observed in Section 4.2.3.1 for the computed MMSE equalizer, Section 4.3.2 for the LMS equalizer and Section 4.4.2 for the RLS equalizer, the presence of interference can cause wide variations in the received signal spectrum which translate into very large eigenvalue spreads, i.e., ill-condition, for the received signal autocorrelation matrix. It has also been observed that this eigenvalue spread is typically much larger for the interference case than for the stationary noise case. Moreover, the autocorrelation matrix of a fractionally spaced equalizer is known to exhibit very small eigenvalues (e.g., see Table 4.1) [40],[75],[100],[101],[121].

Section 4.3.2.1 has demonstrated that the prohibitively slow MSE convergence of the LMS DFE is one consequence of this large eigenvalue spread in the presence of interference. In addition, finite-precision effects can be problematic for a digitally implemented receiver which attempts to achieve a performance gain in the presence of interference, since such a situation entails a fractionally spaced equalizer and interference, both of which can aggravate the eigenstructure of the received signal.

In general, an ill-conditioned autocorrelation matrix, i.e., with a large eigenvalue spread, magnifies the performance sensitivity to errors in calculations such as those involved with an adaptive algorithm which either explicitly, e.g., computed Wiener solution, or implicitly, e.g., LMS or RLS algorithm, inverts the autocorrelation matrix (see Appendix E on page 231). Equation (4.55) demonstrates that deviations in the autocorrelation matrix, e.g., due to quantization errors, translate into deviations in the filter coefficient or output MSE through components along its eigenvectors, with resulting errors being magnified by the reciprocal of the corresponding eigenvalues. Thus, errors, e.g., due to finite-precision effects, in the direction of eigenvectors with small corresponding eigenvalues are strongly magnified. Such offending errors can originate in either the algorithm, e.g., CIR and interference plus noise autocorrelation estimation errors involving the computed MMSE equalizer (see Section 4.2.3), or the digital implementation, e.g., quantization errors due to finite-precision representation. This section considers the latter source of errors on the performance of a practical adaptive equalizer.

A digitally implemented adaptive filter, e.g., equalizer, incurs finite-precision effects due to quantization [15],[48]. Specifically, quantization errors are introduced by the analogue to digital conversion of the input signals and the limited number of bits, i.e., finite word length, used to represent the internal quantities of the adaptive algorithm. These quantiza-

tion errors result in a performance degradation from that of the ideal infinite precision case. Moreover, this performance degradation encompasses the two issues of numerical accuracy and numerical stability.

Numerical accuracy is determined by how close the steady state performance of a digitally implemented adaptive filter can approach the ideal infinite-precision performance. Numerical accuracy is thus a function of the precision, i.e. number of bits, used to implement the computation and representation of the internal quantities of the adaptive algorithm. Consequently, numerical accuracy is related to the cost of implementation.

The notion of numerical accuracy implicitly assumes numerical stability. Numerical stability is satisfied when the performance difference between the digital implementation and the infinite-precision design is bounded at all times during both the transient and steady state operation of the adaptive filter. In other words, numerical stability implies that the adaptive filter energy remains finite, i.e., its coefficients do not "blow-up". Numerical stability is especially important for a continuously adaptive algorithm whose digital implementation may be vulnerable to the accumulation of quantization errors due to overflows in the internal representations. Since numerical stability is highly dependent on the particular adaptive algorithm, an increase in the accuracy of the representation, i.e., word length, of these algorithmic quantities may only delay the onset of instability but does not change the inherent stability of the adaptive algorithm [48].

The first scheme of the computed MMSE equalizer (see Section 4.2) is not a continuously iterative algorithm. Upon obtaining sufficiently accurate CIR and interference plus noise autocorrelation estimates (which can be found iteratively, e.g., RLS CIR estimation, and with relatively rapid convergence), the MMSE equalizer is computed via the Wiener solution. The latter part of equalizer computation is inherently stable. However, its numerical accuracy is quite sensitive to the eigenvalue spread of the autocorrelation matrix. For example, it has been shown that the performance degradation due to numerical inaccuracy with a given digital precision increases sharply if the eigenvalue spread exceeds a threshold [83]. Thus, in order to ensure adequate performance gains in the presence of interference which is associated with higher eigenvalue spreads, a greater precision implementation is required. Moreover, the estimation of the CIR and interference plus noise may also require a higher precision, especially under low interference conditions.

A resolution test chart featuring several groups of horizontal and vertical lines of varying thicknesses. Each group is accompanied by a numerical value indicating the resolution. The values include 1.0, 1.1, 1.25, 1.4, 1.6, 1.8, 2.0, 2.2, 2.5, 3.2, 3.6, 4.0, 4.5, and 5.0. Some values are accompanied by smaller text, possibly indicating units or standards.

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
1 - STANDARD REFERENCE MATERIAL 1010a
ANSI and ISO TEST CHART No. 21

As demonstrated in Section 4.3, the directly adapted LMS DFE exhibits extremely slow MSE convergence due to the LMS algorithm's sensitivity to the large eigenvalue spreads, in particular the dominant small eigenvalues, associated with an interference disturbance. Moreover, the steady state performance of a digitally implemented LMS equalizer can also be adversely affected by such dominant small eigenvalues. It is known that the numerical accuracy, i.e., deviation from the infinite-precision performance, of a digitally implemented LMS equalizer is inversely proportional to the step size⁴ [15],[39],[48]. Because there can exist dominant small eigenvalues which can amplify the adverse effect of a small step size on the numerical accuracy of a practical LMS equalizer, proper operation in the presence of interference may require a higher precision implementation.

While the MSE convergence of the directly adapted RLS equalizer has been shown in Section 4.4 to be insensitive to the eigenstructure of the received signal, the same cannot be assumed for the numerical stability of the RLS algorithm which also implicitly inverts the autocorrelation matrix. RLS algorithms, especially the computationally fast schemes, are notorious for their numerical instability [15],[48],[52]. Specifically, the numerical instability of the RLS algorithm stems from the subtraction in the recursion of the autocorrelation matrix inverse of Equation (4.81) which can result in a negative-definite matrix due to the accumulation of quantization errors. In the case of the computationally fast RLS schemes, the numerical instability is more severe due to hyperbolic rotational transformations which are very sensitive to the accumulation of quantization errors, especially along the eigenvector corresponding to the greater than unity eigenvalue [15]. These numerical instabilities can be exacerbated when the autocorrelation matrix is nearly singular, i.e., ill-conditioned. Thus, the numerical stability of a practical RLS equalizer operating in the presence of interference may require a higher precision implementation.

The use of excess transmission and receiver bandwidths requires the implementation of a fractionally spaced equalizer. It is known that the input signal autocorrelation matrix of a T/d -spaced equalizer has a fraction, $1/d$, of its eigenvalues which are significant while the other fraction, $(d-1)/d$, of eigenvalues are close to zero (down around the ambient noise level) [40],[75],[100],[101],[121]. As discussed earlier, this larger eigenvalue spread can render the digitally implemented fractionally spaced equalizer, especially in the pres-

4. Note that this is in contention with the infinite-precision case where the steady state excess MSE is proportional to the step size (see Equation (F.15) on page 236) [48].

ence of interference, more susceptible to finite-precision effects and degrade its numerical accuracy and the numerical stability of its adaptive algorithm.

It is known that the presence of very small eigenvalues can result in an adaptive filter's MSE being fairly insensitive to changes in its coefficients corresponding to these small eigenvalues [100]. Thus, numerical instability can arise as quantization errors accumulate along the directions of eigenvectors corresponding to these small eigenvalues (see Equation (4.55)). In this case, the coefficients can grow alarmingly large without the performance, e.g., in terms of the output MSE, being adversely affected until the coefficients overflow, i.e., "blow-up". Thus, the use of fractionally spaced equalizers associated with wide receiver bandwidths can also require higher digital precision in implementation.

4.7 Summary

A computed MMSE DFE which employs estimates of the CIR and interference plus noise autocorrelation has been demonstrated to function in the presence of interference and noise. An expression for the upper bound of the computed DFE excess MSE due to CIR and interference plus noise autocorrelation estimation errors has been derived as Equation (4.52). This excess MSE upper bound has been shown to be very accurate in the presence of noise. However, this excess MSE expression tends to produce rather loose upper bounds in the presence of interference. This existence of loose bounds in the presence of interference is due to the reliance of the excess MSE upper bound expression on the assumption that the autocorrelation and corresponding error matrices are diagonal. This assumption results in the excess MSE upper bound being sensitive to the large equalizer energies associated with the presence of interference.

The excess MSE formulation provides insight into how interference affects the computed MMSE DFE performance and the inherent differences in equalizer performance in the presence of interference and in stationary noise. Moreover, the excess MSE analysis shows that *the directly adapted RLS DFE performs better than the computed MMSE DFE whose performance is limited by its CIR and interference plus noise autocorrelation estimation errors.*

In addition, a perturbation analysis illustrates the sensitivity of the computed MMSE equalizer performance to small inaccuracies in the autocorrelation matrix and cross-correlation vector. This corroborates simulation results which indicate that *the computed*

MMSE equalizer performance requires very accurate estimates of the CIR and interference plus noise autocorrelation. Moreover, this sensitivity of the computed MMSE equalizer performance to estimation errors is exacerbated by the fact that the interference plus noise autocorrelation estimation depends on the accuracy of the CIR estimation.

The perturbation analysis also demonstrates that the eigenvalue spread of the autocorrelation matrix plays an important role in the sensitivity of the computed MMSE equalizer performance. It has been observed that *the eigenvalue spread, or condition number, of the autocorrelation matrix is typically orders of magnitude larger in the presence of interference than in stationary noise.* This eigenvalue spread, or condition number, of the autocorrelation matrix also has ramifications on the numerical accuracy and stability of a digitally implemented adaptive equalizer which is subject to finite-precision effects. In order to maintain its performance, *an adaptive fractionally spaced equalizer operating in the presence of inter-user interference may require a greater precision implementation.*

Simulation and calculation results show that *the directly adapted LMS DFE exhibits prohibitively slow MSE convergence in the presence of interference due to its sensitivity to the eigenvalue spread of the equalizer input signal autocorrelation matrix and the existence of a significantly lower MMSE in interference than in an equivalent amount of stationary noise.* Furthermore, it has been demonstrated that the extremely slow convergence of the LMS DFE in the presence of interference is caused by one or two very slow-decaying significant eigenmodes which have weights that are significant relative to the MMSE. Thus, *the LMS algorithm for training the DFE in the presence of interference does not realize the full potential reduction in MMSE within a reasonable convergence time.*

Alternatively, simulation and calculation results indicate that *the directly adapted RLS DFE, at the expense of higher computational complexity, exhibits much faster MSE convergence in the presence of interference than the directly adapted LMS DFE.* In fact, the MSE convergence of the RLS DFE in the presence of interference is practically as fast as that in the presence of stationary noise. These results indicate that the MSE convergence of the RLS DFE is essentially insensitive to the eigenstructure of its input signal, even in the presence of interference. Consequently, *the RLS algorithm should be employed to train the DFE in an interference-limited quasi-static multipath fading environment.* In order to reduce the computational effort under slowly fading conditions, the DFE adaptation algorithm could switch to normalized LMS after RLS convergence.

Chapter 5 Adaptive DFE and System Performance

5.1 Introduction

This chapter investigates the steady state performance of an adaptive fractionally spaced DFE in the presence of interference and noise. The DFE is either directly adapted with the RLS algorithm or computed via the Wiener solution with CIR and interference plus noise autocorrelation estimates. The performance criteria are the DFE output SDR, bit error rate (BER) and outage probability. The SDR and BER are averaged over the detected data sequence and the ensemble of independent quasi-static fading channel realizations.

Section 5.2 presents computer-simulated average SDR performance results which indicate that the directly adapted RLS DFE performs better than the computed MMSE DFE in the presence of strong interference and negligible ambient noise. This performance difference exists because the computed MMSE DFE requires very accurate estimates of the channel and disturbance parameters while the RLS DFE does not use such explicit estimates of the CIR and interference plus noise autocorrelation. These simulation results corroborate the analytical result of Section 4.5 on page 157 that the directly adapted RLS DFE offers superior performance and thus should be the preferred adaptive equalizer for a typical interference-limited quasi-static multipath fading channel environment.

The ideal DFE performance results of Chapter 3 are based on the MMSE DFE output SDR. Moreover, the adaptive DFE performance results of Chapter 4 and Section 5.2 are measured in terms of the DFE output MSE convergence and its average output SDR, respectively. While such performance results are important, it is of greater interest to the system designer to determine the average BER and outage probability. Consequently, Section 5.3 presents computer simulation results in terms of such system performance criteria to corroborate the main results of Chapter 3 and Section 5.2.

Section 5.3.2 considers the performance of a directly adapted RLS DFE and a computed MMSE DFE in the presence of a dominant CCI user and negligible ambient noise. These results demonstrate that the average BER and outage performance of a fractionally spaced DFE in the presence of a dominant interferer and negligible ambient noise can be signifi-

cantly superior to that in an equivalent amount of stationary noise. This performance gain is greatest for a receiver with perfect knowledge of the channel and disturbance. However, an adaptive receiver which has to estimate the channel and disturbance parameters can still realize a significant performance gain in interference relative to that in stationary noise. Moreover, system performance results further confirm the conclusion of Section 4.5 on page 157 and Section 5.2 that the directly adapted RLS DFE performs better than the computed DFE in the presence of strong interference and negligible ambient noise.

Section 5.3.3 investigates the performance of a directly adapted RLS DFE and a computed MMSE DFE in the presence of a dominant CCI user and ambient noise. It is observed that the performance gain in interference diminishes as the ambient noise power increases. In fact, a performance gain in the presence of a dominant interferer over that in an equivalent amount of stationary noise is possible only if the interference power is greater than the noise power, i.e., $\text{INR} > 0$ dB. Moreover, the performance advantage of the RLS DFE over the computed DFE is achievable only if $\text{INR} > 20$ dB. Otherwise, the presence of significant noise power reduces the effectiveness of an equalizer to mitigate the interference and results in equally poor system performance with either equalization scheme.

Section 5.3.4 considers the average BER and outage performance of a directly adapted RLS DFE in the presence of one CCI user, two ACI users and ambient noise with a constraint of $\text{SIR} = 10$ dB. These results confirm the observation of Section 3.5.3 on page 70 that in order to achieve a desired performance with a fixed overall interference level, it is best not to increase the frequency reuse factor in order to reduce the CCI at the expense of drastically decreasing the radio capacity. Rather, it is preferable to increase the channel spacing, at the expense of a slight decrease in the radio capacity, in order to avoid much of the ACI. An RLS DFE can then be used to mitigate the strong CCI and remaining ACI.

Section 5.3.5 investigates the performance of a directly adapted RLS DFE in the presence of one CCI user, two ACI users and ambient noise with the average CCI power fixed at $\text{SCCIR} = 10$ dB. Consistent with the results of Section 3.5.4 on page 75, it is observed that the use of a wide receiver bandwidth can improve the system performance in negligible ambient noise. This performance improvement through better interference suppression is more significant at smaller channel spacings which allow for sufficient spectral overlap of the ACI with the desired signal. While maintaining a desired performance, the use of a wide receiver bandwidth can either increase the radio capacity through a reduction in channel spacing or allow the equalizer to operate in a noisier environment.

5.2 Adaptive DFE SDR Performance

5.2.1 DFE Output MSE and Average SDR

The ideal DFE output MMSE of Equation (3.49) on page 50 is an optimum statistical performance measure. In practice, the performance criterion is the time-averaged mean squared error, $\langle MSE \rangle$, at the output of the adaptive DFE during the data mode after initial convergence. In this case, the $\langle MSE \rangle$ replaces the MMSE in the expression for the optimum SDR of Equation (3.61) on page 53 to yield the average SDR:

$$SDR = \frac{1 - \langle MSE \rangle}{\langle MSE \rangle} = \frac{1}{\langle MSE \rangle} - 1 \quad (5.1)$$

with the DFE output MSE being (time) averaged over the $N_d = 100$ QPSK data symbols and (ensemble) averaged over the $N = 1000$ independent random channel realizations:

$$\langle MSE \rangle = \frac{1}{NN_d} \sum_{n=0}^{N-1} \sum_{k=N_t}^{N-1-N_d+N_t-1} |a_n(kT) - y_n((k+D)T)|^2 \quad (5.2)$$

where $a_n(kT)$ is the k th transmitted data symbol in the n th trial,
 $y_n((k+D)T)$ is the corresponding DFE output signal
 and D is the optimally chosen equalizer delay

5.2.2 Computed MMSE DFE and Directly Adapted RLS DFE

The average SDR performance of a DFE(5.2) in the presence of one CCI user and negligible ambient noise is investigated as a function of both the SIR and training overhead. The SDR performance is displayed in Figure 5.1 to Figure 5.4. The DFE is either adapted directly with the RLS algorithm (curves: (5) and (6)) or computed under various assumptions about the receiver's knowledge of the CIR and interference (curves (2), (3), (4), (7) and (8)). Specifically, curve (2) assumes exact knowledge of the CIR and interferer CIR while curves (3) and (4) employ a sample estimate of the interference autocorrelation (see Section 4.2.2.1 on page 108) with exact knowledge of the CIR and an RLS CIR estimate, respectively. Furthermore, curves (7) and (8) assume the interference is receive-filtered stationary noise with exact knowledge of the CIR and an RLS CIR estimate, respectively. As a performance upper bound, the optimum DFE SDR is also ideally calculated from exact knowledge of the CIR and interferer CIR (curve (1)).

Figure 5.1 and Figure 5.2 present the DFF SDR performance as a function of the SIR for a training overhead of $N_t = 15$ and $N_t = 60$, respectively. Curves (1), (2) and (7) are independent of the training overhead since they assume exact knowledge of the CIR and interference. Furthermore, under weak interference conditions, i.e., $\text{SIR} > 10$ dB, the ideally simulated performance of curve (2) matches the ideally calculated performance of curve (1). However, due to the DFE's susceptibility to decision feedback error propagation at high interference levels, i.e., $\text{SIR} < 10$ dB, the simulated performance of curve (2) can be up to several dB in SIR worse than the ideally calculated performance of curve (1).

The simulated performance of curve (7) is worse by about 10 and 8 dB in SIR at high and low levels of interference, respectively, than the simulated performance of curve (2). This demonstrates the potential performance advantage of providing the DFE with explicit knowledge of the interference rather than assuming the interference is just receive-filtered stationary noise. Furthermore, in Figure 5.1 with a training overhead of $N_t = 15$ symbols, the simulated performance represented by curve (8) is several dB in SIR worse than that of curve (7). The exception is at low SIR values where the simulated performance is the same for both cases since the CIR estimation error is swamped by the strong interference. However, in Figure 5.2 with a longer training period of $N_t = 60$ symbols, curves (7) and (8) are virtually identical, indicating that the RLS CIR estimation error is negligible for this case when interference autocorrelation estimation is not performed.

The performance improvement of curve (4) over that of curve (8) is only a few dB in SIR under high interference conditions for the case in Figure 5.1 with a training overhead of $N_t = 15$ symbols. Moreover, the simulated performance of curve (4) is marginally worse than that of curve (8) under low interference conditions. This degraded DFE SDR performance in weak interference is attributed to the fact that in estimating the interference autocorrelation, the estimation gain is completely nullified by the estimation error.

On the other hand, the DFE SDR performance as represented by curve (3) is noticeably improved over that of curve (4). *This indicates the importance of an accurate CIR estimate in the interference autocorrelation estimation.* Specifically, this DFE performance gain is about 5 and 3 dB in SIR under high and low interference conditions, respectively. Furthermore, under high interference conditions, the DFE SDR performance as displayed by curve (3) is within 5 dB in SIR of the ideally computed DFE performance of curve (2), while it is over 6 dB in SIR better than that of curve (7).

Again, these simulated performance gains realized at significant interference levels are absent under low interference conditions of $SIR > 20$ dB. It is known that the computation of the optimum filter coefficients via the normal equations (see Section 3.4 on page 47) requires a fairly accurate estimate of the autocorrelation matrix due to the matrix inversion operation [48]. Moreover, it has been shown in Section 4.2.4 on page 126 that this Wiener solution can be quite sensitive, via the eigenvalue spread of the autocorrelation matrix, to small estimation errors. It appears that the interference autocorrelation estimate is not sufficiently accurate to allow the computed DFE to exploit the cyclostationarity of weak interference. Furthermore, it has also been previously noted that equalization in interference requires a higher coefficient accuracy than that in noise [94].

In the operating region of interest where there is significant interference, i.e., low SIR of around 10 dB or less, curves (3) and (4) in Figure 5.1 indicate that the performance of a DFE computed with CIR and interference autocorrelation estimates can be degraded by an inaccurate CIR estimate. On the other hand, the approach of directly adapting the DFE with the RLS algorithm seems to yield better performance. Specifically, while the RLS DFE SDR performance as indicated by curves (5) and (6) is still about 3 dB in SIR worse than that of curve (3), its performance is nevertheless about 3 dB in SIR better than that of curve (4). The RLS DFE with a value of λ just less than unity yields marginally better performance due to its slightly faster convergence, especially under high SIR conditions.

However, when the interference is negligible the SDR performance of the RLS DFE degrades as in the case of the computed DFE. This unimpressive performance for the RLS DFE under high SIR conditions is due to its incomplete convergence after $N_t = 15$ symbol periods. Moreover, as shown by curves (5) and (6) in Figure 5.2, increasing the training overhead to $N_t = 60$ symbols only improves slightly the performance of the RLS DFE under high SIR conditions as its convergence is still rather slow in this case.

Under high interference conditions in Figure 5.2, the SDR performance of the RLS DFE (curves (5) and (6)) is better by about 4 dB in SIR than that of the computed DFE (curve (4)). In fact, the performance of the RLS DFE with $\lambda = 0.98$ (curve (5)) matches that of curve (3). Furthermore, curves (7) and (8) are virtually identical, indicating that the RLS CIR estimation error is negligible with $N_t = 60$ training symbols. This, together with curves (3) and (4), implies that the computed DFE performance (curve (4)) can be limited by the effect of the LS CIR estimation error on the interference autocorrelation estimation.

Figure 5.1 Simulated SDR performance for an RLS and a computed MMSE DFE(5,2) in the presence of noise or in the presence of one CCI user for a training overhead of 15 symbols

Figure 5.2 Simulated SDR performance for an RLS and a computed MMSE DFE(5,2) in the presence of noise or in the presence of one CCI user for a training overhead of 60 symbols

Thus, the interference autocorrelation estimation used by the computed DFE requires an accurate CIR estimate. Consequently, the approach of directly adapting the DFE with the RLS algorithm can yield superior performance to that of computing the DFE which requires the explicit estimation of the autocorrelation matrix in terms of the CIR and interference. Also of note is that an LMS DFE, albeit requiring a prohibitively high training overhead of over 20,000 symbols, can also surpass the performance of the computed DFE. This is again due to the fact that the LMS DFE, like the RLS DFE, does not require an explicit estimation of the CIR and interference autocorrelation.

The DFE SDR performance as a function of the training overhead, N_T , for SIR = 0 and 10 dB is shown in Figure 5.3 and Figure 5.4, respectively. Within experimental error, the calculated and simulated SDR performance curves (1), (2) and (7) are independent of the training overhead since they assume exact knowledge of the CIR and interference. Furthermore, other observations made earlier regarding the SDR performance curves in Figure 5.1 and Figure 5.2 also apply here. From the curves in Figure 5.3 and Figure 5.4, it is observed that increasing the training overhead generally improves the DFE SDR performance in an interference-limited quasi-static multipath fading channel environment.

The DFE SDR performance of curve (3) approaches the ideal performance of curve (2) with a training overhead of about 100 and 200 symbols for SIR = 0 and 10 dB, respectively. However, the DFE SDR performance of curve (4) is worse than that of curve (3) by over 2 dB in SDR. Moreover, curves (7) and (8) yield practically the same SDR performance with a minimum training overhead of about 50 symbols. That is, the RLS CIR estimation error is essentially negligible. From these observations, it is again evident that the performance of a realistically computed DFE (curve (4)) is limited by its interference autocorrelation estimation which requires a very accurate CIR estimate.

The RLS DFE SDR performance of curves (5) and (6) is superior to that of a realistically computed DFE (curve (4)). These results for high interference conditions indicate that the SDR performance of a $\lambda = 0.98$ RLS DFE (curve (5)) with a minimum training overhead of about 30 symbols matches or surpasses that of a DFE computed with perfect CIR knowledge (curve (3)). Thus, it is inferred that an RLS DFE is superior to a computed DFE which requires explicit estimation of the CIR and interference autocorrelation.

Figure 5.3 Simulated SDR performance for an RLS and a computed MMSE DFE(5,2) in the presence of noise or in the presence of one CCI user (SIR = 0 dB) as a function of training overhead

Figure 5.4 Simulated SDR performance for an RLS and a computed MMSE DFE(5,2) in the presence of noise or in the presence of one CCI user (SIR = 10 dB) as a function of training overhead

5.3 System Performance

5.3.1 Average Bit Error Rate and Outage Probability

A standard performance measure of a digital communications system operating in a multipath fading environment is the bit error rate (BER) averaged over the fading channel statistics. In this simulation study, the symbol error rate is (time) averaged over the N_d data symbols per frame and (ensemble) averaged over a large number, N , of independent random channel realizations. With the assumption that for QPSK data modulation the bit error rate is half the symbol error rate¹, the average BER is computed from simulation as:

$$BER = \frac{1}{2NN_d} \sum_{n=0}^{N-1} \sum_{k=N_t}^{N_d+N_t-1} [a_n(kT) \oplus \hat{a}_n(kT)] \quad (5.3)$$

where $a_n(kT)$ is the k th transmitted QPSK data symbol in the n th independent trial, $\hat{a}_n(kT)$ is the k th detected QPSK data symbol in the n th independent trial and $\oplus$ is the exclusive-OR logical operator.

As discussed in Section 3.6 on page 81, symbol errors tend to occur in bursts for communications in a quasi-static multipath fading channel environment. Thus, the outage probability is a more meaningful performance measure in this case. Specifically, the outage probability is computed from simulation as:

$$\begin{aligned} P_{out} &= Prob(BER_{frame} > P_{oc}) \\ &= Prob\left(\frac{1}{2N_d} \sum_{k=N_t}^{N_d+N_t-1} [a_n(kT) \oplus \hat{a}_n(kT)] > P_{oc}\right) \end{aligned} \quad (5.4)$$

where BER_{frame} is the BER of a data frame for each independent trial in an ensemble and P_{oc} is the abscissa value representing the outage criterion for a data frame.

5.3.2 In the Presence of a Dominant Interferer and Negligible Noise

The average BER and outage performance of a $T/2$ -spaced DFE(5,2) in the presence of a dominant CCI user and negligible additive ambient noise, e.g., SNR = 100 dB, are investigated as functions of the SIR. The system performance is obtained from simulation over an ensemble of 5000 to 20,000 independent quasi-static fading channel realizations. For

1. This assumes the use of Gray coding whereby adjacent symbols in the signal constellation differ in only one bit position such that one symbol error most likely leads to one bit error [65],[98].

each trial, detection errors are counted over the $N_d = 100$ QPSK data symbols which follow the $N_t = 100$ training symbols. Moreover, the $T = 2$ -spaced MPDP-1 multipath profile channel with 3 equal-strength paths and 100% SRRC transmit and receive filtering are used (see Section 3.2 on page 28). The average BER results are displayed in Figure 5.5 while the outage probability results are shown in Figure 5.6 to Figure 5.8.

The DFE is either adapted directly with the RLS algorithm with $\lambda = 0.98$ (see Section 4.4 on page 149) in stationary noise or in CCI (e.g., curves (1) and (5) of Figure 5.5, respectively) or optimally computed (see Section 4.2 on page 94) with sample estimates of the CIR and/or CCI autocorrelation (curves (2), (4) and (6)). Specifically, curve (2) assumes the CCI is receive-filtered stationary noise and employs an RLS CIR estimate (see Section 4.2.1 on page 94) while curves (4) and (6) employ a sample estimate of the CCI autocorrelation (see Section 4.2.2.1 on page 108), with an RLS CIR estimate and exact knowledge of the CIR, respectively. As MMSE DFE performance bounds (see Section 3.4 on page 47) in the presence of CCI or an equivalent amount of receive-filtered stationary noise, curve (7) assumes exact knowledge of both the CIR and co-CIR while curve (3) assumes exact knowledge of the CIR, respectively.

The average BER performance as a function of the SIR is shown in Figure 5.5. The performance in noise (curve (1)) is the worst while the performance in CCI which the receiver assumes to be stationary noise (curves (2) and (3)) is better by about 2 dB in SIR. The fact that the performance of curves (2) and (3) are practically identical implies that RLS CIR estimation error is negligible with a training overhead of 100 symbols. Comparing curves (3) and (7) indicates that the ideal MMSE DFE can achieve a performance advantage of over 10 dB in SIR in the presence of strong interference and negligible additive noise relative to the case when the interference is an equivalent amount of stationary noise. Moreover, comparing curves (2) and (4) shows that accounting for the CCI in the adaptive DFE receiver can yield a performance gain of over 5 dB in SIR.

These results demonstrate that the average BER performance of a fractionally spaced DFE in CCI and negligible additive noise can be significantly superior to that in an equivalent amount of stationary noise. This performance gain is greatest for a receiver with perfect knowledge of the channel and disturbance. However, an adaptive receiver which has to estimate explicitly (e.g., computed MMSE DFE) or implicitly (e.g., directly adapted RLS DFE) the channel and disturbance parameters can still achieve a significant performance gain in interference relative to that in stationary noise.

Figure 5.5 Average BER for an RLS DFE and a computed MMSE DFE as a function of SIR due to a dominant CCI user with negligible additive noise for a training overhead of 100 symbols

Also of importance is the relative performance of a computed MMSE DFE employing explicit estimates of the CIR and CCI autocorrelation (curve (4)) and a directly adapted RLS DFE (curve (5)). This performance comparison indicates that the average BER performance under strong interference conditions of the RLS DFE is better by over 2 dB in SIR than that of the computed DFE. Thus, in the low SIR region of interest, the RLS DFE is shown to perform better than the computed DFE which is limited by its CCI autocorrelation estimation. Note that for $\text{SIR} > 10$ dB, the performance of the RLS DFE is only marginally superior to that of the computed DFE. This degraded RLS DFE performance is due to the fact that with a fixed training overhead of 100 symbols the RLS DFE does not fully converge under these lower CCI conditions. The use of a longer training sequence restores the performance advantage of the RLS DFE.

The outage performance for $\text{SIR} = 0$ dB and 10 dB is shown in Figure 5.6 and Figure 5.7, respectively. These outage probability curves again demonstrate the superior performance available under all outage criteria in the presence of strong CCI and negligible additive noise relative to that in an equivalent amount of stationary noise. Moreover, the outage probability results are consistent, in terms of the relative performance ranking of the various disturbance conditions and receiver specifications, with the DFE output SDR results of Section 5.2.2 and the average BER results of Figure 5.5.

For instance, comparing the outage performance of curves (4) and (5) indicates that the RLS DFE performs significantly better than the realistically computed DFE under these strong interference conditions of interest. Furthermore, the RLS DFE (curve (5)) performs practically as well as the computed DFE with exact knowledge of the CIR (curve (6)). In fact, with $\text{SIR} = 0$ dB and for outage criteria below 10%, the outage performance of the RLS DFE (curve (5)) is marginally better than that of the computed DFE with exact CIR information (curve (6)).

Figure 5.8 depicts the outage performance in the presence of strong CCI, i.e., $\text{SIR} = 0$ or 10 dB, and negligible additive noise of the RLS DFE (curves (2) and (4), respectively) and the computed DFE (curves (1) and (3), respectively). These outage probability curves (just curves (4) and (5) in Figure 5.6 and Figure 5.7) emphasize the performance advantage under strong interference conditions of the directly adapted RLS DFE over the computed DFE which employs CIR and CCI autocorrelation estimates.

Figure 5.6 Outage probability for an RLS DFE and a computed MMSE DFE with SIR = 0 dB due to a dominant CCI user with negligible additive noise and a training overhead of 100 symbols

Figure 5.7 Outage probability for an RLS DFE and a computed MMSE DFE with SIR = 10 dB due to a dominant CCI user with negligible additive noise and a training overhead of 100 symbols

Figure 5.8 Outage probability for an RLS DFE and a computed MMSE DFE as a function of SIR due to a dominant CCI user with negligible additive noise and a training overhead of 100 symbols

5.3.3 In the Presence of a Dominant Interferer and Noise

It is demonstrated in Section 5.3.2 that significant performance gains in the presence of interference relative to that in an equivalent amount of stationary noise are achievable when the receiver does not have to contend with any additive ambient noise. This section investigates the effect of appreciable ambient noise on the average BER and outage performance of a $T/2$ -spaced DFE(5,2) in the presence of a strong CCI user with SIR = 0 or 10 dB. For ambient noise levels of SNR = 0 dB to 60 dB and CCI levels of SIR = 0 or 10 dB, the corresponding SINR values are listed in Table 3.1 on page 63.

As with Section 5.3.2, this section also considers both the directly adapted RLS DFE and the computed MMSE DFE employing CIR and CCI autocorrelation estimates. Moreover, the system parameters are the same as that in Section 5.3.2 while the system performance is obtained from simulation over an ensemble of 5000 independent quasi-static fading channel realizations. The average BER performance is displayed in Figure 5.9 to Figure 5.11 while the outage performance is depicted in Figure 5.12 to Figure 5.14.

The average BER performance as a function of the SNR for SIR = 0 and 10 dB is shown in Figure 5.9 and Figure 5.10, respectively. Curve (1) models the inter-user disturbance as receive-filtered stationary noise. On the other hand, curves (2) and (3) essentially represent the inter-user disturbance of stationary interference as seen by the receiver. Curves (4) to (7) considers the inter-user disturbance as cyclostationary interference. By comparing the performance curves with SNR > 40 dB such that the ambient noise is negligible, it is confirmed that the equalizer performance is not interference-limited. In particular, the average BER performance gain in CCI is maximum for INR > 40 dB.

Specifically, by comparing curves (1) and (5) for the case with negligible ambient noise, i.e., SNR > 40 dB, performance gains in the presence of strong CCI over that in the equivalent stationary noise case are observed. Moreover, if the receiver, e.g., computed MMSE DFE, in the presence of strong CCI is designed under the assumption that the CCI is stationary noise (curve (2)), then the performance gain in CCI relative to that of curve (1), which actually experiences the disturbance of stationary noise, is not nearly as significant. However, if the receiver does account for the CCI explicitly (i.e., computed MMSE DFE of curve (4)) or implicitly (i.e., directly adapted RLS DFE of curve (5)), then the performance gain is considerably greater.

Figure 5.9 Average BER for an RLS DFE and a computed MMSE DFE as a function of SNR with SIR = 0 dB due to a dominant CCI user for a training overhead of 100 symbols

Figure 5.10 Average BER for an RLS DFE and a computed MMSE DFE as a function of SNR with SIR = 10 dB due to a dominant CCI user for a training overhead of 100 symbols

The presence of significant ambient noise can drastically reduce this performance gain in CCI relative to that in an equivalent amount of stationary noise. In fact, a performance gain in CCI is not achievable if $\text{INR} < 0$ dB, regardless of the SIR value. Conversely, a performance gain in CCI over that in noise is possible only if the CCI is of greater strength than the noise, i.e., $\text{INR} > 0$ dB. Thus, the equalizer performance is indeed noise-limited.

A performance gain in CCI is only available if the overall disturbance has sufficient cyclostationarity for the fractionally spaced equalizer to exploit. Moreover, for a given (sufficiently wide) bandwidth, this cyclostationarity is reduced if the number of interferers and/or the stationary noise component of the disturbance is increased. As an increasing amount of stationary noise is introduced to the receiver, the optimum equalizer response will have to attempt to minimize not only the interference but also the additive noise. Thus, the equalizer increasingly compromises its interference mitigation in order to also minimize the noise enhancement. As the SNR decreases, the equalizer performance tends to converge to the worst-case performance in stationary noise of curve (1).

In addition, curves (1), (4), (5) and (7) of Figure 5.9 and Figure 5.10 are plotted in Figure 5.11 to highlight the performance comparison of the two adaptive equalizers for $\text{SIR} = 0$ and 10 dB. Comparing curves (2) and (3) for the $\text{SIR} = 0$ dB case or curves (6) and (7) for the $\text{SIR} = 10$ dB case again confirms that the RLS DFE yields superior performance to the computed DFE in the presence of strong CCI and negligible ambient noise, i.e., $\text{SNR} > 40$ dB. However, this performance advantage of the RLS DFE over the computed DFE vanishes as the additive noise power becomes appreciable relative to the interference power, i.e., $\text{INR} < 20$ dB. In fact, the high noise level associated with $\text{INR} < 20$ dB reduces the effectiveness of an equalizer to mitigate the interference. Thus, the resulting poor performance of both adaptive equalizers approaches that of the worst case in stationary noise.

These results are mirrored in the outage probability curves of Figure 5.12 to Figure 5.14 which compare the outage performance of these two adaptive equalizers. For instance, the outage performance of the two adaptive equalizers in Figure 5.12 and Figure 5.13 are practically identical for $\text{INR} < 20$ dB. However, the outage probability of the RLS DFE is consistently lower than that of the computed DFE for $\text{INR} > 20$ dB. This performance difference between the two adaptive equalizers is noticeable for the relevant outage criteria below 10%. Figure 5.14 also depicts this superior outage performance of the RLS DFE under low additive noise conditions.

Figure 5.11 Average BER for an RLS DFE and a computed MMSE DFE as a function of SNR with SIR = 0 or 10 dB due to a dominant CCI user for a training overhead of 100 symbols

Figure 5.12 Outage probability for an RLS DFE and a computed MMSE DFE as a function of SNR with SIR = 0 dB due to a dominant CCI user for a training overhead of 100 symbols

Figure 5.13 Outage probability for an RLS DFE and a computed MMSE DFE as a function of SNR with SIR = 10 dB due to a dominant CCI user for a training overhead of 100 symbols

Figure 5.14 Outage probability for an RLS DFE and a computed MMSE DFE as a function of SNR and SIR due to a dominant CCI user for a training overhead of 100 symbols

5.3.4 In the Presence of Interferers with SIR = 10 dB and Noise

This section considers the performance of a directly adapted $T/2$ -spaced RLS DFE(5,2) in the presence of one CCI and two (i.e., upper and lower) ACI users. The average BER and outage performance, simulated over an ensemble of 1000 independent quasi-static fading channels, are investigated as functions of the SNR and normalized channel spacing, $\tilde{f}_s$, which determines the relative contributions of the ACI and CCI to the overall interference, constrained to SIR = 10 dB. This hypothetical disturbance scenario is the same as that in Section 3.5.3 on page 70. A worst-case ACI reduction factor, $q_a = 1$, is assumed and 100% SRRC $T/4$ -spaced transmit and receive filtering is used.

Figure 5.15 displays the average BER as a function of the normalized channel spacing for different SNR values. The performance in an equivalent amount of stationary noise is also displayed as dotted horizontal lines. The SNR < 20 dB curves depict fairly poor performance for all normalized channel spacings. In this case, the significant ambient noise severely hampers the ability of the equalizer to filter the SIR = 10 dB interference.

As the SNR increases, the equalizer is better able to mitigate the interference and produces a commensurate improvement in the system performance. Moreover, the composition of the interference affects the system performance. As the normalized channel spacing is reduced from 2.0 (with no ACI and one CCI user) to a minimum of 1.247 (with no CCI and 2 ACI users), the system performance initially degrades and then improves. The worst performance is at $\tilde{f}_s = 1.45$ when the overall interference consisting of two ACI and one CCI users is most noise-like while the best performance is roughly at either $\tilde{f}_s = 2.0$ or $\tilde{f}_s = 1.247$ where the equalizer can better mitigate the interference. As was discussed in Section 3.5.3 on page 70, it is preferable to operate at a channel spacing close to $\tilde{f}_s = 2.0$, at the expense of a slight decrease in the radio capacity, in order to avoid much of the ACI. The RLS DFE can then be used to mitigate the relatively strong CCI and remaining ACI.

Figure 5.16 and Figure 5.17 show the average BER and outage probability (for an outage criterion of 1%), respectively, as functions of the SNR for selected values of the normalized channel spacing. Note that the performance for a given amount of interference is slightly better at $\tilde{f}_s = 2.0$ (no ACI, one CCI user) than at $\tilde{f}_s = 1.247$ (no CCI, two ACI users) when the ambient noise is negligible, i.e., SNR > 30 dB. However, this performance difference vanishes when the ambient noise becomes appreciable, i.e., SNR < 30 dB.

Figure 5.15 RLS DFE average BER as a function of normalized channel spacing and SNR, for a normalized receiver bandwidth of 1 and SIR = 10 dB

Figure 5.16 RLS DFE average BER as a function of SNR and normalized channel spacing, for a normalized receiver bandwidth of 1 and SIR = 10 dB

Figure 5.17 RLS DFE outage probability, for an outage criterion of 0.01, as a function of SNR and normalized channel spacing, for a normalized receiver bandwidth of 1 and SIR = 10 dB

5.3.5 In the Presence of Interferers with SCCIR = 10 dB and Noise

This section considers the performance of a directly adapted RLS DFE in the presence of ambient noise, one CCI and two (i.e., upper and lower) ACI users. Unlike the last section, this section fixes the average CCI power at 10 dB below the average desired signal power, i.e., SCCIR = 10 dB. The system performance is determined by the ability of the equalizer to handle, in addition to the fixed amount of CCI, the increasing amount of ACI which is manifested at the receiver as the normalized channel spacing, $\hat{f}_s$, is reduced. This interference mitigation and consequent performance improvement through the use of a wider receiver bandwidth is studied. The average BER and outage performance, simulated over an ensemble of 1000 independent quasi-static fading channels, are investigated as functions of the SNR and normalized channel spacing, $\hat{f}_s$.

A 100% excess bandwidth (i.e., $\bar{B}_t = 1$) SRRC transmit filter is used in all cases. $T/4$ -spaced transmit and receive filters and a $T/2$ -spaced DFE(5,2) are employed with a normalized receiver bandwidth of $\bar{B}_r = 1$ (i.e., 100% excess bandwidth), while $T/8$ -spaced transmit and receive filters and a $T/4$ -spaced DFE(9,2) are used with a normalized receiver bandwidth of $\bar{B}_r = 2$ (i.e., 300% excess bandwidth). Moreover, the system parameters of this section are the same as that in Section 3.5.4 on page 75 except that a nominal case ACI reduction factor, $q_a = \sqrt{3}$, is used here.

Figure 5.18 displays the SINR as a function of the normalized channel spacing, $\hat{f}_s$, and the SNR (see Section 3.2.1 on page 31). Note as the channel spacing is reduced, the SINR decreases due to the additional ACI present at the receiver input. In addition, the SINR is lower for a wider receiver bandwidth since it lets in more ACI. For instance, at a normalized channel spacing, $\hat{f}_s = 1.75$, the SACIR = 42.44 dB and 19.72 dB with $\bar{B}_r = 1$ and $\bar{B}_r = 2$, respectively. However, when the normalized channel spacing is reduced to $\hat{f}_s = 1.0$, the SACIR = 14.31 dB and 10.68 dB with $\bar{B}_r = 1$ and $\bar{B}_r = 2$, respectively.

Figure 5.19 displays the average BER as a function of SNR for $\bar{B}_r = 1$ or $\bar{B}_r = 2$, and for $\hat{f}_s = 1.0$ to 1.75. As discussed earlier, when the ambient noise is appreciable, i.e., SNR < 20 dB, there is little difference in the uniformly poor performance of all the curves. In this case, the amount of ACI introduced to the receiver as a function of $\hat{f}_s$ is insignificant relative to the larger noise power. Thus, the use of a wide bandwidth fractionally spaced equalizer to mitigate interference is fruitless in this case when the stationary noise is the dominant impairment.

Figure 5.18 SINR as a function of the normalized channel spacing, receiver bandwidth and SNR. The CCI power is fixed at $SCCIR = 10$ dB while a nominal ACI reduction factor is assumed.

Figure 5.19 RLS DFE average BER as a function of SNR, normalized channel spacing and normalized receiver bandwidth, with SCCIR = 10 dB

Comparison of the curves for a given receiver bandwidth (i.e., the odd and even numbered curves for $\bar{B}_r = 1$ and $\bar{B}_r = 2$, respectively) demonstrates that performance in the presence of negligible ambient noise (i.e., $\text{SNR} > 40$ dB) degrades as the channel spacing is decreased. That is, reducing the channel spacing introduces more ACI to the receiver which leads to worse performance at this lower SIR level. This performance degradation is due to not only an increase in interference power but also an increase in the number of significant interferers that the equalizer has to mitigate.

It is known that an increase in receiver bandwidth can provide the equalizer with more flexibility to suppress the interference [29],[94],[95]. Comparing the $\bar{B}_r = 1$ and $\bar{B}_r = 2$ curves for a given channel spacing indicates that a performance gain is available under low ambient noise conditions. Thus, an adaptive fractionally spaced DFE can benefit from using a wide receiver bandwidth when the stationary noise is not appreciable.

This benefit from the use of a wide receiver bandwidth can result in an increase in the radio capacity via a reduction in the channel spacing while maintaining a desired performance. For instance, the performance in the presence of negligible ambient noise of curve (2) with $\hat{f}_s = 1.0$ and $\bar{B}_r = 2$ is no worse than that of curve (3) with $\hat{f}_s = 1.25$ and $\bar{B}_r = 1$. Thus, with the use of a wider receiver bandwidth the performance has not degraded, actually improved slightly, even though the channel spacing has been decreased from $\hat{f}_s = 1.25$ to $\hat{f}_s = 1.0$. Using Equation (3.39) on page 43 or Figure 3.18 on page 80, it can be shown that this reduction in the channel spacing corresponds to a 25% increase in the radio capacity. However, this improvement does incur the expense of implementing an equalizer with a higher sampling rate associated with the wider receiver bandwidth.

This performance gain with the use of a wider receiver bandwidth is greater at smaller channel spacings where there exists sufficient spectral overlap of the ACI and desired signal to allow the equalizer to mitigate the interference more effectively, despite the higher ACI power present. While there exists some performance gains with the use of a wider receiver bandwidth at larger channel spacings, these gains are not as large, for the same required performance, as those at lower channel spacings. This reduction in the incremental capacity gains with decreasing channel spacing is a consequence of a positive second-order derivative of the radio capacity with respect to the channel spacing (see Equation (3.36) on page 42).

For instance, the performance in the presence of negligible ambient noise of curve (6) with $\tilde{f}_s = 1.5$ and a wider receiver bandwidth of $\bar{B}_r = 2$ is still worse than that of curve (7) with $\tilde{f}_s = 1.75$ and $\bar{B}_r = 1$. Using Equation (3.39) on page 43 or Figure 3.18 on page 80, it can be shown that this channel spacing reduction from $f_c = 1.75$ to $f_c = 1.5$ results in a radio capacity increase of 16.67%. However, in order to maintain the performance of curve (7), the channel spacing cannot be reduced as far down as $\tilde{f}_c = 1.5$ when using $\bar{B}_r = 2$. Thus, the radio capacity increase in this case is actually less than 16% when using a wider receiver bandwidth of $\bar{B}_r = 2$ to maintain the performance of curve (7).

Figure 5.20 and Figure 5.21 display the average BER and outage probability (for an outage criterion of 1%), respectively, as a function of SNR for $f_s = 1.0$ or $f_s = 1.75$, and $\bar{B}_r = 1$ or $\bar{B}_r = 2$. The performance in an equivalent amount of stationary noise is also displayed as curve (1) for $\tilde{f}_s = 1.0$ and curve (4) for $\tilde{f}_s = 1.75$. It is observed that the performance in stationary noise is significantly worse than that in interference due to one CCI and two ACI users. Furthermore, the performance of curve (1) is worse than that of curve (4) because the former is subjected to more noise since its equivalent ACI is stronger due to a smaller channel spacing of $\tilde{f}_s = 1.0$.

The performance in negligible ambient noise of curves (2) and (3) with $f_s = 1.0$ is appreciably worse than that of curves (5) and (6) with $\tilde{f}_s = 1.75$. The reason is because at the smaller channel spacing of $\tilde{f}_s = 1.0$, the receiver is exposed to more ACI and the resulting SINR is lower by roughly a couple of dB (see Figure 5.18). Also, the performance improvement in interference (curves (5) and (6)) over that in an equivalent amount of stationary noise (curve (4)) is much larger at the larger channel spacing of $\tilde{f}_s = 1.75$. This is because there is much less ACI at the larger channel spacing, implying that the bulk of the disturbance power is due to the dominant CCI. This results in greater cyclostationarity in the disturbance signal which aids the interference suppression ability of the equalizer.

Comparison of curves (2) and (3) shows that a performance gain from the use of a wider receiver bandwidth of $\bar{B}_r = 2$ is relatively significant at the smaller channel spacing of $\tilde{f}_s = 1.0$ where there exists greater spectral overlap between the ACI and the desired signal. However, comparison of curves (5) and (6) indicates that there is little performance advantage by using a wider receiver bandwidth at the larger channel spacing of $f_s = 1.75$. The performance gains in interference or through the use of a wide receiver bandwidth are nullified as the ambient noise power increases, i.e., $\text{SNR} < 10$ dB in this case.

Figure 5.20 RLS DFE average BER as a function of SNR, normalized channel spacing and normalized receiver bandwidth, with SCCIR = 10 dB

Figure 5.21 RLS DFE outage probability, for an outage criterion of 0.01, as a function of SNR, normalized channel spacing and normalized receiver bandwidth, with SCCIR = 10 dB

Figure 5.22 depicts the outage performance for $\bar{f}_s = 1.0$ with $\bar{B}_r = 1$ or $\bar{B}_r = 2$ in the presence of appreciable (i.e., SNR = 20 dB) and negligible (i.e., SNR = 40 dB) ambient noise. Comparing curves (2) and (3) for SNR = 20 dB, or curves (5) and (6) for SNR = 40 dB, confirms that performance gains resulting from the use of a wide receiver bandwidth are possible at all outage criteria. In addition, these gains are more significant at lower ambient noise levels.

Moreover, it is observed that the performance of curve (3) (with SNR = 20 dB and $\bar{B}_r = 2$) and curve (5) (with SNR = 40 dB and $\bar{B}_r = 1$) are practically the same at all outage criteria. This indicates that the use of a wider receiver bandwidth can allow the system to operate in the presence of higher ambient noise power while maintaining the same performance and radio capacity. In this case, the use of $\bar{B}_r = 2$ allowed the system to operate in an appreciable amount, viz., SNR = 20 dB, of ambient noise.

Figure 5.23 displays similar outage probability curves for $\bar{f}_s = 1.75$. In this case with less ACI due to a larger channel spacing, the performance gains with the use of a wide receiver bandwidth is not as significant as those for the case with more ACI due to a smaller channel spacing as shown in Figure 5.22. In fact, Figure 5.23 shows that there does not exist any performance gains through the use of a wider receiver bandwidth for the important outage criteria of 1% or less. Note also in both Figure 5.22 and Figure 5.23, the outage performance in the presence of interference is significantly better than that in an equivalent amount of stationary noise.

Thus, the use of a wider receiver bandwidth can result in two potential benefits while maintaining a desired system performance. As was just discussed, one benefit is the ability to operate in the presence of stronger ambient noise. The other benefit is the realization of a radio capacity gain under negligible ambient noise conditions. This latter benefit was discussed earlier with reference to Figure 5.19.

Figure 5.22 Outage probability for an RLS DFE as a function of normalized receiver bandwidth and SNR for a normalized channel spacing of 1.0

Figure 5.23 Outage probability for an RLS DFE as a function of normalized receiver bandwidth and SNR for a normalized channel spacing of 1.75

5.4 Summary

Average SDR, BER and outage performance results, obtained from simulation over an ensemble of quasi-static fading channel realizations, demonstrate that *a conventional adaptive fractionally spaced DFE can achieve considerable performance gains in the presence of strong interference and negligible ambient noise over that in an equivalent amount of stationary noise*. Moreover, these performance results confirm that, in an interference-limited quasi-static fading channel environment, *a DFE which is adapted directly with an RLS algorithm performs better than a computed MMSE DFE which requires explicit estimates of the CIR and interference plus noise autocorrelation*.

Unfortunately, *the performance gain in the presence of interference over that of the equivalent stationary noise case diminishes as the ambient noise power increases*. For instance, the performance gain in the presence of a dominant interferer is possible only if the interference power is greater than the noise power. Moreover, the performance advantage of the RLS DFE over the computed MMSE DFE is achievable only if the ambient noise power is at least about 20 dB below the interference power. Otherwise, the significant noise power affects the equalizer's ability to mitigate the interference and results in equally poor performance for both adaptive equalizers.

The use of a wide receiver bandwidth can improve the system performance in the presence of ACI, CCI and negligible ambient noise. This performance improvement through better interference suppression is more significant at smaller channel spacings which allow for sufficient spectral overlap of the ACI with the desired signal. *While maintaining a desired performance, the use of a wide receiver bandwidth can either increase the radio capacity through a reduction in channel spacing or allow the equalizer to operate in an environment with appreciable ambient noise*.

In order to achieve a desired performance with a fixed amount of interference due to a dominant CCI user and two ACI users, it is best not to increase the frequency reuse factor to reduce the CCI at the expense of drastically decreasing the radio capacity. Rather, it is preferable to increase the channel spacing, at the expense of a slight decrease in the radio capacity, to avoid much of the ACI. An RLS DFE can then be used to mitigate the strong CCI and remaining ACI.

Chapter 6 Conclusions

This thesis research has demonstrated the feasibility and performance advantage of a conventional adaptive fractionally spaced decision feedback equalizer operating in the presence of inter-user interference in a quasi-static multipath fading channel environment.

Simulation results, taken over an ensemble of multipath fading channel realizations, have shown that the average SDR and outage performance for an ideally computed DFE in the presence of inter-user interference and negligible ambient noise is superior to that in an equivalent amount of stationary noise. Simulation results have also demonstrated that a conventional adaptive fractionally spaced DFE can achieve average BER and outage performance gains in the presence of strong interference and negligible ambient noise. However, these performance gains diminish considerably as the ambient thermal noise becomes appreciable with a decreasing SNR. In fact, such performance gains can be achieved only if the interference is of greater strength than the ambient noise.

It has been observed that, while performance gains are compromised by the presence of strong stationary noise or a large number of interferers, the use of a wider receiver bandwidth can result in improved interference suppression as long as there is sufficient spectral overlap between the ACI and the desired signal. The use of a wide-bandwidth DFE results in either an increase in radio capacity through a reduction in channel spacing or the capability to operate in a noisier environment while maintaining a desired system performance.

A computed MMSE DFE which employs estimates of the CIR and interference plus noise autocorrelation has been demonstrated to function in the presence of inter-user interference and ambient noise. An expression for the upper bound of the computed DFE excess MSE due to CIR and interference plus noise autocorrelation estimation errors has been derived and shown to be very accurate in the presence of noise. However, this excess MSE expression tends to produce rather loose upper bounds in the presence of interference. Nevertheless, the excess MSE analysis has been used to infer the superior performance of the directly adapted RLS DFE relative to the computed MMSE DFE. Moreover, simulation results have confirmed that the average SDR, BER and outage performance in an interference-limited quasi-static fading channel environment of a system employing an RLS DFE is better than that with a computed MMSE DFE.

A perturbation analysis has illustrated the sensitivity of the computed MMSE equalizer performance to small inaccuracies in the autocorrelation matrix and cross-correlation vector. Corroborating simulation results have also indicated that the computed MMSE equalizer performance requires very accurate estimates of the CIR and interference plus noise autocorrelation. In addition, this perturbation analysis has demonstrated that the performance of an adaptive MMSE equalizer is highly sensitive to the eigenvalue spread of its input autocorrelation matrix. Moreover, it has been observed that the eigenvalue spread of the received signal autocorrelation matrix is typically orders of magnitude larger in the presence of interference than in stationary noise. Thus, this larger eigenvalue spread of the input signal autocorrelation matrix of an adaptive fractionally spaced equalizer operating in the presence of inter-user interference may necessitate a greater precision digital implementation in order to achieve sufficient numerical accuracy and stability.

Simulation and calculation results have shown that the directly adapted LMS DFE exhibits prohibitively slow MSE convergence in the presence of inter-user interference due to: (1) its sensitivity to the eigenvalue spread of the equalizer input signal autocorrelation matrix; and (2) the existence of a significantly lower MMSE in interference than in an equivalent amount of stationary noise. Furthermore, it has been revealed that the extremely slow convergence of the LMS DFE in the presence of interference is caused by one or two very slow-decaying eigenmodes which have weights that are significant relative to the MMSE. Thus, it is concluded that the LMS algorithm for training the DFE in the presence of interference does not realize the full potential reduction in MMSE within a reasonable convergence time.

Simulation and calculation results have indicated that the directly adapted RLS DFE, at the expense of higher computational complexity, exhibits much faster MSE convergence in the presence of interference than the directly adapted LMS DFE. In fact, the MSE convergence of the RLS DFE in the presence of interference is practically as fast as that in the presence of stationary noise. These results confirm that the MSE convergence of the RLS DFE is essentially insensitive to the eigenstructure of its input signal, even in the presence of interference. Consequently, it is concluded that the RLS algorithm should be employed to train the DFE in an interference-limited quasi-static multipath fading environment. In order to reduce the computational effort under slowly fading conditions, it is suggested that the DFE adaptation algorithm could switch to normalized LMS after RLS convergence.

6.1 Recommendations for Further Research

The following are some suggestions for further research stemming from this thesis work.

It is desirable to obtain an approximate expression or bound on the system performance as measured in terms of the average BER or outage probability. Mathematical analysis not only removes the need for prolonged and expensive computer simulation but can also provide insight into the system performance. However, an accurate performance analysis for a multipath fading system is intractable due to the residual ISI in the decision-point variable which complicates the averaging over the fading channel statistics. Performance bounds exist for a DFE operating in either non-fading or multipath fading environments [23],[81]. Under quasi-static channel conditions, a common approach is to calculate the BER or outage for a given set of static multipath channel responses and then resort to a Monte-Carlo simulation over an ensemble of the independently fading channel realizations. Such calculations can involve the computation of moments of the interference in the decision random variable [5],[8],[21],[32],[60],[86],[117].

It is of interest to consider the performance in a rapidly time-varying environment of the directly adapted RLS DFE and the computed MMSE DFE which employs decision-directed (e.g., LMS) CIR tracking and (e.g., sliding window) update of the interference plus noise autocorrelation. Most investigations into tracking in a rapidly time-varying channel environment either assume flat fading or model the inter-user interference as additive stationary noise [30],[31],[59],[68],[84],[85],[99],[132],[135]. Other studies assume that the inter-user interference is either known at the receiver or varies so slowly that it can be assumed to be quasi-static [21],[36],[46],[58],[94],[104],[119]. Thus, there seems to be a dearth of research into the tracking problem in an interference-limited multipath fading environment. Further to this decision-directed adaptation issue is the problem of decision error propagation in the tracking loop of both the adaptive equalizer and the adaptive CIR and interference plus noise autocorrelation estimators.

Alternative receiver structures can also be investigated. For instance, it is useful to study some form of a multi-user receiver employing maximum likelihood sequence detection which is known to perform better than the MMSE DFE receiver considered in this thesis [125]. Moreover, the use of antenna diversity in a flat fading channel has been shown to improve interference suppression and capacity [131],[132]. Thus, the use of antenna

diversity, perhaps in conjunction with a wide-bandwidth equalizer as suggested in [94], could be investigated for operation in a multipath fading environment.

Another issue is that the inter-user interference has non-Gaussian statistics and may not be fully characterized by first- and second-order statistical measures. However, the adaptive receiver approach of Wiener filtering considered in this thesis processes up to second-order statistics, viz., correlation functions. Thus, it is of interest to study whether or not higher-order statistical signal processing can yield better performance than Wiener filtering for digital communications in the presence of inter-user interference in a quasi-static multipath fading environment [37],[78],[87],[88]. Similarly, a non-linear receiver employing neural-network signal processing can also be investigated [70],[113]. Moreover, finite-precision effects of a digital implemented adaptive fractionally spaced decision feedback equalizer have been briefly discussed in this thesis. It would be informative to determine, via computer simulation or hardware implementation, the achievable performance of such a digitally implemented receiver.

This thesis research has assumed a fairly simplistic multipath fading channel model with equally spaced independently fading paths of equal average strength. It would be useful for system performance evaluation to employ realistic channel models which are more typical of indoor wireless environments. Another assumption made in this thesis is that the users' TDMA bursts are synchronized such that the interference does not vary within a time slot. When this is not the case, a user can be adversely affected by the increased interference due to another user who initiates its transmission burst within the same time slot. Thus, an appropriate TDMA burst start-up strategy is required to minimize the effect of these abrupt increases in the interference power. An alternative approach is to treat this as an interference or channel mismatch and consider it as a problem of estimating and tracking a rapidly time-varying channel. The agility and robustness of the DFE adaptation or CIR and interference correlation estimation could be investigated.

Appendix A Variance of a BLS CIR Estimator

The following develops the expression for the estimation variance of an unbiased BLS CIR estimator as formulated in Equation (4.14) on page 99. This result is well known for BLS CIR estimation in the presence of noise [18],[48],[66]. In this thesis, this result is applied to BLS CIR estimation in the presence of interference plus noise. Using Equation (4.7) on page 98 and Equation (4.13) on page 99, the BLS estimate for the p th T -spaced CIR phase can be written as:

$$\hat{h}_p = (A^* A)^{-1} A^* \left(A h_p + \sum_{l=1}^{n_i} A^{(l)} h_p^{(l)} + v_{r,p} \right) \quad (\text{A.1})$$

By taking the expectation of the BLS CIR estimate with respect to the interference and noise processes which have zero mean values, it is evident that the BLS CIR estimate is unbiased, as the second and third terms vanish:

$$E[\hat{h}_p] = (A^* A)^{-1} (A^* A) E[h_p] = h_p \quad (\text{A.2})$$

Thus, for a given training sequence of length $N_t \geq 2N_c - 1$, on average, the same unbiased BLS CIR estimate is obtained in the presence of additive noise and/or interference. That is, BLS CIR estimation in interference and in noise are equivalent in the mean sense.

However, they may be different in the mean square sense. That is, for a given training sequence, the CIR error covariance matrix may be different for the cases in interference and in noise. Furthermore, the choice of the training sequence affects the variance of the CIR estimation error for both cases, while the interferer CIRs can also affect the CIR error variance in the case of interference.

The CIR estimation error can be characterized by its covariance (or correlation for an unbiased estimator) matrix:

$$C_{e_{h_r}} = E[e_{h_r} e_{h_r}^*] \quad (\text{A.3})$$

where the CIR estimation error vector is defined as:

$$e_{h_r} = \hat{h}_p - h_p \quad (\text{A.4})$$

and the expectation is taken with respect to the noise and interference (i.e., interferer data)

Using Equation (A.1), the CIR estimation error vector can be written as:

$$\mathbf{e}_{h_p} = (\mathbf{A}^* \mathbf{A})^{-1} \mathbf{A}^* \left(\sum_{l=1}^{n_i} \mathbf{A}^{(l)} \mathbf{h}_p^{(l)} + \mathbf{n}_{r,p} \right) \quad (\text{A.5})$$

The CIR estimation error correlation matrix can now be written as:

$$\mathbf{C}_{e_{h_p}} = E \left[(\mathbf{A}^* \mathbf{A})^{-1} \mathbf{A}^* \left(\sum_{l=1}^{n_i} \mathbf{A}^{(l)} \mathbf{h}_p^{(l)} + \mathbf{n}_{r,p} \right) \left(\sum_{l=1}^{n_i} \mathbf{h}_p^{(l)*} \mathbf{A}^{(l)*} + \mathbf{n}_{r,p}^* \right) \mathbf{A} (\mathbf{A}^* \mathbf{A})^{-1} \right] \quad (\text{A.6})$$

Since it is assumed that the noise and interference are mutually uncorrelated, the cross terms vanish. It is further assumed that the noise samples are uncorrelated (in fact, independent), therefore the CIR estimation error correlation matrix can be simplified as:

$$\mathbf{C}_{e_{h_p}} = (\mathbf{A}^* \mathbf{A})^{-1} \mathbf{A}^* \sum_{l=1}^{n_i} E [\mathbf{A}^{(l)} \mathbf{h}_p^{(l)} \mathbf{h}_p^{(l)*} \mathbf{A}^{(l)*}] \mathbf{A} (\mathbf{A}^* \mathbf{A})^{-1} + \sigma_n^2 (\mathbf{A}^* \mathbf{A})^{-1} \quad (\text{A.7})$$

In general, the uncorrelated interference data is convolved with interferer CIRs with finite memory. Consequently, the interference samples at the output of the interferer CIRs are correlated and the $N_e \times N_e$ autocorrelation matrix of the interference received vector is non-diagonal. Moreover, it can be shown that the (i, j) th element of this interference autocorrelation matrix, $R_{i,p}(i, j)$, is:

$$R_{i,pp}(i, j) = \sum_{l=1}^{n_i} \{ E [\mathbf{A}^{(l)} \mathbf{h}_p^{(l)} \mathbf{h}_p^{(l)*} \mathbf{A}^{(l)*}] \}_{ij} = \sigma_i^2 \sum_{l=1}^{n_i} \sum_{k=0}^{N_i-1} h_{k+p/d-l}^{(l)} h_{k+p/d-j}^{(l)*} \quad (\text{A.8})$$

Note that this is the T -spaced equivalent of the (i, j) th element of the interference autocorrelation matrix as formulated in the T/d -spaced MMSE DFE solution in Equation (3.52) on page 51 or in the T/d -spaced interference autocorrelation estimation in Equation (4.23) on page 106. It is essentially this correlation in the interferer CIRs which the DFE can exploit to decrease its output MSE. Besides the difference in the sampling rate, the other difference between these versions of the interference autocorrelation is the matrix size, or equivalently the autocorrelation lag. In general, the size of the autocorrelation matrix is determined by the data span of interest in the relevant problem. Specifically in the MMSE DFE formulation, the interference autocorrelation spans, in T/d -spaced lag intervals, the size of the feedforward DFE filter, N_f , while in the interference plus noise

autocorrelation estimation, the interference plus noise autocorrelation spans, in T/d -spaced lag intervals, the correlation length, N_c , of the CIR estimator output error sequence. In this case of BLS CIR estimation, the interference autocorrelation spans, in T -spaced lag intervals, the N_c observation data samples.

A suitable measure of the quality of the unbiased BLS T/s -spaced CIR estimate, $\hat{h}$, is the norm tap error (NTE) which can be defined as the sum of the individual CIR tap error variances, or equivalently, the trace, $tr(\cdot)$, of the CIR estimation error correlation matrix, normalized by the actual CIR energy which can be arbitrarily chosen to be unity, i.e., $|\hat{h}|^2 = 1$ with $|h_p|^2 = 1/s$ for T/s phases $p = 0, 1, \dots, s-1$:

$$NTE = \frac{\sum_{p=0}^{s-1} \sum_{k=0}^{N_c-1} E[\hat{h}_{k+p/s} - h_{k+p/s}]^2}{\sum_{p=0}^{s-1} \sum_{k=0}^{N_c-1} |h_{k+p/s}|^2} = \frac{\sum_{p=0}^{s-1} E[|e_{h_p}|^2]}{\sum_{p=0}^{s-1} |h_p|^2} = tr(C_{e_h}) \quad (A.9)$$

where C_{e_h} is the estimation error covariance matrix of the overall T/s -spaced CIR.

An equivalent measure of CIR estimation quality is the signal to CIR estimation noise ratio (SER), which is defined, for unit variance training symbols, as:

$$SER = \frac{1}{NTE} = \frac{1}{tr(C_{e_h})} \quad (A.10)$$

From Equation (A.7) and Equation (A.8), the NTE for a T -spaced CIR estimate can be expressed as:

$$NTE = tr(C_{e_h}) = tr((A^*A)^{-1}A^*R_A(A^*A)^{-1} + \sigma_n^2(A^*A)^{-1}) \quad (A.11)$$

From this last equation, it is evident that the quality of the BLS CIR estimate, as measured by the NTE for this present case of time-invariant CIR estimation, is independent of the desired CIR. Furthermore, in the presence of a given amount of noise, the NTE is a function of just the training sequence. However, in the presence of interference, it is a function of both the training sequence and the interferer CIRs. It is not immediately obvious whether the NTE is greater or less in an equal amount of interference or noise. In the case for interference, this seems to depend on the amplitude and phase relationship of the tap

coefficients of the interferer CIRs which determine the interference autocorrelation matrix, R_I .

For a given N_c and choice of $N_t \geq 2N_c - 1$, there exists at least one optimum training sequence which minimizes the NTE. Such optimum training sequences can be found by an exhaustive search of all 2^{N_t} BPSK candidate sequences or, equivalently, matrices A . Furthermore, for a chosen optimum training sequence, the inverse of the training sequence autocorrelation matrix, $(A^* A)^{-1}$, can be precomputed to reduce on-line receiver computational effort [18].

If the interference autocorrelation matrix is assumed to be diagonal, i.e., $R_I = \sigma_I^2 I$, the NTE expression reduces to:

$$NTE = (\sigma_I^2 + \sigma_n^2) \text{tr}((A^* A)^{-1}) \quad (\text{A.12})$$

It can be shown that an efficient (i.e., minimum NTE) BLS CIR estimate can be obtained with a training sequence whose autocorrelation matrix is diagonal [48],[66]. In this case, the CIR estimation errors for each CIR tap are uncorrelated and, assuming unit variance training symbols, the NTE for a T -spaced CIR estimate can be expressed as:

$$NTE = \frac{(\sigma_I^2 + \sigma_n^2) N_c}{N_t - N_c + 1} \quad (\text{A.13})$$

Appendix B Excess MSE Analysis of a Computed MMSE DFE

In order to develop the expression for the excess MSE in Equation (4.47) on page 113, expressions of the form $E[\tilde{R}R^{-1}\tilde{R}]$, $E[\tilde{R}R^{-1}\tilde{p}]$ and $E[\tilde{p}^*R^{-1}\tilde{p}]$ in the first, second and third terms, respectively, need to be derived. For the sake of notational simplicity, let $R_{ij} = R(i, j)$, $R_{ij}^{-1} = R^{-1}(i, j)$, $\hat{R}_{ij} = \hat{R}(i, j)$ and $\tilde{R}_{ij} = \tilde{R}(i, j)$. The autocorrelation error matrix of the DFE can be written as a partitioned matrix as in Equation (D.1) on page 227. However, since it will be assumed that only the diagonal matrix elements contribute to the excess MSE, the off-diagonal error matrix, $\tilde{R}_{ra}$, is not significant. Furthermore, the lower diagonal error matrix spanning the feedback filter is zero-valued, i.e., $\tilde{R}_{aa} = 0$. Thus, the focus of this development will be on the upper diagonal error matrix spanning the forward filter, $\tilde{R}_{rr}$. For notational simplicity, this upper diagonal autocorrelation error matrix for the DFE will be denoted as $\tilde{R}_{rr} = \tilde{R}$, as is the case for the linear equalizer.

B.1 First Term of the Excess MSE

Considering the first term of Equation (4.47) on page 113, the (i, j) th element of $E[\tilde{R}R^{-1}\tilde{R}]$ spanning the DFE forward filter is of the following form:

$$\{E[\tilde{R}R^{-1}\tilde{R}]\}_{ij} = \sum_k \sum_l E[\tilde{R}_{ik}\tilde{R}_{lj}] R_{kl}^{-1} \quad (\text{B.1})$$

Recall that R and thus $\tilde{R}$ are Hermitian matrices (i.e., $R_{ij} = R_{ji}^*$ and $\tilde{R}_{ij} = \tilde{R}_{ji}^*$). Furthermore, if the elements of the error matrix, $\tilde{R}_{ij}$, are assumed to be uncorrelated for all i and j , then Equation (B.1) can be simplified as:

$$\{E[\tilde{R}R^{-1}\tilde{R}]\}_{ij} = \begin{cases} \sum_k E[|\tilde{R}_{ik}|^2] R_{kk}^{-1} & i = j, (k = l) \\ E[\tilde{R}_{ij}^2] R_{ji}^{-1} & i \neq j, (k \neq l) \end{cases} \quad (\text{B.2})$$

On the other hand, a more restrictive assumption is that all the autocorrelation and error matrices are diagonal. Thus, Equation (B.1) can be further simplified as only one term results from the summations since $i = j = k = l$:

$$\{E[\tilde{R}R^{-1}\tilde{R}]\}_{ii} = E[|\tilde{R}_{ii}|^2]R_{ii}^{-1} = \frac{E[|\tilde{R}_{ii}|^2]}{R_{ii}} = \frac{\text{Var}[\hat{R}_{ii}]}{\text{Var}[x_i]} \quad (\text{B.3})$$

where the last equality holds since the input signal to the equalizer, x_i , and its autocorrelation error, $\tilde{R}_{ii}$, are assumed to be zero-mean processes, the latter assumption implying that $\hat{R}_{ii}$ is an (asymptotically) unbiased estimator. Moreover, since each T -spaced phase of the input signal, x_i , is stationary (and thus each T -spaced phase of its autocorrelation matrix, R , is Toeplitz), by summing over all d phases of the fractionally spaced equalizer, the first term of Equation (4.47) on page 113 can be approximated as:

$$\underline{w}_o^* E[\tilde{R}R^{-1}\tilde{R}]\underline{w}_o = |\underline{w}_o|^2 \frac{\sum_{p=0}^{d-1} \text{Var}(\tilde{R}_{pp})}{\sum_{p=0}^{d-1} \text{Var}(x_p)} \quad (\text{B.4})$$

Using Equation (D.12) on page 229, the (i, k) th element of autocorrelation error matrix in Equation (B.1) can be expressed as:

$$\tilde{R}_{ik} = \tilde{R}_{h,ik} + \hat{R}_{i+n,ik} - \beta_e R_{i+n,ik} \quad (\text{B.5})$$

The desired received signal autocorrelation error, $\tilde{R}_{h,ik}$, is uncorrelated with the interference plus noise autocorrelation, $R_{i+n,ik}$. It is also uncorrelated with the estimated interference plus noise autocorrelation, $\hat{R}_{i+n,ik}$, since the latter is not a function of the CIR estimation error which is assumed to be small. Thus, the correlation of the received signal autocorrelation error in Equation (B.5) can be written as the following sum:

$$E[\tilde{R}_{ik}\tilde{R}_{lj}] = E[\tilde{R}_{h,ik}\tilde{R}_{h,lj}] + E[\tilde{R}_{i+n,ik}\tilde{R}_{i+n,lj}] \quad (\text{B.6})$$

That is, this covariance of the received signal autocorrelation is composed of a desired channel component and an interference plus noise component. Consequently, these two component terms are derived separately as follows.

The derivation of the first component of Equation (B.6) for the desired channel is as follows. Substituting Equation (D.8) on page 229 into the first term of Equation (B.6) yields the following expression for the covariance of the desired channel autocorrelation:

$$\begin{aligned}
E[\tilde{R}_{h,ik}\tilde{R}_{h,lj}] &= \sum_m h_{m-i} \sum_n h_{n-j} E[e_{h,m-k}^* e_{h,n-l}^*] \\
&+ \sum_m h_{m-i} \sum_n h_{n-j}^* E[e_{h,m-k}^* e_{h,n-l}] \\
&+ \sum_m h_{m-k}^* \sum_n h_{n-l} E[e_{h,m-i} e_{h,n-j}^*] \\
&+ \sum_m h_{m-k}^* \sum_n h_{n-j}^* E[e_{h,m-i} e_{h,n-l}]
\end{aligned} \tag{B.7}$$

where the BLS estimation errors for the N_c CIR tap coefficients are assumed to be mutually uncorrelated:

$$E[e_{h,i} e_{h,j}^*] = \left(\frac{NTE}{N_c}\right) \delta_{ij} \tag{B.8}$$

Thus, Equation (B.7) can be upper bounded as follows:

$$E[\tilde{R}_{h,ik}\tilde{R}_{h,lj}] \leq 2 \left(\frac{NTE}{N_c}\right) \left[\sum_m (h_{m-(i-k)} h_{m-(j-l)}^* + h_{m-(i-k)} h_{m+(j-l)}) \right] \tag{B.9}$$

Furthermore, with the assumption of diagonal matrices, Equation (B.9), representing the MSE of the BLS CIR estimator (or equivalently the estimation variance for the unbiased estimator), simplifies as:

$$Var[\hat{R}_{h,ii}] = E[|\tilde{R}_{h,ii}|^2] \leq 4 \left(\frac{NTE}{N_c}\right) |h|^2 \tag{B.10}$$

Then substituting into Equation (B.4) and Equation (B.5) results in the following upper bound for the desired channel autocorrelation portion of the first term of Equation (4.47) on page 113:

$$\begin{aligned}
\mathbf{w}_o^* E[\tilde{\mathbf{R}}_h \mathbf{R}^{-1} \tilde{\mathbf{R}}_h] \mathbf{w}_o &\leq \frac{4|\mathbf{w}_o|^2 \left(\frac{NTE}{N_c}\right) |h|^2}{\sum_{p=0}^{d-1} R_{pp}} = \frac{4|\mathbf{w}_o|^2 \sum_{p=0}^{d-1} R_{i+n,pp}}{N_c \sum_{p=0}^{d-1} R_{pp}}
\end{aligned} \tag{B.11}$$

where the last equality assumes the CIR energy is normalized to unity, i.e., $|h|^2 = 1$, and uses the expression for the BLS CIR estimation NTE from Equation (4.14) on page 99 with the (average) interference plus noise variance being:

$$\sum_{p=0}^{d-1} R_{i+n,pp} = \sigma_i^2 + \sigma_n^2 \quad (\text{B.12})$$

The derivation of the second component of Equation (B.6) for the interference plus noise is as follows. Substituting Equation (D.12) on page 229 into the second term of Equation (B.6) yields the following covariance of the interference plus noise autocorrelation.

$$E[\tilde{R}_{i+n,ik}\tilde{R}_{i+n,lj}] = E[(\hat{R}_{i+n,ik} - \beta_e R_{i+n,ik})(\hat{R}_{i+n,lj} - \beta_e R_{i+n,lj})] \quad (\text{B.13})$$

With the assumption of diagonal matrices, Equation (B.13) becomes the estimation variance of the unbiased, i.e., $\beta_e = 1$, sample estimator of the interference plus noise variance, $\hat{R}_{i+n,ii}$:

$$\begin{aligned} \text{Var}[\hat{R}_{i+n,ii}] &= E[|\tilde{R}_{i+n,ii}|^2] = E[|\hat{R}_{i+n,ii} - R_{i+n,ii}|^2] \\ &= E[|\hat{R}_{i+n,ii}|^2] - R_{i+n,ii}^2 \end{aligned} \quad (\text{B.14})$$

It is shown in Appendix C on page 220 that for the case of an unbiased interference plus noise variance estimator, Equation (B.14) can be upper bounded as:

$$\text{Var}[\hat{R}_{i+n,ii}] \leq \frac{4}{N_e} \sum_{l=0}^{N_f-1} |R_{i+n,il}|^2 \quad (\text{B.15})$$

Note that this result is of the same form as that of Equation (4.35) on page 110.

Finally, substituting into Equation (B.4) and Equation (B.5) results in the following upper bound for the interference plus noise sample autocorrelation portion of the first term of Equation (4.47) on page 113:

$$\underline{w}_o^* E[\tilde{R}_{i+n} R^{-1} \tilde{R}_{i+n}] \underline{w}_o \leq \frac{4|\underline{w}_o|^2 \sum_{p=0}^{d-1} \sum_{l=0}^{N_f-1} |R_{i+n,pl}|^2}{\frac{d-1}{N_e} \sum_{p=0} R_{pp}} \quad (\text{B.16})$$

Therefore, combining Equation (B.11) and Equation (B.16) results in the following upper bound for the first term of Equation (4.47) on page 113:

$$\underline{w}_n^* E [R_h R^{-1} \tilde{R}_h] \underline{w}_n \leq \frac{4 |\underline{w}_n|^2 \sum_{p=0}^{d-1} R_{l+n,pp}}{N_c \sum_{p=0}^{d-1} R_{pp}} \left(1 + \frac{\sum_{p=0}^{d-1} \sum_{l=0}^{N_c-1} |R_{l+n,pl}|^2}{\sum_{p=0}^{d-1} R_{l+n,pp}} \right) \quad (\text{B.17})$$

Substituting Equation (4.49) on page 114 for the input SINR to the equalizer, $SINR_{in}$, yields the resulting first two terms in the excess MSE upper bound expression of Equation (4.52) on page 115:

$$\underline{w}_n^* E [\tilde{R} R^{-1} \tilde{R}] \underline{w}_n \leq \frac{4 |\underline{w}_n|^2}{N_c (1 + SINR_{in})} \left(1 + \frac{\sum_{p=0}^{d-1} \sum_{l=0}^{N_c-1} |R_{l+n,pl}|^2}{\sum_{p=0}^{d-1} R_{l+n,pp}} \right) \quad (\text{B.18})$$

Note that under the assumption of diagonal matrices, this upper bound may be looser since the approximation of Equation (B.4) is also an upper bound. That is, the assumption of diagonal matrices removes the possibility of the cancellation of cross-terms resulting from the presence of off-diagonal elements in the matrices. This can be thought of as a form of the Schwartz' inequality. If the equalizer coefficients are relatively small in amplitude (or energy), then the influence of the cancellation of the cross-terms is not that significant and the resulting upper bound on the excess MSE can be fairly tight. On the other hand, if the equalizer coefficients are relatively large in amplitude (or energy), then the consequence of not accounting for the relatively large cross-terms can be quite significant and thus results in a loose upper bound on the excess MSE. It is demonstrated in Section 4.2.3.1 on page 115 that the former case is representative of a noise disturbance while the latter case is typical of an interference disturbance.

B.2 Second Term of the Excess MSE

In the second term of Equation (4.47) on page 113, the expected value of the column vector can be written as:

$$E [\tilde{R} R^{-1} \tilde{p}] = E [\tilde{R}_h R^{-1} \tilde{p}] + E [\tilde{R}_{l+n} R^{-1} \tilde{p}] \quad (\text{B.19})$$

where, according to Equation (D.14) on page 230, the cross-correlation error vector, $\tilde{p} = \tilde{e}_h$, is a direct function of the BLS CIR estimation error. Under the assumption of

small CIR estimation error, the interference plus noise autocorrelation error matrix, $\tilde{R}_{i+n}$, does not depend on the CIR estimation error and is thus independent of the cross-correlation error vector, $\tilde{p} = \tilde{e}_h$. Consequently, the second term in Equation (B.19) vanishes and the i th element of the column vector $E[\tilde{R}R^{-1}\tilde{p}]$ spanning the DFE forward filter is of the following form:

$$\{E[\tilde{R}R^{-1}\tilde{p}]\}_i = \sum_k h_{k-i} \sum_l R_{il}^{-1} E\{e_{h,k-i}^* e_{h,D-i}\} + \sum_k h_{k-j}^* \sum_l R_{jl}^{-1} E\{e_{h,k-i} e_{h,D-i}\} \quad (\text{B.20})$$

Since the BLS estimation errors for the N_c CIR tap coefficients are assumed to be mutually uncorrelated (see Equation (B.8)), Equation (B.20) can be upper bounded as follows:

$$\{E[\tilde{R}R^{-1}\tilde{p}]\}_i \leq \left(\frac{NTE}{N_c}\right) \sum_k (h_{k-i} R_{j(D+j-k)}^{-1} + h_{k-j}^* R_{j(D+i-k)}^{-1}) \quad (\text{B.21})$$

If it is further assumed that the matrices are diagonal, then Equation (B.21) becomes:

$$\{E[\tilde{R}R^{-1}\tilde{p}]\}_i \leq \left(\frac{NTE}{N_c}\right) R_{ii}^{-1} (h_{D-i} + h_{D-i}^*) = 2 \left(\frac{NTE}{N_c}\right) R_{ii}^{-1} \text{Re}\{h_{D-i}\} \quad (\text{B.22})$$

Equation (B.22) can be further developed by recognizing that the optimally delayed, time-reversed CIR samples comprise the cross-correlation vector, p , from Equation (3.54) on page 52 and using the expression for the BLS CIR estimation NTE from Equation (4.14) on page 99:

$$\{E[\tilde{R}R^{-1}\tilde{p}]\}_i \leq \frac{2R_{i+n} \text{Re}\{p_i\}}{N_c R_{ii}} \quad (\text{B.23})$$

Substituting Equation (4.49) on page 114 for the input SINR to the equalizer, SINR_{in} , and then substituting into the second term of Equation (4.47) on page 113 results in the following lower bound representing the third term in the excess MSE upper bound expression of Equation (4.52) on page 115:

$$-2\text{Re}\{\underline{w}_n^* E[\tilde{R}R^{-1}\tilde{p}]\} \geq \frac{-4\text{Re}\{\underline{w}_n\} \text{Re}\{p\}}{N_c (1 + \text{SINR}_{in})} \quad (\text{B.24})$$

Being a negative contribution to the excess MSE, this term could be neglected to ensure a strict (but looser) upper bound for the excess MSE. However, for typical channel conditions, especially in noise, this lower bound term does contribute significantly to the overall

upper bound on the excess MSE. In the case with interference, this negative component of the excess MSE tends to be relatively small as compared to the first (i.e., $4|\underline{w}_{off}|^2$) term due to the CIR estimation error (see Section 4.2.3.1 on page 115).

B.3 Third Term of the Excess MSE

Using Equation (D.14) on page 230, the third term of the excess MSE expression of Equation (4.47) on page 113 can be written as:

$$E[\tilde{\underline{p}}^* R^{-1} \tilde{\underline{p}}] = \text{tr}(R^{-1} E[\tilde{\underline{p}} \tilde{\underline{p}}^*]) = \text{tr}(R^{-1} E[\tilde{\underline{e}}_h \tilde{\underline{e}}_h^*]) \quad (\text{B.25})$$

Assuming that the BLS estimation errors for the N_c CIR tap coefficients are mutually uncorrelated (see Equation (B.8)) and using the expression for the BLS CIR estimation NTE from Equation (4.14) on page 99 results in the following:

$$E[\tilde{\underline{p}}^* R^{-1} \tilde{\underline{p}}] = \frac{\sum_{p=0}^{d-1} R_{i+n,pp}}{N_e} \text{tr}(R^{-1}) = \frac{\sum_{p=0}^{d-1} R_{i+n,pp}}{N_e} \sum_l \lambda_l^{-1} \quad (\text{B.26})$$

where λ_l are the eigenvalues of the input signal autocorrelation matrix, R

Further assuming diagonal matrices and substituting Equation (4.49) on page 114 for the input SINR to the equalizer, SINR_{in} , the fourth term of the excess MSE expression of Equation (4.52) on page 115 is obtained:

$$E[\tilde{\underline{p}}^* R^{-1} \tilde{\underline{p}}] = \frac{N_f \sum_{p=0}^{d-1} R_{i+n,pp}}{N_e \sum_{p=0}^{d-1} R_{pp}} = \frac{N_f}{N_e (1 + \text{SINR}_{in})} \quad (\text{B.27})$$

Appendix C Variance of an Interference plus Noise Sample Variance Estimator

The following is a derivation of an upper bound for the estimation variance of the unbiased interference plus noise sample variance (i.e., autocorrelation with zero lag) estimator as formulated in Equation (B.14) on page 216. Similar calculations for the variance of sample correlation estimators have been previously described [11],[116]. Without loss of generality, let the autocorrelation lag indices be $i = j = 0$ for both error components in the sample variance estimator. Substituting Equation (4.36) on page 111 into the unbiased estimation variance expression of Equation (B.14) on page 216 yields:

$$\text{Var}[\hat{R}_{i+n,ii}] = \frac{1}{N^2} \sum_{e=m=0}^{N_c-1} \sum_{n=0}^{N_c-1} E[e_{r,m} e_{r,m}^* e_{r,n}^* e_{r,n}] - R_{i+n,ii}^2 \quad (\text{C.1})$$

where the squared variance of the interference plus noise for the i th T -spaced phase of the equalizer can be expressed as:

$$R_{i+n,ii}^2 = (\sigma_i^2 + \sigma_n^2)^2 \quad (\text{C.2})$$

and the m th ($m = N_c - 1, \dots, N_t - 1$) error at the BLS CIR estimator output is:

$$e_{r,m} = a_m^* e_h - i_m - n_m \quad (\text{C.3})$$

with the interference written as:

$$i_m = \sum_{l=1}^{n_i} a_m^{(l)*} h^{(l)} = \sum_{l=1}^{n_i} \sum_{p=0}^{N_t-1} a_{m-p}^{(l)*} h_p^{(l)} \quad (\text{C.4})$$

Let the second-order (i.e., variance) and fourth-order moments of the l th user data symbol be defined respectively as:

$$\sigma_d^2 = E[|a_k^{(l)}|^2] \quad (\text{C.5})$$

and

$$\sigma_{|d|^2}^2 = E[|a_k^{(l)}|^4] \quad (\text{C.6})$$

Moreover, let the l th user CIR autocorrelation with lag α be defined as:

$$\rho_a^{(l)} = \sum_{p=0}^{N_c-1} h_p^{(l)} h_{p+a}^{(l)*} \quad (C.7)$$

Using Equation (C.2), Equation (C.4), Equation (C.7) and the assumption that the user data and noise samples are mutually independent, the variance of the interference ($l = 1, \dots, n_i$) plus noise for the i th T -spaced phase can be expressed as:

$$R_{i+n, ii} = \sigma_i^2 + \sigma_n^2 = \sum_{l=1}^{n_i} \rho_o^{(l)} + \sigma_n^2 \quad (C.8)$$

Consider the first term of Equation (C.1) which represents the second moment of the sample variance estimator:

$$E[|\hat{R}_{i+n, ii}|^2] = \frac{1}{N^2} \sum_{e,m=0}^{N_c-1} \sum_{n=0}^{N_c-1} E[e_{r,m} e_{r,m}^* e_{r,n}^* e_{r,n}] \quad (C.9)$$

Substituting Equation (C.3) into this expression produces up to $3^4 = 81$ terms. However, with the assumptions of small CIR estimation error and mutually independent and zero-mean samples of the user data, $a^{(l)}$, noise, n_k , and CIR estimation error, e_h , all terms of order $O(e_h^3)$ and $O(e_h^4)$ are negligible and all terms with a single factor of user data, noise and CIR estimation error have zero expectation. Thus, Equation (C.9) contains only 20 fourth-order moment terms, of which 2 terms are for the individual interference and noise components, denoted C_i and C_n , respectively, 6 terms are the cross-products of the interference and noise variances, denoted C_{in} , and 12 terms contain the effects of the CIR estimation error, denoted C_e .

In order to simplify the mathematics, the case with only interference and negligible noise is considered. The general case with interference plus noise can be subsequently obtained by incorporating the noise component into the interference plus noise correlation terms of the derived variance expression. With this removal of the noise component in the derivation, Equation (C.9) contains only one term for the fourth-order moment of the interference, C_i , and 6 terms involving the CIR estimation error, C_e :

$$E[|\hat{R}_{i+n, ii}|^2] = C_i + C_e \quad (C.10)$$

The fourth-order moment interference term in Equation (C.10) can be expressed as:

$$C_i = \frac{1}{N_e^2} \sum_l \sum_{m,n,p,q,r,s} h_p^{(l)} h_q^{(l)*} h_r^{(l)*} h_s^{(l)} E [a_{m-p}^{(l)*} a_{m-q}^{(l)} a_n^{(l)} a_{n-s}^{(l)*}] \quad (C.11)$$

where the l -indexed summation is over the n_l independent interferers,

the m - and n -indexed summations are over the N_e autocorrelation error samples

and the p -, q -, r - and s -indexed summations are over the N_c CIR tap coefficients.

The expectation in Equation (C.11) is non-zero if and only if the user data symbol subscripts are: (1) equal in fours (i.e., all equal), denoted as C_{1234} , or (2) equal in pairs, with different possible subscript combinations denoted as C_{12} , C_{13} and C_{14} . Since case (1) is a subset of case (2), the occurrence of type C_{1234} in each of the three combinations of case (2) should be subtracted from the case (2) terms in order to avoid double counting C_{1234} . If this subtraction is done, then all four terms in these two cases are mutually exclusive and comprise the fourth-order moment interference component of Equation (C.10):

$$C_i = C_{1234} + C_{12} + C_{13} + C_{14} \quad (C.12)$$

The first term of Equation (C.12) with four equal subscripts, i.e., $p = q, r = s$ and $m - p = n - r$, is obtained from Equation (C.11) and can be written as:

$$\begin{aligned} C_{1234} &= \frac{1}{N_e^2} \sum_l \sum_{m,n,p,r} |h_p^{(l)}|^2 |h_r^{(l)}|^2 E [|a_k^{(l)}|^4] \\ &= \frac{\sigma_{|d|^2}^2}{N_e^2} \sum_l \sum_{m,n} \sum_p |h_p^{(l)}|^2 |h_{p+n-m}^{(l)}|^2 \end{aligned} \quad (C.13)$$

where Equation (C.6) and $r = p + n - m$ have been substituted into the last equation. A further substitution of the lag index $\delta = n - m$, which ranges from 0 to N_c of the CIR components indexed by p results in:

$$C_{1234} = \frac{\sigma_{|d|^2}^2}{N_e^2} \sum_l \sum_{\delta} (N_e - |\delta|) \sum_p |h_p^{(l)}|^2 |h_{p+\delta}^{(l)}|^2 \quad (C.14)$$

Assuming $N_e \gg N_c$, or equivalently $N_e \gg |\delta|$, and employing Equation (C.7) leads to the following inequality:

$$\begin{aligned} C_{1234} &\leq \frac{\sigma_{|d|^2}^2}{N_e} \sum_l \sum_p |h_p^{(l)}|^2 \sum_{\delta} |h_{p+\delta}^{(l)}|^2 \\ &\leq \frac{\sigma_{|d|^2}^2}{N_e} \sum_l (\rho_0^{(l)})^2 \end{aligned} \quad (C.15)$$

The second term of Equation (C.12) with paired equal subscripts, i.e., $p = q$, $r = s$ but $m - p \neq n - r$, is expressed as:

$$C_{12} = \frac{1}{N_e^2} \sum_l \sum_m \sum_{n,p,r} |h_p^{(l)}|^2 |h_r^{(l)}|^2 E[|a_{m-p}^{(l)}|^2] E[|a_{n-r}^{(l)}|^2] - \sigma_d^4 C_{1234} \quad (\text{C.16})$$

where the C_{1234} term is subtracted to avoid its double counting in the summation over m and n . Using Equation (C.5) and Equation (C.7), and recognizing that $\sum_{m,n} 1 = N_e^2$ yields:

$$C_{12} = \sigma_d^4 \left\{ \sum_l (\rho_0^{(l)})^2 - C_{1234} \right\} \quad (\text{C.17})$$

The third term of Equation (C.12) with paired equal subscripts, i.e., $m - p = n - r$, $m - q = n - s$ but $m - p \neq m - q$, and with the substitutions of $r = p + n - m$ and $s = q + n - m$, can be expressed as:

$$C_{13} = \frac{1}{N_e^2} \sum_l \sum_m \sum_{n,p,q} h_p^{(l)} h_q^{(l)*} h_{p+n-m}^{(l)*} h_{q+n-m}^{(l)} E[|a_{m-p}^{(l)}|^2] E[|a_{m-q}^{(l)}|^2] - \sigma_d^4 C_{1234} \quad (\text{C.18})$$

Using Equation (C.5) and Equation (C.7), and a further substitution of the lag index $\delta = n - m$, which can range from $-N_c$ to N_c of the CIR components indexed by p and q results in:

$$C_{13} = \sigma_d^4 \left\{ \frac{1}{N_e^2} \sum_l \sum_{\delta} (N_e - |\delta|) |\rho_{\delta}^{(l)}|^2 - C_{1234} \right\} \quad (\text{C.19})$$

Assuming $N_e \gg N_c$, or equivalently $N_e \gg |\delta|$, leads to the following inequality:

$$\begin{aligned} C_{13} &\leq \sigma_d^4 \left\{ \frac{1}{N_e} \sum_l \sum_{\delta} |\rho_{\delta}^{(l)}|^2 - C_{1234} \right\} \\ &\leq \sigma_d^4 \left[\frac{1}{N_e} \sum_l \left((\rho_0^{(l)})^2 + 2 \sum_{\delta=1}^{N_c-1} |\rho_{\delta}^{(l)}|^2 \right) - C_{1234} \right] \end{aligned} \quad (\text{C.20})$$

In an analogous manner to the calculation of C_{13} , it can be shown that the fourth term of Equation (C.12) with paired equal subscripts, i.e., $m - p = n - s$, $m - q = n - r$ but $m - p \neq m - q$, is approximately equal to C_{13} :

$$C_{14} \approx C_{13} \quad (\text{C.21})$$

Thus substituting Equation (C.14), Equation (C.17), Equation (C.19) and Equation (C.21) into Equation (C.12) yields the fourth-order moment interference term of Equation (C.10):

$$C_i \leq \sigma_d^4 \left[\frac{4}{N_e} \sum_l \sum_{\delta=1}^{N-1} |p_\delta^{(l)}|^2 + \left[1 + \frac{1}{N_e} \right] 2 + (\sigma_d^4 - 3) \sigma_{[d]}^2 \right] \sum_l (p_0^{(l)})^2 \quad (C.22)$$

The 6 terms which involve the CIR estimation error, C_e , of Equation (C.10) can be expressed as:

$$\begin{aligned} C_e = \frac{1}{N_e^2} \sum_l \sum_{m,n} \{ & \sum_{p,q,r,s} h_p^{(l)} h_q^{(l)*} E[a_{m-p}^{(l)*} a_{m-q}^{(l)}] E[a_{n-r} a_{n-s}^*] E[e_{h,r}^* e_{h,s}] \\ & + \sum_{p,q,r,s} h_r^{(l)*} h_s^{(l)} E[a_{n-r}^{(l)} a_{n-s}^*] E[a_{m-p}^* a_{m-q}] E[e_{h,p} e_{h,q}^*] \\ & + \sum_{p,q,r,s} h_p^{(l)} h_r^{(l)*} E[a_{m-p}^{(l)*} a_{n-r}^{(l)}] E[a_{m-q} a_{n-s}^*] E[e_{h,q}^* e_{h,s}] \\ & + \sum_{p,q,r,s} h_q^{(l)*} h_s^{(l)} E[a_{m-q}^{(l)*} a_{n-s}^{(l)}] E[a_{m-p}^* a_{n-r}] E[e_{h,p} e_{h,r}^*] \\ & + \sum_{p,q,r,s} h_p^{(l)} h_s^{(l)} E[a_{m-p}^{(l)*} a_{n-s}^{(l)*}] E[a_{m-q} a_{n-r}] E[e_{h,q}^* e_{h,r}^*] \\ & + \sum_{p,q,r,s} h_q^{(l)*} h_r^{(l)*} E[a_{m-q}^{(l)} a_{n-r}^{(l)}] E[a_{m-p}^* a_{n-s}^*] E[e_{h,p} e_{h,s}] \} \end{aligned} \quad (C.23)$$

The expectations in the first two terms of Equation (C.23) are non-zero if and only if $r = s$ and $p = q$. Using Equation (4.14) on page 99 and Equation (C.5), each of these two terms can be written as:

$$C_{e,1} = C_{e,2} = \frac{\sigma_d^4}{N_e^2} \left(\frac{\sigma_i^2}{N_e} \right) \sum_l \sum_{m,n} \sum_{p,r} |h_p^{(l)}|^2 \quad (C.24)$$

Using Equation (C.7) and Equation (C.8) while recognizing that $\sum_{m,n} 1 = N_e^2$ and $\sum_r 1 = N_e$ yields:

$$C_{e,1} = C_{e,2} = \sigma_d^4 \frac{N_e}{N_e^2} \sum_l (p_0^{(l)})^2 \quad (C.25)$$

The expectations in the third and fourth terms in Equation (C.23) are non-zero if and only if $q = s$, $m = n$ and $p = r$. Using Equation (4.14) on page 99 and Equation (C.5), each of these two terms can be written as:

$$C_{e,3} = C_{e,4} = \frac{\sigma_d^4}{N_e^2} \left(\frac{\sigma_i^2}{N_e} \right) \sum_l \sum_m \sum_{p,q} |h_p^{(l)}|^2 \quad (\text{C.26})$$

Using Equation (C.7) and Equation (C.8) while recognizing that $\sum_m 1 = N_e$ and $\sum_q 1 = N_e$ yields:

$$C_{e,3} = C_{e,4} = \sigma_d^4 \frac{N_e}{N_e^2} \sum_l (\rho_0^{(l)})^2 = \frac{1}{N_e} C_{e,1} \quad (\text{C.27})$$

Assuming $N_e \gg 1$, these two terms are negligible as compared to the first two terms:

$$C_{e,3} = C_{e,4} = \frac{1}{N_e} C_{e,1} \ll C_{e,1} \quad (\text{C.28})$$

Similarly, the fifth and sixth terms in Equation (C.23) are also negligible. Thus, the C_e term, involving the CIR estimation error, of Equation (C.10) can be written as:

$$C_e = 2\sigma_d^4 \frac{N_e}{N_e^2} \sum_l (\rho_0^{(l)})^2 \quad (\text{C.29})$$

Therefore, substituting Equation (C.22) and Equation (C.29) into Equation (C.10) and grouping terms results in a simplified expression for the second moment of the sample variance estimator, $E[|\hat{R}_{i+n,ii}|^2]$:

$$\begin{aligned} E[|\hat{R}_{i+n,ii}|^2] &\leq \sigma_d^4 \frac{4}{N_e} \sum_l \sum_{\delta=1}^{N_f-1} |\rho_\delta^{(l)}|^2 \\ &\quad + \sigma_d^4 \left[1 + \frac{1}{N_e} [2(1+N_e) + (\sigma_d^{-4} - 3)\sigma_{|d|^2}^2] \right] \sum_l (\rho_0^{(l)})^2 \end{aligned} \quad (\text{C.30})$$

In order to generalize to the interference plus noise case, the noise is incorporated as an additive component to the interference via Equation (C.8). Moreover, with application to the computed MMSE DFE, the interference plus noise autocorrelation lag index, δ , ranges up to $N_f - 1$ in order to span the forward filter of the DFE. Hence, Equation (C.30) for the i th T -spaced phase of the DFE becomes:

$$\begin{aligned}
 E[|\hat{R}_{i+n,ii}|^2] &\leq \sigma_d^4 \frac{4}{N_c} \sum_{\delta=1}^{N_f-1} |R_{i+n,i\delta}|^2 + \\
 &\quad + \sigma_d^4 \left[1 + \frac{1}{N_c} [2(1+N_c) + (\sigma_d^4 - 3)\sigma_{id}^2] \right] R_{i+n,i0}^2
 \end{aligned}
 \tag{C.31}$$

Employing the fact that $\sigma_d^4 = 1$ for normalized data symbols and substituting Equation (C.31) into the unbiased estimation variance expression of Equation (C.1) yields the following upper bound:

$$\text{Var}[\hat{R}_{i+n,ii}] \leq \frac{4}{N_c} \left[\sum_{\delta=1}^{N_f-1} |R_{i+n,i\delta}|^2 + \frac{1}{2} [1 + N_c - \sigma_{id}^2] R_{i+n,i0}^2 \right]
 \tag{C.32}$$

For a relatively large transmission bandwidth, e.g., greater than 100% excess bandwidth, the CIR spans a couple of T -spaced tap coefficients. Thus, under the reasonable assumption that $N_c = 1 + \sigma_{id}^2$, Equation (C.32) simplifies to the result of Equation (B.14) on page 216:

$$\text{Var}[\hat{R}_{i+n,ii}] \leq \frac{4}{N_c} \sum_{\delta=0}^{N_f-1} |R_{i+n,i\delta}|^2
 \tag{C.33}$$

The summation indexed by δ ranges through all lags spanned by the DFE forward filter with the autocorrelation products summed over all columns along the zeroth ($i = 0$) row of the interference plus noise autocorrelation matrix.

Appendix D Autocorrelation Error Matrix and Cross-Correlation Error Vector of a Computed MMSE DFE

Expressions for the autocorrelation error matrix, $\tilde{\mathbf{R}}$, and the cross-correlation error vector, $\tilde{\mathbf{p}}$, in terms of the CIR and interference plus noise autocorrelation estimation errors are required in the development of the excess MSE expression in Appendix A on page 209. Without loss of generality, the following formulation is for a T -spaced DFE. In order to generalize to a fractionally spaced DFE, the following results are taken to represent one T -spaced phase of the overall fractionally spaced DFE and the excess MSE is obtained by summing over all d phases within a baud interval of the equalizer.

Following Equation (3.50) on page 50, the autocorrelation error matrix of the DFE can be written as a partitioned matrix:

$$\tilde{\mathbf{R}} = \hat{\mathbf{R}} - \mathbf{R} = \begin{bmatrix} \tilde{\mathbf{R}}_{rr} & \tilde{\mathbf{R}}_{ra} \\ \tilde{\mathbf{R}}_{ra}^* & \tilde{\mathbf{R}}_{aa} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{R}}_{rr} & \tilde{\mathbf{R}}_{ra} \\ \tilde{\mathbf{R}}_{ra}^* & 0 \end{bmatrix} \quad (\text{D.1})$$

The off-diagonal correlation (of the received signal and decision feedback data) matrix, $\mathbf{R}_{ra}$, elements consists of optimally delayed, time-reversed CIR samples (see Equation (3.53) on page 51). Thus, the corresponding error matrix, $\tilde{\mathbf{R}}_{ra}$, contains CIR estimation error samples of order $O(e_h)$. However, since it is assumed that only the diagonal matrix elements contribute to the excess MSE, this off-diagonal error matrix is assumed to be zero. One result of this assumption is that the CIR estimation error effects resulting from the correlation of the received signal with the decision feedback data do not manifest themselves in the excess MSE. Moreover, from the discussion in Appendix A on page 209, the excess MSE upper bound can be even looser for the case of a DFE since more off-diagonal terms are excluded from the excess MSE expression.

Under the assumption of diagonal matrices, it is sufficient to consider just the autocorrelation matrix, $\mathbf{R}_{rr}$, of the received signal, r_k , spanning the feed-forward filter taps, $\underline{w}_f$, of the DFE. Since the desired and interferer data and noise samples are mutually uncorrelated, the received input signal autocorrelation matrix consists of the sum of the autocorrelation matrices for the desired received signal, $\mathbf{R}_h$, and interference plus noise, $\mathbf{R}_{i+n}$:

$$\begin{aligned}
 R_{rr} &= E\{r_k r_k^*\} = R_h + R_{u,n} \\
 &= HH^* + \sigma_u^2 \sum_{l=1}^n H^{(l)} H^{(l)*} + \sigma_n^2 R_n
 \end{aligned} \quad (D.2)$$

where the received signal is expressed as:

$$r_k = Hq_k + \sum_{l=1}^n H^{(l)} q_k^{(l)} + u_k \quad (D.3)$$

and the channel matrix is of the following form:

$$H = \begin{bmatrix} \underline{h} & & & z_{N_f-1}^* \\ & \underline{h} & & \dots \\ & & \dots & z_1^* \\ z_{N_f-1} & \dots & z_1 & \underline{h} \end{bmatrix} = \begin{bmatrix} h_0 & h_1 & \dots & h_{N_f-1} & 0 & 0 & \dots & 0 \\ 0 & h_0 & h_1 & \dots & h_{N_f-1} & 0 & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 \\ 0 & \dots & 0 & h_0 & h_1 & \dots & \dots & h_{N_f-1} \end{bmatrix} \quad (D.4)$$

with z_N being an $N \times 1$ column vector of zeroes.

Furthermore, associated with the channel matrix is a corresponding channel error matrix:

$$E_h = \begin{bmatrix} \underline{e}_h & & & z_{N_f-1}^* \\ & \underline{e}_h & & \dots \\ & & \dots & z_1^* \\ z_{N_f-1} & \dots & z_1 & \underline{e}_h \end{bmatrix} = \begin{bmatrix} e_{h,0} & e_{h,1} & \dots & e_{h,N_f-1} & 0 & 0 & \dots & 0 \\ 0 & e_{h,0} & e_{h,1} & \dots & e_{h,N_f-1} & 0 & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & 0 \\ 0 & \dots & 0 & e_{h,0} & e_{h,1} & \dots & \dots & e_{h,N_f-1} \end{bmatrix} \quad (D.5)$$

where $\underline{e}_h$ is the BLS CIR estimation error vector as defined in Equation (A.4) on page 209.

Thus, the estimated autocorrelation matrix of the desired received signal can be written as:

$$\hat{R}_h = \hat{H}\hat{H}^* = (H + E_h)(H^* + E_h^*) = HH^* + HE_h^* + E_h H^* + E_h E_h^* \quad (D.6)$$

where the last term is of order $O(e_h^2)$ and is thus negligible if the CIR estimation error is assumed to be small.

Consequently, the autocorrelation error matrix of the desired received signal can be expressed as the following zero-mean matrix:

$$\tilde{R}_h = HE_h^* + E_h H^* \quad (D.7)$$

whose (i, j) th element is:

$$\tilde{R}_h(i, j) = \sum_k (h_{k-i} e_{h, k-j}^* + e_{h, k-i} h_{k-j}^*) \quad (\text{D.8})$$

The interference plus noise autocorrelation matrix, R_{i+n} , is obtained from the autocorrelation matrix, R_{e_r} , of the BLS CIR estimation output error, e_r , spanning the N_e observed training symbols (see Equation (4.20) on page 105):

$$R_{e_r} = (NTE)I + R_{i+n} \quad (\text{D.9})$$

where the BLS CIR estimation norm tap error can be expressed as in Equation (4.14) on page 99.

An asymptotically unbiased sample interference plus noise autocorrelation estimator (see Equation (4.36) on page 111) is used, with an average value for the (i, j) th element being:

$$E[\hat{R}_{e_r}(i, j)] = \beta_e E[R_{e_r}(i, j)] = \beta_e [R_{i+n}(i, j) + (NTE) \delta_{ij}] \quad (\text{D.10})$$

where the sample autocorrelation estimation bias factor is (see Equation (4.33) on page 110):

$$\beta_e = 1 - \frac{\lceil i-j \rceil}{N_e} \quad (\text{D.11})$$

and δ_{ij} is the Kronecker delta function.

Under the assumption of small CIR estimation error, the NTE diagonal term is negligible. Furthermore, since $\hat{R}_{i+n}$ is an asymptotically unbiased estimator, it is reasonable to assume that:

$$\hat{R}_{i+n} = \beta_e R_{i+n} + \tilde{R}_{i+n} \quad (\text{D.12})$$

where the zero-mean correlation error matrix, $\tilde{R}_{i+n}$, has an (i, j) th element as:

$$\tilde{R}_{i+n}(i, j) = \frac{1}{N_e} \sum_{k=0}^{N_e-1-\lceil (i-j)/d \rceil} e_{r, k-i/d} e_{r, k-j/d}^* - \beta_e R_{i+n}(i, j) \quad (\text{D.13})$$

$$\begin{aligned}
&= \frac{1}{N_i} \sum_{k=0}^{N-1-i} e_{i,k+1,d} e_{i,k+1,d}^* \\
&\quad - \beta_e \left[\sigma_e^2 \sum_{l=1}^n \sum_{k=0}^{N-1-i} h_k^{(l)} e_{i,k+1,d} h_k^{(l)*} e_{i,k+1,d}^* + \sigma_n^2 R_n((i-j)-d) \right]
\end{aligned}$$

Thus, the (i, j) th element of the overall received signal autocorrelation error matrix, $R_{r,i}$, is the sum of Equation (D.8) and Equation (D.13).

In the Wiener solution of the MMSE DFE, the non-zero part of cross-correlation vector, p , which spans the DFE forward filter consists of optimally delayed, time-reversed samples of the desired CIR (see Equation (3.54) on page 52). Thus, the cross-correlation error vector consists of the corresponding vector of BLS CIR estimation tap errors:

$$\tilde{p} = \tilde{e}_h = [e_{h,D} \ e_{h,D-1} \ \cdots \ e_{h,D-N_i+1}]^T \quad (\text{D.14})$$

where D is the optimally chosen equalizer delay in symbol periods and the BLS estimation error for the i th CIR tap coefficient is:

$$e_{h,i} = \hat{h}_i - h_i \quad (\text{D.15})$$

Appendix E MSE Perturbation Analysis of a Wiener Filter

It is desired to analyze the sensitivity of the Wiener filter output MSE to small inaccuracies in its autocorrelation matrix and cross-correlation vector. Let the perturbation in the equalizer output MSE be (see Equation (4.37) on page 111):

$$\delta J = E [\delta \underline{w}^* R \delta \underline{w}] \quad (\text{E.1})$$

while from Equation (3.49) on page 50, the equalizer output MSE is:

$$J = 1 - \underline{w}^* \underline{p} = 1 - \underline{p}^* R^{-1} \underline{p} \quad (\text{E.2})$$

In order to obtain an upper bound on the increase in MSE, the Schwartz' inequality is applied to the matrix/vector norm as follows:

$$\delta J = E [|\delta \underline{w}^* R \delta \underline{w}|] \leq E [||R|| ||\delta \underline{w}||^2] = E \left[||R|| ||\underline{w}||^2 \frac{||\delta \underline{w}||^2}{||\underline{w}||^2} \right] \quad (\text{E.3})$$

It is useful to express the relative perturbation in the Wiener coefficient vector, $\underline{w}$, in terms of the relative perturbations in autocorrelation matrix, R , and cross-correlation vector, $\underline{p}$.

Consider the following actual and perturbed (e.g., Wiener) systems of equations:

$$R \underline{w} = \underline{p} \quad (\text{E.4})$$

and

$$(R + \delta R) (\underline{w} + \delta \underline{w}) = \underline{p} + \delta \underline{p} \quad (\text{E.5})$$

Combining these two matrix equations results in:

$$\delta \underline{w} = (I + R^{-1} \delta R)^{-1} R^{-1} \delta \underline{p} - (I + R^{-1} \delta R)^{-1} R^{-1} \delta R \underline{w} \quad (\text{E.6})$$

Applying the triangle and Schwartz' inequalities yields:

$$\begin{aligned} ||\delta \underline{w}|| &\leq ||(I + R^{-1} \delta R)^{-1} R^{-1} \delta \underline{p}|| + ||(I + R^{-1} \delta R)^{-1} R^{-1} \delta R \underline{w}|| \\ &\leq ||(I + R^{-1} \delta R)^{-1}|| ||R^{-1}|| ||\delta \underline{p}|| + ||(I + R^{-1} \delta R)^{-1}|| ||R^{-1}|| ||\delta R|| ||\underline{w}|| \end{aligned} \quad (\text{E.7})$$

Define the following perturbation, assuming that δR is sufficiently small:

$$P_{\Delta} = \|R^{-1}\| \|\delta R\| < 1 \quad (\text{E.8})$$

Moreover, from Schwartz' inequality results:

$$\|R^{-1}\delta R\| \leq \|R^{-1}\| \|\delta R\| = P_{\Delta} < 1 \quad (\text{E.9})$$

Furthermore, suppose the norm is defined such that $\|I\| = 1$. Therefore, $I + R^{-1}\delta R$ is non-singular and it can be shown that [89]:

$$\|(I + R^{-1}\delta R)^{-1}\| \leq (1 - P_{\Delta})^{-1} \quad (\text{E.10})$$

Thus, substituting Equation (E.8) into Equation (E.7) produces:

$$\frac{\|\delta \underline{w}\|}{\|\underline{w}\|} \leq \frac{1}{1 - P_{\Delta}} \left(\|R^{-1}\| \frac{\|\delta \underline{p}\|}{\|\underline{p}\|} + \|R^{-1}\| \|\delta R\| \right) \quad (\text{E.11})$$

and applying Schwartz' inequality to Equation (E.4) yields:

$$\frac{1}{\|\underline{w}\|} \leq \frac{\|R\|}{\|\underline{p}\|} \quad (\text{E.12})$$

which can then be substituted into Equation (E.11) to get the following result [89]:

$$\frac{\|\delta \underline{w}\|}{\|\underline{w}\|} \leq \frac{\|R\| \|R^{-1}\|}{1 - \|R\| \|R^{-1}\| \|\delta R\|} \left(\frac{\|\delta \underline{p}\|}{\|\underline{p}\|} + \frac{\|\delta R\|}{\|R\|} \right) \quad (\text{E.13})$$

The condition number of the autocorrelation matrix, R , is defined as:

$$\kappa(R) = \|R\| \|R^{-1}\| \quad (\text{E.14})$$

Therefore Equation (E.8) becomes:

$$P_{\Delta} = \kappa(R) \frac{\|\delta R\|}{\|R\|} \quad (\text{E.15})$$

and Equation (E.13) becomes:

$$\frac{\|\delta \underline{w}\|}{\|\underline{w}\|} \leq \frac{\kappa(R)}{1 - \kappa(R) \frac{\|\delta R\|}{\|R\|}} \left(\frac{\|\delta \underline{p}\|}{\|\underline{p}\|} + \frac{\|\delta R\|}{\|R\|} \right) \quad (\text{E.16})$$

Substituting Equation (E.16) into Equation (E.3) gives the desired expression for the perturbation in the MSE in terms of the autocorrelation matrix condition number, $\kappa(R)$, and relative perturbations in the autocorrelation matrix, R , and the cross-correlation vector, p :

$$\delta J \leq E \left[\|R\| \|w\|^2 \left[\frac{\kappa(R)}{1 - \kappa(R)} \frac{\|\delta R\|}{\|R\|} \left(\frac{\|\delta p\|}{\|p\|} + \frac{\|\delta R\|}{\|R\|} \right) \right]^2 \right] \quad (\text{E.17})$$

Note that for small average perturbations, δR , in the matrix, R , Equation (E.17) is approximately:

$$\delta J \leq E \left[\|R\| \|w\|^2 \kappa^2(R) \left(\frac{\|\delta p\|}{\|p\|} + \frac{\|\delta R\|}{\|R\|} \right)^2 \right] \quad (\text{E.18})$$

Let the norm be defined as the spectral norm [48],[89]:

$$\|R\| = \|R\|_2 = (\max \lambda(R^* R))^{1/2} \quad (\text{E.19})$$

Since the autocorrelation matrix is Hermitian, (i.e., $R^* = R$), thus:

$$\max \lambda(R^* R) = \max \lambda(R^2) = \lambda_{\max}^2 \quad (\text{E.20})$$

where $\lambda_{\max}$ is the maximum eigenvalue of R .

Thus, Equation (E.19) results in:

$$\|R\| = \lambda_{\max} \quad (\text{E.21})$$

and

$$\|R^{-1}\| = \lambda_{\min}^{-1} \quad (\text{E.22})$$

Therefore, when using the spectral norm, the condition number is just the eigenvalue spread of the autocorrelation matrix:

$$\kappa(R) = \frac{\lambda_{\max}}{\lambda_{\min}} \quad (\text{E.23})$$

Thus, from Equation (E.18), it is evident how for a given perturbation in R and p , the eigenvalue spread of the autocorrelation matrix can magnify the perturbation in the MSE.

Appendix F MSE Convergence Analysis of the LMS Algorithm

The following is a summary of the derivation of the MSE convergence of the LMS algorithm [48],[52],[53],[120]. The MSE as a function of the k th symbol time instant can be expressed as a sum of the excess MSE, $XSMSE(kT)$, and the minimum MSE, $MMSE$

$$\begin{aligned} MSE(kT) &= XSMSE(kT) + MMSE \\ &= tr[RK(kT)] + (1 - \rho^* R^{-1} \rho) \end{aligned} \quad (F.1)$$

where $K(kT)$ is the $(N_f + N_b) \times (N_f + N_b)$ autocorrelation matrix of the DFE tap coefficient error vector (compare with Equation (4.37) on page 111).

A linear rotational transformation is employed to simplify the formulation of the MSE function by removing the cross-product terms due to the Hermitian autocorrelation matrix R . Specifically, R can be diagonalized as:

$$U^* R U = \Lambda \quad (F.2)$$

where U is a unitary matrix whose column vectors are the eigenvectors, u_i , of R and Λ is a real-valued diagonal matrix of the corresponding eigenvalues, λ_i , of R

The unitary similarity transformation matrix U can also operate on the matrix $K(kT)$:

$$U^* K(kT) U = S(kT) \quad (F.3)$$

where $S(kT)$ is, in general, not a diagonal matrix.

Substituting Equation (F.2) and Equation (F.3) into the excess MSE expression in Equation (F.1) and using the fact that $U^* U = I$, the time evolution of the excess MSE can be expressed as:

$$XSMSE(kT) = tr[\Lambda S(kT)] = \sum_{i=0}^{N_f + N_b - 1} \lambda_i S_{ii}(kT) = \underline{\lambda}' \underline{s}(kT) \quad (F.4)$$

where the vector $\underline{s}(kT)$ is defined as:

$$\underline{s}(kT) = diag(S(kT)) \quad (F.5)$$

Thus, only the diagonal elements of the matrix $S(kT)$ influence the excess MSE. In addition, its time evolution can be expressed as a first-order difference equation:

$$s((k+1)T) = Bs(kT) + \beta^2(MMSE)\underline{\lambda} \quad (F.6)$$

where the (i, j) th element of the real, symmetric, positive definite matrix B is:

$$B_{ij} = \begin{cases} (1 - \beta\lambda_i)^2 + \beta^2\lambda_i^2 & i = j \\ \beta^2\lambda_i\lambda_j & i \neq j \end{cases} \quad (F.7)$$

Therefore, by substituting the solution of this difference equation into Equation (F.4), the excess MSE can be expressed as a sum of a transient and a steady state term:

$$\begin{aligned} XSMSE(kT) &= \underline{\lambda}'B^k[s(0) - \beta^2(MMSE)(I - B)^{-1}\underline{\lambda}] \\ &\quad + \beta^2(MMSE)\underline{\lambda}'(I - B)^{-1}\underline{\lambda} \end{aligned} \quad (F.8)$$

where the first transient term can be written as:

$$XSMSE_{tr}(kT) = \underline{\lambda}'B^k[s(0) - s(\infty)] \quad (F.9)$$

and the second steady term can be written as:

$$XSMSE(\infty) = \underline{\lambda}'s(\infty) = \beta^2(MMSE)\underline{\lambda}'(I - B)^{-1}\underline{\lambda} \quad (F.10)$$

with, assuming $w(0) = \varphi$, the i th component of the initial vector $s(0)$ being (see Equation (F.3)):

$$s_i(0) = E[u_i^* w_o w_o^* u_i] \quad (F.11)$$

To simplify further the excess MSE expression by removing the cross-product terms due to the matrix B (since $S(kT)$ is not a diagonal matrix), the matrix B can also be diagonalized as:

$$G'BG = C \quad (F.12)$$

where G is a matrix whose column vectors are the eigenvectors, g_i , of the matrix B and C is a diagonal matrix of the corresponding eigenvalues, c_i , of the matrix B

Therefore, the excess MSE can also be expressed as:

$$XSMSE(kT) = GC^kG'[\underline{\lambda}'s(0) - XSMSE(\infty)] + XSMSE(\infty) \quad (F.13)$$

$$= \sum_{i=0}^{N_f+N_b-1} \gamma_i c_i^k + XSMSE(\infty)$$

where the transient excess MSE consists of a sum of weighted exponential terms with weights:

$$\gamma_i = \underline{\lambda}' \underline{g}_i \underline{g}_i' [s(0) - s(\infty)] \quad (\text{F.14})$$

and the steady state excess MSE can also be expressed as (compare with Equation (F.10)):

$$XSMSE(\infty) = \underline{\lambda}' s(\infty) = (MMSE) \left[\frac{\sum_{i=0}^{N_f+N_b-1} \beta \lambda_i / (2 - \beta \lambda_i)}{1 - \sum_{i=0}^{N_f+N_b-1} \beta \lambda_i / (2 - \beta \lambda_i)} \right] \quad (\text{F.15})$$

As an aside, note that the steady state excess MSE, or the misadjustment if normalized by the MMSE, is non-zero if the step size is non-zero. In order to minimize this misadjustment of the LMS algorithm, the step size should be kept as small as possible after convergence. However, tracking considerations may require a larger step size. In general, there exists a trade-off between the misadjustment and the tracking lag error [48],[52],[127].

Finally, substituting Equations (F.13) and (F.15) into Equation (F.1) yields the expression for the time evolution of the MSE for the LMS algorithm:

$$MSE(kT) = \sum_{i=0}^{N_f+N_b-1} \gamma_i c_i^k + \frac{MMSE}{1 - \sum_{i=0}^{N_f+N_b-1} \beta \lambda_i / (2 - \beta \lambda_i)} \quad (\text{F.16})$$

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