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(Signed) *Zakaria M. El-Ramly*

PERMANENT ADDRESS:

.....272 Woodfield Drive...
Ottawa, Ontario.
.....
.....K2G 3W6.....

DATED...*Oct 7th*.....1975

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Supervisor: H. J. Rainbird

INVESTIGATION OF THE DEVELOPMENT
OF THE TRAILING VORTEX SYSTEM
BEHIND A SWEPT-BACK WING

by

E. ElRamly
E. ElRamly, B.Sc., M.Eng.

A thesis submitted to the Faculty of
Graduate Studies in partial fulfilment
of the requirement for the degree of
Doctor of Philosophy.

Faculty of Engineering
Carleton University
Ottawa, Ontario

12 June, 1975

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The undersigned hereby recommend to the Faculty of Graduate Studies & Research, Carleton University, acceptance of the thesis, "Investigation of the Development of the Trailing Vortex System behind a Swept-back Wing", submitted by Z. ElRamly, B.Sc., M.Eng., in partial fulfilment of the requirements of the degree of Doctor of Philosophy in Aeronautical Engineering.

M. J. Ramiak
Thesis Supervisor

H. H. Paramanathan
Chairman, Department of Mechanical
and Aeronautical Engineering.

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ABSTRACT

Measurements of the trailing vortex system behind a 35° sweptback wing have been made in a low speed wind tunnel at a mean chord Reynolds number, $R_{\bar{c}}$, of about 0.5×10^6 with vortex Reynolds numbers, Γ_∞/ν , up to about 0.25×10^6 . Spanwise and chordwise loading on the main wing was obtained from pressure distributions, and the character of the boundary layer flow was explored using oil-dot flow visualization and limited hot wire measurements. Mean flow properties of the vortex field have been measured, using a non-nulling 5-hole probe, at two stations $2\frac{1}{2}$ and 5 main wing spans downstream (17.5 and $35\bar{c}$ respectively). Using a simple strain-gauged balance, the induced rolling moment on two trailing wings (of representative relative scale and aircraft geometry), which intercepted the main wing vortex system, was also measured. Maximum induced rolling moments as well as variation with lateral and vertical position were found.

At the $2\frac{1}{2}$ span station it was found that the shear layers, resulting from separation of the turbulent boundary layers on the main wing, had not fully rolled up and consequently the circulation around a circuit near the 'core' was considerably below (about 45% that derived from the corresponding wing spanwise loading. Moving downstream to the 5-span station it was found that no further appreciable roll up of the shear layers occurred when judged by the circulation around the core. This suggests that the shear layers are already effectively fully rolled up at the $2\frac{1}{2}$ span station even though not in an axi-symmetric form.

Comparisons of the measured rolling moments with simple strip theory estimates, based on the measured flow field, and on a line vortex model with strength equal to the rolled up core circulation, have been made and show good agreement. Furthermore, there is very little difference between the induced rolling moments measured at the two downstream stations, which is consistent with the detailed flow field measurements.

Measurements made with simulated engines and with cold air injection representing the jet efflux, showed no appreciable change on the rolled up part of the shear layer "the core". The spiralling of the still unrolled connecting shear layer around the core has, however, been delayed by mounting the engines, especially at the high lift coefficient. The induced rolling moment on the small trailing wing has decreased by about 10%, for $\alpha = 11^\circ$, between the two measuring stations $2\frac{1}{2}b$ and $5b$, as a result of engine mounting. The effect of the simulated jet exhaust, by cold air injection, is small within the downstream measuring range available.

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NOMENCLATURE

- a - constant of proportionality in Squire's (27)* model
 a - radius at which $V_\theta = V_{\theta\max}$
 A - area
 AR - wing aspect ratio
 b - wing span
 c - wing local chord
 $\bar{c}$ - wing mean chord
 C_D - wing drag coefficient, $D / \frac{1}{2} \rho U_\infty^2 S$
 C_ℓ - rolling moment coefficient, $R.M. / (\frac{1}{2} \rho U_\infty^2 \cdot S_t \times b_t)$
 C_L - local lift coefficient, $Lift / unit\ span / \frac{1}{2} \rho U_\infty^2 c$
 $\bar{C}_L$ - overall lift coefficient, $L / \frac{1}{2} \rho U_\infty^2 S$
 C'_L - modified lift coefficient, $L / \frac{1}{2} \rho U_\infty^2 b^2$
 C_p - pressure coefficient, $P - P_{st\infty} / q_\infty$
 C_p - total pressure function, defined by equation III-3
 C_{p_s} - suction box pressure coefficient, defined in Appendix I
 d - distance downstream to a certain degree of roll up
 d_N - exit diameter of simulated jet engine
 D - wing drag
 K_a, K_c, K_o, K_s, K_v - working section calibration parameter, defined in Appendix I
 L - wing lift
 M - Mach number
 P or P_{st} - static pressure
 P_{atm} - ambient or atmospheric pressure

- P - dynamic head parameter, defined by equation III-4
 P_{C_1}, P_{C_2} - tunnel speed sensing pressures
 P_1, P_5 - pressures as measured by the 5-hole probe
 P_o - total pressure
 P_{av} - average of P_1 to P_5
 q - dynamic head, $\frac{1}{2} \rho U_\infty^2$
 Q - function defined by equation III-1
 Q - volume flow rate
 r, θ, z - cylindrical coordinates with positive z along downstream axis
 r_c - core radius; see local text
 R_c - Reynolds number based on mean chord length, $\rho_\infty U_\infty \bar{c} / \mu_\infty$
 $R.M.$ - rolling moment
 S - wing planform area
 t - time
 T - temperature
 U_∞ - free stream velocity
 V_x, V_y, V_z - velocity components along the x, y, z coordinates
 W - mass flow rate
 x, y, z - Cartesian axis, x vertical, y spanwise and z in the downstream direction
 α - angle of attack, or pitch angle
 γ - yaw angle
 ϵ - eddy viscosity

- θ - misalignment angle
- ϕ - roll angle
- Γ - circulation, $\int_0^{2\pi} V_\theta \cdot r \cdot d\theta$
- Γ_c - circulation around a circuit properly chosen around rolled up viscous core
- ΔC_p - local surface loading $C_{p_L} - C_{p_u}$
- μ - absolute viscosity of air
- ν - laminar kinematic viscosity of air
- ν_t - turbulent kinematic viscosity, eddy viscosity
- ρ - mass density of air
- τ - non-dimensional time

Subscripts

- Q - refers to working section centerline values
- F - refers to fan flow
- j - refers to jet flow
- L - refers to lower surface
- m - refers to mean or mixed values
- o - refers to total values
- r - refers to wing root values
- t - refers to wing tip or trailing wing
- u - refers to upper surface
- ∞ - refers to free stream

CHAPTER I

BACKGROUND

In recent years there has been renewed interest and activity (both theoretical and experimental) aimed at understanding the fluid mechanics of trailing vortices generated by aircraft. The aim of all this work is, of course, to reduce the hazards resulting from light aircraft being upset by interaction with trailing vortices from heavily loaded aircraft. If trailing vortex break-up decay can be enhanced then flight spacings can safely be reduced with subsequent improvement in airfield and air-space utilization.

In November 1972 the present author wrote a report⁽¹⁾ in which he very carefully surveyed (but did not critically review) the problem of trailing vortices shed from high aspect ratio straight or swept wings. In the past two and a half years the interest in the problem was still growing and a great number of investigations were contributed in this area. However, because of the complexity of the problem no major break-throughs in the state of knowledge in the area have been achieved, although the initial roll up and formation of the vortex is probably better understood now.

In what follows the previous survey⁽¹⁾ is updated and a general review of the problem is attempted; discussion will fall logically into four sections, namely:

- a) The initial formation and roll up of the vortex system
- b) Steady motion and gradual decay of vortex system
- c) Vortex rapid decay and disintegration, and
- d) Vortex dissipation and control.

Experimental work will be considered, however, those wind tunnel investigations comparable to the present investigation will receive fuller analysis in Chapter VII.

1-1 Initial Formation of the Vortex System and Its Roll Up

Until quite recently, the analysis of aircraft wakes and initial roll up depended upon a simplified model presented by Spreiter and Sacks^{(11)*}. Based on the preservation of lift impulse, and assuming elliptic span loading, the distance between the two vortices, after roll up, is calculated by Spreiter and Sacks and they give the classical separation distance of $\frac{\pi}{4} b$. The streamwise distance required for complete roll up is also derived in Ref. (11)* using Kaden's^{(3)*} analysis, of 1931, for the roll up of a sheet of semi-infinite breadth. The core size can again be calculated using Spreiter and Sacks result, which, however, contains three unconfirmed assumptions. These are: 1) all the shed vorticity wraps into compact cores, 2) the vorticity is uniformly distributed in the core, and 3) the induced drag is equal to the kinetic

* An asterisk beside the reference number, ()*, refers to reference numbers in the author's report "Aircraft Trailing Vortices: A Survey of the Problem", Report #ME/A 72-1, Nov. 1972.

energy per unit length of the vortex motion in the wake. Experimental measurements, both flight and wind tunnel, show the trailing vortex center just inboard but very close to the tip, so the separation distance as predicted by Spreiter and Sacks is not accurate. The concept of a fully rolled up vortex is rather artificial since a residue of the connection sheet is always present but investigators usually use, unjustifiably, any seeming symmetry in tangential velocity as an indication of a full roll up state. Calculations based on this theory (Ref. 11*), generally predict core diameters more than a factor of two larger than observed, in flight tests^{(52,81,82)*} or wind tunnel measurements^{(12,31,93,94)*}; again, Spreiter and Sacks define the core as the central region in which viscous effects are present while investigators usually define the core as the radius at which the tangential velocity reaches its maximum value, thus direct comparison is improper. Hancock^{(19)*}, with similar assumptions to those of Spreiter and Sacks, but conserving the angular momentum rather than the energy, predicts an even larger core size.

An earlier theory by Betz^{(20)*}, that uses the laws of conservation of momentum in potential flow to relate the structure of the vortex sheet behind an isolated wing tip (isolated half span) to the structure of a single, fully-developed vortex, was made popular again when in 1971 Donaldson^{(2)*} demonstrated that the method gives a more accurate description of the radial distribution of vorticity

and tangential velocity in rolled up vortices than that of Spreiter and Sacks^{(11)*}, for elliptically loaded wings. Mason and Marchman^{(102,103)*} generalized Betz's method, using the same arguments and assumptions but represented the bound vorticity distribution on the wing by a Fournier series rather than by an elliptical distribution. Brown⁽²⁾ used Betz's method to calculate the downwash field for the case of parabolic wing loading and compared it to the elliptic loading case. He found that the effect of wing span loading is important in regard to the peak rotational speeds developed in the vortex system, and that the axial velocity at the vortex center could either be less or greater than flight speed depending on the ratio of profile drag to induced drag.

The Betz argument assumes that the vortex roll up proceeds monotonically from tip to wing center, so that the vorticity shed at the trailing edge near the wing tip goes into the center of the vortex located at the spanwise centroid of vorticity. Difficulties arise when calculating the roll up of vortices that trail behind wings with arbitrary span-load distribution, when the loading has a pronounced local dip, and multiple vortex pairs can form, for example with some degree of flap deflection.† For such cases, Rossow⁽³⁾ gives two rules for

† It may be worth noting here that where a very steep change of spanwise loading occurs (in a distance, say, less than the local chord of the wing) the basic assumptions of high aspect ratio wing theory are likely to be violated.

subdividing the vortex sheet into separate segments and for identifying the beginning points of roll up for each segment as 1) Vortex roll up sites are located at maxima of sheet strength and at abrupt changes in sheet strength, 2) the edges of the segment of vortex sheet that rolls into a vortex occur where the sheet strength vanishes or change sign, or where the sheet strength is at a minimum. He uses these rules in a simplified form of Betz equations to calculate the structure of vortices that trail behind wings with arbitrary spanwise loading; however, he makes an approximation by ignoring the interaction of the vortices with one another, along with ignoring variations in axial velocity and, of course, viscous effects. Donaldson et al⁽⁴⁾ also looked at the problem of modifying Betz's theory for multiple vortex roll up, for aircraft wakes with a representation of some degree of flap deflection. In their approach the number of vortices is found by dividing the vorticity up according to where the minima of $\left|\frac{d\Gamma}{dy}\right|$ occurred. Each vortex sheet roll up is then calculated by assuming that the Betz invariants are preserved in detail; but without allowing for the interaction of vortices on each other. The reasonable agreement between the initial tangential velocity profiles in the vortex wakes of DC-7, DC-9 and C-141 aircraft, as measured by NAPEC^{(81)*} using tower fly-by technique, with those calculated by Donaldson et al, using their modified Betz method, led them to conclude that the primary processes which determine the roll up and early history of wake vortices are inviscid. They also

concluded that because of the inviscid character of these wakes turbulence and its diffusion have little to do with determining the initial velocity profiles except, perhaps, in regions where there is a possibility for entrainment of axial momentum [for example if the outboard engine centerline, for wing mounted engines, is close to the flap vortex center]. Yates⁽⁵⁾ investigate the basic validity of the Donaldson⁽⁴⁾ hypothesis by calculating the initial in plane acceleration of the shed vortex sheet. His numerical results [for a C-141 take-off and DC-9 landing configuration] generally confirm the validity of the Donaldson hypothesis and provide additional information on the relative rates of vortex roll up.

The Betz method, or the modifications to it discussed above, does not treat the transition or intermediate stages between the initial vortex sheet behind the wing and the final rolled up vortex structure. They simply use the three conservation relations for two dimensional vortex systems to relate the span-load distribution to the fully-developed vortex structure. Since 1935 several investigators have tried to numerically calculate the detailed roll up of the vortex system by discretization of the assumed vortex sheet trailing behind the wing, even though, these numerical methods are in great dispute^{(5-7)*}. [The vortex sheet is an idealization for the case of inviscid flow behind high aspect ratio wings. In the real case, the boundary layer on the upper and lower surfaces of the wing, with its distributed vorticity, together with the velocity

discontinuity, or the vorticity, associated with lift, form a shear layer of finite but small thickness, rather than a vortex sheet, which trails behind the wing. The flow near the wing tip is highly three dimensional and the vorticity vector there is far from being in the streamwise direction.] In 1935 Westwater^{(4)*} simplified the problem of inviscid roll up behind an elliptically loaded wing by ignoring the bound vortex and treating the trailing vortices as infinite (in both directions) so that the problem becomes a two-dimensional one. Using a step by step process, he replaced the continuous sheet by a finite number of line vortices of equal total 'streamwise' vorticity. In 1973 Clements and Maull⁽⁶⁾ used essentially Westwater's model to calculate the roll up of a vortex sheet as if generated by a wing with loading represented by a Fourier series. They found that improvements of the order of 13-15% in the strength of the tip vortices may be achieved at a given downstream distance (time) for incurred induced drag penalties as low as 5% (as a result of using other than elliptic loading). More realistic approaches to the numerical calculation of the roll up allowing for some of the three dimensional aspects of the problem are those of Butter and Hancock^{(8)*}, Hackett and Evans^{(9)*} and Labrujere⁽⁷⁾. In their approaches the finite upstream length of the trailing vortices and the effect of the bound vortex system (representing the wing) were taken into account. Moreover, Labrujere⁽⁷⁾ represented the behavior of the vortex sheet at the trailing edge using the relations derived by Mangler and Smith⁽⁸⁾.

Hackett and Evans^{(9)*} presented examples to show the effect of sweep, lift coefficient and incidence, the effect of approaching the ground, and the effect of wind tunnel wall constraint [wind tunnel wall effects were represented by a few layers of image vortex sheets].

Examination of the roll up computational results cited above, reveals that most of these authors encountered an irregular roll up of the vortex sheet in the tip region. Hackett and Evans^{(9)*}, for example, found that with a short step size, 0.01 semispan, for equal strength vortices, the vortex sheet has crossed itself, which is physically impossible in a real flow; they warn against trying to look in great detail near the singularity associated with the sheet edge (and core). In Nielsen and Schwind^{(10)*} calculation of vortex roll up if two vortices come closer than a prescribed distance compatible with the accuracy of calculation based on the axial interval size, they are combined at their center of gravity and the calculation continued downstream. Kuwahara and Takami⁽⁹⁾ found that by including an "artificial viscous core" term in the equation for induced velocity with a suitably chosen value of the "artificial viscosity coefficient" the irregularities that normally occur in the tip region can be smoothed out. Bloom and Jen⁽¹⁰⁾ applied Kuwahara and Takami's proposal to a number of aerodynamic configurations and found that the theoretical estimate of the vortex core location compared well with the spanwise position measurement, if a proper value of the artificial viscosity is used.

Discretization of the vortex sheet model by finite elements, as used by Mokry and Rainbird⁽¹¹⁾, resulted in a more stable behavior than the Westwater's^{(4)*} array of line vortices. They suggest that for a suitable distribution of vortex sheet elements and for appropriate time increments, the roll up process can be followed over sufficiently large times without the need for artificial viscosity^(9,10) as reported by Mokry and Rainbird's investigation was initiated to calculate the roll up of the vortex sheet in a rectangular wind tunnel, having the characteristics of the Carleton University test facility and the size of wing used in the experimental investigation to be reported here. Their model is basically two dimensional and utilizes the concept of the influence (Green's) function (analogous to that used by Mokry⁽⁴⁴⁾) to allow for the influence of the wind tunnel solid wall. Figs. (1-1 and -2) show a comparison of their calculated vortex roll up histories in free air and in the wind tunnel. Fig. (1-1) shows results for an elliptic load distribution, $\Gamma \sin \theta$. With the exception of downwash, which is clearly constrained by the working section walls, the roll up patterns seem to differ very little in the initial stage. Later, however, the wind tunnel spiral tends to deform in a flattened manner according to the constraining 3:1 rectangular walls. The wind tunnel distortion effect is more pronounced, Fig. (1-2), for the load distribution⁽⁶⁾

$$\Gamma = (\sin \theta + \sqrt{2}/10) \sin 5\theta \quad (10\% \text{ induced drag penalty}).$$

FIG. (1-1) TIME HISTORY OF A VORTEX SHEET
 $\Gamma_{\infty} \sin(\theta)$ "ELLIPTIC LOADING"
 FROM REF. (11)

FIG. (1-2) TIME HISTORY OF A VORTEX SHEET
 10% DRAG PENALTY
 FROM REF. (11)

reviewed different theories explaining the vortex breakdown phenomenon. Vortex breakdown, or bursting as it is sometimes called, has long been observed over slender wings in wind tunnels and in flight, but has only been recently observed in trailing vortices behind aircraft with high aspect ratio wings (18-20). Parks (48)* described a new axial flow core instability associated with trailing vortices with high axial velocity, which he calls trailing jet-vortex pair. This axial instability might be related to vortex breakdown. Olsen (53)* has noticed the axial flow instability in his towing tank experiments, behind a high aspect ratio wing. The instability drastically enlarges the core, reducing the tangential velocities, but does little to change the motion far from the core.

The theory for hydrodynamic instability was first introduced by Crow (45)*. In his analysis, Crow idealized the wake as a pair of nearly parallel inviscid vortex lines, and the analysis gives the maximally unstable long and short wavelengths and their amplification rate. The effects of finite core as well as the effect of axial flow in the core has been considered in references (46-48, 122, 123)*.

Existing stability theories seem useful in predicting the unstable disturbance wavelengths, however, they do not explain how the disturbances started nor the time of final breakdown. Factors involved in initiating disturbances and breakdown may include atmospheric turbulence, wake turbulence, periodic lift

circulation Γ_0 around a circuit distant from the vortex core, i.e. $\epsilon = a \Gamma_0$ where "a" is constant. The solution for turbulent flow is then similar to laminar flow with $(\nu + \epsilon)$ replacing ν . Experimental measurements^{(13,23,28)*} show that the value of the eddy viscosity required to make the laminar-like solution fit experimental measurements in turbulent vortices, varies with both radial and axial directions. Flight tests^{(34,78,79)*} give different values of the constant "a". Owen^{(33)*} suggested a model for a turbulent trailing vortex in which he avoids the postulation of eddy viscosity. His solution shows that "a", in Squire's analysis, should not be constant, but should vary with the age of the vortex and should asymptotically reach a constant value. Experimental results show a better fit to Owen's solution. A very good complete discussion of the above mentioned solutions can be found in Ref. (22)* by Hall (1966).

Hoffmann and Joubert^{(32)*} attempted a similarity solution, on lines similar to that taken in boundary layer theory, for incompressible, turbulent line vortices. They established that the inner region for fully turbulent vortices, like the law of the wall region for bounded turbulent flows, is separated into three regions. These are:

- i) an "eye" of solid body rotation
- ii) a transition between the solid body rotation and the logarithmic circulation, and
- iii) a region in which circulation varies logarithmically with radius.

The logarithmic distribution of circulation has been confirmed not only with wind tunnel measurements^{(16,32)*} and by the smaller scale flight test results of Ref.(16)* but also by the FAA measurements obtained with current jet transports^{(88)*}.

Using a fundamentally different argument from that of Hoffmann and Joubert^{(32)*}, Saffman⁽¹²⁾ presented a theory which argues that the turbulent vortex has a triple structure. There is an outer region for $r > r_1$ (r_1 is the radius of maximum tangential velocity) with a logarithmic distribution of circulation, and for $r < r_1$ an inner region and viscous core in both of which the motion is close to solid body rotation. The theory predicted that $r_1 \sim (\nu \Gamma_1 t^2)^{\frac{1}{2}}$, where Γ_1 is the circulation at r_1 , ν is the kinematic viscosity, and t denotes vortex age.

Two questions have to be answered if the motion of trailing vortices is to be understood. The first is concerned with the axial motion in the core. The second has to do with the turbulent structure of the core.

Batchelor^{(29)*} has shown that both axial velocity excess and deficit are possible in the core, of a laminar vortex, depending on the profile drag of the assumed generating wing. Moore and Saffman⁽¹³⁾ found that the perturbation of axial velocity, again for laminar flow, can be either away from the wing or towards the wing, depending on the distribution of loading near the tip of the wing; for elliptic loading, the perturbation is towards the wing. For turbulent vortices, Saffman⁽¹²⁾, using an argument full of approximations, found

that the axial velocity is to leading order uniform across the core. Wind tunnel measurements, for example Ref. (12,30,31,90,91,93,103)* show all kinds of values for velocity excess and deficit.

Some investigators question the existence of turbulence within the core. Pictures of smoke introduced into trailing vortices from towers^{(52,81,82)*} suggest that the cores of trailing vortices are laminar (however, the pictures show vortices that have been 'cut' by the tower and the effect of cutting the vortex is not known). Crow^{(109)*} questions whether a simple vortex can sustain turbulence at all. Owen^{(33)*} suggests that the turbulence in the core is sustained by the swirl of circumferential velocity; however, Donaldson^{(43)*} argues that axial flow is a necessary condition to sustain turbulence.

To the best of the author's knowledge, there are only three sets of turbulence measurements in vortex cores in wind tunnel. Two of the measurements were performed at the same facility^{(90)*}, (the annular tunnel at McGill University), but were generated by a split wing, and the question of how representative the generated vortex to the trailing vortex is still to be answered. The other set of measurements is that of Chigier and Corsiglia^{(31)*} (in the NASA Ames 7-by 10 Foot Wind Tunnel) behind a rectangular and a CV-990 model wings. They measured turbulence intensity (for the axial component) as high as 13%; however, they reported later⁽¹⁵⁾ that the vortex was not stationary but was meandering

in the tunnel, and this must have contributed to the high turbulence intensity measured.

Snedeker⁽¹⁶⁾ made a comparison of Poppleton^{(90)*} measurements with computations based on Donaldson's invariant modeling method. He concluded that two dimensional, axially symmetric model equations do not adequately describe the development of the turbulence characteristics of Poppleton's vortex; and that description of the vortex requires a theory which accounts for a strongly coupled axial flow.

An interesting phenomenon that could happen to trailing vortices is the overshoot in circulation (a strange situation in which the circulation rises above Γ_∞ , where Γ_∞ is the circulation very far from vortex center, and then falls back to Γ_∞ at large r . Govindaraju and Saffman^{(41)*} were the first to present an analysis in which they were trying to prove that the circulation distribution in a turbulent vortex is very likely to acquire a maximum value which is greater than Γ_∞ (i.e. develop overshoot in circulation). Donaldson et al^{(43)*} also found a circulation overshoot possible in their analysis of the effect of turbulent shear on the decay of a single vortex. Donaldson et al^{(43)*} discussed this overshoot, which results in instability in the Rayleigh sense, and they suggested that perhaps it is the instability that is inherent in the overshoot in the circulation well outside the core of a turbulent trailing vortex, that is responsible for the "doughnut" shaped rings sometimes observed about the trailing vortices in the wake of a jet aircraft. Graham et al⁽¹⁷⁾ reported observing circulation overshoot in

their measurements; behind the split wing in the McGill annular tunnel (this work will be discussed again later in this Chapter).

1-3 Vortex Decay and Disintegration

There are three mechanisms known by which a trailing vortex system can decay or disintegrate, these are: a) viscous dissipation, b) vortex breakdown, and c) mutual or hydrodynamic instability.

Viscous dissipation, and we include here any dissipation caused by small-scale turbulence that interacts with the velocity gradients in the vortex to produce local Reynolds stresses, is a very slow process. In fact it is too slow to the extent that it has led some investigators to question if there is indeed any turbulence in the core, since a turbulent vortex core should have a high rate of diffusion. Nielsen and Schwind^{(10)*} studied the motion of a vortex pair in a quiet atmosphere with no buoyancy; a case that represents a vortex with extreme persistence. The available mechanisms for decay are the viscous dissipation and the interaction between the two vortices, when they grow in size to the extent that their cores start 'touching' and 'overlapping'. The analysis shows that the interaction between the vortices appears not to present a promising mechanism for rapid decay of trailing vortices.

The problem of "vortex breakdown" has been the subject of extensive analytical and experimental work. Hall^{(61)*} recently

reviewed different theories explaining the vortex breakdown phenomenon. Vortex breakdown, or bursting as it is sometimes called, has long been observed over slender wings in wind tunnels and in flight, but has only been recently observed in trailing vortices behind aircraft with high aspect ratio wings⁽¹⁸⁻²⁰⁾. Parks^{(48)*} described a new axial flow core instability associated with trailing vortices with high axial velocity, which he calls trailing jet-vortex pair. This axial instability might be related to vortex breakdown. Olsen^{(53)*} has noticed the axial flow instability in his towing tank experiments, behind a high aspect ratio wing. The instability drastically enlarges the core, reducing the tangential velocities, but does little to change the motion far from the core.

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variation, Benard cell-type structure arising from buoyancy, and core characteristics^{(72)*}. Another difficulty with stability theories is that they assume the vortices are doubly infinite in extent; they cannot be used to calculate the growth of perturbations deliberately introduced at the wings of the aircraft. Bilanin and Widnall⁽²¹⁾ modified the theory for sinusoidal instability to include the effect of forcing (differentially oscillating flaps to keep lift constant).

Subscale measurements^{(53,55,59)*}, often with Reynolds number 10^3 below flight values, in towing tanks, show all kinds of instabilities, long wave instability, short wave instability, and axial flow instability, as well as vortex breakdown. The experiment of Bilanin and Widnall⁽²¹⁾ is particularly interesting. Sinusoidal instability of the trailing vortices from a model wing, in a ship towing tank, were induced by differentially oscillating flaps so as to keep total lift constant. Flap oscillation shifts the centroid of vorticity in the wake and results in a sinusoidal axial perturbation to the circulation of the trailing vortex pair. Measured amplification rates qualitatively agree with their theoretical predictions. Vortex breakdown was also observed to occur along trailing vortices undergoing sinusoidal instability near but ahead (towards the wing) of the position of maximum separation. Axisymmetric pressure gradients imposed along the vortex core by the other sinusoidally deformed vortex were responsible for the observed changes in core diameter.

Flight tests designed to describe the characteristics of vortex behavior and their instability were reported by

Tombach⁽¹⁹⁾, Chevalier⁽¹⁸⁾ and Jones and Chevalier⁽²⁰⁾. The tests were basically observations and photographic analysis of the trailing vortex system generated by a light aircraft (Cessna 170 for the tests reported by Tombach and DeHavilland Beaver DHC-2 and a Beachcraft T-34B for tests reported by Chevalier); smoke grenades located near the wing tips made the vortices visible. The tests revealed similar conclusions and because of the definitive nature of these conclusions some of them are summarized below:

- 1) The results of the flight test investigations have provided additional experimental verification of the existence of vortex wake instability predicted by the Crow theory. The results also show rough agreement of the long wavelength instability as predicted by theory.
- 2) The vortices were never observed to decay away due to viscous or turbulent dissipation, but were always destroyed by some form of instability.
- 3) Two modes of instability were observed. One was a localized "bursting" of the smoke-marked core of an individual vortex, usually with no apparent effect on the adjacent vortex. The other instability was the sinuous "Crow" instability of both vortices which resulted in their linking into vortex rings.
- 4) There is a clear correlation between wake lifetime and atmospheric turbulence. The life of the wake is

dramatically shortened by even small amounts of natural wind turbulence.

- 5) The majority of the wakes were observed to roll onto their sides to some degree.
- 6) Flight tests made with the DeHavilland Beaver during calm conditions with the elevator oscillated at the appropriate frequency showed a marked reduction in the time it took the vortices to dissipate. Oscillating the airplane's rudder and ailerons did not produce an appreciable change in the total vortex dissipation time.

The question of scale is of relevance to the experiments described above. The relationship between energy in the wake and that in the atmosphere should have some effect on determining the degree of atmospheric dominance of the wake motion. Tombach⁽¹⁹⁾ argues that the common mode of decay in very small scale (or low Reynolds number) experiment is the core bursting mode, while for experiments with large transport aircraft it appears to be the linking mode. Additional testing using aircraft with large differences in size are needed to obtain more detailed understanding of the instability.

Another factor that determines the life of vortices is the interaction with the atmosphere. Besides the fact that atmospheric turbulence might trigger the hydrodynamic instability, relative buoyancy affects the vertical transport of the vortices and their horizontal separation. The effects also depend intimately upon the mixing between the wake and the environment.

There are several models that describe the motion of buoyant vortex wakes^{(40,73,74,76)*,(22)}, however, all are idealized models and include unconfirmed assumptions. Unfortunately, there are no experimental results available that can be used to check any of these models.

1-4 Vortex Dissipation and Control

Several attempts have been made to accelerate the dissipation or disintegration of trailing vortices, or to reduce the hazard in encountering their wakes, by trying to spread the tight cores of high vorticity and consequently reducing the maximum tangential velocities. Experimental approaches include,

- a) axial mass flow injection near the core^{(90,91,102-107)*,(23,24)},
- b) change of wing tip configuration^{(99,100,118)*,(24-27,37)},
- c) drooping the wing tip^{(97,98)*} (this can lead to breaking the vortex into two vortices),^{(94,120)*,(24,26,27)}
- d) use of drag devices.

Experimental results show that mass flow injection near the core can result in an appreciable increase in the core size and a reduction of the maximum swirl component of velocity. Several explanations have been offered for the effect of mass flow injection on the vortex system. Reference (90)* explains the effect of injection through the core as effectively to "age" the vortex prematurely, while reference (102)* explains the effect as primarily that of introducing large amounts of turbulence into the core. Kantha's et al^{(107)*} explanation is quite different. In the vicinity of highly loaded wing tip the tangential and axial velocities in the vortex core are of the same order of

magnitude. Under this condition the vortex appears quite stable and thus persists far downstream. Vorticity diffusion can be enhanced by either sufficiently increasing or decreasing the core axial velocity. If the core axial velocity is sufficiently decelerated it is possible for downstream turbulence to propagate upstream via inertial waves to dissipate the core. On the other hand, when the axial velocity is increased to be larger than the tangential velocity, the core behaves as a turbulent jet and thus spreads relatively quickly.

An approximate theory has been developed, by Graham et al⁽¹⁷⁾, for the decay of the vortex in the presence of a small-increment jet or wake by assuming that the jet dominates the flow and provide a scalar eddy viscosity which may be used to predict the growth of the vortex. The maximum rotational velocity $V_{\max}$ is shown to be related to the momentum flux increment or, decrement $|J|$, the circulation Γ_{∞} and downstream distance z by

$$V_{\max} = 0.35 \Gamma_{\infty} \left(\frac{\rho U_{\infty}^2}{|J| z} \right)^{\frac{1}{3}}.$$

Graham et al estimated that for a Boeing 747, at take-off and landing speeds, if 10% of the total thrust of the four JT9D Pratt and Whitney engines is used for vortex modification the time for the maximum tangential velocity to decay to a value of 17.25 ft/sec is reduced to 9.15 minutes, compared to 10 minutes required with no modification.

Although experimental results clearly show that axial injection does modify the velocity profile and results in a lower

tangential velocity, there is no experimental evidence that a vortex wake which has been modified by a jet is actually less hazardous at a given downstream position than one which is unmodified. Snederker^{(121)*} did limited experimental work in which a simple wing- torque meter was used to measure the induced rolling moment in a vortex wake with and without axial mass injection. The rolling moment as measured by the torque-meter is only slightly affected, even with injection velocity as high as 18 times the free stream velocity (note that conclusion depends on relative size of device and vortex core).

Mass flow injection has been tried not only axially downstream but also facing upstream⁽²³⁾ ("forward blowing"), facing downward, deflected jet and with jet and blown flaps on the wing tip⁽²⁴⁾. A comparison of the different injection methods was given by Reference (24) and will be presented later in this section.

In an attempt to reduce the vortex hazard by modifying the tip such as to diffuse the vortex core, investigators have tried several tip shapes. As a matter of fact so many tip shapes and modifications have been tried that it is hard to think of a shape, conventional or unconventional, that has not been tried. However, most of the results are, in the author's opinion, inconclusive, either because insufficient data were collected or because very unrepresentative basic wing configurations were used. Investigations aiming at assessing the effect of changing the tip shape on the vortex system must include, at least, a) lift measurements, or still better, spanwise loading, and

b) quantitative flow field measurements far enough downstream to avoid looking at variations in the initial phase of roll up. Table I-1 shows a listing of some experimental work aiming at changing the vortex system by changing the tip, the technique used for measurements and the general conclusions. The table also includes experimental work in which drag or drogue devices were used in an effort to spread the core and reduce the maximum tangential velocity. Again, assessment of such devices is not possible without measuring the associated drag penalty; even if they prove to be effective in spreading the core. Also, a change in core size and tangential velocity profile alone should not be used as a measure of hazard reduction of a vortex system, unless the new structure results in a faster decay. When some of the techniques that showed effectiveness in spreading the core in a wind tunnel were tried with full scale aircraft (1,94)*, there was very little change noted in the vortex system far downstream.

Reference (126)* presents unpublished work of Crow in which the motion of an airplane flying parallel to a vortex wake and at various positions relative to the wake was calculated using strip theory. A danger region was defined as the region where the calculated roll rate exceeded the maximum possible roll rate using full control at the same flight speed in still air. Results for a Boeing 737 following a Boeing 747 show that the danger region is essentially independent of core size until the core diameter approaches the span of the following aircraft.

TABLE (1-1)

Reference	Basic Wing	Tip Modification	Drag Device	Measuring Technique	Conclusion
Pisiali and Tranks (99)*	Straight, untapered semi-span $R_c = 8$, NACA 0012	Rounded square tip, blunt square, 45° swept forward, 45° swept aft, 60° swept forward, and a cusp tip.		Flow visualization, Oil smoke and surface flow with pigmented oil. (Wind tunnel)	Square planform tip generates the most outboard and more concentrated tip vortex.
McCormick and Padakannaya (97,98)*	Straight, untapered semi-span $R_c = 4.5$	Drooping 45% of the semi-span. Droop angles of 0°, 70°, 80°, 90°, and 110°.		Vortex meter, up to 2 1/2 chords downstream. (Wind tunnel)	For droop angles of 90° or less, two distinct vortices are generated. A droop angle of 90° appears to result in minimum value of induced velocity in the combined vortex system.
Scheiman and Shivers (100)*	Straight, untapered semi-span.	Leading edge disk flow spoiler, a trailing-edge disk flow spoiler, a porous wing-span extension, and a tip adjustable air jet-sheet ejector.		Tuft grid photographic, overall aerodynamic forces measurements. (Wind tunnel)	No tip configuration was tested that resulted in any appreciable vortex core position change with respect to the basic configuration.
Marchman and Uzel (118)*	Straight, untapered semi-wing, $R_c = 12$, NACA 0012		Crossed blade behind the wing tip (3 sizes), fence on the upper and lower surface at tip.	5-hole probe measurements at 10 and 30 chords downstream. (Wind tunnel)	Reduction in maximum tangential velocity and spread of core.

TABLE (1-1) Continued

Reference	Basic Wing	Tip Modification	Drag Device	Measuring Technique	Conclusion
Corsiglia et al (94)*	Straight, untapered $R = 9$, 0.13C thick.		Dissipator plate at wing tip.	Smoke-tuft grid - hot wire. (Wind tunnel)	Reduction in max. tangential velocity, increase in core size.
Scheiman et al (120)*	Straight, untapered NACA 0012, span 7 ft. chord 2 ft.		Wind sock, solid cone and a small vane down- stream of the wing tip.	Tuft grid and smoke visualization. Force measurements. (Wind tunnel)	Most successful object is the vane. Decrease vorticity in the core but max. tangential velocity and core size are the same.
Kirkman et al (24)	F-747 aircraft model.	Forward, downward and rearward blowing jets, de- flected jet and jet and blown flaps at tip.	Cabled drogue cone, spline, vortex genera- tor, spoiler dissipator and trailing edge flap.	Flow visualization, arrays of hot film probes. Force measurement. (towing tank)	See main text.
Borke and Moffitt (25)	Straight, untapered semi- wing, $R = 4.2$, NACA 0012.	Ogee tip (designed to eliminate the separation vortex near the tip).		Force measurements, triaxial hot wire probe. (Wind tunnel)	Max. tangential velocity in the vortex of Ogee tip is 25% of the rectangular wing.
Thompson (27)	Straight, untapered semi- wing, $R = 4$, NACA 0012.	Inboard flap, out- board flap. Rec- tangular, semi- circular, delta and reversed delta tip extension. Circular end plate extension, fully perforated, inboard perforations and outboard perfora- tions.	Upper surface spoiler, lower surface spoiler, tip face spoiler, and trailing edge tip spoiler.	Hydrogen bubble visualization technique. (Towing tank)	Increase in core size. Reduction in axial and tan- gential velocities. Most effective modifications are the perforated tip extension, the circular end plate, and the upper surface tip spoiler.

TABLE (1-1) Continued

Reference	Basic Wing	Tip Modification	Drag Device	Measuring Technique	Conclusion
Patterson (26)	Douglas C-54 aircraft.		Spline drogue devices.	Chase Piper Cherokee aircraft.	The vortex of C-54 may be attenuated with spline drogue device to be considered non- hazardous to smaller aircraft.
Earl (37)	Swept 35°, half- wing 1/3 taper ratio, $R = 7$, symmetrical section no twist.	3 sharp edged extension; 70° Delta planform, hyper- bolic leading edge, and hyperbolic trailing edge. Rect- angular endplate 17 tip chords high.		Surface flow visualization, detailed surface pressure plotting, induced rolling moment on representative trailing wing, and total pressure measurements.	The tip extensions spread the core size but did not effect the induced rolling moment on the trailing wing. Rectangular end- plate reduced the induced rolling moment by about 15-20%, but the exact nature of the vortex structure was inconclusive.

The experimental work of Ref. (24) is probably the most comprehensive in evaluating the effectiveness of various candidate aircraft-wing devices for attenuation of trailing vortices. A 0.03-scale model of a Boeing 747 aircraft was used at the Hydro-nautics Ship Model Basin. The evaluation was based on vortex velocity distribution, made by hot film probes, and the calculated induced rolling moment on a small following aircraft; Gates Learjet. Overall lift and drag measurements on the generating aircraft model were also made to assess the effects of each device investigated on the performance characteristics. Eleven devices were tried, six active devices and five passive devices. The active devices included a forward blowing jet, a downward blowing jet, a rearward blowing jet, a deflected jet and a jet flap all mounted at the wing tip. The passive devices included a cabled drogue cone, spoiler dissipator, trailing edge flap and splines all mounted at or near the tip, and an array of vortex generators mounted on both the upper and lower surfaces of the basic aircraft. In cruise conditions, the vortex generators were found to be the most effective among the devices investigated from the standpoint of reducing the induced velocities of the trailing vortices. The added drag from the vortex generators was about 50 percent of the Boeing 747 in cruise condition (with loss of lift of about 5 percent). The Blown flap, which was the most effective among the active devices, added less than 10 percent to the drag with no measurable loss in lift, but was somewhat less effective in reducing vortex velocities.

In Flaps-30 conditions, none of the devices investigated produced a significant reduction in the induced velocities. Anyhow, based on calculation, a small following aircraft (Gates Learjet) will be unable to counteract the induced rolling moment using maximum aileron control when flying through the core center of a trailing vortex generated by the Boeing 747 in the cruise condition at a distance of 4.42 kilometers downstream; even if the 747 is equipped with the best of the devices among those investigated by Ref.(24).

It is interesting to notice that a spline drogue device, which Ref.(26) claims to be effective in reducing vortex hazard, could result in increase of drag as high as 280 percent as measured by Ref.(24), depending on its size. This should stress the importance of collecting a proper set of measurements to obtain conclusive answers from a test.

1-5 Experimental Measurements and Subscale Modeling of Aircraft Trailing Vortices

Aircraft trailing vortices do not lend themselves easily to laboratory investigation, the main problem being the distance downstream and the usual difficulties with Reynolds number simulation. Another obvious problem is wall interference, although this can be accounted for approximately, wall effects are, however, sometimes hard to assess especially if one is studying hydrodynamic instability of the Crow type, for example. Simulation of atmospheric conditions, such as temperature

distribution or the relative scale of turbulence is clearly very difficult if not impossible.

In any test facility the relative motion between the test model and the working fluid can be achieved either by towing the model through stationary working fluid (towing tank and moving-wing (89)*, (26)) or, through blowing the working fluid over a stationary model (wind and water tunnel). The first mechanism is suitable for studying phenomenon that move with the surrounding fluid, i.e. far field vortex phenomenon, while the second mechanism is suitable for studying phenomenon stationary with respect to the wing, i.e. near field phenomenon.

The importance of actual flight tests is quite clear; however, the techniques of flight testing also present a problem. The techniques usually used are:

- a) Using probe aircraft: the probe aircraft either follows the generating aircraft^{(77,78)*} or traverses its vortex system^{(34,79)*}. The generating aircraft is usually equipped with smoke generating devices to mark the trailing vortices or the tests are done at high altitude where condensation trails exist. The probe aircraft is usually instrumented with pressure and flow angle measuring devices located on a boom at the nose to measure the mean vortex characteristics^{(77,79)*} or with instruments to measure the probe aircraft response (pitch velocity, yaw velocity,, angle of attack, bank angle,, normal acceleration^{(80)*}

Yet in some experiments no special instruments were used at all and the pilots' impressions of the severity of the disturbances are taken^{(78)*}.

There are problems associated with this technique: the probe aircraft cannot be too close to the generating aircraft, for safety and control reasons, and the near flow field therefore cannot be studied. Also very far from the generating aircraft the vortex is not visible enough to be penetrated accurately. Another problem is that the pilot of the probe aircraft has to rely on his own judgement to find the vortex core and it is highly probable that he might miss it. Another factor is the short time scale and rapid response instrumentation needed if the probe aircraft is to be flown transversely through the vortex system (note that the total time to traverse through the vortex system laterally might only be of the order of $\frac{1}{2}$ sec.)

- b) The tower fly by technique: different experiments were performed using ground mounted instrumentation. These instruments could be a tuft grid^{(16)*}, smoke bombs^{(81,82)*} or velocity sensors^{(16,81,82)*}. Ground effects are naturally always present in these experiments and the effect of the tower on the vortex system (the tower acts as an obstacle in the flow field) is unknown. In some of these experiments the technique depends on the slow lateral drift of the vortex system across the tower, in light lateral wind conditions^{(81)*}. Hence it is difficult to determine

accurately the time (age) and distance from the generating aircraft at which the vortex encounter is made.

- c) Remote measuring of the vortex system: this technique includes the Laser Doppler^{(83)*}, Doppler Radar^{(84)*}, and acoustic radar detection techniques^{(85,86,101)*}. These systems are still under development; however, they will eventually offer a very good tool for vortex detection, interference problems are not present and the measurements are direct.
- d) Instrumenting the aircraft to measure its own vortex system: this technique was used by McCormick et al^{(16)*}. A vortex meter was supported on the aircraft and vorticity surveys were made in the proximity of the trailing edge of its wing. Clearly such techniques are limited to the very near vortex field.

Although the opportunities for research offered by the conventional wind tunnel are restricted to the near wake, because of the length of working section normally available, nonetheless, several wind tunnel facilities have been actively involved in trailing vortex studies. However, comparison of one researcher's test data with that of another, or with theoretical models, on a meaningful basis is always a problem. The wide range of tests has led to a very wide range of results with no basis to compare them to each other or to full-scale flight test results. The main reason seems to be the failure of most researchers to measure

enough basic parameters to completely define the vortex flow field and the characteristics of the generating wing. We should not expect to be able to fully define a test condition by merely giving the generating wing angle of attack and Trefftz-plane mean flow measurements. The generating wing loading and the nature of the boundary layer and its separation from the wing surface are essential information as is the Reynolds number scale. Another problem is that most investigators use wings of unrepresentative configuration. Straight, untapered wings seem to be popular, presumably because they are easier to machine or to calculate their loading. However, the new generation of commercial aircraft which represent the hazard have swept tapered wings and as long as the effect of sweepback, taper and the nature of separation at the tip are not known, results of straight wings cannot be used with any confidence. For unswept wings, Grow^{(15)*} conducted some tests for aspect ratios from 2.0 to 6.0 and taper ratios from 0 to 1.0. The half-wings used were of NACA 0012 section and tests were performed at $R_c = 3.5 \times 10^5$. The results show that the circulation and the maximum tangential velocity of the rolled up vortex increase with increasing aspect ratio and taper ratio. However, these measurements were made at approximately 4 chords downstream and it could very well be that the vortex was still rolling up.

The classical question of Reynolds number simulation and the validity of applying wind tunnel results to full scale is

always asked, even with properly conducted tests. Ref.(16)* reported results of flight tests using a U.S. Army O-1 aircraft and a Piper Cherokee ($R_C > 3 \times 10^6$) and of wind tunnel tests for a $\frac{1}{12}$ scale model of the O-1 wing ($R_C = 3.5 \times 10^5$). The results show that the vorticity in the vortex system for a full scale aircraft was reported to be approximately a third of that for the model. Also, at corresponding downstream distances, the shear layer behind the full scale aircraft appears to be more completely rolled up. The radial distribution of circulation of the vortex changes from an exponential form at the model Reynolds number to a logarithmic form for the full scale. However, these conclusions are based on vorticity probe measurements. The diameter of the vanes was $\frac{3}{8}$ in. for the wind tunnel measurements and $\frac{3}{4}$ in. for the full scale measurements, and a factor of 6 difference in spacial resolution is thus present.

Graham et al⁽¹⁷⁾ explain that the apparent, but in their opinion unlikely, scale effects on the decay of trailing vortices is the result of two factors. Firstly, the drag coefficient of the wing is less at high Reynolds numbers, and secondly, some momentum flux from the jet engines or propellers may be gathered into the vortices in full scale case. The two factors will result in reducing the momentum deficit in flight conditions compared to wind tunnel tests, and consequently they would have a slower rate of decay.

Rorke and Moffitt⁽²⁵⁾ conducted an experimental investigation in a wind tunnel to determine the important scaling parameters for the flow in the core region of a vortex generated by a rectangular wing tip. Using two geometrically similar wings (with chords of 26.0 and 4.25 inches) and running at different tunnel speeds, it was possible to obtain Reynolds numbers ranging from 4.4×10^5 to 7.0×10^6 at Mach numbers of 0.2, 0.5 and 0.6. Their results show that the measured vortex core diameter to chord ratios, peak tangential velocity ratios, and axial velocity ratios are functions only of wing lift coefficient and elapsed time from vortex formation, and appear to be independent of Mach number and Reynolds number. In an effort to verify and extend the work of Rorke and Moffitt, Marchman and Marshall⁽²⁸⁾ conducted a wind tunnel study by examining the flow downstream of a single square tipped NACA 0012 wing at various angles of attack over a range of downstream distances and free stream speeds (free stream dynamic head between 0.54 and 2.2 inches of water). The resulting data indicates that vortex age is not a self-sufficient scaling parameter but a free-stream velocity influence also exists at higher angles of attack which cannot be explained in terms of Reynolds number or Mach number. However, the author finds it difficult to accept that at a free stream dynamic head of 0.54 inches of water one can measure the vortex induced velocities, using inclined manometers, accurately enough to answer a question of such high sensitivity.

In order to increase the available distance downstream in a wind tunnel Corsiglia et al⁽¹⁵⁾ used two relatively small geometrically similar rectangular wings (96 in. and 32 in. span, NACA 0015 section, aspect ratio 5.33, square cut tip) in the large NASA Ames 40- by 80-foot wind tunnel. The tunnel has no screens, and only a small contraction ratio, and the high turbulence level resulted in a distorted vortex path; the trailing vortices were not stationary and the amplitude of the disturbance increased with distance downstream. A rotating arm apparatus allowed a continuous and constant speed traverse through the vortex. The three dimensional hot wire probe used was supported at the tip of the rotating arm. From a series of velocity distributions in the vicinity of the vortex core, obtained as the vortex meandered across the traverse path, the traverses that came closest to passing through the vortex center were selected. Measurements were made up to 31 spans downstream for the small wing at a chord Reynolds number of 0.3×10^6 . Peak tangential velocities normalized by free-stream velocity and lift coefficient decreased from 0.8 at the trailing edge to 0.6 at 31 span lengths downstream while the circulation within the core (where V_θ is maximum) remained constant. Measured tangential velocity distributions had the same functional form as that determined by Hoffman and Joubert^{(32)*}. The results of Corsiglia et al⁽¹⁵⁾ using the instantaneous traverse gave values of peak velocities higher than their previous results^{(12,31)*} in the NASA Ames 7- by 10-foot wind tunnel behind a half-wing of similar

configuration. The difference is attributed to the fact that a fixed probe and time-averaged hot wire data were collected in the 7- by 10-foot tunnel. Measurements^(29,30) made with a rapid scanning laser anemometer, again in the 7- by 10-foot wind tunnel, correlate well with the 40- by 80-foot tunnel measurements. Measurements by Mason and Marchman^{(102,103)*} using a 5-hole pressure probe in a low-turbulence wind tunnel (behind a half-rectangular wing, square cut tip, NACA 0012 section) are lower in core radius and velocity than those made using the rapid scanning hot wire⁽¹⁵⁾ and laser anemometer^(29,30), but higher than fixed hot wire probe measurements^{(12,31)*}. Corsiglia et al⁽¹⁵⁾ ascribe the difference between their instantaneous measurements and those of Mason and Marchman's as possibly due to averaging in the presence of vortex meander. However, vortex meander was reported only by Corsiglia et al⁽¹⁵⁾ and they are not justified in assuming that it would be present in other facilities. Also, Mason and Marchman half-wing has an aspect ratio of 12 and NACA 0012 section while Corsiglia et al used a wing of aspect ratio 5.33 and NACA 0015 section and direct comparison is thus not justified. Even for similar wings at the same Reynolds number comparing results of wings and half-wings is not necessarily correct, especially when no care is given to root conditions. The spanwise loading could have a dip near the root, as a result of interaction with tunnel wall boundary layer, and the effect of such a dip on the tip vortex is not as yet well known.

To produce a stable single vortex whose position in the tunnel remains independent of velocity, angle of attack and distance downstream, Hoffman and Joubert^{(32)*} used a "differential" aerofoil. This consisted of a 6 in. chord wing spanning the tunnel vertically, the lower half being mounted at an angle of incidence equal and opposite to that of the upper half. The single vortex generated is near the tunnel centerline and wall interference is minimum. A similar arrangement was used at McGill Aerodynamics Laboratory in the open-return, circular tunnel. Some of the most complete measurements in a trailing vortex were collected at McGill by Poppleton^{(90,91)*}, Phillips⁽¹⁴⁾ and Graham et al⁽¹⁷⁾. The measurements included the static pressure and total pressure, together with sufficient hot-wire measurements to determine all components of the Reynolds stress tensor. The problem of such a set-up is that the initial formation and roll up of the vortex cannot be studied. Also, how representative the generated vortex, by the "differential" aerofoil, to an aircraft trailing vortices is still to be answered.

deVries⁽³¹⁾ conducted what is probably the most comprehensive wind tunnel investigation of the development of the vortex wake. The flow behind a 30° untapered sweptback wing of aspect ratio 5 was measured at a test Reynolds number, based on chord length, of 1.45×10^6 . Three types of measurements were carried out, a) a total head survey (viscous wake), b) yaw measurement (side-wash variation across shear layer), and c) vortex indicator measurements (vortex wake). Also measured was the detailed wing

loading together with surface flow visualization using tufts and oil pigment. It was found that 60 percent of the trailing vorticity is concentrated in the tip vortex leaving the wing tip upper surface. Proceeding downstream (up to 4.2 chords from the tip trailing edge), the remaining vorticity due to the wing diffuses into the surrounding air, without rolling up into the tip vortex. Because deVries' measurements properly define the flow field and the loading on the generating wing, it was possible to compare the measurements to theoretical calculation of the deformation of the vortex sheet behind the wing⁽³²⁾ and weak points in the theoretical model were identified. deVries⁽³¹⁾ model is still not representative of current transport wings as it has a low aspect ratio of 5 and no taper or taper ratio of 1 (current transport aircraft have aspect ratio between 6.4 - 8.8 and typical taper ratio of about 3). The measurements were restricted to the very near field; less than one span downstream, due to the short working section available.

Several investigators^{(13,15,16)*}, beside deVries⁽³¹⁾, have remarked that the integrated rolled up vorticity in the wake as measured does not seem to add up to the shed vorticity calculated from high aspect ratio wing theory. For example, Dosanjh et al^{(13)*} measured only 58 percent of that expected from theoretical calculation (at model Reynolds number $R_c = 10,000$), while McCormick et al^{(16)*} measured only 16 percent at flight Reynolds number $R_c = 3 \times 10^6$.

Towing tanks have long been used to study the stability of trailing vortices using mostly flow visualization techniques (53-55,59)*, (21). Recently measurements⁽³³⁾ of the tangential and axial velocities have been made in the wake of wings being towed under water, using 2-D scanning laser velocimeter, and up to 200 spans downstream. Three different wings, rectangular, triangular and swept wing, were used so that different spanwise loading could be tested without adding devices. Test chord Reynolds number was 2.43×10^5 . The results identify two separate flow regions for the dependence of vortex tangential velocity on downstream distance. The first, a "plateau" region, with little, if any, change in maximum tangential velocity, extends from wake roll up to downstream distances as great as 100-spans, depending on span loading and angle of attack. This is followed by a decay region in which the maximum tangential velocity decreases with downstream distance at a rate nominally proportional to the inverse one-half power.

Iversen⁽³⁴⁾ found that if the downstream dependence of vortex maximum rotational velocity is presented as a vortex velocity scaling parameter, $V_{\max} b / \Gamma_0 R$, vs a distance scaling parameter, $(Z/b) (\Gamma_0 / U_\infty b) (R)^2 f(\Gamma_0 / v)$, the scale model and flight data collapse to a single curve. Ciffone⁽³⁵⁾ employed these scaling parameters to plot vortex maximum rotational velocity in the plateau region versus a corresponding downstream duration of this region obtained from the experimental data of Refs. (33,36). The line joined through the experimental points

has a slope of -1. This implies that from the near wake velocity measurements in the plateau region we can determine the end of the plateau region and then apply, say, a $-\frac{1}{2}$ decay region to calculate the far flow field. However, a proper definition of roll up is necessary to ensure that velocity data are not used during roll up. The Reynolds number parameter, $f(\Gamma_0/\nu)$, was developed in Ref. (34). For $\Gamma_0/\nu > 3 \times 10^5$, $f(\Gamma_0/\nu) = 1$, and is of the order 1 for most ground based tests. The relation between the end of the plateau region $(\frac{x}{b})_p$ and the maximum tangential velocity in the plateau region

$(\frac{V_{max}}{U_\infty})_p$ is then simplified to

$$(\frac{x}{b})_p = 47/R_\infty (\frac{V_{max}}{U_\infty})$$

1-6 Conclusion

In conclusion, the opportunities for research offered by the trailing vortices problem are enormous. Theoretical work should continue, especially in the area of the initial formation and roll up. More well-planned, carefully executed, laboratory experiments and observations should be made. Detailed flight experiments rather than limited flight observations should be performed. The tower fly-by, one major source of information, is a questionable technique because of the tower 'cutting' the vortex. One might quote J.H. Olsen's comment on the results

of the Aircraft Wake Turbulence Symposium in 1970, "Pandora's box has just opened. More questions have appeared than have been answered. Low speed aerodynamics is once again an area with pressing practical problems". Even though this comment was made five years ago, and many research papers have been contributed in this area since that time, the problem is still far from being properly understood.

CHAPTER II

ON THE EXPERIMENTAL APPROACH TO THE PROBLEM

2-1 The Need for Experimental Work

The need for more experimental and theoretical work (especially experimental) to understand and to try to reduce the hazard from trailing vortices should be clear from the survey⁽¹⁾ and the present introduction. The amount of experimental work available, compared to the complexity of the problem, is very small and most of it is incomplete. By incomplete experimental work we mean studies in which there are not enough parameters measured to properly define the flow field and make the results useful for checking flow models and theoretical calculation, or for assessing the effect of certain devices on the flow field. For example, an experimental investigation aimed at describing the vortex roll up without measuring the spanwise loading on the generating wing is incomplete because the roll up is (according to high aspect ratio wing theory) a strong function of the gradient of the spanwise loading.

The opportunities for research offered by the conventional wind tunnel are restricted to the near wake, because of the length of working section normally available. Nonetheless, systematic wind tunnel experiments must answer some key questions before theoretical work on trailing vortices can proceed much further.

At the time of planning for the present experimental work the only set of wind tunnel measurements (known to the author) behind a wing representative of current transport aircraft was that of Chigier and Corsigalia^{(31)*}. In their experiments the model (a semispan model of a CV-990) has a swept, tapered, twisted wing with fuselage, open engine nacelles, antishock bodies, and flaps. Such a model is very representative of a real wing but it results in a complicated flow field and it is very difficult to assess the contribution of the different components on the flow field. The measurements included the three components of velocity only and extended to 12 chords downstream. A complete, well-planned set of measurements behind a simplified, but still largely representative, wing is needed and should provide valuable information on the near wake behind current transport aircraft.

Another point that is interesting and has not been explored as yet is the effect of the jet engine exhaust and engine pylons on the vortex and its roll up, for the case of aircraft with wing-mounted engines. If a wing model can be built that has removable engines, then it will be possible to study the vortex system without engines mounted, i.e. a clean configuration, then add the engines and study the engine effect.

The effect of engine pylons on the initial formation is to interrupt the boundary layer on the lower surface of the wing, even causing local separation. This might result in

either local thickening of the free shear layer leaving the wing trailing edge and therefore a spread of the core and reduced maximum tangential velocity; or the free shear layer might roll up into more than a pair of trailing vortices each with strength less than if it rolls up into one pair (flight tests conducted by NAE,^{(77)*} Canada, show secondary vortices behind the CV-880; wind tunnel measurement^{(31)*} with a CV-990 semi-wing model also shows secondary vortices).

Also the high velocity, high temperature, jet exhaust can affect the vortex system in at least two ways:

- i) The high velocity jet exhaust may act in a similar way to axial mass injection. Experimental results show a drastic change (decrease) in the maximum tangential velocity as well as increase in the core size (which does not necessarily mean a reduction of hazard on following aircraft, as will be discussed later) when high velocity air is injected within or near the vortex core. The jet exhaust, from the outboard engine (on a 4-engined wing mounted layout) especially, might act in a similar way.
- ii) The high temperature jet exhaust, if entrained within the core, may result in higher rate of diffusion due to heating. However, it is very doubtful that any heating effects can be detected within the available test section length as the jet exhaust must be entrained first within the core before it starts affecting it.

2-2 Wing Configuration

A survey of wing configurations (planforms) used with current jet transports (see Appendix IV) shows that, for the same family of aircraft, the planforms do not differ very much from each other. A configuration similar to that of Boeing 747 (which is very similar to the Boeing 707) is chosen for the present study. The model has straight leading and trailing edges aspect ratio of 7, $\frac{1}{4}$ chord sweep of 35° , taper ratio of 3, and provision for 2 simulated engines, with air injection, mounting at several spanwise positions. Flaps, ailerons, antishock bodies, etc., are not to be represented to avoid complicating the flow field and masking engine effects. Although removable flap tracks may be added at a later stage, the present configuration is thought to be reasonably representative of cruise conditions.

The wing without engines mounted is similar to the Boeing 727 clean wing case. With one engine mounted, at the proper spanwise location, the model is similar to the Lockheed L-1011 or DC-10 wing. To achieve higher mean chord Reynolds number a half wing is chosen. Inviscid high aspect ratio wing theory predicts, for elliptic spanwise loading, a vortex spacing of $\pi/4$ the wing span. Also, to minimize tunnel wall interference effects the tip vortex from the half wing should be close to the tunnel centerline, consequently a semispan of 21 in. is chosen (tunnel width is 30 in.). To avoid unrepresentative

end effects (wing root loadings) which could result from the interaction of the wing pressure field and the working section side wall boundary layer, distributed suction at the wing root is applied (details of suction wall design is given in Appendix II).

2-3 Engine Simulation

From practicality considerations the jet exhaust is simulated using cold high pressure air supplied through the wing to properly simulated engines with single exit nozzle and a no through flow nacelle. The installation chosen for the model represents a B-707 or B-747; however, when simulating the engine we will take the B-747 as more representative of present technology.

The general requirements for engine simulation are:

1. Spanwise position and depth below wing chordal plane must be correctly simulated to insure that the jet exhaust is in the right relative position to the shear layers leaving the wing trailing edge. However, other spanwise positions are available for possible modification of engine position or simulation of other aircraft.
2. Scale (size) of total jet efflux must be simulated. Since a single nozzle is used on the model to represent a mixed exhaust from a turbo fan, some average values must be used.
3. The jet mixing (and dissipation) will depend strongly on the jet velocity ratio (properly defined mixed jet

velocity ratio, i.e. $\frac{V_m}{V_\infty}$, but since the full scale jet is at an elevated temperature, which will not be represented, it might be more appropriate to keep $\rho_m V_m^2 / \rho_\infty V_\infty^2$ at the right ratio.

4. Simulation of the thrust equal drag condition associated with a steady level cruise condition.

Since the requirement for simulating the jet efflux size might conflict with the simulation of the thrust of the engine, some kind of compromise must be used.

Details of the engine simulation used are presented in Appendix VI.

2-4 Flow Field Measurements

For the flow field to be fully defined in low speed flow, the following parameters should be measured:

- 1) The three components of mean velocity.
- 2) Total pressure.
- 3) Static pressure (need not be measured in low speed flow if the total pressure and local velocities are known).
- 4) Turbulence quantities (the six Reynolds stress components and their spectra and scales).
- 5) Temperature (only if the hot jet exhaust is simulated).

The first three parameters can be measured simultaneously using a calibrated 5-hole pressure probe. A minimum probe size

is desirable for reasonable spatial resolution; maximum acceptable size would be about $\frac{1}{8}$ " (Ref. 31* measurements suggest that the core size is about 2% of the span, or a core size of 0.84 inch for our experiment), but a smaller size is possible and, of course, desirable.

A Laser Doppler Anemometer system was considered for turbulence, and mean velocity measurements; however, after thorough consideration the idea was rejected (see Appendix V).

2-5 Rolling Moment Measurements

Investigators used to describe achievement in vortex dissipation as a reduction in maximum tangential velocity or as an increase in core size. However, core size can represent a small part of the flow encountered by a following airplane (core radius is typically of the order of 0.02 span of the generating aircraft). The irrotational flow outside the core is not affected by the redistribution of vorticity within the core; it depends on the circulation Γ_c (the amount of streamwise vorticity actually rolled up in a concentrated core). Since the wing of a following airplane sees the whole flow field, reduction of the induced velocity over a small fraction of the wing would not effectively reduce the rolling moment.

A better way of describing the effect of changing the core structure would be to directly measure the change of the induced rolling moment imposed on a certain (arbitrary) wing

placed at a certain distance, preferably as far downstream as possible. Since any wing to be used will have arbitrary dimensions and will be placed at a downstream station ($35\bar{c}$ or 5 spans maximum) too close to the generating wing to represent a real flight case, the results of the rolling moment measurements should be interpreted as an overall qualitative description of the effect of the factor considered and not as description of hazard.

To represent extremes in vortex hazard, two intercepting trailing wings are thought of; a small straight wing representative of a light aircraft, (Cessna 175), behind a large commercial transport (Boeing 707), which will also simulate a small commercial transport (deHavilland DHC-6) behind a Jumbo Jet (Boeing 747), and a second larger swept-back wing, representative of a medium commercial transport (Boeing 737) behind a Jumbo Jet (Boeing 747). A simple strain gauge roll balance system could be used to measure the induced rolling moment, the system being provided with x-y traverses to monitor variations with position and to find the maximum induced rolling moment.

2-6 Wing Loading and Surface Flow Visualization

For any trailing vortex measurements to be complete, and useful, the measurements should include generating wing loading, especially if modification to the flow field through changing the tip, mounting of engines or use of drag devices is considered.

If the spanwise loading is not known (the assumption of elliptic loading is usually used) the measurements are of limited use in checking flow models or roll up calculations. When considering a method of altering the flow field there is no sense in discussing the effect of any device on the vortex without knowing its effect also on the generating wing lift distribution, and drag. Moreover, in the case of low speed wind tunnels, when the Reynolds number is relatively small, it is important to check the nature of the boundary layer and its separation from the wing surface. Laminar separation and/or extensive tip separation could result in a dispersed vortex structure which is not representative of full-scale flight conditions.

Surface pressure plotting, although time-consuming, is the only means of measuring the chordwise and spanwise loading on the generating wing. Since changing the wing tip and engine position is considered and to check the performance of the suction system, as many measuring stations as practically possible is recommended. Even then minor perturbations to the local wing pressure distribution due to say pylons or flap tracks will not be easily measured.

At the mean chord Reynolds number expected (0.5×10^6) the boundary layer on the generating wing might be expected to be laminar. To obtain the right nature of boundary layer, i.e. "turbulent", a tripping wire, or sand band, near the leading edge can be used. Of course the relative scale of the boundary

layer will still be about twice as thick as in the flight case. Surface flow visualization, using, say, the oil dot technique, can help throw some light on the nature of the boundary layers (specifically the direction of the surface shear stress) and their separation.

2-7 Test Section Length and Wind Tunnel Modification

It is clear that the trailing vortex hazard problem is typically a far field problem hence maximum distance obtainable downstream is very important. Modification of the Carleton 20" x 30" wind tunnel by replacing the first diffuser with two extra 8 feet working sections in tandem was proposed. With the new test section, 22 feet long, it is possible to make measurements up to 5 main wing spans, 35 mean chords, downstream of the main (or generating) wing, which is itself placed in the middle of the first section, away from test section entrance effects. Since the vortices are pressure gradient sensitive the test section was made divergent to allow for the boundary layer growth. Appendix I gives the details of tunnel modification and subsequent calibration.

2-8 Details of Work Program

The test program for this experimental work includes the following:

1) Modification and re-calibration of the 20" x 30" wind tunnel.

2) Design and supervision of manufacturing of the equipment needed for this experiment, which included:

- a) Main wing and suction system
- b) Two trailing wings
- c) Trailing wings x-y traversing gear and roll balance system
- d) 5-hole pressure probe
- e) Probe x-y traversing mechanism
- f) Two simulated engines
- g) Control systems for the traversing mechanisms
- h) Centerline locating device.

Fig.(3-1) shows an isometric sketch of the long working section with the different equipment mounted; the detail of the experimental set-up is given in Chapter III while Table (3-3), p.80, supplies the list of working drawings.

3) Calibration of the 5-hole probe, the detail of which is given in Appendix III.

4) Measurements of the wing loading on the main wing using pressure plotting and study of the nature of the boundary layer with and without transition wires. The choice of the angles of attack is to follow from an examination of the wing loading and associated flow visualization.

5) Flow field measurements clean wing configuration, with two different tips, square cut and half body of revolution,

but no engine mounted, including rolling moment measurements. The flow field measurements are to be made at two chosen angles of attack and at two measuring stations, $2\frac{1}{2}$ and 5 main wing spans downstream of the main wing. The 5 spans station is the most downstream station at which measurements can be made without interference from the first corner flow; the $2\frac{1}{2}$ spans station was chosen as half the distance to the 5 spans station. The clean wing configuration measurements should be thoroughly made as they represent the basic measurements as well as being the reference case for the rest of the measurements.

6) Measurements with engines mounted; this is basically rolling moment measurements with detailed flow field measurements only when appreciable change in the maximum induced rolling moment is measured. To determine the source of influence, the measurements are to be repeated three times; first with engines mounted and no injection, to determine if it is a pylon or exhaust effect, then with air injection through both engines and finally with injection through the outboard engine only to determine if the inboard engine has any effects.

2-9 Limitations on Experimental Work

The experimental set-up in its present form is capable of supplying enough data for defining the roll up and the start of steady motion of trailing vortices as well as the engine effect

on the flow. The author, because of the time limit on the project, is forced to confine his measurements to two axial stations and two angles of attack (except when it is feasible, timewise, or necessary to obtain variations with angle of attack), and to exclude turbulence measurements. The measurements are made with the utmost care to obtain accurate enough data to define the main features of the flow field and to properly direct future measurements. However, much more data is to be collected, especially closer to the wing, before proper understanding of the problem is achieved. (See Appendix VII for details of error analysis).

CHAPTER III

EXPERIMENTAL SET-UP

3-1 Wind Tunnel and Long Test Section

The experiments have been carried out in the 20" x 30" low speed return circuit wind tunnel at Carleton University, with the modified 22 ft. long test section, here called the long test section. Essentially zero pressure gradient at a maximum test section speed of about 180 ft/sec. has been achieved by a combination of divergence of the top and bottom walls (20 in. dimension) and small tailored corner fillets.

In spite of the drastic loss of circuit efficiency resulting from the substitution of the long test section for the normal first diffuser, the flow in the working section, following the 12:1 contraction, is still of very good quality - total pressure uniformity better than 0.5% and streamwise component of turbulence less than 0.1%.

A schematic of the working section layout is shown in Fig. (3-1) while Fig. (3-2) shows a cross section in the long test section. The two sides of the test section were made out of plexiglass for viewing and to leave the option of using a Laser Doppler Anemometer available. The four U-channels are used to facilitate the mounting of equipment; and on the upper two channels $\frac{1}{4} \times \frac{1}{2}$ " steel rails are mounted parallel to each other and to the centerline of the test section, as accurate as practically possible. The rails were used to mount the traversing mechanism and the centerline locating device.

FIG.(3-2) CROSS SECTION IN LONG TEST SECTION

3-2 Centerline Locating Device

This device, Fig. (3-3), was built to physically define the centerline of the long test section and is especially needed because of the divergence of the test section. The two concentric cylinders, which are the basic part of the device, can be positioned through a three dimensional adjustable linkage so that the centerline of the inner cylinder coincides with the centerline of the long test section. Before measurements are taken at any axial station, the centerline locating device is mounted upstream of the measuring probe, and the probe is positioned in an accurately sized hole in the center of the inner cylinder. In this way a reference position (0, 0) is established.

To compensate for machining tolerance (straightness and parallelism of the rails, parallelism of the two side walls and flushness of the channel-plexiglass junction), a survey of the tunnel inside dimensions and the position of the center of the centerline locating device, with respect to one of the side walls and the floor, was made using a stick micrometer. Measurements were made at 34 axial stations, 8 inches apart. The maximum difference between the test section actual centerline, i.e. halfway between each two opposite walls, and the centerline as located by the locating device, was 0.089". These differences were recorded and used for defining the actual centerline of the test section for the flow field measurements.

FIG. (3-3) CENTERLINE LOCATING DEVICE

3-3 Main Wing and Suction System

The large half-wing (herein called the main, or generating, wing), see Table (3-1), was machined in the Mechanical Workshop, Faculty of Engineering, Carleton University (as was the rest of the equipment reported in Sections 3-1 to 3-6). Since the wing is of constant sectional shape and not twisted, it was possible to machine it as a cone, having as a cross section the wing section, and to cut the wing out of the cone to the required dimensions after all the machining has been completed. Stress relieved 7075 aluminum alloy was used to minimize machining deformation.

Table (3-1): Geometry of Main (Half-) Wing:

Semi-Span:	21.0 in.
Average Chord:	6.00 in.
Aspect Ratio:	7.0
Taper Ratio:	1/3
1/4c Sweepback:	35°
Wing Twist:	0°
Wing Section:	ONERA Transonic Calibration Model 'Peaky' Section: Thickness/Chord Ratio 12% (Streamwise). Uncambered.
Tip Form:	Square Cut or Half-Body of Revolution
Transition Fixing:	None (some Tests with Transition Wires, 0.012 in., and Sand Roughness Strip, 0.0057 - 0.0072 in.)

Position of Pressure Plotting Stations:

Station	1	2	3	4	5	6	7
Fraction of							
Semi-Span	0.071	0.143	0.214	0.286	0.357	0.429	0.476
Station	8	9	10	11	12	13	14
Fraction of							
Semi-Span	0.571	0.643	0.714	0.786	0.857	0.929	0.988

Possible Engine Positions (Fraction of Semi-Span):

Inboard Engine	0.25 ,	0.33 ,	0.40 ,	0.50
Outboard Engine	0.60 ,	0.66 ,	0.75 ,	0.80

The major problem in designing the main wing was in trying to accommodate the pressure plotting tubes and the high pressure air supply to the engines within the wing, especially near the tip. The wing has 42 pressure plotting tubes (brass tubes $\frac{1}{16}$ " or $\frac{3}{32}$ " O.D.) let in to the surface in machined grooves and epoxed to the wing before the final surface finish. The chordwise distribution of the tubes is shown in Fig.(3-4a); the concentration of the pressure plotting tubes near the leading edge is dictated by the nature of the expected flat plate like load distribution, which peaks near the leading edge. The last (downstream tube) on both the upper and lower surfaces extends only to $0.5 \frac{b}{2}$ then the trailing edge gets too thin to accommodate the pressure plotting tubes. The pressure holes are 0.030 in. diameter and are drilled at 14 spanwise stations (see Table (3-1)). The most inboard station is $\frac{1}{2}$ " from the root (or about $0.16 \times$ root chord) while the most outboard is $\frac{1}{4}$ " from the square cut tip (or about $0.08 \times$ tip chord) to monitor the steep spanwise loading gradient near the tip. Measurements at any station can be made by covering the other 13 station pressure holes with Monokote tape strips 0.002 in. thick.

The high pressure air is supplied to the two engines through 2 - $\frac{3}{8}$ " O.D. copper tubes each has four outlets corresponding to the different positions of the engines (Table (3-1)). Fig.(3-4a) shows cross sections in the wing at $0.33 \frac{b}{2}$ and $0.8 \frac{b}{2}$, while Fig.(3-4b) shows the wing lower surface, surface pressure measuring stations, and engine positions.

FIG. (3-4-) CROSS SECTION IN MAIN WING

Fig. (3-4b) MAIN WING LOWER SURFACE, SURFACE PRESSURE MEASURING STATION AND ENGINE POSITIONS

The base suction system, which also serves as the incidence gear, is simply an aluminum box 15.5" x 14" x 4" deep mounted between two of the U-channels in the first section of the long test section, in a special cut-out in the plexiglass. One of the 15.5" x 14" sides of the suction box, which is flush with the tunnel side wall, is made out of porous bronze ($\frac{1}{4}$ " thick grade D Porosint sintered bronze - see Appendix II for details) and is the resistive part of the base suction. The main wing is mounted at the root on an 11 in. sintered bronze circular disc that can rotate in a matched hole in the sintered bronze plate. Angular graduation at the edge of the disc will give reading of the angle of attack to within $\pm 0.1^\circ$, but the aerodynamic zero (overall no-lift angle) has to be found. The suction box is supplied with suitable pneumatic connections for the pressure plotting tubes, high pressure engines, air and suction fans. Fig.(3-5) shows a view of the suction box with one side removed. Four Dustbane Model PC-1 vacuum cleaners were adequate to handle the required suction flow rate through the sintered bronze plate at maximum wind tunnel speed.

3-4 Engines

In designing the engines the prime concern was simulating the jet exit plane position, engine axis relative to wing chord, engine maximum diameter, and pylon position relative to the wing and the engine; the Boeing 747 configuration was simulated.

ATION

WING MOUNTED IN SINTERED BRONZE DISC

FIG(3-5) MAIN WING AND SUCTION BOX

SUCTION BOX WITH COVER REMOVED

The exit area was taken as the equivalent area defined in Chapter II - see Appendix VI for details. Since the intake air was not simulated, it was important to fair the engine nose to avoid stagnating the air over a large frontal area; and to avoid boundary layer separation over the outside surface of the engine steep gradients were avoided - the result was an engine that is relatively much longer than the scaled engine. The inboard and outboard engine pylons typically have little difference and the two simulated engines with their pylons were therefore made identical, for machining convenience.

The engines, made of brass, consist of four basic parts, the body, nose, liner and the pylon, Fig.(3-6). The body, liner and nose are screwed together, and the pylon is soft soldered to the body. High pressure air flows from the wing to the pylon, to the gap between the liner and the body, then it is throttled before entering the liner. The throttling is achieved through matched holes in the liner and nose; by turning the nose a variable degree of throttling can be obtained. The liner, which is acting as a nozzle, has an area ratio of 2.3. Four screens (open area ratio 0.54 and pressure loss coefficient $K = 1.6$) are placed before the contraction entrance to reduce the turbulence from the main supply or generated in the air passages. A total pressure probe, 0.036 in. O.D. stainless tube, is placed $\frac{1}{2}$ " downstream of the screens and monitors the air total pressure, or when properly calibrated

FIG. (3-6) SIMULATED JET ENGINE

with respect to exit static pressure, the jet exit velocity. The total pressure probe is connected to $\frac{1}{16}$ " brass tubing let in the wing and is running parallel to the high pressure air tube and has outlets at the same spanwise position.

3-5 Intercepting Trailing Wings and Rolling Moment Balance

The geometries of the two intercepting trailing wings used are given in Table (3-2). The small straight wing is made of a special fiber material (Phenolic CP905) while the larger sweptback wing is made out of mahogany. The wings were sting mounted, horizontally (i.e. parallel to the main wing plane), from a traversing (x, y) strut, at a nominal incidence of zero. The trailing wings with their traversing mechanism were built for this experiment.

Table (3-2): Geometry of Intercepting Wings

	Small Straight Wing	Larger Sweptback Wing
Span (Relative Span)	10.0 in. (0.24)	20.0 in. (0.48)
Average Chord	$1\frac{1}{3}$ in.	2.35 in.
Aspect Ratio	$7\frac{1}{2}$	$8\frac{1}{2}$
Taper Ratio	1	$\frac{1}{3}$
1/4c Sweepback	0°	35°
Wing Twist	0°	0°
Wing Section	NACA 64 ₂ 015	NACA 64 015
Tip Form	Square Cut	Square Cut
Transition Fixing	Sand Roughness Strip (0.0057-0.0072 in.) to 15% Chord (both surfaces)	Sand Roughness St. (0.0057-0.0072 in.) to 15% Chord (both surfaces)

The traversing mechanism is simply a carriage that travels laterally on two U-channels driven by a rack and pinion mechanism; the scanning speed can be controlled by adjusting the voltage to the D-C motor driving the pinion (2-10 in./min.). The position of the carriage is indicated by a 10 turn potentiometer; however, because of the backlash in the system lateral position can only be determined within 0.050 in. The carriage carries a vertical strut that can be driven with a similar mechanism. An antibacklash pinion on the vertical strut mechanism reduces the uncertainty in vertical position to 0.020 in. The strut carries the sting, supported on two bearings, and a cantilevered beam $5" \times \frac{1}{2}" \times \frac{1}{8}"$ at the sting base serves as a roll balance. Two strain gauges mounted on the cantilever and connected in a half bridge circuit can measure the induced rolling moment on the wing. Fig.(3-7a) shows a photograph of the trailing wing mechanism while a block diagram of the system is shown in Fig.(3-7b). Static calibration of the system is performed using a calibration arm and dead weights that can be mounted on the sting in place of the wing. The system was checked and it does not respond to axial force, thrust or drag, or normal force, i.e. mean lift on the wing. The natural frequency in roll with the small wing mounted is about 20 Hz.

The trailing wing mechanism can be positioned at any of the two downstream stations - $2\frac{1}{2}$ and 5 main wing spans

FIG. (3-7b) BLOCK DIAGRAM OF ROLL BALANCE SYSTEM

FIG. (3-7a) TRAILING WINGS & TRAVERSING MECHANISM

downstream of the main wing mean $\frac{1}{4}$ chord - by replacing an 8 inch strip of the tunnel floor. The two trailing wings can be scanned laterally until either of the wing tips reaches the side walls, and vertically 6 in. above and below the centerline.

3-6 5-Hole Probe and Traversing Mechanism

The non-nulling blunted conical 5-hole probe, used in this experimental work, was designed to be as small as practically possible for maximum resolution and minimum interference with the vortical flow. The probe, Fig.(3-8), consists of a brass head, a 5 in. long stem, 0.093 in. outside diameter ($\frac{3}{32}$ in. brass tubing drilled to increase the inside diameter to 0.078 in.), and a steel base. The brass head was first machined as a cylinder 0.093 in. outside diameter. In one flush end of the cylindrical head 5 holes of 0.013 in. diameter were drilled; one in the center and two opposite vertically and horizontally. Concentric with these small holes, 5 holes, 0.028 in., were drilled on the other side. Five hyperdermic needles, 0.028 in. O.D. - 5.25 in. long, were fitted in the head and soft soldered. After fitting the hyperdermic tubing in the stem and soft soldering the head, the conical surface, a 60° included angle blunted cone, was machined with the axis of the cone coinciding with the axis of the stem. The stem-head assembly was mounted in the base with the axis of the stem and the

Details of Probe Head

Fig(3-8) 5-Mole Pressure Probe

plane through the centerline of holes 1, 5 and 2 parallel to the mounting surface of the base. In this way the base defines the zero roll plane of the probe. Two locating pins in the base will insure that the probe axis is normal to the scanning direction. Details of the aerodynamic calibration of the 5-hole probe can be found in Appendix III.

The x-y traversing mechanism was built to scan the 5-hole probe, or any other instrumentation, through the tunnel. The mechanism can slide, manually, on the two upper rails in the test section and can be positioned anywhere downstream of the main wing down to the end of the long test section. The horizontal mechanism consists of a 5-26 R.P.M. geared D.C. motor that drives, through antibacklash gears, a 20 threads per inch horizontal shaft, which in turn drives the horizontal slide guided by a faired horizontal rail. The horizontal slide carries a similar vertical mechanism that drives the vertical slide guided by a vertical faired rail. The scanning speed of both slides can be controlled between 0.25 - 1.3 inch per min. The position of each of the slides is determined from a ten turn potentiometer, driven by a 50:1 precision gear reducer, to within 0.015 in. The motor, gears, gear reducer, and potentiometer of each mechanism were housed in a faired housing and were arranged to minimize blockage effects as far as possible, Fig. (3-9a). The maximum scanning capability of the mechanism is shown in Fig.(3-9b); the lateral scanning capability can be increased

FIG.(3-9a) PROBE TRAVERSING MECHANISM

FIG.(3-9b) SCANNING LIMITS OF TRAVERING MECHANISM

by rotating the mechanism 180°. Vertically, the maximum limit of ± 5 in. from the working section centerline is a handicap and should be corrected for future work by replacing the vertical drive shaft and guide rail (it was not expected that measurements would be needed more than 5 in. from the center, as will be explained later).

To minimize upstream disturbances of the traversing mechanism, on the 5-hole probe measurements, the probe was not mounted directly on the vertical slide, an extension arm about 5 in. long was mounted on the vertical slide and the probe mounted on the extension arm. With this arrangement the probe tip was about 9 in. upstream of the vertical slide, or about 5 in. upstream of the most upstream point in the mechanism. Even with these precautions, some small upstream mean flow disturbances coming from the traversing mechanism, were measured. To increase the accuracy of the results, these small disturbances were allowed for by taking empty tunnel measurements at each measuring station (more details are given in Appendix I).

3-7 Control System for the Traversing Mechanism

A control system was designed to control the motion of the two x-y traversing mechanisms; the trailing wing mechanism and the probe mechanism. The main function of the control system is to facilitate continuous scanning without taking

the risk of damaging the system by driving into the mechanical limits. The system, with two identical channels, is basically a very stable 2.5 volts d.c. power supply feeding the two potentiometers, of either of the x-y mechanisms, and the corresponding analogue outputs which is a function of the mechanism position. Moreover, the control system controls the variable, controllable, power supply to the motors. Upper and lower limits on the x and y mechanism position can be adjusted and if the mechanism reaches any of the limits the power to the drive motor is shut off (or reverses if desired). Manual shut-off or reverse of the power to the motors is possible from manual switches. Another feature of the control system is the digital read out; 2 Weston 1230 series digital panel meters $3\frac{1}{2}$ digits, are used to display the relative mechanism position. A variable power supply in series with each d.v.m is used to offset the meter reading; the displayed reading could thus be adjusted to read zero when the mechanism (measuring probe to be accurate) is at the tunnel centerline. The resolution of the ten turn potentiometers used is 0.009%, an order of magnitude smaller than the resolution of the digital voltmeters; consequently, the accuracy in reading the position is determined by the accuracy of the meters, or the plotters, plus any mechanical backlash in the system. The specified accuracy of the Weston meters is 0.1% of the reading $\pm$ one digit; this is equivalent to 0.015 in.

Fig.(3-10a) shows the traversing mechanism control system front and rear panels; the circuit for the system was designed and built in the Electrical Workshop, Faculty of Engineering, Carleton University, and is diagrammatically shown in Fig.(3-10b).

3-8 Other Equipment

As in low speed wind tunnels of the type used in this work, the normal operating reference quantity is the pressure difference measured between a wall total pressure tube within the settling chamber (P_{C_1}) and a wall static pressure hole just upstream of the working section (P_{C_2}), ahead of possible interference effects due to the presence of a model within the working section. This pressure difference between these two holes (known as the speed sensing holes) is commonly called the contraction pressure difference ($P_{C_1} - P_{C_2}$). Since the contraction pressure difference is used as a reference quantity, its measurement is usually performed by a highly accurate and sensitive instrument which is normally installed in a permanent manner. The Carleton wind tunnel makes use of two variable capacitance differential pressure transducers, made by the Rosemount Engineering Company, Model 831A4, Serial numbers 476 and 477, with a range of ± 1 psid of pressure difference, giving a nominal output of ± 2.5 v. d.c. The transducer output, in the range of interest - less than 0.25 psid, is proportional to the input pressure difference

REAR PANEL

FRONT PANEL

FIG. (3-10a) TRAVERSING MECHANISM CONTROL SYSTEM

Fig. (3-10b) DIAGRAM OF CONTROL SYSTEM

to better than 0.1%. Two DISA type 55D31 digital voltmeters, $3\frac{1}{2}$ digits, are used for reading the transducer output. The DISA voltmeter has an accuracy of 0.1% of full scale ± 1 digit; at tunnel maximum speed ($P_{C_1} - P_{C_2}$) produces an output higher than 500 millivolt and the accuracy of measuring ($P_{C_1} - P_{C_2}$), on the ± 1.0 volt range, is thus better than 0.4%.

The two Rosemount transducers were also used for measuring the 5-hole probe 5 pressures, relative to P_{C_2} ; however, the output of the transducers was recorded on the x-channel of two Hewlett Packard "high speed" x-y recorders model 7044A; the y-channels were driven by the analogue output from the appropriate position potentiometer. The accuracy of the recorders is $\pm 0.2\%$ of full scale and it is possible to read the charts to within $\pm 0.1\%$ (the recorders use 10" x 15" chart paper).

To measure the induced rolling moment on the trailing wings, the two strain gauges, mounted on the roll balance system, were connected to a Philips direct-reading strain measuring AC bridge type PT 1200, in a half bridge circuit. The bridge has 10 measuring ranges, dynamic measurements up to 1250 c/s, zero drift, after heating up period of 90 minutes, less than 2% per hour, and DC output for oscilloscope and recording instruments. The output from the bridge was displayed either on the DISA digital voltmeter or on the x-channel of the Hewlett Packard recorder. A low pass filter

(0.35 sec. time constant) was used before the recorder; the DVM has its own filtering system and 0.3 sec. time constant was used. The calibration of the combined roll measuring and read out system was performed before and after each test, using the calibration arm and dead weights; the linearity of the system was checked and is better than 1%; hysteresis is also less than 1% of full scale.

Fig.(3-11a) shows a schematic diagram of the overall experimental set-up; Fig.(3-11b) is a photograph of the long test section with most of the equipment mounted and Table (3-3) gives a listing of the mechanical working drawings.

Table (3-3): List of Working Drawings

Wind Tunnel Modification	Z.E. MWT-1 to Z.E. MWT-3
Main Wing and Suction System	Z.E. W-1 to Z.E. W-7
Straight Trailing Wing	Z.E. TW-5
Swept Trailing Wing	Z.E. TW-6
Trailing Wing Traverse and Roll Balance System	Z.E. TW-1 to Z.E. TW-4 and Z.E. TW-7
5-Hole Probe	Z.E. P-1 and Z.E. P-2
Probe x-y Traversing Mechanism	Z.E. TM-1 to Z.E. TM-3
Centerline Locating Device	Z.E. TM-4
Simulated Engines	Eng-1 to Eng-8
Traversing Mechanisms Control System	Designed by Peter Manashe, Electronic Workshop.

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FIG. (3-11a) SCHEMATIC DIAGRAM OF EXPERIMENTAL SET-UP

Fig. (3-11a) Long Test Section with Equipment Mounted.

CHAPTER IV

WING LOADING AND SURFACE FLOW VISUALIZATION

The first set of measurements made were logically those of the main wing loading, basically to check the spanwise loading and to choose the appropriate angles of attack for the rest of the measurements. The main wing has 42 pressure plotting tubes let in to the surface in spanwise grooves and pressure distributions can be taken at 14 spanwise stations using the Aero-lab multi-tube manometer (50 tubes). The most inboard measuring station is $1\frac{1}{2}$ in. from the root or about 0.16 root chord; however, the most outboard station is only $\frac{1}{4}$ in. from the square cut tip (or about 0.08 tip chords) to monitor the steep gradient there. The stations are equally spaced except for station 7 which was too close to one of the engine outlet stations and so was displaced $\frac{1}{2}$ in. to avoid measuring very local pylon effects. Measurements at any station can be made by covering the other 13 station pressure holes; 1 in. strips of Monokote tape 0.002 in. thick were used for that purpose and in addition are very suitable for surface flow visualization. The static holes are 0.030 in. diameter, a typical depth would be the thickness of the brass tubes or 0.015 in.

Before any measurements were made the pressure holes for the 14 stations were covered and the system was carefully checked for leakage. During the measurements the uncovered station was always covered and the system checked for leaks before a new station was uncovered. This way a continuous check on leaks was provided.

4-1 Data Reduction

A computer program was written to reduce the data. At any station the measured pressure from the 42 pressure tubes together with P_{C_1} , P_{C_2} and the suction box pressure were recorded, from the multi-tube manometer, and fed to the data reduction program. The program calculates the undisturbed free stream dynamic pressure, velocity and static pressure from $(P_{C_1} - P_{C_2})$ and the tunnel calibration parameters allowing for the wall suction correction, as explained in Appendices I and II (the undisturbed free stream values used are the average of the corresponding values upstream and downstream of the suction box). The program then calculates the upper and lower surfaces pressure coefficients, wing loading, local lift coefficient and normalized (with respect to lift coefficient) chordwise loading. These parameters are defined as follows:

$$\text{Upper Surface Pressure Coefficient } C_{P_u} = \frac{P_u - P_{st_\infty}}{q_\infty} \quad \text{IV-1}$$

$$\text{Lower Surface Pressure Coefficient } C_{P_L} = \frac{P_L - P_{st_\infty}}{q_\infty} \quad \text{IV-2}$$

$$\text{Chordwise Loading } \Delta C_p = C_{P_L} - C_{P_u} = \frac{P_L - P_u}{q_\infty} \quad \text{IV-3}$$

$$\text{Local Lift Coefficient } C_L = \int_0^1 \Delta C_p d(z/c) \quad \text{IV-4}$$

where

$$q_{\infty} = \frac{(P_{C_1} - P_{C_2})}{K_V} \times K_C \quad \text{IV-5}$$

$$P_{st_{\infty}} - P_{C_2} = (P_{C_1} - P_{C_2}) \times K_S + q_{\infty}(1-K_C) \quad \text{IV-6}$$

and K_C is the suction correction factor.

All pressures are measured with reference to P_{C_2} ; in this way the absolute pressure is not required, except to calculate the Reynolds number.

Figs.(4-1a-d) show a sample of the pressure distributions and chordwise loading results for different angles of attack and measuring stations. The pressure coefficients are accurate to within ± 0.004 , the lift coefficient estimated accuracy is ± 0.01 and the normalized loading accuracy will depend on the lift coefficient (see Appendix VII). The reproducibility of the loading is dependent mainly on the accuracy of setting the angle of attack which is $\pm 0.1^\circ$ and results in less than 0.01 increment in lift coefficient. By repeating the measurements at stations 10 and 11, resetting the angle of attack every time, the reproducibility of within ± 0.01 in lift coefficient was checked.

The overall lift coefficient, $\bar{C}_L$, at any angle of attack, is determined by integrating, using the trapezoidal rule, the local loading on the wing and dividing by the average wing chord $\bar{c}$,

Fig. (4-1a).

Pressure Distribution and Chordwise Loading

Fig. (4-1b) Pressure Distribution and Chordwise Loading

Fig.(4-1c) Pressure Distribution and Chordwise Loading

Fig 4-10 Pressure Distribution and Chordwise Loading

$$\bar{C}_L = \frac{\int_0^1 C_L \times c \, d(Y/b/2)}{\bar{c}},$$

IV-7

and is estimated to be accurate to better than 0.01.

The spanwise loading $C_L \cdot c$ is normalized by dividing by $\bar{C}_L \cdot \bar{c}$, the overall lift coefficient times the average chord.

4-2 Aerodynamic Zero Angle of Attack

Since the main wing is symmetrical and has no twist, the aerodynamic zero angle of attack can be determined from the pressure distribution, i.e. when the upper and lower surface pressure distributions are identical. The aerodynamic zero was first found from the pressure distribution at station number 10, which is at the wind tunnel centerline, the overall lift coefficient was then measured and found to have a small non-zero value of 0.007, equivalent to less than 0.1° .

If the wing is mechanically perfect, with no twist or camber, at zero angle of attack the lift coefficient must be zero over the whole wing. Any variation in the local lift coefficient is the result of machining imperfections or wind tunnel mean flow angularity. Variations in the local lift coefficient, around the overall value of 0.006, is less than ± 0.006 (see Table (4-1)), and is equivalent to less than $\pm 0.1^\circ$ about the average value of about 0.1° . This only means that the mean flow angularity in the test section must be better than $\pm 0.1^\circ$, and that mechanical imperfections in the wing are very small.

Table (4-1): Spanwise Variation in Local C_L at $\alpha = 0$

<u>Station No.</u>	<u>C_L</u>
1	0.011
2	0.011
3	0.004
4	0.004
5	0.006
6	0.005
7	0.000
8	0.005
9	0.006
10	0.000
11	0.008
12	0.000
13	0.000
14	0.012
$\bar{C}_L$	0.006

4-3 Wing Maximum Lift Coefficient

Swept wings do not stall in the same sense as high aspect ratio straight wings; however, at high angles of attack leading edge separation starts along the tip edge and near the tip and creeps inboard and affects a larger portion of the span as the angle of attack increases. The tip leading edge separation is sometimes called tip stall and this term will be adopted here.

The model, main wing, has no twist or progressive camber, and consequently it was expected to tip stall at a relatively low angle of attack. Aircraft do not usually fly with stalled

tips (extensive leading edge separation) and it is therefore important not to operate the model at angles of attack higher than the onset of tip stall.

Fig. (4-2) shows the variation of the normalized chordwise loading at tip station "14" with angle of attack. From the figure it is clear that $\alpha = 12.8^\circ$ represent the limiting angle of attack before marked tip stall, as seen from the loss of the suction peak. The overall lift coefficient at $\alpha = 12.8^\circ$, with full root suction and half body of revolution tip (from now on referred to as the tip for simplicity), was found to be $\bar{C}_L = 0.77$.

4-4 Boundary Layer Tripping

Since no tip stall was apparent up to $\alpha = 12.8^\circ$ it was expected that the lift curve slope should be linear up to $\alpha = 12.8^\circ$; and for a typical cruise lift coefficient of about 0.36, the angle of attack should be about 6.25° . However, at $\alpha = 6.25^\circ$ the measured overall lift coefficient was found to be 0.46, which means that the lift curve shape is not linear up to $\alpha = 12.8^\circ$. The sources of nonlinearity, which would result in loss of lift at the high angles of attack, could either come from leading edge separation near the tip, which was not apparent, or from a trailing edge separation over a high percentage of the wing chord.

To ensure that the boundary layers on the main wing are turbulent and avoid laminar separation, it was decided to use

boundary layer tripping. Thin transition wires were considered more convenient because of the presence of the pressure plotting holes.

The size of the transition wire should be chosen such that its drag is just enough to produce an increment in the momentum thickness of the laminar boundary layer upstream of the wire which will increase its value to the minimum for fully developed turbulent layer. Preston⁽³⁸⁾ recommended a minimum figure, for wire Reynolds number, based on experimental evidence for producing transition at a wire which is placed close to the leading edge, of 600. Two wires 0.012 in. in diameter, giving an approximate Reynolds number of 1000 at tunnel maximum speed, were used on the wing upper and lower surfaces. The position of the wire is also critical and should be carefully selected. On the lower surface the wire must be placed downstream of the "stagnation" point (because of the sweep effect there is no stagnation point, the equivalent point, defining an attachment line, is the point of minimum velocity or highest pressure), otherwise one might end up tripping the upper surface boundary layer twice and not tripping the lower surface boundary layer at all. On the upper surface one should avoid putting the transition wire in the steep favorable pressure gradient before the suction peak because its stabilizing effect might cancel the effect of the wire. Inspecting figures (like 4 -1a to d), which were chosen to represent the full angle of attack range, a wire position between

the 5% and 7.5% pressure holes, along the full span of the wing, will always be downstream of the "stagnation" point on the lower surface and the suction peak on the upper surface.

Wing loading measurements, with the transition wire, showed almost no change in the chordwise or spanwise loading at $\alpha = 6.25^\circ$ from the no wire case, the maximum variation in local C_L is about 0.01 and on the overall $\bar{C}_L$ the change is only 0.002, i.e. less than the uncertainty of the measurement. This is the expected behavior since the wire is only changing the nature of the boundary layer and is not expected to alter the loading. However, trailing wing measurements (to be discussed in the next chapter) showed unexpected response beyond main wing angles of attack of 12° , namely a decrease in maximum induced rolling moment with increasing main wing incidence. At this stage it was decided to conduct some surface flow visualization to help understand the nature of the boundary layer and its separation on the main wing.

4-5 Surface Flow Visualization

The oil dot technique with TiO_2 pigment was used for surface flow visualization: The viscosity of the oil used and the size and spacing of the oil dots is just a matter of experimentation. However, the size of the dots should be homogeneous and consistent and the height of the dot should be kept as low as possible to keep it within the laminar sublayer. In interpreting the flow visualization results one should always keep in mind that the oil

streaks follow the direction of the shear stress vector at the surface and in no way represent the external or inviscid flow.

Some flow visualization results with the transition wire are shown in Fig.(4-3), for both the upper and lower surfaces at several angles of attack. On the upper surface, separation and reattachment lines upstream and downstream of the transition wire are clear at all angles of attack. As the trailing edge is approached and the surface shear stress becomes low in magnitude, the oil traces become shorter and its spanwise component increases. This effect increases with angle of attack and at $\alpha = 12.8^\circ$ the oil traces are essentially parallel to the trailing edge over more than 30% of the chord near the trailing edge. Leading edge separation near the tip, "tip stall", is not evident at $\alpha = 6.25^\circ$. At $\alpha = 10.3^\circ$ very limited tip stall is clear; however, its inboard extent increases rapidly with α and is visible over about 40% of the span at $\alpha = 12.8^\circ$. On the lower surface the picture is much less complicated, the oil traces are very stream-wise in direction except, of course, along the attachment line and very near the tip.

Without the transition wire, amazingly enough, the surface flow visualization, Fig.(4-4), is very little different from that with the transition wire. A short laminar separation bubble, due to the large leading radius and consequent steep adverse pressure gradient of the peaky section used, is present and seems to be giving turbulent boundary layer conditions downstream of

FIG.(4-3)

**MAIN WING SURFACE
FLOW VISUALIZATION
With Transition Wire**

FIG.(4-3) Continued

FIB(4-3) Concluded

FIG.(4-4)

**MAIN WING SURFACE
FLOW VISUALIZATION**

No Transition Fixing

FIG (4-4) Continued

UPPER SURFACE

FIG(4-4) Concluded

bubble reattachment. The transition wire was probably sitting in the bubble and consequently not affecting the boundary layer development very much. Tip stall starts showing up at $\alpha = 9.5^\circ$ and again it extends over about 40% of the span at $\alpha = 12.8^\circ$.

Attempts to use sand strips as alternative transition fixing devices resulted in deterioration of the flow over the wing. Fig-(4-5a) shows the flow visualization with sand strip (0.0057-0.0072 in.) extending to 15% local chord on the upper and lower surfaces at $\alpha = 12.8^\circ$; Fig.(4-5b) is for the same conditions but with a smaller sand strip, between 5% and 7.5% chord.

The conclusion from the upper surface flow visualization was that the laminar separation bubble near the leading edge was "naturally" giving turbulent boundary layer conditions downstream of bubble re-attachment, without need for other transition devices. Adding transition devices was just increasing the drag on the wing but not changing the nature of the boundary layer.

4-6 On the Boundary Layer on the Main Wing

To confirm the flow visualization finding some limited hot wire measurements (vertical scans) were made, at $\alpha = 6.25^\circ$ and 11° . The measurements extended over the root, intermediate and tip stations on both the lower and upper surface, between 0.37 and 0.65 local chord with the wire mounted from the traversing mechanism. Because of the qualitative nature of the measurements and because the hot wire was not calibrated the results are not presented here.

**FIG.(4-5) MAIN WING SURFACE
FLOW VISUALIZATION**

Transition Fixed with Sand band

However, the results confirm, with no doubt, that the boundary layers on both the upper and lower surface are turbulent (at least for $\alpha = 6.25^\circ$ and 11°). Presumably there is sweep instability which is responsible for the turbulent state of the boundary layer on the lower surface.

4-7 Effect of Tip on Wing Loading

Figs.(4-6a and b) show a comparison of the chordwise loading at tip station "14", with the half-body of revolution tip (with tip) and with the square cut tip (without tip), for $\alpha = 6.25^\circ$ and 12.8° respectively. The corresponding flow visualization for $\alpha = 6.25^\circ$ is shown in Fig.(4-7). Because of the nature of the loading, at the tip the boundary layer on the lower surface curls up around the tip towards the upper surface. Without the tip, or even with the tip at high angles of attack, the lower surface boundary layer will separate at the wing tip, at least over part of the tip chord, giving rise to a vortex over the wing upper surface at the tip. The vortex induces extra suction on the upper surface and results in a lift increment (see Fig.(4-6)) (similar to vortex induced loads on delta wings). Tip effects were found to be very local, no measurable change in chordwise loading at station 13 was found and the flow visualization does not show any difference except very near the tip.

Since tip effects are very local and small and because the chord at the tip is small anyhow, the difference between the spanwise loading with and without tip is negligible. However, we should not disregard the fact that even though the tip does

Fig(4-6a) Effect of Tip on Chordwise Loading at station 14.

Fig. 6W Effect of Tip on Chordwise Loading at station 14

not change the wing loading it does change the nature of the early separating shear layer from the wing, which could very well be forming the very core on the vortex.

4-8 Effect of Wing Root Suction on Loading

The best way to examine the effect of root suction is to look at the chordwise loading, preferably at high angle of attack, i.e. high loading. Fig.(4-8) shows a comparison of the normalized spanwise loading with and without suction. The dip in the curve with no suction, at the root, is the result of the interaction with the test section side wall boundary layer, and is eliminated by applying suction. Even though the most inboard station "No.1" is well outside the test section side wall boundary layer (about 1 in. thick approaching the wing) its local lift coefficient has increased by about 7% by applying suction. Up to station 3 ($2y/b$ about 0.2) some root suction effects can still be seen, but the increase in the overall lift coefficient is only 0.01 or about 1.5%.

The chordwise loading and the upper surface pressure distribution at station "1", with and without suction, are shown in Fig.(4-9). The minimum pressure coefficient is lower than the design suction box value of -2 over about 5% of root chord, on the upper surface. However, the difference in pressure is much too small to result in any appreciable flow, from the suction box to the test section, across the resistive porous bronze.

Fig4.8 Effect of Root Suction on Main Wing Spanwise Loading

**Fig (4-9) Effect of Wing Root Suction on Pressure Distribution
at Root Station "1"**

4-9 Main Wing Loading

The results of the main wing loading* up to and beyond the onset of tip and leading edge separation ("tip stall") are summarized in Figs.(4-10) to (4-13); with no engine mounted, no transition fixing, full root suction, and with the tip on (this will always be the test condition of the main wing during tests - unless otherwise specified).

Fig.(4-10) shows the variation of the local lift coefficient, at consecutive spanwise stations, with angle of attack. Each C_L vs α curve, representing the variation at a certain station, is shifted from the next curve an increment $\Delta\alpha$ proportional to the two stations spanwise separation distance. Thus the family of C_L vs α curves, for the 14 stations, generate another family of curves for the variation of local lift coefficient with spanwise position, at different angles of attack. Also included in Fig.(4-10) is the variation of the overall $\bar{C}_L$ with α . The C_L vs α curves are linear up to $\alpha = 4^\circ$ to 5° then there is a slight non-linear increase in lift up to $\alpha = 8^\circ$ to 10° . The overall $\bar{C}_L$ vs α curve is linear up to $\alpha = 5^\circ$ ($\bar{C}_L = 0.36$) and the non-linear increase in lift is up to $\alpha = 8.4^\circ$ ($\bar{C}_L = 0.6$). The source of non-linear increase in lift is not known, but similar behavior has, of course, been noticed with delta wings with leading edge separation. Beyond $\alpha = 8^\circ$ "tip stall" (which is evident in the flow visualization - Fig.(4-4)) progressively unloads the outboard stations, but a serious decrease in overall lift curve slope is

* Results of Main Wing Loading, and other results to be presented, are available and will be supplied by the author upon request.

Angle of Attack — Degrees
Fig 4-10 Local & Overall Lift Coefficient

not evident until $\alpha = 11^\circ$ ($\bar{C}_L = 0.74$). Highest local loading occurs around stations 9 and 10 ($2y/b$ about 0.7) away from both root and tip effects.

The normalized spanwise loading results (Fig.(4-11)), for α up to 8° ($\bar{C}_L = 0.58$), form a single curve (there is no wing twist) and when compared with elliptic loading indicate (as expected) higher loading near the wing tip. At higher incidence progressive "tip stall" unloads the outboard sections ($\frac{2y}{b}$, 0.6 + 0.9) and transfers the load to the root.

The chordwise loading (Fig.(4-12)) is "flat-plate-like" (the sections are symmetrical) from about 0.35 to 0.85 of the semi-span (stations 5 to 12). Due to the low Reynolds number of the tests, $R_c = 0.5 \times 10^6$, viscous effects are quite strong and reduce the chordwise loading to very low values near the trailing edge, $z/c > 0.8$ (the loading based on inviscid calculation for NACA 0012 2-dimensional aerofoil is included for comparison). The center "kink-effect" results in flattened loadings near the root, extending out about one root chord length, i.e. to about 40% semi-span (station 4). Similarly, the "tip-effect" extends inboard about one tip chord (about 15% semi-span, station 13) giving more peaky loading, at least prior to tip stall.

A complete wing loading, at $\alpha = 6.25^\circ$ ($C_L = 0.46$), is shown in Fig.(4-13) and highlights most of the loading characteristics described above.

FIG. (4-11) Spanwise Loading - Main Wing

Fig. (4-12) Normalized Chordwise Loading

$$\Delta c_p = \frac{p_u - p_l}{\frac{1}{2} \rho U_\infty^2}$$

$$\bar{C}_L = 0.46$$

$$\alpha = 6.25^\circ$$

Conditions

No Transition Fixing
Full Suction
With Tip
No Engines

Fig 4-13 Main Wing Loading

4-10 Incidence for Flow Field Measurements

As mentioned in Chapter II, extensive lengthy flow field measurements are limited to two angles of attack because of the time limit on the project. Since the present main wing configuration represents steady level cruise condition, it is only logical to reproduce cruise $\bar{C}_L$, consequently $\alpha = 5^\circ$ ($\bar{C}_L = 0.36$) was chosen. The other angle of attack chosen is $\alpha = 11^\circ$, at which the highest overall lift coefficient is generated without extensive tip stall.

4-11 Lifting Characteristics of Trailing Wings

To ensure that the two trailing wings do not have any peculiar lift characteristics, which might result in unexplained response to induced rolling moment, their overall loading characteristics were measured. The wind tunnel overhead balance system was used for the measurements with a single mounting strut and balance wire.

Fig.(4-14) shows the variation of the overall lift coefficient for the two trailing wings, as well as the main wing, with angle of attack. The low value of the maximum lift coefficient, even with transition fixed, is the result of the low test Reynolds number based on mean chord (1.1×10^5 for the straight wing and 2.0×10^5 for the swept wing). Drag and pitching moment measurements together with surface flow visualization did not reveal any irregular behavior of the trailing wings.

Fig. 4-14 Lifting Characteristics - Main & Trailing Wings

CHAPTER V

INDUCED ROLLING MOMENT MEASUREMENTS

Before discussing the rolling moment measurements, it should be stressed again that because the trailing wings are too close to the generating wing to represent realistic flight conditions (although far downstream in terms of usually available wind tunnel test section length), the results should be interpreted as having an overall qualitative nature and not as an assessment and description of hazard. The induced rolling moment measurement is a simple single overall measurement which gives a logical and easy means of comparing vortices and assessment of decay in relation to hazard to following aircraft.

Detailed induced rolling moment measurements were made at incidence angles of 5° , 6.25° and 11° ($\bar{C}_L$ of 0.36, 0.46 and 0.74 respectively); however, maximum induced roll moments were measured at a few more incidences up to a maximum of $\alpha = 12.8^\circ$. With measurements taken at the two downstream stations $2\frac{1}{2}$ and 5b it was hoped that some kind of a gross decay parameter will be determined. The effect of the tip ($\frac{1}{2}$ body of revolution) on the induced rolling moment as well as the variation with free stream dynamic pressure were also investigated.

5-1 Transition Fixing and Main Wing Out Measurements

Before fixing the transition on the trailing wings it was noticed that induced rolling moment measurements were sensitive to trailing wing incidence. The trailing wings were usually set at a nominal zero angle of attack but small incidence was possible

If the wings were perfectly symmetrical about the median plane, small incidence should not add to the induced rolling moments. Inverting the wings, maintaining the same incidence, did not change the measured rolling moments, and asymmetry was excluded as a factor. Flow visualization on the trailing wings, Fig.(5-1), revealed that this irregular behavior is possibly associated with meandering of the laminar separation point. The peculiar behavior of the trailing wings was rectified by fixing transition (using sand band strip, 0.0057 - 0.0072 in., to 15% chord, both surfaces). Fig.(5-1) shows a comparison of the surface flow visualization with and without transition, at position of maximum induced roll at the 5b station for $\alpha = 6.25^\circ$. The improvement in flow behavior over the wings, when transition is fixed, is clearly evident.

Any residual swirl in the free stream flow, or any inherent imperfection in machining the trailing wings, could result in an induced rolling moment; even though the wings might not be in a vortical flow. To remove this non-zero value, induced rolling moments measurements were taken with the main wing either removed or set at zero incidence. The correction (referred to as the main wing out correction) amounted to at least 10% of the maximum induced rolling moment, and without it the results would have been greatly in error.

5-2 Data Reduction

The sequence in which data were collected and reduced is as follows:

Upper Surface

Lower Surface

Without Transition Fixing

Upper Surface

Lower Surface

With Transition Fixing

Wing C_L at Position of Max. Induced Roll - 5b Stn - $\alpha = 6.25^\circ$

FIG. (B-1a.)

SURFACE FLOW VISUALIZATION

With & Without Transition Fixing - Straight Wing

Wing ζ at Position of Max. Induced Roll - 5b Stn - $\alpha = 6.25^\circ$

FIG. (B-1b)

SURFACE FLOW VISUALIZATION
With & Without Transition Fixing - Swept Wing

- 1) after choosing the proper ranges on the x-y plotter and strain gauge indicator, the calibration of the position channels on the recorder is checked against the position digital voltmeters,
- 2) the tunnel is then turned on and allowed to warm up until the temperature stabilizes, to eliminate temperature effects on the strain gauges. The strain gauge indicator was usually left on overnight to avoid zero drift.
- 3) A lateral scan is first made with the trailing wing at the working section center (vertically), where the vortex is expected to be found. From this scan the lateral position of the vortex core is roughly found (maximum induced roll position).
- 4) A vertical scan at the lateral position of maximum induced roll is then made and the vertical position of maximum roll is found.
- 5) The maximum induced roll and its position is accurately found by successive repetition of steps 3 and 4; however, two trials are usually sufficient (vortex center is, vertically, near the tunnel center and the lateral variation of induced rolling moment is very flat near the maximum).
- 6) To obtain detailed rolling moment measurements several scans at different lateral positions, properly spaced, are made. Scans were always made in the same direction.
- 7) With the main wing set at zero incidence (or removed) a vertical scan is made at the tunnel centerline, to remove

roll moment induced as a result of manufacturing error and residual swirl. No significant lateral variation of this induced roll was observed with main wing at $\alpha = 0$.

- 8) Calibration of the overall roll measuring system with calibration arm and weights is performed immediately after the tunnel is turned off and while the system is still warm.

Rolling moments are measured from the main wing out reference line and non-dimensionalized to give rolling moment coefficients

$$C_l = \frac{\text{R.M.}}{\frac{1}{2} \rho U_\infty^2 S_t \times b_t}$$

where S_t and b_t are the trailing wing area and span respectively. The estimated accuracy of the rolling moment coefficient measurements is about $\pm 3\%$ at maximum tunnel speed.

5-3 Rolling Moment Results

Typical rolling moment results measured on the trailing wings at two downstream locations ($2\frac{1}{2}$ and 5 main wing spans) are shown in Figs. (5-2 to -4). The main wing for these measurements was in the clean configuration (no engine mounted), full root suction and with tip on; and tests were performed at tunnel maximum speed.

Fig. (5-2) shows a sample of the measured variation of induced rolling moment, on both wings, with lateral position in the

Fig.15-2) VARIATION OF INDUCED ROLLING MOMENT WITH POSITION
(LATERAL PLANE AT VARIOUS VERTICAL POSITIONS)

working section. Assuming the trailing vortex is axisymmetric, and the wing is also symmetric, the maximum induced roll will be encountered when the vortex and trailing wing centers coincide. The minimum induced roll (maximum negative) occurs when the vortex is at any of the trailing wing tips. Fig.(5-2a) shows the lateral variation of induced roll on the small straight wing (10 in. span); at main wing $\alpha = 5^\circ$ at the 5b station. The maximum induced roll is at about $y = 4.6$ in. outboard of the working section centerline, the minimum (maximum negative) induced roll is at about $y = -0.4$ in., or $\frac{1}{2}$ a trailing wing span from the maximum. Results for other main wing angles of attack and at the $2\frac{1}{2}$ b station are very similar in characteristics to Fig.(5-2a). The swept wing, because of its larger span (20 in.), just misses the maximum induced roll (vortex center) at the $2\frac{1}{2}$ b station, Fig.(5-2b), due to the outboard wing tip entering the wall boundary layer. At the 5b station, because of the inboard drift of the vortex, the maximum induced roll is encountered, as shown in Fig.(5-2c), for main wing at $\alpha = 11^\circ$.

Variation of rolling moment coefficient, on the two trailing wings, with vertical position in the working section for various lateral stations is shown in Figs.(5-3a-d) copied directly from the x-y recorder output, also shown is the main wing out traverse. Some asymmetry is noticed in the vertical scans and is more pronounced at the $2\frac{1}{2}$ b station than at the 5b station, and with the swept wing than with the straight wing. The asymmetry suggests that the vortex is not axisymmetric, or not fully rolled up, which was not at all expected, since, even at the $2\frac{1}{2}$ b station (17 mean

Fig (5-34) VARIATION OF INDUCED ROLLING MOMENT WITH POSITION

(VERTICAL PLANE AT VARIOUS LATERAL POSITIONS)

Fig. (5-3b) VARIATION OF INDUCED ROLLING MOMENT WITH POSITION
(VERTICAL GRAB AT VARIOUS LATERAL POSITIONS)

Fig(5-3c) VARIATION OF INDUCED ROLLING MOMENT WITH POSITION

(ROLLING MOMENT INDUCED BY MAIN WING)

Fig.(5-3d) VARIATION OF INDUCED ROLLING MOMENT WITH POSITION
(VERTICAL SCARS AT VARIOUS LATERAL POSITIONS)

chord downstream of the main wing) all previous trailing vortex measurements in wind tunnels showed a relatively earlier roll up (12,14,15,16)*. The position at which maximum induced rolling moment is encountered drifts inboard and upward between the two measuring stations. This should give an indication of the vortex position; however, because of the flat rolling moment response near the peak value quantitative results should not be quoted (the position of the vortex center should be more accurately defined from the flow field measurements, discussed in the next chapter).

The maximum rolling moment coefficient $C_{l_{\max}} = \left(\frac{\text{R.M.}_{\max}}{\frac{1}{2} \rho U_{\infty}^2 S_t b_t} \right)$,

Fig. (5-4), increases with overall wing $\bar{C}_L$ up to $\alpha = 11^\circ$, $\bar{C}_L = 0.74$, and exhibits a slight non-linearity (due to more complete roll up at higher $\bar{C}_L$'s). However, once tip stall becomes dominant, and the vortex structure more dispersed, the roll moment induced on the small wing falls rapidly ($\bar{C}_L > 0.75$). The swept wing does not experience such a rapid fall off of roll moment, because of the large span and relatively small fraction of the span that is affected by the vortex dispersion compared to the outer "inviscid" part of the flow. No significant attenuation in maximum induced rolling moment is observed between the two downstream measuring stations, even though the distance from the main wing has doubled (see also Figs. (5-3a,c) and (5-6)).

To check if there is any dependence of the induced rolling moment coefficient on the free stream velocity (dynamic pressure), roll moment measurements on the straight wing were made at the 5b

Fig.5-4 Maximum Induced Rolling Moment on Trailing Wing Versus Overall Lift Coefficient of Main Wing

station, for main wing $\alpha = 6^\circ$, at varying wind tunnel speed, Fig.(5-5). Within the uncertainty of the measurements there is no dependence of the maximum induced rolling moments on tunnel free stream dynamic head.

5-4 Effect of Main Wing Tip on Induced Rolling Moment

The variation of the induced rolling moment on the trailing wings with the main wing "half body of revolution" tip on and off is shown in Fig.(5-6). The figure shows a vertical scan at the lateral position of maximum induced roll, at main wing $\alpha = 6.25^\circ$, at both the $2\frac{1}{2}b$ and $5b$ stations. The main wing tip did not affect the induced rolling moment, or its rate of decay, as there is no measurable difference at both measuring stations. It should be recalled that the tip did not change the measured main wing loading or its spanwise distribution, but did change the flow at the tip and the early separated shear layers.

Further results with much larger changes to wing tip shape will be reported by Earl⁽³⁷⁾.

FIG (5-5) EFFECT OF FREE STREAM DYNAMIC HEAD ON MAXIMUM INDUCED ROLLING MOMENT

Fig.(5-6) EFFECT OF MAIN WING TIP ON
INDUCED ROLLING MOMENT
(VERTICAL SCANS AT LATERAL POSITION OF MAX. INDUCED ROLL)

CHAPTER VI

FLOW FIELD MEASUREMENTS

By the flow field measurements we mean the 5-hole non-nulling probe measurements, which include the three components of local mean velocity and the local total pressure. Flow field measurements were taken at the two downstream measuring stations $2\frac{1}{2}$ and 5 main wing spans, with the probe tip located axially at the position of the straight trailing wing $\frac{1}{4}$ chord line; and at main wing angles of attack $\alpha = 5^\circ$ and 11° ($\bar{C}_L = 0.36$ and 0.74).

To remove free stream irregularities, traversing mechanism upstream effects and any probe misalignment with the free stream, main wing out measurements were also taken, at the two stations, and properly allowed for.

Because the flow field measurements (total pressure loss coefficient, normalized tangential velocity and normalized axial velocity) have revealed some interesting new characteristics of the flow, the results are presented, in this chapter, in their raw "as measured" form, to allow for interpretations different from those of the author. Manipulation and fuller discussion of the results will be delayed to Chapter VIII.

6-1 Measuring Technique and Data Reduction

The 5 pressures sensed by the 5-hole probe, $P_1 + P_5$, together with the contraction pressure P_{C_1} were measured relative to the contraction pressure P_{C_2} . Since only two transducers

were available, pneumatic switching was used with P_1 , P_2 and P_3 connected, through a 3-way valve, to one transducer (# 476) and P_1 , P_4 and P_{C_1} connected to the other (# 477). The output from the transducers was connected to the x-y plotter and the DISA digital voltmeters in parallel, thus discrete measurements or scanning was possible. However, when scanning three scans were required to collect the data, P_1 to P_3 , for each traverse. Care was taken to ensure that scanning was always made in the same direction, vertically and laterally, to avoid backlash and consequent angular misalignment error. Scanning rate was maintained below 1 inch/min, a rate that was found slow enough compared to the response of the 5-hole probe and measuring system. The x-y plotters were cross calibrated, each test, against the position and pressure measuring digital voltmeters. ($P_{C_1} - P_{C_2}$) was maintained constant and checked periodically during all the traverses.

The data reduction program takes as input the 5-hole pressures ($P_1 - P_{C_2}$) to ($P_3 - P_{C_2}$) as measured of the plotter chart, ($P_{C_1} - P_{C_2}$) and the x and y coordinates of the data point. Using the 5-hole probe calibration parameters the program calculates (as outlined in Appendix III) the local velocity V and its components V_x , V_y and V_z all normalized to the free stream velocity (allowing for root suction effects - as explained before), as well as the total pressure loss

coefficient $\frac{(P_{o_{local}} - P_{o_{\infty}})}{\frac{1}{2} \rho U_{\infty}^2}$. The normalized velocity components

are then corrected by subtracting the small main wing out non-uniformities at the same measuring point (main wing out surveys were made at the two stations and the results (corrections) reduced to a set of equations that describe these non-uniformities in the three velocity components to within $\pm 0.3\%$ of the free stream velocity).

The uncertainties in the total pressure loss coefficient is estimated at ± 0.005 for small coefficients but will increase to ± 0.01 for loss coefficients higher than about 0.15 (see Appendix VII for details). For the normalized velocity components the results are accurate to better than 0.008.

A technique that is very often used by investigators, when making measurements in trailing vortices, is to first find the vortex center (zero cross plane velocity) and then traverse laterally and/or vertically through the center, assuming the flow is axisymmetrical, the one or two traverses will give all the required data. A similar approach was attempted for this investigation. From the center "pitot" hole pressure, the region of high total pressure loss, which forms the vortex core, can be found, then it is easy to locate the center; where $V_x = V_y = 0$, or when $P_1 = P_2$ and $P_3 = P_4$ (except for the possible small probe misalignment and free stream irregularities, which is not important because of the steep velocity gradient near the center). Fig. (6-1) shows the measured pressures $(P_1 - P_{C_2})$ to

$(P_s - P_{C_2})$, as copied directly from the x-y plotter, for a lateral scan passing through the vortex center at the $2\frac{1}{2}b$ station and main wing $\alpha = 11^\circ$.[‡] The calculated total pressure loss coefficient (which actually defines the viscous region) for the scan shown in Fig.(6-1), and for a similar vertical scan, is shown in Fig.(6-2a). The total pressure loss is not symmetrical around the center; moreover, the humps in the curves suggest that the shear layers, separating from the wing have not fully rolled up. Consequently, it was decided that detailed flow field surveys rather than two scans should be made.

6-2 Flow Field Survey Results

Since the total pressure loss defines the lossy "viscous" regions of the flow those results will be presented first. Figs.(6-3a-d) show the contours of equal total pressure loss for the four flow field surveys made, ($\alpha = 5^\circ$ and 11° at both the $2\frac{1}{2}b$ and $5b$ stations); while Figs.(6-2a-c) show the lateral and vertical variation of the total pressure loss coefficient, for scans passing through the center of the vortex, for the cases surveyed. The shear layers, separating from the wing and formed from the upper and lower surface boundary layers, can still be distinctly seen curling up around a rolled up core with high total pressure loss, even up to the $5b$ station ($35\bar{c}$ downstream of the main wing). The "humps" in the total pressure loss curves, Fig.(6-2), are, as was suggested before, the result of crossing those still unrolled free shear

[‡] Measurements made using a high response pressure probe showed that vortex meander is less than $0.02b$, at a frequency of 1-2 c.p.s. A pass filter (0.35 sec. time constant) was used on the input to the plotter to filter high frequency noise.

FIG (6-1) SAMPLE OF 5-HOLE PROBE MEASURED PRESSURES

(P - P)

FIG (6-1) CONTINUED

TOTAL PRESSURE LOSS COEFFICIENT

FIG. (6-2) FLOW FIELD MEASUREMENTS

FIG. (6-2C) FLOW FIELD MEASUREMENTS

2y/b From Wing Root

Fig. (6-3) Contours of equal total pressure loss Coefficient
 $-(P_0 - P_\infty) / \frac{1}{2} \rho U_\infty^2$

Fig. (6-3) Contours of equal total pressure loss Coefficient

$$-(P - P_\infty) / \frac{1}{2} \rho V_\infty^2$$

layers. Unfortunately, because of the limited vertical scanning capability of the traversing mechanism, ± 5 in. from the working section centerline, it was not always possible to trace the whole viscous region of the flow, especially for $\alpha = 11^\circ$ where the still "unrolled up" shear layer was completely missed at the 5b station (previous experimental measurements suggested that the core is likely to be about 2% of the wing span, or 0.8 in. for our model wing, and it was considered that we would have enough scanning capability!).

Another feature of the flow that is clear from Figs. (6-2) and (6-3) is the faster rate of diffusion of the viscous region for $\alpha = 5^\circ$ than for $\alpha = 11^\circ$. Contours of equal total pressure loss show relatively more spread between the $2\frac{1}{2}b$ station and 5b station, for $\alpha = 5$ than for $\alpha = 11$. The region of high total pressure loss, in Fig. (6-2), which define the core of the vortex, also show a similar behavior. The maximum total pressure loss coefficient decreased from 0.7 to 0.53, between the two measuring stations, for $\alpha = 11$; for $\alpha = 5$ a relatively larger decrease occurred, from 0.39 to 0.20.

From the total pressure loss we cannot deduce much about the degree of roll up or its development between the two measuring stations; however, the still unrolled up part of the free shear layer seems to be drifting downward, leaving the vortex more axisymmetric.

The normalized tangential velocity variation is shown in Figs. (6-4a-b). Fig. (6-4a) shows lateral scans through the

FIG (8-4a) NORMALIZED TANGENTIAL VELOCITY (LATERAL SCANS)

FIG (8-4b) NORMALIZED TANGENTIAL VELOCITY (VERTICAL SCANS)

vortex center so the tangential velocity equals the V_x component of velocity, while Fig. (6-4b) shows vertical scans so the V_y component represents the tangential velocity. The asymmetry and humps in the tangential velocity curves are explained if compared to the corresponding total pressure loss contours; the humps result from crossing the still unrolled up free shear layers. The tangential velocity profile is more symmetrical at the far downstream station 5b than at the $2\frac{1}{2}b$ station, as a result of the downward drift of the unrolled up free shear layer leaving the central "core" part more axisymmetric. The tangential velocity profile is also more symmetrical laterally than vertically in accordance with the total pressure loss profile.

The $V_{\theta \max}$ for $\alpha = 5^\circ$ decrease from about $0.2 U_\infty$ to $0.12 U_\infty$ between the $2\frac{1}{2}b$ and 5b station, while the corresponding decrease for $\alpha = 11^\circ$ is only from $0.26 U_\infty$ to $0.22 U_\infty$. The core size, see Table (6-1), p.15 defined as the radius at which $V_\theta = V_{\theta \max}$, shows a higher rate of growth for $\alpha = 5^\circ$ than for $\alpha = 11^\circ$, which agrees with the faster rate of diffusion noticed from the total pressure loss contours for $\alpha = 5^\circ$.

The axial velocity profiles, for scans passing through the vortex center, show a velocity deficit ($V_z < U_\infty$) at the two measuring stations and at the two angles of attack, Fig. (6-5a and b). The velocity deficit ($U_\infty - V_z$) amounts to about 20% U_∞ for $\alpha = 11^\circ$ at the $2\frac{1}{2}b$ station and decreases to about 15% U_∞ at the 5b station; for $\alpha = 5^\circ$ the deficit is about 11% at the $2\frac{1}{2}b$ station and

FIG. (6 - 5a) NORMALIZED AXIAL VELOCITY
"LATERAL SCANS"

**Fig(6-5 b) Normalized Axial Velocity
(Vertical Scans)**

decreases to less than $4\% U_\infty$ at the $5b$ station. Again the $\alpha = 5^\circ$ case shows a faster decay or diffusion than the $\alpha = 11^\circ$ case. The axial velocity profile is very similar to the total pressure loss profile, Fig.(6-2), and their minima coincide with each other and with the vortex center, within experimental error.

The vortex center, as determined from the position of $V_\theta = 0$, Fig.(6-4), moves inboard and upward as the vortex travels from the $2\frac{1}{2}b$ to the $5b$ station, a behavior that was noticed before from the maximum induced rolling moment measurements. Table (6-1) gives the vortex position, core size, normalized maximum tangential velocity and maximum axial velocity deficit; the accuracy of the core size (as defined above) is within 0.1 in., but much higher accuracy can be obtained by finer analysis of the data.

Fig.(6-6) is an interesting figure; it shows the component of the velocity vector in the cross flow plane, for $\alpha = 11^\circ$ at the $2\frac{1}{2}b$ station. The cross flow exhibits most of the characteristics discussed above, in particular, the effect of the unrolled up shear layer is clear below and inboard of the working section centerline.

6-3 Effect of Tip

The effect of the main wing tip on the flow field was examined by comparing a lateral scan through the vortex, at $\alpha = 11^\circ$ at the $2\frac{1}{2}b$ station, with and without the tip. No

Fig. (6-6) Component of Velocity Vector in Cross Flow Plane

measurable effect on $(P_1 - P_{C_2})$ to $(P_8 - P_{C_2})$ was found, and the recorded traces could not be differentiated. In the light of the rolling moment results (which also showed no effect of the tip) this point was not persuaded any further in this investigation.

TABLE (6-1)

FLOW FIELD RESULTS

	Vortex Center in. from ζ	Core Size in	Max. Tangential Velocity V_θ/U_∞	Axial Velocity Deficit $(U_\infty - V_z)/U_\infty$						
	Lat'l Vertical Scan	Lat'l Vertical Scan	Outboard Above Center	Below Center	Lat'l Scan Vertical Scan					
$\frac{1}{2}b$ station	4.67	-0.12	0.4	0.5	0.194	0.2	0.189	0.199	0.119	0.105
$a = 5$										
$\bar{C}_L = 0.36$ $\frac{1}{2}b$ station	4.2	0.3	0.8	0.9	0.124	0.124	0.122	0.122	0.035	0.028
$a = 11$										
$\frac{1}{2}b$ station	4.24	-0.47	0.5	0.6	0.272	0.251	0.267	0.243	0.203	
$\bar{C}_L = 0.74$ $\frac{1}{2}b$ station	3.24	0.17	0.8	0.8	0.218	0.218	0.223	0.214	0.154	0.153

CHAPTER VII

DISCUSSION OF THE CLEAN WING CONFIGURATION RESULTS

7-1 Main Wing Loading and the Circulation Distribution

Before discussing the flow field results, let us reconsider the main wing loading. From the spanwise loading (Fig.(4-11), page 113) we can, from high aspect ratio wing theory, calculate the circulation distribution on the main wing, which is proportional to the local loading $c C_L$. Such circulation distribution, normalized to the root circulation Γ_r , is shown in Fig.(7-1) for $\alpha = 5^\circ$ and 11° ($\bar{C}_L = 0.36$ and 0.74 respectively). The distribution of streamwise vorticity in the vortex sheet (shear layer) leaving the wing trailing edge, assuming (unrealistically) that no roll up into a tip vortex has occurred, due to separation along the tip edge, is given by $d\Gamma/dy$ and is also shown in Fig.(7-1), (values of $d\Gamma/dy$ have been determined graphically from smoothed curve through data points). More loading is concentrated near the tip for $\alpha = 5^\circ$ than for $\alpha = 11^\circ$ because the limited leading edge separation near the tip for $\alpha = 11^\circ$ (which is apparent from the surface flow visualization Fig.(4-4)) tends to unload the tip stations. As a result of the relatively higher loading at the tip for $\alpha = 5^\circ$ than for $\alpha = 11^\circ$ proportionately more vorticity would be shed near the tip for $\alpha = 5^\circ$.

The slope of the circulation distribution $d\Gamma/dy$, shows no change of sign, Fig.(7-1), hence from the inviscid point of view^(3,4) only one vortex is expected to be produced as a result

FIG.(7-1) SPANWISE DISTRIBUTION OF CIRCULATION ON MAIN WING AND OF TRAILING VORTICITY (ASSUMING NO ROLL-UP ON WING UPPER SURFACE)

of roll up. The vorticity shed at the position where $d\Gamma/dy$ is maximum, that is at the tip, should end up, or form, the vortex center.

7-2 The Roll Up of the Trailing Vortex

At any downstream station the viscous wake is composed of the shear layers resulting from the separation of the boundary layers on the wing upper and lower surface. Inspecting Figs. (6-3a-d), which give the total pressure loss contours for the two angles of attack considered at the two downstream stations $2\frac{1}{2}$ and 5b, shows that the shear layers have not fully rolled up. A viscous region with high total pressure loss is clear and could be treated as the vortex core, however, the connecting shear layer is still clearly visible spiraling around that core. It should be interesting to calculate the degree of roll up and its development between the two measuring stations. This can be done by calculating the circulation around a properly chosen circuit in the cross flow plane surrounding the core. The problem is in defining the core region and choosing an integration circuit around it, avoiding as much as possible intercepting the spiraling shear layer or enclosing it within the integration circuit. The edge of the viscous region can be defined, in a manner similar to boundary layer theory, by, say, the 1% total pressure loss contour. Figs. (7-2a and b) also show the integration circuits chosen for determining Γ_c . These circuits were chosen to be as close as possible to the core so as to obtain higher

2y/b From Wing Root

Fig. (7-2a) Contours of equal total pressure loss Coefficient
 $-(R - R_\infty) / \frac{1}{2} \rho u^2$

Fig. (7-2b) Contours of equal total pressure loss Coefficient

$$-(P_t - P_{t0}) / \frac{1}{2} \rho u_\infty^2$$

accuracy because of the higher induced velocities near the core, while at the same time enclosing all vortical fluid within the viscous core region.

At the $2\frac{1}{2}b$ station the circulation around the core (Γ_c), as calculated above, amounted to only 41% of the wing root circulation Γ_r (as calculated from the measured wing loading) at $\alpha = 5^\circ$ and to 48% at $\alpha = 11^\circ$. Furthermore, moving downstream to the 5b station resulted in very little further roll up (concentration of vorticity in a core) as judged by the small increase in core circulation to 43% of the root circulation for $\alpha = 5^\circ$ and 51% for $\alpha = 11^\circ$. Even this small increase in the core circulation, of about 3% Γ_r , between the two measuring stations cannot be judged with any confidence as a real measure of further roll up because of the arbitrariness in crossing the spiraling (unrolled) shear layers. Also we should remember that the maximum induced rolling moment on the small trailing wing did not show a measurable change between the $2\frac{1}{2}b$ and 5b stations, which would not have been the case if the roll up was still proceeding. Consequently, we can say that essentially there has been no further (or very little) roll up between the two measuring stations, even though the distance from the generating wing has doubled. The shear layers were already, almost, fully developed or rolled up, at the $2\frac{1}{2}b$ station, but not in the classical sense of fully rolled up, i.e. as an axisymmetric vortex. Before elaborating more on the last statement, let us

review some of the other experimental investigations in which the circulation around the core has been measured.

Dosanjh et al^{(13)*} reported that in their wind tunnel measurements behind a rectangular half wing of aspect ratio 7 and NACA 0009 section, only 58% of the expected theoretical circulation around a two-dimensional NACA 0009 aerofoil was measured. The test Reynolds number, based on wing chord, was 10,000 and measurements extend up to 8 chords downstream. Considerable model testing was performed by Grow (in a wind tunnel), as reported in Refs. (15 and 16)*, for wings of aspect ratio from 2.0 to 6.0 and taper ratio from 0 to 1.0, untwisted and unswept with constant NACA 0012 airfoil sections, at a Reynolds number based on the midchord of approximately 3.5×10^5 . Measurements were made of the vortex geometry at approximately 4 chord lengths downstream of the trailing edge. At this station the vortex sheet seems to be completely rolled up into discrete vortices. In general, Grow found that the strength of the vortex is less than one would predict on the basis of midspan circulation about the wing (uncorrected values are almost 50% of those predicted). The experimental results of McCormick, Tangeler and Sherrieb^{(16)*} behind an Army O-1 aircraft and a 1/12 scale model of its wing shows that, at a distance of only 20% of the midchord behind the trailing edge a well defined tip vortex already exists. At less than a chord length downstream, 90% of the measurable circulation of the

vortex system is contained in the tip vortices. The magnitude of the streamwise vorticity in the vortex system, measured using a vorticity meter, for a full scale aircraft is approximately a third of that of the model (and is about 16% of the theoretical total strength of the vortex). Also, at corresponding downstream distances, the vortex sheet behind the full-scale aircraft appears to be more completely rolled up. This the authors explain may be the result of the vorticity in the vortex sheet for the full-scale aircraft falling below the sensitivity threshold of the vortex-meter. deVries⁽³¹⁾ measurements, behind a 30° swept back untapered wing of aspect ratio 5, show that at the trailing edge, about 60% of all the trailing vorticity is already concentrated in a vortex, which originates from the separation line along the streamwise tip. This tip vortex flows downstream approximately parallel to the undisturbed free stream, without significant variation of its strength. The chord Reynolds number for deVries's test was 1.45×10^6 and the measurements extended up to 4 chords downstream of the tip trailing edge (slightly more than one wing span if measured from the wing mean chord 1/4 chord point). Smith and Lazzeroni⁽⁴⁸⁾ also reported that at the trailing edge (of a rectangular half wing, $Re = 4$, test $Re = 0.6 \times 10^6$) the strength of the tip vortex was nearly half of the entire circulation.

The above reported measurements, including our own, span about two orders of magnitude in Reynolds number. Still they all show that more or less only about 50% of the trailing

vorticity given by high aspect ratio wing theory is concentrated in the core. This evidence appears to preclude the possibility that the incomplete roll up (i.e. the fact that part of the shear layer leaving the wing did not roll up or proceed to roll up around the concentrated core) is a low Reynolds number effect and is related to the fact that the boundary layers on the wing are relatively thicker than the flight case.

It is unfortunate that at this stage we do not have measurements very close to the wing trailing edge. However, the measurements of deVries⁽³¹⁾ are to some extent similar to ours and seem to be of such a good quality and are complete enough to justify using them in conjunction with our measurements.

deVries⁽³¹⁾ measurements show that the part of the trailing (streamwise) vorticity not concentrated in a tip vortex leaves the wing in the form of vorticity (shear) layer. The inboard part moves downwards and the vorticity diffuses slowly into the surrounding air flow, whereas the outboard part of the shear layer moves in a helical path around the tip vortex, diffusing its vorticity into the surrounding air without concentrating it into the core of the tip vortex. Fig. (7-3a), which is Fig. 5 of Ref. (32), shows the development of contours of equal total head up to 4 chords downstream of the tip trailing edge, from deVries measurements. Inspecting Figs. (7-2a -2b), which show the development of the contours of equal total pressure loss between the $2\frac{1}{2}b$ and $5b$ stations ($17.5 \bar{c}$ to $35 \bar{c}$), it is clear that very much the same picture is still proceeding in

(a) Contour lines of constant total head in planes perpendicular to the free-stream. The wake location is not corrected for tunnel wall constraint.

$$\frac{\Gamma}{1/2 C_L c U_\infty}$$

(b) Measured and calculated spanwise lift distribution.

(c) Downstream variation of the vorticity distribution in the tip vortex and the spanwise circulation distribution on the wing in the tip region.

FIG. (7-3) SOME OF REFERENCE(32) RESULTS

our experiments at greater downstream distances. The inboard part of the shear layers is drifting downward and the outboard part spiraling around the core, but very little, or no, increase in the core strength is measured. The present author suggests that the roll up develops as follows:

"A tip vortex is rolled up above the wing upper surface near the tip; in an analogous way to that on sharp leading edge slender delta wings. The strength of this tip vortex depends on the spanwise loading, or, more specifically, concentration of local loading near the tip, and on the nature of the boundary layer separation near and on the tip. At the tip trailing edge a large fraction of all the trailing vorticity (probably around 40% - 60% for elliptic-like loading, without extensive leading edge separation) is already concentrated in a tip vortex. That part of the trailing vorticity not concentrated in a tip vortex leaves the wing trailing edge in the form of a free shear layer (vorticity layer); however, it seems that very little further concentration of vorticity into the tip vortex occurs. The free shear layer (which might also be called a connecting layer) drifts downward and spirals around the tip vortex slowly diffusing and leaving the concentrated tip vortex more axisymmetric."

The above conclusion is supported by several investigators^{(12,15,16,31)*,(28,30)} observations that the trailing vortex rolls up much faster than expected, from high aspect ratio inviscid theories, and within a very few chord

lengths downstream. Their conclusions are usually based on the apparent symmetry in measured tangential velocity, which is actually the result of the downward drift of the connecting shear layer leaving the induced velocities from the tip vortex more symmetric, at least for measurements in the lateral horizontal plane.

The measured circulation around the rolled up vortex core is equal to the calculated circulation around a certain wing section, A, say. According to Stokes theorem, and assuming the roll up to proceed uniformly so that the vorticity shed at the tip ends up in the core center, then the vorticity shed outboard of section A will form the core (by the core we mean that portion of the shear layer that is concentrated in an axisymmetric form). For the $\alpha = 5^\circ$ case in our experiment, section A corresponds to about $0.93 b/2$ or about one $\frac{1}{2}$ tip chord from the tip (see Fig.(7-1)), while for the $\alpha = 11^\circ$ case A corresponds to 1.2 tip chords from the tip. It is interesting to notice that for $\alpha = 11^\circ$ a relatively higher percentage of the root circulation was measured around the core than for $\alpha = 5^\circ$ (51% vs 43% at the 5b station), even though for $\alpha = 11^\circ$ relatively less load was concentrated near the tip as a result of the tip "stall". However, as a result of the tip stall (with some leading edge separation) the separated boundary layers are more effective in concentrating the vorticity shed over a larger fraction of the span into the tip vortex. From the variation of the local lift coefficient with α (Fig.(4-10) page (113)) and from the flow

visualization (Fig.(4-4), page 100) it is clear that some leading edge separation near the tip is evident for $\alpha = 11^\circ$ and not for $\alpha = 5^\circ$. deVries^(31,32) measurements show that for $\alpha = 10^\circ$, $\bar{C}_L = 0.5$, 60% of the circulation is concentrated in the tip vortex and that section A is located at about $\frac{1}{3}$ tip chord from the tip (see Fig.(7-3b,c). The stronger tip vortex in deVries' case is due to the relatively higher loading near the tip (the swept wing used has no taper, but has a washout of 4° , linearly distributed along the span).

It is possible now to explain why the half body of revolution tip did not affect the flow field behind the wing. The tip effects were noticed to be very local (see Fig.(4-7), page 108); no tip effects were noticed in the pressure distribution at station 13 which is $0.07 b/2$ from the wing tip. The affected part of the flow, very near the tip edge, ends in the core center and is diffused before reaching the first measuring station. If measurements are taken very near the trailing edge some tip effects on the detailed vortex structure would probably be seen due to changes of the separation lines along the streamwise tip. Earl⁽³⁷⁾ has seen some tip effects on the flow field at the 5b measuring station, but with more drastic changes of tip configuration.

Methods for calculating the roll up of vortex sheets^{(4,8,9,10)*, (7,9,10,11)} usually assume inviscid flow and a planar sheet leaving the wing trailing edge. Since it is concluded above that a tip vortex (which rolls up above the

wing upper surface near the tip and along the leading edge) leaves the wing trailing edge, with a substantial fraction of the vorticity already concentrated in it, the assumption of planar sheet leaving only the trailing edge is greatly in error, especially very near the wing. Also, because of the finite thickness of the shear layer and its diffusion, its roll up around the concentrated core will proceed more slowly and it spirals around the core rather than merges in it. Hence, it seems that such viscous effects cannot be neglected when calculating the roll up in real fluid flows. Even the most realistic approach to the three dimensional inviscid roll up calculation given by Labrujere⁽⁷⁾ which explicitly allows for the 'bound' vorticity distribution on the wing, showed discrepancies between the calculated and measured shape of the 'vortex sheet' from deVries' set up. However, the comparison was made only up to 4 chords downstream of the tip trailing edge, where the inviscid roll up model is least accurate.

7-3 Effect of Tunnel Wall Interference on Roll Up

To ensure that wind tunnel wall interference is not interfering with the roll up and delaying it, Mokry and Rainbird⁽¹¹⁾ calculated vortex roll up histories in free air and in a rectangular wind tunnel working section of the same proportions as the present experiment. The roll up model used is basically two-dimensional and inviscid; the interest being in wall interference rather than an accurate representation of

§ Also similar calculations by Lind^{##} and Rom et al^{###}.

I.A. Lind, "A Nonlinear Vortex Lattice Method Applicable to Three Dimensional Wing Systems with Rolled Up Vortex Wakes in Low Subsonic Flow. Forces and Moments on Some Flat Wings and on High Lift Wing," KTH Aero Rpt. F I 43, May 1973.

J. Rom et al, "On the Calculation of Non-Linear Aerodynamic Characteristics and the Near Vortex Wake," ICAS Paper No. 74-27, Aug. 1974.

roll up. Fig.(7-4) shows a comparison of the calculated vortex sheet roll up histories in free air and in a wind tunnel having the characteristics and wing size of the experimental work reported here, for elliptic loading. The roll up appears to differ very little, with the exception of the position of the core and connecting sheet, which is clearly constrained by the working section walls. One would intuitively expect that because the connecting vortex sheet is closer to the core, for roll up within wind tunnel walls, the roll up would proceed faster rather than be delayed.

7-4 Location of Vortex Center

The location of the vortex center indirectly gives an idea about the degree of vortex roll up. According to Betz's theory the center of gravity of the streamwise vorticity distribution remains at a constant distance from the plane of symmetry. Consequently, for streamwise vorticity distributions that will result in one pair of trailing vortices only, as the roll up proceeds monotonically from the tip inboard, the center of the rolled up part has to drift inboard to keep the center of gravity at a constant lateral position. For elliptically loaded wings, if we assume complete roll up, the separation distance of the pair of trailing vortices will asymptotically reach the classical value of $\frac{3}{4}b$. Fig.(7-5) shows the position of the 'center' of the rolled up part of the vortex sheet, for elliptic loading, in the free and constrained case, from

(VALUE OF τ CORRESPONDING TO PRESENT EXPERIMENT) FIG. (7-4) INFLUENCE OF WIND TUNNEL WALLS ON ROLL-UP OF VORTEX SHEET (FROM REF. 11)

STATION	$\alpha = 5^\circ$	$\alpha = 11^\circ$
2.5 b	0.33	0.7
5 b	0.65	1.34

FIG.(7-5) COMPARISON OF THE POSITION OF VORTEX
CENTER WITH TWO DIMENSIONAL ROLL-UP CALCULATION

Mokry and Rainbird's⁽¹¹⁾ results. Also included on the figure is the measured position of the center of the roll up portion of the shear layer for $\alpha = 5^\circ$ and 11° at the two downstream measuring stations. To the author's surprise, the calculated and measured values of lateral position of the vortex center are in reasonable agreement, with the measured values being more on the outboard side.[†] Such agreement was not expected because of the oversimplification of the inviscid roll up model and the differences in spanwise loading shape. For the vertical position of the vortex center the measured values are well below those calculated. However, the theoretical model starts with a vortex sheet at the working section centerline, while actually the trailing edge of the swept-back wing is below the working section centerline (see Fig.(7-2a,b) giving the position of the wing trailing edge at $\alpha = 5^\circ$, 11° , while in Fig.(7-5) the position of tip trailing edge is marked on the vertical axis). If a reasonable correction is applied, namely that equal to the distance by which the tip trailing edge is below the working section centerline, then the experimental data tend to agree with the calculation, especially for $\alpha = 5^\circ$. For $\alpha = 11^\circ$ the measured data thus corrected are still above the calculated values; probably a smaller correction should have been applied because the origin of the tip vortex is closer to the tip leading edge, as a result of the leading edge separation.

In Mokry and Rainbird's⁽¹¹⁾ model the roll up starts with a planar vortex sheet at the origin and does not represent the [†] It should be interesting to calculate the degree of roll up (i.e. percentage of root circulation that is concentrated in the rolled core) from Mokry and Rainbird's model and compare it to the present measurements. The rate at which roll up proceeds between the 2**b** 5b station should probably be the basis for comparison.

vortex resulting from separation along the tip edge. However, because in the experiments the mean chord $\frac{1}{4}$ chord point of the main wing is taken as the origin for downstream distance, the wing tip trailing edge is about $0.22 b$ downstream and the inviscid model represents some degree of roll up at the tip trailing edge. In addition the model neglects the finite thickness of the shear layer and replaces it with an infinitesimally thin sheet which would probably result in a faster roll up than in a real case. Thus the inviscid model will catch up and represent the actual roll up over some region. This region probably happens to be over our measuring range and the agreement between Mokry and Rainbird's model and our own measurement is to some extent accidental.

The inboard drift of the vortex center with increasing downstream distance has been reported by several investigators using ground based facilities. Except for the water tank photographic results reported by Spreiter and Sacks^{(11)*}, which confirmed their finding that the separation distance for elliptic loading is $\frac{\pi}{4}b$, most other measurements reported a larger separation distance or less inboard drift. Page and Simmons^{(14)*} results in a wind tunnel show that the vortex center, as defined by the maximum vorticity position, is about $0.13 b/2$ from the tip, 13 chords downstream. The flight and wind tunnel results reported by McCormick et al^{(16)*}, also from vorticity-meter measurements, show the vortex center within $0.06 b/2$ from the tip, but these measurements were taken

less than a chord downstream. Dosanjh et al^{(13)*} found that the vortex center reaches the asymptotic value of $0.08 b/2$ from the tip for downstream distance larger than 6 chord lengths. Chigier and Corsiglia^{(31)*} measurements, behind a rectangular and a CV-990 model wings show an inboard drift of the vortex center less than $0.10 b/2$ for both wings; while deVries⁽³¹⁾ measurements behind a swept wing show the center at about $0.08 b/2$ from the tip. Our own measurements show a maximum inboard drift of $0.13 b/2$ at 5 spans (35 mean chords) downstream; it also shows that for $\alpha = 5^\circ$ and the same downstream distance that the vortex center is more outboard than for $\alpha = 11^\circ$ indicating less degree of roll up, as confirmed with the measured circulation. It is interesting to notice that in all the above reported investigations the circulation around the rolled up vortex was reported to be much less than the value expected based on high aspect ratio wing theory (except for the work of Chigier and Corsiglia^{(31)*} in which the circulation has not been reported).

An important conclusion from the comparison, and agreement, of the lateral vortex position with Mokry and Rainbird's model is that the degree of roll up, as indicated by the model, for $\alpha = 5^\circ$, $\bar{C}_L = 0.36$, at the $5b$ station is the same as for $\alpha = 11^\circ$, $\bar{C}_L = 0.74$, at the $2\frac{1}{2}b$ station. This follows from the fact that the distance from the wing to stations having similar degrees of rolling up is inversely proportional to the overall lift coefficient (notice that approximately doubling $\bar{C}_L$ resulted in

halving of the distance). This last conclusion was suggested before by Spreiter and Sacks and will be discussed again in the next section.

7-5 Vortex Induced Velocity Field

The vortex induced flow field, presented as normalized tangential and axial velocities in Figs. (6-4 and 5), exhibits the same characteristics as the viscous wake (total pressure loss contours). The incomplete roll up of the shear layers is reflected on the distribution of tangential and axial velocities as kinks or humps on the curves. The position of the kinks in the axial and tangential velocities corresponds to crossing the outboard spiraling part of the shear layer, as appear in the lateral scans, or to the downward drifting part of the shear layer, as is clear from the vertical scans.

Another feature of the induced velocities is the asymmetry around the vortex axis, which is the result of the viscous wake "vortical region" asymmetry. Figs. (7-6a and b) show the variation of the tangential velocity with distance, measured from the vortex center, for vertical and lateral scans. Also included in the figure is the velocity induced by a Rankine vortex ($V_\theta \sim \frac{1}{r}$) with strength equal to the average 'core' circulation, Γ_c , calculated from the flow fields at the $2\frac{1}{2}b$ and $5b$ stations. The increase in symmetry between the $2\frac{1}{2}b$ station and the $5b$ station should not be interpreted as a result of more complete roll up. However, as the connecting

3

OF/DE

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FIG. (7-6a) NORMALIZED TANGENTIAL VELOCITY
(LATERAL SCAN THROUGH VORTEX CENTER)

DISTANCE FROM VORTEX CENTER
 FIG. (7-6b) NORMALIZED TANGENTIAL VELOCITY
 (VERTICAL SCAN THROUGH VORTEX CENTER)

shear layer spirals around the rolled up part and drifts downward, it stretches and together with its diffusion it becomes weaker and its effect on the induced velocities decreases. Also, the downward drift of the shear layers results in more symmetry about the lateral plane and the kinks in the tangential velocities become far away from the center.

Such kinks and asymmetry have appeared in most experimental work^{(31,90,102)*},^(14,15,28,30,45,46), even though not necessarily recognized and reported on. Chigier and Corsiglia^{(31)*} reported different maximum tangential velocities from normal and spanwise scans up to 12 chords downstream. Phillips⁽¹⁴⁾ noticed kinks in the tangential velocity profile up to 45 chords downstream, but because the maximum tangential velocity varied by only $\pm 2\%$ he concluded that the roll up was almost complete. From the present experiment for $\alpha = 5^\circ$ at the 5b station (35 $\bar{c}$) the variation in the maximum tangential velocity between lateral and vertical scans is less than $\pm 1\%$, see Table (6-1); still from the total pressure loss contours, Fig.(7-2), the vortex is not fully rolled up. Kinks in the velocity profile of Ref.(15) were taken as an indication of secondary vortices shed from the wing tip! The kinks showed up for runs made at axial^o stations closest to the wing (where the spiraling shear layer is still strong). Marchman and Marshall⁽²⁸⁾ noted that at stations near the wing ($z/c = 2,5$) the vortex did not appear as symmetrical as at stations downstream. They attribute that to the highly three-dimensional nature of the flow in the immediate wing wake

and the ensuing difficulty in measuring the resulting flow components. However, asymmetry in their measured tangential velocity profiles can still be seen up to the $z/c = 25$ station. The axial velocity profile at $z/c = 30$ (their Fig.(11)) shows a kink that could be the result of crossing the spiraling shear layer outboard of the vortex center. Moreover, they presented only lateral scans, and the lateral scans are usually more symmetrical than the vertical scans. Kinks and asymmetry are very clear in the results presented by Orloff⁽³⁰⁾ using a laser doppler velocimeter. The kinks appear in both the tangential and axial velocity profiles outboard of the vortex center, suggesting that it is the effect of crossing the spiraling connecting shear layers. Orloff noted an asymmetry of the tangential velocity profiles for distance, r/b , greater than 0.05. This same phenomenon has been observed by Chigier⁽⁴⁵⁾ and Legins⁽⁴⁶⁾, with a consistent pattern of higher circumferential velocities (for a given r/b) below the wing than are encountered above the wing. This is, of course, the effect of the added induced velocities by the connecting shear layer below the vortex center, and can be easily visualized from the present experiments by examining Fig.(6-6) which gives the velocity vector, in the cross-flow planes.

From the discussion presented above we can conclude, with some confidence, that the shear layers were not completely rolled up in most wind tunnel experiments reported and known to the author. Asymmetry and kinks in the tangential and axial velocity

profiles could be, in most cases, interpreted as the result of crossing the still unrolled shear layers, if the viscous wake is visualized as presented by the contours of equal total pressure loss in Figs. (7-2a,b). The kinks, or humps, in the velocity profiles are of such small magnitude and vary in relative position, depending on the degree of roll up, that it is difficult to accept them without other evidence of incomplete roll up. Also far downstream the still unrolled shear layers are weak and drift downward far from the rolled central part and so unless traverses are extended far from the core the vortex will look symmetrical. It is with the help of the extensive and accurate total pressure measurements that we were able to visualize the viscous wake and drew our conclusions. It is recommended that such total pressure measurements be made in conjunction with trailing vortex measurements to define the limits of the viscous wake.

Comparison of the normalized tangential velocity profiles for the two angles of attack measurements seems to be difficult, Fig. (7-6). The maximum tangential velocity normalized to the free stream velocity, or to the free stream velocity and the main wing lift coefficient, has different values for each α for both measuring stations. Moreover, the maximum tangential velocity decays faster between the two measuring stations for $\alpha = 5^\circ$, $\bar{C}_L = 0.36$, than for $\alpha = 11^\circ$, $\bar{C}_L = 0.74$ (from $0.195 U_\infty$ to $0.123 U_\infty$ for $\alpha = 5^\circ$ as compared to from $0.259 U_\infty$ to $0.218 U_\infty$ for $\alpha = 11^\circ$). The faster decay for $\alpha = 5^\circ$ was also

noticed from the axial velocity profiles and the total pressure loss. Logan^{(30)*} has noticed similar trends in his measurements, the axial and tangential velocity decayed faster for $\alpha = 4^\circ$ than for $\alpha = 12^\circ$ between his two stations at $z/c = 10$ and 26).

It should be realized that the vortex generated for $\alpha = 5^\circ$ is different in structure, at least the original structure, than the vortex generated for $\alpha = 11^\circ$. For $\alpha = 5^\circ$ there seems to be no leading edge separation; the separation is along the trailing edge and perhaps along the tip edge, and the lift curve slope for the tip station 14 ($2y/b = 0.988$) is still linear. For $\alpha = 11^\circ$ leading edge separation near the tip is evident and the origin of the tip vortex probably starts there. The importance of this point is not in the length difference between two origins, but in the difference of the nature of the early separating shear layers which form the core. Also, the thickness of the boundary layers leaving the wing is different for both angles of attack, being presumably thicker for $\alpha = 11^\circ$, and the rate of diffusion of the tip vortex and connecting shear layer is different (this is clear from the extent of the viscous wake at the two angles of attack, see Fig. (7-2)). Consequently, we should not expect that we will be able to easily compare the velocity profiles.

According to high aspect ratio inviscid roll up models (either Spreiter and Sacks^{(11)*} or Betz's^{(20)*} model) the rolled up wake consists of an irrotational flow field and a region in which the streamwise vorticity is non-zero. Spreiter and Sacks^{(11)*} model assumes a uniformly distributed vorticity over the core which has a radius $r_c = 0.086 b$ (for elliptic

loading). Betz's ^{(20)*} model, as approximated by Donaldson ^{(2)*}, has a distributed vorticity over a radius $\frac{r}{b} = \frac{1}{3}$. Irrespective to the model, the core size is a constant fraction of the span depending on the load distribution but not on loading intensity; i.e. lift coefficient. The vorticity in the rolled up vortex and consequently the induced velocities are, however, proportional to the root circulation Γ_r . Now

$$\Gamma_r = \frac{K L}{\rho U_\infty b} = K \frac{\bar{C}'_L}{\rho U_\infty b} \times \frac{1}{2} \rho U_\infty^2 b^2 = \frac{K}{2} \bar{C}'_L U_\infty b$$

where K is a factor depending on the shape of the spanwise load distribution and $\bar{C}'_L$ is the modified lift coefficient based on wing span, $\bar{C}'_L \equiv L / \frac{1}{2} \rho U_\infty^2 b^2$.

The induced tangential velocity at the edge or outside the core, V_θ , is given by

$$V_\theta = \frac{\Gamma_r}{2\pi r} = \frac{K}{2} \bar{C}'_L U_\infty \frac{b}{r} \cdot \frac{1}{2\pi}$$

$$\frac{V_\theta}{U_\infty} = \frac{K}{4\pi} \bar{C}'_L \cdot \left(\frac{b}{r}\right)$$

for $\frac{V_\theta}{U_\infty} = \frac{K}{4\pi} \frac{\bar{C}'_L}{AR} \cdot \left(\frac{b}{r}\right)$]

hence

$$\frac{V_\theta}{U_\infty \bar{C}'_L} = \frac{K}{4\pi} \left(\frac{b}{r}\right)$$

Within the core, and assuming fully rolled up axisymmetric conditions, the $\frac{\Gamma}{\Gamma_r}$ distribution is a function of (r/b) only

$$\text{i.e.} \quad \frac{\Gamma}{\Gamma_r} = \text{fn} \left(\frac{r}{b} \right)$$

$$\text{and} \quad v_\theta = \frac{\Gamma}{2\pi r} = \frac{\Gamma_r}{2\pi r} \cdot \frac{\Gamma}{\Gamma_r}$$

or, as before

$$\frac{v_\theta}{U_\infty \bar{C}_L'} = \frac{K}{4\pi} \left(\frac{b}{r} \right) \cdot \frac{\Gamma}{\Gamma_r} = \frac{K}{4\pi} \text{fn} \left(\frac{b}{r} \right)$$

The above argument applies only to rolled up wake and does not include any viscous effects on the roll up or the effects of viscous diffusion. The idea behind the exercise is to try to find proper scaling parameters to compare experimental results, even in an overall sense. If roll up is not complete one might expect that the same scaling parameters would still apply, assuming comparison is made at distances having the same degree of roll up.

Spreiter and Sacks^{(11)*} argue that the distance d , in terms of wing spans from the trailing edge to stations having similar degrees of rolling up of the trailing vortex sheet, is proportional to $R/\bar{C}_L'$ (or $\frac{1}{\bar{C}_L'}$). Hence, the requirements for proper comparison of measured tangential velocities, from the inviscid argument, are:

- same shape of the normalized load distribution (or proper allowance for the factor "K" introduced above),
- that comparison be made at downstream distances in terms of wing span " d/b " proportional to $1/\bar{C}_L'$ or $\bar{C}_L'/R$,
- distances from vortex center be normalized to wing span, and
- that the tangential velocity be normalized to the free stream velocity and modified lift coefficient $\bar{C}_L'$.

From the preceding section, and bearing in mind its limitations, we have concluded that, based on vortex center position, the shear layer for $\alpha = 5^\circ$, $\bar{C}_L = 0.36$, at the 5b station is at the same degree of roll up as for $\alpha = 11^\circ$, $\bar{C}_L = 0.74$, at the $2\frac{1}{2}b$ station. The difference in the spanwise loading will vary the factor K in proportion to the variation in the ratio of the local lift coefficient to the overall lift coefficient, however, the variation is less than 4% ($\frac{C_L}{\bar{C}_L} = 1.36$ for $\alpha = 11^\circ$ and 1.3 for $\alpha = 5^\circ$). Fig. (7-7) shows a comparison of the axial and tangential velocity profiles normalized to the free stream velocity and main wing $\bar{C}_L$, for $\alpha = 5^\circ$ at the 5b station and $\alpha = 11^\circ$ at the $2\frac{1}{2}b$ station (since we are comparing profiles for the same wing, same Re , the overall lift coefficient is proportional to the modified lift coefficient). The agreement in the tangential velocity profile is very good except near the center where the differences in the early history of the shear layers produces different detail core structure. The agreement not only confirms that the two vortices have a similar degree of roll up, but also that the induced velocities are proportional to the modified lift coefficient (actually it only proves that it is proportional to the lift, since the tests were performed at the same free stream velocity, with the same wing). McCormick et al^{(16)*}, Orloff⁽³⁰⁾, and Corsiglia, Schwind and Chigier⁽¹⁵⁾ also found the peak tangential velocity to be proportional to the lift coefficient, for the same or

FIG.(7-7) COMPARISON OF VELOCITY PROFILE FOR VORTEX
SYSTEM HAVING SIMILAR DEGREE OF ROLL-UP

geometrically similar wings. The axial velocity profiles do not agree except in their general shape. The inviscid argument presented is in a sense two-dimensional and does not consider axial velocity variation; hence, it is not applicable for axial velocity comparison. The axial velocity deficit is associated with the profile drag^{(23)*}, which is clearly relatively higher for $\alpha = 11^\circ$ with the leading edge separation near the tip than for $\alpha = 5^\circ$ with no separation and hence results in a higher axial velocity deficit.

It is realized that firm conclusions cannot be drawn from comparing just two velocity profiles. However, it is hoped that the importance of proper scaling of experimental results is demonstrated.

The faster decay of the tangential velocity, axial velocity deficit, and total pressure loss for $\alpha = 5^\circ$ than for $\alpha = 11^\circ$ between the two measuring stations can be explained in two different ways. First, the different rate of decay could be ascribed to the difference in the initial formation of the tip vortex, giving different mean shear stress distributions, as discussed before. The vortex system, including the rolled up part and the connecting shear layer, is more diffused for $\alpha = 11^\circ$ than for $\alpha = 5^\circ$ and this may have resulted in slower decay. Orloff et al⁽³³⁾ has noticed that diamond wings, with triangular span loading, yield a lower concentration of vorticity in the core and diminished circumferential velocities in the adjacent regions than those produced by swept or rectangular wings. However, the diffused vortex has a slower rate of decay

and at 100 span lengths downstream the velocity profiles are quite similar. The second explanation can be made also in the light of Orloff et al⁽³³⁾ towing tank results. From measurements made in the wake of wings being towed under water they identified two characteristic flow regions. The first, a "plateau" region, with little, if any, change in maximum tangential velocity, extends from wake roll up to downstream distances as great as 100-span lengths, depending on span loading and angle of attack. This is followed by a decay region in which the maximum tangential velocity decreases with downstream distance at a rate nominally proportional to the inverse one half-power. For $\alpha = 11^\circ$, $\bar{C}_L = 0.74$, the vortex system is in a fuller degree of roll up than for $\alpha = 5^\circ$, $\bar{C}_L = 0.36$, and is probably closer to the plateau region in which there is little change in maximum tangential velocity; that could indicate slower decay.

7-6 Vortex Core

There have been several interpretations to which part of the vortex system is considered as the vortex core, all of which assume an axisymmetric vortex. For a vortex system that is not axisymmetric, because of the incomplete roll up, we can talk of the core as the rolled up part of the shear layers. Even this region is not easy to define when the spiraling shear layers are close but not actually rolled up into the concentrated part.

The logical definition of a vortex core is that it is the region in which viscous effects are present, or in which the axial component of vorticity is non zero, which is the definition used by Spreiter and Sacks^{(11)*} and by Batchelor^{(29)*} for example. Jordan⁽⁴⁷⁾ denotes by a "core" that part of the vortex where it differs from a Rankine vortex (i.e. where V_θ is not proportional to $1/r$); which is the same as the above definition if the vortex is axisymmetric. The most often used, and probably the least meaningful, definition of core is that it is the region between the maximum tangential velocity points^{(16,31,103)*}, (15,30). The reason for the popular use of that definition is the argument presented by McCormick et al^{(16)*} that the product of the maximum tangential velocity $V_{\theta\max}$ and the radius at which this $V_{\theta\max}$ occurs, the so-called core radius "a", is a constant.

In the present experiments the core 'diameter' defined by the viscous region (say the 1% total pressure loss contour) is about 2.8 in. or 0.067 b at the $2\frac{1}{2}b$ station, for $\alpha = 5^\circ$, and diffuses to about 4 in. or 0.095 b at the 5b station (see Fig. (7-2)). For $\alpha = 11^\circ$ due to the more diffused shear layer, and because the spiraling shear layer now merges with the central part, the core size is about 5.8 in., 0.14 b, at the $2\frac{1}{2}b$ station and about 8 in., 0.19 b, at the 5b station; twice as large as for $\alpha = 5^\circ$. Alternatively, if the core is defined as where V_θ is maximum, much smaller core size is obtained;

about 0.5* in., 0.01 b at the $2\frac{1}{2}b$ station and 0.8 in., 0.02 b, at the 5b station, for both α 's (see Table (6-1)). The circulation around this (inner) 'core' is about half of the circulation around the viscous core for $\alpha = 5^\circ$, but only about one third for $\alpha = 11^\circ$; moreover, it is not constant but increases with distance downstream (faster than the observed increase in the circulation around the viscous core). We can think of the core, as defined by $V_{\theta\max}$, as the core of the viscous core.

7-7 Induced Rolling Moment on Trailing Wings

A comparison of the measured rolling moment with values calculated from the measured flow field can supply an independent cross check on both measurements. Fig.(7-8) shows such a comparison for the straight wing at the two measuring stations and main wing lift coefficients; it shows the variation of the induced rolling moment with vertical position at the lateral position of maximum induced rolling moment. The solid line with fluctuations represents the measured induced rolling moment coefficient as copied directly from the x-y plotter, but with the zero reference adjusted to allow for main wing-out correction as discussed in Chapter V. The solid circles represent the induced rolling moments calculated from the measured flow field (also corrected for main wing-out) assuming frozen flow, i.e.

* There is a serious lack of spatial resolution here because of the probe size, 0.093 in. O.D.

Fig. (7-8) Variation of Induced Rolling Moment on Straight Wing with Position
 "Vertical Scan at Lateral Position of Max Induced Roll"

that the presence of the trailing wing does not alter the vortex system or its position. The induced loads, and rolling moments, on the trailing wing, are calculated using a simple strip theory and assuming the sectional lift slope is the same as the overall lift slope together with limiting the maximum lift to the maximum measured value from the overhead balance, see Fig. (4-14) page 119. The broken line is a similar calculation but with a simple infinite line vortex of strength equal to the calculated circulation around the viscous core replacing the vortex system and is located at its center. Fig. (7-9) shows the spanwise distribution of induced angles of attack on the trailing wing, when its center coincides with the vortex center at the 5b station, as calculated from the measured flow field and from the line vortex of strength Γ_c . Fig. (7-9) is actually $\arctan \left(\frac{V_\theta}{U_\infty} \right)$, as presented in Fig. (7-6a), and should explain most of the observed characteristics of Fig. (7-8).

Naturally the calculated induced rolling moment from the measured flow field agrees better with the measured values than does the line vortex calculation. However, both calculations show better agreement for $\alpha = 5^\circ$, $\bar{C}_L = 0.36$, than for $\alpha = 11^\circ$, $\bar{C}_L = 0.74$. The difference between the measured rolling moment and that calculated using the measured flow field is due to the combined effect of using a strip theory and assuming that the sectional lift curve slope is the same as the overall.

The difference between the calculations using a vortex line model compared to the measured flow field, is largely because the vortex core is not represented in the former. Fig.(7-9) shows that the result of not representing the core is artificially higher induced angles of attack. For $\alpha = 11^\circ$ the viscous core size is about 8 in. or 0.8 trailing wing span, but only half that value for $\alpha = 5^\circ$ explaining the better agreement for $\alpha = 5^\circ$. Also, there is apparently a better agreement between measured rolling moments (and the rolling moment calculated assuming a line vortex) above the vortex center than below the center. Careful inspection of Fig.(7-8) shows that the maximum measured induced rolling moment occurs when the trailing wing chord plane is above the vortex center, because of the asymmetry of the vortical flow, and results in the apparent asymmetry in agreement.

The effect of the relative lateral position of the wing and the vortex on the agreement between measured and calculated (using the line vortex model with the adjusted core circulation Γ_c) induced rolling moment is shown in Fig.(7-10). The discrepancies are highest when the vortex center is at the wing center or tip, again because of the unrepresented viscous core. The result of not representing the core is artificially higher induced velocities, and rolling moments, on either side of the vortex center but in opposite directions. If the vortex

FIG.(7-9) INDUCED DOWNWASH ANGLES ON SMALL TRAILING WING
(5b STATION)

FIG.(7-10) COMPARISON OF MEASURED AND CALCULATED ROLLING MOMENT COEFFICIENT.

is far from the wing center the positive and negative induced rolling moments, near the vortex center, tend to cancel. When the vortex center is near the wing center they add up. However, with the vortex center at the wing tip, the wing sees induced velocities in one sense only; overestimated at the tip where the roll arm is largest.

Some of the characteristics of the induced rolling moment measurements noticed in this experiment were also noticed by Smith and Lazzeroni⁽⁴⁸⁾ in their wind tunnel experiment of a rectangular wing in a vortical wake. For example, the asymmetry of the induced rolling moment above and below the vortex center which they also found to be the result of the incomplete roll up of the shear layers. In section (5-1) it was pointed out that there was some dependence of the measured rolling moment on trailing wing angle of attack; this was also noticed by Ref. (48).

Finally, because of the good agreement between the measured rolling moment and the calculated values from the simple vortex model with the adjusted core circulation " Γ_c ", the induced rolling moment (which is a simple single measurement) could be used as an indirect measure of the degree of roll up or for assessing different modifications to the main wing of its tip. Earl⁽³⁷⁾ has used the roll balance and trailing wing mechanism with success to investigate the effect of wing tip shape on the vortex system. No refinement to the theoretical calculation scheme is intended at this stage because the whole idea behind

the present investigation is to examine the feasibility of using a trailing wing to assess, in an overall way, a vortex flow field and to simply study the effect of any modification to it, from the point of view of a trailing aircraft. The wind tunnel simulation misrepresents the real flight situation because of the same constant free stream velocity (cruise speed) over the main wing (leading aircraft) as over the trailing wing (following aircraft). Large transport aircraft, which the main wing represents, usually cruise about 4 times faster than light aircraft, as represented by the small trailing wing. Thus, the downwash angles, see Fig.(7-9) for example, are much higher in the real flight situation due to the lower speed of the trailing aircraft and the induced rolling moments are higher (very roughly as the speed ratio) than those presented in Fig.(7-8).

CHAPTER VIII

EFFECT OF SIMULATED JET ENGINES ON THE VORTEX FLOW FIELD

So far, in this thesis, measurements with the simulated jet engines have not been presented. There are two reasons for delaying the discussion of the simulated engine results to this point. First, the flow field characteristics for the clean wing configuration must be studied so that effects due to engine pylons and nacelles or the simulated jet exhaust can be identified. Secondly, the author knows of no similar measurements, i.e. flow field measurements with simulated jet engines, to compare with the present measurements and in a sense, therefore, these measurements stand by themselves. Chigier and Corsiglia^{(31)*} have reported hot wire measurements behind a semispan model of a CV-990 aircraft. Their model has simulated open engine nacelles, antishock bodies, and flaps; however, it is not possible in their case to identify the effects of the different components on the flow field.

As mentioned in Chapter III and Appendix VI the engines were designed to simulate a Boeing 747 with JT9D engines in the cruise condition (with a mixed jet exit velocity to free stream velocity ratio of 1.5). However, the high pressure air capacity in the Engineering building, Carleton University, was not enough to supply the two engines with air at the maximum tunnel speed of about 175 f.p.s. while maintaining the required velocity ratio (even though tests were performed during off working hours). The maximum tunnel speed at which there was still enough high pressure air capacity to correctly simulate jet exit conditions

was about 115 f.p.s, giving a mean chord Reynolds number of about 0.34×10^6 . To avoid comparing measurements taken at different speeds, and Reynolds number, measurements and comparisons were made as follows:

a) with the two engines mounted but with no air injection, tests were made at maximum tunnel speed and compared to the clean wing configuration results. Effects of engine pylons and nacelles on main wing loading, flow field, and induced rolling moment on the small trailing wing are thus detected.

b) tests for condition (a) above are then repeated at a free stream velocity of about 115 f.p.s (the maximum possible speed for jet exhaust simulation). A reference condition is thus available for studying the effects of the simulated jet exhaust. It is also possible to study possible speed effects by comparing results of (a) and (b).

c) measurements with simulated jet exhaust (cold air simulation only), at a free stream velocity of about 115 f.p.s, are compared to the no injection case; (b), above.

For the measurements reported here the inboard engine was at the spanwise position $2y/b = 0.4$ and the outboard engine was at the $2y/b = 0.66$ position.

8-1 Calibration of the Simulated Jet Engines

Calibration of the simulated engine flow was necessary not only to determine the jet exit velocity but also to insure the uniformity of the flow at the exit plane. The engines have a total pressure probe built in, see Fig.(3-6), but calibration is necessary because of the variation of the base (jet exit) pressure with angle of attack.

To check the flow uniformity at the jet exit plane a total pressure probe (hyperdermic tubing 0.028 in. O.D.) was diametrically scanned laterally and vertically about $\frac{1}{8}$ in. downstream from the exit plane. The total pressure (from the external and internal probes) was measured relative to a working section wall static pressure across the jet exit plane. The total pressure uniformity is expressed as the ratio of the externally measured total pressure to the internal probe measured total pressure, for the wing at zero angle of attack. The four screens (open area ratio 0.54 and pressure loss coefficient $k = 1.6$) originally installed did not remove the non-uniformities in the entering flow, generated mostly at the 90° bend at the exit from the wing feed tubes and in the pylon, and the flow at the exit was of unacceptable quality. By adding three extra-fine mesh screens (100 wires per inch, open area ratio 0.25) the non-uniformity in the total pressure was brought down to less than $\pm 4\%$, Fig.(8-1), which is quite adequate for the present engine simulation.

Calibration of the jet exit velocity was made using $\frac{1}{16}$ in. pitot-static probe. The probe was aligned with the engine axis,

FIG. (8-1) TOTAL PRESSURE UNIFORMITY AT JET EXIT PLANE

for the two angles of attack $\alpha = 5^\circ$ and 11° , with its pitot tip about $\frac{1}{8}$ in. downstream of the exit plane. Measurements were made of $(P_{O_j} - P_{st_{base}})$, $(P_{t_{probe}} - P_{C_2})$ and $(P_{C_1} - P_{C_2})$ at a tunnel speed of 115 f.p.s, (where P_{O_j} and $P_{st_{base}}$ are the total and static pressure, respectively, as measured by the pitot-static probe at the jet exit plane, and $P_{O_{probe}}$ is the total pressure as measured by the engine total pressure probe). Since $P_{O_j} = P_{O_{probe}}$ (from Fig.(8-1) it was possible to calibrate $(P_{st_{base}} - P_{C_2})$ versus $(P_{C_1} - P_{C_2})$ and angle of attack. From the requirement that $V_j = 1.5 U_\infty$, the tunnel calibration parameter K_v , and $(P_{st_{base}} - P_{C_2})$ calibration we can determine the required $(P_{t_{probe}} - P_{C_2})$ for a given $(P_{C_1} - P_{C_2})$.

Table (8-1) shows the variation of $(P_{st_{base}} - P_{C_2})/(P_{C_1} - P_{C_2})$ and $(P_{O_{probe}} - P_{C_2})/(P_{C_1} - P_{C_2})$ with α as well as the wing lower surface pressure coefficient (normalized to $(P_{C_1} - P_{C_2})$) at chordwise positions corresponding to both the inboard and outboard engines exit plane.

$(P_{t_{probe}} - P_{C_2})$ was monitored during the tests (for the outboard engine) and its variation was within $\pm 4\%$, corresponding to variation in V_j/U_∞ of less than $\pm 3\%$.

	Inboard Engine			Outboard Engine		
	$\alpha=0$	$\alpha=5$	$\alpha=11$	$\alpha=0$	$\alpha=5$	$\alpha=11$
$(P_{O_{probe}} - P_{C_2}) / (P_{C_1} - P_{C_2})$	2.52	2.68	2.76	2.47	2.6	2.67
$(P_{st_{base}} - P_{C_2}) / (P_{C_1} - P_{C_2})$	0.12	0.28	0.36	.07	0.20	0.27
$(P_L - P_{C_2}) / (P_{C_1} - P_{C_2})$	-0.19	0.01	0.44	-0.21	0.05	0.35

Table (8-1): Engine Calibration Parameters

8-2 Wing Loading

When considering the effect of the engine pylon and nacelle or air injection on the wing loading, measurements were taken first at pressure plotting stations adjacent to the pylon, then by proceeding inboard and outboard as long as changes in loading were significant. With the two engines mounted but with no injection the effects of the pylon and nacelle on the wing loading were found to be more than local, at least for $\alpha = 5^\circ$. Table (8-2) shows a comparison of local lift coefficient C_L with and without engines mounted. Pylon-nacelle effects are more or less local for $\alpha = 11^\circ$, and the overall lift coefficient shows almost no change. However, for $\alpha = 5^\circ$ the effects are less localized and a general decrease in loading is noticed and results in a reduction of the overall lift coefficient from 0.36 to 0.325, or about 10% (Note that the values for the clean wing

configuration, $\alpha = 5^\circ$, have been interpolated from Fig.(4-10); all other values are measured values). Also included in Table (8-2) are the measured values for the local lift coefficient at stations adjacent to the two engines (5,6; 9,10) at the reduced speed with and without air injection. Considering that at the reduced speed, and dynamic head, the accuracy of measurements decreases and the uncertainties in local C_L 's are thus higher, we can conclude that the air injection did not affect the spanwise loading significantly. Figs. (8-2a to d) show a comparison of the local pressure distribution, with and without injection, for stations 5, 6, 9 and 10; no effects of air injection can be distinguished.

Table (8-2): Effect of Engines and Air Injection on Main Wing Loading

(Local and Overall Lift Coefficient)

Station No.	2y/b	$\alpha = 11^\circ$				$\alpha = 5^\circ$			
		Max. Speed Clean Wing	Reduced Speed With Engine	Reduced Speed With in-jection		Max. Speed Clean Wing	Reduced Speed With Engine	Reduced Speed With in-jection	
1	0.071	0.696	0.708			0.32	0.304		
2	0.143	0.719	0.725			0.33	0.317		
3	0.214	0.749	0.746			0.34	0.323		
4	0.286	0.747	0.765			0.355	0.327		
5	0.357	0.762	0.766	0.766	0.771	0.36	0.336	0.347	0.359
	0.4	"Position of Centerline of Inboard Engine"							
6	0.429	0.773	0.721	0.729	0.698	0.36	0.327	0.344	0.337
7	0.476	0.771	0.774			0.37	0.343		
8	0.571	0.767	0.793			0.37	0.336		
9	0.643	0.765	0.755	0.75	0.744	0.395	0.353	0.364	0.385
	0.66	"Position of Centerline of Outboard Engine"							
10	0.714	0.75	0.725	0.69	0.688	0.41	0.326	0.347	0.326
11	0.786	0.754	0.761			0.40	0.350		
12	0.857	0.732	0.761			0.395	0.348		
13	0.929	0.655	0.658	0.651		0.355	0.315		
14	0.988	0.451	0.471			0.23	0.20		
Over-all		0.734	0.736			0.36	0.325		
						Interpolated Values

Fig. (8-2) PRESSURE DISTRIBUTION WITH SIMULATED ENGINES
(a) STATION 5

Fig. (8-2) PRESSURE DISTRIBUTION WITH SIMULATED ENGINES
(b) STATION 6

Fig. (8-2) PRESSURE DISTRIBUTION WITH SIMULATED ENGINES

(c) STATION 9

Fig. (8-2) PRESSURE DISTRIBUTION WITH SIMULATED ENGINES

(d) STATION 10

The effect of engine mounting on the shape of the spanwise loading shown in Fig.(8-3), compared to the clean wing configuration. For $\alpha = 11^\circ$ there is very little decrease in the normalized spanwise loading between $2y/b = 0.4$ and 0.75 , i.e. close to the engine pylons, with relatively higher loading near the tip. For $\alpha = 5^\circ$, however, the decrease in normalized loading close to the pylons appears as increase in loading near the root.

8-3 Flow Field Measurements

Ideally speaking, to examine the effect of the simulated engines on the flow field we should map the complete cross flow plane and generate, in particular, the total pressure loss contours, similar to Figs.(6-3a to d). These cross flow surveys are valuable because they define the extent of the viscous wake, and the regions of high total pressure (jet exhaust), but are very time consuming to obtain. Single lateral scans through the vortex center were instead made, and from the 5-hole probe measurements the total pressure loss coefficient and the axial and tangential velocity profiles were determined. Figs.(8-4 to 8-6) summarize the flow field results for $\alpha = 11^\circ$, while Figs.(8-7 to 8-9) summarize those for $\alpha = 5^\circ$. The results are presented such that, part (a) of each figure shows the comparison, at the maximum tunnel speed, of the clean wing configuration with that of engines mounted, but no air injection, part (b) of each figure is a comparison with engine mounted, no injection, at the

FIG. (8-3) EFFECT OF ENGINES ON MAIN WING SPANWISE LOADING

maximum tunnel speed and at the reduced speed for jet simulation of about 115 f.p.s., and part (c) of each figure is a comparison with and without injection, at the reduced speed. Parts (a) to (c) are results for the 5b station; the results for the $2\frac{1}{2}b$ station are all presented in part (d) of Figs. (8-4 to 8-9). From the comparison made with engines, but no injection, at the two free stream velocities (b) of Figs. (8-4 to 8-9) it is clear that there are no speed effects, i.e. tangential and axial velocities normalized to U_∞ and total pressure loss normalized to q_∞ are not functions of U_∞ . Hence, measurements with engines, no injection, were not repeated at the $2\frac{1}{2}b$ station for the high tunnel speed.

The total pressure loss coefficient profiles, Figs. (8-4 and 8-7), suggest that the engines and air injection did not affect the rolled up 'core' part of the vortical wake. The size and shape of the region of high total pressure loss did not change appreciably. The maximum total pressure loss coefficient shows some variation, but this could result from missing the vortex center even by a very small distance because of the steep gradient there. The position of the vortex center (where the total pressure is minimum) was found to vary slightly for the different configurations tested and some of the profiles presented in Figs. (8-4 and 8-7) were shifted (maximum of $\frac{1}{4}$ in.) to match the vortex centers and facilitate comparison. The connecting shear layer, which appears as a kink or bump in the total pressure loss coefficient profile,

LATERAL POSITION (IN. OUTBOARD OF W.S. ξ)

FIG. (8-4) TOTAL PRESSURE LOSS COEFFICIENT ($\alpha = 11^\circ$)

FIG.(8-4) CONTINUED

FIG. (8-5) AXIAL VELOCITY PROFILE ($\alpha=11^\circ$)

FIG. (8-6) TANGENTIAL VELOCITY PROFILE ($\alpha=11^\circ$)

FIG. (8-6) CONCLUDED

FIG.(8-7) TOTAL PRESSURE LOSS COEFFICIENT ($\alpha=5^\circ$)

LATERAL POSITION (IN. OUTBOARD OF W.S. C)

FIG. (8-8) AXIAL VELOCITY PROFILE ($\alpha=5^\circ$)

LATERAL POSITION (IN. OUTBOARD OF W.S. ξ)

FIG. (8-9) TANGENTIAL VELOCITY PROFILE ($\alpha=5^{\circ}$)

FIG. (8-9) CONCLUDED

has, however, been affected by mounting the engines and their pylons even without air injection, especially for $\alpha = 11^\circ$, Fig.(8-4a). The pylons are bound to add extra drag and will interrupt the boundary layer on the wing lower surface and these effects are reflected on the viscous wake. The effect of air injection on the connecting shear layer is also more pronounced for $\alpha = 11^\circ$ than for $\alpha = 5^\circ$. At the $2\frac{1}{2}b$ station, for $\alpha = 11^\circ$, and at the 5b station for both $\alpha = 5^\circ$ and 11° , the bump in the total pressure loss profile that marks the spiralling shear layer around the rolled up part has now disappeared, but conclusions about the nature of its disappearance are not possible without detailed flow surveys.

Two more complete total pressure loss surveys with air injection were made at the $2\frac{1}{2}b$ station using a single total pressure probe (stainless steel tube 0.040 in. O.D.). For these supplementary measurements the probe was mounted from the sting of the trailing wing traversing mechanism. The local total pressure was measured relative to the contraction total pressure P_{C_1} for higher sensitivity. A newly available Rosemount differential transducer, serial number 586, with a more limited differential pressure range of ± 0.5 psi, was used for measuring $(P_O - P_{C_1})$. The new transducer has double the sensitivity of the transducers used before, serial numbers 476 and 477, thus compensates somewhat for the loss of sensitivity at the lower

tunnel speed with air injection. The misalignment angle θ (see Appendix III) seldom exceeds 15° and thus an angularity correction is not necessary for the measured total pressure, especially since the results are more or less of a qualitative nature. The total pressure loss coefficient contours generated from the cross flow plane survey, at the $2\frac{1}{2}b$ station, are shown in Figs. (8-10 and 8-11) for $\alpha = 11^\circ$ and 5° respectively, compared to the corresponding clean wing configuration case. The drastic effect that the air injection (and of course the pylon-nacelle effects that cannot be separately identified) has on the still unrolled up shear layer is quite evident. However, there is almost no effect of injection on the concentrated rolled up part of the core flow, as noticed before from the single scans (Figs. (8-4 and 8-7)). Contours of total pressure excess (dotted contours) marking the exhaust high pressure air can still be distinctly seen for both engines. It appears that the high total pressure air from both engines has not mixed significantly with the free shear layers leaving the wing or with each other. The connecting shear layer even though distorted in shape still seems to be of the same intensity and very roughly the same size as of the clean wing configuration case. Because of the relatively higher induced cross flow velocities for $\alpha = 11^\circ$ than for $\alpha = 5^\circ$ the high pressure air exhaust from both engines has obviously drifted more outboard for $\alpha = 11^\circ$ than for $\alpha = 5^\circ$. For $\alpha = 11^\circ$ the high pressure air from the outboard engine is spiralling around the rolled up

2y/b From Wing Root

Fig. (8-10) Contours of equal total pressure loss Coefficient

$$-(P_t - P_{t\infty}) / \frac{1}{2} \rho u_\infty^2$$

2y/b From Wing Root

Fig. (9-11) Contours of equal total pressure loss Coefficient

$$-(P - P_m) / \frac{1}{2} \rho u_\infty^2$$

part: at the $2\frac{1}{2}b$ station it can be seen just outboard of the rolled up part (see Figs(8-10) and (8-4d)). As it moves to the 5b station the high pressure air is not fully entrained into the rolled up part but rather has spiralled and drifted more outboard as can be seen from the total pressure excess region, Fig.(8-4c). The disappearance of the bump outboard of the vortex center for $\alpha = 5^\circ$ at the 5b station suggests that the high pressure air is shifting outboard and spiralling around the rolled up part similar to $\alpha = 11^\circ$ case but at a slower rate. It is hoped (and it is probable) that the presence of the high total pressure air region around the rolled up core will enhance its decay at increasing distances downstream.

The axial velocity profile, Figs.(8-5 and 8-8), although having scatter higher than desirable, show characteristics similar to those noticed from the total pressure loss coefficient profiles. The region of large axial velocity deficit, which corresponds to the core or the rolled up region, is more or less unaffected by the engines mounting or by air injection. Outside this region some effects due to mounting the engines and the air injection can be seen, more noticeably for $\alpha = 11^\circ$. The presence of regions of axial velocity excess outboard of the "core" is particularly interesting. The axial velocity excess is present for the $\alpha = 11^\circ$ case at both the $2\frac{1}{2}b$ and the 5b stations, even without air injection. Air injection expands and intensifies the region of axial velocity excess.

The tangential velocity profiles with engines mounted with and without air injection, compared to the clean wing configuration, are shown in Figs. (8-6a-d) for $\alpha = 11^\circ$ and Figs. (8-9a-d) for $\alpha = 5^\circ$. With engine mounted, no air injection, for the $\alpha = 5^\circ$ case, there is a general decrease in the tangential velocity, corresponding to the decrease in the overall lift coefficient, and is more clear at the 5b station, Fig. (8-9a). At the $2\frac{1}{2}b$ station the decrease in the tangential velocity, for $\alpha = 5^\circ$, is not clear only because the velocity profile is more symmetrical. Inspection of Fig. (8-9d) shows that although inboard of the vortex center the tangential velocity seems to have not changed, outboard of the vortex center the decrease is quite clear. Again for $\alpha = 5^\circ$, there seems to be almost no effect of air injection on the tangential velocity. This is explained, at least at the $2\frac{1}{2}b$ station, from Fig. (8-11); the jet efflux is too far from the lateral plane through the center to affect it. For $\alpha = 11^\circ$, at the $2\frac{1}{2}b$ station, the tangential velocity profile is more symmetrical when engines are mounted (as noted for $\alpha = 5^\circ$); moreover, the tangential velocities near the vortex center are higher than for the clean wing configuration, which is possibly related to the higher loading near the tip and the thinner initial shear layers present. At the 5b station, for $\alpha = 11^\circ$, because of the outboard drift of the spiralling shear layers when engines are mounted (Fig. (8-4a)), the tangential velocity outboard of the vortex center decreased accordingly;

but it is increasing again further outboard as the shear layers are approached again. The effect of air injection for the present engine positions, on the tangential velocity is small (as noted for $\alpha = 5^\circ$); however, at the $2\frac{1}{2}b$ station there is an increase in tangential velocity near the center possibly because of tighter viscous region (see Fig.(8-1c)).

8-4 Induced Rolling Moment on Small Trailing Wing

The effect of engines on the flow field is best described and assessed in terms of its effect on the induced rolling moment on the trailing wing. Measurements of the induced rolling moment on the small wing were made for $\alpha = 5^\circ$ and 11° at both the $2\frac{1}{2}b$ and $5b$ stations with and without injection at the maximum tunnel speed and at the reduced speed for air injection. Fig.(8-12) shows some of the results for $\alpha = 11^\circ$, presented as a vertical scan through the lateral position of maximum induced roll for each case. Also included in Fig.(8-12) is the level of maximum induced rolling moment coefficient for the clean wing configuration, which, it will be recalled, did not vary between the $2\frac{1}{2}b$ and $5b$ stations. Fig.(8-13a) is similar to Fig.(8-12) but for $\alpha = 5^\circ$ and Fig.(8-13b), also for $\alpha = 5^\circ$, shows a comparison of the lateral variation of the induced rolling with and without injection. There is very little effect of air injection on the induced rolling moment, apart from the small variation of the position of maximum induced roll (which is an indication of the vortex center). This last conclusion is in agreement with the flow field results;

FIG. (8-12) INDUCED ROLLING MOMENT COEFFICIENT ON SMALL WING

($\alpha=11^\circ$, WITH ENGINES MOUNTED)

Note: Traces from original x-y plotter output; different scales is the result of different free stream dynamic head.

FIG.(8-13) INDUCED ROLLING MOMENT ON SMALL WING

($\alpha=5^\circ$, WITH ENGINES)

+ See note, FIG.(8-12)

the tangential velocity profiles showed little effect with injection.

For the $\alpha = 5^\circ$ case the maximum induced rolling moment coefficient showed very little change between the $2\frac{1}{2}b$ and $5b$ stations and is about 0.031 with engines mounted ($\bar{C}_L = 0.325$) compared to 0.035 for the clean wing configuration ($\bar{C}_L = 0.36$); i.e. the change is roughly proportional to the change in $\bar{C}_L$ (notice that the spanwise loading has changed but only slightly).

For $\alpha = 11^\circ$ at the upstream station the maximum induced rolling moment coefficient with engines mounted is slightly higher than for the clean wing configuration, reflecting the effect of the higher induced tangential velocities. Between the $2\frac{1}{2}b$ and the $5b$ stations the maximum induced rolling moment coefficient, with engine mounted, has decreased by slightly more than 10%. This decrease in maximum induced rolling moment is the result of the outboard drift of the shear layer, as explained in the previous section. Between the $2\frac{1}{2}b$ and $5b$ stations the rolled up part of the shear layers "the core" naturally diffuses; the connecting shear layer, however, spirals around the core and merges with it. It seems that the effect of the engines "the pylons" is to somehow interrupt or delay the spiralling of the shear layers around the core and in effect retard the roll up. The core for $\alpha = 11^\circ$ is of comparable width to the span of the small trailing wing and any diffusion of the core should be detected by the trailing wing.

The shape of the induced rolling moment coefficient curves below the vortex centerline shows clear variation between the $2\frac{1}{2}b$ and $5b$ station for each angle of attack. This is, of course, due to the effect of the roll up of the connecting shear layers, which do not seem to be rolling as uniformly as was noticed with the clean wing configuration (see Figs.(8-10,-11) for comparison). Also, at the same station the shape of the induced rolling moment curves, again below vortex centerline, is quite different for $\alpha = 5^\circ$ than for $\alpha = 11^\circ$. Inspection of Figs.(8-10,8-11) shows the great difference in the relative position of the connecting shear layer to rolled up "core" region for $\alpha = 5^\circ$ and 11° ; the difference in induced rolling moment coefficient logically follows.

From the flow field results of lateral scans through the vortex center, made with the 5-hole probe, the maximum induced rolling moment coefficient was calculated, as discussed in Chapter VII. A comparison of the measured and calculated values of the maximum induced rolling coefficient is given in Table (8-3) for all cases tested. The agreement, at the very least in the trend, gives confidence in both the trailing wing measurements and the flow field measurements. The two measurements are completely independent of each other and performed at different times and agreement is definitely encouraging.

For $\alpha = 11^\circ$ the measured rolling moment coefficient shows higher values for the high tunnel speed (about 170 f.p.s.) than

for the low tunnel speed (115 f.p.s.), and is consistent for both the $2\frac{1}{2}b$ and $5b$ stations. The tangential velocity profiles, Fig. (8-6b) did not show any speed effect; and consequently the calculated rolling moment coefficient is the same for both speeds. A check was made on the main wing loading at station 13 ($y/b/2 = 0.93$) to see if there is any Reynolds number effect on the loading and the nature of the leading edge separation near the tip, but revealed no change in the loading. At $\alpha = 11^\circ$ the induced downwash angles on the trailing wing exceeds the maximum angle of attack before two dimensional stall over about 10% of the span, see Fig. (7-9). In all probability it is the characteristic and response of the trailing wing that changed with Reynolds number, especially since no similar effects were noticed for $\alpha = 5^\circ$, when the induced downwash angles are relatively small.

Table (8-3): Maximum Induced Rolling Moment Coefficient on Small Wing

			Clean Wing	With Engines Max. Speed	With Engines Reduced Speed	With Air Injection
$2\frac{1}{2}b$ Station	$\alpha = 5^\circ$	Measured	0.034	0.030	0.030	0.031
		Calculated	0.039		0.033	0.034
	$\alpha = 11^\circ$	Measured	0.067	0.069	0.065	0.066
		Calculated	0.074		0.074	0.075
$5b$ Station	$\alpha = 5^\circ$	Measured	0.036	0.031	0.032	0.033
		Calculated	0.038	0.032	0.034	0.034
	$\alpha = 11^\circ$	Measured	0.067	0.062	0.058	0.057
		Calculated	0.078	0.067	0.067	0.063

CHAPTER IX

CONCLUSIONS AND RECOMMENDATION FOR FUTURE WORK

Conclusions

An experimental set-up has been designed and built to study the problem of aircraft trailing vortices. Wind tunnel measurements of the trailing vortex behind a 35° sweptback wing have been made at moderate lift coefficients of 0.36 and 0.74, and at mean chord Reynolds number of about 0.5×10^6 . The measurements include wing loading and detailed flow field measurements, as well as induced rolling moments on two trailing wings. From measurements made at two stations $2\frac{1}{2}$ and 5 span lengths downstream of the main wing mean chord point, the following was concluded:

- 1) For the main wing, a turbulent boundary layer condition existed on upper and lower surfaces, without the use of any tripping device. Progressive "tip stall" developed beyond an incidence angle of about 8° , $\bar{C}_L = 0.6$, but a serious decrease in lift curve slope is not evident until about $\alpha = 11^\circ$, $\bar{C}_L = 0.74$.
- 2) At the $2\frac{1}{2}$ span station it was found that the shear layers, resulting from separation of the turbulent boundary layers on the main wing, had not fully rolled up and consequently the circulation around a circuit near the "core" was considerably below (about 45%) that derived from the corresponding wing span loading. Moving downstream to the

5-span station it was found that no further appreciable roll up of the shear layers occurred when judged by the circulation around the core. This suggests that the shear layers are already effectively fully rolled up at the $2\frac{1}{2}$ span station even though not in an axisymmetric form.

- 3) The vortex system for $\alpha = 5^\circ$, $\bar{C}_L = 0.36$, at the 5b station is believed to be in a similar degree of roll up as for $\alpha = 11^\circ$, $\bar{C}_L = 0.74$, at the $2\frac{1}{2}$ b station. The center of the rolled up part of the shear layer is in the same lateral position for the two systems and the induced tangential velocity profiles normalized to the free stream velocity and main wing $\bar{C}_L$ are in fairly good agreement.
- 4) The maximum tangential velocity, axial velocity deficit, and total pressure loss coefficient decay rates, between the $2\frac{1}{2}$ b and 5b stations, are relatively higher for $\alpha = 5^\circ$ than for $\alpha = 11^\circ$. The difference in decay rates could either be due to the difference in the early history of the separating shear layers, or the flow for the $\alpha = 11^\circ$ case is approaching the recently identified "plateau" region (in which there is little decay of maximum tangential velocity) faster than for $\alpha = 5^\circ$ case.
- 5) There was no measurable difference in the flow field, or trailing wing measurements, between the vortex generated from the main wing with a square cut tip or with a half body

of revolution tip; even though the nature of boundary layer separation near the tip edge appeared to have changed.

- 6) There is no appreciable difference between the induced rolling moment measured at the two downstream stations, which is consistent with the detailed flow field measurements.
- 7) Comparison of measured rolling moments with simple strip theory estimates, based on the measured flow field, and on line vortex model with strength equal to the rolled up core circulation, have been made and show good agreement, especially at the low $\bar{C}_L$ of 0.36, when the viscous core size is still a small fraction of the trailing wing span.
- 8) The induced rolling moment on trailing wings has proved to be a valuable tool in trailing vortex measurements. Not only do such measurements give a practical assessment of the resulting vortex hazard and of gross changes in core structure, but also they give an easy single measurement that can indicate the degree of shear layer roll up.
- 9) Reduction in maximum tangential velocity, or increase in core size as defined by the maximum tangential velocity, cannot be used as a sole measure of reduction in hazard without evidence that the new structure has a faster rate of decay. For example, the maximum tangential velocity for $\alpha = 5^\circ$ decayed from about $0.2 U_\infty$ to $0.12 U_\infty$ between the two measuring stations but the measured induced rolling moment did not show any corresponding change.

- 10) Mounting of two simulated engines on the wing lower surface did not affect the rolled up part of the flow; nor did it affect the loading near the main wing tip. However, the still unrolled connecting shear layer seems to be spiralling around the core at a slower rate and is further away from it compared to the clean wing configuration.
- 11) As a result of mounting the engines, and the interruption it causes to the connecting shear layer, the maximum induced rolling moment on the small wing has decreased by about 10% between the $2\frac{1}{2}b$ and the $5b$ stations, for $\alpha = 11^\circ$.
- * 12) No speed effects have been noticed on the main wing loading or its vortex system. The induced rolling moment coefficient on the small wing, at $\alpha = 11^\circ$, has, however, shown some speed effect (reduction of induced rolling moment coefficient with reducing speed) which is most probably a low Reynolds number effect on the lifting characteristic of the trailing wing.
- 13) Air injection through the simulated engines has resulted in no appreciable change on the induced rolling moment coefficient on the small trailing wing. The injection has resulted in small changes in the core position and in the relative position of the spiralling shear layers to it.
- 14) For $\alpha = 5^\circ$ with engines mounted, there is a decrease in the maximum induced rolling moment coefficient and in the induced tangential velocities as compared to the clean wing configuration. This decrease, however, is the consequence

of the decrease in wing loading, overall lift coefficient, and should stress the importance of measuring the loading especially if a change in wing configuration is planned.

Recommendation for Future Work

The problem of aircraft trailing vortices is still far from being properly understood and research work, both theoretical and experimental, is needed over the different phases of vortex development, from the initial formation until it is practically decayed or disintegrated. The initial formation and roll up of the vortex system, in particular, needs more attention as the description of the further development will hinge on properly representing the rolled up vortex. Almost all methods of vortex roll up calculation are inviscid and assume that all the vorticity shed from the wing trailing edge rolls up finally into a concentric axisymmetric core. The experimental results presented in this report suggest that only about 45% of the vorticity shed from the wing trailing edge, according to high aspect ratio theory, is concentrated into a core (this result was noticed by several other investigators but to a much lesser downstream extent). The shear layers were practically fully rolled up but not in axisymmetric form. Further investigation into the reasons for the incomplete roll up and the effect of the spanwise loading and the boundary layer separation near the tip on the degree of roll up and on the

strength of the tip vortex leaving the wing is recommended. This point is suitable for wind tunnel investigation and can be investigated by changing the tip configuration.

A theoretical model of roll up that will take into consideration the tip vortex generated by separation from the 'streamwise' tip edge, and viscous effects (even in a very crude sense) to allow for the finite thickness of the shear layer is also needed and can best be developed in conjunction with experimental work.

Until the complete fluid dynamics of the trailing vortex problem are much better understood, it is recommended that, when doing experimental work in ground based facilities, the detailed loading and boundary layer characteristics on the generating wing be included in the measurements. Total pressure measurement is also a valuable, and easy to measure, parameter and should be measured to define the full extent of the viscous wake.

To complete and continue the experimental work reported here, the traversing mechanism scanning capabilities should be increased, vertically to reach the bottom of the connecting shear layer, and horizontally to reach the inboard side wall. Also, the installation of a digital data acquisition system is recommended to speed up the data gathering and data reduction process for the flow field measurements. The following points, in order of importance, are recommended to conclude this work:

- 1) Complete the measurements made at the $2\frac{1}{2}b$ and $5b$ stations, for the clean wing configuration, to include all the viscous wake up to the inboard wall (plane of symmetry) to account for all the vorticity shed from the wing trailing edge.
- 2) Measurements close to the main wing, probably first as close behind the trailing edge as possible to follow the roll up from the tip trailing edge. The strength of the tip vortex leaving the tip is particularly of interest.
- 3) Detailed measurements with engines mounted starting close to the wing to determine more conclusively the effect of pylons on the roll up. Measurements at different engine positions, especially for outboard engines, with and without injection, are also recommended.
- 4) At high angle of attack (12.8° for example) extensive leading edge separation is clear; measurements just downstream of the trailing edge of the tip to determine the strength of the tip vortex could be valuable in understanding the relative effect of loading and separation near the tip on the strength of the tip vortex.
- 5) Flow field measurements with different tips (the tips used by D. Earl are probably a good start since loading and trailing wing measurements are available) could also clarify the effect of leading edge separation on the tip vortex.

- 6) The addition of high lift devices to simulate other than cruise condition is also important, especially with simulated jet exhaust. The rolled up vortex will probably be more inboard, say with simulated flaps, and interaction of the jet exhaust with the vortex system will probably be more pronounced.

APPENDIX I

WIND TUNNEL MODIFICATION AND CALIBRATION

Since the first diffuser and the test section, in the 20" x 30" Carleton University low speed continuous closed circuit wind tunnel, were replaced by a long test section, it was expected that the tunnel characteristics and calibration would be changed. A thorough study of the new tunnel characteristics was, therefore, essential. As a result of replacing the first diffuser the circuit losses increased substantially, and the tunnel fan stalled at the original blade setting. Tests were performed at several fan blade settings to determine the maximum possible tunnel speed, with still a margin before stalling, since the tunnel was tested empty. The maximum speed was found to be about 181 f.p.s at blade setting 2[‡] and the centerline streamwise turbulence level $(\bar{u}^2)^{\frac{1}{2}}/U$ was about 0.1% (the maximum tunnel speed with the first diffuser was about 270 f.p.s at blade setting 3).

In an attempt to improve the tunnel performance, a screen was installed after the first corner ($K = 1.6$, open area ratio 0.54), to improve the quality of flow going to the fan. The screen increased the circuit losses, which was clear from the decrease of the maximum tunnel speed at all blade settings tested. Moreover, the screen did not improve the flow quality

[‡] Numbered settings on blade hub and boss. Blade pitch manually adjustable.

in the test section, the turbulence level did not show any measurable change.

It was decided to use blade setting $1\frac{1}{2}$, to avoid stalling the fan after loading the tunnel. The maximum empty tunnel speed at this blade setting is about 176 f.p.s and the centerline streamwise turbulence level $(\bar{u}^2)^{\frac{1}{2}}/U$ is about 0.1%. All results presented in this report were taken with the fan blade setting number of $1\frac{1}{2}$.

The wind tunnel calibration program involved the following:

1. Check and correction of the working section axial pressure gradient.
2. Calibration of the working section centerline flow characteristics as a function of tunnel speed. This includes
 - a) Variation of centerline dynamic pressure with tunnel speed, i.e. q_c as a function of $(P_{C_1} - P_{C_2})$.
 - b) Variation of centerline static pressure with tunnel speed, i.e. $(P_\infty - P_{C_2})$ as a function of $(P_{C_1} - P_{C_2})$.
 - c) Variation of centerline total pressure with tunnel speed $(P_{C_1} - P_0)$ as a function of $(P_{C_1} - P_{C_2})$.
 - d) Variation of the difference between atmospheric (room) pressure and tunnel reference static pressure with tunnel speed, i.e. $(P_{atm} - P_{C_2})$ as a function of $(P_{C_1} - P_{C_2})$.
(This quantity, $P_{atm} - P_{C_2}$, does not need to be known accurately since it is only used for Reynolds number calculations.)

3. Check of the flow uniformity at four axial stations in the working section; this includes:

a) Total pressure flow uniformity, i.e. variation of

$$\frac{P_o - P_{C_1}}{q_c} \quad \text{with vertical and horizontal position.}$$

b) Mean flow angularity; pitch and yaw angles.

4. Effect of the wing root suction scheme on centerline flow characteristics, namely centerline dynamic pressure and static pressure, and on flow uniformity in the working section.

5. Measurement of the longitudinal component of turbulence.

6. Survey of tunnel inside dimension to define tunnel working section centerline, as explained in the main text.

1- Axial Pressure Gradient Along Long Test Section

Trailing vortices are pressure gradient sensitive, hence, since the test section is reasonably long, it was essential to avoid the pressure gradient generated in a constant section as a result of the boundary layer growth along the four walls. Measurements of the displacement thickness development along the original test section were available from previous laboratory results. These measurements were extrapolated to give the displacement thickness development along the three sections of the long test section. Correction for the boundary layer growth was allowed for by diverging the tunnel top and

bottom walls (slope of 0.23°) leaving the two side walls parallel to maintain a reference for the test section. Measurements of the axial pressure gradient along the long test section, using 45 of the static pressure holes available at the tunnel top wall centerline, showed an overcorrection for the boundary layer growth with a resultant positive pressure change of about 5% of the centerline dynamic pressure along the full working section. With wooden corner fillets expanding with downstream distance, the axial pressure change was reduced to an average of about 0.25% overall of the centerline dynamic pressure, as can be seen from Fig.(I-1). This figure shows the axial pressure along the long test section normalized to the centerline dynamic pressure q_c .

2- Calibration of Working Section Centerline Flow Characteristics

Calibration of centerline flow characteristics was achieved through the use of a standard NPL-type pitot-static tube $\frac{1}{4}$ " O.D. The tube was mounted from the bottom of the tunnel with the head coinciding with the tunnel centerline as close as was practically possible, and the tube nose midway along the first section. The contraction pressure difference $(P_{C_1} - P_{C_2})$ was monitored by one of the two Rosemount pressure transducers, namely #477. The other transducer, #476, was used to monitor the other quantities of interest, i.e., $(P_o - P_\infty)$ or q_c , $(P_{C_2} - P_\infty)$ and $(P_{C_1} - P_o)$ each versus

$(P_{C_1} - P_{C_2})$ in separate runs at various fan speeds.

$(P_{C_2} - P_{atm})$ was monitored on the Aerolab multitube manometer.

Fig.(I-2) shows a schematic of the set-up during calibration, while Fig.(I-3) shows the results of the calibration. For values of $(P_{C_1} - P_{C_2}) > 200$ millivolt, which corresponds to $U_G > 100$ f.p.s, these results can be simplified to:

$$K_V \equiv \frac{P_{C_1} - P_{C_2}}{P_O - P_\infty} = 0.938$$

$$K_S \equiv \frac{P_{C_2} - P_\infty}{P_{C_1} - P_{C_2}} = 0.076$$

$$K_O \equiv \frac{P_{C_1} - P_O}{P_{C_1} - P_{C_2}} = 0.0056$$

$$K_a \equiv \frac{P_{C_2} - P_{atm}}{P_{C_1} - P_{C_2}} = 0.02 \text{ (from the multitube manometer readings)}$$

3- Tunnel Flow Uniformity

A blunted conical head 5-hole pressure probe, 0.093" O.D., was used to check the working section flow uniformity. The probe was mounted on an extension arm which in turn was mounted on the vertical slide of the traversing mechanism. With this arrangement, the probe tip was about 5" upstream of most upstream points in the traversing mechanism, Fig.(I-4). Four axial stations were carefully checked, namely halfway along

the first and second test sections and at the two measuring stations $2\frac{1}{2}b$ and $5b$, Fig. (I-2). At each station the centerline of the tunnel was found with the help of the centerline locating device. At least three horizontal traverses and three vertical traverses were made at each station. With measurements taken every inch, at least 180 points were checked at each station. Care was taken to scan, vertically and horizontally, in the same direction always.

The five pressures of the 5-hole probe together with the contraction pressure P_{C_1} were connected to the high pressure sides of the two Rosemount differential pressure transducers, three to each transducer, and could be switched pneumatically one after the other. P_{C_2} was connected to the low pressure side of the two transducers. The six quantities measured ($P_1 - P_{C_2}$, $P_2 - P_{C_2}$ ----- $P_5 - P_{C_2}$ and $P_{C_1} - P_{C_2}$) together with the probe calibration were sufficient to define the quantities of interest. The total pressure nonuniformity can be expressed as

$$\frac{[(P_5 - P_{C_2}) - (P_{C_1} - P_{C_2})]_{C_L} - [(P_5 - P_{C_2}) - (P_{C_1} - P_{C_2})]_{x,y}}{(P_{C_1} - P_{C_2})/K_V},$$

the yaw angle as

$$\frac{[(P_1 - P_{C_2}) - P_5 - P_{C_2}]}{\frac{(P_{C_1} - P_{C_2})}{K_V} \times \delta_Y}, \quad (\text{for small yaw angles} < 2^\circ)$$

and the pitch angle as

$$\frac{[(P_1 - P_{C_2}) - (P_2 - P_{C_2})]}{\frac{(P_{C_1} - P_{C_2})}{K_V} \times S_P} \quad (\text{for small pitch angles } < 2^\circ)$$

where S_y and S_p are the yaw and pitch sensitivity of the probe respectively. For a tunnel speed of about 150 f.p.s., with a system sensitivity of 1 millivolt, the resolution was 0.2% for total pressure nonuniformity and 0.05° for pitch and yaw angles, which is smaller than that obtained from the calibration functions of the probe.

From the measurements at the four axial stations it was found that:

- a) The maximum total pressure nonuniformity, or in other terms, the change in stagnation (total) pressure deviation, i.e. $\frac{\Delta P_0}{\frac{1}{K_V}(P_{C_1} - P_{C_2})}$, over each of the

four cross-sections surveyed is less than 0.5%. (Of course this applies only outside the wall boundary layers).

- b) The maximum mean flow angularity measured, relative to the flow direction at the centerline, is 0.3° in pitch and 0.7° in yaw, at the four stations surveyed. Moreover, the mean flow angularity retained almost the same pattern at the four stations surveyed; this

suggested that the flow angularity is probably the result of some traversing mechanism blockage effect propagating upstream of the mechanism.

- c) There is a static pressure variation of about 5% of the centerline dynamic head. The variation is mainly in the vertical direction (the lateral variation is less than 0.5%), with the static pressure increasing toward the tunnel upper wall. Again this suggests that it is an upstream effect of the traversing mechanism rather than flow nonuniformity in the tunnel. This effect is probably from the vertical traversing mechanism housing which travels laterally with the probe. Fig. (I-4) shows this static pressure variation and the relative position of the probe to the mechanism.

Some measurements were made of the static pressure, relative to the contraction pressure, using the N.P.L.-type pitot tube with the traversing mechanism removed. The measurements are shown in Fig. (I-4) compared to those taken with 5-hole probe, mounted on the traversing mechanism. The vertical variation in the static pressure, measured with the pitot tube is less than 0.5% of the free stream (centerline) dynamic head, compared to 5% measured with the 5-hole probe mounted on the traversing mechanism. Consequently, it is concluded that the flow nonuniformity measured with

the 5-hole probe is mostly a traversing mechanism upstream effect, rather than empty tunnel flow non-uniformities.

To remove any working section flow nonuniformity and traversing mechanism upstream effects, as well as any misalignment of the probe axis with the tunnel axis, empty tunnel, main wing out, measurements were taken at each measuring station and properly allowed for in the flow field measurements.

4- Effect of Wall Suction on Flow Characteristics in the Working Section

The effect of wing root wall suction on flow characteristics was studied at an axial station half-way into the second test section, i.e. about 7 feet downstream of the suction box. At the time of the test the suction box was mounted in its place; however, the wing with its disc was not yet mounted (see Fig. (I-2)).

The effect of the wall suction on the total pressure uniformity and flow angularity was studied, in the same manner as described in the preceding section, at suction box pressure coefficient (defined as $C_{p_s} \equiv (P_{sb} - P_\infty)/q_c$) of - 2.0. The results showed no appreciable change in the total pressure uniformity or in the flow angularity as a result of applying the suction, which means that the wall suction did not disturb the flow uniformity at the station examined. The results also

showed no change in the total pressure as a result of wall suction, which is expected.

To study the effect of suction on the centerline dynamic pressure, and on the tunnel calibration factors K_v and K_s , the NPL-type pitot static tube was mounted half-way into the second test section, with its measuring head coinciding with the tunnel centerline, as accurately as practically possible. The tunnel was operated at different values of fan speed, $(P_{C_1} - P_{C_2})$, and different degrees of suction were applied at each speed. A summary of the results is presented in Fig. (I-5), which shows the variation of the ratio of the dynamic pressures, with and without suction, with the suction pressure coefficient C_{p_s} . Since the total pressure did not change with and without suction, any reduction in the dynamic pressure must result in a corresponding increase in the static pressure. Fig. (I-1) shows the effect of suction on the static pressure along the test sections, at a suction pressure coefficient of -2. The increase in the static pressure of $0.015 q_c$ downstream of the suction box is equal to the decrease in dynamic pressure, at the same suction coefficient. In defining the reference free stream velocity and static pressure at the wing position in subsequent parts of the experiment the average of the values upstream and downstream of the suction box were used.

Fig. (I-1) Axial Pressure Gradient Along Test Section

"Carleton 20' x 30' Wind Tunnel"

FIG. (I-2) CALIBRATION SET-UP

Fig. (I-2) Long Test Section Calibration Parameters
 "Carleton 28'x38' Wind Tunnel"

Fig (I-4) Relative Variation of Static Pressure Coefficient with Vertical Position.

"Halfway Along First Section".

Fig(I-5) Effect of Suction on $(P_0 - P_{\infty})$ "At Mid-Point of Second Section"

APPENDIX II

DESIGN AND CALIBRATION OF ROOT SUCTION SYSTEM

1- Suction System Design

To avoid unrepresentative end effects, for the root mounted wing, which could result from the interaction of the wing pressure field and the side wall boundary layer, distributed suction is applied over an area surrounding the root. The root chord is 9" and the suction area is 14" x 15.5".[#]

Rainbird⁽³⁹⁾ reviewed the criteria used for calculating the suction requirement; he presented them as:

- a) removal of 1% of working section mass flow.
- b) $V_n/U_\infty = 1\%$ over the suction area (empty tunnel) where V_n is the normal component of velocity outside the boundary layer.
- c) removal of 1 to 3 times the approaching turbulent boundary layer deficit mass flow, i.e. up to $3\rho_\infty U_\infty \delta^* h$, where h is the vertical height of the porous area.
- d) removal of 2 to 5 times $\rho_\infty U_\infty \delta^* t$, where t is the maximum aerofoil thickness (for 2D inserts).

Rainbird⁽³⁹⁾ rejects (a) and (b) above because they do not include a parameter that depend on the approaching side-wall boundary layer conditions, say δ^* . Hence, condition (c), i.e. the removal of up to $3\rho_\infty U_\infty \delta^* h$ was used for design purposes.

[#] Limited suction of this kind does not, of course, avoid the downstream interaction at the inner (root) part of the separated shear layer with the working section sidewall boundary layer.

Now δ^* , at the suction box position, is approximately 0.12" (from previous laboratory results), at 175 f.p.s, the maximum tunnel speed, the required volume flow is up to

$$(3 \times 175 \times \frac{0.12}{12} \times \frac{14}{12}) \text{ or } 367 \text{ c.f.m.}$$

To insure that the direction of the suction flow is not to reverse locally then the suction box pressure must always be lower than the lowest pressure in the flow field near the wing root at the highest C_L used. It was estimated at the time of the design that the minimum C_p , at the root, would be about -2. Hence to maintain suction unidirectional the wall pressure drop must be at least (at tunnel maximum speed)

$$\begin{aligned} \Delta P &= P_\infty - P_{sb} = 2 \times \frac{1}{2} \rho_\infty U_\infty^2 \\ &= 2 \times \frac{0.00238}{2} \times 175^2 = 73 \text{ lbf/ft}^2 \end{aligned}$$

$\frac{1}{4}$ " thick grade D Porosint[‡] sintered bronze has a resistance that would give the required flow rate over the suction area (15.5" x 14") at the required pressure loss.

2- Suction System Calibration

The suction system was calibrated, in the wind tunnel, before the main wing was mounted. The objective of the

[‡] "Sintered Products Ltd.", Hamilton Rd., Sutton-in-Ashfield, England.

calibration was to establish the minimum flow rate required to reduce the boundary layer thickness to its minimum value, governed by the sintered bronze surface roughness, while maintaining the pressure loss coefficient across the suction plate at a value less than -2 to insure unidirectional suction (the pressure loss coefficient will be called the suction box pressure coefficient and is defined as $(P_{sb} - P_{\infty}) / \frac{1}{2} \rho_{\infty} U_{\infty}^2$).

Two Preston tubes, one just upstream and the other just downstream of the suction box, together with boundary layer rake downstream of the box, were used for the calibration, see Fig.(I-2). Fig.(II-1) shows the variation of the momentum thickness, the ratio of the downstream Preston tube reading with and without suction, as well as the suction box pressure coefficient with suction flow rate. It is clear from the figure that increasing the flow rate to more than 220 c.f.m. will not decrease the momentum thickness appreciably any more; if it was not for the suction box pressure coefficient the suction flow rate would have been limited to 180 c.f.m. Four Dustbane vacuum cleaners, model PC-1, were required to achieve the required suction box pressure coefficient and flow rate.

Looking back now at the different criteria for calculating the suction requirement, (a) to (c) mentioned before, we find that,

- a) 0.5% of the working section flow rate is removed compared to the recommended 1%.
- b) $\frac{V_n}{U_\infty}$ is 1.5% compared to the recommended 1%.
- c) $1.8(\rho_\infty U_\infty \delta^* h)$, is removed, right in the middle of the, 1-3 times the approaching turbulent boundary layer deficit mass flow, range recommended.

Also checked were the effect of suction on the free stream velocity and on the test section flow uniformity, as mentioned in Appendix I.

Fig (II-1) Suction System Calibration

APPENDIX III

5-HOLE PRESSURE PROBE CALIBRATION

The five-hole pressure probe used in this experimental work, Fig.(3-8), was calibrated in the wind tunnel in the constant area test section No.2. The tunnel has a total pressure uniformity better than 0.5% of the free stream dynamic pressure. The probe was mounted on one inch cubic block which is supported and can rotate on the sting of the overhead tunnel balance, Fig.(III-1). Thus the probe has two degrees of angular freedom, the first is in a vertical plane passing through the tunnel centerline and can be achieved by pitching the balance arm (the arm pitch angle will be referred to as α). The second degree of angular freedom is achieved by rotating (yawing) the block in the pitch plane (this yaw angle will be referred to as γ). The arm pitch angle, α , can be set as close as $\pm 0.1^\circ$; however, the accuracy in setting the block yaw angle γ is about $\pm 0.4^\circ$. The zero pitch and yaw angles are set mechanically by insuring that the probe axis is parallel to the tunnel floor and side wall respectively. The probe was calibrated, in the range $\alpha = 35^\circ$ to -35° and $\gamma = 25^\circ$ to -25° , by fixing γ and changing α through the calibration range. The free stream velocity during the test was about 145 f.p.s; however, tests were made at other speeds and confirmed the finding of Ref.(40) that Reynolds number effects on the calibration are negligible.

The 5 pressures $P_1 + P_2$ and the contraction pressure P_{C_1} were all measured referenced to the contraction pressure P_{C_2} . The dimensionless pressure parameters used in the subsequent presentation of data are defined in the following way:

$$Q = \frac{P_2 - P_1}{P_5 - P_{av}} \quad \text{III-1}$$

$$R = \frac{P_3 - P_4}{P_5 - P_{av}} \quad \text{III-2}$$

$$C_P = \frac{P_{s0} - P_5}{\frac{1}{2} \rho U_\infty^2} \quad (\text{total pressure parameter}) \quad \text{III-3}$$

$$P = \frac{P_5 - P_{av}}{\frac{1}{2} \rho U_\infty^2} \quad (\text{dynamic head parameter}) \quad \text{III-4}$$

where $P_{av} - P_{C_2} = (P_1 + P_2 + P_3 + P_4) - \Delta P_{C_2}/4$ and P_{s0} is the local total pressure.

The misalignment angle, θ , which is defined as the angle the velocity vector makes with the probe axis, and the roll angle, ϕ , the angle the velocity vector and probe axis plane make with a reference plane passing through the probe axis, can be calculated from α , γ (see Fig.(III-1)) using the relations

$$\cos \theta = \cos \alpha \times \cos \gamma \quad \text{III-5}$$

$$\tan \phi = \frac{\sin \gamma \cos \alpha}{\sin \alpha} \quad \text{III-6}$$

The reference plane $\phi = 0$, for our case is the base plane of the probe mount block; this supplies an easy to align plane for setting the probe for measurements after it is calibrated.

Calibration Data Reduction

In reducing the calibration data a technique very similar to that of Wickens⁽⁴¹⁾ was used. The functions $\tan^{-1}(\frac{Q}{R})$ and $\sqrt{Q^2 + R^2}$ were plotted against ϕ and θ respectively; also the total pressure parameter C_p and the dynamic head parameter P were plotted versus θ . These results are shown in Figs.(III-2a-d) for the four quadrants, and indicate a satisfactory collapse with the parameters ϕ and θ , up to θ slightly higher than 30° (half the cone angle). It should be noted that the data for the four quadrants could be reduced or collapsed to one set (after allowance for a zero roll angle) with a small sacrifice in accuracy; however, this simplification was not used in this work for the sake of better accuracy.

A least square technique was used to fit a polynomial for the four different functions. The highest order required, for best fit, was never higher than 4. Figs.(III-2a-d) show the data and the fitted curves. Wickens used a straight line to fit $\tan^{-1}(Q/R)$ vs ϕ , it is clear from his figure A3 that a higher order curve could have represented his data much better and reduced the error considerably.

The use of a blunted conical probe has eliminated the cyclic variation of dynamic pressure parameter "P" with roll angle and "P" was found to be a function of the misalignment angle θ only.

Calculation of Flow Components

A data reduction program was written which uses the calibration data to determine the characteristics of a flow which may have local variation of velocity, direction and total head.

The five measured pressures $P_1 \rightarrow P_5$, referred to P_{C_2} , are used to determine the test values of Q and R from which the angles θ and ϕ can be determined. Having determined θ the value of C_p and P are determined from the calibration functions. The dynamic head and the total pressure can be found from equations (III-3) and (III-4). Non-dimensionalizing to the free stream dynamic pressure q_∞ we get

$$\frac{q}{q_\infty} = \frac{V^2}{U_\infty^2} = (P_5 - P_{av}) \times P / q_\infty \quad \text{III-7}$$

$$(P_{50} - P_{C_2}) / q_\infty = \frac{P_5 - P_{C_2}}{q_\infty} + C_p \times \frac{q}{q_\infty} \quad \text{III-8}$$

The velocity components, non-dimensionalized to the free stream velocity (normal to and along the probe axis - see Fig. (III-1)) are as follows

$$v_x/u_\infty = \frac{V}{U_\infty} \sin\theta \cos\phi \quad \text{III-9}$$

$$v_y/u_\infty = \frac{V}{U_\infty} \sin\theta \sin\phi \quad \text{III-10}$$

$$v_z/u_\infty = \frac{V}{U_\infty} \cos\theta \quad \text{III-11}$$

The signs of $(P_2 - P_1)$ and $(P_4 - P_3)$, i.e. Q and R, are used to determine the appropriate quadrant and to assign the sign of v_x and v_y .

By feeding calibration pressures back into the data reduction program it was possible to assess the overall accuracy. For θ less than 20° the accuracy in determining the normalized velocity was better than 1% with directional accuracy of better than 1° . For $20^\circ < \theta < 30^\circ$ the accuracy drops to 2.5% and 2° angular accuracy respectively.

In measuring the flow field the misalignment angle θ seldom exceeded 15° , consequently, it can be claimed that the accuracy in measuring the flow field velocity is better than 1%; the total pressure, relative to P_{C_1} and normalized to q_∞ , is accurate to within 0.5%.

FIG. (III-1) CALIBRATION SET-UP FOR 5-HOLE PROBE

FIG (x-2 a) 5-HOLE PRESSURE PROBE CALIBRATION

VARIATION OF ROLL ANGLE ϕ WITH $\tan^{-1} Q/R$

FIG(III-2 b) 5 — HOLE PRESSURE PROBE CALIBRATION

VARIATION OF MISALIGNMENT ANGLE θ WITH $\sqrt{Q^2 + R^2}$

FIG (II - 2c) 5 - HOLE PRESSURE PROBE CALIBRATION

VARIATION OF TOTAL PRESSURE FUNCTION C_p WITH θ

FIG (II-2d) 5 - HOLE PRESSURE PROBE CALIBRATION

VARIATION OF DYNAMIC HEAD FUNCTION P' WITH θ

APPENDIX IV

AIRCRAFT WING CONFIGURATIONS

A limited survey of aircraft wing configurations was made in order to select a wing configuration representative of current subsonic transport aircraft. For the benefit of the reader, Table (IV-1) is included, which summarizes the overall or main configuration data of some current large transport aircraft. It is interesting to notice the similarity of wing configuration of aircraft of the same family. Included in Table (IV-1), also, is the selected configuration of the main half-wing and the swept trailing wing.

Table (IV-2) includes the wing configurations of light aircraft as well as the selected configurations of the small trailing wing.

TABLE (IV-1)
TRANSPORT AIRCRAFT WING CONFIGURATION
(Extracted from "Janes" All the World Aircraft)

Aircraft	A	Span	Wing Surface Area	$\frac{1}{4}$ Sweep	Chord	Taper Ratio	Root Chord	Tip Chord	Wing Thickness	Engine Mounting
Lockheed C-5A	7.75	222.7	6200	25°	25°	2.97	45.5	15.33	NACA 0012 Mod NACA 0011 Mod	4-Wing Mounted
Boeing 747	6.96	195.7	5500	37.5°	37.5°	4.0	53.33	13.33	13.44% inboard 7.8% outboard	4-Wing Mounted
Boeing 707-320	7.06	145.75	3010	35°	35°	3.65	33.84	9.25		4-Wing Mounted
Douglas DC-8 Super62	7.32	148.42	2927	30°	30°	4.3	31.7	7.3		4-Wing Mounted
BAC VC10-1100 Series	7.49	146.0	2851	32.5	32.5		33.5		13% inboard 9.75% outboard	4-Rear Fuselage Mounted
Douglas DC-10	6.8	155.33	3550	35°	35°					2-Wing Mounted 1-Tail Mounted
Lockheed L-1011	6.95	155.33	3755	35°	35°	3.34	34.33	10.25		2-Wing Mounted 1-Tail Mounted
Trident 3-B	6.43	98.0	9493	35°	35°		15.2	mean	9.8% mean	3-Rear Fuselage Mounted
Boeing 727-100	7.67	108.0	1700	32°	32°	3.29	25.25	7.67	9% to 13%	3-Rear Fuselage Mounted
Boeing 737	8.83	93.0	980	25°	25°	2.95	15.5	5.25	av 12.85% mean 11.6%	2-Wing Mounted
Douglas DC-9-10	8.25	89.42	934.3	24°	24°					2-Rear Fuselage Mounted
BAC-one-eleven	8.0	88.5	1003	20°	20°	3.11	16.4	5.27	12.5% root 11% tip	2-Rear Fuselage Mounted
Caravelle 11R	8.02	112.5	1579	20°	20°	2.83	20.75	7.33	NACA 65 ₁ 212	2-Rear Fuselage Mounted
Main Wing	7.0	42"		35°	35°	3.0	9"	3"	ONERA Peak 12%	
Swept Trailing Wing	8.5	20"		35°	35°	3.0	3.525"	1.175"	64 ₂ -015	

TABLE (IV-2)
 LIGHT AIRCRAFT WING CONFIGURATIONS
 (Extracted from Aviation Week and Space Technology)

A/C	Section	W lb (max)	b	R or S
Cessna 206 (Stationair)	A2412 (mod.)	3600	35' 10"	7.63
Biai-Marchetti S.208	63 ₃ 618 (root) 63 ₃ 415 (tip)	2976	36' 10"	7.04
Grumman-American AA5 Traveller	64 ₂ 415 modified	2200	31' 6"	7.1
Cessna 175	2412	2450	36' 2"	7.52
Cessna Turbo 310 (twin)	23018 (root) 23009 (tip)	5500	36' 11"	S = 179 ft ²
Beech A60 Duke	23016.5 (root) 23010.5 (tip)	6775	39' 3"	S = 212.9 ft ²
Beech Sierra	63A2415	2750	32' 9"	7.5
Beech E90 King Air	23014.1 (mod.) (root) 23012 (tip)	10,100	50' 6"	S = 293.9 ft ²
Piper PA-39CR Twin Comanche	64 ₂ A215	3600	36'	7.28
de Havilland Canada DHC-6 Twin Otter	NACA 6A Series NACA 0016 (mod.)	7320	65'	10
Small Trailing Wing	64 ₂ 015 NACA		10"	7.5

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APPENDIX V

EVALUATION OF LASER-DOPPLER SYSTEM

"For Use in the 20" x 30" Wind Tunnel to Measure the Trailing Vortex Flow Field"

Since the start of the program, the possibility of using the laser-doppler anemometer for measuring the velocity flow field in the trailing vortex system was left open. The long test section walls are made out of plexiglass to allow using the system at any time. The two major advantages of the laser-doppler system over conventional systems are:

- 1) The system does not interfere with the flow it is measuring, except for seeding, so it is especially useful in flow situations that are sensitive to the presence of a probe (or interfering pressure field), such as trailing vortex flow.
- 2) It is an absolute measuring device, no calibration is needed.

The two major disadvantages of the laser system are:

- 1) Unless a three-dimensional unit is used, extracting all 6 components of the turbulent field is quite a laborious job.
- 2) There is no means of recognizing low total pressure (or vortical) regions, except after analysing the results - even then it might be difficult.

The author, after attending two courses on Laser Anemometry at Imperial College, London, careful study of the literature on

Laser-Doppler anemometer systems, and trying a very simple one dimensional unit, advised against using it. In a report to the Mechanical Engineering Department, Carleton University (in February 1973), the author stated the reasons for his decision as follows:

1. Most of the systems successfully developed are of a bench size and usually the working section is traversed rather than the laser system. There have been some systems developed for wind tunnel, at Arnold (AEDC)⁽⁴²⁾ for example, however, the development of these systems involved a working team and a large amount of money. Also, to the best of the author's knowledge, almost all the systems developed were one or two dimensional units, to get three dimensional measurement is several orders of magnitude more difficult.
2. Signal processing of the laser-doppler receiver output is really tricky, in fact most of the skill in laser-doppler anemometer is in signal processing. In consequence each application is a new application which needs its own development before being reliable. This is the result of the signal being dependent on the scattering particles size and concentration, which is highly variable from one application to another. As a result if a decision is made to use the laser-doppler in the wind tunnel one should expect that most of the time will be directed to the development of the laser system rather than to the aerodynamic problem at hand.

3. Most of the measurements planned (trailing vortex measurement) are new measurements, i.e. there is no available results in the literature to check the measurements against; as a result we should try to give our measurements as much confidence as we can to make it widely acceptable.
4. A laser-doppler system, working in the forward scatter mode, and its traversing mechanism is going to be too bulky to be mounted and dismounted very often. If the wind tunnel is to be used by more than one person, with the consequence of having to exchange the working section, which is certainly the case, using the laser system will make exchanging the working sections quite a laborious job.

In the past two years a few more Laser-Doppler systems have been developed for use in wind tunnels for trailing vortex measurements. Interested readers are particularly referred to the 2-dimensional system developed at NASA Ames Research Center by Grant and Orloff⁽⁴³⁾. To demonstrate the utility of the laser system, detailed measurements of mean velocity distributions have been in the vortices generated by a square-tipped wing mounted in the NASA Ames 7- by 10-foot wind tunnel⁽³⁰⁾. The vortex generated by this wing at 12° angle of attack has previously been studied by Chigier and Corsiglia^(31*,15) and a comparison with the laser velocimeter is given in Ref.(29).

APPENDIX VI ENGINE SIMULATION

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As mentioned in the main text, for engine simulation the B-747 case is simulated. Since a single exit nozzle is used to represent the fan and jet exit, the first step in the simulation will be to calculate an appropriate mixed jet velocity and a representative exit area.

For the Pratt and Whitney Engine JT9D-3A, used with the B-747 aircraft, the following internal pressures and temperatures were obtained* for the sea level take-off thrust of 43,500 lbs.

P_t (psi)	14.7	22.4	20.9
t_t (°F)	59	130	850
V_j (fps)		885	1190
W (lbs/sec)		1249	247

Assuming isentropic expansion through the nozzle, the conditions at the nozzle exit and the nozzle area can be calculated. The fan exit area can also be calculated. The calculations give:

$$\text{Nozzle area } A_j = 6.3 \text{ ft}^2$$

$$\text{Fan area } A_f = 18.8 \text{ ft}^2$$

* PNA OI 200 "The Aircraft Gas Turbine Engine and its Operation", published by Pratt and Whitney Aircraft, Aug. 1970.

To calculate the reference, single, exit area, two assumptions are made:

1) The momentum must be conserved, i.e. the momentum of the mixed stream must be equal to the summation of the momenta of the two individual streams.

$$W_m \times V_m = W_j V_j + W_F V_F$$

$$(1249 + 247)V_m = 247 \times 1190 + 1249 \times 885$$

$$V_m = 936 \text{ ft/sec}$$

2) The density of the mixed stream is taken as the weighted mixed density

$$\bar{\rho}_m = \frac{\rho_F \times W_F + \rho_j W_j}{W_F + W_j}$$

after substituting we get

$$\bar{\rho}_m = 0.068 \text{ lbs/ft}^3$$

and the reference or single mixed area is

$$A_m = \frac{W_m}{\bar{\rho}_m V_m} = \frac{1496}{0.068 \times 936} = 23.5 \text{ ft}^2$$

The basic assumption made is that the mixed area A_m remains constant, i.e. does not change with operating conditions. This assumption should be reasonable, particularly for high bypass engines, since A_m is dominated by fan exit conditions.

For cruise conditions, say at 35,000 ft. alt.

$$M = 0.80 \quad T_\infty = -65.8^\circ\text{F} \quad a(\text{speed of sound}) = 1000 \text{ ft/sec}$$

$$U_\infty = 800 \text{ ft/sec.}$$

The net thrust developed per engine (from manufacturer's data) is 9964 lb_f and the total air flow is 645 lb/sec.

The thrust can be expressed as

$$\text{Thrust} = \frac{W_m (V_m - V_\infty)}{g} + \text{pressure thrust}$$

Allowing 15% as pressure thrust, and substituting, we get

$$V_m = 1220 \text{ ft/sec}$$

$$\text{or} \quad V_m/U_\infty = 1.525$$

From V_m , A_m and the total air flow $\bar{\rho}_m$ is calculated

$$\bar{\rho}_m = \frac{W}{V_m A_m} = 0.02244 \text{ lbs/ft}^3$$

$$\text{at 35,000 ft. alt. } \rho_\infty = 0.0228 \text{ lbs/ft}^3$$

Hence simulating $\bar{\rho}_m V_m^2 / \rho_\infty V_\infty^2$ is equivalent to simulating

$$\frac{V_m}{V_\infty}$$

Model Engine Sizing

$$\frac{V_m}{U_\infty} = 1.525, \text{ for cruise.}$$

Tunnel maximum speed (during tests) = 170 ft/sec

$$V_m = 1.525 \times 170 = 260 \text{ ft/sec}$$

If the mixed area is scaled down directly then

$$A_{m_{\text{model}}} = A_{m_{\text{full scale}}} \times \left(\frac{1}{\text{length scale}} \right)^2$$

$$A_{m_{\text{model}}} = 23.5 \times \left(\frac{42}{196 \times 12} \right)^2 \text{ ft}^2$$

$$= 1.08 \text{ sq.in.}$$

and the exit diameter would be 1.175 in.

Now to simulate thrust = drag, the cruise C_L is about 0.36 and assuming lift/drag ratios of 16, then

$$C_D = 0.0225$$

$$T = D$$

$$2 \times A_m \times \rho_\infty V_m (V_m - 0) = C_D \times \frac{1}{2} \rho U_\infty^2 \times S$$

$$2 \times A_m \times \rho_\infty U_\infty^2 \times 1.525^2 = C_D \times \frac{1}{2} \rho U_\infty^2 \times S$$

$$A_m = C_D \times \frac{S}{4 \times (1.525)^2}$$

$$= 0.305 \text{ sq.in.}$$

$$d_m = 0.62 \text{ in.}$$

about half the size required for simulating the mixed jet efflux.

If the thrust is calculated neglecting the fact that there is no flow through the nacelle, then

$$2 \times A_m \times \rho_\infty V_m (V_m - U_\infty) = C_D \times \frac{1}{2} \rho U_\infty^2 \times S$$

$$A_m = \frac{C_D \times S}{4 \times 1.525 \times 0.525}$$

$$= 0.885 \text{ sq.in.}$$

$$d_N = 1.06 \text{ in.}$$

just 10% smaller than the size of the jet efflux calculated above (1.175 in). Because of the small difference, the latter size of $d_N = 1.06$ in. was used for designing the model engine.

The volume flow rate through the simulated engine will be

$$Q = \frac{260 \times 0.885}{144} = 1.6 \text{ c.f.s}$$

$$= 96 \text{ c.f.m}$$

$$\text{Mach No. at exit} = \frac{260}{1160} = 0.236$$

$$\text{Hence } \frac{P_o}{P_\infty} = \frac{1}{0.962} = 1.038$$

$$\text{or } P_o = 1.038 P_\infty$$

By monitoring the total pressure before the nozzle exit, a measure of the exit velocity is available (for air at "room" temperature).

APPENDIX VII

ERROR ANALYSIS

The basic instruments used in the measurements and their accuracy are:

- 2 Rosemount Transducers - linearity $< 0.1\%$ (for output less than 600 M.V.)
- 2 hp x-y plotters - accuracy $\pm 0.2\%$ of full scale
- 2 DISA digital DVM - accuracy $\pm 0.1\%$ of full scale ± 1 digit
- 2 Weston DVM - accuracy $\pm 0.1\%$ of the reading ± 1 digit
- 1 Aero-lab multitube manometer $\pm 0.02"$ of liq.
- 1 Philips strain measuring bridge - linearity $\pm 1\%$ of full scale

Most of the experiments were made at tunnel maximum speed and measurements are usually normalized to the working section free stream dynamic head " q_∞ " or free stream velocity " U_∞ ".

A max. tunnel speed $U_\infty \approx 180$ f.p.s.

$q_\infty \approx 0.25$ psi

≈ 9 in. manometer liq.

≈ 600 m.volt (output from #477 Rosemount transducer)

and the uncertainty in q_∞ , which is also the uncertainty in the free stream total pressure relative to P_{C_2} " $P_{O_\infty} - P_{C_2}$ ", is

$< \pm 0.4\%$ when measured by the DISA DVM or the x-y plotter.

$< \pm 0.2\%$ when measured by the multitube manometer.

At the reduced tunnel speed of 115 f.p.s. (maximum speed for proper simulation of jet exhaust) the uncertainty in q_∞ increases to $\pm 1\%$ when measured by the DISA DVM and to $\pm 0.5\%$ when measured by the multitube manometer.

Wing Loading:

The pressure coefficient is defined as

$$C_p = \frac{P - P_{st}}{q_\infty}$$

From the tunnel calibration we have

$$(P_{st} - P_{C_2}) = K_S (P_{C_1} - P_{C_2}) = 0.076 (P_{C_1} - P_{C_2})$$

or

$$P_{st} = P_{C_2} + 0.076 (P_{C_1} - P_{C_2})$$

Small error in $(P_{C_1} - P_{C_2})$ will not have appreciable effects on P_{st} and the accuracy of P_{st} is determined by the accuracy of measuring P_{C_2} .

Now, $(P - P_{C_2})$ can be measured, on the multitube manometer, to within ± 0.02 in. liquid, or about ± 0.002 of q_∞ ($q_\infty \approx 10$. in liquid). The uncertainty in measuring q_∞ , on the multitube manometer, is about $\pm 0.2\%$; consequently, the maximum uncertainty in C_p is ± 0.004 . The local lift coefficient, calculated from the integration of pressure coefficient, will have uncertainty less than the ± 0.004 uncertainty in measuring the pressure coefficient, as error will tend to cancel when integrating. However, the uncertainty in local lift coefficient will still be higher than the uncertainty in measuring q_∞ or ± 0.002 ,

plus the uncertainty in setting the angle of attack. The angle of attack can be set to within $\pm 0.1^\circ$; for lift curve slope of about 0.07 per degree the resulting uncertainty is ± 0.007 . The total uncertainty in local lift coefficient is then about ± 0.01 . The uncertainty in the overall lift coefficient is liable to be less than the uncertainty in local lift coefficient, 0.01, but still higher than the uncertainty due to angle of attack setting, 0.007.

At the reduced speed the uncertainty in lift coefficient (local or overall) as the result of uncertainty in angle of attack setting is still within ± 0.007 . The uncertainty in q_∞ is, however, increased to $\pm 0.5\%$ and the total uncertainty in lift coefficient is now within ± 0.015 .

Induced Rolling Moment Coefficient:

The uncertainty in the rolling moment coefficient measurement includes:

- a) Phillip strain measuring bridge linearity of $\pm 1\%$ of full scale.

The analogue output of the bridge for any scale is 0-1 volt.

The output from the bridge was usually around 600 m.v. at the maximum induced rolling moment; hence the accuracy of the output is better than $\pm 2\%$ of the maximum induced value (even for the reduced tunnel speed the output is still around 600 m.v. since a more sensitive scale could be used).

- b) DISA DVM or hp x-y plotter reading accuracy. Bridge output of about 600 m.v. can be measured by the DVM or the plotter to within ± 2 m.v. (on the 0-1V scale) or $\pm 0.4\%$.
- c) q_∞ uncertainty or $\pm 0.4\%$ for full speed and $\pm 1\%$ for the reduced speed.

The estimated accuracy of the rolling moment coefficient measurement is then about $\pm 3\%$ of the maximum induced rolling moment, at the maximum tunnel speed, and about 3.5% at the reduced tunnel speed.

Flow Field Measurement:

In measuring the flow field the misalignment angle seldom exceeded 15° , consequently, the accuracy in measuring the misalignment angle θ is better than 1° and the normalized velocity V/U_∞ is better than $\pm 1\%$.

$$\text{Now } \frac{V_z}{U_\infty} = \frac{V}{U_\infty} \cos \theta$$

The uncertainty in V_z/U_∞ (normalized axial velocity) is then the uncertainty in V/U_∞ or about $\pm 1\%$ because the uncertainty in $\cos \theta$ is less than $\pm 0.3\%$, even for θ as large as 15° .

$$\text{Also } \frac{V_x}{U_\infty} = \frac{V}{U_\infty} \sin \theta \cos \phi$$

$$\text{and } \frac{V_y}{U_\infty} = \frac{V}{U_\infty} \sin \theta \sin \phi$$

For scans through the vortex center, the tangential velocity component is actually V_x for lateral scans and V_y for vertical scans. The roll angle ϕ is very close to 90° for vertical scans

and to 0° for lateral scans, because the radial velocity component is very small, hence

$$\frac{V_\theta}{U_\infty} \approx \frac{V}{U_\infty} \sin \theta$$

The maximum uncertainty in $\frac{V_\theta}{U_\infty}$ occurs when θ is at its maximum value, because the probe accuracy decreases with increasing the misalignment angle. The maximum value for θ is about 15° and corresponds to the point where V_θ is maximum for main wing $\alpha = 11^\circ$. The maximum uncertainty in $\frac{V_\theta}{U_\infty}$ is then about ± 0.015 , corresponding to the 1° uncertainty in θ . Far from the vortex center θ is small, typically 4° , and the uncertainty in θ is less than ± 0.02 (see Fig. (III-2b)); corresponding to ± 0.004 . Another source of error in the measurements is due to the main wing out correction. For practical considerations main wing out corrections ($\Delta V/U_\infty$, $\Delta V_y/U_\infty$, $\Delta V_x/U_\infty$, $\Delta \alpha$, $\Delta \gamma$) were curve fitted for data reduction. The fitting is accurate to better than 0.003 for velocity and 0.1° for angles. The uncertainty in the normalized tangential velocity far from the vortex center is then about 0.008.

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