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MEASUREMENT, CHARACTERIZATION AND MODELLING
OF
800/900 MHZ MOBILE RADIO CHANNELS
FOR DIGITAL COMMUNICATIONS

by

ROBERT JAMES CUTHBERT BULTITUDE

A thesis submitted to the faculty of Graduate Studies
and Research in partial fulfillment of the requirements
for the degree of

Doctor of Philosophy

Department of Electronics
Faculty of Engineering
Carleton University
Ottawa, Canada
February 1987
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ABSTRACT

This thesis is concerned with the characterization and modelling of 800/900 MHz mobile radio channels for digital communications. Based on a preliminary set of measurements which were performed as part of the thesis proposal, a mathematical model is developed into which measured mobile radio channel characteristics may be substituted, leading to the prediction of error rates on a DPSK system operating with different transmission bandwidths. A system for the measurement of radio channel impulse response estimates, developed as part of the thesis is described. Experimental data recorded during impulse response estimate measurements in downtown Ottawa are presented. These data are used in the model to predict system performance. Results obtained using novel techniques which extend prediction capabilities to the frequency selective case are then compared with well-known flat fading predictions for narrow transmission bandwidths. The excellent correspondence is taken as confirmation of the newly developed techniques.

Analysis results show that urban mobile radio channels with high powered centrally located base stations having elevated antennas can be modelled as quasi-wide-sense stationary channels with almost uncorrelated scattering. Results also show that frequency-selective fading takes place for transmission bandwidths above about 38 kHz, after which error rates reach an irreducible level as a function of receive-signal to noise ratio unless equalization techniques are implemented.

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GLOSSARY OF SYMBOLS AND ABBREVIATIONS

XVI

AGC	automatic gain control
$a(t)$	arbitrary signal
a, a	polynomial coefficient, index
α	attenuation factor
$\alpha(t)$	time-variant attenuation factor
B, B, B_N	bandwidth, noise bandwidth
BW	bandwidth
B_c	channel correlation (or coherence) bandwidth
b, b	polynomial coefficient, index
β	normalized received signal envelope
γ	signal to noise ratio on a fading channel
$\gamma(\Delta f)$	real part of the channel spaced frequency correlation function
CDF	cumulative distribution function
CRC	Communications Research Centre
CW	continuous wave
c, c	polynomial coefficient; index; subscript-carrier
DPSK	differential phase shift keying
dB	decibel
d, d	subscript; index
$\delta()$	impulse function
Δ	denotes incremental shift, as in Δt or Δf
E	energy per bit
$E\{ \}$	expected value
FET	field effect transistor
F_{eq}	equivalent system noise figure
f_b	PRBS bit rate
GMSK	Gaussian minimum shift keying
$g(), G()$	arbitrary functions
$h(\tau)$	radio channel impulse response
$h(\tau, t)$	time-variant radio channel impulse response
$H(f)$	radio channel transfer function
$H(f; t)$	time-variant radio channel transfer function
$H_N(f)$	noise filter transfer function

$H_R(f)$	receive filter transfer function
$H_T(f)$	transmit filter transfer function
I	representation for the in-phase channel
I_0	Bessel function of the 1st kind and order zero
I_i	the i th integral
IF	intermediate frequency
ISI	inter-symbol interference
j	the square root of -1
K	Boltzman's constant
θ	phase angle
M	radio channel covariance matrix
MIP	multipath intensity profile
m_i	element of M - moments of q , the decision variable
N, N	noise power
N_0	two-sided noise power spectral density
ν	Doppler frequency
$P(\cdot)$	cumulative distribution function
P_e	probability of error
P_e^0	probability of error under nonfading conditions
$p(\cdot)$	probability density function
$p_{ab c}$	probability of error conditioned on the sequence "abc"
PRBS	pseudo-random binary sequence
PSK	phase shift keying
Q	Hermitian matrix for the decision variable; quadrature channel representation
QWSS	quasi-wide-sense stationary
q	decision variable
RF	radio frequency
R_i	the i th residue
R_H	channel transfer function correlation function
R_h	channel impulse response correlation function
$R(t)$	bandpass representation for the received signal
$r(t)$	complex envelope low pass equivalent received signal
S	signal power
S_M	multipath spread
S_D	Doppler spread
S/N	signal-to-noise ratio

$S_H(\nu)$	Doppler power spectrum	xviii
$S(r; \nu)$	channel scattering function	
$S(t)$	bandpass representation for the transmitted signal	
s	seconds	
$S_n(f)$	noise power spectral density	
$s(t)$	complex envelope low pass equivalent for the transmitted signal	
$\sigma(\Delta f)$	imaginary part of the spaced frequency correlation function	
T	transmitted symbol duration	
T_b	scaled bit period	
T_C	channel correlation (or coherence) time	
T_{ant}	antenna brightness temperature	
T_{eff}	effective system noise temperature	
t_0	sampling instant	
τ	time delay	
U	delayed received signal at the sampling instant	
u	dummy variable	
V	undelayed received signal at the sampling instant	
v_{th}	decision threshold	
W	transmitted symbol bandwidth; Watt	
WSS	wide-sense stationary	
WSSUS	wide-sense stationary uncorrelated scattering	
$\Psi(\cdot)$	characteristic function	
ψ	phase angle	
ω	$2\pi f$	
$x(t)$	input signal	
$y(t)$	output signal	
$z(t)$	complex envelope low pass representation for white noise at the receiver input	

NOTES

- (1) **Bold Face** letters represent bandpass signals
- (2) Upper case letters denote frequency domain representations
- (3) Lower case letters denote time domain representations.

CHAPTER I

INTRODUCTION

1.1 Introduction

As a result of overcrowding in the VHF and 450 MHz mobile radio communications bands, regulatory authorities world wide have recently allocated blocks of spectrum in the 800/900 MHz band for mobile radio service. Since the present trend in mobile radio communications is advancing towards ~~digital~~ digital transmission, it is anticipated that digital modulation techniques will be common in this new band. It is therefore desirable that 800/900 MHz mobile radio propagation channels be fully characterized for digital communications. This requirement for characterization can be seen from two perspectives.

From the perspective of regulatory authorities, a knowledge of channel characteristics is necessary so that frequency allocations, channel bandwidths, and channel spacings can be set down in an optimal fashion. Without such knowledge, channelling schemes currently being used in the lower frequency bands for voice communications will most likely be applied in the newly allocated band, resulting in possible inefficient use of the available spectrum. This is especially true since it is anticipated that initially, digital communications will be implemented by digital modulation of audio tones before FM modulation using existing technology from VHF and 450 MHz equipment.

The second perspective from which it is perceived that a complete channel characterization is required is that of the digital equipment and mobile radio system design engineer. Mobile radio channels are randomly dispersive in both time and frequency. Therefore, when the duration of a transmitted symbol is anywhere near the [1] fading correlation time for the channel, an increase in the probability of error (P_e) will result due to time-selective fading caused by Doppler effects as a result of vehicular motion. Similarly, if the bandwidth of a transmitted symbol is close to the coherence bandwidth of the channel, an increase in P_e results from frequency-selective fading due to multipath propagation. On such channels therefore, there exists an optimum combination of digital symbol rate and symbol shape that can be chosen to minimize P_e . Even if the time-bandwidth product of transmitted symbols is chosen so as to make the channel nonselective, flat fading causes an effective signal to noise power ratio (S/N) degradation which must be accounted for in system design. Allowance for such phenomena can only be made if a model is available which accurately describes the expected probability distribution of the envelope of signals received over a mobile radio link, and the amount of dispersion that can be expected.

From both perspectives, it is clear that accurate characterization and modelling of

800/900 MHz mobile radio channels is a necessity. The required information for this characterization includes: a knowledge of the statistics for signal fading, of the dispersive nature of the medium, and of the additive noise that can be expected in a mobile radio environment. This thesis project is concerned with measurement, characterization and modelling of the fading and dispersive properties of the 800/900 MHz mobile radio environment. The models selected as appropriate are used to predict the performance of proposed 800/900 MHz digital mobile radio systems.

1.2 Background

Investigations in the past that were concerned with mobile radio communications fall into five broad categories. These include: received signal envelope fading measurements, mobile channel impulse response measurements, channel modelling, digital communications system performance predictions, and digital system performance measurements. This section gives an overview of some of the work that has been carried out in each of these categories. References are given so that more detailed information can be obtained from the literature.

1.2.1 Envelope fading measurements

It appears that the earliest work toward characterization of the 800/900 MHz portion of the spectrum for mobile radio service was carried out by Young [2], in 1952. This work involved the experimental determination of the envelope fading distribution for a CW signal received at a mobile unit (10 kHz BW) driven through urban and suburban sections of New York City. Transmission was from a fixed base station at an elevated downtown location.

Results from the experiment were reported to indicate that whenever measurements were confined to a relatively short distance (150-300 m) the fading distribution tended strongly to follow a Rayleigh curve. The seven points plotted on the curve presented in the referenced paper, however, show that agreement with the Rayleigh curve is only good for signal amplitudes between 6 dB above, and 15 dB below the median. The goodness of fit deteriorates for weak signal levels, where, from a systems design perspective, accuracy is most important.

Similar measurements were made by Jakes and Reudink [3] in a suburban area near Murray Hill, New Jersey. Their results show that the fading distribution follows a Rayleigh

law for signal levels in the vicinity of the median and deviations increase towards both ends of the fading range. CW fading measurements were also made by Rustako [4], Okamura et al. [5] and Shepherd [6]. All results are compared with the Rayleigh distribution, and in most cases show substantial deviations for weak signals, and for very strong signals.

In addition to the rapid fading over short distances investigated in the previously referenced papers, it has been found that there is a slower component of variation in the mean signal level which can be measured over longer distances and wider areas. Mobile measurements of this fading component in the 800/900 MHz frequency band have been made by Black and Reudink [7] and by Okumura et al. [5]. As in the lower frequency mobile radio bands, results show that slow fading of the mean signal level is lognormally distributed. This description of slow fading statistics is generally accepted in mobile radio work.

1.2.2 Impulse response measurements

In 1950, a simple experiment to investigate time dispersion on a 450 MHz mobile radio channel was reported by Young and Lacy [8]. RF pulses $0.5 \mu s$ long were transmitted from the top of a tall building in downtown New York City. Echo patterns received and displayed on an oscilloscope in a car moving between 15 and 25 km/hr were automatically photographed in a motion picture sequence and later viewed to permit interpretation and limited statistical analyses. Results show that excess delays (propagation time in excess of that for the line-of-sight path) on the order of $8-10 \mu s$ are common. Analysis of actual propagation dynamics, however, was not possible due to the poor time resolution of the system and the lack of phase information.

The most complete equivalent low pass impulse response measurements on mobile channels were reported by Cox [9, 10, 11, 12, 13, 14, 15] in the early 1970's. In these experiments a 10 Mb/s, 511 bit pseudo-random binary sequence was phase reversal modulated onto a 70 MHz RF carrier, then upconverted to 910 MHz for transmission through an amplifier and antenna system having a 10 MHz bandwidth. Reception at a mobile unit was through a vertical monopole and 10 MHz bandwidth receiver with an IF output at 70 MHz. The IF output was split into quadrature channels and each channel was applied to a sliding correlator. It is readily shown (Chapter IV) that the outputs from such correlators are good estimates of the in-phase and quadrature components of the complex envelope equivalent low pass representation for the impulse response of the propagation channel.

In the referenced papers, Cox analyzed data measured in both urban and suburban areas of the United States. Results included: the presentation of multipath intensity profiles (Chapter II); the determination of typical values of excess delays (5 to 7 μ s for suburban and 9 to 10 μ s for urban environments); calculation of statistical multipath spreads; calculation of cumulative distributions for the impulse response envelope at fixed delays on consecutive impulse response estimates; computation of Doppler spreads; presentation of channel scattering functions; analysis of bandwidths at which frequency correlation function magnitudes dropped by 10 and 50 percent (correlation bandwidths), calculation of excess delay distributions; and calculation of multipath spread/path loss correlations. Although a large amount of information on channel characteristics was made available by this work, no material was published concerning the development of a channel model based on the measurement results. It was shown, however, that for both urban and suburban environments, there are some areas in which it is reasonable to describe mobile channels in accordance with Bello's Gaussian wide-sense stationary uncorrelated scattering model [16]. Only one recent publication [17] by Glance and Greenstein bases portions of digital system performance calculations on results reported by Cox.

The response of an urban radio channel at 488, 1280, and 2920 megahertz to 100 ns pulse transmissions was measured and analyzed by Turin et al. [18, 19]. These experiments were in support of work on an automatic vehicle monitoring system rather than for digital communications purposes. Stationary measurements were therefore adequate and no determination of frequency dispersion was made. An urban propagation model was developed for the vehicle monitoring application.

Hubbard et al. [20] made wideband impulse response measurements using equipment similar to that used by Cox. Their pseudo-noise probing signal was centred at 8.5 GHz and transmission was from a fixed base station to a mobile receiver. Measurements were made only on a limited scale in Boulder, Co. to demonstrate the measurement system. Correlation bandwidths for 50 percent magnitude decorrelation computed for a few urban measurements were reported to be on the order of 500 kHz.

Nielson [21] used a wideband pseudo-noise system with a centre frequency of 1370 MHz to measure impulse response estimates for mobile channels in downtown San Francisco and in surrounding urban, suburban, and rural areas. Narrowband measurements were also made to facilitate investigations of time variability, and power of the received

signal. Analyses of measured results included: determination of amplitude-delay statistics for the identifiable paths from recorded impulse response estimates; computation of maximum delay widths at various received power thresholds; path number statistics, inter-arrival time statistics; multipath spread statistics; cursory investigation of the effects of moving reflectors; and the computation of received signal power spectral densities as well as spatial, amplitude, and phase correlations from the narrowband (CW transmission) measurements. No attempts at channel modelling or performance predictions were reported.

Bajwa and Parsons [22] reported the results of a channel measurement program in the U.K. which duplicated the work of Cox, but at a frequency of 436 MHz. An example suggesting the type of performance predictions that could be made based on the measurement results was given. Actual use of their measured data to make such predictions however, was not reported.

1.2.3 Channel modelling

Aside from rough comparisons between the Rayleigh distribution and envelope fading distributions derived from mobile measurements, the first attempt to actually model mobile channel characteristics was reported by Ossanna [23] in 1964. A rough physical model of the urban environment based upon the geometry of reflections from plane vertical reflectors was developed. Then by consideration of fading produced by vehicular motion through the standing wave pattern created by interaction between a single dominant reflection and the incident wave, a theoretical fading spectrum for the received signal was calculated. Comparison was made with the results from mobile CW measurements in New Providence, New Jersey at 838 MHz. Agreement between experimental and theoretical spectra was found to be quite good, except for a measured high energy content at low frequencies that was absent from theoretical results.

In 1965, Gilbert [24] made a theoretical investigation of energy reception (E and H field combined) in mobile radio. A model was developed for the superposition of vertically polarized plane waves near a vertical reflecting surface. This study was made in order to compare received signal dynamics for total energy reception against signal fading statistics at the output of a receiving system which responds only to the electric field. Energy density distribution functions, spatial field correlations, and energy density power spectra at a moving point were computed. No comparisons were made with measured data.

The first completely statistical study of mobile radio reception was reported by Clarke [25] in 1968. The statistical characteristics of the fields and signals at a moving vehicle were deduced from a scattering propagation model. Amplitude and phase distributions and spatial correlations were calculated. Whenever possible, theoretical predictions were compared with the measurement results of others. There was considered to be sufficient agreement as a result of these comparisons to indicate the viability of the modelling approach, and agreement was reported to improve when allowances were made for the nonstationary character of mobile radio signals.

Schmid [26] developed a simple semi-empirical model which permits the probability of occurrence of multipath propagation over irregular terrain to be predicted. Direct and multipath received powers were assumed to have Gaussian probability distributions. Satisfactory agreement with measured data is reported, and results indicate that for a fixed transmitter-receiver separation, multipath component amplitudes decrease with increasing excess delay; and for constant excess delay values, the occurrence of multipath propagation increases as the transmitter-receiver separation is increased.

In a 1972 publication [27] Gans used geometrical considerations to derive the statistical properties of mobile radio propagation. The power spectrum of the channel transfer function was derived and then used to determine amplitude and phase probability distributions, field correlations versus time and space at both the mobile and the base station, level crossing rates, fade durations, and the power spectrum of random FM. The application of derived statistical properties in the design of standard diversity systems was demonstrated. No experimental measurements were made, and no attempt was made to compare theoretical results with experimental results reported by others.

*Based on dispersion experiments at 488, 1280, and 2920 MHz, Turin et al. [18, 19] modified a basic "path summation" model to include Gaussian small sector, and lognormal wide area fading distributions, as well as a modified Poisson path arrival time distribution. Simulations using the derived model were reported to be able to accurately reproduce the measured behavior of an experimental automatic vehicle monitoring system.

Suzuki [28] analyzed path strength statistics by attempting to fit Rayleigh, lognormal, Nakagami (Chapter II) and Rice distributions to Turin's experimental path strength data. Further modifications were also made to Turin's arrival time distribution. Results

were reported to show that the distribution of received signal amplitudes tends to be Nakagamian for the earliest received signal components and changes to lognormal as the excess delay becomes large. Satisfactory comparison with measured results was reported for the modified Poisson arrival time model.

1.2.4 Performance analysis

An overview of the effects of Rayleigh fading on voice and data transmission (binary FSK) was reported by Arretlondo [29] et al. in 1977. This work is based on consideration of the envelope of a fading received signal at 850 MHz. With reference to digital communications, bit and word error rate and error burst density are reported to be determined by an error rate of 0.5 during fades into noise. Error burst frequency and duration are cited as being wholly dependent upon fade interval and fade duration. Possible improvements as a result of diversity, bit inhibition, and message interlacing are discussed.

Adachi et al. [30] analyzed the performance of a Manchester coded FSK system in experiments using a Rayleigh fading simulator. They noted that in the non-quasistationary case (when symbol durations are close to the channel correlation time due to higher Doppler spreads) the increased fading rate can be interpreted as a reduction in the fading range (sharper probability density function for E/N_0) of the Rayleigh model used in the fading simulator. This results due to the fact that even though the fading rate is higher, the probability that the signal is high rather than faded is greater, leading to a lower probability of error. This can also be interpreted as a sort of inherent time diversity, where integration of the received signal is over time periods when the fading is partially decorrelated. It was also pointed out, however, that this reduction in error rate due to faster fading is bounded by the increase in random FM noise at very high fading rates, which, in turn, increases P_e . To avoid this increase in random FM, but still obtain the same sharpening of the E/N_0 probability density function (pdf), a switched antenna diversity technique was proposed. No measurements were made to verify the predicted error-rate improvement.

French [31] investigated the performance of a 1.8 kb/s direct digital FSK system operating in a mobile environment at 162 MHz. Back-to-back receiver to transmitter bench tests were made in which error performance as a function of signal to noise ratio was measured. An exponential relationship was then fit to the measured data. This

represented actual equipment (rather than theoretical) performance under nonfading conditions. Then consecutive averaging of the empirical relationship over Rayleigh envelope fading, lognormal mean signal variations and measured ignition noise distributions were carried out by means of numerical integration. Comparison of performance predictions with digital system performance measurements made in the urban sections of London, England are generally good. Another publication by French [32] regarding digital mobile radio-operation at 462 MHz predicts the existence of a critical bit rate below which data can be transmitted error free, and above which the probability of error is high and roughly independent of bit rate. This critical bit rate is reported to be dependent upon carrier frequency, vehicle speed and mean signal level. French also [33] investigated error distributions and block error rates with the objective of providing a basis for the selection of error correcting codes

Charash [34] modelled multipath signals as being comprised of "L" resolvable fading paths, with the k th path associated with signal strength α_k , modulation delay τ_k , and phase shift θ_k , all assumed to be constant over a single symbol duration. The joint probability distribution of these three random variables was found, assuming

- a) θ_k is uniform on $[0, 2\pi]$,
- b) τ_k is zero outside some interval $[A, B]$, and
- c) α_k has a Nakagami probability distribution

Although originally α_k was assumed to be dependent upon k , this dependence was dropped in order to simplify the mathematics required for performance analyses. Based upon this channel model, a likelihood equation was derived for optimum Bayes detection for the case of fixed multipath delays, and for the case of random delays. Difficulty, however, was reported in finding a decision rule for general Nakagami amplitude fading, and a Rayleigh approximation was used as an alternative. Separate decision rules and error probability equations were then derived for the fixed and random delay cases. Results show predicted probability of error sensitivity to:

- a) the number of received multipath components,
- b) the amount of fading (defined in [34]),
- c) the multipath spread

Generally, for fixed delays, P_e was found to decrease as the number of radio paths increased, and increase as the amount of fading increases, with greater sensitivity to the amount of fading at high S/N. It was predicted that S/N degradation due to multipath propagation would increase by 1 to 1.5 dB for every doubling of the multipath spread in the case of random delays.

Glance and Greenstein [17] analyzed the problem of frequency-selective fading in a cellular mobile radio system. The modulation was assumed to be PSK, with cosine roll-off transmitted symbol spectral shaping, and space diversity with maximal ratio combining. Signal transmission was assumed to suffer from random frequency-selective Rayleigh multipath fading and lognormal shadow fading. Cochannel interference was considered and was also assumed to be influenced by fading. Monte Carlo simulations were employed to obtain average bit error rate distributions for a variety of assumed channel conditions. Results include evaluation of the influence of multipath spread, impulse response shape, symbol spectrum shape, diversity combiner imperfections, the order of diversity, and modulation type (2 or 4 PSK) on system performance. These results show that degradation due to multipath propagation is small so long as the multipath spread is less than about 0.2 times the transmitted symbol period. For greater multipath spreads, performance degradation increases rapidly in a manner that depends upon the shape of the transmitted spectrum. Quadrature PSK is reported to result in better performance than binary PSK when the ratio of multipath spread to the equivalent 2-PSK symbol duration exceeds the order of 0.3.

1.2.5 Performance measurements

Daikoku et al [35] have measured the performance of a 920 MHz mobile system using GMSK modulation in suburban areas of Tokyo and Yokosuka, Japan. Their work is concerned with probability of error measurements from a bench system with simulated Rayleigh fading for coherent and differential detection. Comparisons were made with error rate measurements from field trials at different vehicle speeds. Good comparison is shown between simulation and experimental results. An increase in irreducible error rate is shown as the vehicle speed increases. The differential and coherent detection schemes appear to have resulted in equivalent performance.

French [31, 32, 33] also made digital system performance measurements at 462 MHz. This work has already been discussed in Section 1.2.3.

1.3 Objectives

It is the objective of this thesis to make small sector channel characterization measurements and predict performance for a binary DPSK digital mobile radio communication system operating in the 800/900 MHz frequency band. Theoretical analysis of system performance using tractable mathematical models incorporating measured channel characteristics is conducted. Comparison of accepted flat fading performance curves with predictions made using new results of this work is used to verify techniques. System performance predictions for operation under frequency-selective fading conditions, made possible for the first time through the work of this thesis, are then presented. Ultimately, it should be possible that simulations using the methods developed as part of this thesis project can be used to predict system performance over complete digital mobile radio coverage areas.

1.4 Organization

Chapter II is concerned with the characterization and classification of fading, dispersive radio communication channels. It is in this chapter that explanation and definition of the channel characteristics discussed in Section 1.2 can be found. It is shown how the channel characteristics are interrelated and used for channel classification. This chapter provides a basis for determination of the necessary measurements and type of performance analysis to be made under varied channel conditions.

Chapter III discusses one of the major contributions of the thesis. After an introduction, methods for calculating error probabilities for binary DPSK on flat fading channels and frequency-selective channels are discussed. Individual sections of this chapter specifically include descriptions of the methods by which measured channel characteristics can be included in mathematical analyses in order to predict digital system performance.

Chapter IV describes the equipment and instrumentation that was used for the experiments. The pseudo-noise probing system is described, and problems that were encountered during implementation of specific parts of the system are discussed. A section is also included concerning the capabilities of the measurement system with regard to range resolution, unambiguous range, and various sampling modes that are available to the user.

Measurements, data analyses and results are discussed in Chapter V. First, the transmitter installation, transmit and receive locations and field measurement procedures are described. Then, the channel classification and analyses procedures developed in Chapters II and III are combined with the results of channel characterization measurements and used to classify and model an 800/900 MHz urban mobile radio channel. Next, data analyses techniques are outlined, and the results of different steps in analyses leading to classification of the measured channel are presented. After classification of the channel, digital system performance computations are outlined, and performance predictions are presented. A discussion of results is included at the end of the chapter.

A summary, discussion, and conclusions of the thesis, as well as a section concerning topics for further investigation can be found in Chapter VI.

CHAPTER II

CHARACTERIZATION AND CLASSIFICATION OF FADING, DISPERSIVE, DIGITAL RADIO COMMUNICATION CHANNELS

2.0 Introduction

Fading dispersive radio propagation channels are often modelled as random, time-variant linear filters, the input and output ports of which are effectively the transmit and receive antennas of the communication system under investigation. The statistical properties of the complex envelope low pass equivalent impulse response for the channel are described by a number of different time and frequency correlation functions that are used as descriptors for channel classification.

In this chapter, representations [36] for the channel impulse response and correlation functions are reviewed. Interrelationships among these channel descriptors are explained and widely used fading channel classifications are defined. Probability distributions appropriate to fading mobile radio channels are also reviewed in the last section of the chapter.

2.1 Complex envelope low pass representation of bandpass signals

In this Chapter, and in Chapter III, complex envelope low pass representation of bandpass signals is employed. The purpose of this section is to attempt to prevent misinterpretations that can occur in dealing with this representation. Such misinterpretations are easily made, especially in the calculation of powers.

Consider the bandpass signal,

$$A(t) = |a(t)| \cos[\omega_c t + \theta(t)] \quad (2-1)$$

Now define

$$a(t) = |a(t)| e^{j\theta(t)}$$

After expansion of the cosine term in eqn(2-1), one can write

$$A(t) = |a(t)| \cos \theta(t) \cos \omega_c t - |a(t)| \sin \theta(t) \sin \omega_c t$$

It is easy to show from this equation that

$$A(t) = \text{Re}\{a(t)e^{j\omega_c t}\}. \quad (2-2)$$

The signal $a(t)$ is referred to as the complex envelope low pass equivalent for the bandpass signal $A(t)$. The term "complex envelope" is used because $a(t)$ has the conventional envelope of $A(t)$, but also has a phase $\theta(t)$ (with respect to the carrier frequency) associated with it. Hence, it is a complex envelope.

It is noteworthy that the power of $a(t)$ is twice that of $A(t)$, as shown in Appendix A-1. In addition, it should be noted that time-frequency domain relationships for low pass equivalents are the same as those for their corresponding bandpass signals.

2.2 Channel characterization

2.2.1 Complex envelope equivalent low pass representation of the impulse response of a radio channel

Let the bandpass signal transmitted from a hypothetical radio transmitter [36] be represented as

$$S(t) = \text{Re}\{s(t)e^{j\omega_c t}\}, \quad (2-3)$$

where,

$s(t) = |s(t)|e^{j\theta(t)}$ is the complex envelope low pass equivalent for $S(t)$.

Assume that the propagation environment is such that a continuum of replicas of the transmitted signal arrive at the receiver with random delays and amplitudes, both of which are functions of time. Then the received bandpass signal can be written as

$$R(t) = \int_{-\infty}^{\infty} \alpha(r,t) S(t-r) dr, \quad (2-4)$$

where, α represents the attenuation of the signal components at delay, r , and time, t .

By substitution for $S(t)$ from equation (2-3), and rearranging terms, $R(t)$ can be written as

$$R(t) = \text{Re} \left\{ \int_{-\infty}^{\infty} \alpha(\tau; t) s(t - \tau) e^{-j\omega_c \tau} d\tau \times e^{j\omega_c t} \right\}. \quad (2-5)$$

Now, since the integral in this equation represents the complex envelope low pass equivalent for the received signal $r(t)$, which is from theory, the convolution of $s(t)$ with the low pass equivalent of the channel impulse response, it is clear that

$$h(\tau; t) = \alpha(\tau; t) e^{-j\omega_c \tau}, \quad (2-6)$$

is the complex-envelope low pass equivalent for the impulse response of the channel. This function represents the response of the channel at time t to an impulse applied at time $t - \tau$.

2.2.2 The autocovariance of the complex low pass envelope equivalent channel impulse response

If $h(\tau; t)$ is assumed to be wide-sense stationary (WSS) its autocovariance function can be written as

$$R_h(\tau_i, \tau_j; t_i, t_j) = R_h(\tau_i, \tau_j; \Delta t) = \frac{1}{2} E \{ h^*(\tau_i; t) h(\tau_j; t + \Delta t) \}, \quad (2-7)$$

where the factor of one half is a result of the complex envelope notation.

Further, if the attenuation and phase shift of the propagation path associated with delay "i" is uncorrelated with those parameters associated with the path having delay "j", the channel is referred to as a wide-sense stationary uncorrelated scattering (WSSUS) channel. For uncorrelated scattering at τ_i and τ_j ,

$$R_h(\tau_i, \tau_j; \Delta t) = 0, i \neq j,$$

and

$$R_h(\tau; \Delta t) = \frac{1}{2} E \{ h^*(\tau; t) h(\tau; t + \Delta t) \}. \quad (2-8)$$

Then the correlation among simultaneous variations of the channel impulse response at different delay times is given by eqn(2-8) with $\Delta t = 0$, as

$$R_A(\tau) = \frac{1}{2} E\{|h(\tau; t)|^2\}. \quad (2-9)$$

$R_A(\tau)$ is referred to by various names, including:

- (a) the power delay profile
- (b) the multipath intensity profile
- (c) the delay-power spectrum
- or, (d) the power impulse response

for the channel. In this work $R_A(\tau)$ will be referred to as the multipath intensity profile (MIP). The range of over which $R_A(\tau)$ is nonzero is called the multipath spread (S_M) of the channel.

2.2.3 The autocovariance of the complex envelope low pass equivalent channel transfer function

The equivalent low pass time-variant transfer function of a random time varying propagation channel is given by

$$H(f; t) = \int_{-\infty}^{\infty} h(\tau; t) e^{-j2\pi f\tau} d\tau, \quad (2-10a)$$

or, by substitution from eqn(2-6) for $h(\tau; t)$,

$$H(f; t) = \int_{-\infty}^{\infty} \alpha(\tau; t) e^{-j2\pi f_s \tau} e^{-j2\pi f \tau} d\tau. \quad (2-10b)$$

If $h(\tau; t)$, and hence, $H(f; t)$, is random, the complex autocorrelation of $H(f; t)$ can be written as

$$R_H(f_i, f_j; t_i, t_j) = \frac{1}{2} E\{H^*(f_i; t_i) H(f_j; t_j)\}.$$

Then, if the WSS property of $h(r; t)$ is carried through the Fourier transform, R_H is independent of t . Further, if there is uncorrelated scattering [36,37], the function is independent of f ; and one can write

$$R_H(\Delta f; \Delta t) = \frac{1}{2} E\{H^*(f; t) H(f + \Delta f; t + \Delta t)\}. \quad (2-11a)$$

It is readily shown [36] that this function can also be written as

$$R_H(\Delta f; \Delta t) = \int_{-\infty}^{\infty} R_A(\tau; \Delta t) e^{-j2\pi \Delta f \tau} d\tau. \quad (2-11b)$$

If Δt in eqn(2-11a) is set equal to zero, the autocorrelation function for simultaneous variations in the channel transfer function at different frequencies is given by

$$R_H(\Delta f; 0) = \frac{1}{2} E\{H^*(f) H(f + \Delta f)\}, \quad (2-12a)$$

or equivalently,

$$R_H(\Delta f) = \int_{-\infty}^{\infty} R_A(\tau) e^{-j2\pi \Delta f \tau} d\tau. \quad (2-12b)$$

This function is called the spaced frequency, or two frequency correlation function. As a result of the Fourier Transform in equation (2-12b), $R_H(\Delta f)$ vanishes at $\Delta f = 1/S_M$. This frequency separation is called the coherence bandwidth, B_C , of the channel.

It is useful to note that if there are correlations among scattered signals, the dependence upon f results in asymmetries in $R_H(\Delta f; 0)$. These asymmetries can readily be seen in the results of computations using the methods outlined in Bultitude [37]. Inspection of the computed functions can therefore be used as an indicator of the degree to which there is correlation among multipath components.

2.2.4 Characterization of channel time variations

The time variations of a radio propagation channel result in Doppler broadening and perhaps a Doppler shift of spectral components transmitted over the channel.

In order to relate the Doppler effects on the transmitted signal to time variations on the channel, it is convenient to employ the Fourier transform of $R_H(\Delta f, \Delta t)$ with respect to Δt , given by

$$S_H(\Delta f, \nu) = \int_{-\infty}^{\infty} R_H(\Delta f, \Delta t) e^{-j2\pi\nu\Delta t} d\Delta t, \quad (2-13)$$

where ν represents Doppler frequency.

For a single spectral position in the channel transfer function, $\Delta f = 0$, and one can write

$$S_H(0, \nu) = \int_{-\infty}^{\infty} R_H(0, \Delta t) e^{-j2\pi\nu\Delta t} d\Delta t \quad (2-14)$$

By definition, $S_H(\nu)$ is a power spectrum, and for a single spectral component, gives the received signal intensity as a function of Doppler frequency. $S_H(\nu)$ is referred to as the Doppler spectrum of the channel at a particular frequency, and the range over which it is nonzero is called the Doppler spread (S_D), or fading bandwidth of the channel. $S_H(\nu)$ characterizes the rapidity of signal fading.

The function $R_H(0, \Delta t)$ in eqn(2-14) can be obtained from eqn(2-11a) if Δf is set equal to zero, and is given by

$$R_H(0, \Delta t) = \frac{1}{2} E \{ H^*(t) H(t + \Delta t) \}. \quad (2-15)$$

This function is called the spaced time correlation function. The range, Δt over which it is nonzero is referred to as the correlation time, T_C , of the channel. Through the Fourier transform relationship in eqn(2-14), it is clear that T_C is the reciprocal of S_D .

2.2.5 The channel scattering function

For a WSSUS channel, $R_A(\tau, \Delta t)$ can be related to $S_H(\Delta f, \nu)$ by two Fourier transforms, as follows.

Define a new function

$$S(\tau, \nu) = \int_{-\infty}^{\infty} R_h(\tau; \Delta t) e^{-j2\pi\nu\Delta t} d\Delta t. \quad (2-16)$$

If a Fourier transform is now taken with respect to τ , one can write

$$\begin{aligned} F\{S(\tau, \nu)\}_\tau &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_h(\tau; \Delta t) e^{-j2\pi\nu\Delta t} d\Delta t e^{-j2\pi\Delta f\tau} d\tau \\ &= \int_{-\infty}^{\infty} R_H(\Delta f, \Delta t) e^{-j2\pi\nu\Delta t} d\Delta t \\ &= S_H(\Delta f, \nu). \end{aligned}$$

This implies that

$$S(\tau, \nu) = \int_{-\infty}^{\infty} S_H(\Delta f, \nu) e^{j2\pi\Delta f\tau} d\Delta f \quad (2-17)$$

Thus, from eqn(2-13),

$$S(\tau, \nu) = \int_{-\infty}^{\infty} R_H(\Delta f; \Delta t) e^{-j2\pi\nu\Delta t} e^{j2\pi\Delta f\tau} d\Delta\tau d\Delta f \quad (2-18)$$

$S(\tau, \nu)$ is referred to as the scattering function for the channel. This function provides a measure of the average power of the random processes which perturb the channel as a function of multipath delay (τ) and Doppler frequency (ν).

2.3 Classification of fading channels

If receiver front end noise is neglected, the equivalent low pass signal at the input to the receiver after transmission over a fading channel can be represented [36] as

$$r'(t) = \int_{-\infty}^{\infty} H(f, t) S(f) e^{j2\pi f t} df, \quad (2-19)$$

where the prime indicates that the receiver noise has been neglected, and $S(f)$ is the spectrum of the low pass modulating signal $s(t)$.

If $s(t)$ is a series of pulses which modulate f_c at a rate $1/T$, two types of distortion can be introduced by the random transfer function of the channel, $H(f, t)$. These include.

- (a) Frequency selectivity, which subjects different spectral components of $S(f)$ to different gains and phase shifts,

and,

- (b) Variations in received signal strength caused by time variations of $H(f, t)$

On mobile radio channels, frequency selectivity is a result of multipath propagation. Time variability results from the combined effects of changes in shadow loss (due to diffraction by natural or man made features on the earth's surface), and Doppler phenomena due to vehicular motion. Frequency-selective distortion is dependent upon S_M and B_C . Time variability is a function of S_D and T_C .

The influence of a propagation channel on a digital signal transmitted over it is a function of the relationship between the time-bandwidth product of transmitted symbols, and the correlation properties of the channel. For example, if the symbol duration (T) is selected such that T is significantly larger than S_M , negligible inter-symbol interference will be introduced by the channel.

Classification and modelling of fading radio channels is in accordance with their effect on transmitted signals. The following sections of this chapter outline commonly used fading channel classifications. These classifications apply to digital channels, the statistics of which are nonstationary, but which may be regarded as wide-sense stationary (WSS) for short time and frequency intervals, which are greater than the durations, and bandwidths respectively of transmitted symbols. Because of their quasi-stationary nature, these channels are referred to as quasi-wide-sense stationary (QWSS) channels. Quantitatively, for a QWSS channel [1]:

$$W \ll \frac{1}{\delta_{max}} \quad \text{and} \quad (T + S_M) \ll \frac{1}{\sigma_{max}}$$

where, W and T represent the transmission bandwidth and symbol duration respectively, and δ_{max} and σ_{max} are the maximum rates at which $R_H(\Delta f; \Delta t)$ fluctuates in the f and t domains. These criteria almost certainly apply for digital land mobile radio communications, as will be verified in Chapter V through the examination of channel characterization measurement results.

2.3.1 Flat/flat fading channels

A fading radio channel is referred to [1] as a flat/flat fading channel, or slowly varying, nonselective channel [36] if the bandwidth of transmitted symbols is much less than B_C , and the duration of transmitted symbols is much less than T_C .

If the flat-fading criteria are satisfied, all spectral components of the transmitted signal fade in unison. Therefore, within the bandwidth of $S(f)$, the fading statistics of the channel are independent of frequency, and if $s(t)$ is assumed to be narrowband (compared with f_c), $H(f; t)$ can be assumed to be constant over the bandwidth W . Under these conditions, the channel transfer function can be removed from the integral in eqn(2-19). Additionally, since $s(t)$ is a low pass signal, $H(f; t)$ can be assumed to be constant at its value for $f = 0$, and, on completion of the integration, eqn(2-19) can be rewritten as

$$r'(t) = H(0; t)s(t)$$

Then, by substitution in eqn(2-10b) to obtain $H(0; t)$, one can write

$$r'(t) = \int_{-\infty}^{\infty} \alpha(\tau; t) e^{-j2\pi f_c \tau} d\tau \times s(t). \quad (2-20)$$

If the transmitted symbol characteristics are such that $WT = 1$, in the frequency nonselective case, $T \gg S_M$ and received multipath signal components are nonresolvable. Then, since for a practical receiver, all the multipath components are received within one resolvable unit of τ , the integral in eqn(2-20) disappears, and the phase term becomes time dependent due to phase cancellation of multipath components with delay separations within the resolution time of the receiver, giving

$$r'(t) = \alpha(t) e^{-j\theta(t)} s(t). \quad (2-21)$$

If now the low pass envelope equivalent noise contribution $z(t)$ from the receiver front end is added, the low pass equivalent for the total signal at the receiver input can be written

$$r(t) = \alpha(t)e^{-j\phi(t)}s(t) + z(t).$$

A further characteristic of a flat-fading channel is that the amplitude and phase of the received signal are constant over at least one symbol interval. For the consideration of probability of error per bit, the time dependence can therefore be dropped from the channel impulse response representation, and over any given symbol interval,

$$r(t) = \alpha e^{-j\phi} s(t) + z(t). \quad (2-22)$$

For any particular digital modulation scheme standard derivations for error probability, P_e , on a nonfading channel with constant gain, α , exist throughout the literature [36, 38]. For applicability in the slowly fading nonselective environment under consideration, where α varies in a slow random fashion over two or three symbol intervals, a simple extension [32, 36] to the resulting relationship for P_e can be made. This is done by averaging P_e over the probability density function for the variations in S/N resulting from the dynamic behavior of α . In general, expressions for P_e on a nonfading system take the form

$$P_e^0(\Gamma) = KG(\Gamma), \quad (2-23)$$

where,

K is a constant,

$G(\Gamma)$ represents a function which is peculiar to the modulation scheme,

and,

$$\Gamma = \alpha^2 E/N_0.$$

$P_e^0(\Gamma)$ can be viewed as a conditional probability, conditioned on α being fixed. To obtain the probability of error when α is random, averaging over the pdf for Γ results in the equation

$$P_e = \int_0^{\infty} P_e^0(\Gamma) p_r(\Gamma) d\Gamma \quad (2-24)$$

For all but the most well known distributions for α , a closed form expression for $p_r(\Gamma)$ is difficult to obtain, and eqn(2-24) must be evaluated numerically.

2.3.2 Time-selective/frequency-flat fading channels

This classification is given to digital radio channels which carry digital signals for which the duration of transmitted symbols is greater than T_C , but symbol bandwidths are much smaller than B_C . Under these conditions, the amplitude and phase relationships of the received signal can be expected to change over a single symbol duration. Inter-symbol interference, however, can be expected to be negligible.

Since this type of channel is frequency nonselective, the relationship for the complex low pass envelope of the received signal given by equation (2-22) remains valid. For cases where fading is described by non-Gaussian statistics, the mathematical relationships involved in calculations for probability of error appear to be intractable. However, if $h(\tau; t)$ can be assumed to be a complex narrowband Gaussian process [39] error probability equations can be derived. For most of the common modulation types, the derivation follows the general routine of first reducing the expression for the signal at the input to the decision circuitry of the receiver to a Hermitian quadratic form [39, 40]. The pdf for this signal can then be derived using the channel and transmitted signal correlation functions, and a decision rule can be established. For time-selective/frequency-flat channels, it can be shown [39] that for rectangular symbols, the increase in S/N required for a desired P_e as a result of fading is a function of the ratio of T to T_C .

2.3.3 Frequency-selective/time-flat fading channels

When the bandwidth of transmitted symbols is greater than B_C , but the total duration of all symbols which cause inter-symbol interference is less than T_C , the channel is called a frequency-selective/time-flat channel.

For this type of channel propagation statistics are independent of time over at least one symbol duration. For the consideration of error probabilities per symbol, the time dependence in the channel transfer function can therefore be dropped and one can write

$$r(t) = \int_{-\infty}^{\infty} H(f)S(f)e^{j2\pi ft}df + z(t), \quad (2-25)$$

where again, the mathematics become intractable unless $H(f)$ is assumed to be a complex narrowband Gaussian process. Under this assumption, the pdf of the quadratic form at the input to the decision device in the receiver can be derived [41] and a decision rule can be formulated. It can be shown [41] that for this type of channel, and rectangular data symbols, S/N degradation due to fading is a function of the ratio of symbol bandwidth to B_C .

2.3.4 Time-selective/frequency-selective fading channels

If the duration of transmitted symbols is greater than T_C , and in addition, the bandwidth of transmitted symbols is greater than B_C , the channel is said to be simultaneously time, and frequency selective. If fading on the channel is characterized by Gaussian statistics, calculations of error probability can be made in a similar manner [1] to those calculations for a time-flat/frequency-selective channel. In this case, however, the moments required for derivation of the pdf for the signal at the decision circuit input involve the channel scattering function, and a cross ambiguity function between the received signal and mark and space waveforms.

It has been shown [1] that for doubly selective channels there is an optimum symbol rate at which P_e is minimum. This symbol rate is dependent upon the spread factor ($S_M \times S_D$) of the channel. Minimum P_e occurs when the normalized symbol rate,

$$R_s = \frac{1}{2T(S_D/S_M)^{1/2}} \quad (2-26)$$

is slightly greater than unity. If the time-bandwidth product of transmitted symbols is not too large and B_C is several orders of magnitude greater than S_D , the minimum is very broad. Over the range of symbol rates spanned by the minimum P_e , the channel can be modelled as a flat/flat fading channel. For lower symbol rates the channel will behave as if it were purely time-selective, and for higher symbol rates it behaves as if it were purely frequency-selective.

2.4 Statistical distributions appropriate to multipath fading on radio channels

Statistical models for multipath fading are generally developed using one of two different approaches. Often, a simplified physical model of the propagation environment is described and equations for the statistics are developed, based upon the assumptions of the physical model. For verification, results are then compared with results derived through the analysis of measured data. The second approach involves empirically fitting known probability distributions (often derived for other applications) to those computed from measured data.

If the first type of model is considered for use, care must be taken to ensure that the physical assumptions upon which the model is based are not seriously violated in the environment of proposed operation. If the empirical type of model is to be employed, the use of extrapolations to predict signal behaviour outside the region of operation for which the original data were measured must be closely monitored. Often measured variances apply well to data collected in the median signal range, but produce erroneous extrapolations when applied at the tails of the distribution. In fact, it is possible [42] for several of the well known distributions to provide similar fits to the same measured data near the rms value of the received signal.

The following paragraphs present a discussion of the origin of the probability distributions that have been previously proposed for use in modelling multipath fading. This information is presented here so that the suitability of these distributions for mobile radio work can be assessed (Chapter V) once the characteristics of the radio channel under study have been determined.

2.4.1 The Rayleigh distribution

The Rayleigh distribution was derived by Lord Rayleigh [43] in 1880 in response to a published work by Verdet in which it was concluded that the resultant of n vibrations of unit amplitude and arbitrary phase approaches a definite value, n , when n is a very large number. Lord Rayleigh showed that this result is inaccurate, and that no matter how large n becomes, the value of the resultant is random. Further, he showed that for large n (no specific bounds were given), if n unit vectors of the same frequency and arbitrary

phase are compounded, the probability of a resultant intermediate in amplitude between r and $(r + dr)$ is given by

$$p(r)dr = \frac{2}{n} e^{-r^2/n} r dr. \quad (2-27)$$

It follows that the corresponding cumulative distribution function (CDF) is

$$\begin{aligned} P(r \leq R) &= \int_0^R \frac{2}{n} e^{-r^2/n} r dr \\ &= 1 - e^{-R^2/n} \end{aligned} \quad (2-28)$$

Lord Rayleigh's paper also shows that the expectation of power, (time average or power for an ergodic process) has a value of n . Further, it was shown that the basic exponential law is not altered even if the components have different amplitudes, provided that there are a large number of waves of each amplitude.

It is common in current literature [44] to find a derivation of the Rayleigh probability density function based on the assumption that, from central limit theory considerations, for large n , the resultant of n component waves can be decomposed into two orthogonal Gaussian random processes, which have zero mean, and the same standard deviation. Lin [42], however, has shown that the constraints of the complex Gaussian model are unnecessary. His work shows that the relationship between the Rayleigh pdf and its CDF is not unique. Physically, this means that if a received signal envelope CDF derived from measurements follows a Rayleigh curve, it does not necessarily mean that there are a large number of interfering waves, or that the complex Gaussian decomposition is accurate. The relationship is, however, unique under the conditions:

- (i) The amplitude and phase of each component wave are statistically independent.
- (ii) The phase of each component wave is a random variable which is uniformly distributed on $[0, 2\pi]$.

2.4.2 The Rice distribution

When there is a direct line of sight or specularly reflected component which contributes to the received signal, the envelope probability distribution is altered from that

which would be received in the absence of such a steady signal component. If there is a single steady component and a large number of multipath signals, the situation is analogous to the case of a single sine wave being received through additive Gaussian noise. Rice [45] has shown that for the case of a sine wave plus Gaussian noise, the probability of the resultant amplitude lying between r and $(r + dr)$ is

$$p(r)dr = \frac{r}{\Psi} e^{-(r^2 + B^2)/2\Psi} J_0\left(\frac{rB}{\Psi}\right) dr, \quad (2-29)$$

where,

J_0 is the Bessel function of zero order and imaginary argument,

B is the amplitude of the steady signal,

and,

$\Psi = \frac{R_0^2}{2}$, R_0 is the rms value of r .

2.4.3 The Nakigami (or m) distribution

In the early 1940's Nakigami et al [46] performed an experiment in which the fading of HF signals received over long propagation paths was monitored by connecting the same fading input to both the horizontal and vertical deflection plates of a cathode ray tube. The received signal amplitude probability distribution was then determined by measuring the emulsion density of CRT photographs taken after carefully controlled (for statistical stationarity) monitoring periods. The distributions were then plotted on log-log paper and compared with the results of a mathematical expression, the form of which was derived by inspection. Agreement between the measured and theoretical distributions was found to be reasonable, and after normalization and a change of variables, the corresponding probability density function was written as

$$p(r)dr = \frac{2m^m r^{2m-1} e^{-(m/\Omega)r^2}}{\Gamma(m)\Omega^m} dr, \quad (2-30)$$

where,

$\Omega = \overline{r^2}$ is the time average power of the received signal,

m is the inverse of the normalized variance of $\bar{r}^2$. That is

$$m = \frac{(\bar{r}^2)^2}{(\overline{r^2} - \bar{r}^2)} \geq \frac{1}{2},$$

and is referred to as the shape factor.

Nakigami was subsequently able to express eqn(2-30) in the form of a Hankel integral. Later, in 1953, he mathematically derived this integral form based on the assumption that the received signal is composed of $n(>1)$ component waves, the amplitudes and phases of which are both random variables. It was also shown that the Nakigami distribution can be applied to the case of a random superposition of random vectorial elements.

The Nakigami probability density function of eqn(2-30) can be shown to be a general expression for other well known density functions. For $m=1$, the Rayleigh probability density function is obtained. For $m=1/2$, eqn(2-30) describes a one-sided Gaussian density function. Finally, it can be shown [46,34] that the Nakigami expression can approximate both the Rice and the lognormal density functions over certain domains, given appropriate bounds on the parameter values.

CHAPTER III

PREDICTION OF DIGITAL SYSTEM PERFORMANCE USING MEASURED RADIO CHANNEL CHARACTERISTICS

3.1 Introduction

The derivation of mathematical expressions into which channel characteristics obtained from field measurements can be inserted in order to predict the performance of proposed digital systems is the subject of this chapter. In particular, analytical methods for the prediction of the performance of binary differential phase shift keyed (DPSK) transmission systems on flat/flat and frequency-selective fading channels is outlined. Section 3.2 explains a model for the DPSK system. This system has been implemented for performance measurements that will be performed as an extension to this thesis. The system model is employed in the mathematical analyses for both flat and frequency-selective fading channels. The derivation of an equation for the transmission of DPSK over an ideal channel and reception with the receiver modelled in Section 3.2 is presented in Section 3.3. Section 3.4 is concerned with the performance analysis for flat fading channels. The mathematical development for frequency-selective fading channels is outlined in Section 3.5 and a summary of the chapter is included as Section 3.6.

3.2 Transmitter and receiver models

The DPSK system discussed in this section was designed and implemented for the confirmation of predictions resulting from work conducted during the thesis project concerning capacity limitations imposed by mobile radio propagation channels. The system hardware was therefore designed so as to have minimal distorting effects on the encoded baseband signal, and to thus permit the determination of distortion caused by the propagation channel. As a result, the system is non-optimal from a communications engineering viewpoint, but will allow experiments to be conducted in order to achieve the desired confirmation of predictions. It should be noted, however, that the techniques used in this chapter are applicable for the prediction of performance on any system, so long as the actual system parameters (filter characteristics) are specified.

The transmitter consists of a data source followed by a DPSK modulator, a bandpass filter, a wideband linear power amplifier and an antenna. The system was designed for the transmission of binary DPSK at switch selectable data rates between 1 kb/s and 1 Mb/s. The filter employed at the modulator output is assumed to have a linear phase characteristic, and has a fixed 3 dB bandwidth of 18 MHz, or 2 percent of the carrier frequency of 890 MHz. The transmit filter bandwidth is therefore much wider than any

intended transmission bandwidth, and this, coupled with the linear phase characteristics, allows distorting effects of the transmitter to be omitted from consideration. Thus, the magnitude of the transmit filter transfer function, $|H_T(f)|$, is set equal to unity, and its phase characteristic is assumed to be linear in the performance analyses. The transmitter model is shown in Fig. 3.1(a).

The signal received at the mobile unit is filtered by a 2 percent bandpass filter, and downconverted to the first IF frequency (60 MHz) of a commercially available telemetry receiver, the IF section of which has a 5-pole bandpass L-C filter with a 3 dB bandwidth of 3 MHz. Again, since the bandwidth of this section is much wider than that of the transmitted signal, no intersymbol interference effects are created by the receiver.

At the input to the receiver demodulator, white Gaussian noise is added after filtering in a crystal filter so as to have an equivalent bandwidth of $2/T$ Hz. This bandwidth was chosen since most of the energy of the pseudorandom data signal proposed for transmission during experiments is confined to bandwidth of $2/T$ Hz about the carrier frequency. Results are therefore easily related to those which would be obtained if the noise and the data signal were passed through the same filter. The added noise is assumed to be at power levels much in excess of the receiver front end noise, so the latter can be neglected in the analyses. It is important to note that this added noise has a different bandwidth from that of the received signal, and this is reflected in the analyses by different limits on integral relationships associated with the signal and noise terms. The two-sided noise power spectral density is denoted N_0 .

After the addition of noise, the received signal is split into in-phase and quadrature channels. A delay and multiply operation is followed by an active low pass 4-pole Butterworth filter having a bandwidth which is 12 times the data rate. The outputs from the low pass filter on each channel are then multiplied together to form the input to the decision circuitry. This signal is passed through one more fourth order active Butterworth low pass filter having a bandwidth which is 1.5 times the rate before application to the decision device. It is clear that in this receiver, there is practically no bandlimiting applied to the received signal. Fig. 3.1(b) is a block diagram of the receiver model.

Fig. 3.1(a) Transmitter model block diagram.

Fig. 3.1(b) Receiver model block diagram.

3.3 Probability of error for DPSK transmission on nonfading channels

The equation for probability of error for the transmission of DPSK on ideal channels is derived in many textbooks. Most derivations, however, apply to receivers with matched filters and are not appropriate for the receiver model of Section 3.2. This section therefore summarizes a 1959 derivation of probability of error published by Cahn [47] which can be applied to the receiver model under consideration. The referenced paper shows that the probability of error can be derived by considering phasor representations of the present and previously received symbols.

Represent the received bandpass signal, with power, S , at the input to the demodulator as

$$R(t) = \sqrt{2S} \cos[\omega_c t + \theta(t)]. \quad (3-3)$$

Let the previously received symbol be represented by the phasor R_1 , and let its phase difference in the presence of noise from its unperturbed phase $\theta(t)$ be equal to $\Delta\theta$, as in Fig. 3.2. Represent the present received symbol by the phasor R_2 , perturbed from its ideal phase (either $\theta(t)$ or $-\theta(t)$ for binary DPSK) by noise components n_1 and n_2 , which are in-phase, and orthogonal with R_1 , respectively. Then, regardless of the unperturbed phase of R_2 , the probability of error is the probability that R_2 crosses the decision boundary orthogonal to R_1 , as shown in the figure. Since only the noise component n_2 can effect this transition, the required probability, given the phase difference $\Delta\theta$ of R_1 from $\theta(t)$ is given by

$$P(\text{error} | \Delta\theta) = P(n_2 < -\sqrt{2S} \cos \Delta\theta). \quad (3-4)$$

In the presence of zero mean white Gaussian noise of power N , eqn(3-4) can be written

as

$$P(\text{error} | \Delta\theta) = \frac{1}{\sqrt{2\pi N}} \int_{-\sqrt{2S} \cos \Delta\theta}^{\infty} e^{-n_2^2/2N} dn_2 \quad (3-5)$$

Fig. 3.2 Phasor representation for received DPSK signals.

The unconditional probability of error is then

$$P_e^o = \int_{-\pi}^{\pi} P(\text{error} | \Delta\theta) p(\Delta\theta) d(\Delta\theta) \quad (3-6)$$

where the superscript "o" indicates that this is the probability of error for a non-fading channel. Since this is the classical case of a sine wave in the presence of white Gaussian noise, $p(\Delta\theta)$ is given by Rice [45] as

$$p(\Delta\theta) = \frac{1}{2\pi} e^{-S/N} [1 + \sqrt{4\pi S/N} \cos \Delta\theta e^{(S/N) \cos^2 \Delta\theta} \Phi(\sqrt{2S/N} \cos \Delta\theta)], \quad (3-7)$$

where,

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-x^2/2} dx.$$

Cahn has evaluated the integral of eqn(3-6) and found the expression for probability of error to be

$$P_e^o = \frac{1}{2} e^{-S/N} \quad (3-8)$$

The additive noise has two-sided power spectral density N_0 in the receiver model employed in this thesis, and the receive noise filter has a bandwidth of $2/T$. The total noise power, N , is therefore $4N_0/T$. Then, since the power of the received signal during one bit period is equal to E/T , the exponent S/N of eqn(3-8) is equal to $E/4N_0$, and the probability of error can be written as

$$P_e^o = \frac{1}{2} e^{-E/4N_0} \quad (3-9)$$

3.4 Probability of error for the transmission of DPSK on flat/flat fading channels

In standard textbooks the probability of error is derived in terms of the transmit power. In order to make the equations such that measured propagation data can be employed in performance predictions, however, the probability of error must be in terms of the received signal power. This also makes resulting equations independent of transmission loss, and therefore applicable for any location with respect to the transmitter once the free space and diffraction losses for the direct path have been calculated. The inclusion of normalization procedures in the derivation of the error equations to achieve these objectives is discussed in this section.

Consider the complex envelope low pass equivalent for the signal at the input to the demodulator,

$$r(t) = \int_{-\infty}^{\infty} S(f)H_T(f)H(f;t)H_R(f)e^{j2\pi ft}df + n(t), \quad (3-10)$$

where complex envelope low pass equivalent representation has been employed throughout, and

$S(f)$ represents the transmit signal spectrum,

$H_T(f)$, $H(f)$, and $H_R(f)$ represent the transmitter, channel and receiver transfer functions respectively, and

$n(t) = r(t) * h_N(t)$ represents the added noise at the input to the demodulator.

It has been shown in Section 2.3.1 that under flat fading conditions there are several simplifications that can be made in eqn(3-10) to reduce the expression for the off-air signal to the product of the complex envelope low pass equivalent for the impulse response of the radio channel and the transmitted signal waveform. This representation is not altered if $|H_T(f)| = |H_R(f)| = 1$ over a bandwidth which is large compared with that of $S(f)$, as in Fig. 3.1. In addition, for the consideration of probability of error per symbol, the channel time dependence can be dropped, since conditions can be assumed to be time-invariant over at least one symbol interval. The input to the demodulator can then be written as

$$\begin{aligned} r(t) &= \alpha e^{-j\phi} s(t) + n(t) \\ &= \alpha e^{-j\phi} |s(t)| e^{j\theta(t)} + n(t). \end{aligned}$$

where $s(t)$ is the complex envelope low pass equivalent for the bandpass signal $S(t)$, and α and ϕ represent the channel gain and phase shift for the symbol period under consideration.

Even though channel conditions can be assumed constant over one symbol duration, α is a random variable and varies between adjacent symbols, causing random changes in the received signal to noise ratio. Let the energy of the off-air signal at the receiver input be denoted as E . Then the average envelope of the off-air signal can be written as

$$E\{\alpha\} |s(t)| = \sqrt{2E/T}, \quad (3-11)$$

Thus, at the sampling instant t_0 , substitution into eqn(2-22) as rewritten above gives

$$r(t_0) = \beta \sqrt{2E/T} e^{j(\theta - \phi)} + n(t_0), \quad (3-12a)$$

where,

$$\beta = \frac{\alpha}{E\{\alpha\}}, \quad (3-12b)$$

The power of the received signal envelope is given by

$$S = 2\beta^2 \frac{E}{T} \quad (3-13a)$$

and for noise envelope power spectral density $2N_0$, with bandwidth $2/T$, the noise power is given by

$$N = \frac{8N_0}{T} \quad (3-13b)$$

Substitution in eqn(3-8) therefore yields

$$P_e^* = \frac{1}{2} e^{-\beta^2 E / 4N_0} \quad (3-14)$$

It is clear that under non-fading conditions, $\beta = 1$, and P_e^* is as in eqn(3-9).

Under fading conditions, P_e^* must be averaged over the probability density function for $\beta^2 \frac{E}{4N_0}$ to give the average probability of error, P_e . That is

$$P_e = \int_0^\infty P_e^* p_\gamma(\gamma) d(\gamma) \quad (3-15)$$

where,

$$\gamma = \beta^2 E / 4N_0.$$

If α has a Rayleigh probability density function, the pdf for γ is given (Appendix A-2) by

$$p_\gamma(\gamma) = \frac{1}{\xi} e^{-\gamma/\xi}, \quad (3-16)$$

where

$$\xi = \frac{E}{4N_0} E\{\beta^2\}. \quad (3-17)$$

Let $\eta = E/4N_0$. Then substitution in eqn(3-15) gives

$$P_t = \frac{1}{2} \int_0^\infty e^{-\eta\beta^2} \times \frac{e^{-\eta\beta^2/\eta E\{\beta^2\}}}{\eta E\{\beta^2\}} d(\eta\beta^2)$$

Since η is deterministic,

$$\eta E\{\beta^2\} = E\{\eta\beta^2\}.$$

Then, if x is defined as $x \equiv \eta\beta^2$,

$$P_t = \frac{1}{2} \int_0^\infty e^{-x} \times \frac{e^{-x/E\{x\}}}{E\{x\}} dx.$$

Finally, since α , and thus, β is quasi-wide-sense-stationary, $E\{\eta\beta^2\} = E\{x\} = \xi$ is a constant. Therefore,

$$P_t = \frac{1}{2\xi} \int_0^\infty e^{-x(\xi+1)/\xi} dx.$$

This integral is readily evaluated to give

$$P_e = \frac{1}{2(1 + \xi)} \quad (3-18)$$

for the probability of error under flat fading conditions.

3.4.1 Example of performance calculations for a flat fading channel

Although measurement results are discussed in detail in Chapter V, some results from measurements over a short section of street in downtown Ottawa are given here to exemplify the application of the flat fading error probability equations discussed in the foregoing paragraphs.

Figure 3.3 shows a time series of data derived from pseudo-noise impulse response measurements. These data represent variations in the amplitude, α of the complex envelope low pass equivalent for the impulse response of the measured channel. The cumulative distribution function for this time series is shown in Fig. 3.4, and clearly corresponds well with a Rayleigh distribution. Figure 3.5 shows envelope fading data which follow variations of the random variable β of eqn(3-12b). These data were obtained from the data of Fig. 3.4 through normalization with respect to $E\{\alpha\}$. The second moment, $E\{\beta^2\}$, of this time series has been computed to be equal to 1.264. Substitution of this value in equations (3-17) and (3-18) results in the flat fading error probability curve shown in Fig. 3.6. This represents performance of the system over a flat Rayleigh fading channel with envelope variance, 0.264, or -6.9 dB. Also shown in Fig. 3.6 is the performance characteristic for a non-fading channel as given by eqn(3-9). The degradation under fading conditions can be seen to be significant for average bit energy to noise power spectral density ratios greater than 10 dB. It will be seen in Section 3.5 that this degradation increases as a function of transmission bandwidth, for bandwidths greater than that at which the channel becomes frequency selective.

3.5 Probability of error on frequency-selective/time-flat fading channels

The derivation presented in this section is lengthy and is difficult to follow without segmentation. For this reason, the section has been divided into six subsections which

Fig. 3.3 Time series of envelope fading data derived from pseudo-noise impulse response measurements in downtown Ottawa.

Fig. 3.4 Cumulative probability distribution function for the time series of envelope values shown in Fig. 3.3.

Fig. 3.5 Normalized time series of envelope values corresponding to Figure 3.3.

Fig. 3.6 Predicted error performance for DPSK on ideal (non-fading) and flat fading channels. The receiver bandwidth used in the analyses was $2/T$, and the flat fading data were derived from the time series of Figure 3.5.

discuss different steps in the derivation. The general procedure follows that published by Bello [41]. There are significant differences, however, due to the different receiver structure, and partitioning of the equations so that complex correlation functions derived from propagation measurement data can be used in the analyses. First, by tracing low pass equivalents of the received signal through the receiver structure (Fig. 3.1b), it is shown that the decision variable can be written in a Hermitian quadratic form. Then, after taking note that under Rayleigh fading conditions, $H(f)$ is a zero mean Gaussian random process, an expression derived by Turin is employed to write the characteristic function for the decision variable. This leads through integration to an expression for the conditional probability that the decision variable is less than or equal to the decision threshold. The conditionality for this probability and the moments needed for its computation are symbol sequence dependent. Expressions are therefore derived for the moments of the decision variable into which symbol sequence values as well as spaced frequency correlation functions derived from pseudo-noise channel impulse response measurements can be substituted. The moment expressions so derived are then substituted into the conditional probability expression for the decision variable to yield conditional probability of error equations. Finally, the conditioning is removed by taking note of statistical symmetries and summing conditional probabilities over a finite set of symbol sequences representing all possible combinations.

3.5.1 Derivation of a Hermitian quadratic representation for the decision variable

Again consider the complex envelope low pass equivalent for the signal at the demodulator input,

$$r(t) = \int_{-\infty}^{\infty} S(f)H_T(f)H(f;t)H_R(f)e^{j2\pi ft}df + n(t),$$

where complex envelope low pass equivalent representation has been employed throughout, and the notation is the same as that in Section 3.4.

In accordance with Section 3.2, let $|H_T(f)| = |H_R(f)| = 1$, over a bandwidth which is much larger than that of $S(f)$ and assume the filters have linear phase characteristics. In addition, since the channel under consideration is time flat, the time dependence can be

dropped from the channel transfer function representation. The input to the demodulator can then be represented as

$$r(t) = \int_{-\infty}^{\infty} S(f)H(f)e^{j2\pi ft} df + n(t), \quad (3-19)$$

where retention of the infinite limits on the integral is justified by the fact that $S(f)$ diminishes at a bandwidth that is much smaller than that of the transmit and receive filters.

Let $r(t)$ be written in terms of its real and imaginary parts as

$$r(t) = r_I(t) + jr_Q(t).$$

Without loss of generality, perfect local oscillator synchronization with the received signal can be assumed. Then, at the input to the I channel low pass filter in the model of Fig. 3.1(b), the in-phase signal can be written as

$$I'(t) = \operatorname{Re}\{r(t)e^{j\omega_c t}\} \cos \omega_c t$$

where the loss in the power splitter has been neglected for convenience. Upon trigonometric expansion,

$$I'(t) = \frac{1}{2}r_I(t)[1 + \cos 2\omega_c t] - \frac{1}{2}r_Q(t)\sin 2\omega_c t.$$

Then after the low pass filter,

$$I(t) = \frac{1}{2}r_I(t) = \frac{1}{2}\operatorname{Re}\{r(t)\}.$$

Similarly, the input to the quadrature channel multiplier can be written as

$$Q(t) = -\frac{1}{2}r_Q(t) = -\frac{1}{2}\operatorname{Im}\{r(t)\}.$$

At time $(t - T)$, denote the low pass equivalent for the received signal as

$$r(t - T) = r_I(t - T) + jr_Q(t - T).$$

Then,

$$I(t - T) = \frac{1}{2}r_I(t - T) = \frac{1}{2}\text{Re}\{r(t - T)\},$$

and,

$$Q(t - T) = -\frac{1}{2}r_Q(t - T) = -\frac{1}{2}\text{Im}\{r(t - T)\}.$$

The output from the I channel multiplier is then

$$q_I(t) = \frac{1}{4}r_I(t)r_I(t - T).$$

Similarly,

$$q_Q(t) = \frac{1}{4}r_Q(t)r_Q(t - T),$$

is the output from the Q channel multiplier. The summer output is then

$$q(t) = \frac{1}{4}[r_I(t)r_I(t - T) + r_Q(t)r_Q(t - T)],$$

which can readily be rewritten as

$$q(t) = \frac{1}{8}[r(t)r^*(t - T) + r^*(t)r(t - T)]. \quad (3 - 20)$$

Denote the waveforms $r(t - T)$ and $r(t)$ at the sampling instant, (t_0) , by U and V respectively. Then using eqn(3-19),

$$U = r(t_0 - T) = \int_{-\infty}^{\infty} S(f)H(f)e^{-j2\pi fT} e^{j2\pi f t_0} df + n(t_0 - T) \quad (3-21a)$$

and,

$$V = r(t_0) = \int_{-\infty}^{\infty} S(f)H(f)e^{j2\pi f t_0} df + n(t_0). \quad (3-21b)$$

From eqn(3-20), the decision variable is then simply written as

$$q = \frac{1}{8}[U^*V + UV^*]. \quad (3-22)$$

Equation (3-22) has the general form

$$q = [a|U|^2 + b|V|^2 + cU^*V + cUV^*].$$

with $a = b = 0$, and $c = 1/8$. The coefficient "c" can be normalized without effecting the resultant probability equations since the normalization has equal effects on the signal and noise powers. Then the quadratic for q can be written as

$$q = W^T Q W,$$

where,

$$W = \begin{pmatrix} U \\ V \end{pmatrix}$$

and

$$Q = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Since $Q^* = Q^T$, Q is a Hermitian matrix, and eqn(3-22) is a Hermitian quadratic form.

3.5.2 A general expression for the probability that the decision variable is less than or equal to zero

From Turin [49], the characteristic function for the decision variable, q , provided that $H(f)$ is a zero mean Gaussian random process (Rayleigh envelope fading on the channel), is given by

$$\Psi_q(jx) = |I - jxM^*Q|^{-1} \quad (3-23)$$

where I is the identity matrix and M is the covariance matrix for q , given by

$$M = E\{(W - \bar{W})^*(W - \bar{W})^T\}$$

For the zero mean case $\bar{W} = 0$, and

$$\begin{aligned} M &= E\{W^*W^T\} \\ &= E\left\{\begin{pmatrix} |U|^2 & U^*V \\ UV^* & |V|^2 \end{pmatrix}\right\} \end{aligned}$$

Therefore,

$$\begin{aligned} M^* &= E\left\{\begin{pmatrix} |U|^2 & UV^* \\ U^*V & |V|^2 \end{pmatrix}\right\} \\ &= \begin{pmatrix} m_{11} & m_{01}^* \\ m_{01} & m_{00} \end{pmatrix} \end{aligned} \quad (3-24)$$

Then substitution for M^* and Q in eqn(3-23) yields

$$\Psi_q(jx) = [(1 - jxm_{01}^*)(1 - jxm_{01}) + x^2 m_{00}m_{11}]^{-1} \quad (3-25)$$

By definition,

$$p_q(q) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Psi_q(jx) e^{-jxq} dx$$

As a simplification, let $z = jx$. Then

$$p_q(q) = \frac{1}{j2\pi} \int_{-\infty}^{\infty} \Psi_q(z) e^{-zq} dz \quad (3-28)$$

Upon substitution for jx in eqn(3-25) and expansion, $\Psi_q(z)$ can be written as

$$\Psi_q(z) = [az^2 - bz + 1]^{-1},$$

where

$$a = |m_{01}|^2 - m_{00}m_{11},$$

and

$$b = m_{01} + m_{01}^* = 2\text{Re}\{m_{01}\}$$

The roots of this quadratic can be written as

$$r_1 = \frac{b + \sqrt{b^2 - 4a}}{2a}$$

and

$$r_2 = \frac{b - \sqrt{b^2 - 4a}}{2a}$$

Substitution therefore gives

$$\Psi_q(z) = [a(z - r_1)(z - r_2)]^{-1}$$

Then from eqn(3-26)

$$p_e(q) = \frac{1}{j2\pi} \int_{-\infty}^{\infty} \frac{e^{-qz}}{a(z-r_1)(z-r_2)} dz. \quad (3-27)$$

The residue theorem can be applied in the evaluation of this integral if the contour along the imaginary axis is closed at infinity in such a manner that it can be shown that the integral along the contour vanishes, except when $\operatorname{Re}\{z\} = 0$. For $q < 0$, the exponent in the numerator of the integrand is positive. Therefore for $\operatorname{Re}\{z\}$ large, e^{-qz} , and thus the integrand, tends to zero if $\operatorname{Re}\{z\} < 0$. The contour must therefore be closed in the left hand side of the complex plane. The integrand is also well behaved at $q = 0$. Since the matrix M is Hermitian, it is positive semidefinite, and its determinant is positive. Therefore, $a < 0$, and r_1 and r_2 must be real and on opposite sides of the imaginary axis. Application of the residue theorem thus results in two different possible equations for $p_e(q)$, as follows

Case 1 $r_2 < 0$:

$$p_e(q) = \frac{1}{j2\pi} \times j2\pi R_2, q \leq 0, \quad (3-29)$$

where R_2 represents the residue at r_2 . It is easily shown that this residue is given by

$$R_2 = \frac{e^{-r_2 q}}{a r_2 (r_2 - r_1)}, q \leq 0$$

Then from eqn(3-29)

$$p_e(q) = \frac{e^{-r_2 q}}{a(r_2 - r_1)}, q \leq 0$$

Finally, by definition

$$P(q \leq v_{th}) = \int_{-\infty}^{v_{th}} \frac{e^{-r_2 q}}{a(r_2 - r_1)} dq$$

Thus, since $r_2 < 0$, for a decision threshold (v_{th}) equal to zero, this integral can easily be evaluated to give

$$P(q \leq 0) = -\frac{1}{ar_2(r_2 - r_1)}, \quad (3-30a)$$

where a, r_1 and r_2 are dependent upon the transmitted symbol sequence, $s(t)$, and hence, $P(q \leq 0)$ is conditional upon $s(t)$.

Case 2. $r_1 < 0$:

Identical operations to those above show that

$$P(q \leq 0) = -\frac{1}{ar_1(r_1 - r_2)}. \quad (3-30b)$$

3.5.3 General expressions for the moments of the decision variable

The moments m_{ij} from eqn(3-24), required for the calculation of the roots, r_1 and r_2 , depend upon the transmitted symbol sequence. It should be noted therefore, that in the equations immediately following, $S(f)$, is sequence dependent. This dependency is included in the analysis as part of the final step in the moment calculations of Section 3.5.5. General expressions for the moments are derived in the following paragraphs.

Consider first the moment $m_{00} = \overline{|V|^2}$. Substitution for V from eqn(3-21b) yields

$$m_{00} = \frac{1}{2} \left[\int_{-\infty}^{\infty} S(f_1) \overline{H(f_1)} e^{j2\pi f_1 t_0} df_1 + \overline{n(t_0)} \right] \times \left[\int_{-\infty}^{\infty} S^*(f_2) \overline{H^*(f_2)} e^{-j2\pi f_2 t_0} df_2 + \overline{n^*(t_0)} \right]$$

ie

$$m_{00} = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(f_1) S^*(f_2) \overline{H(f_1)} \overline{H^*(f_2)} e^{j2\pi t_0(f_1 - f_2)} df_1 df_2$$

$$+ \frac{1}{2} E\{n(t_0)\} \int_{-\infty}^{\infty} S^*(f_2) \overline{H^*(f_2)} e^{-j2\pi f_2 t_0} df_2$$

$$+ \frac{1}{2} E\{n^*(t_0)\} \int_{-\infty}^{\infty} S(f_1) \overline{H(f_1)} e^{j2\pi f_1 t_0} df_1$$

$$+ \frac{1}{2} \overline{n(t_0)n^*(t_0)}, \quad (3-31)$$

where both the bars and $E\{\}$ have been used to represent expectation; and the factors of one half result from the complex envelope low pass notation.

The second and third terms in eqn(3-31) are equal to zero since it is reasonable to assume that random variations of the channel and those of the noise are uncorrelated. In addition, the term $\frac{1}{2} \overline{H(f_1)H^*(f_2)}$ is the spaced frequency correlation function for the channel, $R_H(f_2 - f_1)$. Finally, the term $\frac{1}{2} \overline{n(t_0)n^*(t_0)}$ can be recognized as the variance of the noise at the output from the receive filter, and is given [36] by

$$R_n(0) = \int_{-B_N/2}^{B_N/2} S_s(f) |H_N(f)|^2 df,$$

where $|H_N(f)| = 1, |f| \leq B_N/2$ is the noise filter transfer function, and from [36], $S_s(f) = 2N_0$ is the power spectral density† for the complex envelope low pass equivalent noise at the demodulator input. Therefore if f_2 is mapped into $(f_1 + \Delta f)$, and f_1 is mapped into f , (Appendix A-3) equation (3-31) can be rewritten as

$$m_{00} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(f) S^*(f + \Delta f) R_H(\Delta f) e^{-j2\pi \Delta f t_0} df d\Delta f + 2N_0 B_N.$$

Let

$$X(\Delta f) = \int_{-\infty}^{\infty} S(f) S^*(f + \Delta f) df. \quad (3-32)$$

Then,

$$m_{00} = \int_{-\infty}^{\infty} X(\Delta f) R_H(\Delta f) e^{-j2\pi \Delta f t_0} d\Delta f + 2N_0 B_N. \quad (3-33)$$

To facilitate the determination of reasonable finite limits for the integral of eqn(3-32) based upon the multipath spread of the channel, it is necessary to convert the equation to its time domain equivalent. This can be achieved as follows.

† In Bello [41], $S_s(f) = 4N_0$ since the $\frac{1}{2}$ is omitted from the correlation representations

Substitution of the Fourier integrals for $S(f)$ and $S^*(f + \Delta f)$ gives

$$S(f)S^*(f + \Delta f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s(t_1)s^*(t_2)e^{j2\pi f(t_2-t_1)}e^{j2\pi \Delta f t_2} dt_1 dt_2$$

Then,

$$X(\Delta f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s(t_1)s^*(t_2)e^{j2\pi f(t_2-t_1)}e^{j2\pi \Delta f t_2} dt_1 dt_2 df.$$

But,

$$\int_{-\infty}^{\infty} e^{j2\pi \beta f} df = 0, \beta \neq 0, \text{ or } \delta(\beta), \beta = 0. \quad (3-34)$$

Thus,

$$X(\Delta f) = \int_{-\infty}^{\infty} s(t_1) dt_1 \int_{-\infty}^{\infty} s^*(t_2) e^{j2\pi \Delta f t_2} \delta(t_1 - t_2) dt_2,$$

and the time domain equivalent of eqn(3-32) is

$$X(\Delta f) = \int_{-\infty}^{\infty} |s(t)|^2 e^{j2\pi \Delta f t} dt \quad (3-35)$$

By consideration of the multipath spread on the channel, finite limits can easily be determined for this integral. Then, by substitution of each symbol sequence of interest for $s(t)$ the integral can be evaluated (Section 3.5.5) for the calculation of m_{00} using eqn(3-33).

Next consider $m_{11} = \overline{|U|^2}$. Substitution for U from eqn(3-21a) gives

$$m_{11} = \frac{1}{2} \left[\int_{-\infty}^{\infty} e^{-j2\pi f_1 T} S(f_1) \overline{H(f_1)} e^{j2\pi f_1 t_0} df_1 + \overline{n(t_0 - T)} \right]$$

$$\times \left[\int_{-\infty}^{\infty} e^{j2\pi f_2 T} S^*(f_2) \overline{H^*(f_2)} e^{-j2\pi f_2 t_0} df_2 + \overline{n^*(t_0 - T)} \right].$$

Then after mapping f_2 into $f_1 + \Delta f$, and f_1 into f ,

$$m_{11} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(f) S^*(f + \Delta f) R_H(\Delta f) e^{j2\pi \Delta f (T - t_0)} df d\Delta f + 2N_0 B_N.$$

Again, substitute $X(\Delta f) = \int_{-\infty}^{\infty} S(f) S^*(f + \Delta f) df$

Then,

$$m_{11} = \int_{-\infty}^{\infty} X(\Delta f) R_H(\Delta f) e^{j2\pi \Delta f (T - t_0)} d\Delta f + 2N_0 B_N, \quad (3-86)$$

where $X(\Delta f)$ is given by eqn(3-35).

Finally, consider the cross moment $m_{01} = \overline{VU^*}$. Substitution for U and V from equations (3-21a) and (3-21b) gives

$$m_{01} = \frac{1}{2} \left[\int_{-\infty}^{\infty} S(f_1) \overline{H(f_1)} e^{j2\pi f_1 t_0} df_1 + n(t_0) \right] \\ \times \left[\int_{-\infty}^{\infty} e^{j2\pi f_2 T} [S^*(f_2) \overline{H^*(f_2)}] e^{-j2\pi f_2 t_0} df_2 + n^*(t_0 - T) \right].$$

Then with $f_2 = f_1 + \Delta f$, $f_1 = f$,

$$m_{01} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(f) S^*(f + \Delta f) R_H(\Delta f) e^{-j2\pi \Delta f t_0} e^{j2\pi (f + \Delta f) T} df d\Delta f + \frac{1}{2} \overline{n(t_0) n^*(t_0 - T)}.$$

Let $Y(\Delta f) = \int_{-\infty}^{\infty} S(f) S^*(f + \Delta f) e^{j2\pi f T} df$. The integral in this equation can then be written as

$$I = \int_{-\infty}^{\infty} Y(\Delta f) R_H(\Delta f) e^{j2\pi \Delta f (T - t_0)} d\Delta f.$$

The second term in the equation is the autocorrelation function of the noise at the demodulator input, and is given [36] by

$$R_n(T) = \int_{-\infty}^{\infty} S_n(f) |H_N(f)|^2 e^{j2\pi f T} df,$$

and for noise bandwidth $B_N = \frac{2}{T}$

$$R_n(T) = \int_{-1/T}^{1/T} 2N_0 e^{j2\pi fT} df = 0.$$

$$m_{01} = \int_{-\infty}^{\infty} Y(\Delta f) R_H(\Delta f) e^{j2\pi \Delta f (T-t_0)} d\Delta f = 0. \quad (3-37)$$

A time domain equivalent for $Y(\Delta f)$ can be derived as follows. As before, substitution of the Fourier integrals for $S(f)$ and $S(f + \Delta f)$ can be used to show that

$$Y(\Delta f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s(t_1) s^*(t_2) e^{j2\pi f(t_2 - t_1 + T)} df e^{j2\pi \Delta f t_2} dt_1 dt_2.$$

Then using eqn(3-34)

$$Y(\Delta f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s(t_1) s^*(t_2) \delta(t_2 - t_1 + T) e^{j2\pi \Delta f t_2} dt_1 dt_2$$

This integral has value only when $t_2 = t_1 - T$. This implies that

$$Y(\Delta f) = \int_{-\infty}^{\infty} s(t) s^*(t - T) e^{j2\pi \Delta f (t - T)} dt, \quad (3-38)$$

in which $s(t)$ again represents a specific symbol sequence.

3.5.4 Error probability equations.

Let the subscripts a, b , and c each take on the binary values "0", or "1". Then various symbol sequences can be denoted by adding the appropriate subscripts to $s(t)$. Multipath spreads measured on 800/900 MHz mobile radio channels (Chapter V) indicate that only intersymbol interference from adjacent symbols would occur at practical transmission rates. Therefore, in the DPSK case, due to the differential encoding, four consecutive symbols must be considered for each conditional error probability calculated using eqn(3-30). A symbol stream of interest might then be represented, for example, as $s_{abcc}(t)$ and equations (3-35) and (3-38) would be written as

$$X(\Delta f) = \int_{-\infty}^{\infty} |s_{abc}(t)|^2 e^{j2\pi \Delta f t} dt,$$

and

$$Y(\Delta f) = \int_{-\infty}^{\infty} s_{abc}(t) s_{abc}^*(t-T) e^{j2\pi \Delta f (t-T)} dt$$

respectively.

In the receiver model of Fig. 3.1 (b), it is clear that, on an ideal channel, if there is a phase inversion between the present and previously received symbol, q , the decision variable at the output from the summer will be negative. Thus, in the presence of noise and selective fading, the decoded signal is in error if q is greater than zero, given that there was a transition in the symbol sequence at the output from the transmitter encoder. An error will also be made in the converse situation, if q is less than zero, given that there was no transition in the transmitter encoder output. Let the absence of a transition in the decoder output (transmitted signal) be denoted by the sequence "abbc", and let the presence of a transition be represented by "ab $\bar{b}$ c". Table III-1 gives reference numbers for the sixteen different possible combination of this four symbol sequence.

In accordance with the error criteria stated above, the conditional probability of error, given that there was no transition in the transmitted sequence is given by

$$P(\text{error} | abbc) = P[q \leq 0 | s(t) = s_{abc}(t)]. \quad (3-39)$$

Similarly, the conditional probability of error given that there was a transition is

$$P(\text{error} | ab\bar{b}c) = P[q > 0 | s(t) = s_{ab\bar{b}c}(t)]. \quad (3-40)$$

Assuming equi-probable data symbols, the unconditional probability of error is therefore given by

$$P_e = \frac{1}{2}(P_{e0} + P_{e1}), \quad (3-41a)$$

where

$$P_{e0} = \frac{1}{8} \sum_{a=0}^1 \sum_{b=0}^1 \sum_{c=0}^1 P_{(error|abbc)}, \quad (3-41b)$$

and,

$$P_{e1} = \frac{1}{8} \sum_{a=0}^1 \sum_{b=0}^1 \sum_{c=0}^1 P_{(error|a\bar{b}\bar{b}c)}. \quad (3-41c)$$

For phase reversal keying, a reduction in the number of computations can be achieved if it is noted [41] that

$$P(error|abcd) = P(error|\bar{a}\bar{b}\bar{c}\bar{d}).$$

By consideration of this symmetry, it can be determined with reference to Table III-1 that the following equalities exist among the conditional probabilities of error:

$$P_1 = P_7$$

$$P_2 = P_6$$

$$P_3 = P_5$$

$$P_4 = P_8$$

$$P_9 = P_{15}$$

$$P_{10} = P_{14}$$

$$P_{11} = P_{13}$$

and,

$$P_{12} = P_{16}$$

where $P_i = P(\text{error} | \text{sequence No. } i)$.

In addition to the above listed equalities, there are symmetries associated with the moment calculations that further reduce computational requirements. First, since $|s_0(t)| = |s_1(t)|$, where the subscripts indicate $s(t)$ is the time-waveform corresponding to a binary

TABLE III-1SYMBOL SEQUENCE COMBINATIONS

SEQUENCE No.	SEQUENCE
	abbc
1	0000
2	0001
3	0110
4	1000
5	1001
6	1110
7	1111
8	0111
	abbc
9	0010
10	0011
11	0100
12	1010
13	1011
14	1100
15	1101
16	0101

1, or a binary 0, the moments m_{00} and m_{11} are the same for every sequence. Secondly, it is required that inter-symbol interference from adjacent symbols only be considered. The time domain integrations of equations (3-35) and (3-38) are therefore only over two symbol durations centered at the end of the second symbol in each four symbol sequence. As a result, after alignment of delayed and non-delayed sequences, and definition of the integration limits, the moments for sequence No. 1 are identical with those for sequence No. 2, and those for sequence No. 9 are identical with those for sequence No. 10. In addition, after derivation of the reduced set of moment equations, it can be shown that P_4, P_9, P_{11} and P_{12} can be evaluated using combinations of moments for sequences No. 1 and No. 3. The result is that only the moments: $m_{001}, m_{111}, m_{011}$ and m_{013} are required for the calculation of all sixteen partial probabilities. These four moments will be referred to as the primary moments, and equations that can be applied in their evaluation are derived in Section 3.5.5, following.

3.5.5 Moment expressions for specific symbol sequences

It remains now to derive equations for $X(\Delta f)$ and $Y(\Delta f)$ for the primary moments. This can be achieved through consideration of the time waveforms $s(t)$ and $s^*(t - T)$ for the associated sequences.

Consider first the moment m_{00} , associated with sequence No. 1, (0000), and let this moment be denoted m_{001} . Using equation (3-33),

$$m_{001} = \int_{-\infty}^{\infty} X_1(\Delta f) R_H(\Delta f) e^{-j2\pi \Delta f t_0} d\Delta f + 2N_0 B_N,$$

where, from eqn(3-35),

$$X_1(\Delta f) = \int_{-\infty}^{\infty} |s_{0000}(t)|^2 e^{j2\pi \Delta f t} dt,$$

and the subscript "1" identifies $X(\Delta f)$ with symbol sequence number one.

In the system under consideration, a binary "0" implies that the transmitted phase, $\theta(t)$, is π radians. A schematic for defining the integration limits can therefore be drawn as in Fig. 3.7. In addition, the magnitude, $|s(t)|$, of the complex low pass envelope of the transmitted signal is equal to $\sqrt{2E/T}$. Then, if intersymbol interference from adjacent symbols only is considered,

Fig. 3.7 Schematic diagram used for defining the integration limits in equation (3-35) for sequence No. 1, (0000).

$$X_1(\Delta f) = \int_{-T}^T \frac{2E}{T} e^{j2\pi\Delta f t} dt.$$

This integral is easily evaluated to give

$$X_1(\Delta f) = \frac{2E}{\pi\Delta f T} \sin 2\pi\Delta f T \quad (3-42)$$

Let the complex envelope low pass representation for the spaced frequency correlation function of the channel be written in terms of its real and imaginary components as

$$R_H(\Delta f) = \frac{1}{2}[\gamma(\Delta f) + j\sigma(\Delta f)]. \quad (3-43)$$

After the exponential is expressed in trigonometric form, m_{001} can therefore be rewritten as

$$m_{001} = \frac{1}{2} \int_{-\infty}^{\infty} X_1(\Delta f) [\gamma(\Delta f) + j\sigma(\Delta f)] [\cos 2\pi\Delta f t_0 - j \sin 2\pi\Delta f t_0] d\Delta f + 2N_0 B_N.$$

Then, if sampling is performed in the middle of the symbol duration, at $t_0 = \frac{T}{2}$,

$$\begin{aligned} m_{001} = & \frac{1}{2} \int_{-\infty}^{\infty} X_1(\Delta f) [\gamma(\Delta f) \cos \pi\Delta f T + \gamma(\Delta f) \sin \pi\Delta f T] \\ & + jX_1(\Delta f) [\sigma(\Delta f) \cos \pi\Delta f T - \gamma(\Delta f) \sin \pi\Delta f T] d\Delta f + 2N_0 B_N. \end{aligned} \quad (3-44)$$

Now consider the symmetry of the imaginary part of the integrand in eqn(3-44). From eqn(3-42), $X_1(\Delta f)$ is symmetrical about $\Delta f = 0$. In addition, if there is uncorrelated scattering, $\sigma(\Delta f)$ is anti-symmetrical about $\Delta f = 0$, with positive values for $\Delta f > 0$. Then, since $\cos \pi\Delta f T$ is symmetrical about Δf , the first component of the imaginary part integrates to zero. In the second component, $\gamma(\Delta f)$ is symmetrical for uncorrelated scattering, and since $\sin \pi\Delta f T$ is anti-symmetrical, the second component of the imaginary part also integrates to zero. Equation (3-44) can therefore be rewritten as

$$m_{001} = \frac{1}{2} \int_{-\infty}^{\infty} X_1(\Delta f) [\gamma(\Delta f) \cos \pi\Delta f T + \sigma(\Delta f) \sin \pi\Delta f T] d\Delta f + 2N_0 B_N.$$

Substitution from eqn(3-42) for $X_1(\Delta f)$ now gives

$$m_{001} = \frac{E}{\pi \Delta f T} \int_{-\infty}^{\infty} [\gamma(\Delta f) \cos \pi \Delta f T + \sigma(\Delta f) \sin \pi \Delta f T] \sin(2\pi \Delta f T) d\Delta f + 2N_0 B_N, \quad (3-45)$$

where $\gamma(\Delta f)$ and $\sigma(\Delta f)$ can be derived from pseudo-noise measurements on the channel under investigation and the uncorrelated scattering assumption can be verified through an examination of the symmetry of these functions. The moment m_{001} can be evaluated using numerical integration since $X_1(\Delta f)\gamma(\Delta f)$ and $X_1(\Delta f)\sigma(\Delta f)$ are fairly rapidly diminishing functions of Δf for practical channels.

Now consider the moment m_{111} . From eqn(3-36) one can write

$$m_{111} = \int_{-\infty}^{\infty} X_1(\Delta f) R_H(\Delta f) e^{j2\pi \Delta f (T-t_0)} d\Delta f + 2N_0 B_N.$$

With $t_0 = \frac{T}{2}$, substitution from eqn(3-43) for $R_H(\Delta f)$ now gives

$$m_{111} = \frac{1}{2} \int_{-\infty}^{\infty} X_1(\Delta f) [\gamma(\Delta f) + j\sigma(\Delta f)] e^{j\pi \Delta f T} d\Delta f + 2N_0 B_N$$

Again, it can be shown that the imaginary parts integrate to zero. Therefore, using eqn(3-42) for $X_1(\Delta f)$, m_{111} can be written as

$$m_{111} = \frac{E}{\pi \Delta f T} \int_{-\infty}^{\infty} [\gamma(\Delta f) \cos \pi \Delta f T - \sigma(\Delta f) \sin \pi \Delta f T] \sin(2\pi \Delta f T) d\Delta f + 2N_0 B_N. \quad (3-46)$$

For the computation of m_{011} , the function $Y_1(\Delta f)$ must be evaluated. Using eqn(3-38) for $Y(\Delta f)$,

$$Y_1(\Delta f) = \int_{-\infty}^{\infty} s_{0000}(t) s_{0000}^*(t-T) e^{j2\pi \Delta f (t-T)} dt.$$

Reference to Fig. 3.8 shows that if only adjacent symbol interference is considered, integration limits can be defined to yield

Fig. 3.8 Schematic diagram used for defining the integration limits for equation (3-38) for sequence No. 1, (0000).

Fig. 3.9 Schematic diagram used for defining the integration limits for equation (3-38) for sequence No. 3, (0110).

$$Y_1(\Delta f) = \int_{-T}^T \frac{2E}{T} e^{j2\pi\Delta f(t-T)} dt \\ = X_1(\Delta f) e^{-j2\pi\Delta f T}$$

Then from eqn(3-37)

$$m_{011} = \int_{-\infty}^{\infty} X_1(\Delta f) R_H(\Delta f) e^{-j2\pi\Delta f t_0} d\Delta f,$$

and, for $t_0 = \frac{T}{2}$, substitution for $X_1(\Delta f)$ and $R_H(\Delta f)$ gives

$$m_{011} = \frac{E}{\pi\Delta f T} \int_{-\infty}^{\infty} [\gamma(\Delta f) \cos \pi\Delta f T + \sigma(\Delta f) \sin \pi\Delta f T] \sin(2\pi\Delta f T) \\ + j[\sigma(\Delta f) \cos \pi\Delta f T - \gamma(\Delta f) \sin \pi\Delta f T] \sin((2\pi\Delta f T) d\Delta f + 0. \quad (3-47)$$

The remaining primary moment for which an equation must be derived is m_{013} . With reference to Fig. 3.9, and eqn(3-38) the equation for $Y_3(\Delta f)$ can be written as

$$Y_3(\Delta f) = \left[\int_{-T}^0 -\frac{2E}{T} e^{j2\pi\Delta f t} dt + \int_0^T \frac{2E}{T} e^{j2\pi\Delta f t} dt \right] \times e^{-j2\pi\Delta f T}$$

The two integrals in this equation are readily evaluated and combined to give

$$Y_3(\Delta f) = \frac{j2E}{\pi\Delta f T} (1 - \cos 2\pi\Delta f T) \times e^{-j2\pi\Delta f T}$$

Finally, upon substitution in eqn(3-37) for m_{013} ,

$$m_{013} = \frac{E}{\pi\Delta f T} \int_{-\infty}^{\infty} (1 - \cos 2\pi\Delta f T) [\gamma(\Delta f) \sin \pi\Delta f T - \sigma(\Delta f) \cos \pi\Delta f T] \\ + j(1 - \cos 2\pi\Delta f T) [\gamma(\Delta f) \cos \pi\Delta f T + \sigma(\Delta f) \sin \pi\Delta f T] d\Delta f + 0. \quad (3-48)$$

Moments associated with each of the other data sequences in Table III-1 can be constructed from the primary moments given by equations (3-45) to (3-48). The moments for each of the possible combinations are listed in Appendix A-4.

3.5.6 Example of performance calculations for a frequency-selective fading channel

The impulse response data from which parameters used in the example in Section 3.4.1 were derived can now be substituted into equations developed in this Section in order to make performance predictions for systems operating with wider transmission bandwidths. Figure 3.10 shows examples of pseudo-noise impulse response measurement recordings for the channel. Although normal measurement procedure is to record such impulse response estimates at 3.97 cm intervals along the street, the three examples shown in the figure are taken from non-consecutive portions of a data file and were recorded at spatial separations of 40 cm. The figure shows that multipath delays in the area of the city where these measurements were made ranged up to about 2.5 microseconds and that the impulse response function of the channel varied rapidly with distance along the street. The non-minimum phase characteristic should also be noted. This made it necessary to consider symbols on either side of the symbol of interest during intersymbol interference computations. The spaced frequency correlation function, $R_H(\Delta f)$, computed via a procedure described in Bultitude [37] is shown in Fig. 3.11. The real and imaginary curves shown in the figure correspond to $\gamma(\Delta f)$, and $\sigma(\Delta f)$, respectively of eqn(3-43). The symmetry of the functions attests (Section 2.2.3) to the validity of the uncorrelated scattering assumption. These functions were substituted into equations (3-35) to (3-38) in order to calculate the primary moments. These moments were then used to calculate the moments for all sixteen sequences listed in Table III-1. For each sequence the probability of the decision variable 'q' being on the wrong side of the decision threshold was then calculated using eqn(3-30). Finally, equation (3-41) was employed to calculate the unconditional probability of error from the sixteen conditional partial probabilities. This computation was made for selected data rates, and selected average values of E/N_0 , with $B_N = 2/T$, which is appropriate for the receiver model of Fig. 3.1. Results are in Fig. 3.12, which shows that the error probability becomes independent of E/N_0 for higher data rates, and that the irreducible error probability increases as the data rate increases. Also shown in the figure is the flat fading curve, reproduced from Fig. 3.6. Comparison with the 9 kb/s curve is excellent, confirming that the performance prediction equations of Section 3.4 and those of this section yield the same results for narrow transmission bandwidths. This is considered to be partial confirmation of the validity of new work presented in this thesis. From Fig. 3.12 it can be concluded that the measured channel behaves as a flat fading channel for data rates up to about 10 kb/s. Above that rate the channel is frequency

Fig. 3.10 Impulse response estimates for a mobile radio channel in downtown Ottawa.

Fig. 3.11 Spaced frequency correlation function for the channel on which the impulse response estimates of Fig. 3.10 were measured.

Fig. 3.12 Predicted error performance for DPSK transmission for the channel on which the impulse response estimates of Fig. 3.10 were measured. The flat fading curve is reproduced from Fig. 3.6, and was calculated from envelope fading data derived from the same impulse response estimates.

selective, and system error performance is dominated by intersymbol interference effects rather than by noise.

3.6 Summary

In this chapter equations have been derived, and techniques have been outlined for the application of channel characteristics interpreted from pseudo-noise propagation measurements in the prediction of digital system performance. First, it was shown how Rayleigh envelope fading data derived from pseudo-noise measurements can be applied in the prediction of performance for systems operating with bandwidths at which the measured channels are nonselective. Results in an example show that, compared with performance on an ideal channel, there is significant performance degradation for narrow bandwidth systems operating on 800/900 MHz mobile channels. After a lengthy derivation, equations for the calculation of the performance of wider bandwidth systems operating on the same channels were formulated. These equations are novel results of this thesis, as they include provisions, not hitherto made, for the application of complex spaced frequency correlation functions derived from propagation measurements in the predictions. Finally, an example was given in which data derived from the same measurement results as those used in the flat fading example were used for predicting performance at higher data rates. Measured data from the example were also used to support Gaussian and uncorrelated scattering assumptions that were made in the derivation.

For narrow transmission bandwidths, results obtained using the flat fading equations were almost identical with those resulting from equations derived for frequency-selective channels. This is considered to be confirmation of the validity of new work presented in this thesis.

CHAPTER IV

EQUIPMENT AND INSTRUMENTATION

4.1 Introduction

Equipment and instrumentation that was used to perform measurements discussed in Chapter V is described in this chapter. Section 4.2 describes the pseudo-noise channel probing system and includes a summary level discussion of the data collection subsystem. Section 4.3 outlines some of the more important pseudo-noise system capabilities and parameters.

Since the instrumentation is extensive, detailed explanations of all circuits and their operation would be too lengthy for this report. A number of design memos and calibration procedures for different parts of the systems have been written, however, and are referenced in the text.

4.2 The pseudo-noise channel probing system

The pseudo-noise channel probing system is similar to that described by Cox [9]. Figure 4.1 is a block diagram of the transmitter. A 10 Mb/s pseudo-random binary sequence (PRBS) is used to phase (BPSK) modulate a 70 MHz CW carrier. After up-conversion to 910 MHz the resulting spread spectrum signal is bandlimited to 20 MHz, amplified to 10 watts, and transmitted via an omnidirectional antenna. If measurements are made within a 10 km radius of the base laboratory, a commercial 9 element vertically polarized colinear array is used for transmitting. This antenna is mounted approximately 30 m above ground level in a clutter free environment. For portable installations, a laboratory constructed and well characterized thick monopole antenna (Section 4.2.3) is employed.

The off-air signal is received via a laboratory constructed wideband monopole mounted above a 5.04 m long track affixed to the roof of an eight meter long motor home (Figure 4.2). The antenna is moved along the track by two synchronous motors and a pulley system in order to simulate mobile operation.

The receive system block diagram is shown in Figure 4.3. After 20 MHz bandpass filtering and preamplification, the received signal is down-converted to 350 MHz and applied to a commercial telemetry receiver for further amplification, bandpass filtering, and down-conversion to 70 MHz. The 70 MHz telemetry receiver output is then applied to a two channel sliding correlator (Section 4.2.1). The input to the correlator is split

Fig. 4.1 Pseudo noise system transmitter block diagram.

Fig. 4.2 The mobile measurement laboratory showing the receive antenna track.

Fig 4.3 Pseudo noise system receiver block diagram

into two channels, and each channel is applied to an SRA-1 double balanced mixer, for multiplication with a 70 MHz BPSK signal generated by modulation of a 70 MHz carrier with a PRBS identical to that generated at the transmitter, but at a frequency 1 kHz (selectable) lower. The correlator mixers are followed by integrators implemented with low pass filters, the 3 dB cut-off frequencies of which are switch selectable for compatibility with the selectable local PRBS frequency offset. Since the 70 MHz carrier input to one of the local BPSK modulators is phase shifted by 90 degrees, the correlator outputs are in phase quadrature. As a result of total transmit and receive system synchronization to phase coherent rubidium frequency standards, these quadrature outputs represent good estimates [50] of the low pass equivalents for the real (I) and imaginary (Q) parts of the impulse response of the propagation channel between the transmitter and the receiver. These output waveforms allow a $.05\mu s$ resolution of received multipath components when the system is properly aligned. The I and Q signals are digitized, passed to an LSI 11/23 microprocessor based data collection system (Section 4.2.4) and recorded on 9 track magnetic tape for later processing on a mainframe computer.

The time scale for most of the data recorded during field measurements is relative, with reference to the earliest received signal. If absolute transmit-receive propagation times are of interest, however, absolute timing can be obtained through proper adjustment of the synchronizer circuit (Figure 4.4) with the receiver at a known distance from the transmit antenna.

The functions of most of the circuits in the pseudo-noise system are readily understood through study of the block diagrams presented in the foregoing paragraphs. For this reason, detailed explanations of the circuitry have been omitted. The following paragraphs, however, detail specific problems that were encountered during implementation, and the methods by which these problems were overcome. The data collection subsystem is described in Section 4.2.4.

4.2.1 The receiver correlator

Figure 4.5 is a block diagram of the sliding correlator used in the receiver subsystem. The initial implementation of this circuit resulted in several undesirable features of the output I and Q waveforms, even when an ideal 70 MHz BPSK input directly from the transmitter modulator was applied. There was considerable cross-talk between the I and

*MIXER PORT PADS NOT SHOWN

Fig. 4.4 Pseudo noise receiver synchronization circuitry

Fig. 4.5 Receiver two channel sliding correlator block diagram.

Q channels, the outputs were noisy and contained dc offsets, it was difficult to obtain exactly 90 degrees phase shift between the two channels, identical channel gains were hard to achieve, and the output waveforms were found to be asymmetrical. The crosstalk was removed by housing I and Q channel electronics in separate boxes. Noise, and dc biases were reduced by selection of appropriate cut-off frequencies for the low pass filter/integrators (Figure 4.6) and the addition of offset null circuits to all IC modules (Figure 4.7). To obtain the desired 90 degree phase shift, the voltage controlled phase shifter shown in Figure 4.5 was used to replace the original 90 degree power splitter. The I and Q waveforms were then adjusted to be exactly 90 degrees out of phase by varying circuit parameters and the phase shifter control voltage to make their Lissajous pattern circular. Channel gains were made identical by careful selection and measurement of circuit components. The channel asymmetries were found to be the result of overdriving the SRA-1 double balance mixers, and considerable effort was expended to find optimum RF and LO drive levels. It was found that circuit adjustments had to be made in a definite sequence in order to produce the desired results. A receiver correlator alignment procedure [51] was subsequently written. This procedure takes an average of two days to perform and must be run through before each major series of channel characterization experiments. When properly aligned, the correlator produces quadrature output correlation triangles (given an ideal 70 MHz BPSK applied to the input power splitter) which are identical in shape and amplitude. A hardware sum of the squares ($V_I^2 + V_Q^2$) output from the circuit is monitored during alignment. The performance criterion set for the circuit is less than 5 percent peak pulse variation at this output. Results after sampling and squaring the digitally recorded I and Q voltages, however, have been found to always be better than the 5 percent figure for the output from the hardware squarer, and to be acceptable for data analysis purposes.

4.2.2 Transmitter and receiver bandpass characteristics and correlator output waveform distortion

It was found [52] that unless the bandpass characteristics of the transmit and receive subsystems are maintained relatively flat (± 2 dB) and very symmetrical about their centre frequencies, the symmetry of the correlator output waveforms is distorted, even if the correlator circuitry has been properly aligned using an ideal 70 MHz BPSK input, as discussed in Section 4.2.1. Alignment procedures were therefore developed and followed to ensure spectral symmetry throughout the pseudo-noise probing system.

Fig. 4.6 Receiver correlator low pass filter/integrator.

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Fig. 4.7 Receiver correlator audio processing circuit schematic diagram (one channel).

First, the power of the 10 MHz PRBS input to the SRA-1 70 MHz modulator was adjusted by suitably padding the IF port. The 70 MHz LO signal power was also adjusted and suitably padded to prevent IF-LO feedthrough. These adjustments lead to spectral symmetry of the 70 MHz BPSK signal into the 910 MHz up-conversion stage. Similarly, drive levels to this upconverter were adjusted for symmetry at the bandpass filter input. The final transmitter adjustments involved swept frequency measurements of the tunable bandpass filter characteristics to allow tuning for a symmetrical bandpass transfer function in the desired frequency band. By padding the power amplifier output with a 40 dB attenuator, the transmit signal at the antenna input was also monitored and found to be spectrally symmetrical.

Receive system alignment began by proper tuning of the input bandpass filter with the aid of swept frequency measurements. The previously adjusted symmetrical output from the padded transmitter was then applied to the receive system input, and the preamplifier output spectrum was observed to be symmetrical for the full range of preamp attenuator settings. Drive levels to the receiver SRA-1 910/350 MHz downconverter were next adjusted for a symmetrical output centred on 350 MHz.

The bandpass characteristics of the Microdyne telemetry receiver were also measured and adjusted for optimum performance of the channel probing system. The RF tuner section of this receiver was found to have an overcoupled frequency response characteristic. This allows the receiver to be used for two channel operation, with each channel centred in one of the response peaks that exist near the upper and lower edges of the tuner pass-band. Since two channel operation is not required in the pseudo-noise probing system, the band edge filters in the tuner were retuned to obtain a single tuned response centred on 350 MHz. It was found that a symmetrical frequency response could be maintained for received signal powers up to -30 dBm. Above this received power level, operating characteristics of the FET amplifiers in the tuner shift such that their drive point impedances skew the tuned circuit frequency response characteristics.

Swept frequency measurements were also made of the Microdyne receiver IF frequency response. AGC slope and gain controls were adjusted to yield best symmetry for received power levels up to -30 dBm. Swept frequency measurements for the combined receiver RF tuner and IF section also showed that total receiver bandpass symmetry could be maintained for received power levels up to -30 dBm.

After the measurements and adjustments discussed above were made, tests performed with the transmitter and receiver connected via a coaxial cable which simulated an "ideal" channel showed that the waveforms at the output from the correlator were perfectly symmetrical, as required.

The linearity of the total receive system as a function of received signal power was measured. Linear operation was found to occur for received signal powers between -35 dBm and -80 dBm. A calibration procedure was also written and performed so that correlator waveform amplitudes could be referenced to received signal powers.

4.2.3 The laboratory constructed wideband monopole antennas

Originally it was planned to employ a DB Products model DB 725 omnidirectional colinear whip array as the receive antenna. Since the antenna is required to move along a track, it was mounted on a circular ground plane, 1 wavelength in diameter which moves with the antenna in order to provide a fixed operating environment. After correlator alignment problems (Section 4.2.1) and system bandpass asymmetries (Section 4.2.2) were remedied, asymmetry of the correlator output waveforms was still observed during line of sight off-air tests using the whip antenna. Replacement of this antenna with a laboratory constructed quarter wave monopole antenna with 4 sloping quarter wavelength radials (Figure 4.8) removed the observed asymmetry. Subsequent feedpoint impedance measurements showed input characteristics for the two antennas as shown in Figures 4.9 (monopole) and 4.10 (DB 725). The commercial antenna input impedance variation over the frequency range of interest is clearly unacceptable. Due to the movement of the antenna along the mobile laboratory roof, the monopole with the sloping radials could not provide an antenna pattern that was insensitive to antenna position. For this reason, solid ground plane antennas were investigated. The design selected for use in the system (Figure 4.11) is a monopole 0.234 wavelengths in length and .02 wavelengths thick. This antenna is made of brass stock and is mounted on a groundplane using a teflon washer for insulation, and a brass SMA connector. The ground plane is aluminum and is 1 wavelength in diameter. The final design was measured to have a relatively flat impedance characteristic (Figure 4.12) over the frequency band of interest, and off-air system tests showed an acceptable reduction in correlator output waveform asymmetries. At a mounting height of 27 cm above the antenna track, in-situ radiation pattern measurements (Figure 4.13) were found to be omnidirectional within ± 3 dB at a number of antenna positions along

Fig. 4.8 Laboratory constructed quarter wavelength monopole antenna with quarter wavelength sloping radials.

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IMPEDANCE COORDINATES—50-OHM CHARACTERISTIC IMPEDANCE

Fig. 4.10 Measured feedpoint impedance characteristic for DB Products Model DB 725 mobile whip antenna.

Fig. 4.17 Laboratory constructed wideband monopole antenna.

Fig. 4.12 Feedpoint impedance characteristic for the laboratory constructed wideband monopole antenna shown in Fig. 4.11.

Fig. 4.13 A superposition of five wideband monopole receive antenna radiation patterns measured with the antenna mounted in place on the mobile laboratory. The five measurements are for different stationary antenna positions at 1 m intervals along the antenna track.

the track, and at frequencies across the 890 to 920 MHz band. In the vertical plane, the antenna was measured in an anechoic chamber to have a pattern as shown in Fig. 4.14.

4.2.4 The data collection subsystem

The data collection subsystem employed to record the I and Q channel outputs from the correlator consists of an A/D converter, synchronization circuitry, a central controlling LSI 11/23 microprocessor, a floppy disk unit, and a magnetic tape unit. Under programmed control, the I and Q channel outputs from the correlator are sampled at subintervals determined by the synchronization circuitry and the sampled data are then transferred to magnetic tape for permanent storage. The tape records so-created can be transferred directly to a mainframe computer for data analysis.

The data collection subsystem was designed and constructed by a graduate level engineer under the supervision of the author. His reports [53] form a detailed explanation of the circuits and the overall system. For this reason, only a summary level discussion of the system operation and capabilities is given here.

Synchronization in the system is derived from the difference frequency between the outputs of two frequency synthesizers which are slaved to the receiver rubidium standard. The difference frequency is divided by the number of bits (N) in the transmitted PRBS to provide a synchronization pulse at the end of each PRBS word. The synchronization pulse is used to trigger a real time clock which forms part of the microprocessor system. This clock generates an overflow pulse at the end of a programmed clock count which triggers a 100 μ s two channel A/D sampling trigger sequence. Sampled data are initially stored in a software buffer then transferred directly to a 72 kbyte/s 9-track magnetic tape recorder system. Since the tape system cannot afford random access capabilities for checking recorded data in the mobile lab at the recording site, a floppy disk system with programmed playback capabilities is available for data checking. The controlling software for the overall data collection system is also stored on floppy disk.

The real time executive program for the subsystem is extensive and affords considerable flexibility with regard to the sampling rate, the portion of each impulse response estimate to be sampled, the spatial interval along the antenna track at which samples are to be recorded, and the number of estimates to be recorded during a single antenna run. Upon receiving a run command from the operator at a control console, the program runs

Fig. 4.14 Vertical plane radiation pattern for the wideband monopole antenna measured in an anechoic chamber. The gain at the zero dB reference point is 2.85 dBi.

through a series of tests to ensure that all selected collection parameters are compatible with physical limitations of the system, such as buffer storage size, the length of the antenna track, and the speed at which the antenna moves for the current experiment. If test results show that input parameters have not been selected properly, an error message is printed at the control console and the program stops.

If the parameter tests are positive, the program asks for a file number, waits for an input from the console, and transfers control to a routine for conversion of the input file number from ASCII to RADIX 50, as required by the mag tape unit. On return from the conversion routine, a file header is composed which consists of an information block 128 bytes long, containing the date, time, and all data collection parameters pertinent to the file about to be recorded. The data in the header is then formatted to be compatible with the mainframe computer bit structure, and written onto magnetic tape.

After the data header is written on tape, the collection of data from the correlator outputs is initialized. The program signifies that it is ready to collect data by printing "READY" on the console terminal. When the user enters yes, a reply, "EXECUTING NOW" is returned to the terminal, and the real time clock and system counters are started. Data collection parameters are moved to the appropriate computer registers, and the console keyboard is disabled. The program waits in a two statement software loop until a real time clock (RTC) overflow occurs and triggers A/D conversion sequence to start on channel one of the A/D input multiplexer. The program waits for an end of conversion signal from the A/D converter, then checks for conversion errors. If an error has occurred, data acquisition is stopped, and an error message is printed on the terminal. If no errors are sensed, the 12 bits of sampled data are transferred to a register for truncation to the most significant 8 bits, formatted for mainframe computer compatibility, and transferred to a software buffer for intermediate storage. Data conversion is then started on channel two.

After the designated number of samples per impulse response estimate have been recorded, all data from the software buffer are transferred to magnetic tape, and an end of file mark is written on the tape. If tape write errors occur, data acquisition is stopped and an error message is printed. If there are no errors, the program waits for a period of time corresponding to the user specified spacing between recorded impulse response estimates, and the recording process is repeated.

When the user designated number of impulse response estimates have been recorded, the keyboard at the console is re-enabled, the bell is sounded, and the message "END OF RECORDING" is printed.

4.3 Impulse response measurement system parameters and capabilities

4.3.1 Range resolution

The pseudo-noise probing system output spectrum follows a $[\sin(x)/x]$ envelope, the width of the main lobe of which is dependent upon the bit rate (f_b) of the modulating PRBS. Since a 10 Mb/s modulation is employed, this main lobe is 20 MHz wide and the envelope falls relatively steeply at the 20 MHz band edges. Since the receiver IF circuit is 20 MHz wide, range resolution for the system is given by

$$\Delta R = \frac{3 \times 10^8 \text{ m/s}}{20 \times 10^6 \text{ sec}^{-1}} = 15 \text{ m}, \quad (4-1)$$

or a propagation delay of $0.05 \mu\text{s}$.

4.3.2 Unambiguous measurement range

If the excess delay of multipath components beyond that of the earliest received signal (line of sight path, if one exists) is longer than the period of the PRBS modulating the probe signal, long delayed components from one impulse response estimate show up in consecutively measured impulse response estimates as multipath signals with ambiguous delay times. The maximum unambiguous range is therefore given by:

$$R_{max} = \frac{N \times 3 \times 10^8}{f_b}, \quad (4-2)$$

where N is the number of bits in the transmitted PRBS. The choice of N is optional, and can be 31, 63, 127, or 255. For measurements reported in this thesis, a PRBS length of 127 bits was employed, leading to an R_{max} of 3.8 km.

4.3.3 Sampling rate

The sliding correlator in the receiver outputs correlation waveforms that are scaled versions of those which would result were the actual 910 MHz probing signal used in the correlation. From theory, the correlation triangle for a PRBS should be reduced to an amplitude near zero, given a 1 bit (T_1) time skew between reference and "sliding" sequences. For the sliding correlator implementation, each bit in the receiver generated PRBS is offset by ($T_2 - T_1$) in time from that in the corresponding transmitted sequence, where T_1 is the transmit PRBS bit period, and T_2 is the local receiver PRBS bit period. Thus the number of bit periods required for a T_1 second skew is

$$\frac{T_1}{(T_2 - T_1)},$$

and the time required is:

$$T_b = \frac{T_1}{(T_2 - T_1)} \times T_2$$

In the sequel, this is referred to as one scaled bit period.

Since the minimum detectable propagation delay is $.05\mu s$, the minimum detectable scaled delay between correlation peaks in a multipath environment is

$$\tau_{min} = .05\mu s \times f_b / \Delta f, \quad (4-3)$$

where $\Delta f = \frac{1}{T_2 - T_1}$.

This means that the minimum number of samples (n_{min}) that must be taken per scaled bit period in order to take full advantage of the 20 MHz bandwidth is given by

$$n_{min} = \frac{2T_b}{\tau_{min}}, \quad (4-4)$$

where the 2 is a result of Nyquist considerations. For $f_b = 10$ Mb/s and $\Delta f = 1$ kHz,

$$n_{min} = 4.$$

4.3.4 Constraints on T_s dictated by the spatial interval within which a single impulse response estimate must be sampled

For channel characterization measurements it is desired to sample the collection of instantaneous phase and amplitude relationships that exist among the multiple received signals as the receive antenna moves throughout a local multipath environment. It is therefore necessary that the phase of each signal that arrives at the antenna remain relatively constant throughout the recording interval for a single impulse response estimate. The received signal phase variations caused by the motion of the antenna will be maximum for signals received from the direction of antenna travel and will be equal to 360 degrees per wavelength of antenna motion. The antenna speed must therefore be such that sampling within a time interval equal to $N \times T_s$ is performed over a spatial interval that is small compared with a wavelength (31 cm). If a value of one tenth wavelength is assumed to be a good sampling distance, T_s is constrained to be less than or equal to 1 ms if an antenna speed of 0.028 m/s is employed, and N is 127. In the system employed for measurements reported in this thesis, $T_s = 1.002$ ms.

4.3.5 Required spatial interval between impulse response estimate recordings

To characterize a mobile channel it is required to measure impulse responses at spatial intervals that are close enough to enable the sampled data to provide a reasonable representation of the typical range of phase relationships that can be expected among multipath signals. Since the maximum rate of phase change that can be encountered is 360 degrees per wavelength of antenna travel, Nyquist considerations show that impulse response estimates should be recorded at spatial intervals that are less than half a wavelength. During the measurements reported in this thesis, sampling was at intervals slightly larger than one tenth of a wavelength.

CHAPTER V

MEASUREMENTS, DATA ANALYSIS, AND RESULTS

5.0 Introduction

In this chapter, the channel classification and analysis procedures studied and developed in chapters II and III are combined with the results of channel characterization measurements and used to classify an 800/900 MHz urban mobile radio channel with regard to the effect it would have on digital communications. In accordance with the resulting classification, models are then used in conjunction with measured characteristics to predict the performance of a digital (DPSK) communication system operating on the measured channel. Section 5.1 describes the field measurements in which the channel characterization data were recorded. The subsections of section 5.3 discuss data analysis techniques and contain the results of different steps in analysis procedures leading to the channel classifications of Section 5.4. Performance analysis techniques and performance predictions are presented in Section 5.5. Section 5.6 is a summary of the chapter.

5.1 Field measurements

Measurements were conducted along a one city block section of Metcalfe Street, between Laurier and Gloucester Avenues, in downtown Ottawa. This section of street is at a range of 533.75 m in an East North East direction from the transmitter. Line of sight from the measurement area (Fig. 5.1) to the transmitter was blocked by several tall office towers.

The transmitter was housed in a maintenance room in the penthouse on the top of a 24 story office tower, centrally located in the urban core. The transmit antenna (a wideband monopole) was mounted on a steel mast above the penthouse, at an elevation of approximately 78 m above ground level. A block diagram of the installation is shown in Fig. 5.2.

The mobile laboratory housing the receiver was moved in a leap frog fashion so that the complete block along the west side of Metcalfe street was covered in eight measurement sections. In each mobile lab location 128 equally spaced impulse response estimate recordings were made during a three minute antenna run (Chapter 4) at an antenna speed of 0.028 m/s. The data were recorded on nine track magnetic tape, and analysed on a mainframe computer as outlined below.

POWER AT Pt. "A"	- 11.5 dBm
POWER AMPLIFIER GAIN	50 dB
TRANSMISSION LINE LOSS	- 4.3 dB
CONNECTOR LOSS	- 2 dB
POWER DELIVERED TO ANTENNA	33.2 dBm
ANTENNA GAIN	2.8 dBi
EFFECTIVE RADIATED POWER	36 dBm

Fig. 5.2 Transmitter installation block diagram.

5.2 Data analyses and field measurement results

5.2.1 Transmission loss to the measurement area

The free space power budget for the radio path between the transmit antenna and the intersection of Metcalfe St with Gloucester Ave, the first mobile lab measurement location is shown in Table V-I. The table shows the peak received power to be 6.8 dB (measured as discussed in Section 5.2.3) below that which would have been received in a free space environment. It should be noted, however, that Doppler analyses (not presented in this report) indicate that there were several multipath components separated by delay time intervals which were less than the time resolution of the measurement system at the delay time corresponding to this power. The 6.8 dB figure in the table may not therefore be accurate, as phase cancellation could have created a false minimum in the received signal. Such ambiguities cannot be resolved by the measurement system that was used in the experiments. Transmission loss information, however is superfluous to the objectives of the thesis. The power budgets for each of the other mobile lab positions in the measurement area were similar. Received powers differed within the range of -5 and +2 dB.

5.2.2 Measured impulse response estimates

Consider a fixed linear filter having an impulse response $g(t)$. An arbitrary input, $x(t)$ to the filter would produce an output $y(t)$ given by

$$y(t) = \int_{-\infty}^{\infty} x(t-u)g(u)du. \quad (5-1)$$

If both sides of eqn(5-1) are multiplied by $x^*(t-r)$, one can write

$$y(t)x^*(t-r) = \int_{-\infty}^{\infty} x(t-u)x^*(t-r)g(u)du. \quad (5-2)$$

Now, if $x(t)$ is a wide-sense stationary random process, expected values can be taken on both sides of the equation to give

$$R_{yx}(r) = \int_{-\infty}^{\infty} R_{xx}(r-u)g(u)du$$

TABLE V-1

POWER BUDGET CALCULATIONS

Range	534	m
Elevation angle	8.2	deg
Slant range	539	m
Free-space Loss	- 86	dB
Transmit antenna input power	32.2	dBm
Antenna gains	- 5.6	dB
Receive antenna output power	- 48.2	dBm
Transmission losses	- 11.	dB
Free space input power to Rx. bandpass filter	- 59.2	dBm
Correlator pre-amp control voltage for 4 V peak	6.27	V
Effective input power to the Rx. bandpass filter	- 66	dBm
Shadow/Diffraction loss estimation	- 6.8	dB
LNA Gain	20.0	dB
Preamplifier attenuator setting	0	dB
Down-conversion loss	- 6.0	dB
Connector losses	- 2	dB
Microdyne receiver gain	43	dB
Carrier power at 70 MHz receiver output	- 11	dBm
Noise floor at receiver output (measured)	- 45	dBm
C/N at peak receiver power	34	dB

$$=R_{xx}(\tau) * g(\tau). \quad (5-3)$$

where R_{xx} and R_{xy} represent autocorrelation and cross correlation functions for the random processes $x(t)$ and $y(t)$.

It is clear that if $x(t)$ is white noise,

$$R_{xy}(\tau) = \delta(\tau) * g(\tau) = g(\tau).$$

That is, the cross correlation function $R_{xy}(\tau)$ is identical to the response of the filter at time t to an impulse applied to its input at time $(t - \tau)$.

In the pseudo-noise channel measurement system (Chapter IV) implemented as part of this thesis project, an almost white (flat spectrum over a large part of the measurement bandwidth) stationary pseudo-random process $y'(t)$ received over a radio propagation channel is cross-correlated in a complex baseband correlator with a second pseudo-random process, $x'(t)$ which is identical to the random process that was transmitted at time $(t - \tau)$, where τ is the propagation delay of the channel. With the reasonable assumption that tropospheric radio channels are linear, this cross-correlation product can be taken [50] as an estimate of the impulse response, $h(\tau)$, of the radio channel.

For channels on which there is terminal motion, the channel impulse response is time-variant, and is denoted $h(\tau; t)$. This function was measured for the channel under study in this thesis by recording discrete samples of $h(\tau)$ estimates measured over repetitive, short, evenly spaced time (distance) intervals as the measurement system receive antenna (Chapter IV) moved along its track.

Figure 5.3 shows the first impulse response estimate from each of the eight street sections where measurements were recorded. The average impulse response for each section is shown in Fig. 5.4. The ordinates in each of these figures are power scales, being equal to the sum of the squares of the in-phase and quadrature channel correlator output voltages. The abscissas represent time delay with respect to an arbitrary zero reference that was different for each antenna run. The figures show that multiple signals were received over the entire city block. Minimum path delay separations were within the $.05\mu s$ time

Fig. 5.3 The first impulse response estimate recorded on each of eight sections of street.

Fig. 5.4 Averages of 128 impulse response estimates recorded along eight sections of street.

resolution of the measurement system, and multipath spreads ranged between two and four microseconds. Every tenth impulse response estimate recorded for the first antenna run is plotted in Appendix B-1. Reference to this appendix shows that there were significant, rapid variations in the structure of the channel impulse response along this 5.04 m street section. The non-minimum phase characteristics in some of the plots support the inclusion of precursors (ISI from the previously transmitted symbol) in the analyses of Section 3.5.5. Rapid variations such as those of the channel impulse response estimates reported here have deterred the development of hardware equalization techniques for the improvement of digital system performance on mobile radio channels.

5.2.3 Estimation of received power

With the probing system transmitter and receiver connected back-to-back in the laboratory, the pseudo-noise modulated signal at the output from the transmitter was adjusted to pre-determined power levels between -50 dBm and -90 dBm. At each power level, the voltage controlled attenuator at the 70 MHz input to the receiver correlator was adjusted so that the maximum in-phase amplitude at the peak of the pulse at the correlator output was four volts. The value of the attenuator control voltage was recorded for each transmitter output power. This procedure enabled the construction of a gain control voltage/received power calibration chart for the system. During field experiments, the attenuator control voltage was set by running the antenna down the entire length of its track and ensuring that the peak voltage of the correlator output pulse was four volts. Comparison of the attenuator control voltages used for the field measurements with the calibration chart was then used to obtain an estimate of the maximum received power at the delay corresponding to the largest peak in recorded impulse response estimates. This method is fairly accurate so long as the spectrum of the off-air signal recorded in the field is not too badly distorted by multipath propagation. Measurements in a line-of-sight environment, in the absence of multipath propagation showed that the received power estimated in this manner was equal to the calculated free space value.

5.2.4 Envelope fading results

Envelope fading statistics can be derived from measured pseudo-noise impulse response estimates through tracing envelope variations in discrete spectral lines of their Fourier transforms (channel transfer functions) for consecutive recording intervals. The

frequency resolution of transfer functions computed for work presented in this thesis was 156.25 kHz, since 512 point Fast Fourier transforms were computed from impulse response estimates sampled at delay intervals of $\tau = 0.125 \mu s$. It should be noted that the dc component in the transforms was found to be too noisy to be applied in any of the analyses. This is a result of suppressed carrier transmission in the measurement system. The first spectral line in the transforms was therefore at a frequency of 156.25 kHz from midband.

A time series of envelope values for the 156.25 kHz spectral line was computed for each of the eight street sections in the measurement area. These time series (Appendix B-2) invariably exhibited mean value variations, which would normally be attributed to shadowing. However, since the lengths of the measurement sections were short (5.04 m), it is believed that diffraction loss along the transmit path to all antenna positions during each measurement run would have been almost constant. Apparent variations in the mean of computed envelope time series were therefore attributed to finite sampling and are considered to be valid effects of multipath fading, rather than changes in shadowing. For this reason, no filtering was applied to remove these variations before further analysis.

Since the average received power, and receive system gain were not identical on each street section, the mean values of computed envelope time series varied from street section to street section. Therefore, to reduce analysis complexity, and because unity mean envelope time series are required for the flat fading performance calculations, the envelope time series were adjusted to have a mean value of unity. The cumulative distribution functions for these time series and parameters of the best fit Rayleigh distributions were computed. Figure 5.5 shows these distribution functions plotted on the same set of axes as a median-adjusted Rayleigh distribution. It can be seen that correspondence with the Rayleigh function ranged from fair to excellent. Table V-II indicates that variations in the variance parameters were small. Figure 5.6 shows the CDF resulting from the concatenation of envelope fading data from the eight sections of street. Reference to Section 5.2.5.1 shows that the main lobe of the spaced time correlation function for time series measured on the street sections under study almost always decreased to zero, or below after 3 data points from its peak. Every third data point in the concatenated file was therefore taken to be an independent sample. The number of independent samples ($8 \times 128 \times 1/3$) was then substituted in the Kolmogorov-Smirnov equations [54] in order to determine the statistical significance of the CDF for the concatenated file. It found that there is 99 percent probability that the true envelope fading distribution for the

TABLE V-II

ENVELOPE FADING DISTRIBUTION PARAMETERS

Street section	Variance of the experimental data	Variance (Ω)* for the best-fit Rayleigh distribution
1	1.264	1.29
2	1.246	1.24
3	1.2	1.54
4	1.26	1.34
5	1.24	1.37
6	1.24	1.49
7	1.25	1.32
8	1.17	1.54

$$* \Pr(r)dr = \frac{2r}{\Omega} e^{-r^2/\Omega} dr$$

Fig. 5.5 Cumulative distribution functions for the received signal envelope on each of eight sections of street.

Fig. 5.6 The envelope cumulative distribution function resulting from the concatenation of data from files recorded on eight different street sections.

measurement area lies within plus or minus eight percent of the experimentally determined distribution in Fig. 5.6. Since comparison with the Rayleigh distribution in the same figure is excellent, it is concluded that the Rayleigh distribution shown (variance=1.42) is an appropriate model for envelope fading in the measurement area. It should be noted, however, that the results of measurements in other parts of Ottawa show that the Rician distribution is more appropriate due to the existence of a specular reflection, or line of sight path from the transmitter. The analysis of such cases is left to be done as an extension to the work of this thesis.

5.2.5 Spaced time channel correlation functions

In accordance with Section 2.2.3, if the transfer function of the channel, $H(f, t)$, is wide-sense stationary, and if there is uncorrelated scattering, the autocorrelation of the transfer function can be written as

$$R_H(\Delta f, \Delta t) = \frac{1}{2} E\{H^*(f; t) H(f + \Delta f; t + \Delta t)\} \quad (5-4)$$

Now if Δf is set equal to zero, the autocorrelation function for variations in the channel transfer function at a single frequency can be written as

$$R_H(0; \Delta t) = \frac{1}{2} E\{H^*(f; t) H(f; t + \Delta t)\} \quad (5-5)$$

This function is the spaced time correlation function. Although closed form expressions for $H(t)$ cannot be derived from experimental data for substitution into eqn(5-5), $R_H(0; \Delta t)$ can be estimated using time-series analyses techniques. The mathematical form of the estimate employed in this thesis is given in Appendix C-1. Since the speed of the antenna during the experiments was .028 m/s, and 128 (actually 127, since the first one was recorded at $t=0$) impulse response estimates were recorded during a 3 minute interval, the separation between recordings, or sampling resolution, was 3.97 cm.

Variations in the spectral line at 156.25 kHz from midband in the channel transfer functions were again traced in order to compute $R_H(0; \Delta t)$. The autocorrelation functions for individual street sections were used in determining the degree to which the measured channel can be considered to be statistically stationary, as discussed in Section 5.3. The

complex autocorrelation function for the file resulting from the concatenation of data from each of the eight measurement sections is shown in Fig. 5.7. The lag axis in the figure is presented in terms of distance along the street so that results can be interpreted for any vehicle speed. Due to the larger number of data points (1056) the confidence level for this function is greater than that for the individual street sections. It can be seen that the function is oscillatory and that correlation is nonzero over relatively large distances. The real part (which is assumed to be of most significance) is seen to decrease below a value of 50 percent, however, over a distance of only about 10 or 12 cm.

5.2.6 Spaced frequency channel correlation functions

If Δt is set equal to zero in eqn(5-4), that is, the covariance is computed for simultaneous variations of the channel transfer function at different frequencies, one can write

$$R_H(\Delta f; 0) = \frac{1}{2} E\{H^*(f; t) H(f + \Delta f; t)\}. \quad (5-6)$$

This function is the spaced frequency correlation function. The mathematical form of the estimate used to obtain this function from experimentally derived time-series is given in Appendix C-2. The frequency resolution of the computed functions is 156.25 kHz, as for the Fourier transforms from which they were calculated.

Spaced frequency correlation functions for each of the eight measurement sections were computed. The computed function for the first vehicle length beginning at the corner of Gloucester and Metcalfe is shown in Fig. 5.8. The corresponding plots for the other seven sections of street in this measurement area are shown in Appendix B-3. There is little interpretation of these functions that can be offered. It is noteworthy, however, that the real parts of these functions are always nearly symmetrical, while the imaginary parts are nearly anti-symmetrical. This verifies that the uncorrelated scattering assumption (Section 2.2.3) is reasonable for the channel under study. The oscillatory nature of the plots results from the discontinuities in the impulse response estimates. It is believed that these oscillations would not present in the frequency correlation functions for more densely packed urban areas such as New York City [12], as the impulse response functions of mobile radio channels in such areas are more continuous.

Fig. 5.7 Spaced time correlation function for the concatenation of data from eight street sections.

Fig. 5.8 Spaced frequency correlation function computed from data recorded along the first 5 m of street in the measurement area.

5.3 Consideration of the nonstationary character of the measured channels

In a general way, [56] a random process is said to be stationary if its statistical properties are invariant to translation of the index parameter for the process. In particular, a random process is said to be stationary in the wide sense, if it has a finite variance and its autocorrelation function is independent of the index parameter. A considerable body of literature for the characterization of random phenomena is available for wide-sense stationary processes, or for processes which can be considered to be wide-sense stationary over translations of the index parameter that are of practical importance. In order to exploit this body of theory for application in the prediction of digital radio system performance, it is necessary to determine the bandwidths and time periods over which measured channels can be classified as wide-sense stationary for the purpose of digital system error rate computations. Channels which are strictly nonstationary, but which can be assumed to be wide-sense stationary for restricted mathematical computations are referred to as being quasi-wide-sense stationary (Section 2.3).

5.3.1 Stationarity in the time domain

In the time domain the stationarity of a channel can be assessed by plotting spaced time correlation functions for variations in the channel transfer function at a particular frequency, with contiguous time reference values until significant dissimilarities can be seen among the functions. Figure 5.9 shows correlation plots for receive antenna positions separated by 19.85 cm near the centre of the first, fourth and eighth street sections in the measurement area. Each figure represents the autocorrelation function of a time-series of 40 data points (158.8 cm) on either side of its reference, for antenna displacements out to 39.8 cm. The similarity of the plots for different reference points indicates that it is reasonable to consider the measured channel to be wide-sense stationary over at least three measurement points, or 11.91 cm of antenna travel. For a vehicle travelling at a speed of 100 km/hr, this represents a time interval of 4.3 ms.

5.3.2 Stationarity in the frequency domain

Using similar methods to those applied in the time domain, the sensitivity of statistics to translations of the index parameter in the frequency domain can be determined from plots of spaced frequency correlation functions $R_H(\Delta f; 0)$, centred on contiguous reference

Fig. 5.9 Spaced time correlation functions for receive antenna positions separated by 19.58 cm near the centre of the 1st (left), 4th (middle), and 8th (right) street sections.

frequencies. Figure 5.10 shows superimposed spaced frequency correlation functions for reference frequencies separated by 156.25 kHz for data measured in the first, fourth, and eighth street section. The curves in each figure show only minor variations in the correlation functions for frequency shifts up to 312.5 kHz.

5.4 Classification of the measured channel

With reference to Section 2.3 of Chapter II, a digital radio communication channel can be classified as quasi-wide-sense stationary if symbol durations and transmission bandwidths are limited to time and frequency intervals over which there is negligible change in the channel autocorrelation function $R_H(\Delta f; \Delta t)$. In the previous two sections of this chapter, changes in $R_H(0; \Delta t)$ and $R_H(\Delta f; 0)$ have been examined independently. In the time domain, it was found that the spaced time correlation functions changed very little for spatial intervals ranging to about 12 cm along the street. Table V-III shows corresponding time intervals for different vehicle speeds. It can be seen from the table that even at vehicle speeds as high as 100 km/hr, the time interval for quasi-wide-sense stationary behavior is 4.3 ms. Inversion yields the corresponding rectangular symbol rate as 232 symbols/sec. This represents a lower bound on the transmission rates for which mathematical analyses for wide-sense stationary random processes can be applied in the prediction of performance for digital signalling over the measured channel.

In the frequency domain, little change in the mid-band region of spaced frequency correlation functions was evident for frequency shifts up to about 325 kHz. This represents an upper bound on transmission rates for which statistical analysis techniques for wide-sense stationary processes can be applied. In summary, the measured channel is QWSS for :

$$232 \text{ Hz} \leq W \leq 325 \text{ kHz},$$

where W is the transmission bandwidth.

Now that the region over which the channel under study is QWSS has been defined, it remains to classify the channel with respect to its dispersion characteristics. In accordance with the definitions of Section 2.3, this classification can also be effected through an examination of the behavior of the channel autocorrelation functions.

In the strict sense, reference to Section 2.3.1 shows that a digital communication channel is referred to as nonselective in the time domain, or time-flat if the total duration

Fig. 5.10 Space frequency correlation functions computed for three contiguous reference frequencies² from data recorded in each of the 1st (left), 4th (middle) and 8th (right) street sections.

TABLE V-III

QWSS TIME DURATION FOR SELECTED
VEHICLE SPEEDS

Vehicle speed (km/hr.)	Time for which channel is QWSS (ms)
20	21.16
40	10.8
60	7.2
80	5.4
100	4.3

of transmitted symbols, plus the multipath spread on the channel is much less than the correlation time, T_C , being the time interval over which the function $R_H(0; \Delta t)$ is nonzero. There is difficulty with this definition, however, if one considers the complex oscillatory nature of the time autocorrelation functions for the measured channels. Without windowing, Fig. 5.7 for the concatenated file for all eight measured street sections shows that correlation remains nonzero for spatial intervals exceeding 600 cm. For a vehicle travelling at 100 km/hr, this corresponds to a time interval of 220 ms. If the description "much less than" is arbitrarily taken to mean one tenth to permit computations, this indicates that the measured channel is time flat for transmission rates as low as 4.63 symbols/sec. The point at which the real part of the functions decreases below 0.5 (Section 5.2.5), however, defines a more conservative lower bound to transmission rates for flat fading, and is easier to determine, since it can be read from curves representing autocorrelation functions for individual street sections. Reference to Fig. 5.9 shows that almost invariably this occurs at about two data points on either side of the reference, or at a spatial separation along the street of about 7.8 cm from the reference. Table V-IV lists corresponding time intervals for vehicles travelling at different speeds. Again using 100 km/hr as the maximum speed of a vehicle in an urban area, T_C , as defined here, is about 2.81 μ ms. For rectangular data symbols, this corresponds to a data rate of 355 bits/s. Thus, through application of the one-tenth interpretation of "much less than", the measured channels can be classified as time-flat for data rates greater than 355 kb/s.

A digital radio channel is classified as frequency nonselective, or frequency flat if the bandwidth of transmission on the channel is maintained to be much less than the bandwidth over which the spaced frequency correlation function for the channel is nonzero. In this case also there are interpretation problems, since the functions derived from measurements are complex and oscillatory. As with the time domain analysis, for the purposes of this work, the correlation bandwidth will be taken as that frequency separation from the reference frequency at which the real part of the spaced frequency correlation function falls below a value of 0.5. Examination of the plots in Fig. 5.10 shows that this occurs at a frequency shift between 195 kHz and 315 kHz from the reference. The ten percent rule then indicates that, on average, the measured channels should be frequency nonselective for transmission bandwidths below 24 kHz. The results of the performance analyses in Section 5.5 will be compared with this "arbitrarily" established upper bound, and a more quantitative definition of correlation bandwidth will be offered as one of the results of this

TABLE V-IV

CHANNEL CORRELATION TIMES FOR
SELECTED VEHICLE SPEEDS

Vehicle speed (km/hr)	Correlation time (ms)
20	14.0
40	7.0
60	4.7
80	3.5
100	2.8

thesis.

Based upon the boundaries established in the preceding paragraphs as a result of considering the statistics of field data, for reasonable urban vehicle speeds, the measured channel can be classified as nonselective for transmission bandwidths between 3.5 kHz and 24 kHz. For bandwidths below 3.5 kHz, the channel is time-selective, and performance is limited by distortion due to Doppler and shadowing effects. For bandwidths above 24 kHz, performance is limited by intersymbol interference due to multipath propagation. As a result of considering measured channel characteristics presented in this thesis, it does not appear probable that urban digital mobile radio channels can be doubly selective for practical vehicle speeds.

5.5 Performance analyses

It has been shown in Section 5.4 that the urban mobile radio channel under study can be classified either as a flat, or a frequency-selective/time-flat fading channel for practical vehicle speeds and data rates. The distinction between flat and frequency-selective fading is dependent upon the transmission bandwidth. Performance predictions for both of these channel types can be made using the theory developed in Chapter III for frequency-selective channels. In this Chapter, however, as in the examples of Chapter III, data analyses techniques for both types of channels will be used so that results from application of the two methods for lower transmission bandwidths (data rates) can be compared. Another objective is to determine the bandwidth at which the channels become frequency selective by examination of the performance predictions to identify the lowest data rate at which irreducible error effects are evident. In this manner, a practical definition of the frequency correlation properties necessary for flat fading behavior can be obtained.

5.5.1 Data analyses procedures

Identical procedures were used to process the field measurement data recorded on each of the eight sections of street in the measurement area. For each section, each of the 128 recorded impulse response estimates was Fourier transformed to yield the transfer function for the channel between the transmitter and the receive antenna position at which the recording was made. The series of envelope values at a frequency spacing of 156.25 kHz from midband was used in the flat fading computations outlined in the following

subsections. The time series of 128 envelope and phase values at each spectral position was also correlated with the time series at the 156.25 kHz spectral position to obtain the basic spaced frequency correlation function for each section. These functions were used in performance calculations for digital transmission on frequency-selective channels, also outlined below.

5.5.2 Performance computations for flat fading channels

For each section of street, the envelope files for the 156.25 kHz spectral position in the channel transfer function were adjusted have unity mean, thus producing files of β [Ref. eqn(3-12b)]. These data are plotted in Appendix B-2. The variance parameters of the these time series of data points; as well as that for the best fit Rayleigh distributions were computed, and are displayed in Table V-II of Section 5.2.3. It can be seen from the table that there are small differences between the Rayleigh, and the experimental variances, since the experimental results differ slightly from theoretical Rayleigh distributions, as has been shown in Fig. 5.5 (Section 5.2.3). The choice of best fit Rayleigh or experimental variance figures for use in probability of error calculations has an influence on performance predictions. For this reason, performance curves computed using both figures are presented in the results. After computation of the variances, equations (3-17) and (3-18) were used to compute error performance as a function of average E/N_0 .

5.5.3 Performance computations for frequency-selective fading channels

The envelope fading results in Section 5.2.2 show that the Rayleigh distribution is a good model for envelope fading on the measured channels. In addition, the conditions set forth in Section 2.4.1 for a unique relationship between a Rayleigh CDF and its corresponding zero-mean complex Gaussian random process are supported by the symmetry of the experimental spaced frequency correlation functions (uncorrelated scattering) and by intuitive reasoning that a uniform phase distribution is appropriate for the environment under study. It is therefore considered reasonable to model the random transfer function of the measured channel as a zero-mean complex Gaussian random process. The mathematical development for performance predictions on frequency-selective channels presented in Section 3.5 is therefore applicable and was used for error rate calculations for DPSK ($2/T$ Hz bandwidth) transmission on the measured channel. Procedures that were followed are outlined below.

To prepare the experimental spaced frequency correlation functions for use in performance analyses, a number of processing procedures were necessary. First, since data from the dc component in computed channel transfer functions was unreliable, the 156.25 kHz spectral position in the channel transfer function was used as the reference frequency for correlation computations. The $\Delta f = -156.25 \text{ kHz}$ data point in resulting correlation functions was therefore missing. Based upon examination of the unprocessed correlation functions (Appendix B-3), it was considered reasonable to assume perfect symmetry over a 312.5 kHz bandwidth centered on the reference frequency. The correlation value at $\Delta f = -156.25 \text{ kHz}$ was therefore set equal to the value computed for $\Delta f = +156.25 \text{ kHz}$. In addition, it was found that the 156.25 kHz resolution of the experimental spaced frequency correlation functions is not sufficient to permit an accurate computation of the moments for the decision variable from equations (3-45) to (3-48) using numerical integration. For this reason the data was interpolated to obtain a resolution of 4 kHz. After each step in the interpolation, the interpolated data were compared with the experimental data to ensure accurate representation of the experimentally derived functions. One final processing step was necessary. Since it was found during the calculation of the moments that, due to weighting by the sinc functions in the integrands of the moment equations, for symbol rates less than about 100 kb/s, the portions of the correlation functions for $|\Delta f| > 500 \text{ kHz}$ contributed little to the integration results. After interpolation, the functions were therefore chopped at 500 kHz on either side of the reference frequency. Figure 5.11 shows the interpolated function for the first of the eight street sections, along with the experimental function. It can be seen from the figure that the interpolated function is an accurate representation of the original data.

The integrands for the moments represented by equations 3.45 to 3.48 were computed after preparation of the spaced frequency correlation functions. Figure 5.12 shows a plot of the integrand for m_{00} . It is clear that the contribution for $|\Delta f| > 500 \text{ kHz}$ is negligible. The integrations were next performed numerically to obtain the signal terms for the moments. The noise term $2N_0B_N$ was then added to the moments and the probability of error was computed for different values of E/N_0 using eqn(3-41).

5.5.4 Performance analysis results

This section discusses performance predictions for the transmission of DPSK with a bandwidth of $2/T \text{ Hz}$ on the measured channel.

Fig. 5.11 Comparison of the interpolated spaced frequency correlation function from the 1st street section with the original correlation data.

Fig. 5.12 The real and imaginary parts of the integrand in eqn (3-45) for a data rate of 38 kb/s.

Figure 5.13 shows the results for the first street section, and is identical with the results presented in Figures 3.11 and 3.12, except for an added flat fading curve. Figure 5.13(a) shows the interpolated spaced frequency correlation function. Probability of error predictions are shown in Figure 5.13(b). The curve marked "FLAT (MEASURED)" was computed using the variance of the time series of β values calculated by adjustment of the experimental data to have a mean value of unity. The curve marked "FLAT (RAYLEIGH)" was computed using the variance for the best fit Rayleigh distribution. For this particular street section, it can be seen that the best fit Rayleigh, and measured results are identical. It appears also from the figure that irreducible error rate effects begin to influence the curves at bit rates slightly below 9 kb/s (18 kHz transmission bandwidth). It is interesting to note that the real part in the spaced frequency correlation function decreases to a value of 0.5 at a value of $|\Delta f|$ slightly greater than ten times the 18 kHz bandwidth limit, or at about 195 kHz (as read directly from the data file). The symmetry of the function in Figure 5.13 (a) is also noteworthy, as previously discussed in Section 5.2.5.

Figure 5.14 shows results for the second street section. In this case also, the "measured" and "Rayleigh" flat fading curves are identical. Again it is seen that irreducible error rate effects begin to take place for transmission rates close to 9 kb/s. For this street section, however, the spaced frequency correlation function is not so symmetrical, and the correspondence between correlation and flat-fading bandwidth is not the same. The real part of the spaced frequency correlation function can be seen to decrease to the 50 percent level at 275 kHz from midband.

Results for the third street section are shown in Fig. 5.15. Again, the flat/frequency-selective fading boundary is shown to be at a transmission rate of about 9 kb/s. In this figure the "measured" and "Rayleigh" flat fading results are slightly different. The "measured" curve shows greater correspondence with the 9 kb/s frequency selective curve. This is reasonable, since parameters for these two curves were derived from the same field data. The spaced frequency correlation curves in Fig. 5.15(a) are fairly symmetrical, but as for those corresponding to the second street section, have a lower slope in the midband region than the curves for the first section. The real part drops to a correlation figure of 0.5 at about 225 kHz from midband.

Results for the remaining street sections are shown in figures 5.16 to 5.20. The

Fig. 5.13(a) Interpolated spaced frequency correlation function for the 1st street section.

Fig. 5.13(b) Predicted DPSK system performance characteristics for the 1st street section.

Fig. 5.14(a) Interpolated spaced frequency correlation function for the 2nd street section.

Fig. 5.14(b) Predicted DPSK system performance characteristics for the 2nd street section.

Fig. 5.15(a) Interpolated spaced frequency correlation function for the 3rd street section.

Fig. 5.15(b) Predicted DPSK system performance characteristics for the 3rd street section.

Fig. 5.16(a) Interpolated spaced frequency correlation function for the 4th street section.

Fig. 5.16(b) Predicted DPSK system performance characteristics for the 4th street section.

Fig. 5.17(a) Interpolated spaced frequency correlation function for the 5th street section.

Fig. 5.17(b) Predicted DPSK system performance characteristics for the 5th street section.

Fig. 5.18(a) Interpolated spaced frequency correlation function for the 6th street section.

Fig. 5.18(b) Predicted DPSK system performance characteristics for the 6th street section.

Fig. 5.19(a) Interpolated spaced frequency correlation function for the 7th street section.

Fig. 5.19(b) Predicted DPSK system performance characteristics for the 7th street section.

Fig. 5.20(a) Interpolated spaced frequency correlation function for the 8th street section.

Fig. 5.20(b) Predicted DPSK system performance characteristics for the 8th street section.

performance characteristics on all street sections can be seen to be similar, with irreducible error effects beginning to show at transmission rates of about 9 kb/s (bandwidths of about 18 kHz). The spaced frequency correlation functions, however, have a variety of different shapes and slopes in the midband region. Table V-V lists the positive and negative frequency separations from midband at which the real parts of the correlation functions have values of 0.5. The average of the frequency shifts listed in this table is 238 kHz. The bandwidth of 18 kHz corresponding to the 9 kb/s transmission rate which the performance results show is the maximum for flat fading, is eight percent of this value. It is therefore concluded that a slight modification to the ten percent value assumed in Section 5.4 is required.

Designers of modern commercial digital mobile radio systems typically specify that the maximum error rate should be 1×10^{-3} . Reference to results presented in this section shows that performance levels off at about that error rate for data rates just below 10 kb/s (38 kHz transmission bandwidth). The corresponding average bit energy to noise power spectral density ratio (E/N_0) is 36 dB. For operation in thermal noise over a system with a diffraction loss of, say 10 dB, along the transmission path, a transmit antenna gain of 9 dB, a transmit power at the input to the antenna of 20 W, and a receive system equivalent noise temperature (Fig. 5.21) of 6960 Kelvin, transmit range would therefore be limited to 860 meters. If the transmission bandwidth were kept below 18 kHz, so that fading was nonselective, a 4 dB E/N_0 gain could be achieved, increasing the transmit range to 1.4 km. These range figures are reasonable for cellular mobile radio systems. DPSK transmission with single level signalling and bandwidths up to near 38 kHz is therefore concluded to be possible for cellular systems operating within urban environments. For systems operating without a cellular structure, the requirement of a high signal to noise ratio for acceptable error performance would result in severe coverage limitations.

5.6 Summary and discussion

In this chapter, the results of field measurements that were conducted to determine the characteristics of 910 MHz urban mobile radio channels, and performance predictions based on the measured data were discussed. The measurement setup was described as one in which a wideband pseudo-noise probing signal was transmitted from the top of a tall office tower over a non-line-of-sight-urban radio path to a moving receive antenna which ran along a track mounted on the roof of a laboratory vehicle. Channel impulse

TABLE V-V

BANDWIDTHS FOR WHICH THE REAL PART OF THE
SPACED FREQUENCY CORRELATION FUNCTION DECREASES TO 0.5

Street section	Negative frequency (kHz)	Positive frequency (kHz)
1	195	202
2	275	334
3	220	246
4	178	198
5	256	246
6	183	173
7	295	234
8	315	259

RECEIVE SYSTEM NOISE TEMPERATURE CALCULATION:

$$\text{Feeder loss} = L = [\text{line} + \text{connector losses}] = 4 \text{ dB} = 2.5$$

$$\begin{aligned} F_{eq} &= L + \frac{F_{Rn}}{L-1} \\ &= 2.5 + \frac{10-1}{0.4} \\ &= 25. \end{aligned}$$

$$T_{eq} = (25 - 1) \times 290 = 6960 \text{ Kelvin,}$$

where,

F_{eq} represents the equivalent receive system noise figure

T_{eq} represents the equivalent system noise temperature.

For an antenna with a typical brightness temperature of 220 Kelvin the receive system noise temperature would then be

$$T_{sys} = T_{eq} + T_{ant} = 7180 \text{ Kelvin.}$$

Receiver input noise power is thus

$$N = K \times T_{sys} \times W = -114 \text{ dBm,}$$

where, K is Boltzman's constant $= 1.38 \times 10^{-23}$.

Fig. 5.21 Mobile radio system configuration and receiver noise power calculation.

response estimates were recorded for single passages of the antenna along its track with the mobile lab parked in eight contiguous positions along one side of a busy street. These eight street sections covered one complete city block.

Analysis of the data began with an examination of recorded impulse response estimates. These recordings showed that multiple replicas of the transmitted signal were recorded along the entire city block. Minimum time delay separations were within the $0.05\mu\text{s}$ time resolution capabilities of the measurement system, and multipath spreads ranged between two and four microseconds. The structure of the channel impulse response varied rapidly over a significant amplitude range, and occasionally exhibited non-minimum phase characteristics.

Sampled series of envelope values at a single frequency in the equivalent low pass transfer function of the channel for consecutive antenna positions were next derived from fast Fourier transforms of the impulse response estimates. These series of envelope values were then used to compute cumulative distribution functions for envelope fading on each of the eight street sections. The correspondence of computed distribution functions with median-adjusted Rayleigh distributions ranged from fair to excellent. The distribution computed from the concatenation of data from all eight street sections showed excellent correspondence with a Rayleigh distribution for which the variance parameter was equal to 1.42. Due to the larger size of the concatenated file compared with that for the individual street sections, it was practical to make tests regarding the statistical significance of its corresponding cumulative distribution function. The Kolmogorof-Smirnov equations were used to compute confidence limits. It was found that there is a 99 percent probability that the true distribution for envelope fading in the measurement area lies within plus or minus eight percent of the experimentally derived curve. It was therefore concluded that it is reasonable to model the envelope fading process along the city block under study as having a Rayleigh amplitude distribution. This conclusion confirmed the applicability of equations derived in this thesis.

Next, complex spaced time channel correlation functions were computed from the data measured on each street section. The purpose of these computations was to note variations in the computed correlation functions centered on different antenna reference positions, and thus, determine the time interval, at practical vehicle speeds, over which random variations on the measured channel are quasi-wide-sense stationary. It was found

that the channel is quasi-wide-sense stationary over spatial intervals of about 12 cm along the street. This distance was translated into a time interval of 4.3 ms for a vehicle travelling at 100 km/hr, the maximum anticipated in an urban area. For rectangular data symbols, this was calculated to be the bit period for a transmission rate of 232 bits/sec.

In a similar analysis to that for the spaced time correlation computations, complex spaced frequency correlation functions for the channel were computed, and used to determine the bandwidth over which the measured channel is quasi-wide-sense stationary. This bandwidth was found to be about 325 kHz. Based on results summarized in this and the preceding paragraph, for vehicle speeds up to 100 km/hr, the measured channel was classified as quasi-wide-sense stationary for transmission bandwidths (W) within the interval $232 \text{ Hz} \leq W \leq 325 \text{ kHz}$.

Since it was determined that the bandwidth over which the measured channel is quasi-wide-sense stationary is one which is practical to use for mobile system data transmission, it was concluded that techniques developed in Chapter III of this thesis are applicable for the prediction of the performance of systems proposed for operation on the channel. The bandwidths for which flat fading and frequency selective techniques are applicable were therefore determined next.

In Chapter II, based upon information from reference documents, bandwidths within the flat fading region were reported to be those which are much greater than the reciprocal of the correlation time, and much less than the correlation bandwidth of the channel. This definition presented two problems. First, the descriptions "much less than" and "much greater than" had to be interpreted quantitatively. Secondly, practical numerical definitions for correlation time and correlation bandwidth had to be ascertained, since experimentally determined correlation functions were oscillatory and did not tend to zero in a well behaved manner, rendering the theoretical definitions of chapter II to be of little use. In order to allow performance analyses to proceed, "much less than" and "much greater than" were arbitrarily interpreted to mean one tenth, and ten times, respectively, and verification of these assumptions was left until the performance results for frequency selective channel computations could be examined. In addition, after some deliberation, it was considered reasonable to define correlation bandwidth to be that bandwidth at which the real part of the spaced frequency correlation function drops to a value of 50 percent. Similarly, correlation time was defined as that time lag at which the real part

of the spaced time correlation function decreased to a value of 0.5. It was left as part of the thesis project to verify the practicality of these definitions also. Based upon the one-tenth, and 50 percent figures, it was determined that nonselective fading would occur for transmission bandwidths between 3.5 kHz and 24 kHz.

After characterization of the measured channel, performance calculations were made for a DPSK system with a $2/T$ Hz bandwidth operating with data rates for which the channel is nonselective, as well as those for which the channel is frequency selective. The time selective case was not considered, since the channel would be time selective only for transmission rates which are lower than those which are practical. The doubly selective case was considered to be improbable for urban mobile radio channels.

For each street section, the probability of error as a function of the average ratio of bit energy to noise power spectral density was computed for transmission at flat fading bandwidths, and at several bandwidths within the frequency selective range. Two flat fading computations were made for each section. These calculations differed with regard to the fading variance parameter employed. For one of the calculations, the variance of experimentally determined envelope fading data adjusted to have unity mean was used directly. For the second calculation, the variance of the Rayleigh distribution which best fit the experimentally determined envelope fading distribution was used. The difference between results was small for all street sections, and varied in direct relation to a qualitative determination of the goodness of fit of the Rayleigh distribution to the experimental results.

The data rates for which frequency selective calculations were made ranged from 9 kb/s (18 kHz bandwidth), in one octave steps to 78 kb/s. It was found in all cases that little evidence of irreducible error effects was exhibited by the 9 kb/s curves. It was therefore concluded that the upper bandwidth for flat fading is near 18 kHz. In addition, comparison of the 9 kb/s curves with the flat fading curves was excellent. Since the flat fading calculations are widely known and accepted to be correct, this was considered to be verification of the new work presented in this thesis with regard to the calculation of performance on frequency selective channels using experimentally determined channel characteristics. The frequency selective results also showed that the irreducible error rate increases as the transmission bandwidth increases.

The 9 kb/s boundary for flat fading ascertained from the frequency selective per-

formance results was compared with the average frequency separation from midband at which the real part of the spaced frequency correlation function decreased to 50 percent. This comparison showed that the ten percent ratio for the flat fading bandwidth assumed in Section 5.4 should be modified to be eight percent. Also, since at 9 kb/s the channel was a flat-fading channel in all cases, while the correlation functions had various shapes and degrees of symmetry, it was concluded that the exact shape of the correlation function is of a lesser significance. Finally, it was found that it could not be assumed that if the correlation function had a lesser slope (decreased to 50 percent at a greater frequency separation from midband) the irreducible error rate for a certain transmission bandwidth would be lower. No determination of the validity of the time selective boundary assumed in this report was effected, since no time selective calculations were made.

Performance prediction results showed that for any particular transmission rate, performance on all eight street sections where measurements were made would be similar. This could be interpreted as an indication that averaging of results from small sector measurements to obtain performance predictions for larger coverage areas need not be complicated.

As a final step in the analysis and interpretation of results, the typical modern design specification that digital mobile radio systems should operate with error rates less than 1×10^{-3} was considered in relation to the computed DPSK system performance results. It was found that on the city block investigated, the specified maximum error rate was achievable for transmission bandwidths up to about 38 kHz without equalization or error correction, for operation with average bit energy to noise power spectral density ratios greater than 36 dB. For a system with assumed practical receiver noise characteristics it was found that with a 20 W input to a transmit antenna with 9 dB gain in the direction of the receiver, a monopole receive antenna (0 dB gain), and 10 dB diffraction loss along the transmission path, the maximum transmission range for a transmission bandwidth of 38 kHz is about 860 meters. A 4 dB gain is achieved under flat fading conditions, so that at transmission bandwidths below about 18 kHz, the maximum transmission range is extended to be 1.4 km. These ranges are practical for cellular systems, but show that without a cellular structure, digital mobile radio system coverage areas are of limited size for 800/900 MHz systems operating in urban environments.

CHAPTER VI

SUMMARY, DISCUSSION AND CONCLUSIONS

6.1 Introduction

The material presented in this chapter is a consolidation and discussion of the background, preparatory analyses and results of this thesis project. Section 6.2 is a summary and discussion. Conclusions are presented in Section 6.3, and topics for further investigation are the subject of Section 6.4

6.2 Summary and discussion

A significant portion of radio spectrum in the 800/900 MHz frequency band has recently been allocated for land mobile communications. Since it is anticipated that digital transmission will be desired in this new band, it is necessary to characterize channel behavior in order to support modelling for regulatory and system design purposes.

Envelope fading as well as complex impulse response measurements have been made on 800/900 MHz mobile channels in the past by others. Although limited modelling has been reported, the results of such measurements have not hitherto been applied in a comprehensive effort to develop and verify models for channel behaviour from which system performance can be predicted. In this thesis, system performance predictions have been made using models adapted from research on other types of channels and verified as appropriate to the channels under investigation through field measurements.

Since mobile radio channels exhibit severe fading, and are dispersive, past research regarding such channels was studied as a background to this project. It was found that extensive modelling of both fading and dispersion has been performed for the purpose of predicting digital system performance on ionospheric channels. During this work, a number of statistical descriptors and classifications for fading channels were developed for the assessment of the suitability of such channels for digital communications. In addition, it was shown that if channel parameters satisfy certain bounding relationships, various analysis procedures can be applied in order to predict the performance of specific digital communication systems operating over the channels of interest. Based on suggestions made by Cox [11] it was considered probable that the ionospheric models could be applied to 800/900 MHz mobile channels. This was verified by field measurements during the proposal stage of this project. It was therefore set as a research objective to measure small sector characteristics of urban mobile channels and modify existing models to reflect

measured channel characteristics in order to predict performance for a specific digital system.

The field measurements that were made during the proposal stage showed that the channels of interest could be classified as either flat, or as frequency-selective/time-flat fading channels. A number of researchers in the field of mobile radio communications have made performance predictions based on measured or assumed envelope fading characteristics for systems operating with bandwidths for which the channels can be classified as flat fading. These results have been verified as correct. Previous to the work presented in this thesis, however, no performance predictions have been made for systems operating with bandwidths for which the channels are frequency-selective. Since digital system performance characteristics on flat and frequency-selective channels overlap for narrow transmission bandwidths, it was considered that verification of new results for the frequency-selective case could be achieved through a comparison of results for narrow bandwidths. It was also considered to be desirable, however, that, as an extension to the thesis project, a digital system be constructed so that simultaneous characterization and performance measurements could be made. In this way, additional verification of the results could be obtained through the comparison of actual performance measurements with performance predictions. For this reason, a significant amount of effort was expended during the thesis project on the selection, and development of a digital communication system to the stage at which it was certain that the system would operate on the channels of interest. The system which was developed for this work is a DPSK system which uses a unique complex structure in the receiver demodulator. Rather than being designed for optimum performance, the system was designed so as to introduce minimum distortion of received digital symbol elements, thus allowing performance to be determined by the channel characteristics, rather than those of the receiver. In this way, it was considered that the results of this work would be more generally applicable.

It was found that the minimum distortion criterion required that receiver filter bandwidths be left extremely wide. In order to measure system performance as a function of average bit energy to noise ratio in planned experiments white Gaussian noise will be added at the input to the demodulator. Since the demodulator filters are also much wider than necessary, a problem arose as to the appropriate choice of a bandwidth for the additive noise. Since the digital signal transmitted over the system will be modulated with a pseudo-random sequence, most of the energy per bit is contained within a bandwidth

of $2/T$ about the centre frequency. The additive noise will therefore be processed so that it will have an equivalent bandwidth of $2/T$. The receiver model employed in the system performance analyses presented in this thesis is based upon the system described in the foregoing paragraphs.

As a result of the differences between the receiver model employed for work in this thesis, and those employed for predictions on the ionospheric channels, a new derivation of mathematical equations for the probability of error had to be developed. Additional differences exist as a result of the requirement in this work to partition equations so that complex correlation functions derived from channel characterization measurements could be included in the analyses. This is the first time that experimentally derived, rather than assumed functions have been employed in such computations. It was an objective of this work to derive the required mathematical relationships using a minimum of arbitrary assumptions regarding the channel characteristics. It was found, however, that in order to achieve mathematical tractability, it had to be assumed that the channels under investigation could be modelled as Gaussian quasi-wide-sense stationary uncorrelated scattering channels. It was verified, however, through the examination and analysis of measured channel characteristics that all criteria associated with this assumption were satisfied. Equations that were derived during the thesis project were used in an example which showed that results are reasonable and system performance trends are as anticipated.

Existing theory for flat fading channels had also to be modified for applicability to the chosen receiver model, and so that experimentally derived channel characteristics could be used in probability of error equations. This work involved special consideration of the wide bandwidth of the receiver proposed for verification experiments. In addition, a normalization procedure that resulted in equations which included the statistics of envelope fading data derived from pseudo-noise channel impulse response estimate measurements was developed. Data from the same experiments as those used in the frequency-selective performance example were also used for flat fading calculations. It was found that agreement between performance characteristics calculated via these two different techniques for narrow transmission bandwidths was excellent. This provided the desired verification of equations and techniques for the prediction of digital system performance on frequency-selective channels developed in this thesis.

In addition to the development of methods for the prediction of the performance

of digital systems on 800/900 MHz mobile radio channels, it was also an objective of this thesis project to develop techniques for the measurement and characterization of such channels, and to apply these techniques in the analysis of measured data. For this purpose, a pseudo-noise channel probing system was developed for the measurement and recording of urban mobile radio channel impulse response estimates.

The channel impulse response measurement system was used to record impulse response estimates for a mobile radio channel in the urban core of Ottawa. The probing signal was transmitted from the top of a tall office tower over a non-line-of-sight path to a moving receive antenna which ran along a track affixed to the roof of a laboratory vehicle in order to simulate mobile operation. Channel impulse response estimates were recorded for single passages of the antenna along its track with the mobile lab parked in eight contiguous parking positions along one side of a busy city street. These eight street sections covered one complete city block.

Analysis of the data began with an examination of recorded impulse response estimates. These recordings showed that multiple replicas of the transmitted signal were received along the entire city block. Minimum radio path delay separations were within the $.05\mu\text{s}$ time resolution capabilities of the measurement system, and multipath spreads ranged between two and four microseconds. The structure of the channel impulse response varied rapidly over a significant amplitude range, and occasionally exhibited non-minimum phase characteristics. The non-minimum phase characteristics were accounted for in equations for performance prediction by consideration of the possibility of intersymbol interference from the symbol following the symbol of interest, as well as the previously transmitted symbol.

Sampled series of envelope values at a single frequency in the equivalent low pass transfer function of the channel for consecutive antenna positions were next derived from fast Fourier transforms of the impulse response estimates. Such data have not been previously derived from pseudo-noise measurements, and the techniques that were employed are a novel contribution of this thesis. The envelope time series were used to compute cumulative distribution functions for fading on each of the street sections. The correspondence of computed distribution functions with Rayleigh distributions adjusted to have the same medians ranged from fair to excellent. Parameters of the best fit Rayleigh distributions varied little, however, for data from different street sections. A distribution function

computed from the concatenation of data from all eight street sections showed excellent correspondence with a Rayleigh distribution for which the variance parameter was equal to 1.42, or 1.5 dB. Confidence limits were computed for this distribution which showed that there is 99 percent probability that the true distribution for envelope fading along the measured city block lies within plus or minus eight percent of the experimentally determined curve. It was therefore concluded that it is reasonable to model the envelope fading process along the city block where measurements were made as having a Rayleigh distribution. This conclusion confirmed the applicability of equations derived in this thesis for flat fading performance calculations. In addition, since correlation functions showed that the uncorrelated scattering assumption is applicable, the unique correspondence between Rayleigh envelope fading and Gaussian channel characteristics reported by Lin [43] was used to show that the channel transfer function could be modelled as a complex Gaussian random process.

Next, spaced time correlation functions were computed from the data measured on each street section. The purpose of these computations was to enable the observation of variations in the computed correlation functions for different antenna reference positions. In this way, the time interval at practical vehicle speeds over which random variations on the measured channel can be considered quasi-wide-sense stationary were determined. It was found that the channel is wide-sense stationary over spatial intervals of about 12 cm along the street. This distance was translated into a time interval of 4.3 ms for a vehicle travelling at 100 km/hr, the maximum anticipated in an urban area. For rectangular data symbols, this was calculated to be the symbol duration corresponding to a transmission rate of 232 bits/sec.

In a similar analysis to that for the spaced time correlation functions, complex spaced frequency correlation functions for the channel were computed and used to determine the bandwidth over which the measured channel could be considered to be quasi-wide-sense stationary. This bandwidth was found to be about 325 kHz. Based on results summarized in this and the preceding paragraph, for vehicle speeds up to 100 km/hr, the measured channel was classified as quasi-wide-sense stationary for transmission bandwidths between 232 Hz and 325 kHz. The techniques developed to extract time and frequency correlation data from pseudo-noise channel measurements are novel results of this thesis, as are the manner in which they were employed to characterize the channel.

Since it was determined that the bandwidth over which the measured channel could be considered to be quasi-wide-sense stationary is one which is practical to use for mobile system data transmission, it was concluded that performance evaluation techniques developed in this thesis are applicable to the measured channel. The bandwidths for which flat fading and frequency-selective techniques are applicable were therefore determined next.

Based upon information from reference documents, it was determined that bandwidths within the flat fading region are those which are much greater than the reciprocal of the correlation time, and much less than the correlation bandwidth of the channel. This definition presented two problems. First, the descriptions "much less than" and "much greater than" had to be interpreted quantitatively. Secondly, practical numerical definitions for correlation time and correlation bandwidth had to be ascertained, since experimentally determined correlation functions were oscillatory, rendering the theoretical definitions to be of little use. In order to allow performance analyses to proceed, "much less than" and "much greater than" were arbitrarily interpreted to mean one tenth and ten times respectively. In addition, after some deliberation, it was considered reasonable to define correlation bandwidth as that bandwidth at which the real part of the spaced frequency correlation function drops to a value of 50 percent. The real part of this function was chosen for use in the definition because it provides the most significant contribution to the moments in the probability of error equations. This is because the imaginary part is smaller and being antisymmetrical, integrates to zero in some of the equations. The same definition was applied to the time statistics for the channel. That is, correlation time was defined as that time lag at which the real part of the spaced time correlation function decreased to a value of 0.5. It was left as an objective of the thesis project to verify the practicality of all "arbitrarily" chosen definitions. Based upon the one-tenth and 50 percent figures, it was determined that nonselective (flat) fading would occur for transmission bandwidths between 3.5 kHz and 24 kHz.

After characterization of the measured channel, performance calculations were made for a DPSK system operating over a similar channel with transmission bandwidths corresponding to the flat and frequency-selective fading ranges. Two flat fading calculations were made for each of the eight street sections where measurements were made. For one of the calculations the variance of experimentally determined envelope fading data was used directly. For the second calculation, the variance parameter of the Rayleigh distribution

which best fit the experimentally determined envelope fading distribution was used. The difference between computed results was small for all street sections, and varied in direct relation to a qualitative determination of the goodness of fit of the Rayleigh distribution to the experimental results.

The data rates for which frequency-selective performance calculations were made ranged between 9 kb/s (18 kHz transmission bandwidth) and 78 kb/s in one octave steps. It was found that in all cases, little evidence of irreducible error affects caused by frequency-selective fading were exhibited by the 9 kb/s curve. It was therefore concluded that the upper bandwidth for flat fading is near 18 kHz. In addition, it was found that the irreducible error rate due to intersymbol interference increases monotonically with transmission rate for transmission bandwidths greater than that corresponding to the flat fading boundary.

The 9 kb/s boundary for flat fading ascertained from the frequency-selective performance results was compared with the average frequency separation from midband at which the real part of the computed spaced frequency correlation functions decreased to a value of 0.5. This comparison showed that the ten percent ratio arbitrarily assumed for the calculations should be modified to be eight percent.

Results showed that digital system performance would be similar on all eight street sections in the measurement area. It had been anticipated that averaging techniques would have to be developed to predict wide area performance characteristics from small sector measurements. The similarity of results on the city block where measurements were made suggests, however, that such averaging may not be necessary. Determination of the requirement for averaging, and averaging methods are left as a topic for further investigation.

In order to demonstrate the practical utility of the results of this thesis, the typical modern design specification that digital mobile radio systems should operate with error rates less than 1×10^{-3} was considered in relation to computed DPSK system performance results. It was found that on the city block where measurements were made, the specified maximum error rate was achievable for transmission bandwidths up to about 38 kHz without equalization or error correction, for operation with average bit energy to noise ratios greater than 36 dB. For a system with assumed practical receiver noise characteristics

($T_{\text{ant}} = 6960 \text{ K}$), it was found that with a 20 W input to a transmit antenna with 9 dB gain in the direction of the receiver, a monopole receive antenna (0 dB gain) and 10 dB diffraction loss along the transmission path, the maximum coverage range for a transmission bandwidth of 38 kHz is about 860 meters. A 4 dB gain is achieved under flat fading conditions so that at transmission bandwidths less than about 18 kHz, the transmission range is extended to 1.4 km. These ranges are practical for cellular systems, but show that without a cellular structure, digital mobile radio coverage areas are of limited size for 800/900 MHz systems operating in urban environments.

6.3 Conclusions

As a result of the work performed during this thesis project, it can be concluded that 800/900 MHz mobile radio channels can be modelled as quasi-wide-sense stationary uncorrelated scattering channels. Further, it can be concluded that it is possible to predict the performance of digital systems operating on these channels with data rates (transmission bandwidths) which are sufficiently high that performance is limited by intersymbol interference rather than by noise. In addition, new techniques and equations for making such predictions have been developed.

The objective of this thesis project was to develop and verify modelling and prediction methods, rather than to present sufficient data to permit general design guideline for mobile systems to be set down. The development of such guidelines is a natural extension of the project and would involve extensive field measurements and considerable subsequent analysis. As a result of applying measurement, data analysis, and prediction techniques, many of which have been newly developed as part of this thesis project, to data collected over a limited coverage area, however, the general characteristics of the channels that were studied can be stated in more detail than has previously been possible.

A summary of generalized 800/900 MHz urban mobile radio channels can be written as follows. First, such channels are randomly time-variant and dispersive in both the time and frequency domains. At practical vehicle speeds (below 100 km/hr) and data transmission rates (above several hundred bits per second), however, frequency dispersion is not a problem, and channel capacity is limited by intersymbol interference effects resulting from multipath propagation. Minimum time delay separations between received multipath signal components can be expected to be less than $.05 \mu\text{s}$. On average, in a

city similar to Ottawa, it can be expected that multiple replicas of the transmitted signal will be received over time intervals ranging up to $4\text{-}\mu\text{s}$. Based on material in reference documents, it can be concluded that this latter quantity varies from city to city, and can be expected to be as great as $10\text{ }\mu\text{s}$ in larger urban areas. In some locations, urban channels exhibit non-minimum phase characteristics. This necessitates the inclusion of impulse response precursors in intersymbol interference analyses. The impulse response characteristics of the channels of interest vary rapidly with receive position along a city street. Over small sectors the Rayleigh distribution is believed to provide an adequate description of envelope fading for modelling purposes where there is no line of sight to the transmitter, or received specular component. It is conjectured that a Rician distribution is more appropriate in the latter cases. Under Rayleigh fading conditions, the complex Gaussian decomposition can be applied if it can be shown that scattering is reasonably uncorrelated. This was found to be the case on the channel that was measured. Channel characteristics are quasi-wide-sense stationary over distances of travel of about 12 cm and over frequency bandwidths of about 325 kHz. This characteristic allows the application of analyses procedures appropriate to wide-sense stationary random processes to be applied in the prediction of the performance of mobile digital systems operating with transmission capacities within the specified range.

A conclusion of this thesis is that the bandwidth boundary for flat fading performance is about eight percent of the frequency separation from midband at which the real part of the spaced frequency correlation function for the channel decreases to a value of 0.5. If it can be assumed that the area in Ottawa chosen for experiments is typical of urban environments for medium sized cities, it can also be concluded from performance calculation results that an error rate of 1×10^{-3} can be achieved without equalization or error correction for transmission bandwidths up to about 38 kHz, and average bit energy to noise ratios greater than 36 dB. A 4 dB gain can be achieved for bandwidths narrower than the flat fading boundary of about 18 kHz.

Through the use of techniques developed in this thesis, coupled with extensive channel characterization measurements in the area of interest it should be possible to predict the performance of proposed digital systems on 800/900 MHz mobile radio channels before licensing or construction. This is a significant addition to the resources currently available to design engineers and regulatory authorities. Extensions to this work that would provide further contributions are discussed as topics for further investigation in the following, final

section of this chapter.

6.4 Topics for further investigation

During the course of the work for this thesis project, a number of topics for research were considered which could not be investigated within a reasonable period of time for inclusion in this thesis. These topics are discussed individually in the subsections to follow.

6.4.1 Performance predictions for data rates above 100 kb/s

It was found during the analyses of data used for performance predictions presented in this thesis that with data rates above 100 kb/s (specifically 156 kb/s), predicted performance is better than that for narrower transmission bandwidths at low bit energy to noise ratios. The 156 kb/s curve in Fig. 6.1 is an example. It is probable that this is the result of programming inaccuracies, or some function of the frequency resolution of experimentally derived correlation functions. These characteristics should be investigated more closely in order to determine the reason for the unexpected behavior.

6.4.2 Bit error rate measurements

As discussed in the summary section of this chapter, it is considered desirable to conduct simultaneous characterization and performance measurements on 800/900 MHz mobile channels for transmission bandwidths in the frequency-selective range. In this way, additional verification of the frequency-selective performance predictions could be achieved. In addition, the effects of impulsive noise, and non-Gaussian channel behavior, not studied during the thesis project could be assessed. It is conceivable that correction factors could be derived if necessary to permit application of the Gaussian model for predicting the performance of systems operating on radio channels in environments, the models for which would result in a requirement for analyses which are not mathematically tractable.

6.4.3 Performance analyses based upon envelope measurements

The measurements that were performed as part of this thesis project involved very complex measurement and recording systems that took several years to develop. Such equipment would probably not be available to practicing engineers and systems designers.

Fig. 6.1 Predicted DPSK system performance characteristics for the 1st street section, showing the curve for a data rate of 156 Kb/s.

For this reason, it is desirable to determine the effects of using envelope correlation data that could be measured with less sophisticated equipment in place of the complex functions employed for analyses in this thesis. Some preliminary work has been done which indicates that predictions based on envelope correlations differ only slightly, and conservatively from those based on complex data. It remains to design a simple propagation measurement system which can be used as a low-budget means for obtaining channel characteristics necessary to make good approximate performance predictions. Such a system could be utilized by design engineers to make propagation surveys in the region of interest before commencing system design work.

6.4.4 Prediction of performance over large coverage areas

Performance predictions made in this thesis are applicable to small coverage sectors to which communications are primarily effected by multipath propagation. Within such small sectors, it was found that performance characteristics over even shorter sub-sectors are similar enough that averaging is not required. For the prediction of performance over larger areas, however, techniques will have to be developed for the prediction of average wide-area performance from small sector measurements. At least three approaches are possible. System performance predictions such as those presented in this thesis could simply be averaged for a large number of small sectors. Alternatively, measured channel characteristics could be averaged, and predictions could be based on the average characteristics. The third approach would be to concatenate data files from all small sector measurements into one large file for a complete coverage area. Predictions would then be made based upon the characteristics of this "super" data file. No matter which approach is taken, allowances would have to be made for the inclusion of variations in the mean of the received signal due to changes in diffraction loss on the channel.

6.4.5 Performance predictions for areas where a specular component is received

Neither the flat fading, nor the frequency-selective channel analyses techniques developed in this thesis are applicable to channels over which a direct signal from the transmitter, or a specular reflection is received. In the flat fading case, an extension to include such conditions would involve substitution of a Rician or Nakagami distribution for the Rayleigh model used in developing the probability of error equation. In the frequency-selective case, the quadratic form for the decision variable would have to be modified to

reflect nonzero-mean values for the sampled waveforms at the demodulator output. This work would be straight forward but time consuming.

6.4.6 Consideration of frequency dispersion

A further extension to the work of this thesis could be made in the form of modifications to the mathematics for the inclusion of frequency dispersion effects. Such an extension could be based on the work of Bello [1], and would result in prediction techniques which are applicable to mobile channels operating in environments where vehicle speeds are greater than those which are typical in urban areas. Examples include rural, marine, aeronautical, and satellite channels.

6.4.7 The investigation of optimum modulation schemes, and receiver structures

Mathematical relationships developed in this thesis for performance prediction are based on a specific modulation scheme and receiver structure. Logical extensions to this work would include repeating the analyses for different modulation schemes and more efficient receivers. The effects of pulse shaping, and the optimum choice of receiver filters could be studied.

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By definition, the average power † of the bandpass signal $A(t)$ is given by

$$\begin{aligned} P_A &= \frac{1}{T} \int_{-\infty}^{\infty} A^2(t) dt \\ &= \frac{1}{T} \int_{-\infty}^{\infty} [\operatorname{Re}\{a(t)e^{j2\pi f_c t}\}]^2 dt, \end{aligned} \quad (A-1-1)$$

where $a(t)$ is the complex envelope low pass equivalent for $A(t)$.

Using the definition $\operatorname{Re}(z) = \frac{1}{2}(z + z^*)$, one can write

$$P_A = \frac{1}{2} \int_{-\infty}^{\infty} |a(t)|^2 dt + \frac{1}{2} \int_{-\infty}^{\infty} |a(t)|^2 \cos[4\pi f_c t + 2\theta(t)] dt, \quad (A-1-2)$$

where, $\theta(t)$ is the original phase angle of $A(t)$.

The modulating function, $|a(t)|$, varies slowly relative to the rapid variation exhibited by the cosine function in the second integral of eqn(A-1-2). The net area contributed by the second integral is therefore small relative to that contributed by the first integral, and one can write the approximate relationship

$$P_A \approx \frac{1}{2T} \int_{-\infty}^{\infty} |a(t)|^2 dt, \quad (A-1-3)$$

from which it is clear that the power of the complex low pass envelope is twice that of the bandpass signal.

† Proakis, J.G., "DIGITAL COMMUNICATIONS", McGraw Hill, New York, 1983, pp. 105-108.

APPENDIX A-2
DERIVATION OF A PROBABILITY DENSITY FUNCTION
FOR $\gamma = \beta^2 E/4N_0$ WHEN β HAS A RAYLEIGH DISTRIBUTION†

From Proakis†, if Y is a quadratic function of X , given by

$$Y = aX^2 + b,$$

then the probability density function for Y can be derived from the equation

$$p_Y(y) = \frac{p_X(\sqrt{(y-b)/a})}{2a\sqrt{(y-b)/a}} + \frac{p_X(-\sqrt{(y-b)/a})}{2a\sqrt{(y-b)/a}} \quad (A-2-1)$$

Therefore if β has a Rayleigh distribution,

$$p_\beta(\beta) = \frac{2\beta}{E\{\beta^2\}} e^{-\beta^2/E\{\beta^2\}}, \beta \geq 0, \quad (A-2-2)$$

and, using eqn(A-2-1),

$$p_\gamma(\gamma) = \frac{p_\beta(\sqrt{\gamma/(E/4N_0)}) + p_\beta(-\sqrt{\gamma/(E/4N_0)})}{2(E/4N_0)\sqrt{\gamma/(E/4N_0)}}, \quad (A-2-3)$$

where

$$\gamma = \beta^2 E/4N_0.$$

Then, after noting that $p_\beta(\beta) = 0$, for $\beta < 0$, and substitution for γ , it is easy to show that

$$p_\gamma(\gamma) = \frac{1}{\xi} e^{-\gamma/\xi}, \quad (A-2-4)$$

where,

$$\xi = \frac{E}{4N_0} E\{\beta^2\}.$$

† PROAKIS, J.G., "DIGITAL COMMUNICATIONS", McGraw Hill, New York, 1983, page 14.

APPENDIX A-3
TRANSFORMATION OF VARIABLES
IN EQUATION (3-31)

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Given the equation

$$m_{00} = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(f_1) S^*(f_2) R_H(f_2 - f_1) e^{j2\pi f_1 t_0} df_1 df_2 \quad (A-3-1)$$

from Section 8.5.3, a change of variables can be performed from the f_1 and f_2 domains to the f and Δf domains respectively, as follows.

The variable f_1 can be written as a function of the new variables, f and Δf as

$$f_1 = \phi_1(f, \Delta f) = f.$$

Similarly, f_2 can be written as

$$f_2 = \phi_2(f, \Delta f) = f + \Delta f.$$

Then it is possible to formulate the Jacobian of the transformation,

$$J = \frac{\partial(f_1, f_2)}{\partial(f, \Delta f)} = \begin{vmatrix} \frac{\partial f_1}{\partial f} & \frac{\partial f_1}{\partial \Delta f} \\ \frac{\partial f_2}{\partial f} & \frac{\partial f_2}{\partial \Delta f} \end{vmatrix}.$$

Upon evaluation of the partial differentials,

$$J = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1.$$

Then in accordance with Kreyszig †, eqn(A-3-1) can be rewritten in terms of f and Δf as

$$m_{00} = \frac{1}{2} \int \int S(f) S^*(f + \Delta f) R_H(\Delta f) e^{-j2\pi \Delta f t_0} \times |J| df d\Delta f. \quad (A-3-2)$$

where the limits of integration must be converted to those appropriate to the new variables.

† Kreyszig, E., "ADVANCED ENGINEERING MATHEMATICS", John Wiley & Sons, 2nd Ed, New York, 1972, page 333.

For $f = f_1$:

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and

when $f_1 = -\infty, f = -\infty$,

when $f_1 = \infty, f = \infty$.

For $\Delta f = f_2 - f_1$,

and

when $f_2 = \infty, \Delta f = \infty$,

when $f_2 = -\infty, \Delta f = -\infty$.

Therefore, substitution of the appropriate limits, and the value $|J| = 1$ gives

$$m_{00} = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(f) S^*(f + \Delta f) R_H(\Delta f) e^{-j2\pi \Delta f t_0} df d\Delta f.$$

(A-3-3)

APPENDIX A-4
SUMMARY OF MOMENT EQUATIONS
(UNCORRELATED SCATTERING ASSUMED)
($t_0 = \frac{T}{2}$)

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For All Sequences

$$m_{00} = \frac{E}{\pi \Delta f T} \int_{-\infty}^{\infty} [\gamma(\Delta f) \cos \pi \Delta f T + \sigma(\Delta f) \sin \pi \Delta f T] \sin(2\pi \Delta f T) d\Delta f + 2N_0 B_N \quad (A-4-1)$$

$$m_{11} = \frac{E}{\pi \Delta f T} \int_{-\infty}^{\infty} [\gamma(\Delta f) \cos(\pi \Delta f T - \sigma(\Delta f) \sin(\pi \Delta f T)] \sin(2\pi \Delta f T) d\Delta f + 2N_0 B_N \quad (A-4-2)$$

For sequences 1,2,6,7,(11),(12),(13),(16).
[brackets indicate the moments are the negative of those listed]

$$m_{01} = \frac{E}{\pi \Delta f T} \int_{-\infty}^{\infty} \sin 2\pi \Delta f T [\gamma(\Delta f) \cos \pi \Delta f T + \sigma(\Delta f) \sin \pi \Delta f T] \\ + j \sin 2\pi \Delta f T [\sigma(\Delta f) \cos \pi \Delta f T - \gamma(\Delta f) \sin \pi \Delta f T] d\Delta f + 0 \quad (A-4-3)$$

For sequences 3,4,5,8,(9),(10),(14),(15)

$$m_{01} = \frac{E}{\pi \Delta f T} \int_{-\infty}^{\infty} [1 - \cos 2\pi \Delta f T] [\gamma(\Delta f) \sin \pi \Delta f T - \sigma(\Delta f) \cos \pi \Delta f T] \\ + j [1 - \cos 2\pi \Delta f T] [\gamma(\Delta f) \cos \pi \Delta f T + \sigma(\Delta f) \sin \pi \Delta f T] d\Delta f + 0 \quad (A-4-4)$$

APPENDIX B-1

**IMPULSE RESPONSE ESTIMATES FOR
THE FIRST STREET SECTION IN THE
MEASUREMENT AREA**

(a) No. 1

(d) No. 30

(b) No. 10

(e) No. 40

(c) No. 20

(f) No. 50

Fig. B-1.1 Selected impulse response estimates from the first street section.

(g) No. 60

(j) No. 90

(h) No. 70

(k) No. 100

(i) No. 80

(l) No. 120

Fig. B-1.1 Impulse response estimates (cont'd).

APPENDIX B-2

ENVELOPE TIME SERIES FOR THE EIGHT
STREET SECTIONS IN THE MEASUREMENT AREA

(a) Street section No. 1.

(b) Street section No. 2.

(c) Street section No. 3.

(d) Street section No. 4.

Fig. B-2.1 Envelope time series for the eight street sections in the measurement area.

(e) Street section No. 5.

(f) Street section No. 6.

(g) Street section No. 7.

(h) Street section No. 8.

Fig. B-2.1 Envelope time series (cont'd).

Fig. B-3.1 Spaced frequency correlation function for the second street section.

Fig. B-3.2 Spaced frequency correlation function for the third street section.

Fig. B-3.3. Spaced frequency correlation function for the fourth street section.

Fig. B-3.4 Spaced frequency correlation function for the fifth street section.

Fig. B-3.5 Spaced frequency correlation function for the sixth street section.

Fig. B-3.6 Spaced frequency correlation function for the seventh street section.

Fig. B-3.7 Spaced frequency correlation function for the eighth street section.

3 of/de 3

APPENDIX C-1

ESTIMATION OF SPACED TIME CORRELATION FUNCTIONS

By definition, the autocovariance of any complex random time waveform, $V(t) = V_I(t) + jV_Q(t)$, is given by

$$\gamma_V(\Delta t) = E\{[V(t) - E\{V(t)\}][V^*(t + \Delta t) - E\{V^*(t + \Delta t)\}]\}. \quad (C-1-1)$$

Let the sampled equivalent of $V_I(t)$ be represented by a_i , and let the sampled equivalent of $V_Q(t)$ be b_i . Then, if the time increment, Δt is spanned by h samples, an estimate for $\gamma_V(\Delta t)$ is given† by

$$\hat{\gamma}_V(h) = \frac{1}{N-h} \sum_{i=1}^{N-h} (a_i - \bar{a})(a_{i+h} - \bar{a}_{i+h}) + (b_i - \bar{b})(b_{i+h} - \bar{b}_{i+h}) + j[(a_i - \bar{a})(b_{i+h} - \bar{b}_{i+h}) - (b_i - \bar{b})(a_{i+h} - \bar{a}_{i+h})], \quad (C-1-2)$$

where N is the number of elements in the sampled time series equivalent for $V(t)$, and the overlines represent estimated means given by

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{N+h} x_{i+h}$$

For calculation of spaced time correlation functions for the measured channels, the complex channel transfer values at 156.25 kHz from midband were read from files containing the Fourier transforms of a series of measured impulse response estimates. This gave a time series of complex values for a single spectral position in the channel transfer functions, from which $R_H(0; \Delta t)$ was estimated by replacing $V(t)$ in eqn(C-1-2) by $H(1.5625\text{kHz}; t)$.

Note that the measured time series data had small nonzero means, whereas it was assumed that the channel transfer function had a mean value of zero in the derivation of the equations for probability of error. The mean in the sampled series is a result of the small sample sizes, and was accounted for as in eqn(C-1-1). Justification for the zero mean assumption in the derivations lies in the Rayleigh envelope fading distributions, and the sample means should not be taken as an indication the the derivations are not valid. For large sample sizes, these means would converge to zero.

† Fuller, W.A., "INTRODUCTION TO STATISTICAL TIME SERIES", John Wiley & Sons, New York, 1976, chapter 6, page 236.

APPENDIX C-2

ESTIMATION OF SPACED FREQUENCY CORRELATION FUNCTIONS

By definition, the cross covariance of any two complex random processes x and y is given by

$$\gamma_{xy} = E\{(x - E\{x\})(y^* - E\{y^*\})\}.$$

Let $x = (a + jb)$ and $y = (c + jd)$. Then

$$E\{x\} = E\{a\} + jE\{b\},$$

and

$$E\{y^*\} = E\{c\} - jE\{d\}.$$

The first term in γ_{xy} then becomes

$$(x - E\{x\}) = [(a - E\{a\}) + j(b - E\{b\})].$$

Similarly,

$$(y^* - E\{y^*\}) = [(c - E\{c\}) - j(d - E\{d\})].$$

Let

$$A = a - E\{a\},$$

$$B = b - E\{b\},$$

$$C = c - E\{c\},$$

and,

$$D = d - E\{d\}.$$

Then

$$\gamma_{xy} = E\{AC\} + E\{BD\} - jE\{AD\} + jE\{BC\}.$$

Expanding the first term,

$$E\{AC\} = E\{(a - E\{a\})(c - E\{c\})\} = \gamma_{AC}.$$

In the analysis of a time series of measured data points, the cross covariance function γ_{AC} can be estimated † by

$$\hat{\gamma}_{AC} = \frac{1}{n} \sum_{i=1}^n (a_i - \bar{a})(c_i - \bar{c}),$$

where, n is the number of data points in the series, a_i represents the i th data point, and $\bar{a}$ represents the average value of a , or $\bar{a} = \frac{1}{n} \sum_{i=1}^n a_i$. The other terms in the equation for γ_{xy} can therefore be estimated by

$$\hat{\gamma}_{BD} = \frac{1}{n} \sum_{i=1}^n (b_i - \bar{b})(d_i - \bar{d}),$$

$$\hat{\gamma}_{AD} = \frac{1}{n} \sum_{i=1}^n (a_i - \bar{a})(d_i - \bar{d}),$$

and,

$$\hat{\gamma}_{BC} = \frac{1}{n} \sum_{i=1}^n (b_i - \bar{b})(c_i - \bar{c}).$$

Collecting terms, one can now write

$$\hat{\gamma}_{xy} = [(\hat{\gamma}_{AC} + \hat{\gamma}_{BD}) + j(\hat{\gamma}_{BC} - \hat{\gamma}_{AD})]$$

An estimate of the cross correlation † of x and y is then given by

$$\hat{R}_{xy} = \frac{\hat{\gamma}_{xy}}{[\hat{\gamma}_{xx} \hat{\gamma}_{yy}]^{\frac{1}{2}}},$$

where

$$\hat{\gamma}_{xx} = (\hat{\gamma}_{AA} + \hat{\gamma}_{BB}),$$

and,

$$\hat{\gamma}_{yy} = (\hat{\gamma}_{CC} + \hat{\gamma}_{DD}).$$

In the calculation of the spaced frequency correlation functions, the time series of measured impulse response estimates were Fourier transformed using fast Fourier transform techniques to derive corresponding time series of channel transfer functions. Let $[H_i(f_1) \mid i = 1 \dots n]$ be the series of n complex transfer function values at a particular frequency, f_1 . Similarly, let $[H_i(f_1 + \Delta f) \mid$

† Fuller, W.A. "INTRODUCTION TO STATISTICAL TIME SERIES", John Wiley & Sons, New York, 1976, chapter 6, page 262.

† Beckman, P., "ELEMENTS OF APPLIED PROBABILITY THEORY", Harcourt, Brace, & World, Inc, New York, 1978, page 138.

$i = 1 \dots n$] be the series of transfer values at a second frequency, $(f_1 + \Delta f)$. Then, if one makes the substitutions $x = H(f_1)$ and $y = H(f_1 + \Delta f)$, it is clear that

$$a_i = \text{Re}\{H_i(f_1)\},$$

$$b_i = \text{Im}\{H_i(f_1)\},$$

$$c_i = \text{Re}\{H_i(f_1 + \Delta f)\},$$

and,

$$d_i = \text{Im}\{H_i(f_1 + \Delta f)\}.$$

The spaced frequency correlation function, $R_H(\Delta f; 0) = \frac{1}{2} E\{H(f_1)H^*(f_1 + \Delta f)\}$ (see eqn (2.12(a))) can be therefore be calculated using techniques outlined in this appendix. With the exceptions of the spaced frequency correlation functions computed for different references in Section 5.3.2, all such functions in this thesis were computed with $f_1 = +156.25$ kHz from midband.

END

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FIN