The History, Mechanics, and Strength of Stone Buttresses in Canada

by

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Abstract

Stone buildings found across Canada were constructed in a different era according to different engineering methodologies than new buildings. These buildings are important representations of the history of the country and should be preserved for future generations. One of the major concerns with these structures is the out-of-plane strength of the walls under earthquake loads. Many of these buildings, notably churches, include buttresses which were originally included to improve the out-of-plane strength of the walls. Current Codes and Standards in Canada do not provide guidance for engineers to assess the out-of-plane strength of walls with buttresses. A survey of churches with buttresses in Ottawa was conducted, acquiring the different sizes and dimensions of buttresses in the downtown core. An Applied Element Method (AEM) software was then used to recreate existing experimental data on the out-of-plane and in-plane strength of stone masonry walls, and the model was modified to analyze the behaviour of buttresses. The results from the modelling program are related back to historic and modern analysis methods obtained from a thorough literature review.

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Chapter 1: Introduction

1.1 Background

Stone masonry has proven to have an unrivalled longevity and durability when compared to other construction materials, with examples across the world dating back centuries and even millennia. The design knowledge to build these structures was passed down through the generations with a constant goal of safety and stability, while also pushing the limits to design grander, more awe-inspiring structures. The remaining stone buildings that have survived to date are physical reminders from a previous era of construction and should be protected and maintained for future generations.

The past century has shown monumental change in the field of structural engineering. The development of engineering as a profession, development of building codes, limit states design, stress analyses of indeterminate structures, and finally more recently the widespread use of structural analysis tools are all major developments which have completely defined the field of structural engineering. There have been a few attempts to relate the old methodology of designing masonry buildings with the modern practices, however many of these attempts are from the mid-twentieth century and were written prior to structural analysis and computer aided design (CAD) software. Modern building codes also lack information and guidance on how to assess the strength of these buildings, even though this construction typology forms a significant proportion of our current building stock. For example, based on a 2014 survey, 11.2% of buildings in central Ottawa are unreinforced masonry ((Sabbagh, 2014)).

Another major recent shift in structural engineering is the focus on seismic analysis. The field of seismology has experienced significant advancements, including recording of seismic

events and prediction of future events, allowing for the structural engineer to have access to a typical response spectrum for the buildings location. In addition, advanced laboratory setups have allowed for precise experiments of many different systems under simple pushover tests, cyclical quasi-static loading, and also true dynamic tests on shake tables which are able to recreate seismic ground displacements in all three axes. These experiments and knowledge on how different materials fail have provided input for building codes worldwide.

Stone buildings were built before these advancements, and are not explicitly designed to meet modern seismic codes. Despite this, there are countless examples of buildings which have survived centuries or even millennia in high seismic zones such as Italy and Mexico. At the same time there are examples of masonry buildings which have failed during seismic events, with dangerous consequences. This high variability and unknown response to earthquakes has resulted in concern for these buildings, especially considering that the majority of mass masonry buildings are located in downtown cores of cities where population densities are typically the highest. In order to properly protect the public during future seismic events we need to be able to assess, with modern engineering principles and mathematics, which of these buildings are safe and which require reinforcing.

Based on current codes and standards in Canada, there are limited resources for engineers to assess existing stone masonry buildings. The lack of guidance for assessing plain walls is compounded when looking at various masonry features such as buttresses. This lack of peer-reviewed, code-approved methodologies makes the simple solution that these antiquated building systems are inadequate, and designing reinforcing solutions out of modern materials such as steel or concrete.

One of the main concerns with stone buildings is the out-of-plane failure of the walls, which is extremely dangerous to the public. Historically, one of the methods used to reinforced walls in the out-of-plane direction were buttresses. There are many existing buildings around the city of Ottawa which feature buttresses, including some which were added as part of repair/retrofit projects. It is important that our national codes and standards offer methodologies to standardize the analysis of stone buildings, including buttresses.

1.2 Research Objectives and Scope

The main objective of this thesis is to quantify the out-of-plane strength of stone walls with buttresses to improve the resources available for engineers working on stone buildings. This was achieved with a focus on the buildings in Ottawa, Canada, by analyzing the existing buildings, historical references, and performing a literature review on relevant modern research. An Applied Element Method (AEM) software was then used to recreate existing experimental data on the out-of-plane and in-plane strength of stone masonry walls, and the model was modified to analyze the behaviour of buttresses. The results from the modelling program were related back to the various analysis methods recommended in the historical and modern references.

Chapter 2 looks at the practical information about stone masonry: the various types of stone masonry, good construction practices, historical methods of design, and the past performance including deterioration and buttress retrofit case studies. A survey of Ottawa buildings was also conducted, including true wall cross sections and the sizes of buttresses located within the downtown core.

Chapter 3 reviews the current state of engineering regarding stone masonry, looking at the Canadian building codes and standards and performing a literature review on the out-of-plane strength of plain walls and buttresses.

Chapter 4 describes the AEM software used in the analysis, focussing on the underlying theories and assumptions which make up the software including a verification of the basic mortar-stone interface.

Chapter 5 outlines all of the modelling performed for basic walls and walls with buttresses, including comparisons to existing testing.

Chapter 6 analyzes the buttress models in comparison to the various analysis procedures reviewed in Chapter 3.

Chapter 7 summarizes the conclusions and limitations of this research and provides recommendations for future work.

Appendices are included as supplementary information and include reference tables from literature and the output from the models tested in Chapter 5.

Chapter 2: Historic Stone Construction

The first course of action when analyzing any material is understanding its basic nature, construction and long term performance. Since there are minimal new stone constructions today, it is imperative to look back to the times when these buildings were commonplace. There is much to be learned from our existing building stock which can shed light on the best methods to analyze, repair and strengthen these structures.

With the goal of assessing walls with buttresses, it is first required to understand the behaviour of a plain wall and then build upon this basis to include buttress reinforcements. There are three general categories of historic information which provide insight into stone behaviour: the way that the buildings were actually constructed and the construction specifications (Section 2.1), the way that designers historically applied engineering principles and sized the building elements (Section 2.2), and learning from structural failures (Section 2.3). This Chapter concludes looking specifically at buttresses, Section 2.4, including the typical locations of buttress reinforcements of Canadian structures, a survey of buttress sizes in downtown Ottawa, and case studies of previous projects where buttresses were added to existing structures for reinforcement.

2.1 Canadian Stone Construction

Stone buildings are found from coast to coast through Canada, constructed in a variety of shapes and sizes. Canada's stone building stock is relatively new, in comparison to most other countries, and has significant influences from both Europe and the United States. In order to understand stone construction in Canada, first it is important to look at Canada's history to characterize

the common construction timelines for stone buildings. Section 2.1.1 digs deeper into this Canadian history, looking at the factors and circumstances surrounding stone construction in Canada, focussing specifically on Ontario and the Capital Region. The typical construction of Canadian stone buildings is examined in Section 2.1.2, including vertical cross-sections from stone walls in Ottawa. Finally, this Section concludes with construction specifications which were published in Masonry Treatises at the time of the construction of the majority of Canadian structures (Section 2.1.3).

2.1.1 History of Stone Construction in Canada

For thousands of years people have been building structures on the ground that is now Canada. The indigenous peoples each had unique traditions and construction methodologies, typically constructing with wood, snow or other organic materials, for more information see Mills and Kalman (2007). The earliest known masonry buildings in the country were constructed by the French in the 1600s, as the settlements became more permanent. General trends for the early history of the country involve competition between the various factions for control of the abundant natural resources. Canada, while first explored by Europeans at similar times as modern-day United States and Mexico, was slower to be colonized due to the extremely harsh environment. Famously, when Samuel de Champlain founded Quebec City in 1608, 20 out of the 27 men died during that first winter (Toupin, 2014).

The earliest masonry buildings in the country can be found in the province of Quebec from the late 1600s when the French were constructing permanent settlements. Old Quebec City, the capital of New France, was built strategically on the top of an escarpment on the St. Lawrence River and still features surviving 17th century stone fortifications to protect the region from the English. Stone buildings line the streets within the city walls, see Figure 2.1, and 135 hectares of the City are designated a UNESCO heritage site for being “the finest example of a colonial fortified town north of Mexico” (UNESCO, 2020a). Atlantic Canada, as the entryway into the St. Lawrence, also has some remaining fortifications from the early 1700s such as Fort Anne. In this region, early brick buildings were common, as bricks were placed in the hulls of the ships from Europe as ballasts and offloaded in Canada, replaced with valuable furs and other trading

goods for the journey back to Europe.

Figure 2.1: Stone Buildings Line the Streets of Old Quebec City. (Marrs, June 2019)

Ontario and Manitoba also have a few surviving buildings from the 1700s, however these are rare. These buildings were mostly fortifications to protect trading routes. A great example of the competition between the English and the French inspiring the construction of masonry structures is the Prince of Wales Fort in Churchill, Manitoba. The remote Fort was constructed by the Hudson's Bay Company to protect the valuable fur trade routes from the French. Construction began in 1731 with a team of 24 tradesmen and labourers from England, and it took over 40 years to construct the Fort in the harsh conditions off the shores of Hudson's Bay (Parks Canada, 2017).

The late 1700s saw a number of changes which significantly impacted the future of the country. The 1763 Treaty of Paris awarded most of France's lands in Canada to Britain. Then, the American Revolution saw an exodus of British Loyalists to the north, where Britain granted the Loyalists land in Ontario. This was the first widespread settlement of Ontario, and substantially

increased the European populations west of the Ottawa River/St. Lawrence junction.

John Simcoe, the first Lieutenant-Governor of the newly created Upper Canada (now Ontario), was charged with choosing their new capital. Surveying the northern shores of Lake Ontario in his ship the *HMS Mississauga*, he founded the new City of York (Toronto) in 1793 (Dendy and Kilbourn, 1986).

Simcoe had chosen his town-site not for pastoral tranquillity, however, but because he expected war. The Thirteen Colonies had been irretrievably lost to republicanism. Simcoe had fought the Americans during the Revolution and he expected it would be only a matter of time until Britain would have to fight them again. Simcoe made Toronto his capital because its natural harbour and protected bay could be fortified against attack by water, and because it afforded a naval base from which to control Lake Ontario. Though isolated from civilization and supplies, the surrounding region could be farmed to grow most necessities; the great pines of the Don Valley would provide splendid masts and ships' timber. Toronto's unique advantage, however, was the long arm of the peninsula: a sandbar that had been piling up over some 7,000 years as the current of the Niagara River cut into the highlands of Scarborough and arced back to form an unusually well protected harbour. (Dendy and Kilbourn, 1986)

As the new capital of Upper Canada, the new government buildings that were being constructed in Toronto were of stone, see Figure 2.2. Stone vernacular buildings were also becoming more common, with stone mills and houses appearing in the 1830s in the Rideau Lakes area, Kingston, Hamilton, Guelph and London (McIlwraith, 1997).

Following the war of 1812, Great Britain reinforced the border with the United States with stone fortifications, armouries, and other miscellaneous buildings, many of which survive to today. This includes the Rideau Canal, a UNESCO World Heritage site, which was built between 1826 and 1832 as a secure supply route between Kingston and Montreal (UNESCO, 2020b). The Rideau Canal was the largest civil engineering project undertaken at the time in Canada, and many of the structures survive to date including the Commissionaire's Building in downtown Ottawa, which is the oldest stone building in the Capital. The 202 km long canal

Figure 2.2: King Street facing Church Street, 1835, watercolour by John Howard. Reproduced from Dendy and Kilbourn (1986)

with its various dams, locks, lock stations, store houses and fortifications was a project which required thousands of skilled masons, who were mostly French-Canadians and stonemasons brought over from the British Isles.

The small town created at the junction between the Rideau Canal and the Ottawa River was named Bytown, after Lt. Colonel By who oversaw the construction the Canal, and would eventually become the Capital of Canada. The initial growth of the city was largely due to the logging industry, where timber could be floated down the Ottawa River to the St. Lawrence for sale in the larger Montreal or Quebec City markets. The growth of the region was slow, with the combined populations of Bytown, Perth and Richmond estimated as high as 9,000 by the late 1840s (Young, 1995).

The singular Province of Canada, in British North America, was formed in 1841 combining Upper and Lower Canada. A singular capital was required, and originally Kingston was chosen, however was soon found to be inadequate (Young, 1995). There were political issues and tensions between the English and the French under a single government, and when Montreal became the capital in 1844 these tensions resulted in rioting and finally the burning of the legislative building in 1849. Representing a compromise, Queen Victoria chose Ottawa, the small, bustling lumber town on the border of the two regions as the capital in 1857. The con-

troversial choice needed to be solidified by government buildings, and ground was broken in 1859 for the construction of the new Parliament buildings. The construction of the Parliament buildings was the beginning of the major growth in the Ottawa region, and are some of the oldest stone structures in the Capital.

Figure 2.3: The Library of Parliament Under Construction, Ottawa, photographed in the early 1870s. Reproduced from Young (1995)

At the same time as the new capital was being constructed, the province was growing. The population of Ontario passed one million in the 1850s and twice that number by 1891 (McIlwraith, 1997). Similarly, the number of houses rose from 146,000 in 1851 to more than 350,000 in 1881. The trend to build the new houses out of brick or stone rather than wood, a material which was prevalent throughout the country, was influenced by a number of factors including the large-scale fires which were occurring around the world. This large increase in population and new constructions in the latter half of the 19th century represents the majority of the mass masonry buildings that exist today.

The large number of stone churches scattered throughout the Ontario landscape con-

structed at this time can owe their architecture to England's gothic revival in the mid-1800s, which helped to highlight the beauty of stone construction and thrust it back into popularity (McIlwraith, 1997). Gothic architecture is characterized with high, pointed arches and stepped buttresses. A new generation of British architects brought the gothic revival style to Ontario, well timed for the large increase in populations immediate need of showy buildings.

One of the first reinforced concrete structures in Ottawa, Tabaret Hall at the University of Ottawa, was built in 1904, and marked the beginning of the decline of mass masonry construction. Steel and reinforced concrete offered larger spans, thinner walls and larger windows. Buildings were also getting taller; the rise of the skyscraper highlighting the limits of mass masonry construction. This limitation is highlighted in the Monadnock building in Chicago, a mass brick building seventeen storeys (66 m tall) completed in 1881, where the interior load bearing walls are 1.8 m thick at the bottom.

There was a period in the first half of the 1900s of "transitional masonry" structures, where the exterior walls are self-supporting mass masonry with an internal steel or concrete structure. The new Centre Block of Parliament building is included in this category, after the original burnt down in 1916, constructed with a steel frame clad in mass stone.

These "transitional masonry" structures came at a time when there was a transition in engineering principals to focus on elastic theory rather than the empirical methods of the previous centuries. A number of unique and interesting buildings were constructed in this era, as designers were experimenting with the new technologies and materials. The mass masonry exteriors of these buildings can be more susceptible than their predecessors since they are taller, frequently not carrying the mass of the interior floors, and there are less interior dividing walls due to the larger floor spans.

In modern times in Canada it is almost entirely unseen for a new structure be constructed of mass masonry. The only major masonry projects involve the restoration, with areas of reconstruction, of heritage structures. While these materials are no longer used for new constructions, there is an existing building stock from the various stages of Canada's growth that need to be protected and maintained. These buildings are representative of the social and political landscape at the time of their construction, and provide an insight into the history of the country.

2.1.2 Typical Stone Construction

Within the general label of stone construction there lies a wide variety of different types of construction which impact the overall structural response. A first major difference is that stone masonry can be constructed with or without mortar, the latter referred to as *dry-stack*. In Canada, dry-stack construction is typically limited to garden walls, retaining walls, short-span arch bridges, or other decorative purposes (Shaw-Rimington, 2016). While the presence of this type of construction is acknowledged, the vast majority of Canadian structures and therefore the focus of this research is mortared masonry. The mortar performs a number of functions, one of which is to ensure that the force from the stones above is distributed evenly to the stones below by adding material which will fill in any imperfections in the stones and reduce sharp contact surfaces where pressures would concentrate. This provides some margin of error for the mason to not have to make the joints perfect. In imperfect dry-stack construction, the stones meet at localized pressure points and increase the likelihood of the stone cracking or spalling. A famous example of ‘perfect’ dry-stack stonework where each stone is carved exactly to meet its neighbours are many of the Incan ruins in Peru, see Figure 2.4.

Figure 2.4: Dry-Stack Incan Ruins, Cuzco, Peru. (G. Marrs, Aug 2017)

Typical Canadian stone building construction is mortared masonry, comprising of an outer wythe and an inner wythe with a rubble core between them, with wall thicknesses at least 550 mm thick (22 inches). A wythe is defined as a continuous vertical section of masonry one unit in thickness, for other definitions refer to the Glossary at the end. The rubble core is usually

filled with small stones, stone chips and spalls from the stone cutting process and packed with mortar. Found all around the world, this construction is typically called “multi-wythe stone” or “three-leaf stone” (English Heritage, 2012). Mortared stone is classified first by the care employed in dressing or finishing the stone, ashlar or rubble masonry, and second by whether the horizontal joints are more or less continuous. Masonry can be further defined by the finish of the face, however this is an aesthetic decision and is not considered of structural importance for the purposes of this research. Detailed descriptions of different face finishes can be found in Baker (1889).

Ashlar Masonry

Ashlar masonry is comprised of cut stones with smoothly dressed bed and mortar joints less than one-half inch thick (Roca et al., 2019). Ashlar is the best quality of stone masonry, and is employed in most important structures. It is used for piers, abutments, arches and, in general, for works in which great strength and stability are required. A slight roughness of the joint surfaces, such as left by tool marks, is desirable as it increases the adhesion of the mortar. If the bed surface is concave, the pressure is distributed to the edges and they are liable to spall. If the bed surface is convex, the stone will not be stable until the mortar has set which usually results in open joints on the face. Ashlar masonry is further subdivided by the horizontal joints as coursed and uncoursed, see Figure 2.5 below.

Figure 2.5: Left: Coursed Ashlar, Right: Uncoursed Ashlar. (Marrs, Aug and Sept 2019)

There exists a middle category between ashlar masonry and rubble masonry referred to

historically as *Squared-Stone Masonry* (Baker, 1889). This type is comprised of roughly squared stones with roughly dressed bed joints, with mortar joints greater than one-half inch. The nuance of the differing mortar joint thickness has been lost over time, and this type of construction in Canada is generally referred to as either ashlar or rubble, depending on the squareness of the stones.

Rubble Masonry

Rubble Masonry is comprised of roughly squared or unsquared stones used as they come to the site with minor additional preparation other than the removal of very acute angles and excessive projections (Roca et al., 2019). Due to the random shapes and sizes, thick mortar joints are required to fit the stones together. This is generally considered a lower grade of masonry due to the high reliance upon the mortar. Rubble masonry is much less expensive to construct than ashlar (Trautwine, 1872). Rubble masonry can be further subdivided by the horizontal joints as coursed and random, see Figure 2.6 below, however the differences are more nuanced and have less of an overall impact on aesthetics in comparison to ashlar stonework.

Figure 2.6: Left: Coursed Rubble, Right: Random Rubble. (Marrs, July and Oct 2019)

A final note regarding the typologies of stone walls is that the multiple wythes need not be constructed of the same material. In fact, ashlar walls are typically only finished with ashlar stones on the exposed face of the wall, with the interior wythe and core material typically comprised of cheaper masonry, such as rubble stone or brick. See Figure 2.7 below for an image

showing the rubble core behind an ashlar facing.

Figure 2.7: Rubble core behind ashlar facing. (JCAL, May 2017)

The two, inner and outer, wythes should be connected regularly by bond stones (also referred to as headers), or mechanical connectors, see Figure 2.8. Bond stones or headers are the general term which describe either through-stones or key stones. Through-stones traverse the entire section of the wall, and physically connect to two wythes with the friction from the weight above. Key stones extend into the core but not into the opposing wythe, tying the two wythes together by applying gravity loads into the core increasing the friction forces and in turn the bond strength between the wythes.

Mechanical connectors can also be used to create a tension tie between the wythes, the strength of this type of connection is dependent on the bond between the anchor and the wythe rather than the self-weight of the masonry.

Typical historic construction consists of either bond stones or cramp anchors, with helical ties and grouted anchors being modern solutions. Key stones are significantly more common than through-stones, since they are less expensive to construct.

Figure 2.8: Sketch of Bond Stones and Mechanical Connectors

An extremely well-built new wall under construction is shown in Figure 2.9. There is a minimal gap between the backs of the wythes, requiring only a small core. The construction also includes long through-stones at every three feet. Note that even in this well-constructed wall, the back faces of the stones are not finished. The rounded backs of the stones is typical for all stone constructions.

Figure 2.9: Stone Wall Under Construction Showing Through-Stones. Reproduced from Shaw-Rimmington (2016)

The well-built wall of Figure 2.9 is in comparison to a more standard historic construction with a larger core and no through-stones shown in Figure 2.10. These stone ruins are located near Ottawa, at the location of a historic lime kiln.

Figure 2.10: Ruins of a stone building near Ottawa. (Marrs, Sept 2020)

As part of this research, the vertical cross-sections from four pre-1900 stone walls were able to be recorded. It is uncommon to see full vertical cross sections of existing walls since, while fully intact, the interior of the wall is completely hidden from the inspector. Even during major repairs, typically only one wythe is deconstructed in order to access and consolidate the core (see Section 2.3.1 for more information on stone deterioration). However, in circumstances such as demolition or major alterations these walls can be dismantled which provides access to the cross-section. The walls shown in Figure 2.11 are all located in Ottawa; two foundation walls and two above grade walls. The line drawings were produced by rectifying the images and tracing the stones in AutoCAD. Note that this is the cross-section at a singular vertical plane in the wall, the cross-section will always vary along the length of the wall especially in rubble construction.

Figure 2.11: Wall Sections from pre-1900s Ottawa Stone Buildings (left is exterior)

In all of the walls, there are no through-stones present and only a few large key stones. Each wall has a different quantity and size of key stones, average stone size, and core size. This demonstrates the great variability between different walls, and that it is valuable to record walls which are dismantled to evaluate and record any trends, standard details and construction outliers in order to better inform future projects.

2.1.3 Masonry Treatises

Good practices for the construction of stone buildings can be found in historic treatises, dating back to the early 19th century. These treatises provide proper construction practices that were published at the time of the construction of the majority of Canadian structures, and can provide

insight into the factors which the original builders considered important. These are clues to the variables which modern engineers should be considering in the assessment of these structures, and are good starting points for future research and testing. Unfortunately, due to the limited quantity of modern testing, the effects of these factors have not been adequately researched. One of these sets of general rules are listed below, from the textbook titled *A Treatise on Masonry Construction*, originally published by Ira Baker in the United States in 1889 and reprinted ten times by 1914 (Baker, 1889).

1. The largest stones should be used in the foundation to give the greatest strength and lessen the danger of unequal settlement.
2. A stone should be laid upon its broadest face.
3. Larger stones should be placed in lower courses, the thickness of the courses decreasing gradually toward the top of the wall.
4. Stratified stones should be laid upon their natural bed.
5. Masonry should be built in courses perpendicular to the pressure it is to bear.
6. To bind the wall together laterally, a stone in any course should break joints with or overlap the stone in the course below. (*do not have stacked vertical joints*)
7. To bind the wall together transversely, there should be a considerable number of headers extending from the front to the back of thin walls or from the outside to the interior of thick walls.
8. The spaces between the back ends of adjoining stones should be as small as possible, and these spaces should be filled with mortar.
9. The rougher the stones, the better the mortar should be. The principal object of the mortar is to equalize the pressure; and the more nearly the stones are reduced to closely fitting surfaces, the less important is the mortar.

While noting the importance of header stones these guidelines only include terminology such as “a considerable number of headers” and recommending through-stones for thin walls

and key stones for thick walls. Further information would have been included in project specifications, such as minimum size and spacing of headers, however this guidance does not appear to be consistent through sources. Listed below are some of the recommended specifications for header stones found in these older texts, as well as some additional guidelines for header stones.

- There should not be less than one header for every five square feet of surface of wall. (Siebert and Biggin, 1896)
- Headers shall, in all cases, extend into the wall two thirds the thickness, and preferably entirely through the wall. (Siebert and Biggin, 1896)
- It is frequently specified that one fourth or one fifth of the mass shall be headers. (Baker, 1889)
- Headers shall occupy one-fifth of face of wall. (Howe, 1915)
- Headers in the heart of the wall shall be the same size as shown in face, so arranged that a header in a superior course shall not be laid over a joint. (Howe, 1915)
- Bond stones extending through the wall and uniformly distributed shall be provided to the extent not less than 10 per cent of the area, and there shall be at least one bond stone for every eight stretchers. (US Department of Commerce, 1924)

Once a wall is constructed, it is impossible to tell which stones are headers. This makes it difficult for an engineer examining these buildings today to assess how they were constructed, and it was also difficult at the time of construction to perform quality control. To quote from Baker (1889): “A trick of masons is to use *blind-headers*, or short stones that look like headers on the outside but do not go deeper into the wall than the adjacent stretchers. When a course has been put on top of these, they are completely covered up; and, if not suspected, the fraud will never be discovered unless the weakness of the wall reveals it.”

This difficulty for modern engineers to assess these structures extends beyond simple walls. There is a large portion of the shaping and placing of stones, the art of *stereotomy*, that is invisible to an inspector trying to assess an existing stone building. Every stone had to be sized to fit into its specific position in a masonry structure, chosen and executed by a mason.

There are many treatises and books written on stereotomy, on the art of being able to properly size and shape a three-dimensional stone to fit perfectly into complex curves, arches, vaults, etc., including but not limited to Nicholson (1828), Dobson (1849), and Siebert and Biggin (1896). These documents are filled with sketches and helpful geometric guides for masons to properly cut stones of any geometry prescribed by the designer. The 1896 Treatise includes a sketch of elevations and plans to teach apprentice masons how to carve buttress stones based on drawings, reproduced in Figure 2.12. The simple buttress profile and description included here matches many of the buttresses constructed around Ottawa, refer to Section 2.4.2.

Figure 2.12: Elevations and plans for cutting the stones of a simple gothic buttress.
Reproduced from Siebert and Biggin (1896)

However, this graphic does not provide information of how the buttresses were connected to the actual wall. Similar to the discussion regarding headers, the extent that the buttress is keyed with the actual wall is unknown unless an opening is made in the wall itself. See Figure 2.13 for the side profile of a buttress in Ottawa. In this case, the wall does not continue behind the buttress, instead the core thickens to meet the buttress stones. The buttress stones are not significantly deeper than the face of the wall, and there is minimal keying into the core. More data needs to be gathered to determine if this is a typical construction detail.

Figure 2.13: Buttress in Ottawa with outer wythe removed for core consolidation.
(Marrs/JCAL Oct 2019)

The purpose of this section is to leave the reader with the understanding that there will always be portions of masonry structures that are hidden behind the visible wythe of stonework. The stone structures in Canada are built by different masons at different time periods with different access to resources. No two buildings are the same, and while past experience can help guide engineers to reasonable hypotheses on the construction of a building it is valuable for major projects to create pilot openings in order to gain information on the construction and core condition.

2.2 Historical Stone Design

The vast majority of unreinforced masonry buildings in Canada were built between 1850 and 1900. This is before building codes were implemented in Canada, engineering became a profession, or the mass production of building materials regulated material properties. One of the recurring concepts in research papers is classifying historic mass masonry buildings as *non-engineered*, since they were not specifically designed for seismic capacity. While that is true in the most basic sense that the calculations as required by modern building codes for seismic design were not performed on historic buildings, it is not reasonable to state that there was no engineering judgement incorporated into the design.

This Section looks into the published information from historic engineers discussing the mathematics and design considerations for stone masonry structures. This summary is in no way a complete investigation into every historic publication written on mass masonry design. Overall, there is a general lack of information published specifically on the strength of stone walls, however, there is information available on the design of stone retaining walls, gravity dams, and buttresses. The published information on retaining walls and dams is included in Section 2.2.1 and the analysis of arches and buttresses is included in Section 2.2.2.

2.2.1 Masonry Wall Design

The only published historical data regarding the out-of-plane strength of masonry walls relates to retaining walls and dams. Looking back at the historical texts, these problems were typically twofold: how to calculate the applied lateral loads from the soil pressure or water pressure, and how to design the wall accordingly. This research is the basis for modern retaining wall design, including the estimation of lateral earth pressure and water surcharge.

Retaining walls, due to their abundance, were typically designed according to empirical rules, which had been developed by long-term trial and error. Much of the uncertainty of these original designs were based on the lateral loads imposed by the retained soil, rather than the strength of the wall. These rules were typically published in tables, some of which are included in Appendix A. Analyzing these rules, it is clear that the basis for many of these rules is simply

calculating the overturning moment of the wall.

The textbook *A Treatise on Masonry Construction* by Baker (1889), provides detailed information on how to design stone dams. This text recommends analyzing the section as a cantilever along a unit length. The masonry retaining walls or dams can fail in one of three ways, however, “it is admitted that by far the greater number of failures of dams is due to defects in the foundation”.

1. By sliding along a horizontal joint or shearing along any section.
2. By overturning about the front of a horizontal joint.
3. By the crushing of the masonry either at the downstream edge of a horizontal section when the reservoir is full or at the upstream edge when the reservoir is empty.

For a wall to fail by sliding, the lateral force of the water above the sliding plane (H) must be greater than the mass of the masonry above the sliding plane multiplied by the coefficient of friction, see Equation 2.1. “It is unreasonable to apply a safety factor to this equation since the value of the coefficient of friction, μ , is approximated, so the safety is meaningless and fully dependent on this estimated value” (Baker, 1889). Various tables are published on the possible coefficients of friction for masonry, included in Appendix A.

$$H < W_{above} \mu \quad (2.1)$$

where: H is the horizontal force, per unit length of the wall;

W_{above} is the self-weight of the masonry above the joint; and

μ is the coefficient of friction.

There are conservative assumptions included in this sliding equation. If the back face of the wall is sloped, then the vertical component of the water or soil pressure is excluded. The equation also assumes the dam will fail by the sliding of one stone upon another along a horizontal joint through the entire thickness of the wall. First, the stones are laid with mortar which gives a considerable resistance through cohesion in addition to the frictional resistance. Second, masonry dams are built of random rubble masonry, the stones of which interlock in

every direction; and hence the tendency to slide is resisted by the shearing strength of the individual stones as well as by friction. If the dam is built of coursed masonry, which is very improbable, the courses should be inclined downward toward the up-stream side.

For a wall to fail by overturning, there is both an algebraic solution and a graphical solution proposed. The algebraic solution is simply the overturning moment, with the weight of the wall acting at the COG with overturning at the toe, see Equation 2.2.

$$H < \frac{W_{above} \bar{x}}{h_{force}} \quad (2.2)$$

where: $\bar{x}$ is the distance from the COG to the toe; and

h_{force} is the height at which the horizontal force is applied.

The vertical component of the water or soil pressure is also neglected in this equation. As with the other studies, there is a focus on the best way to determine the COG, which for modern engineers can be easily acquired through drafting software. Computed this way, the factor of safety against overturning for the New Croton Dam, the next to the largest masonry dam in the world, varies between 2.07 and 3.68 ((Baker, 1889)).

The crushing of the masonry is an additional consideration for overturning. This is calculated algebraically by determining the stress profile at the base of the wall. This is shown in Equation 2.3 is provided to assist the designers.

$$P = \frac{4w}{l} - \frac{6W\bar{x}}{l^2} + \frac{6M}{l^2} \quad (2.3)$$

where: P is the pressure at the toe in pounds per unit area;

w is the weight of masonry in pounds per cubic foot;

l is the length of the base of the cross-section;

W is the weight in pounds of a 1 ft wall section; and

M is the applied overturning moment from the external lateral force.

By limiting the pressure at the toe, the maximum allowable moment can be calculated. In basic terms, this equation is summing the axial stress due to the self-weight, the decrease in

pressure at the toe due to the negative moment from the self-weight (since the COG is typically closer to the heel due to the batter), and adding the stress due to the external lateral forces.

2.2.2 Arches and Buttresses

The arch and the buttress are inextricable tied together, as the lateral thrust at the base of arches and vaults was the original motivation for including buttresses in many structures. Similar to the previous discussion on retaining wall design, a big focus of the research was calculating the applied load, and proportioning a “safe buttress” rather than calculating the maximum strength of the buttresses. There are detailed books which summarize the history of arch analysis, focussing on important designers from the past and their theories and assumptions, including Gillmor and Heyman (1973), Benvenuto and Benvenuto (1991), Boothby (2015a), and Roca et al. (2019). Due to the significant wealth of information on this topic, the focus of the following section is the 19th century graphic static method, which would have been known and potentially used during the construction of the majority of Canadian stone buildings.

Before the 1800s, there are a number of influential engineers and mathematicians who published various methods of analyzing arches and designing appropriate abutments, see summarized histories in Heyman (1972), and Ochsendorf et al. (2004). Reading through the historic manuscripts, a reader can easily become lost in the multiple variables and mathematical expressions which do not follow modern nomenclature. Once the process of these old mathematic protocols is understood it becomes clear that a large portion of the design was determining the approximate centre of gravity and then using distances to determine lever arms (Gillmor and Heyman, 1973). The use of lever arms and force vectors is the precursor to the modern calculation of moments. These methods both treat the buttress or abutment as a rigid block, subject to overturning, with the majority of the process focussed on acquiring the magnitude and orientation of the thrust.

Figure 2.14 shows two examples of determining the arch thrust and required buttress sizes, (a) from De La Hire (1712) and (b) from Coulomb (1773) .

Figure 2.14: Pre-1800s Methods of Analyzing Buttresses, reproduced from Heyman (1972)

The processes of analyzing arches and buttresses was continually refined. A few decades after the publication of Coulomb's vault theory it was confirmed that this theory provided accurate approximations for the horizontal thrust of an arch, while the values provided by La Hire's method earlier in the 18th century overestimated the lateral thrust (Roca et al., 2019). La Hire's method, however, had been used to design arches, vaults and buttresses for a hundred years and had proven to be adequate for long-term construction. The designers in the 1800s therefore based a safety factor off of the difference between the two thrust calculations, obtaining a factor of 1.9, which was frequently rounded to 2.

In the 19th century the analysis of masonry buildings was dominated by graphic statics, and the determination of the *line of thrust* (Boothby, 2015b). The line of thrust is the location through which the resultant force passes. Graphic statics is the process of determining the resultant of the forces, idealized as vectors, at critical cross-sections and visually being able to determine the eccentricity of the resultant force. Static theory had been long developed, and it was known that as long as the resultant of the force passed within the middle third of the cross section then the entire section would be under compression.

There are many references which discuss the method of acquiring the line of thrust, includ-

ing Trautwine (1872), Kidder (1886), and Boothby (2015b). A full worked through example of calculating the line of thrust for a buttress can be found in the handbook *The architect's and builder's pocket-book: A handbook for architects, structural engineers, builders, and draughtsmen* by Frank E. Kidder (Kidder, 1886).

A pier or buttress may be considered stable when the forces acting upon it do not cause it to rotate or “tip over,” or any courses of stones or brick of masonry to slide on its bed. When a pier has to sustain only a vertical load, it is evident that the pier must be stable... It is only when the pier receives a thrust such as that from a rafter or an arch, that its stability must be considered. In order that there may be no rotation, we must have the condition that the moment of the thrust of the pier about any point in the outside of the pier shall not exceed the moment of the weight of the pier about the same point. (Kidder, 1886)

Figure 2.15: Graphic Demonstrating the Two Recommended Methods to Analyze Piers, (a) Mass Moments and (b) Graphic Statics. Redrawn from Kidder (1886)

The two recommended methods are shown for a simple pier in Figure 2.15. For the mass moment option, (a), it is simply that the moment from the weight of the buttress around point b_1 must be greater than the moment from the thrust. To achieve equilibrium, Equation 2.4 must be true. Note that rather than breaking the applied thrust T into a vertical and horizontal component, as is more common in modern analyses, the line of the force is extended and the

perpendicular distance (ie. lever arm) between the thrust and the toe is measured.

$$T \times a^1 b_1 = W \times t/2 \quad (2.4)$$

where: T is the applied thrust;

$a^1 b_1$ is the dimension between points a^1 and b_1 , representing the lever arm of the thrust T at point b_1 ;

W is the weight of the pier; and

t is the thickness of the pier.

However, this does not provide any factor of safety against toe crushing. It is recommended to move the rotation point away from the outer edge, a minimum of $0.25t$, represented on the graphic as point b . Equation 2.4 is then modified to Equation 2.5. An interesting note is that halving the lever arm of the weight, W , from $t/2$ to $t/4$, is similar to applying a safety factor of 2.

$$T \times ab = W \times t/4 \quad (2.5)$$

where: ab is the dimension between points a and b , representing the lever arm of the thrust T at point b .

For the graphic statics method, (b) in Figure 2.15, typically the process involves having known forces (externally applied thrust T and self-weight W). Drawing these two forces as vectors with lengths proportional to their magnitudes, the resultant force can be drawn. The line of the thrust T is extended until reaching the vertical plane of the centroid of the pier, shown as point d in Figure 2.15. The resultant force is then plotted at point d and extended to the base of the pier. Where the resultant meets the bottom of the pier this is the eccentricity at the base, labelled as e . Together the extended line from the external thrust, points Td , and the line from the resultant, points de , create the line of thrust. If the line of thrust exceeds $0.33t$ at any point, then tension will develop at the inner edge and it is assumed that this will initiate cracking and the subsequent failure of the section.

2.3 Performance of Stone Construction

There is much to be learned based on the past performance of stone construction. This section begins by looking at the natural deterioration of stone in Section 2.3.1, examines the behaviour of stone masonry in a seismic event in Section 2.3.2, and concludes by looking at previous retrofits of masonry buildings with buttresses in Section 2.3.3.

2.3.1 Stone Deterioration

In mass stone construction, as with other modern materials, the main cause of deterioration is the presence of water, exacerbated by salts and freeze-thaw conditions. There are many studies on the durability of mortars and stones, and the curious reader is directed towards the technical report published by Historic Environment Scotland (Wiggins, 2018). This report discusses the function of mortars and the long term deterioration at the molecular level.

For the purposes of this report, overarching concepts will be discussed. First is that stone masonry walls will get wet. They are exposed to the exterior, and although most stone buildings were designed with large roof overhangs or cornices, winds will still drive the rain against the wall. When the wall gets wet, the moisture is pulled out of the stone units into the mortar since the pore size of the mortars are typically much larger than the pores of the stones. The moisture can then travel through the mortar through capillary action to the face of the wall where it evaporates. As the moisture evaporates, any soluble salts within the wall recrystallize and over time this process damages the exterior surface of the mortar. Evaporation of the moisture can also result in the loss of the lime binder of the mortar over time. This long-term deterioration of the mortar joints is by design, and the outer inch of the mortar joint is often referred to as “sacrificial mortar”. Re-

Figure 2.16: Mortar Deterioration, from English Heritage (2012)

pointing is the act of removing this deteriorated mortar and reinstalling new mortar, and should be considered as a regular maintenance activity for masonry structures. Figure 2.16 reproduces a graphic depicting the deterioration of mortar over time, published by English Heritage (2012).

If repointing is neglected, the deterioration of the mortar will continue deeper into the wall, and eventually into the core of the wall. When the lime binder washes out of the core of the wall then, without adequate bond stones, it can result in the wall essentially acting as two much thinner walls with a void space between. Figure 2.17 shows examples of walls with washed out cores. This situation is dangerous, and can lead to the collapse of the outer wythe.

(a) (Marrs/JCAL Nov 2019)

(b) (JCAL Nov 2016)

Figure 2.17: Walls With Washed Out Cores

The deterioration of the mortar is especially prevalent in Canada, where masonry buildings are expected to withstand numerous freeze-thaw cycles each year and de-icing salts are used along public walkways and roadways, including directly adjacent to buildings. The exten-

sive use of de-icing salts adds to the soluble salt content within the wall and in turn the rate of deterioration. This deterioration, however, is expected and can be mitigated through regular inspections of the structure and repointing when required.

As part of the repointing process masons should remove (rake out) all of the deteriorated mortar from the joints until good mortar is reached. This may require multiple inches of raking out, depending on the extent of the deterioration. If good mortar is reached before the core, then this is an indication that the core is still in good condition. If the deteriorated mortar extends into the core, then it is likely that areas of the core have deteriorated. Another indication of a deteriorated core is bulging of the outer wythe. If the core is deteriorated, it may be required to remove stones from the exterior wythe in order to access the core to consolidate it (Cooke, 2020). Grouting the core of the wall is also an option that does not require localized removal of the outer wythe, but this process is blind and there is a danger of the grout exiting the wall in undesirable locations or not filling all of the voids (Cooke, 2020).

Regular repointing and maintenance for stone buildings has remained a constant task for masons for since stone buildings began being constructed with mortar.

A major change to this process occurred in the 20th century with the introduction of cementitious mortars. Cementitious mortars are harder, stronger, and much less porous than the historic lime mortars. Using these cementitious mortars for repointing resulted in essentially a moisture barrier, where the moisture could no longer evaporate out the mortar joint. This results in the stone being the most porous and therefore easiest route for the water to evaporate. The same process of salt crystallization occurs, except now on the surface of the stone instead of in the mortar resulting in spalling, cracking, and overall expedited deterioration of the stone units, see Figure 2.18. Unlike mortar, stones cannot be easily repaired. In addition, the moisture transferring from the mortar to the stones within the wall results in deterioration which is invisible from the face of the wall since the pointing mortar remains unchanged. It is possible in this situation for the core of the wall to deteriorate while the front pointing mortar appears in good condition. Similar long term results occur when concrete or other cementitious materials are cast immediately adjacent to stone masonry.

Figure 2.18: Deterioration of Stone Resulting from the use of Cementitious Mortars.
(Marrs/JCAL, Aug 2019)

2.3.2 Structural Performance

Throughout history, the collapse of mass masonry buildings was sudden, devastating, and well-recorded. Similar to modern days where engineers visit areas following earthquakes to record the damage and inspect the effectiveness of various structural systems, historically engineers used these failures as guides, especially when pushing the safe and known construction limits.

The global failure of a structure was uncommon, unless initiated by extreme conditions, leading to the introduction of the “five-minute rule” for arches and other masonry components, such as flying buttresses (Heyman, 1995). That is to say, if the work stands for five minutes after the removal of the false work, it will stand for 500 years. This theorem is in relation to dead load only, and in reference to the safe theorem and line of thrust. It takes five minutes for the arch structure to settle under its own weight, and find a stable line of thrust to support the load. The upper limit of 500 years is in relation to the deterioration of the material. However, this does not account for foundation settlement from the extremely high gravity loads or natural disasters such as fires or earthquakes. Heyman recommends replacing the ‘five-minute rule’ with the ‘generation rule’ (+/-20 years) for the structure as a whole, in order to account for these longer term loads. This is similar to the “grandfather clause” which exists in many building codes, where a structure has performed adequately for a set period of time, and there are no

major renovations, alterations, or noticeable changes/deterioration of the structure itself, then the structure is acceptable.

This clause, however, is debatable when considering earthquakes. Especially considering that Ottawa has not yet experienced an earthquake with a magnitude that is equivalent to the “design” earthquake, these 100 years old structures cannot be guaranteed to withstand a design-level seismic event. During the stone wall experiments for the West Block rehabilitation project, the ground motion from the 2010 Central Canada earthquake was used, scaled up by a factor of 183 in order to meet the design values (Sorour, 2010).

In addition, masonry is typically regarded as a poor building material for high seismic zones, and even in Ottawa which is a relatively moderate seismic risk, new unreinforced masonry buildings may only be constructed on Class A or B soils which are both rock-type bearing surfaces. In a seismic event, generally speaking, the seismic waves travel through the ground from the epicentre causing the ground to vibrate. The movement of the ground is transferred to the building through the foundation, but the mass of the building above ground resists this motion through its inertia. It is this inertial force of the upper structure resisting the ground motion that results in the lateral forces on the structure. Mass masonry buildings can be more susceptible to seismic loads than buildings made of other materials since they are extremely heavy, resulting in higher inertial forces. In addition, stone structures rely on their own weight to resist lateral loads which can vary if there is a vertical acceleration component to the earthquake.

The best way to predict the behaviour of masonry structures in earthquakes is to review the performance in previous earthquakes. The study of post-earthquake structural responses is a significant topic on its own, see Bruneau (1994), Lagomarsino (1999), Lamontagne (2008), Turner et al. (2012) and Sorrentino et al. (2014). The general themes from the performance of stone buildings in previous earthquakes as published in these numerous papers are summarized in the following list:

- In-plane or global response of the structure results in shear cracks in the walls, and is most likely to occur when the floors are well connected to the walls and can transfer the lateral movement of the building to the shear walls.

- Out-of-plane failure, sometimes referred to as local failure, typically results in the collapse of a wall due to excessive displacements.
- Unrestrained masonry elements such as parapets and chimneys are extremely susceptible.
- Poor mortar quality is commonly noted on collapsed sections.
- Upper story walls are susceptible to out-of-plane failures, typically attributed to the lack of overburden.
- Anchors attaching walls to roofs and upper floors have shown unreliable performance.
- Multi-wythe walls improperly bonded along their collar joint (due to no or discontinuous mortar at the backs of the stones) are extremely vulnerable to out-of-plane failure.

Two specific earthquakes are also analyzed below in more detail; the Molise, Italy, earthquake in 2002 and the 2010-2011 earthquakes in Christchurch, New Zealand.

Molise, Italy, 2002 (Decanini et al., 2004)

Decanini et al. (2004), studied the performance of masonry buildings in Molise, Italy, that experienced two moderate earthquake events a day apart, the first $M_L = 5.4$ and the second $M_L = 5.3$ with a depth of 15-20 km. Three general categories of buildings were analyzed following the two earthquakes; historic buildings in the town centre (constructed of mass stone walls, masonry vault floors at ground level, wood floors above), the buildings of late 19th to mid 20th century (mass masonry with floors typically fireproof iron and masonry jack-arches), and buildings constructed in the last 40 years with mixed materials (reinforced concrete frames, with concrete slab floors). “With some exceptions, the older historical masonry buildings did notably better. The new developments built at the margins of the old towns over the last 100 years proved to be extremely vulnerable.”

The article discusses possible reasons for the historic town centre buildings to have performed better. One of the main reasons could be the underlying strata. The historic town centres were built on hilltops for defensive purposes, which are typically firm limestone, while as the towns expanded they built down into the valleys on poorer clay, which is more prone to amplify

the earthquake vibrations. Second is the original configuration, craftsmanship and current condition of the older stone and brick masonry. “In San Giuliano there are a number of buildings in the historic core that date back centuries, but were in good condition going into the earthquake. These generally suffered little damage during the shaking.”

Poor maintenance and poor core materials in the walls were noted as indications of poor seismic performance. This was exemplified by the failure of many buildings in poor condition and those which had been abandoned for an extended period of time. The mortar in many of these walls had typically deteriorate to loose sand, and there was an overall lack of bond stones, see Figure 2.19.

There was a final focus in the article on the effects of incompatible materials. “The Molise earthquake was particularly damaging to masonry buildings that had been remodelled with heavy reinforced concrete floors.”

Figure 2.19: The inner wythe of the wall where the exterior wythe has fallen off. There is little evidence of bond stones and many of the internal stones are undressed river stone mixed with broken bricks or other rubble. Reproduced from Decanini et al. (2004)

Christchurch, New Zealand, 2010 (Ingham and Griffith (2010), and Turner et al. (2012))

Ingham and Griffith (2010), and Turner et al. (2012)) studied the performance of unreinforced masonry buildings the city of Christchurch following a number of earthquakes in 2010-2011, starting with the Darfield Earthquake in September of 2010 and numerous serious aftershocks into 2011. In these earthquakes the response of the mass masonry structures in New Zealand is directly relatable to the Canadian landscape since the masonry structures were built in the same era (1850s-1900) with similar construction typologies. For this study, the authors do not differentiate between mass stone and brick construction, however it was noted that before the earthquakes, 76% of the buildings were brick and the remainder stone.

The first earthquake of September 4, 2010, was a Mw 7.1, over 35 km from the central business district (CBD). In the first earthquake, 21% of the mass masonry structures were “red-tagged”, indicating that the buildings were not accessible, while 47% received “green tags” which were safe for public use. On February 22nd, 2011, there was a Mw 6.2 earthquake less than 10 km from the CBD, and another close proximity Mw 6.0 on June 13, 2011. After the February earthquake, the number of “red-tagged” buildings increased to 82%, and only 1% were “green-tagged”. After the June earthquakes, 68% of the buildings had been demolished.

Based on the articles, it is clear that the subsequent aftershocks and cumulative damage significantly weakened the masonry structures. For example the Christchurch Cathedral, a stone church of the gothic revival style reminiscent of many Ottawa churches, “suffered insignificant damage during the 2010 earthquake. The only damage was to ornamental stone details.” (Anagnostopoulou and Bruneau, 2010). After the February earthquake, the building experienced a number of collapses including top of the bell tower above the buttresses and front gable wall.

There were a number of retrofits performed for masonry buildings between the earthquakes, notably the installation of masonry anchors to the floor and roof diaphragms. The poor performance of adhesive anchors was observed in the subsequent earthquakes. Through-wall anchors with plates on the exterior had varying success, see Figure 2.20 where some walls had collapsed leaving the anchors attached to the structure, some walls had visibly pulled against the anchor partially failing but leaving the wall intact, and some having been fully successful.

(a) Failed gable wall anchor (b) Damaged gable wall anchor (c) Successful gable wall anchors

Figure 2.20: Gable Wall Anchor Performance in the Christchurch, NZ, Earthquakes, reproduced from Turner et al. (2012)

It is not clear if, at the time that these reinforcing techniques were implemented, the masonry was repointed and the cores of the walls were reconsolidated as required. The detrimental effect of the subsequent earthquakes indicates ongoing and cumulative damage to the masonry, and it is likely that there was internal cracking and other mechanisms occurring within masonry resulting in strength loss.

2.3.3 Buttress Retrofits

One of the most interesting parts of stone construction is the evolution of the structures over time as the needs and requirements of the community changed. Mass masonry structures were not built for temporary purposes, so as cities grew, the need for larger buildings with new uses frequently resulted in the renovation and modification of these structures. It is a safe assumption for any engineer working with these historic structures that there have been previous repair, renovation and alteration campaigns over the generations.

An interesting article by Clifton (2008), discusses the historical and modern structural interventions of cathedrals in the UK, with examples from the authors' personal experience. One of the case studies is the retro choir at the Wells Cathedral. The slender columns of the choir support unbalanced loads from flying buttresses supporting the high vault above. These columns are known to support lateral loads and, based on Clifton's analysis, the columns have a safety factor just over 1, which is reasonable since this structure has been standing unchanged for 900 years. Looking at the history of the structure, Clifton discovered that every Cathedral Architect,

Engineer, or master mason working on the cathedral throughout history had also expressed concerns about this system, and had even set up various forms of monitoring systems. He concludes by saying that many of the current concerns are “merely the continuation of worries of the past generations”.

While our concerns with historic buildings may be consistent with past generations, our methods of strengthening and repair have evolved. This introduces the problem of incompatible restoration practices, since many recent restoration campaigns include steel and reinforced concrete. The American publication *Techniques for the Seismic Rehabilitation of Existing Buildings* (FEMA, 2006), includes an entire chapter on unreinforced masonry, including a table outlining seismic deficiencies and potential rehabilitation techniques for bearing walls. Every recommended solution is either of steel, concrete, or wood. For insufficient out-of-plane strength, the document recommends improving the connection between the floors/roof and the wall, adding steel reinforcing in the core of the wall, or applying a concrete wall or a fibre composite wall overlay. Notably, the only line of the chapter which mentions the condition of the masonry is: “In typical stone masonry and brick masonry situations, the wall will be adequate to serve as the backside form. When the wall is particularly thin or in poor condition, the contractor will have to take care to brace the masonry to resist the force of the applied shotcrete.” (FEMA, 2006)

Based on the damage to buildings in previous earthquakes, there is a significant risk that the addition of these extremely stiff elements will contribute negatively to the seismic behaviour. In addition, the presence of concrete directly adjacent to stone masonry significantly affects on the moisture transfer through the wall and can cause serious deterioration. The incompatibility of modern materials and stone masonry is discussed at length in a number of reference texts, notably Roca et al. (2019) and English Heritage (2012). The long-term concern of using these new materials in historic structures is highlighted in the following quote.

All over Europe we have repaired, strengthened and stiffened our major masonry buildings with concrete. Is this a good thing? Is it durable? What happens to our lovely flexible masonry buildings in the next 900 years after they have been stiffened and tied together? (Clifton, 2008)

But what is the alternative? The standards of safety of today are different than they were 100 years ago, tightening over the decades with each subsequent building code. When a deficiency is noted, engineers working on these structures need to be able to strengthen them with reinforcing measures that have been confirmed by modern engineering principles.

Where known out-of-plane strength is a concern, there is the possibility, as has been implemented in the past, of using the buttress as a reinforcing element. A number of buildings over the course of history have been reinforced with buttresses, and the list below only includes some of the most famous, and some of the most relevant. The first two case studies, the Colosseum and the Hagia Sophia, are frequently cited when discussing buttress interventions as they were implemented on structures that were known to have significant value at the time that they were reinforced. Based on significant deliberation, the buttress was chosen to be the best solution to preserve and strengthen these structure for future generations. The third case study, St. Brigid's, is a local Ottawa example of adding reinforced concrete buttresses to a church. The final example, the Church of Santiago Apóstol in Peru, is a recent project where strengthening the church was performed using local materials and traditional methods.

The Colosseum, Rome, Italy (ca. 70 AD)

The Colosseum structure survived the fall of the Roman empire and over a dozen centuries of various uses including as a shelter complete with cooking fires, stables, and gun-powder storage (Jokilehto, 1986). By the beginning of the 19th century the structure was in a bad state of repair. There had been a serious earthquake at the beginning of the 18th century which caused partial collapse, and another earthquake in the first years of the 19th century which further endangered the structure, notably the remaining sections of the outer ring had cracked and was out of plumb. Raffaele Stern led a major stabilization project in 1806-1807 building brick buttresses to stop the lateral movement. This solution allowed the stabilization of the structure, and froze the historic arches in a state of mid-collapse, see Figure 2.21.

(a) Accepted Proposal for the Buttress Consolidation of the East Wall. Reproduced from Jokilehto (1986)

(b) Photograph of the Completed Buttress and Frozen Arches. Reproduced from Roca et al. (2019)

Figure 2.21: Colosseum stabilizing buttresses, 1806-1807

Hagia Sophia, Istanbul, Turkey (ca. 532 AD)

The Hagia Sophia, an icon of Istanbul, was constructed between 532 and 537 AD with a main dome 31.2 m in diameter and 55.6 m tall (Roca et al., 2019). By the 16th century, the structure had deteriorated significantly and “faced imminent collapse, already leaning about 1.5 cubits to one side” (Mark and Cakmak, 1993). A cubit is generally accepted to mean approximately 18 inches, indicating that the structure was leaning around 680 mm according to this record. The Ottoman architect Mimar Sinan was tasked with reinforcing the structure, and built an impressive buttressing system at the exterior of the basilica to support the thrust of the internal vaulted structures, see Figure 2.22.

Figure 2.22: Hagia Sophia buttressing system built by Sinan in the 16th century. Reproduced from Roca et al. (2019)

St. Brigid Church, Ottawa, Canada (ca. 1889 AD)

Noticeable deformation of the sidewalls of St. Brigid's Church (now St. Brigid's Centre for the Arts) in the Byward Market in Ottawa was discovered in the 1960s, at the same time as major construction works were being implemented on the main road adjacent to the structure (Jokinen, 1987). It is hypothesized that the installation of the new sewer system and rain water catchment changed the water content of the ground, destabilizing the footings. This area of the city has a large overburden of clay, making it likely that the foundations were bearing on moisture-susceptible materials. The softening of the ground in addition to the eccentric stress profile under the footing (due to the applied lateral load on the sidewalls/stone buttresses from the roof truss) resulted in the walls tilting outwards. It is important to note that the original buttresses on this building were the most slender buttresses recorded based on the survey of buttresses around the city of Ottawa (see Section 2.4.2).

The solution of installing large reinforced concrete buttresses at the location of the existing stone buttresses was chosen, stabilizing the wall rather than rebuilding it, see Figure 2.23. The new buttresses were constructed along with new footings, deeper than the original and acting as a partial underpinning. The connection at the top of the wall between the stone and roof

truss was also reinforced.

(a) Detail of the New Buttresses. Reproduced from Jokinen (1987)

(b) Photograph of the Completed Concrete Buttresses. (Marrs, Nov 2019)

Figure 2.23: Colosseum stabilizing buttresses, 1806-1807

Church of Santiago Apóstol, Kuñotambo, Peru (ca. 1681 AD)

More recently, the Church of Santiago Apóstol was damaged by previous earthquakes and lack of maintenance (Cancino et al., 2020). This church is built of adobe and other traditional materials. The interior maintains the original decorative wall paintings, which were noted to have significant historical value so any repair/reinforcing solution needed to conserve the existing interior walls. A collaborative effort between the Getty Conservation Institute, the Pontificia Universidad Católica del Perú and the University of Minho studied and modelled the structure to determine the best reinforcing solutions. There was also the additional consideration that this church was located remotely in Peru's Andes Mountains.

The solutions derived were all of locally available materials, using traditional reinforcing systems. The foundations and walls were repaired, repointed, and consolidated. New buttresses were added to the exterior and the existing damaged buttresses were reconstructed. A new roof was constructed and attached to the top of the walls with timber-reinforced connections. The new buttresses were keyed into the walls with timber reinforcements to ensure that they would act as a composite section, as shown in Figure 2.24.

(a) Axonometric view of the church showing planned seismic interventions. Structural elements enlarged. Lateral walls have reconstructed buttresses connected with timber keys. Drawing by Giorgos Karanikoloudis, © 2016 J. Paul Getty Trust.

(b) Timber reinforcements and horizontal geomesh placed in one of the new buttresses. Photograph by Juan Carlos Mellado, © 2017 J. Paul Getty Trust and Dirección Desconcentrada de Cultura Cusco, Ministerio de Cultura del Perú.

Figure 2.24: Timber Reinforcement of New Adobe Buttresses. Reproduced from Cancino et al. (2020)

To conclude, buttresses are a valid reinforcing measure that has been implemented numerous times throughout history. Adding buttresses will not be the best solution for all projects, notably where the exterior form of the structure is considered to hold value. There are a number of other solutions that may be more attractive to a designer depending on the specific circumstances surrounding the structure in question. The purpose of highlighting these projects is to begin the conversation of compatible reinforcing techniques, and strengthening a structure using the same structural material with which it was constructed.

2.4 Buttresses

This Section first looks at the history and purpose of buttresses and finishes the Chapter with a survey of churches around the downtown core of Ottawa (Section 2.4.2).

The buttress was originally included in structures to provide increased out-of-plane strength of walls. Buttresses act as an additional mass to reinforce and stabilize the walls in a singular direction, typically to resist an externally applied lateral force such as from arches or soil pressure. The masonry buttress became more common in the Roman era with the construction of arched and vaulted structures. As the arches and vaults got larger, the lateral thrust they exerted on the exterior walls became too high for a standard wall. The most iconic buttresses are the flying buttresses from the gothic period (circa 1150-1450) (Heyman, 1995). At this time, the churches were growing in size from approximately 50 ft to over 200 ft tall. The buttress form of an increase in wall section for the earlier and shorter churches needed to reach incredible thicknesses in order to resist the forces for such tall churches, highlighted in the sketch in Figure 2.25, showing a mass buttress and equivalent flying buttress.

Figure 2.25: From Hudson and Cosgrove (2019): (a) A stone arch supported on high piers. On the left side, stability is ensured by the use of a large buttress. On the right side, the thrust is channelled down only the necessary part of the buttress. (b) Use of this principle illustrated by the flying buttresses at Reims Cathedral, France.

Figure 2.25 also shows another structural detail, the pinnacle. The weight from the pinnacle ensures that the friction force at the location that the lateral thrust meets the pier is high enough that the buttress does not fail by sliding.

2.4.1 Typical Buttress Locations

Buttresses are typically found at specific locations around buildings where additional reinforcing is required. In Canada, these locations can be generally categorized as the sidewalls of buildings that have peaked roofs, the corners of towers/at corners, around circular masonry features, on either side of arched doorways, and on walls with a significant number of windows. Photos of each type are included in Figures 2.26 to 2.30 below.

(a) Ottawa First Baptist Church

(b) St. Albans Anglican Church

Figure 2.26: Buttresses Along Sidewalls. (Marrs, Nov 2019)

(a) St. Patrick's Basilica

(b) Christ Church Cathedral

Figure 2.27: Buttresses At Towers. (Marrs, Apr 2020 and Aug 2020)

(a) Museum of Nature

(b) Notre-Dame Cathedral Basilica

Figure 2.28: Buttresses At Circular Masonry Features. (Marrs, Aug 2020 and Dec 2020)

(a) St. Andrew's Presbyterian Church

(b) Dominion University College

Figure 2.29: Buttresses At Doorways. (Marrs, March 2020 and Aug 2020)

(a) Connaught Building

(b) Old College of Bytown Building

Figure 2.30: Buttresses Between Windows. (Marrs, Sept 2020 and Dec 2020)

2.4.2 Survey of Ottawa Churches

Scattered around the downtown core of Ottawa are dozens of churches dating from the early 1800s to the middle of the 20th century. The buttresses measured for this survey are buttresses along the sidewalls of mass masonry buildings.

The methodology of this survey was measuring the buttress at grade and extrapolating these results. The average stone size was first measured, and for rubble stone where the stones vary significant in height the average height of three stones was taken. The dimensions of the foundation, and first tier of the buttress were typically able to be measured. The height of the buttress was calculated simply by multiplying the number of courses by the average height of three stones. This results in a good approximation of the height, but it is not perfectly accurate. For the purposes of this research, to acquire the average heights and sizes of buttresses found around the city, this accuracy was adequate. To not overstate the accuracy, the acquired dimensions are not shown directly on the line drawings produced for the buttresses. However, the buttresses are drawn on a 250mm x 250mm grid for approximate sizing, and all of the buttresses are printed at the same scale.

The summarized values for the height of the buttress, depth of the main/lowest section of the buttress and width of the buttress are included in Table 2.1. The full results including scaled cross sections of the buttress survey are included in Table 2.2. These parameters provided reasonable ranges for the buttress models in Chapter 5. The other important note which factored into the models is that the buttress stones are typically large ashlar stones, on average 600 x 300 x 300 mm ($l \times w \times h$), while the stones for the walls are typically rubble with significant variations in size.

Table 2.1: Summary of Ottawa Buttress Survey

	Buttress Height (m)	Main Section Depth (mm)	Buttress Width (mm)
Range	2 - 9.6	420 - 1180	430 - 1090
Average	6.06	738	670
Median	6.25	690	615
Mode	4.8	660	610
Used in Chapter 5	3.6 - 9.6	300, 600, 900	600

Table 2.2: Survey of Ottawa Buttresses

	Allsaints Event Space	Bell Street Methodist Church	Centretown United

<u>Church Information:</u>			
Year	1900 ¹	1910 ²	1906 ³
Neighborhood	Sandy Hill	Centretown	Centretown
Address	317 Chapel St	384 Arlington Ave	507 Bank St
Material	Stone	Brick	Stone
Avg. Unit Size (<i>l x w x h</i>)	460 x 280 x 270 mm	200 x 95 x 68	600 x 290 x 290 mm
<u>Buttress Dimesions:</u>			
Height (m)	2.5, 7.1	1.4, 4.8, 8.5	1.5, 6.7, 8.6
Depth (mm)	1070, 990	520, 320	650, 600, 400
Width (mm)	1090	630	600

¹ www.heritagetrust.on.ca/en/places-of-worship/AllSaintsAnglicanChurch

² www.heritagetrust.on.ca/en/places-of-worship/BellStreetMethodistChurch

³ www.heritagetrust.on.ca/en/places-of-worship/CentretownUnitedChurch

Survey of Ottawa Buttresses (cont'd)

Church Information:

Year	1872 ¹	1904 ²	1909 ³
Neighborhood	Centretown	Glebe	New Edinburgh
Address	439 Queen St	109 Fourth Ave	39 Dufferin Rd
Material	Stone	Brick	Stone
Avg. Unit (<i>l x w x h</i>)	- x - x 235	215 x 100 x 62	550 x 270 x 320

Buttress Dimesions:

Height (m)	2.0, 3.2, 6.0	1.2, 4.8, 6.6	5.3, 7.2
Depth (mm)	1030, 830	1180-650*, 650	550, 270
Width (mm)	1020, 620	650	550

* Depth is sloped linearly.

¹ www.heritagetrust.on.ca/en/places-of-worship/ChristChurchCathedral

² www.heritagetrust.on.ca/en/places-of-worship/FourthAvenueBaptistChurch

³ www.heritagetrust.on.ca/en/places-of-worship/MacKayUnitedChurch

Survey of Ottawa Buttresses (cont'd)

Ottawa Baptist
Evangelical Ottawa First
Baptist Peace Tower
Church Church Information:

Year	1919 ¹	1878 ²	1901 ³
Neighborhood	Byward Market	Centretown	Centretown
Address	284 King Edward Ave	140 Laurier Ave W	343 Bronson Ave
Material	Brick	Stone	Stone

Buttress Dimesions:

Height (m)	1.2, 3.2, 5.1	2.2, 6.2, 9.6	1.5, 2.1, 4.4, 6.5
Depth (mm)	655, 440	800, 400	810, 600, 390
Width (mm)	540	600	610
Avg. Unit (l x w x h)	215 x 100 x 62	560 x 280 x 260	610 x 300 x 300

¹ <https://www.heritagetrust.on.ca/en/places-of-worship/ÉgliseÉvangéliqueBaptisted'Ottawa>² <https://www.heritagetrust.on.ca/en/places-of-worship/FirstBaptistChurch>³ <https://www.heritagetrust.on.ca/en/places-of-worship/PeaceTowerChurch>

Survey of Ottawa Buttresses (cont'd)

St. Albans
Anglican Church[†] St. Andrew's
Presbyterian Church St. Bartholomew's
Anglican Church Church Information:

Approx. Year	1867 ¹	1872 ²	1868 ³
Neighborhood	Sandy Hill	Centretown	New Edinburgh
Address	454 King Edward Ave	82 Kent St	125 MacKay St
Material	Stone	Stone	Stone

Buttress Dimesions:

Height (m)	0.9, 3.2	0.8, 4.2, 6.9, 8.0	0.3, 2.0
Depth (mm)	970, 830	940, 620, 160	625
Width (mm)	1050, 770	620	610
Avg. Unit (<i>l x w x h</i>)	- x - x 300 mm	600 x 290 x 290 mm	- x - x 200 mm

[†] St. Albans was built on a slope, only the southern (shorter) buttresses were accessible to be measured.

¹ www.heritagetrust.on.ca/en/places-of-worship/StAlbansAnglicanChurch

² www.heritagetrust.on.ca/en/places-of-worship/StAndrewsPresbyterianChurch

³ www.heritagetrust.on.ca/en/places-of-worship/StBartholomewsAnglicanChurch

Survey of Ottawa Buttresses (cont'd)

St. Giles
Presbyterian Church St. John the
Evangelist St. Luke's
Lutheran Church Information:

Year	1928 ¹	1891 ²	1915 ³
Neighborhood	Glebe	Centretown	New Edinburgh
Address	174 First Ave	154 Somerset St W	326 McKay St
Material	Brick	Brick	Brick

Buttress Dimesions:

Height (m)	0.6, 2.2, 5.0	0.8, 4.9	3.1, 4.2
Depth (mm)	480, 430	660	420, 310
Width (mm)	430	640	620
Avg. Unit (<i>l x w x h</i>)	200 x 100 x 58	210 x 100 x 70	200 x 100 x 62

¹ www.heritagetrust.on.ca/en/places-of-worship/StGilesPresbyterianChurch² www.heritagetrust.on.ca/en/places-of-worship/StJohnEvangelistChurch³ www.heritagetrust.on.ca/en/places-of-worship/St.Luke'sLutheranChurch

Survey of Ottawa Buttresses (cont'd)

St. Paul
Lutheran Church St. Peter & St. Paul's
Anglican Church Trillium
Hall Church Information:

Year	1889 ¹	1880 ²	1911 ³
Neighborhood	Sandy Hill		Centretown
Address	210 Wilbrod St	152 Metcalfe St	381 Kent St
Material	Stone	Brick	Brick
<u>Buttress Dimensions:</u>			
Height (m)	1.5, 4.8	0.8, 4.8	1.2, 5.4, 6.9
Depth (mm)	820, 720	790	660, 470
Width (mm)	610	540	660
Avg. Unit Size (<i>l x w x h</i>)	- x - x 300	210 x 100 x 60	205 x 95 x 60

¹ www.heritagetrust.on.ca/en/places-of-worship/StPaulsEvangelicalLutheranChurch² www.heritagetrust.on.ca/en/places-of-worship/StGeorgesAnglicanChurch³ www.heritagetrust.on.ca/en/places-of-worship/ChineseCommunityCentre

Chapter 3: Guidance for Assessing Masonry

Looking deeper into the available modern guidance for assessing stone masonry in Canada, there are the applicable building codes and standards, and also published research papers. The information contained within the Canadian codes and standards is included in Section 3.1, and the literature review is divided into research on the out-of-plane wall behaviour (Section 3.2), research on buttresses (Section 3.3), and major research programs in Canada (Section 3.4).

3.1 Canadian Building Codes and Standards

In Canada, structures are designed according to limit states design. There are both serviceability and ultimate limit states (SLS and ULS) that must be considered. ULS refers to mechanisms affecting the safety of the structure, for example exceeding the load carrying capacity, overturning, and sliding. SLS includes deflections, vibrations, permanent deformations, cracking, and, in general, issues that affect the everyday use of the building.

This study focusses on the ULS design of masonry structures. In ULS design, the anticipated loads on the structure are increased with load factors and the strength of the materials are decreased with resistance factors. A building and its structural components shall be designed to have sufficient strength and stability so that the factored resistance is greater than the factored loads.

The two main documents which govern the design and analysis of stone masonry buildings in Canada are the National Building Code of Canada (NBCC) (NRC, 2015) and the standard for the Design of Masonry Structures, CSA S304-14 (CSA, 2014). The NBCC sets out technical provisions for the design and construction of new buildings. This is the foundational document

for design and construction in Canada, with all other codes and standards building off of it. The standard CSA S304-14 specifies requirements for the structural design of unreinforced, reinforced, and prefabricated masonry structures and components in accordance with the limit states design method of the NBCC.

3.1.1 National Building Code of Canada (NRC, 2015)

The National Building Code of Canada describes the safety requirements of new constructions and the determination of loads for all structures, new and existing. The methods to determine the design loads for a structure are generally independent of the construction material. For this discussion, the limitations that the NBCC places on mass masonry buildings are interesting since these limitations make it so that many of the existing buildings are not “code compliant”.

According to the NBCC, unreinforced masonry shall not be used where the seismic hazard index is greater than or equal to 0.35.

$$\text{Seismic Hazard Index} = I_E F_a S_a(0.2) \quad (3.1)$$

where: I_E is the importance factor, typically = 1.0;

F_a is the site coefficient, = $F(0.2)$; and

$S_a(0.2)$ is the 5% damped spectral response acceleration, as a ratio to gravitational acceleration.

See Table 3.1 for the values of the seismic hazard index for different soil classes in central Ottawa, respectively. Note that according to this clause, unreinforced masonry buildings are only allowable in Ottawa for Class A and B soils, which are both rock type classifications. This indicates that many of the existing buildings around the city, notably in the Byward Market which has clay overburden over bedrock that was frequently not excavated, would not be permitted to be constructed today.

In addition, there are height restrictions for unreinforced masonry buildings. If the seismic hazard index is less than 0.2, then the maximum building height above grade is 30 m. Regions with a seismic hazard index greater than 0.2, as it is in Ottawa for all soil classes, the

Table 3.1: Ottawa Seismic Hazard Index By Soil Classification.

City	Soil Classification				
	A	B	C	D	E
Ottawa (City Hall)					
F_a	0.69	0.77	1.00	1.02	1.09
$I_E F_a S_a(0.2)$	0.303	0.338	0.439	0.448	0.479

maximum building height is limited to 15 m above grade. Under these limitations a number of existing buildings around the city including the Parliament buildings would not be permitted to be constructed today.

3.1.2 CSA S304-14 Design of Masonry Structures (CSA, 2014)

This Standard specifies requirements for the structural design of unreinforced, reinforced, and prefabricated masonry structures and components in accordance with the limit states design method of the National Building Code of Canada (CSA, 2014). This Standard does not specifically apply to existing buildings, but best practice for engineers is to reference this document. In addition, the Standard specifically states that it does not apply to the structural design of rough (rubble) stone masonry (CL 1.6).

These brief descriptions of the contents of the Standard immediately highlight the lack of guidance for Canadian engineers when working on stone buildings. For the purposes of this thesis, the procedures described within the Standard for unreinforced masonry will be applied to the analysis of existing stone structures.

Definitions

The following definitions can be found within the Standard, and are summarized below for clarification.

A COLUMN is a stand-alone vertical compression member. When the column is contained within a wall, it is called a PIER. If the column is built integrally with the wall and interacts with the wall to resist out-of-plane loads it is a PILASTER. A BUTTRESS is a pilaster with a cross-section that increases from the top to bottom. The wall between these members can be assumed to span horizontally between them to carry the lateral loads.

Resistance Factors

Resistance factor for masonry shall be taken as $\phi_m = 0.60$.

The effective stiffness, $(EI)_{eff}$, used in determining slenderness effects on the capacity of a wall or column shall be reduced by a factor of $\phi_e = 0.65$ for unreinforced masonry.

Specified Strength

The design compressive strength of the masonry, f'_m , shall be determined based on the results of masonry prism tests or shall be assigned based on unit and mortar tests. *Annex D* of the Standard provides guidance on the construction and testing protocol of the masonry prisms and *Annex C* of the Standard provides calculations to acquire f'_m from these tests. A minimum of 5 tests are to be conducted, with this number increasing to a minimum of 10 tests if the first 5 have a coefficient of variation greater than 15%. There are also tabulated values specifying the strength of solid brick masonry with either Type S or Type N mortar, with unit strengths ranging from 15 - 90 MPa, and the compressive strength for concrete block masonry with various mortars and grouts. There is no additional guidance for stone masonry.

Similarly, the design flexural tensile strength is specified for clay brick, concrete brick, and block. There is no additional guidance for stone masonry and instead testing is recommended, with the same testing requirements as for the compressive strength. The axial tensile strength is taken as zero for unreinforced masonry.

The design modulus of elasticity should be acquired by testing, but can be assumed as $E_m = 850f'_m$, but not greater than 20,000 MPa.

The recommendation for testing is likely in response to the focus on new constructions, where the materials are being sourced for the project and an additional 10 units can be purchased for the testing. This is much more complicated for existing buildings, where this testing would require samples be removed from the structure for destructive testing.

Axial Load and Out-of-Plane Bending in Walls and Columns

The factored axial load resistance, P_r , of compression members, neglecting any lateral effects, is equal to:

$$P_{r(max)} = 0.80(0.85\phi_m f'_m A_e) \quad (3.2)$$

where; $A_e = A$ when voids do not exceed 25% of the area.

Since URM walls have no effective tensile strength, they must be designed for lateral loads in such a way as to prevent cracking. In the standard this is done by limiting the eccentricity, e , to less than e_{limit} , where e_{limit} is 0.33 times the direction perpendicular to the bending axis for rectangular walls and columns, or is 0.5 times the distance from the centroid of the section to the extreme compression fiber for non-rectangular walls and columns. See Equation 3.3 for a rectangular section.

$$e = M_{fp}/P_f \leq e_{limit} = t/3 \quad (3.3)$$

where: e is the eccentricity;

M_{fp} is the factored primary moment caused by lateral loads;

P_f is the factored axial load;

e_{limit} is the eccentricity at which tension develops in the section; and

t is the thickness of the wall or column perpendicular to the bending axis.

This can be rearranged, with the wall geometry and axial load known, to solve for the moment at which tension develops, denoted $M_{f,tension}$, see Equation 3.4.

$$M_{f,tension} = \frac{t}{3}P_f \quad (3.4)$$

The above Equation 3.3 can be applied to walls with a slenderness ratio, kh/t , less than 10. Once this ratio exceeds 10, then slenderness effects need to be considered. The slenderness ratio should never exceed 30. There are two methods included in the code to account

for slenderness: the $P\delta$ method and the moment magnifier method.

The $P\delta$ method adds a $P_f \delta_f$ term to the calculation of the total moment, which must be less than the moment at which tension develops, see Equation 3.5.

$$M_{f,tension} = M_{fp} + P_f \delta_f \quad (3.5)$$

where: $P_f \delta_f$ is the additional moment due to $P\delta$ effects;

P_f is the factored axial load; and

δ_f is the lateral deflection under the factored lateral and axial loads.

There is limited guidance provided to calculate δ_f . However, the Standard states that the following rigidity coefficient must be included, see Equation 3.6.

$$\phi_e (EI)_{eff} / (1 + 0.5\beta_d) \quad (3.6)$$

where: $(EI)_{eff}$ is the effective stiffness, $(EI)_{eff} = 0.4 E_m I_o$;

E_m is the modulus of elasticity of the masonry;

I_o is the gross moment of inertia perpendicular to the bending axis; and

β_d is the ratio of factored dead load moment to total factored moment.

The moment magnifier method increases the applied moment based on Euler's buckling, according to Equation 3.7.

$$M_{f,tension} = M_{fp} \frac{C_m}{\left(1 - \frac{P_f}{P_{cr}}\right)} \quad (3.7)$$

where: $C_m = 0.6 + 0.4M_1/M_2 \leq 0.4$;

M_1/M_2 shall be determined by calculating end moments, when $e_{end} \geq 0.1t$; or

M_1/M_2 shall be equal to 1.0, when $e_{end} \leq 0.1t$ or $>50\%$ of M_{fp} is from lateral loads

$$P_{cr} = \frac{\pi^2 \phi_e (EI)_{eff}}{(1 + 0.5\beta_d)(kh)^2}$$

Shear in Walls and Columns

The factored in-plane shear resistance, $V_{r,ip}$, shall be taken as Equation 3.8.

$$V_{r,ip} = \phi_m (\nu_m b_w d_v) + 0.25 (P_d) \gamma_g \leq 0.4 \phi_m \sqrt{f'_m} b_w d_v \gamma_g \quad (3.8)$$

where: $\nu_m = 0.16 \left(2 - \frac{M_f}{V_f d_v} \right) \sqrt{f'_m};$

$\frac{M_f}{V_f d_v}$ shall not be less than 0.25 or greater than 1;

b_w is the width of the wall or column;

d_v is the effective depth for shear calculations, $= 0.8l_w$;

P_d is the dead axial load, taken as 0.9 times the dead load; and

γ_g is taken as 1.0 for solid masonry, A_e/A_g for other types.

The factored out-of-plane shear resistance, $V_{r,oop}$, shall be taken as Equation 3.9.

$$V_{r,oop} = \phi_m \left[0.16 \sqrt{f'_m} A_e + A_e + 0.25 P_d \right] \leq 0.4 \phi_m \sqrt{f'_m} A_e \quad (3.9)$$

The factored sliding shear resistance, $V_{r,sliding}$, shall be taken as Equation 3.10.

$$V_{r,sliding} = 0.16 \phi_m \sqrt{f'_m} A_{uc} + \phi_m \mu P_1 \quad (3.10)$$

where; A_{uc} is the uncracked portion of the effective cross-sectional area

μ is the friction coefficient, $= 1.0$ for a masonry-to-masonry or $= 0.7$ for a masonry-to-smooth concrete or bare steel; and

P_1 is the factored dead load above the sliding plane.

There are no explicit design recommendations for walls with buttresses. In Chapter 6, Section 6.2.2, when the buttress models are compared against these CSA calculations, all of the above methods are checked for their applicability.

3.2 Previous Research: Out-of-Plane Behaviour

With the goal of quantifying the out-of-plane strength of walls with buttresses, first the existing research on the out-of-plane behaviour of standard walls is examined. This research is widespread geographically, looking at local building typologies and weaknesses. The limited connections between these diverse research campaigns has resulted in the information being spread over dozens of research articles and the obfuscation of the true picture of the current research status. This has previously been addressed by Ferreira et al. (2015b) in the 24-page article “Analysis of the Out-Of-Plane Seismic Behaviour of Unreinforced Masonry: A Literature Review”, providing a thorough history of research, experimental works and modelling procedures. This previously published literature review is complimentary to the following section, and the curious reader is directed there for additional information.

The wide variety of existing experimental work that has been taken place can be divided into Plain Wall Behaviour (Section 3.2.1), and Composite Wall Behaviour (Section 3.2.3). Plain walls refer to free-standing walls, without any influence of the side walls. In experimental programs, these are built alone or for in-situ testing the wall segment may be saw-cut from the rest of the building. Composite walls include sidewalls, or test set-ups can include steel frames to impose boundary conditions on free-standing walls. Composite action is a beneficial response that increases the strength of the masonry wall through internal arching within the masonry and potential two-way bending, depending on the distance between the side walls.

The boundary condition at the top of the wall is also extremely important, and varies throughout the experimental campaigns. Common construction in Canada involves wooden floors or roofs bearing directly into notches in the masonry. Churches and other vaulted spaces typically have large timber roof trusses simply bearing on the exterior walls. Neither of these construction methods provide significant restraint at the top of the wall. All top boundary conditions are included in this literature review, since reinforcing the top connection detail (ie. between the wall and the roof) is a frequently recommended reinforcing detail. However, it should be noted that for a standard historic construction it is conservative to analyze the exterior, load-bearing masonry walls as cantilevers.

A final note prior to examining other research campaigns is the widespread influence on modern engineers by the work of Jacques Heyman in the mid-20th century. “The Stone Skeleton” by Heyman (1966), is an extremely influential and highly cited paper, resulting in the publication of a 150-page book under the same title (Heyman, 1995). This text dives into the history of masonry construction, with descriptions on how the various stone components of a structure work independently, as well as together. The three basic masonry assumptions outlined in this work are frequently reproduced, and are the basis for many analytical procedures:

1. Masonry has no tensile strength,
2. Stresses are low in comparison to the crushing strength that masonry has effectively an unlimited compressive strength, and
3. Sliding failure does not occur.

The first assumption is conservative, neglecting the weak tensile strength of historic mortars and accounting for situations where the mortar may be deteriorated or missing. The second neglects the possibility of masonry crushing, the applicability of which is a focus of some studies (ie. Orduna and Lourenço (2003), Block et al. (2006), Gilbert (2007), and Portioli et al. (2015)). The final assumption, introduced in reference to arches, is based on the fact that the normal stresses and therefore Coulomb friction values are so high that stonework typically fails as a geometric instability. Notable exemptions of this final assumption, pointed out by Heyman, include at the top of walls or buttresses where lateral loads are applied with limited mass above, hence the use of pinnacles in historic constructions. Many researchers recommend having a separate check to see if failure by sliding is possible, rather than including friction and sliding in analytical calculation (see Giuffre (1996), Lourenço (1996), and D’Ayala and Speranza (2003)).

3.2.1 Rigid Body Mechanisms

Simplified rigid body mechanics have been applied to cantilevered mass masonry walls as a means of acquiring the peak overturning and sliding forces for the out-of-plane strength of masonry walls. This is a force-based method for analyzing the failure of the wall, based on the geometry and mass of the wall resisting overturning. Although extremely simple, this method

has consistently shown to be accurate in approximating the out-of-plane failure load of masonry walls. Heyman has been attributed with identifying this methodology for masonry structures (Heyman, 1966), however this is truly inspired by the methods of the past. These equations are compiled through static mechanics, and do not require sophisticated software or computing power to analyze.

Cantilevered Walls

Assessing a mass masonry wall as a cantilever is a conservative way to neglect poor connection details at the top of the wall, or to assess parapet walls. Most historic masonry buildings were not constructed with proper floor or roof diaphragms, resulting in the individual elements being susceptible to local out-of-plane failures.

A major study into the out-of-plane capacity of cantilevered, plain masonry walls was performed by Giuffrè and Cerdini in the 1990s where the stone wall was compared to the ideal wall, represented by a singular rigid body (Giuffrè (1996) and Ceradini (1992)). The study focused on simple mechanics: the overturning of the wall around the bottom corner based on forces through the centre of gravity (F_R), refer to Figure 3.1. There is also the “worst case” scenario where the wythes separate. The much lower force required to overturn a wall with a vertical shear plane is described in Figure 3.1 as F'_R .

Figure 3.1: Simplified rigid-body failure mechanism and force-displacement relationship of cantilevered walls. Reproduced from Giuffrè (1996)

The flaws with this idealization are apparent: the wall is not a singular stone instead is comprised of much smaller stones mortared together to create a larger mass. There are also

multiple wythes, which must be able to transfer shear between them in order to act as a singular unit. Ceradini focussed on this problem, performing multiple experiments on double-wythe walls varying the quantity of bond stones. The experiments were small double-wythe walls built of dry-stack blocks constructed on a tilting table. The tilt of the table was increased until the wall failed, the slope of the tilting table easily converting into the lateral acceleration/force as a portion of its mass. These experiments are published in Ceradini's thesis, in Italian, and are frequently referenced in literature. The results show a clear relationship between the number of bond stones and the out-of-plane strength of the wall, at a minimum when there are no bond stones so the wythes act completely independently (F'_R). Ceradini concludes that "*the [bond stones] have the task of supporting the smaller stones, making their weight participant to the stabilizing moment*".

The maximum horizontal force a wall can withstand through its centre of gravity before overturning is described in Equations 3.11 and 3.12. Note that this overturning force is independent of the height of the wall. As the wall height increases the self-weight increases in proportion to the overturning moment caused by the horizontal force. Therefore, theoretically, with a point load applied at the centre of gravity, there will be no difference in the wall overturning force (F) for two walls with different heights.

$$F_R \left(\frac{H}{2} \right) = \gamma b H \left(\frac{b}{2} \right) ; F_R = \gamma b^2 \quad (3.11)$$

where: F_R is the maximum horizontal overturning force for a solid wall;

$H/2$ is the height that the force is applied;

γ is the average density of the wall; and

b is the dimension perpendicular to the bending axis.

$$F'_R \left(\frac{H}{2} \right) = 2\gamma(b/2)H \left(\frac{b}{4} \right) ; F'_R = \frac{F_R}{2} \quad (3.12)$$

where: F'_R is the maximum horizontal overturning force for a wall with two distinct wythes.

Giuffrè assumes that below a certain height to thickness ratio of the wall, the wall will fail

by sliding rather than by overturning. The limiting height of the wall is $h_{sliding} = \frac{b}{2\mu}$, where: μ is the coefficient of friction. Above this height the wall will fail by overturning, below this height the wall will fail by sliding according to Equation 3.13.

$$F_S = \mu\gamma bH \quad (3.13)$$

where: F_S is the maximum horizontal sliding force.

A 2011 study by de Felice built on the work of Giuiffre and Ceradini by modelling the walls in a Distinct Element Modelling (DEM) software to confirm the results previously developed analytically/experimentally (De Felice, 2011). The ability to insert any geometry into the computer model allowed de Felice to vary the shape of the stones, from the perfectly squared ashlar as examined in Guiffre and Ceradini's work to true wall sections pulled from a database of typical masonry wall sections. This database had been previously compiled by a research group of the Polytechnic of Milan (Binda et al. 1999; 2003). The database includes photographs and drawings of masonry walls surveyed by the group, including information regarding the stone and mortar characteristics. A set of 30 wall sections of varying stone shapes and sizes representative of buildings around Italy were recreated in the DEM software by de Felice from the database and were tested under both quasi-static and dynamic analysis. The acceleration-displacement curves from the DEM tests are included in Figure 3.2. They are plotted as a percentage of the acceleration associated with a perfect rigid body (F_R). Both the perfect rigid body response and the response of two separate wythes are plotted as dashed lines, with the majority of the walls falling within these bounds. All sections showed a lower strength and displacement capacity than the rigid body, by an average of 25% and 35% respectively. Where the wythes were not properly connected together, the displacement capacity was reduced below the lower bound.

In addition to reconfirming Guiffre and Ceradini's work that there is a direct correlation between the percentage of headers and the out-of-plane capacity of walls, the other conclusion of de Felice's research is the comparison between the quasi-static analysis and the dynamic analysis. For well bonded walls, the hinge at the base typically formed at the same location during both tests. However, for walls with limited headers, there was a significant loss of strength and

Figure 3.2: Out-of-plane acceleration-displacement capacity curves. Reproduced with modification from De Felice (2011)

displacement capacity. The effects of the dynamic tests were more severe than the static tests, with the wythes ultimately detaching and causing collapse of the external wythe. For most of the tests (80%), the quasi-static overturning acceleration was less than the acceleration required during the dynamic tests. However, for 20% of the tests where there were inadequate headers and the wall failed by wythe separation it was shown that the wythes could separate at lower accelerations in the dynamic tests than the static tests.

A final study by Szyszka et al. (2018) further adds to this series of experimental testing by incorporating high speed cameras to capture the failure of the walls with varying headers.

The result of these works is the simplified representation of the out-of-plane behaviour of masonry walls by a bilinear curve, based solely on an infinite initial stiffness and the overturning equilibrium of the cantilevered wall around the base.

Restrained Walls

The above formulations for cantilevered walls can be modified to include a restraint at the top of the wall. The top can be assumed to be rigidly restrained, simply supported, or flexibly restrained. Rigid restraint, locking the top of the wall from displacement and rotation, has been used in laboratory experiments but is unlikely to be representative of true boundary conditions unless the wall segment in question is a lower storey and restrained by masonry above. Typi-

cally the lower sections of walls are thicker and are supporting a higher axial load so are less susceptible to damage than the upper storeys. Simply supported restraints can represent a good diaphragm connection, such as in situations where it has been reinforced, and is commonly used in laboratory experiments. Flexible restraints are the closest representation to the flexible diaphragm/weak connections that are common at the tops of these walls, and are the focus of recent research.

Simply supported walls have been shown through laboratory experiments to fail with a horizontal crack at mid-height (ABK Joint Venture, 1984). Rigid body mechanics have been applied with a simple static equilibrium calculation similar to the cantilevered wall to assess this form of cracking.

Rigid body mechanics for simply supported walls were investigated by Hendry (1997) as a method for simplifying the highly nonlinear behaviour of masonry walls. A graphic from the textbook “Design of Masonry Structures” is included in Figure 3.3 showing the rigid body formulation. The associated lateral force is described in Equation 3.14.

$$q = \frac{8\sigma t(t-a)}{h^2} \quad (3.14)$$

where: q is the applied uniformly distributed lateral load;

σ is the self-weight in load per unit area ($= \gamma h$);

t is the thickness of the wall; and

a is the deflection of the wall at mid-height.

The maximum force associated with the onset of cracking and lateral displacement occurs prior to cracking, when $a = 0$. This procedure assumes perfectly rigid wall sections, with the resulting point loads acting at the corners of the wall, and applies the self-weight at the top corner along with any applied gravity loads. This results in an unconservative calculation since having the self-weight acting at the inner corner rather than at the COG increases the stabilizing moment.

Flexibly restrained walls have recently been the focus of a number of research papers, in an attempt to improve the strength of masonry walls which would otherwise need to be

Figure 3.3: Simplified rigid-body failure mechanism of walls simply supported at top and bottom. Reproduced from Hendry (1997)

assessed as cantilevered. This type of analysis includes a spring at the top of the wall with an axial stiffness simulating an element restraining the walls, such as a tie-rod or a diaphragm. This results in eight possible failure mechanisms (four outwards and four inwards), depicted by Casapulla et al. (2017) and shown in Figure 3.4.

Figure 3.4: Simplified rigid-body failure mechanisms of flexibly restrained walls. Reproduced from Casapulla et al. (2017)

Gabellieri et al. (2012) looked at these failure mechanisms and the equations of motion

relating to each of the four mechanisms. An algorithm was developed for the numerical integration of these equations, and walls of varying sizes were subjected to a variety of ground motions, however the numerical results were not able to be compared to existing testing.

3.2.2 Semi-Rigid Body Mechanics

The concept of Semi-Rigid Body Mechanics pushes back on the assumption that the hinge around which the overturning moment is calculated is infinitesimally small, at the exact edge of the wall. If the hinge has an actual thickness, then the lever arm of the stabilizing moment from the self-weight is lowered and so is the force required for failure.

Priestley (1985) first attempted to quantify the width of the hinge by incorporating concrete principles and rectangular stress areas. This method used the known behaviour of concrete applied directly to masonry. The force is assumed to be distributed along the cracking plane. At the onset of cracking (maximum horizontal force) the stress distribution is triangular, 0 at the inner/cracking edge of the wall and at a maximum at the exterior edge. The reaction force R acts at a distance $t/3$ from the exterior edge. Beyond this stress, following the zero tensile strength assumption, the width over which the reaction force is distributed shrinks to the point where it converts to a rectangular stress approximation, shown in Figure 3.5. The width of the rectangular stress zone a is calculated as the reaction force over $0.85f'_m$.

The maximum inertial distributed load at the onset of cracking is described in Equation 3.15. Analyzing the failure of a simply supported wall according to this methodology results in a curvilinear response, an example of which is shown in Figure 3.6.

$$w_{in} = \frac{8}{h^2} \left(\sigma + \frac{W}{2} \right) \left(\frac{t}{6} \right) \quad (3.15)$$

where: w_{in} is the maximum inertial distributed load;

σ is the externally applied vertical load;

W is the self-weight of the wall; and

t is the thickness of the wall.

Doherty (2000) has largely been cited as expanding upon the bilinear relationship of Rigid

Figure 3.5: Concrete approximation for the rigid-body failure mechanism of simply supported walls. Reproduced from Priestley (1985)

Body Mechanics, into the trilinear relationship that has been denoted as “Semi-Rigid Body Mechanics”. Through an extensive research program of simply-supported single-wythe brick walls, it was shown that the experimental behaviour did not perfectly match the bilinear relationship. Figure 3.6 shows the response from all free vibrations responses with no overburden compared with the bilinear rigid body force-displacement relationship.

Figure 3.6: Experimental Force-Displacement Relationship Compared with Rigid Body Mechanics. Reproduced from Doherty (2000)

The experimental behaviour of the single-wythe brick walls showed that the walls can deform significantly, diverging from the hypothetical bilinear relationship. Doherty et al. (2002) states that these non-linear deformations of the mortar joints result in hinges having finite dimensions rather than occurring at the exact edge of the masonry. This shortens the length of the lever arm from the self-weight of the wall, lowering the restoring moment during rocking. The resulting curvilinear response incited the adoption of a trilinear approximation, with an initial slope K_{in} , a force plateau between Δ_1 and Δ_2 , and a third negative slope following the rigid body equation beyond Δ_2 , see Figure 3.7. This relationship is defined by the displacements Δ_1 and Δ_2 , as a percentage of the total displacement at instability (Δ_f). Doherty (2000) recommends the values described in Table 3.2 for the displacements, depending on the level of deterioration of the mortar joint.

Figure 3.7: Trilinear “Semi-Rigid” Force-Displacement Relationship Approximation.
Reproduced from Doherty et al. (2002)

Table 3.2: Displacement factors for the trilinear “Semi-Rigid” $F - \Delta$ relationship. Reproduced from Doherty (2000)

State of degradation at cracked joint	Δ_1/Δ_f	Δ_2/Δ_f
None	6%	28%
Moderate	13%	40%
Severe	20%	50%

3.2.3 Composite Wall Behaviour

The lack of connection between walls and sidewalls, and walls and diaphragms results in the early failure of the structure due to local out-of-plane collapse. When these connections are ensured, then the out-of-plane loads can be transferred through the diaphragm to engage the full capacity of the in-plane walls, referred frequently in literature as “box-like” behaviour. Many authors have discussed the efficiency of this “box-like” behaviour of masonry structures (D’Ayala and Speranza (2003), Valluzzi (2007), Simsir et al. (2004) Gabellieri et al. (2012), and Restrepo Vélez et al. (2014)).

D’Ayala and Speranza (2003) published a list of probable out-of-plane failure mechanisms, including the effect of the sidewalls. This study is highly cited in literature and the diagrams have been reproduced in reference to experimental tests and reviewing damages following earthquakes, refer to Figure 3.8. The publication also includes load factor equations for the overturning failure, based on a large variety of variables including the number of internal bearing walls parallel to the front wall, presence of openings, etc. Understanding the ways that the front wall could fail are important in order to design appropriate reinforcement measures.

Figure 3.8: Out-of-plane collapse mechanisms of historic masonry. Reproduced with modification from D’Ayala and Speranza (2003)

Restrepo Vélez et al. (2014) and Vaculik et al. (2014) built off of the work of D'Ayala and Speranza (2003) with experimental tilt-table tests on 1:5 scale dry stone masonry wall specimens, including full return walls at varying connection strengths and a variety of diaphragms in order to recreate the predefined collapse mechanisms as proposed by D'Ayala and Speranza (2003). In general, the analytical proposed equations provided a very good estimation of the collapse load over 41 tests, with a mean ratio of calculated result/test result of 0.95 and a coefficient of variation of 0.13.

Casapulla and Argiento (2016) looked at methods of including the side walls into a 2D problem, building on the rigid body mechanisms with flexible top restraints. The effect of the side walls was included as an additional force at the toothed corner connection that must be overcome in order for the front wall to separate. The corner connection is only typically held together by friction from the masonry self-weight. The force at the corner is therefore a triangularly distributed load (maximum at the base), which they simplified into a horizontal point load (F_p) at $1/3$ rd the height, see Figure 3.9.

Figure 3.9: Simplified rigid-body failure mechanisms of flexibly restrained walls. Reproduced from Casapulla and Argiento (2016)

In this Figure 3.9, the self-weight is denoted as P , overburden denoted as Q , restraining force at the top of the wall denoted as F_Q , restraining forces by tie-rods as T , and the vertical and horizontal thrust components from a vault or arch included as V_H and V_V . The lateral loads caused by inertial forces are prefixed with the activation load factor α . This activation load factor is a constant that neglects any dynamic amplification. The results were compared against experimental tilting table tests, with a 1.6% error for the free standing facade and 5.6% error for the three-wall system.

3.2.4 Previous Experiments

There are a number of experimental programs testing unreinforced single-wythe brick masonry walls which, while extremely informative in that field, do not represent multi-wythe stone construction. Even though the testing is not directly applicable, it is important to discuss the influential large-scale experimental campaign on the dynamic out-of-plane strength of masonry walls that took place in the United States in the 1980s by the ABK Joint Venture (ABK Joint Venture (1981) and ABK Joint Venture (1984)). These tests remain to date as the primary source for American guidelines for the seismic design and evaluation of unreinforced masonry such as FEMA 356 (FEMA, 2000). A wide variety of walls were tested, reinforced and unreinforced, of both clay brick and concrete block at a wide variety of slenderness ratios and axial loads. The walls were tested with displacement-controlled actuators at the top and bottoms of the walls, to represent the effects of the floors (simply supported at both ends). The results found that most URM walls crack horizontally at mid-height and near their base, collapsing once the mid-height crack opened to the point of instability. These studies concluded by summarizing the maximum slenderness (h/t) values for walls depending on their location within the structure. For example, the first storey of a multi-storey building has should have a maximum slenderness of 20, while the top storey should have a maximum slenderness of 9. A conclusion of note from these studies is the increased allowable slenderness ratio for buildings with cross walls, in the previous example a building with cross walls could have a maximum slenderness of 14 for the top storey.

Similarly, an extensive research campaign in Australia examined single-wythe brick walls

subjected to out-of-plane static push tests, free vibration tests and shaking table tests (Doherty, 2000). Specifically, the focus of these experiments were to look at the ‘weak links’ in the seismic load path of unreinforced masonry buildings, notably the limited force capacity of the connections between the walls and floors/roofs and the out-of-plane failure of the upper storeys of URM buildings. Material tests were conducted; however it was noted that the ultimate behaviour of the URM walls were not dependent on the brickwork material properties. The results from the experimental campaign resulted in the trilinear “semi-rigid” response, and compared the results to the simple rigid body formulations, and the concrete approximations as described by Priestley (1985).

Figure 3.10: Force-Displacement Curve Compared with Analytical. Reproduced from Doherty (2000)

In addition to these large-scale experiments, there are a number of smaller laboratory experimental campaigns using tilt tables to apply horizontal accelerations, actuators to apply point or line loads, airbags/water bags to apply pressures, or shake table to recreate ground motions. Tilt table experiments have been common in masonry testing as it easily applies a horizontal acceleration to the mass of the wall without the need for a shake table. The first tilt table tests noted were performed by Ceradini (1992) to test the effectiveness of header bricks on two-wythe walls. These were then further studied by Szyszka et al. (2018) where a large number of small-scale tilting table tests were performed with simple outputs and failures captured by high-speed cameras.

For the line load/point load experiments, the loads are either applied at the centre of gravity to represent horizontal forces from seismic loads or at the top to calculate the pushover

moment resistance. Tumialan et al. (2003) tested double-wythe brick and terracotta non-loadbearing walls of a decommissioned building out-of-plane to failure, with failure in good agreement with rigid body modelling. Dusi and Mezzi (2007) performed extensive laboratory tests on brick wall panels, including diagonal compression tests on brick masonry squared panels, shear compression tests on brick masonry rectangular panels, out-of-plane tests on brick masonry large panels, shear and flexural tests on complex 3D elements. Ferreira et al. (2015a) built six full-scale stone walls and tested three by a point load at the top and the other three by airbag testing, at varying degrees of superimposed axial load. Vermeltfoort and Martens (2016) removed masonry wallettes from existing buildings ranging from 95-3 years old and tested them in a laboratory for mechanical properties.

Applying a uniform pressure through a system of airbags has been suggested by Priestley (1985) to create a realistic representation of the lateral seismic forces. Typically, these experiments measure the force applied to the wall as the reaction force of the frame to which the airbags are connected. In New Zealand, Derakhshan (2011) performed a comprehensive quasi-static testing on full-scale, one-way spanning, two-leaf and three-leaf brick walls subjected to out-of-plane forces applied with airbags. As mentioned, three of full-scale stone walls built by Ferreira et al. (2015a) were tested in the lab with airbags.

Shaking table tests provide good information on the dynamic response of walls, with base accelerations representing either basic frequencies, synthetic ground motions or reproductions of actual time histories. Griffith et al. (2007) performed small scale shaking tests in the lab on single-wythe brick walls. Meisl et al. (2007) performed shake table experiments on three-wythe brick walls with regular headers, Bothara et al. (2009) performed small scale shaking table tests, and Magenes et al. (2012) had full-scale shaking table tests. Elmenhawawi et al. (2010c) tested eight full-scale multi-wythe stone walls on a single DOF shaking table under synthetic and actual time histories of increasing amplitude, following in-plane testing, to allow the walls to incur progressive damage.

Due to the high degree of complexity and wide range of possible material properties, it is impractical to construct exact replicas of historic URM bearing walls (Calvi et al., 1996). In-situ testing is a practical option which solves many of the issues associated with new constructions

of URM walls. Dizhur et al. (2009) performed in-situ testing with airbags on non-load bearing terracotta partition walls. Derakhshan (2011) tested in-situ panels from three multi-wythe brick buildings in New Zealand with airbag uniform out-of-plane pressures, having saw-cut on either side to separate the test wall from the rest of the structure. The different wall panels had varied top boundary conditions, including some tests with the existing wood framing and a concrete ring beam. Ismail and Ingham (2012) conducted in-situ tests on six single-wythe brick walls, saw-cut to exhibit one-way bending only. Three tests were basic walls the other three included near-surface mounted vertical twisted steel bars as a potential retrofit strategy. Ferreira et al. (2016) performed unidirectional and bidirectional tests using a system of airbags on multi-wythe dry stone walls damaged in the 1998 earthquakes which struck the Azores archipelago. Fonti et al. (2017) tested over a dozen walls from three buildings condemned following the 2009 earthquake in l'Aquila City. Some experiments showed very similar results to the rigid body rocking behaviour, especially when there was a plaster coating applied, at least for the first few cycles. Over time the stiffness and strength deteriorated along with cracking. Many of the tests also had large displacement plateaus, where the force remained constant but the wall exhibited significant deformation.

Other experimental campaigns referenced for this research which tested full-scale or composite wall systems are listed below:

- Griffith et al. (2007) tested eight full-scale single-wythe brick walls with returns under cyclic airbag forces. The variation between the walls included different wall geometries and openings.
- Dolce et al. (2008) tested 1:1.5 scaled full masonry buildings on shaking tables to test the efficacy of base isolation systems. The unreinforced model experienced partial collapse at 0.24g NPA with damage to the upper corners as well as activation of an out-of-plane collapse mechanism due to a complete disconnection of the walls without compromising the vertical load capacity of the structure. As the test continued the other three walls failed simultaneously.
- Arêde et al. (2008) performed in-situ testing on buildings damaged during the Azores

earthquake, resulting in the loss of the roof but the sidewalls were still standing. An interesting test set-up based on a self-equilibrated scheme where the back wall, built into the ground with half the unrestrained height, was used as the reaction force for the actuator. Over the course of this experimental program, a number of buildings were tested and published (ie. Costa (2002)). Strengthening schemes were included, and the walls were analyzed theoretically, similarly to the process described by Casapulla and Argiento (2016). The basic wall strength was predicted with only a 2.4% error, while the strengthened walls were estimated to be between 10% and 20% weaker than the experimental results, supporting the applicability of simple calculations for point load tests.

- Mazzon et al. (2009) built and tested two two-storey stone masonry buildings on shake tables subject to a natural ground acceleration. Separation of the wythes and out-of-plane failure at the corners of the building were noticed, tested up to a maximum PGA of 0.45g, with the first cracking occurring at 0.25g.
- Tomaževič and Weiss (2010) performed 1:5 scaled shaking table tests on full buildings, with the goal of finding the q behaviour factor as required by Eurocode 8. The small buildings were built with single wythe small ‘brick’ stones, including ties at the connections, with different internal configurations.
- A major European and Italian joint research program tested full-scale stone buildings on a shaking table, driven by the introduction of new seismic regulations by Eurocode 8. This research is published across a number of reports, journals and conference papers including Magenes et al. (2011) and Magenes et al. (2012). This testing suggests that the enhancement of the seismic performance of stone masonry buildings relates more to the improvement of the roof or floor-to-wall connection, than to a high in-plane stiffness of the floors since the experiments where stiff concrete floors were added resulted in crushing of the masonry.
- Costa et al. (2015) reported on the EUCENTRE shake table tests, recreating the response numerically using rigid-body dynamics.
- Vintzileou et al. (2015) tested a 1:2 scaled model of a full historic three-leaf stone ma-

sonry building with timber floors on a shaking table. A scheme of intervention techniques was adopted, comprising (a) grouting of walls using a hydraulic lime based grout, (b) enhancement of the diaphragm action of floors and (c) connection between diaphragms and bearing walls. Grouting resulted to an increase of the bearing capacity of masonry walls in shear and in out-of-plane bending, as well as to a modification of the sequence of their failure mechanisms. The stiffening of the floors and their connections to the walls also led to significant improvement of the overall response of the structure, more specifically, to substantial reduction of the out-of-plane deformations of masonry walls.

- Chourasia et al. (2016) performed full-scale shake table testing on URM brick masonry buildings modelled after the typical Indian layouts, including repeating tests with repairs, and also comparing the results with brick models built with bond-beams and corner reinforced concrete vertical reinforcement.

3.3 Previous Research: Buttresses

Based on the literature review, there has not been any testing done on walls which contain buttresses.

There is a research program from MIT looking at the theoretical analysis of buttresses, in collaboration with researchers of the Politécnica de Madrid and University of Lecce, see Ochsendorf et al. (2004) and Ochsendorf and De Lorenzis (2008). These papers formulate equations using statics that modern engineers can use. They looked first at the buttress as if it were a monolithic section, where the full mass was resisting overturning. Using simple statics calculating the overturning force is possible since the system is determinate. However, it was hypothesized that in reality there will be an angled fracture plane, and that not all of the mass would participate in the overturning. This was supported by historical analysis, where some designers from the 1800s assumed a 45° fracture plane, see Figure 3.11 (a).

This recent research treated the angle of the fracture plane as unknown. The plane was assumed to be straight, and that the lower end of the fracture at failure corresponds with the pivot point around which overturning will occur. For the crack progression, it is assumed that

the crack is formed on the inner face and spreads downward towards the overturning hinge. These assumptions allow for determining static equations, however it is noted that in actual failures the fracture plane would follow mortar joints. Figure 3.11 (b) shows the free body diagram of the recommended analysis method.

Figure 3.11: Analyzing Buttresses with Fracture Planes, reproduced from Ochsendorf et al. (2004)

If there were no fracture plane included, ie. Figure 3.11 (b-a), the stabilizing moment, M_s , for the buttress is:

$$M_s = W_b x_b + V b \quad (3.16)$$

where: W_b is the weight of the buttress;

x_b is the dimension from the COG to the compression edge;

V is the externally applied vertical load; and

b is the base dimension perpendicular to the bending axis.

This stabilizing moment, M_s , is reduced by the portion of the mass which fractures and remains behind, (W_c). The reduced stabilizing moment is at equilibrium with the externally applied moment at the point of fracturing, shown in Figure 3.11 (b-b).

$$M_s - W_c \left(\frac{2b}{3} \right) = H h \quad (3.17)$$

where; $W_c = b e \gamma / 2$

e is the height of the fracture;

γ is the average masonry density;

H is the externally applied horizontal load; and

h is the height of the externally applied horizontal load.

Substituting in M_s and W_c gives

$$W_b x_b + V b - \frac{b^2 e \gamma}{3} - H h = 0 \quad (3.18)$$

Equation 3.18 provides the general overturning equation for a buttress of any shape. There are two unknowns: the fracture height e and the collapse load H . To solve for these two variables, solely the bottom (fractured) portion of the buttress is examined, shown in Figure 3.11 (b-c). It is assumed that there is 0 tensile strength, so the stress profile at e would be triangular, 0 at the point of cracking. This allows the resultant force from the above mass to be at the known location $b/3$ due to elastic theory. Applying moment equilibrium to Figure 3.11 (b-c), this results in the following Equation 3.19.

$$W_b b + V b - \frac{b^2 e \gamma}{2} - 3 H h = 0 \quad (3.19)$$

Combining Equations 3.18 and 3.19 to eliminate e results in a quadratic equation that gives the value of the collapse load.

To use these equations, the authors make it clear that the line of thrust must be contained

everywhere within the masonry, and that this mechanism should be examined at all critical sections of the buttress. The analyst must also verify that sliding failure will not occur ($H < (W + V)f_s$), where W is the weight of the buttress above the level being considered and f_s is the coefficient of friction (typically 0.6-0.7) (Ochsendorf et al., 2004).

Rectangular Buttresses

The previous equations can be adapted to account for buttresses of various profiles. For rectangular buttresses, the formulation is further simplified (Ochsendorf et al., 2004). Three dimensionless parameters were introduced:

$$\zeta = \frac{e}{h} \qquad \mu = \frac{h}{h_b} \qquad \psi = \frac{V}{W_b}$$

where; ζ is the parameter relating the fracture height to height of the applied load;

μ is the parameter relating the height of the applied load to the buttress height; and

ψ is the parameter relating the externally applied vertical load to the weight of the buttress.

Simplifying the above Equations 3.18 and 3.19 for a rectangular section (ie. $W_b = b h_b \gamma$, $x_b = b/2$, etc.) and substituting the above dimensionless parameters results in Equation 3.20 for the ζ value at overturning, denoted as ζ_{OV} .

$$\zeta_{OV}^2 - \left(\frac{1}{2} + \frac{3}{2\mu} + \frac{3\psi}{\mu} \right) \zeta_{OV} + \left(\frac{\psi + 1}{\mu} \right) = 0 \quad (3.20)$$

This research program also looked into verifying their results with small-scale laboratory tests, published in Ochsendorf and De Lorenzis (2008). Rectangular buttresses 72.8 cm tall and 18.2 cm at its base were built with plywood blocks coated with sand to increase friction. Two tests were performed, with externally applied axial loads of 0 and 5 N. A string was connected to the mid-height of the buttress, and the horizontal load was slowly increased.

In both tests failure occurred by overturning with a fracture plane. The fracture plane predicted in the above equations presented a reasonable approximation of the true fracture plane, however it was noted that the size, orientation, and layout of the blocks will influence the actual fracture shape. The predicted failures were within 6% of the experimental data.

3.4 Previous Research: Canadian Context

In Canada, there have been limited research programs looking specifically at the response of stone masonry walls. As previously mentioned, the Canadian Standard for Masonry Design (CSA, 2014) focuses mostly on CMU construction, and applying concrete principles broadly to “unreinforced masonry”, whether CMU, brick veneer, multi-wythe brick, or stone construction. Despite this lack of research, there is a portion of the existing building stock which falls into this mass masonry typology, many over 100 years old. A number of studies have attempted to calculate the seismic risk of the current building stock, looking at specific typologies from specific locations in Canada, such as Karbassi (2010). At this point in their lifespans these structures have likely had a major renovation, or require a major renovation in order to stay functional and continue to be useable spaces. For example at Parliament Hill in Ottawa, the West Block of Canada (circa 1865, 1878, 1909) recently underwent a full-scale, \$863 million rehabilitation/modernization project, including masonry restoration and upgrading for seismic design. Due to the amount of unknowns with regards to stone masonry strength and behaviour, this project incited the largest experimental program to date in Canada, see Section 3.4.1. The other major stone masonry research program looks at the unique Prince of Wales Fort in northern Manitoba, where the 300 year old walls have begun to deteriorate at a much faster rate in the past decades including some sections of collapse, see Section 3.4.2.

3.4.1 West Block Experimental program

One of the most important experimental testing programs of stone masonry walls in Canada took place prior to the restoration of the West Block of Parliament, in Ottawa. The goal of the program was to obtain as much information as possible in terms of material and wall properties, with a specific focus on the out-of-plane strength and the concern that the wythes could separate in a seismic event. The program tested the effectiveness of several through-the-wall anchors to tie the two wythes of masonry together. The wide variety of tests performed were published in a series of publications. Significant information was contained within the PhD dissertation (Sorour, 2010), with other publications including: Sorour et al. (2011), Elmenshawi

et al. (2010a), Elmenshawi et al. (2010b), and Elmenshawi et al. (2010c).

The walls of the West Block consist of a sandstone outer wythe in a sneck bond pattern, limestone inner wythe in a running bond pattern and a rubble core. The original walls constructed in 1865, and to a lesser extent the 1878 and 1909 walls, had a lack of keystones. Part of the project included extensive masonry repairs, including the dismantle, rebuild and core consolidation of the top of the walls where the core had deteriorated. The deterioration was caused by moisture infiltration due to a poor roof detail and lack of maintenance at this level. There was also some damage to the interior due to the roof fire of 1916. The other location of extensive water damage was at the first storey weather band, which also required sections of dismantle and rebuild with core consolidation (Cooke, 2020).

Eight stone walls were built and tested by professional heritage masons, representing the typical upper storey of the exterior walls of the West Block. The walls measured 2 m wide, 2.75 m tall and 0.54 m thick. The thickness represents the lower end of wall thicknesses found throughout the structure, which ranges from 1.5m on the ground floor to 0.54m on the upper floors. The width of 2m was chosen in correlation with the heritage masons to properly represent the sneck bond pattern, and the height was chosen based on the limitations of the testing setup. To test the effects of the various wythe connections; two of the walls were built plain similar to existing conditions, two were built with grouted (Cintec) anchors installed during construction, one with Cintec anchors installed post construction, one with helical (Helifix) anchors, one with stainless steel cramps, and one was built with extra-large bond stones overlapping across the core.

The walls were all put through a series of tests where the damage was accumulated over time. First were the simple loading tests to acquire the elasticities. The elastic modulus was measured as both displacement over the entire wall and displacement of the middle third. The results showed that for the plain walls the elastic modulus was relatively constant no matter the point of reference, while for all of the reinforced walls (including the wall with bond stones) this value significantly varied. This indicates that the reinforcing measures cause localized stiffness around the location of the wythe connection. The result is an elastic modulus between 1600 and 2300 MPa when calculated as a secant modulus at a stress level of 0.55 MPa. All of the

walls experienced a drop in the elastic modulus after the in-plane tests, with no further drop after the out-of-plane testing.

The results from the in-plane testing shows that, in general, the strengthening methods had limited effect on the overall behaviour of the walls. From Sorour et al. (2011): *“the overall behaviour of the walls was dominated by the behaviour of the stone masonry”*.

Following the damage from the in-plane testing, the walls were then subjected to six different out-of-plane dynamic loadings. The base displacement time histories were chosen as a real earthquake recorded in 2002 on Parliament Hill and a synthetic record developed to match the short period end of the uniform hazard spectrum (UHS) for Ottawa with a probability of 2% being exceeded in 50 years. The original earthquake signal was very low with a PGA of 0.00192g, so was scaled by a factor 183 in order to meet the PGA of 0.353g as per the NBCC. The records were applied sequentially at 60%, 100% and 110% for six tests total.

Unfortunately, due to equipment errors, there are no result time-histories available from these experiments besides visual observations. All of the walls survived these tests, and none of the collar joints cracked or failed. The walls did flex under the loads, and most walls lost superficial mortar especially in areas previously damaged by the in-plane tests. Two walls had singular stones become loose, but they remained in the wall unable to be removed after the tests and were located at the intersection between the ‘X’ diagonal cracking from the in-plane tests. As for the different out-of-plane strengthening methods, *“none of the strengthening schemes provided a significant difference to wall behaviour”* (Elmenshawi et al., 2010c).

All of these results are in line with the theory that the geometry and boundary conditions of the wall govern the out-of-plane response. As long as the wythes are acting together, the strengthening scheme cannot increase the out-of-plane strength since it is dictated by the mass and geometry of the wall.

It is also important to note that these were newly constructed walls built by professional masons. The core had recently been properly constructed and the collar joints would have still been intact. It would have been interesting to compare the results of the anchors with a deteriorated wall, where the core had contained voids or was washed out. Without a solid core, the plain walls may have failed by wythe separation and it would be valuable to determine if

the different anchors or bond stones would provide a solid connection between wythes even with deteriorated mortar. This testing shows the efficacy of a well built and well consolidated wall, highlighting the importance of proper maintenance and deep repointing including core consolidation as required.

3.4.2 Prince of Wales Fort Experimental program

The Prince of Wales Fort (ca. 1731-1771), in northern Manitoba has experienced harsh weather conditions for almost 300 years, but recent significant deterioration and deformations in the walls have been noticed, with some areas of localized collapse. *“During the past decade the rate of deterioration has accelerated, specifically in the north wall. Climate change is thought to be the cause of the acceleration. Rising temperatures have shifted the thermal gradient within the earthen rampart, and it is thought that the high volumes of water draining through the walls during spring and summer have washed out much of the degraded mortar.”* (Fontaine and Elliot, 2008)

A stabilization project for the structure was initiated in 2003. Researchers from the University of Calgary and the University College Dublin have analyzed the structure, publishing their findings in Isfeld and Shrive (2015) and Isfeld et al. (2016).

The walls themselves of this structure have been modified over time. The stone underfoot in the north of Canada is an exposed, hard granite or quartzite bedrock which makes up the Canadian Shield and is not easy to quarry or cut to size. The initial rush to build the fort had the first iteration of walls completely constructed of split boulders. Over time, the workers then set out to remove the outer wythe and work the split boulders into ashlar stones. This partial reconstruction while the core was intact resulted in negligible key stones. The stones in the core were left rounded.

With speed of the deterioration and localized collapse, there was a research focus to determine whether the wall, constructed with rounded core stones, was inherently unstable. A series of tests were run using the software *LMGC 90*, modelling simplified wall sections to determine their behaviour with various cohesion values, see Figure 3.12.

Figure 3.12: Prince of Wales Fort Wall Sections

The modelling was intended to be only a starting point in determining if it is possible to achieve stability in these inherently unstable cross-sections and if so, does a relationship exist between the cohesion ratio and the normal cohesion component at stability.

While the cohesion of the core was shown to be an important factor for stability of the wall type constructed, it was noted that the geometry was also important. Stones can wedge and lock against each other, so angular stones produce more stable cores than rounded ones. This is typically referred to as “stone interlocking” in research. For the different core geometries tested the cohesion required for stability varied significantly.

In order to practically increase the cohesion within the walls the prospect of grouting was examined. The modelling which included grouting the core showed an improvement in wall behaviour. Grouting the wall in the model resulted in increased cohesion and friction between the stones and also stiffened the wall, lessening the displacement of the wall and increasing the strength.

These studies are important in the Canadian context as they are two of the largest experimental programs to date, on two buildings with significant heritage value. This section

highlighted what other Canadian researchers are focussing on, which in both cases was the out-of-plane strength of stone walls. Both of these research programs looked at methods to prevent wythe separation, with mechanical connectors for the West Block project and grouting the core for the Prince of Wales Fort project. The concern for wythe separation is a major theme in this current thesis, and is explored in more depth in Chapter 5: Modelling Masonry.

Chapter 4: The Applied Element Method

Modelling unreinforced masonry is a major focus of modern research. Developing software and modelling protocols which will give engineers the ability to verify these large, three-dimensional systems is necessary for ensuring the safety of built heritage. For this research, multiple simple out-of-plane pushover tests were run using the software *Extreme Loading for Structures*, the results of which are published in Chapter 5. The software *Extreme Loading for Structures* was designed by Dr. Hatem Tagel-Din and uses the Applied Element Method (AEM) for the analysis (Tagel-Din, 1998). This Chapter focusses on the background theory behind the AEM software. This software was chosen primarily because of its proven applicability to masonry structures, with built-in functions for masonry walls that allow for unit cracking, joint failure, and element separation.

4.1 The Basis of AEM

The following information is summarized from Hatem Tagel-Din's Ph.D Thesis, "*A new efficient method for nonlinear, large deformation and collapse analysis of structures*" from the University of Tokyo, 1998 (Tagel-Din, 1998).

In commercially available software there are two main groups of background theory. The first has its basis in the Finite Element Method (FEM), where a material is assumed to be continuous and is divided into a number of elements where the stress within each small element is easily definable. The overall response of a member is simply the sum of all of the small elements. This is an extremely accurate method for determining exact stress states and locations of high stress where failure will begin to occur. However, when looking at the collapse of a struc-

ture where elements may break and separate or have extremely high deformations, there are significant inputs required from the designer including a known fracture plane location which complicates the modelling of masonry.

The second basis of structural analysis software is discrete element method (DEM). This type of software is extremely common for civil engineering and geotechnical purposes. The theory is that the media consists of discrete elements, connected by normal and shear springs at contact surfaces. This is the basis of collapse modelling, and is good at looking at overall element interactions, but not the exact stress states since the elements are considered rigid. The accuracy at regular stress levels is less than FEM, but its ability to predict collapse is much higher and includes the possibility of progressive failure.

The goal of AEM is to have a method which is accurate in the small deformation range, as FEM, but is also able to predict the collapse behaviour of structures. “To develop a new efficient and accurate technique which can follow structural behavior from zero loading, crack initiation and propagation, separation of structural members and till total collapse in reasonable time, reliable accuracy and with relatively simple material models” (Tagel-Din, 1998).

4.2 Basic Element Formulation

The model works by assuming the study region is composed of discrete elements connected together with a series of normal and shear springs. The stresses and deformations of the material are fully represented within the springs (K_n and K_s). The springs are located at contact points which are distributed around the element edges. The basic element springs are found between smaller subdivided elements of a larger member, such as small steel elements within a beam. Each spring represents a defined area of the element, so the overall stiffness (equal to EA/L for normal forces) has been defined for the smaller area in Equations 4.1 and 4.2 below.

$$K_n = \frac{EdT}{a} \quad (4.1)$$

$$K_s = \frac{GdT}{a} \quad (4.2)$$

where: d is the distance between springs;

T is the thickness of the element;

a is the length of the representative area; and

E and G are the Young's and shear modulus of the material, respectively.

A contact surface area is made up of many springs, with the same properties. The element itself is rigid, as previously described, with the deformations occurring in the surrounding springs. To demonstrate the situation in 2D, each rigid body has three degrees of freedom, see Figure 4.1. The three degrees of freedom are denoted as u_1 , u_2 , and u_3 , for the left element in Figure 4.1

Figure 4.1: Element shape, contact point and degrees of freedom. Reproduced from Tagel-Din (1998)

The stiffness matrix of each element depends on the contact spring stiffnesses (K_n and K_s) as well as the location of the springs. The global stiffness matrix K_G is simply the sum of the stiffness matrices from the individual element springs. The global stiffness matrix is therefore “an average stiffness matrix for the element according to the stress situation around the element”

(Tagel-Din, 1998). The Poisson's ratio for confined elements is also included in the stiffness matrix. The equation governing the reaction of the element is:

$$[K_G][\Delta] = [F] \quad (4.3)$$

When performing a load-control test, the applied load vector $[F]$ is known before the analysis. Load control has the disadvantage that it cannot follow post peak behaviour of structures.

When performing a displacement-control test, the load is applied by applying a virtual displacement at one or more degrees of freedom. This technique is able to follow post peak behaviour, but its application is limited to cases where the load is applied to a limited number of points, as controlling the displacement at a number of locations can lead to unrealistic results.

For uniform materials, the user is able to define either the size or number of elements, and the program distributes them throughout the defined volume of the overall element. The spring locations are distributed along the contact surface depending on the user-defined number of springs per element, except for reinforced concrete where springs need to be applied at the location of the reinforcing bars. Two springs are created at each reinforcing bar, one for the concrete and one for the rebar. It is assumed that the strain of these two springs is equal.

4.2.1 Effect of Number of Springs

The number of springs chosen between two elements can have a significant impact on the accuracy of results, especially when the elements are large in comparison to the overall structure size. The number of springs between two elements does not affect the shear or axial loading (perfectly normal or shear behaviour at the interface) which is represented by the overall area of the interface. However, there is a significant effect on any rotation effects which are resisted by a combination of shear springs together with normal springs.

Tagel-Din (1998) performed a series of tests to compare the theoretical rotation stiffness K_r^t with the actual calculated rotational stiffness K_r^n . The number of springs was varied to calculate the percent error.

Table 4.1: Relation between the number of connecting springs and the calculated error in rotational stiffness. Reproduced from Tagel-Din (1998)

Number of connecting springs (2n)	2	4	6	8	10	20
Percent error $\left[\left(\frac{K_r^n}{K_r^t} - 1 \right) * 100\% \right]$	25%	6.3%	2.8%	1.6%	1.0%	0.25%

The effects of the number of springs are less severe when the element size is small, since the relative rotation between adjacent elements also becomes small.

4.2.2 Effect of Element Size

In addition to providing better results with less springs, reducing the element size overall provides more accurate results when looking at elastic materials. Using large element sizes leads to an increased structural stiffness, which in turn results in lower calculated displacements and higher failure loads.

This effect was proven by Tagel-Din (1998) by comparing the software results of a perfectly elastic cantilevered column with an applied lateral load with the simple static equations. The results showed that having a single element representing the width of the cantilever resulted in a 30% error on the calculated displacement, while dividing the width into five or more elements reduced the error to less than 1%. However, this drastically increased the CPU time required for the analysis. Halving the number of springs with the same number of elements halved the required CPU time. It was concluded that using a large number of elements together with relatively few connecting springs leads to very high accuracy with a reasonable analysis time.

Looking at the stress distribution along the base of the cantilever, the normal stress distribution was unaffected by the number of elements or springs. The shear stress across the joint on the other hand varies significantly depending on the number of elements, and requires numerous elements to properly represent the theoretical distribution. The theoretical shear stress is parabolic, zero at either end and maximum at the midpoint of the width. The software assumes that the shear stress along one element is constant, so for a single element the shear distribution is uniform. Increasing the number of elements makes the distribution closer to the theoretical

value, in a “stepped pyramid” which follows the theoretical parabola. Even with 24 elements across the base the calculated shear stress distribution is very similar to the theoretical parabola, but cannot recreate the change in shear stress along a single element and will therefore always have a stepped pattern.

4.2.3 Incorporating Non-Linear Behaviour

Non-linear behaviour has been previously reproduced in many available software. The non-linear behaviour incorporated in the AEM software is cracking, crack propagation and element separation/collapse. This is defined by the failure criteria. A typical failure criteria is the Mohr-Coulomb’s failure criteria, which is calculated from normal and shear springs (not principal stresses) and is reasonable when applied to a material physically divided into individual units, such as masonry buildings. However, this failure criteria has limitations when applied to uniform materials, such as reinforced concrete, where the structure is only virtually divided into elements and cracking will be diagonal rather than following element boundaries.

The technique incorporated into the software calculates the principal stresses at each spring location. The principal stress (σ_p) value is then compared with the material tensile resistance, accounting for the failure criterion. When σ_p exceeds the failure criterion, the normal and shear spring forces are redistributed in the next increment by applying them in the reverse direction. These redistributed forces are transferred to the element centre as a force and moment, which are in the next increment applied to the whole structure.

4.3 Masonry Modelling

The previous section discussed generally the formulation of elements and springs, and how the AEM software works. These general formulations apply to uniform materials. For uniform materials the material properties are easily applied to the shear and normal springs and all deformation and stresses are concentrated at the springs. In masonry construction, whether historic mass masonry, brick masonry, or modern CMU masonry, the material is non-uniform and both the mortar material as well as the material of the units need to be modelled. This

means that the model needs to properly account for the failure of the stone, the failure of the mortar, and the interaction between the two.

The basis for the masonry material model, the designers of the AEM software consulted the PhD thesis, published in 1996, titled “*Computational Strategies for Masonry Structures*” by Dr. Paulo Lourenço, who is an influential practicing engineer and professor at the University of Minho focussing on the behaviour of historic mass masonry constructions (Lourenço, 1996). An in-depth review unit-mortar interfaces is provided in this thesis, along with methods for how to incorporate this behaviour into computer models.

In general, the experimental testing available which looks directly at the interface behaviour looks at two failure modes (denoted mode I and mode II failures). The first is a pure tensile failure (mode I) and the second is a pure shear failure (mode II). These two basic failure modes are simple to study independently and there are various standards which provide guidance in order to standardize this testing. However, in reality the failure of masonry assemblies is a combination of these two modes, which Lourenço summarized into 5 five basic failure types: (1) joint debonding/cracking, (2) sliding along the bed or head joints, (3) cracking of units under direct tension, (4) diagonal tensile cracking of the units under high compression and shear, and (5) masonry “crushing”, which is essentially the splitting of units. Failures (1) and (2) are dependent on the mortar properties, while (3) relates to the unit properties, and (4) & (5) are combined failure of the mortar and unit. Any model which is to predict masonry behaviour needs to be able to properly account for all five of these failure modes. In order to model these failure modes, Lourenço proposed that yielding can only occur if the stresses σ satisfy the general yield criterion:

$$f(\sigma, \bar{\sigma}(\kappa)) = 0 \quad (4.4)$$

where: $\bar{\sigma}(\kappa)$ is a function of the scalar κ which accounts for the material softening.

Multiple yield criterion are required in order to account for all five failure modes. Lourenço proposed a failure model which consists of a tension cut-off for tensile stresses, the Coulomb friction model for intermediate compressive and shear stresses, and an elliptical cap, see Figure 4.2.

The results were compared against in-plane tests on brick shear walls with good correlation.

Figure 4.2: Proposed model for unit-mortar interfaces. Reproduced from Lourenço (1996)

For the masonry model included in the AEM software *Extreme Loading for Structures*, the yield criterion proposed by Lourenço are used, with some simplifications (Mayorca and Meguro, 2003). The model includes tension cut-off failure (f_1), Coulomb friction (f_2), and compression cap (f_3). Some simplifications of the AEM model include neglecting the dilatancy (the effect of becoming more viscous) of mortar when sheared. Cohesion and tension softening are assumed to be related and the degradation of one results in the degradation of the other. The modified failure envelope is reproduced in Figure 4.3 below. See Mayorca and Meguro (2003) for the background behind Figure 4.3 and further verification of the software for masonry models.

Figure 4.3: Proposed model for unit-mortar interfaces. Reproduced from Mayorca and Meguro (2003)

In order to incorporate the material properties of both the mortar and the unit into the material failure, the masonry elements themselves also need to be formulated differently than a

standard, uniform material. The material properties of both the mortar and units can be inputted to the software, both of which affect the response of the model (as previously described by the five (5) failure methods and yield criterion). The masonry model incorporated into the software models each unit as a solid volume, tied together with “equivalent” springs representing the behaviour and stiffness of both the unit and the mortar. These “equivalent” springs are based on individual mortar and unit springs in series, as shown in Figure 4.4. The equivalent normal and shear springs are denoted Kn_{eq} and Ks_{eq} , and are described in the following Equations 4.5 and 4.6.

Figure 4.4: Brick-mortar combined spring. Reproduced from Mayorca and Meguro (2003)

$$\frac{1}{Kn_{eq}} = \frac{a - th}{E_b t d} + \frac{th}{E_m t d} \quad (4.5)$$

$$\frac{1}{Ks_{eq}} = \frac{a - th}{G_b t d} + \frac{th}{G_m t d} \quad (4.6)$$

where: a is the length of the representative area;

th is the mortar joint thickness;

t is the thickness of the representative area;

d is the distance between springs;

E_b and E_m are the Young's modulus of the brick and mortar, respectively; and

G_b and G_m are the shear modulus of the brick and mortar, respectively.

These equivalent springs provide the ability to incorporate both the elasticities of the brick and mortar. However, it is important to verify the overall behaviour, since typically in experimental testing the elastic and shear moduli are published for the masonry assembly, and the individual material properties need to be back calculated.

4.4 AEM Verification

While there are previous studies which have compared masonry experimental data to the software, such as Mayorca and Meguro (2003), it is important for any new user to ensure that they are aware of the assumptions the software is making, and how the different features relate to the background theory. The following verifications were performed as part of this study on a simple two stone stack, with the bottom stone completely fixed and a displacement-controlled load applied to the top stone. This allowed for the springs at the interface of the two stones to be tested for axial and shear failures (Mode I and Mode II failures, respectively).

The stones were modelled as 300 x 300 x 300 mm. The mortar joint thickness was unable to be individually defined. For the following experiments a basic set of material properties were used, based on the above combined spring equations and available codes and literature. Refer to Table 4.2 for more information.

To test the software, both the automatic masonry wall function and user-defined 8-node objects were used. The bottom stone was fully fixed against any movement, and the top stone was fixed against rotation to ensure that pure sliding failure would occur. A displacement controlled static test was performed, resulting in the following hysteresis.

4.4.1 Axial Loads

The simple stone stack was tested to failure under both compressive and tensile axial loads. See Figures 4.5 and 4.6 for the compressive and tensile response of the mortar joint, respectively. The multiple curves in each test show the response of the model varying the time step, highlighting the importance of having a very short time step in order for the model to accurately

Table 4.2: Basic Material Properties for Software Verification

Material Property	Model
Stone Properties:	
Compressive Strength, f_{cb}	40.0 MPa
Tensile Strength, f_{tb}	4.0 MPa
Shear Strength, τ_b	1.0 MPa
Elastic Modulus, E_b	1 800 MPa
Shear Modulus, G_b	600 MPa
Mortar Properties:	
Compressive Strength, f_{cm}	2.0 MPa
Tensile Strength, f_{tm}	0.2 MPa
Shear Strength, τ_m	0.04 MPa
Elastic Modulus, E_m	600 MPa
Shear Modulus, G_m	140 MPa
Assembly Properties:	
Density, γ	2.31 g/cm ³
Friction Coefficient, μ	0.8

predict the response.

For the compression test, the compressive strength of the mortar (f_{cm}) was modelled as 2.0 MPa over an area of 300 x 300 mm resulting in a compressive strength of 180 kN or 18.36 tons. This is plotted in Figure 4.5 as the horizontal dashed line.

Similarly, the tensile strength of the mortar (f_{tm}) was modelled as 0.2 MPa resulting in a tensile strength of 18 kN or 1.84 tons. Including the self-weight of the top stone requires a tensile force of 1.90 tons to break the joint.

Figure 4.5: Stone-mortar joint compressive behaviour

Figure 4.6: Stone-mortar joint tensile behaviour

The important result from these examples is that the axial response, both failure load and stiffness, is governed by the mortar properties. Once the mortar springs reach their peak stress, both the mortar and stone elements fail simultaneously.

4.4.2 Shear Loads

Theoretical shear behaviour from experimental investigations (Mode II failure) have shown that the shear strength corresponding to slip along a bed joint is strongly related to the combined shear and compressive stresses. This relationship is commonly represented by Coulomb friction.

$$\tau = \tau_0 + \mu \sigma_n \quad (4.7)$$

where: τ is the joint shear strength;

τ_0 is the shear bond strength at zero compressive stress;

μ is the coefficient of friction; and

σ_n is the compressive stress normal to the bed joint.

The first test involved cyclical loading to develop the full hysteretic behaviour of the joint under solely its own weight. This is shown in Figure 4.7.

Figure 4.7: Stone-mortar joint shear behaviour

The upper peak is equal to $\tau_0 + \mu \sigma_n$ (Equation 4.7), the lower equilibrium lines are equal to the friction portion of the above equation ($\mu \sigma_n$) and the sloped line is equal to Ks_{eq} as provided by Equation 4.6. Once the initial shear springs are broken, pure friction behaviour governs the response.

To further test the shear behaviour, three different tests were run at varying normal

stresses: 0.1 MPa, 0.5 MPa and 1.0 MPa. The stress was applied three different methods: (1) the density of the units was increased, (2) a lumped mass was applied, and (3) a vertical external load was applied. The software did not differentiate between these three methods and provided identical results. The software results are almost exactly the theoretical values.

Table 4.3: Initial and Residual Shear Forces; Comparing Theoretical and Software Results

σ_n (MPa)	W (tons)	Theoretical		Software	
		τ (tons)	τ_{fric} (tons)	τ (tons)	τ_{fric} (tons)
0.1	0.918	8.65	0.67	8.66	0.67
0.5	4.589	11.33	3.35	11.35	3.35
1.0	9.177	14.68	6.70	14.69	6.70

With the basics of the AEM software described and the mechanics of the masonry modelled verified, this software will now be used to model the out-of-plane response of walls and buttresses.

Chapter 5: Modelling Masonry

With the basis of the AEM software previously described in Chapter 5, the software *Extreme Loading for Structures* was used to model stone walls and buttresses. This Section first confirms the overall mechanics of the model against real fracture plane data, in Section 5.1, and expands upon this to explore the effects of varying the different material and geometric properties. The model is then compared to previous out-of-plane experiments in Section 5.2. Finally, the calibrated model is modified to include a buttress, see Section 5.3.

A limitation of this section is the lack of experimentation. The model is calibrated for existing testing on the out-of-plane strength of stone walls, however the applicability to expand this model to include buttresses without any experiments to confirm the model is a point of discussion, see Chapter 7. Nevertheless, the predicted trends are valuable indicators for further research.

5.1 Modelling Mass Masonry Walls

Properly modelling the behaviour of stone masonry walls is a focus of modern research. For this study, the out-of-plane wall pushover model was first compared against previous fracture plane experiments to ensure that the model was properly predicting the failure mechanisms for differing cross-sections, see Section 5.1.1. The models predicted effects of modifying the various material properties and different wall geometries are provided in Sections 5.1.2 and 5.1.3.

The discussions in this Section all refer to the theoretical rigid body overturning, which was described in detail in Chapter 3. The process of acquiring the rigid overturning moment,

denoted $M_{OV,rigid}$, is repeated below in Equation 5.1.

$$M_{OV,rigid} = \frac{\gamma h w t^2}{2} \quad (5.1)$$

where: γ is the material density;

h is the height of the wall;

w is the width of the wall, the dimension parallel to the bending axis; and

t is the thickness of the wall, the dimension perpendicular to the bending axis.

Note that $\gamma h w t$ is simply the weight of the wall, W .

The moment-displacement slope of the rigid overturning equation is simply the maximum moment at 0 displacement, decreasing to 0 moment at the maximum displacement which is when the COG is directly above the pivot point. This can be described in equation form with the moment as a function of the top displacement by:

$$M_{OV,rigid} = W \left(\frac{t - \Delta_{top}}{2} \right) \quad (5.2)$$

where: Δ_{top} is the displacement of the top of the wall;

The minimum rigid overturning moment, when the wythes fail separately, is simply half of the $M_{OV,rigid}$, as proposed by Giuffre (1996), and described in Chapter 3.

5.1.1 Modelling Out-of-Plane Failure Patterns

To verify the software against existing experimental failure modes, the previous experiments varying the layout and number of header stones were recreated. Giuffre (1996) and Ceradini (1992) published experimental and analytical results on the failure of masonry walls depending on their cross section and number of bond stones, recreated by De Felice (2011) and finally by Szyszka et al. (2018). Due to the high quantity of data relating to these experiments, this can provide a good reference to ensure that the model is properly accounting for the different failure modes and geometric considerations.

A basic wall typology of 3.9 m tall, 0.5 m thick and 1 m wide was used based on the

previous testing. The walls were comprised of 30 rows of stone, the rows either consisting of full header courses or two-wythe courses. Fifteen different wall header layouts were modelled to match the previous experiments. The previous experiments conducted in this first were on dry-stack small scale models, built on a tilting table. There were no external vertical loads applied for any of the previous tests. For all of the following tests, the dimensions and standard material properties were taken in reference to De Felice (2011), shown in Figure 5.1 and Table 5.1.

Figure 5.1: Basic Dimensions (Wall “14” Shown), (a) Individual Units, (b) Entire Wall

Table 5.1: Basic Material Properties for Wall Tests

Material Property	Value
Density (γ)	2.31 g/cm ³
Friction Coefficient (μ)	0.7
<u>Mortar:</u>	
Tensile Strength ($f_{t,m}$)	0 MPa
Comp. Strength ($f_{c,m}$)	1 MPa
Shear Strength (τ_m)	0 MPa
Elastic Modulus (E_m)	900 MPa
Shear Modulus (G_m)	400 MPa
<u>Masonry Unit:</u>	
Tensile Strength ($f_{b,m}$)	4 MPa
Comp. Strength ($f_{b,m}$)	40 MPa
Shear Strength (τ_b)	1 MPa
Elastic Modulus (E_b)	8000 MPa
Shear Modulus (G_b)	3200 MPa

De Felice (2011) tested these walls using the distinct element method, applying an increasing lateral acceleration in steps. A comparison of the failure modes between the work of De Felice (2011) and the results from this study show clear similarities, see Table 5.2. The failure modes shown in this graph were from the tilting table experiments, which is a good method to apply a slowly increasing lateral acceleration.

Table 5.2: Comparison of Failure Methods from De Felice (2011), Varying Number and Location of Headers

Experimental, from De Felice (2011)													
$M_{max,exp}$ Wall Headers													
	96.7%	96.7%	96.7%	94.4%	94.4%	94.3%	89.9%	89.8%	84.5%	62.8%	55.2%	58.2%	51.6%
	1	2	3	4	5	6	7	8	9	10	11	12	13
	50%	33%	23%	20%	17%	13%	10%	7%	7%	7%	3%	3%	3%
AEM Model													
$M_{max,mod}$													
	91.2%	90.1%	89.2%	88.5%	83.1%	85.5%	85.1%	80.2%	81.4%	60.8%	71.0%	68.2%	52.7%

For this specific experiment, having 3/30 through-stones (equivalent to 10% of the area) ensured that the wall failed as a unit. While some of the 2/30 tests failed uniformly, the response is extremely sensitive to the location of those two stones and typically failed with the separation of the wythes.

As part of this study a comparison was made of different loading protocols. The following four different loadings were used:

1. Tilting table,
2. Displacement-controlled pushover at the top of the wall,
3. Displacement-controlled pushover at the centre of gravity (COG), and
4. Triangularly distributed force-controlled pushover.

Loading scenarios 2 and 3, being displacement-controlled tests, provide the opportunity to see the post-peak response of the wall which has proven to be important based on previous experimental testing. The tilting table method and the triangularly distributed force are both force-controlled loading methods and do not provide a clear peak value or any post-peak behaviour. For these tests, the peak was chosen as the force at which the top displacement of the wall began to displace non-linearly.

See Figure 5.2 for the moment-displacement curves for all 15 tests from the two displacement-controlled tests and Figure 5.3 for the peak values of all four pushover methods. For the tilting table and triangular force methods, the peak value was chosen as the rotation at which the top of the wall began to displace non-linearly. The graph represents all of the tests, with the darker markers representing the tests with $\geq 10\%$ of the area to be header stones. In Chapter 2 when describing the typical construction of these walls, it was noted that while there was a wide variation in recommended bond stone quantities, the minimum recommendation was 10%.

Figure 5.2: Moment-Displacement Curves of Double-Wythe Walls

Figure 5.3: Moment-Displacement Peak Values of Double-Wythe Walls Comparing Testing Method. Dark markers for tests with headers $\geq 10\%$ area

Table 5.3 below lists the maximum moment value and associated percent difference from the maximum rigid overturning for all of the models that were run. The results show that 85% of all the models were within the range prescribed by static pushover analysis, with the lower end of the results corresponding to the models which failed by wythe separation. None of the models perfectly matched the strength predicted by the rigid overturning equations (11.04 kN.m).

Table 5.3: Peak Moment Values from Double-Wythe Tests, Comparison of Pushover Method

Wall Label	Header Percent (%)	Tilt Table M_{max} (kN.m)	COG Displacement M_{max} (kN.m)	Top Displacement M_{max} (kN.m)	Triangular Force M_{max} (kN.m)
1	50	9.74	10.07	8.62	
2	33	9.66	9.95	8.45	
3	23	9.21	9.84	8.27	
4	20	9.18	9.77	8.07	
5	17	8.86	9.17	7.68	8.65
6	13	8.90	9.44	7.48	7.62
7	10		9.40	6.89	7.62
8	6.7	8.03	8.86	6.41	7.00
9	6.7	7.83	8.98	6.03	6.80
10	6.7	6.39	6.71	5.01	5.36
11	3.3	5.73	7.84	4.23	
12	3.3	7.03	7.53	5.38	
13	3.3	5.90	5.82	5.07	
14	0	4.86	4.86	3.81	3.91

From these models the different pushover methods can also be compared. Overall, the different pushover methods resulted in a 25% variation of overturning moment.

- Top Displacement-Controlled: These models consistently provided the lowest wall overturning moment of all the models performed, and are considered a lower-bound estimate.
- COG Displacement-Controlled: These models consistently provided the highest wall overturning moment of all the models performed, and are considered a higher-bound estimate.
- Tilt Table: The tilt table method resulted in maximum overturning moments within the range of the top and COG displacement-controlled models, however, being a force-controlled model there was no consistent way to output the maximum overturning moment.
- Triangular Distributed Force: The triangularly distributed force resulted within the range of the top and COG displacement-controlled models, however, being a force-controlled model there was no consistent way to output the maximum overturning moment.

Based on these results, for all of the following models only the top and COG displacement controlled loading protocols were run, representing an approximate higher and lower bound estimate of overturning resistance.

5.1.2 Effect of Varying Material Properties

It was important to test the effect of varying the material properties in the software to better understand the properties that the model predicts as having a major overall impact on the response.

Two basic walls were modelled, “Wall 7” and “Wall 14” from the previous analyses, representing walls with 10% and 0% header stones. These wall sections were chosen to represent a typical wall and a “lower-bound” wall where there is no connection between the wythes. For all of the following models, the basic dimensions and standard material properties of the walls are shown previously in Figure 5.1 and Table 5.1, with only the highlighted variable being modified. The following models were all run with a displacement-controlled test at the COG.

The purpose of this modelling is primarily as a comparison, to see how the model predicts varying the material properties will affect the overall response of the system, for walls with and without a reasonable number of headers. For that reason, the maximum overturning moment is frequently compared to the theoretical rigid overturning moment, see previous Equation 5.1.

5.1.2.1 Density, γ

Referencing the rigid overturning equation, see Equation 5.1, theoretically the density of the material should be directly proportional to the overturning strength of the wall.

The moment-rotation response of the two wall sections was obtained for densities ranging from 1.8 to 3.0 g/cm³, and the results are published in Figure 5.4 and Table 5.4. The density of 2.31 g/cm³, which was the value used for the basic walls, is bolded in the graph. As expected, there was a linear correlation between the increase in peak strength and increase in density, with a correlation R^2 value of 0.99, see Figure 5.5.

Figure 5.4: Model Response Varying Density ("Wall 14")

Figure 5.5: Linear Correlation Between Density and Overturning Moment ("Wall 14")

Table 5.4: Peak Moment, Varying Density (γ).

Density (g/cm ³)	Overturning Moment (kN.m)	
	"Wall 14" No Headers	"Wall 7" 10% Headers
1.8	3.38	5.94
1.9	3.54	6.25
2.0	3.71	6.53
2.1	3.87	6.81
2.2	4.02	7.10
2.31	4.19	7.42
2.4	4.33	7.67
2.5	4.48	7.95
2.6	4.63	8.25
2.7	4.78	8.51
2.8	4.93	8.81
2.9	5.08	9.11
3.0	5.22	9.37

5.1.2.2 Coefficient of Friction, μ

The coefficient of friction does not factor into any of the overturning equations, however it was important to test to see if a higher friction value would affect the overturning strength of the wall. A basic value of 0.7 had been used for the original models in agreement with the Canadian Masonry Standard (CSA, 2014). The coefficient of friction was varied from 0.5 to 0.9, and modifying this value resulted in negligible effects on the overall behaviour of the walls, with identical moment-rotation graphs. This is expected since the wall failed by overturning, where sliding was not occurring.

For shorter walls which fail by sliding the friction coefficient would affect the shear strength as per the verification tests performed in this study at the end of Chapter 4. The theoretical limiting height, according to Giuffrè is $H_{sliding} = \frac{b}{2\mu}$, which is equal to a height of 357 mm for walls with a base thickness of 500 mm and μ value of 0.7.

5.1.2.3 Tensile Strength of Mortar, $f_{t,m}$

The typical wall section was modelled with a tensile strength of 0 MPa. This is a conservative assumption and is representative of historic mortars which have a very low strength. The Canadian Masonry Standard (CSA, 2014) recommends a mortar tensile strength of 0 MPa for historic masonry structures, and 10% the compressive strength for modern construction.

The mortar tensile strength was varied from 0 to 2.0 MPa, resulting in negligible effects for the overall behaviour of the walls. This is an interesting result since having additional tensile resistance would theoretically require a higher lateral force before the onset of cracking and overturning, however these results show that this effect is negligible.

5.1.2.4 Shear Strength of Mortar, τ_m

Similar to the tensile strength of mortar, the shear strength (or cohesion, as referred to in Coulomb's friction equation) was modelled with a value of 0 MPa to perform a lower-bound analysis. Similar to the friction coefficient, it is expected with the overturning failure to have minimal effect on the overturning moment resistance of the wall.

The mortar shear strength was varied from 0 to 0.5 MPa. As expected, varying the shear strength of the mortar resulted in negligible effects for the overall behaviour of the walls, with identical moment-rotation graphs.

5.1.2.5 Mortar Stiffness, E_m and G_m

The stiffness of the wall, while not a factor in many of the historic references, is discussed in the Canadian Masonry Standard (CSA, 2014) in the discussion of slenderness effects. These values are not well published, and the input for the AEM software includes both mortar and stone axial and shear stiffnesses. In the Canadian Masonry Standard the shear modulus is approximated as 40% of the elastic modulus based on a Poisson's ratio, (ν), of 0.2 (CSA, 2014).

$$\frac{E}{G} = 2(1 + \nu) \quad (5.3)$$

$$E = 2(1 + 0.2) * G$$

$$\frac{1}{2.4}E \approx 0.4E = G$$

Modifying the shear modulus had an effect on the overall response of the wall, with the stiffer walls corresponding to higher peak overturning moments. It is hypothesized that the stiffer walls displaced less before cracking, resulting in a higher restoring moment. The force-displacement graphs for the different mortar moduli tested are included in Table 5.5 for values of E_m ranging between 400 MPa to 1400 MPa, and the $G_m = 0.4E_m$. The percent difference shown is in comparison to the theoretical rigid overturning moment.

Looking at this comparison, the wall without any headers had a more significant variation in response due to the increase in stiffness than the wall with 10% headers. This is a reasonable result since the wall without any headers fails as, essentially, two directly adjacent walls with half the thickness. In this situation the slenderness effects are doubled, and therefore the stiffness would have a higher impact.

Table 5.5: Force-Displacement Curves of Walls Varying Mortar Stiffness, E_m and G_m

5.1.2.6 Stiffness of Stone, E_b and G_b

Unlike the shear modulus of the mortar, the stiffness of the stone did not effect the behaviour of the wall segment. This is the expected outcome, since the stones themselves are not deforming rather the mortar is cracking and crushing allowing the wall as a unit to bend. This is represented in the model by the springs in series equation, where the weaker spring represents the majority of the deformation of the system as a whole. Hypothetically, this effect becomes less severe as the mortar and stone stiffnesses approach the same value, however in practice this would never occur when considering historic mortars.

5.1.3 Modelling Walls with a Core

With the promising correlation between the double-wythe modelling results and existing published data, these models were expanded to the wall typology more common in Canada. This construction includes a stone outer and inner wythe, separated by a rubble core. Since the thickness of the wall remains constant, including a core decreases the thickness of the outer

wythes. This section looks at both walls with full through-stones and walls with key stones that traverse the core but not the whole wall. The core was modelled with the same mortar and stone properties as the rest of the wall. The goal of this incremental modelling, continually refining the typical wall model is to move closer to realistic wall constructions in Canada.

Walls with Through Stones

The walls 1 through 14 from the previous section were recreated, with the change that the regular courses were divided into thirds of the wall thickness, rather than halves. The headers were left as full through-stones extending fully through the wall cross-section. They were modelled by displacement-controlled pushover at the COG, and the moment-displacement pushover curves are plotted in Figure 5.6. This type of construction is unrealistic for stone construction in Canada, instead key stones rather than through stones is the typical detail. A comparison of the peak moment values from the double wythe tests, these tests for walls with a core and through stones, and the subsequent tests for walls with a core and key stones can be found in Section 5.1.4.

Figure 5.6: Moment-Displacement Curves of Walls with a Solid Core

Walls with Key Stones

To better represent Canadian walls the previous tests were modified, changing the through stones to key stones extending $2/3^{\text{rds}}$ through the wall. Some cross sections are included in Figure 5.7 below. Since the walls are no longer symmetrical, each pushover test (top and COG displacement-controlled tests) was run in each direction.

Figure 5.7: Diagrams of Walls with Solid Cores

Following the same loading protocol as in the previous section, the walls were modelled under two displacement-controlled tests, one at the top of the wall and one at the COG, as well as using the tilt table method. Figure 5.8 shows the moment-rotation curves for the two displacement controlled models. The dashed line refers to the top displacement-controlled models and the solid line refers to the COG displacement-controlled tests. Once again the graph is divided into darker lines representing the walls that had $\geq 10\%$ header area, while the lighter lines represent wall sections with $\leq 10\%$ headers.

The peak moment-rotation points of all three testing methods are included in Figure 5.9. Similar to the above graph, the darker markers representing the walls that had $\geq 10\%$ header area, while the lighter markers represent wall sections with $\leq 10\%$ headers. The results of these models are promising, since they show that Guiffre's static analysis based on only double-wythe walls still represents walls with cores, as long as there is an appropriate number of headers.

These walls are denoted as "Solid Core" walls since the core was modelled as a singular stone rather than many smaller stones, which is explored next. The difference in core stone size

can be seen in Figures 5.7 and 5.10.

Figure 5.8: Moment-Rotation Curves of Walls with a Solid Core

Figure 5.9: Moment-Rotation Peak Values of Walls with a Solid Core

Walls with Rubble Core

This previously modelled approximation of the multi-wythe walls using a solid core is, while better than neglecting the core, not entirely applicable since true cores are built up with smaller stones and mortar. In order to test the effect of using small pieces of rubble in the core three wall sections were chosen to be ran, varying the size of the core material. See Figure 5.10 below for a sketch of the three wall sections modelled under varying core material sizes, representing 50% headers, 10% headers, and 0% headers. The core material size displayed in the

figure below is 42.5mm x 32.5mm ($t \times h$).

Figure 5.10: Diagrams of Walls with Rubble Cores

The core stone size in the original solid core tests was one third the thickness of the wall, 170 mm, and the height of one course, 130 mm. In reality the stones in the core vary significantly in size, and are irregular throughout the wall. However, it is a safe assumption that the stone chips will be larger than 25 mm x 25 mm. Sixteen different core material sizes were modelled for the maximum moment capacity, listed in Table 5.6.

For the wall section with 50% header stones, reducing the size of the core material resulted in an increase in strength. This is assumed to be due to the increased number of friction planes, with the surplus of headers adequately able to confine the smaller core pieces. For the wall section with 10% header stones, there was an initial increase in strength when dividing the core into large horizontal pieces, however after the core stones were divided in half horizontally, adding another vertical shear plane, the maximum moment resistance dropped. At 21.25 mm x 16.25 mm (the core stone divided both vertically and horizontally into 8 pieces), the wall saw the minimum moment resistance, lowering 10% from the initial solid core value. The wall section without any headers was the most impacted by reducing the size of the core material. As with the wall with 10% headers, there were some sizes which resulted in an increase in moment resistance, however with the smallest stones there was a 30% reduction in strength. This is shown graphically in Figure 5.11, where the moment resistance is plotted as a fraction of the “solid core” moment resistance.

This modelling shows that the increased complexity of the analysis when including the

Table 5.6: Peak Moment Values from Tests with Rubble Core, Varying Rubble Size

Core Stone Size, $t \times h$ (mm x mm)	Core Stone Size (mm ²)	M_{max} by Header Area		
		50% (kN.m)	10% (kN.m)	0% (kN.m)
170 x 130	22100	9.48	6.39	2.83
170 x 65	11050	10.76	6.64	2.35
170 x 32.5	5525	10.13	6.54	3.20
170 x 16.5	2763	9.79	6.57	3.01
85 x 130	11050	10.76	5.98	2.35
85 x 65	5525	10.70	6.24	3.03
85 x 32.5	2763	10.05	6.19	2.79
85 x 16.5	1381	9.70	6.22	2.51
42.5 x 65	2763	10.39	5.94	2.45
42.5 x 32.5	1381	9.90	5.94	2.27
42.5 x 16.5	691	9.62	5.88	2.16
28.33 x 32.5	921	9.80	5.74	2.13
28.33 x 21.67	614	9.65	5.77	2.08
21.25 x 32.5	691	9.75	5.72	2.05
21.25 x 21.67	460	9.61	5.78	1.95
21.25 x 16.25	345	9.55	5.76	2.00

Figure 5.11: Moment-Rotation Peak Values of Double-Wythe Walls with a Solid Core

rubble core can increase or decrease the out-of-plane strength of the wall section depending on the header geometry. For the following modelling, only a solid core was included since the accuracy of the rubble core model was unconfirmed and may be over-estimating the strength. Physical testing should be done to confirm the effect of smaller core stone sizes, however it is logically unlikely that it would increase the strength of the wall.

5.1.4 Modelling Comparison

The main wall models were previously run with three general conditions: double wythe walls with through stones, walls with a core and through stones, and finally walls with a core a key stones. The same geometry and wall labelling was maintained throughout the modelling, and the results of all are summarized in Table 5.7. The COG displacement-controlled models are compared.

Table 5.7: Peak Moment Values from All Wall Models

Wall Label	Header Percent (%)	Double	<u>Walls With A Core</u>	
		Wythe Walls M_{max} (kN.m)	Through-Stones M_{max} (kN.m)	Key Stones M_{max} (kN.m)
1	50	10.07	9.52	9.34
2	33	9.95	9.47	9.03
3	23	9.84	9.43	8.74
4	20	9.77	9.24	-
5	17	9.17	9.21	8.01
6	13	9.44	8.59	-
7	10	9.40	8.44	6.85
8	6.7	8.86	8.24	6.98
9	6.7	8.98	8.06	-
10	6.7	6.71	7.89	-
11	3.3	7.84	7.06	5.89
12	3.3	7.53	6.29	5.48
13	3.3	5.82	4.60	4.56
14	0	4.86	2.71	2.71

These results correlate well with the stone construction practices discussed in Chapter 2. The strength reduction for the walls with a core and through stones is $\approx 5\%$ when there $>20\%$ headers, $\approx 10\%$ when there 10% headers, and more significantly divergent results when there are less headers. This maximizes when there are no header stones, and the wall is essentially modelled as three adjacent walls with a thickness $= t/3$. For the walls with key stones rather than through stones, the strength of the wall sees a further reduction as expected. The strength reduction for the walls with a core and key stones is $\approx 10\%$ when there are $>20\%$ headers, and $\approx 28\%$ when there are 10% headers.

5.2 Modelling Experimental Data

With the various functions of the software previously analyzed, it was possible to model existing experiments on stone walls.

5.2.1 Modelling Laboratory Testing

First, the experimental results by Ferreira et al. (2015a) were recreated, published in “*Experimental characterization of the out-of-plane performance of regular stone masonry walls, including test setups and axial load influence*”. This is a good experiment to begin with since the walls were built and tested in the laboratory, with plenty of information on the individual material properties, testing protocols and wall response. They tested 6 different stone walls, 1.3 m long, 0.65 m thick and 2.5 m tall. Each wall was constructed slightly differently, varying the header locations and percentages of header stones/surface area. There were two different testing setups: the first tested three walls subjected to uniform out-of-plane pressure with airbags; the second tested another three walls with displacements applied at the top. Within each of the testing protocols the three walls had different superimposed axial loads: 0 kN, 52 kN, and 140 kN. These pre-compression values were chosen for a masonry weight of $\gamma = 21 \text{ kN/m}^3$ and typical floor loading for the main floor wall of a single storey, two storey, and three storey traditional Mediterranean house, respectively. However, the actuator which was applying the vertical load was displacement-controlled so the actual applied axial force varied significantly over the course of the test as the wall moved, see Figure 5.12 below. For the modelling the axial load was applied as a constant value, distributed uniformly to the top stones.

The material properties of the stone and mortar are published in the paper, from testing 17 stone samples and 12 mortar cubes. See Table 5.8 below for the published material properties and the properties used in the model.

The results from the tests were compared to rigid overturning in the original study, which is also included here. The overturning moment, M_{OV} , based on rigid mechanics can be expressed

Figure 5.12: Vertical pre-compression load histories. Reproduced from Ferreira et al. (2015a)

Table 5.8: Material Properties, from Ferreira et al. (2015a)

Material Property	Published (CoV)	Model
Stone Properties:		
Compressive Strength, f_{cb}	43.83 MPa (16%)	43.83 MPa
Tensile Strength, f_{tb}	3.37 MPa (14%)	3.37 MPa
Shear Strength, τ_b	-	3.37 MPa
Elastic Modulus, E_b	13 850 MPa (25%)	13 850 MPa
Shear Modulus, G_b	-	5 540 MPa*
Mortar Properties:		
Compressive Strength, f_{cm}	1.28 MPa (3%)	1.28 MPa
Tensile Strength, f_{tm}	0.53 MPa (12%)	0.53 MPa
Shear Strength, τ_m	-	0.13 MPa
Elastic Modulus, E_m	-	600 MPa [†]
Shear Modulus, G_m	-	240 MPa*
Assembly Properties:		
Density, γ	21 kN/m ³	2.14 g/cm ³
Friction Coefficient, μ	-	0.7

* G was approximated as $0.4 E$.[†] E_m was based on the realistic values derived from the material properties section, chosen to suit experimental data.

as the following:

$$M_{OV} = \frac{Vt}{2} - V\Delta + \frac{Wt}{2} - \frac{W\Delta}{2} \quad (5.4)$$

where: V is the externally applied vertical load at the centre of the wall; t is the thickness of the wall;

Δ is the top displacement; and

W is the weight of the wall.

This equation includes the reduction in overturning moment resistance due to the top displacement of the wall. The maximum overturning resistance occurs when there is no displacement at the top, calculated for each of the superimposed vertical loads below:

$$M_{OV,0kN} = (21)(2.5)(0.65)(1.3)(0.65/2) + 0 = 14.42kN.m$$

$$M_{OV,50kN} = (21)(2.5)(0.65)(1.3)(0.65/2) + 50 * 0.65/2 = 30.67kN.m$$

$$M_{OV,142kN} = (21)(2.5)(0.65)(1.3)(0.65/2) + 142 * 0.65/2 = 46.15kN.m$$

The top displacement, Δ , at which the overturning moment goes to 0 can be acquired by rearranging Equation 5.4 and setting $M_{OV}=0$:

$$\Delta = \left(\frac{Vt}{2} + \frac{Wt}{2} \right) / \left(V + \frac{W}{2} \right)$$

$$\Delta_{OV,0kN} = \left(\frac{Wt}{2} \right) / \left(\frac{W}{2} \right) = t = 650mm$$

$$\Delta_{OV,50kN} = \left(\frac{Vt}{2} + \frac{Wt}{2} \right) / \left(V + \frac{W}{2} \right) = 425mm$$

$$\Delta_{OV,142kN} = \left(\frac{Vt}{2} + \frac{Wt}{2} \right) / \left(V + \frac{W}{2} \right) = 369mm$$

The exact stone layouts for each wall were published for each of the six tests (Ferreira et al., 2015a). The cross-sections were simplified for the modelling, see the actual and modelled cross section of Wall “PA1” shown in Figure 5.13. Each of the different wall sections were individually modelled.

Figure 5.13: Experiment and Modelled Wall Sections.

The results of the experiment pushover tests as published by Ferreira et al. (2015a) are included in Figure 5.14 below. The cyclical results could not be pulled from these graphs and superimposed with the modelling results, so the original graphs are included here. There is a clear increase in strength with the applied axial load and a negative stiffness after reaching the peak moment.

Figure 5.14: Moment-rotation curves for (a) airbag tests and (b) line load tests. Reproduced from Ferreira et al. (2015a)

Beginning with the airbag tests, in order to represent the uniformly distributed load the

walls were loaded by displacement-control at the centre and the top. This could have been done with force-control, similar to the triangular distributed load previously modelled, in order to get a more realistic displaced shape, but the post-peak behaviour would not have been obtained. In addition, the displacement of the walls over the height during the tests was published in the original study and the wall displaced linearly, with only a fracture at the base. The model output moment-displacement curves are plotted against the envelope of the experimental data, shown on the graph as the thick black line, see Figure 5.15. Overall, the results from the airbag tests for the two lower axial loads correlate extremely well with the experimental results, with fair results for the highest load test, see Table 5.9.

Figure 5.15: Moment-rotation curves from model for airbag tests, compared against experiment force envelopes from Ferreira et al. (2015a)

The 142 kN model was unable to meet the same displacement as the original experiment. A possible explanation for this poor fit is that in the experiment the actuator which produced the axial load was attached to the setup and likely provided additional lateral restraint, and therefore displacement capacity.

For the line load tests, the model also had a good approximation of the overall behaviour of the walls, but less accurate than the airbag tests. For these tests, the models were loaded with

a horizontal displacement-controlled load at the top course. It is important to note that for the test without any superimposed axial load, this actually resulted in the top course simply sliding off. This test was instead modelled as with a displacement-controlled load at the second course. The model moment-displacement curves are plotted against the envelope of the experimental data, shown on the graph as the thick black line, see Figure 5.16.

Figure 5.16: Moment-rotation curves from model for line load tests, compared against experiment force envelopes from Ferreira et al. (2015a)

The peak moment results from all of the tests are summarized in Table 5.9. The different test loading protocols in the model both resulted in very similar peak moments, while the experimental test with a line load at the top course was generally stronger than the airbag tests.

Table 5.9: Comparison of Peak Moment Values to Experimental Data (Ferreira et al., 2015a)

Test Label	Superimposed Axial Load	Experimental M_{max} (kN.m)	Analytical M_{max} (kN.m)	Percent Difference
PA1	0 kN	14.68	14.22	-2.5%
PA2	52 kN	28.30	28.48	+0.6%
PA3	140 kN	47.35	47.66	+0.7%
PF1	0 kN	15.35	14.31	-6.8%
PF2	52 kN	33.16	28.54	-13.9%
PF3	140 kN	56.90	49.46	-13.1%

5.2.2 Modelling In-Situ Testing

The model was also verified against in-situ out-of-plane wall testing. The study by Ferreira et al. (2016) on dry-stack stone buildings that had been previously damaged in an earthquake provides input for a low quality of masonry construction. The study also looked at reinforcing the walls with steel mesh adhered to the surface with a thick cement plaster, but only the plain wall denoted ‘CN-04’ in the study was recreated here.

The wall section studied was 1.38 m wide, 0.8 m thick and 2.6 m tall, built of a dry-stack local basalt stone. Based on the photographs provided in the paper, the stones are typically not very tall and extremely irregular with large stones framing openings, see Figure 5.17. In order to use a regular stone size and still capture the behaviour of the smaller stones, a stone course height of 104 mm was chosen for the model (25 courses to make 2.6 m). Based on the photographs the wall is typically three-wythes, with significant overlapping of the smaller stones. The chosen stone size was 345 x 300 x 104 mm, with the other material properties described in Table 5.10. The only material property published was the approximate density of the masonry. For most of the other material properties, they were approximated as 0 to represent dry-stack construction.

Figure 5.17: In-Situ Wall Geometry, (a) outer face and (b) cross section from Ferreira et al. (2016), (c) AEM model cross-section

Table 5.10: Material Properties, from Ferreira et al. (2016)

Material Property	Published	Model
Stone Properties:		
Compressive Strength, f_{cb}	-	43.83 MPa*
Tensile Strength, f_{tb}	-	3.37 MPa*
Shear Strength, τ_b	-	3.37 MPa*
Elastic Modulus, E_b	-	13 850 MPa*
Shear Modulus, G_b	-	5 540 MPa*
Mortar Properties:		
Compressive Strength, f_{cm}	-	1.28 MPa
Tensile Strength, f_{tm}	-	0 MPa [†]
Shear Strength, τ_m	-	0 MPa [†]
Elastic Modulus, E_m	-	10 MPa
Shear Modulus, G_m	-	2.5 MPa [†]
Assembly Properties:		
Density, γ	15.7 kN/m ³	1.60 g/cm ³
Friction Coefficient, μ	0.7	0.7

* From basic material properties and previous experimental results.

[†] Approximated to 0 to represent dry-stack masonry.

The results of the models show good correlation with the experimental data, in terms of overall strength. See Figure 5.18 for the equivalent force-displacement curves of the walls. The model predicts that the wall loses strength faster than in the experimental testing, although it is likely due to internal mechanisms occurring within the wall. As the wall is experiencing large displacements and rocking, there will be smaller stones and others which fall, move, slide, and grind together. It is also possible for rubble walls to experience “shaped interlock”, where the stones which are displacing lock together and result in an increase in stiffness and strength after significant rocking has already taken place, shown in the results of Fonti et al. (2017). The results from the AEM model do show a minor increase in strength and stiffness over the course of the cyclical load increments, visible in comparison to the results of the simple pushover test, however this is minor and does not occur during the rocking process.

Figure 5.18: In-Situ Failure Force-Displacement Curves, numerical response compared with Ferreira et al. (2016)

5.2.3 Modelling In-Plane Testing

With the goal of expanding the above models to include buttresses, it is also important to look at the in-plane behaviour of masonry walls since the buttress as a structural element will be acting in its strong axis. All of the above out-of-plane testing resulted in overturning failure of the walls, however with deep buttresses there is the possibility of sliding to occur. Modelling existing experimental testing to verify the behaviour of the modelled walls under in-plane friction forces provides reliability that the model will properly represent sliding failures.

The experiments conducted by Casapulla and Argiento (2018) were chosen to be recreated since they were dry stack stone walls, with published coefficients of friction and density values. The test setup included every other end stone being anchored to a steel rod, which was then pulled away from the wall. This ensured that the forces were transferred from the anchored stones to the adjacent stones purely through friction. All of these elements were recreated in the model. Two cases were examined where the loading was applied at the mid-height of the rod, with a 0 N and 320 N vertical load uniformly distributed over the top of the wall, respectively.

The material properties used in the analysis are described below. Most are published in the experimental results or are reasonable assumptions based on dry-stack behaviour, however it is important to note the shear moduli used. Since these walls do not have any mortar, there

should be no stiffness at the mortar joint besides that provided by friction but that is not possible to be modelled using the ‘wall’ function in the software. Setting the mortar stiffness to 0 also causes significant numerical errors and does not allow the model to run properly. This was investigated thoroughly while modelling the out-of-plane in-situ testing of the dry-stack stone wall, and setting the mortar shear stiffness to 250 tons/m² (or 2.5 MPa) allowed the software to run the model while having extremely flexible joints. The shear modulus of the stone was then calibrated to the experimental results based on the “springs in parallel” equation used in the creation of the modelling software (See Chapter 4). The overburden for the second case was applied by adding lumped masses to the upper row of eight blocks, 40 N each (0.004 tons).

Table 5.11: In-Plane Model Material Properties

Material Property	<i>kN</i> base	<i>ton</i> base	Notes
γ	13.1 kN/m ³	1.34 tons/m ³	from experimental results
μ	0.52	0.52	from experimental results
$f_{t,m}$	0 MPa	0 tons/m ²	dry-stack
τ_m	0 MPa	0 tons/m ²	dry-stack
G_m	2.5 MPa	250 tons/m ²	dry-stack
G_b	21.6 MPa	2200 tons/m ²	to match experiment results

The results of the model show good correlation with the experimental data, in terms of overall strength and failure pattern. Figure 5.19 compares the cracking and failure modes of the experimental and numerical results of the test with overburden. While there are minor variations, in general the model successfully recreated the failure modes.

The model also showed good similarities with the force-displacement curves of the experimental results, see Figure 5.20. The experimental data had scatter due to the stick-slip behaviour of the mortar joints which came out very smooth in the numerical response since the displacement interval was quite small (5×10^{-4} mm). As mentioned previously, it is valuable when running this software to have a lower displacement interval to ensure that there is no initial force peak, which was noted in the model verification section in Chapter 4.

Figure 5.19: In-Plane Failure Mode Comparison, (a) from Casapulla and Argiento (2018) (b) from AEM Model

Figure 5.20: In-Plane Failure Force-Displacement Curves, numerical response compared with Casapulla and Argiento (2018)

5.3 Modelling Buttresses

The modelling of plain walls was expanded to include the effects of buttresses. As was described in Chapter 2, walls with buttresses exist throughout the city of Ottawa yet there is a significant lack of guidance for how to calculate the strength of these systems. The variables that were examined in this model were the wall-to-buttress connection, the wall width, the wall height, and buttress depth. All of the dimensions and variable ranges tested were chosen based on the survey conducted of buttresses in downtown Ottawa, found in Chapter 2. The majority of the stone buttresses recorded had ashlar stones approximately 600 mm x 300 mm x 300 mm ($l \times w \times h$) making up the buttresses and rubble stone walls. This was recreated in the software, with the wall stone courses being half the height of the buttress stones (150 mm). This difference in wall and buttress stone height was noted in Chapter 2 where the walls are typically rubble stone of varying heights while the buttress is typically constructed of regular ashlar stones.

As was discussed in Chapter 2, it is difficult to predict the shear strength of the connection between a buttress and the wall behind. For that reason, two different situations were modelled: first where the buttress is perfectly keyed with the outer wythe of the wall and the second where the buttress is disconnected from the wall. This represents upper and lower bound conditions for the connection between the buttress and the wall.

The height of the walls and buttresses were varied according to the buttresses found in Ottawa, between 3600 and 9600 mm. While most buttresses stopped short of the top of the wall, for this analysis they were modelled to be the same height.

The wall width is also investigated. Side wall buttresses are typically positioned between windows, so this variable investigates the effects of including the wall flanges in the analysis.

The following material properties in Table 5.12 were used for all models. These material properties were all chosen to be within the reasonably expected range, based off the sensitivity analysis with input from the previous experiments.

All buttresses were tested by displacement-controlled tests at the COG and at the third course from the top. The exact top was unable to be used since it resulted in simply pushing off the top course.

Table 5.12: Material Properties for Buttrass Testing

Material Property	Model
Stone Properties:	
Compressive Strength, f_{cb}	40 MPa
Tensile Strength, f_{tb}	4 MPa
Shear Strength, τ_b	1 MPa
Elastic Modulus, E_b	8 000 MPa
Shear Modulus, G_b	3 200 MPa
Mortar Properties:	
Compressive Strength, f_{cm}	1 MPa
Tensile Strength, f_{tm}	0 MPa
Shear Strength, τ_m	0 MPa
Elastic Modulus, E_m	900 MPa
Shear Modulus, G_m	400 MPa
Assembly Properties:	
Density, γ	2.3 g/cm ³
Friction Coefficient, μ	0.7

5.3.1 Disconnected Buttrasses

Beginning with the lower-bound analysis where the buttress is not keyed into the wall, refer to Figure 5.21 for the typical cross-section. There are two standard 300mm tall courses alternated along the height. The diagram shows 12 courses, or the 3600 mm tall buttress.

The following models all refer back to the rigid overturning moment equation. Since there is no shear connection between the wall and the buttress for the disconnected tests, the wall and buttress each rotate separately. The variables in these equations are as follows:

b_w is the width of the wall;

t_w is the thickness of the wall;

b_b is the width of the buttress; and

d_b is the depth of the buttress.

The overturning graphs for the basic walls at 3600 mm, 6600 mm, and 9600 mm are included in Figures 5.23 to 5.25. Note for the shortest 3600 mm wall the strength is actually higher than the overturning moment. This is presumed to be due to a combined sliding and overturning failure, note the force plateau where pure sliding was occurring.

Figure 5.21: Layout for Disconnected Buttresses.

$$W_w = b_w t_w h \gamma$$

$$W_b = b_b d_b h \gamma$$

$$M_{OV, rigid} = M_w + M_b$$

$$M_{OV, rigid} = W_w \left(\frac{t_w - \Delta_{top}}{2} \right) + W_b \left(\frac{d_b - \Delta_{top}}{2} \right)$$

where: M_w is the wall stabilizing moment;
 M_b is the buttress stabilizing moment;
 Δ_{top} is the top displacement.

Maximum at $\Delta_{top} = 0$

Minimum at $\Delta_{top} = \max(t_w, d_b)$

Figure 5.22: Rigid Overturning of Disconnected Buttresses.

Figure 5.23: Moment-Displacement Curve of Basic Disconnected 3600 mm Buttress.

Figure 5.24: Moment-Displacement Curve of Basic Disconnected 6600 mm Buttress

Figure 5.25: Moment-Displacement Curve of Basic Disconnected 9600 mm Buttress

5.3.1.1 Varying Height, h

The standard disconnected buttress was tested at heights ranging from 3600 to 9600 mm, at intervals of 300 mm or equivalent to one course height. The individual results of the tests can be found in Appendix B, and the summary graphs are included in Figures 5.26 and 5.27. It was interesting to look at both the actual peak overturning moments and equivalent lateral force of the tests in comparison with the rigid overturning equations since, theoretically, the lateral force at overturning is independent of the height of the wall. In order to make it so that all of the tests could be compared to the theoretical overturning force, an equivalent lateral force at the top of the wall was calculated for the tests, $F_{eq} = M_{OV,test}/h_{wall}$.

Figure 5.26: Maximum Overturning Moment, Disconnected Buttresses, Varying Height

Looking at these graphs, it is clear that there is an effect caused by the height of the buttress that is not covered by the simple rigid overturning formulas. In addition, there is a significant strength reduction between the top-displacement controlled tests and COG-displacement controlled tests, as was noted previously during the standard wall tests. This strength reduction is more severe for the taller buttresses.

Figure 5.27: Equivalent Lateral Force, Disconnected Buttresses, Varying Height

5.3.1.2 Varying Buttress Depth, d_b

The depth of the buttress was varied from 300 mm to 900 mm, with 600 mm being the “standard” wall section previously discussed. The “thin” and “deep” buttresses were tested for models 3600 mm, 6600 mm, and 9600 mm tall. The individual results of the tests can be found in Appendix B, and the summary graphs are included in Figure 5.28.

Similar results are apparent from these results as per the height tests. It is clear that there is an effect caused by the height of the buttress, and there is a significant strength reduction between the top-displacement controlled tests and COG-displacement controlled tests. The non-linear change between the 300 mm buttress and 900 mm buttress was predicted by the rigid overturning equations, indicating that the approximated mechanism of looking at the wall and buttress separate and simply adding the overturning moments of the two together is similar to what is predicted by the model.

In addition, this clearly indicates that there are slenderness effects at play. Looking at the COG tests, none of the thin buttresses 300 mm deep reached the theoretical rigid overturning strength, while the 3600 mm tall buttress exceeded the predicted strength for 600 mm depth and greatly exceeded the predicted strength for the 900 mm depth. The 6600 mm tall buttress

was also very close to the predicted value for the 900 mm deep section.

Figure 5.28: Equivalent Lateral Force, Varying Buttress Depth, d_b

5.3.1.3 Varying Wall Width, b_w

The width of the wall was varied from the standard 1800 mm to 600 mm. At 600 mm wall width, there is no visible face wall since the wall and buttress widths are the same. When the wall width is larger than the buttress, the wall and buttress create a “T” section, with the wall overhang referred to as flanges. The individual results of these models can be found in Appendix B, and the summary graphs are included in Figure 5.29.

As with the previous tests, the correct approximation of the difference in strength between the 1800 mm walls and 600 mm walls reinforces the concept of analyzing the walls and buttresses separately and simply adding their overturning moments together for disconnected buttresses.

Figure 5.29: Equivalent Lateral Force, Varying Wall Width, b_w

5.3.2 Keyed Buttresses

The possibility of having a good shear connection between the wall and buttress was also examined as an upper bound condition. The typical cross-section of the connected buttresses is included in Figure 5.30. There are two standard 300mm tall courses, which are alternated along the height of the model. The diagram shows 12 courses, or the 3600 mm tall buttress, but the taller buttresses follow the same pattern.

The following models all refer back to the rigid overturning equation, as discussed previously. Since there is a shear connection between the wall and the buttress for the keyed tests, the wall and buttress rotate together. This is depicted along with the overturning equations in Figure 5.31. The overturning graphs for the walls with and without wall flanges at 3600 mm, 6600 mm, and 9600 mm are included in Figures 5.32, 5.33, and 5.34, with both the rigid overturning from the disconnected and keyed analyses plotted.

Figure 5.30: Layout for Keyed Buttresses

$$W_w = b_w t_w h \gamma$$

$$W_b = b_b d_b h \gamma$$

$$M_{OV, rigid} = M_w + M_b$$

$$M_{OV, rigid} = W_w \left(d_b + \frac{t_w - \Delta_{top}}{2} \right) + W_b \left(\frac{d_b - \Delta_{top}}{2} \right)$$

Maximum at $\Delta_{top} = 0$

Minimum at $\Delta_{top} = t_w + d_b$

Figure 5.31: Rigid Overturning of Keyed Buttresses

Figure 5.32: Overturning Moment for 3600 mm Keyed Buttresses

Figure 5.33: Overturning Moment for 6600 mm Keyed Buttresses

Figure 5.34: Overturning Moment for 9600 mm Keyed Buttresses

Table 5.13: Overturning Moment Comparison of Keyed and Disconnected Buttresses

Height (mm)	Maximum Overturning Moment (kN.m)			
	No Wall Flanges, $b_w = 600$ mm		Including Wall Flanges, $b_w = 1800$ mm	
	Disconnected	Keyed	Disconnected	Keyed
3600	19.78	26.57	37.29	52.89
6600	29.70	44.73	59.46	117.92
9600	36.10	64.59	72.95	152.93

These results indicate that there is a substantial increase in strength when the buttress is keyed with the wall, but it does not reach near the theoretical value for rigid body overturning of a fully connected buttress. All of the models resulted in peak moments approximately halfway between the disconnected and connected rigid approximations. This indicates that examining the cross section of a buttress, including the wall behind, as failing by rigid overturning is not an accurate method of analysis.

There are some hypotheses that could account for the high rigid overturning moment, highlighted in the previous research. Elastic deformations occur prior to cracking, resulting in the COG being closer to the point of overturning than the rigid equations predict. The wall is also fracturing, with a portion of the mass not included in the overturning portion. When the wall and buttress were failing separately, the portion of the mass was small since both the wall and buttress individually lost a small portion. With the sections connected, the amount of mass lost is substantially greater. In addition, neglecting this portion of the mass would move the COG of the mobilized portion closer to the overturning point. Both of these theories, in addition to comparing the results from these models to the available guidance, are further examined in the next Chapter - Buttress Analysis.

Chapter 6: Analysis

This Chapter will discuss the results of the modelling in comparison to the background information and literature review from Chapters 2 and 3. First, a discussion regarding the general results of the wall modelling can be found in Section 6.1, followed by a comparison of the buttress model results with a variety of analysis methods previously discussed in Chapter 3 (Section 6.2).

6.1 Wall Analysis

There were a wide range of walls modelled over the course of this research, with the basic wall calibrated against real out-of-plane experimental testing and then expanded upon to see what would happen with minor changes. There was a sensitivity analysis performed on the various material properties and different wall section geometries were examined to include the effects of a rubble core.

The main outcome from these models is that there was a significant decrease in the overturning moment resistance for walls with limited or no header stones. This was consistent for the double-wythe walls with through-stones, the walls with a core and through-stones, and the walls with a core and key stones. The Table summarizing the results of these tests, previously published as Table 5.7, is recreated in Table 6.1. This new Table shows the percent difference from the models in comparison to the “Wall 1” model where every other stone is a header.

This comparison shows the significant reduction in strength for walls without adequate bond stones, either through-stones or key stones. This strength loss is more than 50%, across all wall cross-section geometries modelled. Through-stones are generally more effective at ensur-

Table 6.1: Peak Moment Values from All Wall Tests, with Percent Difference from “Wall 1”

Wall Label	Header Percent (%)	Double Wythe Walls	Walls With A Core	
		M_{max} (kN.m)	Through-Stones M_{max} (kN.m)	Key Stones M_{max} (kN.m)
1	50	10.07	9.52	9.34
2	33	9.95 -1.2%	9.47 -0.5%	9.03 -3.3%
3	23	9.84 -2.3%	9.43 -0.9%	8.74 -6.4%
4	20	9.77 -3.0%	9.24 -2.9%	
5	17	9.17 -8.9%	9.21 -3.3%	8.01 -14.2%
6	13	9.44 -6.3%	8.59 -9.8%	
7	10	9.40 -6.7%	8.44 -11.3%	6.85 -26.7%
8	6.7	8.86 -12.0%	8.24 -13.4%	6.98 -25.3%
9	6.7	8.98 -10.8%	8.06 -15.3%	
10	6.7	6.71 -33.4%	7.89 -17.1%	
11	3.3	7.84 -22.1%	7.06 -25.8%	5.89 -36.9%
12	3.3	7.53 -25.2%	6.29 -33.9%	5.48 -41.3%
13	3.3	5.82 -42.2%	4.60 -51.7%	4.56 -51.2%
14	0	4.86 -51.7%	2.71 -71.5%	2.71 -71.0%

ing that the wall fails as a unit. The worst case scenario where there are no headers the walls are modelled effectively as separate walls adjacent to each other with uninterrupted vertical shear planes.

In practice, a situation where a vertical shear plane may be present within a wall is when there is significant deterioration and the core has washed out, see Section 2.3.1. This mortar/core deterioration has also been noted to many damaged masonry buildings in earthquakes. The significant loss of strength with this form of deterioration highlights the benefits of regular maintenance and ensuring that the core is properly consolidated.

For supplemental strength, there are various mechanical wythe connectors available to be used in retrofits, as discussed in Chapter 2. These are typically stainless steel and form a rigid connection between the wythes. Different types of mechanical connectors were tested in the West Block experiments (See Section 3.4.1). During these experiments there were no differences noted in wall behaviour between the basic walls, walls with headers, and walls with mechanical connectors. This is contrary to this modelling programme which showed that neglecting headers should result in a significant reduction in strength. It is hypothesized that this consistent experimental response is due to the excellent construction of these walls, with properly built cores and solid mortar, so the walls did not need to rely on the additional connectors.

These findings highlight that perhaps the simplest way to ensure the strength of stone walls would be to ensure, through regular and thorough maintenance, that the core is consolidated. It would be beneficial to test in a laboratory the deteriorated condition of walls without headers, with headers, and with mechanical wythe connectors to see the long term effects of these various systems in comparison to a simple regular maintenance program.

As a final note to the plain wall modelling, the sensitivity analysis showed that there are limited material properties that affect the overturning moment resistance of the walls. Notably, there are negligible effects of varying the compressive, tensile or shear strength of the mortar within reasonable ranges for historic mortars. This reinforces the original theory, as proposed by Heyman (1966), that it is reasonable to assume that masonry has an infinite compressive strength, zero tensile strength, and that sliding does not occur. The material properties which affect the behaviour of the walls were the density, which directly affects the weight and stabilizing moment, and the stiffness. It is hypothesized that the less stiff walls displaced more before cracking, reducing the distance between the COG and the overturning point and therefore reducing the stabilizing moment.

6.2 Buttness Analysis

This section will compare the results of the buttness experiments with a variety of analysis methods previously discussed. There do not exist reliable recommendations for buttness strength calculations, so all of the different methods discussed in Chapter 3, including the in-plane and out-of-plane recommendations from the Canadian Masonry Standard (CSA, 2014), are compared to the model results. Since none of the analysis methods reviewed include the influence of wall flanges, the walls without flanges will be assessed. The effect of flanges is included briefly at the end. Sample calculations and descriptions of the methods used are described below for the 6600 mm tall connected buttness. The material properties and geometries for these analyses can be found in Table 5.12 and Figures 5.21 and 5.30, for the disconnected and keyed buttnesses respectively.

The different failure mechanisms are divided into Single Body Statics (Section 6.2.1) which includes rigid and elastic overturning, the CSA (2014) proposed methods of analysis (Section 6.2.2), both in-plane and out-of-plane calculations, and Fracture Plane Analyses (Section 6.2.3).

For all of the different methods of analysis, both the predicted failure value from the equations and the design value is examined. This allows for a thorough comparison of the different analysis methods, since these methods use different design reductions such as safety factors and reduction factors.

6.2.1 Single Body Statics

Single body statics refers to analyzing the buttness as a singular body overturning. Within this category both rigid and elastic overturning are examined, as well as the graphic statics method used in the 19th century.

6.2.1.1 Rigid Overturning

Rigid overturning is the historic method of analysis and is still frequently included in modern research. The equations for the rigid overturning of the disconnected and keyed but-

tresses are repeated in Equations 6.1 and 6.2. These were previously shown in Section 5.1.1, Equation 5.2, and modified slightly to represent the disconnected and keyed buttresses in Section 5.3. Historically, the rigid overturning value was calculated, and then a safety factor of 2 was applied (see Section 2.2.2). The rigid overturning moment of the buttresses are denoted as $M_{OV,rigid,disconnected}$ and $M_{OV,rigid,keyed}$.

$$M_{OV,rigid,disconnected} = W_w \left(\frac{t_w - \Delta_{top}}{2} \right) + W_b \left(\frac{d_b - \Delta_{top}}{2} \right) \quad (6.1)$$

$$M_{OV,rigid,keyed} = W_w \left(d_b + \frac{t_w - \Delta_{top}}{2} \right) + W_b \left(\frac{d_b - \Delta_{top}}{2} \right) \quad (6.2)$$

Sample calculations are included below for the 6600mm buttress.

$$M_{OV,rigid,disconnected} = (0.6)(0.6)(6.6)(22.55) \left(\frac{0.6 - 0}{2} \right) + (0.6)(0.6)(6.6)(22.55) \left(\frac{0.6 - 0}{2} \right)$$

$$M_{OV,rigid,disconnected} = 32.15 \text{ kN.m}$$

$$M_{OV,rigid,keyed} = (0.6)(0.6)(6.6)(22.55) \left(0.6 + \frac{0.6 - 0}{2} \right) + (0.6)(0.6)(6.6)(22.55) \left(\frac{0.6 - 0}{2} \right)$$

$$M_{OV,rigid,keyed} = 64.30 \text{ kN.m}$$

The rigid overturning formulation in comparison to the disconnected and keyed buttresses are included in Figures 6.1 and 6.2. For the keyed buttresses, this method overestimated the peak overturning moment for all heights tested; however, with the safety factor of 2 as was used historically in design, the design strength equals approximately 75-80% of the maximum overturning moment. For the disconnected buttresses, the effect of increasing the height resulted in a nonlinear curve where the strength for the shortest buttresses (COG tests) was higher than predicted by rigid overturning. At the higher end of buttress heights, including the safety factor of 2 resulted in design strengths 88% of the overturning strength.

Figure 6.1: Rigid Overturning of Keyed Buttresses

Figure 6.2: Rigid Overturning of Disconnected Buttresses

6.2.1.2 Elastic Overturning

The possibility of examining the walls as elastic rather than rigid was also explored. The need to look at the walls as elastic rather than rigid was highlighted by the effects of varying the height of the walls. While the rigid approximation provided a reasonable prediction of overturning moment of the shorter walls, the taller walls were found to fail at lateral forces lower than what was predicted by rigid overturning. It is likely that this is because the walls are so tall that the lateral force causes the walls to bend as per an elastic cantilever prior to overturning. This pre-overturning displacement lowers the distance between the COG and the overturning tow, and therefore the predicted M_{OV} . The theoretical displacement of a cantilever with a horizontal load somewhere along the height at h_{force} is well published, and included in Figure 6.3. For this analysis the elastic deformation due to both bending and shear was included.

Figure 6.3: Elastic Displacement

For the following equations:

$\Delta(y)$ is the elastic deformation at y ;

y is the vertical location along the height;

H is the horizontal force;

h is the total height;

h_{force} is the height of the horizontal force;

E is the modulus of elasticity;

I is the moment of inertia; and

G is the shear modulus.

for $(0 \leq y \leq h_{force})$:

$$\Delta(y) = \frac{H y^2}{6 E I} (3 h_{force} - y) + \frac{6 H y}{5 G A}$$

for $(h_{force} \leq y \leq h)$:

$$\Delta(y) = \frac{H h_{force}^2}{6 E I} (3 y - h_{force}) + \frac{6 H h_{force}}{5 G A}$$

Including these elastic deformations, the elastic overturning moment, $M_{OV, elastic}$ can be calculated as Equation 6.3.

$$M_{OV, elastic} = W_{bot} (d_b - \bar{x}_{bot}) + W_{top} (d_b - \bar{x}_{top}) \quad (6.3)$$

where: W_{bot} is the weight of the bottom segment, below h_{force} ; and

W_{top} is the weight of the top segment, between h_{force} and h .

With the two formulas governing the displaced shape, above and below the height of the lateral force, it is necessary to divide the wall into 2 segments at the height of the lateral force to acquire the lateral displacement of the COG of these two segments. In order to find the displaced COG, the displacement equations above for $\Delta(y)$ were integrated with respect to y to get the area under the curve. The area represents the displaced area. Dividing this area by the height of the wall segment (h_{force} for the bottom segment or $h - h_{force}$ for the top segment) provides the average horizontal displacement, $\bar{x}$, which is the location of the COG. The procedure for the bottom segment of the wall is described in Equation 6.4 with the final solution shown in Equation 6.5.

$$\bar{x}_{bot} h_{force} = \int_0^{h_{force}} \frac{3 H y^2 h_{force}}{6 E I} - \frac{H y^3}{6 E I} + \frac{6 H y}{5 G A} \quad (6.4)$$

$$\bar{x}_{bot} h_{force} = \frac{3 H y^3 h_{force}}{(3) 6 E I} - \frac{H y^4}{(4) 6 E I} + \frac{6 H y^2}{(2) 5 G A} \Big|_0^{h_{force}}$$

$$\bar{x}_{bot} = \left(\frac{H h_{force}^4}{6 E I} - \frac{H h_{force}^4}{24 E I} + \frac{3 H h_{force}^2}{5 G A} \right) / h_{force}$$

$$\bar{x}_{bot} = \frac{H h_{force}^3}{8 E I} + \frac{3 H h_{force}}{5 G A} \quad (6.5)$$

This $\bar{x}_{bot}$ is the location of the COG for the bottom section to account for elastic deformation.

The procedure for the top segment of the wall is described in Equation 6.6 with the final

solution shown in Equation 6.7.

$$\begin{aligned}
 \bar{x}_{top}(h-h_{force}) &= \int_{h_{force}}^h \frac{3Hyh_{force}^2}{6EI} - \frac{Hh_{force}^3}{6EI} + \frac{6Hh_{force}}{5GA} \quad (6.6) \\
 \bar{x}_{top}(h-h_{force}) &= \frac{3Hy^2h_{force}^2}{(2)6EI} - \frac{Hh_{force}^3y}{6EI} + \frac{6Hh_{force}y}{5GA} \Big|_{h_{force}}^h \\
 \bar{x}_{top}(h-h_{force}) &= \left(\frac{Hh^2h_{force}^2}{4EI} - \frac{Hh_{force}^3h}{6EI} + \frac{6Hh_{force}h}{5GA} \right) - \left(\frac{Hh_{force}^4}{4EI} - \frac{Hh_{force}^4}{6EI} + \frac{6Hh_{force}^2}{5GA} \right) \\
 \bar{x}_{top} &= \left(\frac{Hh_{force}^2}{12EI}(3h^2 - 2hh_{force} - h_{force}^2) + \frac{6Hh_{force}}{5GA}(h-h_{force}) \right) / (h-h_{force}) \\
 \bar{x}_{top} &= \frac{Hh_{force}^2}{4EI} \left(h + \frac{h_{force}}{3} \right) + \frac{6Hh_{force}}{5GA} \quad (6.7)
 \end{aligned}$$

This $\bar{x}_{top}$ is the location of the COG for the top section to account for elastic deformation.

The force at which the wall begins to overturn can be approximated as H_{OV} :

$$H_{OV} = \frac{M_{OV,rigid}}{h_{force}} \quad (6.8)$$

Substituting H_{OV} into Equations 6.5 and 6.7, gives the predicted elastic deformation of the wall at overturning.

$$\bar{x}_{bot,OV} = \frac{H_{OV}h_{force}^3}{8EI} + \frac{3H_{OV}h_{force}}{5GA} \quad (6.9)$$

$$\bar{x}_{top,OV} = \frac{H_{OV}h_{force}^2}{4EI} \left(h + \frac{h_{force}}{3} \right) + \frac{6H_{OV}h_{force}}{5GA} \quad (6.10)$$

With these initial displacements known, they can be subbed into Equation 6.3 to calculate the $M_{OV,elastic}$.

A further consideration is that these equations only consider elastic deformation. In ma-

sonry and concrete structures which crack, it is common procedure to use $0.4EI$ as the stiffness in order to account for this cracking. A safety factor of 2 was also considered, to be consistent with the rigid analysis.

Sample calculations are included for the 6600mm keyed buttress:

$$H_{OV} = \frac{M_{OV}}{h_{force}} = \frac{64.31kN.m}{5.85m} = 10.99 kN$$

$$\bar{x}_{bot} = \frac{H h_{force}^3}{8 E I} + \frac{3 H h_{force}}{5 G A} = \frac{(10990)(5850)^3}{8(1000)(8.64 \times 10^{10})} + \frac{3(10990)(5850)}{5(250)(720000)} = 3.40 mm$$

$$\bar{x}_{top} = \frac{H h_{force}^2}{4 E I} \left(h + \frac{h_{force}}{3} \right) + \frac{6 H h_{force}}{5 G A}$$

$$\bar{x}_{top} = \frac{(10990)(5850)^2}{4(1000)(8.64 \times 10^{10})} \left(6600 + \frac{5850}{3} \right) + \frac{6(10990)(5850)}{5(250)(720000)} = 9.73 mm$$

$$M_{OV,elastic} = A h_{force} \gamma (t/2 - \bar{x}_{bot}) + A (h - h_{force}) \gamma (t/2 - \bar{x}_{top})$$

$$= (1.2)(0.6)(5.85)(22.55)(1.2/2 - 0.0034) + (1.2)(0.6)(6.6 - 5.85)(22.55)(1.2/2 - 0.0097)$$

$$M_{OV,elastic} = 63.86 kN.m$$

$$M_{SF=2,elastic} = \frac{M_{OV,elastic}}{SF} = \frac{63.86}{2} = 31.93kN.m$$

The elastic overturning formulations in comparison to the disconnected and keyed buttress model results are included in Figures 6.4 and 6.5. The reduction provided by accounting for the initial displacement is one method to account for slenderness. For the keyed buttresses, where the entire thickness of the wall and buttress are considered combined, there were only minor differences when including the elastic analysis in comparison to the rigid analysis. For the disconnected buttress, the elastic approximation using $0.4EI$ predicted fairly accurately the nonlinear loss of strength with increased slenderness.

Figure 6.4: Elastic Overturning of Keyed Buttresses

Figure 6.5: Elastic Overturning of Disconnected Buttresses

6.2.1.3 Graphic Statics

The historic method of graphic statics was assessed in comparison to the model results. The purpose of following the line of thrust through the entire buttress is to ensure that at section reductions the line of thrust stays within the masonry, which is irrelevant for our purposes since the cross-section is rectangular.

Since the section is rectangular, this process is as simple as ensuring that the slope of the resultant force from H and W is equal to the geometric slope of the resultant of $t/3$ and h , see Figure 6.6. The maximum allowable moment from the graphic statics method, denoted M_{GS} , can be calculated as follows, and the results are plotted in Figures 6.7 and 6.8.

Due to equivalent triangles:

$$\tan \theta = \frac{h}{t/3} = \frac{W}{H}$$

$$M_{GS} = Hh = \frac{Wt}{3}$$

For the 6600 mm keyed buttress:

$$M_{GS} = \frac{(107.16)(1.2)}{3} = 42.86 \text{ kN.m}$$

Figure 6.6: Graphic Statics Equivalent Triangles

For the disconnected buttresses, t is the thickness of a single part (600 mm) and it is assumed that the half of the horizontal force is taken by each portion. A sample calculation is included for the 6600 mm disconnected buttress:

$$\tan \theta = \frac{h}{t/3} = \frac{W}{H/2}$$

$$M_{GS} = \frac{2Wt}{3} = \frac{2(53.58)(600)}{3} = 21.43 \text{ kN.m}$$

Figure 6.7: Graphic Statics for Keyed Buttresses

Figure 6.8: Graphic Statics for Disconnected Buttresses

6.2.2 CSA S304-14 Analysis

The Canadian Masonry Standard, CSA (2014), provides methods to analyze masonry walls both in-plane and out-of-plane. These methods are described in detail in Section 3.1.2. Both of these methods were examined since, while the wall is acting out-of-plane with lateral loads, the buttress itself is technically acting as a wall in-plane.

6.2.2.1 Out-of-Plane Analysis

Rather than stating a method for determining the out-of-plane strength of walls and columns, the code instead states that tension is not allowed to be developed for unreinforced masonry. The equations provided are typically to determine if the applied moment is acceptable, which for this analysis are reversed in order to determine the maximum allowable moment. There are various reduction factors included in the equations. The following equations are used both with and without these reduction factors in order to compare the predicted strength with the other methods analyzed, along with design strengths. In order to satisfy the requirement that tension is not allowed to be developed, the following equation must be true:

$$e = \frac{M}{P} \leq \frac{t}{3} \quad \text{or for factored loads} \quad e = \frac{M_f}{P_f} \leq \frac{t}{3}$$

Based on this limitation, M_r the moment resistance of the walls, must be greater than the moment at which tension develops in the walls. Since this is not a true moment resistance, the moment that these equations provide will be denoted as $M_{tension}$.

$$M_r > \frac{t}{3}P = M_{tension} \quad \text{or for factored loads} \quad M_r > \frac{t}{3}P_f = M_{f,tension}$$

The axial load of the modelled walls is only due to the self-weight. According to the NBCC (NRC, 2015), the dead load should be factored by 0.9 since it is contributing to the strength, so lowering the load will provide in a more conservative result. Sample calculations for the 6600 mm wall are:

$$P = Ah \gamma = (1.2)(0.6)(6.6)(22.55) = 107.2 \text{ kN}$$

$$P_f = 0.9P = 0.9(107.2) = 96.44 \text{ kN}$$

$$M_{tension} = \frac{t}{3}P = \frac{1.2}{3}(107.2) = 42.87 \text{ kN.m}$$

$$M_{f,tension} = \frac{t}{3}P_f = \frac{1.2}{3}(96.44) = 38.58 \text{ kN.m}$$

For short walls where slenderness does not apply, the moment directly caused by lateral loads, M_{fp} can be equal to $M_{tension}$. Slenderness effects need to be considered when $kh/t \geq 10$, which for the 6.6m keyed buttress is equal to 11 so must be considered. Note that k is equal to 2 for cantilevered walls.

Moment Magnifier Method

First using the moment magnifier method, this method increases the primary moment previously calculated based on the ratio of axial load to Euler's buckling load. Since the axial load in this situation is contributing to the moment, for the factored calculations the self-weight should be factored by 1.4, $P_f = 150 \text{ kN}$. The parameter C_m for this analysis is 1.0 since the moment is caused entirely by lateral loads applied on the section, rather than fixed end moments. While the code does not provide any guidance for determining E_m , a value of 1000 MPa was used in accordance with the Italian code and previous studies.

$$M_{f\text{ tot}} = M_{fp} \frac{C_m}{\left(1 - \frac{P_f}{P_{cr}}\right)}$$

The total moment, $M_{f\text{ tot}}$, must be less than $M_{tension}$. Sample calculations for the 6600 mm keyed buttress are included below. For the disconnected buttresses, the following process was performed for the wall and buttress separately and then summed together.

$$(EI)_{eff} = 0.4 E_m I_o = 0.4(1000)(8.64 \times 10^{10}) = 3.46 \times 10^{13} \text{ N.mm}^2$$

$$P_{cr} = \frac{\pi^2 \phi_e (EI)_{eff}}{(1 + 0.5\beta_d)(kh)^2} = \frac{\pi^2 0.65 (3.46 \times 10^{13})}{(1 + 0.5(0))(2 \times 6600)^2} = 1272 \text{ kN}$$

$$M_{tension} = M_{fp} \frac{1}{\left(1 - \frac{150}{1272}\right)}$$

The $M_{tension}$ is constant from the above equations neglecting slenderness, but the allowable moment from lateral forces, M_{fp} , is reduced to include the magnifying terms. For comparison purposes with the $P\delta$ method, this allowable moment from lateral forces, M_{fp} , is denoted as M_{MMag} .

$$M_{MMag} = \frac{M_{tension}}{\left(1 - \frac{P}{P_{cr}}\right)} = \frac{42.87}{\left(1 - \frac{107.2}{1272}\right)} = 39.26 \text{ kN.m}$$

$$M_{f,MMag} = \frac{M_{f,tension}}{\left(1 - \frac{P_f}{P_{cr}}\right)} = \frac{38.58}{\left(1 - \frac{150}{1272}\right)} = 34.03 \text{ kN.m}$$

Pδ Method

Second analyzing the wall with the $P\delta$ method, this method increases the self-weight moment by approximating the lateral deflection. The process is similar to that for the elastic walls, using a horizontal force of $P_H = M_{tension}/h$ and approximating the lateral deflection using $(EI)_{eff}$. Including sample calculations for the 6600 mm keyed buttress:

$$P_H = \frac{M_{tension}}{h} = \frac{42.87}{6.6} = 6.50 \text{ kN}$$

$$P_{Hf} = \frac{M_{f,tension}}{h} = \frac{38.58}{6.6} = 5.84 \text{ kN}$$

$$\delta = \frac{PL^3}{3(EI)_{eff}} = \frac{(6500)(6600)^3}{3(3.46 \times 10^{13})} = 18.01 \text{ mm}$$

$$\delta_f = \frac{PL^3}{3\phi_e(EI)_{eff}} = \frac{(5840)(6600)^3}{3(0.65)(3.46 \times 10^{13})} = 24.94 \text{ mm}$$

$$M_{tension} = M_{fp} + P\delta \leq M_{tension} \quad \text{or for factored loads}$$

$$M_{f,tension} = M_{fp} + P_f\delta_f \leq M_{f,tension}$$

As with the moment magnifier method, the $M_{tension}$ is constant from the above equations neglecting slenderness, but the allowable moment from lateral forces, M_{fp} , is reduced to include the magnifying terms. For comparison purposes with the moment magnifier method, this allowable moment from lateral forces, M_{fp} , is denoted as M_{pDelta} .

$$M_{pDelta} = M_{tension} - P\delta = 42.87 - 107.2(0.01801) = 40.94 \text{ kN.m}$$

$$M_{f,pDelta} = M_{f,tension} - P_f\delta_f = 38.58 - 150(0.02494) = 34.84 \text{ kN.m}$$

The CSA analysis in comparison to the keyed and disconnected buttress model results are included in Figures 6.9 and 6.10.

Figure 6.9: CSA Analysis of Keyed Buttresses

Figure 6.10: CSA Analysis of Disconnected Buttresses

For the keyed buttresses, the CSA method of analysis proved extremely accurate at predicting the failure strength. Including slenderness effects, the $P\Delta$ (unfactored) had a good correlation with the modelling results. The moment magnifier analysis over-estimated the strength reduction due to slenderness, resulting in unrealistic values for the lower-bound disconnected buttress models. The slenderness equation, kh/t , equals 30 at a height of 9000 mm, at which point the factored moment magnifier equation goes to 0. This is not representative of the modelling data, where there was still an increase in overturning moment resistance between the 9000 mm and 9600 mm models. There are buttresses of this height found within the City of Ottawa that have been standing for over 100 years so clearly attributing a factored strength of 0 to these systems is illogical. The $P\Delta$ analysis provides much more realistic values and has a good prediction of the slenderness effects, for both the keyed and disconnected buttresses.

6.2.2.2 In-Plane Analysis

Looking at a buttress cross-section, it can theoretically be analyzed as a wall in-plane. See Section 3.1.2 for a detailed description on the in-plane analysis of masonry walls from the Canadian Masonry Standard (CSA, 2014).

$$V_r = \phi_m (v_m b_w d_v) + 0.25 (P_d) \gamma_g \leq 0.4 \phi_m \sqrt{f'_m} b_w d_v \gamma_g$$

$$d_v = 0.8 l_w = 0.8(1200) = 960 \text{ mm}$$

$$v_m = 0.16 \left(2 - \frac{M_f}{V_f d_v} \right) \sqrt{f'_m}$$

For the shear strength of masonry, v_m , there are many values which are undefined in the Standard for stone masonry. For this analysis f'_m is assumed to be 5 MPa. The shear capacity of the wall is also dependent on the applied shear and moment. The term $\frac{M_f}{V_f d_v}$ is bounded in the code as a minimum of 0.25 and maximum of 1, so instead of inserting hypothetical applied moment and shears a bounded analysis will be performed.

$$v_{m,max} = 0.16(2 - 0.25)\sqrt{f'_m} = 0.63 \text{ MPa}$$

$$v_{m,min} = 0.16(2 - 1)\sqrt{f'_m} = 0.36 \text{ MPa}$$

$$V_{r,max} = 0.6(0.63)(600)(960) + 0.25(96.44)(1) = 242 \text{ kN}$$

$$V_{r,min} = 0.6(0.36)(600)(960) + 0.25(96.44)(1) = 149 \text{ kN}$$

For these walls, the overturning moment failure condition will be reached significantly before the shear failure. Shear failure occurred at 149 kN while overturning failure occurred at 6.5 kN. It is not explicitly said in the in-plane analysis section that overturning needs to be considered as well. Therefore it is clear that buttresses cannot be treated as in-plane walls.

6.2.3 Fracture Plane Analysis

6.2.3.1 Method proposed by Ochsendorf and De Lorenzis (2008)

As one of the only published modern methods of analyzing buttresses, it was important to compare the analytical results with the strength calculations from Ochsendorf and De Lorenzis (2008). The concept behind the equations proposed by Ochsendorf and De Lorenzis (2008) is that as the buttress fails, there is a non-negligible amount of masonry which separates from the buttress and does not contribute to the overturning moment resistance.

The full procedure and methodology is included in Section 3.3.

$$\zeta = \frac{e}{h} \quad \mu = \frac{h}{h_b} \quad \psi = \frac{V}{W_b}$$

where: e is the height of the fracture plane;

h is the height of the applied load;

h_b is the height of the buttress;

V is the externally applied vertical load; and

W_b is the weight of the buttress.

For our analysis, the height of the wall and the height of the buttress are equal making $\mu = 1$ and there is no externally applied load making $\psi = 0$. The following quadratic equation is solved for ζ_{OV} , to determine the height of the diagonal fracture plane, e .

$$\zeta_{OV}^2 - \left(\frac{1}{2} + \frac{3}{2\mu} + \frac{3\psi}{\mu} \right) \zeta_{OV} + \left(\frac{\psi + 1}{\mu} \right) = 0$$

$$\zeta_{OV}^2 - \left(\frac{1}{2} + \frac{3}{2}\right)\zeta_{OV} + 1 = 0$$

$$\zeta = 0.979$$

$$H_{OV} = \frac{W_b}{h} \left(\frac{1 + \psi}{3\mu\zeta} - \frac{1}{6} \right) = \frac{107.2}{6.6} \left(\frac{1}{3(1)(0.755)} - \frac{1}{6} \right) = 4.06 \text{ kN}$$

$$M_{OV} = H_{OV} h = 26.8 \text{ kN.m}$$

Note that $\zeta \approx 1$, indicating that the fracture plane extends almost to the top corner. The model fracture did not extend beyond 50% of the height for any of the tests, however a standardized value could not be obtained since the slope of the diagonal fracture varied significantly depending on stone size, header location and location of horizontal force. This resulted in a low prediction of the overturning strength, since it was predicting a much larger loss of mass than the software. The results are plotted in Figure 6.11.

6.2.3.2 45° Fracture Plane

As was noted in the historical texts and revisited by Ochsendorf and De Lorenzis (2008), a simplified method of including the loss of material can be to assume that the fracture plane will be 45°. This simplifies the above process since e is equal to t , and the weight of the top and bottom sections can be simply calculated.

$$W_{top} = \gamma A(h - e) = 87.8 \text{ kN}$$

$$W_{bot} = \gamma A e / 2 = 9.7 \text{ kN}$$

$$M_{OV} = W_{top}(t/2) + W_{bot}(t/3) = 56.6 \text{ kN.m}$$

This process, being similar to rigid overturning only neglecting a portion of the mass which is not mobilized, was likely used alongside a safety factor of 2 (Ochsendorf and De Lorenzis, 2008). The two fracture analysis methods are plotted in comparison to the keyed and disconnected buttresses, included in Figures 6.11 and 6.12.

Figure 6.11: Fracture Analysis of Keyed Buttresses

Figure 6.12: Fracture Analysis of Disconnected Buttresses

Overall, the fracture analyses did not have accurate approximations of the failure strengths of the buttresses modelled. These methods do not include any slenderness accommodations, so the predicted response varying height was linear. The equations provided by Ochsendorf and De Lorenzis (2008), over estimate the angle of the fracture plane, predicting that the fracture reaches the upper corner for all tests. This results in underestimating the amount of mass that is mobilized and, in turn, underestimating the strength. The 45° fracture plane, on the other hand, over estimated the strength for the keyed buttresses and was approximately accurate for the disconnected buttresses, but did not account for the non-linear curve due to slenderness. Interestingly, for the keyed buttresses, applying a safety factor of 2 to the 45° fracture plane provides similar results to the equations recommended by Ochsendorf and De Lorenzis (2008).

6.2.4 Buttresses with Wall Flanges

The buttress typology that was analyzed for this research, buttresses along the side walls of churches, are typically located between large windows. These buttresses do not extend fully between the windows, instead have a certain amount of wall on either side, see Figure 6.13. There are no recommendations in literature to examine these composite sections, so a thorough analysis such as was completed for the buttresses neglecting the wall flanges could not be completed.

Figure 6.13: Buttresses with Wall Flanges, Typical. (Marrs, Nov 2019)

The buttress and wall section that was modelled includes 600 mm of wall on either side of the buttress. The peak overturning moment of the buttresses with wall flanges are compared to the buttresses without in Table 6.2 below. This Table shows the comparison of the COG displacement-controlled tests, with similar results for the top displacement-controlled tests.

Table 6.2: Comparison of Buttress Models With and Without Wall Flanges

Buttress Height (mm)	Keyed Buttresses		Disconnected Buttresses	
	No Wall Flanges	With Wall Flanges	No Wall Flanges	With Wall Flanges
	$b_w = 600$ mm M_{max} (kN.m)	$b_w = 1800$ mm M_{max} (kN.m)	$b_w = 600$ mm M_{max} (kN.m)	$b_w = 1800$ mm M_{max} (kN.m)
3600	26.57	61.27 (+131%)	19.78	37.29 (+89%)
6600	44.73	117.92 (+164%)	29.70	59.46 (+100%)
9600	64.59	137.05 (+112%)	36.10	72.65 (+101%)

Generally, including the wall flanges doubled the strength of the section. For the disconnected buttresses, the almost exact doubling of the overturning moment is expected since, with the geometry used, including the 600 mm wall flanges doubles the mass of the wall. For the keyed buttresses the mass is also doubled, but examining the cross section as a “T” shows that the mass is being added further from the overturning point, moving the COG of the entire section into the wall and therefore increasing the stabilizing moment. There is, however, no guidance available to compare these results to. Without experiments proving that the mass of the wall is mobilized along with the buttress, this increase in strength cannot be relied upon in practice.

6.2.5 Summary

In summary, the rigid overturning method significantly overestimated the buttress overturning moment. This method over simplifies the process and neglects many of the internal mechanisms, bending and cracking, which are occurring. The elastic overturning method, using an effective stiffness, $0.4EI$, has a fairly good prediction of the COG displacement tests, but is not conservative for the top displacement tests. The graphic statics method is accurate at predicting the strength of the keyed buttresses, but is less accurate for the disconnected buttresses since it is a linear method and the disconnected buttress models experienced a significant non-linear

response.

The CSA out-of-plane analysis was the best method looked at in this section to predict the behaviour of the buttresses. This is interesting since the method of limiting the eccentricity to $t/3$ is the same as the graphic statics method, which would have been used originally in the design of buttresses around Canada in the 19th century. It is not clear if this historic analysis method is the inspiration for the Canadian Masonry Code, or if simply the theory of not letting the masonry experience tension is the foundation for both. Either way, these two methods produce the exact same (unfactored) results. The inclusion of slenderness with the $P\Delta$ method increases the accuracy of the graphic statics method, especially for the disconnected buttresses.

The fracture plane analysis methods were not very accurate at predicting the overturning strength of the buttress models. The strength calculation as proposed by Ochsendorf and De Lorenzis (2008), predicted the fracture plane to extend to the top of the buttress which significantly overestimated the mass neglected in the overturning moment resistance calculation and therefore under estimated the strength. It is possible that not including any external vertical loads caused this error. The 45° fracture plane was inaccurate at predicting the overturning strength of the buttress models, overestimating the strength. Also, the fracture plane analysis methods are linear methods and do not account for any slenderness effects.

The results of the modelling show that slenderness effects need to be considered. Slenderness effects are not required to be considered according to the CSA analysis under a slenderness ratio, kh/t , of 10. The keyed buttresses ($t = 1.2$ m) exceed this ratio at a height of 6 m. At this height, the $P\Delta$ method predicts a 4% strength reduction due to slenderness, and the moment magnifier method predicts a 10% strength reduction. The process of acquiring the reduced allowable strength including either $P\Delta$ or moment magnifier effects is simple and conservative, even for the walls with lower slenderness ratios. Based on these results, including slenderness effects at all slenderness ratios is good practice. This process can be simple for designers if the Standard included design stiffnesses and methods to calculate the deflection for the $P\Delta$ analysis.

Chapter 7: Conclusions

Stone buildings found across Canada were constructed in a different era with different engineering methodologies than today. These buildings are important representations of the history of the country and should be preserved for future generations. History has shown that stone buildings have wide ranging responses to seismic activity and can have extremely dangerous failures. Therefore, it is important to be able to properly assess the strength of these buildings to ensure a level of safety for the public.

The overarching finding of this research is that a major source of strength loss for plain walls and walls with buttresses is the occurrence of a vertical shear plane within the wall. This strength loss is more than 50% in plain walls, and slenderness effects are magnified. In practice, a vertical shear plane can occur within a wall when the core is significantly deteriorated. This mortar/core deterioration has also been noted to many damaged masonry buildings in earthquakes.

This research has shown that a significant strength increase can be obtained by ensuring that the walls are properly constructed and the cores are consolidated. In the strength assessment of stone walls there should be an increased focus on the construction practices and maintenance to ensure that the walls are as strong as they can be prior to looking into additional strengthening procedures.

Regarding the analysis of buttresses, while there is no specific inclusion in the CSA Masonry Standard for buttresses, the methodology prescribed for the determining the out-of-plane strength of walls provides good correlation with the buttress modelling results (CSA, 2014). This method of limiting tension is similar to the historic line of thrust method for analyzing buttresses, with the addition of slenderness effects. The results of the modelling show that slen-

derness effects need to be considered, and including slenderness effects at all slenderness ratios is good practice instead of solely including them above a certain slenderness ratio.

The shortfall of the CSA methodology is the lack of guidance in the standard to assist designers in quantifying the stiffness of stone walls without having to rely on testing. It should be a priority to include acceptable design stiffnesses for various masonry walls, accounting for the construction typology (ie. ashlar or rubble stone) to assist designers and standardize this practice across the country.

The historic method of approximating the strength of walls or buttresses by the rigid overturning moment significantly overestimates the strength of the walls, especially for the more slender walls. This method is not recommended unless it is used as a quick preliminary check, including a conservative safety factor of 2.

Including wall flanges for buttresses significantly affects the overturning strength of the wall. However, further research is required to confirm that the mass of the wall is mobilized alongside the buttress to take advantage of this increase in strength. Neglecting the wall flanges is an extremely conservative approach.

Limitations to this research include the limited testing of stone masonry walls, especially walls with buttresses. The Canadian stone wall typology including core material should be further tested, including the effects of varying core stone size and number of key stones since this was shown to have a significant impact on the overall response of the modelled walls. The basic wall was able to be compared to the existing experimental data, with good correlation, but the buttress modelling was an extrapolation of this data and should be confirmed with experimental testing.

Suggested future research in this field focusses mainly on testing. Laboratory testing and in situ testing on the specific typologies of Canadian stone walls can provide valuable input for the expected stiffnesses of the local structures. This testing could include walls with and without buttresses, to confirm the modelling data.

Finally, it would be beneficial for plain walls to be tested with and without deterioration, with the various wythe connectors as were tested in the West Block experiments (See Section 3.4.1). During these previous experiments there were no differences noted in wall behaviour

between the basic walls, walls with headers, and walls with mechanical connectors. It is hypothesized that this consistent response is due to the excellent construction of these walls, with properly built cores and solid mortar, so the walls did not need to rely on the additional connectors. These mechanical wythe connectors are being added to many structures across Canada, yet these experiments showed that there was no difference in the overall behaviour of walls in good condition and the long-term effects of having modern materials within the wall is unknown. The suggested experiments would determine whether these mechanical connectors are sufficient to hold the wythes together once the wall begins to deteriorate. The effects of mechanical connectors could also then be compared to a proper maintenance program.

Glossary

The term *masonry* has developed and changed over the centuries but originally referred to stone and mass brick constructions. In the 19th century when the use of concrete was just beginning, concrete was actually used by engineers following the same principles as stone constructions. Original uses for concrete were massive, monolithic sections. These were expensive, considering the cost of the cement. Later unreinforced concrete constructions had large rubble stones dropped into the formwork to increase the volume at a low cost, referred to as rubble concrete. Designers at the time used the geometries and empirical formulas from stone masonry design and applied them to these unreinforced concrete structures. With the advancement of reinforced concrete and reinforced concrete masonry unit design (CMU), the distinction had to be made between unreinforced and reinforced masonry. In modern engineering mass stone, mass brick, terra cotta, adobe, veneers, cinder blocks, concrete masonry units, and unreinforced concrete are all collected under the term *unreinforced masonry* (URM).

This non-specific terminology has created issues with the analysis of these structures. Every single one of the above-mentioned materials behaves differently, yet in Canada they are all covered under a single design code, the *Design of Masonry Structures* CSA (2014).

The focus of this study is on the *mass masonry* subgroup of URM, namely mass stone construction. While many research paper and studies typically use the URM terminology, it is the decision of the author to use the term mass masonry. Applying the unreinforced masonry label to stone constructions indicates that the design is lacking steel, as if including reinforcing steel would have improved the design, which is inaccurate.

The following Glossary includes only the terminologies which are used within this document, and is not an exhaustive list of all stone industry terms. The sources for the definitions

include the Standards and Guidelines for the Conservation of Historic Places in Canada, the Mason Contractors Association of America, and the Marble Institute of America.

<i>abutment</i>	A solid stone “springer” at the lowest point of an arch or vault. See <i>buttress</i> .
<i>anchor</i>	A corrosion resistant metal fastener used for securing dimension stone to a structure or adjacent stone units.
<i>anisotropic</i>	Having properties, either visual or mechanical, that differ based on the direction in which they are measured. All stones are anisotropic to some degree, but the sedimentary stones typically have the greatest degree of anisotropy.
<i>arch</i>	The curved or pointed construction over a doorway or opening which supports its own weight as well as the weight of the masonry work above it.
<i>ashlar</i>	A stone façade of generally square or rectangular units having sawed or dressed beds.
<i>bed</i>	The top or bottom of a joint, see <i>bedding plane</i> . Faced bedded stones are placed such that the natural horizontal surface forms the vertical face of the work. Edge bedded stones are placed such that the natural horizontal layers are placed perpendicular to the face of the work.
<i>bed joints</i>	Horizontal mortar joint on which masonry is placed.
<i>bedding plane</i>	Plane of sedimentary stone in the position of its original formation. This plane may be horizontal, coincident with mountain slopes, or random.
<i>bond</i>	Pattern of joints in successive courses.
<i>bond stone</i>	Stone or masonry unit that projects back from the facing wall into a backup wall. A bond stone is designed to tie the two wythes together. A bond stone may or may not project completely through the two wythes.
<i>buttress</i>	Any external prop or support built to steady a structure by opposing its outward thrusts, a projecting support built into or against the outside of a masonry wall.
<i>cantilever</i>	A structural member supported at only one end.
<i>cap, capital</i>	The culminating stone at the top of a column or pilaster, often carved.
<i>course</i>	A horizontal range of stone units the length of a wall.

<i>cramp</i>	A U-shaped metal anchor used to hold two adjacent units of stone together.
<i>cut stone</i>	Stone that has been fabricated to a finished size and configuration and ready to be installed.
<i>dressings</i>	The shaping and squaring of natural stone blocks for storage and shipment.
<i>dry stack</i>	In rubble masonry construction, a self-supporting wall erected without mortar.
<i>face</i>	The exposed surface of a wall. Also the surface of a masonry unit to be exposed in finished work.
<i>grout</i>	A mixture of cementitious material and water, with or without sand, proportioned to produce a plastic consistency without segregation of the constituents.
<i>head joints</i>	The vertical mortar joint between ends of masonry units. Often called cross joint.
<i>in-situ</i>	This term means 'in place' and refers to the existing materials in the location where they were found.
<i>jamb</i>	Vertical sides of an opening such as the side of a door or window.
<i>joint</i>	Joints are the spaces between stone, bricks, concrete blocks, or glass blocks that are filled with mortar or grout. Mortar joints can be made in a series of different fashions.
<i>lime</i>	A white caustic alkaline substance consisting of calcium oxide, made by adding water to quicklime and used in traditional building methods to make plaster, mortar, and limewash.
<i>lime mortar</i>	Composed of lime and an aggregate such as sand, mixed with water. The soft and porous properties provide certain advantages when working with softer building materials such as natural stone and terracotta.
<i>lintel</i>	A horizontal stone spanning over the opening of a door, window, or other opening that acts as a beam carrying the weight of the wall above it. See also <i>arch</i> .

<i>masonry</i>	That which is built by a mason; anything constructed of the materials used by masons, such as stone, brick, tiles, or the like.
<i>monolithic</i>	Shaped from a single block of stone, in contrast to a unit that was created by using multiple units of stone.
<i>mortar</i>	A mixture of a binder and fine aggregate used in setting stone units or filling joints between stone units. The binder may consist of masonry cement, or hydraulic cement with lime (and possibly other admixtures).
<i>paring</i>	Process of applying a coat of mortar to masonry construction, especially used for masonry walls. Also, the cement mortar coat itself.
<i>pilaster</i>	A shallow, engaged pier or column projecting from a wall.
<i>plumb</i>	Exactly vertical. Measured with a plumb line.
<i>porous</i>	Materials ability to absorb water having many small openings.
<i>rubble</i>	Dimension stone consisting of units that may be highly irregular or partly trimmed or squared.
<i>setting</i>	The installation of dimension stone units.
<i>stereotomy</i>	The art and science of cutting stones into particular shapes.
<i>stretcher</i>	A stone whose greatest dimension lies parallel to the face of the wall.
<i>veneer</i>	A non-structural facing of masonry, interior or exterior, serving as ornamentation and a weather barrier.
<i>wythe</i>	A continuous vertical section of masonry one unit in thickness. A wythe may be independent of, or interlocked with, the adjoining wythe.

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Appendices

Appendix A: Reference Tables

A.1 Modern National Standard References

Table A.1: Physical Requirements of Building Stone, from the Canadian Masonry Standard, (CSA, 2014). (Table 3.8, pg. 154)

Group of Stone	Maximum Absorption by Weight, %	Minimum Density, kg/m ³	Minimum Compressive Strength, MPa	Minimum Modulus of Rupture, MPa	ASTM Standard
Limestone	12	1760	12	2.9	C568
	7.5	2160	28	3.4	
	3	2560	55	6.9	
Sandstone	8	2000	27.6	2.4	C616
	3	2400	68.9	6.9	
	1	2560	137.9	13.9	
Granite	0.4	2560	131	10.4	C615
Marble	0.2	2300-2800	52	7.0	C503

Table A.2: Default lower-bound masonry properties, from ASCE 41-06, 2006.

Property	Masonry Condition		
	Good	Fair	Poor
Compressive Strength (MPa)	6.21	4.41	2.07
Young's modulus	$550f_c$	$550f_c$	$550f_c$
Flexural tensile strength (MPa)	0.14	0.07	0.00
Shear strength (MPa) Masonry with a running bond	0.19	0.14	0.09
Fully grouted masonry other than running bond	0.19	0.14	0.09
Partially grouted or ungrouted masonry other than running bond	0.08	0.06	0.03

Table A.3: Italian Building Code: mechanical properties of masonry and multiplication factors, (Borri et al., 2018), (IMIT, 2009).

	Comp Strength (MPa)		Shear Strength (MPa)		Young's Modulus (MPa)		Shear Modulus (MPa)		Mulitplication Factors		
	Min	Max	Min	Max	Min	Max	Min	Max	A	B	C
Irregular stone	1.0	2.0	.018	.032	690	1050	230	350	1.5	1.3	1.5
Roughly cut	2.0	3.0	.035	.051	1020	1440	340	480	1.4	1.2	1.5
Cut and dressed	2.6	3.8	.056	.074	1500	1980	500	660	1.3	1.1	1.3
Irregular softstone	1.4	2.2	.028	.042	900	1260	300	420	1.5	1.2	1.3
Squared softstone	2.0	3.2	.040	.080	1200	1620	400	500	1.6		1.2
Squared hardstone	5.8	8.2	.090	.120	2400	3300	800	1100	1.2		1.2
Brickwork (lime)	2.6	4.3	.050	.130	1200	1800	400	600	1.27		1.3
Hollow brick	5.0	8.0	.080	.170	2500	5600	875	1400	1.2		

Where Multiplication Factor A is for good quality mortar, B is if there are brick courses tying the wythes together, and C is if there are other transverse connections (header stones, tie rods, etc.).

A.2 Historic References

Table A.4: Weight of Building Stones, (Trautwine, 1872).

Kind of Stone	Pounds per Cubic Foot		
	Min.	Max.	Mean.
Granites	161	178	167
Limestones	146	174	158
Marbles	157	180	170
Sandstones	127	151	139
Slates	160	175	174

Table A.5: Allowable Stresses in Stonework, from (Baker, 1889).

	Masonry Description	tons per square foot
<u>Examples:</u>		
Piers of the large churches in Europe	Monolithic, varies	<30
Pont-y-Prydd 140 ft. span bridge, Wales (1750)	Rubble limestone, lime mortar	20.7
FAILED former bridge, same location	Unknown	64
Column 4 ft square, tested by Rennie	Rubble limestone	22
Saltash Bridge piers	Granite	9
Brooklyn Bridge towers	Granite	28.5
Brooklyn Bridge	Limestone	10
Rookery Building columns 3 ft square, Chicago	Granite	30
Washington Monument	Marble	25.4*
St. Louis Bridge piers, before completion	Limestone	38
St. Louis Bridge piers, after completion	Limestone	19
FAILED Niagara Suspension Bridge towers	Limestone	36 [†]
South Street Bridge, Philadelphia	Rubble limestone	15.7
Quaker Bridge Dam, design value	Rubble, hydraulic cement mortar	16.67
<u>Recommended values:</u>		
Concrete		5 to 15
Rubble		10 to 15
Squared stone		15 to 20
Limestone ashlar		20 to 25
Granite ashlar		30

* Includes wind load.

[†] It was noted that the masonry was not well executed.

Table A.6: Allowable Stresses in Stonework, from (Kidder, 1886).

Stonework	Tons per square foot	MPa
Rubble walls, irregular stones	3	0.29
Rubble walls, irregular stones	2 1/2	0.24
Rubble walls, irregular stones	5 to 16	0.48 to 1.54
Dimension-stone, square, in cement mortar:		
Sandstone and limestone	10 to 20	0.96 to 1.92
Granite	20 to 40	1.92 to 3.84
Dressed stone, with 3/4" dressed joints, in Portland-cement mortar:		
Granite	60	5.76
Marble or limestone, best	40	3.84
Sandstone	30	2.88

The height of the columns should not exceed eight times the least diameter, unless the least diameter is sufficiently greater than necessary for the strength of the material used.

Table A.7: Crushing Strength of Cubes of Stone, from (Trautwine, 1872).

Kind of Stone	Ultimate Crushing Strength			
	Pounds per Square Inch		Tons per Square Foot	
	Min.	Max.	Min.	Max.
Trap Rocks of N.J.	20,000	24,000	1,440	1,730
Granite	12,000	21,000	860	1,150
Marble	8,000	20,000	580	1,440
Limestone	7,000	20,000	500	1,440
Sandstone	5,000	15,000	360	1,080

Table A.8: Crushing Strength of Masonry, from (Kidder, 1886).

Kind of stone	Crushing strength, lb per sq in
Granites:	
Blue, Fox Island, Me.	14 875
Gray, Vinal Haven, Me.	13 000-18 000
Westerly, R. I.	15 000
Rockport and Quincy, Mass.	17 750
Milford, Conn.	22 600
Staten Island, N. Y.	22 250
East St. Vloud, Minn.	28 000
Gunnison, Col.	13 000
Red, Platte Canon, Col.	14 875
Limestones:	
Glens Falls, N. Y.	11 475
Joliet, Ill.	12 775
Bedford, Ind.	6 000-10 000
Salem, Ind.	8 625
Red Wing, Minn.	23 000
Stillwater, Minn.	10 750
Sandstones:	
Dorchester, N. B. (brown)	9 150
Mary's Point, N. B. (dark brown)	7 700
Connecticut brown stone	7 000-13 000
Longmeadow, Mass. (reddish brown)	7 000-14 000
Longmeadow, Mass.	12 000
Little Falls, N. Y.	9 850
Medina, N. Y.	17 000
Potsdam, N. Y. (red)	18 000-42 000
Cleveland, Ohio	6 800
North Amherst, Ohio	6 212
Berea, Ohio	8 000-10 000
Hummelstown, Pa.	12 810
Fond du Lac, Minn.	8 750
Fond du Lac, Wis.	6 237
Manitou, Col. (light red)	6 000-11 000
St. Vrain, Col. (hard laminated)	11 505
Marbles:	
Lee, Mass.	22 900
Rutland, Vt.	10 746
Montgomery Co., Pa.	10 000
Colton, Cal.	17 783
Italy	12 156

Table A.9: Coefficient of Elasticity of Stone, from (Trautwine, 1872).

Material	Pounds per Square Inch
Haverstraw Freestone *	950,000
Portland Stone (oolite limestone)†	1,530,000
Marble†	2,500,000
Portland Granite‡	5,500,000
Slate†	7,000,000
Grafton Limestone‡	8,000,000
Richmond Granite‡	13,000,000

* U.S. testing-machine, Watertown, Mass.

† Tredgold, as quoted by Stoney.

‡ History of St. Louis Bridge, pp. 324-28.

Table A.10: Comparison of Building Laws, from (Kidder, 1886).

Materials	Allowable pressures in tons per sq ft						
	Boston, 1915	Buffalo, 1909	New York, 1917	Chicago, 1916	St. Louis, 1907	Philadelphia, 1914	Denver, 1898
Granite, cut	60-72		72	43			40
Marble and limestone, cut	40		43-50	29			
Sandstone, hard cut	30		29	29			12
Hard-burned brick in Portland-cement mortar	18-20	12	18	12.5	21.5		40
Hard-burned brick in natural-cement mortar		9	15	10.5		15	9
Hard-burned brick in cement-and-lime mortar	12-14		11.5		11.5	12	
Hard-burned brick in lime mortar	6-8	6	8	7	11	8	8
Pressed brick in Portland-cement mortar		12		18			
Pressed brick in natural-cement mortar		9					12
Rubble stone in natural-cement mortar		5	8			10	12

A.3 Coefficients of Friction for Masonry

Table A.11: Coefficients of Friction for Masonry, from (Baker, 1889).

Description of the Masonry	Coefficient
Soft limestone on soft limestone, both well dressed	0.75
Brick-work on brick-work, with slightly damp mortar	0.75
Hard brick-work on hard brick-work, with slightly damp mortar	0.70
Point-dressed granite on like granite	0.70
Point-dressed granite on well-dressed granite	0.65
Common brick on common brick	0.65
Common brick on hard limestone	0.65
Hard limestone on hard limestone, with moist mortar	0.65
Concrete blocks on like concrete blocks	0.65
Fine cut granite on pressed concrete blocks	0.60
Well-dressed granite on well-dressed granite	0.60
Polished limestone on polished limestone	0.60
Well-dressed granite on like granite, with fresh mortar	0.50
Common brick on common brick, with wet mortar	0.50
Polished marble on common brick	0.45
Point-dressed granite on gravel	0.60
Point-dressed granite on dry clay	0.50
Point-dressed granite on sand	0.40
Point-dressed granite on moist clay	0.33
Well-dressed limestone on wrought iron	0.50
Well-dressed limestone on wrought iron, wet	0.25
Limestone on oak, flatwise	0.65
Limestone on oak, endwise	0.40

Table A.12: Coefficients and Angles of Friction, from (Kidder, 1886).

Kind of surface	Coefficient of friction, $\tan\phi$
Granite, limestone and marble:	
Soft dressed upon soft dressed	0.70
Hard dressed upon hard dressed	0.55
Hard dressed upon soft dressed	0.65
Stone, brick or concrete:	
Masonry upon masonry	0.65
Masonry upon wood (with the grain)	0.60
Masonry upon wood (across the grain)	0.50
Masonry upon dry clay	0.50
Masonry upon wet or moist clay	0.33
Masonry upon sand	0.40
Masonry upon gravel	0.60
Soft stone upon steel or iron	0.40
Hard stone upon steel or iron	0.30

Appendix B: Software Output - Buttresses

B.1 Keyed Buttresses

B.1.1 Varying Buttress Height

Table B.1: Buttress Model Response Varying Height

4800 mm

$$M_{OV,r,keyed} = 46.77 \text{ kN.m}$$

$$M_{OV,r,sep} = 23.28 \text{ kN.m}$$

$$M_{max,COG} = 32.60 \text{ kN.m}$$

$$M_{max,top} = 31.89 \text{ kN.m}$$

5400 mm

$$M_{OV,r,keyed} = 52.62 \text{ kN.m}$$

$$M_{OV,r,sep} = 26.31 \text{ kN.m}$$

$$M_{max,COG} = 39.98 \text{ kN.m}$$

$$M_{max,top} = 33.10 \text{ kN.m}$$

6000 mm

$$M_{OV,r,keyed} = 39.25 \text{ kN.m}$$

$$M_{OV,r,sep} = 29.23 \text{ kN.m}$$

$$M_{max,COG} = 44.70 \text{ kN.m}$$

$$M_{max,top} = 39.25 \text{ kN.m}$$

6600 mm

$$M_{OV,r,keyed} = 64.31 \text{ kN.m}$$

$$M_{OV,r,sep} = 32.15 \text{ kN.m}$$

$$M_{max,COG} = 44.73 \text{ kN.m}$$

$$M_{max,top} = 42.91 \text{ kN.m}$$

7200 mm

$$M_{OV,r,keyed} = 70.15 \text{ kN.m}$$

$$M_{OV,r,sep} = 35.08 \text{ kN.m}$$

$$M_{max,COG} = 53.59 \text{ kN.m}$$

$$M_{max,top} = 46.35 \text{ kN.m}$$

7800 mm

$$M_{OV,r,keyed} = 76.00 \text{ kN.m}$$

$$M_{OV,r,sep} = 38.00 \text{ kN.m}$$

$$M_{max,COG} = 58.11 \text{ kN.m}$$

$$M_{max,top} = 48.09 \text{ kN.m}$$

8400 mm

$$M_{OV,r,keyed} = 81.85 \text{ kN.m}$$

$$M_{OV,r,sep} = 40.93 \text{ kN.m}$$

$$M_{max,COG} = 62.07 \text{ kN.m}$$

$$M_{max,top} = 53.18 \text{ kN.m}$$

9000 mm

$$M_{OV,r,keyed} = 87.69 \text{ kN.m}$$

$$M_{OV,r,sep} = 43.85 \text{ kN.m}$$

$$M_{max,COG} = 64.49 \text{ kN.m}$$

$$M_{max,top} = 56.15 \text{ kN.m}$$

9600 mm

$$M_{OV,r,keyed} = 93.54 \text{ kN.m}$$

$$M_{OV,r,sep} = 46.77 \text{ kN.m}$$

$$M_{max,COG} = 64.59 \text{ kN.m}$$

$$M_{max,top} = 59.78 \text{ kN.m}$$

B.2 Disconnected Buttresses

B.2.1 Varying Wall and Buttress Height

Table B.2: Buttress Model Response Varying Height

B.2.2 Varying Buttress Depth

Table B.3: Buttress Model Response Varying Depth

6600 mm Buttresses

Depth = 300 mm

$$M_{OV,rigid} = 52.25 \text{ kN.m}$$

$$M_{OV,elastic} = 50.60 \text{ kN.m}$$

$$M_{max,COG} = 47.03 \text{ kN.m}$$

$$M_{max,top} = 35.09 \text{ kN.m}$$

Depth = 600 mm

$$M_{OV,rigid} = 64.31 \text{ kN.m}$$

$$M_{OV,elastic} = 62.49 \text{ kN.m}$$

$$M_{max,COG} = 59.46 \text{ kN.m}$$

$$M_{max,top} = 46.15 \text{ kN.m}$$

Depth = 900 mm

$$M_{OV,rigid} = 84.40 \text{ kN.m}$$

$$M_{OV,elastic} = 82.71 \text{ kN.m}$$

$$M_{max,COG} = 83.71 \text{ kN.m}$$

$$M_{max,top} = 63.89 \text{ kN.m}$$

9600 mm Buttresses

Depth = 300 mm

$$M_{OV,rigid} = 76.00 \text{ kN.m}$$

$$M_{OV,elastic} = 68.84 \text{ kN.m}$$

$$M_{max,COG} = 58.07 \text{ kN.m}$$

$$M_{max,top} = 40.88 \text{ kN.m}$$

Depth = 600 mm

$$M_{OV,rigid} = 93.54 \text{ kN.m}$$

$$M_{OV,elastic} = 85.67 \text{ kN.m}$$

$$M_{max,COG} = 72.95 \text{ kN.m}$$

$$M_{max,top} = 52.72 \text{ kN.m}$$

Depth = 900 mm

$$M_{OV,rigid} = 122.8 \text{ kN.m}$$

$$M_{OV,elastic} = 115.5 \text{ kN.m}$$

$$M_{max,COG} = 101.2 \text{ kN.m}$$

$$M_{max,top} = 77.09 \text{ kN.m}$$

B.2.3 Varying Wall Width

Table B.4: Buttress Model Response Varying Wall Width

6600 mm Buttresses

Width = 600 mm

$$M_{OV,rigid} = 32.15 \text{ kN.m}$$

$$M_{OV,elastic} = 31.25 \text{ kN.m}$$

$$M_{max,COG} = 29.70 \text{ kN.m}$$

$$M_{max,top} = 23.14 \text{ kN.m}$$

Width = 1800 mm

$$M_{OV,rigid} = 64.31 \text{ kN.m}$$

$$M_{OV,elastic} = 62.49 \text{ kN.m}$$

$$M_{max,COG} = 59.46 \text{ kN.m}$$

$$M_{max,top} = 46.15 \text{ kN.m}$$

9600 mm Buttresses

Width = 600 mm

$$M_{OV,rigid} = 46.77 \text{ kN.m}$$

$$M_{OV,elastic} = 42.83 \text{ kN.m}$$

$$M_{max,COG} = 36.10 \text{ kN.m}$$

$$M_{max,top} = 26.49 \text{ kN.m}$$

Width = 1800 mm

$$M_{OV,rigid} = 93.54 \text{ kN.m}$$

$$M_{OV,elastic} = 85.67 \text{ kN.m}$$

$$M_{max,COG} = 72.95 \text{ kN.m}$$

$$M_{max,top} = 52.72 \text{ kN.m}$$