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NAME OF SUPERVISOR/NOM DU DIRECTEUR DE THÈSE M.K. Sundaresan

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REVIEW OF THE CURRENT STATUS OF THE WEINBERG-SALAM MODEL
OF WEAK AND ELECTROMAGNETIC INTERACTIONS

by

KATE DICKINSON

A thesis submitted to the Faculty of Graduate Studies
in partial fulfillment of the requirements for the
degree of Master of Science.

Carleton University .

Ottawa, Canada

May 1978

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The undersigned recommend to the Faculty of
Graduate Studies acceptance of the thesis "Review
of the Current Status of the Weinberg-Salam Model of
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Kate Dickinson, in partial fulfillment of the requirements
for the degree of Master of Science.

.....M. K. Sundaresan.....

Thesis Supervisor

.....John Kember.....
for Chairman, Department of Physics

Carleton University

ABSTRACT

The predictions of the Weinberg-Salam model of weak and electromagnetic interactions are compared with recent experimental developments. Although it has had considerable success, the model is found to be incompatible with certain experimental results. The possibility of extending the Weinberg-Salam model within the $SU(2) \otimes U(1)$ gauge group is investigated, but none of the suggested alternatives satisfy all the present experimental and theoretical constraints. Two models based on larger gauge groups are examined; one of these, a left-right symmetric $SU(2) \otimes SU(2) \otimes U(1)$ model, is able to explain all the present weak interaction data.

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INTRODUCTION

The first unified theory of weak and electromagnetic interactions was proposed in 1967 by S. Weinberg¹ and independently by A. Salam.² The model is based on the gauge group $SU(2) \otimes U(1)$,³ and was later expanded, for experimental and theoretical reasons, to incorporate a fourth quark: the charmed quark.⁴ This later version will be referred to simply as the Weinberg-Salam (or W-S) model. Quarks and leptons in this model are assigned to left-handed doublets and right-handed singlets of weak isospin as follows:

$$\begin{pmatrix} u \\ d_c \end{pmatrix}_L, \begin{pmatrix} c \\ s_c \end{pmatrix}_L, \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, u_R, d_R, c_R, s_R, e^-_R, \mu^-_R$$

where d_c and s_c are the Cabibbo rotated states:

$$d_c = d \cos \theta_c + s \sin \theta_c, s_c = -d \sin \theta_c + s \cos \theta_c$$

and θ_c is the Cabibbo angle.

The single parameter of the model is the $SU(2)$ and $U(1)$ mixing angle, known as the Weinberg angle, θ_w . The neutrinos have no right-handed components and are, therefore, massless. A consequence of this exact mass degeneracy is that a Cabibbo-type rotation in the lepton sector has no physical meaning. This ensures that μ - and e -lepton numbers are separately conserved. In fact, muon number, electron number and CP are automatically conserved by all mass terms and gauge interaction terms in the

Lagrangian.³

The discovery of weak interactions mediated by neutral currents in 1973⁵ and of charm in 1974⁶ have provided support for the gross features of the W-S model. The aim of the present work is to examine this model in the light of the many recent experimental developments and attempt a critical review of its current status as a viable model of the weak and electromagnetic interactions. Thus, the thesis is basically one of an expository nature. The results of the study are presented in a number of chapters. The presentation of the significant experimental results and their comparison with the W-S model predictions occupy the first three chapters. In Chapter I, the neutrino scattering data obtained since 1975 are discussed. The two main categories are (anti) neutrino-electron elastic scattering and muon (anti) neutrino-nucleon scattering. The latter category is divided into elastic and deep-inelastic inclusive processes. The formalism is outlined: derivations of the cross-section formulae can be found in Appendix A. The evidence for new quarks and leptons is discussed in Chapter II. Chapter III describes the experiments searching for parity-violating effects due to neutral current weak interactions: at the time of writing, results are available from the heavy atom experiments only.

It is found that the W-S model is compatible with the neutral current neutrino scattering data for an average value of $\sin^2 \theta_W \approx 0.33$, where θ_W is the Weinberg angle. It predicts

the presently accepted charged current neutrino-nucleon results with the possible exception of the observed rate for events with three muons in the final state. The W-S model is not consistent with the evidence presented in Chapter II, or with the interpretation of the data from the atomic physics experiments. In Chapter IV, I have proposed various alternative $SU(2) \otimes U(1)$ models in an attempt to satisfy all of the present experimental and theoretical constraints. None of these models is completely successful. The existing constraints would therefore imply that $SU(2) \otimes U(1)$ will have to be abandoned in favour of a larger gauge group. Two examples are examined: an ambidextrous $SU(2)_L \otimes SU(2)_R \otimes U(1)$ model and a model based on $SU(3) \otimes U(1)$. The former is found to fit the data rather well. Chapter IV concludes with a summary of the present situation and a description of the important areas for further experimental study.

CHAPTER I

NEUTRINO AND ANTINEUTRINO SCATTERING

Elastic (Anti) Neutrino-Electron Scattering

This process involves the weak interaction alone, and so, although very difficult to measure, is amenable to unambiguous theoretical analysis. In any $SU(2) \otimes U(1)$ gauge model, the differential cross sections involve two parameters, the vector and axial-vector coupling constants g_V and g_A :

$$\frac{d\sigma}{dE_e}(\nu_\mu, \bar{\nu}_\mu) = \frac{G_F^2 m_e}{2\pi} [(g_V \pm g_A)^2 + (g_V \mp g_A)^2 (1 - \frac{E_e}{E_\nu})^2 - \frac{m_e E_e}{E_\nu^2} (g_V^2 - g_A^2)]$$

where E_ν (E_e) is the energy of the incident neutrino (recoil electron) and m_e is the electron mass. Muon neutrino scattering is a purely neutral current process, but for electron neutrinos, charged as well as neutral currents contribute and $g_V(g_A)$ is replaced by $G_V(G_A)$ where, assuming $\mu - e$ universality:

$$G_V = 1 + g_V$$

$$G_A = 1 + g_A$$

In the Weinberg-Salam model:

$$g_V = -\frac{1}{2} + 2\sin^2\theta_w$$

$$g_A = -\frac{1}{2}$$

θ_w being the Weinberg angle.

If the electron coupling is pure vector (V) or pure axial vector (A), $\nu - e$ scattering will conserve parity. In $SU(2) \otimes U(1)$, neutrino and anti-neutrino cross sections will then be equal. In particular, for $\nu_\mu - e$ scattering, either $g_A = 0$ (pure V) or $g_V = 0$ (pure A). The former situation arises in those $SU(2) \otimes U(1)$ models in which the electron's left- and right-handed weak isospin assignments are the same. Parity conservation would be expected in the Weinberg-Salam model only if $\sin^2 \theta_w = 0.25$, in which case $g_V = 0$. Other significant values of the mixing angle are $\sin^2 \theta_w = 0$ (pure V - A) and $\sin^2 \theta_w = 0.5$ (pure V + A).

Experimental data exist for the first three of the following processes:

$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$$

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$

$$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$$

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

The data for $\bar{\nu}_e - e$ scattering, although not incompatible with V - A, favour the Weinberg-Salam model with $0.20 \leq \sin^2 \theta_w \leq 0.37$.² During 1975, data were acquired on $\nu_\mu - e$ and $\bar{\nu}_\mu - e$ total scattering cross sections by the Aachen-Padova collaboration at CERN. The recent analysis of their results³⁻⁵ together with those from Gargamelle on $\bar{\nu}_\mu - e$

scattering⁶ are shown in Figure 1 which has been taken from Reference 5. The conclusions are:

- Pure V - A is excluded at the 3 standard deviation level.

- Pure V + A is admitted.

- A pure V or pure A parity conserving interaction, although not definitely excluded, has a probability of only about 2%.

- The favoured solution, within $SU(2) \otimes U(1)$, is the Weinberg-Salam model with $\sin^2 \theta_W = 0.38 \pm 0.10$.

- $SU(2) \otimes U(1)$ with a Higgs boson with isospin $I_\phi > 1/2$ is excluded with $> 99\%$ confidence.⁵

Taking $I_\phi = 1/2$, Figure 2 (taken from Reference 5) gives g_V and g_A for different values of I_3^R and $\sin^2 \theta_W$, I_3^R being the third component of the right-handed electron's weak isospin. The shaded regions are those allowed by total $\nu - e$ scattering cross sections. The data admit two values of I_3^R : the Weinberg-Salam assignment $I_3^R = 0$ which is favoured, and the improbable pure V solution $I_3^R = -1/2$.

To retain a parity conserving electron coupling and be consistent with $\nu - \bar{\nu}$ differences, it is necessary to consider gauge groups larger than $SU(2) \otimes U(1)$. This is discussed further in Chapter IV.

Figure 1

ν_e and $\bar{\nu}_e$ total cross sections, as measured by Aachen-Padova and Gargamelle, from Ref. 5. Straight lines delineate simple V, A interactions, the curve is for the W-S model.

$$I_\phi = 1/2$$

★ = Aachen-Padova solution

Figure 2 Bounds on V and A coupling constants provided by $\nu_\mu e$, $\bar{\nu}_\mu e$ and $\bar{\nu}_e e$ total cross sections, from Ref. 5. Vertical bars indicate allowed range of $\sin^2 \theta_W$ for different I_3 assignments to right-handed electron, with Higgs isospin $I_\phi = 1/2$. Ratio of electron energies admits sectors around $V + A$ axis (All errors: 1 s.d.)

(Anti) Neutrino-Nucleon Scattering

Formalism

The following expressions summarize those derived in Appendix A.

Elastic Scattering:

The differential cross section for the elastic and quasi-elastic scattering of neutrinos by nucleons can be given in terms of three form factors:

$$\frac{d\sigma}{dQ^2}(\nu, \bar{\nu}) = \frac{G_F^2}{32\pi M^2 E^2} \{ Q^2 [8M^2 W_1 - (Q^2 + 4M^2) W_2] + (Q^2 - 4ME)^2 W_2 \pm 2Q^2(Q^2 - 4ME) W_3 \}$$

having omitted second-class currents and terms involving the mass of the scattered lepton. E is the incident neutrino energy, and M is the target nucleon mass. The right-hand side is multiplied by $\cos^2 \theta_c$ in the case of quasi-elastic (i.e. charged current) scattering, θ_c being the Cabibbo angle. The form factors W_1 , W_2 and W_3 are functions of $Q^2 = -q^2 \geq 0$ only. For quasi-elastic scattering they can be related, by CVC, to the form factors G_E , G_M and G_A which are themselves conveniently described by dipole form. For elastic (i.e. neutral current) scattering, W_1 , W_2 and W_3 are assumed to be similarly related to the form factors G_E^0 , G_M^0 and G_A^0 which are taken to be proportional to the charged current form factors:

$$G_E^0(Q^2) = \frac{1}{2} (\alpha + \gamma) G_E(Q^2)$$

$$G_M^0(Q^2) = \frac{1}{2} (\alpha + \frac{0.88}{4.7} \gamma) G_M(Q^2)$$

$$G_A^0(Q^2) = \frac{1}{2} (\beta + \delta) G_A(Q^2)$$

α , β , γ and δ are model dependent parameters defined in Appendix A. In the Weinberg-Salam model:

$$\alpha = 1 - 2\sin^2\theta_w$$

$$\beta = 1$$

$$\gamma = -2\sin^2\theta_w$$

$$\delta = 0$$

The processes of practical interest are the following:

$$\nu_\mu + p \rightarrow \nu_\mu + p$$

$$\bar{\nu}_\mu + p \rightarrow \bar{\nu}_\mu + p$$

$$\nu_\mu + n \rightarrow \mu^- + p$$

$$\bar{\nu}_\mu + p \rightarrow \mu^+ + n$$

therefore, $(\bar{\nu})$ ν will hereafter indicate a muon (anti) neutrino.

Deep-inelastic Inclusive Scattering:

At high energy and momentum transfer, scaling is expected and the differential cross sections can be written in terms of the scaling variables, $x = Q^2/2M\nu$, $y = \nu/E$, where $\nu = E - E'$ is the energy transferred to the hadron system in the lab frame:

$$\frac{d^2\sigma(\nu, \bar{\nu})}{dx dy} = \frac{G_F^2 ME}{\pi} \{ y^2 x F_1(\nu, \bar{\nu}) N + (1 - y) F_2(\nu, \bar{\nu}) N + (1 - y/2) y x F_3(\nu, \bar{\nu}) N \}$$

For a given interaction and in the scaling region, the structure functions F_1 , F_2 and F_3 depend only on x . The main predictions of scaling are that the cross sections rise linearly with (anti) neutrino energy and that the x and y dependences are independent of (anti) neutrino energy. In the quark parton model scaling follows naturally since the incident neutrino is viewed as scattering elastically off free quarks with negligible masses, and x is identified with the fractional nucleon momentum carried by the initial quark in the infinite momentum-frame. The following relations are obtained for the structure functions:

$$2xF_1 = F_2$$

$$F_2 = q(x) + \bar{q}(x)$$

$$-xF_3 = q(x) - \bar{q}(x)$$

where $q(x)$ and $\bar{q}(x)$ are the fractional nucleon momenta carried

by left- and right-handed quarks respectively. The differential cross sections become:

$$\frac{d^2\sigma^{\nu}}{dx dy} = \frac{G_F^2 ME}{\pi} \{q(x) + \bar{q}(x) (1-y)^2\}$$

$$\frac{d^2\sigma^{\bar{\nu}}}{dx dy} = \frac{G_F^2 ME}{\pi} \{q(x) (1-y)^2 + \bar{q}(x)\}$$

Specific expressions for neutral or charged current inclusive scattering by proton (p), neutron (n) or isoscalar ($\frac{1}{2}(p+n)$) targets are obtained by writing $q(x)$ and $\bar{q}(x)$ in terms of $u(x)$, $d(x)$ etc., the distributions of the u, d, etc. quarks in the proton. As an example, for neutrino-proton inclusive scattering in the W-S model, neglecting the effects of charmed quarks:

$$q(x) (\nu p \rightarrow \mu^- X) = x[d(x) \cos^2\theta_c + s(x) \sin^2\theta_c]$$

$$\bar{q}(x) (\nu p \rightarrow \mu^- X) = x\bar{u}(x).$$

$$q(x) (\nu p \rightarrow \nu X) = \frac{1}{4}x[u(x) (1 - \frac{4}{3}x_W^2) + [d(x) + s(x)] (-1 + \frac{2}{3}x_W^2) + \bar{u}(x) (\frac{4}{3}x_W^2) + [\bar{d}(x) + \bar{s}(x)] (-\frac{2}{3}x_W^2)]$$

$$\bar{q}(x) (\nu p \rightarrow \nu X) = \frac{1}{4}x[u(x) (-\frac{4}{3}x_W^2) + [d(x) + s(x)] (\frac{2}{3}x_W^2) + \bar{u}(x) (-1 + \frac{4}{3}x_W^2) + [\bar{d}(x) + \bar{s}(x)] (1 - \frac{2}{3}x_W^2)]$$

where $x_W = \sin^2\theta_W$. Assuming charge symmetry invariance the corresponding expressions for neutron targets are obtained from

the above by interchanging $u(x)$ and $d(x)$, $\bar{u}(x)$ and $\bar{d}(x)$. For large Q^2 , the distributions of asymptotically free quarks become functions of both x and Q^2 ,⁷ and as Q^2 increases, the quarks tend to concentrate towards lower values of x . Violation of scaling is also expected if Q^2 reaches the threshold value associated with the production of a heavy quark. In the region just above threshold, the effective mass m_i of the heavy quark is no longer negligible and x is usually replaced by the "slow rescaling" variable $z_i = x + m_i^2/2MEy$. Well above threshold, z_i reduces to x and scaling is recovered. Of the expressions given above for the Weinberg-Salam model, $q(x) (\nu p \rightarrow \mu^- X)$ gets an additional contribution above charm threshold:

$$q(x, z_c) (\nu p \rightarrow \mu^- X) = x[d(x) \cos^2 \theta_c + s(x) \sin^2 \theta_c] \\ + z_c [d(z_c) \sin^2 \theta_c + s(z_c) \cos^2 \theta_c]$$

The Developments of Recent Years

The experimental data available prior to 1975 on (anti) neutrino-nucleon scattering⁸⁻¹¹ were adequately explained by the 3-quark W-S model with $0.3 \leq \sin^2 \theta_w \leq 0.5$.¹⁵ SU(2) $\otimes$ U(1) models with more than 4 quarks and with right-handed coupling had been considered¹²⁻¹⁶, to explain, for example, such effects as the $\Delta I = 1/2$ rule in non-leptonic weak decays and CP non-invariance.

During 1975, ν and $\bar{\nu}$ scattering events with two muons in the final state were observed.¹⁷ The opposite-sign dimuons

were compatible with the production and subsequent semi-leptonic decay of charmed quarks in the Weinberg-Salam model:

$$\nu + N \rightarrow \mu^- + c + \dots$$

$$\hookrightarrow \mu^+ + \nu + \dots$$

$$\bar{\nu} + N \rightarrow \mu^+ + \bar{c} + \dots$$

$$\hookrightarrow \mu^- + \bar{\nu} + \dots$$

To explain the equal sign dimuon events, the W-S model required the associated production of charm:

$$\nu + N \rightarrow \mu^- + c + \bar{c} + \dots$$

$$\hookrightarrow \mu^- + \bar{\nu} + \dots$$

$$\bar{\nu} + N \rightarrow \mu^+ + c + \bar{c}$$

$$\hookrightarrow \mu^+ + \nu + \dots$$

to occur at the level of ~1%.¹⁸ The possible inadequacy of the W-S model was substantiated by experimental evidence suggesting the existence of heavy quarks beyond charm and/or heavy leptons.¹⁹ This is discussed in Chapter II. Alternatives to the Weinberg-Salam model were seriously considered.²⁰⁻²²

In 1976, the HPWF collaboration²³ and the CLEF group²⁴,

both at Fermilab, reported evidence of a large departure from scaling in deep-inelastic charged current anti-neutrino scattering. It was found that as E increased beyond about 30 GeV, $d^2\sigma^{\bar{\nu}N}/dx dy$ changed from the approximate $(1-y)^2$ shape observed at low energy, and acquired a flat component especially noticeable at high y . The effect was localized in the small x region: $x \leq 0.15$. Correspondingly, a substantial increase with energy in both $\langle y \rangle^{\bar{\nu}N}$ and the total cross-section ratio $\sigma^{\bar{\nu}N}/\sigma^{\nu N}$ was observed. The first of these effects came to be referred to as the "high- y anomaly". Results obtained from the 15' Fermilab bubble chamber did not show any anomalous behaviour.²⁵ No significant departure from scaling was observed for neutrino scattering by any of the above experiments.

In the Weinberg-Salam model, threshold effects would be expected from the production of charmed quarks. The additional contributions to the $\bar{\nu}$ cross-section above charm threshold would be small, arising from the transitions $\bar{s} \cos^2\theta_c \rightarrow \bar{c}$ and $\bar{d} \sin^2\theta_c \rightarrow \bar{c}$ which are Cabibbo and/or sea suppressed. Furthermore, this effect sets in below 30 GeV and does not increase between 30 and 80 GeV.²⁶ Unless the relative importance of the sea quarks could be significantly increased in some way²⁷, the W-S model is unable to explain the high- y anomaly.²⁸ The anomalous behaviour could be understood easily by assuming the existence of a new quark with mass ≈ 6 GeV, coupled to a valence quark via a right-handed current. Different models

containing these* and other features were proposed and their predictions compared with the experimental data available on neutrino-nucleon scattering.²⁹ Elastic scattering data³⁰ could be adequately explained by the W-S model with $0.25 \leq \sin^2 \theta_W \leq 0.30$ ³¹ and more recent data from Gargamelle is consistent with this.³²

At Fermilab, during 1977, the HPWF and CITF groups observed a total of 14 deep-inelastic, neutrino-nucleon scattering events in which three muons (with charges --+) were present in the final state.³³ This corresponds to an average relative rate of $\sigma(3\mu)/\sigma(\mu) \sim 4 \times 10^{-4}$. The average energy of the trimuon system was ~ 100 GeV although there were two exceptional events with energy ~ 250 GeV. There was no conclusive evidence for antineutrino-induced trimuons.

In mid 1977, the preliminary results of (anti) neutrino scattering experiments were reported from CERN³⁴ and from the 15' bubble chamber at Fermilab.³⁵ Somewhat surprisingly, the data did not confirm the earlier results from Fermilab. In particular, the CDHS collaboration at CERN, with substantially improved statistics, found no evidence for anomalous behaviour at high y ,³⁶ and both equal³⁷ and opposite³⁴ sign dimuon production consistent with the decay of charmed quarks. The results from the other experiments are not

* In general, the hadronic charged current of models with such couplings differs from that of the W-S model only above the new quark threshold(s) whereas the neutral current differs at all energies.

inconsistent with this. The CDHS results ("This Expt") are compared with the earlier Fermilab data (HPWF 1 and HPWF 2) in Figure 3, taken from Reference 36. The data for neutral current interactions were consistent with the W-S model if $0.22 \leq \sin^2 \theta_W \leq 0.26$,³⁸⁻⁴¹ as shown in Figure 4 (taken from Reference 39).

Two neutrino-induced trimuon events, however, were observed by the CDHS group,⁴² although this rate is lower than expected on the basis of the Fermilab data. A possible anti-neutrino trimuon event was also observed, the relative rate corresponding to the three events being $\sigma(3\mu)/\sigma(\mu) \sim 4 \times 10^{-5}$.

The main suggestions for mechanisms for trimuon production are shown in Figure 5. Mechanisms 1) - 3) are possible in the W-S model but the calculated rates are too low to explain the number of observed events. However, the possibility of a type 1) mechanism in which the quarks are produced diffractively cannot yet be ruled out.⁴⁷ In calculating the rate for mechanism 2), gauge invariance requires that the diagrams in which the radiation occurs from the quark lines also be included. Although the contribution from this mechanism is significant, it predicts trimuons with energy < 100 GeV which is rather low, particularly for the 2 exceptional events. Mechanisms 4) - 7) require new heavy leptons or quarks or both. With the presently available statistics, it is not possible to rule

Figure 3 The ratio of antineutrino and neutrino charged-current total cross sections as a function of neutrino energy, taken from Ref. 36. Indicated errors are statistical only.

Figure 4 Graphical illustration of the left-handed (Q) vs right-handed ($\bar{Q}$) contributions to the neutral current neutrino cross sections, from Ref. 39. Q and $\bar{Q}$ are defined in Appendix A. The relative strength of the sea quark contribution: $\int x(\bar{u} + \bar{d} + 2\bar{s})dx / \int x(u + d)dx$, was assumed to be 0.10. The experimental point is shown together with the $V - A$, $V + A$, pure V , pure A and $W-S$ model predictions.

1) Associated charm. Refs. 43, 47

2) Radiative. Refs. 43, 55

3) Internal Drell-Yan. Refs. 43, 53

4) Lepton cascade. Refs. 43, 44-6, 48-50, 54-8

Figure 5 Suggested mechanisms for trimuon production.

out any of these possibilities. The trimuon rates predicted by mechanisms 4) - 6) have been calculated for a variety of heavy particle masses, couplings and branching ratios.

Agreement with the observed rate is possible with:

$$\begin{aligned} m(M^-) &\approx 7 - 8 \text{ GeV} \\ m(M^0) &\approx 2 - 4 \text{ GeV} \\ m(b) &\approx 4.75 \text{ GeV (see Chapter II)} \\ m(t) &\approx 6 - 8 \text{ GeV} \end{aligned}$$

Certain characteristics of the events, for example, angular distributions of the muons, are not so sensitive to model-dependent assumptions and these favour mechanisms 4) and 6) over 5). A recent search for heavy charged leptons in an experiment using the 15' Fermilab bubble chamber⁵⁹ has set the following lower limits for the mass of the M lepton:

$$\begin{aligned} m(M^-) &\geq 7.5 \text{ GeV} \\ m(M^+) &\geq 9 \text{ GeV} \end{aligned}$$

assuming that the $\nu_\mu - M$ coupling is full-strength.

Mechanism 6) requires large flavour-changing neutral current coupling which would be difficult to incorporate into an $SU(2) \otimes U(1)$ model without getting strangeness- and charm-changing neutral currents as well.

Mechanism 7) involves the direct production of three leptons by the incident neutrino. In Reference 51, the case is considered in which this occurs via approximately local structure induced by the very massive leptons L^0, L^- ($m(L) > 100 \text{ GeV}$), as shown in Figure 5.

By charge conjugating the appropriate particles in the diagrams of Figure 5, all mechanisms allow $\bar{\nu}$ -induced trimuons. The lack of definite observation of such events in practice, is attributed largely to the different ν and $\bar{\nu}$ beam fluxes. The $\bar{\nu}$ flux is lower than that of ν and is peaked towards lower energies.

Exceptionally energetic trimuon events are improbable in any of the Figure 5 mechanisms with the possible exceptions of mechanism 7) or the internal Drell-Yan process that puts nearly all the beam energy into the trimuon system.⁵³ Other possible mechanisms with little or no missing energy are shown in Figure 6.

Trimuons produced via mechanism 8) would have a unique 3μ invariant mass, however, the present exceptional events have different invariant masses. Mechanism 9) requires $m(t) \geq 10 \text{ GeV}$ to yield muon energies comparable with those of the exceptional events.

In the most recent experiment by the CDHS group at CERN, a neutrino-induced tetramuon event (with charges $--++$) was observed.⁶⁰ Some possible mechanisms are shown in

8) Lepton cascade. Refs. 43, 46

9) Quark cascade. Ref. 43

10) Coherent electromagnetic production of heavy lepton. Ref. 43

11) Electromagnetic production of charged Higgs meson. Ref. 43

Figure 6 Additional possibilities for trimuon production.

Figure 7, these are simply extensions of the trimuon mechanisms 1), 2) and 4) - 6). Diagrams in which the photon emerges from the quark lines also contribute to mechanism 13) and are necessary for gauge invariance. The kinematics of the event would seem to favour either this process or 14) and 16) but are not very probable for mechanisms 12) or 15). However, on the basis of a single event, none of these possibilities can be excluded. V. Barger et al.⁶¹ have estimated the maximal relative rate $\sigma(4\mu)/\sigma(\mu)$ for mechanisms 14), 15), 13) and 12) to be of the order of 10^{-5} , 10^{-6} , 10^{-7} and 10^{-9} respectively, using the following mass values:

$$\begin{aligned} m(t) &= 8 \text{ GeV} \\ m(b) &= 4.75 \text{ GeV} \\ m(M^-) &= 8 \text{ GeV} \\ m(M^0) &= 1.5 \text{ GeV} \end{aligned}$$

However, the rates are relatively insensitive to particle masses.

Of the diagrams in Figure 7, only mechanism 14) and possibly 16) can predict pentamuon events. The relative rate $\sigma(3\mu^-, 2\mu^+)/\sigma(\mu)$ has been estimated for 14) to be of order 10^{-6} .⁶¹

Neutrino scattering data can be used to determine the coupling constants of the hadronic neutral current independently

12) Charm plus associative charm. Refs. 60, 61

13) Charm plus radiative. Refs. 60, 61

14) Lepton cascade. Ref. 61

Figure 7 Possible mechanisms for tetramuon production.

15) Quark cascade. Ref. 61

16) Mixed lepton quark cascade

Figure 7 Continued

of specific gauge models.⁶² Recent analyses^{63,64} have been able to show that there are only two possible solutions for these coupling constants, and they are given in Table I.

Data on the following processes were used:

$$\nu(\bar{\nu}) + N \rightarrow \nu(\bar{\nu}) + \chi^{65}$$

$$\nu(\bar{\nu}) + p \rightarrow \nu(\bar{\nu}) + p^{66}$$

$$\nu(\bar{\nu}) + N \rightarrow \nu(\bar{\nu}) + \pi^-(\pi^+) + \chi$$

$$\nu(\bar{\nu}) + N \rightarrow \nu(\bar{\nu}) + \pi^0 + \chi^{67}$$

N is used here to represent an isoscalar target.

Table I

	α	β	γ	δ
A	0.45 ± 0.14	0.92 ± 0.14	-0.35 ± 0.15	0.12 ± 0.15
B	0.92 ± 0.14	0.45 ± 0.14	0.12 ± 0.15	-0.35 ± 0.15

Possible solutions for the coupling constants α , β , γ , δ , from Reference 63. γ here is a factor 3 smaller than the definition given in Appendix A.

Thus, of the $SU(2) \otimes U(1)$ models with the standard left-handed coupling of the u and d quarks, only two types are consistent with these data. These are models in which the right-handed couplings of the u and d quarks ($I_{3R}^{u,d}$) are:

$$A \quad I_{3R}^u \approx 0 \text{ and } I_{3R}^d \approx 0$$

$$B \quad I_{3R}^u > 0 \text{ and } I_{3R}^d \approx 0$$

These results calculated with the average value of $\sin^2 \theta_w = 0.33$ are shown in Figure 8, which is taken from Reference 63.

Figure 8 The ranges of $2I_{3R}^u$ and $2I_{3R}^d$ for solutions A and B, from Ref. 63.

Summary of the Present Situation

If the conclusions drawn from the CDHS data are correct, (and doubt of this has been expressed⁶⁸), then full-strength right-handed coupling between u quarks and heavy quarks with mass less than about 8 GeV is ruled out. The data do not rule out reduced strength coupling or other types of right-handed charged current coupling, such as that between two heavy quarks or between a heavy quark and a light sea quark. Furthermore, neither solution A nor B above permit significant right-handed coupling of the d quark within $SU(2) \otimes U(1)$ models.

The W-S model is consistent with the type A solution and can, with the possible exception of the observed trimuon rate, explain all $\nu_\mu(\bar{\nu}_\mu) - N$ scattering results with $\sin^2\theta_W \approx 0.25$. This value of the mixing angle, however, is at variance with that determined from $\nu_\mu(\bar{\nu}_\mu) - e^-$ elastic scattering, i.e. $\sin^2\theta_W \approx 0.38$. New results on $\nu - e$ scattering are expected soon from Fermilab.

Higher statistics data on trimuon production will soon be available from the most recent CDHS experiment at CERN.⁶⁹ The characteristics of these events, such as energy and angular distributions, transverse muon momenta etc. will help distinguish between different modes of production. Discrimination may also be possible on the basis of the differing predictions for other kinds of multilepton

phenomena. In particular, the lepton cascade and mixed quark lepton cascade, the two most favoured mechanisms (4) and 6) of Figure 5), differ significantly in their predictions for the following processes:

i) μe dilepton events.^{57,58} These events can be identified in bubble chamber experiments. The dominant contributions to ν -induced μe events for mechanisms 4) and 6) are shown in Figure 9. As the e^- comes from quite different places in each case, its properties can be used to discriminate between these two alternatives.

ii) $\bar{\nu}$ -induced trimuons.⁵⁷ In mechanism 4), the muon charges are $++$ -, the charge conjugates of those obtained in ν -induced events. Mechanism 6) involves neutral current coupling and so predicts muons with charges $--+$ in both ν - and $\bar{\nu}$ -induced trimuon events. Similarly, the $\bar{\nu}$ analogues of the diagrams in Figure 5 lead to quite different dilepton processes:

$$\bar{\nu} N \rightarrow \mu^+ e^+ X \quad (\text{or } \mu^+ \mu^+ X) \quad \text{mechanism 4)}$$

$$\bar{\nu} N \rightarrow \mu^+ e^- X \quad (\text{or } \mu^+ \mu^- X) \quad \text{mechanism 6)}$$

iii) Tetramuons. The heavy lepton cascade i.e. mechanism 14) of Figure 7 allows more than one combination of muon charges: it predicts that $----$ and $--++$ occur with equal probability. In the mixed quark lepton cascade (and the other mechanisms of Figure 7) only $--++$ tetramuons are expected.

Mechanism 4): Lepton cascade.

Mechanism 6): Mixed lepton quark cascade.

Figure 9 Dominant diagrams for $\nu N \rightarrow \mu^- e^- X$

CHAPTER II

NEW QUARKS AND LEPTONS

There is evidence for the existence of new heavy fermions that is more direct than that described in the previous section. It has been obtained in electron-positron annihilation experiments at SPEAR and DESY and in proton-nucleus collision experiments at Fermilab.

 e^+e^- Annihilation

The simplest way to express the results of these experiments is in terms of the quantity R , defined as the ratio of the total cross section for producing hadrons to that for producing muon pairs. Naive theory predicts $R = \sum q_i^2$ where q_i is the charge of the i th particle in units of the positron's charge, and the summation is over quarks and heavy leptons - assuming that the latter frequently decay to hadrons.

It has been found that R exhibits three distinct regions over the SPEAR centre of mass energy range $2.4 \text{ GeV} \leq E_{\text{CM}} \leq 8 \text{ GeV}$, as shown in Figure 10.¹ Below 3.5 GeV R is approximately constant at a value near 2.5 which is compatible with the prediction $R = 2$ from the 3 quark Weinberg-Salam model. Above 5 GeV, R is again approximately constant at ≥ 5 . Between these two regions, there is a complicated transition region. Including the contribution of charmed quarks, the W-S model predicts $R = 3 \frac{1}{3}$. This

suggests that there are other thresholds in the transition region, resulting in an additional contribution of approximately 2 units to R .²

Figure 10 R versus centre of mass energy in coarse steps, as measured in e^+e^- annihilation at SPEAR, from Ref. 1.

In exploring the 4 GeV region more fully, events were observed at SPEAR³ and later at DESY⁴ that could not be explained on the basis of charmed particle decays but were consistent only with the production and decay of a pair of new particles: a heavy charged lepton (τ^-) with mass ~ 1.8 GeV and its antiparticle^{5,6}:

$$e^+ + e^- \rightarrow \tau^+ + \tau^-$$

The τ may be a sequential lepton or possess the same lepton number as the electron or the muon. However, a search for heavy lepton production and decay via the process:

$$\nu_\mu + N \rightarrow L^\pm + X$$

$$L \rightarrow e^\pm + \nu + \bar{\nu}$$

using the 15' Fermilab bubble chamber, has set an upper limit on the $\nu_\mu - \tau$ coupling strength of 0.025 relative to the $\nu_\mu - \mu$ coupling.⁷ Assuming that τ is sequential and has its own neutrino ν_τ , the leptonic decay modes are:

$$\tau^- \rightarrow \nu_\tau + e^- + \bar{\nu}_e, \quad \tau^- \rightarrow \nu_\tau + \mu^- + \bar{\nu}_\mu$$

$$\tau^+ \rightarrow \bar{\nu}_\tau + e^+ + \nu_e, \quad \tau^+ \rightarrow \bar{\nu}_\tau + \mu^+ + \nu_\mu$$

The results from SPEAR favour V - A over V + A coupling of

τ and ν_τ but are not able to say anything about coupling intermediate between $V - A$ and $V + A$.⁶ The possibility has been considered that the presence of ν_τ and $\bar{\nu}_\tau$ in the Fermilab neutrino beam may contribute to the anomalous effects observed there.⁸

No evidence for neutral heavy leptons has been found in $e^+ e^-$ annihilations. The results of searches for narrow neutral resonances in 2π channels at SPEAR imply that such particles, if they exist, are heavier than 1.2 GeV.⁹

Proton-Nucleus Collisions

In recent experiments at Fermilab, the spectrum of muon pairs produced by incident 400 GeV protons was measured, and new structure was observed in the 9 - 10 GeV region.¹⁰ The structure was later resolved into two narrow peaks, denoted by T and T' at 9.5 and 10.0 GeV respectively, and possibly a third T'' at 10.4 GeV¹¹ (see Figure 11). By analogy with $\psi(3.1)$, $\psi(3.7)$ etc., the enhancements T , T' and the possible T'' are generally believed to be bound states of a new quark (denoted by b in the literature) and its antiquark. A charge of $-1/3$ ^{11,12} and an effective mass of 4.6 GeV¹³ for the b quark are consistent with the data, although $Q = +2/3$ cannot be definitely ruled out.

Figure 11 Dimuon spectrum in the 10 GeV region from Ref. 11. The plot shows the excess of the data over a continuum fit of the form: $(d^2\sigma/dm dy)_{y=0} = Ae^{-bm}$. Errors shown are statistical only. The solid curve is the 3-peak fit; the dashed curve is the 2-peak fit.

The experiments have provided no information on the handedness of the b quark. The τ family is also expected to show up in sufficiently energetic e^+e^- annihilation, although there are practical difficulties.¹⁴

The production of a massive lepton pair in hadron-hadron collisions is described in the quark-parton model as taking place mainly via quark-antiquark annihilation - the so-called Drell-Yan mechanism.¹⁵ The dimuon production data from Fermilab have been used as an additional means of analyzing the quark distributions in the proton via the Drell-Yan model.¹⁶ In particular, the data are sensitive to

the sea quark distributions which are difficult to obtain from lepton-hadron scattering data.

A recent search for heavy neutral leptons resulting from 400 GeV proton-nucleus collisions at Fermilab has produced a null result: no evidence was found for heavy neutral leptons produced unaccompanied by muons with momentum ≥ 10 GeV.¹⁷

CHAPTER III

PARITY NON-CONSERVATION

Parity Non-Conservation in Atoms

It was suggested in 1965 that a possible, but difficult, way to investigate weak neutral currents would be to look for parity violating effects arising from the weak interaction between the electron and proton in hydrogen.¹ The interest in atomic parity experiments has since increased,² following developments in weak interaction theory and the discovery of weak neutral currents in neutrino interactions. In principle, the results of such experiments can be used to determine completely the right-handed coupling of the electron and the u and d quarks, and hence distinguish between different gauge models of weak interactions.

The nucleon-nucleon and electron-electron weak couplings are assumed to have a negligible effect on atomic energy levels, and as the parity conserving part of the electron-nucleon weak interaction is masked by electromagnetic interactions, the experiments measure only the parity non-conserving part of the electron-nucleon weak neutral current. If the electron-quark coupling is described by vector and axial-vector coupling constants v_e , v_q and a_e , a_q , the parity violating Hamiltonian, H_{PV} , will contain two types of terms, proportional to $v_e a_q$ and

$a_e v_q$, where the indices e and q refer to electron and quark respectively. In light atoms the magnitudes of the two terms are comparable, but in heavy atoms, the term involving $a_e v_q$ is considerably enhanced and provides the dominant contribution. The reason for this is that the quark neutral vector couplings sum coherently and v_q gets multiplied by the number of valence quarks in the nucleus (sea quarks are generally ignored). The term involving the quark axial coupling, a_q , depends on the spin of the nucleus and, therefore, never gets very large.³ The coupling constants depend on the specific model chosen. In general, for a fermion i , placed in a left-handed (right-handed) weak isospin multiplet with third component of weak isospin I_{3L}^i (I_{3R}^i):

$$a_i = -2 (I_{3L}^i - I_{3R}^i)$$

$$v_i = 2 (I_{3L}^i + I_{3R}^i) - 4 q_i \sin^2 \theta_w$$

where q_i is the charge of the fermion.⁴

In the Weinberg-Salam model:

$$a_e = 1, v_u = 1 - \frac{8}{3} \sin^2 \theta_w, v_d = 1 + \frac{4}{3} \sin^2 \theta_w$$

$$v_e = -1 + \frac{4}{3} \sin^2 \theta_w, a_u = -1, a_d = +1$$

(1)

where indices u and d refer to up and down quarks respectively.

An alternative notation, more common in the literature, is to express H_{PV} in terms of electron-nucleon, rather than electron-quark coupling by writing:

$$H_{PV} = - \frac{G_F}{\sqrt{2}} [C_{1p} \bar{e} \gamma_\mu \gamma_5 e \bar{p} \gamma^\mu p + C_{1n} \bar{e} \gamma_\mu \gamma_5 e \bar{n} \gamma^\mu n + C_{2p} \bar{e} \gamma_\mu e \bar{p} \gamma^\mu \gamma_5 p + C_{2n} \bar{e} \gamma_\mu e \bar{n} \gamma^\mu \gamma_5 n]$$

In the Weinberg-Salam model⁵:

$$\begin{aligned} C_{1p} &= \frac{1}{2} (1 - 4 \sin^2 \theta_w) , & C_{1n} &= -\frac{1}{2} \\ C_{2p} &= \frac{1}{2} (1 - 4 \sin^2 \theta_w) g_A , & C_{2n} &= -\frac{1}{2} (1 - 4 \sin^2 \theta_w) g_A \end{aligned} \quad (2)$$

where g_A , the ratio of axial to vector coupling, is ~ 1.25 . For $\sin^2 \theta_w = 0.25$, only one of these coupling constants contributes:

$$\begin{aligned} C_{1p} &= 0 , & C_{1n} &= -\frac{1}{2} \\ C_{2p} &= 0 , & C_{2n} &= 0 \end{aligned} \quad (3)$$

The parity violating Hamiltonian causes mixing of atomic energy levels, but because of the short range of the weak interaction, only s- and p- states are significantly affected. Using first-order perturbation theory, the amplitude for mixing between two states denoted by $|ns\rangle$ and

$|n'p\rangle$ is:

$$\frac{\langle n'p | H_{PV} | ns \rangle}{E_{n'p} - E_{ns}} \quad (4)$$

The extent of the mixing is determined experimentally by measuring the interference between an allowed transition and a transition which is forbidden in the absence of the level mixing. In most of the experiments proposed so far, the former is a magnetic dipole transition, or an electric dipole transition induced by an external static electric field, while the latter is an electric dipole transition. As a typical case, consider an allowed M1 transition and a weak induced E1 transition, from state $|i\rangle$ to state $|f\rangle$, $|i\rangle$ and $|f\rangle$ having the same parity and angular momentum. The parity violating signal, R , is proportional to:

$$\frac{\langle f | M1 | i \rangle^* \langle f | E1 | i \rangle}{|\langle f | M1 | i \rangle|^2 + |\langle f | E1 | i \rangle|^2} \sim \frac{\langle f | E1 | i \rangle}{\langle f | M1 | i \rangle}$$

where

$$\langle f | E1 | i \rangle = \sum_n \frac{\langle f | H_{PV} | n \rangle \langle n | E1 | i \rangle}{E_f - E_n} + \sum_{n'} \frac{\langle f | E1 | n' \rangle \langle n' | H_{PV} | i \rangle}{E_i - E_{n'}}$$

allowing for mixing in both the initial and the final states.

Heavy Atoms ($Z > 50$)

Calculations for heavy atoms are based on the "independent particle model": only one electron at a time interacts with the nucleus via H_{pv} . The single electron matrix element $\langle n'p | H_{pv} | ns \rangle$, in the non-relativistic limit, behaves like Z^3 and so considerable enhancement is expected for large Z . This enhancement arises from the coherent effect of the nucleons and from the fact that the density and velocity of the valence electron near the nucleus both grow like Z . The correction due to relativistic effects gives a further enhancement factor of 5 - 10 in heavy atoms.⁶ However, although the experiments would seem easier to do for heavy atoms, the interpretation of the results is obscured by the difficulty of estimating the electron wave functions in such complex systems. Explicit calculations have been done for Cs, Tl, Bi and Pb.⁷ The latter two cases are further complicated by the presence of more than one valence electron.⁸

The quantity that these experiments measure is the "weak vector charge" of the nucleus, Q_w , defined by:

$$Q_w = a_e v_q$$

For a nucleus with Z protons and N neutrons:

$$\begin{aligned} Q_w &= a_e [(2v_u + v_d)Z + (v_u + 2v_d)N] \\ &= 2[C_{1p}Z + C_{1n}N] \end{aligned}$$

Using equations (1) or (2), the Weinberg-Salam model predicts:

$$Q_W = (1 - 4 \sin^2 \theta_W)Z - N$$

For arbitrary Z and N , this expression cannot be zero for any value of $\sin^2 \theta_W$. Moreover, except for a few very light atoms, Q_W is negative for any value of $\sin^2 \theta_W$. The value 0.25 is particularly interesting as in this case only the neutrons contribute:

$$Q_W (\sin^2 \theta_W = 0.25) = -N$$

The experiments at Washington⁹ and Oxford¹⁰ both looked for interference between an allowed M1 transition and a weak induced E1 transition in bismuth ($Z = 83$, $N = 126$). A laser beam, with wavelength close to that corresponding to the transition, traversed bismuth vapour and any rotation in the plane of polarization of the light was measured. The two groups used the same technique, but different transitions: $^4S_{3/2} \rightarrow ^2D_{3/2}$ and $^4S_{3/2} \rightarrow ^2D_{5/2}$ respectively. In most models the rotation is expected to be very small ($< 10^{-6}$ radians), the W-S model in particular predicts $R \sim -10^{-7}$ for both transitions.¹¹ The most recent results reported are:

$$\begin{array}{ll} R = (-0.7 \pm 3.2) \times 10^{-8} & \text{Washington}^9 \\ R = (2.7 \pm 4.7) \times 10^{-8} & \text{Oxford}^{10} \end{array}$$

In terms of Q_W :

$$-159.2 \leq Q_W \leq -126, \quad \text{W-S, } 0.35 \geq \sin^2 \theta_W \geq 0.25$$

$$-50 \leq Q_W \leq +75 \quad \text{Experimental}$$

Despite the rather large errors and theoretical uncertainty involved in the calculations, there seems to be serious discrepancy with the W-S model. Q_W is sensitive to possible right-handed coupling of the electron or u, d quarks. If the electron is assigned to a right-handed doublet together with a neutral lepton, $I_{3R}^e = -\frac{1}{2}$ and $Q_W = 0$ for any value of $\sin^2 \theta_W$. If the u quark is assigned to a doublet $(\begin{smallmatrix} u \\ b \end{smallmatrix})_R$, where b has charge $-1/3$, then $I_{3R}^u = \frac{1}{2}$ and Q_W for bismuth has the opposite sign as compared with the W-S model:

$$132.8 \leq Q_W \leq 166 \quad \text{W-S} + (\begin{smallmatrix} u \\ b \end{smallmatrix})_R, \quad 0.35 \geq \sin^2 \theta_W \geq 0.25$$

Similarly, if the d quark has right-handed coupling of the form $(\begin{smallmatrix} t \\ d \end{smallmatrix})_R$, where t has charge $2/3$, then $I_{3R}^d = -1/2$ and Q_W for bismuth has the same sign but is about three times larger in magnitude than in the W-S model:

$$-494.2 \leq Q_W \leq -461 \quad \text{W-S} + (\begin{smallmatrix} t \\ d \end{smallmatrix})_R, \quad 0.35 \geq \sin^2 \theta_W \geq 0.25$$

These values of Q_W correspond to full-strength right-handed

coupling. The possibility of right-handed electron and u quark coupling is considered in more detail in Chapter IV.

A different technique is being employed for the experiments involving cesium ($Z = 55$, $N = 78$) and thallium ($Z = 81$, $N = 124$).¹² Here, the ratio R is increased by a factor of $\sim 10^4$ by using a "forbidden" M1 transition, i.e. one that involves a change of principal quantum number. The transitions are $6s_{1/2} \rightarrow 7s_{1/2}$ for cesium and $6p_{1/2} \rightarrow 7p_{1/2}$ for thallium. In both cases the transition is excited by circularly polarized laser light, and a difference in the cross-sections for left-handed and right-handed polarizations is looked for. Results from these experiments are expected soon.

Light Atoms

In light atoms, particularly the single-electron atoms, hydrogen and deuterium, the atomic physics calculations are relatively simple allowing unambiguous interpretation of experimental results. An additional advantage is that it should be possible to measure the 4 coupling constants; C_{1p} , C_{1n} , C_{2p} and C_{2n} , rather than a linear combination of the first two as is presently the case in heavy atom experiments.

The greatest parity mixing occurs between $nS_{1/2}$ and $nP_{1/2}$ due to the close spacings of these levels. However, the parity mixing as defined in (4), has been calculated to

be $\sim 10^{-11}$ in the absence of external fields.¹³ The effect can be enhanced to $\sim 10^{-10}$ by applying a magnetic field to further reduce the energy difference between the levels.¹⁴ At the level crossings¹⁵ the energy difference is taken to be $\frac{1}{2} \Gamma_p$, where Γ_p is the natural width of the $nP_{1/2}$ state: other line broadening effects are considered negligible with the present experimental precision.¹³ Different level crossings involve different linear contributions of C_{1p} , C_{1n} and C_{2p} , C_{2n} and for this reason each coupling constant can, in principle, be determined.

The experiments currently in progress¹⁶⁻¹⁸ are investigating $2S_{1/2} - 2P_{1/2}$ mixing in hydrogen, the choice of initial state is due to the availability of intense metastable 2S beams. The transitions that interfere with the weak induced E1 transition are either an E1 transition induced by an external static electric field E_s ^{16,17}, or, an M1 transition.¹⁸

In one of the experiments of the first type, for example,¹⁶ the E1 transitions are microwave $2S \rightarrow 2S$ transitions. The magnetic field is chosen for the 550 gauss level crossing at which the parity violating effect is expected to be $R \sim 2.5 C_{2p} E_s \times 10^{-6}$. E_s cannot be made too large because of quenching effects.

The Weinberg-Salam model predicts $R \approx 0$ for $\sin^2 \theta_w \approx 0.25$, ie for this value of the mixing angle the W-S model predicts a null result from this experiment.

Alternative Processes

An experiment is being undertaken at SLAC to look for evidence of parity non-conservation in three different types of electron-nucleon interactions: deep inelastic scattering, elastic scattering and Δ electroproduction.¹⁹ The electrons are longitudinally polarized. Parity violating weak neutral currents will result in a dependence of the electron-nucleon scattering cross sections on the electron's helicity, arising from interference between weak and electromagnetic amplitudes. If the experiments prove sufficiently sensitive, (a few parts in 10^5 at $Q^2 = 1 \text{ GeV}^2$) the results will be able to distinguish between various $SU(2) \otimes U(1)$ models and between various classes based on larger gauge groups. A fourth type of electron-nucleon interaction has also been considered and this is the scattering of a longitudinally polarized neutron beam from atomic electrons.²⁰ However, it is not known whether this method will prove to be practically feasible.

These experiments and those in atomic physics are looking for weak-electromagnetic interference in electron-nucleon interactions. Parity violating effects arising from the interference of weak and strong amplitudes have also been considered.²¹ The experimental possibilities include various kinds of nucleon-nucleus^{20,22,23} and nucleon-nucleon^{22,24,25} polarized scattering. The calculations are simpler in the

latter case and the estimated asymmetries are, in general, larger. In particular, the expected asymmetries in polarized $n - d$ and $p - p$ elastic scattering, neglecting electromagnetic effects, have been estimated to be $\sim 10^{-8}$ and 10^{-7} respectively.²⁴ Experimental data²⁶ are available for $p - p$ scattering where the asymmetry is measured to be $(0.1 \pm 1.4) \times 10^{-7}$. The discrepancy is considered largely to be a consequence of neglecting repulsion in the strong potential. These effects are considerably smaller than the asymmetry expected in polarized electron-nucleon scattering. Another disadvantage is that the experimental results will be more difficult to interpret unambiguously because the strong interaction is not as well understood as the electromagnetic interaction.

It may be possible to detect an effect of parity nonconserving weak neutral currents in superfluid $^3\text{He-B}$. This effect is a permanent electric-dipole-moment density directed along the characteristic rotation axis. In an ordinary atomic or molecular system, an electric dipole moment will exist only if the electron-nucleon weak coupling violates both parity conservation and time reversal invariance. This fact, together with experimental measurements of the electric dipole moments of atoms and molecules, has been used to set an upper limit on the presence of such couplings.²⁷ In superfluid $^3\text{He-B}$, however, the Cooper pairs exhibit spontaneously broken spin-orbit symmetry, which means that

$\vec{S}$ and $\vec{L}$ are separately conserved quantities. In this case, provided $\vec{S}$ and $\vec{L}$ are non-zero, coupling that violates parity, but not time reversal invariance, will result in an electric dipole moment. Preliminary estimates²⁸ predict a dipole moment of $\sim 10^{-12}$ e.cm⁻² which, although extremely small, may result in effects that are observable with currently available experimental techniques.

CHAPTER IV

ALTERNATIVE MODELS

The existing weak interaction data, discussed in the previous three chapters, places a number of constraints upon gauge models. The results are summarized below:

1) $\nu - e$ elastic scattering cross sections favour the minimal $SU(2) \otimes U(1)$ (i.e. Weinberg-Salam) electron couplings with $\sin^2 \theta_w \approx 0.38$. Right-handed electron coupling in $SU(2) \otimes U(1)$ has a probability of $\sim 2\%$.

2 i) The behaviour of $\bar{\nu} - N$ deep-inelastic charged current scattering cross section rules out full-strength right-handed coupling of the u quark with a quark of mass < 8 GeV.

ii) Multimuron events have been observed in $\nu - N$ deep-inelastic scattering. In particular, for neutrino-induced trimuons, relative rates $\sigma(3\mu)/\sigma(\mu) \sim 10^{-4}$ have been observed.

iii) The $(\bar{\nu}) \nu - N$ neutral current interactions are adequately explained by Weinberg-Salam type couplings with $\sin^2 \theta_w \approx 0.25$.

iv) The weak hadronic neutral current coupling constants have been almost uniquely determined. In particular, within $SU(2) \otimes U(1)$, there are only two types

of u, d couplings compatible with scattering data:

$$A \quad I_{3R}^u \approx 0 \quad \text{and} \quad I_{3R}^d \approx 0$$

$$B \quad I_{3R}^u > 0 \quad \text{and} \quad I_{3R}^d \approx 0$$

3) A new quark (b) with mass ≈ 4.6 GeV has been discovered. In the following, the b quark is assumed to have a charge of $-1/3$. Also, there is a new charged lepton (τ^-) with mass ≈ 1.8 GeV. Pure V - A coupling between τ and its neutrino is favoured over pure V + A.

4) Parity-violating effects in heavy atoms have been reported to be considerably smaller than the Weinberg-Salam predictions.

In addition, there are the following standard experimental constraints:

5) Quark-lepton universality i.e. the fact that $G_F \sec \theta_c = G_\mu$, where θ_c is the Cabibbo angle.¹

6) μ - e universality, in particular, the equality of ν_μ - μ and ν_e - e couplings.

7) The absence of right-handed currents which involve only light fermions.

8) The absence to order αG_F and the suppression to order G_F^2 of neutral strangeness- and charm-changing

currents.²

9) Strong suppression of processes like $\mu \rightarrow e\gamma$ ³, $\mu \rightarrow 3e$ ⁴ and $\bar{\mu}N \rightarrow eN$ ⁵ that violate the separate conservation of μ - and e -lepton numbers.

10) The requirement that the weak contributions to the anomalous magnetic moments of the electron and muon be sufficiently small. The limits on the weak contributions are set by the discrepancy between the experimental values and the theoretical electromagnetic contributions.⁶

11) Predicted CP violating effects, e.g. the neutron electric dipole moment, must be in agreement with present experimental bounds.⁷

There are also the following theoretical restrictions on gauge models:

12) A general requirement is that the experimental constraints 5) - 11) be met naturally, i.e. for arbitrary values of the parameters of a given model.⁸

13) Cancellation of triangle anomalies. This can be done within the quark and lepton sectors separately by having an analogous right-handed current for each left-handed current. Alternatively, if the appropriate fermion

charges, Q_i , sum to zero, the anomalies can be cancelled.

For $SU(2) \otimes U(1)$ models, the condition reduces to:

$$\sum_{i_L} Q_{i_L} = \sum_{i_R} Q_{i_R}$$

summing over all left-handed (L) and right-handed (R) fermion charges.⁹

14) The assumption is made that quarks have charge $+2/3$ or $-1/3$ and leptons have charge 0 or -1 .

15) Quark - lepton symmetry is a desirable feature (particularly from the point of view of constructing superunified theories of the weak, electromagnetic and strong interactions).

As has been seen, the Weinberg-Salam model is compatible with the previous constraints with the exception of 2 ii), 3) and 4). Various alternative models will now be considered; their quark and lepton multiplet assignments are given in Table II. The eigenstates of the weak gauge group are in general not mass eigenstates, with the result that mixing occurs between fermions with the same charge and handedness. For simplicity, all mixing angles have been set to zero in Table II. As the W-S model has been so successful, 3 extensions are proposed: model A with left-handed currents only and models B and C with

Table II

$SU(2) \otimes U(1)$: Model A

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \quad \begin{pmatrix} M^0 \\ M^- \end{pmatrix}_L$$

$$M_R^0, e_R^-, \mu_R^-, \tau_R^-, M_R^-$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \quad \begin{pmatrix} v \\ g \end{pmatrix}_L$$

$$u_R, c_R, t_R, v_R, d_R, s_R, b_R, g_R$$

$SU(2) \otimes U(1)$: Model B

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \quad \begin{pmatrix} E^0 \\ E^- \end{pmatrix}_L \quad \begin{pmatrix} M^0 \\ M^- \end{pmatrix}_L \quad \begin{pmatrix} T^0 \\ T^- \end{pmatrix}_L$$

$$\begin{pmatrix} E^0 \\ E^- \end{pmatrix}_R \quad \begin{pmatrix} M^0 \\ M^- \end{pmatrix}_R \quad \begin{pmatrix} T^0 \\ T^- \end{pmatrix}_R \quad E_R^-, M_R^-, T_R^-$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$u_R, c_R, t_R, d_R, s_R, b_R$$

$SU(2) \otimes U(1)$: Model C

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

$$E_L^0, M_L^0, T_L^0 \\ E_L^-, M_L^-, T_L^-$$

$$\begin{pmatrix} E^0 \\ E^- \end{pmatrix}_R \quad \begin{pmatrix} M^0 \\ M^- \end{pmatrix}_R \quad \begin{pmatrix} T^0 \\ T^- \end{pmatrix}_R$$

$$e_R^-, \mu_R^-, \tau_R^-$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L$$

$$g_L, b_L, t_L$$

$$\begin{pmatrix} u \\ g \end{pmatrix}_R \quad \begin{pmatrix} c \\ b \end{pmatrix}_R$$

$$d_R, s_R, t_R$$

Table II cont'd.

$SU(2) \otimes SU(2) \otimes U(1)$: Model D

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \begin{pmatrix} E^0 \\ L^- \end{pmatrix}_L \quad \begin{pmatrix} E^0 \\ e^- \end{pmatrix}_R \begin{pmatrix} M^0 \\ \mu^- \end{pmatrix}_R \begin{pmatrix} T^0 \\ \tau^- \end{pmatrix}_R \begin{pmatrix} L^0 \\ L^- \end{pmatrix}_R$$

$$M_L^0, L_L^0, T_L^0$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} c \\ s \end{pmatrix}_L \begin{pmatrix} t \\ b \end{pmatrix}_L \begin{pmatrix} \nu \\ g \end{pmatrix}_L \quad \begin{pmatrix} u \\ g \end{pmatrix}_R \begin{pmatrix} c \\ d \end{pmatrix}_R \begin{pmatrix} t \\ s \end{pmatrix}_R \begin{pmatrix} \nu \\ b \end{pmatrix}_R$$

$SU(3) \otimes U(1)$: Model E

$$\begin{pmatrix} \nu_e \\ e^- \\ E^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \\ M^- \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau^- \\ T^- \end{pmatrix}_L \quad \begin{pmatrix} E^0 \\ E^- \\ e^- \end{pmatrix}_R \begin{pmatrix} M^0 \\ M^- \\ \mu^- \end{pmatrix}_R \begin{pmatrix} T^0 \\ T^- \\ \tau^- \end{pmatrix}_R$$

$$E_L^0, M_L^0, T_L^0$$

$$\begin{pmatrix} u \\ d \\ b \end{pmatrix}_L \begin{pmatrix} c \\ s \\ g \end{pmatrix}_L \quad t_L, \nu_L \quad \begin{pmatrix} t \\ b \\ d \end{pmatrix}_R \begin{pmatrix} \nu \\ g \\ s \end{pmatrix}_R \quad -u_R, c_R$$

both left- and right-handed currents. None of these models satisfy all the constraints and other $SU(2) \otimes U(1)$ models are even less likely.

Models D and E are examples of models based on other gauge groups: $SU(2) \otimes SU(2) \otimes U(1)$ and $SU(3) \otimes U(1)$ respectively, model D is found to fit the data rather well. Larger gauge groups, for example, $SU(3) \otimes SU(3)$ ¹⁰, are not considered here.

The standard experimental constraints, in particular 9) - 11), have been included for completion; the present treatment is brief and qualitative in nature, although the results of relevant calculations found in the literature will be mentioned.

For the purposes of comparing the models of Table II, the mechanism for CP violation and μ -, e- number non-conservation considered here is that of fermion mass mixing. The latter effect is analogous to the non-conservation of separate $(u)_L$ and $(c)_L$ quark numbers due to the familiar Cabibbo mixing of the d and s quarks (or equivalently, the u and c quarks). An alternative mechanism, for gauge models with at least two complex Higgs doublets, is via the exchange of Higgs bosons. CP invariance will in general be spontaneously broken with such a Higgs sector and μ -, e- number non-conservation will follow if the most general gauge-invariant coupling between the Higgs particles and the leptons is allowed for.¹¹ The observable effects of this "milliweak" interaction, if it occurs, could be

comparable with present experimental limits. In particular, the contribution to the neutron electric dipole moment has been estimated by Weinberg¹² to be $\sim 10^{-24}$ e.cm. Bjorken and Weinberg¹³ have calculated upper limits on the $\mu \rightarrow e\gamma$ branching ratio of $\sim (\alpha/\pi)^3$, i.e. 10^{-8} and on the relative rate for $\mu N \rightarrow e N$ of $\sim 10^{-8} - 10^{-9}$. However, this last result depends strongly on the mass and couplings chosen for the Higgs bosons. Muon number non-conservation, and the process $\mu N \rightarrow e N$ in particular, is discussed further at the end of this chapter.

All of the models considered here include the b quark and τ lepton. As with e and μ , τ is coupled to its own (massless) neutrino, ν_τ , via a left-handed current. Each model has been constructed to satisfy constraints 13) and 14). The properties of each model are discussed below and summarized in Table III.

SU(2) $\otimes$ U(1)

Models A, B and C, like the W-S model, are based on SU(2) $\otimes$ U(1). This gauge group has four bosons: W^1 , W^2 , W^3 and B. Of the bosons of definite mass, two are charged ($W^\pm$) and two are neutral (Z and the photon A):

$$W^{\pm\mu} = (W^{1\mu} \mp iW^{2\mu})/\sqrt{2}$$

$$Z^\mu = \cos\theta_W W^{3\mu} - \sin\theta_W B^\mu$$

$$A^\mu = \sin\theta_W W^{3\mu} + \cos\theta_W B^\mu$$

where $\tan\theta_W = g'/g$, g and g' being the coupling strengths of the $SU(2)$ and $U(1)$ groups respectively.

The coefficient of Z for left- and right-handed fermions is proportional to:

$$I_3 - 2Q\sin^2\theta_W$$

where $I_3 = +1/2$ ($-1/2$) for top (bottom) doublet members and $Q = I_3 + Y/2$.

The masses of W and Z are related by:

$$M_Z = \kappa M_W \sec\theta_W$$

where κ is determined by the Higgs sector and:

$$M_W = 37.5 \operatorname{cosec}\theta_W$$

The mechanism by which the vector bosons and the fermions of the W - S model acquire mass is described in Appendix B. The procedure outlined there serves as the prototype for other models, including those based on larger gauge groups.

Model A

The simplest way to incorporate more fermions into

the W-S model is to add more left-handed doublets and the corresponding right-handed singlets. This model has 4 new leptons and the same number of new quarks. A single Higgs doublet is sufficient to give mass to the charged leptons, another doublet is required for a massive M^0 (see Appendix B). The $(M^0 M^-)_L$ doublet is added to provide a mechanism for trimuon events, i.e. via the production and subsequent cascade decay of the M^- as described in Chapter I.¹⁴ For this to be possible, M^- must be heavier than M^0 and large mixing must occur between μ^- and M^- to provide sufficient $\nu_\mu - M^-$ coupling to explain the observed trimuon rate. This large mixing is allowed provided it is sufficiently symmetric with respect to μ and e and also with respect to quarks and leptons as required by the universality constraints 5) and 6).¹⁵ This means that universality is achieved by appropriate choices of mixing angles, i.e. at the expense of constraint 12).

A further consequence of mixing in the lepton sector is that μ - and e -lepton numbers are not separately conserved. The heavy lepton M^0 serves as an intermediate state in the $\mu \rightarrow e$ transition, allowing such processes as $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$. However, the rates for these processes involve the ratio $(M_{M^0}^2/M_W^2)$ which results in substantial suppression, independent of the values of mixing angles,

as $M_W \gg M_{M0}$. Explicit calculations for a model with 6 leptons in left-handed doublets: $(\nu_e e^-)$, $(\nu_\mu \mu^-)$ and $(L^0 L^-)_L$ with $M_{L0} \sim 10$ GeV result in rates for $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ and $K \rightarrow e\mu$ that are well below the present experimental limits.¹⁶

As in the case of the Weinberg-Salam model, the neutral currents in this model are naturally flavour diagonal to $O(G_F^2)$.¹⁷ In the quark sector there are $4 \times 4 = 16$ complex mixing angles. After absorbing the maximum number of phases by redefining quark fields, there remain 6 real mixing angles and 3 potentially CP violating phases.¹⁸ However, CP violating effects are suppressed due to the occurrence of the mass ratio $(\Delta M_q^2/M_W^2)$, where ΔM_q^2 is the difference of the squares of two quark masses and is of the order of a few GeV^2 .¹⁹

The doublet $(\nu g)_L$ has been included to eliminate triangle anomalies in a way that ensures quark-lepton symmetry. Finally, with left-handed currents only, this model fails to explain the apparent absence of enhanced parity violation in heavy atoms.

Model B

The lepton sector of this model is quasi-vector like, each lepton appearing with left- and right-handedness except the 3 naturally massless neutrinos. There are 3 heavy neutral leptons and, including τ , 4 new charged

leptons. Higgs doublets are sufficient to provide the lepton masses. There are two new quarks, but as in model A, quarks are assigned to left-handed doublets and right-handed singlets only: there are no right-handed charged hadronic currents.²⁰

Although not definitely ruled out, this model is unlikely on the basis of constraint 1). Mixing between μ_L^- and M_L^- permits $\nu_\mu - M_L^-$ coupling which, as in model A, can explain trimuon events if the mixing is large enough.²¹ The determination of $\sin^2 \theta_w$ from ratios of neutral current cross sections is unaffected by this mixing.²²

As the electron current is pure vector, there is no enhanced parity violation in heavy atoms in this model, to lowest order.

Quark-lepton and $\mu - e$ universality are imposed in an unnatural way, by requiring that the mixing between left-handed fermions is symmetric with respect to μ and e and quarks and leptons.

The neutral hadronic current is flavour diagonal, but the neutral leptonic current is not because the $Q = -1$ right-handed leptons do not all have the same isospin assignments.⁸ Hence, this model does not belong to the class discussed in Reference 19 for which there exists a natural suppression mechanism for the effects of fermion mixing. Nor is there suppression as in Cheng & Li's model

in which the singlets are neutral rather than charged leptons and naturality is imposed via a special property of the Higgs system.²³

A flavour diagonal neutral lepton current in this model requires either unnaturally small mixing among right-handed leptons or the imposition of discrete or global symmetries to prevent such mixing.²⁴ A further requirement for this mixing to be very small is to satisfy constraints 9) and 10), by suppressing the contributions to the associated processes that arise from LR (see Figure 12), and RL graphs.²⁵

Figure 12 Left-Right (LR) contribution to the anomalous magnetic moment of the muon.

CP violation is allowed in this model in both lepton and quark sectors. However, with only left-handed quark doublets, the electric dipole moment of the neutron is expected to be very small, of the order of 10^{-30} e.cm.²⁶ This model does not possess any kind of quark-lepton symmetry, either in the numbers of quark and lepton flavours or in their assignment to isospin multiplets.

Model C

This model has the same left-handed doublets as the Weinberg-Salam model with the addition of $(\nu_\tau \tau^-)_L$. Six new heavy leptons are assigned to right-handed doublets. There are two new $Q = -1/3$ quarks with right-handed couplings to the u and c quarks, and a new $Q = 2/3$ quark that mixes with the right-handed u quark:²⁷

$$u_R \rightarrow (u \cos \phi + t \sin \phi)_R$$

$$t_R \rightarrow (-u \sin \phi + t \cos \phi)_R$$

The mixing angle ϕ is restricted by constraint 2iv) to lie in the range $0.800 \leq \cos^2 \phi \leq 0.992$.²⁸

Mixing among the $Q = -1/3$ quarks is restricted by the observed lack of anomalous behaviour in $\nu - N$ scattering cross sections. In the lepton sector, $\mu^- - M^-$ mixing (or

$\nu_\mu - M^0$ mixing) must occur to allow the production of trimuons.

This model can predict parity violating effects in heavy atoms that are an order of magnitude smaller than (and of opposite sign to) the W-S model, due to the right-handed coupling of the u quark. For example, with $\cos^2 \phi = 0.8$:

$$74.4 \leq Q_W \leq 107.6 \quad 0.35 \geq \sin^2 \theta_W \geq 0.25$$

which is just consistent with the present experimental results.²⁹

Again, mixing must be symmetric between μ and e and between quarks and leptons to guarantee universality. However, the remaining experimental constraints: 7) - 11), provide even stronger restrictions on-mixing parameters. To satisfy 7) and 8), it is necessary to assume that no mixing occurs between light and heavy quarks except u_R and t_R . However, if on the basis of constraints 5), 9) and 10), the light and heavy leptons are assumed not to mix, the model would no longer have the coupling assumed necessary for trimuon production.

The above assumptions rule out the possibility of CP violating effects arising from mixing angles, except for among the heavy leptons. As in the case of model B, this model is not quark-lepton symmetric.

The previous discussion shows that the present outlook for $SU(2) \otimes U(1)$ as the gauge group for weak and electromagnetic interactions is not very good. Model B is perhaps the most successful, but very unattractive from the quark-lepton symmetry point of view. The only problem encountered for model A is the experimental lack of parity violation in heavy atoms. This has provided the motivation for many authors to (re) consider models based on higher gauge groups which have no parity violating (i.e. VA) terms in the neutral current. The minimum gauge group with this feature would be $SU(2) \otimes U(1) \otimes U(1)$.³⁰ However, the absence of VA terms in the neutral current is accidental in this gauge unless the $SU(2) \otimes U(1)$ belongs to a larger group, such as $SU(3)$, or the $U(1)$ belongs to another $SU(2)$ with a discrete symmetry existing between the two $SU(2)$ groups.^{31,32} Ma³¹ and Pakvasa³² have considered both possibilities for gauge models with 2 neutral gauge bosons such that the neutral current coupling of the usual quarks and leptons has the form $VV + AA$. The conclusion is that there is only one such model not excluded by the present neutrino data. This is the model based on the ambidextrous (i.e. left-right symmetric) $SU(2)_L \otimes SU(2)_R \otimes U(1)$ gauge group of which model D is an example. The $SU(3) \otimes U(1)$ model considered here, model E, is not of the $VV + AA$ type.

$SU(2) \otimes SU(2) \otimes U(1)$

This gauge group has been discussed previously in the literature, both with³³ and without³⁴ natural left-right symmetry.

Model D

There are four heavy neutral leptons in this model, 2 heavy charged leptons (including τ), and 4 new heavy quarks: 2 with $Q = +2/3$, 2 with $Q = -1/3$. The $Q = +2/3$ quarks appear in a fixed sequence for each handedness, the right-handed $Q = -1/3$ quarks are cyclically rotated with respect to the left-handed $Q = -1/3$ quarks for complete generality. Complete left-right symmetry between elementary fermions could be achieved by introducing right-handed massless neutrinos or by lifting the restriction that the existing left-handed neutrinos be naturally massless.

There are four charged gauge bosons ($W_L^\pm, W_R^\pm$) and three neutral bosons (Z_V, Z_A and γ or A). A natural discrete left-right symmetry is imposed on the Lagrangian which yields only 2 independent gauge couplings: g for both $SU(2)$ groups and g' for the $U(1)$ group. In this model, parameters in the Higgs sector are chosen so that $M_{W_R} \gg M_{W_L}$ with no $W_R - W_L$ mixing. The photon and the neutral bosons can be written in terms of W_{3L}, W_{3R} and the $U(1)$ gauge boson B as follows:

$$Z_A^\mu = (W_{3L}^\mu - W_{3R}^\mu)/\sqrt{2}$$

$$Z_V^\mu = \cos\theta_2 (W_{3L}^\mu + W_{3R}^\mu)/\sqrt{2} - \sin\theta_2 B^\mu$$

$$A^\mu = \sin\theta_2 (W_{3L}^\mu + W_{3R}^\mu)/\sqrt{2} + \cos\theta_2 B^\mu$$

where $\tan\theta_2 = \sqrt{2}g'/g$: θ_2 is the analogue of the Weinberg angle in $SU(2) \otimes U(1)$.

The coefficients of Z_A and Z_V for each left-, right-handed fermion are proportional to:

$$\pm I_3/2\sqrt{2} \quad \text{for } Z_A$$

$$(I_3 - Q\sin^2\theta_2)2\sqrt{2}\cos\theta_2 \quad \text{for } Z_V$$

where $I_3 = +1/2$ ($-1/2$) for top (bottom) doublet members and $Q = I_{3L} + I_{3R} + Y/2$.

The masses of W_L , Z_A and Z_V are related by:

$$M_{Z_A}^2 = M_{W_L}^2 (1 + \epsilon)$$

$$M_{Z_V}^2 = M_{W_L}^2 (1 - \epsilon) \sec^2\theta_2$$

and

$$M_{W_L}^2 = \frac{e^2}{2\sqrt{2}G_F \sin^2\theta_2}$$

where $-1 \leq \epsilon \leq 1$ is related to the fermion masses through the Higgs structure.³⁵

The effective neutrino-nucleon and neutrino-electron neutral couplings of this model reduce to those of the W-S model in the limit $\epsilon = 0$, and $\sin^2 \theta_2 = 2\sin^2 \theta_W$.³⁶ For the neutral current data taken as a whole, the best values for these parameters are $\epsilon \approx 0$ and $\sin^2 \theta_2 \approx 0.5$.³⁷ These values have been found³⁸ to give good agreement with constraint 2iv). They correspond to $M_{Z_A} \approx 75$ GeV which is lower than the mass of the neutral vector boson of the W-S model; the latter is ≈ 88 GeV for $\sin^2 \theta_W = 0.25$.

With $M_{W_R} \gg M_{W_L}$, the right-handed charged current interactions are suppressed at low energies as required by experiment.³⁹ Trimuon events could occur via the production and decay of a heavy quark and/or heavy lepton.

The two neutral bosons, Z_V and Z_A , couple to vector and axial-vector currents respectively. The electron-nucleon coupling is, therefore, naturally parity conserving and no parity-violating effects are expected in atoms, to order G_F . However, this does not mean that neutrino and anti-neutrino cross sections are the same, as in $SU(2) \otimes U(1)$ models with parity conservation. The apparent parity violation suggested

by $\sigma^V \mp \bar{\sigma}^V$ is a consequence of the fact that the neutrino (anti-neutrino) is purely left-handed (right-handed) and its neutral current can couple to both Z_V and Z_A . Parity violation in this model⁴⁰ arises purely from the result of spontaneous breakdown of gauge symmetry, the same mechanism that provides boson and fermion masses, as described in Appendix B. To lowest order, the strength of neutral parity-violating processes is proportional to $G_F \propto \ln(M_{W_R}/M_{W_L})^2$.⁴¹

The neutral hadronic current is naturally flavour-conserving, the condition for this is that left-handed (right-handed) quarks be assigned to $SU(2)_L$ ($SU(2)_R$) doublets. Attempts have been made to calculate quark mixing angles in terms of quark masses.⁴² A calculation in the 6-quark ambidextrous $SU(2)_L \otimes SU(2)_R \otimes U(1)$ model has been done by H. Fritzsch.⁴³

As right-handed charged currents are suppressed, μ - and e-lepton number violating processes will result mainly from mixing in the left-handed sector in a similar fashion to model A. The weak contribution to the anomalous magnetic moment of the muon has been calculated in the 4- and 6-lepton versions of the $SU(2)_L \otimes SU(2)_R \otimes U(1)$ model. In these cases, the experimental limit is found not to place any constraint on neutral gauge boson or heavy lepton masses.⁴⁴

Left- and right-handed mixing angles will contain

CP-violating phases. In general, a quark sector consisting of $2N$ quarks (N with $Q = +2/3$, N with $Q = -1/3$) has a total number of $N(N-1)$ independent and arbitrary phases.⁴⁵ For model D with 8 quarks, both the left- and right-handed mass matrices contain 6 potentially CP-violating phases. With manifest left-right symmetry, i.e. identical left-handed and right-handed mixing, $N(N-1)$ is replaced by $(N-1)(N-2)/2$, which as expected, is the same as for purely left-handed $SU(2) \otimes U(1)$ models. In this case, therefore, models A and D would have the same number (3) of CP-violating phases.

Apart from the differences between the quark and lepton sectors arising from the imposed masslessness of the neutrinos, this model is quark-lepton symmetric.

$SU(3) \otimes U(1)$

Earlier treatments of $SU(3)$ and $SU(3) \otimes U(1)$, including models other than the model E type, can be found in Reference 46.

Model E

This is a model of the type discussed originally by Lee and Weinberg.⁴⁷ The fermions are assigned to both left- and right-handed triplets (3 representations of $SU(3)$), except for the $Q = +2/3$ quarks, which occur in

singlets for one handedness, and extra left-handed lepton singlets present if the neutrinos are the only massless leptons. A discrete symmetry can be imposed such that the neutrinos are naturally massless. There are two new $Q = -1/3$ quarks, two new $Q = +2/3$ quarks, four new $Q = -1$ leptons and 3 new heavy neutral leptons.

A discrete symmetry is imposed which results in fermions of two types: either with even or odd parity under the symmetry operation S . The known fermions (including τ and ν_τ but not the b quark) have the same parity which is designated as even. Mixing can occur between fermions of the same charge, handedness and S parity, i.e. effectively among the following combinations:

$$u_L - c_L, b_L - g_L - t_R - \nu_R, b_R - g_R, E_L^- - M_L^- - T_L^-,$$

$$E_R^0 - M_R^0 - T_R^0, \text{ and } E_R^- - M_R^- - T_R^-.$$

However, an additional discrete symmetry is imposed to prevent mixing between b and g and between E^-, M^- and T^- . The lightest of the neutral heavy leptons (generally taken to be E^0), will be absolutely stable.

The gauge group $SU(3) \otimes U(1)$ has nine bosons, denoted by V_a^μ , $0 \leq a \leq 8$. In model E, those of definite mass are the four charged bosons ($V^\pm, W^\pm$) and five

neutral bosons (Z, Y, U, $\bar{U}$ and γ or A). V, W, Y, Z and A are given by:

$$V^{\pm\mu} = (V_4^{\mu} \mp iV_5^{\mu})/\sqrt{2}$$

$$W^{\pm\mu} = (V_1^{\mu} \mp iV_2^{\mu})/\sqrt{2}$$

$$Y^{\mu} = (V_3^{\mu} - \sqrt{3} V_8^{\mu})/2$$

$$Z^{\mu} = \cos\theta_3(\sqrt{3} V_3^{\mu} + V_8^{\mu})/2 - \sin\theta_3 V_0^{\mu}$$

$$A^{\mu} = \sin\theta_3(\sqrt{3} V_3^{\mu} + V_8^{\mu})/2 + \cos\theta_3 V_0^{\mu}$$

where $\tan\theta_3 = g'/\sqrt{3} g$, g and g' being the coupling strengths of the SU(3) and U(1) groups respectively.

W, V and U couple the first and second, first and third, and second and third fermions in a column respectively. Weak diagonal neutral current couplings occur through both Y and Z: the former is pure axial-vector and the latter has no neutrino term.

The coefficients of Y and Z couplings relative to those of W, for fermions of each handedness are:

$$(I_3 - 3y')/\sqrt{2} \quad \text{for Y}$$

$$\sqrt{3} (I_3 + y' - Q\sin^2\theta_3)/\sqrt{2} \cos\theta_3 \quad \text{for Z}$$

where $I_3 = +1/2, -1/2, 0$ and $y' = 1/6, 1/6, -1/3$ for the first, second and third triplet members and $Q = I_3 + y + y'$.

The masses of Y , Z and W are related by:

$$M_Z^2 = M_W^2 4\eta^2 / 3\cos^2\theta_3$$

$$M_Y^2 = M_W^2 4(1 - \eta^2)$$

where $0 \leq \eta^2 \leq 1$ is related to fermion masses through the Higgs structure.⁴⁸

The neutral current interactions involving the light fermions (including the τ lepton) are mediated by the Z and Y bosons which automatically conserve all quark and lepton flavours to order αG_F . There are no non-diagonal neutrino neutral currents. (The U bosons do not couple to neutrinos). A good fit with most neutral current data is possible with $\eta^2 \approx 0.85$, $\sin^2\theta_3 \approx 0.25$.⁴⁹ However, as this model predicts $\beta = \delta$, it is not consistent with constraint 2iv).³⁸

The charged current interactions of the same light fermions are mediated by $W^\pm$, and the couplings are the same as those in the Weinberg-Salam model (with the addition of $(\nu_\tau - \tau)_L$): they satisfy both quark-lepton and $u - e$ universality. There is no $(u\ b)_R$ coupling via

any of the charged gauge bosons.

An important feature of this model is that in neutrino reactions, heavy quark and heavy lepton production necessarily occur together, via exchange of the V boson. At sufficiently high energies, the following reactions would be expected:

$$\nu_\mu + d_R \rightarrow M^- + t_R \quad \text{i)}$$

$$\nu_\mu + d_R \rightarrow M^- + \bar{\nu}_R \quad \text{ii)}$$

process ii) is made possible by $t_R - \nu_R$ mixing.

Similarly:

$$\bar{\nu}_\mu + u_L \rightarrow M^+ + b_L \quad \text{iii)}$$

As there is no $b_L - g_L$ mixing, the only neutrino reaction producing the g quark is:

$$\bar{\nu}_\mu + c_L \rightarrow M^+ + g_L \quad \text{iv)}$$

However, in neutrino-nucleon interactions, the contribution to heavy lepton production from process iv) is negligible as it occurs off sea quarks only.

Trimuon events would result from the subsequent

cascade decay of the M leptons, the dominant decay modes being:

$$M^- \rightarrow M^0 + \bar{\nu}_\mu + \mu^-$$

$$L \rightarrow \mu^- + \mu^+ + E^0$$

and

$$M^+ \rightarrow \bar{M}^0 + \nu_\mu + \mu^+$$

$$L \rightarrow \mu^+ + \mu^- + \bar{E}^0$$

M^0 decays to the stable E^0 (via $E^0 - M^0$ mixing) instead of ν_μ as is the case for models A - D. The thresholds associated with heavy quark production impose strong kinematic constraints on this mechanism for trimuon production. Assuming that the heavy quarks have mass ≥ 5 GeV, it has been shown that the visible hadron energy of one trimuon event lies outside the kinematic region allowed by this mechanism.⁵⁰

In all models, it has been assumed that b , the $T(9.5)$ quark has $Q = -1/3$. In this case, a new $Q = +2/3$ quark must have mass > 5.5 GeV.⁵¹ The main contribution to ν_μ -induced trimuon events comes from process 1). For $M_t > 5.5$ GeV, the observed trimuon rate can be achieved for $M_t \approx 7$ GeV. (This is less than the lower limit of 7.5 GeV established for full-strength

conventional $\nu_\mu - M^-$ coupling).

The contribution to the total inclusive ν and $\bar{\nu}$ cross sections from processes i) - iv) would be small ($< 1\%$),⁵¹ and hence, there would be no sizeable high- y anomaly in anti-neutrino cross sections.

The slow muons from the semi-leptonic decay of the heavy quark, if detected, would contribute to both right and wrong sign multilepton events and also tetramuons. For example:

$$M^- \rightarrow \mu^- \mu^- \mu^+ E^0 \bar{\nu}_\mu$$

$$t \rightarrow d E^0 \mu^+$$

The tetramuon events would be expected typically to have two relatively fast muons of opposite charge, and two slower muons,⁵² which is not inconsistent with the single tetramuon event reported to date (see Chapter I). Reference 51 contains a calculation of the contributions to the rates of ν_μ - and $\bar{\nu}_\mu$ -induced multilepton production from this mechanism, assuming in each case that the heavy quark involved is the T (9.5) constituent. The calculated event rates are found to be within easy reach of experiments at Fermilab and CERN.

The weak electron-nucleon interaction is mediated by the Y and Z neutral bosons. As Y exchange is parity-

conserving and the coupling of the electron to the Z boson is pure vector, this model predicts no enhancement of parity non-conservation in heavy atoms, in agreement with present experimental results.

If E^0 and M^0 mix, as is required in this model for trimuon production, processes such as $\mu \rightarrow e\gamma$ will occur in much the same manner as described by Cheng and Li⁵³, but with no LR or RL terms. The expected branching ratio for $\mu \rightarrow e\gamma$ for example, is $\sim 10^{-10}$.⁴⁷ By a suitable choice of parameters in the Higgs sector, this model can predict acceptably small values for the weak contributions to the anomalous magnetic moments of μ and e . CP-violating effects can occur, for typical values of the parameters involved, the neutron is expected to have an electric dipole moment $\leq 10^{-24}$ e.cm.⁴⁷

As presented in Table II, model E is not quark-lepton symmetric. However, the asymmetry could be removed if required, by including 4 new quarks and assigning them to a triplet and singlet of each handedness.

Summary and Future Outlook

The Weinberg-Salam model can at present explain all neutral current (anti) neutrino scattering data with an average value of $\sin^2\theta_w \approx 0.33$. It is also consistent with the (more recent) deep-inelastic charged current

Table III

Model	Constraint														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	✓	✓	✓	x	?	?	✓	✓	✓	✓	✓	x	✓	✓	✓
B	?	✓	✓	✓	?	?	✓	✓	✓	✓	✓	x	✓	✓	x
C	✓	x	✓	✓	✓	✓	✓	✓	✓	✓	✓	x	✓	✓	x
D	✓	✓	✓	✓	?	?	✓	✓	✓	✓	✓	✓	✓	✓	✓
E	✓	x	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	x

Properties of models given in Table II in terms of the constraints discussed in the text.

✓: Agreement

?: Possible inconsistency or reason(s) for lack of complete agreement

x: Disagreement

neutrino-nucleon scattering results, apart from the difficulty in accounting for the observed trimuon events. The W-S model cannot account for the evidence for new heavy leptons and quarks that has been obtained from e^+e^- annihilation and pN collision experiments. Furthermore, with only left-handed currents, the W-S model predicts much larger parity violating effects in heavy atoms than are presently reported from the bismuth experiments.

Attempts to extend the W-S model within the framework of $SU(2) \otimes U(1)$ (with the conventional quark and lepton charges), in the form of models A, B and C has been shown to lead to serious difficulties. A more elegant solution would be to extend the $SU(2) \otimes U(1)$ gauge group itself to provide more degrees of freedom. That this can be done relatively easily is shown by models D and E which are examples of models based on the gauge groups $SU(2) \otimes SU(2) \otimes U(1)$ and $SU(3) \otimes U(1)$ respectively.

When greater statistics are available on multilepton events, it should be possible to determine the mode(s) of production and to obtain more precise values for the masses of any new heavy particles involved (and to fix the charge of the T (9.5) quark).

Further information on the couplings between the familiar quarks and leptons will be provided by the results

of searches for parity violating effects in weak interactions as has been discussed in Chapter III. In particular, the experiments investigating the electron-nucleon neutral current coupling hope to distinguish between $SU(2) \otimes U(1)$ models and between various models based on larger gauge groups. The expected asymmetries in polarized electron-proton deep-inelastic scattering are shown in Figure 13.⁵⁴ Table IV gives expressions for the parameters C_{1p} , C_{1n} , C_{2p} and C_{2n} , which can be measured in light atom parity violation experiments.⁵⁵

The experimental upper bounds on muon-number violating processes, such as $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ and $\mu N \rightarrow eN$ provide strong restrictions on the couplings that can occur within specific gauge models. An experiment currently running at LAMPF is designed to detect the decay $\mu \rightarrow e\gamma$ if it occurs with a branching ratio, R_γ , of at least 1×10^{-10} , or to yield an upper bound of about 5×10^{-11} . This would be a significant advance over the present bound of 1.1×10^{-9} .³

Of the above three processes, recent calculations have shown that muon conversion may be the most sensitive test for muon-number non-conservation for a large class of gauge models which allow such processes to occur.⁵⁶ For the models considered here, the only possibility for

Figure 13 The asymmetry in deep-inelastic polarized electron-proton scattering at $x = 1/3$ as a function of y for the models discussed in the text. Curves calculated by Cahn and Gilman in Ref. 54. The scaling variables x and y are defined in Appendix A.

Table IV

Parameter				
Model	C_{1p}	C_{2n}	C_{2p}	C_{2n}
A	$K(1-4\sin^2\theta_w)/2$ $0.0 \rightarrow -0.2$	$-K/2$ -0.5	$K(1-4\sin^2\theta_w)(F+D)/2$ $0.0 \rightarrow -0.25$	$-K(1-4\sin^2\theta_w)(F+D)/2$ $0.0 \rightarrow +0.25$
B	0	0	$K(1-2\sin^2\theta_w)(F+D)$	$-K(1-2\sin^2\theta_w)(F+D)$ $-0.625 \rightarrow -0.375$
C	$K(1+2\cos^2\phi-4\sin^2\theta_w)/2$ $0.906 \rightarrow 0.706$	$-K(1-\cos^2\phi)/2$ -0.047	$K(1-4\sin^2\theta_w) \times$ $(F+D-2F\cos^2\phi)/2$ $0.0 \rightarrow -0.096$	$-K(1-4\sin^2\theta_w) \times$ $(F+D+(F-D)\cos^2\phi)/2$ $0.0 \rightarrow +0.178$
D	0	0	0	0
E	0	0	$4K'(\cos^2\theta_3-2/3)F$ $+ 0.227$	$2K'(\cos^2\theta_3-2/3)(F-D)$ -0.107

Expressions for the coupling parameters relevant to parity violation in atoms, adapted from Ref. 55.

$K = M_W^2 \sec^2\theta_w/M_Z^2$, $F+D = g_A$. Numerical values correspond to: $K=1$, $K'=1$, $F=0.425$, $D=0.825$, $\sin^2\theta_w = 0.25 \rightarrow 0.35$, $\cos^2\phi = 0.906$ and $\cos^2\theta_3 = 0.8$ as applicable.

the transition $\mu \rightarrow e$ arising from mixing in the lepton sector, is via an intermediate neutral lepton. In this case, the relative rate for $\mu N \rightarrow e N$, R_N , may be a factor of 10 or more larger than the branching ratio for $\mu \rightarrow e \gamma$. This enhancement over the expected value of $R_N/R_Y \sim \alpha/\pi$ is due to large logarithmic contributions to the amplitude for $\mu N \rightarrow e N$ which, in addition, sum coherently over all quarks in the nucleus. These effects occur at the amplitude level and are squared in calculating the transition rate. The ratio R_N/R_Y is a strongly model dependent quantity so that a simultaneous detection of these processes would give important information on the structure of quarks and leptons. The only drawback of muon conversion from a theoretical point of view is that it involves the weak interactions of quarks which automatically implies some uncertainty in rate estimates. From an experimental viewpoint, muon conversion is easier to perform than either $\mu \rightarrow e \gamma$ or $\mu \rightarrow 3e$ because only a single particle need be detected. The experimental signature of conversion is very clear: a monoenergetic electron with energy $\sim m_\mu = 105$ MeV. This is a factor of 2 greater than the maximum energy of an electron resulting from ordinary muon decay.

The present upper bound of 4×10^{-10} was obtained with a sulphur target at SIN.⁵ This experiment can be repeated with other targets. An experiment recently

proposed for TRIUMF⁵⁷, however, is designed to be a factor of 10^2 more efficient than this at detecting muon conversion and hopes to push the present upper limit of 1.6×10^{-8} obtained using a copper target⁵⁸ down to $10^{-11} - 10^{-12}$. The ultimate limit will depend on the main background processes of ordinary decay ($\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$) with the nucleus taking all the momentum of the neutrinos, and radiative capture ($\mu N \rightarrow \nu_\mu N' \gamma$) followed by $\gamma \rightarrow e^+ e^-$ conversion in the target.

A search for the process $\mu N \rightarrow e N$ may also provide information on the Higgs sector. There is the possibility of a Higgs boson being coherently produced via $\mu \rightarrow e$ conversion in the Coulomb field of the nucleus:

$$\mu^- + N \rightarrow e^- + \phi^0 + N$$

The cross section for this process, calculated using the model of Bjorken & Weinberg¹³, is rather small even for the highest Fermilab energies ($< 10^{-40} \text{ cm}^2/\text{nucleon}$ in $^{208}_{82}\text{Pb}$ with $m_\phi \approx 10 \text{ GeV}$).⁵⁹ However, the leptonic decays of the Higgs boson:

$$\phi^0 \rightarrow e^- \mu^+, \quad \phi^0 \rightarrow \mu^- \mu^+$$

would provide a distinctive signal. Replacing μ by ν_μ

could lead to such processes as:

$$\nu_\mu + N \rightarrow e^- + \phi^+ + N$$

$$\hookrightarrow \nu_\mu + \mu^+$$

and

$$\nu_\mu + N \rightarrow \tau^- + \phi^+ + N$$

If $m_\phi > 5$ GeV, (see Appendix B), the latter interaction would be difficult to detect ($\lesssim 10^{-40}$ cm²/nucleon) but a possible signature is provided by the following leptonic decays:

$$\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau$$

$$\phi^+ \rightarrow \tau^+ \nu_\mu$$

$$\hookrightarrow \ell^+ \nu_\ell \bar{\nu}_\tau$$

This would lead to opposite sign dilepton events with no hadronic energy.

Finally, as model D is by far the most successful of the alternatives considered here, several other experimental consequences of this model will be mentioned. To be consistent with the present neutral current data; $\epsilon \approx 0$ and $\sin^2 \theta_2 \approx 0.5$. For these values, the neutral

vector boson Z_A of model D is ~ 10 - 20 GeV lighter than the $SU(2) \otimes U(1)$ neutral vector boson. This enhances the possibility of detecting its effects in e^+e^- annihilation experiments in the PETRA and PEP energy regime. An interesting point is that the couplings of Z_V to charged lepton pairs (e^+e^- , $\mu^+\mu^-$ etc.) involve $\cos 2\theta_2$, which vanishes for $\sin^2 \theta_2 = 0.5$. Thus, only Z_A would be expected to show up as a peak in the dimuon effective mass distribution in pp collisions at the proposed Isabelle energies.³⁷ Recently, Gupta and Roy⁶⁰ have derived some general inequalities for inclusive neutral current cross sections in ambidextrous $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models using the naive quark-parton model. When the coupling constants C_1 and C_2 , and the total ν and $\bar{\nu}$ inclusive neutral current cross sections are available for protons and neutrons separately, it will be possible to use these results to distinguish between $SU(2) \otimes U(1)$ models and ambidextrous $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models, such as model D.

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APPENDIX A

(ANTI) NEUTRINO SCATTERING CROSS SECTION FORMULAE

Elastic (Anti) Neutrino-Electron Scattering

For elastic neutrino-electron scattering, the effective coupling is¹:

$$L(\bar{\nu}e \rightarrow \nu e) = \frac{G_F}{\sqrt{2}} [\bar{\nu}_\mu \gamma_\alpha (1 - \gamma_5) \nu_\mu \bar{e} \gamma^\alpha (g_V + g_A \gamma_5) e \\ + \bar{\nu}_e \gamma_\alpha (1 - \gamma_5) \nu_e \bar{e} \gamma^\alpha (G_V + G_A \gamma_5) e]$$

g_V and g_A are the vector and axial-vector coupling constants and G_V and G_A include the charged current contribution to $\nu_e - e$ scattering. Assuming $\mu - e$ universality:

$$G_V = 1 + g_V, \quad G_A = 1 + g_A$$

A straight-forward calculation results in the following expressions for the unpolarized differential cross section in the lab frame:

$$\frac{d\sigma}{dE_e}(\nu_\mu, \bar{\nu}_\mu) = \frac{G_F^2 m_e}{2\pi} [(g_V \pm g_A)^2 + (g_V \mp g_A)^2 (1 - \frac{E_e}{E_\nu})^2 - \frac{m_e E_e}{E_\nu^2} (g_V^2 - g_A^2)]$$

where $E_\nu(E_e)$ is the energy of the incident neutrino (recoil electron) and m_e is the electron mass. The upper signs refer

to ν_μ , the lower to $\bar{\nu}_\mu$. Similarly:

$$\frac{d\sigma}{dE_e}(\nu_e, \bar{\nu}_e) = \frac{G_F^2 m_e}{2\pi} [(G_V \pm G_A)^2 + (G_V \mp G_A)^2 (1 - \frac{E_e}{E_\nu})^2 - \frac{m_e E_e}{E_\nu} (G_V^2 - G_A^2)]$$

In $SU(2) \otimes U(1)$ gauge models:

$$g_V = \frac{1}{2} (I_{3L}^e + I_{3R}^e + 2 \sin^2 \theta_w), \quad g_A = \frac{1}{2} (I_{3L}^e - I_{3R}^e)$$

where $\kappa = \frac{M_Z}{M_W} \cos \theta_w$, and $I_{3L,R}^e$ are the third components of left-handed, right-handed weak isospin of the electron.

In the Weinberg-Salam model with $\kappa = 1$, $I_{3L}^e = -\frac{1}{2}$ and $I_{3R}^e = 0$:

$$g_V = -\frac{1}{2} + 2 \sin^2 \theta_w, \quad g_A = -\frac{1}{2}$$

$$G_V = \frac{1}{2} + 2 \sin^2 \theta_w, \quad G_A = \frac{1}{2}$$

(Anti) Neutrino-Nucleon Scattering

Elastic and Quasi-Elastic Scattering²⁻⁵

For quasi-elastic (i.e. charged current) $\nu - N$ scattering, the effective coupling is:

$$L(\nu_\mu N \rightarrow \mu N') = \frac{G_F}{\sqrt{2}} [\bar{u}(p') \gamma_\mu (1 - \gamma_5) u(p) \bar{u}(P') \{ \gamma^\mu (g_V + 2Mf_V - g_A \gamma_5) - (P + P')^\mu f_V \} u(P)]$$

Time reversal invariance implies that the form factors g_V , g_A and f_V are real. Second class currents have been omitted. Kinematic variables are defined in Figure A.1. The unpolarized differential cross section in the lab frame takes the following form:

$$\frac{d^2\sigma^{\nu, \bar{\nu}}}{dE' dq^2} = \frac{G_F^2}{4\pi ME^2} \{ p_\mu p'_\nu + p_\nu p'_\mu - g_{\mu\nu} p \cdot p' \mp i\epsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta \} W^{\mu\nu}$$

The upper sign refers to ν , the lower to $\bar{\nu}$. Terms proportional to the mass of the scattered lepton have been neglected.

Figure A.1 Charged current muon neutrino-nucleon scattering in the lab frame.

$$\begin{aligned}
W^{\mu\nu} = & \frac{1}{2} \{ (P^\mu P'^\nu + P^\nu P'^\mu) [(g_V + 2Mf_V)^2 + g_A^2 + f_V^2 (P \cdot P' + M^2) - 2Mf_V (g_V + 2Mf_V)] \\
& + (P^\nu P'^\mu + P'^\nu P^\mu) [f_V^2 (P \cdot P' + M^2) - 2Mf_V (g_V + 2Mf_V)] \\
& - g^{\mu\nu} [P \cdot P' ((g_V + 2Mf_V)^2 + g_A^2) - M^2 ((g_V + 2Mf_V)^2 - g_A^2)] \\
& - 2i(g_V + 2Mf_V)g_A \epsilon^{\nu\mu\alpha\beta} P_\alpha P'_\beta \} \quad (A.1)
\end{aligned}$$

Contracting $W^{\mu\nu}$ with the leptonic part gives:

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{dE'dq^2} = \frac{G_F^2}{2\pi M^2 E^2} \{ A(q^2) + B(q^2)(q^2 + 4ME) + C(q^2)(q^2 + 4ME)^2 \} \quad (A.2)$$

and

$$\frac{d\sigma^{\nu,\bar{\nu}}}{dq^2} = \frac{G_F^2}{8\pi M^2 E^2} \{ A(q^2) + B(q^2)(q^2 + 4ME) + C(q^2)(q^2 + 4ME)^2 \} \quad (A.3)$$

where

$$A(q^2) = \frac{1}{4}q^2(q^2 - 4M^2)g_A^2 + \frac{1}{4}q^2(q^2 + 4M^2)[g_V^2 + q^2f_V^2] + 2q^4Mg_Vf_V$$

$$B(q^2) = q^2g_A(g_V + 2Mf_V)$$

$$C(q^2) = \frac{1}{4}(g_V^2 - q^2f_V^2 + g_A^2)$$

The charged current form factors $G_E^\pm$, $G_M^\pm$ and $G_A^\pm$ are defined as follows:

$$G_E^\pm = 2(g_V - \frac{Q^2}{2M}f_V) \quad , \quad G_M^\pm = 2(g_V + 2Mf_V) \quad , \quad G_A^\pm = g_A$$

They are functions of q^2 or $Q^2 = -q^2 \geq 0$.

Inverting the above relationships:

$$g_V = \frac{G_E^+ + \tau G_M^+}{2(1 + \tau)}, \quad f_V = \frac{G_M^+ - G_E^+}{4M(1 + \tau)}, \quad \text{where } \tau \equiv Q^2/4M^2$$

In terms of Q^2 , G_E^+ , G_M^+ and G_A^+ :

$$A(Q^2) = M^2 Q^2 [(1 + \tau) G_A^{+2} - \frac{1}{4} G_E^{+2} + \frac{1}{4} G_M^{+2}]$$

$$B(Q^2) = \frac{Q^2}{2} G_A^+ G_M^+$$

$$C(Q^2) = \frac{1}{4} [G_A^{+2} + \frac{(G_E^{+2} + \tau G_M^{+2})}{4(1 + \tau)}]$$

and equation (A.3) becomes:

$$\begin{aligned} \frac{d\sigma^{v,\bar{v}}}{dQ^2} = & \frac{G_F^2}{8\pi M^2 E^2} \{ M^2 Q^2 [(1 + \tau) G_A^{+2} - \frac{1}{4} G_E^{+2} + \frac{1}{4} G_M^{+2}] \\ & + \frac{Q^2}{2} G_A^+ G_M^+ (Q^2 - 4ME) + \frac{1}{4} [G_A^{+2} + \frac{(G_E^{+2} + \tau G_M^{+2})}{4(1 + \tau)}] (Q^2 - 4ME)^2 \} \end{aligned} \quad (A.4)$$

For elastic (i.e. neutral current: $\nu N \rightarrow \nu N$) scattering;

G_E^+ , G_M^+ and G_A^+ are replaced by the neutral current form factors G_E^0 , G_M^0 and G_A^0 respectively.

The conserved vector current (CVC) hypothesis states that the weak charged vector currents belong to the same isospin triplet as the isovector part of the electromagnetic current. If this hypothesis holds, the charged current form factors

$G_E^\pm$ and $G_M^\pm$ are related to the Sachs electromagnetic form factors G_E and G_M as follows:

$$G_E^\pm = 2 \cos\theta_c G_E$$

$$G_M^\pm = 2 \cos\theta_c G_M$$

The factor $\cos\theta_c$ takes into account the d - s quark Cabibbo mixing. Writing $G_A^\pm \equiv \cos\theta_c G_A$, equation (A.4) for the charged current cross section becomes:

$$\begin{aligned} \frac{d\sigma^{\nu, \bar{\nu}}}{dQ^2} &= \frac{\cos^2\theta_c G_F^2}{8\pi M^2 E^2} \{ M^2 Q^2 [(1 + \tau) G_A^2 - G_E^2 + \tau G_M^2] \\ &\quad + Q^2 G_A G_M (Q^2 - 4ME) + \frac{1}{4} [G_A^2 + \frac{(G_E^2 + \tau G_M^2)}{1 + \tau}] (Q^2 - 4ME)^2 \} \end{aligned} \quad (A.5)$$

Both quasi-elastic and elastic scattering can be described by the form factors $W_1(Q^2)$, $W_2(Q^2)$ and $W_3(Q^2)$. For quasi-elastic scattering they are defined as:

$$W_1 = (1 + \tau) G_A^2 + \tau G_M^2$$

$$W_2 = G_A^2 + \frac{(G_E^2 + \tau G_M^2)}{1 + \tau}$$

$$W_3 = -2G_M G_A$$

For elastic scattering the analogous definitions are:

$$W_1 = (1 + \tau) G_A^{02} + \frac{\tau}{4} G_M^{02}$$

$$W_2 = G_A^{02} + \frac{(G_E^{02} + \tau G_M^{02})}{4(1 + \tau)}$$

$$W_3 = - G_M^0 G_A^0$$

In terms of the W's:

$$A(Q^2) = M^2 Q^2 [2W_1 - (1 + \tau)W_2]$$

$$B(Q^2) = - \frac{Q^2}{2} W_3$$

$$C(Q^2) = \frac{1}{4} W_2$$

From equations (A.2) and (A.3), the differential cross sections in the lab frame become:

$$\begin{aligned} \frac{d\sigma^{\nu, \bar{\nu}}}{dQ^2} &= \frac{\lambda G_F^2}{32\pi M^2 E^2} \{ Q^2 [8M^2 W_1 - (Q^2 + 4M^2)W_2] + (Q^2 - 4ME)^2 W_2 \\ &\quad \pm 2Q^2(Q^2 - 4ME)W_3 \} \end{aligned} \quad (A.6)$$

and

$$\frac{d^2\sigma^{\nu, \bar{\nu}}}{dE' dQ^2} = \frac{\lambda G_F^2}{2\pi M} \frac{E'}{E} \left\{ \frac{Q^2}{2EE'} W_1 + \left(1 - \frac{Q^2}{4EE'}\right) W_2 \mp \frac{(E + E')}{M} \frac{Q^2}{4EE'} W_3 \right\} \quad (A.7)$$

where $\lambda = \begin{cases} \cos^2 \theta_c & \text{for quasi-elastic scattering } \nu N \rightarrow \ell N' \\ 1 & \text{for elastic scattering } \nu N \rightarrow \nu N. \end{cases}$

and E , E' and Q^2 are related by $Q^2 = 2M(E' - E)$.

The processes of interest are:

$$\nu_\mu + p \rightarrow \nu_\mu + p$$

$$\bar{\nu}_\mu + p \rightarrow \bar{\nu}_\mu + p$$

and

$$\nu_\mu + n \rightarrow \mu^- + p$$

$$\bar{\nu}_\mu + p \rightarrow \mu^+ + n$$

The form factors G_E , G_M and G_A are conventionally described by dipole forms as follows:

$$G_E(Q^2) = \left(1 + \frac{Q^2}{M_V^2}\right)^{-2}$$

$$G_M(Q^2) = (1 + \mu_p - \mu_n) \left(1 + \frac{Q^2}{M_V^2}\right)^{-2} = 4.7 \left(1 + \frac{Q^2}{M_V^2}\right)^{-2}$$

$$G_A(Q^2) = g_A(0) \left(1 + \frac{Q^2}{M_A^2}\right)^{-2} = 1.24 \left(1 + \frac{Q^2}{M_A^2}\right)^{-2}$$

μ_p and μ_n are the anomalous magnetic moments of proton and neutron. The squares of the vector and axial masses are

$M_V^2 = 0.74 \text{ GeV}^2$ and $M_A^2 \approx 1 \text{ GeV}^2$ respectively.⁶

The isovector parts of the neutral and charged weak currents are members of the same isospin triplet. The neutral current form factors G_E^0 , G_M^0 and G_A^0 are, therefore, related to the charged current form factors and hence to G_E , G_M and G_A :

$$G_E^0(Q^2) = \frac{1}{2} (\alpha + \gamma) G_E(Q^2)$$

$$G_M^0(Q^2) = \frac{1}{2} \left(\alpha + \left(\frac{1 + \mu_p + \mu_n}{1 + \mu_p - \mu_n} \right) \gamma \right) G_M(Q^2) = \frac{1}{2} \left(\alpha + \frac{0.88}{4.7} \gamma \right) G_M(Q^2)$$

$$G_A^0(Q^2) = \frac{1}{2} (\beta + \delta) G_A(Q^2)$$

The parameters α , β , γ and δ are model dependent and are the coefficients of the various components of the valence quark (i.e. u and d) contributions to the weak hadronic neutral current J_μ^{nc} :

$$J_\mu^{\text{nc}}(u,d) = \alpha V_\mu^3 - \beta A_\mu^3 + \frac{1}{3} \gamma V_\mu^0 - \delta A_\mu^0 \quad (\text{A.8})$$

V_μ and A_μ are vector and axial-vector components; the superscripts 3 and 0 are used to denote isovector and isoscalar parts respectively.

The weak hadronic neutral current in $SU(2) \otimes U(1)$ gauge theories can be written as:

$$J_{\mu}^{nc} = \frac{1}{2} \sum_L \bar{\psi}_L \tau_3 \gamma_{\mu} (1 - \gamma_5) \psi_L + \frac{1}{2} \sum_R \bar{\psi}_R \tau_3 \gamma_{\mu} (1 + \gamma_5) \psi_R - 2 \sin^2 \theta_w J_{\mu}^{em} \quad (A.9)$$

where the sums are over all left-handed (L) and right-handed (R) doublets in a particular model, $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and J_{μ}^{em} is the electromagnetic current:

$$J_{\mu}^{em} = \sum_i q_i \bar{\psi}_i \gamma_{\mu} \psi_i$$

Here the sum is over different quark flavours in the model,

q_i being the charge of the i th quark.

The various components of the neutral current in the u, d sector are defined as:

$$V_{\mu}^3 = \frac{1}{2} (\bar{u} \not{\partial}) \tau_3 \gamma_{\mu} \begin{pmatrix} u \\ d \end{pmatrix}, \quad A_{\mu}^3 = \frac{1}{2} (\bar{u} \not{\partial}) \tau_3 \gamma_{\mu} \gamma_5 \begin{pmatrix} u \\ d \end{pmatrix}$$

$$V_{\mu}^0 = \frac{1}{2} (\bar{u} \not{\partial}) \gamma_{\mu} \begin{pmatrix} u \\ d \end{pmatrix}, \quad A_{\mu}^0 = \frac{1}{2} (\bar{u} \not{\partial}) \gamma_{\mu} \gamma_5 \begin{pmatrix} u \\ d \end{pmatrix}$$

or, in terms of single quark wave functions:

$$V_{\mu}^3 = \frac{1}{2} (\bar{u} \gamma_{\mu} u - \bar{d} \gamma_{\mu} d), \quad A_{\mu}^3 = \frac{1}{2} (\bar{u} \gamma_{\mu} \gamma_5 u - \bar{d} \gamma_{\mu} \gamma_5 d)$$

$$V_{\mu}^0 = \frac{1}{2} (\bar{u} \gamma_{\mu} u + \bar{d} \gamma_{\mu} d), \quad A_{\mu}^0 = \frac{1}{2} (\bar{u} \gamma_{\mu} \gamma_5 u + \bar{d} \gamma_{\mu} \gamma_5 d)$$

In any model, the electromagnetic current of the u and d quarks

has the form:

$$\begin{aligned} J_{\mu}^{\text{em}}(u,d) &= \frac{2}{3} \bar{u} \gamma_{\mu} u - \frac{1}{3} \bar{d} \gamma_{\mu} d \\ &= \frac{1}{2} (\bar{u} \ d) \left(\tau_3 + \frac{1}{3} \right) \begin{pmatrix} u \\ d \end{pmatrix} \end{aligned}$$

$$\text{i.e. } J_{\mu}^{\text{em}}(u,d) = V_{\mu}^3 + \frac{1}{3} V_{\mu}^0$$

In $SU(2) \otimes U(1)$ gauge models, with the standard weak isospin assignments for the left-handed u and d quarks:

$$\alpha = \frac{1}{\kappa^2} (1 - 2 \sin^2 \theta_W + I_{3R}^u - I_{3R}^d)$$

$$\beta = \frac{1}{\kappa^2} (1 - I_{3R}^u + I_{3R}^d)$$

$$\gamma = \frac{1}{\kappa^2} (-2 \sin^2 \theta_W + 3I_{3R}^u + 3I_{3R}^d)$$

$$\delta = \frac{-\epsilon}{\kappa^2} (I_{3R}^u + I_{3R}^d)$$

where $I_{3R}^{u,d}$ are the third components of right-handed weak isospin, and ϵ represents the relative ratio of the isoscalar to isovector axial-vector current matrix elements.

In the Weinberg-Salam model with $\kappa = 1$, $I_{3R}^{u,d} = 0$:

$$\alpha = 1 - 2 \sin^2 \theta_W, \beta = 1, \gamma = -2 \sin^2 \theta_W, \delta = 0.$$

The specific expressions for various other $SU(2) \otimes U(1)$ models are given in Table II of Reference 5.

Deep-Inelastic Inclusive Scattering⁷

The processes under consideration are both charged current ($\nu N \rightarrow \ell X$), and neutral current ($\nu N \rightarrow \nu X$), where X represents some final hadron state. For scattering in the lab frame, the scaling variables are defined as:

$$x = \frac{Q^2}{2M\nu} \quad 0 \leq x \leq 1$$

$$y = \frac{\nu}{E} \quad 0 \leq y \leq 1$$

where $\nu = E - E'$ is the energy transfer to the hadron system.

In the limit $Q^2 \rightarrow \infty$ such that x is fixed, the scaling behaviour is:

$$W_1(Q^2) \rightarrow F_1(x)$$

$$\frac{\nu}{M} W_2(Q^2) \rightarrow F_2(x) \quad (A.10)$$

$$\frac{\nu}{M} W_3(Q^2) \rightarrow F_3(x)$$

Equation (A.7) becomes:

$$\frac{d^2\sigma^{\nu, \bar{\nu}}}{dx dy} = \frac{G_{FME}^2}{\pi} \{ y^2 x F_1(\nu, \bar{\nu}) N + (1-y) F_2(\nu, \bar{\nu}) N \mp (1-y/2) y x F_3(\nu, \bar{\nu}) N \} \quad (A.11)$$

Assuming charge symmetry invariance and for scattering

from isoscalar targets: $F_i^{\nu N} = F_i^{\bar{\nu} N}$, $i = 1, 2, 3$.

Expressions for the F_i 's can be calculated using the quark-parton model of deep-inelastic lepton-hadron scattering.⁸⁻¹⁰

In this model the interaction is viewed in the infinite momentum frame of the hadron, which is treated as a collection of co-moving point-like partons (quarks). For sufficiently large Q^2 , the impulse approximation is assumed valid, i.e. the incident lepton scatters from an essentially free quark, independently of the rest (see Figure A.2). The total cross section is therefore an incoherent sum over lepton-quark scattering cross sections.

Each quark has a finite fraction ξ_i of the longitudinal hadron momentum. For ξ_i not too small ($\xi_i \geq 1/P$) the approximation $p_i = \xi_i P$ is reasonable, where $p_i(P)$ is the initial quark (hadron) 4-momentum. The phenomenon of scaling is expected at high energies where the quark and hadron masses may be neglected with respect to the kinematic variables. Then the amplitude for finding a given quark distribution is independent of P in the high P limit (and for $\xi_i \geq 1/P$) but depends only on the ξ_i 's of the quarks and their transverse momenta.

Figure A.2 Parton model picture of deep-inelastic lepton-hadron scattering.

In the limit of negligible masses, the conservation of 4-momentum at the quark vertex requires:

$$(\xi P + q)^2 = 0$$

where ξ is the fractional momentum of the struck quark.

$P \cdot q$ is usually evaluated in the lab frame where $P \cdot q = M\nu$.

Hence, $\xi = \frac{Q^2}{2M\nu}$ and is identified with the scaling variable x .

As the energy increases, the threshold may be reached for the production of a heavy quark, one whose mass is not negligible in the "rescaling" region just above the threshold. In this region scaling is violated and x is no longer the fractional momentum of the initial quark. Neglecting all masses except that of the heavy quark m :

$$(\xi P + q)^2 = m^2$$

$$\text{i.e.} \quad \xi = x + \frac{m^2}{2Mv}$$

$z \equiv x + m^2/2Mv$ is known as the slow rescaling variable.¹¹

To obtain expressions for the structure functions F_1 , F_2 and F_3 , we begin with the hadronic tensor $W^{\mu\nu}$ written as:

$$W^{\mu\nu} = (P^\mu - q^\mu \frac{P \cdot q}{q^2}) (P^\nu - q^\nu \frac{P \cdot q}{q^2}) 2W_2 - (g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) 2M^2 W_1 - i\epsilon^{\nu\mu\alpha\beta} q_\alpha P_\beta W_3 \quad (\text{A.12})$$

This form can be obtained from equation (A.1) but also follows naturally from arguments of gauge invariance and relativity in the case of unpolarized scattering. It describes scattering from a hadron with 4-momentum P and mass M . For scattering from a free point particle (i.e. a quark) with fractional momentum ξ and negligible mass:

$$P \rightarrow \xi P, W_1 \rightarrow 1 + Q^2/2M^2, W_2 \rightarrow 2, W_3 \rightarrow -2$$

and

$$W^{\mu\nu} \rightarrow \left\{ 4\xi^2 \left(p^\mu - q^\mu \frac{P \cdot q}{q^2} \right) \left(p^\nu - q^\nu \frac{P \cdot q}{q^2} \right) + \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) (q^2 - 2M^2) \right. \\ \left. + 2\xi i \epsilon^{\nu\mu\alpha\beta} q_\alpha p_\beta \right\} M^2 \delta[(\xi P + q)^2] \quad (A.13)$$

As $-q^2$ and $P \cdot q = Mv \rightarrow \infty$ such that $x = -q^2/2Mv$ remains finite, the hadron mass M may be neglected:

$$\delta[(\xi P + q)^2] \approx \frac{1}{2Mv} \delta(\xi - x)$$

To retrieve the hadronic tensor $W^{\mu\nu}$, the contribution (A.13) from a single quark must be summed over all quarks present in the hadron. If $f_L(\xi)d\xi$ and $f_R(\xi)d\xi$ are the number of left- and right-handed quarks with fractional momentum between ξ and $\xi + d\xi$:

$$W^{\mu\nu} = \int_0^1 \frac{d\xi}{\xi} \{ f_L(\xi) + f_R(\xi) \} \left[4\xi^2 \left(p^\mu - q^\mu \frac{P \cdot q}{q^2} \right) \left(p^\nu - q^\nu \frac{P \cdot q}{q^2} \right) \right. \\ \left. + \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) q^2 \right] + [f_L(\xi) - f_R(\xi)] [2\xi i \epsilon^{\nu\mu\alpha\beta} q_\alpha p_\beta] \frac{M}{2v} \delta(\xi - x)$$

The extra $1/\xi$ results from normalization factors.

$$\begin{aligned} \therefore W^{\mu\nu} = & \{ [f_L(x) + f_R(x)] [4x(p^\mu - q^\mu \frac{p \cdot q}{q^2}) (p^\nu - q^\nu \frac{p \cdot q}{q^2}) + \frac{1}{x} (g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) q^2] \\ & + [f_L(x) - f_R(x)] [2i\epsilon^{\nu\mu\alpha\beta} q_\alpha p_\beta] \} M/2\nu \end{aligned}$$

Comparing this with equation (A.12) for $W^{\mu\nu}$, we see that:

$$2W_1 = f_L(x) + f_R(x)$$

$$\frac{\nu W_2}{M} = x[f_L(x) + f_R(x)]$$

$$\frac{-\nu W_3}{M} = f_L(x) - f_R(x)$$

In the scaling region equations (A.10) are applicable:

$$\therefore 2F_1(x) = f_L(x) + f_R(x)$$

$$F_2(x) = x[f_L(x) + f_R(x)]$$

$$-F_3(x) = f_L(x) - f_R(x)$$

The structure functions, therefore, satisfy the following relations:

$$2xF_1 = F_2 \tag{A.14}$$

$$-xF_3 = F_2$$

where $\eta = +1$ (-1) for scattering from left-handed (right-handed) quarks.

Writing $q(x) = xf_L(x)$ and $\bar{q}(x) = xf_R(x)$, the fractional momenta carried by left-handed and right-handed quarks respectively:

$$F_2 = q(x) + \bar{q}(x)$$

$$-xF_3 = q(x) - \bar{q}(x)$$

Using these expressions together with the Callan-Gross relation, (A.14), equation (A.11) becomes:

$$\begin{aligned} \frac{d^2\sigma^v}{dx dy} &= \frac{G_F^2 ME}{\pi} \{q(x) + \bar{q}(x) (1-y)^2\} \\ \frac{d^2\sigma^{\bar{v}}}{dx dy} &= \frac{G_F^2 ME}{\pi} \{q(x) (1-y)^2 + \bar{q}(x)\} \end{aligned} \tag{A.15}$$

As the quark coupling varies from model to model, for a given interaction $q(x)$ and $\bar{q}(x)$ are model dependent. All models incorporate the basic u , d , s and c quarks and a symmetric sea:

$$\bar{u}(x) = \bar{d}(x) = \bar{s}(x) = \bar{c}(x) = \dots = \xi(x)$$

$$u(x) = u_v(x) + \xi(x)$$

$$d(x) = d_v(x) + \xi(x)$$

$u(x)$, $d(x)$ etc. are defined as the probabilities of finding a u , d etc. quark with fractional momentum x , in the proton. For the neutron, assuming charge symmetry invariance:

$$u(x) \leftrightarrow d(x) \quad , \quad \bar{u}(x) \leftrightarrow \bar{d}(x)$$

For an isoscalar target, i.e. one with equal numbers of protons and neutrons:

$$u(x), d(x) \rightarrow \frac{1}{2} [u(x) + d(x)]; \quad \bar{u}(x), \bar{d}(x) \rightarrow \frac{1}{2} [\bar{u}(x) + \bar{d}(x)]$$

The expressions for $q(x)$ and $\bar{q}(x)$ are given below for (anti) neutrino scattering from a proton target with Weinberg-Salam model couplings and an $SU(3)$ symmetric sea:

$$q(x, z_c) (vp \rightarrow \mu^- X) = x[d(x) \cos^2 \theta_c + s(x) \sin^2 \theta_c] \\ + \theta_c z_c [d(z_c) \sin^2 \theta_c + s(z_c) \cos^2 \theta_c]$$

$$\bar{q}(x) (vp \rightarrow \mu^- X) = x\bar{u}(x).$$

θ_c is a step function which vanishes below the threshold for charm production and z_c is the slow rescaling variable defined earlier, it reduces to x well above threshold.

$$q(x) (\bar{\nu}p \rightarrow \mu^+ X) = xu(x)$$

$$\begin{aligned} \bar{q}(x, z_c) (\bar{\nu}p \rightarrow \mu^+ X) = & x[\bar{d}(x) \cos^2 \theta_c + \bar{s}(x) \sin^2 \theta_c] \\ & + \theta_c z_c [\bar{d}(z_c) \sin^2 \theta_c + \bar{s}(z_c) \cos^2 \theta_c] \end{aligned}$$

$$\begin{aligned} q(x) (\nu p \rightarrow \nu X) = & \frac{1}{4} x [u(x) (1 - \frac{4}{3} x_w)^2 + [d(x) + s(x)] (-1 + \frac{2}{3} x_w)^2 \\ & + \bar{u}(x) (\frac{4}{3} x_w)^2 + [\bar{d}(x) + \bar{s}(x)] (-\frac{2}{3} x_w)^2] \end{aligned}$$

$$\begin{aligned} \bar{q}(x) (\nu p \rightarrow \nu X) = & \frac{1}{4} x [u(x) (-\frac{4}{3} x_w)^2 + [d(x) + s(x)] (\frac{2}{3} x_w)^2 \\ & + \bar{u}(x) (-1 + \frac{4}{3} x_w)^2 + [\bar{d}(x) + \bar{s}(x)] (1 - \frac{2}{3} x_w)^2] \end{aligned}$$

where $x_w = \sin^2 \theta_w$.

$$q(x) (\bar{\nu}p \rightarrow \bar{\nu} X) = q(x) (\nu p \rightarrow \nu X)$$

$$\bar{q}(x) (\bar{\nu}p \rightarrow \bar{\nu} X) = \bar{q}(x) (\nu p \rightarrow \nu X)$$

Integrating equations (A.15) over y :

$$\frac{d\sigma^{\nu}}{dx} = \frac{G_F^2 M E^4}{3\pi} \{3q(x) + \bar{q}(x)\}$$

$$\frac{d\sigma^{\bar{\nu}}}{dx} = \frac{G_F^2 M E^4}{3\pi} \{q(x) + 3\bar{q}(x)\}$$

(A.16)

Defining $Q = \int_0^1 q(x) dx$ and $\bar{Q} = \int_0^1 \bar{q}(x) dx$: the total fractional

momenta carried by left- and right-handed quarks, and integrating equations (A.16) over x , the total cross sections are obtained:

$$\sigma^V = \frac{G_F^2}{3\pi} (3Q + \bar{Q})$$

$$\sigma^{\bar{V}} = \frac{G_F^2}{3\pi} (Q + 3\bar{Q})$$

Then the ratio $R = \sigma^{\bar{V}}/\sigma^V$ is a function of Q and $\bar{Q}$ only:

$$R = \frac{Q + 3\bar{Q}}{3Q + \bar{Q}}$$

Alternatively, in terms of B where:

$$B = \frac{-\int_0^1 x F_3(x) dx}{\int_0^1 F_2(x) dx}$$

$$= \frac{\int_0^1 [q(x) - \bar{q}(x)] dx}{\int_0^1 [q(x) + \bar{q}(x)] dx}$$

$$\text{then } R = \frac{2 - B}{2 + B}$$

If the contribution from right-handed quarks is negligible, and this is to be expected at low energies:

$$B = 1, \quad R = 1/3 \quad \text{i.e. } \sigma^V = 3\sigma^{\bar{V}}$$

APPENDIX B

THE $SU(2) \otimes U(1)$ GAUGE GROUP AND THE WEINBERG-SALAM MODELLocal $SU(2) \otimes U(1)$ Gauge Invariance

The Lagrangian density for a free Dirac spinor with mass m is:

$$L = \bar{\psi}(x) (i\gamma^\mu \partial_\mu - m) \psi(x)$$

To introduce a notation that will be useful later, we write:

$$\psi_L = \frac{1}{2} (1 - \gamma_5) \psi ; \quad \psi_R = \frac{1}{2} (1 + \gamma_5) \psi$$

$$\therefore \quad \psi = \psi_L + \psi_R$$

having used the sign convention of Bjorken and Drell.¹ The free Lagrangian density becomes:

$$L = i\bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + i\bar{\psi}_R \gamma^\mu \partial_\mu \psi_R - m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

A local $U(1)$ gauge transformation has the form:

$$\phi(x) \rightarrow \phi'(x) = \exp(-i\theta(x))\phi(x) \tag{B.1}$$

for a field $\phi(x)$.

The infinitesimal transformation is:

$$\delta\phi(x) = -i\theta(x)\phi(x)$$

For SU(2), a local gauge transformation has the form:

$$\phi(x) \rightarrow \phi'(x) = \exp(-i\vec{L}\cdot\vec{\theta}(x))\phi(x) \quad (B.2)$$

where $\phi(x)$ is a column vector, and $\vec{L}$ is the appropriate matrix representation of SU(2). For a doublet; $\vec{L} = \frac{1}{2}\vec{\tau}$, where $\vec{\tau}$ are the Pauli spin matrices. The infinitesimal transformation is:

$$\delta\phi(x) = -i\vec{L}\cdot\vec{\theta}(x)\phi(x)$$

The condition for local gauge invariance is $\delta L = 0$. Terms in the Lagrangian which depend only on the fields are invariant under the transformations (B.1) and (B.2), although terms involving the gradient ∂_μ of a field are not. Consider first the U(1) gauge transformation - equation (B.1). A covariant derivative $D_\mu\phi(x)$ is introduced such that $D_\mu\phi(x)$ transforms in the same way as $\phi(x)$.

$$\text{i.e. } D_\mu\phi(x) \rightarrow \exp(-i\theta(x)) D_\mu\phi(x) \quad (B.3)$$

Then, if $\partial_\mu\phi(x)$ appears in the Lagrangian L only as a part of $D_\mu\phi(x)$, all terms in L will be invariant under local U(1) gauge

transformations. $D_\mu \phi(x)$ is constructed by introducing a vector field, $B_\mu(x)$, with:

$$D_\mu \phi(x) = (\partial_\mu - ig'B_\mu)\phi(x)$$

where g' is the coupling constant associated with the $U(1)$ group. The field B_μ transforms in such a way as to ensure (B.3). For local $SU(2)$ gauge invariance, the procedure is similar but now a triplet of vector fields $\vec{W}_\mu(x)$ is introduced (i.e. one for each dimension of the algebra). The gradient $\partial_\mu \phi(x)$ is then replaced by the covariant derivative:

$$(\partial_\mu - ig\vec{L} \cdot \vec{W}_\mu)\phi(x)$$

where g is the coupling constant associated with the $SU(2)$ group. For generality, the factor groups of $SU(2) \otimes U(1)$ are assumed to have different coupling constants.

The Weinberg-Salam Model

In the Weinberg-Salam model, the lepton fields are those of the electron (e^-), its neutrino (ν_e), the muon (μ^-) and its neutrino (ν_μ). The same symbol is used, in general, for both the particle and its field. In what follows e^- and ν_e alone are considered; the μ^- and ν_μ case is similar. The

quark sector will be discussed later. A left-handed isospin is defined by placing ν_e and e^- in a doublet:

$$L = \frac{1}{2} (1 - \gamma_5) \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$$

which transforms as a spinor representation of the weak isospin group $SU(2)_L$. The electron (but not the neutrino) is placed in an $SU(2)_L$ singlet:

$$R = \frac{1}{2} (1 + \gamma_5) e^- = e^-_R$$

The doublet is assigned a "hypercharge" $Y = -1$ and the singlet $Y = -2$ so that the charge Q satisfies:

$$Q = I_{3L} + Y/2$$

for all particles.

The group $SU(2)_L \otimes U(1)$ is made into the gauge symmetry of the model by introducing the vector gauge fields $\vec{W}_\mu$ and B_μ as described above.

The Lagrangian density can now be written as:

$$L = L(\text{gauge}) + L(\text{leptons}) \quad (B.4)$$

with:

$$L(\text{gauge}) = -\frac{1}{4} (\partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu + g \vec{W}_\mu \times \vec{W}_\nu)^2 - \frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu)^2 \quad (\text{B.5a})$$

$$L(\text{leptons}) = \bar{R} i \gamma^\mu (\partial_\mu + i g' B_\mu) R + \bar{L} i \gamma^\mu (\partial_\mu + i g' B_\mu - i g \frac{\vec{\tau} \cdot \vec{W}_\mu}{2}) L \quad (\text{B.5b})$$

Explicit mass terms (which are of the form $-m^2 B_\mu B^\mu$) for the vector fields cannot appear in L as they violate local gauge invariance. For the same reason, bare mass terms for the electron are not allowed. The conclusion is that gauge invariance is impossible unless the quanta of the $\vec{W}_\mu$ and B_μ fields (the vector gauge bosons) and the electron are massless. However, to correspond to the situation found in nature, the theory should have only one conserved quantity (electric charge) and, therefore, only one massless gauge boson (the photon). In addition, it should be able to accommodate a non-zero electron mass. The next section introduces a mechanism which will be used to achieve the required masses in a gauge invariant way.

Spontaneous Breakdown of Symmetry

A "spontaneously broken"² symmetry is one which is a symmetry of the Lagrangian but not of the minimum energy state (i.e. the ground state). Physically, spontaneous breakdown of symmetry (SBS) occurs when the ground state is degenerate and one state is arbitrarily chosen. This situation is quite common in many-particle systems. For example, the Hamiltonian

describing a ferromagnet has rotational invariance, but the ground state has the spins aligned in some arbitrary direction. Each different direction defines a distinct ground state, in the limit of an infinite size system.

In field theory, the ground state is the vacuum. To achieve SBS, a scalar field is first introduced into the Lagrangian density. For a single self-interacting complex scalar field ϕ , the contribution is:

$$L(\text{scalars}) = \frac{1}{2} (\partial^\mu \phi^* \partial_\mu \phi) - V(\phi) \quad (\text{B.6})$$

The potential $V(\phi)$ is given by:

$$V(\phi) = \frac{1}{2} \mu^2 \phi^* \phi + \frac{1}{4} \lambda (\phi^* \phi)^2$$

The Lagrangian is invariant under the global phase transformations:

$$\phi(x) \rightarrow \exp(i\alpha) \phi(x) \quad (\text{B.7})$$

with α a real constant. The term in λ is a self-interaction, λ being positive to ensure the existence of a minimum energy state. If μ^2 is also positive, the minimum potential occurs at $\phi = 0$. In this case, the symmetry of the Lagrangian is also a symmetry of the ground state and $-\frac{1}{2} \mu^2 \phi^* \phi$ is an

ordinary mass term. However, if $\mu^2 < 0$, the potential will have a local maximum at $\phi = 0$, the minima now occurring when:

$$|\phi| = (-\mu^2/\lambda)^{1/2}.$$

Solutions with different phases are distinct and are, therefore, not invariant under the phase transformations, equation (B.7), that are a symmetry of the Lagrangian.

For a quantized field, these solutions for ϕ correspond to the vacuum expectation values (vev's) of the field:

$$\text{i.e. } |\langle 0|\phi|0\rangle| = (-\mu^2/\lambda)^{1/2}$$

Writing $\phi = \phi_1 + i\phi_2$, the vev's of the real scalar fields ϕ_1 and ϕ_2 can be chosen to be:

$$\langle 0|\phi_1|0\rangle = (-\mu^2/\lambda)^{1/2}, \quad \langle 0|\phi_2|0\rangle = 0$$

without any loss of generality. The vacuum state $|0\rangle$ is not invariant under the transformations (B.7). It is now convenient to define a new field ϕ'_1 :

$$\phi'_1 = \phi_1 - \langle 0|\phi_1|0\rangle$$

massless boson. This boson must be neutral and couple to the electron with coupling constant e (the electric charge), for it to be identified as the photon. The gauge symmetry will be spontaneously broken in such a way that a subgroup $U(1)^{\text{em}}$ generated by the operator $Q = I_3 + Y/2$ remains an unbroken local gauge symmetry. From Goldstone's theorem we would, therefore, expect $4 - 1 = 3$ massless scalar bosons. The scalar field ϕ is chosen to be a complex doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1^+ + i\phi_2^+ \\ \phi_1^0 + i\phi_2^0 \end{pmatrix}$$

as 4 real fields are sufficient to give 3 Goldstone bosons.

As will be seen later, the above choice is influenced by the lepton content of the Weinberg-Salam model.

Incorporating $SU(2) \otimes U(1)$ gauge invariance in equation (B.6), ϕ contributes a term to the Lagrangian:

$$L(\text{scalars}) = \frac{1}{2} \left| \left(\partial_\mu - ig' B_\mu - ig \frac{\vec{\tau} \cdot \vec{W}_\mu}{2} \right) \phi \right|^2 - \frac{1}{2} \mu^2 \phi^\dagger \phi - \frac{1}{4} \lambda (\phi^\dagger \phi)^2 \quad (\text{B.8})$$

The ϕ field can couple with fermions in a gauge invariant way with a contribution to the Lagrangian of the following form:

massless boson. This boson must be neutral and couple to the electron with coupling constant e (the electric charge), for it to be identified as the photon. The gauge symmetry will be spontaneously broken in such a way that a subgroup $U(1)^{\text{em}}$ generated by the operator $Q = I_3 + Y/2$ remains an unbroken local gauge symmetry. From Goldstone's theorem we would, therefore, expect $4 - 1 = 3$ massless scalar bosons. The scalar field ϕ is chosen to be a complex doublet:

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The ϕ field can couple with fermions in a gauge invariant way with a contribution to the Lagrangian of the following form:

$$L(\text{inter}) = -f_e [\bar{L}\phi_R + \bar{R}\phi^\dagger_L] \quad (\text{B.9})$$

$$= -f_e [(\bar{\nu}_e \bar{e}^-)_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \text{h.c.}]$$

In the absence of SBS, i.e. if ϕ has zero vev, the Lagrangian density:

$$L' = L + L(\text{scalars}) + L(\text{inter})$$

given by equations (B.4), (B.5), (B.8) and (B.9), conserves isospin and hypercharge. For spontaneous breakdown of gauge symmetry, we let μ^2 be negative and choose the real neutral component, ϕ_1^0 , of ϕ to be the one that develops a non-zero vev:

$$\langle \phi \rangle = \langle 0 | \phi | 0 \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} ; \quad v = (-\mu^2/\lambda)^{1/2} \text{ is real.} \quad (\text{B.10})$$

After an appropriate redefinition of fields, we would expect a massive scalar boson (from ϕ_1^0) and 3 Goldstone bosons (from ϕ^+ , ϕ_2^0). The most convenient procedure is to make a transformation to the Unitary gauge in which the particle content of the model is apparent. Effectively ϕ is replaced everywhere by its vev, (B.10). $L(\text{gauge})$ and $L(\text{leptons})$ remain unchanged, $L(\text{inter})$ becomes:

$$L(\text{inter}) = -f_e v [\bar{e}_L e_R + \bar{e}_R e_L] = -f_e v \bar{e} e \quad (\text{B.11})$$

Thus, the electron has acquired a mass $m_e = f_e v$. The neutrino remains massless primarily because there are no right-handed neutrino fields. Even if there were, the neutrino does not couple to ϕ^0 and would still not acquire mass. Thus, a model with heavy neutral leptons would need a different doublet of scalar fields. L (scalars) yields the following term quadratic in the redefined vector meson fields:

$$\frac{1}{8} v^2 [(g' B_\mu - g W_\mu^3)^2 + g^2 ((W_\mu^1)^2 + (W_\mu^2)^2)] \quad (\text{B.12})$$

The charged fields $W_\mu^\pm$ are defined as:

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2) \quad (\text{B.13a})$$

and from (B.12) we see that the quanta of these fields have acquired a mass:

$$M_W = \frac{1}{2} g v \quad (\text{B.13b})$$

The neutral vector fields of definite mass are:

$$Z_\mu = \frac{g W_\mu^3 - g' B_\mu}{(g^2 + g'^2)^{1/2}} \quad (\text{B.14a})$$

and

$$A_\mu = \frac{g' W_\mu^3 + g B_\mu}{(g^2 + g'^2)^{1/2}}$$

and the masses are:

$$M_Z = \frac{1}{2}v (g^2 + g'^2)^{1/2}, \quad M_A = 0 \quad (\text{B.14b})$$

There is also a neutral scalar field with mass $-2\mu^2$, known as a Higgs boson or Higgs scalar.⁴ However, the 3 expected Goldstone bosons have been absorbed into the vector gauge fields (when the latter were redefined): they are responsible for the longitudinal components of the 3 massive vector bosons. The would-be Goldstone bosons are often referred to as "ghosts". The number of degrees of freedom in terms of particle helicities remains the same, and in the limit $\langle\phi\rangle \rightarrow 0$, the longitudinal component of each massive vector boson becomes a pure scalar mode.

L (leptons), given by (B.5b), can be written in terms of $W_\mu^\pm$, Z_μ and A_μ . The part associated with the neutral fields is:

$$\begin{aligned} & \frac{-gg'}{2(g^2 + g'^2)^{1/2}} [2\bar{R}\gamma^\mu R + \bar{L}\gamma^\mu L - \bar{L}\gamma^\mu \tau_3 L] A_\mu \\ & + \frac{1}{2(g^2 + g'^2)^{1/2}} [g'^2(2\bar{R}\gamma^\mu R + \bar{L}\gamma^\mu L) + g^2(\bar{L}\gamma^\mu \tau_3 L)] Z_\mu \end{aligned} \quad (\text{B.15})$$

The A_μ part of the above is:

$$\begin{aligned} & \frac{-gg'}{(g^2 + g'^2)^{1/2}} [\bar{e}_R \gamma^\mu e_R + \bar{e}_L \gamma^\mu e_L] A_\mu \\ &= \frac{-gg'}{(g^2 + g'^2)^{1/2}} [\bar{e} \gamma^\mu e] A_\mu \end{aligned} \quad (B.16a)$$

Thus, A_μ couples to the electric current $\bar{e} \gamma^\mu e$ and is identified as the photon field. The electric charge e is given by:

$$e = \frac{gg'}{(g^2 + g'^2)^{1/2}}$$

The Weinberg angle θ_W is defined by $\tan \theta_W = g'/g$, so:

$$e = g \sin \theta_W = g' \cos \theta_W \quad (B.17)$$

$$\text{and: } M_Z = M_W / \cos \theta_W$$

Equations (B.14a) become:

$$Z_\mu = \cos \theta_W W_\mu^3 - \sin \theta_W B_\mu$$

$$A_\mu = \sin \theta_W W_\mu^3 + \cos \theta_W B_\mu$$

θ_W is therefore the mixing angle of the neutral vector

gauge fields.

The neutral coupling between the leptons and Z_μ , from (B.15), can now be written:

$$\begin{aligned} & \frac{g}{2\cos\theta_W} [\sin^2\theta_W (2\bar{L}\gamma^\mu R + \bar{L}\gamma^\mu L) + \cos^2\theta_W (\bar{L}\gamma^\mu \tau_3 L)] Z_\mu \\ &= \frac{g}{2\cos\theta_W} [\bar{L}\gamma^\mu \tau_3 L + 2\sin^2\theta_W \bar{e}\gamma^\mu e] Z_\mu \end{aligned} \quad (B.16b)$$

The part of L (leptons) associated with the charged fields is:

$$\begin{aligned} L(\text{charged}) &= \frac{g}{2} [\bar{L}\gamma^\mu L (\tau_1 W_\mu^1 + \tau_2 W_\mu^2)] \\ &= \frac{g}{2\sqrt{2}} [\bar{L}\gamma^\mu (\tau_1 + i\tau_2) L W_\mu^+] + \frac{g}{2\sqrt{2}} [\bar{L}\gamma^\mu (\tau_1 - i\tau_2) L W_\mu^-] \\ &= \frac{g}{\sqrt{2}} [\bar{L}\gamma^\mu \tau_+ L] W_\mu^+ + \text{h.c.} \end{aligned} \quad (B.16c)$$

where

$$\tau_\pm = \frac{1}{2}(\tau_1 \pm i\tau_2).$$

L (leptons) given by equations (B.16):

$$\begin{aligned} L(\text{leptons}) &= -e[\bar{e}\gamma^\mu e] A_\mu + \frac{g}{2\cos\theta_W} [\bar{L}\gamma^\mu \tau_3 L + 2\sin^2\theta_W \bar{e}\gamma^\mu e] Z_\mu \\ &\quad + \frac{g}{\sqrt{2}} [\bar{L}\gamma^\mu \tau_+ L] W_\mu^+ + \text{h.c.} \end{aligned}$$

describes the weak and electromagnetic coupling of the electron and its neutrino. To completely specify the lepton sector of the W-S model, extra terms are added with $e^- \rightarrow \mu^-$ and $\nu_e \rightarrow \nu_\mu$.

There are 2 constraints, equations (B.17), between the parameters g , g' and θ_W . One more constraint can be obtained by considering the exchange of one charged vector boson in the limit of low momentum transfer. This relates the Fermi coupling constant G_F to g and/or g' and the masses of the vector bosons. The charged coupling, described by (B.16c), can be written as:

$$L(\text{charged}) = \frac{g}{2\sqrt{2}} L^\mu W_\mu^+ + \text{h.c.} \quad (\text{B.18})$$

by defining the charged currents:

$$L^{\pm\mu} = 2\bar{L}\gamma^\mu \tau_\mp L$$

In the low momentum transfer region; $q^2 \ll M_W^2$, and the W propagator can be approximated by $g_{\mu\nu}/M_W^2$. Replacing $W_\mu^\pm$ in (B.18) by:

$$\frac{g_{\mu\nu}}{M_W^2} \frac{g}{2\sqrt{2}} L^{\pm\nu} = \left(\frac{g}{2\sqrt{2} M_W^2} \right) L_\mu^\pm$$

the exchange of one charged boson is described by:

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$$L \text{ (charged)} = \left(\frac{g}{2\sqrt{2} M_W} \right)^2 L^{-\mu} L_{\mu}^{+} + \text{h.c.}$$

• Comparing this with the definition of G_F :

$$L \text{ (charged)} = \frac{G_F}{\sqrt{2}} L^{-\mu} L_{\mu}^{+} + \text{h.c.}$$

we have that:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

Using equation (B.13b):

$$\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}$$

indicating that v is very large (≈ 248 GeV for the value of G_F given overleaf).

Using equation (B.17) for g gives:

$$M_W^2 = \frac{\sqrt{2} e^2}{8G_F \sin^2 \theta_W}$$

and hence:

$$M_Z^2 = \frac{\sqrt{2} e^2}{8G_F \sin^2 \theta_W \cos^2 \theta_W}$$

The above expressions yield the following lower limits on

the vector boson masses:

$$M_W \geq 37.5 \text{ GeV} ; M_Z \geq 75.0 \text{ GeV}$$

having used $G_F = 1.01 \times 10^{-5} M_p^{-2}$, where M_p is the mass of the proton.

The apparent weakness of the weak interactions is due to the large masses of the intermediate vector bosons; $W^\pm$ and Z .

The electron-Higgs boson coupling, from equations (B.9) and (B.11), has strength:

$$\begin{aligned} f_e &= m_e/v \\ &= m_e (\sqrt{2} G_F)^{1/2} \\ &\approx 2.06 \times 10^{-6} \end{aligned}$$

The coupling of ϕ_1^0 to the muon will be stronger by the factor m_μ/m_e , but still very weak.

By considering the stability of the vacuum, Weinberg has shown⁵ that there is an effective lower bound of order $\alpha G_F^{-1/2}$ on the masses of Higgs bosons. For the Higgs boson of the W-S model, and with $\sin^2 \theta_W = 0.33$, this lower bound is 4.89 GeV.

The quark sector of the W-S model can be treated in a similar fashion, although two Higgs scalar doublets are required in order for the top members (u and c) as well as the bottom members (d_c and s_c) of the isospin doublets to gain mass. The doublets are chosen to be

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \text{and} \quad \tilde{\phi} = \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix} \quad \text{with} \quad \langle \phi^+ \rangle = 0, \quad \langle \phi^0 \rangle = v$$

as in the lepton case.

The mechanism that has been outlined here for giving masses to the $SU(2) \otimes U(1)$ vector bosons and to the fermions of the W-S model can be applied to other gauge models of the weak and electromagnetic interactions. The general procedure is summarized below:

- Choose a gauge group for the weak and electromagnetic interactions. Let the dimension of the group be N .

- Construct a Lagrangian density L , invariant under local gauge transformations, thus introducing N massless vector gauge fields.

- Choose the fermions and their representations.

- Choose the representation and charges of the scalar Higgs fields.

- Incorporate the Higgs couplings into L in a gauge invariant way.

- Allow spontaneous breaking of gauge symmetry by having the minimum of the Higgs self-interaction potential occur at a non-zero vev of a neutral Higgs field(s). This leaves the vacuum invariant under, say, an M dimensional subgroup.

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- Transform to the Unitary gauge and read off masses of particle eigenstates. There will be M massive Higgs scalars and M gauge bosons that remain massless. $N-M$ would-be Goldstone bosons ("ghosts") provide the longitudinal components of the $N-M$ vector gauge bosons that acquire mass.

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V. Barger & R.J.N. Phillips, Phys. Lett. 73B, 91 (1978)

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K. Kleinknecht in Proc. 17th International Conference
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(Rutherford Laboratory 1974)

2. The bound on neutral charm-changing currents is
comparable to that on neutral strangeness-
changing currents:

G.J. Feldman et al., Phys. Rev. Lett. 38, 1313 (1977)

J. Steinberger "Some Recent Results in High-Energy
Neutrino Physics":

Lectures presented at the Summer Institute, Cargèse,
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There is at present no phenomenological reason to require
diagonal neutral currents for flavours other than charm
and strangeness. $SU(2) \otimes U(1)$ models whose neutral
currents conserve strangeness but not other flavours
have also been considered:

K. Kang & J.E. Kim, Phys. Rev. D16, 568 (1977)

S. Barshay, Phys. Lett. 66B, 246 (1977)

3. The upper limit on the branching ratio of $\mu^+ \rightarrow e^+ \gamma$
relative to the decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ is:

1.1×10^{-9} from SIN:

H. Povel et al., Phys. Lett. 72B, 183 (1977)

3.6×10^{-9} from TRIUMF:

P. Depommier et al., Phys. Rev. Lett. 39, 1113 (1977)

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S.M. Korenchenko et al., Zh. Eksp. Teor. Fiz. 70, 3 (1976)
[Sov. Phys. JETP 43, 1 (1976)]
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 4×10^{-10} from SIN using a sulphur target:
A. Badertscher et al., Phys. Rev. Lett. 39, 1385 (1977)
6. The greater constraint is on the weak contribution to the muon anomalous magnetic moment. For the muon,
 $a_\mu(\text{exp}) - a_\mu(\text{em}) = (4 \pm 22) \times 10^{-9}$, where $a_\mu = \frac{1}{2} (g_\mu - 2)$.
The latest value for $a_\mu(\text{exp})$ was obtained by:
J. Bailey et al., Phys. Lett. 67B, 225 (1977)
and $a_\mu(\text{em})$ has been calculated to eighth order by:
J. Calmet et al., Rev. Mod. Phys. 49, 21 (1977)
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W.B. Dress et al., Phys. Rev. D15, 9 (1977)
This is expected to be improved to the level of $\sim 10^{-26}$ in the near future.
8. The concept of naturality was introduced by:
S.L. Glashow & S. Weinberg, Phys. Rev. D15, 1958 (1977)
They determine the conditions for naturally flavour-conserving neutral currents to be:
 - 1) Fermions of a given charge and handedness have the same weak I and I_3 .

2) Fermions of given charge receive their mass either from a gauge-invariant bare mass term or from their couplings with a single neutral Higgs field (but not from both).

M.S. Chanowitz et al., Nucl. Phys. B128, 506 (1977) have determined which gauge groups satisfy these conditions. $SU(2) \otimes U(1)$ in particular, is considered by:

F.E. Paige et al., Phys. Rev. D15, 3416 (1977)

Although naturally diagonal neutral currents have a certain aesthetic appeal, it is pointed out by:

H. Fritzsch in Proc. Orbis Scientiae (1977)

that this may not be a desirable feature in models with light and heavy quarks.

9. For more information on triangle anomalies, see:

T. Hagiwara & E. Takasugi, Phys. Rev. D15, 3232 (1977),
App. D

S.L. Adler in "Lectures on Elementary Particles and Quantum Field Theory", (MIT Press, Cambridge, Mass. 1970)

S.B. Treiman, R. Jackiw and D.J. Gross in "Lectures on Current Algebra and its Applications", (Princeton University Press, Princeton, New Jersey, 1972)

10. $SU(3) \otimes SU(3)$ has been considered recently by:

F. Gürsey et al., Phys. Lett. 60B, 177 (1976)

Y. Achiman, Phys. Lett. 70B, 187 (1977)

J.D. Bjorken & K.D. Lane, SLAC-PUB-1996 (1977)

P. Minkowski, Berne preprint (1977)

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T.D. Lee, Phys. Rev. D8, 1226 (1973)

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S. Weinberg, Phys. Rev. Lett. 37, 657 (1977)

The non-conservation of muon number resulting from the same mechanism has been discussed by:

J.D. Bjorken & S. Weinberg, Phys. Rev. Lett. 38, 622 (1977)

G.C. Branco, Phys. Lett. 68B, 455 (1977)

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T.K. Gaisser & A. Halprin, Phys. Rev. D17, 159 (1978)

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13. J.D. Bjorken & S. Weinberg in Ref. 11

14. Trimuon production in the 6-quark, 6-lepton version of model A is discussed by, for example:

B.W. Lee & R.E. Shrock, Fermilab-Pub-77/21-THY (to be published in Phys. Rev.), Phys. Rev. D16, 1444 (1977)

C.H. Albright, R.E. Shrock & J. Smith, Fermilab-Pub-77/81-THY (1977)

15. Such a mixing scheme is considered in an $SU(2) \otimes U(1)$ model with the same left-handed doublets as model A, by:
V. Barger et al., Phys. Rev. D16, 2141 (1977), Section VII

16. B.W. Lee & R.E. Shrock, Phys. Rev. D16, 1444 (1977)

The authors determine the set of conditions that an $SU(2) \otimes U(1)$ model must satisfy to guarantee strong natural suppression of μ and e lepton number non-conservation. See also:

T-P Cheng & L-F Li, Phys. Rev. D16, 1565 (1977)

17. F.E. Paige et al., in Ref. 8

18. In general for an $SU(2) \otimes U(1)$ model with $2N$ quarks in LH doublets and RH singlets, the number of phases is given by $(N - 1)(N - 2)/2$. The extension from $N = 2$ (W-S model) to $N = 3$ to admit CP violating phases was first considered by:

M. Kobayashi & T. Maskawa, Progr. Theor. Phys. 49, 652 (1973)

See also:

L. Wolfenstein in Proc. Intern. Neutrino Conf. Aachen (1976)

H. Fritzsch in Ref. 8

J. Ellis et al., Nucl. Phys. B131, 285 (1977)

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The 7th Hawaii Topical Conf. In Particle Physics, Aug. 1977

J.D. Bjorken, K. Lane & S. Weinberg, Phys. Rev. D16, 1474 (1977)

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- 23. T-P Cheng & L-F Li, Phys. Rev. Lett. 38, 381 (1977)
Phys. Rev. D16, 1425 (1977)

See also:

Proc. Orbis Scientiae (1977)

J.D. Bjorken et al., in Ref. 20

S.B. Treiman, F. Wilczek & A. Zee, Phys. Rev. D16, 152
(1977)

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by:

C.H. Albright et al., Ref. 14

They assume that mixing of e-type with μ - or τ -type
leptons and mixing of leptons between singlets and
doublets is prevented.

- 25. More details can be found in:

C.H. Albright et al., Ref. 14

V. Barger et al., Ref. 22

- 26. This estimate is quoted by:

H. Fritzsch in Ref. 8

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P.Q. Hung & J.J. Sakurai, Phys. Lett. 72B, 208 (1977)
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For similar attempts to explain these results, see:
P.Q. Hung & J.J. Sakurai, Ref. 28
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E. Ma, Phys. Lett. 65B, 468 (1976),
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$$M_{W_L} = M_{W_R} = M_{Z_A} = 37.5 \operatorname{cosec} \theta_W$$

$$M_{Z_V} = M_W (\sec 2\theta_W)^{1/2}$$

J.C. Pati & A. Salam in Ref. 34

See also:

S.L. Glashow in Ref. 20

The Higgs sector of model D is taken from:

R.N. Mohapatra & D.P. Sidhu, Brookhaven preprint:

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37. R.N. Mohapatra & D.P. Sidhu in Ref. 35

38. P. Langacker & D.P. Sidhu, Brookhaven preprint:

"Neutral Current Constraints on Gauge Models of Weak & Electromagnetic Interactions" (1977)

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R.N. Mohapatra & D.P. Sidhu, *Phys. Rev. D* 16, 2843 (1977)

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41. Q. Shafi & Ch. Wetterich in Ref. 33
42. A. Zee, Phys. Rev. D13, 713 (1976)
 H. Fritzsch, Phys. Lett. 70B, 436 (1977)
 S. Weinberg, Harvard preprint: HUTP-77/A057 (1977)
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 "Theory of Flavour Mixing", (1977) (To be published in
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- M. Yoshimura, Prog. Theor. Phys. 57, 237 (1977)
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- B.W. Lee, R.E. Shrock & S. Weinberg, Fermilab-Pub-77/48-THY (1977)
48. The ω and λ of Ref. 47 have been replaced here by $\sin^2 \theta_3$ and $(1 - n^2)/n^2$ respectively.
49. R.M. Barnett, SLAC-PUB-1961 (1977)
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R.N. Cahn & G.L. Kane, Phys. Lett. 71B, 348 (1977)
56. W.J. Marciano & A.I. Sanda, Phys. Rev. Lett. 38,
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G. Altarelli et al., Nucl. Phys. B125, 285 (1977)
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Virginia Collaboration:
A.L. Carter, private communication
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APPENDIX A

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APPENDIX B

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17 [REDACTED] 79

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