RAKE RECEIVER PERFORMANCE IN DIRECT
SEQUENCE SPREAD SPECTRUM SYSTEMS
OPERATING AT HIGH DATA RATES

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Abstract

Direct sequence spread spectrum transmission is an option for both current and next generation wireless systems. In code division multiple access scenarios, the entire spectrum can be time-shared among many high data rate users to mitigate the effects of multiple-access interference. Alternatively, resources can be shared simultaneously between one high data rate user and several low data rate users with lower power requirements. In such situations, the limiting factor is self-interference due to high autocorrelation sidelobes. These result from multipath propagation and imperfect autocorrelation properties of code sequences over a single symbol interval. While an increase in power can mitigate interference due to other users, it cannot counter the effects of self-interference.

New analytical upper bounds on the probability of bit error under conditions of uncorrelated Rayleigh fading with equal average power for multipath groups are derived for Rake reception with varying numbers of fingers, taking the effects of self-interference into account. While these bounds can be predicted with little complexity, an accurate estimate of bit error rate performance is often desired for purposes of efficient system design.

Work in this thesis includes a more comprehensive analysis than that available in the literature, of the problem of self-interference in direct sequence spread spectrum systems operating at high data rates. Performance estimates in terms of bit error rate as a function of signal-to-noise ratio at the receiver are derived for different

Rake receiver configurations. Semi-analytical techniques previously reported in the literature by Lehnert and Pursley to analyse multiple-access interference in non-fading environments are extended to apply to the problem of estimating self-interference in fading environments. These techniques avoid the Gaussian approximation for the interference, which can lead to inaccurate performance estimates. Additionally, from channel impulse response estimates measured in urban microcellular environments it has been observed that time-delayed replicas of the transmitted signal have unequal average powers in general and the fading is not always uncorrelated. The semi-analytical approach is therefore also used to analyse performance for arbitrary fading conditions and compare estimated performance with results obtained when standard channel models that assume equal average powers and independent fading are used. Deficiencies in the results from application of standard models are reported.

Knowledge of performance deficiencies and reasons for them can be used for better allocation of resources such as transmit power for different operating modes that use, for example different spreading factors, and for different radio channel conditions, such as different numbers of multipath groups and their associated unequal relative average powers.

To enable coherent detection in many current systems using Rake receivers, a pilot sequence is code-multiplexed with the data for the purpose of estimating the complex amplitudes of received multipath groups. As a result of both self-interference and thermal noise, errors can occur in the estimates of these amplitudes. Again, the semi-analytical technique can be used to permit consideration of Rake estimation errors in the analysis of performance for different channel conditions and receiver configurations. An increase in power of the data signal can cause an increase in interference to the pilot channel thereby increasing the magnitude of errors on channel estimates, while an increase in the power of the pilot signal will reduce the channel estimation error while enhancing the interference to the data. The semi-analytical

approach is used in the thesis to determine a suitable power level difference between the data and pilot signals. In cases where channel estimation errors are significant, analysis results indicate a degradation in performance with the inclusion of Rake fingers for multipath groups having powers below a certain threshold relative to the power of the strongest multipath group, concomitant with poorer channel estimates. Hence the requirement for a finger activation threshold. The resulting reduction in the number of active Rake fingers has the additional advantage of decreasing power consumption in the mobile terminal.

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List of Symbols

α_{KS}	Measure of goodness-of-fit for a distribution using the Kolmogorov-Smirnov test
$a(t)$	Data spreading signal
a_q	Data spreading sequence
$a^p(t)$	Pilot spreading signal
a_q^p	Pilot spreading sequence
A_p	Pilot signal power
A_d	Data signal power
β	Self-interference component in the expression for bit error rate
$b(t)$	Data signal
b_q	Data sequence
$b^p(t)$	Pilot signal
b_q^p	Pilot data sequence
$C_a(\cdot)$	Discrete aperiodic autocorrelation function of the spreading sequence
D_1	Correlation receiver decision statistic
D_{Rake}	Rake receiver decision statistic
$D_{Rake,norm}$	Normalised Rake receiver decision statistic

$\mathcal{E}\{X\}$	Expected value of random variable X
$\mathcal{E}_n\{Y\}$	Average value of a time-series Y
e_k	Error in the estimate of the k th multipath group amplitude
E_b	Energy per bit
$E_{b,data}$	Energy per bit of the data symbol
$E_{b,pilot}$	Energy per bit of the pilot symbol
$f(\cdot)$	Sum of the continuous time partial autocorrelation functions of the chip waveform over a chip period
γ_k	Relative delay in chips of the k th multipath group relative to the first, rounded down to the nearest integer value
$g(\cdot)$	Difference between the continuous time partial autocorrelation functions of the chip waveform over a chip period
h_i	Complex amplitude of multipath group having time-delay τ_i
$h_{k,i}$	Product of the absolute magnitudes of multipath groups having time-delays τ_k and τ_i
K	Number of received multipath groups
$K_\nu(\cdot)$	ν th order modified Bessel function of the second kind
K_{Rice}	Ricean-K factor
λ_u	Ratio between the variance of self-interference calculated using the semi-analytical technique and the variance of the upper Gaussian bound on self-interference
L	Number of Rake fingers
$M_{k,i}$	Mutual interference on Rake fingers having time-delays τ_k and τ_i
N_d	Data spreading factor
N_i	Number of independent samples in a time-series

N_p	Pilot spreading factor
N_u	Number of subintervals used in the semi-analytical technique
ϕ_k	Phase offset with respect to the carrier of the multipath group having time-delay τ_k
$\psi(t)$	Chip waveform
$p_T(t)$	Rectangular pulse waveform with duration T
P_e	Probability of bit error
P_E^L	Lower bound on the probability of bit error
P_E^U	Upper bound on the probability of bit error
$Q(\cdot)$	Gaussian-Q function
ρ_c	Correlation coefficient
$r(t)$	Equivalent low pass received signal
$R_a(\tau), \widehat{R}_a(\tau)$	Continuous-time partial cross-correlation functions of the delayed and the reference spreading waveforms
$R_\psi(\tau), \widehat{R}_\psi(\tau)$	Continuous-time partial autocorrelation functions of the chip waveforms
σ_I^2	Variance of the interference
σ_{Rake}^2	Variance of thermal noise in the Rake receiver
σ_u^2	Variance of the upper bound on the self-interference calculated using the semi-analytical technique
τ_i	Excess time-delay of i th multipath group
$\tau_{k,i}$	Difference between time-delays τ_k and τ_i
$\theta_{k,i}$	Phase difference between multipath groups having time-delays τ_k and τ_i

Δ	Relative average power between two received multipath groups
T_c	Chip period
T_d	Data symbol period
T_p	Pilot symbol period
$u_i(c)$	Upper bound on pdf of self-interference, conditioned on $C_a(1) = c$.
u_i	Unconditional upper bound on pdf of self-interference
$v_i(c)$	Lower bound on pdf of self-interference, conditioned on $C_a(1) = c$.
v_i	Unconditional lower bound on pdf of self-interference
$V_{k,i}$	Cross-correlation between interfering binary pilot and data sequences with time-delays τ_k and τ_i , respectively
$W_{k,i}$	Cross-correlation between interfering binary data sequences with time-delays τ_k and τ_i , respectively
$Y_{k,i}$	Cross-correlation between interfering binary pilot sequences with time-delays τ_k and τ_i , respectively

List of Acronyms

AWGN	Additive white Gaussian noise
APDP	Average power delay profile
BER	Bit error rate
BPSK	Binary phase shift keying
CDF	Cumulative distribution function
CDMA	Code division multiple access
DSSS	Direct sequence spread spectrum
ESR	Experimentally derived system response estimate
GSC	Generalised selection combining
i.i.d.	Independent and identically distributed
IRE	Impulse response estimate
ISI	Intersymbol interference
LOS	Line-of-sight
MAI	Multiple-access interference
MGF	Moment generating function
MPG	Multipath group
MRC	Maximal ratio combining

NLOS	Non-line-of-sight
pdf	Probability density function
PN	Pseudo-noise
SAT	Semi-analytical technique
SC	Selection combining
SI	Self-interference
SNR	Signal-to-noise ratio
WCDMA	Wideband code division multiple access
WSS	Wide-sense stationary

Chapter 1

Introduction

A growing need for mobile wireless systems that offer higher data rate services such as internet access and video, and better spectrum efficiency has led to the specification of advanced wireless systems based on direct sequence spread spectrum technology. Wideband code division multiple access (WCDMA), for instance, is an air-interface standard that falls under the umbrella of the third generation of wireless systems, and is based on direct sequence spread spectrum (DSSS) technology. A service specification of WCDMA is to enable the delivery of data rates up to 2 Mbps to a single user on the downlink channel. The evolution of WCDMA, known as high speed downlink packet access, is expected to increase peak data rates to 8-10 Mbps. The use of high data rates in a multi-user, urban propagation environment poses unique problems not encountered previously in DSSS systems intended primarily for voice and low data rate traffic.

1.1 Background

It is well-known that on multipath fading channels, a Rake receiver can be used to improve bit error rate (BER) performance [1]. In this section, a concise introduction to Rake reception in DSSS systems will be presented. Self-interference (SI) and its effect on Rake receiver performance will also be discussed.

Figure 1.1: Block diagram representation of a transmitter and receiver linked by a radio channel.

Figure 1.1 shows a block diagram representation of a transmitter and receiver linked by a radio channel. Time-delayed replicas of the transmitted signal arrive at the receiver as a result of reflections from, diffraction around, or scattering by interacting objects in the common volume of the transmit and receive antennas. As a result of changes in the physical makeup of the propagation environment (e.g. movement of objects into and out of the common volume) and motion of the transmitter, receiver or interacting objects, each time-delayed replica can vary in amplitude and phase as time progresses. If, as a result, variations of the resolvable signals at the receiver are uncorrelated to some degree, these can be combined to mitigate the effects of fading.

The propagation channel can be modelled as a linear filter and therefore characterised using an impulse response. Associated with each radio path is a peak in the impulse response having a time-delay $\tau_i(t)$ and attenuation factor $h_i(t)$, both of which can vary with time, t . In practical situations where the receiver has a finite bandwidth, it is not possible to determine the true impulse response of the channel. Instead an experimentally derived system response estimate (ESR) is calculated by

transmitting a known pseudo-noise (PN) sequence and cross-correlating the received signal with a locally generated reference. The vector sum of all signals received within the delay resolution of the receiver is referred to as a multipath group (MPG) in this thesis. Associated with each MPG is a time-delay and complex amplitude.

In the case of a DSSS system, the transmitted signal is multiplied by a pre-determined chip sequence. This is known as spreading. The data signal to be transmitted, $b(t)$, can be written as $b(t) = \sum_{q=-\infty}^{\infty} b_q p_{T_d}(t - qT_d)$, where $\{b_q\}$ is a sequence of elements from the set $\{-1, 1\}$, T_d is the period of the data symbol, $p_{T_d}(t)$ is a rectangular pulse, i.e. $p_{T_d}(t) = 1$ for $0 \leq t \leq T_d$ and $p_{T_d}(t) = 0$, otherwise. The spreading signal is given by $a(t) = \sum_{q=-\infty}^{\infty} a_q \psi(t - qT_c)$, where $\{a_q\}$ is a sequence of elements from the set $\{-1, 1\}$ and T_c is the chip period. Also,

$$\frac{1}{T_d} \int_0^{T_d} a^2(t) dt = 1. \quad (1.1.1)$$

The transmitted spread spectrum signal for a single user with BPSK modulation can be written as $s(t) \cos \omega_c t$, where $s(t) = \sqrt{\frac{2E_b}{T_d}} b(t) a(t)$ and ω_c is the carrier frequency. The chip waveform $\psi(t)$ is the rectangular pulse function $p_{T_c}(t)$, and $T_d/T_c = N_d$ is the number of chips per symbol interval or the data spreading factor. Energy per bit is denoted E_b . The equivalent low pass received signal consists of K MPGs and can be written as

$$r(t) = \sum_{k=1}^K |h_k| e^{j\phi_k} s(t - \tau_k) + n(t), \quad (1.1.2)$$

where $\tau_k = \tau_k(t)$ and $|h_k| = |h_k(t)|$ are the time-delay and amplitude of the k th MPG, respectively. The argument t of $h_k(t)$ and $\tau_k(t)$ has been dropped under the assumption that h_k is constant between successive channel sampling instants. The phase offset with respect to the carrier of the k th MPG is represented by $\phi_k = \omega_c \tau_k + \varphi_k$ and φ_k is the phase shift that results from interactions with objects along

the propagation path. Also the parameter $n(t)$ represents additive white Gaussian noise (AWGN) with two-sided power spectral density N_0 .

Figure 1.2: Block diagram of the Rake section at the front end of a receiver.

Figure 1.2 shows the block diagram of the Rake section at the front end of a receiver. Its purpose is to combine time-delayed versions of the transmitted signal. Rake receivers used in DSSS systems coherently combine and despread time-delayed versions of the transmitted signal.

Time-delays and complex amplitudes of resolvable components that comprise the received signal are first estimated by cross-correlating a known pilot sequence transmitted along with the data with a locally generated pilot reference. This process is

commonly referred to as channel estimation and through it each Rake finger or diversity branch is matched to one time-delayed version of the transmitted signal. Under the assumption that time-delays and amplitudes of MPGs can be estimated with no error, the output of a Rake receiver having L despreading fingers can be written as

$$D_{Rake} = \sum_{i=1}^L \int_{\tau_i}^{\tau_i+T_d} \text{real}(r(t) h_i^* a(t - \tau_i)) dt, \quad (1.1.3)$$

where the i th Rake finger multiplies the MPG having delay τ_i by its complex conjugate, h_i^* , and despreads the sequence at that delay by multiplying it with the local reference sequence. Equation (1.1.3) can be expanded by the substitution of (1.1.2) for $r(t)$,

$$D_{Rake} = \sqrt{E_b T_d / 2} \left(\sum_{i=1}^L |h_i|^2 \right) + \sqrt{E_b T_d / 2} \left[\sum_{i=1}^L \sum_{k=1, k \neq i}^K h_{k,i} W_{k,i} \cos \theta_{k,i} / T_d \right] + \eta_{Rake}, \quad (1.1.4)$$

where $h_{k,i} = |h_k| |h_i|$ and the second summation term represents the total SI that results from multipath propagation. The cross-correlation between interfering binary sequence k , and the reference sequence i , is denoted by

$$W_{k,i} = \int_{\tau_i}^{T_d + \tau_i} b(t - \tau_k) a(t - \tau_k) a(t - \tau_i) dt, \quad (1.1.5)$$

and as previously $T_d = N_d T_c$. It is considered that, to a good approximation, the phase term $\theta_{k,i} = \phi_k - \phi_i$ is assumed to be uniformly distributed on $[0, \pi]^1$. This is because wavelengths are relatively short at the operating frequencies of interest, and a small change in position of anything in the common volume of the transmit and receive antennas can introduce a phase change as high as π .

¹In the open literature it is often assumed that the phase is uniformly distributed on $[0, 2\pi)$ but for the purpose of obtaining the pdf of the SI, only the range of principal values for the inverse cosine function is considered.

The random variable η_{rake} that results from AWGN input $n(t)$ to the Rake receiver can be written as $\eta_{rake} = \sum_{p=1}^L \eta_{rake,p}$, where from (1.1.2) and (1.1.3),

$$\eta_{rake,p} = \int_{\tau_p}^{\tau_p+T_d} \text{real} (n(t) h_p^*(t) a(t - \tau_p)) dt. \quad (1.1.6)$$

Let the noise $n(t) = n_r(t) + jn_i(t)$ and MPG amplitude $h_p = h_{p,r} + jh_{p,i}$. Equation (1.1.6) can be rewritten as

$$\eta_{rake,p} = \int_0^{T_d} [n_r(t + \tau_p) h_{p,r} + n_i(t + \tau_p) h_{p,i}] a(t) dt. \quad (1.1.7)$$

The variance of $\eta_{rake,p}$ can be written as

$$\begin{aligned} \mathcal{E} \{ \eta_{rake,p}^2 \} &= \mathcal{E} \left\{ \int_0^{T_d} \int_0^{T_d} a(t) a(u) [n_r(t + \tau_p) h_{p,r} + n_i(t + \tau_p) h_{p,i}] \times \right. \\ &\quad \left. [n_r(u + \tau_p) h_{p,r} + n_i(u + \tau_p) h_{p,i}] dt du \right\}. \end{aligned} \quad (1.1.8)$$

Since $\mathcal{E} \{ n_r(t) n_r(u) \} = \mathcal{E} \{ n_i(t) n_i(u) \} = \frac{N_0}{2} \delta(t - u)$ and $\mathcal{E} \{ n_r(t) n_i(u) \} = 0$,

$$\mathcal{E} \{ \eta_{rake,p}^2 \} = \int_0^{T_d} a^2(t) \left[h_{p,r}^2 \frac{N_0}{2} + h_{p,i}^2 \frac{N_0}{2} \right] dt. \quad (1.1.9)$$

Also, since $a^2(t) = 1$, (1.1.9) reduces to

$$\mathcal{E} \{ \eta_{rake,p}^2 \} = \frac{N_0 T_d}{2} |h_p|^2. \quad (1.1.10)$$

Using the same approach it can readily be seen that $\mathcal{E} \{ \eta_{rake,p} \eta_{rake,q} \} = 0, p \neq q$. Therefore the term η_{rake} has zero-mean and variance $\frac{N_0 T_d}{2} \sum_{p=1}^L |h_p|^2$.

One implication of higher data rates is the reduction of spreading factor N_d or the number of chips per symbol interval when the chip rate or the bandwidth is fixed. This results in greater SI. In a CDMA system, closed-loop power control on

the downlink can be used to negotiate a higher transmit power from the base-station for a desired high data rate user to mitigate the effects of MAI from other users. A higher transmit power cannot however mitigate the effects of SI, because the powers of all MPGs increase proportionately.

As mentioned earlier, in CDMA systems, a known pilot signal can be used to estimate the channel. In order to do this, pilot data spread with a sequence of known pilot chips can be transmitted in parallel with the data. The use of a code-multiplexed pilot sequence can however lead to additional SI which in turn affects both the channel estimation and data detection.

The transmitted pilot data signal can be written as $b^p(t) = \sum_{q=-\infty}^{\infty} b_q^p p_{T_p}(t - qT_p)$ where $\{b_q^p\}$ is a binary sequences of elements from the set $\{-1, 1\}$; $p_{T_p}(t) = 1$ for $0 \leq t \leq T_p$, $p_{T_p}(t) = 0$ otherwise; T_p represents the period of the pilot symbol. The pilot spreading signal is $a^p(t) = \sum_{q=-\infty}^{\infty} a_q^p \psi(t - qT_c)$, where $\{a_q^p\}$ is a binary sequence of elements from the set $\{-1, 1\}$. Under the assumption that the energy per bit of the pilot is the same as the energy per bit of the data, the transmitted spread spectrum signal for a single user with BPSK modulation, can be written as $s'(t) \cos \omega_c t$,

$$s'(t) = \sqrt{\frac{2E_b}{T_d}} b(t) a(t) + \sqrt{\frac{2E_b}{T_p}} b^p(t) a^p(t). \quad (1.1.11)$$

The equivalent low pass received signal consists of K MPGs and can be written as

$$r(t) = \sum_{k=1}^K |h_k| e^{j\phi_k} s'(t - \tau_k) + n(t). \quad (1.1.12)$$

For the purpose of channel estimation, the received signal can be cross-correlated with the known pilot data and chip sequence to give

$$c(t) = r(t) * b^p(t) a^p(t). \quad (1.1.13)$$

At lag τ_1 , (1.1.13) can be written as

$$c(t, \tau_1) = c_1(t) = \int_{\tau_1}^{T_p + \tau_1} r(t) b^p(t - \tau_1) a^p(t - \tau_1) dt. \quad (1.1.14)$$

If $c_1(t)$ exceeds a pre-determined noise threshold level, it can be considered that there exists an MPG having time-delay τ_1 . By substituting (1.1.2) in (1.1.14), $c_1(t)$ can be rewritten as

$$c_1(t) = \sqrt{2E_b T_p} |h_1| e^{j\phi_1} + \sqrt{\frac{2E_b}{T_p}} \left(\sum_{k=2}^K |h_k| e^{j\phi_k} V_{k,1} + \sqrt{\frac{T_p}{T_d}} \sum_{k=2}^K |h_k| e^{j\phi_k} Y_{k,1} \right) + \eta_p, \quad (1.1.15)$$

where η_p is the result of thermal noise input to the correlation receiver and ϕ_k is the phase of the k th MPG. The second and third summation terms represent SI from the pilot and data, respectively. In the first summation, $V_{k,1}$ represents the result of cross-correlation between the pilot sequence having time-delay τ_k and the local reference for the pilot sequence having time-delay τ_1 or

$$V_{k,1} = \int_{\tau_1}^{T_p + \tau_1} b^p(t - \tau_1) a^p(t - \tau_1) b^p(t - \tau_k) a^p(t - \tau_k) dt. \quad (1.1.16)$$

In the second summation, $Y_{k,1}$ represents the result of cross-correlation between the data sequence having time-delay τ_k and reference pilot sequence having time-delay τ_1 or

$$Y_{k,1} = \int_{\tau_1}^{T_p + \tau_1} b^p(t - \tau_1) a^p(t - \tau_1) b(t - \tau_k) a(t - \tau_k) dt. \quad (1.1.17)$$

It has been assumed that the pilot and data chip sequences have been designed in a way that they are perfectly orthogonal when the relative time-delay is 0 or

$$\int_{\tau_i}^{T_p + \tau_i} a^p(t - \tau_i) a(t - \tau_i) dt = 0, \quad (1.1.18)$$

and

$$\int_{\tau_i}^{T_d + \tau_i} a^p(t - \tau_i) a(t - \tau_i) dt = 0. \quad (1.1.19)$$

Normalising (1.1.15) by $\sqrt{2E_b T_p}$ gives the estimated channel coefficient $\hat{h}_1$ as

$$\hat{h}_1 = h_1 + e_1, \quad (1.1.20)$$

where

$$e_1 = \frac{1}{N_p T_c} \left(\sum_{k=2}^K |h_k| e^{j\phi_k} V_{k,1} + \sqrt{\frac{N_p}{N_d}} \sum_{k=2}^K |h_k| e^{j\phi_k} Y_{k,1} \right) + \frac{\eta_p}{\sqrt{2E_b T_p}}. \quad (1.1.21)$$

Multipath propagation can thus cause a degradation in performance both directly as a result of interference on the data, and indirectly by its effect on channel estimation.

To estimate the BER performance of a Rake receiver it is necessary to determine the SI to both the data and the pilot. Since the sequences are random, one approach would be to determine the probability distributions for the SI. In a majority of publications in the open literature it has been assumed that the SI can be modelled as Gaussian as this assumption yields mathematical tractability. The effect of this assumption on Rake receiver BER performance estimation is however not understood.

1.2 Objectives

The objectives of the work reported in this thesis include:

- The determination of a method to predict Rake receiver BER performance on DSSS cellular radio downlinks, which includes the consideration of fading, multipath-related SI and channel estimation errors, without resorting to computationally-intensive simulations or making unrealistic channel and interference assumptions in order to achieve mathematical tractability.
- Application of the newly developed method to prediction of BER performance on urban microcells characterised using measured data, and comparison of BER with that achievable using methods and models previously available.
- The formulation of recommendations for the efficient selection of MPGs to be despread in a Rake receiver in order to achieve the best performance under practical operating conditions.

1.3 Contributions

- New analytical expressions for upper bounds on BER performance on fading channels with multipath related SI were derived. However, to achieve mathematical tractability it was found necessary to retain three possibly non-realistic assumptions, including a Gaussian model for interference (from a constraint that MPGs must be separated by integral multiples of a chip period), an assumption of equal average MPG powers and an assumption of uncorrelated MPG fading.
- To eliminate all assumptions and increase the efficiency of simulations, a new semi-analytical BER estimation technique based on [2], which dealt with MAI, was developed and extended to apply in the SI case.
- The complexity involved in calculating the SI using the semi-analytical technique when there are a large number of MPGs was reduced through the use of

a Gaussian approximation for the SI having the same variance as that of the SI calculated using the semi-analytical technique.

- In order to also avoid the requirement for standard fading model assumptions, a method was specified for application of the new semi-analytical technique in the estimation of average Rake BER performance based on time series of either simulated or measured fading channel impulse response functions.
- By making Rake BER performance estimates using the new semi-analytical technique in conjunction with both standard fading models and measured data for Ottawa urban microcellular channels, deficiencies in performance predictions made on the basis of standard models were determined. Recommendations were also made for more power efficient Rake receiver design.
- The new semi-analytical technique was applied to study the effect of channel estimation (i.e. Rake finger weights) errors on BER performance. This allowed the investigation of different pilot/data signal power tradeoffs under more realistic interference and channel fading conditions than previously possible. Recommendation of approaches for the determination of which of available Rake fingers should be combined using either maximal ratio combining or generalised selection combining are made.

1.4 Organisation of the thesis

The literature review in Chapter 2 presents background information on multipath fading models and diversity systems, a summary of relevant research on Rake receivers and key developments in the areas of performance estimation of Rake receivers in DSSS systems. In Chapter 3, background material on the measurement system and data processing techniques for both measured and simulated data used in the

semi-analytical technique to predict Rake receiver BER performance is presented. In Chapter 4, analytical expressions for upper bounds on BER performance of a Rake receiver are derived using simplifying assumptions about the fading and multipath characteristics of the radio channel and the statistics of the interference. The development of the semi-analytical technique for estimating Rake receiver BER performance in a multipath fading channel is presented in Chapter 5. This technique is used to estimate performance for different Rake receiver combining schemes. Furthermore, the validity of the assumptions made in Chapter 4 is discussed and their effect on performance prediction is shown to be significant. In Chapter 6, the effect of errors in channel estimation on Rake receiver performance is studied. Finally in Chapter 7, the work in this thesis is summarised and recommendations for future work are made.

Chapter 2

Literature review

2.1 Introduction

There are many papers available in the literature that report methods for the prediction of Rake receiver performance. These predictions are often made on the basis of channel models, which themselves have been developed with inaccuracies to facilitate mathematical tractability. Further, they most often use additional approximations that involve trade offs between simplicity of computation and accuracy of results. Sometimes the consequences of such compromises are serious, yet have not been properly reported or evaluated to reveal inadequacies of the reported methods. There is therefore a requirement to avoid the problems inherent with channel modelling. It also appears that there is a need for the evaluation of SI and its effects on BER performance of DSSS systems operating at high data rates.

Older generations of spread spectrum systems [3] are used primarily for voice communication with lower data rate requirements. Next generation systems on the other hand are intended for both voice and data transmission at variable data rates. Wideband CDMA systems for instance are designed to transmit up to a maximum of 2 Mbps on the downlink.

Both multiple access interference (MAI) and SI place significant limitations on

the BER performance of high data rate CDMA systems. While both MAI and SI are significant, the latter is often neglected from performance analyses because it is assumed to be negligible compared with MAI. However while it is possible to reduce the effects of MAI in a CDMA system, SI places a significant limitation on Rake receiver performance.

For example, a simple technique to achieve reliable high data rate transmission to a desired user, with a fixed downlink transmitted power, is to increase the relative power transmitted to the desired user. This reduces the MAI to the desired high data rate user. In the event of more than one high data rate user, a simple technique to circumvent the problem of MAI is to time-share the entire spectrum among users, with each user assigned the exclusive use of the spectrum over a specified time slot. In both the above cases, the problem that remains is that of SI since the energy of the interference increases with that of the signal when transmit power is increased. As explained in Chapter 1, SI is the result of multipath propagation and reduced isolation or increased cross-correlation sidelobe levels between time-delayed code sequences. With an increase in data rate for a fixed chip rate there is a reduction in the number of chips per symbol interval. This in turn results in increased SI when a Rake receiver operates on a channel that exhibits a significant amount of multipath propagation, because the cross-correlation of shorter sequences results in higher sidelobe levels.

As explained in Chapter 1, a sequence of pilot bits, time- or code-multiplexed with the data, can be used to estimate time-delays and complex amplitudes of multipath groups (MPGs) at the receiver, which are required for coherent detection. An MPG is the vector sum of all signals received within the delay resolution of the receiver. Errors in MPG amplitude estimates, commonly referred to as channel estimation errors, result from interference caused by the cross-correlation between the reference pilot sequence and any data or pilot sequences that have non-zero relative time-delays in excess of the chip interval. The average SNR of a fading MPG usually decreases

with increasing time-delay relative to the MPG with the largest SNR. The interference from stronger MPGs can therefore be significant. As a result, the accuracy of MPG amplitude estimates decreases with increasing time-delay. Another source of channel estimation error is thermal noise in the receiver.

As a result of the sources of error discussed in the previous paragraph, an important consideration is the determination of a power threshold for MPGs such that if an MPG falls below this threshold level, it should not be combined by the Rake receiver. Combining such an MPG would at worst degrade performance. At best it would lead to no significant improvement in performance at the expense of increased hardware complexity.

Sections 2.2-2.4 provide a review of work in the open literature pertinent to channel modelling, SI and channel estimation.

There are numerous techniques described in the open literature that can be used to accurately determine and track the time-delays of received MPGs intended for combining. In the treatment of the problem of channel estimation in this thesis it is therefore assumed that the Rake is able to accurately determine the time-delays of received MPGs. In the remainder of this thesis, channel estimation errors refer only to errors in the estimates of complex MPG amplitudes.

In Section 2.2, different diversity combining techniques that can be implemented using Rake receivers are described and a review of techniques used to estimate performance of diversity combining schemes under various channel conditions is presented. This is followed by Section 2.3 in which the application of Rake receivers in DSSS systems is reviewed with a focus on literature concerning the effects of SI. Finally, in Section 2.4 papers that deal with the effects of channel estimation errors on performance are reviewed.

2.2 Diversity combining and its effect on receiver performance

The Rake concept was first proposed by Price and Green in [1] as a method to achieve diversity by weighting time-delayed replicas of the transmitted signal arriving at the receiver and recombining them after aligning them in time. Receivers that use this concept can mitigate the effects of fading if the fading of resolvable MPGs is uncorrelated to some extent [4]. In recent years there have been a number of publications that address the performance of diversity combining schemes under different fading conditions. The following terminology is used herein to describe commonly used diversity combining techniques that can be implemented through the use of Rake receivers [5], [6]

- Maximal ratio combining (MRC) describes a technique in which all received MPGs are weighted in proportion to their SNR and combined coherently.
- Equal gain combining (EGC) describes a technique in which a subset of all received MPGs are weighted equally and combined coherently.
- Selection combining (SC) describes a technique in which only the MPG with the largest SNR is used. Strictly speaking there is no signal combining when this approach is taken.
- Generalised selection combining (GSC) describes a technique in which a subset, usually the largest in amplitude, of available MPGs is selected. Each selected MPG is weighted in proportion to its SNR and combined coherently.

The performance of a diversity combining scheme can be assessed either in terms of its ability to reduce the probability of deep fades or to improve the BER on the associated radio link. The performance of diversity combining techniques that can be

implemented using Rake receivers has been investigated in a number of publications. Such publications usually utilise a model to describe the fading distribution of received signals. Fading models are a result of the consideration that the signal received on a multipath channel is a vector sum of multiple component signals.

In [7], Rayleigh showed that if n unit vibrations of equal pitch and random phase are summed vectorially, the probability of a resultant intermediate in amplitude between r and $r + dr$ is $\frac{2}{n}e^{-\frac{r^2}{n}}rdr$. The probability of a resultant of amplitude less than r is therefore given by

$$P[r \leq R] = \int_0^R \frac{2}{n}e^{-\frac{r^2}{n}}rdr = 1 - e^{-\frac{r^2}{n}}. \quad (2.2.1)$$

Rayleigh also demonstrated that the probability is not altered if the components have different amplitudes, provided there are a large number of waves. A commonly-used model to derive the Rayleigh distribution is by generating a complex signal in which the quadrature components are zero-mean Gaussian.

For situations in which one or several received components has a deterministic phase, a model developed by Rice to describe the distribution of a sine wave in Gaussian noise is applied. In [8], Rice showed that for the case of a sine wave in Gaussian noise, the probability that the resultant amplitude lies between r and $r + dr$ is

$$p(r)dr = \frac{r}{\Psi}e^{-(r^2+B^2)/(2\Psi)}I_0\left(\frac{rB}{\Psi}\right)dr, \quad (2.2.2)$$

where I_0 is the zero order Bessel function, B is the amplitude of the steady signal and $\Psi = \frac{R_0^2}{2}$, where R_0 is the rms value of r . A commonly-used model to derive the Rice distribution is by generating a complex signal in which the quadrature components are Gaussian with an equal non-zero mean. Sometimes this equation is expressed in terms of K_{Rice} , where $K_{Rice} = r^2/B^2$.

Another fading model, the Nakagami model [9], was derived from the analysis of a series of statistical studies of HF signals received over long distances for short time intervals. The probability density function (pdf) for the envelope R , was deduced from the data to be

$$f_R(r) = \frac{2m^m r^{2m-1}}{\Gamma(m) \Omega^m} e^{-(m/\Omega)r^2}, \quad (2.2.3)$$

where $\Omega = \mathcal{E}_n \{R^2\}$, is the power of the received signal averaged over time and

$$m = \frac{\mathcal{E} \{r^2\}^2}{\mathcal{E} \{r^2 - \mathcal{E} \{r^2\}\}^2} \geq \frac{1}{2} \quad (2.2.4)$$

is the inverse of the normalised variance of R^2 .

The Rayleigh pdf provides a suitable model for a non-line-of-sight propagation path between the transmitter and receiver. The Ricean pdf on the other hand can be used to model the envelope fading statistics of an MPG that consists of a coherent replica of the transmitted signal along multipath signals having random phase. It is therefore a suitable model for cases in which there is either a line-of-sight between the transmitter and receiver or a specular reflection is received. As will be demonstrated in Chapter 5 with the use of measured data, the variance of the signal envelope is greater in Rayleigh fading conditions than it is under Ricean fading conditions. The Nakagami pdf reduces to the Rayleigh pdf for $m = 1$ and can be used to approximate the Ricean pdf [10]. For $\frac{1}{2} \leq m < 1$, the Nakagami pdf can be used to model fading situations where the envelope variations are greater than those for the Rayleigh pdf, thereby allowing for greater flexibility in modelling. The Rayleigh pdf has been used extensively as a fading model in the literature as it lends itself well to solving analytical expressions for the BER. However, as with any model, those described above provide only an approximation to the fading characteristics encountered on many urban microcellular channels. Examples of the goodness-of-fit of such models

to cumulative distribution functions estimated from measured data can be found in Chapter 3.

Closed-form expressions for BER performance with MRC on a Rayleigh fading channel can be found in [11]. This was the first work to eliminate the i.i.d. assumption. More recently in [12] and [13] a closed-form expression was derived for the received SNR in the GSC scheme for i.i.d. MPG fading. Using this expression, it was demonstrated that the average SNR after combining is upper bounded by the average SNR for MRC and lower bounded by the average SNR for SC. The work in [12] and [13] was extended in [14] in which a simple closed-form expression for the average SNR with GSC was found for a non-i.i.d. Rayleigh fading channel with unequal average MPG SNRs. Numerical results presented in [14] demonstrate that the total average SNR after combining decreases with an increase in the decay parameter of the average power delay profile, which is an average of the squared magnitudes of impulse response estimates (IREs). As a result, the i.i.d. case studied in [12] and [13], for which the average branch SNRs are equal, gives the highest average SNR after combining.

In [5], Kong et al. demonstrated that the BER performance of the GSC scheme approaches the BER performance of the MRC scheme with higher rate decays of the average power delay profile. This is expected as the fraction of the total energy captured with a fixed number of branches increases with an increase in the decay rate of the multipath delay profile. Kong also reported that with an increase in the number of combined MPGs, performance of the GSC scheme approaches that of the MRC scheme.

Alouini and Simon in [15] analysed the performance of GSC for Nakagami- m fading [16] of MPGs. Bit error rate performance results indicate an expected improvement in BER with an increase in the number of branches from two to four when there are four received MPGs. They also showed that BER performance for

a fixed number of received MPGs having equal average SNR, and a fixed number of branches, improves with increasing fading figure m , or reduced severity of fading. An interesting result in [15] is that a reduction in fading figure decreases the relative improvement in performance obtained when additional MPGs are combined. For instance increasing the number of combined MPGs from two to three would result in a smaller performance improvement for $m = 1/2$ than it would for $m = 2$. This is a result of the increased severity of fading when $m = 1/2$ compared with $m = 2$. For $m = 1/2$, the likelihood of a null in the envelope of the combined signal would be greater than the likelihood of a null in the envelope of the combined signal for $m = 2$ for a fixed number of branches and received MPGs.

Alouini and Simon compared the performance of coherent receivers using GSC with that of those using MRC on Nakagami- m fading channels in [6]. It was found that the gain in performance using MRC over that achieved using GSC is a constant, independent of the SNR, but dependent on the number of branches in GSC. Furthermore they reported that the gain using GSC over that achievable with SC is also constant independent of SNR, but dependent on the number of branches used in GSC. An observation in this paper, closely related to that in [15], is that for channels with severe fading, performance with GSC is closer to MRC than for channels in which the fading is less severe. This is because when the fading of MPGs is very severe, GSC ignores MPGs which if combined would only lead to small improvements in performance. Recall that the improvement in performance decreases with a reduction in the fading figure or increased severity of fading. On the other hand, when the fading is less severe, the difference between MRC and GSC performance results from ignoring MPGs which are significant sources of energy.

In [17], Win and Kostic investigated the effects of signal bandwidth in spread spectrum systems operating in dense multipath environments using GSC. Results demonstrate that, for a channel with a fixed maximum time-delay, as the bandwidth

increases the average SNR of a single MPG decreases as a result of increased time resolution. Therefore, a fixed number of fingers capture less energy with increased bandwidth. Also, the average combined SNR increases with an increase in the number of diversity branches for a fixed bandwidth.

Work reported in the above-mentioned publications focuses on performance estimation for diversity systems when the envelope fading of resolvable replicas of the transmitted signal is uncorrelated. In reality, envelope variations of replicas of the transmitted signal that are separated in either space and/or time at the receiving terminal can be correlated. Under such conditions, it is expected that there would be a degradation in performance when compared with the uncorrelated case.

One of the first papers in which BER performance under correlated fading conditions was studied was [11] in which Pierce and Stein derived the average error probabilities for BPSK modulation with MRC and BFSK with EGC. The statistics of the correlated received signal components were assumed to be Rayleigh and identically distributed. Two example calculations are presented in [11] using series approximations for the analytical expressions for the probability of bit error. No general interpretation of results were presented or could be made.

In [18], Aalo estimated the BER performance of MRC over a Nakagami fading channel on which the envelope fading of received MPGs was assumed to be correlated. The correlation was modelled as being either constant or decaying exponentially with increasing time-delay. Results indicate a degradation in performance with increased correlation among the branches. Also, consistent with the findings in [15] for uncorrelated fading, Aalo found that performance under correlated fading conditions for a fixed number of received MPGs improves with increasing fading figure m , or reduced severity of fading.

In [19], Alouini and Simon extended the work of [18] by deriving expressions for the pdf and moment generating function (MGF) of the combined signal when using MRC.

In addition to the two cases of identically distributed branches¹ with exponential and constant correlations in time-delay, they derived the pdfs for two correlated branches with non-identically distributed branches (each branch has the same fading figure but different average SNR) with arbitrary correlation. The pdf and MGF derived in [19] can be used to compute the probability of bit error. However such results were not presented.

More recently in [20], Veeravalli presented a general approach for analysing the performance of digital signalling with diversity reception on channels for which the fading of received MPGs is correlated. While the approach of [18] uses an approximation to the pdf of the combined signal, the approach in [20] utilises a complex Gaussian model [21]. The fading correlation among resolvable replicas of the transmitted signal in space or time was modelled as either following a Bessel function or decaying exponentially with space or time. Results for BER performance estimation in this paper are consistent with expectations that increased correlation causes degradation in performance. An example of how this knowledge can be used in practice is demonstrated by considering an antenna array with two elements spaced such that the received signals at the elements are uncorrelated. If a third element is placed between the two, such that the received signal is correlated with the signal received at other two elements, the improvement in performance over the two-element case would be smaller than if the element were placed so that the received signal is uncorrelated with the signal received at the other two elements. The trade off, however, is in the increase in size of the array for the latter case compared with the former.

¹Under the assumption that the complex amplitudes and time-delays of replicas of the transmitted signal arriving at the receiver can be estimated with no error, the fading statistics as well as the instantaneous amplitude of each diversity branch, or simply branch, are identical to those of the replica it is assigned to.

The analyses of the performance of systems in which diversity combining is employed reported in [16]-[20] can be applied to estimate receiver performance in situations in which thermal noise is the main source of signal distortion. The reported methods can also be applied when the interference to each diversity branch (assumed to have the same average SNR) can be modelled as Gaussian with the same variance. However, in the case of interference on real-world radio channels (on which MPGs probably do not have equal average SNR), it is reasonable to expect that, as a result of SI, the interference to a weaker MPG will be more significant than that to a stronger MPG. Under such circumstances, certain findings in the above publications are therefore not valid. For instance, the observation in [6] that the gain of GSC over that provided by SC or the gain of MRC over that provided by GSC is independent of the combined average received SNR will not be true.

2.3 The effect of interference on Rake receiver performance in DSSS systems

Rake receivers are used extensively in DSSS systems, where replicas of the transmitted signal can be resolved in time at the receiver. The resolution is achieved as a result of code sequences with low autocorrelation sidelobes that spread data symbols over a larger bandwidth. Techniques for estimating the BER performance of Rake receivers in a DSSS system are similar to those used in other diversity systems as discussed above. If interference is modelled as Gaussian, its variance can be added to that of the noise. Bit error rates can then be calculated using the Gaussian-Q function [16]. In situations where interference cannot be modelled as Gaussian a more complicated treatment of the problem is required. The remainder of this section is a review of literature that deals with techniques for BER performance estimation taking interference into account.

In a comprehensive tutorial and research paper [22], Turin investigated the use

of spread spectrum techniques for combatting the effects of multipath propagation on radio channels in high data rate applications. The first part of the paper discusses models for multipath propagation. The later part of the paper investigates optimal and sub-optimal receiver combining strategies in situations where the delays or strengths of MPGs are not known a priori. Different decision mechanisms are also discussed. The essence of combining laws according to Turin is scaling the signals on different branches in proportion to their credibility. Maximal ratio combining is one example of this approach.

Turin's paper was one of the first to discuss the effects of multipath induced interference. Intersymbol interference (ISI) occurs when the time interval between two successive symbol transmissions is less than the maximum multipath delay on the channel. In such situations equalisation techniques can be used to cancel the effect of the delayed symbol for small spreading factors or when the symbol period, T_d , is not much larger than the chip period, T_c . In the case of a Rake receiver, when $T_d \gg T_c$, Turin makes the argument that it would be unlikely for a symbol delayed by more than a symbol interval to be time aligned with an activated Rake finger. This delayed symbol would therefore not be despread and no ISI would result. To assess the impact of ISI on performance, an equivalent noise analysis was used where the interference was modelled as Gaussian and the symbol period was assumed to be much larger than the chip period. It was also assumed that the fading of equal average powered MPGs could be modelled as Rayleigh. Performance results indicate that for a specified BER the required SNR per MPG decreases exponentially with an increase in the number of groups when MRC is used.

Performance comparisons between GSC (which Turin refers to as a DRAKE) and MRC, using simplified theoretical estimates that neglect considerations of interference, reveal a constant difference in average required SNR of about 3 dB for all values of probability of error greater than 10^{-5} at both low (78.7 kbps) and high data

rates (787 kbps) for a fixed spreading factor of 127, with performance in the low data rate case better than the high rate case. This trend is consistent with the findings in [6].

An interesting comparison made in Turin's paper is that between simulated results and performance estimates made using the simplified theoretical models for DRAKE performance. A database of IREs from measurements was used for the simulation. A set of random data bits was spread using a 127 chip maximal-length shift register sequence and convolved with the set of channel IREs. The DRAKE decision scheme was simulated. The resulting BER was compared with simplified theory. Theoretical performance estimates indicate the required average SNR at a BER of 10^{-4} is about 7 dB less than that required based on the simulation results. This optimism is explained by Turin as the result of the following factors:

- Actual urban fading is more severe than Rayleigh fading;
- The fading of MPGs can be correlated in practice;
- Multipath group powers have smaller variances in local areas than on a global basis. The simplified theory assumes larger global variances apply everywhere, thereby leading to optimistic path diversity gain and thus lower probability of error;
- Simplified theoretical estimates neglect the effects of ISI.

Similar considerations motivated the work in this thesis.

Cheun derived expressions in [23] for probability of error for DSSS systems using BPSK and QPSK modulation with Rake reception. The fading of received MPGs was modelled as Rayleigh and i.i.d. The analysis accounts for the effects of SI when random spreading sequences are used. For spreading factors greater than 64 the interference was modelled as having a Gaussian distribution [24]. Performance results

are presented for different numbers of MPGs and Rake fingers, for EGC, SC and GSC under two Rake finger assignment schemes. In one scheme, the Rake combines a subset of MPGs with the largest average SNR. In the other scheme, the Rake selects the subset with the largest instantaneous SNR. Performance results indicate the latter scheme performs better, but it is difficult to implement in practice since there is a finite time-delay between channel estimation and application of the channel estimates to combine received MPGs. Performance results for SC indicate that the BER increases with an increase in the number of received MPGs, resulting in increased SI. Results for GSC show that for a fixed number of received MPGs, performance improves with an increase in the number of fingers less than or equal to the number of received MPGs.

To avoid the Gaussian approximation, in [25] Hwang and Lee analysed the performance of Rake receivers used in DSSS systems operating at high data rates using a simulation-based technique that averages performance over random spreading codes. They assumed i.i.d. Rayleigh fading for all MPGs for both MRC and EGC. The cases investigated in [25] were those in which the number of fingers is the same as the number of MPGs. In the case when there are two received MPGs the relative time-delay for the second MPG is uniformly distributed over either $[T_c, 2T_c)$ or $[T_d, T_d + T_c)$, where T_c and T_d are the chip and data symbol periods, respectively. In the first case the interference terms for the two Rake fingers are correlated as the sequence cross-correlation terms are almost identical, while in the latter case they are uncorrelated because the sequence cross-correlation terms are in general not the same. Performance results indicate that the probability of bit error in the uncorrelated case is lower than that for the correlated case. Also results indicate, as would be expected, an improvement in performance with increasing spreading factors. This is the result of a reduction in SI with an increase in spreading factor. Furthermore, the difference between results obtained using the Gaussian approximation for interference (using the

equations in [23]) and the simulated result increases with a reduction in spreading factor for $\text{BER} > 10^{-6}$.

In [26], Boujemaa and Siala investigated the effect of SI on Rake receiver performance. A simplification in the expression for probability of error was achieved by assuming that received MPGs arrive at time-delays that are integer multiples of the chip period apart from one another. The result of this assumption is that SI can be modelled as a binomial random variable (later in this thesis it will be shown that simulation results based on this assumption result in overestimating the SI). The authors also compared BER results from simulation with results obtained using both Gaussian and binomial models for interference for MRC. The BER from simulation compares well with the BER estimated with the use of the binomial model, for all values of spreading factor considered. As well, there is good agreement between the BER from simulation and the BER using the Gaussian approximation for spreading factors of 16 and 32. The Gaussian approximation however is not valid for spreading factors less than 8. Also, as would be expected, Rake receiver BER performance improves with an increase in spreading factor.

In [27], Guenach and Vandendorpe analysed the BER performance of a Rake receiver using MRC in a WCDMA downlink. Simulation-based BER performance results are presented for the situation in which six MPGs are received and combined by the user of interest. It was assumed that fading of MPGs was Rayleigh distributed and relative average powers and time-delays were fixed according to a pre-determined channel model. The system was utilised by three active users and as a result there was both SI and multi-user interference. For the case in which the channel was estimated perfectly, i.e., there were no errors in estimating either the amplitudes or the time-delays of received MPGs, results indicate an improvement in BER performance for the user of interest with an increase in spreading factor employed by the desired user or that employed by the other two users. When the spreading factor of the desired user

is increased, SI decreases thereby resulting in an improvement in BER performance. It was also found that as the spreading factor of other users increases, the multi-user interference to the desired user decreases when the energy per data bit is fixed. This is because the power of the interference over the symbol duration of the desired user decreases with an increase in spreading factor of the interfering users' signals. The effect of channel estimation errors for both time- and code-multiplexed data and pilot sequences was also investigated. Results indicate improved BER performance with an increase in the number of pilot symbols over which MPG amplitudes are averaged for both the code- and time-multiplexed cases. Since interference by the pilot to the data is significant in the code-multiplexed case, the authors proposed cancelling this known interference prior to detecting the data. A larger averaging period would then contribute to more accurate amplitude estimates and hence improved interference cancellation. This is an additional benefit to the improvement in performance gained from increased accuracy of channel estimates used for combining. Also, for the same averaging length, performance for the code-multiplexed case is worse than that for the time-multiplexed case for the same pilot averaging length. However, the trade off, which is a reduction in the number of data bits transmitted over a fixed time interval in the time multiplexed situation, was not investigated. Such an investigation should have been conducted in order to facilitate a fair comparison.

In [25] it is claimed that for values of BER expected to be in the range $> 10^{-4}$, the Gaussian approximation is not valid for low spreading factors. In this thesis it will be shown that there exists a Gaussian assumption that is only useful for predicting the performance of a correlation receiver when there are a large number of MPGs. The Gaussian assumption cannot be used in general to predict Rake receiver BER performance. Additionally assumptions such as that of uncorrelated fading and of equal average SNRs of received MPGs are not often valid as will be demonstrated in this thesis through the analysis of propagation measurements. In addition, MRC is

not always possible, for example, in a propagation environment that changes between channel estimation intervals with the addition or disappearance of MPGs and/or for reasons of limited availability of hardware resources (Rake fingers). It is useful therefore to estimate performance when GSC is used so that the number of Rake fingers can be less than the number of received MPGs.

Thus none of the techniques reported in the cited papers were considered adequate for use in achieving the objectives of the thesis. Also while the simplicity of simulation-based approaches makes them attractive, it is expected that the use of such techniques for BER estimation using measured data would be both time-consuming and computationally intensive.

In [2], Lehnert and Pursley estimated the effects of MAI on performance in a DSSS system for random spreading sequences when a correlation receiver is used on a non-fading channel. The interference due to 2^{N_d} possible sequences of length N_d was divided into N_d classes leading to tractable expressions for BER. Other simulation-based approaches would scale exponentially in complexity requiring estimates of interference due to a significant number of possible combinations of sequence delays of length N_d . The complexity of the approach in [2], by comparison, scales as N_d . This made it possible to obtain distributions for inter-chip interference over a symbol interval, for different spreading factors. It is considered that this approach might also be useful in achieving tractable expressions for the BER in cases where SI is taken into account.

Application of the approach discussed in the foregoing paragraph to estimate performance when a Rake receiver is used would involve extending the analysis to account for unequal average powers, fading and correlations among different MPGs. Also, depending on the relative time-delay between two combined MPGs and the spreading factors, the interference to the fingers aligned with these MPGs may be highly correlated for small relative time-delay, or uncorrelated for large relative time-delay. As a

result, performance for arbitrary delays can be bounded using this approach. Bounds on the BER performance of a Rake receiver are derived in this thesis. Since measured data, discussed in Chapters 3 and 5 herein, indicate that the fading statistics of resolvable MPGs do not always conform to commonly-used distribution functions, a semi-analytical technique based on this approach provides an attractive alternative.

2.4 Channel estimation

A Rake receiver requires knowledge of the complex amplitudes of received MPGs, in order that time-delayed replicas of the transmitted signal can be coherently combined using either MRC or GSC. For this purpose a time-multiplexed or code-multiplexed pilot sequence can be used. A correlation receiver can extract MPG delay, phase and amplitude information by cross-correlating the pilot with a reference sequence that is generated locally in the receiver. The pilot channel is, however, subject to the same multipath effects as the data sequence. Errors in channel estimation as a result of MAI, SI and thermal noise therefore must be taken into account when predicting link BER performance. In the remainder of this section, past research in estimating the degradation in the BER performance as a result of errors in channel estimation, first for diversity systems in general and then more specifically for DSSS systems, is summarised. Using this knowledge, BER estimation techniques can be applied to determine the impact of combining MPGs at large excess time-delays which in general have lower average SNR. Literature pertinent to the development of algorithms that are designed to exclude noisy MPGs in Rake combining is therefore also discussed.

The effect of channel estimation errors resulting from decorrelation between the pilot and data as a result of frequency selective fading in a frequency division multiplexed system in which the pilot and data occupy different frequency bands was studied by Bello and Nelin in [28] for a system employing frequency diversity. In this paper, analytical expressions for error probabilities are presented for both coherent

and non-coherent detection schemes and MRC. It was assumed that the fading of the quadrature components of the different tones carrying the same data signal was Gaussian and that all tones had equal average power. Performance results indicate an increase in probability of bit error for a fixed order of diversity as the decorrelation between the pilot and data increases. Also, BER results for any order of diversity asymptotically approach the performance results for no diversity with a reduction in correlation between pilot and data.

In [29], Gans also investigated the effect of Gaussian errors in MRC branch weighting factors as a result of frequency or time decorrelation between the pilot and data. Results indicate, as in [28], an increase in amplitude and phase error for each branch with a reduction in correlation between the pilot and data. An interesting finding reported in this paper is that small errors in channel estimation reduce the ability of an MRC scheme to eliminate deep fades. These errors, on the other hand, have a lesser impact on the median SNR. As a result, if deep fades are infrequent, small errors have little effect on system performance.

Based on expressions in [29], Tomiuk et al. [30] introduced general relationships for the output SNR distribution and BER performance of non-ideal MRC. The analysis reported in [30] assumes that the statistics of the weighting errors are Gaussian, and the fading of the diversity branches is i.i.d. Analytical expressions for the BER for DPSK with non-ideal MRC are derived and used to estimate performance. It was found that the rate at which the BER decreases with increasing SNR for imperfect combining is the same as the rate at which the BER decreases with increasing SNR for the case of no diversity (single branch case). This rate was reported to be $1/\Gamma$, where Γ is the mean branch SNR, assumed equal for all branches. As the SNR increases channel weighting errors become negligible. For perfect combining it is known that the asymptotic BER decays as $1/\Gamma^L$ where L is the number of branches [11].

Recently, in [31], Singh et al. derived BER expressions for binary coherent orthogonal signalling with MRC when there are errors in estimates of complex MPG amplitudes. In this work, the weighting errors were assumed to be the result of additive channel noise. For large branch SNRs it was found that the probability of symbol error degrades by a factor of 2^L compared with perfect channel estimation, where L is the number of diversity branches.

In [32], Benthin and Kammeyer investigated the effect of channel estimation errors on the uplink of a CDMA system. Pilot symbols, code-multiplexed with the data, were averaged over time to minimise the influence of noise. The averaging period had to decrease in accordance with increasing mobility in the channel in order that changes in the channel MPG amplitudes could be tracked with some accuracy. Hence there was a trade off in the averaging time interval. Received MPGs were assumed to be i.i.d. and Rayleigh distributed. The interference to the data from the pilot was modelled as Gaussian. Bit error rate results obtained from simulations are presented for optimal choices of averaging period, vehicle speeds and ratio of data to pilot power, for a fixed spreading factor for both pilot and data and a fixed number of users. Results for the different parameter choices were used to determine the best achievable symbol error rate as a function of the SNR, which deteriorates with increasing vehicular speed and is lower bounded by the situation in which the channel is estimated perfectly. An interesting finding reported in this paper is that the sensitivity of the symbol error rate to the pilot averaging interval is greater than its sensitivity to the ratio of data to pilot power level.

In [33], Ko and Choi also estimated the BER performance of a Rake receiver (using MRC) over a Rayleigh fading channel with imperfect channel estimation. On fading channels, errors in channel estimation can be attributed to the Doppler spread on the channel, receiver carrier offset with respect to nominal transmitted signal centre frequency caused by unknown local oscillator frequency instabilities and to thermal

noise. All these phenomena were simulated. Multipath interference from all users (including the desired user) was modelled as Gaussian. Results indicate performance degradation with increasing Doppler shifts for a fixed pilot symbol averaging period. Also, it is shown that the averaging length that minimises BER decreases with an increase in Doppler shifts. This is expected as with increasing mobility there is a reduction in coherence time.

On real-world radio channels, MPG amplitudes most often do not have equal average powers, but their powers generally decrease with increasing time-delay with respect to the strongest MPG. As a result it is expected that the degradation in the accuracy of MPG amplitude estimation as a result of SI and thermal noise would increase with increasing relative time-delay with respect to that of the strongest MPG. To avoid this situation, Fukumoto et al. [34], addressed the problem of selecting only MPGs with the largest SNRs in the case where the pilot and data are code-multiplexed. Using a simulation, two selection thresholds to be applied sequentially were determined. The first threshold was used to select MPGs having an SNR within a certain range below the maximum. The second threshold rejected noisy MPGs among those selected by the first threshold. These thresholds were determined from simulation. All simulations were performed with equal pilot and data spreading factors of 64. Standard ITU-R channel models described in [35] with a fixed number of MPGs were used. Based on performance results, two thresholds were derived to determine the validity of an MPG. The resulting combining rule was that an MPG should be combined if it has a power greater than 5 dB below the strongest MPG and at least 3 dB above the mean noise level.

In [36], Bejjani et al. implemented an algorithm that first eliminates the consideration of data in the time slots corresponding to valid MPGs from channel impulse response estimates (IREs). Time-delays of these MPGs are determined by non-coherent averaging of IREs over a time slot that contains six pilot symbols. What remains in

the IRE after eliminating valid MPGs is assumed to be due to noise and interference. Samples of the residual are then used to compute the statistics of the noise, knowledge of which in turn is used to determine an MPG rejection threshold. Multipath effects were neglected and the noisy MPGs were assumed to be the result of thermal noise and MAI, both modelled as Gaussian. Statistics of the noise are computed periodically and the resulting threshold varies with time. Performance in terms of required SNR per branch for a fixed BER as a function of mobile speed was estimated and compared with results from [34]. Performance for fixed thresholds of 3 dB, 6 dB and 9 dB, below the strongest MPG was also estimated for fixed data and pilot spreading factors of 128 and 256, respectively. Results show the best performance is obtained when the variable threshold is used.

In [37], Elders-Boll demonstrated a simplified dynamic threshold rule for MPG selection and presented the argument that the selection technique reported in [36] required too much complexity as a result of discarding valid MPGs prior to determining the statistics of the noise. The algorithm presented in [37] simplifies the process of determining a threshold. The rationale for using this algorithm is based on the fact that the number of valid MPGs in an IRE is small. The threshold can therefore be based on knowledge of the average power delay profile over some particular period, without first having to identify valid MPGs in order to estimate noise statistics. Simulation results indicate an improvement over performance achieved when using algorithms presented in [34] and [36].

In [38], Ling investigated the influence of code rate, pilot to data power ratio and order of diversity (m) on Rake receiver BER performance for a Rayleigh fading channel on which MPGs have equal average SNR. It was assumed that the total interference could be modelled as Gaussian. Only MAI was considered as it was assumed that the effect of SI on performance was negligible. The receiver used MRC. Channel estimation was accomplished using a code-multiplexed pilot. Ling found

that the SNR required to achieve the best BER performance decreases monotonically with code rate for the values of $m=1, 2$ and 4 considered. Also for low code rates, the performance of a pilot-assisted coherent detection scheme degrades as m increases. This is because as m increases, the signal to interference ratio per MPG decreases thus leading to greater inaccuracies in channel estimates. The benefit from diversity combining is outweighed by the degradation due to channel estimation errors. For higher code rates however there is a benefit to diversity combining.

A review of the literature has thus shown that while the approaches of Bejjani [36] and Elders-Boll [37] have been used to determine an adaptive threshold for discriminating between MPGs and noise, the effects of relative power levels of the pilot and data signals and their spreading factors on channel estimation errors causing BER performance degradation still needs to be investigated. Furthermore, the above-mentioned approaches are difficult to implement in real-time in a mobile terminal.

2.5 Summary

A review of literature pertinent to Rake receiver performance in DSSS systems has been presented in this chapter. This has revealed that most early analyses of diversity combining schemes used in Rake receivers in spread spectrum systems neglected to account for the effects of interference on performance. In the DSSS systems under consideration in this thesis, interference can be of two kinds, SI and MAI. Much of the work reported in the literature ignores the effect of SI on performance because in multi-user environments, it can often be neglected in comparison with the MAI. However, while interference cancellation and power control techniques can be used to mitigate the effects of MAI on the downlink, these techniques cannot be used to reduce the effects of SI without sacrificing diversity gain. Self-interference can therefore remain a significant influence causing performance degradation in DSSS systems operating at high data rates. As well, with the trend towards higher data

rates resulting in lower spreading factors (in systems such as WCDMA which have been allocated a fixed bandwidth) SI is more severe.

Some of the research conducted during the last two decades has taken the effects of SI into account under the assumption that this interference can be modelled as Gaussian. The use of the Gaussian approximation simplifies the process of BER estimation. It is however not well understood how closely this model for interference compares with reality. More recently, therefore, simulations have been used extensively when it is desired to take the effects of SI into account. A number of authors have used simulations employing an equal average power assumption for a fixed number of Rayleigh fading MPGs and MRC in the receiver. It is well known, however, that the average powers of received MPGs decay with increasing time-delay. In addition, as a result of limited hardware resources in the mobile terminal, it is seldom practical or possible to implement MRC. The utility of such simulations is therefore questionable.

The trend towards a requirement for higher rates of data transmission over increasingly scarce spectrum resources requires more accurate models for interference in order to better predict performance for efficient system design. Efficient techniques must therefore be developed to use these models in conjunction with an assumption of GSC rather than MRC, and either multipath fading models that more accurately reflect real-world channels or data measured on a fading channel.

While performance predictions are often made under the assumption that the receiver can estimate the complex amplitudes of MPGs for the adjustment of Rake finger weights with no error, this is seldom the case in real-world situations. It would therefore be impossible to design an efficient system without taking into consideration the effect of channel estimation errors on performance. It is expected that the likelihood of errors in estimating the complex amplitude of an MPG increases as its average power decreases, as a result of SI from stronger MPGs and thermal noise.

Such errors result in incorrectly weighted Rake fingers which cause an increase in BER. The development of methods for accurately predicting BER performance when SI is significant would also facilitate improved studies of the trade offs between combining all available MPGs, and combining only those that are despread under the best SNR conditions to avoid such penalties.

This literature review has revealed that a number of papers present analytical expressions and BER performance results for Rake receivers when channel estimation errors are taken into account. Most, however, use a Gaussian model for the distribution of channel estimation errors and neglect the effect of SI. The effect of these assumptions on performance prediction is not well understood. One exception that used simulations rather than making the usual assumptions was a recent paper in which the authors investigated the effect of channel estimation errors on Rake receiver performance in DSSS systems when a channel model with fixed average MPG powers and data and pilot spreading factors are assumed. The conclusions, however, are not general enough because they used a fixed channel model.

Chapter 3

Data extraction for measurement- and simulation-based BER performance estimation

Measured and simulated data are both used as a basis for Rake receiver BER performance estimates made using the semi-analytical technique developed in Chapter 5. The techniques used to process measured data from downtown Ottawa are described in this chapter.

The impulse response of a channel can be estimated by transmitting a pseudo-noise (PN) sequence of bits, sampling the received signal and cross-correlating it offline with a reference sequence identical to the transmitted one. Time-series of received signal data were measured before this thesis project began by researchers at Communications Research Centre in Ottawa [10], [39] using a 5 Mchips/second channel sounder. Time-series of impulse response estimates extracted from these data were used to estimate the probability of bit error for low data rate applications. The 5 Mchips/second sounder had a sounding spectrum with a $\frac{\sin x}{x}$ characteristic with its first null at ± 5 MHz. This results in a flat region of the sounding spectrum centred at midband with a 3 dB bandwidth of about 4 MHz. Thus for application to studies involving a system with 3.84 MHz bandwidth, the results from the measurements

cannot be considered as good estimates to the channel impulse response. However, since the sounder is a spread spectrum system, it was considered reasonable to filter its spectrum at 3.84 MHz and process the corresponding time domain results as approximations to those that would be realised in a WCDMA receiver with the same bandwidth. Since the measurement results were not good approximations to the channel impulse response they are herein referred to as experimentally derived system response estimates (ESRs).

Time-series of ESRs processed to determine the relative time-delays and complex amplitudes of MPGs were used to obtain realistic BER performance estimates for Rake receivers operating in urban microcellular environments. Comparisons between Rake receiver BER performance estimated using a Rayleigh i.i.d. channel model and that estimated based on measured data were made. The goodness-of-fit of the fading cumulative distribution function (CDF) of each MPG to either a Rayleigh or Ricean model was then determined. Only measured time-series in which at least the strongest MPGs was either Rayleigh or Ricean were selected from the measured data pool for use in these comparisons. This enabled a study of the effects of correlation and unequal average powers of MPGs on performance, with the knowledge that the fading models for both measured and simulated data were either almost the same or distinctly different. The Kolmogorov-Smirnov (KS) test [40] was used to determine the goodness-of-fit for the models.

3.1 Measurement setup

The system used for the measurements employed a fixed transmitter situated so as to simulate a microcellular base-station. The receiver was mounted in a mini-van. During the measurements, the mini-van moved along different urban street sections at typical urban traffic speeds of 50 km/hr or less. A known pseudo-noise maximal length sequence of length 511, BPSK modulated on a carrier at a centre frequency of

1.9 GHz, was transmitted continuously at a rate of 5 Mchips/second. The received signal was sampled at 10 Msamples/second. Four sequence lengths consisting of a total of 4088 samples were recorded every 2.2 ms at the receiver for subsequent offline processing to extract ESRs. The sequence sampling rate of 450 Hz is about five times the maximum expected Doppler shift. For the purposes of noise reduction, the four sequence lengths could be averaged. However the requirement to preserve the phase information led to the use of only one out of every four sequences to determine each complex ESR. As a final step each ESR was filtered with a raised cosine filter having a bandwidth of 3.84 MHz in order to make the filtered result similar to the signal that would be received on the downlink of a WCDMA system having the same bandwidth.

3.2 Data processing

Figure 3.1 shows the locations of the street sections (indicated by arrows) where 2.2 s measurement runs were made. Runs were constrained to such short intervals because of limited storage capacity in the data collection system.

A characterisation of the stochastic characteristics of the measured radio channels requires the assumption of ergodicity so that ensemble averages, on which much of statistical theory is based, can be estimated from time averages. The validity of such an assumption was assessed for every measurement run by noting that if a stochastic process is Gaussian and wide-sense stationary (WSS), then it is also ergodic [41]. Thus, as in [42] checks were made to ensure the Gaussian WSS model was appropriate for the measured data. Briefly, Gaussianity was assessed through the Central Limit Theorem by observing the number of distinct eigenvalues in the covariance matrix for the time-series of complex amplitudes of a given MPG. The WSS nature of the channel process was assessed by applying the RUNs test [41], with 20 samples per bin and a 95% confidence level, to the time-series of a spectral line close to midband. The longest series of measured data recorded during each run that exhibited conformance

Figure 3.1: Street sections along which measurements were made.

was subsequently used as a basis for the determination of envelope fading distributions for the MPGs. Data series that did not conform were discarded, since non-stationary characteristics could have biased any fading distributions estimated from them. In addition, if there was no basis for an assumption of ergodicity, any analysis and modelling results would not be representative of anything but the single series of data that was analysed, rather than the associated type of radio channel. It was found that WSS intervals coincided with intervals over which at least the strongest MPG could be modelled as either Rayleigh or Ricean.

3.3 Multipath group identification and normalisation

A set of 80 time-series was recorded during measurements in downtown Ottawa. Each ESR in each time-series was then filtered with a raised cosine filter of 3.84 MHz bandwidth and a roll-off factor of 0.22. A non-coherent average of the squared magnitudes of the ESRs in each time-series was then computed. This average is referred to in the literature as an average power delay profile (APDP) [43].

As will be shown in Chapter 5, MPGs of interest in the APDP were those which had a power relative to the main peak in the ESR of greater than -20 dB. The choice of this threshold was based on the results of Section 5.4.6 in which it is demonstrated that in urban microcells no significant improvement in Rake receiver BER performance can be obtained by combining MPGs that differ in average power by greater than 20 dB. Figure 3.2 shows an example of the APDP of a time-series recorded along a non-line-of-sight (NLOS) section of street.

For the sampling rate of 10 Msamples/second, there were two samples per chip. It was found that because of this adjacent samples in the ESRs are correlated (with a coefficient of correlation $\rho_c \geq 0.9$). Additionally as a result of the filtering to a bandwidth of 3.84 MHz, it was found that the correlation was high among three adjacent samples in the ESR. For this reason only energy received at separations greater than three sampling periods (or 300 ns) was identified as belonging to distinct MPGs. Since the Rake receiver exploits time diversity, significant gains in performance can be made by combining MPGs that are uncorrelated.

Based on these considerations, five MPGs were identified for the APDP in Figure 3.2. A common characteristic of APDPs derived from data measured in urban microcellular environments, exemplified in Figure 3.2, was the reduction in MPG

Figure 3.2: The APDP estimated from a set of ESRs recorded along a NLOS section of street.

power relative to the strongest MPG (in this case MPG 2) with an increase in relative time-delay from it.

Prior to using data in conjunction with the semi-analytical technique for Rake receiver BER prediction, the entire time-series was normalised by the mean of the total received signal energy. This was done to ensure the total received energy had a mean value of 1 for all time-series so that E_b/N_0 could be adjusted for each case. The normalised energy of the combined signal at time instant n , for L Rake fingers and K received MPGs, can therefore be written as

$$|h(n)|^2 = \frac{\sum_{i=1}^L |h'_i(n)|^2}{\frac{1}{M} \sum_{m=1}^M \sum_{i=1}^K |h'_i(m)|^2}, \quad (3.3.1)$$

where $h'_i(n)$ is the i th MPG from the n th set of a total of M sets of MPGs. The

denominator in (3.3.1) is the average over the time-series of length M of the total coherently combined signal energy in a Rake receiver having K fingers. The normalised amplitude of the i th MPG can be written as

$$h_i(n) = \frac{h'_i(n)}{\sqrt{\frac{1}{M} \sum_{m=1}^M \sum_{i=1}^K |h'_i(m)|^2}}. \quad (3.3.2)$$

3.4 Modelling of fading MPGs

Later in Chapter 5, measured data are used in conjunction with the semi-analytical technique and the resulting BER performance is compared with BER performance estimates using this technique and simulated fading data that are Rayleigh and i.i.d.

In order to determine if fading MPGs could be modelled as Rayleigh random variables, the experimental CDF for N_i independent samples¹ of an MPG from the measured data was estimated from first principles using the definition of probability. The theoretical CDF for the Rayleigh distribution is

$$P[r \leq R] = \int_0^R \frac{2}{n} e^{-\frac{r^2}{n}} r dr = 1 - e^{-\frac{r^2}{n}}, \quad (3.4.1)$$

where R is the envelope.

To determine whether the observed data fits the specified model, in this case the Rayleigh distribution, the maximum value D_{obs} of the absolute difference D between the measured and Rayleigh CDFs is determined. The goodness-of-fit measure [40], [44], [45] is then given by

$$\alpha_{KS} = P(D > D_{obs}) = Q_{KS} \left(\left[\sqrt{N_i} + 0.12 + \frac{0.11}{\sqrt{N_i}} \right] D \right), \quad (3.4.2)$$

¹The RUNs test described in [41] was used to determine the number of independent samples from the time-series of the data.

where

$$Q_{KS}(\lambda) = 2 \sum_{j=1}^{\infty} (-1)^{j-1} e^{-2j^2\lambda^2}. \quad (3.4.3)$$

A larger value of α_{KS} (close to 1) indicates that the Rayleigh model is a good fit to the data.

In order to test if the data can be modelled as Ricean, the Ricean-K factor, K_{Rice} , is first estimated from measured data as [46]

$$K_{Rice} = \frac{\sqrt{1-\vartheta}}{1-\sqrt{1-\vartheta}}, \quad (3.4.4)$$

where $\vartheta = \sigma_{|h_i(n)|^2}^2 / (\mathcal{E}_n \{|h_i(n)|^2\})^2$, can be calculated using the variance $\sigma_{|h_i(n)|^2}^2$, and average value $\mathcal{E}_n \{|h_i(n)|^2\}$ from the data. A hypothetical Rice pdf is then calculated using this value of K_{Rice} ,

$$f_R(r) = \frac{2(K_{Rice} + 1)r}{\Omega} \exp\left(-K_{Rice} - \frac{(K_{Rice} + 1)}{\Omega} r\right) I_0\left(2\sqrt{\frac{K_{Rice}(K_{Rice} + 1)}{\Omega}} r\right), \quad (3.4.5)$$

where $I_0(\cdot)$ is the 0th-order modified Bessel function of the first kind and $\Omega = \mathcal{E}\{R^2\}$, where R is again the envelope $|h(n)|$. A CDF is calculated from this pdf and compared with the estimated CDF using measured data. Once again the two CDFs are compared using the KS-test described in [40].

Table 3.1 lists the relative powers and α_{KS} values for the MPGs that correspond to the APDP shown in Figure 3.2. Figure 3.3(a) shows a theoretical Rayleigh CDF with the CDF of the strongest MPG (2). The value of $\alpha_{KS} = 0.84$ confirms a reasonably good fit to the data. In Figure 3.3(b), a set of Rayleigh random variables generated in MATLAB is compared with the theoretical Rayleigh CDF. This results in a very good fit with $\alpha_{KS} = 1$. Figure 3.3(c) shows the CDF of MPG 3 compared with the Rayleigh CDF. The value of $\alpha_{KS} = 0.09$ indicates a poor fit. Figure 3.3(d) gives an

(a) CDF of MPG 2 compared with a Rayleigh CDF, $N_i = 55, \alpha_{KS} = 0.84$.

(b) CDF of a Rayleigh distributed random variable generated in MATLAB, $N_i = 1000, \alpha_{KS} = 1$.

(c) CDF of MPG 3 compared with a Rayleigh CDF, $N_i = 55, \alpha_{KS} = 0.09$.

(d) CDF of MPG 1 compared with a Ricean CDF, $N_i = 64, \alpha_{KS} = 0.62$.

Figure 3.3: CDFs of measured and simulated data compared with Rayleigh or Ricean CDFs.

MPG	Average relative power level (dB)	Fading model	α_{KS}	N_i
1	-12.3	Rice ($K_{Rice}=1.7$ dB)	0.62	64
2	0	Rayleigh	0.84	55
3	-7.9	Rayleigh	0.09	55
4	-15.1	Rayleigh	0.40	64
5	-19.1	Rayleigh	0.003	77

Table 3.1: Average relative power levels, fading distributions, α_{KS} and N_i for the time-series of absolute magnitudes of five received MPGs measured along a NLOS section of street.

appreciation of the degree of similarity between a Ricean model with $K_{Rice} = 1.7$ dB and a CDF estimated from measured data when $\alpha_{KS} = 0.62$.

This concludes a description of the measurement setup and processing techniques for both measured and simulated data which will be revisited in the next few chapters.

Chapter 4

Analytical bounds on the performance of Rake receivers

In this chapter, analytical upper bounds on probability of bit error for a Rake receiver operating in a Rayleigh fading environment are derived in the form of a single integral expression that can be easily evaluated numerically. In the case of a correlation receiver, which is the same as a Rake receiver with one finger, a closed-form analytical expression for a loose upper bound on the probability of bit error is derived in terms of the incomplete Bessel function.

To yield tractability, the following three assumptions are made:

1. The relative time-delay between any two received multipath groups (MPGs) is assumed to be an integer multiple of the chip period.
2. Received MPGs are assumed to have equal average SNR.
3. The fading of MPGs is assumed to be Rayleigh and i.i.d.

It is obvious that the first assumption is not true in general. The use of measured data shows that the second and third assumptions also are seldom valid. The analytical expressions made possible by these assumptions, however, enable quick

performance estimation and allow for comparison between Rake receiver BER performance for different parameters such as the number of received MPGs, number of Rake fingers and data rates. More importantly they indicate an upper bound on the achievable BER. Later, in Chapter 5, a semi-analytical technique is developed for performance estimation which enables the elimination of assumptions 1-3, and leads to better accuracy than that previously reported by others, but adds significant complexity. Using the semi-analytical technique, the effects of the first assumption on performance estimation is investigated and discussed. Also with the use of measured data, the effects of the second and third assumptions on performance prediction is discussed.

4.1 System model

In Section 1.1 it was shown that the equivalent low pass complex representation of a signal received on a radio channel can be written as

$$r(t) = \sum_{k=1}^K |h_k| e^{j\phi_k} s(t - \tau_k) + n(t). \quad (4.1.1)$$

In the case of a Rake receiver with L despreading fingers and MRC, if the time-delays and amplitudes of MPGs are estimated with no error, the decision variable can be written as

$$D_{Rake} = \sum_{i=1}^L \int_{\tau_i}^{\tau_i + T_d} \text{real}(r(t) h_i^* a(t - \tau_i)) dt, \quad (4.1.2)$$

where the i th Rake finger multiplies the MPG having delay τ_i by its complex conjugate and despreads the sequence at the same delay τ_i , relative to the first sequence. If the number of fingers L , is less than the number of received MPGs, K , the combining is GSC, while if $L = K$, it is MRC. By substituting (4.1.1) in (4.1.2), the decision

statistic D_{Rake} can easily be expanded to

$$D_{Rake} = \sqrt{E_b T_d / 2} \left(\sum_{i=1}^L |h_i|^2 \right) + \sqrt{E_b T_d / 2} \left[\sum_{i=1}^L \sum_{k=1, k \neq i}^K h_{k,i} W_{k,i} \cos \theta_{k,i} / T_d \right] + \eta_{Rake}, \quad (4.1.3)$$

where $h_{k,i} = |h_k| \cdot |h_i|$ and the second summation term represents the total SI resulting from multipath interference. The cross-correlation of interfering binary sequences k and i is denoted by

$$W_{k,i} = \int_{\tau_i}^{\tau_i + T_d} b(t - \tau_k) a(t - \tau_k) a(t - \tau_i) dt. \quad (4.1.4)$$

It is considered reasonable to assume that the phase term $\theta_{k,i} = \phi_k - \phi_i$ is uniformly distributed on $[0, \pi]$. It was shown in Chapter 1 that the random variable η_{rake} that results from AWGN input $n(t)$ to the Rake receiver is zero-mean Gaussian with variance $\frac{N_0 T_d}{2} \sum_{i=1}^L |h_i|^2$.

To simplify the expression for probability of error, (4.1.3) is normalised by $T_c \sqrt{E_b / (2T_d)}$ to give

$$D_{Rake, norm} = N_d \sum_{i=1}^L |h_i|^2 + \left[\sum_{i=1}^L \sum_{k=1, k \neq i}^K |h_k| |h_i| \frac{W_{k,i}}{T_c} \cos \theta_{k,i} \right] + \frac{1}{T_c} \sqrt{\frac{2T_d}{E_b}} \eta_{Rake}. \quad (4.1.5)$$

The variance of the normalised noise term is $\sigma_{Rake}^2 = \frac{N_0 N_d^2}{E_b} \sum_{i=1}^L |h_i|^2$. The second summation in (4.1.5) represents the total SI as a result of multipath propagation. The interference between a received sequence with time-delay, τ_k , and the weighted reference sequence with time-delay, τ_i , is $|h_k| |h_i| \frac{W_{k,i}}{T_c} \cos \theta_{k,i}$. Note that

$$|h_k| |h_i| \frac{W_{k,i}}{T_c} \cos \theta_{k,i} \leq |h_k| |h_i| \frac{W_{k,i}}{T_c} \quad (4.1.6)$$

is always true. Finally, under the assumption that the relative time-delays between received MPGs are integer multiples of the chip period, T_c , the sequence cross-correlation, $\frac{W_{k,i}}{T_c}$, can be modelled using a binomial random variable. The product of two random sequences is a random sequence in which the probability that a bit is +1 is the same as the probability that it is -1. The sequence cross-correlation is therefore a random variable, say X , that has a pdf given by

$$f_X(x) = \begin{cases} C(N_d, \frac{x+N_d}{2}) 2^{-N_d}, & x = -N_d, -N_d + 2, \dots, N_d - 2, N_d \\ 0, & \text{elsewhere,} \end{cases} \quad (4.1.7)$$

where the function $C(n, k)$ represents the binomial coefficient nC_k . In usual notation X is distributed according to $B(N_d, p)$, where $p = 0.50$.

Figures 4.1(a) and 4.1(b) show the binomial CDF and a Gaussian CDF with zero mean and variance, N_d , for $N_d = 11$ and $N_d = 63$, respectively. Clearly, the difference between the Gaussian CDF and binomial CDF decreases with an increase in N_d . Hence it is reasonable to expect that the statistics of the interference can be modelled as Gaussian for the larger value of $N_d = 63$, but only under the assumption that the relative time-delay between received MPGs are integer multiples of chip period, T_c . This approximation and the resulting estimate of probability of bit error will decrease in accuracy as N_d decreases. For the purpose of deriving expressions for probability of bit error in this chapter, however, it will be assumed that the statistics of the interference can be modelled as Gaussian for all values of N_d .

The use of a Gaussian random variable to describe the statistics of the interference is convenient as it can be simply added to the variance of the Gaussian noise under the assumption that the noise and interference are independent. From (4.1.2), the probability of bit error, P_e , can then be written as

(a) $N_d = 11$ (b) $N_d = 63$

Figure 4.1: Comparison of the CDF of a binomial distribution, $B(N_d, 0.5)$ with the CDF of a Gaussian distribution having zero-mean and variance N_d .

$$P_e = Q \left(\frac{N_d \sum_{i=1}^L |h_i|^2}{\sqrt{\sigma_I^2 + \sigma_{Rake}^2}} \right), \quad (4.1.8)$$

where $\sigma_{Rake}^2 = \frac{N_0 N_d^2}{E_b} \sum_{i=1}^L |h_i|^2$ is the variance of the noise, σ_I^2 is the variance of the interference. The Gaussian-Q function can be written as [16]

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-w^2/2} dw. \quad (4.1.9)$$

The technique by which σ_I^2 is calculated will be demonstrated later in Sections 4.1.1 and 4.1.2. If the number of Rake fingers is less than the number of received MPGs, in general the statistics of the combined signal are independent of the statistics of the interference. This implies that the variance σ_I^2 is independent of the signal $N_d \sum_{i=1}^L |h_i|^2$. If the number of fingers is the same as the number of MPGs, then the statistics of the interference are not independent of the statistics of the signal because all MPG amplitudes are included in the interference component. Both these cases are considered in deriving expressions for the probability of bit error.

4.1.1 Case 1: Number of fingers less than number of MPGs

Let the complex amplitudes of received MPGs be denoted $h_k, k = 1 \dots K$, where K is the number of received MPGs having equal average powers. Absolute magnitudes of each MPG, written as $|h_k| = \sqrt{\alpha_{2k-1}^2 + \alpha_{2k}^2}$ where the α_i 's are assumed to be zero-mean Gaussian with standard deviation σ , are Rayleigh distributed. Then the total combined signal component (for L Rake fingers), $y = \sum_{m=1}^L |h_m|^2 = \sum_{i=1}^{2L} \alpha_i^2$, has a Chi-squared pdf,

$$f_Y(y) = \begin{cases} \frac{1}{\sigma^n 2^{n/2} \Gamma(n/2)} y^{n/2-1} e^{-y/2\sigma^2}, & y \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad (4.1.10)$$

with $n = 2L$ degrees of freedom [16].

If the modulation is BPSK, the probability of error is given by the Gaussian-Q function averaged over the fading and can be written as

$$P_e = \int_0^\infty Q\left(\frac{N_d y}{\sqrt{\frac{N_0 N_d^2 y}{2E_b} + \sigma_I^2}}\right) f_Y(y) dy, \quad (4.1.11)$$

where σ_I^2 is the variance of the interference. Using Craig's formula [47],

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(\frac{-x^2}{2 \sin^2 \phi}\right) d\phi. \quad (4.1.12)$$

Note that now the limits of the definite integral do not include the argument of the Gaussian-Q function. Substituting the Chi-squared pdf for $f_Y(y)$ (4.1.11) can therefore be re-written as

$$P_e = \int_{y=0}^\infty \int_0^{\pi/2} \frac{\kappa}{\pi} \exp\left(\frac{-N_d^2 y^2}{2\left(\frac{N_0 N_d^2 y}{2E_b} + \sigma_I^2\right) \sin^2 \phi}\right) y^{L-1} \exp\left(\frac{-y}{2\sigma^2}\right) d\phi dy, \quad (4.1.13)$$

where

$$\kappa = \frac{1}{\sigma^{2L} 2^L \Gamma(L)} \quad (4.1.14)$$

and $\Gamma(\cdot)$ is the Gamma function. A loose upper bound on P_e , denoted $\bar{P}_e$ can be obtained by substituting the value $\phi = \pi/2$, at which the exponential function in (4.1.12) is maximised.

$$\bar{P}_e = \int_0^\infty \frac{\kappa}{2} \exp\left(\frac{-N_d^2 y^2}{2\left(\frac{N_0 N_d^2 y}{2E_b} + \sigma_I^2\right)}\right) y^{L-1} \exp\left(\frac{-y}{2\sigma^2}\right) dy. \quad (4.1.15)$$

Since h_1 and h_2 are independent, the variance of the interference, as before σ_I^2 , on Rake finger 1 from MPG 2, can be written as

$$\sigma_I^2 = \sigma_{|h_1|}^2 \sigma_{|h_2|}^2 N_d, \quad (4.1.16)$$

where $\sigma_{|h_1|}^2$ and $\sigma_{|h_2|}^2$ are the variances of $|h_1|$ and $|h_2|$, respectively. It is easy to show that the variance of the total SI where L of K received MPGs are combined coherently can be written as

$$\sigma_I^2 = 4N_d \sum_{i=1}^L \sum_{j=i+1}^L \sigma_{|h_i|}^2 \sigma_{|h_j|}^2 + N_d \sum_{i=1}^L \sum_{j=L+1}^K \sigma_{|h_i|}^2 \sigma_{|h_j|}^2. \quad (4.1.17)$$

The first summation in the above equation is the SI from the first L MPGs under the assumption that the SI on the i th Rake finger from the k th MPG is the same as the SI on the k th Rake finger from the i th MPG. Later, in Section 5.3, it will be shown that this assumption gives an upper bound on the SI. The second summation is the SI from the remaining $L - K$ MPGs that are not assigned to any finger. Using this expression for σ_I^2 , (4.1.15) can then be integrated numerically.

In the case of a correlation receiver ($L = 1$), a closed form expression for the probability of bit error can be derived as follows. Equation (4.1.15) can be rewritten as

$$\bar{P}_e = \int_0^\infty \frac{\kappa}{2} \exp\left(\frac{-\gamma_b y^2}{y+a}\right) y^{L-1} \exp\left(\frac{-y}{2\sigma^2}\right) dy, \quad (4.1.18)$$

where $\gamma_b = E_b/N_0$ and $a = \frac{2\gamma_b \sigma_I^2}{N_d^2}$. Then, rearranging terms and using $z = y + a$,

$$\bar{P}_e = \frac{\kappa}{2} \exp\left(2a\gamma_b + \frac{a}{2\sigma^2}\right) \int_a^\infty \exp\left(-\left(\gamma_b + \frac{1}{2\sigma^2}\right)z - \frac{\gamma_b a^2}{z}\right) dz \quad (4.1.19)$$

for $L = 1$. The above expression can be upper bounded by extending the range of the integral

$$\bar{P}_e \leq \bar{\bar{P}}_e = \frac{\kappa}{2} \exp\left(2a\gamma_b + \frac{a}{2\sigma^2}\right) \int_0^\infty \exp\left(-\left(\gamma_b + \frac{1}{2\sigma^2}\right)z - \frac{\gamma_b a^2}{z}\right) dz \quad (4.1.20)$$

Equation (4.1.20) can be expressed in closed-form [48] as

$$\bar{\bar{P}}_e = \kappa \exp\left(2a\gamma_b + \frac{a}{2\sigma^2}\right) \sqrt{\frac{s}{t}} K_1\left(2\sqrt{st}\right), \quad (4.1.21)$$

where $K_\nu(\cdot)$ is the ν th order modified Bessel function of the second kind, $s = \gamma_b a^2$, and $t = \gamma_b + \frac{1}{2\sigma^2}$.

Figure 4.2 shows the bounds obtained by numerical integration of (4.1.15) with $N_d = 31, 127$ and 511 , and by use of the closed-form expression in (4.1.21) when one of two received MPGs is combined in the receiver.

Figure 4.2: Comparison of estimated BER for a Rake receiver using GSC ($L=1$, $K=2$), for $N_d = 31, 127$ and 511 . Performance is estimated by numerical integration or a closed-form approximation to the integral.

Although these bounds are loose, it can be observed that as N_d increases, the effects of SI, which results in the error floors in $\bar{\bar{P}}_e$ and $\bar{P}_e$, are reduced. Also, the

upper bounds using (4.1.21) approach the better upper bound using (4.1.15) with an increase in spreading factor at low values of E_b/N_0 . This is because the lower limit of the integration, $a = \frac{2\gamma_b\sigma_I^2}{N_d^2}$, decreases with an increase in N_d , thereby bringing the upper bound (4.1.21) closer to the true value of the integral, (4.1.19).

4.1.2 Case 2: Number of fingers the same as number of MPGs

In this case the statistics of the interference are not independent of the signal statistics. As a result an expression for the variance of the SI must involve the signal term in (4.1.2). The probability of error, expressed in terms of the Gaussian Q-function as before, can be written as

$$P_e = \int_0^\infty Q\left(\frac{N_d y}{\sqrt{\frac{N_0 N_d^2 y}{2E_b} + \sigma_I^2}}\right) f_Y(y) dy, \quad (4.1.22)$$

where σ_I^2 is the variance of the interference.

As demonstrated in Section 4.1.1, an upper bound on P_e , denoted $\bar{P}_e$ can be written as

$$\bar{P}_e = \int_0^\infty \frac{\kappa}{2} \exp\left(\frac{-N_d^2 y^2}{2\left(\frac{N_0 N_d^2 y}{2E_b} + \sigma_I^2\right)}\right) y^{(n/2)-1} \exp\left(\frac{-y}{2\sigma^2}\right) dy. \quad (4.1.23)$$

In what follows, the variance of the interference, σ_I^2 , is expressed in terms of y . For any two real numbers, $|h_1|$ and $|h_2|$ it is always true that

$$(|h_1|^2 + |h_2|^2)^2 \geq (2|h_1||h_2|)^2 \quad (4.1.24)$$

Also in general it is easy to show that

$$\frac{2(L-1)}{L} \left(\sum_{i=1}^L |h_i|^2 \right)^2 \geq \sum_{m=1}^L \sum_{n=m+1}^L (2|h_m||h_n|)^2. \quad (4.1.25)$$

If the total signal energy is

$$N_d y = N_d \sum_{i=1}^L |h_i|^2 \quad (4.1.26)$$

then using (4.1.25), the variance of the interference which is modelled as a Gaussian random variable can be upper bounded using the approximation

$$\sigma_I^2 = \frac{2(L-1)N_d y^2}{L}. \quad (4.1.27)$$

Figure 4.3 shows the upper bounds obtained by numerically integrating (4.1.23) when the number of received MPGs are two, three, four and five, all of which are combined in the receiver, for a spreading factor $N_d = 31$. There is no error floor for $P_e > 10^{-3}$. Also the results indicate, as would be expected, diminishing returns as the number of received MPGs, or order of diversity, is increased.

Figure 4.3: Comparison of BER estimates for a Rake receiver using MRC, when $L = 2, 3, 4$ or 5 , for a spreading factor $N_d = 31$.

4.2 Comparison of analytical bounds on BER with BER estimates from simulation

In order to compare the upper bounds on BER determined using the analytical expressions for GSC and MRC with an accurate estimate of BER using the same assumptions of equal average power and chip-spaced, Rayleigh fading MPGs with a Gaussian assumption for the SI, the first step was to generate an array of i.i.d. Rayleigh random variables in MATLAB. The normalised amplitude of the i th MPG from the n th set of a total of M sets of MPGs can be written as

$$h_i(n) = \frac{h'_i(n)}{\sqrt{\frac{1}{M} \sum_{m=1}^M \sum_{i=1}^K |h'_i(m)|^2}}, \quad (4.2.1)$$

where $h'_i(n)$ represents the i th un-normalised MPG from the array. The instantaneous probability of error can be estimated as

$$P_E(n) = Q \left(\frac{N_d \sum_{i=1}^L |h_i(n)|^2}{\sqrt{\sigma_I^2 + \sigma_{Rake}^2}} \right), \quad (4.2.2)$$

where σ_{Rake}^2 is the variance of the noise. The SI, modelled as Gaussian, has zero mean with variance σ_I^2 expressed in terms of the fixed MPG amplitudes $h_i(n)$, $i = 1 \dots K$, as

$$\sigma_I^2 = 4N_d \sum_{i=1}^L \sum_{j=i+1}^L |h_i(n)|^2 |h_j(n)|^2 + N_d \sum_{i=1}^L \sum_{j=L+1}^K |h_i(n)|^2 |h_j(n)|^2. \quad (4.2.3)$$

The average BER can be written as

$$\mathcal{E}_n(P_E) = \frac{1}{M} \sum_{n=1}^M P_E(n). \quad (4.2.4)$$

(a) GSC, $L = 1, K = 2$. (b) MRC, $L = 2, K = 2$.

Figure 4.4: Comparing Rake receiver BER estimates from simulation with analytical upper bounds on BER estimates for GSC and MRC.

Figure 4.4(a) shows analytical upper bounds on BER estimates when there are two received MPGs having equal average SNR, only one of which is processed by the Rake receiver for spreading factors, $N_d = 7, 11$ and 31 . Clearly these upper bounds on BER do not provide an accurate assessment of performance and are very loose. Figure 4.4(b) shows analytical upper bounds on BER when there are two received MPGs, both of which are combined by the Rake. The upper bounds on BER approach the BER results predicted using the SAT with an increase in spreading factor from 7 to 31 . This is simply because over the range of BER considered, the effect of SI on performance decreases with an increase in spreading factor. In both the above cases, the discrepancy between the upper bounds on BER and the true BER is a result of the approximation used in (4.1.15) and the upper bound on the variance of the interference.

4.3 Summary

In this chapter new analytical expressions for upper bounds on the probability of bit error for a Rake receiver were derived under the assumption that the fading of received MPGs having equal average SNR is Rayleigh and i.i.d. Additionally it was assumed that the relative time-delay between any two received MPGs is an integer multiple of the chip period.

As a result of the use of upper bounds on both the Gaussian-Q function and the Gaussian SI, the analytical expressions for Rake receiver BER gave performance bounds that significantly overestimated the required SNR to achieve a desired BER when compared with simulation results that used exact values for both the Gaussian-Q function and the SI (assumed to be Gaussian as well). It is however recognised that the assumptions of equal average power for received MPGs and chip-spaced MPGs used for the purposes of simulation are unrealistic. For this reason it is necessary to consider an alternate approach to predicting Rake receiver BER performance as

it is not understood how these assumptions affect Rake receiver BER performance predictions.

Chapter 5

Rake receiver BER performance estimation

In this chapter, the theoretical framework for a semi-analytical analysis of Rake receiver performance on a fading multipath channel is developed. The effect of self-interference (SI) on Rake receiver BER performance is studied. Since random sequences are considered, SI is non-deterministic and varies from one symbol to the next. It is therefore required to determine the pdf or bounds on the pdf of the SI. The basic technique to determine bounds on the pdf follows closely that in [2], in which Lehnert and Pursley analysed the performance of a correlation receiver in the presence of multiple-access interference (MAI) in a non-fading environment for a DSSS system. It was assumed that the signal received from each transmitter had the same power. Channel multipath effects were not considered. The work in this thesis extends the work in [2] by analysing a Rake receiver operating on a fading channel with MPGs having unequal average powers. Performance results are presented in terms of probability of bit error as a function of average received SNR per bit.

In the first stage of the analysis, bounds on the pdf of SI are derived. The SI depends on the data spreading factor, the number of received MPGs and their powers, as well as the number of Rake fingers.

Bounds on the probability of bit error are expressed in terms of the Gaussian-Q

function, and depend on the spreading factor, power at the output of the Rake receiver (hereinafter referred to as the combined signal power), thermal noise and bounds on the pdf for interference. Upper and lower bounds on this pdf are used to estimate upper and lower bounds, respectively, on the probability of bit error using numerical integration. The average of bounds on the probability of bit error can be estimated as a sample average for a time-series of channel states, each defined by a set of complex MPG amplitudes. A semi-analytical procedure is used because the complexity of the expression for the pdf of the SI is such as to require numerical integration. In addition, if real-world channel information is to be used, it is not always possible to model fading distributions in closed-form to enable analytical averaging.

It should be noted that the procedure detailed in the next section on estimating bounds on the pdf of the interference and the resulting probability of bit error is almost identical to the procedure in [2], which was developed from the work in [49] and [50]. However, an error in [2] is identified and its effect on the probability of bit error estimates is shown to be significant. Other differences are modifications to the expressions for the continuous-time partial cross-correlations of the delayed and the reference sequences to reflect the fact that there is only a single user, and extensions to the pdfs of SI to allow consideration of cases when interfering signals (MPGs) have different powers. Finally, the use of multiple Rake fingers required consideration of aspects of mutual interference not encountered in the systems involving only correlation (single finger) receivers studied in the cited papers.

5.1 Self-interference for a correlation receiver

In Section 1.1 it was shown that the complex equivalent low pass representation for the signal received on a multipath channel can be written as

$$r(t) = \sum_{k=1}^K |h_k| e^{j\phi_k} s(t - \tau_k) + n(t). \quad (5.1.1)$$

It was also shown that the output of a spread spectrum correlation receiver, the reference sequence of which is aligned with the MPG at time-delay $\tau_k = \tau_1$, can be written as

$$D_1 = \int_{\tau_1}^{\tau_1 + T_d} \text{real}(r(t) h_1^* a(t - \tau_1)) dt, \quad (5.1.2)$$

which was expanded to

$$D_1 = \sqrt{E_b T_d / 2} (|h_1|^2) + \sqrt{E_b T_d / 2} \left[\sum_{k=2}^K |h_k| |h_1| W_{k,1} \cos \theta_k / T_d \right] + \eta_1, \quad (5.1.3)$$

where $W_{k,1}$ is the sequence cross-correlation between spreading sequences having time-delays τ_k and τ_1 and it is considered reasonable to assume that $\theta_k = \phi_k - \phi_1$ is uniformly distributed on $[0, \pi]$. The second term in (5.1.3) represents the total SI from sequences received at the time-delays associated with $K - 1$ MPGs. This interference is the result of non-zero cross-correlation between the time-delayed sequences and the reference despreading sequence aligned in time with the first MPG. The noise component η_1 was shown to be Gaussian with zero-mean and variance $N_0 T_d |h_1|^2 / 2$ which results from AWGN input $n(t)$ to the correlation receiver. Expression (5.1.3) can be normalised by $T_c \sqrt{\frac{E_b}{2T_d}}$ to give a modified decision variable

$$D_{1,norm} = N_d |h_1|^2 + \left[\sum_{k=2}^K |h_k| |h_1| \frac{W_{k,1}}{T_c} \cos \theta_k \right] + \frac{1}{T_c} \sqrt{\frac{2T_d}{E_b}} \eta_1. \quad (5.1.4)$$

The summation in (5.1.4) represents the total SI.

In what follows it will be shown that the pdf of $|h_k| |h_1| \frac{W_{k,1}}{T_c}$ conditioned on the autocorrelation of the spreading sequence can be written as the sum of a binomially

distributed random variable and a uniformly distributed random variable. The pdf of $|h_k| |h_1| \frac{W_{k,1}}{T_c} \cos \theta_k$ is then derived and bounds on the BER are calculated by numerically averaging the Gaussian-Q function over variations in the bounds on the pdf of the SI as described in [2].

To begin, the sequence cross-correlation term

$$W_{k,1} = \int_{\tau_1}^{T_d + \tau_1} b(t - \tau_k) a(t - \tau_k) a(t - \tau_1) dt, \quad (5.1.5)$$

can be rewritten as

$$W_{k,1} = b_{\lfloor t/T_d \rfloor - \lfloor \tau_{k,1}/T_d \rfloor - 1} R_a(\tau_{k,1} - \lfloor \tau_{k,1}/T_d \rfloor T_d) + b_{\lfloor t/T_d \rfloor - \lfloor \tau_{k,1}/T_d \rfloor} \widehat{R}_a(\tau_{k,1} - \lfloor \tau_{k,1}/T_d \rfloor T_d), \quad (5.1.6)$$

where $\tau_{k,1} = \tau_k - \tau_1$. The first term in this equation represents the SI due to the previous symbol and the second term represents the interference due to the current symbol. Also, $R_a(\tau)$ and $\widehat{R}_a(\tau)$ are the continuous-time partial cross-correlation functions of the time-delayed and reference sequences. They are defined in a manner similar to their definition in [49], the modification being to allow the interfering sequence to be a time-delayed version of the desired sequence, as

$$R_a(\tau) = \int_0^{\tau} a(t - \tau) a(t) dt \quad (5.1.7)$$

and

$$\widehat{R}_a(\tau) = \int_{\tau}^{T_d} a(t - \tau) a(t) dt \quad (5.1.8)$$

for $0 \leq \tau \leq T_d$.

These integrals can be expressed in terms of the discrete aperiodic autocorrelation function $C_a(\zeta)$ of the spreading sequence and the continuous-time partial autocorrelation functions $R_\psi(\tau)$ and $\widehat{R}_\psi(\tau)$ of the chip waveform. The function $C_a(\zeta)$ is

dependent upon the signature sequence and is given by

$$C_a(\zeta) = \begin{cases} \sum_{r=t'}^{t'+N_d-1+\zeta} a_r a_{r+\zeta} & 1 - N_d \leq \zeta < 0 \\ \sum_{r=t'}^{t'+N_d-1-\zeta} a_r a_{r+\zeta} & 0 \leq \zeta \leq N_d - 1 \\ 0 & |\zeta| \geq N_d, \end{cases} \quad (5.1.9)$$

where $t' = \lfloor t/T_c \rfloor$.

The functions $R_\psi(\tau)$ and $\widehat{R}_\psi(\tau)$ are defined as in [2], [50] for $0 \leq \tau \leq T_c$ by $R_\psi(\tau) = \int_0^\tau \psi(t)\psi(t+T_c-\tau)dt$ and $\widehat{R}_\psi(\tau) = \int_\tau^{T_c} \psi(t)\psi(t-\tau)dt$, where $\psi(t)$ is the chip waveform defined in Section 1.1. For $\tau > T_c$ or $\tau < 0$, $R_\psi(\tau) = \widehat{R}_\psi(\tau) = 0$.

$R_a(\tau)$ and $\widehat{R}_a(\tau)$ can therefore be rewritten as

$$R_a(\tau_k) = C_a(\gamma_k - N_d)\widehat{R}_\psi(\tau_k - \gamma_k T_c) + C_a(\gamma_k + 1 - N_d)R_\psi(\tau_k - \gamma_k T_c) \quad (5.1.10)$$

and

$$\widehat{R}_a(\tau_k) = C_a(\gamma_k)\widehat{R}_\psi(\tau_k - \gamma_k T_c) + C_a(\gamma_k + 1)R_\psi(\tau_k - \gamma_k T_c), \quad (5.1.11)$$

where $\gamma_k = \lfloor (\tau_k - \tau_1)/T_c \rfloor$ is the relative delay in chips of the k th MPG relative to the first, rounded down to the nearest integer value, and the residual $\tau_k - \tau_1 - \gamma_k T_c = S_k$ is assumed to be uniformly distributed on $(0, T_c)$. From (5.1.6), (5.1.10) and (5.1.11), $W_{k,1}$ can be rewritten as

$$W_{k,1} = [b_{-\delta_{k,1}-1}C_a(\gamma_k - N_d) + b_{-\delta_{k,1}}C_a(\gamma_k)]\widehat{R}_\psi(S_k) + [b_{-\delta_{k,1}-1}C_a(\gamma_k + 1 - N_d) + b_{-\delta_{k,1}}C_a(\gamma_k + 1)]R_\psi(S_k), \quad (5.1.12)$$

where $\delta_{k,1} = \lfloor (\tau_{k,1})/T_d \rfloor - \lfloor t/T_d \rfloor$. Equation (5.1.9) can now be used to rewrite (5.1.12)

as

$$W_{k,1} = \begin{bmatrix} \sum_{j=t'}^{t'+\gamma_k-1} b_{-\delta_{k,1}-1} a_{j-\gamma_k+N_d} a_j + \sum_{j=t'+\gamma_k}^{t'+N_d-1} b_{-\delta_{k,1}} a_{j-\gamma_k} a_j \\ \sum_{j=t'}^{t'+\gamma_k} b_{-\delta_{k,1}-1} a_{j-\gamma_k-N_d} a_{j+1} + \sum_{j=t'+\gamma_k}^{t'+N_d-2} b_{-\delta_{k,1}} a_{j-\gamma_k} a_{j+1} \end{bmatrix} \begin{bmatrix} \widehat{R}_\psi(S_k) + \\ R_\psi(S_k), \end{bmatrix} \quad (5.1.13)$$

where $t' = \lfloor t/T_c \rfloor$.

The next step in the derivation for the bounds on the pdf of the SI is to simplify (5.1.13). The simplification is best explained with the use of Figure 5.1 which shows a reference and an interfering sequence of chips. If chips r1 and r2 have the same sign, then the interference from i1 to r1 should be added to the interference from i1 to r2. Otherwise if chips r1 and r2 do not have the same sign, the interference from i1 to r1 should be subtracted from the interference from i1 to r2.

Figure 5.1: Time-delayed chip sequences that spread a data signal cause SI.

In order to simplify (5.1.13), it can first be rewritten as

$$W_{k,1} = b_{-\delta_{k,1}-1} \sum_{j=t'}^{t'+\gamma_k-1} a_{j-\gamma_k+N_d} \left(a_j \widehat{R}_\psi(S_k) + a_{j+1} R_\psi(S_k) \right) + b_{-\delta_{k,1}} \sum_{j=t'+\gamma_k}^{t'+N_d-2} a_{j-\gamma_k} \left(a_j \widehat{R}_\psi(S_k) + a_{j+1} R_\psi(S_k) \right) + b_{-\delta_{k,1}} a_{t'+N_d-\gamma_k-1} a_{N_d-1} \widehat{R}_\psi(S_k) + b_{-\delta_{k,1}-1} a_{t'+N_d-\gamma_k-1} a_0 R_\psi(S_k). \quad (5.1.14)$$

Then a set of $N_d + 1$ random variables Z_j can be defined as

$$Z_j = \begin{cases} b_{-\delta_{k,1}-1} a_{j-\gamma_k+N_d} a_j & j = 0, \dots, \gamma_k - 1 \\ b_{\delta_{k,1}} a_{j-\gamma_k} a_j & j = \gamma_k, \dots, N_d - 2 \\ b_{\delta_{k,1}} a_{N_d-\gamma_k-1} a_{N_d-1} & j = N_d - 1 \\ b_{\delta_{k,1}-1} a_{N_d-\gamma_k-1} a_0 & j = N_d. \end{cases} \quad (5.1.15)$$

For any γ_k in the set $\{0, \dots, N_d - 1\}$, the random variables Z_j are mutually independent and satisfy $P(Z_j = 1) = P(Z_j = -1) = 1/2$. Then using the fact that $a_j = \pm 1$ for all j , (5.1.14) can be reduced to

$$W_{k,1} = \sum_{j=0}^{N_d-2} Z_j \left(\widehat{R}_\psi(S_k) + a_j a_{j+1} R_\psi(S_k) \right) + Z_{N_d-1} \widehat{R}_\psi(S_k) + Z_{N_d} R_\psi(S_k). \quad (5.1.16)$$

To simplify the notation, define

$$f(S_k) = \widehat{R}_\psi(S_k) + R_\psi(S_k) \quad (5.1.17)$$

and

$$g(S_k) = \widehat{R}_\psi(S_k) - R_\psi(S_k). \quad (5.1.18)$$

The term $f(S_k)$ is the sum of the continuous time partial autocorrelation functions of the chip waveform over a chip period while the term $g(S_k)$ is the difference between the continuous time partial autocorrelation functions of the chip waveform over a chip period. For future reference note also that for a rectangular chip waveform,

$$f(S_k) = T_c \quad (5.1.19)$$

and

$$|g(S_k)| \leq T_c. \quad (5.1.20)$$

If A is the set of all nonnegative integers i less than $N_d - 1$ such that $a_i a_{i+1} = 1$ and B is the set of all nonnegative integers i less than $N_d - 1$ such that $a_i a_{i+1} = -1$, (5.1.16) can be rewritten as

$$W_{k,1} = \sum_{j \in A} Z_j f(S_k) + \sum_{j \in B} Z_j g(S_k) + Z_{N_d-1} \widehat{R}_\psi(S_k) + Z_{N_d} R_\psi(S_k). \quad (5.1.21)$$

By replacing $\sum_{j \in A} Z_j$ with X_k and $\sum_{j \in B} Z_j$ with Y_k , (5.1.21) can in-turn be rewritten as

$$W_{k,1} = X_k f(S_k) + Y_k g(S_k) + P_k \widehat{R}_\psi(S_k) + Q_k R_\psi(S_k), \quad (5.1.22)$$

where $P_k = Z_{N_d-1}$ and $Q_k = Z_{N_d}$. Since the random variables Z_j for $0 \leq j \leq N_d$ are mutually independent, the random variables X_k, Y_k, P_k and Q_k are mutually independent. Additionally, sets $A, B, \{N_d - 1\}$ and $\{N_d\}$ are disjoint. The random variables $W_{k,1}, 2 \leq k \leq K$, are also mutually independent. The pdf for $W_{k,1}$ can then be determined from the pdfs of the four terms in (5.1.22).

Since X_k is the sum of a set of numbers each of which is either +1 or -1, the density function for the random variable X_k is given by

$$f_{X_k}(j) = \begin{cases} C(n_A, \frac{j+n_A}{2}) 2^{-n_A}, & j = -n_A, -n_A + 2, \dots, n_A - 2, n_A \\ 0, & \text{elsewhere,} \end{cases} \quad (5.1.23)$$

where n_A is the cardinality of the set A . The function $C(n, k)$ represents the binomial coefficient nC_k . Similarly the density function for the set Y_k is given by

$$f_{Y_k}(j) = \begin{cases} C(n_B, \frac{j+n_B}{2}) 2^{-n_B}, & j = -n_B, -n_B + 2, \dots, n_B - 2, n_B \\ 0, & \text{elsewhere,} \end{cases} \quad (5.1.24)$$

where n_B is the cardinality of the set B . The complexity of the problem has now been reduced from a consideration of 2^{N_d} possible outcomes to only $4(n_A + 1)(n_B + 1)$, where $n_A + n_B = N_d - 1$.

From an examination of (5.1.16) it is clear that $W_{k,1}$ depends on the spreading sequence $\{a_r\}$ through the discrete aperiodic autocorrelation function $C_a(\zeta)$ evaluated at $\zeta = 1$. Using (5.1.9)

$$C_a(1) = \sum_{r=t'}^{t'+N_d-2} a_r a_{r+1} = \sum_{r=t'}^{t'+N_d-2} c_r, \quad (5.1.25)$$

where $c_r = a_r a_{r+1}$, $t' = \lfloor t/T_c \rfloor$ and $c_r = 1$ if $a_r = a_{r+1}$ and $c_r = -1$ otherwise. Since $P(a_r = 1) = P(a_r = -1) = 1/2$, $P(c_r = 1) = P(c_r = -1) = 1/2$. The density function for the discrete random variable $C_a(1)$ is then given by

$$f_{C_a(1)}(j) = \begin{cases} C(N_d - 1, \frac{j+N_d-1}{2}) 2^{1-N_d}, & j = 1 - N_d, 3 - N_d, \dots, N_d - 3, N_d - 1 \\ 0, & \text{elsewhere.} \end{cases} \quad (5.1.26)$$

It is also easy to show that $C_a(1) = n_A - n_B$. Also recall that $n_A + n_B = N_d - 1$. The result of sequence cross-correlation is conditioned on the value of $C_a(1)$ which, given N_d , uniquely determines n_A and n_B . With this knowledge, the pdfs of X_k and Y_k can be found using (5.1.23) and (5.1.24), respectively, and using these, the pdf of $W_{k,1}$ can be determined, once again conditioned on $C_a(1)$. This conditioning on $C_a(1)$ is the simplification that forms the basis for the analysis.

Referring to (5.1.3), the SI is the summation of $K - 1$ terms, each of which is the product of two MPG amplitudes and the term representing the cross-correlation between the code sequences aligned in time with the two MPGs. The procedure to estimate the interference terms that comprise the summation is described below for $k = 2$. The method for determining terms $W_{k,1}$ for $k = 3 \dots K$, is the same.

From (5.1.4) the interference term for $k = 2$ can be written as $H_2 = |h_2| |h_1| I_2 \cos \theta_2$, where $I_2 = W_{2,1}/T_c$ and $|h_2|$ and $|h_1|$ are the magnitudes of the two received MPGs. The first MPG, with amplitude h_1 , is the desired signal which is coherently processed by the correlation receiver, while the second MPG with amplitude h_2 causes the interference.

From (5.1.19) and (5.1.20), $|T_c^{-1} f(S_2)| \leq 1$ and $|T_c^{-1} g(S_2)| \leq 1$ are always true. Also from (5.1.23) and (5.1.24), $|X_2| \leq n_A$ and $|Y_2| \leq n_B = N_d - 1 - n_A$. Thus $||X_2 f(S_2) + Y_2 g(S_2)| T_c^{-1}| \leq N_d - 1$. Additionally since P_k and Q_k are either ± 1 , using (5.1.17) and (5.1.18), it can be shown that the term $[P_k \widehat{R}_\psi(S_k) + Q_k R_\psi(S_k)]$ is

either $\pm f(S_2)$ or $\pm g(S_2)$. As a result, the random variable I_2 satisfies $|I_2| \leq N_d$ and from (5.1.22) has the form $I_2 = [wf(S_2) + zg(S_2)]T_c^{-1}$, where w and z are integers and the sum of a sequence of bits of length n_w and n_z , respectively. Also, $n_w + n_z = N_d$. From (5.1.19), under the assumption of a rectangular chip waveform, $f(s) = T_c$ for s in the interval $(0, T_c)$, and I_2 can be written as

$$I_2 = w + zg(S_2)T_c^{-1}. \quad (5.1.27)$$

The pdf of I_2 can be written for the two possible cases, namely, $z = 0$ and $z \neq 0$, conditioned on $C_a(1) = c$, as follows.

$z = 0$: In this case $f_{I_2}(x|C_a(1) = c)$ is binomially distributed and can be calculated using (5.1.23) for sequence length n_w . It can be written as

$$f_{I_2}(x|C_a(1) = c; z = 0) = \sum_{i=-N_d}^{N_d} P(x = i|c) \delta(x - i), \quad (5.1.28)$$

where

$$P(x = i|c) = \begin{cases} C(n_w, \frac{i+n_w}{2}) 2^{-n_w}, & i = -n_w, -n_w + 2, \dots, n_w - 2, n_w \\ 0, & \text{elsewhere.} \end{cases} \quad (5.1.29)$$

Also, the condition $z = 0$ requires that n_z is even. As a result n_w is even if N_d is even and n_w is odd if N_d is odd. Depending on whether N_d is odd or even, f_{I_2} is nonzero at odd or even integer values, respectively.

$z \neq 0$: In this case it is necessary to consider the pdf of $g(S_2)T_c^{-1}$. It was earlier assumed that S_k was uniformly distributed on $(0, T_c)$. Also using (5.1.20) it can be stated that $g(S_2)T_c^{-1}$ is uniformly distributed on the interval $(-1, 1)$. The pdf of the second term, $d = zg(S_2)T_c^{-1}$ on the right hand side of (5.1.27), which is the product of a discrete random variable z and a continuous random variable $g(S_2)T_c^{-1} = U$, can then be written as

$$f_d(u|c) = \sum_{\substack{i=-n_z \\ i \neq 0}}^{n_z} P(z=i|c) \frac{1}{|i|} f_U\left(\frac{u}{i}\right), \quad (5.1.30)$$

where $f_U(\cdot)$ is the pdf of a random variable, U , that is uniformly distributed on the interval $(-1, 1)$, and $P(z=i|c)$ can be written as

$$P(z=i|c) = \begin{cases} C(n_z, \frac{i+n_z}{2}) 2^{-n_z}, & i = -n_z, -n_z+2, \dots, n_z-2, n_z \\ 0, & \text{elsewhere.} \end{cases} \quad (5.1.31)$$

Figure 5.2: The pdf of $d = zg(S_2)T_c^{-1}$ where z is distributed according to $B(7, 0.50)$ and $g(S_2)T_c^{-1}$ is uniformly distributed on $(-1, 1)$.

The pdf $f_d(u|c)$ for an arbitrarily chosen value of $n_z = 7$ is shown (not to scale) in Figure 5.2. For odd values of n_z , $f_d(u|c)$ is constant between odd integer values of u . For even values of n_z , and $z \neq 0$, $f_d(u|c)$ is constant between even integer values of u . Also since the terms on the right hand side of (5.1.27) are independent conditioned on $C_a(1) = c$, the pdf of I_2 can be written as

$$f_{I_2}(x|c) = P(w=x|c) * f_d(x|c). \quad (5.1.32)$$

It is important to note the exclusion of the limit values of -1 and 1 when defining $f_U(\cdot)$. If these values were included, in addition to a uniform distribution, the convolution in (5.1.32) would give a series of weighted unit impulses at odd values of the index

for N_d odd and even values of the index at N_d even. This would be incorrect as it would result in an area greater than unity for the pdf $f_{I_2}(x|c)$. This was incorrectly identified as a closed interval in [2].

The pdf of the random variable w can be calculated using (5.1.23) for a sequence length n_w . For $z \neq 0$, the pdf of I_2 is therefore the convolution of a pdf that consists of weighted unit impulses and a pdf that has a constant value over intervals as shown in Figure 5.2. The result of the convolution in (5.1.32) thus has the same form as the pdf of d and can be written as

$$f_{I_2}(x|C_a(1) = c; z \neq 0) = \sum_{i=-N_d}^{N_d} 2P(x = i|c) f_2(x - i + 1), \quad (5.1.33)$$

where $P(x = i|c) = f_{I_2}(y|c)\delta(y - i)$, $i = -N_d + 1, \dots, N_d - 1$, and $f_2(x) = 1/2$ for $0 < x < 2$, $f_2(x) = 0$ for $x \leq 0$ or $x \geq 2$. It is important to note the scaling factor, 2, in (5.1.33). In [2], the scaling factor is 1. The effect of this scaling error on performance will be demonstrated in Section 5.2.

Referring to (5.1.4), the next step is to determine the pdf of $H_2 = |h_2||h_1|I_2 \cos \theta_2$. Note that in [2], it was assumed that $|h_2| = |h_1| = 1$. In order to account for the multipath problem that assumption can no longer be made. It is shown in Appendix A that if a random variable U , is uniformly distributed on an interval (m, n) , where m and n are both positive, the pdf of the random variable $H_2 = |h_2||h_1|U \cos \phi$, where ϕ is uniformly distributed on $[0, \pi]$, can be written as

$$f_{H_2}(x|i = (m+n)/2) = \begin{cases} \frac{1}{\pi h_{1,2}(n-m)} \ln \left[\frac{n}{m} \frac{(1 + \sqrt{1 - x^2/(nh_{1,2})^2})}{(1 + \sqrt{1 - x^2/(mh_{1,2})^2})} \right], & 0 \leq |x| \leq h_{1,2}m \\ \frac{1}{\pi h_{1,2}(n-m)} \ln \left[\frac{nh_{1,2}}{|x|} \left(1 + \sqrt{1 - x^2/(nh_{1,2})^2} \right) \right], & h_{1,2}m \leq |x| \leq h_{1,2}n \\ 0, & |x| > h_{1,2}n, \end{cases} \quad (5.1.34)$$

where $h_{1,2} = |h_1||h_2|$. This pdf was not derived in [2]. Figure 5.3(a) shows the pdf

f_{H_2} determined using (5.1.34) for arbitrarily chosen $m = 5$ and $n = 7$. In other words, the random variable U is uniformly distributed between 5 and 7.

Also it is shown in Appendix A that if U is a positive constant k , the pdf of the random variable $H_2 = |h_2| |h_1| U \cos \phi$, where ϕ is uniformly distributed on $[0, \pi]$, can be written as

$$f_{H_2}(x|i=k) = \begin{cases} \frac{1}{k\pi h_{1,2} \sqrt{1-x^2/(h_{1,2}k)^2}}, & |x| < h_{1,2}k \\ 0, & |x| \geq h_{1,2}k \end{cases} \quad (5.1.35)$$

Figure 5.3(b) shows the pdf $f_{H_2}(\cdot)$ using (5.1.35) for arbitrarily chosen $k = 7$.

Equations (5.1.34) and (5.1.35) describe the pdf of $H_2 = |h_2| |h_1| U \cos \phi$ depending on whether U is uniformly distributed or a constant, respectively. The next step is to determine bounds on the pdf of H_2 from (5.1.34) and (5.1.35). This is required as it is difficult to determine a closed-form solution for an integral expression that involves the Gaussian-Q function with the pdf of interference from (5.1.34) and (5.1.35) as its argument. Instead bounds on the probability of bit error are determined by numerical integration using the bounds on the pdfs.

Figure 5.4 shows the conditional density function $f_{H_2}(x; i = (m+n)/2)$ for $|h_1| = |h_2| = 1$, and I_2 uniformly distributed between $m = 5$ and $n = 7$. Upper and lower bounds on the pdf $f_{H_2}(x; i = (m+n)/2)$ are indicated for each integer interval within the range of x . An upper bound on the pdf over an interval corresponds to the maximum value of the pdf over that interval, while the lower bound corresponds to the minimum value of the pdf over the interval. Tighter bounds can be defined by further dividing each integer interval into smaller subintervals. Upper and lower bounds on $f_{H_2}(x|i; c)$ denoted by $f_{H_2}^U(x|i; c)$ and $f_{H_2}^L(x|i; c)$, respectively, are presented in [2] and in Appendix B of this thesis.

If each integer interval is divided into N_u subintervals, the vectors $f_{H_2}^U(x|i; c)$, $i = -N_d, \dots, N_d$, and $f_{H_2}^L(x|i; c)$, $i = -N_d, \dots, N_d$, are the upper and

(a) Probability density function, $f_{H_2}(x; i = (m + n)/2)$, for $m = 5$ and $n = 7$, for U uniformly distributed on (m, n) .

(b) Probability density function, $f_{H_2}(x; k)$ for $k = 7$, for U a positive constant.

Figure 5.3: Probability density function, $f_{H_2}(\cdot)$.

Figure 5.4: Upper and lower bounds on the pdf $f_{H_2}(x; i = (m+n)/2)$, for $m = 5$ and $n = 7$.

lower bounds on the pdf $f_{H_2}(x|i; c)$, $i = -N_d, \dots, N_d$, calculated at $2N_dN_u + 1$ values of H_2 . Conditioned on $C_a(1) = c$, $N_d - 1$ vectors can be written for both upper and lower bounds, $u(c)$ and $v(c)$, respectively. Each vector,

$$u_i(c) = f_{I_2}(i|c)f_{H_2}^U(x|i; c) \quad (5.1.36)$$

and

$$v_i(c) = f_{I_2}(i|c)f_{H_2}^L(x|i; c), \quad (5.1.37)$$

has length $2N_dN_u + 1$, where each integer interval is divided into N_u subintervals. Upper and lower bounds on the interference u_i and v_i , respectively, for $i = -N_dN_u, \dots, +N_dN_u$, can then be obtained by averaging over the distribution for $C_a(1)$ given by (5.1.26),

$$u_i = \sum_{c=1-N_d}^{N_d-1} u_i(c) P(C_a(1) = c), \quad (5.1.38)$$

$$v_i = \sum_{c=1-N_d}^{N_d-1} v_i(c) P(C_a(1) = c). \quad (5.1.39)$$

Equations (5.1.36) and (5.1.37) give upper and lower bounds on the pdf of one interfering term within the summation that represents the total interference in (5.1.4) conditioned on $C_a(1) = c$. The procedure to determine bounds on the pdf for additional interferers is identical. In what follows, $u_i^{k,1}(c)$ and $v_i^{k,1}(c)$ are used to denote upper and lower bounds, respectively, on the pdf of interference due to the k th MPG. Conditioned on $C_a(1) = c$, the terms $u_i^{k,1}(c)$, $k = 2, \dots, K$, are independent of one another. This is because the cross-correlations between each time-delayed sequence and the reference sequence are independent of one another. The same is true for $v_i^{k,1}(c)$, $k = 2, \dots, K$. The upper bound on the pdf for the total interference $u_i(c)$ due to $K - 1$ interferers conditioned on $C_a(1) = c$ can therefore be written as

$$u_i(c) = u_i^{2,1}(c) * u_i^{3,1}(c) * \dots * u_i^{K,1}(c) \quad (5.1.40)$$

and the lower bound on the pdf for the total interference $v_i(c)$ due to $K - 1$ interferers conditioned on $C_a(1) = c$ can be written as

$$v_i(c) = v_i^{2,1}(c) * v_i^{3,1}(c) * \dots * v_i^{K,1}(c). \quad (5.1.41)$$

Unconditional bounds u_i and v_i can be obtained by removing the dependence on $C_a(1)$ through the summations (5.1.38) and (5.1.39). Unconditional upper and lower bounds u_i and v_i respectively, on the pdf of interference in a correlation receiver for the case in which there are two MPGs having unit amplitude for a spreading factor $N_d = 7$ are shown in Figures 5.5(a) and 5.5(b) for $N_u = 5$ and $N_u = 50$ subintervals, respectively. Clearly the difference between the bounds decreases with

a larger number of subintervals. The bounds on the pdf for positive values of the sequence cross-correlation have been shown. For negative values of interference, the bounds are a mirror image about the y-axis of the bounds for positive values of interference.

Figure 5.6 shows the bounds on the interference with $N_d = 7, N_u = 50$ calculated using the semi-analytical technique (SAT) developed above, compared with a Gaussian pdf with variance $\sigma_I^2 = N_d$ (see Chapter 4). Also shown in Figure 5.6 is a Gaussian pdf with variance equal to the variance of the upper bound on the pdf of the interference, σ_u^2 , calculated using the semi-analytical technique. While the Gaussian pdf with $\sigma_I^2 = \sigma_u^2$ shows fair agreement with the bounds on interference estimated using the SAT, the Gaussian pdf with $\sigma_I^2 = N_d$, as used in previous publications, has a larger variance.

Given the upper bound discussed above, an upper bound on average probability of error can then be written as

$$P_E^U = \sum_{i=-(K-1)N_dN_u}^{(K-1)N_dN_u} u_i P_e(N_d|h_1|^2, iN_u^{-1}), \quad (5.1.42)$$

The probability of bit error P_e can be written in terms of the Gaussian-Q function as [2]

$$\begin{aligned} P_e(N_d|h_1|^2, iN_u^{-1}) &= \frac{1}{\sqrt{2\pi}} \int_{\frac{N_d|h_1|^2 + iN_u^{-1}}{\sigma}}^{\infty} e^{-w^2/2} dw \\ &= Q((N_d|h_1|^2 + iN_u^{-1})/\sigma), \end{aligned} \quad (5.1.43)$$

where h_1 denotes the MPG amplitude corresponding to time-delay τ_1 , and $\sigma^2 = \frac{N_d^2 N_0 |h_1|^2}{E_b}$ is the variance of the noise term in (5.1.4). In (5.1.42) u_i is the value of the upper bound on the pdf that corresponds to the largest value of interference in the interval $[iN_u^{-1}, (i+1)N_u^{-1}]$, $-N_dN_u \leq i < N_dN_u$, in the case where there

(a) $N_d = 7$ and $N_u = 5$. (b) $N_d = 7$ and $N_u = 50$.

Figure 5.5: Upper and lower bounds on SI for two (equal amplitude) MPGs when a correlation receiver is used.

Figure 5.6: Upper and lower bounds on SI for two (equal amplitude) MPGs , $N_d = 7$ and $N_u = 50$, when a correlation receiver is used compared with Gaussian pdfs.

are two received MPGs. The argument iN_u^{-1} corresponds to the maximum value of the interference in the interval $[iN_u^{-1}, (i+1)N_u^{-1}]$ since the value of the Gaussian-Q function, upon which P_e depends, decreases monotonically with increasing argument.

A lower bound on performance can be determined similarly by substituting v_{i+1} , the value of the lower bound on the pdf that corresponds to the smallest value of interference in the interval $[iN_u^{-1}, (i+1)N_u^{-1}]$, $-N_dN_u \leq i < N_dN_u$, in place of u_i in (5.1.42). Since the bounds decrease monotonically, a lower bound on average probability of error can then be written as

$$P_E^L = \sum_{i=-(K-1)N_dN_u}^{(K-1)N_dN_u} v_{i+1} P_e(N_d|h_1|^2, (i+1)N_u^{-1}). \quad (5.1.44)$$

The following three examples present BER results estimated using the semi-analytical procedure. In the case of a correlation receiver that combines one of two

(a) Bounds on the BER for two received MPGs, $N_d = 7, N_u = 5$ and $N_d = 7, N_u = 50$.

(b) Bounds on the BER for two received MPGs, different spreading factors $N_d = 7, 11, 31$ and $N_u = 50$.

(c) Bounds on the BER for two received MPGs having unequal power, $N_d = 7, N_u = 50$.

Figure 5.7: Upper and lower bounds on BER when only one of two received MPGs is processed in a correlation receiver.

received MPGs, Figure 5.7(a) shows bounds on the probability of bit error as a function of SNR per bit for spreading factors, $N_d = 7$ for both $N_u = 5$ and $N_u = 50$. As would be expected, the bounds on BER approach each other with an increase in the number of subintervals, N_u , from 5 to 50.

Figure 5.7(b) shows bounds on the estimated BER for different spreading factors of 7, 11 and 31 with $N_u = 50$, for an identical correlation receiver that processes only one of two received MPGs having equal power. Clearly as the spreading factor increases the SI decreases and BER improves and approaches the BER estimate for the case where there is no SI.

Figure 5.7(c) shows bounds on the BER when there are two received MPGs having unequal amplitudes h_1 and h_2 when the spreading factor $N_d = 7$ and $N_u = 50$. Only the MPG with amplitude h_1 is processed in the correlation receiver. Clearly, the BER degrades as the amplitude of h_2 relative to $h_1 = 1$ increases from 0.5 to 2.

In this section the SAT was developed to estimate bounds on the BER performance of a correlation receiver in a non-fading environment. It is a computationally efficient technique for estimating bounds on the BER, while taking the effects of SI into account. For the simple non-fading cases considered in this section, it was demonstrated that SI can result in a significant error floor.

While the results presented in this section for different numbers and powers of MPGs are intuitively pleasing, it is necessary to confirm that the BER estimated using the SAT is accurate. For this reason, BER estimates are validated using simulation in the next section.

5.2 Simulation-based validation of the SAT

In this section, BER results obtained using the SAT described in the previous section for estimating the BER performance of a correlation receiver are verified by

simulation. A simulation was conducted to determine the distribution for the cross-correlation of random sequences multiplied by a phase term that represents the interference term in a correlation receiver. The distribution is compared with upper and lower bounds on the distribution of the interference calculated using the SAT. The simulated BER is also compared with upper and lower bounds on the BER estimated using the SAT.

The simple case where the two MPGs have unit amplitude was first considered. For the case of a correlation receiver, there will be one interference term of the form $H_2 = I_2 \cos \phi$, where I_2 represents the sequence cross-correlation normalised by the chip period and ϕ is the phase difference between the two MPGs.

If the two received MPGs have time-delays τ_1 and τ_2 , the time-delay τ_2 of the second MPG relative to the time-delay τ_1 of the first, rounded down to the nearest integer value can be written as $\gamma_2 = \lfloor (\tau_2 - \tau_1) / T_c \rfloor$, where T_c is the chip period. While for the SAT it was assumed that $S_2 = \tau_2 - \tau_1 - \gamma_2 T_c$ is uniformly distributed on $(0, T_c)$, for the purpose of simulation each chip interval was divided into N_s subintervals and S_2/T_c was allowed any value from the set $\{0, \frac{1}{N_s}, \dots, \frac{N_s-1}{N_s}\}$ with probability $1/N_s$. An arbitrary (low) spreading factor of $N_d = 7$ was used because this results in significant SI. For each iteration in the simulation, a pair of random spreading sequences was generated and the cross-correlation value determined.

Referring to Figure 5.5(b), it can be seen that the minimum value of the bounds on the pdf of SI is a little less than 10^{-5} . A total of 10^5 pairs of random sequences were therefore generated with the expectation that the estimated pdf of the cross-correlation should be accurate for values of the pdf greater than 10^{-3} , and decrease in accuracy for smaller values. Each cross-correlation result was multiplied by a total of 100 randomly chosen values of $\cos \phi$ where ϕ was generated to be uniformly distributed between 0 and π to determine the pdf of the interference term $H_2 = I_2 \cos \phi$.

Figure 5.8(a) shows the bounds on the pdf of the interference estimated using the

SAT with $N_u = 20$, compared with the simulated distribution for the cross-correlation H_2 , with $N_s = 10$. Clearly there is a discrepancy. This discrepancy is the result of N_s being too low, which restricts the value of the cross-correlation to $\pm 2k/N_s$, where $k \in \{0, 1, 2, \dots, \frac{N_d N_s}{2}\}$. If N_s is too low, the pdf of I_2 does not accurately model a uniform distribution. As a result the pdf of $H_2 = I_2 \cos \phi$, for $I_2 = k$, can be written as

$$f_{H_2}(x|I_2 = k) = \begin{cases} \frac{1}{k\pi\sqrt{1-x^2/(k)^2}}, & |x| < k \\ 0, & |x| \geq k \end{cases} \quad (5.2.1)$$

and is shown in Figure 5.9 for $k = 1$. The peaks in the simulated pdf occur at even integer multiples of $\frac{1}{10}$.

The average bounds on BER can be estimated by numerically integrating (5.1.42) and (5.1.44) using upper and lower bounds on the pdf of the SI estimated using the SAT. The simulated pdf f_{H_2} can similarly be used to estimate the BER by numerical integration

$$P_E = \sum_{x=x_{min}}^{x_{max}} f_{H_2}(x) P_e(N_d, x), \quad (5.2.2)$$

where x_{max} and x_{min} are the maximum and minimum values of the SI, H_2 . The probability of bit error $P_e(N_d, x)$, is expressed in terms of the Gaussian-Q function as

$$P_e(N_d, x) = Q\left(\frac{N_d + x}{\sigma}\right), \quad (5.2.3)$$

where N_d is the spreading factor and σ^2 is the variance of the noise.

Figure 5.8(b) shows upper and lower semi-analytical bounds on the BER and the simulated estimate of BER with $N_s = 10$. While the simulated result lies within the semi-analytical bounds for larger values of BER, there is a discrepancy for smaller values of BER.

(a) Simulated pdf of interference ($N_s = 10$) compared with semi-analytical bounds on the pdf of interference ($N_u = 20$) when there are two received MPGs having equal amplitude, one of which is used in a correlation receiver. The spreading factor was chosen to be $N_d = 7$.

(b) Upper and lower bounds on BER using the SAT ($N_u = 20$) compared with a simulated BER estimate ($N_s = 10$) when there are two received MPGs having equal amplitude, one of which is used in a correlation receiver. The spreading factor was chosen to be $N_d = 7$.

Figure 5.8: Simulated pdf of SI and resulting BER estimates compared with semi-analytical bounds on pdf of SI and bounds on the BER.

Figure 5.9: Probability density function of $H_2 = I_2 \cos \phi$, for $I_2 = 1$ and ϕ uniformly distributed on $[0, \pi]$.

It is expected that increasing N_s in the simulation would result in a more accurate estimate of BER as it would model the realistic scenario of $S_2 = \tau_2 - \tau_1 - \gamma_2 T_c$ being uniformly distributed on $(0, T_c)$ more accurately. Figure 5.10(a) shows the bounds on the pdf of the interference estimated using the SAT using $N_u = 20$, compared with the simulated distribution for the cross-correlation H_2 , with $N_s = 1000$. In this case the simulated pdf lies between the bounds, as required, except at values of $f_{W_2}(x) < 10^{-4}$, which is a result of the finite sample size.

Figure 5.10(b) shows upper and lower semi-analytical bounds on the BER for $N_u = 20$ with the simulated estimate of BER using $N_s = 1000$. In this case the simulated result lies within the semi-analytical bounds for all values of BER. Also shown in Figure 5.10(c) are upper and lower semi-analytical bounds on the BER for $N_u = 20$ and $N_u = 50$ compared with the simulated estimate of BER using $N_s = 1000$. Clearly as N_u increases from 20 to 50 the bounds approach the simulated result.

Figure 5.10(d) shows the BER comparison for a correlation receiver that processes

(a) Simulated pdf of interference using $N_s = 1000$ compared with semi-analytical bounds on the pdf of interference ($N_u = 20$).

(b) Upper and lower bounds on BER using the SAT ($N_u = 20$) compared with a simulated BER estimate using $N_s = 1000$.

(c) Upper and lower bounds on BER using the SAT ($N_u = 20$ and $N_u = 50$) compared with a simulated BER estimate using $N_s = 1000$.

(d) Upper and lower bounds on BER using the SAT ($N_u = 20$) compared with a simulated BER estimate using $N_s = 1000$. Lehnert and Pursley's expressions are not modified to correct the scaling error.

Figure 5.10: Simulated pdf of SI and resulting BER estimates compared with semi-analytical bounds on pdf of SI and bounds on the BER when a correlation receiver is used to process only one of two received MPGs.

only one of two received MPGs when the scaling factor in (5.1.33) is 1 instead of 2, as in [2]. Clearly this is an error.

Figure 5.11(a) shows the BER comparison for a correlation receiver that processes only one (h_1) of two MPGs (h_1, h_2) having unequal power, $h_1 = 1, h_2 = 2$ and $h_1 = 1, h_2 = 0.50$. Clearly the simulated BER lies between the bounds on the BER estimated using the SAT with $N_u = 50$. This simulation validates the extension to Lehnert and Pursley's technique presented in this thesis that allows consideration of unequal MPG amplitudes.

Figure 5.11(b) shows the BER comparison for a correlation receiver that processes one of three MPGs having equal power of 1. The spreading factor was chosen to be $N_d = 3$, because the large number of correlations (approximately 10^8) that would be required in a simulation for a spreading factor of $N_d = 7$ made this simulation impractical. Once again the simulated result lies between the bounds on the BER estimated using the SAT with $N_u = 50$. This simulation further verifies the process of convolution in the SAT that is used in the event that there is more than one interfering MPG.

From the above presented cases, it can be accepted that the SAT provides reliable bounds on BER. By increasing the number of subintervals N_u , the bounds approach the true BER result. As well, $N_u = 50$ provides reasonable bounds on the BER.

It is considered of interest that the SAT based prediction of BER for the simple case of two received MPGs and a correlation receiver took about 5% of the time required to run a simulation to arrive at the same result. It was considered impractical to conduct a simulation for a larger number of received MPGs using a spreading factor of 7 or more. The process of validation therefore served to highlight the reduction in computational complexity that can be achieved using the SAT when compared with simulation.

(a) Simulated pdf of interference ($N_s = 1000$) compared with semi-analytical bounds on the pdf of interference ($N_u = 20$) when there are two received MPGs having unequal amplitude, only one of which is processed in a correlation receiver.

(b) Upper and lower bounds on BER using the SAT ($N_u = 20$) compared with a simulated BER estimate ($N_s = 1000$) when there are three received MPGs having equal amplitude, only one of which is processed in a correlation receiver.

Figure 5.11: Simulated BER estimates compared with semi-analytical bounds on the BER performance of a correlation receiver.

5.3 Estimation of Rake receiver BER performance in a fading environment

In Section 5.1, the technique for analysing the BER performance of a correlation receiver in a non-fading environment was presented. This technique is next extended to analyse the performance of a Rake receiver (correlation receiver with multiple despreading fingers) in a multipath, fading environment.

In the case of a Rake receiver with L despreading fingers and MRC, if the time-delays and complex amplitudes of MPGs are estimated with no error, the decision variable has the same form as (5.1.2) and can be written as

$$D_{Rake} = \sum_{i=1}^L \int_{\tau_i}^{\tau_i+T_d} \text{real}(r(t) h_i^* a(t - \tau_i)) dt, \quad (5.3.1)$$

where the i th Rake finger multiplies the MPG having time-delay τ_i by its complex conjugate and despreads the sequence at the same time-delay. The decision variable D_{Rake} can be expanded to

$$D_{Rake} = \sqrt{E_b T_d / 2} \left(\sum_{i=1}^L |h_i|^2 \right) + \sqrt{E_b T_d / 2} \left[\sum_{i=1}^L \sum_{k=1, k \neq i}^K h_{k,i} W_{k,i} \cos \theta_{k,i} / T_d \right] + \eta_{Rake}, \quad (5.3.2)$$

where $h_{k,i} = |h_k| \cdot |h_i|$ and the second summation term represents the SI resulting from multipath interference. The cross-correlation of interfering binary sequences k and i , denoted by $W_{k,i}$ can be written as,

$$W_{k,i} = \int_{\tau_i}^{T_d + \tau_i} b(t - \tau_k) a(t - \tau_k) a(t - \tau_i) dt. \quad (5.3.3)$$

As before, it is reasonable to assume that the phase term $\theta_{k,i} = \phi_k - \phi_i$ is uniformly distributed on $[0, \pi]$, ϕ_k is the carrier phase offset of the k th MPG. It was shown in

Chapter 1 that the random variable η_{rake} , that results from AWGN input $n(t)$ to the Rake receiver, is zero mean Gaussian with variance $\frac{N_0 T_d}{2} \sum_{i=1}^L |h_i|^2$.

For the sake of convenience, the sum of the interference to the i th Rake finger from the k th MPG, $h_{k,i} W_{k,i} \cos \theta_{k,i} / T_d = J_1$, and the interference to the k th Rake finger from the i th MPG, $h_{i,k} W_{i,k} \cos \theta_{i,k} / T_d = J_2$, can be defined as the mutual SI, $M_{k,i}$, on Rake fingers i and k and written as

$$M_{k,i} = h_{k,i} W_{k,i} \cos \theta_{k,i} / T_d + h_{i,k} W_{i,k} \cos \theta_{i,k} / T_d = J_1 + J_2, \quad (5.3.4)$$

where $h_{k,i} = h_{i,k}$ and $\cos \theta_{k,i} = \cos \theta_{i,k}$. If $\sigma_{J_1}^2$ and $\sigma_{J_2}^2$ are the variances of J_1 and J_2 respectively, then $\sigma_{J_1}^2 = \sigma_{J_2}^2$.

Figure 5.12: Two Rake fingers used to despread two received MPGs.

Figure 5.12 shows two time-delayed replicas of the transmitted spread spectrum signal having time-delays τ_1 and τ_2 . Two Rake fingers having time-delays τ_1 and τ_2 are used to despread the replicas with time-delays τ_1 and τ_2 , respectively. The sequence cross-correlation $W_{2,1}$ which results from the interference from the second sequence (2) to the first sequence (1) over the symbol interval T_d is

$$W_{2,1} = \int_{\tau_1}^{T_d+\tau_1} b(t-\tau_1) a(t-\tau_1) a(t-\tau_2) dt, \quad (5.3.5)$$

and can be rewritten as

$$W_{2,1} = \int_{\tau_1}^{\tau_2} b(t-\tau_1) a(t-\tau_1) a(t-\tau_2) dt + \int_{\tau_2}^{T_d+\tau_1} b(t-\tau_1) a(t-\tau_1) a(t-\tau_2) dt. \quad (5.3.6)$$

The sequence cross-correlation $W_{1,2}$ which results from the interference from sequence 2 to sequence 1 over the symbol interval T_d can similarly be written as

$$W_{1,2} = \int_{\tau_2}^{T_d+\tau_1} b(t-\tau_2) a(t-\tau_1) a(t-\tau_2) dt + \int_{T_d+\tau_1}^{T_d+\tau_2} b(t-\tau_2) a(t-\tau_1) a(t-\tau_2) dt. \quad (5.3.7)$$

Also for $0 < \tau_2 < T_d$, $b(t-\tau_2) = b(t-\tau_1)$ in the interval $[\tau_2, \tau_1 + T_d]$. The second integral in (5.3.6) is therefore the same as the first integral in (5.3.7). The distributions of J_1 and J_2 will therefore be correlated. Bounds on the variance of $M_{2,1}$ can be obtained as follows. When $\tau_2 - \tau_1 > T_d$, the distributions of J_1 and J_2 are independent. As a result, $\sigma_{M_{2,1}}^2 = 2\sigma_{J_1}^2$. But as $\tau_2 - \tau_1 \rightarrow 0$, $M_{2,1} \rightarrow 2J_1$, so $\sigma_{M_{2,1}}^2 \rightarrow 4\sigma_{J_1}^2$. As a result the following inequality holds for the variance of $M_{2,1} = J_1 + J_2$:

$$2\sigma_{J_1}^2 \leq \sigma_{M_{2,1}}^2 \leq 4\sigma_{J_1}^2. \quad (5.3.8)$$

These bounds can be used to place limits on the effects of mutual interference in a Rake receiver. Without this the SI would have to be conditioned on the relative time-delay between two MPGs which would result in great complexity.

Later, in Section 5.4.5, it will be shown that in the fading case when the difference in average power, Δ , between two combined MPGs is small (the case $\Delta = 0$ dB

is studied), there is a noticeable though small difference between the upper bound on BER estimated when it is assumed that $W_{2,1}$ and $W_{1,2}$ are the same, and the upper bound on BER estimated when it is assumed that $W_{2,1}$ and $W_{1,2}$ are not the same. For the difference in average power of 0 dB, however, in urban microcellular environments it was found that the received MPGs are spaced no more than a chip period apart for the case of a system with a chip rate of 3.84 Mchips/second. As a result it is considered reasonable to assume that $W_{2,1}$ and $W_{1,2}$ are the same. For greater average differences in power, there is no noticeable difference between the upper bound on BER estimated when it is assumed that $W_{2,1}$ and $W_{1,2}$ are the same, and the upper bound on BER estimated when it is assumed that $W_{2,1}$ and $W_{1,2}$ are not the same. In both cases, the assumption that $W_{2,1} = W_{1,2}$ results in an upper bound on the mutual interference. As a result, the mutual interference term in the case of L Rake fingers can be written as $2 \sum_{i=1}^L \sum_{k=i+1}^L M_{k,i}$.

Therefore the Rake decision variable (5.3.2) normalised by $T_c \sqrt{E_b/(2T_d)}$ can be written as

$$D_{\text{Rake,norm}} = N_d \sum_{i=1}^L |h_i|^2 + 2 \sum_{i=1}^L \sum_{k=i+1}^L M_{k,i} + \sum_{i=1}^L \sum_{k=L+1}^K h_{k,i} \frac{W_{k,i}}{T_c} \cos \theta_{k,i} + \frac{1}{T_c} \sqrt{\frac{2T_d}{E_b}} \eta_{\text{Rake}}. \quad (5.3.9)$$

Distributions for the interference component can be calculated as before using (5.1.36) to (5.1.41). The probability of bit error can then be evaluated using the Gaussian-Q function

$$\begin{aligned} P_e \left(N_d \sum_{i=1}^L |h_i|^2, \beta \right) &= \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-w^2/2} dw \\ &\quad \frac{N_d \sum_{i=1}^L |h_i|^2 + \beta}{\sigma_{\text{Rake}}} \\ &= Q \left(\left(N_d \sum_{i=1}^L |h_i|^2 + \beta \right) / \sigma_{\text{Rake}} \right), \end{aligned} \quad (5.3.10)$$

where h_i denotes the MPG corresponding to time-delay i and β is the SI. The variance

of the noise term, σ_{Rake}^2 , in this case is $\frac{N_0 N_d^2}{E_b} \sum_{i=1}^L |h_i|^2$. Bounds on the probability of bit error can be estimated using (5.1.42) and (5.1.44).

Equations (5.1.42) and (5.1.44) give upper and lower bounds, respectively, on the BER for a non-fading channel. The SAT can be easily extended to determine bounds on BER for a fading channel. From (5.1.42), the upper bound on probability of bit error at time instant n , $P_E^U(n)$ can be written as

$$P_E^U(n) = \sum_{i=-L(K-1)N_d N_u}^{L(K-1)N_d N_u} u_i P_e \left(N_d \sum_{j=1}^L |h_j(n)|^2, iN_u^{-1} \right). \quad (5.3.11)$$

The instantaneous probability of bit error is given by

$$\begin{aligned} P_e \left(N_d \sum_{j=1}^L |h_j(n)|^2, iN_u^{-1} \right) &= \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-w^2/2} dw \\ &\quad \frac{N_d \sum_{j=1}^L |h_j(n)|^2 + iN_u^{-1}}{\sigma_{Rake}} \\ &= Q \left(\left(N_d \sum_{j=1}^L |h_j(n)|^2 + iN_u^{-1} \right) / \sigma_{Rake} \right), \end{aligned} \quad (5.3.12)$$

where $h_j(n)$ represents the amplitude of the MPG corresponding to time-delay τ_j from the n th set of MPGs. The lower bound can be derived similarly. For M sets of MPGs in a time-series, the upper bound on the probability of error can therefore be averaged as

$$\mathcal{E}_n \{P_E^U\} = \frac{1}{M} \sum_{n=1}^M P_E^U(n). \quad (5.3.13)$$

5.4 Application of the SAT to estimate bounds on BER for example cases

Now that the SAT has been developed, verified and extended, the objective is to give examples of the effects of various assumptions often used in work that can be found

in the literature for estimating BER performance of a Rake receiver. This is done by using the expressions derived in Sections 5.1 and 5.3 in the semi-analytical approach to derive estimates of BER performance without the cited simplifying assumptions and then comparing results with those obtained when the simplifying assumptions are used.

In Section 5.4.1, BER performance results obtained under the assumption that received MPGs are separated in time by integer multiples of chip period are compared with performance results obtained from using the SAT described in Section 5.1 for a correlation receiver. The SAT does not require the assumption of integer multiples of chip period. It is shown that this assumption leads to inaccuracies in BER performance estimates. This is one example that demonstrates the need for an analysis method such as the SAT.

In Section 5.4.2, BER performance results obtained using the Gaussian assumption for the interference that was introduced in Section 4.1 are compared with BER performance results obtained using the SAT, both for a correlation receiver. An alternate Gaussian approximation derived empirically from the bounds on SI determined using the SAT is discussed in Section 5.4.3 for the fading case. The relevance of the independent fading assumption on performance is discussed in Section 5.4.4 and the influence of relative time-delay between received MPGs on performance is discussed in Section 5.4.5. In Section 5.4.6, the effect on estimated performance of the equal average SNR assumption for received MPGs is discussed by comparing BER performance estimated using this assumption with BER performance estimates made using measured data. Finally in Section 5.4.7 BER performance results obtained using the SAT in conjunction with an APDP derived from measured data are compared with performance results obtained using the SAT with the ITU-R vehicular A channel model.

5.4.1 The impact of simplifying the model for MPG time-delays

The difference in BER performance for the scenario in which time-delays of received MPGs relative to the first received MPG are assumed to be integer multiples of the chip period, and the more realistic scenario in which the time-delays are non-integer multiples of the chip period is investigated for the non-fading case.

As discussed in Section 4.1, in the first scenario, since the time-delays are assumed to be integer multiples of chip period apart, the interference which is the product of two random binary sequences can be precisely modelled as a binomial random variable, making this a popular assumption. The value of this interference is an integer between $-N_d$ and $+N_d$. The probability of bit error, P_E , can be written as

$$P_E = \sum_{x=-N_d}^{N_d} P(\beta = x) Q\left(\frac{N_d \sum_{i=1}^L |h_i|^2 + x}{\sigma}\right), \quad (5.4.1)$$

where $\sigma^2 = \frac{N_d^2 N_0}{E_b} \sum_{i=1}^L |h_i|^2$ is the variance of the noise term in (5.1.4), and β is the interference.

In the second scenario, the pdf of the interference must be modelled as described in Section 5.1 and bounds on the BER calculated using the SAT involving (5.1.42) and (5.1.43). This process was demonstrated in Sections 5.1 and 5.3. Figure 5.13 shows the performance for the two cases with different spreading factors, $N_d = 11, 31$ and 63 and $N_u = 50$. Upper and lower bounds on BER estimated using the SAT are shown for $N_d = 63$. Only upper bounds are shown for $N_d = 11$ and $N_d = 31$. Lower bounds on BER for $N_d = 11$ and $N_d = 31$ were shown in Figure 5.7(b) and found to lie very close to the upper bounds on BER.

Clearly, the simplified model for interference (scenario 1) overestimates the BER

Figure 5.13: Estimates for BER performance of a correlation receiver when the time-delays between received MPGs is assumed to be an integer multiple of the chip period (scenario 1) and when the time-delays between received MPGs are in general non-integer multiples of the chip period (scenario 2) for $N_d = 11, 31$ and 63 .

relative to the more realistic model (scenario 2) for all the considered values of spreading factor. This is because in the second scenario, interference over a chip interval can be partially cancelled out when chip transitions of the interfering sequence occur within a chip interval of the reference sequence. The discrepancy between the BER estimated using the simplified time-delay model and the bounds on the BER estimated using the semi-analytical approach increases with a reduction in spreading factor. This is because the value of probability of bit error at which the error floor occurs (as a result of SI) is greater for the low spreading factor of 11 than it is for the other higher spreading factors (31, 63) for which performance results are shown. It is expected that the comparisons in this section will also hold true for fading channels.

5.4.2 Gaussian models for the self-interference

A simplification to the case considered in the previous section is the use of a Gaussian model for the SI. It was earlier shown in Section 4.1 that the Gaussian pdf can be used to approximate a binomial pdf. If the SI is modelled as Gaussian with zero mean and variance σ_I^2 , in the case of the Rake receiver with L fingers, the probability of bit error, P_E , can be written as

$$P_E = Q \left(\frac{N_d \sum_{i=1}^L |h_i|^2}{\sqrt{\sigma_I^2 + \sigma_{Rake}^2}} \right), \quad (5.4.2)$$

where σ_{Rake}^2 is the variance of the noise. In Section 5.4.1, the interference was modelled as a binomial random variable for chip-spaced MPGs. Also it was shown in Section 4.1 that the Gaussian CDF with variance N_d approximates a binomial CDF used to model the SI in a correlation receiver in the case where there are two received MPGs having unit amplitude, for a spreading factor N_d . The approximation improves with an increase in N_d . Recall from (4.1.17) that in general, a Gaussian model for the total SI has zero mean with variance σ_I^2 expressed in terms of the fixed MPG amplitudes $h_i, i = 1, \dots, K$, as

$$\sigma_I^2 = 4N_d \sum_{i=1}^L \sum_{j=i+1}^L |h_i|^2 |h_j|^2 + N_d \sum_{i=1}^L \sum_{j=L+1}^K |h_i|^2 |h_j|^2, \quad (5.4.3)$$

where the first summation represents the total SI resulting from MPGs time aligned with any of the despreading fingers and the second summation represents the total SI from uncombined MPGs. For the case where $L = 1, K = 2$ and $|h_1| = |h_2| = 1$, $\sigma_I^2 = N_d$.

The interference can also be modelled as Gaussian with the same variance as the variance of the upper and lower bounds on the interference estimated using the SAT. The variance σ_u^2 , of the upper bound u_i , on the interference can be written as

$$\sigma_u^2 = \sum_{i=-L(K-1)N_dN_u}^{L(K-1)N_dN_u} \left(\frac{i}{N_u} \right)^2 u_i. \quad (5.4.4)$$

The variance of the lower bound v_i , on the interference can similarly be written as

$$\sigma_v^2 = \sum_{i=-L(K-1)N_dN_u}^{L(K-1)N_dN_u} \left(\frac{i}{N_u} \right)^2 v_i. \quad (5.4.5)$$

Figure 5.14 shows upper and lower bounds on the pdf of interference calculated using the SAT with $N_u = 50$ for the case in which one of two received MPGs, each having unit amplitude, is processed for $N_d = 31$. Also shown are Gaussian pdfs having variance $\sigma_I^2 = N_d$ and $\sigma_I^2 = \sigma_u^2$. Figure 5.6 showed the same comparison when $N_d = 7$.

Figure 5.14: Comparison of Gaussian approximations for the statistics of the interference and bounds on the pdf for the interference calculated using the SAT. The spreading factor was $N_d = 31$.

Figure 5.15 shows bounds on the probability of bit error for spreading factors $N_d = 7$ and $N_d = 31$, calculated using the SAT with $N_u = 50$ when one of two received MPGs, both having unit amplitude, is processed in the receiver (note that upper and

Figure 5.15: Bounds on the probability of bit error for two received MPGs having equal power, when a correlation receiver is used, compared with BER estimated using Gaussian approximations for the pdf of the interference.

lower bounds for $N_d = 31$ coincide). Figure 5.16 shows bounds on the probability of bit error for $N_d = 7$ calculated using the SAT when only one of two received MPGs having unequal amplitudes is processed in the receiver. Shown for comparison in the same figures are the estimated probabilities of bit error using Gaussian models for the SI. One Gaussian model has the same variance as u_i , calculated using (5.4.4), while the other Gaussian model has variance N_d .

Referring to Figure 5.14, it is obvious that estimated BER performance when it is assumed that $\sigma_I^2 = N_d$ should be worse than estimated BER performance when it is assumed that $\sigma_I^2 = \sigma_u^2$. This is because the variance (indicated by the width of the pdf) in the former case exceeds the variance in the latter case. Consequently, the probability that the estimated SI lies in the interval $[0 - \epsilon, 0 + \epsilon]$, $\epsilon \rightarrow 0$, is smaller when $\sigma_I^2 = N_d$ than when $\sigma_I^2 = \sigma_u^2$. It is not as straightforward to make such comparisons between the Gaussian pdfs and the bounds on the pdf of the interference shown

Figure 5.16: Bounds on the probability of bit error for two received MPGs having unequal power, when a correlation receiver is used, compared with BER estimated using Gaussian approximations for the pdf of the interference. The spreading factor was $N_d = 7$.

in Figure 5.14 to predict the difference in the estimated BER resulting from the use of the latter two models for interference. However, referring to Figure 5.14, it can be seen that the Gaussian pdf with $\sigma_I^2 = \sigma_u^2$ shows fair agreement with the bounds derived using the SAT for small arguments. For larger arguments, this Gaussian pdf decays more rapidly than the Gaussian pdf with $\sigma_I^2 = N_d$. This was found to be true for all values of spreading factor and numbers of received and combined MPGs.

As would be expected from considerations of pdf shown in Figure 5.14 BER estimates (Figure (5.15)) made using the Gaussian approximation for interference with $\sigma_I^2 = \sigma_u^2$ provide optimistic estimates of performance compared with the true bounds on the BER estimated using the semi-analytical approach. Also the BER estimated using the Gaussian approximation for interference with $\sigma_I^2 = N_d$ provides pessimistic estimates of performance compared with the true bounds on the BER. It can also

be seen from Figure 5.15 that the BER estimates made using the Gaussian approximations increase in accuracy with an increase in spreading factor from 7 to 31. It is important to note that the reason for this improvement in accuracy is the decreasing significance of SI for the given channel conditions with an increase in spreading factor. It would be incorrect, however, to assume in general that the Gaussian approximation for interference becomes more accurate with an increase in spreading factor. This is because the Gaussian approximation is strictly only valid at high spreading factors if the relative time-delay between any two received MPGs is an integer multiple of the chip period.

For a small spreading factor, for example, $N_d = 7$, it would be useful to know if the Gaussian model for interference is ever valid. It is well known from the Central Limit Theorem that the sum of a large number of independent random variables has a Gaussian pdf. It is therefore expected that performance results predicted using a Gaussian approximation for interference should be valid when there are a large number of MPGs. Figures 5.17(a), 5.17(b) and 5.17(c) show the probability of bit error as a function of received SNR per bit for the following three scenarios and a spreading factor $N_d = 7$:

1. One of three received MPGs is processed,
2. One of five received MPGs is processed,
3. One of seven received MPGs is processed,

respectively. All MPGs were assumed to have equal power. Bit error rate performance results obtained using the Gaussian approximation for the interference with variance $\sigma_I^2 = \sigma_u^2$ approach the performance results obtained using the SAT with an increase in the number of received MPGs. In contrast if $\sigma_I^2 = (K - 1)N_d$ is used to approximate the variance of the Gaussian model for the interference, the resulting performance

Figure 5.17: Comparison of bounds on BER performance estimated using the SAT with $N_u = 50$ for a correlation receiver with different numbers of received MPGs (K) and $N_d = 7$ with BER estimated using two different variances for the interference (assumed to be Gaussian).

estimates are not as accurate because they do not approach the semi-analytical result with an increase in the number of received MPGs.

It is important to note that based on the results presented in this section, the Gaussian approximation with variance given by (5.4.4) can only be used once the bounds on the interference have been calculated using the SAT, and the variance estimated. In the next section an empirical relationship between σ_u^2 and (N_d, K, L) is derived. This is of interest because there is an advantage to using the Gaussian approximation for interference in cases where it is valid in BER calculations since it avoids the complex computations that are involved in using the SAT to determine bounds on the pdf of the interference.

5.4.3 Estimation of σ_u^2 using knowledge of N_d, K and L

An alternate approximation for the variance of a Gaussian model for the interference will be derived using the SAT in this section. In Section 5.4.2, it was demonstrated that the statistics of the interference can be approximated with a Gaussian pdf having the same variance, σ_u^2 , as the bounds on the pdf for the interference, u_i , calculated using

$$\sigma_u^2 = \sum_{i=-L(K-1)N_dN_u}^{L(K-1)N_dN_u} \left(\frac{i}{N_u} \right)^2 u_i. \quad (5.4.6)$$

While it is difficult to demonstrate mathematically that this Gaussian approximation results in a BER estimate that is more optimistic than the bounds on the BER obtained using the SAT (since there is no analytical expression for the bounds on the interference), it was found that the Gaussian pdf with $\sigma_I^2 = \sigma_u^2$ decays more rapidly with increasing arguments as shown in Figure 5.14 for $N_d = 31$. For small values of the abscissa, however, the Gaussian pdf shows fair agreement with the bounds. This was found to be true for different spreading factors, numbers of received MPGs and

Rake fingers studied.

The variances of both the upper and lower bounds on the SI increase with either the number of MPGs or the spreading factor. It is therefore proposed to investigate the ratio between the variance, σ_u^2 , of the upper bound on the interference derived using the SAT and the variance $\sigma_I^2 = 4N_d \sum_{i=1}^L \sum_{j=i+1}^L |h_i|^2 |h_j|^2 + N_d \sum_{i=1}^L \sum_{j=L+1}^K |h_i|^2 |h_j|^2$, (from Chapter 4) of the upper Gaussian bound on interference for K received MPGs and L Rake fingers,

$$\lambda_u = \frac{\sigma_u^2}{4N_d \sum_{i=1}^L \sum_{j=i+1}^L |h_i|^2 |h_j|^2 + N_d \sum_{i=1}^L \sum_{j=L+1}^K |h_i|^2 |h_j|^2}. \quad (5.4.7)$$

Figure 5.18: The ratio (λ_u) between the variance of SI computed using the SAT with $N_u = 50$ and σ_I^2 , as a function of spreading factor, N_d , for K received and L combined MPGs, each having unit amplitude.

Figure 5.18 shows λ_u as a function of spreading factor N_d for L Rake fingers and K received MPGs, each having unit amplitude. Clearly there is little variation in λ_u for a fixed value of N_d for different L and K . Also $0.32 < \lambda_u < 0.34$.

In a similar fashion, the ratio between the variance, σ_v^2 , of the lower bound on the interference derived using the SAT and the variance σ_f^2 for K received MPGs and L Rake fingers, can be written as

$$\lambda_v = \frac{\sigma_v^2}{4N_d \sum_{i=1}^L \sum_{j=i+1}^L |h_i|^2 |h_j|^2 + N_d \sum_{i=1}^L \sum_{j=L+1}^K |h_i|^2 |h_j|^2}. \quad (5.4.8)$$

It was found that $0.31 < \lambda_v < 0.335$. It is interesting to note that the range of values of λ_u and λ_v compare well with the mean of $\cos^2 \theta_{1,2} [(1 - \mu_{1,2})^2 + \mu_{1,2}^2] = 0.33$, which is the normalised SI term in [24] derived without the conditioning on $C_a(1)$. The reason for this similarity is not understood at the present time.

Figures 5.19(a), 5.19(b) and 5.19(c) show the BER as a function of SNR per bit for $N_d = 7$ for a correlation receiver with two, five and nine received non-fading MPGs, each having unit amplitude, and a value of $\lambda = 0.33$. This value of λ falls within the range of λ_u , from Figure 5.18. The Gaussian approximation in this case improves the accuracy of BER estimates with an increase in the number of MPGs from two to nine.

It is of interest to determine if this Gaussian approximation is valid for real-world fading scenarios. Figure 5.20 shows the BER corresponding to a set of data measured along a non-line-of-sight (NLOS) section of street¹. There were $K = 5$ received MPGs with the relative powers listed in Table 5.1. For the situation in which $L = 1$, a Gaussian approximation for the interference with $\lambda = 0.33$ gives a BER estimate that lies between the bounds on the BER estimated using the SAT. For $L = 5$, $\lambda = 0.33$ gives a BER estimate that is lower than the bounds on the BER obtained using the SAT.

Figures 5.21(a) and 5.21(b) show bounds on the BER performance of a Rake

¹The use of the SAT in conjunction with measured data will be revisited in greater detail in Section 5.4.6.

Figure 5.19: Bounds on correlation receiver BER estimates for nine received MPGs using the SAT with $N_u = 50$ compared with BER estimates using a zero-mean Gaussian approximation with $\sigma_I^2 = (K - 1) \lambda N_d$, $\lambda = 0.33$, $N_d = 7$, for the statistics of the interference.

Figure 5.20: Rake receiver BER estimates made using two different Gaussian approximations, with $\lambda = 0.33$, compared with BER estimated using the SAT with $N_u = 50$ for $L = 1, K = 5$ and $L = 5, K = 5$.

receiver estimated using the SAT and BER estimates made using the Gaussian assumption with $\lambda = 0.33$. The BER results presented in Figure 5.21(a) for $L = 1$ and $L = 3$ Rake fingers use the data that forms the basis for Table 5.3 on page 125, recorded along a line-of-sight (LOS) section of street. In this case the Rake receiver processes either only one or all three received MPGs. The BER results presented in Figure 5.21(b) for $L = 1$ and $L = 2$ Rake fingers use the data that form the basis

MPG	relative power (dB)
1	0
2	-0.2
3	-7.8
4	-11.1
5	-12.1

Table 5.1: Average relative power levels for a time-series of ESRs recorded along an NLOS section of street.

(a) LOS, $K = 3$ received MPGs.(b) NLOS, $K = 5$ received MPGs.

Figure 5.21: Bounds on the BER estimated using the SAT with $N_u = 50$ compared with BER estimated using a Gaussian approximation.

for Table 5.2 on page 120, recorded along an NLOS section of street. In this case the Rake receiver processes either only one or the strongest two received MPGs. In both cases, the Gaussian approximation gives reasonably accurate estimates of BER for the $L = 1$ case. For $L > 1$, BER estimates using the Gaussian approximation are optimistic compared with the bounds on BER estimated using the SAT. This is because the Gaussian pdf decays more rapidly than the bounds on the pdf of the SI estimated using the SAT for larger arguments which are relevant for the lower error floor that comes into play when the signal energy is increased relative to the SI.

Based on the results in this section, it is conjectured that the Gaussian approximation can be used to estimate BER for a correlation receiver ($L = 1$) in urban environments for the system under consideration as the number of received MPGs is typically large. Use can also be made of the knowledge that the Gaussian approximation gives optimistic Rake receiver BER estimates for $L > 1$.

5.4.4 The influence of correlation between fading MPGs on Rake receiver BER performance

In this section the coefficient of correlation among fading MPGs, and the effect of correlation on Rake receiver BER performance is investigated by employing the SAT in conjunction with measured data from downtown Ottawa.

The expression for the correlation coefficient ρ_c corresponding to the variations in amplitudes of two MPGs, h_1 and h_2 , can be written as

$$\rho_c = \frac{\mathcal{E}_n \{ [|h_1(n)| - \mathcal{E}_n \{|h_1(n)|\}] [|h_2(n)| - \mathcal{E}_n \{|h_2(n)|\}] \}}{\sqrt{\sigma_{|h_1(n)|}^2 \sigma_{|h_2(n)|}^2}}, \quad (5.4.9)$$

where $\mathcal{E}_n \{X\}$ denotes the time average of X , and σ_X^2 denotes its variance. The foregoing equation uses the absolute values of the amplitudes of the MPGs. This is because under the assumption that channel MPGs are estimated with no error, the result of multiplying a channel MPG with its complex conjugate yields the absolute value of the amplitude of the MPG.

To begin the analysis of measured data, the coefficient of correlation in the fading of all pairs of MPGs that had a power greater than -20 dB relative to the strongest MPG were determined. Figure 5.22(a) shows the experimental CDF for the correlation coefficient ρ_c which was found to vary between +0.6 and -0.69. The probability that ρ_c lies between ± 0.20 is 0.7.

Since a Rake receiver is used to maximise diversity gain, it is obvious that in a situation where there are three received MPGs that are i.i.d., and only two available Rake fingers, the best BER performance would be achieved by combining the two fingers that are least correlated (the most favourable situation would be if $\rho_c = -1$ between the selected MPGs). However, since this would require prior knowledge of the correlations between the different MPGs and the guarantee that the future can be predicted from the past for a reasonable period of time, it is not understood if this

(a) CDF for correlation coefficients calculated from the set of recorded ESRs.

(b) Upper bounds on Rake receiver BER calculated using the SAT with $N_u = 50$ for spreading factor $N_d = 7$ and $K = 2$ received MPGs having correlation coefficient $\rho_c = \pm 0.2, \pm 0.6$, when both MPGs are combined.

Figure 5.22: CDF for correlation coefficients from measurement and the effect of correlation on Rake receiver BER estimates.

approach is practical.

The effect of correlation on Rake receiver BER performance is investigated using the SAT and a simple channel model in which there are two received MPGs. Figure 5.22(b) shows upper bounds on the BER as a function of SNR per bit for different values of ρ_c between two Rayleigh fading MPGs having equal average power, both of which are combined in the receiver for $N_d = 7$. To achieve a BER greater than 10^{-3} , the difference between the required SNR per bit for $\rho_c = 0.2$ and $\rho_c = -0.2$ is less than 1 dB. As would be expected performance is best for $\rho_c = -0.6$ and worst for $\rho_c = 0.6$.

5.4.5 The effect of the time-delay between MPGs on performance

In Section 5.3, the concept of mutual SI between two Rake fingers was discussed. An upper bound on the variance of the mutual SI was obtained by assuming that the two terms on the right hand side of (5.3.4) are identical, while a lower bound on the variance of the SI was obtained by assuming that the two terms on the right hand side of (5.3.4) are not the same. To reiterate, physically, as the relative time-delay between two received MPGs, denoted by h_1 and h_2 , decreases, it becomes increasingly more reasonable to assume that the corresponding mutual SI is given by $M_{2,1} = J_1 + J_2 = 2J_1 = 2h_{2,1} \frac{W_{2,1}}{T_c} \cos \theta_{2,1}$. For simplicity, when the mutual interference is given by (5.3.4) with $J_1 = J_2$, the relevant SI terms are referred to as being correlated and when the mutual interference is given by (5.3.4) with $J_1 \neq J_2$, the SI terms are referred to as being uncorrelated.

Figure 5.23(a) shows the difference in upper bounds on BER (estimated using the SAT with $N_u = 10$) between the uncorrelated and correlated mutual interference situations for spreading factors $N_d = 7, 11$ when there are two Rayleigh MPGs having equal average SNR. The difference in BER between the correlated and uncorrelated

Figure 5.23: Upper bounds on Rake receiver BER for different spreading factors, N_d , different relative average powers, Δ , and MRC ($K = 2, L = 2$) when the mutual SI is correlated and uncorrelated.

cases increases slightly with a reduction in spreading factor. This is because the effect of SI increases with a reduction in spreading factor. For spreading factors greater than 11, it was found that the difference in BER between the correlated and uncorrelated cases is insignificant when the received MPGs have equal average SNR.

Figure 5.23(b) shows upper bounds on the BER performance of a Rake receiver for a spreading factor $N_d = 7$, when two received MPGs with a difference in average power, $\Delta = 0, 10$ and 20 dB, are processed in the receiver, for both the correlated and uncorrelated cases. Clearly as the difference between the average powers of the two MPGs increases, the difference in BER for the cases where the mutual SI terms are uncorrelated and correlated decreases². This results from a reduction in total SI as Δ increases.

Using this knowledge it can be deduced that the mutual SI can be approximated by

²For $\Delta = 20$ dB, upper bounds on BER performance for both the correlated case and the uncorrelated case coincide.

$$M_{2,1} = 2J_1 = 2h_{2,1} \frac{W_{2,1}}{T_c} \cos \theta_{2,1}. \quad (5.4.10)$$

This can be explained with the use of Figure 5.24. The mutual SI between MPGs 1 and 2 can be approximated using (5.4.10) as the relative time-delay between them is small. The mutual interference between MPGs 1 and 7 can also be approximated using (5.4.10) since the greater relative time-delay between MPGs 1 and 7 is consistent with a greater difference in relative power and Figure 5.23(b) shows the difference between the uncorrelated and correlated mutual SI assumption on BER performance is very small in this case.

Figure 5.24: Impulse response estimate model for a channel in which there are seven resolvable time-delayed replicas of the transmitted signal at the receiver.

From the performance results discussed in this section and observations from measured data, it can be stated that if the APDP of the channel decreases monotonically, as the relative time-delay between two received MPGs increases, the difference in BER for correlated and uncorrelated mutual SI decreases. To reduce the computation required to generate BER bounds, the mutual SI can therefore be modelled as correlated. For small time-delays between MPGs, the difference between the upper bounds on BER for the correlated and uncorrelated cases increases but the resulting BER would be closer to the more pessimistic estimate of the two, as the mutual SI terms are in fact correlated. Hence for the remainder of this thesis the terms that

comprise the mutual SI are assumed to be correlated.

5.4.6 The effect of unequal average MPG powers on Rake receiver BER performance in a fading environment

An equal average SNR assumption for received MPGs was made in Chapter 3 to derive analytical expressions for Rake receiver performance when there is multipath fading. This assumption is often used in the literature to estimate the performance of diversity systems using different analytical techniques under a variety of channel conditions and serves as a common reference. For instance in [20], Veeravalli presented a general approach for analysing the BER performance of BPSK with diversity reception on a channel for which it was assumed that the fading of MPGs was correlated and that all diversity branches had equal average SNR. The equal average SNR assumption is popular because performance results from using this assumption can be easily compared with performance estimates made using closed-form expressions for diversity combining in [16] for the case of i.i.d. fading of MPGs. However this case is only a simple approximation for more complex real-world channel characteristics.

In practice it is seldom true that received MPGs have equal average SNRs. In this section therefore the SAT is used to study the effect of having MPGs with unequal average SNR on the BER performance of a Rake receiver. Also, since it was shown in the previous sections that upper and lower bounds on BER estimated using the SAT with $N_u = 50$ are close to each other and are a good approximation to the true BER, only upper bounds on BER with this value of N_u will be presented in what follows. It is considered that these upper bounds present good approximations to the true BER.

In the first simulation using fading distributions generated in MATLAB, the BER is estimated for a Rake receiver that processes either one or both of two received MPGs having unequal average powers. Bit error rate performance estimates made

using time-series of ESRs from measured data are then compared with performance estimates made using a channel model in which received MPGs have equal average SNR and Rayleigh uncorrelated fading statistics. Performance estimation errors that result from using idealised channel models are noted. For instance it will be shown that the use of an idealised channel model in which received MPGs are assumed to have equal average power results in pessimistic estimates of SI relative to a more realistic case. It should also be noted that for the purpose of comparison, when measured data were used to estimate Rake receiver BER performance, it was required that the pdf of at least the strongest MPG in the time-series could be modelled using either a Rayleigh or a Ricean pdf. This was done for purposes of comparison with a Rayleigh i.i.d. model that is often used in the literature. As a result of this requirement the number of consecutive ESRs used was generally smaller than the total number of ESRs in a measurement run. It should however be emphasised that the SAT can be used to estimate Rake receiver BER performance over the entire time-series regardless of its length or pdfs of the individual MPGs. Note also the measured data analysed in this thesis were chosen to demonstrate the capabilities of the SAT. There was no attempt to make a generalised characterisation of the data recorded over the entire microcellular region.

A set of two data arrays, having a difference in average power Δ , a correlation coefficient $\rho_c \approx 0$, and Rayleigh fading statistics, was generated to represent the envelopes of the received MPGs. The Rake receiver structure that was considered had $L = 1$ or 2 fingers and the spreading factor considered was $N_d = 7$. This is considered to be a low value which results in some of the severest SI.

Figure 5.25(a) shows BER for different values of Δ when only the strongest MPG (1) is processed (GSC) or both received MPGs (2) are combined (MRC). The cases in which the MPGs are combined give the best performance particularly when they have equal average power. This situation maximises the diversity gain. The worst

(a) Rake receiver BER estimates for $N_d = 7$, when one (1) or both (2) received MPGs having an average difference in power of $\Delta = 0$ dB, 6 dB or 20 dB, are used. Both MPGs have Rayleigh fading statistics.

(b) Rake receiver BER estimates when one or both received MPGs with average difference in power Δ are used. The total average received E_b/N_0 is 20 dB. Both MPGs have Rayleigh fading statistics.

Figure 5.25: Rake receiver BER estimates when one or both received MPGs with average difference in power Δ are used.

result is obtained when only one of two received MPGs, having equal average power, is processed. This is because the SNR of the stronger MPG is at a minimum when MPGs have equal average power and the SI is at its maximum value. Clearly, there is an improvement in performance for $N_d = 7$ in the scenario when both received MPGs with $\Delta = 20$ dB are combined over the scenario when only the stronger of the two received MPGs is processed. Later in this section, however, it will be shown that for a real-world channel, MPGs with a difference in power of 20 dB from the stronger peak do not have a significant impact on the BER. This will be revisited in greater detail later in this section when the BER is estimated using measured data.

Figure 5.25(b) shows the BER as a function of Δ for a fixed spreading factor, $N_d = 7$ and average $E_b/N_0 = 20$ dB. As before when Δ increases BER improves for $L = 1$ because of the increasing strength of the MPG that is used in the Rake. The SI from the second MPG is also reduced. For $L = 2$, however, BER degrades as Δ increases because of the reduction in diversity gain with increasing Δ .

The SAT is next used to compare Rake receiver BER performance estimates made using an equal average MPG power assumption with performance estimates made using measured data. One LOS and one NLOS case are considered.

Bit error rate performance for an NLOS section of street

Rake receiver BER performance along an NLOS section of street is first estimated. The measurement interval over which ESRs were estimated was 0.84 s long. This was the longest time interval over which the strongest MPG could be reasonably modelled using a Rayleigh pdf. From the average power delay profile shown in Figure 5.26 it can be seen that there were five MPGs having relative average powers greater than -20 dB compared with the strongest MPG. It should be made clear that these MPG amplitudes are for fixed time-delays. Clearly the average powers of MPGs relative to the strongest MPG decrease with increasing time-delay. Table 5.2 lists the relative powers of the five MPGs and the pdf, either Rayleigh or Ricean, that provides the

MPG	Average relative power level (Δ dB)	Fading model	α_{KS}
1	-12.3	Rice ($K_{Rice}=1.7$ dB)	0.62
2	0	Rayleigh	0.84
3	-7.9	Rayleigh	0.09
4	-15.1	Rayleigh	0.40
5	-19.1	Rayleigh	0.003

Table 5.2: Average relative power levels, fading models and α_{KS} for the time-series of absolute magnitudes of five received MPGs measured along an NLOS section of street.

best fit to their fading distributions.

Figure 5.26: The average power delay profile corresponding to a set of ESRs recorded along an NLOS section of street.

Figure 5.27(a) shows BER performance results obtained using the measured data (MD) that forms the basis for Table 5.2 for different numbers of Rake fingers, $L = 1, 2, 3, 4$ and 5. In each case the L MPGs having the highest average SNRs are combined. Figure 5.27(b) shows BER performance results obtained using the same set of measured data but neglecting the effects of SI. The results in Figure 5.27(a) indicate

(a) BER estimates using data recorded along an NLOS section of street (MD) and using simulated data (EP) in which all MPGs had equal average power. The spreading factor is $N_d = 7$. Performance estimates are shown for different numbers of combined MPGs.

(b) BER estimates neglecting the effects of SI, using data recorded along an NLOS section of street (MD) compared with BER estimates using simulated data in which MPGs have equal average power (EP).

Figure 5.27: Rake receiver BER performance estimates using data recorded along an NLOS section of street.

a significant improvement in performance between the $L = 1$ case and the $L = 2$ case where $\Delta = 8$ dB. Combining additional MPGs does not significantly improve performance. This is the result of both diminishing returns with an increase in the order of diversity, first discussed in Section 4.1.2, and the decrease in average SNR of additional MPGs.

An alternate combining strategy would be to combine the L MPGs that have the strongest instantaneous power. It is expected that over the time interval for the data under consideration, the average BER using this instantaneous combining strategy would not be much different from the average BER when MPGs having the strongest average power are combined. This is because it was observed from the measured data that MPGs in real-world channels have unequal average powers that decrease almost exponentially with increasing time-delay from the strongest MPG. For this reason the weaker of two MPGs on average will seldom have an instantaneous power that is greater than the stronger MPG. As a result of not using this combining strategy, there exists a continuous fading distribution over the time-series interval to allow a fair comparison with a channel model that has the same number of MPGs with equal average power, each of which is Rayleigh distributed.

Also shown in Figure 5.27(a) is the BER performance of a Rake receiver that combines all five received MPGs when they are modelled as having equal average power (EP). Five data arrays, each representing an MPG with Rayleigh fading statistics, were generated using MATLAB. This is one model that could be used by system designers in the absence of measured data. The BER performance under these conditions is very close to (actually slightly better than) the BER performance obtained using measured data for $E_b/N_0 < 10$ dB ($P_e > 10^{-3}$), both predicted using the SAT. The reason for the slight improvement can be explained with the use of Figure

5.28(a). This figure shows the total combined MPG power $|h(n)|^2 = \frac{\sum_{i=1}^L |h'_i(n)|^2}{\frac{1}{M} \sum_{m=1}^M \sum_{i=1}^K |h'_i(m)|^2}$,

where $h'_i(n)$ is the complex amplitude of the i th MPG at time instant n , for the set of measured data (MD-NLOS) and for the equal average power (EP) data set. The total signal power, $|h(n)|^2$ for the MD-NLOS case spends more time below the average value of 1. As a result of the reduced severity of fading of the combined signal in the EP case, in the absence of SI it is expected that the BER in the EP case would be better than the BER in the MD-NLOS case. Figure 5.27(b) shows the BER for the two cases in the absence of SI. Performance results confirm expectations. It is also interesting to note the increased rapidity of fluctuations of $|h(n)|^2$ in the EP case compared with the MD-NLOS case. This is because in the measured data, adjacent samples of the total signal energy are correlated.

Referring to Figure 5.27(a), the effects of SI result in an error floor at $P_e \approx 2 \times 10^{-4}$ for the EP case. This is because the interference in the case where all MPGs have equal average power is greater than in the MD-NLOS case. A simple explanation for this is the fact that when the sum of two numbers $|h_1|$ and $|h_2|$ is a constant, their product, $|h_1||h_2|$, the determining factor in SI terms, is maximised when they are equal. Figure 5.28(b) shows the variance of the upper bound on the interference σ_u^2 for both the measured data and the simulated data. Clearly the average value of σ_u^2 in the MD-NLOS case is lower than the average value of σ_u^2 in the EP case. The SI is therefore greater in the EP case than it is in the MD-NLOS case.

It should be noted that the correlation coefficients (5.4.9) estimated from the measured data were found to vary between -0.17 and +0.15. In other words they were close to zero. The correlation coefficients estimated from simulated data were almost equal to zero. Based on results presented in Section 5.4.4 it can therefore be stated that there was no bias resulting from the correlation among MPGs between the measured data and the simulated data.

Bit error rate performance for an LOS section of street

Rake receiver BER performance for an LOS section of street is presented next.

(a) Total combined signal power for measured data (MD-NLOS) compared with total combined signal power from simulated data (EP).

(b) Variance of the SI at each time instant n when measured data (MD-NLOS) is used and variance of SI when simulated data for MPGs (EP) is used.

Figure 5.28: Total combined signal power and variance of the interference for both measured data recorded along an NLOS section of street and simulated data in which MPGs have equal average power.

MPG	Relative power level (dB)	Fading model	α_{KS}
1	-18.1	Rice ($K_{Rice}=6.5$ dB)	0.10
2	0	Rice ($K_{Rice}=16.2$ dB)	0.98
3	-10.6	Rayleigh	0.05

Table 5.3: Average relative power levels, fading models and associated α_{KS} for the time-series of envelopes of three received MPGs measured along an LOS section of street.

Figure 5.29 shows the APDP for a measurement interval that lasted 1 second. There were three MPGs with powers that exceeded a threshold of 20 dB below the strongest MPG. The fading statistics of these MPGs are presented in Table 5.3 and the correlation coefficients between pairs of the identified MPGs are presented in Table 5.4.

Figure 5.29: The average power delay profile corresponding to a set of ESRs recorded along an LOS section of street.

Figure 5.30(a) shows performance results estimated from the use of measured data (MD) for different numbers of Rake fingers, $L = 1, 2$ and 3. Each analysis was conducted for the case where L MPGs with the highest average powers are combined. The results indicate a significant improvement in performance when two Rake fingers

(a) Rake receiver BER performance estimates using data recorded along an LOS section of street (MD) and using simulated data (EP) in which all MPGs had equal average power. The spreading factor is $N_d = 7$. Performance estimates are shown for different numbers of combined MPGs.

(b) Rake receiver BER estimates, neglecting the effects of SI, using data recorded along an LOS section of street (MD-LOS) compared with BER estimates using simulated data in which MPGs have equal average power (EP).

Figure 5.30: Rake receiver BER performance estimates using data recorded along an LOS section of street.

MPG	1	2	3
1	1	-0.42	0.00
2	-0.42	1	0.22
3	0.00	0.22	1

Table 5.4: Coefficients of correlation for fading of three received MPGs measured along an LOS section of street.

are processed instead of one, which in this case differ in average power by about 11 dB. The reduction in BER is a result of diversity gain from combining the second MPG that has a coefficient of correlation $\rho_c = -0.42$ with the first MPG. Combining additional MPGs did not significantly affect performance. The impact of correlated MPGs on BER performance was discussed in Section 5.4.4.

Also shown in Figure 5.30(a) is the performance of a Rake receiver that combines all three received MPGs from an equal average power (EP) model. Three arrays, each representing an MPG envelope with Rayleigh pdf, were simulated using MATLAB. The Rayleigh pdf was selected as it is often used in the literature to model the fading of MPGs. Performance estimates made using simulated data are pessimistic compared with performance estimates made using measured data for MRC. This overall performance improvement in the measured data scenario can be attributed to the fact that the strongest MPG has Ricean fading statistics. This was also observed by Bultitude and Leslie for DBPSK on LOS channels in [51] with no diversity.

Figure 5.31(a) shows the variance of the upper bound on the interference σ_u^2 for both the measured data under LOS conditions (MD-LOS) and the simulated data for MPGs each having equal average power (EP). Clearly the average value of σ_u^2 in the MD-LOS case is smaller than the average value of σ_u^2 in the EP case. As a result the total SI is greater for the EP case.

Figure 5.31(b) shows the total combined MPG power $|h(n)|^2$ for the set of measured data (MD-LOS) and for the simulated data set with each MPG having equal average power (EP). The total signal energy for the MD-LOS case fluctuates over

(a) Variance of the SI at each time instant n using measured (MD-LOS) and simulated data (EP).

(b) Total combined signal power computed from measured data (MD-LOS) compared with total combined signal power computed from simulated data (EP).

Figure 5.31: Total combined signal power and variance of the interference for both measured data recorded along an LOS section of street and simulated data in which MPGs have equal average power.

a smaller range than the EP case as a result of the strong Ricean MPG present in the former. As a result, in the absence of interference, the BER performance of a Rake receiver in the MD-LOS conditions would be better than the BER performance of a Rake receiver in the EP case. Figure 5.30(b) shows the BER for the two cases MD-LOS and EP in the absence of SI and confirms expectations.

The results presented in this section demonstrate the inaccuracies in BER performance prediction that can occur when using the equal average power assumption for Rayleigh fading MPGs that are i.i.d. For instance in the LOS case in which the fading of the strongest MPG was Ricean, performance was significantly better than performance under an equal average power assumption for MPGs with Rayleigh fading distributions. So if the fading was assumed to be Rayleigh and it was actually Ricean, the required SNR would be lower than that designed to achieve a desired BER.

In general, a channel model that uses an equal average power assumption for received MPGs leads to pessimistic estimates of SI when compared with a channel model that has the same number of MPGs with average powers that decrease with increasing time-delay relative to the strongest MPG. Additionally, based on BER results in Figure 5.27(a), it can be said that the equal average power assumption for a fixed number of received MPGs, each of which is i.i.d. and Rayleigh distributed, does not provide a reliable upper bound on expected BER performance for a Rake receiver using MRC with the same number of received MPGs in a real-world propagation environment.

Finally, it was observed in both the LOS and NLOS cases that as a result of diminishing returns with an increase in order of diversity, an MPG with power level lower than about -20 dB relative to the strongest MPG, does not affect BER performance. It is also expected that provided the channel is estimated with no error, when there are additional MPGs having relative power levels between 0 and -20 dB relative

MPG	Relative power level (dB)
1	0
2	-1
3	-9
4	-10
5	-15
6	-20

Table 5.5: Average relative power levels for the ITU-R channel model.

to the strongest MPG, combining an MPG that has a power level of -20 dB or less relative to the strongest MPG will not degrade performance. This is because in Rake combining schemes such as GSC and MRC, the weakest MPG has an insignificant combining weight or scaling factor relative to the stronger MPGs. When processing ESRs therefore for work reported herein, a -20 dB threshold power level relative to the strongest MPG was applied to identify valid MPGs.

5.4.7 Estimation of Rake receiver BER performance using the ITU-R vehicular A channel model

In [52], a channel impulse response model has been proposed by the ITU-R for terrestrial links. This model (known as the ITU-R vehicular A channel model but referred to herein as the ITU-R model for simplicity) was developed to provide a better channel model than one that uses the equal average power assumption for received MPGs. In this section BER estimates made using this model are compared with BER estimates made using an average MPG power model derived from measurements in downtown Ottawa.

The ITU-R model with MPGs having the relative average powers listed in Table 5.5 is prescribed for radio channels with small rms-delay spreads.

In order to compare Rake receiver BER performance results obtained using the

MPG	Relative power level (dB)
1	0
2	-5.2
3	-8.3
4	-10.8
5	-13.4
6	-15.6
7	-17.8
8	-19.8

Table 5.6: Average relative power levels for the strongest eight MPGs from the entire set of ESRs recorded in downtown Ottawa.

ITU-R channel model with expected BER performance results obtained using measured data, an APDP was computed for each time-series of ESR data from downtown Ottawa. Received MPGs that had a relative average power level of -20 dB or greater were identified for each time-series and ordered in decreasing order of their magnitude. This ordering was done because it is expected that a Rake receiver with $L \leq K$ fingers would typically use a strategy that combines L MPGs having the strongest average powers. An average was then computed for all MPGs as it is expected that the resulting average power levels would be observed at each Rake finger over the entire microcellular region. Relative power levels of the strongest eight MPGs are listed in Table 5.6. Comparing Tables 5.5 and 5.6 it can be seen that there are eight MPGs with average power relative to the strongest MPG greater than or equal to -20 dB in the measured data case and six in the ITU-R model.

Fading in the ITU-R model of each MPG was assumed to have a Rayleigh pdf. From measured data it was observed that this is not always the case. However for purposes of comparison, eight sets of uncorrelated Rayleigh random variables were generated using MATLAB with the average powers in Table 5.6. Even if all MPGs are Rayleigh it will be shown that the BER estimates for the two models will differ significantly.

Figure 5.32 shows upper bounds on Rake receiver BER performance results based on both the ITU-R model and the APDP for $L = 1$ and $L = 2$ Rake fingers.

Figure 5.32: Upper bounds on Rake receiver BER estimates using the ITU-R vehicular A channel model (ITU-R) and the average powers of the eight strongest MPGs (APDP) for $L = 1$ and $L = 2$ received MPGs.

Clearly, there is significant discrepancy between the BER performance results predicted using the ITU-R model and that predicted using the APDP for the GSC cases with $L = 1$ and $L = 2$. For the $L = 1$ case, use of the ITU-R model results in pessimistic BER performance estimates compared with BER performance estimates using the APDP model. This is because in the ITU-R model the second MPG has a greater power level than in the APDP model. As a result there will be greater signal power and a smaller amount of SI in the APDP case compared with the ITU-R case. Similarly for the $L = 2$ case, combining the strongest two MPGs that differ in power by 1 dB results in optimistic BER performance predictions when the ITU-R model is used compared with BER performance predictions using the APDP model for $L = 2$ in which the strongest two MPGs differ in power by about -5 dB. In the ITU-R case,

the $L = 2$ scenario results in greater signal energy (and smaller SI) compared with the APDP case.

While the BER results for the two models for $L = 1$ and $L = 2$ differ, it should be noted that one way to guarantee accurate BER estimates would be to use measured data that is representative of the propagation environment of interest.

5.5 Summary and discussion

In this chapter the theoretical framework has been developed for estimating Rake receiver BER performance in a multipath fading environment using measured data. This was accomplished by correcting, modifying and extending Lehnert and Pursley's technique [2] for estimating BER performance of a correlation receiver in a multi-user, non-fading environment in which there is no multipath propagation. The ability to use measured data for Rake receiver BER performance prediction was one of the objectives of this thesis. This capability is provided by the SAT which involves a computationally efficient technique for determining the distribution for the SI. These distributions can then be used to determine the BER.

The ability of the SAT to provide useful bounds on Rake receiver BER was validated using simulation. During the process of validation it was recognised that an accurate estimation of SI by simulation required careful consideration and modelling of non-chip-spaced MPG time-delays. The process of validation also served to highlight the reduction in computation time and resources that was achieved using the SAT compared with a simulation-based technique. For example, the use of the SAT to predict BER performance of a correlation receiver for a spreading factor of 7 and two received MPGs resulted in a 95% reduction in computation time when compared with simulation. A simulation to accurately determine Rake receiver BER performance in real-world channels would take an inordinate amount of time.

Once the SAT was validated it was used to study the impact of simplifying assumptions about the channel, often used in the literature on Rake receiver BER performance.

A common assumption is that received MPGs have equal average power. This assumption is often used for mathematical tractability in deriving closed-form expressions for the probability of bit error in the literature. It is also a useful model when nothing is known about the channel. With the use of measured data from downtown Ottawa however, it was observed that the powers of MPGs always decrease with an increase in relative time-delay from the strongest MPG. This is typical of an urban microcellular environment. The SAT was used to predict BER performance for both the idealised case and the real-world situation. It was demonstrated an equal average power model predicts a larger amount of SI than exists in real-world situations. As a result, BER estimates using the equal average power model would be worse than BER estimates using measured data.

It is often assumed that received MPGs are separated in time by integer multiples of the chip period. It was shown in Chapter 4 that this assumption validates the use of the Gaussian model for SI. It is well-known however, that received MPGs are not chip-spaced and the use of the SAT to predict Rake receiver BER in this situation results in a more optimistic estimate of BER performance than the situation where MPGs are chip-spaced.

System design based on either or both of the above-mentioned idealised models for the channel would result in overestimating the SNR required to achieve a desired BER. The resulting requirement for an increase in transmit power would reduce the efficiency of utilisation of scarce spectrum resources. The errors in BER performance prediction that result from using idealised channel models highlight the need for improved channel models based on measured data that reflect real-world propagation conditions.

A step in that direction is the use of the ITU-R vehicular A channel model in which MPGs that are Rayleigh distributed and independent have powers that decrease with increasing time-delays from the strongest MPG. It is believed that this model was extracted from a large database of measurements in typical microcellular environments. Its usefulness lies in the fact that it provides system designers a common reference model to base system performance and design on. It is however obvious that this model does not exemplify any one particular microcellular scenario. Also, while this model cannot be used in conjunction with analytical expressions for BER it can be used with the SAT to predict Rake receiver BER performance. The use of the Rayleigh fading model is however inconsistent with findings based on the use of measured data from downtown Ottawa. From measured data it was observed that fading of MPGs could not always be modelled as Rayleigh. A Ricean model was at times a better fit to a MPG, particularly in LOS scenarios. In this case, BER performance was significantly better than that predicted using a Rayleigh model. Additionally using measured data it was observed that an independent fading assumption was not always correct. The fading of MPGs was at times correlated; a coefficient of correlation as high as $\rho_c = 0.6$ was observed in one instance. An example BER estimation using the SAT revealed that for MRC with two received MPGs having equal average powers, $\rho_c = 0.6$ can result in the requirement of 4 dB more power to achieve a BER of 10^{-3} compared with the $\rho_c = -0.6$ case.

While measured data provides an accurate picture of the channel at the time the measurements were made, an ESR does not represent a general channel model for the section of street along which the measurement was made. In order that a channel model for any section of street be extracted from measured data (along that street) a statistically significant number of measurements would have to be conducted prior to characterisation. For an urban microcell, channel characterisation would require a statistically significant number of measurements along the different NLOS and LOS

sections of street. Further generalisation to urban microcellular environments would require even more measurements in different cities.

The work in this thesis only utilised measured data that could be characterised using Rayleigh or Ricean pdfs (for purposes of comparison with commonly-used assumptions in the literature). It should however be noted that the SAT can also be used to process time-series data that cannot be modelled using standard distribution functions. Current BER prediction techniques do not consider these scenarios which (as was observed from measured data) occur very often. The only other alternative to performance predictions in such scenarios is simulation-based techniques which are computationally prohibitive. The SAT therefore provides a needed alternative.

An interesting finding reported in this chapter is the applicability of a Gaussian model with a reduced variance compared with the variance that approximates the variance of a binomial distribution, to accurately estimate BER for a correlation receiver. This approximation is useful as it significantly reduces the amount of computation required to obtain BER performance estimates for a correlation receiver in a fading environment.

Finally, based on the use of data measured in downtown Ottawa to estimate BER in Sections 5.4.6 and 5.4.4, it can be stated that the relative improvement in Rake receiver BER performance on the downlink is greatest when two Rake fingers are processed instead of only one. There is a smaller relative improvement in BER performance when three MPGs are processed instead of two. Based on the BER results shown in Figures 5.27(a), 5.30(a) and 5.33, which shows Rake receiver BER performance for the data that forms the basis for Table 5.6, a recommendation for Rake receiver design is the provision for at least three Rake fingers in a receiver. It should be noted that this is under the assumption that channel estimation is error-free. In Chapter 6 it will be shown that the number of Rake fingers used could be less than three depending on the impact of channel estimation errors on performance.

Figure 5.33: Upper bounds on Rake receiver BER estimates for a spreading factor $N_d = 7$ using simulated data for MPGs having average SNRs listed in Table 5.6, for $L = 1, 2, 3, 4$ and 6.

Chapter 6

The effect of channel estimation errors on Rake receiver performance

In this chapter the SAT developed in Chapter 5 is extended to enable BER performance prediction for a Rake receiver under conditions where MPG amplitudes are incorrectly estimated. The influences of different relative power levels between the pilot and data signal, relative powers of received MPGs, pilot and data spreading factors and diversity combining technique on Rake receiver BER performance are investigated. It will be shown that the use of MRC does not always result in a reduction in BER when compared with GSC.

6.1 Rake receiver performance

In Section 1.1 it was shown that the complex equivalent low pass representation for the signal received on a multipath channel can be written as

$$r(t) = \sum_{k=1}^K |h_k| e^{j\phi_k} s(t - \tau_k) + n(t), \quad (6.1.1)$$

where $h_k = h_k(n)$, and the time index n has been omitted for clarity. In order to estimate the complex MPG amplitudes $h_k, k = 1, \dots, K$, pilot data spread with a

sequence of known pilot chips can be transmitted in parallel with the data. The transmitted pilot data signal can be written as $b^p(t) = \sum_{q=-\infty}^{\infty} b_q^p p_{T_p}(t - qT_p)$ where $\{b_q^p\}$ is a binary sequence of elements from the set $\{-1, 1\}$; $p_{T_p}(t) = 1$ for $0 \leq t \leq T_p$, $p_{T_p}(t) = 0$ otherwise; T_p represents the period of the pilot symbol. The pilot spreading signal is $a^p(t) = \sum_{q=-\infty}^{\infty} a_q^p \psi(t - qT_c)$, where $\{a_q^p\}$ is a binary sequence of elements from the set $\{-1, 1\}$. The chip waveform $\psi(t)$ is the rectangular pulse function $p_{T_c}(t)$, T_c is the chip period. The integer $N_p = T_p/T_c$ is the pilot spreading factor.

It was also shown in Chapter 1 that the transmitted spread spectrum signal for a single user with BPSK modulation, can be written as $s'(t) \cos \omega_c t$, where

$$s'(t) = \sqrt{\frac{2E_b}{T_d}} b(t) a(t) + \sqrt{\frac{2E_b}{T_p}} b^p(t) a^p(t). \quad (6.1.2)$$

The received signal consisting of K MPGs, written as

$$r(t) = \sum_{k=1}^K |h_k| e^{j\phi_k} s'(t - \tau_k) + n(t), \quad (6.1.3)$$

can be cross-correlated with the known pilot data and chip sequences to give

$$c(t) = r(t) * b^p(t) a^p(t). \quad (6.1.4)$$

The result of this cross-correlation at lag τ_1 can be written as

$$c(t, \tau_1) = c_1(t) = \int_{\tau_1}^{T_p + \tau_1} r(t) b^p(t - \tau_1) a^p(t - \tau_1) dt. \quad (6.1.5)$$

If $c_1(t)$ exceeds a pre-determined threshold, it can be considered that there exists a valid MPG having time-delay τ_1 . Equation (6.1.5) can be easily expanded to

$$c_1(t) = \sqrt{2E_b T_p} |h_1| e^{j\phi_1} + \sqrt{\frac{2E_b}{T_p}} \left(\sum_{k=2}^K |h_k| e^{j\phi_k} V_{k,1} + \sqrt{\frac{T_p}{T_d}} \sum_{k=2}^K |h_k| e^{j\phi_k} Y_{k,1} \right) + \eta_p, \quad (6.1.6)$$

where η_p is the result of thermal noise input to the correlation receiver and ϕ_k is the phase of the k th MPG. The first and second summations represent SI from the pilot and data, respectively. In the first summation, $V_{k,1}$ represents the result of cross-correlation between the pilot sequence having time-delay τ_k and reference pilot sequence having time-delay τ_1 or

$$V_{k,1} = \int_{\tau_1}^{T_p+\tau_1} b^p(t-\tau_1) a^p(t-\tau_1) b^p(t-\tau_k) a^p(t-\tau_k) dt. \quad (6.1.7)$$

In the second summation, $Y_{k,1}$ represents the result of cross-correlation between the data sequence having time-delay τ_k and reference pilot sequence having time-delay τ_1 or

$$Y_{k,1} = \int_{\tau_1}^{T_p+\tau_1} b^p(t-\tau_1) a^p(t-\tau_1) b(t-\tau_k) a(t-\tau_k) dt. \quad (6.1.8)$$

It should be noted that in (6.1.6) it has been assumed that the pilot and data spreading sequences have been designed in a way that they are perfectly orthogonal when the relative time-delay is zero or

$$\int_{\tau_i}^{T_p+\tau_i} a^p(t-\tau_i) a(t-\tau_i) dt = 0. \quad (6.1.9)$$

This design requirement yields a significant reduction in the effects of SI due to the strongest MPG ($k = 1$) from the pilot to the data and vice versa. Normalising (6.1.6) by $\sqrt{2E_b T_p}$ gives the estimated channel coefficient $\hat{h}_1$ as

$$\hat{h}_1 = h_1 + e_1, \quad (6.1.10)$$

where

$$e_1 = \frac{1}{N_p T_c} \left(\sum_{k=2}^K |h_k| e^{j\phi_k} V_{k,1} + \sqrt{\frac{N_p}{N_d}} \sum_{k=2}^K |h_k| e^{j\phi_k} Y_{k,1} \right) + \frac{\eta_p}{\sqrt{2E_b N_p T_c}}. \quad (6.1.11)$$

In the case of a Rake receiver with L despreading fingers, the decision variable has the same form as (5.3.1) and can be written as

$$D_{\text{Rake}} = \sum_{i=1}^L \int_{\tau_i}^{T_d + \tau_i} \text{real} \left(r(t) \hat{h}_i^* a(t - \tau_i) \right) dt, \quad (6.1.12)$$

where the i th Rake finger multiplies the MPG having delay τ_i by the complex conjugate of the estimated MPG amplitude, $\hat{h}_i^*$, and despreads the sequence at the same delay τ_i . The decision variable normalised by $T_c \sqrt{\frac{E_b}{2T_d}}$ has the same form as (5.3.9) and can be written as

$$\begin{aligned} D_{\text{Rake, norm}} = & N_d \sum_{i=1}^L \text{real}(\hat{h}_i \hat{h}_i^*) + \sum_{i=1}^L \sum_{k=1, k \neq i}^K \hat{h}_{k,i} \frac{W_{k,i} \cos \theta_{k,i}}{T_c} + \\ & \sqrt{\frac{T_d}{T_p}} \sum_{i=1}^L \sum_{k=1, k \neq i}^K \hat{h}_{k,i} \frac{X_{k,i} \cos \theta_{k,i}}{T_c} + \frac{1}{T_c} \sqrt{\frac{2T_d}{E_b}} \eta_d \sum_{i=1}^L \hat{h}_i^*, \end{aligned} \quad (6.1.13)$$

where the first summation represents the desired signal component. The cross-correlation between the interfering binary data sequence k and the data reference sequence i is denoted by $W_{k,i} = \int_{\tau_i}^{T_d + \tau_i} b(t - \tau_k) a(t - \tau_k) a(t - \tau_i) dt$. The cross-correlation between interfering binary pilot sequence k , and the data reference sequence i is denoted by $X_{k,i} = \int_{\tau_i}^{T_d + \tau_i} b^p(t - \tau_k) a^p(t - \tau_k) a(t - \tau_i) dt$. The phase difference between the i th estimated Rake finger coefficient and the k th MPG is $\theta_{k,i}$ and $\hat{h}_{k,i} = |\hat{h}_k| |\hat{h}_i^*|$. Interference terms $X_{k,i}$ and $W_{k,i}$ can be calculated using the SAT described in Chapter 5. The thermal noise η_d is zero-mean Gaussian with variance $N_0 T_d / 2$. As a result the last summation in (6.1.13) is Gaussian with variance $\frac{N_0 N_d^2}{E_b} \sum_{i=1}^L |\hat{h}_i|^2$. The probability of bit error can be estimated as before using (5.3.11) and (5.3.13).

In general, the energy per bit for the pilot symbol $E_{b,pilot}$ does not have to be the same as the energy per bit for the data symbol $E_{b,data}$. The total energy per bit E_b can be constrained as

$$E_b = E_{b,data} + E_{b,pilot}, \quad (6.1.14)$$

The ratio $E_{b,pilot}/E_{b,data}$ is denoted by ρ . Also $E_{b,data} = A_d T_d$ and $E_{b,pilot} = A_p T_p$, where A_d and A_p are the powers of the transmitted data and pilot signals, respectively. The transmitted spread spectrum signal can then be written as $s''(t) \cos \omega_c t$, where

$$s''(t) = \left(\sqrt{\frac{2E_{b,data}}{T_d}} b(t) a(t) + \sqrt{\frac{2E_{b,pilot}}{T_p}} b^p(t) a^p(t) \right). \quad (6.1.15)$$

The result of the cross-correlation at lag τ_1 can be written as

$$\begin{aligned} c_1(t) = & \sqrt{2E_{b,pilot}T_p} |h_1| e^{j\phi_1} + \sqrt{\frac{2E_{b,pilot}}{T_p}} \sum_{k=2}^K |h_k| e^{j\phi_k} V_{k,1} + \\ & \sqrt{\frac{2E_{b,pilot}}{T_p}} \left(\sqrt{\frac{T_p E_{b,data}}{T_d E_{b,pilot}}} \sum_{k=2}^K |h_k| e^{j\phi_k} Y_{k,1} \right) + \eta_p \end{aligned} \quad (6.1.16)$$

Normalising (6.1.16) by $\sqrt{2E_{b,pilot}T_p}$ gives the estimated channel coefficient $\hat{h}_1$ as

$$\hat{h}_1 = h_1 + e_1, \quad (6.1.17)$$

where

$$e_1 = \frac{1}{N_p T_c} \left(\sum_{k=2}^K |h_k| e^{j\phi_k} V_{k,1} + \sqrt{\frac{N_p E_{b,data}}{N_d E_{b,pilot}}} \sum_{k=2}^K |h_k| e^{j\phi_k} Y_{k,1} \right) + \frac{\eta_p}{\sqrt{2E_{b,pilot}N_p T_c}}. \quad (6.1.18)$$

The thermal noise term η_p is zero-mean Gaussian with variance $N_0 T_p$. As a result the last term in (6.1.18) is zero-mean with variance $(1 + \rho) N_0 / 2\rho E_b$. Also, based on the discussion presented in Chapter 4, the variance of the cross-correlation between any

two sequences having length N_p can be upper-bounded as Gaussian with variance N_p . Independent Gaussian distributed arrays with this variance can be generated in MATLAB to simulate the error.

The decision variable for the Rake receiver normalised by $T_c \sqrt{\frac{E_{b,data}}{2T_d}}$ has the same form as (6.1.13) and can be written as

$$D_{Rake, norm} = N_d \sum_{i=1}^L \text{real}(h_i \hat{h}_i^*) + \sum_{i=1}^L \sum_{k=1, k \neq i}^K \hat{h}_{k,i} \frac{W_{k,i} \cos \theta_{k,i}}{T_c} + \sqrt{\frac{\rho T_d}{T_p}} \sum_{i=1}^L \sum_{k=1, k \neq i}^K \hat{h}_{k,i} \frac{X_{k,i} \cos \theta_{k,i}}{T_c} + \frac{1}{T_c} \sqrt{\frac{2T_d}{E_{b,data}}} \eta_d \sum_{i=1}^L \hat{h}_i^* \quad (6.1.19)$$

The probability of bit error can once again be expressed in terms of the Gaussian-Q function

$$P_e(\beta) = \frac{1}{\sqrt{2\pi}} \int_{\frac{N_d \sum_{i=1}^L \text{real}(h_i \hat{h}_i^*) + \beta}{\sigma_{Rake}}}^{\infty} e^{-w^2/2} dw = Q \left(\frac{N_d \sum_{i=1}^L \text{real}(h_i \hat{h}_i^*) + \beta}{\sigma_{Rake}} \right) \quad (6.1.20)$$

where the variance of the noise term, σ_{Rake}^2 , in this case is $\frac{N_0 N_d^2}{E_{b,data}} \sum_{i=1}^L |\hat{h}_i|^2$, and β is the interference. The pdf of β is determined using the SAT and the probability of bit error can be estimated using (5.3.11) and (5.3.13).

6.2 Rake receiver BER performance estimates

In this section the effect of errors in MPG amplitude estimates on Rake receiver BER performance on a fading channel is studied using the SAT with $N_u = 50$. As explained in Section 6.1 these errors are caused by both thermal noise and SI. The factors that influence BER performance are average received SNR per bit, data and pilot spreading factors and their relative powers and the characteristics of the radio channel.

In Figures 6.1(a), 6.1(b) and 6.1(c) Rake receiver BER performance is shown for both GSC ($L=1$) and MRC ($L=2$) for a channel in cases where there are two Rayleigh fading MPGs, with differences in their average powers of $\Delta = 6$ dB, $\Delta = 8$ dB and $\Delta = 20$ dB. The pilot and data spreading factors were chosen to be 127 and 7, respectively, and the ratio of pilot to data powers, $A_p/A_d = 0.01$.

While the three above-mentioned values of Δ are chosen to illustrate the effect of average received SNR per bit on BER, for the cases in which the channel is incorrectly and correctly estimated, the effect of Δ on BER performance of a Rake with MRC and GSC will also be investigated. The pilot spreading factor of 127 was chosen as it is typically much higher than the data spreading factor which in this case was chosen to be 7. It is also expected that, in general, the power of the data signal should be greater than the power of the pilot signal. The power ratio $A_p/A_d = 0.01$ was chosen for this reason but later in this chapter it will be shown that this quantity has an insignificant impact on BER over a large range of values.

Performance results for $\Delta = 6$ dB in Figure 6.1(a) indicate that for the parameters ($A_p/A_d = 0.01$, $N_p = 127$, $N_d = 7$) under consideration, MRC gives better BER performance than GSC for all values of average SNR per bit. Also shown for comparison in the same figure are BER performance results when there are no errors in channel estimation. Clearly there is significant degradation in BER performance for both GSC and MRC for the cases in which there are channel estimation errors, compared with the cases when the channel is estimated with no errors.

Rake receiver BER performance results for $\Delta = 8$ dB in Figure 6.1(b) indicate a slight degradation in BER performance when $L = 2$ compared with the $L = 1$ case, for $E_b/N_0 < 11$ dB, when there are errors in channel estimation. This indicates that the effect of channel estimation errors on performance at low values of SNR is significant enough to make any potential benefit from combining a second MPG with a relative power level of -8 dB negligible. For larger values of average SNR per bit,

Figure 6.1: Rake receiver BER estimates when there are two received MPGs with a relative average power level of Δ , and $L = 1$ or $L = 2$ Rake fingers. The pilot to data power ratio, $A_p/A_d = 0.01$. The data and pilot spreading factors are $N_d = 7$ and $N_p = 127$, respectively.

or $E_b/N_0 > 11$ dB, MRC results in improved BER performance compared with GSC.

Finally BER performance results for $\Delta = 20$ dB in Figure 6.1(c) indicate that there is no potential benefit to be realised by using MRC when the second MPG has a very low relative power. In fact there is a degradation in performance when the weaker MPG is combined compared with the case when only the stronger MPG is used because of the errors in the estimate of the weaker MPG.

The pilot sequence cross-correlation in (6.1.18) was modelled as Gaussian with variance N_p and $0.33N_p$. It was shown in Chapter 5, for sequence length N_p , a Gaussian pdf with variance N_p resulted in an overestimation of the effects of SI. It was also shown that for sequence length N_p , a Gaussian pdf with variance $0.33N_p$ was a good approximation to the true pdf of the SI near the mean and the Gaussian pdf decayed more rapidly than the pdf of the SI in the tails. It was also demonstrated that BER estimates using this Gaussian approximation were more optimistic than BER estimates using the SAT. Figure 6.1(d) shows BER as a function of received SNR per bit for $\Delta = 20$ dB when the sequence cross-correlation is modelled as Gaussian with variance λN_p , $\lambda = 0.33$ and $\lambda = 1$. The true BER estimates for both GSC and MRC lie between the two curves for $L = 1$ and $L = 2$, respectively, and in both cases, MRC gives worse performance than GSC.

The effect of Δ , A_p/A_d and E_b/N_0 on Rake receiver BER performance

Figure 6.2(a) shows BER estimates as a function of the difference in average power, Δ , between two received MPGs for $L = 1$ and $L = 2$ Rake fingers, for average received $E_b/N_0 = 10, 20$ dB and $A_p/A_d = 0.01$. In the case when $E_b/N_0 = 10$ dB for $\Delta < 7.5$ dB, combining both received MPGs results in an improvement in BER performance when compared with the case when only one MPG is processed. For $\Delta > 7.5$ dB, combining both MPGs results in worse BER performance compared with the case when only one MPG is processed. For $E_b/N_0 = 20$ dB, the maximum value of Δ at which there are gains to be realised when MRC is used compared with

(a) BER as a function of Δ for $E_b/N_0 = 10, 20$ dB.

(b) BER as a function of Δ for $A_p/A_d = 0.1, 0.01$.

Figure 6.2: Rake BER estimates as a function of Δ , when there are two received MPGs and $L = 1, 2$.

GSC is about 11 dB. The increase of this threshold maximum value of Δ from 7.5 dB for $E_b/N_0 = 10$ dB to 11 dB for $E_b/N_0 = 20$ dB is the result of a reduction in channel estimation errors with an increase in E_b/N_0 , and an associated increase in $E_{b,pilot}$.

It should be emphasised that while the difference in performance between the $L = 1$ and $L = 2$ cases for a fixed value of E_b/N_0 is small, the important conclusion is that the $L = 2$ case does not yield a significant improvement in performance over the $L = 1$ case for either value of E_b/N_0 .

Figure 6.2(b) shows BER estimates as a function of the difference in average power, Δ , between two received MPGs for $L = 1$ and $L = 2$ Rake fingers, for average received $E_b/N_0 = 15$ dB and $A_p/A_d = 0.1, 0.01$. Clearly there is a small improvement in BER performance for $L = 1$ when A_p/A_d increases from 0.01 to 0.1 as a result of an improvement in the accuracy of the channel estimates. The same is true for $L = 2$. The maximum value of Δ for which the BER for $L = 2$ is better than the BER for $L = 1$ increases from about 9 dB to about 13 dB as A_p/A_d is increased from 0.01 to 0.1. This is the result of a reduction in the amplitude estimation error for the weaker MPG 2 with an increase in the power of the pilot signal relative to the data signal.

It is expected that these thresholds obtained for the case of two received MPGs can be generalised to the cases in which there are more than two received MPGs for urban microcells. This is based on the observation from measured data that MPG powers decrease with an increase in time-delay relative to the strongest MPG. The effect of the total SI from a number of MPGs on any one particular MPG will therefore be dominated by the effect of the strongest MPG in the interfering set.

Figure 6.3(a) shows the BER performance for both GSC and MRC, when $E_b/N_0 = 10$ dB and $\Delta = 6$ dB, for different values of the ratio A_p/A_d . The pilot and data spreading factors are 127 and 7, respectively. Clearly BER performance in the $L=2$ case is better than performance in the $L=1$ case, indicating that MRC is a better choice over the range of A_p/A_d values considered. Also these results indicate that

Figure 6.3: Rake receiver BER performance as a function of the pilot to data power ratio, A_p/A_d when there are two received MPGs with a difference in average power of Δ dB and $L = 1$ or $L = 2$ Rake fingers, for average received $E_b/N_0 = 10$ dB. The data spreading factor is $N_d = 7$ and the pilot spreading factor is $N_p = 127$.

there is an optimal value of $A_p/A_d \approx -13$ dB that gives the lowest BER. For larger values of A_p/A_d , performance degrades as a result of increased interference to the data from the pilot and a smaller data signal power, while for smaller values of A_p/A_d , performance degrades as a result of the increasing inaccuracy of MPG amplitude estimates. It should be noted however, that BER performance is not very sensitive to A_p/A_d over the range of values considered.

Figure 6.3(b) shows BER performance for GSC and MRC, when $E_b/N_0 = 10$ dB and $\Delta = 20$ dB, for different values of the ratio A_p/A_d . The pilot and data spreading factors are 127 and 7, respectively. In this case, BER performance for $L = 1$ is better than performance in the $L = 2$ case, indicating that using only the stronger of the two MPGs gives better performance.

Figure 6.3(c) shows BER performance for both GSC and MRC, when $E_b/N_0 = 10$ dB and $\Delta = 8$ dB, for different values of power ratio, A_p/A_d . The pilot and data spreading factors are 127 and 7, respectively. In this case BER performance for $L = 2$ is better than BER performance for $L = 1$, for $A_p/A_d > -17$ dB. For $A_p/A_d < -17$ dB, the pilot power is low enough to result in poorer BER performance for the $L = 2$ case than the $L = 1$ case. This is the result of the effect of both interference from the data on the weaker MPG (2) and the normalised thermal noise contribution in (6.1.18) which increases with a reduction in pilot power.

Rake receiver BER performance estimated using measured data

A time-series of ESRs recorded along an NLOS section of street was used to determine Rake receiver BER performance taking the effects of channel estimation errors into account. Table 6.1 lists the relative average power levels of the five strongest received MPGs.

Figure 6.4 shows Rake receiver BER performance for the ESR with average relative MPG powers listed in Table 6.1 for $N_p = 127$, $N_d = 7$ and $A_p/A_d = 0.01$. Performance results for both the situation where the channel is estimated with no errors (labelled

MPG	relative power (dB)
1	0
2	-0.2
3	-7.8
4	-11.1
5	-12.1

Table 6.1: Average relative power levels of valid MPGs in a time-series of ESRs recorded along an NLOS section of street.

“no error”) and the situation where there are channel estimation errors (labelled “error”) are shown for different numbers of Rake fingers, L . For the error case, clearly, there is an improvement in performance that can be achieved when two Rake fingers are processed instead of one. There is a small performance degradation for $L = 3$ compared with $L = 2$. This is consistent with the BER results in Figure 6.3(c) for $\Delta = 8$ dB. Also, there is degradation in performance for the $L = 4$ and $L = 5$ cases, with BER performance for the latter being worse than that for the former. For the no error case, as would be expected, performance improves with an increase in L from one to five. Clearly the difference in BER between the no error and error cases is significant.

The relative effects of thermal noise and interference on channel estimation errors

The effect of pilot and data spreading factors, the ratio of pilot to data power levels and thermal noise on MPG amplitude estimation errors exemplified in the example cases discussed earlier in this section can be better understood by examining the terms in (6.1.18).

In the case where there are two received MPGs with a relative difference in average power of 20 dB, the effect of SI from the stronger MPG (1) to the Rake finger that despreads the weaker MPG (2) will be greater than the effect of SI from MPG 2 to the Rake finger that despreads MPG 1. The error, e_2 , in the estimate of the amplitude of MPG 2 can be written as

Figure 6.4: Rake receiver BER performance corresponding to a time-series of ESRs with MPGs having relative average powers shown in Table 6.1, for $L = 1, 2, 3, 4$ and 5 . The data spreading factor is $N_d = 7$ and the pilot spreading factor is $N_p = 127$. The ratio of the pilot to data power is $A_p/A_d = 0.01$.

$$e_2 = \frac{1}{N_p T_c} \left(|h_1| e^{j\phi_1} V_{1,2} + \sqrt{\frac{N_p}{\rho N_d}} |h_1| e^{j\phi_1} Y_{1,2} \right) + \frac{\eta_p}{\sqrt{2E_{b,pilot} N_p T_c}}. \quad (6.2.1)$$

The variance of the error, $\sigma_{e_2}^2$, can be written as the sum of three terms, corresponding to the terms in (6.2.1),

$$\sigma_{e_2}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2, \quad (6.2.2)$$

where $\sigma_1^2 = \frac{\sigma_{|h_1|}^2}{N_p}$ is the SI from the pilot to itself, $\sigma_2^2 = \frac{\sigma_{|h_1|}^2 A_d}{N_p A_p}$ is the interference from the data to the pilot and $\sigma_3^2 = \frac{N_0(1+\rho)}{2E_b \rho}$ is the normalised thermal noise contribution. Recall that $\rho = \frac{E_{b,pilot}}{E_{b,data}} = \frac{A_p N_p}{A_d N_d}$.

Figure 6.5 shows σ_1^2 , σ_2^2 and σ_3^2 for $N_p = 127$, $N_d = 7$ as a function of E_b/N_0 , for $A_p/A_d = 0.1$ and $A_p/A_d = 0.01$. The variance of $|h_1|$ was estimated from simulated

Figure 6.5: The effect of the pilot to data power ratio, A_p/A_d , on the variance of the SI and the thermal noise terms in (6.2.2). The data spreading factor is $N_d = 7$ and the pilot spreading factor is $N_p = 127$.

data and found to be $\sigma_{|h_1|}^2 = 0.21$ for $\Delta = 20$ dB. Clearly at low SNR, the errors in e_2 are mainly a result of thermal noise. At high SNR the errors are dominated by the effect of SI from the data to the pilot.

As is obvious from its definition, σ_1^2 does not vary with A_p/A_d or SNR. The variance σ_3^2 in general decreases with an increase in the received SNR and the variance σ_2^2 that represents the power of the SI caused by the data to the pilot remains constant with changes in the received SNR. Also as A_p/A_d decreases from 0.1 to 0.01, both the SI from the data to the pilot and the normalised thermal noise contribution increase. As a result there will be an increase in the variance $\sigma_{e_2}^2$.

Figure 6.6 shows σ_1^2, σ_2^2 and σ_3^2 for $A_p/A_d = 0.1$ as a function of E_b/N_0 , for $N_p = 127$ and $N_p = 63$. Clearly, as N_p decreases from 127 to 63, σ_1^2 (SI from the pilot), σ_2^2 (interference from the data to the pilot) and σ_3^2 increase. Also, once again it can be seen that channel estimation errors are dominated by σ_3^2 at low values of

Figure 6.6: The effect of the pilot spreading factor, N_p on the variance of the SI and the thermal noise terms in (6.2.2) for $A_p/A_d = 0.1$.

SNR and by σ_2^2 at high SNR values.

From Figures 6.5 and 6.6, it is clear that interference caused by the data to the pilot is significant and cannot be mitigated by an increase in SNR. For lower values of N_p , the SI due to the pilot on itself will be significant and also presents a limitation on the accuracy of channel estimation.

6.3 Summary and Discussion

In this chapter, the influence of the relative power levels between the pilot and data, diversity combining technique and pilot and data spreading factors on performance were investigated.

In Section 6.2 it was shown that an increase in pilot signal energy results in an improvement in channel estimates. An increase in data signal energy results in a reduction in the pilot signal energy or poorer channel estimates. It was therefore

expected that there would be an optimal value of the pilot to data power ratio which minimises the BER. From the results in Section 6.2 for the case in which there are two received MPGs, for $N_p = 127, N_d = 7$, there is an optimal value of $A_p/A_d \approx -13$ dB. However, BER performance is not very sensitive to A_p/A_d over a wide range of values, $-25 \text{ dB} < A_p/A_d < 0 \text{ dB}$. Clearly, fluctuations in this ratio will not affect BER performance significantly.

It was also shown that with a reduction in A_p/A_d for a fixed value of E_b/N_0 , performance for MRC becomes worse than performance for GSC due to the increase in the magnitude of the error in estimating the amplitude of the weaker MPG. As would be expected, the difference in average power, Δ , at which BER performance for MRC is worse than performance for GSC increases with an increase in A_p/A_d . Also as the received E_b/N_0 increases, the BER for both GSC and MRC decreases. Finally the relative average power threshold between two MPGs at which BER performance for MRC is worse than performance for GSC increases with an increase in E_b/N_0 .

Most importantly it was shown that the effect of channel estimation errors on the BER performance of a Rake receiver was significant. An analysis of the channel estimation error term for an MPG reveals that while the error is dominated by the effects of thermal noise at low values of SNR, the effects of SI become more significant with an increase in SNR. As a result the magnitude of these errors cannot be reduced simply by increasing the SNR or increasing the number of Rake fingers.

With the use of measured data it was shown that combining an MPG with an average power level of -8 dB relative to the strongest MPG, led to a degradation in Rake receiver BER performance when compared with the best BER performance that could be achieved for that set of measured data. Referring to the average power levels of MPGs relative to the strongest MPG shown in Table 5.6, it can be seen that for a system with the parameters considered, on average no more than two MPGs satisfy this criterion for the set of measured data recorded in downtown Ottawa. Hence a

Rake receiver design that allows for two Rake fingers would be suitable on average for the microcellular environment and system under consideration. In other words the GSC scheme gives better performance than the MRC scheme. In the situation where the Rake receiver is part of a battery-powered handheld communication device, the reduction in processing translates to an increase in battery life and a reduction in cost.

Since parameters such as the pilot and data spreading factors and the pilot to data power ratio are pre-determined at the system level, it is possible to use an adaptive decision rule that combines only MPGs at time-delays where this -8 dB threshold is exceeded based on a running APDP calculation in the receiver.

When a time-multiplexed pilot scheme is used, an important consideration is the coherence time of the channel. Although such schemes have not been investigated in this thesis, it would be interesting to investigate the trade offs between time- and code-multiplexed strategies. It is reasonable to expect that when the coherence time is very small, a code-multiplexed pilot would be more beneficial to performance as it provides more frequent updates to the channel estimates. It was however shown in this thesis that current BER requirements of 10^{-3} for high data rates cannot be met in downtown Ottawa with the code-multiplexed channel scheme described in this thesis. Another consideration is the distance of the receiver from the transmitter. Since the code-multiplexed pilot is power controlled along with the data signal and has a fixed power relative to it, it provides more reliable channel estimates at cell boundaries compared with a common time-multiplexed pilot.

Chapter 7

Conclusions and recommendations for future work

A growing need for high data rate services for an ever increasing user base will soon lead to the requirement for systems designers to more efficiently utilise scarce spectrum resources. Performance prediction plays a major role in the design of wireless systems and better predictions, as well as improved receiver design are the key to better efficiency. These considerations were the motivation for work conducted during this thesis project.

During the initial literature survey, it was found that most reported DSSS link BER prediction methods depend upon idealised radio channel models that lead to mathematical tractability. At the same time, such methods can lead to inaccuracies that result in predicting a requirement for greater SNRs than those that are actually required to achieve target BER performance, or to a requirement for excessive receiver complexity. Work during this thesis project led to the extension and further development of a technique for radio link performance analysis, based on work reported earlier by others that focused on predicting degradation caused by multiple-access interference in a non-fading environment. This technique, which is semi-analytical, is more computationally intensive than approaches that utilise simplifying assumptions about the fading and average powers of received MPGs. It however allows

for the consideration of either more realistic channel models, or the incorporation of measured time varying radio channel characteristics in prediction routines, without resorting to brute force simulation which can be more computationally intensive. In addition, it accounts for the effects of SI caused by multipath propagation, without constraining multipath delays to be integer multiples of chip durations, for conformance with a Gaussian interference model. The new technique can be applied to investigate errors caused by interference to both received data and pilot information, enabling additional studies concerning the effects of channel estimation errors.

One result from the thesis work is, for the first time, the reporting of a computationally efficient sequence of procedures for accurately assessing DSSS link BER performance on fading radio channels that allows for consideration of all natural radio channel impairments. Although not considered in this thesis, the same methods and their extensions can also be used in their original application to incorporate considerations for the effects of multiple-access interference.

The new semi-analytical procedures were applied to study performance that would be achieved on urban microcellular mobile channels that were characterised through the analysis of wideband propagation measurements. Results were compared with those that would be reported based on the use of idealised channel models, and previously used BER analysis methods. Benefits of the new approach and the implications of resulting newly available information were then discussed and considered in the development of recommendations for Rake receiver design. It is considered that these recommendations would lead to decreased receiver complexity, as well as better BER performance under real-world microcellular urban mobile radio channel operating conditions.

A more detailed summary of important findings from each chapter of the thesis follows.

In Chapter 4, new analytical expressions for upper bounds on BER performance

of a Rake receiver were presented. These expressions were derived using simplifying assumptions about the propagation channel and a Gaussian model for the interference. An upper bound on the SI was derived, and was found to be very loose as a result of the approximations to both the SI and the Gaussian-Q function. A technique for performance prediction that could use measured data and did not necessitate the use of simplifying assumptions was therefore required.

The development of a semi-analytical technique was reported in Chapter 5, with the objective of applying it to accurately predict Rake receiver performance based on measured data. The semi-analytical technique was validated through simulation and it was concluded that the approach was correct. Using the semi-analytical technique it was found that the effect of SI on BER performance was significant and necessary to consider when studying Rake receiver performance.

The semi-analytical technique was also used to assess the impact of simplifying assumptions related to both the propagation channel and the interference statistics on performance. This was done by comparing Rake receiver BER estimates made using the assumptions with estimates made using measured data and the semi-analytical technique for estimating the statistics of the interference. It was concluded that the use of a Gaussian model for the interference that reflected an upper bound on the variance significantly overestimates the BER for all values of received SNR. It was also found that use of the semi-analytical technique with a channel model that assumes there are K received MPGs with equal average power results in overestimation of the total interference power when compared with that estimated using a time-series of K MPGs from measured data. This is because MPGs in general do not have equal average powers. It can be concluded therefore that use of either of the above-mentioned assumptions to make predictions for a practical system design would lead to reporting the requirement for a larger transmit power than is actually required to achieve a desired BER.

From subsequent work using the semi-analytical technique it was concluded, based on consideration of the average of received MPG power levels determined from data recorded in downtown Ottawa, that a Rake receiver with three fingers would be sufficient to exploit the diversity offered by the channel on the downlink, if the system bandwidth were 3.84 MHz as currently proposed for 3G systems. The gains in performance from using additional Rake fingers would not be as significant. Additional data from urban microcellular environments could be examined in order to determine if this finding can be generalised to some class of urban centres.

In Chapter 6, using the semi-analytical technique developed in Chapter 5, the effect of errors in estimating MPG amplitudes on Rake receiver BER performance, was investigated for a system with a code-multiplexed pilot. This was done with the objective of suggesting improvements in the design of practical Rake receivers used in WCDMA systems.

While in Chapter 5 it was shown that Rake receiver performance always improves when the number of fingers is increased, up to the number of received MPGs, in Chapter 6, it was shown that for the system that was considered during the thesis work, combining MPGs that have a power level of -8 dB relative to the strongest MPG result in performance degradation. This is the result of errors in estimating the complex amplitudes of the MPGs with lower average relative powers. Reducing the pilot to data power ratio, for fixed pilot and data spreading factors and MPG amplitudes, leads to a reduction in the accuracy of the channel estimate and a degradation in performance, while increasing it leads to a degradation in performance resulting from SI to the data. Also, reducing the pilot spreading factor results in greater SI both to the data and the pilot itself, thereby degrading the quality of channel estimation and hence the BER, while reducing the data spreading factor increases the interference both to the data and the pilot thereby increasing the BER. Finally, it was demonstrated with the use of measured data for the system under consideration

MPGs	ρ_c
1 and 2	0.57
2 and 3	0.09
1 and 3	0.02

Table 7.1: Correlation coefficients among three received MPGs.

that, as a result of channel estimation errors, a BER of lower than 5×10^{-2} cannot be achieved. Channel estimation therefore places a significant limitation on Rake receiver performance in DSSS systems operating at high data rates.

Recommendations for future work

In Chapter 5, the effect of an uncorrelated fading assumption for received MPGs having equal average power, on Rake receiver performance, was studied. While the correlation coefficient between any two MPGs represented in the measured data from downtown Ottawa has a mean value of 0, values of the correlation for time-series were found to vary between ± 0.6 . In a practical situation where hardware resources in a receiver are limited, knowledge of correlation among received MPGs could be used to combine only those that are least correlated. For instance if there were only two Rake fingers in a receiver and three received MPGs having equal average power with correlation coefficients as shown in Table 7.1, then clearly combining MPGs 1 and 3 would give the best performance.

A practical implementation of the above proposed idea would however require estimation of channel information for a period of time prior to MPG selection for combining. Under the assumption that the correlation does not vary with time or at least until the next estimation interval, a performance gain could be achieved. While this has not been investigated in the thesis, it is considered that it could be an approach to achieving performance improvements. One possibility for future work is to investigate if, given limited hardware complexity in a receiver, knowledge of

correlation could be used to combine MPGs that are least correlated.

Application of the semi-analytical technique to determine the effects of MAI on performance in the absence of power control is straightforward and could be used in conjunction with work reported here but is not discussed in this thesis. A suitable model for the effect of power control on signal levels could be incorporated into the semi-analytical technique and BER performance could then be evaluated taking the effects of both SI and MAI into account. Bit error rate performance estimates made using the Gaussian approximation discussed previously improve in accuracy when the interference increases for a fixed combined signal power. The approximation could therefore be used to predict Rake receiver performance in situations where there is MAI. An investigation of modelling both MAI and SI for use in estimating Rake receiver BER performance is an area in which additional work could be done.

Also, the derivation of a tighter upper bound on BER performance that does not rely on the techniques presented in Chapter 5 of this thesis would be desirable as it would eliminate the time-consuming SI calculations that are required for performance estimation.

The significance of channel estimation errors on BER performance when a Rake receiver with MRC is used indicates that this is possibly not the best combining scheme. Determining an optimal Rake combining scheme for high data rate DSSS systems operating in real-world channels could be one area of possible research.

During the course of this work in this thesis it was realised that cross-correlation sidelobes for two sequences that have time-aligned chip boundaries can be greater than the sidelobes for two sequences offset from each other by, say, half a chip period. This is a result of interference cancellation over a chip interval. It is considered that knowledge of this might be useful in finding means for allowing more users in a CDMA system where the number of orthogonal codes for the downlink is limited.

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Appendix A

The pdf of $Z = Y \cos X$ where X is uniformly distributed on $[0, \pi]$

Case 1: Y is a constant.

Let $Y = k$. The pdf of

$$Z = k \cos X = g(X) \quad (\text{A.1})$$

where X is uniformly distributed on the interval $[0, \pi]$, and k is constant can be determined as follows.

The pdf $f_Z(z)$ can be written as

$$f_Z(z) = \sum_i \frac{f_X(x_i)}{|g'(x_i)|}. \quad (\text{A.2})$$

Also,

$$g'(X) = -k \sin X = -k \sqrt{1 - \left(\frac{Z^2}{k^2}\right)}. \quad (\text{A.3})$$

Using the above equations, $f_Z(z)$ can be written as

$$f_Z(z) = \begin{cases} \frac{1}{k\pi \sqrt{1 - \left(\frac{z^2}{k^2}\right)}}, & |z| < k \\ 0, & |z| \geq k. \end{cases} \quad (\text{A.4})$$

Case 2: Y is uniformly distributed on $[a, b]$.

Let $V = \cos X$. The pdf of V can be written as

$$f_V(v) = \begin{cases} \frac{1}{\pi\sqrt{1-v^2}} & |v| < 1 \\ 0 & \text{elsewhere.} \end{cases} \quad (\text{A.5})$$

The pdf of $Z = YV$, can be determined as follows.

Using an auxiliary random variable $W = Y$, the joint pdf $f_{ZW}(z, w)$ can be written as

$$f_{ZW}(z, w) = \frac{1}{|w|} f_{YV}\left(w, \frac{z}{w}\right) \quad (\text{A.6})$$

The pdf $f_Z(z)$ can then be written as

$$f_Z(z) = \begin{cases} \frac{1}{\pi(b-a)} \int_a^b \frac{1}{\sqrt{w^2 - z^2}} dw, & 0 \leq |z| \leq a \\ \frac{1}{\pi(b-a)} \int_z^b \frac{1}{\sqrt{w^2 - z^2}} dw, & a \leq |z| \leq b \\ 0 & |z| > b, \end{cases} \quad (\text{A.7})$$

which can be simplified to

$$f_Z(z) = \begin{cases} \frac{1}{\pi(b-a)} \ln \left[\frac{b}{a} \frac{(1 + \sqrt{1 - z^2/b^2})}{(1 + \sqrt{1 - z^2/a^2})} \right] & 0 \leq |z| \leq a \\ \frac{1}{\pi(b-a)} \ln \left[\frac{b}{|z|} \left(1 + \sqrt{1 - z^2/b^2} \right) \right] & a \leq |z| \leq b \\ 0 & |z| > b. \end{cases} \quad (\text{A.8})$$

Appendix B

Numerical computation of the bounds on the pdf of interference

In this Appendix, upper and lower bounds on the pdf for interference are computed. It should be noted that the approach is identical to the work of Lehnert and Pursley in [2] and has been reproduced here with minor modifications, such as for unequal MPG powers, so as to be consistent with the work in this thesis. The notation used in what follows is consistent with that in Chapter 4.

To evaluate the interference or, $W_{k,1}$ for one interferer or $K = 2$, the interference due to multipath group $k = 2$ can be written as $H_2 = |h_2| |h_1| I_2 \cos \phi$, where $I_2 = W_{2,1}/T_c$.

If the random variable I_2 is uniformly distributed on an interval $[m, n]$, where m and n are both positive, the density function for the random variable $H_2 = |h_2| |h_1| I_2 \cos \phi$ can be written as

$$f_{H_2}(x; i = (m+n)/2) = \begin{cases} \frac{1}{\pi h_{12}(n-m)} \ln \left[\frac{n}{m} \frac{(1 + \sqrt{1 - x^2/(nh_{12})^2})}{(1 + \sqrt{1 - x^2/(mh_{12})^2})} \right], & 0 \leq |x| \leq h_{12}m \\ \frac{1}{\pi h_{12}(n-m)} \ln \left[\frac{nh_{12}}{|x|} \left(1 + \sqrt{1 - x^2/(nh_{12})^2} \right) \right], & h_{12}m \leq |x| \leq h_{12}n \\ 0, & |x| > h_{12}n, \end{cases} \quad (\text{B.1})$$

where $h_{12} = |h_1| |h_2|$. If the random variable I_2 is a positive constant k , the density

function for the random variable H_2 can be written as

$$f_{H_2}(x; i = k) = \begin{cases} \frac{1}{k\pi h_{12} \sqrt{1-x^2/(h_{12}k)^2}}, & |x| < h_{12}k \\ 0, & |x| \geq h_{12}k. \end{cases} \quad (\text{B.2})$$

The above expressions for the pdf of H_2 were derived in Appendix A. Since expressions for probability of error require numerical integration over the pdf for the interference, it is required to derive upper and lower bounds on the continuous pdfs which can then be numerically integrated over the range of possible values for interference.

Upper and lower bounds on $f_{H_2}(x|i; c)$, denoted by $f_{H_2^U}(x|i; c)$ and $f_{H_2^L}(x|i; c)$ are calculated over the range of possible interference values $-N_d, \dots, +N_d$. Each integer subinterval is further divided into N_u subintervals; the larger the value of N_u , the closer the bounds are to each other and to the true value. The upper bound $f_{H_2^U}(x|i; c)$ can be denoted by $u_j(i; c)$ where the index j varies between $-N_d N_u$ and $+N_d N_u$. Also, $f_{H_2^L}(x|i; c)$ can be re-written as $v_j(i; c)$. The pdfs for H_2 when $i = 6$ (even) and $i = 7$ (odd) are shown in Figure B.1 and Figure B.2, respectively, for positive values of arguments.

The pdfs are symmetric about the origin for negative values of the argument. When i is odd, the pdf is an increasing function for arguments less than i , and zero otherwise. When i is even, the pdf is an increasing function for arguments less than $i - 1$ and decreasing for arguments greater than $i - 1$ and less than $i + 1$. It is zero elsewhere. This is taken into account when defining bounds for the interference.

For i odd, since the function $p_{H_2|\alpha}$ is strictly increasing on the interval $[0, i)$, a lower bound can be defined as

$$v_j(i; c) = \begin{cases} N_u^{-1} f_{H_2}(j N_u^{-1}; i), & 0 \leq j \leq i h_{12} N_u - 1 \\ 0, & i h_{12} N_u \leq j \leq N_d N_u - 1. \end{cases} \quad (\text{B.3})$$

Figure B.1: Probability density function, $f_{H_2}(x; i = (m+n)/2)$, for $m = 5$ and $n = 7$.

Also an upper bound can be defined as

$$u_j(i; c) = \begin{cases} N_u^{-1}[f_{H_2}(jN_u^{-1}; i) + f_{H_2}((j+1)N_u^{-1}; i)]/2, & 0 \leq j \leq ih_{12}N_u - 2 \\ \frac{1}{2} - \sum_{k=0}^{ih_{12}N_u - 2} v_k(i; c), & j = ih_{12}N_u - 1 \\ 0, & ih_{12}N_u \leq j \leq N_dN_u - 1. \end{cases} \quad (\text{B.4})$$

For i even and positive, the function f_{H_2} is strictly increasing on the interval $[0, i-1]$, and strictly decreasing on the interval $[i-1, i+1]$. Also, for $i = 0$, $f_{H_2}(x|i; c) = \lim_{\varepsilon \rightarrow 0+} f_{H_2}(x; \varepsilon, 1)$. The function $f_{H_2}(x; \varepsilon, 1)$ is strictly decreasing on the interval $[\varepsilon, 1]$ for $0 < \varepsilon < 1$. Lower bounds for $i > 0$ can then be defined as

Figure B.2: Probability density function, $f_{H_2}(x; i = k)$ for $k = 7$.

$$v_j(i; c) = \begin{cases} N_u^{-1} f_{H_2}(j N_u^{-1}; i); i - 1, i + 1), & 0 \leq j \leq (i - 1) h_{12} N_u - 1 \\ N_u^{-1} f_{H_2}((j + 1) N_u^{-1}; i - 1, i + 1), & (i - 1) h_{12} N_u \leq j \leq (i + 1) h_{12} N_u - 1 \\ 0, & (i + 1) h_{12} N_u \leq j \leq N_d N_u - 1. \end{cases} \quad (\text{B.5})$$

Upper bounds can similarly be defined as

$$u_j(i; c) = \begin{cases} N_u^{-1} f_{H_2}((j + 1) N_u^{-1}; i - 1, i + 1), & 0 \leq j \leq (i - 1) h_{12} N_u - 1 \\ N_u^{-1} f_{H_2}(j N_u^{-1}; i - 1, i + 1), & (i - 1) h_{12} N_u \leq j \leq (i + 1) h_{12} N_u - 1 \\ 0, & (i + 1) h_{12} N_u \leq j \leq N_d N_u - 1. \end{cases} \quad (\text{B.6})$$

For $i = 0$,

$$v_j(0; c) = \begin{cases} N_u^{-1} \lim_{\varepsilon \rightarrow 0^+} f_{H_2}(N_u^{-1}(j+1); \varepsilon, 1), & 0 \leq j \leq h_{12}N_u - 1 \\ 0 & h_{12}N_u \leq j \leq N_dN_u - 1 \end{cases} \quad (\text{B.7})$$

and

$$u_j(0; c) = \begin{cases} \frac{1}{2} - \sum_{k=1}^{N_u-1} v_k(0; c), & j = 0 \\ N_u^{-1} \lim_{\varepsilon \rightarrow 0^+} p_{H_2}(N_u^{-1}j; \varepsilon, 1) & 1 \leq j \leq h_{12}N_u - 1 \\ 0 & h_{12}N_u \leq j \leq N_dN_u - 1 \end{cases} \quad (\text{B.8})$$

are the lower and upper bounds, respectively. Also $v_{-j-1}(i; c) = v_j(i; c)$ and $u_{-j-1}(i; c) = u_j(i; c)$ for $0 \leq j \leq N_dN_u - 1$.