Models for Optimization of Network Topology and Performance in Wireless Mesh Networks

by

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Abstract

This dissertation presents models for the optimization of network topology and performance in Wireless Mesh Networks (WMNs). We examined the problems of network connectivity and network planning for optimizing the topology. Efficient routing of Internet traffic was considered for optimizing the performance.

We presented a methodology for accurately calculating the network connectivity in WMNs. The methodology takes into account dependencies between links of geometrically co-located nodes, as well as the reduced connectivity of boundary nodes. Closed form expressions for the upper bound for the network connectivity in a chain topology, as well as a triangular lattice topology were derived. A comparison study between models that assume independent links' connectivities and our model shows that the commonly used assumption of link independence almost always underestimates the network connectivity. We also show that the effect of boundary nodes on the network connectivity can not be usually eliminated by assuming an infinitely large network.

The optimal network plan in terms of node class selection and number of nodes

required to achieve the given service requirements in a WMN was investigated. The network is said to be optimal if it minimizes the network implementation cost while achieving the applications' requirements. To tackle this problem, we developed an analytical model and methodology for WMN planning optimization. The model takes into account dependencies between links of co-located nodes, the effect of boundary nodes' reduced connectivity, in addition to fading and shadowing effects. The proposed methodology resulted in a qualitative and quantitative description of the optimal WMN planning policy.

Furthermore, we proposed a dynamic routing policy for routing Internet traffic from mesh nodes to multiple Internet gateways simultaneously. The system model for the WMN takes into account the effects of the wireless channel. Stochastic programming was used to find the optimal policy for the route controller. Our performance evaluation shows that utilizing multiple Internet gateways simultaneously results in a much more efficient use of resources. The performance of the proposed optimal routing policy was found superior compared to the performance of other policies found in the literature.

to mom and dad...

for who they are and what they did

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List of Acronyms

AODV	Ad Hoc On Demand Distance Vector routing protocol
AODV-ST	AODV - Spanning Tree routing protocol
ATM	Asynchronous Transfer Mode
CCTV	Closed Circuit Television
DSR	Dynamic Source Routing protocol
DiffServ	Differentiated Services Architecture
ENT	Effective Number of Transmissions metric
ETT	Estimated Transmission Time metric
ETX	Expected Transmission Count metric
FSMC	Finite State Markov Channel

GPS	Global Positioning System
iAware	The interference Aware metric
IEEE	Institute of Electrical and Electronics Engineers
IETF	Internet Engineering Task Force
IG	Internet Gateway
IntServ	Integrated Services
IP	Internet Protocol
LAN	Local Area Network
LQSR	Link Quality Source Routing
MAC	Media Access Control
MANET	Mobile Ad Hoc Network
mETX	modified ETX metric
MIC	Metric of Interference and Channel-switching
ML	Minimum Loss metric

MN	Mesh Node
MPI	Message Passing Interface
MR-LQSR	Multi Radio LQSR
OLSR	Optimized Link State Routing
OSPF	Open Shortest Path First
QoS	Quality of Service
RTT	Round Trip Time
SNR	Signal to Noise Ratio
TDMA	Time Division Multiple Access
TCP	Transmission Control Protocol
VoIP	Voice over IP
WCETT	Weighted Cumulative ETT
WiFi	Wireless Fidelity
WLAN	Wireless Local Area Network

WMN	Wireless Mesh Network
WSN	Wireless Sensor Network

Chapter 1

Introduction

1.1 Motivation

Wireless Mesh Networks (WMNs) are considered key technologies that may play a fundamental role in the future pervasive computing scenario. Enhancing and optimizing the topology and performance of such networks is therefore essential.

Compared to wired infrastructure-based and ad hoc networks, wireless mesh networks levy a number of unique challenges. In WMNs, the radio transmission medium often used is characterized by limited bandwidth, dynamic changes to link capacity, and asymmetrical wireless links. WMNs are usually deployed over wide geographic areas, typically providing metropolitan-sized coverage. Consequently, routes in most cases transverse several hops. Moreover, the network deployments are comprised of a wireless backbone of mesh nodes communicating back-haul traffic, combined with edge or access networks that serve stationary and mobile users. Furthermore, most

applications of WMNs impose high-bandwidth and low delay-requirements.

The work in this dissertation focuses on two design aspects that together contribute to the efficient deployment of wireless mesh networks. First, optimizing the topology subject to budget and coverage constraints, and second, optimizing the routing of Internet traffic subject to traffic demands and application performance. In optimizing the topology, we examine the problem of network connectivity by ensuring that all nodes in a WMN can communicate with each other at any point of time. We also consider the problem of optimally planning a network's topology by determining the configuration aspects, such as mesh nodes type, the number of nodes, and the nodes' locations required to achieve the applications' requirements at the lowest networking cost. When the optimal topology is discovered and deployed, it is vital to use the available network resources efficiently. Since Internet packets constitute the common-case traffic in WMNs, we solve the problem of optimizing the routing of Internet traffic.

1.2 Research Contributions

This research concentrated on enhancing the efficiency and performance of WMN deployments by optimizing and planning their topology, as well as effectively utilizing the Internet gateways in WMNs. In order to do so, we first devised a methodology for estimating the network connectivity in WMNs. Our methodology is the first to take

into account the effect of both: interdependence in connectivity of spatially co-located nodes, as well as the effect of the reduced connectivity of network boundary nodes. The resulting connectivity model hence provides better estimate of network connectivity. The developed connectivity model was used to develop an analytical model and methodology for finding the optimal network plan in WMNs; one that minimizes the network implementation cost while achieving the application matrix requirements. Lastly, we provided an optimal routing protocol that improves the performance of WMNs by dynamically routing mesh nodes' Internet traffic to several Internet gateways simultaneously. The proposed routing strategy results in efficient use of network resources while satisfying the applications' throughput and delay requirements.

1.3 Organization

Following this Introduction, Chapter 2 reviews the background associated with wireless mesh networks; the chapter also introduces the concepts of network connectivity, network planning, and routing in WMNs. Chapter 3 is a review of the state of the art relevant to network connectivity models, planning of WMN topologies, and routing methodologies specific to WMNs; the problem statement is given at the end of this chapter. Chapter 4 presents the methodology and models derived to accurately estimate the network connectivity in wireless mesh networks. Chapter 5 presents a

methodology to optimally plan a WMN's topology. Chapter 6 presents a methodology used for the optimal routing of Internet traffic in WMNs. Chapter 7 gives some conclusions and suggestions for future research.

Chapter 2

Background

2.1 Introduction to Wireless Mesh Networks (WMNs)

The deployment of traditional fixed-line broadband service can be difficult and expensive, especially in large communities, or highly dispersed areas. Cellular networks operate in licensed bands, hence the spectrum acquisition costs make the deployment of such networks expensive. On the other hand, Wireless Local Area Networks (WLAN) or Wireless Fidelity (Wi-Fi), use the unlicensed Industrial, Scientific and Medical (ISM) band, making their deployment much easier. Nonetheless, attempts to use Wi-Fi hotspots for citywide or regional coverage have been hampered by the need for wiring each access point to the Internet.

The technology of Wireless Mesh Networks (WMNs) is quickly revolutionizing

the way wireless hi speed IP connectivity is provided. This is contributed to the cost-effective way in which the technology addresses the problem of last mile connectivity. WMNs extend the range of WLANs beyond few meters to large scale outdoor coverage that can span a whole neighborhood or city. Therefore, WMNs will help improve services for users, increase overall efficiency, and enhance productivity, through providing a seamlessly integrated wireless network capable of providing high speed broadband services, while minimizing capital, deployment, and operational expenses.

WMNs are dynamically self-organized self-configured networks composed of mesh routers and mesh clients. All communication between mesh routers, as well as between mesh clients and mesh routers, is wireless, and is typically based on 802.11 technology. Figure 2.1 on the following page depicts a typical WMN.

Mesh routers automatically establish and maintain mesh connectivity among themselves, forming a wireless backbone that carries the backhaul traffic. We will use the term *mesh node* or simply *node* to refer to a mesh router. Mesh nodes operate both, as hosts, and as routers forwarding packets on behalf of other mesh nodes and clients that are not within the direct range of their destinations. Mesh nodes are mostly stationary with very limited mobility. They are typically installed in locations where mains power sources are readily available, such as buildings' roofs and lighting poles, hence enabling them to be constantly powered. Nonetheless, some WMNs (e.g., battlefield deployments) will consist of mobile mesh nodes; such nodes are typically

battery powered. Usually, a handful of the mesh nodes have a direct wired connection to the Internet; these nodes serve as Internet gateways that can communicate Internet traffic to and from the rest of the backbone mesh nodes, and subsequently to and from the mesh clients. We will also use the term *gateway mesh node*, *Internet gateway*, or simply *gateway* to refer to a mesh router that is directly connected to the Internet.

Mesh clients on the other hand are wireless devices, such as laptops, IP phones, cameras, or even other wireless gateways (e.g., gateways of sensor networks), that communicate directly with mesh nodes in their radio range. Mesh clients can be mobile or stationary. Additionally, they might very well be battery operated.

Figure 2.1: A typical Wireless Mesh Network

2.2 Applications of WMNs

WMNs are primarily designed to provide an enhanced wireless communication infrastructure; they are capable of delivering rich content of data, voice, and video to mobile users. Moreover, the topologies typically used in the deployment of WMNs facilitate locating users and objects (i.e., wireless mesh clients) in the network's coverage area. There are numerous applications of wireless mesh networks in safety, security, education, real estate, and transportation to name a few [1]. In this section, we discuss some of the common WMN applications.

Wireless Broadband Access WMNs connect distant buildings, enable broadband access for under-served areas, and support deployment of hotspots. Internet gateway access, Voice over IP (VoIP), real time audio and video, video on demand, and many other services will hence be possible to a large number of mobile and wireless users. WMNs will also enable communication in areas where cellular coverage is not available (e.g., tunnels, underground parking garages).

Support Wireless Sensor Networks WMNs will be used to support and bridge underlying Wireless Sensor Networks (WSNs) that monitor and control the environment. WSNs are employed in industrial applications to monitor equipment; in energy applications to monitor resources and energy consumption; in environmental applications to monitor and control air and water quality; in civic

infrastructure to monitor structural conditions and seismic activity of buildings and roads; and in biomedical and health care applications to monitor health related conditions, as well as the the daily activities of physically, and cognitively challenged people.

Security and Safety Applications WMNs enable real-time situational awareness for law enforcement usage by supporting devices such as wireless IP-based Closed Circuit Television (CCTV) surveillance cameras and perimeter security sensors. WMNs can also be used to monitor and protect valuable assets normally located in public spaces (e.g., electronic and communications equipment, signs, displays, bus shelters) making them vulnerable to theft and vandalism. Moreover, WMNs can be easily and quickly setup at times of emergencies and security breaches.

Location-Based Services Location-based tourist services promote city attractions or exhibits and provide entertainment information to tourists and guests. Likewise, WMNs can locate safety, maintenance, and other personnel who are constantly on the move, resulting in effective job dispatch and assignment schedules. More importantly, WMNs can be used to locate people in distress and emergency situations.

Wireless-Enabled Mobile Agent Applications Can enhance the efficiency and productivity of inspectors, field workers, social workers, municipal police, parking attendants, and other field-based workers (e.g., safety officers can download amber alerts while in the field).

Municipal Transport Solutions Improve operation and prevent traffic problems with real-time data and video information exchange. Moreover, unlike navigation information attained via the Global Positioning System (GPS) technology, communication in WMNs is bi-directional, and is not restricted to having an un-obstructed view of the sky.

Today, many authorities, such as Beaverton Police [2], universities, such as the University of North Carolina [3], and cities, such as Dublin, Ohio [2], have already deployed large scale wireless mesh networks.

2.3 Network Connectivity

Network connectivity is a fundamental property of wireless networks. It is integral to providing reliable applications to end users. Network connectivity is also a vital metric that is frequently utilized in the design and network planning process, as well as in describing and comparing networks.

In cellular networks, a mobile device is said to be *connected* if it has a wireless

link to at least one base station. In wireless mesh networks, the situation is more complicated. Traffic is often communicated in a multi-hop fashion, hence mesh nodes can be intermediate routers that relay traffic to other mesh nodes. Each mesh node hence contributes to the connectivity of the entire network.

In WMNs, network connectivity often refers to the probability that all mesh nodes (forming the wireless backbone) can communicate with each other at any given time. In this work, we will use the term *network connectivity* to refer to the probability with which nodes in a WMN are connected; a node is said to be *connected* if has at least one path to every node in the network.

2.4 Planning of WMNs

Vendors of WMNs offer an increasing selection of mesh nodes that vary in capabilities and cost. There surely exists a tradeoff between cost and performance; mesh nodes capable of providing greater network capacity, and longer communication ranges come at higher cost. Moreover, different types of mesh nodes can outperform one another, depending on the topology and networking scenario in which they are deployed. It is hence vital for network planning teams to identify which types of mesh nodes to use, and the number of nodes to deploy, under which conditions so that the required performance is achieved at the lowest possible cost.

Unfortunately, the configuration and planning aspects of wireless mesh networks,

including the selection of node types, the required node density, and the actual placement of nodes, are mostly accomplished based on past experience as well as trial and error experimentation. This results in networks that are ultimately far from optimal.

2.5 Routing in WMNs

Providing Quality of Service (QoS) support for the diverse applications of WMNs, including voice, video, and data, is imperative to the success of WMNs. Efficient routing can help in maximizing the usage of the available bandwidth, as well as minimizing the delay of data packets, and is therefore integral for providing the QoS demands of WMN applications.

Despite of the numerous routing protocols that have been proposed for wireless networks, the unique characteristics of wireless mesh networks demand new and better dynamic routing solutions [4]. Similar to most wireless networks, the radio transmission medium often used in WMNs is characterized by limited bandwidth, asymmetric links, and dynamic changes to link capacity. However, unlike most wireless networks, the network deployment is typically comprised of stationary nodes forming a wireless backbone that exchange back-haul traffic, combined with edge or access networks that serve stationary and mobile users.

An optimal routing protocol for WMNs must capture five features [4] [5]. Firstly, a routing path must be selected by considering multiple performance metrics, some

or all of which must be related to link quality. Secondly, the routing protocol must be fault tolerant; link failures must be quickly detected and new paths quickly established to avoid service disruption. Thirdly, the routing protocol should be able to effectively perform load balancing. Fourthly, the routing process must be scalable, that is, setting up new paths, monitoring existing and alternate paths, and switching to better paths should be fast. Finally, the routing protocol must adaptively support mesh routers and clients; while mesh routers are typically neither mobile nor power constrained (i.e., not requiring ad hoc routing optimizations), mesh clients usually are, and hence need the support of ad-hoc type routing protocols.

Chapter 3

Related Work

3.1 Review of Connectivity Models

In wireless networks, connectivity is often expressed in terms of individual links' connectivities. To the best of our knowledge, all previous work embeds, whether implicitly or explicitly, either or both of the following assumptions:

1. Inter-node links' connectivities are independent of one another,
2. Border nodes have the same connectivity as the interior nodes in a network.

The first assumption implies that the probability that any given node in a network is connected (i.e., has at least one path to each node in the network) is independent of that of the other nodes in the same network. As a direct consequence, the connectivity of all nodes in a network are independent of one another.

Stuedi et al. [6] alludes that such assumption was motivated by an experimental study in [7]. In [7], Zhou et al. investigated the impact of radio irregularity and discovered that the variation in the received signal strength is largely random in the case of wireless sensor networks. Hence, one can directly infer that wireless links are often asymmetric; this is applicable and even more evident when the nodes of a WMN have different transceivers. Nonetheless, asymmetry does not infer independency under any circumstance. It is therefore not obvious whether assuming independence in links connecting spatially co-located nodes is accurate.

Furthermore, most researchers avoided the effect of reduced node connectivity of border nodes by assuming that the networks examined were infinitely large. Border nodes have less neighbors, and hence, less chances of being connected. The embedded assumption here is that the effect of such nodes on the overall network connectivity in an infinitely large network is negligible.

Although these simplifications are adopted by many researchers such as [6], [8], [9], [10], and [11], the impacts of such assumptions were never examined.

3.2 Review of Network Planning Methodologies

Some industry initiatives such as [12] and [13] aim at developing systematic WMN planning methodologies. However, these solutions suffer from important drawbacks. Firstly, they do not result in optimal node allocations. Secondly, they do not allow for

an optimal selection of different nodes from distinctive vendors. Thirdly, they are not very flexible in that they provide little or no capability for modifying the parameters pertaining to different physical environments and deployment sites. Lastly, they are mostly proprietary solutions that shield end users from understanding and modifying the models and policies used to derive the node allocation plan. Hence, such tools are often inadequate in characterizing the tradeoffs between performance and cost for various node classes, leading in many cases to decisions that could have been dramatically improved.

Sikora et al. [14] consider a communication system with a linear topology, where there is a single source node, a single destination node, and multiple relay nodes placed equidistantly between the source and destination. Given a global constraint on bandwidth, they determine the number of hops that achieve the desired end-to-end data rate with the least transmission power. Their analysis is based on information theory, and assume Time Division Multiple Access (TDMA) multihop transmission where nodes do not cooperate. Moreover, the work presented in [14] did not consider any two dimensional topologies. This could be due to the complexity of the analysis resulting from two dimensional connectivity patterns.

Bai et al. [15] tackled a related, but different problem. They evaluate the efficiency of regular deployments in terms of the number of sensors needed to provide coverage and connectivity in a wireless sensor network. They also propose an optimal

deployment pattern to achieve both full coverage and 2-connectivity in wireless sensor networks.

3.3 Review of WMN Routing Paradigms

One of the major characteristics that differentiate WMNs from ad hoc networks is the fact that the common case traffic in WMNs take place between mesh nodes and gateways nodes. Typically, each mesh node finds a path to one gateway node, and forwards to it all of its egress Internet traffic; this is an artifact of most routing protocols. It is hence foreseen that the chosen paths and gateways can quickly become congested, causing delays in the network and limiting the overall network capacity.

Multipath routing allows the establishment of multiple paths between a single source and a single destination node. Multipath routing is typically proposed in order to increase the reliability of data transmission (i.e., fault tolerance) or to provide load balancing [16]. However, if multiple paths are used simultaneously to route data, the aggregate bandwidth available to the applications will increase. Also, since there is more bandwidth available, a shorter end-to-end delay can be achieved.

To predict and select the best route(s) from a source to a destination, the path selection process of a routing algorithm applies a *routing metric* to the various paths in a network. The routing metric evaluates the various paths, hence determining the best route(s) for packets.

First in Section 3.3.1, we provide a summary of routing metrics that can be used in WMN routing algorithms, specially those designed to enhance the performance of WMNs. Section 3.3.2 then provides an overview of routing proposals and methodologies that use multipath routing and load balancing techniques.

3.3.1 Routing Metrics

Most routing protocols for wired and ad hoc networks choose to use hop count as their routing metric. In MANETs, the highly mobile nature of users comprising the network nodes necessitate finding new paths rapidly, whereas high-quality routes may not be found in due time [17]. However, this might not be the case for WMNs. We have discussed in Chapter 2 some of the unique characteristics of WMNs, most notable of which is that backbone mesh routers are usually stationary, and can be permanently powered. Hence, mobility and energy conservation are no longer concerns for the routing process. Additionally, the diverse QoS requirements imposed by the various WMN applications, coupled with the highly unstable nature of the wireless medium, led to the evolution of quality-aware routing metrics. Such metrics take into account the conditions of the wireless medium, as well as the conditions of the network. An overview and discussion of the most prominent of these metrics follows.

Expected Transmission Count (ETX)

This first quality-aware metric proposed was ETX [18]. The metric resembles the expected number of transmissions, including retransmissions, needed to successfully transmit a packet across a link.

ETX is derived from measurements of the underlying packet loss probability in both the forward, p_f , and reverse directions, p_r . The 802.11 protocol requires that for a transmission to be successful, the packet must be successfully acknowledged. Hence, the probability that the packet transmission over a single hop is not successful, p , is

$$p = 1 - (1 - p_f) \cdot (1 - p_r).$$

The 802.11 MAC will retransmit a packet whose transmission was not successful. Let the probability that the packet will be successfully delivered from a point to its neighbor after k attempts be denoted by $s(k)$. Then,

$$s(k) = p^{k-1} \cdot (1 - p).$$

The expected number of transmissions required to successfully deliver a packet from a point to its neighbor is hence

$$ETX = \sum_{k=1}^{\infty} k \cdot s(k) = \frac{1}{1 - p}.$$

The path metric is the sum of the ETX values for each link in the path. The routing protocol selects the path with the minimum path metric.

To calculate p_f and p_r , each node periodically (once every second) sends out a broadcast probe packet. Broadcast packets are not retransmitted by the 802.11 MAC. Nodes track the number of probes received from each neighbor during a sliding time window (10 seconds) and include this information in their own probes. Nodes can calculate p_r directly from the number of probes they receive from a neighbor in a time window; to calculate p_f , they use the information received about themselves from a neighbor.

ETX suffers from two main drawbacks. Firstly, broadcasts are performed at the network basic rate, so ETX can not distinguish between links with different bandwidths. Secondly, probes are smaller than typical data packets, therefore, ETX will underestimate the loss rates, resulting in deteriorated performance, unless the network is operating at low rates.

Estimated Transmission Time (ETT), and Weighted Cumulative ETT (WCETT)

Draves et al. recognize that one of the reasons for the reduction in total capacity often seen in wireless mesh networks is due to interference between simultaneous transmissions [19]. Hence, they devise a metric for choosing a high-throughput path between a source and destination.

The ETX metric proposed in [18] assumes that the probability that a given packet

is lost in transmission is independent of its size, and is independent and identically distributed. The ETX metric is also bidirectional; the metric in one direction is the same as the metric in the opposite direction. Moreover, ETX considers loss rates on the links, and not their bandwidths. Also, the metric does not give any preference to channel-diverse paths, resulting in choosing sub-optimal paths in networks composed of nodes furnished with multiple radios (operating on different frequencies, or tuned to use non-interfering channels).

To address ETX's shortcomings, Draves et al. propose two metrics. First, ETT, which is essentially a bandwidth adjusted ETX, given by

$$ETT = ETX \cdot \frac{S}{B},$$

where S denotes the size of the packet, and B is the total link capacity. ETX is calculated as was previously proposed in [18]. Since most 802.11 radios automatically select the bandwidth for every packet (auto-rate feature), B will need to be obtained from the MAC layer, or alternatively measured. In [19], B is measured by periodically sending two back-to-back probe packets, calculating the bandwidth, and communicating it back to the sender. This is only an estimate since several factors that might affect the packet delivery times are ignored.

Based on ETT, the path metric WCETT was also proposed, and is given by

$$WCETT = (1 - \beta) \cdot \sum_{i=1}^n ETT_i + \beta \cdot \max_{1 \leq j \leq k} X_j.$$

WCETT has two components. The first term resembles the estimated end-to-end delay experienced by a packet traveling along that n -hop path. The second term captures the reduction in throughput due to interference among links that operate on the same channel. Assuming that the system has a total of k channels, X_j is defined as

$$X_j = \sum_{\text{Hop } i \text{ on channel } j} ETT_i .$$

Hence, the second term in the WCETT equation resembles the total path throughput dominated by the bottleneck channel (i.e., having the largest X_j). Lastly, β is a tunable parameter that allows to tradeoff the first term (reflecting total air time consumed along the path), with the second term (reflecting the set of hops that will have the most impact on the throughput of that path).

The authors implement a Link Quality Source Routing (LQSR) protocol, called MR-LQSR, that utilizes the WCETT metric. They show that MR-LQSR outperforms the ETX and hop count metrics in networks equipped with multiple heterogeneous radios. LQSR is derived from Dynamic Source Routing (DSR) [20], and includes all the basic DSR functionality, but is implemented at layer 2.5 instead of layer 3, and supports link quality metrics.

The two proposed metrics suffer mainly from two shortcomings. Firstly, the derivation of ETT does not account for available bandwidth, instead it uses the

total link capacity. Secondly, when multiple channels are used in the network deployment, inter-flow and intra-flow interference must be avoided [17]. Intra-flow interference occurs when different nodes transmitting packets from the same flow interfere. Inter-flow interference is the interference suffered among concurrent flows. While WCETT considers intra-flow interference, it neither guarantees shortest paths, nor avoids inter-flow interference [21]. Not avoiding inter-flow interference may lead to WCETT choosing routes in congested areas [17].

Other Metrics

The three metrics discussed so far, namely, ETX, ETT, and WCETT, are among the most widely used link-aware metrics. Many other more sophisticated metrics are proposed in the literature. Campista et al. provides a comparison of the various metrics in [17]. We give a brief overview of some of the more elaborate metrics below.

The Metric of Interference and Channel-switching (MIC) [21] is based on the ETT metric, but, addresses both inter-flow, as well as intra-flow interference. Each node takes into account the number of interfering nodes in the neighborhood in order to estimate inter-flow interference.

The Minimum Loss (ML) [22] metric similar to the ETX metric rely on probing to compute the delivery ratio. However, unlike ETX, it finds the route with the lowest end-to-end loss probability by multiplying the delivery ratios of the links in

the reverse and forward directions.

The modified ETX (mETX) and Effective Number of Transmissions (ENT) [23] metrics were proposed to address fast link quality variation, where metrics based on average values fail or may result in prohibitive control overhead. mETX and ENT consider the standard deviation in addition to link-quality average values to project physical-layer variations onto routing metrics.

The interference Aware (iAware) [24] metric uses Signal to Noise Ratio (SNR) and Signal to Interference and Noise Ratio (SINR) to continuously capture neighboring interference variations onto routing metrics. The iAWARE metric estimates the average time the medium is busy due to transmissions from interfering neighbors. Thus, iAWARE considers intra-flow interference, inter-flow interference, medium instability, and data-transmission time.

Discussion

A performance study comparing the hop count, ML, ETX, and ETT metrics was carried out in [17]. Results confirmed that the hop count although resulted in paths with least hops, performed poorly in terms of loss rate, network delay, and throughput, since it is not aware of link quality variation. ML had the lowest packet loss rate. ETT was the only metric that estimated RTT, and hence resulted in the least network delay. Moreover, although the hop count metric resulted in high throughput paths for

shorter distances, it performed quite poorly compared to ETX and ETT for longer distances.

It is therefore evident that each of the hop metrics tested performed best for the criteria that it was designed for; none of the metrics performed best in all QoS criteria (packet loss rate, delay, and throughput). However, the various QoS metrics were implemented and assessed using the link state Optimized Link State Routing (OLSR) protocol [25], running on a mesh network composed of 9 indoor nodes. It is unclear whether the qualitative results obtained will hold if these metrics were assessed using different routing protocols, or with different mesh network topologies.

Finally, Campista et al. points out that most routing protocol implementations still favor metrics with simpler designs such as ETX and ETT [17].

3.3.2 Multipath Routing and Load Balancing

Krishna et. al. define two types of load balancing in [26], *path load balancing*, and *gateway load balancing*. Path load balancing aims at improving network performance and reliability by distributing traffic among a set of diverse paths. Gateway load balancing on the other hand aims at distributing the traffic among a set of gateways in the wireless mesh network; typically, one of several gateways is chosen as the egress point for flows originating from the network.

The Ad Hoc On Demand Distance Vector - Spanning Tree (AODV-ST) routing

protocol proposed in [26] is based on the AODV protocol [27], and is optimized for the common case traffic flow from mesh nodes to gateway nodes. To achieve gateway load balancing, an access router first selects one of the discovered gateways as its default gateway. The default gateway is the one with which the relay can achieve the highest capacity (as determined by the routing metric). The access router typically use its default gateway as the egress point for all flows it initiates. Each access router also monitors the quality of the best path to each of its gateways. The best path is simply the shortest path on the spanning tree computed for that gateway. The path quality is monitored using a simple Round Trip Time (RTT) probing tool. The gateway with the least-delay is designated as the least-loaded gateway. When an access relay detects that its least-loaded gateway and its default gateway are different, it infers the existence of congestion in the network on the path leading to its default gateway; all new flows initiated by that router will be directed to the least-loaded gateway.

Liao et. al. [28] present a multi-path routing protocol for finding a route that satisfies a bandwidth constraint in mobile ad hoc networks. The protocol is based on a bandwidth reservation scheme, where the original bandwidth requirement is split into multiple sub-bandwidth requirements, and each sub-path is responsible for one sub-bandwidth requirement. The protocol uses the bandwidth information at each node in an on-demand fashion during the route discovery process. However, the bandwidth allocation along various paths might be far from optimal.

Choi et. al. [29] tackle the problem of balancing the Internet traffic load among multiple gateways by heuristically dividing the entire network into partitions, and applying load balancing between partitions as well as within the partitions. The goal is to minimize the numbers of sessions rejected by Internet gateways due to congestion. A tree-style single path routing, where Internet gateways form the roots of the trees is assumed. Each node can only have one path towards an Internet gateway (uplink), while it may have several downlink paths from the same gateway. Partitioning the entire network is based on a clustering metric that combines hop-count and network load. Load balancing across neighboring partitions need to be performed in order rectify situations where the load at a gateway exceed's its capacity. This is done by identifying a suitable node in such partition, and re-associating it with another less loaded partition.

Traffic loads and capacities might vary significantly in WMNs, hence, continually re-running relatively complicated procedures as those proposed in [29] might prove to be impossible. The overhead of the messaging between the nodes appear to be significant, and should be assessed, specially for WMNs with dynamic users. Moreover, Krishna et. al. [26] suggest that balancing load across gateways is probably more effective than load balancing along the paths.

Bejerano et. al. [30] explain that using multiple paths in ad hoc networks to achieve higher bandwidth may not be as straightforward as in wired networks because

of radio interference that occurs in the wireless medium. Transmissions from a node along one path may interfere with transmissions from a node along another path, thereby limiting the achievable throughput. When choosing multiple paths, it is hence important to choose paths that are as independent as possible to ensure the least inter-path interference. The authors also explain that the allocation granularity, that is the smallest unit of information allocated to each path is important in traffic allocation. While a per packet granularity will distribute the packets evenly among the paths, packets might arrive out of order, and reordering them is an issue that needs to be dealt with, possibly at the transport layer.

In [31], Nandiraju et. al. propose a technique whose goal is to balance Internet traffic among Internet gateway nodes in WMNs. This is achieved in two phases. In the first phase, each node discovers its primary gateway by listening to periodic beacons sent by gateway nodes in the network. Each node then registers itself to the gateway that is the least number of hops away, and may keep the information of a secondary gateway for purposes of load balancing. Each node also announces to other intermediate nodes the list of Internet gateways it is aware of. In the second phase, each gateway node continuously monitors its queue length. Once the queue length exceeds a certain threshold, it identifies a set of source nodes contributing most to the traffic load, and notifies them to look for alternative less congested gateways. Each of those highly active source nodes then request an association from their secondary

gateway. If the queue length of the chosen gateway is below the congestion threshold, the association is accepted. If a secondary gateway was also congested, or if the active source did not have any secondary gateways to try, a new request is sent to solicit gateway responses. The primary gateway is used until a new association is successful.

The technique proposed in [31] is advantageous in its simplicity and ease of implementation. However, the main goal is to achieve fairness across the available gateways. The technique relies on changing the point of attachment, or gateway association, whenever a threshold is reached. The proposal however does not discuss how this threshold is identified. Moreover, the proposal does not discuss how many active sources should be requested to seek alternate gateways. An inherent assumption of this proposal is that the gains achieved from sending the traffic to an alternate gateway must exceed the losses resulting from keeping the association with the primary congested gateway. This might not be the case if the secondary gateway is considerably slower, or much further away, than the original congested gateway.

Discussion

Pearlman et al. [32] show that path load balancing provides negligible performance improvement in wireless multi-hop networks because of route coupling of candidate paths between common endpoints. Route coupling is a result of the geographic proximity of the candidate paths. This can lead to self-interference between those paths

and can therefore adversely impact performance.

In WMNs, gateway load balancing will most likely result in greater performance gains than path load balancing since route coupling of the paths to different gateways from an endpoint is expected to be minimal in such well-planned topologies [26]. However, all proposals encountered in the literature achieve gateway load balancing by changing a node's gateway association in response to the predicted performance of several gateways that it monitors. A node's throughput will hence remain limited by the throughput of the path to the single selected gateway. Moreover, frequent route-flapping due to transient changes to the link conditions may inhibit performance gains.

3.4 Problem Statement

The goal of this dissertation is to improve the efficiency in wireless mesh network deployments. We approach this problem from two complementary prospects: optimizing the topology, and optimizing the routing of Internet traffic.

Accurately modeling the network connectivity is integral to optimizing a WMN's topology. This work hence aims at reducing inaccuracies resulting from previous connectivity models by devising a methodology for finding the network connectivity while taking into account dependencies between links of geometrically co-located nodes, as well as the reduced connectivity of boundary nodes.

The configuration and planning aspects of WMNs is also a fundamental component in optimizing a network's topology. However, little work addressed this problem. This dissertation aims at finding the optimal number and class of nodes required to achieve the given applications' requirements for the lowest deployment cost. A node class is generally characterized by its capabilities, such as transmitted power and receiver sensitivity, and its cost. The service requirements are described in terms of communication range, minimum network connectivity, and achievable throughput.

Once the network is in place, it is important to ensure efficient use of its resources. Internet traffic exchanged between mesh nodes and gateways constitute the majority of traffic in WMNs. When a node associates itself with a single gateway for all of its Internet traffic, this gateway can easily become a bottleneck. In this dissertation,

we aim at improving the performance of WMNs by optimally routing a node's Internet traffic to several gateways simultaneously. This is expected to maximize the utilization of the overall network capacity, as well as minimize the delays in WMNs.

Chapter 4

Connectivity Model

As previously discussed in Section 2.3, *network connectivity* refers to the probability with which nodes in a WMN are connected; a node is said to be *connected* if there exists at least one path between that node and every other node in the network.

In this chapter, we start by presenting a methodology to derive a closed form upper bound for the node connectivity in wireless networks taking into account dependencies in nodes' and links' connectivity. Section 4.1 defines the problem. We apply the devised methodology and evaluate our model for two interesting network topologies. First, Section 4.2 presents the derivation of the closed form expression, as well as present results for a simplistic yet practical chain topology. Section 4.3 presents the derivation of the closed form expression, as well as present results for a more elaborate network whose topology is based on the optimal triangular lattice. In Section 4.4, we extend the presented methodology to include the effect of the reduced connectivity of border nodes in the network connectivity of triangular lattice topologies.

4.1 Problem Definition

A network formed by a set of nodes located in an area $\mathbf{A}$ is said to be fully connected if and only if there is at least one path between each pair of nodes in $\mathbf{A}$. For a path to exist between a pair of wireless nodes, all links along the path must be connected. Hence, in wireless networks, connectivity is usually expressed in terms of the probability of individual links' connectivities.

In radio links, the probability that a link exist between two nodes depend among other things on the euclidian distance between the two nodes. Hence, link connectivity in one direction implicitly depends on that of the opposite direction. The amount of correlation, or the extent of dependency between the two unidirectional links is affected by several factors such as multi-path fading, shadowing, and the difference between the nodes' transmitting powers.

Hence, we present a methodology for finding a closed form expression for the connectivity in wireless networks while taking into account inter-node links' dependencies. We also evaluate the accuracy of the independence assumption.

A wireless network is composed of nodes that are allocated in an area $\mathbf{A}$ according to the topology $\mathbf{T}$. Define the following:

- $\mathbb{L}$ to be the set of wireless nodes in $\mathbf{A}$. $\mathbb{L} = \{1, 2, \dots, L\}$.
- $\mathbb{D}_i \subset \mathbb{L}$ to be the set of nodes that are neighbors of *node* $i \in \mathbb{L}$, that is, nodes

that are one-hop away from *node i*. *Node j* is said to be a neighbor of *node i*, or one hop-away from *node i*, if the path from *i* to *j* transverses exactly one wireless link.

- l_{ij} to be the event that a link exists between *node i* and *node j*, where $i, j \in \mathbb{L}$.
- r_{ij} to be the probability that a link exists between *node i* and *node j*, that is $P(l_{ij})$, $\forall i, j \in \mathbb{L}$, where $P(e)$ is the probability of event e happening.

4.1.1 Basic Assumptions

Some realistic assumptions were made, these are:

- We consider interference as a capacity-affecting factor that reduces the capacity of a link instead of lowering the probability of connectivity.
- Nodes are equidistantly distanced apart, and that $\mathbf{A}$ has homogenous shadowing and fading conditions. Hence, $P(l_{ij}) = r_{ij} = r$, $\forall i \in \mathbb{L}$ and $\forall j \in \mathbb{D}_i$.
- Symmetric bidirectional links that are fully dependant. That is, $P(l_{ij}) = P(l_{ji})$ $\forall i, j \in \mathbb{L}$.
- $P(l_{ij}) = 0 \quad \forall j \notin \mathbb{D}_i$, i.e., *node i* can only be connected through one or more of its neighbors (i.e., nodes in $\mathbb{D}_i$).

The first assumption is valid, and follows from the findings of many researchers such as [33] and [34]. The condition of equidistantly located nodes is necessary for topology

optimality [35]. The assumption of homogenous shadowing and fading conditions is usually valid for a confined space that possess similar topological properties. The third assumption follows from the second assumption, and we conjecture that it yields more accurate results compared to the common assumption of totally independent bidirectional links since the correlation between bidirectional links is very high. The effect of the fourth assumption is negligible since at best, the received power at a node is inversely proportional to d^4 , where d is the internode distance.

4.1.2 Upper Bound for Connectivity

Finding the exact connectivity of a wireless network is very costly in terms of computational complexity. Instead, we find the probability that none of the nodes in the network are isolated, $P(\text{no node isolated})$. A node is not isolated if it is connected to at least one of the other nodes in the network. Therefore,

$$P(\text{no node isolated}) = P(n_1, n_2, \dots, n_L), \quad (4.1)$$

where n_i is a random variable that denotes whether *node i* is isolated or not, that is,

$$n_i = \begin{cases} 1 & \text{if node } i \text{ is not isolated with probability } P(n_i) \\ 0 & \text{if node } i \text{ is isolated with probability } 1 - P(n_i), \end{cases} \quad (4.2)$$

where

$$P(n_i) \equiv P(\text{at least one link } (l_{ij} : \forall j \in D_i) \text{ is connected}). \quad (4.3)$$

Since a situation may arise where one or more islands of nodes are formed causing the network to be partitioned, $P(\text{no node isolated})$ is an upper bound for the connectivity of a network, that is,

$$P(\text{network connected}) \leq P(\text{no node isolated}). \quad (4.4)$$

According to Erdős and Rényi [36], this bound for connectivity is tight for nodes whose topology are purely random. Intuitively, this bound is as tight, or even tighter for a network of fixed topology, and where nodes are placed equidistantly apart.

4.1.3 Link and Node Independence

In this chapter, we compare our computed upper bound for connectivity, $P(\text{no node isolated})$, that takes into account links' dependencies, with the claimed upper bound, $P^*(\text{no node isolated})$ commonly utilized in the literature, and which assumes links' independence. The probability that none of the nodes in a given network are isolated, assuming independent links, is given by

$$\begin{aligned} P^*(\text{no node isolated}) &= \prod_{\forall i \in \mathbb{L}} P(n_i) \\ &= \prod_{\forall i \in \mathbb{L}} (1 - P(\overline{n_i})) \\ &= \prod_{\forall i \in \mathbb{L}} (1 - P(\text{all links } (l_{ij} : \forall j \in D_i) \text{ not connected})) \\ &= \prod_{\forall i \in \mathbb{L}} \left(1 - \prod_{\forall j \in \mathbb{D}_i} P(\overline{l_{ij}}) \right), \end{aligned} \quad (4.5)$$

where $\bar{e}$ denotes the complement of event e ; the first line follows from applying the assumption of node independence on (4.1); the second line follows the complement rule; the third line follows from applying the complement rule on (4.3); the fourth line follows from applying the assumption of link independence.

4.2 Connectivity in a Chain Topology

4.2.1 Problem Formulation

To demonstrate the devised methodology, we start by applying it to a network whose topology, $\mathbf{T}$, is composed of a chain of nodes similar to the one shown in Figure 4.1. Such network topology is widely found in rural areas; wireless nodes are typically mounted on electricity and lighting poles along highways, and network traffic is communicated back and forth in a point-to-point fashion. This topology is widely utilized by many researchers, such as in [10] and [37], for its simplicity and ease of modeling.

Figure 4.1: Chain Topology

The goal is to derive the joint probability in (4.1) for this network. This probability

is given by

$$\begin{aligned}
P(n_1, n_2, \dots, n_L) &= P(n_1|n_2, \dots, n_L) \cdot P(n_2, \dots, n_L) \\
&= P(n_1|n_2) \cdot P(n_2, \dots, n_L) \\
&= P(n_1|n_2) \cdot P(n_2|n_3) \cdots P(n_{L-1}|n_L) \cdot P(n_L), \quad (4.6)
\end{aligned}$$

where the first line follows from the definition of conditional probability; the second line is due to the fact that n_1 depends only on n_2 and is independent of all other nodes' connectivities; the third line is obtained by iterating these steps.

Furthermore,

$$P(n_i|n_{i+1}) = \frac{P(n_i, n_{i+1})}{P(n_{i+1})} \quad \text{for } i < L \quad (4.7)$$

also using the definition of conditional probability.

Hence, to find $P(n_i|n_{i+1})$, we need to derive the terms $P(n_i, n_{i+1})$ and $P(n_{i+1})$ for the various nodes. We define A_i as the event that node i is not isolated. There are three cases to consider.

Case 1: $i = 1$

node 1 is only neighbored by *node 2*, hence,

$$A_1 = l_{12}. \quad (4.8)$$

node 2 has two neighbors, *node 1* from one side, and *node 3* from the other, hence,

$$A_2 = (l_{21} \cap l_{23}) \cup (l_{21} \cap \overline{l_{23}}) \cup (\overline{l_{21}} \cap l_{23}). \quad (4.9)$$

Therefore,

$$\begin{aligned}
A_1 \cap A_2 &= l_{12} \cap \left((l_{21} \cap l_{23}) \cup (l_{21} \cap \overline{l_{23}}) \cup (\overline{l_{21}} \cap l_{23}) \right) \\
&= (l_{12} \cap l_{21} \cap l_{23}) \cup (l_{12} \cap l_{21} \cap \overline{l_{23}}) \cup (l_{12} \cap \overline{l_{21}} \cap l_{23}) \\
&= (l_{12} \cap l_{21} \cap l_{23}) \cup (l_{12} \cap l_{21} \cap \overline{l_{23}}) \\
&= (l_{12} \cap l_{23}) \cup (l_{12} \cap \overline{l_{23}}), \tag{4.10}
\end{aligned}$$

where the second line is obtained by using the principle of distributivity; the third and fourth lines follow from the assumption of symmetric bidirectional links, that is, $l_{12} \equiv l_{21}$, and hence $l_{12} \cap \overline{l_{21}} = \emptyset$.

The probability that *node* 1 and *node* 2 are not isolated is given by

$$\begin{aligned}
P(n_1, n_2) &= P(A_1 \cap A_2) \\
&= P\left((l_{12} \cap l_{23}) \cup (l_{12} \cap \overline{l_{23}})\right) \\
&= P(l_{12} \cap l_{23}) + P(l_{12} \cap \overline{l_{23}}) \\
&= P(l_{12})P(l_{23}) + P(l_{12})P(\overline{l_{23}}) \\
&= r^2 + r(1 - r), \tag{4.11}
\end{aligned}$$

since l_{12} is independent of l_{23} and $P(\overline{l_{23}}) = 1 - P(l_{23}) = 1 - r$.

Similarly,

$$\begin{aligned}
P(n_2) &= P(A_2) \\
&= P\left((l_{21} \cap l_{23}) \cup (l_{21} \cap \overline{l_{23}}) \cup (\overline{l_{21}} \cap l_{23})\right) \\
&= P(l_{21})P(l_{23}) + P(l_{21})P(\overline{l_{23}}) + P(\overline{l_{21}})P(l_{23}) \\
&= r^2 + r(1-r) + (1-r)r \\
&= r^2 + 2r(1-r).
\end{aligned} \tag{4.12}$$

Case 2: $2 \leq i \leq L-2$

Here, *nodes* i and $i+1$ will both have a neighboring node on each side, hence A_i and A_{i+1} will be of the same form as A_2 :

$$A_i = (l_{ii-1} \cap l_{ii+1}) \cup (l_{ii-1} \cap \overline{l_{ii+1}}) \cup (\overline{l_{ii-1}} \cap l_{ii+1}) \tag{4.13}$$

$$A_{i+1} = (l_{i+1i} \cap l_{i+1i+2}) \cup (l_{i+1i} \cap \overline{l_{i+1i+2}}) \cup (\overline{l_{i+1i}} \cap l_{i+1i+2}). \tag{4.14}$$

Therefore,

$$\begin{aligned}
A_i \cap A_{i+1} &= (l_{ii-1} \cap l_{ii+1} \cap l_{i+1i+2}) \cup \\
&\quad (l_{ii-1} \cap l_{ii+1} \cap \overline{l_{i+1i+2}}) \cup \\
&\quad (l_{ii-1} \cap \overline{l_{ii+1}} \cap l_{i+1i+2}) \cup \\
&\quad (\overline{l_{ii-1}} \cap l_{ii+1} \cap l_{i+1i+2}) \cup \\
&\quad (\overline{l_{ii-1}} \cap l_{ii+1} \cap \overline{l_{i+1i+2}}),
\end{aligned} \tag{4.15}$$

where the simplifications in the above equation follow from the symmetric bidirectional links assumption as discussed in Case 1. Hence for Case 2,

$$\begin{aligned}
P(n_i, n_{i+1}) &= P(A_i \cap A_{i+1}) \\
&= P(l_{ii-1})P(l_{ii+1})P(l_{i+1i+2}) + \\
&\quad P(l_{ii-1})P(l_{ii+1})P(\overline{l_{i+1i+2}}) + \\
&\quad P(l_{ii-1})P(\overline{l_{ii+1}})P(l_{i+1i+2}) + \\
&\quad P(\overline{l_{ii-1}})P(l_{ii+1})P(l_{i+1i+2}) + \\
&\quad P(\overline{l_{ii-1}})P(l_{ii+1})P(\overline{l_{i+1i+2}}) \\
&= r^3 + 3r^2(1-r) + r(1-r)^2,
\end{aligned} \tag{4.16}$$

and

$$\begin{aligned}
P(n_{i+1}) &= P(A_{i+1}) \\
&= r^2 + 2r(1-r).
\end{aligned} \tag{4.17}$$

Case 3: $i = L - 1$

node $L - 1$ will also have one neighbor on each side, and hence, A_{L-1} will be of the same form of A_i of Case 2:

$$A_{L-1} = (l_{L-1L-2} \cap l_{L-1L}) \cup (l_{L-1L-2} \cap \overline{l_{L-1L}}) \cup (\overline{l_{L-1L-2}} \cap l_{L-1L}). \tag{4.18}$$

node L will have only one neighbor, and hence A_L will be of the same form of A_1 , that is,

$$A_L = l_{LL-1}. \tag{4.19}$$

$A_{L-1} \cap A_L$ will take the same form of $A_1 \cap A_2$, that is,

$$A_{L-1} \cap A_L = (l_{L-1L-2} \cap l_{L-1L}) \cup (\overline{l_{L-1L-2}} \cap l_{L-1L}), \quad (4.20)$$

where the simplifications in the above equation follows again from the symmetric bidirectional links assumption as discussed in Case 1.

Hence,

$$\begin{aligned} P(n_{L-1}, n_L) &= P(A_{L-1} \cap A_L) \\ &= P(A_1 \cap A_2) \\ &= r^2 + r(1-r), \end{aligned} \quad (4.21)$$

and

$$\begin{aligned} P(n_L) &= P(A_L) \\ &= r. \end{aligned} \quad (4.22)$$

Substituting in (4.7) yields:

$$P(n_i | n_{i+1}) = \begin{cases} \frac{r^2 + r(1-r)}{r^2 + 2r(1-r)} & \text{if } i = 1; \\ \frac{r^3 + 3r^2(1-r) + r(1-r)^2}{r^2 + 2r(1-r)} & \text{if } 2 \leq i \leq L-2; \\ \frac{r^2 + r(1-r)}{r} & \text{if } i = L-1. \end{cases} \quad (4.23)$$

Finally, by substituting in (4.6), the probability that none of the nodes in a chain topology of L nodes are isolated is given by

$$P(n_1, n_2, \dots, n_L) = \frac{r^2 + r(1-r)}{r^2 + 2r(1-r)} \cdot \left(\frac{r^3 + 3r^2(1-r) + r(1-r)^2}{r^2 + 2r(1-r)} \right)^{L-3} \cdot r. \quad (4.24)$$

4.2.2 Model Evaluation

We computed the probability that none of the nodes are isolated for chain networks composed of 5, 20, and 50 nodes using our developed closed form solution that takes into account internode links' dependencies. We also compared our results to the case when independent node connectivity is assumed. We note that the internode distance across the various sized-networks is the same, hence, as the number of nodes in a network increase, so does the area $\mathbf{A}$; this is to keep the probability of link connectivity, r , constant across the different sized networks.

In assuming independence of links and nodes in a chain topology, (4.5) simplifies to

$$P^*(no\ node\ isolated) = r^2 \cdot (1 - (1-r)^2)^{L-2}, \quad (4.25)$$

which follows from the fact that both *node* 1 and *node* L have only one neighbor, and hence only one link. Each of the rest of the $L - 2$ nodes have two neighbors, and hence two links.

Figure 4.2: Probability that none of the nodes are isolated in a chain topology

Figure 4.2 plots three sets of results corresponding to three networks of 5, 20, and 50 nodes. For each set, we plot our closed form solution given by (4.24), and its independent link counterpart given by (4.25). Few observations are noteworthy. Firstly, the probability that none of the nodes in the network are isolated, $P(\text{no node isolated})$, increases non-linearly with increasing probability of link connectivity, r . Secondly, for a given value of r , $P(\text{no node isolated})$ decreases as the size of the network increase; this is expected since the larger the network, the greater the chances of at least one node becoming isolated. Moreover, the computed $P(\text{no node isolated})$ always exceeds the claimed upper bound

$P^*(no\ node\ isolated)$. Since relaxing the embedded assumption of link independence resulted in $P(no\ node\ isolated) > P^*(no\ node\ isolated)$, this signifies that $P^*(no\ node\ isolated)$ is an erroneous upper bound for connectivity, and that there are cases where the network connectivity can exceed $P^*(no\ node\ isolated)$ resulting in

$$P^*(no\ node\ isolated) < P(network\ connected) \leq P(no\ node\ isolated).$$

We hence established a new upper bound for the connectivity in a chain topology that can be computed using (4.24).

Figure 4.3 on the following page plots the percentage deviation in $P(no\ node\ isolated)$ due to the assumption of independent links, that is,

$$\%Deviation = \frac{P(no\ node\ isolated) - P^*(no\ node\ isolated)}{P(no\ node\ isolated)}. \quad (4.26)$$

We observe that the difference between the two bounds decreases as r increase. Thus, the assumption of independence in node connectivity is valid only in very high link connectivity conditions ($r > 0.9$). It is also evident that the deviation increases as the size of the network increase. The methodology can hence be used to quantify the deviation resulting from the independent links assumption, as well as identify the critical range of r at which the error is unacceptably high for a given application.

Figure 4.3: Percentage deviation in $P(\text{no node isolated})$ due to assuming link independence in a chain topology

4.3 Connectivity in a Triangular lattice

4.3.1 Problem Formulation

We now apply our methodology to find the connectivity in a network whose topology, $\mathbf{T}$, is a triangular lattice. This topology is obtained when nodes are placed on the vertices of an equilateral triangular lattice, or equivalently, at the centers of regular hexagons. Figure 4.4 on the next page demonstrates such topology by depicting the

nodes constituting the first two tiers. It was previously proven that the triangular lattice topology results in the minimum number of nodes needed to achieve full coverage of a plane [35].

Figure 4.4: Nodes in a Triangular Lattice Topology

Again, the goal is to derive the joint probability in (4.1) for this network. We use an approach similar to the one we used in finding the connectivity in a chain topology in Section 4.2.

We can re-write (4.1) as

$$\begin{aligned}
 P(n_1, n_2, \dots, n_L) &= P(n_1 | n_2, \dots, n_L) \cdot P(n_2, \dots, n_L) \\
 &= \prod_{i=1}^{L-1} P(n_i | n_{i+1}, \dots, n_L) \cdot P(n_L), \quad (4.27)
 \end{aligned}$$

where the first line follows from the definition of conditional probability, and the second line is obtained by iteratively applying the definition of conditional probability.

- Define $\mathbb{L}_i \subset \mathbb{L}$ to be the set of nodes that have not yet been considered (i.e., whose marginal probability was not yet computed). Then,

$$\mathbb{L}_i = \{i + 1, i + 2, \dots, L\},$$

where the nodes in $\mathbf{T}$ are numbered in sequence from the center outwards, starting with *node* 1 for the center node as in Figure 4.6 on page 51.

- Define the set $\mathbb{Q}_i \subseteq \mathbb{D}_i$ to be the set of all neighboring nodes of i that have not yet been considered (i.e., whose marginal probability was not yet computed). Then,

$$\mathbb{Q}_i = \mathbb{L}_i \cap \mathbb{D}_i.$$

- Define $q_\xi^i \in \mathbb{Q}_i$ to indicate the ξ th element in $\mathbb{Q}_i$, and $N \equiv |\mathbb{Q}_i|$.

Hence, (4.27) simplifies to

$$P(n_1, n_2, \dots, n_L) = \prod_{i=1}^{L-1} P(n_i | n_{q_1^i}, \dots, n_{q_N^i}) \cdot P(n_L), \quad (4.28)$$

which follows from the assumption that *node* i can only be connected through one or more of its neighbors, that is, n_i depends only on nodes in $\mathbb{Q}_i$.

Furthermore,

$$P(n_i | n_{q_1^i}, \dots, n_{q_N^i}) = \frac{P(n_i, n_{q_1^i}, \dots, n_{q_N^i})}{P(n_{q_1^i}, \dots, n_{q_N^i})} \quad \text{for } i < L \quad (4.29)$$

also using the definition of conditional probability.

Figure 4.5: Location of various node types

Depending on the location of *node i* in the topology, the number of elements in the set Q_i will be different. Therefore, the quotient resulting from (4.29) will differ depending on the location of *node i*. Because of the assumption of equal network-wide link connectivity, only the number of elements in Q_i matters. The exact node id is irrelevant. In order to simplify matters, we leverage on the geometry and recursive decomposition properties of the triangular lattice topology. Similar to most research, we also ignore the border effect of the outer most tier (nodes in the outer tier only have three or four neighbors); in a way, we assume that the network exists in an infinitely large topology, so every node has 6 neighbors. Hence, there are

Figure 4.6: Neighbors and Dependencies for various node types

four cases to consider; refer to Figures 4.5 and 4.6.

Case 1: *node 1*

We start by first considering the node located at the center of the topology, that is *node 1*. $\mathbb{D}_1$ similar to $\mathbb{D}_i$ for all nodes i will be comprised of the six nodes surrounding *node i*. Since *node 1* is the first node to be considered in the computation, $\mathbb{L}_1$ will contain all nodes in the topology except this center node. Hence, $\mathbb{Q}_1 = \mathbb{L}_1 \cap \mathbb{D}_1 = \mathbb{D}_1$. That is, $\mathbb{Q}_1$ is comprised of all six nodes neighboring *node 1*, and $|\mathbb{Q}_1| = 6$. There are 12 link dependencies as denoted by the dashes in Figure 4.6.

Case 2: nodes of type β

We find that for each node of type β , Q_i is comprised of four of the six nodes neighboring it, that is $|Q_i| = 4$. The two remaining nodes are not elements of L_i (they were already considered in the previous term and hence eliminated from the consequent terms), and therefore are not elements in Q_i . There are 7 link dependencies as denoted by the dashes in Figure 4.6 on the previous page.

Note that when determining the marginal probability of the first node in a tier, it might be rather intuitive to think that it has one additional element of its D_i neighbors included in its Q_i (i.e., L_i for the first node can include one additional node than the rest). However, this effect will be exactly offset by the fact that the last node in the tier (the one adjacent to the first one from the other side) will have one less element of its D_i included in its Q_i . For example, if we considered that the first node in tier 1 has $|Q_i| = 5$, then the four next nodes will have $|Q_i| = 4$, and the sixth node will have $|Q_i| = 3$. The effect of this on the joint probability of the six nodes constituting tier 1 is exactly equivalent to assuming that $|Q_i| = 4$ for all six nodes, given that the link connectivity is the same for all links involved.

Case 3: nodes of type γ

For nodes of type γ , we find that Q_i is comprised of three of the six neighboring nodes, that is $|Q_i| = 3$. The three remaining nodes are not elements of L_i (they were already considered in the computation), and are consequently not elements in Q_i .

There are 5 link dependencies as denoted by the dashes in Figure 4.6 on page 51.

Case 4: *node L*

Lastly, we note that we need to find $P(n_L)$ in order to solve (4.28). However, no dependencies need to be considered since we do not need to find *node L*'s marginal probability.

For a triangular lattice topology whose number of tiers, T , is greater than 1, there are exactly the following occurrences of the above node types:

- *node 1*: 1 occurrence;
- node type β : $6T - 1$ occurrences;
- node type γ : $\sum_{t=1}^T 6(t - 1) = 3T(T - 1)$ occurrences.
- *node L*: 1 occurrence.

Re-writing (4.28) in terms of *nodes* 1, β , γ , and L yields

$$\begin{aligned}
 P(n_1, n_2, \dots, n_L) &= P(n_1 | n_2, \dots, n_7) \cdot \\
 &\quad P(n_\beta | n_{q_1^\beta}, n_{q_2^\beta}, n_{q_3^\beta}, n_{q_4^\beta})^{6T-1} \cdot \\
 &\quad P(n_\gamma | n_{q_1^\gamma}, n_{q_2^\gamma}, n_{q_3^\gamma})^{3T(T-1)} \cdot \\
 &\quad P(n_L).
 \end{aligned} \tag{4.30}$$

Finally substituting (4.29) into each of the conditional probabilities in (4.30) yields

$$\begin{aligned}
 P(n_1, n_2, \dots, n_L) &= \frac{P(n_1, n_2, \dots, n_7)}{P(n_2, \dots, n_7)} \cdot \\
 &\quad \left(\frac{P(n_\beta, n_{q_1^\beta}, n_{q_2^\beta}, n_{q_3^\beta}, n_{q_4^\beta})}{P(n_{q_1^\beta}, n_{q_2^\beta}, n_{q_3^\beta}, n_{q_4^\beta})} \right)^{6T-1} \cdot \\
 &\quad \left(\frac{P(n_\gamma, n_{q_1^\gamma}, n_{q_2^\gamma}, n_{q_3^\gamma})}{P(n_{q_1^\gamma}, n_{q_2^\gamma}, n_{q_3^\gamma})} \right)^{3T(T-1)} \cdot \\
 &\quad P(n_L). \tag{4.31}
 \end{aligned}$$

4.3.2 Computational Complexity

The complexity in computing the joint probability grows exponentially with the number of nodes in the network; the computational complexity in calculating $P(n_i, \dots, n_j)$ is $O(c^{j-i+1})$, where the constant c represents the number of events preventing a node in a given topology from becoming isolated. Also note that c grows exponentially with the topology, that is $c = 2^{d_{\mathbf{T}}} - 1$, where $d_{\mathbf{T}}$ = degree of a node in topology $\mathbf{T}$ ($d_{\mathbf{T}} = 6$ for a triangular lattice topology).

The manual computation of (4.31) is quite cumbersome. Hence, we developed a computer program and utilized a cluster of processors to automate and speed up the process. Due to the complexity of the resulting computation, we have only included the closed form expression of the denominator of the first term in (4.31). Refer to Appendix A for the rest of the computations.

$$\begin{aligned}
P(n_2, \dots, n_7) = & \\
& 2r^3q^{27} + 249r^4q^{26} + 6582r^5q^{25} + 76365r^6q^{24} + \\
& 505440r^7q^{23} + 2253024r^8q^{22} + 7432948r^9q^{21} + \\
& 19215999r^{10}q^{20} + 40387200r^{11}q^{19} + \\
& 70736989r^{12}q^{18} + 105022794r^{13}q^{17} + \\
& 133752441r^{14}q^{16} + 147303096r^{15}q^{15} + \\
& 141017040r^{16}q^{14} + 117684414r^{17}q^{13} + \\
& 85685772r^{18}q^{12} + 54372282r^{19}q^{11} + \\
& 29981259r^{20}q^{10} + 14295006r^{21}q^9 + 5851269r^{22}q^8 + \\
& 2035656r^{23}q^7 + 593769r^{24}q^6 + 142506r^{25}q^5 + \\
& 27405r^{26}q^4 + 4060r^{27}q^3 + 435r^{28}q^2 + 30r^{29}q + r^{30}, \tag{4.32}
\end{aligned}$$

where $q = 1 - r$. While understanding the resulting computation given in (4.32) is not straight forward, we will give few brief insights. We note that the last term represents the probability of one extreme event occurring, that is when all 36 links (6 links/node $\times$ 6 nodes) are connected. Since 6 out of the 36 inter-node links are common (i.e., dependant), the probability of such event occurring is r^{30} . Similarly, the first term represents the probability of the other extreme event, i.e., when the 6 nodes are connected just enough for none of them to be isolated. This event occurs

when only links between 3 pairs of adjacent nodes (i.e., nodes 2-3, 4-5, 6-7 or 7-2, 3-4, 5-6) are connected, and the remaining 27 links are not connected. Since such event can occur in two different ways, the resulting probability is $2r^3q^{27}$. The middle terms in (4.32) represent the probabilities of all other combinations between the two discussed extremes.

4.3.3 Model Evaluation

We present the probability that none of the nodes are isolated for networks composed of 1, 3, and 5 tiers found using our generated closed form solution that takes into account internode links dependencies. We compare our results to computations based on the common assumption of internode link independence. We again note that the internode distance across the various sized-networks is the same, hence, as the number of tiers (or nodes) in a network increase, so does the area **A**.

In assuming independence of links and nodes in a triangular lattice topology, (4.5) simplifies to

$$P^*(no\ node\ isolated) = \left(1 - (1 - r)^6\right)^{1+3T(T+1)}. \quad (4.33)$$

The simplification is due to the fact that all nodes in the topology have 6 neighbors; the total number of nodes is $1 + \sum_{t=1}^T 6t = 1 + 3T(T+1)$ (1 added for the center node).

Figure 4.7 on the next page plots three sets of results corresponding to the three

Figure 4.7: Probability that none of the nodes are isolated in a triangular lattice topology

sized networks. For each set, we plot our closed form solution given by the computation resulting from (4.31), and its independent link counterpart given by (4.33). We first observe that $P(\text{no node isolated})$ increases with increasing probability of link connectivity, r , as expected. Furthermore, for a given value of r , $P(\text{no node isolated})$ decreases as the size of the network increase. Similar to the case of the chain topology, a larger sized network enhances the chances of at least one node in the network of becoming isolated. Lastly, the computed $P(\text{no node isolated})$ again exceeds the claimed upper bound $P^*(\text{no node isolated})$. Therefore, we proved that there are cases when

the network connectivity can exceed $P^*(no\ node\ isolated)$, and hence established a new upper bound that can be computed using (4.31).

Figure 4.8: Percentage deviation in $P(no\ node\ isolated)$ due to assuming link independence in a triangular lattice topology

Figure 4.8 plots the percentage deviation in $P(no\ node\ isolated)$ due to the assumption of independent links and nodes connectivities. The percentage deviation is calculated according to (4.26). It is evident that the deviation decreases with increasing values of r ; it is almost negligible for $r \geq 0.6$. It is also evident that the error increases as the network size increase. Hence, for a given network size, we can

quantify the deviation resulting from the assumption of link and node independence, and identify the critical range of r at which the error is unacceptable for a given application.

4.4 Modeling the Finite Network Boundary

In the literature, the effect of reduced node connectivity of border nodes was usually avoided by assuming that the networks examined were infinitely large. Border nodes have less neighbors, and hence, less chances of being connected. The embedded assumption here is that the effect of such nodes on the overall network connectivity in an infinitely large network is negligible.

In this section, we argue against that. We build upon the methodology presented earlier in this chapter and include in our connectivity model the effect of the reduced connectivity of border nodes in a triangular lattice topology (i.e., in addition to the effect of interdependent nodes). The derivation of the expressions are presented in Section 4.4.1. We evaluate our extended model, and investigate the impact of ignoring the effect of border nodes on the overall network connectivity in Section 4.4.2.

4.4.1 Extended Connectivity Model

Figure 4.9: Location of various node types in the finite topology

We now extend the model presented in Section 4.3 to take into account the effect of a finite network. In such network, nodes belonging to the outer tier (i.e., nodes at the border of the network) will have reduced connectivity due to the reduced number of neighbors.

Figure 4.9 shows the locations of the various node types in a finite triangular lattice topology whose number of tiers, T is equal to 3. The characteristics of nodes 1, β , and γ are identical to those described in Section 4.3. However, two

Figure 4.10: Neighbors and Dependencies for additional node types

additional node types, λ and λ° , will need to be considered in the computation of the joint probability, $P(n_1, n_2, \dots, n_L)$. Therefore, in addition to the four cases described in Section 4.3.1, we describe below an additional fifth case resulting from the two new node types.

Case 5: nodes of type λ and λ°

Consider nodes comprising the outer tier of a finite triangular lattice. We find that these nodes can only have either three neighbors (i.e., nodes λ° with $|\mathbb{D}_i| = 3$), or four neighbors (i.e., nodes λ with $|\mathbb{D}_i| = 4$). Out of these three or four neighbors, only one (the node that follows in the same tier) remains an element in $\mathbb{L}_i$, the rest were already considered (i.e., their marginal probabilities were computed). Hence, $|\mathbb{Q}_i| = 1$ for nodes of type λ and λ° . Therefore, there is a single link dependency as denoted by the dash in Figure 4.10.

Note that we do not need to find *node L*'s marginal probability, and hence no dependencies need to be considered. However, *node L* only have three neighbors (i.e., $|\mathbb{D}_i| = 3$) in this finite topology, and will therefore result in a $P(n_L)$ less than that of

an infinite topology.

For a finite triangular lattice topology of $T > 1$ tiers, there are exactly the following occurrences of the various node types (and combinations), each of which will have a different marginal probability:

- *node 1*: 1 occurrence;
- node type β : $6(T - 1)$ occurrences;
- node type γ : $\sum_{t=2}^{T-1} 6(t - 1) = 3T^2 - 9T + 6$ occurrences;
- node type λ followed by λ : $6(T - 2)$ occurrences;
- node type λ followed by λ° (or L): 6 occurrences;
- node type λ° followed by λ : 5 occurrences.
- *node L*: 1 occurrence.

Hence, re-writing (4.28) in terms of *nodes* 1, β , γ , λ° , λ , and L yields

$$\begin{aligned}
 P(n_1, n_2, \dots, n_L) &= P(n_1 | n_2, \dots, n_T) \cdot \\
 &\quad P(n_\beta | n_{q_1^\beta}, n_{q_2^\beta}, n_{q_3^\beta}, n_{q_4^\beta})^{6(T-1)} \cdot \\
 &\quad P(n_\gamma | n_{q_1^\gamma}, n_{q_2^\gamma}, n_{q_3^\gamma})^{3T^2-9T+6} \cdot \\
 &\quad P(n_\lambda | n_{q_1^\lambda})^{6(T-2)} \cdot P(n_\lambda | n_{q_1^{\lambda^\circ}})^6 \cdot P(n_{\lambda^\circ} | n_{q_1^\lambda})^5 \cdot \\
 &\quad P(n_L).
 \end{aligned} \tag{4.34}$$

Finally substituting (4.29) into each of the conditional probabilities in (4.34) yields

$$\begin{aligned}
 P(n_1, n_2, \dots, n_L) &= \frac{P(n_1, n_2, \dots, n_7)}{P(n_2, \dots, n_7)} \cdot \\
 &\quad \left(\frac{P(n_\beta, n_{q_1^\beta}, n_{q_2^\beta}, n_{q_3^\beta}, n_{q_4^\beta})}{P(n_{q_1^\beta}, n_{q_2^\beta}, n_{q_3^\beta}, n_{q_4^\beta})} \right)^{6(T-1)} \cdot \\
 &\quad \left(\frac{P(n_\gamma, n_{q_1^\gamma}, n_{q_2^\gamma}, n_{q_3^\gamma})}{P(n_{q_1^\gamma}, n_{q_2^\gamma}, n_{q_3^\gamma})} \right)^{3T^2-9T+6} \cdot \\
 &\quad \left(\frac{P(n_\lambda, n_{q_1^\lambda})}{P(n_{q_1^\lambda})} \right)^{6(T-2)} \cdot \left(\frac{P(n_\lambda, n_{q_1^{\lambda^\circ}})}{P(n_{q_1^{\lambda^\circ}})} \right)^6 \cdot \left(\frac{P(n_{\lambda^\circ}, n_{q_1^\lambda})}{P(n_{q_1^\lambda})} \right)^5 \cdot \\
 &\quad P(n_L). \tag{4.35}
 \end{aligned}$$

The expressions resulting from computing (4.35) are presented in Appendix B due to their complexity and length.

4.4.2 Model Evaluation

Figure 4.11: Probability that none of the nodes are isolated in a finite triangular lattice topology

We first plot in Figure 4.11 our closed form expression resulting from (4.35). In the same figure, the closed form expression resulting from (4.31) which takes into account the effect of link dependence but ignored the effect of a finite boundary is given for comparison. It is evident that $P(\text{no node isolated})$ is erroneously higher when an infinite boundary is assumed. This is contributed to the fact that the outer tier nodes are only surrounded by three or four neighbors (versus six when an infinite boundary is assumed), therefore causing a significant reduction in connectivity.

Figure 4.12: Percentage deviation in $P(\text{no node isolated})$ due to assuming an infinite boundary

Figure 4.12 plots the percentage deviation in $P(\text{no node isolated})$ due to neglecting the effect of boundary nodes, that is the difference between (4.35) and (4.31). We observe that the deviation only drops to 10% when $r \geq 0.75$. Figure 4.12 also shows that the percentage deviation due to this assumption increases rapidly with the size of the network. Hence, the assumption that the effect of border nodes is negligible for infinitely large networks is inaccurate.

Chapter 5

Optimal Network Planning

In this chapter, we propose a methodology to find the optimal network plan in terms of node class selection and number of nodes required to achieve the given service requirements in a wireless mesh network. The network is said to be optimal if it minimizes the network implementation cost while achieving the application matrix requirements. In this work, these requirements are described in terms of communication range, minimum network connectivity, and achievable throughput. A node class is generally characterized by its capabilities, such as transmitted power and receiver sensitivity, and its cost.

Section 5.1 gives a brief overview of the system description and the problem being investigated. Section 5.2 presents the detailed methodology for finding the optimal number and class selection of mesh nodes. We apply the devised methodology and present the results for two common topologies: the chain topology in Section 5.3, and the triangular lattice topology in Section 5.4.

5.1 System Description

5.1.1 Model Description and Problem Statement

Typically, the goal of any network planning team looking into deploying a WMN, is to provide a network capable of warranting its mesh clients a *minimum level of service requirements*, while minimizing the cost of deploying the network. This is achieved by allocating an optimal number of nodes from the optimal *node class* such that the *total network cost* is minimized.

A *node class* is characterized by its capabilities and unit price. The capabilities of a node is usually related to its radio's specifications, such as transmit power, p_t , and receiver sensitivity, γ .

The *total network cost* maybe determined by many factors: they typically include direct expenses such as cost of deployed nodes, installation, and power consumption, but may also include cost associated with other considerations such as potential interference and maintenance.

Define $\mathcal{R}_{min}$ to be the network connectivity requirement. That is, at any given time, all nodes in the WMN can communicate with each other with a probability at least equal to $\mathcal{R}_{min}$.

The goal of this chapter is to devise a methodology to find:

1. the optimal number of mesh nodes to deploy, and
2. the optimal class selection of mesh nodes, such that

a given network connectivity requirement, $\mathfrak{R}_{min}$, is achieved while minimizing the total network cost.

5.1.2 Modeling Network Connectivity

In order to optimize the number of nodes and class selection for a given network connectivity requirement, $\mathfrak{R}_{min}$, we need to assess the network connectivity, $\mathfrak{R}$, for any network configuration. Network connectivity, $\mathfrak{R}$, is usually given by the probability that a network is connected. A network formed by a set of nodes located in space $\mathbf{S}$ is said to be fully connected if and only if there is a path between each pair of nodes in $\mathbf{S}$ [8].

We will use the connectivity model and closed form solutions presented in Chapters 4 which take into account dependencies existing between links of geometrically co-located nodes, as well as the effect of a finite network size, to estimate the network connectivity, $\mathfrak{R}$.

5.2 Problem Formulation

A wireless mesh network is composed of nodes that are allocated in space $\mathbf{S}$ according to topology $\mathbf{T}$. Define the following:

- $\mathbf{X}$ to be the set of all mesh node classes available for the planning team to choose from.
- L to be the number of nodes in $\mathbf{S}$, and $\mathbb{L}$ is the set of indices $\mathbb{L} = \{1, 2, \dots, L\}$.
- r_{ij} to be the probability that a link exists between *node i* and *node j*, $\forall i, j \in \mathbb{L}$.

5.2.1 Basic Assumptions

1. All nodes in a network belong to the same node class $x \in \mathbf{X}$; that is, all nodes in the topology possess identical characteristics and capabilities (e.g., transmit power, receive sensitivity).
2. Mesh nodes are placed equidistantly apart. Nodes that are neighbors, or one-hop away, are separated by internode distance d .
3. $\mathbf{S}$ has homogenous shadowing and fading conditions. Hence, $r_{ij} = r$, $\forall i, j \in \mathbb{L}$ where *node i* and *node j* are one-hop away.
4. *node i* can only be connected through one or more of its neighbors. Hence, $r_{ij} = 0$, $\forall i, j \in \mathbb{L}$ where *node i* and *node j* are not one-hop away.

5. All mesh nodes in the network are always operational.
6. All nodes use omni-directional antennas with transmitter and receiver gain equal to 1. The system is an ideal one, with system loss equal to 1.

The first assumption is a valid one since it is often the case that network planning teams choose to deploy a specific type of mesh nodes from the same vendor throughout the network. The second assumption is a necessary condition for topology optimality [35]. The third assumption is valid for a confined space that possess similar topological properties. The effect of the fourth assumption is negligible since at best, the received power at a node is inversely proportional to d^4 , where d is the internode distance. The fifth assumption is imposed to isolate our problem from issues involving fault tolerance, which is a different problem on its own. The last assumption is for sake of normalization.

5.2.2 Cost Function

We model the *total network cost*, C_x , where $x \in \mathbb{X}$ is the node class deployed, as a function of the number of nodes, n_x , and the unit price, u_x . In this work, we set C_x to reflect the financial cost of the nodes, that is,

$$C_x = n_x \cdot u_x. \quad (5.1)$$

u_x is known for all $x \in \mathbb{X}$, and will have a direct influence on the total network cost. n_x on the other hand influences both the network cost, C_x , as well as the network connectivity, $\mathfrak{R}$.

5.2.3 Network Connectivity Formulation

As alluded to earlier, we will use the tight upper bound $P(\text{no node isolated})$ to denote the connectivity, $\mathfrak{R}$, of a given network, that is,

$$\mathfrak{R} \leq P(\text{no node isolated}). \quad (5.2)$$

The probability that no node is isolated is a function of the probability that a link exists between any two neighbors, r , and the number of nodes, n , constituting the network, that is,

$$P(\text{no node isolated}) = f(r, n), \quad (5.3)$$

where $f(r, n)$ was derived for the chain and triangular lattice topologies in Chapter 4, and is given by (4.24) and (4.35) respectively.

The number of nodes in a network, n , is a function of the space, $\mathbf{S}$, and the internode distance, d , that is,

$$n = g(\mathbf{S}, d), \quad (5.4)$$

where the function $g(\mathbf{S}, d)$ is defined by the geometry of the topology $\mathbf{T}$.

The probability, r , that a link exists between two nodes that are one-hop away is given by

$$r = P[p_r(d) > \gamma], \quad (5.5)$$

where $p_r(d)$ is the random variable representing the received signal power of any link of length d . γ is the minimum signal threshold level needed to achieve a given throughput; the value of γ is dependant on the physical layer technology of the node's receiver.

The received power at a certain distance is affected by several components including pathloss, shadowing, and multipath fading [38]. All connectivity related work prior to [8] did not include the effect of shadowing. However, the stochastic behavior of the shadowing component can reduce a network's connectivity. Thus, in this work, we use the log-normal shadowing model [38], [39]; a widely used model that captures the various effects of the wireless channel. The measured loss, $L(d)$, in dB at a distance d from the transmitter, is

$$L(d) = \bar{L}(d_o) + 10\alpha \log \left(\frac{d}{d_o} \right) + \epsilon_\sigma, \quad (5.6)$$

where d_o is a reference distance for which we have a mean measured loss, $\bar{L}(d_o)$, measured in dB. α is the pathloss exponent particular to the propagation scenario. ϵ_σ is a zero mean Gaussian random variable with standard deviation σ ; it represents the shadowing deviation and is measured in dB.

Hence, the received signal power in dBm at distance d from the transmitter is

$$p_r(d) = p_t - \bar{L}(d_o) - 10\alpha \log\left(\frac{d}{d_o}\right) + \epsilon_\sigma, \quad (5.7)$$

where p_t is the transmitter power in dBm.

Combining Equations (5.5) and (5.7) yields

$$\begin{aligned} r &= P[p_r(d) > \gamma] \\ &= P\left[p_t - \bar{L}(d_o) - 10\alpha \log\left(\frac{d}{d_o}\right) + \epsilon_\sigma > \gamma\right] \\ &= P\left[\epsilon_\sigma > \gamma - p_t + \bar{L}(d_o) + 10\alpha \log\left(\frac{d}{d_o}\right)\right] \\ &= \frac{1}{2} \operatorname{erfc}\left(\frac{\gamma - p_t + \bar{L}(d_o) + 10\alpha \log\left(\frac{d}{d_o}\right)}{\sigma\sqrt{2}}\right), \end{aligned} \quad (5.8)$$

where $\operatorname{erfc}(\cdot)$ denotes the complementary error function defined as

$$\operatorname{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-\frac{t^2}{2}} dt. \quad (5.9)$$

5.2.4 Optimal Number of Nodes

Our first goal is to find the optimal number of nodes, n^* , that minimizes the cost function defined by (5.1), while satisfying the network connectivity requirement. This directly translates into finding the minimum number of nodes such that $\mathfrak{R} \geq \mathfrak{R}_{min}$.

From Equations (5.2), (5.3), (5.4), (5.5), and (5.8), we deduce that

$$\mathfrak{R} = f(p_t, d, \gamma, \alpha, \sigma, \mathbf{S}). \quad (5.10)$$

p_t and γ are parameters specific to node class $x \in \mathbb{X}$ being deployed, that is,

$$x = h(p_t, \gamma), \quad (5.11)$$

where $h(\cdot)$ is given in data sheets of mesh nodes. The pathloss component, α , the shadowing deviation represented by σ , and the space $\mathbf{S}$ are constants specific to the deployment site. Consequently, for the sake of node deployment optimization, Equation (5.10) can be expressed as

$$\mathfrak{R} = f(d, x). \quad (5.12)$$

Therefore, given class x is deployed, the optimal internode distance d_x^* , is given by

$$d_x^* = \{\max d : \mathfrak{R} \geq \mathfrak{R}_{min}\}, \quad (5.13)$$

and the optimal number of nodes, n_x^* , is

$$n_x^* = g(\mathbf{S}, d_x^*). \quad (5.14)$$

The optimal number of nodes is found numerically by iterating d until maximum d that achieves $\mathfrak{R} \geq \mathfrak{R}_{min}$. We choose to use a bisection search algorithm because of its reduced computational complexity. Procedure 1 on the next page is used to find the optimal number of nodes, n_x^* , for each $x \in \mathbb{X}$.

Procedure 1 Finding the Optimal Number of Nodes

```

1: procedure OPTIMALNUMNODES( $x, d_L, d_R, \mathfrak{R}_{min}$ )
2:   while  $d_L - d_R > 2\varepsilon$  do
3:      $d \leftarrow (d_L + d_R)/2$ 
4:     calculate  $r$  using (5.8)
5:     calculate  $n$  using (5.4)
6:     calculate  $\mathfrak{R}$  using (5.3)
7:     if  $\mathfrak{R} - \mathfrak{R}_{min} > 0$  then
8:        $d_R \leftarrow d$ 
9:     else
10:       $d_L \leftarrow d$ 
11:    end if
12:  end while
13:   $d \leftarrow (d_L + d_R)/2$ 
14:  calculate  $n$  using (5.4)
15:  return  $n$ 
16: end procedure

```

where ε is the tolerance used in the bisection search, d_L is the largest possible internode distance, and d_R is the smallest possible internode distance. The search space of the bisection search algorithm is bounded by d_L and d_R .

5.2.5 Optimal Node Class Selection

The second goal is to find the optimal node class, x^* , out of the set of available node classes, $\mathbb{X}$, that minimizes the cost function defined by (5.1) for a given $\mathfrak{R}_{min}$.

The optimal class selection is given by

$$x^* = \{k : C_k \in \arg \min_{x \in \mathbb{X}}(C_x) \text{ and } \mathfrak{R} \geq \mathfrak{R}_{min}\}. \quad (5.15)$$

Finding the optimal class selection given by (5.15) requires knowledge of the unit price, u_x , for each $x \in \mathbb{X}$. This price is determined by the market value. We hence derive the node selection policy for choosing between class $x \in \mathbb{X}$, and a given base class $i \in \mathbb{X}$.

For base class $i \in \mathbb{X}$, the optimal cost when using optimal node allocation is given by

$$C_i = n_i^* \cdot u_i. \quad (5.16)$$

Similarly, the optimal cost for any class $x \in \mathbb{X}$ is given by

$$C_x = n_x^* \cdot u_x. \quad (5.17)$$

According to (5.15), for a given $\mathbf{T}$, class x is selected rather than class i if

$$C_x < C_i. \quad (5.18)$$

Substituting (5.16) and (5.17) in (5.18) yields

$$n_x^* \cdot u_x < n_i^* \cdot u_i. \quad (5.19)$$

We can infer from (5.19) that the policy for selecting between a pair of classes is of switch-over type. Let $\xi = \frac{n_i^*}{n_x^*}$, then the threshold occurs when

$$u_x = \xi \cdot u_i. \quad (5.20)$$

From (5.19) and (5.20), we conclude that class x is chosen over class i if the normalized unit price of class x is less than ξ . That is,

$$\frac{u_x}{u_i} < \xi. \quad (5.21)$$

Given base class $i \in \mathbb{X}$, the normalized unit price thresholds, u_x/u_i , for the remaining $x \in \mathbb{X}$ is found using Procedure 2.

Procedure 2 Obtaining the Switch-Over Thresholds

```

1: procedure SELECTTHRESHOLDS( $i, \mathbb{X}, d_L, d_R, \mathcal{R}_{min}$ )
2:    $n_i^* \leftarrow \text{OPTIMALNUMNODES}(i, d_L, d_R, \mathcal{R}_{min})$ 
3:   for all  $x \in \mathbb{X} \setminus \{i\}$  do
4:      $n_x^* \leftarrow \text{OPTIMALNUMNODES}(x, d_L, d_R, \mathcal{R}_{min})$ 
5:      $\xi[x] = n_i^*/n_x^*$ 
6:   end for
7:   return  $\xi$ 
8: end procedure

```

5.3 Chain Topology

To demonstrate the devised methodology, we start by applying it to a network whose topology, $\mathbf{T}$, is composed of a chain of nodes similar to the one shown in Figure 5.1.

Figure 5.1: Chain Topology

In such topology, the space, $\mathbf{S}$, can be characterized by the end-to-end communication range, D . The number of nodes in a chain topology required to cover space $\mathbf{S}$ is given by

$$n = g(\mathbf{S}, d) = \left\lceil \frac{D}{d} \right\rceil - 1. \quad (5.22)$$

5.3.1 Experimental Setup

The model parameters pertaining to the characteristics of the deployment site were selected as follows: $\alpha = 4$, $\sigma = 9$ dB, $D = 10$ km, and $d_o = 100$ m. The environmental parameters' values were chosen to resemble values typically measured in shadowed urban environments [6]. The end-to-end distance resembles a fairly long chain network. The reference path loss, $\bar{L}(d_o)$, was calculated according to the free

space path loss formulation, where the wavelength was set to 0.125 m (corresponding to 2.4 GHz carrier frequency) [38].

Without loss of generality, we construct a set of 6 node classes:

$$\mathbb{X} = \{x_1, x_2, x_3, x_4, x_5, x_6\},$$

where a node class is identified by its transmitted power, p_t , and receive threshold, γ , in accordance with (5.11). Let x_1 be the base class with the least transmitting power; $p_t = 24\text{ dBm}$ for x_1 . Node classes are ordered in increasing transmit power; p_t for x_2, x_3, x_4, x_5 , and x_6 was set in respective order to 1.2, 1.4, 1.6, 1.8, and 2.0 times that of x_1 . All node classes were set to use the same receive sensitivity; γ was constrained to -89 dBm in order to achieve 11 Mbps using $802.11g$ modulation.

5.3.2 Evaluation

Optimal Number of Nodes

We first compute the optimal number of nodes, n_x^* , by executing the OPTIMALNUMNODES procedure for each of the six classes $x \in \mathbb{X}$. The procedure parameters were set to: $d_L = D$, $d_R = d_o$, $\mathcal{R}_{min} = 95\%$, and $\varepsilon = 1$. Note that (5.22) defines the function in (5.4) for a network with a chain topology. Figure 5.2 shows the results obtained. We observe that n^* has a non-linear negative relationship with the node class (transmit power).

Figure 5.2: Optimal number of nodes, n^* , in a Chain Topology

Class Selection Policy

Figure 5.3 shows the class selection policy, given base class x_1 . The results were obtained by executing the `SELECTTHRESHOLDS` procedure with $i = x_1$ (rest of the parameters was set as discussed earlier). The intersections of the dotted line with the various curves resemble the normalized unit price thresholds underneath which it is optimal to select that node class over x_1 . For instance, it is optimal to select x_3 rather than x_1 as long as the unit price of x_3 is at most 1.83 times that of x_1 . We also note that the thresholds increase non-linearly with node class.

Figure 5.3: Optimal node class selection in a Chain Topology

Effect of $\mathcal{R}_{min}$ and γ

Figure 5.4 shows the effect of varying the minimum network connectivity requirement, $\mathcal{R}_{min}$, on the optimal number of nodes. We plot two sets of results, the first for nodes with $p_t = 24 \text{ dBm}$ and $\gamma = -89$ to achieve 11 Mbps (similar to x_1), and the other for nodes also with $p_t = 24 \text{ dBm}$, but, with $\gamma = -76$ to achieve 54 Mbps using 802.11g modulation. We observe that as $\mathcal{R}_{min}$ increases, the number of additional nodes needed increases non-linearly.

Figure 5.4: Effect of $\mathcal{R}_{min}$ on n^* in a Chain Topology

In order to investigate the effect of achievable throughput on the optimal number of nodes, we varied the receive threshold, γ , for x_1 . Figure 5.5 shows the results when $\mathcal{R}_{min}$ is set to 95%. The plot is annotated with typical throughput values achieved at various γ using 802.11g modulation. We observe that the optimal number of nodes increase non-linearly with γ (or achievable throughput requirement).

Figure 5.5: Effect of γ on n^* in a Chain Topology

5.4 Triangular Lattice Topology

We now apply our methodology to find the connectivity in a network whose topology, $\mathbf{T}$, is a triangular lattice. This topology is obtained when nodes are placed on the vertices of an equilateral triangular lattice, or equivalently, at the centers of regular hexagons. Figure 5.6 demonstrates such topology by depicting the nodes constituting the first two tiers. It was previously proven that the triangular lattice topology results in the minimum number of nodes needed to achieve full coverage of a plane [35], [40].

Figure 5.6: Triangular Lattice Topology

In a triangular lattice topology, the space $\mathbf{S}$ is characterized by the sum of the areas, A , of the individuals hexagons. That is, all the shaded regions in Figure 5.6b.

Let H be the area of a hexagon of side length s . This area (dotted region in

Figure 5.6b on the preceding page) is given by

$$H = \frac{3\sqrt{3}}{2}s^2 = \frac{\sqrt{3}}{2}d^2, \quad (5.23)$$

where d is the internode distance.

Consequently, the least number of nodes needed to possibly cover area A is

$$n' = \frac{A}{H} = \frac{2A}{d^2\sqrt{3}}. \quad (5.24)$$

However, a triangular lattice topology must subsume

$$1 + \sum_{i=1}^t 6i = 3t^2 + 3t + 1 \text{ nodes}, \quad (5.25)$$

where $t = 1, 2, 3, \dots$ resembles the number of tiers in the topology. Accordingly, the minimum number of tiers, T , required to cover area A is

$$T = \{\min t : 3t^2 + 3t + 1 - n' \geq 0\}. \quad (5.26)$$

Therefore, the number of nodes in a triangular lattice topology required to cover space $\mathbf{S}$ is given by

$$n = g(\mathbf{S}, d) = 3T^2 + 3T + 1. \quad (5.27)$$

5.4.1 Experimental Setup

The model parameters pertaining to the characteristics of the deployment site were selected as follows: $\alpha = 4$, $\sigma = 9 \text{ dB}$, $A = 5 \text{ km}^2$, and $d_o = 100 \text{ m}$. The environmental parameters' values were chosen to resemble typical values measured in shadowed urban environments [6]. The area A resembles a fairly large two dimensional network. The reference path loss, $\bar{L}(d_o)$, was calculated according to the free space path loss formulation, where the wavelength was set to 0.125 m (corresponding to 2.4 GHz carrier frequency) [38].

We use the same set, $\mathbb{X}$, of six node classes used in the evaluation of the chain topology (Section 5.3.1). All node classes were set to use the same receive sensitivity, γ , which was constrained to -76 dBm in order to achieve 54 Mbps using $802.11g$ modulation.

5.4.2 Results

Optimal Number of Nodes

We compute the optimal number of nodes, n_x^* , by executing the `OPTIMALNUMNODES` procedure for each of the six classes $x \in \mathbb{X}$. We set the procedure parameters to: $d_L = \sqrt{\frac{2A}{3\sqrt{3}}}$, $d_R = d_o$, $\mathfrak{R}_{min} = 99.99\%$, and $\varepsilon = 1$. d_L was chosen to reflect the maximum diameter of a hexagon of area A . Also note that (5.27) defines the function in (5.4) for a network with a triangular lattice topology. Figure 5.7 shows the results obtained. We again observe that n^* decreases non-linearly with node class (or transmit power).

Figure 5.7: Optimal number of nodes, n^* , in Triangular Lattice Topology

Class Selection Policy

Figure 5.8 shows the class selection policy given base class x_1 . The results were obtained by executing the `SELECTTHRESHOLDS` procedure with $i = x_1$ (the rest of the parameters were set as previously discussed). The intersections of the dotted line with the various curves resemble the normalized unit price threshold, underneath which it is optimal to select that node class over x_1 . For instance, it is optimal to select x_3 rather than x_1 as long as the unit price of x_3 is at most 2.78 times that of x_1 . We also note that the thresholds increase non-linearly with node class.

Figure 5.8: Optimal node class selection in Triangular Lattice Topology

Effect of $\mathcal{R}_{min}$ and γ

Figure 5.9 shows the effect of varying the minimum connectivity requirement, $\mathcal{R}_{min}$, on the optimal number of nodes. We plot two sets of results, the first for nodes with $p_t = 24 \text{ dBm}$ and $\gamma = -76$ (to achieve 54 Mbps; similar to x_1), and the other for nodes also with $p_t = 24 \text{ dBm}$, but whose $\gamma = -89$ (to achieve 11 Mbps using 802.11g modulation). The stepwise behavior observed is due to the fact that nodes in a triangular lattice topology are only added in additional tiers, that is tier t adds $6t$ nodes.

Figure 5.9: Effect of $\mathcal{R}_{min}$ on n^* in Triangular Lattice Topology

In order to investigate the effect of achievable throughput on the optimal number of nodes, we varied the receive threshold, γ , for x_1 . Figure 5.10 shows the results when $\mathcal{R}_{min}$ is set to 99.99%. The plot is annotated with typical throughput values achieved at various γ using 802.11g modulation. We observe that as γ (or achievable throughput requirement) increase, the optimal number of nodes required increases non-linearly.

Figure 5.10: Effect of γ on n^* in Triangular Lattice Topology

Chapter 6

Optimal Local Routing of Internet Traffic

Figure 6.1: A Simple Wireless Mesh Network

In this chapter, we develop an optimal routing policy that tackles the problem of maximizing the throughput, and minimizing the delay in WMNs.

Since it is often the case that several gateways exist at geographically independent locations in a WMN, and hence several independent disjoint paths will likely exist from each mesh node to several distinct gateways, we propose that mesh nodes distribute their Internet traffic among several gateways simultaneously, instead of forwarding all the Internet traffic to a single gateway. This will result in an increase in the utilization of the overall network capacity, a more efficient and balanced use of resources, and should mitigate congestion and delays in a WMN.

Figure 6.1 depicts a simple WMN composed of four mesh nodes (MN), two of which are connected directly to the Internet, and are hence Internet Gateways (IG). MN1 and MN2 each have two distinct paths to IG1 and IG2. We hence propose that each mesh node (e.g. MN1 or MN2), distribute its Internet traffic on the available gateways (IG1 and IG2) simultaneously.

In order to achieve our goal, we need to address two problems. Firstly, we must devise a system model that captures the essential components and unique characteristics of WMNs. Then, we must find the policy that can optimally and dynamically distribute a mesh node's Internet traffic among the available gateways. The challenges in this two-fold problem are largely due to the wireless channel characteristics of the links connecting the mesh nodes.

Next in Section 6.1 we show via a simple simulation study that using multiple

paths, and distributing a mesh node's Internet traffic among several gateways improves the performance of WMNs. In Section 6.2, we present our system model for the WMN. Consequently, we use an analytical model based on stochastic dynamic programming to find the optimal routing policy in Section 6.3. Section 6.4 presents a case study where a mesh node's traffic is distributed among two gateways. In Section 6.5, we examine the performance of the optimal policy.

6.1 Load Distribution on Multiple Gateways

In this section, the goal is to show that distributing Internet traffic among multiple gateways will likely result in performance gains, hence establishing the motivation for this work.

Consider two network topologies:

1. A mesh node, MN , requesting data from an Internet Gateway, IG , that is two-hops away, as in Figure 6.2a.
2. A mesh node, MN , simultaneously requesting data from two Internet gateways, $IG1$ and $IG2$, each of which is two-hops away, as in Figure 6.2b.

We use the OPNET Modeler simulator [41] to compare the performance of the discussed topologies. The physical layer of all the nodes in the simulations implement 802.11g, the transmit powers of the transceivers were set to 0.005W, the packet

Figure 6.2: Simulated Topologies

reception power threshold was set to -95dBm, the internode distance was 400m, and the data rate for all the nodes except *IG2*, was set to 11Mbps. We limited the data rate for *IG2* to 5.5Mbps in order to induce the possible asymmetrical conditions of the Internet gateways.

In our simulations, *MN* sends a request of 1024 bytes, and the inter-arrival time of the requests follows an exponential distribution with a mean of 0.008s. The Internet gateways upon receiving a request, respond with an object of 2050 bytes. Hence, the nominal throughput assuming no delays and infinite channel capacity is

$$\text{Nominal Throughput} = \frac{2050 \cdot 8}{0.008} = 2050000 \text{ bps.}$$

In the two gateway topology (Figure 6.2b), the mesh node sends the requests according to two policies: Round-Robin (RR) where the requests are sent in an equally interleaved fashion to *IG1* and *IG2*; and, Weighted Delay (WD) where the requests

Figure 6.3: Simulation results for the one and two Internet gateway scenarios

are sent to the gateways in proportion to the relative packet delays experienced from each gateway.

Figure 6.3 plots the throughput as a function of simulation time. We observe that the throughput when the requests were distributed among two Internet gateways was much greater than when it was only sent to a single gateway. This infers that distributing the Internet traffic among multiple gateways will likely result in significant performance gains.

Figure 6.3 also shows that the throughput resulting from using the WD policy is greater than the throughput resulting from using the RR policy. This suggests that

distributing the Internet traffic equally among heterogenous gateways is not optimal, and there should exist a policy that can dynamically and optimally distribute the load among heterogenous gateways.

6.2 Modeling the WMN

6.2.1 Capturing the Effects of the Wireless Channel

The wireless channel is susceptible to a variety of transmission impairments. Noise and interference is caused by other radio sources transmitting in the same or nearby frequency bands. Moreover, path loss, multipath fading, and shadowing can greatly impede the radio signals.

Path loss refers to the attenuation of radio waves as they pass through matter, or when they disperse in free space, resulting in decreased signal strength as the distance between the sender and receiver increases [42]. Multipath fading refers to interference resulting at a receiver when two or more versions of the same signal arrive at slightly different times due to reflection off the ground and other objects [43]. Shadow fading refers to the random variability in the received signal power due to the existence of obstructions (e.g. large objects) between the sender and receiver.

Furthermore, pathloss and shadow fading in a wireless medium may cause signals from some users in the system to be hidden, resulting in collisions that can not be

easily detected; this problem is known as the hidden terminal problem [44].

The effect of the impediments outlined above is a reduction in the data rates of wireless links. The extent of the performance limitations are mostly determined by the propagation environment. In a WMN, each path from a source mesh node to a gateway will typically transverse several other mesh nodes connected by wireless links. Due to the characteristics of the wireless channel, the conditions of the links and nodes in the path to the gateway, as well as the conditions of the gateway itself will vary, affecting the data rate achieved between the two end-points (i.e., mesh nodes and Internet gateways).

In this work, we use the *probability of packet transmission completion* to denote the effective data rate between a mesh node and an Internet gateway. The better the conditions of the wireless links connecting the mesh node to that Internet gateway, the higher the *probability of packet transmission completion* of a gateway, and therefore, the higher the data rate that can be achieved between the two end points.

6.2.2 Queuing System Model

We model the entire path from a source mesh node to an Internet gateway by a single server having its own queue. Therefore, for M paths connecting a source mesh node to M unique Internet gateways, we model the network by the queuing system in Figure 6.4. The servers are heterogenous in the sense that each server

Figure 6.4: Queuing System Model for WMNs

$i \in \mathbb{M} = \{1, 2, \dots, M\}$ has a different *probability of packet transmission completion*. The conditions of the links and nodes along the path to gateway i is incorporated in the *probability of packet transmission completion* of server i .

In deployments of mesh networks, the *probability of packet transmission completion* is likely to be different for the different gateways due to variations in the quality of the routes' links. For instance, the *probability of packet transmission completion* pertaining to a route connecting a mesh node to a gateway located inside a room is likely to be higher than that pertaining to a route connecting a mesh node to an outdoor gateway that is affected by weather conditions.

We note that several QoS-aware metrics such as the hop count, Minimum Loss [22], Expected Transmission Count (ETX) [18], and Expected Transmission Time (ETT) [19], were proposed to capture and evaluate the performance of routing paths in wireless networks. Such metrics can be used to estimate the *probability of packet*

transmission completion values for the servers in our model. This mapping is however outside the scope of this work. Instead, our goal is to use the various paths information to optimally and dynamically distribute a mesh node Internet traffic to the available gateways.

The source mesh node is denoted by a *Route Controller* in Figure 6.4. In our model, we slot the time into equal intervals of length Δt . Let $T = \{1, 2, \dots\}$ denote the set of decision epochs of the system. At each time slot, $t \in T$, a packet arrives to the *Route Controller* according to the Bernoulli distribution with parameter q , and is assigned to one of the M queues. Arrivals are assumed to be independent of the system state. We will assume that the buffers associated with each of the M servers are of the same size $B - 1$, where $B - 1 > 1$. At the same slot, a packet departs (completes service) from server $i \in \mathbb{M}$ with probability p_i . The transmission completion of a packet during a time slot is independent of packet transmission completions during previous slots, and of that of other servers in the system. Each server does not idle, unless its queue is empty.

We define:

- $u(t) \in \mathbb{U} = \{0, 1\}$ the arrival of a packet to the *Route Controller* during interval $(t, t + \Delta t]$, hence

$$u(t) = \begin{cases} 1 & \text{if a packet arrives with prob } q \\ 0 & \text{if no packet arrives with prob } 1 - q. \end{cases} \quad (6.1)$$

For each server $i \in \mathbb{M}$, and slot $t \in T$, we also define:

- $x_i(t) \in \mathbb{X} = \{0, 1, 2, \dots, B\}$ the queue size at time t (including the currently serviced packet).
- $y_i(t)$ the number of packets completed transmission in the interval $(t, t + 1]$, hence

$$y_i(t) = \begin{cases} 1 \cdot \mathbb{I}_{\{x_i(t) > 0\}} & \text{with prob } p_i \\ 0 & \text{with prob } 1 - p_i, \end{cases} \quad (6.2)$$

where $\mathbb{I}_{\{\cdot\}}$ is the indicator function. The time a packet takes to depart from server i follows a geometric distribution with mean $\Delta t/p_i$, the number of packets departing from the system during a time slot Δt follows a binomial distribution with mean $\sum_{i=1}^M p_i$.

6.3 Finding the Optimal Routing Policy

The goal of this section is to find an optimal routing policy for the *Route Controller*, one that maximizes the overall system throughput and minimizes the average queuing delay. We used an approach based on Markov Decision Processes (MDPs) [45] [46].

MDPs extend Markov chains to include actions and rewards. We define $\mathbf{s}(t)$ as the system state at time $t \in T$. We also define $\mathbf{a}(\mathbf{s})$ as the action taken when the system is in state $\mathbf{s}$. Let S be the state space, A be the action space, $P_{\mathbf{ss}'}(\mathbf{a}) = \Pr(\mathbf{s}(t+1) = \mathbf{s}' | \mathbf{s}(t) = \mathbf{s}, \mathbf{a}(\mathbf{s}) = \mathbf{a})$ be the state transition probability, and $R(\mathbf{s}, \mathbf{a})$ be the immediate reward if action $\mathbf{a}$ is taken when in state $\mathbf{s}$. The system can hence be described by the tuple $(T, S, A, P_{\mathbf{ss}'}(\mathbf{a}), R(\mathbf{s}))$.

6.3.1 State Set

The state space S is the set of all possible states. $\mathbf{s}(t) \in S$ is the system's state at time $t \in T$; it is represented by a vector comprised of the queue sizes for the M servers in the system, as well as the outcome of the arrival process:

$$\mathbf{s}(t) = (x_1(t), x_2(t), \dots, x_M(t), u(t)). \quad (6.3)$$

$S = \{\mathbb{X} \times \mathbb{U}\}^M$ is finite due to the assumption of finite buffers' size, and the assumption that at most one packet can arrive at any time slot.

6.3.2 Action Set

The action space A is the set of all possible actions. $\mathbf{a}(\mathbf{s}) \in A$ is the action taken when in state $\mathbf{s}$. The action taken at each time slot corresponds to the assignment of the arriving packet (if any) to one of the M servers. Hence, $\mathbf{a}(\mathbf{s})$ can be defined as

$$\mathbf{a}(\mathbf{s}) = \{a_1(\mathbf{s}), a_2(\mathbf{s}), \dots, a_i(\mathbf{s}), \dots, a_M(\mathbf{s})\}, \quad (6.4)$$

subject to

$$a_i(\mathbf{s}) \in \{0, 1\} \quad \text{and} \quad \sum_{i=1}^M a_i(\mathbf{s}) = 1 \cdot u(t).$$

The first constraint results from the assumption that any server can be allocated at most one packet during any time slot. The second constraint ensures that if a packet has arrived during a time slot, it must be assigned to exactly 1 of the M servers; otherwise, no packet is assigned to any of the servers.

6.3.3 Reward Function

The reward function is used to determine the optimal routing policy for the *Route Controller*. Our objective is to maximize the system throughput while minimizing the average queuing delay. We define a weight factor denoted by σ that reflects the significance of minimizing the delay in the optimal policy. Hence, the reward function

$R(\mathbf{s}, \mathbf{a})$ is given by

$$R(\mathbf{s}, \mathbf{a}) = \sum_{i=1}^M y_i - \frac{\sigma}{B} \sum_{i=1}^M \frac{x_i + u a_i}{p_i}. \quad (6.5)$$

Note that the time index will be suppressed from now on to simplify the notation, for example y_i in (6.5) refers to $y_i(t)$. The positive component of the reward function maximizes the throughput of the system, and is given by the sum of the individual throughputs of each server in the system. The negative component of the reward function minimizes the average delay. This reward function, $R(\mathbf{s}, \mathbf{a})$, will produce a policy that maximizes throughput for a given σ ; that is, lower values of σ will result in optimal policies that favor throughput over minimizing the delay, while higher values for σ will result in optimal policies that favor minimizing the delay over maximizing throughput.

This MDP-based approach is very flexible in that different optimization objectives (e.g., minimize network jitter) can be easily realized by simply altering the reward function to reflect the new optimization objective.

6.3.4 Transition Probability Function

$P_{ss'}(\mathbf{a})$ is the probability that action $\mathbf{a}$ in state $\mathbf{s}$ at time t will lead to state $\mathbf{s}'$ at time $t + 1$. Following (6.3) and (6.4), $P_{ss'}$ can be expressed as

$$\begin{aligned} P_{ss'}(\mathbf{a}) &= \Pr(\mathbf{s}(t+1) = \mathbf{s}' | \mathbf{s}(t) = \mathbf{s}, \mathbf{a}(t) = \mathbf{a}) \\ &= \Pr(x'_1, \dots, x'_M, u' | x_1, \dots, x_M, u, a_1, \dots, a_M). \end{aligned} \quad (6.6)$$

The queue size, x_i , evolution is given by

$$x'_i = \min \left([x_i - y_i]^+ + u a_i, B \right), \quad (6.7)$$

where $[e]^+$ is equal to e when $e \geq 0$, and 0 otherwise. The arrival of a packet, u , depends only on the previous arrival, that is, $Pr(u'|s) = Pr(u'|u) = P_{uu'}$.

In Appendix C, we derive the state transition probability; accordingly, we can write (6.6) as follows

$$P_{ss'}(\mathbf{a}) = \prod_{i=1}^M P_{x_i x'_i}(u, a_i) \cdot P_{uu'}. \quad (6.8)$$

$P_{uu'}$ is the arrival transition probability, and was derived in Appendix C. It is given by

$$P_{uu'} = \begin{cases} 1 - q & \text{if } u' = u = 0 \\ q & \text{if } u' = 1 \text{ \& } u = 0 \\ q & \text{if } u' = u = 1 \\ 1 - q & \text{if } u' = 0 \text{ \& } u = 1. \end{cases} \quad (6.9)$$

$P_{x_i x'_i}(u, a_i)$ is the queue state transition probability, and was derived in Appendix D. It is given by

$$P_{x_i x'_i}(u, a_i) = \begin{cases} 1 - p_i & \text{if } x'_i = x_i = B \text{ \& } a = 1 \text{ \& } u = 1 \\ p_i & \text{if } x'_i = x_i + a_i - 1 \leq B \text{ \& } x_i > 0 \text{ \& } u = 1 \\ 1 - p_i & \text{if } x'_i = x_i + a_i \leq B \text{ \& } x_i > 0 \text{ \& } u = 1 \\ 1 & \text{if } x'_i = a_i \text{ \& } x_i = 0 \text{ \& } u = 1 \\ p_i & \text{if } x'_i = x_i - 1 \leq B \text{ \& } x_i > 0 \text{ \& } u = 0 \\ 1 - p_i & \text{if } x'_i = x_i \leq B \text{ \& } x_i > 0 \text{ \& } u = 0 \\ 1 & \text{if } x'_i = x_i = 0 \text{ \& } u = 0 \\ 0 & \text{otherwise.} \end{cases} \quad (6.10)$$

6.3.5 Solving the MDP

In this work, we consider an infinite-horizon model for the MDP. The goal is to find a rule at each decision time point that optimizes the system's performance. Hence, the solution to a Markov decision process is a function that maps states to actions, and can be expressed as a policy π . Combining a policy with a Markov decision process fixes the action taken in each state, resulting in a Markov chain.

The performance of a system is expressed as a certain *utility function*. We will use the total expected discounted reward over an infinite horizon as our utility function. The goal is to determine the policy π among all policies, that maximize the value of the utility function $V^\pi(\mathbf{s})$. The optimality equation is given by

$$V^*(\mathbf{s}) = \max_{\mathbf{a} \in A} \left[R(\mathbf{s}, \mathbf{a}) + \gamma \sum_{\mathbf{s}' \in S} P_{\mathbf{s}\mathbf{s}'}(\mathbf{a}) V^*(\mathbf{s}') \right], \quad (6.11)$$

where $V^*(\mathbf{s})$ is the maximal discounted *value function* obtained when the optimal policy, π^* , is used, that is,

$$V^*(\mathbf{s}) = \sup_{\pi} V^\pi(\mathbf{s}). \quad (6.12)$$

We solve the optimality equation using value iteration. Value iteration uses successive approximations; an arbitrary bounded function, $V_0(\mathbf{s})$, is first defined, $V_n(\mathbf{s})$ is then computed from $V_{n-1}(\mathbf{s})$ for $n = 1, 2, \dots$, where $V_n(\mathbf{s})$ is given by

$$V_n(\mathbf{s}) = \max_{\mathbf{a} \in A} \left[R(\mathbf{s}, \mathbf{a}) + \gamma \sum_{\mathbf{s}' \in S} P_{\mathbf{s}\mathbf{s}'}(\mathbf{a}) V_{n-1}(\mathbf{s}') \right].$$

V_n converges to V^* as $n \rightarrow \infty$, and a nearly optimal policy is obtained [47]. The iterative procedure is repeated until

$$\|V_{n+1} - V_n\| \leq (1 - \gamma)\gamma^{-1}\epsilon, \quad (6.13)$$

where $\|v\| = \sup_{s \in S} |v(s)|$. It was shown in [48] that when the condition in (6.13) is met, then $\|V^* - V_{n+1}\| \leq \epsilon$, that is, V_{n+1} is an ϵ -approximation of V^* , and a 2ϵ -optimal policy is obtained.

6.4 Case Study: Two Servers

We use the approach presented earlier to model the case when the system is composed of two servers, i.e. when $M = 2$. Hence there are only three possible actions at each time slot: $A = \{(0, 0), (0, 1), (1, 0)\}$, where $\mathbf{a} = (a_1, a_2)$ corresponds to the assignment of a packet to Server 1 denoted by a_1 , and to Server 2 denoted by a_2 . The two-server case will result in simplified policies, that are easier to visualize and evaluate.

6.4.1 Homogenous Servers

We first use the proposed approach to model a system of two symmetrical homogenous servers. This denotes the case when there exists two unique routes leading to two different Internet gateways, and the conditions of the wireless links, intermediate nodes, and Internet gateway in one route is identical to the other. The optimal policy for such case is intuitive, and will hence enable us to verify our proposed approach.

The system parameters were chosen as follows: $B_1 = B_2 = 20$, $q = 0.7$, $p_1 = p_2 = 0.4$, and $\sigma = 0.1$. The model is solved using value iteration to determine the optimal policy. The value iteration parameters were set as follows: $\gamma = 0.95$, $\epsilon = 0.01$.

The resulting optimal policy for two homogenous servers with the same packet transmission completion probabilities ($p_1 = p_2 = 0.4$) and with $q = 0.7$ is shown in Figure 6.5. The policy shown here is for the case when a packet arrives to the system during each time slot (i.e., $u(t) = 1$, $\forall t \in T$). In the other case when a packet

does not arrive (i.e., $u(t) = 0$), the action is simply $\mathbf{a} = (0, 0)$, that is, no packet is assigned to either servers. In Figure 6.5, the label ‘1’ denotes the action $\mathbf{a} = (1, 0)$ corresponding to the decision that the packet is assigned to Server 1. Similarly, the label ‘2’ denotes the action $\mathbf{a} = (0, 1)$ corresponding to the decision that the packet is assigned to Server 2. We observe that the optimal policy is equivalent to Join Shortest Queue (JSQ), which is intuitive and agrees with findings in [49], [50], and [51].

Figure 6.5: The Optimal Policy for two homogenous servers; $p_1 = p_2 = 0.4$, $q = 0.7$

6.4.2 Heterogenous Servers

We now consider two heterogenous servers, that is servers whose packet transmission completion probabilities are different. This denotes the more realistic case when two distinct Internet gateways are reachable from a mesh node, but the conditions along

one route is different than the other.

For example, when the *transmission completion probability* of a packet in Server 1, is less than that of Server 2 ($p_1 < p_2$), as in Figures 6.6a and 6.6b, the optimal policy favors assigning the arriving packets to Server 2 rather than Server 1 since it is less probable for Server 1 to have a packet completed its transmission during a time slot. This bias is illustrated by a larger darker area that correspond to action $(0, 1)$ (i.e., assigning the packet to Server 2). This bias increases as the difference between p_1 and p_2 increase (compare Figure 6.6a to Figure 6.6b).

Figure 6.6: Optimal Policies for two heterogeneous servers

6.5 Performance Evaluation

The performance of the optimal policy was studied using discrete-event simulation. The length of the time slot (Δt) was chosen to be 1.09 ms, hence 1 simulation second constitutes around 917 time slots; this value corresponds to a peak data rate of 11Mbps assuming 1500-byte packets. The sizes of the buffers were set to $B_1 = B_2 = 20$. At each time slot, a packet may arrive with probability q , a packet may leave Server 1 with probability p_1 , and a packet may leave Server 2 with probability p_2 .

In our evaluation, we considered the following metrics:

- **Throughput:** The number of packets departing the system every second.
- **Delay:** The average queueing delay of a packet.
- **Buffers Occupancy:** The percentage of the total buffers' capacity occupied.
- **Drops:** The percentage of arriving packets dropped due to buffer overflow.

We repeated each simulation, each time with a different seed, until the standard deviation of the samples became fairly small. It was hence determined that 10 repetitions were sufficient. Similarly, it was also determined that 500 s of simulation, that is around 458,500 time slots, were enough to ensure that the system has reached a steady state.

We calculated the 98% confidence intervals for the various means, and found that the widths of the intervals were very small (i.e., the samples hardly deviated from

the mean). We hence did not plot the y-error bars since they were too narrow to be observed on the plots. In subsequent sections, all metric values resemble the mean of 10 simulation repetitions.

6.5.1 Effect of Offered Load

Homogenous Servers

Figure 6.7: Effect of Load - homogenous servers; $p_1 = p_2 = 0.4$

We first consider the case of two homogenous servers with $p_1 = p_2 = 0.4$, and where the offered load is varied through the probability of packet arrival, q . Figure 6.7a

shows the effect of the load on the average throughput. We observe that the throughput increases linearly with the offered load up to the point where q is around 0.8. For $q > 0.8$, the throughput is maintained at the same constant level.

The average capacity of the system can be estimated as follows

$$\text{Average Capacity} = \frac{1}{\Delta t} \sum_{i=1}^M p_i. \quad (6.14)$$

For the case being considered, where $p_1 = p_2 = 0.4$ and Δt is 1.03 msec, the capacity is $917 \times 0.8 = 733$ pkts/second. This is consistent with the maximum throughput attained when $q = p_1 + p_2 = 0.8$ as shown in Figure 6.7a. When $q > p_1 + p_2$, the finite buffers become full, and the system starts to drop the arriving packets; the throughput stays constant at that (maximum) level.

Figure 6.7b shows that for lower values of offered load, the queueing delay increases linearly as the load increases. The delay however starts to increase rapidly as $q \rightarrow p_1 + p_2$. This is due to the fact that as the buffers become increasingly occupied, the average delay experienced by a packet increases.

Figure 6.7c follows the same trend; the buffers occupancy increases linearly with the load, reaching around 10% at $q = 0.7$; the buffers fills rapidly beyond this point, and the mean of the buffers' occupancy becomes 90% at $q = 0.9$.

Similarly, Figure 6.7d shows that no drops occur for values of $q \leq 0.7$, for values of $q > 0.7$, the percentage of packets dropped increases rapidly as the buffers have no more room for the arriving packets.

Heterogenous Servers

Figure 6.8: Effect of Load - heterogenous servers; $p_1=0.2$, $p_2=0.6$

We now consider the case of two heterogenous servers with $p_1=0.2$ and $p_2=0.6$. Figure 6.8 reveals the same trends discussed in the analysis of the homogenous servers case. The values of the various metrics obtained in the heterogenous case are similar to those obtained in the homogenous case.

This signifies that the effect of the offered load on the performance of a system composed of heterogenous servers is similar to that composed of homogenous servers with similar characteristics, i.e., equal buffer sizes, equal load, and where the combined servers' performance (i.e., $\sum_{i=1}^M p_i$) is equal.

6.5.2 Effect of Server Performance

In this section, we examine the effect of the servers' performance on the overall performance of the system. This is equivalent to examining the effect of the quality of the wireless channels (or alternatively the effect of congestion in any node along the paths to the gateways) on the overall performance of the network. In order to do so, we fixed the load, q , as well as and the *packet transmission completion probability* of Server 1, p_1 , and varied the *packet transmission completion probability* of Server 2, p_2 . We chose to set q to 0.7, and p_1 to 0.2.

Figure 6.9a shows the system's throughput as p_2 is varied between 0.5 and 1.0. We observe that the throughput increases slightly as p_2 increases from 0.5 to 0.6 and remains almost constant for $p_2 \geq 0.6$. This is due to fact that 0.5 is the minimum value of p_2 required in order for the servers to sustain the offered load (i.e., in order for $q = p_1 + p_2$ to hold). Given the system is stable, and that the offered load is within the system's capacity, the maximum possible throughput is equal to the offered load. The average offered load is given by

$$\text{Average Load} = q \times \frac{1}{\Delta t}. \quad (6.15)$$

Figure 6.9: Effect of Server Performance; $q=0.7$, $p_1=0.2$

For the case being considered, the average load is $0.7 \times 917 = 642$ pkts/sec. According to Figure 6.9a, this is consistent with the average overall throughput achieved when $p_2 > 0.5$. The throughput here is limited by the offered load, hence increasing either servers' performance further will have no effect on the overall system's throughput.

Figure 6.9b plots the relation between delay and server performance. We find that the delay always decreases as p_2 increases. This is expected since each packet will

spend less time in the queue as the *probability of packet transmission completion* of a server increases. Again, we notice that the delay decreases rapidly as p_2 increases from 0.5 to 0.6, then the delay decreases linearly in a much slower rate with increasing p_2 . The rapid decrease in the delay from the point where $p_2 = 0.5$ to the point where $p_2 = 0.6$ is due to the fact that the system quickly stabilizes as the servers' performance measures up to the load, i.e., $p_1 + p_2 > q$, driving the queue lengths down, and eliminating any drops.

Figures 6.9c and 6.9d confirms that both the percentage of percentage of buffers occupied, and percentage of packets dropped, follow the same trend as that of the delay. This is due to the discussed inherent dependency between the metrics.

6.5.3 Comparison of Policies

We compare the optimal policy, we notate and refer to as MDP, to the Round-Robin fair queueing (RR), Join Shortest Queue (JSQ) [49], and Weighted Join Shortest Queue (WJSQ) [52] policies. While the RR policy assigns each arrived packet to the servers in equal portion, in order, and without priority, the JSQ policy assigns the arrived packet to the server with the shortest queue length (x_i), and the WJSQ policy assigns the new packet to the the server with the smallest ratio of queue length to speed. We assume that the speed of the server can be estimated by the *probability of packet transmission completion*, hence the WJSQ heuristic assigns the packet to the server with the smallest ratio x_i/p_i .

To ensure fairness, the outcomes of the arrival processes were identical within a simulation set, where a simulation set consists of four simulations pertaining to the four examined policies.

Homogenous Servers

We first consider the case of two homogenous servers, where $q=0.7$ and $p_1=p_2=0.4$. Figure 6.10a shows that the throughput is almost the same for all four policies. This is expected since the policies yield similar decisions for the case of homogenous servers. Note that JSQ and WJSQ are essentially identical policies when the servers are homogenous, and their result hence coincide (we will hence only refer to JSQ in

Figure 6.10: Comparison of policies - homogenous servers; $q=0.7$, and $p_1=p_2=0.4$ (the subsequent discussion).

Figure 6.10b plots the average queueing delay of a packet with the different policies. We observe similar delay when either the MDP or the JSQ policies were used; this delay is less than the delay experienced with the RR policy. The average queueing delay of a packet with RR was 7.30 msec, while it was 5.41 msec with either MDP or JSQ. This is contributed to the fact that unlike in the RR scheme, the assignment of

a packet with the MDP and JSQ policies takes into account the state of the queues.

This is also evident in Figure 6.10c, which show that the buffers are always more occupied with the RR policy in comparison to MDP and JSQ. On average, 9.92% of the buffers are occupied using the MDP or JSQ policies, while 13.38% of the buffers are occupied using the RR policy.

Figure 6.10d shows the percentage of total arriving packets that get dropped. We observe that no packets are dropped when either the MDP or JSQ policies were used. In the RR case, few packets were dropped sporadically due to transient over-flow of the buffers.

Heterogenous Servers

We now consider the case of heterogenous servers, with $q=0.7$, $p_1=0.2$, and $p_2=0.8$. Figure 6.11a compares the throughput attained with the four policies. We find that the throughput is the same for the MDP, JSQ, and WJSQ policies. However, the value of the throughput obtained with the RR policy was significantly lower. The average throughput using MDP, JSQ, or WJSQ was around 650 pkts/sec, while it was only 384.32 pkts/sec using the RR policy. The average actual load was found to be 659.57 packets/sec, hence, MDP, JSQ, and WJSQ were able to match the throughput to the offered load by assigning most of the load to the faster server. In the RR case, only half of the load was assigned to the faster server; since the *packet transmission*

Figure 6.11: Comparison of policies - heterogenous servers; $q=0.7$, $p_1=0.2$, $p_2=0.8$

completion probability of the slower server is very low (almost negligible), using the RR policy in such configuration will limit the total achievable throughput to barely more than half the offered load.

Figure 6.11b plots the queueing delay. We find that the delay attained using MDP was the lowest, followed by WJSQ, followed by JSQ, followed by RR. Figure 6.12 focuses on on the results of JSQ, WSJQ, and MDP only, since RR's delay

Figure 6.12: Magnified version of Figure 6.11b

was significantly higher. The overall average packet queuing delay was 1.53 msec for MDP, 2.55 msec for WJSQ, 3.01 msec for JSQ, and 53.22 msec for RR. This trend was expected since MDP optimizes for both throughput and delay. The WJSQ heuristic on the other hand performs better than JSQ since it takes the relative speed of the servers into account, along with the relative queue sizes. RR performs worst since its assignment decisions are independent of the state of the servers.

Table 6.1: Average Queue Sizes, $q=0.7$, $p_1=0.2$, and $p_2=0.8$

Policy	$\bar{x}_1$	$\bar{x}_2$
MDP	0.30	1.03
WJSQ	0.70	0.94
JSQ	0.90	0.80
RR	19.39	0.45

Figure 6.11c plots the buffers occupancy for the various policies. The queues when using MDP were the least occupied, with an average occupancy of 3.34%, whereas the queues when using RR were the most occupied with an average of 49.59%. WJSQ and JSQ fell in the middle with averages of 4.11% and 4.25% respectively. This can be explained by examining the average queue sizes given in Table 6.1. Combining the results in Table 6.1 with those given in Figure 6.11c, we deduce that using the RR policy will result in the slower server queue almost full at all times, while the faster server queue almost empty. We also infer that the MDP policy is most efficient in terms of buffers occupancy, allowing the system to be withstanding to greater variances in traffic patterns.

Figure 6.11d shows that MDP, JSQ, and WJSQ policies do not result in any drops. However, the drops resulting from the RR policy averaged 20.37%. As discussed above, this is explained by examining the average queue sizes given in Table 6.1. We find that when the RR policy was used, Server 1's queue quickly filled up leading to the observed packet drops.

Chapter 7

Conclusions

7.1 Conclusions

In this dissertation, we tackled the problem of improving the efficiency in wireless mesh network deployments through two complementary prospects: optimizing the topology, and optimizing the routing of Internet traffic.

We first presented a methodology for finding a closed form upper bound for the connectivity in WMNs. Our solution is the first to take into account the effect of both: interdependence in connectivity of spatially co-located nodes, as well as the effect of the reduced connectivity of network boundary nodes. We used this methodology to find the connectivity in two distinct topologies, namely a chain topology, and the optimal triangular lattice topology.

We analyzed the impact and validity of the commonly utilized assumption of link independence. Results obtained show that there are cases where the connectivity

exceeds $P^*(no\ node\ isolated)$ that assumes independent links between nodes by a significant amount. Hence, the assumption of independent links between co-located nodes is inaccurate, and our derived expression for connectivity serve as a more accurate upper bound for network connectivity. We also examined the impact of assuming infinite network boundary. Results show that for most cases, this assumption overestimates the network connectivity by a significant amount, and is hence inaccurate. For network planners, this assumption gives a false sense of security, that is, nodes are thought to be connected while in reality they are not. We also showed that the deviation resulting from either assumptions increases with the size of the network; this deviation only becomes negligible at very high values of link connectivity that are practically hard to achieve. The developed methodology can similarly be applied to any network topology with equal distance node allocation. The qualitative results obtained will generally hold for similar topologies.

We also presented a methodology for finding the optimal number and class selection of mesh nodes that achieve some given minimum level of service requirements for the lowest cost. To the best of our knowledge, our work is the first to tackle such problem. The presented methodology is believed to be a valuable contribution particularly to network design teams looking into deploying wireless mesh networks. We used the methodology to find the optimal number, and node class (out of a set of

available node classes) in both, the chain topology, and the triangular lattice topology. The developed methodology can however be easily adapted to any topology. The optimal class will depend on the costs associated with each of the classes considered. However, we showed that the optimal class selection policy for choosing between any pair of node classes is of switch-over type. The methodology also allows network planners to draw more accurate quantitative conclusions, such as: the additional number of nodes required to increase the reliability; the additional nodes required to increase the network throughput; or, how the optimal class selection will change when the applications' requirements change.

An optimal routing protocol for WMNs was also presented in this dissertation. The protocol is optimal in that it maximizes the throughput and minimizes the delay of Internet packets in WMNs. The proposed protocol dynamically routes each mesh node's Internet traffic to several Internet gateways simultaneously, rather than to a single gateway, thus increasing the utilization of the overall network capacity, and reducing congestion and delays in the WMN. The WMN was modeled as a queuing system of heterogeneous servers; the model captures the effects of the wireless channel. Stochastic dynamic programming was used to find and characterize the optimal policy for the *Route Controller*. We showed that the optimal routing policy for a mesh node distributing its Internet traffic among two gateways simultaneously is of switch-over type. Results show that the optimal policy performs better than the Round-Robin

policy in terms of throughput and delay. The throughput resulting from the optimal policy was similar to that of the Join Shortest Queue and Weighted Join Shortest Queue policies. However, the optimal policy resulted in the least packet delays.

7.2 Summary of Contributions

This dissertation provided the following contributions:

Connectivity Model

1. A methodology for modeling network connectivity in WMNs taking into account the effect of interdependence in connectivity of spatially co-located nodes, as well as the effect of network boundary nodes' reduced connectivity.
2. Derivation of accurate closed form upper bounds for the connectivity in chain topologies, and triangular lattice topologies.
3. Examination of the validity of the assumption of independent links between co-located nodes, and the assumption of infinite network boundary. Proof is provided showing that the assumptions are inaccurate.
4. Computation of the error resulting from the assumption of independent links and the error resulting from the assumption of infinitely large network boundaries.

5. Computation of critical ranges of link connectivity at which the error resulting from either assumption is unacceptable for a given application.

Network Planning

6. A methodology was developed for finding the optimal number and class selection of mesh nodes that achieve the given minimum level of service requirements for the lowest cost.
7. Proved that the optimal class selection policy for choosing between any pair of node classes is of switch-over type.
8. Provided methods that allow for quantitative conclusions regarding the network planning aspects of WMNs.

Optimal Routing

9. An optimal routing protocol that improves the performance of WMNs by dynamically routing mesh nodes' Internet traffic to several Internet gateways simultaneously.
10. An analytical model and methodology based on Markov Decision Processes to determine the optimal policy for the assignment of packets to heterogeneous servers (Internet gateways) in WMNs. The effects of the wireless channel on the quality of the wireless links are captured in the system model, and are hence reflected on the routing decisions.

11. Proved that the optimal packet assignment policy performs better than Round Robin, Join Shortest Queue, and Weighted Join Shortest Queue Policies in terms of throughput, delay, and drops in WMNs with Internet gateways.

7.3 Future Research

Several avenues for future work exist, most prevailing of which include:

1. The network connectivity model proposed assumes symmetric bidirectional links that are fully dependant, that is, $P(l_{ij}) = P(l_{ji}) \forall i, j \in \mathbb{L}$. In practice, shadowing and fading effects are not homogenous throughout the deployment environment. This means that links l_{ij} and l_{ji} although not independent as previously discussed, are correlated, where the degree of correlation affects the amount of dependency between them. The amount of correlation or dependance is determined by the deployment environment. Extending the proposed connectivity model to include the effect of partially-dependant links will result in more accurate estimates of network connectivity.
2. The network planning methodology can be enhanced by allowing network planners to alter the topological characteristics of the deployment site. For instance, add obstacles, or designate areas where mesh nodes can not be deployed. Such hurdles will ultimately affect the optimal network plan (i.e., number of nodes,

types of nodes, and locations of nodes). Such enhancement to the presented model will result in more flexible and accurate planning of WMNs.

3. The performance results for the optimal routing policy was based on per-packet granularity, that is, for each arriving packet, the *Route Controller* decides to which of the the Internet gateways the packet should be routed. This might however result in complexities including out-of-order packets, and other security-related issues. Per-flow or per-connection routing policies, where the controller routes all packets pertaining to a flow or connection to the same Internet gateway will probably resolve such problems, but, is likely to result in sub-optimal use of network resources. It is therefore useful to investigate how the granularity of the routing policy affects the system performance, as well as identify traffic loading conditions where a fine/coarse policy is desirable.
4. More elaborate models for the wireless channels, such as the Finite State Markov Channel (FSMC) model [53], can be incorporated in modeling the WMN in Chapter 6.

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Appendix A

Computation of the Connectivity in Triangular Lattice Topologies with Infinite Boundaries

The probability that none of the L nodes in a triangular lattice topology are isolated was derived in Section 4.3.1 and was given by

$$\begin{aligned} P(n_1, n_2, \dots, n_L) &= \frac{P(n_1, n_2, \dots, n_7)}{P(n_2, \dots, n_7)} \cdot \\ &\quad \left(\frac{P(n_\beta, n_{q_1^\beta}, n_{q_2^\beta}, n_{q_3^\beta}, n_{q_4^\beta})}{P(n_{q_1^\beta}, n_{q_2^\beta}, n_{q_3^\beta}, n_{q_4^\beta})} \right)^{6T-1} \cdot \\ &\quad \left(\frac{P(n_\gamma, n_{q_1^\gamma}, n_{q_2^\gamma}, n_{q_3^\gamma})}{P(n_{q_1^\gamma}, n_{q_2^\gamma}, n_{q_3^\gamma})} \right)^{3T(T-1)} \cdot \\ &\quad P(n_L), \end{aligned} \tag{A.1}$$

where $T > 1$ is the total number of tiers constituting the topology.

We developed a computer program to automate the process of computing the terms constiuting (A.1). We utilized a cluster of processors running on Sharcnet [54] in order to speed up the computation. The Message Passing Interface (MPI) [55] was used for passing messages between the parallel processes.

We present the resulting closed form expressions below:

$$\begin{aligned}
P(n_1, n_2, \dots, n_7) = & \\
& 87r^4q^{26} + 3678r^5q^{25} + 52913r^6q^{24} + 395652r^7q^{23} + 1906956r^8q^{22} + \\
& 6632116r^9q^{21} + 17782062r^{10}q^{20} + 38333088r^{11}q^{19} + 68331612r^{12}q^{18} + \\
& 102688686r^{13}q^{17} + 131860854r^{14}q^{16} + 146018842r^{15}q^{15} + 140287383r^{16}q^{14} + \\
& 117339336r^{17}q^{13} + 85551290r^{18}q^{12} + 54329784r^{19}q^{11} + 29970633r^{20}q^{10} + \\
& 14292982r^{21}q^9 + 5850993r^{22}q^8 + 2035632r^{23}q^7 + 593768r^{24}q^6 + \\
& 142506r^{25}q^5 + 27405r^{26}q^4 + 4060r^{27}q^3 + 435r^{28}q^2 + 30r^{29}q + r^{30} \tag{A.2}
\end{aligned}$$

$$\begin{aligned}
P(n_2, \dots, n_7) = & 2r^3q^{27} + 249r^4q^{26} + 6582r^5q^{25} + 76365r^6q^{24} + 505440r^7q^{23} + \\
& 2253024r^8q^{22} + 7432948r^9q^{21} + 19215999r^{10}q^{20} + 40387200r^{11}q^{19} + \\
& 70736989r^{12}q^{18} + 105022794r^{13}q^{17} + 133752441r^{14}q^{16} + 147303096r^{15}q^{15} + \\
& 141017040r^{16}q^{14} + 117684414r^{17}q^{13} + 85685772r^{18}q^{12} + 54372282r^{19}q^{11} + \\
& 29981259r^{20}q^{10} + 14295006r^{21}q^9 + 5851269r^{22}q^8 + 2035656r^{23}q^7 + \\
& 593769r^{24}q^6 + 142506r^{25}q^5 + 27405r^{26}q^4 + 4060r^{27}q^3 + 435r^{28}q^2 + \\
& 30r^{29}q + r^{30}
\end{aligned} \tag{A.3}$$

$$\begin{aligned}
P(n_\beta, n_{q_1^\beta}, n_{q_2^\beta}, n_{q_3^\beta}, n_{q_4^\beta}) = & 33r^3q^{20} + 997r^4q^{19} + 9354r^5q^{18} + 46800r^6q^{17} + 154422r^7q^{16} + \\
& 372721r^8q^{15} + 697345r^9q^{14} + 1047321r^{10}q^{13} + 1290285r^{11}q^{12} + \\
& 1321145r^{12}q^{11} + 1132166r^{13}q^{10} + 813790r^{14}q^9 + 489634r^{15}q^8 + \\
& 245072r^{16}q^7 + 100942r^{17}q^6 + 33649r^{18}q^5 + 8855r^{19}q^4 + \\
& 1771r^{20}q^3 + 253r^{21}q^2 + 23r^{22}q + r^{23}
\end{aligned} \tag{A.4}$$

$$\begin{aligned}
P(n_{q_1^\beta}, n_{q_2^\beta}, n_{q_3^\beta}, n_{q_4^\beta}) = & \\
& r^2 q^{19} + 94 r^3 q^{18} + 1521 r^4 q^{17} + 9469 r^5 q^{16} + 35126 r^6 q^{15} + \\
& 91008 r^7 q^{14} + 177912 r^8 q^{13} + 273943 r^9 q^{12} + 340707 r^{10} q^{11} + \\
& 347256 r^{11} q^{10} + 292110 r^{12} q^9 + 203070 r^{13} q^8 + 116220 r^{14} q^7 + \\
& 54260 r^{15} q^6 + 20349 r^{16} q^5 + 5985 r^{17} q^4 + 1330 r^{18} q^3 + \\
& 210 r^{19} q^2 + 21 r^{20} q + r^{21}
\end{aligned} \tag{A.5}$$

$$\begin{aligned}
P(n_{q_1^\gamma}, n_{q_2^\gamma}, n_{q_3^\gamma}) = & \\
& 2 r^2 q^{17} + 130 r^3 q^{16} + 1399 r^4 q^{15} + 6781 r^5 q^{14} + 20415 r^6 q^{13} + \\
& 43565 r^7 q^{12} + 70439 r^8 q^{11} + 89518 r^9 q^{10} + 91234 r^{10} q^9 + \\
& 75270 r^{11} q^8 + 50336 r^{12} q^7 + 27128 r^{13} q^6 + 11628 r^{14} q^5 + \\
& 3876 r^{15} q^4 + 969 r^{16} q^3 + 171 r^{17} q^2 + 19 r^{18} q + r^{19}
\end{aligned} \tag{A.6}$$

$$\begin{aligned}
P(n_{q_1^\gamma}, n_{q_2^\gamma}, n_{q_3^\gamma}) = & \\
& 11 r^2 q^{14} + 224 r^3 q^{13} + 1201 r^4 q^{12} + 3614 r^5 q^{11} + 7378 r^6 q^{10} + \\
& 11080 r^7 q^9 + 12735 r^8 q^8 + 11410 r^9 q^7 + 8005 r^{10} q^6 + 4368 r^{11} q^5 + \\
& 1820 r^{12} q^4 + 560 r^{13} q^3 + 120 r^{14} q^2 + 16 r^{15} q + r^{16}
\end{aligned} \tag{A.7}$$

$$P(n_L) = 6 r q^5 + 15 r^2 q^4 + 20 r^3 q^3 + 15 r^4 q^2 + 6 r^5 q + r^6 \tag{A.8}$$

where $q = 1 - r$, and denotes the probability that a link is not connected.

Appendix B

Computation of the Connectivity in Triangular Lattice Topologies with Finite Boundaries

The probability that none of the L nodes in a finite triangular lattice topology are isolated was derived in Section 4.4.1 and was given by

$$\begin{aligned}
 P(n_1, n_2, \dots, n_L) = & \frac{P(n_1, n_2, \dots, n_7)}{P(n_2, \dots, n_7)} \cdot \\
 & \left(\frac{P(n_\beta, n_{q_1^\beta}, n_{q_2^\beta}, n_{q_3^\beta}, n_{q_4^\beta})}{P(n_{q_1^\beta}, n_{q_2^\beta}, n_{q_3^\beta}, n_{q_4^\beta})} \right)^{6(T-1)} \cdot \\
 & \left(\frac{P(n_\gamma, n_{q_1^\gamma}, n_{q_2^\gamma}, n_{q_3^\gamma})}{P(n_{q_1^\gamma}, n_{q_2^\gamma}, n_{q_3^\gamma})} \right)^{3T^2-9T+6} \cdot \\
 & \left(\frac{P(n_\lambda, n_{q_1^\lambda})}{P(n_{q_1^\lambda})} \right)^{6(T-2)} \cdot \left(\frac{P(n_\lambda, n_{q_1^{\lambda^\circ}})}{P(n_{q_1^{\lambda^\circ}})} \right)^6 \cdot \left(\frac{P(n_{\lambda^\circ}, n_{q_1^\lambda})}{P(n_{q_1^\lambda})} \right)^5 \cdot \\
 & P(n_L), \tag{B.1}
 \end{aligned}$$

where $T > 1$ is the total number of tiers constituting the topology.

We developed a computer program to automate the process of computing the terms constituting (B.1). We utilized a cluster of processors running on Sharcnet [54] in order to speed up the computation. The Message Passing Interface (MPI) [55] was used for passing messages between the parallel processes.

The six expressions (i.e., numerators and denominators) constituting the first three fractions in (B.1) are identical to (A.2), (A.3), (A.4), (A.5), (A.6), and (A.7). We present the closed form expressions constituting the remaining terms of (B.1) below:

$$P(n_\lambda, n_{q_1^\lambda}) = rq^6 + 15r^2q^5 + 33r^3q^4 + 35r^4q^3 + 21r^5q^2 + 7r^6q + r^7 \quad (\text{B.2})$$

$$P(n_{q_1^\lambda}) = 4rq^3 + 6r^2q^2 + 4r^3q + r^4 \quad (\text{B.3})$$

$$\begin{aligned} P(n_\lambda, n_{q_1^{\lambda^\circ}}) &= P(n_{\lambda^\circ}, n_{q_1^\lambda}) \\ &= rq^5 + 11r^2q^4 + 19r^3q^3 + 15r^4q^2 + 6r^5q + r^6 \end{aligned} \quad (\text{B.4})$$

$$\begin{aligned} P(n_{q_1^{\lambda^\circ}}) &= P(n_L) \\ &= 3rq^2 + 3r^2q + r^3 \end{aligned} \quad (\text{B.5})$$

where $q = 1 - r$, and denotes the probability that a link is not connected.

Appendix C

Derivation of State Transition Probability

The goal is to derive the state transition probability, $P_{ss'}(\mathbf{a})$, given by

$$\begin{aligned} P_{ss'}(\mathbf{a}) &= Pr(\mathbf{s}(t+1)=\mathbf{s}' | \mathbf{s}(t)=\mathbf{s}, \mathbf{a}(t)=\mathbf{a}) \\ &= Pr(x_1(t+1)=x'_1, \dots, x_M(t+1)=x'_M, u(t+1)=u' | x_1(t)=x_1, \\ &\quad \dots, x_M(t)=x_M, u(t)=u, a_1(t)=a_1, \dots, a_M(t)=a_M). \end{aligned} \quad (\text{C.1})$$

Applying Bayes' theorem to (C.1) yields

$$\begin{aligned} P_{ss'}(\mathbf{a}) &= Pr(x'_1, \dots, x'_M | x_1, \dots, x_M, u, a_1, \dots, a_M) \cdot \\ &\quad Pr(u' | x'_1, \dots, x'_M, x_1, \dots, x_M, u, a_1, \dots, a_M). \end{aligned} \quad (\text{C.2})$$

By applying Bayes' theorem again to the first part of (C.2), the joint probability of the queue size can be given by

$$\begin{aligned}
 & Pr(x'_1, \dots, x'_M | x_1, \dots, x_L, u, a_1, \dots, a_M) \\
 &= Pr(x'_1 | x'_2, \dots, x'_M, x_1, \dots, x_M, u, a_1, \dots, a_L) \cdot \\
 & Pr(x'_2 | x'_3, \dots, x'_M, x_1, \dots, x_L, u, a_1, \dots, a_L) \dots \\
 & Pr(x'_M | x_1, \dots, x_M, u, a_1, \dots, a_L).
 \end{aligned} \tag{C.3}$$

The evolution of the queue size (x_i) is given by

$$x'_i = \min \left([x_i - y_i]^+ + u a_i, B \right). \tag{C.4}$$

Hence, the queue size at the next slot, x'_i , depends on the current queue size (x_i), whether a departure from queue i occurred in the interval $(t, t + 1]$ (y_i), and whether a packet arrived in the the interval $(t, t + 1]$ (u) and was assigned to the i th queue (a_i), and is independent on anything else. Hence

$$Pr(x'_i | \mathbf{s}) = Pr(x'_i | x_i, u, a_i) = P_{x_i x'_i}(u, a_i). \tag{C.5}$$

The arrival of a packet, u , depends only on the previous arrival and is independent of everything else. That is

$$Pr(u' | \mathbf{s}) = Pr(u' | u) = P_{uu'}. \tag{C.6}$$

Using (C.3), (C.5), and (C.6), Equation (C.2) reduces to

$$P_{\mathbf{ss}'}(\mathbf{a}) = \prod_{i=1}^M P_{x_i x'_i}(u, a_i) \cdot P_{uu'}. \tag{C.7}$$

The state transition probability will be the product of the queues' state transition probability, and the arrival transition probability, where the queues' state transition probability is the product of the individual servers' queue state transition probabilities.

Figure C.1: Markov Transition Probability for the Arrival Process

Following the definition of u in (6.1), the arrival process can be expressed by the Markov chain in Figure C.1. Hence, the arrival transition probability, $P_{uu'}$, is given by

$$P_{uu'} = \begin{cases} 1 - q & \text{if } u' = u = 0 \\ q & \text{if } u' = 1 \text{ \& } u = 0 \\ q & \text{if } u' = u = 1 \\ 1 - q & \text{if } u' = 0 \text{ \& } u = 1. \end{cases} \quad (\text{C.8})$$

In Appendix D, we derive an expression for the queue state transition probability, $P_{x_i x'_i}(u, a_i)$.

Appendix D

Derivation of Queue State Transition Probability

The queue size evolves according to

$$x_i(t+1) = \min \left([x_i(t) - y_i(t)]^+ + u(t) a_i(t), B \right). \quad (\text{D.1})$$

The queue transition probability is given by

$$P_{x_i x'_i}(u, a_i) = \Pr(x_i(t+1) = x'_i | x_i(t) = x_i, u(t) = u, a_i(t) = a_i). \quad (\text{D.2})$$

Substituting (D.1) in (D.2) yields the transition probabilities corresponding to the boundary state, and the non-boundary states, which are derived in D.1 and D.2 respectively.

D.1 Boundary State

When $[x_i(t) - y_i(t)]^+ + u(t) a_i(t) > B$ then

$$x_i(t+1) = B,$$

and

$$\begin{aligned} P_{x_i B}(u, a_i) &= \Pr([x_i(t) - y_i(t)]^+ + u(t) a_i(t) > B \mid x_i(t) = x_i, \\ &\quad u(t) = u, a_i(t) = a_i) \\ &= \Pr([x_i - y_i(t)]^+ + u a_i > B). \end{aligned} \quad (\text{D.3})$$

The queue transition probability is related to the departure probability as follows

$$P_{x_i B}(u, a_i) = \Pr(y_i(t) < x_i + u a_i - B). \quad (\text{D.4})$$

The departure process is assumed to be Bernoulli with parameter p_i for queue i and is given by

$$y_i(t) = \begin{cases} 1 \cdot \mathbb{I}_{\{x_i(t) > 0\}} & \text{with prob } p_i \\ 0 & \text{with prob } 1 - p_i, \end{cases} \quad (\text{D.5})$$

where $\mathbb{I}_{\{\cdot\}}$ is the indicator function.

Since $u(t) \in \mathbb{U} = \{0, 1\}$, and $\sum_1^M a_i(t) = 1 \cdot u(t)$, there are two cases to consider:

1. if $u = 1$, (D.4) reduces to

$$P_{x_i x'_i}(u=1, a_i) = \Pr(y_i(t) < x_i + a_i - B), \quad (\text{D.6})$$

and the queue state transition probability in this case will be

$$P_{x_i B}(u=1, a_i) = \begin{cases} 1 - p_i & \text{if } x_i = B \text{ \& } a_i = 1 \\ 0 & \text{otherwise,} \end{cases} \quad (\text{D.7})$$

due to the assumptions that $x_i(t) \in \mathbb{X} = \{0, 1, 2, \dots, B\}$ and $a_i(t) \in \{0, 1\}$.

2. if $u = 0$, (D.4) reduces to

$$P_{x_i B}(u=0, a_i) = Pr(y_i(t) < x_i - B), \quad (\text{D.8})$$

and the queue state transition probability in this case will be

$$P_{x_i B}(u=0, a_i) = 0, \quad (\text{D.9})$$

due to the same aforementioned assumptions.

D.2 Non-Boundary States

When $[x_i(t) - y_i(t)]^+ + u(t) a_i(t) \leq B$ then

$$x_i(t+1) = [x_i(t) - y_i(t)]^+ + u(t) a_i(t),$$

and

$$\begin{aligned} P_{x_i x'_i}(u, a_i) &= Pr([x_i(t) - y_i(t)]^+ + u(t) a_i(t) = x'_i \mid x_i(t) = x_i, \\ &\quad u(t) = u, a_i(t) = a_i \\ &= Pr([x_i - y_i(t)]^+ + u a_i = x'_i). \end{aligned} \quad (\text{D.10})$$

Then $P_{x_i x'_i}(u, a_i)$ is related to the arrival probability as follows

$$P_{x_i x'_i}(u, a_i) = Pr(y_i(t) = x_i + u a_i - x'_i). \quad (D.11)$$

Hence,

1. if $u = 1$, (D.11) reduces to

$$P_{x_i x'_i}(u=1, a_i) = Pr(y_i(t) = x_i + a_i - x'_i), \quad (D.12)$$

and the queue state transition probability in this case will be

$$P_{x_i x'_i}(u=1, a_i) = \begin{cases} p_i & \text{if } x'_i = x_i + a_i - 1 \text{ \& } x_i > 0 \\ 1 - p_i & \text{if } x'_i = x_i + a_i \text{ \& } x_i > 0 \\ 1 & \text{if } x'_i = a_i \text{ \& } x_i = 0 \\ 0 & \text{otherwise.} \end{cases} \quad (D.13)$$

2. if $u = 0$, (D.11) reduces to

$$P_{x_i x'_i}(u=0, a_i) = Pr(y_i(t) = x_i - x'_i), \quad (D.14)$$

and the queue state transition probability in this case will be

$$P_{x_i x'_i}(u=0, a_i) = \begin{cases} p_i & \text{if } x'_i = x_i - 1 \text{ \& } x_i > 0 \\ 1 - p_i & \text{if } x'_i = x_i \text{ \& } x_i > 0 \\ 1 & \text{if } x'_i = x_i = 0 \\ 0 & \text{otherwise.} \end{cases} \quad (D.15)$$

D.3 Summary

The results obtained in (D.7), (D.9), (D.13), and (D.15) can be summarized as follows

$$P_{x_i x'_i}(u, a_i) = \begin{cases} 1 - p_i & \text{if } x'_i = x_i = B \text{ \& } a = 1 \text{ \& } u = 1 \\ p_i & \text{if } x'_i = x_i + a_i - 1 \leq B \text{ \& } x_i > 0 \text{ \& } u = 1 \\ 1 - p_i & \text{if } x'_i = x_i + a_i \leq B \text{ \& } x_i > 0 \text{ \& } u = 1 \\ 1 & \text{if } x'_i = a_i \text{ \& } x_i = 0 \text{ \& } u = 1 \\ p_i & \text{if } x'_i = x_i - 1 \leq B \text{ \& } x_i > 0 \text{ \& } u = 0 \\ 1 - p_i & \text{if } x'_i = x_i \leq B \text{ \& } x_i > 0 \text{ \& } u = 0 \\ 1 & \text{if } x'_i = x_i = 0 \text{ \& } u = 0 \\ 0 & \text{otherwise.} \end{cases} \quad (\text{D.16})$$