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**THE INFLUENCE OF LEADING-EDGE GEOMETRY ON
PROFILE AND SECONDARY LOSSES IN TURBINE
CASCADES**

by

Michael William Benner

A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

in Aerospace Engineering

Ottawa-Carleton Institute for Mechanical & Aerospace Engineering

Department of Mechanical & Aerospace Engineering

Carleton University

Ottawa, Ontario, Canada

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**The undersigned recommend to
the Faculty of Graduate Studies and Research
acceptance of the thesis**

**The Influence of Leading-Edge Geometry on Profile and Secondary Losses in
Turbine Cascades**

submitted by

**Michael William Benner, B.Eng.
in partial fulfilment of the requirements for
the degree of Doctor of Philosophy**

External Examiner

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July 2003**

Dedicated to my Wife

Karen

to my Daughter

Emma Joyce

and my angel Mother.

Abstract

This thesis presents detailed experimental results of the midspan and secondary flows from two large-scale, low-speed linear turbine cascades. The airfoils for the two cascades differ mainly in their leading-edge geometries. Detailed flow field measurements were made upstream and downstream of the cascades using three- and seven-hole pressure probes and static pressure distributions were measured on the airfoil surfaces. The data were supplemented with extensive surface oil flow visualization. Measurements were made at three different values of incidence: 0, +10 and +20 degrees.

For the off-design profile losses, significant discrepancies were observed between the present measurements and the predictions from the most recent available correlation (Moustapha *et al.*, 1990). Based on data from the present experiment and cases from the open literature, a revised off-design profile loss correlation has been developed that is significantly more successful than the existing correlations. The new correlation includes leading-edge wedge angle as an influential and additional correlating parameter.

The results from the secondary flow measurements at the design incidence suggest that the leading-geometry has only a minor influence on the secondary losses, whereas, the loading on the forward part of the airfoil appears to be of primary importance. At off-design incidence, the most recent off-design secondary loss correlation (Moustapha *et al.*, 1990) includes leading-edge diameter as an influential correlating parameter. The correlation predicts that the secondary losses for the airfoil with the larger leading-edge diameter are lower at off-design incidence; however, the opposite is observed experimentally. The loss results at high positive incidence have also highlighted some serious shortcomings with the conventional method of loss decomposition. An empirical prediction method for secondary

losses has been developed to replace the conventional one.

In addition to the experimental study, a computational investigation was conducted using a CFD code that is currently widely used in turbomachinery design. The objective was to determine the degree to which the code is capable of predicting off-design profile losses. For cases at moderate values of incidence, the correct loss trend was predicted; however, the code did not predict the trailing-edge separation that occurred at high incidence.

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List of Symbols

A	= area
A_{prim}	= area of the primary region (Equation 8.3)
A_s	= suction surface area
A^+	= 26 = constant in Baldwin and Lomax turbulence model
AVR	$= \frac{\int_0^s (u_2)_{MS} dy'}{\int_0^s (u_1)_{MS} dy'} =$ axial velocity ratio (Equation 5.1)
C	= airfoil chord length
CFL	= Courant, Friedrichs and Lewy number
CR	$= \frac{\cos \alpha_1}{\cos \alpha_2} =$ convergence ratio
C_{cp}	= 1.6 = constant in Baldwin and Lomax turbulence model
C_d	= boundary layer dissipation coefficient (Equation 8.19)
C_f	$= \frac{\tau_w}{1/2 \rho u_e^2} =$ skin friction coefficient
C_{Kleb}	= 0.3 = constant in Baldwin and Lomax turbulence model
C_L	= lift coefficient based on vector mean velocity (Equation 8.16)
C_p	$= \frac{P - P_{CL}}{\frac{1}{2} \rho V_{CL}^2} =$ static-pressure coefficient; specific heat at constant pressure (Section 4.3.1)

Cp_0	$= \frac{P_{0_{local}} - P_{0_{CL}}}{\frac{1}{2} \rho V_{CL}^2} = \text{total-pressure coefficient}$
$Cp_{0_{(total)}}$	$= \frac{P_{0,2}'' - P_{0,1}''}{\frac{1}{2} \rho V_{CL}^2} = \text{total-pressure loss coefficient based on the inlet}$ centreline dynamic pressure
$Cp_{0_{WT}}$	$= \text{wind-tunnel total pressure calibration coefficient (Equation 3.4)}$
Cq_{WT}	$= \text{wind-tunnel dynamic pressure calibration coefficient (Equation 3.5)}$
C_{SKE}	$= \frac{SKE}{\frac{1}{2} \rho V_{CL}^2} = \text{non-dimensional secondary kinetic energy}$
C_T	$= \text{tangential loading coefficient based on inlet dynamic pressure (Equation 7.1)}$
Cv	$= \text{specific heat at constant volume (Section 4.3.1)}$
Cx	$= \text{airfoil axial chord length}$
C_{Γ_s}	$= \frac{\Gamma}{V_{CL} C} = \text{non-dimensional secondary circulation}$
C_{ω_s}	$= \frac{\omega_s C}{V_{CL}} = \text{non-dimensional streamwise vorticity}$
d	$= \text{airfoil leading-edge diameter}$
$\mathbf{D}$	$= \text{artificial dissipation operator (Section 4.4.4)}$
e	$= \text{internal energy}$
E	$= \text{total internal energy}$
$F(y)$	$= y \omega \left[1 - e^{(-y^+/A^+)} \right] = \text{function used to determine length and velocity}$ scales for the wall shear layer in the Baldwin and Lomax turbulence model
F_{Kleb}	$= \text{Klebanoff intermittency factor (Equation 4.55)}$
F_t	$= \text{tangential loading non-dimensionalized by dynamic pressure}$ based on the vector mean velocity (Equation 8.9)
F_{wake}	$= \text{constant in Baldwin and Lomax turbulence model}$
h	$= \text{span}$

H	$= \frac{\delta^*}{\theta} =$ shape factor; total internal energy
$\mathbf{H}$	$=$ flux vector (Equation 4.2)
i	$= \alpha_1 - \alpha_{1_{design}} =$ incidence, in degrees
k	$= 0.41 =$ Von Karman's constant; thermal conductivity
K	$= 0.0168 =$ constant in Baldwin and Lomax turbulence model
K_D	$=$ diffusion number
l	$=$ mixing length
$\dot{m}$	$=$ mass flow rate
$\mathbf{n}_I, \mathbf{n}_J, \mathbf{n}_K$	$=$ unit vectors normal to the surfaces of constant X_I, X_J and X_K , respectively (Section 4.4.2)
N_{fan}	$=$ fan speed, in RPM
P	$=$ static pressure
P_0	$=$ total pressure
q	$= \frac{1}{2} \rho V^2 =$ dynamic pressure
Q	$= \left(\Delta C_{p_{max}} \frac{q_{CL}}{q_{mean}} \frac{Cx}{s^*} \right)^{1.25} \left(1 - \frac{x_{min}}{Cx} \right) =$ quality factor
$\mathbf{Q}$	$=$ solution vector (Equation 4.2)
Pr	$= \frac{C_p \mu}{k} =$ Prandtl number
$\mathbf{r}$	$=$ cell face diagonal vector (Section 4.4.2)
$\mathbf{R}$	$=$ residual vector (Equation 4.34)
Re	$= \frac{\rho V_{CL} C}{\mu} =$ Reynolds number
s	$=$ blade pitch or spacing
s^*	$=$ cross-passage distance (measured in tangential direction) at axial location

	of $\Delta C_{p_{max}}$
S	= suction-surface coordinate
$\dot{S}$	= entropy generation rate (Equation 8.18)
$\dot{S}_a$	= entropy generation rate per unit surface area
SKE	= $\frac{1}{2}\rho (v_{sec}^2 + w_{sec}^2)$ = secondary kinetic energy
SSL	= suction-surface length
t_{max}	= airfoil maximum thickness
tet	= trailing-edge thickness
t	= time
T	= temperature
u, v, w	= velocity components in axial, pitchwise and spanwise directions, respectively
u_τ	= $\sqrt{\frac{\tau_w}{\rho}}$ = friction velocity
u^+	= $\frac{u}{u_\tau}$ = law-of-the-wall coordinate
v_{sec}, w_{sec}	= components of secondary velocity vector
V	= velocity
We	= leading-edge wedge angle, in degrees
x_{min}	= axial distance from leading-edge plane to minimum suction side pressure
x	= axial coordinate measured from the leading-edge plane
x'	= axial location measured from the trailing-edge plane
X_I, X_J, X_K	= non-orthogonal coordinates that define in physical space the edges of a finite volume cell (Section 4.4.2)
y	= normal distance to the nearest surface

y', z	= local pitchwise and spanwise co-ordinates
y^+	$= \frac{u_\tau y}{\nu}$ = law-of-the-wall coordinate
Y	$= \frac{P_{01}'' - P_{02}''}{q_2''}$ = mass-averaged total-pressure loss coefficient (Section 3.9.1)
Z	= Ainley and Mathieson's loading parameter (Equation 8.15)
Z_{TE}	= spanwise distance of the S_4 separation line from the endwall at the trailing edge
α	= flow angle measured from the axial direction, in degrees; scaling factor for artificial dissipation (Section 4.4.4)
$\alpha^{(k)}$	= Runge-Kutta time integration coefficient for stage k
β	= aerofoil metal angle measured from the axial direction, in degrees; artificial compressibility parameter (Section 4.3.2)
δ	= boundary layer thickness
δ^*	$= \int_0^\delta \left(1 - \frac{V}{V_e}\right) dz$ = boundary-layer displacement thickness
$\Delta C p_{max}$	= maximum non-dimensional cross-passage pressure difference
$\varepsilon_I, \varepsilon_J, \varepsilon_K$	= smoothing coefficients for implicit residual smoothing
ϕ^2	$= \frac{(V_2'')^2}{(V_{2is}^2)''}$ = mass-averaged kinetic-energy coefficient
γ	= stagger angle measured from the axial direction, in degrees; specific heat ratio (Section 4.3.1); weighting factor for Runge-Kutta time integration
Γ_s	$= \int_0^{h/2} \int_0^s \omega_s \cos \alpha_2'' dy' dz$ = secondary circulation
$\kappa^{(2)}$	= scaling constant for second-order artificial dissipation term
$\kappa^{(4)}$	= scaling constant for fourth-order artificial dissipation term
λ	= spectral radius of the inviscid flux Jacobian
μ	= dynamic viscosity

ν	= kinematic viscosity; pressure gradient switch for artificial dissipation (Equation 4.32)
θ	$= \int_0^\delta \left(1 - \frac{V}{V_e}\right) \frac{V}{V_e} dz$ = boundary-layer momentum thickness
ρ	= density
$\bar{\tau}$	= stress tensor (Section 4.3)
ω	= vorticity
Ω	= cell volume (Section 4.4.2)
ψ_T	= Zweifel loading parameter (Equation 8.11)

Subscripts

C	= convective
C1	= static tap location upstream of wind-tunnel contraction (Figure 3.1)
C2	= static tap location downstream of wind-tunnel contraction (Figure 3.1)
CL	= centreline value at the inlet boundary layer traverse plane
D	= diffusive
e	= boundary-layer edge value
inner	= inner layer
l	= laminar
max	= maximum
m	= based on vector mean velocity
MS	= midspan
outer	= outer layer
pro	= profile

sec	= secondary
t	= turbulent
tip	= tip-leakage
w	= wall value
x, y, z, s	= components in axial, pitchwise, spanwise and streamwise directions, respectively
1, 2	= cascade inlet and outlet
+, -	= associated with positive and negative streamwise vorticity, respectively

Superscripts

v	= viscous
w	= wall value
$/$	= pitchwise mass-averaged value
$//$	= refers to a value that has been mass-averaged over a reference of area that covers one airfoil pitch and 50% of the span (Section 3.9.1)

Chapter 1

Introduction

1.1 Motivation

A turbine airfoil is usually designed to produce minimum losses at its design operating condition. Additional losses, often referred to as off-design losses, occur because conditions in the turbine deviate from those at the design point. Such deviations are common and they can occur for several reasons. For instance, in aircraft engines, operation at idle, take-off and cruise all result in changes in the operating conditions for the turbines, particularly the low-pressure stages. Furthermore, any deviation in the compressor and/or turbine efficiency from their target values will result in the turbine operating away from its design point. Up-rating an existing engine to produce more power or thrust, by increasing the engine mass flow for example, can also cause the turbine to operate away from its original design operating condition.

The change in operating conditions can take a number of different forms, including deviations in Reynolds number and Mach number from their design values. Perhaps the most significant effect on losses occurs when the incidence differs from its design value.

This body of work is concerned primarily with the effects of such off-design incidence on the behaviour of the flow field and on the generation of losses.

1.2 Thesis Objectives

The objectives of this work were both fundamental and applied in nature.

The fundamental objective was to gain a better physical understanding of the influence of leading-edge geometry on both profile and secondary losses, at design and off-design values of incidence. Profile loss is the loss generated well away from the endwall whereas secondary losses arise from the flow in the endwall region in the absence of tip clearance. An understanding of the origins of these losses is important in a couple of ways. It is often used to provide guidance in the aerodynamic design of turbines and it is a necessary condition for designing turbine airfoils that are more tolerant to off-design incidence. Also, the physics of the flow must be understood such that the appropriate airfoil and flow parameters can be used to correlate the losses.

The applied objectives were focused on loss prediction at off-design incidence. Predicting airfoil losses is an important capability for turbine aerodynamicists. It is performed in all phases of turbine design, in engine growth studies and in engine development programs. Empirical loss correlations and meanline analysis are essential tools for loss prediction. For this reason, one of the main applied objectives of this research is to review and improve these correlations.

Results from the present study on profile losses at off-design incidence have suggested that an important parameter was missing from the currently-available profile loss correlations at off-design incidence. Using data obtained from the literature, the present author subsequently revised this correlation by including the missing parameter, (see Chapter 5).

A number of correlations are available for predicting secondary losses at off-design incidence. However, based on the experimental results of this work it appears that the effect of certain geometric parameters on secondary losses is not fully understood. Furthermore, some problems with the conventional subdivision of the losses into profile and secondary components have been observed. An improved empirical prediction method for secondary losses has been developed and it is based on a new loss breakdown scheme that is more consistent with the observed flow physics.

Another tool that is used, often in parallel with meanline analysis, is a Navier-Stokes solver. By comparison with empirical loss correlations, Navier-Stokes solvers are relatively new tools that have not been validated for all geometries of flow. Consequently, the designer often does not know the quality of the loss predictions. Based on the review of previous literature, there appears to be very few comparisons of loss predictions to experimental measurements, especially for off-design values of incidence. Therefore, the second applied objective was to evaluate the ability of a CFD code currently widely used in turbomachinery design, employing a popular, simple and robust turbulence model to predict off-design losses.

1.3 Approach

Both the fundamental and applied objectives of this research were achieved mainly through experimental measurements; however, numerical simulations were also performed. The experimental measurements were made at low subsonic conditions using a low-speed cascade wind tunnel with a variable-incidence cascade test section. Flow visualization was used to gain additional insight into the physics of the flow. The experimental results were also used to develop an improved correlation for off-design profile losses and a new prediction method for secondary losses.

The computational tool that was used during this research program was a substantially modified version of a three-dimensional Navier-Stokes solver developed by Dawes (1985). The code, as developed by Dawes, was originally intended to be used to predict transonic flows in rotating turbomachinery. Therefore, the original code could not be directly used to simulate the experiments carried out in this work, that is, low-speed linear cascade testing.

1.4 Overview of Thesis

This thesis presents the results of an experimental study that has examined the influence of leading-edge geometry on the flow behaviour and loss generation in the midspan and near-endwall regions (in the absence of tip clearance) of turbine cascades. Based on the data from these experiments, as well as cases from the open literature, an improved off-design profile loss correlation and a new empirical prediction for secondary losses are introduced. Computations were also performed to assess the degree to which a currently widely used Navier-Stokes code is capable of predicting off-design losses.

An overview of the literature related to the profile and secondary losses in axial turbines at both design and off-design values of incidence is presented in Chapter 2. A number of references that were particularly relevant to the author's research are discussed in more detail together with the presentation of results later in the thesis.

The experimental work was performed in a low-speed wind tunnel using a variable incidence cascade test section. Data was gathered mainly with the use of pressure probes. Chapter 3 details the experimental set-up, instrumentation and procedures.

The computational part of this work was originally planned to be completed with a Navier-Stokes solver (BTOB3D) developed by Dawes. The original code, however, could not be used directly to simulate the experiments. Chapter 4 describes the original Dawes

code and the modifications which were made to it.

Chapter 5 presents the experimental and computational results related to the profile loss component. As well, an improved off-design profile loss correlation is described. Chapters 6 through 8 present experimental measurements related to the secondary flows. Chapter 6 examines the influence of leading-edge geometry on the secondary flow behaviour at the design incidence, whereas the off-design incidence results are presented in Chapter 7. A new empirical method for predicting secondary losses is detailed in Chapter 8. Finally, the conclusions from the current work and recommendations for future studies are presented in Chapter 9.

Chapter 2

Review of Previous Work

2.1 Introduction

The present research has considered a number of topics related to the profile and secondary losses in axial turbines at both design and off-design values of incidence. The current chapter provides an overview of the previous literature that is related to these topics. All the key references are cited and summarized. However, a number of references that are particularly relevant to the author's research are discussed in more detail together with the presentation of results later in the thesis. Thus, there are sections in Chapters 5 through 8 that effectively supplement the present chapter.

It should be mentioned that there is a significant body of recent research concerned with profile losses in axial turbines at low Reynolds numbers. Under these conditions, the location of boundary layer transition plays an important part in the behaviour of the blade surface flow and thus in the profile loss behaviour. Such low Reynolds number flows arise most frequently in the low-pressure turbines in aircraft gas turbine engines. The present study has focused on the higher Reynolds number flows that are typical of both the high-

pressure turbines in aircraft engines and, more generally, the turbines in land-based gas turbine engines. Consequently, the research on low-Reynolds number flows in turbines is not reviewed here.

The chapter begins with a description of the terminology used in this work. Section 2.2.2 gives a description of the conventional loss breakdown used for axial-flow turbines. Following this, a brief description of the flow in an airfoil passage is presented. A summary of the investigations that have focused on profile and secondary losses is given in Sections 2.4. Sections 2.5 and 2.6 briefly discuss the most important findings from these investigations. Section 2.7 discusses the widely-used design and off-design profile and secondary loss correlations.

2.2 Nomenclature

2.2.1 Terminology for Incidence and Cascade Geometry

The definition of incidence for turbine airfoils is summarized in Figure 2.1. The difference between design inlet flow angle and inlet metal angle is often referred as design, minimum loss or optimum incidence, i_{des} . The off-design incidence, i , is generally defined as the difference between the inlet flow angle and the inlet metal angle. For profile loss correlations at off-design incidence, it is useful to define effective incidence, i_{eff} , which is the difference between the inlet flow angle and the design inlet flow angle.

The terminology used for defining the airfoil and cascade geometry is shown in Figure 2.2.

Note: All angles are measured relative to the axial direction

Figure 2.1: Incidence terminology for axial-flow turbines.

2.2.2 Loss Terminology

Aerodynamic losses occur when useful mechanical energy is converted into internal energy through an irreversible process. As a result, some measure of irreversibilities is required to quantify losses. Since most turbomachines are very nearly adiabatic, entropy production is a direct measure of irreversibilities, and consequently, it would seem to be a logical measure of loss. Furthermore, entropy, unlike many other quantities, is independent of whether it is viewed from a rotating or stationary airfoil row (Denton, 1993). Nevertheless, entropy generation is infrequently used in experimental work as a measure of the losses since it cannot be measured directly. There are several alternative parameters, usually based on

Figure 2.2: Terminology for airfoil and cascade geometry.

measured total pressures, that give an indirect measure of entropy production for stationary airfoil rows.

The mass-averaged total-pressure loss coefficient is probably the most commonly used parameter for measuring losses since it is easy to calculate from experimental cascade measurements. This coefficient is defined as the loss in mass-averaged total pressure through the airfoil row normalized with the mass-averaged dynamic pressure at the outlet of the airfoil row:

$$Y = \frac{P_{01}'' - P_{02}''}{q_2''} \quad (2.1)$$

where P_0 and q are the total and dynamic pressures, respectively. The subscripts 1 and 2 refer to the inlet and outlet of the airfoil row. In this work, the superscript double-prime (′′) denotes a mass-averaged quantity. For determining losses in turbomachinery flows, mass-averaging, as opposed to area- or arithmetic-averaging, is necessary because it produces integrated results that are consistent with the conservation equations. The data reduction procedure that was used to obtain the values of the mass-averaged pressure loss coefficient is described in Section 3.9.

Another parameter that is sometimes used to quantify losses is the mass-averaged kinetic energy coefficient. It is defined as

$$\phi^2 = \frac{(V_2^2)''}{(V_{2is}^2)''} \quad (2.2)$$

where V_{2is} is the isentropic outlet velocity and is obtained from an isentropic expansion to the same final pressure as the actual process. Although the kinetic energy coefficient cannot be measured experimentally, it is a convenient parameter for design purposes. Conversions

between ϕ^2 and Y are easily made using

$$Y = \frac{\left[1 - \frac{\gamma-1}{2}M_2^2 \left(\frac{1}{\phi^2} - 1\right)\right]^{-\frac{\gamma}{\gamma-1}} - 1}{1 - \left(1 + \frac{\gamma-1}{2}M_2^2\right)^{-\frac{\gamma}{\gamma-1}}} \quad (2.3)$$

where M_2 is the Mach number at the outlet of the airfoil row and γ is the specific heat ratio. For incompressible flows, Equation 2.3 reduces to

$$Y = \frac{1}{\phi^2} - 1. \quad (2.4)$$

2.2.3 Loss Components

Traditionally, total-pressure losses in turbines are divided into individual components that are superimposed linearly: this is known as the conventional loss breakdown scheme. Empirical correlations are then used to predict the losses for each loss component. The basic components are usually identified as follows:

- Profile losses:** losses generated in the airfoil suction and pressure surface boundary layers in a uniform two-dimensional flow. The extra loss due to the sudden area increase at the finite trailing-edge thickness of the airfoil is usually included in the profile losses.
- Secondary losses:** losses associated with the endwall boundary layers and the various vortical structures in the endwall region in the absence of tip clearance (e.g., horseshoe vortex, corner vortex, etc.).
- Tip leakage losses:** losses arising from the leakage of flow over the tips of rotor blades.

A few salient points are noteworthy regarding the basic loss components. First, the secondary and tip-leakage losses are categorized as three-dimensional loss components since significant spanwise and pitchwise flow occurs. Second, in two-dimensional cascades such

as the ones used in the present study, the measured midspan loss is usually assumed to be representative of the profile losses notwithstanding that the midspan flow is never precisely two-dimensional. The present work deals with only the profile and secondary loss components.

As will be discussed in Chapter 7, the data from the present study indicate that under certain conditions the conventional scheme for subdividing the losses gives physically misleading results. An alternative scheme that appears to be more consistent with the observed physics is proposed in Chapter 8.

2.3 General Description of Flow in an Airfoil Passage in the Absence of Tip Clearance

A major focus of the present work is the study of secondary flows: the complex three-dimensional flows near the endwall of turbine airfoil passages in the absence of tip clearance. These flow structures would typically be found at the hub wall of rotor airfoils and the hub and tip walls of stator airfoils. The terminology often used to describe endwall flows (i.e., “secondary”) is unfortunate in that it leaves the impression that these flows are of secondary importance. On the contrary, the losses produced by secondary flows are significant: they can account for as much as 30-50% of the total loss in a blade or stator row (e.g., Sharma and Butler, 1987).

The physics of secondary flows have been studied extensively since the early 1940’s, and as a result, the basic flow in the endwall region is fairly well understood. Nonetheless, as noted by both Sieverding (1985*b*) and Langston (2001) in their excellent reviews of secondary flows, the picture continues to evolve. Sjolander (1975) proposed one of the earliest pictures of the flow field in this region, and since then, refinements to his basic

Figure 2.3: Endwall flow structures, reproduced from Wang *et al.* (1997).

picture have emerged (e.g., Langston *et al.*, 1977; Sharma and Butler, 1987; Hodson and Dominy, 1987a; Wang *et al.*, 1997). As will be discussed further in Chapter 6, the present author's interpretation of the flow behaviour is largely consistent with the picture proposed by Wang *et al.* (1997). A brief description of this model is given here, highlighting only the major structures that are observed in the endwall region. A more detailed discussion is presented in Chapter 6.

Based on flow visualization results obtained using multiple smoke wires and laser light, Wang *et al.* (1997) proposed the endwall flow picture shown in Figure 2.3. As can be observed, the flow field is composed of a complex system of vortical structures. One of these vortices is referred to as the passage vortex: it is the largest and often the most

vigorous vortex in this region. The data from the current study suggest that it contributes significantly to the secondary losses, as will be discussed in Chapter 6.

The passage vortex seems to be composed mainly of fluid from the inlet endwall boundary layer. Some of the endwall boundary layer fluid that approaches the airfoil row encounters an adverse pressure gradient in the leading-edge region which causes the boundary layer to separate. This separated fluid then rolls up into a horseshoe vortex that wraps around the leading edge with suction- and pressure-side legs. The cross-passage pressure gradient causes the pressure-side leg to migrate across the airfoil passage along the endwall towards the suction surface of the adjacent airfoil. It has been well established through measurement that the pressure-side leg of the horseshoe vortex evolves into the passage vortex.

Because the passage vortex is a structure that originates in the leading-edge region, a valid question is whether the leading-edge geometry has a significant influence on the strength of this vortex. As will be shown in the next section, there is very little published data on the influence of leading-edge geometry on secondary losses. The present study was conducted, in part, to help provide some physical insights into this relationship.

2.4 Summary of Experimental Investigations

Measurement of losses in cascades have been made for many decades and have resulted in continual progress in the understanding of the flow physics. This has led to improved aerodynamic design of turbines, which usually takes the form of either reduced stage/airfoil counts or higher stage efficiencies, or a combination of both. Therefore, for the purposes of this work (e.g., developing updated empirical prediction methods) the measurements made on turbine airfoils of recent design are the most relevant. Consequently, the review

of previous work has concentrated on those studies that have been published since the mid-70's.

A summary of the experimental investigations that have focused on profile losses and secondary flows in turbine cascades is presented in Table 2.1. This table shows that there is very little data on the influence of leading-edge geometry on loss components at either design or off-design incidence. The table has also proved to be useful for identifying those geometric or aerodynamic parameters that have a significant influence on the loss components. This facilitated the development of the new empirical prediction method for secondary losses, which is discussed in Chapter 8.

As can be seen, details of each investigation are presented under six main column headings:

- 1) Reference,
- 2) loss component investigated,
- 3) measurement details,
- 4) parameters investigated,
- 5) conditions for experimental investigations, and
- 6) comments.

The studies are tabulated according to the “loss component investigated” (i.e., column 2): those which emphasized midspan losses are listed first, followed by those which examined secondary flows. Within each group, the studies are ordered chronologically. For cases where data are presented for both loss components, the component which is the primary focus of the study is given in bold typeface. The last column presents a brief summary of the major findings for each experiment. This column also gives information regarding the type and in certain cases the identity of the cascade. For example, the investigation of Langston *et al.* (1977) was performed in a linear cascade with the “UTRC/VPI&SC” cascade geometry. From the table, it can easily be observed that this particular cascade geometry has been

widely used for secondary flow studies. Finally, the abbreviations used in the table are listed below:

ann	: annular	Ma	: Mach number
AP	: airfoil passage	meas	: measured
AR	: aspect ratio (h/C)	obj	: objective
BL	: boundary layer	press	: pressure
casc	: cascade	pro	: profile
CE	: contoured endwall	Re	: Reynolds number
CFTR	: cold flow turbine rig	red	: reduction
cmp	: compare	ReS	: Reynolds stress
d/s	: downstream	s/C	: pitch-to-chord ratio
decr	: decrease	S, sec	: secondary
diff	: different	SFC	: secondary flow control
grad	: gradient	sig	: significantly
i	: incidence	SS	: suction surface
IBLS	: inlet boundary layer skew	t	: trailing-edge thickness
IBLT	: inlet boundary layer thickness	TE	: trailing edge
incr	: increase	TKE	: turbulence kinetic energy
LCI	: loss component investigated	TL	: tip leakage
LD	: loading distribution	Tu	: inlet turbulence intensity
LE	: leading edge	turb	: turbine
LEG	: leading-edge geometry	Unst	: unsteadiness
lin	: linear	viz	: visualization
M, mid	: midspan	τ	: tip clearance

2.5 Profile Loss Investigations

2.5.1 Experiments at Transonic Conditions

From Table 2.1, it can be seen that over half of the profile loss investigations were conducted at transonic Mach numbers; that is, conditions commonly observed in the engine

Table 2.1: Summary of experimental investigations.

Reference	LCI	Measurement Details										Parameters Investigated										Conditions for Experimental Investigations			Comments
		Location of Flow Field Measurement Within d/s of AP	Aerofoil surface static press.	Hot wire	Press. probe	Flow Viz.	Laser	i	Re	Ma	Tu	s/c	IBLT	IBLS	AR	LD	LEG	SFC	Unst.	Incidence [deg.]	Re x 10 ³	Ma _{ss}	Tu [%]		
Arnov <i>et al.</i> (1975)	M	✓	✓	-	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-20.0 to +20.0 0.0 to +15.0	-	-	-	• Measured profile losses for two different lin. casc. (45 and 105 deg. of turning) over a range of i.	
Graham & Kost (1979)	M	-	✓	✓	-	Schli- eren	-	✓	✓	-	-	-	-	-	-	-	-	-	-	-20 to +8	8.6 ₂	0.7 to 1.4	≈1.0	• Meas. d/s flow field in two cases.; one of which has flat-backed aerofoils.	
Joslyn & Dring (1983)	M	-	✓	-	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-45 to +5	-	low subsonic	-	• No loss data presented.	
Kock <i>et al.</i> (1986)	M	-	✓	-	✓	Schli- eren	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	3.5-8.0 ₂	0.8-1.1	≈1.0	• Meas. made of the transonic flow through a lin. turb. casc. in 4 wind-tunnels.	
Patterson & Hoeger (1986)	M	✓	✓	✓	-	oil neutral	-	✓	-	-	✓	-	-	-	-	-	-	-	-	Design	1.5-10.0 ₂	0.78	4.0	• Meas. made in 3 LPT stator casc. of different flow turning & LD; • Incr mid. losses with decr. Re; • no data presented.	
Williamson & Moustapha (1986)	M, S	-	✓	✓	-	oil	-	✓	-	✓	-	-	-	-	-	-	-	-	-	Design	90.0 ₂ at Ma _{ss}	0.65-1.04	1.7	• Meas. performance of an ann. casc. of stators; • OD has contoured endwall (Russian kink).	
Dietrichs <i>et al.</i> (1987)	M	-	✓	✓	-	Schli- eren	-	✓	✓	-	-	-	-	-	-	-	-	-	-	-10 to +20	-	0.6-1.3	-	• Mainly Schlieren and Ma distr. are presented; • no integrated quantities (incl. losses) are given.	
Hohneisel <i>et al.</i> (1987)	M	✓	✓	✓	-	-	-	-	-	-	✓	-	-	-	-	-	-	-	-	-37.7 to +12.3	1.0-10.0 ₂	0.30-0.95	1.0-8.0	• Examined 3 lin. casc. of LPT aerofoils (T104, T105, T106); • all cases have same duty but diff. LD; • BL meas. at various locations along SS.	
Vijayaraghavan & Kavanaugh (1988)	M	-	✓	✓	-	✓	-	✓	✓	-	-	-	-	-	-	-	-	-	-	-7, Design, +7	3.0-7.0 ₁	low subsonic	0.8-6.4	• Mid. losses insensitive to changes in i & Tu.	
Detemple-Laake (1988)	M, S	-	-	-	-	oil & Schli- eren	-	✓	✓	-	-	-	-	-	-	-	-	-	-	-10, +10, +20, +25	-	0.8-1.3	-	• No loss data presented, mainly flow viz.	
Goobie <i>et al.</i> (1989)	M	-	✓	✓	-	oil	-	✓	-	-	-	-	-	-	-	-	-	-	-	-15, Design, +15	1.0-3.0 ₁	low subsonic	1.5	• I was found to have large effect on blade loading in LE region.	
Tremblay <i>et al.</i> (1990)	M	-	✓	✓	-	oil	-	✓	-	-	-	-	-	-	-	-	-	-	-	-25 to +25 ΔI = 5	1.0-3.0 ₁	low subsonic	1.0	• For i > 15°, steep rise in mid. losses; • small increase in losses for i < 0°.	
Rodger <i>et al.</i> (1992)	M	-	✓	✓	-	oil	-	✓	-	-	-	-	-	-	-	-	-	-	-	-20, 0, 20	3.0 ₁	low subsonic	0.3	• Studied the effect of AVR on mid. losses	
Meer <i>et al.</i> (1992)	M	✓	✓	-	✓	Schli- eren	-	-	✓	✓	-	-	-	-	-	-	-	-	-	Design	4.5-20.0 ₂	0.6 to 1.2	4.1	• At transonic Mach No. TE loss begins to dominate. BL losses are less significant.	
Moustapha <i>et al.</i> (1993)	M, S	-	✓	✓	-	oil & Schli- eren	-	-	✓	-	-	-	-	-	-	-	-	-	-	Design	2.0 ₂ at Ma _{ss}	0.3-1.3	0.2	• Examined the performance of a transonic stator in a lin. casc.; • results compared with same stator in ann. casc. and stage environment.	
Whitehouse <i>et al.</i> (1993)	M	-	✓	✓	-	oil	-	✓	-	✓	-	-	-	-	-	-	-	-	-	-20 to +20	3.0 ₁	low subsonic	0.3, 3.0	• Also investigated the effect of AVR and LEG; • Tu had little effect on mid. losses.	
Kind <i>et al.</i> (1998), Abbott (1993), Serjak (1995)	M	-	✓	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-10 to +15	3.0 ₁	low subsonic	0.3	• Examined the influence of surface roughness on the performance of the L33 lin. casc.;	
Jouni <i>et al.</i> (2001)	M	-	✓	✓	-	-	-	-	✓	-	-	-	-	-	-	-	-	-	-	-10.0, 0.0, +4.5, +10.0, +14.5	4.0-10.0 ₂	0.5-1.2	4.0	• Cascade tested: H5A; • at subsonic Ma, losses begin to incr. at Re = 6.0 × 10 ⁵ ; • losses at trans. Ma are closely related to base press. behaviour.	
Jouni <i>et al.</i> (2002)	M	-	✓	✓	-	-	-	-	✓	-	-	-	-	-	-	-	-	-	-	-10.0, 0.0, +4.5, +10.0, +14.5	4.0-10.0 ₂	0.5-1.2	4.0	• Examined two cases that differed mainly in LEG: (1) Baseline, H5A; (2) H5B, same service as (1) but β ₁ is 10° larger; • incidence sensitivity for H5B was no different than H5A	
Langston <i>et al.</i> (1977)	S	✓	✓	✓	-	oil	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	10 ₂	low subsonic	1.0	• UTRC/VP1&SU casc.; • low AR -> sec. flows merge at mid.; • presents loss growth through casc.	
Marchal & Sieverding (1977)	S	✓	✓	✓	-	smoke, oil	-	✓	-	-	✓	-	-	-	-	-	-	-	-	Design	stator 2.6, rotor 2.82	incomp	0.5	• Examined flow field in a stator and rotor casc.; • 2 different IBLTs were tested for stator case.	
Graziani <i>et al.</i> (1980)	M, S	-	-	✓	-	oil	-	-	-	-	✓	-	-	-	-	-	-	-	-	Design	8.8 ₂	low subsonic	1.0	• UTRC/VP1&SU casc.; • focus of investigation is on surface heat transfer rates but flow viz. also performed.	
Kopper <i>et al.</i> (1980)	M, S	-	✓	✓	-	oil	-	-	-	-	-	-	-	-	-	-	-	-	✓	Design	18.0 ₂	0.85	2.6	• Examined "Russian kink" in a stator casc.; h ₂ /h ₁ =0.83; • meas. 17% red. in overall loss comp. to casc. w/ planar walls.	
Langston (1980)	S	✓	✓	✓	-	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	Design	10 ₂	low subsonic	1.0	• UTRC/VP1&SU casc.; • presents endwall velocity profiles within passage	

Reference	LCI	Measurement Details						Parameters Investigated										Conditions for Experimental Investigations				Comments		
		Location of Flow Field Measurement Within d/s of AP	Aerofoil surface static press.	Hot wire probe	Press. probe	Flow Viz.	Laser	i	Re	Ma	Tu	s/c	IBLT	IBLS	AR	LD	LEG	SFC	Unst	Incidence [deg.]	Re x 10 ⁵		Ma _{2s}	Tu [%]
Severding & Wilpulte (1981)	S	✓	✓	-	✓	-	-	-	✓	-	-	-	-	-	-	-	-	-	-	Design	2.6-11.0, 18.0 ₂	0.1, 0.6, 0.8	0.5, 1.0, 1.0	• Examined loss growth through casc. for 3 Ma; • loss contours show little effect of Ma on sec. losses.
Hunter (1982)	S	-	✓	-	✓	oil	-	-	-	-	-	✓	-	-	-	-	-	-	-	Design	4.39, 3.29 ₁	incomp.	-	• CFTR test; measurements made d/s of stator and rotor in single-stage turbine.
Bridger & Romey (1983)	S	-	✓	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	2.6 ₁	0.74	-	• Examined the flow field d/s of a nozzle guide vane (no rotor present) in an ann. casc.
Severding & Van den Bosch (1983)	S	-	-	-	-	smoke	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	-	incomp.	-	• Examined details of the sec. flow structures.
Camus <i>et al.</i> (1984)	S	-	✓	✓	✓	oil	-	✓	✓	-	-	-	-	-	-	-	-	-	-	-10, 0, 5	3.6-10.6	0.7-1.2	-	• No incidence data presented; • mainly effects of Ma; • very little flow field data presented.
Gaugler & Russell (1984)	S	-	-	-	-	smoke, oil	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	0.7-3.2 ₁	incomp.	-	• Geometry examined was nozzle guide vane.
Severding <i>et al.</i> (1984)	S	✓	✓	-	-	smoke	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	0.98 ₁	incomp.	0.8	• Examined velocity profiles and total pressure distributions within and d/s of an ann. casc. of stator aerofoils.
Moore & Ransmayr (1984)	S	-	✓	✓	✓	-	-	-	-	-	-	✓	-	-	-	-	-	-	-	Design	5.8 ₂	low subsonic	-	• UTRCVPI&SU casc.; • examined two leading-edge shapes: circle & wedge.
Moore & Smith (1984)	S	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	5.8 ₂	low subsonic	-	• UTRCVPI&SU casc.; • Examined sec. flow trajectories using ethylene detection.
Moore & Adhye (1985)	S	✓	✓	✓	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	Design	5.2 ₂	low subsonic	0.2	• UTRCVPI&SU casc.; • AR < 1 so sec. flows merge at mid. at x/Cx=1.1.
Goldstein & Chen (1985)	S	-	-	-	-	oil	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	-	incomp.	-	• Focus is mainly on film cooling effectiveness but some oil flow viz presented.
Moustapha <i>et al.</i> (1985)	S	-	✓	✓	✓	oil	-	-	✓	-	-	-	-	-	-	-	-	-	-	Design	3.0-6.0 ₁	low subsonic	4.0	• Examined the flow field in both a lin. and ann. casc. of rotor blades of identical profile; • examined two higher losses in ann. casc.
Moustapha & Williamson (1985)	S	-	✓	✓	✓	oil	-	-	✓	-	-	-	-	-	-	-	-	-	-	Design	'90 ₂	0.67-1.20	1.7	• Examined two axisymmetric outer endwall pro. in an ann. casc.; • lower losses were meas. w/ 'Russian kink' pro. over complete Ma range.
Sonoda (1985)	S	-	✓	✓	-	smoke	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	0.48 ₂	incomp.	0.2	• Examined details of the sec. flow structures.
Chan & Dixon (1986)	S	-	✓	✓	-	✓ (real)	-	-	-	-	✓	-	-	-	-	-	-	-	-	Design	3.5 ₂	-	-	• IBLT had no effect on sec. losses
Lilek (1986)	S	✓	✓	✓	✓	✓	-	✓	-	-	-	-	-	-	-	-	-	-	-	Design	Stator 14 ₂ , Rotor 10 ₂	low subsonic	2.0	• Examined stator and impulse rotor casc.; • insufficient data to obtain loss coefficients.
Atkins (1987)	S	-	✓ (baseline only)	✓ (slot mesh)	-	-	-	-	-	-	-	✓	-	-	-	-	-	-	✓	Design	4.0 ₁	incomp.	-	• Examined various axisymmetric outer endwall profiles; • obs. 10% red. in overall losses with 'Russian kink' profile
Bridger <i>et al.</i> (1987)	S	-	-	-	-	-	L2F	-	-	-	-	-	-	-	-	-	-	-	✓	Design	-	transonic	-	• L2F measurements in the endwall region of a single-stage cold flow turbine; • examined physics of unsteady flow interaction.
Hodson & Donnin (1987a)	S,M	✓	✓	✓	✓	oil	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	2.9 ₂	subsonic	0.5	• Performed detailed traverses within and d/s of aerofoil passage using 5-hole probe
Hodson & Donnin (1987b)	S,M	-	✓	✓	✓	oil	-	✓	✓	-	-	✓	-	-	-	-	-	-	-	-20 to +9	1.5-6.0 ₂	subsonic	0.5	• Sec. losses incr. with incr. Re, i, and s/c; • sec. losses were independent of IBLT.
Moore <i>et al.</i> (1987)	S	-	✓	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	5.2 ₂	subsonic	0.3	• UTRCVPI&SU casc.; • turbulence measurements made with 2-wire (X) probe to obtain full ReS tensor.
Yamamoto (1987a)	S	✓	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	2.8 ₂	low subsonic	0.5	• Part I: examined stator casc
Yamamoto (1987b)	S	✓	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	1.8 ₂	low subsonic	0.5	• Part II: examined rotor casc.
Zunino <i>et al.</i> (1987)	S	✓	✓	✓	✓ (real)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	3.1 ₂	low subsonic	0.4	• High TKE and ReS associated with passage vortex.

Reference	LCI	Measurement Details					Parameters Investigated										Conditions for Experimental Investigations				Comments				
		Location of Flow Field Measurement Within d/s of AP	Aerofoil surface static press.	Hot wire	Press. probe	Flow Viz.	Laser																		
								i	Re	Ma	Tu	s/c	IBLT	IBLS	AR	LD	LEG	SFC	Unst	Incidence [deg.]		Re x 10 ⁵	Ma ₉₀	Tu [%]	
Gregory-Smith & Graves (1983) Gregory-Smith <i>et al.</i> (1988a)	S	✓	✓	-	✓	oil	-	-	-	✓	-	-	-	-	-	-	-	-	-	Design	5.0 ₂	low subsonic	1.4	- No clear trend of sec. loss with IBLT.	
Gregory-Smith <i>et al.</i> (1988b)	S	✓	✓	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	5.0 ₂	low subsonic	1.4	- Measured TKE and components of Tu.	
Han <i>et al.</i> (1988)	S	-	✓	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	4.5 ₂	-	-	- Examined the effect of straight lean on losses in ann. casc. observed a sig. effect on losses.	
Mobarak <i>et al.</i> (1988)	S, M	-	✓	-	✓	-	-	-	✓	-	-	✓	-	-	-	-	-	-	-	Design	2.2-5.5 ₂	0.2 - 0.5	2-3	- Also investigated the effect of s/c on mid. & sec. losses.	
Yamamoto & Nouse (1988)	S, M	✓	✓	-	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-53 to +7	1.8 ₂	low subsonic	-	- For i <23° loss generation was mainly due to flow on pass. surface.	
Niehuis <i>et al.</i> (1989)	S	-	✓	-	✓	oil	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	-	subsonic	-	- strength of passage vortex incr. with increasing i.	
Yaras & Sjölander	TL, S, M	-	✓	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	4.3 ₁	low subsonic	1.5	- Examined the flowfield d/s of an ann. casc. of low turning LPT stators.	
Harrison (1990)	S	✓	✓	-	✓	oil	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	8.7 ₂	low subsonic	0.4	- TL losses incr. with i/c;	
Hebert & Tiederman (1990)	S	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	0.7 ₁	incomp.	2.3	- sec. losses decr. with incr. i/c.	
Perdichizzi (1990)	S, M	-	✓	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	-	0.2 - 1.5	-	- Performed detailed traverses within and d/s of aerofoil passage;	
Walsh & Gregory-Smith (1987) Walsh & Gregory-Smith (1990)	S	✓	✓	-	✓	oil	-	-	-	-	-	-	✓	-	-	-	-	-	-	Design	4.0 ₂	low subsonic	-	- measured endwall shear stress.	
Ajlout & Dixon (1992)	S	-	✓	✓	✓	oil, smoke	-	-	-	✓	-	-	-	-	-	-	-	-	-	Design	3.7 ₂	low subsonic	0.7 - 3.8	- Investigated effect of simulated rotor wakes on sec. flow structure in a turbine stator.	
Gregory-Smith & Biesinger (1992)	S	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	4.0 ₂	low subsonic	-	- Sec. loss becomes smaller fraction of total losses as Ma no. incr.	
Gregory-Smith & Cleak (1992)	S	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	4.0 ₂	low subsonic	low, high	- IBLT has a sig. effect on sec. losses.	
Harrison (1992)	S	✓	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	8.7 ₂	low subsonic	0.4	- Very little effect of Tu on sec. losses.	
Jabbari <i>et al.</i> (1992)	S	-	-	-	-	oil	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	1.2 ₁	incomp.	-	- Very little effect of Tu on sec. losses.	
Chan <i>et al.</i> (1993)	TL, S, M	-	✓	✓	-	✓	-	-	-	✓	-	-	-	-	-	-	-	-	-	Design	4.3 ₁	low subsonic	1.5	- For i little effect of Tu on sec. losses and turbulence flow field.	
Dossena <i>et al.</i> (1993)	S	-	✓	-	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	Design	3.7 ₂	low subsonic	0.9	- Investigated the influence of compound and straight lean on losses in a lin. casc. Compound lean case produces lowest mixed-out losses.	
Perdichizzi & Dossena (1993)	S, M	-	✓	-	✓	oil	-	✓	-	-	-	-	-	-	-	-	-	-	-	-30 to +30	8.4 ₂	subsonic	0.9	- Examined details of the sec. flow structures; included mass transfer measurements.	
Doerfler & Amsacke (1994)	S	-	✓	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	-	0.7	-	- No effect of IBLT on secondary losses.	
Han <i>et al.</i> (1994)	S	✓	✓	-	✓	helium bubble	-	-	-	-	-	-	-	-	-	-	-	-	-	-60 to +35	8.4 ₂	subsonic	0.9	- Large effect of s/c and i on turbulence field.	
Rama Murty & Venkatrayulu (1994)	S, M	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	6.4 ₂	0.26	-	- Sec. losses incr. with incr. i;	
Sauer & Wolf (1994)	S	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	- no clear trend between sec. losses and s/c;	
Moore & Gregory-Smith (1995)	S	✓	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	4.0 ₂	low subsonic	4.5	- Examined the effect of streamwise and cross-streamwise oriented endwall fences on sec. flows.	
																									- no integrated quantities presented.
																									- Investigated the influence of compound and straight lean on losses in a lin. casc. A reduction in losses is observed for the case with compound lean.
																									- Sec. losses incr. for both +i and -i;
																									- mid. losses not representative of pro. losses due to strong sec. flows
																									- Examined turbulence flow field with emphasis on near-endwall and SS BL

Reference	LCI	Measurement Details					Parameters Investigated										Conditions for Experimental Investigations				Comments				
		Location of Flow Field Measurement Within d/s of AP	Aerofoil surface static press.	Hot wire probe	Press. probe	Flow Viz.	Laser	i	Re	Ma	Tu	s/c	IBLT	IBLS	AR	LD	LEG	SFC	Unst	Incidence [deg.]		Re x 10 ³	Ma _∞	Tu [%]	
																								Re x 10 ³	Tu [%]
Weiss & Foltner (1995)	S	-	✓	-	✓	-	oil	-	-	-	-	-	-	-	-	-	✓	-	-	-	Design	5.0 ₂	0.59	4.0	• front-loaded casc. produces higher sec. losses
Moore & Gregory-Smith (1996)	S	✓	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	4.0 ₂	low subsonic	5.0	• Measurements were made of the intermittency near endwall, PS & SS.
van de Wall <i>et al.</i> (1996)	S	-	-	-	-	-	✓	-	-	-	-	-	-	-	-	-	-	-	✓	-	Design	-	incomp.	-	• Examined the influence of a simulated vortex (both stationary and moving) on the flowfield d/s of a lin. casc.
Wang & Han (1995)	S	-	✓	✓	✓	oil	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	6.4 ₂	0.26	-	• Described the flow physics for a lin. turbine casc. with compound lean.
Duden & Foltner (1997)	S	✓	✓	-	✓	✓	-	✓	-	-	-	-	-	-	-	-	-	-	-	-	Design	12, 5.0 ₂	subsonic	1.4-6.8	• Examined two cases of T106 profile: // & diverging endwall; case with tapered endwall (more low. loaded) has sigly higher losses; Ma & Re small effect on losses
Schnaus & Foltner (1997)	S	✓	✓	✓	✓	-	✓	-	-	-	-	-	-	-	-	-	-	✓	-	-	Design	5.0 ₂	0.59	4.0	• Examined the influence of divergence and contouring of a single endwall on the losses for T106 casc.;
Wang <i>et al.</i> (1997)	S	-	-	-	-	smoke	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Design	0.27 ₂	incomp.	0.2	• incr. divergence causes radial press. grad. and larger losses.
Harland <i>et al.</i> (1998)	S	✓	✓	✓	✓	-	✓	-	-	-	-	-	-	-	-	-	-	✓	-	-	Design	4.0 ₂	incomp.	5.0	• Based on smoke flow viz. using laser light sheet, developed a new model describing sec. flows.
Michelassi <i>et al.</i> (1998)	S	✓	✓	✓	✓	-	✓	-	-	-	-	-	-	-	-	-	-	✓	-	-	Design	1.8 ₁	low subsonic	2.0	• Examined nonaxisymmetric endwall profiling in Durham casc.; obj. was to red. static press. non-uniformities on the endwall d/s of casc.
Dossena <i>et al.</i> (1999)	S	✓	✓	✓	✓	-	✓	-	-	-	-	-	-	-	-	-	-	✓	-	-	Design	10.8 ₂	subsonic	0.5	• Examined the flow field within and d/s of a lin. casc. w/ low-turning LPT stators with highly divergent CEs.
Duden & Foltner (1999)	S	✓	✓	✓	✓	oil	-	✓	-	-	-	-	-	-	-	-	-	✓	-	-17.7, 0, +12.3	-	subsonic	3.6, 3.3, 2.5	T106 casc. with CE tested by Duden <i>et al.</i> , is examined at off-design t.	
Duden <i>et al.</i> (1999)	S	✓	✓	✓	✓	-	✓	-	-	-	-	-	-	-	-	-	-	✓	-	-	Design	5.0 ₂	0.59	3.3	• CE sig. reduces loss sensitivity to +1.
Kang <i>et al.</i> (1999)	S	✓	✓	✓	✓	-	✓	-	-	-	-	-	-	-	-	-	-	✓	-	-	Design	6.0, 12.0 ₂	incomp.	0.6	• Baseline: T106; examined 3 stacking arrangements with axisym. contouring to outer endwall; inner wall is diverging; overall loss remains unchanged.
Yan <i>et al.</i> (1999)	S	✓	✓	✓	✓	oil	LDV	-	-	-	-	-	-	-	-	-	-	✓	-	-	Design	5.0 ₂	incomp.	5.0	• Measurements made mainly in LE region; results suggest that strength of LE vortex is sensitive to Re.
Aunapu <i>et al.</i> (2000)	S	✓	✓	✓	✓	-	-	-	-	-	-	-	-	-	-	-	-	✓	-	-	Design	1.8 ₁	incomp.	8.0	• Examined nonaxisymmetric endwall profiling in a stator casc.; observed 20% reduction in mixed-out secondary loss.
Harland <i>et al.</i> (2000)	S	✓	✓	✓	✓	oil	-	✓	-	-	-	-	-	-	-	-	-	✓	-	-	Design	4.0 ₂	incomp.	5.0	• Examined GE CFE-50 rotor profile in a casc. simulator; SFC is achieved using a diff. arrangements of endwall jets.
Sauer <i>et al.</i> (2001)	S	✓	✓	✓	✓	-	✓	-	-	-	-	-	-	-	-	-	-	✓	-	-	Design	4.5 ₂	0.2	<1.0	• Examined non-axisymmetric endwall profiling in Durham casc. (rotor aerobol); observed 1/3 reduction in mixed-out sec. loss.
Ingram <i>et al.</i> (2002)	S	✓	✓	✓	✓	-	✓	-	-	-	-	-	-	-	-	-	-	✓	-	-	Design	4.0 ₂	incomp.	5.0	• Examined T106 casc. with a profile mod. in the LE region close to the endwall (LE bulb); observed 50% red. in sec. loss.
Treiber <i>et al.</i> (2002)	S	✓	✓	✓	✓	-	✓	-	-	-	-	-	-	-	-	-	-	✓	-	-	Design	4.0 ₂	incomp.	5.0	• Examined non-axisymmetric endwall profiling in Durham casc. (rotor aerobol); observed 24% red. in mixed-out sec. loss.
	S	✓	✓	✓	✓	-	✓	-	-	-	-	-	-	-	-	-	-	✓	-	-	Design	-	0.5	-	• Examined the effect of compound lean in an ann. casc.; case with compound lean has lower losses between 0.3 and 0.5 Cx d/s of TE; sign. higher losses at 1.0 Cx d/s of TE.

Note: $\xi = 1 - \phi^2$
total = total profile loss
ref = reference conditions, $M_{exit} = 0.92$ and $Re_{exit} = 1.6 \times 10^6$

Figure 2.4: Contributions to profile losses, reproduced from Mee *et al.* (1992).

environment. From the literature, it is clear that compressibility effects become particularly important in the transonic Mach number regime because a trailing-edge shock system begins to develop. The flow fields and the loss mechanisms in transonic turbines are extremely complex because of this shock system and its interaction with the airfoil surface boundary layers. Figure 2.4 shows the effect of Mach number on the overall profile losses. It can be seen that non-linear variations in the profile loss occur in the transonic regime.

The complex flow behaviour under transonic conditions is an important aspect of turbine aerodynamics which is not well understood. However, the investigations conducted in the

current work have been limited to low-speed testing. Investigations at low-speed conditions have been successfully used in the past to gain important insights into the aerodynamic and geometric parameters which influence profile losses and to provide useful experimental data for design purposes. Furthermore, it is common to use data from low-speed testing as the basis for the development of correlations. Corrections are then applied to account for factors such as compressibility effects (e.g., Ainley and Mathieson, 1951; Dunham and Came, 1970; Kacker and Okapuu, 1982). In fact, Corriveau (2003) is currently incorporating compressibility effects into the present author's profile loss correlation (see Section 5.3.5) based on his measurements in transonic turbine cascades conducted in the High Speed Wind Tunnel at Carleton University.

2.5.2 Low-Speed Experiments

A review of the low-speed profile loss data assembled by Moustapha *et al.* (1990) and a survey of more recent published data has been performed by the present author. A "cascade database" was created and subsequently used to develop an improved correlation (see Section 5.3.5). It is worth noting that the references to all cases in the database are included in Table 2.1.

The survey indicated a need for additional and more detailed experimental results, particularly for turbine airfoils of recent design. For example, of the eighteen cascade data sets listed by Moustapha *et al.* (1990) only five were published in the previous decade. Only five additional cascade data sets have been published since the work of Moustapha *et al.* Furthermore, in a number of the cascade data sets, important details of the airfoil geometry or flow conditions were omitted, thus limiting the value of the data. Because of the limited number of recent and sufficiently detailed experimental results, some of the experimental loss measurements included in the current database were taken from older blade designs.

Figure 2.5: Typical variation of profile losses with incidence at low-speed conditions.

In this section, a description of the more recent experimental studies is given.

As an aid to this discussion, a typical variation of profile losses with incidence is shown in Figure 2.5. Generally a small increase in losses occurs at negative incidence, and a sharper rise for positive incidence.

In a number of investigations, although midspan losses were presented, the emphasis of the study was on the effects of incidence on secondary losses (e.g., Hodson and Dominy, 1987b; Yamamoto and Nouse, 1988; Perdichizzi and Dossena, 1993; Ramana Murty and Venkatrayulu, 1994; Duden and Fottner, 1999).

Other investigations have emphasized mainly the effect of incidence on profile losses. Vijayaraghavan and Kavanagh (1988) measured midspan losses over a narrow range of

incidence. The losses were found to vary only slightly with change in incidence probably because of the limited range of incidence investigated. The remaining investigations discussed in this section were conducted in the Low-Speed Turbomachinery Laboratory at Carleton University. It is worth noting that the present experimental investigation was performed in the same facility.

Goobie *et al.* (1989), Tremblay *et al.* (1990) and Rodger *et al.* (1992) investigated the effect of incidence on midspan losses: the same airfoil design was used for each investigation. Tremblay *et al.* (1990) concluded that, for attached flow, the variation in losses with incidence was a result of loss production which occurred in the leading-edge region. At high values of positive incidence, where trailing-edge separation began to appear, the trailing-edge losses became significant.

Whitehouse *et al.* (1993) performed a study to investigate the effects of leading-edge geometry on the profile losses of a turbine cascade at off-design incidence. The turbine cascade used in their experiment was aerodynamically similar to the one used by Goobie *et al.* (1989), Tremblay *et al.* (1990) and Rodger *et al.* (1992) but differed in leading-edge geometry. The airfoil was designed according to the correlation of Moustapha *et al.* (1990), with the intent of reducing losses at off-design incidence. However, it was found that the change in leading-edge geometry did not reduce the airfoil sensitivity to losses at off-design incidence as expected, which suggested that the effect of leading-edge geometry was not correctly accounted for in the correlation of Moustapha *et al.* (1990). This prompted the present author to re-examine the influence of leading-edge geometry on profile losses. It was found that a significantly better correlation could be obtained by including an additional and more influential leading-edge geometric parameter, as discussed in Chapter 5. The present author also subsequently used the same two cascades to investigate the influence of leading-edge geometry and incidence on secondary losses.

2.6 Secondary Loss Investigations

From Table 2.1 it can be seen that most studies of secondary flows have been conducted at the design incidence, and this work is described first. Following this, the few available studies of secondary losses at off-design incidence are summarized.

Many of the design-incidence investigations were conducted to help clarify details of the flow structures in the endwall region (e.g., Sjolander, 1975; Langston *et al.*, 1977; Sieverding and Van den Bosche, 1983; Sonoda, 1985; Hodson and Dominy, 1987a; Sharma and Butler, 1987; Jabbari *et al.*, 1992; Wang *et al.*, 1997). Some studies were concerned with secondary loss production mechanisms (e.g., Moore and Adhye, 1985; Gregory-Smith *et al.*, 1988a; Harrison, 1990) while other investigations examined the effect of various aerodynamic and geometric parameters on secondary losses. The major findings from these experimental investigations are summarized below.

A number of experimental studies examined the effects of inlet boundary layer thickness and skew on secondary losses. Sharma and Butler (1987) reviewed many past experimental studies and concluded that secondary losses were insensitive to inlet boundary layer thickness. This conclusion was also arrived at by a number of other authors (e.g., Chen and Dixon, 1986; Hodson and Dominy, 1987b; Chan *et al.*, 1994). Gregory-Smith *et al.* (1988a) found no clear relationship between secondary losses and inlet boundary layer thickness whereas Sauer and Wolf (1994) observed some dependence but only for very thin layers. Both Walsh and Gregory-Smith (1990) and Sauer and Wolf (1994) observed a significant influence of inlet boundary layer skew on secondary losses.

Hodson and Dominy (1987b) and Perdichizzi and Dossena (1993) investigated the effect of blade loading on secondary flows. Changes in blade loading were achieved through variations in pitch-to-chord ratio. Hodson and Dominy (1987b) found that as the pitch-to-

chord ratio increased (i.e., as blade loading increased) the production of secondary losses increased. On the other hand, the results of Perdichizzi and Dossena (1993) indicated that there was no clear trend in the secondary losses with pitch-to-chord ratio. Weiss and Fottner (1995) investigated the influence of loading distribution, and found that a front-loaded airfoil produced significantly higher secondary losses by comparison with a more rear-loaded one. The results from the present study provide some additional insights into the effect of loading distribution on the secondary flow.

Several investigations were conducted to examine the effects of compressibility on secondary flows (e.g., Sieverding and Wilputte, 1981; Camus *et al.*, 1984; Moustapha *et al.*, 1993; Perdichizzi, 1990; Duden and Fottner, 1997). Sieverding and Wilputte (1981) and Duden and Fottner (1997) examined the variation in losses from low to high subsonic Mach numbers, and over this range, only small variations in secondary losses were observed. Perdichizzi (1990) and Moustapha *et al.* (1993) examined the three-dimensional flow downstream of transonic turbine cascades: measurements were made from low subsonic to highly transonic Mach numbers ($Ma_{2is} > 1.3$). For transonic Mach numbers, Perdichizzi (1990) found that the flow behaviour in the endwall region became significantly more complex due to the interaction of the shocks with the endwall boundary layer. However, there was no observable influence of Mach number on the strength of the secondary flow field. This was somewhat surprising given that the secondary losses decreased significantly in the transonic flow range. Evidently, further investigations are required to clarify the physics of this complex flow.

Gregory-Smith and Cleak (1992) and Adjout and Dixon (1992) investigated the effects of free-stream turbulence on secondary losses and found that it had very little effect. A number of others have examined the turbulence flow field in the endwall region (e.g., Moore *et al.*, 1987; Zunino *et al.*, 1987; Gregory-Smith *et al.*, 1988b; Gregory-Smith and Biesinger,

1992). Finally, Harrison (1990) and Moore and Gregory-Smith (1996) measured the state of the endwall boundary layers inside the blade passage and found substantial regions of laminar flow.

Only one investigation has focused on the effect of leading-edge geometry on the secondary losses at the design incidence (Moore and Ransmayr, 1984). In their experiment, the conventional circular leading edge of the “baseline” cascade was modified by attaching a sharp, wedge-shaped extension. They observed very little difference in losses between the two cases.

There have been very few recent studies concerned with the effect of off-design incidence on secondary losses. Moustapha *et al.* (1990) performed a literature survey and found that of the fourteen cascade test results taken from the open literature, only two were airfoil sections of recent design (i.e., Hodson and Dominy, 1987b and Yamamoto and Nouse, 1988). From Table 2.1, it can be seen that only three additional investigations have been performed since the work of Moustapha *et al.* (e.g., Perdichizzi and Dossena, 1993; Ramana Murty and Venkatrayulu, 1994; Duden and Fottner, 1999). Perdichizzi and Dossena (1993) measured secondary losses at off-design incidence for three cascades with different pitch-to-chord ratios while Duden and Fottner (1999) examined the secondary flows in a cascade with highly divergent endwalls.

From these experiments, it was evident that the strength of the secondary flows and the peak losses increased with increasing incidence. All investigators concluded that the behaviour of the secondary flow field was related to the observed changes in airfoil loading in the leading-edge region. However, the mechanisms causing the changes in the secondary losses are not yet fully understood. Moreover, the results suggest that the leading-edge geometry may play a role in the behaviour of the secondary flows at off-design incidence. There are no data available in the open literature related to the influence of leading-edge

geometry on secondary losses at off-design incidence.

This review of previous work has shown that there is very little published data on the influence of leading-edge geometry on secondary losses at either design or off-design incidence. The present study was undertaken, in part, to provide some insight into the relationship between the leading-edge geometry and secondary losses over a range of values of incidence.

2.7 Empirical Prediction Methods

In the early stages of design, when detailed airfoil geometries are not known, meanline analysis and empirical loss correlations continue to be important tools for the turbine aerodynamicist. Therefore, there is an on-going need to review and improve these correlations.

Probably the most widely used empirical loss system for axial turbines is that due to Ainley and Mathieson (1951). The Ainley and Mathieson system is a complete system because it includes correlations for all the components of the loss, and for both design and off-design conditions. It was subsequently modified by Dunham and Came (1970) to reflect the improved understanding of some aspects of the flow, notably the secondary flows. Further improvements to the design-point correlations were presented by Kacker and Okapuu (1982). Craig and Cox (1971) and Traupel (1977) have also proposed complete loss systems, but these are not as widely used as those based on Ainley and Mathieson's work. It is important to note that no aspect of the leading-edge geometry appears in any of these design-point loss correlations.

In 1990, Moustapha *et al.* (1990) reviewed the available correlations for both profile and secondary losses at off-design conditions and compared them with a range of recent measurements. The authors concluded that the existing correlations do not adequately account

for recent improvements in airfoil design and are no longer as reliable as they once were. Based on the substantial database of measurements that they had collected, the authors devised new correlations for the profile and secondary losses. These were significantly more successful at correlating the data than the Ainley and Mathieson correlations. Drawing on the work of Mukhtarov and Krichakin (1969), and unlike Ainley and Mathieson, Moustapha *et al.* included the leading-edge diameter as a correlating parameter. The new off-design correlations were intended for use with the Kacker and Okapuu design-point correlations to form a complete loss system. During the course of the present work, some significant shortcomings in the Moustapha *et al.* correlations have been identified. Among the results to be presented in this thesis are improved correlations that supersede those proposed by Moustapha *et al.*.

2.8 Conclusions

The review of previous work has shown that there is little information in the literature related to the effects of leading-edge geometry on profile losses at both the design and off-design incidence. The most recent profile loss correlations indicate that leading-edge geometry is an important correlating parameter only at off-design incidence.

In the past several decades, a significant number of experimental studies (approximately 70) have focused on secondary flows. As a result of this work, a much clearer picture of the secondary flow structures has evolved. Furthermore, many of the important geometric and aerodynamic parameters that influence secondary losses have been identified, at least for the design incidence. It appears that the leading-edge geometry is one of the few exceptions.

A number of authors have noted significant changes in secondary losses at off-design incidence; however, the mechanisms that cause these changes do not appear to be fully

understood. In addition, there have been no studies that have examined the effects of leading-edge geometry on secondary losses at off-design incidence. The leading-edge diameter does, however, appear as an important correlating parameter in the most recent off-design secondary loss correlation.

Several of the gaps that have been identified here in our understanding of both the profile and secondary flows in axial turbines are the focus of the present thesis.

Chapter 3

Experimental Set-Up and Measurement Procedures

3.1 Introduction

The experimental part of this research program was performed in the Low Speed Turbomachinery Laboratory (LSTL) at Carleton University. All measurements were made in a variable-incidence linear cascade test section which was supplied with air from the newer of two wind-tunnels in the LSTL. The test section was designed and manufactured by Goobie (1989) and the wind-tunnel commissioned by Rodger (1992). Both have been used with success for over a decade: primarily for the study of the effects of off-design incidence, surface roughness and trailing-edge damage on profile losses (e.g., Rodger, 1992; Whitehouse, 1993; Abbott, 1993; Serjak, 1995). Two linear cascades, designated as LS2 and LS3, have been examined in this work. The first two sections of this chapter describe the wind-tunnel, test section and cascade geometries.

All flow field measurements were made with pressure probes and static pressure distributions were measured with airfoils instrumented with static taps. The data acquisition systems and procedures used for collecting pressure data are described in Section 3.4. All pressure transducers and pressure probes were calibrated in house. The pressure standard used for transducer calibration is discussed in Section 3.5 and the probe calibration rigs are detailed in Section 3.6. Section 3.7 describes the geometry and accuracy of the pressure probes and the cascade testing techniques are outlined in Section 3.8. Finally, Section 3.9 details the data reduction procedures.

3.2 Wind tunnel

An open-circuit, blower-type wind tunnel was used for these experiments. A detailed description of the wind-tunnel is given by Rodger (1992) and as such only its salient features are presented here.

Figure 3.1 shows a schematic layout of the wind-tunnel. The wind-tunnel uses a centrifugal fan, manufactured by Daltec Industries, Model number IE17P, as the wind source. The fan is belt driven (gear ratio of 1.0) by a 4-pole, 40 HP AC motor and is capable of delivering a maximum flow rate of about 5 kg/sec at a total pressure rise of about 12 cm of water. The rotational speed of the fan is set by specifying an input frequency to a Mitsubishi variable frequency drive (or VFD). The relationship between fan speed, N_{fan} and input frequency is:

$$N_{fan} = 30f_{input} \quad (3.1)$$

where the units for N_{fan} and f_{input} are RPM and Hz, respectively.

As shown in Figure 3.1, ambient air enters the centrifugal fan through a bell mouth inlet. It is then compressed and accelerated as it passes through the impeller. Immediately

Figure 3.1: **Wind-tunnel schematic.**

downstream of the impeller, the non-uniform fan-delivery air enters a square, 280 mm honeycomb conduit where the swirl in the flow is removed. Prior to entering the settling chamber, the flow is diffused through a straight-walled, wide-angle diffuser which has an area ratio of 5.6 and divergence angle of 19 degrees. The diffuser contains five evenly-spaced screens which are used to prevent or reduce the extent of boundary layer separation. Four

screens in the settling chamber help attenuate any mean flow non-uniformities and promote the dissipation of large-scale turbulence. The nearly-stagnant flow from the settling chamber is accelerated into the test section through a smooth contraction of rectangular cross-section (area ratio of 14:1). The contraction is instrumented with two static taps: one located at the inlet (P_{C1}) and the other at the outlet (P_{C2}), as shown in Figure 3.1. The contraction pressure difference, $P_{C1} - P_{C2}$, and the static pressure, P_{C2} , are used to monitor the wind-tunnel dynamic and static pressure, respectively, as will be further discussed in Section 3.4.2.2. They were also used to provide reference pressures for all differential pressure transducers.

With the test section and cascade geometries used in this work, the maximum attainable velocity from the wind-tunnel was approximately 30 m/s so that the conditions were essentially incompressible. The freestream turbulence intensity just downstream of the test-section inlet is about 0.4% (Mahallati, 2003).

3.3 Test Section and Cascades

All measurements were made in the linear cascade test section shown schematically in Figure 3.2. The inlet to the test section has a rectangular cross-section which is 672 mm high and 203 mm wide. Immediately downstream of the inlet, two adjustable boundary layer bleed slots are used to reduce the boundary layer thickness on the endwalls. Further downstream, a five-airfoil linear cascade is mounted on a turntable which can be rotated to allow the cascade to be tested over a wide range of inlet flow angles (total range of about 60 degrees). The angular orientation of the cascade can be adjusted to within ± 0.2 degrees using a digital protractor. The turntable serves as the back endwall. The front endwall is made of plexiglass to allow flow visualization and is mounted on a moveable frame for easy access to instrumentation and control surfaces. As shown in Figure 3.2, the test section

Figure 3.2: Schematic of variable-incidence test section.

is designed with seven control surfaces: side flaps, tailboards, bypass flaps and a floating wall. The side flaps and tailboards are used primarily for adjusting the inlet uniformity and outlet periodicity. The bypass flaps mainly influence the mass flow through the cascade and the floating wall is used to adjust the height of the test section as the incidence is varied.

Two cascades, designated as LS2 and LS3, were examined in this work and their respective geometries are summarized in Figure 3.3. The airfoil section in the LS2 cascade represents the midspan section of a Pratt & Whitney Canada, PW100-series power turbine blade. The LS2 cascade has been used in the past mainly for studies of off-design profile losses (e.g., Goobie, 1989; Tremblay, 1989; Rodger, 1992). After the development of the

Parameters	LS2	LS3
Blade span, h [mm]	203.7	203.7
Blade spacing, s [mm]	110.7	110.7
True chord, C [mm]	162.8	162.3
Axial chord, C_x [mm]	150.0	149.4
Stagger angle, γ [deg]	23.1	21.6
$t_{\max}/C$ [%]	18.0	19.6
Inlet metal angle, β_1 [deg]	29.3	25.5
Outlet metal angle, β_2 [deg]	57.5	57.5
Design inlet flow angle, α_1 [deg]	28.4	28.4
Design outlet flow angle, α_2 [deg]	57.7	57.7
Leading-edge diameter, d [mm]	9.4	16.7
Leading-edge wedge angle, We [deg]	52.4	43.0
Trailing-edge thickness, t_{et} [mm]	4.2	4.2

Figure 3.3: Airfoil and Cascade Geometries.

Moustapha *et al.* (1990) correlation, LS3 cascade was manufactured to examine further the influence of leading-edge geometry on the off-design profile losses (Whitehouse, 1993). The two airfoil sections were designed for the same Zweifel loading ($\psi_{des}=0.90$) using the same turbine-profile design system.

As shown in Figure 3.3, LS3 has a leading-edge diameter roughly twice that of LS2. To maintain similar lifing requirements, which required the cross-sectional metal areas to be essentially the same, the leading-edge wedge angle for LS3 was reduced to 43.0 degrees from the value of 52.4 degrees used for LS2. In addition, the stagger angle was reduced from the value of 23.1 degrees for LS2 to 21.6 degrees for LS3. It is important to note that Korakianitis and Papagiannidis (1993) have observed an influence of stagger angle on the loading distribution.

Figure 3.3 also shows the locations of the flow field measurements. For all values of incidence, the detailed measurements of the inlet boundary layer were made at 1.2 axial chord lengths upstream of the leading edge. Traverses were made at 0.5 axial chord lengths upstream of the leading edge to verify inlet uniformity. All downstream measurements were made at 0.4 axial chord lengths downstream of the trailing edge unless otherwise noted. A more detailed discussion on cascade testing techniques is given in Section 3.8.

To measure airfoil surface static pressures, one airfoil in each of the cascades was instrumented over half of its span with 11 (LS2) and 12 (LS3) chordwise rows of static taps. Each row consists of 43 static taps of 0.51-mm-diameter: 25 on the suction surface, 17 on the pressure surface and one base pressure tap. The airfoil surface static tap locations are given in Rodger (1992) for LS2 and Whitehouse (1993) for LS3.

3.4 Pressure Acquisition Systems

Two pressure acquisition systems were used during the course of this experimental work. The Sciometric-based system, which was used for the LS3 measurements, has been used extensively for over a decade in the LSTL (e.g., Tremblay, 1989; Rodger, 1992; Whitehouse, 1993). For the LS2 measurements, the Sciometric-based system was replaced with a new pressure acquisition system, which is referred to as the PS9010-based system. It was commissioned by the present author and Mahallati (2003). The salient features of both pressure acquisition systems are described in Section 3.4.1.

With the PS9010-based system, a significant reduction in data collection time was realized, and it is for this reason the Sciometric-based system was replaced. This will be further discussed in Section 3.4.2 where the data-acquisition procedures are detailed.

3.4.1 Apparatus

Figure 3.4 shows a schematic of the Sciometric-based system which is comprised of four main components:

- Scanivalve pressure multiplexer (scanivalve),
- two capacitive-type differential pressure transducers,
- Sciometric Instruments electronic measurement system, and
- DOS-based desktop computer.

The Scanivalve pressure multiplexer is a pneumatic switching device that allows a single transducer to be sequentially connected to a maximum of 48 independent input pressure ports. The multiplexer is controlled through the parallel port on the desktop computer.

The differential pressure transducers, which are detailed in Table 3.1, were used to convert the pressure signals into analog signals. The characteristics of these transducers have

Figure 3.4: Experimental set-up with Sciometric-based pressure acquisition system.

been well established by previous investigators - they have been shown to possess minimal zero-drift, excellent linearity and long-term stability. Therefore, transducer calibrations were performed over part of their input ranges and the resultant calibration curves are presented in Appendix A. The pressure standard used for these calibrations is described in Section 3.5.

The analog output signal from the pressure transducers is converted into a digital signal using the Sciometric Instruments Model 81/902 Electronic Measurement System (EMS). This system is comprised of 8 analog input channels, a multiplexer, variable-gain amplifier and a sequential analog-to-digital (A/D) converter. The A/D converter digitizes the analog signal with 12-bit-plus-sign (0.024% input full scale) resolution at a sampling rate of 5 samples/sec/channel. The full-scale input range can be adjusted with a software-controlled, variable-gain amplifier. For this work, all measurements were made with the input range set to ± 5 V. The corresponding resolution of the A/D conversion is ± 1.2 mV which is better than the transducer accuracy.

Following the A/D conversion, the data acquired during one sampling period are stored in internal memory on board the EMS. Upon completion of the sampling, the pressure signal (in Volts) is uploaded to the desktop computer through the RS232 serial port. It is then converted into Engineering units using the calibration coefficients of the transducers.

Table 3.1: **Pressure Transducer Specifications**

Manufacturer	Model No.	Serial No.	Input Range [kPa]	Output Range [V]
MKS	Baratron 220CD	79508-1	± 13.6	± 10.0
Rosemount	831A3	567	± 3.4	± 2.5

The new data acquisition set-up consists of a Pressure Systems Model 9010 Pneumatic Pressure Scanner, a tank of compressed nitrogen and a Windows-based desktop computer as shown in Figure 3.5. The enlargement in Figure 3.5 shows the external layout of the scanner. The PS9010 is a self-contained pressure acquisition module; that is, the transducers, multiplexer, signal conditioner and analog-to-digital (A/D) converter are integrated into a single unit. It also incorporates a pneumatically-actuated manifold that enables on-line rezeroing of the transducers. High-pressure nitrogen, which is stored in a cylinder and regulated to about 760 kPa, is used as the pneumatic-actuator pressure source.

The PS9010 has a dedicated piezoresistive differential transducer¹ for each of its 16 input pressure channels. In a single PS9010 unit, transducers with different pressure ranges may be used. For the unit used in this work, transducers 1-8 have input pressure ranges of ± 5.0 kPa; transducers 9-16 were intended to be used in experiments with significantly higher pressure levels than the current one. Each transducer has been pressure calibrated by the manufacturer over a temperature range of 0-50 degrees C at 10 degree intervals and the resultant calibration coefficients are stored in nonvolatile memory contained within each transducer. A temperature sensor mounted on each transducer ensures that the appropriate set of pressure calibration coefficients is used.

The analog signals from the transducers are sampled at a rate of

$$\dot{S} = \frac{48}{N_{CH}} \quad (3.2)$$

where N_{CH} is the number of channels scanned and the units for sampling rate are samples/second/channel. The A/D converter produces a digital signal which is 15-bit accurate (or ± 0.15 Pa). This signal is subsequently processed by a microprocessor (within the PS9010) using the pressure calibration data and the transducer's temperature to produce

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¹Model Number DH200, manufactured by Pressure Systems, Inc.

Figure 3.5: Experimental set-up with PS9010-based pressure acquisition system.

a “calibrated” output signal in engineering units. With this arrangement the manufacturer guarantees an uncertainty in the measured output pressure of less than ± 7.5 Pa immediately after rezeroing the transducers (Pressure Systems, Inc., 1998). To reduce uncertainty in the measured pressures, the output pressure signals from the PS9010 were post-processed with transducer calibration coefficients measured by the present author and Mahallati (2003).

From 1997 to 2001, numerous transducer calibrations were performed to verify the accuracy and long-term stability of the PS9010-based pressure acquisition system. Typical transducer calibration curves are presented in Appendix A and suggest that the uncertainty interval is considerably smaller than the value specified by the manufacturer. Provided that the zero offset is adjusted after every 10 measurement points, the uncertainty in a measured pressure was determined to be ± 1.5 Pa with 95% confidence over a calibration range of ± 1250 Pa. This includes the effects of hysteresis, non-linearity, long-term stability as well as the uncertainty in the calibration working standard (i.e., micromanometer). For the calibration of the PS9010 pressure transducers, a new micromanometer was designed and manufactured by the present author and Mahallati (2003). This allowed calibrations to extend over the range of pressures encountered during experimental testing. Details of this new micromanometer are discussed in Section 3.5.

3.4.2 Procedures

3.4.2.1 Data Acquisition Time

As indicated earlier in Section 3.4.1, a component of the Sciometric-based system is a Scanivalve pressure multiplexer. The use of such a device requires, for each measured port pressure, a wait time before data sampling can begin. That is, when a scanivalve switches

ports a pressure transient is generated in the pneumatic system² due to differences in port pressures. The time required for pressure transients to dissipate in the pneumatic system is referred to as wait time and was set conservatively to 20 seconds/port for these experiments. It was determined by monitoring the output voltage of the transducer as the system was subjected to a step change in pressure.

In addition to the wait time, there is an interval of time over which the measured pressure must be sampled, that is, the “sampling” time. It is the time required to achieve an accurate, long-term statistical average of the measured pressure. For this work, the sampling time was set to 25 seconds/port at a sampling rate of slightly more than 5 samples/sec for a total of 125 samples. The appropriate sampling time was determined by calculating the cumulative average of the transducer output voltage. The combined wait and sampling time results in significant data acquisition time.

The total data acquisition time (wait plus sampling) for one seven-hole probe measurement was approximately 5.25 minutes. Note that one probe measurement is the sampling and waiting time for all 7 holes. As will be discussed later in this chapter, the number of probe measurements needed for good spatial resolution of the flow field was approximately 400; therefore, the data acquisition time was of the order of 35 hours.

As discussed in Section 3.4.1, the PS9010 pressure acquisition system has a pressure transducer dedicated to each pressure port, thus, obviating the need for waiting time between pressure port measurements. During any probe movement, however, pressure transients are generated in the pneumatic system and a waiting time was required. For all production measurements, the waiting time following a probe movement was set conservatively to 8 seconds; it was determined using the same procedure as outlined earlier.

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²Pneumatic system refers to the sealed arrangement that connects the transducer to the measured pressure.

For the PS9010 pressure acquisition system, it was necessary to specify the number of data samples rather than the data sampling time. By examining the cumulative average of a pressure signal, it was determined that 300 samples were sufficient to achieve a good statistical average of the signal. Note that for three-hole probe measurements, which require a 4-channel scan³, a 25 second sampling time was required for 300 samples. For seven-hole probe measurements, a 50 second sampling time was required.

The total data acquisition time for a 400-point, seven-hole probe traverse was approximately 6.5 hours with the PS9010 system. By comparison to the Sciometric-based system, which required 35 hours for the same traverse, a fivefold reduction in data acquisition time was realized.

3.4.2.2 Data Sampling Technique

In both systems, the differential pressures transmitted to the computer from the data acquisitions were used to define pressure coefficients of the form:

$$Cp_{port} = \frac{P_{port} - P_{C2}}{P_{C1} - P_{C2}} \quad (3.3)$$

where the subscript port refers to a particular probe hole or airfoil static tap. With both pressure acquisition systems, the numerator and denominator of Equation 3.3 were measured using separate transducers.

During a given sampling period, the data acquisition system scans all active channels (transducers) sequentially, and this process, referred to as a measurement cycle, is repeated for the specified sampling time (Sciometric system) or for the specified number of samples (PS9010).

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³Three holes and reference dynamic pressure.

With the Sciometric system, there were only two active channels: channel 1, which measured the differential port pressure, $P_{port} - P_{C2}$, and channel 2, which measured the tunnel contraction pressure difference. For the PS9010 system, the user can activate a maximum of 7 channels for differential port measurements; the eighth channel monitors the contraction pressure difference. Each measurement cycle consists of: $P_1 - P_{C2}$, $P_2 - P_{C2}$, $\dots$, $P_{N-1} - P_{C2}$, $P_{C1} - P_{C2}$ where N is the number of active channels.

For each measurement cycle in a given sampling period, the port pressure(s) were non-dimensionalized by the contraction pressure difference. All pressure coefficients formed during a given sampling period were arithmetically averaged to give one representative value, $\overline{Cp_{port}}$ ⁴. This technique has been used successfully by Yaras (1990).

3.5 Pressure Standard for Transducer Calibration

The pressure standard used for the calibration of the transducers used in the Sciometric-based system was a Dwyer Instruments Hook-Gauge with a full-scale range of 0-50 mm of H₂O (± 500 Pa at 20 degrees C). Based on the author's experience, the uncertainty in the measured differential height is ± 0.05 mm of H₂O (i.e., ± 0.5 Pa at 20 degrees C) with 95% confidence. Note that the manufacturer suggests uncertainties of ± 0.025 mm of H₂O when the instrument is used by a skilled operator.

A new micromanometer was designed and manufactured to allow calibrations to extend over the range of pressures encountered during experimental testing. This working calibration standard was used for the PS9010-based pressure acquisition system, and a photograph of it is shown in Figure 3.6. The new micromanometer was designed with a full-scale range of 0-30 cm of H₂O and a differential-height measurement uncertainty equal to that of the

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⁴For the sake of clarity, the overline has been omitted in all subsequent references to Cp_{port}

Figure 3.6: Photograph of “new” micromanometer.

Hook-gauge.

The design incorporated a number of features that help reduce sources of error that are common in other types of manometers. For instance, the micromanometer was designed with large diameter water columns (approximately 65 mm) to keep the effect of capillary attraction to a minimum. Spirit-levels and screw-type levelling adjustments were used in the design to ensure verticality of the water columns. Accurate positioning of the measuring rods relative to the base plate was performed with gauge blocks, and this ensured positioning tolerances of ± 0.025 mm of H_2O .

It should be noted that distilled water was used in both the Hook-Gauge and the new micromanometer; therefore, the uncertainty in density was estimated to be better than $\pm 1 \text{ kg/m}^3$.

3.6 Probe Calibration Rigs

All work related to the calibration of multi-hole pressure probes was performed using two in-house-designed calibration rigs. The first system was used early in this research program, during the preparation and measurement phases of the LS3 experiments. It had been used extensively by various investigators and is described in detail by Rodger (1992) and Sadr (1996); therefore, only a brief description is given here. A schematic of the set-up is shown in Figure 3.7. It consists of a circular nozzle, a settling chamber, a two-axis turntable system and three Shop-Vac blowers (not shown) which were used as the wind source.

In this rig, ambient air is compressed and accelerated through the blowers after which it is discharged into the settling chamber through a flexible hose. The settling chamber, which has a cross section of 27×28 cm, was manufactured with a flow distributor plate, one

Figure 3.7: Schematic of original calibration rig.

honeycomb screen and four fine-mesh screens. The air flow from the settling chamber enters a bell mouth and is accelerated smoothly to atmosphere through a circular nozzle which has an exit diameter of 55 mm. With this set-up, the maximum jet velocity was approximately 35 m/s.

As shown in Figure 3.7, the settling chamber is instrumented with a static tap, P_{C1} , located downstream of the last screen. The total pressure of the jet flow was inferred from P_{C1} through calibration and the jet static pressure was taken as atmospheric pressure - an assumption verified through static pressure probe measurements. The total pressure and turbulence intensity profiles of the jet flow were documented by Sadr (1996) over a range of jet-velocities from 8 to 38 m/s. He determined from pitot-tube measurements that the inviscid region was approximately 44 mm in diameter at the calibration measurement plane; that is, at an axial location two nozzle diameters downstream of the nozzle exit plane. The turbulence intensity of the inviscid core was $2.9 \pm 0.4\%$ at the same axial location.

The relative flow direction between the jet and the probe axis was varied with a motorized turntable mechanism designed by Yaras (1987). This turntable mechanism has two rotational degrees of freedom which allows the angular position of the probe to be varied ± 90 degrees in the pitch and yaw directions. The probe tip is located at the centre of rotation; therefore, its absolute position is essentially invariant. The turntable mechanism uses two stepper motors, which operate at 200 steps per motor shaft revolution. Each motor drives a 180:1 worm gear so that the angular position of the probe is resolved to 0.01 degrees per motor step. Backlash in both worm gears was measured and found to be negligible. The motors are driven by motor controllers which receive instructions, in the form of TTL voltage signals, from the computer through the parallel port. With this system, the accuracy of the angular positioning is estimated conservatively to be ± 0.05 degrees.

As previously noted, this calibration rig had been used successfully over a number of years; however, the present author found that it was conducive to probe fouling, particularly on days when the relative humidity was high. Probes would frequently become fouled with a black, greasy substance which likely originated from the carbon brushes in the blower motors. To mitigate this problem, the present author designed and manufactured a new open-circuit, suction-type set-up which is shown schematically in Figure 3.8. With this set-up, air is drawn into the working-section through the same nozzle as described earlier and exhausted through a bell mouth into a flexible hose. The hose is connected to a throttle valve that is located at the inlet to a radial vacuum blower. Commercial air filter panels surround the nozzle inlet to help both attenuate entry disturbances and prevent dust particles from entering the working-section. With this set-up, the maximum jet velocity was approximately 45 m/s.

Detailed flow field measurements were made with a seven-hole probe to characterize the jet-flow. The measurements were repeated over a range of axial positions beginning from the nozzle exit plane to approximately 1.6 nozzle-diameters or 90 mm downstream of the nozzle exit plane. The results from this investigation are presented in Appendix B, and they show a large inviscid core (approximately 20 mm in diameter) and excellent uniformity over the entire axial extent of the measurements. Within the inviscid core, the variations in total and static pressure were $\pm 0.1\%$ of centreline dynamic pressure; the flow angularity varied by ± 0.4 degrees. Mahallati (2003) measured the turbulence intensity and observed that it was essentially constant at $0.4 \pm 0.1\%$ over a range of jet velocities from 1 to 45 m/s. Locating the probe within the core was guaranteed given the size of the core and the positioning accuracy of the probe (approximately ± 5 mm).

In this set-up, the reference total pressure, P_{C1} , was measured from within the air-filter enclosure, and as shown in Figure 3.8, the reference static pressure, P_{C2} , was measured from

Figure 3.8: Schematic of new calibration rig.

a static tap located just upstream of the nozzle exit plane. Calibrations of both pressures were carried out using pitot- and static-tubes over a large range of velocities (1 to 45 m/s) and axial positions (from the nozzle exit plane to 2 nozzle-diameters further downstream). For the total pressure, the calibration results indicated that P_{C1} and the measured pitot pressure were equal, within transducer accuracy. The static pressure calibration was insensitive to the jet-flow velocity and showed only marginal pressure variations with axial position (i.e., less than $\pm 0.25\%$ of centreline dynamic pressure).

3.7 Pressure Probes

3.7.1 Introduction

Both single- and multi-hole probes were used for flow field measurements in the present experiment. A pitot-tube, which is described in Section 3.7.2, was used to measure the total pressure distribution of the endwall boundary layer upstream of the cascade. A three-hole probe was used to inspect both the midspan inlet uniformity, outlet periodicity and it was also used to obtain the midspan losses at the downstream measurement plane. The three-dimensional flow field downstream of the cascade was measured with a seven-hole probe. The seven-hole probe was preferred over the more commonly used five-hole probe for a few reasons. First, the seven-hole probe can be used over a much larger range of flow misalignment angles (typically up to 75 degrees, Everett *et al.*, 1983). Furthermore, the tubes in the seven-hole probe form a natural bundle so that it is easier to manufacture and it produces no additional probe blockage by comparison with the five-hole probe.

The three- and seven-hole probes were used in the non-nulling mode; therefore, each probe required calibration over a range of flow angles. The calibration procedures as well as the geometry and measurement accuracy for these two types of probes are described in

Sections 3.7.3 and 3.7.4, respectively.

3.7.2 Pitot-tube

A pitot-tube with an inner diameter of 0.33 mm and outer diameter of 0.64 mm was used to measure the endwall boundary layer at the reference location located 1.2 Cx upstream of the cascade. The diameter of the pitot-tube was about 3% of the boundary layer thickness, therefore, good resolution of the boundary layer was achieved. The pitot-tube also has a square, sharp-edged tip which is circular in cross-section. The tip is located 25 mm upstream of the 6-mm-diameter stem to help reduce possible interference effects. The inner wall at the tip is bevelled to decrease the probe sensitivity to flow misalignment.

In low Mach number flows, such as those encountered in this work, a number of factors can influence the total pressure sensed by a pitot-tube, these include: Reynolds number, turbulence, wall proximity, shear and flow misalignment. Although corrections exist for these effects, none have been applied to the present measurements. It should be noted that the error due to Reynolds number effects is only significant if the Reynolds numbers based on inner and outer tube diameter are lower than 200 and 400, respectively (Macmillan, 1954). Even at the lowest flow velocities in the current experiment, the probe-based Reynolds numbers exceeded these levels. Therefore, the influence of Reynolds numbers on the measured total pressure is probably small. The influence of flow misalignment on the measured total pressure is also likely to be small because the spanwise variation in flow angle was measured to be no more than 3 degrees at the inlet measurement location.

Based on repeatability tests, the error in total pressure is estimated to be about 0.5% of the reference dynamic pressure.

3.7.3 Three-hole Probe

3.7.3.1 Geometry

For this research, a three-hole probe was used to characterize the midspan flow field up- and downstream of the cascade. Because the flow was essentially two-dimensional at the midspan plane, the three-hole probe was well-suited for these measurements.

The three-hole probe was designed by Goobie (1989) and has been used extensively by numerous researchers (e.g., Goobie, 1989; Tremblay, 1989; Rodger, 1992; Whitehouse, 1993). The probe geometry was described only briefly by Goobie (1989); a more complete description follows.

Figure 3.9 shows a schematic drawing of the three-hole probe along with the pressure-port numbering convention and angle definitions. The probe consists of a 67 mm long stainless steel square channel section (probe body) and three stainless steel hypodermic tubes, each with an inner and outer diameter of 0.33 mm and 0.64 mm, respectively. The three tubes are soldered side-by-side along their entire length. The tubes extend 25 mm beyond the probe body and the tips of each of the two outer tubes are machined at a 45 degree angle with respect to the centre tube. The probe tip has a thickness of 0.64 mm and a width of 2.0 mm which is approximately 50% the trailing-edge thickness for these measurements.

3.7.3.2 Calibration Procedure, Data Reduction and Uncertainties

The probe was calibrated in 0.5 degree steps over a range of ± 10 degrees of yaw misalignment using the set-up previously described in Section 3.6. For a given calibration, the probe Reynolds number based on probe width (Re_w) was kept constant. The calibration

Figure 3.9: Three-hole probe geometry.

data were reduced in the form of pressure coefficients according to the method of Lewis (1966). A local linear interpolation scheme was used on the calibration coefficients to extract the flow angle, dynamic and static pressure. The calibration procedure and the resultant calibration data are presented in Appendix C along with an estimate of the expanded uncertainty for each of the data points.

For all results reported herein, the expanded uncertainty defines an interval having a level of confidence of approximately 95%. This uncertainty was estimated by combining each of the elemental sources of error using the *law of propagation of uncertainty*, also referred to as the “root-sum-of-squares” (RSS) method, (Taylor and Kuyatt, 1994). Both precision and bias errors (e.g., instrument calibration errors) are included in the uncertainty estimates, and as such, the approach is identical to an Nth Order uncertainty estimate, as described by Moffat (1982). The most significant source of error, the repeatability error, was determined by performing more than thirty repeated calibrations over a one-year period. Other sources of error, such as uncertainty in both pressure-transducer and nozzle calibration coefficients, were also included. It is important to note that the probe calibration was checked regularly throughout the experiments and was found to be repeatable within the estimated uncertainty intervals presented in Appendix C.

The measurement uncertainty of yaw angle, dynamic and total pressure in an unknown flow field is comprised of uncertainties in both the probe calibration and in the actual cascade measurements. The uncertainty due to errors in the probe calibration process was estimated by making repeated measurements in an known flow (i.e., jet-flow in the calibration rig) with the calibrated probe. The yaw angle, dynamic and total pressure were extracted from each of the repeated probe pressure measurements and compared to the actual values. The uncertainty results presented in Table 3.2 are based on approximately 1200 variable-angle measurements; that is, the data from twenty-nine repeated calibrations. It is worth noting

Table 3.2: **Three-hole probe measurement uncertainties arising from the calibration process.**

Flow Field Parameter	Expanded Uncertainty Intervals
Total Pressure [% of reference dynamic pressure]	± 0.3
Dynamic Pressure [% of reference dynamic pressure]	± 0.5
Flow Angle [degrees]	± 0.5

that the measurement errors presented in Table 3.2 do not include interpolation errors because the “measurements” points are taken at the same angles as the calibration data points. In Table 3.2, the total and reference dynamic pressure are defined as $P_{0,local} - P_{C1}$ and $P_{C1} - P_{C2}$, respectively.

The interpolation error was estimated by removing alternate calibration data points (i.e., doubling the calibration step size) from the calibration set, and in turn, using this set to infer the flow parameters at the missing points. The flow parameters inferred at the missing points were then compared to actual ones. The results from this analysis showed that the interpolation error was negligible relative to the repeatability error - a consequence of the calibration step size and linearity of the calibration curves. Additional errors related to measurements in the cascade wind-tunnel are discussed in Section 3.8.1.

3.7.3.3 Reynolds Number Effects on Probe Calibration

In the cascade experiments, the mean downstream velocities varied considerably (from approximately 30 m/s to 42 m/s) with incidence due to changes in mean blade passage acceleration. Locally within the flow field, such as in airfoil wakes, the flow velocity was observed to be as low as 19 m/s. Due to these variations in flow velocity, the pressure probes experienced a fairly wide range of Reynolds numbers. To gain a better understanding of the

sensitivity of the three-hole probe calibration to Reynolds number variations, calibrations were performed for Reynolds numbers ranging from 1300 to 5400 (10 m/s to 42 m/s). The calibration coefficients at a Reynolds number of 5400 were used to extract the flow angle, dynamic and static pressure from each of the remaining data sets.

Figure 3.10a, b and c show in order, the error in yaw angle, dynamic and total pressure for Reynolds numbers of 1300, 2600 and 3900. Also shown are the uncertainty intervals from Table 3.2; that is, for measurements and calibration data taken at the same Reynolds number. As can be seen from Figure 3.10a and c, the accuracy of the flow angle and total pressure measurements does not appear to be influenced by Reynolds number variations within the investigated range. In contrast, Figure 3.10b shows that the dynamic pressure is sensitive to Reynolds number variations with a systematic (or bias) error of approximately 5% of reference dynamic pressure for the smallest Reynolds number tested.

Over the range of incidence investigated in this experiment, the mean probe Reynolds number varied from approximately $Re_w=3900$ to 5400 and the three-hole probe cascade measurements were reduced with a calibration in the middle of this range. This procedure introduces an additional estimated uncertainty in dynamic pressure of $\pm 0.25\%$; that is, half of the 0.5% average error for the data taken at $Re_w = 3900$, as shown in Figure 3.10b.

Combining the uncertainty for the “off-calibration” Reynolds number effect with the results in Table 3.2 gives an uncertainty in dynamic pressure of $\pm 0.6\%$. Note that uncertainties were combined according to the RSS technique. The uncertainty in total pressure and flow angle are unchanged from the values shown in Table 3.2.

Figure 3.10: Effect of Reynolds number on three-hole probe: (a) flow angle, (b) dynamic pressure and (c) total pressure.

3.7.4 Seven-hole Probe

3.7.4.1 Geometry

Two seven-hole probes of identical design (Amrud, 1985), designated as SHP1 and SHP2, were used throughout this experiment. The earlier LS3 measurements were taken with the SHP2 probe which had never been previously used. Following the completion of these measurements, an experimental study was undertaken to evaluate the measurement uncertainties for both probes. The results from this study showed that SHP2 had comparatively more scatter in the raw probe-hole pressure data than SHP1 notwithstanding that the tip geometries were identical, within machining tolerances. For this reason, the more recent LS2 measurements were taken with the SHP1 probe. This probe has also been used extensively by several other investigators (e.g., Danias, 1987; Yaras, 1990; Cao, 1993; Hetu, 1993; DeCecco, 1995).

Figure 3.11 shows a schematic drawing of the seven-hole probes along with the overall dimensions and the pressure-port numbering convention. Each probe consists of seven stainless steel tubes of equal inner and outer diameters, 0.33 mm and 0.64 mm, respectively. As noted earlier, the tubes are arranged in a natural bundle with six tubes fitting exactly around a single centre one. The bundle is contained within a brass tube and the interstices at the tip are filled with solder. Each probe has a diameter of 2.4 mm, which is approximately 60% of the trailing-edge thickness for the LS2 and LS3 cascades, and a conical tip with a nominal semi-apex angle of 30 degrees.

3.7.4.2 Calibration Procedure, Data Reduction and Uncertainties

For the SHP2 probe, calibration data was collected in 5 degree steps for all combinations of pitch and yaw up to 50 degrees of misalignment relative to the probe axis. The calibration

Figure 3.11: Seven-hole probe geometry.

range for the SHP1 probe was reduced to 40 degrees of misalignment in both the pitch and yaw directions, which was found to be sufficient for measuring the three-dimensional flows downstream of the test cascades. The maximum flow misalignment angle encountered during the LS3 testing was approximately 25 degrees. It should be noted that the SHP1 probe was calibrated using the faster PS9010-based pressure acquisition system; therefore, the angular spacing of the calibration grid was reduced to 2 degree intervals. The effect of calibration grid density on measurement uncertainty is discussed below.

For each of the two probes, calibration data is then reduced into a series of calibration coefficients according to the method of Gallington (1980) as referred to by Gerner and Maurer (1981). The calibration coefficients can be used once an interpolation scheme has been established. This scheme allows flow parameters (i.e., flow angle, dynamic and total pressure) to be determined from any combination of calibration coefficients within the

calibration range. A wide variety of numerical schemes, of varying degrees of accuracy, have been used. For example, numerous investigators have fitted polynomial surfaces (third- and fourth-order) to the calibration data using the method of least squares (e.g., Gallington, 1980; Everett *et al.*, 1983; Gerner *et al.*, 1984; Yaras, 1990). Everett *et al.* (1983) have shown that these “global” polynomial functions can produce large local deviations from the calibration data so that the expanded error for flow angles can exceed ± 1.5 degrees for pitch and yaw misalignment angles in the range of ± 30 degrees. Another commonly used approach, which appears to be considerably more accurate (Zilliac, 1993), is local piecewise interpolation using a finite number of calibration data points. This approach has been used by many investigators with varying orders of interpolation: bi-quadratic (Yaras, 1996); bi-cubic (Rediniotis *et al.*, 1993) and bi-quintic (Zilliac, 1993);. In the past few years, a number of other techniques have been developed. Among these is a two-step procedure by Wenger and Devenport (1999) in which error look-up tables are used in conjunction with a traditional least-squares surface fit of the calibration data. Also, a neural-network-based calibration technique has been developed by Rediniotis and Vijayagopal (1999) that reduces data reduction times by an order of magnitude relative to interpolation schemes. The present author used a local piecewise bi-linear interpolation scheme on the calibration data itself to extract flow parameters. A description of the calibration procedure, adapted from Gallington (1980), and data from repeated calibrations are presented in Appendix D.

An estimate of the measurement uncertainty due to errors in the probe calibration process was made by performing measurements in the calibration rig with a calibrated probe, as was done for the three-hole probe. Flow angles (pitch and yaw) and pressures (total and dynamic) were extracted from each of the repeated probe pressure measurements and compared to the actual values. The uncertainty results presented in Table 3.3 are based on approximately 2200 variable-angle measurements and are valid for both SHP1 and SHP2. It is important to note that uncertainties are dependent on the slopes of the calibration

Table 3.3: Seven-hole probe measurement uncertainties arising from the calibration process.

Flow Field Parameter	Expanded Uncertainty Intervals
Total Pressure [% of reference dynamic pressure]	± 0.3
Dynamic Pressure [% of reference dynamic pressure]	± 0.9
Flow Angle [degrees]	± 0.5

Table 3.4: Seven-hole probe measurement uncertainties arising from interpolation errors.

Flow Field Parameter	Uncertainty Intervals	
	SHP1	SHP2
Total Pressure [% of reference dynamic pressure]	± 0.1	± 0.6
Dynamic Pressure [% of reference dynamic pressure]	± 0.2	± 0.7
Flow angles [degrees]	± 0.1	± 0.2

curves, in other words, they are functions of the flow misalignment angles. Therefore, the values presented in Table 3.3 are representative or “averaged” values over the range of measured flow angles.

The procedure for estimating the interpolation errors was identical to the one used for the three-hole probe. More specifically, alternate data points were removed from a calibration set (i.e., doubling the calibration step size), and from this set, values were estimated at the missing points. The difference between the inferred and actual flow parameters provided an estimate of the interpolation error. These error values are summarized in Table 3.4 for both the SHP1 and SHP2 probes.

In the experiment, the mean probe Reynolds number varied from 4700 to 6500 for the

Table 3.5: Seven-hole probe measurement uncertainties with contributions from calibration repeatability errors, interpolation errors and Reynolds number variations.

Flow Field Parameter	Uncertainty Intervals	
	SHP1	SHP2
Total Pressure [% of reference dynamic pressure]	± 0.3	± 0.7
Dynamic Pressure [% of reference dynamic pressure]	± 1.2	± 1.4
Flow angles [degrees]	± 0.5	± 0.6

range of incidences investigated. The calibration of the seven-hole probe was performed at a Reynolds number in the middle of this range. To quantify the error incurred from “off-calibration” Reynolds effects, the calibration of SHP1 was tested over the aforementioned range. The investigation showed that the flow angle and total pressure measurements were insensitive to Reynolds number variations. For the dynamic pressure measurements, the Reynolds number effect introduced an average uncertainty of $\pm 0.80\%$ of reference dynamic pressure. It should be noted that the Reynolds number sensitivity of SHP2 was expected to be the same as SHP1 because of the similarity of the probe tip geometry.

The combined measurement uncertainties for the seven-hole probe, which include calibration repeatability errors, interpolation errors and Reynolds number variations, are given in Table 3.5. Note again that uncertainties were combined according to the RSS technique.

3.8 Cascade Testing Techniques

3.8.1 Wind-Tunnel Operating Point and Calibration

The calibration procedure for the wind-tunnel involved correlating the total and dynamic pressures at the inlet⁵ centreline of the cascade with the contraction pressure difference. From this procedure, the total pressure and dynamic pressure calibration coefficients were measured. The total pressure calibration coefficient, $Cp_{0_{WT}}$, is defined as

$$Cp_{0_{WT}} = \frac{P_{0_{CL}} - P_{C1}}{P_{C1} - P_{C2}} \quad (3.4)$$

where $P_{0_{CL}}$ is the total pressure at the centreline of the inlet traverse location. The dynamic pressure calibration coefficient, $C_{q_{WT}}$, is expressed as

$$C_{q_{WT}} = \frac{P_{0_{CL}} - P_1}{P_{C1} - P_{C2}} \quad (3.5)$$

where P_1 is the static pressure at the inlet traverse plane. The total pressure was measured with the pitot-tube described in Section 3.7.2. The static pressure was measured with a static pressure tap located on the endwall at the inlet traverse location.

Results from this procedure indicated that the dynamic pressure calibration coefficient varied considerably with incidence: from approximately 0.88 at 0 degrees to 0.67 at +20 degrees. This variation is attributed to changes in bleed flows upstream of the inlet traverse location. Based on repeatability tests, the dynamic pressure calibration coefficient was estimated to be accurate within ± 0.004 . The total pressure calibration coefficient was invariant with incidence and accurate to within ± 0.001 .

The wind-tunnel operating point was determined from the Reynolds number based on

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⁵Inlet is located 1.2 axial chords as shown in Figure 3.3.

airfoil chord, that is,

$$Re = \frac{\rho V_{CL} C}{\mu} \quad (3.6)$$

where V_{CL} is the velocity at the inlet centreline location, ρ and μ are the density and kinematic viscosity of air, respectively. The density and viscosity were obtained from the inlet static pressure and the settling chamber stagnation temperature. The inlet centreline velocity is given by

$$V_{CL} = \sqrt{\frac{2(P_{C1} - P_{C2}) C_{qWT}}{\rho}}. \quad (3.7)$$

The Reynolds number was kept constant at $3.0 \pm 0.03 \times 10^5$ during all measurements except for some tests where Reynolds number effects were investigated.

3.8.2 Inlet and Outlet Flow Quality

Prior to making detailed cascade measurements in this experiment, great care was taken to ensure uniform inflow and periodic outflow. The procedures used to achieve these required conditions are well established and have been described in detail by previous investigators (e.g., Rodger, 1992; Whitehouse, 1993; Abbott, 1993); therefore, only a brief description is given here. Typical flow quality results are also presented in this section.

The uniformity of the cascade inlet flow was measured in both the spanwise and pitchwise directions. In the spanwise direction, uniformity was determined with a pitot-tube at the boundary layer traverse location which is located 120% C_x upstream of the leading edges (refer to Figure 3.3 for traverse locations). The traverse extended across approximately 70% of the blade span beginning at the front endwall. Typical results are presented in Figure 3.12, and as shown, the total pressure in the freestream was essentially uniform, with variations of less than 0.5% of the centreline dynamic pressure. Details of the inlet losses and integral boundary layer parameters are given in Chapters 6 and 7. It is worth

Figure 3.12: Spanwise total pressure distribution at cascade inlet.

noting that the spanwise uniformity of the freestream total pressure was largely a function of the wind-tunnel flow quality and could not be adjusted with the test-section control surfaces. The thickness of the endwall boundary layers is varied by adjusting the boundary layer bleed slot openings. These openings were kept at a fixed width for all measurements except for some test cases where inlet boundary layer effects were investigated.

The pitchwise uniformity of the inlet flow was controlled primarily with the side flaps and was evaluated with three-hole probe traverses at midspan in a plane 50% C_x upstream of the cascade leading edge. Initially, the top and bottom flaps were positioned according to settings documented by previous investigators. The inlet flow was then traversed and the position of the flaps fine tuned: this procedure was repeated until the inlet flow angle varied

by no more than ± 0.5 degrees across the inlet. The present author, like Tremblay (1989), found that for moderate values of incidence (i.e., less than 10 degrees) 4 or 5 iterations were sufficient to achieve flow uniformity. At higher incidence, a strong coupling existed between the top and bottom flaps and as such, achieving uniformity was considerably more difficult - as many as 25 iterations were required.

Typical results of the pitchwise traverses for +10 degrees of incidence are presented in Figures 3.13a, b and c, which show in order, flow angle, dynamic pressure and total pressure distributions. As illustrated, the uniformity of flow angle (± 0.5 degrees) and total pressure coefficient (± 0.002) were quite satisfactory across the entire inlet. The dynamic pressure distribution over the bottom half of the cascade, where all detailed downstream measurements were made, was uniform to within $\pm 1\%$ of the reference dynamic pressure. From the middle of the cascade to the top, a progressive reduction in dynamic pressure was observed. This behaviour has also been observed by others (e.g., Rodger, 1992; Abbott, 1993; Mahallati, 2003) and occurs at positive values of incidence when the mean acceleration through the cascade is reduced. For these conditions, the pressure difference between the inlet and outlet (i.e., atmosphere) is small and the top flap becomes less effective at controlling the flow. To improve the effectiveness of the top flap for high angles of incidence (i.e., +10 and +20 degrees), a throttling screen was mounted at the outlet of the tailboards to raise the test section static pressure. Without the throttling screen, inlet uniformity could not be achieved. It is worth noting that Mahallati (2003) recently added a flow control arrangement to the top of the test section which has significantly improved the inlet flow uniformity without the need for the throttling screen. The effect of the variation of dynamic pressure on the outlet periodicity is discussed below.

The outlet flow periodicity was controlled mainly with the tailboards and was evaluated with three-hole probe traverses at midspan in a plane $40\% C_x$ downstream of the cascade.

Figure 3.13: Inlet flow uniformity at +10 degrees of incidence: (a) flow angle, (b) dynamic pressure and (c) total pressure.

To achieve outlet periodicity, the outlet flow was traversed and appropriate changes were made to the tailboard settings based on the traverse results. This process was continued until the exit flow angle varied by less than ± 0.5 degrees between blade passages at which point, the periodicity was considered acceptable. The tailboard settings were observed to influence the inlet flow uniformity for each of the test cases investigated, therefore, iterations between inlet and outlet were necessary. For high values of incidence, as many as 5 iterations were needed to achieve the proper flow conditions. Figures 3.14a, b and c show typical pitchwise variations of outlet flow angle, dynamic pressure and total pressure for +10 degrees of incidence, respectively. From the three plots it can be seen that very good periodicity has been achieved: the exit flow angles are observed to be within ± 0.5 degrees from passage to passage; the total pressure deficit associated with each wake is of similar depth and width; and the freestream dynamic pressure levels in each passage are comparable. It is interesting to note that the outlet periodicity seems to be insensitive to the non-uniform variation in dynamic pressure that was observed at the inlet of the cascade.

3.9 Data Reduction

3.9.1 Measured and Mixed-out Loss Coefficients

The loss data presented here is expressed mainly in terms of the mass-averaged total-pressure loss coefficient, Y . This coefficient is obtained using the mass-averaged total-pressure coefficient, Cp_0'' . The total-pressure coefficient is defined as

$$Cp_0 = \frac{P_{0_{local}} - P_{0_{CL}}}{\frac{1}{2} \rho V_{CL}^2} \quad (3.8)$$

(a)

(b)

(c)

Figure 3.14: Outlet flow periodicity at +10 degrees of incidence: (a) flow angle, (b) dynamic pressure and (c) total pressure.

where V is the magnitude of the flow velocity. The mass-averaged values of Cp_0 are calculated from

$$Cp_0'' = \frac{\iint \rho u(y', z) Cp_0(y', z) dy' dz}{\iint \rho u(y', z) dy' dz} \quad (3.9)$$

where the integrals are evaluated over a reference area which covers one blade pitch and 50% of the span and u is the axial component of the flow velocity. The denominator in Equation 3.9 is simply the mass flow rate over the reference area. As a means of verifying the data quality, the outlet mass flow measured downstream of the cascade over the reference area was compared to the inlet mass flow over the same area. For all cases, there was a deficit in mass flow at the downstream measurement plane by comparison with the inlet mass flow. At 0 and +10 degrees of incidence, the mass flow rates differed by no more than 2.5%. At +20 degrees of incidence when trailing-edge separation was present, the difference between the inlet and outlet mass flows was as high as 6%. There are a few possible reasons for the discrepancies between the inlet and outlet mass flows. Based on some of the flow visualization results, it appears that there were differences in the size of front- and back-endwall secondary flow structures and is presumably due to differences in the thickness of the inlet endwall boundary layers. The deficit in mass flow is attributed, in part, to asymmetry in the size of the secondary flow structures. Also, the inability to resolve the outlet endwall boundary layers with the seven-hole probe is another potential reason for discrepancies to arise between the inlet and outlet mass flows.

The overall total-pressure loss coefficient is defined as

$$Y_{total} = (Cp_{0_1}'' - Cp_{0_2}'') \left(\frac{q_{CL}}{q_2} \right)'' \quad (3.10)$$

where Cp_0'' is obtained from Equation 3.9 and the dynamic pressure, q , is defined as $q = 1/2 \rho V^2$. The subscripts 1 and 2 refer to the inlet boundary layer traverse location and the

outlet measurement plane, respectively.

The downstream mass-averaged quantities in Equation 3.10 were evaluated from the seven-hole probe measurements made at a plane located 0.4 axial chord lengths downstream of the trailing edge. In this plane, the measurement grid extended 1.287 airfoil spacings in the pitchwise direction (142.5 mm) and 0.700 airfoil heights in the spanwise direction (142.5 mm). The spacing between measurement points was 7.5 mm in both the pitchwise and spanwise directions, for a total of 400 grid points. For the measurements that were made at the traverse plane located 0.1 axial chord lengths downstream of the trailing edge (see Section 6.2.3.2), the measurement grid in the spanwise direction was reduced to 0.526 airfoil heights and the spacing was decreased to 3.75 mm in both directions, for a total of 1161 points.

The error in the mass-averaged loss coefficient due to grid density effects was estimated by removing alternate data points in both the pitchwise and spanwise directions (i.e., doubling the grid spacing from 3.75 to 7.5 mm) from the data taken at the traverse plane located 0.1 axial chord lengths downstream of the trailing edge, and in turn, evaluating the loss coefficient for each of the two grids. The difference in the values of the loss coefficients was very small (i.e., $\Delta Y_{total} < 0.001$), thus indicating that the value of loss coefficient was essentially “grid-independent” for the coarser of the two grids (i.e., grid with measurement intervals of 7.5 mm). Presumably this conclusion holds for the measurements taken at the traverse plane located 0.4 axial chord lengths downstream of the trailing edge since the flow structures are larger due to the effects of diffusion (see Chapter 6).

The midspan total-pressure loss coefficient is obtained in a similar manner to Equation

3.10. First, the pitchwise mass-averaged total-pressure coefficient, Cp_0' , is calculated from

$$Cp_0' = \frac{\int_0^s \rho u(y', z) Cp_0(y', z) dy'}{\int_0^s \rho u(y', z) dy'} . \quad (3.11)$$

Equation 3.11 is then applied at midspan and converted into a total-pressure loss coefficient with the following relation

$$Y_{midspan} = Cp_{0' midspan} \left(\frac{q_{CL}}{q_2} \right)' . \quad (3.12)$$

The total-pressure loss coefficients that have been defined up to this point refer to the “measured” losses, that is, the losses that have been incurred between the inlet plane and the measurement plane. Note that for this current study, the measurement plane is located at 40% axial chord lengths downstream of the trailing edge. Because the flow at the measurement plane is still spatially non-uniform, additional losses will be generated further downstream due to mixing. At the downstream location where the flow is completely uniform, the flow is said to be mixed out and the losses that have been incurred up to that location are referred to as the mixed-out losses. The mixed-out loss coefficient is a convenient quantity for comparing cascade results because it is in principle independent of the location of the downstream measurement plane. It is for this reason that mixed-out loss coefficient is used for correlation purposes. It should be noted that the mixed-out losses are unrepresentative of the losses generated in the engine environment since variations in the flow will inevitably still exist at the entry of the next airfoil row.

The mixing losses are calculated by applying the continuity and momentum equations to a control volume that extends over one full pitch from the measurement plane to the fully mixed-out plane and from the endwall to midspan. Mixing is assumed to occur at incompressible conditions and at constant area until uniform flow conditions have been

achieved.

It should be noted that the endwall shear stress is not included in the momentum balance notwithstanding that the control volume extends to the endwall. Yaras and Sjolander (1989) examined the influence of the wall shear stress term on the mixing losses by calculating the mixed-out losses based measurement planes located at various downstream axial locations. They concluded that the influence of the wall shear stress term was small beyond about 40% axial chord lengths downstream of the trailing edge. Moore and Adhye (1985), who performed secondary loss measurements in the UTRC/VPI&SU cascade, arrived at a similar conclusion. The mixing calculations in the current study were based on measurements that were made at 40% axial chord downstream of the trailing edge. Therefore, it seems reasonable to neglect the endwall shear stress in light of the results of Moore and Adhye (1985) and Yaras and Sjolander (1989).

The resultant expression for the overall mixed-out total pressure has been derived by Harrison (1992), and is given below:

$$P_{0_{mixed}} = \frac{1}{A_{mixed}} \left\{ \iint (P_2 + \rho u_2^2) dy' dz \right\} + \frac{\rho}{2} \left\{ \left(\frac{\iint u_2 v_2 dy' dz}{\iint u_2 dy' dz} \right)^2 + \left(\frac{\iint u_2 w_2 dy' dz}{\iint u_2 dy' dz} \right)^2 - \frac{1}{A_{mixed}^2} \left(\iint u_2 dy' dz \right)^2 \right\} \quad (3.13)$$

where v and w are the tangential and spanwise components of the flow velocity, respectively. A_{mixed} is the area of the control volume exit plane measured perpendicular to the axial direction and the integrals in Equation 3.13 are evaluated over one full airfoil pitch and 50% of the span.

Before converting Equation 3.13 into a total-pressure loss coefficient, the equation was

re-expressed in terms of quantities that are output from the probe data reduction program. To do this, it was necessary first to subtract the inlet static pressure from both sides of the equation and then non-dimensionalize by the inlet centreline dynamic pressure. That is,

$$\begin{aligned} \frac{P_{0_{mixed}} - P_{CL}}{\frac{1}{2} \rho V_{CL}^2} = & \frac{1}{A_{mixed}} \left\{ \iint C_{p_2} dy' dz + 2 \iint \left(\frac{u_2}{V_{CL}} \right)^2 dy' dz \right\} + \\ & \left(\frac{\iint \left(\frac{u_2}{V_{CL}} \right) \left(\frac{v_2}{V_{CL}} \right) dy' dz}{\iint \left(\frac{u_2}{V_{CL}} \right) dy' dz} \right)^2 + \\ & \left(\frac{\iint \left(\frac{u_2}{V_{CL}} \right) \left(\frac{w_2}{V_{CL}} \right) dy' dz}{\iint \left(\frac{u_2}{V_{CL}} \right) dy' dz} \right)^2 - \\ & \frac{1}{A_{mixed}^2} \left(\iint \left(\frac{u_2}{V_{CL}} \right) dy' dz \right)^2 \end{aligned} \quad (3.14)$$

where C_{p_2} is the measured static pressure coefficient, and is defined as

$$C_{p_2} = \frac{P_2 - P_{CL}}{\frac{1}{2} \rho V_{CL}^2} . \quad (3.15)$$

The mixed-out total-pressure coefficient may be expressed as

$$C_{p0_{mixed}} = \frac{P_{0_{mixed}} - P_{CL}}{\frac{1}{2} \rho V_{CL}^2} - 1 \quad (3.16)$$

where the first term on the right-hand side of the equation is given by Equation 3.14. Finally, the overall mixed-out total-pressure loss coefficient can be written in terms of Equation 3.16:

$$Y_{total_{mixed}} = (C_{p0,1''} - C_{p0_{mixed}}) \left(\frac{q_{CL}}{q_{mixed}} \right) \quad (3.17)$$

where q_{mixed} is defined as

$$q_{mixed} = \frac{1}{2} \rho (u_{mixed}^2 + v_{mixed}^2 + w_{mixed}^2)$$

and the mixed-out velocity components are given by

$$\begin{aligned} u_{mixed} &= \frac{1}{A_{mixed}} \iint u_2 dy' dz , \\ v_{mixed} &= \frac{\iint u_2 v_2 dy' dz}{\iint u_2 dy' dz} , \text{ and } w_{mixed} = \frac{\iint u_2 w_2 dy' dz}{\iint u_2 dy' dz} . \end{aligned}$$

An expression for the mixed-out profile losses has been derived in Appendix E. For incompressible flows, the resultant equation is given by

$$Y_{P_{mixed}} = \frac{P_{0CL} - P_{0mixed}}{q_m} = \frac{1 - \int_0^1 Cp_2 d(y/s) + 2(I_1^2 - I_2)}{\left(\frac{u_{mixed}^2 + v_{mixed}^2}{V_{CL}^2} \right)} - 1 . \quad (3.18)$$

where Cp_2 is given by Equation 3.15, and the variables I_1 and I_2 are defined, in order, as

$$\begin{aligned} I_1 &= \int_0^1 \left(\frac{u_2}{V_{CL}} \right) d(y/s) , \\ I_2 &= \int_0^1 \left(\frac{u_2}{V_{CL}} \right)^2 d(y/s) \end{aligned}$$

and the mixed-out x- and y-components of velocity (i.e., u_{mixed} and v_{mixed}) are given by

$$\begin{aligned} u_{mixed} &= \int_0^1 u_2 d(y/s) \quad \text{and} \quad . \\ v_{mixed} &= \frac{\int_0^1 (v_2)(u_2) d(y/s)}{\int_0^1 u_2 d(y/s)} . \end{aligned}$$

The uncertainties in the mass-averaged total pressure loss coefficients were estimated mainly from repeated measurements as well as from observed variations in flow quantities from airfoil to airfoil. For cases in which the boundary layers remained largely attached (i.e., at 0 and +10 degrees of incidence) the loss coefficients were estimated to be accurate to within ± 0.0035 with a 95% level of confidence. At +20 degrees of incidence, the uncertainty in the total pressure loss coefficients was significantly higher because of the presence of trailing-edge separation. The uncertainty was estimated to be ± 0.04 with a 95% level of confidence.

It should be noted that the discrepancies between the measured inlet and outlet mass flows that were quoted earlier seem to have little influence on values of the mass-averaged total pressure loss coefficients. For a given case, the sensitivity of the loss coefficient to errors in mass flow was determined by perturbing the velocity at each measurement point in the data set by a fixed amount. The losses were then calculated for the modified data set and compared to the losses for the baseline case. For example, when the outlet velocity was changed by an amount that corresponds to a 2.5% change in the half-passage mass flow rate, the resulting loss coefficient only changed by 0.002, which is less than the estimated uncertainty in the loss coefficients due to all factors.

3.9.2 Streamwise Vorticity and Secondary Kinetic Energy

A knowledge of the streamwise vorticity and secondary kinetic energy distributions is useful for studying the three-dimensional structures downstream of the blade row.

The axial and tangential components of vorticity were calculated from the grid data at the downstream traverse plane using

$$\omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}$$

and

$$\omega_y = \frac{1}{u} \left(v \omega_x + \frac{1}{\rho} \frac{\partial P_0}{\partial z} \right). \quad (3.19)$$

Equation 3.19 is derived from the incompressible Euler equations (Gregory-Smith *et al.*, 1988a) and Yaras and Sjolander (1990) examined its validity in viscous flows. They found that the error in vorticity was very small except near the edge of a vorticity core where the velocity gradient was changing most rapidly and the viscous effects were most important. Even there, however, the errors were small enough to justify the use of Equation 3.19.

The streamwise component of vorticity is calculated from

$$\omega_s = \omega_x \cos \alpha_2'' + \omega_y \sin \alpha_2''$$

where the mass-averaged exit flow angle, α_2'' , is defined as

$$\alpha_2'' = \tan^{-1} \left(\frac{\iint \rho u_2 v_2 dy' dz}{\iint \rho u_2^2 dy' dz} \right).$$

The non-dimensional streamwise vorticity, C_{ω_s} , is given as

$$C_{\omega_s} = \frac{\omega_s C}{V_{CL}} \quad (3.20)$$

where C is the airfoil chord.

An integrated quantity that is used as an indicator of the strength of the secondary flow field is the secondary circulation, Γ_s . It is related to the streamwise vorticity through the following expression

$$\Gamma_s = \int_0^{h/2} \int_0^s \omega_s dA_s$$

where dA_s is the differential area of a plane measured perpendicular to the streamwise

direction, that is,

$$dA_s = \cos \alpha_2'' dy' dz .$$

The non-dimensional secondary circulation, C_{Γ_s} , is defined as

$$C_{\Gamma_s} = \frac{\Gamma_s}{V_{CL} C} . \quad (3.21)$$

Secondary kinetic energy is defined as the kinetic energy associated with the velocity components in a plane perpendicular to the mass-averaged exit flow angle, α_2'' . Mathematically, this may be expressed as

$$SKE = \frac{1}{2} \rho (v_{sec}^2 + w_{sec}^2)$$

where

$$v_{sec} = -u_2 \sin \alpha_2'' + v_2 \cos \alpha_2'' \quad \text{and} \quad w_{sec} = w_2 .$$

The non-dimensional secondary kinetic energy, C_{SKE} , is simply

$$C_{SKE} = \frac{v_{sec}^2 + w_{sec}^2}{V_{CL}^2} . \quad (3.22)$$

Chapter 4

Navier-Stokes Numerical Algorithm, BTOB3D

4.1 Introduction

In the mid-1980s, an intensive effort was underway in the CFD and turbomachinery communities to develop three-dimensional Navier-Stokes codes for application to turbomachinery flows. A product of this effort was the BTOB3D code, a generalized turbomachinery flow solver developed by Dawes (1985).

Since the inception of the BTOB3D code, the predictive ability of the code has been demonstrated through extensive testing over a wide range of turbomachinery flows. Table 4.1 summarizes the various test cases for which computations have been compared to experimental data. Included in the table are the following details:

- 1) geometry of the test case;
- 2) specific flow parameters that were compared;
- 3) whether midspan or three-dimensional flow was simulated

- 4) whether tip leakage was simulated, and
- 5) the reference.

Table 4.1: BTOB3D Test Cases

Turbomachinery Case	Flow Parameter(s) Investigated	Tip Cl.	2D/3D	Reference
Off-design transonic compressor rotor	Mach no. distributions	no	2D	Dawes (1986 <i>b</i>)
Transonic linear turbine cascade	Mach no. distributions	no	3D	Dawes (1986 <i>c</i>)
High-speed centrifugal compressor	Mach no. distributions	no	3D	Dawes (1987 <i>a</i>)
Transonic compressor rotor	Mach no. distributions	yes	3D	Dawes (1987 <i>b</i>)
Transonic linear compressor cascade	Mach no. distributions & losses	no	2D	Dawes (1988)
High-speed radial inflow turbine	Velocity distributions	yes	3D	Dawes (1988)
Off-design transonic linear compressor cascade	Boundary layer profile & losses	no	2D	Dawes (1990)
Subsonic linear turbine cascade	Static pressure distributions, losses & swirl angle	no	3D	Dawes (1990)
Subsonic linear compressor cascade	Losses & swirl angle	yes	3D	Storer & Cumpsty (1993)
Subsonic linear turbine cascade	Static pressure distributions, losses & deviation	no	2D	Saroch (1996)
Subsonic linear turbine cascade	Static pressure distributions & losses	yes	3D	DeCecco (1995)
Transonic linear turbine cascade	Static pressure distributions, deviation & losses	no	3D	Islam (1999)

As can be seen from Table 4.1, the majority of validation studies have been conducted more than a decade ago and since that time major advancements in numerical techniques have occurred. A number of papers have recently appeared in the open literature which describe BTOB3D-based codes that have been substantially modified to incorporate state-

of-the-art numerical techniques (e.g., TRANScode (Emmerson, 1998), Stage 3D (Emunds *et al.*, 1999) and 3DNS (Gummer *et al.*, 2001)). In all cases the modifications have focused on improving the accuracy and speed of convergence of the original BTOB3D code. To perform the computational part of the present work, a state-of-the-art Navier-Stokes solver was required.

At the onset of this research in the mid-1990's, a number of alternatives had been considered for Navier-Stokes codes. Commercial codes were the most attractive option because they are well validated and continually updated with the most recent numerical algorithms. However, commercial codes were unaffordable whereas the original BTOB3D code was available in the Department. To conduct this computational study, it was necessary to update the code to reflect recent improvements in numerical techniques. As well, further modifications to the code were necessary because in its original form, it could not be used directly to simulate the incompressible experiments for this research, as discussed in Chapter 1.

4.2 Overview of Modifications

To accommodate both low-speed flows and linear cascade geometries, two changes were made to the original BTOB3D code. First, the original code was modified to handle both compressible and incompressible flows, as detailed in Section 4.3.2. Second, the present author reworked a major portion of the original code so that the solver could accommodate much more arbitrary geometric models. This modification eliminated the inherent grid limitations of the BTOB3D code such as the inability to accurately model the tip gap. The user now has much greater flexibility in specifying the location of physical and computational boundaries, as described in Section 4.6. This shift towards a more “general-purpose” flow solver proved to be beneficial for a number of other reasons.

First, it enabled the author to verify the proper functioning of the code following the extensive modifications that had been made. In other words, the code could be used to perform simulations of simple flow geometries for which there were either exact solutions or substantial experimental data such as flow over a backward-facing step and flow through a 90 degree duct. Secondly, with a generalized flow solver, it was no longer necessary to use the original BTOB3D mesh configuration. As will be discussed in Section 4.4.1, the single-block H-mesh configuration chosen by Dawes, by nature, produced highly skewed grids in the leading and trailing edge regions of turbine airfoils. Turner *et al.* (1993) and Chung and Baek (1992) have shown that numerical solutions are sensitive to grid-skew where meshes with higher grid-skew produce solutions with higher numerical error. Therefore, numerical errors related to poor grid quality could be reduced with a more generalized flow solver.

Several modifications were made to the original BTOB3D to improve the solution accuracy.

First, it was found that the spatial discretization scheme for the viscous terms produced significant errors even with levels of grid clustering well within those stipulated by Dawes (Sjolander, 1994). As a result, the present author implemented a second-order accurate discretization scheme for the viscous terms (e.g., Jayaram and Jameson, 1988; Muller and Rizzi, 1988) to reduce discretization errors.

Second, in the original code the viscous terms were simplified using the thin-layer assumption¹. This assumption is satisfactory for attached and mildly separated flows but it is invalid for largely separated flows. In a number of off-design cases that were to be simulated in the present work, particularly those at high positive incidence, large separations were expected to occur (e.g., Perdichizzi and Dossena, 1993; Rodger *et al.*, 1992). Therefore, it was necessary to remove the thin-layer assumption.

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¹i.e., omission of the streamwise diffusion terms

Finally, the artificial dissipation model in the BTOB3D code has been shown to produce excessive dissipation for certain viscous simulations. The present author has implemented a more recent model which produces inherently less dissipation. Section 4.4.4 of this chapter discusses the original and improved artificial dissipation models.

A number of changes were made to improve the efficiency of the original BTOB3D code. The modified version of the code has the capability of handling either two- or three-dimensional mesh geometries. The original BTOB3D code was strictly a three-dimensional code which required a minimum of 7 cells in the third dimension (usually the spanwise direction). For this work, a significant reduction in computational effort was realized because a majority of the simulations were performed in the 2-D mode. The time-marching scheme has also been modified. The original time-marching scheme was replaced with a scheme which has been shown to be as robust as the Dawes algorithm, but effects a two-fold reduction in computational effort, as discussed in Section 4.4.5. Modifications to the local time-stepping technique have also been made to eliminate potential convergence problems on fine meshes.

The remaining part of this chapter focuses on describing the current version of the BTOB3D code. First, however, the compressible and incompressible flow equations are given.

4.3 Governing Equations for Compressible Flow

The BTOB3D code developed by Dawes (1985) is a numerical algorithm for solving the three-dimensional, Reynolds-averaged, Navier-Stokes equations. For the algorithm, Dawes uses a finite-volume approach where the domain of the flow is first divided into a large number of subdomains. The governing equations are then discretized by applying the

integral form of the conservation laws to each subdomain. That is,

$$\frac{\partial}{\partial t} \oint_{\Omega} \mathbf{Q} + \oint_A (\mathbf{H}_c + \mathbf{H}_v) \cdot \mathbf{n} dA = 0 \quad (4.1)$$

where Ω is the volume of the subdomain and A is the closed surface that bounds Ω . $\mathbf{Q}$ is the solution vector, $\mathbf{H}_c$ and $\mathbf{H}_v$ are the convective (or inviscid) and viscous (or diffusive) flux vectors, respectively.

In the original code, the Navier-Stokes equations were cast in a rotating frame of reference and expressed in polar coordinates. With the modified code, the Navier-Stokes equations are cast in a stationary frame of reference using Cartesian coordinates; therefore, $\mathbf{Q}$, $\mathbf{H}_c$ and $\mathbf{H}_v$ are given as

$$\mathbf{Q} = \begin{bmatrix} \rho \\ \rho v_x \\ \rho v_y \\ \rho v_z \\ \rho E \end{bmatrix} \quad \mathbf{H}_c = \begin{bmatrix} \rho \mathbf{V} \\ \rho v_x \mathbf{V} \\ \rho v_y \mathbf{V} \\ \rho v_z \mathbf{V} \\ \rho H \mathbf{V} \end{bmatrix} \quad \mathbf{H}_v = \begin{bmatrix} 0 \\ -\bar{\tau} \cdot \mathbf{i} \\ -\bar{\tau} \cdot \mathbf{j} \\ -\bar{\tau} \cdot \mathbf{k} \\ -\bar{\tau} \cdot \mathbf{V} + k \nabla T \end{bmatrix} \quad (4.2)$$

with the absolute velocity: $\mathbf{V} = v_x \mathbf{i} + v_y \mathbf{j} + v_z \mathbf{k}$

total internal energy: $E = e + \frac{1}{2} \mathbf{V} \cdot \mathbf{V}$

total enthalpy: $H = E + \frac{p}{\rho}$

and stress tensor: $\bar{\tau} = \begin{vmatrix} -p + \tau_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & -p + \tau_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & -p + \tau_{zz} \end{vmatrix}$

As mentioned in Section 4.2, it was necessary to remove the thin-layer assumption used in the original code; thus, all streamwise partial derivatives (i.e., $\frac{\partial}{\partial x}$) are retained in the viscous stress terms. These terms are expressed as

$$\tau_{xx} = \mu \left(2 \frac{\partial v_x}{\partial x} + \lambda \nabla \cdot \mathbf{V} \right) \quad (4.3)$$

$$\tau_{xy} = \mu \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) \quad (4.4)$$

$$\tau_{xz} = \mu \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right) \quad (4.5)$$

$$\tau_{yx} = \tau_{xy} \quad (4.6)$$

$$\tau_{yy} = \mu \left(2 \frac{\partial v_y}{\partial y} + \lambda \nabla \cdot \mathbf{V} \right) \quad (4.7)$$

$$\tau_{yz} = \mu \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right) \quad (4.8)$$

$$\tau_{zx} = \tau_{xz} \quad (4.9)$$

$$\tau_{zy} = \tau_{yz} \quad (4.10)$$

$$\tau_{zz} = \mu \left(2 \frac{\partial v_z}{\partial z} + \lambda \nabla \cdot \mathbf{V} \right) \quad (4.11)$$

$$(4.12)$$

where μ and λ are the first and second coefficients of viscosity. According to the Stokes hypothesis, λ is taken to be $-\frac{2}{3}\mu$.

Equations 4.1 and 4.2 represent the conservation of mass, x-momentum, y-momentum and z-momentum and energy, in that order. Two additional equations are used to close the system of equations: an equation of state, which is described below; and, the two-layer eddy-viscosity turbulence model developed by Baldwin and Lomax (1978). The turbulence model is described in Section 4.7.

4.3.1 Fluid Properties

It has been assumed that the working fluid is air; therefore, the ideal gas law is used as the equation of state. This equation, normally written in terms of p , ρ and T , can also be expressed in terms of total internal energy:

$$p = \rho(\gamma - 1) \left(E - \frac{1}{2} \mathbf{V} \cdot \mathbf{V} \right) \quad (4.13)$$

where $\gamma = Cp/Cv$ (i.e., specific heat ratio).

The fluid viscosity and thermal conductivity are determined from the Sutherland formulae (White, 1991):

$$\begin{aligned} \frac{\mu}{\mu_0} &\approx \left(\frac{T}{T_0} \right)^{\frac{3}{2}} \left(\frac{T_0 + S_\mu}{T + S_\mu} \right) \\ \frac{k}{k_0} &\approx \left(\frac{T}{T_0} \right)^{\frac{3}{2}} \left(\frac{T_0 + S_k}{T + S_k} \right) \end{aligned}$$

$$\text{where for air} \quad \mu_0 = 1.716 \times 10^{-5} \text{ N} \cdot \text{s/m}^2, \quad k_0 = 0.0241 \text{ W/m} \cdot \text{K}$$

$$T_0 = 273 \text{ K}, \quad S_\mu = 111 \text{ K} \quad \text{and} \quad S_k = 194 \text{ K}$$

Other thermodynamic properties such as specific heats are assumed to be invariant with temperature.

4.3.2 Modified Equations for Incompressible Flows

It is fairly well known that many Euler/Navier-Stokes solvers, which are normally quite efficient at compressible conditions, have problems converging to steady solutions at low

Mach numbers. These convergence problems occur even though there is usually no change in the complexity of the flow physics as the Mach number decreases into the incompressible regime. In contrast, however, changes do occur to the mathematical properties of the governing equations. It is these changes and the choice of numerical algorithm that cause many solvers to become inefficient or in fact unstable at low Mach numbers.

BTOB3D and many other Euler and Navier-Stokes solvers that are in use today (e.g., Davis *et al.*, 1987; Liu and Jameson, 1993; Chima, 1985) rely on the hyperbolic character of the governing differential equations to achieve convergence to steady-state. These solvers progress the solution to steady-state using the time derivative terms in the governing equations (i.e., time-marching algorithms).

The inherent problem with the use of time-marching algorithms for solving low-speed flows resides with the disparity between the propagation rate of pressure (acoustic) waves and waves that are convected at the fluid speed. That is, with decreasing Mach number (ratio of convective speed to acoustic speed) the speed of pressure waves dominates over the fluid speed. As discussed by Rizzi and Eriksson (1985), it is this disparity in the wave speeds that causes the system of hyperbolic equations to be ill-conditioned (i.e., excessively “stiff”).

The most commonly used method for solving this stiffness problem is to modify or precondition the differential equations to eliminate the disparity in wave speeds. Clearly the preconditioning that is chosen should not affect the accuracy of the steady-state solution. It is for this reason that the preconditioning is performed on the time derivative terms in Equation 4.1. Modifying the time derivative term does not influence the steady solution because $\frac{\partial}{\partial t} \rightarrow 0$ as the solution approaches steady-state.

An excellent review of preconditioning methods is given by Turkel (1987). The preconditioning method that was implemented in this work was developed by Chorin (1967)

and is often referred to as the method of artificial compressibility. This technique has been used successfully in a wide range of applications (e.g., Choi and Merkle, 1985; Rizzi and Eriksson, 1985; Kwak *et al.*, 1984). It was also implemented by Walker and Dawes (1990) in a later version of BTOB3D.

In the method of artificial compressibility, the time derivative term in the continuity equation is replaced by an artificial time derivative term. That is, $\frac{\partial \rho}{\partial t}$ is replaced by $\rho_0 \beta \frac{\partial p}{\partial t}$ where ρ_0 is the constant density of the flow and β is a free parameter, which is often referred to as the artificial compressibility parameter. Implementing these changes into equations 4.1 and 4.2 gives the “incompressible” form of the governing equations:

$$\begin{bmatrix} \beta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \frac{\partial}{\partial t} \oint_{\Omega} \begin{bmatrix} p \\ \rho_0 v_x \\ \rho_0 v_y \\ \rho_0 v_z \end{bmatrix} + \oint_A \begin{bmatrix} \rho_0 \mathbf{V} \\ \rho_0 v_x \mathbf{V} - \bar{\tau} \cdot \mathbf{i} \\ \rho_0 v_y \mathbf{V} - \bar{\tau} \cdot \mathbf{j} \\ \rho_0 v_z \mathbf{V} - \bar{\tau} \cdot \mathbf{k} \end{bmatrix} \cdot \mathbf{n} dA = 0 \quad (4.14)$$

The artificial compressibility parameter, β , is the parameter which is used to control the convergence rate. It is typically set to be proportional to either the local velocity squared or the square of some meaningful reference velocity, i.e., $\beta = r \mathbf{V} \cdot \mathbf{V}$ or $\beta = r v_{ref}^2$ where r is a proportionality constant. Chang and Kwak (1984) and Rizzi and Eriksson (1985) found that the optimal value of r was within the range of 0.1-10. Walker and Dawes (1990) performed a 1-D inviscid wave propagation analysis and found that the optimal value of r was 2.14. Based on observations made throughout the course of this work, it was determined that the optimal value of r was within the range of 1.0-3.0. Its specific value, within this range, seemed to be a function of the geometry of the flow.

4.4 Numerical Formulation

4.4.1 Computational Mesh

Of the many popular grid configurations (e.g., C-mesh, O-mesh, O-H mesh and H-mesh), Dawes selected the structured H-mesh for its simplicity and flexibility (Dawes, 1986*a*). As illustrated in Figure 4.1 the H-mesh is constructed from the intersection of meridional, quasi-orthogonal and streamwise surfaces. The intersection points of these three surfaces are referred to as grid points or cell corners and their positions in space were expressed in cylindrical coordinates in the original code. This network of grid points is used to divide the domain into hexahedral subdomains or cells.

In theory, any arbitrary cell shape can be used with a finite volume approach. In practice, however, the more generalized the cell shape, the more floating point operations and computer memory are required per cell. In the mid-1980's when BTOB3D was being developed, computational resources were limited by comparison with today's computers. Therefore, Dawes imposed geometric limitations on the quasi-orthogonal and streamwise surfaces to constrain cell shapes and, in effect, limit computational resources and "running" costs. He did, nonetheless, recognize a number of shortcomings with this particular arrangement. Perhaps the most serious being the regions of highly-skewed cell shapes near the leading- and trailing-edges of the turbine airfoils as shown in the insets of Figure 4.2. It is clear from the figure that the grid skew problem arises because the cross-stream grid lines are constrained to reside on planes of constant x .

For the present work, the original code was modified to accommodate any structured hexahedral mesh. This freedom allowed the author to use higher quality meshes, albeit at the expense of memory resources. As mentioned earlier, the shift towards a more generalized solver also allowed the author to easily verify the proper functioning of the

Figure 4.1: Surface definitions for original BTOB3D H-mesh (reproduced from De-Cecco, 1995).

code by simulating simple flows for which there were exact solutions.

Figure 4.3 shows an example of a typical mesh which was used for the simulations performed in this work. The mesh generator was developed by Zavitz (2000) and as can be seen, the mesh arrangement is considerably different than the one used by Dawes. To

Figure 4.2: **Example of an original BTOB3D H-mesh for a turbine passage.**

begin, the airfoil is positioned inside the computational domain where the mesh is allowed to extend through it. To simulate the airfoil itself, the mesh cells within its boundaries are identified as “blocked” or “dead” cells. The so-called blocked-cell approach, which is the topic of further discussion in Section 4.6, was introduced to allow more flexibility in modelling complex geometries. Another advantage of using blocked cells, although not directly pertinent to the present work, is in modelling tip-leakage flows. The original version of the Dawes code was incapable of handling sharp-cornered blade tips. As shown in the quasi-orthogonal view of Figure 4.1, the blade near the tip is required to be tapered from

Figure 4.3: **Example of a new BTOB3D H-mesh.**

a finite thickness to zero at the tip. Also, the original code could not resolve the flow in the clearance space. The introduction of the blocked-cell approach eliminates these two shortcomings. Perhaps the most important advantage of the new mesh arrangement, for this work, is the improvement in grid quality. By comparing Figures 4.2 and 4.3, it is evident that cell orthogonality around the airfoil has improved with the new mesh arrangement. The largest improvement can be observed in the leading- and trailing-edge regions of the

Figure 4.4: Typical finite-volume cell.

airfoil.

4.4.2 Cell Geometry

Figure 4.4 shows a typical finite volume cell and as discussed earlier, the geometry of the hexahedral cells is defined by the coordinates of the grid points or cell nodes. The flow variables themselves are defined at the cell centre, and for this reason the numerical formulation is often referred to as a cell-centred scheme. To assist in the discussions to

follow, it is useful to identify cell surfaces by defining a local non-orthogonal coordinate system as shown in Figure 4.4. The non-orthogonal coordinates X_I , X_J and X_K define in physical space the cell edges. The cell faces are defined by surfaces of constant X_I , X_J or X_K , which are identified using the following nomenclature

- Surfaces of constant X_I : North/South surfaces,
- Surfaces of constant X_J : East/West surfaces,
- Surfaces of constant X_K : Up/Down surfaces.

The North, East and Up surfaces are defined by the positive outward I, J, and K vectors, respectively.

It is the opinion of this author that one of the advantages of the finite-volume method is that, unlike the finite-difference method, there is a closer relationship between the conservation laws and the mesh geometry. That is, for the finite-volume method, all conservation equations are applied in physical space. There is neither a need to define a coordinate transformation from physical to computational space nor is there a need to explicitly calculate the metric coefficients and the Jacobian of the transformation matrix. Instead, the equivalent of the nine metric coefficients and the Jacobian, specifically, the three components of each of the cell face surface areas, A_I , A_J and A_K and the cell volume are calculated from principles of geometry. It is these ten geometric parameters that are used directly in the conservation equations to calculate mass, momentum and energy flux² through the various cell faces. The cell-face surface-area vectors are defined as

$$\begin{aligned}
 \mathbf{A}_I &= A_I \mathbf{n}_I, \\
 \mathbf{A}_J &= A_J \mathbf{n}_J, \\
 \mathbf{A}_K &= A_K \mathbf{n}_K
 \end{aligned}
 \tag{4.15}$$

---

²per unit volume

where the unit vectors $\mathbf{n}_I$, $\mathbf{n}_J$ and $\mathbf{n}_K$ are normal to the surfaces of constant X_I , X_J and X_K , respectively. To use the surface area vectors in the conservation equations it is necessary to express the vectors in Equation 4.15 in terms of x, y and z components:

$$\begin{aligned}\mathbf{A}_I &= A_{Ix}\mathbf{i} + A_{Iy}\mathbf{j} + A_{Iz}\mathbf{k} , \\ \mathbf{A}_J &= A_{Jx}\mathbf{i} + A_{Jy}\mathbf{j} + A_{Jz}\mathbf{k} , \\ \mathbf{A}_K &= A_{Kx}\mathbf{i} + A_{Ky}\mathbf{j} + A_{Kz}\mathbf{k} .\end{aligned}\tag{4.16}$$

The components of area of a cell face are calculated from a cross product of the diagonals (Vinokur, 1989). For example, for cell face, X_J , with vertices $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$ and $\mathbf{r}_4$ (see Figure 4.5):

$$\mathbf{A}_J = \frac{1}{2}(\mathbf{r}_3 - \mathbf{r}_1) \times (\mathbf{r}_4 - \mathbf{r}_2) .\tag{4.17}$$

It should be noted that Equation 4.17 is exact only if the four vertices are coplanar.

The volume of a hexahedron can be calculated in several different ways. Dawes used an expression that took advantage of the constraints that he had imposed on the mesh geometry. An expression for generalized nonorthogonal cells (Hung and Kordulla, 1984) was used in the present version of the code. With reference to Figure 4.6, the cell volume is given as

$$\Omega = \frac{1}{3}(\mathbf{A}_{1234} + \mathbf{A}_{5678} + \mathbf{A}_{4378}) \cdot (\mathbf{r}_3 - \mathbf{r}_5) .\tag{4.18}$$

Equation 4.18 is derived by dividing the hexahedron into six tetrahedrons which share the main diagonal as a common edge. It is an exact relationship if the four grid points on each cell face are coplanar.

Figure 4.5: Cell-face diagonals used for calculation of surface area, A_i .

4.4.3 Finite-Volume Discretization of the Governing Equations

In the finite-volume method, the flow domain is divided into a large number of smaller subdomains and the conservation equations are applied to each. The resulting system of equations is solved by first evaluating the integrals in Equation 4.1 with a discrete approximation (i.e., spatial discretization). The resulting set of ordinary differential equations is then integrated in time until a steady-state solution has been achieved.

The spatial discretization can take a number of forms depending on the choice that is

Figure 4.6: Geometry used for calculation of cell volume, Ω

made for the storage location of the flow variables. There are essentially two alternatives: the cell-centred scheme where the flow variables are stored at the cell centre or the cell-vertex scheme where the flow variables are stored at the cell corners or grid points. For the original BTOB3D code, Dawes used a cell-centred discretization scheme. Since the development of BTOB3D, Swanson and Radespiel (1991) have investigated the relative merits of each scheme. Based on a one-dimensional analysis using Taylor series expansions, they concluded that the cell-vertex scheme exhibited smaller discretization errors on stretched meshes.

However, their simulations for flow over a transonic airfoil showed that the methods gave almost identical results even for coarse meshes. Based on these findings, the present author retained the cell-centred discretization scheme that was originally implemented by Dawes.

To approximate or discretize the conservation equations over a cell, it is assumed that the flow variables within the cell and the fluxes at the cell face are spatially uniform. With these assumptions, Equation 4.1 can be rewritten in semi-discrete form

$$\Omega_{i,j,k} \frac{d}{dt}(\mathbf{Q}_{i,j,k}) + \sum_{l=1}^6 (\mathbf{H}_c + \mathbf{H}_v) \cdot \mathbf{n}_l A_l = 0 \quad (4.19)$$

where $\Omega_{i,j,k}$ is the cell volume. The net flux of mass, momentum and energy out of the cell volume is computed by summing up the fluxes over all the cell faces, that is, $\sum_{l=1}^6 (\mathbf{H}_c + \mathbf{H}_v)_l \cdot \mathbf{n}_l A_l$. To perform the summation in Equation 4.19, the convective and viscous fluxes of mass, momentum and energy need to be estimated at the cell faces. These fluxes require different calculation procedures, each of which are explained below.

To achieve second-order spatial accuracy, the convective flux at a particular face, l , is estimated by arithmetically averaging the flow variable(s) taken from the two cells that share the face, l . The choice of the flow variables to be used in the averaging can influence the steady solution as discussed by Holmes and Tong (1985). For the continuity equation, Dawes averaged the product $\rho \mathbf{V}$; therefore, the estimate of mass flux at the cell face l , is

$$\dot{m} = \frac{1}{2} [(\rho \mathbf{V})_A + (\rho \mathbf{V})_B] \cdot \mathbf{n}_l A_l$$

where subscripts A and B refer to the cell centres neighbouring the cell face, l . The momentum flux, $\dot{\mathbf{G}}$ and energy flux, $\dot{T}$, were estimated, respectively, as

$$\dot{\mathbf{G}} = \frac{1}{2} \dot{m} (\mathbf{V}_A + \mathbf{V}_B),$$

$$\dot{T} = \frac{1}{2}\dot{m}(h_A + h_B) .$$

Jameson *et al.* (1981) and Holmes and Tong (1985) used a slightly different procedure which produced less oscillations than the procedure implemented by Dawes. From the solutions which will be presented in Chapter 5, it will be shown that the presence of oscillations was minimal even with lower than recommended levels of artificial dissipation. Given these findings, the present author used the same scheme as Dawes.

To evaluate the viscous flux term, H_v , an approximation is first required for the spatial velocity derivatives in the stress tensor, Equation 4.2. These derivatives may be evaluated either by applying Gauss' formula to the control volume or assuming a local transformation to computational coordinates and applying the chain rule (Jameson, 1997). In the original Dawes code, the latter scheme was implemented using a mesh transformation that was first-order accurate for meshes where the ratio of adjacent cells were in a range of 0.7 to 1.3 (Saroeh, 1996). With this scheme, the present author could not reproduce the analytic solution of a simple laminar validation test case, even with levels of grid clustering well within the limits specified by Dawes. Consequently, the present author implemented the Gauss formula for the calculation of the spatial derivatives. With this implementation, second-order accuracy of the viscous terms is guaranteed on smoothly varying meshes (Swanson and Turkel, 1997).

Application of the Gauss formula to a control volume allows the spatial derivative of a scalar to be converted into a surface integral involving the scalar itself. To illustrate mathematically, consider as an example the gradient, $\frac{\partial v_x}{\partial y}$. Applying the Gauss formula yields

$$\oint_{\Omega} \frac{\partial v_x}{\partial y} d\Omega = \oint_A v_x \mathbf{j} \cdot \mathbf{n} dA \quad (4.20)$$

where Ω is the cell volume and A is the surface boundary. Assuming that the gradient

is spatially uniform within the cell and that the x-component of velocity, v_x , is spatially uniform on each of the cell surfaces, then Equation 4.20 can be approximated as

$$\overline{\frac{\partial v_x}{\partial y}} = \frac{1}{\Omega_{i,j,k}} \sum_{l=1}^6 v_{x,l} A_{y,l} \quad (4.21)$$

where $\overline{\frac{\partial v_x}{\partial y}}$ is the volumetric average of the gradient, $A_{y,l}$ is the y-component of area and $v_{x,l}$ is the x-component of velocity at cell face l . The x-component of velocity is estimated by arithmetically averaging the values of v_x taken from the two cells that share the face l . The other velocity gradients which appear in the shear stress tensor are calculated in a similar manner.

The cell-centred shear stress tensor can be calculated using Equation 4.3 after having approximated the spatial gradients. To calculate the viscous flux vector, the shear stress tensor needs to be approximated at cell faces. The cell-face quantities are determined by arithmetically averaging the shear stress tensors at contiguous cells.

4.4.4 Artificial Dissipation

The artificial dissipation model used in the original Dawes code was based on one developed by Jameson *et al.* (1981) in which low- and high-levels of artificial dissipation are added adaptively to the semi-discrete form of the governing equations (i.e., Equation 4.19). The low-level, fourth-difference dissipation is used to prevent odd-even point decoupling. In contrast, the more rigorous, second-difference dissipation is added to suppress the appearance of oscillations in regions containing severe pressure gradients such as at shock waves and stagnation points. Blending of the second and fourth-difference artificial dissipation terms is performed with a suitable switching function which keeps dissipation to levels that will not contaminate the solution. In regions of the flow away from strong

pressure gradients, this dissipation model will not affect the asymptotic accuracy of the discretization scheme and therefore, second-order accuracy is retained.

The model used by Dawes is referred to as a scalar isotropic dissipation model. It was originally developed for inviscid flow problems where computational meshes were typically constructed with cell aspect ratios of the order of unity. For mesh cells with these aspect ratios, the dissipation is isotropic; that is, it is scaled equally in all directions. However, for mesh cells with aspect ratios of the order 100 to 1000, such as those typically found in the boundary layer of high Reynolds number flows, this isotropic scaling can create excessive dissipation (Mackenzie, 1995).

As discussed previously, the computational modelling performed in this study was the simulation of the midspan flow of linear turbine cascades operating at relatively high Reynolds numbers, (i.e., of the order of 10^5). The objective of these simulations was to assess the loss prediction capability of the code; therefore, it was necessary to capture the boundary layer development accurately. To do so with a manageable number of mesh cells and an appropriate spatial resolution of the flow field, the aspect ratio of surface cells ranged from 100 to 300. In an effort to improve this cell-aspect-ratio problem, the present author replaced the original, isotropic model used by Dawes with the scalar anisotropic dissipation model developed by Swanson and Turkel (1987) which scales the dissipation with the cell aspect ratio. This model has been shown to produce a significant improvement in accuracy for high aspect ratio meshes relative to the isotropic model used by Dawes. (e.g., Swanson and Turkel, 1987; Martinelli and Jameson, 1988). Details of this model are presented below.

With the addition of artificial dissipation, the semi-discrete form of the Navier-Stokes equations (4.19) become

$$\Omega_{i,j,k} \frac{d}{dt}(\mathbf{Q}_{i,j,k}) + \sum_{l=1}^6 (\mathbf{H}_c + \mathbf{H}_v) \cdot \mathbf{n}_l \mathbf{A}_l - \mathbf{D}_{i,j,k}(\mathbf{Q}) = 0 \quad (4.22)$$

where the total dissipation is defined as

$$\mathbf{D}_{i,j,k}(\mathbf{Q}) = (\mathbf{D}_I^2 - \mathbf{D}_I^4 + \mathbf{D}_J^2 - \mathbf{D}_J^4 + \mathbf{D}_K^2 - \mathbf{D}_K^4) \mathbf{Q}_{i,j,k} . \quad (4.23)$$

Equation 4.23 shows that the total dissipation is composed of second and fourth differences (superscripts) in each of the local coordinate directions (subscripts).

The second- and fourth-difference contributions for the I-direction are determined by

$$\mathbf{D}_I^2(\mathbf{Q}_{i,j,k}) = \nabla_I \left[\alpha_{i+\frac{1}{2},j,k} \varepsilon_{i+\frac{1}{2},j,k}^{(2)} \Delta_I \right] \mathbf{Q}_{i,j,k} , \quad (4.24)$$

$$\mathbf{D}_I^4(\mathbf{Q}_{i,j,k}) = \nabla_I \left[\alpha_{i+\frac{1}{2},j,k} \varepsilon_{i+\frac{1}{2},j,k}^{(4)} \Delta_I \nabla_I \Delta_I \right] \mathbf{Q}_{i,j,k} \quad (4.25)$$

where Δ_I and ∇_I are forward and backward difference operators in the I-direction, respectively. The scaling factor, $\alpha_{i+\frac{1}{2},j,k}$, provides the directional anisotropy for varying cell aspect ratios. It is defined as

$$\alpha_{i+\frac{1}{2},j,k} = \frac{1}{2} \left[(\overline{\lambda_I})_{i,j,k} + (\overline{\lambda_I})_{i+1,j,k} \right] \quad (4.26)$$

where $(\overline{\lambda_I})_{i,j,k}$ is related to the spectral radii (i.e., maximum values) of the flux Jacobian matrix associated with the Euler equations in the three coordinate directions. It is expressed as

$$(\overline{\lambda_I})_{i,j,k} = (\lambda_I)_{i,j,k} \left[1 + \left(\frac{(\lambda_J)_{i,j,k}}{(\lambda_I)_{i,j,k}} \right)^\varsigma + \left(\frac{(\lambda_K)_{i,j,k}}{(\lambda_I)_{i,j,k}} \right)^\varsigma \right] \quad (4.27)$$

where the exponent ς is generally taken between 1/2 and 2/3. The spectral radius for the I-direction is given as

$$(\lambda_I)_{i,j,k} = \left| \mathbf{V}_{i,j,k} \cdot (\mathbf{A}_I)_{i+\frac{1}{2},j,k} \right| + c \left| (\mathbf{A}_I)_{i+\frac{1}{2},j,k} \right| \quad (4.28)$$

where c is the local speed of sound.

It should be noted that the spectral radii for the “incompressible” equations of motion (i.e., Equation 4.14) are different from those given in Equation 4.28. For the I-direction, the spectral radius is defined as

$$\begin{aligned}
 (\lambda_I)_{i,j,k} &= \left| \mathbf{V}_{i,j,k} \cdot (\mathbf{A}_I)_{i+\frac{1}{2},j,k} \right| \\
 &+ \left[\left(\mathbf{V}_{i,j,k} \cdot (\mathbf{A}_I)_{i+\frac{1}{2},j,k} \right)^2 + \beta^2 \left| (\mathbf{A}_I)_{i+\frac{1}{2},j,k} \right|^2 \right]^{\frac{1}{2}}.
 \end{aligned} \tag{4.29}$$

Each of the contributions from the J- and K-directions are calculated in a similar manner.

The coefficients $\varepsilon^{(2)}$ and $\varepsilon^{(4)}$ in Equations 4.24 and 4.25 use the pressure field adaptively to sense sharp pressure gradients such as shocks and stagnation points. They are defined as

$$\varepsilon_{i+\frac{1}{2},j,k}^{(2)} = \kappa^{(2)} \min \left[0.5, \frac{1}{2} (\nu_{i,j,k} + \nu_{i+1,j,k}) \right] \tag{4.30}$$

$$\varepsilon_{i+\frac{1}{2},j,k}^{(4)} = \max \left[0.0, \kappa^{(4)} - \varepsilon_{i+\frac{1}{2},j,k}^{(2)} \right] \tag{4.31}$$

where

$$\nu_{i,j,k} = \left| \frac{p_{i-1,j,k} - 2p_{i,j,k} + p_{i+1,j,k}}{p_{i-1,j,k} + 2p_{i,j,k} + p_{i+1,j,k}} \right| \tag{4.32}$$

and $\kappa^{(2)}$ and $\kappa^{(4)}$ are user-defined constants. Typical values for these constants range from $\frac{1}{4}$ to $\frac{1}{2}$ for $\kappa^{(2)}$ and $\frac{1}{64}$ to $\frac{1}{32}$ for $\kappa^{(4)}$ (Swanson and Turkel, 1997). To limit the effects of artificial dissipation, these values are set just large enough so that spatial oscillations do not appear in the solution. For the simulations that were carried out in this work, $\kappa^{(2)}$ and $\kappa^{(4)}$ were set to values of 0.25 and 0.006, respectively.

4.4.5 Time Integration

The discretization of the spatial derivatives as outlined in Section 4.4.3 transforms the governing equations (Equation 4.1) into a system of coupled ordinary differential equations. These differential equations can be written in the form

$$\frac{d(\mathbf{Q}_{i,j,k})}{dt} + \mathbf{R}_{i,j,k}(\mathbf{Q}) = 0, \quad (4.33)$$

where $\mathbf{Q}_{i,j,k}$ and $\mathbf{R}_{i,j,k}(\mathbf{Q})$ are the solution and residual vectors for a given cell, respectively.

The residual vector is defined as

$$\mathbf{R}_{i,j,k}(\mathbf{Q}) = \frac{1}{\Omega_{i,j,k}} \left[\sum_{l=1}^6 (\mathbf{H}_c + \mathbf{H}_v) \cdot \mathbf{n}_l A_l - \mathbf{D}_{i,j,k}(\mathbf{Q}) \right]. \quad (4.34)$$

The algorithm which is used to integrate Equation 4.33 in time requires that the residual vector be split into three separate terms: a convection term, physical diffusion term and a artificial dissipation term. As a result, the residual is expressed as

$$\mathbf{R} = \mathcal{L}_C \mathbf{Q} + \mathcal{L}_D \mathbf{Q} + \mathcal{L}_{AD} \mathbf{Q} \quad (4.35)$$

where

$$\begin{aligned} \mathbf{R} &= \mathbf{R}_{i,j,k}(\mathbf{Q}), \\ \mathcal{L}_C \mathbf{Q} &= \frac{1}{\Omega_{i,j,k}} \left[\sum_{l=1}^6 \mathbf{H}_c \cdot \mathbf{n}_l A_l \right], \\ \mathcal{L}_{PD} \mathbf{Q} &= \frac{1}{\Omega_{i,j,k}} \left[\sum_{l=1}^6 \mathbf{H}_v \cdot \mathbf{n}_l A_l \right], \text{ and} \\ \mathcal{L}_{AD} \mathbf{Q} &= \frac{-1}{\Omega_{i,j,k}} [\mathbf{D}_{i,j,k}(\mathbf{Q})]. \end{aligned}$$

Subscripts $\mathcal{C}$, $\mathcal{PD}$ and $\mathcal{AD}$ refer to the convection, physical diffusion and artificial dissipation terms, respectively. Note that the cell indices (i,j,k) on $\mathbf{Q}$ have been suppressed for clarity.

A time-marching algorithm is used to numerically integrate this system of equations in time until the solution has reached steady state, i.e., $\frac{d\mathbf{Q}}{dt} \approx 0$. Because the steady-state solution is of primary interest, the main requirement for the algorithm is to remove the transient solution as fast as possible, and thus, maximize the convergence rate.

The time-stepping algorithm developed by Dawes was a two-step explicit, one-step implicit scheme which is described in detail by Dawes (1986*a*). It was based on a Beam and Warming (1978) implicit scheme but the solution algorithm is in fact identical to a two-stage Runge-Kutta method with implicit residual smoothing (Jameson and Baker, 1983). Note that the implicit residual smoothing technique will be discussed in Section 4.5.3.

Within the last two decades, a considerable amount of work has been done to improve the performance of multi-stage methods (Jameson, 1985) and they have been used extensively by a number of different investigators (e.g., Kunz and Lakshminarayana, 1993; Arnone and Swanson, 1993; Turner and Jennions, 1993). For the purpose of improving the efficiency of the original Dawes code, the present author implemented a state-of-the-art multistage explicit time-marching scheme. This scheme is described below.

As mentioned earlier, the time-marching algorithm that was implemented in the Dawes code belongs to the Runge-Kutta (R-K) class of time-integration schemes. Unlike the standard R-K integration methods which were intended for temporal accuracy, the class of multistage methods which are used in steady-state solvers are designed to have the required stability and temporal damping characteristics for rapid convergence. Swanson and Turkel (1997) have described in detail various multistage schemes which are in use in present-day steady-state solvers and they all follow the same general formulation. An m -stage explicit Runge-Kutta scheme may be expressed as

$$\mathbf{Q}^{(0)} = \mathbf{Q}^n$$

$$\begin{aligned}
\mathbf{Q}^{(1)} &= \mathbf{Q}^n - \alpha^{(1)} \Delta t \mathbf{R}^{(0)} \\
&\vdots \\
\mathbf{Q}^{(k)} &= \mathbf{Q}^n - \alpha^{(k)} \Delta t \mathbf{R}^{(k-1)} \\
&\vdots \\
\mathbf{Q}^{(m)} &= \mathbf{Q}^n - \alpha^{(m)} \Delta t \mathbf{R}^{(m-1)} \\
\mathbf{Q}^{n+1} &= \mathbf{Q}^{(m)}
\end{aligned} \tag{4.36}$$

where the superscripts n and k denote, in order, the time level $n\Delta t$ and the stage number.

The residual function at stage k is given by

$$\mathbf{R}^{(k)} = \frac{1}{\Omega} \left(\mathcal{L}_C \mathbf{Q}^{(k-1)} + \mathcal{L}_{\mathcal{PD}} \mathbf{Q}^{(0)} + \sum_{r=1}^k \gamma_{kr} \mathcal{L}_{\mathcal{AD}} \mathbf{Q}^{(r-1)} \right) \tag{4.37}$$

The coefficients $\alpha^{(k)}$ and the weighting factors γ_{kr} are chosen to give optimal hyperbolic and parabolic stability limits, respectively.

For the Dawes code, a four-stage Runge-Kutta scheme was used with the following time-stepping parameters and weighting factors

$$\begin{array}{lllll}
\alpha^{(1)} = \frac{1}{4} & \gamma_{11} = 1.0 & \gamma_{21} = 0.0 & \gamma_{31} = 0.0 & \gamma_{41} = 0.0 \\
\alpha^{(2)} = \frac{1}{3} & & \gamma_{22} = 1.0 & \gamma_{32} = 1.0 & \gamma_{42} = 1.0 \\
\alpha^{(3)} = \frac{1}{2} & & & \gamma_{33} = 0.0 & \gamma_{43} = 0.0 \\
\alpha^{(4)} = 1 & & & & \gamma_{44} = 0.0
\end{array}$$

A number of salient points are to be noted. First, with these weighting factors, the artificial dissipation terms are evaluated at the first and second stages and frozen for the remaining stages. The physical diffusion terms are evaluated only at the first stage, as indicated in Equation 4.37. Both the present author and Swanson and Radespiel (1991) have observed that the single evaluation does not affect the stability of the scheme; therefore, computational effort is reduced.

This four-stage scheme has been used by a number of investigators (e.g., Swanson and Turkel, 1987; Arnone and Swanson, 1993) and it is significantly more efficient by comparison to the two-stage scheme used by Dawes.

Table 4.2 shows a comparison of the computational effort required per timestep for the four-stage scheme used by the present author and the original two-stage scheme. Also included in the table is a comparison of the CFL numbers (hyperbolic stability limits) for the two schemes.

Table 4.2: Comparison of Two- and Four-Stage Time-Stepping Schemes

Scheme	Number of Evaluations per Time Step			CFL
	Convection	Physical Diffusion	Artificial Dissipation	
Two-Stage	2	1	1	1
Four-stage	4	1	2	$2\sqrt{2}$

The results from Table 4.2 imply that, by using the four-stage R-K scheme, there is at least a 50% reduction in CPU time to reach a given convergence level. That is, the time step, as measured by the CFL number, is three times larger for the four stage scheme while there is less than a two-fold increase in the number of flux evaluations. Note that the CFL number limits quoted in Table 4.2 have been calculated from a stability analysis which was performed on a 1-D, linear-wave equation (Swanson and Turkel, 1997).

4.5 Convergence Acceleration Techniques

4.5.1 Introduction

Three numerical techniques have been used by Dawes to accelerate convergence to the steady-state solution. These techniques are

- 1) local time stepping,
- 2) implicit residual smoothing, and
- 3) multigrid method.

The multigrid-acceleration technique did not function properly for both the original and modified versions of the code. As a result, a description of the technique will not be presented; however, the remaining two techniques are described below.

4.5.2 Local Time Stepping

With local time stepping the solution at each cell is advanced by the maximum time step as allowed by the local stability criteria. This technique ensures that temporal disturbances (e.g., pressure waves) are propagated out of the computational domain in the shortest period of time. For viscous flow calculations, the local time step must be chosen to satisfy both the convective and diffusive stability limits. In the original code, Dawes did not incorporate a diffusive stability limit. Without this limit, the present author observed convergence deterioration for simulations that were performed with fine mesh densities. For the modified code, both stability criteria were required to be satisfied; therefore, the maximum allowable time step was specified as

$$\Delta t = \min(\Delta t_C, \Delta t_D) \quad (4.38)$$

where Δt_C and Δt_D represent the convective and diffusive time step limits, respectively.

The expressions for the convective and diffusive time step limits are derived from linear stability analyses performed on the hyperbolic (inviscid) and parabolic forms of Equation 4.1, respectively (Swanson and Turkel, 1997). Note that the parabolic form of the governing equation is obtained by eliminating the convective terms. The convective time step limit is given by

$$(\Delta t_C)_{i,j,k} = \text{CFL} \left(\frac{\Omega}{\lambda_I + \lambda_J + \lambda_K} \right)_{i,j,k} \quad (4.39)$$

where λ_I , λ_J and λ_K are the spectral radii of the flux Jacobian matrix for the inviscid governing equations and CFL is the Courant, Friedrichs and Lewy (CFL) number for the four-stage Runge-Kutta scheme. As noted earlier in Section 4.4.4, the spectral radii for the compressible and “incompressible” equations are calculated according to Equations 4.28 and 4.29, respectively. In the original Dawes code, the velocity scale used in the calculation of the time step was based on the stagnation speed of sound. As such, the time step did not account for locally varying wave speeds.

The diffusive time step limit is defined as

$$(\Delta t_D)_{i,j,k} = \frac{1}{K_d} \left(\frac{\Omega}{\lambda_I^v + \lambda_J^v + \lambda_K^v} \right)_{i,j,k} \quad (4.40)$$

where K_d is a diffusion number and it is analogous to the CFL number for the convective time step limit; it was set to 2.5 for this work. λ_I^v , λ_J^v and λ_K^v are the spectral radii of the flux Jacobian matrix for the parabolic form of the governing equations. The spectral radius for the I direction is approximated by

$$(\lambda_I^v)_{i,j,k} = \frac{\Omega_{i,j,k}}{\mu_{i,j,k} (\mathbf{A}_I \cdot \mathbf{A}_I)_{i+\frac{1}{2},j,k}} \quad (4.41)$$

The spectral radii for the J- and K-directions are calculated in a similar manner.

4.5.3 Implicit Residual Smoothing

Implicit residual smoothing is a commonly used technique for extending the stability bounds of the Runge-Kutta class of time-stepping schemes. The original scheme was developed by Jameson (1983) and since that time a number of different variations have followed. An excellent summary on the evolution of this technique is provided by Swanson and Turkel (1997).

At the same time that Jameson was developing the implicit residual smoothing technique, Dawes was working on an implicit step for his explicit time-marching scheme. Dawes was motivated to include an implicit step for the same reason Jameson developed implicit residual smoothing; that is, to help remove stability limitations from the explicit part of the scheme. The implicit step which is described in detail by Dawes (1985), was based on a preprocessed form of the Beam-Warming (Beam and Warming, 1978) implicit scheme.

The present author compared the implicit algorithm developed by Dawes to the original implicit smoothing technique by Jameson and it was found that the two schemes are identical. As such, the implicit routine as developed by Dawes was used to perform residual smoothing in the modified version of the code. A description of this technique is given below.

For three-dimensional flows, the implicit residual smoothing is applied in the following form,

$$(1 - \varepsilon_I \delta_{II})(1 - \varepsilon_J \delta_{JJ})(1 - \varepsilon_K \delta_{KK}) \overline{\mathcal{R}}_{i,j,k}^{(k)} = \mathcal{R}_{i,j,k}^{(k)} \quad (4.42)$$

where the quantities δ_{II} , δ_{JJ} and δ_{KK} are the second-difference operators in the I, J and K directions, respectively. The second-difference in the I direction acting on $\overline{\mathcal{R}}_{i,j,k}^{(k)}$ is defined as

$$\delta_{II} \overline{\mathcal{R}}_{i,j,k}^{(k)} = \overline{\mathcal{R}}_{i-1,j,k}^{(k)} - 2\overline{\mathcal{R}}_{i,j,k}^{(k)} + \overline{\mathcal{R}}_{i+1,j,k}^{(k)} \quad (4.43)$$

The second-differences in the J- and K-directions are defined in a similar manner. The quantities ε_I , ε_J and ε_K are smoothing coefficients which will be described in further detail below. The unsmoothed residual at stage k of the Runge-Kutta integration scheme, $\mathcal{R}_{i,j,k}^{(k)}$, is defined by

$$\mathcal{R}_{i,j,k}^{(k)} = \alpha^{(k)} \Delta t_{i,j,k} \mathbf{R}_{i,j,k}^{(k)} \quad (4.44)$$

where $\mathbf{R}_{i,j,k}^{(k)}$ is given by Equation 4.37. $\mathcal{R}_{i,j,k}^{(k)}$ is the smoothed residual at stage k and it is determined by solving three tridiagonal systems of equations, one for each of the coordinate directions. Following the recommendation of Jameson and Baker (1983), the residual smoothing was evaluated only on the even stages of the four-stage Runge-Kutta scheme.

The smoothing coefficients which appear in Equation 4.42 are the parameters that control the stability of the scheme. In the Jameson scheme, these coefficients were user-defined constants and provided that they were made sufficiently large, stability was guaranteed for any time step size. The expression for unconditional stability was

$$\varepsilon \geq \frac{1}{4} \left[\left(\frac{\text{CFL}}{\text{CFL}^*} \right)^2 - 1 \right] \quad (4.45)$$

where CFL and CFL* are the CFL numbers for the smoothed and unsmoothed schemes, respectively.

Although the time-stepping scheme can be made stable for any time step size (i.e., any CFL number), the level of temporal damping is reduced as the CFL number is increased. As such, the optimal time step size, from the view point of maximizing convergence rate, occurs only at relatively modest values of CFL numbers. For all the simulations which were performed during the course of this work, the optimum CFL-to-CFL* ratio was approximately 2.25. With this ratio, the smoothing coefficients were equal to about unity.

With implicit residual smoothing, Swanson and Turkel (1997) have shown that the stability bounds (i.e., CFL number) can be increased by a factor of two.

4.6 Boundary Conditions

4.6.1 Introduction

Once a computational domain has been constructed, conditions must be imposed at the physical and computational boundaries to uniquely define the physical problem. A variety of boundary conditions are available for use in the Dawes code. At the computational boundaries, inflow, outflow and periodic conditions may be specified. The physical boundary conditions consist of both slip³ and no-slip solid surfaces. Each of these conditions is described later in this section. First, a brief discussion is given on the flexibility of handling boundaries with the modified version of the code. Following this, the numerical approach for imposing boundary conditions is described.

As mentioned in the introduction to this chapter, the original Dawes code was written for the simulation of rotating turbomachinery. Accordingly, there were limitations on the geometry of the physical domain and the general arrangement of the computational domain. As an example, with reference to Figure 4.1, the boundaries for the first and last meridional mesh surfaces were required to consist of the blade suction, pressure and periodic surfaces. Also, the first and last streamwise surfaces were required to be the hub and casing walls. With these constraints, the original code could not be used to simulate more generalized flow geometries, even ones as simple as flow over a flat plate or flow over a backward-facing step.

To address this problem, the present author restructured the Dawes code to allow

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³Boundary condition implemented by the present author.

complete flexibility in imposing the number, location and type of boundaries. In the modified version of the code, the physical and computational boundaries may be located on any mesh surface or portion thereof. A solid body may be modelled within the computational domain by identifying a volume of contiguous cells as “dead” cells around which solid surface boundaries are defined. The turbine airfoil shown in Figure 4.3 is an example of a solid body which has been modelled within the computational domain.

4.6.2 Numerical Approach

The modified version of the Dawes code employs the phantom or ghost cell concept to impose boundary conditions, as shown in Figure 4.7 for a two-dimensional mesh. A phantom

Figure 4.7: Two-dimensional mesh with phantom cells.

cell is a fictitious cell that is contiguous with a boundary cell and is located outside the flow field. Flow variables are stored at the centre of the phantom cells and fluxes at the boundary

faces are calculated in a similar manner to internal cells. The values of the flow variables within the phantom cells are specified to satisfy the imposed boundary condition. The main advantage of the phantom cell approach is that the spatial discretization technique does not need to be altered for near-boundary cells.

4.6.3 Inflow, Outflow and Periodic Boundaries

According to the method of characteristics, the number of independent flow variables to be imposed at the inlet and outlet phantom cells must equal the number of characteristics that enter the computational domain. Those characteristics that leave the computational domain must be extrapolated from the interior (Rizzi and Eriksson, 1984).

Subsonic inflows have four incoming characteristics requiring four flow variables to be imposed. These four variables can be specified in two ways. First, total temperature, total pressure and two flow angles are fixed. The two flow angles, which are denoted as pitch and yaw, each define velocity components in the x-y and x-z planes, respectively (refer to Figure 4.4). For cascade simulations, the pitch and yaw angles give the flow tangential and spanwise components of velocity, respectively. The fifth variable, static pressure, is linearly extrapolated from the interior of the domain. Alternatively, total temperature and the three velocity components can be specified.

At subsonic outflow boundaries, the only variable that needs to be imposed is the static pressure; all remaining flow variables are linearly extrapolated from the interior of the computational domain. For supersonic outflows, all flow variables are extrapolated from the interior.

In both the original and modified versions of the code, the periodic boundaries must be conformal. That is, a cell surface which has been identified as a periodic boundary must

have an identical “twin” surface which is located one period away. The values of the flow variables in a phantom cell adjacent to periodic boundary are prescribed from the periodic interior cell values.

4.6.4 Solid Surface Boundaries

Solid surface boundaries may be treated as either slip (inviscid) or no-slip walls. Inviscid surfaces must satisfy the condition of no convective flux through the boundary. Mathematically, this condition is satisfied by setting the velocity component normal to the wall to zero. Numerically, the phantom cell velocity components are prescribed so that the cell-face velocities satisfy the impermeability condition. The normal momentum equation is applied at the wall to update the phantom cell pressure. That is,

$$\left(\frac{\partial p}{\partial n}\right)_w = \left(\frac{\rho \mathbf{V} \cdot \mathbf{V}}{r}\right)_w \quad (4.46)$$

where r is the local radius of curvature and the subscript w refers to wall quantities. The density at the wall is extrapolated from the interior of the domain.

Viscous solid surfaces must simultaneously satisfy the no-slip and the no-convective flux conditions. These conditions result in a specification of zero velocity at the surface. The phantom cell velocity components are defined by an antisymmetric reflection of the known velocity at the cell centre adjacent to the wall.

An estimate of the wall shear stress is necessary to calculate the diffusive flux at a viscous solid surface. For laminar flows, the wall shear stress is derived from the known fluid viscosity and the velocity gradient at the wall. The velocity gradient, in turn, is approximated by the known velocity at the wall and the known velocities in cells adjacent to the wall. For turbulent flows, the code calculates the shear stress in one of two ways.

If the mesh is sufficiently fine so that the first cell centre lies within the laminar sublayer ($y^+ \leq 4$) then the shear stress calculation follows the same procedure as was used for laminar flows. If $y^+ > 4$ then the wall shear stress is calculated with wall functions (e.g., Launder and Spalding, 1974). The numerical implementation of wall functions in the Dawes code is detailed by Islam (Islam, 1999). In the present study, all simulations were performed with grid densities that were fine enough so that wall functions were not required.

The assumption of zero normal pressure gradient at the wall is used to extrapolate the phantom cell pressure⁴.

For compressible flows, the temperature at the wall may be specified or the adiabatic condition may be imposed; in the latter case, the normal temperature gradient is zero.

4.7 Baldwin-Lomax Turbulence Model

The Dawes code solves the Reynolds-averaged Navier-Stokes equations, as discussed in Section 4.3. These equations have the same form as the laminar flow equations except that the stress tensor includes additional unknown terms, often referred to as Reynolds stresses. Closure for this system of equations is obtained by devising relationships between the Reynolds stresses and known flow variables (White, 1991).

In the Dawes code, closure is achieved with the Boussinesq eddy-viscosity approximation which states that the Reynolds shear stresses are computed as the product of an eddy viscosity and mean-flow field gradients. With this approximation, the Reynolds stresses are eliminated from the averaged Navier-Stokes equations; the effects of turbulence are

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⁴The assumption of zero normal pressure gradient at the wall is a reasonable assumption for high Reynolds number flows.

simulated with the eddy viscosity, μ_t . The resulting effective fluid viscosity becomes

$$\mu = \mu_l + \mu_t \quad (4.47)$$

where the subscripts l and t refer to laminar and turbulent values, respectively. For turbulent flows with non-negligible heat transfer, an effective thermal conductivity must also be specified

$$k = k_l + k_t \quad (4.48)$$

where the turbulent thermal conductivity k_t is given by

$$Pr_t = \frac{C_p \mu_t}{k_t} \quad (4.49)$$

Pr_t is the turbulent Prandtl number, and it is assumed to have a value of 0.9 (White, 1991).

During the development of the BTOB3D code, Dawes had considered a number of turbulence models with levels of complexity varying from simple mixing-length models to two-equation transport models. He concluded that the limited ability of two-equation models to predict flow phenomena such as shock-boundary layer interaction and regions of gross recirculation did not warrant the increase in computational expense. Dawes preferred the simplicity of the Baldwin and Lomax (1978) mixing-length model notwithstanding that it was unable to capture upstream history effects.

Since the development of the Dawes code, the Baldwin-Lomax turbulence model has become the most extensively used turbulence model in the turbomachinery community. This was evident from the computations that were presented at the 1994 ASME/IGTI Turbo Expo “CFD Code Assessment in Turbomachinery” forum. In this forum, code developers and users performed blind calculations of a transonic fan rotor with tip-clearance (Sjolander, 1994). Twelve researchers from academia and NASA research laboratories presented their results; nine of which used the Baldwin-Lomax turbulence model (Chima, 1995). Further

evidence of the model's popularity comes from its widespread use in industry (e.g., Emunds *et al.*, 1999; Woinowsky-Krieger *et al.*, 1999).

Despite the popularity of this mixing-length model, very few studies have been conducted to evaluate the loss prediction ability of the Baldwin-Lomax turbulence model in a state-of-the-art Navier-Stokes code, particularly at off-design incidence. The computational part of this work was conducted to address this issue.

The form of the Baldwin-Lomax turbulence model which was used in this work is similar to that described in the original paper. Therefore, only a brief description of the model is presented here. Also, some of the numerical implementation details will be described as they are often omitted in the open literature.

Most algebraic turbulence models, such as the Cebeci-Smith model (Cebeci and Smith, 1974), depend on accurately locating the edge of the boundary layer to determine turbulent length and velocity scales. The Baldwin-Lomax model has been designed to eliminate this dependence: a feature that is particularly attractive for turbomachinery flows because of the inherent difficulty in locating the outer edge of the boundary layer. In the Baldwin-Lomax turbulence model, the length scale is determined from a function which is created from the product of vorticity and distance normal to the nearest wetted surface. In effect, the distribution of vorticity is used to eliminate the need to define an outer edge (Baldwin and Lomax, 1978).

The Baldwin-Lomax turbulence model is a two-layer model; that is, the eddy viscosity is given by different expressions in each layer.

For the inner layer, the Prandtl-Van Driest formulation is used

$$(\mu_t)_{inner} = \rho l^2 |\omega| \quad (4.50)$$

where the mixing length, l , is given by

$$l = ky \left[1 - e^{(-y^+/A^+)} \right], \quad (4.51)$$

y is defined as the normal distance to the nearest surface and A^+ is usually given a value of 26. A number of authors have used the distance along grid-lines rather than normals to surfaces to calculate y (e.g., Turner and Jennions, 1993). In the modified version of the code, normal distances are provided as part of the mesh geometry input file. This input file is created from a grid generation program developed by Zavitz (2000).

In the original Dawes code, the Van Driest damping function, $1 - e^{(-y^+/A^+)}$, was omitted. This function is necessary for meshes that are constructed with near-wall cell centres that have values of $y^+ < 50$. The damping function was included in the modified code because relatively fine mesh densities were required to achieve grid independence, as will be discussed in Chapter 5.

The normalized wall distance, y^+ , which appears in Equation 4.51, is defined as

$$y^+ = \frac{y \sqrt{\rho_w \tau_w}}{\mu_w} \quad (4.52)$$

where the subscript w denotes wall values. The magnitude of the vorticity, ω is defined as

$$|\omega| = \sqrt{\left(\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} \right)^2 + \left(\frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y} \right)^2 + \left(\frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z} \right)^2} \quad (4.53)$$

In the outer layer, the eddy viscosity is given by

$$(\mu_t)_{outer} = K C_{cp} \rho F_{wake} F_{Kleb} y \quad (4.54)$$

where the Klebanoff intermittency factor, F_{Kleb} is defined as

$$F_{Kleb} = \left[1 + 5.5 \left(\frac{C_{Kleb} y}{y_{max}} \right)^6 \right]^{-1} \quad (4.55)$$

and

$$F_{wake} = \min \left(\frac{y_{max} F_{max}}{\frac{C_{wk} y_{max} U_{dif}^2}{F_{max}}} \right) \quad (4.56)$$

The second expression in Equation 4.56 is commonly neglected because it has been observed to cause convergence problems (Heidegger *et al.*, 1999). Both versions of the BTOB3D code calculate F_{wake} with the first expression only.

The quantities F_{max} is the maximum value of the function

$$F(y) = y|\omega| \left[1 - e^{(-y^+/A^+)} \right] \quad (4.57)$$

and y_{max} is the value of y at which F_{max} occurs. y_{max} and F_{max} constitute the length and velocity scales for the wall shear layer, respectively. The function, $F(y)$, was constructed to produce a maximum at a location within the layer that correlates with the boundary layer thickness. It should be noted that a number of authors have observed multiple peaks in the function $F(y)$ (e.g., Swanson and Turkel, 1997; Chima *et al.*, 1993). For low Reynolds flows with favourable pressure gradients, Chima *et al.* (1993) showed that spurious maxima can occur at the edge of the viscous sublayer. Using a peak within the viscous sublayer or buffer layer would produce an incorrect outer layer length scale. To eliminate this problem, F_{max} is taken as the maximum value of $F(y)$ for values of $y^+ > 30$. An upper limit on the turbulent length scale is specified to avoid nonphysically large eddy viscosities. This limit is normally set to 10% of an appropriate reference length scale (e.g., axial chord).

The model constants are defined as

$$\begin{array}{lll}
A^+ & = 26 & C_{Kleb} = 0.3 & K = 0.0168 \\
k & = 0.41 & C_{cp} = 1.6 &
\end{array}$$

Having defined expressions for the eddy viscosity in the inner and outer layers, the composite distribution for a boundary layer is given by

$$\mu_t = \min [(\mu_t)_{inner}, (\mu_t)_{outer}] \quad (4.58)$$

For wake flows, the eddy viscosity distribution is given by the outer formulation and the Van Driest damping factor is set to unity (Baldwin and Lomax, 1978). In the original and current versions of the code, a nonstandard approach was followed for modelling wakes. That is, the values of F_{max} and y_{max} were taken from the suction and pressure surfaces near the trailing-edge; Turner and Jennions (1993) followed the same approach.

To apply the Baldwin-Lomax formulation in three-dimensions, the eddy viscosity for a given cell-centre is based on the length scale associated with the nearest no-slip surface.

4.8 Conclusions

The original BTOB3D code developed by Dawes (1985) has been substantially modified. Many of the modifications were made so that the code could be used to simulate the incompressible linear cascade experiment that was conducted in the present study. Additional modifications were made to improve the solution accuracy and to reduce convergence times. Furthermore, the modified code is no longer restricted for use with flows that are attached or mildly separated flows since the thin-layer assumption was removed. The modified version of the code has been described in detail in this chapter. Chapter 5 presents the results of the computations from the modified code.

Chapter 5

The Influence of Leading-Edge Geometry on Profile Losses

5.1 Introduction

As discussed in Chapter 2, there is very little data on the effects of leading-edge geometry on profile and secondary losses both at design and off-design incidence. Furthermore, the most recent design (Kacker and Okapuu, 1982) and off-design correlations (Moustapha *et al.*, 1990) for profile losses suggest that leading-edge geometry is an influential parameter only at off-design incidence. To verify the Kacker and Okapuu and Moustapha *et al.* correlations and to investigate further the influence of the leading-edge geometry on the profile losses as well as secondary losses for turbine airfoils, the present experimental study was initiated. The investigation was performed in the large-scale, low-speed wind tunnel using both the LS2 and LS3 test cascades described in Chapter 3. This chapter focuses on the profile losses whereas Chapters 6 through 8 examine the secondary losses.

In this chapter, the design-incidence results for LS2 and LS3 are presented first because they represent the baseline cases against which the results at off-design incidence are compared. The results at off-design incidence are then presented in Section 5.2.3. Following this, Section 5.3 presents a general discussion of the influence of leading-edge geometry on profile losses. Sections 5.3.3 and 5.3.4 discuss, in order, the importance of the leading-edge geometry for the prediction of profile losses at design and off-design incidence. A new off-design loss correlation is then presented in Section 5.3.5. Section 5.4 presents off-design loss predictions from a CFD code that is currently widely used in turbomachinery design, which employs a popular, simple and robust turbulence model. The predictions were compared with experimental results with the objective of determining the degree to which the code is capable of predicting off-design losses. To the author's knowledge, there have been very few published studies that have made these types of comparisons, notwithstanding that the tool continues to be widely used in the design environment. Finally, the last section presents the conclusions of this chapter.

5.2 Experimental Results

5.2.1 Sources of Data

The midspan measurements for the LS2 and LS3 cascades that are presented in this chapter were obtained in the linear cascade test section shown schematically in Figure 3.2. In the present study, three values of incidence were investigated: 0, +10 and +20 degrees.

The airfoil loading data for LS2 at design and off-design incidence were obtained from a study of the effects of axial velocity ratio (AVR) on the midspan aerodynamic behaviour of a turbine cascade (Rodger, 1992). AVR is a measure of the average streamtube convergence (or divergence) through the cascade; therefore, it provides an approximate measure of the

two-dimensionality of the flow. A value of unity indicates that the flow is approximately two-dimensional. For an incompressible flow, AVR is simply the ratio of the area-averaged outlet to inlet midspan axial velocities; that is,

$$AVR = \frac{\int_0^s (u_2)_{MS} dy'}{\int_0^s (u_1)_{MS} dy'} \quad (5.1)$$

where u_1 and u_2 are the inlet and outlet axial velocities, respectively; y' is the pitchwise distance; s is the airfoil spacing and the subscript MS refers to the airfoil midspan.

Rodger (1992) used an arrangement of endplates for controlling the flow convergence or divergence. With these endplates, the effective aspect ratio of the cascade was reduced to approximately 1.0 (from 1.25), and the thickness of the inlet endwall boundary layer was substantially decreased. The loading measurements for LS3 were obtained in the present study, and therefore, correspond to somewhat higher values of endwall boundary layer thickness.

The loss coefficients presented in this chapter were obtained from several sources, these include: LS2 measurements from Tremblay (1989) and Rodger (1992), LS3 measurements from Whitehouse (1993) and Abbott (1993), as well as measurements for both cascades from the present study. The results from the earlier studies are presented here for a few reasons. First, they include some cases that were not examined in the present study, most notably, cases at negative incidence. And second, for cases where repeat measurements have been made, the scatter in the losses indicates the uncertainty present in the measurements.

In the studies of Rodger (1992) and Whitehouse (1993), the AVR was varied at each incidence to provide data over a range which included $AVR = 1.0$. The loss coefficients for these cases that are used here are the interpolated values at $AVR = 1.0$. In the present study for 0 and +10 degrees of incidence, the results are as two-dimensional as

possible since AVR was essentially unity for both cascades, at least to within the limits of the experimental uncertainty ($\partial AVR = \pm 0.025$). At +20 degrees of incidence, the axial velocity ratios for both LS2 and LS3 were considerably greater than unity, (i.e., $AVR = 1.05$ and 1.04 , respectively). Because the losses quoted from the studies of Rodger (1992) and Whitehouse (1993) were values interpolated to $AVR = 1$, any observed difference in the loss results between the present and earlier studies are due to differences in AVR and/or measurement uncertainty.

As noted earlier, the normal turbulence intensity at the cascade inlet is about 0.4% and all measurements presented in this chapter were obtained under these conditions. The influence of inlet turbulence intensity has been examined by Whitehouse (1993) with the LS3 cascade. The turbulence intensity was increased to about 1.5% using a turbulence-generating grid (Mahallati, 2003), but this was found to have very little effect on the loss behaviour of this airfoil: at both design and off-design incidence, differences in measured losses were close to the uncertainty in the measurements.

5.2.2 Results at Design Incidence

Figure 5.1 shows the measured variations in the midspan airfoil pressure as well as the corresponding values of the axial velocity ratio (AVR) for both LS2 and LS3. It can be seen from the figure that LS2 is slightly more forward loaded than LS3. This is consistent with the fact that LS2 has a higher stagger angle than LS3 (23.1 degrees versus 21.6 degrees, as shown in Figure 3.3). The suction peak has moved forward from about 55% C_x for LS3 to 45% C_x for LS2 and the magnitude of the suction peak has increased for LS2. A change in the slope of the pressure distribution at about 80% C_x is observed for LS3 and appears to correspond to the reattachment point of a small laminar separation bubble which starts at about 60% C_x . Results from the suction surface oil flow visualization, which are presented

Figure 5.1: Measured midspan surface pressure distributions for LS2 and LS3 at design incidence.

in the next chapter, also indicate the presence of this separation bubble. In contrast, the deceleration is quite uniform for LS2 on the aft portion of the suction surface. This may indicate that the size of the bubble is smaller or perhaps non-existent for LS2. Tremblay (1989) and Rodger (1992) performed surface flow visualization for LS2 and their findings seem to be in agreement with the measured surface static pressures; that is, transition seems to occur naturally over a distance from about 60 to 80% C_x .

Finally, both profiles experience a leading-edge velocity spike on the suction side of the airfoil, although in the measured results (Figure 5.1) this was evident only for LS3. Figure 5.2 shows the suction-surface velocity distributions for both profiles as predicted

Figure 5.2: Predicted flow around leading edge at the design incidence.

using a blade-to-blade potential flow code developed by Kind (1973), and as can be seen, the leading-edge velocity spike is stronger for the LS3 profile than for the LS2 profile. The difference in the overspeeds seems to be related to the discontinuity in curvature at the point where the leading-edge circle meets the rest of the profile (i.e., the blend point), as will be discussed later.

From Table 5.1, it can be seen that the measured and mixed-out profile losses for LS3 are approximately 20% larger than those for LS2. Although the absolute difference between the two loss coefficients is quite small, the measured values are consistent, within ± 0.002 , with

Table 5.1: Profile losses at the design incidence for LS2 and LS3.

FLOW QUANTITY	LOCATION	CASCADE	
		LS2	LS3
Y_{profile}	OUTLET (Traverse plane: 40% Cx downstream of trailing edge.)	0.021	0.025
	MIXED-OUT	0.022	0.026

those measured by previous investigators (e.g., for LS2, Goobie (1989), Tremblay (1989), and Rodger (1992), and for LS3, Whitehouse (1993) and Abbott (1993)). Therefore, the difference in the midspan losses appears to be genuine. It is noteworthy that the predicted losses from the Navier-Stokes solver are identical for both profiles, as discussed in Section 5.4.3. This indicates that the measured loss difference may be related to boundary layer transition since the computations were performed assuming turbulent flow from the leading edge. Further discussion of the loss behaviour at the design incidence is given in Section 5.3.3.

5.2.3 Results at Off-Design Incidence

Figure 5.3 shows the measured profile loss coefficients for the two cascades as a function of incidence. The corresponding predictions using the Ainley and Mathieson (AM) and the Kacker and Okapuu/Moustapha *et al.* (KOM) loss systems are also shown. The AM values are essentially the same for the two airfoils since the only relevant parameter which varied was the maximum thickness-to-chord ratio and it changed by only a small amount. While the KOM system matches the data somewhat better than AM near design incidence, KOM significantly under-predicts the losses observed for LS2 and LS3 at positive incidence.

Figure 5.3: Variation of midspan loss coefficients with incidence.

A number of points are noteworthy concerning the scatter amongst the cases where there are repeat data points. From Figure 5.3, it can be seen that the loss coefficients at 0 and +10 degrees of incidence obtained from the present study and the earlier studies are in excellent agreement. At +20 degrees of incidence, the losses measured in the present study for LS3 were in satisfactory agreement with the results of Abbott (1993). On the other hand, the present results for both LS2 and LS3 were considerably higher than those quoted in the studies of Whitehouse (1993) and Rodger (1992), as can be seen in the figure. This result was unexpected given that the AVR values were larger in the present study. That is, when separation is present the imposed favourable pressure gradient, which is implied by $AVR > 1.0$, would tend to shift the separation point closer to the trailing edge. This

would lead to lower profile losses than would be the case for more closely two-dimensional flow. However, this is clearly not the case with the present results. It is conceivable that the difference in the loss coefficients is simply due to measurement uncertainty. When trailing-edge flow separation is present, the uncertainty in losses is significantly higher ($\partial Y_p = \pm 0.04$) than for cases with attached flow. It is worth noting that Mahallati (2003) estimated similar uncertainty intervals for cases where trailing-edge flow separation was present.

At +20 degrees of incidence, the measurements made in the present study show that LS3 has higher losses than LS2. Again, it is possible that the difference is simply the result of measurement uncertainty. However, considering the results of this study and those of the earlier studies, there seems to be no indication that a larger leading-diameter reduces the sensitivity of the airfoils to off-design incidence, as suggested by Moustapha *et al.* (1990). This indicates a need to re-examine the influence of leading-edge geometry on off-design profile losses.

For use in later discussion, some additional airfoil pressure measurements are presented here.

Figure 5.4 shows the variation in the airfoil pressure distributions at positive incidence for LS3. As incidence increases, a velocity overshoot appears on the suction side at the leading edge and, whereas at design incidence the flow is accelerated up to about midchord, at higher values of incidence the flow is decelerated along the full length of the suction surface. Figure 5.5 shows the pressure distributions for the two airfoils at +20 degrees of incidence. The mismatch in static pressures along most of the suction surface is probably the result of the difference in the axial velocity ratios for the two sets of measurements. However, the difference in the suction peaks near the leading edge of the airfoils seems to be too large to explain in this way. Nor does the uncertainty in the incidence angle seem adequate to explain the difference. In fact, the suction peak would have been expected to

Figure 5.4: Effect of positive incidence on airfoil pressure distributions for LS3.

be milder for LS3 because of its larger leading-edge diameter. Instead, some other factor is evidently amplifying the suction peak.

As already noted, the agreement between the present measurements and the Moustapha *et al.* correlation was somewhat poor. This prompted a re-examination of the correlation and the results of this are presented next.

Figure 5.5: Comparison of airfoil pressure distributions at +20 degrees of incidence.

5.3 Design and Off-Design Profile Loss Correlations

5.3.1 Introduction

The most recent design-point profile loss correlation (Kacker and Okapuu, 1982) indicates that the leading-edge geometry does not influence profile losses. The good agreement observed between the measured design-point losses and those predicted by Kacker and Okapuu seem to confirm that profile losses are insensitive to leading-edge geometry, at least at the design incidence. In contrast, the results at off-design incidence show significant discrepancies with the Moustapha *et al.* correlation. As will be shown later, the Moustapha

et al. correlation for off-design profile losses has been modified to include the effect of the leading-edge wedge angle. This resulted in a noticeable improvement in agreement with the present experimental data as well as with the data from the literature for which the wedge angle was available. The next section examines the physical relationship between the leading-edge geometry and the profile losses. Sections 5.3.3 and 5.3.4 attempt to explain, in order, why (1) losses at the design point are insensitive to leading-edge geometry, and (2) why the wedge angle should appear in the off-design correlation. Finally, Section 5.3.5 presents the new off-design loss correlation.

5.3.2 Physical Relationship between Losses and Leading-Edge Geometry

The importance of the details of the leading-edge geometry has been recognized for some time. For example, Stow (1985) discusses the redesign of the leading edge of a turbine airfoil to remove a “spike”, or local overshoot, in velocity very close to the leading edge. Stow does not explicitly identify the origin of the spike, but it appears to be associated with the blend point where the leading-edge circle meets the rest of the suction-side profile. It is plausible that a pressure disturbance would occur at this point since there must be a discontinuity in curvature there.

In his detailed examination of the boundary layer on a turbine rotor blade, Hodson (1985) likewise noted the occurrence of a sharp velocity overshoot at the blend point. As Hodson observed, this overshoot may often be missed in airfoil loading measurements since it is of short extent and can easily fall between static taps. This is almost certainly the case for the present measurements with LS2 and LS3. Hodson found that the bubbles quickly reattached since the velocity overshoot occurred in a region of overall favourable pressure gradient. In fact, he concluded that transition was not necessarily completed in the bubbles and that the subsequent favourable pressure gradient could in any case cause

relaminarization of the boundary layer.

More recently, Walraevens and Cumpsty (1995) specifically examined the leading-edge flow using a plate mounted in a wind tunnel. The incidence and the freestream turbulence were varied and both circular and elliptic leading edges were considered. Separation occurred at the same point, a short distance upstream of the blend point, under most conditions. The authors explicitly link the likelihood of separation to the magnitude of the discontinuity in curvature at the blend point. Thus, the appearance of separation was delayed to a higher angle of incidence for the elliptic leading edge because of its milder discontinuity in curvature. Raising the level of freestream turbulence shortened the separation bubble somewhat but, for the circular leading edge in particular, did not prevent the appearance of the separation bubble or change the location of the separation. Boundary-layer measurements made downstream of the bubbles showed a substantial increase in momentum thickness as the extent of the bubble increased. This loss of momentum would be due to the mixing that occurs in the free shear layer as well as the fact that the layer reattached turbulent. It should be noted that for all of the cases considered, the bubbles were reattaching in a region of adverse pressure gradient.

The profile losses are taken to include the loss production in the airfoil-surface boundary layers as well as at the trailing edge. With some simplifying assumptions, Denton (1993) relates the profile losses to the boundary layer parameters at the trailing edge and the trailing edge conditions as follows:

$$Y = \frac{2\theta}{w} + \left(\frac{\delta^* + t}{w} \right)^2 - \frac{C_{Pb}t}{w} \quad (5.2)$$

where t is the trailing-edge thickness and w is the passage width. The first term accounts for the direct loss production in the boundary layers. The momentum thickness in this term is particularly influenced by the location of boundary-layer transition as well as by

the mixing at the edge of any separation bubbles which may be present. The second term is the sudden-expansion loss at the trailing edge, where the blockage associated with the boundary-layer displacement thickness contributes to the effective change in area. The final term accounts for the retarding force which would be exerted by a low base pressure. The influence of the leading-edge geometry on the profiles losses should thus be explained mainly through its influence on the boundary-layer parameters at the trailing edge. This influence would be expected to make itself felt mainly up to the onset of trailing-edge separation, at which point the last two terms begin to dominate the losses. The boundary layers on the suction surface of turbomachinery airfoils are normally turbulent by the time they reach the trailing edge. Thus, although the onset of separation is influenced by the thickness of the incoming boundary layer, the angle of incidence at which separation begins should be determined mainly by the airfoil pressure distribution. Therefore, any conclusions about the influence of the leading-edge geometry on losses are expected to hold mainly for negative incidence and for positive incidence up to the onset of significant trailing-edge separation.

In the light of Equation 5.2 and the earlier experimental studies, the following picture is proposed for LS2 and LS3.

5.3.3 Loss Behaviour at Design Incidence

At the design incidence, both profiles experience a leading-edge velocity spike as noted earlier, and from Figure 5.2, it is clear that the spikes are directly related to the two blend points. The blend point for LS3 is further from the leading edge, consistent with the smaller wedge angle. As noted earlier, the wedge angle was chosen to give roughly the same maximum thickness for the two profiles. However, on geometric grounds the smaller wedge angle will also tend to give a larger discontinuity in curvature at the blend point.

The approximate relationship between the wedge angle and the curvature discontinuity

Figure 5.6: Relationship between wedge angle and curvature discontinuity.

at the blend point is demonstrated in Figure 5.6 with the simple profile geometry of a leading-edge circle and a circular arc. Both cases are shown with the same leading-edge circle radius, R_1 and maximum thickness, T_{max} . The blend point is the point where the leading-edge circle meets the rest of the profile. For the geometry shown in Figure 5.6, as well as turbine airfoils in general, continuity in slope is forced at the blend point. The location of the blend point along the leading-edge circle is fixed by the wedge angle, We . Once the leading-edge radius, wedge angle and maximum thickness are specified and the constraint of slope continuity at the blend point is imposed then the radius of the circular arc, and thus, the magnitude of the curvature discontinuity at the blend point is known. Bearing this in mind and considering the two cases in Figure 5.6, it is evident that the

case with the smaller wedge angle will tend to produce a larger discontinuity in curvature between the leading-edge circle and the circular arc.

The larger curvature discontinuity for LS3 is also apparent from the stronger velocity spike in Figure 5.2. Immediately after the spike, the strong adverse pressure gradient may be sufficient to separate the boundary layer which will be laminar at that point. Normally transition would occur quickly in a laminar separation bubble, which then allows it to reattach. However, since the overall pressure gradient quickly reverts to strongly favourable, a transitional or even laminar separation bubble should be able to reattach here. Furthermore, even if transition had commenced it might well be followed by relaminarization due to the acceleration. The value of the acceleration parameter K for both cascades is in fact higher than the suggested critical value for relaminarization (e.g., Mayle, 1991) of about 3×10^{-6} for a short length after the velocity spike. In any event, as mentioned earlier, surface flow visualization conducted on the suction surface strongly suggested the presence of a laminar separation bubble on the rearward part of the suction surface for LS3. Evidently, this indicates that the boundary layer was still, or again laminar far aft of the leading edge. Thus, while separation bubbles might have been present in the leading-edge region, they were evidently very short and did not trigger transition. For both reasons, they would have had relatively little effect on the boundary-layer development, and therefore, on the losses generated on the airfoil surfaces. This may explain why the design incidence profile loss correlations (e.g., Ainley and Mathieson, 1951; Dunham and Came, 1970; Kacker and Okapuu, 1982; Craig and Cox, 1971; Traupel, 1977) do not need to include leading-edge geometry as a correlating parameter.

One salient point is noteworthy concerning the small difference in losses between LS2 and LS3 at the design incidence. Based on the previous arguments, it seems unlikely that the difference in leading-edge geometry can explain this loss difference. Instead some other

factor seems to be affecting the losses generated on the airfoil surfaces. As mentioned earlier, the surface flow visualization and the static pressure variation over the rearward part of the suction surface seem to indicate the presence of a laminar separation bubble for LS3. In contrast, transition seems to occur naturally for LS2 over the same region. As a result, one may expect higher loss production for LS3 due to the comparatively higher levels of mixing that would accompany the reattachment of the boundary layer. Evidently, this provides a plausible explanation for the slightly higher measured losses for LS3. However, it should be noted that it is entirely unclear why the boundary layer behaviour on the rearward part of the suction surface differs between LS2 and LS3.

5.3.4 Loss Behaviour at Off-Design Incidence

As discussed in the last section, the leading-edge overspeed seems to have very little effect on the losses at the design incidence. However, the picture changes as the incidence becomes positive. As seen from Figure 5.4, at positive incidence a normal overspeed develops at the leading edge. At +20 degrees of incidence a short separation bubble was also apparent from surface flow visualization, as can be seen in Figure 5.7. Furthermore, once the incidence exceeds about +10 degrees the pressure gradient is adverse over essentially the whole of the suction surface. As a result of the adverse pressure gradient, the leading-edge separation will begin to have a greater influence on the losses: the bubble will tend to be longer, with greater loss production, and transition will probably be necessary for its reattachment. In turn, a geometric factor which tends to make the bubble larger will also tend to increase the losses. Potential-flow calculations for the flow in the leading-edge region of both airfoils are shown in Figure 5.8 for a small range of positive incidence angles. It is seen that at a given incidence the velocity spike continues to be stronger for LS3 than for LS2, whereas one would have expected the larger diameter of LS3 to moderate the overshoot. The airfoil-surface measurements also showed consistently higher suction peaks for LS3, as indicated

Figure 5.7: Suction surface oil flow visualization showing leading-edge separation bubble at +20 degrees of incidence.

in Figure 5.4. Thus, it appears that the curvature discontinuity continues to influence the flow by amplifying the normal suction peak at the leading edge. For these reasons, one would likewise expect the curvature discontinuity to continue to influence the profile losses, at least up to the appearance of significant separation at the trailing edge.

For negative incidence, the discontinuity in curvature at the pressure-side blend point comes into play to encourage the occurrence of a pressure-side separation bubble. This bubble is expected to reattach under most circumstances, but the mixing at the edge of this and the likely turbulent state of the subsequent boundary layer will increase the loss production on the pressure surface. Again, these losses will be influenced by the severity of the discontinuity in curvature through its influence on the appearance and chordwise extent of the separation bubble.

Figure 5.8: Predicted flow around leading edge - Effect of incidence.

To summarize, it is believed, as has been suggested by a few other researchers, that the strength of the discontinuity in curvature at the leading-edge blend points will influence the trend in losses at off-design values of incidence. Obviously, it would be very inconvenient to have the curvature discontinuity appear explicitly in the profile-loss correlation. However, it is suggested that the value of the leading-edge wedge angle is a reasonable, approximate measure of the curvature discontinuity: larger values of wedge angle will tend to produce smaller discontinuities, and vice versa. Therefore, the use of the wedge angle in the revised correlation is interpreted as a convenient alternative to using the curvature discontinuity

directly.

5.3.5 New Off-Design Profile Loss Correlation

The new correlation is a revised version of the correlation of Moustapha *et al.* (1990). The latter correlation was based on data from 19 cascades. Unfortunately, the values of the leading-edge wedge angles were not quoted for all of these cascades. Therefore, the present correlation is based on a subset of 8 of the cascades from the earlier database together with data for LS2, LS3 and the three cascades of Perdichizzi and Dossena (1993), for a total of 13 cascades. Table 5.2 summarizes the cascade database, and as can be seen, data for a very wide range of wedge angles and ratios of leading-edge diameter-to-spacing are included. Those sources of data which are in the open literature are identified in the first column of the table.

To demonstrate the improvements achieved with successive correlations, the data are compared with the Ainley and Mathieson, the Moustapha *et al.*, and finally, the new correlation.

Figure 5.9 compares the incremental losses predicted by the Ainley and Mathieson correlation with the measured values, expressed as changes to the kinetic-energy coefficient. It should be noted that in all the figures presented in this section, the values of $|\Delta\phi_p^2|$ which exceeded 0.12 are plotted along the edges of the figures. Ainley and Mathieson's correlation was based on cascade data obtained in the 1940's and 50's whereas the measured values are mostly from cascades of recent design. As might be expected, the Ainley and Mathieson correlation does not reflect the improvements in turbine design and tends to over-predict the off-design losses.

Moustapha *et al.* correlated the incremental losses with the incidence, the channel

Table 5.2: Experimental data for off-design profile loss correlation.

Reference	i_{eff} [degrees]	We_1 [degrees]	d/s [-]	β_1 [degrees]	β_2 [degrees]	$\Delta\phi_p^2$ [-]
Aronov et al. (1975) Series B	20.0	34.5	0.100	40.0	65.5	0.0560
	10.0	34.5	0.100	40.0	65.5	0.0080
	0.0	34.5	0.100	40.0	65.5	0.0000
	-10.0	34.5	0.100	40.0	65.5	0.0040
	-20.0	34.5	0.100	40.0	65.5	0.0080
Aronov et al. (1975) Series B	15.0	12.5	0.039	-20.0	65.5	0.0150
	5.0	12.5	0.039	-20.0	65.5	0.0045
	0.0	12.5	0.039	-20.0	65.5	0.0000
Hodson & Dominy (1987b)	0.0	25.0	0.057	38.8	54.2	0.0000
	8.6	25.0	0.057	38.8	54.2	0.0211
	-20.3	25.0	0.057	38.8	54.2	0.0301
Vijayaraghavan & Kavanagh (1988)	-7.0	46.0	0.105	52.0	66.3	0.0019
	0.0	46.0	0.105	52.0	66.3	0.0000
	7.0	46.0	0.105	52.0	66.3	0.0028
Yamamoto & Nouse (1988)	0.0	29.0	0.267	49.8	63.5	0.0000
	-4.2	29.0	0.267	49.8	63.5	0.0034
	-11.7	29.0	0.267	49.8	63.5	0.0171
	-28.3	29.0	0.267	49.8	63.5	0.0238
	-45.8	29.0	0.267	49.8	63.5	0.0432
	-54.2	29.0	0.267	49.8	63.5	0.0560
LS2, Tremblay (1989)	-25.0	52.4	0.085	29.3	57.5	0.0040
	-20.0	52.4	0.085	29.3	57.5	0.0033
	-15.0	52.4	0.085	29.3	57.5	0.0034
	-10.0	52.4	0.085	29.3	57.5	0.0023
	-5.0	52.4	0.085	29.3	57.5	0.0000
	0.0	52.4	0.085	29.3	57.5	0.0000
	5.0	52.4	0.085	29.3	57.5	0.0045
	10.0	52.4	0.085	29.3	57.5	0.0050
LS2, Rodger (1992)	15.0	52.4	0.085	29.3	57.5	0.0091
	-20.0	52.4	0.085	29.3	57.5	0.0028
	0.0	52.4	0.085	29.3	57.5	0.0000
LS3, Whitehouse (1993)	20.0	52.4	0.085	29.3	57.5	0.1078
	-20.0	43.0	0.151	25.5	57.5	0.0014
	-10.0	43.0	0.151	25.5	57.5	0.0045
	0.0	43.0	0.151	25.5	57.5	0.0000
	5.0	43.0	0.151	25.5	57.5	0.0037
	10.0	43.0	0.151	25.5	57.5	0.0098
	15.0	43.0	0.151	25.5	57.5	0.0446
LS3, Abbott (1993)	20.0	43.0	0.151	25.5	57.5	0.1058
	-20.0	43.0	0.151	25.5	57.5	0.0027
	-10.0	43.0	0.151	25.5	57.5	0.0004
	0.0	43.0	0.151	25.5	57.5	0.0000
	10.0	43.0	0.151	25.5	57.5	0.0049
Perdichizzi & Dossena (1993) $s/C=0.58$	20.0	43.0	0.151	25.5	57.5	0.1648
	-60.0	57.1	0.281	13.9	75.5	0.0170
	-30.0	57.1	0.281	13.9	75.5	0.0080
	0.0	57.1	0.281	13.9	75.5	0.0000
	25.0	57.1	0.281	13.9	75.5	0.0010
	35.0	57.1	0.281	13.9	75.5	0.0090

Reference	i_{eff} [degrees]	We_1 [degrees]	d/s [-]	β_1 [degrees]	β_2 [degrees]	$\Delta\phi_p^2$ [-]
Perdichizzi & Dossena (1993) $s/C=0.73$	-60.0	57.1	0.223	13.9	75.5	0.0055
	-30.0	57.1	0.223	13.9	75.5	-0.0005
	0.0	57.1	0.223	13.9	75.5	0.0000
	25.0	57.1	0.223	13.9	75.5	0.0020
	35.0	57.1	0.223	13.9	75.5	0.0085
Perdichizzi & Dossena (1993) $s/C=0.87$	-60.0	57.1	0.187	13.9	75.5	-0.0020
	-30.0	57.1	0.187	13.9	75.5	-0.0010
	0.0	57.1	0.187	13.9	75.5	0.0000
	25.0	57.1	0.187	13.9	75.5	0.0125
LS2, Present data	0.0	52.4	0.085	29.3	57.5	0.0000
	10.0	52.4	0.085	29.3	57.5	0.0086
	20.0	52.4	0.085	29.3	57.5	0.1635
LS3, Present data	0.0	43.0	0.151	25.5	57.5	0.0000
	10.0	43.0	0.151	25.5	57.5	0.0073
	20.0	43.0	0.151	25.5	57.5	0.1840
Pratt & Whitney Canada in house	10.0	25.0	0.110	53.9	70.7	0.0087
	0.0	25.0	0.110	53.9	70.7	0.0000
	-10.0	25.0	0.110	53.9	70.7	0.0025
Pratt & Whitney Canada in house	11.1	18.5	0.055	60.9	67.5	0.0730
	7.1	18.5	0.055	60.9	67.5	0.0130
	2.0	18.5	0.055	60.9	67.5	0.0020
	-3.9	18.5	0.055	60.9	67.5	0.0030
	-8.9	18.5	0.055	60.9	67.5	0.0040
Pratt & Whitney Canada in house	7.1	8.5	0.055	65.9	67.5	0.0930
	3.1	8.5	0.055	65.9	67.5	0.0090
	-2.0	8.5	0.055	65.9	67.5	0.0020
	-7.9	8.5	0.055	65.9	67.5	0.0090
	-12.9	8.5	0.055	65.9	67.5	0.0180

acceleration and the ratio of the leading-edge diameter to the spacing using the incidence parameter

$$\chi' = \left(\frac{d}{s}\right)^{-1.6} \left(\frac{\cos \beta_1}{\cos \beta_2}\right)^{-2.0} [\alpha_1 - \alpha_{1,des}] .$$

The resulting correlation is shown in Figure 5.10 and the evaluation of its accuracy is shown in Figure 5.11. Comparison with Figure 5.9 shows that it is significantly more successful than the Ainley and Mathieson correlation, although it still noticeably over- or under-predicts the losses for some cascades.

The new correlation is shown in Figure 5.12 and the corresponding evaluation in Figure 5.13. The new incidence parameter takes the form

Figure 5.9: Evaluation of Ainley and Mathieson profile loss correlation.

$$\chi = \left(\frac{d}{s}\right)^{-0.05} We^{-0.2} \left(\frac{\cos \beta_1}{\cos \beta_2}\right)^{-1.4} [\alpha_1 - \alpha_{1,des}] .$$

As indicated, the influence of the leading-edge diameter is now much smaller and a significant dependence on wedge angle has been introduced. The revised correlation is seen to be considerably more successful in correlating the data. Even the data at large positive incidence seem to be quite well correlated. However, the rapid rate of increase in the losses makes the collapse of the data appear slightly better than it actually is. This is evident from Figure 5.13 where the largest differences between predictions and measurements are seen to occur at large values of incidence (that is, for large values of $|\Delta\phi_p^2|$). Nevertheless, the correlation must be regarded as excellent. For use in computations, the following

Figure 5.10: Moustapha *et al.* correlation for profile losses at off-design incidence.

polynomials have been fitted to the data:

$$\Delta\phi_p^2 = \frac{0.178}{1 + 540.6 e^{-1.263\chi}} - 3.3 \times 10^{-4} \quad (5.3)$$

for $\chi \geq 0$ and

$$\Delta\phi_p^2 = 1.358 \times 10^{-4} \chi^2 - 8.720 \times 10^{-4} \chi \quad (5.4)$$

for $\chi < 0$.

Like the Moustapha *et al.* correlation, the new one is expressed in terms of a change to the kinetic-energy coefficient, ϕ^2 , because that parameter varies more weakly with Mach number than does, for example, the total-pressure loss coefficient, Y . However, conversions between ϕ^2 and Y are easily made using Equations 2.3 and 2.4 given in Chapter 2.

Figure 5.11: Evaluation of Moustapha *et al.* correlation.

5.4 Computational Results

5.4.1 Introduction

To the author's knowledge, there has been only one documented study (Boyle, 1994) that has examined the ability of a Navier-Stokes solver to predict off-design losses. Unfortunately, Boyle made no direct comparisons of the measured and predicted midspan losses: the computational results were only compared with predictions from off-design loss correlations. Based on this apparent lack of previous comparisons, it is unclear how well CFD codes predict off-design losses. The objective of the present computational investigation was to evaluate the ability of a code currently widely used in turbomachinery design, employing a popular, simple and robust turbulence model, to predict off-design profile losses. This data

Figure 5.12: Improved correlation for profile losses at off-design incidence.

will be useful to turbine aerodynamicists since they rely heavily on computational tools to provide estimates of off-design losses, particularly during the design of growth engines. It is worth noting that the original intent of the computational investigation was to examine the secondary losses as well, but unfortunately, this part of the investigation was not performed due to time constraints.

The computational results presented in this chapter were obtained using the LS2 and LS3 cascade geometries. The computations were carried out at conditions similar to the experiment: the Reynolds number based on inlet velocity and chord length was specified as 3.0×10^5 and incompressible conditions were assumed. As mentioned in Chapter 4, the original version of the Dawes code could not be run at incompressible conditions. Therefore, previous comparisons made at Carleton University between Dawes code computations

Figure 5.13: **Evaluation of improved correlation.**

and low-speed experimental results (e.g., DeCecco, 1995; Saroch, 1996) were obscured by compressibility effects in the computations. For the present computations, it is also worth noting that the code was run in “2-D” mode so that by definition, $AVR = 1$, and in addition the boundary layers were assumed turbulent from the leading edge. The comparisons between the experimental and computational results are presented in Section 5.4.3.

Before performing the simulations of the flows for LS2 and LS3, a grid refinement study was carried out to establish grid density guidelines for accurate resolution of the flow field. Two cases were examined: laminar and turbulent flow over a flat plate. These were chosen because there are substantial experimental data against which the predictions can be verified. Furthermore, these cases also enabled the author to check the proper functioning of the code following the extensive modifications that had been made. In both

cases, the simulations were conducted with a computational domain 1.2 m in length and 0.3 m in height; the leading-edge of the plate was located 0.2 m downstream of the inlet of the computational domain. An inviscid wall boundary condition was specified upstream of the leading edge.

It was also necessary to examine the influence of grid density on the cascade loss predictions, primarily because cascade flows are considerably more complex than the flow over a flat plate. Furthermore, the loss coefficient tends to be a parameter that is sensitive to grid density effects. For these reasons, the midspan flow for LS3 was examined for two grids of different density. The first grid was generated with a density that followed the guidelines established for the flat plate. A grid of significantly different density was then examined and the results were compared to assess whether a grid independent solution had been achieved.

While grid density effects are important, it is equally necessary to establish sound criteria for assessing whether a solution has “converged”. The convergence criteria used in the course of this work were based on the experience of DeCecco (1995), Islam (1999) and Grosvenor (2000).

5.4.2 Effects of Grid Density

For the case of laminar flow over a flat plate, three different grids of varying mesh density were examined: the streamwise and cross-stream dimensions of each of the grids were 261×81 , 261×101 , and 261×121 . For each case, Table 5.3 summarizes important characteristics of the grid, most notably, the distance of the first cell centre away from the plate as well as the number of cells in the boundary layer. The edge of the boundary layer was defined as the cross-stream location at which the streamwise velocity was 99% of the freestream velocity. Also, the values of the grid expansion ratio, GER , are included in

Table 5.3: Comparison of the grid geometric parameters for the three grids used in the simulations of laminar flow over a flat plate.

Grid density	GER_{CS}	GER_S	No. of nodes in boundary layer at $x/L = 0.50$	Distance of first node from the wall (y/L)
261x81	1.05	1.03	9	1.54E-04
261x101	1.05	1.03	19	5.75E-05
261x121	1.05	1.03	34	2.16E-05

Note: node is defined here as a cell centre.

the table. GER is defined as

$$GER = \frac{L_{cell,i+1}}{L_{cell,i}} \quad (5.5)$$

where $L_{cell,i+1}$ and $L_{cell,i}$ are the lengths of two adjacent grid cells in a specified coordinate direction. Subscripts S and CS refer to the streamwise and cross-stream directions, respectively. All distances given in table are non-dimensionalized by the plate length, L . Finally, x and y are, in order, the streamwise coordinate as measured from the plate leading edge and the cross-stream coordinate as measured from the surface.

From Table 5.3, it can be seen the expansion ratios used in both the streamwise and cross-stream directions are relatively small by comparison with the upper limit of 1.3 suggested by Dawes. It is worth noting that a few laminar flat-plate simulations were performed with higher values of GER , and it was found that the Blasius solution could be reproduced for values GER as large as 1.3.

For each of three grids, Figure 5.14 shows the predicted velocity profiles at an axial location midway along the plate. Also shown in the figure is the velocity profile obtained from the Blasius solution. As can be seen in Figure 5.14c, excellent agreement is achieved between the computations and the Blasius solution, which provides evidence that the

(a)

(b)

(c)

Figure 5.14: Comparison of laminar velocity profile predictions for three different grids: (a) 261×81 grid, (b) 261×101 grid and (c) 261×121 grid.

Table 5.4: Comparison of mesh parameters and near-wall spatial resolution details for the three grids used in the simulations of turbulent flow over a flat plate.

Grid density	GER_{CS}	GER_S	No. of nodes in boundary layer	No. of nodes at $y^+ < 4$	No. of nodes at $y^+ < 30$	Distance of first node from the wall	
						y/L	y^+
261x81	1.05	1.03	20	0	2	1.54E-04	7.51
261x121	1.05	1.03	53	2	12	5.75E-05	0.90
261x161	1.05	1.03	92	12	39	3.05E-06	0.13

Note: Node refers to a cell centre.

modified code was functioning properly. Also, a comparison of the a and b parts of the figure indicates that a minimum of 19 nodes is required in the boundary layer to reproduce the Blasius velocity profile.

The influence of grid density on the prediction of turbulent flow over a flat plate was examined with three grids of different density: 261×81, 261×121, 261×161. These simulations were performed at a Reynolds number of 1.0×10^6 based on plate length. Table 5.4 summarizes the overall grid characteristics and commonly quoted information related to the near-wall grid density. Listed in the table are the number of nodes in the boundary layer as well as the number of nodes with y^+ values less than 4 (i.e., within the laminar sublayer) and less than 30. It should be noted that y^+ is defined as

$$y^+ = \frac{u_\tau y}{\nu}$$

where $u_\tau = \sqrt{\frac{\tau_w}{\rho}}$ and τ_w is the wall shear stress. Also given in the table are the values of y/L and y^+ for the first node from the wall. These data were taken at an axial location midway along the plate (i.e., $x/L = 0.5$). The corresponding velocity profile predictions, which are plotted in terms of law-of-the-wall coordinates (y^+, u^+), are shown in Figure 5.15.

The non-dimensional velocity, u^+ , is defined as

$$u^+ = \frac{u}{u_\tau}$$

where u is the local flow velocity. Also included in the figure is a law-of-the-wall velocity profile as obtained by Thompson (1967). As can be seen, there is very good agreement between the predictions and the Thompson profile for the two finer grids (parts b and c of the figure). Therefore from Table 5.4, it appears that a minimum of two grid cells centred within the laminar sublayer and approximately 50 cells in the boundary layer are required for a grid independent solution. Heidegger *et al.* (1999) also examined the effects of grid density using the case of turbulent flow over a flat plate. They used the “ADPAC” code, a Navier-Stokes solver developed at NASA for turbomachinery geometries. With the Baldwin and Lomax turbulence model, they found that the near-wall grid densities required for grid independence were similar to the present results.

To investigate the influence of grid density on the prediction of cascade flows, simulations were performed with two grids of different densities: 266×203 and 266×123 using the LS3 cascade at the design incidence. The grids were constructed such that the distances between the suction surface and the adjacent cell centres were kept constant. This implied that the cross-stream GER was higher for the coarser mesh ($GER_{coarse\ mesh} = 1.17$ versus $GER_{fine\ mesh} = 1.07$). At an axial location near the trailing edge, two cell centres were located in the suction surface laminar sublayer for both grids.

The predicted distributions of airfoil surface static pressures are presented in Figure 5.16. The two distributions are nearly indistinguishable which indicates that grid independence has been achieved for the static pressure field. Moreover, the distribution appears to be quite smooth in the leading- and trailing-edge regions. The appearance of spatial oscillations in the static pressure distribution is usually an indicator that the axial grid density is too

Figure 5.15: Comparison of turbulent velocity profile predictions for three different grids: (a) 261x81 grid, (b) 261x121 grid and (c) 261x161 grid.

Figure 5.16: The influence of grid density on the predicted airfoil surface static pressure distributions for LS3 at the design incidence.

coarse (DeCecco, 1995).

The influence of grid density on the midspan total-pressure loss coefficient, Y_p , was also examined. The difference in the loss coefficients between the two grids was less than 0.001, which again suggests that a grid independent solution had been achieved.

5.4.3 Comparison of Experimental and Computational Results

Figure 5.17 presents the predicted and measured variations of midspan loss with incidence for LS2 and LS3. The most striking feature in the figure is the discrepancy

Figure 5.17: Comparison of the predicted and measured midspan loss variations with incidence ($x/C_x=1.40$).

between the predicted and measured loss at +20 degrees of incidence. The low values of predicted loss at this incidence suggest that the suction surface boundary layer remains attached along the entire surface. The predicted surface static pressure distribution, which is shown in Figure 5.18, confirms this result as there is diffusion on the suction surface right up to the trailing edge.

It is also important to note that, unlike the experimental case in which there was a leading-edge separation bubble, no such separation bubble was predicted at +20 degrees of incidence. This implies that the onset of trailing-edge separation will tend to occur at a larger value of incidence for the computations since the predicted suction surface boundary layer will be thinner than the layer in the experimental case. To determine the incidence

Figure 5.18: Predicted and measured surface static pressure distribution at +20 degrees of incidence ($x/C_x=1.40$).

value at which separation would be predicted, an additional simulation was carried out for LS3 at +30 degrees of incidence. The predicted loss coefficient for this case is shown in Figure 5.17. As can be seen, a more rapid rise in the loss coefficient occurs between +20 and +30 degrees but the predictions at +30 degrees are still considerably lower than the experimental values at +20 degrees. From a vector plot of the computed velocity field, it was found that the suction surface flow separated at about 80% C_x for the case at +30 degrees. The boundary layer did, however, reattach near the trailing edge, which may explain why the predicted losses were still significantly smaller than those observed experimentally.

Based on the results from this analysis, one must conclude that the BTOB3D code performs poorly for the prediction of trailing-edge separation. There seem to be two

plausible explanations for this shortcoming. First, the lack of predicted separation at the leading edge, and the resulting thinner boundary layer would make the computed boundary layer more resistant to separation. Of course, this cannot be attributed to the Baldwin and Lomax model since it is a laminar separation in the experiment. The inclusion of a transition model would probably result in a laminar separation bubble being predicted at the leading edge. This would result in a better prediction of the boundary layer thickness by comparison with the current assumption of turbulent flow from the leading edge. However, the fact that at +30 degrees of incidence a reattaching separation bubble was being predicted, whereas the experimental flow was fully separated even at +20 degrees, suggests that the Baldwin and Lomax model results in boundary layers that appear to be much too resistant to separation.

It should be mentioned that Menter (1992) also found that the Baldwin and Lomax performs poorly for separated flows. He examined four turbulence models (e.g., a zero-equation model - Baldwin and Lomax (1978), a half-equation model, a one-equation model and a two-equation model) using the case of an adverse pressure gradient flow with separation. He found that the three transport-equation based models predicted separation significantly better than the Baldwin and Lomax model. An obvious question one may ask is why the Baldwin and Lomax was used in this study given that other turbulence models had been shown to predict separated flows more accurately. First, the Baldwin and Lomax continues to be widely used in turbomachinery design, and there seems to be a general lack of awareness of how well the model performs at predicting off-design losses. Moreover, at the time when this study was undertaken there were no commercial codes available to the author: The BTOB3D code, which incorporates only the Baldwin and Lomax turbulence model, was the only accessible code at the time.

Turning now to the results at 0 and +10 degrees of incidence. By comparison with the results at +20 degrees of incidence, the agreement between the predicted and measured

losses is significantly better for the lower incidence cases. Nonetheless, it can be seen that the Dawes code tends to over-predict the losses for these cases: a slightly larger discrepancy was observed at the design incidence. This poorer agreement at the design incidence was probably the result of the suction surface boundary layer being laminar up to the suction peak in the experiment whereas the simulations were performed assuming turbulent flow from the leading edge. This again highlights one of the major shortcomings of the Dawes code, and indeed most other CFD codes: that is, the absence of a transition model. At +10 degrees of incidence, better agreement was observed between the measured and predicted losses since the suction surface layer was turbulent over most of the surface.

5.5 Conclusions

Measurements have been made of the off-design profile losses for two turbine airfoils which differ primarily in their leading-edge geometries. The recent correlation of Moustapha *et al.* predicted that the airfoil with the larger leading-edge diameter would be considerably less sensitive to incidence. However, the losses were very similar for the two airfoils over a wide range of incidence angles. This prompted a re-examination of the influence of the leading-edge geometry on profile losses. It was found that a significantly better correlation could be obtained by including the leading-edge wedge angle as an additional and more influential correlating parameter. It is believed that the off-design behaviour of the airfoil is influenced by the magnitude of the discontinuity in curvature at the blend points where the leading-edge circle joins the rest of the profile. The value of the wedge angle appears to give an approximate and convenient indication of the magnitude of this discontinuity in curvature.

The present results were obtained exclusively from airfoils employing leading-edge circles. It would appear that an elliptic leading edge allows an airfoil to be designed with

milder curvature jumps at the blend points. This probably explains the reduced sensitivity to incidence which is usually attributed to elliptic leading edges.

A computational investigation was performed to evaluate the ability of the combination of a state-of-the-art Navier-Stokes solver and Baldwin and Lomax turbulence model to predict off-design losses. For cases where the suction surface flow remained attached (i.e., moderate values of incidence), the correct loss trend with incidence was predicted. The code, however, did not predict the trailing-edge separation that occurred at high incidence.

Finally, although the present work has focused exclusively on the influence of the leading-edge geometry on the off-design profile losses, it is recognized that there are additional factors which should be investigated. These include the Reynolds number, the turbulence intensity and the airfoil loading distribution.

Chapter 6

The Influence of Leading-Edge Geometry on Secondary Losses at the Design Incidence

6.1 Introduction

Secondary flows in turbine cascades have been the subject of extensive experimental research in the past decades. Several excellent reviews have appeared. Research from the 1950's to the mid-1980's was summarized by Sieverding (1985*b*). Gregory-Smith (1997) and Langston (2001) have reviewed the more recent research on secondary flows. Based on these reviews, it is clear that there is still a requirement for an improved understanding of a number of aspects of the physics, including in particular the influence of the inlet boundary layer separation process (i.e., the horseshoe vortex formation) on the flow and loss behaviour in the endwall region.

The flow at the junction of a cylindrical body and flat plate, and the resulting horseshoe vortex formation, has been studied in great detail (e.g., Baker, 1980; Eckerle and Langston, 1987; Pierce and Tree, 1990; Devenport and Simpson, 1990). The flow and geometric parameters that affect the structure and dimensions of the horseshoe vortices are therefore reasonably well understood (Ballio *et al.*, 1998). However, for these flows the pressure field imposed on the incoming boundary layer is a function of the size of the obstacle only. In the case of the turbomachinery-airfoil-endwall junction, the upstream pressure field becomes a function of the magnitude and distribution of the loading of the airfoil itself. The effect of this is likely to be particularly strong in turbine airfoil rows because of their high airfoil loadings. To the author's knowledge, only one investigation has focused on the development of the horseshoe vortex in turbine cascades (Moore and Ransmayr, 1984), and therefore, the physics of this flow are not as well understood. The geometric and flow parameters that affect the characteristics of the vortex have yet to be fully identified.

This chapter presents the results of an experimental study of secondary flows that was performed in the wind-tunnel set-up described in Chapter 3. The LS2 and LS3 airfoil designs, which differ mainly in their leading-edge geometries, have been investigated. This chapter concentrates on providing insight into the effect of leading-edge geometry on the endwall flow field and loss behaviour at the design incidence. The subsequent chapter addresses the effects of off-design incidence. Detailed flow field measurements were made upstream and downstream of the cascade and static pressure distributions were measured on the airfoil surfaces. The data were supplemented with surface oil flow visualization.

6.2 Experimental Results

6.2.1 Inlet Endwall Boundary Layers

The inlet endwall boundary layer parameters are given in Table 6.1, and as can be seen, the thickness of the layer for LS2 is slightly larger than for LS3. This arises from a small difference in the endwall boundary-layer bleed flows upstream of the two cascades. The location of the bleed slots is shown in Figure 3.2.

6.2.2 Blade Static Pressure Distributions

The strength of secondary flows downstream of a turbine have been shown to depend strongly on both the overall airfoil loading (e.g., Marchal and Sieverding, 1977; Hodson and Dominy, 1987a; Perdichizzi and Dossena, 1993) and the loading distribution (e.g., Weiss and Fottner, 1995). To provide background for the subsequent discussions, the blade pressure distributions are presented first.

Figure 6.1 shows the measured variations in the midspan airfoil pressure for LS2 and LS3. It is worth noting again that the airfoil loading data for LS2 were obtained from a study of the effects of axial velocity ratio (AVR) on the midspan aerodynamic behaviour of a turbine cascade (Rodger *et al.*, 1992). The loading measurements for LS3 were obtained in the present study.

An important difference between the two pressure distributions shown in Figure 6.1 is that LS2 is slightly more forward loaded than LS3. This is consistent with LS2 having a larger stagger angle than LS3, as noted in the previous chapter. Therefore, the present measurements include the effects of more than the leading-edge geometry.

Table 6.1: Summary of Experimental Results.

LOCATION	FLOW QUANTITY	CASCADE	
		LS2	LS3
INLET	δ^* [mm]	2.6	2.2
	θ [mm]	1.9	1.5
	H	1.39	1.41
	C_{Po}'' inlet	-0.033	-0.027
OUTLET (Traverse Plane: 40% Cx downstream of trailing-edge.)	C_{Po}'' total	-0.090	-0.083
	C_{Po}' profile	-0.047	-0.052
	C_{Po}'' secondary	-0.043	-0.031
	Y_{total}	0.043	0.041
	$Y_{profile}$	0.021	0.025
	$Y_{secondary}$	0.022	0.016
	$C_{\Gamma+}$	0.196	0.159
	$C_{\Gamma-}$	-0.110	-0.086
	C_{SKE}''	0.013	0.008
	α_2'' [deg.]	-53.6	-52.6
	q_2/q_{CL}''	2.09	2.02
	$\alpha'_{midspan}$ [deg.]	-54.1	-52.9
	q_2/q_{CL}' midspan	2.18	2.09
	AVR	0.98	0.99
	Quality factor, Q	5.06	4.50
MIXED-OUT	C_{Po}'' total	-0.108	-0.096
	C_{Po}' profile	-0.048	-0.054
	C_{Po}'' secondary	-0.060	-0.042
	Y_{total}	0.053	0.048
	$Y_{profile}$	0.022	0.026
	$Y_{secondary}$	0.031	0.023
	q_2/q_{CL}''	2.04	1.98
	q_2/q_{CL}' midspan	2.18	2.08

Figure 6.1: Measured midspan surface pressure distributions for LS2 and LS3 at design incidence (reproduced from Figure 5.1).

6.2.3 Physics of the Endwall Flow at the Design Incidence

6.2.3.1 Introduction

Extensive work has been performed since the 1940's on secondary flows and has provided a basis for a number of detailed descriptions of the endwall flow within the blade passage at the design incidence (e.g., Sieverding, 1985b; Hodson and Dominy, 1987a; Sharma and Butler, 1987). Amongst these descriptions, there is fairly broad agreement about the

details of the flow field. However, some questions still remain. To further advance the understanding of the endwall flow, Wang *et al.* (1997) provided additional data in the form of oil and smoke flow visualization. They gave a detailed interpretation of the flow physics inside the blade passage based on their data; the present author's interpretation is presented below as background for the subsequent discussions of the effects of leading-edge geometry (present chapter) and incidence (Chapter 7) on secondary flows.

6.2.3.2 Interpretation of the Flow Field

Figure 6.2 and 6.3 show, in order, the oil-film flow visualization obtained on the endwall and suction surface at the design incidence. The corresponding interpretation of these results is shown in Figures 6.4a and b. Downstream flow field measurements made in this work were also used to interpret the inside-passage flow physics. These figures will be discussed below.

Many of the commonly observed features of the turbine endwall flow are found in the present flow. Ahead of the leading edge there are two major separation lines which are associated with the horseshoe vortex. These are designated S_1 and S_2 on Figure 6.4a; the pressure and suction side legs are subscripted "p" and "s" respectively. The primary separation line, S_1 , is the separation of the inlet endwall boundary layer. The secondary separation line, S_2 , is generally regarded as the "lift-off" line of the horseshoe vortex (Sieverding, 1985*b*). Initially the secondary separation is quite strong, as evident from the strong convergence of the surface shear stress vectors. However, as the separation line is traced across the passage, it weakens until it becomes a line of weak boundary layer convergence. The separation lines S_{1s} and S_{2s} merge as they round the suction side of the leading edge. As suggested by a number of investigators, it is at this point that the suction-side leg of the horseshoe vortex is driven off the endwall by the cross-passage flow. The other

Figure 6.2: Endwall flow visualization at the design incidence.

Figure 6.3: Suction surface flow visualization at the design incidence.

Figure 6.4: Interpretation of the (a) endwall, and (b) suction surface flow visualization at the design incidence.

major separation line which is visible in Figure 6.2 is that of the endwall boundary layer in the suction corner, S_3 . S_3 seems to originate at the point of intersection of the suction-side leg of S_1/S_2 with the suction surface. There are also two regions of reattaching flow. The first, marked R_1 , is a reattachment or saddle point. The second, although barely visible, is a reattachment line, R_2 , near the corner at the blade pressure surface. Except perhaps near the leading edge, it is suspected that the fluid which reattaches at R_2 originates in the freestream (Sjolander, 1975).

As observed by a number of investigators (e.g., Sonoda, 1985; Hodson and Dominy, 1987a; Sharma and Butler, 1987; Jabbari *et al.*, 1992; Wang *et al.*, 1997) some of the inlet endwall boundary layer fluid can end up at the suction surface, at separation line, S_3 , without having taken part in the separations, S_1 or S_2 . This region is indicated on Figure 6.4a with double-lined arrows. As will be discussed below, this fluid seems to be associated with one of the dominant high-loss regions in the downstream flow field.

Turning to the airfoil suction surface, Figure 6.3 shows the surface oil-film flow visualization. The corresponding interpretation of the oil patterns is given in Figure 6.4b. The scales on this figure indicate spanwise and chordwise distances, with the suction-surface scale being non-dimensionalized by the suction-surface length (SSL).

There appear to be two major separation lines on the suction surface in the endwall region: S_5 , an apparent extension of S_{2s} , and S_4 . S_4 is the most commonly observed separation line and it is associated with the passage vortex whereas S_{2s} is the separation line for the suction-side leg of the horseshoe vortex. Figure 6.3 shows a finite distance between the starting points of the S_{2s} and S_4 separation lines. This supports the earlier interpretation that some of the inlet endwall boundary layer fluid was not involved in the inlet endwall boundary layer separation. Based on similar observations, Hodson and Dominy (1987a) and Denton (1993) described this fluid as being “funnelled” between the passage

vortex and suction-side leg of the horseshoe vortex.

In contrast to the observations of Hodson and Dominy (1987a), in the present flow the suction-side leg of the horseshoe vortex does not appear to remain close to the suction surface with downstream distance since the S_{2s} separation line seems to end fairly quickly. Some evidence of the suction-side leg of the horseshoe vortex (scouring of the oil film and a separation line) can be seen on the blade just downstream of the leading edge and for a short distance along the midspan side of the funnelled boundary-layer fluid. The trace of this vortex is weak; it is slightly more obvious on the endwall side of the blade (e.g., around $z/h = 0.9$). It can be observed that the scouring of the oil gradually weakens downstream of where the passage vortex meets the suction surface. The only plausible explanation for this seems to be that the passage vortex interacts with the suction side leg of the horseshoe vortex, thus forcing it off the suction surface. Presumably, the suction side leg of the horseshoe vortex wraps itself around the passage vortex, as has been observed by many investigators (e.g., Sieverding and Van den Bosche, 1983; Moore and Smith, 1984; Sonoda, 1985; Sharma and Butler, 1987; Wang *et al.*, 1997). The separation line S_5 occurs where the suction-surface boundary layer meets the funnelled fluid. Finally, there are also two regions of attaching flow. Line R_3 represents the reattachment of the endwall fluid which separated along S_3 . As in many experiments, it was possible to identify the resulting corner vortex in the present downstream measurements. The R_4 reattachment line is discussed further below.

The details of the surface flow behaviour within the region bounded by the S_4 and S_5 lines were difficult to discern with surface oil flow visualization alone because the region was so small. Therefore, pressure probe measurements were made near the trailing edge.

Figure 6.5 gives the resulting, tentative interpretation. The region between separation lines S_4 and S_5 is believed to be occupied, in part, by the funnelled endwall boundary layer fluid. The present author also believes that some of the suction-surface fluid from beneath

Figure 6.5: Interpretation of the flow structures in the endwall region.

the passage vortex is convected into this region. Therefore, R_4 is the reattachment line that divides the funnelled boundary-layer flow from the suction-surface fluid that has been convected from beneath the passage vortex. In the region bounded by R_4 and S_5 , the surface flow visualization showed very little evidence of either flow convergence on the S_5 separation line or flow divergence from the R_4 reattachment line. For this reason, it appears that the vortical structure, referred to as SS2, is quite weak. The vortex in the region bounded by S_4 and R_4 , which is denoted as SS1, appears to be considerably smaller than SS2 but quite intense. This interpretation is based on measurements that are presented below and the smoke flow visualization results of Wang *et al.* (1997).

Figures 6.6 and 6.7 show contour plots of total-pressure deficit overlaid with colour floods of positive and negative streamwise vorticity, respectively.

Figure 6.6: Line contours of total-pressure coefficient superimposed on flood contours of positive streamwise vorticity for LS3 at the design incidence ($x/C_x=1.10$).

The measurements presented in these figures were taken for LS3 at 0.1 axial-chord lengths downstream of the trailing edge. These contour plots were found to be useful for identifying the location and characteristics (e.g., size, strength and direction of rotation) of the vortical structures. Vorticity can be used to visualize the motion of the flow since it is convected with the fluid particles. It should be noted that for the selected coordinate system, the passage vortex contains positive streamwise vorticity.

From Figure 6.6, it can be seen that the endwall region is occupied predominantly by

Figure 6.7: Line contours of total-pressure coefficient superimposed on flood contours of negative streamwise vorticity for LS3 at the design incidence ($x/C_x=1.10$).

fluid with positive streamwise vorticity. Within this region, there are two cores of high vorticity. The one closest to midspan is coincident with a region of high loss and is quite intense: it is typically identified as the passage vortex. The other vorticity core, which is closer to the endwall, is weaker and associated with relatively low-loss fluid. To the present author's knowledge, this flow feature has not been observed previously.

The negative vorticity region shows a number of distinctive features, including a small region of intense vorticity beneath the passage vortex, and partially overlapping this, a region of high-loss fluid. The present author believes that this loss core is associated with

the funnelled boundary layer fluid. It is also believed that, as mentioned earlier, the SS1 vortex is composed mainly of suction surface fluid. This fluid originates in the endwall suction surface corner (at R_3) and is driven towards the S_4 separation line by the cross flows induced by the passage vortex. The interaction appears to be quite strong from the surface flow visualization. Evidently, the boundary layer that originates at R_3 , is being strongly accelerated as it makes its way towards S_4 , and as a result, the vorticity production is high (the resultant vorticity is negative in sign). The region of high negative vorticity beneath the passage vortex, which is observed in Figure 6.7, is likely the roll-up of this vorticity sheet. It is worth noting that the present author has considered the possibility that the vorticity core was composed of trailing-shed vorticity, as has been suggested by a number of investigators. However, surface static pressure measurements presented in the following chapter indicate very little spanwise variation in the loading. Also, pressure probe measurements made slightly upstream of the trailing edge showed evidence of a core of high negative vorticity being present already in the vicinity of the S_4/S_5 separation lines.

The final feature in the negative vorticity field is the vortex located at the endwall suction surface corner. As expected, this region is characterized by high losses and elevated levels of negative vorticity.

6.2.4 Mass-Averaged and Mixed-Out Results

The mass-averaged and mixed-out results for LS2 and LS3 are presented in Table 6.1.

As seen from the table, the overall total-pressure losses for LS2 and LS3 are very similar. The profile losses are taken as the losses measured at midspan where the influence of the secondary flow is small. The secondary loss coefficient is then taken as the difference between the overall loss coefficient and the profile loss coefficient. Thus, it is implicitly assumed that the profile losses generated near the endwall are the same as those at midspan. This is the

conventional loss breakdown and it seems to give reasonable results at design incidence. However, it will be shown in Chapter 7 that the conventional loss breakdown can result in misleadingly low values of the secondary loss coefficient at off-design incidence.

The mass-averaged results presented in Table 6.1 show that the measured and mixed-out secondary losses are higher for LS2. Also shown in the table are the values of the “quality factor”, Q . This parameter was proposed by Marchal (1980), as referred to in Sieverding (1985*b*), to allow designers to qualitatively assess the effects of airfoil loading and loading distribution on secondary losses. Marchal argued that an increase in the cross-passage pressure gradient or a shift in the loading distribution toward the leading-edge would produce higher secondary losses. The higher value of Q for LS2 is mainly due to the airfoil being more forward-loaded, as can be seen in Figure 6.1. The increase in secondary loss for LS2 is thus consistent with the higher value of Q . Weiss and Fottner (1995) investigated the effect of loading distribution on secondary losses and also found that a more forward-loaded airfoil (higher Q) produced higher losses and overall stronger secondary flows. In their investigation, the two cascades had identical leading-edge geometries; the main geometric difference was the stagger angle - the front-loaded cascade having a larger stagger angle.

In addition to the higher losses, the overall strength of the secondary flow field was also larger for LS2. Three parameters indicate this: the mass-averaged secondary kinetic energy (C_{SKE}'') and the absolute values of the positive and negative circulation ($C_{\Gamma+}$ and $C_{\Gamma-}$), all of which were larger for LS2. The secondary kinetic energy is significant since one of the dominant mechanisms for loss generation is mixing (Denton, 1993). Through this process, secondary kinetic energy is eventually lost if it is not used in a downstream rotor for work extraction. Therefore, secondary kinetic energy may be regarded as a potential loss source and a cascade with stronger secondary flows will ultimately generate more loss, as is the case for LS2.

Table 6.2: Energy audit from the measurement plane to the mixed-out plane.

Description	LS2	LS3	LS3-LS2
"Diffusion" loss **	-0.006	-0.005	0.000
+ Δ Secondary KE	-0.013	-0.008	0.005
= Mixing loss	-0.018	-0.013	0.005
Mixing loss	-0.018	-0.013	0.005
+ Measured loss	-0.090	-0.083	0.007
= Mixed-out loss	-0.108	-0.096	0.012

** Diffusion loss = Δ Primary KE + Static Pressure Recovery

NOTE: All quantities are non-dimensionalized by inlet centreline dynamic pressure

Table 6.2 summarizes the results of an “energy audit” from the measurement plane to the mixed-out plane. All loss values are normalized on the inlet dynamic pressure. The mixing losses are divided into two components. The first is described as the “diffusion” loss and is the portion of the primary kinetic energy that is lost between the measurement and mixed-out planes. The remaining portion of the primary kinetic energy is recovered as static pressure. The second term is the dissipation of secondary kinetic energy. Two observations are worth making: first, the diffusion losses are nearly identical for the two cascades; and second, the difference in the mixing losses can be mostly attributed to the difference in the measured secondary kinetic energies.

The remainder of this chapter examines the downstream traverses in more detail and presents a comparison of the measured secondary losses with empirical predictions.

6.2.5 Total Pressure Distributions

Figures 6.8a and b show contour plots of the total-pressure deficits at the downstream measurement plane for LS2 and LS3, respectively. Darker shading indicates higher losses.

(a)

(b)

Figure 6.8: Line contours of total-pressure coefficient for (a) LS2 and (b) LS3 at the design incidence ($x/C_x=1.40$).

In broad terms, the total-pressure deficit distributions for LS2 and LS3 are very similar. However, there are some differences. Starting near the endwall with the corner vortex, both the pitchwise extent of the high loss region and the peak loss magnitude are larger for LS2. Moreover, for LS2, the size of the loss cores associated with the SS1 and passage vortices are larger and have penetrated further toward midspan. The size of the loss cores must, to some extent, be related to the strength of the vortices. Data presented below support this assertion.

6.2.6 Positive Vorticity Field

Figures 6.9 and 6.10 show the contour lines from Figures 6.8a and b superimposed with colour floods illustrating the regions of positive streamwise vorticity for LS2 and LS3, respectively. As mentioned, the passage vortex has positive vorticity.

As can be seen, both flows are largely dominated by regions of positive streamwise vorticity with nearly equal area. However, the vorticity levels in this region are comparatively higher for LS2; thus, the circulation or overall strength of the positive vorticity field is greater.

The present author believes that most of the fluid of positive streamwise vorticity originates from the inlet endwall boundary layer. The other sources of vorticity mostly produce vorticity of the opposite sense. These sources include: the endwall region aft of separation line S_{1p} ; the region of the suction surface between the endwall and the S_4 separation line; and the portion of the inlet endwall boundary layer that becomes the suction side leg of the horseshoe vortex. The negative vorticity produced by these sources will reduce, through mixing, the overall circulation associated with the fluid of positive vorticity. Thus, it is conceivable that the positive vorticity levels for LS3 are lower than those of LS2 because of stronger mixing with the endwall and suction surface boundary

Figure 6.9: Line contours of total-pressure coefficient superimposed on flood contours of positive streamwise vorticity for LS2 at the design incidence ($x/C_x=1.40$).

layer fluid. However, this does not seem likely since such mixing would also generate losses and the losses for LS3 are lower than those of LS2. A more plausible explanation is simply that LS2 produces a stronger positive streamwise vorticity field. Since the positive vorticity is believed to result from the separation of the inlet endwall boundary, any factor that influences the extent of this separation should also affect the generation of positive vorticity. Another recent experiment, by Sauer *et al.* (2001), also highlights the importance of the inlet endwall boundary layer separation. The authors found that the secondary loss in a cascade of high-turning turbine airfoils could be significantly reduced by modifying the

Figure 6.10: Line contours of total-pressure coefficient superimposed on flood contours of positive streamwise vorticity for LS3 at the design incidence ($x/C_x=1.40$).

airfoil shape over the first $20\%C_x$ in the airfoil-endwall junction.

The process of horseshoe vortex formation in turbine cascades is not fully understood owing to the scarcity of detailed measurements in the leading-edge region. This contrasts with the flow at the junction of a cylindrical body and flat plate, which has been investigated extensively. A number of the studies have looked at the influence of leading-edge shape and size on the strength of the horseshoe vortex. For example, Mehta (1984) and Kubendran *et al.* (1986) showed experimentally that for the flow around the junction of a non-lifting body and a flat plate, simulating a wing/fuselage junction, the circulation associated with

the horseshoe vortex was related to the shape of the leading edge. In both experiments, the strength increased significantly with nose (or leading-edge) bluntness. It appears that the strength of the vortex is closely related to the pressure disturbance produced by the obstacle: a larger pressure disturbance generates a stronger vortex.

For the simple junction flows, the pressure disturbance is solely a function of the leading-edge shape and size. However, the pressure disturbance produced by a lifting body, such as a turbine airfoil, is more complex. The present author believes that the pressure field on the endwall upstream of the leading-edge is substantially influenced by the loading in the leading-edge region. This hypothesis is based on off-design incidence data from the current study, which are presented in Chapter 7, and from Langston *et al.* (1977). Both experiments show that the saddle point of separation moves towards midpitch as the blade loading distribution shifts forward. The important point is that the effective obstacle size seen by the oncoming endwall boundary layer seems to be strongly related to the loading on the forward part of the airfoil, and only secondarily on the thickness of the leading edge. This argument provides a plausible explanation for why the more forward-loaded LS2 cascade generates a stronger positive vorticity field than LS3, even though LS2 has the smaller leading-edge diameter.

6.2.7 Negative Vorticity Field

Figures 6.11 and 6.12 show the corresponding contours of negative streamwise vorticity for LS2 and LS3.

The most prominent feature is the core of negative vorticity associated with the SS1 vortex. This vortex and its neighbouring passage vortex appear to be nearly identical in size and peak vorticity magnitude for the respective airfoils. Therefore, like the passage vortices, the strength of the SS1 vortex for LS2 is significantly stronger than for LS3. The

Figure 6.11: Line contours of total-pressure coefficient superimposed on flood contours of negative streamwise vorticity for LS2 ($x/C_x=1.40$).

vortex is approximately the same size for both cases, but the peak vorticity level for LS2 is approximately 40% higher than for LS3.

As mentioned earlier, the SS1 vortex seems to be composed of fluid from the thin boundary layer on the suction surface between the corner vortex reattachment line, R_3 , and the passage vortex separation line, S_4 . The strong cross flows induced on the suction surface by the nearby passage vortex, together with the apparent thinness of the boundary layer, leads to high surface shear stress. Consequently, the production of negative streamwise vorticity on the surface will also be high. This vortical fluid is convected towards the S_4

Figure 6.12: Line contours of total-pressure coefficient superimposed on flood contours of negative streamwise vorticity for LS3 ($x/C_x=1.40$).

separation line where it rolls up into the SS1 vortex. Since the vorticity in the SS1 vortex is largely generated by the action of the passage vortex, one would expect the strength of the two vortex structures to be related. In other words, a stronger passage vortex will lead to a stronger SS1 vortex, as is the case for LS2 as compared to LS3.

6.2.8 Downstream Secondary Kinetic Energy Field

Figures 6.13 and 6.14 show contour lines of total-pressure deficit along with colour floods of secondary kinetic energy for LS2 and LS3 respectively. As can be seen, there are two

Figure 6.13: Line contours of total-pressure coefficient superimposed on flood contours of secondary kinetic energy for LS2 at the design incidence ($x/C_x=1.40$).

regions of fluid with relatively high secondary kinetic energy: one located immediately adjacent to the endwall, extending over approximately 30% of the pitch, and another located between the passage and the SS1 vortex. In both cases, there is comparatively little secondary kinetic energy associated with the vortex cores themselves.

The main difference between the two cases is that the peak levels of secondary kinetic energy are comparatively higher for LS2. Near the endwall, the peak level for LS2 is approximately 30% greater than for LS3, and for the region between the passage vortex and the SS1 vortex, the difference is even greater at approximately 60%. The most likely

Figure 6.14: Line contours of total-pressure coefficient superimposed on flood contours of secondary kinetic energy for LS3 at the design incidence ($x/C_x=1.40$).

explanation for the latter difference lies with the strength of the vortical structures. The results presented earlier clearly showed that the two vortices for LS2 are much stronger than those for LS3, while there was no significant difference in their proximity. Therefore, the induced secondary velocities and corresponding secondary kinetic energies would be expected to be higher for LS2, as observed.

The stronger secondary kinetic energy near the endwall for LS2 is somewhat surprising since it was noted earlier that the cross-passage pressure difference is lower for LS2 on the rearward part of the airfoil. The most probable explanation lies with the passage vortex.

As shown in Figures 6.9 and 6.10, the passage vortex for LS2 is comparatively stronger than the one for LS3. Because the downstream endwall region lies partly beneath the passage vortex, the higher secondary kinetic energy is likely induced by the stronger passage vortex. Therefore, in this part of the blade passage it is the passage vortex rather than the cross-passage pressure difference that seems to play the dominant role in determining the near-endwall secondary kinetic energy. The higher velocities near the endwall should also increase the entropy generation in the endwall boundary layer.

As noted earlier, the secondary kinetic energy is itself a potential source of losses, with the losses being generated through mixing as the flow proceeds downstream. As pointed out in connection with Tables 6.1 and 6.2, the increase in losses from the measurement plane to fully mixed-out conditions correlated well with the mass-averaged secondary kinetic energies obtained at the measurement plane and LS2 did indeed give the higher mixing losses.

6.3 Comparison Of Measured Secondary Losses With Empirical Predictions

Since empirical loss correlations continue to play an important role in the design process, there is an ongoing need to review and improve these correlations. This section compares the present results to a few of the most widely used correlations for secondary losses: Kacker and Okapuu (1982), which is based on Ainley and Mathieson (1951) and Dunham and Came (1970); Craig and Cox (1971); and Traupel (1977). The comparisons are given in Table 6.3.

The correlations are seen to predict significantly higher losses than those measured: in all cases the predicted losses are at least twice as large as the measured. Similar discrepancies between cascade results and the correlations have been noted before. As discussed by Sieverding (1985*a*), the loss correlations derived from cascade data are scaled or “calibrated”

to reproduce stage efficiencies derived from rig or engine data. It appears that most of the differences between the correlation predictions and the present cascade results are due to this calibration process, since the present measured secondary losses are comparable with those observed in similar turbine cascades. Given that these loss systems have been shown to predict turbine efficiency reasonably well, there are evidently additional and significant loss generating mechanisms in the engine environment that are not captured in cascade testing. Thus, cascade results such as the present ones should be used mainly to gain physical insights and to establish trends due to variations in particular aerodynamic or geometric parameters.

As shown in Table 6.3, the correlations predict no significant differences in the losses for LS2 and LS3. This is to be expected since the correlations do not account for differences in loading distributions, which are thought to be the main source of the differences in the measured losses.

Table 6.3: Comparison of measured and predicted secondary losses.

Source	Secondary loss coefficient, $Y_{\text{secondary}}$	
	LS2	LS3
Measured, (mixed-out)	0.031	0.023
Kacker & Okapuu	0.093	0.091
Craig & Cox	0.071	0.072
Measured, (40% C_x downstream of trailing-edge)	0.022	0.016
Traupel*	0.062	0.063

* - Loss coefficient is a function of downstream distance, and assumed value of endwall skin friction coefficient is $C_f = 0.003$.

6.4 Conclusions

Detailed measurements have been made of the secondary flows for LS2 and LS3 at the design incidence. The LS2 airfoil represents the midspan section of a power turbine blade of fairly recent design. The LS3 airfoil was designed with a larger leading-edge diameter to investigate the effect of leading-edge geometry on the profile and secondary losses. The LS3 airfoil also has a slightly smaller stagger angle; therefore, it was more aft loaded than LS2.

The measured and mixed-out secondary losses were found to be higher for LS2. The detailed flow field measurements showed that the streamwise vorticity and secondary kinetic energy fields were also stronger for this airfoil. It is argued that the higher loss generation for LS2 is mainly the result of a stronger passage vortex. The increased strength of this vortex combined with its close proximity to the suction surface and endwall would be expected to produce thinner boundary layers and larger boundary-layer edge velocities, which in turn should increase the loss production near the walls. In addition, the stronger passage vortex seemed to induce a stronger separation vortex on the suction side of the airfoil. The higher secondary velocities in the region where these two vortices interacted resulted in higher downstream mixing losses for the baseline airfoil.

The greater size and strength of the passage vortex for the LS2 airfoil is attributed to this airfoil's front-loaded pressure distribution and the resulting stronger upstream pressure field seen by the endwall boundary layer approaching the leading edge. This effect seemed to dominate over any influence of the leading-edge diameter, which was smaller for LS2. It is tentatively concluded that in turbine blade rows the leading-edge geometry has at best a minor influence on the secondary flow field, and the corresponding secondary losses, at least for the design incidence and for conventional leading-edge geometries. On the other hand, the loading on the forward part of the airfoil appears to be of primary importance.

Chapter 7

Secondary Losses at Off-Design Incidence

7.1 Introduction

In Chapter 2 it was noted that the most extensive studies conducted on secondary flows have been performed almost exclusively at the design incidence. By contrast, there has been comparatively little research on turbine secondary flows at off-design incidence (e.g., Hodson and Dominy, 1987b; Yamamoto and Nouse, 1988; Perdichizzi and Dossena, 1993). These investigations have provided a number of insights into the physics of the endwall flow at off-design conditions. They have demonstrated that there is an increase in the degree of three-dimensionality as incidence increases. However, a number of questions remain. Furthermore, while overall losses were generally evaluated, few attempts were made to use these results to verify or improve the secondary loss correlations used in design.

The first part of this chapter presents the results from an experimental study of the

effects of incidence on secondary flows. The investigation was performed in the large-scale, low-speed wind tunnel with the LS3 test cascade described in Chapter 3. Results are presented from the detailed flow field measurements that were made upstream and downstream of the cascade and static pressure distributions that were measured on the blade surfaces for three different values of incidence: 0, +10 and +20 degrees. The data were supplemented by extensive surface oil flow visualization. The ultimate goal of the study was to improve the off-design correlations for secondary losses, as has been done for the profile losses (Chapter 5). However, such improvements must be built on a sound understanding of the physics of the flow. Therefore, the first few sections of this chapter concentrate on the physical insights into the relationship between the behaviour of secondary flows and incidence which have been gained from the measurements and the flow visualization. Subsequently, the effect of incidence on the detailed downstream flow field behaviour and the mass-averaged results are discussed. The measurements identify some serious shortcomings at off-design incidence of the traditional decomposition of the losses into components.

The most recent correlation for secondary losses at off-design incidence (Moustapha *et al.*, 1990) includes the leading-edge diameter as a parameter that influences the losses. In the second and final part of this chapter, the LS2 loss results are presented and compared with those of LS3 to verify the Moustapha *et al.* correlation and to investigate the influence of leading-edge geometry on the off-design behaviour of turbine airfoils.

7.2 Inlet Endwall Boundary Layers

The inlet endwall boundary layer parameters for both LS2 and LS3 are given in Table 7.1, and as can be seen, the thickness of the layer is slightly larger at off-design incidence. As pointed out in the previous chapter, the small difference arises from variations in the endwall boundary-layer bleed flows upstream of the cascade. From inviscid flow theory, the

Table 7.1: Inlet boundary layer parameters for LS2 and LS3 at design, +10 and +20 degrees of incidence.

LOCATION	FLOW QUANTITY	Incidence [deg.]					
		LS2			LS3		
		0.0	10.0	20.0	0.0	10.0	20.0
INLET	δ' [mm]	2.59	2.60	3.44	2.16	2.78	2.88
	θ [mm]	1.86	1.88	2.49	1.53	2.01	2.09
	H	1.39	1.38	1.38	1.41	1.38	1.38
	$C_{Po}''_{inlet}$	-0.033	-0.033	-0.044	-0.027	-0.036	-0.037

strength of the secondary flow field at the outlet of the cascade is related to the distribution of vorticity and the overall circulation in the inlet endwall boundary layer. The overall circulation of the layer was invariant for all testing since the freestream velocity at the inlet boundary layer traverse plane was essentially fixed (i.e., constant Reynolds number at the inlet plane). The distributions of vorticity were essentially the same for all cases as well.

7.3 Blade Static Pressure Distributions

Once the endwall flow enters the blade passage, the cross-passage pressure difference (that is, the blade loading) plays a major role in driving the secondary flow. Therefore, as background to the later discussions of the physics of the endwall flow and secondary loss behaviour, the blade pressure distributions are presented first.

Preliminary measurements showed that the blade surface pressures varied slowly in the spanwise direction. Therefore, the final measurements were made at only 6 or 7 of the 12 available rows of taps. Repeatability tests showed that for a particular incidence most pressure coefficients were repeatable to within ± 0.02 . However, it was found that near a sharp suction peak the pressure coefficient could differ from previous measurements by up to 0.1. Since the pressure coefficients were about -3.0 to -4.0 at such suction peaks, the

Figure 7.1: Effect of incidence on midspan airfoil pressure distributions for LS3 (reproduced from Figure 5.4).

relative uncertainty is still reasonably small.

Figure 7.1 shows the variations in the midspan airfoil pressures for 0, +10 and +20 degrees of incidence. Figure 7.2 shows the near-endwall ($z/h = 0.05$) pressures for 0 and +20 degrees as well as the corresponding curves at midspan. The curves for +10 degrees have been omitted for the sake of clarity. It can be seen from Figure 7.1 that incidence variation has a significant effect on the pressures, particularly on the suction side of the airfoil. At the design incidence, the suction surface flow is accelerated up to about midchord. As incidence increases, a velocity overshoot appears on the suction side near the leading edge,

Figure 7.2: Effect of incidence on endwall airfoil pressure distributions for LS3.

and downstream of the leading edge, a significant reduction in acceleration is observed. At +10 degrees of incidence, there is essentially zero pressure gradient up to about midchord after which the flow is decelerated until the trailing edge. At +20 degrees, the flow is decelerated along the suction surface until about 80% of axial chord at which point a breakdown in diffusion occurs. Flow visualization, which will be presented later, verifies that the subsequent region of constant pressure is a region of two-dimensional separation.

The near-endwall pressure distributions, given in Figure 7.2, show similar trends to those at midspan; however, there are some differences. Firstly, the suction surface pressure coefficients are higher near the endwall up to about 2/3 of axial chord for both values of

Figure 7.3: Effect of incidence on midspan blade pressure differences for LS3.

incidence. This is presumably due to the lower dynamic pressure in the endwall boundary layer. Also, at +20 degrees of incidence there is diffusion on the suction surface right up to the trailing edge. It appears that near the endwall, the interaction between the endwall flow and the suction-surface boundary layer prevents the trailing-edge separation that is present at midspan. This has a significant effect on the total losses, as will be discussed later.

Figure 7.3 shows the airfoil pressure differences at midspan for the three values of incidence. The curves show that as incidence increases, the loading distribution changes from mid- to forward-loaded. They also show that the blade pressure difference for +10 and +20 degrees of incidence is larger than that at design incidence for only about the first 20%

Table 7.2: Variation of midspan tangential loading coefficient with incidence.

Incidence [deg.]	Tangential loading coefficient, C_T
0	2.28
+10	2.07
+20	1.74

axial chord. Evidently, for most of the passage length, the pressure difference driving the secondary flows is largest at the design incidence. The tangential loading coefficient based on inlet dynamic pressure, C_T , is used to quantify the variation in loading with incidence. It is defined as

$$C_T = \int_0^1 \Delta C_p d(x/Cx) \quad (7.1)$$

and it represents the area underneath one of the ΔC_p curves plotted in Figure 7.3. The values of the coefficients are presented in Table 7.2, and as can be seen, the airfoil loading does indeed decrease with increasing incidence.

7.4 Inside-Passage Flow Field Behaviour

7.4.1 Endwall Flow Visualization

Endwall flow visualization was performed at 0, +10 and +20 degrees of incidence with the LS3 cascade, and a consistent trend in the flow pattern behaviour was observed with increasing incidence. Therefore, the results are presented just for the extreme case of +20 degrees. Figure 7.4a and b show, in order, the oil-dot and oil-film flow visualization obtained on the endwall at +20 degrees of incidence; the corresponding interpretation of these results is shown in Figure 7.5b. The interpretation at the design incidence is shown in Figure 7.5a, and is included here for reference.

(a)

(b)

Figure 7.4: Endwall flow visualization for LS3 at $i = +20$ degrees: (a) dot technique, and (b) film technique.

Figure 7.5: Interpretation of the endwall flow visualization for LS3 at (a) design incidence, and (b) $i = +20$ degrees.

Figure 7.5 indicates that there are a number of effects of increasing incidence on the oil-film patterns. The most significant changes occur in the location of the primary and secondary separation lines. As shown in Figure 7.3, an increase in incidence has a strong effect on the blade loading, particularly in the leading-edge region. The pressure field on the endwall upstream of the leading edge must also be influenced substantially. Therefore, it is not surprising that the endwall boundary layer would be significantly affected. With this in mind, the changes in the endwall flow behaviour are examined in more detail.

As the incidence is increased, the stagnation streamline of the incoming endwall boundary layer (and the resulting saddle point) is shifted towards midpitch. Due to the change in blade loading, the inlet endwall boundary evidently “sees” a larger effective obstacle as it approaches the leading edge of the airfoil. As a result, the separation lines are shifted upstream. The results of Langston *et al.* (1977) and Perdichizzi and Dossena (1993) also showed a change in the position of the saddle point with incidence. In contrast, Hodson and Dominy (1987b) did not observe such a shift. It is suggested that the movement of the saddle point may be related to the ratio of leading-edge diameter to spacing. The LS3 cascade and that used by Perdichizzi and Dossena (1993) have leading-edge-diameter to spacing ratios which are roughly 2.5 and 4 times larger respectively than that of the cascade used by Hodson and Dominy (1987b). It appears that the effective blockage may be amplified at off-design incidence by a larger leading-edge diameter. This may in turn have implications for the respective off-design behaviours of the secondary losses.

Considering next the primary separation line, for both positive values of incidence the pressure-side leg of the primary separation line, S_{1p} , remains quite distinct as it is traced across the blade passage. Also, the point of intersection of S_{1p} with the suction surface moves closer to the leading edge with increasing incidence. The same behaviour was observed by Hodson and Dominy (1987b). At +20 degrees, S_{1p} has moved far enough forward to nearly

intersect separation line S_{2s} . Clearly, the amount of inlet boundary layer fluid which is funnelled between S_{1p} and S_{2s} decreases with increasing incidence. Instead, it appears that progressively more of the inlet endwall boundary fluid is convected into the suction-side leg of the horseshoe vortex. In addition, with the increasing loading in the leading-edge region the suction-side leg of the horseshoe vortex should experience higher acceleration (and therefore stretching) as it wraps around the leading edge. This would make the vortex more intense, although it would also reduce its cross section. Stronger convergence of the surface shear stress vectors on the S_{2s} separation line seems to confirm that the intensity of this leg of the horseshoe vortex has increased with incidence.

The endwall flow visualization also indicated that there was progressively stronger convergence of the surface shear stress vectors on the S_{2p} separation line with increasing incidence. S_{2p} was quite distinct across the entire blade passage for both positive values of incidence, unlike at design where the line weakened as the suction surface was approached. The more distinct appearance of the S_{2p} separation line suggests that, like the suction-side leg, the pressure-side leg of the horseshoe vortex becomes more intense with increasing incidence. This is presumably the result of a similar stretching effect. The flow visualization on the suction surface of the blade will provide additional evidence that the pressure-side leg of the horseshoe vortex becomes stronger with incidence.

Finally, Figure 7.5 indicates that downstream of the secondary separation line, S_{2p} , the endwall shear stress vectors are rotated closer to the pitchwise direction as the incidence increases; this was particularly clear from the oil-dot flow visualization. This stronger cross-channel flow is somewhat surprising since it was noted earlier that the blade loading, and therefore the cross-channel pressure difference, is in fact reduced on the rearward part of the blade as incidence increases. The most likely explanation for the behaviour of the endwall flow lies with the passage vortex. It was noted that the primary and secondary separation

lines were better defined at positive incidence and that the horseshoe and passage vortices were thought to be stronger as a result of stretching. The downstream endwall region lies partly under the strengthened passage vortex and the stronger cross-flow is likely being induced by the vortex. Thus, in this part of the blade passage the passage vortex rather than the cross-passage pressure difference seems to play the dominant role in determining the direction, and probably also the magnitude, of the endwall shear stress vectors. This presumably influences the losses generated in this region.

7.4.2 Suction Surface Flow Visualization

Figure 7.6 shows the suction-surface oil-film flow visualization for +10 degrees of incidence. Suction-surface oil-film and oil-dot flow visualization for +20 degrees are shown in Figures 7.7a and b, respectively. The flow visualization results using oil-dot technique have been included here because certain features are more easily seen using this technique.

From Figure 7.6, it can be seen that the passage-vortex separation line, S_4 , has moved forward on the blade surface and towards midspan by comparison with its location at the design incidence (Figure 6.3). This is consistent with the shift in the S_{1p} separation line on the endwall. At the trailing edge, S_4 is located at roughly 20% of span. As noted by Sharma and Butler (1987), the increased penetration towards midspan of the S_4 separation line must, to some extent, indicate that the passage vortex has become a larger structure.

Unlike at design incidence, the suction-side leg of the horseshoe vortex could not be observed on the blade surface at +10 degrees. That is, there was no scouring of the oil film on the midspan side of the funnelled endwall boundary layer fluid. As suggested by Sieverding (1985*b*), the inability to trace the vortex on the suction surface may be a result of the strong stretching of the vortex in the streamwise direction and the consequent reduction in its size. Like at design incidence, it is also quite probable that the horseshoe vortex is

Figure 7.6: Suction-surface oil flow visualization for LS3 at $i = +10$ degrees.

quickly forced off the suction surface by the passage vortex, resulting in a very short S_{2s} separation line.

The S_5 separation line is roughly parallel to the passage-vortex separation line, S_4 , up to about 60% suction surface length (SSL). At this point the two lines begin to diverge, with the divergence becoming more rapid at about 75% SSL. At the trailing edge, the S_5 separation line extends to roughly 40% of the blade span. Hodson and Dominy (1987b) observed a similar movement in the S_5 separation line. The behaviour of the S_5 separation line may simply be the result of the increase in size of the passage vortex with incidence. However, the kink in the separating line at about 75% SSL could conceivably be due to bursting of the passage vortex.

The surface shear stress in the region between the S_4 and S_5 separation lines is also

Figure 7.7: Suction-surface oil flow visualization for LS3 at $i = +20$ degrees: (a) film technique and (b) dot technique.

different at +10 degrees compared with design. It undergoes a noticeable reduction on the rearward part of the suction surface, starting at the point where the S_4 and S_5 separation lines begin to diverge. This reduction in shear stress is consistent with the flow being diffused as a result of the diverging separation lines.

Finally, the midspan flow has also undergone some changes as a result of the increase in incidence. Figure 7.6 shows the presence of a short, laminar separation bubble very close to the leading edge, just downstream of the suction surface overspeed. A leading-edge separation bubble is also believed to be present at design incidence (Section 5.3.3) due to the discontinuity in surface curvature at the leading-edge blend point. However, at design incidence the bubble reattaches in a region of favourable pressure gradient and the boundary layer is subsequently relaminarized. At +10 degrees, the bubble reattaches in a region of adverse pressure gradient and the subsequent boundary layer apparently remains turbulent. Therefore, in contrast to the midspan flow at the design incidence, there is no longer a laminar separation bubble on the rearward part of the suction surface.

Further changes in the flow behaviour occur at +20 degrees. For example, Figure 7.7a indicates that the origin of the S_4 separation line has moved even further forward toward the leading edge. There is again little evidence of the suction-side leg of the horseshoe vortex. Perhaps the most noticeable change at +20 degrees is that there no longer appears to be any funnelled inlet endwall boundary fluid present on the blade surface. This is largely consistent with the endwall flow visualization (Figure 7.5) which showed that the region of funnelled fluid was very narrow at this incidence. As a consequence, the S_5 separation line is not present on the suction surface. Near midspan, the oil over a majority of the rearward part of the suction surface shows very little movement, indicating the presence of trailing-edge separation. Near the trailing edge, a small region of reverse flow is present as can be seen in Figure 7.7b from the upstream movement of the oil dots. These midspan

results are consistent with the static pressure measurements which indicated a breakdown of diffusion in the same region. This region of separated flow also seems to constrain the spanwise migration of the passage-vortex separation line. Interestingly, there is a fairly abrupt bend in the separation line at about 85% SSL: bursting of the passage vortex again appears to be a possibility.

A final observation is made based on the suction-surface oil-dot patterns. As indicated with arrows in Figure 7.7b, surface shear stress vectors from within the trailing-edge separation region can be seen converging on the S_4 separation line. These markings are believed to be caused by the cross-flows induced by the SS1 vortex.

7.5 Downstream Flow Field Behaviour

7.5.1 Total Pressure Field

As background to the discussion of the mass-averaged results, contour plots of total-pressure deficits at the downstream measurement plane are presented first. As well, these downstream results provide further evidence to support some of the inferences that were made in the discussion of the flow visualization.

Figures 7.8 and 7.9 show the downstream total-pressure contours for the off-design cases at +10 and +20 degrees of incidence, respectively. Also included in each figure are the results for the design-incidence case, which has been reproduced from Figure 6.8b. Darker shading indicates higher losses. The contour plot for the design incidence case is reproduced in each figure because it is the baseline case against which the results at off-design are compared.

There is a significant influence of increasing incidence on the behaviour of near-endwall total-pressure field with increasing incidence. The results at +10 degrees, by comparison

Figure 7.8: Contours of total-pressure coefficient for LS3 at (a) design incidence (reproduced from Figure 6.8b), and (b) $i = +10$ degrees ($x/C_x=1.40$).

Figure 7.9: Contours of total-pressure coefficient for LS3 at (a) design incidence (reproduced from Figure 6.8b), and (b) $i = +20$ degrees ($x/C_x=1.40$).

with those at the design incidence, show that the loss cores associated with the passage vortex and the funnelled boundary layer are considerably larger and have shifted toward midspan. A reduction in the magnitudes of the loss peaks can also be observed. At +20 degrees of incidence and at midspan, both the pitchwise extent of the blade wake and the magnitude of the peak loss have increased substantially. This result is consistent with trailing-edge flow separation which was observed in both the flow visualization and the blade static pressure distributions. In contrast to the high losses at midspan, near the endwall for the first 10% of airfoil height, the loss levels are similar to those at the lower incidence values. It is this disparity in losses that causes the conventional loss breakdown scheme to produce physically implausible results at high values of incidence, as will be discussed later. From Figure 7.9b, it can also be observed that the size and peak magnitude of the loss core associated with the passage vortex have increased substantially. Also, at +20 degrees of incidence there no longer appears to be a loss core which is identifiable with the funnelled boundary layer fluid. Clearly these downstream flow field measurements seem to be consistent with observations made earlier on the behaviour of the endwall flows.

As a final observation, it can be seen that the size and peak magnitude of the loss core in the endwall corner decreases with increasing incidence. This result was unanticipated since the endwall flow visualization indicated that the corner separation was becoming larger with incidence. That is, stronger convergence of the surface shear stress vectors on the S_3 separation line was evident with increasing incidence. Also, the separation line was observed to shift away from the suction surface as incidence increased.

7.5.2 Positive Vorticity Field

Figures 7.10 and 7.11 show the total-pressure coefficient contour lines from Figures 7.8 and 7.9 superimposed with colour floods illustrating the regions of positive streamwise

vorticity for LS3 at 0, +10 and 0 and +20 degrees of incidence, respectively. Figure 7.12 shows the circulation associated with the fluid of positive vorticity (referred to hereafter as positive circulation) and quantifies the results presented in Figures 7.10 and 7.11. Also shown is the peak magnitudes of positive vorticity for the three values of incidence.

In general terms, the positive vorticity fields are similar: for the three cases, two cores of vorticity are observed. For each incidence, the vorticity core located closest to midspan is associated with a total-pressure loss core (and also a core of low static pressure). In contrast, the vorticity core near the endwall is located in a region where there is no discernible total-pressure or static pressure core. Based on streamline patterns calculated from the secondary velocity fields, it appears that fluid particles from within the near-endwall vorticity core are convected into the one closer to midspan where they eventually roll up into a vortex.

A number of observable changes occur in the positive vorticity field as incidence increases. For instance at +10 degrees, the core closer to midspan has increased significantly in strength since it is similar in size to the design incidence case, but has much higher levels of vorticity. In contrast, the strength of the core near the endwall has diminished and continues to weaken with increasing incidence. At +20 degrees of incidence, the near-endwall core is barely visible. The strength of the vorticity core closest to midspan remains fairly constant from +10 to +20 degrees: the vorticity levels have decreased (peak has decreased from 8.0 to 6.5, as shown in Figure 7.12), but the vorticity is spread over a larger area.

The overall strength of the positive vorticity field is quantified in Figure 7.12. The figure shows an increase in the strength of the vorticity field from the design incidence to +10 degrees of incidence. From +10 to +20 degrees, the trend differs from the one observed from 0 to +10 degrees in that the positive circulation remains nearly constant. This was somewhat unexpected since the strength of the positive vorticity field has been shown to be directly proportional to the flow turning in the blade row, at least for inviscid flows. As

Figure 7.10: Line contours of total-pressure coefficient superimposed on flood contours of positive streamwise vorticity for LS3 at (a) design incidence (reproduced from Figure 6.10), and (b) $i = +10$ degrees ($x/Cx=1.40$).

Figure 7.11: Line contours of total-pressure coefficient superimposed on flood contours of positive streamwise vorticity for LS3 at (a) design incidence (reproduced from Figure 6.10), and (b) $i = +20$ degrees ($x/Cx=1.40$).

Figure 7.12: Variation of positive circulation and peak vorticity magnitudes with incidence.

will be discussed further below, the invariance in the strength of the vorticity field between +10 and +20 degrees is probably the result of enhanced mixing.

In connection with Figures 7.10 and 7.11 and the influence of incidence on the behaviour of the positive vorticity field, a few additional observations are worth making. At the design incidence, it can be seen that fluid with relatively strong vorticity continues to be convected into the vorticity cores from the endwall. Evidently, the rolling up of the inlet endwall boundary layer is still incomplete at this axial location. At +10 degrees of incidence, the inlet endwall layer roll-up appears to be nearly complete as fluid with only weak vorticity is being convected from the endwall into the vorticity cores. The fluid of positive vorticity has largely been removed from the endwall at +20 degrees of incidence; therefore, the roll-up process is essentially complete. Finally, the vorticity cores closest to midspan are coincident

with their associated loss cores for both 0 and +10 degrees of incidence whereas at +20 degrees of incidence, the loss core clearly does not coincide with the vorticity core. A number of other investigators have observed a similar behaviour (e.g., Marchal and Sieverding, 1977; Gregory-Smith and Graves, 1983; Hodson and Dominy, 1987b), but its physical significance is not immediately clear.

The remainder of this section focuses on providing physical insights that will help explain the behaviour of the positive vorticity field with incidence.

As discussed in Chapter 6, most of the positive vorticity that appears within and downstream of the blade passage is believed to originate from the inlet endwall boundary layer. Evidently, the fluid particles within this layer can follow very different paths depending on their pitchwise and spanwise locations. Some of the inlet endwall fluid becomes a part of the suction-side leg of the horseshoe vortex while some of it separates along S_3 , and is funnelled between the suction and pressure-side legs of the horseshoe vortices. The following discussion concentrates on the behaviour of the remaining inlet boundary layer fluid, that is, the fluid that becomes involved in the passage vortex.

The formation and evolution of the passage vortex has a significant influence on the downstream flow field behaviour, as discussed in Chapter 6. Many studies have detailed the development of this vortical structure within turbine cascades; therefore, the basic flow physics are reasonably well understood. Nevertheless, some questions still remain: the results presented earlier in this section shed light on some of these issues, as will be discussed further below.

In Figures 7.10 and 7.11, the positive vorticity and coincident high-loss core closest to midspan correspond to the passage vortex. It has been well established through measurement that passage vortex evolves from the pressure-side leg of the horseshoe vortex. Therefore, the strength of the passage vortex at the downstream measurement plane would

tend to increase as the strength of the pressure-side leg of the horseshoe vortex increases. The flow visualization results presented earlier indicated that the pressure-side leg had become more intense with increasing incidence. This is consistent with the observed trend in the strength of the vorticity core in the downstream measurement plane, at least between design and +10 degrees of incidence. From +10 to +20 degrees of incidence, essentially no change in the strength of the positive vorticity field was observed in the downstream measurement plane, yet from the flow visualization results the passage vortex had clearly intensified. Some other physical mechanism seems to be causing the vorticity strength to decrease as it evolves downstream.

The destruction of vorticity, and hence reduction in vortex strength occurs through mixing of two regions of vortical fluid of opposite signs. Therefore, for reasons which will be discussed below, it is postulated that the vortex strength remains unchanged because the interaction (or mixing) between the passage vortex and negative-vorticity fluid increases with incidence. This would also help explain the substantial growth of the loss core between +10 and +20 degrees of incidence. It is recognized, however, that the growth of the loss core from +10 to +20 degrees is probably not solely due to the increase in mixing. Inevitably the passage vortex has also entrained some of the high-loss fluid from within the trailing-edge separation region. As will be discussed further in Section 7.7, the interaction between the midspan and secondary flows violates one of the main assumptions in the conventional loss breakdown scheme; that is, that the profile and secondary loss components are independent.

The increase in mixing between the passage vortex and the negative-vorticity fluid may occur for a few reasons. First, it is conceivable that the substantially increased streamwise deceleration between +10 and +20 degrees, which was shown in Figure 7.1, results in an increase in the duration of the interaction between the passage vortex and fluid with negative vorticity. This would tend to reduce the strength of the vortex. Furthermore, if vortex

bursting does occur, significantly higher levels of turbulence would be expected downstream of the bursting location, which in turn, would enhance mixing. It is worth noting that the streamwise velocity in the core of the passage vortex for +20 degrees of incidence is approximately half of the design-incidence value.

Discussion is now directed to the vorticity core closest to the endwall. The behaviour of this vorticity core is believed to depend on the strength of the pressure-side leg of the horseshoe vortex. This assertion is based partially on the work of Sjolander (1975) who appears to be one of the few investigators who has detailed the endwall boundary layers in the region of the S_{1p} or S_{2p} separation lines. Evidently near the endwall, there is a dividing streamline below which fluid is entrained into the pressure-side leg of the horseshoe vortex. Sjolander (1975) noted that the spanwise location of this dividing streamline is related to the strength of local adverse pressure gradient field and the inlet boundary layer thickness. It is worth noting again that the strength of the horseshoe vortex in a turbine is believed to be related to the strength of the upstream adverse pressure gradient field, as is the case with other junction flows. As the adverse pressure gradient field becomes stronger, the dividing streamline will tend to move outward from the endwall into the freestream: that is, more boundary layer fluid becomes involved in the separation.

As previously discussed, there is evidence from both the surface flow visualization and downstream measurements that the pressure-leg of the horseshoe vortex strengthens with incidence. If the vortex is strong enough to entrain only part of the inlet endwall layer then the remaining boundary fluid of relatively higher momentum and total pressure will convect over top of the pressure-side leg of the horseshoe vortex. This vortical fluid will then follow a path with relatively higher curvature than the freestream fluid and it will remain close to the endwall until it reaches the suction surface. The region of positive vorticity closest to the endwall in Figures 7.10 and 7.11 is believed to be composed of this endwall fluid.

When this fluid reaches the suction surface, it will then begin to convect towards the S_4 separation line. Mixing is likely to occur with the suction surface layer which has vorticity of the opposite sign, so vorticity cancellation will occur. This may explain why there are two cores of vorticity and not a continuous band from the endwall to the pressure-side leg of the horseshoe vortex.

7.5.3 Negative Vorticity Field

Figures 7.13 and 7.14 show colour floods of negative streamwise vorticity for LS3 at 0, +10 and 0 and +20 degrees of incidence, respectively, again superimposed on contour lines of total-pressure coefficient from Figures 7.8 and 7.9. The corresponding plot of negative circulation and peak vorticity magnitude is shown in Figure 7.15.

Starting with the flow near the endwall, the corner vortex appears to be weakening with increasing incidence. As seen from Figures 7.13 and 7.14, the peak magnitude and size of the vorticity core diminish with increasing incidence; the peak magnitude of the total-pressure deficit decreases as well. These results were unexpected since the endwall flow visualization showed evidence that the vortex was becoming stronger with incidence, as discussed previously. Based on these observations, one must infer that the physical process that is effecting an apparent decrease in the strength of the corner vortex is occurring between the trailing-edge and the measurement plane. It is suggested that downstream of the trailing edge, in the absence of the suction surface, the passage vortex entrains the suction-corner fluid (i.e., the corner vortex) and convects it away from the endwall. Based on the behaviour of the peak magnitude and size of the vorticity core observed in Figures 7.13 and 7.14, it is evident that more high-loss, suction-corner fluid is removed from the near-endwall region with increasing incidence. Again, it is conceivable that the duration of the interaction between the endwall fluid and the passage vortex increases with incidence

Figure 7.13: Line contours of total-pressure coefficient superimposed on flood contours of negative streamwise vorticity for LS3 at (a) design incidence (reproduced from Figure 6.12), and (b) $i = +10$ degrees ($x/Cx=1.40$).

Figure 7.14: Line contours of total-pressure coefficient superimposed on flood contours of negative streamwise vorticity for LS3 at (a) design incidence (reproduced from Figure 6.12), and (b) $i = +20$ degrees ($x/Cx=1.40$).

Figure 7.15: Variation of negative circulation and peak vorticity magnitudes with incidence.

due to the reduction in streamwise velocities. More suction-corner fluid will be convected away as the duration of the interaction increases.

Figures 7.13 and 7.14 show a vorticity core located within the wake and on the midspan-side of the passage vortex for the three values of incidence. For design and +10 degrees of incidence, a loss core can also be seen in the same region as the vorticity core; at +20 degrees of incidence, there is no longer a loss core present. Evidently, the two features can appear independently of one another, which again suggests that they identify two distinct structures. As discussed previously, the loss core is believed to consist mainly of funnelled boundary layer fluid whereas the core of vortical fluid, denoted as the SS1 vortex, originates from the boundary layer on the suction surface between the endwall and the S_4 separation line.

Finally, the strength of the positive and negative vorticity fields follow similar trends with increasing incidence: they both intensify from design to a maximum at +10 degrees. This result is consistent with the present author's belief that the strength of the negative vorticity field is largely determined by the strength of the passage vortex.

7.6 Mass-Averaged Total-Pressure Losses

Figures 7.16 and 7.17 show the influence of incidence on the overall, Y_{total} , and the midspan total-pressure loss coefficients, $Y_{midspan}$, respectively. The results at midspan are reproduced here for convenience as they will be referenced in the following discussion. Table 7.3 lists the loss coefficients along with other aerodynamic parameters, such as, total-pressure coefficient, secondary kinetic energy, circulation, axial velocity ratio and exit flow angle.

Figure 7.16: Influence of incidence on measured and mixed-out total-pressure losses.

Figure 7.17: Influence of incidence on measured midspan total-pressure losses.

From Figure 7.16, the measured overall total-pressure loss coefficients are seen to rise gradually from 0 to +10 degrees after which the losses increase more rapidly from +10 to +20 degrees.

A similar trend is observed for the midspan losses, and as pointed out earlier, the rapid increase in loss from +10 to +20 degrees is largely the result of the onset of trailing-edge separation on the suction side of the airfoil. Because the trailing-edge separation extends over a large percentage of the span (approximately 60% span at the trailing edge), it will have significant influence on the overall losses, as observed.

Figure 7.16 also shows the mixed-out overall total-pressure loss coefficients. As can be seen, the mixing losses progressively increase with incidence: at +20 degrees, the losses are approximately four times larger than the value at design incidence. The large mixing losses at +20 degrees were somewhat surprising since it was argued earlier that the reduction in vorticity strength, and the growth of the passage vortex loss core from +10 to +20

Table 7.3: Summary of experimental results.

LOCATION	FLOW QUANTITY	Incidence [deg.]		
		0.0	10.0	20.0
OUTLET (Traverse Plane: 40% Cx downstream of trailing-edge.)	C_{Po}'' total	-0.083	-0.090	-0.184
	C_{Po}' profile	-0.052	-0.052	-0.215
	C_{Po}'' secondary	-0.031	-0.038	0.031
	Y_{total}	0.041	0.058	0.192
	$Y_{profile}$	0.025	0.032	0.220
	$Y_{secondary}$	0.016	0.026	-0.028
	$C_{\Gamma+}$	0.159	0.200	0.204
	$C_{\Gamma-}$	-0.086	-0.011	-0.087
	$C\omega_{s+}$ (peak)	6.6	8.0	6.6
	$C\omega_{s-}$ (peak)	-6.4	-9.0	-6.2
	C_{SKE}''	0.008	0.015	0.021
	α_2'' [deg.]	-52.6	-51.0	-47.3
	q_2/q_{CL}''	2.02	1.54	0.96
	$\alpha'_{midspan}$ [deg.]	-52.9	-51.4	-45.0
	$q_2/q_{CL midspan}'$	2.09	1.62	0.98
	AVR	0.99	1.01	1.04
MIXED-OUT	C_{Po}'' total	-0.096	-0.112	-0.231
	C_{Po}' profile	-0.054	-0.054	-0.242
	C_{Po}'' secondary	-0.042	-0.057	0.011
	Y_{total}	0.048	0.075	0.266
	$Y_{profile}$	0.026	0.034	0.265
	$Y_{secondary}$	0.023	0.041	0.002
	q_2/q_{CL}''	1.98	1.49	0.87
	$q_2/q_{CL midspan}'$	2.08	1.61	0.91

Table 7.4: Energy audit from the measurement plane to the mixed-out plane for LS3 at 0, +10, and +20 degrees of incidence.

Description	i=0	i=10	i=20
"Diffusion" loss **	-0.005	-0.007	-0.026
+ Δ Secondary KE	-0.008	-0.015	-0.021
= Mixing loss	-0.013	-0.022	-0.047
Mixing loss	-0.013	-0.022	-0.047
+ Measured loss	-0.083	-0.090	-0.184
= Mixed-out loss	-0.096	-0.112	-0.231

** Diffusion loss = Δ Primary KE + Static Pressure Recovery

NOTE: All quantities are non-dimensionalized by inlet centreline dynamic pressure

degrees was largely due to more mixing taking place upstream of the measurement plane. An energy audit has been conducted between the measurement and mixed-out plane to help shed light on this unexpected result: Table 7.4 summarizes the results of this energy audit. As mentioned earlier, all loss values in the table are normalized on the inlet dynamic pressure.

The results from this audit raise a number of points of discussion. In the first place, the diffusion losses are nearly identical for 0 and +10 degrees of incidence. Therefore, the difference in the mixing loss is attributed mostly to the difference in the measured secondary kinetic energies, as was the case for LS2 and LS3 at the design incidence. The picture changes as incidence increases from +10 to +20 degrees. From Table 7.4 it is seen that the diffusion losses increase significantly between +10 and +20 degrees whereas a comparatively smaller change in secondary kinetic energy is observed. Consequently, the difference in the diffusion losses accounts for more than 75% of the increase in mixing loss.

It was noted earlier in Chapter 6 that the diffusion losses represent the loss in primary kinetic energy due to mixing; therefore, a flow with highly non-uniform primary velocity magnitudes will tend to produce high diffusion losses. The large increase in diffusion losses observed between +10 and +20 degrees of incidence is most certainly the result of trailing-edge separation on the suction side of the airfoil. For all other cases that were examined, the airfoil surface layers were attached and the results showed nearly identical diffusion losses. Interestingly, the strength of the secondary flows was also very different in each of these cases and again, the diffusion losses remained invariant. In summary, it would seem that the increase in mixing loss between +10 and +20 degrees is largely the effect of trailing-edge separation since the diffusion losses have increased significantly by comparison to the change in the secondary kinetic energy levels.

7.7 Conventional Loss Breakdown

For correlation purposes, the results in Figure 7.16 would normally be decomposed into profile and secondary loss components. In a two-dimensional cascade such as the present one, the profile losses would be obtained from the results at midspan where the influence of the secondary flow on the blade surface flow is small. The secondary loss coefficient would then be taken as the difference between the overall loss coefficient and the profile loss coefficient. Thus, it is implicitly assumed that near the endwalls the loss generation on the blade surfaces and at the trailing edge is essentially the same as that observed at midspan.

Figure 7.18a and b show the overall and midspan (profile) total-pressure losses as a function of incidence for both LS3 and LS2, respectively. According to the conventional loss breakdown, the secondary loss component is simply the difference between these two. It is apparent that in these terms the secondary losses decrease for incidence values greater than +10 degrees in both cases. In fact, the “secondary losses” are negligible at about

(a)

(b)

Figure 7.18: Conventional loss breakdown at off-design incidence: (a) LS3, (b) LS2.

+20 degrees for LS3. A similar behaviour is observed for the LS2 cascade: the “secondary losses” actually become negative at incidence values above about +17 degrees of incidence. Clearly there are strong secondary flow structures present and generating losses at these conditions. Therefore, the loss coefficients resulting from the conventional decomposition are physically misleading.

From detailed flow field results presented earlier it is evident that for high values of incidence the midspan loss no longer provides a good estimate for the losses generated in the blade boundary layers across the full span. At +20 degrees, the suction surface flow visualization and the blade pressure distributions indicate that there is a large region of separated flow away from the endwalls on the rearward part of the suction surface. Consequently, the profile losses are much higher than at design. On the other hand, in the endwall region where the blade surface flow is influenced by the secondary flow, the suction surface flow is largely attached and shows high surface shear stresses. Clearly this boundary layer fluid has not become involved in the two-dimensional separation. As a result, the losses generated on the blade surfaces in the endwall region are substantially less than at midspan. Thus, the conventional loss breakdown will over-estimate the profile loss component in the endwall region, and lead to a misleadingly low value of the secondary loss component, as observed.

These data indicate that there are inherent problems at large positive values of incidence with the conventional scheme for subdividing the losses. An alternative scheme that is more consistent with the observed physics is presented in the next chapter.

7.8 Influence of Leading-Edge Geometry

As noted in the introduction to this chapter, the most recent correlation for secondary losses at off-design incidence (Moustapha *et al.*, 1990, or MKT) suggests that the leading-edge diameter-to-chord ratio is a parameter that influences the losses. Their correlation predicts that the secondary losses vary inversely with leading-edge diameter-to-chord ratio. Moustapha *et al.* argue that the leading-edge diameter influences the losses by affecting the development of the horseshoe vortex. The LS2 results are presented in this section and compared with those of LS3 to investigate the influence of leading-edge geometry on secondary losses at off-design incidence. As mentioned previously in Chapter 3, the leading-edge diameter of LS3 is approximately twice that of LS2.

The results from the downstream traverses showed that the LS2 flow field behaviour with incidence (e.g., distributions of streamwise vorticity and total-pressure coefficient) was very similar to the LS3 results presented earlier in this section. Because there was no evidence of any new flow physics from the detailed flow field results, the section will focus solely on the integrated loss results.

Figure 7.19 shows the effect of incidence on the mixed-out overall total-pressure loss coefficients for LS2 and LS3. As can be seen, the loss behaviour for both cascades is very similar. For both cascades, the overall losses increase gradually up to about +10 degrees of incidence after which a more rapid rise is observed from +10 to +20 degrees. The large increase in loss between +10 and +20 degrees of incidence is due to the onset of trailing-edge separation on the suction side of the airfoil, as noted in Section 7.6. The trailing-edge separation has a significant effect on the overall losses because it extends over a large fraction of the span.

From Figure 7.19 it can be seen that the largest difference between the overall losses for

Figure 7.19: Effect of incidence on the mixed-out overall total-pressure loss coefficients for LS2 and LS3.

LS2 and LS3 occurs at +20 degrees of incidence. Again, it appears that the difference can largely be explained by the midspan loss behaviour. Figure 5.3 showed that the midspan loss coefficient at +20 degrees for LS3 was greater than that for LS2 by about 0.04. The overall losses for LS3 are greater than those of LS2 by about the same amount, as can be observed from Figure 7.19. At +10 degrees of incidence, the overall losses for LS2 and LS3 are the same, and at the design incidence, LS2 has slightly higher overall losses than LS3. As a result, the overall losses for the LS2 cascade seem to be slightly less sensitive to incidence than those of LS3. However, bearing in mind the uncertainties in the measured losses, one must conclude that the leading-edge geometry has, at best, a secondary effect on the overall losses at off-design incidence.

Figure 7.20: Effect of incidence on the mixed-out secondary loss coefficients for LS2 and LS3.

Figure 7.20 compares the incremental secondary losses (i.e., $Y_{sec\ off-design} - Y_{sec\ design}$) predicted by MKT with the measured values obtained from the conventional method of loss decomposition for both LS2 and LS3. From the perspective of evaluating the influence of leading-edge diameter effects on the secondary losses, less consideration should be given to the data at +20 degrees due to the shortcomings of the conventional method of loss decomposition at high values of incidence, as noted in the previous section.

The differences in the secondary losses for LS2 and LS3 that are predicted by MKT are largely due to the difference in the airfoil leading-edge diameters. As can be seen, the MKT correlation predicts that the secondary losses for the LS3 profile are lower at off-design incidence due to its larger leading-edge diameter. However, it should be noted that the predicted loss difference between LS2 and LS3 is quite small (i.e., less than 0.01) over a

large portion of the plotted incidence range.

The MKT correlation predicts that the secondary loss would be lower for LS3 than for LS2, while the opposite was observed experimentally for both +10 and +20 degrees of incidence. The loss differences are comparable with the experimental uncertainties, but even given this, it seems probable that the incremental losses for LS3 are larger than those of LS2. Therefore, the data suggest that the secondary losses are lower at off-design incidence for an airfoil with a smaller leading-edge diameter. In Section 8.2, it will be shown that the incremental secondary loss results for LS2 and LS3 obtained with a new method of loss decomposition are consistent with this conclusion.

7.9 Conclusions

One of the objectives of this chapter was to provide physical insights into the relationship between the behaviour of secondary flows and incidence. To achieve this end, detailed flow field measurements were made upstream and downstream of the cascades, and these data were supplemented by extensive surface oil flow visualization. A second objective was to investigate the influence of airfoil leading-edge diameter on off-design secondary losses. Some problems with the conventional subdivision of the losses into profile and secondary components were also pointed out.

The airfoil surface static pressure measurements showed that the midspan pressure distribution changed from mid- to front-loaded as incidence increased. This had a significant effect on the endwall surface oil flow patterns, particularly in the forward part of the passage. The primary and secondary separation lines were shifted upstream and the saddle point moved towards midpitch. This effect has been observed by some but not in others. As a result, it was postulated that the upstream pressure field imposed on the inlet endwall

boundary layer is influenced at off-design incidence by the leading-edge diameter. This seems to have implications for the off-design secondary losses: it was observed that the secondary losses were more sensitive to incidence variation for the cascade with the larger leading-edge diameter (i.e., LS3).

The most recent off-design secondary loss correlation (Moustapha *et al.*, 1990) includes leading-edge diameter as an influential correlating parameter. The correlation predicts that the secondary loss sensitivity to incidence variation increases as the airfoil leading-edge diameter is reduced. Using the current experimental results as a basis, it must be concluded that the correlation does not correctly capture the leading-edge diameter effect on off-design secondary losses.

The data presented in this chapter have highlighted some serious shortcomings with the conventional method of loss decomposition. With this method, the secondary loss is generally obtained by subtracting the profile loss from the total measured loss in the endwall region. This assumes that the loss generation on the airfoil surface next to the secondary flow is similar to that in the undisturbed surface flow. This assumption is perhaps reasonable at design incidence; however, when it is applied to the present results at +20 degrees, it leads to negligible or even negative secondary losses. This is obviously misleading and unphysical. The next chapter presents a new method of loss decomposition that is more consistent with the observed physics.

Chapter 8

Empirical Prediction of Secondary Losses

8.1 Introduction

Despite its wide use in meanline analyses, the conventional loss breakdown scheme described in the previous chapter is based on a number of assumptions that are known to be physically unsatisfactory. One of these assumptions states that the loss generated in the airfoil surface boundary layers is uniform across the span. The arguments made in the previous chapter indicate that this assumption causes the scheme to produce erroneous values of the secondary loss component at large positive incidence.

This chapter describes an improved empirical prediction method for secondary losses that can be used at either design or off-design incidence. The prediction method is based on a new breakdown scheme that is described in Section 8.2. To use the new breakdown scheme, a correlation for the spanwise location of the S_4 separation line at the trailing edge

is required. One such correlation exists (Sharma and Butler, 1987), however, it was based on a small database. An improved correlation has been developed from a considerably larger database, and it is detailed in Section 8.3. A modified secondary loss correlation was also developed for use in the new breakdown scheme, and it is described in Section 8.4. It was based on a substantial database of measurements from the current work and from the open literature. The same database was used to evaluate the new correlation as well as the most recent published secondary loss correlations for design and off-design values of incidence.

8.2 New Loss Breakdown Scheme

As a starting point, the new loss breakdown scheme is developed with the conventional, simplistic notion that the total losses can be subdivided into profile, secondary and tip-leakage loss components. That is,

$$Y_{total} = Y_{pro} + Y_{sec} + Y_{tip}. \quad (8.1)$$

where Y_{total} , Y_{pro} , Y_{sec} and Y_{tip} are the total, profile, secondary and tip-leakage loss components, respectively. The secondary loss component would be obtained at zero tip clearance, as is done in the conventional scheme.

One of the main differences between the conventional and new loss breakdown schemes is the approach that is taken for estimating the profile and secondary loss components: the one proposed here is believed to be more consistent with the observed physics. To assist in the following discussion, Figure 8.1 shows a schematic picture of a typical suction surface oil-film pattern.

As discussed earlier, there are two regions on the suction surface where the losses generated in the boundary layers are clearly quite different. These two regions are shown

Figure 8.1: Suction surface definition drawing for new loss breakdown scheme.

in Figure 8.1 and are denoted as the primary and secondary regions. From the figure it can be seen that the primary region is bounded by the S_4 lines, and the leading and trailing edges. The loss generated in the primary region is determined mainly by the airfoil pressure distribution and Reynolds number (and Mach number for transonic flows). On the other hand, the loss generation in the secondary region is believed to be influenced mainly by the strength of the passage vortex and probably, to a lesser extent, on the chordwise pressure distribution. Evidently, the factors that influence loss generation in these regions are quite different; therefore, there is no valid justification for assuming similar blade surface losses in the two regions, even at the design incidence.

In the new scheme, the profile loss is defined as the loss generated in surface boundary layers in the primary region. That is, in the region where the blade surface boundary layers

are largely uninfluenced by the secondary flows. To obtain the expression for the profile loss it is first assumed that, within the primary region, the average loss per unit airfoil surface area can be represented by the midspan loss per unit suction surface area. This may be expressed mathematically as,

$$\overline{Y}_a \approx \frac{Y_{midspan}}{A_s} \quad (8.2)$$

where $\overline{Y}_a$ is the average loss per unit airfoil surface area. The suction surface area, A_s , is approximated by $A_s \approx h Cx$, where h and Cx are the airfoil span and axial chord, respectively. The profile loss is obtained by simply multiplying Equation 8.2 by the surface area of the primary region. Based on the simplified representation of the suction surface flow patterns, the area of the primary region, A_{prim} , is

$$A_{prim} = h Cx - 2 \left(\frac{1}{2} Z_{TE} Cx \right). \quad (8.3)$$

where Z_{TE} is the spanwise distance of the S_4 separation line from the endwall at the trailing edge. Z_{TE} is a function of the size of the passage vortex (Sharma and Butler, 1987). The profile loss may then be expressed as

$$\begin{aligned} Y_{pro} &= Y_{mid} \left(\frac{A_s}{A_{prim}} \right) \\ &= Y_{mid} \left(1 - \frac{Z_{TE}}{h} \right). \end{aligned} \quad (8.4)$$

It is envisaged that the midspan loss would be obtained from a profile loss correlation such as Kacker and Okapuu (1982) at the design point, and Equations 5.3 and 5.4 at off-design incidence. The secondary loss would then be the difference between the overall loss and the profile loss. As a result, the secondary loss includes, not only the endwall loss, but also the loss generated in the airfoil surface boundary layers within the secondary region. Evidently, a benefit of this definition is that it avoids the difficulty of having to separate the airfoil surface boundary layer loss in the secondary region from the overall loss generated in the

endwall region.

With the new loss breakdown scheme the overall loss coefficient is written as

$$Y_{total} = Y_{mid} \left(1 - \frac{Z_{TE}}{h} \right) + Y_{sec}. \quad (8.5)$$

where the first term on the right-hand side of the equation represents the airfoil profile loss. To predict the total loss coefficient with Equation 8.5 in the early stages of design, it is necessary to provide a correlation, not only for Y_{mid} and Y_{sec} , but also $\frac{Z_{TE}}{h}$. Correlations for $\frac{Z_{TE}}{h}$ and Y_{sec} have been developed during the course of this work. The new penetration depth and secondary loss correlations are detailed in Sections 8.3 and 8.4, respectively.

Given the flow visualization results presented earlier, it is clear that the triangular representation of the secondary region presented in Figure 8.1 is approximate. In the first place, the experimental results from this work indicate that the point where the S_4 separation first intersects the suction surface varies according to the loading distribution. For the new loss breakdown scheme, the S_4 separation line is assumed to begin at the leading edge, as shown in Figure 8.1. Secondly, it is assumed that the boundary of the secondary region is defined by the S_4 separation line. At low values of incidence, the S_5 separation line may be a better approximation for the secondary region's boundary. Obviously, there is some room to refine the new loss breakdown scheme; nonetheless, the new scheme as it has been proposed, is believed to be a significant improvement. Unlike the conventional scheme, it produces physically plausible values of the secondary loss coefficient over a wide range of incidence, as is shown below.

Figure 8.2 shows a comparison of the secondary losses for LS3 as obtained from the conventional and new loss breakdown schemes. As already noted, the conventional loss breakdown scheme produces secondary loss values for large values of positive incidence that are inconsistent with the observed physics. With the new scheme, the results at design

Figure 8.2: Comparison of secondary losses obtained with conventional and new loss breakdown methods.

and +10 degrees are similar to those obtained with the conventional scheme. However, a significant difference is observed at +20 degrees. The secondary loss values obtained with the new scheme are all higher than the corresponding “conventional” values since they include the boundary layer loss that has been generated in the secondary region. The secondary losses obtained with the new scheme now produce a trend with incidence that is physically reasonable.

Figure 8.3 compares the measured values of the incremental secondary loss for LS2 and LS3 as obtained using the new loss breakdown scheme. It can be seen that the off-design secondary losses for LS2 are smaller than those of LS3 at both +10 and +20 degrees of incidence. This is consistent with the results from the conventional loss breakdown, and

Figure 8.3: Comparison of the mixed-out secondary losses at off-design incidence for LS2 and LS3 as obtained from the new loss breakdown scheme.

again, it suggests that secondary losses are lower at off-design incidence for an airfoil with a smaller leading-edge diameter.

8.3 Correlation for the Penetration Depth of the S_4 Separation Line

8.3.1 Motivation and Sources of Data

To estimate the overall losses with the new loss breakdown scheme, a correlation for the penetration depth of the S_4 line is required, as discussed earlier. Sharma and Butler (1987)

Table 8.1: Experimental data for penetration depth correlation.

Reference	Actual Z_{TE}/h	Predicted Z_{TE}/h	F_t	CR	h/C	δ^*/h	α_1 [deg.]	γ [deg.]
Present data, LS3	0.15	0.15	2.38	1.45	1.25	0.011	28.4	21.6
Present data, LS3	0.20	0.19	2.85	1.25	1.25	0.014	38.4	21.6
Present data, LS3	0.20	0.23	3.26	0.98	1.25	0.014	48.4	21.6
Hodson and Dominy (1987a)	0.15	0.13	2.42	1.33	1.82	0.006	38.8	19.6
Hodson and Dominy (1987b)	0.18	0.15	2.94	1.32	1.82	0.006	38.8	19.6
Hodson and Dominy (1987b)	0.18	0.16	2.90	1.15	1.82	0.006	47.4	19.6
Langston et al. (1977)	0.33	0.30	4.60	1.61	0.81	0.012	45.3	34.9
Graziani et al. (1980)	0.25	0.30	4.64	1.60	0.81	0.003	45.4	34.9
Moore and Adhye (1985)	0.35	0.31	4.61	1.61	0.81	0.023	45.3	34.9
Marchal and Sieverding (1977)	0.14	0.12	1.93	2.65	0.83	0.011	0.0	42.5
Marchal and Sieverding (1977)	0.17	0.17	2.70	2.48	0.79	0.021	30.0	38.5
Jilek (1986)	0.06	0.06	1.49	3.86	1.20	0.006	0.0	46.0
Jilek (1986)	0.28	0.26	4.67	1.41	1.20	0.009	57.8	12.5
Walsh and Gregory-Smith (1987)	0.14	0.17	4.39	2.03	1.79	0.015	42.8	36.1
Harrison (1990)	0.23	0.22	4.22	1.94	1.08	0.009	40.0	37.0
Weiss and Fottnner (1995)	0.12	0.12	3.91	1.75	3.00	0.008	37.7	38.5
Weiss and Fottnner (1995)	0.11	0.12	3.75	1.75	3.00	0.008	37.7	30.7
Perdichizzi and Dossena (1993)	0.21	0.21	2.28	3.88	0.91	0.058	13.9	50.1
Yaras (1992)	0.09	0.12	1.27	1.48	0.81	0.018	0.0	37.2

developed such a correlation for use in their semi-empirical model for estimating secondary losses. Due to the scarcity of flow visualization data at the time, their correlation was based on a small database. It consisted of cascade data from 10 test cases, of which 4 were for the same cascade geometry (Langston *et al.*, 1977) but varying inlet boundary layer thicknesses. Since the development of the Sharma and Butler correlation, additional flow visualization data have been published in the open literature. The present author has compiled data from 19 linear cascade experiments and this data suggests that important parameters may be missing from the Sharma & Butler correlation. The expanded data set consists of cases from the Sharma & Butler database, recent cases that were available in the open literature, and also those from the present work. The cascade database is summarized in Table 8.1. The parameters that are included in the database are those which are believed to be the most important in influencing the penetration depth. A detailed discussion of the relationship between penetration depth and these parameters is presented in the following

section. Finally, it should be noted that the database only includes turbine airfoils of modern design.

8.3.2 Factors Influencing Penetration Depth

The starting point for developing a correlation is identifying the geometric and flow parameters that influence the dependent variable most. In this work, the dependent variable is the penetration depth of the S_4 separation line at the trailing edge, Z_{TE} . The passage vortex size is believed to determine this length; as such, the parameters that need to be identified are those that influence the size of the passage vortex. The choice of correlating parameters was based initially on the hypothesis that the passage vortex size was determined by the total airfoil loading, the loading distribution, the channel convergence and the inlet endwall boundary layer thickness. Before discussing the relationship between these parameters and the penetration depth, the Sharma and Butler correlation is presented since it is referred to frequently in this section.

The Sharma and Butler penetration depth correlation is expressed as

$$\frac{Z_{TE}}{C_x} = 0.15 \frac{\Delta\alpha}{\sqrt{CR}} + f\left(\frac{\delta_1}{h}\right) \quad (8.6)$$

where C_x is the airfoil axial chord, δ_1 and h are the inlet boundary layer thickness and airfoil height, respectively. $\Delta\alpha$ is the flow turning in radians, and is an approximate measure of the airfoil loading. The convergence ratio, CR , is an approximate measure of the channel acceleration; it is defined as

$$CR = \frac{\cos \alpha_1}{\cos \alpha_2} . \quad (8.7)$$

The influence of the inlet boundary layer is captured by the function $f\left(\frac{\delta_1}{h}\right)$, and it is

expressed as

$$f\left(\frac{\delta_1}{h}\right) = 1.4\frac{\delta_1}{h} - 2.73\left(\frac{\delta_1}{h}\right)^2 + 1.77\left(\frac{\delta_1}{h}\right)^3. \quad (8.8)$$

As noted earlier, one of the parameters that is believed to influence the penetration depth is the total airfoil loading. For axial turbomachines, the total airfoil loading is largely determined by the total flow turning and the pitch-to-chord ratio and as a result, both should appear as correlating parameters. From Equation 8.6, it can be seen that Sharma and Butler included flow turning as a correlating parameter but the effect of pitch-to-chord was neglected. Since their work, the influence of pitch-to-chord on the three-dimensionality of the endwall flows has been investigated experimentally by Hodson and Dominy (1987b) and Perdichizzi and Dossena (1993). Both groups of investigators observed a growth in the passage vortex with increased pitch-to-chord ratio. In the present correlation the influence of total flow turning and pitch-to-chord ratio are reflected in the tangential loading parameter, F_t , which is defined as

$$F_t = 2\left(\frac{S}{Cx}\right) \cos^2 \alpha_m (\tan \alpha_1 - \tan \alpha_2) \quad (8.9)$$

where α_m is the mean vector angle through the airfoil row and may be expressed as

$$\tan \alpha_m = \frac{1}{2} (\tan \alpha_1 + \tan \alpha_2). \quad (8.10)$$

The parameter, F_t , represents the tangential force per unit length non-dimensionalized by dynamic pressure based on the vector mean velocity. The more familiar Zweifel loading parameter, ψ_T , differs from F_t only in that ψ_T is non-dimensionalized by the dynamic pressure based on the outlet dynamic pressure. Mathematically, the two loading parameters are related as follows

$$\psi_T = F_t \left(\frac{\cos^2 \alpha_2}{\cos^2 \alpha_m} \right). \quad (8.11)$$

It is worth noting that during the development of the correlation, a few different definitions

of loading parameters were evaluated as correlating parameters, including a tangential loading coefficient based on inlet dynamic pressure and the Zweifel loading parameter. The loading coefficient defined in Equation 8.9 gave the best agreement between predicted penetration depth and that observed experimentally.

Weiss and Fottner (1995) investigated the effect of loading distribution on the endwall flows and observed that the penetration depth was slightly larger for a forward-loaded airfoil. Korakianitis and Papagiannidis (1993) argued that the chordwise distribution of the loading is influenced mainly by the stagger angle. They observed that an airfoil tends to become more forward loaded as the stagger angle increases. Although the loading distribution may have a somewhat weak effect on the penetration depth, the stagger angle is retained as a correlating parameter.

To the author's knowledge, no experiments have been conducted to investigate specifically the influence of channel convergence on secondary flows. Nonetheless, it is argued that increasing channel convergence, which results in increased flow acceleration, will tend to reduce the size of the passage vortex through vortex stretching. Based on these arguments, it is expected that the penetration depth should decrease with increasing channel convergence. Evidently, Sharma and Butler arrived at a similar conclusion as the convergence ratio appears as a parameter in their correlation.

The inlet endwall boundary layer thickness has also been shown to have an influence on the penetration depth of the S_4 separation line (Graziani *et al.*, 1980). Using data compiled from the open literature, Sharma and Butler found that the variation in penetration depth due to changes in inlet boundary layer thickness was largely independent of cascade geometry. Results obtained during the development of the present correlation seem to be in agreement with their conclusions, as will be discussed further below.

An initial attempt was made to correlate the penetration depth based on the parameters

described in the preceding paragraphs. The new correlation was somewhat better than the Sharma and Butler correlation; nevertheless, a moderate level of scatter was present suggesting that one or more significant parameters was missing. Subsequently, it was found that the quality of the correlation was improved by including the airfoil aspect ratio as a correlating parameter. The physical relationship between aspect ratio and penetration depth is not immediately evident, notwithstanding that the correlation indicates that it is an important parameter.

The present author believes that the aspect ratio captures two separate physical effects, each of which influences the penetration depth of the S_4 separation line. Changing the aspect ratio by either varying airfoil chord length or height produces these different effects. The influence of a reduction in aspect ratio due to an increase in chord length is considered first. As discussed earlier, the passage vortex strength seems to be influenced mainly by the total loading and the loading distribution. If the chord length is increased while maintaining a given total airfoil loading¹ then the passage vortex strength will remain invariant. Evidently, increasing the airfoil chord also effects an increase in the channel length. Presumably, the passage vortex becomes larger with increasing chord length since the vortex will tend to entrain more fluid as it evolves through a longer passage. A reduction in aspect ratio due to a decrease in height seems to have a different physical effect on the penetration depth of the S_4 separation line. As the airfoil height is decreased the blockage produced by the endwall flows becomes a large proportion of the total flow area. Thus, the effective flow convergence will tend to increase, which as argued earlier, reduces the penetration depth of the S_4 separation line.

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¹To maintain a given airfoil loading while increasing the chord requires a proportional increase in the airfoil spacing.

8.3.3 New Correlation

For the new correlation, the penetration depth of the S_4 separation line was assumed to vary as follows:

$$\frac{Z_{TE}}{h} = a (F_t)^u (CR)^v \left(\frac{h}{C}\right)^w (1 + b \sin^x \gamma) + y \left(\frac{\delta^*}{h}\right)^z \quad (8.12)$$

where

- Z_{TE}/h = non-dimensional penetration depth,
- CR = convergence ratio, defined in Equation 8.7,
- F_t = tangential loading coefficient, defined in Equation 8.9,
- h/C = airfoil aspect ratio,
- δ^*/h = non-dimensional inlet endwall boundary layer displacement thickness,
- γ = stagger angle in degrees, and
- $a, b, u, v,$
 w, x, y, z = empirical coefficients.

A few points are noteworthy regarding the form of the new correlation. Like the Sharma and Butler correlation, the influence of the inlet endwall boundary layer displacement thickness on the penetration depth was assumed to be independent of cascade loading and flow convergence. The alternative was to include the boundary layer parameter as a multiplier to the remaining correlating parameters. However, if δ^*/h was a multiplier, it would make $Z_{TE}/h \rightarrow 0$ as $\delta^*/h \rightarrow 0$ which is clearly incorrect. That is, even for the hypothetical case of zero thickness at the inlet, the endwall boundary layers would continue to develop inside the passage, which in turn, would result in growth of the secondary flows. It is worth noting that the quality of the correlation was somewhat better with the form given in Equation 8.12. Finally, the influence of the stagger angle in Equation 8.12 was assumed to take the form $(a + b \sin^x \gamma)$ allowing the correlation to apply for impulse turbines (i.e., $\gamma = 0.0$).

Having assumed a suitable form for the correlation, the problem then is to determine the values of the empirical coefficients in Equation 8.12 that give the best agreement between the

predicted and measured penetration depths. Establishing the values of these coefficients was treated as a problem in optimization. The process of optimizing was performed with a code written by Sjolander (2001), and is based on a genetic algorithm optimization technique. Islam and Sjolander (1999) used this code with apparent success for the development of a correlation for deviation in axial turbines. The fundamentals of the genetic algorithm technique and details of the code are described by Islam (1999).

To demonstrate the improvements achieved with the new correlation, the experimental data are compared first with the Sharma and Butler correlation. Figure 8.4 compares the predicted values of the penetration depth with the measured values for all the cases in Table 8.1. The solid line represents perfect agreement between the predicted and measured values; the dashed lines represent a ± 0.025 error band on the predicted penetration depth. In Figure 8.4, it is seen that the agreement between the predicted and measured values is rather poor since a majority of the predictions fall outside the plotted error band. Moreover, a number of the predictions deviate significantly from the measured values indicating that one or more important parameters has probably been omitted.

The expression for the new penetration depth correlation is given in Equation 8.13. The empirical coefficients were obtained using the genetic-algorithm optimization code discussed earlier.

$$\frac{Z_{TE}}{h} = \frac{0.10(F_t)^{0.79}}{\sqrt{CR} \left(\frac{h}{C}\right)^{0.55}} + 32.70 \left(\frac{\delta^*}{h}\right)^2 \quad (8.13)$$

One salient point is to be noted regarding Equation 8.13. The results from the optimization process indicated that the penetration depth was insensitive to stagger angle, so it was omitted in the final form of the correlation. This result was not completely unexpected given that Weiss and Fottner (1995) showed only a 1% change in penetration depth for a fairly significant change in stagger angle (approximately 8 degrees).

Figure 8.4: Evaluation of the penetration depth correlation of Sharma and Butler.

Figure 8.5 shows the evaluation diagram for the new penetration depth correlation. As can be seen, the agreement between the measured and predicted values of penetration depth is quite satisfactory. Evidently, the new correlation is a significant improvement over the Sharma and Butler correlation (Figure 8.4).

8.4 Secondary Loss Correlation

8.4.1 Motivation, Sources of Data and Range of Validity

As discussed above, the definition of secondary loss in the new loss breakdown scheme now includes the losses generated in the thin boundary layer that develops between the

Figure 8.5: Evaluation of the new penetration depth correlation.

endwall and the S_4 separation line. Therefore, a correlation is required for predicting the newly defined secondary loss component. The remaining part of this section details the sources of data that were used for the correlation while the subsequent section describes the parameters that are believed to influence secondary losses. The new correlation is presented in the last section.

The new secondary loss correlation was developed from a database of 34 experimental cases, which are summarized in Table 8.2. Included in the data set are cases from the present experiment as well as recent cases from the open literature (within the past several decades) for which sufficient geometric and aerodynamic information were given, or could be estimated with acceptable accuracy. Therefore, the new correlation accounts for recent improvements in airfoil design and incorporates improvements in the understanding of the

Table 8.2: Experimental data for secondary loss correlation.

Case No.	Reference	Description	Y _{secondary}		Location of Measurement Plane, x' / Cx	Z _{ref} /h Predicted	C _t	F _t	Z	s/Cx	Δα [deg.]	CR	h/Cx	δ*/h	α ₂ [deg.]	γ [deg.]
			Measured	Predicted												
1	Present Data	LS3, i=0 deg.	0.026	0.034	0.40 *	0.15	0.0031	2.39	5.40	0.741	81.0	1.45	1.363	0.011	52.6	21.6
2	Present Data	LS3, i=10 deg.	0.047	0.040	0.40 *	0.19	0.0030	2.86	6.66	0.741	89.4	1.25	1.363	0.014	51.1	21.6
3	Present Data	LS3, i=20 deg.	0.063	0.050	0.40 *	0.23	0.0030	3.27	8.97	0.741	95.7	0.98	1.363	0.014	47.3	21.6
4	Present Data	LS2, i=0 deg.	0.034	0.035	0.40 *	0.15	0.0030	2.40	5.48	0.738	82.0	1.48	1.358	0.013	53.6	23.1
5	Present Data	LS2, i=10 deg.	0.047	0.040	0.40 *	0.19	0.0031	2.93	6.75	0.738	91.7	1.31	1.358	0.013	53.3	23.1
6	Present Data	LS2, i=20 deg.	0.042	0.051	0.40 *	0.24	0.0029	3.36	8.92	0.738	97.4	1.01	1.358	0.017	49.0	23.1
7	Hodson and Dornier (1987a)	Design	0.029	0.030	0.42 *	0.13	0.0034	2.42	6.84	0.600	92.9	1.33	1.934	0.006	54.1	19.6
8	Hodson and Dornier (1987b)	+22% s/C	0.034	0.030	0.42 *	0.15	0.0034	2.94	6.83	0.732	92.6	1.32	1.934	0.006	53.8	18.2
9	Hodson and Dornier (1987b)	-18% s/C	0.021	0.030	0.42 *	0.11	0.0034	1.99	6.84	0.492	93.0	1.33	1.934	0.006	54.2	20.6
10	Hodson and Dornier (1987b)	i=+8.6 deg.	0.044	0.034	0.42 *	0.16	0.0033	2.90	8.47	0.600	101.5	1.15	1.934	0.006	54.1	19.6
11	Hodson and Dornier (1987b)	i=-20.3 deg.	0.019	0.025	0.42 *	0.08	0.0036	1.61	4.51	0.600	72.0	1.59	1.934	0.006	53.5	19.6
12	Hodson and Dornier (1987b)	Thick inlet boundary layer (IBL)	0.028	0.032	0.42 *	0.13	0.0032	2.42	6.83	0.600	92.7	1.32	1.934	0.013	53.9	19.6
13	Marchal and Sieverding (1977)	Nozzle, thin IBL	0.035	0.031	0.30	0.12	0.0044	1.93	5.42	0.983	67.8	2.65	1.130	0.011	67.8	42.5
14	Marchal and Sieverding (1977)	Nozzle, thick IBL	0.034	0.039	0.30	0.16	0.0036	1.92	5.55	0.983	68.3	2.70	1.130	0.038	68.3	42.5
15	Marchal and Sieverding (1977)	Rotor	0.041	0.039	0.30	0.17	0.0040	2.70	7.54	0.875	99.6	2.48	1.006	0.021	69.6	38.5
16	Harrison (1990)		0.040	0.035	0.23 *	0.23	0.0032	4.31	7.65	1.036	106.5	1.85	1.351	0.009	65.5	37.0
17	Weiss and Fottner (1995)	T104 Front-loaded	0.015	0.020	0.50 *	0.12	0.0025	3.91	7.20	0.969	100.9	1.75	3.833	0.008	63.2	38.5
18	Weiss and Fottner (1995)	T106 Rear-loaded	0.012	0.020	0.50 *	0.12	0.0025	3.75	7.20	0.929	100.9	1.75	3.490	0.008	63.2	30.7
19	Perdichizzi (1990)	M2is=0.3; i =20 deg.	0.041	0.042	1.10	0.19	0.0031	2.42	5.38	0.889	79.3	2.03	1.482	0.042	61.9	45.2
20	Perdichizzi and Dossena (1993)	M2is=0.7; s/C=0.58; i=0 deg.	0.046	0.038	0.50	0.20	0.0027	1.87	8.48	0.942	88.8	3.73	1.470	0.058	74.9	50.1
21	Perdichizzi and Dossena (1993)	M2is=0.7; s/C=0.73; i=0 deg.	0.038	0.038	0.50	0.22	0.0026	2.42	8.16	1.185	88.1	3.57	1.470	0.058	74.2	50.1
22	Perdichizzi and Dossena (1993)	M2is=0.7; s/C=0.87; i=0 deg.	0.042	0.041	0.50	0.26	0.0025	3.17	7.19	1.412	85.5	3.08	1.470	0.058	71.6	50.1
23	DeCecco (1995)	LS1	0.053	0.044	0.48 *	0.12	0.0038	1.27	2.47	0.754	47.4	1.48	1.020	0.018	47.4	37.2
24	Gregory-Smith et al. (1988)	Thin IBL	0.040	0.028	0.29 *	0.18	0.0029	4.69	8.14	1.091	110.3	1.92	2.611	0.021	67.5	35.9
25	Gregory-Smith et al. (1988)	Natural IBL	0.034	0.030	0.29 *	0.19	0.0028	4.69	8.14	1.091	110.3	1.92	2.611	0.027	67.5	35.9
26	Gregory-Smith et al. (1988)	Thick IBL	0.044	0.039	0.29 *	0.28	0.0018	4.69	8.14	1.091	110.3	1.92	2.611	0.061	67.5	35.9
27	Yan et al. (1999)		0.012	0.016	0.26	0.06	0.0056	1.59	7.42	0.919	74.0	3.62	2.626	0.006	74.0	47.3
28	Kopper et al. (1981)	Planar endwalls	0.036	0.034	0.28	0.18	0.0036	3.25	10.32	2.347	78.7	5.08	1.282	0.025	78.7	66.4
29	Atkins (1987)	Planar endwalls	0.034	0.042	0.50	0.24	0.0033	3.77	7.28	0.870	100.0	1.50	1.206	0.009	59.5	37.8
30	Chen and Dixon (1986)	δ*/C=0.0404	0.020	0.023	0.91	0.11	0.0034	2.75	7.00	0.732	98.8	1.87	2.654	0.014	64.0	27.2
31	Moharak et al. (1988)	Cascade No. 3	0.018	0.027	0.86	0.10	0.0030	1.54	7.23	0.780	79.7	3.34	1.489	0.028	72.7	38.6
32	Yamamoto and Nouse (1987a)	Part I - Stator	0.019	0.032	0.28	0.12	0.0033	1.76	5.24	0.942	64.8	2.63	1.230	0.023	67.7	38.7
33	Yamamoto and Nouse (1987b)	Part II - Rotor	0.051	0.044	0.23	0.20	0.0024	3.63	7.35	0.846	101.8	1.61	1.387	0.020	61.7	44.7
34	Yamamoto and Nouse (1988)	i=7.2 deg.	0.064	0.075	0.24	0.36	0.0041	5.49	10.75	0.846	117.0	1.09	1.387	0.038	60.0	44.7

physics of secondary flows that have been made since the early 1980's.

As can be seen in Table 8.2, the database consists of cases that cover a wide range of incidence values (-23 to +20 degrees), acceleration levels (nozzle to near-impulse airfoils), aspect ratios (0.79 to 3.0), flow turning (47 to 110 degrees) and inlet endwall boundary layer displacement thickness (from 0.6% to 6.0% of blade height).

The new loss breakdown scheme, like the old one, assumes that the secondary flows from each of the endwalls do not merge at midspan. This limits the range of validity of the correlation to preclude low-aspect ratio geometries; that is, cases where the secondary flows are most likely to merge. Unfortunately, a precise value for the lower limit of aspect ratio cannot be specified since the penetration of the secondary flows is also a function of, amongst other parameters, the airfoil loading. A number of cases from the open literature for which the secondary flows have merged at midspan have been omitted from the database (e.g., Graziani *et al.*, 1980; Moore and Ransmayr, 1984, all data in Mobarak *et al.*, 1988, except the case with the highest aspect ratio and Yamamoto and Nouse, 1988, at positive incidence and high negative incidence).

The secondary loss values that appear in Table 8.2 were evaluated with the new loss breakdown scheme using predicted penetration depth values and the experimental values of mixed-out overall and midspan losses. Unfortunately there were a number of cases in the open literature for which insufficient loss data were given (e.g., Langston *et al.*, 1977; Moore and Adhye, 1985; Jilek, 1986). In a number of other cases the mixing losses were estimated because the mixed-out losses were not quoted. The procedure that was followed to arrive at these estimates is detailed below. It should be noted that the sixth column in Table 8.2 gives the location of the downstream measurement plane. For cases where measurements were made at multiple axial locations, the furthest downstream location is quoted. If mixed-out loss values have been given then an asterisk is used to identify the case. Also, the values of

non-dimensional penetration depth, as predicted using Equation 8.13, are included in Table 8.2.

Based on the results from this work and the work of Moore and Adhye (1985), Hodson and Dominy (1987a), Hodson and Dominy (1987b) and Harrison (1990), an approximate expression for the overall mixing losses for cases at the design incidence is

$$\frac{\Delta Y_{total(mixing)}}{Y_{total(measured)}} = \begin{cases} (1 - 2x'/Cx) & \text{for } x'/Cx \leq 0.5 \\ 0.0 & \text{for } x'/Cx > 0.5 \end{cases} \quad (8.14)$$

where x'/Cx is the non-dimensional axial location of the measurement plane as measured from the trailing-edge plane. With this approximation, mixing is assumed to be complete at 50% axial chord lengths downstream of the trailing edge. If losses were quoted at multiple measurement planes then the mixing losses were estimated from the measurements made at the furthest downstream plane, thus, minimizing any approximation error.

For the mixing that occurs at midspan, the results of Goobie *et al.* (1989) suggest that the wakes are nearly completely mixed out by 20% axial chord lengths downstream of the trailing-edge. Therefore, the measured midspan losses for all cases in the database provide reasonable estimates of the fully-mixed out losses since all measurement locations were at least 20% axial chord lengths downstream of the trailing edge.

8.4.2 Factors Influencing Secondary Losses

Dunham (1970) and Sieverding (1985a) performed detailed reviews of the twenty or so secondary loss correlations that have been proposed in the past half century. Based on these reviews, there appears to be a number of factors that influence endwall losses: these are total airfoil loading, flow acceleration, size of the endwall surface area and the vorticity in the inlet endwall boundary layer. Moreover, based on the results from the present study

and the work of Weiss and Fottner (1995) and Marchal (1980) as referred to by Sieverding (1985*b*), it also seems that loading distribution is an important factor.

The precise influence that each of these factors has on endwall loss generation is still generally not fully understood - this is undoubtedly an indication of the complexity of the endwall flows. Consequently, there is very little agreement in the literature concerning the geometric and aerodynamic parameters that are important in capturing the factors that influence loss generation. This is particularly the case for airfoil loading, as will now be discussed.

In his review of secondary loss correlations, Dunham (1970) evaluated the quality of some of the more widely-used secondary loss correlations, by comparing the predictions to a comprehensive set of experimental data. Although the data set was comprised of results from a number of different sources, a majority of the comparisons were made against the results of Wolf (1961). Nonetheless, Dunham conclusively showed that the loading parameter devised by Ainley and Mathieson (1951) produced the best agreement between predictions and the then-available experimental data. Therefore, Dunham and Came (1970) retained the Ainley and Mathieson loading parameter in their secondary loss correlation, as did Kacker and Okapuu (1982).

Ainley and Mathieson's loading parameter, Z , takes the following form

$$Z = \left(\frac{C_L}{s/c} \right)^2 \frac{\cos^2 \alpha_2}{\cos^3 \alpha_m} \quad (8.15)$$

where C_L is the lift coefficient based on the vector mean velocity. The lift coefficient may be expressed as

$$C_L = 2 \frac{s}{c} (\tan \alpha_1 + \tan \alpha_2) \cos \alpha_m. \quad (8.16)$$

where α_m is the vector mean gas angle (defined in Equation 8.10). It is worth noting

that the lift coefficient, which is a measure of airfoil loading, is directly proportional to pitch-to-chord ratio, as shown in Equation 8.16. Substituting Equation 8.16 into 8.15 gives

$$Z = 4(\tan \alpha_1 + \tan \alpha_2)^2 \frac{\cos^2 \alpha_2}{\cos \alpha_m} . \quad (8.17)$$

As can be seen from Equation 8.17, the Ainley and Mathieson loading parameter depends only on the inlet and outlet flow angles and not on the pitch-to-chord ratio. This result is somewhat surprising given the premise that secondary losses are dependent on airfoil loading. The Craig and Cox correlation, on the other hand, predicts a strong dependence of secondary losses on pitch-to-chord ratio. More recently, Hodson and Dominy (1987b) and Perdichizzi and Dossena (1993) examined the influence of pitch-to-chord ratio on the secondary loss behaviour. Hodson and Dominy found that the secondary losses increased significantly with increasing pitch-to-chord ratio (i.e., increased airfoil loading) while Perdichizzi and Dossena observed no clear trend.

Based on the correlations (e.g., Ainley and Mathieson, 1951; Craig and Cox, 1971) and the more recent experimental studies (e.g., Hodson and Dominy, 1987b; Perdichizzi and Dossena, 1993), it is clear that the various groups have observed different sensitivities of secondary loss production to variations in pitch-to-chord ratio. Evidently more data are necessary to help improve our understanding of the influence of pitch-to-chord ratio on secondary losses. Nonetheless, in light of what is now known about the influence of loading distribution, the present author believes that there is a plausible explanation for the different sensitivities that have been observed.

It is postulated that the importance of pitch-to-chord ratio in terms of secondary loss production is dependent on the loading distribution of the airfoil. For example, a rear-loaded airfoil may be less sensitive to loading variations by comparison with more front-loaded airfoils. It is entirely possible that the airfoils tested by Craig and Cox (1971)

were designed according to a different loading-distribution philosophy than those tested by Ainley and Mathieson (1951) and Wolf (1961). As will be discussed in the next section, the correlation developed in this work does not seem to show a dependence of secondary losses on the pitch-to-chord ratio.

An important source of loss in the endwall region comes from the entropy that is produced in the endwall boundary layers within, upstream and downstream of the airfoil row. For an airfoil with an aspect ratio of unity, Denton (1993) estimated that two-thirds of the total secondary loss was generated within the endwall boundary layers. To gain some insight into the parameters that influence the endwall boundary losses it is useful to examine an expression for the total-pressure loss coefficient as derived from the concepts of entropy generation.

To begin, the entropy generation rate for the endwall boundary layer can be expressed as (Denton, 1993):

$$\dot{S}_{endwall} = \frac{\rho}{T} \int_0^{Cx} \int_0^w C_d u_e^3 dy dx \quad (8.18)$$

where C_d is the boundary layer dissipation coefficient, u_e is the velocity at the edge of the boundary layer, x and y are the axial and pitchwise coordinates, respectively. The flow is assumed incompressible with negligible temperature variations. The dissipation coefficient can be interpreted as a dimensionless form of the viscous shear work, and for a two-dimensional boundary layer, it is expressed as

$$C_d = \int_0^\delta \frac{\tau}{\rho u_e^2} \frac{d}{dy} \left(\frac{u}{u_e} \right) dy \quad (8.19)$$

where τ is the shear stress, u is the local flow velocity, and ρ is the fluid density.

A few salient points are worth making in connection with Equation 8.18. First, as discussed by Denton (1993), the variation of the dissipation coefficient is known only in very

general terms since the characteristics of the endwall layer have not been well documented, particularly downstream of the S_2 separation line where the layer is very thin (Langston *et al.*, 1977). However, based on the work of Harrison (1990), it seems that a constant value for the dissipation coefficient can be assumed as a rough approximation. Second, the dependence of the entropy generation rate on the cube of the boundary layer edge velocity is very important in that it shows that areas of high velocity will tend to dominate loss production. Due to the strong cross flows induced by the passage vortex, the largest edge velocities within the airfoil passage probably exist in the region of the endwall that lies underneath this vortex. Therefore, any mechanism that increases the strength of the vortex will tend to increase entropy production in the endwall boundary layers. Based on the arguments presented in Chapter 6, a factor that seems to be important in influencing the strength of the passage vortex is the airfoil loading in the forward part of the passage. Evidently, this provides some justification for including a loading coefficient and stagger angle (an indicator of loading distribution) as correlating parameters.

From Equation 8.18, it can be seen that the endwall boundary losses are related to the surface area, as one might expect. The parameter that captures the influence of the size of the endwall surface area may be determined by converting Equation 8.18 into a mass-averaged loss coefficient. This coefficient may be thought of as the contribution of the endwall losses to the total secondary pressure loss coefficient, and can be expressed as

$$Y_{endwall} = \frac{T \dot{S}_{endwall}}{\dot{m} \left(1/2V_2^2\right)} \quad (8.20)$$

for incompressible flows. In Equation 8.20, $\dot{m}$ is the mass flow rate through the passage; it may be written as

$$\dot{m} = \rho (h S)(V_2 \cos \alpha_2) \quad (8.21)$$

and V_2 is flow velocity at the exit of the airfoil row. Substituting Equations 8.18 and 8.21

into 8.20 and simplifying gives

$$Y_{endwall} = \frac{2 \int_0^{Cx} \int_0^w C_d \left(\frac{V_e}{V_2} \right)^3 dy dx}{(h S) \cos \alpha_2}. \quad (8.22)$$

The integrand in Equation 8.22 is a non-dimensional form of the entropy generation rate per unit surface area, and is denoted as $\dot{S}_a$. If an average entropy generation rate is assumed over the endwall surface area, then Equation 8.22 can be rewritten as

$$Y_{endwall} = \frac{2 \bar{\dot{S}}_a \int_0^{Cx} \int_0^w dy dx}{(h S) \cos \alpha_2}. \quad (8.23)$$

If we now make the simplification that the airfoils are infinitely thin then the suction to pressure surface pitchwise distance, w , can be replaced with the airfoil pitch. Equation 8.23 then simplifies to

$$Y_{endwall} = 2 \bar{\dot{S}}_a \left(\frac{Cx}{h \cos \alpha_2} \right). \quad (8.24)$$

From Equation 8.24, it can be seen that the influence of the endwall surface area on loss coefficient is captured by the term in brackets. An important and commonly observed result, at least for cases where the aspect ratio is greater than two (Kacker and Okapuu, 1982), is the inverse dependence of the endwall losses on the airfoil height.

The most widely-used secondary loss correlations (e.g., Mukhtarov and Krichakin, 1969; Craig and Cox, 1971; Traupel, 1977; Ainley and Mathieson, 1951; Dunham and Came, 1970; Kacker and Okapuu, 1982), show that channel acceleration has a strong influence on secondary losses. In each of these correlations, the losses vary as the inverse of the convergence ratio, a parameter which is an approximate measure of the channel acceleration. To the author's knowledge, there have been no studies that have examined specifically the influence of channel acceleration on secondary flows; however, significant variations do occur

with off-design incidence. Thus, the results from the present experiment are believed to provide some insights into the influence of channel acceleration on secondary loss generation.

This relationship is physically complex primarily because many of the loss generating mechanisms are influenced by the acceleration of the flow through the passage. First, the influence of acceleration on the secondary kinetic energy of the SS1 and passage vortices is considered. In the case of a vortex that is evolving in an accelerating flow, the secondary kinetic energy of the vortex increases due to vortex stretching. The vortex stretching is not itself an entropy generating process, but in a cascade there is no mechanism to recover the secondary kinetic energy so that it will eventually be dissipated due to mixing (Denton, 1993). Based on these arguments one must conclude that if the flow acceleration is reduced, vortex stretching should be decreased, which in turn, should produce lower mixing losses. This trend contradicts the correlations, which indicate an increase in secondary losses as the flow acceleration is reduced. Evidently, there are other loss generating mechanisms that mask the effects of vortex stretching on the losses.

In Sections 7.5.2 and 7.5.3, it was argued that the reduction in both the positive and negative streamwise vorticity levels observed as incidence increased from +10 to +20 degrees was due to the reduced acceleration. The reduction in acceleration seems to cause an increase in the interaction between the passage vortex and the suction surface and endwall boundary layers, thus, providing a mechanism for generating entropy. Moreover, the measurements presented earlier showed that the SS1 and passage vortices were coaxial with a region of streamwise momentum deficit. Based on arguments presented by Denton (1993), the rate of entropy generation that results from the mixing out of this wake will be influenced by flow acceleration. If a case is considered where the wake is accelerated as it convects downstream, then the cross-stream velocity gradient within the wake will be attenuated. Therefore, the entropy generation rate, which is proportional to the square

of this velocity gradient (Denton, 1993), will tend to increase as the flow acceleration is reduced.

In summary, there is unanimity amongst the most widely-used secondary loss correlations concerning the effect of channel acceleration on secondary losses. Channel acceleration does, however, appear to influence several entropy production mechanisms. For example, if one considers a reduction in flow acceleration then secondary kinetic energy levels will tend to decrease, thus resulting in lower mixing losses. In contrast, the interaction between the endwall and suction surface boundary layer and the mixing of the vortex wake will both be amplified, thereby enhancing entropy production. Since the correlations show an increase in loss with reduced flow acceleration, it must be concluded that the latter two mechanisms combined are the dominant sources of entropy production.

The influence of inlet endwall boundary layer thickness on secondary losses has been the subject of numerous experimental studies. Evidence from many of these studies (e.g., Marchal and Sieverding, 1977; Chen and Dixon, 1986; Sharma and Butler, 1987; Hodson and Dominy, 1987b; Chan *et al.*, 1994) suggests that the secondary losses are insensitive to the thickness of the inlet endwall layer. In contrast, Dunham (1970) reviewed data from ten different studies and concluded that secondary losses were dependent on inlet boundary layer thickness: the largest sensitivity of secondary losses to thickness variations occurred for very thin layers. Others (e.g., Wolf, 1961; Came, 1973; Sauer and Wolf, 1994) have reported a rising trend in secondary loss with increasing thickness up until a “critical” thickness beyond which there was no further dependence.

There appear to be a few possible explanations for the differing observed loss behaviours. First, it is possible that the boundary layer thickness (or some measure of it, such as displacement or momentum thickness) is not the most relevant parameter for capturing the relationship between the inlet endwall boundary layer and secondary losses. A second

possibility is that there is some other important, “uncontrolled” inlet boundary layer parameter that confounds the influence of the inlet boundary layer thickness. Some of the insights gained from the present work as well as the results of Sauer and Wolf (1994) seem to support the latter postulate, as will now be discussed.

The following argument is based primarily on the notion that the production of streamwise vorticity from the vorticity contained in the inlet endwall boundary layer has an important role in the production of entropy within and downstream of the airfoil row. This idea seems to be widely accepted since the passage vortex originates from the vorticity contained within the inlet endwall boundary layer. The secondary kinetic energy associated with this vortex as well as the interaction of the vortex with the suction and endwall boundary layers appear to be significant sources of entropy production in the endwall region.

In turbines, one of the main mechanisms of streamwise vorticity production occurs through the deflection of pre-existing vorticity (i.e., vorticity within the inlet endwall boundary layer). This process is essentially inviscid in nature, and it has been shown that the production of streamwise vorticity at a particular spanwise location is directly proportional to the magnitude of the vorticity and the amount of flow turning (Hawthorne, 1954). Therefore, the influence that the inlet endwall layer has on the passage vortex strength is dependent mainly on the distribution of vorticity and the overall circulation in the inlet endwall boundary layer. The latter quantity, per unit streamwise distance, is simply the “jump” in velocity across the layer. It is invariant if the Reynolds number based on inlet velocity is fixed, as would be the case in experiments that are examining solely the influence of boundary layer thickness. In most turbine cascade experiments, the inlet endwall layers are turbulent (using the quoted values of shape factor as a basis) so that most of the streamwise vorticity is produced deep within the layer where the velocity gradients are largest (i.e., within the laminar sublayer and the log-law region). As a result, it is the

characteristics of this region that will tend to have an effect on secondary losses. The streamwise vorticity production due to the gradients in the outer layer will be negligible, by comparison. For these reasons the present author believes that the skin friction coefficient, which is an indicator of the magnitude of the velocity gradients in the inner layer, may be an important correlating parameter for secondary losses. The results of Hodson and Dominy (1987b) and Sauer and Wolf (1994) seem to support this hypothesis, as will be discussed below. First, however, it should be noted that the values of the inlet endwall skin friction coefficient, C_f , are given in Table 8.2 for each of the cases in the database. Estimates of C_f were made using a correlation based on Coles' law of the wake (Equation 6-120 in White, 1991):

$$C_f = \frac{0.3}{e^{1.33H} (\log_{10} Re_\theta)^{1.74+0.31H}} \quad (8.25)$$

where H is the measured boundary layer shape factor, Re_θ is the Reynolds number based on momentum thickness, and C_f is defined as

$$C_f = \frac{\tau_w}{1/2 \rho u_e^2}. \quad (8.26)$$

In Equation 8.26, τ_w is the wall shear stress, and u_e is the flow velocity at the edge of the boundary layer.

As mentioned in the preceding paragraph, the results of Hodson and Dominy (1987b) and Sauer and Wolf (1994) seem to indicate some dependence of secondary losses on the inlet endwall skin friction coefficient. In the Hodson and Dominy study, the inlet endwall boundary layer thickness was doubled and a small reduction in secondary losses was observed. As shown in Table 8.2, the estimated values of the skin friction coefficients for the thin and thick boundary layers were 0.0036 and 0.0032, respectively. The skin friction coefficient for the thin boundary layer case was slightly larger than the corresponding thick case; consequently, the velocity gradients in the inner layer would tend to be larger. The

higher measured secondary losses for the thin layer case is consistent with the proposed hypothesis since it is the case with the larger skin friction coefficient. The results of Sauer and Wolf (1994) also suggest that the gradients in the inner layer influence the secondary losses. With their set-up, they were able to vary the boundary layer thickness and the near-wall velocity gradients independently. Presented in the paper were two cases that had identical inlet layer displacement thicknesses but different velocity gradients in the inner layer. For the case with smaller near-wall gradients, lower endwall losses were measured. Based on the results from the two research groups, the velocity gradients in the inner-layer region and the associated vorticity strength seem to influence the secondary losses.

It may also be argued that the inlet boundary layer displacement thickness-to-airfoil height (δ^*/h) should appear as a correlating parameter for secondary losses. This can be best explained if one considers the influence of the secondary kinetic energy on the secondary loss coefficient. From previous discussions, it was concluded that the passage vortex generates secondary kinetic energy in amount which is related to the flow turning and the circulation in the inner region of the inlet boundary layer. This secondary kinetic energy is eventually dissipated through mixing and must be considered as a source of loss. However, from the view point of a mass-averaged secondary loss coefficient, the amount of mass associated with this secondary kinetic energy is also important. The contribution of the secondary kinetic energy to the secondary loss coefficient may be expressed as

$$\Delta Y_{secondary} = \frac{\dot{m}_{pv} SKE}{\dot{m}} \quad (8.27)$$

where $\dot{m}_{pv}$ is the fraction of mass flow through the passage vortex tube that contains fluid of high secondary kinetic energy and $\dot{m}$ is the mass flow through the passage. The mass flow, $\dot{m}_{pv}$, may be expressed in terms of the inlet boundary layer displacement thickness since it has been concluded that the passage vortex is comprised mainly of fluid from the

inlet endwall boundary layer. That is,

$$\dot{m}_{pv} = a \rho V_{CL} \cos \alpha_1 \delta^* s \quad (8.28)$$

where a is the fraction of the mass flow through the passage vortex tube that contains fluid of high secondary kinetic energy. The mass flow through the passage may be written as

$$\dot{m} \approx \rho V_{CL} \cos \alpha_1 (h s) \quad (8.29)$$

The approximation arises because the displacement thickness is assumed to be small relative to the airfoil height. Substituting Equations 8.28 and 8.29 into 8.27 gives

$$\Delta Y_{secondary} = \left(a \frac{\delta^*}{h} \right) (SKE) . \quad (8.30)$$

The important conclusion here is that the overall secondary losses may depend, at least to some degree, on the displacement thickness-to-height ratio.

8.4.3 New Correlation

The new secondary loss correlation was assumed to take the following form

$$Y_{secondary} = (F_{loading})^a (\cos^b \gamma) (CR)^c \left(\frac{h \cos \alpha_2}{Cx} \right)^d \left[w + x (C_f)^e \tanh \left(f \frac{\delta^*}{h} \right) \right] . \quad (8.31)$$

Table 8.3 defines the parameters in Equation 8.31 and it describes qualitatively the physical effect that each parameter is intended to measure. The variables a , b , c , d , e , f , w , x are empirical coefficients, and their values were determined using the same optimization procedure previously described in Section 8.3.3. Equation 8.31 is of similar form to the Dunham and Came (1970) and Kacker and Okapuu (1982) correlations. However, there is

Table 8.3: List of correlating parameters for new secondary loss correlation.

Parameter	Definition	Description
F_{loading}	Aerofoil loading coefficient: $F_t = 2(s/Cx)\cos^2\alpha_m(\tan\alpha_1 - \tan\alpha_2)$, $F_t/(s/Cx) = 2\cos^2\alpha_m(\tan\alpha_1 - \tan\alpha_2)$, $Z = 4(\tan\alpha_1 + \tan\alpha_2)^2\cos^2\alpha_2/\cos\alpha_m$	Measure of total aerofoil loading.
$\cos\gamma$	/	Indicator of aerofoil loading distribution.
CR	Convergence ratio: $CR = \cos\alpha_1/\cos\alpha_2$	Indicator of flow acceleration.
$\frac{Cx}{h(\cos\alpha_2)}$	/	Captures influence of endwall surface area.
C_f	Skin friction coefficient: $C_f = \frac{0.3}{e^{1.33H}(\log_{10}Re_\theta)^{1.74+0.31H}}$	Indicator of vorticity strength in the inner layer of inlet endwall boundary layer.
δ'/h	Displacement thickness-to-aerofoil height	Indirect measure of ratio of mass flow in the endwall region to the passage mass flow rate.

one notable difference: the earlier correlations ignore the effect of the inlet endwall boundary layer on the secondary losses. This was done because details of the endwall boundary layer are typically not known in the early stages of design. Finally, the effect of the displacement thickness-to-airfoil height ratio in Equation 8.31 is expressed in terms of a hyperbolic tangent function because it produces a similar loss trend to the ones observed by Came (1973) and Hultsch and Sauer (1979).

The optimization procedure was performed for three cases, each with the same form as Equation 8.31 but with different loading parameters. In the first case, the tangential loading coefficient, F_t , which is a function of the pitch-to-axial chord ratio and the inlet and outlet flow angles (Table 8.3), was used as the measure of total airfoil loading. For the second case, the loading parameter in Equation 8.31 was taken as

$$F_{\text{loading}} = \frac{F_t}{s/Cx} \quad (8.32)$$

such that the loading parameter is only a function of the inlet and outlet flow angles.

Finally, the optimization was also performed with Ainley and Mathieson's loading coefficient because it has had documented success in capturing the influence of total airfoil loading on secondary losses (Dunham and Came, 1970). The quality of the correlations was assessed using evaluation diagrams, as was done for the work on the penetration depth correlation.

Several observations are worth making in connection with the optimization results.

It was observed that the secondary losses were only weakly dependent on the total airfoil loading. Therefore, no particular difference in the quality of the correlation was apparent with any of the three loading parameters, at least to within the uncertainty of the correlation. Consequently, the influence of the total airfoil loading was omitted in the final form of the correlation. A tentative explanation for this result is given later.

A second observation concerns the influence of the skin friction coefficient. The initial results from the optimizer indicated that the secondary losses were inversely dependent on the skin friction coefficient, as shown in the following expression:

$$Y_{secondary} = \frac{0.038 + 0.09 \tanh(0.98 \delta^*/h)}{(C_f)^{0.28} (\cos \gamma)^{0.47} (CR) \left(\frac{h \cos \alpha_2}{Cx} \right)^{0.58}}. \quad (8.33)$$

Subsequently, however, the optimizer was run with the skin friction coefficient omitted as a correlating parameter, and no degradation in the quality of the correlation was observed by comparison with the earlier run. With the omission of the skin friction, the expression for the secondary losses was

$$Y_{secondary} = \frac{0.038 + 0.41 \tanh(1.20 \delta^*/h)}{\sqrt{\cos \gamma} (CR) \left(\frac{h \cos \alpha_2}{Cx} \right)^{0.55}}. \quad (8.34)$$

Interestingly, the only coefficients that are significantly different between the correlations are those associated with the inlet endwall boundary layer. Although the significance of this

is not immediately clear, it may be because δ^*/h and C_f are not completely independent. In any case, Equation 8.34 is used as the basis for the final correlation which is presented at the end of this section. Equation 8.33 would also be somewhat inconvenient to use in meanline analyses since the skin friction coefficient is typically not known in the early stages of design.

To demonstrate the improvements achieved with the new correlation, the experimental data are compared first with the predictions of the Kacker and Okapuu (1982) (KO) and Moustapha *et al.* (1990) (MKT) correlations using the **conventional loss breakdown**. The KO and MKT correlations were used to predict the design incidence and off-design incidence losses, respectively. Figure 8.6a compares the predicted values of the secondary losses with the measured values for all the cases in Table 8.2. Predicted values of $Y_{secondary}$ that exceeded 0.14 were plotted along the top edge of the figure. As can be seen, the agreement between the predictions and the cascade measurements is poor: the KO-MKT correlations significantly over-predict most of the loss measurements. This result was not completely surprising since it is known that the predicted losses are scaled or “calibrated” to reproduce stage efficiencies derived from rig or engine data, as previously discussed in Section 6.3. The large disparity between the measured cascade losses and the predicted losses reinforces the observations made in Section 6.3, namely that there appear to be additional and significant loss generating mechanisms in the engine environment that are not captured in cascade testing.

Since the predictions from the KO-MKT correlations correspond to engine-representative loss levels, the comparison presented in Figure 8.6a is not very meaningful for evaluating the degree to which the correlations accurately predict losses in a cascade environment. A more appropriate comparison is to convert the engine-representative losses to cascade-representative losses by an engine-to-cascade scaling factor. Unfortunately, this scaling

(a)

(b)

Figure 8.6: Evaluation of the Kacker and Okapuu/Moustapha *et al.* secondary loss correlations: (a) engine-representative, and (b) cascade-representative loss levels.

factor is not normally quoted in published correlations, as is the case with Kacker and Okapuu (1982). However, the ratio of the average of the measured losses to the average of the predicted losses for all cases in the cascade database is probably a reasonable estimate. Figure 8.6b compares the scaled predicted losses to the measured cascade losses, and as noted in the figure, the value of the engine-to-cascade scaling factor was 0.23. It can be seen that the agreement now is considerably better, however, there are still numerous data points that fall outside the ± 0.0075 error band. Furthermore, the shortcomings of the conventional loss breakdown scheme that were described in the previous chapter are also evident in this figure. That is, a few of the off-design incidence cases have negligible or even negative measured losses.

The evaluation diagram for the new correlation (Equation 8.34) is shown in Figure 8.7. As can be seen, a majority of the predictions lie inside the error band, and therefore, the agreement between the predictions and the experimental data is regarded as satisfactory. The largest discrepancies occur for the cases with high values of positive incidence. The appearance of trailing-edge flow separation tends to increase the levels of uncertainty for these cases. Nonetheless, with the present prediction method there is considerably less scatter than the combination of conventional loss breakdown and KO-MKT correlations (Figure 8.6b).

As discussed earlier, the results from the optimizer showed that, within the uncertainty of the correlation, the secondary losses were essentially independent of the total airfoil loading. This was unexpected since the correlations available in the open literature (e.g., Craig and Cox, 1971; Traupel, 1977; Kacker and Okapuu, 1982) show the contrary. This apparent inconsistency seems to arise due to the relatively narrow range of airfoil loading values present in the database.

The influence of airfoil loading on secondary losses may be examined in more detail with

Figure 8.7: Evaluation of the new secondary loss correlation (Equation 8.34).

the Wolf (1961) secondary loss correlation. This particular correlation forms the basis for Traupel's correlation, and was chosen because it is based on an extensive set of data from tests that examined solely the influence of airfoil loading and channel acceleration. The cascade geometries that were examined had identical pitch-to-chord ratios, and therefore, the data does not include the effects of this parameter. In the original form of the correlation, the losses were correlated with the total flow turning and the convergence ratio. The present author has recast the correlation in terms of the loading parameter $F_t/(s/Cx)$ and the inverse of the convergence ratio, as shown in Figure 8.8.

To gain a better sense of the variation of secondary losses that might be expected for the range of loading values in the database, the cases from the present data set are plotted in Figure 8.8 and the grey region shows the area within which most of the data points fall.

Figure 8.8: The influence of convergence ratio and total airfoil loading on secondary losses (based on the secondary loss correlation of Wolf, 1961).

The maximum variation in losses for the range of loading levels in the current database occurs at a flow acceleration level that corresponds to $1/CR \approx 0.6$. At this acceleration level, one might expect a maximum secondary loss variation of $\Delta Y_{secondary} \approx 0.005$ for the range of loading levels in the database, which is well within the uncertainty of the current loss correlation. Therefore, it is likely that the optimizer has not identified an influence of airfoil loading simply because the secondary losses are relatively insensitive to loading differences, at least for loading levels that are common in turbines of recent design.

In Chapter 6 it was concluded that the loading distribution has a significant influence

on secondary losses and that a more front-loaded airfoil tends to generate higher secondary losses. Furthermore, it was noted that the loading distribution is determined mainly by the stagger angle: an airfoil tends to become more forward-loaded as the stagger angle increases. The new correlation is consistent with the conclusions of Chapter 6 in that the influence of the loading-distribution parameter ($\cos \gamma$) is significant. The parameter appears in the denominator of Equation 8.33 indicating that an increase in stagger angle (more forward loading) produces higher losses.

For the influence of the channel acceleration, the results from the optimizer suggest that the secondary losses vary linearly with the inverse of the convergence ratio. This linear relationship is also observed in the Ainley and Mathieson, Dunham and Came, and Kacker and Okapuu correlations.

In Section 8.4.2, it was suggested that the dissipation in the endwall boundary layers is a significant contributor to the secondary losses and for this reason, it was postulated that the size of the endwall surface area affects the secondary losses. In the present correlation, the parameter that captures the influence of the endwall surface area is a function of the airfoil height-to-axial chord ratio. By comparison, the Dunham and Came, Kacker and Okapuu, and Craig and Cox correlations depend on the airfoil height-to-chord ratio.

A number of experimental studies have examined the influence of aspect ratio on secondary losses (e.g., data compiled by Dunham, 1970 and Atkins, 1987). In these studies the aspect ratio variations were achieved exclusively through changes to the airfoil height, and the results strongly suggest that the secondary losses vary as the inverse of the airfoil height, at least for large aspect ratios (i.e., $h/C > 2.0$). Based on these results, Dunham and Came correlated the secondary losses with the inverse of the airfoil aspect ratio. This dependence was retained in the Kacker and Okapuu correlation for airfoils with aspect ratios greater than 2.0. For low aspect ratio airfoils (i.e., $h/C < 2.0$), the results of Rogo (1968)

Figure 8.9: The influence of aspect ratio on secondary losses for LS2.

and Okapuu (1974) showed that the secondary losses were less sensitive to aspect ratio, and therefore, Kacker and Okapuu modified the dependence on aspect ratio accordingly.

Figure 8.9 shows the influence of aspect ratio on secondary losses for the LS2 cascade using the current correlation (Equation 8.34), the Dunham and Came correlation (inverse aspect ratio dependence), and the Kacker and Okapuu correlation (Dunham and Came correlation with modified dependence for low aspect ratios). From the figure it can be seen that the loss trend with aspect ratio for $h/C < 2.0$ is nearly identical for the new correlation and the Kacker and Okapuu correlation. For aspect ratios greater than 2.0, the loss trend in the new correlation differs considerably from the commonly-observed inverse aspect ratio dependence, and therefore, Equation 8.34 should only be used for values of aspect ratio less than 2.0. This result was not completely surprising given that the current database consists

mostly of low aspect ratio airfoils. For aspect ratios greater than 2.0, an expression for secondary losses has been developed based on Equation 8.34 but incorporating the inverse aspect ratio dependence. The final secondary loss expressions are then

$$Y_{secondary} = \begin{cases} \frac{0.038 + 0.41 \tanh(1.20 \delta^*/h)}{\sqrt{\cos \gamma} (CR) (h/C)^{0.55} \left(\frac{C \cos \alpha_2}{C_x} \right)^{0.55}} & \text{for } h/C \leq 2.0, \\ \frac{0.052 + 0.56 \tanh(1.20 \delta^*/h)}{\sqrt{\cos \gamma} (CR) (h/C) \left(\frac{C \cos \alpha_2}{C_x} \right)^{0.55}} & \text{for } h/C > 2.0. \end{cases} \quad (8.35)$$

8.5 Conclusions

An empirical prediction method for secondary losses has been developed. The method is based on a loss breakdown scheme that is more consistent with the observed flow physics than the conventional scheme.

In the new loss breakdown scheme, the profile loss is defined as the loss generated in the portion of the airfoil surface boundary layers which is largely uninfluenced by the secondary flows. The secondary loss is then simply the difference between the total loss and the profile loss. With the new scheme, the secondary loss component includes the loss generated in the airfoil surface boundary layers where they are affected by the secondary flow. Because the secondary loss definition in the new scheme differs from that in the conventional scheme, a new secondary loss correlation for design and off-design values of incidence was devised. It was based on a database of cascade measurements from the present study and from recent cases in the open literature.

Predictions from the most recent design and off-design incidence secondary loss correlations (Kacker and Okapuu, 1982; Moustapha *et al.*, 1990, respectively) were compared to the measured cascade data. For all cases in the database, the measured cascade data were significantly over-predicted. However, both correlations have been shown to reproduce loss

data from engine or rig tests reasonably well. It is believed that the large discrepancies between the predictions and the cascade measurements are due to the calibration or scaling that is necessary to make up for the additional losses in the endwall regions that are present in the engine environment.

Evidently, there is a need for further secondary loss experiments to investigate the effects of phenomena that are present in the engine environment but absent in cascade testing. These effects include the influence of wake- or vortex-induced unsteadiness on secondary losses. Another area that has received very little attention to date is the influence of disk- and shroud-cavity purge flows on the mainstream secondary flow behaviour.

Chapter 9

Conclusions and Recommendations

9.1 Conclusions

The present research has considered a number of topics related to profile and secondary losses in axial turbines. This section presents the overall conclusions from the present research: more detailed conclusions for each of the specific topics that were considered are presented in Chapters 5 through 8. Recommendations for future research are detailed in Section 9.2.

The review of previous work has shown that there is little information in the literature related to the effects of leading-edge geometry on profile and secondary losses at both design and off-design incidence. Furthermore, the most recent profile and secondary loss correlations suggest that leading-edge geometry is an important correlating parameter only at off-design incidence. The main objective of the present thesis was to investigate the influence of leading-edge geometry on the loss behaviour of turbine airfoils.

To achieve these objectives an experimental study was conducted in a low-speed wind

tunnel using a variable-incidence cascade test section. Two cascades, designated as LS2 and LS3, were examined. Detailed flow field measurements were made upstream and downstream of the cascades and static pressure were measured on the airfoil surfaces. In addition to the measurements, extensive surface oil flow visualization was conducted. The airfoil section in the LS2 cascade represents the midspan section of a Pratt & Whitney Canada, PW100-series power turbine blade. The LS3 cascade was manufactured to examine the influence of leading-edge geometry on the off-design profile losses. It has a leading-edge diameter roughly twice that of LS2. Also, the stagger angle is slightly smaller for LS3; therefore, it was more aft loaded than LS2. The two airfoil sections were designed for the same inlet and outlet design conditions using the same turbine-profile design system.

The most recent off-design profile loss correlation (Moustapha *et al.*, 1990) predicts that LS3 is considerably less sensitive to incidence due to its larger leading-edge diameter. However, the measured losses were very similar for LS2 and LS3 over a wide range of incidence angles. This prompted a re-examination of the influence of the leading-edge geometry on profile losses. A significantly better off-design profile loss correlation was obtained by including the leading-edge wedge angle as an additional and more influential correlating parameter. The present author believes that the off-design behaviour of the airfoil is influenced by the magnitude of the discontinuity in curvature at the blend points where the leading-edge circle joins the rest of the profile. The value of the wedge angle appears to give an approximate and convenient indication of the magnitude of this discontinuity in curvature.

The present profile loss results indicate that the reduction or elimination of the curvature discontinuity at the blend points will produce airfoils that are more tolerant to off-design incidence. An elliptic leading edge, which is commonly used in design, allows an airfoil to be designed with milder curvature jumps at the blend points than the more conventional

circular leading edge. The use of higher-order curves in the leading-edge region could eliminate the curvature discontinuity all together.

In addition to the experimental investigation on profile losses, a computational study was carried out using a CFD code that is currently commonly used in turbomachinery design. The objective was to determine the capability of the code to predict off-design profile losses. For cases at moderate values of incidence, the correct loss trend was predicted; however, the code did not predict the trailing-edge separation that occurred at high incidence.

As noted above, there is very little data on the effects of leading-edge geometry on secondary losses at both the design and off-design incidence. At the design incidence, the experimental results showed that the secondary losses for LS2 are higher than those of LS3, and this is believed to be due mainly to a stronger passage vortex. The greater strength of the passage vortex for the LS2 airfoil is attributed to this airfoil's front-loaded pressure distribution and the resulting stronger upstream pressure field seen by the endwall boundary layer approaching the leading edge. This effect seemed to dominate over any influence of the leading-edge diameter, which was smaller for LS2. It is tentatively concluded that in turbine blade rows the leading-edge geometry has at best a minor influence on the secondary flow field and the corresponding secondary losses, at least for the design incidence and for conventional leading-edge geometries. On the other hand, the loading on the forward part of the airfoil appears to be of major importance.

The most recent off-design secondary loss correlation (Moustapha *et al.*, 1990) includes leading-edge diameter as an influential correlating parameter. The correlation predicts that the secondary losses for the LS3 profile are lower at off-design incidence due to its larger leading-edge diameter; however, the opposite is observed experimentally. Therefore, it must be concluded that the correlation does not correctly capture the leading-edge diameter effect on off-design secondary losses.

The loss results at off-design incidence have highlighted some serious shortcomings with the conventional method of loss decomposition. One of the assumptions with this method is that the loss generation on the airfoil surface next to the secondary flow is similar to that in the undisturbed surface flow. This assumption is perhaps reasonable at design incidence; however, when it is applied to the present results at high values of incidence, it leads to negligible or even negative secondary losses. This is obviously misleading and unphysical. An empirical prediction method for secondary losses has been developed to replace the conventional loss breakdown scheme. The method is based on a loss breakdown scheme that is more consistent with the observed flow physics than the conventional scheme.

Predictions from the most recent design and off-design incidence secondary loss correlations (Kacker and Okapuu, 1982; Moustapha *et al.*, 1990, respectively) were compared to cascade data obtained from the present research and from the open literature. For all cases, the measured cascade data were significantly over-predicted. However, both correlations have been shown to reproduce loss data from engine or rig tests reasonably well. It is believed that the large discrepancies between the predictions and the cascade measurements are due to the calibration or scaling that is necessary to make up for the additional losses in the endwall regions that are present in the engine environment.

9.2 Recommendations for Future Work

The present study has uncovered a number of future research directions that may help improve the current understanding of loss generation in axial turbines at design and off-design conditions.

For the profile loss component, the present work has focused exclusively on the influence of the leading-edge geometry on the off-design losses. However, the present investigation

was carried out at incompressible conditions with a fixed Reynolds number and turbulence intensity: parameters that are known to influence profile losses at the design incidence. By contrast, the importance of these parameters at off-design incidence is still unclear due to a lack of data.

In the present experiment the conditions at the inlet of the cascade were steady. However, in the engine environment, the relative motion of airfoil rows results in inlet conditions that are unsteady. This motion causes the wakes, vortices and pressure field of the upstream airfoil row to interact with the downstream airfoil row. The latter form of unsteadiness arises due to the tangential variation in static pressure at the outlet of the upstream airfoil. It becomes more important in high-pressure turbine stages of aircraft engines because the blade passage Mach numbers are typically transonic. The influence of blade-wake interaction on profile losses at the design incidence has recently been investigated by Mahallati (2003) in the test section used by the present author. However, the importance of this form of interaction at off-design values of incidence is unknown. Arguments presented by Hodson and Dawes (1998) suggest that pressure-field interaction may play an important role in the generation of profile losses; however, this has yet to be verified experimentally.

The results from the present work showed that there are significant sources of loss in the **endwall regions** that are present in the engine environment but absent in linear cascades. This provides the motivation for research into the effects of wake- or vortex-induced unsteadiness on secondary losses. Another area that has received very little attention to date is the influence of disk- and shroud-cavity purge flows on the mainstream secondary flow behaviour. The recent results of McLean *et al.* (2001) have shown that cavity purge flows can seriously degrade the performance of turbine stages.

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Appendix A

Transducer Calibration Curves

The appendix presents the calibration curves for the pressure transducers used in the Sciometric- and PS9010-based pressure acquisitions. The calibration curves for the transducers used in the Sciometric-based system (i.e., the Baratron and Rosemount transducers) are shown in Figure A.1. As noted in Chapter 3, these transducers were calibrated using a Dwyer Instruments Hook-Gauge as the pressure calibration standard. The calibration curves for the 8 transducers used in the PS9010-based system are shown in Figures A.2 (channels 1-4) and A.3 (channels 5-8). The new micromanometer that was described in Section 3.5 was used as pressure calibration standard for these transducers.

Figure A.1: Pressure transducer calibration curves for Baratron and Rosemount Transducers.

Figure A.2: Pressure transducer calibration curves for PS9010, Channels 1-4.

Figure A.3: Pressure transducer calibration curves for PS9010, Channels 5-8.

Appendix B

Characterization of the Jet-Flow in the “New” Calibration Rig

This appendix presents the flow characteristics of the jet-flow in the “new” calibration rig. Figure B.1 shows profiles of flow angle, total-pressure coefficient and dynamic-pressure coefficient over a range of axial positions from the nozzle exit plane to 1.6 nozzle-diameters downstream of the nozzle exit plane. It should be noted that the total-pressure and dynamic-pressure coefficients are defined as

$$C_{p0} = \frac{P_{0_{local}} - P_{C1}}{P_{C1} - P_{C2}} \quad \text{and}$$
$$C_q = \frac{P_{0_{local}} - P_{local}}{P_{C1} - P_{C2}},$$

respectively. The reference total pressure, P_{C1} , was measured from within the air-filter enclosure and the reference static pressure, P_{C2} , was measured from a static tap located just upstream of the nozzle exit plane.

Figure B.1: Flow quality in “new” calibration rig.

Appendix C

Three-Hole Probe Calibration Procedure

This appendix describes the procedure used to calibrate the three-hole probe. The probe was calibrated in 0.5 degree steps over a range of ± 10 degrees of yaw misalignment using the set-up described in Section 3.6.

The pressure data from the three probe holes (or ports) are used to determine the following non-dimensional coefficients according to the method of Lewis (1966):

$$\begin{aligned} K_1 &= \frac{P_1 - P}{q} \\ K_{12} &= \frac{P_1 - P_2}{q} \\ K_{13} &= \frac{P_1 - P_3}{q} \\ K_{\phi 2} &= \frac{P_1 - P_2}{P_1 - P_3} \\ K_{\phi 3} &= \frac{P_1 - P_3}{P_1 - P_2} \end{aligned} \tag{C.1}$$

Figure C.1: **Three-hole probe port numbering.**

where P and q are the local static and dynamic pressure, respectively, and the pressure indices correspond to the port numbering shown in Figure C.1.

Figures C.2 to C.6 present the calibration data for the three-hole probe used during this study. The error bars for each of the data points provide a 95% confidence estimate of the uncertainty in the calibration process and they were obtained through repeated probe calibrations.

The following procedure is used to determine the flow quantities in an unknown flow. First, the sign of the flow angle is determined by comparing P_2 and P_3 . The magnitude of the flow misalignment angle is obtained from either $K_{\phi 2}$ or $K_{\phi 3}$ depending on the sign of the misalignment angle. As can be seen in Figures C.2 and C.3, the scatter in the $K_{\phi 2}$ data is less at negative flow angles, while at positive angles, the scatter is less for $K_{\phi 3}$ data.

Based on these trends, it is evident that better accuracy is obtained by using the $K_{\phi 2}$ and $K_{\phi 3}$ calibration coefficients for negative and positive flow misalignment angles, respectively. Once the flow angle is determined, the local dynamic pressure is extracted using either the K_{12} or the K_{13} calibration coefficients. From Figures C.4 and C.5 it can be seen that K_{12} curve is better suited for negative flow angles since there is slightly less scatter, while K_{13} curve is more suitable for positive flow misalignment angles. Finally, the static pressure is obtained using the K_1 curve and the known values of the flow misalignment angle and dynamic pressure.

Figure C.2: Variation of $K_{\phi 2}$ with yaw angle.

Figure C.3: Variation of $K_{\phi 3}$ with yaw angle. Figure C.4: Variation of K_{12} with yaw angle.

Figure C.5: Variation of K_{13} with yaw angle. Figure C.6: Variation of K_1 with yaw angle.

Appendix D

Seven-Hole Probe Calibration Procedure

This appendix describes the calibration procedure of the seven-hole probe. DeCecco (1995) has given a detailed description of the calibration procedure and most of the description provided here was drawn from his work. The calibration procedure was developed by Gallington (1980) as referred to by Gerner and Maurer (1981), and is valid only for incompressible flows.

Low-Angle Regime

For small misalignment angles, which are typically less than 25 degrees relative to the probe axis, the flow is expected to remain attached over the entire surface of the probe tip. The central port (port 7), as shown in Figure D.1, senses the highest pressure, and pressure information from all seven holes is used to determine the following non-dimensional

Figure D.1: Seven-hole probe port numbering scheme.

parameters:

$$\begin{aligned}
 C_{\alpha_1} &= \frac{P_4 - P_1}{P_7 - \bar{P}_{1-6}} \\
 C_{\alpha_2} &= \frac{P_3 - P_6}{P_7 - \bar{P}_{1-6}} \\
 C_{\alpha_3} &= \frac{P_2 - P_5}{P_7 - \bar{P}_{1-6}}
 \end{aligned} \tag{D.1}$$

where $\bar{P}_{1-6}$ is the average of the six surrounding peripheral pressures. It approximates the local static pressure whereas P_7 approximates the local total pressure; therefore, the denominator in the above expressions is referred to as the “pseudo-dynamic pressure”. The numerator in each non-dimensional parameter is based on the difference between diametrically opposed port pressure readings. This implies that each coefficient is most sensitive to flow angularity in three separate planes, each 60 degrees apart, passing through the probe axis.

Of the three coefficients, only two are required to uniquely define the flow direction. By equally weighing each coefficient of Equation D.1 along the tangential coordinate axes shown in Figure D.2, the following angle coefficients are defined:

Figure D.2: Tangential coordinate system for low-angle regime (reproduced from DeCecco, 1995).

$$\begin{aligned}
 C_{\alpha} &= \frac{2C_{\alpha_1} + C_{\alpha_2} - C_{\alpha_3}}{3} \\
 C_{\beta} &= \frac{C_{\alpha_2} + C_{\alpha_3}}{\sqrt{3}}.
 \end{aligned}
 \tag{D.2}$$

By calibrating the probe in a known flow, the local pitch angle, α , yaw angle, β , total-pressure coefficient, C_0 , and dynamic pressure coefficient, C_q , can all be related to the two non-dimensional coefficients of Equation D.2:

$$\begin{aligned}
\alpha &= f_1(C_\alpha, C_\beta) \\
\beta &= f_2(C_\alpha, C_\beta) \\
C_0 &= f_3(C_\alpha, C_\beta) \\
C_q &= f_4(C_\alpha, C_\beta)
\end{aligned}
\tag{D.3}$$

where the total- and dynamic-pressure coefficients are defined as follows:

$$\begin{aligned}
C_0 &= \frac{P_7 - P_0}{P_7 - \bar{P}_{1-6}} \\
C_q &= \frac{P_7 - \bar{P}_{1-6}}{P_0 - P}
\end{aligned}
\tag{D.4}$$

The low-angle-regime procedure outlined above is used as long as the central port reads the highest pressure, which is typical of flow misalignment angles of less than 25 degrees. Figures D.3 to D.6 and D.7 to D.10 and present the calibration data for the low-angle regime for SHP1 and SHP2, respectively. In each figure, a contour plot of the calibration data is presented to show the smoothness of the data. Beneath each of the contour plots, data along lines of $\alpha = 0$ and $\beta = 0$ are shown in the x-y plots. Data from four repeated calibrations are shown for SHP2, and the amount of scatter gives a sense of the repeatability error in the calibration coefficients (i.e., C_α , C_β , C_0 , and C_q).

Figure D.3: C_α calibration data for SHP1, sector 7.

Figure D.4: C_β calibration data for SHP1, sector 7.

Figure D.5: C_0 calibration data for SHP1, sector 7.

Figure D.6: C_q calibration data for SHP1, sector 7.

Figure D.7: C_α calibration data for SHP2, sector 7.

Figure D.8: C_β calibration data for SHP2, sector 7.

Figure D.9: C_0 calibration data for SHP2, sector 7.

Figure D.10: C_q calibration data for SHP2, sector 7.

High-Angle Regime

At large flow misalignment angles the flow will tend to separate over the downstream portions of the probe. The pressure ports in the separated region are insensitive to changes in flow angularity. Therefore, it necessary to redefine calibration coefficients so they contain only the port pressures in the attached flow region. This is done by grouping the pressure ports into six, 60 degree wedge-shaped sectors each containing the central port and three adjacent peripheral ports. Figure D.11 shows the resultant division of angular space.

In the high-angle regime, Gallington (1980) found that it was more appropriate to use a polar coordinate system, shown in Figure D.12, to define the calibration coefficients:

Figure D.11: Division of angular space (reproduced from Gerner *et al.*, 1984).

Figure D.12: Polar coordinate system used for high-angle regime (reproduced from DeCecco, 1995).

$$\begin{aligned}
 C_{\theta_n} &= \frac{P_n - P_7}{P_n - \left(\frac{P_{n-} + P_{n+}}{2} \right)} \\
 C_{\phi_n} &= \frac{P_{n-} - P_{n+}}{P_n - \left(\frac{P_{n-} + P_{n+}}{2} \right)}
 \end{aligned}
 \tag{D.5}$$

where the subscripts n , $n+$, and $n-$ represent the peripheral-pressure-port number, and the adjacent ports clockwise and counterclockwise to port n , respectively. These coefficients can be related to the pitch angle, θ , roll angle, ϕ , total pressure coefficient, C_0 , and dynamic pressure coefficient, C_q , as follows:

$$\begin{aligned}
\theta_n &= f_1(C_{\theta_n}, C_{\phi_n}) \\
\phi_n &= f_2(C_{\theta_n}, C_{\phi_n}) \\
C_{0_n} &= f_3(C_{\theta_n}, C_{\phi_n}) \\
C_{q_n} &= f_4(C_{\theta_n}, C_{\phi_n})
\end{aligned} \tag{D.6}$$

where the total- and dynamic-pressure calibration coefficients are given by:

$$\begin{aligned}
C_0 &= \frac{P_n - P_0}{P_n - \left(\frac{P_{n-} + P_{n+}}{2} \right)} \\
C_q &= \frac{P_n - \left(\frac{P_{n-} + P_{n+}}{2} \right)}{P_0 - P}.
\end{aligned} \tag{D.7}$$

Similar to the low-angle regime, the functions in Equation D.6 are determined by calibrating the probe in a known flow. Figures D.13 to D.16 and D.17 to D.20 present the calibration data for Sector 4 for SHP1 and SHP2, respectively. Data from two repeated calibrations are shown in the x-y plots for SHP2. The smoothness of the calibration data is similar for the other high-angle sectors as is the data scatter.

Figure D.13: C_ϕ calibration data for SHP1, sector 4.

Figure D.14: C_θ calibration data for SHP1, sector 4.

Figure D.15: C_0 calibration data for SHP1, sector 4.

Figure D.16: C_q calibration data for SHP1, sector 4.

Figure D.17: C_ϕ calibration data for SHP2, sector 4.

Figure D.18: C_θ calibration data for SHP2, sector 4.

Figure D.19: C_0 calibration data for SHP2, sector 4.

Figure D.20: C_q calibration data for SHP2, sector 4.

Appendix E

Mixing Analysis for Profile Losses

This appendix details the derivation of the expression for the mixed-out profile losses that is presented in Equation 3.18. The following analysis has been adapted from Mahallati (2003) and is included here for the sake of completeness.

The derivation begins by defining the control volume (CV) shown in Figure E.1. As can be seen, the inlet and outlet control surfaces are located at the outlet measurement plane and the mixed-out plane, respectively. At the inlet plane, the static pressure is spatially non-uniform as is the magnitude and direction of the flow velocity. At the mixed-out plane, all quantities are uniform by definition. The upper and lower control surfaces are spaced one full airfoil pitch apart so that the boundaries are periodic. As such, there is no mass or momentum flux through the upper and lower surfaces.

The control volume analysis presented here assumes that the flow is incompressible and that mixing occurs at constant area. First, the mixed-out axial velocity is calculated by applying the conservation of mass to the CV:

$$\begin{aligned}\int_{in} d\dot{m} &= \int_{out} d\dot{m} \\ \int_0^s \rho u_2 dy &= \int_0^s \rho u_{mixed} dy\end{aligned}$$

Figure E.1: Control volume used for derivation of mixed-out profile losses.

$$\int_0^s u_2 dy = u_{mixed} s$$

$$u_{mixed} = \frac{1}{s} \int_0^s u_2 dy \quad (E.1)$$

where the subscripts 2 and *mixed* refer to the locations of the measurement and mixed-out planes, respectively.

The mixed-out static pressure is calculated from the x-component of momentum conservation.

$$\sum F_x = \int_{out} u d\dot{m} - \int_{in} u d\dot{m}$$

$$\int_0^s P_2 dy - \int_0^s P_{mixed} dy = \int_0^s u_{mixed}(\rho u_{mixed} dy) - \int_0^s u_2(\rho u_2 dy) . \quad (E.2)$$

Substituting Equation E.1 into the first term on the right hand side of Equation E.2 gives

$$\int_0^s P_2 dy - P_{mixed} s = \rho s \left[\frac{1}{s} \int_0^s u_2 dy \right]^2 - \rho \int_0^s (u_2)^2 dy$$

$$P_{mixed} = \int_0^1 P_2 d(y/s) - \rho \left[\int_0^1 u_2 d(y/s) \right]^2 + \rho \int_0^1 (u_2)^2 d(y/s) . \quad (E.3)$$

The pitchwise velocity is calculated from the y-component of momentum conservation.

$$\sum F_y = \int_{out} v d\dot{m} - \int_{in} v d\dot{m}$$

$$0 = \int_0^s (-v_{mixed})(\rho u_{mixed} dy) - \int_0^s (-v_2)(\rho u_2 dy)$$

$$0 = -v_{mixed} u_{mixed} s + \int_0^s v_2 u_2 dy \quad (E.4)$$

The mixed-out y-component of velocity is obtained by substituting Equation E.1 into

Equation E.4 and rearranging:

$$v_{mixed} = \frac{\int_0^1 (v_2)(u_2) d(y/s)}{\int_0^1 u_2 d(y/s)} . \quad (E.5)$$

The velocity at the mixed-out plane is then calculated from the two components of velocity (i.e., Equations E.1 and E.5).

$$V_{mixed}^2 = u_{mixed}^2 + v_{mixed}^2 = \left[\int_0^1 u_2 d(y/s) \right]^2 + \left[\frac{\int_0^1 (v_2)(u_2) d(y/s)}{\int_0^1 u_2 d(y/s)} \right]^2$$

Non-dimensionalizing the above equation by the square of inlet centreline velocity, V_{CL}^2 , gives

$$\left(\frac{V_{mixed}}{V_{CL}} \right)^2 = \left[\int_0^1 \left(\frac{u_2}{V_{CL}} \right) d(y/s) \right]^2 + \left[\frac{\int_0^1 \left(\frac{v_2}{V_{CL}} \right) \left(\frac{u_2}{V_{CL}} \right) d(y/s)}{\int_0^1 \left(\frac{u_2}{V_{CL}} \right) d(y/s)} \right]^2 . \quad (E.6)$$

For an incompressible flow, the total pressure at the mixed-out plane may be written as $P_{0mixed} = P_{mixed} + \frac{1}{2}\rho V_{mixed}^2$. By substituting the expression for static pressure from Equation E.3, the total pressure at the mixed-out plane can be expressed as

$$P_{0mixed} = \int_0^1 P_2 d(y/s) - \rho \left[\int_0^1 u_2 d(y/s) \right]^2 + \rho \int_0^1 (u_2)^2 d(y/s) + \frac{1}{2}\rho V_{mixed}^2 .$$

Subtracting the mixed-out total pressure, P_{0mixed} , from the inlet centreline total pressure, P_{0CL} , and then dividing through by the inlet centreline dynamic pressure gives

$$\begin{aligned} \frac{P_{0CL} - P_{0mixed}}{\frac{1}{2}\rho V_{CL}^2} &= \left[\int_0^1 \frac{P_{CL} - P_2}{\frac{1}{2}\rho V_{CL}^2} d(y/s) + 1 \right] + 2 \left[\int_0^1 \left(\frac{u_2}{V_{CL}} \right) d(y/s) \right]^2 - \\ &\quad 2 \int_0^1 \left(\frac{u_2}{V_{CL}} \right)^2 d(y/s) - \left(\frac{V_{mixed}}{V_{CL}} \right)^2 \end{aligned}$$

$$= 1 - \int_0^1 C_{p_2} d(y/s) + 2(I_1^2 - I_2) - \left(\frac{V_{mixed}}{V_{CL}} \right)^2 \quad (\text{E.7})$$

where C_{p_2} is the measured static pressure coefficient; I_1 and I_2 are given by

$$I_1 = \int_0^1 \left(\frac{u_2}{V_{CL}} \right) d(y/s)$$

$$I_2 = \int_0^1 \left(\frac{u_2}{V_{CL}} \right)^2 d(y/s) .$$

and $\left(\frac{V_{mixed}}{V_{CL}} \right)^2$ is given by Equation E.6. The mixed-out profile loss coefficient, $Y_{p_{mixed}}$, is obtained by dividing Equation E.7 by $(V_{mixed}/V_{CL})^2$:

$$Y_{p_{mixed}} = \frac{P_{0_{CL}} - P_{0_{mixed}}}{P_{0_{mixed}} - P_{mixed}} = \frac{1 - \int_0^1 C_{p_2} d(y/s) + 2(I_1^2 - I_2)}{\left(\frac{V_{mixed}}{V_{CL}} \right)^2} - 1 . \quad (\text{E.8})$$