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RESOURCE MANAGEMENT AND SERVICE DIFFERENTIATION IN WDM NETWORKS

BY

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To My Wife
-Maryam-
For Her Love, Support, and Patience

Abstract

In this thesis, we present our recent research work in the area of optical Wavelength Division Multiplexing (WDM) networks, an important area which promises to be the focus of ongoing and future research. A WDM network consists of optical wavelength routing nodes interconnected by point-to-point fiber links in an arbitrary topology. WDM allows simultaneous transmission of multiple wavelengths within a single fiber. Although each wavelength can have a transmission capacity up to ten gigabit per second, some traffic streams may require lower bandwidth. To utilize the wavelengths more efficiently, traffic *grooming* is deployed to share the capacity of a wavelength among users with different bandwidth granularities. In WDM networks, the Routing and Wavelength Assignment (RWA) problem is an important issue.

Service Differentiation, Quality of Service (QoS), and Fairness Management are still open research problems in WDM grooming/non-grooming networks. The main contribution of this work is to formulate these problems for both grooming and non-grooming networks in the framework of the Markov Decision Process theory to determine the optimal dynamic wavelength allocation policies. Due to the huge size of the state space for realistic networks, the optimal policy determination is not possible in practice. Hence, heuristic algorithms are proposed with close performance to the optimal solution and lower computational complexity than the optimal. Another contribution of this proposal is to estimate the blocking performance of the system in order to dimension the network links. This work is a fundamental research study on resource management and has great value in future QoS-aware WDM networks.

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Abbreviations

ACA	Asynchronous Criticality Avoidance
ADM	Add-Drop Multiplexing
AR	Adaptive Routing
CAC	Call Admission Control
CP	Complete Partitioning
CS	Complete Sharing
DLE	Dynamic Lightpath Establishment
FAR	Fix Alternative Routing
FF	First Fit
FR	Fix Routing
FSC	Fiber Switch Capable
GMPLS	Generalized Multi Protocol Label Switching
GoS	Grade of Service
IETF	Internet Engineering Task Force
ILP	Integer Linear Programming
L2SC	Layer-2 Switch Capable
LCP	Least Cost Path
LSC	Lambda Switch Capable
MC	Markov Chain
MDP	Markov Decision Process
MPLS	Multi Protocol Label Switching

MT	Multi Threshold
OADM	Optical Add-Drop Multiplexing
OC	Optical Carrier
OEO	Optical-Electrical-Optical
OSPF	Open Shortest Path First
OXC	Optical Cross Connect
PSC	Packet Switch Capable
PXC	Photonic Cross Connect
QoS	Quality of Service
RSVP	Resource ReSeRvation Protocol
SADM	SONET Add-Drop Multiplexing
SDH	Synchronous Digital Hierarchy
SLE	Static lightpath Establishment
SONET	Synchronous Optical Network
TDM	Time Division Multiplexing
TSI	Time Slot Interchanger
WC	Wavelength Converter
WDM	Wavelength Division Multiplexing

Notation

Notation	Description
ℓ	Class of call
λ_ℓ	Arrival rate of class ℓ
μ_ℓ	Service rate of class ℓ
W	Number of Wavelengths
m	Number of reserved wavelengths
n_ℓ	Number of class ℓ calls
i, j, k	Temporary variables
S	State Space
s_t	State at time t
t	Time
Λ	Set of available wavelengths
q	Number of available wavelengths
D_ℓ	Departure operator for class ℓ
A_ℓ	Arrival operator for class ℓ
ε	Set of possible events
e	Event
a	Action
$\mathcal{A}$	Set of actions
P_a	Policy operator when applying action a
ν	Random sampling rate

Notation	Description
β_ℓ	Weighting factor assigned to class ℓ
$R(\cdot)$	One-step reward function
$\mathbf{B}$	Vector of weighting factors
$C(\cdot)$	One-step cost function
$\Pi = \{\pi_1, \pi_2, \dots\}$	Policy sequence
γ	Discount factor
$\Pi_o(\cdot)$	Optimal policy
n	Time step (stage)
$E[\cdot]$	Expected value
$V(\cdot)$	Value function
$P_{ss'}^a$	Transition probability
F	Temporary variable
f	A function
Φ	Switching curve
p_u^ℓ	Probability of having u calls from class ℓ
ρ	Traffic intensity
N	Number of nodes
h	Number of hops
c_h	Class of calls with hop-count h
$\boldsymbol{\tau} = (\tau_1, \tau_2, \dots)$	Vector of thresholds
l_x	Link between node x and node $x + 1$
B_x	Blocking probability of class c_x
Θ_x	A recursive function in x recursion
f_r	Fairness ratio
$\mathcal{S}_x$	Set of wavelengths dedicated to class c_x
T	Total number of time slots on each wavelength
K	Total number of subclasses of class c_h

Notation	Description
c_h^k	Subclass k of class c_h
λ_h^k, μ_h^k	Arrival and service rates of subclass c_h^k
t^k	Required time-slots for subclass c_h^k
n_h^k	Number of c_h^k calls currently in system
α_k	Weighting factor assigned to c_h^k
$\mathcal{W}_{a=1}^d$	A subset of wavelengths with action $a = 1$
B_h^k	Blocking probability of subclass c_h^k
$n_h^{k,w}$	Number of c_h^k calls on wavelength w
$\pi(s)$	Product form formula
$G(.)$	Normalizing Constant
Ω_W	System State Space

Chapter 1

Introduction

Without doubt computer communication networks have pervaded every aspect of human life in recent times most especially with the ongoing explosive growth of the Internet, which continues to define the way we work and interact on a global scale. Of prime importance to this present way of communications is a robust network architecture that is quite capable of meeting present demands as well as scaling to meet future ones. This is where the importance of optical fiber communications comes to the fore. Optical fiber communication architectures present the highest data transfer rates with the lowest bit error rates.

1.1 Literature Review on Wavelength Division Multiplexing Networks

Wavelength Division Multiplexed (WDM) technology is an important area which promises to be the focus of ongoing and future research. WDM allows simultaneously transmission of multiple wavelengths within a single fiber. Two examples of a WDM networks are the broadcast-and-select and the wavelength-routed architecture. Broadcast-and-select is based on a passive star coupler device connected to several

nodes in a star topology. Basically a signal received on one port is split and broadcast to all ports. Networks are simple and have natural multicasting capabilities. In this research work, we are not focusing on broadcast-and-select networks.

A wavelength-routed WDM network consists of optical wavelength routing nodes interconnected by point-to-point fiber links in an arbitrary topology. End nodes with a limited number of optical transmitters and receivers are attached to the routing nodes. A message arriving on any of the input links at some wavelength can be routed to any one of the output link at the same wavelength, without requiring any Electro-optic conversion, by controlling optical switches, demultiplexers and multiplexers. In a transparent network, which is known as the all-optical network, the signal remains in the optical domain when it leaves the source, until the destination.

In WDM networks, signals transmitted from end users can be converted from electrical signals to optical, and then back to electrical at each node in the network. This is called Optical-to-Electrical-to-Optical (OEO) conversion. Another name for “OEO networks” is “opaque networks” because the light signal must be stopped and converted electrically.

The optical path that the signal traverses can be a WDM channel that is also referred to as a *lightpath*. In WDM wavelength-routed networks, each optical path must be established with a specific wavelength between each source-destination pair. This is known as *wavelength continuity constraint* and can be relaxed by using Wavelength Converters (WCs) at intermediate nodes [1, 2]. In WDM networks, lightpaths should be set up or released in real-time. Therefore, service providers can order lightpaths dynamically based on demand and network conditions. In this case, the lightpath

provisioning should be done quickly and also in a way that the overall network performance is optimized while the network operational expense is minimized [3].

1.1.1 Generalized Multiprotocol Label Switching (GMPLS)

GMPLS is considered as the standardized open and interoperable control plane in optical networks to dynamically provision resources and to provide network survivability using protection and restoration techniques. A brief review of the Internet Engineering Task Force (IETF) standardization on GMPLS [4–6] will be given below.

Such a network includes devices such as routers, switches, add-drop multiplexers (ADMs), optical cross-connects (OXC), etc. The focus of GMPLS is on the control plane of various layers. GMPLS decomposes the control plane into two parts; the signaling plane and the routing plane. It supports five interfaces: packet switch capable (PSC) interface; layer-2 switch capable (L2SC) interface; time-division multiplex capable (TDM) interface; lambda switch capable (LSC) interface; and fiber-switch capable (FSC) interfaces.

PSC forwards data based on the content of the packet header. L2SC can forward data based on the content of the frame/cell header. TDM forwards data based on the assigned time slot. LSC forwards data based on the wavelength on which the data is received. FSC forwards data based on the position of the data in the physical space. A circuit can be established only between, or through, interfaces of the same type. Depending on the particular technology being used for each interface, different circuit names can be used, e.g., SDH circuit, lightpath, etc. In the context of GMPLS all these circuits are referenced by a common name: label switched path (LSP).

A node can support only one type of interface or a combination of these five

interfaces. A number of existing protocols such as OSPF, RSVP, and CR-LDP are being extended to work with GMPLS [7]. As will be clear from this discussion, we will focus more on LSC and TDM interfaces of the GMPLS in this thesis.

1.1.2 Wavelength-Routed WDM Networks

In this case, each connection uses the entire wavelength to be established. The network consists of switching fabrics connected by fiber links to form an arbitrary physical topology. Each end-user is connected to an active switch via a fiber link. The combination of an end-user and its corresponding switch is referred to as a network node. Each node is equipped with a set of transmitters and receivers, both of which may be wavelength tunable. A transmitter at a node sends data into the network and a receiver receives data from the network.

The basic mechanism of communication in a wavelength-routed network is the lightpath. The intermediate nodes in the fiber path route the lightpath in the optical domain using their active switches. The end-nodes access the lightpath with transmitters and receivers that are tuned to the wavelength on which the lightpath operates. In the absence of any wavelength conversion device, a lightpath is required to be on the same wavelength channel throughout its path in the network.

A fundamental requirement in a wavelength-routed optical network is that two or more lightpaths traversing the same fiber link must be on different wavelength channels so that they do not interfere with one another [8]. In wavelength-routed WDM networks, we can use Static Lightpath Establishment (SLE) where a set of lightpaths are set up all at once and remain in the network for a long time. We can also use Dynamic Lightpath Establishment (DLE), where lightpaths are set up for each connection on a request basis as it arrives and released after some finite

time [9]. SLE with the wavelength-continuity constraint can be formulated as an Integer Linear Program (ILP) in which the objective function is to minimize the number of lightpaths [10].

The DLE problem is more difficult to analyze. Therefore, many heuristic algorithms such as Random, Least-Used, Most-Used, and First-Fit wavelength assignments have been already proposed to assign wavelengths in WDM networks with dynamic traffic [10]. The objective of these algorithms is typically to minimize the overall call blocking probability or maximize the overall utilization in a single-class network. Some analytical studies have been performed in [11–14] to calculate the blocking probabilities or network utilization for different assignment strategies. The results of these studies show that because of the wavelength continuity constraint, the blocking probability in the network is high. To improve the blocking performance, we can use wavelength converters.

A wavelength converter converts an incoming wavelength to a different outgoing wavelength. There are different categories of wavelength converters as follows.

- *Full conversion:* Any wavelength conversion is possible and so any incoming wavelength can be converted to any outgoing wavelength.
- *Limited conversion:* Wavelength conversion is restricted so that only some of the incoming wavelengths can be converted to some other outgoing wavelengths.
- *Sparse Wavelength Conversion:* Networks are comprised of a mix of nodes having full wavelength conversion and no wavelength conversion.

In [11–13, 15], the impact of wavelength converters on blocking performance in wavelength-routed networks has been studied. Because of the high cost of wavelength

converters, it is impractical to employ full-wavelength conversion at all nodes. In [15], the impact of sparse-wavelength conversion has been studied. In [16–19], various optimal algorithms for allocating wavelength converters are investigated. The effect of limited-wavelength conversion has been presented in [20, 21]. These studies show that the blocking performance in sparse-wavelength conversion (or limited wavelength conversion) networks can be close to blocking performance in the full wavelength conversion networks but the cost is lower. Multifiber multiwavelength wavelength-routed networks have been shown to offer blocking performance similar to networks that employ limited or sparse-wavelength conversion [22, 23].

1.1.3 WDM Grooming Networks

In wavelength-routed networks, although each wavelength can have a transmission capacity up to ten gigabit per second (e.g., OC-48 or OC-192), some traffic streams may require lower bandwidth (e.g., OC-3 or OC-12). To utilize the wavelengths more efficiently, traffic *grooming* is deployed to share the capacity of a wavelength among users with different bandwidth granularities by multiplexing/demultiplexing the lower rate traffic onto high capacity wavelengths. Therefore, the bandwidth of each wavelength may be divided into a set of time slots. Depending on the bandwidth requirement of a traffic demand, one or multiple time slots can be used to accommodate the traffic. In WDM grooming networks, nodes are equipped with Add/Drop Multiplexers (ADMs). Each node can have wavelength converter (WC) and time-slot interchanger (TSI) devices [24]. TSIs are used to switch a set of time slots from one wavelength to another, whereas WCs convert an incoming wavelength to another outgoing wavelength. In this type of network both LSC and TDM interfaces are employed.

The grooming of traffic can be static or dynamic. In static traffic grooming, the

source-destination pairs whose requirements will be combined are predetermined. In this case, the goal is to minimize network resources such as wavelengths or switches [25–27]. In dynamic traffic grooming, connection requests from different source-destination pairs arrive in a random fashion. A connection is established depending on the existing lightpaths at the time of request arrival. The objective, then, is to minimize the blocking probabilities or maximize wavelength utilization [28–30].

Some surveys on the traffic grooming problem can also be found in [7, 31, 32]. In [7] the traffic grooming problem is defined and a general formulation that captures the features of problem variants is provided. It also presents a comparative survey of the literature and offers a set of research directions. An overview of the traffic grooming problem and survey of some recent work on grooming in SONET over WDM is presented in [31]. Also in [32], various network and node architectures for different traffic-grooming scenarios are compared and discussed.

Another important issue in WDM networks is the Routing and Wavelength Assignment (RWA). In the following paragraphs, we briefly describe this issue.

1.2 Routing and Wavelength Assignment Algorithms

Because of the complexity of the combined wavelength assignment and routing problem, it can be simplified by subdividing them into two separate problems: the routing problem and the wavelength assignment problem.

1.2.1 Routing

Routing algorithms employ Fixed Routing (FR), Fixed Alternate Routing (FAR), and Adaptive Routing (AR) algorithms. FR employs a single route for lightpath

communication while FAR utilizes multiple fixed routes and as a connection request arrives, one route is chosen from a predetermined table. For both algorithms, the routes are predetermined and much simpler to implement than AR. Fixed routing algorithms may show higher blocking probability than adaptive ones [9]. AR can be classified based on global or local information. There are different implementations of AR: Link-State, Distributed routing or Distance-Vector, Least Congested Path (LCP). In the link-state approach, each node must maintain all network state and find a route for a connection request and also all nodes must be informed about the changes in state. Distance-vector approach does not need to keep information of all nodes, but instead each node maintains a routing table, which indicates the next hop and its distance to next destination. In [9], they compare the performance of distance-vector approach and link-state approach via simulation on NSFNET. This comparison shows that link-state approach has shorter stabilizing delay and under low load, lower blocking probability. Distance-vector has advantages in shorter connection set up delays and, under high load, has lower blocking probability. Another algorithm for adaptive routing is LCP where a sequence of routes is pre-chosen. When a connection is requested, the least congested path among the pre-determined routes is selected. High computational complexity of LCP is its disadvantage. It has been shown that using shortest path first and LCP second for tie breaking works better than using LCP alone.

1.2.2 Wavelength Assignment

Current DLE wavelength assignment algorithms can be divided into two categories: Complete Sharing (CS) and Complete Partitioning (CP) [33]. When implementing CS, no wavelength is reserved for any class. In addition, an arriving call will be

accepted if at least one wavelength is available throughout all the hops along its path. When deploying CP policy, each class is assigned a constant number of wavelengths that cannot be used by calls from the other classes. For DLE problem, we attempt to minimize the overall call blocking probability or maximize the overall utilization in a single-class network. Several heuristics have been proposed in the literature [10] as follows:

- Random: One of the available wavelengths is randomly selected.
- First-Fit: All wavelengths are numbered and the lower numbered one is chosen.
- SPREAD/Relative Used: The least used wavelength is selected.
- PACK/Most-Used: The most used wavelength is chosen.
- Least-Loaded: Wavelength with the largest residual capacity on most-loaded link is selected in multi-fiber network.
- Max-Sum: Consider all possible lightpaths with their pre-selected routes in network and attempt to maximize the remaining path capacity after lightpath establishment and can be used in both single or multi-fiber networks.
- Relative Capacity Loss: Based on Max-Sum and choose the wavelength that minimizes the capacity loss on all lightpaths.
- Wavelength Reservation: A given wavelength reserved for a traffic stream, usually for a multi-hop path.
- Protecting Threshold: A single-hop connection is assigned a wavelength only if the number of idle wavelengths on the link is above a given threshold.

All of the above approaches may be used as on-line algorithms and also can mix with different routing approaches. Some more RWA algorithms are presented in [20,34–36]. The routing and wavelength assignment in WDM grooming networks is studied in [37,38].

1.3 Motivation

Most of the mentioned RWA schemes have been proposed for the case when each call occupies the whole wavelength. The objective of these algorithms is typically to minimize the overall call blocking probabilities in a *single-rate* WDM network [10–15]. A few analytical models have been proposed in the literature to compute the blocking probabilities in *multi-rate* WDM networks with grooming capabilities [28,39,40]. In [28], a capacity correlation model is presented for evaluation of blocking probabilities in a multi-hop single wavelength path, whereas [39] discusses the numerical computation of the blocking rates in a two-class network without capacity correlation between the links. An analytical model is provided in [40] to evaluate the blocking performance in WDM-Time Division Multiplexing (TDM) networks.

None of the above mentioned RWA algorithms considers Quality of Service (QoS) and service differentiation in the network. Differentiation involves providing different services, in terms of blocking probabilities, to different users according to their classes of traffic. Our motivation is to propose analytical models to provide service differentiation in WDM networks.

As a special case of service differentiation, we propose fairness control algorithms in order to equalize the blocking probabilities of different traffic classes. The fairness issue in WDM grooming networks is investigated in [41]. In this study, a CAC

mechanism is used to improve the fairness in the network. The CAC algorithm is implemented based on current statistics (i.e., blocking probabilities of different classes in the whole network). This mechanism can be used only after the network passes the transient condition.

1.3.1 Contributions of this Thesis

The main contribution of this thesis is to provide service differentiation and fairness control in both wavelength-routed WDM grooming and non-grooming networks. We take into consideration the dynamic and stochastic nature of the system. The allocation strategies are based on analytical models. We summarize the contributions of this research in the following points:

- 1- Providing service differentiation and fairness control in non-grooming WDM networks with and without WCs. We propose an **optimal dynamic** wavelength allocation scheme for simple non-grooming WDM network. (Chapter 2).
- 2- Showing *analytically* and numerically that the optimal policy has the form of a monotonic non-decreasing switching curve for each class (Appendix A).
- 3- Developing approximations and using the properties of the optimal policy to design a heuristic Multi-Threshold (MT) allocation policy to provide service differentiation and fairness management in an arbitrary non-grooming WDM ring network (Chapter 3).
- 4- Showing *analytically* that an objective function used for fairness determination is unimodal (Appendix B).

- 5- Studying an optimal Call Admission Control (CAC) mechanism in a WDM network with grooming capabilities to provide service differentiation and fairness control (Chapter 4).
- 6- Proposing a heuristic decomposition algorithm with lower computational complexity to implement the CAC scheme in ring networks (Chapter 4).
- 7- Presenting an analytical model to evaluate the blocking performance and determine the required number of wavelengths to meet a predefined GoS for a WDM grooming network (Chapter 5).
- 8- Developing approximations and studying the asymptotic behavior of the blocking probabilities. We provide an *analytical* model for asymptotic behavior in low load traffic (Appendix C).

We have published the research work presented in this thesis in several international conferences [42–49] and technical journals [50, 51].

1.4 Organization of this Thesis

This thesis is organized in four major chapters. This is intended to logically separate the research work studied in this thesis.

Chapter 2 is where we started investigating the area of optimal resource allocation in wavelength-routed WDM networks. We started with a fundamental topology and proposed an optimal dynamic wavelength allocation scheme in order to provide service differentiation in this topology. This problem has been formulated as a Markov Decision Process (MDP) [52]. It has been analytically and numerically shown that the optimal policy has the form of a monotonic non-decreasing switching curve. In

Chapter 3, we use the properties of the optimal policy presented in Chapter 2 and developed approximations and proposed a novel heuristic threshold-based wavelength allocation policy to provide fairness control in WDM ring networks (with and without WCs). A recursive simulation-based algorithm is developed to find the optimal thresholds.

In Chapter 4, we study a Call Admission Control (CAC) mechanism to provide service differentiation in WDM grooming networks. Similar to Chapter 2, using an MDP approach, an optimal dynamic CAC policy is derived for a single-wavelength single-link network. Furthermore, a heuristic decomposition algorithm, with lower computational complexity and very good performance, is proposed for realistic networks. Chapter 5, presents an analytical model to evaluate the blocking performance of a WDM grooming network. In this chapter, we also study the behavior of the system in different regimes, in particular, we investigate the asymptotic behavior of the blocking probabilities in low load traffic. Chapter 6 concludes the thesis and presents suggestions for future research in this area.

Chapter 2

Optimal Dynamic Wavelength Allocation

In this chapter, we investigate service differentiation and fairness problems in a WDM wavelength-routed network with dynamic traffic. First, we define a Markov Decision Process (MDP) model [52] for a 3-node tandem network with the objective of maximizing the weighted sum of class-based utilization. Based on the MDP formulation, the optimal wavelength allocation policy is determined. As shown in many admission control and resource allocation problems in telecommunications, under some conditions the optimal policy of an MDP exists and it is stationary and monotone [53–60]. In [61], the multimodularity, submodularity, and convexity properties are investigated in queuing systems. These properties imply the monotonicity of optimal policy in the context of stochastic control. By using the multimodularity of the cost function and the induction method, we prove that our optimal policy is a non-decreasing monotonic switching curve. Moreover, the Policy Iteration algorithm [62] is deployed to determine numerically the optimal policy. The simulation results, carried out for 3-node tandem network, show that our optimal policy yields significant performance improvement compared to CP and CS policies.

In Section 2.1, we describe the problem and make some assumptions regarding the network. In Section 2.2, the problem is formulated in an MDP framework with the discounted cost model. Section 2.3 shows the multimodularity of the value function and provides the structure of optimal policy. Section 2.4 compares the performance of our proposed allocation algorithm with other standard policies.

2.1 Problem Description

We consider a 3-node tandem topology for a single fiber circuit switched wavelength routed network as depicted in Fig. 2.1(a). This fundamental topology can represent the link-load correlation model [16], [12]. Traffic is divided into three classes. Each class corresponds to a different source-destination pair. Class 1 (respectively, Class 3) consists of the users that use hop H_1 (respectively, H_2); Class 2 includes the customers that use both hops H_1, H_2 . If a WC is deployed at node 2, then a Class 2 call is accepted whenever there is at least one available wavelength in both hops. Without a WC at this node, the same wavelength must be available in both hops to accommodate a Class 2 call. In addition, we make the following assumptions:

- The total number of available wavelengths in the system is W .
- Any arriving call is blocked when all wavelengths along its path are used.
- Blocked calls do not interfere with the system.
- Arrivals of Class ℓ calls, $\ell = 1, 2, 3$, are distributed according to Poisson process with rate λ_ℓ .
- Call holding times of Class ℓ calls are exponentially distributed with mean μ_ℓ^{-1} .

Figure 2.1: 2-hop network topology: (a) For a 3-class system, (b) For a 2-class system.

Wavelength allocation policy is a particular problem related to resource allocation policies. In general, current wavelength allocation strategies are deploying heuristic algorithms such as Complete Sharing (CS) and Complete Partitioning (CP) [33]. When using CS, the global network utilization is high, but it is suboptimal if different classes of users provide different rewards for the same grade of service. When deploying CP policy, each class is assigned a constant number of wavelengths that cannot be used by calls from the other classes. Hence, it supports service differentiation and controls class-based blocking probabilities. However, CP policy may not maximize the overall utilization of the available resources.

To improve the system performance in a dynamic environment, it would be essential to assign a certain number of wavelengths to each class as a function of the current number of customers from different classes. In this chapter, we propose a dynamic wavelength allocation policy, which is called Dynamic Partitioning (DP) hereafter. It consists of determining the appropriate number of wavelengths allocated to each class taking into account the current state of the system, with the objective of maximizing the weighted sum of the number of calls for each class. This approach can take advantage of both CP and CS policies, providing service differentiation while maximizing the available resources.

2.2 Problem Formulation

In this section, we present an MDP approach to the problem described in Section 2.1. In order to model the system, we first define the state of the system and then introduce the appropriate cost function.

2.2.1 Defining the state of the system for a network with WC

Let n_ℓ denote the number of Class ℓ calls in the system and m be the number of wavelengths currently allocated to Class 2 calls to use in both hops H_1 and H_2 . We first formulate the problem when there is a WC at node 2. Thus, for any (n_1, n_2, n_3) , one can derive $i = W - m - n_1$, $j = m - n_2$, and $k = W - m - n_3$ as the numbers of available wavelengths reserved for Class 1, Class 2, and Class 3 users, respectively (refer to Fig. 2.1(a)). Therefore, the four-component vector (i, j, k, m) completely characterizes the system. Let $S = \{(i, j, k, m) | 0 \leq i \leq W - m, 0 \leq j \leq m, 0 \leq k \leq W - m, 0 \leq m \leq W\}$ be the state space of the system and let s_t denote the state of the system at time t . Based on the statistical assumptions, $\{s_t, t \geq 0\}$ is a continuous-time Markov chain whose transitions are either the event of a call request arrival or call departure.

2.2.2 Defining the State of The System for a Network without WC

When removing the WC from node 2, the *wavelength continuity constraint* must be satisfied. Let Λ_1 and Λ_2 denote the sets of available wavelengths on H_1 and H_2 , respectively. Let q denote the number of wavelengths that are available on both H_1 and H_2 (i.e., $q = |\Lambda_1 \cap \Lambda_2|$). Variable q shows the number of Class 2 calls that can

be accepted in the system. In a system without WC, if $q = 0$ then a Class 2 call will be blocked. In this case, the state space of the system is $S' = \{(i, j, k, m, q) \mid 0 \leq i \leq W - m, 0 \leq j \leq q, 0 \leq k \leq W - m, 0 \leq m \leq W, 0 \leq q \leq m\}$, where $i = W - m - n_1$, $j = m - n_2$, and $k = W - m - n_3$.

One can note that for a given q , if $(i, j, k, m, q) \in S'$ then $(i, j, k, m) \in S$. Thus, both systems with and without WC have similar structures. In the following, we only focus on a system with WC.

2.2.3 MDP Description

To simplify the notation, we drop index t from s_t and introduce the following operators:

- $D_\ell : S \mapsto S$, $\ell = 1, 2, 3$; *Departure Operator*, describing the change of the state of the system at a Class ℓ departure time.

$$- D_1 s = (i + 1, j, k, m), D_2 s = (i, j + 1, k, m), D_3 s = (i, j, k + 1, m).$$

- $A_\ell : S \mapsto S$, $\ell = 1, 2, 3$; *Arrival Operator*, describing the change of the state of the system at a Class ℓ arrival time.

$$- A_1 s = ((i - 1)^+, j, k, m), A_2 s = (i, (j - 1)^+, k, m), A_3 s = (i, j, (k - 1)^+, m),$$

where $x^+ = \max(0, x)$ and $s = (i, j, k, m)$.

According to the above operators, we can define the set of possible events $\mathcal{E}$ as

$$\mathcal{E} = \{A_1, A_2, A_3, D_1, D_2, D_3\}.$$

Investigating DP policy involves the determination of wavelength allocation as a function of the current state of the system, $s = (i, j, k, m)$, and the event $e \in \mathcal{E}$.

Figure 2.2: Possible states after the departure of a Class 2 Call: (a) Original state, (b) Final state after taking action $a = 0$, (c) Final state after taking action $a = -1$.

The objective, then, is to maximize the usage of the optical resources. Equivalently, this can be translated into maximizing the weighted sum of the number of calls from different classes. This problem can be formulated as an MDP [63]. Let us describe the model more accurately in the following paragraphs.

- Decision epochs take place only at departure times, when one call terminates and one wavelength becomes available. *The released wavelength may be reserved for the same class or switched to use by calls of other classes.* This is shown in Fig. 2.2 where the initial state is $s(i, j, m)$. In Fig. 2.2(b), the final state of the system is presented after a departure of Class 2 call, when we decide to keep the wavelength for the same class. Fig. 2.2(c) depicts the state of the system when the policy maker decides to reserve the wavelength for Class 1 and Class 3 users. Thus, the decision results in leaving m unmodified (Fig. 2.2(b)) or *decreasing* it by 1 (Fig. 2.2(c)). In a similar fashion, when considering the termination of a Class 1 (or 3) call, the decision will result in *increasing* the value of m by 1, or leaving it unchanged.

- When the system is in state s and the event e has just occurred, the decision maker takes *action* a from a set of possible actions $\mathcal{A}(s, e)$, where

$$\mathcal{A}(s, e) = \begin{cases} \{\emptyset\} & \text{if } e = A_1 \text{ or } A_2 \text{ or } A_3 \\ \{0, +1\} & \text{if } e = D_1 \text{ or } D_3 \\ \{-1, 0\} & \text{if } e = D_2 \end{cases}$$

Let $P_a, a \in \mathcal{A}(s, e)$ be the *Policy Operator* describing the change of state of the system when applying action a after a departure:

$$\begin{aligned} - D_1 P_0 s &= (i+1, j, k, m), D_1 P_1 s = (i, (j+1)^-, (k-1)^+, (m+1)^-), \\ - D_2 P_0 s &= (i, j+1, k, m), D_2 P_{-1} s = ((i+1)^-, j, (k+1)^-, (m-1)^+), \\ - D_3 P_0 s &= (i, j, k+1, m), D_3 P_1 s = ((i-1)^+, (j+1)^-, k, (m+1)^-), \end{aligned}$$

where $x^+ = \max(0, x)$, $x^- = \min(W, x)$ and $s = (i, j, k, m)$.

- This initial continuous-time MDP can be converted into an equivalent discrete time MDP by applying the *uniformization* technique [63]. To do so, we introduce a random sampling rate ν defined as $\nu := W(\mu_1 + \mu_2 + \mu_3) + \lambda_1 + \lambda_2 + \lambda_3$. In the original MDP, the system is observed at the times of actual transitions. After applying uniformization technique, the system may be observed between two consecutive actual transitions which are called ‘fictitious’ or ‘dummy’ transitions. When considering the discrete-time MDP, only one single transition can occur during each time slot. Let s_n denote the state of the system for the equivalent discrete-time MDP during time slot n . A transition can correspond to an event of: 1) Class 1 call arrival or departure, 2) Class 2 call arrival or departure, 3) Class 3 call arrival or departure, and 4) fictitious (or dummy) transition.

2.2.4 The Discounted Cost Problem

To complete the MDP description, we need to define a reward (or cost) function [63]. As mentioned previously, our objective is to determine a wavelength allocation policy that maximizes the weighted sum of n_1 , n_2 , and n_3 . Therefore, we define the one-step reward function which expresses the instant reward when the state of the system is s :

$$R(s) = \beta_1 n_1 + \beta_2 n_2 + \beta_3 n_3 = (\beta_1 + \beta_3)W - (\beta_1 i + \beta_2 j + \beta_3 k + (\beta_1 + \beta_3 - \beta_2)m), \quad s \in S \quad (2.2.1)$$

where β_1 , β_2 , and β_3 are the weights assigned to Classes 1, 2, and 3 users, respectively. Note that one can maximize $R(s)$ by minimizing $\beta_1 i + \beta_2 j + \beta_3 k + (\beta_1 + \beta_3 - \beta_2)m$. Consequently, instead of maximizing reward function (4.2.1), we can minimize one-step cost function defined by:

$$C(s) = \beta_1 i + \beta_2 j + \beta_3 k + (\beta_1 + \beta_3 - \beta_2)m = \mathbf{B} \cdot s^T, \quad s \in S \quad (2.2.2)$$

where vector $\mathbf{B} := (\beta_1, \beta_2, \beta_3, (\beta_1 + \beta_3 - \beta_2))$.

The choice of the weights β_1 , β_2 , and β_3 has an impact on wavelength occupancy by each class and on their respective blocking probabilities. By assigning appropriate values to these weights, we can provide service differentiation based on classes of traffic.

We define policy $\Pi = (\pi_1, \pi_2, \dots, \pi_n)$ for n -stage finite-horizon problem such that π_i is the action applied after the i th event. Based on the one-step cost function, we can apply the results of discounted finite-horizon model [62]. In order to estimate the expected cost under a policy π and at the time step n , we can define n -stage

finite-horizon γ -discounted value function as [62]:

$$V_n^\pi(s) = E^\Pi \left[\sum_{l=0}^{n-1} \gamma^l C(s_l) \mid s_0 = s \right] \quad (2.2.3)$$

where $0 < \gamma < 1$ is the discount factor, $\Pi = (\pi_1, \pi_2, \dots, \pi_n)$ is the allocation policy and E^Π denotes the conditional expected cost given that the initial state is s while the decision maker applies the policy Π . The optimal policy $\Pi_n(s)$ and the minimum value function $V_n(s)$ are given by:

$$\Pi_n(s) = \arg \min_{\Pi} V_n^\Pi(s) \text{ and } V_n(s) = \min_{\Pi} V_n^\Pi(s).$$

The optimal value function and the optimal policy can be computed by using the following recursive scheme, known as the relative value iteration algorithm [62].

$$V_{n+1}(s) = \min_{\Pi} [C(s) + \gamma \sum_{s'} P_{ss'}^a V_n(s')] \quad (2.2.4)$$

where $P_{ss'}^a = P(s_{n+1} = s' \mid s_n = s, \Pi_n = a)$ is the transition probability to jump from state s to state s' when applying action a .

Using the uniform sampling rate ν , introduced in the uniformization procedure, the transition probabilities can be written [64]:

$$P_{ss'}^a \times \nu = \begin{cases} \lambda_1 & \text{if } s' = A_1 s, \\ \lambda_2 & \text{if } s' = A_2 s, \\ \lambda_3 & \text{if } s' = A_3 s, \\ n_1 \mu_1 & \text{if } s' = D_1 P_0 s, a = 0, \\ n_1 \mu_1 & \text{if } s' = D_1 P_1 s, a = 1, \\ n_2 \mu_2 & \text{if } s' = D_2 P_0 s, a = 0, \\ n_2 \mu_2 & \text{if } s' = D_2 P_{-1} s, a = -1, \\ n_3 \mu_3 & \text{if } s' = D_3 P_0 s, a = 0, \\ n_3 \mu_3 & \text{if } s' = D_3 P_1 s, a = 1, \\ F & \text{if } s' = s, \end{cases}$$

where $F = W(\mu_1 + \mu_2 + \mu_3) - n_1\mu_1 - n_2\mu_2 - n_3\mu_3$. Replacing $P_{ss'}^a$ in (4.2.2) yields:

$$\begin{aligned} V_{n+1}(s) = C(s) + \frac{\gamma}{\nu} & \left[\lambda_1 V_n(A_1s) + \lambda_2 V_n(A_2s) + \lambda_3 V_n(A_3s) + F V_n(s) + \right. \\ & \mu_1 n_1 \min\{V_n(D_1P_0s), V_n(D_1P_1s)\} + \\ & \mu_2 n_2 \min\{V_n(D_2P_0s), V_n(D_2P_{-1}s)\} + \\ & \left. \mu_3 n_3 \min\{V_n(D_3P_0s), V_n(D_3P_1s)\} \right]. \quad (2.2.5) \end{aligned}$$

From the above equation, it can be noticed that at a Class 1 (respectively, Class 3) call termination time, the optimal action is $a = 0$, if $V_n(D_1P_0s) < V_n(D_1P_1s)$ (respectively, $V_n(D_3P_0s) < V_n(D_3P_1s)$), and $a = 1$, otherwise. Similarly, after a Class 2 departure, the policy maker takes action $a = 0$ if $V_n(D_2P_0s) < V_n(D_2P_{-1}s)$, and $a = -1$, otherwise.

2.3 Structure of the Optimal Policy

In this section, we will prove that for the discounted cost problem, the optimal allocation policy is a switching curve of threshold type. We first show this property for two classes of users, which is less complex and easier to present. In what follows, we use the term *2-class system* when system includes Classes 1 and 2. *Proposition 2.1*, which is based on value iteration and induction method, proves that the optimal value function for a 2-class system is multimodular. In *Theorem 2.1*, using multimodularity of value function, we show that optimal policy is a monotonic non-decreasing switching curve. In *Proposition 2.2* and *Theorem 2.2*, we generalize the results of the 2-class system for the original problem with three classes.

Consider a 2-class system shown in Fig. 2.1(b). Let m be the number of wavelengths allocated to Class 2 calls. Therefore, the state of the system can be expressed

by $s = (i, j, m)$. We consider similar definition and operators for this system. The optimal value function for this case can be shown by:

$$\begin{aligned} V_{n+1}^*(s) = & \mathbf{B}^*.s^T + \gamma[\lambda_1 V_n^*(A_1 s) + \lambda_2 V_n^*(A_2 s) + F^* \times V_n^*(s) + \\ & \mu_1 n_1 \min\{V_n^*(D_1 P_0 s), V_n^*(D_1 P_1 s)\} + \\ & \mu_2 n_2 \min\{V_n^*(D_2 P_0 s), V_n^*(D_2 P_{-1} s)\}] \end{aligned} \quad (2.3.1)$$

where $F^* = (W(\mu_2 + \mu_1) - n_2 \mu_2 - n_1 \mu_1)$, vector $\mathbf{B}^* := (\beta_1, \beta_2, (\beta_1 - \beta_2))$ and $\mathbf{B}^*.s^T$ is the one-step cost.

If the value function has some properties such as *convexity*, *submodularity*, or more generally *multimodularity*, then its computation time will be remarkably decreased and we can show that the associated optimal policy will have a simple switching curve. To show the multimodularity of the value function, we first introduce a class of multimodular functions, $\mathcal{F}$, as follows [61].

Definition 2.1: A function $f \in \mathcal{F}$ such that $f : \mathbb{N}^3 \mapsto \mathbb{R}$ is multimodular, if for all $(i, j, m) \in \mathbb{N}^3$:

$$f(i, j, m) + f(i, j + 1, m + 1) \leq f(i + 1, j, m) + f(i - 1, j + 1, m + 1), \quad (2.3.2)$$

$$f(i, j, m) + f(i - 1, j, m + 1) \leq f(i, j - 1, m) + f(i - 1, j + 1, m + 1), \quad (2.3.3)$$

$$f(i, j, m) + f(i - 1, j + 1, m) \leq f(i, j, m - 1) + f(i - 1, j + 1, m + 1). \quad (2.3.4)$$

To interpret the multimodularity of f , as an example we rewrite (2.3.2) as follows,

$$f(i - 1, j + 1, m + 1) - f(i, j + 1, m + 1) \geq f(i, j, m) - f(i + 1, j, m). \quad (2.3.5)$$

This equivalent inequality shows that function f has monotonic increasing differences. In the following, we show that the value function is multimodular and satisfies inequalities (2.3.2)-(2.3.4). This leads us to show that the optimal policy is a monotonic switching curve.

2.3.1 Multimodularity of the Value Function and Monotonicity of the Optimal Policy

Proposition 2.1: The value function for a 2-class system, $V_{n+1}^*(i, j, m)$, is multimodular in i , j , and m .

Proof: The proof of this proposition, which is based on induction, is given in Appendix A.

We now derive the structure of the optimal policy and show that it is non-decreasing switching curve. More specifically, we will prove that after a termination of Class ℓ call, there is a non-decreasing switching curve which partitions the state space into two regions. One is *transferring region* (i.e., for all states belonging to this region, the optimal policy transfers the released wavelength to the other class), whereas the other is *keeping region* (i.e., in all states within this region, the optimal policy keeps the released wavelength to Class ℓ calls). As an illustrative example, the structure of the optimal policy is shown in Fig. 2.3.

Theorem 2.1: For each class of users in a 2-class system, the optimal wavelength allocation policy is a monotonic non-decreasing switching curve.

Proof: To prove this theory, we consider the two following cases.

Case 1: Optimal policy after a Class 1 departure: Suppose that the state of the system is $s = (i, j, m)$ and a Class 1 call terminates. We first claim that if the optimal action is to keep the released wavelength for Class 1 (i.e., $a = 0$), then for all states $s' = (i', j, m)$ with $i' < i$, the optimal action will be equal to zero, as well, which is in agreement with intuition. Because of taking action $a = 0$ as optimal action in state s , it can be seen that: $V_n^*(i+1, j, m) - V_n^*(i, j+1, m+1) < 0$. By using multimodularity

of $V_n^*(s)$, inequality (2.3.2), one can show that

$$\begin{aligned} V_n^*(i, j, m) - V_n^*(i - 1, j + 1, m + 1) &\leq \\ V_n^*(i + 1, j, m) - V_n^*(i, j + 1, m + 1) &< 0. \end{aligned} \quad (2.3.6)$$

The above inequality shows that if a Class 1 call departs while system state is $(i - 1, j, m)$, then the policy maker again selects action $a = 0$.

Now suppose that the system's state is $s'' = (i, j', m)$ with $j' > j$ (i.e., the number Class 2 users in state s'' is smaller than the one in state $s = (i, j, m)$). In this case, we can use (2.3.3) to show that:

$$\begin{aligned} V_n^*(i + 1, j + 1, m) - V_n^*(i, j + 2, m + 1) &\leq \\ V_n^*(i + 1, j, m) - V_n^*(i, j + 1, m + 1) &< 0. \end{aligned} \quad (2.3.7)$$

Therefore, we prove that if the state of the system is $s = (i, j, k)$ and the controller takes action $a = 0$ after a Class 1 termination as the optimal action, then for all states $s' = (i', j, m)$ and $s'' = (i, j', m)$ with $i' < i$ and $j' > j$, action $a = 0$ will be taken as the optimal action.

Case 2: Optimal policy after a Class 2 departure: An analogous result can be proved for Class 2.

As an example, one can consider Fig. 2.3(a). Suppose that the state of the system is $s = (3, 4, 5)$ and action $a = 0$ is taken in this state, then for all states with $i' < 3$ and $j' > 4$, the optimal action is $a = 0$.

We now add Class 3 users to the problem and determine analytically the properties of the optimal policy. The problem formulation can be extended for a 3-class system by generalizing *Definition 2.1*, *Theorem 2.1*, and *Proposition 2.1*.

Figure 2.3: Optimal Policy for a 2-class system.

Definition 2.2: A function $f \in \mathcal{F}$ such that $f : \mathbb{N}^4 \mapsto \mathbb{R}$ is multimodular, if for all $(i, j, k, m) \in \mathbb{N}^4$:

$$\begin{aligned} f(i, j, k, m) + f(i, j + 1, k - 1, m + 1) &\leq f(i + 1, j, k, m) \\ &\quad + f(i - 1, j + 1, k - 1, m + 1), \end{aligned} \quad (2.3.8)$$

$$\begin{aligned} f(i, j, k, m) + f(i - 1, j, k - 1, m + 1) &\leq f(i, j - 1, k, m) \\ &\quad + f(i - 1, j + 1, k - 1, m + 1), \end{aligned} \quad (2.3.9)$$

$$\begin{aligned} f(i, j, k, m) + f(i - 1, j + 1, k, m + 1) &\leq f(i, j, k + 1, m) \\ &\quad + f(i - 1, j + 1, k - 1, m + 1), \end{aligned} \quad (2.3.10)$$

$$\begin{aligned} f(i, j, k, m) + f(i - 1, j + 1, k - 1, m) &\leq f(i, j, k, m - 1) \\ &\quad + f(i - 1, j + 1, k - 1, m + 1). \end{aligned} \quad (2.3.11)$$

Proposition 2.2: $V_{n+1}(i, j, k, m)$ is multimodular in i, j, k , and m .

Proof: The proof is similar to that of *Proposition 1*. It is given in Appendix B.

Using the result of *Proposition 2.2*, we prove that the optimal policy is monotonic.

Theorem 2.2: The optimal policy for a 3-class system is a monotonic nondecreasing switching curve.

Proof: The major contribution of this section will be proved in this part. Suppose that the state of the system is $s = (i, j, k, m)$ and a Class 2 call departs from the network and the optimal action is $a = 0$, then we can see that: $V_n(i, j + 1, k, m) - V_n(i + 1, j, k + 1, m - 1) < 0$. According to multimodularity of $V_n(s)$, inequality (2.3.9), one can derive:

$$\begin{aligned} V_n(i, j, k, m) - V_n(i + 1, j - 1, k + 1, m - 1) &\leq \\ V_n(i, j + 1, k, m) - V_n(i + 1, j, k + 1, m - 1) &< 0 \end{aligned} \quad (2.3.12)$$

This illustrates that if a Class 2 call departs while the state of the system is $(i, j - 1, k, m)$, then the optimal action is again $a = 0$. As a result, the optimal action remains the same (i.e., $a = 0$) for all states $s' = (i, j', k, m)$ with $j' < j$. Now suppose that the state of the system is $(i + 1, j, k, m)$, then by using (2.3.8), we get the following inequality

$$\begin{aligned} V_n(i + 1, j + 1, k, m) - V_n(i + 2, j, k + 1, m - 1) &\leq \\ V_n(i, j + 1, k, m) - V_n(i + 1, j, k + 1, m - 1) &< 0, \end{aligned}$$

which confirms that the controller takes action $a = 0$ as the optimal action in state $s'' = (i', j, k, m)$ with $i' > i$. Using (2.3.10), it can be shown that for all states $s''' = (i, j, k', m)$ with $k' > k$, action $a = 0$ will be taken as the optimal action. An analogous proof can be provided for Classes 1 and 3 calls. This property shows that for each class of users, the optimal policy is non-decreasing switching curve.

Hence, the optimal policy is described by three switching curves; each switching curve corresponds to one of the classes. For Class 1 users, we can show that for each (j, k, m) , there exists a minimum value for i , such that $V_n(i + 1, j, k, m) <$

$V_n(i, j+1, k-1, m+1)$. This value is the boundary between *transferring* and *keeping* regions.

Formally, we define Φ_1 , Φ_2 , and Φ_3 as the switching curves for Classes 1, 2, and 3, respectively.

$$\Phi_1 = \min_i \{(i, j, k, m) \in S : V_n(i+1, j, k, m) < V_n(i, j+1, k-1, m+1)\},$$

$$\Phi_2 = \min_j \{(i, j, k, m) \in S : V_n(i, j+1, k, m) < V_n(i+1, j, k+1, m-1)\},$$

$$\Phi_3 = \min_k \{(i, j, k, m) \in S : V_n(i, j, k+1, m) < V_n(i-1, j+1, k, m+1)\}.$$

Recursively, we can determine the sequence of n stage value functions, $\{V_1(s), V_2(s), \dots, V_n(s)\}$, and the limit of this sequence when n goes to infinity. Lippman in [65] shows that $V(s) := \lim_{n \rightarrow \infty} V_n(s)$ exists and it is the solution of the infinite horizon discounted cost problem. Besides, $V(s)$ is the unique solution to the dynamic programming equation given by (4.2.3). Note that the switching curve structure of the optimal policy holds for the infinite horizon discounted case.

2.3.2 Computational Complexity and Numerical Result

The optimal policy can be derived numerically by implementing Policy Iteration algorithm [62]. This algorithm has two phases:

- Policy determination phase which returns the structure of the optimal policy in $O(|S|^3)$ steps where $|S|$ is the total number of states of the system [66]. It can be shown that for a 3-class system with W wavelengths $|S| = O(W^5)$.
- Policy improvement phase which optimizes the determined policy from the previous phase in few iterations. For instance, the policy improvement phase of a 3-class 20-wavelength network with $\gamma = 0.9$ converges in 5 iterations.

Without loss of generality, we assume that $\mu_1 = \mu_2 = \mu_3 = 1$, unless otherwise stated. In this part, we will show the structure of the optimal policy for 2- and 3-class systems which are derived from Policy iteration algorithm.

For a 2-class system, we consider the following cases with these parameters: $W = 10$, $\lambda_1 = \lambda_2 = 5$, $\beta_1 = 1$, $m = 5$; The structure of the optimal policy is plotted for two cases: 1) for $\beta_2 = 0.5$ (Fig. 2.3), and 2) for $\beta_2 = 0.1$ (Fig. 2.4). In both cases, as we have already proved, the optimal policy has the well-known structure of the switching curve, separating the state space into two domains. One domain for which the optimal action is $a = +1$ (depicted by '+') and the other one for which the optimal decision is $a = 0$ (depicted by '0'). Figs. 2.3(a) and 2.4(a) show the optimal policy after a Class 1 call terminates. In Figs. 2.3(b) and 2.4(b), when a Class 2 call departures, the switching curve separates the states whose optimal action is $a = -1$ (depicted by '-') from the states where the optimal action is $a = 0$. Furthermore, the switching curve is monotonic non-decreasing.

Figure 2.4: Optimal Policy for a 2-class system.

The comparison of Figs. 2.3 and 2.4 illustrates the sensitivity of the switching curve to the weighting factors β_1 and β_2 . For $\beta_1 = 1$, when β_2 is changed from 0.1

to 0.5, more priority is given to Class 2 calls. As a consequence, the system is more likely to reserve a larger number of wavelengths to this class; either by transferring one wavelength from Class 1 (at a Class 1 call termination), or by maintaining the number of wavelengths to Class 2 calls (at a Class 2 call termination).

Now we show the structure of the optimal policy for three classes of users. Fig. 2.5 depicts the optimal policy for $W = 10$, $k = 4$, $\lambda_1 = \lambda_2 = \lambda_3 = 5$, $\beta_1 = 1, \beta_2 = 0.1$, $\beta_3 = 0.1$, at a Class 1 call termination time.

Figure 2.5: Optimal Policy for 3 classes of traffic. $\beta_2 = 0.1$

Figure 2.6: Optimal Policy for 3 classes of traffic. $\beta_2 = 0.5$

In this figure, each cube represents action $a = 1$. Note that the policy is a monotonic 3D switching curve, dividing the state space into two subsets. The structure of the policy reflects the fact that the three classes of calls are competing for the available wavelengths. In Fig. 2.6, we only change the value of β_2 from 0.1 to 0.5, and calculate the optimal policy for the same parameters as in Fig. 2.5. Comparison of Figs. 2.5 and 2.6 shows that by increasing β_2 , the decision maker gives more resources to Class 2 users (i.e., more cubes are represented).

2.4 Simulation Results and Performance Comparison

In this section, we compare the performance of our proposed DP policy, with those of CS and CP policies. The simulation results are carried out for different classes of traffic with different weights.

In order to implement CP policy, one can divide the total number of wavelengths, W , into two parts; Let M be the number of wavelengths dedicated to Class 2 and $W - M$ be the number of wavelengths reserved to Classes 1 and 3. Note that M is a constant value. Using Erlang's B formula, we can compute p_u^ℓ , the probability of having u users of Class ℓ in the system.

$$p_u^\ell = \frac{\frac{1}{u!} \left(\frac{\lambda_\ell}{\mu_\ell} \right)^u}{\sum_{x=0}^{U_\ell} \frac{1}{x!} \left(\frac{\lambda_\ell}{\mu_\ell} \right)^x}$$

where U_ℓ is the total number of wavelengths allocated to Class ℓ . One can notice that $p_{U_\ell}^\ell$ is the probability that all dedicated wavelengths to Class ℓ are busy (i.e., $p_{U_\ell}^\ell$ is the blocking probability of Class ℓ calls). Using p_{W-M}^1 , p_M^2 , and p_{W-M}^3 we can derive the expected number of calls of each class in the system:

$$N_1(M) = \frac{\lambda_1}{\mu_1} (1 - p_{W-M}^1), N_2(M) = \frac{\lambda_2}{\mu_2} (1 - p_M^2), N_3(M) = \frac{\lambda_3}{\mu_3} (1 - p_{W-M}^3).$$

We also define M^* as:

$$M^* := \arg \max_{M \in \{1, \dots, W-1\}} \beta_1 N_1(M) + \beta_2 N_2(M) + \beta_3 N_3(M). \quad (2.4.1)$$

To compare the performance of DP, CP, and CS policies, we simulate the system by deploying the optimal policy implemented in Section 2.3 to evaluate the performance metric of DP policy. For CP, the simulation result is carried out for two independent

$M/M/(W - M^*)/(W - M^*)$ queues associated with Class 1 and Class 3 and one $M/M/M^*/M^*$ queue related to Class 2. Finally, we simulate the system without any allocation policy to evaluate the performance of CS policy. We use OPNET modeler for all of our simulations. OPNET is graphical and sequential even-driven-based simulation tool which makes the verification and validation parts very easy and straightforward.

Figure 2.7: Performance comparison of DP, CP and CS policies.

We first study the system by considering a 2-class system. Figs. 2.7 and 2.8 depict the average-time reward function $(\beta_1 n_1 + \beta_2 n_2)$ versus the total offered load ρ , where $\rho = \lambda_1/\mu_1 + \lambda_2/\mu_2$. In both examples, the parameters are set as follows: $W = 10$, $\lambda_1 = \lambda_2$, $\beta_1 = 1$, and $6 \leq \rho \leq 40$. The difference between the two examples lies in the value of β_2 . In Fig. 2.7, we set $\beta_2 = 0.1$. It shows that for low load, all the policies have similar performance. As the load increases, DP policy shows much better performance, in particular when compared to CS policy. Fig. 2.8 illustrates

the performance of the system when $\beta_2 = 0.5$. Note that in this case, β_2 is closer to β_1 , compared with that in Fig. 2.7. One can observe that CP and DP policies yield to similar performance, which is still better than CS policy. When β_2 is close to β_1 , the priority assigned to Class 2 calls is close to the one assigned to Class 1 calls. Therefore, calls from both classes are equally competing for the access to the wavelengths.

Figure 2.8: Performance comparison of DP, CP and CS policies.

To determine the relative performance improvement of DP policy when compared to CS policy, we calculate the relative performance ratio, $(DP_p - CS_p)/CS_p$, where DP_p and CS_p represent the performance of DP and CS policies, respectively. This quantity is plotted versus the load (Fig. 2.9). For $\beta_1 = 1$ and $\beta_2 = 0.1$, DP policy has significantly better performance, up to 75% for heavy load, whereas for $\beta_2 = 0.5$, the best improvement is only equal to 25%.

Another important performance metric is the weighted sum of blocking rates.

Figure 2.9: Relative performance comparison.

Applying DP and CS policies, we simulate and determine this quantity for a system with $W = 10$, $\lambda_1 = \lambda_2$, $\beta_1 = 1$, $\beta_2 = 0.1$, and $6 \leq \rho \leq 40$. The relative performance improvement is plotted in Fig. 2.10. We can see that DP policy have higher performance, up to 45% for intermediate offered load (e.g., $\rho \simeq 15$), which is a fact observed in networks and is in agreement with intuition.

One potential problem in wavelength-routed WDM networks is the following unfairness issue: calls which need to pass more hops from source to destination, experience more blocking than calls requiring fewer hops [67]. We implemented a CS policy for a 3-class system with the following parameters: $W = 10$, $\lambda_1 = \lambda_2 = \lambda_3$, $\rho = \lambda_1/\mu_1 + \lambda_2/\mu_2 + \lambda_3/\mu_3$, and $9 \leq \rho \leq 45$. The simulation result depicted in Fig. 2.11 shows that Class 2 calls experience more blocking than Class 1 and Class 3 calls. We apply the DP policy to improve the unfairness problem in the network. Through the simulation, we observe that for $\beta_1 = \beta_3 = 1$ and $\beta_2 = 1.95$, all the

Figure 2.10: Relative performance comparison between DP and CS policies.

classes can experience the same blocking probabilities (Fig. 2.11). Another approach to equalize the blocking probabilities is to use CP policy to partition the wavelengths among the users of different classes. In this particular example, we dedicate half of the wavelengths to Class 2 calls and the other half to Classes 1 and 3. Fig. 2.11 shows the blocking probabilities while we deploy CS, CP and DP policies. It depicts that by using the CS policy, Class 1 and 3 traffic experience less blocking than Class 2. Note that by using CP and DP policies, all three classes of users experience the same blocking rates. Moreover, the performance of DP policy is always better than CP policy, especially for low and intermediate loads. Qualitatively, we can conclude that when the different classes of calls are differentiated with priorities, then DP policy is the most efficient policy.

Figure 2.11: Performance comparison of DP, CP and CS policies for three classes of users.

2.5 Verification and Validation of Simulation Models

Verification is concerned with building the model right. It is utilized in the comparison of the conceptual model to the computer representation that implements that conception. Validation is concerned with building the right model. Validation is deployed to determine that a model is an accurate representation of the real system [68]. To verify the model one can make a flow diagram which includes each logically possible action that a system can take when an event occurs, and follow the model logic for each action for each event type. Since we have used OPNET to simulate the system, the verification and validation of the simulation is simple.

2.5.1 OPNET Modeler

OPNET Modeler [69] is a tool to model, simulate, design and study communication networks, devices, protocols, and applications. Graphical editors and object-oriented modeling approach mirror the structure of networks, so the system intuitively maps to the model. Modeler is based on a series of hierarchical editors that directly parallel the structure of real networks.

- “The Project Editor graphically represents the topology of a communications network. Networks consist of node and link objects, configurable via dialog boxes. Drag and drop nodes and links from the editor’s object palettes to build the network, or use import and rapid object deployment features. Use objects from OPNET’s extensive Model Library, or *customize palettes to contain your own node and link models*. The Project Editor provides geographical context, with physical characteristics reflected appropriately in simulation of both wired and mobile/wireless networks. Use the protocol menu to quickly configure protocols and activate protocol specific views.
- The Node Editor captures the architecture of a network device or system by depicting the flow of data between functional elements, called ”modules.” Each module can generate, send, and receive packets from other modules to perform its function within the node. Modules typically represent applications, protocol layers, algorithms and physical resources, such as buffers, ports, and buses. Modules are assigned process models (developed in the Process Editor) to achieve any required behavior.

- The Process Editor uses a powerful finite state machine (FSM) approach to support specification, at any level of detail, of protocols, resources, applications, algorithms, and queuing policies. States and transitions graphically define the progression of a process in response to events. Each state of a process model contains C/C++ code, supported by an extensive library of functions designed for protocol programming. Each FSM can define private state variables and can make calls to code in user-provided libraries. FSMs are dynamic and can be spawned (by other FSMs) during simulation in response to specific events. Dynamic FSMs dramatically simplify specification of protocols that manage a scalable number of resources or sessions. One can use the Process Editor to *develop entirely new process models*, or use the models in OPNET Technologies' Model Library as a starting point. OPNET's process modeling framework supports full multi-threading and use of parallel computing architectures."

2.5.2 DP implementation using OPNET Modeler

In our simulation we design and develop entirely new process model to implement DP policy. To do so, we first developed C code to find the optimal policies introduced in Section 2.3 and then used the code in the new process model. Figs. 2.12 and 2.13 represent the node level and process level of the OPNET model we have designed for implementing DP policy for a three-node tandem network. As can be seen in fig. 2.12, in node level we have used three random generators, p_0, p_1 and p_2, for generating Poisson arrivals for three classes of users. These random generators are from OPNET Library. We also design our new process model called server_glob to implement the DP policy. Fig. 2.13 shows the process level of server_glob in more details. When a user arrives, we go from **idle** state to **serve** state through link **ARRIVAL**. In this

state, we recognize the class of user and depending on availability of wavelengths for that class, we accept or reject the user. When a user leaves the system we go from **idle** state to **Depart** state through link **CHANGE**. In **Depart** state, we look at the lookup table that we have calculated off-line using MDP and decide to keep the release wavelength for the same class or transfer it to other classes. Using OPNET, we have done the following verification steps:

- Examine the model output for reasonableness under a variety of settings of the input parameters and collect a variety of output statistics.
- Make the computer code as self-documenting as possible.
- Give a precise definition of every variable used, and a general description of the purpose of each major section of code.

As an aid in the validation process, we have validated model assumptions and compared the model input-output transformations to corresponding input-output transformations for a simple case of the real system.

One can see that the verification and validation of the simulation model is quite straightforward when using OPNET modeler.

In Chapters 3, 4 and 5, we also use OPNET modeler to simulate the network. Similar to the simulations of this chapter, we combine our own models with the models from OPNET library.

Figure 2.12: OPNET model. Node Level

Figure 2.13: OPNET model. Process Level for server_glob

Chapter 3

Multi-Threshold Wavelength Allocation

As seen in Chapter 2, implementation of an MDP-based DP allocation policy requires exponential time, and as a result, it is infeasible to use it for realistic networks. Hence, we use the properties of DP policy to devise a simpler heuristic allocation algorithm. In Chapter 2 when DP policy is used, we observe that: 1) Optimal actions are taken based on *available wavelengths* for each class. 2) Wavelengths are *shared* as much as possible and by adjusting threshold m , they are dynamically partitioned between classes.

We consider these two facts and propose a Multi-Threshold (MT) allocation scheme to achieve fairness and provide service differentiation. Threshold-based RWA schemes are introduced for dynamic wavelength-routed mesh networks in [67], [70]. In [67], a basic routing method such as Fixed Routing (FR) and Fixed Alternative Routing (FAR), combined with a threshold-based allocation technique, is used in order to obtain the same grade of service on a multi-hop route as on a single-hop. In [70], a routing and signaling protocol called Asynchronous Criticality Avoidance (ACA)

is developed to improve the blocking probabilities in mesh networks. ACA mechanism marks a set of wavelength channels as critical if the available channels between a source-destination pair is less than a predefined threshold. The critical channels information is announced to the other nodes in the network so that the other source-destination pairs avoid the use of these critical channels. The determination of the thresholds has not been studied in [70].

The purpose of this chapter is to develop a threshold-based wavelength allocation mechanism to support fairness and service differentiation in non-grooming WDM ring networks. This chapter is the continuation of Chapter 2. We modify some the assumptions of the previous chapter and then define a Multi-Threshold (MT) wavelength allocation policy to equalize the class-based blocking probabilities in an unidirectional WDM ring network. To determine the optimal thresholds of MT policy, we then define an objective function to be minimized. Using the properties of the objective function, a recursive algorithm is developed to find the optimal thresholds.

In Section 3.1, we describe the network model and modify some of the definitions and assumptions of Chapter 2. In Section 3.2, we introduce the MT allocation scheme and define an objective function which measures the differences between class-based blocking probabilities. Section 3.3 describes our recursive simulation-based optimization algorithm. Section 3.4 compares the performance of the proposed algorithm with other standard algorithms.

3.1 Network Model

We consider a non-grooming unidirectional WDM ring network with N nodes. Each node is equipped by Optical Add/Drop Multiplexers (OADM). Each link carries one

fiber with W wavelengths. The network includes $N(N - 1)$ classes of traffic streams characterized by their source-destination. We modify the definition of classes as follow: A call from Class C_h^r , $r = 1, 2, \dots, N$ and $h = 1, 2, \dots, N - 1$, originates at node r and passes through h hops from origin to destination. In addition to the assumptions stated in Section 2.1, we assume that classes with the same hop-count (i.e., the number of hops used from origin to destination) are assigned the same weighting factor (priority), and the same arrival rate (i.e., $\lambda_{C_h^1} = \lambda_{C_h^2} = \dots = \lambda_{C_h^N}$, $h = 1, 2, \dots, N - 1$, where h is the hop-count). Therefore, we can merge classes with the same hop-count into a single class. In the following, we refer to classes with hop-count h as Class c_h . We consider two types of networks in terms of WC usage: networks with WCs in all nodes and network without any WC. In a WDM network without WCs, *wavelength continuity constraint* is considered.

We also make the following assumptions.

- Arrivals of class c_h calls are distributed according to a Poisson process with rate λ_h . Calls of class c_h are uniformly distributed among nodes. The call holding time of this class is exponentially distributed with mean μ_h^{-1} . Unless otherwise stated, we assume $\mu_h^{-1} = 1$.
- The total offer load per link is $\rho_L = \sum_{h=1}^{N-1} (h\lambda_h/\mu_h)$.
- Any arriving call from class c_h is blocked when no wavelength is available along its path.
- Blocked calls do not interfere with the system.

In such a system, since calls from different classes compete for access to the network, fairness management and resource sharing control become very challenging issues.

In this case, calls with smaller hop counts (respectively, higher arrival rates) may starve calls with greater hop-counts (respectively, lower arrival rates). Therefore, the system is unfair in the sense that there is a noticeable difference between the blocking probabilities experienced by different classes of users. Here, the objective of fairness is to keep this difference as small as possible, so that all classes with different hop-counts experience the same blocking probability.

Our goal, then, is to design a threshold-based allocation mechanism which achieves the fairness objective and maximizes overall utilization. In the following sections, we introduce a Multi-Threshold allocation policy to improve fairness in a WDM ring network.

3.2 Multi-Threshold Wavelength Allocation

As mentioned earlier in Chapter 2, CS policy provides high utilization and poor service differentiation (in particular, fairness) whereas CP achieves the fairness but the overall utilization deteriorates. This reduction in utilization results directly from putting up barriers against sharing wavelengths.

In this section, we propose a Multi-Threshold (MT) allocation scheme which takes advantage of both CP and CS policies. When MT policy is used, while all classes of users share the wavelengths, classes with higher hop-counts are protected from the ones with lower hop-counts on each link of the ring network.

We define $\boldsymbol{\tau} = (\tau_1, \tau_2, \dots, \tau_{N-1})$ a vector representing the thresholds of MT allocation where τ_i , $i = 1, 2, \dots, N-1$, is the threshold associated with class c_i traffic on each link of the ring network. To ensure that calls with higher hop-counts are protected from the ones with lower hop-counts, we assume that $\tau_1 \geq \tau_2 \geq \dots \geq \tau_{N-1}$. Since

calls from class c_{N-1} experience the highest blocking probability in the ring, they are always accepted in the system. As a result, we set $\tau_{N-1} = 0$. Let l_n , $n = 1, 2, \dots, N$, be the link between node n and node $n+1$ and ω_n denote the number of available wavelengths on link l_n . An arriving call of class c_i will be assigned an available wavelength only if ω_n on all the links along its path is greater than or equal to τ_i .

As an example, consider a 4-node unidirectional ring network shown in Fig. 3.1. One can note that on each link, the total number of available wavelengths W is partitioned into three sets. When the number of free wavelengths on a link is greater or equal to threshold τ_1 , then all three classes of calls share the available wavelengths on that link. If this number is between τ_1 and τ_2 , then only class c_1 calls will be dropped. Furthermore, if the number of free wavelengths is less than τ_2 on a link, then both c_1 and c_2 calls will be blocked and c_3 calls are the only ones allowed to use the wavelength resources on that particular link. As shown in Fig. 3.1, available wavelengths on links l_1 and l_2 are greater than τ_1 . Thus, all classes of calls which pass through l_1 and l_2 can occupy free wavelengths of these links. Since $\tau_2 < \omega_3 < \tau_1$, only c_1 calls, which use link l_3 as the part of their route, will be blocked. On links l_4 , we have $\omega_4 < \tau_2$ and as a result, only c_3 calls are allowed to occupy an idle wavelength on this link.

The next step towards MT allocation implementation is to determine the threshold vector $\boldsymbol{\tau}$, so that all classes of calls obtain the same Grade of Service (GoS) in terms of blocking probabilities. To do so, we define the following objective function

$$f(\boldsymbol{\tau}) = \sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} |B_i(\boldsymbol{\tau}) - B_j(\boldsymbol{\tau})| \quad (3.2.1)$$

where $B_i(\boldsymbol{\tau})$ is the class c_i blocking probability associated with $\boldsymbol{\tau}$. Since this objective function measures the differences between class-based blocking probabilities, by

Figure 3.1: A 4-node unidirectional ring network.

minimizing that, we can achieve fairness. To determine $B_i(\tau)$ and compute $f(\tau)$, we have developed a simulation program in OPNET. In order to obtain an accurate value of f , the simulation experiment continues until the steady-state regime is reached. In the following section, we develop an efficient algorithm to minimize the objective function and determine the optimal τ

3.3 Recursive Optimization Algorithm

The optimization procedure can take advantage of structural properties of the objective function (such as monotonicity or multimodularity) to improve the computation time [61]. In *Theorem 1*, we prove that the objective function f has only one unique minimum and it does not have local minima. According to this property and the fact that $W \geq \tau_1 \geq \tau_2 \geq \dots \geq \tau_{N-1} \geq 0$, we propose a gradient-like optimization algorithm in order to determine the appropriate values for the thresholds.

Figure 3.2: An illustrative example to show the minimization algorithm for a function with one unique global minimum.

Definition 3.1: A function is *unimodal* if it decreases to a minimum and then increases. A unimodal function has a single minimum.

Theorem 3.1: The objective function f is unimodal.

Proof: The proof is given in Appendix B.

Fig. 3.2 depicts an illustrative example of the objective function, $f(\tau_1, \tau_2)$, as a function of τ_1 and τ_2 . To find the global minimum, one can first search for a minimum when $\tau_2 = 0$. This is done by incrementally increasing τ_1 . After finding the optimal τ_1 , denoted by $\tau_1^{\tau_2=0}$, and reaching the minimum at $(\tau_1^{\tau_2=0}, 0)$, we increase τ_2 by 1 and search for another minimum in the neighborhood of $(\tau_1^{\tau_2=0}, 1)$. As can be seen in Fig. 3.2, we again need to increase $\tau_1^{\tau_2=0}$ to reach a minimum at $(\tau_1^{\tau_2=1}, 1)$. The reason is that at point $(\tau_1^{\tau_2=0}, 1)$, decreasing of τ_1 results in increasing of f and as a result, we need to increase τ_1 in order to get to the minimum. This procedure continues until the global minimum is reached at point $(\tau_1^{\tau_2=4}, 4)$. Therefore, we can see that

the following inequity is satisfied

$$\tau_1^{\tau_2=4} \geq \tau_1^{\tau_2=3} \geq \tau_1^{\tau_2=2} \geq \tau_1^{\tau_2=1} \geq \tau_1^{\tau_2=0} \quad (3.3.1)$$

We generalize our search for $N-1$ classes as follows.

Let $\boldsymbol{\tau}_{\tau_{i+1}, \dots, \tau_{N-1}}^i = (\tau_1^{*i}, \tau_2^{*i}, \dots, \tau_i^{*i}, \tau_{i+1}, \dots, \tau_{N-1})$ be the optimal value of $\boldsymbol{\tau}$ when the last $N-i-1$ thresholds are fixed. In other words

$$\boldsymbol{\tau}_{\tau_{i+1}, \dots, \tau_{N-1}}^i = \arg \min_{\tau_1, \tau_2, \dots, \tau_i} f(\tau_1, \dots, \tau_i, \dots, \tau_{N-1}) \quad (3.3.2)$$

For simplicity, index of $\boldsymbol{\tau}_{\tau_{i+1}, \dots, \tau_{N-1}}^i$ is no longer mentioned in notation. In the following, we introduce a recursive algorithm, Θ_{i+1} , which receives $\boldsymbol{\tau}^i$ and computes $\boldsymbol{\tau}^{i+1}$.

$$\Theta_{i+1}(\boldsymbol{\tau}^i) = \boldsymbol{\tau}^{i+1} \quad (3.3.3)$$

Clearly, by using an initial threshold vector $\boldsymbol{\tau}^0$ and running this algorithm for $N-1$ times, we can obtain $\boldsymbol{\tau}^{N-1}$. Similar to inequality (3.3.1), we can show that when we start our search from initial value $\boldsymbol{\tau}^0 = (0, 0, \dots, 0)$, then

$$\tau_1^{*i+1} \geq \tau_1^{*i}, \tau_2^{*i+1} \geq \tau_2^{*i}, \dots, \tau_i^{*i+1} \geq \tau_i^{*i} \quad (3.3.4)$$

Because of this important property, if the initial value is $\boldsymbol{\tau}^0 = (0, 0, \dots, 0)$, we only need to increase τ_i to find the minimum. Fig. 3.3 depicts the block diagram of this recursive algorithm in the $(i+1)^{\text{th}}$ recursion.

As can be seen, Θ_{i+1} is expressed based on Θ_i , the algorithm's i^{th} recursion. Note that Θ_0 is an identity function (i.e., the input and output of Θ_0 are the same, $\Theta_0(\boldsymbol{\tau}) = \boldsymbol{\tau}$). By initializing the recursive algorithm with $\boldsymbol{\tau}^0 = (0, 0, \dots, 0)$, we can recursively compute the optimal threshold vector $\boldsymbol{\tau}^{N-1}$. For instance, in the first recursion where $\boldsymbol{\tau} = \boldsymbol{\tau}^0 = (0, 0, \dots, 0)$, we increase τ_1 until a minimum is reached at

Figure 3.3: Block Diagram of the recursive minimization function Θ_{i+1} .

point $\tau^1 = (\tau_1^*, 0, \dots, 0)$. In the second recursion where $\tau = \tau^1$, we first increase τ_2 and then for each value of τ_2 , we apply Θ_1 in order to find a minimum for that particular τ_2 . Increasing τ_2 and applying Θ_1 continues until a minimum is found at point $\tau^2 = (\tau_1^*, \tau_2^*, \dots, 0)$. Recursively, the global minimum and the associated optimal threshold vector τ^{N-1} will be determined.

More precisely, this optimization problem can be solved by the following procedure.

Step 1: Set $i = 0$ and $\tau_1^*, \tau_2^*, \dots, \tau_i^*, \dots, \tau_{N-1}^* = 0$.

Step 2: Set $\tau^i = (\tau_1^*, \tau_2^*, \dots, \tau_i^*, \dots, \tau_{N-1}^*)$.

Step 3: Use τ^i as the input of Θ_{i+1} and compute τ^{i+1} .

Step 4: If $\tau^{i+1} = \tau^i$, then stop the recursion. τ^i is the optimal value. Otherwise, $i = i + 1$ and go to *Step 2*.

We implemented the above procedure for a 4-node 40-wavelength unidirectional ring network *without* WCs for $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 30$. To illustrate our recursive simulation-based algorithm, Fig. 3.4 shows the values of τ_1, τ_2, B_1, B_2 and B_3 in different steps of the procedure. In order to calculate an accurate f , the simulation continues for 500,000 arrivals in each run. As it is presented, after a few steps the algorithm converges and we get $\tau^3 = (6, 1, 0)$. After applying MT allocation scheme

with the optimal thresholds, all three classes of users gain almost the same GoS, $B_1 \approx B_2 \approx B_3 \approx 0.075$.

Figure 3.4: Evaluation of τ_1 and τ_2 according to the recursive simulation-based algorithm for a 4-node 40-wavelength ring network *without* WCs. Between two consecutive dot-lines 500,000 arrivals are generated.

3.4 Performance Comparison

In this section, we compare the performance of our proposed wavelength allocation scheme, with CS and CP policies. The simulation results are carried out for 4-node and 8-node unidirectional ring networks with and without WCs. The performance metric used in the numerical comparison deals with fairness and blocking probabilities.

Suppose that c_i and c_j calls experience the highest and lowest blocking probabilities, respectively, then we define fairness ratio as $f_r := B_i/B_j$. The closer the f_r to

1, the better the fairness is. Also in our simulations, the arrival rate of class c_i is set inversely proportional to its hop-count (i.e., $\lambda_1 = 2\lambda_2 = \dots = (N-1)\lambda_{N-1}$). Hence, on each link, the expected wavelength request rate of each class is the same.

In order to employ CP policy, one can partition the total number of available wavelengths into separate sets, each of which is dedicated to one of the origin-destination pairs. Note that non-overlapping origin-destination pairs can share the same set of wavelengths. For example, consider the 4-node ring network depicted in Fig. 3.1. Since class c_1 calls do not share any link, they can use the same set of wavelengths. Class c_2 calls passing through $\{l_1, l_2\}$ and also c_2 calls using $\{l_3, l_4\}$ can share the same set of wavelengths. Similarly, c_2 calls using $\{l_2, l_3\}$ and c_2 calls passing through $\{l_4, l_1\}$ can use one set. Four different sets are required for *overlapping* c_3 calls. As a result, the total wavelengths is partitioned into seven sets; set $\mathcal{S}_1$ is dedicated to class c_1 calls, set $\mathcal{S}_2$ is assigned to c_2 calls passing through $\{l_1, l_2\}$ and $\{l_3, l_4\}$. Set $\mathcal{S}_3$ is allocated to c_2 calls using $\{l_2, l_3\}$ and $\{l_4, l_1\}$. Four sets, $\mathcal{S}_4$ through $\mathcal{S}_7$, are needed for c_3 calls. Let m_j denote the number of wavelengths in set $\mathcal{S}_j$. Note that all sets associated with class c_i have the same number of wavelengths. Therefore, we have $m_2 = m_3$ and $m_4 = m_5 = m_6 = m_7$ and $W = \sum_{j=1}^7 m_j$.

Using Erlang's B formula, we can compute p_j , the blocking probability of a call which uses wavelengths of set $\mathcal{S}_j$ by

$$p_j = \frac{\frac{1}{m_j!} \left(\frac{\lambda}{\mu}\right)^{m_j}}{1 + \sum_{n=1}^{m_j} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n} \quad (3.4.1)$$

where λ/μ is traffic density to wavelengths of $\mathcal{S}_j$. We can find appropriate values for m_j in a way that $p_1 = p_2 = \dots = p_7$.

We simulate the system by deploying the recursive algorithm implemented in

Table 3.1: Class-based blocking probabilities and fairness ratios for a 4-node 40-wavelength ring network without WCs.

ρ	Policy	B_1	B_2	B_3	f_r	τ^3
20	CS	.000008	0.00021	0.00064	80.0	-
20	MT	0.00049	0.00045	0.00043	1.14	(4,1,0)
25	CS	0.00033	0.00795	0.02473	75.0	-
25	MT	0.01410	0.01561	0.01505	1.10	(5,1,0)
30	CS	0.00201	0.04344	0.13807	67.7	-
30	MT	0.07360	0.07063	0.07750	1.10	(6,1,0)
35	CS	0.00502	0.10198	0.29933	59.6	-
35	MT	0.13509	0.15496	0.17947	1.32	(6,1,0)
40	CS	0.00859	0.13096	0.37806	44.0	-
40	MT	0.16722	0.16821	0.19403	1.16	(6,1,0)

Section 3.3 to calculate the performance metric of MT allocation policy. For implementing CP, the above procedure is used. Finally, the system is simulated without any allocation policy to find the performance of CS.

We first study the fairness objective in the ring network shown in Fig. 3.1 when MT, CP and CS policies are used. Table 3.1 shows the class-based blocking probabilities for a 40-wavelength ring network without WCs. As Table 3.1 reports, deploying MT policy results in a fairness ratio close to 1. As expected, when CS policy is used the fairness ratio is high, $f_r \in [44, 80]$. For low traffic load, the blocking rates are relatively small and the competition for access to the network is low. Conversely, when the offered load is high, calls from different classes compete for utilizing resources. As a result, by increasing traffic load, τ_1^{*3} increases to block more c_1 calls in order to protect c_2 and c_3 calls.

Now we compare MT and CP policies. This comparison is based on the number of wavelengths required to provide a certain GoS. By using (3.4.1), we computed $m_1 = 13$, $m_2 = m_3 = 8$ and $m_4 = m_5 = m_6 = m_7 = 6$. Therefore, 53 wavelengths ($W = 13 + 2 \times 8 + 4 \times 6$) is required for CP implementation, which shows 32.5% growth in terms of network resource cost. For instance when $\rho = 30$ and CP is used,

Table 3.2: Class-based blocking probabilities and fairness ratios for a 4-node 40-wavelength ring network with WCs in all nodes.

ρ	Policy	B_1	B_2	B_3	f_r	τ^3
25	CS	0.00137	0.00257	0.00400	2.91	-
25	MT	0.00280	0.00156	0.00259	1.79	(1,0,0)
30	CS	0.01245	0.02361	0.03522	2.82	-
30	MT	0.02547	0.01609	0.02425	1.58	(1,0,0)
35	CS	0.03990	0.07491	0.10968	2.74	-
35	MT	0.07935	0.05349	0.07911	1.48	(1,0,0)
40	CS	0.07695	0.14370	0.20794	2.70	-
40	MT	0.15311	0.10807	0.15417	1.42	(1,0,0)
45	CS	0.11677	0.21677	0.30387	2.60	-
45	MT	0.22698	0.16142	0.23046	1.42	(1,0,0)

we get $f_r = 1.2$ and $B_1 = 0.084$, $B_2 = 0.070$ and $B_3 = 0.072$.

We also investigate the impact of using WCs on fairness. Table 3.2 presents the performance of MT and CS policies for the above example when all nodes are equipped by WCs. As reported in this table, even if MT policy is not used, utilizing WCs considerably improves the fairness ratio.

Comparison of Tables 3.1 and 3.2 illustrates that: 1) for systems without WCs, using MT allocation policy results in a very good fairness ratio (i.e., $f_r \approx 1$). 2) Deploying WCs not only decreases the overall blocking probabilities but also improves the fairness ratio. For example, the overall blocking rate of the system using MT allocation without WCs is equal to 0.0733 for $\rho = 30$ whereas this metric for the network with WCs is 0.0226. 3) In both cases, with and without WCs, MT allocation mechanism is able to improve the fairness ratio. One can see that $f_r \in [1.42, 1.79]$ and $f_r \in [1.10, 1.32]$ for the networks with and without WCs, respectively. 4) The optimal thresholds in the network with WCs are smaller than the ones in the network without WCs.

We now implement and study the performance of MT and CS policies in a 8-node 110-wavelength unidirectional WDM ring network. Table 3.3 (respectively, 3.4)

Table 3.3: Class-based blocking probabilities and fairness ratios for a 8-node 110-wavelength ring network without WCs.

	$\rho = 600$		$\rho = 700$		$\rho = 800$	
	CS	MT	CS	MT	CS	MT
B_1	.00000	.01450	.00000	.07205	.00000	.16216
B_2	.00000	.00847	.00000	.06887	.00080	.12974
B_3	.00071	.01423	.00688	.08841	.01715	.17925
B_4	.00518	.01098	.04304	.06650	.10057	.15347
B_5	.01326	.00819	.10927	.06420	.24370	.14202
B_6	.01887	.01137	.15298	.08567	.33907	.18776
B_7	.02030	.01241	.16109	.08973	.36291	.20718
f_r	> 1000	1.77	> 1000	1.39	> 1000	1.59
τ^7	(19, 8, 4, 1, 0, 0, 0)		(20, 9, 4, 1, 0, 0, 0)		(21, 9, 4, 1, 0, 0, 0)	

Table 3.4: Class-based blocking probabilities and fairness ratios for a 8-node 110-wavelength ring network with WCs in all nodes.

	$\rho = 700$		$\rho = 800$		$\rho = 900$	
	CS	MT	CS	MT	CS	MT
B_1	.00192	.00617	.01388	.04728	.03271	.11702
B_2	.00387	.00633	.02617	.04705	.06183	.11821
B_3	.00567	.00341	.03941	.02665	.09055	.06720
B_4	.00749	.00474	.05026	.03476	.11968	.08634
B_5	.00886	.00598	.06159	.04172	.14601	.10720
B_6	.01037	.00672	.07388	.05029	.17114	.12922
B_7	.01210	.00849	.08473	.05841	.19707	.14428
f_r	6.30	2.49	6.10	2.19	6.02	2.14
τ^7	(2, 1, 0, 0, 0, 0, 0)		(2, 1, 0, 0, 0, 0, 0)		(2, 1, 0, 0, 0, 0, 0)	

presents the class-based blocking probabilities and fairness ratios of the system with WCs (respectively, without WCs). It can be observed from Table 3.3 that when CS is deployed the fairness ratio is above 1000 for all traffic loads. When MT allocation is implemented, the fairness ratio is remarkably improved, $f_r \in [1.39, 1.77]$. As Table 3.4 shows, using WCs in the system plays an important role in fairness improvement. Similar to the 4-node network case, Table 3.3 confirms that in order to preserve wavelengths for calls with higher hop-counts, τ_1^{*7} and τ_2^{*7} will increase when the offered load increases. Comparison of Tables 3.1 and 3.3 shows that when the number of nodes increases in ring networks without WCs, using MT allocation scheme becomes more crucial.

According to the above numerical results, MT wavelength allocation scheme may block one class of users to preserve wavelengths for other classes. In fact, we reduce the blocking rates of calls with higher hop-counts at the expense of increasing the blocking rates of ones with lower hop-counts .

Chapter 4

Call Admission and Fairness Control in WDM Grooming Networks

The purpose of this chapter is to develop a call admission control to provide fairness control and service differentiation in WDM grooming networks. We first define a Markov Decision Process (MDP), with the objective of maximizing class-based utilization in a single-wavelength single-link network. Based on MDP formulation, a CAC scheme can be determined as a function of current capacity usage on each wavelength. The Policy Iteration algorithm can be deployed to determine the optimal CAC policy [62]. Since implementing an MDP-based CAC mechanism is very difficult for a complex network topology, we develop approximations and make some assumptions to extend the result of the single-wavelength single-link network for applying to multi-wavelength multi-link tandem and ring networks. Thus, we concentrate on developing an estimation of CAC scheme which provides accurate results with lower computational complexity.

The rest of the chapter is organized as follows. In Section 4.1, we introduce the network model. Section 4.2 deals with MDP formulation for a single-wavelength

single-link network and the discounted reward function associated with the problem in the infinite horizon case. Section 4.3 shows the structure of optimal policy and Section 4.4 describes our proposed heuristic algorithm. Section 4.5 compares the performance of the proposed policy with other standard policies.

4.1 Traffic Grooming capabilities

We now add the grooming capabilities to the ring network model presented in Chapter 3 and address the associated fairness problem. Consider a unidirectional WDM grooming ring (or tandem) network with N nodes. Each node is equipped by OADM and SADMs without TSI and WC devices. Each link carries one fiber with W wavelengths and each wavelength consists of T time slots. The network includes $N(N-1)$ Origin-Destination (O-D) pairs, indexed by $d = 1, 2, \dots, d_{max}$ and $d_{max} \leq N(N-1)$.

- Total arrival of class c_h is uniformly distributed among all O-D pairs which includes h hops from origin to destination.
- Class c_h includes K subclasses of traffic, $c_h^1, c_h^2, \dots, c_h^K$, which differ by their bandwidth (time-slot) requirements. Subclass c_h^k requires t^k time slots to be established.
- Arrivals of subclass c_h^k calls are distributed according to a Poisson process with rate λ_h^k . Let $\lambda_h = \sum_{k=1}^K \lambda_h^k$ denote the total arrival rate of class c_h .
- Call holding time of subclass c_h^k is exponentially distributed with mean $1/\mu_h^k$.
- Any arriving call of subclass c_h^k is blocked when no wavelength has t^k available time slots along its path.

- Blocked calls do not interfere with the system. Nodes are equipped by non-blocking grooming switches.

When CS policy is employed in a WDM grooming ring network, besides the fairness issue based on hop-counts studied in chapters 2 and 3, there will be another fairness problem based on bandwidth requirement. In the latter case, within each class, subclasses of calls with lower bandwidth requirements experience less blocking probabilities than the ones with higher bandwidth requirements. As a result, the system is unfair in the sense that there is a big difference between the blocking probabilities experienced by different subclasses of users within each class.

In the following sections, we formulate this problem in an MDP framework. Since the MDP-based formulation for a ring network is computationally infeasible, we first formulate the problem for a single-link (i.e., single-class) single-wavelength network to derive the optimal policy. By using the properties of the optimal policy and developing approximations, we propose a quite simple heuristic algorithm to improve the fairness within each class and also in the whole network.

4.2 MDP Formulation for a single-link single-wavelength path

Consider a single-link path which has only one wavelength to carry traffic streams. Note that this system only handles class c_1 calls. Let n_1^k denote the number of c_1^k calls currently in the system. One can note that a K -component vector, $(n_1^1, n_1^2, \dots, n_1^K)$, can completely characterize the system. Thus, the state space of the system is $S = \{(n_1^1, n_1^2, \dots, n_1^K) \mid \sum_{k=1}^K t^k n_1^k \leq T\}$.

Similar to Chapter 2, we introduce the following operators and formulate the

problem under an MDP framework:

- $A_k : S \mapsto S, k = 1, 2, \dots, K$: *Arrival operator*, describing the change of system state at the arrival time of a c_1^k user,

$$A_k s = (n_1^1, n_1^2, \dots, n_1^k + 1, \dots, n_1^K).$$

- $D_k : S \mapsto S, k = 1, 2, \dots, K$: *Departure operator*, describing the change of system state at the departure time of a c_1^k user,

$$D_k s = (n_1^1, n_1^2, \dots, n_1^k - 1, \dots, n_1^K).$$

Investigating CAC policy involves the determination of connection admission as a function of the current state s and event $e \in \{A_k, D_k, k = 1, 2, 3, \dots, K\}$.

Actions take place only at arrival times. If the wavelength has sufficient free space, the connection may be accommodated immediately, or it can be rejected to preserve the capacity for the users of the other subclasses. Let $a \in \{0, 1\}$ be the action taken after an arrival. Then, the decision maker accepts the arriving call if $a = 1$. Otherwise, $a = 0$ and the call will be rejected.

Let $P_a, a = 0, 1$ be the *policy operator* to describe the change of system state when applying action a :

- $A_k P_a s = (n_1^1, n_1^2, \dots, n_1^k + a, \dots, n_1^K).$

This initial continuous-time MDP can be converted into an equivalent discrete-time MDP by applying the uniformization technique [63]. To do so, we introduce a random sampling rate ν

$$\nu := \sum_{k=1}^K \left(\left\lfloor \frac{T}{t^k} \right\rfloor \mu_1^k + \lambda_1^k \right)$$

where $\left\lfloor \frac{T}{t^k} \right\rfloor$ represents the maximum number of class c_k calls that can be carried by a wavelength. The intuitive meaning of the sampling rate is as follows. In the original continuous-time MDP, the system is observed at the times of actual transitions. After applying uniformization technique and sampling the system with rate ν , the system may be observed between two consecutive actual transitions which are called ‘fictitious’ or ‘dummy’ transitions. When considering the discrete-time MDP, only one single transition can occur during each time slot. By sampling the system with rate ν and considering the equivalent discrete-time MDP, it can be seen that only one single transition can occur during each time slot.

We define the one-step reward function as:

$$R(s) = \sum_{k=1}^K \alpha_k (t^k n_1^k), \quad s \in S \quad (4.2.1)$$

where $(t^k n_1^k)$ represents the subclass c_1^k slot utilization and α_k is the weighting factor assigned to this subclass. The choice of the weighting factor, α_k , has an impact on the slot occupancy rates by different subclasses and their respective blocking probabilities. Note that when $\alpha_k = 1$ for all $k = 1, 2, \dots, K$, then the optimal policy aims at maximizing the total utilization of the wavelength. In the following paragraphs, we present the determination of the optimal CAC policy.

Based on the one-step reward function, we can apply the results of discounted cost model with the finite horizon. In order to estimate the expected reward under policy $\Pi = \{\pi_1, \pi_2, \dots, \pi_n\}$ and at the time step n , we can define the optimal value function $V_{n+1}(s)$ as:

$$V_{n+1}(s) = \max_{\Pi} V_n^{\Pi}(s) = \max_{\Pi} [R(s) + \gamma \sum_{s'} P_{ss'}^{\Pi} V_n(s')] \quad (4.2.2)$$

where $P_{ss'}^\pi := P(s_{n+1} = s' \mid s_n = s, \pi_n = \pi)$ is

$$P_{ss'}^\pi \times \nu = \begin{cases} \lambda_1^k & \text{if } s' = A_k P_0 s, \pi = 0, \\ \lambda_1^k & \text{if } s' = A_k P_1 s, \pi = 1, \\ n_1^k \mu_1^k & \text{if } s' = D_k s, \\ \nu - \sum_{k=1}^K (n_1^k \mu_1^k + \lambda_1^k) & \text{if } s' = s, \end{cases}$$

Replacing $P_{ss'}^\pi$ in (4.2.2) yields (without loss of generality, we assume that $\nu = 1$):

$$V_{n+1}(s) = R(s) + \gamma \left\{ \sum_{k=1}^K \lambda_1^k \max\{V_n(A_k P_0 s), V_n(A_k P_1 s)\} + \sum_{k=1}^K \mu_1^k n_1^k V_n(D_k s) + \left[1 - \sum_{k=1}^K (n_1^k \mu_1^k + \lambda_1^k)\right] V_n(s) \right\}. \quad (4.2.3)$$

From the above equation, it can be noticed that at a c_1^k arrival time, the optimal action is $a = 1$ if $V_n(A_k P_1 s) \geq V_n(A_k P_0 s)$ and $a = 0$, otherwise.

As pointed out in chapter 2, the limit of sequence $\{V_1(s), V_2(s), \dots, V_n(s)\}$ can be determined when n goes to infinity. This limit is the unique solution of the infinite horizon discounted cost problem.

4.3 Structure of the optimal policy

The Policy Iteration algorithm [62] is implemented to numerically compute the optimal policy. This section is devoted to study the effect of the weighting factor, α_k , and the traffic characteristics on the optimal CAC structure.

Consider a system that only includes class c_1 with two subclasses c_1^1 and c_1^2 . Subclasses c_1^1 and c_1^2 include OC-12 and OC-48 streams, respectively. Assume that the capacity of each wavelength is OC-192. Therefore, we can divide each wavelength into 16 time slots of length OC-12 (i.e., $T = 16$, $t^1 = 1$, $t^2 = 4$). A subclass c_1^2 call is accepted whenever at least four time slots are unused in the wavelength, whereas class c_1^1 calls are accommodated when only one time slot is available. For this system,

the optimal value function is:

$$\begin{aligned}
 V_{n+1}(s) = & R(s) + \gamma \left[\lambda_1^1 \max\{V_n(A_1 P_0 s), V_n(A_1 P_1 s)\} \right. \\
 & + \lambda_1^2 \max\{V_n(A_2 P_0 s), V_n(A_2 P_1 s)\} \\
 & + \mu_1^1 n_1^1 V_n(D_1 s) + \mu_1^2 n_1^2 V_n(D_2 s) \\
 & \left. + (1 - \mu_1^1 n_1^1 - \mu_1^2 n_1^2 - \lambda_1^1 - \lambda_1^2) V_n(s) \right]. \quad (4.3.1)
 \end{aligned}$$

In the following, we study the impact of arrival rates and weighting factors on the optimal policy.

4.3.1 Impact of the weighting factors on the optimal policy

In order to investigate the effect of the weighting factors on the CAC, we fix $\alpha_1 = 1$, $\lambda_1^1 = 4\lambda_1^2 = 8$ and depict the CAC policy for two values of α_2 . Figs. 4.1(a) and (b) illustrate the structure of the CAC policy at a class c_1^1 arrival time for $\alpha_2 = 2$ and 3, respectively. The optimal policy accepts all c_1^1 arrival calls in these two situations.

Note that the CAC policy has a structure of generalized switching curve. Fig. 4.1(a) illustrates that there exists a set of states in which the decision maker rejects c_1^1 calls to reserve the remaining capacity for accommodating incoming c_1^2 calls. We define these states, such as $(12, 0)$, $(8, 1)$, $(4, 2)$ and $(0, 3)$, as *threshold states*. In *threshold states*, there are only four time slots available and by accepting a c_1^1 call in these states, a future c_1^2 call is most likely to be blocked. Moreover, when the available capacity is not enough to carry a c_1^2 call (e.g., states $(9, 1)$, $(5, 2)$), the decision maker tends to increase the class c_1^1 utilization by accepting c_1^1 calls.

By increasing α_2 from 2 to 3, the policy maker rejects c_1^1 calls not only in *threshold states*, but also in some other states on the neighborhood of these states. For instance,

Figure 4.1: Representation of optimal CAC policy. Sign "*" refers to a boundary state where the system is already full and no action can be taken.

if a c_1^1 arrival is rejected in state $(9, 1)$, then after a departure of either class, there will be enough time slots to carry a c_1^2 call.

4.3.2 Impact of the arrival rates on the optimal policy

We now set $\alpha_1 = \alpha_2 = 1$, $\lambda_1^1 = 8$ and vary λ_1^2 . This case deals with maximizing the system utilization. Figs. 4.1(c) and (d) depict the optimal actions at an arrival epoch of a c_1^1 call, for $\lambda_1^2 = 2$ and 8, respectively. Note that regardless of the value of λ_1^2 , the CAC mechanism always accommodates an arriving c_1^2 call. Fig. 4.1(c) shows that the CAC policy accepts all c_1^1 calls. Therefore, the optimal policy is a CS policy for this traffic distribution. In Fig. 4.1(d), the CAC policy is plotted for $\lambda_1^1 = \lambda_1^2$ and $t^2 = 4t^1$. Note that the expected slot request rate of class c_1^2 is four times higher than c_1^1 (i.e., $\lambda_1^2 t^2 = 4\lambda_1^1 t^1$). As a result, the decision maker rejects some of the class c_1^1 arrivals to reserve the capacity for class c_1^2 . One can see that in *threshold state* $(12, 0)$, the CAC decides to accept an arrival of class c_1^1 call while in other *threshold*

states $(8, 1)$, $(4, 2)$, $(0, 3)$, it rejects c_1^1 arriving calls. According to equation (4.3.1), we can interpret this behavior as follows. When the system's state is $(13, 0)$, the rate of terminating a c_1^1 user from the system is $n_1^1 \mu_1^1 = 13$ while in states $(9, 1)$, $(5, 2)$ and $(1, 3)$, the termination rates are 9, 5 and 1, respectively. Therefore, in state $(13, 0)$, the probability of having a 4-slot free space is higher than states $(9, 1)$, $(5, 2)$ and $(1, 3)$ and consequently, the chance of losing a c_1^2 call in the future is lower. Thus, the CAC policy accepts a c_1^1 call in *threshold state* $(12, 0)$.

From the above numerical results, we can conclude that the weighting factors can be set to control the class-based utilization. Therefore, the decision maker can provide differentiated services in terms of some network criteria such as GoS or fairness.

4.4 Heuristic Decomposition Algorithm

Obviously, for a large and realistic network, implementation of an MDP-based CAC mechanism is practically infeasible. Thus, we propose a four-step decomposition algorithm to extend the results of a single-link single-wavelength network to multi-link multi-wavelength tandem and ring networks.

Step 1: For each link, say link l , partition the set of available wavelengths into separate subsets, each of which is dedicated to one of O-D pairs using link l . Let W_d denote the total number wavelengths in the subset dedicated to O-D pair d . Note that non-overlapping O-D pairs can share the same set of wavelengths. Let I_l denote the number of the O-D pairs that use link l as part of their routes, then equality $W_1 + W_2 + \dots + W_{I_l} = W$ must be satisfied. In this paper, we consider two following examples; The first example is a 3-node tandem network shown in Fig. 4.2. In this case, the set of available wavelengths is divided into two different subsets; one of

Figure 4.2: A 2-hop network topology.

which is dedicated to O-D 2, and the other one to O-D 1 and O-D 3. (Since O-D 1 and O-D 3 use separate links, they can share the same subset of wavelengths.) For instance for $\lambda_1 = 2\lambda_2$ (i.e., the arrival rates per O-D pairs 1, 2 and 3 are the same), we dedicate half of the wavelengths to O-D 2 and the other half to the other pairs, $W_1 = W_2 = W_3 = W/2$. As the second example, we consider a 4-node unidirectional ring network depicted in Fig. 4.3. One can note that in this example there are three sets of non-overlapping paths; one set of (non-overlapping) one-hop paths, {O-D 1, O-D 2, O-D 3, O-D 4}, and two sets of non-overlapping two-hop paths, {O-D 5, O-D 7} and {O-D 6, O-D 8}. As a result, the total wavelengths can be partitioned into three subsets; one subset is dedicated to O-D 1 through O-D 4, and one subset to O-D 5 and O-D 7, and the last subset is assigned to O-D 6 and O-D 8. If the arrival rate per each O-D pair is the same, one can set $W_d = W/3$, $d = 1, 2, \dots, 8$.

Step 2: Assume that the total offered load to O-D pair d is uniformly distributed among the W_d wavelengths reserved to this pair. Calls from O-D pair d belong to class c_h passing through h hops. Therefore, the arrival rate of subclass c_h^k for each of the W_d wavelengths is given by: λ_h^k/W_d .

Step 3: Compute the CAC policy presented in Section 2.2 with respect to λ_h^k/W_d .

Step 4: When a c_h^k call arrives to O-D pair d , using the CAC policy computed in *Step 3*, we determine the optimal action for each of the W_d wavelengths, individually. If the optimal action for a wavelength is “acceptance”, $a = 1$, we put that wavelength

Figure 4.3: A 4-node ring network.

in subset $\mathcal{W}_{a=1}^d$. If the subset $\mathcal{W}_{a=1}^d$ is empty, then the call will be rejected. Otherwise, one wavelength is selected randomly from subset $\mathcal{W}_{a=1}^d$, to carry the arrival call.

The proposed heuristic decomposition algorithm has two main advantages: 1) It masks the interferences among different classes by isolating them from each other. 2) It overcomes the complexity of the implementation of an MDP-based CAC mechanism, by considering each wavelength separately.

4.5 Performance comparison

In this section, the performance of the proposed heuristic algorithm without CAC implementation (i.e., using only *Step 1* and *Step 2* of the proposed algorithm) and with CAC implementation (i.e., using all steps of the proposed algorithm) is studied for 3-node tandem and 4-node ring networks shown in Figs. 4.2 and 4.3, respectively.

Here, we assume that the capacity of each wavelength, T , is 16 time slots. We will compare the performance of our proposed algorithm, with those of CS and CP policies.

We define $\rho_h = \sum_{k=1}^K \lambda_h^k / \mu_h^k$ as the total offered load to class c_h and B_h^k as the blocking performance of subclass c_h^k calls. Suppose that $c_h^{k_1}$ and $c_h^{k_2}$ calls experience the highest and lowest blocking probabilities in the network, then we define fairness ratio as $f_r := B_h^{k_1} / B_h^{k_2}$.

According to Fig. 4.2, O-D 1 (respectively, O-D 3) consists of the calls that use link l_1 (respectively, link l_2), whereas O-D 2 includes the customers that use both links l_1 and l_2 . Furthermore, when the arrival rates per O-D pairs 1, 2 and 3 are the same, the network can be decomposed into 3 independent O-D pairs with $W_1 = W_2 = W_3 = W/2$.

We first study the fairness of a system with two wavelengths and two classes of users, c_1 and c_2 . Within each class there are two subclasses, c_1^1, c_1^2, c_2^1 and c_2^2 . We assume that $t^1 = 1, t^2 = 4$ and $\lambda_1 = 2\lambda_2$. We dedicate one of the wavelengths to O-D 2 and the other one to O-D 1 and O-D 3. We set the arrival rate of class c_h^k , $h = 1, 2$ inversely proportional to its time slot requirement. Hence, within class c_h , the expected slot request rates of the two subclasses are the same (i.e., $t^1\lambda_h^1 = t^2\lambda_h^2$, $h = 1, 2$). We first simulated the system without using CAC policy. Fig. 4.4 depicts the class-based blocking probabilities versus offered load. One can notice that when the CAC is not used, the blocking probability of class c_h^2 is about 10 times higher than class c_h^1 . We will show that by applying our proposed CAC with appropriate values of α_1 and α_2 , the fairness objective will be met. Assuming $\alpha_1 = 1$, for each offered load we find a relevant value of α_2 through the simulation, so that the blocking rates of c_h^1 and c_h^2 calls become equal. For each offered load, we also obtain the

Figure 4.4: Class-based blocking probabilities with and without CAC.

structure of the optimal policy. Although the value of α_2 depends on the offered load, we have noticed that the structure of the optimal policy, that results in equal blocking probabilities, remains the same as long as $\lambda_h^1 = 4\lambda_h^2$ holds. For instance for $\lambda_h^1 = 4\lambda_h^2 = 8$, we came up with the CAC policy depicted in Fig. 4.1(a). We used this CAC policy and simulated the system for different offered loads. Fig. 4.4 shows the class-based blocking probabilities versus the offered load, when implementing this CAC mechanism. It can be observed that the blocking probabilities of both classes are very close to each other. In fact, we reduce the blocking rate of c_h^2 calls at the expense of increasing the blocking rate of c_h^1 calls and a deterioration of the overall blocking probabilities in the system as illustrated in Fig. 4.5.

Figure 4.5: Overall call blocking probabilities with and without CAC.

Now subclasses c_1^3 and c_2^3 is added to the network. Assuming $t^3 = 8$, we investigate the fairness ratio when $\lambda_h^1 = 4\lambda_h^2 = 8\lambda_h^3$. Similar to the previous case, we obtain the appropriate CAC policy in order to achieve a fairness ratio close to 1. Table 4.1 (respectively, Table 4.2) shows the blocking probabilities of c_h^1 , c_h^2 and c_h^3 calls (i.e., B_h^1 , B_h^2 and B_h^3 , $h = 1, 2$) for a system with $W = 2$ (respectively, $W = 10$).

Table 4.1: Blocking performance for a 2-wavelength tandem network with/without CAC

ρ	CAC	B_1^1, B_2^1	B_1^2, B_2^2	B_1^3, B_2^3	f_r
2.750	without	0.011	0.076	0.248	22.5
2.750	with	0.132	0.144	0.178	1.34
3.347	without	0.018	0.108	0.322	17.8
3.347	with	0.192	0.201	0.211	1.09
4.125	without	0.028	0.153	0.395	14.1
4.125	with	0.233	0.269	0.271	1.16
5.500	without	0.053	0.236	0.522	9.84
5.500	with	0.361	0.343	0.350	1.05
6.875	without	0.075	0.311	0.632	8.42
6.875	with	0.387	0.439	0.419	1.13

Table 4.2: Blocking performance for a 10-wavelength tandem network with/ without CAC

ρ	CAC	B_1^1, B_2^1	B_1^2, B_2^2	B_1^3, B_2^3	f_r
20.62	without	0.000	0.008	0.098	> 1000
20.62	with	0.029	0.031	0.048	1.65
27.50	without	.0007	0.039	0.267	381
27.50	with	0.088	0.132	0.117	1.5
34.37	without	0.003	0.104	0.465	155
34.37	with	0.139	0.209	0.241	1.73

Since each O-D pair is dedicated the same number of wavelengths, the blocking probability experienced by each subclass is the same in all O-D pairs. Columns c_h^1 , c_h^2 and c_h^3 show the class-based blocking probabilities. Comparison of these tables illustrates that:

- For systems without CAC, the fairness ratio increases by increasing the number of wavelengths.
- The difference between the fairness ratios with and without CAC implementation is larger for the 10-wavelength system in Table 4.2.
- In all the cases, the CAC mechanism is able to maintain the fairness ratio close to 1. One can see that $f_r \in [1.05, 1.34]$ for $W = 2$ and $f_r \in [1.5, 1.73]$ for $W = 10$.

We now compare the performance of the proposed MDP-based CAC with CP policy. This comparison is based on the number of wavelengths required to provide a certain grade of service. For a given offered load ρ , we apply the MDP-based CAC and determine B_h^k . When applying CP, subclass c_h^k is allocated a fixed number of wavelengths, w_h^k . The blocking probability experienced by subclass c_h^k with traffic

intensity $\rho_h^k = \lambda_h^k / \mu_h^k$ is given by the Erlang's B formula:

$$E(b, \rho_h^k) = \frac{(\rho_h^k)^b / b!}{\sum_{n=0}^b (\rho_h^k)^n / n!},$$

where $b = w_h^k \cdot t^k / T$ is the total number of subclass c_h^k calls that can be simultaneously carried by w_h^k wavelengths. Using Erlang's B formula, we can calculate the minimum number of wavelengths so that $E(w_h^k \cdot t_k / T, \rho_h^k) < B_h^k$.

As listed in Table 4.2, for a 10-wavelength system with MDP-based CAC and a total offered load $\rho = 27.50$, the set of class-based blocking probabilities are $B_h^1 = 0.088$, $B_h^2 = 0.132$ and $B_h^3 = 0.117$. When applying CP policy, the minimum number of wavelengths required by class c_h^1, c_h^2 and c_h^3 are 2, 2 and 3, respectively. Therefore, 7 wavelengths is required for each O-D pair. As O-D 1 and O-D 2 (respectively, O-D 2 and O-D 3) use link l_1 (respectively, link l_2) as part of their routes, then we need 14 wavelengths in total, which shows 40% growth in terms of network resource cost.

We also used CS policy and conducted simulations to evaluate the performance of the system. Table 4.3 reports the blocking performance of a 10-wavelength 2-class (including 2 subclasses within each class) 3-node tandem network with the same parameters used in Table 4.2. Here, a connection with less hop counts is more likely to be accepted than one with more hop counts: One can see that class-based blocking probabilities of class c_1 are lower than the ones of c_2 . Comparison of Tables 4.2 and 4.3 shows that our proposed method efficiently improves the fairness. As an

Table 4.3: Blocking performance for a 10-wavelength tandem network using CS policy

ρ	B_1^1	B_1^2	B_1^3	B_2^1	B_2^2	B_2^3
20.62	.000	0.000	0.016	.000	0.002	0.102
27.50	.000	0.004	0.104	.000	0.039	0.410
34.37	.000	0.020	0.271	.001	0.151	0.722

example, we compare the result of our proposed policy with CS policy when the total offered load is $\rho = 27.50$. When CS policy is used, B_1^1 and B_2^1 are almost zero while $B_1^3 = 0.104$ and $B_2^3 = 0.410$. By applying the CAC, we have $B_1^1 = B_2^1 = 0.088$ and $B_1^3 = B_2^3 = 0.117$.

We now show that our proposed CAC policy can be used to improve the global utilization. We consider a system with the following parameters: $\lambda_h^1 = \lambda_h^2 = \lambda_h^3$, $t^1 = 1$, $t^2 = 4$, $t^3 = 8$, $\alpha_1 = \alpha_2 = \alpha_3 = 1$, $W_1 = W_2 = W_3 = W/2$ and $W = 2$. Table 4.4 presents the utilization of the system with and without CAC. It can be observed that for all traffic loads both utilization and fairness ratio are improved. For low traffic load, the blocking rates are relatively small, in particular for c_h^1 and c_h^2 , and as a result, the utilization is approximately the same with or without CAC mechanism. For high load (e.g., $\rho = 8$), the wavelengths are almost full regardless of CAC implementation. For medium loads, $\rho = 4$ and 6, the use of MDP-based CAC results in an improvement of the utilization (e.g., up to 12.9% for $\rho = 4$).

Table 4.4: Wavelength Utilization for a 2-wavelength tandem network with/ without CAC

ρ	CAC	B_1^1, B_2^1	B_1^2, B_2^2	B_1^3, B_2^3	Util.	f_r
1.5	without	0.048	0.088	0.254	0.043	5.29
1.5	with	0.052	0.092	0.250	0.044	5.00
4.0	without	0.083	0.222	0.502	0.208	6.04
4.0	with	0.302	0.220	0.450	0.235	2.04
6.0	without	0.109	0.419	0.741	0.554	6.79
6.0	with	0.384	0.546	0.685	0.603	1.78
8.0	without	0.193	0.619	0.894	0.875	4.63
8.0	with	0.575	0.581	0.861	0.887	1.49

As the last case study, we consider the 4-node unidirectional ring network depicted in Fig. 4.3. Assume that $t^1\lambda_h^1 = t^2\lambda_h^2 = t^3\lambda_h^3$, $\lambda_1 = \lambda_2 = \lambda_3$, and $t^1 = 1$, $t^2 = 4$, $t^3 = 8$. We first deploy the CS policy and carry out the simulation to evaluate the performance

of the system. Table 4.5 shows the blocking rates of a 15-wavelength 3-class (with 3 subclasses within each class) 4-node unidirectional ring network when CS policy is deployed. One can notice that the blocking probabilities of c_1^1 , c_1^2 and c_1^3 are lower than c_2^1 , c_2^2 and c_2^3 , respectively. We implemented our heuristic decomposition algorithm to improve the fairness problem for this network. Table 4.6 reports the result of our method. In Table 4.5, when CS policy is deployed and $\rho = 20$, $B_1^1 = B_2^1 \simeq 0$, while $B_1^3 = 0.036$ and $B_2^3 = 0.175$. After using our MDP-based CAC policy, we have $B_1^1 = B_2^1 = 0.029$ and $B_1^3 = B_2^3 = 0.033$, which indicates the remarkable improvement in terms of fairness.

Table 4.5: Blocking performance for a 15-wavelength ring network using CS policy

ρ	B_1^1	B_1^2	B_1^3	B_2^1	B_2^2	B_2^3
20	.000	0.000	0.036	.000	0.007	0.175
25	.000	0.006	0.135	.000	0.045	0.429
30	.000	0.021	0.275	.000	0.13	0.654

Table 4.6: Blocking performance for a 10-wavelength ring network with/without CAC

ρ	CAC	B_1^1, B_2^1	B_1^2, B_2^2	B_1^3, B_2^3	f_r
20	without	0.000	0.007	0.085	> 1000
20	with	0.029	0.028	0.033	1.17
25	without	.0003	0.026	0.198	660
25	with	0.075	0.082	0.83	1.1
30	without	.0013	0.063	0.351	270
30	with	0.126	0.155	0.162	1.28

According to the above numerical results, the CAC mechanism may block one subclass of users to preserve the capacity for other subclasses. This may degrade the overall blocking performance in the system. Therefore, it is a tradeoff between improving fairness and minimizing the overall blocking probabilities.

Chapter 5

Link Dimensioning in WDM Grooming Networks

In this chapter, considering the correlation between slot occupancies, we present an analytical model to evaluate the blocking performance for a WDM grooming network. In [71], the problem of dimensioning WDM networks under traffic growth is considered. In this study, the network supports single-rate streams. Here, assuming that there are high-rate and low-rate streams of calls, we determine the minimum number of wavelengths required to meet a predefined GoS. The high-rate calls occupy the whole wavelength and the low-rates require only a portion of a wavelength. The objective is to design an algorithm to determine the required number of wavelengths in order to meet a specified GoS. We show that when implementing the Random assignment strategy, the system can be modeled as a Markov Chain which has a product-form solution [72–74]. Nevertheless, this solution may be inefficient because of its computational complexity, even for moderate sized networks. One way to reduce the numerical complexity is to develop approximations and study the behavior of the system in different regimes [75, 76]. As a future work, we will investigate the asymptotic behavior of the blocking probabilities.

The rest of this chapter is organized as follows: In Section 5.1, we formulate the problem as a Markov process and propose an approximate solution to compute the blocking probabilities. Section 5.2 deals with the asymptotic behavior of the system for low load traffic. Section 5.3 presents the performance of the proposed method when we use it to dimension the network.

5.1 Problem Formulation

Using the network model introduced in Chapter 4, we first consider a single-link multiple-wavelength path. Each link carries one fiber with W wavelengths and each wavelength consists of T time slots. One can notice that this system supports only class c_1 which includes subclasses $c_1^1, c_1^2, c_1^3, \dots, c_1^K$. Subclass c_1^k requires t^k time-slots, $t^k \leq T$, to be established. We define the following notation.

- $n_1^{k,w}$, $k = 1, 2$, $w = 1, 2, \dots, W$, is the number of subclass c_1^k calls currently carried by wavelength w .
- $1/\lambda_1^k$, is the mean interarrival time of subclass c_1^k calls.
- $1/\mu_1^k$, is the mean holding time of subclass c_1^k calls.

Since we consider only class c_1 calls, we drop the class index from notation in this chapter.

We first study a simple model with two wavelengths and two subclasses. The problem can be modeled as a Markov Chain and the system state can be characterized by a four-component vector: $s = (n^{1,1}, n^{2,1}, n^{1,2}, n^{2,2})$. The state space in this case can be expressed as:

Figure 5.1: Markov Chain Representation of the system with $W = 2$, $t^1 = 1$, $t^2 = T = 4$. (a) The original Model, (b) The simplified model after considering symmetric property

$$\Omega_2 \equiv \{(n^{1,1}, n^{2,1}, n^{1,2}, n^{2,2}) \in N^4 \mid t^1 n^{1,1} + t^2 n^{2,1} \leq T, \quad t^1 n^{1,2} + t^2 n^{2,2} \leq T\} \quad (5.1.1)$$

Let s_t be the system state at time t . Then $\{s_t, t \geq 0\}$ is a Markov Chain (MC) from which we can compute the stationary slot occupancy probabilities. Fig. 5.1(a) depicts the MC of a system with $W = 2$, $t^1 = 1$, $t^2 = T = 4$. Due to the symmetric nature of the MC shown in Fig. 5.1(a), it can be easily shown that the stationary probability of this MC can be determined from the stationary probability of a simpler MC depicted in Fig. 5.1(b). Thus, instead of solving the MC showed in Fig. 5.1(a), we can determine the stationary slot occupancy probabilities, and in particular the blocking probabilities, from the MC depicted in Fig. 5.1(b). Note that all transition rates are twice as the ones in Fig. 5.1(a).

In order to calculate the probabilities for the MC in Fig. 5.1(b), we first examine the reversibility (*Kolmogorov's*) criterion [77].

Kolmogorov's criterion states that a Markov process is time reversible if and only if

$$r(s_1 \rightarrow s_2)r(s_2 \rightarrow s_3) \cdots r(s_n \rightarrow s_1) = r(s_1 \rightarrow s_n)r(s_n \rightarrow s_{n-1}) \cdots r(s_2 \rightarrow s_1) \quad (5.1.2)$$

where $r(s_i \rightarrow s_j)$ is the transition rate from state s_i to state s_j [12]. This indicates that cyclic paths are bidirectional and the overall transition rate along a cycle is the same in either direction.

From Fig. 5.1(b), we can see that some cyclic sequences such as the sequence of states:

$$\{(0, 0, 0, 0), (1, 0, 0, 0), (1, 0, 0, 1), (0, 0, 0, 1)\}$$

do not meet the reversibility criterion. Thus, the system is not time-reversible and does not have the product form solution.

To overcome this problem, we modify some of the transition rates in a way that the MC becomes time-reversible. Then, we can obtain the product form solution for the modified time-reversible MC. We introduce the following transition rate modifications to the MC depicted in Fig. 5.1(b):

$$\begin{cases} r((n^{1,1}, 0, 0, 1) \rightarrow (n^{1,1} + 1, 0, 0, 1)) = \lambda^1 & \text{if } n^{1,1} = 0, 1, 2, 3 \\ r((4, 0, n^{1,2}, 0) \rightarrow (4, 0, n^{1,2} + 1, 1)) = \lambda^1 & \text{if } n^{1,2} = 0, 1, 2, 3 \\ r((n^{1,1}, 0, 0, 0) \rightarrow (n^{1,1}, 0, 0, 1)) = \lambda^2 & \text{if } n^{1,1} = 0, 1, 2, 3 \end{cases} \quad (5.1.3)$$

We have found the following product form solution for the modified MC as

$$\pi(s) = \frac{1}{G(2)} \frac{\rho_1^{(n^{1,1}+n^{1,2})} \rho_2^{(n^{2,1}+n^{2,2})}}{n^{1,1}! n^{2,1}! n^{1,2}! n^{2,2}! \times 2^{(n^{1,1}+n^{2,1}+n^{1,2}+n^{2,2})}} \quad (5.1.4)$$

where $s = (n^{1,1}, n^{2,1}, n^{1,2}, n^{2,2})$, $\rho_k = \lambda^k / \mu^k$ is the traffic intensity of subclass c^k stream, $\pi(s)$ is the probability that the system state is s and $G(2)$ is the normalizing constant.

The generalization of the analysis to $W > 2$ wavelengths yields a Markov Chain of W -dimension with the following state space:

$$\Omega_W \equiv \{n^{1,w}, n^{2,w} \geq 0, t^1 n^{1,w} + t^2 n^{2,w} \leq T, w = 1, 2, \dots, W\} \quad (5.1.5)$$

Because of the Random assignment strategy, the symmetry of this MC still holds. Using the same idea as in Fig. 5.1, we can extract a 2-dimensional truncated MC from the original W -dimensional MC based on the symmetry, whose transition rates are W times larger than the ones of the original MC. Similar to (5.1.3), we can modify the transition rates of the truncated MC so that the final 2-dimensional MC becomes time-reversible. The following equation gives the product form solution in steady state for a network with $W > 2$:

$$\pi(s) = \frac{1}{G(W)} \prod_{w=1}^W \frac{\rho_1^{n^{1,w}} \cdot \rho_2^{n^{2,w}}}{n^{1,w}! \cdot n^{2,w}! \cdot W^{(n^{1,w} + n^{2,w})}} \quad \text{for } s \in \Omega_W \quad (5.1.6)$$

where $s = (n^{1,1}, n^{2,1}, n^{1,2}, n^{2,2}, \dots, n^{1,W}, n^{2,W})$, $\pi(s)$ is the probability that the system state is s and the normalizing constant, $G(W)$, is:

$$G(W) = \sum_{s \in \Omega_W} \left(\prod_{w=1}^W \frac{\rho_1^{n^{1,w}} \cdot \rho_2^{n^{2,w}}}{n^{1,w}! \cdot n^{2,w}! \cdot W^{(n^{1,w} + n^{2,w})}} \right) \quad (5.1.7)$$

5.2 Asymptotic Behavior for Light Load Traffic

In this section, we study the behavior of the system when it is fed with light traffic in order to dimension the network links. From (5.1.5), one can note that the cardinality

of Ω_W is given by 6^W . Therefore, the calculation of the normalizing constant is no longer trivial and requires considerable computation, when the number of wavelengths increases [75, 76, 78].

The objective here is to determine the minimum number of wavelengths to satisfy the GoS in terms of the overall blocking probability. We assume that the GoS takes a small value, for example 10^{-3} . Thus, we can restrict our analysis to a system with light traffic, or equivalently $\frac{\rho_2}{W} < \frac{\rho_1}{W} \ll 1$. Using this assumption, we can say that the term $\frac{(\frac{\rho_1}{W})^{n^{1,j}}}{n^{1,j}!}$, for $n^{1,j} > 1$, is negligible when compared to $(\frac{\rho_1}{W})$. Accordingly, it may be omitted in the calculation of $G(W)$. Similarly, term $\frac{(\frac{\rho_2}{W})^{n^{2,j}}}{n^{2,j}!}$, for $n^{2,j} > 1$, can be neglected.

In this part, we show the approximation of $G(W)$, for $W = 2$. Using (5.1.7), let rewrite $G(2)$ as:

$$\begin{aligned} G(2) &= \sum_{s \in \Omega_2} \frac{(\frac{\rho_1}{2})^{n^{1,1}} \cdot (\frac{\rho_2}{2})^{n^{2,1}}}{n^{1,1}! \cdot n^{2,1}!} \times \frac{(\frac{\rho_1}{2})^{n^{1,2}} \cdot (\frac{\rho_2}{2})^{n^{2,2}}}{n^{1,2}! \cdot n^{2,2}!} \\ &= \underbrace{\sum_{\{n^{1,2} + 4n^{2,2} \leq 4\}} \frac{(\frac{\rho_1}{2})^{n^{1,2}} \cdot (\frac{\rho_2}{2})^{n^{2,2}}}{n^{1,2}! \cdot n^{2,2}!}}_I + \sum_{k=1, \dots, 4} \frac{(\frac{\rho_1}{2})^k}{k!} \cdot I + (\frac{\rho_2}{2}) \cdot I \quad (5.2.1) \end{aligned}$$

In (5.2.1), we can remove the terms $\frac{(\frac{\rho_1}{2})^k}{k!}$ for $k > 1$. Hence, we can rewrite (5.2.1) as:

$$G(2) \simeq I \times \left(1 + \frac{\rho_1}{2} + \frac{\rho_2}{2}\right) \quad (5.2.2)$$

We now decompose I by considering the possible values of $(n^{1,2}, n^{2,2})$ so that $(n^{1,2} + 4n^{2,2} \leq 4)$

$$I = 1 + \frac{\rho_1}{2} + \frac{(\frac{\rho_1}{2})^2}{2!} + \frac{(\frac{\rho_1}{2})^3}{3!} + \frac{(\frac{\rho_1}{2})^4}{4!} + \frac{\rho_2}{2} \simeq 1 + \frac{\rho_1}{2} + \frac{\rho_2}{2} \quad (5.2.3)$$

We derive a simple approximation of $G(2)$:

$$G(2) \simeq \left(1 + \frac{\rho_1}{2} + \frac{\rho_2}{2}\right)^2 \quad (5.2.4)$$

Using the assumption $\frac{\rho_1}{2} + \frac{\rho_2}{2} \ll 1$ and the fact that $e^x \simeq 1 + x$ for $x \ll 1$, we conclude:

$$G(2) \simeq e^{\rho_1 + \rho_2} \quad (5.2.5)$$

Appendix C shows that in general for $W > 2$, for low load regime $G(W) \simeq e^{\rho_1 + \rho_2}$ and consequently, the stationary probability $\pi(s)$ in (5.1.6) can be approximated by:

$$\pi(s) \simeq \frac{1}{\exp(\rho_1 + \rho_2)} \prod_{w=1}^W \frac{\left(\frac{\rho_1}{W}\right)^{n^{1,w}} \cdot \left(\frac{\rho_2}{W}\right)^{n^{2,w}}}{n^{1,w}! \cdot n^{2,w}!} \quad (5.2.6)$$

The above equation estimates the product form solution under the asymptotic condition for low load traffic when $W > 2$. It shows that in this regime, we do not need to compute the normalizing constant which remarkably decreases the numerical computation time.

5.3 Performance Evaluation

This section is devoted to the numerical computation of the blocking probabilities (BP). As shown in [10], the First-Fit wavelength assignment scheme is a simple and efficient candidate to apply in WDM networks. For comparison, we implement both Random and First-Fit assignment schemes and compare them with the results of section 5.1 and 5.2.

For the case of two wavelengths when deploying the Random assignment scheme, we use the generator matrix Q^* to compute the stationary probability for the MC shown in Fig. 5.2(a). The following equations lead us to calculate the exact solution

for the Random assignment.

$$\pi(s)Q^* = 0, \pi(s)e = 1 \quad (5.3.1)$$

We have also applied the above numerical method to calculate the exact blocking probabilities for First-Fit assignment as well.

In Fig. 5.2, we plot the blocking probabilities calculated for the Random and First-Fit versus the traffic load ($\rho = \rho_1 + \rho_2$) as follows.

- Exact value of the blocking rate (Random assignment) calculated by (5.3.1).
- Exact value of the blocking rate (First-Fit) calculated by (5.3.1).
- Approximate value of the blocking rate (Random) calculated by product form (5.1.4).
- Approximate value of the blocking rate (Random) calculated by (5.2.6).

Fig. 5.2 illustrates that the First-Fit policy has better performance than the Random specially for the low load traffic. It also depicts that the product form solution (5.1.4) is a lower bound for the Random assignment. In the area of interest, the product form solution is an almost indistinguishable upper bound of the First-Fit assignment. It also shows that the result of the approximate solution (5.2.6) gives a very good estimate of (5.1.4).

To evaluate a 3-wavelength system, we simulate the system using OPNET. Fig. 5.3 shows the simulation results for this case. The accuracy of the approximations, as mentioned previously, holds also for the 3-wavelength system.

Since the First-Fit assignment provides better performance in terms of utilization and blocking probabilities (see Figs. 5.2,5.3), it is more likely to be the policy implemented in practice. For a GoS of the order 10^{-3} or 10^{-5} , the formula (5.2.6) is a

Figure 5.2: Overall blocking probabilities for a 2-wavelength link

good approximation of the blocking probability when the First-Fit assignment is used. Therefore, we use (5.2.6) to determine $W_{\min}$, the minimum number of wavelengths to meet a specified GoS.

We plot $W_{\min}$ in Fig. 5.4 as a function of the total offered traffic load in logarithm scale. Fig. 5.5 provides the comparison of $W_{\min}$ for two GoSs, $\text{GoS}=10^{-3}$ and $\text{GoS}=10^{-5}$. Let ρ be the total offered traffic, and f is a mapping: $f : \rho \mapsto W_{\min}$. Fig. 5.5 illustrates that:

- As ρ increases, the slope of f decreases and the average traffic carried per wavelength increases. This is referred to as the economies of scale. For $\text{GoS}=10^{-3}$, the system requires 8 wavelengths to accommodate a total traffic of 5 Erlang. Thus, in the average a wavelength carries $(5/8) \sim 0.625$ Erlang. For the same GoS, if we increase the traffic load to 35 Erlang, then we only need 21 wavelengths. In this case, each wavelength carries $(35/21) \sim 1.67$ Erlang traffic

Figure 5.3: Overall blocking probabilities for 3-wavelength link

load.

- By increasing GoS from 10^{-3} to 10^{-5} for the same load, the slope of f increases as well. This shows that a system with more stringent GoS is more demanding in terms of resources.
- In addition, we also investigate a mathematical interpolation model of the mapping f that provides the minimum number of wavelengths required for a given total load and a given GoS. For GoS= 10^{-3} (respectively, GoS= 10^{-5}), our mathematical interpolation is $W_{min}(\rho) = 1 + 3.4\sqrt{\rho}$ (respectively, $W_{min}(\rho) = 1 + 4.4\sqrt{\rho}$).

To verify the accuracy of this dimensioning method, we consider the system for the worst case scenarios. They consist of the thresholds in Fig. 5.4 at which the number of wavelengths is increased. For each threshold, we simulate the system

Figure 5.4: Minimum number of wavelengths to satisfy $\text{GoS}=10^{-3}$

when the First-Fit and Random assignments are employed. Table 5.1 reports the overall blocking probabilities experienced by the system for both schemes. As we expected, our algorithm provides a good estimation of the number of wavelengths to meet the GoS, when the First-Fit assignment is used. But it underestimates this number when using the Random assignment.

Table 5.1: Performance Evaluation for Proposed Algorithm ($\text{GoS}=10^{-3}$)

w	Offered Load(Erlang)	BP of First-Fit	BP of Random
2	0.138	$.9 \times 10^{-3}$	1.14×10^{-3}
3	0.575	$.64 \times 10^{-3}$	1.9×10^{-3}
4	1.299	$.49 \times 10^{-3}$	2.6×10^{-3}
5	2.276	$.5 \times 10^{-3}$	3.78×10^{-3}
6	3.475	$.56 \times 10^{-3}$	5.06×10^{-3}
7	4.871	$.72 \times 10^{-3}$	6.2×10^{-3}

Figure 5.5: Minimum number of wavelengths to satisfy $\text{GoS}=10^{-3}$ and $\text{GoS}=10^{-5}$

Chapter 6

Conclusions and Future Work

In this chapter, we briefly review the contributions of this thesis, and suggest some directions for future research work.

6.1 Summary of Contributions

In this thesis, we take into account the dynamic nature of the WDM networks and proposed dynamic wavelength allocation policies in order to provide service differentiation and fairness control in both wavelength-routed WDM grooming and non-grooming networks. These allocation strategies are based on analytical models.

In the first contribution of this work, an **optimal dynamic** wavelength allocation scheme is proposed to support service differentiation and fairness control in wavelength-routed WDM networks with and without WCs. The optimal wavelength allocation policy maximizes the weighted sum of users of all classes. Consequently, this method is able to provide differentiated services in the network. It has been analytically and numerically shown that the optimal policy has the form of a monotonic non-decreasing switching curve for each class. Simulation results verify that the optimal allocation policy provides service differentiation while maintaining a high

level wavelength utilization. Using the properties of the MDP-based optimal policy, we have developed approximations and proposed a heuristic Multi-Threshold wavelength allocation scheme to support fairness management in an arbitrary WDM ring network. When the Multi-Threshold heuristic scheme is deployed, while all classes of users share the wavelengths, classes with higher hop-counts are protected from the ones with lower hop-counts on each link of the ring network.

Then, we add the grooming capabilities to the WDM networks and design an optimal dynamic CAC policy to support service differentiation and fairness control in the system. Furthermore, we propose a heuristic decomposition algorithm with lower computational complexity and very good performance. Simulation results compared the performance of the proposed approach with those of CS and CP policies. It has been observed that we can achieve substantial improvement in terms of fairness ratio and utilization.

6.2 Future work

In this thesis we proposed and designed several dynamic resource management techniques to provide service differentiation in WDM networks. As a future work, one can investigate the hardware/software implementation of these algorithms in practical WDM networks.

In this thesis we have not considered survivability of the network. A network is survivable if it is capable of continuing to serve the end users in case of network failures (i.e., node failure, link failure or protocol failure). Since the interruption of a high-speed optical connection results in a huge waste of data information and bandwidth, survivability of WDM networks is an important issue.

6.3 Survivability of WDM networks

Network survivability can be achieved by employing protection mechanisms such as traffic monitoring and fault localization along with restoration mechanisms to switch over the traffic in the case of failure [79]. In a survivable network, some spare capacity is dedicated to each working path so that traffic influenced by the failure can move to the spare path. Generally, there are two protection operations: dedicated protection and shared protection. In the dedicated protection, a source node splits the signal in two identical copies which is transited on two different lightpaths and the receiver chooses the better signal. This mechanism needs 100% of spare capacity and in the case of failure, there is no signaling required and the recovery time is very short. In shared protection, the protection path is reserved and shared by multiple working paths. In this case, the protection path is considered under path disjoint constraint [80]. The major interest here is to provide a cost-effective spare capacity reservation to meet a certain level of survivability. Important parameters that would have to be considered while comparing WDM protection techniques are spare capacity allocation, switching speed for connection recovery, cost of the protection equipment, and implementation complexity [81–83]. In order to determine the spare capacity allocation one can use mathematical programming or heuristics algorithms to minimize the spare resources (capacity). It has been shown that Integer Programming formulation is NP-hard. Therefore, relaxation methods are widely used to approximate Integer Programming [79,83]. For a comprehensive study on architecture and survivability, the reader is referred to [79].

Our work can be extended to provide service differentiation in WDM survivable networks when we use shared protection. In this case, by using dynamic wavelength

allocation we can utilize the resources more efficiently.

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Appendix A

Multimodularity of The Value Function

A.1 Proof of Proposition 2.1: Multimodularity of $V_n^*(s)$

To simplify the notation, the following intermediate operators are introduced:

$$J_n^{+1}(i, j, m) = \min \left\{ V_n^*(i + 1, j, m), V_n^*(i, j + 1, m + 1) \right\}, \quad (\text{A.1.1})$$

$$J_n^{-1}(i, j, m) = \min \left\{ V_n^*(i, j + 1, m), V_n^*(i + 1, j, m - 1) \right\}. \quad (\text{A.1.2})$$

One can interpret J_n^{+1} and J_n^{-1} as the costs just after event e and exactly before taking action a . Using these operators, the dynamic programming equation (2.3.1) can be rewritten as:

$$V_{n+1}^*(s) = \mathbf{B}^*.s^T + \gamma/\nu \left[\lambda_1 V_n^*(A_1 s) + \lambda_2 V_n^*(A_2 s) + \mu_1 n_1 J_n^{+1}(s) + \mu_2 n_2 J_n^{-1}(s) + (W(\mu_1 + \mu_2) - n_1 \mu_1 - n_2 \mu_2) V_n^*(s) \right]. \quad (\text{A.1.3})$$

We will prove that $V_{n+1}^* \in \mathcal{F}$ by induction in three steps. In *Step 1*, we show that $V_0^* \in \mathcal{F}$. Assuming $V_n^*(s) \in \mathcal{F}$, we establish in *Step 2*, that both J_n^{+1} and $J_n^{-1} \in \mathcal{F}$. Finally, in *Step 3*, we conclude that $V_{n+1}^* \in \mathcal{F}$.

Step 1: One can easily verify that $V_0^* = \mathbf{B}^*.s^T$ satisfies inequalities (2.3.2)-(2.3.4) and as a result $V_0^* \in \mathcal{F}$.

Step 2: Assuming that $V_n^*(s) \in \mathcal{F}$, one can show that both J_n^{+1} and $J_n^{-1} \in \mathcal{F}$. Let us first prove that $J_n^{+1} \in \mathcal{F}$. In doing so, we first show that J_n^{+1} satisfies (2.3.2) and

$$\begin{aligned} J_n^{+1}(i, j, m) + J_n^{+1}(i, j+1, m+1) \leq \\ J_n^{+1}(i+1, j, m) + J_n^{+1}(i-1, j+1, m+1) \end{aligned} \quad (\text{A.1.4})$$

is valid for all four possible values of $J_n^{+1}(i+1, j, m)$ and $J_n^{+1}(i-1, j+1, m+1)$ as follows.

Case 1: Assume that $V_n^*(i+2, j, m) < V_n^*(i+1, j+1, m+1)$ and $V_n^*(i, j+1, m+1) < V_n^*(i-1, j+2, m+2)$. According to J^+ operator, it can be noticed that

$$\begin{aligned} J_n^{+1}(i+1, j, m) &= \min\{V_n^*(i+2, j, m), V_n^*(i+1, j+1, m+1)\} = V_n^*(i+2, j, m), \\ J_n^{+1}(i-1, j+1, m+1) &= \min\{V_n^*(i, j+1, m+1), V_n^*(i-1, j+2, m+2)\} = V_n^*(i, j+1, m+1). \end{aligned}$$

Using (A.1.1), we can deduce:

$$\begin{aligned} J_n^{+1}(i, j, m) + J_n^{+1}(i, j+1, m+1) \leq \\ V_n^*(i+1, j, m) + V_n^*(i+1, j+1, m+1) \end{aligned} \quad (\text{A.1.5})$$

On the other hand, the multimodularity of $V_n^*(s)$ implies that:

$$\begin{aligned} V_n^*(i+1, j, m) + V_n^*(i+1, j+1, m+1) \leq \\ V_n^*(i+2, j, m) + V_n^*(i, j+1, m+1). \end{aligned} \quad (\text{A.1.6})$$

According to the assumption of this case, the right hand side of inequality (A.1.6) is equal to:

$$\begin{aligned} V_n^*(i+2, j, m) + V_n^*(i, j+1, m+1) = \\ J_n^{+1}(i+1, j, m) + J_n^{+1}(i-1, j+1, m+1). \end{aligned} \quad (\text{A.1.7})$$

By combining (A.1.5), (A.1.6), and (A.1.7), we get:

$$\begin{aligned} J_n^{+1}(i, j, m) + J_n^{+1}(i, j+1, m+1) \leq \\ J_n^{+1}(i+1, j, m) + J_n^{+1}(i-1, j+1, m+1). \end{aligned} \quad (\text{A.1.8})$$

Case 2: The assumption for this case is:

$$J_n^{+1}(i+1, j, m) = V_n^*(i+2, j, m), \quad J_n^{+1}(i-1, j+1, m+1) = V_n^*(i-1, j+2, m+2).$$

Using (A.1.1), we derive the following inequality:

$$\begin{aligned} J_n^{+1}(i, j, m) + J_n^{+1}(i, j+1, m+1) &\leq \\ V_n^*(i+1, j, m) + V_n^*(i, j+2, m+2). \end{aligned} \quad (\text{A.1.9})$$

Similar to the previous case, the multimodularity of the value function implies that:

$$\begin{aligned} V_n^*(i+1, j, m) + V_n^*(i+1, j+1, m+1) &\leq \\ V_n^*(i+2, j, m) + V_n^*(i, j+1, m+1), \end{aligned} \quad (\text{A.1.10})$$

$$\begin{aligned} V_n^*(i, j+1, m+1) + V_n^*(i, j+2, m+2) &\leq \\ V_n^*(i+1, j+1, m+1) + V_n^*(i-1, j+2, m+2). \end{aligned} \quad (\text{A.1.11})$$

After summing (A.1.10) and (A.1.11), we obtain:

$$\begin{aligned} V_n^*(i+1, j, m) + V_n^*(i, j+2, m+2) &\leq \\ V_n^*(i+2, j, m) + V_n^*(i-1, j+2, m+2), \end{aligned} \quad (\text{A.1.12})$$

$$\begin{aligned} V_n^*(i+2, j, m) + V_n^*(i-1, j+2, m+2) &= \\ J_n^{+1}(i+1, j, m) + J_n^{+1}(i-1, j+1, m+1). \end{aligned} \quad (\text{A.1.13})$$

Combining (A.1.9), (A.1.12), and (A.1.13) implies (A.1.4).

Case 3: The assumption is:

$$J_n^{+1}(i+1, j, m) = V_n^*(i+1, j+1, m+1), \quad J_n^{+1}(i-1, j+1, m+1) = V_n^*(i-1, j+2, m+2).$$

Using (A.1.1), we can show that:

$$\begin{aligned} J_n^{+1}(i, j, m) + J_n^{+1}(i, j+1, m+1) &\leq \\ V_n^*(i, j+1, m+1) + V_n^*(i, j+2, m+2). \end{aligned} \quad (\text{A.1.14})$$

From multimodularity of $V_n^*(s)$ and the assumption of this case, it can be concluded that:

$$\begin{aligned}
 & V_n^*(i, j+1, m+1) + V_n^*(i, j+2, m+2) \leq \\
 & V_n^*(i+1, j+1, m+1) + V_n^*(i-1, j+2, m+2), \\
 & V_n^*(i+1, j+1, m+1) + V_n^*(i-1, j+2, m+2) = \\
 & J_n^{+1}(i+1, j, m) + J_n^{+1}(i-1, j+1, m+1). \tag{A.1.15}
 \end{aligned}$$

By combining (A.1.14) and (A.1.15), it can be shown that inequality (A.1.4) is satisfied.

Case 4: The assumption for this case is:

$$J_n^{+1}(i+1, j, m) = V_n^*(i+1, j+1, m+1), J_n^{+1}(i-1, j+1, m+1) = V_n^*(i, j+1, m+1).$$

Using (A.1.1) and the assumption of case 4, we can obtain:

$$\begin{aligned}
 & J_n^{+1}(i, j, m) + J_n^{+1}(i, j+1, m+1) \leq \\
 & V_n^*(i, j+1, m+1) + V_n^*(i+1, j+1, m+1), \\
 & V_n^*(i, j+1, m+1) + V_n^*(i+1, j+1, m+1) = \\
 & J_n^{+1}(i+1, j, m) + J_n^{+1}(i-1, j+1, m+1). \tag{A.1.16}
 \end{aligned}$$

So far, we have proved that for all possible cases, inequality (A.1.4) is satisfied. An analogous proof can be applied (by induction with respect to n) to get the following inequalities:

$$\begin{aligned}
 & J_n^{+1}(i, j, m) + J_n^{+1}(i-1, j, m+1) \leq J_n^{+1}(i, j-1, m) + J_n^{+1}(i-1, j+1, m+1), \\
 & J_n^{+1}(i, j, m) + J_n^{+1}(i-1, j+1, m) \leq J_n^{+1}(i, j, m-1) + J_n^{+1}(i-1, j+1, m+1).
 \end{aligned}$$

Hence, we established that $J_n^{+1}(s)$ and similarly $J_n^{-1}(s)$ are multimodular.

Step 3: Since $V_{n+1}(s)$ is a linear combination of $J_n^{+1}(s)$, $J_n^{-1}(s)$, and $V_n(s)$ with positive coefficients, then $V_{n+1}(s)$ is also multimodular.

A.2 Proof of Proposition 2.2: Multimodularity of $V_n(s)$

Let us generalize the intermediate operators as:

$$J_n^{+1}(i, j, k, m) = \min \left\{ V_n(i+1, j, k+1, m), V_n(i, j+1, k, m+1) \right\} \quad (\text{A.2.1})$$

$$J_n^{-1}(i, j, k, m) = \min \left\{ V_n(i, j+1, k, m), V_n(i+1, j, k+1, m-1) \right\} \quad (\text{A.2.2})$$

By substituting these operators in the dynamic programming equation given in (4.2.3), we can see that:

$$V_{n+1}(s) = C(s) + \frac{\gamma}{\nu} \left[\lambda_1 V_n(A_1 s) + \lambda_2 V_n(A_2 s) + \lambda_3 V_n(A_3 s) + \mu_1 n_1 J_n^{+1}(s) + \mu_2 n_2 J_n^{-1}(s) + \mu_3 n_3 J_n^{+1}(s) + F V_n(s) \right] \quad (\text{A.2.3})$$

Similar to the proof of *Proposition 1*, it can be shown by induction that $V_{n+1} \in \mathcal{F}$. We can easily show that $V_0 = C(s) \in \mathcal{F}$. Assume $V_n(s) \in \mathcal{F}$ and one can establish that both $J_n^{+1}, J_n^{-1} \in \mathcal{F}$. As a result, it can be shown that $V_{n+1} \in \mathcal{F}$. Let us prove that J_n^{+1} satisfies equations (2.3.8)-(2.3.11). We first show that:

$$J_n^{+1}(i, j, k, m) + J_n^{+1}(i, j+1, k-1, m+1) \leq J_n^{+1}(i+1, j, k, m) + J_n^{+1}(i-1, j+1, k-1, m+1) \quad (\text{A.2.4})$$

is valid for all four possible values of $J_n^{+1}(i+1, j, k, m)$ and $J_n^{+1}(i-1, j+1, k-1, m+1)$. Let us provide more details for one of these four cases.

Case 1: Assume that: $V_n(i+2, j, k+1, m) < V_n(i+1, j+1, k, m+1)$ and $V_n(i, j+1, k, m+1) < V_n(i-1, j+2, m+2)$, then:

$$J_n^{+1}(i+1, j, k, m) = V_n(i+2, j, k+1, m) \\ , J_n^{+1}(i-1, j+1, k-1, m+1) = V_n(i, j+1, k, m+1)$$

Using (A.2.1), we can derive:

$$J_n^{+1}(i, j, k, m) + J_n^{+1}(i, j+1, k-1, m+1) \leq V_n(i+1, j, k+1, m) + V_n(i+1, j+1, k, m+1) \quad (\text{A.2.5})$$

$V_n(s) \in \mathcal{F}$ implies that:

$$\begin{aligned} V_n(i+1, j, k+1, m) + V_n(i+1, j+1, k, m+1) \leq \\ V_n(i+2, j, k+1, m) + V_n(i, j+1, k, m+1) \end{aligned} \quad (\text{A.2.6})$$

According to the assumption of this case, the right hand side of inequality (A.2.6) is equal to:

$$\begin{aligned} V_n(i+2, j, k+1, m) + V_n(i, j+1, k, m+1) = \\ J_n^{+1}(i+1, j, k, m) + J_n^{+1}(i-1, j+1, k-1, m+1) \end{aligned} \quad (\text{A.2.7})$$

By combining (A.2.5), (A.2.6), and (A.2.7), it can be shown that:

$$\begin{aligned} J_n^{+1}(i, j, k, m) + J_n^{+1}(i, j+1, k-1, m+1) \leq \\ J_n^{+1}(i+1, j, k, m) + J_n^{+1}(i-1, j+1, k-1, m+1) \end{aligned} \quad (\text{A.2.8})$$

The proof of *Case 2*, *3*, and *4* is similar to *Case 2*, *3*, and *4* of *Proposition 1*, respectively.

Correspondingly, it can be shown that that J_n^{+1} satisfies equations (2.3.9)-(2.3.11) as well. To this end, we established that $J_n^{+1}(s) \in \mathcal{F}$. In much the same way, we can show that $J_n^{-1}(s) \in \mathcal{F}$. Since $V_{n+1}(s)$ is a linear combination of $J_n^{+1}(s)$, $J_n^{-1}(s)$, and $V_n(s)$ with positive coefficients, then $V_{n+1}(s)$ is also multimodular.

Appendix B

Properties of The Objective Function f

B.1 Proof of Theorem 3.1: $f(\boldsymbol{\tau})$ is a unimodal function

Let e_i denote a vector with all entries zero except for a 1 in its i th entry. Define operator δ^i as follows

$$\delta^i g(\boldsymbol{\tau}) = g(\boldsymbol{\tau} + e_i) - g(\boldsymbol{\tau})$$

We consider a 4-node network and prove that $f(\boldsymbol{\tau})$ has a single minimum. First, we present some properties of the system and then using these properties we show that the sign of $\delta^1 f(\boldsymbol{\tau})$ changes from negative to positive only once for all possible values of $\boldsymbol{\tau}$. It shows that f is unimodal.

Property 1: By fixing threshold τ_2 and increasing τ_1 , the blocking rate of class c_1 increases and the blocking rates of c_2 and c_3 decrease. Equivalently, we can say

$$\delta^1 B_1(\boldsymbol{\tau}) > 0, \quad \delta^1 B_2(\boldsymbol{\tau}) < 0 \quad \text{and} \quad \delta^1 B_3(\boldsymbol{\tau}) < 0.$$

and also

$$|\delta^1 B_1(\boldsymbol{\tau})| > |\delta^1 B_2(\boldsymbol{\tau})| > |\delta^1 B_3(\boldsymbol{\tau})|.$$

Property 2: By fixing threshold τ_1 and increasing τ_2 , the blocking probability of

class c_2 increases while the ones of c_1 and c_3 decrease

$$\delta^2 B_2(\boldsymbol{\tau}) > 0, \quad \delta^2 B_1(\boldsymbol{\tau}) < 0 \quad \text{and} \quad \delta^2 B_3(\boldsymbol{\tau}) < 0.$$

and

$$|\delta^2 B_2(\boldsymbol{\tau})| > |\delta^2 B_3(\boldsymbol{\tau})| > |\delta^2 B_1(\boldsymbol{\tau})|.$$

We first fix τ_2 and derive $\delta^1 f(\boldsymbol{\tau})$ for different scenarios and show that its sign changes from negative to positive only once. Each scenario starts with an initial situation and ends at a final position.

Scenario 1: In this scenario, the initial situation is when $B_1 < B_2 < B_3$. This can be (but not necessarily) associated with $(\tau_1, \tau_2) = (0, 0)$.

Case 1: In this case, we assume that

$$B_1(\boldsymbol{\tau}) < B_2(\boldsymbol{\tau}) < B_3(\boldsymbol{\tau}), \quad B_1(\boldsymbol{\tau} + e_1) < B_2(\boldsymbol{\tau} + e_1) < B_3(\boldsymbol{\tau} + e_1)$$

According to 3.2.1 and the assumption of this case, one can note that

$$\delta^1 f(\boldsymbol{\tau}) = -2[\delta^1 B_3(\boldsymbol{\tau}) + \delta^1 B_1(\boldsymbol{\tau})] < 0$$

Case 2: In this case, we assume that

$$B_1(\boldsymbol{\tau}) < B_2(\boldsymbol{\tau}) < B_3(\boldsymbol{\tau}), \quad B_2(\boldsymbol{\tau} + e_1) < B_1(\boldsymbol{\tau} + e_1) < B_3(\boldsymbol{\tau} + e_1)$$

We have

$$f(\boldsymbol{\tau}) = B_2(\boldsymbol{\tau}) - B_1(\boldsymbol{\tau}) + B_3(\boldsymbol{\tau}) - B_1(\boldsymbol{\tau}) + B_3(\boldsymbol{\tau}) - B_2(\boldsymbol{\tau}) = 2B_3(\boldsymbol{\tau}) - 2B_1(\boldsymbol{\tau})$$

Similarly,

$$f(\boldsymbol{\tau} + e_1) = 2B_3(\boldsymbol{\tau} + e_1) - 2B_2(\boldsymbol{\tau} + e_1)$$

Therefore

$$\delta^1 f(\boldsymbol{\tau}) = 2B_3(\boldsymbol{\tau} + e_1) - 2B_3(\boldsymbol{\tau}) - 2B_2(\boldsymbol{\tau} + e_1) + 2B_1(\boldsymbol{\tau})$$

According to the general assumption

$$\delta^1 f(\boldsymbol{\tau}) < 2B_3(\boldsymbol{\tau} + e_1) - 2B_3(\boldsymbol{\tau}) - 2B_2(\boldsymbol{\tau} + e_1) + 2B_2(\boldsymbol{\tau}) = 2[\delta^1 B_3(\boldsymbol{\tau}) - \delta^1 B_2(\boldsymbol{\tau})] > 0$$

Since $\delta^1 B_3(\boldsymbol{\tau}) - \delta^1 B_2(\boldsymbol{\tau}) > 0$, we cannot decide about the sign of $\delta^1 f(\boldsymbol{\tau})$.

Case 3: The assumption for this case is

$$B_2(\boldsymbol{\tau}) < B_1(\boldsymbol{\tau}) < B_3(\boldsymbol{\tau}), \quad B_2(\boldsymbol{\tau} + e_1) < B_1(\boldsymbol{\tau} + e_1) < B_3(\boldsymbol{\tau} + e_1)$$

We have

$$\delta^1 f(\boldsymbol{\tau}) = 2[\delta^1 B_3(\boldsymbol{\tau}) - \delta^1 B_2(\boldsymbol{\tau})]$$

Since $\delta^1 B_3(\boldsymbol{\tau}) > \delta^1 B_2(\boldsymbol{\tau})$, one can note that $\delta^1 f(\boldsymbol{\tau}) > 0$.

Case 4: The assumption for this case is

$$B_2(\boldsymbol{\tau}) < B_1(\boldsymbol{\tau}) < B_3(\boldsymbol{\tau}), \quad B_2(\boldsymbol{\tau} + e_1) < B_3(\boldsymbol{\tau} + e_1) < B_1(\boldsymbol{\tau} + e_1)$$

We have

$$\delta^1 f(\boldsymbol{\tau}) = 2[B_1(\boldsymbol{\tau} + e_1) - B_3(\boldsymbol{\tau}) - B_2(\boldsymbol{\tau} + e_1) + B_2(\boldsymbol{\tau})]$$

From the assumption, one can see that $B_1(\boldsymbol{\tau} + e_1) > B_3(\boldsymbol{\tau} + e_1)$ and as a result

$$\delta^1 f(\boldsymbol{\tau}) > 2[\delta^1 B_3(\boldsymbol{\tau}) - \delta^1 B_2(\boldsymbol{\tau} + e_1)] > 0$$

Case 5: The assumption for this case is

$$B_2(\boldsymbol{\tau}) < B_3(\boldsymbol{\tau}) < B_1(\boldsymbol{\tau}), \quad B_2(\boldsymbol{\tau} + e_1) < B_3(\boldsymbol{\tau} + e_1) < B_1(\boldsymbol{\tau} + e_1)$$

We have

$$\delta^1 f(\boldsymbol{\tau}) = 2[\delta^1 B_1(\boldsymbol{\tau}) - \delta^1 B_2(\boldsymbol{\tau} + e_1)] > 0$$

It can be noticed that in this scenario the sign of $\delta^1 f(\boldsymbol{\tau})$ can be changed only once. It can happen in either *Case 2* or *Case 3*.

Scenario 2: The initial condition for this scenario is $B_1 < B_3 < B_2$.

Case 1: The assumption for this case is

$$B_1(\boldsymbol{\tau}) < B_3(\boldsymbol{\tau}) < B_2(\boldsymbol{\tau}), \quad B_1(\boldsymbol{\tau} + e_1) < B_3(\boldsymbol{\tau} + e_1) < B_2(\boldsymbol{\tau} + e_1)$$

We have

$$\delta^1 f(\boldsymbol{\tau}) = 2[\delta^1 B_2(\boldsymbol{\tau}) - \delta^1 B_1(\boldsymbol{\tau} + e_1)] < 0$$

Case 2: The assumption for this case is

$$B_1(\boldsymbol{\tau}) < B_3(\boldsymbol{\tau}) < B_2(\boldsymbol{\tau}), \quad B_3(\boldsymbol{\tau} + e_1) < B_1(\boldsymbol{\tau} + e_1) < B_2(\boldsymbol{\tau} + e_1)$$

We have

$$f(\boldsymbol{\tau}) = 2[B_2(\boldsymbol{\tau}) - B_1(\boldsymbol{\tau})]$$

$$f(\boldsymbol{\tau} + e_1) = 2[B_2(\boldsymbol{\tau} + e_1) - B_3(\boldsymbol{\tau} + e_1)]$$

Therefore

$$\delta^1 f(\boldsymbol{\tau}) = 2[\delta^1 B_2(\boldsymbol{\tau}) - B_3(\boldsymbol{\tau} + e_1) + B_1(\boldsymbol{\tau})] < 2[\delta^1 B_2(\boldsymbol{\tau}) - \delta^1 B_3(\boldsymbol{\tau})] < 0$$

Case 3: The assumption for this case is

$$B_3(\boldsymbol{\tau}) < B_1(\boldsymbol{\tau}) < B_2(\boldsymbol{\tau}), \quad B_3(\boldsymbol{\tau} + e_1) < B_1(\boldsymbol{\tau} + e_1) < B_2(\boldsymbol{\tau} + e_1)$$

We have

$$\delta^1 f(\boldsymbol{\tau}) = 2[\delta^1 B_2(\boldsymbol{\tau}) - \delta^1 B_3(\boldsymbol{\tau})] < 0$$

Case 4: The assumption for this case is

$$B_3(\boldsymbol{\tau}) < B_1(\boldsymbol{\tau}) < B_2(\boldsymbol{\tau}), \quad B_3(\boldsymbol{\tau} + e_1) < B_2(\boldsymbol{\tau} + e_1) < B_1(\boldsymbol{\tau} + e_1)$$

We have

$$\delta^1 f(\boldsymbol{\tau}) > 2[\delta^1 B_2(\boldsymbol{\tau}) - \delta^1 B_3(\boldsymbol{\tau})]$$

Since $\delta^1 B_2(\boldsymbol{\tau}) - \delta^1 B_3(\boldsymbol{\tau}) < 0$, we cannot decide about the sign of $\delta^1 f(\boldsymbol{\tau})$.

Case 5: The assumption for this case is

$$B_3(\boldsymbol{\tau}) < B_2(\boldsymbol{\tau}) < B_1(\boldsymbol{\tau}), \quad B_3(\boldsymbol{\tau} + e_1) < B_2(\boldsymbol{\tau} + e_1) < B_1(\boldsymbol{\tau} + e_1)$$

We have

$$\delta^1 f(\boldsymbol{\tau}) = 2[\delta^1 B_1(\boldsymbol{\tau}) - \delta^1 B_3(\boldsymbol{\tau})] > 0$$

For this scenario the sign of $\delta^1 f(\boldsymbol{\tau})$ can be changed either in Case 4 or Case 5.

Scenario 3: The last scenario starts with $B_1 < B_3 < B_2$.

Case 1: The assumption for this case is

$$B_1(\boldsymbol{\tau}) < B_3(\boldsymbol{\tau}) < B_2(\boldsymbol{\tau}), \quad B_1(\boldsymbol{\tau} + e_1) < B_2(\boldsymbol{\tau} + e_1) < B_3(\boldsymbol{\tau} + e_1)$$

We have

$$f(\boldsymbol{\tau}) = 2[B_2(\boldsymbol{\tau}) - B_1(\boldsymbol{\tau})]$$

$$f(\boldsymbol{\tau} + e_1) = 2[B_3(\boldsymbol{\tau} + e_1) - B_1(\boldsymbol{\tau} + e_1)]$$

We have

$$\delta^1 f(\boldsymbol{\tau}) = 2[B_3(\boldsymbol{\tau} + e_1) - B_2(\boldsymbol{\tau}) - \delta^1 B_1(\boldsymbol{\tau} + e_1)]$$

Note that $B_3(\boldsymbol{\tau} + e_1) < B_3(\boldsymbol{\tau}) < B_2(\boldsymbol{\tau})$. As a result $B_3(\boldsymbol{\tau} + e_1) - B_2(\boldsymbol{\tau}) < 0$ and consequently, $\delta^1 f(\boldsymbol{\tau}) < 0$.

Case 2: The assumption for this case is the same one as *Case 1* of *Scenario 1*. Therefore, we can follow the cases in *Scenario 1*

So far we proved the objective function is unimodal in τ_1 , i.e., when τ_2 is constant, say $\tau_2 = \sigma$, the $f(\tau_1, \sigma)$ has only one minimum. This reaches at the local optimum point $(\tau_1^{\tau_2=\sigma}, \sigma)$. Equivalently, we can say

$$(\tau_1^{\tau_2=\sigma}, \sigma) = \arg \min_{\tau_1} f(\tau_1, \sigma) \quad (\text{B.1.1})$$

In a very similar manner, one can show that, when τ_1 is fixed, the objective function is unimodal in τ_2 . As a result, we can say that $f(\tau_1, \tau_2)$ is unimodal in both τ_1 and τ_2 .

Appendix C

Approximation of $G(W)$

To calculate $G(W)$ for a system carrying light traffic, we can assume that $\frac{\rho_1}{W} \ll 1$ and $\frac{\rho_2}{W} \ll 1$. One can notice that by using this assumption the terms $(\frac{\rho_1}{W})^{n^{1,j}}/n^{1,j}!$ for $n^{1,j} > 1$ are negligible when compared to $(\frac{\rho_1}{W})$ and may be omitted in the calculation of $G(W)$. Similarly, $(\frac{\rho_2}{W})^{n^{2,j}}/n^{2,j}!$ for $n^{2,j} > 1$ can be omitted in the approximation of $G(W)$.

Let $\Pi_k(s) = \prod_{j=k}^W \frac{(\rho_1/W)^{n^{1,j}} \cdot (\rho_2/W)^{n^{2,j}}}{n^{1,j}! \cdot n^{2,j}!}$. From (5.1.7), we can write:

$$\begin{aligned}
 G(W) = \sum_{s \in \Omega_W} \Pi_1(s) &= \sum_{\{s \in \Omega_W | n^{1,1} = n^{2,1} = 0\}} \Pi_2(s) + \sum_{k=1, \dots, 4} \frac{(\frac{\rho_1}{W})^k}{k!} \left(\sum_{\{s \in \Omega_W | n^{1,1} = k\}} \Pi_2(s) \right) \\
 &\quad + \left(\frac{\rho_2}{W} \right) \left(\sum_{\{s \in \Omega_W | n^{2,1} = 1\}} \Pi_2(s) \right) \\
 &\simeq \underbrace{\sum_{\{s \in \Omega_W | n^{1,1} = 0, n^{2,1} = 0\}} \Pi_2(s)}_{I_1} + \left(\frac{\rho_1}{W} \right) \underbrace{\left(\sum_{\{s \in \Omega_W | n^{1,1} = 1\}} \Pi_2(s) \right)}_{I_2} \\
 &\quad + \left(\frac{\rho_2}{W} \right) \underbrace{\left(\sum_{\{s \in \Omega_W | n^{2,1} = 1\}} \Pi_2(s) \right)}_{I_3} \quad \text{C.0.1}
 \end{aligned}$$

In the above equations, we have removed the terms $\frac{(\rho_1/W)^k}{k!}$ which are negligible

when $k > 1$. In (C.0.1), the three terms I_1, I_2 and I_3 are equal:

$$G(W) \simeq \sum_{s \in \Omega_W} \Pi_2(s) \left(1 + \frac{\rho_1}{W} + \frac{\rho_2}{W}\right)$$

Similar to the previous decomposition, We now decompose $\Pi_2(s)$ by considering the possible values of $(n^{1,2}, n^{2,2})$ and by introducing $\Pi_3(s)$:

$$G(W) \simeq \sum_{s \in \Omega_W} \Pi_3(s) \left(1 + \frac{\rho_1}{W} + \frac{\rho_2}{W}\right)^2$$

Iteratively, we derive a simple approximation of $G(W)$ as:

$$G(W) \simeq \left(1 + \frac{\rho_1}{W} + \frac{\rho_2}{W}\right)^W$$

Using the assumption of light traffic and the fact that $e^x \simeq 1 + x$ when $x \ll 1$, we conclude that:

$$G(W) \simeq e^{\rho_1 + \rho_2} \tag{C.0.2}$$