INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

ProQuest Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600

UMI[®]

NOTE TO USERS

This reproduction is the best copy available.

UMI^{*}

Diagonal Tension Buckling and Post-Buckling of Fibre-Metal Laminate Beams

By

Thor Normand Alexandre Jodoin, B.A.Sc.

A thesis submitted to the Faculty of Graduate Studies and Research

In partial fulfillment of the requirements for the degree of:

Master of Applied Science

Ottawa-Carleton Institute of Mechanical and Aerospace Engineering

Department of Mechanical and Aerospace Engineering

Carleton University

Ottawa, Ontario

Canada

April 2002

**National Library
of Canada**

**Acquisitions and
Bibliographic Services**

**385 Wellington Street
Ottawa ON K1A 0N4
Canada**

**Bibliothèque nationale
du Canada**

**Acquisitions et
services bibliographiques**

**385, rue Wellington
Ottawa ON K1A 0N4
Canada**

Your file Votre référence

Our file Notre référence

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-72066-7

Canada

**The undersigned recommend to
The Faculty of Graduate Studies and Research
acceptance of the thesis**

**Diagonal Tension Buckling and Post-Buckling of
Fibre-Metal Laminate Beams**

**Submitted by
Thor Normand Alexandre Jodoin, B.A.Sc.
In partial fulfillment of the requirements for
the degree of Master of Applied Science**

A handwritten signature in black ink, appearing to read 'D. Hasnady', written over a horizontal line.

Thesis Co-Supervisor

A handwritten signature in black ink, appearing to read 'W. J. R.', written over a horizontal line.

Thesis Co-Supervisor

A handwritten signature in black ink, appearing to read 'Robert Bell', written over a horizontal line.

Chair, Department of Mechanical and Aerospace Engineering

**Carleton University
April 2002**

ABSTRACT

As part of a larger ongoing collaborative project between the National Research Council Canada (NRC), Bombardier Aerospace and Carleton University, various GLARE (GLASS REinforced aluminum) web materials were tested to determine their performance while in a state of diagonal tension. Of importance was to evaluate the validity of the methodology developed by the National Advisory Committee for Aeronautics (NACA) for aluminum alloys, when applied to predicting the yielding and failure of GLARE-webs. Each test specimen is an 83 in x 31 in (2.06 m x 0.79 m) flat beam emulation of an aircraft stiffened thin skin. The diagonal tension effect is simulated in the test specimen by applying an in-plane shear load. Under increasing shear stress in the beam, the web material buckles elastically, then forms permanent buckles, and finally suffers a critical failure. The tests were conducted using the Institute for Aerospace Research's Multi-Actuator Test Platform, a new facility for testing large aircraft components and sub-assemblies. Varying thickness of GLARE 3, GLARE 4, and 2024-T3 aluminum were tested. The results have shown that an unmodified NACA-based methodology can be used to predict GLARE web stress/strain reactions up until its yielding point. However, more detailed studies into the plastic buckling behavior of GLARE will have to be conducted to accurately predict critical failures.

ACKNOWLEDGEMENTS

- To my supervisor, **Professor Paul V. Straznicky**, for selecting me to pursue a Masters degree in the field of Aerospace Engineering, for having faith in my abilities, and for providing a wealth of constructive advice that will help to guide me through the rest of my career.
- To my co-supervisor, **Dr. Cheung J. Poon**, for providing me with a challenging and diverse environment from which to do my thesis.
- To **Bombardier Aerospace**, for providing the funding and materials for this project.
- To my good friend and fellow graduate student, **Jeremy Laliberté**, for helping to maintain my objectivity, for keeping me focused on the problems at hand, and for providing me with endless information and perspectives about the field of FMLs.
- To **Bruce Shaver, John Rogers, and Jean Lalonde**, for your wealth of practical experience and for teaching me the realities of Large Scale Testing. I could not have finished without your guidance and help.
- To **Jim Keller, Charles Wilson, and Michel Delannoy**, for your invaluable help in instrumenting the test beams, and supporting the various data acquisition methods used through-out this project.
- To **Edward Lukas**, for your skill in constructing the test specimen beams, under difficult and challenging circumstances.
- To Sensei's **Derek Kossatz, Dave Decarie, Claire Terrillon, and Chris Sherlok**, and all the students of the **Douvris Martial Arts Centre - Ogilvie Rd.**, for providing me with an outlet for my frustrations, a forum for focusing my self-discipline, and an opportunity to improve myself outside the realm of my graduate studies.
- To the rest of the staff at the Montreal Road Campus of the National Research Council, Institute for Aerospace Research, Structures Materials Propulsion Laboratory, it has been a joy and a privilege to work with you all.
- Last but not least, to my parents, **Normand and Tordis Jodoin**, for their unending support and encouragement.

TABLE OF CONTENTS

ABSTRACT	I
ACKNOWLEDGEMENTS	II
TABLE OF CONTENTS	III
LIST OF TABLES.....	VII
LIST OF FIGURES.....	VIII
NOMENCLATURE	X
1. INTRODUCTION	1
2. LITERATURE REVIEW	4
2.1 SHEAR RESISTANT CONSTRUCTION	4
2.2 PURE DIAGONAL TENSION (WAGNER THEORY)	7
2.2.1 <i>Principles of Wagner Theory</i>	8
2.2.2 <i>Secondary Bending Stresses</i>	16
2.2.3 <i>Diagonal Tension Angle</i>	17
2.2.4 <i>Diagonal Tension Shear Modulus</i>	19
2.2.5 <i>Effects of Flange Flexibility</i>	19
2.3 INCOMPLETE DIAGONAL TENSION (NACA THEORY).....	21
2.3.1 <i>Principles of NACA Theory</i>	22
2.3.2 <i>Strain-Based Diagonal Tension Angle</i>	25
2.3.3 <i>Critical Buckling Shear Stress</i>	26
2.3.4 <i>Effective Shear Modulus</i>	27
2.3.5 <i>Portal Frame Effect</i>	29
2.3.6 <i>Web Stress Empirical Corrections</i>	31
2.3.7 <i>Web Stiffener Empirical Corrections</i>	33
2.4 POST-NACA DEVELOPMENTS.....	36
2.4.1 <i>The Grumman Modifications</i>	36
2.4.1.1 <i>Initial Buckling</i>	38
2.4.1.2 <i>Diagonal Tension Angle</i>	38
2.4.1.3 <i>Allowable Shear Stress</i>	38
2.4.1.4 <i>Allowable Web Stiffener Stress Against Forced Crippling</i>	39
2.4.1.5 <i>Structures With Integrated Stiffeners</i>	39
2.4.1.6 <i>Sheet With Lands</i>	40
2.4.1.7 <i>Fastener Loads</i>	41
2.4.1.8 <i>Effective Shear Modulus</i>	41
2.4.2 <i>The Boeing SST Developments</i>	42
2.4.3 <i>Northrop's Corrections for Composite Beams</i>	45
2.4.4 <i>Diagonal Tension in Fibre Metal Laminates</i>	45
2.5 GLARE.....	46
2.5.1 <i>A Brief History of FMLs</i>	46
2.5.2 <i>Properties of GLARE</i>	48
2.5.3 <i>SLC's Evaluation of Diagonal Tension in GLARE</i>	52
3. THESIS OBJECTIVES	53
4. SPECIMEN DESIGN AND MANUFACTURE.....	54

4.1 BASIC DESIGN RESTRICTIONS	54
4.2 DESIGN ANALYSIS.....	56
4.3 FINAL SPECIMEN DESIGN	59
5. GENERAL TEST SET-UP	61
5.1 THE MULTI-ACTUATING TEST PLATFORM	61
5.2 INSTRUMENTATION	65
5.2.1 Strain Gauges.....	65
5.2.2 LVDT's.....	66
5.2.3 Photoelastic Coating.....	67
6. GENERAL TEST PROCEDURES.....	69
6.1 BEAM INSTALLATION	69
6.2 SPECIMEN TEST	70
6.2.1 Test Safety.....	72
7. OBSERVATIONS (TESTS 1 TO 4).....	74
7.1 INITIAL PREDICTIONS	74
7.1.1 Material Properties.....	75
7.1.2 Initial Buckling Estimates	76
7.1.3 Permanent Deformation.....	76
7.1.4 Critical Failure Modes.....	78
7.2 TEST OBSERVATIONS AND PROTOCOL EVOLUTION	79
7.3.1 Test #1: 0.050" 2024-T3 Aluminum.....	80
7.3.2 Test #2: GLARE 3 – 3/2	83
7.3.3 Test #3: GLARE 3 – 3/2	89
7.3.4 Test #4: GLARE 4 – 3/2	92
7.4 SUMMARY OF TEST RESULTS	95
7.5 BEAM DISASSEMBLY	97
7.5.1 Disassembly Procedure.....	97
7.5.2 Post-Disassembly Observations.....	99
8. THE CARLETON MINI-BEAM TEST	102
8.1 INTRODUCTION	102
8.2 ANALYSIS.....	103
8.3 TEST SET-UP.....	104
8.3.1 The Carleton Diagonal Tension Mini-Beam	104
8.3.2 The Test Frame	104
8.3.3 The Loading Arrangement	105
8.3.4 Strain Gauges.....	106
8.3.5 Laptop Computer	107
8.4 PROCEDURE.....	107
8.5 RESULTS AND DISCUSSION	109
8.6 CONCLUSIONS	112
9. CORRECTIONS TO THE NRC-IAR PREDICTIONS	113
9.1 DEFLECTION ESTIMATES	113
9.2 PORTAL FRAME EFFECT AND SHEAR MODULUS PLASTICITY	119
10. THE ELASTIC STRAIN SURVEY	125
10.1 OBJECTIVES.....	125
10.1.1 Events of Interest.....	126
10.2 INSTRUMENTATION.....	126
10.2.1 Linear Variable Differential Transducers (LVDTs).....	126
10.2.2 Strain Gauges.....	128
10.2.3 Data Acquisition Systems.....	129

10.3 TEST PROCEDURE	130
10.4 PRE-TEST PREDICTIONS	132
10.5 SURVEY RESULTS	133
11. DISCUSSION	137
12. CONCLUSIONS	141
13. RECOMMENDATIONS FOR FUTURE WORK	142
REFERENCES	143
APPENDIX A: SAMPLE MATHCAD BEAM DESIGN CALCULATIONS	148
A.1 INPUT VALUES	148
A.1.1 Main Web Properties	148
A.1.2 Flange Properties	148
A.1.3 Web Stiffener Properties	149
A.1.4 Beam Properties	150
A.2 FORMULA LIMITATIONS CHECK	151
A.3 CRITICAL SHEAR STRESS	152
A.4 LOADING RATIO	154
A.5 DIAGONAL TENSION FACTOR (K)	155
A.6 DIAGONAL TENSION ANGLE (α)	155
A.7 PORTAL FRAME EFFECT	156
A.7.1 Basic Equations	156
A.7.2 Initial Buckling Correction (when $k = 0$, and $\tau < 14$ ksi)	156
A.7.3 Portal Effects for Varying Diagonal Tension	157
A.8 ALLOWABLE STIFFENER STRESSES	160
A.8.1 Average and Maximum Web Stiffener Stress	160
A.8.2 Column Failure	161
A.8.2.1 Column Yield Stress	161
A.8.2.2 Centroidal Column Stress Check	161
A.8.3 Forced Crippling Failure	162
A.8.4 Natural Crippling Failure	162
A.8.5 General Elastic Stability of Web and Uprights	162
A.9 WEB DESIGN	163
A.9.1 Allowable Shear Stress	163
A.9.2 Permanent Web Buckling	164
A.10 FLANGE STRENGTH	165
A.10.1 Primary Bending Moments	165
A.10.2 Primary Bending Stresses	165
A.10.2.1 Upper (Compressive) Flange Bending Stresses	165
A.10.2.2 Lower (Tensile) Flange Bending Stresses	165
A.10.2.3 Average Axial Flange Loads Due to Bending	166
A.10.2.4 Axial Flange Loads Due to Pure Bending	166
A.10.2.5 Axial Flange Loads Due to Diagonal Tension Field	166
A.10.2.6 Total Average Axial Flange Loads	166
A.10.3 Secondary Bending Moments	167
A.10.4 Secondary Bending Stresses	167
A.10.5 Maximum Flange Bending Stresses	167
A.11 FASTENER DESIGN	168
A.11.1 Bearing and Fastener Properties	168
A.11.1.1 Web Bearing Properties	168
A.11.1.2 Web Stiffener Bearing Properties	169
A.11.1.3 Flange Bearing Properties	169
A.11.1.4 Aluminum Rivet Strength Properties	170
A.11.1.5 Flange Bolt Shear Strength Properties	170

A.11.1.6 Stiffener/Flange Bolt Shear Strength Properties.....	170
A.11.2 Web to Flange Fastener Loads.....	170
A.11.3 Web to Web Stiffener River Loads.....	171
A.11.3.1 Web to Web Stiffener Shear Effects.....	171
A.11.3.2 Web to Web Stiffener Tension Effects.....	172
A.11.4 Web Stiffener to Flange Fastener Loads.....	173
A.11.5 Loading Flange Corner Loads.....	174
A.11.6 Support Flange Corner Loads.....	175
A.12 FAILURE MODE ASSESSMENT.....	176
APPENDIX B: GENERAL NRC-IAR TEST BEAM DESIGN	178
B.1 GENERAL CONSTRUCTION	178
B.2 FASTENER ARRANGEMENTS	182
APPENDIX C: NRC-IAR TEST BEAM DRAWINGS.....	184
APPENDIX D: TEST BEAM TORQUE PATTERN	199
APPENDIX E: TEST BEAM STRAIN GAUGE PLACEMENTS	201
APPENDIX F: REVISED MARGIN OF SAFETY GRAPHS	206
APPENDIX G: TEST #1 (0.050" 2024-T3 ALUMINUM) RESULTS	209
APPENDIX H: TEST #2 (GLARE 3 – 3/2) RESULTS	216
APPENDIX I: TEST #3 (GLARE 3 – 3/2) RESULTS	222
APPENDIX J: TEST #4 (GLARE 4 – 3/2) RESULTS	228
APPENDIX K: CARLETON MINI-BEAM RESULTS	234
APPENDIX L: TEST #5 (0.071" 2024-T3 ALUMINUM) RESULTS.....	239

LIST OF TABLES

TABLE 1: STANDARD GLARE GRADES [20]	49
TABLE 2: MATERIAL THICKNESS ORIGINALLY INTENDED FOR EXPERIMENT	56
TABLE 3: NACA VS. NRC TEST SPECIMEN COMPARISON	59
TABLE 4: SELECTED MATERIAL PROPERTY DATA FOR ALUMINUM COMPONENTS [26]	75
TABLE 5: SELECTED MATERIAL PROPERTY DATA FOR FIBRE-METAL LAMINATE COMPONENTS	75
TABLE 6: INITIAL WEB BUCKLING LOADS FOR EACH MATERIAL	76
TABLE 7: PERMANENT BUCKLING LOADS FOR EACH EXPERIMENT MATERIAL TYPE	77
TABLE 8: PREDICTED FAILURES FOR STIFFENED BEAMS WITH 7075-T6 WEB STIFFENERS	79
TABLE 9: SUMMARY OF TESTS COMPLETED	79
TABLE 10: SUMMARY OF INITIAL BUCKLING	95
TABLE 11: SUMMARY OF FAILURE RESULTS	96
TABLE 12: CARLETON BEAM R1/R3 WEB TENSILE STRAIN COMPARISON	111
TABLE 13: CARLETON BEAM R2/R4 WEB TENSILE STRAIN COMPARISON	112
TABLE 14: REVISED INITIAL BUCKLING COMPARISON	139
TABLE 15: WEB SHEAR YIELDING COMPARISON	139
TABLE 16: WEB TENSILE YIELDING COMPARISON	139
TABLE 17: REVISED WEB FAILURE COMPARISON	140

LIST OF FIGURES

FIGURE 1: COMPOSITION LAY-UP OF GLARE 3 – 3/2 [1]	1
FIGURE 2: DIAGONAL TENSION IN A CANADAIR CT-114 TUTOR.	2
FIGURE 3: BENDING STRESSES IN AN IDEALIZED WING STRUCTURE.....	5
FIGURE 4: BOUNDARY CONDITIONS FOR A PLATE EDGE	6
FIGURE 5: SHEAR BUCKLING COEFFICIENTS [1].....	6
FIGURE 6: THE PRINCIPLE OF DIAGONAL TENSION	8
FIGURE 7: SIMPLIFIED MULTI-BAY BEAM UNDER SHEAR	9
FIGURE 8: DIAGONAL WEB STRESS ELEMENT	9
FIGURE 9: HORIZONTAL WEB STRESS ELEMENT	10
FIGURE 10: FLANGE FORCE BALANCE	11
FIGURE 11: VERTICAL WEB STRESS ELEMENT	12
FIGURE 12: WEB STIFFENER FORCE BALANCE	13
FIGURE 13: FIXED-END PRISMATIC BEAM REACTIONS UNDER A UNIFORMLY DISTRIBUTED LOAD [9]	14
FIGURE 14: SINGLE AND DOUBLE WEB STIFFENER CONSTRUCTIONS.....	15
FIGURE 15: FLANGE BENDING MOMENTS	16
FIGURE 16: FLANGE SECONDARY BENDING MOMENTS [1].....	16
FIGURE 17: VERTICAL WEB STRESSES AND FLANGE FLEXIBILITY	20
FIGURE 18: WEB STRESS AND SECONDARY MOMENT CORRECTION COEFFICIENTS [10].....	21
FIGURE 19: NACA TYPE I-25 TEST BEAM [10].....	22
FIGURE 20: STRESS SYSTEMS IN A PROGRESSING STATE OF DIAGONAL TENSION	23
FIGURE 21: STRESS DEVELOPMENT IN DIAGONAL TENSION WEBS	24
FIGURE 22: GRAPHS FOR CALCULATING CRITICAL BUCKLING STRESS [10]	27
FIGURE 23: SHEAR MODULUS PLASTICITY CORRECTION FOR 2024-T3 ALUMINUM [10].....	28
FIGURE 24: NACA TYPE I-25 TEST BEAM (H = 26.1 IN.) [11].....	29
FIGURE 25: PORTAL FRAME ACTION [12].....	29
FIGURE 26: PLASTICITY CORRECTION FOR 2024-T3 AND 7075-T6 ALUMINUM [10]	31
FIGURE 27: ALLOWABLE VALUES OF NOMINAL WEB SHEAR STRESS [10]	32
FIGURE 28: RATIO OF MAXIMUM STRESS TO AVERAGE STRESS IN WEB STIFFENERS [10].....	33
FIGURE 29: FAILURE OF WEB STIFFENER BY FORCED CRIPPLING [10]	34
FIGURE 30: GRUMMAN TEST BEAM AND LOAD FRAME [14].....	37
FIGURE 31: WEB STIFFENER WITH LAND.	40
FIGURE 32: BOEING SHEAR TEST BEAM AND LOAD FRAME [16].	42
FIGURE 33: DATA SCATTER OF NACA TESTS AND INFLUENCE ON ALLOWABLE MATERIAL LIMITS [17].....	43
FIGURE 34: STIFFENER STRESS CONCENTRATION DUE TO BAY CONFIGURATION OF TEST SPECIMEN [15]	44
FIGURE 35: CRACK BRIDGING IN FIBRE METAL LAMINATES [21]	50
FIGURE 36: FML AND ALUMINUM CRACK GROWTH BEHAVIOR UNDER FUSELAGE LOADING [21]	50
FIGURE 37: NORMALIZED ABSORBED ENERGY VERSUS IMPACT ENERGY [22]	51
FIGURE 38: NACA TYPE I-25 WAGNER-BEAM TEST SPECIMEN [10].....	55
FIGURE 39: BASIC BAY LAYOUT OF THE TEST BEAM.....	55
FIGURE 40: ORIGINAL MARGIN OF SAFETY GRAPH FOR THE 0.050" ALUMINUM BEAM	58
FIGURE 41: NRC-IAR TEST BEAM.	59
FIGURE 42: IMAGE OF MAT PLATFORM.....	61
FIGURE 43: GENERAL TEST SET-UP	62
FIGURE 44: ANTI-TWIST SYSTEM.....	63
FIGURE 45: LOADING ARRANGEMENT	64
FIGURE 46: TEST BEAM #4 STRAIN GAUGE LOCATIONS	66
FIGURE 47: LVDT PLACEMENTS.....	66
FIGURE 48: PHOTOELASTIC COATING ON TEST BEAM #1	68
FIGURE 49: TYPICAL FLEXTES DISPLACEMENT PROFILE.....	71
FIGURE 50: PERMANENT DEFORMATION STRESSES FOR SHEAR BUCKLED SHEETS [7].....	77
FIGURE 51: TEST #1 STRAIN GAUGE LOCATIONS	80
FIGURE 52: DISPLACEMENT PROFILE FOR TEST #1	81

FIGURE 53: PHOTOELASTIC COATING RESULTS FOR TEST BEAM #1	81
FIGURE 54: TEST BEAM #1 AT -40,000 LBF. APPLIED LOAD	82
FIGURE 55: TEST #1 MAXIMUM WEB TENSION STRAINS	83
FIGURE 56: TEST #2 CENTRAL BAY STRAIN GAUGE LOCATIONS	84
FIGURE 57: TEST #2 SUPPORT END STRAIN GAUGE LOCATIONS	84
FIGURE 58: PHOTO-ELASTIC RESULTS FOR TEST BEAM #2	85
FIGURE 59: FAILURE OF TEST BEAM #2 AFTER AN APPLIED LOAD OF ~51,000LBF	86
FIGURE 60: TEST #2 MAXIMUM WEB TENSION STRAINS	86
FIGURE 61: TEST #2 SUPPORT END FLANGE STRAINS	87
FIGURE 62: FLANGE STRESS DISTRIBUTION	88
FIGURE 63: TEST #3 SET-UP WITH LEXAN SHIELDS	89
FIGURE 64: TEST#3 STRAIN GAUGE LOCATIONS	90
FIGURE 65: DISPLACEMENT PROFILE FOR TEST #3	90
FIGURE 66: TEST #3 MAXIMUM WEB TENSION STRAINS	91
FIGURE 67: TEST #3 SUPPORT END FLANGE STRAINS	92
FIGURE 68: TEST #4 MAXIMUM WEB TENSION STRAINS	93
FIGURE 69: TEST #4 SUPPORT END FLANGE STRAINS	94
FIGURE 70: GLARE 4 LAY-UP ERROR	94
FIGURE 71: PERMANENT BUCKLES IN BEAM #2	99
FIGURE 72: CRACK IN BEAM #2 FLANGE	100
FIGURE 73: LOCAL BUCKLING OF BEAM #3 FLANGE	100
FIGURE 74: MARGIN OF SAFETY GRAPH FOR THE CARLETON TEST BEAM	103
FIGURE 75: THE CARLETON MINI-BEAM	104
FIGURE 76: CARLETON MINI-BEAM TEST SET-UP	105
FIGURE 77: CARLETON MINI-BEAM STRAIN GAUGE LOCATIONS	106
FIGURE 78: CARLETON BEAM DEFLECTIONS	109
FIGURE 79: CARLETON BEAM FLANGE STRAINS	110
FIGURE 80: CARLETON BEAM MAXIMUM WEB TENSION STRAINS	111
FIGURE 81: DEFLECTION PROFILE OF TEST BEAM #1 AT 39,954 LBF.	114
FIGURE 82: VARYING CROSS-SECTIONS OF THE NRC TEST BEAM	114
FIGURE 83: VARYING CROSS-SECTIONS OF THE NRC TEST BEAM	115
FIGURE 84: RE-CALCULATED TEST #1 PREDICTED BEAM DEFORMATION UNDER VARIOUS LOADS	117
FIGURE 85: TEST #1 LVDT DISPLACEMENT READINGS	118
FIGURE 86: ORIGINAL PORTAL FRAME EFFECT PREDICTION FOR THE 0.050" ALUMINUM BEAM	120
FIGURE 87: ORIGINAL MARGIN OF SAFETY GRAPH FOR THE 0.050" ALUMINUM BEAM	120
FIGURE 88: TEST #1 VERTICAL SHEAR STRESS WITH ORIGINAL STRESS PREDICTION	121
FIGURE 89: REVISED PORTAL FRAME EFFECT PREDICTION FOR THE 0.050" ALUMINUM BEAM	122
FIGURE 90: REVISED MARGIN OF SAFETY GRAPH FOR THE 0.050" ALUMINUM BEAM	123
FIGURE 91: REVISED TEST #1 PRINCIPAL WEB STRAINS	124
FIGURE 92: LVDT'S IN THE SUPPORT MOUNT	126
FIGURE 93: LVDT WELDED TRUSS ARM CONFIGURATION	127
FIGURE 94: POSSIBLE SUPPORT MOUNT VS. BEAM ROTATION	128
FIGURE 95: SKETCH SHOWING STRAIN GAUGE LOCATIONS	129
FIGURE 96: PROGRAMMED FLEXTEST DISPLACEMENT PROFILE	131
FIGURE 97: MARGIN OF SAFETY GRAPH FOR THE 0.071" 2024-T3 TEST BEAM	132
FIGURE 98: TEST #5 LVDT DISPLACEMENT READINGS	133
FIGURE 99: TEST #5 LVDT SUPPORT MOUNT CORRECTION	134
FIGURE 100: TEST #1 CORRECTED LVDT DISPLACEMENTS	135
FIGURE 101: TEST #5 SUPPORT FLANGE STRAINS	136
FIGURE 102: TEST #1 0.050" 2024-T3 ALUMINUM PRINCIPAL STRAINS	137
FIGURE 103: TEST #2 GLARE 3 - 3/2 PRINCIPAL WEB STRAINS	138
FIGURE 104: TEST #4 GLARE 4 - 3/2 PRINCIPAL WEB STRAINS	138
FIGURE 105: EXPLODED VIEW OF THE NRC-IAR TEST BEAM	178
FIGURE 106: WEB STIFFENER PLACEMENT AND ORIENTATION	181
FIGURE 107: WEB STIFFENER CONSTRUCTIONS	181
FIGURE 108: GENERAL FASTENER ARRANGEMENTS	182

NOMENCLATURE

A_f	Area of flange
A_{stf}	Area of stiffener
A_{stfe}	Effective area of stiffener
b, d	Bay width
C_1	Angle factor
C_2	Web stress coefficient
C_3	Stiffener moment coefficient
C_{stf}	Maximum stiffener stress coefficient
d_c	Effective bay width (between fastener lines)
d_{land}	Land width
e	Eccentricity
E	Modulus of elasticity
E_e	Effective elastic modulus
E_f	Flange elastic modulus
E_{stf}	Stiffener elastic modulus
E_{web}	Web elastic modulus
F_c	Force on compressive flange
F_f	Diagonal tension force acting on flange
F_{Sall}	Nominal allowable web shear stress
F_{stf}	Force on stiffener
F_T	Force on tensile flange
G	Shear modulus
G_{DT}	Diagonal tension shear modulus
G_e	Effective shear modulus
G_{IDT}	Incomplete diagonal tension shear modulus
G_{PDT}	Pure diagonal tension shear modulus
G_{web}	Web shear modulus
h	Height
h_c, h_e	Effective bay height (between fastener inner lines)

h_{stf}	Height of stiffener
I	Second moment of inertia
I_C	Moment of inertia of compressive flange about beam neutral axis
I_n	Flange moment of inertia
I_T	Moment of inertia of tensile flange about beam neutral axis
I_{web}	Web moment of inertia
K, K_s	Buckling coefficient
k	Diagonal tension factor
K_{SC}	Stiffener stress concentration
k_{SS}	Simple support coefficient
L	Length
L_P	Portal frame length
M	Bending moment
M_{12}, M_{21}	Reaction moment
M_{mid}	Moment above mid-bay
M_{stf}	Moment above web stiffener
P	Applied load
q	Shear flow
Q	Static moment of inertia
Q_0	Load distribution
R_1, R_2	Reaction force
R_d	Restraint coefficient along flanges
R_h	Restraint coefficient along web stiffeners
t	Thickness
t_F	Flange thickness
t_{land}	Land thickness
t_{stf}, t_U	Web stiffener thickness
V	Shear load
W	Work
x	Horizontal distance
y	Distance from neutral axis

α	Diagonal tension angle
δ	Deflection
δ_1	Web deflection under unit load
δ_2	Flange deflection under unit load
δ_{bending}	Deflection due to bending
ϵ	Tensile strain
ϵ_{fl}	Flange strain
ϵ_{stf}	Web stiffener strain
γ	Shear strain
γ_{IDT}	Incomplete diagonal tension shear defection
ν	Poisson's ratio
ρ	Radius of gyration
σ	Stress
σ_c	Compressive stress
σ_{cy}	Compressive yield stress
$(\sigma_{\text{FC}})_D$	Force crippling allowable for double web stiffeners
$(\sigma_{\text{FC}})_S$	Force crippling allowable for single web stiffeners
σ_{fl}	Diagonal tension stress acting on flange
σ_{stf}	Stress on web stiffener
σ_{su}	Shear ultimate stress
σ_t	Horizontal stress
σ_t	Tensile stress
$\sigma_{t, \text{allow}}$	Allowable tensile yield stress
σ_{ty}	Tensile yield stress
σ_{uts}	Ultimate tensile stress
σ_y	Vertical stress
τ	Shear stress
τ', τ_{web}	Web shear stress with portal frame effect
τ_{allow}	Allowable shear yield stress

τ_{CR}	Critical shear stress
$\tau_{CR, \text{elastic}}$	Elastic critical shear stress
τ_{DT}	Diagonal tension shear stress
τ_{PS}	Pure shear stress
ωd	Flange flexibility factor

1. INTRODUCTION

Fibre-metal laminates (FMLs) are increasingly being used in the design of secondary and tertiary aircraft structural components, such as cargo bay floors, bulkheads, access hatches and wing flaps. However, the constant effort to make aircraft stronger and lighter has now led to a variety of studies to determine the feasibility of constructing primary aircraft structures using this new class of material. Foremost in these studies, is determining the potential applications of GLARE (GLAss REinforced aluminum): a laminate composed of alternating layers of thin aluminum sheets and unidirectional or cross-ply glass-fiber/epoxy (Figure 1).

Figure 1: Composition Lay-up of GLARE 3 – 3/2 [1]

As part of a larger ongoing collaborative FML project between the National Research Council Canada (NRC), Bombardier Aerospace and Carleton University, efforts are being made to understand the behaviour of GLARE in aircraft structures, and its construction properties.

In general, an aircraft structure is optimized to minimize its weight. This means that certain web areas of the fuselage and wing spars are allowed to undergo a state of elastic buckling, where visible folds that develop in the web material leave no permanent deformations in the over-all structure when the loads are removed (Figure 2). While in this buckled state, the aircraft skin is still capable of carrying load through tension-loaded bands of material located between the buckles. This condition is referred to as a state of diagonal tension.

Figure 2: Diagonal Tension in a Canadair CT-114 Tutor.

The diagonal tension theory has evolved since its conception from the late 1920's and has been enhanced by several aviation research and industrial establishments, to allow for various isotropic materials and manufacturing methods. However, it was not known if the calculation methods used to determine diagonal tension could be applied to orthotropic materials such as GLARE. Based on this concern, Bombardier and NRC established a research project to investigate the diagonal tension phenomenon in GLARE-web flat stiffened beams and stiffened cylinders.

This thesis presents the development and the initial results of the flat stiffened beam portion of the collaborative project. Provided is a literature review of the origins and evolution of the NACA-based diagonal tension theory, followed by a statement of the thesis objectives, and a description of the NRC specimen design. A description of the original test set-up and procedures is given, followed by an overview of the observations and changes made during the first tests, and resulting investigation to better understand the diagonal tension theory. Results of this investigation are discussed and recommendations are made for continuing work outside the time constraints of this thesis.

2. LITERATURE REVIEW

Based on analytical theories developed by Herbert Wagner in the late 1920's, NACA (National Advisory Committee for Aeronautics) researcher Paul Kuhn, in the mid-1930's to early-1950's, developed a semi-empirical design procedure to determine diagonal tension effects in aluminum aircraft structures. This semi-empirical method was later modified and expanded by several aviation companies such as Grumman, Boeing and Northrop to account for other materials.

Since then, an understanding of the problem has been developed for isotropic materials such as aluminum and titanium, as well as some quasi-isotropic composite materials. However, there is no clear indication of how these design methodologies will apply in predicting the allowable limits of the diagonal tension effects for orthotropic materials such as GLARE.

The following sections will detail the evolution of the NACA methodology for the diagonal tension phenomenon. A brief proof of the original Wagner and Kuhn theories will be given, followed by a history of additions and enhancements leading to the currently used semi-empirical methodology.

2.1 Shear Resistant Construction

In the pioneering days of aircraft design, the principles of civil and nautical engineering had a heavy influence on aeronautical engineering. Early metallic aircraft were often designed with shear resistant skins, over truss-like structures. It was assumed that the shear loads would be carried by the aircraft's skin (Figure 3), and that the compressive or tensile loads would be carried by the framing structure.

Figure 3: Bending stresses in an idealized wing structure

Under these assumptions, the buckling of the skin would mean the failure of the structure. Therefore, the structure would be designed to withstand loads up to the buckling stress of the material. The elastic buckling stress τ_{CR} for a thin rectangular plate in shear (with a Poisson's ratio of ~ 0.3) is calculated by the following equation [1]:

$$\tau_{CR} = K E \left(\frac{t}{b} \right)^2 \quad (2.1)$$

The term K , is a theoretically determined constant based on the boundary conditions of the plate: either simply supported, where the edge is free to rotate, but not allowed to deflect normal to the plane of the plate (Figure 4a), or fixed and fully clamped, where the edge is prevented from rotating or deflecting (Figure 4b). The term t is the thickness of the web material, and b is the width of the bay as shown in Figure 5.

Figure 4: Boundary conditions for a plate edge

Based on the edge conditions and the geometry of the web (or panel), the constant K can be determined from Figure 5.

Figure 5: Shear buckling coefficients [2]

Bending stresses through the panel cross-sections are calculated by Equation 2.2, where M is the bending moment in the panel, y is the distance from the neutral axis, and I is the second moment of inertia of the panel.

$$\sigma = \frac{M y}{I} \quad (2.2)$$

Shear stresses through the web cross-section are calculated by Equation 2.3, where V is the shear load acting through the panel, Q is the static moment of inertia of the panel, and t is the thickness at the viewed section.

$$\tau = \frac{q}{t} = \frac{V Q}{I t} \quad (2.3)$$

2.2 Pure Diagonal Tension (Wagner Theory)

In 1929, German Professor Herbert Wagner realized that the aircraft design methods of the time were too conservative, and introduced the concept of diagonal tension, a theory he developed while working for the Rohrbach Metal Airplane Company. Using conventional solid mechanics analysis he showed that, as a shear load is applied to a structure, the buckling only removes the compressive load carrying capacity of the web material. Wagner's original proof for his theory ([3], [4], and [5]) is convoluted. However, other publications and reports, such as Kuhn's summary of Wagner's method [6] and various aircraft structures texts (such as [2], [7], and [8]), provide a clearer description of Wagner's ideas.

2.2.1 Principles of Wagner Theory

Under a shear load, Wagner envisioned that a single web bay would react no differently than a simplified truss structure as shown in Figure 6a, with two diagonal struts A and B . After loading the structure by the load P , strut A is loaded in tension while strut B has buckled and failed under the compressive load (Figure 6b). Applying this concept to the stresses in a web panel, what remains is a uniform tension field working in a diagonal direction, without any compressive stresses (Figure 6c).

Figure 6: The Principle of Diagonal Tension

The stresses in the web, flanges and stringers can then be determined as follows: Consider the multi-bay cantilever beam in Figure 7 (which represents an idealized flat fuselage section or wing spar), with height h , bay width d and a web skin thickness t . As the load P is applied to end of the beam, after buckling has occurred the diagonal tension stresses in the web are developed at an angle α from the bottom flange.

Figure 7: Simplified multi-bay beam under shear

As shown in Figure 7, it can be assumed that the vertical shear force V is equal to the load applied.

$$V = P \quad (2.4)$$

Considering that the flanges contribute little to the shear resistance, the average shear stress τ in the web skin can be estimated as:

$$\tau = \frac{P}{ht} \quad (2.5)$$

To determine the diagonal stress in the web, let us investigate a section (Figure 8) taken from the centre of one of the bays: a diagonally positioned rectangular element 1234, angled parallel to the diagonal web stress σ_t , with shear force τ acting in the vertical direction along line 35.

Figure 8: Diagonal web stress element

A force balance is taken between the shear load along the line 35, and the diagonal tension load along the line 34:

$$\sigma_t \cdot (\overline{34} \cdot t) \cdot \sin \alpha = \tau \cdot (\overline{35} \cdot t) \quad (2.6)$$

Rearranging the terms, we can find the diagonal tension stress in the web material:

$$\sigma_t = \frac{\tau}{\sin \alpha} \cdot \left(\frac{\overline{35}}{\overline{34}} \right) = \frac{\tau}{\sin \alpha \cos \alpha} \quad (2.7)$$

Which can be simplified to the final equation:

$$\sigma_t = \frac{2\tau}{\sin 2\alpha} \quad (2.8)$$

Considering the same element, this diagonal tension stress can also be translated into a horizontal stress component σ_x , as illustrated in Figure 9:

Figure 9: Horizontal web stress element

A force balance is taken between the diagonal tension load along the line 34, and the horizontal load along the line the line 35:

$$\sigma_x \cdot (\overline{35} \cdot t) = \sigma_t \cdot (\overline{34} \cdot t) \cdot \cos \alpha \quad (2.9)$$

Rearranging the terms, we can find the horizontal stress in the web material:

$$\sigma_x = \sigma_t \cdot \cos \alpha \cdot \left(\frac{\overline{34}}{\overline{35}} \right) = \sigma_t \cdot \cos^2 \alpha \quad (2.10)$$

Substituting in the value of σ_t from equation (2.8), the horizontal web stress can be simplified and expressed as:

$$\sigma_x = \frac{\tau}{\tan \alpha} \quad (2.11)$$

This horizontal web stress has an influence on the loads carried by the beam's framing structure. Tensile and compressive forces in the flanges (F_T , F_C) are a combined result of the bending stresses of the beam and the tensile stress σ_x caused by the buckled web. To analyze this reaction, we consider a cross-section taken at distance x along the beam from the load introduction point, as shown in Figure 10.

Figure 10: Flange force balance

To find the force F_C in the compression-bearing flange, a moment balance is taken from the point of the cross-sectional force in the tensile flange:

$$Px = F_C h - \frac{\sigma_x t h^2}{2} \quad (2.12)$$

Rearranging the terms, the force in the compression flange is:

$$F_C = \frac{Px}{h} + \frac{\sigma_x t h}{2} \quad (2.13)$$

Substituting in the value of σ_x from equation (2.11), the force in the compression-bearing flange can be simplified and expressed as:

$$F_c = \frac{Px}{h} + \frac{P}{2 \tan \alpha} \quad (2.14)$$

A force balance is then performed to determine the reaction F_T in the tensile flange:

$$F_T - F_c + \sigma_x t h = 0 \quad (2.15)$$

Resolving this balance gives the force in the tension-bearing flange as:

$$F_T = \frac{Px}{h} - \frac{P}{2 \tan \alpha} \quad (2.16)$$

In both (2.14) and (2.16), the force component on flanges caused by diagonal tension is:

$$F_f = -\frac{P}{2 \tan \alpha} \quad (2.17)$$

The stress component in the flanges caused strictly by the diagonal tension field in the web is therefore:

$$\sigma_f = -\frac{\tau h t}{2 A_f \tan \alpha} \quad (2.18)$$

By rearranging element 1234, seen in Figure 11, we can now determine the vertical web stress component σ_y caused by the diagonal tension field.

Figure 11: Vertical web stress element

A force balance is taken between the diagonal tension load along the line 34, and the vertical load along the line 46:

$$\sigma_y \cdot (\overline{46} \cdot t) = \sigma_t \cdot (\overline{34} \cdot t) \cdot \sin \alpha \quad (2.19)$$

Rearranging the terms, we can find the vertical stress in the web material:

$$\sigma_y = \sigma_t \cdot \sin \alpha \cdot \left(\frac{\overline{34}}{\overline{46}} \right) = \sigma_t \cdot \sin^2 \alpha \quad (2.20)$$

Substituting in the value of σ_t from equation (2.8), the vertical web stress can be simplified and expressed as:

$$\sigma_y = \tau \tan \alpha \quad (2.21)$$

As with the horizontal web stress, the vertical web stress has an influence on the loads carried by the beam flanges and web stiffeners. The tensile nature of the vertical web stress means that the upper and lower flanges are being pulled together, causing a compressive load on the web stiffeners. This effect is extended over the entire length of the beam, where the bay width d , seen in Figure 12 is represented by the length L of the beam from Figure 13.

Figure 12: Web stiffener force balance

To calculate the additional frame stresses, the following assumptions are made:

- The web stiffeners are rigid.
- The flanges act as continuous beams supported by the web stiffeners.
- The ends of the flanges, between the web stiffeners, are fixed and locked against any rotation.

Under these assumptions, it can be shown that the moment and end reactions would be similar to the beam shown in Figure 13.

$$M_{12} = -M_{21} = \frac{q_0 L^2}{12} \quad (2.22)$$

$$R_1 = R_2 = -\frac{q_0 L}{2} \quad (2.23)$$

Figure 13: Fixed-end prismatic beam reactions under uniformly distributed load [9]

For the web stiffeners in the middle of the beam, this means that the reacting compressive load F_{stf} supports half the forces induced by each adjacent bay.

$$F_{stf} = \sigma_y \cdot (t \cdot d) \quad (2.24)$$

Substituting in the value of σ_y from equation (2.21), the force in the web stiffener can be simplified and expressed as:

$$F_{stf} = \tau t d \tan \alpha \quad (2.25)$$

The stress in the web stiffeners is determined by dividing the force in the stiffener by its effective cross-sectional area, defined below.

$$\sigma_{stf} = \frac{F_{stf}}{A_{stf,e}} \quad (2.26)$$

For beams with a single stiffener construction, having only one face of the web supported, causes an imbalance in the load introduction into the stiffeners (Figure 14a). This means that the effective area of the supporting stiffener is reduced.

Figure 14: Single and double web stiffener constructions

The effective area is thus determined by dividing the original area of the stiffener by a ratio between the eccentricity e (the distance between the neutral axis of the stiffener and the web material) and the radius of gyration ρ of the stiffener's cross-section.

$$A_{sf_e} = \frac{A_{sf}}{1 + \frac{e^2}{\rho^2}} \quad (2.27)$$

Beams with a double stiffener construction (Figure 14b) are assumed to have a balanced loading between the two sides of the web, therefore the effective area of the stiffeners is equal to the original area of the stiffener:

$$A_{sf_e} = A_{sf} \quad (2.28)$$

With the effective area of the web stiffener calculated, and by substituting equation (2.25) into (2.26), the stress in the stiffener can then be determined.

$$\sigma_{sf} = \frac{\tau d t \tan \alpha}{A_{sf_e}} \quad (2.29)$$

2.2.2 Secondary Bending Stresses

Due to the vertical stresses (σ_y) exerted by the web, the flanges of the beam are affected by two bending moments. The primary bending moment (Figure 15a) is distributed over the entire length and depth of the beam, due to the applied load. The secondary bending moment (Figure 15b) affects only each individual flange and is due to the diagonal tension web stresses pulling the flanges together between the web stiffeners.

Figure 15: Flange bending moments

Based on the earlier assumption that the flanges at the web stiffeners are fixed and locked against any rotation (as was shown in Figure 13) the secondary moment can be viewed as a series of fixed-end beams equal to the number of individual bays (Figure 16).

Figure 16: Flange secondary bending moments [1]

The moment M_{sf} above the web stiffeners can be calculated by entering the appropriate values into equation (2.21).

$$M_{sf} = \frac{(\sigma_y \cdot t) \cdot d^2}{12} \quad (2.30)$$

Substituting in the value of σ_y from equation (2.20), the secondary moment at the web stiffener can be expressed as:

$$M_{sf} = \frac{\tau t d^2 \tan \alpha}{12} \quad (2.31)$$

The moment M_{mid} at the mid-bay, is half the value of the stiffener moment and is of opposing magnitude.

$$M_{mid} = -\frac{\tau t d^2 \tan \alpha}{24} \quad (2.32)$$

2.2.3 Diagonal Tension Angle

Up until this point the diagonal tension angle α , has been an arbitrary value in all of the equations. To predict the maximum stress values in the beam α should ideally be 45° , which would simplify all the equations shown. However, an attempt to better approximate this angle can be made by considering the internal energy balance. For a single bay in a beam, the internal work can be expressed by the equation:

$$W = \frac{\sigma_t^2}{2 E_{web}} d h t + \frac{\sigma_{sf}^2}{2 E_{web}} A_{sf} h + \frac{\sigma_f^2}{2 E_{web}} A_f d \quad (2.33)$$

Substituting in the values, from equation (2.8), (2.17) and (2.27), the work equation can be expressed as follows:

$$W = \frac{d t h \tau^2}{2 E_{web}} \left(\frac{4}{\sin^2 2\alpha} + \frac{d t}{A_{sf}} \tan^2 \alpha + \frac{t h}{4 A_f} \frac{1}{\tan^2 \alpha} \right) \quad (2.34)$$

This work equation is then differentiated with respect to α

$$\frac{dW}{d\alpha} = \frac{d t h \tau^2}{2 E_{web}} \left(8 \frac{\cos 2\alpha}{\sin^3 2\alpha} + \frac{d t}{A_{sff_c}} \frac{\sin \alpha}{\cos^3 \alpha} - \frac{t h}{2 A_f} \frac{\cos \alpha}{\sin^3 \alpha} \right) \quad (2.35)$$

Setting (2.33) to zero and simplifying, results in the following:

$$0 = 8 \tau \frac{\cos 2\alpha}{\sin^3 2\alpha} + \frac{\tau d t}{A_{sff_c}} \frac{\sin \alpha}{\cos^3 \alpha} - \frac{\tau t h}{2 A_f} \frac{\cos \alpha}{\sin^3 \alpha} \quad (2.36)$$

This equation can now be rearranged in the following manner:

$$0 = \left(\frac{2 \tau}{\sin 2\alpha} \right) \frac{4 \cos 2\alpha}{\sin^2 2\alpha} + \left(\frac{\tau d t \tan \alpha}{A_{sff_c}} \right) \frac{1}{\cos^2 \alpha} - \left(\frac{\tau t h}{2 A_f \tan \alpha} \right) \frac{1}{\sin^2 \alpha} \quad (2.37)$$

The expressions inside the brackets can be replaced by the stress values of equation (2.8), (2.17) and (2.27):

$$0 = (\sigma_t) \frac{4 \cos 2\alpha}{\sin^2 2\alpha} + (\sigma_{sff}) \frac{1}{\cos^2 \alpha} - (\sigma_f) \frac{1}{\sin^2 \alpha} \quad (2.38)$$

From which, the expression for α can be obtained:

$$\tan^2 \alpha = \frac{\sigma_t - \sigma_f}{\sigma_t - \sigma_{sff}} \quad (2.39)$$

Substituting in the values, from equation (2.8), (2.17) and (2.27), the diagonal tension angle can be expressed in terms of the beam's geometry:

$$\tan^4 \alpha = \frac{1 + \frac{h t}{2 A_f}}{1 + \frac{d t}{A_{sff_c}}} \quad (2.40)$$

Ideally α is assumed to be 45° , to maximize the shear and tension effects in the web. But in practice, most aerospace structures produce diagonal tension angles around 40° .

2.2.4 Diagonal Tension Shear Modulus

After the beam has initially buckled and diagonal tension field has developed, the web material does not have the same shear carrying capacity as it did when it was a flat sheet. Using a similar process for calculating the internal energies of the web material, as was done to determine the diagonal tension angle, the shear modulus G_{PDT} for pure diagonal tension can be determined.

$$G_{PDT} = \frac{E_{web}}{\frac{4}{\sin^2 2\alpha} + \frac{d t \tan^2 \alpha}{A_{stf}} + \frac{h t \cot^2 \alpha}{2 A_f}} \quad (2.41)$$

Put in terms of the resulting stresses, this equation can also be expressed as:

$$G_{PDT} = \frac{E_{web}}{2 \left(2 - \frac{\sigma_f}{\sigma_t} - \frac{\sigma_{stf}}{\sigma_t} \right)} \quad (2.42)$$

2.2.5 Effects of Flange Flexibility

Wagner did not just leave his theories on paper. A few empirically based corrections were made after preliminary tests in Germany and were supported by further tests conducted for NACA in the early 1930's. These corrections accounted for flexibility in the flanges since they would tend to deform between the web stiffeners, as shown in Figure 17.

Figure 17: Vertical web stresses and flange flexibility

As the flanges deformed, the maximum tensile stress in the web would increase, but the deformation would also reduce the secondary moment developed over the web stiffeners. To account for these reactions, Wagner developed the flange flexibility factor ωd .

$$\omega d = d \sin \alpha \sqrt{\left(\frac{1}{I_T} + \frac{1}{I_C} \right) \frac{t}{4h}} \quad (2.43)$$

Where, I_C and I_T are the moments of inertia of the compressive and tensile flanges, about the neutral axis of the beam. Several tests that Wagner conducted showed that most beams typical of aircraft structures formed their diagonal tension field at angles greater than 40° . With this in mind, the flange flexibility factor was simplified to the following equation:

$$\omega d \approx 0.7d \sqrt{\frac{t}{h(I_T + I_C)}} \quad (2.44)$$

Once the flange flexibility was determined, the correction coefficients were cross-referenced to the empirically determined graph in Figure 18.

Figure 18: Web stress and secondary moment correction coefficients [10]

These coefficients would then be applied into the following modified equations:

$$\sigma_t = (1 + C_2) \cdot \frac{2\tau}{\sin 2\alpha} \quad (2.45)$$

$$M_{sf} = C_3 \cdot \frac{\tau t d^2 \tan \alpha}{12} \quad (2.46)$$

2.3 Incomplete Diagonal Tension (NACA Theory)

Paul Kuhn, a researcher at NACA, followed the theories developed by Wagner, and began more extensive studies into diagonal tension during the early 1930's. By the early 1940's, he had determined that Wagner's methods produced conservative results when used in the design of aircraft structures. He found that the buckled web of an aircraft beam was still capable of carrying compressive loads, and that this ability diminished as the shear load was increased. This development prompted another decade of work to analyze and confirm this theory (Figure 19). Kuhn summarized this body of work by May of 1952, describing the basic theory [10] and the experimental evidence

used to support his theories [11]. The following section will outline the basics of his methodology.

Figure 19: NACA Type I-25 Test Beam [10]

2.3.1 Principles of NACA Theory

The diagonal tension field does not develop until after a critical shear level has been reached and the beam has initially buckled. Therefore, the shear stress driving diagonal tension τ_{DT} would be calculated by the total shear applied τ less the critical shear value τ_{CR} required to buckle the beam.

$$\tau_{DT} = \tau - \tau_{CR} = \tau \left(1 - \frac{\tau_{CR}}{\tau} \right) \quad (2.47)$$

This meant that the total shear introduced into a beam could be divided between the proportion k of shear influencing diagonal tension, and the remaining proportion acting in a state of pure shear τ_{PS} . The proportion k , is known as the diagonal tension factor.

$$\tau = \tau_{DT} + \tau_{PS} \quad (2.48)$$

$$\tau_{DT} = k\tau \quad ; \quad \tau_{PS} = (1-k)\tau \quad (2.49)$$

Therefore, as a beam is loaded in shear, it progresses through three different states. When load is first applied, the beam reacts as a shear resistant structure, until the initial buckling occurs (Figure 20a). Until buckling occurs, k is assumed to be zero. As the shear load increases beyond the buckling point, then the level of diagonal tension begins to steadily increase (Figure 20b). When the shear load is high enough to make the pure shear in the beam insignificant then a state of pure diagonal tension has been reached (Figure 20c).

Figure 20: Stress systems in a progressing state of diagonal tension

Taken from equation (2.47), the diagonal tension factor can be calculated as:

$$k = 1 - \frac{\tau_{CR}}{\tau} \quad (2.50)$$

After multiple tests, Kuhn expressed the diagonal tension factor by the empirical equation:

$$k = \tanh\left(0.5 \cdot \log_{10} \frac{\tau}{\tau_{CR}}\right) \quad (2.51)$$

Equation (2.50) and (2.51) imply that it is impossible to reach a pure diagonal tension state. The highest levels of diagonal tension are only possible when the applied shear stress is several hundred times greater than the critical shear stress. Most aircraft structures would be expected to fail before this loading ratio reached 60 to 80. This means that most aircraft structures under shear load will fail while in a state of incomplete diagonal tension. From this principle, stresses in the web skin element can be broken down into those caused by the pure shear reaction (σ_{PS}), and those developed under diagonal tension (σ_{DT}).

$$\sigma_{DT} = \frac{2k\tau}{\sin 2\alpha} \quad ; \quad \sigma_{PS} = (1-k)\tau \sin 2\alpha \quad (2.52)$$

The combination of these two sets of stresses results in both tensile and compressive reactions within the web material

$$\sigma_t = \sigma_{DT} + \sigma_{PS} \quad ; \quad \sigma_c = -\sigma_{PS} \quad (2.53)$$

Substituting in the values of equation (2.32) into (2.33), we get the tensile and compressive stresses in the web.

$$\sigma_t = \frac{2k\tau}{\sin 2\alpha} + (1-k)\tau \sin 2\alpha \quad (2.54)$$

$$\sigma_c = -(1-k)\tau \sin 2\alpha \quad (2.55)$$

Figure 21 illustrates the combination of stresses, from Diagonal Tension (DT) and Pure Shear (PS) components, for developing the stress in Incomplete Diagonal Tension (IDT).

Figure 21: Stress development in diagonal tension webs

These changes in the web stresses carry through and change the stresses experienced by the flanges and web stiffeners.

$$\sigma_{sf} = \frac{k \tau \tan \alpha}{\frac{A_{sf}}{dt} + 0.5 \cdot (1 - k)} \quad (2.56)$$

$$\sigma_f = \frac{k \tau \cot \alpha}{\frac{2 A_f}{ht} + 0.5 \cdot (1 - k)} \quad (2.57)$$

2.3.2 Strain-Based Diagonal Tension Angle

Kuhn modified Wagner's diagonal tension angle equation (2.39), by replacing the stress terms with those of strain:

$$\tan^2 \alpha = \frac{\epsilon - \epsilon_f}{\epsilon - \epsilon_{sf}} \quad (2.58)$$

The strain values for the web tension ϵ , flanges ϵ_f and web stringers ϵ_{sf} can be expressed as:

$$\epsilon_f = \frac{\sigma_f}{E_{web}} \quad ; \quad \epsilon_{sf} = \frac{\sigma_{sf}}{E_{web}} \quad ; \quad \epsilon = \frac{(\sigma_t - \nu \sigma_c)}{E_{web}} \quad (2.59)$$

Substituting in values from equations (2.54), (2.55), (2.56), and (2.57), provides the following expressions:

$$\varepsilon = \frac{\tau}{E} \left[\frac{2k}{\sin 2\alpha} + (1-k)(1+\nu)\sin 2\alpha \right] \quad (2.60)$$

$$\varepsilon_{sf} = \frac{k \tau \tan \alpha}{E \left[\frac{A_{sf}}{d t} + 0.5 \cdot (1-k) \right]} \quad (2.61)$$

$$\varepsilon_f = \frac{k \tau \cot \alpha}{E \left[\frac{2 A_f}{h t} + 0.5 \cdot (1-k) \right]} \quad (2.62)$$

The calculation of the diagonal tension angle α now becomes an iterative process. An initial value of α is selected using 40° , 45° , or by calculation using equation (2.40). The strains are then calculated using equation (2.60), (2.61), and (2.62) and then entering the values into equation (2.58) to find a new value of α . The value of α is valid when the angle used to calculate the strains equals the α calculated from (2.58).

2.3.3 Critical Buckling Shear Stress

Kuhn expanded the expression for the shear required to initially buckle the beam structure. This equation takes into account the thickness t of the web and the effective height h_c and effective width d_c of the bay section as defined by the constraining rivet or bond-lines of the framing structure.

$$\tau_{CR,elastic} = k_{SS} E_{web} \left(\frac{t}{d_c} \right)^2 \left[R_h + \frac{1}{2} (R_d - R_h) \left(\frac{d_c}{h_c} \right)^3 \right], \quad \text{if } (d_c < h_c) \quad (2.63a)$$

$$\tau_{CR,elastic} = k_{SS} E_{web} \left(\frac{t}{h_c} \right)^2 \left[R_d + \frac{1}{2} (R_h - R_d) \left(\frac{h_c}{d_c} \right)^3 \right], \quad \text{if } (d_c > h_c) \quad (2.63b)$$

Equations (2.63) contains three empirical constants: the Simple Support Coefficient k_{ss} , and the two Restraint Coefficients, which describe the boundary conditions along the edges of the web stiffeners R_h , and the edges of the flanges R_d . These values are determined from the empirical data shown in Figure 22.

Figure 22: Graphs for calculating critical buckling stress [10]

2.3.4 Effective Shear Modulus

When buckling occurs within the web, the effective stiffness of the material decreases as the diagonal tension factor k increases. This effect is determined by calculating the influence on the shear modulus by the state of diagonal tension. This shear modulus of diagonal tension, G_{DT} , is based on the geometry of the beam section and the elastic modulus of the web material. This equation is a modified version of pure diagonal tension shear modulus (2.41).

$$G_{DT} = \frac{E_{web}}{\frac{4}{\sin^2 2\alpha} + \frac{\tan^2 \alpha}{\frac{A_{sf}}{dt} + 0.5(1-k)} + \frac{\cot^2 \alpha}{\frac{2A_f}{ht} + 0.5(1-k)}} \quad (2.64)$$

This is then converted into the shear modulus of incomplete diagonal tension, G_{IDT} , through the ratio:

$$G_{IDT} = \frac{G_{web}}{1 + k \left(\frac{G_{web}}{G_{DT}} - 1 \right)} \quad (2.65)$$

The above modulus is only valid within the elastic limits of the web material. Once the web starts to yield the effective shear modulus must be adjusted to account for the plastic deformation of the material. The NACA references only provide the plasticity correction for 24S-T3 aluminum, now called 2024-T3 (Figure 23). This data was compiled from unpublished sources from several U.S. aeronautical companies of the 1930's and 40's.

Figure 23: Shear Modulus Plasticity Correction for 2024-T3 Aluminum [10]

2.3.5 Portal Frame Effect

NACA's standard test beam construction consisted of a sheet of web material framed by large flanges and divided into bays, by web stiffeners. The end-bays of the test beam were either doubled or tripled in web thickness to allow for a proper development of the shear flow (Figure 24).

Figure 24: NACA Type I-25 test beam ($h = 26.1$ in.) [11]

During the early experiments, it was discovered that the large flanges used to frame the test beams, were absorbing some of the shear load away from the web material. The concept of "portal frame effect" was developed to account for this loss in shear load. The portal of a diagonal tension beam is defined within the length L_p of the test section between the thickened end-bays (Figure 25).

Figure 25: Portal Frame action [12]

Within this section, it is assumed that the deflections of the flanges and web are determined independently of each other. In this assumption, the flanges carry all the bending stresses and the web carries all the shear stresses. From this, the deflection δ_1 of the web under a unit shear load is expressed as:

$$\delta_1 = \frac{L_p}{h_e t G_{IDT}} \quad (2.66)$$

The deflection δ_2 of the flanges (or portal frame) under a unit of applied load is determined by using the elastic modulus E_f and the moment of inertia I_f for one flange as if it were independent from the rest of the structure:

$$\delta_2 = \frac{L_p^3}{24 E_f I_f} \quad (2.67)$$

These two terms are then combined in a ratio to express the difference between the total shear stress introduced into the beam as a whole τ , and the shear stress absorbed by the web alone τ' .

$$\frac{\tau'}{\tau} = \frac{1}{1 + \frac{\delta_1}{\delta_2}} \quad (2.68)$$

Substituting (2.66) and (2.67) into (2.68) provides the equation:

$$\left(\frac{\tau'}{\tau} \right)_{PFE} = \frac{1}{1 + \frac{24 E_f I_f}{L_p^2 h_e t G_{IDT}}} \quad (2.69)$$

This equation is used as a rough approximation to account for the shear load carried by the flanges, but is considered unreliable for predicting the strength limits of a beam design beyond the yielding point of the web. For reliable but conservative strength

estimates, it is better to calculate the shear load distribution by using standard shear flow calculations.

2.3.6 Web Stress Empirical Corrections

During testing it was found that empirical adjustments had to be made to the base theory to properly predict the stresses within the web material. This included a plasticity correction for the critical shear stress, a calculation for determining the maximum shear developed in the web, and a maximum stress allowable used to determine the failure of the web material.

In most cases, beams that develop diagonal tension are thin enough that the critical buckling shear stress occurs within the elastic limits of the web material. However, it appears that NACA developed corrections for the case that a beam might contain a web thick enough to yield prior to buckling. Empirical data was collected to correct for plasticity in 2024-T3 aluminum (24S-T3) and Alclad 7075-T6 aluminum (75S-T6) alloys (Figure 26).

Figure 26: Plasticity Correction for 2024-T3 and 7075-T6 Aluminum [10]

By equation (2.5) it is assumed that a uniform stress is developed across the web material. In reality, this just represents an average of stresses in the web, ranging from a minimum value at the flange boundary, to a maximum in the centre of the web. The maximum shear stress in the web is calculated by a modified version of equation (2.45):

$$\tau_{\max} = \tau (1 + k^2 C_1) (1 + k C_2) \quad (2.70)$$

As in (2.45), the constant C_2 is found by following the Wagner method of calculating the flange flexibility factor ω using (2.44) and using the graph in Figure 18. C_1 is determined by using the following equation:

$$C_1 = \frac{1}{\sin 2\alpha} - 1 \quad (2.71)$$

Allowable shear stresses, which predict the stress limit for web failure, were empirically determined and were based on the style of flange attachment to the web. Graphs for 75-T6 and 24-T3 (shown in Figure 27) were created by NACA.

Figure 27: Allowable Values of Nominal Web Shear Stress [10]

2.3.7 Web Stiffener Empirical Corrections

As with the web material, empirical corrections were made to account for failures of, and stresses in the web stiffeners. This includes the maximum stress developed in the stiffener, the correction to the column buckling stress, and stress allowable for localized buckling effects.

The maximum stress in the web stiffeners (or uprights, as referred to in the NACA reports) is determined by the ratio of the bay width d and the height of the web stiffeners h_{sf} , plotted for different values of the diagonal tension factor k developed in the web (Figure 28).

Figure 28: Ratio of Maximum Stress to Average Stress in Web Stiffeners [10]

This graph can be expressed by the equation:

$$C_{sf} = \frac{\sigma_{sf_{max}}}{\sigma_{sf}} = 0.64k \left(\frac{d}{h_{sf}} \right) - 0.775k - 0.64 \left(\frac{d}{h_{sf}} \right) + 1.775 \quad (2.72)$$

The maximum stress is then determined by multiplying the stress concentration with the stiffener stress determined by equation (2.72).

$$\sigma_{stf_{max}} = C_{stf} \sigma_{stf} \quad (2.73)$$

Another concern with the web stiffeners is the possibility of forced crippling. This occurs when the force generated by the shear buckles in the web causes the attached legs of the web stiffeners to fail by local buckling (Figure 29).

Figure 29: Failure of web stiffener by forced crippling [10]

Single web stiffener designs have a higher tolerance against forced crippling due to the reduced resisting stiffness when compared against the clamped effect of the double stiffener design. A double stiffener has higher out-of-plane stresses concentrating between the rivet pitches. To predict the forced crippling allowables, NACA empirically determined equations for two types of material:

For stiffeners made from 2024-T3 aluminum, the following equations are used:

$$(\sigma_{FC})_D = (21000 \text{ psi}) k^{\frac{2}{3}} \left(\frac{t_{sf}}{t} \right) \quad \text{for double stiffeners} \quad (2.74)$$

$$(\sigma_{FC})_S = (26000 \text{ psi}) k^{\frac{2}{3}} \left(\frac{t_{sf}}{t} \right) \quad \text{for single stiffeners} \quad (2.75)$$

For stiffeners made from 7075-T6 aluminum the following equations are used:

$$(\sigma_{FC})_D = (26000 \text{ psi}) k^{\frac{2}{3}} \left(\frac{t_{sf}}{t} \right) \quad \text{for double stiffeners} \quad (2.76)$$

$$(\sigma_{FC})_S = (32500 \text{ psi}) k^{\frac{2}{3}} \left(\frac{t_{sf}}{t} \right) \quad \text{for single stiffeners} \quad (2.77)$$

2.4 Post-NACA Developments

Since the NACA research, several more attempts have been made to analyze the diagonal tension phenomenon. Some follow the NACA methodology while others develop their theories by different methods.

2.4.1 The Grumman Modifications

In November of 1968, Robert T. Ratay, an employee of the Grumman Aircraft Engineering Corporation, released a technical memorandum [13] listing modifications to the diagonal tension theory. This memo represented the beginning of a limited attempt to streamline and extend the Kuhn's methodology, to handle other materials and constructions not considered or tested by NACA. This project was a result of a contract issued by NASA's Manned Spacecraft Centre, and was prompted by changes in aircraft design philosophy and manufacturing techniques. Key to this was the development of chemical milling, a technique, which had been used extensively to reduce the weight of the Orbiting Astronomical Observatory (OAO) and the Lunar Module (LM) spacecraft. Research was done on eighteen flat test specimens representative of the current spacecraft construction techniques [14]. Specimens were made of very thin (0.028", 0.009" and 0.005"), chemically milled 7076-T6 aluminum alloy web sheets with 7075-T6 aluminum web stiffeners. Four of these specimens were fatigue tested, and fourteen specimen were statically loaded to refine the diagonal tension theory (Figure 30).

1. Test panel
2. Tie-down to floor
3. Loading arm
4. Counter weight for loading arm
5. Lateral support for loading arm
6. Lateral supports for test panel

Figure 30: Grumman Test Beam and Load Frame [14]

For the type of test specimens being used, Grumman identified that the NACA method was typically conservative by 16% for web failures, and 13% for localized stiffener failures, but did correctly determine if either a web or stiffener failure would occur. Determining column failure of the stiffeners was still considered to be too conservative, but no attempt was made to modify this prediction method. Grumman's conclusion was that the NACA method, with modifications, was adequate for determining the ultimate load capacity of a stiffened beam design, with an acceptable built-in conservatism. With this in mind, Grumman made a few additions and changes to the NACA equations, as shown in the following sections.

2.4.1.1 Initial Buckling

Grumman was not convinced that the NACA calculation properly accounted for the flexural and rotational stiffness of the web stiffeners, so a different method was used for determining the initial buckling shear stress. Using data from studies conducted by K. C. Rockey, and I. T. Cook, Grumman developed a new series of empirical data curves [13] to determine the buckling coefficient K_S used in the standard formula.

$$\tau_{CR} = \frac{K_S \pi^2}{12(1-\nu^2)} \cdot E \left(\frac{t}{h_c} \right)^2 \quad (2.78)$$

Grumman appears to be the only reference to use this methodology, as later references still show preference the NACA calculation, but with modifications to determining the value of K_S . In general, the Grumman methods used for calculating τ_{CR} requires interpolation of the data between graphs and is cumbersome at best.

2.4.1.2 Diagonal Tension Angle

For determining α , Grumman recognized that the NACA method was a cumbersome trial and error process. Their solution was to develop new graphs [13] for specific beam geometries and to develop their own computer software to calculate the angle.

2.4.1.3 Allowable Shear Stress

Allowable shear stress curves in the NACA work were only provided to determine the critical failure of 2024-T3 and 7075-T6 Alclad webs. However, Grumman chose to define web failure based on a yielding criterion, and developed a new set of

generic equations to predict the potential tensile or shear yielding of any isotropic web material:

$$\sigma_{t,allow} = 0.9 \sigma_{ty} \left[1 + \frac{1}{2} \left(\frac{\sigma_{us}}{\sigma_{ty}} - 1 \right)^2 \right] \left[\frac{1}{2} + (1-k)^3 \left(\frac{\sigma_{su}}{\sigma_{ty}} - \frac{1}{2} \right) \right] \quad (2.79)$$

$$\tau_{allow} = 0.9 \sigma_{su} \left(\frac{\sigma_{ty}}{\sigma_{us}} \right) \left[1 + \frac{1}{2} \left(\frac{\sigma_{us}}{\sigma_{ty}} - 1 \right)^2 \right] \left[\frac{1}{2} + (1-k)^3 \left(\frac{\sigma_{su}}{\sigma_{ty}} - \frac{1}{2} \right) \right] \quad (2.80)$$

2.4.1.4 Allowable Web Stiffener Stress Against Forced Crippling

NACA equations are only valid for stiffeners made from 2024-T3 and 7075-T6, so new equations were developed to handle other materials. For single stiffener constructions:

$$(\sigma_{FC})_S = 26 \sigma_{cy} k^{\frac{2}{3}} \left(\frac{t_{stf}}{t} \right)^{\frac{1}{3}} \left(\frac{0.00182}{\sqrt{\frac{\sigma_{cy}}{E_c} + 0.002}} \right) \quad (2.81)$$

For double stiffener constructions:

$$(\sigma_{FC})_D = \frac{21}{26} (\sigma_{FC})_S \quad (2.82)$$

But these new equations were only valid if:

$$k^{\frac{2}{3}} \left(\frac{t_{stf}}{t} \right)^{\frac{1}{3}} \leq 1.3 \quad (2.83)$$

2.4.1.5 Structures With Integrated Stiffeners

NACA's work was based on designs that had riveted or bolted structures. A structure that had fully integrated or bonded stiffeners would produce the same results.

For these structures, the allowable web stress calculations would become more conservative, the forced crippling of the stiffeners would no longer be a concern, but column failure would still be considered a valid failure mode.

2.4.1.6 Sheet With Lands

One limitation of the NACA method was that it did not account for web skins with lands behind the stiffeners, as shown in Figure 31.

Figure 31: Web Stiffener with Land.

Lands were believed to improve stiffener strength. Grumman did not have the experimental evidence, but tentatively suggested that the following steps be taken when analyzing a structure with this feature:

- When determining forced crippling allowable stiffener stresses: DO NOT include the land thickness with the stiffener thickness t_{sf} .
- When determining column failure of the web stiffener: DO NOT include the land properties when calculating the moment of inertia I_{sf} , or the radius of gyration ρ .
- When calculating rivet loads along the stiffener, DO NOT include the land in calculating the effective stiffener area.

- For all other calculations, the effective area of the stiffener should be written as:

$$A_{stf_e}^* = A_{stf_e} + (t_{land} - t) d_{land} \quad (2.84)$$

2.4.1.7 Fastener Loads

Grumman provided a more extensive analysis of the rivet and bolt loads required for a diagonal tension structure. These methods are described in more detail in Appendix A.

2.4.1.8 Effective Shear Modulus

Grumman used a rewritten version of NACA's equation (2.65):

$$G_{IDT} = \frac{G_{web}}{(1-k) + \frac{k}{2(1+\nu)} \left\| \frac{4}{\sin^2 2\alpha} + \frac{\tan^2 \alpha}{\frac{A_{stf_e}}{d t} + 0.5(1-k)} + \frac{\cot^2 \alpha}{\frac{2 A_f}{h t} + 0.5(1-k)} \right\|} \quad (2.85)$$

This would then be used to determine the effective shear deflection of the web material.

$$\gamma_{IDT} = \frac{\tau}{G_{IDT}} \quad (2.86)$$

2.4.2 The Boeing SST Developments

Research was started in the early-1970's to refine the theory for the design of the titanium structures, which dominated the Boeing Super-Sonic Transport (SST) project [15]. The primary reasons were to account for the strict performance and economic requirements, which demanded a high level of structural efficiency (economic value per pound of weight savings). New allowables for titanium were to be developed using the test rig shown in Figure 32.

Figure 32: Boeing Shear Test Beam and Load Frame [16].

This study was conducted to take advantage of titanium's greater strength-to-weight advantage, as compared to an aluminum-based structure, and to develop the material allowables for use in the NACA methodology. However, titanium also seemed to exhibit other characteristics not evident in other materials: highly anisotropic properties, variations in the elastic modulus as a function of stress level, and a low shear-to-axial strength ratio. Most of the Boeing SST developments were in refining a design solution modified directly into a semi-empirical computer program. However, some of the published data was helpful in showing additions and modifications to the original NACA methodology. All of the equations used incorporate the Grumman modifications, plus a few more additions.

Efforts were made to tightly control the tests to reduce the data scatter and to refine the accuracy of the new allowables. Some of the empirical data needed to be refined, as it was determined that the NACA data (which included some 200 test beams) contained too high a scatter and gave an unacceptable level of conservatism (Figure 33).

Figure 33: Data scatter of NACA tests and influence on allowable limits [17]

One of the additions was in noting the stress concentration effects for beams with different numbers of test bays. This had an impact on the stress development in the web stringers.

Figure 34: Stiffener stress concentration due to configuration of test specimen [15]

The average compressive stress in a stiffener was calculated by the equation:

$$\sigma_{sf} = \left(\frac{k \tau \tan \alpha}{\frac{A_{sf_e}}{d t} + 0.5(1-k)} \right) \left(\frac{A_{sf_e}}{A_{sf}} K_{sc} \right) \quad (2.87)$$

The maximum compressive stress in the stiffener was calculated by the equation:

$$\sigma_{sf_{max}} = \sigma_{sf} \left[1 + 0.775(1-k) \left(1 - \frac{0.8d}{h_u} \right) \right] K_{sc} \quad (2.88)$$

Calculation of the critical buckling shear stress favoured the original NACA calculation but with an alternate method of determining the simply supported coefficient:

$$k_{ss} = 4.9 + 3.5 \left(\frac{d_c}{h_c} \right)^2 \quad \text{if } d_c < h_c \quad (2.89a)$$

$$k_{ss} = 4.9 + 3.5 \left(\frac{h_c}{d_c} \right)^2 \quad \text{if } d_c > h_c \quad (2.89b)$$

2.4.3 Northrop's Corrections for Composite Beams

In 1985, Northrop Corporation, Aircraft Division released a report that includes modifications for using the Modified NACA methodology for anisotropic materials, such as aircraft composites [18]. Although intended for use in designing curved sections, key equations can be adjusted for use in flat stiffened beams. However, none of the calculations for determining the diagonal tension effects were modified to account for anisotropic properties.

2.4.4 Diagonal Tension in Fibre Metal Laminates

No previous work has been found which gives a reliable method for calculating the diagonal tension effect in orthotropic materials, such as GLARE. However, some efforts were made by the Structural Laminates Company and the Technical University of Delft in an attempt to pursue this topic. These efforts will be described in the following section describing brief history and properties of GLARE.

2.5 GLARE

Advances in aerospace engineering structures focus on the evolution of design, and the development and application of new materials to be used in these structures. One such material is GLARE (GLAss REinforced aluminum): a laminate composed of alternating layers of thin aluminum sheets and glass-fibre/epoxy. GLARE represents an emergence of a new class of materials, called Fibre-Metal Laminates (FMLs).

2.5.1 A Brief History of FMLs

FMLs were conceived in the 1950s by Fokker Aircraft, during the development of the Fokker F-27. It was found that a bonded laminated structure of thin aluminum sheets could prevent rapid fatigue crack growth that would normally occur in a construction made from a monolithic material. Studies by ALCOA (ALuminum Company Of America) in the 1960's, strengthened this concept [19]. In the 1970s, British and American researchers were studying the benefits of adding bonded composite reinforcements to metallic structures. Based on this progress, Fokker conducted further tests, which introduced composite fibres (such as carbon and nylon) into the bondline between thin (1mm thick) aluminum sheets to reinforce the laminate structure. This increased the crack growth resistance of the laminate and showed promise in structural applications, but the high cost of manufacturing these materials made them uneconomical for industrial use. Questions concerning the overall durability and quality control of these technically interesting materials remained as an academic problem.

The Technical University of Delft continued to research these problems, and by the early 1980s had developed the first commercial FML product, as an optimized laminated semi-finished sheet. This laminate material, called ARALL (Aramid Reinforced ALuminum Laminate), was based on unidirectional aramid fibres and thin (0.3mm) 2024-T3 aluminum layers. Driven by the Delft research, this first generation of commercially available FMLs was produced as a combined effort by ALCOA (for the aluminum), the Dutch chemical company AKZO (for the aramid fibers), and 3M (for the epoxy adhesive).

ARALL was optimized primarily for wing skins, and found its first certified application in 1988, as a number of wing hatch replacements for the Fokker 50 aircraft. ARALL's first design application was won by ALCOA for the large cargo doors of the newly developed Boeing C-17 military transport aircraft. Design studies showed that a 26% weight saving was achieved by ARALL compared to a conventional aluminum skin. Unfortunately, ARALL's usage was still limited due to the expense of manufacturing the required aircraft components: a cost of eight to ten times greater than an optimized aluminum based structure. As a result, only about thirty C-17 transports were produced with the ARALL cargo door. Trials by Aerospatiale on an Airbus A320 shell in 1988, found that ARALL could not be used in aircraft fuselage applications due to the low compressive properties of the aramid fibres.

From this, the second generation of FMLs was developed in late 1980's by replacing the aramid fibres with glass. This new material, called GLARE, was not only better optimized for fuselage design, but was also developed into several uniaxial (GLARE 1 and GLARE 2) and biaxial (GLARE 3, GLARE 4, GLARE 5 and GLARE 6)

variants. In 1991, a partnership was made between ALCOA and AKZO to commercialize this new material. The Structural Laminates Company (SLC), based in Pennsylvania, was created to produce this material, while agreements were made with Delft University to continue research, development and marketing. Due to its properties, GLARE has since found applications in the cargo bay floors of the Boeing 777, the bulkhead of the Bombardier Learjet 125, and a number of structures in the Airbus A380, currently under development.

2.5.2 Properties of GLARE

GLARE is currently available in six different standard grades. They are all based on thin aluminum layers combined with built-up prepreg layers of unidirectional S-glass fibres embedded in an epoxy adhesive. The grade and sub-set of GLARE is defined by the orientation and number of prepreg layers between the aluminum sheets, as shown in Table 1. Each grade and sub-set provides a different list of advantages for use in design.

Table 1: Standard GLARE Grades [20]

GLARE Grade	Sub Set	Metal Sheet Thickness [mm] & Alloy	Prepreg orientation in each fibre layer	Main Beneficial Characteristics
GLARE 1	-	0.3-0.4 7475-T761	0 / 0	Fatigue, strength, yield stress
GLARE 2	A	0.2-0.5 2024-T3	0 / 0	Fatigue, strength
	B	0.2-0.5 2024-T3	90 / 90	Fatigue, strength
GLARE 3	-	0.2-0.5 2024-T3	0 / 90	Fatigue, impact
GLARE 4	A	0.2-0.5 2024-T3	0 / 90 / 0	Fatigue, strength in 0° direction
	B	0.2-0.5 2024-T3	90 / 0 / 90	Fatigue, strength in 90° direction
GLARE 5	-	0.2-0.5 2024-T3	0 / 90 / 90 / 0	Impact
GLARE 6	A	0.2-0.5 2024-T3	+45 / -45	Shear, off-axis properties
	B	0.2-0.5 2024-T3	- 45 / +45	Shear, off-axis properties

* All aluminum rolling directions in standard laminates are in the same orientation; the rolling direction is defined 0°, the transverse rolling direction is defined as 90°.

** The number of orientation in this column is equal to the number of prepreg layers (each nominally 0.127mm thick) in each fibre layer.

A coding system is used to define the grade, sub-set and thickness of the laminate.

An example of a basic designation would be GLARE 4B – 3/2. This is defined as a GLARE laminate consisting of 3 aluminum layers and 2 fibre layers, with a 90 / 0 / 90 fibre orientation within each fibre layer.

The combination of thin aluminum layers and fibre layers results in a number of improvements over a comparable aluminum sheet, with regards to fatigue behaviour, impact resistance, corrosion, and fire resistance. The improved fatigue behaviour is the primary benefit of all FML's over monolithic materials. A crack that forms in the aluminum layers is restricted in its propagation through the thickness of the material due to the fibre/resin layers. The fibres also act to transfer loads by "bridging" them over the cracks in the aluminum layers (Figure 35).

Figure 35: Crack bridging in fibre metal laminates [21]

This causes the stress intensity at the crack tips to be reduced. When the crack is formed under fatigue loading, the fibres do not fail because the shear in the bondline creates a controlled delamination. This means that under realistic fatigue loading conditions, crack growth in FMLs can be ten to a hundred times slower than in monolithic aluminum structures (Figure 36).

Figure 36: FML and aluminum crack growth behavior under fuselage loading [21]

Some GLARE variants also tend to have a greater resistance to impact because more failure modes are available than in monolithic materials, to absorb the impact energy: elasticity of the fibre layers, delamination, matrix cracking, fibre fracture, and the plastic deformation of the aluminum layers. Figure 37 shows that GLARE 5, which was reportedly optimized against impact, absorbs the least amount of energy for a given amount of impact energy.

Figure 37: Normalized absorbed energy versus impact energy [22]

GLARE has a high corrosion resistance when compared to aluminum. This is due to alternating layers of metal and epoxy. Corrosive agents only penetrate through the first layer of aluminum, but do not affect the underlying layer of fibre/epoxy. In this way the remaining structure is protected.

GLARE is also highly flame resistant for almost the same reasons. Under a standard FAA fire test, a high intensity flame will easily penetrate through an aluminum skin, but is contained by the GLARE skin of comparable thickness. This is because, while the outer layer of aluminum may be melted away, the layers of adhesive will begin to char and form an insulating carbon barrier, which protects the rest of the material.

2.5.3 SLC's Evaluation of Diagonal Tension in GLARE

In early 1997, SLC released a paper, which outlined equations for calculating the stability of GLARE structures [23]. Included in this document, was a general theoretical overview for determining diagonal tension in GLARE. However, these corrections only affected the initial buckling calculation, and had not been experimentally proven. By 2000, an attempt was made to investigate the applicability of the modified NACA methodology for use in stiffened GLARE shear beams [24]. Two 1 meter by 1 meter test panels were constructed using GLARE 3 – 5/4 (0.118 in) (3.0 mm) and GLARE 4 – 5/4 (0.138 in) (3.5 mm) skins, and were tested using the picture frame method. Predictions were made with hand calculations and finite element analysis. Unfortunately, the test panels proved to be too thick for the application of the NACA methodology. It was not possible to study the post-buckling behaviour of GLARE. However, the project did serve to validate the applied finite element methods. During the same year, plasticity effects of GLARE panels loaded in pure shear were also studied, but only for the simply supported boundary condition [25]. In all of these studies, it was found that GLARE can be treated as an isotropic material for most cases, up until the proportionality limit of the material has been reached. However, once the material has entered a fully plastic state, the direction of the principal stresses can have a large impact on the carrying capacity of the material due to the orthotropic load transfer into the glass fibres. Despite these studies, a need still exists to investigate the post-buckling behaviour of GLARE in a built-up structure.

3. THESIS OBJECTIVES

The original objective of this thesis (as proposed in May 2000) was to determine if the NACA-based methodology for calculating diagonal-tension effects could accurately predict the stresses and failures of flat beam structures using GLARE webs. If necessary and appropriate, new empirical correction factors, data graphs, and equations were to be generated to allow for future design of these GLARE structures, and to calibrate Bombardier's computer models. To achieve this, a standardized test beam was to be designed, and ten specimens utilizing varying thickness and grades of GLARE 3, GLARE 4, and 2024-T3 aluminum, were to be constructed. The design of the test specimen was to ensure a web-specific critical failure. This was essential, as the data would be used for determining the design allowables for GLARE-based beam structures loaded in diagonal tension. These test specimens were then to be loaded and failed using a large-scale test facility provided by the National Research Council of Canada's (NRC) Institute for Aerospace Research, Structures, Materials and Propulsion Laboratories (IAR-SMPL). However, the focus of the research was changed due to complications experienced during the first half of the test program. These difficulties were based on stress concentrations and distributions not described by the various diagonal tension theories, which contributed to the premature failures of the test specimen flanges. The objective was modified as follows: to better understand the NACA-based diagonal-tension equations and their effects and application in a research test program. This document outlines the work completed to fulfill the original objectives, and the final results, corrections, and procedures leading to a rescoping of the project that is continuing.

4. SPECIMEN DESIGN AND MANUFACTURE

The replication of the stiffened beam design used by NACA was complicated because the original NACA engineering drawings were not available. Therefore, the NRC stiffened beam design was developed through examination of the photos provided in the NACA technical notes, and by referring to basic aircraft and load frame design practices. The methodology used for designing the test beam was based on the NACA method presented in Section 2.3. Other references were not available until later.

All the equations and empirical data graphs were converted into a more usable format. The procedures were checked with MathCAD and then transferred to an Excel spreadsheet. The use of the parametric Excel program aided greatly in the final design analysis by accounting for all the different effects of material selection and geometry and the desired failure modes. A detailed set of the original MathCAD design calculations can be found in Appendix A. However, several design restrictions had to be considered.

4.1 Basic Design Restrictions

It was decided, that the design would emulate the NACA diagonal tension tests, as was outlined in NACA Technical Notes 2661 and 2662, and use the web dimensioning of NACA's Type I-25, small-size Wagner-beam test specimens (81.75in x 26.1in) shown in Figure 38.

Figure 38: NACA Type I-25 Wagner-Beam Test Specimen [10].

The resulting layout is shown in Figure 39. The web consists of three 20" wide internal bays, bordered by 10" wide end-bays with thicker webs. The boundaries of the bays are defined by the centrelines of the innermost rows of fasteners.

Figure 39: Basic bay layout of the test beam

Using NACA's example and to ensure the closest results to web-specific failures/effects, the web stiffeners are made from 1" x 3/4" x 1/8" aluminum 7075-T6 extruded angles. The design also has to ensure a fixed-end support condition, and the free (loading) end of the beam must be stiff enough to provide a good load transfer and even shear distribution. The design has to be as versatile as possible, meaning that all components between the test specimens are standardized. The only variable for each of the test specimens is the web material and web thickness. This standardized design attempts to accommodate the sheet thickness of aluminum and GLARE listed in Table 2:

Table 2: Material thickness originally intended for experiment

Material	Thickness
Aluminum 2024-T3	0.032in, 0.050in, 0.071in, 0.090in.
GLARE 3 & 4	0.85mm, 1.40mm, 1.65mm, 1.95mm, 2.50mm (0.033in, 0.055in, 0.065in, 0.077in, 0.098in)

4.2 Design Analysis

With the basic geometry selected, the design was refined to account for the desired critical failure mode of the web. Although not the primary desired failure mode, forced crippling of the web stiffeners was allowed to occur to provide information related to the web material properties. The design was also to avoid any other undesired specimen failures, such as column buckling of the web stiffeners, flange failures, and/or fastener failures.

A critical component of the test beam design was the selection of materials and geometry for the tensile and compressive flanges. Due to the high bending stresses and the additional internal stresses caused by diagonal tension, the flanges needed to have a

sufficient thickness and high yield strength to ensure that web material failed first. Unfortunately, the flanges could not be designed to be excessively large or stiff as the portal frame effect was also to be considered. Based on Kuhn's work [10] [11], the portal frame effect was seen to emerge in a research environment, due to the large built-up nature of the flanging structures used to test the flat beams. Although not typically a concern in normal aircraft sections, (due to the limited size of the frames and stringers), the larger flanges used in flat beam tests have the capacity to absorb a significant portion of the shear flow through the cross-section of the specimen.

Based on NACA's analysis it was determined that, after buckling occurred, the portion of shear absorbed by the flanges would begin to gradually increase. After the web material completely yielded, the flanges would rapidly absorb most of the shear load as the web loses all of its load carrying capacity. Optimization of the flanges was done through the Excel program and by developing a Margin of Safety graph. The term "Margin of Safety" refers to the degree of reliability that a structure can withstand or realize above or below its minimum guaranteed requirements. The applied or calculated design stress of a component is then compared to the allowable stress of the material being used. This is expressed in the following equation:

$$\text{Margin of Safety} = M.S. = \frac{\text{allowable stress}}{\text{applied stress}} - 1 \quad (4.1)$$

Depending on the criteria used in (4.1), the numerical value provides a means to determine points of structural failure, yielding, instability, deformation, etc. These points are realized when the margin of safety reaches "zero" or "negative" values.

Through this method, graphs like Figure 40 were used to monitor all the critical components of the beam design, such as the impact of Portal Frame Effect on the shear load into the web (Shear, Corr Shear) and the diagonal tension factor k (DT Factor, Corr DT Factor), the permanent buckling point of the material (Permanent Buckle MS), the yielding point of the flanges (Flange Yield MS), the forced crippling of the web stiffeners (Forced Crippling MS), and the failure of the web (Web Fracture MS).

Figure 40: Original Margin of Safety Graph for the 0.050" Aluminum Beam

Using this method, different geometries of standard angle extrusions, and varying grades of steel and aluminum were analyzed. The final flange size and type was selected when all of the different test specimens were predicted to fail, either by web fracture or by forced crippling of the web stiffeners.

4.3 Final Specimen Design

After several trial configurations, a final design was established which seemed to be effective for test beams using GLARE or 2024-T3 aluminum webs ranging in thickness from 0.050" to 0.100" (Figure 41).

Figure 41: NRC-IAR Test Beam.

The NRC test beam has approximately the same web size as the NACA design, but the overall framing structure is substantially bulkier and made from a stronger alloy to allow for the thicker web materials and to force the web-specific critical failure. Table 3 shows a comparison between the NACA test specimen and the NRC design.

Table 3: NACA vs. NRC Test Specimen Comparison

Test Beam Size	81.75" x 26.1"	83" x 31"
Web Material	2024-T3 Aluminum	2024-T3 Aluminum, GLARE 3, GLARE 4
Web Thickness	0.010" to 0.016", 0.040"	0.050" to 0.100"
Flange Size	2" x 2" x 3/16"	4" x 3" x 1/4"
Flange Material	2024-T3 Aluminum Angle	7075-T6 Aluminum Angle
Load Range	6.3kip to 12.7kip	37kip to 74kip (predicted)
Failure Mode	Web, Flange, Column, Forced Crippling	Web (preferred), Forced Crippling

Ten specimens were manufactured utilizing resources from the NRC and from Ryerson University:

- One 0.050" 2024-T3 Aluminum
- One 0.071" 2024-T3 Aluminum
- Two GLARE 3 – 3/2
- Two GLARE 3 – 4/3
- Two GLARE 3 – 5/4
- Two GLARE 4 – 3/2

A detailed listing and description of all the specimen components can be found in Appendix B. The detailed manufacturing drawing can be found in Appendix C.

5. GENERAL TEST SET-UP

5.1 The Multi-Actuating Test Platform

All tests for this project are conducted within the Multi-Actuating Test (MAT) Platform, a new large-scale test facility developed by NRC-IAR-SMPL. This test structure is a large self-reacting steel space frame with the capacity to add additional structures, mounts, hard-points, and reinforcements to accommodate any tests that can be arranged within its boundaries. The diagonal tension stiffened FML flat beams are the first project to use this new facility, and occupy the greater portion of one face of the platform (Figure 42). Additional columns and bracing were required to stiffen the platform for the purposes of the experiment. Plexiglas and Lexan shields are placed to protect personnel and equipment from flying debris, in the event of a catastrophic failure.

Figure 42: Image of MAT Platform

The test beams are attached through the support flanges to an interface plate, which is fixed to four clamp-on loading blocks (Figure 43). The blocks are attached to a structural member of the MAT Platform, which is reinforced to provide extra stiffness and strength to the test frame. An additional structural member is secured ahead of the test specimen to provide more stiffness to the test frame as well as to provide a mount for the specimen anti-twist system.

Figure 43: General test set-up

The anti-twist system consists of two sets of guides attached to the loading end flange, as seen in Figure 44. Each of these guides contains a pair of 2" cam followers, which ride along the outside surface of a straight square steel rod. This system restricts the movement of the beam within one plane of motion.

Figure 44: Anti-Twist System

The loading arrangement (Figure 45) consists of six key components: the lower universal joint, the load actuator, the external actuator LVDT, the load cell swivel, the load cell, and the upper universal joint. The pair of universal joints allows for the free motion of the loading arrangement, and prevents off-axis bending in the load actuator and load cell. The load actuator has a 150 kip capacity, with a 10 in maximum stroke. The load cell has a 200 kip capacity, but has been calibrated for a 100 kip limit. The load cell swivel was designed to prevent the possibility of twisting moments on the load cell. The exterior LVDT has a 10 in maximum displacement, and is attached between the actuator housing and the load cell swivel. The signal from this LVDT is used to control the displacement of the actuator.

Figure 45: Loading Arrangement

5.2 Instrumentation

Eight additional LVDTs and twelve strain gauges compose the instrumentation for the test. All electronic data was fed into two computers: one handling the test control and one handling the data acquisition requirements for the test. The final test data output was stored in a tabular numeric (volts) format, which, along with instrument calibration information, was processed later to provide the results.

5.2.1 Strain Gauges

Twelve strain gauges are attached to various locations on the test specimen (Figure 46). Two strain gauge rosettes are attached to the web surface of the central bay, and two more are attached to the web of the rear bay. The rosettes are paired and are set at back-to-back locations. The data from the rosettes is used to determine the onset of initial web buckling, the diagonal tension angle and the web stresses. Four uniaxial strain gauges are attached along the neutral axis lines of the top and bottom flanges and on the inboard web stiffeners. The data from these uniaxial gauges are used to determine the stresses in those components. Four additional uniaxial gauges are mounted near the fixed-end of the beam, in a region where horizontal flange bending is anticipated. These gauges are used to monitor potential yielding and to warn of any integrity loss in the framing structure, as cross-sectional yielding of the top/bottom flanges would result in an invalid test beyond that event.

Figure 46: Test beam #4 strain gauge locations

5.2.2 LVDT's

Nine LVDTs are arranged along the bottom surface of the test specimen. One of the LVDTs is directly connected to the loading actuator and is used as the reference for displacement control. The remaining eight LVDTs are suspended along a support mount, which is connected to the interface plate (Figure 47).

Figure 47: LVDT placements

The LVDT data is used to determine the overall specimen deflection, the specimen stiffness, and the effective shear modulus of the web material. Data is also utilized to determine if there is any slippage at the specimen-to-frame interface, and/or if there is any rotation or relative movement in the test frame between the specimen-to-frame and the actuator-to-frame interfaces.

5.2.3 Photoelastic Coating

For a limited number of tests, a photoelastic coating was used to assist in determining the onset of initial buckling and to provide a quantitative picture of the buckling mode shape in the post-buckled specimen. For this purpose, a 20" x 17" x 0.010" sheet of photoelastic coating is applied to the central bay. Two holes are cut into the coating to accommodate the strain gauge rosettes. Observation of the coating is documented by using a camera attached with a polarizing lens and connected to a television set with a VCR recorder. A digital read-out, indicating load, is been placed in view of the video camera. Still images are captured from the video footage and used for strain analysis.

Figure 48: Photoelastic coating on Test Beam #1

6. GENERAL TEST PROCEDURES

6.1 Beam Installation

Before installation into the MAT platform, the test beam was inspected and all interfacing surfaces were ground-down to ensure secure fits. All bolts connecting the web to the beam flanges were adjusted to finger-tightness and then strain gauges were applied. The beam was then loosely fastened to the platform, and the anti-twist assembly was attached. This ensured the proper in-plane alignment of the beam. Checks were made at this stage and any shimming between the rear support flanges and interface plate was performed as needed. When final alignment was achieved, all beam-to-platform interface bolts were then torqued to the maximum recommended level of 90 ft-lbs.

All the bolts in the beam flange structure of the test specimen were then torqued to a maximum of 90ft-lbs. This torquing procedure was done in three stages of 30 ft-lbs, 60 ft-lbs, and 90 ft-lbs, following a specific pattern. This pattern can be found in Appendix D. All LVDTs were then installed, and all wires from the LVDTs and strain gauges were attached and then connected to the data acquisition system. All the signals to the data acquisition systems were then checked, and adjustments were made as necessary.

6.2 Specimen Test

Just before testing, all strain gauges and LVDT signals were again checked and calibrated and the loading arrangement was attached to the beam. The front-end (tare) weight of beam had been previously calculated (~325 lbs for the thinner test specimens) and this value was preloaded by the load actuator into the beam. The test was controlled through a computer with MTS FlexTest software. Data acquisition was handled on a separate computer using LabView software.

The displacement profile for the tests (Figure 49), had a ramp rate of 0.020"/min for 25 minutes (0.5" displacement), followed by a ramp rate of 0.200"/min for 18 minutes (4.1" displacement). A ramp rate of 2.0" per 10 minutes would then be used, until either a total displacement of 7.1" had been reached or a specimen failure had occurred. However, the data acquisition system read the data channels in series: 40 channels were available, but only 29 channels were active. It took 2.5 seconds to read all 40 channels. To ensure data reliability, dwelling periods had been incorporated into the load profile. Between each one-minute block, a 10 second dwell was provided to acquire 3 or more sets of readings from all gauges and LVDTs at each static displacement point. These static points would then serve as references when processing the post-test data. This dwelling procedure added approximately 10 minutes to the maximum test time. Total maximum test time was estimated at 1 hour and 08 minutes, for an actuator displacement of 7.1 inches. Events such as initial buckling and first-to-second buckling mode change would occur during the first ramp setting, while the second and third ramp settings would be used until failure.

Figure 49: Typical FLEXTest Displacement Profile

LVDT's #1 and #2, located nearest the loading actuator had a recording range of 4 inches. To allow for the recording of the whole range of movement anticipated at their position on the specimen, they had to be manually offset to a new position, 2 inches higher, during the test. This was done when the specimen deflection reached approximately 3 inches, during a manual hold of the test loading, and reference readings from the LVDT's were taken immediately before and after the hold. A correction was then made during the post-test processing of the data.

It was determined that the test would be stopped if any one of the following conditions occurred:

- Following a major failure (fracture of web, flange, or stiffener).
- Following evidence of an unacceptable hazard to the test platform or personnel. (i.e. if unsafe conditions developed within the test hardware, specimen, software, etc.).
- A hold, for safety check purposes, would be initiated if the test reached 4.1 inches in displacement; a decision would then be made to either proceed towards 7.1 inches of displacement or to stop the test. Once the actuator reached the maximum displacement of 7.1 inches, the test would be stopped.

Upon stopping the test, the FlexTest system would then be switched to manual control, and the actuator ramped down to a zero applied load state.

6.2.1 Test Safety

A catastrophic failure of the specimen would release high levels of stored energy that could propel test hardware or specimen fragments away from the immediate test area, thereby creating a hazard for unprotected personnel. The test hardware was therefore designed, selected and assembled to handle anticipated test loading. However, the possibility of a failure in the test hardware still exists, particularly as a result of a catastrophic failure of the specimen. To mitigate the risk of personnel injury during testing, the following safety guidelines were enforced:

- As the applied load reached 30 kip, all spectators would leave the test area, and test personnel would be moved to an area in the “shadow” of a large structural member of the MAT Platform and further protected by transparent Lexan shields.
- If absolutely necessary, designated test personnel would be allowed near the test specimen under load; however, active loading would be stopped and the applied load would not be within 25% of the predicted specimen failure load. Any activity of this nature would be at the discretion of the Facility Manager or his designate.
- Video monitors are placed within a safe area and distance to allow for remote viewing of the specimen during loading and all cameras are controlled remotely from a safe location away from the immediate test area.

7. OBSERVATIONS (Tests 1 to 4)

Four specimens were tested during the summer of 2001. However, the results were plagued with the premature failure of the flanges before the predicted critical failure of the web material. It soon became evident that efforts were needed to better understand the Diagonal Tension phenomenon by re-analyzing the prediction equations and understanding the test data. The following chapter will show the original design predictions, and outline the observations and protocol changes made during each of these four tests.

7.1 Initial Predictions

As mentioned in Chapter 4, the predictions used to design the test beams were primarily based on references [7], [10], and [11]. Access to other references was limited at the time. The prime influences on the earlier design equations were NACA's empirically determined graphs, specific to the 24S-T and 75S-T aluminum alloys of the 1940's. This empirical information does not affect calculations made within the elastic loading range of the beam design, but is critical for determining the plasticity effects and critical failure modes of the structure. Such material-specific information does not exist for GLARE materials in a diagonal tension application. Therefore, 24S-T aluminum alloy data was used when calculating the plastic and failure values for the GLARE web beams, to provide a baseline for comparison.

7.1.1 Material Properties

The following material properties were used for the initial design analysis. Aluminum properties for the web, web stiffeners, and the top/bottom flanges were provided from MIL-HDBK-5H [26] and are summarized in Table 4. Material properties for GLARE 3 and GLARE 4 were calculated based on values provided by a Structural Laminates Company brochure and through the use of laminate ply theory, summarized in Table 5. When calculating the stiffened beam stress analysis, the weaker longitudinal transverse values for aluminum and GLARE were used.

Table 4: Selected material property data for aluminum components [26]

	2024-T3 Aluminum [ksi]	7075-T6 Aluminum Extrusion $t < \frac{1}{4}"$ [ksi]	7075-T6 Aluminum Extrusion $\frac{1}{4}" \leq t < \frac{1}{2}"$ [ksi]
E	10,500	10,400	10,400
$\sigma_{ult,L}$	65	82	85
$\sigma_{ult,LT}$	64	79	82
$\sigma_{ty,L}$	48	74	77
$\sigma_{ty,LT}$	43	70	72
$\sigma_{cy,L}$	40	74	77
$\sigma_{cy,LT}$	46	76	78
G	4000	4000	4000
σ_{tu}	40	44	45
σ_{bru}	106	117	121
σ_{bry}	75	97	101

Table 5: Selected material property data for fibre-metal laminate components

	GLARE 3 - 3/2 [ksi]	GLARE 3 - 4/3 [ksi]	GLARE 3 - 5/4 [ksi]	GLARE 4 - 3/2 [ksi]
E_L	8,444	8,289	8,202	8,351
E_{LT}	8,444	8,289	8,202	7,427
$\sigma_{ult,L}$	104.0	105.3	106.9	148.9
$\sigma_{ult,LT}$	103.8	103.8	105.4	88.0
$\sigma_{ty,L}$	44.2	43.7	43.4	51.1
$\sigma_{ty,LT}$	41.0	40.0	39.7	37.0
G	3380	3380	3380	3380

7.1.2 Initial Buckling Estimates

The incomplete diagonal tension state is initiated by web buckling of the stiffened beam. The applied loads required to induce web buckling (Table 6) in each of the beam designs were calculated based on the stiffness and thickness of the web material, and on the geometry of the stiffened beam design. See Appendix B for sample calculations:

Table 6: Initial web buckling loads for each material

Main Web Type	Thickness [in (mm)]	Actuator Load [lbf]
0.050" 2024-T3 Al	0.050 (1.27)	~ 1220
0.071" 2024-T3 Al	0.071 (1.80)	~ 3223
GLARE 3 – 3/2	0.056 (1.40)	~ 1287
GLARE 3 – 4/3	0.077 (1.95)	~ 3185
GLARE 3 – 5/4	0.100 (2.50)	~ 5912
GLARE 4 – 3/2	0.065 (1.65)	~ 2020

7.1.3 Permanent Deformation

Based on the analysis procedure provided in [7], a check was made to find the web shear required to deform the web material and to create permanent buckles. This buckling limit was determined based on the beam geometry, on the compressive elastic modulus and compressive yield strength of the web material, and then using Figure 50. The load values required to cause permanent buckling in each of the beam designs are listed in Table 7.

Figure 50: Permanent deformation stresses for shear buckled sheets [7]

Table 7: Permanent buckling loads for each experiment material type

Web Material	Thickness [in-(mm)]	Load [lbf]	Permanent Deformation [%]
0.050" 2024-T3 Al	0.050 (1.27)	~ 7012	~ 36.1
0.071" 2024-T3 Al	0.071 (1.80)	~ 11247	~ 26.4
GLARE 3 - 3/2	0.056 (1.40)	~ 6207	~ 32.7
GLARE 3 - 4/3	0.077 (1.95)	~ 9382	~ 22.9
GLARE 3 - 5/4	0.100 (2.50)	~ 12678	~ 16.3
GLARE 4 - 3/2	0.065 (1.65)	~ 7083	~ 26.4

7.1.4 Critical Failure Modes

Several different failure modes were checked for in each of the beam designs. The following seven failure modes were viewed as the most critical:

- **Web Shearing:** The web fails by shearing when the applied shear stress exceeds the allowable shear ultimate strength of the stiffened flat beam design.
- **Column Buckling:** The web stiffeners fail by Johnson-Euler primary column buckling.
- **Forced Crippling:** The force generated by the shear buckles in the web causes the attached legs of the web stiffeners to fail by local buckling.
- **Flange Yielding:** The extreme fibres of the flanges experience stresses which exceed the yield strength of the flange material. For the stiffened flat beams, this will occur by compressive yielding in the upper compressive flange.
- **Fastener Shearing:** The fasteners fail when forces transmitted throughout the stiffened beam exceed the in-shear strength.
- **Bearing Failure:** Any fastener holes within the material of the stiffened beam yield under the bearing load generated by forces transmitted throughout the flanges, web stiffeners, and web.
- **PFE Minimum:** The Portal Frame Effect causes the shear generated in the web to decrease as the frame absorbs more load. If this decrease in web shear occurs before any failure modes, then the methodology used predicts no failure in the web or web stiffeners.

Neglecting potential fastener and bearing failures, the predicted failure conditions in each of the designed beams are listed in Table 8 (See Appendix B for sample calculations):

Table 8: Predicted failures for stiffened beams with 7075-T6 web stiffeners

Web Material	Mode of Failure	Actuator Load [kip]	Diagonal Tension [%]
0.050" 2024-T3 Al	Web Shearing	~ 37.5	~ 62.1
0.071" 2024-T3 Al	Web Shearing	~ 52.0	~ 53.3
GLARE 3 – 3/2	Web Shearing	~ 43.0	~ 62.7
GLARE 3 – 4/3	Web Shearing	~ 57.5	~ 54.7
GLARE 3 – 5/4	Forced Crippling	~ 68.5	~ 48.1
GLARE 4 – 3/2	Web Shearing	~ 49.0	~ 58.8

7.2 Test Observations and Protocol Evolution

During the course of the first three tests, the set-up and procedure was modified for each test, in an effort to explain unexpected events and to analyze differences between experimental and calculated results. The set-up and procedure described in Chapters 5 and 6 are the result of the modifications made over the course of these initial tests and represents the common protocol used for Tests 3 and 4 (Table 9).

Table 9: Summary of Tests Completed

Test	Web Material	Thk. [in]	Test Date
#1	2024-T3 Aluminum	0.050"	Fri. 11-May-01
#2	GLARE 3 – 3/2	0.056"	Wed. 07-Jun-01
#3	GLARE 3 – 3/2	0.056"	Fri. 13-Jul-01
#4	GLARE 4 – 3/2	0.066"	Tue. 31-Jul-01

The following sections give a brief account of the protocols used in each test and the modifications that were made in an effort to understand the nature of the diagonal tension beams.

7.3.1 Test #1: 0.050" 2024-T3 Aluminum

This beam was to serve as the only aluminum baseline specimen to confirm the validity of the NACA methodology. The strain gauge configuration included two rosette pairs placed in the central bay, and four uniaxial gauges placed on the stiffeners and flanges surrounding the central bay (Figure 51).

(a) Front Face – Centre Bay

(b) Rear Face – Centre Bay

Figure 51: Test #1 strain gauge locations

The displacement profile for this test (Figure 52) was much slower than the one shown in Chapter 5. Due to the uncertainty of the beam's response during loading, this slow profile would ensure high data resolution for subsequent analysis.

Figure 52: Displacement profile for Test #1

A photoelastic coating was also applied to one face of the central bay web. A video camera connected to a computer with image capture software recorded the web buckle formation and strain development (Figure 53).

Figure 53: Photoelastic coating results for Test Beam #1

It was also noticed that from approximately 10,000 lbf. of applied load onward, the diagonal tension buckles in the web were clearly visible to the naked eye (Figure 54).

Figure 54: Test Beam #1 at ~40,000 lbf. applied load

However, at an applied load of approximately 25,000 lbf. it was noticed that the test beam was absorbing load at a much slower rate than expected. At approximately 35,000 lbf., strain readings from the central bay web did not appear to be high enough to lead towards a web failure. Deflection readings from the actuator LVDT were also much higher than expected, and discoloration and a noticeable deflection difference was observed on the flanges near the support end of the beam. A final decision was made to stop the test at an applied load of approximately 40,000 lbf., due to concerns that the flanges were yielding and that the web was no longer carrying sufficient load to induce a critical web failure. After processing the data, it was found that the web had started to yield at ~25,100 lbf at approximately 3100 μ strain. The maximum tensile strain reached

in the web was 11,639 μ strain, at an applied load of 39,954 lbf. (Figure 55) and a tip deflection of 2.304 inches. 2024-T3 aluminum has an ultimate tensile breaking strain of approximately 18% elongation (180,000 μ strain).

Figure 55: Test #1 Maximum web tension strains

7.3.2 Test #2: GLARE 3 – 3/2

Despite the results of the first test, it was decided to proceed with testing of the GLARE beams. Two major changes were made to the test protocol following the preliminary results of Test #1. A closer examination of the data, revealed that the MAT platform was deflecting during the test and causing the LVDT attached to the load actuator to read higher than anticipated. Based on this information, the MAT Platform was reinforced with large steel C-channel cross beams. The next modification involved the configuration of the strain gauges. One rosette gauge pair (R1/R3) was moved to monitor a different corner of the central bay web (Figure 56) and four uniaxial strain

gauges were added to monitor the potential yielding of the flanges near the support end of the beam (Figure 57).

Figure 56: Test #2 Central Bay Strain Gauge Locations

Figure 57: Test #2 Support End Strain Gauge Locations

The displacement profile remained unchanged, and a photoelastic coating was again applied to the central bay web of the beam. Due to budget restrictions, this was the last application of a photoelastic coating for any test beams in the project. The photoelastic observations showed that despite the increased thickness of the web material (0.056" total thickness) the buckles appeared to be more defined than in the aluminum test beam (Figure 58). The coating delaminated off the test beam above the applied load of 41,000lbf as the surface strains exceeded the design limit of the coating adhesive.

Figure 58: Photo-elastic results for Test Beam #2

As the applied load of 30,000 lbf, was exceeded, the flanges began to exhibit deformation and discoloration similar to what was observed in Test #1. The displacement profile reached its programmed maximum of 4 inches (as read by the actuator LVDT) with no occurrence of a critical failure. A decision was then made to manually increase the displacement of the actuator until a failure occurred. At applied load of 50,895 lbf and a tensile web strain of 45,193 μ strain, a combined failure of web and flanges was achieved (Figure 59) at a tip deflection of 5.971 inches. GLARE 3 has an ultimate tensile breaking strain of approximately 4.5% elongation (45,000 μ strain).

After processing the data, it was found that the web had started to yield at $\sim 19,900$ lbf at a strain of ~ 3100 μ strain.

Figure 59: Failure of Test Beam #2 after an applied load of $\sim 51,000$ lbf.

Figure 60: Test #2 Maximum web tension strains

Examination of the rear flange strain data revealed that after the web yielded, the flanges began to react as independent bending members and that the web was no longer providing a load transfer in that region of the beam. This effect can be seen in Figure 61, where before the web yields at ~19,900 lbf, the top flange is acting in pure compression and the bottom flange is acting in pure tension (Figure 62a). After the web starts to yield, the inside fibres of the flanges begin to reverse their stress flow direction. This continues until after ~27,000 lbf., from which both the top and bottom flanges were then bending as independent structures (Figure 62b).

Figure 61: Test #2 support end flange strains

Figure 62: Flange stress distribution

This effect had not been predicted nor mentioned in the original NACA documents. Furthermore, the strain data showed the outside fibre strain of the compressive flange (U5) exceeded the compressive yield strain of $7100 \mu\text{strain}$, just prior to the beam's failure. This lead to some question, as to whether the web or the flanges failed first.

7.3.3 Test #3: GLARE 3 – 3/2

Due to the large unexpected energy release of the previous test failure, clear Lexan shields were added to the test set-up and stricter safety protocols were enforced to protect all observers and operators (Figure 63). Unfortunately, light reflections off the Lexan shields made it difficult to get quality photographs.

Figure 63: Test #3 Set-up with Lexan Shields

Strain gauge arrangements were again modified. The uniaxial gauges monitoring the flanges (U5 to U8) were moved closer to the apparent yielding region, and one rosette pair (R1/R3) was moved into the centre of the rear bay to monitor strain levels in that area. The placement of this rosette pair was done in response to a concern that the shear stresses in the beam might not be distributing evenly across the beam, and that the central bay of the test specimen might not be seeing the maximum strain of the structure (Figure 64).

Figure 64: Test#3 Strain Gauge Locations

The displacement profile was also modified to increase the speed of testing and to account for the higher deflections recorded from the actuator LVDT (Figure 65). This modification shortened the maximum test time by one third.

Figure 65: Displacement profile for Test #3

Beam #3 appeared to behave similar to Beam #2. As 30,000 lbf was exceeded, the flanges again began to exhibit deformation and discoloration. The flanges suddenly failed at an applied load of 47,941 lbf (Figure 66) with a tip deflection of 5.582 inches. After processing the data, it was found that the web had started to yield at ~19,900 lbf at a strain of ~3100 μ strain. Unfortunately it was also found that the rosettes had started disbonding from the web between 40,000 lbf and 45,000 lbf, so no usable web strain data was recorded at the moment of beam failure. No clear explanation could be found for what would cause the gauges to disbond prematurely.

Figure 66: Test #3 maximum web tension strains

Again, examination of the rear flange strain data revealed that, after the web yielded, the flanges began to react as independent bending members (Figure 67). As with test #2, the strain data shows the outside fibre strain of the compressive flange (U5) exceeded the compressive yield strain of 7100 μ strain, just prior to the beam failure. Due

to the apparent web failure of test #2, it was argued that the flange failure in this test might not be representative of all the test beams. From this decision, very few changes were made for the next test.

Figure 67: Test #3 support end flange strains

7.3.4 Test #4: GLARE 4 – 3/2

Based on the flange failures of the previous two tests, some suggestions were made that the position of the universal ball-joints might be causing loading problems. As a result, the ball-joint was removed, disassembled and inspected. Finding no evidence of damage or locking, the ball-joint was re-lubricated, reassembled and reinstalled. The same strain gauge pattern was used, but a different type of adhesive was applied. The same test profile as Test #3 was also used.

As the applied load of 35,000 lbf, was exceeded, the flanges began to exhibit deformation and discoloration similar to what was observed in all previous tests. The flanges of test beam failed at an applied load of 51,267 lbf, at a tensile web strain of 32,511 μ strain (Figure 68) and at a tip deflection of 5.599 inches. GLARE 4 has an ultimate tensile breaking strain of approximately 4.5% elongation (45,000 μ strain). After processing the data, it was found that the web had started to yield at $\sim$ 21,500 lbf at a strain of $\sim$ 4100 μ strain.

Figure 68: Test #4 maximum web tension strains

As with previous tests, examination of the rear flange strain data revealed that after the web yielded, the flanges began to react as independent bending members (Figure 69).

Figure 69: Test #4 support end flange strains

It was later found that there had been a manufacturing error in the lay-up of the GLARE 4 web material, where Figure 70a was the expected configuration and Figure 70b was the material received. This meant that the GLARE 4 -3/2 beams were no longer representatives of that material. The stiffness balance in the x and y direction more closely resembled a GLARE 3 material, and one face of the sheet was now stiffer than the other.

Figure 70: GLARE 4 lay-up error

7.4 Summary of Test Results

With the mounting difficulties of the premature flange failures, it was decided to analyze the data collected so far. It was necessary to determine if any modifications to the design predictions were required to account for the flange failures and unusually high displacement values. Table 10 lists a summary of the predicted and experimental load values for initial buckling. From this table it can be seen that the original NACA –based predictions are within 10% of the experimental values for most of the beams.

Table 10: Summary of Initial Buckling

Test	Web Material	Thk. [in]	Calc IB [lbf]	Test IB [lbf]	Diff. [%]
#1	2024-T3 Aluminum	0.050	1,216	1,191	- 2.06
#2	GLARE 3 – 3/2	0.056	1,349	1,698	25.87
#3	GLARE 3 – 3/2	0.056	1,349	1,430	6.00
#4	GLARE 4 – 4/3	0.066	2,104	2,071	- 1.57

Predicted failure modes, load values, and tip displacements for each of the four beams do not agree with the tests as shown in Table 11. Each test was dominated by the unexpected flange failures. From the strain gauge data, it was surmised that the flange failure was caused when the top compressive flange completely yielded through the thickness of its horizontal leg (the top ¼” of the flange). After this component had yielded, the flange lost the majority of its load carrying capacity and stability. The vertical leg then force crippled under the transferred load.

Table 11: Summary of Failure Results

Test	Web Material	Thk. [in]	Calc Fail [lbf]	Calc Disp	Mode	Test Fail [lbf]	Test Disp	Mode
#1	2024-T3 Aluminum	0.050	37,825	1.941"	Web	39,954	2.304"	N/A
#2	GLARE 3 – 3/2	0.056	44,000	3.162"	Web	50,895	5.971"	Web / Flange
#3	GLARE 3 – 3/2	0.056	44,000	3.162"	Web	47,941	5.582"	Flange
#4	GLARE 4 – 4/3	0.066	49,000	2.442"	Web	51,267	5.599"	Flange

For the high tip displacements, suspicions were raised that the bolted fastening of the flanges might be causing bearing failures and hole elongations, which led to some of the additional displacement. To collect further information, two of the test beams were disassembled.

7.5 Beam Disassembly

Test Beam #1 and Test Beam #2 had the framing structures removed, to inspect the damage to the flanges and to view the condition of the bolt-holes in the web. A common procedure was used to disassemble both beams, and a general visual inspection was made to establish a possible explanation for the discrepancy in the measured versus calculated deflections.

7.5.1 Disassembly Procedure

The following general procedure was used while disassembling the two test beams:

- Place the test beam on the ground in an upright vertical position.
- Loosen and remove all bolts from the steel support flanges.
- Remove the steel support flanges.
- Remove the rear steel filler plates.
- Loosen and remove all bolts from the steel loading flanges.
- Remove the steel loading flanges.
- Remove the forward steel filler plates.
- Carefully move the test beam onto a pair of wooden horse supports, and place it into a horizontal position. Make sure that the top and bottom flanges firmly rest on the wooden horse supports.
- Remove all bolts from the steel web stiffener straps.
- Remove steel web stiffener straps and all underlying shims.

- Select one flange, and loosen and remove all the remaining nuts and washers. Leave all bolts in place to minimize movement or collapse of the specimen. Repeat process with the remaining flange.
- Start removing bolts from one flange, then the other one.
- Special care should be taken when loosening nuts and removing bolts from the top and bottom flange assemblies, as a large amount of residual energy is still stored in the web and flanges.
- Once all bolts have been extracted, carefully remove the flanges lying on top of the beam.
- Remove the underlying aluminum filler plates.
- The web should now be lying freely on top of the remaining flanges and filler plates.

7.5.2 Post-Disassembly Observations

Both beams were permanently deformed. The GLARE 3 – 3/2 beam (with its large rivet line failure) had the greatest deformations. As shown in Figure 71, after the flanges had been removed from both beams, permanent buckles were clearly seen to have formed within the web material.

Figure 71: Permanent buckles in Beam #2

A closer look at the failed flanges for the GLARE 3 – 3/2 beam, revealed large cracks in the vertical legs of the flanges, as seen in Figure 72. These cracks followed one set of bolt holes in the flanges, for the steel stringer stiffeners.

Figure 72: Crack in Beam #2 flange

In the opposite corners to the cracks, the flanges had locally buckled leaving a single large out-of-plane bulge in the vertical leg, as seen in Figure 73. This bulge was centred over the bolt holes for the steel stringers.

Figure 73: Local buckling of Beam #3 flange

On the webs, noticeable hole elongations (0.020" to 0.030") were found above and below the central bay of the test beams. Smaller hole elongations (0.005" to 0.015") were found in the bolt holes above and below the bays, to either side of the central bay. Bearing damage was found in the corner bolt holes, underneath the loading and support flanges. Small cracks (1 mm to 3 mm) were also found around a handful of holes on the bottom the central bay, underneath the tensile flange.

The post-disassembly observations initiated a study of the effects of different styles of flange attachment on a diagonal tension beam, with the hope that it might provide more insight for the future NRC tests.

8. THE CARLETON MINI-BEAM TEST

8.1 Introduction

Since the initial testing by NRC-IAR showed that the beams was proving more flexible than theory suggested, independent testing was conducted at Carleton University to better understand the diagonal tension theory. The Carleton University test used a miniature diagonal tension beam that had been built previously. It was originally intended for use as a mechanical engineering undergraduate lab experiment. The test loaded the beam strictly within the elastic limits, allowing for repetitive tests with the same specimen. Of key interest was the concern that the NRC beams used bolts for securing the flanges and that the manufacturing of the test beams had resulted in varying levels of fit between the bolts and bolt holes. For this reason, three different cases were studied:

- Fully Riveted Structure, denoted as “Rivet”
- Riveted Stiffeners, Bolted Flanges, tight fit, denoted as “TF Bolt”
- Riveted Stiffeners, Bolted Flanges, clearance fit, denoted as “LF Bolt”

The last two cases altered the restraining conditions along the flange of the beam. The test was conducted to determine if fastening method and manufacturing tolerances had an impact on the validity of the NACA predictions.

8.2 Analysis

A method of analysis, similar to the NRC-IAR test beams, was used to predict the properties of the Carleton Mini-beam. Adjustments were made to account for the single stiffener design of the Carleton beam, and a new Margin of Safety graph was generated (Figure 74).

Figure 74: Margin of Safety Graph for the Carleton Test Beam

Based on Figure 74, the applied load of 1000lbf was set as the maximum for the tests. At this load level, the beam would be loaded below the potential permanent buckling point, to keep the test within the elastic range and not to risk any plastic deformation of the specimen. This load level also prevented yielding and damaging the flanges.

8.3 Test Set-up

8.3.1 The Carleton Diagonal Tension Mini-Beam

The Carleton test specimen (Figure 75) is a 2024-T3 aluminum, 31" x 10" beam, with a 0.032" web, and has stiffeners supporting only one side of the web. The three stiffeners split the web surface into two 10in x 10in central bays and two smaller 10" x 5" end-bays. The end bays contain no tripling material.

Figure 75: The Carleton mini-beam

8.3.2 The Test Frame

The Test Frame is a 48" x 44" self-reacting structure constructed from 6" x 4" x ½" steel square tubing. The beam is mounted on the inside of the test structure. A hole has been drilled at the top of the frame to allow the loading arrangement to slip through and connect to the beam (Figure 76).

Figure 76: Carleton Mini-Beam Test set-up

8.3.3 The Loading Arrangement

The loading arrangement for the test consists of three parts: the screw actuator, the load cell and the connecting clevis. The actuator is a manually operated Duff-Norton Co. Model M3554-27 LC. Each full rotation of the operating wheel raises or lowers the actuator approximately 0.02 mm. This is connected to a GSE Inc. Model 5410 5kip force transducer. The load cell is connected to a Fluke 8840A multi-meter and a HP 6213A power supply. Finishing the arrangement is a steel clevis, which fits over the loading plate of the beam.

8.3.4 Strain Gauges

Nine strain gauges were attached to various locations on the test specimen (Figure 77). Two strain gauge rosettes were attached to the web surface of each of the central bays. The rosettes are paired and are set at back-to-back locations. The data from the rosettes is used to determine the onset of initial web buckling, the diagonal tension angle and the web stresses. Three uniaxial strain gauges were attached along the web stiffeners, and two uniaxial strain gauges are attached to the outside surfaces of the top and bottom flanges in-line with the central web stiffener. The data from these uniaxial gauges is used to determine the stresses in those structures.

Figure 77: Carleton Mini-Beam Strain Gauge Locations

8.3.5 Laptop Computer

A Compaq Armada 4120 Laptop Computer (w/P120 processor) was used for direct data entry into an Excel spreadsheet file. The Excel file was arranged to average and zero values recorded during the test and to convert data into engineering units. All the data was graphed and compared against theoretical values.

8.4 Procedure

The following was the general procedure followed for all cases of the experiment.

Each case was conducted at least three times:

- All power sources supporting the strain gauge and load cell equipment are switched on and allowed to warm-up for at least 20 minutes.
- The actuator is raised/lowered into a position that allows a smooth and easy insertion of the loading pin.
- Once the loading pin is in place, the actuator is raised until a load just starts to register on the load cell.
- The load cell, strain gauges and dial gauges are then calibrated and zeroed.
- The load cell voltage is read and entered into the laptop spreadsheet program. The equivalent load value (in lbf.) is then calculated.
- The dial gauges on the actuator and on the underside of the specimen are then read and the values entered into the laptop spreadsheet. Values are converted from millimeters to inches.

- Reading of the strain gauges involves observing the signal and noting the maximum and minimum values for a particular strain gauge. This data is entered into the spreadsheet to be averaged and processed.
- After reading all gauges, the actuator is raised until an increment of ~100lbf has been reached. All instruments are read and data recorded as previously mentioned. The auto-generated strain graphs are checked regularly to reduce the risk of permanently deforming the test specimen.
- This procedure is repeated until a maximum load of ~1000lbf.
- The actuator is then lowered to release the load on the test specimen, until the load cell registers a zero reading. The loading pin is then unlocked and removed.

8.5 Results and Discussion

A prime focus was made on three sets of data from the tests: the beam deflections, the flange strains, and the principal tensile web strains. From this data, it would be determined if the type of fastenings would have an impact on these values. Some interesting results were found.

The changes in flange fastening did not appear to affect the deflection of the beam, as can be seen in Figure 78. Tests 2 and 3 of the riveted case (Rivet), the tight fit bolted case (TF Bolt) and the loose fit bolted case (LF Bolt) all follow the same deflection path. Only the first test of each case showed any discrepancy, but this can be explained by initial slippage and settling of the beam while it is loaded the first time.

Figure 78: Carleton Beam Deflections

As with the deflections, all the flange strains appear to follow the same strain path regardless of the type of fasteners, see Figure 79.

Figure 79: Carleton Beam flange strains

Differences were found in the web tensile strains. As can be seen in Figure 80, it appears that an increase in web strain occurs as the flanges fastening conditions are “loosened” from riveted to loose fit bolted. It was also noted that the web strains from both bays were higher than the predicted web strain. This could be caused by the two-bay construction of the beam. As was implied by [15] and from the graph shown in Figure 34 (Section 2.4.2), the number of bays that make up the test beam can impact the stress distribution. Although it is shown from this reference that the web stiffener stresses would be lower due to the limited number of bays, it is unknown if this would be balanced by higher stresses in the web. A possible cause might be from the force used to torque the bolted test cases: the highly torqued bolts may have produced a greater clamping force than exerted by the riveted test case.

Figure 80: Carleton Beam maximum web tension strains

Despite this discrepancy, quantitative data can still be collected by taking a closer look at the web tensile strain values shown in Table 12 and Table 13.

Table 12: Carleton Beam R1/R3 Web Tensile Strain Comparison

Applied Load (lbf)	Micro-Strain	Applied Load (lbf)	Micro-Strain	Applied Load (lbf)	Micro-Strain
0	0	0	0.00	0	0.00
104	78	81	3.46	84	7.55
202	149	160	7.42	160	7.48
301	229	241	5.06	247	7.59
405	313	339	8.40	349	11.44
501	411	443	7.66	454	10.47
602	540	565	4.70	575	6.52
703	661	684	3.50	690	4.41
802	776	800	3.18	807	4.04
899	895	912	1.88	920	2.71
1001	1017	1024	0.71	1036	1.94

Table 13: Carleton Beam R2/R4 Web Tensile Strain Comparison

0	0	0	0.00	0	0.00	0.00
104	30	34	12.57	39	26.74	11.18
202	64	75	16.42	79	23.88	6.03
301	110	126	14.64	136	23.85	7.44
405	171	205	19.70	219	27.77	6.32
501	264	308	16.95	324	22.82	4.78
602	407	445	9.34	458	12.58	2.87
703	554	584	5.54	592	6.92	1.28
802	694	723	4.10	729	5.03	0.88
899	839	855	2.01	863	2.91	0.88
1001	987	990	0.29	1001	1.47	1.16

8.6 Conclusions

The following conclusions were arrived at within the elastic limits of the Carleton Test Beam:

- There is little difference between the deflections and flange strains between riveted, tight fit bolted, and loose fit bolted conditions.
- The bolted cases show a maximum 19% to 28% increase in web strains over the riveted case.
- There was no more than an 11% maximum difference between tight fit bolted and loose fit bolted web tensile strains.

9. Corrections to the NRC-IAR Predictions

Tests #1 to #4 showed discrepancies in the original predictions for web strain and beam deflection. The deflection estimates were too stiff, and the web strain estimates didn't properly reflect the plastic deformation of the web material. As the literature search continued, more information became available and the predictions were reviewed and revised. In addition to the changes in equations and approach listed throughout Section 2.4, some corrections were made in the calculations for determining the deflection profile of the beam, and in the application of the portal frame, and plasticity effects.

9.1 Deflection Estimates

The original equation used to predict the deflection along the entire length of the beams, was taken from the Portal Frame calculation found in NACA-TN-2661 [10]:

$$\delta = \frac{PL^3}{24EI} + \frac{PL}{G_e h_e t} \quad (9.1)$$

Where the effective shear modulus G_e , accounts for the buckling and plasticity effects during deflection. Equation (9.1) resulted in a very stiff prediction for the deflection of the beams. But a further literature search uncover Kuhn's earlier beam deflection equations from NACA-TN-1364 [27]:

$$\delta = \frac{PL^3}{3EI} + \frac{PL}{G_e h_e t} \quad (9.2)$$

This equation produced better results by estimating a more flexible beam, but there were still some discrepancies, and no explanation for the "S" shaped bending profile witnessed in the tests (Figure 81).

Figure 81: Deflection Profile of Test Beam #1 at 39,954 lbf.

Based on this problem, a more detailed analysis was made of the displacement along the length of the test beam. For the revised equations, the beam was broken down to reflect its three distinct geometries (Figure 82 & Figure 83):

Figure 82: Varying cross-sections of the NRC test beam

Figure 83: Varying cross-sections of the NRC test beam

From this division it was assumed that only the test section (Figure 83a) would undergo buckling and yielding, while the remaining sections (Figure 83b & Figure 83c) would be shear resistant and elastic. The base equation for the flexural and shear end-tip deformation was taken from an energy method calculation for combined shear and bending deflection in a beam [9], given as:

$$\delta = \frac{PL^3}{3EI} \left(1 + \frac{3E}{10G} \cdot \frac{h^2}{L^2} \right) \quad (9.3)$$

When the terms in equation (9.3) are multiplied out, we get:

$$\delta = \frac{PL^3}{3EI} + \frac{PLh^2}{10GI} \quad (9.4)$$

This equation was used to describe the deflection in the tripler regions and in the support end. To account for the differing materials in the structure, the EI and GI terms were calculated as:

$$EI = E_{web} I_{web} + E_{tripler} I_{tripler} + E_{flange} I_{flange} + \dots \quad (9.5)$$

$$GI = G_{web} I_{web} + G_{tripler} I_{tripler} + G_{flange} I_{flange} + \dots \quad (9.6)$$

For the test section, modifications were made to equation (9.4). First, it was assumed that the flange supports the bending load and the web only the shear load (these were the conditions used by Kuhn). Therefore, the first term in equation (9.4) would be the bending deflection of the flanges and the second term would be the shear deflection of the web. Since the test section of the beam is expected to buckle and yield, G is replaced with G_e . From this, the equation is re-written as:

$$\delta = \frac{PL^3}{3EI} + \frac{PLh_e^2}{10G_e I_{web}} \quad (9.7)$$

Where:

EI is the combined effect of the tripler and the flanges in the test section about the Neutral Axis.

G_e is the effective shear modulus as calculated from (2.65) and Figure 23.

h_e is the effective height of the web between the inner flange rivet lines.

With this in mind, the term I_{web} can be replaced with:

$$I_{web} = \frac{h_e^3 t}{12} \quad (9.8)$$

Substituting (9.8) into (9.7), we get:

$$\delta = \frac{PL^3}{3EI} + 1.2 \frac{PL}{G_e h_e t} \quad (9.9)$$

To account for the change in moment along the beam and away from the support end, the bending component of the deflection calculations is replaced with:

$$\delta_{bending} = \frac{P}{6EI} (4L^2x - Lx^2 - x^3) \quad (9.10)$$

Each section is calculated separately, within its defined boundaries, and added to the deflection of the section that preceded it. Using this method reproduced the “S” deflection curvature of the beam (Figure 84), as was witnessed during the testing.

Figure 84: Re-calculated Test #1 Predicted Beam Deformation with Various Loads

As can be seen from Figure 84, the displacements in the loading-end tripled region are all at the same value. This was confirmed visually during the tests. This also means that the reading from LVDT #1 and the LVDT from the load actuator (Ld-Disp.) should be reading close to the same value. However, the results shown in Figure 85 did not support the visual evidence.

Figure 85: Test #1 LVDT Displacement Readings

Efforts were made to quantify this discrepancy by monitoring dial gauges placed at the base of the loading arrangement, and to the vertical support of the MAT platform used to mount the test beam. Unfortunately, no correction factor could be found that would fully accounted for the differences in the displacement readings. It was determined that the Ld-Disp. reading was to be considered unreliable for monitoring the tip-deflection of the test beams. This left LVDT's #1 to #8 as the lone sources for monitoring the deflections of the beam. However, some concerns were raised that the support that mounted the LVDTs might be deflecting during testing and altering the readings. This concern would be addressed later, during the elastic range testing of the 0.071" 2024-T3 Aluminum Test Beam (Test #5).

9.2 Portal Frame Effect and Shear Modulus Plasticity

One of the most confusing concepts with the diagonal tension theory has been the application of the Portal Frame Effect. Although the basic theory and reasoning is easily understood, the application of the equations shown in Section 2.35 is never clearly described. No examples of calculations have been found which show the Portal Frame Effect or the impact of plasticity on a buckled research beam: all found examples show idealized elastic airframes. During the design of the test beam, the description of Portal Frame Effect was taken literally as a removal of stresses from the web material, and the effect was calculated using the same G_e value as for determining beam deflections. From this, the corrected shear through the web was calculated as:

$$\tau_{web} = \tau \left(\frac{\tau'}{\tau} \right)_{PFE} \quad (9.11)$$

The corrected web shear value would then be used to recalculate all remaining effects: such as, increasing the load required to initially buckle the beam, but reducing the effective diagonal tension factor. The use of the plastic G_e term in equation (2.69) also predicted a sudden reduction of portal frame effect and therefore dramatically reduced the load through the web, once the material reached a fully plastic state (Figure 86).

Figure 86: Original Portal Frame Effect Prediction for the 0.050" Aluminum Beam

This meant that, when the web completely lost its capacity to carry load, the stresses in all of the components of the beam would appear to recede (Figure 87).

Figure 87: Original Margin of Safety Graph for the 0.050" Aluminum Beam

It was later discovered that this had the unwanted effect of giving an unrealistic prediction of Load-Stress curves, as seen in Figure 88.

Figure 88: Test #1 Vertical Shear Stress with Original Stress Prediction

A new approach was taken for determining the impact of the portal frame effect. To start, the unmodified shear modulus of incomplete diagonal tension G_{IDT} from equation (2.65) was used to determine the shear absorption of the flanges. This provided a linear but reducing portal frame effect (Figure 89), without the sudden drop in shear stresses seen in Figure 86.

Figure 89: Revised Portal Frame Effect Prediction for the 0.050" Aluminum Beam

The new method for determining the portal frame effect, combined with additional equations from Section 2.4, lead to a new Margin of Safety Graph for each of the test beams (Figure 90). The new equations included the use of the Grumman web yielding criteria from equations (2.79) and (2.80), and a revision to the Forced Crippling equation from (2.81). The resulting graph now shows a clear sequence of events and failures, without the Portal Frame Effect Minimum mentioned in Section 7.1.4, and as witnessed in Figure 87.

Figure 90: Revised Margin of Safety Graph for the 0.050" Aluminum Beam

The plasticity effects were then handled by applying the effective shear modulus (with plasticity correction) to the predicted web shear. For compressive and tensile strains, an effective elastic modulus was estimated:

$$E_e = E \left(\frac{G}{G_e} \right) \quad (9.12)$$

Final web strains were then determined by taking the web stresses of the beam and dividing them by the respective effective moduli: G_e or E_e .

$$\gamma = \frac{\tau}{G_e} \quad , \quad \varepsilon = \frac{\sigma}{E_e} \quad (9.13)$$

The resulting predictions were realistic and comparable to the test data (Figure 91).

Figure 91: Revised Test #1 Principal Web Strains

10. THE ELASTIC STRAIN SURVEY

A decision was made to address the concerns of possible distortions in the beam deflection readings and to better understand the unexpected flange failures. This would be achieved by having the 0.071" 2024-T3 aluminum beam fitted with additional strain gauges and reconfiguration of the LVDT placements. This extra instrumentation would allow for a more detailed analysis of the diagonal tension phenomenon within the elastic limits of the beam design.

10.1 Objectives

This test loaded the 0.071" 2024-T3 aluminum beam several times within the elastic limits to produce a strain survey of the structure. This data was used to calibrate the NRC and Bombardier FEA models, and the NACA methodology hand-calculations, within the elastic range. The data from this test was combined with collected 0.050" and 0.071" aluminum stress-strain tensile test data to fine-tune the prediction methods. To meet this objective, forty strain gauge signals were added to the general set-up for this specific test. The additional signals (mostly in the form of rosettes) were placed to provide a view of the strain field of the central bay and to present a cross-section of the shear flow through the beam. An extra LVDT supporting structure was added to determine if there was any rotation between the beam and LVDT support mount interface locations on the MAT Platform.

10.1.1 Events of Interest

- **Initial web buckling:** The incomplete diagonal tension state is initiated by web buckling of the stiffened beam. The applied load required to induce web buckling is based on the stiffness and thickness of the web material, and on the geometry of the stiffened beam design.
- **Hysteresis effects:** As the test is cycled, shifts and changes in the structure mean that strain and displacement data will not necessarily be the same between loading and unloading actions.

10.2 Instrumentation

Changes in the basic test set-up were made for the purposes of the strain survey.

10.2.1 Linear Variable Differential Transducers (LVDTs)

Eight LVDTs were still arranged along the bottom surface of the test specimen. However, one of the LVDTs was directly connected to the loading actuator and seven LVDTs were suspended along the support mount (Figure 92).

Figure 92: LVDT's in the Support Mount.

The ninth LVDT was suspended from a welded truss arm that was attached to the base of the beam by the outermost bolt connections to the interface plate. This kept the arm referenced to the Neutral Axis of the test beam's initial starting position. The arm extended out and, supporting the LVDT, monitored the displacement of the LVDT support mount. (Figure 93)

Figure 93: LVDT Welded Truss Arm Configuration.

This data was used to determine if the LVDT support mount was rotating away from the initial base position of the Test Beam (Figure 94). The concern was that the possible existence of this type of rotation would distort the LVDT readings along the support mount.

Figure 94: Possible Support mount vs. Beam rotation.

10.2.2 Strain Gauges

Twenty-eight strain gauges (twelve uniaxial and sixteen rosettes) were attached to various locations on the test specimen. On the web, two strain gauge rosettes were attached to the centre of the rear bay, two were attached to the centre of the forward bay, and ten were attached to the various locations of the central bay. The rosettes were paired and set at back-to-back locations (Figure 95). The data from the rosettes was used to determine the onset of initial web buckling, the diagonal tension angle and the web stresses. Two rosette strain gauges were attached along the neutral axis lines of the top and bottom flanges. Four uniaxial strain gauges were attached to the outside and inside extreme fibres of the flanges. Two uniaxial strain gauges were attached on the inboard web stiffeners. The data from these gauges was used to determine the stresses in those structures. Two uniaxial gauges were connected near the fixed-end of the beam, in a region where horizontal flange bending was anticipated. These gauges were used to monitor potential yielding and to warn of any integrity loss in the framing structure (cross-sectional yielding of the top/bottom flanges will result in an invalid test beyond that event). Four additional uniaxial gauges are connected on the inside flange fibres

near the outside web stiffeners. These gauges will be used to monitor stress concentrations in these regions. The data will then be used to calibrate and assess NRC-IAR's FEA model.

Figure 95: Sketch showing Strain Gauge Locations

10.2.3 Data Acquisition Systems

The strain gauge signals were split between two different data acquisition systems. Thirty signals (U1 to U12, R1/R10, R3, R7, R9/R16) were connected to the same HP/Labview system that had been used for the previous four tests. The remaining thirty signals (R2/R11, R4/R12, R5/R13, R6/R14, R8/R15) were attached to the new InstruNet system. These gauges were in place to map the strain field of the central bay web. The signal for the load actuator was split between the two systems, to coordinate the post-processing of the data.

10.3 Test Procedure

Control of the test was conducted through a computer with MTS FlexTest software. Data acquisition was handled on two separate computers. The displacement profile had a ramp-up rate of 0.020"/min for 10 minutes (0.2" displacement), followed by a ramp-down rate of 0.020"/min for 10 minutes (approx. 0" displacement). However, the LabView data acquisition system read the data channels in series: 50 channels were available, but only 39 channels were active in that system. It took 3.5 seconds to read all 50 channels. To ensure data reliability, dwelling periods were incorporated into the load profile. Between each one-minute block, a 20 second dwell was provided to acquire 5 or more sets of readings from all gauges and LVDTs at each static displacement point. The static points would serve as references when processing the post-test data. This dwelling procedure added approximately 6 minutes to the maximum test time. Total minimum test time was estimated at 26 minutes, for a single test cycle. Four cycles were run for this test, giving a minimum test time estimated at 1 hour and 46 minutes (Figure 96).

Figure 96: Programmed FLESTest Displacement Profile

During the unloading portion of the test profile, manual input was used to offset the displacement of the specimen to compensate for the effects of hysteresis and settling. This was accomplished by monitoring the load signal so that a negative load was not applied on the test beam. This resulted in a differing starting displacement for the subsequent test cycle.

As part of the test protocol, it was agreed that the test would be stopped if any one of the following conditions occurred:

- Following a major specimen failure (fracture of web, flange, or stiffener).
- Following any evidence of permanent deformation in the test beam.
- If the load signal exceeded the maximum of 11,000lbf. or a minimum of -500lbf.
- Following evidence of an unacceptable hazard to the test platform or personnel. (i.e. if unsafe conditions develop with the test hardware, specimen, software, etc.).

Upon stopping the test, the FlexTest system would then be switched to manual control, and the actuator ramped down to a zero applied load state.

10.4 Pre-test Predictions

Following the corrections and additions outlined in Section 9, a new Margin of Safety graph was created for the 0.071" 2024-T3 aluminum stiffened beam. From Figure 97, the initial buckling load was estimated to occur at 3220 lbf. In addition, the potential for permanent buckling was predicted to occur at ~12,000 lbf. Therefore, it was decided that the test would not exceed 10,000 lbf.

Figure 97: Margin of Safety graph for the 0.071" 2024-T3 Test Beam

The deflection estimates were also calculated. It was predicted that initial web buckling would occur at an actuator displacement of 0.059 inches. The maximum applied load of 10,000 lbf. would occur at a displacement of 0.209 inches.

10.5 Survey Results

Of first concern was the possibility of the LVDT support mount deflecting during the test. The data collected is shown in Figure 98. After undergoing four cycles, the displacement readings show good repeatability.

Figure 98: Test #5 LVDT Displacement Readings

From this data, LVDT#1 and LVDT#8 were isolated to see how much the LVDT support mount deflected (Figure 99). A linear fit was taken of the LVDT #8, giving the equation:

$$y = 240649 x \quad (10.1)$$

where y is the applied load P , and x is the displacement of the LVDT support mount δ_{LB} .

From this, (10.1) can be re-written as:

$$P = 240649 \delta_{LB} \quad (10.2)$$

Figure 99: Test #5 LVDT Support mount Correction

Or, in terms of deflection:

$$\delta_{LB} = 4.155 \times 10^{-6} P \quad [in] \quad (10.3)$$

However, this equation is only valid at the position of LVDT#1 (71.75 in from the interface). To make the equation generic for this beam design, (10.3) is multiplied by $x / 71.75$, (where x is the distance along the beam from the interface) giving:

$$\delta_{LB} = 5.792 \times 10^{-8} P x \quad [in] \quad (10.4)$$

The equation (10.4) can now be used to adjust the LVDT#1 to LVDT#8 data for the earlier four tests. This slight shift results in a closer comparison between the predicted and test values up to the yielding point of the web, as can be seen in Figure 100. However, even though Test #1 was made with a 2024-T3 aluminum web, there seemed to be some differences in the plastic behaviour predicted by NACA in the 1940's, and the performance of the material as it exists today.

Figure 100: Test #1 Corrected LVDT Displacements

The next objective was to try to find insight into the stress concentrations being formed in the corners of the test section. Figure 101 shows the strain data collected from the flange gauges placed near the support end of the beam. From this graph it can be seen that the outside fibres (of) of the upper (U02) and lower (U01) flanges have strains greater than predicted (Upper_of, Lower_of). While the inside fibres (if) for the upper (U09) and lower (U11) flanges have strains less than predicted (Upper_if, Lower_if). However, this is inconclusive, as it has been observed from the previous four tests that the diagonal tension field is not fully developed until after second mode buckling has set in.

Figure 101: Test #5 Support Flange Strains

11. DISCUSSION

With insights from the Elastic Strain Survey and the Carleton Mini-Beam Test, data from Test #1 through Test #4 were re-analyzed. Web data for the 0.050" 2024-T3 aluminum test beam (Figure 102) shows a good comparison with the predicted results, even partially into the plastic region of web material. In the plastic range, there is a deviation due to the NACA plasticity correction.

Figure 102: Test #1 0.050" 2024-T3 Aluminum Principal Strains

The predictions for GLARE are only valid up to the yield point of the material as GLARE has a different plastic behaviour than aluminum (Figure 103 and Figure 104). It is noted that the 24S-T (2024-T3) aluminum plastic correction data were used for the GLARE predictions because no comparable data exists for GLARE at this time.

Figure 103: Test #2 GLARE 3 - 3/2 Principal Web Strains

Figure 104: Test #4 GLARE 4 - 3/2 Principal Web Strains

Table 14, Table 15, and Table 16 show the comparison of events within the elastic limits for each of the first four tests. Most predictions are within approximately 10% of the test values. This gives some evidence that the modified NACA methodology can be used for predicting GLARE behaviour below its yield point, as proposed by Grumman. Some of this deviation may be due to the loose-fit bolted construction of the flanges as seen in the Carleton Mini-Beam Test (which could account for a 19% to 28% increase in web strains), balanced by the inherent 13% to 16% conservatism of the prediction equations.

Table 14: Revised Initial Buckling Comparison

1	2024-T3 Al	0.050	1259	1191	-5.41
2	GLARE 3 – 3/2	0.056	1399	1698	21.39
3	GLARE 3 – 3/2	0.056	1399	1430	2.24
4	GLARE 4 – 3/2	0.066	1931	2071	7.24

Table 15: Web Shear Yielding Comparison

1	2024-T3 Al	0.050	3071	22848	27698	21.23
2	GLARE 3 – 3/2	0.056	1836	13767	14680	6.63
3	GLARE 3 – 3/2	0.056	1836	13767	14683	6.66
4	GLARE 4 – 3/2	0.066	1598	13425	13536	0.83

Table 16: Web Tensile Yielding Comparison

1	2024-T3 Al	0.050	3772	23431	25101	7.13
2	GLARE 3 – 3/2	0.056	3853	22148	19931	-10.01
3	GLARE 3 – 3/2	0.056	3853	22148	19964	-9.86
4	GLARE 4 – 3/2	0.066	4137	23355	21510	-7.90

By contrast, Table 17 shows large differences in the mode and load values for the failure of the test beams. The failure of the compressive and tensile flanges seems to preempt any other failure modes.

Table 17: Revised Web Failure Comparison

1	2024-T3 Al	0.050	37662	39954	N / A	6.08
2	GLARE 3 – 3/2	0.056	42675	50895	Web/Flng	19.26
3	GLARE 3 – 3/2	0.056	42675	47941	Flange	12.34
4	GLARE 4 – 3/2	0.066	50219	51567	Flange	2.09

Therefore, the modified NACA methodology cannot accurately predict the failure of the NRC test beams. Unfortunately, no solution or prediction method has been found to account for the stress concentrations that have been forming in the corners of the beam test section. This leads to two possible arguments: either the modified NACA method is still too conservative for estimating the web stress/strain reactions, or more effort is required to improve the corrections that describe the plastic behaviour of aluminum and GLARE. Thus, under its current configuration, the NRC test beam will not produce a web specific failure in the remaining five test specimens. It is expected that specimen flanges will yield and locally fail before sufficient strain can be developed to fracture the web.

12. CONCLUSIONS

A large amount of useful information was collected in this work, and this resulted in a better understanding of the diagonal tension phenomenon. The review of the diagonal tension methodology resulted in a correction in the NACA deflection equation, and an understanding of the use of the portal frame equations. By using the existing hand-calculation methods within the elastic range of the beam design, it is possible to estimate the diagonal tension reaction of GLARE beams. The modified NACA methodology has been shown to provide excellent predictions for the GLARE and aluminum specimens, up to the Grumman criterion for determining the yielding of the web material. However, the Carleton Mini-Beam Test did show that some discrepancies might be occurring in the web strain distribution due to the loose-fit bolted construction of the NRC test beams. It was also found that the historical empirical data used for determining the plasticity effects in 2024-T3 aluminum needs to be revised. However, more data needs to be collected to properly predict GLARE's reaction within the plastic range and to estimate the allowables for a web failure. This has lead to some differences of opinion regarding the mode of failure to be used in the design of GLARE structures, between the yielding criteria presented by Grumman and the failure criteria presented by NACA.

13. RECOMMENDATIONS FOR FUTURE WORK

Here are the recommendations for future work:

- New effective shear modulus data could be extracted from the GLARE 3- 3/2 tests, thereby providing a plasticity correction for the diagonal tension analysis.
- An opportunity exists with the well-instrumented aluminum beam used in Test #5. This beam can still be tested to failure, to provide a better insight into the NRC-IAR Test beam performance, and greater knowledge of the diagonal tension phenomenon.
- Repairing or modifying the current beam design to produce a web specific failure will be costly in resources, time, and manpower, as the compressive and tensile flanges would probably have to be replaced. An alternative to this would be to change the method for introducing the load into the beams. Replacing the loading-end flange of the beam with another support-end flange, and introducing the load through a method similar to Grumman or Boeing could accomplish this.

REFERENCES

1. Asundi, A.; Choi, Alta Y. N.; ***Fiber Metal Laminates: An Advanced Material for Future Aircraft***; Journal of Materials Processing Technology, Vol. 63, No. 1, pg 384-394; January 1997.
2. Peery, David J.; Azar, Jamal J.; ***Aircraft Structures***; 2nd edition; McGraw-Hill: 1982.
3. Wagner, Herbert; ***Flat Sheet Metal Girders with Very Thin Metal Web, Part I – General Theories and Assumptions***; NACA-TM-604; Washington: February 1931.
4. Wagner, Herbert; ***Flat Sheet Metal Girders with Very Thin Metal Web, Part II – Sheet Metal Girders with Spars Resistant to Bending, Oblique Uprights – Stiffness***; NACA-TM-605; Washington: February 1931.
5. Wagner, Herbert; ***Flat Sheet Metal Girders with Very Thin Metal Web, Part III – Sheet Metal Girders with Spars Resistant to Bending, The Stress in Uprights – Diagonal Tension Fields***; NACA-TM-606; Washington: February 1931.
6. Kuhn, Paul; ***A Summary of Design Formulas for Beams Having Thin Webs in Diagonal Tension***; NACA-TN-469; Washington: August 1933.
7. Bruhn, E. F.; ***Analysis and Design of Flight Structures***; Tri-State Offset: 1973.
8. Megson, T. H. G.; ***Aircraft Structures for Engineering Students***; 3rd edition; Arnold Publishers: 1999.
9. Popov, Egor P.; ***Engineering Mechanics of Solids***; Prentice-Hall Inc.: 1990.
10. Kuhn, Paul; Peterson, James P.; Levin, L. Ross; ***A Summary of Diagonal Tension, Part I – Methods of Analysis***; NACA-TN-2661; Langley Field: May 1952.
11. Kuhn, Paul; Peterson, James P.; Levin, L. Ross; ***A Summary of Diagonal Tension, Part II – Experimental Evidence***; NACA-TN-2662; Langley Field: May 1952.
12. Kuhn, Paul; Chiarito, Patrick T.; ***The Strength of Plane Web Systems in Incomplete Diagonal Tension***; NACA Wartime Report L-367; Washington: 1942.
13. Ratay, Robert T.; ***Instability and Failure Analysis of Flat Stiffened Plates Under Shear (Modified NACA Method)***; Structural Mechanics Note No.14; Grumman Aircraft Engineering Corporation: November 1968.
14. Tsongas, Alexander G.; Ratay, Robert T.; ***Investigation of Diagonal-Tension Beams with Very Thin Stiffened Webs***; Contract NAS 9-8284; Grumman Aerospace Corporation: July 1969.

15. Musgrove, Max D.; Mortensen, Robert E.; ***Development of Titanium Structural Element Allowables for the Boeing SST***; AIAA/ASME 11th Structures, Structural Dynamics and Materials Conference: April 1970.
16. Mello, Raymond M.; Sherrer, Robert E.; Musgrove, Max D.; ***Intermediate Diagonal Tension Filled Shear Beam Development for the Boeing SST***; AIAA/ASME 12th Structures, Structural Dynamics and Materials Conference, (Anaheim, California / April 19-21, 1971), Paper No. 71-340: April 1971.
17. Barevics, Vilis M.; Hoy, J. Douglas; Sherrer, Robert E.; ***SST Technology Follow-On Program - Phase I, Intermediate Shear Beam Analyses***; Boeing Company Report No. FAA-SS-72-11; Seattle: May 1972.
18. Deo, R. B.; Agarwal, B. L.; Madenci, E.; ***Design Methodology and Life Analysis of Postbuckled Metal and Composite Panels***; Northrop Corporation, Aircraft Division Report AFWAL-TR-85-3096, Vol. 1; Hawthorne: December 1985.
19. Kaufman, J. G.; ***Fracture Toughness of 7075-T6 and -T651 Sheet, Plate, and Multilayered Adhesive-Bonded Panels***; Journal of Basic Engineering, pg 503-507: September 1967.
20. (Ed.) Vlot, Ad; (Ed.) Gunnink, Jan Willem; ***Fibre Metal Laminates: an introduction***; Kluwer Academic Publishers: 2001.
21. Vlot, A.; Vogesang, L. B.; de Vries, T. J.; ***Towards Application of Fibre Metal Laminates in Large Aircraft***; Aircraft Engineering and Aerospace Technology: An International Journal; Vol. 71; No. 6; pg. 558-570; 1999
22. Laliberté, J.; Poon, C.; Straznicky, P. V.; ***Durability Testing of Fibre-Metal Laminates***; IAR Laboratory Report Number LTR-ST-2231; National Research Council Canada: 1999.
23. van Wimersma, B.; van Oost, R. C.; Roebroeks, G.; Gunnick, J. W.; ***Stability of GLARE Structures - Calculation Method***; Structural Laminates Company, Report: TD-R-97-001: February 1997.
24. Wittenberg, T. C.; de Boer, A.; ***Design of FML Shear Panels for Ultra-High Capacity Aircraft***; Delft University of Technology: 2000.

25. Wittenberg, T. C.; van Baten, T. J.; ***Plastic Buckling Analysis of Flat Rectangular FML Plates Loaded in Shear***; Advanced Materials and Processes: Technology and Economics, Proceedings of the 32nd International SAMPE Technical Conference, Boston, Nov 5-9, 2000, pg 673-686: November 2000.
26. Authors Unknown; ***Military Handbook: Metallic Materials and Elements for Aerospace Vehicle Structures***; MIL-HDBK-5H; U.S.A. Dept. of Defense: December 1998.
27. Kuhn, Paul; Peterson, James P.; ***Strength Analysis of Stiffened Beam Webs***; NACA-TN-1364; Washington: July 1943.

Additional Diagonal Tension literature, read for completion, but not referenced in this report:

- Kuhn, Paul; ***Loads Imposed on Intermediate Frames of Stiffened Shells***; NACA-TN-687; Washington: February 1939.
- Kuhn, Paul; ***Investigations on The Incompletely Developed Plane Diagonal-Tension Field***; NACA Report 697; Langley Field: March 1940.
- Kuhn, Paul; ***Ultimate Stresses Developed by 24S-T Sheet in Incomplete Diagonal Tension***; NACA-TN-833; Washington: December 1941.
- Kuhn, Paul; Moggio, Edwin M.; ***The Longitudinal Shear Strength Required in Double-Angle Columns of 24S-T Aluminum Alloy***; WR-RB-3E08; NACA Wartime Report L-472; Washington: May 1943.
- Levin, L. Ross; ***Ultimate Stresses Developed by 24S-T and Alclad 75S-T Aluminum-Alloy Sheet in Incomplete Diagonal Tension***; NACA-TN-1756; Washington: November, 1948.
- Levin, L. Ross; Sandlin, Charles W. Jr.; ***Strength Analysis of Stiffened Thick Beam Webs***; NACA-TN-1820; Washington: March 1949.
- Stien, Manuel; Fralich, Robert W.; ***Critical Shear Stress of Infinitely Long, Simply Supported Plate With Transverse Stiffeners***; NACA-TN-1851; Washington: April 1949.
- Upson, Ralph H.; Phelps, George M.; Liu, Tung-Sheng; ***Equal-Strength Design of Tension-Field Webs and Uprights***; NACA-TN-2548; Washington: January 1952.
- Kuhn, Paul; ***Stresses in Aircraft and Shell Structures***; McGraw-Hill Book Company: December 1957.

- Robinson, John; ***Computerised Diagonal Tension Analysis***; Journal of the Royal Aeronautical Society; Vol. 69: September 1965.
- Rockey, K. C.; ***Plate Girder Design, Flange Stiffness and Web Plate Behaviour***; Unknown Source: November 1968.
- Mansfield, E. H.; ***Load Transfer via a Wrinkled Membrane***; Proceedings of the Royal Society of London, Series A, Mathematical and Physical Sciences, Vol. 316, No. 1525: April 1970.
- Mansfield, E. H.; Baxter, P. S. A.; ***Analogy Between Wrinkled Membranes and Very Thin Plates in Flexure***; The Journal of Strain Analysis, Vol. 6, pg 204-209: July 1971.
- Kaminski, B. E.; Ashton, J. E.; ***Diagonal Tension Behavior of Boron-Epoxy Shear Panels***; Journal of Composite Materials, Vol. 5, p 533-558: October 1971.
- Agarwal, B. L.; ***Postbuckling Behaviour of Composite Shear Webs***; AIAA Journal, Vol. 19, No. 7: July 1981.
- Stienmann, D. J.; ***Tension-Field Theory***; Proceedings of the Royal Society of London, Series A, Mathematical and Physical Sciences, Vol. 429, No. 1876: May 1990.

Additional Fibre Metal Laminate (FML) literature, read for completion, but not referenced in this report:

- Johnson, W. S.; ***Damage Tolerance Evaluation of Adhesively Laminated Titanium***; Journal of Engineering Materials and Technology, Vol. 105, pg. 182-187: July 1983.
- Lawcock, Glyn; Ye, Lin; Mai, Yiu-Wing, Mai; ***Novel Fiber Reinforced Metal Laminates for Aerospace Applications - A Review, Part I - Background and General Mechanical Properties***; SAMPE Journal, Vol. 31, No. 1, pg 23-31: February 1995.
- Verolme, J. L.; van Wimersma, B.; Gunnink, J. W.; ***Prediction of Stress-Strain Curves of GLARE***; Structural Laminates Company, Report: TD-R-96-004: May 1996.

Miscellaneous literature, read for completion, but not referenced in this report:

- Rivello, Robert M.; ***Theory and Analysis of Flight Structures***; McGraw-Hill: 1969.
- Authors Unknown; ***Astronautics Structures Manual***; Vol. I, NASA TM X-73305; Marshall Space Flight Center: October 1969.
- Reithmaier, Larry; Leavell, Stuart; Bungay, Stanley; ***Standard Aircraft Handbook***; 5th Edition; TAB/AERO Books, McGraw-Hill: 1991
- Oberg, Erik; Jones, Franklin D.; Horton, Holbrook L.; Ryffel, Henry H.; ***Machinery's Handbook***; 25th Edition; Industrial Press Inc., New York: 1996.
- Flabel, Jean-Claude; ***Practical Stress Analysis for Design Engineers***; 1st Edition; Lake City Publishing, Hayden Lake: 1997.
- Nemeth, Starnes; ***The NASA Monographs on Shell Stability Design Recommendations***; NASA/TP-1998-206290; Langley Research Center, 1998.

APPENDIX A: Sample MATHCAD Beam Design Calculations

The following section displays the MATCAD 2000 format used to check the diagonal tension equations before implementing them into an Excel spreadsheet.

A.1 Input Values

A.1.1 Main Web Properties

$E_{web} := 10500000 \cdot \text{psi}$	<---	2024-T3 (Sheet) Modulus of Elasticity, [26] pg.3-70
$\sigma_{y_{web}} := 43000 \cdot \text{psi}$	<---	2024-T3 (Sheet) Yield Tensile Strength, [26] pg.3-70
$\sigma_{uts_{web}} := 64000 \cdot \text{psi}$	<---	2024-T3 (Sheet) Ultimate Tensile Strength, [26] pg.3-70
$G_{web} := 4000000 \cdot \text{psi}$	<---	2024-T3 (Sheet) Shear Modulus, [26] pg.3-70
$t := 0.050 \cdot \text{in}$	<---	Web Thickness
$h := 31 \cdot \text{in}$	<---	Web Height
$l := 83 \cdot \text{in}$	<---	Web Length

A.1.2 Flange Properties

$E_{flg} := 10400000 \cdot \text{psi}$	<---	7075-T6 (Extruded) Modulus of Elasticity, [26] pg.3-260
$\sigma_{cy_{flg}} := 77000 \cdot \text{psi}$	<---	7075-T6 (Extruded) Yield Comp. Strength, [26] pg.3-260
$\sigma_{uts_{flg}} := 85000 \cdot \text{psi}$	<---	7075-T6 (Extruded) Ultimate Tensile Strength, [26] pg.3-260
$t_{flg} := 0.25 \cdot \text{in}$	<---	Flange Thickness
$a_{flg} := 4 \cdot \text{in}$	<---	Flange Height (Vertical Leg)
$b_{flg} := 3 \cdot \text{in}$	<---	Flange Width (Horizontal Leg)
$l_{tb_{flg}} := 83 \cdot \text{in}$	<---	Top/Bottom Flange Length

$l_{ld_flg} = 30.375 \cdot \text{in}$ <--- Loading Flange Length

$A_{flg} := (a_{flg} + b_{flg} - t_{flg}) \cdot$ <--- Single Flange Area

$A_{flg} = 1.688 \text{ in}^2$

$y_{flg} := a_{flg} - \left[\frac{t_{flg}[2 \cdot (a_{flg} - t_{flg}) + b_{flg}] + (a_{flg} - t_{flg})^2}{2 \cdot (a_{flg} + b_{flg} - t_{flg})} \right]$ <--- Distance to Centroid from extreme inside fibre

$y_{flg} = 2.764 \text{ in}$

$I_{flg} := \frac{t_{flg} y_{flg}^3 + b_{flg} (a_{flg} - y_{flg})^3 - (b_{flg} - t_{flg}) (a_{flg} - y_{flg} - t_{flg})^3}{3}$ <--- Single Flange Second Moment of Inertia through centroid, about bending plane perpendicular to web

$I_{flg} = 2.769 \text{ in}^4$

A.1.3 Web Stiffener Properties

$E_{stf} := 10400000 \cdot \text{psi}$ <--- 7075-T6 (Extruded) Modulus of Elasticity, [26] pg.3-360

$\sigma_{y_stf} := 74000 \cdot \text{psi}$ <--- 7075-T6 (Extruded) Yield Tensile Strength, [26] pg.3-360

$\sigma_{uts_stf} := 82000 \cdot \text{psi}$ <--- 7075-T6 (Extruded) Ultimate Tensile Strength, [26] pg.3-360

$t_{stf} := 0.125 \cdot \text{in}$ <--- Stiffener Thickness

$a_{stf} := 1 \cdot \text{in}$ <--- Stiffener Height (Perpendicular-to-Web Leg)

$b_{stf} := 0.75 \cdot \text{in}$ <--- Stiffener Width (Parallel-to-Web Leg)

$l_{stf} := 23 \cdot \text{in}$ <--- Stiffener Length

$A_{stf} := (a_{stf} + b_{stf} - t_{stf}) \cdot t_{stf}$ <--- Single Stiffener Area

$A_{stf} = 0.203 \text{ in}^2$

$x_{stf} := \frac{t_{stf}[2 \cdot (a_{stf} - t_{stf}) + b_{stf}] + (a_{stf} - t_{stf})^2}{2 \cdot (a_{stf} + b_{stf} - t_{stf})}$ <--- Distance to Centroid from extreme inside fibre

$$x_{\text{stf}} = 0.332 \text{ in}$$

$$I_{\text{ws}} := \frac{t_{\text{stf}} \cdot (a_{\text{stf}} - x_{\text{stf}})^3 + b_{\text{stf}} \cdot x_{\text{stf}}^3 - (b_{\text{stf}} - t_{\text{stf}}) \cdot (x_{\text{stf}} - t_{\text{stf}})^3}{3} \quad <--- \quad \text{Single Stiffener Second Moment of Inertia}$$

$$I_{\text{ws}} = 0.020 \text{ in}^4$$

$$I_{\text{stf}} := I_{\text{ws}} + A_{\text{stf}} \cdot \left(\frac{t}{2} + x_{\text{stf}} \right)^2 \quad <--- \quad \text{Single Stiffener Second Moment of Inertia through web centroid, about bending plane parallel to web}$$

$$I_{\text{stf}} = 0.046 \text{ in}^4$$

$$\rho_{\text{stf}} := \sqrt{\frac{I_{\text{stf}}}{A_{\text{stf}}}} \quad <--- \quad \text{Stiffener Radius of Gyration}$$

$$\rho_{\text{stf}} = 0.474 \text{ in}$$

$$Z_{\text{stf}} := \frac{I_{\text{stf}}}{\frac{t}{2} + x_{\text{stf}}} \quad <--- \quad \text{Single Stiffener Section Modulus about web mid-plane}$$

$$Z_{\text{stf}} = 0.128 \text{ in}^3$$

$$h_u := 21.75 \cdot \text{in} \quad <--- \quad \text{Effective Stiffener Height}$$

$$\text{ksi} := 1000 \cdot \text{psi}$$

A.1.4 Beam Properties

$$h_c := 23 \cdot \text{in} \quad <--- \quad \text{Spacing of exposed web between T/B flanges}$$

$$h_e := h - 2 \cdot (a_{\text{flg}} - y_{\text{flg}}) \quad <--- \quad \text{Distance between T/B flange centroids}$$

$$h_e = 28.528 \text{ in}$$

$$h_r := 25 \cdot \text{in} \quad <--- \quad \text{Distance between inner T/B flange rivet lines}$$

$$d := 20 \cdot \text{in} \quad <--- \quad \text{Spacing of stiffeners}$$

$$d_c := 19.25 \cdot \text{in} \quad <--- \quad \text{Spacing of exposed web between stiffeners}$$

$t_{trip} := 0.090\text{-in}$ <--- Tripler Plate thickness

$I_{tf} := 2 \cdot \left[I_{flg} + A_{flg} \left(\frac{h_e}{2} \right)^2 \right]$ <--- Moment of Inertia of the Tensile Flange about the neutral axis of the web.

$$I_{tf} = 692.211 \text{ in}^4$$

$I_{cf} := I_{tf}$ <--- Moment of Inertia of the Compressive Flange about the neutral axis of the web.

$$I_{cf} = 692.211 \text{ in}^4$$

$I_{web} := \frac{t \cdot h^3}{12} + 4 \left[\frac{t_{trip} a_{flg}^3}{12} + t_{trip} a_{flg} \left[\frac{(h - a_{flg})}{2} \right]^2 \right]$ <--- Web Moment of Inertia

$$I_{web} = 388.489 \text{ in}^4$$

A.2 Formula Limitations Check

The NACA Diagonal Tension analysis can only be used if the following conditions are met:

(1) The stiffener should not be too thin ($t_{stf} / t > 0.6$):

$$\frac{t_{stf}}{t} = 2.500 \quad \text{<--- [7] pg.C11.16}$$

(2) The stiffener spacing should be within the following range ($0.2 < d / h < 1.0$):

$$\frac{d}{h} = 0.645 \quad \text{<--- [7] pg.C11.16}$$

(3) Tests conservative and valid for the following range ($200 < h / t < 1500$):

$$\frac{h}{t} = 620.000 \quad \text{<--- [7] pg.C11.16}$$

If all three conditions are met, then the NACA diagonal tension method may be used.

A.3 Critical Shear Stress

The critical shear stress can be described as the stress at which web buckling first occurs.

This value is determined by the following formula:

$$\tau_{cr_elastic} := K_{ss} \cdot E_{web} \cdot \left(\frac{t}{dc} \right)^2 \cdot \left[Rh + \frac{1}{2} \cdot (Rd - Rh) \cdot \left(\frac{dc}{hc} \right)^3 \right] \quad \text{---- [7] pg.C11.16}$$

The equation contains three empirical constants: the Simple Support Coefficient (K_{ss}), and the two Restraint Coefficients (Rh , Rd). These geometry driven values help describe the support conditions around a single bay within the beam. These values are determined by cross-referencing the appropriate empirical data graphs ([10] pg.106).

The following ratios are used to find K_{ss} , Rh , & Rd :

$$\frac{dc}{hc} = 0.837 \qquad \frac{t_{stf}}{t} = 2.500 \qquad \frac{t_{flg}}{t} = 5.000$$

For the Simple Support Coefficient, the curve of the NACA graph ([10] pg.106) was analyzed and converted into the following best-fit polynomial equation:

$$K_{ss}(x) := -11.9281 \cdot x^6 + 38.9894 \cdot x^5 - 46.2908 \cdot x^4 + 25.6364x^3 - 4.3207 \cdot x^2 + 1.5338 \cdot x + 4.8198$$

From this equation, the Simple Support Coefficient can be determined by referencing the ratio of dc/hc :

$$K_{ss} := K_{ss} \left(\frac{dc}{hc} \right)$$

$$K_{ss} = 7.305$$

For the Restraint Coefficients, the curve of the NACA graph ([10] pg.106) was analyzed and converted into the following best-fit equation:

$$R(x) := \begin{cases} 1 \cdot x & \text{if } x < 1.1 \\ (0.0204 \cdot x^6 - 0.2423 \cdot x^5 + 1.1069 \cdot x^4 - 2.3339 \cdot x^3 + 1.8300 \cdot x^2 + 0.952 \cdot x - 0.3205) & \text{if } 1.1 \leq x < 3 \\ 1.62 & \text{if } x \geq 3 \end{cases}$$

Restraint Coefficient along uprights

$$R_h := R\left(\frac{t_{stf}}{t}\right)$$

$$R_h = 1.586$$

Restraint Coefficient along flanges

$$R_d := R\left(\frac{t_{flg}}{t}\right)$$

$$R_d = 1.620$$

From these values, the elastic critical shear buckling stress ($\tau_{cr_elastic}$) for the web can be determined:

$$\tau_{cr_elastic} := K_{ss} \cdot E_{web} \cdot \left(\frac{t}{dc}\right)^2 \left[R_h + \frac{1}{2} \cdot (R_d - R_h) \cdot \left(\frac{dc}{hc}\right)^3 \right]$$

$$\tau_{cr_elastic} = 826.035 \text{ psi}$$

This value for critical shear is now corrected for plasticity by referencing the appropriate NACA empirical data graph. For this correction, the curve of the graph ([10] pg.107) was analyzed and converted in the following best fit equation:

$$\tau_{cr_elastic} := \frac{\tau_{cr_elastic}}{\text{ksi}}$$

$$T_6 := -2.59178 \cdot 10^{-9} \quad T_5 := 8.57715 \cdot 10^{-7} \quad T_4 := -1.07558 \cdot 10^{-4} \quad T_3 := 6.78923 \cdot 10^{-3}$$

$$T_2 := -2.35743 \cdot 10^{-1} \quad T_1 := 4.61292 \quad T_0 := -19.1162$$

$$\tau_{cr}(x) := \begin{cases} (1 \cdot x) & \text{if } x < 16 \\ (T_6 \cdot x^6 + T_5 \cdot x^5 + T_4 \cdot x^4 + T_3 \cdot x^3 + T_2 \cdot x^2 + T_1 \cdot x + T_0) & \text{if } x \geq 16 \end{cases}$$

Therefore the following correction is made:

$$\tau_{cr} := \tau_{cr}(\tau_{cr_elastic}) \cdot \text{ksi}$$

$$\tau_{cr} = 826.035 \text{ psi}$$

By this value for critical shear stress, web buckling should start to occur at the following shear load (Q_{cr}):

$$Q_{cr} := \tau_{cr} \cdot (h_e \cdot t)$$

$$Q_{cr} = 1178.247 \text{ lbf}$$

A.4 Loading Ratio

According to the NACA-TN-2662 report, beams of a type similar to NRC's, failed at a maximum load of 37.0kip. However, NRC-IAR-SMPL's tests will be using over twice the web thickness as that used by NACA. Therefore, potential failure loads may reach over 50kip. Due to the nature of the diagonal tension design process, multiple values (at 500lbf increments) has be checked until an applied load is found which induces a initial mode of failure. The following applied load has been predicted to precede the load required for the first onset of failure:

$$P := 37500\text{-lbf} \quad \text{<--- Applied Load}$$

By the NACA theory, it is assumed that the shear load (Q) is equal to the introduced force load (P) and is constant throughout the web.

$$Q := P \quad \text{<--- Shear Load}$$

$$Q = 3.750 \times 10^4 \text{ lbf}$$

Therefore the web shear stress (τ) is as follows:

$$\tau := \frac{Q}{h_e \cdot t} \quad \text{<--- Shear Stress, [7] pg.C11.16}$$

$$\tau = 2.629 \times 10^4 \text{ psi}$$

From this, the loading ratio is determined:

$$\frac{\tau}{\tau_{cr}} = 31.827 \quad \text{<--- Loading Ratio, [7] pg.C11.16}$$

A.5 Diagonal Tension Factor (k)

The loading ratio can now be used to determine the diagonal tension factor (k). The diagonal tension factor determines what percentage of complete diagonal tension the web is undergoing (i.e. k = 1 for complete diagonal tension, k = 0 for no diagonal tension):

$$k := \tanh\left(0.5 \cdot \log\left(\frac{\tau}{\tau_{cr}}\right)\right) \quad \text{<--- Diagonal Tension Factor, [10] pg.18}$$

$$k = 63.598 \%$$

A.6 Diagonal Tension Angle (α)

The Diagonal Tension Angle is the angle from the horizontal, at which the web buckles form. This angle can be approximated from the following equation:

$$\alpha := \operatorname{atan}\left[\left(\frac{1 + \frac{h \cdot t}{4 \cdot A_{flg}}}{1 + \frac{d \cdot t}{2 \cdot A_{stf}}}\right)^{\frac{1}{4}}\right] \quad \text{<--- Diagonal Tension Angle, [10] pg.10}$$

$$\alpha = 0.657 \text{ rad or } \alpha = 37.669 \text{ deg}$$

This equation is only being used for first approximation. Actual values of α will be measured during the experiments, based on visual inspection and on the flange, stiffener and web strain data.

A.7 Portal Frame Effect

A.7.1 Basic Equations

In a normal aircraft structure, it is customary to assume that the web will bear the full amount of any applied shear. However, for research applications, the test frame is usually large enough to carry a portion of the induced shear. The following basic equation is used to calculate for the test frame's effect on the shear carried by the web:

$$PFE := \frac{1}{1 + \frac{\delta 1}{\delta 2}} \quad <--- \quad \text{Portal Frame Effect, [10] pg.97}$$

Where:

$$\delta 1 := \frac{1}{h \cdot t \cdot G} \quad <--- \quad \text{Shear Web Deflection under Unit Load, [10] pg.97}$$

$$\delta 2 := \frac{l^3}{24 \cdot E \cdot I_g (2 \cdot I_{fg})} \quad <--- \quad \text{Portal Frame Deflection under Unit Load, [10] pg.97}$$

A.7.2 Initial Buckling Correction (when $k = 0$, and $\tau < 14$ ksi)

From zero load to the critical buckling load, the portal frame effect remains constant and at its most beneficial level. However, this effect can still influence the result of the applied load necessary to initially induce web buckling. Up until the onset of diagonal tension, it can be assumed that all material properties remain uniform, and that the web has not yet plastically deformed. Based on these assumptions, the following simplified analysis can be made:

$$\delta 1 := \frac{1}{h \cdot t \cdot G_{web}} \quad <--- \quad \text{Unbuckled Shear Web Deflection under Unit Load}$$

$$\delta 1 = 7.644 \times 10^{-8} \frac{s^2}{kg}$$

$$\delta 2 := \frac{l^3}{24 \cdot E_{flg2} \cdot I_{flg}} \quad <--- \quad \text{Portal Frame Deflection under Unit Load}$$

$$\delta 2 = 2.362 \times 10^{-6} \frac{s^2}{kg}$$

$$PFEcr := \frac{1}{1 + \frac{\delta 1}{\delta 2}} \quad <--- \quad \text{Portal Frame Effect for the unbuckled flat beam}$$

$$PFEcr = 96.865 \%$$

With this correction, the applied load required to induce web buckling can be adjusted:

$$Q_{cr} := \frac{\tau_{cr} \cdot (h \cdot t)}{PFEcr} \quad <--- \quad \text{Corrected Critical Shear Load}$$

$$Q_{cr} = 1216.381 \text{ lbf}$$

A.7.3 Portal Effects for Varying Diagonal Tension

Once a diagonal tension field has started to develop and as the applied load to the beam is increased, the effective shear modulus of the web material (G_e) begins to degrade. This alteration is influenced by two factors: (1) the shear modulus change due to the diagonal tension field, and (2) the shear modulus correction due to plasticity.

The shear modulus developed due to a "controlled" state of diagonal tension is calculated from the following equation:

$$Gdt := \frac{E_{web}}{\frac{4}{\sin(2\alpha)^2} + \frac{\tan(\alpha)^2}{\left(\frac{2 \cdot A_{stf}}{d \cdot t}\right) + 0.5 \cdot (1 - k)} + \frac{\cot(\alpha)^2}{\left(\frac{4 \cdot A_{flg}}{h \cdot t}\right) + 0.5 \cdot (1 - k)}} \quad <--- \text{ Shear Modulus for "Controlled" Diagonal Tension, [10] pg.22}$$

$$Gdt = 1.856 \times 10^6 \text{ psi}$$

This value is then used to determine the correction to the web materials shear modulus due to diagonal tension:

$$Gidt_G := \frac{1}{1 + k \cdot \left(\frac{G_{web}}{Gdt} - 1\right)} \quad <--- \text{ Shear Modulus Ratio for Incomplete Diagonal Tension, [10] pg.21}$$

$$Gidt_G = 57.652\%$$

The value for the effective shear modulus is now corrected for plasticity by referencing the appropriate NACA empirical data graph. For this correction, the curve of the graph ([10] pg.117) was analyzed and converted in the following best fit equation:

$$A6 := -0.35555 \quad A5 := 4.81757 \quad A4 := -26.34937 \quad A3 := 74.41340$$

$$A2 := -114.72156 \quad A1 := 91.68225 \quad A0 := -28.69518$$

$$Ge_Gid(x) := \begin{cases} 1 & \text{if } x \leq 1.4 \\ (A6 \cdot x^6 + A5 \cdot x^5 + A4 \cdot x^4 + A3 \cdot x^3 + A2 \cdot x^2 + A1 \cdot x + A0) & \text{if } x > 1.4 \end{cases}$$

Therefore the following value is determined:

$$Ge_Gid\left(\frac{\tau}{10 \cdot \text{ksi}}\right) = 47.920\% \quad <--- \text{ Shear Modulus Ratio Plasticity Correction}$$

The ratios for incomplete diagonal tension and for plasticity correction are then used to calculate the ratio:

$$Ge_G := Gidt_G \cdot Ge_Gid\left(\frac{\tau}{10 \cdot \text{ksi}}\right) \quad <--- \text{ Effective Shear Modulus Ratio, [7] pg.C11.19}$$

$$Ge_G = 0.276$$

The effective shear modulus of the web is now calculated by multiplying the shear modulus of the web material:

$$G_e := G_{web} \quad \text{<--- Effective Shear Modulus}$$

$$G_e = 1.105 \times 10^6 \text{ psi}$$

This value can now be used to calculate the Portal Frame Effect during a diagonal tension state:

$$\delta_1 := \frac{1}{h \cdot t \cdot G_e} \quad \text{<--- Unbuckled Shear Web Deflection under Unit Load}$$

$$\delta_1 = 2.767 \times 10^{-7} \frac{\text{s}^2}{\text{kg}}$$

$$\delta_2 := \frac{I^3}{24 \cdot E_{flg} \cdot I_{flg}} \quad \text{<--- Portal Frame Deflection under Unit Load}$$

$$\delta_2 = 2.362 \times 10^{-6} \frac{\text{s}^2}{\text{kg}}$$

$$PFE := \frac{1}{1 + \frac{\delta_1}{\delta_2}} \quad \text{<--- Portal Frame Effect under diagonal tension}$$

$$PFE = 89.513 \%$$

The Portal Frame Effect is now used to correct the values for web shear and diagonal tension factor:

$$\tau := \tau \cdot PFE \quad \text{<--- Corrected Web Shear Stress}$$

$$\tau = 2.353 \times 10^4 \text{ psi}$$

$$k := \tanh\left(0.5 \cdot \log\left(\frac{\tau}{\tau_{cr}}\right)\right) \quad \text{<--- Corrected Diagonal Tension Factor}$$

$$k = 62.144 \%$$

A.8 Allowable Stiffener Stresses

A.8.1 Average and Maximum Web Stiffener Stress

The average stress induced through the web stiffeners can be calculated by the following formula:

$$\sigma_{\text{stf}} := \frac{k \cdot \tau \cdot \tan(\alpha)}{\left(\frac{2 \cdot A_{\text{stf}}}{d \cdot t} \right) + 0.5 \cdot (1 - k)} \quad \leftarrow \text{Average Web Stiffener Stress, [10] pg.19}$$

$$\sigma_{\text{stf}} = 1.896 \times 10^4 \text{ psi}$$

Maximum stress within the stiffeners is calculated by multiplying the average web stiffener stress by a stress concentration factor. This factor is normally referenced from an empirical data graph ([10] pg.110). Since, for all NRC stiffened beams, the following ratio is constant:

$$\frac{d}{h_u} = 0.920 \blacksquare$$

The following stress concentration equation can be used for all web thickness variations of the design:

$$SC := 0.64 \cdot k \cdot \left(\frac{d}{h_u} \right) - 0.775 \cdot k - 0.64 \cdot \left(\frac{d}{h_u} \right) + 1.775$$

Where x is replaced by the value of the diagonal tension factor k:

$$SC = 1.071 \blacksquare \quad \leftarrow \text{Stress Concentration}$$

$$\sigma_{\text{stf_max}} := SC \cdot \sigma_{\text{stf}} \quad \leftarrow \text{Maximum Stiffener Stress}$$

$$\sigma_{\text{stf_max}} = 2.030 \times 10^4 \text{ psi}$$

A.8.2 Column Failure

A.8.2.1 Column Yield Stress

The effective column length of the web stiffeners can be determined by the following equation:

$$l_e := \begin{cases} h_u & \text{if } d > 1.5 \cdot h \\ \frac{h_u}{\sqrt{1 + k^2 \cdot \left(3 - 2 \cdot \frac{d}{h_u}\right)}} & \text{if } d \leq 1.5 \cdot h \end{cases} \quad \leftarrow \text{Effective Web Stiffener column length, [10] pg.46}$$

$$l_e = 18.073 \text{ in}$$

The critical yield stress of the web stiffeners is calculated as follows:

$$\sigma_{cs} := 0.05 \cdot \sqrt{\sigma_{y_stf} \cdot E_{stf}} \quad \leftarrow \text{Web Stiffener critical yield stress, [7] pg.C11.14}$$

$$\sigma_{cs} = 4.386 \times 10^4 \text{ psi}$$

A.8.2.2 Centroidal Column Stress Check

A check is now made to determine if the web stiffeners will buckle under its compressive loading conditions:

$$\frac{h_u}{2 \cdot \rho_{stf}} = 22.960 \quad \leftarrow \text{Column slenderness ratio, [7] pg.C11.20}$$

$$\sigma_c := \sigma_{cs} - \frac{\sigma_{cs}^2}{4 \cdot \pi^2 \cdot E_{stf}} \cdot \left(\frac{h_u}{2 \cdot \rho_{stf}} \right)^2 \quad \text{Johnson-Euler Column Equation, [7] pg.C11.20}$$

$$\sigma_c = 4.139 \times 10^4 \text{ psi}$$

$$MS_{cs} := \frac{\sigma_c}{\sigma_{stf}} - 1 \quad \text{Column Stress Margin of Safety, [7] pg.C11.20}$$

$$MS_{cs} = 1.183$$

A.8.3 Forced Crippling Failure

Forced crippling is a localized buckling failure of the web stiffener span directly connected to the main web.

$$\sigma_{\text{stf_fc}} := 26000 \cdot k^{\frac{2}{3}} \cdot \left(\frac{t_{\text{stf}}}{t} \right)^{\frac{1}{3}} \cdot \text{psi}$$

<--- Force Crippling Limit for 7075-T6 double upright, [7] pg.C11.18

$$\sigma_{\text{stf_fc}} = 2.570 \times 10^4 \text{ psi}$$

$$\text{MS}_{\text{fc}} := \frac{\sigma_{\text{stf_fc}}}{\sigma_{\text{stf_max}}} - 1$$

<--- Force Crippling Margin of Safety, [7] pg.C11.20

$$\text{MS}_{\text{fc}} = 0.266$$

A.8.4 Natural Crippling Failure

Natural crippling failure is not considered a controlling factor in the design process ([7] pg.C11.18)

A.8.5 General Elastic Stability of Web and Uprights

Previous test experience indicates that general elastic stability is not a controlling factor in the design process ([7] pg.C11.18).

A.9 Web Design

A.9.1 Allowable Shear Stress

Two corrective constants are required to calculate the allowable shear stresses within the main web. The constant C_1 can be calculated by the following equation:

$$C_1 := \frac{1}{\sin(2 \cdot \alpha)} - 1 \quad \text{<--- Angle Factor } C_1, [10] \text{ pg.27}$$

$$C_1 = 0.034$$

The remaining constant, C_2 , (as well as a future used C_3) is referenced from an empirical data graph ([10] pg.112). To find this constant, the flange flexibility factor, ω_d , must first be calculated:

$$\omega_d := 0.7 \cdot d \cdot \sqrt[4]{\frac{t}{(I_{cf} + I_{tf}) \cdot h_e}} \quad \text{<--- Flange Flexibility Factor, [10] pg.12}$$

$$\omega_d = 0.470$$

Since all ω_d values are less than 1 for all of NRC's test thicknesses, it can be assumed that the C_2 and C_3 stress concentration factors are constant for all cases. Therefore $C_2 := 0$ and $C_3 := 1$. With the C_1 and C_2 constants, the maximum web shear can be determined:

$$\tau_{\max} := \tau \cdot (1 + k^2 \cdot C_1) \cdot (1 + k \cdot C_2) \quad \text{<--- Maximum Web Shear, [10] pg.44}$$

$$\tau_{\max} = 2.384 \times 10^4 \text{ psi}$$

To determine the allowable web shear stress, an empirical data graph ([7] pg.C11.28) was referenced. To calculate the appropriate value, the curve of the graph was analyzed and converted into the following best-fit equation:

$$\tau_{\text{all}}(x) := 89188 \cdot x^6 - 306294 \cdot x^5 + 414627 \cdot x^4 - 286399 \cdot x^3 + 114882 \cdot x^2 - 34262 \cdot x + 31005$$

$$\tau_{all} := \tau_{all}(k) \cdot \psi_i \quad <--- \quad \text{Allowable Web Shear Stress}$$

$$\tau_{all} = 2.393 \times 10^4 \text{ psi}$$

A check is now made to determine the margin of safety for the main web:

$$MS_{web} := \left(\frac{\tau_{all}}{\tau_{max}} \right) - 1 \quad <--- \quad \text{Web Shear Margin of Safety}$$

$$MS_{web} = 0.004$$

A.9.2 Permanent Web Buckling

The following is a check to see if permanent shear buckles have formed in the main web.

By using the ratio in an empirical data graph ([7] pg.C11.54):

$$\frac{4 \cdot \tau_{cr} \cdot 10^8}{E_{web} \cdot \sigma_{y_{web}}} \cdot \psi_i = 0.732 \bullet$$

All of NRC's beams fall in a range between 0.2 and 4.0.

The approximate slope for this range is:

$$PB(x) := 4.4043x^{-0.8174} \quad <--- \quad \text{Critical Shear multiplier for permanent buckling}$$

The shear stress required to form permanent buckles can now be determined:

$$PB \left(\frac{4 \cdot \tau_{cr} \cdot 10^8}{E_{web} \cdot \sigma_{y_{web}}} \cdot \psi_i \right) = 5.685 \bullet$$

$$\tau_{pb} := PB \left(\frac{4 \cdot \tau_{cr} \cdot 10^8}{E_{web} \cdot \sigma_{y_{web}}} \cdot \psi_i \right) \cdot \tau_{cr} \quad <--- \quad \text{Shear Stress for permanent web buckling}$$

$$\tau_{pb} = 4695.850 \text{ psi}$$

$$P_{pb} := \frac{\tau_{pb} \cdot h_e \cdot t}{PFE_{cr}} \quad <--- \quad \text{Applied Load Required for permanent web buckles}$$

$$P_{pb} = 6914.895 \text{ lbf}$$

$$MSpwb := \frac{\tau_{pb}}{\tau} - 1$$

$$MSpwb = -0.800$$

<--- Permanent Buckling Margin of Safety

A.10 Flange Strength

A.10.1 Primary Bending Moments

$$M := P \cdot l$$

$$M = 3.112 \times 10^6 \text{ in-lbf}$$

<--- Primary Bending Moment, [7]
pg.C11.22

$$Mdt := k \cdot M$$

$$Mdt = 1.934 \times 10^6 \text{ in-lbf}$$

<--- Diagonal Tension Bending Moment,
[7] pg.C11.22

$$Mpb := (1 - k) \cdot M$$

$$Mpb = 1.178 \times 10^6 \text{ in-lbf}$$

<--- Pure Bending Moment, [7]
pg.C11.22

A.10.2 Primary Bending Stresses

A.10.2.1 Upper (Compressive) Flange Bending Stresses

$$\sigma_{bu_o} := \frac{-Mpb \cdot h}{2 \cdot (l_{cf} + l_{tf} + l_{web})} - \frac{Mdt \cdot h}{2 \cdot (l_{cf} + l_{tf})}$$

$$\sigma_{bu_o} = -3.196 \times 10^4 \text{ psi}$$

<--- Outer Prime Bending Stress, [7]
pg.C11.22

$$\sigma_{bu_i} := \frac{-Mpb \cdot (h - a_{flg})}{2 \cdot (l_{cf} + l_{tf} + l_{web})} - \frac{Mdt \cdot (h - a_{flg})}{2 \cdot (l_{cf} + l_{tf})}$$

$$\sigma_{bu_i} = -2.783 \times 10^4 \text{ psi}$$

<--- Inner Prime Bending Stress, [7]
pg.C11.22

A.10.2.2 Lower (Tensile) Flange Bending Stresses

$$\sigma_{bl_o} := -\sigma_{bu_o}$$

$$\sigma_{bl_o} = 3.196 \times 10^4 \text{ psi}$$

$$\sigma_{bl_i} := -\sigma_{bu_i}$$

$$\sigma_{bl_i} = 2.783 \times 10^4 \text{ psi}$$

A.10.2.3 Average Axial Flange Loads Due to Bending

$$P_{pb_avg} := \left(\frac{I_{cf} + I_{tf}}{I_{cf} + I_{tf} + I_{web}} \right) \cdot \left(\frac{M_{pb}}{h_e} \right) \quad <--- \quad \text{Average Flange Load due to pure bending, [7] pg.C11.22}$$

$$P_{pb_avg} = 3.225 \times 10^4 \text{ lbf}$$

$$P_{dt_avg} := \frac{M_{dt}}{h_e} \quad <--- \quad \text{Average Flange Load due to diagonal tension, [7] pg.C11.22}$$

$$P_{dt_avg} = 6.780 \times 10^4 \text{ lbf}$$

A.10.2.4 Axial Flange Loads Due to Pure Bending

$$P_c := -P_{pb_avg} - P_{dt_avg} \quad <--- \quad \text{Upper Axial Flange Load due to pure bending, [7] pg.C11.22}$$

$$P_c = -1.001 \times 10^5 \text{ lbf}$$

$$P_t := -P_c \quad <--- \quad \text{Lower Axial Flange Load due to pure bending, [7] pg.C11.22}$$

$$P_t = 1.001 \times 10^5 \text{ lbf}$$

A.10.2.5 Axial Flange Loads Due to Diagonal Tension Field

$$P_{dtf} := -\frac{k \cdot Q \cdot PFE}{2} \cdot \cot(\alpha) \quad <--- \quad \text{Axial Flange Load due to diagonal tension, [7] pg.C11.22}$$

$$P_{dtf} = -1.351 \times 10^4 \text{ lbf}$$

A.10.2.6 Total Average Axial Flange Loads

$$P_{uf_total} := P_c + P_{dtf} \quad <--- \quad \text{Total Average Upper Axial Flange Load due to diagonal tension, [7] pg.C11.22}$$

$$P_{uf_total} = -1.136 \times 10^5 \text{ lbf}$$

$$P_{lf_total} := P_t + P_{dtf} \quad <--- \quad \text{Total Average Lower Axial Flange Load due to diagonal tension, [7] pg.C11.22}$$

$$P_{lf_total} = 8.654 \times 10^4 \text{ lbf}$$

A.10.3 Secondary Bending Moments

$$M_{sb_stf} := \frac{1}{12} \cdot k \cdot \tau \cdot t \cdot d^2 \cdot C3 \cdot \tan(\alpha)$$

<--- Secondary Bending Moment of flange above web stiffeners, [7] pg.C11.22

$$M_{sb_stf} = 1.882 \times 10^4 \text{ in-lbf}$$

$$M_{sb_mid} := \frac{1}{2} \cdot M_{sb_stf}$$

<--- Secondary Bending Moment of flange between web stiffeners, [7] pg.C11.22

$$M_{sb_mid} = 9408.538 \text{ in-lbf}$$

A.10.4 Secondary Bending Stresses

$$\sigma_{sb_o} := \frac{M_{sb_stf} \cdot (a_{flg} - y_{flg})}{2 \cdot I_{flg}}$$

<--- Secondary Bending Stress at extreme outside fibre of flange above web stiffeners, [7] pg.C11.22

$$\sigma_{sb_o} = 4199.759 \text{ psi}$$

$$\sigma_{sb_i} := \frac{M_{sb_stf} \cdot (y_{flg})}{2 \cdot I_{flg}}$$

<--- Secondary Bending Stress at extreme inside fibre of flange above web stiffeners, [7] pg.C11.22

$$\sigma_{sb_i} = 9390.473 \text{ psi}$$

A.10.5 Maximum Flange Bending Stresses

$$\sigma_{uf_omax} := \sigma_{bu_o} + \sigma_{sb_o}$$

<--- Max Upper Flange Bending Stress at extreme outside fibre, [7] pg.C11.22

$$\sigma_{uf_omax} = -2.776 \times 10^4 \text{ psi}$$

$$\sigma_{uf_imax} := \sigma_{bu_i} - \sigma_{sb_i}$$

<--- Max Upper Flange Bending Stress at extreme inside fibre, [7] pg.C11.22

$$\sigma_{uf_imax} = -3.722 \times 10^4 \text{ psi}$$

$$\sigma_{lf_imax} := \sigma_{bl_i} - \sigma_{sb_i}$$

<--- Max Lower Flange Bending Stress at extreme inside fibre, [7] pg.C11.22

$$\sigma_{lf_imax} = 1.844 \times 10^4 \text{ psi}$$

$$\sigma_{lf_omax} := \sigma_{bl_o} + \sigma_{sb_o}$$

<--- Max Lower Flange Bending Stress at extreme outside fibre, [7] pg.C11.22

$$\sigma_{lf_omax} = 3.616 \times 10^4 \text{ psi}$$

$$MSuf_o := \left| \frac{\sigma_{cy_flg}}{\sigma_{uf_omax}} \right| - 1 \quad <--- \quad \text{Upper Flange, Outer Fibre Margin of Safety}$$

$$MSuf_o = 1.774$$

$$MSuf_i := \left| \frac{\sigma_{cy_flg}}{\sigma_{uf_imax}} \right| - 1 \quad <--- \quad \text{Upper Flange, Inner Fibre Margin of Safety}$$

$$MSuf_i = 1.069$$

$$MSlf_i := \left| \frac{\sigma_{cy_flg}}{\sigma_{lf_imax}} \right| - 1 \quad <--- \quad \text{Lower Flange, Inner Fibre Margin of Safety}$$

$$MSlf_i = 3.175$$

$$MSlf_o := \left| \frac{\sigma_{cy_flg}}{\sigma_{lf_omax}} \right| - 1 \quad <--- \quad \text{Lower Flange, Outer Fibre Margin of Safety}$$

$$MSlf_o = 1.130$$

A.11 Fastener Design

A.11.1 Bearing and Fastener Properties

A.11.1.1 Web Bearing Properties

Bearing properties of 2024-T3 aluminum sheet with a thickness of $t = 0.050\text{in}$:

$$\sigma_{bys_web} := 75000\text{psi} \quad <--- \quad \text{2024-T3 (Sheet) Bearing Yield Strength, [26] pg.3-70}$$

$$P_{3/16web} := \sigma_{bys_web} \cdot t \cdot \frac{3}{16} \cdot \text{in} \quad <--- \quad \text{3/16" Hole Bearing Limit}$$

$$P_{3/16web} = 703.125\text{lbf}$$

$$P_{1/2web} := \sigma_{bys_web} \cdot t \cdot \frac{1}{2} \cdot \text{in} \quad <--- \quad \text{1/2" Hole Bearing Load}$$

$$P_{1/2web} = 1875.000\text{lbf}$$

A.11.1.2 Web Stiffener Bearing Properties

Bearing properties of 7075-T6 aluminum extruded angle with a thickness of $t_{stf} = 0.125$ in:

$$\sigma_{bys_stf} := 97000 \text{ psi} \quad <--- \quad \text{7075-T6 (Extruded) Bearing Yield Strength, [26] pg.3-360}$$

$$P_{316stf} := \sigma_{bys_stf} \cdot t_{stf} \cdot \frac{3}{16} \cdot \text{in} \quad <--- \quad \text{3/16" Hole Bearing Limit}$$

$$P_{316stf} = 2273.438 \text{ lbf}$$

A.11.1.3 Flange Bearing Properties

Based on the observed trend of other materials, a value of two-thirds the steel flange's ultimate tensile strength will be taken as a *rough estimate* of the ultimate shear strength.

$$\sigma_{bys_flg} := 101000 \text{ psi} \quad <--- \quad \text{7075-T6 (Extruded) Bearing Yield Strength, [26] pg.3-360}$$

$$P_{316flg} := \sigma_{bys_flg} \cdot t_{flg} \cdot \frac{3}{16} \cdot \text{in} \quad <--- \quad \text{3/16" Hole Bearing Limit}$$

$$P_{316flg} = 4734.375 \text{ lbf}$$

$$P_{12flg} := \sigma_{bys_flg} \cdot t_{flg} \cdot \frac{1}{2} \cdot \text{in} \quad <--- \quad \text{1/2" Hole Bearing Load}$$

$$P_{12flg} = 1.263 \times 10^4 \text{ lbf}$$

$$P_{12_sptflg} := 44000 \text{ psi} \cdot \frac{1}{2} \cdot \text{in} \cdot \frac{1}{2} \cdot \text{in} \quad <--- \quad \text{1/2" Hole Bearing Load}$$

$$P_{12_sptflg} = 1.100 \times 10^4 \text{ lbf}$$

A.11.1.4 Aluminum Rivet Strength Properties

Rivet selection taken from weakest materials properties of 2017-T4, 2024-T4, and 2117-T4 rivet types.

$$P_{ss_riv} := 860 \text{ lbf} \quad <--- \quad \text{2117-T4 Al, 1/8" Rivet, Single Shear Strength, [I] pg.8-11}$$

A.11.1.5 Flange Bolt Shear Strength Properties

AN bolts will be used for fastening all flange components.

$$\sigma_{ss_bolt} := 75000 \text{ psi} \quad <--- \quad \text{AN Bolt Shear Strength, [7] pg.D1.3}$$

$$P_{ss_fb} := \sigma_{ss_bolt} \cdot \pi \cdot \left(\frac{1}{4} \cdot \text{in} \right)^2 \quad <--- \quad \text{AN8 (1/2") Bolt Single Shear Limit}$$

$$P_{ss_fb} = 1.473 \times 10^4 \text{ lbf}$$

A.11.1.6 Stiffener/Flange Bolt Shear Strength Properties

AN bolts will be used for fastening all web stiffener to flange components.

$$\sigma_{ss_188ss} := 75000 \text{ psi} \quad <--- \quad \text{AN3 (3/16") Shear Strength}$$

$$P_{ss_sf} := \sigma_{ss_188ss} \cdot \pi \cdot \left(\frac{3}{32} \cdot \text{in} \right)^2 \quad <--- \quad \text{AN3 (3/16") Bolt Shear Limit}$$

$$P_{ss_sf} = 2070.874 \text{ lbf}$$

A.11.2 Web to Flange Fastener Loads

$$R_f := \frac{Q}{h_r} \cdot (1 + 0.414k) \quad <--- \quad \text{Load per inch of web to flange bolts, [7] pg.C11.18}$$

$$R_f = 1885.912 \frac{\text{lbf}}{\text{in}}$$

$$R_f \cdot 1.5 \text{ in} = 2828.868 \text{ lbf} \quad <--- \quad \text{Average Flange Bolt Pitch Load}$$

$$MS_{wf} := \frac{2 \cdot P_{ss_fb}}{R_f \cdot 1.5 \text{ in}} - 1 \quad <--- \quad \text{Margin of Safety for Web to Flange Bolt Reactions}$$

$$MS_{wf} = 9.411$$

$$MS_{wf_wbrg} := \frac{2 \cdot P_{12web}}{R_f \cdot 1.5 \text{ in}} - 1 \quad <--- \quad \text{Margin of Safety for Web to Flange, Web Bearing}$$

$$MS_{wf_wbrg} = 0.326$$

$$MS_{wf_fbrg} := \frac{2 \cdot P_{12flg}}{R_f \cdot 1.5 \text{ in}} - 1 \quad <--- \quad \text{Margin of Safety for Web to Flange, Flange Bearing}$$

$$MS_{wf_fbrg} = 7.926$$

A.11.3 Web to Web Stiffener River Loads

A.11.3.1 Web to Web Stiffener Shear Effects

$$R_{total} := \frac{2 \cdot \sigma_{cs} \cdot Z_{stf} \cdot h_u}{a_{stf} \cdot l_e} \quad <--- \quad \text{Total Web to Web Stiffener Shear Load, [7] pg.C11.18}$$

$$R_{total} = 1.349 \times 10^4 \text{ lbf}$$

A total of 29 rivets per double stiffener is being used to connect the web stiffeners to the web. The total shear resistance of rivets can be calculated as follows:

$$Priv_{total} := 29 \cdot (2 \cdot P_{ss_riv}) \quad <--- \quad \text{Total Web Stiffener Rivet Shear Resistance}$$

$$Priv_{total} = 4.988 \times 10^4 \text{ lbf}$$

A check must now be made to see how much resistance is offered by the rivets versus the applied loading condition.

$$\frac{Priv_{total}}{R_{total}} = 3.698 \quad <--- \quad \text{Resistance vs. Load Ratio}$$

In the event that the shear resistance of the rivets does not equal or exceed the applied load, a correction must be made to recalculate the Allowable Column Stress (as determined earlier by the Johnson-Euler column equation). This correction is determined

by referencing an empirical data graph ([10] pg.116). To calculate the appropriate value, the curve of the graph was analyzed and converted into the following best-fit equation:

$$\sigma_{c_crct}(x) := \begin{cases} 0.99 & \text{if } x \geq 1 \\ (0.0968x^3 - 0.4163x^2 + 0.5781x + 0.7311) & \text{if } x < 1 \end{cases}$$

Note: The empirical data indicates a large degree of variability for this correction.

From this, the correcting multiplier can be calculated and the adjusted Allowable Column Stress determined:

$$\sigma_{c_crct}\left(\frac{\text{Priv_total}}{R_{\text{total}}}\right) = 0.990$$

$$\sigma_c := \sigma_c \cdot \sigma_{c_crct}\left(\frac{\text{Priv_total}}{R_{\text{total}}}\right) \quad \leftarrow \text{Corrected Johnson-Euler Column Strength}$$

$$\sigma_c = 4.098 \times 10^4 \text{ psi}$$

The margin of safety against gross buckling of the web stiffeners is then recalculated:

$$MS_{cs} := \frac{\sigma_c}{\sigma_{stf}} - 1 \quad \leftarrow \text{Recalculated Column Stress Margin of Safety}$$

$$MS_{cs} = 1.162$$

A.11.3.2 Web to Web Stiffener Tension Effects

The combined tensile strength of the rivets must be great enough to prevent the web stiffeners from lifting off the surface of the buckled web. The required tensile strength per rivet is calculated as follows:

$$TSR := 0.15t \cdot \sigma_{y_web} \quad \leftarrow \text{Required Tensile Strength per inch of rivets (for double web stiffeners), [7] pg.C11.19}$$

$$TSR = 322.500 \frac{\text{lb} \cdot \text{f}}{\text{in}}$$

Since the rivets will be installed into the web stiffeners at a pitch of 0.5in, the required tensile strength per rivet can be calculated as:

$$TSR_{0.5\text{ in}} = 161.25 \text{ kbf} \quad \leftarrow \text{Required Tensile Strength per Rivet}$$

The Margin of Safety for the web to web stiffener rivets is as follows:

$$\sigma_{ty_riv} := 43500 \text{ psi} \quad \leftarrow \text{2117-T4 Tensile Yield Strength, [1]}$$

$$P_{ty_riv} := \sigma_{ty_riv} \cdot \pi \cdot \left(\frac{1}{16} \cdot \text{in} \right)^2 \quad \leftarrow \text{2117-T4 Tensile Yield limit load for 1/8" rivet}$$

$$P_{ty_riv} = 533.825 \text{ lbf}$$

$$MS_{wvs} := \frac{P_{ty_riv}}{TSR_{0.5\text{ in}}} - 1 \quad \leftarrow \text{Web to Web Stiffener Rivet Margin of Safety}$$

$$MS_{wvs} = 2.311$$

A.11.4 Web Stiffener to Flange Fastener Loads

A check must be made to ensure that there is sufficient fastener strength to keep the web stiffeners connected to the flange. The force loaded at the ends of the web stiffeners can be calculated as follows:

$$P_{stf} := \sigma_{stf} \cdot 2 \cdot A_{stf} \quad \leftarrow \text{Web Stiffener End Load, [7]} \\ P_{stf} = 7701.79 \text{ kbf} \quad \text{pg.C11.19}$$

A total of four cap screws will be used to secure each end of the web stiffeners to the flanges.

The total load resistance of the cap screws is calculated as follows:

$$P_{cap} := 4 \cdot (2 \cdot P_{ss_sf}) \\ P_{cap} = 1.657 \times 10^4 \text{ lbf}$$

The Margin of Safety for the web stiffener to flange fastener loads is calculated as follows:

$$MScap := \frac{P_{cap}}{P_{stf}} - 1 \quad <--- \quad \text{Web Stiffener to Flange Margin of Safety}$$

$$MScap = 1.151$$

$$MSwsf_{fbrg} := \frac{8 \cdot P_{316flg}}{P_{stf}} - 1 \quad <--- \quad \text{Margin of Safety for Web to Flange, Steel Flange Bearing}$$

$$MSwsf_{fbrg} = 3.918$$

$$MSwsf_{sbrg} := \frac{8 \cdot P_{316stf}}{P_{stf}} - 1 \quad <--- \quad \text{Margin of Safety for Web to Flange, Web Stiffener Bearing}$$

$$MSwsf_{sbrg} = 1.361$$

A.11.5 Loading Flange Corner Loads

The corners of the loading flange must transfer a load equal to the output of the actuator.

$$P = 3.750 \times 10^4 \text{ lbf} \quad <--- \quad \text{Applied Load}$$

Four 3/8" bolts are used to secure each corner. The total shear resistance of these bolts is calculated as follows:

$$P_{cmr} := 4 \cdot (2 \cdot P_{ss_{fb}}) \quad <--- \quad \text{Loading Corner, Bolt Arrangement Shear Resistance}$$

$$P_{cmr} = 1.178 \times 10^5 \text{ lbf}$$

The Margin of Safety for the loading corner bolt arrangements is calculated as follows:

$$MS_{cmr} := \frac{P_{cmr}}{P} - 1 \quad <--- \quad \text{Loading Corner, Bolt Arrangement Margin of Safety}$$

$$MS_{cmr} = 2.142$$

$$MS_{cmr_{fbrg}} := \frac{8 \cdot P_{12flg}}{P} - 1 \quad <--- \quad \text{Loading Corner, Flange Bearing Margin of Safety}$$

$$MS_{cmr_{fbrg}} = 1.693$$

A.11.6 Support Flange Corner Loads

The corner bolt arrangements at the support flange must be able to transfer the axial loading conditions of the upper and lower flanges. The maximum axial flange force is always experienced by the upper (compressive) flange:

$$|P_{uf_total}| = 1.136 \times 10^5 \text{ lbf}$$

An array of 8 bolts is used to resist the fixed support loads. The total shear resistance of these bolts is calculated as follows:

$$\begin{aligned} P_{sprt} &:= 8 \cdot (2 \cdot P_{ss_fb}) &<--- &\text{Support Corner, Bolt Arrangement} \\ P_{sprt} &= 2.356 \times 10^5 \text{ lbf} &&\text{Shear Resistance} \end{aligned}$$

The Margin of Safety for the loading corner bolt arrangements is calculated as follows:

$$\begin{aligned} MS_{sprt} &:= \frac{P_{sprt}}{|P_{uf_total}|} - 1 &<--- &\text{Support Corner, Bolt Arrangement} \\ MS_{sprt} &= 1.075 &&\text{Margin of Safety} \end{aligned}$$

$$\begin{aligned} MS_{sprt_fbrg} &:= \frac{20 \cdot P_{12flg}}{|P_{uf_total}|} - 1 &<--- &\text{Support Corner, T/B Flange Bearing} \\ MS_{sprt_fbrg} &= 1.223 &&\text{Margin of Safety} \end{aligned}$$

$$\begin{aligned} MS_{sprt_sfbrg} &:= \frac{20 \cdot P_{12_sprtflg}}{|P_{uf_total}|} - 1 &<--- &\text{Support Corner, Support Flange} \\ MS_{sprt_sfbrg} &= 0.937 &&\text{Bearing Margin of Safety} \end{aligned}$$

A.12 Failure Mode Assessment

The four main failure modes identified in this structure are (a) web stiffener column buckling, (b) web stiffener force crippling, (c) web shearing, and (d) top/bottom flange yielding. These failure modes will be checked by equating the following terms with their respective Margins of Safety:

$$\begin{aligned}\text{Stiffener_Column_Buckle} &:= \text{MS_cs} \\ \text{Stiffener_Force_Cripple} &:= \text{MS_fc} \\ \text{Web_Shear} &:= \text{MS_web} \\ \text{Flange_Yield} &:= \text{MSuf_i}\end{aligned}$$

Secondary failure modes are identified as (e) fastener shearing, and (f) bearing failure. These failure modes will be checked by equations with the following terms with their respective Margins of Safety:

$$\begin{aligned}\text{Web_to_Flg_Bolts} &:= \text{MSwf} \\ \text{Web_to_Flg_FlgBear} &:= \text{MSwf_fbrg} \\ \text{Web_to_Flg_WebBear} &:= \text{MSwf_wbrg} \\ \text{Web_to_Web_Stf_TSR} &:= \text{MSwws} \\ \text{Web_Stf_to_Flg_CapScrews} &:= \text{MScap} \\ \text{Web_Stf_to_Flg_FlgBear} &:= \text{MSwsf_fbrg} \\ \text{Web_Stf_to_Flg_StfBear} &:= \text{MSwsf_sbrg} \\ \text{Loading_Corner_Bolts} &:= \text{MScmr} \\ \text{Loading_Corner_FlgBear} &:= \text{MScmr_fbrg} \\ \text{Support_Corner_Bolts} &:= \text{MSsprt} \\ \text{Support_Corner_FlgBear} &:= \text{MSsprt_fbrg} \\ \text{Support_Corner_SprtFlgBear} &:= \text{MSsprt_sfbrg}\end{aligned}$$

Primary failure has occurred by the first mode to reach a MS of less than or equal to zero.

Therefore, under the given conditions:

$$\begin{aligned}P &= 3.750 \times 10^4 \text{ lbf} \\ k &= 62.144\%\end{aligned}$$

The following Margins of Safety have been determined:

Primary Components

Stiffener_Column_Buckle = 1.162

Stiffener_Force_Cripple = 0.266

Web_Shear = 0.004

Flange_Yield = 1.069

Fasteners

Web_to_Flg_Bolts = 9.411

Web_to_Web_Stf_TSR = 2.311

Web_Stf_to_Flg_CapScrews = 1.151

Support_Corner_Bolts = 1.075

Loading_Corner_Bolts = 2.142

Material Bearing Properties

Web_to_Flg_FlgBear = 7.926

Web_to_Flg_WebBear = 0.326

Web_Stf_to_Flg_FlgBear = 3.918

Web_Stf_to_Flg_StfBear = 1.361

Loading_Corner_FlgBear = 1.693

Support_Corner_FlgBear = 1.223

Support_Corner_SprtFlgBear = 0.937

APPENDIX B: General NRC-IAR Test Beam Design

B.1 General Construction

Figure 105 shows the general components of the NRC-IAR test beam.

Figure 105: Exploded view of the NRC-IAR test beam

The following is a brief description of the components identified in the above figure:

- **The Web** of the stiffened beam consisted of either aluminum or GLARE materials. The experiments focused on the specific properties of the various web materials under diagonal tension.

- **The Forward and Rear Tripler Plates** were used to reinforce the extreme ends of the main web, and to help ensure an even shear load distribution throughout. The tripler plates were made of 0.090" thick aluminum sheet.
- **The Aluminum Filler Plates** lay underneath the top and bottom flanges. Filler plate thickness was equal to the forward/rear tripler plates. This ensured a continuous contact surface between the top/bottom flanges and the stiffened plate. It should be noted that the NACA documents do not indicate such a component. However, since NACA's tests did not exceed web thickness over 0.045", it was surmised that the steel flanges were either bent or machined to accommodate the thickness changes. The planned use of web thickness over 0.045" could have caused manufacturing difficulties, therefore a level connecting surface was considered to be preferable.
- **The Top and Bottom Flanges** were cut from 4" x 3" x 1/4" 7075-T6 aluminum angle. The flanges ensured a clamped boundary condition and also provided good anchoring points and contact surfaces for the anti-buckling system. These flanges were unique to each built specimen.
- **The Forward and Rear Steel Filler Plates** were cut from 1/4" thick steel plate. The plates were used to provide a continuous contact surface for the Loading and Built-in Fixed Support Flanges. The stiffened beam design required that the forward/rear flanges overlap the top/bottom flanges. Therefore, the filler plates were used to fill

the gap between the forward/rear flanges and the web. An observation of the NACA photos, seemed to indicate a similar approach.

- **The Free End Loading Flange** was made from 4" x 3" x 3/8" steel angle. The Loading Flange was chamfered at the edges to avoid interference with the top/bottom flange fillets. These flanges were unique to each built specimen.
- **The Fixed End Support Flanges** was made from 8" x 6" x 1/2" steel angle. These flanges assisted in maintaining the clamped boundary condition and helped to act as a connecting interface with the test frame. As with the loading flange, the edges were chamfered to avoid interference with the top/bottom flange fillets. These flanges were reusable between select beams. Four sets of flanges were available during testing.
- **The Web Stiffeners** were cut from 1" x 3/4" x 1/8" extruded 7075-T6 aluminum angle. Based on the NACA technical notes, this configuration appeared to represent the most common web stiffener type utilized during the NACA experiments. This size was selected in an attempt to focus NRC's test results within the NACA data scatter, while still encouraging web-specific failures/effects during the experiments. See Figure 106 for the web stiffener orientation.

Figure 106: Web stiffener placement and orientation

- The Steel Stiffeners, Steels Shims, and Aluminum Shims** were used to interface the web stiffeners with the top and bottom flanges. The steel stiffeners were made from 6 3/8" x 5/8" x 1/8" AISI 4130 steel sheet. A steel shim was used to level the interfacing surface from the web stiffener to the flange (see Figure 107a). The steel shims were made from 3 1/2" x 5/8" x 1/8" AISI 4130 steel sheet. Interior web stiffeners required an additional shim to level the interfacing surface (see Figure 107b). These shims were made from 3 1/2" x 5/8" x 0.090" 2024-T3 aluminum sheet.

Figure 107: Web stiffener constructions

B.2 Fastener Arrangements

Observation of the NACA photos clearly indicates a square pattern of fasteners in the corners of the beam, a staggered, double row of fasteners along the edges of the plate, and riveted web stiffeners with a bolted, joggled overlap onto the top/bottom flanges. A similar arrangement was used for the NRC stiffened beams (see Figure 108).

Figure 108: General fastener arrangements

Due to the limitations of NRC's in-house manufacturing abilities, bolted fasteners was used along the flanged edges of the stiffened beam, and was used to secure the web stiffener to flange interface. Aluminum rivets was used to secure the web stiffeners to the main and tripled web.

Using the available working area on the flanges and stiffener, 1/2" bolts was used for all flange/plate connections, 3/16" rivets to secure all the web stiffeners to the web, and AN3 (3/16") steel bolts for the web stiffener-to-flange interface. From these dimensions pitch calculations have been checked and incorporated into the final part dimensions. The effective web dimensions are defined by the boundaries created by the innermost row of rivets. As such, the basic web dimensions and those of the additional components have been sized to maintain the desired bay sizes.

REVISIONS
DESCRIPTION
DATE

ISSUED FOR REFERENCE

DATE	10/1/81	BY	W. J. H. / J. H. H.
REVISION	1	DESCRIPTION	Top/Bottom Filler Plate
REVISION	2	DESCRIPTION	Top/Bottom Filler Plate
REVISION	3	DESCRIPTION	Top/Bottom Filler Plate
REVISION	4	DESCRIPTION	Top/Bottom Filler Plate
REVISION	5	DESCRIPTION	Top/Bottom Filler Plate
REVISION	6	DESCRIPTION	Top/Bottom Filler Plate
REVISION	7	DESCRIPTION	Top/Bottom Filler Plate
REVISION	8	DESCRIPTION	Top/Bottom Filler Plate
REVISION	9	DESCRIPTION	Top/Bottom Filler Plate
REVISION	10	DESCRIPTION	Top/Bottom Filler Plate

APPENDIX D: Test Beam Torque Pattern

Order	Bolt	30	60	90	Order	Bolt	30	60	90	Order	Bolt	30	60	90	Order	Bolt	30	60	90
1	B20				48	B33				95	C01				142	N02			
2	P20				49	B06				96	O02				143	N01			
3	I01				50	P34				97	F01				144	D02			
4	I02				51	P06				98	L02				145	B10			
5	A01				52	B34				99	L01				146	P30			
6	Q42				53	B15				100	F02				147	P10			
7	Q01				54	P25				101	P04				148	B30			
8	A42				55	P15				102	B36				149	B12			
9	B02				56	B25				103	B04				150	P28			
10	P38				57	B16				104	P36				151	P12			
11	P02				58	P24				105	A05				152	B28			
12	B38				59	P16				106	A38				153	A20			
13	A02				60	B24				107	Q05				154	Q23			
14	Q41				61	A11				108	Q38				155	Q20			
15	P01				62	Q32				109	A04				156	A23			
16	B39				63	Q11				110	A39				157	Q03			
17	B01				64	A32				111	Q04				158	A40			
18	P39				65	A12				112	Q39				159	A03			
19	Q02				66	Q31				113	B05				160	Q40			
20	A41				67	Q12				114	B35				161	A08			
21	P11				68	A31				115	P05				162	A35			
22	B29				69	Q21				116	P35				163	Q08			
23	B11				70	A22				117	B18				164	Q35			
24	P29				71	A21				118	P22				165	B14			
25	E01				72	Q22				119	P18				166	P26			
26	M02				73	G01				120	B22				167	P14			
27	M01				74	K02				121	P13				168	B26			
28	E02				75	K01				122	B27				169	P03			
29	A07				76	G02				123	B13				170	B37			
30	Q36				77	B09				124	P27				171	B03			
31	Q07				78	P31				125	A19				172	P37			
32	A36				79	P09				126	Q24				173	B08			
33	A17				80	B31				127	Q19				174	P32			
34	Q26				81	A09				128	A24				175	P08			
35	Q17				82	Q34				129	Q13				176	B32			
36	A26				83	Q09				130	A30				177	A18			
37	A16				84	A34				131	A13				178	Q25			
38	Q27				85	A10				132	Q30				179	Q18			
39	Q16				86	Q33				133	B17				180	A25			
40	A27				87	Q10				134	P23				181	A15			
41	A06				88	A33				135	P17				182	Q28			
42	Q37				89	J01				136	B23				183	Q15			
43	Q06				90	H02				137	Q14				184	A28			
44	A37				91	H01				138	A29				185	B19			
45	B07				92	J02				139	A14				186	P21			
46	P33				93	O01				140	Q29				187	P19			
47	P07				94	C02				141	D01				188	B21			

APPENDIX E: Test Beam Strain Gauge Placements

REV	DATE
1	10/10/10

ISSUED FOR REFERENCE	
PROJECT NO.	CLARE 3 3/2
STRAIN GAUGE LOCATIONS	STRAIN GAUGE LOCATIONS
STRAIN GAUGE TYPE	STRAIN GAUGE TYPE
STRAIN GAUGE SIZE	STRAIN GAUGE SIZE
STRAIN GAUGE MATERIAL	STRAIN GAUGE MATERIAL
STRAIN GAUGE MANUFACTURER	STRAIN GAUGE MANUFACTURER
STRAIN GAUGE INSTALLER	STRAIN GAUGE INSTALLER
STRAIN GAUGE DATE	STRAIN GAUGE DATE
STRAIN GAUGE LOCATION	STRAIN GAUGE LOCATION
STRAIN GAUGE DESCRIPTION	STRAIN GAUGE DESCRIPTION
STRAIN GAUGE COMMENTS	STRAIN GAUGE COMMENTS

APPENDIX F: Revised Margin of Safety Graphs

0.050" Aluminum: Primary Component Failure

GLARE 3 - 3/2: Primary Component Failure

GLARE 4 - 3/2: Primary Component Failure

Carleton Mini-Beam: Primary Component Failure

0.071" Aluminum: Primary Component Failure

APPENDIX G: Test #1 (0.050" 2024-T3 Aluminum) Results

Test #1 - 0.050" 2024-T3 Aluminum Displacement over Beam Length

Test #1 - 0.050" 2024-T3 Aluminum Corrected Displacement over Beam Length

Test #1 - 0.050" 2024-T3 Aluminum Deflection @ 39,954 lbf.

Test #1 - 0.050" 2024-T3 Aluminum Flange Strains

Test #1 - 0.050" 2024-T3 Aluminum Web Stiffener Strains

Test #1 - 0.050" 2024-T3 Aluminum Mohr's Circle Center Check

Test #1 - 0.050" 2024-T3 Aluminum Mohr's Circle Center Check (Initial Buckling)

NOTE:

Rosette 2 and
Rosette 4 are back-to-
back pairs and are
located on the centre
of the web

Rosette 1 and
Rosette 3 are back-to-
back pairs and are
located between the
centre of the web and
the upper corner of
the central bay

Test #1 - 0.050" 2024-T3 Aluminum Principal Web Strains, ϵ_1

NOTE:

Rosette 2 and Rosette 4
are back-to-back pairs
and are located on the
centre of the web

Rosette 1 and Rosette 3
are back-to-back pairs
and are located between
the centre of the web
and the upper corner of
the central bay

Test #1 - 0.050" 2024-T3 Aluminum Principal Web Strains, ϵ_2 Test #1 - 0.050" 2024-T3 Aluminum Maximum Shear Strains, $(\gamma/2)_{\max}$

Test #1 - 0.050" 2024-T3 Aluminum Vertical Shear Strains, $(\gamma/2)_{xy}$

Test 1 - 0.050" 2024-T3 Aluminum - Principal Strains

Test #1 - 0.050" 2024-T3 Aluminum Diagonal Tension Angle, α

APPENDIX H: Test #2 (GLARE 3 – 3/2) Results

Test #2 (GLARE 3- 3/2) Displacement over Beam Length

Test #2 (GLARE 3- 3/2) Corrected Displacement over Beam Length

Test #2 (GLARE 3- 3/2) Flange Strains

Test #2 (GLARE 3- 3/2) Support-End Flange Strains

Test #2 (GLARE 3- 3/2) Web Stiffener Strains

Test #2 (GLARE 3 - 3/2) Mohr's Circle Center Check

Test #2 (GLARE 3 - 3/2) Mohr's Circle Center Check (Initial Buckling)

Test #2 (GLARE 3- 3/2) Principal Web Strains, ϵ_1

Test #2 (GLARE 3- 3/2) Principal Web Strains, ϵ_2 Test #2 (GLARE 3- 3/2) Maximum Shear Strains, $(\gamma/2)_{\max}$

Test 2 - GLARE 3 - 3/2 - Principal Strains

Test #2 (GLARE 3- 3/2) Diagonal Tension Angle, α

APPENDIX I: Test #3 (GLARE 3 – 3/2) Results

Test #3 (GLARE 3- 3/2) Displacement over Beam Length

Test #3 (GLARE 3- 3/2) Corrected Displacement over Beam Length

Test #3 (GLARE 3- 3/2) Flange Strains

Test #3 (GLARE 3- 3/2) Support-End Flange Strains

Test #3 (GLARE 3- 3/2) Web Stiffener Strains

Test #3 (GLARE 3 -3/2) Mohr's Circle Center Check (Initial Buckling)

Test #3 (GLARE 3- 3/2) Principal Web Strains, ϵ_1 Test #3 (GLARE 3- 3/2) Principal Web Strains, ϵ_2

Test #3 (GLARE 3- 3/2) Maximum Shear Strains, $(\gamma/2)_{\max}$

NOTE:

Rosette 2 and Rosette 4 are back-to-back pairs and are located on the centre of the web

Rosette 1 and Rosette 3 are back-to-back pairs and are located on the centre of the inboard web

Test 3 - GLARE 3-3/2 - Principal Strains

Test #3 (GLARE 3- 3/2) Diagonal Tension Angle, α

APPENDIX J: Test #4 (GLARE 4 – 3/2) Results

Test #4 (GLARE 4- 3/2) Displacement over Beam Length

Test #4 (GLARE 4- 3/2) Corrected Displacement over Beam Length

Test #4 (GLARE 4- 3/2) Flange Strains

Test #4 (GLARE 4- 3/2) Support-End Flange Strains

Test #4 (GLARE 4- 3/2) Web Stiffener Strains

Test #4 (GLARE 4 - 3/2) Mohr's Circle Center Check (Initial Buckling)

Test #4 (GLARE 4 - 3/2) Principal Web Strains, ϵ_1 Test #4 (GLARE 4 - 3/2) Principal Web Strains, ϵ_2

Test #4 (GLARE 4- 3/2) Maximum Shear Strains, $(\gamma/2)_{\max}$

Test 4 - GLARE 4-3/2 - Principal Strains

Test #4 (GLARE 4- 3/2) Diagonal Tension Angle, α

APPENDIX K: Carleton Mini-Beam Results

Carleton Beam Tip Deflection

Carleton Beam Average U1 Web Stiffener Strain

Carleton Beam Average U2 Web Stiffener Strain

Carleton Beam Average U3 Web Stiffener Strain

Carleton Beam Average Flange Strains

Carleton Beam Average Mohr's Circle Center Check

Carleton Beam Average Principal Tensile Strain

Carleton Beam Average Principal Compressive Strain

Carleton Beam Average Maximum Shear Strain

APPENDIX L: Test #5 (0.071" 2024-T3 Aluminum) Results

Test #5 - 0.071" 2024-T3 Aluminum Displacement over Beam Length

Test #5 - 0.071" 2024-T3 Aluminum Displacement - LVDT #1 vs. LVDT #8

Test #5 - 0.071" 2024-T3 Aluminum Web Stiffener Strains **Test #5 - 0.071" 2024-T3 Aluminum Central Bay Flange Strains**

Test #5 - 0.071" 2024-T3 Aluminum Support End Flange Strains **Test #5 - 0.071" 2024-T3 Aluminum Beam Corner Concentrations**

Test #5 - 0.071" 2024-T3 Aluminum 3 Bay Mohr's Circle Centers **Test #5 - 0.071" 2024-T3 Aluminum 3 Bay Principal Tensile Strains**

Test #5 - 0.071" 2024-T3 Aluminum 3 Bay Principal Compression Strains **Test #5 - 0.071" 2024-T3 Aluminum 3 Bay Maximum Shear Strain**

Test #5 - 0.071" 2024-T3 Aluminum Central Bay Mohr's Circle Centers **Test #5 - 0.071" 2024-T3 Aluminum Central Bay Principal Tensile Strain**

Test #5 - 0.071" 2024-T3 Aluminum Central Bay Principal Compressive Strain **Test #5 - 0.071" 2024-T3 Aluminum Central Bay Maximum Shear Strain**

Test #5 - 0.071" 2024-T3 Aluminum
Central Bay Vertical Shear Flow (Portal Frame Effect)