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Simulation of High Frequency Plasma Oscillations within Hall Thrusters

by

**Aaron Kombai Knoll,
B. Eng.**

A thesis submitted to the Faculty of Graduate Studies and Research

in partial fulfilment of the requirements for the degree of

Master of Applied Science

Ottawa-Carleton Institute for Mechanical and Aerospace Engineering

Department of Mechanical and Aerospace Engineering

Carleton University

Ottawa, Ontario

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Simulation of High Frequency Plasma Oscillations within Hall Thrusters

submitted by

Aaron Kombai Knoll, B. Eng.

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the degree of Master of Applied Science**

Thesis Supervisor

Chair, Dept. of Mechanical and Aerospace Engineering

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Abstract

The purpose of this project is to study high frequency plasma oscillations that occur within Hall thrusters. This is approached in two ways: through a numerical simulation, and experimental research conducted on a laboratory Hall thruster. This study seeks to address an inherent problem with existing Hall thruster simulations related to the electron transport process. The goal of the current project is to determine how significant high frequency plasma oscillations are to the electron transport.

Experimental research on high frequency plasma oscillations was carried out at the Stanford University Plasma Dynamics Lab. The experimental data gathered at Stanford forms a basis from which to compare the simulation results. In general, the trends of the simulation parameters agree with experiment. However, there were notable discrepancies in terms of the ion velocity and electron density. Despite this shortcoming, the simulation was successful at capturing the effects of high frequency plasma density oscillations.

Acknowledgments

I am grateful for the support and encouragement of my supervisor Professor Tarik Kaya, and for the guidance I received during my work at Stanford from Professor Mark Cappelli. A large portion of credit also goes to Dr. Eduardo Fernandez who developed the original version of the Hall thruster simulation code that this thesis relied upon heavily. These three individuals are truly the giants whose shoulders I stood upon during this project. I believe that their combined support can be best summed up in a quote by John Irving: “For what I may have managed to get right, the credit belongs to them; if there are errors, the fault is mine.”

I owe my parents incalculable thanks for laying the groundwork. The variety of my background and life experiences has brought me where I am. Most of all, thanks to my wife Linda who has been there to support me and keep me focussed. You helped me more than you know.

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Nomenclature

i. Nomenclature

<u>Symbol</u>	<u>Definition</u>
B	Magnetic induction vector
C	Total current density
D	Displacement current density
e	Electron charge
E	Electric field strength
H	Magnetic field intensity
J	Current density
k	Boltzmann's constant
m	Mass
N	Particle number density
P	Pressure
q	Particle charge
Q	Heat flux
R	Force vector
r	Radial displacement
T	Temperature
t	Time
v	Velocity
V	Peculiar velocity
V_d	Discharge voltage

Nomenclature

ν_{SN}	Collision frequency for momentum transfer
z	Axial displacement
ϵ	Permiativity
ϵ_0	Permittivity of free space
η	Charge density
θ	Azimuthal displacement
κ	Permiativity dyadic
τ	Mean time between collisions
Φ	Potential
ψ	Pressure tensor
ω_{ce}	Electron cyclotron frequency
ω_{pe}	Plasma frequency

Subscripts:

r	Radial coordinate
z	Axial coordinate
θ	Azimuthal coordinate

Superscripts:

e	Electrons
i	Ions
n	Neutral particles

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Section 1: Introduction

1.0 Introduction

The goal of this project is to study the high frequency plasma oscillations that occur within Hall thrusters. The motivation for this work arises from challenges encountered when attempting to predict the performance of Hall thrusters mathematically or by simulation. It has been found that the conductivity of the plasma within a Hall thruster is substantially higher than traditional plasma physics models suggest. These traditional models are based on the macroscopic electron momentum equations as derived from the collisional Boltzmann equation. This problem is addressed by existing Hall thruster simulations by using an experimentally determined correction factor for the anomalous electron transport. However, this coefficient is highly dependent on the geometry and operating regime of the thruster. The anomalous transport coefficient severely limits the flexibility and usefulness of these simulations.

It has been suggested in many research studies that the anomalous electron transport is related to high frequency oscillations that occur in the plasma density. These oscillations are not random and tend to occur at specific frequencies corresponding with natural instabilities of the plasma. This project will focus on predicting these oscillations and quantifying their significance on the plasma conductivity within the Hall thruster. Plasma oscillations occurring between 1MHz and 500MHz are the focus of the work conducted during this project.

The first component of this project involved experimental investigation of plasma oscillations within Hall thrusters. This experimental work was carried out at the Stanford University Plasma Dynamics Lab (SPDL). Langmuir probes were used to directly observe the plasma density fluctuations at the exit plane of a Hall thruster. The main challenge to this experimental work was an inherent impedance mismatch between the plasma probes and the coaxial cables connecting the probes to the data acquisition equipment. The impedance mismatch caused the high frequency components of the signal to be filtered out. This problem was resolved by designing high frequency impedance matching electronics. The final experimental setup was successful at measuring frequency oscillations up to 500 MHz. The experimental results were analysed using power spectral density plots.

The second component of this project involved a Hall thruster simulation that was designed to capture the high frequency plasma oscillations that occur within Hall thrusters. The development of this simulation was aided by Dr. Eduardo Fernandez with participation from Stanford University. This simulation has two main differences from traditional Hall thruster models. First, it solves the governing equations in two dimensions along the azimuth and axial coordinate directions. Second, the simulation makes no use of an anomalous electron transport coefficient. Most existing simulations are 1-dimensional or 2-dimensional in the radial-axial plane. The new coordinate system was selected so that instabilities in the plasma that propagate azimuthally could be captured. The assumption of axis symmetry used in traditional Hall thruster models may

explain why these oscillations have not been previously reproduced. This simulation uses a hybrid particle-in-cell solver. The electrons are treated as a continuum, and the ions and neutrals are treated as discrete particles.

The simulation was run under a number of different operating regimes. The results of this study indicate that the simulation was indeed able to capture high frequency plasma oscillations. The basic frequencies at which these oscillations occurred closely matched experimental observations collected in this and other studies. These results are promising because they suggest that a simulation in this coordinate system has the potential to replicate the anomalous electron transport phenomena. The plasma density oscillations computed by the simulation were used to statistically determine the anomalous electron transport coefficient found in traditional Hall thruster simulations. The agreement between the simulation results and the experimentally determined coefficient were good. The oscillation data was also used to statistically predict the electron current. The results appeared reasonable and in good agreement with experimental values of discharge current.

Despite the successful predictions of plasma density oscillations, many of the time averaged plasma parameters differed from experimental values. The cause of this discrepancy was linked to unusual ‘spikes’ that occur within the plasma potential field. The likely explanation for these unusual features is that the electron energy equation was not solved explicitly during the simulation. Rather, electron temperatures were obtained

from experiment in order to simplify the model. Including the electron energy equation may help to damp the plasma potential spikes. This was not attempted during this project and is an important item for future investigation.

This thesis starts by providing a brief overview of the Hall thruster in section 2. Next, a review of available literature is discussed in section 3. This includes literature related to the operating characteristics of Hall thrusters, plasma oscillations, anomalous electron transport, and Hall thruster simulation techniques. Section 4 presents the experimental work that was conducted at Stanford University related to measuring the high frequency plasma oscillations within Hall thrusters. Section 5 describes the Hall thruster simulation that was developed during this project. Section 6 presents the results of the simulation and compares these results with experimental data. Section 7 summarizes the conclusions gained as a result of this project. Finally, section 8 recommends tasks that were not attempted during this project but were deemed important for future investigation.

Section 2: Overview of Hall-field Thrusters

2.0 Overview of Hall-field Thrusters

This section provides a basic introduction to Hall-field thrusters and explains how they work. Hall thrusters are a type of stationary plasma propulsion device used for spacecraft applications. The Hall thruster develops its thrust from the momentum of ions which are emitted from the device at high velocities. The concept of the Hall thruster is not new. Hall thrusters have been used for spacecraft applications since the early 1960's in the former Soviet Union. Research into Hall thrusters in North America is a recent development by comparison. North America has traditionally focussed its research efforts into gridded ion thrusters. However, comparable performance levels have been demonstrated between gridded ion thrusters and Hall thrusters. The Hall thruster offers a desirable balance between thrust and power requirements.

Electrical propulsion devices such as Hall thrusters have substantially higher performance characteristics than traditional chemical rocket propulsion. The thermal efficiency of the Hall thruster is typically above 50% with an I_{sp} between 1500s and 2000s. This compares with an I_{sp} of approximately 400s for a chemical propulsion system. The savings in propellant mass are enormous. For a given delta velocity requirement the increased performance makes the difference between a few hundred kilograms of propellant compared with just a couple kilograms for the Hall thruster. However, the thrust produced by electrical propulsion devices are far smaller than their chemical propulsion counterparts. Modern Hall thrusters exert a maximum force of just a couple

microNewtons, compared with hundreds of Newtons for chemical propulsion systems. This technology is well suited for satellite station keeping applications and deep space exploration.

The basic concept behind the Hall thruster is as follows. The propellant is ionized by a field of electrons that are contained within the thruster. The electrons are trapped within the thruster by perpendicular electric and magnetic fields. Once the propellant particles are ionized they are rapidly accelerated by the electric field and leave the thruster at tens of kilometres per second. The ions are far more massive than the electrons and are virtually unaffected by the magnetic field. The electrons that are created by the ionization process are trapped by the containment field and subsequently ionize more propellant particles in a cascade process. It should be noted that the Hall thruster can only operate in vacuum and near vacuum conditions. A vacuum chamber is required to experimentally investigate the Hall thruster on earth.

A schematic view of a Hall thruster is shown in figure 2.1. The propellant gas is injected through small holes in a metallic anode plate. A small portion of the propellant is diverted toward the cathode. The function of the cathode is to release electrons that trigger the ionization process. An electric field is established between the anode and cathode by keeping the anode plate at a positive potential of a couple hundred volts relative to the cathode. The cavity within the Hall thruster is known as the acceleration channel. The walls to the acceleration channel act to contain the plasma in the radial

direction. The channel walls are usually made out of a ceramic non-conductive material. The material is selected to be resistant to corrosion and to withstand extreme temperatures. The plasma within a Hall thruster is known to be a highly corrosive substance. The final component of the Hall thruster is the electromagnets. The electromagnets are oriented so that the magnet along the thruster axis is opposite to the magnets around the circumference. This creates a magnetic field in the radial direction of the thruster.

Figure 2.1. Schematic Diagram of a Hall Thruster

A schematic diagram of the Hall thruster that shows the basic electric power loops is shown in figure 2.2. The cathode is typically held at ground potential. The cathode releases electrons that travel toward the anode. The plasma has a resistance based on the

conductivity across the magnetic field lines. The discharge current and power requirements of the Hall thruster are established based on the resistance of the plasma. The conductivity therefore is a function of both the operating characteristics of the thruster and magnetic field strength. A separate circuit supplies power to the electromagnets. The power requirements for the electromagnets are far smaller than for the discharge loop.

Figure 2.2. Electric Circuit Loops within a Hall Thruster

The propellant used for the vast majority of Hall thrusters is Xenon. This propellant was selected for its relatively large ion mass, which acts to increase the thrust of the device. Another advantage is that Xenon is a non-reactive substance that can be easily stored for

long periods. Also, electrons are easily ionized from the outer electron shells of Xenon. Other propellants have also been considered for use with Hall thruster. In particular, solid Bismuth thrusters are currently being investigated in many studies. The work in this project will focus on Xenon Hall thrusters.

A photograph of an experimental Hall thruster is shown in figure 2.3. This Hall thruster has been used extensively for studies at Stanford University, and was the thruster selected for the experimental component of this project. This figure shows the Hall thruster disconnected from the experimental test setup. The following image (figure 2.4) shows the thruster in operation.

Figure 2.3. Stanford Hall Thruster

Figure 2.4. Stanford Hall Thruster in Operation

Section 3: Review of Available Literature

3.0 Review of Available Literature

This section provides a summary of the literature that was reviewed in preparation for this project. The literature can be divided into four topic areas. The first is the operating characteristics of the Hall thruster, described in section 3.1. This topic involves the steady state operating characteristics including the power, discharge voltage, current, and thrust. This sub-section establishes the dependence of the operating characteristics on the thruster geometry and operating regime. It also shows experimental measurements taken within the plasma discharge of a standard Hall thruster. The material discussed in section 3.1 is important because it provides experimental data to compare with simulation results. It also helps define initial conditions and boundary conditions for the simulator. Finally, it identifies regimes of operation for which the Hall thruster is physically capable of achieving a stable discharge.

The second topic area is plasma oscillations within Hall thrusters, described in section 3.2. This topic involves the various categories of oscillations that occur within the Hall thruster. It identifies what oscillations can be expected within each operating regime. Finally, it provides experimental data to characterize the oscillation properties with changes in discharge voltage and location within the thruster. This material is important because it provides experimental data to compare with simulation results. Also, it will help to correctly identify and classify oscillations that are observed in the simulation.

Finally, it will help suggest time step sizes and time capture windows appropriate for the simulator.

The third topic area is anomalous electron transport, described in section 3.3. This topic is concerned with one of the most significant obstacles to Hall thruster simulation: the enhanced drift of electrons toward the anode as compared to classical theory. Experimental results are provided to quantify this phenomenon. This provides a basis of comparison to see how well the simulation reproduces the anomalous electron transport effect.

The final topic area is Hall thruster simulation, described in section 3.4. This topic involves different approaches to numerically modelling the Hall thruster. It also describes some of the challenges encountered with previous simulation attempts. Results are presented from a typical two-dimensional Hall thruster simulation and compared to experimental observations. This material is needed because it provides the background and framework for developing a simulation within this project. It also identifies the challenges and hurdles that need to be addressed by the current work.

3.1 Operating Characteristics of Hall Thrusters

A number of Hall thruster configurations have been developed and extensively characterised for flight applications and laboratory research. Hall thrusters are generally

categorized according to their outer diameter. The name of the thruster indicates the outer diameter of the device in millimetres. For instance, an SPT-100 thruster has an outer diameter of 100mm. The most important performance parameters for Hall thrusters are the specific impulse (I_{sp}), the overall efficiency (η), the thrust, and the power requirements. The performance parameters of a number of different Hall thrusters are provided in table 3.1.

Thruster Name	Specific Impulse [s]	Overall Efficiency [%]	Thrust [N]	Power [kW]
SPT-50	2000	40	0.019	0.3
SPT-70	2000	45	0.040	0.7
SPT-100	1600	50	0.080	1.4
SPT-140	1700	60	0.290	4.5

Table 3.1. Performance Parameters for Various Hall Thruster Configurations

It can be observed from the previous table that the efficiency and the thrust both tend to increase at higher outer diameters. However, the power requirements of these thrusters also increase significantly.

There are several design parameters besides the outer diameter that affect the performance of the Hall thruster. In particular the axial gradient in the magnetic field and the chamber length are known to influence the performance characteristics. A study

conducted by Ehedo and Escobar [1] looks in depth at the significance of each of these parameters on the overall performance of a Hall thruster.

Every Hall thruster has the capacity to operate under several operating regimes. An operating regime is defined as a range of discharge current versus applied magnetic field under which sustained operation of the thruster is possible. Efforts were made in early Russian studies, Tilinin [2], to divide the operating regimes into categories. The categories were established according to plasma oscillations that could be observed under each range of applied voltage. Tilinin reported the operating regimes for a 90 mm outer diameter Hall thruster as indicated on the current versus magnetic field plot shown in figure 3.1.

Figure 3.1. Operating Regimes of a Hall Thruster [2]

The characteristics of each operating regime shown in the previous figure can be described as follows. The first regime (I) is called the collisional conductivity regime. This regime is distinguished by the fact that both high and low frequency oscillations within the plasma cannot be experimentally observed. If oscillations do exist within the plasma, they are weak enough to occur below the measurement threshold of experimental equipment. The collisional conductivity regime is so called because the motion of the electrons toward the anode is thought to be driven primarily by the collisions between the electrons and neutral particles, ions, and the channel walls.

The second regime (II) is known as the regular electron drift wave regime. This regime is characterised by the occurrence of a plasma oscillation that propagates in the azimuthal direction known as the “spoke” mode drift wave. This wave commonly occurs between 20 kHz and 60 kHz.

The third regime (III) is known as the transition regime. This regime is further divided into two sections: IIIa and IIIb. Within the IIIa regime, the predominant plasma oscillation is of relatively low frequency in the range of 1 kHz to 20 kHz. This oscillation was commonly referred to as the “loop”, or “circuit” oscillations in Russian literature. In more recent studies by Boeuf and Garrigues [3] it has been called the “breathing mode” oscillation. The IIIb regime is somewhat similar to the IIIa regime, but is distinguished by a sudden rise in the level of higher frequency oscillations between 0.5

MHz to 10 MHz. The IIIb regime also has a significant increase in the medium frequency plasma oscillations between 70 kHz to 500 kHz. These oscillations were referred to as “transit-time” oscillations by Tilinin [2].

The fourth regime (IV) is known as the optimum regime. In this regime many of the Hall thruster parameters reach their maximum value. In particular, the proportion of the total discharge current that is caused by the ion transport is maximized. In this regime the breathing mode oscillations and the spoke oscillations decrease. This is accompanied by a moderate increase for many of the medium to high frequency plasma oscillations.

The fifth regime (V) is known as the regime of macroscopic instability. There is a notable jump in all thruster parameters at the start of this regime (Va). This regime is characterized by the sudden re-emergence of the breathing mode oscillations. These low frequency oscillations are so violent that they can be observed visually, and often cause the discharge to be completely extinguished.

The final regime (VI) is known as the magnetic saturation regime. In this regime the increase of the magnetic field has little effect on the parameters of the Hall thruster. The medium and high frequency oscillations reach their highest values in this regime.

In addition to the applied magnetic field, the voltage difference between the anode and cathode also has a significant impact on the operating characteristics of the Hall thruster.

In studies performed by the University of Michigan [4] and at the French research facility of PIVOINE [5, 6, 7] the influence of applied voltage on the performance parameters of Hall thrusters was documented. The performance parameters of interest included the discharge current, thrust, efficiency, and specific impulse. Each of these parameters has been plotted as a function of applied voltage in figures 3.2, 3.3, 3.4, and 3.5 below. The thruster used in the University of Michigan studies was a 5 kW class thruster with a 169 mm diameter acceleration channel and a nominal specific impulse of 2200s [4]. This thruster was tested at three different flow rates: 58 sccm, 79 sccm, and 105 sccm. The thruster used in the PIVOINE studies was a standard SPT-100 type Hall thruster: 1 kW class thruster with a 100 mm channel diameter and a nominal specific impulse of 1600s [5].

Figure 3.2. Discharge Current versus Applied Voltage Characteristics

Figure 3.3. Thrust versus Applied Voltage Characteristics

Figure 3.4. Total Efficiency versus Applied Voltage Characteristics

Figure 3.5. Specific Impulse versus Applied Voltage Characteristics

Studies have also been performed to document the parameters of a Hall thruster as they vary along the length of the device. These measurements are typically conducted at the mean radius of the acceleration channel. One such study was conducted by Stanford University by N. B. Meezan et al. [8]. The parameters that were investigated during this study included the following:

- Magnetic field strength
- Plasma potential
- Electric field strength
- Electron temperature
- Ion velocity
- Neutral and electron density

The axial profiles of these parameters have been provided in their original form below (see figures 3.6 – 3.12). The Hall thruster used in these experiments was a custom built low power device. The acceleration channel of this thruster was 90mm in diameter, 11mm in width, and 80 mm in length.

Figure 3.6. Axial Profile of Radial Magnetic Field [8]

Figure 3.7. Axial Profile of Electric Field Strength [8]

Figure 3.8. Axial Profile of Plasma Potential [8]

Figure 3.9. Axial Profile of Electron Temperature [8]

Figure 3.10. Axial Profile of Ion Velocity [8]

Figure 3.11. Axial Profile of Neutral Xenon Density [8]

Figure 3.12. Axial Profile of Electron Density [8]

3.2 Plasma Oscillations within Hall Thrusters

Plasma oscillations refer to a fluctuation of the plasma properties such as a change in the density and temperature. These oscillations are an inherent phenomenon within Hall thrusters and they exist within all modes of operation. Depending on the nature of the plasma oscillations, they can propagate through the plasma in various directions and at various speeds. Some of these oscillations propagate predominantly along the axis of the

thruster and are referred to as axial waves. Other types of oscillations propagate mostly around the circumference of the thruster and are known as azimuthal waves. Table 3.2 below summarizes the major categories of plasma oscillations that exist within Hall thrusters.

<u>Frequency range</u>	<u>Name</u>	<u>Propagation direction</u>	<u>Reference to more information</u>
1kHz – 20kHz	Breathing mode oscillations	Axial	Boeuf and Garrigues [3]
5kHz – 25kHz	Rotating spoke oscillations	Azimuthal	E. Y. Choueiri [9]
20kHz – 60kHz	Gradient-induced oscillations	Azimuthal	E. Y. Choueiri [9]
70kHz – 500kHz	Transient-time oscillations	Axial	Y. Esipchuk et al. [10]
0.5MHz – 500MHz	High frequency oscillations	Varies	A. Litvak [11]
~1GHz	Electron cyclotronic oscillations	Varies	V. I. Baranov et al. [12]
100MHz – 10GHz	Langmuir oscillations	Varies	V. I. Baranov et al. [12]

Table 3.2. Types of Oscillations within Hall Thrusters

Many experimental studies have been conducted to determine the characteristics of the lower frequency plasma oscillations (less than 1 MHz). However, relatively little experimental data exists for the high frequency plasma oscillations. One of the reasons for the relative lack of experimental data is the technical challenges associated with measuring these oscillations [13].

The type of plasma oscillations that exist when running a Hall thruster depend on a number of factors. These include the magnetic field strength, applied voltage, propellant flow rate, and the geometric parameters of the thruster. A change in any of these parameters results in significant changes to the properties of the oscillations. The plasma oscillations can also vary with location along the axis of the acceleration channel and downstream into the plasma plume. A concise summary of the plasma oscillations that exist at various operating conditions of the Hall thruster was presented in a study by E. Y. Choueiri [9]. This summary is given in table 3.3 below. The relative intensities of each oscillation mode are given on a scale of 1 to 10, with 1 being the smallest and 10 being the largest amplitude.

Regime	<u>I</u>	<u>II</u>	<u>IIIa</u>	<u>IIIb</u>	<u>IV</u>	<u>V</u>	<u>VI</u>
1 – 20kHz	1	1	8	8	3	10	4
20 – 60kHz	0	6	0	4	2	0	0
20 – 100kHz	1	5	4	6	7	6	4
70 – 500kHz	1	4	4	7	7	6	8
2 – 5MHz	1	3	3	4	5	1	1
0.5 – 10MHz	1	3	3	4	5	5	6
10 – 400MHz	1	3	2	3	4	5	5

Table 3.3. Characteristics of Plasma Oscillations within Hall Thrusters [9]

There have been several studies conducted to characterize the properties of the oscillations at various operating conditions and at various locations along the axis of the thruster. One such study was conducted at Stanford University in 2001 by Chesta et al. [14]. The Hall thruster used in these experiments was a custom built low power device with an acceleration channel 90mm in diameter, 11mm in width, and 80 mm in length.

The measurements were collected with a set of negatively biased electrodes (Langmuir probes) inserted directly into the plasma. The results of this study were presented in the form of dispersion maps of the magnitude of the electron density oscillations within the plasma. This data has been included for reference below. The first set of images (figures 3.13 to 3.16) show the relative magnitude of the oscillations as a function of the discharge voltage at four locations along the axis of the thruster: 12.7mm, 0mm, -12.7mm, and -25.4mm. The zero position for these axial measurements is at the exit of the acceleration channel.

Figure 3.13. Spectral Map as a Function of Discharge Voltage ($x = 12.7$ mm) [14]

Figure 3.14. Spectral Map as a Function of Discharge Voltage ($x = 0$ mm) [14]

Figure 3.15. Spectral Map as a Function of Discharge Voltage ($x = -12.7$ mm) [14]

Figure 3.16. Spectral Map as a Function of Discharge Voltage ($x = -25.4$ mm) [14]

A similar set of results was collected during the study that showed the dependence of the plasma oscillations on the position along the axis of the thruster. A second set of images (figures 3.17 to 3.21) show the relative magnitude of the oscillations as a function of the axial position of the probe for five discharge voltage conditions: 184V, 161V, 128V, 100V, and 86V.

Figure 3.17. Spectral Map as a Function of Axial Position (Voltage = 184 V) [14]

Figure 3.18. Spectral Map as a Function of Axial Position (Voltage = 161 V) [14]

Figure 3.19. Spectral Map as a Function of Axial Position (Voltage = 128 V) [14]

Figure 3.20. Spectral Map as a Function of Axial Position (Voltage = 100 V) [14]

Figure 3.21. Spectral Map as a Function of Axial Position (Voltage = 86 V) [14]

For the purpose of clarity, the data presented in the intensity plots of figures 3.13 through 3.16 has been extracted to form two-dimensional plots of the amplitude versus frequency of the oscillations. These plots were all made at the 150V discharge condition, which corresponded to the region of greatest activity. These new plots clearly illustrate the frequencies at which the plasma oscillations are greatest at four axial locations: 12.7mm, 0mm, -12.7mm, and -25.4mm.

Figure 3.22. Plasma Oscillation Intensity at $x = 12.7\text{mm}$ (150V discharge)

Figure 3.23. Plasma Oscillation Intensity at $x = 0.0\text{mm}$ (150V discharge)

Figure 3.24. Plasma Oscillation Intensity at $x = -12.7\text{mm}$ (150V discharge)

Figure 3.25. Plasma Oscillation Intensity at $x = -25.4$ mm (150V discharge)

High frequency plasma oscillations occur in the range of approximately 500 kHz to 500MHz. There is less experimental data available on high frequency plasma oscillations than for the low frequency oscillations. However, high frequency oscillations are known to have a significant impact on the overall performance of the Hall thruster [15]. The impact of the High frequency oscillations on the Hall thruster performance will be discussed in more detail in section 3.3.

High frequency plasma oscillations exhibit the same dependence on the operating parameters of the Hall thruster as the low frequency oscillations. Although less experimental data exists, there have been some efforts to characterize the behaviour of the high frequency oscillations at various operating conditions. One such study was conducted at Ecole Polytechnique in France by Guerrini and Michaut [16]. These

experiments were performed on a standard SPT-50 type thruster with an internal diameter of 28 mm, and a channel length of 25mm. The measurements were made using a spectrum analyser to detect rapid changes in the electromagnetic radiation emissions from the Hall thruster discharge. Their experimental observations have been given below for reference purposes.

Figures 3.26 though 3.28 demonstrate how the frequency spectra of the plasma oscillations changes with a variation in the voltage, propellant flow rate, and radial magnetic field. Each peak represented on these spectral density plots corresponds to a favoured mode of oscillation within the plasma.

Figure 3.26. High Frequency Spectra for two Discharge Voltages [16]

Figure 3.27. High Frequency Spectra for two Xenon Flow Rates [16]

Figure 3.28. High Frequency Spectra for two Radial Magnetic Fields [16]

3.3 Anomalous Electron Transport

Early in the development of Hall thrusters it was discovered that the electrons within the discharge tended to drift toward the anode faster than could be predicted with classical transport models. This phenomenon was termed anomalous electron transport. Anomalous electron transport plays a major role in modern numerical simulations of Hall thrusters. In practical terms, it is an ad-hoc factor that has been introduced into simulations to produce reasonable results.

A study was conducted at Stanford University by N. B. Meezan et al. [8] to experimentally measure the cross-field mobility of electrons within a Hall discharge. The methodology followed was to reduce the cross field electron mobility to a simple expression based on the time averaged plasma properties that could be measured within the thruster. This formula was based on the momentum equation for weakly ionized plasma.

The equation shown below (see equation 3.1) was derived in a paper by Meezan [8], and shows how the momentum equation can be related to measurable plasma properties. This equation relates the electron mobility to the following measurable parameters: electron number density, electric field strength, and magnetic field strength. An inverse proportionality was established between the electron transport and the Hall parameter

($\omega_{ce}\tau$). The Hall parameter was experimentally determined using time averaged data from the Hall thruster, and subsequently compared to the value predicted from classical electron theory.

$$J_z^e = eN^e v_z^e = eN^e \left(\frac{E_z}{B_r} \right) \frac{v_{SN}}{\omega_{ce}} = eN^e \left(\frac{E_z}{B_r} \right) \frac{1}{\omega_{ce}\tau} \quad (3.1)$$

Where:

- J_z^e = Current in the axial direction
- e = Electron charge
- N^e = Electron number density
- v_z^e = Electron drift velocity
- E_z = Axial electric field strength
- B_r = Radial magnetic field strength
- v_{SN} = Momentum transfer collision frequency for neutral-electron collisions
- ω_{ce} = Electron cyclotron frequency
- τ = Mean time between collisions

Classical magnetohydrodynamic analysis predicts the cross-field transport of electrons based on collision scattering. Equations 3.2 through 3.5 below were taken from a book by E. H. Holt and R. E. Haskell [17]. These equations predict the cross field electron movement parallel to the axis of the Hall thruster.

$$J_z^e = \left(\frac{\sigma}{1 + \left(\frac{\sigma B_r}{n_e e} \right)} \right) E_z \quad (3.2)$$

Where:

$$\frac{1}{\sigma} = \frac{1}{\sigma_{ei}} + \frac{1}{\sigma_{en}} \quad (3.3)$$

$$\sigma_{ei} = \frac{1}{N^e Q_{ei} \sqrt{8kT/\pi m^e}} \quad (3.4)$$

$$\sigma_{en} = \frac{1}{N^n Q_{en} \sqrt{8kT/\pi m^e}} \quad (3.5)$$

B_r = Radial magnetic field strength

E_z = Axial electric field strength

N^e = Number density of electrons

e = Electron charge

N^n = Number density of neutral atoms

Q_{ei} = Collision cross section of electron-ion collisions

Q_{en} = Collision cross section of electron-neutral collisions

k = Boltzmann's constant

T = Temperature

m^e = Electron mass

The paper by Meezan [8] compares the experimental Hall parameter to the classical prediction. Figure 3.29 shows the results as they were presented in this paper. It can be

observed from this figure that the experimental values depart significantly from the classical prediction toward the exit of the discharge channel.

Figure 3.29. Comparison of the Measured and Classical Values of the Inverse Hall Parameter [8]

Another important finding of this paper was that the electron mobility is driven to a large extent by the plasma fluctuations that exist within Hall thrusters. This same conclusion was echoed in earlier studies conducted in the former Soviet Union by V. I. Baranov et al. [12]. The relation between electron mobility and plasma oscillations is particularly significant to developing successful Hall thruster simulations. It implies that if the simulation does not capture the plasma oscillations it will under estimate the mobility of the electrons. For instance, the azimuthal oscillations that exist within Hall thrusters can not be captured by existing one-dimensional simulations and two-dimensional simulations in the radial-axial plane. These simulations then require an artificial modifier to account for the anomalous electron transport.

3.4 Hall Thruster Simulation

Work on Hall thruster simulators generally fall into two categories: modelling the plume of the thruster, and modelling the interior of the channel. The motivation for studying the thruster plume is to study the effects of sputtering deposition on spacecraft materials and critical spacecraft components. The motivation for modelling the interior of the channel is to optimize the performance of the thruster, realistically model space conditions for performance predictions, and develop a tool to aid with experiments. This project is concerned primarily with modelling the performance characteristics of the Hall thruster. Therefore, this section will focus on efforts to numerically model the interior of the discharge channel.

At the current time, the biggest obstacle to developing reliable simulations of Hall thrusters is the poorly understood electron conductivity from the anode toward the cathode of the thruster [8]. The approach to date has been to introduce an artificial factor, obtained from experiments, to correct for the anomalous electron transport phenomenon. This limits the usefulness of the simulation as this factor is highly sensitive to the operating parameters of the thruster, and makes it virtually impossible to predict the performance of new thruster geometries or operating regimes.

There are two numerical approaches to modelling plasma within Hall thrusters. The first method is to model the plasma as a continuum by solving the governing

magnetohydrodynamic equations including the continuity, momentum, and energy equations. A variety of mathematical techniques exist for discretizing the partial differential equations that describe the plasma. Two examples from literature include using the finite difference method, as illustrated in the work by J. M. Fife [18], or by using the finite element method as shown in the work by S. Roy and B. P. Pandey [19].

The second method to simulate plasma is to model it as discrete particles. This involves tracking each particle and statistically calculating the properties at every time step. In practice, the particles are represented as groups of many particles called ‘super-particles’. The advantage of this is that only the information associated with each super-particle needs to be stored and updated in the computer. This technique is described in more detail in the work by Fife [18].

In a hybrid simulation both methods of modelling plasma are used simultaneously. The electrons are modelled as a continuum and the ions and neutral particles are modelled using discrete particle techniques. There are also some examples in literature of purely discrete particle formulations for both the electrons and heavy particles. For an example of a purely discrete particle simulation see the work by M. Hirakawa [15].

The continuum equations that are used to model the electrons are given below. These equations were taken from a paper by E. Chessa et al. [20]. These include the continuity

(3.6), momentum (3.7), and energy (3.8) equations of the electrons. These equations will be described in more detail in section 5 of this thesis.

$$\frac{\partial N^e}{\partial t} + \nabla \cdot (N^e \mathbf{v}^e) = \alpha_i N^e N^n \quad (3.6)$$

$$N^e m^e \frac{\partial \mathbf{v}^e}{\partial t} + N^e m^e \mathbf{v}^e \cdot \nabla \mathbf{v}^e = -e N^e \mathbf{E} - e N^e \mathbf{v}^e \times \mathbf{B} - N^e m^e \bar{\nu}_{ei} (\mathbf{v}^e - \mathbf{v}^i) - N^e m^e \bar{\nu}_{en} (\mathbf{v}^e - \mathbf{v}^n) \quad (3.7)$$

$$\frac{3}{2} N^e k \frac{\partial T^e}{\partial t} = e N^e \mathbf{v}^e \cdot \mathbf{E} - \alpha_i N^e N^n \varepsilon_i \quad (3.8)$$

Where:

- N^e = Number density of electrons
- N^n = Number density of neutrals
- $\mathbf{v}^e$ = Electron velocity vector
- $\mathbf{v}^i$ = Ion velocity vector
- $\mathbf{v}^n$ = Neutral Xenon velocity vector
- α_i = Volumetric rate constant for ionization
- m^e = Electron mass
- e = Electron charge
- $\mathbf{E}$ = Electric field vector
- $\mathbf{B}$ = Magnetic field vector
- $\bar{\nu}_{ei}$ = Electron-ion momentum transfer collision frequency
- $\bar{\nu}_{en}$ = Electron-neutral momentum transfer collision frequency
- k = Boltzmann constant

T^e = Electron temperature

ϵ_i = Ionization energy of Xenon

The discrete particle equations used to model the ions and neutrals are given below. These equations were taken from the work by Fife [18]. The general matrix form of the equations is given in (3.9). The field force terms for the ions (3.10) and neutral particles (3.11) are also supplied.

$$\frac{d}{dt} \begin{bmatrix} r \\ z \\ \dot{r} \\ h \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \dot{r} \\ \dot{z} \\ \frac{F_r}{m} + \frac{h^2}{r^3} \\ \frac{rF_\theta}{m} \\ \frac{F_z}{m} \end{bmatrix} \quad (3.9)$$

For Ions:

$$\mathbf{F}_{ion} = e\mathbf{E} \quad (3.10)$$

For Neutral Xenon:

$$\mathbf{F}_{neutral} = 0 \quad (3.11)$$

Where:

r = Radial position

z = Axial position

$\mathbf{F}$ = Field force vector on particles

h = Angular momentum per unit mass

m = Particle mass

e = Particle charge

In simulations that use both the discrete and continuum equations, a technique needs to be employed to couple the two sets of equations together. A technique that is often used is the particle-and-cell method described by Fife [18]. This technique relates the discrete properties of the heavy particles to an equivalent set of values on the nodes of a computational grid. The area ratio of the rectangle defined by the particle and grid corner to the overall area of the cell determines the weight that is assigned to each node. This process is described graphically in figure 3.30 below.

Figure 3.30. Graphical Representation of the Particle-in-cell Method [18]

Examples can be found in literature of both one-dimensional and two-dimensional extensions of the governing differential equations. The computational domain for one-dimensional simulations is along the axis of the thruster. The computational domain for most two-dimensional simulations is in the radial-axial coordinate plane. This computational plane is illustrated in figure 3.31. The limitation of this computational

plane is that it is not capable of capturing the azimuthal plasma oscillations. As indicated in section 3.3, this also implies that the axial transport of electrons will be underestimating without the aid of an experimental correction factor. Examples of this type of simulation can be found in the work by E. Fernandez [21].

As an alternative to the traditional radial-axial computational plane, simulations in the azimuthal-axial plane have also been constructed. This alternative computational domain is illustrated in figure 3.32. The advantage of this computational domain is its capability to resolve the majority of the plasma oscillations, as most oscillations occur primarily in the axial and azimuthal directions. Examples of this type of simulation can be found in the work by J. C. Adam et al. [22].

Figure 3.31. Computational Domain used in Typical Hall Thruster Simulations (grid on right hand side taken from Fife [18])

Figure 3.32. Alternative Computational Domain for Hall Thruster Simulations

The results of a typical radial-axial Hall thruster simulation are provided below. These results were presented in a recent paper from Stanford University by M. K. Allis et al. [23]. Figures 3.33 through 3.39 compare the following simulated parameters to experimental measurement: plasma potential, electron temperature, axial ion velocity, axial neutral velocity, neutral density, plasma density, and the current-voltage profile of the thruster. They illustrate some of the challenges associated with reliably predicting Hall thruster operation.

It is apparent from these results that the greatest difference between simulated and experimental results occurs near the anode. Also, the simulated plasma and neutral densities have the greatest difference from experimental measurements. It can also be observed that the simulation becomes less reliable at lower discharge voltages (see figure 3.39).

Figure 3.33. Simulated versus Experimental results of the Plasma Potential [23]

Figure 3.34. Simulated versus Experimental Results of the Electron Temperature [23]

Figure 3.35. Simulated versus Experimental Results of the Axial Ion Velocity [23]

Figure 3.36. Simulated versus Experimental Results of the Axial Neutral Velocity [23]

Figure 3.37. Simulated versus Experimental Results of the Neutral Density [23]

Figure 3.38. Simulated versus Experimental Results of the Plasma Density [23]

Figure 3.39. Simulated versus Experimental Current-Voltage Profile [23]

Section 4: High Frequency Plasma Probing Experiment

4.0 High Frequency Plasma Probing Experiment

This section describes the experiments that were conducted during this project to investigate high frequency plasma oscillations within Hall thrusters. These experiments were carried out in collaboration with Stanford University using the facilities available in the Stanford Plasma Dynamics Lab (SPDL). The Hall thruster selected for these experiments was the Stanford Hall Thruster (SPT), which is a custom built low power Hall thruster. This thruster has a channel diameter of 90mm, a channel width of 11mm, and a length of 80mm. The performance characteristics of this thruster have been well characterized by previous experimental work [8] [14] [20].

The objective of the experimental work carried out at Stanford University was to develop an experimental setup capable of measuring plasma density fluctuations at frequencies from 1MHz to 500MHz. The main challenge to this task was the impedance mismatch between the plasma probes and the cables connecting the probes to data acquisition equipment located outside of the vacuum chamber. Reflected noise resulting from this impedance mismatch acted to filter the high frequency portion of the signal. The solution used in this study was to design high frequency impedance matching circuitry which was mounted within the vacuum chamber close to the plasma probes.

Despite its necessity, the high frequency signal conditioning circuitry introduced a new challenge to the experimental setup. The components of the high frequency circuit were not designed to operate in vacuum conditions. Several of these components could not dissipate heat at a sufficient rate in a vacuum. Without mediating action they would overheat and be destroyed. This challenge was addressed by designing an enclosure for the high frequency electronics to dissipate the thermal energy through radiation alone.

The experimental apparatus developed during this project was successful at measuring high frequency plasma oscillations up to 500MHz. The data collected in this experiment may be important for a number of reasons. First, it has been conjectured that the high frequency plasma oscillations within a Hall thruster are connected to the anomalously high mobility of electrons in the axial direction. This data may provide evidence to confirm or refute this connection. Second, the data collected in this experiment can be compared to numerical plasma simulations to see if similar high frequency density changes are captured numerically. This could in turn lead to a better understanding of the underlying physics behind these oscillations. Finally, the data will be useful in determining how plasma oscillations within Hall thrusters may interfere with RF communications equipment on a spacecraft.

The experimental setup developed during this project is described in section 4.1. Next, the design of the signal conditioning electronic circuit is explained in section 4.2.

Thermal design considerations are discussed in section 4.3. Finally, the experimental results are presented in section 4.4.

4.1 Experimental Setup

A schematic of the experimental setup can be found in figure 4.1. The experimental setup consists of a plasma probe inserted through a slot in the side of the Stanford Hall Thruster. The probe unit is composed of three separate Langmuir probes separated by small azimuth, axial, and radial offsets. The probe unit is connected to a signal conditioning device that resides on the inside of the vacuum chamber. The signal conditioning electronics are connected to the probe unit and external data acquisition equipment by six coaxial cables. The length of coaxial cable between the probe unit and signal conditioning circuitry is minimized in order to reduce signal degradation. The experimental setup was designed so that the thruster could be actively translated in the axial direction by a stepper motor. This allows the properties down the entire length of the acceleration channel to be characterized.

Figure 4.1. Experimental Test Setup

Four mechanical components were designed and constructed for this test setup. These include the probe holder stand, the Langmuir probe assembly, the electronics casing assembly, and the Stanford Hall Thruster mounting platform. The first of these components is the probe holder stand. This was used to hold the Langmuir probes and position them within the acceleration channel of the Hall Thruster. The stand was bolted directly to the base-plate within the vacuum chamber, and could be manually adjusted on three axes to correctly position the probe. A solid model rendering of this component can be found in figure 4.2. A picture of the manufactured component can be found in figure 4.3.

Figure 4.2. Probe Holder Stand Component

Figure 4.3. Probe Holder Stand

The next component is the Langmuir probe assembly which is shown in figure 4.4. A Langmuir probe is simply a length of conducting wire that is inserted directly into plasma. In this case the probe tip was composed of three separate Langmuir probes, separated by an offset in azimuth, axial, and radial directions of the Hall thruster. The purpose of having three probes was to characterize the speed and direction of the high frequency plasma waves within the Hall thruster. The probe was constructed with alumina ceramic tubing and tungsten wire.

Figure 4.4. Langmuir Probe Assembly

The electronics casing assembly is shown in figure 4.5. The purpose of this component was to encase the signal conditioning circuit and dissipate the power of the electronic components by radiating this energy from the body surface. A picture of the manufactured component can be found in figure 4.6.

Each coaxial cable connection is clearly labelled on the side of the electronics holder. The three 'Input' channels connect directly to the Langmuir probe assembly through a short length of coaxial cable. The three 'Output' channels connect to the data acquisition system through ports on the walls of the vacuum chamber. The 'Power' input was connected to a DC power supply set to between 5V and 10V. The ground of the power supply can be set to any level relative to the vacuum chamber (true ground) in order to bias the potential of the Langmuir probes. For this experiment the Langmuir probes were biased to the vacuum chamber ground potential.

Figure 4.5. Electronics Casing Assembly

Figure 4.6. Signal Conditioning Electronics

The Hall thruster mounting platform is shown in figure 4.7. The purpose of this component is to correctly orient the thruster within the vacuum chamber so that the probe can be placed in the acceleration channel.

Figure 4.7. Stanford Hall Thruster Mounting Platform

The next component of the test setup is the Stanford Hall Thruster itself. This thruster was not designed during this project, but is being used for the experimental trials. This thruster has been used extensively for experimental work at Stanford University in the past. The reason this thruster was selected for the current project arises, in part, because of a slot along the acceleration channel of the thruster. This slot allows the Langmuir probes to be inserted through the side into the acceleration channel of the thruster. This is assumed to cause fewer disturbances to the flow than approaching the acceleration channel from the axial direction. The Stanford Hall Thruster is shown disconnected in figure 4.8.

Figure 4.8. Stanford Hall Thruster

The complete setup of all components within the vacuum chamber is shown below. Four images are provided. The first image (figure 4.9) shows what the vacuum chamber looks like without any of the experimental components installed. This can be used for reference purposes, and to identify which components are permanent fixtures within the vacuum chamber. The second image (figure 4.10) shows all the components assembled within the vacuum chamber. The third image (figure 4.11) shows a picture of the vacuum chamber from the outside. The final image (figure 4.12) provides a size reference for the experimental apparatus, and shows the author of this report within the vacuum chamber.

Figure 4.9. Empty Vacuum Chamber

Figure 4.10. Full Experimental Setup

Figure 4.11. Plasma Dynamics Lab Large Vacuum Chamber

Figure 4.12. Size Reference for Experimental Setup

The final components of the test setup were located outside of the vacuum chamber. These included a signal generator, a DC power supply, and an Oscilloscope (used in this setup as the data acquisition device). These three components are shown in figure 4.13. The signal generator was used for test purposes only; to ensure that all channels of the probe were operational before sealing the vacuum chamber.

Figure 4.13. DC Power Supply, Signal Generator, and Oscilloscope

4.2 Signal Conditioning Electronics

A three channel signal conditioning electronic device was designed and constructed during the course of this project. The purpose of this device was to amplify and match the impedance of the signal coming from the Langmuir probes to the coaxial cables used to transmit this signal to the data acquisition system. This device was installed inside the

vacuum chamber as close to the probes as possible in order to limit the decay of the signal between the probes and the conditioning electronics.

The signal conditioning electronics were necessary due the high frequency nature of the signals being investigated. Basic electronic theory demonstrates that a mismatch in impedance between a transmitter, cable, and receiver acts to filter the high frequency components of the signal. The impedance mismatch causes the signal to attenuate and phase shift before it reaches the data acquisition system. Previous experimental work conducted at Stanford and Princeton University has confirmed the need for signal conditioning for measuring high frequency plasma oscillations [13].

The basic design concept for the signal conditioning electronics was taken from a study conducted by Princeton University involving a similar experimental investigation of high frequency plasma oscillations [11] [13]. Several modifications from the original concept were made and will be discussed later in this section. The original form of the signal conditioning electronics circuit presented in the Princeton paper is shown in figure 4.14.

Figure 4.14. Original Concept for the Frequency Conditioning Electronics (Princeton University) [13]

An electronics schematic of the high frequency signal conditioning circuit can be found in figure 4.15. Each component of this circuit was selected for its ability to function between the ranges of 1MHz to 500MHz. This circuit was first prototyped on a breadboard setup to establish a working configuration. Next, this circuit was constructed three times in a parallel, once for each Langmuir probe, on an electronics circuit board. This circuit was then mounted within the electronics casing assembly.

Figure 4.15. High Frequency Electronics Configuration

This circuit has been modified from the original concept by incorporating the idea of a virtual ground. This allows the probe to be biased to a specified potential relative to true ground (Hall thruster ground potential). The purpose of each component of the signal conditioning circuit is as follows. First, the Intersil EL5196ACS Operational Amplifier was used to amplify the signal by a factor of two and reduce the output impedance to virtually zero. The 50 Ohm resistor, which is connected in series to the operational amplifier, was used to match the impedance of the signal to the 50 Ohm coaxial cable.

The 1:1 Pulse CX2060 transformer was used to balance the voltage signal emitted by the transformer at zero volts (or the virtual ground value) and decouple the operational amplifier from the plasma probe. The 51.1 KOhm resistor was used as a load resistor for the 1:1 transformer. Finally, the placement of the two capacitors was recommended in the product documentation for the Intersil EL5196ACS Operational Amplifier, and appears to extend the operating frequency range of the amplifier.

A simple test was conducted to demonstrate how the signal from the Langmuir probes could be improved, in the case of impedance mismatch, by passing the signal through the high frequency conditioning electronics. A signal generator was attached to the probe and used to create a sinusoidal signal. The impedance of the probe was deliberately mismatched with the line by introducing a resistor in series with the probe. This setup is shown schematically in figure 4.16.

Figure 4.16. Impedance Mismatch Test Setup

The output from the probe was connected to the conditioning electronics as well as directly to the oscilloscope. The amplitude of each signal was tested over a range of frequencies. This test was conducted for three different values of impedance mismatch: 50, 150 and 250 Ohms. This test demonstrated that the need for the signal conditioning increased with the amount of impedance mismatch between the Langmuir probe and the coaxial cable. The results of this experiment are shown in figure 4.17. The gain factor in this graph is a simple amplitude ratio of the conditioned and non-conditioned signals.

Figure 4.17. Mismatched Impedance Test Results

The performance of the signal conditioning electronics was characterized over a range of frequencies. To accomplish this, a signal generator was connected to each channel of the Langmuir plasma probe in turn. The signal generator was set to a 0.3V amplitude sine

wave for a range of frequencies from 1MHz to 15MHz. The peak to peak voltage of the output signal was measured at each step with an oscilloscope. Ideally, the gain of the signal conditioning electronics should be constant over this range. However, this turned out not to be the case. This is likely caused by some inductance in the circuit (from wires and resistors) as well as impedance mismatch between the Langmuir probes and the length of coaxial cable connecting the probes to the signal conditioning circuit. The results from this test are shown in figure 4.18.

Figure 4.18. Electrical Box Operating Characteristics

The performance characteristics of the high frequency signal conditioning electronics were used to generate gain function plots for each channel of the Langmuir probe. A sixth order polynomial was calculated in Matlab to best fit the experimental observations using the least squares method. The gain function curves and corresponding formulas can be found in figure 4.19 and table 4.1.

Figure 4.19. Electrical Box Gain Functions

Gain = P(1)*X^N + P(2)*X^(N-1) + ... + P(N)*X + P(N+1)							
	P(1)	P(2)	P(3)	P(4)	P(5)	P(6)	P(7)
Gain A	-1.3811e-041	6.7967e-034	-1.2461e-026	1.0502e-019	-4.1277e-013	7.2578e-007	1.2205
Gain B	2.8924e-042	-7.1319e-035	-5.8386e-029	1.2386e-020	-9.426e-014	2.5096e-007	1.2518
Gain C	9.133e-042	-3.6835e-034	5.2411e-027	-3.1681e-020	7.9913e-014	-4.4062e-008	1.5927

Table 4.1. Gain Function Parameters

The amplitude of experimentally observed plasma oscillations can be corrected by simply dividing by the gain factor, calculated with the previous formula, at the corresponding frequency at which they occur. The performance of the signal conditioning electronics was only characterized up to 15 MHz because this was the limit of the signal generator. However, plasma oscillations were observed well beyond this 15MHz boundary. No attempt has been made to correct the data above 15MHz. This may contribute to experimental error in the collected results.

4.3 Thermal Considerations

The electrical components used in the signal conditioning circuit dissipate a significant amount of power. In atmospheric conditions this power is removed primarily by convection and conduction to the air surrounding the electronics. However, in a vacuum the power is dissipated by radiation alone at a slower rate. This is a problem because if no mediating action was taken the electronic components would overheat and be destroyed. This problem was addressed in the study conducted by Princeton University using forced gas convection. Nitrogen gas was fed in an open loop configuration from outside of the vacuum chamber to an enclosed volume within the vacuum chamber that contained the conditioning electronics [13]. A different approach was used in this project. The electronics were mounted on the surface of a copper casing assembly that dissipated the thermal energy through radiation.

A component wise power dissipation analysis was conducted on the electronics. The power dissipation was found to be dominated by only two components: the operational amplifier, and the 1:1 transformer. The maximum power dissipation for each component was determined to have the following values:

- Intersil EL5196ACS Operational Amplifier: 81mW
- 1:1 Pulse CX2060 Transformer: 0.8mW

The total power produced by the complete circuit is therefore 245.4mW (each component was used three times). Equation 4.1 was used to calculate the surface temperature of the electronics casing assembly assuming radiation within a black body enclosure.

$$q = \epsilon_1 \sigma A_1 (T_1^4 - T_2^4) \quad (4.1)$$

From this equation the average surface temperature of the copper was found to be 30.2C. The temperature on the surface of the electronic components will be higher than this average value, due to 3-dimensional dissipation of the power within the plate. A more thorough analysis was conducted, leading to the conclusion that the temperature on the surface of the operational amplifiers would be approximately 30.4C. This temperature was well below the maximum operating conditions of these components.

4.4 Experimental Observations

Only a preliminary investigation of high frequency plasma oscillations was attempted during this project. The primary motivation for these tests was to determine if the experimental setup was working, and if it was capable of supporting further study related to high frequency plasma oscillations.

The probes were positioned approximately 5 cm downstream of the exit plane of the plasma thruster for this test. This position was selected because it had a much milder thermal environment than the inside of the acceleration channel. The operating conditions of the Hall thruster were set at 210V anode potential, 2.1 Amp driving current, and a peak magnetic field strength of approximately 100 Gauss.

An oscilloscope was used to capture the experimental data. The oscilloscope used in this trial was capable of capturing 10000 data points at a time, with a user selectable time capture window. Data was collected at various time capture window settings during the operation of the thruster. However, only two of these capture windows proved to be useful for the subsequent data analysis. These were set at 10 μ s and 40 μ s.

The data was processed by performing a power spectral density analysis in Matlab. The power spectral density plots help to identify favoured frequencies of oscillation in the

plasma. Peaks in these plots correspond with natural instabilities that occur within the plasma. Further discussion of these results can be found in section 6 of this thesis.

The power spectral density plots for each port of the probe unit are shown in figures 4.20 through 4.25. It can be observed from these plots that most of the plasma oscillations occur in the 1 – 50MHz range, and again at approximately 90MHz and 230MHz. However, the 90MHz and 230MHz features may be caused by electrical noise within the signal conditioning electronics, and are probably of no interest.

Figure 4.20. Port A, 40 μ s Capture Window

Figure 4.21. Port B, 40 μ s Capture Window Figure 4.22. Port C, 40 μ s Capture Window

Figure 4.23. Port A, 10 μ s Capture Window Figure 4.24. Port B, 10 μ s Capture Window

Figure 4.25. Port C, 10 μ s Capture Window

Section 5: Hall Thruster Simulation

5.0 Hall Thruster Simulation

This section describes the numerical model that was developed to simulate the Hall thruster. The code for the Hall thruster simulation was developed primarily by Dr. Eduardo Fernandez (Eckerd College, Florida) with participation from Stanford University. The code was adapted by the author to suit the purposes of this project. The major changes that were implemented include the following:

- An adaptive time-step was implemented to improve computational performance and numerical stability.
- The original Fortran Code was integrated into a custom designed simulation environment with visualization and multi-threaded capabilities.
- The operating parameters of the Hall thruster were set to user defined variables in order to collect results for various operating regimes. These variables include:
 - Discharge voltage
 - Magnetic field strength
 - Electron temperature
- Virtual plasma probes were added to the simulation to collect statistical oscillation data at various locations within the Hall thruster.

The Hall thruster simulation described in this section is a Hybrid particle-in-cell solver. The electrons are modelled as a continuum using the governing equations developed in section 5.1. The electron continuum equations are solved by the finite difference method described in section 5.2. The ions are modelled as collections of discrete particles known as super-particles, described in section 5.3. The ion and electron equations are coupled by the particle-in-cell technique described in section 5.3. The boundary conditions and particle-wall interactions are described in section 5.4 and 5.5 respectively. The simulation is solved in two-dimensions in the azimuth and axial directions.

The following procedure is used to advance the simulation through each time step. First, the discrete ion distribution is related to values on a computational grid using the particle-in-cell method. Second, the electron continuum equations are solved to determine the potential field distribution. Third, the electric field strength and electron velocities are deduced from the potential field solution. Fourth, the ions and neutral particles are advanced in time based on the forces produced by the electric field. Finally, statistical methods are used to predict ionizing collisions between electrons and neutral particles, and the results of particle-wall interactions.

It should be noted that the Hall thruster simulation discussed in this section does not converge to a final solution. This is a property of Hall thruster simulations in general. Rather than dealing with a deterministic field solution, the results of these types of simulations are largely chaotic. Statistical methods are employed to evaluate the

simulation results. Assuming that the simulation is carried out over an infinite period of time, the solution will keep changing in a chaotic fashion and never reach steady state. This is because the ion portion of the numerical model is based on a random collection of discrete particles. The electron solution, in turn, is tied to this chaotic field of ions.

5.1 Governing Equations

Plasma can be mathematically described by a set of properties associated with a small volume of substance. The volume under consideration should contain a large number of particles, but have dimensions far smaller than any physical lengths of interest. The first property is the number density represented by the symbol $N^{(S)}$, where the superscript (S) refers to the species under consideration. The number density is defined as the number of particles per unit volume of the material. In simple plasma only two species are considered: electrons and a single type of ion. The second property is the charge density (η) defined by equation 5.1. The charge density is related to the number density of each species multiplied by the particle charge.

Charge Density:

$$\eta = \sum_s N^{(s)} q^{(s)} \quad (5.1)$$

Where:

η = Charge density

N = Particle number density

q = Particle charge

The third property is the velocity. The velocity of each particle is described individually by a vector quantity ($\mathbf{v}^{(s)}$). The average velocity of a given species is represented by the symbol $\langle \mathbf{v}^{(s)} \rangle$. The fourth property is the current density ($\mathbf{J}$). Current density is defined as the net rate at which charge flows through a unit area, and is mathematically described by equation 5.2 for simple plasma. Current density, like velocity, is a vector property and has components in each of the coordinate axis.

Current Density:

$$\mathbf{J} = e(N^i \langle \mathbf{v}^i \rangle - N^e \langle \mathbf{v}^e \rangle) \quad (5.2)$$

Where:

$\mathbf{J}$ = Current density vector

e = Electron charge

N = Particle number density

$\langle \mathbf{v} \rangle$ = Average particle velocity vector

The first set of equations used to describe plasma are derived from the forces that the negative and positive particles exert on each other, and the consequences of these interactions. These equations are known as the electrodynamic equations, and they include a subset known as Maxwell's equations. The formulas that follow were taken from a text by Holt and Haskell [17] on this subject.

The continuity of charge equation (5.3) can be derived by integrating the charge leaving a given volume of space across a bounding surface. This equation simply states that the current leaving a given volume of space must equal the rate at which the charge density changes through time.

Continuity of Charge:

$$\frac{\partial \eta}{\partial t} = -\nabla \cdot \mathbf{J} \quad (5.3)$$

Where:

η = Charge density

$\mathbf{J}$ = Current density vector

Maxwell's equations are experimental laws that have been derived for four integral quantities of electromagnetism: magnetic flux, electric current flux, magnetomotive force, and electromotive force. These quantities are defined over a surface and bounding curve as shown in figure 5.1.

Figure 5.1. Integral over a Closed Surface [17]

The first of Maxwell's equations is given by Faraday's law, which relates the rate of change of the magnetic field to the electromotive force produced around its boundary. Faraday's law is provided in integral form by equation 5.4. By applying Stokes' theorem to the right hand side of equation 5.4 and differentiating, we obtain the differential form of Faraday's law given by equation 5.5.

Faraday's Law:

$$\frac{d}{dt} \oint_S (\mathbf{B} \cdot \mathbf{n}) dA = - \oint_C \mathbf{E} ds \quad (5.4)$$

$$\nabla \times \mathbf{E} = - \dot{\mathbf{B}} \quad (5.5)$$

Where:

$\mathbf{B}$ = Magnetic induction vector

$\mathbf{E}$ = Electric field intensity vector

The second of Maxwell's equations is given by Ampere's law, which relates the rate of change of the electric field to the magnetomotive force produced around its boundary. Ampere's law is given by equation 5.6. By applying Stokes' theorem and differentiating we obtain the differential form of Ampere's law given by equation 5.7. A new term is introduced in this equation known as the total current density represented by the symbol $\mathbf{C}$. The total current density is the sum of the current density ($\mathbf{J}$) plus the rate of change of the displacement current density. The displacement current is related to the properties of a dielectric medium, specifically the permittivity constant. The formula to obtain the total current density is given by 5.8. The displacement current is defined by equation 5.9.

Ampere's Law:

$$\oint_s (\mathbf{C} \cdot \mathbf{n}) dA = \oint_L \mathbf{H} ds \quad (5.6)$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \dot{\mathbf{D}} \quad (5.7)$$

Where:

$$\mathbf{C} = \mathbf{J} + \dot{\mathbf{D}} \quad (5.8)$$

$$\mathbf{D} = \kappa \cdot \mathbf{E} \quad (5.9)$$

$\mathbf{C}$ = Total current density vector

$\mathbf{H}$ = Magnetic field intensity vector

$\mathbf{J}$ = Current density vector

$\mathbf{D}$ = Displacement current density vector

κ = Permittivity dyadic

$\mathbf{E}$ = Electric field intensity vector

If we take the divergence of the equations used to describe Faraday's and Ampere's laws (5.5) & (5.7), and apply the continuity of charge equation (5.3), we can derive equations 5.10 and 5.11 respectively. These equations define the divergence of the magnetic induction field and electric displacement field. Maxwell's equations are usually summarized as four differential formula: (5.5), (5.7), (5.10), and (5.11).

$$\nabla \cdot \mathbf{B} = 0 \quad (5.10)$$

$$\nabla \cdot \mathbf{D} = \eta \quad (5.11)$$

Where:

$\mathbf{B}$ = Magnetic induction vector

$\mathbf{D}$ = Displacement current density vector

It is convenient to formulate Maxwell's equations in terms of potential functions. The field properties of the medium can subsequently be obtained by differentiating these functions. In general, this process can be accomplished by formulating Maxwell's equations in terms of two quantities: a vector potential and scalar potential field. However, if we consider an electrostatic field in which the field properties do not change with time, we can reduce Maxwell's equations to a formula based on a single scalar potential function (ϕ). The result is known as Poisson's equation given by 5.12. The electric field strength can be deduced by taking the gradient of the potential function as shown in equation 5.13.

Poisson's Equation:

$$\nabla^2 \phi = -\frac{\eta}{\epsilon} \quad (5.12)$$

$$\mathbf{E} = -\nabla \phi \quad (5.13)$$

Where:

ϕ = Scalar potential function

η = Charge density

ε = Permittivity of the medium

$\mathbf{E}$ = Electric field intensity vector

The second set of equations used to describe plasma are based on the concept of plasma kinetic theory. The most significant of these equations is the Boltzmann equation which will be described below. The formulas that follow were taken from a text by Holt and Haskell [17] on this subject.

Plasma kinetic theory relies on the concept of six-dimensional velocity space, often referred to as molecular phase space. The molecular phase space representation of plasma is composed of two 3-dimensional domains: the configuration space and velocity space. Configuration space, shown in figure 5.2, represents the physical location of particles within a given volume of ordinary 3-dimensional space. Velocity space, shown in figure 5.3, represents the velocity of every molecule within the small volume element dr defined in configuration space.

Figure 5.2. Configuration Space [17]

Figure 5.3. Velocity Space [17]

Two scalar functions are important when describing particles within molecular phase space. The first is the number density (N) which is defined as the number of particles per unit volume of configuration space. The second is the velocity distribution function (f)

which is defined as the density of particles within the volume element dc shown in figure 5.3.

When dealing with molecular phase space the concept of pressure and temperature need to be rigorously defined. In order to do this we employ the definition of peculiar velocity. Peculiar velocity is defined as the relative motion of each particle relative to the average velocity within a specific region of configuration space. The mathematical definition of peculiar velocity is given by equation 5.14.

Peculiar Velocity:

$$\mathbf{V} = \mathbf{v} - \langle \mathbf{v} \rangle \quad (5.14)$$

Where:

$\mathbf{V}$ = Peculiar velocity vector

$\mathbf{v}$ = Particle velocity vector

$\langle \mathbf{v} \rangle$ = Average particle velocity vector

The pressure of plasma is defined as the average rate at which momentum is transferred across a differential surface element in configuration space per unit area. Pressure is defined mathematically by equation 5.15 in terms of the peculiar velocity. As opposed to a classical definition of pressure, the pressure defined by equation 5.15 is a vector quantity. The term $\langle \mathbf{V}\mathbf{V} \rangle$ is a dyadic that is commonly used to define the *pressure tensor* represented by the symbol (Ψ). The equation for the plasma pressure in terms of the

pressure tensor is given by equation 5.16. The definition of the pressure tensor is given by equation 5.17.

Definition of Pressure:

$$\mathbf{P} = Nm\mathbf{n} \langle \mathbf{V}\mathbf{V} \rangle \quad (5.15)$$

$$\mathbf{P} = \mathbf{n}\Psi \quad (5.16)$$

Where:

$$\Psi = Nm \langle \mathbf{V}\mathbf{V} \rangle \quad (5.17)$$

$\mathbf{P}$ = Pressure vector

N = Particle number density

m = Particle mass

$\mathbf{n}$ = Surface normal vector

$\langle \mathbf{V}\mathbf{V} \rangle$ = Peculiar velocity dyadic

Ψ = Pressure tensor

Similar to the definition of pressure, the plasma temperature is also given as a vector quantity. The temperature is related to the kinetic energy of the particles that are transferred through the medium. Another way of looking at the temperature is to consider the hidden kinetic energy (heat energy) of the plasma particles that is not related to the average molecular velocity: the peculiar velocity. Temperature is directly proportional to the peculiar velocity dyadic and is defined by equation 5.18. By comparing equations 5.15 and 5.18 for the case of uniform particle distribution, we can

conclude that the relationship between the plasma temperature and pressure is given by the well known equation of state for an ideal gas (equation 5.19).

Definition of Temperature:

$$\mathbf{T} = \frac{m}{3k} \langle \mathbf{V}\mathbf{V} \rangle \quad (5.18)$$

$$P = NkT \quad (5.19)$$

Where:

$\mathbf{T}$ = Temperature vector

m = Particle mass

k = Boltzmann's constant

$\langle \mathbf{V}\mathbf{V} \rangle$ = Peculiar velocity dyadic

P = Pressure scalar

T = Temperature scalar

N = Particle number density

It is possible to deduce all of the plasma properties if the velocity distribution function (f) were known. One method to determine the velocity distribution function is to solve for the time variation of this function starting from known initial conditions. The resulting time variation relation is known as the Boltzmann equation, and is given by 5.20. In this equation ($\mathbf{R}$) represents the force per unit mass on the particles. The right hand side of the Boltzmann equation results from particle collisions that occur within the plasma. The particle collision term will be discussed in more detail in section 5.3.

Boltzmann Equation:

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} + R_i \frac{\partial f}{\partial v_i} = \int [(\tilde{f}f^B - ff^B)gp] dp d\psi dc^B \quad (5.20)$$

Where:

- f = Velocity distribution function
- $\mathbf{v}$ = Velocity vector
- $\mathbf{R}$ = Force vector
- $\tilde{f}$ = Velocity distribution function (after collision)
- f^B = Velocity distribution function of colliding particles (before collision)
- $\tilde{f}^B$ = Velocity distribution function of colliding particles (after collision)

In general, the Boltzmann equation provides more information than is needed to deduce the physical properties of the plasma. Rather than solving the Boltzmann equation directly, it is more convenient to use the Boltzmann equation to derive the time variation of the state variables directly. This is accomplished by multiplying the Boltzmann equation by a given function (Φ) and integrating over velocity space for each species. This leads to the third set of plasma equations: the macroscopic equations.

Following the approach outlined above, the continuity equation is obtained by setting (Φ) equal to 1. It can be shown that the result is given by equation 5.21 [17]. The superscript (S) represents the species under consideration. This equation neglects gains and losses due to ionization or re-attachment processes.

Continuity Equation:

$$\frac{\partial N^{(s)}}{\partial t} + \frac{\partial}{\partial x_i} (N^{(s)} \langle v_i^{(s)} \rangle) = 0 \quad (5.21)$$

Where:

N = Particle number density

$\mathbf{v}$ = Particle velocity vector

The momentum equation can be derived by setting Φ equal to $m^{(s)}\mathbf{v}^{(s)}$, and following the same method that was used to derive the continuity equation. It can be shown that the result is given by equation 5.22 [17]. The term on the right hand side of equation 5.22 represents momentum gain from collisions between charged and uncharged particles. Collisions between oppositely charged particles are also important, particularly when considering the ion momentum equation, and will be considered in more detail in section 5.3.

Momentum Equation:

$$\frac{\partial \langle v_i^{(s)} \rangle}{\partial t} + \langle v_j^{(s)} \rangle \frac{\partial \langle v_i^{(s)} \rangle}{\partial x_j} + \frac{1}{N^{(s)}m^{(s)}} \frac{\partial \Psi_{ij}^{(s)}}{\partial x_j} - \frac{q^{(s)}}{m^{(s)}} [E_i + (\langle \mathbf{v}^{(s)} \rangle \times \mathbf{B})_i] = v_{SN} (\langle v_i^N \rangle - \langle v_i^{(s)} \rangle) \quad (5.22)$$

Where:

$\mathbf{v}$ = Particle velocity vector

N = Particle number density

m = Particle mass

Ψ = Pressure tensor

- q = Particle charge
 $\mathbf{E}$ = Electric field intensity vector
 $\mathbf{B}$ = Magnetic induction vector
 ν_{SN} = Collision frequency for momentum transfer

The energy equation can be derived by setting Φ equal to $\frac{1}{2}m^{(S)}\mathbf{v}^{(S)}\mathbf{v}^{(S)}$, and following the same method that was used to derive the continuity and momentum equations. It can be shown that the result is given by equation 5.23 [17]. Three terms appear on the right hand side of the energy equation. The first term represents the rate of kinetic energy addition due to particle collisions. The second term is related to the rate of change of the particle momentum due to collisions. The final term is related to the number of particles produced or removed due to ionization/reattachment processes. A new variable is introduced in the energy equation called the *heat flux vector* ($\mathbf{Q}$). The heat flux vector is defined by equation 5.24.

Energy Equation:

$$\begin{aligned}
 & \frac{3}{2} \frac{\partial P^{(S)}}{\partial t} + \frac{3}{2} \frac{\partial P^{(S)} \langle v_i^{(S)} \rangle}{\partial x_i} + \Psi_{ij}^{(S)} \frac{\partial \langle v_j^{(S)} \rangle}{\partial x_i} + \frac{\partial Q_i^{(S)}}{\partial x_i} = \dots \\
 & \dots = M_{coll}^{(S)} - [m^{(S)} N^{(S)} \nu_{SN} (\langle v_i^N \rangle - \langle v_i^{(S)} \rangle) \langle v_j^{(S)} \rangle] + \frac{1}{2} m^{(S)} \langle v_j^{(S)} \rangle \langle v_j^{(S)} \rangle S_{coll}^{(S)}
 \end{aligned}
 \tag{5.23}$$

Where:

$$Q_i^{(S)} = \frac{1}{2} N^{(S)} m^{(S)} \langle V_i^{(S)} V_j^{(S)} V_j^{(S)} \rangle
 \tag{5.24}$$

$\mathbf{P}$	=	Pressure vector
$\mathbf{v}$	=	Particle velocity vector
Ψ	=	Pressure tensor
$\mathbf{Q}$	=	Heat flux vector
$M_{coll}^{(s)}$	=	Rate of change of kinetic energy due to collisions
m	=	Particle mass
N	=	Particle number density
ν_{SN}	=	Experimentally determined collision coefficient
$S_{coll}^{(s)}$	=	Rate of change of particle density due to collisions
$\mathbf{V}$	=	Peculiar velocity vector

A set of working equations will now be derived for the electron continuum within a Hall thruster. The coordinate system used for these equations is illustrated in figure 5.4. The governing equations will be reduced to two-dimensions in the axial/azimuth coordinate plane (z - θ).

Figure 5.4. Hall Thruster Computational Coordinate System

Four state variables will be considered in this model. These include the electron density, scalar potential, and electron velocity in the axial and azimuth directions. The electron temperature will be assumed to be constant in the azimuth direction and will be obtained from experimental measurements. This simplification greatly reduces the computational complexity of the model because the energy equation does not need to be solved. The magnetic field will be assumed to act in the radial direction (purely due to the Hall thruster electromagnets) and will also be obtained by experimental measurements.

The first working equation is obtained by solving the conservation of charge formula (5.3) in the azimuth-axial plane, and imposing the definition of current in simple singly charged plasma (5.2). Additionally, by assuming quasi-neutrality, the density of the ions is set equal to the density of the electrons and both will be represented by the symbol (N). The result is given by equation 5.25. The unknowns in this equation include various ion terms, which will be calculated separately, and the electron velocity and density.

Continuity of Charge in the Azimuth-Axial Plane:

$$\begin{aligned} <v_z^i> \frac{\partial(N)}{\partial z} + N \frac{\partial <v_z^i>}{\partial z} + \frac{1}{r} \left[<v_\theta^i> \frac{\partial(N)}{\partial \theta} + N \frac{\partial <v_\theta^i>}{\partial \theta} \right] - \dots \\ \dots - <v_z^e> \frac{\partial(N)}{\partial z} - N \frac{\partial <v_z^e>}{\partial z} - \frac{1}{r} \left[<v_\theta^e> \frac{\partial(N)}{\partial \theta} + N \frac{\partial <v_\theta^e>}{\partial \theta} \right] = 0 \end{aligned} \quad (5.25)$$

Where:

$<\mathbf{v}^i>$ = Ion velocity vector

N = Plasma number density

r = Radius of azimuth-axial plane

$\langle \mathbf{v}^e \rangle$ = Electron velocity vector

The electron velocities can be obtained by considering the electron momentum equation (5.22) in the azimuth and axial directions. The following assumptions were used to simplify the momentum equation:

- Steady state electron drift velocity.
- Mean thermal velocity is much greater than the electron drift velocity.
- Negligible contribution to the momentum equation due to gradients in the electron temperature field.

The resulting equations are given by (5.26) and (5.27) for the axial and azimuthal directions respectively.

Electron Momentum Equation in the Axial and Azimuth Directions:

$$v_{SN} \langle v_z^e \rangle = -\frac{T^e k}{N m^e} \frac{\partial(N)}{\partial z} - \frac{e}{m^e} E_z + \frac{e B_r}{m^e} \langle v_\theta^e \rangle \quad (5.26)$$

$$v_{SN} \langle v_\theta^e \rangle = -\frac{T^e k}{N^e m^e} \frac{1}{r} \frac{\partial(N)}{\partial \theta} - \frac{e}{m^e} E_\theta - \frac{e B_r}{m^e} \langle v_z^e \rangle \quad (5.27)$$

Where:

v_{SN} = Collision frequency for momentum transfer

$\langle \mathbf{v}^e \rangle$ = Electron velocity vector

T^e = Electron temperature

k = Boltzmann's constant

- N = Plasma number density
 m^e = Electron mass
 e = Electron charge
 $\mathbf{E}$ = Electric field intensity vector
 B_r = Radial magnetic field strength
 r = Radius of azimuth-axial plane

By substituting equation 5.26 into 5.27, we can solve for the axial and azimuthal electron drift velocities in terms of the electric field, plasma density, and known field properties. The result is given by equations 5.28 and 5.29.

$$\langle v_z^e \rangle = \frac{\left[-\frac{T^e k}{Nm^e v_{SN}} \frac{\partial(N)}{\partial z} - \frac{e}{m^e v_{SN}} E_z - \frac{ekB_r T^e}{N^e (m^e)^2 v_{SN}^2} \frac{1}{r} \frac{\partial(N)}{\partial \theta} - \frac{e^2 B_r E_\theta}{(m^e)^2 v_{SN}^2} \right]}{\left[1 + \left(\frac{e^2 B_r^2}{(m^e)^2 v_{SN}^2} \right) \right]} \quad (5.28)$$

$$\langle v_\theta^e \rangle = \frac{\left[-\frac{T^e k}{Nm^e v_{SN}} \frac{1}{r} \frac{\partial(N)}{\partial \theta} - \frac{e}{m^e v_{SN}} E_\theta - \frac{ekB_r T^e}{N^e (m^e)^2 v_{SN}^2} \frac{\partial(N)}{\partial z} - \frac{e^2 B_r E_z}{(m^e)^2 v_{SN}^2} \right]}{\left[1 + \left(\frac{e^2 B_r^2}{(m^e)^2 v_{SN}^2} \right) \right]} \quad (5.29)$$

The electric field can be related to a single scalar potential function by equation (5.13). The solution of this equation for the electric field in the axial and azimuthal directions is given by equations 5.30 and 5.31.

$$E_z = -\frac{\partial \phi}{\partial z} \quad (5.30)$$

$$E_{\theta} = -\frac{1}{r} \frac{\partial \phi}{\partial \theta} \quad (5.31)$$

The working equations listed above (5.25 – 5.31) form a closed set of equations from which the properties of the Hall thruster can be deduced. However, the properties of the plasma will depend on the radius (r). It is more useful to express the equations so that the plasma properties can be solved for all radial locations simultaneously. In order to accomplish this, we will follow the technique illustrated in the work by Fife [18]. First, we consider the electron momentum equation along the magnetic field lines in the radial direction. The following assumptions are used to simplify the momentum equation:

- Steady electron velocity in the radial direction.
- Negligible momentum transfer by electron-neutral and electron-ion collisions.
- Negligible contribution to momentum due to ionization/recombination processes.
- Electron velocity in the radial direction is much larger than in the axial direction.
- Negligible variation of radial electron velocity in the azimuth direction.

The simplified form of the radial electron momentum equation is given by equation 5.32. By assuming constant electron density in the radial direction, we can integrate this expression to obtain equation 5.33. This equation is particularly significant because it provides a constant plasma parameter (ϕ^*) along any magnetic field line, and therefore a constant parameter at any radius. By formulating the working equations in terms of ϕ^* instead of ϕ , we can reduce the equations to two-dimensions perpendicular to the

direction of the magnetic field lines. Post processing the results will allow us to deduce the plasma properties at any radius.

Electron Conduction Parallel to Magnetic Field Lines:

$$\frac{\partial(N^e k T^e)}{\partial r} = N^e e \left(\frac{\partial \phi}{\partial r} \right) \quad (5.32)$$

$$\phi = \phi^* + \frac{T^e k}{e} \ln(N^e) \quad (5.33)$$

Where:

N^e = Electron number density

k = Boltzmann's constant

T^e = Electron temperature

e = Electron charge

ϕ = Scalar plasma potential

ϕ^* = Constant parameter along magnetic field lines

One of the simplifications used to reduce the radial electron momentum equation was to neglect the variation of the radial electron velocity in the azimuth direction. In the study conducted by Fife [18] this was a good assumption because the properties of the Hall thruster were assumed to be axially symmetric. However, in this project variations of the plasma properties are considered in both the axial and azimuthal directions. The relaxation of this assumption results in an additional term within the momentum equation as shown in equation 5.34. For the time being, this additional term will be ignored.

However, the computational significance of this term is unknown and should be investigated in future work.

Additional Term in the Momentum Equation:

$$\phi = \phi^* + \frac{T^e k}{e} \ln(N^e) + \int \frac{m^e}{re} \langle v_\theta^e \rangle \frac{\partial \langle v_r^e \rangle}{\partial \theta} dr \quad (5.34)$$

5.2 Discretization and Time Step Methodology

The solution methodology at each time step of the simulation is shown schematically in figure 5.5. The solution procedure is divided into two main components: the heavy particle model and the electron continuum. For the heavy particle model, the positions of the ions and neutral particles are first updated. Next, the neutral particles are ionized according to the ionization rate calculated from field parameters. Finally, new neutral particles are injected at the anode, and the ion and neutral distributions are interpolated to a computational grid. The heavy particle model will be discussed in more detail in section 5.3. For the electron continuum, the governing electron equations are solved across the computational grid for the plasma potential. Next, the plasma potential is used to deduce the electric field strength. Finally, the field properties obtained from the heavy particle model and electron continuum are used to determine the time step size for the next iteration.

Figure 5.5. Simulation Flowchart

The time step size for the next iteration is computed using the stability requirement shown in equation 5.35. The time step is limited by the inverse of the plasma frequency. The formula to compute the plasma frequency from known field parameters is given by equation 5.36. This stability requirement was discussed in a paper by J. C. Adam et al. [22], for a similar hall thruster simulation.

Simulation Time Step:

$$\Delta t < 1.0 / \omega_{pe} \quad (5.35)$$

Where:

$$\omega_{pe} = \sqrt{\frac{N^e (e)^2}{m^e \epsilon_0}} \quad (5.36)$$

 Δt = Simulation time step ω_{pe} = Plasma frequency N^e = Electron number density e = Electron charge m^e = Electron mass ϵ_0 = Permittivity of free space

A set of equations will now be derived in order to solve the electron continuum equations using the finite differencing technique. First, the governing equations are reformulated as a single expression in terms of the plasma potential function ϕ^* , which is constant along the magnetic field lines. This is accomplished by substituting equation 5.33 into the electric field equations (5.30 & 5.31). The field equations are then solved simultaneously with the electron momentum equations (5.28 & 5.29) and the continuity of charge equation (5.25). The result can be expressed in the form of equation 5.37, where the A_i 's are constant coefficient terms.

Form of the Plasma Potential Equation:

$$A_1 \frac{\partial^2 \phi^*}{\partial z^2} + A_2 \frac{\partial \phi^*}{\partial z} + A_3 \frac{\partial \phi^*}{\partial \theta} + A_4 \frac{\partial^2 \phi^*}{\partial \theta^2} + A_5 = 0 \quad (5.37)$$

Where:

$$\phi^* = \text{Plasma potential function (constant along magnetic field lines)}$$

The coefficients used in 5.37 are defined by equations 5.38 through 5.42 below. All the terms appearing on the right hand side of these equations are obtained from the field parameters at the previous time step.

Coefficient Terms in the Plasma Potential Equation:

$$A_1 = \left(\frac{-N \cdot e}{m^e v_{SN}} \right) \left/ \left(1 + \frac{e^2 B_r^2}{(m^e)^2 (v_{SN})^2} \right) \right. \quad (5.38)$$

$$A_2 = \frac{\left[\frac{-e}{m^e v_{SN}} \left(\frac{\partial N}{\partial z} \right) + \frac{e^2 B_r}{(m^e)^2 (v_{SN})^2 r} \left(\frac{\partial N}{\partial \theta} \right) \right]}{\left[1 + \frac{e^2 B_r^2}{(m^e)^2 (v_{SN})^2} \right]} + \dots$$

$$+ \dots + \frac{\left[\frac{N \cdot e}{m^e (v_{SN})^2} \frac{\partial v_{SN}}{\partial z} \left(1 - \frac{e^2 B_r^2}{(m^e)^2 (v_{SN})^2} \right) + \frac{2N(e)^3 B_r}{(m^e)^3 (v_{SN})^3} \frac{\partial B_r}{\partial z} - \frac{2N(e)^2 B_r}{(m^e)^2 (v_{SN})^3 r} \frac{\partial v_{SN}}{\partial \theta} \right]}{\left[1 + \frac{e^2 B_r^2}{(m^e)^2 (v_{SN})^2} \right]^2} \quad (5.39)$$

$$A_3 = \frac{\left[\frac{-e^2 B_r}{(m^e)^2 (v_{SN})^2 r} \left(\frac{\partial N}{\partial z} \right) - \frac{e}{m^e v_{SN} (r)^2} \left(\frac{\partial N}{\partial \theta} \right) \right]}{\left[1 + \frac{e^2 B_r^2}{(m^e)^2 (v_{SN})^2} \right]} + \dots$$

$$+ \dots + \frac{\left[\left(\frac{e \cdot N}{m^e (v_{SN})^2 (r)^2} \frac{\partial v_{SN}}{\partial \theta} - \frac{(e)^2 N}{(m^e)^2 (v_{SN})^2 r} \frac{\partial B_r}{\partial z} \right) \left(1 - \frac{e^2 B_r^2}{(m^e)^2 (v_{SN})^2} \right) + \frac{2NB_r e^2}{(m^e)^2 (v_{SN})^3 r} \frac{\partial v_{SN}}{\partial z} \right]}{\left[1 + \frac{e^2 B_r^2}{(m^e)^2 (v_{SN})^2} \right]^2} \quad (5.40)$$

$$A_4 = \left(\frac{-N \cdot e}{m^e v_{SN} (r)^2} \right) / \left(1 + \frac{e^2 B_r^2}{(m^e)^2 (v_{SN})^2} \right) \quad (5.41)$$

$$A_5 = \frac{\left[\frac{-N \cdot k}{m^e v_{SN}} \ln(N) \frac{\partial^2 T}{\partial z^2} - \frac{k}{m^e v_{SN}} (1 + \ln(N)) \frac{\partial N}{\partial z} \frac{\partial T}{\partial z} + \frac{e B_r k}{(m^e)^2 (v_{SN})^2 r} (1 + \ln(N)) \frac{\partial N}{\partial \theta} \frac{\partial T}{\partial z} \right]}{\left[1 + \frac{e^2 B_r^2}{(m^e)^2 (v_{SN})^2} \right]} + \dots$$

$$+ \dots + \frac{\left[\frac{N \cdot k \ln(N)}{m^e (v_{SN})^2} \frac{\partial T}{\partial z} \frac{\partial v_{SN}}{\partial z} \left(1 - \frac{e^2 B_r^2}{(m^e)^2 (v_{SN})^2} \right) + \frac{2kN(e)^2 B_r \ln(N)}{(m^e)^3 (v_{SN})^3} \frac{\partial T}{\partial z} \frac{\partial B_r}{\partial z} - \frac{2e B_r k N \ln(N)}{(m^e)^2 (v_{SN})^3 r} \frac{\partial v_{SN}}{\partial \theta} \frac{\partial T}{\partial z} \right]}{\left[1 + \frac{e^2 B_r^2}{(m^e)^2 (v_{SN})^2} \right]^2} + \dots$$

$$+ \dots + \left(\langle v_z^i \rangle \frac{\partial N}{\partial z} + N \frac{\partial \langle v_z^i \rangle}{\partial z} + \frac{1}{r} \left(\langle v_\theta^i \rangle \frac{\partial N}{\partial \theta} + N \frac{\partial \langle v_\theta^i \rangle}{\partial \theta} \right) \right) \quad (5.42)$$

Where:

- N = Plasma number density
- e = Electron charge
- m^e = Electron mass
- v_{SN} = Collision frequency for momentum transfer
- B_r = Radial magnetic field strength
- r = Radius of azimuth-axial plane
- T = Electron temperature
- k = Boltzmann constant
- $\langle v_z^i \rangle$ = Axial component of ion velocity
- $\langle v_\theta^i \rangle$ = Azimuth component of ion velocity

The derivative terms in the plasma potential equation (5.37) can be approximated using a 2nd order central difference scheme. The central difference expressions are given by equations 5.43 through 5.46. These equations were taken from a text on Computational Fluid Dynamics by Hoffmann and Chiang [24]. The term f in these equations represents an arbitrary function that can be replaced by the plasma potential function ϕ^* . The subscripts i and j indicate the computational grid location in the axial and azimuthal directions respectively. These finite difference formulas are also used to approximate the plasma state variable derivatives found in equations 3.38 through 5.42.

2nd Order Central Difference Equations:

$$\frac{\partial^2 f}{\partial z^2} = \frac{f_{i-1,j} - 2f_{i,j} + f_{i+1,j}}{(\Delta z)^2} \quad (5.43)$$

$$\frac{\partial f}{\partial z} = \frac{f_{i+1,j} - f_{i-1,j}}{2(\Delta z)} \quad (5.44)$$

$$\frac{\partial f}{\partial \theta} = \frac{f_{i,j+1} - f_{i,j-1}}{2(\Delta \theta)} \quad (5.45)$$

$$\frac{\partial^2 f}{\partial \theta^2} = \frac{f_{i,j-1} - 2f_{i,j} + f_{i,j+1}}{(\Delta \theta)^2} \quad (5.46)$$

Where:

$$f = \text{Arbitrary function}$$

Substituting the finite difference expressions into equation 5.37, we obtain the finite difference form of the plasma potential equation given by 5.47 below.

Finite Difference Form of the Plasma Potential Equation:

$$\begin{aligned} & \left(\frac{-2A_1}{(\Delta z)^2} - \frac{2A_4}{(\Delta \theta)^2} \right) \cdot \phi_{i,j}^* + \left(\frac{A_1}{(\Delta z)^2} + \frac{A_2}{2(\Delta z)} \right) \cdot \phi_{i+1,j}^* + \left(\frac{A_1}{(\Delta z)^2} - \frac{A_2}{2(\Delta z)} \right) \cdot \phi_{i-1,j}^* + \dots \\ & \dots + \left(\frac{A_3}{2(\Delta \theta)} + \frac{A_4}{(\Delta \theta)^2} \right) \cdot \phi_{i,j+1}^* + \left(\frac{-A_3}{2(\Delta \theta)} + \frac{A_4}{(\Delta \theta)^2} \right) \cdot \phi_{i,j-1}^* = -A_5 \end{aligned} \quad (5.47)$$

Where:

$$\phi^* = \text{Plasma potential function (constant along magnetic field lines)}$$

By evaluating equation 5.47 at every point on the computational grid, we develop a system of $N1 \cdot N2$ equations with $N1 \cdot N2$ unknowns; where $N1$ and $N2$ are the number of grid points in the axial and azimuthal directions respectively. This system of equations can be written in the form of a block tri-diagonal matrix equation that can be solved efficiently using linear algebra computational techniques. The matrix form of the plasma potential equation is given by 5.48.

[illegible]

(5.48)

The B_i coefficient terms that appear in 5.48 are defined by equations 5.49 through 5.53 below. The C_i and D_i coefficients are established from the boundary conditions to the system of equations and will be derived in section 5.4.

$$B_1 = \frac{-2A_1}{(\Delta z)^2} - \frac{2A_4}{(\Delta \theta)^2} \quad (5.49)$$

$$B_2 = \frac{A_1}{(\Delta z)^2} + \frac{A_2}{2(\Delta z)} \quad (5.50)$$

$$B_3 = \frac{A_1}{(\Delta z)^2} - \frac{A_2}{2(\Delta z)} \quad (5.51)$$

$$B_4 = \frac{A_3}{2(\Delta \theta)} + \frac{A_4}{(\Delta \theta)^2} \quad (5.52)$$

$$B_5 = \frac{-A_3}{2(\Delta \theta)} + \frac{A_4}{(\Delta \theta)^2} \quad (5.53)$$

5.3 Heavy Particle Model

The ions and neutral particles are modelled using the discrete kinetic method. This method involves tracking the position and velocity of each particle. Ideally, every atom within the Hall thruster domain should be tracked independently. In practice the number of atoms is too great for a time efficient simulation. Instead, the collective motion of a group of particles is calculated. These particles are known as super-particles. A schematic representation of a collection of super-particles occupying an arbitrary volume (V) is shown in figure 5.6. Assume that the number of super-particles within this domain is given by (n). Also, the number of discrete particles per super-particle is given by (p). Then the number density within the domain can be calculated by equation 5.54. Inversely, the average number of particles per super-particle can be calculated by

equation 5.55. The Hall thruster simulation used in this study is composed of approximately 2,500,000 super-particles. Each super-particle subsequently represents a collection of several million discrete particles.

Figure 5.6. Super-particle Representation of Plasma

$$N = \frac{1}{V} \sum_{i=1}^n p_i \quad (5.54)$$

$$\bar{p} = \frac{N \cdot V}{n} \quad (5.55)$$

Where:

N = Particle number density

V = Volume

p_i = Number of particles per super-particle

$\bar{p}$ = Average number of particles per super-particle

n = Number of super-particles within the volume

In order to evaluate the equations governing the heavy particle motion, it is necessary to know the plasma field properties at the location of the particle. However, these field

properties are often provided on a computational grid which is not aligned with the particle itself. Also, the collective properties of the particles need to be correctly distributed over a computational grid in order to solve the electron continuum equations. The method used to connect the computational grid solution to the discrete particle distribution is the particle-in-cell (PIC) technique. This method is described in more detail in the work by Fife [18].

A schematic representation of the PIC technique is shown in figure 5.7. The formula used to linearly interpolate the field properties at the location of the particle is given by equation 5.56. The formulas used to assign nodal values from a discrete particle solution are given by equations 5.57 through 5.60.

Figure 5.7. Particle-in-Cell Interpolation Schematic

Particle-in-Cell Interpolation Functions:

$$f_p = f_1(1-\xi)(1-\eta) + f_2(\xi)(1-\eta) + f_3(\xi)(\eta) + f_4(1-\xi)(\eta) \quad (5.56)$$

$$f_1 = f_p(1-\xi)(1-\eta) \quad (5.57)$$

$$f_2 = f_p(\xi)(1-\eta) \quad (5.58)$$

$$f_3 = f_p(\xi)(\eta) \quad (5.59)$$

$$f_4 = f_p(1-\xi)(\eta) \quad (5.60)$$

Where:

f = Arbitrary function

ξ = Distance function going from 0 to 1

η = Distance function going from 0 to 1

The neutral particles are unaffected by the electric and magnetic fields within the Hall thruster. Additionally, inter-particle collisions and collisions between neutral particles and ions are neglected because of a small collision frequency compared with residence time within the thruster. Collisions between neutral particles and electrons are assumed to contribute a negligible amount of momentum. Then, by Newton's first law, the velocity of the neutral particles remains constant. The position of the neutral particles can be derived using a first order Taylor series expansion. The resulting equations of motion in each coordinate direction are given by 5.61 through 5.63.

Neutral Particle Equations of Motion:

$$r' = r + (v_r \cdot \Delta t) \quad (5.61)$$

$$\theta' = \theta + \left(\frac{v_\theta \cdot \Delta t}{r} \right) \quad (5.62)$$

$$z' = z + (v_z \cdot \Delta t) \quad (5.63)$$

Where:

- r = Position in the radial direction
- v_r = Velocity in the radial direction
- Δt = Time step size
- θ = Angular displacement in the azimuth direction
- v_θ = Velocity in the azimuth direction
- z = Position in the axial direction
- v_z = Velocity in the axial direction

The ions are affected by both the electric and magnetic fields. The force acting on the ions is given by the Lorentz force equation (5.64). However, in the case of a Hall thruster the force on the ions produced by the electric field is dominant, and the magnetic field can be neglected. Inter-particle collisions and collisions of ions with neutral particles are also neglected. Additionally, the momentum imparted by collisions with electrons is assumed to be negligible. Then, the equations of motion can be derived by a first order Taylor series expansion of Newton's laws of motion. The resulting equations are given by 5.65 through 5.70.

Lorentz Force Equation:

$$m\mathbf{R} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (5.64)$$

Where:

- m = Ion mass
- $\mathbf{R}$ = Force vector per unit mass

- q = Ion charge
 $\mathbf{E}$ = Electric field vector
 $\mathbf{v}$ = Velocity vector
 $\mathbf{B}$ = Magnetic field vector

Ion Particle Equations of Motion:

$$v_r = \text{constant} \quad (5.65)$$

$$v'_\theta = v_\theta + \frac{qE_\theta \Delta t}{m} \quad (5.66)$$

$$v'_z = v_z + \frac{qE_z \Delta t}{m} \quad (5.67)$$

$$r' = r + (v_r \cdot \Delta t) \quad (5.68)$$

$$\theta' = \theta + \left(\frac{v_\theta \cdot \Delta t}{r} \right) \quad (5.69)$$

$$z' = z + (v_z \cdot \Delta t) \quad (5.70)$$

Where:

- v_r = Velocity in the radial direction
 v_θ = Velocity in the azimuth direction
 q = Ion charge
 E_θ = Electric field strength in the azimuth direction
 Δt = Time step size
 m = Ion mass
 v_z = Velocity in the axial direction
 E_z = Electric field strength in the axial direction
 r = Position in the radial direction
 θ = Angular displacement in the azimuth direction
 z = Position in the axial direction

At each time step neutral particles are introduced at the anode boundary of the thruster. The position in the azimuth and radial directions is established randomly. The velocity in each coordinate direction is determined using the Maxwell particle distribution function. The Maxwell particle distribution is derived from a solution to the Boltzmann equation (5.20) assuming steady conditions and neglecting any field force terms. The resulting function is given by equation 5.71. This particle distribution function depends on the average temperature of the particles. The general trends of this function are shown in figure 5.8, where the Maxwellian distribution function is plotted versus the peculiar velocity at various temperatures.

Maxwellian Particle Distribution:

$$f = N \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-[mV_i V_i / 2kT]} \quad (5.71)$$

Where:

- f = Velocity distribution function
- N = Particle number density
- m = Particle mass
- k = Boltzmann constant
- T = Average particle temperature
- V = Peculiar velocity

The probability that a particle will have a given peculiar speed is given by equation 5.72. This probability equation was derived from the Maxwellian particle distribution (5.71) in a text by Holt and Haskell [17].

Maxwellian Probability Density:

$$4\pi \cdot f \cdot V_i V_i = \left(\frac{2}{\pi}\right)^{1/2} N \left(\frac{m}{kT}\right)^{3/2} V_i V_i e^{-(mV_i V_i / 2kT)} \quad (5.72)$$

Where:

- f = Velocity distribution function
- N = Particle number density
- m = Particle mass
- k = Boltzmann constant
- T = Average particle temperature
- V = Peculiar velocity

The Maxwellian probability distribution function (5.72) is used to statistically calculate the velocity of each neutral particle entering the Hall thruster domain. This probability function is also used to establish the velocity of each neutral particle within the Hall thruster domain when the simulation begins. The Maxwellian probability density results in a non-symmetric distribution shown in figure 5.9, where the probability density is plotted versus the peculiar velocity at various temperatures.

Figure 5.8. Maxwellian Distribution of Peculiar Velocities

Figure 5.9. Maxwellian Distribution of Peculiar Speeds

Neutral particles are transformed into ions by ionizing collisions with electrons. As the simulation progresses the mass of the neutral super-particles is reduced and new ion super-particles are created. The ‘children’ ion particles are initialized with the same position and velocity as the ‘parent’ neutral particle. The frequency of ion formation is determined by the local rate of ionization (R_i). The method used to calculate the ionization rate is taken from a paper by Ahedo et al. [25]. The ionization rate formula used in this paper is given by equation 5.73.

$$R_i = N^i N^n \bar{\sigma}_i \cdot \bar{c}_e \quad (5.73)$$

Where:

$$\bar{\sigma}_i = \sigma_{i0} \left(1 + \frac{T^e \cdot k \cdot E_i}{(k \cdot T^e + E_i)^2} \right) e^{\left(\frac{-E_i}{k \cdot T^e} \right)} \quad (5.74)$$

$$\bar{c}_e = \sqrt{8 \cdot k \cdot T^e / \pi \cdot m^e} \quad (5.75)$$

R_i = Ionization rate

N^i = Ion number density

N^n = Neutral particle number density

$\bar{\sigma}_i$ = Average ionization cross-section

$\bar{c}_e$ = Proportionality coefficient

σ_{i0} = Experimentally determined constant: $5 \times 10^{-20} m^2$

T^e = Electron temperature

E_i = Energy for primary ionization

k = Boltzmann constant

m^e = Electron mass

The Monte-Carlo statistical technique is used to determine which neutral particles ionize at each time step. The Monte-Carlo method is described schematically in figure 5.10. A random number is generated for each neutral super-particle and compared to a probability function. If the random number is less than the probability, then a new ion super-particle is created and the mass of the neutral particle is reduced. The mass of the newly created ion, and mass decrease of the neutral super-particle, are calculated consistently based on the ionization rate (5.73). The neutral super-particle is removed from the simulation once its mass is depleted. The probability function is calculated based on a user defined constant: $const_p$. For the simulation conducted in this study, the $const_p$ term was set equal to 0.004.

Figure 5.10. Monte-Carlo Technique for Predicting Neutral Ionization

5.4 Boundary Conditions and Imposed Field Properties

The boundary conditions for the electron continuum within the Hall thruster are shown in figure 5.11. The physical placement of the computational plane is shown on the left hand side of this figure. The computational plane has been extended as a rectangle on the right to help illustrate each of the four boundary conditions. The physical interpretation of each boundary is as follows. Boundary CD corresponds to the anode region of the thruster. Next, boundary AB represents the cathode region. Finally, boundary CA and DB are periodic with a 2π periodicity. These periodic boundaries are imaginary and have no physical equivalence in the actual Hall thruster.

Figure 5.11. Electron Continuum Boundary Conditions

The boundary condition for the anode of the thruster is given by equation 5.76. This equation simply states that the potential of the anode is constant and equal to the

discharge voltage V_d . Using equation 5.33, we can reformulate this expression in terms of the radius independent potential ϕ^* . The result is given by equation 5.77. By comparing this expression to the governing matrix equation (5.48) we can identify that the term D_1 within the matrix equation is given by equation 5.78 below.

Boundary CD : Anode

$$\phi_{1,j} = V_d \quad (5.76)$$

$$\phi^* = V_d - \frac{T^e k}{e} \ln(N^e) \quad (5.77)$$

$$D_1 = V_d - \frac{T^e k}{e} \ln(N^e) \quad (5.78)$$

Where:

- ϕ = Plasma potential
- V_d = Discharge voltage
- ϕ^* = Constant plasma potential along magnetic field lines
- T^e = Electron temperature
- k = Boltzmann constant
- e = Electron charge
- N^e = Plasma number density

The boundary conditions for the cathode region of the Hall thruster are given by equations 5.79 through 5.82. These equations state that the velocity of the electrons and ions leaving the hall thruster domain are normal to the boundary. Additionally, these

equations ensure that the electrons and ions do not accelerate in the axial direction as they enter and leave solution domain respectively.

Boundary AD : Cathode

$$\langle v_\theta^i \rangle = 0 \quad (5.79)$$

$$\langle v_\theta^e \rangle = 0 \quad (5.80)$$

$$\frac{\partial \langle v_z^i \rangle}{\partial z} = 0 \quad (5.81)$$

$$\frac{\partial \langle v_z^e \rangle}{\partial z} = 0 \quad (5.82)$$

Where:

$$\langle v_\theta^i \rangle = \text{Ion velocity in the azimuth direction}$$

$$\langle v_\theta^e \rangle = \text{Electron velocity in the azimuth direction}$$

$$\langle v_z^i \rangle = \text{Ion velocity in the axial direction}$$

$$\langle v_z^e \rangle = \text{Electron velocity in the axial direction}$$

If we apply the cathode boundary conditions to the continuity of charge equation (5.25), we can derive the expression found in equation 5.83. This equation demonstrates that the velocity of the electrons entering the Hall thruster equals the velocity of the ions leaving. By substituting the expression for the electron velocity (5.28), and reformulating in terms of the radius independent potential ϕ^* , we can derive equation 5.84.

$$\langle v_z^i \rangle = \langle v_z^e \rangle \quad (5.83)$$

$$\left[\left(\frac{e}{m^e v_{SN}} \right) \frac{\partial \phi^*}{\partial z} + \left(\frac{e^2 B_r}{(m^e)^2 v_{SN}^2 r} \right) \frac{\partial \phi^*}{\partial \theta} + \left(\frac{k \ln(N^e)}{m^e v_{SN}} \frac{\partial T^e}{\partial z} \right) \right] \left/ \left[1 + \left(\frac{e^2 B_r^2}{(m^e)^2 v_{SN}^2} \right) \right] \right. = \langle v_z^i \rangle \quad (5.84)$$

Where:

$\langle v_z^i \rangle =$ Ion velocity in the axial direction

$\langle v_z^e \rangle =$ Electron velocity in the axial direction

$e =$ Electron charge

$m^e =$ Electron mass

$\nu_{SN} =$ Collision frequency for momentum transfer

$\phi^* =$ Constant plasma potential along magnetic field lines

$B_r =$ Radial magnetic field strength

$r =$ Position in the radial direction

$k =$ Boltzmann constant

$N^e =$ Plasma number density

$T^e =$ Electron temperature

The next step is to formulate equation 5.84 as a finite difference expression. However, we cannot use a central difference formula for the derivatives in the axial direction because the boundary is at the right hand extent of the domain. Instead, we choose a 1st order backward difference formula given by equation 5.85.

1st Order Backward Difference Equation:

$$\frac{\partial f}{\partial z} = \frac{f_{i,j} - f_{i-1,j}}{\Delta z} \quad (5.85)$$

Where:

$f =$ Arbitrary function

The general form of the finite difference formula derived from equation 5.84 is given by equation 5.86 below. The coefficient terms used in this formula are provided by equations 5.87 through 5.91. Comparing this expression to the governing plasma potential matrix equation (5.48), the coefficient terms C_1 through C_5 correspond to the same terms in the matrix equation.

Finite Difference Formulation at the Cathode:

$$C_1\phi_{i,j}^* + C_2\phi_{i,j+1}^* + C_3\phi_{i,j-1}^* + C_4\phi_{i-1,j}^* = -C_5 \quad (5.86)$$

Coefficient Terms:

$$C_1 = \frac{1}{\Delta z} \left(\frac{e}{m^e v_{SN}} \right) \left/ \left(1 + \frac{e^2 B_r^2}{(m^e)^2 v_{SN}^2} \right) \right. \quad (5.87)$$

$$C_2 = \frac{1}{2(\Delta\theta)} \left(\frac{e^2 B_r}{(m^e)^2 v_{SN}^2 r} \right) \left/ \left(1 + \frac{e^2 B_r^2}{(m^e)^2 v_{SN}^2} \right) \right. \quad (5.88)$$

$$C_3 = \frac{-1}{2(\Delta\theta)} \left(\frac{e^2 B_r}{(m^e)^2 v_{SN}^2 r} \right) \left/ \left(1 + \frac{e^2 B_r^2}{(m^e)^2 v_{SN}^2} \right) \right. \quad (5.89)$$

$$C_4 = \frac{-1}{\Delta z} \left(\frac{e}{m^e v_{SN}} \right) \left/ \left(1 + \frac{e^2 B_r^2}{(m^e)^2 v_{SN}^2} \right) \right. \quad (5.90)$$

$$C_5 = \left(\frac{k \ln(N^e)}{m^e v_{SN}} \frac{\partial T^e}{\partial z} \right) \left/ \left(1 + \frac{e^2 B_r^2}{(m^e)^2 v_{SN}^2} \right) \right. - \langle v_z^i \rangle \quad (5.91)$$

The periodic boundary conditions are implicitly accounted for in the formulation of the governing plasma potential matrix equation (5.48). For each point that falls outside of the periodic boundary, the corresponding point along the opposing boundary was selected. In this way the electron solution wraps around the periodic boundary.

The Hall thruster simulation relies on two field properties that are obtained from experiment: the temperature and magnetic field profiles. An analytical function is used in both cases to approximate the experimental observations. The analytical function that was selected closely resembles the well know Gaussian distribution formula (5.92). The coefficients of the Gaussian distribution were tailored to approximate the experimental observations. The resulting expressions for the magnetic field and temperature profile are given by equations 5.93 and 5.94 respectively.

Gaussian Distribution:

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)} \quad (5.92)$$

Radial Magnetic Field Profile:

$$B_r = B_{\max} \left(4.7619 \times 10^{-2} + 9.5238 \times 10^{-1} e^{-(z-0.075)^2/(0.04)^2} \right) \quad (5.93)$$

Electron Temperature Profile:

$$T^e = T_{\max} \left(0.6373 + 0.3627 e^{-(z-0.075)^2/(0.04)^2} \right) \quad (5.94)$$

Where:

B_r = Radial magnetic field strength

$B_{\max}$ = Peak value of the magnetic field strength

T^e = Electron temperature

$T_{\max}$ = Peak value of the temperature field

The magnetic field approximation has been plotted in comparison to experimental measurements in figure 5.12. It can be observed from this figure that the analytical

approximation departs from experimental observation for much of the Hall thruster domain. The significance of this discrepancy on the simulation results is unknown and is cited for future work.

The temperature field approximation has been plotted in comparison to experimental measurements in figure 5.13. This figure compares the temperature profiles at three operating points of the Hall thruster. It can be observed that the approximation closely follows experimental measurement in the 100V discharge condition. However, the approximation departs significantly from experiment in the 150V and 200V operating regimes. This is particularly apparent in the anode region of the thruster. The significance of this discrepancy is unknown and has also been cited for future work.

Figure 5.12. Radial Magnetic Field Profile

Figure 5.13. Electron Temperature Profile

5.5 Heavy Particle Boundary Interactions

The heavy particles interact with six boundaries within the Hall thruster simulation. The first two boundaries are the inner and outer channel walls of the Hall thruster. The third boundary is the anode of the Hall thruster. The fourth boundary is the exit plane of the solution domain. The final two boundaries are periodic boundaries at azimuth offsets of 0 and 2π respectively. The two periodic boundaries are imaginary and have no physical significance within the Hall thruster.

The neutral particles are assumed to reflect diffusely from all solid surfaces within the Hall thruster. The first surface is the outer walls of the Hall thruster channel. Figure 5.14 schematically shows a neutral particle undergoing a diffuse reflection from the outer boundary. The expressions that describe this interaction are given by equations 5.95 through 5.97. The neutral particle remains at the same azimuth offset at which the collision occurred. The radial location is set to the radius of the channel. The angle of the reflection is randomly assigned between 0 and π . The velocity of the particle is statistically assigned based on the Maxwellian distribution (5.71).

Figure 5.14. Diffuse Particle Reflection from Outer Wall

Outer Wall Diffuse Reflection:

$$V_{\theta} = \cos(\theta) \cdot V_{tot} \quad (5.95)$$

$$V_r = -\sin(\theta) \cdot V_{tot} \quad (5.96)$$

$$\theta = rand\{0 \dots \pi\} \quad (5.97)$$

Where:

V_θ = Velocity of particle in the azimuth direction

θ = Angle of reflection

V_r = Velocity of particle in the radial direction

V_{tot} = Total particle velocity calculated statistically from the Maxwellian distribution

V_z = Velocity of particle in the axial direction calculated statistically from the Maxwellian distribution

The diffuse reflection of neutral particles at the inner channel walls follows the same form as the outer wall interaction. Figure 5.15 shows a schematic representation of a neutral particle undergoing a diffuse reflection from the inner channel wall. The expressions that govern this interaction are given by equations 5.98 through 5.100.

Figure 5.15. Diffuse Particle Reflection from Inner Wall

Inner Wall Diffuse Reflection:

$$V_{\theta} = -\cos(\theta) \cdot V_{tot} \quad (5.98)$$

$$V_r = \sin(\theta) \cdot V_{tot} \quad (5.99)$$

$$\theta = rand\{0 \dots \pi\} \quad (5.100)$$

Where:

V_{θ} = Velocity of particle in the azimuth direction

θ = Angle of reflection

V_r = Velocity of particle in the radial direction

V_{tot} = Total particle velocity calculated statistically from the Maxwellian distribution

V_z = Velocity of particle in the axial direction calculated statistically from the Maxwellian distribution

The neutral particles also undergo diffuse reflection from the anode of the thruster. This is similar to the injection of neutrals at the anode described in section 5.3. When the neutral particle crosses the anode it is removed from the simulation and a new neutral particle is injected at the same location at the anode. The velocity and orientation of the neutral particle is statistically calculated based on the Maxwellian distribution (5.71). When the neutral particles cross the cathode boundary they are simply removed from the simulation.

The periodic boundaries are enforced at each time step by checking for neutral particles and ions at azimuth offsets below 0 or above 2π . If this condition exists, then quantity 2π is added or subtracted from the azimuth offset to satisfy the periodicity.

Ions interact with the same boundaries as the neutral particles. One key difference is that when the ion interacts with a solid boundary it is assumed to undergo an electron reattachment process and become a neutral particle. When the ion crosses the anode or channel walls it is removed from the simulation and a neutral particle is diffusely reflected at the same location as the ion. The neutral reflection is performed using the methods described above. Ideally, for each ion that interacts with a solid boundary a new neutral particle is formed. However, to prevent small neutral super-particles from rapidly accumulating within the Hall thruster domain the following technique is employed. A random number scheme is used so that only 1% of the ions that collide with the boundary are reflected back as neutral particles. However, the mass of the reflected neutral particle is 100 times greater than the original ion. When the ions cross the cathode boundary they are simply removed from the simulation.

Section 6: Results and Discussion

6.0 Results and Discussion

This section presents the results that were obtained from the Hall thruster simulation and compares them to experimental measurements. The simulation trials that were conducted during this project are described in section 6.1. The geometric properties of the simulated Hall thruster were set equal to those of the Stanford Hall thruster. This was done to facilitate direct comparison between the simulated and experimental results. The Stanford Hall thruster is a custom built low power device with the following geometric parameters:

- Channel diameter: 90mm
- Channel width: 11mm
- Channel length: 80mm

The results are divided into two main components: the time averaged simulation properties, and the unsteady plasma oscillation characteristics. The time averaged simulation results include the axial distribution of various plasma parameters including the ion velocity, plasma potential, plasma density, and neutral density. Also, 2-dimensional snapshots are provided of the simulation parameters captured at a particular instant of time. The time averaged simulation results are presented in section 6.2.

The unsteady plasma oscillations are evaluated by expressing the plasma parameters in the frequency domain. This analysis allows favoured frequencies of plasma oscillations to be observed. The oscillations are tracked along the axis of the thruster in order to determine where the oscillations tend to occur. The plasma oscillations are described in section 6.3.

The final component to this section deals with the contribution of the plasma oscillations to the electron conduction properties of the plasma. This is described in section 6.4. By statistically analysing the plasma oscillations it is possible to evaluate the electron transport within the plasma. Additionally, it is possible to compute the anomalous electron transport term that appears in traditional Hall thruster simulations. It was concluded that high frequency plasma oscillations have a significant impact on the plasma conduction. Additionally, it was found that the electron conduction predicted by the simulation is in good agreement with experimental observation.

6.1 Summary of Simulation Trials

Twelve separate simulation trials were performed during this project. Each trial represented a distinct set of discharge voltage and magnetic field strength operating conditions. The values of discharge voltage and magnetic field strength were assigned for each trial as shown in table 6.1.

		Peak Magnetic Field Strength			
		50 Gauss	100 Gauss	150 Gauss	200 Gauss
Discharge Voltage	100V	Run_A1	Run_A2	Run_A3	Run_A4
	150V	Run_B1	Run_B2	Run_B3	Run_B4
	200V	Run_C1	Run_C2	Run_C3	Run_C4

Table 6.1. Simulation Run Naming System

It is well known that the plasma oscillation characteristics within Hall thrusters depend heavily on which regime the thruster is operating. The values of discharge voltage and magnetic field strength were selected in table 6.1 in order to observe the plasma oscillation characteristics under a wide range of Hall thruster operating regimes. The operating regimes of the Hall thruster are shown in figure 6.1. Unfortunately, it is not possible to determine which regime the Hall thruster is running in advance of the simulation. This is because the operating regime is based on the discharge current and magnetic field strength. The discharge current is not imposed by the simulation but establishes itself naturally based on the performance of the thruster. The discharge voltage values were assigned in table 6.1 based on knowledge of similar Hall thruster performance.

Figure 6.1. Operating Regimes of the Hall Thruster

The simulation time window for runs A1 through A4 was two microseconds. This small simulation duration was chosen to minimize the time that the simulation took to complete, and still enabled oscillations between 1MHz and 500MHz to be captured. This time window allowed the simulation to resolve two complete wave lengths of 1MHz frequency. Due to time constraints, the simulation time window for runs B1 through C4 was set to one microsecond. This simulation time window still allowed a wave length of 1MHz frequency to be captured. However, the accuracy of the lower frequency measurements may be diminished. The computational grid size for all simulation runs was 50 by 50.

Each simulation run took an average of 2 weeks to complete. On a single computer the 12 simulation trials would have taken approximately 6 months to run. In order to

dramatically reduce the time needed to complete this project, the trials were run on 6 separate computers. As a result, the simulation component of this project took approximately one month to complete. The performance statistics for the computers used to run these simulations were as follows:

- Processor: Pentium 4, 3.0 GHz
- Memory: 1000.0 MB RAM
- Operating System: Windows XP

6.2 Steady State Simulation Results

In order to compare the simulation and experimental results the average value of various plasma parameters was plotted along the axis of the thruster. These parameters were averaged over the duration of the simulation. The parameters that were considered included the axial ion velocity, plasma potential, plasma density, and neutral density. It should be noted that the time frame for this simulation was likely too small for good averages of most simulation parameters. Most existing Hall thruster simulations use a time window of a couple hundred microseconds, which is 100 times greater than that of this simulation. However, many conclusions can still be gained by looking at the plots of the average plasma parameters.

The first plasma parameter that will be considered is the axial ion velocity. This parameter is the most important aspect of the Hall thruster performance. This is because

the momentum of the ions creates the force that the Hall thruster exerts on the spacecraft. The velocity of the ions exiting the thruster is directly related to the specific impulse (I_{sp}) of the propulsion system.

The axial ion velocity predicted by the simulation is compared to experimental observations in figure 6.2. In general, the axial distribution of the ion velocity follows the same trend as indicated by experiment. The ion velocity increases in an exponential manner in the region of the highest magnetic field strength of the thruster. However, two important discrepancies between experiment and simulation can be observed. First, the velocity of the ions exiting the Hall thruster is overestimated in the case of the 100V and 200V discharge conditions, and underestimated in case of the 150V condition. It is possible that a larger simulation time frame would increase the ion exit velocity in the 150V condition. However, an increase in the simulation time would not decrease the exit velocity in the 100V and 200V conditions. The second discrepancy is a substantial negative ion velocity in the mid-channel location of the thruster. This negative ion velocity continues all the way to the anode in the case of the 200V discharge condition. This negative ion velocity is not reflected by experimental observations.

Figure 6.2. Comparison of Simulated and Experimental Axial Ion Velocity [8]

The model for the ion component of the simulation is purely kinetic and driven entirely by the gradients in the electric field. The overestimation of the ion exit velocity results from a more fundamental problem within the simulation. This problem is related to oscillations that occur in the potential field solution. A time-trace of the plasma potential at the exit to the Hall thruster is shown in figure 6.3. It can be observed from this figure that the plasma oscillation reach magnitudes of over 800 volts. Within the peak magnetic field region of the thruster these oscillations can be even more dramatic and reach amplitudes of thousands of volts. Similar plasma potential oscillations do occur within actual Hall thrusters. However, the magnitudes of these oscillations do not reach these large values. The overestimation of the ion velocity is driven by these unusually large

gradients in the potential field solution. These plasma oscillations will be discussed in more detail in section 6.3.

Figure 6.3. Time Trace of Plasma Potential, $B=100$ Gauss, $V_d=100$ V

The axial ion velocity has been plotted for each of the 12 simulation trials in figure 6.4 through 6.7 below. It can be observed that the largest overestimation of the ion velocity occurs for the 200 Gauss, 200V discharge condition. It will be shown in section 6.3 that the plasma oscillations under these conditions are also the greatest. The most accurate prediction of the axial ion velocity occurs when the imposed magnetic field is 50 Gauss. This corresponds to the conditions where the plasma oscillations are the smallest.

Figure 6.4. Axial Ion Velocity, B=50 Gauss

Figure 6.5. Axial Ion Velocity, B=100 Gauss

Figure 6.6. Axial Ion Velocity, B=150 Gauss

Figure 6.7. Axial Ion Velocity, B=200 Gauss

The next parameter under consideration is the plasma potential. The simulation results are compared against experiment in figure 6.8. It can be observed from this figure that the simulation predicts the plasma potential quite well in the anode region of the thruster. However, as we approach the cathode the simulation results rapidly become chaotic. This departure from experiment is caused by spikes in the plasma potential field that become dominant near the exit of the Hall thruster. These anomalous spikes overshadow the mean value of the potential field. It can be observed from a time-trace of the potential field (figure 6.3) that these spikes are short duration events and can be easily filtered from the results. This is accomplished by ignoring potential values that are above a reasonable limit. A much better estimate of the mean value of the potential field can be determined when this filtering technique is applied to the results. Figure 6.9 shows the revised comparison between the simulation and experimental results.

Figure 6.8. Comparison of Simulated and Experimental Plasma Potential [8]

Figure 6.9. Comparison of Filtered Simulated and Experimental Plasma Potential [8]

The average value of the plasma potential field is plotted for each of the 12 simulation trials in figure 6.10 through 6.14. It can be observed that there is some jaggedness to the curves within these graphs. This reflects the limitation of the small simulation time window over which the results were averaged.

Figure 6.10. Plasma Potential, B=50 Gauss

Figure 6.11. Plasma Potential, B=100 Gauss

Figure 6.12. Plasma Potential, B=150 Gauss

Figure 6.13. Plasma Potential, B=200 Gauss

The next parameter under consideration is the electron number density. The simulated electron number density has been compared to experimental values in figure 6.14. It can be observed from this plot that there is a considerable difference between simulated and experimental results. This difference is particularly notable in the anode region of the thruster. In the experimental results the electron density tends to decrease toward the anode. In contrast, the simulation indicates that the electron density increases toward the anode. This trend is in apparent contradiction to the ionization rate formula, which predicts the ionization rate based on the electron temperature and should be highest in the vicinity of the exit plane. This abnormal rise in the electron density is most likely due to a strong negative flow of ions (toward the anode) that can be observed in figures 6.5

through 6.7. A higher concentration of ions around the anode necessitates a higher electron density, given the requirement of local plasma neutrality. This negative flow of ions does not occur in experimental observations.

Figure 6.14. Comparison of Simulated and Experimental Electron Density [8]

The average electron number density has been plotted for each of the 12 simulation trials in figures 6.15 through 6.18 below. The most accurate prediction of the electron density occurs when the discharge voltage is set to 100V. The largest discrepancy between simulated and experimental results occurs for the 200V discharge conditions.

Figure 6.15. Electron Number Density, $B=50 \text{ Gauss}$ Figure 6.16. Electron Number Density, $B=100 \text{ Gauss}$

Figure 6.17. Electron Number Density, $B=150 \text{ Gauss}$ Figure 6.18. Electron Number Density, $B=200 \text{ Gauss}$

The final parameter that will be considered is the neutral Xenon density. A plot of the experimental measurements can be found in figure 6.19. In comparison, a plot of the average simulation results can be found in figures 6.20 through 6.23. A direct comparison is not appropriate in this case because the Xenon flow rate does not correspond between simulation and experiment. However, it can be observed that the simulation results accurately reflect the trends found experimentally for all 12 simulation trials.

Figure 6.19. Experimental Neutral Density [8]

Figure 6.20. Neutral Number Density, B=50 Gauss

Figure 6.21. Neutral Number Density, B=100 Gauss

Figure 6.22. Neutral Number Density, $B=150 \text{ Gauss}$ Figure 6.23. Neutral Number Density, $B=200 \text{ Gauss}$

Additional insight into the simulation can be gained by looking at 2-dimensional plots of the plasma parameters across the computational grid at a particular point of time. Two trials have been shown below. The first represents conditions where the plasma oscillations are mild. For this trial the magnetic field strength is 50 Gauss and the discharge voltage is 150V. The second trial represents conditions where the plasma oscillations are violent and dominate the solution. For the second trial the magnetic field strength is 150 Gauss and the discharge voltage is 200V. All of the 2-dimensional plots have been captured at one microsecond instant in the simulation.

The plasma parameters that have been plotted include the plasma potential, axial electron velocity, azimuthal electron velocity, axial ion velocity, azimuthal ion velocity, and the plasma density. Regular patterns can be observed in certain plasma parameters in the case of mild oscillations. This is especially apparent with the azimuthal electron velocity (figure 6.26) and axial ion velocity (figure 6.27). When the plasma oscillations become violent the plasma parameters appear to become more chaotic for all plasma parameters (figures 6.30 through 6.35).

Figure 6.24. Plasma Potential, $B=50$ Gauss, $V_d=150V$ Figure 6.25. Axial Electron Velocity, $B=50$ Gauss, $V_d=150V$ Figure 6.26. Azimuthal Electron Velocity, $B=50$ Gauss, $V_d=150V$

Figure 6.27. Axial Ion Velocity, $B=50$ Gauss, $V_d=150V$ Figure 6.28. Azimuthal Ion Velocity, $B=50$ Gauss, $V_d=150V$ Figure 6.29. Plasma Density, $B=50$ Gauss, $V_d=150V$

Figure 6.30. Plasma Potential, $B=150$ Gauss, $V_d=200V$ Figure 6.31. Axial Electron Velocity, $B=150$ Gauss, $V_d=200V$ Figure 6.32. Azimuthal Electron Velocity, $B=150$ Gauss, $V_d=200V$

Figure 6.33. Axial Ion Velocity, $B=150$ Gauss, $V_d=200V$ Figure 6.34. Azimuthal Ion Velocity, $B=150$ Gauss, $V_d=200V$ Figure 6.35. Plasma Density, $B=150$ Gauss, $V_d=200V$

Additional 2-dimensional plots of the plasma parameters for all 12 simulation trials can be found in appendix A. All of these plots were obtained at the one microsecond instant of the simulation.

6.3 High Frequency Simulation Results

All of the plasma parameters within the simulation, with the notable exception of the neutral density, fluctuate at high frequencies. These fluctuations from the mean value are known collectively as plasma oscillations. The parameter that is focussed on in this section is the plasma density oscillations. There are two reasons for concentrating on the plasma density alone. First, these oscillations are the easiest to observe experimentally and have historically been the main focus of research. Second, there is a direct link between plasma oscillations and the conductivity of the plasma that will be elaborated on in section 6.4.

In order to analyse the high frequency oscillations we will transform the plasma density signal, recorded at a single point in space, into the frequency domain. This is accomplished by using the Fourier transform. The continuous Fourier Transform is shown mathematically by equation 6.1. The Fourier transform is a useful mathematical technique that is used to decompose a signal into a function based on sinusoids. The resulting function reveals favoured frequencies of oscillation within the signal.

Continuous Fourier Transform:

$$f(v) = F_t[f(t)](v) = \int_{-\infty}^{\infty} f(t) e^{-2\pi i v t} dt \quad (6.1)$$

The discrete form of the Fourier Transform is given by equation 6.2. This function will be used to transform the plasma density signal into a set of discrete sinusoidal functions.

Discrete Fourier Transform:

$$F_n = \sum_{k=0}^{N-1} f_k e^{-2\pi i n k / N} \quad (6.4)$$

The Power Spectral Density (PSD) is the magnitude of the signal power expressed in the frequency domain. This is equivalent to the modulus of the discrete Fourier Transform, which may have complex and real components. It is convenient to express the PSD in a logarithmic scale in units of Decibels. The formula used to compute the PSD is given by equation 6.3. The term (A) in this equation represents the amplitude of the complex modulus in the frequency domain. The term (A_{ref}) is a reference value. In this study the reference value was set to the average value of the plasma density signal.

Power Spectral Density:

$$PSD_{dB}(f) = 10 \cdot \log \left(\frac{A}{A_{ref}} \right) \quad (6.3)$$

The graphs that follow show the PSD distribution of the plasma density measured at the channel exit (figures 6.36 through 6.47). These plots were constructed for all 12 simulation trials. For many of the trials there are clear peak values that can be observed

in the PSD plots. These peak values represent favoured frequencies of oscillation in the plasma density signal. The frequencies at which these peaks occur correspond with natural instabilities in the plasma. In general, it can be observed that new modes of oscillation become apparent at higher values of the magnetic field conditions. Another interesting observation is that when the discharge voltage is at 150V and the magnetic field is small there are no peaks in the PSD plot. This indicates that the plasma oscillations in these conditions are similar to random turbulence with an exponential decrease in magnitude at higher frequencies. No coherent structures or distinct favoured frequencies can be observed under these conditions.

Figure 6.36. Power Spectral Density, B=50 Gauss, $V_d=100V$

Figure 6.37. Power Spectral Density, $B=100$ Gauss, $V_d=100V$ Figure 6.38. Power Spectral Density, $B=150$ Gauss, $V_d=100V$

Figure 6.39. Power Spectral Density, B=200 Gauss, $V_d=100V$ Figure 6.40. Power Spectral Density, B=50 Gauss, $V_d=150V$

Figure 6.41. Power Spectral Density, $B=100$ Gauss, $V_d=150V$ Figure 6.42. Power Spectral Density, $B=150$ Gauss, $V_d=150V$

Figure 6.43. Power Spectral Density, $B=200$ Gauss, $V_d=150V$ Figure 6.44. Power Spectral Density, $B=50$ Gauss, $V_d=200V$

Figure 6.45. Power Spectral Density, $B=100$ Gauss, $V_d=200V$

Figure 6.46. Power Spectral Density, $B=150$ Gauss, $V_d=200V$

Figure 6.47. Power Spectral Density, B=200 Gauss, $V_d=200V$

For the case of the 200 Gauss magnetic field strength, interesting structures can be observed at frequencies above 50 MHz. For the 100V discharge condition peaks in the PSD plot occur at frequencies up to approximately 200MHz. For the 150V discharge condition peaks can be observed up to approximately 100MHz. Finally, for the 200V discharge condition peaks can be observed all the way to 500MHz.

Figure 6.48. Power Spectral Density: 1 - 500MHz, B=200 Gauss, $V_d=100V$

Figure 6.49. Power Spectral Density: 1 - 500MHz, B=200 Gauss, $V_d=150V$

Figure 6.50. Power Spectral Density: 1 - 500MHz, B=200 Gauss, $V_d=200V$

The next PSD plot was taken from the experimental work of Guerrini et al. [16] (figure 6.51). It is interesting to note the remarkable resemblance to the results given by the simulation. The peak values in the PSD plot appear to occur at the same basic frequencies, although the magnitudes of these peaks are smaller than predicted by the simulation.

Figure 6.51. Power Spectral Density from Guerrini et al. [16]

The PSD plot shown in figure 6.52 was obtained from the experimental work performed during this project. The signal in this case is not a measure of the plasma density directly, but the voltage obtained from a Langmuir probe kept at ground potential. Probe theory indicates that this voltage signal is directly proportional to the density of the plasma. Again, it can be observed that very similar structures occur in the PSD plot at the same basic frequencies.

Figure 6.52. Experimental Power Spectral Density

By comparing the PSD plots obtained by simulation and experiment the following conclusions were reached. First, it was concluded that the simulation could successfully reproduce the high frequency plasma density oscillations observed experimentally. However, the magnitudes of these oscillations do not correspond with experiment. The likely cause for this discrepancy is the anomalously high spikes that occur in the plasma potential field (see figure 6.3). These large spikes in the plasma potential appear to influence all parameters within the simulation. The likely explanation for these spikes is that the electron energy equation is not solved explicitly in order to solve for the potential field. The electron energy equation could add damping to the system and would drive down the amplitude of these oscillations. This change to the governing equations has been cited for future work.

The next set of graphs shows how the PSD of the plasma oscillations changes at different axial locations within the thruster. These graphs were constructed for all 12 simulation trials and are shown in figure 6.53 through 6.64. It is clear from these graphs that the oscillations become more coherent near the channel exit (where the magnetic field is highest) and in the immediate vicinity of the anode. This is most notable when the magnetic field strength is at its highest. This is consistent with experimental observations reported in studies by Choueiri [9].

Figure 6.53. Axial Variation of Power Spectral Density, $B=50$ Gauss, $V_d=100V$

Figure 6.54. Axial Variation of Power Spectral Density, B=100 Gauss, $V_d=100V$ Figure 6.55. Axial Variation of Power Spectral Density, B=150 Gauss, $V_d=100V$

Figure 6.56. Axial Variation of Power Spectral Density, B=200 Gauss, $V_d=100V$

Figure 6.57. Axial Variation of Power Spectral Density, B=50 Gauss, $V_d=150V$

Figure 6.58. Axial Variation of Power Spectral Density, B=100 Gauss, $V_d=150V$ Figure 6.59. Axial Variation of Power Spectral Density, B=150 Gauss, $V_d=150V$

Figure 6.60. Axial Variation of Power Spectral Density, B=200 Gauss, $V_d=150V$ Figure 6.61. Axial Variation of Power Spectral Density, B=50 Gauss, $V_d=200V$

Figure 6.62. Axial Variation of Power Spectral Density, B=100 Gauss, $V_d=200V$ Figure 6.63. Axial Variation of Power Spectral Density, B=150 Gauss, $V_d=200V$

Figure 6.64. Axial Variation of Power Spectral Density, $B=200$ Gauss, $V_d=200$ V

6.4 Contribution of Plasma Oscillations to Electron Mobility

The goal of this section is to relate the high frequency plasma oscillations to the electron conduction properties of the plasma. This is done in order to quantify how the plasma oscillations contribute to the electron transport. Also, it provides a means to predict the anomalous electron transport term that appears in traditional Hall thruster simulations.

The electron current density is defined as the rate at which charge is transported across a unit area. This is defined mathematically by equation 6.4. It can be seen from this equation that the electron current is directly related to the axial velocity of the electrons.

Electron Current Density:

$$J_i = -N^e e < v_z^e > \quad (6.4)$$

Where:

$$\begin{aligned} J_i &= \text{Current density} \\ N^e &= \text{Electron number density} \\ e &= \text{Electron charge} \\ < v_z^e > &= \text{Axial electron velocity} \end{aligned}$$

The axial velocity of electrons can be obtained from the electron momentum equation. The electron momentum equation has been simplified in the axial and azimuthal directions by equations 6.5 and 6.6 respectively. These equations consider a small volume of plasma in which the properties are homogenous and steady.

Electron Momentum Equation

$$< v_z^e > = \frac{-e \cdot E_x}{m^e v_{SN}} - \frac{e < v_\theta^e > B_r}{m^e v_{SN}} \quad (6.5)$$

$$< v_\theta^e > = \frac{e < v_z^e > B_r}{m^e v_{SN}} \quad (6.6)$$

Where:

$$\begin{aligned} < v_z^e > &= \text{Axial electron velocity} \\ < v_\theta^e > &= \text{Azimuthal electron velocity} \\ e &= \text{Electron charge} \\ E_z &= \text{Axial electric field strength} \\ m^e &= \text{Electron mass} \end{aligned}$$

ν_{SN} = Momentum transfer collision frequency

B_r = Radial magnetic field strength

Equation 6.6 can be substituted into 6.5 to solve for the axial electron velocity. The resulting equation is given by 6.7, which incorporates the definition of the electron cyclotron frequency (6.9). Assuming that the electron cyclotron frequency is much larger than the momentum transfer collision frequency ($\omega_{ce} \gg \nu_{SN}$), the electron velocity can be approximated by equation 6.8.

Cross-field Electron Velocity:

$$\langle v_z^e \rangle = \frac{E_z}{B_r} \frac{\omega_{ce} \nu_{SN}}{(\nu_{SN}^2 + \omega_{ce}^2)} \quad (6.7)$$

$$\langle v_z^e \rangle \cong \frac{E_z}{B_r} \frac{\nu_{SN}}{\omega_{ce}} = \frac{E_z}{B_r} \frac{1}{\omega_{ce} \tau} \quad (6.8)$$

Where:

$$\omega_{ce} = \frac{E \cdot B}{m^e} \quad (6.9)$$

ω_{ce} = Electron cyclotron frequency

τ = Mean time between momentum transfer collisions

Incorporating the equation for the electron velocity (6.8) we can rewrite the axial current equation as 6.10. This equation shows the inverse relation between the current and the Hall parameter which is defined as $(\omega_{ce} \tau)$. The Hall parameter is equivalent to the anomalous electron transport term that appears in traditional Hall thruster simulations,

and is typically assigned a value of 16 (for Bohm conduction) or is determined from experiment.

Axial Current Density:

$$J_z = N^e e \frac{E_z}{B_r} \frac{1}{\omega_{ce} \tau} \quad (6.10)$$

The inverse Hall parameter can be established statistically based on random oscillations within the plasma density. The statistical equation for the inverse Hall parameter is given by equation 6.11. This equation was developed mathematically in a paper by Yoshikawa et al. [26]. A study conducted by Meezan et al. [8] demonstrates the agreement of this equation with experimental findings.

Statistical Inverse Hall Parameter:

$$\frac{1}{\omega_{ce} \tau} = \frac{\pi}{4} \frac{\langle (N^e - \langle N^e \rangle)^2 \rangle}{\langle N^e \rangle^2} \quad (6.11)$$

The inverse Hall parameter was computed at each axial location for all 12 simulation trials. The following two graphs (figure 6.65 and 6.66) show a comparison of the simulated results with the experimental observations conducted by Meezan et al. [8] for 100V and 200V discharge conditions. It can be observed in both cases that the prediction of the inverse Hall parameter at the channel exit is in good agreement with experiment. In the case of the 200V discharge, the simulation results closely follow the experiment near the anode and cathode, but depart in the region in between. However, in the 100V

operating condition the simulation results diverge from the experimental results near the anode. It should be noted that this measure of the electron mobility is based only on the high frequency components of the plasma oscillations. There are also other factors that influence the electron mobility; for instance lower frequency large amplitude oscillations that were not captured by these simulations. These results are encouraging because they show that the high frequency oscillations can be used to account for nearly all of the electron transport in certain regions of the thruster. This clearly shows the significance of plasma oscillations that occur in the 1MHz to 500MHz range in the electron transport process.

Figure 6.65. Comparison of Experimental and Simulated Inverse Hall Parameter, 100V

Figure 6.66. Comparison of Experimental and Simulated Inverse Hall Parameter, 200V

The axial distribution of the inverse Hall parameter is shown for all 12 simulation trials in figures 6.67 through 6.68. It can be concluded from these plots that the high frequency oscillations have a significant impact on the conduction properties of the plasma.

Figure 6.67. Simulated Inverse Hall Parameter, 50 Gauss Magnetic Field

Figure 6.68. Simulated Inverse Hall Parameter, 100 Gauss Magnetic Field

Figure 6.69. Simulated Inverse Hall Parameter, 150 Gauss Magnetic Field

Figure 6.70. Simulated Inverse Hall Parameter, 200 Gauss Magnetic Field

The inverse Hall parameter can be used to estimate the current flow through the Hall thruster by equation 6.10. All the parameters appearing in this equation are specified by the operating conditions except for the electron density, which was set at $4 \times 10^{17} \text{ [1/m}^3\text{]}$ from experiment. The resulting voltage versus current characteristic curves are compared with experiment in figure 6.71. The results appear reasonable and follow the experimental trends.

Figure 6.71. Voltage versus Current Profile

The current is plotted against the magnetic field in figure 6.72. Recall that the operating regime of the Hall thruster is established based on its location in the magnetic field versus current curve. Superimposed on this graph are the experimental measurements of an actual Hall thruster showing the various operating regimes. Based on characteristics of

this curve we can form conclusions about which operating regime the simulation was capturing. Table 6.2 shows which operating regime each simulation trial was likely capturing.

	<u>50 Gauss</u>	<u>100 Gauss</u>	<u>150 Gauss</u>	<u>200 Gauss</u>
100V	III or IV	III or IV	III or IV	III or IV
150V	I or II	III	III or IV	IV
200V	I or II	II	III	III or IV

Table 6.2. Operating Regimes of the Simulation Trials

Figure 6.72. Magnetic Field Strength versus Current Profile

Section 7: Conclusions

7.0 Conclusions

This section summarizes the conclusions that were gained during this project. The objective of this project was to study high frequency plasma oscillations that occur within Hall thrusters and evaluate their significance on the transport of electrons. This was accomplished by measuring plasma oscillations within a laboratory Hall thruster and conducting a numerical simulation capable of reproducing these oscillations. It was concluded as a result of this work that high frequency plasma oscillations indeed have a significant impact on the electron transport process. Additionally, it was found that a statistical evaluation of the plasma oscillations, obtained through the numerical simulation, could be used to make a reasonable estimate of the anomalous electron transport phenomenon in certain regions of the Hall thruster.

The Hall thruster simulation was conducted at 12 different operating points representing distinct sets of discharge voltage and magnetic field strength conditions. The operating points for the simulation trials were selected in order to observe the oscillation characteristics under a number of operating regimes of the Hall thruster. The overall accuracy of the simulation model was investigated by comparing the average axial distribution of various plasma parameters with experimental data. These parameters included the axial ion velocity, plasma potential, electron density, and neutral density. The general trends of the simulation parameters were in fairly good agreement with experiment. However, a number of important discrepancies were observed between the

experimental and simulation results. First, the axial ion velocity was overestimated by as much as three times. Second, there was a strong negative flow of ions that was not observed experimentally. Finally, the negative ion flow contributed to abnormally high plasma density toward the anode of the thruster.

The discrepancies between the experimental and simulation results were determined to be the result of large spikes that occurred in the potential field solution. These spikes created unusually large gradients in the local electric field strength that drove the ion velocity to unrealistically high values. These local field gradients also caused the ions to flow towards the anode under certain operating conditions. Despite this shortcoming, the simulation was able to make very good predictions of the axial variation in plasma potential and neutral density.

Statistical analysis of the plasma density oscillations was conducted on the simulation results. This analysis revealed certain favoured frequencies of oscillation corresponding to natural instabilities of the plasma. The frequency at which these oscillations occurred closely matched experimental results collected in this study. Similar high frequency oscillations have also been reported in experimental studies by Litvak [13] and Guerrini [16]. The plasma oscillations were analysed at various locations along the axis of the thruster. The results indicate that the oscillations were most coherent near the peak magnetic field region and in the immediate vicinity of the thruster anode. This trend agrees with experimental observation.

The statistical data collected on the plasma oscillations was used to evaluate the anomalous electron transport coefficient, also known as the inverse Hall parameter. The agreement between the experimentally measured inverse Hall parameter and the simulation results was good, particularly in the region near the exit of the thruster. The prediction of the electron transport given by the simulation results was better than could be obtained by classical theory or the Bohm conductivity model. The inverse Hall parameter was subsequently used calculate the current flow through the Hall thruster. The resulting current, voltage, and magnetic field strength characteristics of the simulated Hall thruster were in good agreement with experimental trends.

Section 8: Suggestions for Future Work

8.0 Suggestions for Future Work

This section presents tasks that were not addressed during this project, but were deemed to be important if work in this area is continued. The most important of these tasks is to include the electron energy equation in the plasma simulation. One of the main challenges experienced in the simulation was anomalous ‘spikes’ in the plasma potential field. These anomalous features were found to influence all parameters within the simulation. Adding the energy equation may act to dampen these oscillations and improve the accuracy of the simulation results. This reasoning is based on the assumption that adding additional equations to the system will increase the correspondence between the numerical model and physical reality, thereby adding a physical means to damp the potential field solution. However, it is not clear from the energy equation which terms will act to damp the plasma potential. It therefore remains to be seen whether adding the energy equation will in fact resolve the spikes that occur in the potential field.

The next recommendation is to evaluate the significance of a missing term that appears in the plasma potential equation cited in section 5.2. This term is the integral expression that appears in equation 5.34. This equation has been repeated below for reference. This extra term arises because of the coordinate system that was selected for this simulation (azimuth-axial plane). The significance of including this term is unknown and may also prove to lessen the large amplitude spikes in the potential solution. As a first order

estimate, the relative size of this parameter may be extrapolated from the simulation results produced during this project.

$$\phi = \phi^* + \frac{T^e k}{e} \ln(N^e) + \int \frac{m^e}{re} \langle v_\theta^e \rangle \frac{\partial \langle v_r^e \rangle}{\partial \theta} dr \quad (5.34)$$

The next task that was deemed important for future investigation is the dependence of the simulation results on the computational grid size. A grid size of 50 by 50 elements was used for the simulation trials in this project. Due to time constraints no other grid size was attempted. A 50 by 50 grid size may prove to be too coarse to obtain accurate results. Alternatively, this grid may in fact be too fine. A grid that contains fewer nodes may be sufficient for these simulations and may dramatically reduce the time needed to complete the computations.

Another task that would be very informative is to run the simulation for a couple hundred microseconds. In this project the simulation was only run to a maximum of 2 microseconds. If the simulation was run for this long duration then the effects of both low and high frequency oscillations would be captured. It would be interesting to see how well the simulation could reproduce experimental results under these conditions. It should be noted that this simulation would take a long time to complete with current technology restrictions. With the current simulation running on a Pentium 4 3GHz computer it would take over four years to complete a single simulation trial. However, improvements to computer technology will soon make this goal attainable. Also, there is

a potential to run the simulation in parallel over a cluster of computers to reduce the computation time considerably. Perhaps the most promising option is to reformulate the mathematics of this simulation in order to optimize the simulation run time.

Future work is also needed on the experimental aspects of this project. In particular, it would be useful to have experimental high frequency plasma oscillation data under all conditions that were investigated during the simulation trials. Also, experimental data collected at numerous axial locations along the thruster would be valuable to compare against simulation results. The experimental results collected during this project involved a single operating point and all results were collected at the exit plane of the thruster.

A summary of all future work items discussed in this section are tabulated below for convenience and reference:

- Include the electron energy equation in the Hall thruster simulation.
- Evaluate the significance of the missing term found in the plasma potential equation.
- Investigate the grid size dependence of the Hall thruster simulation.
- Complete a simulation of a couple hundred microsecond duration.
- Gather experimental data of high frequency plasma oscillations under all conditions investigated by simulation trials during this project.
- Gather experimental data of high frequency plasma oscillations at numerous locations along the axis of the thruster.

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Appendix A: Simulation Snapshots

Appendix A: Simulation Snapshots

The 2-dimensional plots that appear in this section were obtained from the results of the 12 simulation trials conducted during this project. The parameters that are shown in these plots include the plasma potential, electron velocity, ion velocity, and plasma density. The data was taken at the one microsecond instant in the simulation.

Run A1: B=50 Gauss, $V_d=100V$

Figure A.1. Plasma Potential, B=50 Gauss, $V_d=100V$

Figure A.2. Axial Electron Velocity, B=50 Gauss, $V_d=100V$

Figure A.3. Azimuthal Electron Velocity, $B=50$ Gauss, $V_d=100V$ Figure A.4. Axial Ion Velocity, $B=50$ Gauss, $V_d=100V$ Figure A.5. Azimuthal Ion Velocity, $B=50$ Gauss, $V_d=100V$

Figure A.5. Plasma Density, $B=50$ Gauss, $V_d=100V$ **Run A2: $B=100$ Gauss, $V_d=100V$** Figure A.6. Plasma Potential, $B=100$ Gauss, $V_d=100V$ Figure A.7. Axial Electron Velocity, $B=100$ Gauss, $V_d=100V$

Figure A.8. Azimuthal Electron Velocity, $B=100$ Gauss, $V_d=100V$ Figure A.9. Axial Ion Velocity, $B=100$ Gauss, $V_d=100V$ Figure A.10. Azimuthal Ion Velocity, $B=100$ Gauss, $V_d=100V$

Figure A.11. Plasma Density, $B=100$ Gauss, $V_d=100V$ **Run A3: $B=150$ Gauss, $V_d=100V$** Figure A.12. Plasma Potential, $B=150$ Gauss, $V_d=100V$ Figure A.13. Axial Electron Velocity, $B=150$ Gauss, $V_d=100V$

Figure A.14. Azimuthal Electron Velocity, $B=150$ Gauss, $V_d=100V$ Figure A.15. Axial Ion Velocity, $B=150$ Gauss, $V_d=100V$ Figure A.16. Azimuthal Ion Velocity, $B=150$ Gauss, $V_d=100V$

Figure A.17. Plasma Density, $B=150$ Gauss, $V_d=100V$ **Run A4: $B=200$ Gauss, $V_d=100V$** Figure A.18. Plasma Potential, $B=200$ Gauss, $V_d=100V$ Figure A.19. Axial Electron Velocity, $B=200$ Gauss, $V_d=100V$

Figure A.20. Azimuthal Electron Velocity, $B=200$ Gauss, $V_d=100V$ Figure A.21. Axial Ion Velocity, $B=200$ Gauss, $V_d=100V$ Figure A.22. Azimuthal Ion Velocity, $B=200$ Gauss, $V_d=100V$

Figure A.23. Plasma Density, $B=200$ Gauss, $V_d=100V$ **Run B1: $B=50$ Gauss, $V_d=150V$** Figure A.24. Plasma Potential, $B=50$ Gauss, $V_d=150V$ Figure A.25. Axial Electron Velocity, $B=50$ Gauss, $V_d=150V$

Figure A.26. Azimuthal Electron Velocity, $B=50$ Gauss, $V_d=150V$ Figure A.27. Axial Ion Velocity, $B=50$ Gauss, $V_d=150V$ Figure A.28. Azimuthal Ion Velocity, $B=50$ Gauss, $V_d=150V$

Figure A.29. Plasma Density, $B=50$ Gauss, $V_d=150V$ **Run B2: $B=100$ Gauss, $V_d=150V$** Figure A.30. Plasma Potential, $B=100$ Gauss, $V_d=150V$ Figure A.31. Axial Electron Velocity, $B=100$ Gauss, $V_d=150V$

Figure A.32. Azimuthal Electron Velocity, $B=100$ Gauss, $V_d=150V$ Figure A.33. Axial Ion Velocity, $B=100$ Gauss, $V_d=150V$ Figure A.34. Azimuthal Ion Velocity, $B=100$ Gauss, $V_d=150V$

Figure A.35. Plasma Density, $B=100$ Gauss, $V_d=150V$ **Run B3: $B=150$ Gauss, $V_d=150V$** Figure A.36. Plasma Potential, $B=150$ Gauss, $V_d=150V$ Figure A.37. Axial Electron Velocity, $B=150$ Gauss, $V_d=150V$

Figure A.38. Azimuthal Electron Velocity, $B=150$ Gauss, $V_d=150V$ Figure A.39. Axial Ion Velocity, $B=150$ Gauss, $V_d=150V$ Figure A.40. Azimuthal Ion Velocity, $B=150$ Gauss, $V_d=150V$

Figure A.41. Plasma Density, $B=150$ Gauss, $V_d=150V$ **Run B4: $B=200$ Gauss, $V_d=150V$** Figure A.42. Plasma Potential, $B=200$ Gauss, $V_d=150V$ Figure A.43. Axial Electron Velocity, $B=200$ Gauss, $V_d=150V$

Figure A.44. Azimuthal Electron Velocity, $B=200$ Gauss, $V_d=150V$ Figure A.45. Axial Ion Velocity, $B=200$ Gauss, $V_d=150V$ Figure A.46. Azimuthal Ion Velocity, $B=200$ Gauss, $V_d=150V$

Figure A.47. Plasma Density, $B=200$ Gauss, $V_d=150V$ **Run C1: $B=50$ Gauss, $V_d=200V$** Figure A.48. Plasma Potential, $B=50$ Gauss, $V_d=200V$ Figure A.49. Axial Electron Velocity, $B=50$ Gauss, $V_d=200V$

Figure A.50. Azimuthal Electron Velocity, $B=50$ Gauss, $V_d=200V$ Figure A.51. Axial Ion Velocity, $B=50$ Gauss, $V_d=200V$ Figure A.52. Azimuthal Ion Velocity, $B=50$ Gauss, $V_d=200V$

Figure A.53. Plasma Density, $B=50$ Gauss, $V_d=200V$ **Run C2: $B=100$ Gauss, $V_d=200V$** Figure A.54. Plasma Potential, $B=100$ Gauss, $V_d=200V$ Figure A.55. Axial Electron Velocity, $B=100$ Gauss, $V_d=200V$

Figure A.56. Azimuthal Electron Velocity, $B=100$ Gauss, $V_d=200V$ Figure A.57. Axial Ion Velocity, $B=100$ Gauss, $V_d=200V$ Figure A.58. Azimuthal Ion Velocity, $B=100$ Gauss, $V_d=200V$

Figure A.59. Plasma Density, $B=100$ Gauss, $V_d=200V$ **Run C3: $B=150$ Gauss, $V_d=200V$** Figure A.60. Plasma Potential, $B=150$ Gauss, $V_d=200V$ Figure A.61. Axial Electron Velocity, $B=150$ Gauss, $V_d=200V$

Figure A.62. Azimuthal Electron Velocity, $B=150$ Gauss, $V_d=200V$ Figure A.63. Axial Ion Velocity, $B=150$ Gauss, $V_d=200V$ Figure A.64. Azimuthal Ion Velocity, $B=150$ Gauss, $V_d=200V$

Figure A.65. Plasma Density, $B=150$ Gauss, $V_d=200V$ **Run C4: $B=200$ Gauss, $V_d=200V$** Figure A.66. Plasma Potential, $B=200$ Gauss, $V_d=200V$ Figure A.67. Axial Electron Velocity, $B=200$ Gauss, $V_d=200V$

Figure A.68. Azimuthal Electron Velocity, $B=200$ Gauss, $V_d=200V$ Figure A.69. Axial Ion Velocity, $B=200$ Gauss, $V_d=200V$ Figure A.70. Azimuthal Ion Velocity, $B=200$ Gauss, $V_d=200V$

Figure A.71. Plasma Density, $B=200$ Gauss, $V_d=200V$