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**Calculation of the
Strange Axial-Vector Meson Decay Constants
and
Extraction of the $^3P_1 - ^1P_1$ Mixing Angle**

by

Brian F. Phelps

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of
Master of Science

Ottawa Carleton Institute for Physics
Department of Physics, Carleton University
Ottawa, Ontario, Canada
December, 1995

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conjugation. The parity operator operates on a quantum state to invert the spatial 'handedness' (corresponding to the operation $\vec{r} \rightarrow -\vec{r}$), and intrinsic 'handedness' of the state. By convention, quarks have intrinsic handedness $+1$, and antiquarks -1 . The numbers are multiplicative, giving overall parity with possible eigenvalues:

$$P = \pm 1.$$

For mesons [1, p. 127]:

$$P = (-1)^{L+1}. \quad (1.2)$$

The charge conjugation operator operates on a quantum state to convert a particle to its antiparticle and vice-versa. The possible charge conjugation eigenvalues, like parity, are:

$$C = \pm 1,$$

and like parity are also multiplicative. C is well-defined for self-conjugate mesons only [1, p. 129]:

$$C = (-1)^{L+S}. \quad (1.3)$$

However, for mesons which are not self-conjugate, that is, which have non-identical quark and antiquark flavours, C is no longer a good quantum number. The only two remaining quantum numbers are J^P . This system of labelling cannot distinguish between two spin states of $S = 0, 1$ with the same total angular momentum J , which we mentioned earlier. Whenever there exist multiple but indistinguishable quantum states, they can mix. In this case, the two spin states of same J^P mix to the degree specified by the 'mixing angle' θ . The two 'unmixed states' which for purposes purely of notation we *label* with C , are related through θ to the two 'mixed' states. The unmixed states are internal to our meson model, while the mixed states are physically observable.

In working with the unmixed states we will use two standard notations: the

Abstract

The (3P_1) and (1P_1) charged weak decay constants, f_{K_1} , are calculated for the K_1 strange axial mesons using the mock-meson quark model. We obtain:

$$\left. \begin{aligned} f_{K_1}(^1P_1) &= -0.026 \pm 0.015 \\ f_{K_1}(^3P_1) &= +0.30 \pm 0.22 \end{aligned} \right\} \text{for } K_1(1270)$$

$$\left. \begin{aligned} f_{K_1}(^1P_1) &= -0.028 \pm 0.015 \\ f_{K_1}(^3P_1) &= +0.31 \pm 0.24 \end{aligned} \right\} \text{for } K_1(1400)$$

where the 'error' reflects the theoretical uncertainty in our values which is dominated by our lack of knowledge of the K_1 wavefunction. The (3P_1) - (1P_1) mixing angle θ_{K_1} is extracted using these decay constants in expressions for tau decay widths and comparing them to experimental measurements. The tau decays considered are:

$$\tau^+ \longrightarrow \bar{\nu}_\tau + K_1^+(1270)$$

$$\tau^+ \longrightarrow \bar{\nu}_\tau + K_1^+(1400),$$

using the branching ratio measurements made by the TPC/Two-Gamma collaboration. Our calculation gives the solutions:

$$(-35^\circ \lesssim \theta_{K_1} \lesssim +5^\circ), (10^\circ \lesssim \theta_{K_1} \lesssim 50^\circ).$$

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I wish to thank Dr. Steve Godfrey for his ongoing guidance for the duration of this project, Dr. Mike Doncheski for giving me an overview of the underlying mathematics, Harry Blundell for proofreading the document at various stages, Sherry Towers for helpful conversations on the experimental side of things, Greg Stewart for his help in typesetting the document in $\text{\LaTeX}$, and Matthew Jones for his help in the always very obscure PAW plotting package.

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Chapter 1

Introduction

1.1 The Standard Model

The Standard Model [1, 2] of particle physics is the most fundamental description of matter and energy known at this time. It offers a scheme for classifying all known fundamental particles and describing their interactions. Not all the commonly known particles are fundamental. For example, the particles of the nucleus, the proton and neutron, are not truly fundamental particles, since they are known to be constructed from smaller particles, the quarks, which are given below. The proton has quark content uud and the neutron has quark content udd . The quarks come in doublets. So far, three doublets are known:

$$\left(\begin{array}{c} u \\ d \end{array} \right) \left(\begin{array}{c} c \\ s \end{array} \right) \left(\begin{array}{c} t \\ b \end{array} \right) \Bigg\} \text{Quarks}$$

Quarks have spin $\frac{1}{2}$. The quarks in the upper row have electric charge $+\frac{2}{3}$, and the quarks in the lower row have electric charge $-\frac{1}{3}$. Each quark has an antiquark of the same spin but opposite charge. The quarks also possess a 'colour charge', which can be one of three 'colours' R, G, B (red, green, and blue). The antiquarks possess an 'anti-colour charge', one of $\bar{R}, \bar{G}, \bar{B}$ (anti-red, anti-green, and anti-blue).

The remaining particles from atomic physics, the electrons, which orbit the nucleus, appear to be fundamental. They belong to the class of particles called leptons. The leptons also come in three doublets:

$$\left\{ \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \right\} \text{Leptons}$$

They have spin $\frac{1}{2}$. The leptons in the 'up' position are called neutrinos, and have no electric charge. The leptons in the 'down' position are the electron, the muon, and tauon. They have electric charge -1 . Each lepton has an anti-lepton of the same spin but opposite charge. A doublet of quarks along with the corresponding doublet of leptons are referred to as a generation, or family of fermions.

The final class of fundamental particles is the gauge bosons, which are the 'carriers' of the four fundamental forces of nature: the electromagnetic, strong, weak, and gravitational forces. They are classified according to the force they mediate:

$$\left. \begin{array}{ll} \gamma(\text{photon}) & \text{electromagnetic force} \\ g_1, \dots, g_8(\text{gluons}) & \text{strong force} \\ W^\pm, Z^0 & \text{weak force} \\ \text{graviton} & \text{gravitation} \end{array} \right\} \text{Gauge Bosons}$$

The electromagnetic, strong, and weak gauge bosons have spin 1. The graviton has spin 2. The photon possesses no electric charge, the gluons carry the colour charge but not electric charge, and the weak gauge bosons possess the electric charge indicated.

Hadrons, introduced below, are composite particles that are built primarily of quarks. A quark has never been observed in isolation: they are always seen bound in a quark-gluon system. The reason for this is the nature of the force between them, the strong force, mediated by the gluon. The strong force, unlike the electromagnetic force, displays two different characters depending on the interquark

separation distance. At short distances it displays a Coulomb-like potential, while at large distances it displays a linear potential, which increases in strength with separation. Hence there is no finite 'escape energy' for the quark. It may seem, though, that a quark could still be separated from the other quarks in the hadron to some finite distance, say, a metre. This, however, being energetically unfavourable, never happens. As energy is supplied to remove one quark from a quark-antiquark pair (for example), the potential energy of the system increases so that at some point it becomes energetically favourable for a new quark-antiquark pair to be spontaneously created from the vacuum. The new quarks pair up with the original quarks, and the potential energy of the system is reduced. A quark can therefore never be freed from the system. This phenomenon is referred to as quark confinement.

As of yet, only two types of quark-gluon systems have been unambiguously observed: 2-quark and 3-quark systems. The 2-quark systems, actually consisting of a quark and antiquark, are called mesons. The 3-quark systems are called baryons. Together they are called hadrons.

$$\left. \begin{array}{ll} q\bar{q} & \text{mesons} \\ qqq & \text{baryons} \end{array} \right\} \text{hadrons}$$

The quarks in such a system experience the strong force, which, as explained above, is mediated by gluons. The gluons continually travel back and forth from one quark to another, carrying the 'message' of the strong force, resulting in the interquark potential consisting of the colour Coulomb attraction or repulsion at short separation plus the linear attraction at large separation. QCD (quantum chromodynamics) is the theory of quark and gluon interaction. It is one of the three 'gauge theories' of the Standard Model. Few gauge theory problems possess an exact solution. Usually the best that one can do is to perform a quantum mechanical perturbation calculation to the desired order of accuracy. In QCD, this only works at high energies, where the quarks are at small separation distances and the confining force is weak enough that perturbative methods apply. This is known as the 'hard' QCD regime.

However, in hadrons the quarks are at larger separation distances corresponding to lower energy scales. The confining force becomes too strong for the perturbative methods to be applicable. This is known as the 'soft' QCD regime. Here, one must resort to computationally intensive numerical methods.

The mesons are the systems of interest here. For the phenomenological calculations of this work we model the meson as a 2-quark bound system: experiencing a QCD-motivated Coulomb plus linear spherically symmetric central force. We use a variant of the quark model where the meson is conveniently described by a combination of relativistic and nonrelativistic physics. The nonrelativistic physics is in fact similar to that describing the hydrogen atom. The 2-body system of masses m_q and $m_{\bar{q}}$ with position operators $\vec{r}_q$ and $\vec{r}_{\bar{q}}$ is reduced to the usual 1-body system of reduced mass:

$$\mu = \frac{m_q m_{\bar{q}}}{m_q + m_{\bar{q}}}$$

and relative position vector:

$$\vec{r} = \vec{r}_q - \vec{r}_{\bar{q}}.$$

This reduced system is then described with a wavefunction, which is a product of various specific component wavefunctions. The familiar components from atomic physics are the radial momentum wavefunction, the orbital angular momentum wavefunction (with spherical harmonic eigenfunctions), and the spin angular momentum wavefunction. The additional components, which are specific to quark systems are the flavour and colour wavefunctions.

The wavefunction is a solution to the Schrodinger wave equation, which in this case equates the operation of the total meson Hamiltonian on the wavefunction, to the meson mass times the wavefunction:

$$H|\Phi\rangle = M|\Phi\rangle \quad (1.1)$$

This equation, when written in matrix form, leads to the mass matrix, whose eigenvalues are the masses of the mesons. We describe the spin-dependent meson

Hamiltonian and the mass matrix in more detail in Appendix A

The mathematics of group theory is particularly helpful in formulating the last four wavefunction components. It also provides rules for combining the orbital angular momentum and spin angular momentum wavefunctions into one total angular momentum wavefunction. For while the quark and antiquark, as internal components of the system, each possess orbital and spin degrees of freedom, the meson system as observed externally possesses just one total angular momentum: its spin, the sum of all internal angular momenta.

The colour wavefunctions specify combinations of the 3 quark colours described above. $SU(3)$ symmetry is the group detailing their construction. As far as has been observed, it is an exact symmetry. As dictated by the group, the three colours and three anticolours combine to yield nine colour wavefunctions for quark-antiquark systems. In six of them, two of the three 'colour charges' are unbalanced, and hence visible. In two of them, all the colour charges are balanced, yet they are not invariant under colour permutations. These eight colour wavefunctions comprise the colour $SU(3)$ -octet. In the remaining one, all three colour charges are perfectly cancelled and symmetrical. This is the colour $SU(3)$ -singlet. No meson has ever been observed to possess an overall colour charge. Therefore the appropriate colour wavefunction for the meson is the singlet wavefunction, which will be given in §2.2.

The flavour wavefunctions specify the quark content. The three most common quark varieties or 'flavours' are the u , d , and s (up, down, and strange). The u and d quarks from the first quark doublet are the lightest in mass, and hence are called 'light' quarks. Because of their near mass-degeneracy, many hadron properties can be described using the $SU(2)$ isospin symmetry (in analogy to the regular spin symmetry), where the u and d quarks form the isospin doublet:

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{array}{l} \text{isospin} = +\frac{1}{2} \\ \text{isospin} = -\frac{1}{2} \end{array}.$$

The s quark, being significantly heavier than the u or d , is next in mass. It possesses

the additive quantum number of strangeness:

$$s \text{ strangeness} = -1.$$

The corresponding antiquarks have opposite isospin and strangeness. The mesons these quarks and antiquarks compose are called the 'light' mesons, and possess overall isospin and strangeness according to their u , d , and s content. Often one makes the simplifying assumption that all three flavours have the same mass. Then $SU(3)$ symmetry becomes the valid group for flavour wavefunction construction. In a fashion similar to colour symmetry, the three flavours and three ant Flavours combine to yield nine flavour wavefunctions, which comprise an $SU(3)$ octet and singlet. The flavour wavefunction will be given in §2.2.

The angular momentum wavefunctions are constructed exactly as in atomic physics. The quark and antiquark, being leptons, each have spins:

$$s = \frac{1}{2}$$

$$\bar{s} = \frac{1}{2}$$

and they orbit in the spherically symmetric central-force system $q\bar{q}$. Hence $SU(2)$ symmetry is the group governing spin wavefunction construction, and $O(3)$ symmetry governs orbital angular wavefunction construction. Orbital angular momentum is quantised to integer values only:

$$L = 0, 1, 2, \dots$$

Next, the Clebsch-Gordon rules of group theory are employed to add the spin wavefunctions of the q and the $\bar{q}$, and the orbital angular wavefunctions to yield the total angular momentum wavefunction. First the individual spins s and $\bar{s}$ are

added to give total quark-pair spin with the allowed values:

$$S = 0, 1.$$

Then L and S are added to give total angular momentum or meson spin J , restricted to integer values only:

$$J = 0, 1, 2, \dots$$

Mesons are integer-spin particles.

The final wavefunction component is the radial momentum function. As in atomic physics, any given superposition of the eigenfunctions is completely independent of the other four wavefunction components, which were specified according to the rules of group theory. For a given Hamiltonian, solving the Schrodinger wave equation to yield a radial wavefunction expanded in a well-known eigenfunction basis is a standard problem in quantum mechanics. Unfortunately in meson physics, the Hamiltonian is not well known. The choice of Hamiltonian depends on the specific $q\bar{q}$ model one is working with. Later we will briefly describe one model, and give the radial wavefunction solutions.

Taken together as a product, the five wavefunction components form the total nonrelativistic bound state wavefunction. We continue with the construction of the meson state by next considering the relativistic side of the picture. All three quark flavours are light enough in mass to move at relativistic speeds. Free relativistic spin- $\frac{1}{2}$ particles are described by the Dirac spinors, which are the well-known solutions to the free-particle Dirac wave equation.

To combine the relativistic and nonrelativistic physical pictures into one meson model, we apply the 'mock-meson' ansatz, which assumes that the meson state may be expressed as the free $q\bar{q}$ Dirac spinors weighted by the bound-state wavefunction, summed over momentum space, and normalized in accordance with the Lorentz-invariant convention. The technical details will be given in §2.2.

In addition to the group symmetries and wave-mechanic solutions, there are two important discrete symmetries that apply to mesons. They are parity and charge

conjugation. The parity operator operates on a quantum state to invert the spatial 'handedness' (corresponding to the operation $\vec{r} \rightarrow -\vec{r}$), and intrinsic 'handedness' of the state. By convention, quarks have intrinsic handedness +1, and antiquarks -1. The numbers are multiplicative, giving overall parity with possible eigenvalues:

$$P = \pm 1.$$

For mesons [1, p. 127]:

$$P = (-1)^{L+1}. \quad (1.2)$$

The charge conjugation operator operates on a quantum state to convert a particle to its antiparticle and vice-versa. The possible charge conjugation eigenvalues, like parity, are:

$$C = \pm 1,$$

and like parity are also multiplicative. C is well-defined for self-conjugate mesons only [1, p. 129]:

$$C = (-1)^{L+S}. \quad (1.3)$$

However, for mesons which are not self-conjugate, that is, which have non-identical quark and antiquark flavours, C is no longer a good quantum number. The only two remaining quantum numbers are J^P . This system of labelling cannot distinguish between two spin states of $S = 0, 1$ with the same total angular momentum J , which we mentioned earlier. Whenever there exist multiple but indistinguishable quantum states, they can mix. In this case, the two spin states of same J^P mix to the degree specified by the 'mixing angle' θ . The two 'unmixed states' which for purposes purely of notation we *label* with C , are related through θ to the two 'mixed' states. The unmixed states are internal to our meson model, while the mixed states are physically observable.

In working with the unmixed states we will use two standard notations: the

J^{PC} notation giving J (meson spin), P (parity), and C (charge conjugation):

$$|J^{PC}\rangle \quad (1.4)$$

and the Spectroscopist's notation giving n (radial excitation), L (orbital angular momentum), S (total quark spin), and J (total angular momentum, or meson spin):

$$n^{2S+1}L_J \quad (1.5)$$

(S for $l = 0$, P for $l = 1$, D for $l = 2$, ...).

In constructing the meson state we have described concepts from both meson statics and dynamics. Meson statics is the study of properties that are described by fixed quantum numbers, such as those in notations (1.4) and (1.5) as well as the flavour-related quantum numbers of isospin and strangeness. The quantum numbers of meson statics are well-known and tabulated. Meson dynamics, however, is the study of the functions appearing in the relevant quantum-mechanical equation of motion, in this case the Schrodinger equation (1.1). The Hamiltonian may be solved to yield the meson radial wavefunction and the physical meson mass. The meson Hamiltonian and radial wavefunction are the key functions that form the basis of meson dynamics. Neither is currently well known. QCD-motivated relativistic Hamiltonians have appeared in the literature [3], but nonrelativistic Hamiltonians are still being investigated [4, 5, 3, 6, 7]. Similarly, several meson radial wavefunctions have appeared in the literature (see §3.1), but there is currently little indication of which is best. Meson dynamics in general is at this time not well-known.

Particle decay is a principal topic in particle physics because it reveals so much of the underlying particle dynamics. A given particle i may decay through one of many potential decay reactions. The specific decay reaction a :

$$i \longrightarrow 1 + 2 + 3 + \dots \quad (1.6)$$

occurs with a specific probability, the 'partial decay width' Γ_a . A large number of partial decay widths have been experimentally measured [8]. If the underlying particle dynamics is known, though, they may also be calculated from theory. Consider decay (1.6). We take the complete hadron state $|i\rangle$ as given by the phenomenological model. In the case where i is a meson with broken- $SU(3)$ flavour symmetry, then as described earlier the 'unmixed' states which were *labelled* with $C = -1$ and $C = +1$ mix with the mixing angle θ_i to form the 'mixed states'. For each unmixed state $|i\rangle$ we may calculate the corresponding unmixed 'decay amplitude' A for decay (1.6):

$$|i\rangle \Rightarrow A.$$

The unmixed amplitudes are related through θ_i to the physically meaningful mixed amplitudes:

$$A \xrightarrow{\theta_i} A(\text{physical}),$$

which, for example, can take the form:

$$A(\text{physical}) = A(^3P_1) \cos \theta_i + A(^1P_1) \sin \theta_i$$

By definition, a quantum amplitude squared equals a probability. In this case the partial decay width is the probability of interest:

$$\Gamma_a \propto |A(\text{physical})|^2$$

The expression for partial decay width will contain the unknown mixing angle θ_i . A comparison of phenomenological expressions of partial decay widths calculated from a given meson model, with the corresponding experimental values, would enable extraction of the mixing angle.

1.2 The Strange Axial Meson Decay Amplitude and Mixing Angle

The K_1 is a meson that may be described by the physics we have mentioned in §1.1. It possesses spin

$$J = 1,$$

making it a type of 'vector' particle, and parity

$$P = +1,$$

making it, more specifically, an 'axial vector' particle. It also has s or $\bar{s}$ quark content. Hence the K_1 is a strange axial vector meson. There are two states which may be expressed in either notation (1.4) or (1.5). Due to the total quark spin restriction $S = 0, 1$ and relation (1.3), the two notations obey the following equivalence:

$$|1^{++}\rangle \equiv |^3P_1\rangle \quad (1.7)$$

$$|1^{+-}\rangle \equiv |^1P_1\rangle \quad (1.8)$$

They mix with mixing angle θ_{K_1} .

In this report we both calculate the K_1 decay amplitude and extract the K_1 axial vector meson mixing angle θ_{K_1} . The method is as follows. First the charged weak decay amplitudes for the two unmixed states:

$$\begin{cases} K_1^+(^3P_1) \\ K_1^+(^1P_1) \end{cases}$$

are calculated using our meson phenomenology model. Then they are mixed by an

unknown K_1 mixing angle θ_{K_1} to yield the amplitudes for the two physical states:

$$\begin{cases} K_1^+(1270) \\ K_1^+(1400), \end{cases}$$

so named because they have mass 1270 and 1400 MeV respectively. From the square of the amplitudes we form expressions for the partial decay widths:

$$\begin{cases} \Gamma(1270) \\ \Gamma(1400) \end{cases}$$

for the tauon decay reactions:

$$\tau^+ \longrightarrow \bar{\nu}_\tau K_1^+(1270)$$

$$\tau^+ \longrightarrow \bar{\nu}_\tau K_1^+(1400)$$

Finally, the ratio of the partial decay width expressions is compared to the ratio of experimental partial decay width values:

$$\left\{ \frac{\Gamma(1270)}{\Gamma(1400)} \right\} (\theta_{K_1}) = \left\{ \frac{\Gamma(1270)}{\Gamma(1400)} \right\}_E$$

and the equation is solved to yield θ_{K_1} . We have chosen to work with partial decay width ratios instead of individual partial decay widths. The radial wavefunctions appearing in the decay amplitudes cancel to a large degree in the ratio. Since the meson radial wavefunctions are not very well known, this cancellation is useful. It enables us, in principle, to extract the mixing angle to a degree of precision higher than that to which the radial wavefunctions are known.

The experimental partial decay width values have been measured by the TPC/Two-Gamma Collaboration [9]. CLEO and ALEPH [10, 11, 12] also have $\tau \rightarrow K_1 \nu_\tau$ decay data.

The motivation behind calculating the K_1 decay amplitudes is that they can

be used in more complicated calculations that require that same hadron matrix element. The motivation behind calculating θ_{K_1} is that it is the angle which diagonalizes the mass matrix. The off-diagonal matrix elements which cause mixing come from the spin-orbit term of the Hamiltonian, $H_{s.o.}^-$, described in Appendix A. The $H_{s.o.}^-$ piece receives contributions from the colour magnetic interaction and the Thomas Precession terms in the central-force confining potential. Hence the size of θ_{K_1} gives information about $H_{s.o.}^-$, which provides insights into the relative importance of these two contributions. This approach to understanding the $H_{s.o.}^-$ piece will contribute to knowledge of the meson Hamiltonian, and will hence be key to improvements in meson phenomenology.

Let us review the past work that has been done on the calculation of θ_{K_1} . Carnegie *et al* [13], in 1977, actually obtained the value:

$$\theta_{K_1} = (41 \pm 4)^\circ$$

as the optimum fit to experimental data for strong decays of the K_1 mesons. They performed a least-squares fit of $K_1(1270)$ and $K_1(1400)$ partial decay widths to flavour- $SU(3)$ parameters, extracting θ_{K_1} . Colglazier *et al* [14], in 1971, and Lipkin [15], in 1977, showed that theoretically, if the isospin $\frac{1}{2}$ members of the two spin-state octets are degenerate before mixing, $SU(3)$ -symmetry breaking always leads to the maximal mixing:

$$\theta_{K_1} = 45^\circ.$$

If this were true, then tauon decay would lead to production of $K_1(1270)$ and $K_1(1400)$ in ratio 1 to 1, up to correction factors. However, the TPC/Two-Gamma Collaboration data, of 1994, differs from this simplified picture, with tauon decay favouring $K_1(1400)$ production, with little evidence for $K_1(1270)$ production. This led Suzuki [16, 17], in 1994, to use the TPC/Two-Gamma Collaboration data in a variety of related phenomenological calculations to extract θ_{K_1} . We, in an approach complementary to Suzuki's, have made phenomenological calculations based on our meson model, and compared them with the TPC/Two-Gamma Collaboration data

to extract θ_{K_1} .

The organization of this report is as follows. In chapter 2 we develop the method of calculating the decay amplitude and the mixing angle θ_{K_1} . We begin with the mock-meson phenomenological model. Then, following the procedure already mentioned, we perform calculations for both the relativistic and non-relativistic limit cases. The non-relativistic limit case is done to yield simpler amplitude expressions than the relativistic case, thus lending a greater degree of insight in our study. Note that once we calculate an amplitude, we work with the ‘formfactor’, the numerical factor of the amplitude. In chapter 3 we use our method to calculate numerical results. We discuss the general form of the quark-antiquark reduced mass radial wavefunction in momentum space, and then present the various specific meson models that were tested, which include quark masses and simple-harmonic oscillator eigenfunction and expansion parameters. We also present the four different definitions of K_1 mock-meson mass that were tested. Next, we give the results: tables of the formfactors and then the solutions to the tauon decay width ratio relations, the mixing angles θ_{K_1} . We present a discussion of our work, and in chapter 4 give our conclusions from this study. Appendix A gives details of the nonrelativistic spin-dependent meson Hamiltonian, and Appendix B gives details on the numerical methods that were employed.

Note that in this report the standard units of particle physics are used, with

$$c = 1$$

$$\hbar = 1$$

Hence, energy, mass, and momentum are all quoted in units of GeV .

Chapter 2

Method

2.1 The Mock-Meson Method and the Relativised Quark Model

As stated in the introduction, we will model a meson using the ‘mock-meson’ model. It is not a rigorous theory, but a practical phenomenological model useful for calculation of meson properties. Although mesons can in principle be described by quantum chromodynamics (QCD), calculations from first principles are only now starting to become reliable for heavy quark systems, with reliable calculations for light quarks still to be seen. As a consequence the mock-meson model will for now provide a useful description of hadron properties. It has been useful in describing hadron coupling including electromagnetic, weak, and strong couplings. This approach [18, 19, 3, 20, 21, 22] assumes that the matrix element of a real hadron can be approximated by the following prescription:

1. Express the physical matrix M in terms of Lorentz invariants with scalar coefficients A .
2. With each hadron H of mass M_H , associate a mock hadron $\tilde{H}$ consisting of free quarks with the wavefunction of the bound quarks in H .

3. Calculate $\tilde{M}$, the mock matrix element, in terms of free quark amplitudes.
4. If, as is the case in many simple circumstances, $\tilde{M}$ has the same form as M , take $A = \tilde{A}$.

This is essentially our mock-meson approach written more precisely. In the next section we will write the mock-meson state mathematically. As stated earlier, it is formulated in terms of both relativistic and nonrelativistic quantum mechanics: a pair of free fermion Dirac spinors weighted by the nonrelativistic bound state wavefunctions, summed over momentum space, and normalised by a normalising factor.

The inclusion of the free quark Dirac spinors introduces some relativistic effects, although the model is not rigorously relativistic. For this reason, we call this relativistic extension of the quark model the 'relativised' quark model. This prescription will allow us to formulate hadron amplitudes for hadrons consisting of the light quark flavours, which because of their light mass, move at relativistic speeds within the bound state. We will also express these hadron amplitudes in the non-relativistic limit, bringing us back to the standard quark model. Such expressions will be mathematically simpler, and may offer clearer insights into dependence on various model parameters.

2.2 The Mock-Meson Prescription

The general approach to the problem has been qualitatively presented. We now express the words of the previous section mathematically. The mock-meson state for centre of mass momentum $\vec{K}$ is:

$$|\tilde{M}(\vec{K})\rangle = \sqrt{2\tilde{M}} \int d^3p \Phi(\vec{p}) \chi_{s\bar{s}} \phi_{q\bar{q}} \phi_{colour} |q \bar{q}\rangle \quad (2.1)$$

Let us describe each factor:

- $q, \bar{q}$ are the quark-antiquark relativistic spinors. They are functions of their own momentum and spin:

$$q = q(\vec{k}, s) \quad (2.2)$$

$$\bar{q} = \bar{q}(\vec{k}, \bar{s}) \quad (2.3)$$

- ϕ_{colour} is the colour factor wavefunction of the quark-antiquark pair. The normalized colour singlet wavefunction is [2, p. 67]:

$$\phi_{colour\,singlet} = \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b}). \quad (2.4)$$

Summing over the 3 colours $\{r\bar{r}, g\bar{g}, b\bar{b}\}$ results in the factor 3, giving us an overall colour factor of $\sqrt{3}$. We may drop the explicit form of the colour wavefunction, keeping only this colour factor.

- $\phi_{q\bar{q}}$ is the flavour wavefunction of the quark-antiquark pair. The particular meson we will be studying, K_1^+ , has the flavour wavefunction [8, p. 1200]:

$$\phi_{K_1^+} = -u\bar{s}, \quad (2.5)$$

meaning that in (2.1),

$$q = q_u \quad (2.6)$$

$$\bar{q} = \bar{q}_s, \quad (2.7)$$

with the flavour factor -1 .

- $\chi_{s\bar{s}}$ is the non-relativistic spin wavefunction for the quark-antiquark pair. The quark and antiquark are spin $s = \bar{s} = \frac{1}{2}$ particles (fermions), and are combined to yield the meson spin $S = \{0, 1\}$ using the usual rules for combining angular momentum.

- $\Phi(\vec{p})$ is the momentum-space spatial wavefunction. It depends on $\vec{p}$, the internal momentum of q ; i.e.,

$$\begin{cases} \vec{p} & \text{is the momentum of } q \text{ relative to the centre of mass.} \\ -\vec{p} & \text{is the momentum of } \bar{q} \text{ relative to the centre of mass.} \end{cases}$$

In the usual spherical coordinates:

$$\Phi(\vec{p}) = \Phi_{nl}(p)Y_{lm}(\Omega_p). \quad (2.8)$$

The spherical harmonic function $Y_{lm}(\Omega_p)$ and the nonrelativistic spin function χ_{ss} combine in the group product:

$$[Y_{lm}(\Omega_p) \odot \chi_{ss}].$$

- $\tilde{M}$ is the mock-meson mass. It will be further defined in §3.2.

Hence (2.1) becomes [3, 19]:

$$|\tilde{M}(\vec{K})\rangle = -\sqrt{6\tilde{M}} \int d^3p \Phi_{nl}(p)[Y_{lm}(\Omega_p) \odot \chi_{ss}]|q_u(\vec{k}, s)q_s(\vec{k}, \bar{s})\rangle \quad (2.9)$$

where

$$\vec{k} = \frac{m}{m + \tilde{m}} \vec{K} + \vec{p} \quad (2.10)$$

$$\vec{\bar{k}} = \frac{\tilde{m}}{m + \tilde{m}} \vec{K} - \vec{p}, \quad (2.11)$$

and

$$\begin{cases} \vec{K} & \text{is the momentum of the centre of mass.} \\ \vec{p} & \text{has been defined above.} \end{cases}$$

2.3 Formulation of the Formfactor f_{K_1}

2.3.1 The K_1 Formfactor

The next step is to write the weak matrix element

$$A^\mu = \langle 0 | j_W^\mu | \widetilde{M}(\vec{K}) \rangle \quad (2.12)$$

in terms of formfactors. In the J^P notation we write:

$$|\widetilde{M}(\vec{K})\rangle \Longleftrightarrow |1^+\rangle, \quad (2.13)$$

since the K_1 spin is 1 and its parity is positive. The weak current is written as a sum of a vector piece and an axial-vector piece:

$$j_W^\mu = v^\mu - a^\mu. \quad (2.14)$$

Since j^μ therefore transforms as a Lorentz vector and axial vector sum, we must consider all Lorentz vectors and axial vectors that we can construct, using any vectors and tensors relevant to the problem. Examples are the K_1 momentum K^μ and polarization ϵ^μ . We then apply symmetry transformations to see which vectors are allowed. For example, the weak decay amplitude can be written most generally as:

$$A^\mu = \langle 0 | j_W^\mu | 1^+ \rangle = \langle 0 | (v^\mu - a^\mu) | 1^+ \rangle = f_v K^\mu + f_a \epsilon^\mu \quad (2.15)$$

Applying a parity transformation, we find that:

$$\langle 0 | (v^\mu - a^\mu) | 1^+ \rangle = \langle 0 | (P^{-1}P)(v^\mu - a^\mu)(P^{-1}P) | 1^+ \rangle = \langle 0 | (-v^\mu - a^\mu) | 1^+ \rangle. \quad (2.16)$$

But for (2.16) to hold in (2.15) we must have:

$$f_v \equiv 0. \quad (2.17)$$

Hence (2.15) is simplified to:

$$A^\mu = \langle 0 | j_W^\mu | 1^+ \rangle = \frac{i}{(2\pi)^{\frac{3}{2}}} f_{K_1} \epsilon^\mu, \quad (2.18)$$

where the constant of proportionality, f_{K_1} , is the formfactor.

2.3.2 Evaluating the K_1 Formfactor

We will obtain an expression for f_{K_1} using the mock-meson prescription. Starting with (2.12), we substitute in our expression for $|\tilde{M}\rangle$ (2.9), and the standard expression for the weak charged current [2, p. 255]:

$$j_W^\mu = \bar{q} \gamma^\mu \frac{1}{2} (1 - \gamma_5) q \quad (2.19)$$

to obtain:

$$A^\mu = -\sqrt{6\tilde{M}} \int d^3p \Phi_{nl}(p) [Y_{lm}(\Omega_p) \otimes \chi_{ss}] \langle 0 | \bar{q} \gamma^\mu (1 - \gamma_5) q | q_u(\vec{k}, s) \bar{q}_s(\vec{k}, \bar{s}) \rangle. \quad (2.20)$$

We therefore need to evaluate

$$\langle 0 | \bar{q} \gamma^\mu (1 - \gamma_5) q | q_u(\vec{k}, s) \bar{q}_s(\vec{k}, \bar{s}) \rangle \quad (2.21)$$

In this expression q and $\bar{q}$ are given by the standard expansions [23, p. 56]:

$$q = \sum_{\pm s} \int \frac{d^3p}{(2\pi)^{\frac{3}{2}}} \sqrt{\frac{m}{E}} [b(p, s) u(p, s) e^{-ip \cdot x} + d^\dagger(p, s) v(p, s) e^{ip \cdot x}] \quad (2.22)$$

$$\bar{q} = \sum_{\pm \bar{s}} \int \frac{d^3\bar{p}}{(2\pi)^{\frac{3}{2}}} \sqrt{\frac{\bar{m}}{\bar{E}}} [b^\dagger(\bar{p}, \bar{s}) \bar{u}(\bar{p}, \bar{s}) e^{i\bar{p} \cdot x} + d(\bar{p}, \bar{s}) \bar{v}(\bar{p}, \bar{s}) e^{-i\bar{p} \cdot x}]. \quad (2.23)$$

Substituting into equation (2.21) and using the usual anticommutation relations, we obtain:

$$\langle 0 | \bar{q} \gamma^\mu (1 - \gamma_5) q | q_u \bar{q}_s \rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{m\bar{m}}{E\bar{E}}} \{ \bar{v}(\bar{k}, \bar{s}) \gamma^\mu (1 - \gamma_5) u(k, s) \}. \quad (2.24)$$

However, as we saw in §2.3.1, only the axial vector piece of the weak charged current contributes, leaving:

$$\langle 0 | \bar{q} \gamma^\mu (1 - \gamma_5) q | q_u \bar{q}_s \rangle = -\frac{1}{(2\pi)^3} \sqrt{\frac{m\bar{m}}{E\bar{E}}} \{ \bar{v}(\bar{k}, \bar{s}) \gamma^\mu \gamma_5 u(k, s) \}. \quad (2.25)$$

To evaluate this expression we use the explicit forms of the Dirac spinors [2, p. 105]:

$$u(k, s) = \sqrt{\frac{E+m}{2m}} \begin{pmatrix} \phi_s \\ \frac{\vec{k} \cdot \vec{\sigma}}{E+m} \phi_s \end{pmatrix} \quad (2.26)$$

$$v(\bar{k}, \bar{s}) = \sqrt{\frac{\bar{E} + \bar{m}}{2\bar{m}}} \begin{pmatrix} -\frac{\vec{k} \cdot \vec{\sigma}}{E+m} \chi_{\bar{s}} \\ \chi_{\bar{s}} \end{pmatrix} \quad (2.27)$$

(where ϕ_s and $\chi_{\bar{s}}$ are the nonrelativistic 2-component spinors of spin s and $\bar{s}$ respectively), and its Dirac matrices [1, p. 379]:

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (2.28)$$

$$\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \quad (2.29)$$

$$\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (2.30)$$

(where 1 is the 2×2 unit matrix, and $\vec{\sigma}$ are the 2×2 Pauli matrices.) Upon substitution, we obtain:

$$\langle 0 | \bar{q} \gamma^0 (1 - \gamma_5) q | q_u \bar{q}_s \rangle_{ss} = -\frac{1}{(2\pi)^3} \sqrt{\frac{E+m}{2E} \frac{\bar{E}+\bar{m}}{2\bar{E}}} \left\{ \chi_s^T \left[\frac{\vec{k} \cdot \vec{\sigma}}{E+\bar{m}} \frac{\vec{k} \cdot \vec{\sigma}}{E+m} + 1 \right] \phi_s \right\} \quad (2.31)$$

$$\langle 0 | \bar{q} \gamma^a (1 - \gamma_5) q | q_u \bar{q}_s \rangle_{ss} = -\frac{1}{(2\pi)^3} \sqrt{\frac{E+m}{2E} \frac{\bar{E}+\bar{m}}{2\bar{E}}} \left\{ \chi_s^T \left[\frac{\vec{k} \cdot \vec{\sigma}}{E+\bar{m}} \vec{\sigma} + \vec{\sigma} \frac{\vec{k} \cdot \vec{\sigma}}{E+m} \right] \phi_s \right\} \quad (2.32)$$

To evaluate these Dirac brackets, we must include the appropriate quark and antiquark spins s and $\bar{s}$. These are specified by the total angular momentum wavefunction $[Y_{lm} \otimes \chi_{s\bar{s}}]$ in (2.20), denoted:

$$|J, m_J\rangle = [Y_{lm} \otimes \chi_{s\bar{s}}] \quad (2.33)$$

for convenience. The meson, being a quark-antiquark pair as described in the introduction, is illustrated in Figure 2.1. The quark and antiquark each possess intrinsic spin angular momentum of $s = \frac{1}{2}$ and $\bar{s} = \frac{1}{2}$ respectively. For a P -wave meson, they also orbit about the centre of mass with orbital angular momentum $L = 1$. By the group theory rules for $SU(2)$ antiparticle symmetry [2, p. 47], s and $\bar{s}$ are added by the Clebsch-Gordon rules to give the wavefunctions of total spin angular momentum, $\chi_{s\bar{s}}$, in Table 2.1. Next the wavefunctions of orbital angular momentum for a P -wave meson, written in terms of spherical harmonics Y_{lm} , and for convenience written in Cartesian coordinate form, are given in Table 2.2. Then (2.33), using the Clebsch-Gordon rules a second time, gives the wavefunctions of total angular momentum, which appear in Table 2.3.

Figure 2.1: Spin and Orbital Angular Momentum of the $q\bar{q}$ Pair

Table 2.1: χ Spin Angular Momentum Functions

χ_{00}	$=$	$\sqrt{\frac{1}{2}}$	$(\uparrow\downarrow + \downarrow\uparrow)$
χ_{11}	$=$	$-$	$(\uparrow\uparrow)$
χ_{10}	$=$	$\sqrt{\frac{1}{2}}$	$(\uparrow\downarrow - \downarrow\uparrow)$
$\chi_{1,-1}$	$=$		$(\downarrow\downarrow)$

Table 2.2: Y_{lm} Orbital Angular Momentum Functions

Y_{11}	$=$	$-\sqrt{\frac{3}{8\pi}}$	$\frac{1}{p}(p_x + ip_y)$
Y_{10}	$=$	$\sqrt{\frac{3}{4\pi}}$	$\frac{1}{p}p_z$
$Y_{1,-1}$	$=$	$\sqrt{\frac{3}{8\pi}}$	$\frac{1}{p}(p_x - ip_y)$

Table 2.3: Total Angular Momentum Functions

Singlet States: $|1^{+-}\rangle \Longleftrightarrow |(^1P_1)_{m,}\rangle$

$ 1, 1\rangle$	$=$	Y_{11}	χ_{00}
$ 1, 0\rangle$	$=$	Y_{10}	χ_{00}
$ 1, -1\rangle$	$=$	$Y_{1,-1}$	χ_{00}

Triplet States: $|1^{++}\rangle \Longleftrightarrow |(^3P_1)_{m,}\rangle$

$ 1, 1\rangle$	$=$	$\sqrt{\frac{1}{2}}$	Y_{11}	χ_{10}	$-$	$\sqrt{\frac{1}{2}}$	Y_{10}	χ_{11}
$ 1, 0\rangle$	$=$	$\sqrt{\frac{1}{2}}$	Y_{11}	$\chi_{1,-1}$	$-$	$\sqrt{\frac{1}{2}}$	$Y_{1,-1}$	χ_{11}
$ 1, -1\rangle$	$=$	$\sqrt{\frac{1}{2}}$	Y_{10}	$\chi_{1,-1}$	$-$	$\sqrt{\frac{1}{2}}$	$Y_{1,-1}$	χ_{10}

As alluded to in the introduction, the Clebsch-Gordon rules yield, in Table 2.3, two different spin states $S = \{0, 1\}$. These states, the spin singlet and triplet states respectively, are orthogonal. (Each, of course, consists of the three different spatial orientations $m_J = +1, 0, -1$, but these are physically degenerate.) They may be labelled using either the $|J^{PC}\rangle$ or $(^{2S+1}L_J)$ notations of (2.34, 2.35):

$$|1^{++}\rangle \equiv |({}^3P_1)_{m_J}\rangle. \quad (2.34)$$

$$|1^{+-}\rangle \equiv |({}^1P_1)_{m_J}\rangle \quad (2.35)$$

To obtain an expression for the K_1 formfactor f_{K_1} , we must evaluate A^μ in (2.20). We consider the spatial components A^a , $a = (1, 2, 3)$ of the 3-vector $\vec{A}$. Calculating all three components separately serves as a consistency check. Substituting the $|J, m_J\rangle$ expressions from Table 2.3 and the $\langle 0|\bar{q}\gamma^a(1 - \gamma_5)q|q_u\bar{q}_s\rangle_{s\bar{s}}$ expression from (2.32) into (2.20), using the usual explicit forms of the 2-component spinors:

$$\phi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.36)$$

$$\phi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.37)$$

$$\chi_\uparrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.38)$$

$$\chi_\downarrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad (2.39)$$

we obtain:

$$\bar{A}(^3P_1) = + \frac{i}{(2\pi)^{\frac{3}{2}}} \bar{\varepsilon}(\lambda) \frac{6}{(2\pi)^2} \sqrt{\tilde{M}} \int d^3p \sqrt{\frac{E+m}{2E} \frac{\bar{E}+\bar{m}}{2\bar{E}}} \left[\frac{1}{E+m} + \frac{1}{\bar{E}+\bar{m}} \right] \frac{1}{p} p_z^2 i \phi_{n1}(p) \quad (2.40)$$

$$\bar{A}(^1P_1) = - \frac{i}{(2\pi)^{\frac{3}{2}}} \bar{\varepsilon}(\lambda) \frac{3\sqrt{2}}{(2\pi)^2} \sqrt{\tilde{M}} \int d^3p \sqrt{\frac{E+m}{2E} \frac{\bar{E}+\bar{m}}{2\bar{E}}} \left[\frac{1}{E+m} - \frac{1}{\bar{E}+\bar{m}} \right] \frac{1}{p} p_z^2 i \phi_{n1}(p) \quad (2.41)$$

The spatial portion of (2.18) reads:

$$\bar{A} = \frac{i}{(2\pi)^{\frac{3}{2}}} \bar{\varepsilon}(\lambda) f_{K_1} \quad (2.42)$$

where for the K_1 meson in its own rest frame ε^μ is given by [24, p. 381]:

$$\varepsilon^\mu(K_{rest}, \lambda) = (0, \bar{\varepsilon}(\lambda)) \quad (2.43)$$

where:

$$\bar{\varepsilon}(\lambda = \pm 1) = \mp \sqrt{\frac{1}{2}} (1, \pm i, 0) \quad (2.44)$$

$$\bar{\varepsilon}(\lambda = 0) = (0, 0, 1) \quad (2.45)$$

and $\lambda = \pm 1, 0$ is the spin projection along the z -axis.

Finally, comparing (2.41 and 2.40) and (2.42), and performing the trivial angular integration, we obtain the K_1 formfactor f_{K_1} :

$$f_{K_1}(^3P_1) = + \frac{2}{\pi} \sqrt{\tilde{M}} \int_0^\infty dp \sqrt{\frac{E+m}{2E} \frac{\bar{E}+\bar{m}}{2\bar{E}}} \left[\frac{1}{E+m} + \frac{1}{\bar{E}+\bar{m}} \right] p^3 i \phi_{n1}(p) \quad (2.46)$$

$$f_{K_1}(^1P_1) = - \frac{\sqrt{2}}{\pi} \sqrt{\tilde{M}} \int_0^\infty dp \sqrt{\frac{E+m}{2E} \frac{\bar{E}+\bar{m}}{2\bar{E}}} \left[\frac{1}{E+m} - \frac{1}{\bar{E}+\bar{m}} \right] p^3 i \phi_{n1}(p) \quad (2.47)$$

These formulas will be used to generate numerical results in §3. Details of the numerical integration procedure are discussed in Appendix B.

2.4 The Non-relativistic Limit

Taking the nonrelativistic limit of (2.20) is useful for calculating the K_1 properties. The resulting nonrelativistic formfactor expressions will be simpler than the original relativistic expressions, and the dependence on the quark model parameters will be more transparent. In this limit:

$$E = \sqrt{p^2 + m^2} \longrightarrow m$$

$$\bar{E} = \sqrt{p^2 + \bar{m}^2} \longrightarrow \bar{m}$$

and:

$$\vec{p} \longrightarrow 0.$$

$\bar{A}$ becomes:

$$A_{n.r.}^a = -\sqrt{6\tilde{M}} \lim_{\vec{p} \rightarrow 0} \int d^3p \Phi_{nl}(p) [Y_{lm}(\Omega_p) \otimes \chi_{s\bar{s}}] \langle \rangle_{n.r.}^a, \quad (2.48)$$

where:

$$\langle \rangle_{n.r.}^a = -\frac{1}{(2\pi)^3} \left\{ \chi_s^T \left[\frac{\vec{k} \cdot \vec{\sigma}}{2\bar{m}} \vec{\sigma} + \vec{\sigma} \frac{\vec{k} \cdot \vec{\sigma}}{2m} \right] \phi_s \right\} \quad (2.49)$$

We may Fourier transform the expression into coordinate space:

$$A_{n.r.}^a = -\sqrt{6\tilde{M}} (2\pi)^{\frac{3}{2}} \lim_{\vec{r} \rightarrow 0} \int d^3p \frac{e^{+i\vec{p} \cdot \vec{r}}}{(2\pi)^{\frac{3}{2}}} \Phi_{nl}(p) [Y_{lm}(\Omega_p) \otimes \chi_{s\bar{s}}] \langle \rangle_{n.r.}^a \quad (2.50)$$

For the ground state $L = 1$ meson, the integral transforms momentum space terms of the form:

$$p_i Y_{1m}(\Omega_p) \phi_{01}(p).$$

into coordinate space terms of the form:

$$\lim_{\vec{r} \rightarrow 0} \left(-i \frac{\partial}{\partial r_i} \right) \{ Y_{1m}(\Omega_r) R_{01}(r) \}$$

Performing all the tedious algebra and taking the remaining limit, we obtain:

$$\tilde{A}_{n.r.}(^3P_1) = + \frac{i}{(2\pi)^{\frac{3}{2}}} \tilde{\epsilon}(\lambda) \frac{3}{\sqrt{2\pi}} \sqrt{\tilde{M}} \left(\frac{1}{m} + \frac{1}{\tilde{m}} \right) R'_{01}(0) \quad (2.51)$$

$$\tilde{A}_{n.r.}(^1P_1) = - \frac{i}{(2\pi)^{\frac{3}{2}}} \tilde{\epsilon}(\lambda) \frac{3}{2\sqrt{\pi}} \sqrt{\tilde{M}} \left(\frac{1}{m} - \frac{1}{\tilde{m}} \right) R'_{01}(0) \quad (2.52)$$

where $R'_{01}(0)$ is the derivative of the radial wavefunction at the origin. Once again, comparing these with (2.42), we extract the K_1 formfactors f_{K_1} , this time for the nonrelativistic case:

$$f_{K_1}^{n.r.}(^3P_1) = + \frac{3}{\sqrt{2\pi}} \sqrt{\tilde{M}} \left(\frac{1}{m} + \frac{1}{\tilde{m}} \right) R'_{01}(0) \quad (2.53)$$

$$f_{K_1}^{n.r.}(^1P_1) = - \frac{3}{2\sqrt{\pi}} \sqrt{\tilde{M}} \left(\frac{1}{m} - \frac{1}{\tilde{m}} \right) R'_{01}(0) \quad (2.54)$$

These formulas will be used to generate numerical nonrelativistic results in §3. Note that in the flavour- $SU(3)$ limit $f_{K_1}^{n.r.}(^1P_1)$ is zero.

2.5 The Mixed Formfactor

As explained earlier, in perfect $SU(3)$ symmetry the u , d , and s quarks would all have the same mass and we would see distinct $K_1(^3P_1)$ and $K_1(^1P_1)$ states. However, the s quark has a significantly greater mass, breaking $SU(3)$ symmetry, and resulting in mixing of the spin states $K_1(^3P_1)$ and $K_1(^1P_1)$ to yield the physical states $K_1(1400)$ and $K_1(1270)$. The degree of mixing is parameterized by the 'mixing angle' θ_K in a standard orthogonal system rotation:

$$\begin{pmatrix} |K_1(1400)\rangle \\ |K_1(1270)\rangle \end{pmatrix} = \begin{pmatrix} \cos \theta_{K_1} & -\sin \theta_{K_1} \\ \sin \theta_{K_1} & \cos \theta_{K_1} \end{pmatrix} \begin{pmatrix} |K_1(^3P_1)\rangle \\ |K_1(^1P_1)\rangle \end{pmatrix} \quad (2.55)$$

Since our interest is the K_1 formfactors, we write for f_{K_1} :

$$f_{K_1}(1400) = \cos \theta_{K_1} f_{K_1}(^3P_1) - \sin \theta_{K_1} f_{K_1}(^1P_1). \quad (2.56)$$

$$f_{K_1}(1270) = \sin \theta_{K_1} f_{K_1}(^3P_1) + \cos \theta_{K_1} f_{K_1}(^1P_1) \quad (2.57)$$

The next step in the calculation is to take the expressions for $f_{K_1}(1270)$ and $f_{K_1}(1400)$, which contain the spin-state formfactors from our mock-meson model and the unknown mixing angle θ_{K_1} , and substitute them into expressions for observables which we can compare with experiment. The extraction of the mixing angle will then follow. The observables we choose are the partial decay widths Γ , of the reactions:

$$\tau^+ \longrightarrow \bar{\nu}_\tau + K_1^+(1270) : \text{decay width } \Gamma(1270) \quad (2.58)$$

$$\tau^+ \longrightarrow \bar{\nu}_\tau + K_1^+(1400) : \text{decay width } \Gamma(1400) \quad (2.59)$$

The exact details for deriving expressions for $\Gamma(1270)$ and $\Gamma(1400)$ are dealt with next.

2.6 The Decay Width Γ

2.6.1 Derivation of the Decay Width Formula

We find the partial decay widths Γ of $\tau \rightarrow K_1 \nu_\tau$ through a Feynman calculation. To lowest order the process is described by the Feynman diagram in figure (2.2).

Figure 2.2: Leading-Order Process in $\tau^+ \rightarrow \bar{\nu}_\tau + K_1^+$

By following the Feynman rules, we translate this diagram into the transition matrix element M . The process involves the W -boson, so it is a charged-current weak interaction. The appropriate Feynman rules [24, Appendix F] give us the matrix element:

$$-iM = \underbrace{\bar{v}(p, s)}_{\tau^+} \cdot \underbrace{-i \frac{g_W}{2\sqrt{2}} \gamma_\mu (1 - \gamma_5) V_{su}}_{\tau^+ \rightarrow \bar{\nu}_\tau, W^+ \text{ vertex}} \cdot \underbrace{v(p', s')}_{\bar{\nu}_\tau} \quad (2.60)$$

$$\cdot \underbrace{\frac{i}{p'^2 - M_W^2} \left(-g^{\mu\nu} + \frac{p'^\mu p'^\nu}{M_W^2} \right)}_{W\text{-propagator}} \cdot \underbrace{A_\nu}_{K_1^+} \quad (2.61)$$

Note that the entire shaded area is just assigned the factor A_ν , the amplitude first given in (2.20):

$$A_\nu = \frac{i}{(2\pi)^{\frac{3}{2}}} \cdot \frac{g_W}{2\sqrt{2}} \epsilon_\nu f_{K_1} \quad (2.62)$$

(Note that there is an new factor $-\frac{g_W}{2\sqrt{2}}$ in A_ν . This is the W -boson coupling factor, and must appear in both W -vertex factors.)

Following the usual procedure, we obtain:

$$\frac{1}{2} \langle |M|^2 \rangle = \frac{1}{(2\pi)^3} G_F^2 |V_{su}|^2 f_{K_1}^2 \{ [m_\tau^2 + 2M_K^2][m_\tau^2 - M_K^2]/M_K^2 \}, \quad (2.63)$$

the sum-averaged matrix element squared, where G_F is the 'Fermi coupling constant' [24, p. 351]:

$$G_F = \frac{\sqrt{2}}{2^3} \frac{g_W^2}{M_W^2}. \quad (2.64)$$

Since we are considering a decay of the form:

$$i \longrightarrow 1 + 2,$$

$\frac{1}{2}\langle|M|^2\rangle$ appears in the corresponding differential decay width formula:

$$d\Gamma = \frac{1}{2E_1} \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(p_1 + p_2 - p_i) \cdot \frac{1}{2}\langle|M|^2\rangle. \quad (2.65)$$

Performing the usual integration over phase space, we get the partial decay width:

$$\Gamma = \frac{1}{16\pi} G_F^2 |V_{su}|^2 f_{K_1}^2 \{[m_\tau^2 + 2M_K^2][(m_\tau^2 - M_K^2)^2]/m_\tau^3 M_K^2\} \quad (2.66)$$

Γ is the observable that we may compare with experiment to extract the unknown mixing angle θ_{K_1} .

2.6.2 Extraction of the Mixing Angle θ_K from the Ratio $R(\theta_K)$

Each of the decay reactions (2.58) and (2.59) occurs with branching ratio:

$$B(\tau^+ \longrightarrow \bar{\nu}_\tau + K_1^+(1270)) = \Gamma(1270)/\Gamma_{total}$$

$$B(\tau^+ \longrightarrow \bar{\nu}_\tau + K_1^+(1400)) = \Gamma(1400)/\Gamma_{total}$$

where Γ_{total} is the total decay width of the τ^+ . We define the ratio:

$$R(\theta_{K_1}) \equiv \left\{ \frac{B(\tau^+ \longrightarrow \bar{\nu}_\tau + K_1^+(1270))}{B(\tau^+ \longrightarrow \bar{\nu}_\tau + K_1^+(1400))} \right\} = \left\{ \frac{\Gamma(\tau^+ \longrightarrow \bar{\nu}_\tau + K_1^+(1270))}{\Gamma(\tau^+ \longrightarrow \bar{\nu}_\tau + K_1^+(1400))} \right\} (\theta_{K_1}), \quad (2.67)$$

or, in a simpler notation,

$$R(\theta_{K_1}) = \left\{ \frac{\Gamma(1270)}{\Gamma(1400)} \right\} (\theta_{K_1}), \quad (2.68)$$

where our expression for R contains the unknown mixing angle θ_{K_1} . Recall that much of the radial wavefunction dependence is expected to be cancelled in this ratio.

The experimentally measured values of the branching ratios are [9]:

$$B(\tau^+ \rightarrow \bar{\nu}_\tau + K_1^+(1270)) = (0.41_{-0.35}^{+0.41}) \times 10^{-2} \quad (2.69)$$

$$B(\tau^+ \rightarrow \bar{\nu}_\tau + K_1^+(1400)) = (0.76_{-0.33}^{+0.40}) \times 10^{-2}, \quad (2.70)$$

giving us an experimental value for the ratio R :

$$R_E = (0.54_{-0.52}^{+0.61}). \quad (2.71)$$

We extract θ_{K_1} by solving:

$$R(\theta_{K_1}) = R_E. \quad (2.72)$$

See §3.3 for the solutions. First, though, in §3.1 and §3.2, we will discuss the particular radial wavefunctions $\phi(p)$ and mock-meson mass definitions $\tilde{M}$ that were chosen for our calculations.

Chapter 3

Results

3.1 The Radial Wavefunction

The radial wavefunction and the mock-meson mass in the formfactor expressions have not yet been specified. The mock-meson mass will be defined in the next section, §3.2. The K_1 -meson radial wavefunction is specified by choosing a specific quark model, with values for the q and $\bar{q}$ quark masses, and then solving the Schrodinger equation for the nonrelativistic $q\bar{q}$ bound state. This requires knowledge of the $q\bar{q}$ radial potential function $V(r)$. As explained in the introduction, we employ a QCD-motivated Coulomb plus linear potential in describing the nonrelativistic $q\bar{q}$ bound state:

$$V(r) = -\frac{4}{3} \frac{\alpha_s(r)}{r} + br + C, \quad (3.1)$$

where the first term containing the running strong-force coupling constant $\alpha_s(r)$ is the colour-singlet Coulomb term [1, p. 286], and the second is the linear term. See figure 3.1.

The Schrodinger equation has been solved for this potential, and the solution for the ground state $L = 1$ meson has been expressed in a simple harmonic oscillator (SHO) basis [3, 19]:

$$\phi_{01}(p) = \sum_{a=0}^N d_a \phi_{a1}^{H.O.}(p) \quad (3.2)$$

where the $\phi_{a1}^{H.O.}$ are the SHO-basis eigenfunctions:

$$\phi_{a1}^{H.O.}(p) = -i(-1)^a \left[\frac{2a!}{\Gamma(a + \frac{5}{2})} \right]^{\frac{1}{2}} \beta^{-\frac{1}{2}} p e^{-\frac{p^2}{2\beta^2}} L_a^{\frac{1}{2}}\left(\frac{p^2}{\beta^2}\right) \quad (3.3)$$

and the d_a are the expansion coefficients. (Note that we are only considering P -wave mesons with $l = 1$.) The numerical coefficients d_a and the SHO-parameter β are given by solving the Schrodinger equation for specific quark models. As stated in the above paragraph, a specific model includes values for the q and $\bar{q}$ masses. Then solving the Schrodinger matrix equation for the SHO-basis using variational methods yields the numerical SHO-expansion coefficients d_a and the SHO-parameter β . In the meson physics literature, there are several specific quark models and corresponding radial wavefunction solutions. The physics is not well known to the degree that there is one obvious best choice. Hence we choose to try a number of models to investigate the sensitivity of our results to the various wavefunctions.

The $L_a^{\frac{3}{2}}$ are the associated Laguerre polynomials, defined by:

$$L_n^\alpha(x) = \sum_{m=0}^n (-1)^m \binom{n+\alpha}{n-m} \frac{x^m}{m!} \quad (3.4)$$

and generated by the recurrence relation:

$$(j+1)L_{j+1}^\alpha = (-x + 2j + \alpha + 1)L_j^\alpha - (j + \alpha)L_{j-1}^\alpha. \quad (3.5)$$

For the nonrelativistic limit, we need the position-space form:

$$R_{01}(r) = \sum_{a=0}^N d_a R_{a1}^{H.O.}(r) \quad (3.6)$$

where:

$$R_{a1}^{H.O.}(r) = \left[\frac{2a!}{\Gamma(a + \frac{5}{2})} \right]^{\frac{1}{2}} \beta^{\frac{1}{2}} r e^{-\frac{\beta^2 r^2}{2}} L_a^{\frac{1}{2}}(\beta^2 r^2). \quad (3.7)$$

The required derivative is:

$$R'_{01}(r)|_{r=0} = \beta^{\frac{1}{2}} \sum_{a=0}^N d_a \left[\frac{2a!}{\Gamma(a + \frac{5}{2})} \right]^{\frac{1}{2}} L_a^{\frac{3}{2}}(0) \quad (3.8)$$

where:

$$L_n^{\alpha}(0) = \binom{n + \alpha}{n}, \quad (3.9)$$

so:

$$R'_{01}(0) = \frac{8}{3} \frac{1}{\sqrt{\pi}} \beta^{\frac{1}{2}} \sum_{a=0}^N d_a \left[\frac{2a!}{\Gamma(a + \frac{5}{2})} \right]^{-\frac{1}{2}}. \quad (3.10)$$

Figure 3.1: $q\bar{q}$ Potential Function

3.2 Computational Parameters

In order to actually generate numerical results, we need various input parameters for the radial momentum wavefunction (3.2), the formfactors (2.47 and 2.46), and the decay widths (2.66).

- **Wavefunction Parameters** The required parameters are the quark and antiquark masses m and $\bar{m}$, the SHO-parameter β , and the SHO-basis expansion coefficients $d_a, a = (1, \dots, N)$. Six different sets of these parameters were used. They are listed in table 3.1. Note that cases 1 to 5 involve only the ground-level eigenstate, while case 6 is a multi-level eigenstate expansion.
- **Formfactor Parameters** The required parameter is the mock-meson mass $\tilde{M}$. Four different definitions from meson physics were used, and are listed in table 3.2. Currently, it is not known which definition is best, so we have chosen to try this selection of definitions.

Definition 3 is the average total energy of the quarks:

$$\tilde{M} = \langle E \rangle + \langle \bar{E} \rangle, \quad (3.11)$$

where the energy expectations are given by:

$$\langle E \rangle = \int_0^\infty p^2 dp \sqrt{p^2 + m^2} |\phi(p)|^2 \quad (3.12)$$

$$\langle \bar{E} \rangle = \int_0^\infty p^2 dp \sqrt{p^2 + \bar{m}^2} |\phi(p)|^2 \quad (3.13)$$

- **Decay Width Parameters** The parameters required in the decay width formula are the experimentally measured masses [8].

m_τ	$= (1.7771^{+0.0004}_{-0.0005})$	GeV
M_K	$= \begin{cases} (1.273 \pm 0.007) \text{ for } K1(1270) \\ (1.402 \pm 0.007) \text{ for } K1(1400) \end{cases}$	GeV

Table 3.1: Radial Wavefunction Parameters

Case	m (GeV)	$\bar{m}$ (GeV)	β (GeV)	$\{d_a\}$	
1 [22]	0.33	0.55	0.183	d_0	= +1.0
2 [25]	0.33	0.55	0.257	d_0	= +1.0
3 [18]	0.22	0.43	0.322	d_0	= +1.0
4 [18]	0.34	0.51	0.22	d_0	= +1.0
5 [3]	0.22	0.419	0.45	d_0	= +1.0
6a [3] †	0.22	0.419	0.56	d_0	= +0.926694
				d_1	= -0.266297
				d_2	= +0.232671
				d_3	= -0.083948
				d_4	= +0.082276
				d_5	= -0.027971
				d_6	= +0.034434
				d_7	= -0.009186
				d_8	= +0.016019
				d_9	= -0.002233
				d_{10}	= +0.007530
				d_{11}	= +0.001098
6b [3] ‡	0.22	0.419	0.58	d_0	= +0.918846
				d_1	= -0.290787
				d_2	= +0.231243
				d_3	= -0.095134
				d_4	= +0.078715
				d_5	= -0.033684
				d_6	= +0.030623
				d_7	= -0.012549
				d_8	= +0.012835
				d_9	= -0.004545
				d_{10}	= +0.005302
				d_{11}	= -0.000922

† (1P_1) only‡ (3P_1) only

Table 3.2: Mock-Meson Mass Cases

Case	M (GeV)
1: Physical Mass	1.273 (for $K_1(1270)$), 1.402 (for $K_1(1400)$)
2: Sum of Quark Masses	$m + \bar{m}$
3: Average Energy	$\langle E \rangle + \langle \bar{E} \rangle$
4: Kokoski & Isgur Value [26]	1.380

Note: Case 3 reduces to case 2 in the nonrelativistic limit.

3.3 Results

The results for the K_1 formfactor calculations, f_{K_1} , for both the relativistic and nonrelativistic models, appear in tables 3.3 and 3.4 respectively. In the first half of each table we held the mock-meson mass definition fixed at definition 1 while varying the wavefunctions, while in the second half we held the wavefunction fixed at wavefunction 1 and varied the mass definition. These results are plotted in the form of bar graphs in figures 3.2 to 3.9, on scales that give a visualization of the variation. These plots immediately indicate that the formfactors are quite sensitive to our choice of radial wavefunction; in the four cases where the wavefunction varies, the maximum formfactor varies from the minimum by a factor of from roughly 5 to 8. However, the plots also immediately indicate that the formfactors are not very sensitive to the mock-meson mass definition; in the four cases where the mass definition varies, the formfactor variation is small.

Taking the mean and standard deviation for the varying formfactor values from the first half of each table gives us the results for the relativistic model:

$$\left. \begin{aligned} f_{K_1}(^1P_1) &= -0.026 \pm 0.015 \\ f_{K_1}(^3P_1) &= +0.30 \pm 0.22 \end{aligned} \right\} \text{for } K_1(1270)$$

$$\left. \begin{aligned} f_{K_1}(^1P_1) &= -0.028 \pm 0.015 \\ f_{K_1}(^3P_1) &= +0.31 \pm 0.24 \end{aligned} \right\} \text{for } K_1(1400)$$

and the nonrelativistic model:

$$\left. \begin{aligned} f_{K_1}^{n.r.}(^1P_1) &= -0.062 \pm 0.062 \\ f_{K_1}^{n.r.}(^3P_1) &= +0.31 \pm 0.25 \end{aligned} \right\} \text{for } K_1(1270)$$

$$\left. \begin{aligned} f_{K_1}^{n.r.}(^1P_1) &= -0.065 \pm 0.065 \\ f_{K_1}^{n.r.}(^3P_1) &= +0.32 \pm 0.26 \end{aligned} \right\} \text{for } K_1(1400)$$

Table 3.3: K_1 -Meson Formfactors, Relativistic Model

Wavefunction	$M = M(\text{physical})$	
	$K_1(1270)$	$K_1(1400)$
Wfn. 1	$f_{K_1}(^1P_1) = -0.011$	$f_{K_1}(^1P_1) = -0.012$
	$f_{K_1}(^3P_1) = +0.083$	$f_{K_1}(^3P_1) = +0.087$
Wfn. 2	$f_{K_1}(^1P_1) = -0.019$	$f_{K_1}(^1P_1) = -0.020$
	$f_{K_1}(^3P_1) = +0.164$	$f_{K_1}(^3P_1) = +0.172$
Wfn. 3	$f_{K_1}(^1P_1) = -0.030$	$f_{K_1}(^1P_1) = -0.032$
	$f_{K_1}(^3P_1) = +0.275$	$f_{K_1}(^3P_1) = +0.289$
Wfn. 4	$f_{K_1}(^1P_1) = -0.012$	$f_{K_1}(^1P_1) = -0.013$
	$f_{K_1}(^3P_1) = +0.122$	$f_{K_1}(^3P_1) = +0.128$
Wfn. 5	$f_{K_1}(^1P_1) = -0.040$	$f_{K_1}(^1P_1) = -0.042$
	$f_{K_1}(^3P_1) = +0.485$	$f_{K_1}(^3P_1) = +0.509$
Wfn. 6	$f_{K_1}(^1P_1) = -0.046$	$f_{K_1}(^1P_1) = -0.049$
	$f_{K_1}(^3P_1) = +0.645$	$f_{K_1}(^3P_1) = +0.678$
Wavefunction 1		
Mass	$K_1(1270)$	$K_1(1400)$
Definition		
Dfn. 1	$f_{K_1}(^1P_1) = -0.011$	$f_{K_1}(^1P_1) = -0.012$
	$f_{K_1}(^3P_1) = +0.083$	$f_{K_1}(^3P_1) = +0.087$
Dfn. 2	$f_{K_1}(^1P_1) = -0.009$	
	$f_{K_1}(^3P_1) = +0.069$	
Dfn. 3	$f_{K_1}(^1P_1) = -0.010$	
	$f_{K_1}(^3P_1) = +0.075$	
Dfn. 4	$f_{K_1}(^1P_1) = -0.012$	
	$f_{K_1}(^3P_1) = +0.086$	

Table 3.4: K_1 -Meson Formfactors, Nonrelativistic Model

Wavefunction	$M = M(\text{physical})$	
	$K_1(1270)$	$K_1(1400)$
Wfn. 1	$f_{K_1}(^1P_1) = -0.020$	$f_{K_1}(^1P_1) = -0.021$
	$f_{K_1}(^3P_1) = +0.114$	$f_{K_1}(^3P_1) = +0.120$
Wfn. 2	$f_{K_1}(^1P_1) = -0.048$	$f_{K_1}(^1P_1) = -0.050$
	$f_{K_1}(^3P_1) = +0.269$	$f_{K_1}(^3P_1) = +0.283$
Wfn. 3	$f_{K_1}(^1P_1) = -0.153$	$f_{K_1}(^1P_1) = -0.161$
	$f_{K_1}(^3P_1) = +0.670$	$f_{K_1}(^3P_1) = +0.703$
Wfn. 4	$f_{K_1}(^1P_1) = -0.026$	$f_{K_1}(^1P_1) = -0.027$
	$f_{K_1}(^3P_1) = +0.184$	$f_{K_1}(^3P_1) = +0.193$
Mass Definition	Wavefunction 1	
	$K_1(1270)$	$K_1(1400)$
Dfn. 1	$f_{K_1}(^1P_1) = -0.020$	$f_{K_1}(^1P_1) = -0.021$
	$f_{K_1}(^3P_1) = +0.114$	$f_{K_1}(^3P_1) = +0.120$
Dfn. 2		$f_{K_1}(^1P_1) = -0.017$
		$f_{K_1}(^3P_1) = +0.095$
Dfn. 3		n/a
		n/a
Dfn. 4		$f_{K_1}(^1P_1) = -0.021$
		$f_{K_1}(^3P_1) = +0.119$

Figure 3.2: $f_{K_1}(^1P_1)$ for Mass Case 1, Relativistic Model

Figure 3.3: $f_{K_1}(^3P_1)$ for Mass Case 1, Relativistic Model

Figure 3.4: $f_{K_1}(^1P_1)$ for Wavefunction 1, Relativistic Model

Figure 3.5: $f_{K_1}(^3P_1)$ for Wavefunction 1, Relativistic Model

Figure 3.6: $f_{K_1}(^1P_1)$ for Mass Case 1, Nonrelativistic Model

Figure 3.7: $f_{K_1}(^3P_1)$ for Mass Case 1, Nonrelativistic Model

Figure 3.8: $f_{K_1}({}^1P_1)$ for Wavefunction 1, Nonrelativistic Model

Figure 3.9: $f_{K_1}({}^3P_1)$ for Wavefunction 1, Nonrelativistic Model

We take the relativistic values as our formfactor results. The K_1 meson is a light quark system, composed of the light $q\bar{q}$ flavours u and $\bar{s}$ respectively. Hence relativistic effects are significant, and it is the relativistic model which is the more accurate description. Examples of the importance of relativistic effects are the ρ - π , K^*-K , and B^*-B meson mass splittings, which arises due to relativistic corrections [7].

At this point we can make two decisions about how to next proceed with the mixing angle extractions based on what we have learned from our formfactor calculations. First, since the formfactors are quite sensitive to our choice of radial wavefunction, we will extract the mixing angle for all the wavefunctions. Since we have no constraint on our choice of wavefunction, we can place no corresponding constraint on the mixing angle solutions. Second, since the formfactors are not really sensitive to the definition of mock-meson mass, we will constrain ourselves to definition 1, the physical mass, and extract the mixing angle for just this mass definition.

The θ_{K_1} mixing angle solution intervals appear in tables 3.5 and 3.6. There are two valid solution intervals, one with a negative middle value, and one with a positive middle value. For each interval we have given the minimum, middle, and maximum value. Since we could not constrain our choice of wavefunction, we cannot here constrain our choice of solution interval; we must take the widest possible interval given by the two extreme wavefunctions. This is illustrated in figures 3.10 and 3.11. The two 'parabola' curves are the phenomenologically-generated ratio curves, $R(\theta_{K_1})$, for the two extreme wavefunctions. The three horizontal lines are the minimum, middle, and maximum values for the experimental value of the ratio. The shaded bands are the θ_{K_1} solution intervals, the widest possible intervals bounded by the curves for the two extreme wavefunctions.

The uncertainty in choosing a radial wavefunction is here expressed as the uncertainty in the middle θ_{K_1} values:

$$\delta(\theta_{K_1})_{mid.} \sim 5^\circ$$

This is a fairly small variation in mixing angle value. Recall that the corresponding variation in formfactor value was very large. This clearly illustrates that in using the formfactor calculations for the mixing angle extractions, much of the uncertainty in the radial wavefunction contained in the formfactors is factored out in the ratio of partial decay widths. In the case of the nonrelativistic model, with formfactors (2.53) and (2.54), the wavefunction is explicitly cancelled in this ratio. Hence our decision to use a ratio of partial decay widths is rewarded.

However, the experimental error in the ratio of partial decay widths R_E , is here expressed as the total width of the θ_{K_1} solution intervals:

$$(-35^\circ \lesssim \theta_{K_1} \lesssim +5^\circ), (10^\circ \lesssim \theta_{K_1} \lesssim 50^\circ)$$

or an uncertainty of:

$$\delta\theta_{K_1} \sim 20^\circ$$

Because of the current poor precision of R_E , the constraint on θ_{K_1} is weak. The 'experimental' error described here far outstrips the 'phenomenological' error described above. This fact tells us that despite our current lack of knowledge of meson phenomenology, the mock-meson model is a worthy approach for extracting the K_1 -meson mixing angle. When the experimental values for the tau partial decay widths are known to a higher precision, these calculations may be repeated, and will yield the mixing angle to a correspondingly higher precision.

Table 3.5: K_1 -Meson Mixing Angle, Relativistic Model

Wfn.	$M = M(\text{physical})$					
	$\theta_{K_1}(-)$			$\theta_{K_1}(+)$		
	min.	mid.	max.	min.	mid.	max.
Wfn. 1	-32.1	-22.1	+1.4	14.0	37.4	47.5
Wfn. 2	-33.2	-23.1	+0.4	12.9	36.4	46.5
Wfn. 3	-33.5	-23.4	+0.0	12.6	36.0	46.1
Wfn. 4	-34.2	-24.1	-0.6	11.9	35.4	45.5
Wfn. 5	-35.1	-25.0	-1.5	11.0	34.5	44.5
Wfn. 6	-35.7	-25.6	-2.2	10.4	33.8	43.9

Table 3.6: K_1 -Meson Mixing Angle, Nonrelativistic Model

Wfn.	$M = M(\text{physical})$					
	$\theta_{K_1}(-)$			$\theta_{K_1}(+)$		
	min.	mid.	max.	min.	mid.	max.
Wfn. 1	-29.8	-19.7	+3.8	16.3	39.8	49.8
Wfn. 2	-29.8	-19.7	+3.8	16.3	39.8	49.8
Wfn. 3	-27.0	-16.9	+6.6	19.1	42.6	52.7
Wfn. 4	-31.8	-21.7	+1.8	14.3	37.8	47.9

Figure 3.10: $R(\theta_{K_1})$ vs θ_{K_1} for Mass Case 1, Relativistic Model

Figure 3.11: $R(\theta_{K_1})$ vs θ_{K_1} for Mass Case 1, Nonrelativistic Model

Chapter 4

Conclusions

In summary, we have employed the mock-meson model to calculate the K_1 -meson decay constant for both the spin-triplet and spin-singlet states, $f_{K_1}(^3P_1)$ and $f_{K_1}(^1P_1)$, and have used them in a comparison of a phenomenological expression with the experimental value of a partial decay width ratio, to extract the K_1 -meson mixing angle θ_{K_1} .

The decay constant f_{K_1} is proportional to the decay amplitude of the K_1 -meson decaying through the charged weak current to the vacuum:

$$A^\mu = \langle 0 | j_W^\mu | K_1 \rangle = \frac{i}{(2\pi)^{\frac{3}{2}}} f_{K_1} \epsilon^\mu.$$

We obtained values:

$$\left. \begin{aligned} f_{K_1}(^1P_1) &= -0.026 \pm 0.015 \\ f_{K_1}(^3P_1) &= +0.30 \pm 0.22 \end{aligned} \right\} \text{for } K_1(1270)$$

$$\left. \begin{aligned} f_{K_1}(^1P_1) &= -0.028 \pm 0.015 \\ f_{K_1}(^3P_1) &= +0.31 \pm 0.24 \end{aligned} \right\} \text{for } K_1(1400).$$

The partial decay width ratio comparison:

$$\left\{ \frac{\Gamma(1270)}{\Gamma(1400)} \right\} (\theta_K) = \left\{ \frac{\Gamma(1270)}{\Gamma(1400)} \right\}_E$$

for the tau-decay reactions:

$$\begin{cases} \tau^+ \rightarrow \bar{\nu}_\tau K_1^+(1270) \\ \tau^+ \rightarrow \bar{\nu}_\tau K_1^+(1400) \end{cases}$$

yielded solutions:

$$(-35^\circ \lesssim \theta_{K_1} \lesssim +5^\circ), (10^\circ \lesssim \theta_{K_1} \lesssim 50^\circ)$$

In calculating the K_1 -meson decay constant f_{K_1} from the mock-meson model, we found that the calculation is quite sensitive to our choice of radial wavefunction. Currently it is not known which of the radial wavefunctions given in the literature is best. Further studies are necessary to constrain our choice.

However, calculation of the decay constant is relatively insensitive to our choice of definition for mock-meson mass. We hence conclude that the physical meson mass definition is adequate.

In extracting the K_1 -meson mixing angle θ_{K_1} using the previously calculated decay constants, much of the radial wavefunction dependence was cancelled. Some remained, introducing an uncertainty of $\sim 5^\circ$ into the mixing angle results. When further studies in the mock-meson model are done to constrain our choice of radial wavefunction, this source of uncertainty may be reduced.

The main source of uncertainty in our solutions for the mixing angle is the very high experimental uncertainty in the tabulated values of the tau partial decay widths. In the future, when these values are known to a higher precision, these calculations may be repeated to give mixing angle solutions of correspondingly higher precision.

Our approach to generating the mixing angle θ_{K_1} led to two solution intervals:

one positive and one negative. Presumably, we need one more piece of information to differentiate between the two. However, the previously known experimental result of $(41 \pm 4)^\circ$ [13] is positive. Obviously, the new piece of information, when found, must indicate the positive solution interval for θ_{K_1} .

Appendix A

The Nonrelativistic Spin-Dependent Potential

The K_1 mixing angle θ_{K_1} is the angle that diagonalizes the $(^3P_1) - (^1P_1)$ mass matrix, which is found by taking the expectation values of the Hamiltonian. The off-diagonal mixing terms come from the spin-orbit piece $H_{s.o.}^-$ described below. To illustrate the origin of these things, as well as the implications for learning more about the $q\bar{q}$ potential, we write down the nonrelativistic limit of a QCD-based potential including the spin dependent terms:

$$V(\vec{r}) = H_{q\bar{q}}^{conf} + H_{q\bar{q}}^{cont} + H_{q\bar{q}}^{ten} + H_{q\bar{q}}^{s.o.} \quad (\text{A.1})$$

where

$$H_{q\bar{q}}^{conf} = C + br + \frac{\alpha_s(r)}{r} \vec{F}_q \cdot \vec{F}_{\bar{q}} \quad (\text{A.2})$$

is the linear confinement and Coulomb-like interaction,

$$H_{q\bar{q}}^{cont} = -\frac{8\pi}{3} \frac{\alpha_s(r)}{m_q m_{\bar{q}}} \vec{S}_q \cdot \vec{S}_{\bar{q}} \delta^3(\vec{r}) \vec{F}_q \cdot \vec{F}_{\bar{q}} \quad (\text{A.3})$$

is the colour contact interaction,

$$H_{q\bar{q}}^{ten} = -\frac{\alpha_s(r)}{m_q m_{\bar{q}}} \frac{1}{r^3} \left[\frac{3\vec{S}_q \cdot \vec{r} \vec{S}_{\bar{q}} \cdot \vec{r}}{r^2} - \vec{S}_q \cdot \vec{S}_{\bar{q}} \right] \vec{F}_q \cdot \vec{F}_{\bar{q}} \quad (\text{A.4})$$

is the colour tensor interaction, and

$$H_{q\bar{q}}^{s.o.} = H_{q\bar{q}}^{s.o.(cm)} + H_{q\bar{q}}^{s.o.(tp)} \quad (\text{A.5})$$

is the spin-orbit interaction, with

$$H_{q\bar{q}}^{s.o.(cm)} = -\frac{\alpha_s(r)}{r^3} \left(\frac{\vec{S}_q}{m_q m_{\bar{q}}} + \frac{\vec{S}_{\bar{q}}}{m_q m_{\bar{q}}} + \frac{\vec{S}_q}{m_q^2} + \frac{\vec{S}_{\bar{q}}}{m_{\bar{q}}^2} \right) \cdot \vec{L} \vec{F}_q \cdot \vec{F}_{\bar{q}} \quad (\text{A.6})$$

as the colour magnetic piece arising from one-gluon exchange, and

$$H_{q\bar{q}}^{s.o.(tp)} = -\frac{1}{2r} \frac{\partial H_{q\bar{q}}^{conf}}{\partial r} \left(\frac{\vec{S}_q}{m_q^2} + \frac{\vec{S}_{\bar{q}}}{m_{\bar{q}}^2} \right) \cdot \vec{L} \quad (\text{A.7})$$

the Thomas precession term. In these formulas, $\vec{S}_q$ and $\vec{S}_{\bar{q}}$ are the q and $\bar{q}$ spin operators, $\vec{L}$ is the $q\bar{q}$ orbital angular momentum operator, and $\vec{F}_q$ and $\vec{F}_{\bar{q}}$ are the q and $\bar{q}$ $SU(3)$ -colour operators. Note also that in these formulas,

$$\langle \vec{F}_q \cdot \vec{F}_{\bar{q}} \rangle = -4/3$$

for a meson.

The two spin-orbit pieces may be rewritten as:

$$H_{q\bar{q}}^{s.o.(cm)} = \frac{2\alpha_s}{3r^3} \left(\frac{1}{m_q} + \frac{1}{m_{\bar{q}}} \right)^2 \vec{S} \cdot \vec{L} + \frac{2\alpha_s}{3r^3} \left(\frac{1}{m_q^2} - \frac{1}{m_{\bar{q}}^2} \right) \vec{S}_- \cdot \vec{L} \quad (\text{A.8})$$

$$H_{q\bar{q}}^{s.o.(tp)} = -\frac{1}{4r} \frac{\partial H_{q\bar{q}}^{conf}}{\partial r} \left[\left(\frac{1}{m_q^2} + \frac{1}{m_{\bar{q}}^2} \right) \vec{S} \cdot \vec{L} + \left(\frac{1}{m_q^2} - \frac{1}{m_{\bar{q}}^2} \right) \vec{S}_- \cdot \vec{L} \right] \quad (\text{A.9})$$

where

$$\vec{S} = \vec{S}_q + \vec{S}_{\bar{q}}$$

$$\vec{S}_- = \vec{S}_q - \vec{S}_{\bar{q}}.$$

Then the various terms in $H^{s.o.}$ can be rearranged as

$$H^{s.o.} = H_{s.o.}^+ \vec{S} \cdot \vec{L} + H_{s.o.}^- \vec{S}_- \cdot \vec{L} \quad (\text{A.10})$$

where:

$$H_{s.o.}^+ (\vec{S} \cdot \vec{L}) = \left[\frac{2}{3} \frac{\alpha_s}{r^3} \left(\frac{1}{m_q} + \frac{1}{m_{\bar{q}}} \right)^2 - \frac{1}{4r} \frac{\partial H_{qq}^{nf}}{\partial r} \left(\frac{1}{m_q^2} + \frac{1}{m_{\bar{q}}^2} \right) \right] (\vec{S} \cdot \vec{L}) \quad (\text{A.11})$$

$$H_{s.o.}^- (\vec{S}_- \cdot \vec{L}) = \left[\frac{2}{3} \frac{\alpha_s}{r^3} \left(\frac{1}{m_q^2} - \frac{1}{m_{\bar{q}}^2} \right) - \frac{1}{4r} \frac{\partial H_{qq}^{conf}}{\partial r} \left(\frac{1}{m_q^2} - \frac{1}{m_{\bar{q}}^2} \right) \right] (\vec{S}_- \cdot \vec{L}) \quad (\text{A.12})$$

It is the $H_{s.o.}^-$ term that leads to $(^3P_1) - (^1P_1)$ mixing.

The mass matrix is given by:

$$\begin{aligned} M(^3P_2) &= M_0 + \frac{1}{4} \langle H_{cont} \rangle - \frac{1}{10} \langle H_{ten} \rangle + \langle H_{s.o.}^+ \rangle \\ \begin{pmatrix} M(1400) \\ M(1270) \end{pmatrix} &= \begin{pmatrix} M_0 + \frac{1}{4} \langle H_{cont} \rangle + \frac{1}{2} \langle H_{ten} \rangle - \langle H_{s.o.}^+ \rangle & \sqrt{2} \langle H_{s.o.}^- \rangle \\ \sqrt{2} \langle H_{s.o.}^- \rangle & M_0 - \frac{3}{4} \langle H_{cont} \rangle \end{pmatrix} \begin{pmatrix} ^3P_1 \\ ^1P_1 \end{pmatrix} \\ M(^3P_0) &= M_0 + \frac{1}{4} \langle H_{cont} \rangle - \langle H_{ten} \rangle - 2 \langle H_{s.o.}^+ \rangle \end{aligned} \quad (\text{A.13})$$

where the $\langle H_i \rangle$ are the expectation values of the spatial parts of the various terms and M_0 is the center of mass of the multiplet.

Given the experimental values for the meson masses, and the extracted value for θ_{K_1} , one can solve (A.13) for M_0 and the $\langle H_i \rangle$. The $\langle H_{s.o.}^\pm \rangle$ can then be used to gauge the relative importances of the one-gluon-exchange and the confining contributions to the spin-orbit terms of (A.11) and (A.12), hence giving insights into the meson Hamiltonian.

Appendix B

Numerical Methods

The formfactor formulas (2.47 and 2.46), with the relative-momentum wavefunction $\phi(p)$ included, are written:

$$f_{K_1}(^3P_1) = +\frac{2}{\pi}\sqrt{\tilde{M}} \int_0^\infty dp \{\}_1 \beta^{-\frac{5}{2}} p^4 e^{-\frac{p^2}{2\beta^2}} \sum_{a=0}^N d_a \{\}_2 L_a^{\frac{3}{2}}\left(\frac{p^2}{\beta^2}\right) \quad (\text{B.1})$$

$$f_{K_1}(^1P_1) = -\frac{\sqrt{2}}{\pi}\sqrt{\tilde{M}} \int_0^\infty dp \{\}_1 \beta^{-\frac{5}{2}} p^4 e^{-\frac{p^2}{2\beta^2}} \sum_{a=0}^N d_a \{\}_2 L_a^{\frac{3}{2}}\left(\frac{p^2}{\beta^2}\right) \quad (\text{B.2})$$

where we have used $\{\}_1$ and $\{\}_2$ to represent:

$$\{\}_1 = \left\{ \sqrt{\frac{E+m}{2E}} \frac{\bar{E}+\bar{m}}{2\bar{E}} \left[\frac{1}{\bar{E}+m} \pm \frac{1}{\bar{E}+\bar{m}} \right] \right\}, \quad (\text{B.3})$$

with + for $(^3P_1)$ and - for $(^1P_1)$, and:

$$\{\}_2 = \left\{ (-1)^a \left[\frac{2a!}{\Gamma(a+\frac{5}{2})} \right]^{\frac{1}{2}} \right\} \quad (\text{B.4})$$

We make the change of variable:

$$q = \frac{p^2}{2\beta^2} \quad (\text{B.5})$$

$$dq = \frac{1}{\beta^2} p dp \quad (\text{B.6})$$

which gives:

$$f_{K_1}(^3P_1) = +\frac{4\sqrt{2}}{\pi} \sqrt{\tilde{M}} \int_0^\infty dq q^{\frac{3}{2}} e^{-q} \{ \}_1 \beta^{+\frac{5}{2}} \sum_{a=0}^N d_a \{ \}_2 L_a^{\frac{3}{2}}(2q). \quad (\text{B.7})$$

$$f_{K_1}(^1P_1) = -\frac{4}{\pi} \sqrt{\tilde{M}} \int_0^\infty dq q^{\frac{3}{2}} e^{-q} \{ \}_1 \beta^{+\frac{5}{2}} \sum_{a=0}^N d_a \{ \}_2 L_a^{\frac{3}{2}}(2q) \quad (\text{B.8})$$

The integral in the two formulas above is of the form:

$$I = \int_0^\infty x^\alpha e^{-x} f(x) dx \quad (\text{B.9})$$

which is the appropriate form for Gauss-Laguerre quadrature. As with all types of Gaussian quadrature, the integral I is computed from:

$$I = \sum_{j=1}^N w_j f(x_j), \quad (\text{B.10})$$

where

$$\{x_1, x_2, \dots, x_j, \dots, x_N\}$$

are the N abscissas, and

$$\{w_1, w_2, \dots, w_j, \dots, w_N\}$$

are the corresponding weighting factors. The mathematical theory required to generate the abscissas and weights for a given number N and parameter α is quite involved [27]. However, a ready-made numerical routine which performs this function is available as the subroutine **gaulag** in the book *Numerical Recipes in FORTRAN* [27]. We chose to perform 30-point quadrature with **gaulag** input:

$$N = 30$$

$$\alpha = \frac{3}{2}$$

and **gaulag** output:

$$\{x_1, \dots, x_{30}\}$$

$$\{w_1, \dots, w_{30}\}.$$

Computation of the integral using (B.10) followed.

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