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On The Theory of FRESH-DFE and its Application in Digital Cellular Radio Systems

by

Faramarz Hendessi, B. Sc., M. Sc.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for degree of

Doctor of Philosophy

Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University

September, 1994

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the degree of Doctor of Philosophy

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Abstract

Modulated signals are known to be spectrally redundant and this redundancy can be exploited in the design of optimum receiver. This is the main topic of this thesis. It is shown that the structure of such a receiver is system-dependent and related to: channel spacing, signal excess bandwidth, number of available receiver antennas, modulation scheme, and users' symbol rates.

In terms of cyclostationarity, it is shown that modulation schemes can be classified into three categories. Binary Phase-Shift Keying (BPSK), Quadrature Phase-Shift Keying (QPSK), and Off-set QPSK (OQPSK) are representative of these three categories. An optimum receiver can be designed and analyzed for each of these three categories.

A complete analysis of decision feedback equalizer (DFE) in cyclostationary environment is presented. The analysis is based on minimum time averaged mean square error (MTAMSE) criterion. It is shown that the conventional DFE is not an optimum receiver for BPSK and OQPSK signals but is optimum for QPSK signals if and only if all user transmit at an identical symbol rate. Two different structures are suggested for DFE. The first is based on harmonic series representation (HSR) of a cyclostationary random process (CRP) and is called frequency-shift DFE (FRESH-DFE). The second is based on time series representation (TSR) of a CRP and is called generalized DFE (GDFE). It is shown that FRESH-DFE and GDFE are identical in performance, and in the case of "single symbol rate", their structures are also identical. We shall refer to FRESH-DFE (or GDFE) in single symbol rate systems as optimum DFE (ODFE). The results of analysis and simulation presented in this thesis show that the ODFE has a better performance than the conventional DFE, specially in high interference environment.

An optimum multi-antenna receiver is also considered, and it is shown that the bit error rate (BER) can be reduced by increasing the number of antennas. This reduction of BER can be converted into additional capacity if moderate BER is acceptable.

The indoor cellular mobile environment in the presence of high interference and multipath fading is chosen to evaluate the structure, complexity and performance of the optimum receiver. In this evaluation, the co-channel and adjacent channel interference (CCI and ACI) signals are treated separately and it is shown that the effect of ACI can be

removed despite its considerable spectral overlaps with the signal of interest (SOI). It is also demonstrated that the ODFE with the same complexity as the conventional DFE can perform better in multipath fading environment.

The developed concepts were applied to cellular environment. To this end, the thesis studies signal to noise ratio (SNR), signal to interference ratio (SIR) and the capacity of cellular systems. It is shown that the capacity can be increased if moderate BER is chosen as a criterion.

Computer simulation results are presented to support the theoretical results. It is shown that for the same level of complexity, the performance of ODFE is superior to that of the conventional DFE.

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List of Abbreviations and Acronyms

Abbreviations or Acronym	Meaning
ACI	Adjacent-Channel Interference
AM	Amplitude Modulation
ASK	Amplitude-Shift Keying
AWGN	Additive White Gaussian Noise
EB	Equivalent Baseband
BER	Bit-Error Rate
BPSK	Binary Phase-Shift Keying
B.Sc.	Bachelor of Science
CA	Cyclic Autocorrelation
CCA	Conjugate Cyclic Autocorrelation
CCI	Co-Channel Interference
CCSD	Conjugate Cyclic Spectral Density
cdf	cumulative distribution function
CDMA	Code-Division Multiple Access
CIR	Channel Impulse Response
CRP	Cyclostationary Random Process
CSD	Cyclic Spectral Density
dB	decibels
DCRS	Digital Cellular Radio System
DCS	Digital Communication System
DFE	Decision-Feedback Equalizer
DPAM	Differential Pulse Amplitude Modulation
DQPSK	Differential Quadrature Phase Shift Keying
Dr.	Doctor
DSP	Digital Signal Processing
e.g.	example
FDMA	Frequency Division Multiple Access
FIR	Finite Impulse Response
FM	Frequency Modulation

Abbreviations or Acronym	Meaning
FRESH	FRequency SHift
FRESH-DFE	FRequency SHift DFE
FSE	Fractionally Space Equalizer
FSK	Frequency Shift Keying
GDFE	Generalized DFE
GFSE	Generalized FSE
GS	Gear Shifting
HSR	Harmonic-Series Representation
HSR-TVF	HSR-Time Variant Filter
iff	if and only if
iid	independent identically distributed
ISDN	Integrated Services Digital Network
ISI	Inter-Symbol Interference
LCL	Linear-Conjugate-Linear
LCL-FRESH	Linear-Conjugate-Linear FRequency SHift
LMS	Least-Mean Square
LTI	Linear Time Invariant
LTV	Linear Time Variant
MA-DFE	Multi-Antenna DFE
MA-FRESH-DFE	Multi-Antenna FRESH-DFE
M.Sc.	Master of Science
MMSE	Minimum Mean Square Error
MPDP	Multipath Power Delay Profile
MCRS	Mobile Cellular Radio System
MSE	Mean Square Error
MTAMSE	Minimum TAMSE
NLMS	Normalized LMS
ODFE	Optimum DFE
OQPSK	Off-set Quadrature Phase-Shift Keying
OTIF	Optimum Time-Invariant Filters
OTVF	Optimum Time-Variant Filters
pdf	probability density function

Abbreviations or Acronym	Meaning
PAM	Pulse Amplitude Modulation
PPM	Pulse Position Modulation
PSD	Power Spectral Density
PWM	Pulse Width Modulation
QPSK	Quadrature Phase-Shift Keying
RLS	Recursive Least Squares
RMS	Root-Mean-Square
sec.	seconds
SACIR	Signal to ACI Ratio
SCCIR	Signal to CCI Ratio
SHSR-TVF	Sampled HSR-TVF
SIR	Signal to Interference Ratio
SNR	Signal-to Noise Ratio
SINR	Signal to Interference-plus-Noise Ratio
SOI	Signal-Of-Interest
SONI	Signal-Of-No-Interest
SRRC	Square Root Raised Cosine
STDA	Symmetric Version of the Time-Dependent Autocorrelation
STSR-TVF	Sampled TSR-TVF
TAMSE	Time-Average Mean Square Error
TDF	Time-Dependent Filter
TDL	Tapped Delay Line
TDMA	Time Division Multiple Access
TDSA	Time-Dependent Symmetric Autocorrelation
TDSD	Time-Dependent Spectral Density
TIF	Time-Independent Filter
TSR	Time Series Representation
TVF	Time Variant Filter
TSR-TVF	TSR Time Variant Filter
U.C.	University of California
WS	Wide Sense
WSC	Wide Sense Cyclostationary

**Abbreviations
or
Acronym**

Meaning

WSS	Wide Sense Stationary
VLSI	Very Large Scale Integrated

Special Mathematical Notations

Notation	Meaning
π	is the ratio of the circumference of a circle to its diameter.
e	is the base of natural logarithms.
j	is an imaginary number satisfying $j^2 = -1$.
∞	infinity.
$\equiv$	is uniquely equal to.
$\neq$	is not equal to.
$\rightarrow$	tends to.
$\geq$	is greater than or equal to.
$\leq$	is less than or equal to.
$\forall$	for all.
$\delta(t)$	is the notation for Dirac delta function defined by: $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$
$u(t)$	is the notation for step function defined by: $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$
$D_B(f, \alpha)$	is defined by: $u(f + \frac{B - \alpha}{2}) u(-f - \frac{B - \alpha}{2})$
$W_T(t)$	is the ideal lowpass filter with bandwidth $1/T$.
$\lim$	is the limit function.
$\text{Real}(\cdot)$	is the real function.
$\text{Imag}(\cdot)$	is the imaginary function.
$\left[\frac{n}{p} \right]$	is the integer part of the division of n over p .

Notation	Meaning
$[n]_p$	is the remainder of division of n by p .
$[\gamma]_{1/T}$	is the remainder of division of γ by $1/T$.
$\langle x(t) \rangle_t$	is the time average of $x(t)$ defined by: $\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) dt$
$\langle x(n) \rangle_n$	is the time average of $x(n)$ defined by: $\lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N x(n)$
$F(x(t))$	is the Fourier transform of $x(t)$.
$x_c(t)$	is the complex envelope of $x(t)$.
$x_a(t)$	is the analytical signal associated with real signal $x(t)$.
$x_\mu(t)$	is the frequency-shift version of $x_c(t)$: $x_\mu(t) = x_c(t) e^{j2\pi\mu t}$
$\hat{x}(t)$	is the Hilbert transform of $x(t)$.
x^*	is the x conjugate.
$x(t) \otimes y(t)$	is the convolution of $x(t)$ and $y(t)$.
$\underline{x}$	is the vector x .
$\underline{x}^T$	is the transpose of $\underline{x}$.
$\underline{x}^H$	is the conjugate transpose of $\underline{x}$.
$\ \underline{x}\ $	is the norm of vector x defined by: $\sqrt{\underline{x}\underline{x}^H}$
$\tilde{x}(n)$	is the sampled version of $x(t)$.
$\hat{d}(n)$	is the estimated data of $d(n)$.
$E\{x\}$	is the statistic expectation of x .
$m_x(t)$	is the mean of $x(t)$ at the time t .
$\nabla_a^{(n)}(u)$	is the gradient of $y(n)$ according to a at time u .

Notation	Meaning
$H_q^\alpha(f)$	is defined by: $Q(f + \frac{\alpha}{2}) Q^*(f - \frac{\alpha}{2})$ where $Q(f)$ is the Fourier transform of $q(t)$.
$H_{qq^*}^\alpha(f)$	is defined by: $Q(f + \frac{\alpha}{2}) Q(f - \frac{\alpha}{2})$
$S_x^\alpha(f)$	is the CSD of $x(t)$ at frequency f and cyclic frequency α .
$S_{xx^*}^\alpha(f)$	is the CCSD of $x(t)$ at frequency f and cyclic frequency α .
$R_x^\alpha(\tau)$	is the CA of $x(t)$ at time-lag τ and cyclic frequency α .
$\rho_x^\alpha(f)$	is the spectral autocohereence function at frequency f and cyclic frequency α .

Chapter 1

Introduction

1.1 Thesis Motivation

The demand for digital communication services has been rapidly increasing and is expected to continue to increase [16,4]. Digital technology brings many advantages, include inexpensive VLSI/DSP hardware, superior speech quality, reduced transmission noise sensitivity, greater data security and broadband services [4,60]. The high number of users requires more efficient use of the available bandwidth. To accomplish the efficient utilization of the limited bandwidth, researchers are developing low bit rate speech coders [95,101], bandwidth efficient modulation techniques [3,78], multiple access schemes [65], frequency reuse and cell splitting [65]. However, some of these methods cause co-channel, adjacent-channel and inter-symbol interference (CCI, ACI, and ISI) [17]. In general, ACI and CCI limit the system capacity (number of users per unit area) while ISI limits the maximum data rate [2,57]. The presence of these impairments leads to a severe degradation in the performance of a digital communication systems. Since there is a trade-off between these impairments and the system capacity, suitable communication techniques and receiver structures are required to minimize the effects of these impairments and maximize the system capacity.

It is known that increasing receiver knowledge on the properties of the received signals, decreases the error probability. Therefore, for certain value of bit error rate (BER), the capacity of a system can be increased by using any information about the signal-of-interest (SOI) and signal-of-no-interest (SONI) at the receiver.

The following examples illustrate techniques which improve the BER performance by exploiting the detailed structure of the received signal:

- In code division multiple access (CDMA) systems, knowing the users' codes at the receiver decreases the error probability [103].
- Estimating the channel at the receiver decreases the BER [74,75,98].
- Using a channel coding technique increases the redundancy of the transmitted data and lowers BER [20,60].

- Use of multi-antenna (either at the receiver or at transmitter) reduces BER [6,7,102].
- Using time diversity increases the time redundancy of the received signal and consequently increases the knowledge of the receiver about transmitted signal and, thereby, decreases the BER [90].
- Any frequency diversity causes spectral redundancy and, thereby, reduces the effect of channel fading and lowers the BER [84].

The cyclostationary nature of digitally modulated signal [24,26,28,29] is another source of information which produces spectral redundancy and can be used to reduce the error rate or increase the capacity.

Recently much work has been done to design an optimum receiver for cyclostationary signals [30,32,37,47,87,88]. Most of this work considered analog signals. They are too complex to be practical. The motivation behind this thesis is to design an optimum receiver structure for digital communication systems which can exploit the spectral redundancy of digitally modulated signals and suppress the cyclostationary interference.

Another motivation behind this thesis is to design a practical receiver which exploits the spectral redundancy and works in a harsh environment. The mobile cellular radio environment is chosen to evaluate the performance of such a receiver. The reason is that mobile radio channels are inherently time-variant fading multipath transmission media [59,63,84]. Thus, the mobile radio communication is subject to signal dispersion (due to frequency-selectivity) and time-variant signal propagation (due to vehicular motion). These two properties cause ISI and random Doppler fading of the received signal [59,63,66,84].

1.2 Thesis Contributions

The contributions in this thesis in order of appearance are the following.

The main contributions in chapter 3 are:

3.a- *Classifying digitally modulated signals into three categories according to their cyclostationarity.* BPSK, QPSK, and OQPSK are representatives of these three categories.

3.b- *Introducing two parameters to measure the spectral redundancy and spectral*

efficiency of digitally modulated signals. It is shown that there is a trade-off between the spectral redundancy and the spectral efficiency.

The main contributions in chapter 4 are:

4.a- *Analyzing the conventional DFE with the MTAMSE criterion.* It is shown that the conventional DFE is optimum for QPSK single rate systems.

4.b- *Designing and analyzing a generic optimum receiver structure for digitally modulated signals.* The optimum infinite length and finite length receiver are analyzed separately. It is shown that the structure of optimum receiver depends on the following system parameters: channel spacing, signal excess bandwidth, number of available antenna, modulation scheme, and users' symbol rates.

4.c- *Designing and analyzing a multi-antenna optimum receiver.* It is shown that the multi-antenna receiver can achieve very low minimum mean square error (MMSE) if the number of antennas is high.

The main contributions in chapter 5 are:

5.a- *Deriving general formulas for signal to noise ratio (SNR), signal to CCI ratio (SCCIR), signal to ACI ratio (SACIR), and signal to total interference ratio (SIR) in a cellular system.* These formulas are derived by using curve fitting to simulation results.

5.b- *Proposing a new definition for the capacity of a digital cellular radio system in terms of several design parameters.*

5.c- *Designing of a 2-symbol rate cellular system.*

In chapter 6, a computer simulation study of a TDMA/FDM digital cellular radio system (DCRS) applicable to indoor wireless communications has demonstrated that:

6.a- *The performance of FRESH-DFE is superior to that of conventional DFE.*

6.b- *The superiority of FRESH-SFE over conventional DFE increases when higher spectral redundant signals are used.*

6.c- *Multi-antenna receiver can achieve very low BER even though the channel spacing is less than the symbol rate.*

6.d- *The superiority of FRESH-DFE over conventional DFE increases when multi-antenna receiver is used.*

6.e- *The FRESH-DFE and the conventional DFE in 2-symbol rate system perform better when the SOI symbol rate is twice that of the SNOI.*

1.3 Thesis Organization

This thesis describes a receiver and system design based on exploitation of cyclostationary properties of digitally modulated signals. Chapter 2 presents the concepts and basic cyclostationary properties of a random processes. The discrete time cyclostationary random process (CRP) is considered as well as continuous time CRP. Furthermore, the stationary representations of a CRP are considered.

Chapter 3 introduces the cyclostationary nature of digitally modulated signals. It is shown that digitally modulated signals can be classified into three categories. BPSK, QPSK, and OQPSK are used as the representatives of these categories. The spectral redundancy and spectral efficiency of a modulated signal is also considered and two parameters for measuring the spectral redundancy and spectral efficiency are introduced.

The structure of the receiver is designed in chapter 4. The fractionally spaced equalizer (FSE) [8] is considered and developed. It is shown that two different structures, called FREquency SHift FSE (FRESH-FSE) and generalized FSE (GFSE), may be proposed to replace the FSE. On the basis of these structures, two other structures for decision feedback equalizer (DFE), called FRESH-DFE [53] and GDFE, are suggested. In the case of single symbol rate system, it is shown that the structures of FRESH-DFE and GDFE are identical. We refer to this structure as optimum DFE (ODFE). It is also shown that the conventional DFE uses all the spectral redundancy related to the SOI symbol rate while the FRESH-DFE (or GDFE) uses all the spectral redundancy related to all users' symbol rates and carrier frequencies. It is also concluded that the conventional DFE and FRESH-DFE (or GDFE) are equivalent for QPSK single symbol rate systems.

Chapter 5 presents a basic review of mobile cellular radio system (MCRS) wherein the properties of the MCRS channels are reviewed and the channel model used in this thesis is detailed. The frequency reuse and cell splitting are discussed in the context of cellular systems. The interference properties in a MCRS are studied and results which characterizes CCI and ACI in a cellular system are obtained. A new definition for capacity

in terms of several design parameters is introduced and used in determining the overall cellular capacity. Furthermore, a multi-symbol rate system is proposed and it is shown that the impact of CCI is reduced in this system.

Chapter 6 presents the simulation results for investigating the analytical results of chapter 4 and 5. The performance of FRESH-DFE and the conventional DFE are compared. It is shown that FRESH-DFE always has better performance. It is also shown that multi-antenna receiver can achieve low BER even if the channel spacing is smaller than the symbol rate. A 2-symbol rate cellular system is simulated and it is shown that the performance of FRESH-DFE and that of the conventional DFE is better when the SOI symbol rate is twice the SNOI symbol rate. It is also shown that for the same receiver complexity, the ODFE has better performance than the conventional DFE.

Chapter 7 consists of conclusions, discussions and recommendations for further research.

Chapter 2

Back ground :

Cyclostationary Random Process

2.1 Introduction

In this Chapter, the properties of cyclostationary random processes (CRPs) are described. Continuous and discrete time CRPs are considered separately. First, continuous time CRPs are reviewed and the basic functions and definitions applied to these processes are discussed. This is followed by a discussion on sampled (discrete time) CRPs, the basic functions, and definitions applied to these processes.

More details on topics discussed in this chapter are be found in [9,10,15,23 to 42, 44,54,58,62,69,79,86,96,105].

2.2 Continuc us Time CRPs

2.2.1 Basic Functions and Definitions

A continuous time random process, say $x(t)$, is said to be *cyclostationary in the wide sense* if its mean and autocorrelation functions are periodic with a certain period, T : [77]

$$\begin{aligned} m_x(t+T) &= m_x(t) & \forall t \\ R_x(t+T, u+T) &= R_x(t, u) & \forall (t, u) \end{aligned} \quad (2.1)$$

where the mean and autocorrelation are defined as the follows :

$$\begin{aligned} m_x(t) &= E \{ x(t) \} \\ R_x(t, u) &= \frac{1}{2} E \{ x(t) x^*(u) \} \end{aligned} \quad (2.2)$$

where $E\{\bullet\}$ is the expectation function. Equation (2.1) can be re-expressed using the symmetric version of the time-dependent autocorrelation (STDA) function as follows :

$$R_x\left(t + \frac{\tau}{2} + T, t - \frac{\tau}{2} + T\right) = R_x\left(t + \frac{\tau}{2}, t - \frac{\tau}{2}\right) \quad (2.3)$$

Since $R_x(t + \frac{\tau}{2}, t - \frac{\tau}{2})$ is periodic in t with period T for each τ , it can be expressed by the Fourier series representation as :

$$R_x(t + \frac{\tau}{2}, t - \frac{\tau}{2}) = \sum_{\alpha} R_x^{\alpha}(\tau) e^{j2\pi\alpha t} \quad (2.4)$$

where $\{R_x^{\alpha}(\tau)\}$ are the fourier coefficients,

$$R_x^{\alpha}(\tau) = \frac{1}{T} \int_{-T/2}^{T/2} R_x(t + \frac{\tau}{2}, t - \frac{\tau}{2}) e^{-j2\pi\alpha t} dt \quad (2.5)$$

and α ranges over all integer multiples of the fundamental frequency $\frac{1}{T}$.

If a random process contains other cyclostationary processes with different incommensurate periods $\{T_i: i=1, \dots, M\}$ where M is the number of cyclostationary components, it is said to be almost cyclostationary with periods $\{T_i\}$. In this case $R_x^{\alpha}(\tau)$ is defined as :

$$R_x^{\alpha}(\tau) = \lim_{z \rightarrow \infty} \frac{1}{z} \int_{-z/2}^{z/2} R_x(t + \frac{\tau}{2}, t - \frac{\tau}{2}) e^{-j2\pi\alpha t} dt \quad (2.6)$$

where α ranges over all integer multiples of all fundamental frequencies $\{\frac{1}{T_i}\}$. $R_x^{\alpha}(\tau)$ is referred to as the *cyclic autocorrelation (CA)* function, and α is called the *cycle frequency* parameter. Letting $\alpha = 0$ in (2.6) yields the CA at $\alpha = 0$ which is the time average autocorrelation function,

$$R_x^0(\tau) = \langle R_x(t, \tau) \rangle_t \quad (2.7)$$

where $\langle \cdot \rangle_t$ denotes averaging over t .

Similarly, conjugate cyclic autocorrelation (CCA) function is defined as follows :

$$R_{xx^*}^{\alpha}(\tau) = \lim_{z \rightarrow \infty} \frac{1}{z} \int_{-z/2}^{z/2} E \{ x(t + \frac{\tau}{2}) x^*(t - \frac{\tau}{2}) \} e^{-j2\pi\alpha t} dt \quad (2.8)$$

A complex valued time series $x(t)$ is said to exhibit cyclostationarity if either its CA or its CCA is not identically zero for $\alpha \neq 0$.

The time-dependent spectral density (TDSD) function of a random process $x(t)$ can be defined by the Fourier transforms of the STDA,

$$S_x(t, f) = \int_{-\infty}^{\infty} R_x(t + \frac{\tau}{2}, t - \frac{\tau}{2}) e^{-j2\pi f\tau} d\tau \quad (2.9)$$

The representative Fourier coefficients of TDS is given by

$$S_x^\alpha(f) = \lim_{z \rightarrow \infty} \frac{1}{z} \int_{-z/2}^{z/2} S_x(t, f) e^{-j2\pi\alpha t} dt \quad (2.10)$$

and called the *cyclic spectrum* or *cyclic spectral density (CSD)* function. Letting $\alpha = 0$ in (2.10) yields the time average spectral density function,

$$S_x^0(f) = \langle S_x(t, f) \rangle_t \quad (2.11)$$

It is easy to show that CSD and CA functions form a Fourier transform pair [27],

$$S_x^\alpha(f) = \int_{-\infty}^{\infty} R_x^\alpha(\tau) e^{-j2\pi f\tau} d\tau \quad (2.12)$$

Conjugate cyclic spectral density (CCSD) function is defined as the Fourier transform of the CCA,

$$S_{xx}^{\alpha*}(f) = \int_{-\infty}^{\infty} R_{xx}^{\alpha*}(\tau) e^{-j2\pi f\tau} d\tau \quad (2.13)$$

For a real time series random process, the CA and the CSD are equal to the CCA and the CCSD respectively.

The *spectral autocohereence* function for $x(t)$ is defined by

$$\begin{aligned} \rho_x^\alpha(f) &= \frac{S_x^\alpha(f)}{\sqrt{\langle S_x(t, f + \frac{\alpha}{2}) \rangle \langle S_x(t, f - \frac{\alpha}{2}) \rangle}} \\ &= \frac{S_x^\alpha(f)}{\sqrt{S_x^0(f + \frac{\alpha}{2}) S_x^0(f - \frac{\alpha}{2})}} \end{aligned} \quad (2.14)$$

where we have used $\langle S_x(t, f + \frac{\alpha}{2}) \rangle = S_x^0(f + \frac{\alpha}{2})$ and $\langle S_x(t, f - \frac{\alpha}{2}) \rangle = S_x^0(f - \frac{\alpha}{2})$. It is easy to show that $\rho_x^\alpha(f)$ is the time-averaged correlation coefficient for the two frequency

components at frequencies $f + \frac{\alpha}{2}$ and $f - \frac{\alpha}{2}$ [27]. A process $x(t)$ is said to be completely coherent at spectral frequency f and cycle frequency α if and only if (iff) the spectral auto-coherence magnitude is unity, $|\rho_x^\alpha(f)| = 1$. Also a process $x(t)$ is said to be completely incoherent at spectral frequency f and cycle frequency α iff the spectral auto-coherence is zero, $\rho_x^\alpha(f) = 0$. It is conceptually (also from (2.14)) apparent that $\rho_x^0(f) = 1$ for any process $x(t)$ and any f .

2.2.2 The Analytic Signal and Complex Envelope

The *analytic signal* associated with a real waveform $x(t)$ is defined according to :

$$x_a(t) = x(t) + j\hat{x}(t) = \int \gamma(u) x(t-u) du \quad (2.15)$$

where $j = \sqrt{-1}$, $\hat{x}(t)$ is the Hilbert transform of $x(t)$ and $\gamma(t) = \delta(t) + j\frac{1}{\pi t}$. It can be shown that the Fourier transform of $\gamma(t)$ is $F[\gamma(t)] = \Gamma(f) = 2u(f)$, where $u(f)$ is the step function, and therefore $x_a(t)$ has strictly positive frequencies. The CSD and CCSD of the analytic signal $x_a(t)$ can be found from the CSD of $x(t)$ according to : [10]

$$\begin{aligned} S_{x_a}^\alpha(f) &= 4u\left(f + \frac{\alpha}{2}\right)u\left(f - \frac{\alpha}{2}\right)S_x^\alpha(f) \\ S_{x_a x_a^*}^\alpha(f) &= 4u\left(f + \frac{\alpha}{2}\right)u\left(\frac{\alpha}{2} - f\right)S_x^\alpha(f) \end{aligned} \quad (2.16)$$

Inversely, the CSD and CCSD of $x(t)$ can be found from its analytic signal according to : [10]

$$S_x^\alpha(f) = \frac{1}{4} \left[S_{x_a}^\alpha(f) + S_{x_a}^\alpha(-f) + S_{x_a x_a^*}^\alpha(f) + S_{x_a x_a^*}^{-\alpha}(f)^* \right] \quad (2.17)$$

The *complex envelope* of $x(t)$ relative to f_0 is defined as the down-shifted analytic signal :

$$x_c(t) = x_a(t) e^{-j2\pi f_0 t} \quad (2.18)$$

If $x(t)$ is bandlimited to $||f| - f_0| < B/2$, the SCD of the complex signal $x_c(t)$ can be found from the SCD of the analytic signal according to : [10]

$$\begin{aligned} S_{x_c}^\alpha(f) &= S_{x_a}^\alpha(f + f_0) \\ S_{x_c x_c^*}^\alpha(f) &= S_{x_a x_a^*}^{\alpha + 2f_0}(f) \end{aligned} \quad (2.19)$$

Substituting (2.16) in (2.19) yields

$$\begin{aligned} S_{x_c}^{\alpha}(f) &= 4D_B(f, \alpha) S_x^{\alpha}(f+f_0) \\ S_{x_c x_c^*}^{\alpha}(f) &= 4D_B(f, \alpha) S_x^{\alpha+2f_0}(f) \end{aligned} \quad (2.20)$$

where

$$D_B(f, \alpha) = u\left(f + \frac{B-\alpha}{2}\right) u\left(-f - \frac{B-\alpha}{2}\right) \quad (2.21)$$

inversely, the SCD of the real signal $x(t)$ can be found from its complex envelope according to : [10]

$$S_x^{\alpha}(f) = \frac{1}{4} \left[S_{x_c}^{\alpha}(f-f_0) + S_{x_c}^{\alpha}(-f-f_0) + S_{x_c x_c^*}^{\alpha-2f_0}(f) + S_{x_c x_c^*}^{\alpha-2f_0}(f)^* \right] \quad (2.22)$$

2.2.3 Some Examples

2.2.3.1 An Example of a Real Signal

As an example of a real signal, an amplitude modulating (AM) signal is considered. Let $y(t)$ be a zero-mean wide sense stationary (WSS) process with autocorrelation function $R_y(\tau)$ and spectral density function $S_y(f)$. Assume that $x(t)$ is AM modulated by $y(t)$. The carrier frequency is f_0 and the initial phase is ϕ_0 .

$$x(t) = \sqrt{2}y(t) \cos(2\pi f_0 t + \phi_0) \quad (2.23)$$

STDA, TDSD, CA, and CSD functions can be calculated using (2.2), (2.9), (2.5) and (2.12) as follows :

$$\begin{aligned} R_x\left(t + \frac{\tau}{2}, t - \frac{\tau}{2}\right) &= R_y(\tau) (\cos(2\pi f_0 \tau) + \cos(4\pi f_0 t + 2\phi_0)) \\ S_x(t, f) &= \frac{1}{2} (S_y(f-f_0) + S_y(f+f_0)) + \cos(4\pi f_0 t + 2\phi_0) S_y(f) \\ R_x^{\alpha}(\tau) &= R_y(\tau) (\cos(2\pi f_0 \tau) \delta(\alpha) + \frac{1}{2} (e^{j2\phi_0} \delta(\alpha - 2f_0) + e^{-j2\phi_0} \delta(\alpha + 2f_0))) \quad (2.24) \\ S_x^{\alpha}(f) &= \frac{1}{2} (S_y(f-f_0) + S_y(f+f_0)) \delta(\alpha) + \\ &\quad \frac{1}{2} S_y(f) (e^{j2\phi_0} \delta(\alpha - 2f_0) + e^{-j2\phi_0} \delta(\alpha + 2f_0)) \end{aligned}$$

Figure 2.1 shows STDA, TDSD, CA, and CSD functions from (2.24) for

Figure 2.1 (a) The Symmetric version of the Time-Dependent Autocorrelation (STDA), (b) The Time-Dependent Spectral Density (TDSD), (c) The Cyclic Autocorrelation (CA) and (d) The Cyclic Spectral Density (CSD) of the Amplitude Modulation (AM) example.

$$R_y(\tau) = \begin{cases} 1 - |\tau| & |\tau| \leq 1 \\ 0 & \text{elsewhere} \end{cases} \quad (2.25)$$

and $f_0=10$. It is apparent from the figure and (2.24) that

$$S_x^{\pm 2f_0}(f) = \frac{1}{2} S_y(f) e^{\pm j2\phi_0} \quad (2.26)$$

$$S_x^0(f) = \langle S_x(t, f) \rangle_t = \frac{1}{2} (S_y(f-f_0) + S_y(f+f_0))$$

This example clearly shows that the frequency representation of a signal can be viewed as a (complex) height of a surface above the bifrequency (f, α) plane. Therefore some information about the signal can be found at $\alpha \neq 0$ as well as at $\alpha = 0$.

For this example,

$$\rho_x^\alpha(f) = \begin{cases} 1 & \alpha = 0, \pm 2f_0 \quad \forall f \\ 0 & \text{elsewhere} \end{cases} \quad (2.27)$$

Therefore, an AM signal is completely coherent at $\alpha = 0, \pm 2f_0$ and for all values of f .

2.2.3.2 An Example of a Complex Signal

We consider now the complex envelope of the AM signal relative to an arbitrary frequency f_1 . If $x_a(t)$ is the analytic signal and $x_c(t)$ is the complex envelope relative to the frequency f_1 , the following relations exist :

$$x_a(t) = \sqrt{2}y(t) e^{j(2\pi f_0 t + \phi_0)}$$

$$x_c(t) = x_a(t) e^{-j2\pi f_1 t} = \sqrt{2}y(t) e^{j(2\pi \Delta f t + \phi_0)} \quad (2.28)$$

$$x(t) = \text{Real} \{ x_a(t) \} = \text{Real} \{ x_c(t) e^{j2\pi f_1 t} \}$$

where $\Delta f = f_0 - f_1$. In this case, the CSD and the CCSD of $x_a(t)$ and $x_c(t)$ can be found as follows :

$$S_{x_a}^\alpha(f) = 2S_y(f-f_0) \delta(\alpha)$$

$$S_{x_c}^\alpha(f) = 2S_y(\alpha - \Delta f) \delta(\alpha)$$

$$S_{x_a x_a^*}^\alpha(f) = 2S_y(f) e^{j2\phi_0} \delta(\alpha - 2f_0) \quad (2.29)$$

$$S_{x_c x_c^*}^\alpha(f) = 2S_y(f) e^{j2\phi_0} \delta(\alpha - 2\Delta f)$$

The CSD of $x(t)$ can be found from (2.17) (or (2.22)) and (2.29) and the result will be the same as (2.24). If $\Delta f=0$, $x_c(t)$ is called the equivalent baseband (EB) of $x(t)$. In this example, although the EB of $x(t)$ is not a CRP, $x(t)$ is a CRP. This shows that the EB of a signal contains all the information of the signal including its cyclic properties.

2.2.4 Support Area

It follows from the fact that the magnitude of spectral auto coherence function in (2.14) is bounded by unity, the following inequality exists for any f and α ,

$$|S_x^\alpha(f)| \leq \sqrt{\langle S_x(t, f + \frac{\alpha}{2}) \rangle \langle S_x(t, f - \frac{\alpha}{2}) \rangle} \quad (2.30)$$

If $x(t)$ is bandlimited so that

$$\langle S_x(t, f) \rangle_t \equiv 0 \quad \text{for} \quad |f| \leq b \quad \text{or} \quad |f| \geq B \quad (2.31)$$

where $b < B$, then (2.30) reveals that the cyclic spectrum is constrained to satisfy

$$S_x^\alpha(f) \equiv 0 \quad \left\{ \begin{array}{l} \text{for} \quad (|f| \leq b + \frac{|\alpha|}{2} \quad \text{and} \quad \frac{|\alpha|}{2} \geq b + f) \\ \text{or} \quad |f| \geq B - \frac{|\alpha|}{2} \end{array} \right. \quad (2.32)$$

The area of bifrequency (f, α) plane where CSD (or CCSD) is not uniquely zero is called CSD (or CCSD) support area. Figure 2.2 shows this area for different kinds of mean-square bandlimited processes and Figure 2.3 shows the support area for the analytic signal.

2.2.5 Linear Filtering a CRP

2.2.5.1 Time Invariant Filtering

If a CRP, $x(t)$, is filtered by a time invariant linear filter, $h(t)$, its CSD is filtered along the f and α axis. Figure 2.4 shows an example of this filtering. In this example, a CRP is filtered by a linear time-invariant ideal bandpass filter. The CSD of the output is given by : [27,10]

$$S_y^\alpha(f) = H(f + \frac{\alpha}{2}) S_x^\alpha(f) H^*(f - \frac{\alpha}{2}) \quad (2.33)$$

Defining

Figure 2.2 Support area (shaded) for (a) Lowpass CRP (b) Bandpass CRP (c) Highpass CRP.

Figure 2.3 Support area for the analytic signal.

Figure 2.4 Filtering a CRP by an ideal bandpass filter.

Figure 2.5 The Equivalent Baseband (EB) of the figure 2.4.

$$H_h^\alpha(f) = H(f + \frac{\alpha}{2}) H^*(f - \frac{\alpha}{2}) \quad (2.34)$$

and substituting it in (2.33) yields :

$$S_y^\alpha(f) = S_x^\alpha(f) H_h^\alpha(f) \quad (2.35)$$

We refer to $H_h^\alpha(f)$ as *the cyclic filter*. The EB of the cyclic filter is defined based on the EB of the filter by the following equation :

$$\begin{aligned} H_{h_c}^\alpha(f) &= H_c(f + \frac{\alpha}{2}) H_c^*(f - \frac{\alpha}{2}) \\ H_{h_c h_c^*}^\alpha(f) &= H_c(f + \frac{\alpha}{2}) H_c(-f + \frac{\alpha}{2}) \end{aligned} \quad (2.36)$$

Where $H_c(f)$ is the fourier transform of the EB of the filter. Then, the CSD and CCSD of the EB of the output signal $y(t)$ can be found from the EB of the input signal $x(t)$ according to :

$$\begin{aligned} S_{y_c}^\alpha(f) &= S_{x_c}^\alpha(f) H_{h_c}^\alpha(f) \\ S_{y_c y_c^*}^\alpha(f) &= S_{x_c x_c^*}^\alpha(f) H_{h_c h_c^*}^\alpha(f) \end{aligned} \quad (2.37)$$

Figure 2.5 shows the EB filtering of Figure 2.4.

2.2.5.2 Time Variant Filtering

A time variant filtering of CRP $x(t)$ can be expressed by the convolution equation as :

$$y(t) = \int_{-\infty}^{\infty} h(t, u) x(u) du \quad (2.38)$$

If $h(t, u)$ is periodic (or almost periodic) with period T_h , it can be expressed by its Fourier series as follows :

$$h(t, u) = \sum_{\mu} g_{\mu}(t - u) e^{j2\pi\mu u} \quad (2.39)$$

Substituting (2.39) in (2.38) yields :

$$\begin{aligned} y(t) &= \sum_{\mu} \int_{-\infty}^{\infty} g_{\mu}(t - u) x(u) e^{j2\pi\mu u} du \\ &= \sum_{\mu} g_{\mu}(t) \otimes x_{\mu}(t) \end{aligned} \quad (2.40)$$

where $\otimes$ denotes convolution and $x_\mu(u) = x_c(u) e^{j2\pi\mu u}$ is the *frequency-shifted* version of $x_c(t)$. The CSD of the output signal can be found according to : [10,87,88,89]

$$S_y^\alpha(f) = \sum_{\lambda, \mu} G_\mu(f + \frac{\alpha}{2}) S_x^{\alpha+\lambda-\mu}(f - \frac{\mu+\lambda}{2}) G_\lambda(f - \frac{\alpha}{2})^* \quad (2.41)$$

where $G_\mu(f)$ is the Fourier transform of the $g_\mu(t)$.

Equation (2.41) shows that if the CSD of $x(t)$ is uniquely zero for $\alpha \neq 0$, the CSD of $y(t)$ is not uniquely zero for $\alpha \neq 0$. Thus, if a stationary random process passes through a periodic filter, the output will be a CRP with the periodicity of the filter.

Generally, the EB of (2.38) is given according to the following equation called *linear-conjugate-linear* (LCL) filtering : [10,11,12,32,37]

$$\begin{aligned} y_c(t) &= \sum_{\mu} \int_{-\infty}^{\infty} a_\mu(t-u) x_c(u) e^{j2\pi\mu u} du + \sum_{\beta} \int_{-\infty}^{\infty} b_\beta(t-u) x_c^*(u) e^{j2\pi\beta u} du \\ &= \sum_{\mu} a_\mu(t) \otimes x_\mu(t) + \sum_{\beta} b_\beta(t) \otimes x_{-\beta}^*(t) \end{aligned} \quad (2.42)$$

where $x_{-\beta}^*(u) = x_c^*(u) e^{j2\pi\beta u}$ is the *frequency-shifted* version of $x_c^*(t)$ and the Fourier transforms of $a_\mu(t)$ and $b_\beta(t)$ are given by :

$$\begin{aligned} A_\mu(f) &= \frac{1}{2} \Gamma(f+f_0) G_\mu(f+f_0) \\ B_\beta(f) &= \frac{1}{2} \Gamma(f+f_0) G_{\mu+2f_0}(f+f_0) \end{aligned} \quad (2.43)$$

2.2.6 Stationary Representation

2.2.6.1 Real Signal

A continuous time cyclostationary random process $x(t)$ with period T can be represented by a set of jointly stationary processes $\{A_p(t) = 0, \pm 1, \pm 2, \dots\}$ as follows :

$$x(t) = \sum_{p=-\infty}^{\infty} A_p(t) e^{j2\pi p \frac{t}{T}} \quad (2.44)$$

where, $A_p(t)$ is the p th stationary component, and can be found from $x(t)$ by : [23,27]

$$A_p(t) = \left[x(t) e^{-j2\pi p \frac{t}{T}} \right] \otimes w_T(t) \quad (2.45)$$

where $w_T(t)$ is the ideal lowpass filter with bandwidth $1/T$. This representation is referred to as the *harmonic-series representation* (HSR) [26]. Figure 2.6 shows how stationary components of a CRP can be separated. From (2.44) the TDA can be easily calculated as :

$$R_x(t, u) = \sum_{p, q = -\infty}^{\infty} R_{pq}(t - u) e^{j2\pi(pu - qu)/T} \quad (2.46)$$

where

$$R_{pq}(t - u) = E \{ A_p(t) A_q^*(u) \} \quad (2.47)$$

Therefore CA can be found from its stationary components as :

$$R_x^{n/T}(\tau) = \sum_{p = -\infty}^{\infty} R_{p(p-n)}(\tau) e^{j\pi(2p-n)\tau/T} \quad (2.48)$$

For the AM example, $T=1/f_0$ and the stationary components are :

$$A_{\pm 1}(t) = \frac{\sqrt{2}}{2} y(t) e^{\pm j\phi_0} \quad (2.49)$$

Using (2.48) yields the following CA for this AM signal :

$$\begin{aligned} R_x^0(\tau) &= \frac{1}{2} R_y(\tau) \left(e^{j2\pi \frac{\tau}{T}} + e^{-j2\pi \frac{\tau}{T}} \right) \\ R_x^{\pm 2/T}(\tau) &= \frac{1}{2} R_y(\tau) e^{\pm j2\phi_0} \end{aligned} \quad (2.50)$$

which is the same result as (2.24).

2.2.6.2 Analytical Signal and Complex Envelope

The analytical signal associated with $x(t)$ can be represented by a set of jointly stationary processes $\{B_p(t) : p = 0, \pm 1, \pm 2, \dots\}$ as follows :

$$x_a(t) = \sum_{p = -\infty}^{\infty} B_p(t) e^{j2\pi p \frac{t}{T}} \quad (2.51)$$

Figure 2.6 The HSR of a CRP.

The relation between $A_p(t)$ and $B_p(t)$ can be found by substituting (2.44) in (2.15) :

$$\begin{aligned} x_a(t) &= \int \gamma(u) x(t-u) du = \int \gamma(u) \sum_{p=-\infty}^{\infty} A_p(t-u) e^{j2\pi p \frac{t-u}{T}} du \\ &= \sum_{p=-\infty}^{\infty} \left[\int A_p(t-u) \gamma(u) e^{-j2\pi p \frac{u}{T}} du \right] e^{j2\pi p \frac{t}{T}} \end{aligned} \quad (2.52)$$

$$B_p(t) = \int A_p(t-u) \gamma(u) e^{-j2\pi p \frac{u}{T}} du = A_p(t) \otimes \gamma_{-p}(t) \quad (2.53)$$

The stationary representation of the complex envelope of $x(t)$ relative to f_0 can be found by substituting (2.18) in (2.51) :

$$\begin{aligned} x_c(t) &= x_a(t) e^{-j2\pi f_0 t} = \sum_{p=-\infty}^{\infty} [B_p(t) e^{-j2\pi f_0 t}] e^{j2\pi p \frac{t}{T}} \\ x_c(t) &= \sum_{p=-\infty}^{\infty} C_p(t) e^{j2\pi p \frac{t}{T}} \quad \text{where} \quad C_p(t) = B_p(t) e^{-j2\pi f_0 t} \end{aligned} \quad (2.54)$$

In the example 2.2.3.2 the stationary components of the analytic and the envelope signals are :

$$\begin{aligned} B_1(t) &= \sqrt{2} y(t) e^{j\phi_0} \\ C_1(t) &= \sqrt{2} y(t) e^{-j2\pi f_1 t} e^{j\phi_0} \end{aligned} \quad (2.55)$$

2.3 Discrete Time CRPs

A discrete random process can be assumed to be a sampled version of a continuous time random process. If a continuous time CRP (say $x(t)$) with periodicity T is sampled at every T_s , the result will be a discrete time CRP (say $\tilde{x}(n) = x(nT_s)$) with periodicity $\lceil T/T_s \rceil$ (where $\lceil \bullet \rceil$ is the integer value of its argument). Since $\tilde{x}(n/2)$ does not exist for odd integer n , the definition of the symmetric version of the CA (equation (2.6)) can not be directly extended to the discrete time CRP. Therefore, the CA for the discrete time CRP $\tilde{x}(n)$ is defined by :

$$R_x^\alpha(k) = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{-N}^N R_{\tilde{x}}(n+k, n) e^{-j2\pi \alpha (n+k/2) T_s} \quad (2.56)$$

the CSD of a discrete time CRP is defined by the Fourier transform of its CA,

$$S_x^\alpha(f) = \sum_{k=-\infty}^{\infty} R_x^\alpha(k) e^{-j2\pi kT_s f} \quad (2.57)$$

Applying the following equation (called the synchronized averaging identity [27]) to the definition (2.56) and using the fact that $\tilde{x}(n) = x(nT_s)$,

$$\lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{-N}^N R_x(t+nT_s, u+nT_s) \equiv \sum_{m=-\infty}^{\infty} \lim_{z \rightarrow \infty} \frac{1}{z} \int_{-z/2}^{z/2} R_x(t+v, u+v) e^{-j2\pi m v/T_s} dv \quad (2.58)$$

yields the following relationship between the CA of a continuous time CRP and the CA of its sampled version,

$$R_x^\alpha(k) = \sum_{m=-\infty}^{\infty} R_x^{\alpha+m/T_s}(kT_s) e^{j\pi m k} \quad (2.59)$$

Taking the Fourier transform of the above equation yields :

$$S_x^\alpha(f) = \frac{1}{T_s} \sum_{n, m=-\infty}^{\infty} S_x^{\alpha+m/T_s}(f - \frac{m}{2T_s} - \frac{n}{T_s}) \quad (2.60)$$

The above equation shows that $S_x^\alpha(f)$ exhibits the following periodicity properties :

$$S_x^\alpha(f) = S_x^\alpha(f + \frac{1}{T_s}) = S_x^{\alpha+2/T_s}(f) = S_x^{\alpha+1/T_s}(f - \frac{1}{2T_s}) \quad (2.61)$$

This periodicity properties are called *aliasing effects* in α and f [27]. Due to this aliasing effects, the CSD of $x(t)$ is, in general, not obtainable from the CSD of $\tilde{x}(n)$. However, if $x(n)$ has been sampled at a rate higher than the Nyquist rate, its CSD can be recovered from the CSD of sampled version by : [27]

$$\begin{aligned} S_x^\alpha(f) &= S_x^\alpha(f) & |f| < \frac{1}{2} \left[\frac{1}{T_s} - |\alpha| \right] \\ S_x^\alpha(f) &= 0 & |f| \geq \frac{1}{2} \left[\frac{1}{T_s} - |\alpha| \right] \end{aligned} \quad (2.62)$$

Even though in this case, the CA of $x(n)$ can not be obtained from the CSD of the sampled version by the inverse Fourier transform over the bandwidth. In this case, the inversion formula needed is :

$$R_x^\alpha(f) = \int_{-B+|\alpha|/2}^{B-|\alpha|/2} S_x^\alpha(f) e^{j2\pi f t} df \quad (2.63)$$

where $-B$ to B is the bandwidth of $x(t)$. However, if $x(n)$ has been sampled at twice the Nyquist rate, its CA can be recovered from the CSD of sampled version by simply applying the inverse Fourier transform over the bandwidth [27].

2.3.1 Stationary Representation

A discrete time CRP $\tilde{x}(n)$ with period P can be represented by a set of jointly stationary processes $\{Z_k(m) : k = 0, 1, 2, \dots, P-1\}$ as follows : [27]

$$\tilde{x}(n) = \sum_{k=0}^{P-1} Z_k\left(\left\lceil \frac{n}{P} \right\rceil\right) \delta([n]_P - k) \quad (2.64)$$

where $[n]_P$ is the remainder of n divided by P and $Z_k(m)$, called *kth stationary component*, can be found from $\tilde{x}(n)$ by :

$$Z_k(m) = \tilde{x}(mP + k) \quad (2.65)$$

This representation is referred to as the *time-series representation* (TSR) [27]. Simply, each member of this representation is a subsequence of $\tilde{x}(n)$ obtained by sampling $\tilde{x}(n)$ once per period. Figure 2.7 shows how the stationary components of a cyclostationary discrete signal can be provided from the signal. If we are interested only in one set of the stationary components, the switch in Figure 2.7 can be replaced by a sample and hold (Figure 2.8). In this case, the phase of the sampler (initial sampling time) indicates which set of stationary components has been taken.

2.4 Conclusion

In this chapter, the cyclostationary properties of a CRP were presented. The basic definitions for continuous and discrete time CRP were reviewed. Two stationary representation for CRP were detailed. Real CRPs and their complex envelopes were studied and it was found that all cyclic spectra of a real signal can be found from its complex envelope cyclic spectra.

Figure 2.7 Separation of the stationary components of a CRP.

Figure 2.8 Stationary component of interest.

Chapter 3

Cyclic Spectra of Digitally Modulated Signals

3.1 Introduction

In this Chapter, the cyclostationarity of digitally modulated signals are considered. It is shown that the cyclostationary properties of all digital modulation schemes can be classified into three categories. Binary phase shift keying (BPSK), quaternary PSK (QPSK) and Off-set QPSK (OQPSK) are chosen to represent these categories. The spectral redundancy and spectral efficiency of a digital modulated signal are studied and two measurement parameters for measuring the spectral redundancy and spectral efficiency are introduced.

3.2 Digital Modulation Schemes

Digital modulation techniques may be classified into either carrier modulation or non-carrier modulation. Both carrier and non-carrier modulation schemes are sub-divided into two techniques, differential modulation and non-differential modulation. In non-differential techniques, data itself is modulated while in differential techniques, a combination of data is modulated. Differentially modulation techniques can be further divided into shifted and non-shifted techniques. In shifted techniques, the signal constellation is shifted by some phase during the modulation while in non-shifted techniques, the constellation remain the same. Figure 3.1 shows an overview of digital modulation techniques.

3.2.1 Non-Carrier Modulation Schemes

In non-carrier digital communication systems, a sequence of data is carried by a signal pulse which is shaped to avoid ISI. Generally a pulse modulated signal can be represented by the following formula :

$$x(t) = \sum_{n=-\infty}^{\infty} \sum_{m=1}^M \delta_m(n) q_m(t - nT) \quad (3.1)$$

where M is the alphabet size. $\{q_1(t), q_2(t), \dots, q_M(t)\}$ are the pulse shapes, T is the

Figure 3.1 Digital modulation schemes.

symbol rate and $\delta_m(n)$ is defined as follows :

$$\delta_m(n) = \begin{cases} 1 & \text{if } m\text{th symbol is generated in time } nT \text{ (} d(n) = m\text{th symbol)} \\ 0 & \text{otherwise} \end{cases} \quad (3.2)$$

The data sequences $\{d(n)\}$ are assumed to be real and uncorrelated in the rest of this thesis. It is shown in [29] that the CSD of $y(t)$ in this case is :

$$S_x^\alpha(f) = \begin{cases} \frac{1}{MT} \sum_{m=1}^M \sigma_d^2 Q_m(f + \frac{\alpha}{2}) Q_m^*(f - \frac{\alpha}{2}) - \frac{j}{M^2 T} \left[\sum_{m=1}^M Q_m^*(f - \frac{\alpha}{2}) \right] \\ \cdot \left[\sum_{m=1}^M Q_m(f + \frac{\alpha}{2}) \right] \left[1 - \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta(f + \frac{\alpha}{2} - \frac{n}{T}) \right], & \alpha = \frac{k}{T} \\ 0 & \alpha \neq \frac{k}{T} \end{cases} \quad (3.3)$$

Equation (3.3) indicates that the CSD of the signal can be nonzero only at cyclic frequencies equal to multiple of the symbol rate. As an example of non-carrier modulated signals, binary pulse amplitude modulated signals is detailed.

3.2.1.1 Binary Pulse Amplitude Modulation (Binary PAM)

In Binary PAM signal

$$q_m(t) = \begin{cases} q(t) & m = 1 \\ -q(t) & m = 2 \end{cases} \quad (3.4)$$

therefore, a PAM signal can be represented by the following equation :

$$x(t) = \sum_{n=-\infty}^{\infty} d(n) q(t - nT - t_0) \quad (3.5)$$

where $\{d(n)\}$ takes values ± 1 , $q(t)$ is the pulse shape, t_0 is the initial time of $q(t)$ and T is the symbol time. In this case (3.3) is reduced to the following equation :

$$\begin{aligned} S_x^\alpha(f) &= \frac{\sigma_d^2}{T} Q(f + \frac{\alpha}{2}) Q^*(f - \frac{\alpha}{2}) e^{-j2\pi\alpha t_0} \delta(\alpha - \frac{k}{T}) \\ &= \frac{\sigma_d^2}{T} H_q^\alpha(f) e^{-j2\pi\alpha t_0} \delta(\alpha - \frac{k}{T}) \end{aligned} \quad (3.6)$$

where $Q(f)$ is the Fourier transform of $q(t)$, $\sigma_{d_m}^2$ is the variance of the data and k is an integer number. As it is apparent from (3.6), the CSD of $x(t)$ is not equal to zero for $\alpha=m/T$ iff $H_q^{m/T}(f) \neq 0$. On the other hand, $H_q^{m/T}(f) \neq 0$ iff the excess bandwidth of $q(t)$ is greater than $(m-1)100$ percent [28]. Therefore for having a BPAM signal with nonzero CSD at $\alpha=m/T$, a pulse shape with an excess bandwidth larger than $(m-1)100$ percent has to be used. For example, to have a CSD with cyclic spectrum $1/T$, the excess bandwidth has to be greater than 0. Therefore, PAM with zero excess bandwidth pulse shape is stationary.

3.2.2 Carrier Modulation

A carrier modulated signal is either a bandpass version of a non-carrier modulated signal or else it is a frequency modulated (e.g. FSK). The frequency modulation techniques are not considered in this thesis. However, it is shown in [29] that these modulation schemes also generate nonzero CSD at multiple of the symbol rate.

Assuming binary data, a general form of linear carrier modulation schemes can be expressed with the following equation :

$$x(t) = \sum_{n=-\infty}^{\infty} d_r(n) q(t - nT - t_r) \cos(2\pi f_0 t + \phi_r) - \sum_{n=-\infty}^{\infty} d_i(n) p(t - nT - t_i) \sin(2\pi f_0 t + \phi_i) \quad (3.7)$$

where $p(t)$ and $q(t)$ are the pulse shapes of the inphase and quadrature components, respectively. The CSD and CCSD of the complex envelope of the modulated signal relative to the frequency f_1 is given by the following equation :

$$S_{x_c}^{\alpha}(f) = \left(\frac{\sigma_{d_r}^2}{T} H_q^{\alpha}(f - \Delta f) e^{-j2\pi\alpha t_r} + \frac{\sigma_{d_i}^2}{T} H_p^{\alpha}(f - \Delta f) e^{-j2\pi\alpha t_i} \right) \delta\left(\alpha - \frac{k}{T}\right) \\ S_{x_c x_c^*}^{\alpha}(f) = \left(\frac{\sigma_{d_r}^2}{T} H_{qq^*}^{\alpha}(f) e^{-j(2\pi(\alpha - 2\Delta f)t_r - 2\phi_r)} - \frac{\sigma_{d_i}^2}{T} H_{pp^*}^{\alpha}(f) e^{-j(2\pi(\alpha - 2\Delta f)t_i - 2\phi_i)} \right) \delta\left(\alpha - 2\Delta f + \frac{k}{T}\right) \quad (3.8)$$

where $\Delta f = f_0 - f_1$ and is referred to as the complex carrier. This class of signals can be divided into many subgroups depending upon the relationship between $p(t)$, $q(t)$, t_i , t_r , ϕ_r , ϕ_i , $\sigma_{d_r}^2$ and $\sigma_{d_i}^2$. However all these subgroups can be categorized into three categories. In

the first category, the CSD can be non-zero at cyclic frequencies equal to plus and minus the harmonics of the symbol rate and the CCSD can be non-zero at cyclic frequencies equal to twice the complex carrier plus and minus the harmonics of the symbol rate. BPSK is an example of this category. In the second category, the CSD can be nonzero only at cyclic frequencies equal to plus and minus the even harmonics of the symbol rate while the CCSD can be nonzero at cyclic frequencies equal to twice the complex envelope carrier plus and minus the odd harmonics of the symbol rate. OQPSK is an example of these signals. In the third category, the CSD can be non-zero at cyclic frequencies equal to plus and minus the harmonics of the symbol rate while the CCSD is uniquely zero. QPSK is an example of these signals. BPSK, OQPSK and QPSK are considered in details to illustrate these categories.

3.2.2.1 BPSK

A BPSK signal can be considered as a binary PAM modulated by an AM modulator :

$$x(t) = \sum_{n=-\infty}^{\infty} d(n) q(t - nT - t_0) \cos(2\pi f_0 t + \phi_0) \quad (3.9)$$

The CSD and CCSD of the complex envelope of the BPSK signal relative to the frequency f_1 is :

$$\begin{aligned} S_{x_c}^{\alpha}(f) &= \frac{\sigma_d^2}{T} H_q^{\alpha}(f - \Delta f) e^{-j2\pi\alpha t_0} \delta\left(\alpha - \frac{k}{T}\right) \\ S_{x_c x_c^*}^{\alpha}(f) &= \frac{\sigma_d^2}{T} H_{qq^*}^{\alpha}(f) e^{-j(2\pi(\alpha - 2\Delta f)t_0 - 2\phi_0)} \delta\left(\alpha - 2\Delta f + \frac{k}{T}\right) \end{aligned} \quad (3.10)$$

It is apparent from (3.10) that the CSD is nonzero for α equals to plus and minus the symbol rate while the CCSD includes α equals to twice the complex envelope carrier plus and minus the symbol rate and its harmonics.

3.2.2.2 QPSK

In balanced QPSK signals the following relationship exist :

$$P(t) = q(t), \quad t_i = t_r, \quad \phi_i = \phi_r, \quad \sigma_{d_i}^2 = \sigma_{d_r}^2 \quad (3.11)$$

In this case, (3.8) is simplified to :

$$S_{x_r}^{\alpha}(f) = \left(\frac{2\sigma_{d_r}^2}{T} H_q^{\alpha}(f - \Delta f) e^{-j2\pi\alpha t_r} \right) \delta\left(\alpha - \frac{k}{T}\right) \quad (3.12)$$

$$S_{x_r, x_r^*}^{\alpha}(f) \equiv 0$$

It is apparent from (3.12) that the CCSD of a balanced QPSK does not contain any cyclic spectrum related to the carrier frequency.

3.2.2.3 OQPSK

In OQPSK, the time delays t_i and t_r are off from each other by half the symbol rate. The following relationship exists in this case :

$$P(t) = q(t), \quad t_i = t_r + \frac{T}{2}, \quad \phi_i = \phi_r, \quad \sigma_{d_i}^2 = \sigma_{d_r}^2 \quad (3.13)$$

Therefore (3.8) is simplified to :

$$\begin{aligned} S_{x_r}^{\alpha}(f) &= \left(\frac{\sigma_{d_r}^2 (1 + e^{-j\pi\alpha k T})}{T} H_q^{\alpha}(f - \Delta f) e^{-j2\pi\alpha t_r} \right) \delta\left(\alpha - \frac{k}{T}\right) \\ &= \left(\frac{2\sigma_{d_r}^2}{T} H_q^{\alpha}(f - \Delta f) e^{-j2\pi\alpha t_r} \right) \delta\left(\alpha - \frac{2k}{T}\right) \\ S_{x_r, x_i^*}^{\alpha}(f) &= \left(\frac{\sigma_{d_r}^2 (1 - e^{-j\pi\alpha k T})}{T} H_{qq^*}^{\alpha}(f) e^{-j(2\pi(\alpha - 2\Delta f)t_r - 2\phi_r)} \right) \delta\left(\alpha - 2\Delta f + \frac{k}{T}\right) \\ &= \left(\frac{2\sigma_{d_r}^2}{T} H_{qq^*}^{\alpha}(f) e^{-j(2\pi(\alpha - 2\Delta f)t_r - 2\phi_r)} \right) \delta\left(\alpha - 2\Delta f + \frac{2k+1}{T}\right) \end{aligned} \quad (3.14)$$

The above equations show that the CSD of OQPSK contains the even harmonics while the CCSD contains twice the complex carrier plus and minus the odd harmonics of the symbol rate.

3.2.3 Differentially Modulation Techniques

3.2.3.1 Non-Shifted

A non-shifted differentially modulated signal can be viewed as a non-differentially modulated signal with a combinational data sequence. For example, differential BPSK (DBPSK) signal is expressed as :

$$x(t) = \sum_{-\infty}^{\infty} d(n) q(t - nT - t_0) \cos(2\pi f_0 t + \phi_0) \quad (3.15)$$

where $d(n)$ is a data sequence $\{-1, 1\}$ created from the following binary sequence $\{0, 1\}$: [51]

$$c(n) = c(n-1) b(n) \oplus \overline{c(n-1) b(n)} \quad (3.16)$$

where $\oplus$ is an XOR function and $b(n)$ is the binary data sequence $\{0, 1\}$. The CSD and CCSD of DBPSK can be obtained from (3.10) where $\sigma_d^2 = 1$. Therefore DBPSK can be categorized in the same category with BPSK, and this is true for other differentially modulation schemes. For example differential QPSK (DQPSK) is categorized with QPSK, and DOQPSK is categorized with OQPSK.

3.2.3.2 Shifted Schemes

$\pi/4$ -shifted-DQPSK is considered as an example of this family of modulations. $\pi/4$ -shifted-DQPSK can be represented by the following equation :

$$x(t) = \sum_{-\infty}^{\infty} \cos(\theta_n) q(t - nT - t_0) \cos(2\pi f_0 t + \phi_0) - \sum_{-\infty}^{\infty} \sin(\theta_n) q(t - nT - t_0) \sin(2\pi f_0 t + \phi_0) \quad (3.17)$$

where $\theta_n = \theta_{n-1} + \Delta\theta_n$. $\Delta\theta_n$ is the $\pi/4$ shift to the signal constellation and it is calculated according to the Table 3.1 [14]. In this table, b_{1n} and b_{2n} are two transmitted bits at $t=nT+t_0$. Assuming $c_r(n) = \cos(\theta_n)$ and $c_i(n) = \sin(\theta_n)$, equation (3.17) is similar to (3.7). It is easy to show that in this case, the CSD of the signal can be found by the following equation :

Table 3.1 $\pi/4$ -shifting of constellation.

b_{2n}	b_{1n}	$\Delta\theta_n$
0	0	$\pi/4$
0	1	$3\pi/4$
1	1	$5\pi/4$
1	0	$7\pi/4$

$$S_x^a(f) = \frac{2}{T} H_q^a(f - \Delta f) e^{-j2\pi\alpha t_0} S_{c_r}(f + \frac{\alpha}{2}) \delta(\alpha - \frac{k}{T}) \quad (3.18)$$

and the CCSD of the signal is uniquely zero. Therefore $\pi/4$ -shifted-DQPSK can be categorized with QPSK.

3.3 Categorizing The Digital Modulated Signals

It is concluded from previous sections that the complex envelope of all digitally modulated signals can be categorized into three category :

Category 1: Signals which have nonzero CSD at harmonics of the symbol rate and nonzero CCSD at twice the complex envelop carrier plus and minus harmonics of the symbol rate.

Examples: BPSK, DBPSK.

Category 2: Signals which have nonzero CSD at even harmonics of the symbol rate and nonzero CCSD at twice the complex envelop carrier plus and minus odd harmonics of the symbol rate.

Examples: OQPSK, DOPQSK, $\pi/4$ -shifted OQPSK, $\pi/4$ -shifted DOQPSK.

3. Signals which have nonzero CSD at harmonics of the symbol rate and uniquely zero CCSD.

Examples: PAM, PWM, PPM, FSK, QPSK, DQPSK, $\pi/4$ -shifted DQPSK, $\pi/4$ -shifted QPSK.

Therefore studying the cyclostationary properties of BPSK, QPSK, and OQPSK is enough to cover all digital modulation schemes.

In the rest of this chapter, the spectral redundancy and spectral efficiency of these three category are considered.

3.4 Spectral Redundancy

Table 1 summarizes the relationships between the cyclic frequencies with nonzero CSD (or CCSD) and signal excess bandwidth. It is clear from the table that both balanced QPSK and OQPSK signals are cyclostationary iff their excess bandwidths are greater than

Table 3.2 Relations between cyclic frequencies with nonzero CSD (or CCSD) and signal excess bandwidth.

Excess bandwidth(ϵ)	The nonzero CSD (or CCSD) cyclic frequencies	Modulation scheme
$100(k-1)\% < \epsilon \leq 100(k)\%$ $k = 1, 2, \dots$	$\{0, \pm \frac{1}{T}, \pm \frac{2}{T}, \dots, \pm \frac{k}{T}\}, \{2f, 2f \pm \frac{1}{T}, 2f \pm \frac{2}{T}, \dots, 2f \pm \frac{k}{T}\}$	BPSK
	$\left\{ \begin{array}{l} \{0, \pm \frac{2}{T}, \pm \frac{4}{T}, \dots, \pm \frac{k}{T}\}, \{2f \pm \frac{1}{T}, 2f \pm \frac{3}{T}, \dots, 2f \pm \frac{k-1}{T}\} \\ \text{even } k \\ \{0, \pm \frac{2}{T}, \pm \frac{4}{T}, \dots, \pm \frac{k}{T}\}, \{2f \pm \frac{1}{T}, 2f \pm \frac{3}{T}, \dots, 2f \pm \frac{k-1}{T}\} \\ \text{odd } k \end{array} \right.$	QPSK
	$\{0, \pm \frac{1}{T}, \pm \frac{2}{T}, \dots, \pm \frac{k}{T}\}$	QPSK

zero. Nevertheless, BPSK signals have a nonzero CCSD at twice the carrier frequency regardless of their excess bandwidths. However, for all digital signals, the greater is the excess bandwidth, the more the cyclic frequencies with nonzero CSD (or CCSD) exist. On the other hand, the signal excess bandwidth represents the spectral redundancy of the signal. Therefore nonzero CSD and CCSD of the signal can be translated to spectral redundancy. A redundancy factor can be defined as :

$$r_x = \left(\frac{\sum_n \int_{-\infty}^{\infty} |S_x^{\alpha_n}(f)| df + \sum_n \int_{-\infty}^{\infty} |S_{xx}^{\beta_n}(f)| df}{\int_{-\infty}^{\infty} |S_x^0(f)| df} - 1 \right) 100\% \quad (3.19)$$

As it is apparent from (3.19), the presence of any cyclic frequency with nonzero CSD (or CCSD) indicates a spectral redundancy of the signal. The stronger the CSD (or CCSD) is, the more redundant the signal is. As an example, consider a balanced QPSK signal and a BPSK signal. If the excess bandwidth of the signal is zero, the balanced QPSK is a stationary signal and its spectrum has no redundancy. This can be shown by (3.19) as well :

$$r_x = \left(\frac{\int_{-\infty}^{\infty} |S_x^0(f)| df}{\int_{-\infty}^{\infty} |S_x^0(f)| df} - 1 \right) 100\% = 0\% \quad (3.20)$$

However, a BPSK signal with zero excess bandwidth is redundant with a factor 100 percent :

$$\begin{aligned} r_{x_B} &= \left(\frac{\int_{-\infty}^{\infty} |S_x^0(f)| df + \int_{-\infty}^{\infty} |S_{xx}^0(f)| df}{\int_{-\infty}^{\infty} |S_x^0(f)| df} - 1 \right) 100\% \\ &= \left(\frac{\int_{-\infty}^{\infty} |S_x^0(f)| df + \int_{-\infty}^{\infty} |S_x^0(f)| df}{\int_{-\infty}^{\infty} |S_x^0(f)| df} - 1 \right) 100\% = 100\% \end{aligned} \quad (3.21)$$

Generally, a QPSK signal with excess bandwidth $100k\% < \epsilon \leq 100(k+1)\%$ (where k is an integer number) has a redundancy equals to :

$$r_{x_Q} = \left(\frac{\sum_{n=-k-\infty}^k \int_{-\infty}^{\infty} |S_x^{n/T}(f)| df}{\int_{-\infty}^{\infty} |S_x^0(f)| df} - 1 \right) 100\% = \left(\frac{2 \sum_{n=1-\infty}^k \int_{-\infty}^{\infty} |S_x^{n/T}(f)| df}{\int_{-\infty}^{\infty} |S_x^0(f)| df} \right) 100\% \quad (3.22)$$

with the same signal parameter, the redundancy of BPSK would be :

$$\begin{aligned} r_{x_B} &= \left(\frac{\sum_{n=-k-\infty}^k \int_{-\infty}^{\infty} |S_x^{n/T}(f)| df + \sum_{n=-k-\infty}^k \int_{-\infty}^{\infty} |S_{xx}^{2f+\frac{n}{T}}(f)| df}{\int_{-\infty}^{\infty} |S_x^0(f)| df} - 1 \right) 100\% \\ &= \left(1 + \frac{4 \sum_{n=1-\infty}^k \int_{-\infty}^{\infty} |S_x^{n/T}(f)| df}{\int_{-\infty}^{\infty} |S_x^0(f)| df} \right) 100\% \\ &= 2r_{x_Q} + 100\% \end{aligned} \quad (3.23)$$

It can be shown that the spectral redundancies of OQPSK and QPSK signals are equal, (i.e. $r_{x_O} = r_{x_Q}$).

3.5 Spectral Efficiency

The ratio of stationary signal bandwidth to the signal bandwidth when both are carrying the same information is defined as the efficiency factor μ :

$$\mu = \frac{W_0}{W} 100\% \quad (3.24)$$

where W is the signal bandwidth and W_0 is the stationary signal bandwidth. For example, the efficiency factor of a QPSK (or OQPSK) signal with an excess bandwidth $100k\% < \epsilon \leq 100(k+1)\%$ is :

$$\mu_Q = \frac{W_0}{(\epsilon + 1) W_0} 100 = \frac{100}{\epsilon + 1} \% \quad (3.25)$$

and for BPSK signal is :

$$\begin{aligned}\mu_B &= \frac{W_0}{2(\epsilon + 1)W_0} 100 = \frac{100}{2(\epsilon + 1)} \% \\ &= \frac{\mu_Q}{2}\end{aligned}\tag{3.26}$$

There is a trade-off between spectral efficiency and spectral redundancy. A signal with high redundancy is less efficient while a signal with high efficiency has lower redundancy. The efficiency and redundancy have some advantages and disadvantages relative to each other. An efficient signal can carry more information per unit time, but it is easier to recover a redundant signal in an environment impaired by interference, noise and fading. Therefore, redundancy (and spectral efficiency) should be chosen to suit the environment.

3.6 Conclusion

In this chapter, cyclostationary properties of signals have been considered. It has been shown that all digital modulated signals can be classified into three categories. BPSK, QPSK, OQPSK are representative of these three categories. Furthermore, the spectral redundancy and efficiency of a modulated signal are defined and the trade off between spectral redundancy and spectral efficiency is identified. The higher, spectral redundancy yields better performance over severe channels and high interference environment, while higher spectral efficiency yields higher system capacity.

Chapter 4

Receiver Design

4.1 Introduction

In this chapter, the optimum receiver structures, in the sense of exploiting the cyclostationarity, for digital communication systems (DCSs) are obtained and analyzed. It is shown that the structure of these receivers depend on (1) symbol rate, (2) channel spacing, (3) modulation scheme, (4) number of available antennas. The general structure of such a receiver is built on the basis of two stationary representations of the received cyclostationary signal, HSR and TSR. The structure based on HSR is referred to as FRESH-DFE and the structure based on TSR is referred to as GDFE. In addition to the FRESH-DFE and GDFE, the conventional DFE is analyzed in detail and it is shown that the conventional DFE is optimum only for balanced QPSK and single-symbol rate systems.

4.2 Digital Communication System (DCS) Model

The DCS studied in this chapter is a FDMA system operating in a quasi-static channel. Figure 4.1 shows the block diagram of the equivalent baseband model of the DCS. In the figure, the signal of interest (SOI) is indicated by the index zero, CCI signals are indicated by indices $0l$ and ACI signals are indicated by indices either l or $-l$. The negative indices are used for lower ACI signals and the positive indices are used for upper ACI signals. M_c and M_a are the number of CCI and ACI signals respectively. The scales g_l (or g_{0l}) represent distance dependent propagation loss. The pulse shaping filters, $q_l(t)$ (or $q_{0l}(t)$), and the symbol intervals, T_l (or T_{0l}) are usually equal for all the signals, however, we consider the general case that they can be different. We assume also that (1) all the channels are time-invariant over a time frame, (2) the data sequences, $\{d_l\}$ (or $\{d_{0l}\}$), are complex and uncorrelated (3) the noise $n(t)$ is complex white Gaussian, (4) the modulation schemes are in the class of BPSK, OQPSK, and QPSK. Furthermore, some overlapping of SOI and ACI spectra is assumed.

The received signal can be expressed by the following equation :

Figure 4.1 The equivalent baseband of a DCS.

$$r(t) = s_0(t) + s_c(t) + s_a(t) + n(t) \quad (4.1)$$

where $n(t)$ is the additive white Gaussian noise (AWGN), $s_0(t)$, $s_a(t)$ and $s_c(t)$ are the SOI, ACI, and CCI, respectively, and can be represented by the following equations :

$$s_0(t) = \sum_{k=-\infty}^{\infty} d_0^{(r)}(k) p_0^{(1)}(t - kT_0 - t_0^{(r)}) e^{j2\pi\phi_0^{(r)}} + \\ jd_0^{(i)}(k) p_0^{(2)}(t - kT_0 - t_0^{(i)}) e^{j2\pi\phi_0^{(i)}} \quad (4.2)$$

$$s_c(t) = \sum_{l=1}^{M_c} \sum_{k=-\infty}^{\infty} d_{0l}^{(r)}(k) p_{0l}^{(1)}(t - kT_{0l} - t_{0l}^{(r)}) e^{j2\pi\phi_{0l}^{(r)}} + \\ jd_{0l}^{(i)}(k) p_{0l}^{(2)}(t - kT_{0l} - t_{0l}^{(i)}) e^{j2\pi\phi_{0l}^{(i)}} \quad (4.3)$$

$$s_a(t) = \sum_{l=-M_a/2}^{M_a/2} \sum_{k=-\infty}^{\infty} (d_l^{(r)}(k) p_l^{(1)}(t - kT_l - t_l^{(r)}) e^{j2\pi\phi_l^{(r)}} + \\ jd_l^{(i)}(k) p_l^{(2)}(t - kT_l - t_l^{(i)}) e^{j2\pi\phi_l^{(i)}}) e^{j2\pi f_l t} \quad (4.4)$$

where $p_{0l}^{(1)}(t)$, $p_{0l}^{(2)}(t)$, $p_l^{(1)}(t)$, $p_l^{(2)}(t)$, $p_0^{(1)}(t)$, and $p_0^{(2)}(t)$ are defined by :

$$p_{0l}^{(1)}(t) = g_{0l} q_{0l}^{(r)}(t) \otimes c_{0l}(t) \\ p_{0l}^{(2)}(t) = g_{0l} q_{0l}^{(i)}(t) \otimes c_{0l}(t) \\ p_l^{(1)}(t) = g_l q_l^{(r)}(t) \otimes c_l(t) \\ p_l^{(2)}(t) = g_l q_l^{(i)}(t) \otimes c_l(t) \\ p_0^{(1)}(t) = g_0 q_0^{(r)}(t) \otimes c_0(t) \\ p_0^{(2)}(t) = g_0 q_0^{(i)}(t) \otimes c_0(t) \quad (4.5)$$

4.2.1 The Cyclic Spectral Density (CSD) and Conjugate CSD (CCSD) of The Received Signal

Due to the assumption of uncorrelated data sequences, the CSD and CCSD of the received signal can be found by the following equations :

$$S_r^u(f) = S_{s_0}^u(f) + S_{s_c}^u(f) + S_{s_a}^u(f) + N_0 \delta(\alpha) \\ S_{rr}^{u*}(f) = S_{s_0 s_0}^{u*}(f) + S_{s_c s_c}^{u*}(f) + S_{s_a s_a}^{u*}(f) \quad (4.6)$$

where N_0 is the power spectral density (PSD) of the noise and

$$S_{s_0}^{\alpha}(f) = \left(\frac{\sigma_{d_0^{(r)}}^2}{T_0} H_{p_0^{(1)}}^{\alpha}(f) e^{-j2\pi\alpha t_0^{(r)}} + \frac{\sigma_{d_0^{(i)}}^2}{T_0} H_{p_0^{(2)}}^{\alpha}(f) e^{-j2\pi\alpha t_0^{(i)}} \right) \delta\left(\alpha - \frac{k}{T_0}\right)$$

$$S_{s_0 s_0^*}^{\alpha}(f) = \left(\frac{\sigma_{d_0^{(r)}}^2}{T_0} H_{p_0^{(1)} p_0^{(1)*}}^{\alpha}(f) e^{-j(2\pi\alpha t_0^{(r)} - 2\phi_0^{(r)})} - \right.$$

$$\left. \frac{\sigma_{d_0^{(i)}}^2}{T_0} H_{p_0^{(2)} p_0^{(2)*}}^{\alpha}(f) e^{-j(2\pi\alpha t_0^{(i)} - 2\phi_0^{(i)})} \right) \delta\left(\alpha - \frac{k}{T_0}\right) \quad (4.7)$$

$$S_{s_c}^{\alpha}(f) = \sum_{l=1}^{M_c} \left(\frac{\sigma_{d_{0l}^{(r)}}^2}{T_{0l}} H_{p_{0l}^{(1)}}^{\alpha}(f) e^{-j2\pi\alpha t_{0l}^{(r)}} + \frac{\sigma_{d_{0l}^{(i)}}^2}{T_{0l}} H_{p_{0l}^{(2)}}^{\alpha}(f) e^{-j2\pi\alpha t_{0l}^{(i)}} \right) \delta\left(\alpha - \frac{k}{T_{0l}}\right)$$

$$S_{s_c s_c^*}^{\alpha}(f) = \sum_{l=1}^{M_c} \left(\frac{\sigma_{d_{0l}^{(r)}}^2}{T_{0l}} H_{p_{0l}^{(1)} p_{0l}^{(1)*}}^{\alpha}(f) e^{-j(2\pi\alpha t_{0l}^{(r)} - 2\phi_{0l}^{(r)})} - \right.$$

$$\left. \frac{\sigma_{d_{0l}^{(i)}}^2}{T_{0l}} H_{p_{0l}^{(2)} p_{0l}^{(2)*}}^{\alpha}(f) e^{-j(2\pi\alpha t_{0l}^{(i)} - 2\phi_{0l}^{(i)})} \right) \delta\left(\alpha - \frac{k}{T_{0l}}\right) \quad (4.8)$$

$$S_{s_a}^{\alpha}(f) = \sum_{\substack{l=-M_a/2 \\ l \neq 0}}^{M_a/2} \left(\frac{\sigma_{d_l^{(r)}}^2}{T_l} H_{p_l^{(1)}}^{\alpha}(f-f_l) e^{-j2\pi\alpha t_l^{(r)}} + \frac{\sigma_{d_l^{(i)}}^2}{T_l} H_{p_l^{(2)}}^{\alpha}(f-f_l) e^{-j2\pi\alpha t_l^{(i)}} \right) \delta\left(\alpha - \frac{k}{T_l}\right)$$

$$S_{s_a s_a^*}^{\alpha}(f) = \sum_{\substack{l=-M_a/2 \\ l \neq 0}}^{M_a/2} \left(\frac{\sigma_{d_l^{(r)}}^2}{T_l} H_{p_l^{(1)} p_l^{(1)*}}^{\alpha}(f) e^{-j(2\pi(\alpha-2f_l) t_l^{(r)} - 2\phi_l^{(r)})} - \right.$$

$$\left. \frac{\sigma_{d_l^{(i)}}^2}{T_l} H_{p_l^{(2)} p_l^{(2)*}}^{\alpha}(f) e^{-j(2\pi(\alpha-2f_l) t_l^{(i)} - 2\phi_l^{(i)})} \right) \delta\left(\alpha - 2f_l - \frac{k}{T_l}\right) \quad (4.9)$$

From (4.6), we see that the noise is only present on the CSD of the received signal and only effective at $\alpha=0$, suggesting that the CCSD of the received signal is noise free for all the cyclic spectra while the CSD of the received signal is noise free on the nonzero cyclic spectra. It should be made clear at this point that, although there is a noise free cyclic spectra, it does not mean that the data sequence can be estimated noise free. This is

because the CSD or CCSD are the second order statistics of the signal and does not carry the information about data phase.

In the case of balanced BPSK ($\sigma_{d^{(n)}}^2 = 0$) system, (4.7) to (4.9) are simplified to the following equations :

$$S_{s_0}^\alpha(f) = \frac{\sigma_{d_0^{(r)}}^2}{T_0} e^{-j2\pi\alpha t_0^{(r)}} H_{p_0^{(1)}}^\alpha(f) \delta\left(\alpha - \frac{k}{T_0}\right) \quad (4.10)$$

$$S_{s_0 s_0^*}^\alpha(f) = \frac{\sigma_{d_0^{(r)}}^2}{T_0} H_{p_0^{(1)} p_0^{(1)*}}^\alpha(f) e^{-j(2\pi\alpha t_0^{(r)} - 2\phi_0^{(r)})}$$

$$S_{s_c}^\alpha(f) = \sum_{l=1}^{M_c} \frac{\sigma_{d_{0l}^{(r)}}^2}{T_{0l}} H_{p_{0l}^{(1)}}^\alpha(f) e^{-j2\pi\alpha t_{0l}^{(r)}} \delta\left(\alpha - \frac{k}{T_{0l}}\right) \quad (4.11)$$

$$S_{s_c s_c^*}^\alpha(f) = \sum_{l=1}^{M_c} \frac{\sigma_{d_{0l}^{(r)}}^2}{T_{0l}} H_{p_{0l}^{(1)} p_{0l}^{(1)*}}^\alpha(f) e^{-j(2\pi\alpha t_{0l}^{(r)} - 2\phi_{0l}^{(r)})} \delta\left(\alpha - \frac{k}{T_{0l}}\right)$$

$$S_{s_a}^\alpha(f) = \sum_{\substack{l=-M_a/2 \\ l \neq 0}}^{M_a/2} \frac{\sigma_{d_l^{(r)}}^2}{T_l} H_{p_l^{(1)}}^\alpha(f - f_l) e^{-j2\pi\alpha t_l^{(r)}} \delta\left(\alpha - \frac{k}{T_l}\right) \quad (4.12)$$

$$S_{s_a s_a^*}^\alpha(f) = \sum_{\substack{l=-M_a/2 \\ l \neq 0}}^{M_a/2} \frac{\sigma_{d_l^{(r)}}^2}{T_l} H_{p_l^{(1)} p_l^{(1)*}}^\alpha(f) e^{-j(2\pi(\alpha - 2f_l) t_l^{(r)} - 2\phi_l^{(r)})} \delta\left(\alpha - 2f_l - \frac{k}{T_l}\right)$$

If the bit intervals are the same, the CSD of the SOI is corrupted by the CSD of the CCI and ACI signals on all the cyclic spectral components. However, the CCSD of SOI is corrupted on all the cyclic spectra by only the CCSD of CCI signals and it is free of ACI on some cyclic spectra. It does mean that for single symbol rate BPSK systems, there exist some cyclic spectra of the CCSD of SOI where ACI does not exist, however CCI exists at all the cyclic spectra. Again it should be mentioned that the CCI or ACI free cyclic spectra does not mean that the data sequence can be estimated CCI or ACI free.

In the case of conventional QPSK where $\sigma_{d^{(r)}}^2 = \sigma_{d^{(i)}}^2$, $t^{(r)} = t^{(i)}$, $q^{(r)}(t) = q^{(i)}(t)$, $\phi^{(r)} = \phi^{(i)}$, (4.7), (4.8) and (4.9) are simplified to :

$$S_{s_0}^{\alpha}(f) = \frac{2\sigma_{d_0^{(r)}}^2}{T_0} e^{-j2\pi\alpha t_0^{(r)}} H_{p_0^{(1)}}^{\alpha}(f) \delta\left(\alpha - \frac{k}{T_0}\right) \quad (4.13)$$

$$S_{s_0 s_0}^{\alpha*}(f) = 0$$

$$S_{s_c}^{\alpha}(f) = \sum_{l=1}^{M_c} \frac{2\sigma_{d_{0l}^{(r)}}^2}{T_{0l}} H_{p_{0l}^{(1)}}^{\alpha}(f) e^{-j2\pi\alpha t_{0l}^{(r)}} \delta\left(\alpha - \frac{k}{T_{0l}}\right) \quad (4.14)$$

$$S_{s_c s_c}^{\alpha*}(f) = 0$$

$$S_{s_s}^{\alpha}(f) = \sum_{\substack{l=-M_s/2 \\ l \neq 0}}^{M_s/2} \frac{2\sigma_{d_l^{(r)}}^2}{T_l} H_{p_l^{(1)}}^{\alpha}(f-f_l) e^{-j2\pi\alpha t_l^{(r)}} \delta\left(\alpha - \frac{k}{T_l}\right) \quad (4.15)$$

$$S_{s_s s_s}^{\alpha*}(f) = 0$$

Equations (4.13) to (4.15) indicates that, in the case of balanced QPSK there is no information on the CCSD of the received signal. Therefore, if the symbol rates are the same, there is no ACI free cyclic spectrum.

In the case of balanced OQPSK where $\sigma_{d^{(r)}}^2 = \sigma_{d^{(i)}}^2$, $t^{(r)} = (T_0/2) + t^{(i)}$, $q^{(r)}(t) = q^{(i)}(t)$, $\phi^{(r)} = \phi^{(i)}$, (4.7), (4.8) and (4.9) are simplified to :

$$S_{s_0}^{\alpha}(f) = \frac{2\sigma_{d_0^{(r)}}^2}{T_0} e^{-j2\pi\alpha t_0^{(r)}} H_{p_0^{(1)}}^{\alpha}(f) \delta\left(\alpha - \frac{2k}{T_0}\right) \quad (4.16)$$

$$S_{s_0 s_0}^{\alpha*}(f) = \frac{2\sigma_{d_0^{(r)}}^2}{T_0} H_{p_0^{(1)} p_0^{(1)*}}^{\alpha}(f) e^{-j(2\pi\alpha t_0^{(r)} - 2\phi_0^{(r)})} \delta\left(\alpha - \frac{2k+1}{T_0}\right)$$

$$S_{s_c}^{\alpha}(f) = \sum_{l=1}^{M_c} \frac{2\sigma_{d_{0l}^{(r)}}^2}{T_{0l}} H_{p_{0l}^{(1)}}^{\alpha}(f) e^{-j2\pi\alpha t_{0l}^{(r)}} \delta\left(\alpha - \frac{2k}{T_{0l}}\right) \quad (4.17)$$

$$S_{s_c s_c}^{\alpha*}(f) = \sum_{l=1}^{M_c} \frac{2\sigma_{d_{0l}^{(r)}}^2}{T_{0l}} H_{p_{0l}^{(1)} p_{0l}^{(1)*}}^{\alpha}(f) e^{-j(2\pi\alpha t_{0l}^{(r)} - 2\phi_{0l}^{(r)})} \delta\left(\alpha - \frac{2k+1}{T_{0l}}\right)$$

$$\begin{aligned}
S_{s_a}^{\alpha}(f) &= \sum_{\substack{l=-M_a/2 \\ l \neq 0}}^{M_a/2} \frac{2\sigma_{d_l^{(r)}}^2}{T_l} H_{p_l^{(1)}}^{\alpha}(f-f_l) e^{-j2\pi\alpha t_l^{(r)}} \delta\left(\alpha - \frac{2k}{T_l}\right) \\
S_{s_a s_a^*}^{\alpha}(f) &= \sum_{\substack{l=-M_a/2 \\ l \neq 0}}^{M_a/2} \frac{2\sigma_{d_l^{(r)}}^2}{T_l} H_{p_l^{(1)} p_l^{(1)*}}^{\alpha}(f-f_l) e^{-j(2\pi(\alpha-2f_l)t_l^{(r)}-2\phi_l^{(r)})} \delta\left(\alpha - \frac{2k+1}{T_l}\right)
\end{aligned} \tag{4.18}$$

Equations (4.16) to (4.18) show that, the CCSD of the received signal in the balanced OQPSK systems have some ACI free cyclic spectrum.

The above equations show that in a DCS, the CCSD of the received signal includes some information about the SOI and interference signals. The above equations also show that the CSD and CCSD of the SOI are separable from the CCI signals in some cyclic spectrum iff the symbol rate of the SOI is different from the CCI symbol rates. However, the CCSD of the SOI can be separated from ACI signals even though they have the same symbol rate. Balanced QPSK signals are the exceptions. Table 4.1 summarizes the spectral properties of a received signal for the three modulation schemes.

4.3 Receiver Design

4.3.1 Estimating The Entire SOI Waveform

It is well known that the optimum receiver for a stationary signal is a time invariant filter (TIF). However, for a cyclostationary signal, the optimum receiver is a time-variant filter (TVF) [87,88]. The only way to implement an optimal time-variant filter (OTVF) using optimal time-invariant filters (OTIFs) is to filter the stationary components of the signal by OTIFs. Therefore, two structures are found for OTVF due to two representations, HSR and TSR, of a CRP.

4.3.1.1 HSR Time Variant Filter (HSR-TVF)

The HSR-TVF explicitly exploits the spectral correlation that exists in cyclostationary signals to obtain a lower mean square error (MSE). The receiver which use HSR of the signal is called the *linear-conjugate-linear frequency shift* (LCL-FRESH) filter [10,32,37,38]. Figure 4.2 shows the structure of the LCL-FRESH filter. In this figure, $\{\alpha_m\}$ is the set of all the cyclic frequencies for which the CSD of the received signal is nonzero and $\{\beta_n\}$ is the set of all the cyclic frequencies for which the CCSD of the

Table 4.1 A summary of cyclic spectral properties of BPSK, OQPSK and QPSK systems.

Modulation	The CSD	The CCSD
BPSK	<p>1-nonzero at cyclic frequencies equal to multiple of symbol rate.</p> <p>2-noise free at nonzero cyclic frequencies.</p> <p>3-ACI and CCI free in some nonzero cyclic frequencies only for multi-symbol rates systems.</p>	<p>1-nonzero at cyclic frequencies equal to multiple of symbol rate.</p> <p>2-noise free at all the cyclic frequencies.</p> <p>3-CCI free in some nonzero cyclic frequencies only for multi-symbol rates systems and ACI free in some nonzero cyclic frequencies even though for single symbol rates systems</p>
OQPSK	<p>1-nonzero at cyclic frequencies equal to even harmonics of symbol rates.</p> <p>2-noise free at nonzero cyclic frequencies.</p> <p>3-ACI and CCI free in some nonzero cyclic frequencies only for multi-symbol rates systems.</p>	<p>1-nonzero at cyclic frequencies equal to odd harmonics of symbol rates.</p> <p>2-noise free at all the cyclic frequencies.</p> <p>3-CCI free in some nonzero cyclic frequencies only for multi-symbol rates systems and ACI free in some nonzero cyclic frequencies even though for single-symbol rates systems</p>
QPSK	<p>1-nonzero at cyclic frequencies equal to multiple of symbol rate.</p> <p>2-noise free at nonzero cyclic frequency.</p> <p>3-ACI and CCI free in some nonzero cyclic frequencies only for multi-symbol rates systems.</p>	uniquely zero

Figure 4.2 The structure of a receiver using Linear-Conjugate-Linear FRESH (LCL-FRESH) filtering.

Figure 4.3 The structure of a receiver using LCL-FRESH filtering with finite number of frequency shifts.

received signal is nonzero.

The estimated signal can be found by the following equation :

$$\hat{s}_0(t) = \sum_{m=1}^M a_m(t) \otimes [r(t) e^{j2\pi\alpha_m t}] + \sum_{n=1}^N b_n(t) \otimes [r^*(t) e^{j2\pi\beta_n t}] \quad (4.19)$$

where M and N are the maximum number of filters in the linear and conjugate paths respectively. The optimum solution of the LCL-FRESH filters with the criterion of minimum mean square of the error ($e(t) = \hat{s}_0(t) - s_0(t)$) are called *cyclic Wiener filters* and can be found by the following equations [32] :

$$\begin{aligned} \sum_{m=1}^M A_m(f) S_r^{\alpha_k - \alpha_m}(f - \frac{\alpha_m + \alpha_k}{2}) + \sum_{n=1}^N B_n(f) \left[S_{rr^*}^{\beta_n - \alpha_k}(f - \frac{\beta_n + \alpha_k}{2}) \right]^* &= S_{s_0 r}^{\alpha_k}(f - \frac{\alpha_k}{2}) \\ k = 1, 2, \dots, M \\ \sum_{m=1}^M A_m(f) S_{rr^*}^{\beta_k - \alpha_m}(f - \frac{\alpha_m + \beta_k}{2}) + \sum_{n=1}^N B_n(f) S_r^{\beta_k - \beta_n}(-f + \frac{\beta_n + \beta_k}{2}) &= S_{s_0 r^*}^{\beta_k}(f - \frac{\beta_k}{2}) \\ k = 1, 2, \dots, N \end{aligned} \quad (4.20)$$

where $S_{s_0 r}^{\alpha}(f)$ and $S_{s_0 r^*}^{\alpha}(f)$ are the *cyclic cross spectrum* and *conjugate cyclic cross spectrum* between the SOI and the received signal respectively.

The spectrum of error can be found by the following equation [32] :

$$S_e(f) = S_d(f) - \left(\sum_{m=1}^M A_m(f) \left[S_{s_0 r}^{\alpha_m}(f - \frac{\alpha_m}{2}) \right]^* - \sum_{n=1}^N B_n(f) \left[S_{s_0 r^*}^{\beta_n}(f - \frac{\beta_n}{2}) \right]^* \right) \quad (4.21)$$

It is apparent from (4.20) that the best set of $\{\alpha_m\}$ contains all α_m [32] for which either

$$S_r^{\alpha_k - \alpha_m}(f - \frac{\alpha_m + \alpha_k}{2}) \neq 0 \quad (4.22)$$

and α_k for which

$$S_{s_0 r}^{\alpha_k}(f - \frac{\alpha_k}{2}) \neq 0 \quad (4.23)$$

or

$$S_{rr}^{\beta_k - \alpha_m} \left(f - \frac{\beta_k + \alpha_m}{2} \right) \neq 0 \quad (4.24)$$

and β_k for which

$$S_{s_0 r}^{\beta_k} \left(f - \frac{\beta_k}{2} \right) \neq 0 \quad (4.25)$$

and the best set of $\{\beta_n\}$ contains all β_k which either

$$S_{rr}^{\beta_n - \alpha_k} \left(f - \frac{\beta_n + \alpha_k}{2} \right) \neq 0 \quad (4.26)$$

and α_k for which

$$S_{s_0 r}^{\alpha_k} \left(f - \frac{\alpha_k}{2} \right) \neq 0 \quad (4.27)$$

or

$$S_r^{\beta_k - \beta_n} \left(-f + \frac{\beta_n + \beta_k}{2} \right) \neq 0 \quad (4.28)$$

and β_k for which

$$S_{s_0 r}^{\beta_k} \left(f - \frac{\beta_k}{2} \right) \neq 0 \quad (4.29)$$

Since the received signal is not band limited, the optimum number of frequency shifts is infinite. Nevertheless, when the received signal is passed through a *noise limiting filter* only a finite number of frequency shifts will be needed (Figure 4.3). If the SOI and ACI signals are not overlapped, the best choice for this filter is a filter matched to the transmitter pulse shape. However, in the case of overlapping signals, the best noise limiter filter is a filter which passes all the overlapped signals. Nevertheless, a sub-optimum filter which can pass only a portion of ACI signals may be considered.

4.3.1.2 TSR-TVF

Although the FRESH-filter is the optimum linear filter for cyclostationary signals, it is too complex. Another TVF can be built on the basis of TSR of a cyclostationary signal. In this receiver, the signal is sampled after passing through the receiver filter. Figure 4.4 shows the structure of this receiver. In this figure, T_0 is the symbol rate of SOI, $\{T_{0l}\}$ are the incommensurate symbol rates of the CCI signals, $M_{0l} = \lceil T_{0l}/T_s \rceil$, $\{T_l\}$

Figure 4.4 The structure of a receiver using TSR-TVF.

are the symbol rates of the ACI signals, and $M_l = \lceil T_l/T_s \rceil$. Each filter of the branch $\mathfrak{R}_{l,m,f}$ will be updated at $mT_s + nT_l + \xi$ where n is a positive integer and ξ is the sampler delay time. In the case of $S_{s,f_i}(f) = 0$, the conjugate path (c-path) in $\mathfrak{R}_{l,m,f_i}$ will be removed. The linear path (l-path) of $\mathfrak{R}_{l,m,f_i}$ will be removed if T_l is not incommensurate periodicity.

In general, the minimum sampling rate of the receiver has to be greater than the Nyquist rate. However, in order to exploit all periodicities, the minimum sampling rate has to be greater than the least common multiple of the fundamental cyclic frequencies in the received signal.

It can be shown that the TSR-TVF and the HSR-TVF are equivalent [Appendix C].

4.3.2 Estimating the Data Sequence

In a digital communication system, the objective is to extract the data. Therefore, the output of the TSR-TVF and HSR-TVF should be sampled. The sampling rate has to be at least equal to the symbol rate. To understand how a sampler can simplify the structure of TSR-TVF and HSR-TVF, the aliasing effect of a sampler is considered in the following section.

4.3.2.1 The Aliasing Effect of a Sampler

The following equality is valid for any signal $x(t)$ and any L and M :

$$x(t) = \frac{1}{L+M+1} \sum_{m=-L}^M x(t) e^{j2\pi m} \quad (4.30)$$

Multiplying m by nT/T where n is an integer does not change the equality at (4.30):

$$x(t) = \frac{1}{L+M+1} \sum_{m=-L}^M x(t) e^{j2\pi \frac{m}{T} (nT)} \quad (4.31)$$

If $x(t)$ is sampled at the rate $1/T$, the above equation yields :

$$\tilde{x}(n) = x(nT) = \left[\frac{1}{L+M+1} \sum_{m=-L}^M x(t) e^{j2\pi \frac{m}{T} t} \right] \Big|_{t=nT} \quad (4.32)$$

Equation (4.32) shows that sampling a signal with the rate of $1/T$ is equivalent to adding

all versions of the signals shifted by the frequencies m/M where m is an integer.

Now, we assume $x(t)$ is filtered by $h(t)$ and the output is sampled at the rate $1/T$, then

$$y(n) = x(t) \otimes h(t) \Big|_{t=nT} \quad (4.33)$$

The above equation is equivalent to the following equation :

$$\tilde{y}(n) = \left[\frac{1}{L+M+1} \sum_{m=-L}^M x(t) e^{j2\pi m \frac{t}{T}} \otimes h(t) e^{j2\pi m \frac{t}{T}} \right] \Big|_{t=nT} \quad (4.34)$$

Equation (4.34) shows that sampling a filtered signal with the rate of $1/T$ is equivalent to sampling the output of a bank of filters, which filters all frequency shifted versions of the signal (Figure 4.5).

If $x(t)$ is frequency shifted by α and passed through a filter $h(t)$ followed by a sampler with the rate of $1/T$, it can be shown that :

$$\begin{aligned} \tilde{y}(n) &= [x(t) e^{j2\pi\alpha t} \otimes h(t)] \Big|_{t=nT} \\ &= [[x(t) \otimes h(t) e^{-j2\pi\lceil\alpha T\rceil t}] \Big|_{t=nT}] e^{j2\pi\lceil\alpha\rceil_{T^{-1}} nT} \\ &= [x(t) e^{j2\pi\lceil\alpha\rceil_{T^{-1}} t} \otimes h(t) e^{-j2\pi\lceil\alpha T\rceil t}] \Big|_{t=nT} \end{aligned} \quad (4.35)$$

Figure 4.6 illustrates the above equalities. Comparing (4.34) and (4.35) with (4.19) concludes that a filter followed by a sampler is equivalent to the FRESH-filter with the set of frequency shifts $\{0, \pm 1/T, \pm 2/T, \dots\}$. It is clear that, if $x(t)$ is shifted by a frequency μ before entering $h(t)$, the set of equivalent frequency shifts will have an addition shift μ (i.e. $\{\mu, \mu \pm 1/T, \mu \pm 2/T, \dots\}$). Therefore, to achieve an arbitrary frequency shifting, γ , it is enough to shift the frequency by the remainder of the division of γ by T^{-1} , $[\gamma]_{T^{-1}}$.

4.3.2.2 Adding Sampler to The TSR-TVF

Adding the symbol rate sampler to the structure of TSR-TVF creates the structure of sampled TSR-TVF (STSR-TVF) shown in Figure 4.7. In the figure, M_{0l}^0 (or M_l^0) is the least common multiple of M_{0l} (or M_l) and M_0 , and $T_{0l}^0 = M_{0l}^0 T_s$ (or $T_l^0 = M_l^0 T_s$). The structure of STSR-TVF is a general one, and it covers the commensurate periodicities as well as incommensurate periodicities.

Figure 4.5 Sampling the output of a filtered signal.

Figure 4.6 The equivalent procedures in equation (4.35).

Figure 4.7 The structure of Sampled TSR-TVF (STSR-TVF).

As an example, a three symbol rate balanced QPSK system with incommensurate symbol rates 1/2, 1/3, 1/4 Hz is considered. Assuming the symbol rate of SOI is 1/3 Hz. In this case, the following parameters are calculated :

$$\begin{aligned}
 \{T_0 = 3 \text{ sec.}, T_1 = 2 \text{ sec.}, T_2 = 4 \text{ sec.}\} &\Rightarrow T_s = 1 \text{ sec.} \\
 M_0 &= \left\lceil \frac{T_0}{T_s} \right\rceil = 3, M_1 = \left\lceil \frac{T_1}{T_s} \right\rceil = 2, M_2 = \left\lceil \frac{T_2}{T_s} \right\rceil = 4 \\
 M_1^0 &= M_0 \times M_1 = 6, M_2^0 = M_0 \times M_2 = 12 \\
 T_1^0 &= M_1^0 \times T_s = 6 \text{ sec.}, T_2^0 = M_2^0 \times T_s = 12 \text{ sec.}
 \end{aligned} \tag{4.36}$$

Figure 4.8 shows the structure of STSR-TVF for this example where (k Hz, ϕ sec.) represents a sampler with the rate k Hz and initial time ϕ .

In the case of a single symbol rate system, the STSR-TVF is simplified to Figure 4.9. In the case of a single symbol rate balanced QPSK system, STSR-TVF is simplified to Figure 4.10 which is the conventional fractionally space equalizer (FSE). Therefore, STSR-TVF can be considered as the generalized FSE (GFSE). From now on, we will refer to STSR-TVF as GFSE.

4.3.2.3 Adding Sampler to The HSR-TVF

Adding a symbol rate sampler to the HSR-TVF simplifies the structure of HSR-TVF to the one shown in Figure 4.11 called sampled HSR-TVF (SHSR-TVF). Although at first sight, it does not seem to show a great simplification, yet, we will see that the structure of HSR-TVF has become considerably simplified. This can be illustrated if we consider a set of frequency shiftings in HSR-TVF i.e. :

$$\begin{aligned}
 \{\alpha\} &= \left\{ \frac{1}{T_0} \{ \dots, -1, 0, 1, 2, \dots \}, \frac{1}{T_{01}} \{ \dots, -1, 1, 2, \dots \}, \dots, \frac{1}{T_{0M_c}} \{ \dots, -1, 1, 2, \dots \}, \right. \\
 &\quad \left. \frac{1}{T_1} \{ \dots, -1, 1, 2, \dots \}, \dots, \frac{1}{T_{M_a}} \{ \dots, -1, 1, 2, \dots \} \right\} \\
 \{\beta\} &= \left\{ \frac{1}{T_0} \{ \dots, -1, 0, 1, 2, \dots \}, \frac{1}{T_{01}} \{ \dots, -1, 1, 2, \dots \}, \dots, \frac{1}{T_{0M_c}} \{ \dots, -1, 1, 2, \dots \}, \right. \\
 &\quad \left. 2f_1 + \frac{1}{T_1} \{ \dots, -1, 1, 2, \dots \}, \dots, 2f_{M_a} + \frac{1}{T_{M_a}} \{ \dots, -1, 1, 2, \dots \} \right\}
 \end{aligned} \tag{4.37}$$

and the set of frequency shifting in SHSR-TVF are :

Figure 4.8 An example of 5th SR-TVF.

Figure 4.9 The structure of STSR-TVF for single symbol rate systems.

Figure 4.10 The structure of STSR-TVF for balanced QPSK and single symbol rate systems.

Figure 4.11 The structure of SHSR-TVF.

Figure 4.12 The structure of SHSR-TVF for single symbol rate systems.

$$\begin{aligned}
\{[\alpha]_{T_0^{-1}}\} &= \{ \{0\}, \frac{1}{T_{01}^0} \left\{ -\left\lceil \frac{T_{01}}{2} \right\rceil, \dots, -1, 1, \dots, \left\lceil \frac{T_{01}}{2} \right\rceil \right\}, \dots \\
&\frac{1}{T_{0M_c}^0} \left\{ -\left\lceil \frac{T_{0M_c}}{2} \right\rceil, \dots, -1, 1, \dots, \left\lceil \frac{T_{0M_c}}{2} \right\rceil \right\}, \frac{1}{T_1^0} \left\{ -\left\lceil \frac{T_1}{2} \right\rceil, \dots, -1, 1, \dots, \left\lceil \frac{T_1}{2} \right\rceil \right\}, \dots, \\
&\frac{1}{T_{M_d}^0} \left\{ -\left\lceil \frac{T_{M_d}}{2} \right\rceil, \dots, -1, 1, \dots, \left\lceil \frac{T_{M_d}}{2} \right\rceil \right\} \} \\
\{[\beta]_{T_0^{-1}}\} &= \{ \{0\}, \frac{1}{T_{01}^0} \left\{ -\left\lceil \frac{T_{01}}{2} \right\rceil, \dots, -1, 1, \dots, \left\lceil \frac{T_{01}}{2} \right\rceil \right\}, \dots \\
&\frac{1}{T_{0M_c}^0} \left\{ -\left\lceil \frac{T_{0M_c}}{2} \right\rceil, \dots, -1, 1, \dots, \left\lceil \frac{T_{0M_c}}{2} \right\rceil \right\}, [2f_1]_{T_0^{-1}} + \frac{1}{T_1^0} \left\{ -\left\lceil \frac{T_1}{2} \right\rceil, \dots, -1, 1, \dots, \left\lceil \frac{T_1}{2} \right\rceil \right\}, \dots, \\
&[2f_{M_d}]_{T_0^{-1}} + \frac{1}{T_{M_d}^0} \left\{ -\left\lceil \frac{T_{M_d}}{2} \right\rceil, \dots, -1, 1, \dots, \left\lceil \frac{T_{M_d}}{2} \right\rceil \right\} \}
\end{aligned} \tag{4.38}$$

Equations (4.37) and (4.38) show that, the amount and number of frequency shifts in SHSR-TVF are much lower than those for HSR-TVF.

In the case of a single symbol rate system, SHSR-TVF is simplified to Figure 4.12. Comparing Figure 4.9 and Figure 4.12 indicates that, the GFSE is the sampled version of SHSR-TVF, or in other words, SHSR-TVF is an infinite length GFSE. Therefore, SHSR-TVF is an alternative structure for FSE. Since SHSR-TVF is built on the basis of shifting the frequency of the input data, we refer to SHSR-TVF as FRESH-FSE. Although FRESH-FSE and GFSE are equivalent, their structure complexities are different and preference of one over the other depends on the system under consideration.

4.4 Decision Feedback Receiver

4.4.1 Conventional Decision Feedback Equalizer

The conventional fractionally spaced decision feed back equalizer (conventional DFE) is created by adding a feedback path to an FSE. Figure 4.13 shows the structure of conventional DFE. In the figure, $x(t)$ is the received signal after passing through the receiver filter. The feedback is added to decrease the ISI caused by the previous symbols.

There are two criteria that can be used as basis for conventional DFE design (1) the zero forcing criterion and (2) the minimum MSE criterion. The zero forcing DFE and

Figure 4.13 The structure of conventional DFE with infinite length filters.

MMSE-DFE are already analyzed in [1,19,21,22,61,67,70,72,73,80,81,85,93,99]. However, the MMSE-DFE is analyzed in a different way in the following section.

4.4.2 Conventional MMSE-DFE

4.4.2.1 Infinite Length Equalizer

The MMSE-DFE minimizes the time average mean square of the error defined by the following equation :

$$TAMSE = \langle E \{ |b(n) - d_0(n-D)|^2 \} \rangle_n \quad (4.39)$$

where D is the overall delay. To simplify the notations, we remove the index n from $\langle \rangle_n$. Replacing $b(n)$ by $\tilde{y}(n) - f(n)$ in (4.39) and assuming the correct decisions for previous symbols, yields :

$$\begin{aligned} TAMSE = & \langle E \{ |d_0(n-D)|^2 \} \rangle + \langle E \{ |f(n)|^2 \} \rangle + \langle E \{ |\tilde{y}(n)|^2 \} \rangle + \\ & \langle E \{ d_0(n-D)f^*(n) \} \rangle + \langle E \{ d_0^*(n-D)f(n) \} \rangle \\ & - \langle E \{ d_0(n-D)\tilde{y}^*(n) \} \rangle - \langle E \{ d_0^*(n-D)\tilde{y}(n) \} \rangle \\ & - \langle E \{ f(n)\tilde{y}^*(n) \} \rangle - \langle E \{ f^*(n)\tilde{y}(n) \} \rangle \end{aligned} \quad (4.40)$$

Assuming that the data are uncorrelated and the decisions are correct, the first term of the right hand side is :

$$\sigma_{d_0}^2 = \sigma_{d_0^{(r)}}^2 + \sigma_{d_0^{(i)}}^2 \quad (4.41)$$

and 4th and 5th terms are zero.

The optimum feedback and forward filters can be achieved by letting the gradient of TAMSE equal to zero. The gradient of TAMSE according to the feedback filter coefficients can be found by the following equation :

$$\nabla_{a_b}^{TAMSE}(m) = \langle E \{ [f(n) - \tilde{y}(n)] \nabla_{a_b}^{f^*(n)}(m) \} \rangle \quad (4.42)$$

replacing $f(n)$ by :

$$f(n) = \sum_{m=1}^{\infty} d_0(n-D-m) a_b(m) \quad (4.43)$$

and taking gradient of it,

$$\nabla_{a_b}^{f^* (n)} (m) = d_0^* (n - D - m) \quad (4.44)$$

yields :

$$\nabla_{a_b}^{TAMSE} (m) = \left\langle \sum_{k=1}^{\infty} E \{ d_0^* (n - D - m) d_0 (n - D - k) \} a_b (k) \right\rangle - \left\langle E \{ \tilde{y} (n) d_0^* (n - D - m) \} \right\rangle \quad (4.45)$$

The first term of the above equation is nonzero iff $m = k$.

$$\nabla_{a_b}^{TAMSE} (m) = \sigma_{d_0}^2 a_b (m) + \left\langle E \{ \tilde{y} (n) d_0^* (n - D - m) \} \right\rangle \quad (4.46)$$

Replacing $\tilde{y} (n)$ by :

$$\tilde{y} (n) = \int_{-\infty}^{\infty} a_f (u) x (nT_0 - u) d(u) \quad (4.47)$$

yields :

$$\nabla_{a_b}^{TAMSE} (m) = \sigma_{d_0}^2 a_b (m) - \int_{-\infty}^{\infty} a_f (u) \left\langle E \{ d_0^* (n - D - m) x (nT_0 - u) \} \right\rangle d(u) \quad (4.48)$$

substituting $x(t)$ by $x(t) = r(t) \otimes \mathfrak{K}(t)$ and using (4.1) to (4.4) yields :

$$\begin{aligned} E \{ d_0^* (n - D - m) x (nT - u) \} &= \sigma_{d_0^{(r)}}^2 z_0^{(1)} ((D + m) T_0 - t_0^{(r)} - u) e^{j2\pi\phi_0^{(r)}} + \\ &\quad \sigma_{d_0^{(i)}}^2 z_0^{(2)} ((D + m) T_0 - t_0^{(i)} - u) e^{j2\pi\phi_0^{(i)}} \end{aligned} \quad (4.49)$$

where :

$$\begin{aligned} z_0^{(1)} &= p_0^{(1)} (t) \otimes \mathfrak{K} (t) \\ z_0^{(2)} &= p_0^{(2)} (t) \otimes \mathfrak{K} (t) \end{aligned} \quad (4.50)$$

substituting (4.49) in (4.48) yields :

$$\begin{aligned} \nabla_{a_b}^{TAMSE} (m) &= \sigma_{d_0}^2 a_b (m) - \int_{-\infty}^{\infty} a_f (u) (\sigma_{d_0^{(r)}}^2 z_0^{(1)} ((D + m) T_0 - t_0^{(r)} - u) e^{j2\pi\phi_0^{(r)}} + \\ &\quad \sigma_{d_0^{(i)}}^2 z_0^{(2)} ((D + m) T_0 - t_0^{(i)} - u) e^{j2\pi\phi_0^{(i)}}) d(u) \end{aligned} \quad (4.51)$$

Therefore the optimum coefficient for feed back filters are :

$$a_b(m) = \begin{cases} \left(\frac{1}{\sigma_{d_0}^2} \right) \int_{-\infty}^{\infty} a_f(u) (\sigma_{d_0^{(r)}}^2 z_0^{(1)} ((D+m)T_0 - t_0^{(r)} - u) e^{j2\pi\phi_0^{(r)}} + \\ \sigma_{d_0^{(i)}}^2 z_0^{(2)} ((D+m)T_0 - t_0^{(i)} - u) e^{j2\pi\phi_0^{(i)}}) d(u) & m > 0 \\ 0 & m \leq 0 \end{cases} \quad (4.52)$$

$$a_b(m) = \begin{cases} \left(\frac{1}{\sigma_{d_0}^2} \right) \left[a_f(t) \otimes (\sigma_{d_0^{(r)}}^2 z_0^{(1)} (t - t_0^{(r)}) e^{j2\pi\phi_0^{(r)}} + \right. \\ \left. \sigma_{d_0^{(i)}}^2 z_0^{(2)} (t - t_0^{(i)}) e^{j2\pi\phi_0^{(i)}} \right] \Big|_{t = (D+m)T_0} & m > 0 \\ 0 & m \leq 0 \end{cases} \quad (4.53)$$

For BPSK and balanced QPSK, (4.53) is simplified to the following equation :

$$a_b(m) = \begin{cases} \left[a_f(t) \otimes z_0^{(1)} (t - t_0^{(r)}) e^{j2\pi\phi_0^{(r)}} \right] \Big|_{t = (D+m)T_0} & m > 0 \\ 0 & m \leq 0 \end{cases} \quad (4.54)$$

and for OQPSK, it is simplified to :

$$a_b(m) = \begin{cases} \left[a_f(t) \otimes (z_0^{(1)} (t - t_0^{(r)}) + z_0^{(1)} (t - t_0^{(r)} - \frac{T_0}{2})) e^{j2\pi\phi_0^{(r)}} \right] \Big|_{t = (D+m)T_0} & m > 0 \\ 0 & m \leq 0 \end{cases} \quad (4.55)$$

The same procedure has to be repeated to determine the optimum forward filter. The gradient of TAMSE according to $a_f(t)$ in the time u is :

$$\nabla_{a_f}^{TAMSE}(u) = \langle E \{ (\tilde{y}(n) - (d_0(n-D) + f(n))) \nabla_{a_f}^{\tilde{y}(n)}(u) \} \rangle \quad (4.56)$$

From (4.47)

$$\nabla_{a_f}^{\tilde{y}(n)}(u) = x^*(nT_0 - u) \quad (4.57)$$

Replacing the above equation in (4.56) and also considering (4.49), second and 3th term of (4.56) will be :

$$\begin{aligned} \langle E \{ d_0(n-D) \nabla_{a_f}^{\vec{y}^{(n)}}(u) \} \rangle &= \sigma_{d_0^{(r)}}^2 z_0^{(1)*} (DT_0 - t_0^{(r)} - u) e^{-j2\pi\phi_0^{(r)}} + \\ &\quad \sigma_{d_0^{(i)}}^2 z_0^{(2)*} (DT_0 - t_0^{(i)} - u) e^{-j2\pi\phi_0^{(i)}} \end{aligned} \quad (4.58)$$

$$\begin{aligned} \langle E \{ f(n) \nabla_{a_f}^{\vec{y}^{(n)}}(u) \} \rangle &= \sigma_{d_0^{(r)}}^2 e^{-j2\pi\phi_0^{(r)}} \sum_{m=1}^{\infty} z_0^{(1)*} ((D+m)T_0 - u - t_0^{(r)}) a_b(m) + \\ &\quad \sigma_{d_0^{(i)}}^2 e^{-j2\pi\phi_0^{(i)}} \sum_{m=1}^{\infty} z_0^{(2)*} ((D+m)T_0 - u - t_0^{(i)}) a_b(m) \end{aligned} \quad (4.59)$$

The first term of (4.56) is :

$$\begin{aligned} \langle E \{ \tilde{y}(n) \nabla_{a_f}^{\vec{y}^{(n)}}(u) \} \rangle &= \int_{-\infty}^{\infty} a_f(v) \langle E \{ x(nT_0 - v) x^*(nT_0 - u) \} \rangle dv \\ &= \int_{-\infty}^{\infty} a_f(v) \langle R_x(nT_0 - v, nT_0 - u) \rangle dv \end{aligned} \quad (4.60)$$

Using (2.58) yields :

$$\langle R_x(nT_0 - v, nT_0 - u) \rangle = \sum_{m=-\infty}^{\infty} e^{-j\pi\left(\frac{m}{T_0}\right)(v+u)} R_x^{\frac{m}{T_0}}(u-v) \quad (4.61)$$

Replacing (4.61) in (4.60) yields :

$$\begin{aligned} \langle E \{ \tilde{y}(n) \nabla_{a_f}^{\vec{y}^{(n)}}(u) \} \rangle &= \sum_{m=-\infty}^{\infty} \int a_f(v) R_{xx}^{\frac{m}{T_0}}(u-v) e^{-j\pi\frac{m}{T_0}(u+v)} dv \\ &= \sum_{m=-\infty}^{\infty} e^{-j2\pi\frac{m}{T_0}u} \left[a_f(u) \otimes R_{xx}^{\frac{m}{T_0}}(u) e^{j\pi\frac{m}{T_0}u} \right] \end{aligned} \quad (4.62)$$

Substituting (4.59) and (4.62) in (4.56) and tending the gradient to zero yields the following equation for the optimum filters :

$$\begin{aligned}
& \sum_{m=-\infty}^{\infty} e^{-j2\pi \frac{m}{T_0} u} \left[a_f(u) \otimes R_{xx}^{\frac{m}{T_0}}(u) e^{j\pi \left(\frac{m}{T_0}\right) u} \right] = \\
& \sigma_{d_0^{(r)}}^2 e^{-j2\pi \phi_0^{(r)}} \left(z_0^{(1)*} (DT_0 - u - t_0^{(r)}) + \sum_{m=1}^{\infty} z_0^{(1)*} ((D+m)T_0 - u - t_0^{(r)}) a_b(m) \right) + (4.63) \\
& \sigma_{d_0^{(i)}}^2 e^{-j2\pi \phi_0^{(i)}} \left(z_0^{(2)*} (DT_0 - u - t_0^{(i)}) + \sum_{m=1}^{\infty} z_0^{(2)*} ((D+m)T_0 - u - t_0^{(i)}) a_b(m) \right)
\end{aligned}$$

Taking the Fourier transform of (4.63) with respect to u yields :

$$\begin{aligned}
& \sum_{m=-\infty}^{\infty} A_f\left(f + \frac{m}{T_0}\right) S_{xx}^{\frac{m}{T_0}}\left(f + \frac{m}{2T_0}\right) \\
& = \sigma_{d_0^{(r)}}^2 e^{-j2\pi \phi_0^{(r)}} \left[1 + \sum_{m=1}^{\infty} a_b(m) e^{-j2\pi m T_0 f} \right] e^{-j2\pi (DT_0 - t_0^{(r)}) f} Z_0^{(1)*}(f) + \\
& \sigma_{d_0^{(i)}}^2 e^{-j2\pi \phi_0^{(i)}} \left[1 + \sum_{m=1}^{\infty} a_b(m) e^{-j2\pi m T_0 f} \right] e^{-j2\pi (DT_0 - t_0^{(i)}) f} Z_0^{(2)*}(f) \quad (4.64) \\
& = (\sigma_{d_0^{(r)}}^2 e^{-j2\pi [\phi_0^{(r)} + (DT_0 - t_0^{(r)}) f]} Z_0^{(1)*}(f) + \sigma_{d_0^{(i)}}^2 e^{-j2\pi [\phi_0^{(i)} + (DT_0 - t_0^{(i)}) f]} Z_0^{(2)*}(f)) \cdot \\
& (A_b(e^{j2\pi f T_0}) + 1)
\end{aligned}$$

where $A_f(f)$ and $A_b(e^{j2\pi f})$ are the Fourier transforms of $a_f(t)$ and $a_b(t)$ respectively.

Equation (4.64) is simplified to the following equation for BPSK and balanced QPSK :

$$\begin{aligned}
& \sum_{m=-\infty}^{\infty} A_f\left(f + \frac{m}{T_0}\right) S_{xx}^{\frac{m}{T_0}}\left(f + \frac{m}{2T_0}\right) \quad (4.65) \\
& = (\sigma_{d_0}^2 e^{-j2\pi [\phi_0^{(r)} + (DT_0 - t_0^{(r)}) f]} Z_0^{(1)*}(f)) \cdot (A_b(e^{j2\pi f T_0}) + 1)
\end{aligned}$$

and simplified to :

$$\begin{aligned}
& \sum_{m=-\infty}^{\infty} A_f\left(f + \frac{m}{T_0}\right) S_{xx}^{\frac{m}{T_0}}\left(f + \frac{m}{2T_0}\right) \quad (4.66) \\
& = (\sigma_{d_0}^2 e^{-j2\pi [\phi_0^{(r)} + (DT_0 - t_0^{(r)}) f]} Z_0^{(1)*}(f)) (1 + e^{j\pi f T_0}) (A_b(e^{j2\pi f T_0}) + 1)
\end{aligned}$$

for OQPSK.

The minimum TAMSE (MTAMSE) is the correlation between the error and data [88], therefore :

$$\begin{aligned} MTAMSE &= \langle E \{ (d_0(n-D) - b(n)) d_0^*(n-D) \} \rangle \\ &= \sigma_{d_0}^2 - \langle E \{ d_0^*(n-D) \tilde{y}(n) \} \rangle \end{aligned} \quad (4.67)$$

Using (4.49), the MTAMSE which can be reached is equal to the following equation :

$$\begin{aligned} MTAMSE &= \sigma_{d_0}^2 - \int_{-\infty}^{\infty} a_f(u) (\sigma_{d_0^{(r)}}^2 e^{j2\pi\phi_0^{(r)}} z_0^{(1)}(DT_0 - u - t_0^{(r)}) + \\ &\quad \sigma_{d_0^{(i)}}^2 e^{j2\pi\phi_0^{(i)}} z_0^{(2)}(DT_0 - u - t_0^{(i)})) d(u) \\ &= \sigma_{d_0}^2 - \left[a_f(t) \otimes (\sigma_{d_0^{(r)}}^2 e^{j2\pi\phi_0^{(r)}} z_0^{(1)}(t - t_0^{(r)}) + \sigma_{d_0^{(i)}}^2 e^{j2\pi\phi_0^{(i)}} z_0^{(2)}(t - t_0^{(i)})) \right] \Big|_{t=DT_0} \end{aligned} \quad (4.68)$$

4.4.2.2 Finite length

If the FSE and the feedback filter are modeled by finite length tap delay lines, the DFE structure will be as Figure 4.14. In the figure, the output of DFE can be found according to the following equation :

$$b(n) = \underline{a}_f \tilde{x}(nM_0)^T - \underline{a}_b d(n-D-1) \quad (4.69)$$

where $(.)^T$ denotes the transpose of the argument,

$$\begin{aligned} \underline{a}_f^T &= [a_f(0) \ a_f(1) \dots a_f(N_f)] \\ \underline{a}_b^T &= [a_b(1) \ a_b(2) \dots a_b(N_b)] \\ \tilde{x}^T(nM_0) &= [\tilde{x}(nM_0) \ \tilde{x}(nM_0-1) \dots \tilde{x}(nM_0-N_f)] \\ \underline{d}^T(n-D-1) &= [d_0(n-D-1) \ d_0(n-D-2) \dots d_0(n-D-N_b)] \end{aligned} \quad (4.70)$$

where N_f and N_b are the number of taps in the forward and feedback filters, respectively. Defining :

$$\begin{aligned} \underline{W}^T &= [\underline{a}_f^T \quad -\underline{a}_b^T] \\ \underline{V}^T(n) &= [\tilde{x}^T(nM_0) \ \underline{d}^T(n-D-1)] \end{aligned} \quad (4.71)$$

and substituting in (4.69) yields :

Figure 4.14 Finite length conventional DFE.

$$b(n) = \underline{W}^T \underline{V}(n) \quad (4.72)$$

Therefore the optimum Weiner filter coefficients can be found by the following equation [52] :

$$\underline{W}_o = \underline{R}^{-1} \underline{P} \quad (4.73)$$

where

$$\begin{aligned} \underline{R} &= \langle E \{ [\underline{V}^T(n) \underline{V}^*(n)] \} \rangle \\ \underline{P} &= \langle E \{ [\underline{V}^T(n) d_0^*(n-D)] \} \rangle \end{aligned} \quad (4.74)$$

Substituting (4.71) in (4.74) yields :

$$\begin{aligned} \underline{R} &= \begin{bmatrix} \langle E \{ \tilde{x}^T(nM_0) \tilde{x}^*(nM_0) \} \rangle & \langle E \{ \tilde{x}^T(nM_0) d^*(n-D-1) \} \rangle \\ \langle E \{ d^T(n-D-1) \tilde{x}^*(nM_0) \} \rangle & \langle E \{ d^T(n-D-1) d^*(n-D-1) \} \rangle \end{bmatrix} \\ &= \begin{bmatrix} \underline{R}_{\tilde{x}} & \underline{R}_{\tilde{x}d} \\ \underline{R}_{d\tilde{x}} & \sigma_{d_0}^2 I_{N_b \times N_b} \end{bmatrix} \end{aligned} \quad (4.75)$$

where $I_{N_b \times N_b}$ is the N_b by N_b unit matrix. $\underline{R}_{\tilde{x}}$ can be found by :

$$\begin{aligned} \underline{R}_{\tilde{x}}(i, k) &= \langle E \{ \tilde{x}(nM_0 - i - 1) \tilde{x}^*(nM_0 - k - 1) \} \rangle \\ &= \langle \underline{R}_{\tilde{x}}(nM_0 - i - 1, nM_0 - k - 1) \rangle \\ &= \sum_{m=-\infty}^{\infty} \underline{R}_{\tilde{x}}^{\frac{m}{T_0}}((k-i)M_0) e^{j\pi m(k-i)} \\ &= \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \underline{R}_{\tilde{x}}^{\frac{m}{T_0} + \frac{l}{T_s}}((k-i)M_0 T_s) e^{j\pi m(k-i)} e^{j\pi l(k-i)M_0} \\ &= \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \underline{R}_{\tilde{x}}^{\frac{m+lM_0}{T_0}}((k-i)T_0) e^{j\pi(m+lM_0)(k-i)} \end{aligned} \quad (4.76)$$

and $\underline{R}_{\tilde{x}d}$ can be found by :

$$\begin{aligned} \underline{R}_{\tilde{x}d}(i, k) &= \langle E \{ \tilde{x}(nM_0 - i - 1) d^*(n-D-k) \} \rangle \\ &= \sigma_{d_0}^2 z_0^{(1)} ((D+k)T_0 - (i+1)T_s - t_0^{(r)}) e^{j2\pi\phi_0^{(r)}} + \\ &\quad \sigma_{d_0}^2 z_0^{(2)} ((D+k)T_0 - (i+1)T_s - t_0^{(i)}) e^{j2\pi\phi_0^{(i)}} \end{aligned} \quad (4.77)$$

and consequently $\underline{R}_{\tilde{d}\tilde{x}}$

$$\begin{aligned} R_{\tilde{d}\tilde{x}}(i, k) &= \langle E \{ \tilde{d}(n-D-i) \tilde{x}^*(nM_0-k-1) \} \rangle \\ &= \sigma_{d_0^{(r)}}^2 z_0^{(1)*} ((D+i)T_0 - (k+1)T_s - t_0^{(r)}) e^{-j2\pi\phi_0^{(r)}} + \\ &\quad \sigma_{d_0^{(i)}}^2 z_0^{(2)*} ((D+i)T_0 - (k+1)T_s - t_0^{(i)}) e^{-j2\pi\phi_0^{(i)}} \end{aligned} \quad (4.78)$$

On the other hand, the vector $\underline{P}$ can be calculated by the following procedure :

$$\begin{aligned} \underline{P} &= \langle E \{ [\underline{V}^T(n) d_0^*(n-D)] \} \rangle \\ &= \begin{bmatrix} \langle E \{ \tilde{x}^T(nM_0) d_0^*(n-D) \} \rangle \\ \langle E \{ \underline{d}^T(n-D-1) d_0^*(n-D) \} \rangle \end{bmatrix} \\ &= \begin{bmatrix} \underline{P}_{\tilde{x}d} \\ 0_{N_b \times 1} \end{bmatrix} \end{aligned} \quad (4.79)$$

where $0_{N_b \times 1}$ is the N_b by one zero vector and

$$\begin{aligned} \underline{P}_{\tilde{x}d}(i) &= \sigma_{d_0^{(r)}}^2 z_0^{(1)} (DT_0 - (i+1)T_s - t_0^{(r)}) e^{j2\pi\phi_0^{(r)}} + \\ &\quad \sigma_{d_0^{(i)}}^2 z_0^{(2)} (DT_0 - (i+1)T_s - t_0^{(i)}) e^{j2\pi\phi_0^{(i)}} \end{aligned} \quad (4.80)$$

The corresponding MTAMSE can be calculated from the optimum coefficient by the following equation :

$$MTAMSE = \sigma_{d_0}^2 - \underline{P}^* \underline{W}_o \quad (4.81)$$

It is apparent from (4.64) and (4.76) that the conventional DFE does not use the information of the CCSD of the received signal. Furthermore, it does not use the information of incommensurate periodicities of the CSD. Therefore, to obtain the optimum DFE, we should use GFSE or FKESH-FSE as the forward filter of DFE. We shall refer to these structures as generalized DFE (GDFE) and frequency shift DFE (FRESH-DFE).

4.5 FRESH-DFE

Figure 4.15 shows the structure of FRESH-DFE. This structure is created by FRESH-FSE followed by a feedback path. Generally FRESH-DFE can be divided into

Figure 4.15 The structure of FRESH-DFE.

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three parts. The first part is a LCL- FRESH filter, the second part is a baud rate sampler and the third part is a feedback filter. The first part exploits the information of the CSD and CCSD at incommensurate cyclic frequencies, the second part extracts the information at the symbol rate and it's harmonics and the third part removes the residual ISI caused by earlier symbols.

Similar to the conventional DFE, FRESH-DFE minimizes the TAMSE too. In Appendix B, FRESH-DFE is analyzed by the method used for the conventional DFE. The results show that the optimum forward and backward filters must satisfy the following equations :

$$h_b(m) = \begin{cases} \frac{\sigma_{d_0^{(r)}}^2}{\sigma_{d_0}^2} \left[(a_0(t) \otimes z_0^{(1)}(t-t_0^{(r)}) e^{j2\pi\phi_0^{(r)}}) + \right. \\ \left. (b_0(t) \otimes z_0^{(1)*}(t-t_0^{(r)}) e^{-j2\pi\phi_0^{(r)}}) \right] \Big|_{t=(D+m)T_0} + & m > 0 \\ \frac{\sigma_{d_0^{(i)}}^2}{\sigma_{d_0}^2} \left[(a_0(t) \otimes z_0^{(2)}(t-t_0^{(i)}) e^{j2\pi\phi_0^{(i)}}) - \right. \\ \left. (b_0(t) \otimes z_0^{(2)*}(t-t_0^{(i)}) e^{-j2\pi\phi_0^{(i)}}) \right] \Big|_{t=(D+m)T_0} & m \leq 0 \end{cases} \quad (4.82)$$

$$\sum_m \sum_l A_m \left(f + \frac{l}{T_0}\right) S_x^{\left(\frac{l}{T_0} - \alpha_m\right)} \left(f + \frac{l}{2T_0} - \frac{\alpha_m}{2}\right) + \sum_l \sum_l B_l \left(f + \frac{l}{T_0}\right) \left(S_{xx}^{\left(\beta_l - \frac{l}{T_0}\right)} \left(f + \frac{l}{2T_0} - \frac{\beta_l}{2}\right)\right)^* = \quad (4.83)$$

$$(\sigma_{d_0^{(r)}}^2 e^{-j2\pi(\phi_0^{(r)} + (DT_0 - t_0^{(r)})f)} Z_0^{(1)*}(f) + \sigma_{d_0^{(i)}}^2 e^{-j2\pi(\phi_0^{(i)} + (DT_0 - t_0^{(i)})f)} Z_0^{(2)*}(f)) (1 + H_b(e^{j2\pi f T_0}))$$

$$\sum_m \sum_l A_m \left(f + \frac{l}{T_0}\right) S_x^{\left(\frac{l}{T_0} + \alpha_k - \alpha_m\right)} \left(f + \frac{l}{2T_0} - \frac{\alpha_m + \alpha_k}{2}\right) + \quad (4.84)$$

$$\sum_l \sum_l B_l \left(f + \frac{l}{T_0}\right) \left(S_{xx}^{\left(\beta_l - \alpha_k - \frac{l}{T_0}\right)} \left(f + \frac{l}{2T_0} - \frac{\beta_l + \alpha_k}{2}\right)\right)^* = 0 \quad [\alpha_k]_{l_0} \neq 0$$

$$\sum_k \sum_l A_k \left(f + \frac{l}{T_0}\right) S_x^{\left(\frac{l}{T_0} - \alpha_k\right)} \left(f + \frac{l}{2T_0} - \frac{\alpha_k}{2}\right) + \sum_m \sum_l B_m \left(f + \frac{l}{T_0}\right) \left(S_x^{\left(\frac{l}{T_0} - \beta_m\right)} \left(-f - \frac{l}{2T_0} + \frac{\beta_m}{2}\right)\right) = \quad (4.85)$$

$$(\sigma_{d_0^{(r)}}^2 e^{j2\pi(\phi_0^{(r)} - (DT_0 - t_0^{(r)})f)} Z_0^{(1)}(f) - \sigma_{d_0^{(i)}}^2 e^{j2\pi(\phi_0^{(i)} - (DT_0 - t_0^{(i)})f)} Z_0^{(2)}(f)) (1 + H_b(e^{j2\pi f T_0}))$$

$$\sum_k \sum_l A_k \left(f + \frac{l}{T_0} \right) S_{\alpha_k}^{(l)} \left(f + \frac{l}{T_0} - \frac{\beta_l + \alpha_k}{2} \right) + \sum_m \sum_l B_m \left(f + \frac{l}{T_0} \right) \left(S_x^{(\beta_l - \alpha_k - \frac{l}{T_0})} \left(-f - \frac{l}{2T_0} + \frac{\beta_l + \beta_m}{2} \right) \right) = 0 \quad [\beta_l]_{T_0^{-1}} \neq 0 \quad (4.86)$$

It is also shown in Appendix B that the achievable MTAMSE is :

$$MTAMSE = \sigma_{d_0}^2 - \left[a_0(t) \otimes (\sigma_{d_0^{(r)}}^2 e^{j2\pi\phi_0^{(r)}} z_0^{(1)}(t - t_0^{(r)}) + \sigma_{d_0^{(i)}}^2 e^{j2\pi\phi_0^{(i)}} z_0^{(2)}(t - t_0^{(i)})) \right] \Big|_{t=DT_0} - \left[b_0(t) \otimes (\sigma_{d_0^{(r)}}^2 e^{-j2\pi\phi_0^{(r)}} z_0^{(1)*}(t - t_0^{(r)}) - \sigma_{d_0^{(i)}}^2 e^{-j2\pi\phi_0^{(i)}} z_0^{(2)*}(t - t_0^{(i)})) \right] \Big|_{t=DT_0} \quad (4.87)$$

Although all the forward filters do not effect (4.87) directly, their influences in $a_0(t)$ and $b_0(t)$ effect (4.87) indirectly. According to (4.83) to (4.87), FRESH-DFE uses both CSD and CCSD of the received signal.

It is easy to show that equations (4.82), (4.83), and (4.87) are reduced to equations (4.54), (4.65) and (4.68), respectively, for balanced QPSK systems. Thus, the conventional DFE and FRESH-DFE are equivalent for balanced QPSK systems.

4.6 GDFE

Figure 4.16 shows the structure of GDFE. This structure is created by a GFSE followed by a feedback filter. Since the GFSE and FRESH-FSE are equivalent [Appendix C], GDFE and FRESH-DFE are equivalent too. Therefore, the analysis for FRESH-DFE is valid for GDFE. However, it should be noted that the optimum feed-forward filters of FRESH-DFE and GDFE are the frequency-time dual. The optimum feedback filter of GDFE and FRESH-DFE are equivalent.

In the single symbol rate systems, the structure of GDFE and FRESH-DFE will be identical (Figure 4.17). We refer to this structure as the optimum DFE (ODFE). The complexity of ODFE is minimized if the channel spacings are multiple of half the symbol rate (Figure 4.18). In this case the structure of ODFE is almost as simple as DFE.

Figure 4.16 The structure of GDFE.

Figure 4.17 The structure of ODFE.

Figure 4.18 The structure of ODFE in systems with channel spacings multiple of half the symbol rate.

4.7 ODFE

Since ODFE is a special case of FRESH-DFE (or GDFE), the analysis of infinite length FRESH-DFE (Appendix B) is valid for ODF too. However, the finite length ODFE is analyzed in the next section

4.7.1 Finite Length ODFE

The analysis is simpler if we consider the special case in which the channel spacings are multiple of half the symbol rate. However, the analysis can be generalized for any other cases. Figure 4.19 shows the structure of the finite length ODFE. In the figure, the output of DFE can be found according to the following equation :

$$b(n) = \underline{a}_f \tilde{x}(nM_0)^T + \underline{a}_c \tilde{x}(nM_0)^H - \underline{a}_b \underline{d}(n-D-1) \quad (4.88)$$

where $(.)^H$ denotes the conjugate transpose of the argument,

$$\begin{aligned} \underline{a}_f^T &= [a_f(0) \ a_f(1) \dots a_f(N_f)] \\ \underline{a}_c^T &= [a_c(0) \ a_c(1) \dots a_c(N_f)] \\ \underline{a}_b^T &= [a_b(1) \ a_b(2) \dots a_b(N_b)] \\ \tilde{x}^T(nM_0) &= [\tilde{x}(nM_0) \ \tilde{x}(nM_0-1) \dots \tilde{x}(nM_0-N_f)] \\ \underline{d}^T(n-D-1) &= [d_0(n-D-1) \ d_0(n-D-2) \dots d_0(n-D-N_b)] \end{aligned} \quad (4.89)$$

where N_f and N_b are the number of taps in the forward and feedback filters respectively. Defining :

$$\begin{aligned} \underline{W}^T &= [\underline{a}_f^T \ \underline{a}_c^T \ -\underline{a}_b^T] \\ \underline{V}^T(n) &= [\tilde{x}^T(nM_0) \ \tilde{x}^H(nM_0) \ \underline{d}^T(n-D-1)] \end{aligned} \quad (4.90)$$

and substituting in (4.88) yields :

$$b(n) = \underline{W}^T \underline{V}(n) \quad (4.91)$$

Therefore, the optimum Weiner filter coefficients can be found by the following equation :

$$\underline{W}_o = R^{-1} P \quad (4.92)$$

where

Figure 4.19 The finite length ODFE in the system with channel spacings multiple of half the symbol rate.

$$\begin{aligned}
 R &= \langle E \{ [\underline{V}^T(n) \underline{V}^*(n)] \} \rangle \\
 P &= \langle E \{ [\underline{V}^T(n) d_0^*(n-D)] \} \rangle
 \end{aligned}
 \quad (4.93)$$

Substituting (4.90) in (4.93) yields :

$$\begin{aligned}
 R &= \\
 &\begin{bmatrix} \langle E \{ \tilde{x}^T(nM_0) \tilde{x}^*(nM_0) \} \rangle & \langle E \{ \tilde{x}^T(nM_0) \tilde{x}(nM_0) \} \rangle & \langle E \{ \tilde{x}^T(nM_0) d^*(n-D-1) \} \rangle \\ \langle E \{ \tilde{x}^H(nM_0) \tilde{x}^*(nM_0) \} \rangle & \langle E \{ \tilde{x}^H(nM_0) \tilde{x}(nM_0) \} \rangle & \langle E \{ \tilde{x}^H(nM_0) d^*(n-D-1) \} \rangle \\ \langle E \{ d^T(n-D-1) \tilde{x}^*(nM_0) \} \rangle & \langle E \{ d^T(n-D-1) \tilde{x}(nM_0) \} \rangle & \langle E \{ d^T(n-D-1) d^*(n-D-1) \} \rangle \end{bmatrix} \\
 &= \begin{bmatrix} \underline{R}_{\tilde{x}} & \underline{R}_{\tilde{x}\tilde{x}} & \underline{R}_{\tilde{x}d} \\ \underline{R}_{\tilde{x}\tilde{x}}^* & \underline{R}_{\tilde{x}\tilde{x}} & \underline{R}_{\tilde{x}d}^* \\ \underline{R}_{d\tilde{x}} & \underline{R}_{d\tilde{x}}^* & \sigma_{d_0}^2 I_{N_b \times N_b} \end{bmatrix}
 \end{aligned}
 \quad (4.94)$$

$\underline{R}_{\tilde{x}}$, $\underline{R}_{\tilde{x}d}$ and $\underline{R}_{d\tilde{x}}$ can be found by (4.76) (4.77) and (4.78). The same procedure as the one used in (4.76), (4.77) and (4.78) can be used to find $\underline{R}_{\tilde{x}\tilde{x}}^*$, $\underline{R}_{\tilde{x}\tilde{x}}$, $\underline{R}_{\tilde{x}\tilde{x}}^*$, $\underline{R}_{\tilde{x}\tilde{x}}$ and $\underline{R}_{d\tilde{x}}^*$:

$$\begin{aligned}
 \underline{R}_{\tilde{x}\tilde{x}}^*(i, k) &= \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \underline{R}_{\tilde{x}\tilde{x}}^* \left(\frac{m+1M_0}{T_0} \right) ((k-i)T_0) e^{j\pi(m+1M_0)(k-i)} \\
 &= \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left(\underline{R}_{\tilde{x}\tilde{x}} \left(\frac{m+1M_0}{T_0} \right) ((k-i)T_0) \right)^* e^{j\pi(m+1M_0)(k-i)}
 \end{aligned}
 \quad (4.95)$$

$$\underline{R}_{\tilde{x}\tilde{x}}(i, k) = \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \underline{R}_{\tilde{x}\tilde{x}} \left(\frac{m+1M_0}{T_0} \right) ((k-i)T_0) e^{j\pi(m+1M_0)(k-i)} \quad (4.96)$$

$$\begin{aligned}
 \underline{R}_{\tilde{x}\tilde{x}}(i, k) &= \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \underline{R}_{\tilde{x}\tilde{x}} \left(\frac{m+1M_0}{T_0} \right) ((k-i)T_0) e^{j\pi(m+1M_0)(k-i)} \\
 &= \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left(\underline{R}_{\tilde{x}\tilde{x}} \left(\frac{m+1M_0}{T_0} \right) ((k-i)T_0) \right)^* e^{j\pi(m+1M_0)(k-i)}
 \end{aligned}
 \quad (4.97)$$

$$\begin{aligned}
 R_{\tilde{x}d}^*(i, k) &= \sigma_{d_0^{(r)}}^2 z_0^{(1)*} ((D+k)T_0 - (i+1)T_s - t_0^{(r)}) e^{-j2\pi\phi_0^{(r)}} - \\
 &\quad \sigma_{d_0^{(i)}}^2 z_0^{(2)*} ((D+k)T_0 - (i+1)T_s - t_0^{(i)}) e^{-j2\pi\phi_0^{(i)}}
 \end{aligned} \quad (4.98)$$

$$\begin{aligned}
 R_{d\tilde{x}}^*(i, k) &= \sigma_{d_0^{(r)}}^2 z_0^{(1)} ((D+i)T_0 - (k+1)T_s - t_0^{(r)}) e^{j2\pi\phi_0^{(r)}} - \\
 &\quad \sigma_{d_0^{(i)}}^2 z_0^{(2)} ((D+i)T_0 - (k+1)T_s - t_0^{(i)}) e^{j2\pi\phi_0^{(i)}}
 \end{aligned} \quad (4.99)$$

On the other hand the vector $\underline{P}$ can be calculated by the following procedure :

$$\begin{aligned}
 \underline{P} &= \langle E \{ [\underline{V}^T(n) d_0^*(n-D)] \} \rangle \\
 &= \begin{bmatrix} \langle E \{ \tilde{x}^T(nM_0) d_0^*(n-D) \} \rangle \\ \langle E \{ \tilde{x}^H(nM_0) d_0^*(n-D) \} \rangle \\ \langle E \{ \underline{d}^T(n-D-1) d_0^*(n-D) \} \rangle \end{bmatrix} \\
 &= \begin{bmatrix} \underline{P}_{\tilde{x}d} \\ \underline{P}_{\tilde{x}d}^* \\ 0_{N_b \times 1} \end{bmatrix}
 \end{aligned} \quad (4.100)$$

where

$$\begin{aligned}
 \underline{P}_{\tilde{x}d}^*(i) &= \sigma_{d_0^{(r)}}^2 z_0^{(1)*} (DT_0 - (i+1)T_s - t_0^{(r)}) e^{-j2\pi\phi_0^{(r)}} - \\
 &\quad \sigma_{d_0^{(i)}}^2 z_0^{(2)*} (DT_0 - (i+1)T_s - t_0^{(i)}) e^{-j2\pi\phi_0^{(i)}}
 \end{aligned} \quad (4.101)$$

The corresponding MTAMSE can be calculated from the optimum coefficients by the following equation :

$$MTAMSE = \sigma_{d_0}^2 - \underline{P}^* \underline{W}_o \quad (4.102)$$

As it is apparent from (4.92) and (4.94) that the ODFE uses the information of the CCSD of the received signal as well as CSD.

4.8 Multi-Antenna Receiver

Spectral redundancy of digitally modulated signals can be further exploited by

using multi-antenna receiver. Figure 4.20 shows a block diagram of a multi-antenna receiver. In this figure, superscript (l) represents all the related signal to the l th antenna. If L is the number of antenna, there exist L received signals which can be represented by the following equation :

$$r_l(t) = s_0^{(l)}(t) + s_c^{(l)}(t) + s_a^{(l)}(t) + n^{(l)}(t) \quad l = 1, 2, \dots, L \quad (4.103)$$

where $n^{(l)}(t)$, $s_0^{(l)}(t)$, $s_a^{(l)}(t)$ and $s_c^{(l)}(t)$ are the AWGN, SOI, ACI, and CCI at the input of l th antenna respectively, and can be represented by the following equations :

$$s_0^{(l)}(t) = \sum_{k=-\infty}^{\infty} d_0^{(r)}(k) p_0^{(1,l)}(t - kT_0 - t_0^{(r)}) e^{j2\pi\phi_0^{(r)}} + \\ j d_0^{(i)}(k) p_0^{(2,l)}(t - kT_0 - t_0^{(i)}) e^{j2\pi\phi_0^{(i)}} \quad (4.104)$$

$$s_c^{(l)}(t) = \sum_{m=1}^{M_c} \sum_{k=-\infty}^{\infty} d_{0m}^{(r)}(k) p_{0m}^{(1,l)}(t - kT_{0m} - t_{0m}^{(r)}) e^{j2\pi\phi_{0m}^{(r)}} + \\ j d_{0m}^{(i)}(k) p_{0m}^{(2,l)}(t - kT_{0m} - t_{0m}^{(i)}) e^{j2\pi\phi_{0m}^{(i)}} \quad (4.105)$$

$$s_a^{(l)}(t) = \sum_{\substack{m=-M_a/2 \\ l \neq 0}}^{M_a/2} \sum_{k=-\infty}^{\infty} (d_m^{(r)}(k) p_m^{(1,l)}(t - kT_m - t_m^{(r)}) e^{j2\pi\phi_m^{(r)}} + \\ j d_m^{(i)}(k) p_m^{(2,l)}(t - kT_m - t_m^{(i)}) e^{j2\pi\phi_m^{(i)}}) e^{j2\pi f_m t} \quad (4.106)$$

where $p_{0m}^{(1,l)}(t)$, $p_{0m}^{(2,l)}(t)$, $p_m^{(1,l)}(t)$, $p_m^{(2,l)}(t)$, $p_0^{(1,l)}(t)$, and $p_0^{(2,l)}(t)$ are defined by :

$$\begin{aligned} p_{0m}^{(1,l)}(t) &= g_{0m}^{(l)} q_{0m}^{(r)}(t) \otimes c_{0l}^{(l)}(t) \\ p_{0m}^{(2,l)}(t) &= g_{0m}^{(l)} q_{0m}^{(i)}(t) \otimes c_{0l}^{(l)}(t) \\ p_m^{(1,l)}(t) &= g_m^{(l)} q_m^{(r)}(t) \otimes c_l^{(l)}(t) \\ p_m^{(2,l)}(t) &= g_m^{(l)} q_m^{(i)}(t) \otimes c_l^{(l)}(t) \\ p_0^{(1,l)}(t) &= g_0^{(l)} q_0^{(r)}(t) \otimes c_0^{(l)}(t) \\ p_0^{(2,l)}(t) &= g_0^{(l)} q_0^{(i)}(t) \otimes c_0^{(l)}(t) \end{aligned} \quad (4.107)$$

The optimum multi-antenna receiver is a generalization of FRESH-DFE (or GDFE). Figure 4.21 shows an optimum multi-antenna FRESH-DFE (MA-FRESH-DFE).

Figure 4.20 Communication with a multi-antenna receiver.

Figure 4.21 Multi Antenna FRESH-DFE.

In this figure, L FRESH-filter are used in forward filtering to exploit all the spectral redundancy of the L received signals and a backward filter is used to reduce the effect of ISI.

The optimum solution for MA-FRESH-DFE filters can be found by generalizing (4.82) to (4.86) as follows :

$$h_b(m) = \begin{cases} \sum_{l=1}^L \frac{\sigma_{d_0}^{2(l,r)}}{\sigma_{d_0}^2} \left[(a_0^{(l)}(t) \otimes z_0^{(1,l)}(t-t_0^{(r)}) e^{j2\pi\phi_0^{(r)}}) + \right. \\ \left. (b_0^{(l)}(t) \otimes z_0^{(1,l)*}(t-t_0^{(r)}) e^{-j2\pi\phi_0^{(r)}}) \right] \Big|_{t=(D+m)T_0} + & m > 0 \\ \sum_{l=1}^L \frac{\sigma_{d_0}^{2(l,i)}}{\sigma_{d_0}^2} \left[(a_0^{(l)}(t) \otimes z_0^{(2,l)}(t-t_0^{(i)}) e^{j2\pi\phi_0^{(i)}}) - \right. \\ \left. (b_0^{(l)}(t) \otimes z_0^{(2,l)*}(t-t_0^{(i)}) e^{-j2\pi\phi_0^{(i)}}) \right] \Big|_{t=(D+m)T_0} & m \leq 0 \end{cases} \quad (4.108)$$

$$\sum_m \sum_n A_m^{(l)} \left(f + \frac{n}{T_0}\right) S_{x_i}^{(\frac{n}{T_0} - \alpha_m)} \left(f + \frac{n}{2T_0} - \frac{\alpha_m}{2}\right) + \sum_i \sum_n B_i^{(l)} \left(f + \frac{n}{T_0}\right) \left(S_{x_i}^{(\frac{\beta_i}{T_0} - \frac{n}{T_0})} \left(f + \frac{n}{2T_0} - \frac{\beta_i}{2}\right)\right)^* = \\ (\sigma_{d_0}^{2(l,r)} e^{-j2\pi(\phi_0^{(r)} + (DT_0 - t_0^{(r)})f)} Z_0^{(1,l)*}(f) + \sigma_{d_0}^{2(l,i)} e^{-j2\pi(\phi_0^{(i)} + (DT_0 - t_0^{(i)})f)} Z_0^{(2,l)*}(f)) (1 + H_b(e^{j2\pi f T_0})) \quad (4.109)$$

$$\sum_m \sum_n A_m^{(l)} \left(f + \frac{n}{T_0}\right) S_{x_i}^{(\frac{n}{T_0} + \alpha_k - \alpha_m)} \left(f + \frac{n}{2T_0} - \frac{\alpha_m + \alpha_k}{2}\right) + \\ \sum_i \sum_n B_i^{(l)} \left(f + \frac{n}{T_0}\right) \left(S_{x_i}^{(\frac{\beta_i}{T_0} - \alpha_k - \frac{n}{T_0})} \left(f + \frac{n}{2T_0} - \frac{\beta_i + \alpha_k}{2}\right)\right)^* = 0 \quad [\alpha_k]_{T_0} \neq 0 \quad (4.110)$$

$$\sum_i \sum_n A_k^{(l)} \left(f + \frac{n}{T_0}\right) S_{x_i}^{(\frac{n}{T_0} - \alpha_k)} \left(f + \frac{n}{2T_0} - \frac{\alpha_k}{2}\right) + \sum_m \sum_n B_m^{(l)} \left(f + \frac{n}{T_0}\right) S_{x_i}^{(\frac{n}{T_0} - \beta_m)} \left(-f - \frac{n}{2T_0} + \frac{\beta_m}{2}\right) = \\ (\sigma_{d_0}^{2(l,r)} e^{j2\pi(\phi_0^{(r)} - (DT_0 - t_0^{(r)})f)} Z_0^{(1,l)}(f) - \sigma_{d_0}^{2(l,i)} e^{j2\pi(\phi_0^{(i)} - (DT_0 - t_0^{(i)})f)} Z_0^{(2,l)}(f)) (1 + H_b(e^{j2\pi f T_0})) \quad (4.111)$$

$$\sum_k \sum_n A_k^{(l)} \left(f + \frac{n}{T_0}\right) S_{x_i}^{(\frac{n}{T_0} + \beta_i - \alpha_k)} \left(f + \frac{n}{2T_0} - \frac{\beta_i + \alpha_k}{2}\right) + \\ \sum_m \sum_n B_m^{(l)} \left(f + \frac{n}{T_0}\right) \left(S_{x_i}^{(\frac{\beta_i}{T_0} - \alpha_k - \frac{n}{T_0})} \left(-f - \frac{n}{2T_0} + \frac{\beta_i + \beta_m}{2}\right)\right) = 0 \quad [\beta_i]_{T_0} \neq 0 \quad (4.112)$$

where :

$$\begin{aligned} z_0^{(1,l)} &= p_0^{(1,l)}(t) \otimes \mathbf{x}(t) \\ z_0^{(2,l)} &= p_0^{(2,l)}(t) \otimes \mathbf{x}(t) \end{aligned} \quad (4.113)$$

The achievable MTAMSE for MA-FRESH-DFE can be found by generalizing (4.87) as follows :

$$\begin{aligned} MTAMSE &= \sigma_{d_0}^2 - \\ &\sum_{l=1}^L \left[a_0^{(l)}(t) \otimes (\sigma_{d_0^{(1)}}^2 e^{j2\pi\phi_0^{(1)}} z_0^{(1,l)}(t-t_0^{(1)}) + \sigma_{d_0^{(2)}}^2 e^{j2\pi\phi_0^{(2)}} z_0^{(2,l)}(t-t_0^{(2)})) \right] \Big|_{t=DL_0} - \\ &\sum_{l=1}^L \left[b_0^{(l)}(t) \otimes (\sigma_{d_0^{(1)}}^2 e^{-j2\pi\phi_0^{(1)}} z_0^{(1,l)*}(t-t_0^{(1)}) - \sigma_{d_0^{(2)}}^2 e^{-j2\pi\phi_0^{(2)}} z_0^{(2,l)*}(t-t_0^{(2)})) \right] \Big|_{t=DL_0} \end{aligned} \quad (4.114)$$

It is apparent from (4.114) that the MTAMSE of MA-FRESH-DFE can be reduced by increasing the number of antennas. The MTAMSE approaches zero if the number of antenna tends to infinity.

4.9 Structure Complexity

The structures introduced in this chapter might be more complex than needed for practical implementation. These structures might be reduced under certain conditions. For examples :

- The structure of MA-FRESH DFE can be reduced for narrowband channels. In this case, the number of filters in a branch antenna can be reduced because some filters can be simply created from combination of others.

- For OQPSK in a stationary environment (added AWGN), the cyclic spectrum in CCSD of the received signal can be easily transferred into CSD if the real and imaginary parts are synchronized (Figure 4.22). In this case, the structure of ODFE is reduced to the conventional DFE because there is no CCSD to be exploited. However, in the case of cyclostationary environment, this synchronization can only be done if the imaginary and real parts are off-setted by T/2. For example if a QPSK signal exist as an interference, using real-imaginary synchronizer block, transfers the interference QPSK into OQPSK while it is transferring SOI OQPSK into QPSK

- For BPSK signals, ODFE can be simplified into Figure 4.23 [5] if the channel

Figure 4.22 Real-Imag Synchronizer which can be used to reduce the structure of ODFE into the conventional DFE for OQPSK signal.

Figure 4.23 Reduced ODFE structure for BPSK.

spacing is a multiple of half the symbol rate. In this structure, the inputs of the filters are real and therefore the complexity is reduced.

4.10 Conclusion

In this chapter, the optimum receiver structures for DCSs have been obtained and analyzed. It has been shown that the proposed equalizers (FRESH-DFE and GDFE) can exploit the information of the cyclic spectra of the received signal. In addition, the conventional DFE has been analyzed, and it has been shown that the conventional DFE is optimum only for balanced QPSK in a single symbol rate system. A structure for multi-antenna receiver was also proposed and it was shown that the MTAMSE can approach zero if the number of antenna goes to infinity.

Chapter 5

Mobile Cellular Radio Systems

5.1 Introduction

The fundamentals of mobile cellular radio systems (MCRS) are reviewed in this chapter. First, the concepts of cellular radio systems relating to multiple access techniques, frequency reuse and cell splitting are described. This is followed by a review of the properties of digital mobile cellular radio channels with a particular emphasis on multipath fading and modelling. The interference properties in a MCRS are studied and results which characterize CCI and ACI in a cellular system are obtained. A new definition for capacity in terms of several design parameters is introduced and later used in cellular system capacity evaluation. The chapter concludes with an example on a multi rate cellular system design.

5.2 Cellular Radio Concept

The first generation of MCRS employed analog FM modulation technology [66]. The second generation of MCRS employ digital modulation schemes [16,45,65]. The second generation digital cellular radio systems (DCRS) achieve significantly higher capacity than existing analog systems. Furthermore, digital technology brings many advantages including but not limited to inexpensive VLSI/DSP hardware, superior speech quality, reduced transmission noise sensitivity, greater data security and a natural extension to future ISDN implementation [4,60].

One important design objective of a DCRS is to maximize the number of users in a given bandwidth. Several techniques are available to achieve this objective [3,43,91,92]. The multiple access schemes, frequency reuse and cell splitting strategies are the most common.

5.2.1 Multiple Access Schemes

There are essentially three basic multiple access schemes: code division multiple access (CDMA), time division multiple access (TDMA), and frequency division multiple

access (FDMA) [84]. A combination of these multiple access techniques are also used in some cases.

In a TDMA system, each user has access to full bandwidth for a part of the time known as a time slot while in a FDMA system, each user has access to only a part of the bandwidth all the time. In a CDMA system users share the bandwidth simultaneously. Therefore, TDMA and FDMA systems avoid interference by time and frequency filtering respectively, while the CDMA cleans multi user interference by users' codes. FDMA and TDMA are generally regarded as noise limited systems while CDMA is interference limited [91]. Each multiple access scheme has some advantages or disadvantages relative to others, details of which are found in [84].

5.2.2 Frequency Reuse and Cell Splitting

Frequency reuse is the core concept of the cellular system. Users in different geographic locations (different cells) may simultaneously use the same frequency channel provided that they are sufficiently separated in distance. The frequency reuse can tremendously increase the total number of system users. However, if the frequency reuse is not properly designed, serious interference may result. CCI, the co-channel interference due channel reuse, is a major concern in cellular systems.

In a cellular system, regularly shaped cells are grouped into cell clusters, repeated to fill the given geographic service area. The total assigned spectrum, in FDMA or TDMA/FDM system, is divided into K frequency reuse patterns where K is the cluster size. K is chosen to minimize the CCI, and is determined as $K=i^2+im+m^2$ where i and m are integer numbers [63]. Thus, a cellular system can be organized into cluster sizes of one, three, four, seven, nine, twelve or more cells. The smaller the cluster size, the higher is the number of users. However, a smaller cluster size results in higher interference. Figure 5.1 shows two systems with $K=3$ and $K=7$. It is apparent from the figure that under the condition of equal transmit power, CCI and ACI is higher when $K=3$. This is because of shorter reuse distance for $K=3$. CCI is the interference between signals with similar carrier frequencies, whereas ACI is the interference between signals with different carrier frequencies which are close enough to cause overlaps in the spectrum. In fully equipped hexagonal-shaped cellular system, there are always six co-channel interfering cells in the first tier [65] while the number of adjacent channel interfering cells depends on the frequency reuse pattern (cluster size), channel spacing and excess bandwidth. Figure 5.2 shows the

a.1) CCI in a system with cluster size seven.

a.2) ACI in a system with cluster size seven

b.1) CCI in a system with cluster size three.

b.2) ACI in a system with cluster size three.

Figure 5.1 (a) A Cellular Radio System (CRS) with cluster size seven (b) A CRS with cluster size three.

a) received signal by a user in a system with cluster size seven.

b) received signal by a user in a system with cluster size three.

Figure 5.2 Received signal in two different CRS.

spectrum of the signal received by one user in the systems of Figure 5.1. As it is apparent from the figure, the ACI and CCI signals are stronger in the system with cluster size three.

5.3 Digital Mobile Radio Channel Properties

5.3.1 Multipath Channel Properties

Mobile radio communication channels typically are multipath fading channels [16,59,66,97]. In a multipath channel, a transmitted signal arrives at the receiver via several paths with random delays and attenuations. This multipath propagation can be characterized in the time delay domain by multipath power delay profile (MPDP) which describes the average received power as a function of the propagation delay. The maximum propagation delay which MPDP is essentially nonzero is called the multipath delay spread of the channel. It is well known that if the ratio of the root-mean-square (RMS) delay spread to the symbol period is greater than about 0.3, then an adaptive equalization technique must be used to reduce the inter-symbol interference (ISI) caused by neighboring data symbols at sampling time [84]. The inverse of the RMS delay spread is called the "coherence bandwidth" of the channel. Two sinusoids with frequency separation greater than the coherence bandwidth are affected independently by the channel. A fading channel is said to be frequency-selective if its coherence bandwidth is less than the signal bandwidth. In this case, although the signal is severely distorted, the multipath are resolvable and an implicit diversity gain can be achieved [83]. On the other hand, if the coherence bandwidth is large (in comparison to the signal bandwidth), the channel is said to be frequency-nonselective. In this case, the channel exhibits spectrally flat fading in which all signal components fade simultaneously, the multipath rays are unresolvable and equalization is unnecessary [84]. A digital mobile radio channel typically is a frequency-selective fading channel [46,59,97].

5.3.2 Time-Variant Fading Channel Properties

As the signal receiving mobile moves through an approximately stationary spatial pattern, the received signal comes through a time variant-channel. The time variations in the channel are evidenced as a Doppler broadening and Doppler shift of a spectral line. The fading signal power distribution over the range of the Doppler frequencies is called Doppler power spectrum. The range of the Doppler frequencies over which the Doppler power spectrum is essentially nonzero is called the Doppler spread of the channel. The

severity of the time variations in the channel due to multipath fading can be quantified by the RMS Doppler spread. However, the multiplication of the maximum Doppler frequency, f_{max} , by the symbol period time, T , can characterize the severity of a mobile channel. A multipath fading channel with high $f_{max}T$ is called fast fading channel and a multipath fading channel with low $f_{max}T$ is called slow fading channel. Due to slow mobile speed and high symbol rate, the slow fading indoor wireless communication channel does not suffer from significant time variations.

5.3.3 Channel Modelling

The equivalent baseband of a multipath fading channel can be modelled as a complex time-varying signal with random attenuation and phase $g(\tau, t)$, where τ is the propagation time delay and t is the time dependence of the channel : [84]

$$g(\tau, t) = \alpha(\tau, t) e^{-j2\pi f_c \tau} \quad (5.1)$$

where α is the attenuation factor, and f_c is the signal carrier frequency.

Assuming that $g(\tau, t)$ is made up of a finite number of statistically independent propagation paths, (5.1) is written as :

$$g(\tau, t) = \sum_m \alpha_m(t) e^{-j2\pi f_c \tau_m(t)} \delta(\tau - \tau_m(t)) \quad (5.2)$$

where $\alpha_m(t)$ is the m th path attenuation, $\tau_m(t)$ is its propagation delay and $\delta(\tau)$ is dirac delta function.

Assuming a sufficiently large signal bandwidth such that the individual propagation paths can not be resolved, then each component of $g(\tau, t)$ is comprised of a sum of a large number of multipath signals. Therefore, from the central limit theorem [77], each component of $g(\tau, t)$ has independent real and imaginary parts, identically distributed (iid) jointly zero mean Gaussian random processes. In this case, the envelope $|g(\tau, t)|$ at any time instant t is Rayleigh-distributed and the channel is said to be a Rayleigh fading channel.

5.3.3.1 Tapped Delay Line Channel Modelling

The tapped delay line (TDL) channel model composed of evenly spaced tap coefficients represents multipath channel with district number of paths. The TDL should

be sufficiently long to span the significant portion of the channel MPDP. Furthermore, the tap spacing should be fine enough to represent the channel adequately. Specifically, the complex baseband input-output relationship of the frequency selective channel can be written as the following convolution summation : [84]

$$r(k\frac{T}{n}) = \sum_{m=0}^{M-1} g_m(k\frac{T}{n}) s(k\frac{T}{n} - mT_r) \quad (5.3)$$

where $r(kT/n)$ is the channel output, $g_m(kT/n)$ is the m th T/n -spaced channel impulse response (CIR) coefficient, $s(kT/n - mT_r)$ is the channel input signal, T_r is the time resolution, and T is the symbol period. This TDL channel model distinguishes multipath to a time resolution of T_r which has to be less than or equal to $T/(1+\epsilon)$ and a multiple of T/n , where ϵ is the excess bandwidth parameter.

For this discrete channel model, the m th tap coefficient of the TDL can be represented as :

$$g_m(k\frac{T}{n}) = \alpha_m d_m(k\frac{T}{n}) \quad (5.4)$$

where α_m is the RMS attenuation factor of the $(m+1)$ th multipath component, and $d_m(k(T/n))$ is the normalized Doppler fading factor of this path. The normalized Doppler fading factor in a Rayleigh fading environment can be modeled as a sum of weighted sinusoids with random phases and discrete frequencies spanning the Doppler spectrum [59]. Specifically, the fading component of the m th tap coefficient of the TDL channel model is :

$$d_m(k\frac{T}{n}) = \sum_{n=0}^{N_s} d_{m,n} \cos(2\pi\delta_n f_{max} k\frac{T}{n} + \theta_{m,n}) \quad (5.5)$$

where $d_{m,n}$ is the complex-valued amplitude of the n th Doppler frequency component, δ_n is the relative scaling factor for the n th Doppler frequency component, f_{max} is the maximum Doppler frequency, N_s+1 is the number of weighted Doppler sinusoids and $\theta_{m,n}$ is the initial phase of the n th Doppler frequency component at $k=0$.

The relative scaling factor for n th Doppler frequency component is :

$$\delta_n = \cos\left(\pi \frac{n}{2N_s + 1}\right) \quad (5.6)$$

The inphase and quadrature components of the amplitude for the n th Doppler frequency are, respectively : [59]

$$\text{Real}\{d_{m,n}\} = \begin{cases} \sqrt{2} \cos\left(\frac{\pi}{4}\right) & n = 0 \\ 2 \cos\left(\frac{\pi n}{N_s}\right) & 1 \leq n \leq N_s \end{cases} \quad (5.7)$$

and

$$\text{Imag}\{d_{m,n}\} = \begin{cases} \sqrt{2} \sin\left(\frac{\pi}{4}\right) & n = 0 \\ 2 \sin\left(\frac{\pi n}{N_s}\right) & 1 \leq n \leq N_s \end{cases} \quad (5.8)$$

The initial phase $\theta_{m,n}$ is a random, uniformly distributed variable between $-\pi$ and π .

The accuracy of the sum of sines model improves with increasing number of weighted Doppler sinusoids. However, it has been shown that a model using nine discrete Doppler sinusoids yields sufficient accuracy [59].

In the case of low f_{max} that is slow fading, the normalized Doppler fading of the $(m+1)$ th path can be assumed to be constant for a period of the time. In this period of the time, it is comprised of a sum of a large number of signals uniformly distributed between -1 and 1 . Therefore, from central limit theorem, its real and imaginary parts are iid jointly zero mean Gaussian random processes. In this case, the envelope of the m th tap coefficient of the TDL is a Rayleigh distributed random process. Thus, in the case of slow fading and over a short period of time, TDL channel may be modelled as follows :

$$r\left(k\frac{T}{n}\right) = \sum_{m=0}^{M-1} g_m s\left(k\frac{T}{n} - mT_r\right) \quad (5.9)$$

where g_m is a complex random variable with zero mean Gaussian distributed real and imaginary parts.

5.3.3.1.1 3-Rays Model

As an example of TDL channel model, a 3-rays model (Figure 5.3) is considered. In this model, the TDL contains three nonzero complex tap coefficients, (g_m ; $m=1,2,3$), each of which represents a distinct path. The tap coefficients g_m are characterized by the MPDP which is basically a measure of the variance of the channel impulse response (CIR) as a function of the propagation delay. For example, the "mountainous terrain" MPDP of the channel is specified in decibels, dB, relative to the first three paths : [68]

$$g_0 = 0 \text{ dB} , g_1 = -5 \text{ dB} , g_2 = -15 \text{ dB} \quad (5.10)$$

Figure 5.4 shows the MPDP for $T/4$ sampling interval. Regardless of sampling time (T/n) the resolution time is $T_r = T/2$ in this model. Therefore, this model is valid for signals, with less than 100 percent excess bandwidth, sampled at T/n where n is even. This model is used in all simulations reported in this thesis.

5.4 Interference and Noise in Cellular Systems

In a cellular system, mobile position and propagation loss factor determines the SOI, CCI and ACI powers while cluster size affect CCI and ACI powers. Figure 5.5 shows the desired signal, CCI and ACI for a system with a cluster size of three. The following relations can be directly found from the figure :

$$\begin{aligned} r_C^2 &= 9R^2 + r^2 + 6Rr \cos \left(\theta + m \frac{\pi}{3} - \frac{\pi}{2} \right) \\ r_A^2 &= 4R^2 + r^2 + 4Rr \cos \left(\theta + (m-1) \frac{\pi}{3} \right) \end{aligned} \quad (5.11)$$

Where R is the cell size, r is the mobile distance from base station, r_A is the mobile distance from one of the ACI source, r_C is the mobile distance from one of the first tier CCI source. Generally, the following relations are valid in any hexagonal cellular system :

$$r_{cm}^2 = (3K)R^2 + r^2 + 2(\sqrt{3K})Rr \cos \left(\theta + m \frac{\pi}{3} - \text{asin} \left(\frac{3}{\sqrt{3K}} \right) \right) \quad (5.12)$$

$$r_{am}^2 = 4R^2 + r^2 + 4Rr \cos \left(\theta + (m-1) \frac{\pi}{3} \right) \quad (5.13)$$

Where K is the cluster size, r_{cm} is the mobile's distance from m th source of first tier CCI

Figure 5.3 3-rays channel model.

Figure 5.4 MultiPath Delay Profile (MPDP) for 3-rays model.

Figure 5.5 An example of mobile position in a cellular system with cluster size three.

and r_{um} is the mobile's distance from m th source of first tier ACI. If the channel MPDPs used by interference and signal are similar except some attenuation caused by distances, signal to noise ratio (SNR), signal to CCI ratio (SCCIR), the signal to ACI ratio (SACIR), and signal to total interference ratio (SIR) can be easily found. In this case, cell size is not important and for simplicity we assume $R=100$ meters in the rest of this thesis.

5.4.1 SNR in Cellular Systems

The average signal power at the receiver at distance R_1 from the base station is found as :

$$S = \frac{A}{(R_1/R)^\gamma} = \frac{A}{r^\gamma} \quad 1 \geq r \geq \frac{1}{R} = 0.01 \quad (5.14)$$

where γ is the environmentally dependent propagation loss factor and usually lies between 2 and 5, r is the normalized distance and A is the power of the signal at $r=1$. The above equation is valid for R_1 greater or equal to one meter. Therefore the minimum value for r is $1/R$. If the variance of the noise (σ_n^2) is assumed to be constant over the cell, the signal to noise ratio (SNR) at the receiver input can be calculated as follows :

$$\begin{aligned} \frac{S}{N} &= \frac{A}{r^\gamma \sigma_n^2} \\ SNR &= [10\log(A) - 20\log(\sigma_n)] - 10\gamma\log(r) \quad (5.15) \\ SNR &= SNR_{max} - 10\gamma\log(R) - 10\gamma\log(r) = SNR_{min} - 10\gamma\log(r) \quad 1 \geq r \geq \frac{1}{R} \end{aligned}$$

Figure 5.6 shows $SNR-SNR_{min}$ for several values of γ . However, the mobile distance from base station, r , is a random process and its cumulative distribution function (cdf) and probability density function (pdf) can be found as follows :

$$\begin{aligned} F_r(Y) &= p(r < Y) = \frac{\pi Y^2}{\pi} = Y^2 \quad 0 \leq Y \leq 1 \\ f_r(Y) &= \frac{dF}{dY} = 2Y \quad 0 \leq Y \leq 1 \end{aligned} \quad (2.16)$$

where F_r and f_r represent cdf and pdf of random variable x respectively. Therefore, the cdf and pdf of average SNR can be found as follows :

Figure 5.6 SNR-SNR_{min} versus mobile position.

$$F_{SNR}(Y) = p(SNR < Y) = p\left(r > 10^{-\left(\frac{Y - SNR_{min}}{10\gamma}\right)}\right) \quad (5.17)$$

$$F_{SNR}(Y) = 1 - F_r\left(10^{-\left(\frac{Y - SNR_{min}}{10\gamma}\right)}\right) = 1 - 10^{-2\left(\frac{Y - SNR_{min}}{10\gamma}\right)} \quad (5.18)$$

$$f_{SNR}(Y) = \frac{2}{10\gamma} 10^{-2\left(\frac{Y - SNR_{min}}{10\gamma}\right)} \quad SNR_{max} \geq Y \geq SNR_{min} \quad (5.19)$$

Figure 5.7 (a,b) show cdf and pdf of SNR- SNR_{min} for several values of γ . As it is apparent from these curves, the probability of having SNR around SNR_{min} is higher when γ is low. The average SNR over a cell is found by taking another average of SNR as follows :

$$\overline{SNR} = \int_{SNR_{min}}^{SNR_{max}} Y \frac{2}{10\gamma} 10^{-2\left(\frac{Y - SNR_{min}}{10\gamma}\right)} dY \quad (5.20)$$

$$\overline{SNR} = SNR_{min} + 0.9421\gamma$$

As it is apparent from the above equation that the overall average SNR in dB has one to one increase with γ .

5.4.2 Signal to CCI Ratio (SCCIR) in Cellular Systems

The first tier CCI which contains a maximum of 6 CCI signals is most serious interference in cellular systems. The power of this interference, I_c , at the receiver input is found as follows :

$$i_{cm} = \frac{A}{r_{cm}^\gamma}$$

$$I_c = \sum_{m=1}^M i_{cm} = A \sum_{m=1}^6 \frac{1}{r_{cm}^\gamma} \quad (5.21)$$

$$I_c = A \sum_{m=1}^M E \left\{ \frac{1}{(3K + r^2 + 2(\sqrt{3K})r \cos(\theta + m\frac{\pi}{3} - \arcsin(\frac{3}{\sqrt{3K}})))^{\gamma/2}} \right\} \quad r > 0.01$$

where M is the number of active interference and expectation applies on θ . The above

Figure 5.7 (a) cumulative distribution function (cdf) (b) probability density function (pdf) of $SNR-SNR_{min}$ in a cellular system.

average is difficult to calculate, however, the computer simulation and subsequent curve fitting shows that (5.21) can be rewritten as :

$$I_c = \frac{MA}{(3K - 0.5\gamma r^2)^{\frac{\gamma}{2}}} \quad 1 \geq r \geq 0.01 \quad (5.22)$$

Therefore, the signal to first tier CCI ratio can be found according to :

$$\frac{S}{I_c} = \frac{(3K - 0.5\gamma r^2)^{\frac{\gamma}{2}}}{r^\gamma M} \quad (5.23)$$

$$SCCIR = 5\gamma \log(3K - 0.5\gamma r^2) - 10\gamma \log(r) - 10\log(M) \quad 1 \geq r \geq 0.01$$

Figure 5.8 (a) shows the SCCIR versus r and Figure 5.8 (b) shows the SCCIR versus the number of active CCI, M , for several values of γ and K . Maximum and minimum SCCIR can be easily calculated from (5.23) :

$$\begin{aligned} SCCIR_{min} &= 5\gamma \log(3K - 0.5\gamma) - 10\log(M) \\ SCCIR_{max} &= 10\gamma \log(R) + 5\log(3K) - 10\log(M) \end{aligned} \quad (5.24)$$

Table 5.1 summarizes the minimum SCCIR for different values of M , γ and K . As it is apparent from the figures and the table, the minimum SCCIR for $K=3$ is significantly lower than for $K=7$. The cdf and pdf of SCCIR can be found using same method explained in previous section :

$$\begin{aligned} F_{SCCIR}(Y) &= 1 - \frac{3K}{M^{2/\gamma} 10^{Y/5\gamma} + 0.5\gamma} \\ f_{SCCIR}(Y) &= \frac{3KM^{2/\gamma} 10^{Y/5\gamma}}{5\gamma (M^{2/\gamma} 10^{Y/5\gamma} + 0.5\gamma)^2} \quad SCCIR_{max} \geq Y \geq SCCIR_{min} \end{aligned} \quad (5.25)$$

Figure 5.9 (a,b) shows cdf and pdf of SCCIR for several values of γ and for cluster sizes 3 and 7. As it is apparent from these curves, the probability of high SCCIR decreases as γ decreases. The average SCCIR over a cell can be found by using the following integration:

$$\overline{SCCIR} = \int_{SCCIR_{min}}^{SCCIR_{max}} \frac{3YKM^{2/\gamma} 10^{Y/5\gamma}}{5\gamma (M^{2/\gamma} 10^{Y/5\gamma} + 0.5\gamma)^2} dY \quad (5.26)$$

Figure 5.8 SCCIR versus (a) mobile position (b) number of active CCI.

Table 5.1 Minimum SCCIR ($r=1$).

2 active CCI	K=3	K=7
$\gamma=2$	6.0206 dB	10.0000 dB
$\gamma=3$	10.1156 dB	16.3402 dB
$\gamma=4$	13.8917 dB	22.5648 dB
$\gamma=5$	17.3125 dB	28.6690 dB

3 active CCI	K=3	K=7
$\gamma=2$	4.2597 dB	8.2319 dB
$\gamma=3$	8.3547 dB	14.5793 dB
$\gamma=4$	12.1307 dB	20.8039 dB
$\gamma=5$	15.5516 dB	26.9081 dB

4 active CCI	K=3	K=7
$\gamma=2$	3.0103 dB	6.9897 dB
$\gamma=3$	7.1053 dB	13.3299 dB
$\gamma=4$	10.8814 dB	19.5545 dB
$\gamma=5$	14.3022 dB	26.9081 dB

5 active CCI	K=3	K=7
$\gamma=2$	2.0412 dB	6.0206 dB
$\gamma=3$	6.1362 dB	12.3608 dB
$\gamma=4$	9.9123 dB	18.5854 dB
$\gamma=5$	13.3331 dB	24.6896 dB

6 active CCI	K=3	K=7
$\gamma=2$	1.2494 dB	5.2288 dB
$\gamma=3$	5.3444 dB	11.5690 dB
$\gamma=4$	9.1204 dB	17.7936 dB
$\gamma=5$	12.5413 dB	23.8978 dB

Figure 5.9 (a) cdf (b) pdf of SCCIR in cellular systems with cluster sizes three and seven.

Table 5.2 summarizes the average SCCIR over a cell. Comparing Table 5.1 and Table 5.2 yields that the overall average SCCIR values are close to minimum SCCIR values specially when cluster size is three.

5.4.3 Signal to ACI Ratio (SACIR) in Cellular Systems

In addition to mobile position, propagation loss factor and cluster size, the ACI power depends on channel spacing, excess bandwidth and pulse spectrum. The following equation presents the average power of m th ACI at the receiver input :

$$i_{am} = \frac{A_m}{r_{am}^\gamma} \quad (5.27)$$

where A_m is the power in the part of ACI spectrum overlapping SOI. The total ACI power can be found as follows :

$$I_A = \sum_{m=1}^M i_{am} = \sum_{m=1}^M \frac{A_m}{r_{am}^\gamma} \quad (5.28)$$

$$I_A = \sum_{m=1}^M A_m E \left[\frac{1}{(4 + r^2 + 4r \cos(\theta + (m-1)\frac{\pi}{3}))^{\gamma/2}} \right]$$

where M is the number of active ACI. The total ACI, obviously, depends on how channels are assigned. The general form of above equation is not easy to find. However, for the special case where excess bandwidth is less than 100 percent and the channel spacing is equal to the symbol rate (Figure 5.10), (5.28) is simplified as :

$$I_A = A_1 \sum_{m=1}^M E \left[\frac{1}{(4 + r^2 + 4r \cos(\theta + (m-1)\frac{\pi}{3}))^{\gamma/2}} \right] \quad (5.29)$$

where A_1 is the power in the overlapped spectrum between first ACI and SOI. The computer simulation shows that the above equation may be empirically equivalent to the following equation :

$$I_A \cong \frac{MA_1}{(4 - 0.4\gamma r^2)^{0.5\gamma} r^\gamma} \quad (5.30)$$

Table 5.2 Overall cell average of SCCIR.

2 active CCI	K=3	K=7
$\gamma=2$	6.8741 dB	16.0396 dB
$\gamma=3$	11.2760 dB	26.3106 dB
$\gamma=4$	14.9369 dB	34.8527 dB
$\gamma=5$	18.0093 dB	42.0218 dB

3 active CCI	K=3	K=7
$\gamma=2$	4.8776 dB	11.3812 dB
$\gamma=3$	9.2046 dB	21.4774 dB
$\gamma=4$	13.0109 dB	30.3588 dB
$\gamma=5$	16.2635 dB	37.9482 dB

4 active CCI	K=3	K=7
$\gamma=2$	3.7835 dB	8.8283 dB
$\gamma=3$	7.9178 dB	18.4749 dB
$\gamma=4$	11.7489 dB	27.4141 dB
$\gamma=5$	15.0881 dB	35.2056 dB

5 active CCI	K=3	K=7
$\gamma=2$	3.0915 dB	7.2134 dB
$\gamma=3$	7.0203 dB	16.3807 dB
$\gamma=4$	10.8301 dB	25.2703 dB
$\gamma=5$	14.2135 dB	33.1649 dB

6 active CCI	K=3	K=7
$\gamma=2$	2.6138 dB	6.0989 dB
$\gamma=3$	6.3495 dB	14.8154 dB
$\gamma=4$	10.1184 dB	23.6097 dB
$\gamma=5$	13.5234 dB	31.5547 dB

Figure 5.10 A system with channel spacing equals to symbol rate.

and therefore,

$$\frac{S}{I_A} \equiv \frac{(A/A_1) (4 - 0.4\gamma r^2)^{0.5\gamma}}{Mr^\gamma} \quad (5.31)$$

A/A_1 is related to pulse shape and for raised cosine filter [84] with fractional excess bandwidth of ϵ , it is equal to $2/\epsilon$:

$$\begin{aligned} A &= \frac{1}{T} & A_1 &= \frac{\epsilon}{2T} \\ \frac{A}{A_1} &= \frac{2}{\epsilon} \end{aligned} \quad (5.32)$$

Therefore, the SACIR can be represented by the following equation :

$$SACIR \equiv 3 + 5\gamma \log(4 - 0.4\gamma r^2) - 10 \log(M\epsilon) - 10\gamma \log(r) \quad (5.33)$$

Figure 5.11 (a) shows the SACIR versus r when the number of active ACI is maximum. Minimum SACIR can be easily calculated by letting $r=1$ in the above equation :

$$SACIR_{min} \equiv 3 + 5\gamma \log(4 - 0.4\gamma) - 10 \log(M\epsilon) \quad (5.34)$$

Figure 5.11 (b) shows the minimum SACIR versus excess bandwidth for several values of γ and K in the case of all ACI sources are active. It is evident from the figures that SACIR is quite sensitive to excess bandwidth, cluster size, and propagation loss factor. Also, as the mobile position changes between $r=0.01$ and $r=1$ (edge of the cell), SACIR changes up to 120 dB. Table 5.3 summarizes the minimum SACIR for 100 percent excess bandwidth. pdf and cdf and overall average of SACIR can be found in a manner similar to that for SCCIR.

5.4.4 SIR in Cellular Systems

Total interference can be calculated by adding ACI and CCI if they are uncorrelated :

$$I = \frac{MA}{(3K - 0.5\gamma r^2)^{0.5\gamma}} + \frac{MA_1}{(4 - 0.4\gamma r^2)^{0.5\gamma} r^\gamma} \quad 1 \geq r \geq 0.01 \quad (5.35)$$

Therefore,

Figure 5.11 (a) $SACIR_{min}$ and (b) SIR_{min} versus signal excess bandwidth.

$$\frac{S}{I} = \frac{1}{r^\gamma \left(\frac{M_c}{(3K - 0.5\gamma r^2)^{0.5\gamma}} + \frac{M_a \epsilon}{2(4 - 0.4\gamma r^2)^{0.5\gamma}} \right)} \quad 1 \geq r \geq 0.01 \quad (5.36)$$

where M_c and M_a are the number of active CCI and active ACI sources respectively. The SIR can be presented by the following equation :

$$SIR = -10\gamma \log \left(\frac{M_c}{(3K - 0.5\gamma r^2)^{0.5\gamma}} + \frac{M_a \epsilon}{2(4 - 0.4\gamma r^2)^{0.5\gamma}} \right) - 10\gamma \log(r) \quad (5.37)$$

Figure 5.12 (a) shows SIR for maximum number of active interferers. The minimum SIR is obtained from (5.37) by setting $r=1$:

$$SIR_{min} = -10\gamma \log \left(\frac{M_c}{(3K - 0.5\gamma)^{0.5\gamma}} + \frac{M_a \epsilon}{2(4 - 0.4\gamma)^{0.5\gamma}} \right) \quad (5.38)$$

Figure 5.12 (b) shows minimum SIR versus signal excess bandwidth. It is observed that when excess bandwidth is low, CCI is dominant while ACI is significant for high excess bandwidth. Table 5.4 summarizes minimum SIR for 100 percent excess bandwidth when all CCI and ACI are active. The results given in the table indicate that, the minimum SIR is low and even can be negative in the case of cluster size three. The pdf and cdf of SIR can also be found by the method similar to that of SCCIR.

5.5 Cellular System Capacity

The capacity and spectral efficiency of a cellular system has been defined in [49,50,64]. Although these definitions are adequate for analog cellular systems, we propose a better definition for digital cellular systems capacity. We define the capacity of a digital cellular system by the maximum number of bits per second which can be transmitted over a cell area for a given average bit error rate (BER) and total available bandwidth. The maximum number of users over total available bandwidth in a FDMA system can be found according to :

$$N_1 = \frac{W - B + \Delta f}{\Delta f} \quad (5.39)$$

where W is the total available bandwidth, B is the user bandwidth and Δf is the channel spacing. The maximum number of users per cell is $1/K$ of N_1 where K is the cluster size :

Figure 5.12 (a) $SACIR_{min}$ and (b) SIR_{min} versus signal excess bandwidth.

Table 5.3 Minimum SACIR ($r=1$).

$\epsilon=1$	K=3	K=7
$\gamma=2$	0.2700dB	5.0515 dB
$\gamma=3$	1.9259 dB	6.7074 dB
$\gamma=4$	2.8227 dB	7.6042 dB
$\gamma=5$	3.2685 dB	7.8257 dB

Table 5.4 Minimum SIR ($r=1$).

$\epsilon=1, M=6$	K=3	K=7
$\gamma=2$	-2.2724 dB	2.1289 dB
$\gamma=3$	0.3039 dB	5.4804 dB
$\gamma=4$	1.9160 dB	7.2072 dB
$\gamma=5$	2.3208 dB	7.4268 dB

$$N = \frac{N_1}{K} = \frac{W - B + \Delta f}{K\Delta f} \quad (5.40)$$

Therefore the capacity is :

$$C = \max \left(\frac{MN}{AT} \right) \Big|_{BER = b_0, W = W_0} = \max \left(\frac{M(W_0 - B + \Delta f)}{KAT\Delta f} \right) \Big|_{BER = b_0} \quad (5.41)$$

where M is the number of bits per symbol, T is the symbol rate and A is the cell area. With the above definition, the capacity is a function of b_0 and W_0 .

It is apparent from (5.41) that the smaller cell area yields larger capacity. Therefore, in general, the micro-cellular systems have higher capacity than other cellular systems. However, the smaller cell area needs larger number of base stations for the same coverage area. A larger number of base stations leads to greater system complexity.

It is also apparent from (5.41) that the capacity goes to infinity when channel spacing reduces to zero. In this case, either the bit error rate goes to one for single antenna receiver, or infinite number of antennas must be used. For single antenna receiver, the BER is a function of channel spacing, cluster size, excess bandwidth and power of the signal (or SNR), and therefore, the capacity becomes a function of these variables as well.

When the channel spacing is greater than or equal to signal bandwidth,

$$\Delta f \geq B = \frac{1 + \epsilon}{T} \quad (5.42)$$

there is no ACI, and CCI can be suppressed by using large excess bandwidth [80,81]. To suppress the first tier CCI, excess bandwidth should be larger than 600 percent ($\epsilon = 6$). In this case, a very low BER is achievable in single antenna system by increasing the signal power. However, the capacity is lower than $MW_0/KA(1 + \epsilon)$ bits/sec/km². This limitation is calculated by substituting (5.42) in (5.41).

If the channel spacing is less than signal bandwidth, the ACI exists and the BER finally settles to an irreducible value in a single antenna receiver. The amount of this irreducible BER is related to the cluster size, excess bandwidth and channel spacing.

If the channel spacing is fixed to the symbol rate, the capacity is :

$$C = \max \left(\frac{M(W_0 - \epsilon/T)}{KA} \right) \Big|_{BER = b_0} \quad (5.43)$$

Since the ϵ/T is much less than W_0 , the above equation can be approximated to :

$$C \cong \max \left(\frac{MW_0}{KA} \right) \Big|_{BER = b_0} \quad (5.44)$$

It is apparent from the above equation that, for a fixed A , the maximum condition is satisfied when $K=1$. However, the achievable BER with this condition is expected to be very high. For $K=3$, the Table 5.4 shows that the SIR is lower than zero dB and in this condition the BER is still high. For $K=7$, the capacity will be $MW_0/7A$ bits/sec/km² but the SIR is around 5.5 dB (for $\epsilon=1$) which suggests that a reasonable BER is achievable. In the next chapter we will simulate a cellular system with cluster size seven and channel spacing equaling symbol rate to find out what BER can be achieved for $MW_0/7A$ bits/sec/km².

5.5.1 Multi-Symbol Rate Systems

From the previous chapter, we observe that the single symbol rate systems do not create CCI free cyclic spectra while the multi-symbol rate systems do. In this section, we will consider how a multi-symbol rate system can be designed. Figure 5.13 (a) shows that at least three different symbol rates are needed for the first tier CCI signals to be separated in the cyclic spectra. Figure 5.13 (b) shows a design of three symbol rate cellular system. Although the SOI has cyclic spectra free of CCI and ACI, the system management and the receiver structure become complicated.

Consider Figure 5.14 for a two symbol rates system. In this system the number of first tier CCI with same symbol rate is reduced to 2. To keep the signal bandwidth equal, the signal with different excess bandwidth may be used. In this case the channel spacing remain the same. However, the capacity of the system is reduced. For example if 0.5 and 1 Hz are the two chosen symbol rates, the excess bandwidths of $(1+2\epsilon)$ and ϵ will result in the same signal bandwidth for two different symbol rate signals. For channel spacing equals to one, the capacity is reduced to $3MW_0/28A$ bits/sec/km². In next chapter this system is simulated and the results are compared with single symbol rate system.

5.6 Conclusion

Mobile radio cellular systems have been considered in this chapter. Frequency

(a)

(b)

Figure 5.13 (a) multi-symbol rate first tier CCI (b) multi-symbol rate system using three symbol rates.

Figure 5.14 2-symbol rate system (a) cluster 3 (b) cluster 7.

reuse, cell splitting and multiple access techniques have been discussed. The properties of digital mobile radio channels have been investigated. Multipath, and fading channels have been reviewed and the channel models have been considered. Expressions SNR, SCCIR, SACIR, and SIR in cellular system have been derived. The capacity of cellular systems have been defined and analyzed. A multi-symbol rate cellular system has been proposed.

Chapter 6

Simulation Results

6.1 Introduction

In this chapter, results of many simulations experiments to validate the analytical results in chapter 4 and 5 are presented. The performance of FRESH-DFE and conventional DFE are compared and it is shown that FRESH-DFE has better performance. It is also shown that the multi-antenna receiver can achieve low bit error rate even though the channel spacing is smaller than the symbol rate. The 2-symbol rate cellular system shown in Figure 5.14 is also simulated and it is shown that the performances of FRESH-DFE and conventional DFE are better in this system than the system with single symbol rate. It is concluded that for the same receiver complexity, the ODFE has better performance than DFE.

6.2 Adaptive Filters

An adaptive filter is a practical method of implementing an optimal filter that can adjust to slowly changing signal statistics. The adaptive filter schematic which is shown in Figure 6.1, has two inputs, the observed signal, x_k , and the training or desired signal, y_k . The observed signal is filtered and subtracted from training signal to produce an error signal. The adaptive filter is updated to minimize some error criterion, typically mean square error (MSE).

The most common adaptive filter is the finite length impulse response (FIR) least-mean-square (LMS) adaptive filter. The FIR LMS output is given by :

$$\hat{y}_n = \underline{W}_k^T \underline{X}_k \big|_{n=kM} \quad (6.1)$$

where $\hat{y}_n$ is the filter output at the time $n=kM$, M is the ratio of the input data rate to the desired data rate, $\underline{X}_k$ is a vector of observed data within the filter,

$$\underline{X}_k^T = [x_k(1) \ x_k(2) \ \dots \ x_k(N)] \quad (6.2)$$

and $\underline{W}_k$ is the vector of adjustable filter weights at the time k ,

Figure 6.1 An adaptive filter.

$$\underline{W}_k^T = [W_k(1) \ W_k(2) \ \dots \ W_k(N)] \quad (6.3)$$

The weights are adjusted using the LMS algorithm at the time $n=kM$,

$$\underline{W}_{n+1} = \underline{W}_n + 2\mu\epsilon_n X_n^* \quad (6.4)$$

μ is the adaptive step size, and ϵ_n is the error

$$\epsilon_n = y_n - \hat{y}_n \quad (6.5)$$

where y_n is the training or desired signal.

For LMS algorithm, the step size has to satisfy the following condition : [52]

$$0 < \mu < \frac{1}{\|X_n\|^2} \quad (6.6)$$

where $\|X_n\|$ is the norm of input vector at the time n . Since the above condition has to be satisfied at all the time, the step size will be related to the power of the input vector. To avoid this problem, equation (6.4) can be changed to the following equation :

$$\underline{W}_{n+1} = \underline{W}_n + 2\mu\epsilon_n \frac{X_n^*}{\|X_n\|^2} \quad (6.7)$$

This is called normalized LMS (NLMS) [52]. In this case the condition (6.6) is changed to:

$$0 < \mu < 1 \quad (6.8)$$

A smaller step size results in slower convergence but lower minimum MSE (MMSE), while a larger step size yields faster convergence but higher MMSE. This trade-off leads to variable step size [52]. In variable step size LMS algorithm, the step size μ is time-variant and according to a certain rule is started with almost maximum value and decreases to the minimum value approaching convergence. Changing the step size can be function of time or the error. This function can be linear or non-linear. Therefore, there are many possible variable step size algorithms.

One of the most common variable step size rule is gear-shifting (GS) [52]. In GS algorithm, the step size is changed between several values during the convergence time. Figure 6.2 shows an example of GS algorithm which is used in our simulation. P_0 in the

Figure 6.2 An example of Gear Shifting (GS) rule.

figure designates the end of training sequence. At this time the adaptive filter either stop the adaptation or uses output decisions as the desired value. We will represent GS with the following notation :

$$\mu_n = gs(\mu_{max}, \eta_{min}, P_0, n) \quad (6.9)$$

6.3 Adaptive Conventional DFE with Normalized Least-Mean-Square (NLMS) Algorithm

The NLMS updating-equation for the conventional DFE coefficients can be written as :

$$\underline{W}_{n+1} = \underline{W}_n + \mu_n \underline{V}(n) \frac{2\mu_n \epsilon_n \underline{V}(n)^*}{\|\underline{V}(n)\|^2} \quad (6.10)$$

where $\underline{V}(n)$ is defined by (4.71), $\underline{W}_n$ and ϵ_n are defined by :

$$\begin{aligned} \underline{W}_n^T &= [\underline{a}_{f_n}^T \quad -\underline{a}_{b_n}^T] \\ \epsilon_n &= d_0(n-D) - \underline{W}_n^T \underline{V}(n) \end{aligned} \quad (6.11)$$

μ_n is the step size at the time $t = nT_0$. Substituting (4.71) and (6.11) in (6.10) yields :

$$[\underline{a}_{f_{n+1}} \quad -\underline{a}_{b_{n+1}}] = [\underline{a}_{f_n} \quad -\underline{a}_{b_n}] + 2\mu_n \epsilon_n \frac{[\tilde{\underline{x}}(nM_0) \quad \underline{d}(n-D-1)]^*}{\|\tilde{\underline{x}}(nM_0) \quad \underline{d}(n-D-1)\|^2} \quad (6.12)$$

Separating the elements of the above equation yields the following equations for updating the conventional DFE coefficients :

$$\begin{aligned} \underline{a}_{f_{n+1}} &= \underline{a}_{f_n} + 2\mu_n \epsilon_n \frac{\tilde{\underline{x}}^*(nM_0)}{\|\tilde{\underline{x}}(nM_0)\|^2 + N_b \sigma_{d_0}^2} \\ \underline{a}_{b_{n+1}} &= \underline{a}_{b_n} - 2\mu_n \epsilon_n \frac{\underline{d}^*(n-D-1)}{\|\tilde{\underline{x}}(nM_0)\|^2 + N_b \sigma_{d_0}^2} \end{aligned} \quad (6.13)$$

where the following simplification has been used :

$$\begin{aligned}
\left\| \begin{bmatrix} \tilde{x}(nM_0) \\ d(n-D-1) \end{bmatrix} \right\|^2 &= E \left\{ \begin{bmatrix} \tilde{x}(nM_0) \\ d(n-D-1) \end{bmatrix} \begin{bmatrix} \tilde{x}(nM_0) \\ d(n-D-1) \end{bmatrix}^H \right\} \\
&= E \left\{ \begin{bmatrix} \tilde{x}(nM_0) \end{bmatrix} \begin{bmatrix} \tilde{x}(nM_0) \end{bmatrix}^H \right\} + E \left\{ \begin{bmatrix} d(n-D-1) \end{bmatrix} \begin{bmatrix} d(n-D-1) \end{bmatrix}^H \right\} \quad (6.14) \\
&= \left\| \begin{bmatrix} \tilde{x}(nM_0) \end{bmatrix} \right\|^2 + N_b \sigma_{d_0}^2
\end{aligned}$$

Figure 6.3 shows the structure of the adaptive conventional DFE used in simulations. In the conventional DFE two modes of operation exist. In mode one, called training mode, a training sequence is used to adjust the coefficients to achieve an acceptable MSE, and in mode 2, called tracking mode, DFE uses the data decisions to update its coefficients.

The decision device box in this structure is a hard decision device [84] which simply quantizes the real and imaginary parts of the input to two level, -1 and 1 . If BPSK is used, only the real part is used.

6.4 Adaptive Multi-Antenna Conventional DFE with NLMS Algorithm

Figure 6.4 shows the structure of multi-antenna conventional DFE used in the simulation. In this figure, superscript (l) represents the l th antenna branch signals. The NLMS updating equation for this structure can be also represented by (6.10) where :

$$\begin{aligned}
\underline{V}^T(n) &= \left[\tilde{x}^{(1)T}(nM_0) \tilde{x}^{(2)T}(nM_0) \dots \tilde{x}^{(L)T}(nM_0) \underline{d}^T(n-D-1) \right] \\
\underline{W}_n^T &= \left[\underline{a}_{f_n}^{(1)T} \underline{a}_{f_n}^{(2)T} \dots \underline{a}_{f_n}^{(L)T} -\underline{a}_b(n) \right] \quad (6.15) \\
\varepsilon_n &= d_0(n-D) - \underline{W}_n^T \underline{V}(n)
\end{aligned}$$

Substituting (6.15) in (6.10) and separating the elements of the resulted equation yields :

Figure 6.3 Adaptive conventional DFE.

Figure 6.4 Adaptive multi-antenna conventional DFE.

$$\begin{aligned}
 \underline{a}_{f_{n+1}}^{(l)} &= \underline{a}_{f_n}^{(l)} + 2\mu_n \varepsilon_n \frac{\tilde{x}^{(l)*}(nM_0)}{\sum_{m=1}^L \left\| \tilde{x}^{(m)}(nM_0) \right\|^2 + N_b \sigma_{d_0}^2} \quad l = 1, 2, \dots, L \\
 \underline{a}_{b_{n+1}} &= \underline{a}_{b_n} - 2\mu_n \varepsilon_n \frac{\underline{d}^*(n-D-1)}{\sum_{m=1}^L \left\| \tilde{x}^{(m)}(nM_0) \right\|^2 + N_b \sigma_{d_0}^2}
 \end{aligned} \tag{6.16}$$

where μ_n is defined by (6.9).

6.5 Adaptive FRESH-DFE with NLMS Algorithm

The schematic of the adaptive FRESH-DFE used in the simulations is shown in Figure 6.5. Equation (6.10) is also valid for updating the coefficient of this equalizer. However, $\underline{V}(n)$, $\underline{W}_n$ and ε_n are defined by :

$$\begin{aligned}
 \underline{V}^T(n) &= [\underline{X}^{(T)}(n) \quad \underline{d}^T(n-D-1)] \\
 \underline{W}_n^{(1)T} &= [\underline{a}_{0_n}^T \underline{a}_{1_n}^T \dots \underline{a}_{M_n}^T \underline{b}_{0_n}^T \underline{b}_{1_n}^T \dots \underline{b}_{N_n}^T] \\
 \varepsilon_n &= d_0(n-D) - \underline{W}_n^T \underline{V}(n)
 \end{aligned} \tag{6.17}$$

where :

$$\underline{X}^T(n) = [\tilde{x}_0^T(nM_0) \tilde{x}_{\alpha_1}^T(nM_0) \dots \tilde{x}_{\alpha_M}^T(nM_0) \tilde{x}_0^{*T}(nM_0) \tilde{x}_{-\beta_1}^{*T}(nM_0) \dots \tilde{x}_{-\beta_N}^{*T}(nM_0)] \tag{6.18}$$

Substituting (6.17) in (6.10) and separating elements of the resulted equation yields :

$$\begin{aligned}
 \underline{a}_{k_{n+1}} &= \underline{a}_{k_n} + 2\mu_n \varepsilon_n \frac{\tilde{x}_{\alpha_k}^*(nM_0)}{\sum_{m=0}^M \left\| \tilde{x}_{\alpha_m}(nM_0) \right\|^2 + \sum_{m=0}^N \left\| \tilde{x}_{\beta_m}(nM_0) \right\|^2 + N_b \sigma_{d_0}^2} \quad k = 1, \dots, M \\
 \underline{b}_{k_{n+1}} &= \underline{b}_{k_n} + 2\mu_n \varepsilon_n \frac{\tilde{x}_{-\beta_k}(nM_0)}{\sum_{m=0}^M \left\| \tilde{x}_{\alpha_m}(nM_0) \right\|^2 + \sum_{m=0}^N \left\| \tilde{x}_{\beta_m}(nM_0) \right\|^2 + N_b \sigma_{d_0}^2} \quad k = 1, \dots, N
 \end{aligned} \tag{6.19}$$

Figure 6.5 Adaptive FRESH-DFE.

$$a_{h_{n+1}} = a_{h_n} - 2\mu_n \epsilon_n \frac{d^*(n-D-1)}{\sum_{m=0}^M \left\| \tilde{x}_{\alpha_m}(nM_0) \right\|^2 + \sum_{m=0}^N \left\| \tilde{x}_{\beta_m}(nM_0) \right\|^2 + N_b \sigma_{d_0}^2} \quad (6.20)$$

Same as the adaptive conventional DFE, the adaptive FRESH-DFE has two modes, training and tracking and uses the GS given in Figure 6.2.

6.6 Adaptive Multi-Antenna FRESH-DFE (MA-FRESH-DFE) with NLMS Algorithm

Adaptive MA-FRESH-DFE can be found by using an adaptive FRESH-DFE for every antenna. Therefore, $\underline{V}(n)$, $\underline{W}_n$ and ϵ_n are defined by :

$$\begin{aligned} \underline{V}^T(n) &= \left[\underline{X}^{(1)T}(n) \dots \underline{X}^{(L)T}(n) \underline{Y}^{(1)T}(n) \dots \underline{Y}^{(L)T}(n) \underline{d}^T(n-D-1) \right] \\ \underline{W}_n^T &= \left[\underline{w}_n^{(1)T} \underline{w}_n^{(2)T} \dots \underline{w}_n^{(L)T} -\underline{a}_b(n) \right] \\ \epsilon_n &= d_0(n-D) - \underline{W}_n^T \underline{V}(n) \end{aligned} \quad (6.21)$$

where :

$$\begin{aligned} \underline{X}^{(l)T}(n) &= \left[\tilde{x}_0^{(l)T}(nM_0) \tilde{x}_{\alpha_1}^{(l)T}(nM_0) \dots \tilde{x}_{\alpha_M}^{(l)T}(nM_0) \right] \\ \underline{Y}^{(l)T}(n) &= \left[\tilde{x}_0^{*(l)T}(nM_0) \tilde{x}_{-\beta_1}^{*(l)T}(nM_0) \dots \tilde{x}_{-\beta_N}^{*(l)T}(nM_0) \right] \\ \underline{w}_n^{(l)T} &= \left[\underline{a}_{0_n}^{(l)T} \underline{a}_{1_n}^{(l)T} \dots \underline{a}_{M_n}^{(l)T} \underline{b}_{0_n}^{(l)T} \underline{b}_{1_n}^{(l)T} \dots \underline{b}_{N_n}^{(l)T} \right] \end{aligned} \quad (6.22)$$

Substituting (6.21) in (6.10) yields :

$$a_{k_{n+1}}^{(l)} = a_{k_n}^{(l)} + 2\mu_n \epsilon_n \frac{\tilde{x}_{\alpha_k}^{(l)*}(nM_0)}{\sum_{l=1}^L \left[\sum_{m=0}^M \left\| \tilde{x}_{\alpha_m}^{(l)}(nM_0) \right\|^2 + \sum_{m=0}^N \left\| \tilde{x}_{\beta_m}^{(l)}(nM_0) \right\|^2 \right] + N_b \sigma_{d_0}^2} \quad (6.23)$$

$k = 1, \dots, M$
 $l = 1, \dots, L$

$$b_{k_{n+1}}^{(l)} = b_{k_n}^{(l)} + 2\mu_n \varepsilon_n \frac{\tilde{x}_{-\beta_k}^{(l)}(nM_0)}{\sum_{l=1}^L \left[\sum_{m=0}^M \left\| \tilde{x}_{\alpha_m}^{(l)}(nM_0) \right\|^2 + \sum_{m=0}^N \left\| \tilde{x}_{\beta_m}^{(l)}(nM_0) \right\|^2 \right] + N_b \sigma_{d_0}^2} \quad \begin{matrix} k = 1, \dots, N \\ l = 1, \dots, L \end{matrix} \quad (6.24)$$

$$a_{b_{n+1}} = a_{b_n} - 2\mu_n \varepsilon_n \frac{d^*(n-D-1)}{\sum_{l=1}^L \left[\sum_{m=0}^M \left\| \tilde{x}_{\alpha_m}^{(l)}(nM_0) \right\|^2 + \sum_{m=0}^N \left\| \tilde{x}_{\beta_m}^{(l)}(nM_0) \right\|^2 \right] + N_b \sigma_{d_0}^2}$$

6.7 Noise Limiting Filter

It is well known that a filter matched to SOI is the best choice to limit the noise provided there is no ACI. However, in the case of overlapping signal spectra, the best filter is one which passes all the overlapped signal spectra. But passing all the overlapping signals in a single channel spacing system means using an all pass filter before the equalizer which yields an infinite noise power at the equalizer input. Nevertheless, a sub-optimum noise limiter filter which passes spectra of SOI and overlapping interference may be considered. As an example, Figure 6.6 (a) shows a FDMA system which uses a channel spacing equals to the symbol rate. As it is clear from this figure, only first up-channel and down-channel signals are overlapped with SOI. Therefore, the noise limiter filter passes these two signals and partially other ACI signals. Figure 6.6 (b) shows a noise limiter filter for this system which is created by adding three filter responses matched to SOI, first up-ACI and first down-ACI. This filter passes all the CCI signals, and the strongest ACI signals. Therefore there is enough information about CCI and ACI at the input of the equalizer. Figure 6.7 (a) shows that the noise limiter filter can cut the second, third,... overlapping signal. Figure 6.7 (b) shows this filter in dB scale. In all the simulation of this chapter such a noise limiter filter is used.

6.8 Simulated Systems

A cluster size 7 DMCS with channel spacing equals to the symbol rate is simulated to evaluate the performance of DFE and FRESH-DFE. Since the channel spacing is equal to the symbol rate, in a single symbol rate BPSK (or OQPSK) system, the ODFE needs one forward filter in the conjugate path and one forward filter in the linear path

Figure 6.6 (a) A FDMA system with channel spacing equals to the symbol rate (b) Noise limiter filter.

Figure 6.7 (a) The FDMA system and the noise limiter filter (b) The noise limiter filter in dB scale.

($\{\beta\}=\{0\}, \{\alpha\}=\{0\}$). The simulation is run for the worst case that is when the mobile is at the cell boundary, $r=1$, and the channel MPDP for SOI, CCI and ACI signals has three rays at $t=0, T/2, T$ (which T is the symbol rate) regardless of sampling rate.

In the simulations, the following assumptions are made :

1- Regardless of attenuations caused by distances the channel profiles of SOI, CCI and ACI sources are the same.

2- The channels are multipath slow Rayleigh fading and they can be assumed invariant over one frame duration.

3- The 3-Rays "mountainous terrain" MPDP (Figure 5.4) is used to model all the channels.

4- The channels are uncorrelated.

4- All base stations use square root raised cosine with the same transmit power.

6- The cellular system use hexagonal shape cells with identical dimensions.

7- First tier CCI signals are enough to represent CCI effect.

8- The data are uncorrelated with variance of 1.

9- The mobile position is at the boundary, i.e. $r=1$.

10- Running the simulation over 200 channels is sufficient to include the fading effect of the channels.

11- The noise assume to be complex, white and Gaussian with variance N_0 .

12- The equalizers are assumed to be sufficiently large to cover the effect of channel multipath.

13- The header symbols for training in each frame are assumed to be enough to achieve more than 70 percent of MMSE at the end of training.

14- Shadowing is not considered in the simulations.

In addition to the above assumptions, the parameters given in Table 6.1 are used in

Table 6.1 The simulation parameters in all the simulated systems.

Frame length	5000 symbols
Train length	500 symbols
Total number of transmitted data for each run	1,000,000 symbols
Number of taps in forward filters	15 symbols
Number of taps in backward filter	3 symbols
SOI symbol rate	normalized to 1 Hz
SOI excess bandwidth	100 percent
Sampling rate at the input of equalizers	5 Hz
Propagation loss factor	3
Mobile position (r)	at the boundary of cell ($r=1$)
Variance of the real part ($\sigma_{d(r)}^2$)	1
Variance of the imaginary part ($\sigma_{d(i)}^2$)	1
SCCIR (measured)	11.5 dB
SACIR (measured)	7.14 dB
SIR (measured)	5.88 dB

all the simulation. In this table, SCCIR, SACIR, and SIR are the measured values in the simulations. As it is clear, these values are close to the values computed in chapter 5. It is also good to mention that since the number of symbols in one run is 10^6 , the minimum confidence BER is 10^{-4} .

With the above parameters and assumptions the following cases are investigated.

6.8.1 No CCI-No ACI System

As a simple case, first we assume that all the CCI and ACI sources are inactive. In this case, the only effect of the noise is sought. Figure 6.8 (a) shows the BER for such a system. As it is clear from the figure there is no saturation in BER and it can be reduced to zero if E_b/N_0 increases to infinity. Figure 6.8 (b) shows the ratio of conventional DFE BER to the ODFE BER. This ratio can be considered as the BER improvement of ODFE over the conventional DFE. As it is apparent from Figure 6.8 (b) that for $E_b/N_0 > 18$ dB, the BER improvement can be more than 2.5 times for OQPSK. Figure 6.9 (a) shows the MMSE versus E_b/N_0 . In this figure, each point of MMSE curves is obtained by averaging over 200 channels and in each channel by averaging over last 4000 square errors. Figure 6.9 (b) shows the difference between the MMSEs achieved by the conventional DFE and by the ODFE. This amount can be considered as the MMSE improvement of ODFE over conventional DFE. As it is apparent from Figure 6.9 (b), the ODFE can achieve more than 3 dB MMSE improvement over conventional DFE.

Figure 6.10 (a) shows the BER for OQPSK and for two values of excess bandwidth, 100 percent and 300 percent. As it is clear from the figure, the BER in the case of ODFE is improved by an order of magnitude when larger excess bandwidth is used. Corresponding improvement in conventional DFE is much lower. Figure 6.10 (b) shows the BER improvement of ODFE over the conventional DFE.

Figure 6.11 shows an example of the forward filters frequency responses of DFE and ODFE for BPSK and OQPSK systems when $E_b/N_0 = 23$ dB. The figures show that the conjugate path response in ODFE contributes to an improvement in BER and MMSE.

6.8.2 No CCI-6-ACI System

Figure 6.12 (a) shows the BER for the case when all ACI are active and CCI sources are inactive. Irreducible error rates for QPSK and OQPSK are indicated. It means

Figure 6.8 (a) The BER and (b) The BER improvement for no CCI-no ACI system.

Figure 6.9 (a) The MMSE and (b) The MMSE improvement for no CCI-no ACI system.

Figure 6.10 (a) The BER and (b) The BER improvement in no CCI-no ACI systems when OQPSK signals with excess bandwidths of 100 and 300 percent are used.

Figure 6.11 Forward filters frequency responses in no CCI-no ACI systems for (a) BPSK (b) OQPSK.

Figure 6.12 (a) The BER and (b) The BER improvement for no CCI- 6 ACI system.

that QPSK and OQPSK can not cancel ACI in this system while BPSK can. Figure 6.12 (b) shows the BER improvement in this system. The performance of ODFE is better by several dB if more redundant signal (ex. BPSK) is used. Figure 6.13 shows the MMSE and MMSE improvement in this system.

6.8.3 6-CCI-No ACI System

If all CCI are active and ACI sources are inactive, the BER is changed to Figure 6.14 (a). The MMSE and MMSE improvement are shown in Figure 6.15. Comparing Figure 6.14 and Figure 6.15 with Figure 6.12 and Figure 6.13, it is observed that the CCI effect is more serious than ACI when the channel spacing equals the symbol rate.

It can be concluded by comparing Figure 6.12 (a) and Figure 6.14 (a) with Figure 6.8 (a) that the interference causes irreducible BER.

6.8.4 1-CCI-2-ACI System

It is assumed that only one CCI and two ACI, upper channel ACI and lower channel ACI, sources are active. The power of CCI and ACI signals are adjusted to achieve the same SCCIR as in the system with 6 active CCI sources (SCCIR=11.5 dB) and the same SACIR as in the system with 6 active ACI sources (SACIR=7.14 dB). Figure 6.16 and Figure 6.17 show the BER, BER improvement, MMSE and MMSE improvement for this simulation. As it is clear from comparing these figures with Figure 6.14 and Figure 6.15, the performance of ODFE for one strong CCI signals is better than the performance of ODFE for 6 weak CCI signals. The reason is that, the summation of 6 Cyclostationary signals with random phases is close to a stationary signal.

Now we vary the power of CCI and ACI sources to plot the BER and MMSE versus SCCIR and SACIR. Figure 6.18 to Figure 6.27 show the BER, BER improvement, MMSE and MMSE improvement for different values of SCCIR and SACIR, and for different modulation scheme. The E_b/N_0 is equal to 28 dB in all these cases. It is observed from the figures that,

- 1- the BER and MMSE can be reduced by reducing the interference.
- 2- the BER and MMSE are more sensitive to CCI than ACI for a system with channel spacing equals to the symbol rate.

Figure 6.13 (a) The MMSE and (b) The MMSE improvement for no CCI- 6 ACI system.

Figure 6.14 (a) The BER and (b) The BER improvement for 6 CCI- no ACI system.

Figure 6.15 (a) The MMSE and (b) The MMSE improvement for 6 CCI- no ACI system.

Figure 6.16 (a) The BER and (b) The BER improvement for 1 CCI- 2 ACI system.

Figure 6.17 (a) The MMSE and (b) The MMSE improvement for 1 CCI- 2 ACI system.

Figure 6.18 (a) The BER and (b) The BER improvement versus SCCIR for 1 CCI- 2 ACI OQPSK system.

Figure 6.19 (a) The MMSE and (b) The MMSE improvement versus SCCIR for 1 CCI- 2 ACI QPSK system.

Figure 6.20 (a) The BER and (b) The BER improvement versus SACIR for 1 CCI- 2 ACI OQPSK system.

Figure 6.21 (a) The MMSE and (b) The MMSE improvement versus SACIR for 1 CCI- 2 ACI QPSK system.

BER in 1 CCI- 2 ACI system

(a)

BER improvement in 1 CCI- 2 ACI system

(b)

Figure 6.22 (a) The BER and (b) The BER improvement versus SCCIR for 1 CCI- 2 ACI BPSK system.

Figure 6.23 (a) The MMSE and (b) The MMSE improvement versus SCCIR for 1 CCI- 2 ACI BPSK system.

Figure 6.24 (a) The BER and (b) The BER improvement versus SACIR for 1 CCI- 2 ACI BPSK system.

Figure 6.25 (a) The MMSE and (b) The MMSE improvement versus SACIR for 1 CCI- 2 ACI BPSK system.

Figure 6.26 (a) The BER and (b) The MMSE versus SCCIR for 1 CCI- 2 ACI QPSK system.

Figure 6.27 (a) The BER and (b) The MMSE versus SACIR for 1 CCI- 2 ACI QPSK system.

3- In OQPSK system, the BER improvement and MMSE improvement increases when SCCIR decreases while in BPSK system, the BER improvement and MMSE improvement decreases when SCCIR decreases.

6.8.5 6-CCI 6-ACI System

Figure 6.28 and Figure 6.29 show the BER and MMSE for different modulation schemes when all the CCI and ACI sources are active. As it is apparent from the figures, the QPSK technique gives better performance over OQPSK. The reason is that, the ODFE structure for OQPSK is bigger than the ODFE structure for QPSK and this cause slower convergence and more error. However, it is possible to achieve same performance for QPSK and OQPSK if Real-Imaginary synchronizer (Figure 4.22) is used.

The performance is improved if a multi-antenna receiver is used. Figure 6.30 to Figure 6.34 show the BER and MMSE when multi-antennas receivers are used. The branches assumed to be uncorrelated in this simulation. As it is clear from the figure, the BER can be lower than 10^{-5} for a SNR greater than 5 dB if number of antenna increases. These figures also show that the superiority of ODFE over conventional DFE increases when the number of antenna increases.

6.8.6 System with Channel Spacing Equals to Half The Symbol Rate

If the channel spacing decreases by factor of two, the capacity increases by factor of two provided interference does not have a negative influence. Figure 6.35 to Figure 6.39 show the simulation results when the channel spacing equals to half the symbol rate and all the CCI and ACI sources are active. It is observed from these figures that, the BER lower than 10^{-2} (which is speech quality limit [95]) may be achieved with two antenna receiver regardless of modulation scheme. Moreover, the performance of a BPSK system with channel spacing equals to half the symbol rate is as good as the performance of a QPSK system with channel spacing equals to the symbol rate using ODFE. Consequently, the capacities of these two systems are also equal.

6.8.7 2-Symbol Rate System

The 2-symbol rate system shown in Figure 5.14 (b) is simulated. T_0 (the SOI symbol interval) and T_1 are 1 and 2 seconds respectively. The excess bandwidth of the signal with symbol duration of 1 second is 100 percent and for signal with symbol duration of 2

Figure 6.28 (a) The BER and (b) The BER improvement for 6 CCI- 6 ACI system.

Figure 6.29 (a) The MMSE and (b) The MMSE improvement for 6 CCI- 6 ACI system.

Figure 6.30 (a) The BER and (b) The BER improvement for multi-antenna receiver in 6 CCI- 6 ACI OQPSK system.

Figure 6.31 (a) The BER and (b) The BER improvement for multi-antenna receiver in 6 CCI- 6 ACI OQPSK system.

Figure 6.32 (a) The BER and (b) The BER improvement for multi-antenna receiver in 6 CCI- 6 ACI BPSK system.

Figure 6.33 (a) The MMSE and (b) The MMSE improvement for multi-antenna receiver in 6 CCI- 6 ACI BPSK system.

(a)

(b)

Figure 6.34 (a) The BER and (b) The MMSE for multi-antenna receiver in 6 CCI- 6 ACI QPSK system.

Figure 6.35 (a) The BER and (b) The BER improvement for multi-antenna receiver in 6 CCI- 6 ACI OQPSK system when channel spacing is half the symbol rate.

Figure 6.36 (a) The MMSE and (b) The MMSE improvement for multi-antenna receiver in 6 CCI- 6 ACI QPSK system when channel spacing is half the symbol rate.

Figure 6.37 (a) The BER and (b) The BER improvement for multi-antenna receiver in 6 CCI- 6 ACI BPSK system when channel spacing is half the symbol rate.

Figure 6.38 (a) The MMSE and (b) The MMSE improvement for multi-antenna receiver in 6 CCI- 6 ACI BPSK system when channel spacing is half the symbol rate.

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PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
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Figure 6.39 (a) The BER and (b) The MMSE for multi-antenna receiver in 6 CCI- 6 ACI QPSK system when channel spacing is half the symbol rate.

second is 300 percent. With these symbol durations and excess bandwidths, the signal bandwidths are the same. The channel spacing equal to 1 Hz is used. The structure of FRESH-DFE uses the following frequency shifts for this system :

$\{0, 0.5\}$ Hz in Linear path and $\{0, 0.5\}$ Hz in Conjugate path for BPSK.

$\{0\}$ Hz in Linear path and $\{0, 0.5\}$ Hz in Conjugate path for OQPSK.

$\{0, 0.5\}$ Hz in Linear path for QPSK.

Figure 6.40 to Figure 6.45 show the simulation results for this system. Although the FRESH-DFE performance is shown to be better than the conventional DFE (Figure 6.40), the amount of improvement in BER is not significant. The reason is that the total interference at cyclic frequency equal of zero ($\alpha=0$ and $\beta=0$) is reduced comparing with single-symbol rate system.

If T_0 equals 1 second and T_1 is 2 seconds, the structure of FRESH-DFE is reduced and there is no need for 0.5 Hz frequency shifts. However, the increased total interference at $\alpha=0$ and $\beta=0$ yields worse performance compared with the case when time $T_0=1$ and $T_1=2$. Figure 6.46 to Figure 6.50 show the simulation results for two cases (1) $T_0=2$ and $T_1=1$, (2) $T_0=1$ and $T_1=2$. The results indicate that both conventional DFE and FRESH-DFE deliver better performance when the SOI symbol rate is twice SNOI symbol rate. This improvement in BER is greater when two antennas branch receiver is used.

6.9 Receivers with Limited Complexity

In all above simulations, it is assumed that there is no concern about the complexity of the receiver. However, in real world the complexity of the receiver is a main consideration. In equalizers, the number of taps determines complexity. It is proposed to use half the available number of taps in linear path and the other half in conjugate path for BPSK and OQPSK. Figure 6.51 and Figure 6.52 show simulation results for 1-CCI-2 ACI system when the number of available taps is limited. In this simulation, the conventional DFE uses 32 taps (which is equivalent to 8 symbols) for forward filter and 3 taps for feedback filter while ODFE uses 16 taps (4 symbols) in Linear path ($\alpha=0$), 16 taps in Conjugate path ($\beta=0$) and 3 taps in feed back filter. Therefore, the complexity of conventional DFE and ODFE are the same. As it is clear from the figures, the ODFE performance is still superior to the conventional DFE performance.

Figure 6.40 (a) The BER. and (b) The BER improvement for 2-symbol rate system.

Figure 6.41 (a) The MMSE and (b) The MMSE improvement for 2-symbol rate system.

Figure 6.42 (a) The BER and (b) The BER improvement for 2-symbol rate QPSK system.

Figure 6.43 (a) The MMSE and (b) The MMSE improvement for 2-symbol rate OQPSK system.

Figure 6.44 (a) The BER and (b) The BER improvement for 2-symbol rate QPSK system.

Figure 6.45 (a) The MMSE and (b) The MMSE improvement for 2-symbol rate QPSK system.

Figure 6.46 (a) The BER and (b) The BER improvement for 2-symbol rate system and when $T_0=1$ and 2 seconds.

Figure 6.47 (a) The MMSE and (b) The MMSE improvement for 2-symbol rate system when $T_0=1$ and 2 seconds.

Figure 6.48 (a) The BER and (b) The BER improvement for 2-symbol rate BPSK system when $T_0=1$ and 2 seconds.

Figure 6.49 (a) The MMSE and (b) The MMSE improvement for 2-symbol rate BPSK system when $T_0=1$ and 2 seconds.

Figure 6.50 (a) The BER and (b) The BER improvement for 2-symbol rate QPSK system when $T_0=1$ and 2 seconds.

Figure 6.51 (a) The BER and (b) The BER improvement for 1 CCI-2 ACI system when the complexity of DFE and ODFE are the same.

Figure 6.52 (a) The MMSE and (b) The MMSE improvement for 1 CCI-2 ACI system when the complexity of DFE and OFE are the same.

6.10 Conclusion

A cluster 7 cellular system was simulated and following concludes are drawn :

- 1- The performance of FRESH-DFE is superior to that of conventional DFE.
- 2- The superiority of FRESH-DFE over conventional DFE increases when signals with higher redundancy are used. The greater spectral redundancy yields greater superiority of FRESH-DFE over DFE.
- 3- Multi-antenna receiver can achieve very low bit error rate even if the channel spacing is less than the symbol rate.
- 4- The superiority of FRESH-DFE over conventional DFE increases when multi-antenna receiver is used.
- 5- If the optimum receiver is used, the capacity and BER of a system using BPSK with channel spacing equals to half the symbol rate and the capacity and BER of a system using QPSK with channel spacing equals to the symbol rate are the same.
- 6- The performances of FRESH-DFE and conventional DFE in the 2-symbol rate system shown in Figure 5.14 are better when the SOI symbol rate is twice the SNOI symbol rate.
- 7- With same complexity, ODFE can achieve better performance than conventional DFE.

Chapter 7

Conclusions

7.1 Conclusions

This thesis studied digital communication in high interference multipath fading environment and obtained the structure of the optimum receiver with an acceptable performance. The receiver is optimum in the sense that it is able to exploit all the spectral redundancies present in the received signal due to modulation technique. It has been shown that the structure of optimum receiver changes from one system to another as it is related to: (1) channel spacing, (2) signal excess bandwidth, (3) number of available receiver antennas (4) modulation scheme and (5) users' symbol rates.

It has been shown that the modulation schemes can be classified into three categories in the sense of cyclostationarity. BPSK, QPSK, and OQPSK have been used to represent these three categories. An optimum receiver can be designed and analyzed for each of these three modulations without a loss of generality.

The CCI and ACI signals have been considered separately and it has been shown that the interfering effect of ACI can be removed despite its considerable spectral overlap with SOI. In addition to overlapping ACI, multi-symbol rates systems have also been considered and the structure of the optimum receiver in such a case was obtained.

The conventional DFE was fully analyzed in cyclostationary environment by using MTAMSE criterion. It has been shown that the DFE is not an optimum receiver for BPSK and OQPSK signals and it is optimum for QPSK signals if and only if all users' symbol rates are identical.

Two different structures for DFE based on two different stationary representations of a CRP, i.e. HSR and TSR, were introduced. FRESH-DFE was built on HSR and GDFE was built on TSR. It has been shown that FRESH-DFE and GDFE are identical for single symbol rates systems. We referred FRESH-DFE (or GDFE) in single symbol rate systems to as ODFE. The results of analysis and simulation presented in this thesis has shown that the ODFE has better performance than the conventional DFE specially in high interfer-

ence environment.

Digital cellular radio systems were considered to evaluate the performance of designed receiver. The SNR, SCCIR and SIR of a digital cellular radio system has been studied. A definition for the capacity of a digital cellular radio system has been presented and it has been shown that the capacity can be increased if not very low BER is needed.

The simulation results have verified the analytical results. The results have been shown that the performance of FRESH-DFE always is better than that of conventional DFE. It has been also shown that the multi-antenna receiver can achieve low BER even though the channel spacing is smaller than the symbol rate. A 2-symbol rate cellular system has been simulated and it has been shown that the performance of FRESH-DFE and the conventional DFE are better when the SOI symbol rate is twice that of SNOI. It has been also shown that for the same receiver complexity, the ODFE has better performance than the convention DFE.

7.2 Recommendations for Future Research

The followings are some suggestions for future research related to this thesis work:

7.2.1 Using Recursive Least Squares (RLS) Algorithm in Adaptive FRESH-DFE

It is well known that the RLS algorithm provides much faster convergence and therefore it need a fewer number of training data. The frame length can be reduced when the header is reduced. In this case the assumption of time invariance of the channel in one frame time is more closer to reality.

7.2.2 Considering The Effect of Sampling Phase on The Performance of The FRESH-DFE

It is well known that the sampling phase is important when a strong cyclostationary signal is present [1]. However, the importance of sampling phase is reduced when the received signal is more close to stationary [1]. It is strongly recommended to study the effect of sampling phase on the performance of the FRESH-DFE.

7.2.3 Considering The Effect of The Doppler Spread on The FRESH-DFE

It has been described the Doppler effect on multipath channel in chapter 5. The

behavior of channel is more harsh when Doppler frequency increases. It is strongly recommended to study the effect of the Doppler spread on the FRESH-DFE. Since Doppler spread is created by simultaneously frequency shifting of the spectrum with set of frequencies, it seems one can cancel this effect if he can find the best frequency shifts set.

7.2.4 Investigating The Performance and Perhaps New Structure of Optimum DFE in Fast Fading Environment.

A time variant channel in one frame time can be objective of another study. In this case, a new structure for FRESH-DFE can be found which perhaps uses time variant sampler to sample at appropriate times.

7.2.5 Using Channel Estimator Besides ODFE

Another objective of study may be to use a channel estimator with ODFE to reduce the fading effect and consequently reduce the BER.

7.2.6 Investigating The Performance of The FRESH-DFE in Code Division Multiple Access (CDMA) Systems

The performance and perhaps new structure for the FRESH-DFE can be studied when CDMA is used. It is well known that CDMA signals are highly spectral redundant. Therefore, the performance of FRESH-DFE should be much better when CDMA is used.

Appendix A

Some Useful Cyclic Relations

Followings are some useful cyclic relations which used in this thesis :

- The cyclic relations for two complex CRPs $x(t)$ and $y(t)$ are : [10]

$$\begin{aligned}
 R_{xy}^{\alpha}(\tau) &= R_{xy}^{-\alpha}(-\tau)^* \\
 R_{xy}^{\alpha}(\tau) &= R_{yx}^{\alpha}(-\tau) = R_{x^*y}^{-\alpha}(\tau)^* \\
 R_{x^*y}^{\alpha}(\tau) &= R_{xy}^{-\alpha}(\tau)^* \\
 R_x^{\alpha}(\tau) &= R_x^{-\alpha}(-\tau)^* \\
 R_{xx}^{\alpha}(\tau) &= R_{xx}^{\alpha}(-\tau) = R_{x^*x}^{-\alpha}(\tau)^* \\
 S_{yx}^{\alpha}(f) &= S_{xy}^{-\alpha}(f)^* \\
 S_{yx}^{\alpha}(f) &= S_{xy}^{\alpha}(-f) = S_{x^*y}^{-\alpha}(-f)^* \\
 S_{x^*y}^{\alpha}(f) &= S_{xy}^{-\alpha}(-f)^* = S_{yx}^{\alpha}(-f) \\
 S_x^{\alpha}(f) &= S_x^{-\alpha}(f)^* \\
 S_{xx}^{\alpha}(f) &= S_{xx}^{\alpha}(-f) = S_{x^*x}^{-\alpha}(-f)^* \\
 S_{x^*x}^{\alpha}(f) &= S_x^{-\alpha}(-f)^* = S_x^{\alpha}(-f)
 \end{aligned} \tag{A.1}$$

- The CA, CCA, CSD, CCSD of a time delayed version of CRP $x(t)$, i.e. $x(t-\tau)$, are found by multiplying the CA, CCA, CSD, and CCSD of $x(t)$ by $e^{-j2\pi\alpha\tau}$ respectively. [10]

- The following equations shows the cyclic relations between CRP $x(t)$ and $y(t)=x(t)q(t)$ where $q(t)$ is another CRP.

$$\begin{aligned}
 R_y^{\alpha_k}(\tau) &= \sum_m R_x^{\alpha_k - \alpha_m}(\tau) R_q^{\alpha_m}(\tau) \\
 S_y^{\alpha_k}(f) &= \sum_m S_x^{\alpha_k - \alpha_m}(f) \otimes S_q^{\alpha_m}(f)
 \end{aligned}
 \tag{A.2}$$

• The input-output cyclic relations for linear almost periodic system $h(t, u) = \sum_{\mu} g_{\mu}(t-u) e^{j2\pi\mu u}$ are: [10]

$$\begin{aligned}
 S_y^{\alpha}(f) &= \sum_{\lambda, \mu} H_{\mu}(f + \frac{\alpha}{2}) S_x^{\alpha + \lambda - \mu}(f - \frac{\mu + \lambda}{2}) H_{\lambda}(f - \frac{\alpha}{2})^* \\
 S_{yy}^{\alpha*}(f) &= \sum_{\lambda, \mu} H_{\mu}(f + \frac{\alpha}{2}) S_{xx}^{\alpha + \lambda - \mu}(f - \frac{\mu + \lambda}{2}) H_{-\lambda}(-f + \frac{\alpha}{2}) \\
 S_{yx}^{\alpha}(f) &= \sum_{\mu} H_{\mu}(f + \frac{\alpha}{2}) S_x^{\alpha - \mu}(f - \frac{\mu}{2}) \\
 S_{yx}^{\alpha*}(f) &= \sum_{\mu} H_{\mu}(f + \frac{\alpha}{2}) S_{xx}^{\alpha - \mu}(f - \frac{\mu}{2}) \\
 S_{xy}^{\alpha}(f) &= \sum_{\mu} S_x^{\alpha + \mu}(f - \frac{\mu}{2}) H_{\mu}(f - \frac{\alpha}{2})^* \\
 S_{xy}^{\alpha*}(f) &= \sum_{\mu} S_{xx}^{\alpha - \mu}(f + \frac{\mu}{2}) H_{\mu}(-f + \frac{\alpha}{2})
 \end{aligned}
 \tag{A.3}$$

where $x(t)$ and $y(t)$ are the input and output of the system, respectively.

Appendix B

Analysis of The FRESH-DFE

Same as the conventional DFE, FRESH-DFE minimizes the TAMSE too. Taking the gradient of TAMSE according to the backward filter coefficients and setting them to zero, yields:

$$\nabla_{h_b}^{TAMSE}(m) = \sigma_{d_0}^2 h_b(m) + \langle E \{ \tilde{y}(n) d_0^*(n-D-m) \} \rangle \quad (B.1)$$

Replacing $\tilde{y}(n)$ by:

$$\begin{aligned} \tilde{y}(n) = & \sum_k \int_{-\infty}^{\infty} a_k(u) x(nT_0 - u) e^{j2\pi[\alpha_k]_{T_0^{-1}}(nT_0 - u)} d(u) + \\ & \sum_i \int_{-\infty}^{\infty} b_i(u) x^*(nT_0 - u) e^{j2\pi[\beta_i]_{T_0^{-1}}(nT_0 - u)} d(u) \end{aligned} \quad (B.2)$$

yields:

$$\begin{aligned} \nabla_{h_b}^{TAMSE}(m) = & \sigma_{a_0}^2 h_b(m) - \\ & \sum_k \int_{-\infty}^{\infty} a_k(u) \langle E \{ d_0^*(n-D-m) x(nT_0 - u) \} e^{j2\pi[\alpha_k]_{T_0^{-1}}(nT_0 - u)} \rangle d(u) - \\ & \sum_i \int_{-\infty}^{\infty} b_i(u) \langle E \{ d_0^*(n-D-m) (x^*(nT_0 - u)) \} e^{j2\pi[\beta_i]_{T_0^{-1}}(nT_0 - u)} \rangle d(u) \end{aligned} \quad (B.3)$$

Using (4.1) to (4.5):

$$\begin{aligned} E \{ d_0^*(n-D-m) x(nT_0 - u) \} = & \sigma_{d_0^{(r)}, z_0^{(1)}}^2 ((D+m)T_0 - t_0^{(r)} - u) e^{j2\pi\phi_0^{(r)}} + \\ & \sigma_{d_0^{(i)}, z_0^{(2)}}^2 ((D+m)T_0 - t_0^{(i)} - u) e^{j2\pi\phi_0^{(i)}} \\ E \{ d_0^*(n-D-m) x^*(nT_0 - u) \} = & \sigma_{d_0^{(r)}, z_0^{(1)}}^2 ((D+m)T_0 - t_0^{(r)} - u) e^{-j2\pi\phi_0^{(r)}} - \\ & \sigma_{d_0^{(i)}, z_0^{(2)}}^2 ((D+m)T_0 - t_0^{(i)} - u) e^{-j2\pi\phi_0^{(i)}} \end{aligned} \quad (B.4)$$

Replacing (B.4) in (B.3) and considering

$$\begin{aligned} \langle e^{j2\pi [\alpha_k]_{T_0}^{-1} (nT_0 - u)} \rangle &= \begin{cases} 1 & \text{for } [\alpha_k]_{T_0} = 0 \\ 0 & \text{otherwise} \end{cases} \\ \langle e^{j2\pi [\beta_i]_{T_0}^{-1} (nT_0 - u)} \rangle &= \begin{cases} 1 & \text{for } [\beta_i]_{T_0} = 0 \\ 0 & \text{otherwise} \end{cases} \end{aligned} \quad (\text{B.5})$$

yields:

$$\begin{aligned} \nabla_{h_b}^{TMSE}(m) &= \sigma_{d_0}^2 h_b(m) - \\ &\sigma_{d_0}^2 \int_{-\infty}^{\infty} (a_0(u) z_0^{(1)}((D+m)T_0 - t_0^{(r)} - u) e^{j2\pi\phi_0^{(r)}} + \\ &b_0(u) z_0^{(1)*}((D+m)T_0 - t_0^{(r)} - u) e^{-j2\pi\phi_0^{(r)}}) d(u) - \\ &\sigma_{d_0}^2 \int_{-\infty}^{\infty} (a_0(u) z_0^{(2)}((D+m)T_0 - t_0^{(i)} - u) e^{j2\pi\phi_0^{(i)}} - \\ &b_0(u) z_0^{(2)*}((D+m)T_0 - t_0^{(i)} - u) e^{-j2\pi\phi_0^{(i)}}) d(u) \end{aligned} \quad (\text{B.6})$$

Optimum coefficient will be achieved by tending gradient to zero. Therefore the optimum coefficients for the feedback filter are:

$$\begin{aligned} h_b(m) &= \\ &\frac{\sigma_{d_0}^2}{\sigma_{d_0}^2} \int_{-\infty}^{\infty} (a_0(u) z_0^{(1)}((D+m)T_0 - t_0^{(r)} - u) e^{j2\pi\phi_0^{(r)}} + \\ &b_0(u) z_0^{(1)*}((D+m)T_0 - t_0^{(r)} - u) e^{-j2\pi\phi_0^{(r)}}) d(u) + \\ &\frac{\sigma_{d_0}^2}{\sigma_{d_0}^2} \int_{-\infty}^{\infty} (a_0(u) z_0^{(2)}((D+m)T_0 - t_0^{(i)} - u) e^{j2\pi\phi_0^{(i)}} - \\ &b_0(u) z_0^{(2)*}((D+m)T_0 - t_0^{(i)} - u) e^{-j2\pi\phi_0^{(i)}}) d(u) \end{aligned} \quad (\text{B.7})$$

$$h_b(m) = \begin{cases} \frac{\sigma_{d_0^{(r)}}^2}{\sigma_{d_0}^2} \left[(a_0(t) \otimes z_0^{(1)}(t-t_0^{(r)}) e^{j2\pi\phi_0^{(r)}}) + \right. \\ \left. (b_0(t) \otimes z_0^{(1)*}(t-t_0^{(r)}) e^{-j2\pi\phi_0^{(r)}}) \right] \Big|_{t=(D+m)T_0} + \\ \frac{\sigma_{d_0^{(i)}}^2}{\sigma_{d_0}^2} \left[(a_0(t) \otimes z_0^{(2)}(t-t_0^{(i)}) e^{j2\pi\phi_0^{(i)}}) - \right. \\ \left. (b_0(t) \otimes z_0^{(2)*}(t-t_0^{(i)}) e^{-j2\pi\phi_0^{(i)}}) \right] \Big|_{t=(D+m)T_0} \\ 0 \end{cases} \quad \begin{matrix} m > 0 \\ m \leq 0 \end{matrix} \quad (\text{B.8})$$

Equation (B.8) is reduced to:

$$h_b(m) = \begin{cases} \left[(a_0(t) \otimes z_0^{(1)}(t-t_0^{(r)}) e^{j2\pi\phi_0^{(r)}}) + \right. \\ \left. (b_0(t) \otimes z_0^{(1)*}(t-t_0^{(r)}) e^{-j2\pi\phi_0^{(r)}}) \right] \Big|_{t=(D+m)T_0} \\ 0 \end{cases} \quad \begin{matrix} m > 0 \\ m \leq 0 \end{matrix} \quad (\text{B.9})$$

for BPSK systems, to:

$$h_b(m) = \begin{cases} \frac{1}{2} \left[(a_0(t) \otimes (z_0^{(1)}(t-t_0^{(r)}) + z_0^{(1)}(t-t_0^{(i)})) e^{j2\pi\phi_0^{(r)}}) + \right. \\ \left. (b_0(t) \otimes (z_0^{(1)*}(t-t_0^{(r)}) - z_0^{(1)*}(t-t_0^{(i)})) e^{-j2\pi\phi_0^{(r)}}) \right] \Big|_{t=(D+m)T_0} \\ 0 \end{cases} \quad \begin{matrix} m > 0 \\ m \leq 0 \end{matrix} \quad (\text{B.10})$$

for QPSK systems, and so:

$$h_b(m) = \begin{cases} \left[a_0(t) \otimes z_0^{(1)}(t-t_0^{(r)}) e^{j2\pi\phi_0^{(r)}} \right] \Big|_{t=(D+m)T_0} \\ 0 \end{cases} \quad \begin{matrix} m > 0 \\ m \leq 0 \end{matrix} \quad (\text{B.11})$$

for balanced QPSK.

Same procedure has to be used to find the other optimum filters coefficients. The gradient of TAMSE according to the $a_k(t)$ in the time u is:

$$\nabla_{a_k}^{TMSE}(u) = \langle E \{ (\tilde{y}(n) - (d_0(n-D) + f(n))) \nabla_{a_k}^{*y}(n)(u) \} \rangle \quad (\text{B.12})$$

where

$$\nabla_{z_k}^{\vec{y}^{(n)}}(u) = x^*(nT_0 - u) e^{-j2\pi[\alpha_k]_{T_0^{-1}}(nT_0 - u)} \quad (\text{B.13})$$

Replacing (B.13) in (B.12) and also considering (4.1) to (4.5), 2nd and 3th term of (B.12) will be

$$\langle E \{ d_0(n-D) \nabla_{a_k}^{\vec{y}^{(n)}}(u) \} \rangle = \begin{cases} \sigma_{d_0^{(r)}}^2 z_0^{(1)*} (DT_0 - t_0^{(r)} - u) e^{-j2\pi\phi_0^{(r)}} + \\ \sigma_{d_0^{(i)}}^2 z_0^{(2)*} (DT_0 - t_0^{(i)} - u) e^{-j2\pi\phi_0^{(i)}} & [\alpha_k]_{T_0^{-1}} = 0 \\ 0 & [\alpha_k]_{T_0^{-1}} \neq 0 \end{cases} \quad (\text{B.14})$$

$$\langle E \{ f(n) \nabla_{a_k}^{\vec{y}^{(n)}}(u) \} \rangle = \begin{cases} \sigma_{d_0^{(r)}}^2 e^{-j2\pi\phi_0^{(r)}} \sum_{m=1}^{\infty} z_0^{(1)*} ((D+m)T_0 - u - t_0^{(r)}) h_b(m) + \\ \sigma_{d_0^{(i)}}^2 e^{-j2\pi\phi_0^{(i)}} \sum_{m=1}^{\infty} z_0^{(2)*} ((D+m)T_0 - u - t_0^{(i)}) h_b(m) & [\alpha_k]_{T_0^{-1}} = 0 \\ 0 & [\alpha_k]_{T_0^{-1}} \neq 0 \end{cases} \quad (\text{B.15})$$

The first term of (B.12) is:

$$\begin{aligned} & \langle E \{ \tilde{y}(n) \nabla_{a_k}^{\vec{y}^{(n)}}(u) \} \rangle \\ &= \sum_m \int_{-\infty}^{\infty} a_m(v) \langle E \{ x(nT_0 - v) x^*(nT_0 - u) \} e^{j2\pi nT_0[\alpha_m - \alpha_k]_{T_0^{-1}}} e^{j2\pi[u\alpha_k - v\alpha_m]_{T_0^{-1}}} \rangle dv + \\ & \sum_i \int_{-\infty}^{\infty} b_i(v) \langle E \{ x^*(nT_0 - v) x^*(nT_0 - u) \} e^{j2\pi nT_0[\beta_i - \alpha_k]_{T_0^{-1}}} e^{j2\pi[u\alpha_k - v\beta_i]_{T_0^{-1}}} \rangle dv \quad (\text{B.16}) \\ &= \sum_m \int_{-\infty}^{\infty} a_m(v) \langle R_x(nT_0 - v, nT_0 - u) e^{j2\pi nT_0[\alpha_m - \alpha_k]_{T_0^{-1}}} e^{j2\pi[u\alpha_k - v\alpha_m]_{T_0^{-1}}} \rangle dv + \\ & \sum_i \int_{-\infty}^{\infty} b_i(v) \langle R_{x^*x}(nT_0 - v, nT_0 - u) e^{j2\pi nT_0[\beta_i - \alpha_k]_{T_0^{-1}}} e^{j2\pi[u\alpha_k - v\beta_i]_{T_0^{-1}}} \rangle dv \end{aligned}$$

Using (2.58) yields:

$$\begin{aligned} & \langle R_x(nT_0 - v, nT_0 - u) e^{j2\pi nT_0 \{\alpha_m - \alpha_k\}_{T_0^{-1}}} \rangle = \\ & \sum_{l=-\infty}^{\infty} e^{j2\pi \left(\left[\frac{\alpha_m - \alpha_k}{2} \right]_{T_0^{-1}} - \frac{l}{2T_0} \right) (v+u)} R_x \left(\left[\alpha_k - \alpha_m \right]_{T_0^{-1}} + \frac{l}{T_0} \right) (u-v) \end{aligned} \quad (\text{B.17})$$

$$\begin{aligned} & \langle R_{x^*x}(nT_0 - v, nT_0 - u) e^{j2\pi nT_0 \{\beta_i - \alpha_k\}_{T_0^{-1}}} \rangle = \\ & \sum_{l=-\infty}^{\infty} e^{j2\pi \left(\left[\frac{\beta_i - \alpha_k}{2} \right]_{T_0^{-1}} - \frac{l}{2T_0} \right) (v+u)} R_{x^*x} \left(\left[\alpha_k - \beta_i \right]_{T_0^{-1}} + \frac{l}{T_0} \right) (u-v) \end{aligned} \quad (\text{B.18})$$

Replacing (B.17) in (B.16) yields:

$$\begin{aligned} & \langle E \{ \tilde{y}(n) \nabla_{a_k}^{* (n)}(u) \} \rangle \\ &= \sum_m \sum_l \int_{-\infty}^{\infty} a_m(v) e^{-j2\pi \frac{l}{2T_0} (u+v)} e^{j2\pi \left[\frac{\alpha_m + \alpha_k}{2} \right]_{T_0^{-1}} (u-v)} R_x \left(\left[\alpha_k - \alpha_m \right]_{T_0^{-1}} + \frac{l}{T_0} \right) dv + \\ & \sum_i \sum_l \int_{-\infty}^{\infty} b_i(v) e^{-j2\pi \frac{l}{2T_0} (u+v)} e^{j2\pi \left[\frac{\beta_i + \alpha_k}{2} \right]_{T_0^{-1}} (u-v)} R_{x^*x} \left(\left[\alpha_k - \beta_i \right]_{T_0^{-1}} + \frac{l}{T_0} \right) (u-v) dv \\ &= \sum_m \sum_l e^{-j2\pi \frac{l}{T_0} u} \left[a_m(u) \otimes R_x \left(\left[\alpha_k - \alpha_m \right]_{T_0^{-1}} + \frac{l}{T_0} \right) (u) e^{j2\pi \left(\left[\frac{\alpha_m + \alpha_k}{2} \right]_{T_0^{-1}} + \frac{l}{2T_0} \right) u} \right] + \\ & \sum_i \sum_l e^{-j2\pi \frac{l}{T_0} u} \left[b_i(u) \otimes R_{x^*x} \left(\left[\alpha_k - \beta_i \right]_{T_0^{-1}} + \frac{l}{T_0} \right) (u) e^{j2\pi \left(\left[\frac{\beta_i + \alpha_k}{2} \right]_{T_0^{-1}} + \frac{l}{2T_0} \right) u} \right] \end{aligned} \quad (\text{B.19})$$

Changing the variable from l to $l+p$ where either $p = \left\lceil \frac{\alpha_k - \alpha_m}{T_0} \right\rceil$ or $p = \left\lceil \frac{\alpha_k - \beta_i}{T_0} \right\rceil$ yields:

$$\begin{aligned}
& \langle E \{ \tilde{y}^{(n)}(u) \nabla_{\alpha_k}^{(n)*}(u) \} \rangle \\
&= \sum_m \sum_l e^{-j2\pi \frac{l}{T_0} u} \left[a_m(u) \otimes R_{\alpha_k - \alpha_m + \frac{l}{T_0}}(u) e^{j2\pi \left(\frac{\alpha_m + \alpha_k}{2} + \frac{l}{2T_0} \right) u} \right] + \\
& \quad \sum_i \sum_l e^{-j2\pi \frac{l}{T_0} u} \left[b_i(u) \otimes R_{\alpha_k - \beta_i + \frac{l}{T_0}}(u) e^{j2\pi \left(\frac{\beta_i + \alpha_k}{2} + \frac{l}{2T_0} \right) u} \right]
\end{aligned} \quad (\text{B.20})$$

Substituting (B.14), (B.15) and (B.20) in (B.12) and tending the gradient to zero yields the following equation for $[\alpha_k]_{T_0^{-1}} = 0$:

$$\begin{aligned}
& \sum_m \sum_l e^{-j2\pi \frac{l}{T_0} u} \left[a_m(u) \otimes R_{\alpha_k - \alpha_m + \frac{l}{T_0}}(u) e^{j2\pi \left(\frac{\alpha_m + \alpha_k}{2} + \frac{l}{2T_0} \right) u} \right] + \\
& \quad \sum_i \sum_l e^{-j2\pi \frac{l}{T_0} u} \left[b_i(u) \otimes R_{\alpha_k - \beta_i + \frac{l}{T_0}}(u) e^{j2\pi \left(\frac{\beta_i + \alpha_k}{2} + \frac{l}{2T_0} \right) u} \right] \\
&= \sigma_{d_0^{(r)}}^2 e^{-j2\pi \phi_0^{(r)}} \left(\sum_{m=1}^{\infty} z_0^{(1)*} ((D+m)T_0 - u - t_0^{(r)}) h_h(m) + z_0^{(1)*} (DT_0 - t_0^{(r)} - u) \right) + \\
& \quad \sigma_{d_0^{(n)}}^2 e^{-j2\pi \phi_0^{(n)}} \left(\sum_{m=1}^{\infty} z_0^{(2)*} ((D+m)T_0 - u - t_0^{(n)}) h_h(m) + z_0^{(2)*} (DT_0 - t_0^{(n)} - u) \right)
\end{aligned} \quad (\text{B.21})$$

and the following equation for $[\alpha_k]_{T_0^{-1}} \neq 0$:

$$\begin{aligned}
& \sum_m \sum_l e^{-j2\pi \frac{l}{T_0} u} \left[a_m(u) \otimes R_{\alpha_k - \alpha_m + \frac{l}{T_0}}(u) e^{j2\pi \left(\frac{\alpha_m + \alpha_k}{2} + \frac{l}{2T_0} \right) u} \right] + \\
& \quad \sum_i \sum_l e^{-j2\pi \frac{l}{T_0} u} \left[b_i(u) \otimes R_{\alpha_k - \beta_i + \frac{l}{T_0}}(u) e^{j2\pi \left(\frac{\beta_i + \alpha_k}{2} + \frac{l}{2T_0} \right) u} \right] = 0 \quad [\alpha_k]_{T_0^{-1}} \neq 0
\end{aligned} \quad (\text{B.22})$$

Taking the Fourier transform of (B.21) and (B.22) and using $R_{\alpha_k - \alpha}^u(\tau) = R_{\alpha_k}^{-u}(-\tau)$ [Appendix A] yields:

$$\begin{aligned}
& \sum_m \sum_l A_m \left(f + \frac{l}{T_0} \right) S_{\alpha_k - \alpha_m} \left(f + \frac{l}{2T_0} - \frac{\alpha_m}{2} \right) + \sum_i \sum_l B_i \left(f + \frac{l}{T_0} \right) \left(S_{\alpha_k - \beta_i} \left(f + \frac{l}{2T_0} - \frac{\beta_i}{2} \right) \right)^* = \\
& (\sigma_{d_0^{(r)}}^2 e^{-j2\pi(\phi_0^{(r)} + (DT_0 - t_0^{(r)})f}) Z_0^{(1)*}(f) + \sigma_{d_0^{(n)}}^2 e^{-j2\pi(\phi_0^{(n)} + (DT_0 - t_0^{(n)})f}) Z_0^{(2)*}(f)) (1 + H_b(e^{j2\pi f T_0}))
\end{aligned} \quad (\text{B.23})$$

$$\sum_m \sum_l A_m \left(f + \frac{l}{T_0} \right) S_x^{\left(\frac{l}{T_0} + \alpha_k - \alpha_m \right)} \left(f + \frac{l}{2T_0} - \frac{\alpha_m + \alpha_k}{2} \right) +$$

$$\sum_i \sum_l B_i \left(f + \frac{l}{T_0} \right) \left(S_{xx}^{\left(\beta_i - \alpha_k - \frac{l}{T_0} \right)} \left(f + \frac{l}{2T_0} - \frac{\beta_i + \alpha_k}{2} \right) \right)^* = 0 \quad [\alpha_k]_{T_0^{-1}} \neq 0 \quad (\text{B.24})$$

The gradient of TAMSE according to the $b_i(t)$ in the time u is:

$$\nabla_{b_i}^{TMSE}(u) = \langle E \{ (\tilde{y}(n) - (d_0(n-D) + f(n))) \nabla_{b_i}^{y^{(n)}}(u) \} \rangle \quad (\text{B.25})$$

where

$$\nabla_{b_i}^{y^{(n)}}(u) = x(nT_0 - u) e^{-j2\pi [\beta_i]_{T_0^{-1}} (nT_0 - u)} \quad (\text{B.26})$$

Replacing (B.26) in (B.25) and also considering (4.1) to (4.5), 2nd and 3th term of (B.25) will be

$$\langle E \{ d_0(n-D) \nabla_{b_i}^{y^{(n)}}(u) \} \rangle = \begin{cases} \sigma_{d_0^{(r)}}^2 z_0^{(1)} (DT_0 - t_0^{(r)} - u) e^{j2\pi \phi_0^{(r)}} - \\ \sigma_{d_0^{(i)}}^2 z_0^{(2)} (DT_0 - t_0^{(i)} - u) e^{j2\pi \phi_0^{(i)}} & [\beta_i]_{T_0^{-1}} = 0 \\ 0 & [\beta_i]_{T_0^{-1}} \neq 0 \end{cases} \quad (\text{B.27})$$

$$\langle E \{ f(n) \nabla_{b_i}^{y^{(n)}}(u) \} \rangle = \begin{cases} \sigma_{d_0^{(r)}}^2 e^{j2\pi \phi_0^{(r)}} \sum_{m=1}^{\infty} z_0^{(1)} ((D+m)T_0 - u - t_0^{(r)}) h_b(m) + \\ \sigma_{d_0^{(i)}}^2 e^{j2\pi \phi_0^{(i)}} \sum_{m=1}^{\infty} z_0^{(2)} ((D+m)T_0 - u - t_0^{(i)}) h_b(m) & [\beta_i]_{T_0^{-1}} = 0 \\ 0 & [\beta_i]_{T_0^{-1}} \neq 0 \end{cases} \quad (\text{B.28})$$

The first term of (B.25) is :

$$\begin{aligned}
& \langle E \{ \tilde{y}^{(n)}(n) \nabla_{b_i}^{\tilde{y}^{(n)}}(u) \} \rangle \\
&= \sum_k \sum_l e^{-j2\pi \frac{l}{T_0} u} \left[a_m(u) \otimes R_{xx^*}^{(\beta_i - \alpha_k + \frac{l}{T_0})} (u) e^{j2\pi \left(\frac{\alpha_k + \beta_i}{2} + \frac{l}{2T_0} \right) u} \right] + \\
& \sum_m \sum_l e^{-j2\pi \frac{l}{T_0} u} \left[b_i(u) \otimes R_{x^* x^*}^{(\beta_i - \beta_m + \frac{l}{T_0})} (u) e^{j2\pi \left(\frac{\beta_m + \beta_i}{2} + \frac{l}{2T_0} \right) u} \right]
\end{aligned} \quad (B.29)$$

Substituting (B.27), (B.28) and (B.29) in (B.25) and tending the gradient to zero, yields the following equation for $[\beta_i]_{T_0^{-1}} = 0$:

$$\begin{aligned}
& \sum_k \sum_l e^{-j2\pi \frac{l}{T_0} u} \left[a_m(u) \otimes R_{xx^*}^{(\frac{l}{T_0} - \alpha_k)} (u) e^{j2\pi \left(\frac{\alpha_k}{2} + \frac{l}{2T_0} \right) u} \right] + \\
& \sum_m \sum_l e^{-j2\pi \frac{l}{T_0} u} \left[b_i(u) \otimes R_{x^* x^*}^{(\frac{l}{T_0} - \beta_m)} (u) e^{j2\pi \left(\frac{\beta_m}{2} + \frac{l}{2T_0} \right) u} \right] \\
&= \sigma_{d_0^{(r)}}^2 e^{j2\pi \phi_0^{(r)}} \left(\sum_{m=1}^{\infty} z_0^{(1)} ((D+m)T_0 - u - t_0^{(r)}) h_b(m) + z_0^{(1)} (DT_0 - t_0^{(r)} - u) \right) - \\
& \sigma_{d_0^{(i)}}^2 e^{j2\pi \phi_0^{(i)}} \left(\sum_{m=1}^{\infty} z_0^{(2)} ((D+m)T_0 - u - t_0^{(i)}) h_b(m) + z_0^{(2)} (DT_0 - t_0^{(i)} - u) \right)
\end{aligned} \quad (B.30)$$

and the following equation for $[\beta_i]_{T_0^{-1}} \neq 0$:

$$\begin{aligned}
& \sum_k \sum_l e^{-j2\pi \frac{l}{T_0} u} \left[a_k(u) \otimes R_{xx^*}^{(\frac{l}{T_0} + \beta_i - \alpha_k)} (u) e^{j2\pi \left(\frac{\beta_i + \alpha_k}{2} + \frac{l}{2T_0} \right) u} \right] + \\
& \sum_m \sum_l e^{-j2\pi \frac{l}{T_0} u} \left[b_m(u) \otimes R_{x^* x^*}^{(\frac{l}{T_0} + \beta_i - \beta_m)} (u) e^{j2\pi \left(\frac{\beta_i + \beta_m}{2} + \frac{l}{2T_0} \right) u} \right] = 0 \quad [\beta_i]_{T_0^{-1}} \neq 0
\end{aligned} \quad (B.31)$$

Taking the Fourier transform of (B.30) and (B.31) yields:

$$\begin{aligned}
& \sum_k \sum_l A_k \left(f + \frac{l}{T_0} \right) S_{xx^*}^{(\frac{l}{T_0} - \alpha_k)} \left(f + \frac{l}{2T_0} - \frac{\alpha_k}{2} \right) + \sum_m \sum_l B_m \left(f + \frac{l}{T_0} \right) \left(S_{x^* x^*}^{(\frac{l}{T_0} - \beta_m)} \left(-f - \frac{l}{2T_0} + \frac{\beta_m}{2} \right) \right) = \\
& (\sigma_{d_0^{(r)}}^2 e^{j2\pi(\phi_0^{(r)} + (DT_0 - t_0^{(r)})f)} Z_0^{(1)}(f) - \sigma_{d_0^{(i)}}^2 e^{j2\pi(\phi_0^{(i)} + (DT_0 - t_0^{(i)})f)} Z_0^{(2)}(f)) (1 + H_b(e^{j2\pi f T_0}))
\end{aligned} \quad (B.32)$$

$$\begin{aligned}
& \sum_k \sum_l A_k \left(f + \frac{l}{T_0} \right) S_{xx^*}^{(\frac{l}{T_0} + \beta_i - \alpha_k)} \left(f + \frac{l}{2T_0} - \frac{\beta_i + \alpha_k}{2} \right) + \\
& \sum_m \sum_l B_m \left(f + \frac{l}{T_0} \right) \left(S_{x^* x^*}^{(\frac{l}{T_0} + \beta_i - \beta_m)} \left(-f - \frac{l}{2T_0} + \frac{\beta_i + \beta_m}{2} \right) \right) = 0 \quad [\beta_i]_{T_0^{-1}} \neq 0
\end{aligned} \quad (B.33)$$

Using (4.67) and (B.4), the MTAMSE which can be reached is equal to the following equation:

$$\begin{aligned}
 MTAMSE &= \sigma_{d_0}^2 - \int_{-\infty}^{\infty} a_0(u) \left(\sigma_{d_0^{(r)}}^2 e^{j2\pi\phi_0^{(r)}} z_0^{(1)} (DT_0 - u - t_0^{(r)}) + \right. \\
 &\quad \left. \sigma_{d_0^{(i)}}^2 e^{j2\pi\phi_0^{(i)}} z_0^{(2)} (DT_0 - u - t_0^{(i)}) \right) d(u) - \\
 &\quad \int_{-\infty}^{\infty} b_0(u) \left(\sigma_{d_0^{(r)}}^2 e^{j2\pi\phi_0^{(r)}} z_0^{(1)} (DT_0 - u - t_0^{(r)}) - \right. \\
 &\quad \left. \sigma_{d_0^{(i)}}^2 e^{j2\pi\phi_0^{(i)}} z_0^{(2)} (DT_0 - u - t_0^{(i)}) \right) d(u) \\
 &= \sigma_{d_0}^2 - \left[a_0(t) \otimes \left(\sigma_{d_0^{(r)}}^2 e^{j2\pi\phi_0^{(r)}} z_0^{(1)} (t - t_0^{(r)}) + \sigma_{d_0^{(i)}}^2 e^{j2\pi\phi_0^{(i)}} z_0^{(2)} (t - t_0^{(i)}) \right) \right] \Big|_{t=DT_0} \\
 &\quad \left[b_0(t) \otimes \left(\sigma_{d_0^{(r)}}^2 e^{-j2\pi\phi_0^{(r)}} z_0^{(1)*} (t - t_0^{(r)}) - \sigma_{d_0^{(i)}}^2 e^{-j2\pi\phi_0^{(i)}} z_0^{(2)*} (t - t_0^{(i)}) \right) \right] \Big|_{t=DT_0}
 \end{aligned} \tag{B.34}$$

Although all the forward filters does not effect (B.34) directly, their influences in $a_0(t)$ and $b_0(t)$ effect (B.34) indirectly. As it is apparent from (B.8), (B.23), (B.24), (B.32), (B.33) and (B.34), FRESH-DFE uses both CSD and CCSD of the received signal.

Appendix C

Analysis of GDFE

The output of TSR-TVF can be found by the following equation :

$$\begin{aligned}
 \hat{s}_0(n) = & \sum_{l=-\frac{M_a}{2}}^{\frac{M_a}{2}} \sum_{m=0}^{M_l} (x(n) \otimes a_{l,m}(n)) \delta_n(l, m) + \\
 & \sum_{l=1}^{M_c} \sum_{m=0}^{M_{0l}} (x(n) \otimes a_{0l,m}(n)) \delta_n(0l, m) + \\
 & \sum_{l=-\frac{M_a}{2}}^{\frac{M_a}{2}} \sum_{m=0}^{M_l} (x^*(n) e^{j2\pi(2f_l)n} \otimes b_{l,m}(n)) \delta_n(l, m) + \\
 & \sum_{l=1}^{M_c} \sum_{m=0}^{M_{0l}} (x^*(n) \otimes b_{0l,m}(n)) \delta_n(0l, m)
 \end{aligned} \tag{C.1}$$

where

$$\delta_n(l, m) = \begin{cases} 1 & n = m, m \pm M_p, m \pm 2M_p, \dots \\ 0 & \text{Otherwise} \end{cases} \tag{C.2}$$

Defining :

$$\begin{aligned}
 l + \frac{M_a}{2} &= 0l \\
 M_{l + \frac{M_a}{2}} &= M_{0l} \\
 a_{l + \frac{M_a}{2}, m}(n) &= a_{0l, m}(n) \\
 b_{l + \frac{M_a}{2}, m}(n) &= b_{0l, m}(n)
 \end{aligned} \tag{C.3}$$

yields :

$$\begin{aligned}
\hat{s}_0(n) = & \sum_{l=-\frac{M_d}{2}}^{\frac{M_d}{2}+M_c} \sum_{m=0}^{M_l} (x(n) \otimes a_{l,m}(n)) \delta_n(l, m) + \\
& \sum_{l=-\frac{M_d}{2}}^{\frac{M_d}{2}+M_c} \sum_{m=0}^{M_l} (x^*(n) e^{j2\pi(2f_l)n} \otimes b_{l,m}(n)) \delta_n(l, m)
\end{aligned} \tag{C.4}$$

Since $\delta_n(l, m)$ is periodic with period M_l , it can be represented by the following Fourier series :

$$\delta_n(l, m) = \frac{1}{M_l} \sum_{k=0}^{M_l} e^{j2\pi k \frac{(n-m)}{M_l}} \tag{C.5}$$

Substituting (C.5) in (C.4) yields :

$$\begin{aligned}
\hat{s}_0(n) = & \sum_{l=-\frac{M_d}{2}}^{\frac{M_d}{2}+M_c} \sum_{m=0}^{M_l} (x(n) \otimes a_{l,m}(n)) \frac{1}{M_l} \sum_{k=0}^{M_l} e^{j2\pi k \frac{(n-m)}{M_l}} + \\
& \sum_{l=-\frac{M_d}{2}}^{\frac{M_d}{2}+M_c} \sum_{m=0}^{M_l} (x^*(n) e^{j2\pi(2f_l)n} \otimes b_{l,m}(n)) \frac{1}{M_l} \sum_{k=0}^{M_l} e^{j2\pi k \frac{(n-m)}{M_l}} \\
= & \sum_{l=-\frac{M_d}{2}}^{\frac{M_d}{2}+M_c} \sum_{k=0}^{M_l} x(n) e^{j2\pi n \frac{k}{M_l}} \otimes \left[\frac{1}{M_l} \sum_{m=0}^{M_l} a_{l,m}(n) e^{j2\pi m \frac{k}{M_l}} \right] + \\
& \sum_{l=-\frac{M_d}{2}}^{\frac{M_d}{2}+M_c} \sum_{k=0}^{M_l} x^*(n) e^{j2\pi n (\frac{k}{M_l} + 2f_l)} \otimes \left[\frac{1}{M_l} \sum_{m=0}^{M_l} b_{l,m}(n) e^{j2\pi m \frac{k}{M_l}} \right]
\end{aligned} \tag{C.6}$$

$$\begin{aligned}
\hat{s}_0(n) = & \sum_{l=-\frac{M_d}{2}}^{\frac{M_d}{2}+M_c} \sum_{k=0}^{M_l} x(n) e^{j2\pi n \frac{k}{M_l}} \otimes A_{l,k}(n) + \\
& \sum_{l=-\frac{M_d}{2}}^{\frac{M_d}{2}+M_c} \sum_{k=0}^{M_l} x^*(n) e^{j2\pi n (\frac{k}{M_l} + 2f_l)} \otimes B_{l,k}(n)
\end{aligned} \quad (C.7)$$

where $A_{l,k}(n)$ and $B_{l,k}(n)$ are the discrete Fourier transforms (related to the parameter m) of $a_{l,m}(n)$ and $b_{l,m}(n)$ respectively. Defining $\alpha_{k,l} = \frac{k}{M_l}$ yields:

$$\begin{aligned}
\hat{s}_0(n) = & \sum_{l=-\frac{M_d}{2}}^{\frac{M_d}{2}+M_c} \sum_{k=0}^{M_l} x(n) e^{j2\pi n \alpha_{k,l}} \otimes A_{l,k}(n) + \\
& \sum_{l=-\frac{M_d}{2}}^{\frac{M_d}{2}+M_c} \sum_{k=0}^{M_l} x^*(n) e^{j2\pi n (\alpha_{k,l} + 2f_l)} \otimes B_{l,k}(n)
\end{aligned} \quad (C.8)$$

Changing the variable of the summation to $m=kl$ and assuming:

$$\begin{aligned}
\alpha_m &= \alpha_{k,l}, \quad h_m(n) = A_{l,k}(n) \\
\beta_m &= \alpha_{k,l} + 2f_l, \quad g_m(n) = B_{l,k}(n)
\end{aligned} \quad (C.9)$$

yields:

$$\hat{s}_0(n) = \sum_{m=0}^{(M_d+M_c)M_l} x(n) e^{j2\pi n \alpha_m} \otimes h_m(n) + \sum_{m=0}^{(M_d+M_c)M_l} x^*(n) e^{j2\pi n \beta_m} \otimes g_m(n) \quad (C.10)$$

Equation (C.10) represents the sampled version of HSR-TVF. Therefore HSR-TVF is equivalent to the TSR-TVF and its filters are the discrete Fourier transform of the TSR-TVF filters. Therefore the bank of filters in HSR-TVF and the bank of filters in TSR-TVF are time-frequency duals.

Since HSR-TVF and TSR-TVF are time-frequency duals, FSE and GFSE are time-frequency duals too.

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