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TOPICS IN PHENOMENOLOGY
IN
SUPERSYMMETRIC GAUGE THEORIES

by

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A thesis submitted to the Faculty of Graduate Studies in partial
fulfillment of the requirements for the degree of Doctor of Philosophy.

Carleton University

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The undersigned hereby recommend to
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ABSTRACT

Recently there has developed great interest in supersymmetric theories of the fundamental interactions, where the supersymmetry has been broken at low energies. The advent of these theories poses the problem of determining the most effective ways to test these theories, and to find the new particles whose existence they imply. To aid in this search we have undertaken a series of calculations of processes that we felt would have significant "potential" for testing the theory.

We have examined pair production of the gauge and Higgs fermions of supersymmetry in the decays of heavy quarkonia. The processes $^3S_1 \rightarrow \tilde{g}\tilde{g}$, $^3S_1 \rightarrow \tilde{\lambda}\tilde{\lambda}$ or $\tilde{\gamma}\tilde{\gamma}$, and $^1S_0 \rightarrow \tilde{\lambda}\tilde{\lambda}$ or $\tilde{\gamma}\tilde{\gamma}$ are calculated, and representative numerical results are presented for heavy quarkonium systems.

We have calculated the induced one-loop decay of physical Z^0 's into gluino pairs and the Z^0 branching ratio to gluino pairs is calculated. This is compared to gluino pair production in association with quark-antiquark pairs.

The new particles and interactions implied by supersymmetric theories may induce, via radiative corrections, new effective low energy interactions in the "standard" sector of the theory. We calculate the effects of lepton-hadron "contact" interactions thus induced as they appear in the π_{12} decay and in the energy levels of hydrogenic atoms (induced Lamb shift).

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CHAPTER 1 - AN INTRODUCTION TO SUPERSYMMETRY

(1) Introduction

Since the discovery of supersymmetry (GOL.71) (VOL.72) (WES.74) in the early seventies, to the multivaried modern forms (NIL.84), it has been of ever increasing interest to particle physics. This growth has come about despite the lack of direct experimental evidence requiring explanation via supersymmetry.

The progress with gauge theories as the description of fundamental interactions has dominated particle physics and stands virtually unchallenged. The earliest and most successful theory to date is QED which is based on an abelian local gauge theory wherein the Lagrangian is invariant under the gauge group $U(1)$. The theory initially seemed to be plagued with infinities but means were devised whereby reliable calculations were able to be performed (via regularization and renormalization) and the agreement with experiment was spectacular. The basic symmetry principle was extended to non-abelian gauge groups by Yang and Mills (YAN.54), and the modern era of gauge theories was born.

In 1967 and 1968 Weinberg (WEI.67) and Salam (SAL.68) independently proposed models where weak and electromagnetic interactions are unified in a Yang-Mills gauge theory with the intermediate vector bosons $W^{\pm}, Z^0$ and the photon as gauge bosons. Their conceptual "breakthrough" was to attribute the observed dissimilarities between weak and electromagnetic interactions to a spontaneous breakdown of gauge symmetry. They employed what has come to be called the "Higgs Mechanism", after Higgs, who among

others had studied it (ABE.73). This involved making a connection between the massless vector gauge bosons (two degrees of freedom) of a Lagrangian invariant under local gauge transformations and the massless scalar Goldstone bosons (one degree of freedom) one gets from spontaneously broken symmetries, to yield a massive vector boson (3-degrees of freedom). Though the renormalization of these theories was suspected it wasn't until 1971 that 't Hooft ('t HO.71) proved that this symmetry breaking mechanism produced renormalizable formulations of massive Yang-Mills theories.

Non-abelian gauge theories provided the basis for a description of strong interactions. The theory was dubbed Quantum Chromodynamics (GRE.64) (HAN.65) and is based on a non-abelian $SU(3)$ of colour charge. The $SU_C(3)$ is exact and the eight vector bosons (gluons), like the photon from the exact $U(1)$ of Quantum Electrodynamics, are massless. The direct product of these gauge groups, $SU_C(3) \times SU_L(2) \times U(1)_Y$ (with the Weinberg and Salam model as generalized by GIM for the incorporation of Hadrons (GLA.70)) a not quite unified description of strong, electromagnetic and weak interactions, has come to be known as the standard model and, until recently, (UA1.84) (UA2.84) was capable of explaining or was at least consistent with the experimental findings.

Gauge theories are on firm experimental footing as the direct experimental observation of the intermediate vector bosons of the Weinberg and Salam theory has so dramatically demonstrated (UA1.83A) (UA1.83B) (UA1.83C) (UA2.83A) (UA2.83B). What lies beyond the standard model has been a subject of much study and interesting speculation. Perhaps the best known extension has been the imbedding of the standard model in a "unifying" gauge group. The first such scheme was proposed in 1974 by

Georgi and Glashow (GEO.74), who employed the simple lie group $SU(5)$ as the gauge group and the resulting theory has one coupling constant at the unified mass scale and fixes many parameters and relationships that needed to be determined experimentally before. The model has proved remarkably durable and has only recently run into trouble for its proton lifetime predictions which are too short for the new experimental limits (IRV.83). But before the experimental challenge Grand Unified Theories (or GUTS) were questioned on the basis of several weaknesses. The best known is the "Gauge Hierarchy Problem" wherein to get phenomenologically acceptable models the grand unified and weak mass scale are split by $O(10^{12})$. This can be achieved only by fine tuning Higgs sector parameters to 0(25) digit accuracy. Linked to this was the arbitrariness of the Higgs sector which severely reduces the predictive power of the theories. In fact having fundamental scalars, the Higgs bosons, introduced substantial problems with the gauge theory formalism.

One has [For reviews see (ELL.83A)(ELL.83B)(NIL.83)] quadratically divergent contributions to the Higgs propagator (Fig.1.1),

which give a mass shift,

$$\delta m_H^2 = O(\alpha^n) \quad (\Lambda = 0 \text{ (} M_x \text{ or } M_p \text{)})$$

Eq. 1.1

or other mechanisms that tend to pull the Higgs mass to $O(M_x^2)$ or $O(M_p^2)$. Furthermore the question of gravitational interactions and the applicability of the Yang-Mills gauge theory formalism to them appeared ill-fated. So the grand unification was at best incomplete. Ironically the possible solution to these and other problems had been discovered before the problems were posed.

The "possible" solution that we are referring to is supersymmetry.

The trend with gauge theories based on lie groups has been to increase the symmetry which restricts the theory and hence increase the predictive power of the theory. [The key and central problem then becomes the symmetry breaking]. The celebrated theorem of Coleman and Mandula (COL.67) tells us that in more than 1+1 dimensions to have a non-trivial S-matrix the only possible conserved quantities that transform as tensors under the Lorentz group are the following;

The usual space time symmetries	P_μ Energy-Momentum
	$M_{\mu\nu}$ Lorentz-Invariance
Lorentz Invariant Quantum Numbers	Q_i electric charge etc.

It then appears that for an interacting field theory no new symmetries could be added as the theory would become a trivial one with no predictive power whatsoever. But it has been shown (HAA.75) that if one extends the theories based on lie algebras to ones based on graded lie algebras one can still construct a conserved current even in the presence of interactions and hence have a non-trivial theory. Gauge theories based on graded lie algebras are known today as Supersymmetric Gauge Theories. They may well represent the last frontier.

(ii) The Algebra of Supersymmetry

Supersymmetry [(ELL.83A)(ELL.83B)(NIL.83) and Ref. therein] (SUSY) is a new type of symmetry relating bosons to fermions and vice-versa. This comes about as the additional generators, responsible for making a lie algebra a graded lie algebra, are fermionic and thus carry angular momentum. In its simplest form we have the usual 10 generators of the poincare group which obey their familiar commutation rules,

$$\begin{aligned} [P_\mu, P_\nu] &= 0 \\ [M_{\mu\nu}, P_\rho] &= ig_{\mu\rho} P_\nu - ig_{\nu\rho} P_\mu \\ [M_{\mu\nu}, M_{\rho\sigma}] &= g_{\mu\sigma} M_{\nu\rho} + g_{\nu\rho} M_{\mu\sigma} - g_{\mu\rho} M_{\nu\sigma} - g_{\nu\sigma} M_{\mu\rho} \end{aligned} \quad \text{Eq. 1.2}$$

But in addition one has four anticommuting generators, Q_α , sometimes referred to as supertranslations. The new elements, Q_α , have the following commutation rules:

$$\begin{aligned} [Q_\alpha, P_\mu] &= 0, \\ [Q_\alpha, J_{\mu\nu}] &= \frac{1}{2}(\sigma_{\mu\nu})_\alpha^\beta Q_\beta, \\ \{Q_\alpha, Q_\beta\} &= -(\gamma_\mu C)_{\alpha\beta} P_\mu \end{aligned} \quad \text{Eq.1.3}$$

As one can see the Q_α are unaffected by space time translations and they transform as a Dirac spinor under homogeneous Lorentz transformations. The third relation is the main ingredient of the theory and shows that the supertranslations are not independent. The Q_α satisfy the reality condition,

$$Q_\alpha = C_{\alpha\beta} \bar{Q}^\beta \quad \text{Eq.1.4}$$

and hence are components of a Majorana spinor. An equivalent and commonly used representation, is that the Q_α ($\alpha = 1, 2$) are chosen to have the transformation properties of a left-handed Weyl spinor, a $(\frac{1}{2}, 0)$ representation under Lorentz transformations, and its hermitean adjoint is denoted by $\bar{Q}_\beta$, a right-handed Weyl spinor $(0, \frac{1}{2})$. The anticommutator of Q and $\bar{Q}$ transforms as $(\frac{1}{2}, \frac{1}{2})$ under Lorentz transformations and by the Coleman-Mandula theorem there is only one operator that does this, P_μ . Hence

$$[Q_\alpha, P_\mu] = 0$$

$$\{Q_\alpha, \bar{Q}_\beta\} = 2\sigma_{\alpha\beta}^\mu P_\mu, \quad \text{Eq. 1.5}$$

which has an obvious equivalence to the previous formulation.

Returning to the first formulation we look at extensions of SUSY, generated by sets of Majorana spinors $Q_{\alpha j}$, $j = 1, 2, \dots, N$. The third rule of Eq. 1.3 is replaced by,

$$\{Q_{\alpha i}, Q_{\beta j}\} = -\delta_{ij}(\gamma_\mu C)_{\alpha\beta} P_\mu + C_{\alpha\beta} Z_{ij} + (\gamma_5 C)_{\alpha\beta} Z'_{ij} \quad \text{Eq. 1.6}$$

where the generators Z_{ij} and Z'_{ij} are antisymmetric in i, j . These are called central charges and commute with all of the generators of the system. They are only present for $N \geq 1$ and so, as we will see, fall outside the scope of this work.

Only $N \leq 4$ is possible for gauge theories, since each $Q_{\alpha j}$ changes helicity by $\frac{1}{2}$ a unit, and only helicities between ± 1 are allowed:

$$\lambda = +1 \xrightarrow{Q_1} \frac{1}{2} \xrightarrow{Q_2} 0 \xrightarrow{Q_3} -\frac{1}{2} \xrightarrow{Q_4} -1 \quad \text{Eq. 1.7}$$

For theories including gravity we can have helicities between ± 2 and $N \leq 8$:

$$\lambda = +2Q_1^+ + 3/2Q_2^+ + 1Q_3^+ + 1/2Q_4^+ 0Q_5^+ - 1/2Q_6^+ -1Q_7^+ - 3/2Q_8^+ -2 \quad \text{Eq. 1.8}$$

(iii) Sparticles

But for our purposes we need only be concerned with N=1 SUSY which has the following supermultiplets of particles;

Gauge $(1, \frac{1}{2})$

Chiral $(\frac{1}{2}, 0)$

Eq. 1.9

Here also the charges, Q_α , carry no internal quantum numbers, so all partners must share identical quantum numbers with their sparticles.

In N=1 simple SUSY, with the standard model, we have the rich spectroscopy given in Table 1.1. [From Ref. (ELL.83A)]

PARTICLES	SPIN	SPARTICLE	SPIN	MASS
Quark q	$\frac{1}{2}$	Squark $\tilde{q}$	0	$> 0(15)\text{GeV}$
Lepton ℓ	$\frac{1}{2}$	Slepton $\tilde{\ell}$	0	$> 0(15)\text{GeV}$
Photon γ	1	Photino $\tilde{\gamma}$	$\frac{1}{2}$	
Gluon g	1	Gluino $\tilde{\lambda}$	$\frac{1}{2}$	$0(2)\text{GeV}$
$W^\pm$	1	Wino $\tilde{W}$	$\frac{1}{2}$	$> 0(15)\text{GeV}$
Z^0	1	Zino $\tilde{Z}$	$\frac{1}{2}$	
Higgs $H^\pm$	0	Shiggs $\tilde{H}^\pm$	$\frac{1}{2}$	$> 0(15)\text{GeV}$
H^0	0	$\tilde{H}^0$ H	$\frac{1}{2}$	

One needs two Higgs doublets $H_{1,2}$ of opposite hypercharge to cancel triangle anomalies and give masses to the charge $\frac{2}{3}$ and charge $-\frac{1}{3}$ quarks and leptons. These lower limits on the mass of the superpartners are from non-observation of direct production (tree graph level) at e^+e^- machines, and are model dependent.

Simple SUSY and its resulting spectroscopy given, in Table 1.1, exhibit the common feature of supersymmetry models, but we have no evidence for mass-degenerate boson-fermion pairs. So SUSY must be broken at least enough to put the superpartners of the known particles into a sufficiently heavy mass range to be consistent with their experimental absence at presently available energies. This breaking may be spontaneous, dynamical, or by explicit "soft" operators, without losing the desired features of the theory. The numerous schemes for symmetry breaking represent a study in themselves and the excellent reviews of Nilles (NIL.83) & (ELL. 83B) cover this key feature of model building in considerable detail. It is interesting that certain features persist despite the wealth of different scenarios. An intermediate mass scale (Δ) of $O(10^{11})$ GeV is present in many cases and the masses of some of the superpartners are $O(100)$ GeV.

(iv) Couplings

Phenomenologically acceptable models exist with spontaneously broken supergravity and manifest themselves in the low energy theory as softly explicitly broken global supersymmetry. As we are interested in the low energy sector [low energy here implies $O(100 \text{ GeV})$] we will only concern

ourselves with global supersymmetry which looks like an ordinary gauge theory with particular couplings and masses wherein mass scales come from "explicit soft breaking terms" that originate from the aforementioned supergravity theories. With this in mind we proceed to examine the couplings and masses of supersymmetry that we need to explore its phenomenology.

We begin by looking at the general form of couplings of chiral supermultiplets. [Here we follow notations and conventions of (ELL.83A & B)].

$$\phi \equiv (\phi_i \text{ of spin } \frac{1}{2}, S_i \text{ of Spin } 0) \quad \text{Eq. 1.10}$$

As in GUTS it is conventional to work with particles of identical helicities. So we take all fermions left handed;

$$\begin{aligned} f_R &\rightarrow (f^C)_L \\ f_L &\rightarrow f_L \end{aligned} \quad \text{Eq. 1.11}$$

and this leads to more convenient forms for the couplings (this is spelled out in Appendix A). But as the full Lagrangian is hermitian it would also have the right handed parts. The non-gauge couplings of the chiral smultiplets ϕ_i are determined by the superpotential. The superpotential is a polynomial $P(\phi)$ and is at most cubic for the theory to be renormalizable.

$$P = a_{ij}\phi_i\phi_j + b_{ijk}\phi_i\phi_j\phi_k \quad \text{Eq. 1.12}$$

Yukawa interactions involving two fermions are obtained by extracting the fermionic components of two of the ϕ_i and keeping just the spin zero components of what is left.

$$\begin{aligned} \therefore L_y &= (\phi_i \phi_j) \left. \frac{\partial^2 P}{\partial \phi_i \partial \phi_j} \right|_{\phi_k = s_k} \\ &= a_{ij} (\phi_i \phi_j) + b_{ijk} s_k (\phi_i \phi_j) \end{aligned} \quad \text{Eq. 1.13}$$

where by $(\phi_i \phi_j)$ we denote the Lorentz scalar combination of the two fermions.

The first term is the fermion mass matrix and the second is a true Yukawa interaction. Multiple scalar boson interactions are obtained from;

$$V_F = \sum_i \left| \frac{\partial P}{\partial \phi_i} \right|^2 = \left| a_{ij} s_j + b_{ijk} s_j s_k \right|^2 \quad \text{Eq. 1.14}$$

$\phi = s$

Now from the square of the first term on the right-hand side we get the boson (mass)² matrix.

$$(m_B^2)_{ij} = a_{ik} a_{kj}^* = (m_F m_F^*)_{ij} \quad \text{Eq. 1.15}$$

It is equal to the fermion (mass)² matrix. The square of the second term gives the quartic scalar interactions like one gets from a Higgs potential.

The fermion gauge interactions come directly from the standard gauge interactions as N=1 SUSY adds no internal quantum numbers. Recall that for left- and right-handed fermions, $\phi_{L,R}$, in different representations $T_{L,R}^a$ of the gauge group with generators a , we have the following interaction term;

$$g (\bar{\phi}_{L,R} \gamma^\mu T_{L,R}^a \phi_{L,R}) A_\mu^a \quad \text{Eq. 1.16}$$

Then for the scalar partners S_i of the fermions ϕ_i in the chiral multiplet ϕ_i one has;

$$g (S_{L,R}^* \delta^{\mu\nu} T_{L,R}^a S_{L,R}) A_\mu^a \quad \text{Eq. 1.17}$$

and gaugino interactions;

$$\mathcal{L} \equiv \sqrt{2} g (S_{L,R}^{*T} \lambda_{L,R}^a) (\psi_{L,R} \lambda_{L,R}^a) + \text{hc} \quad \text{Eq. 1.18}$$

where $(\psi \lambda)$ is again the Lorentz scalar combination of the two fermions.

Note that superpartner scalars have handedness and thus, for example, the right-handed selectron $\tilde{e}_R$ (or $\tilde{e}_L^c$) would have no $SU(2)_L$ gaugino or gauge interactions. One also has the additional quartic scalar interactions;

$$V_D \equiv \frac{g^2}{2} \sum_{a,s} \left| S_{L,R}^{*T} \lambda_{L,R}^a S_{L,R} \right|^2 \quad \text{Eq. 1.19}$$

which contribute to the full potential,

$$V = V_F + V_D \quad \text{Eq. 1.20}$$

The full interaction Lagrangian and the resulting Feynman rules are given in Appendix A.

(v) Masses

In general, for the mass matrices of the squarks and sleptons, we can have mixing in both flavour and "helicity" (L-R) space. Before SUSY is broken, conventional Higgs-fermion Yukawa interactions derived from a superpotential term,

$$P(\phi) \ni g_{Hqq} q_L q_L^c H, \quad \text{Eq. 1.21}$$

and with the aid of Eq. 1.14 give,

$$V_F \ni |g_{Hqq}|^2 [|\tilde{q}_L H|^2 + |\tilde{q}_L^c H|^2 + |\tilde{q}_L \tilde{q}_L^c|^2] \quad \text{Eq. 1.22}$$

Then using a Higgs vacuum expectation value $\langle 0|H|0 \rangle = v$, such that

$m_q = g_{Hqq} v$, one gets the following contributions to the $(\tilde{q}_L, \tilde{q}_R)$ mass matrix;

$$(\bar{q}_L, \bar{q}_R) \begin{bmatrix} m_{Q^2} & 0 \\ 0 & m_{Q^2} \end{bmatrix} \begin{bmatrix} \tilde{q}_L \\ \tilde{q}_R \end{bmatrix} \quad \text{Eq. 1.23}$$

This mass matrix ($\tilde{U}$) can be diagonalized simultaneously with the conventional quark mass matrix (U) as,

$$Q = \begin{bmatrix} U & V \\ V' & \tilde{U} \end{bmatrix} \quad \text{Eq. 1.24}$$

Q is the extended Cabbibo-Kobayashi-Maskawa matrix where; $U, \tilde{U}, V', V$ are $n_g \times n_g$ (n_g # of (s)quark generations) and V', V are two matrices that describe squark-quark couplings. The squark mass matrix for unbroken SUSY yields the expected result,

$$m_{\tilde{q}_{L,R}} = m_q \quad \text{Eq. 1.25}$$

and this untenable prediction is removed by introducing SUSY breaking (mass)² terms (originating as per earlier discussions) of the general form;

$$m_Q^2 |\tilde{q}|^2 + m_{\tilde{q}}^2 |\tilde{q}|^2 + m_Y^2 |\tilde{Y}|^2 \quad \text{Eq. 1.26}$$

Each of the mass matrices m_Q^2 and $m_{\tilde{q}}^2$ can be a priori a general matrix in flavour and "helicity" space. However careful analysis has shown (ELL. 83A and Ref. therein) that this leads to unacceptable flavour changing neutral current interactions and the gaugino interactions would no longer be flavour diagonal giving catastrophically large contributions to the K_1-K_2 mass matrix. Phenomenological considerations severely constrain the form

of the mass matrices, and the result of more detailed analysis (ELL. 83A and Ref. therein) yields the following flavour diagonalized form;

$$(\tilde{q}_L, \tilde{q}_R) \begin{bmatrix} L^2 \tilde{m}^2 + m_q^2 & A \tilde{m} m_q \\ A \tilde{m} m_q & R^2 \tilde{m}^2 + m_q^2 \end{bmatrix} \begin{bmatrix} \tilde{q}_L \\ \tilde{q}_R \end{bmatrix} \quad \text{Eq. 1.27}$$

where $m_L^2 = L^2 \tilde{m}^2$, $m_R^2 = R^2 \tilde{m}^2$, $L^2 \neq R^2$ in general, $A \sim O(1)$ and $\tilde{m} = O(20 \text{ to } 1000) \text{ GeV}$. For $L^2 \sim R^2$ and $\tilde{m}^2 \sim (100 \text{ GeV})^2$ this reduces to the form used in Chapter 4 for the left-right squark mixing and the left-right slepton mixing contributions for the box diagrams considered therein:

Mass matrices for the spin $\frac{1}{2}$ SUSY fermions --- gauginos and shiggses --- are also non-trivial. But as we make no direct use of them in what follows we will not detail them here.

(vi) Implications and Intentions

So SUSY gives us at least double the number of fundamental particles, with some of the superpartners possibly having masses between 20 and 1000 GeV, and many new couplings. What, then, are the physical implications. An R-symmetry (essentially a charge carried only by superpartners) survives, at least as a discrete symmetry, in most versions of SUSY (NIL. 83) and dictates that superpartners are produced only in association. The constraints on the masses given in Table 1.1 come from non-observation of

pair production at existing e^+e^- machines. As of the time of writing, considerable work has gone into mapping out the phenomenological consequences of SUSY. The new supersymmetric fermions accompanying either the massless gauge bosons (gluinos $\tilde{\lambda}$ with gluons, photinos $\tilde{\gamma}$ with photons) or the scalars of the (extended) Higgs sector (Higgsino $\tilde{H}$ with Higgs) are experimentally attractive; in many theories they are the lightest of the new particles, and in some theories either a photino or neutral Higgsino may be quasi-stable.

Though SUSY is theoretically attractive, lack of experimental evidence poses the problem of determining the most effective ways to test these theories to determine if there are indeed the new particles and couplings that the theory implies. To aid in this search we have undertaken a series of calculations of processes that we felt would have significant "potential", given the present day SUSY folklore, for the observation of either direct production of superpartners or their indirect effects, via radiative corrections, to standard processes. For the former intention we have calculated supersymmetric fermion pair production in quarkonium decay (CAM.83B) and Z^0 decay to two gluinos (CAM.83A) at the one loop level; and for the latter intention we have calculated the lepton-hadron interactions from supersymmetry radiative corrections for the π_{21} decay and the supersymmetric lamb shift (CAM.84). These calculations are detailed in their respective Chapters 2, 3, and 4. A summary of results and conclusions is given in Chapter 5.

CHAPTER 2 - QUARKONIA DECAY (CAM.83B)

(i) Introduction

It may be possible to detect the new particles predicted by supersymmetry in the decays of heavy quarkonium systems. In many theories the lightest of these are the supersymmetry fermions; gluinos $\tilde{\lambda}$, photinos $\tilde{\gamma}$ and Higgsinos $\tilde{H}$. We examined the phase space favoured pair production of these particles in two body decays of heavy quarkonia. This is particularly attractive for the Higgsino production via the decay $^3S_1(Q\bar{Q}) \rightarrow \tilde{H} + \tilde{\gamma}$ as it has the same enhancement, with this coupling proportional to the quark mass, that made Higgs searches with the decay of heavy vector mesons ($^3S_1(Q\bar{Q}) \rightarrow H + \gamma$) so promising (WIL.77). The two photino mode has distinct, but experimentally difficult, signature (quarkonia to nothing) but the gain in phase space with respect to the competitive process, quarkonium $\rightarrow g + \tilde{\lambda} + \tilde{\gamma}$ is offset by an increase in one power of α_{QED} . The advantages of this mode for gluinos are overshadowed by the substantial rates that result from the direct coupling of gluinos to gluons. Campbell, Ellis and Rudaz (CAM.82) have shown that gluino pairs may be produced via the decays in Fig. 2.1.

Fig. 2.1

These will dominate other mechanisms (which are suppressed by the scalar mass $\sim M_S^4$) unless the quarkonium mass happens to be about as large as the scalar mass. This scalar mass suppression is present in all of the rates we calculate but it is also present in all other photino and/or Higgsino production processes in quarkonia decay.

(ii) To Higgsino-Photino Pairs

The two body decay of heavy quarkonium to a Higgsino-photino pair is given in Fig. 2.2.

Fig. 2.2

From the Feynman rules of Appendix A we get the chiral component forms given in Table 2.1. The decay amplitude is then the sum of the amplitudes for these graphs. For notational convenience throughout we will use

$$L = \left(\frac{1 - \gamma_5}{2} \right) \text{ and } R = \left(\frac{1 + \gamma_5}{2} \right) \text{ from our invariant amplitudes onward. This}$$

gives a contribution to the invariant amplitude of,

TABLE 2.1 - LEFT-RIGHT COMPONENT FORMS FOR HIGGSINO-PHOTINO PAIRS

	<p><u>K-VERTEX</u> $-\sqrt{2} g_{\tilde{g}} A_{LK}^* \tilde{\gamma} \frac{(1-\gamma_5)}{2} \Psi_L^i$</p> <p><u>J-VERTEX</u> $\sqrt{2} g_{\tilde{g}} \bar{\Psi}_L \frac{(1-\gamma_5)}{2} \tilde{H} A_L^i$</p>	
	<p><u>K-VERTEX</u> $-\sqrt{2} g_{\tilde{g}} A_{RK}^* \tilde{\gamma} \frac{(1+\gamma_5)}{2} \Psi^i$</p> <p><u>J-VERTEX</u> $\sqrt{2} g_{\tilde{g}} \bar{\Psi}_L \frac{(1+\gamma_5)}{2} \tilde{H} A_R^i$</p>	
	<p><u>K-VERTEX</u> $-\sqrt{2} g_{\tilde{g}} A_{LK}^* \tilde{H} \frac{(1+\gamma_5)}{2} \Psi^i$</p> <p><u>J-VERTEX</u> $\sqrt{2} g_{\tilde{g}} \bar{\Psi}_L \frac{(1+\gamma_5)}{2} \tilde{\gamma} A_L^i$</p>	
	<p><u>K-VERTEX</u> $-\sqrt{2} g_{\tilde{g}} A_{RK}^* \tilde{H} \frac{(1-\gamma_5)}{2} \Psi^i$</p> <p><u>J-VERTEX</u> $\sqrt{2} g_{\tilde{g}} \bar{\Psi}_L \frac{(1-\gamma_5)}{2} \tilde{\gamma} A_R^i$</p>	
PROPAGATORS	LEFT	RIGHT
UNCROSSED	$\frac{i \delta_K^i}{(p_1 - q_1)^2 - M_L^2}$	$\frac{i \delta_K^i}{(p_1 - q_1)^2 - M_R^2}$
CROSSED	$\frac{i \delta_K^i}{(p_1 - q_2)^2 - M_L^2}$	$\frac{i \delta_K^i}{(p_1 - q_2)^2 - M_R^2}$

$$\begin{aligned}
 M_{\gamma H} \sim & -2ig_Y g_H \{ \bar{u}_Y(q_1) L u(p_1) \frac{1}{(p_1 - q_1)^2 - M_L^2} \bar{v}(p_2) L v_H(q_2) \\
 & + \bar{u}_Y(q_1) R u(p_1) \frac{1}{(p_1 - q_1)^2 - M_R^2} \bar{v}(p_2) R v_H(q_2) \\
 & + \bar{u}_H(q_2) R u(p_1) \frac{1}{(p_1 - q_2)^2 - M_L^2} \bar{v}(p_2) R v_Y(q_1) \\
 & + \bar{u}_H(q_2) L u(p_1) \frac{1}{(p_1 - q_2)^2 - M_R^2} \bar{v}(p_2) L v_Y(q_1) \} \quad \text{Eq. 2.1}
 \end{aligned}$$

we set $K = -2ig_Y g_H$ and drop it in what follows to be inserted at the end. In many theories the squark mass is large so we work in the approximation that the momentum exchanged is small compared to the squark mass.

$$\begin{aligned}
 \therefore M_{\gamma H} \sim & \frac{1}{M_L^2} \bar{u}_Y(q_1) L u(p_1) \bar{v}(p_2) L v_H(q_2) \\
 & + \frac{1}{M_R^2} \bar{u}_Y(q_1) R u(p_1) \bar{v}(p_2) R v_H(q_2) \\
 & + \frac{1}{M_L^2} \bar{u}_H(q_2) R u(p_1) \bar{v}(p_2) R v_Y(q_1) \\
 & + \frac{1}{M_R^2} \bar{u}_H(q_2) L u(p_1) \bar{v}(p_2) L v_Y(q_1) \quad \text{Eq. 2.2}
 \end{aligned}$$

The Fierz reordering transformation is,

$$I \otimes I = \frac{1}{4} \{ I \otimes I + \gamma_5 \otimes \gamma_5 + \gamma^\alpha \otimes \gamma_\alpha + \gamma^\alpha \gamma_5 \otimes \gamma_\alpha \gamma_5 + \frac{1}{2} \sigma^{\alpha\beta} \otimes \sigma_{\alpha\beta} \} \quad \text{Eq. 2.3}$$

which gives,

$$\begin{aligned} L \times L &= \frac{1}{4} \{ LILXI + L\gamma_5 L\gamma_5 + L\gamma^\alpha L\gamma_\alpha - L\gamma^\alpha \gamma_5 L\gamma_\alpha \gamma_5 + \frac{1}{2} L\sigma^{\alpha\beta} L\sigma_{\alpha\beta} \} \\ &= \frac{1}{4} \{ 2LXL + \frac{1}{2} L\sigma^{\alpha\beta} \sigma_{\alpha\beta} \} \end{aligned} \quad \text{Eq. 2.4}$$

and

$$R \times R = \frac{1}{4} \{ 2RXR + \frac{1}{2} R\sigma^{\alpha\beta} \sigma_{\alpha\beta} \} \quad \text{Eq. 2.5}$$

applying this to the 2nd and 4th terms in the amplitude gives,

$$\begin{aligned} M_{\gamma H} &\sim \frac{1}{4M_L^2} \{ 2\bar{u}_\gamma(q_1) L v_H(q_2) \bar{v}(p_2) Lu(p_1) \\ &+ \frac{1}{2} \bar{u}_\gamma(q_1) L\sigma^{\alpha\beta} v_H(q_2) \bar{v}(p_2) \sigma_{\alpha\beta} u(p_1) \} \\ &+ \frac{1}{4M_R^2} \{ 2\bar{u}_\gamma(q_1) R v_H(q_2) \bar{v}(p_2) Ru(p_1) + \frac{1}{2} \bar{u}_\gamma(q_1) R\sigma^{\alpha\beta} v_H(q_2) \bar{v}(p_2) \sigma_{\alpha\beta} u(p_1) \} \\ &+ \frac{1}{4M_L^2} \{ 2\bar{u}_H(q_2) R v_\gamma(q_1) \bar{v}(p_2) Ru(p_1) + \frac{1}{2} \bar{u}_H(q_2) R\sigma^{\alpha\beta} v_\gamma(q_1) \bar{v}(p_2) \sigma_{\alpha\beta} u(p_1) \} \\ &+ \frac{1}{4M_R^2} \{ 2\bar{u}_H(q_2) L v_\gamma(q_1) \bar{v}(p_2) Lu(p_1) + \frac{1}{2} \bar{u}_H(q_2) L\sigma^{\alpha\beta} v_\gamma(q_1) \bar{v}(p_2) \sigma_{\alpha\beta} u(p_1) \} \end{aligned}$$

Eq. 2.6

We now assume the quarkonium is sufficiently heavy to be non-relativistic and make this approximation in the $Q\bar{Q}$ part of the amplitude. This will allow us to subsequently use the Van-Royen-Weisskopf formula (ROY.67) to compute the partial decay widths for these processes.

For the non-relativistic reduction we need

$$R = \left(\frac{1 + \gamma_5}{2} \right) = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}; \quad L = \left(\frac{1 - \gamma_5}{2} \right) = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\sigma_{ij} = \begin{bmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{bmatrix} ; \quad \sigma_{0i} = \begin{bmatrix} 0 & -i\sigma_i \\ -i\sigma_i & 0 \end{bmatrix}$$

and with the spinor normalizations suppressed we have:

$$\begin{aligned} \bar{v}(p_2) \left(\frac{1 \pm \gamma_5}{2} \right) u(p_1) &= (x_2^+ \frac{\vec{\sigma} \cdot \vec{p}_2}{E_2 + m}, -x_2^+) \frac{1}{2} \begin{bmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \frac{\vec{\sigma} \cdot \vec{p}_1}{E_1 + m} x_1 \end{bmatrix} \\ &= \mp \frac{1}{2} x_2^+ x_1 \pm x_2^+ \frac{\vec{\sigma} \cdot \vec{p}_2}{E_2 + m} \frac{\vec{\sigma} \cdot \vec{p}_1}{E_1 + m} x_1 \\ &\sim \mp \frac{1}{2} x_2^+ x_1 + O(\vec{p}/m) \end{aligned} \quad \text{Eq. 2.7}$$

$$\begin{aligned} \bar{v}(p_2) \sigma_{ij} u(p_1) &= (x_2^+ \frac{\vec{\sigma} \cdot \vec{p}_2}{E_2 + m}, -x_2^+) \begin{bmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{bmatrix} \begin{bmatrix} x_1 \\ \frac{\vec{\sigma} \cdot \vec{p}_1}{E_1 + m} x_1 \end{bmatrix} \\ &= x_2^+ \frac{\vec{\sigma} \cdot \vec{p}_2}{E_2 + m} \sigma_k x_1 - x_2^+ \sigma_k \frac{\vec{\sigma} \cdot \vec{p}_1}{E_1 + m} x_1 \\ &\sim O(\vec{p}/m) \end{aligned} \quad \text{Eq. 2.8}$$

$$\begin{aligned} \bar{v}(p_2) \sigma_{0i} u(p_1) &= (x_2^+ \frac{\vec{\sigma} \cdot \vec{p}_2}{E_2 + m}, -x_2^+) \begin{bmatrix} 0 & -i\sigma_i \\ -i\sigma_i & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \frac{\vec{\sigma} \cdot \vec{p}_1}{E_1 + m} x_1 \end{bmatrix} \\ &= -i(x_2^+ \frac{\vec{\sigma} \cdot \vec{p}_2}{E_2 + m} \sigma_i \frac{\vec{\sigma} \cdot \vec{p}_1}{E_1 + m} x_1 - x_2^+ \sigma_i x_1) \end{aligned}$$

$$\sim ix_2^+ \sigma_i x_1 + O(\vec{p}/m) \quad \text{Eq. 2.9}$$

With these simplifications and letting the chiral scalars be degenerate ($M_L = M_R$) equation 2.6 becomes;

$$\begin{aligned} M_{\gamma H} \sim & \frac{1}{4M_S^2} \{-u_{\gamma}(q_1) \gamma_5 v_H(q_2) x_2^+ x_1 \\ & + \frac{i}{2} \bar{u}_{\gamma}(q_1) \sigma^{0i} v_H(q_2) x_2^+ \sigma_i x_1 - \bar{u}_H(q_2) \gamma_5 v_{\gamma}(q_1) x_2^+ x_1 \\ & + \frac{i}{2} \bar{u}_H(q_2) \sigma^{0i} v_{\gamma}(q_1) x_2^+ \sigma_i x_1\} \end{aligned} \quad \text{Eq. 2.10}$$

Using Appendix B we can find that,

$$\begin{aligned} \bar{u}_{\gamma}(q_1) \gamma_5 v_H(q_2) &= u(q_1)^* \{\gamma_0 \gamma_5\}_{\alpha}^{\beta} v(q_2)_{\beta} \\ &= v(q_2)_{\beta} \{\gamma_0 \gamma_5\}_{\beta}^{\alpha} u(q_1)^*_{\alpha} \\ &= v(q_2)_{\beta} \{\gamma_5\}_{\beta}^{\gamma} \gamma_0^{\alpha} u(q_1)^*_{\alpha} \\ &= v(q_2)_{\beta} \{\gamma_5\}_{\beta}^{\gamma} \bar{u}(q_1)^{\gamma} \\ &= v(q_2)_{\beta} \{\gamma_5\}_{\beta}^{\gamma} (C^{-1} v(q_1))_{\gamma} \\ &= v(q_2)^T \gamma_5^T C^{-1} v(q_1) \\ &= \bar{u}_H(q_2) C^{-1} \gamma_5^T C^{-1} v_{\gamma}(q_1) \\ &= \bar{u}_H(q_2) \gamma_5 C^{-1} C^{-1} v_{\gamma}(q_1) \end{aligned}$$

$$= - \bar{u}_H(q_2) \gamma_5 \tilde{v}_Y(q_1) \quad \text{Eq. 2.11}$$

There is no explicit minus sign arising from the interchange of the two spinors as we interchange component functions, not operators. In a like fashion we get,

$$\bar{u}_Y(q_1) \sigma^{0i} v_H(q_2) = \bar{u}(q_2) C^{-1} (\sigma^{0i})^T C^{-1} v(q_1)$$

$$\text{But } C^{-1} (\sigma^{0i})^T C^{-1} = \sigma^{0i}$$

$$\therefore \bar{u}_Y(q_1) \sigma^{0i} v_H(q_2) = \bar{u}_H(q_2) \sigma^{0i} \tilde{v}_Y(q_1) \quad \text{Eq. 2.12}$$

This simplifies the amplitude to be,

$$M_{HY} \sim \frac{1}{4M_S^2} \{ \bar{u}_H(q_2) \sigma^{0i} \tilde{v}_Y(q_1) (i\chi_2^+ \sigma_i \chi_1) \}. \quad \text{Eq. 2.13}$$

The contributions from the 3S_1 spin up give the following amplitude:

$$M_{HY} \sim \frac{-i}{4M_S^2} \{ i\bar{u}_H(q_2) \sigma^{02} \tilde{v}_Y(q_1) + \bar{u}_H(q_2) \sigma^{01} \tilde{v}_Y(q_1) \}. \quad \text{Eq. 2.14}$$

Squaring the amplitude gives,

$$|M|^2 \sim \frac{1}{16M_S^4} \{ (-i\tilde{v}_Y(q_1) \sigma^{02} u_H(q_2) + \tilde{v}_Y(q_1) \sigma^{01} u_H(q_2)) \cdot (i\bar{u}_H(q_2) \sigma^{02} \tilde{v}_Y(q_1) + \bar{u}_H(q_2) \sigma^{01} \tilde{v}_Y(q_1)) \}$$

$$\sim \frac{1}{16M_S^4} \left[\text{Tr} \left[\sigma^{02} \frac{\not{A}_2^{+M}}{2M} \sigma^{02} \frac{\not{A}_1^{-m}}{2m} \right] + \text{Tr} \left[\sigma^{01} \frac{\not{A}_2^{+M}}{2M} \sigma^{01} \frac{\not{A}_1^{-m}}{2m} \right] \right]$$

$$\begin{aligned}
 & - i \operatorname{Tr} \left[\sigma_{2M}^{02} \frac{A_2^{+M}}{2M} \sigma_{2m}^{01} \frac{A_1^{-m}}{2m} \right] + i \operatorname{Tr} \left[\sigma_{2M}^{01} \frac{A_2^{+M}}{2M} \sigma_{2m}^{02} \frac{A_1^{-m}}{2m} \right] \\
 |M|^2 & \sim \frac{1}{16M_S^4} \cdot \frac{1}{4Mm} \left\{ \operatorname{Tr} \left[\sigma_{2M}^{02} A_2^{02} A_1^{02} - mM \sigma_{2M}^{01} \sigma_{2m}^{02} \right] \right. \\
 & \quad + \operatorname{Tr} \left[\sigma_{2M}^{01} A_2^{01} A_1^{01} - mM \sigma_{2M}^{02} \sigma_{2m}^{01} \right] \\
 & \quad - i \operatorname{Tr} \left[\sigma_{2M}^{02} A_2^{01} A_1^{02} - mM \sigma_{2M}^{01} \sigma_{2m}^{02} \right] \\
 & \quad \left. + i \operatorname{Tr} \left[\sigma_{2M}^{01} A_2^{02} A_1^{01} - mM \sigma_{2M}^{02} \sigma_{2m}^{01} \right] \right\} \\
 & \text{as } \operatorname{Tr} \left[\sigma_{2M}^{02} A_2^{01} A_1^{02} \right] = \operatorname{Tr} \left[\sigma_{2M}^{01} A_2^{02} A_1^{01} \right]
 \end{aligned}$$

Giving

$$|M|^2 \sim \frac{1}{16M_S^4} \cdot \frac{1}{4Mm} \left\{ \operatorname{Tr} \left[\sigma_{2M}^{02} A_2^{02} A_1^{02} - mM \sigma_{2M}^{02} \sigma_{2m}^{02} \right] \right. \\
 \left. + \sigma_{2M}^{01} A_2^{01} A_1^{01} - mM \sigma_{2M}^{01} \sigma_{2m}^{01} \right\}$$

Eq. 2.15

$$\text{Now } \operatorname{Tr} \left[\sigma_{2M}^{02} \sigma_{2m}^{02} \right] = \operatorname{Tr} \left[\sigma_{2M}^{01} \sigma_{2m}^{01} \right] = -4$$

$$\operatorname{Tr} \left[\sigma_{2M}^{02} A_2^{02} A_1^{02} \right] = 4 \{ 2q_2^t q_1^t - q_1 \cdot q_2 - 2q_2^y q_1^y \}$$

$$\operatorname{Tr} \left[\sigma_{2M}^{01} A_2^{01} A_1^{01} \right] = 4 \{ 2q_2^t q_1^t - 2q_2^x q_1^x - q_2 \cdot q_1 \}$$

$$\therefore |M_{Hy}|^2 \sim \frac{1}{16M_S^4} \cdot \frac{1}{Mm} \{ 2q_2^t q_1^t + 2q_2^z q_1^z + 2Mm \}$$

and the constant that we dropped is

$$|2ig_Y g_H|^2 = 4g_Y^2 g_H^2 = 4e_q^2 g_H^2$$

giving

$$|M_{HY}|^2 = \frac{e_q^2 g_H^2}{2M_S^4} \cdot \frac{1}{M_{HY}} \{q_2^t q_1^t + q_2^Z q_1^Z + M_{HY}^m\} \quad \text{Eq. 2.16}$$

(iii) To Identical Gauginos

The other case to be treated here is the two body decay of heavy quarkonium into identical gauginos. Spin-statistics constrains the final state to be antisymmetric under exchange of the identical (Majorana) particles. The final states are symmetric colour singlets (contracted with δ^{ab}) and hence must be antisymmetric in the combined space and spin wave function. The allowed low lying states are 1S_0 and 3P_0 for $J = 0$ and 3P_1 for $J = 1$. Thus unless there is a parity violating component in the interaction, these are the allowed states that may decay into identical gauginos.

As we are dealing with an effective local interaction (heavy squark mass) the decay of the quarkonium is expected to be proportional to the value of the wave function at the origin. For the P-wave states this vanishes and the relevant quantity is (JAC:26),

$$\frac{1}{2} \left| \frac{\partial}{\partial r} \phi(r), r=0 \right|^2. \quad \text{Eq. 2.17}$$

M_{ONIA}

So the P-waves are strongly suppressed and the 1S_0 is the only parity allowed decay. However, a parity violating component is present if the chiral squarks are not degenerate in mass. With this proviso the 3S_1 can decay into identical gauginos and will be examined.

The decay of quarkonium into two gluinos proceeds via the diagram in Fig. 2.3 and we give the component forms in Table 2.2.

Fig. 2.3

The two photino mode is related to that of the gluinos by a factor for colour and charge which we will use at the end to get the rate for this process. From Table 2.2 we get a contribution to the invariant amplitude of,

$$\begin{aligned}
 M_{\lambda\lambda} \sim & -2ig^2 \{ \bar{u}(q_1) T_{Li}^{ak} u(p_1) \frac{\delta_k^j}{(p_1 - q_1)^2 - M_L^2} \bar{v}(p_2) T_{Lj}^{bi} v(q_2) \\
 & + \bar{u}(q_1) T_{Ri}^{ak} u(p_1) \frac{\delta_k^j}{(p_1 - q_1)^2 - M_R^2} \bar{v}(p_2) T_{Rj}^{bi} v(q_2) \\
 & - \bar{u}(q_2) T_{Li}^{bk} u(p_1) \frac{\delta_k^j}{(p_1 - q_2)^2 - M_L^2} \bar{v}(p_2) T_{Lj}^{ai} v(q_1) \\
 & - \bar{u}(q_2) T_{Ri}^{bk} u(p_1) \frac{\delta_k^j}{(p_1 - q_1)^2 - M_R^2} \bar{v}(p_2) T_{Rj}^{ai} v(q_1) \}
 \end{aligned}
 \quad \text{Eq. 2.18}$$

We have inserted a minus sign between terms for direct and exchange graphs, as we have interchanged identical fermion lines. The colour factors for terms, A,B,C,D are;

TABLE 2.2 LEFT-RIGHT COMPONENT FORMS FOR GLUINO PAIRS

	<p><u>K-VERTEX</u> $-\sqrt{2} g T_{L i}^{a k} A_{L k}^* \bar{\lambda}^a L \psi_i$</p> <p><u>J-VERTEX</u> $\sqrt{2} g T_{L j}^{b i} \bar{\psi}_i R \lambda^b A_L^j$</p>	
	<p><u>K-VERTEX</u> $-\sqrt{2} g T_{R i}^{a k} A_{R k}^* \bar{\lambda}^a R \psi_i$</p> <p><u>J-VERTEX</u> $\sqrt{2} g T_{R j}^{b i} \bar{\psi}_i L \lambda^b A_R^j$</p>	
	<p><u>K-VERTEX</u> $-\sqrt{2} g T_{L i}^{b k} A_{L k}^* \bar{\lambda}^b L \psi_i$</p> <p><u>J-VERTEX</u> $\sqrt{2} g T_{L j}^{a i} \bar{\psi}_i R \lambda^a A_L^j$</p>	
	<p><u>K-VERTEX</u> $-\sqrt{2} g T_{R i}^{b k} A_{R k}^* \bar{\lambda}^b R \psi_i$</p> <p><u>J-VERTEX</u> $\sqrt{2} g T_{R j}^{a i} \bar{\psi}_i L \lambda^a A_R^j$</p>	
PROPAGATORS	LEFT	RIGHT
UNCROSSED	$\frac{i \delta_{ij}^k}{(p_1 - q_1)^2 - M_L^2}$	$\frac{i \delta_{ij}^k}{(p_1 - q_1)^2 - M_R^2}$
CROSSED	$\frac{i \delta_{ij}^k}{(p_1 - q_2)^2 - M_L^2}$	$\frac{i \delta_{ij}^k}{(p_1 - q_2)^2 - M_R^2}$

$$A \quad T_{Li}^{aj} T_{Lj}^{bi}$$

$$B \quad T_{Ri}^{aj} T_{Rj}^{bi}$$

$$C \quad T_{Li}^{bj} T_{Lj}^{ai}$$

$$D \quad T_{Ri}^{bj} T_{Rj}^{ai}$$

and these are just

$$\text{Trace } T^{ab}.$$

Eq. 2.19

It becomes convenient to drop the constant $-2ig^2 \text{Tr}(T^{ab})$, which will be inserted at the appropriate point. We again assume that the squark mass is large and perform the following Fierz rearrangement on the second and fourth spinors;

$$2L \times R = (\gamma_\rho R) \times (\gamma^\rho L).$$

Eq. 2.20

$$\therefore M_{\lambda\lambda} \sim -\frac{1}{2M_L^2} \bar{u}(q_1) \gamma_\rho R v(q_2) \bar{v}(p_2) \gamma^\rho L u(p_1)$$

$$- \frac{1}{2M_R^2} \bar{u}(q_1) \gamma_\rho L v(q_2) \bar{v}(p_2) \gamma^\rho R u(p_1)$$

$$+ \frac{1}{2M_L^2} \bar{u}(q_2) \gamma_\rho R v(q_1) \bar{v}(p_2) \gamma^\rho L u(p_1)$$

$$+ \frac{1}{2M_R^2} \bar{u}(q_2) \gamma_\rho L v(q_1) \bar{v}(p_2) \gamma^\rho R u(p_1)$$

Eq. 2.21

$$M_{\lambda\lambda} = \frac{-1}{2M_L^2} (\bar{v}(p_2) \gamma^\rho L u(p_1)) (\bar{u}(q_1) \gamma_\rho R v(q_2) - \bar{u}(q_1) \gamma_\rho L v(q_2))$$

$$- \frac{1}{2M_R^2} (\bar{v}(p_2) \gamma^\rho R u(p_1)) (\bar{u}(q_1) \gamma_\rho L v(q_2) - \bar{u}(q_1) \gamma_\rho R v(q_2))$$

Eq. 2.22

where we have used $\bar{u}(q_2) \gamma_\rho R/L v(q_1) = \bar{u}(q_1) \gamma_\rho L/R v(q_2)$ (from Appendix B) on the indicated terms.

$$M_{\lambda\lambda} \sim \frac{-1}{2M_L^2} (\bar{v}(p_2)\gamma^\rho L u(p_1))(\bar{u}(q_1)\gamma_\rho \gamma_5 v(q_2)) \\ + \frac{1}{2M_R^2} (\bar{v}(p_2)\gamma^\rho R u(p_1))(\bar{u}(q_1)\gamma_\rho \gamma_5 v(q_2)) \quad \text{Eq. 2.23}$$

The following non-relativistic reduction is common for the parity conserving and parity non-conserving part of the calculation. As before we have suppressed the normalization.

$$\bar{v}(p_2)\gamma^0 u(p_1) \sim \left(\chi_2^+ \frac{\vec{\sigma} \cdot \vec{p}_2}{E_2 + m}, -\chi_2^+ \right) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \chi_1 \\ \frac{\vec{\sigma} \cdot \vec{p}_1}{E_1 + m} \chi_1 \end{bmatrix} \\ \bar{v}(p_2)\gamma^0 u(p_1) \sim O(\vec{p}/m) \\ \bar{v}(p_2)\gamma^0 \gamma_5 u(p_1) \sim \left(\chi_2^+ \frac{\vec{\sigma} \cdot \vec{p}_2}{E_2 + m}, -\chi_2^+ \right) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \chi_1 \\ \frac{\vec{\sigma} \cdot \vec{p}_1}{E_1 + m} \chi_1 \end{bmatrix} \\ \sim \chi_2^+ \chi_1 + O(\vec{p}/m) \\ \bar{v}(p_2)\gamma^i u(p_1) \sim \left(\chi_2^+ \frac{\vec{\sigma} \cdot \vec{p}_2}{E_2 + m}, -\chi_2^+ \right) \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix} \begin{bmatrix} \chi_1 \\ \frac{\vec{\sigma} \cdot \vec{p}_1}{E_1 + m} \chi_1 \end{bmatrix} \\ \sim \chi_2^+ \sigma_i \chi_1 + O(\vec{p}/m) \\ \bar{v}(p_2)\gamma^i \gamma_5 u(p_1) \sim \left(\chi_2^+ \frac{\vec{\sigma} \cdot \vec{p}_2}{E_2 + m}, -\chi_2^+ \right) \begin{bmatrix} \sigma_i & 0 \\ 0 & -\sigma_i \end{bmatrix} \begin{bmatrix} \chi_1 \\ \frac{\vec{\sigma} \cdot \vec{p}_1}{E_1 + m} \chi_1 \end{bmatrix} \\ \sim O(\vec{p}/m) \quad \text{Eq. 2.24}$$

First we look at the parity conserving case with the chiral squark

mass degeneracy. Putting $M_L = M_R = M_S$ gives,

$$M_{\lambda\lambda}^C \sim \frac{1}{2M_S^2} (\bar{v}(p_2) \gamma^\rho \gamma_5 u(p_1)) (\bar{u}(q_1) \gamma_\rho \gamma_5 v(q_2)) \quad \text{Eq. 2.25}$$

and using the non-relativistic reduction,

$$M_{\lambda\lambda}^C \sim \frac{1}{2M_S^2} (\bar{u}(q_1) \gamma_0 \gamma_5 v(q_2)) \chi_2^\dagger \chi_1 \quad \text{Eq. 2.26}$$

In Appendix C we have shown that $\chi_2^\dagger \chi_1 = \sqrt{2}$ for the 1S_0 state. As a check we note that $\chi_2^\dagger \chi_1 = 0$ for the 3S_1 . So from the 1S_0 quarkonium,

$$M_{\lambda\lambda}^C \sim \frac{1}{\sqrt{2} M_S^2} \bar{u}(q_1) \gamma_0 \gamma_5 v(q_2) \quad \text{Eq. 2.27}$$

and

$$\begin{aligned} |M_{\lambda\lambda}^C|^2 &\sim \frac{1}{2M_S^4} \text{Tr} \left[\frac{q_1^\dagger + m}{2m} \gamma_0 \gamma_5 \frac{q_2 - m}{2m} \gamma_0 \gamma_5 \right] \\ &\sim \frac{1}{2M_S^4 m^2} [2q_1^t q_2^t - q_1 \cdot q_2 + m^2] \end{aligned} \quad \text{Eq. 2.28}$$

or

$$|M_{\lambda\lambda}^C|^2 \sim \frac{4g^4}{2M_S^4 m^2} [2q_1^t q_2^t - q_1 \cdot q_2 + m^2] \quad \text{Eq. 2.29}$$

Where we have omitted the colour factor as these will be included when we specialize to individual processes.

For the parity non-conserving piece we retain the chiral squark mass splitting. After the non-relativistic reduction Eq. 2.23 gives,

$$M_{\lambda\lambda}^{NC} \sim \frac{-1}{2M_L^2} \left\{ \frac{1}{2} (\chi_2^{\dagger \sigma i} \chi_1) \right\} \bar{u}(q_1) \gamma_\rho \gamma_5 v(q_2)$$

$$\begin{aligned}
 & + \frac{1}{2M_R} \left[\frac{1}{2} (x_2^{\sigma^i} x_1) \right] \bar{u}(q_1) \gamma_\rho \gamma_5 v(q_2) \\
 & \sim -\frac{1}{4} \left[\frac{1}{M_L^2} - \frac{1}{M_R^2} \right] \bar{u}(q_1) \gamma_\rho \gamma_5 v(q_2) \{x_2^{\sigma^i} x_1\}
 \end{aligned}
 \tag{Eq. 2.30}$$

We note that this parity non-conserving term vanishes if $M_L = M_R$, as required. For the 3S_1 state $x_2^{\sigma^1} x_1 = -1$, $x_2^{\sigma^2} x_1 = -i$ and $x_2^{\sigma^3} x_1 = 0$. With this,

$$M_{\lambda\lambda}^{NC} \sim -\frac{1}{4} \left[\frac{1}{M_L^2} - \frac{1}{M_R^2} \right] \{ -\bar{u}(q_1) \gamma_1 \gamma_5 v(q_2) - i\bar{u}(q_1) \gamma_2 \gamma_5 v(q_2) \}$$

Squaring the amplitude,

$$\begin{aligned}
 |M_{\lambda\lambda}^{NC}|^2 &= \frac{1}{16} \left[\frac{1}{M_L^2} - \frac{1}{M_R^2} \right]^2 \{ \bar{u}(q_1) \gamma_1 \gamma_5 v(q_2) \bar{v}(q_2) \gamma_1 \gamma_5 u(q_1) \\
 &\quad - i\bar{u}(q_1) \gamma_1 \gamma_5 v(q_2) \bar{v}(q_2) \gamma_2 \gamma_5 u(q_1) \\
 &\quad + i\bar{u}(q_1) \gamma_2 \gamma_5 v(q_2) \bar{v}(q_2) \gamma_1 \gamma_5 u(q_1) + \bar{u}(q_1) \gamma_2 \gamma_5 v(q_2) \bar{v}(q_2) \gamma_2 \gamma_5 u(q_1) \} \\
 &= \frac{1}{16} \left[\frac{1}{M_L^2} - \frac{1}{M_R^2} \right]^2 \{ \text{Tr}[\gamma_1 \gamma_5 \left(\frac{\not{q}_2 - m}{2m} \right) \gamma_1 \gamma_5 \left(\frac{\not{q}_1 + m}{2m} \right)] \\
 &\quad - i \text{Tr}[\gamma_1 \gamma_5 \left(\frac{\not{q}_2 - m}{2m} \right) \gamma_2 \gamma_5 \left(\frac{\not{q}_1 + m}{2m} \right)] \\
 &\quad + i \text{Tr}[\gamma_2 \gamma_5 \left(\frac{\not{q}_2 - m}{2m} \right) \gamma_1 \gamma_5 \left(\frac{\not{q}_1 + m}{2m} \right)] + \text{Tr}[\gamma_2 \gamma_5 \left(\frac{\not{q}_2 - m}{2m} \right) \gamma_2 \gamma_5 \left(\frac{\not{q}_1 + m}{2m} \right)] \} \\
 &= \frac{1}{16} \left[\frac{1}{M_L^2} - \frac{1}{M_R^2} \right]^2 \frac{1}{4m^2} \{ \text{Tr}[\gamma_1 \not{q}_2 \gamma_1 \not{q}_1 + m^2 \gamma_1 \gamma_1] + \text{Tr}[\gamma_2 \not{q}_2 \gamma_2 \not{q}_1 + m^2 \gamma_2 \gamma_2] \}
 \end{aligned}$$

$$= \frac{1}{8m^2} \left[\frac{1}{M_L^2} - \frac{1}{M_R^2} \right]^2 [q_1^x q_2^x + q_1^y q_2^y + q_1 \cdot q_2 - m^2]$$

$$= \frac{1}{8m^2} \left[\frac{1}{M_L^2} - \frac{1}{M_R^2} \right]^2 [q_1^t q_2^t - q_1^z q_2^z - m^2] \quad \text{Eq. 2.31}$$

$$= \frac{g^4}{2m^2} \left[\frac{1}{M_L^2} - \frac{1}{M_R^2} \right]^2 [q_1^t q_2^t - q_1^z q_2^z - m^2] \quad \text{Eq. 2.32}$$

Where we have added the non-colour part of our constant in the last line. With equations 2.32, 2.29, and 2.16 we have the amplitudes required to calculate the desired rates. As the parity conserving matrix element (Eq. 2.29) enjoys considerable simplification in the C of M system [$q_1 = (E, p)$, $q_2 = (E, -p)$] we will include this now, as well as picking the photino for the identical gaugino cases, in the following list of our matrix elements.

$$|M(^3S_1 \rightarrow \tilde{\gamma}\tilde{\gamma})|^2 \sim \frac{e^2 g_H^2}{2M_S^4} [q_1^t q_2^t + q_1^z q_2^z + m_{\tilde{H}\tilde{Y}}^2] \quad \text{Eq. 2.33}$$

$$|M(^1S_0 \rightarrow \tilde{\gamma}\tilde{\gamma})|^2 \sim \frac{4e^2 g^4}{M_S^4} \quad \text{Eq. 2.34}$$

$$|M(^3S_1 \rightarrow \tilde{\gamma}\tilde{\gamma})|^2 \sim \frac{e^2 g^4}{2} \left[\frac{M_R^2 - M_L^2}{M_R^2 M_L^2} \right]^2 \frac{[q_1^t q_2^t - q_1^z q_2^z - m_{\tilde{Y}}^2]}{m_{\tilde{Y}}^2} \quad \text{Eq. 2.35}$$

The spinors have been normalized to one ($\bar{u}u = 1$; $u^\dagger u = \frac{E}{m}$) and so the appropriate convention must be matched when we bring in the phase space.

Using the Van-Royen-Weisskopf formula (ROY.67) the partial decay width is,

$$\Gamma(p)_{2S+1S_J} \sim (2\pi)^3 |A|^2 |\psi(0)|^2 \quad \text{Eq. 2.36}$$

where $|A|^2$ is computed with incoming plane waves as,

$$|A|^2 = \Gamma_p = \frac{v}{(2\pi)^3} \sigma_{if} \quad \text{Eq. 2.37}$$

$$\text{with } \sigma_{if} = \frac{1}{4F} \int |M_{if}|^2 d\text{LIPS}(s, p_1, p_2)$$

So the partial decay width is,

$$\begin{aligned} \Gamma(p)_{2S+1S_J} &= \frac{(2\pi)^3}{(2\pi)^3} \frac{|\phi(0)|^2}{4E_a E_b} \int |M_{2S+1S_J}^{if}|^2 d\text{LIPS}(s, p_1, p_2) \\ &= |\phi(0)|^2 \frac{4m_a m_b}{E_a E_b} m_1 m_2 \int |M_{2S+1S_J}^{if}|^2 d\text{LIPS}(s, p_1, p_2) \end{aligned}$$

$$\text{and in the C of M, } \left(\frac{4m_a m_b}{E_a E_b} = \frac{16m_q^2}{S} \right)$$

$$= |\phi(0)|^2 \frac{16m_q^2}{S} m_1 m_2 \int |M_{2S+1S_J}^{if}|^2$$

$$\frac{1}{(2\pi)^2} \delta(s^{\frac{1}{2}} - E_1 - E_2) \frac{1}{2E_1} \frac{1}{2E_2} q_1^2 dq_1 d\Omega_1^{\text{CM}}$$

$$\Gamma(q, \cos\theta_{\text{CM}})_{2S+1S_J} = |\phi(0)|^2 \frac{16m_q^2}{S} m_1 m_2 \int_{-1}^1 |M_{2S+1S_J}^{if}|^2 \frac{q_1}{2\pi} \frac{1}{4\sqrt{s}} d(\cos\theta_{\text{CM}})$$

Eq. 2.38

$$\text{where } q_1 = |\vec{p}_1| \text{ and } |\vec{p}_1|^2 = \frac{[s - (m_1 + m_2)^2][s - (m_1 - m_2)^2]}{4s}$$

$$\text{and } E_1 = (m_1^2 + q_1^2)^{\frac{1}{2}}, E_2 = (m_2^2 + q_1^2)^{\frac{1}{2}} \text{ \& } E_1 + E_2 = \sqrt{s} \text{ and } S = M_{\text{ONIA}}^2 \sim (2m_q)^2.$$

Using the matrix element for Higgsino photino mode (Eq. 2.33) for

$|M_{2S+1S_J}^{if}|^2$ we get,

$$\begin{aligned}
 \Gamma(^3S_1 \rightarrow \tilde{\gamma}\tilde{\gamma}) &\sim |\psi(0)|^2 \frac{16m_q^2}{s} \frac{g_H^2 e_q^2}{2M_S^4} \cdot \frac{q_1}{2\pi} \cdot \frac{1}{4\sqrt{s}} \int (q_1^t q_2^t + q_1^z q_2^z + m_H m_{\tilde{\gamma}}) d(\cos\theta_{CM}^1) \\
 &\sim |\psi(0)|^2 \frac{m_q^2}{(s)^{3/2}} \frac{q_1^2 g_H^2 e_q^2}{\pi M_S^4} \int (E_1 E_2 - q_1^2 \cos^2 \theta_{CM}^1 + m_H m_{\tilde{\gamma}}) d(\cos\theta_{CM}^1) \\
 &\sim \frac{e_q^2 g_H^2 m_q^2}{\pi s^{3/2} M_S^4} 2q_1 \left\{ E_1 E_2 + m_H m_{\tilde{\gamma}} - \frac{q_1^2}{3} \right\} |\psi(0)|^2 \\
 &\sim \frac{e_q^2 g_H^2}{\pi M_S^4 4M_0} 2q_1 \left\{ E_1 E_2 + m_H m_{\tilde{\gamma}} - \frac{q_1^2}{3} \right\} |\psi(0)|^2
 \end{aligned}$$

Eq. 2.39

For the parity conserving two photino process we have immediately,

$$\begin{aligned}
 \Gamma(^1S_0 \rightarrow \tilde{\gamma}\tilde{\gamma}) &\sim \frac{16}{\pi} \frac{m_q^2 |q_1| e_q^4}{(s)^{3/2}} \frac{m_{\tilde{\gamma}}^2}{M_S^4} |\psi(0)|^2 \\
 &\sim \frac{4e_q^4 m_{\tilde{\gamma}}^2}{\pi M_S^4 M_0} q_1 |\psi(0)|^2
 \end{aligned}$$

Eq. 2.40

And the parity non-conserving case gives,

$$\Gamma(^3S_1 \rightarrow \tilde{\gamma}\tilde{\gamma}) = \frac{e_q^4 m_q^2}{\pi s^{3/2}} \left[\frac{M_L^2 - M_R^2}{M_L^2 M_R^2} \right]^2 2q_1 \left\{ E_1 E_2 - m_{\tilde{\chi}}^2 + \frac{q_1^2}{3} \right\} |\psi(0)|^2$$

$$\frac{e_q^4}{4M_0} \left[\frac{M_L^2 - M_R^2}{M_L^2 M_R^2} \right]^2 2q_1 \left[E_1 E_2 - m_\chi^2 + \frac{q_1^2}{3} \right] |\psi(0)|^2. \quad \text{Eq. 2.41}$$

(v) Colour Factors

Our colour factors are determined simply by extending the colour analysis of G. Kane (KAN.77). In general we have,

Fig. 2.4

and the matrix element has $\delta_{ab}/\sqrt{3}$ for the initial wave function and δ_{ac} , δ_{bc} at the quark-photon(ino) vertices.

$$\begin{aligned} \therefore M &= \delta_{ab} \delta_{ac} \delta_{bc} / \sqrt{3} \\ &= \sqrt{3} \end{aligned}$$

$$\text{and } M^2 = 3.$$

Eq. 2.42

The following decay processes are colour identical and so are enhanced by a factor of 3; $\Gamma(^1S_0 \rightarrow \gamma\gamma)$, $\Gamma(^3S_1 \rightarrow \gamma\gamma)$ and $\Gamma(^3S_1 \rightarrow \gamma\gamma)$. For quarks to gluons(inos);

$$M \sim \frac{\delta_{ac}}{\sqrt{3}} \frac{\lambda_{ab}^i}{2} \frac{\lambda_{bc}^j}{2} = \frac{1}{4/3} \text{Tr}(\lambda_{ab}^i \lambda_{bc}^j), \quad \text{Eq. 2.43}$$

with $\text{Tr} \lambda_{ab}^i \lambda_{bc}^j = 0$, $i \neq j$ and $= 2$ if $i=j$,

$$\sigma \sim |M|^2 = \frac{4}{48} \sum_{ij} \delta_{ij} \delta_{ij} = \frac{1}{12} \sum_i \delta_{ii} = \frac{2}{3}, \quad \text{Eq. 2.44}$$

for eight gluons (inos). This is the colour factor for $\Gamma(^1S_0 \rightarrow \tilde{\lambda}\tilde{\lambda})$ and $\Gamma(^3S_1 \rightarrow \tilde{\lambda}\tilde{\lambda})$. The two gluino final states are related to the two photino ones by a factor of charge and colour,

$$\frac{\alpha_s^2}{\alpha^2 Q_j^4} \times \frac{2}{9} \text{ and } m_{\tilde{\gamma}} \rightarrow m_{\tilde{\lambda}}. \quad \text{Eq. 2.45}$$

The three basic widths are;

$$\Gamma(^3S_1 \rightarrow \tilde{\gamma}\tilde{\gamma}) = \frac{3\pi Q^2 \alpha^2}{2\sin^2 \theta_w} \frac{M_0^2}{M_W^2} \frac{1}{M^4 M_0} (2q_1) \left\{ E_1 E_2 + m_{\tilde{\gamma}} m_{\tilde{\gamma}} - \frac{q_1^2}{3} \right\} |\phi(0)|^2, \quad \text{Eq. 2.46}$$

$$\Gamma(^1S_0 \rightarrow \tilde{\gamma}\tilde{\gamma}) = 192\pi Q^4 \alpha^2 \frac{m_{\tilde{\gamma}}^2}{M_S M_0} q_1 |\phi(0)|^2, \quad \text{Eq. 2.47}$$

$$\Gamma(^3S_1 \rightarrow \tilde{\gamma}\tilde{\gamma}) = 12\pi Q^4 \alpha^2 \frac{1}{M_0} \left[\frac{M_L^2 - M_R^2}{M_L^2 M_R^2} \right]^2 2q_1 \left\{ E_1 E_2 - m_{\tilde{\gamma}}^2 + \frac{q_1^2}{3} \right\} |\phi(0)|^2. \quad \text{Eq. 2.48}$$

To get our branching ratios we divide these widths by the leading order decay width to gluons (LEA. 82) :

$$\Gamma(^3S_1 \rightarrow 3G) = \frac{160}{81} \alpha_s^3 (\pi^2 - 9) \frac{|\phi(0)|^2}{M_0^2},$$

$$\Gamma(^1S_0 \rightarrow 2G) = \frac{32}{3} \pi \alpha_s^2 \frac{|\phi(0)|^2}{M_0^2}.$$

Ea. 2.49

(vi) Results

We find for the decays of the heavy quarkonium:

$$R(3S_1 \rightarrow \tilde{H}\tilde{Y}) = \frac{243\pi\alpha^2 Q_q^2}{160\sin^2\theta_W\alpha_s^3} \cdot \frac{1}{(\pi^2-9)} \frac{M_0^3 q_1}{M_W^2 M_S^4} \left\{ E_1 E_2 - \frac{q_1^2}{3} + m_{\tilde{H}\tilde{Y}} \right\} \quad \text{Eq. 2.50}$$

$$\begin{aligned} R(1S_0 \rightarrow \tilde{\chi}\tilde{\chi}) &= \frac{\alpha_s^2}{\alpha^2 \times Q_q^4} \cdot \frac{2}{9} \cdot \left\{ 9 \cdot 2 Q_q^4 \frac{\alpha^2}{\alpha_s^2} \frac{m_{\tilde{\chi}}^2}{M_S^4} M_0 q_1 \right\} \\ &= \frac{4 M_0 m_{\tilde{\chi}}^2 q_1}{M_S^4} \quad \text{Eq. 2.51} \end{aligned}$$

(where the rate to $\tilde{\gamma}\tilde{\gamma}$ is in the bracket ($m_{\tilde{\chi}} \leftrightarrow m_{\tilde{\lambda}}$)).

$$\begin{aligned} R(3S_1 \rightarrow \tilde{\chi}\tilde{\chi}) &= \frac{\alpha_s^2}{\alpha^2 \times Q_q^4} \cdot \frac{2}{9} \cdot \left\{ \frac{3 \cdot 81 \pi Q_q^4}{20} \frac{\alpha^2}{\alpha_s^3} \frac{M_0}{(\pi^2-9)} \right. \\ &\quad \cdot \left[\frac{M_L^2 - M_R^2}{M_L^2 M_R^2} \right]^2 q_1 \left\{ E_1 E_2 - m_{\tilde{\chi}}^2 + \frac{q_1^2}{3} \right\} \Big\} \\ &= \frac{27\pi}{10(\pi^2-9)} [M_L^{-2} - M_R^{-2}]^2 \frac{M_0 q_1}{\alpha_s} \left\{ E_1 E_2 - m_{\tilde{\chi}}^2 + \frac{q_1^2}{3} \right\} \quad \text{Eq. 2.52} \end{aligned}$$

This decay rate is proportional to $(M_L^{-2} - M_R^{-2})^2 = [M_S^{-2} (\Delta M_S^2 / M_S^2)]^2$ with $\Delta M_S^2 \equiv M_R^2 - M_L^2$ and $M_S^2 \equiv M_L^2 \approx M_R^2$. Scaling by this gives,

$$\begin{aligned} R(3S_1 \rightarrow \tilde{\chi}\tilde{\chi}) / (\Delta M_S^2 / M_S^2)^2 &= \frac{27\pi}{10(\pi^2-9)} \alpha_s^{-1} \cdot M_0 q_1 \\ &\quad \cdot (E_1 E_2 - \frac{1}{3} q_1^2 - m_{\tilde{\chi}}^2) / M_S^4 \quad \text{Eq. 2.53} \end{aligned}$$

These ratios have been plotted for typical values of the parameters. The standard constants used were; $M_W \sim 80$ GeV, Q_q the numerical value of the quark charge ($\frac{1}{3}, \frac{2}{3}$), the strong coupling constant $\alpha_s \sim .13$, and the electroweak mixing factor $\sin^2 \theta_W \sim .21$.

In Fig. 2.5 we have plotted $R(^3S_1 \rightarrow H + \tilde{\gamma})$ versus $M_{\tilde{H}}/M_0$ for $M_{\tilde{Y}} = 0$. The $M_{\tilde{H}} = 0$ case is found simply by interchanging $M_{\tilde{H}}$ and $M_{\tilde{Y}}$. The dashed curve is the $\Upsilon(b\bar{b})$ with the scalar mass chosen to be as light as possible consistent with e^+e^- data as of June '83. The other two curves shown are for toponium systems with $M_t = M(t\bar{t})/2 = 20$ GeV and 35 GeV for the dot-dash and solid curves respectively with a scalar mass of $M_S = 80$ GeV $\sim M_W$. To get ratios for other values of the scalar mass the present ones can be scaled by $M_{S,old}^4/M_{S,new}^4$. This figure shows substantial rates for the larger toponium masses in the window accessible to e^+e^- machines before one reaches the Z^0 peak (UA1.83B).

Figure 2.6 is a plot of the ratio $R(^1S_0 \rightarrow \tilde{\chi}\tilde{\chi})$ versus $2M_{\tilde{\chi}}/M_0$ for various values of M_0/M_S . These values can be interpolated by using the fact that the ratio scales as $\sim (M_0/M_S)^4$. The ratio of $R(^1S_0 \rightarrow \tilde{\gamma}\tilde{\gamma})$ can be found from the curves in Figure 2.6 by multiplying by the colour and charge factor $\frac{9}{2} \alpha_q^2 / \alpha_s^2$. The substantial rates evident in Figure 2.6 may be nullified by the difficulty in obtaining the 1S_0 state in quarkonium systems.

In Figure 2.7 we have plotted $R(^3S_1 \rightarrow \tilde{\chi}\tilde{\chi}) / (\Delta M_S^2 / M_S^2)^2$ versus $2M_{\tilde{\chi}}/M_0$; the curves scale as M_0^4 / M_S^4 . Determination of the ratio $R(^3S_1 \rightarrow \tilde{\chi}\tilde{\chi})$ requires a choice of the chiral squark mass splitting. Suzuki's analysis of parity violating nuclear transitions that chiral squark mass splitting would

radiatively induce (SUZ.82) puts stringent constraints on the chiral squark mass splitting. These limits would severely suppress the rates for the present process. However Ellis et al (ELL.82) have argued that the Suzuki constraints could be satisfied with fractional chiral splittings of order unity ($\Delta M_S^2/M_S^2 \sim 0(1)$) provided the squarks of a particular generation (u&c, s&c, b&t) of the same chirality are not split in mass. The rates we consider here would then go essentially unsuppressed. As Figure 2.7 shows the ratio is large for a wide range of values for M_0/M_S . The $\Gamma(^3S_1 \rightarrow \gamma\gamma)$ results are generated from the gluinos rates by multiplying by the factor $\frac{9}{2} \alpha_Q^2 \alpha_q^4 / \alpha_s^2$ for colour and charge.

FIG. 2.5.

FIG. 2.6

FIG. 2.7

CHAPTER 3 - Z^0 DECAY TO GLUINOS (CAM.83A)

(i) Introduction

With a new generation of e^+e^- machines capable of producing a large number of Z^0 's, it may be possible to observe some of the new super particles in their decays. Campbell et al (CAM.82) have suggested that gluino pairs be looked for in e^+e^- annihilation associated with $q\bar{q}$ pairs in four jet events (Fig. 3.1); their calculation is valid for e^+e^- annihilation through both virtual γ 's and Z^0 's.

Fig. 3.1

Later, Nelson and Osland (NEL.82) noted that virtual photon production of gluino pairs can occur at the one loop level without quark jets and hence near threshold this process may dominate the four jet event, which would have very little phase space. In order that their amplitude not vanish, however, parity must be violated. Though electromagnetic couplings conserve parity, the violation may be induced by mass differences between the chiral scalar squarks appearing with their quark partners, in the loop diagrams. This is necessary as the allowed conserved current form factors are further restricted by spin statistics for identical (Majorana) fermions in the final state. (NEL.82)(NIE.82)(KAY.82)(SHR.82). Demanding that the final state be odd under interchange of identical particles, and as it must also have $J=1$, leaves only one invariant amplitude, that which corresponds

to the 3P_1 (axial vector form factor), for the process (see the table in Appendix D). The imaginary intrinsic parity of gluinos dictates that an $L=1$ final state has $J^P=1^+$ and since the original virtual photon has $J^P=1^-$, parity must be violated. To take advantage of the S-channel Z^0 pole we extend the analysis of Nelson and Osland (NEL.82) to the case of Z^0 decay to two gluinos in anticipation of the coming e^+e^- machines, aptly labelled Z^0 factories.

(ii) Spin-Statistics of Identical Gauginos, Form Factors and All That

Since the Z^0 current is not conserved, in addition to the form factors that participate in couplings to conserved currents $(\gamma_\mu, (\gamma_\mu q^2 - 2mq_\mu)\gamma_5, \sigma_{\mu\nu}q^\nu, \sigma_{\mu\nu}q^\nu\gamma_5)$, there will, in general, be two extra form factors (with q_μ and $q_\mu\gamma_5$). Spin statistics for identical final state (Majorana) particles demand that the gluino pair be antisymmetric under interchange. Of the $J=1$ final states for two spinors; 3S_1 , 3P_1 , 1P_1 and 3D_1 , only the 3P_1 is antisymmetric and as was the case earlier (also see Appendix D), it corresponds to the axial vector form factor. The two additional form factors correspond to the $J=0$ final states of two spinors; 1S_0 and 3P_0 , both of which are antisymmetric, hence they are allowed form factors for coupling to the Z^0 . However, if the Z^0 is on mass shell these extra couplings can be eliminated as we show in the following argument. From Appendix D we look at the piece of the weak neutral current induced by the F_3 form factors,

$$J_\mu = \bar{f}[q_\mu (F_3^V + F_3^A \gamma_5)]f \quad \text{Eq. 3.1}$$

The decay of an on mass shell Z^0 to a fermion antifermion pair gives, (see Fig. 3.2),

Fig. 3.2

$$\Gamma \sim |\epsilon^\mu J_\mu|^2$$

$$= (-g^{\mu\nu} + \frac{q^\mu q^\nu}{M^2}) q_\mu q_\nu \cdot \text{Tr}[(F_3^V + F_3^A \gamma_5)$$

$$\cdot (\not{p} + m)(F_3^V + F_3^A \gamma_5)(\not{p} + m)]$$

$$= (-q^2 + \frac{q^2 q^2}{M^2}) \text{Tr}[\quad]$$

$$= 0 \quad \text{for } q^2 = M^2$$

Eq. 3.2

Thus the only contribution to the decay of a physical Z^0 into gluino pairs is through the axial vector form factor.

This form factor receives contributions from the diagrams of Fig. 3.3; wherein there is a relative factor of (-1) between loops of particles of opposite chirality [which we define by the chirality of the unmixed squark line (we have ignored possible mixing between the chiral scalars)].

Fig. 3.3

The factor of minus one can be understood on the general grounds that the axial vector form factor is explicitly parity violating and hence there must be a relative minus sign between the contributions of the right- and left-handed chiral squark. When we write the diagrams of Figure 3.3 in their chiral component form there are only four relevant Feynman graphs and the aforementioned simplification (factor of (-1)) allows us to arrive at the axial vector form factor by only evaluating two. We made the arbitrary choice to calculate the right-handed chiral squark contributions. From Appendix A we get the chiral forms of the right-handed diagrams and display these with the rules for these vertices in Table 3.1. For completeness we give a schematic representation of the opposite chirality contribution.

(iii) The Contribution from Diagram A

Diagram A, Table 3.1 gives; $AMP \sim Z^\mu_{\phi} \tilde{M}^a_{\mu} \phi^b$, where

TABLE 3.1

$$V_s = g_z = \frac{-ie}{\sin\theta_W \cos\theta_W} Y_\mu \left[C_L^f \frac{(1-\gamma_5)}{2} + C_R^f \frac{(1+\gamma_5)}{2} \right]$$

WHERE $C^f = 2\sqrt{2} [T_L^3 - Q \sin^2\theta_W]$ AND ITS VALUES ARE GIVEN FOR A GENERIC QUARK FAMILY IN TABLE 3.2.

DIA B

$$V_s = g_z = \frac{-ie}{\sin\theta_W \cos\theta_W} (2l+q)_\mu C_R, C_R = 2\sqrt{2} (-Q \sin^2\theta_W)$$

WHERE Q IS THE CHARGE OF THE SQUARK.

TABLE 3.2 GENERAL FAMILY COUPLINGS

Particle	Sign in F	Coupling	Net Contribution
"u _L "	+	$2\sqrt{2}(\frac{1}{2} - 2/3 \sin^2 \theta_w)$	$2\sqrt{2}(\frac{1}{2} - 2/3 \sin^2 \theta_w)$
"u _R "	-	$2\sqrt{2}(-2/3 \sin^2 \theta_w)$	$2\sqrt{2}(2/3 \sin^2 \theta_w)$
"d _L "	+	$2\sqrt{2}(-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_w)$	$2\sqrt{2}(-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_w)$
"d _R "	-	$2\sqrt{2}(\frac{1}{3} \sin^2 \theta_w)$	$2\sqrt{2}(-\frac{1}{3} \sin^2 \theta_w)$

$$M_\mu^A \sim (\sqrt{2}g_c)^2 (\mu^{2-\omega})^3 (i)(-i)(-i)(e/\sin \theta_w \cos \theta_w) \cdot \int \frac{d^2 \ell}{(2\pi)^{2\omega}}$$

$$\{ [T_{Rt}^{ai} \frac{(1+\gamma_5)}{2} \frac{-i\delta_t^s}{(\ell+q+m)} \gamma_\mu (C_L^f \frac{(1+\gamma_5)}{2} + C_R^f \frac{(1+\gamma_5)}{2}) \frac{-i\delta_s^k}{\ell+m} T_{Rj}^{bk} \frac{(1-\gamma_5)}{2} \frac{-i\delta_j^i}{(\ell+q-p)^2 + M_B^2}] \}$$

Eq. 3.3

and a factor of $\mu^{2-\omega}$ enters for each vertex:

$$M_\mu^A \sim (\sqrt{2}g_c)^2 (\mu^{2-\omega})^3 (+1) e/(\sin \theta_w \cos \theta_w) T_{Rs}^{ai} T_{Ri}^{bs}$$

$$\cdot \int \frac{d^2 \ell}{(2\pi)^{2\omega}} \{ R \frac{(\ell+q-m)}{(\ell+q)^2 + m^2} \gamma_\mu (C_L^f + C_R^f) \frac{\ell-m}{\ell^2 + m^2} \cdot L \frac{1}{(\ell+q-p)^2 + M_B^2} \}$$

Eq. 3.4

where all quantities are in Euclidean space and C_L^f and C_R^f are coupling constants for chiral squarks.

By the well known properties of chiral projection operators only certain parts of M_μ will survive. Commuting/anticommuting the L&R's through gives;

$$M_\mu^A \sim + 2g_c^2 (\mu^{2-\omega})^3 e/(\sin\theta_\omega \cos\theta_\omega) \left(\frac{1}{2} \delta_a^b \right) \\ \cdot \frac{d^{2\omega} \ell}{(2\pi)^{2\omega}} \{ C_L^f m^2 R_{\gamma_\mu} L + C_R^f R(\gamma_\mu) \gamma_\mu L \} \\ \cdot \left[\frac{1}{(\ell+q)^2 + m^2} \cdot \frac{1}{\ell^2 + m^2} \cdot \frac{1}{(\ell+q-p)^2 + M_B^2} \right] \quad \text{Eq. 3.5}$$

where have used $T_{Ri}^{ai} T_{Ri}^{bs} = T_R(T^a T^b) = \text{Tr}(\frac{\lambda^a}{2} \frac{\lambda^b}{2}) = \frac{1}{2} \delta^{ab}$ and the λ 's are the Gell-Mann matrices.

$$\therefore M_\mu^A \sim K (\mu^{2-\omega})^3 \delta_a^b \int \frac{d^{2\omega} \ell}{(2\pi)^{2\omega}} \cdot \{ C_R^f R(\gamma_\rho \gamma_\mu \gamma_\sigma) L(\ell^\rho \ell^\sigma + q^\rho \ell^\sigma) \\ + C_L^f m^2 R_{\gamma_\mu} L \} \cdot \left[\frac{1}{(\ell+q)^2 + m^2} \cdot \frac{1}{\ell^2 + m^2} \cdot \frac{1}{(\ell+q-p)^2 + M_B^2} \right] \quad \text{Eq. 3.6}$$

and $K = g_c^2 e / \sin\theta_\omega \cos\theta_\omega$.

Calling the denominator I, we have:

$$I = \left[\frac{1}{(\ell+q)^2 + m^2} \cdot \frac{1}{\ell^2 + m^2} \cdot \frac{1}{(\ell+q-p)^2 + M_B^2} \right] \quad \text{Eq. 3.7}$$

We introduce two Feynman parameters and rewrite it as (for example RAM.81),

$$I = 2 \int_0^1 dx \int_0^{1-x} dy \frac{1}{[x(\ell+q)^2 + m^2 + y\ell^2 + m^2 + (1-x-y)(\ell+q-p)^2 + (1-x-y)M_B^2]^3}$$

$$= 2 \int_0^1 dx \int_0^{1-x} dy \frac{1}{\{x^2 + 2x \cdot [(1-y)q - (1-x-y)p] + [(1-x-y)(p^2 - 2q \cdot p + M_B^2) + (1-y)q^2 + (x+y)m^2]\}}$$

Eq. 3.8

$$\text{Let } R_\mu = (1-y)q_\mu - (1-x-y)p_\mu$$

$$M^2 = (1-x-y)(p^2 - 2q \cdot p + M_B^2) + (1-y)q^2 + (x+y)m^2$$

Eq. 3.9

S6

$$I = 2 \int_0^1 dx \int_0^{1-x} dy \frac{1}{[x^2 + 2x \cdot R + M^2]^3}$$

Eq. 3.10

The numerator has three contributions only one of which (the term quadratic in x) gives a divergent loop integration. From Ramond (RAM.81)(See Appendix E for general formula) we have these three integrals evaluated:

$$\underline{1} = \int \frac{d^{2\omega}}{(2\pi)^{2\omega}} \frac{1}{(x^2 + 2x \cdot R + M^2)^3} = \frac{\Gamma(3-\omega)}{(4\pi)^\omega \Gamma(3)} \frac{1}{[M^2 - R^2]^{3-\omega}}$$

$$\xrightarrow{\omega \rightarrow 2} \frac{1}{2(4\pi)^2} \frac{1}{[M^2 - R^2]}$$

Eq. 3.11

$$L = \int \frac{d^{2\omega}}{(2\pi)^{2\omega}} \frac{x_\mu}{(x^2 + 2x \cdot R + M^2)^3} = - \frac{\Gamma(3-\omega)}{(4\pi)^\omega \Gamma(3)} \frac{R_\mu}{[M^2 - R^2]^{3-\omega}}$$

$$\xrightarrow{\omega \rightarrow 2} \frac{-1}{2(4\pi)^2} \frac{R_\mu}{[M^2 - R^2]}$$

Eq. 3.12

$$LL = \int \frac{d^{2\omega}}{(2\pi)^{2\omega}} \frac{x_\mu x_\nu}{(x^2 + 2x \cdot R + M^2)^3}$$

$$= \frac{1}{(4\pi)^{\omega} \Gamma(3)} \left[\frac{R_{\mu} R_{\nu} \Gamma(3-\omega)}{(M^2-R^2)^{3-\omega}} + \frac{1}{2} \frac{\delta_{\mu\nu} \Gamma(2-\omega)}{(M^2-R^2)^{2-\omega}} \right] \quad \text{Eq. 3.13}$$

Now,

$$\Gamma(\epsilon) \sim \left[\frac{1}{\epsilon} + \psi(1) + \frac{1}{2} \epsilon \left(\frac{\pi^2}{3} + \psi^2(1) - \psi'(1) \right) + O(\epsilon^2) \right] \quad \text{Eq. 3.14}$$

$$\text{and } \psi'(1) = \frac{\pi^2}{6}; \psi(1) = -\gamma,$$

γ being the Euler-Mascheroni constant. Furthermore we can expand,

$$\frac{1}{(M^2-R^2)^{2-\omega}} = \frac{1}{(4\pi\mu^2)^{2-\omega}} \left(\frac{4\pi\mu^2}{M^2-R^2} \right)^{2-\omega} = \frac{1}{(4\pi\mu^2)^{2-\omega}} \exp \left\{ (\omega-2) \ln \frac{(M^2-R^2)}{4\pi\mu^2} \right\},$$

about $\omega=2$;

$$\sim 1 + (\omega-2) \ln \frac{(M^2-R^2)}{4\pi\mu^2} + O(\omega-2)^2, \quad \text{Eq. 3.15}$$

where we have dropped the constant of proportionality which will be attended to later. So

$$\begin{aligned} \frac{\Gamma(2-\omega)}{(M^2-R^2)^{2-\omega}} &\stackrel{\omega \rightarrow 2}{\sim} \left[\frac{1}{2-\omega} + \psi(1) + O(2-\omega) \right] \left[1 - (2-\omega) \ln \frac{(M^2-R^2)}{4\pi\mu^2} + O(\omega-2)^2 \right] \\ &\sim \frac{1}{2-\omega} + \psi(1) - \ln \frac{(M^2-R^2)}{4\pi\mu^2} + O(2-\omega). \end{aligned} \quad \text{Eq. 3.16}$$

Substituting this back into Eq. 3.13 gives:

$$\stackrel{\omega \rightarrow 2}{\sim} \frac{1}{2(4\pi)^2} \left[\frac{R_{\mu} R_{\nu}}{M^2-R^2} + \frac{\delta^{\mu\nu}}{2} \left(\frac{1}{2-\omega} - \gamma - \ln \frac{(M^2-R^2)}{4\pi\mu^2} \right) \right], \quad \text{Eq. 3.17}$$

where the constant has been attended to and ω set equal to two in the non-singular piece. With the momentum integral done we can substitute back and we have for the form factor ($\omega=2$ in finite terms):

$$\begin{aligned}
 M_{\mu}^A \sim & K \delta_a^b \cdot 2 \cdot \frac{(\mu^{2-\omega})^3}{2(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \\
 & \cdot [C_R^R(\gamma_\rho \gamma_\mu \gamma_\sigma) L \{ \frac{R^\rho R^\sigma}{(M^2 - R^2)} + \frac{\delta^{\rho\sigma}}{2} (\frac{1}{2-\omega} - \gamma - \ln \frac{(M^2 - R^2)}{4\pi\mu}) \} \\
 & + C_R^R(\gamma_\rho \gamma_\mu \gamma_\sigma) L q^\rho \{ \frac{-R^\sigma}{(M^2 - R^2)} \} + C_L^m{}^2 R_\mu L \{ \frac{1}{M^2 - R^2} \}] \quad \text{Eq. 3.18}
 \end{aligned}$$

Terms multiplying the pole piece $\frac{1}{2-\omega}$ and $(-\gamma)$ are independent of the kinematical variables and thus will cancel in the chiral sum so we drop them here and scale the coupling constant by $(\mu^{2-\omega})^3 \rightarrow 1$ as $\omega \rightarrow 2$. We have

$$\begin{aligned}
 M_{\mu}^A \sim & \frac{K \delta_a^b}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy R [C_R(\gamma_\rho \gamma_\mu \gamma_\sigma) \{ \frac{R^\rho R^\sigma}{M^2 - R^2} - \frac{\delta^{\rho\sigma}}{2} \ln \left(\frac{M^2 - R^2}{4\pi\mu} \right) \} \\
 & + C_R(\gamma_\rho \gamma_\mu \gamma_\sigma) q^\rho \{ \frac{-R^\sigma}{M^2 - R^2} \} + C_L^m{}^2 \gamma_\mu \{ \frac{1}{M^2 - R^2} \}] L \quad \text{Eq. 3.19}
 \end{aligned}$$

(iv) The Contribution From Diagram B

The contribution to the form factor from diagram B, Table 3.1, is;

$$\begin{aligned}
 M_{\mu}^B \sim & (\sqrt{2} g_c)^2 (\mu^{2-\omega})^3 (i)(-i)(-i) e / \sin \theta_\omega \cos \theta_\omega K \int \frac{d^2 \omega}{(2\pi)^{2\omega}} \\
 & \{ T_{Ri}^{at} (\frac{1+\gamma_5}{2}) \frac{-i\delta_i^j}{[-(\cancel{\ell} + \cancel{q} - \cancel{p}) + m]} T_{Rk}^{bj} (\frac{1-\gamma_5}{2}) \frac{-i\delta_k^s}{\ell^2 + M_B^2} C_R(2\ell + q)_\mu \frac{-i\delta_s^t}{(\ell + q)^2 + M_B^2} \} \quad \text{Eq. 3.20}
 \end{aligned}$$

As before a factor of $\mu^{2-\omega}$ enters for each vertex.

$$M_{\mu}^B \sim 2g_c^2 (\mu^{2-\omega})^3 (-1) e / \sin \omega \cos \omega \operatorname{Tr}(T^a T^b) \\ \cdot \int \frac{d^{2\omega} \ell}{(2\pi)^{2\omega}} \left\{ R \frac{\ell^2 + \ell \cdot p + m^2}{(\ell+q-p)^2 + m^2} L \frac{1}{\ell^2 + M_B^2} C_R(2\ell+q)_{\mu} \frac{1}{(\ell+q)^2 + M_B^2} \right\} \quad \text{Eq. 3.21}$$

Then commuting mutually annihilating chiral projection operators through gives,

$$M_{\mu}^B \sim -K (\mu^{2-\omega})^3 \delta^{ab} \int \frac{d^{2\omega} \ell}{(2\pi)^{2\omega}} \left\{ (R \gamma_{\nu} L) (\ell^{\nu} + q^{\nu} - p^{\nu}) \right. \\ \left. C_R(2\ell+q)_{\mu} \cdot \frac{1}{[(\ell+q-p)^2 + m^2][\ell^2 + M_B^2][(\ell+q)^2 + M_B^2]} \right\} \quad \text{Eq. 3.22}$$

where the trace has been done over the Gell-Mann matrices and $K = g_c^2 e / \sin \omega \cos \omega$, as earlier. This is the same denominator as we had before in Eq. 3.7 with the replacements $M \rightarrow m$ and $m \rightarrow M_B$, so we can take the results straight over with the inverse substitution.

$$\therefore I_B = \frac{1}{[(\ell+q-p)^2 + m^2][\ell^2 + M_B^2][(\ell+q)^2 + M_B^2]} \\ = 2 \int_0^1 dx \int_0^{1-x} dy \frac{1}{[\ell^2 + 2\ell \cdot R + M^2]^3} \quad \text{Eq. 3.23}$$

$$\text{with: } R_{\mu} = (1-y)q_{\mu} - (1-x-y)p_{\mu} \quad \text{Eq. 3.24}$$

$$M'^2 = (1-x-y)(p^2 - 2p \cdot q + m^2) + (1-y)q^2 + (x+y)M_B^2 \quad \text{Eq. 3.25}$$

So

$$M_{\mu}^B = -2K(\mu^{2-\omega})^3 \delta^{ab} \int_0^1 dx \int_0^{1-x} dy$$

$$\cdot \int \frac{d^{2\omega}}{(2\pi)^{2\omega}} C_R \left[\frac{2\ell_{\mu} \ell_{\nu} + \ell_{\mu} (q_{\nu} - p_{\nu}) + q_{\mu} \ell_{\nu} + q_{\mu} (q_{\nu} - p_{\nu})}{(\ell^2 + 2\ell \cdot R + M)^3} \right] R_{\gamma}^{\nu L}$$

Eq. 3.26

The momentum integrals to be evaluated are exactly those done previously and the analysis of Equations 3.11 through 3.16 holds giving;

$$M_{\mu}^B = -K(\mu^{2-\omega})^3 \delta^{ab} \frac{C_R}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \cdot R_{\gamma}^{\nu L}$$

$$\cdot \left[\delta_{\mu\nu} \left\{ \frac{1}{2-\omega} - \gamma - \ln \left(\frac{M'^2 - R^2}{4\pi\mu^2} \right) \right\} + \frac{2R_{\mu} R_{\nu}}{[M'^2 - R^2]} \right.$$

$$\left. - \frac{2R_{\mu} (q_{\nu} - p_{\nu}) - q_{\mu} R_{\nu}}{[M'^2 - R^2]} + \frac{q_{\mu} (q_{\nu} - p_{\nu})}{[M'^2 - R^2]} \right]$$

Eq. 3.27

We drop terms independent of the kinematical variables $(\frac{1}{2-\omega}, -\gamma)$ and take the $\omega \rightarrow 2$ limit.

$$M_{H\mu}^B = K \delta^{ab} \frac{C_R}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy R_{\gamma}^{\nu L} \cdot$$

$$\left[- \ln \left(\frac{M'^2 - R^2}{4\pi\mu^2} \right) \delta_{\mu\nu} + \frac{2R_{\mu} R_{\nu} - 2R_{\mu} (q_{\nu} - p_{\nu}) - q_{\mu} R_{\nu} + q_{\mu} (q_{\nu} - p_{\nu})}{(M'^2 - R^2)} \right]$$

Eq. 3.28

(v) Simplification of the Parametric Integrals

As M^2 and M'^2 enter M_{μ}^A (Eq. 3.19) and M_{μ}^B with $-R^2$ it becomes convenient to redefine the variables in terms of these quantities.

$$R^2 = ((1-y)q_\mu - (1-x-y)p_\mu)^2$$

$$= (1-2y+y^2)q^2 + (1-2x-2y+2xy+x^2+y^2)p^2 - 2(1-y)(1-x-y) \cdot p \cdot q$$

So

$$M'^2 - R^2 = q^2 y(1-y) - p \cdot q \cdot 2y(1-x-y) + p^2(x+y-2xy-x^2-y^2)$$

$$+ m^2(1-x-y) + M_B^2(x+y)$$

With $q^2 = s$; $q \cdot p = \frac{s}{2}$; $p^2 = -m_\lambda^2$,

$$M'^2 - R^2 = sxy - m_\lambda^2(x+y)(1-x-y) + (x+y)(M_B^2 - m^2) + m^2.$$

To this point all quantities are in Euclidean space. Continuation to Minkowski space is accomplished by $-m^2 \rightarrow m^2$.

$$M'^2 - R^2 = xys + (x+y)(1-x-y)m_\lambda^2 + (x+y)(m^2 - M_B^2) - m^2$$

$$= \xi$$

Eq. 3.29

Similarly

$$M^2 - R^2 = xys + (x+y)(1-x-y)m_\lambda^2 + (x+y)(M_B^2 - m^2) - M_B^2$$

$$= \xi$$

Eq. 3.30

and $R_\mu = (1-y)q_\mu - (1-x-y)p_\mu$ as before.

With these variables (all quantities are now in Minkowski space).

The parametric integrals for the form factors are:

$$M_\mu^A \sim \frac{K}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \cdot R [C_R \{-\ln(\frac{\xi}{4\pi\mu^2}) - \frac{\delta^{\rho\sigma}}{2}$$

$$+ \frac{(R^\rho - q^\rho)R^\sigma}{\xi} \gamma_\rho \gamma_\mu \gamma_\sigma + C_L \frac{m^2}{\xi} \gamma_\mu] L$$

Eq. 3.31

$$M_{\mu}^B \sim - \frac{KC_{\mu}}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \cdot R \left[\left\{ -\ln \left(\frac{\xi}{4\pi\mu^2} \right) \right\} \delta_{\mu\nu} \gamma^{\nu} \right. \\ \left. + \frac{[2R_{\mu} R_{\nu} - 2R_{\mu} (q_{\nu} p_{\nu}) - q_{\mu} R_{\nu} + q_{\mu} (q_{\nu} - p_{\nu})] \gamma^{\nu}}{\xi} \right] L \quad \text{Eq. 3.32}$$

These expressions are sandwiched between $\bar{u}(p)$ and $v(-(q-p))$ and as they are real on mass shell particles they satisfy the following equations; $\bar{u}(p-m)=0$ and $(-(q-p)+m)v=0$. We are free to (anti) commute the chiral projection operators to either the left or right [terms with the wrong number of gamma matrices have already been dropped] which allows us to use the mass shell condition on either side. This gives further simplification as terms without a γ_{μ} cannot contribute to the axial vector form factor. From this it can be seen that the finite terms from M_{μ}^B do not contribute to the required form factor. As before;

$$R_{\mu} = (1-y)(q_{\mu} - p_{\mu}) + xp_{\mu},$$

$$\therefore \text{On mass shell } R_{\mu} = (1-y+x)m$$

and the numerator;

$$(2R_{\mu} R_{\nu} - 2R_{\mu} (q_{\nu} - p_{\nu}) - q_{\mu} R_{\nu} + q_{\mu} (q_{\nu} - p_{\nu})) \gamma^{\nu},$$

$$\text{On mass shell } \rightarrow 2R_{\mu} (x-y+1)m - 2R_{\mu} m - q_{\mu} (x-y),$$

and hence does not contribute.

In M_{μ}^A terms of the form $+m^2 \gamma_{\mu}$ do contribute but in the finite terms of the form $(R-q) \gamma_{\mu} R$ we again have a simplification.

$$R_{\mu} q_{\mu} = y(p_{\mu} - q_{\mu}) - (1-x)p_{\mu}$$

$$\text{So } (R-A)_{\mu} R = [-y(p-q) - (1-x)p]_{\mu} [(1-y)(p-q) + xp]$$

$$\text{On mass shell } + y(1-y)m_{\lambda}^2 \gamma_{\mu} + xy(s-2m_{\lambda}^2) \gamma_{\mu} + xy m_{\lambda}^2 \gamma_{\mu} \\ - (1-x)(1-x-y)m_{\lambda}^2 \gamma_{\mu} + \text{terms not containing } \gamma_{\mu} \text{ [we have used}$$

$$(q-p) \cdot p = \frac{s}{2} - m_{\lambda}^2]. \text{ This can be rewritten as,}$$

$$(xys - (1-x-y)^2 m_{\lambda}^2) \gamma_{\mu}.$$

Eq. 3.33

This gives

$$M_{\mu}^A \sim \frac{K}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \cdot R \left[C_R \left\{ -\ln \left(\frac{\xi}{4\pi\mu^2} \right) \frac{\gamma_{\rho} \gamma_{\mu} \gamma_{\rho} \delta^{\rho\sigma}}{2} + \frac{(R-A)_{\mu} R}{\xi} \right\} \right. \\ \left. + C_L \frac{m^2}{\xi} \gamma_{\mu} \right] L$$

and selecting out the $\gamma_{\mu} \gamma_5$ piece we have;

$$M_{\mu}^A \sim - \frac{K}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \left\{ C_R \left(-\ln \left(\frac{\xi}{4\pi\mu^2} \right) \right) + C_R \frac{(xys - (1-x-y)^2 m_{\lambda}^2)}{\xi} \right\} \\ + \frac{C_L m^2}{\xi} \gamma_{\mu} \gamma_5$$

Eq. 3.34

Where we have used $\gamma_{\rho} \gamma_{\mu} \gamma^{\rho} = 2\gamma_{\mu}$ (Euclidean space relation necessary) and (anti)commuted the R to the L.

The other contribution gives;

$$M_{\mu}^B \sim - \frac{K C_R}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy R \left\{ -\ln \left(\frac{\xi}{4\pi\mu^2} \right) \gamma_{\mu} \right\} L$$

$$M_{\mu}^B \sim \frac{K C_R}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \left\{ -\ln \left(\frac{\xi'}{4\pi\mu^2} \right) \right\} \gamma_5 \gamma_{\mu} \quad \text{Eq. 3.35}$$

Combining the two results gives,

$$F \sim \frac{K}{4\pi^2} \int_0^1 dx \int_0^{1-x} dy \left[C_R \left\{ \ln(\xi/\xi') - \xi^{-1}(xys + \frac{C_L}{C_R} m^2 - \frac{(1-x-y)^2 m_{\lambda}^2}{m^2}) \right\} \right] \quad \text{Eq. 3.36}$$

where

$$\begin{aligned} \xi &= xys + (x+y)(1-x-y)m_{\lambda}^2 + (x+y)(M_B^2 - m^2) - M_B^2 \\ \xi' &= xys + (x+y)(1-x-y)m_{\lambda}^2 + (x+y)(m^2 - M_B^2) - m^2 \\ K &= g_c^2 e / \sin \theta_w \cos \theta_w \end{aligned} \quad \text{Eq. 3.37}$$

This can be rearranged into two pieces; one proportional to C_R which we call F_1 and the other proportional to C_L , designated F_2 . The parametric integrals to be evaluated are then;

$$F^1 = \int_0^1 dx \int_0^{1-x} dy \left\{ \ln[\xi(s, m_s^2, m_q^2, m_{\lambda}^2) / \xi(s, m_q^2, m_s^2, m_{\lambda}^2)] - \xi^{-1}(s, m_s^2, m_q^2, m_{\lambda}^2) [xys - (1-x-y)^2 m_{\lambda}^2] \right\} \quad \text{Eq. 3.38}$$

$$F^2 = \int_0^1 dx \int_0^{1-x} dy \left\{ -\xi^{-1}(s, m_s^2, m_q^2, m_{\lambda}^2) [m_q^2] \right\} \quad \text{Eq. 3.39}$$

with

$$\begin{aligned} \xi(s, m_s^2, m_q^2, m_{\lambda}^2) &= (x+y)(1-x-y)m_{\lambda}^2 \\ &+ xys + (x+y)(M^2 - m^2) - M^2 + i\epsilon, \end{aligned} \quad \text{Eq. 3.40}$$

and m_q and m_s for chiral quark and chiral squark masses.

(vi) Results

Including the left-handed squark contributions and summing over flavours leads to the following contribution to the width,

$$\left| \sum_q \sum_{L,R} (\pm 1)_{L,R} (C_{L,R}^q F_{L,R}^1 + C_{R,L}^q F_{L,R}^2) \right|^2 \quad \text{Eq. 3.41}$$

So dividing by the width of the Z^0 , (ELL.76B) we find for the ratio

$$R_{\lambda\lambda}(Z) \equiv \Gamma(Z^0 \rightarrow \lambda\lambda) / \Gamma(Z^0 \rightarrow f\bar{f})$$

$$R_{\lambda\lambda}(Z) = (\alpha_s^2 / 2\pi^2) \beta^3$$

$$\times \left| \sum_q \sum_{L,R} (\pm 1)_{L,R} (Q_{L,R}^q F_{L,R}^1 + Q_{R,L}^q F_{L,R}^2) \right|^2$$

$$\times \{ 2N_V + [1 + (1 - 4\sin^2\theta_w)^2] N_L$$

$$+ 3[1 + (1 - 8/3 \sin^2\theta_w)^2] N_{2/3} + 3[1 + (1 - 4/3 \sin^2\theta_w)^2] N_{-1/3} \}^{-1} \quad \text{Eq. 3.42}$$

Examining Table 3.2 in conjunction with the numerator of Eq. 3.42 we see that there are new potential sources of non-cancellation in the sum for the axial vector form factor. As well as splitting between the chiral partners of a given squark, which gave the only contribution in the $\Gamma(\gamma^* \rightarrow \lambda\lambda)$ case (NEL.82), we may also have weak isospin contributions if there are splittings between the doublet quarks, or the doublet squarks.

There is not yet any standard supersymmetric gauge theory. But of particular interest is a class of theories based on local simple supersymmetry (supergravity)(VAN.81)(CRE.82) broken at an "intermediate" mass scale. Typically (CRE.82A)(ELL.82B)(CRE.82B) in these theories the

chiral matter scalars receive a common, chirality and flavour independent, gravitational mass of order the gravitino mass, which is assumed to be approximately the weak mass scale. So for degenerate chiral scalars and doublet squarks the only remaining contribution is the isospin piece between the mass split " u_L " and " d_L " quarks. The only doublet with a big splitting is (t,b) and so the mass splitting between the top and bottom quarks control the rate. For the numerical evaluation we use;

$$\alpha_s = 0.17, \text{ and } \sin^2 \theta_w = .23.$$

For three generations;

$$\frac{\Gamma(Z^0 \rightarrow \lambda\lambda)}{\Gamma(Z^0 \rightarrow f\bar{f})} \approx .9 \times 10^{-4} \beta^3 |F_t - F_b|^2 \quad \text{Eq. 3.43}$$

We will assume that the common chirality and flavour independent mass of the squarks is 200 GeV. With the specified range of physical parameters the parametric integrals were numerically evaluated for several values of the top quark mass and a gluino mass ranging from zero to one-half the Z^0 mass (see Appendix F for an outline of this evaluation (and program)). The ratio of the widths $R_{\lambda\lambda}(Z)$ was plotted as a function of twice the gluino mass for a range of top quark masses in Figure 3.4. The rate saturates for M_t above $M_Z/2$.

We have also plotted the branching ratio for Z^0 decay into $\bar{q}q, \lambda\lambda$ (CAM.82) for comparison. As one may see from Figure 3.4, even with these very conservative numerical assumptions one can get substantial branching fractions into the two gluino final state.

FIG. 3.4

CHAPTER 4 - LEPTON-HADRON INTERACTIONS FROM SUPERSYMMETRY

RADIATIVE CORRECTIONS (CAM.84)

(i) Introduction

The superpartners may not be accessible at present accelerator energies. It is, then, interesting to examine indirect effects of the new particles; these arise at the one loop level where the exchange of the superpartners may induce new, effective interactions between particles of the standard model.

Effects with this origin have already been studied in the literature. One-loop corrections to the lepton anomalous magnetic moment have been examined (FAY.80)(GRI.82)(ELL.82), and put constraints on the slepton and electroweak gauge fermion masses. Radiatively induced strong interaction parity violation has been evaluated, (SUZ.82A)(DUN.83) and puts severe constraints on the mass splitting between left and right chiral squarks. Radiatively induced flavour changing neutral interactions have been calculated in these theories (CAM.83C)(SUZ.82B)(ELL.82'B'), and constrain the masses, mass splittings, and mixing angles for scalars and their coupling to gauge fermions.

We calculate the radiatively induced lepton-nucleon "contact" interactions in supersymmetric theories for the $\pi_{\ell 2}$ decay and the energy levels (Lamb shift) of hydrogenic atoms. Comparison with experiment will then give constraints on the masses of scalar particles and gauge fermions in these theories.

(ii) Mixing and Mass Matrix

From discussions in Chapter 1 we know that the mass squared matrix for the squarks or sleptons will be some $2n \times 2n$ matrix, where n is the number of quarks or leptons of a given charge. In the present work we are not interested in flavour mixing of squarks or sleptons. So in what follows we will suppress flavour (mixing) indices, which would always appear summed over at intermediate stages in our process, giving a result of order unity (GIM-unsuppressed). The mass squared matrix for the chiral scalars may then be written schematically,

$$\begin{bmatrix} \tilde{f}_L^* & \tilde{f}_R^* \end{bmatrix} \begin{bmatrix} A^2 & \Delta^2 \\ \Delta^2 & B^2 \end{bmatrix} \begin{bmatrix} \tilde{f}_L \\ \tilde{f}_R \end{bmatrix} \quad \text{Eq. 4.1}$$

where in the absence of supersymmetry breaking $A^2=B^2=0$, and Δ^2 is the mass of the associated fermion. Depending on how supersymmetry is broken, in general, the diagonal (A^2/B^2), and off diagonal (Δ^2) terms are unrelated in origin, or size and must be kept distinct. In a particular class of models, spontaneously broken minimal supergravity, there is a contribution to A^2 and B^2 equal to the gravitino mass squared and so we may take A^2 and B^2 equal to some large mass squared M_S^2 . This is in accord with the bounds on squark splitting from radiatively induced parity nonconservation (SUZ.82A)(DUN.83). But the contributions to Δ^2 will be from "F terms" in the potential or soft breaking terms derived from them. One then has two parameters describing chiral scalar masses: A large mass parameter M_S^2 giving the approximate mass squared of the scalars ($M_{\tilde{f}_{L,R}}^2 \sim M_S^2 + \Delta^2$); and a, in general smaller, parameter Δ^2 describing the mixing of the chiral

scalars. It is this mixing parameter that enhances the normally helicity suppressed rates that we calculate, while the large mass M_S^2 controls the rate.

For the low energy processes we consider, the momentum transfers in the process will, in general, be much less than the mass scale of the supersymmetric particles circulating in the loops. The resulting amplitude can then be described by a local operator of the light external particles, describing an effective contact interaction (GAI,74)(CAM.83C). By bounding the matrix elements for these operators from comparison with experiment we then get constraints on the supersymmetric content of the theory.

(iii) π_{L2} Decay

Let us first consider constraints from π_{L2} decay^{#1}. Lepton universality in this helicity suppressed weak decay implies that the ratio,

$$R = \Gamma(\pi^+ \rightarrow e^+ \nu_e) / \Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu), \quad 4.2$$

is sensitive to induced pseudoscalar interactions. The lepton-hadron contact interactions from supersymmetry radiative corrections modify the standard (V-A) value because they lead to pseudoscalar currents. The diagrams that contribute to the induced pseudoscalar interaction term in π_{L2} decay are given in Figure 4.1

^{#1} For general considerations and a rough supersymmetry calculation see (SHA.82).

Fig. 4.1

With the $\tilde{W}$ having chiral coupling (like the W), to get the "wrong chiral" amplitude (induced pseudoscalar piece) we need a chiral mass mixing insertion on the slepton line. As we intend to take the photino mass small with respect to the other superpartners circulating in the loops (which will amount to taking $m_{\tilde{\gamma}} \rightarrow 0$ and thus dropping all terms proportional to $m_{\tilde{\gamma}}$) we must also have a chiral mass mixing insertion on the squark line to get the proper chiral couplings. The chiral diagrams that contribute to the pseudoscalar interaction are given in Figure 4.2.

Fig. 4.2

Using Appendix A in conjunction with Fig. 4.2 we can write down the amplitudes that contribute to the pseudoscalar interaction that we are

interested in. The diagram from Fig. 4.2(A) gives;

$$\begin{aligned} \text{AMP} \sim g_2^2 e^2 \int \frac{d^4 k}{(2\pi)^4} & \left[\left(\bar{\psi}_{v_L} \right) \frac{\not{k} + \not{M}_w}{(k^2 - M_w^2)} \psi_u \right] \\ & \cdot \left(\bar{\psi}_{d_R} \right) \frac{\not{k} + \not{M}_Y}{(k^2 - M_Y^2)} \psi_{e_R} \left(\frac{1}{(k^2 - M_{e_R}^2)} \Delta_L^2 \frac{1}{(k^2 - M_{e_L}^2)} \right. \\ & \left. \cdot \frac{1}{(k^2 - M_{d_L}^2)} \Delta_Q^2 \frac{1}{(k^2 - M_{d_R}^2)} \right) \end{aligned} \quad \text{Eq. 4.3}$$

where we have dropped all external momenta as promised earlier; and kept only the consistent chiral pieces. With this next stage we neglect the photino mass as it is expected to be small compared to the other masses of the loop sparticles and exhibit the chiral projection operation explicitly.

$$\begin{aligned} \text{AMP} \sim g_2^2 e^2 \int \frac{d^4 k}{(2\pi)^4} & \left[\left(\bar{\psi}_v \right)_R \frac{\not{k} + \not{M}_w}{(k^2 - M_w^2)} \psi_u \right] \\ & \cdot \left(\bar{\psi}_d \right)_L \frac{\not{k}}{k^2} \psi_e \left(\frac{\Delta_L^2}{(k^2 - M_{e_R}^2)(k^2 - M_{e_L}^2)} \right) \left(\frac{\Delta_Q^2}{(k^2 - M_{q_R}^2)(k^2 - M_{q_L}^2)} \right) \end{aligned} \quad \text{Eq. 4.4}$$

then:

$$\text{AMP} \sim g_2^2 e^2 (\bar{\psi}_v)_R (\psi_u)_L (\bar{\psi}_d)_L (\psi_e)_R \times I, \quad \text{Eq. 4.5}$$

with,

$$I = \int \frac{d^4 k}{(2\pi)^4} \frac{k^2 \epsilon_{\mu\nu}}{k^2 (k^2 - M_w^2)} \frac{\Delta_L^2}{(k^2 - M_{e_R}^2)(k^2 - M_{e_L}^2)} \frac{\Delta_Q^2}{(k^2 - M_{q_R}^2)(k^2 - M_{q_L}^2)} \quad \text{Eq. 4.6}$$

where under symmetric integration we have dropped terms linear in k and $k_\mu k_\nu + k^2 \epsilon_{\mu\nu}$. So with the common large mass for the remaining loop sparticles ($\sim M_S$) we have:

$$I = \int \frac{d^4 k}{(2\pi)^4} \left\{ \frac{\epsilon_{\mu\nu} \Delta^2 \Delta^2 q}{(k^2 - M_S^2)^5} \right\}$$

$$\sim \delta_{\mu\nu} \frac{\Delta^2 \Delta^2 q}{M_S^6}$$

Eq. 4.7

So the amplitude is,

$$\text{AMP} \sim g_2^2 e^2 [O_I] \frac{\Delta^2 \Delta^2 q}{M_S^6},$$

Eq. 4.8

where $O_I = (\bar{\psi}_\nu R \gamma_\mu L \psi_u)(\bar{\psi}_d L \gamma^\mu R \psi_e)$.

Eq. 4.9

Now O_I is of the form $(\gamma_\mu L)x(\gamma^\mu R)$ which under Fierz rearrangement gives $2 R \times L$. So

$$O_I = (\bar{\psi}_\nu \gamma_\mu L \psi_u)(\bar{\psi}_d \gamma^\mu R \psi_e)$$

$$= 2(\bar{\psi}_\nu R \psi_e)(\bar{\psi}_d L \psi_u)$$

Eq. 4.10

And so,

$$\text{AMP} \sim 2g_2^2 e^2 (\bar{\psi}_\nu R \psi_e)(\bar{\psi}_d L \psi_u) \frac{\Delta^2 \Delta^2 q}{M_S^6}$$

Eq. 4.11

which is the sought after pseudoscalar term.

The contribution from Figure 4.2(B) in an analogous manner gives:

$$\text{AMP} \sim g_2^2 e^2 \{(\bar{\psi}_\nu^T) \gamma_\mu L C \bar{\psi}_d^T\} \{(\psi_u^T C^{-1} L \gamma^\mu R \psi_e)\} \times I,$$

Eq. 4.12

where we have used the charge conjugation operator (see Appendix B) to keep the same (anti)particles as in the previous diagram and I is as defined in

Eq. 4.6. Then,

$$\text{AMP} \sim g_2^2 e^2 \{(\bar{\psi}_\nu^T) \gamma_\mu L (C \bar{\psi}_d^T)\} \{(\psi_u^T C^{-1}) \gamma^\mu R \psi_e\} \times I$$

Eq. 4.13

and using the same Fierz rearrangement as before (to get 4.10) gives;

$$\text{AMP} \sim g_2^2 e^2 [2(\bar{\psi}_\nu R \psi_e)(\bar{\psi}_u^T C^{-1} L C^T \bar{\psi}_d^T)] \times I \quad \text{Eq. 4.14}$$

or,

$$\text{AMP} \sim g_2^2 e^2 [2(\bar{\psi}_\nu R \psi_e)(\bar{\psi}_d L \psi_u)] \times I, \quad \text{Eq. 4.15}$$

which is the same as Eq. 4.11, the contribution from Fig. A. Therefore the induced low energy lepton-quark pseudoscalar interaction is,

$$L_{\text{EFF}} \sim g_2^2 e^2 (\bar{\psi}_\nu L \frac{(1+\gamma_5)}{2} \psi_e)(\bar{\psi}_d \gamma_5 \psi_u) \frac{\Delta_{\tilde{T}}^2 \Delta_q^2}{M_S^6}, \quad \text{Eq. 4.16}$$

where M_S^2 is the common chiral scalar mass squared, $\Delta_{\tilde{T}}^2$ the chiral scalar mass squared mixing for the scalar $\tilde{T}$, and g_2/e are the weak/electromagnetic couplings. As the pion is a pseudoscalar particle we may drop the "1"

factor from the $L = \frac{1-\gamma_5}{2}$ as it contributes nothing.

One can parameterize the effective four-fermion local interaction by;

$$L \sim (G_F/\sqrt{2}) [(\bar{\psi}_d \gamma_\mu \gamma_5 \psi_u)(\bar{\psi}_\nu \gamma_\mu (1-\gamma_5) \psi_e) f_{AL}^2 + (\bar{\psi}_d \gamma_5 \psi_u)(\bar{\psi}_\nu (1+\gamma_5) \psi_e) f_{PL}^2] + \text{right-handed terms.} \quad \text{Eq. 4.17}$$

Then the ratio, $R = \Gamma(\pi^+ \rightarrow e^+ \nu_e)/\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu)$, can be written as (SHA.82);

$$R = R_0 \left(1 + \frac{2m_\pi a_p}{m_e a_A} f_{PL}^e\right) / \left(1 + \frac{2m_\pi a_p}{m_\mu a_A} f_{PL}^\mu\right) \quad \text{Eq. 4.18}$$

where R_0 is the ratio from standard weak interactions alone (with radiative corrections included),

$$R_0 = 1.233 \times 10^{-4}.$$

Also, we define:

$$\begin{aligned} \langle 0 | \bar{\psi}_d \gamma_\mu \gamma_5 \psi_\mu | \pi^+(k) \rangle &= k_\mu m_\pi a_A / \sqrt{2m_\pi}, \\ \langle 0 | \bar{\psi}_d \gamma_\mu \psi_\mu | \pi^+(k) \rangle &= m_\pi^2 a_P / \sqrt{2m_\pi}, \end{aligned} \quad \text{Eq. 4.19}$$

and from current algebra we have,

$$a_P/a_A = m_\pi / (m_u + m_d) \approx 14. \quad \text{Eq. 4.20}$$

So (SHA.82)

$$R = R_0 \left(1 + \frac{28m_\pi}{m_e} f_{PL}^e \right) / \left(1 + \frac{28m_\pi}{m_\mu} f_{PL}^\mu \right) \quad \text{Eq. 4.21}$$

Comparison of equations 4.17 and 4.16 gives

$$\frac{G_F}{\sqrt{2}} f_{PL}^e \sim g_2^2 e^2 \Delta_{\tilde{q}\tilde{l}}^2 / M_S^6 \quad \text{Eq. 4.22}$$

To get a numerical feeling for the constraints we take $\Delta_{\tilde{l}}^2 / \Delta_{\tilde{q}}^2$ to be of order the corresponding lepton/quark mass squared, as suggested by supergravity models, and constrain the scalar diagonal mass squared M_S^2 . With

$$\begin{aligned} G_F / \sqrt{2} &= g_2^2 / 8M_W^2, \\ f_{PL}^e &\sim 8e^2 m_l^2 m_q^2 / M_S^6. \end{aligned} \quad \text{Eq. 4.23}$$

Inserting this in the formula for R, then the observed agreement of the measured ratio to the predictions of the standard electroweak theory at the 0.3% level implies, $M_S^2 > 0(5 \text{ GeV})$. This is a far less stringent bound than that given by the rough analysis of Shanker (SHA.82). In his estimates Shanker did not distinguish between the chiral diagonal and mixing terms in the scalar mass squared matrix and ignored the chiral nature of the wino couplings; the bound he quotes is only relevant to the

case of large chiral scalar mixings (comparable to the scalar masses).

(iv) SUSY Induced Lamb Shift

As a second test for lepton-nucleon interactions we consider the energy levels of atomic hydrogen. Lepton-nucleon contact interactions will alter the S-wave energy, contributing an anomalous "Lamb shift" to the splitting between 2s and 2p levels. These interactions will be induced by the diagrams of Figure 4.3.

Fig. 4.3

From theoretical considerations the photino is expected to be the lightest of the superpartners that we consider here. We assume that the inverse photino mass is much smaller than an atomic radius, so the diagrams give an effective local interaction, perturbing only atomic S-waves.

As the superpartners circulating in the loop do not include a sparticle with chiral coupling (like the wino in the π_{12} case), for each diagram one must do the calculation for each choice of chirality of the external fermion lines. This dependence appears in the masses and mixings of chiral scalars to which they couple. For now we suppress the chiral

index on external fermion lines and later insert appropriate values in the L/R scalar masses and mixings.

From Appendix A in conjunction with Figure 4.3 (A) we have the following amplitude:

$$\text{AMP} \sim (-\sqrt{2})^4 e_l^2 e_q^2 \int \frac{d^4 k}{(2\pi)^4} i^4 \left\{ \frac{\bar{\psi}_l (-X+M_\gamma) C \bar{\psi}_q \psi_q C^{-1} (-X+M_\gamma) \psi_l}{(k^2 - M_\gamma^2)^2 (k^2 - M_l^2)^2 (k^2 - M_q^2)^2} \right\} \quad \text{Eq. 4.24}$$

$$\text{AMP} \sim 4e_l^2 e_q^2 \int \frac{d^4 k}{(2\pi)^4} \left\{ \frac{k^2 (\bar{\psi}_l \gamma_\mu C \bar{\psi}_q \psi_q C^{-1} \gamma^\mu \psi_l) + M_\gamma^2 (\bar{\psi}_l C \bar{\psi}_q \psi_q C^{-1} \psi_l)}{(k^2 - M_\gamma^2)^2 (k^2 - M_l^2)^2 (k^2 - M_q^2)^2} \right\} \quad \text{Eq. 4.25}$$

Where under symmetric integration we have dropped terms linear in k and $k_\mu k_\nu \rightarrow k^2 \delta_{\mu\nu}$.

$$\text{AMP} \sim 4e_l^2 e_q^2 \int \frac{d^4 k}{(2\pi)^4} \left\{ \frac{k^2 O_1 + M_\gamma^2 O_2}{(k^2 - M_\gamma^2)^2 (k^2 - M_l^2)^2 (k^2 - M_q^2)^2} \right\} \quad \text{Eq. 4.26}$$

$$\text{where } O_1 = \bar{\psi}_l \gamma_\mu C \bar{\psi}_q \psi_q C^{-1} \gamma^\mu \psi_l, \quad \text{Eq. 4.27}$$

$$O_2 = \bar{\psi}_l C \bar{\psi}_q \psi_q C^{-1} \psi_l \quad \text{Eq. 4.28}$$

Then from Appendix A in conjunction with Figure 4.3(B) we have the other amplitude.

$$\text{AMP} \sim (-\sqrt{2})^4 e_l^2 e_q^2 \int \frac{d^4 k}{(2\pi)^4} i^4 \left\{ \frac{\bar{\psi}_l (-X+M_\gamma) \psi_q \bar{\psi}_q (-X+M_\gamma) \psi_l}{(k^2 - M_\gamma^2)^2 (k^2 - M_l^2)^2 (k^2 - M_q^2)^2} \right\} \quad \text{Eq. 4.29}$$

$$\sim 4e_l^2 e_q^2 \int \frac{d^4 k}{(2\pi)^4} \left\{ k^2 (\bar{\psi}_l \gamma_\mu \psi_q \bar{\psi}_q \gamma^\mu \psi_l) + M_\gamma^2 (\bar{\psi}_l \psi_q \bar{\psi}_q \psi_l) \right\} \quad \text{Eq. 4.30}$$

where we have again used symmetric integration.

$$\sim 4e_x^2 e_y^2 \int \frac{d^4 k}{(2\pi)^4} \left\{ \frac{k^2 O_3 + M_Y O_4}{(k^2 - M_Y^2)^2 (k^2 - M_X^2) (k^2 - M_Q^2)^2} \right\} \quad \text{Eq. 4.31}$$

$$\text{where: } O_3 = \bar{\psi}_X \gamma_\mu \psi_Q \bar{\psi}_Q \gamma^\mu \psi_X \quad \text{Eq. 4.32}$$

$$O_4 = \bar{\psi}_X \psi_Q \bar{\psi}_Q \psi_X \quad \text{Eq. 4.33}$$

Now for the operator induced by the box diagrams the amplitudes for the incoming and outgoing fermion of definite chirality are related by the mass mixing matrix of the chiral sfermions. Also the structure of the four fermion operators is such that if chiral mixing occurs between sleptons then it must also occur in the squark sector to give a non-vanishing matrix element (and vice versa). This was explicitly demonstrated in Figure 4.2 for the π_{12} decay. So the matrix element for chiral mixing is controlled by the smallest sfermion chiral mixing term. As we have stated in the π_{12} case these mixing terms are expected to be small compared to the diagonal terms. Hence in this case, as it is not helicity suppressed, we can expect the contribution from the mixing between left and right chiral scalars to be negligible and so we don't consider them further. Then the non-vanishing chiral combinations to the induced operators are as follows.

$$\begin{aligned}
 O_1 &= \frac{(1+\gamma_5)}{2} C^T \bar{\psi}_q \psi_q C^{-1} \gamma^\mu \frac{(1+\gamma_5)}{2} \psi_\ell \\
 &= \bar{\psi}_\ell \gamma^\mu \frac{(1+\gamma_5)}{2} C^T \bar{\psi}_q \psi_q C^{-1} \gamma^\mu \frac{(1+\gamma_5)}{2} \psi_\ell,
 \end{aligned}
 \tag{Eq. 4.44}$$

Fierz transforming by $(\gamma^\mu L) \times (\gamma_\mu L) = -(\gamma^\mu L) \times (\gamma_\mu L)$ and including a factor of $(-1)^3$ from fermion interchange, we get,

$$\begin{aligned}
 O_1 &= \bar{\psi}_\ell \gamma_\alpha \frac{(1+\gamma_5)}{2} \psi_\ell \psi_q C^{-1} \gamma^\alpha \frac{(1+\gamma_5)}{2} C^T \bar{\psi}_q \\
 O_1 &= \bar{\psi}_\ell \gamma_\alpha \frac{(1+\gamma_5)}{2} \psi_\ell \psi_q C^{-1} \gamma^\alpha C^T \frac{(1+\gamma_5)}{2} \bar{\psi}_q^T,
 \end{aligned}$$

But with, $C^T = -C$; $C^{-1} \gamma^\alpha C^T = -C^{-1} \gamma^\alpha C = \gamma^{\alpha T}$,

$$\begin{aligned}
 O_1 &= \bar{\psi}_\ell \gamma_\alpha \frac{(1+\gamma_5)}{2} \psi_\ell \psi_q \gamma^{\alpha T} \frac{(1+\gamma_5)}{2} \bar{\psi}_q^T \\
 &= -\bar{\psi}_\ell \gamma_\alpha \frac{(1+\gamma_5)}{2} \psi_\ell \bar{\psi}_q \frac{(1+\gamma_5)}{2} \gamma^\alpha \psi_q,
 \end{aligned}$$

where the factor of (-1) is from fermion anticommuting and

$$\frac{(1+\gamma_5)^T}{2} = \frac{(1+\gamma_5)}{2}, \text{ so}$$

$$O_1 = -\bar{\psi}_\ell \gamma_\alpha \frac{(1+\gamma_5)}{2} \psi_\ell \bar{\psi}_q \gamma^\alpha \frac{(1+\gamma_5)}{2} \psi_q.
 \tag{Eq. 4.44}$$

Similarly for O_3 ;

$$\begin{aligned} O_3 &= \bar{\psi}_\ell \frac{(1 \pm \gamma_5)}{2} \gamma_\mu \psi_q \bar{\psi}_q \gamma^\mu \frac{(1 \pm \gamma_5)}{2} \psi_\ell \\ &= \bar{\psi}_\ell \gamma_\mu \frac{(1 \pm \gamma_5)}{2} \psi_q \bar{\psi}_q \gamma^\mu \frac{(1 \pm \gamma_5)}{2} \psi_\ell \end{aligned} \quad \text{Eq. 4.45}$$

Fierz transforming as in the previous case and including $(-1)^3$ for Fermi statistics gives,

$$O_3 = \psi_\ell \gamma_\alpha \frac{(1 \pm \gamma_5)}{2} \psi_\ell \bar{\psi}_q \gamma^\alpha \frac{(1 \pm \gamma_5)}{2} \psi_q \quad \text{Eq. 4.46}$$

The non-zero chiral combinations in O_2 are,

$$O_2 = \bar{\psi}_\ell \frac{(1 \pm \gamma_5)}{2} C T_\psi^\dagger \gamma_q T_C^{-1} \frac{(1 \pm \gamma_5)}{2} \psi_\ell \quad \text{Eq. 4.47}$$

Under Fierz reordering;

$$2L \times R = (\gamma_\rho R) \times (\gamma^\rho L)$$

$$2R \times L = (\gamma_\rho L) \times (\gamma^\rho R)$$

Eq. 4.48

And so,

$$\begin{aligned} O_2 &= -\frac{1}{2} \bar{\psi}_\ell \gamma_\rho \frac{(1 \pm \gamma_5)}{2} \psi_\ell \psi_q T_C^{-1} \gamma^\rho \frac{(1 \pm \gamma_5)}{2} C T_\psi^\dagger \\ &= -\frac{1}{2} \bar{\psi}_\ell \gamma_\rho \frac{(1 \pm \gamma_5)}{2} \psi_\ell \psi_q T_C^{-1} \gamma^\rho C T_\psi^\dagger \end{aligned}$$

But $C^{-1} \gamma^\rho C^T = \gamma^{\rho T}$, then;

$$\begin{aligned} 0_2 &= -\frac{1}{2} \bar{\psi}_\ell \gamma_\rho \frac{(1+\gamma_5)}{2} \psi_\ell \psi_q^T \gamma^{\rho T} \frac{(1+\gamma_5)}{2} \bar{\psi}_q^T \\ &= \frac{1}{2} \bar{\psi}_\ell \gamma_\rho \frac{(1+\gamma_5)}{2} \psi_\ell \bar{\psi}_q \frac{(1+\gamma_5)}{2} \gamma^\rho \psi_q \\ 0_2 &= \frac{1}{2} \bar{\psi}_\ell \gamma_\rho \frac{(1+\gamma_5)}{2} \psi_\ell \bar{\psi}_q \gamma^\rho \frac{(1+\gamma_5)}{2} \psi_q \end{aligned} \quad \text{Eq. 4.49}$$

Finally the 0_4 chiral combinations are;

$$\left\{ \begin{array}{l} 0_4 = -\frac{1}{2} \frac{(1+\gamma_5)}{2} \psi_q \bar{\psi}_q \frac{(1+\gamma_5)}{2} \psi_\ell \end{array} \right. \quad \text{Eq. 4.50}$$

and using the Fierz reordering for 0_2 we have,

$$0_4 = -\frac{1}{2} \bar{\psi}_\ell \gamma_\rho \frac{(1+\gamma_5)}{2} \psi_\ell \bar{\psi}_q \gamma^\rho \frac{(1+\gamma_5)}{2} \psi_q \quad \text{Eq. 4.51}$$

With the correct choice of relative signs for the diagrams we find for $0_K = -0_1 + 0_3$;

$$0_K = \bar{\psi}_\ell \gamma_\alpha \frac{(1+\gamma_5)}{2} \psi_\ell \bar{\psi}_q \gamma^\alpha \psi_q \quad \text{Eq. 4.52}$$

and

$$\begin{aligned} 0_M &= 0_2 - 0_4; \\ 0_M &= \frac{1}{2} \bar{\psi}_\ell \gamma_\alpha \frac{(1+\gamma_5)}{2} \psi_\ell \bar{\psi}_q \gamma^\alpha \psi_q \end{aligned} \quad \text{Eq. 4.53}$$

The relative sign between diagrams may be determined by making chiral scalars degenerate in mass, which gives a parity invariant theory and then demanding that the sum of the contributions from diagrams be parity invariant. In both cases, if we have chirally degenerate scalars, then the coefficients of the chiral pieces of $0_{K,M}$ will have the same coefficients.

and

$$F_{\mu\nu}^a = \partial_\mu B_\nu^a - \partial_\nu B_\mu^a + gf_{bc}^a B_\mu^b B_\nu^c \quad \text{Eq. A-10}$$

and this theory is invariant under gauge-invariant SUSY transformations.

Now from L_M we get the "Gauge-Yukawa" coupling to gauge fermions λ^a . We consider a Dirac fermion $\psi = \psi_L + \psi_R$ whose chiral pieces transform under a gauge group as,

$$\delta\psi_L^i = i\theta^a T_{Lj}^{ai} \psi_L^j; \quad \delta\psi_R^i = i\theta^a T_{Rj}^{ai} \psi_R^j, \quad \text{Eq. A-11}$$

where $T_{L,R}$ are the generators of the representation of $\psi_{L,R}$. And in a supersymmetric theory chiral components of ψ appear in left/right chiral multiplets $(A_{L,R}; \bar{L}_{L,R})$. Then the gauge Yukawa couplings are:

$$L_{g-y} = i/2g [A_{Ri}^* T_{Rj}^{ai} \psi_R^j - \bar{\psi}_{Ri} T_{Rj}^{ai} A_{Rj}] \\ + i/2g [A_{Li}^* T_{Lj}^{ai} \psi_L^j - \bar{\psi}_{Li} T_{Lj}^{ai} A_{Lj}], \quad \text{Eq. A-12}$$

where g is the gauge coupling constant.

Now since,

$$\bar{F}_1 F_2 = \bar{F}_{1L} F_{2R} + \bar{F}_{1R} F_{2L} \\ = \bar{F}_{1R} F_{2L} + \bar{F}_{1L} F_{2R}; \quad \text{Eq. A-13}$$

where $L = \frac{(1-\gamma_5)}{2}$; $R = \frac{(1+\gamma_5)}{2}$ are chirality projection operators, then;

$$L_{g-y} = i/2 g [A_{Ri}^* T_{Rj}^{ai} \psi_R^j - \bar{\psi}_{Ri} T_{Rj}^{ai} A_{Rj}] \\ + i/2 g [A_{Li}^* T_{Lj}^{ai} \psi_L^j - \bar{\psi}_{Li} T_{Lj}^{ai} A_{Lj}] \quad \text{Eq. A-14}$$

Thus we have the following vertices:

$$\int \frac{d^4 k}{(2\pi)^4} \frac{(k^2)^n}{\prod_{i=1}^{\ell} (k^2 - m_i^2)} = \frac{(-1)^{\ell+n+1}}{8\pi^2} \sum_{i=1}^{\ell} \frac{m_i^{2n+2}}{\prod_{\substack{j=1 \\ j \neq i}}^{\ell} (m_j^2 - m_i^2)} \quad \text{Eq. 4.59}$$

valid for $\ell > (n+2)$.

So I_n is;

$$I_0 = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - M_Y^2)^2 (k^2 - M_\ell^2) (k^2 - M_q^2)} \quad , n=0, \ell=4. \quad \text{Eq. 4.60}$$

$$= -\frac{1}{8\pi^2} \left[\frac{1}{(M_\ell^2 - M_Y^2)(M_q^2 - M_Y^2)} + \frac{M_\ell^2 M_q^2}{(M_Y^2 - M_\ell^2)^2 (M_q^2 - M_\ell^2)} + \frac{M_q^2 M_\ell^2}{(M_Y^2 - M_q^2)^2 (M_\ell^2 - M_q^2)} \right]$$

and as there are no singular terms as $M_Y \rightarrow 0$, we are free to use the approximation that was used in the $\pi_{\ell 2}$ decay and take $M_Y \rightarrow 0$. As this integral multiplies M_Y^2 it does not contribute to the amplitude.

$$I_1 = \int \frac{d^4 k}{(2\pi)^4} \frac{k^2}{(k^2 - M_Y^2)^2 (k^2 - M_\ell^2) (k^2 - M_q^2)} \quad n=1, \ell=4 \quad \text{Eq. 4.61}$$

$$= \frac{1}{8\pi^2} \left[\frac{M_Y^2}{(M_\ell^2 - M_Y^2)(M_q^2 - M_Y^2)} + \frac{M_\ell^4 M_q^2}{(M_Y^2 - M_\ell^2)^2 (M_q^2 - M_\ell^2)} + \frac{M_q^4 M_\ell^2}{(M_Y^2 - M_q^2)^2 (M_\ell^2 - M_q^2)} \right]$$

and in the limit, $M_Y \rightarrow 0$;

$$I_1 = \frac{1}{8\pi^2} \left[\frac{\ln(M_\ell^2/M_q^2)}{M_q^2 - M_\ell^2} \right] \quad \text{Eq. 4.62}$$

As in the π_2 case we will assume that the scalars have a large common mass $M_\ell^2 = M_q^2 = M_S^2$. Then

$$\begin{aligned} I_1 &= \frac{1}{8\pi^2} \frac{d/dM_q^2 \ln(M_\ell^2/M_q^2)}{d/dM_q^2 (M_q^2 - M_\ell^2)} \\ \lim_{M_q^2 \rightarrow M_\ell^2 = M_S^2} &= -\frac{1}{8\pi^2} \frac{1}{M_S^2} \end{aligned} \quad \text{Eq. 4.63}$$

$$\therefore \text{AMP} \sim -\frac{e_\ell^2 e_q^2}{2\pi^2} \left[\frac{1}{M_S^2} \right] O^T \quad \text{Eq. 4.64}$$

and $O^T = \bar{\psi}_\ell \gamma_\alpha \psi_\ell \bar{\psi}_q \gamma^\alpha \psi_q$, as before.

We can use this amplitude to evaluate the matrix element for an anomalous "Lamb shift". The non-relativistic limit of O^T with,

$$u = \sqrt{\frac{E+m}{2m}} \begin{bmatrix} \chi \\ \frac{\sigma \cdot p}{E+m} \chi \end{bmatrix} \sim \begin{bmatrix} \chi \\ 0 \end{bmatrix} \quad \text{Eq. 4.65}$$

and

$$\bar{\psi}_\ell \gamma_\alpha \psi_\ell \equiv \bar{u} \gamma_\alpha u \approx \chi^\dagger \chi, \quad \text{Eq. 4.66}$$

yields;

$$O_T \approx \chi_\ell^\dagger \chi_\ell \chi_q^\dagger \chi_q \quad \text{Eq. 4.67}$$

The level shift is; (ITZ.80)

$$\delta E_{n,\ell} = \int d^3r \psi_{n,\ell}^* (\vec{r}) \Delta H_\bullet (\vec{r}) \psi_{n,\ell} (\vec{r}) \quad \text{Eq. 4.68}$$

Our amplitude is identified with,

$$\text{AMP} = i \int d^4p \, e^{ip \cdot x} = - \frac{e_l^2 e_q^2}{2\pi^2} \frac{O^\dagger(p)}{M_S^2}, \quad \text{Eq. 4.69}$$

and must be transformed into configuration space,

$$\Delta H_\bullet(\vec{r}) = - \frac{e_l^2 e_q^2}{2\pi^2} \frac{1}{M_S^2} \delta(r). \quad \text{Eq. 4.70}$$

The $n=2$, $\ell=0$ energy level shift is,

$$\begin{aligned} \Delta E_{\text{LAMB}} &= \int d^3\vec{x} \, \psi^\dagger(\vec{x}) \Delta H_\bullet(x) \psi(\vec{x}) \\ &= - \frac{e_l^2 e_q^2}{2\pi^2} \frac{1}{M_S^2} |\psi(0)|^2 \\ &= - \frac{e_l^2 e_q^2}{2\pi^2} \frac{1}{M_S^2} \frac{(Z\alpha m)^3}{\pi n^3} \end{aligned} \quad \text{Eq. 4.71}$$

where $\psi(0) = \frac{1}{\pi^{1/2}} \left(\frac{mZ\alpha}{n} \right)^{3/2}$

For the 2S hydrogenic wave function we find a level shift of,

$$\begin{aligned} \Delta E_{2S} &\sim -(8/\pi n^3) \frac{5}{3} \frac{m^3}{M_S^2} (n=2) \quad \text{Eq. 4.72} \\ &\sim 9 \times 10^{-13} / M_S^2 \text{ eV } (M_S^2 \text{ in GeV}^2). \end{aligned}$$

Comparing this with the uncertainty in theory (MOH.75) and experiment (AND.76) for the Lamb shift; we find that the squark-slepton mass must be greater than of order 10 GeV, in agreement with limits on scalars from direct searches. Although the induced effect is larger in high Z atoms ($\sim Z^5$), the larger theoretical and experimental uncertainties in the Lamb shift for these atoms result in less stringent constraints on the scalar masses.

CHAPTER 5 - SUMMARY AND CONCLUSIONS

We have calculated processes for the direct production of superpartners and indirect effects of supersymmetry from radiative corrections.

The decay of quarkonium into gauge and Higgs fermions gave potentially encouraging rates for these processes, with a scalar mass not too large. The process $^3S_1 \rightarrow H + \tilde{\gamma}$ gives an attractive search method for Higgsinos given a heavy toponium. The $^1S_0 \rightarrow \tilde{\chi}\tilde{\chi}$ decay is substantial. Similarly, the $^3S_1 \rightarrow \tilde{\chi}\tilde{\chi}$ mode may be large, unless there is mass degeneracy between the chiral scalar quarks. In sum the decays of heavy quarkonium systems should be studied experimentally as a promising source of superpartner production.

The induced one-loop decay of physical Z^0 's into gluino pairs, with very conservative numerical assumptions, gave substantial branching fractions into the two-gluino final state. The rate was controlled by the mass splitting of the t-b doublet. In Figure 3.4 we have also plotted the branching ratio for Z^0 to decay into $\bar{q}q\tilde{\chi}\tilde{\chi}$ for comparison, and Z^0 decay into gluino pairs dominates near threshold. One should also consider that it may be easier experimentally to distinguish two-gluino events than it would be to distinguish four-jet events containing two gluinos.

In both cases considered here we will presumably have to await machines capable of higher energies than presently available today. The advent of a new generation of e^+e^- "Z factories" (such as LEP) would possibly lead to the observation of $Z^0 \rightarrow \tilde{\chi}\tilde{\chi}$ if our estimates of the unknowns (principally the mass scales) have been reasonable. If the toponium mass is below that of the Z^0 but still bigger than twice the mass of the gluinos

or photinos or the sum of Higgsino and photino mass, these too could be observable at a Z^0 factory. For toponium above the Z^0 mass, or other heavier quarkonium we would probably be in the realm of the supercollider (pp or $p\bar{p}$).

Should we be able to generate these superpartners, what would their experimental signature be? In general superpartners will cascade down to the lightest superpartner, which is (quasi)stable. In many models the photino is the lightest superpartner. Gluinos will typically decay to $q\bar{q}\tilde{\gamma}$ and Higgsinos to $f\tilde{f}\tilde{\gamma}$, where f is the heaviest, most electrically charged fermion available. The photino is a neutrino-like object in the sense that it will leave the detector undetected removing energy and momentum. This may lead to the extremely distinctive "Zen events" (ELL.83A), a shower of hadrons (jets) on one side and nothing on the other. The conventional signpost is jets and very high missing E_T or P_T . Indeed some events with high missing P_T (>40 GeV) have been observed and it seems their explanation will require something beyond the standard model (UA1.84B)(UA2.84B).

As well as direct searches it may be possible to detect supersymmetry effects by indirect contributions. Many processes and quantities have been very accurately measured and their degree of agreement with standard model predictions can be judged against the new effective low energy interactions induced by supersymmetric radiative corrections. We considered lepton-nucleon effective low energy interactions induced by one loop radiative corrections with virtual particles which are superpartners of standard model ones. The resulting interactions, which in general could be significant (SHA.82) for superpartner masses of order the weak scale,

appear to give acceptably small anomalous effects for most classes of realistic models.

If effects of supersymmetry from both direct and indirect searches continue to be absent from experimental searches with the next generation of machines the models will become awkward and the relevance to particle physics distant. However expectations have been raised by the results at the SPS (UA1.84B)(UA2.84B). Thus we await further experimental results on SUSY searches with anticipation. SUSY may be shy, but ...

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APPENDIX A - SUSY FEYNMAN RULES

The general Gauge Invariant SUSY Lagrangian (SAL.78)(CAM.82B).

$$L = L_{\text{GAUGE}} + L_{\text{MATTER}} + L_{\text{SCALAR}} + L_{\text{YUKAWA}}, \quad \text{Eq. A-1}$$

where:

$$L_G = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \frac{1}{2} \bar{\chi}^\mu \gamma^\mu (D_\mu^{ab} \chi^b), \quad \text{Eq. A-2}$$

$$L_M = (D_\mu^i A)^* (D_\mu^i A) + i \bar{\psi}_L \gamma^\mu D_\mu^i \psi_L^j - \sqrt{2} g \bar{\chi}^a A_i^* T_{ij}^a \psi_L^j - \sqrt{2} g \bar{\psi}_L^i T_{ij}^a \chi^j A^a \quad \text{Eq. A-3}$$

$$L_Y = -\sum_{ij} \left\{ \left(\frac{\partial^2 W}{\partial A^i \partial A^j} \right) (\psi_L^i)^T C^{-1} \psi_L^j + \left(\frac{\partial^2 W}{\partial A^i \partial A^j} \right)^* (\bar{\psi}_L^j) C (\psi_L^i)^* \right\} \quad \text{Eq. A-4}$$

$$L_S = -\sum_i \left| \frac{\partial W}{\partial A^i} \right|^2 - \frac{1}{2} \sum_a \left(-\frac{1}{2} g A_i^* T_{ij}^a A^j - \xi^a \right)^2 \quad \text{Eq. A-5}$$

where ξ^a appears only for invariant abelian subalgebras,

$$W = c_i A^i + m_{ij} A^i A^j + g_{ijk} A^i A^j A^k \quad \text{Eq. A-6}$$

is the superpotential, a and b are adjoint indices, i and j are complex representation indices (i.e. $(A^i)^* = (A^*)^i$), and;

$$D_\mu A = (\partial_\mu - ig \vec{T} \cdot \vec{B}_\mu) A, \quad \text{Eq. A-7}$$

$$D_\mu \psi_L = (\partial_\mu - ig \vec{T} \cdot \vec{B}_\mu) \psi_L, \quad \text{Eq. A-8}$$

$$(\text{so } D_\mu \psi_R^c = (\partial_\mu + ig \vec{T} \cdot \vec{B}_\mu) \psi_R^c),$$

$$(D_\mu \chi)^a = \partial_\mu \chi^a + g f_{bc}^a B_\mu^b \chi^c, \quad \text{Eq. A-9}$$

and

$$F_{\mu\nu}^a = \partial_\mu B_\nu^a - \partial_\nu B_\mu^a + gf_{bc}^a B_\mu^b B_\nu^c \quad \text{Eq. A-10}$$

and this theory is invariant under gauge-invariant SUSY transformations.

Now from L_M we get the "Gauge-Yukawa" coupling to gauge fermions λ^a . We consider a Dirac fermion $\psi = \psi_L + \psi_R$ whose chiral pieces transform under a gauge group as,

$$\delta\psi_L^i = i\alpha T_{Lj}^{ai} \psi_L^j; \quad \delta\psi_R^i = i\alpha T_{Rj}^{ai} \psi_R^j, \quad \text{Eq. A-11}$$

where $T_{L,R}$ are the generators of the representation of $\psi_{L,R}$. And in a supersymmetric theory chiral components of ψ appear in left/right chiral multiplets $(A_{L,R}; \psi_{L,R})$. Then the gauge Yukawa couplings are:

$$\begin{aligned} L_{g-y} = & i\sqrt{2}g [A_{Ri}^* T_{Rj}^{ai} \psi_R^j - \bar{\psi}_{Ri} \lambda_{Rj}^a T_{Rj}^{ai} A_{Rj}] \\ & + i\sqrt{2}g [A_{Li}^* T_{Lj}^{ai} \psi_L^j - \bar{\psi}_{Li} \lambda_{Lj}^a T_{Lj}^{ai} A_{Lj}], \end{aligned} \quad \text{Eq. A-12}$$

where g is the gauge coupling constant.

Now since,

$$\begin{aligned} \bar{F}_1 F_2 &= \bar{F}_{1L} F_{2R} + \bar{F}_{1R} F_{2L} \\ &= \bar{F}_{1R} F_{2L} + \bar{F}_{1L} F_{2R}; \end{aligned} \quad \text{Eq. A-13}$$

where $L = \frac{(1-\gamma_5)}{2}$; $R = \frac{(1+\gamma_5)}{2}$ are chirality projection operators, then;

$$\begin{aligned} L_{g-y} = & i\sqrt{2} g [A_{Ri}^* T_{Rj}^{ai} \lambda_{Rj}^a \psi_R^j - \bar{\psi}_{Ri} \lambda_{Lj}^a T_{Rj}^{ai} A_{Rj}] \\ & + i\sqrt{2} g [A_{Li}^* T_{Lj}^{ai} \lambda_{Lj}^a \psi_L^j - \bar{\psi}_{Li} \lambda_{Rj}^a T_{Lj}^{ai} A_{Lj}] \end{aligned} \quad \text{Eq. A-14}$$

Thus we have the following vertices:

Fig. A-1

The Euclidean space propagators for the scalars and fermions are;

$$\begin{array}{ccc} \text{---} \Rightarrow \text{---} & & \frac{\delta_k^j}{p^2 + M^2} \\ \text{---} \Rightarrow \text{---} & & \frac{i\delta_b^a}{p + m} \end{array}$$

Fig. A-2

Our metric and other standard conventions are Bjorken and Drell (BJO.67), Euclidean space conventions match Ramond (RAM.81). (See Appendix B for conventions and spinorology). Fig. A-1 contains the basic Feynmann rules that were adapted for each particular calculation.

The presence of Higgsinos as in the quarkonia decay to Higgsinos and photinos gives a different vertex and can be found from L_{YUK} . As outlined briefly in Chapter 1 (Eq. 1.12) a true Yukawa interaction can be derived from the superpotential. In terms of only left-handed components, the relevant term from W is;

$$W \sim g_{YUK} \phi_{H_L} \phi_{q_L}^c \phi_{q_L} \quad \text{Eq. A-15}$$

where ϕ_{H_L} , $\phi_{q_L}^c$, ϕ_{q_L} , the left-handed Higgs, left-handed antiquark and left-handed squark respectively, are the scalars of the chiral multiplets,

$$\begin{bmatrix} \phi_{H_L} \\ \psi_{H_L} \end{bmatrix} \begin{bmatrix} \phi_{q_L}^c \\ q_L^c \end{bmatrix} \begin{bmatrix} \phi_{q_L} \\ q_L \end{bmatrix} \quad \text{Eq. A-16}$$

whose fermionic partners are the left-handed Higgsino (ψ_{H_L}), left-handed antiquark (q_L^c) and the left-handed quark. The vertices are found from,

$$L_Y = - \sum_{ij} \left\{ \left(\frac{\partial^2 W}{\partial A^i \partial A^j} \right) (\psi_L^i)^T C^{-1} \psi_L^j + \left(\frac{\partial^2 W}{\partial A^i \partial A^j} \right)^* (\bar{\psi}_L^j) C (\psi_L^i)^* \right\} \quad \text{Eq. A-4}$$

$$\begin{aligned} L_Y = & -g_{YUK} \{ \phi_{H_L} (q_L^C)^T C^{-1} q_L + \phi_{H_L}^* (\bar{q}_L^C) C (q_L^C)^* \\ & + \phi_{q_L^C} (q_L^C)^T C^{-1} \phi_{H_L} + \phi_{q_L^C}^* (\bar{\psi}_{H_L}) C (q_L^C)^* \\ & + \phi_{q_L} (\psi_{H_L})^T C^{-1} q_L^C + \phi_{q_L}^* (\bar{q}_L^C) C (\psi_{H_L})^* \} \end{aligned} \quad \text{Eq. A-17}$$

As we are only interested in the Higgsino couplings, we drop the terms involving the Higgs couplings and only consider the last four terms in what follows. First, in order, we have:

$$1. \quad \phi_{q_L^C} (q_L^C)^T C^{-1} \psi_{H_L} \equiv - \phi_{q_L^C} (\bar{q}_R^C) \psi_{H_L} \quad \text{Eq. A-18}$$

where we have used, $(\psi_L)^T C^{-1} = -(\bar{\psi}_R^C)$. It is worthwhile detailing what these fields do.

$\phi_{q_L^C}$ annihilates a left-handed antiquark or creates a right-handed squark.

$(\bar{q}_R^C)$ creates a right-handed antiquark or annihilates a left-handed quark.

ψ_{H_L} annihilates a left-handed Higgsino or creates a right-handed antihiggsino.

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$$2. \phi_{q_L}^{*C} (\bar{\phi}_{H_L}) C(q_L)^* \equiv \phi_{q_L}^{*C} (\bar{\phi}_{H_L}) q_R^C,$$

Eq. A-19

where we have used $C(\phi_L^i)^* = \phi_{Ri}^C$.

$\phi_{q_L}^{*C}$ creates a left-handed antiquark and annihilates a right-handed squark.

$(\bar{\phi}_{H_L})$ creates a left-handed Higgsino or annihilates a right-handed antihiggsino.

q_R^C annihilates a right-handed antiquark or creates a left-handed quark.

Similarly

$$3. \phi_{q_L} (\phi_{H_L})^T C^{-1} q_L^C \equiv -\phi_{q_L} (\bar{\phi}_{H_R}^C) q_L^C$$

A-20

$$4. \phi_{q_L}^* (\bar{q}_L^C) C(\phi_{H_L})^* \equiv \phi_{q_L}^* (\bar{q}_L^C) \phi_{H_R}^C$$

A-21

Pulling the chiral projection operators out explicitly from their respective fermions gives;

$$1. -\phi_{q_L}^C (\bar{q}^C) \frac{(1-\gamma_5)}{2} \chi_H$$

A-22

$$2. \phi_{q_L}^{*C} (\bar{\chi}_H) \frac{(1-\gamma_5)}{2} q^C$$

A-23

$$3. -\phi_{q_L} (\bar{\chi}_H) \frac{(1-\gamma_5)}{2} q^C$$

A-24

$$4. \phi_{q_L}^* (\bar{q}^C) \frac{(1+\gamma_5)}{2} \chi_H$$

A-25

and χ_H is a majorana spinor. Using the Appendix from Bjorken and Drell for meson nucleon interactions we have the following Feynman rules:

FIG A-3

and these were used in Table 2.1 for the Higgsino couplings.

APPENDIX B - SPINOROLOGY

The Minkowski space conventions are those of Bjorken and Drell
(BJO.64)(BJO.65):

$$\gamma^0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \gamma^i = \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix}$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma_5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

where $\sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, $\sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ Eq. B-1

and the familiar 2x2 Pauli matrices and

$$1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ is the 2x2 unit matrix. The charge conjugation operation}$$

is

$$C = i\gamma^2\gamma^0 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \text{ Eq. B-2}$$

in this representation.

A free Dirac particle boosted from the rest frame spinor χ is,

$$\phi = N \begin{bmatrix} \chi \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \chi \end{bmatrix} \text{ Eq. B-3}$$

and the Pauli adjoint is,

$$\bar{\phi} = \phi^\dagger \gamma^0 \text{ Eq. B-4}$$

$$\psi \rightarrow \psi^C \equiv C(\bar{\psi})^T$$

$$\bar{\psi} \rightarrow \bar{\psi}^C = -\psi^T C^{-1}$$

and the defining relationship,

$$C\gamma^\mu C^{-1} = -\gamma^{\mu T}$$

$$C = -C^{-1} = -C^+ = -C^T = i\gamma^2\gamma^0$$

Eq. B-11

$$\psi^C = i\gamma^2\psi^*$$

Then for chiral fermions we have; (LAN.81)

$$\psi_{L,R} \rightarrow \psi_{L,R}^C = C(\bar{\psi}_{R,L})^T$$

$$(\bar{\psi}_{L,R}) \rightarrow (\bar{\psi}_{L,R})^C = -\psi_{R,L}^T C^{-1}$$

Eq. B-12

also

$$\psi_{L,R} = C(\bar{\psi}_{R,L}^C)^T$$

$$\bar{\psi}_{L,R} = -\psi_{R,L}^C{}^T C^{-1}$$

Eq. B-13

Majorana spinors can be constructed from Weyl spinors as follows;

$$\eta = \psi_R + \psi_L^C$$

$$\xi = \psi_L + \psi_R^C$$

Eq. B-14

and the Majorana spinor is just a four component representation of Weyl spinor.

$$\text{Then } \eta^C = C(\bar{\eta})^T = \eta$$

$$+ \bar{\eta}^T C = \eta^T$$

$$(\bar{\eta})^T = C^{-1}\eta$$

Eq. B-15

$$\psi \rightarrow \psi^C \equiv C(\bar{\psi})^T$$

$$\bar{\psi} \rightarrow \bar{\psi}^C = -\psi^T C^{-1}$$

and the defining relationship,

$$C \gamma^\mu C^{-1} = -\gamma^{\mu T}$$

$$C = -C^{-1} = -C^+ = -C^T = i \gamma^2 \gamma^0$$

Eq. B-11

$$\psi^C = i \gamma^2 \psi^*$$

Then for chiral fermions we have; (LAN.81)

$$\psi_{L,R} \rightarrow \psi_{L,R}^C \equiv C(\bar{\psi}_{R,L})^T$$

$$(\bar{\psi}_{L,R}) \rightarrow (\bar{\psi}_{L,R}^C) = -\psi_{R,L}^T C^{-1}$$

Eq. B-12

also

$$\psi_{L,R} = C(\bar{\psi}_{R,L}^C)^T$$

$$\bar{\psi}_{L,R} = -\psi_{R,L}^C T C^{-1}$$

Eq. B-13

Majorana spinors can be constructed from Weyl spinors as follows;

$$\eta = \psi_R + \psi_L^C$$

$$\xi = \psi_L + \psi_R^C$$

Eq. B-14

and the Majorana spinor is just a four component representation of Weyl spinor.

$$\text{Then } \eta^C = C(\bar{\eta})^T = \eta$$

$$+ \bar{\eta} C^T = \eta^T$$

$$(\eta^T)^T = C^{-1} \eta$$

Eq. B-15

APPENDIX C - SPINORS AND THEIR EVALUATION IN QUARKONIA DECAY

A spin up quark $u(+)$ = $\begin{pmatrix} \chi^+ \\ 0 \end{pmatrix}$; $\chi^+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\therefore u(+)$ = $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

A spin down quark $u(+)$ = $\begin{pmatrix} \chi^+ \\ 0 \end{pmatrix}$; $\chi^+ = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\therefore u(+)$ = $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

A spin up antiquark $v(+)$ = $\begin{pmatrix} 0 \\ \zeta^+ \end{pmatrix}$; $\zeta^+ = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$ $\therefore v(+)$ = $\begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$

A spin down antiquark $v(+)$ = $\begin{pmatrix} 0 \\ \zeta^+ \end{pmatrix}$; $\zeta^+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\therefore v(+)$ = $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

Now

$$^3S_1 = \begin{matrix} & \text{quark} & \text{antiquark} \\ \uparrow & & \uparrow \end{matrix} ; \uparrow\uparrow ; \frac{\uparrow\uparrow\uparrow\uparrow}{\sqrt{2}} (u(+)\bar{v}(+))$$

So $\chi_2^+ \chi_1^+ = \zeta_+^+ \chi_+^+ = (0 \ -1) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0$

$$x_2^+ \sigma^1 x_1$$

$$x_2^+ \sigma^1 x_1 = \zeta_+^+ \sigma^1 x_1 = (0 \ -1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = -1$$

$$x_2^+ \sigma^2 x_1 = (0 \ -1) \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = -i$$

$$x_2^+ \sigma^3 x_1 = (0 \ -1) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0$$

The $^1S_0 = (++) - (++)/\sqrt{2}$
 $(u(+)v(+)) - (u(+)v(+))$
 (a) (b)

and for identical particles we had contributions from $x_2^+ x_1$, $x_2^+ \sigma^1 x_1$

$$(a) \ \zeta_+^+ x_+ = (1 \ 0) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1$$

$$(b) \ \zeta_+^+ x_+ = (0 \ -1) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = -1$$

$$x_2^+ x_1 = \frac{1}{\sqrt{2}} \zeta_+^+ x_+ - \frac{1}{\sqrt{2}} \zeta_+^+ x_+$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

$$(a) \ \zeta_+^+ \sigma^1 x_+ \quad i=1 \quad (1 \ 0) \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0$$

$$i=2 \quad (1 \ 0) \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0$$

$$i=3 \quad (0 \ -1) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1$$

$$(b) \ \zeta_+^+ \sigma^1 x_+ \quad i=1 \quad (0 \ -1) \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$$

$$i=2 \quad (0 \ -1) \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$$

$$i=3 \quad (0 \ -1) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -1$$

$$\therefore x_2^{+\sigma} x_1 = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0.$$

\therefore For identical particles we had a contribution for 1S_0 of $x_2^{+} x_1 = 1/2$ (parity conserving).

For identical particles we had a contribution for 3S_1 of, $x_2^{+\sigma} x_1 = -1$ and $x_2^{+2} x = -1$ (parity non-conserving).

For non-identical particles, since $x_2^{+} x_1$ cancelled for $M_L = M_R$, there is no contribution for the 1S_0 as $x_2^{+\sigma} x_1 = 0$, as expected.

APPENDIX D - FORM FACTORS AND SPIN STATES

Consider the diagonal matrix element

$$M^\mu = \langle f(p_f, s_f) | J_\mu^{\text{TOT}} | f(p_i, s_i) \rangle \quad \text{Eq. D-1}$$

Then,

$$\begin{aligned} J_\mu^{\text{TOT}} = & [\gamma_\mu F_1^V(q^2) + (\gamma_\mu q^2 - 2m q_\mu) \gamma_5 F_1^A(q^2) \\ & + \frac{i\sigma_{\mu\nu} q^\nu}{2M} (F_2^V(q^2) + F_2^A(q^2) \gamma_5) \\ & + q_\mu (F_3^V(q^2) + F_3^A(q^2) \gamma_5)] \end{aligned} \quad \text{Eq. D-2}$$

is the most general current for weak and electromagnetic interactions with only the first four terms appearing in J_{EM}^μ . The correspondence between form factors and spin states can be determined from the nonrelativistic limit of the matrix element. From Kayser (KAY.82) we have the following table for these nonrelativistic limits for the electromagnetic matrix element.

TERM J_{EM}^{μ}	NON-RELATIVISTIC LIMITS	STATES PRODUCED
γ_{μ}	$\frac{(1+\vec{p}^2)}{12m^2} \vec{n} \cdot \vec{T} - \frac{\vec{p}^2}{6m^2} (3\vec{n} \cdot \vec{p} \vec{T} \cdot \vec{p} - \vec{n} \cdot \vec{T})$	$^3S_1, ^3D_1$
$(q^2 \gamma_{\mu} - 2m i q_{\mu}) \gamma_5$	$\vec{n} \cdot \vec{T} \times \vec{p}$	3P_1
$\sigma_{\mu\nu} q_{\nu}$	$\frac{(1-\vec{p}^2)}{12m^2} \vec{n} \cdot \vec{T} + \frac{\vec{p}^2}{6m^2} (3\vec{n} \cdot \vec{p} \vec{T} \cdot \vec{p} - \vec{n} \cdot \vec{T})$	$^3S_1, ^3D_1$
$\sigma_{\mu\nu} q_{\nu} \gamma_5$	$\vec{n} \cdot \vec{p} S$	1P_1

where $S = \chi_2^+ \chi_1$ and $T = \chi_2^+ \sigma \chi_1$.

APPENDIX E - FEYNMAN PARAMETERIZATION AND DIMENSIONAL REGULARIZATION FORMULA

The general Feynman parameterization and dimensional regularization formulae we needed are (RAM.81):

$$\frac{1}{D_1^{a_1} D_2^{a_2} \dots D_k^{a_k}} = \frac{\Gamma(a_1 + a_2 + \dots + a_k)}{\Gamma(a_1) \Gamma(a_2) \dots \Gamma(a_k)} \int_0^1 \dots \int_0^1 dx_1 \dots dx_k \frac{\delta(1 - x_1 - \dots - x_k) x_1^{a_1-1} \dots x_k^{a_k-1}}{(D_1 x_1 + \dots + D_k x_k)^{a_1 + \dots + a_k}} \quad \text{Eq. E-1}$$

and

$$\int \frac{d^2 \omega_l}{(2\pi)^{2\omega}} \frac{1}{(l^2 + M^2 + 2l \cdot p)^A} = \frac{\Gamma(A-\omega)}{(4\pi)^{\omega} \Gamma(A)} \frac{1}{(M^2 - p^2)^{A-\omega}} \quad \text{Eq. E-2}$$

$$\int \frac{d^2 \omega_l}{(2\pi)^{2\omega}} \frac{l_\mu}{(l^2 + M^2 + 2l \cdot p)^A} = - \frac{\Gamma(A-\omega)}{(4\pi)^{\omega} \Gamma(A)} \frac{p_\mu}{(M^2 - p^2)^{A-\omega}} \quad \text{Eq. E-3}$$

$$\int \frac{d^2 \omega_l}{(2\pi)^{2\omega}} \frac{l_\mu l_\nu}{(l^2 + M^2 + 2l \cdot p)^A} = \frac{1}{(4\pi)^{\omega} \Gamma(A)} \left[p_\mu p_\nu \frac{\Gamma(A-\omega)}{(M^2 - p^2)^{A-\omega}} + \frac{1}{2} \delta_{\mu\nu} \frac{\Gamma(A-1-\omega)}{(M^2 - p^2)^{A-1-\omega}} \right] \quad \text{Eq. E-4}$$

APPENDIX F - NUMERICAL EVALUATION OF THE PARAMETRIC INTEGRAL
(FOR Z^0 DECAY)

It is necessary to evaluate the following parametric integral:

$$F_{f,A} = \int_0^1 dx \int_0^{1-x} dy \{ \ln(\xi(s, m_A^2, m_f^2, m_\lambda^2) / \xi(s, m_f^2, m_A^2, m_\lambda^2)) \\ - \xi^{-1}(s, m_A^2, m_f^2, m_\lambda^2) [xys + m_f^2 - (1-x-y)^2 m_\lambda^2] \} \quad \text{Eq. F-1}$$

with,

$$\xi(s, m^2, m^2, m_\lambda^2) = (x+y)(1-x-y)m_\lambda^2 + xys \\ + (x+y)(M^2 - m^2) - M^2 + i\epsilon \quad \text{Eq. F-2}$$

We will first consider the logarithmic piece of the integrand.

Denote: $^1\xi \equiv xys + (x+y)(1-x-y)m_\lambda^2 + (x+y)(m_A^2 - m_f^2) - m_A^2 + i\epsilon$

$$^2\xi \equiv xys + (x+y)(1-x-y)m_\lambda^2 + (x+y)(m_f^2 - m_\lambda^2) - m_f^2 + i\epsilon \quad \text{Eq. F-3}$$

and define,

$$\eta \equiv \frac{m_\lambda^2}{m_A^2} ; \delta \equiv \frac{s}{m_A^2} ; \epsilon \equiv \frac{m_f^2}{m_A^2} , \quad \text{Eq. F-4}$$

Then,

$$^1\xi = m_A^2 [(x+y)(1-x-y)\eta + xy\delta + (x+y)(1-\epsilon) - 1]$$

$$^2\xi = m_A^2 [(x+y)(1-x-y)\eta + xy\delta + (x+y)(\epsilon - 1) - \epsilon] . \quad \text{Eq. F-5}$$

We then rearrange these expressions to the standard form of a quadratic expression in y :

$$l_{\xi} = m_A^2 [\eta y^2 - y(\eta(1-2x) + \delta x + (1-\epsilon)) + (1-x + \epsilon x - \eta x(1-x))] \quad \text{Eq. F-6}$$

or

$$l_{\xi} = -m_A^2 [\eta y^2 - b_1 y + c_1] \quad \text{with} \quad \text{Eq. F-6}$$

$$b_1 = 1 - \epsilon + \delta x + \eta(1-2x)$$

$$c_1 = 1 - x + \epsilon x - \eta x(1-x)$$

$$\text{So } l_{\xi} = 0 \text{ at } \sqrt{\eta} y = \frac{b_1}{2/\eta} \pm \frac{\sqrt{b_1^2 - 4\eta c_1}}{2/\eta} \quad \text{Eq. F-7}$$

Similarly

$$l_{\xi}^2 = -m_A^2 [\eta y^2 - (\eta(1-2x) + \delta x + (\epsilon-1))y + \epsilon(1-x) + x - \eta x(1-x)] \quad \text{Eq. F-8}$$

$$= -m_A^2 [\eta y^2 - b_2 y + c_2] \quad \text{Eq. F-9}$$

$$\text{with } b_2 = \epsilon - 1 + \delta x + \eta(1-2x)$$

$$c_2 = \epsilon(1-x) + x - \eta x(1-x)$$

So

$$l_{\xi} = m_A^2 \eta [y - y_1^+(x)][y - y_1^-(x)]$$

$$y_1^{\pm}(x) = \frac{b_1 \pm \sqrt{b_1^2 - 4\eta c_1}}{2\eta} \quad \text{Eq. F-10}$$

$$2_{\xi} = -m_A^2 \eta [y - y_2^+(x)] [y - y_2^-(x)]$$

$$y_2^{\pm}(x) = \frac{b_2^2 \pm \sqrt{b_2^4 - 4\eta c_2}}{2\eta}$$

Eq. F-11

Then

$$\ln[1_{\xi}/2_{\xi}] = \ln \left[\frac{(y - y_1^+(x))(y - y_1^-(x))}{(y - y_2^+(x))(y - y_2^-(x))} \right]$$

Eq. F-12

with

$$y_2^{\pm} = \frac{b_2^2 \pm \sqrt{b_2^4 - 4\eta c_2}}{2\eta}$$

$$= \frac{b_2^2}{2\eta} \pm \frac{b_2^2}{2\eta} \left[1 - \frac{4\eta c_2}{b_2^4} \right]^{\frac{1}{2}}$$

Eq. F-13

We then examine the singularities of the integrand for small

$$\eta \equiv \frac{m_{\lambda}^2}{m_A^2}, \quad \epsilon \equiv \frac{m_f^2}{m_A^2}, \quad \delta \equiv \frac{s}{m_A^2}$$

$$(\leq .0526 \quad \leq .25 \quad \leq .2116) .$$

In this case b_2 is -ve so $y_2^+ < 0$, also $y_1^+ \sim \frac{1}{\eta} > 1-x$ so only y_1^- lies in the region $0 < y_1(x) < 1-x$. So

$$\begin{aligned}
 & \int_0^{1-x} dy \ln \left[\frac{1-x}{\epsilon} \right] \lim_{\epsilon \rightarrow 0} \left[\int_0^{y_1^- - \epsilon} (\ln(y_1^- - y) + i\pi) dy \right. \\
 & + \int_{y_1^- + \epsilon}^{1-x} \ln(y - y_1^-) dy \left. \right] + \int_0^{1-x} dy [\ln(y_1^+ - y) + i\pi] \\
 & - \int_0^{1-x} dy \ln(y - y_2^-) - \int_0^{1-x} dy \ln(y - y_2^+) \\
 & = i\pi(1-x+y_1^-) + \lim_{\epsilon \rightarrow 0} \left[\int_0^{y_1^- - \epsilon} dy \ln(y_1^- - y) + \int_{y_1^- + \epsilon}^{1-x} dy \ln(y - y_1^-) \right. \\
 & \left. + \int_0^{1-x} dy \ln(y_1^+ - y) - \int_0^{1-x} dy \ln(y - y_2^-) - \int_0^{1-x} dy \ln(y - y_2^+) \right]
 \end{aligned}$$

Eq. F-14

$$\text{Now: } \int_0^{y_1^-} dy \ln(y_1^- - y) = y_1^- \ln y_1^- - y_1^-$$

Eq. F-15

$$\int_{y_1^-}^{1-x} dy \ln(y - y_1^-) = (1-x-y_1^-) \ln(1-x-y_1^-) - (1-x-y_1^-)$$

Eq. F-16

$$\int_0^{1-x} dy \ln(y_1^+ - y) = y_1^+ \ln(y_1^+) - (y_1^+ - (1-x)) \ln(y_1^+ - (1-x)) - (1-x)$$

Eq. F-17

$$\int_0^{1-x} dy \ln(y - y_2^-) = (1-x-y_2^-) \ln(1-x-y_2^-) - (-y_2^-) \ln(-y_2^-) - (1-x),$$

Eq. F-18

remembering ($y_2^- < 0$).

So

$$\begin{aligned}
 I &= \int_0^{1-x} dy \ln \left[\frac{1-y}{\xi} \right] \\
 &= i\pi(y_1^-) \\
 &\quad + y_1^- \ln(y_1^-) + (1-x-y_1^-) \ln(1-x-y_1^-) - (1-x) \\
 &\quad + y_1^+ \ln(y_1^+) - (y_1^+ - (1-x)) \ln(y_1^+ - (1-x)) - (1-x) \\
 &\quad - [(1-x) - y_2^+ \ln((1-x) - y_2^+) - (y_2^+ \ln(-y_2^+) - (1-x))] \\
 &\quad - [(1-x) - y_2^- \ln((1-x) - y_2^-) - (-y_2^- \ln(-y_2^-) - (1-x))]
 \end{aligned} \tag{Eq. F-19}$$

We drop $(1-x)$ terms which are independent of the kinematical variables and cancel in the chiral sum.

Then,

$$\begin{aligned}
 I &= i\pi(y_1^-) \\
 &\quad + y_1^- \ln(y_1^-) + (1-x-y_1^-) \ln(1-x-y_1^-) \\
 &\quad + y_1^+ \ln(y_1^+) - (y_1^+ - (1-x)) \ln(y_1^+ - (1-x)) \\
 &\quad - ((1-x) - y_2^+) \ln((1-x) - y_2^+) + (-y_2^+) \ln(-y_2^+) \\
 &\quad - ((1-x) - y_2^-) \ln((1-x) - y_2^-) + (-y_2^-) \ln(-y_2^-)
 \end{aligned} \tag{Eq. F-20}$$

Now we evaluate the finite term.

$$\begin{aligned}
 I_F &= - \int_0^{1-x} dy \frac{[xys + m_f^2 - (1-x-y)^2 m_\lambda^2]}{[(x+y)(1-x-y)m_\lambda^2 + xys + (x+y)(m_A^2 - m_f^2) - m_A^2 + i\epsilon]} \\
 &= - \int_0^{1-x} dy \frac{[xys + \epsilon - (1-x-y)^2 \eta]}{[\eta y^2 - b_1 y + c_1]} \\
 &= - \frac{1}{\eta} \int_0^{1-x} dy \frac{[xy\delta + \epsilon - (1-x-y)^2 \eta]}{[y - y_1^+(x)][y - y_1^-(x)]}
 \end{aligned}$$

Eq. F-21

Now

$$\begin{aligned}
 xy\delta + \epsilon - (1-x-y)^2 \eta \\
 = ay^2 + by + c
 \end{aligned}$$

Eq. F-22

with $a \equiv -\eta$

$$b \equiv x\delta + 2\eta - 2\eta x$$

$$c \equiv -\eta x^2 + 2\eta x + \epsilon - \eta$$

Then

$$\begin{aligned}
 \frac{ay^2 + by + c}{(y - y^+)(y - y^-)} &= a + \frac{(b + a(y^+ + y^-))y + (c - ay^+ y^-)}{(y - y^+)(y - y^-)} \\
 &= a + \frac{ey + f}{(y - y^+)(y - y^-)}
 \end{aligned}$$

Eq. F-23

where $e = b + a(y^+ + y^-)$

$$f = c - ay^+ y^-$$

Now

$$\frac{ey+f}{(y-y^+)(y-y^-)} = \frac{g}{(y-y^+)} + \frac{e-g}{(y-y^-)}$$

Eq. F-24

$$\text{where } g = (f+ey^+)/ (y^+-y^-)$$

So

$$\frac{ay^2+by+c}{(y-y^+)(y-y^-)} = a + \frac{g}{(y-y^+)} + \frac{e-g}{(y-y^-)}$$

Eq. F-25

$$\text{where } e = b + a(y^+-y^-)$$

$$f = c - ay^+y^-$$

$$g = (f+ey^+)/ (y^+-y^-)$$

So

$$I_f = \int_0^{1-x} dy \left[a + \frac{g}{y-y_1^+} + \frac{e-g}{(y-y_1^-)} \right]$$

$$= a(1-x) - g[\ln(y_1^+) - \ln(y_1^+ - (1-x))] + (e-g)(\ln(1-x-y_1^-) - \ln(y_1^-))$$

Eq. F-25

$$\text{where } \eta = \frac{\frac{m_A^2}{2}}{\frac{m_A^2}{2}}, \epsilon = \frac{\frac{m_f^2}{2}}{\frac{m_A^2}{2}}, \delta = \frac{s}{2} \text{ as before,}$$

$$\text{and } a = -\eta$$

$$b = x\delta + 2\eta - 2\eta x$$

$$c = -x^2 + 2x + \epsilon - 1$$

$$e = b + a(y_1^+-y_1^-)$$

$$f = c - ay_1^+y_1^-$$

$$g = (f+ey^+)/ (y_1^+-y_1^-)$$

$$\text{and } y_2^{\pm} = \frac{b_2 \pm \sqrt{b_2^2 - 4\eta c_2}}{-2\eta}$$

$$\text{with } b_1 = 1 - \epsilon + \delta x + \eta(1-2x); \quad c_1 = 1 - x + \epsilon x - \eta x(1-x) \\ b_2 = \epsilon - 1 + \delta x + \eta(1-2x); \quad c_2 = \epsilon(1-x) + x - \eta x(1-x)$$

Therefore equation F-1, For $F_{f,A}$ becomes, in terms of real and imaginary pieces;

$$F_{f,A} = F_{f,A}^R + iF_{f,A}^I \quad \text{Eq. F-26}$$

where,

$$F_{f,A}^R = \int_0^1 dx \{ y_1^- \ln(y_1^-) + (1-x-y_1^-) \ln(1-x-y_1^-) + y_1^+ \ln(y_1^+) \\ - (y_1^+ - (1-x)) \ln(y_1^+ - (1-x)) - (1-x-y_2^+) \ln(1-x-y_2^+) \\ - (y_2^+ \ln(-y_2^+) - (1-x-y_2^-) \ln(1-x-y_2^-) + (-y_2^-) \ln(-y_2^-) \\ + \frac{1}{\eta} [-g(\ln(y_1^+) - \ln(y_1^+ - (1-x))) + (\epsilon - g)(\ln(1-x-y_1^-) - \ln(y_1^-))] \} \quad \text{Eq. F-27}$$

$$F_{f,A}^I = \int_0^1 dx \pi y_1^- \quad \text{Eq. F-28}$$

So the integrand of the real part of the form factor is:

$$+ y_1^- \ln(RI1M * y_1^-) + ((1-x) - y_1^-) \ln(RJ1M * ((1-x) - y_1^-)) \\ + y_1^+ \ln(RI1P * y_1^+) + ((1-x) - y_1^+) \ln(RJ1P * ((1-x) - y_1^+)) \\ - y_2^- \ln(RI2M * y_2^-) - ((1-x) - y_2^-) \ln(RJ2M * ((1-x) - y_2^-)) \\ - y_2^+ \ln(RI2P * y_2^+) - ((1-x) - y_2^+) \ln(RJ2P * ((1-x) - y_2^+)) \\ + \frac{1}{\eta} [g(\ln(RJ1P * ((1-x) - y_1^+)) - \ln(RI1P * y_1^+)) \\ + (\epsilon - g)(\ln(RJ1M * ((1-x) - y_1^-)) - \ln(RI1M * y_1^-))]$$

Eq. F-29

where, $RI_2^1 M_p = 1$ for $y_2^- > 0$

$= -1$ for $y_2^- < 0$

$RI_2^1 M_p = 1$ for $y_1^- > (1-x)$

$= -1$ for $y_1^- < (1-x)$

and the integrand for the imaginary part of the form factor is;

$$\begin{aligned} & i\pi \{ y_1^- * \theta((1-x)-y_1^-) \theta(y_1^-) \\ & + y_1^+ * \theta((1-x)-y_1^+) \theta(y_1^+) \\ & - y_2^- * \theta((1-x)-y_2^-) \theta(y_2^-) \\ & - y_1^+ * \theta((1-x)-y_2^+) \theta(y_2^+) \} \end{aligned}$$

Eq. 30

where these expressions have been generalized to include the cases where the parameters are different than originally assumed.

The numerical values assumed for the computation were

$$E_{CM} = 93 \text{ GeV}$$

$$M_s = 200 \text{ GeV}$$

$$M_b = 5 \text{ GeV}$$

$$15 < M_{\tilde{g}} < 60 \text{ GeV (5 GeV steps)}$$

and a gluino mass ranging from zero to one half the Z^0 mass. The program follows.

[illegible]

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Epreuve illisible

```

23200 30 GO TO 40
23300 SJ1M=-1
23300 SJ1M=0
23400 40 CONTINUE
23500 IF (RY1P.LT.0) GO TO 110
23600 RI1P=1
23700 SI1P=1
23800 GO TO 120
23900 110 SJ1P=-1
24000 SJ1M=0
24100 120 CONTINUE
24200 IF (RY2P.LT.0) GO TO 130
24300 RI2P=1
24400 SI2P=1
24500 GO TO 140
24600 130 SJ2P=-1
24700 SJ2M=0
24800 140 CONTINUE
24900 IF (RY2M.LT.0) GO TO 210
25000 RI2M=1
25100 SI2M=1
25200 GO TO 220
25300 210 RI2M=-1
25400 SJ2M=0
25500 CONTINUE
25600 220 IF (RY2M.GT.XX) GO TO 230
25700 RJ2M=1
25800 SJ2M=1
25900 GO TO 240
26000 230 RJ2M=-1
26100 SJ2M=0
26200 240 CONTINUE
26300 IF (RY2P.LT.0) GO TO 310
26400 RI2P=1
26500 SI2P=1
26600 GO TO 320
26700 310 RI2P=-1
26800 SJ2P=0
26900 CONTINUE
27000 320 IF (RY2P.GT.XX) GO TO 330
27100 RJ2P=1
27200 SJ2P=1
27300 GO TO 340
27400 330 RJ2P=-1
27500 SJ2P=0
27600 340 CONTINUE
27700
27800
27900
28000
28100 Z=RY1M*SJ1M+RI1M*RY1P*SJ1P*SJ2P
28200 Z=Z+RY2M*SJ2M+RI2M*RY2P*SJ2P*SJ2P
28300 Z=Z-1416
28400
28500 RA=-RNET
28600 RB=X*RDEL+2.*RNET*XX
28700 RC=(-(XX-2)+2.*X-1.)*RNET+REP
28800 RE=RB+RA*(RY1P-RY1M)
28900 RF=RC-RA*RY1P*RY1M
29000 RG=(RF+RE*RY1P)/(RY1P-RY1M)
29100 Y=RG*(ALOG(RJ1P*(XX-RY1P))-ALOG(RI1P*RY1P))
29200 Y=Y+(RE-RG)*(ALOG(RJ1M*(XX-RY1M))-ALOG(RI1M*RY1M))
29300 Y=Y/RNET
29400
29500 Y=Y+RY1M*ALOG(RI1M*RY1M)
29600 Y=Y+(XX-RY1M)*ALOG(RJ1M*(XX-RY1M))
29700 Y=Y+RY1P*ALOG(RI1P*RY1P)
29800 Y=Y+(XX-RY1P)*ALOG(RJ1P*(XX-RY1P))
29900 Y=Y-RY2M*ALOG(RI2M*RY2M)
30000 Y=Y-(XX-RY2M)*ALOG(RJ2M*(XX-RY2M))
30100 Y=Y-RY2P*ALOG(RI2P*RY2P)
30200 Y=Y-(XX-RY2P)*ALOG(RJ2P*(XX-RY2P))
30300
30400
30500
30600
30700
RETURN
END

```

END

1	0	0	4	8	6
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FIN