Orientation and skill effects in exact and approximate addition

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Abstract

Orientation (horizontal vs. vertical problem presentation) and skill (arithmetic fluency) effects were investigated for exact ($39 + 14 = 53$) and approximate ($39 + 14 \approx 50$) addition problems. Participants solved two-digit addition problems (e.g., $39 + 14$) that were presented both horizontally and vertically. Half of the problems had a carry operation in the units digits (e.g., $39 + 14$), whereas the other half did not (e.g., $22 + 12$). In Experiment 1, participants chose the answer from two alternatives (i.e., the answer and a wrong answer). In Experiment 2, participants responded verbally with exact (e.g., $39 + 14 = 53$) or approximate answers (e.g., $39 + 14 \approx 50$). In both experiments, participants solved horizontal problems more slowly than vertical problems. Further, performance on carry problems was more impaired for low-skill than high-skill participants. Orientation and skill effects did not differentially affect performance of exact and approximate addition.

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Introduction

People are born with the ability to discriminate between small number magnitudes, such that even infants can discriminate one light from two lights (Dehaene, 1997; Dehaene, Dehaene-Lambertz, & Cohen, 1998). This rudimentary number sense develops into a rich numerical knowledge that allows people to perform many different arithmetic tasks. For example, when people see an arithmetic problem (e.g., $24 + 48$, 4×3), they can either provide a numerically precise answer or an approximate answer. In the current research, exact arithmetic problems required precise answers (e.g., $24 + 48 = 72$, $4 \times 3 = 12$), whereas approximate problems required an answer close to the precise answer (e.g., $24 + 48 \approx 70$, $4 \times 3 \approx 10$). According to Dehaene (1997), acquisition of language enables people to make the transition from comparing or approximating quantities to learning the skills necessary to provide exact solutions for arithmetic problems. Butterworth (1999) concurred with Dehaene and discussed how this claim about the role of language in exact versus approximate calculation is supported by biological evidence. In particular, brain regions associated with language learning (left inferior frontal lobe) are activated by exact arithmetic tasks whereas brain regions associated with visual-spatial processing (intraparietal cortex) are activated by approximate arithmetic tasks. Thus, the role of language in mental arithmetic is important when distinguishing between exact and approximate arithmetic.

In addition to differences in brain activation regions, exact and approximate arithmetic differ cognitively. First, exact and approximate arithmetic activate different mental representations, such that exact arithmetic activates verbal codes and approximate arithmetic activates magnitude codes (Dehaene, 1997; Dehaene & Cohen, 1995, 1997;

Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Spelke & Tsivkin, 2001). Second, exact arithmetic requires more calculation and less simplification than approximate arithmetic, as evidenced by different strategies used to solve both types of problems (El Yagoubi, Lemaire, & Besson, 2003; Kalaman & LeFevre, 2004). Third, exact arithmetic is more demanding of mental resources where arithmetic calculations are performed than approximate arithmetic (Kalaman & LeFevre, 2004).

Many of the researchers who investigated differences between exact and approximate arithmetic have developed experiments in which participants solved either single-digit (e.g., $4 + 5$) or multi-digit (e.g., $24 + 48$) addition problems with a variety of response options (e.g., when solving $24 + 48$, choose between 72 and 62, or verbally say “72”; e.g., Dehaene et al., 1999; Duverne, Lemaire, & Michel, 2003; El Yagoubi et al., 2003; Kalaman & LeFevre, 2004; Spelke & Tsivkin, 2001; Stanescu-Cosson, Pinel, van de Moortele, Le Bihan, Cohen, & Dehaene, 2000). Thus, some information is available about how people solve exact and approximate arithmetic problems. Introducing manipulations that are hypothesized to affect the precise and simplified calculations of exact and approximate arithmetic respectively can be useful in further examining differences between both types of arithmetic. Two manipulations of interest are orientation (i.e., horizontal or vertical presentation) and skill (i.e., competency in solving arithmetic problems).

Researchers contrasting exact and approximate arithmetic have not manipulated orientation or skill. In particular, all addition problems used in prior research examining differences between exact and approximate arithmetic were presented horizontally. Through years of schooling, however, children learn to solve exact multi-digit arithmetic

problems in columns (i.e., vertical orientation; Hayes, 1973; Hitch, 1978). Hence, it seems appropriate to study the effects of orientation on arithmetic processing because familiarity with solving problems in a vertical orientation may differentially facilitate exact and approximate calculations. With respect to the skill manipulation, some people are more efficient at solving arithmetic problems than others (Bull & Scerif, 2001). It seems appropriate to study skill to assess whether efficiency in arithmetic problem solving influences the calculation of exact and approximate arithmetic problems. More importantly, manipulations of orientation, skill, and task (i.e., exact and approximate addition) are assumed to influence memory resources that are involved in the calculation of an arithmetic problem. Thus, the goal of this thesis was to examine the effects of orientation and skill on the performance of exact and approximate multi-digit addition problems. The work was framed within Baddeley's (2002) working memory model.

In the remainder of the introduction, the differences between exact and approximate arithmetic mentioned earlier will be discussed in more detail, with hypotheses generated in response to these differences. First, a model of number processing is presented that outlines brain regions and mental representations that are activated when people solve exact and approximate arithmetic problems. Second, research is reviewed that provides support for the claim that exact arithmetic relies on a language system, whereas approximate arithmetic does not. Third, the importance of memory resources for exact and approximate computations is discussed. Differences in strategy selection for both arithmetic tasks are presented next. Finally, hypotheses are generated in response to issues of orientation and skill.

Triple-Code Model of Number Processing

Dehaene (1992; Dehaene & Cohen, 1995, 1997) proposed a triple-code model that accounts for both the cognitive and neuroanatomical processing of numbers. A verbal code, localized in the left-hemispheric perisylvian brain regions, represents numbers as organized word sequences ("five times three, fifteen"). The verbal code accesses rote verbal memory of arithmetic facts. A visual arabic code, localized in the left and right inferior ventral occipito-temporal brain regions, represents numbers as strings of digits (e.g., <4><5>). The visual arabic code is used to process multi-digit operations (e.g., $53 + 38$) and parity judgements (i.e., odd and even status of a number). A magnitude code, localized in the right and left parietal brain regions, represents numbers on a number line (e.g., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5). The magnitude code provides the semantic meaning of numbers (e.g., 50 is larger than 30, 8 is close in distance to 9).

The triple-code model links specific arithmetic tasks to specific codes (Dehaene, 1997; Dehaene & Cohen, 1995). Simple arithmetic problems that require exact answers (e.g., 3×5 , $4 + 2$) use verbal codes. Simple number comparison tasks that do not require exact numerical answers (e.g., Is 5 larger than 4?) use magnitude codes. More complex number problems may involve activation of a variety of codes. For example, if the problem 9×4 does not activate a rote verbal memory of the answer 36, the magnitude code may activate itself by comparing the operands (i.e., 9 and 4), and reorganizing them (4×9) to create an arithmetic fact accessible to the verbal code.

Bilingual Experiments to Support the Triple-Code Model

Dehaene et al. (1999) and Spelke and Tsivkin (2001) showed that different numerical tasks activate different mental codes according to Dehaene's (1992) triple-code model. They tested two hypotheses. Exact arithmetic problems (e.g., $23 + 48 = 71$) should activate verbal codes associated with a language (e.g., English). In contrast, approximate arithmetic problems (e.g., $23 + 48 \approx 70$) should activate magnitude codes that are independent of language. Their participants were Russian-English bilinguals. They used bilingual participants to assess whether verbal language is important when distinguishing exact arithmetic from approximate arithmetic.

In Dehaene et al. (1999), eight Russian-English bilingual participants (3 females and 5 males) received four days of arithmetic training on both addition and multiplication problems. For two days, participants were taught a set of two-digit addition sums with a fixed task (exact or approximate addition) in one (e.g., Russian) of their two speaking languages. Thus, four participants solved exact addition problems and four participants solved approximate addition problems. Following this initial training, participants spent two days being taught multiplication problems (with similar magnitudes as the addition sums) in their second speaking language (e.g., English). Therefore, if a participant was taught a series of exact addition sums in Russian, they also were taught a series of exact multiplication problems in English. The purpose of having participants solve multiplication problems was to provide them with equal exposure to both languages before receiving the experimental addition trials.

After four days of training, participants spent the fifth day solving the experimental addition trials. Participants solved two-digit addition problems by choosing

either an exact answer (for those who were trained with exact addition) or an approximate answer (for those who were trained with approximate addition). Participants solved trained problems (i.e., previously learned) in both languages. Participants solved novel problems (i.e., not previously learned) with similar magnitudes in the language they were trained in. As was the case in training, the addition problems were presented horizontally in symbol format (i.e., $13 + 38 =$), with the possible answers presented horizontally in visual word format (i.e., “fifty-one” vs. “sixty-one”). For the exact addition task, the possible answers were the exact answer (e.g., “fifty-one”) and a wrong answer that was 10 digits from the exact answer (e.g., “sixty-one”). For the approximate addition task, the possible answers were an approximate answer rounded to the nearest decade of the exact answer (e.g., “fifty”), and a wrong answer that was 30 digits above or below the approximate answer (e.g., “eighty”).

Dehaene et al. (1999) found that performance on trained exact addition problems was affected by language switching. For the exact addition task, participants responded more quickly to trained problems when the answers were presented in the same language in which they learned the initial sums than when the answers were presented in the untrained language. In contrast, for the approximate addition task, participants’ performance was similar whether the answers were presented in the trained language or the untrained language. Dehaene et al. concluded that exact addition task performance is dependent on a language system. Further, they concluded that approximate addition task performance is not dependent on a language system.

Dehaene et al. (1999) found that performance on novel addition problems was also affected by task. For the exact addition task, participants responded more slowly

when they solved novel problems than when they solved trained problems in the language they learned the initial sums. This finding suggests that novel exact arithmetic facts were represented as word sequences (independent of numerical magnitude) because different word sequences were presumably activated for novel than trained problems. In contrast, for the approximate addition task, participants' performance was similar whether they solved novel problems or trained problems. This finding suggests that novel approximate arithmetic facts were represented in a magnitude format (independent of language) because language was held constant (i.e., no language switching). Again, Dehaene et al. concluded that performance on exact addition problems is dependent on a language system and requires a verbal code, whereas performance on approximate addition problems is not dependent on a language system and requires a magnitude code (i.e., a non-verbal code).

Spelke and Tsivkin (2001) made similar conclusions as Dehaene et al. (1999). In Experiment 1, Russian-English bilingual participants solved four categories of exact addition problems (adding 54 to each of 12 numbers ranging in value from 47 to 95, adding 63 to each of 12 numbers ranging in value from 39 to 96, base 6 addition, base 8 addition) and two categories of approximation problems (cube root approximations, logarithmic base 2 approximations) by choosing the appropriate answer from two numbers written as words. Consistent with Dehaene et al., for the exact addition tasks, participants responded more slowly when they solved novel problems than when they solved trained problems in the language they learned the initial problems. Further, for the approximation problems, participants' performance was similar whether they solved novel problems or trained problems. Spelke and Tsivkin concluded that approximations

involving logarithms and cube roots depend on magnitude representations independent of language, whereas base 10, base 6, and base 8 addition problems activate verbal codes that are dependent upon a language system.

In contrast to different mental representations being activated for exact and approximate addition, Spelke and Tsivkin (2001, Experiment 2) found no support for different mental representations being activated for exact and approximate multiplication. In Experiment 2, bilingual participants solved exact addition (e.g., $34 + 71 = 105$ versus 115), exact multiplication (e.g., $21 \times 5 = 105$ versus 115), approximate addition (e.g., $34 + 71 \approx 110$ vs. 80) and approximate multiplication problems ($21 \times 5 \approx 110$ vs. 80). For both exact and approximate addition problems, the findings and conclusions were the same as those of Dehaene et al. (1999). In contrast, for both exact and approximate multiplication problems, participants' performance in the trained language exceeded performance in the untrained language. As a result of performance decrements due to language-switching, Spelke and Tsivkin concluded that language is an important factor when distinguishing exact addition from approximate addition but not when distinguishing exact multiplication from approximate multiplication. In addition to language, working memory is another important factor when investigating differences between exact and approximate arithmetic.

Working Memory and Exact and Approximate Addition

Arithmetic is a complex cognitive task that requires working memory resources (Ashcraft, 1995; DeStefano & LeFevre, 2004; LeFevre, DeStefano, Coleman, & Shanahan, in press). Working memory refers to those processes that control, regulate, and maintain specific information for the execution of complex cognitive tasks (Miyake

& Shah, 1999). The majority of research on working memory and arithmetic uses Baddeley's (1986, 2002; Baddeley & Hitch, 1974) multi-component model as a framework for understanding the resources (phonological, visual-spatial, central executive) that are required when people solve exact arithmetic problems (e.g., $3 \times 8 = 24$, $21 + 9 = 30$; e.g., Fürst & Hitch, 2000; Heathcote, 1994; Trbovich & LeFevre, 2003).

Baddeley's (2002) working memory model consists of four components: the central executive, the phonological loop, the visual-spatial sketchpad, and the episodic buffer. The phonological loop provides temporary storage and processing of phonological information. The visual-spatial sketchpad provides temporary storage and processing of visual and spatial information. The episodic buffer is hypothesized to combine information from long-term memory with information from the phonological loop and the visual-spatial sketchpad. The central executive co-ordinates and allocates attentional resources to the three storage systems. A considerable amount of research has supported the use of this model for understanding how people solve arithmetic problems (DeStefano & LeFevre, 2004).

Based on the findings that exact and approximate addition appear to engage in different combinations of mental processes (Dehaene et al., 1999; Spelke & Tsivkin, 2001) and that approximate addition is typically easier than exact addition (Stanescu-Cosson et al., 2000), Kalaman and LeFevre (2004) hypothesized that exact addition would be more demanding of working memory resources than approximate addition. Kalaman and LeFevre used a dual-task paradigm in two experiments to examine differences in the working memory demands of exact and approximate addition. For both experiments, participants solved two-digit addition problems (e.g., $63 + 49$) alone and in

combination with a letter recall task (i.e., remembering four consonants such as XPTZ). In Experiment 1, participants chose an answer from two alternatives (e.g., exact: 112 vs. 122; approximate: 110 vs. 140). In Experiment 2, participants responded verbally with exact or approximate answers. In the dual-task conditions of both experiments, participants memorized the letters, then performed the arithmetic task, and then recalled the letters.

According to dual-task logic, if letter memory and arithmetic require the same working memory resources, then performance on one or both tasks should be worse when they are performed together, as compared to when they are performed alone. Letter recall was used by Ashcraft and Kirk (2001, Experiment 1) and by Seyler, Kirk, and Ashcraft (2003, Experiment 3) to assess the working memory demands of addition and subtraction, respectively. Presumably, letter memory involves both the phonological loop, required to maintain the memory load, and the central executive (see also Seitz & Schumann-Hengsteler, 2000). Central executive resources are required to fuel the rehearsal process and maintain the serial order of letters (Jones, Farrand, Stuart, & Morris, 1995). Kalamian and LeFevre (2004) expected that exact addition would be more affected than approximate addition by a letter load, if letter recall implicates central executive resources.

Recent reviews of working memory and mental arithmetic suggest that calculation demands central executive resources, even when the problems are simple single-digit sums such as $3 + 4$ (DeStefano & LeFevre, 2004; LeFevre et al., in press). Further, problem complexity in the form of more calculations or operations such as carrying or borrowing seem to increase working memory demands (Ashcraft & Kirk, 2001;

DeStefano & LeFevre, 2004; Fürst & Hitch, 2000; Seitz & Schumann-Hengsteler, 2000, 2002; Seyler et al., 2003). On this view, Kalaman and LeFevre (2004) hypothesized that problems involving carrying from the units to the tens (e.g., $37 + 48$) would demand more working memory resources than similar problems that did not require unit carries (e.g., $37 + 41$).

The patterns of results across the two experiments were similar. The consonant load impaired exact and approximate addition performance, but exact addition was affected more. Load also impaired performance on problems with a carry operation in the units more than on problems without a unit carry. Kalaman and LeFevre (2004) concluded that exact addition and carry problems consume more working memory resources than approximate addition or no-carry problems. Further, they also found that participants were slower and made more errors on carry versus no-carry problems for exact addition. In contrast, for approximate addition, there were no significant performance differences between no-carry and carry problems. Given these findings, Kalaman and LeFevre suggested that an important aspect of problem difficulty, the presence of a carry in the units digits, is a more important aspect for exact addition than approximate addition. Thus, participants appear to use different strategies when solving exact versus approximate addition problems.

Strategy Use for Exact and Approximate Addition

A variety of strategies can be used to solve exact and approximate addition problems. Retrieval of any arithmetic answer from memory (e.g., $25 + 25 = 50$) requires minimal processing (Hecht, 2002; LeFevre, Sadesky, & Bisanz, 1996). If an answer to an arithmetic problem can not be retrieved from memory, a strategy with calculation

stages (e.g., for the problem $23 + 42$, add 3 and 2 followed by 2 and 4) may be used (Hitch, 1978). Further, according to Siegler and Shipley's (1995) adaptive strategy choice model, arithmetic strategies are activated based on the type of arithmetic task performed. For exact addition, an example of a strategy that participants have used is decomposition (LeFevre et al., 1996). In decomposition, participants use a known arithmetic fact combined with other arithmetic operations to solve a problem (e.g., $6 + 8 = (6+6) + 2$). For approximate addition, an example of a strategy that participants have used is reformulation (LeFevre, Greenham, & Waheed, 1993). Reformulation involves changing the numbers in an arithmetic problem. One way in which reformulation can occur is by rounding the operands in the arithmetic problem (e.g., $14 + 23 \approx 10 + 20$). Thus exact addition requires more precise calculation stages than approximate addition.

Kalaman and LeFevre (2004) reported strategies participants used to solve multi-digit exact and approximate addition problems. For exact addition problems, participants reported two strategies. Participants reported adding the units digits first followed by adding the tens digits (e.g., for the problem $34 + 19$, they added $4 + 9 = 13$, then $30 + 10 + 10 = 50$, and $50 + 3 = 53$; interim results are shown in italics). Participants also reported adding the tens digits first followed by adding the units digits (e.g., $30 + 10 = 40$, $4 + 9 = 13$, $40 + 13 = 53$). For approximate addition, participants reported three strategies. First, participants reported rounding both operands and then adding them (e.g., $30 + 20 = 50$). Second, participants reported adding the tens digits first followed by performing a rounding operation on the units digits and adjusting the answer if the sum of the units digits was greater than 5 (e.g., $30 + 10 = 40$, $4 + 9 > 5$, therefore $40 + 10 = 50$). Third, participants reported performing a rounding operation on the units digits followed

by adding the tens digits (e.g., $4 + 9 > 5$, $10 + 30 + 10$, answer 50). Kalaman and LeFevre suggested that approximate multi-digit addition is easier than exact multi-digit addition because the approximate strategies involved more simplification (e.g., rounding operands) than exact strategies.

Summary of Differences in Exact and Approximate Addition

According to the research reviewed above, there are both biological and cognitive differences between exact and approximate addition. Exact and approximate addition activate different brain regions. Exact addition activates brain regions responsible for language processing, whereas approximate addition activates brain regions responsible for visual-spatial processing (Butterworth, 1999). Furthermore, different mental representations are accessed when people solve exact and approximate addition problems (Dehaene, 1997; Dehaene & Cohen, 1995). For exact addition, more verbal codes are activated than magnitude codes. For approximate addition, more magnitude codes are activated than verbal codes. Finally, exact addition problems are more difficult to solve than approximate addition problems because of increased working memory demands that involve strategies requiring more interim calculations (i.e., carrying) and less simplification (Kalaman & LeFevre, 2004). Based on these conclusions about exact and approximate arithmetic, hypotheses were generated about how orientation and skill would interact with the task manipulation (i.e., exact vs. approximate arithmetic), taking into account the working memory resources activated for these manipulations along with the task.

Orientation

The orientation of an arithmetic problem refers to its spatial presentation. Two common orientations for arithmetic problems are horizontal and vertical, as shown in Figure 1. Vertical orientation may facilitate calculation because of the familiarity people have with columnar arithmetic. Children typically learn to solve multi-digit addition problems presented vertically. These problems generally require the production of exact answers (e.g., $34 + 48 = 82$). Hayes (1973) suggested that vertical orientation encourages a right-to-left solution strategy (i.e., units addition followed by tens addition). For example, when people solve the problem $34 + 48$, they may add the units digits ($4 + 8$) followed by placing a mark (i.e., 1) above the tens digits, and adding that mark to the sum of the tens digits ($1 + 3 + 4$). Thus, a vertically presented addition problem may be easier to solve than a horizontally presented addition problem because vertical orientation facilitates the use of the standard right-to-left calculation algorithm that people learned and practiced from elementary school on.

HORIZONTAL	VERTICAL
$17 + 18$	$\begin{array}{r} 17 \\ +18 \\ \hline \end{array}$

Figure 1. Examples of orientation of arithmetic problems.

There are few studies examining the effects of orientation on arithmetic problems, and none of these studies have examined the effect of orientation on approximate arithmetic problems (Heathcote, 1994, Experiment 2; Trbovich & LeFevre, 2003). Both Heathcote and Trbovich and LeFevre found that people solved horizontal addition

problems more slowly and less accurately than vertical addition problems. Heathcote concluded that vertical problems are easier to solve than horizontal problems because of the spatial columnar alignment of digits (i.e., the type of alignment people generally use when solving paper and pencil addition problems). Consistent with that analysis, Trbovich and LeFevre suggested that horizontal problems may be more difficult to solve than vertical problems because people use strategies on horizontal problems that require them to retain more information than on vertical problems. For example, the problem $6 + 38$ presented horizontally may be converted to $[6 + 40] - 2$, whereas the same problem presented vertically may be solved with the standard columnar algorithm.

Trbovich and LeFevre (2003) wanted to determine which working memory resources are activated when participants solve addition problems horizontally versus vertically. In their experiment, half of the participants solved two-digit plus one-digit (e.g., $7 + 65$) addition problems horizontally, and the other half solved those problems vertically. A memory load paradigm was used. For both presentation formats, participants solved addition problems with no additional memory load. Half of the participants solved the addition problems with a phonological memory load, and half with a visual-spatial memory load. The difficulty of the load manipulations was varied. For the phonological memory load, participants memorized either a single consonant-vowel-consonant nonword (e.g., gup) or three consonant-vowel-consonant nonwords (e.g., lev fup rul). For the visual-spatial memory load, participants memorized either a pattern of 4 or 8 asterisks.

Trbovich and LeFevre (2003) found different patterns of results for problems solved in combination with either a phonological or a visual-spatial memory load. The

phonological memory load impaired arithmetic performance more for horizontal than vertical problems. The visual-spatial memory load impaired arithmetic performance more for vertical than horizontal problems. Trbovich and LeFevre concluded that different problem orientations invoke different working memory resources. Problems presented horizontally require more phonological resources than problems presented vertically. Problems presented vertically require more visual-spatial resources than problems presented horizontally. Further, Trbovich and LeFevre suggested that strategies used to solve horizontal and vertical problems are in accord with the working memory resources accessed when solving problems in different orientations. Therefore, visual-spatial resources for vertical problems aid in the activation of columnar addition, whereas phonological resources for horizontal problems aid in the activation of strategies requiring storage of intermediate results.

Hypotheses Concerning Orientation and Task (Exact and Approximate Addition)

Exact and approximate addition problems are assumed to activate different mental representations. Specifically, exact addition activates verbal codes and approximate addition activates magnitude codes (Dehaene et al., 1999). Further, horizontal and vertical addition problems activate different working memory resources. Horizontal problems activate phonological resources (i.e., verbal codes) and vertical problems activate visual-spatial resources (Trbovich & LeFevre, 2003). Thus, it was hypothesized that orientation and task should interact. If exact addition relies on a verbal code (Dehaene et al., 1999), it should compete for the same phonological resources as horizontally presented addition problems. If approximate addition relies on a non-verbal code (i.e., magnitude code, Dehaene et al., 1999), it should compete for the same visual-

spatial resources as vertically presented addition problems. Thus, exact addition should be most difficult when these problems are presented horizontally, whereas approximate addition should be most difficult when these problems are presented vertically.

Skill

Some researchers have demonstrated a relation between working memory and skill in arithmetic problem solving (Bull & Scerif, 2001; Geary & Widaman, 1992). Geary and Widaman showed that participants with larger working memory capacities were more competent at performing arithmetic carry operations and retrieving arithmetic facts from memory than participants with smaller working memory capacities. Bull and Scerif showed that children with lower skill in arithmetic had impairments in central executive functioning as compared to more-skilled children. According to Baddeley (1996), the central executive co-ordinates the performance of two simultaneous tasks, switches retrieval strategies, inhibits information, and accesses information from long-term memory. Therefore, people with weak arithmetic skills appear to have less efficient or less effective working memory than people who are more-skilled at arithmetic.

Given these relations between arithmetic skill and working memory abilities, how might skill be related to the solution of exact and approximate arithmetic problems? Exact addition requires more working memory resources than approximate addition (Kalamian & LeFevre, 2004). Further, people with small working memory capacities find arithmetic more difficult than do people with large working memory capacities (Geary & Widaman, 1992). Thus, it was hypothesized that there would be a greater difference between solution of exact and approximate addition for participants who are less-skilled

at solving arithmetic problems than for participants who are more-skilled because exact addition is more demanding of working memory resources than approximate addition.

French, Ekstrom, and Price (1963) developed a test to measure the fluency (i.e., speed and accuracy) with which people solve basic arithmetic problems. The fluency measure includes timed tests which include multi-digit addition, subtraction, and multiplication problems. The number of correct problems across all pages of the test is the arithmetic fluency score. In the population of undergraduate students, the mean fluency score on this test is 80 out of a possible 240 ($SD = 25$; LeFevre, Smith-Chant, Hiscock, Daley, & Morris, 2003). Participants who score above the mean are assumed to be above average in arithmetic fluency (i.e., high-skill), whereas participants who score below the mean are assumed to be below average (i.e., low-skill). In the present research, this test was used to assess arithmetic skill. The test provides a reasonable measure of skill because a variety of complex arithmetic operations are performed, all of which require working memory resources to solve.

The Importance of Problem Difficulty

It was hypothesized that a multi-digit addition problem must be difficult enough for there to be an effect of orientation or skill on the addition task performed. Kalamian and LeFevre (2004) reported no significant differences in errors or response times for no-carry problems across exact and approximate addition. Thus, it appears as though no-carry exact and approximate addition problems require similar amounts of resources from the central executive. The verbal reports that Kalamian and LeFevre reported in Experiment 2 showed that the tens digits were added for both exact and approximate addition tasks. For no-carry and carry approximate addition problems, the units column

was rounded. For no-carry and carry exact addition problems, the units column was added. Researchers have found that a carry operation in the units column makes a multi-digit addition problem more sensitive to central executive interference than no-carry problems (DeStefano & LeFevre, 2004). Thus, exact addition problems that involve a carry should be difficult enough to permit effects of orientation or skill. Further, the larger rounding requirements for approximate addition carry problems will make the task difficult enough to permit effects of orientation or skill.

Summary of Hypotheses and Present Studies

The primary goal of the thesis is to investigate the effects of orientation and skill on adults' solutions to multi-digit exact and approximate addition problems. Main effects of orientation, skill, and task were expected, such that people would solve vertical problems more quickly and accurately than horizontal problems (Trbovich & LeFevre, 2003), more-skilled individuals would solve problems more quickly and accurately than less-skilled individuals (Geary & Widaman, 1992), and people would solve approximate problems more quickly and accurately than exact problems (Kalamian & LeFevre, 2004). Four interactions were predicted. First, it was predicted that the orientation of an addition problem would interact with arithmetic task, such that exact addition performance would be worse for horizontally presented problems (because horizontal problems and exact arithmetic both require phonological resources) whereas approximate addition performance would be worse for vertically presented problems (because vertical problems and approximate arithmetic both require visual-spatial resources). Second, it was predicted that skill and task would interact, such that differences in the performance of exact and approximate addition problems would be greatest for low-skill participants

because of their small working memory capacity compared to high-skill participants. Third, it was predicted that difficulty would interact with (1) orientation and task and (2) skill and task, because carry problems require more central executive resources than no-carry problems. Thus, in both of these three-way interactions, differences were expected to be greater for carry than no-carry problems.

Two experiments were developed to test these hypotheses. Experiment 1 was similar to experiments in which a choice task was used to investigate differences between exact and approximate addition (e.g., Dehaene et al., 1999; Kalaman & LeFevre, 2004; Experiment 1). Participants solved two-digit addition problems horizontally and vertically by choosing between the correct answer and a wrong answer at various distances from the correct answer. It was assumed that the further away the distractor was from the correct answer, the less calculation would be required to solve the problem. Dehaene et al. (1999) and Kalaman and LeFevre (2004, Experiment 1) showed that close distractor problems (exact addition) were more difficult to solve than far distractor problems (approximate addition). Thus, it was assumed that the further away the distractor was from the correct answer, visual-spatial resources would be accessed to solve the problem. Further, it was assumed that the closer the distractor was to the correct answer, phonological resources would be accessed to solve the problem. Experiment 2 was identical to Experiment 1, with the exception that participants provided a verbal response to exact and approximate addition problems presented horizontally and vertically. Regardless of the type of response (i.e., choice or verbal production) similar patterns of results were expected, based on the assumption that differences in exact and

approximate arithmetic performance are at the calculation level and not the response level (Kalamian & LeFevre, 2004).

Experiment 1

Participants solved two-digit addition problems (e.g., $22 + 12$) by choosing the correct answer from two alternatives presented on a computer screen. The two alternatives were a correct answer and a distractor at various distances from the correct answer (e.g., $22 + 12 = 34$ or 38 vs. $22 + 12 = 34$ or 44).

Method

Participants

Twenty-four students (12 males and 12 females) at Carleton University participated in the experiment. Participants ranged in age from 18 to 31 years, with a median age of 20 years. Participants were recruited through posted announcements. Nineteen participants were Introductory Psychology students and received two course credits in exchange for their participation. Five participants were graduate students and received no compensation for their participation.

Apparatus

The addition problems were presented on two 15-inch color monitors controlled by an IBM Intel Pentium III computer, with a central processing unit speed of 866 MHz. A wooden screen measuring 127 cm x 97 cm separated the experimenter's monitor from the participants' monitor. Participants were seated approximately 55 cm in front of the computer monitor. A black cardboard screen surrounded the participants' monitor and provided a viewing area measuring 20 cm x 15 cm. The experiment data set was programmed in E-Prime Version 1.0. Responses to addition problems were recorded

using the Psychology Software Tools' Serial Response Box, with a timing accuracy of 1.25 ms.

Materials

Addition problems. There were two sets of 40 two-digit addition problems (e.g., $22 + 12$; see Appendix A for a listing of all problems). Both sets were identical, with the exception of operand presentation order. For any addition problem, if the larger operand came first in Set A (e.g., $22 + 12$), it was preceded by the smaller operand in Set B (e.g., $12 + 22$). Similarly, if the smaller operand came first in Set A (e.g., $17 + 21$), it was preceded by the larger operand in Set B (e.g., $21 + 17$). Further, half of the problems in each set were presented with the smaller operand first (e.g., $15 + 17$) and half were presented with the larger operand first (e.g., $22 + 12$). None of the operands contained a zero in the units column (e.g., $20 + 38$). Half of the problems in each set involved a unit carry (e.g., $18 + 27$) and half did not (e.g., $22 + 12$). Carry problems consisted of 20 addition combinations of the unit numbers two through nine (i.e., $9 + 2, 8 + 3, 7 + 4, 6 + 5, 6 + 6, 7 + 5, 8 + 4, 9 + 3, 6 + 7, 8 + 5, 9 + 4, 7 + 7, 8 + 6, 9 + 5, 9 + 6, 8 + 7, 9 + 7, 8 + 8, 8 + 9, 9 + 9$). No-carry problems consisted of 20 addition combinations of the unit numbers one through eight (i.e., $1 + 1, 2 + 1, 2 + 2, 3 + 1, 1 + 4, 2 + 3, 1 + 5, 2 + 4, 3 + 3, 1 + 6, 2 + 5, 3 + 4, 1 + 7, 2 + 6, 3 + 5, 4 + 4, 1 + 8, 2 + 7, 3 + 6, 4 + 5$). Half of the sums (i.e., correct answers) in each set were odd (e.g., 45) and half were even (e.g., 34). Each problem had three distractors (i.e., wrong answers) which were close (i.e., 2 or 4 digits), medium (i.e., 10, 12 or 14 digits), or far (i.e., 20, 22, or 24 digits) in distance from the correct answer. Half of the distractors were larger than the correct answer (e.g., $22 + 12$

= 34 vs. 38), and half were smaller (e.g., $17 + 21 = 38$ vs. 36). All distractors and correct answers were two-digit numbers.

Strategy description task. Following the final experimental trial, participants viewed 12 strategy description screens on the monitor (see Appendix B). The strategy description screens and the order of their presentation were identical across participants. For each screen, participants saw two problems from the experiment. Upon viewing the problems, the experimenter said to participants, "How did you solve problems like these?" The participants spoke their responses aloud. Responses were recorded using Microsoft Windows XP Sound Recorder software on an IBM Intel Pentium IV notebook computer. The experimenter typed notes on the notebook computer while the participants spoke. Half of the screens involved carry problems, and half involved no-carry problems. Half of the screens involved horizontally presented problems, and half involved vertically presented problems. Close, medium, and far distractor problems were presented in each of the conditions.

Arithmetic fluency test. After participants completed the strategy description task, they completed a paper and pencil arithmetic fluency test. In the fluency test, participants solved multi-digit arithmetic problems (addition, subtraction, and multiplication) as quickly and accurately as possible.

Math background questionnaire. After participants completed the arithmetic fluency test, they completed the Math Background and Interests Questionnaire (see Appendix C). The questionnaire asked participants about their background, experiences, attitudes, and beliefs about mathematics. The questionnaire also asked participants

whether they had experienced any head injuries (e.g., concussion) or neurological disorders (e.g., seizure).

Design

Participants completed 240 trials. Participants solved every addition problem (80 in total) three times. Participants received two groups (i.e., Set A problems and Set B problems) of 120 trials. For every 120 trials, participants solved each addition problem 3 times (i.e., with all possible distractor situations – close, medium, and far). The two groups of trials were counterbalanced across participants based on order of orientation. Half of the participants ($N = 12$) solved the problems horizontally, followed by solving the same problems vertically. The other 12 participants solved the problems in reverse order (vertical followed by horizontal). Participants were randomly assigned to one of four order groups such that orientation order was counterbalanced with problem sets. Within each group of 120 trials, all addition problems (i.e., carry and no-carry problems) with their possible solutions (i.e., correct answers and distractors) were randomly mixed for each participant.

Procedure

Each participant was tested individually in a quiet, well-lit room with fluorescent lighting. Participants first completed the experimental arithmetic task. Every trial began with a star (e.g., *) that remained on the screen for 1000 ms. Following the star, an addition problem appeared centered on the computer screen with two possible answers below. One possible answer was to the left of center, and the other possible answer was to the right of center. The correct answer was equally located to the left or right of center across all experimental trials. Before participants solved each group of 120 experimental

trials, they received 24 practice trials. Participants were provided with feedback (i.e., correct/incorrect) for the 24 practice trials but not for the 240 experimental trials.

The instructions for every trial in the experiment were the same. Participants were told to place the fingers of their right hand on the serial response box. The index and middle finger of each participant was used to press one of two marked buttons (A, B) on the serial response box. Participants were told to solve all of the problems as quickly and as accurately as possible. Participants were told to press the A button if they felt the answer to the left of center was correct, or press the B button if they felt the answer to the right of center was correct. Once the button was pushed, a 1000 ms delay occurred, followed by the star, followed by a 1000 ms delay, and finally followed by the next problem.

After participants solved all of the problems presented on the computer screen, they completed the strategy description task, the arithmetic fluency test, the Math Background and Interests Questionnaire, and received a written debriefing. For the arithmetic fluency test, participants were given 2 minutes to complete each page of the test. Two of the pages had addition problems, and the other two pages had a mixture of subtraction and multiplication problems. Participants were told to solve all of the problems on the arithmetic fluency test as quickly and as accurately as possible. The experimenter used a stop watch to time each page of the test. Participants were not told their score on the test. The experimental session lasted for approximately 75 minutes.

Results

Arithmetic Fluency

Arithmetic fluency was scored as the total number of problems solved correctly

on the paper and pencil test. In similar samples of undergraduates, the expected mean and standard deviation for the fluency test is 80 and 25, respectively (LeFevre et al., 2003). The mean score on the arithmetic fluency test was 73 ($SD = 35.5$), which did not significantly differ from the expected mean for this population (i.e., 80), $t(23) = -.97, p > .05$. Despite the lack of mean difference, the majority of participants scored below 80 and thus would be considered less-skilled. Participants were divided into low and high skill groups, based on the expected mean. The eight high-skill participants scored 80 or greater, whereas the 16 low-skill participants scored below 80 (see Table 1).

Table 1
Skill Descriptive Statistics for Experiments 1 and 2: Low-Skill Participants Scored Below 80 and High-Skill Participants Scored Above 80 on the Fluency Test

Experiment	Mean		Standard deviation		Minimum score		Maximum score	
	Low	High	Low	High	Low	High	Low	High
1	55.0	109.0	16.6	36.5	22.0	83.0	79.0	190.0
2	54.9	98.6	15.7	7.5	33.0	87.0	79.0	109.0

Math Background and Interests Questionnaire Descriptive Statistics

Sixteen participants reported receiving their elementary school education in Canada and speaking English as their first language. Five participants reported receiving their elementary school education in a country other than Canada (China, Sri Lanka, Turkey, Romania and Panama) and speaking a language other than English as their first language (Cantonese, Tamil, Turkish, Romanian, Spanish). Three participants reported receiving their elementary school education in Canada and speaking Somali as their first language. Hence, for most participants, the acquisition of exact arithmetic skills was

presumably in English. The mean rating for basic arithmetic skill was 5 (moderate) ($SD = 1.5$). This rating is in agreement with the mean on the arithmetic fluency test, such that most participants responded that their basic arithmetic skills were average.

Strategy Descriptions

Twenty-three participants reported that they used a single strategy to solve all problems (i.e., carry and no-carry). The strategy involved adding the units digits first, followed by examining the answer alternatives. For example, for the problem $22 + 12$, the sum of the units digits is $2 + 2 = 4$. If the units digits in the answer alternatives were different (e.g., 34 and 38), participants chose an answer based on comparing the units digits in the answer alternatives to the result of the units addition. If the answer alternatives had the same units digits (e.g., 34 and 54), then participants added the tens digits and chose an answer based on comparing the tens digits in the answer alternatives to the result of the tens addition (e.g., $20 + 10 = 30$; choose the answer 34). Although this was an efficient and useful strategy, it was not what was intended for the distractor manipulation.

For carry problems, participants noted whether a carry operation was performed or not. For medium and far distractor problems, 20 and 12 participants respectively reported adding a unit carry to the tens digits after determining the tens digits would have to be added because of identical units digits in the answer alternatives. For medium distractor problems, one participant transformed one operand for each carry problem into a decade operand (e.g., $16 + 17$ became $13 + 20$). For far distractor problems, 11 participants chose an answer based on comparing operands to the answer alternatives. For example, if an answer alternative was less than an operand (e.g., $14 + 39 = 33$ vs. 53),

the plausible alternative was chosen.

Seven participants reported that vertically presented problems were easier to solve than horizontally presented problems because the columnar alignment of digits made calculation easier than horizontally presented problems. One participant solved all problems by adding both numbers, and then examined the right most answer alternative. If the right most answer alternative was the correct answer, the participant pressed the right response button, otherwise the participant pressed the left response button. Thus, this participant never directly compared the answer alternatives.

From the strategy descriptions, it is clear that participants did not execute a complete two-digit plus two-digit addition algorithm on most problems (e.g., addition of units digits followed by addition of tens digits). This strategy was only reported consistently by participants on the medium distractor problems that had correct units digits. Nonetheless, participants typically added at least two numbers (e.g., addition of units digits only). As described below, these strategy descriptions are consistent with the finding that close distractor problems were the easiest to calculate because they required the addition of the units digits only.

Response Times and Percentage of Errors

Participants completed a total of 5760 trials. Only trials for which participants chose the correct answer were included in the analysis of response times ($n = 5622$ trials). Response times and percentage of errors were analyzed using separate 2 (orientation: horizontal, vertical), $\times$ 2 (difficulty: carry, no-carry) $\times$ 3 (distractor: close, medium, far) $\times$ 2 (skill: low, high) ANOVAs, with repeated measures on the first three factors. The results for both dependent variables are discussed together. Note that, for both

experiments, error bars shown in all figures are the 95% confidence intervals based on the mean-square error values for the displayed interaction (Masson & Loftus, 2003).

All four main effects were significant for response times and two were significant for percentage of errors. First, low-skill participants solved problems more slowly than high-skill participants (2834 vs. 1704 ms), $F(1, 22) = 9.78$, $MSE = 8365659$, $p < .05$. This result was expected, given that the fluency test is an index of the speed of multi-digit calculations. Second, consistent with Trbovich and LeFevre (2003), participants solved horizontally presented problems more slowly than vertically presented problems (2385 vs. 2153 ms), $F(1, 22) = 4.50$, $MSE = 766667$, $p < .05$. Third, consistent with other research on multi-digit addition (e.g., Fürst & Hitch, 2000; Heathcote, 1994, Experiment 1), participants solved carry problems more slowly than no-carry problems (2547 vs. 1992 ms), $F(1, 22) = 35.56$, $MSE = 554555$, $p < .05$ and they made more errors on carry than no-carry problems (3% vs. 2%), $F(1, 22) = 8.41$, $MSE = 10$, $p < .05$. Fourth, as shown in Figure 2, participants solved medium and far distractor problems more slowly than close distractor problems, $F(2, 44) = 45.89$, $MSE = 238755$, $p < .05$, $CI = 195$ ms. This finding is consistent with the strategy descriptions, because participants reported using more calculation for medium distractor problems than close distractor problems. Participants made more errors on medium than on either close or far distractor problems, $F(2, 44) = 14.41$, $MSE = 20$, $p < .05$, $CI = 2\%$, suggesting that medium distractor problems were the most difficult to solve.

Figure 2. Experiment 1. Main effect of Distractor.

Two interactions were significant for response times. First, skill interacted with difficulty, $F(1, 22) = 15.74$, $MSE = 554555$, $CI = 315$ ms, $p < .05$. As shown in the left panel of Figure 3, low-skill participants solved carry problems more slowly than no-carry problems. For high-skill participants, in contrast, the difference between no-carry and carry problems was not significant. Thus, the presence of a carry in the units digits affected low-skill participants more than high-skill participants. Carry problems consume more working memory resources than no-carry problems (Kalamian & LeFevre, 2004). This result suggests that low-skill participants use more working memory resources to solve carry problems than no-carry problems. In contrast, high-skill participants use similar working memory resources to solve no-carry and carry problems.

Figure 3. Experiment 1. Interaction between difficulty and skill.

Second, the interaction between distractor and difficulty was significant, $F(2, 44) = 6.40$, $MSE = 160285$, $CI = 160$ ms, $p < .05$. As shown in Figure 4, there was an effect of difficulty for close, medium, and far distractor problems, but the effect of difficulty was greatest for medium distractor problems. This interaction is consistent with the strategy descriptions. For close distractor problems, participants did not need to add a carry to the tens digits because addition of the units digits resulted in participants scanning the units digits in the response alternatives for the correct answer. For medium distractor problems (e.g., $28 + 15$), when the response alternatives had identical units digits and the tens digits did not (43 vs. 53), participants reported adding a unit carry to the tens digits. For far distractor problems (e.g., $14 + 39$), when the response alternatives had identical units digits and the tens digits did not (e.g., 33 vs. 53), participants either

added a unit carry to the tens digits or made a plausibility judgement between operands and answer alternatives (i.e., 33 is an incorrect answer because it is less than 39). Thus, response times were slowest for medium distractor carry problems because more calculation occurred in this condition than close or far carry conditions.

Figure 4. Experiment 1. Interaction between difficulty and distractor.

Response Times and Percentage of Errors Excluding Close Distractor Problems

Close distractor problems were the easiest to solve because participants added the units digits only, scanned the units digits of the response alternatives (the tens digits were the same), and selected the appropriate answer. It was assumed that close distractor problems would be the most difficult to solve because the small distance between the correct and incorrect answers should have resulted in more calculation compared to

medium and far distractor problems. Thus, close distractor problems are problematic when considering the analysis of response times and percentage of errors because not only were they easy to solve, but also were inconsistent with what was assumed. In order to determine whether significant differences existed between the medium and far distractor problems, separate ANOVAs were performed. Response times and percentage of errors were analyzed using separate 2 (orientation: horizontal, vertical), x 2 (difficulty: carry, no-carry) x 2 (distractor: medium, far) x 2 (skill: low, high) ANOVAs, with repeated measures on the first three factors.

Consistent with the analysis that included close distractor problems, the main effects of difficulty, distractor, and skill were significant for response times and difficulty and distractor effects were significant for percentage of errors. First, participants solved carry problems more slowly than no-carry problems (2780 vs. 2162 ms), $F(1, 22) = 34.28$, $MSE = 475808$, $p < .05$ and they made more errors on carry than on no-carry problems (4% vs. 2%), $F(1, 22) = 6.76$, $MSE = 16$, $p < .05$. Second, participants solved medium distractor problems more slowly than far distractor problems (2550 vs. 2392 ms), $F(1, 22) = 5.04$, $MSE = 209851$, $p < .05$ and they made more errors on medium than far distractor problems (4% vs. 2%), $F(1, 22) = 12.28$, $MSE = 28$, $p < .05$. Thus, far distractor problems were easier than medium distractor problems. Third, low-skill participants solved problems more slowly than high-skill participants (3077 vs. 1865 ms), $F(1, 22) = 10.37$, $MSE = 6038885$, $p < .05$.

Consistent with the analysis that included close distractor problems, there were two significant interactions for response times and no significant interactions for percentage of errors. As shown in the right panel of Figure 3, the interaction between

skill and difficulty was significant, $F(1, 22) = 13.22$, $MSE = 475808$, $CI = 292$ ms, $p < .05$. As shown in Figure 4, the interaction between distractor and difficulty was significant, $F(1, 22) = 11.32$, $MSE = 136101$, $CI = 156$, $p < .05$. There was a greater effect of difficulty for medium distractor problems than far distractor problems. Thus, medium distractor problems were more difficult than far distractor problems.

Discussion

In Experiment 1, the effects of orientation and skill on the performance of problems requiring more versus less calculation were investigated. Participants chose the correct answer from two alternatives. Three distractor situations were assessed. It was assumed that close distractor problems would involve more calculation than medium or far distractor problems. Further, far distractor problems were assumed to involve less calculation than medium distractor problems. The experiment was designed to test three hypotheses. First, it was predicted that close distractor problems would be most difficult to solve when presented horizontally because both horizontal orientation and close distractor problems were assumed to compete for the same phonological resources. In contrast, it was predicted that far distractor problems would be most difficult to solve when presented vertically because both vertical orientation and far distractor problems were assumed to compete for the same visual-spatial resources. This hypothesis was not supported. Although horizontal problems were more difficult than vertical problems (at least in the analysis that included close distractors), replicating previous research, type of task (as indexed by distractor) did not interact with orientation.

The second hypothesis was that skill would interact with task (i.e., type of distractor), such that there would be greater differences in performance between low- and

high-skill participants for close distractor problems than medium or far distractor problems. Again, this hypothesis was not supported, even when a comparison was made only between medium and far distractor problems. Medium distractor problems (which are assumed to require the most exact calculations in this experiment) were solved more slowly and less accurately than far distractor problems (which are assumed to allow for some approximation), but the effect was not differential for high- and low-skill individuals.

Finally, two three-way interactions were predicted, such that the interactions of (1) orientation and task and (2) skill and task would be qualified by the three-way interaction of these variables with difficulty. Although carry problems were more difficult than no-carry problems, the difficulty variable did not interact with the above mentioned two-way interactions. However, difficulty interacted with task, and difficulty interacted with skill. In the interaction between difficulty and task, the greatest differences between no-carry and carry problems occurred for medium distractor problems. Kalamian and LeFevre (2004) also found that difficulty interacted with task in their choice task experiment. They found that the greatest effect of difficulty was for problems with distractors of 10 digits than distractors of 30 digits. Thus, it appears that greater calculation requirements for arithmetic tasks result in greater differences between no-carry and carry problems, presumably because of greater demands on the central executive. In the interaction between skill and task, low-skill participants found carry problems more difficult to solve than no-carry problems. Research has shown that people low in arithmetic skill have weaker central executive functioning (Bull & Scerif, 2001). Thus, low-skill participants found carry problems more difficult to solve than no-carry

problems because their central executive functioning was not as efficient as high-skill participants in processing the carry requirement.

Contrary to the prediction that participants would solve close distractor problems more slowly and less accurately than medium or far distractor problems, participants solved close distractor problems more quickly and accurately than medium or far distractor problems. Close distractor problems were easy because participants adopted an efficient strategy: They added the units digits of each operand, examined the units digits of each answer, and selected a response. This strategy reduced calculation. Although some attempts had been made in designing the stimuli to avoid the use of an efficient strategy (e.g., half of the medium and far distractor problems had response alternatives with the same units digits), the participants nevertheless deduced and adopted the most efficient strategy.

The use of an efficient strategy by participants and methodological limitations may have resulted in inadequate testing of the hypotheses. Although participants solved horizontally presented problems more slowly than vertically presented problems, as was found in other research (Heathcote, 1994, Experiment 2; Trbovich & LeFevre, 2003), this effect was not differential across close, medium, or far distractor problems. Seven participants reported that the columnar alignment of vertical problems made them easier to solve than horizontal problems, independent of orientation. Thus, regardless of the type of arithmetic task (i.e., close vs. far distractor problems), vertical problems will always be easier to solve than horizontal problems because the vertical alignment of operands may foster faster processing than horizontal alignment. The faster processing

of vertical problems could occur because of familiarity in solving arithmetic problems in columns through early schooling.

Although low-skill participants solved problems more slowly than high-skill participants, this effect was not differential across distractor problems. Low-skill participants may find any type of calculation more difficult than high-skill participants because of reduced working memory capacity compared to high-skill participants (Geary & Widaman, 1992). This reduced working memory capacity is evident by greater performance decrements across carry problems for low- than high-skill participants.

Experiment 2 was designed to overcome the limitations of Experiment 1. In Experiment 2, participants provided verbal responses to exact and approximate two-digit addition problems (following Kalamian & LeFevre, 2004). The production task was assumed to force participants to develop strategies unique to exact and approximate addition, without any unwanted heuristics that were specific to the characteristics of the problem categories. Participants produced either an exact answer (e.g., $12 + 22 = 34$) or an answer that was the closest decade to the exact answer (e.g., $12 + 22 \approx 30$). In all other respects, Experiment 2 was identical to Experiment 1.

Experiment 2

Participants solved the same two-digit addition problems as in Experiment 1. In contrast to Experiment 1, however, participants produced either exact or approximate answers to the addition problems.

Method

Participants

Twenty-four students (12 males and 12 females) at Carleton University participated in the experiment. Participants ranged in age from 18 to 35 years, with a median age of 21 years. None of the participants had participated in Experiment 1. Participants were recruited through announcements. Nineteen participants were Introductory Psychology students and received two course credits in exchange for their participation. Five participants were graduate students and received no compensation for their participation.

Apparatus

The apparatus was identical to Experiment 1 with the following changes. In contrast to Experiment 1 where participants' responses were recorded by a serial response box, in Experiment 2, the experimenter recorded responses to addition problems via an IBM compatible keyboard. Participants wore a headset with an attached microphone. The experimenter told participants to place the microphone close to their lips and to speak clearly and loudly. The microphone was connected to a Psychology Software Tools' Serial Response Box, with a timing accuracy of 1.25 ms.

Materials, Design and Procedure

The materials, design, and procedure were identical to Experiment 1, with the following changes. The distractors were not required for Experiment 2. Participants completed 160 experimental trials (vs. 240 experimental trials in Experiment 1). Before participants solved each group (horizontal or vertical) of 80 experimental trials, they received 16 practice trials. Participants were provided with feedback (i.e.,

correct/incorrect) for the practice trials but not for the experimental trials. For each group of 80 experimental trials, participants solved each addition problem twice (i.e., once with an exact answer and once with an approximate answer). Similar to Experiment 1, within each group of 80 experimental trials, all addition problems (i.e., carry and no-carry problems) with their possible solution productions (i.e., exact and approximate answers) were randomly mixed for each participant. Following the final experimental trial, participants viewed 8 strategy description screens (12 strategy description screens in Experiment 1) on the monitor (see Appendix D).

Every trial began with a word (exact or approximate) that remained on the screen for 3000 ms. Following the word, a star (e.g., *) appeared centered on the computer screen for 1000 ms. Following the star, an addition problem appeared centered on the computer screen. If the word said exact, the experimenter instructed participants to speak a precise answer (e.g., $22 + 12 = 34$). If the word said approximate, the experimenter instructed participants to speak an answer to the nearest decade of the exact answer using minimal calculation (e.g., $22 + 12 \approx 30$). Further, for approximate calculations, the experimenter instructed participants to avoid using the following strategy: Adding both operands followed by rounding the sum to the nearest decade of the exact answer (e.g., $22 + 12 = 34$ which rounds to 30). After participants spoke their answer into the microphone, the experimenter recorded the response on a keyboard (followed by a 1000 ms delay), and the word appeared signaling the next trial.

Results

Arithmetic Fluency

The mean score on the arithmetic fluency test was 69.5 ($SD = 24.6$). Participants' fluency scores differed significantly from the expected mean for fluency (i.e., 80) in this population, $t(23) = -2.10, p < .05$. There were no differences in fluency scores across experiments however, $t(46) = .40, p > .05$. As in Experiment 1, eight high-skill participants scored 80 or greater, whereas 16 low-skill participants scored below 80 (see Table 1).

Math Background and Interests Questionnaire Descriptive Statistics

Fifteen participants (vs. 19 participants in Experiment 1) reported receiving their elementary school education in Canada. Of these 15 participants, 13 participants reported English as their first speaking language, with 2 participants who reported a language other than English as their first speaking language (Chinese and French). Nine participants (vs. 5 participants in Experiment 1) reported receiving their elementary school education in a country other than Canada (United States of America, China, Germany, Trinidad, Korea, Scotland, Saudi Arabia, Albania). Of these 9 participants, 5 participants reported English as their first speaking language, with 4 participants who reported a language other than English as their first speaking language (Chinese, Korean, German, Albanian). Consistent with Experiment 1, for most participants, the acquisition of exact arithmetic skills was presumably in English. Further, consistent with Experiment 1, the mean rating for basic arithmetic skill was 5 ($SD = 1.4$). Thus, these descriptive statistics suggest that participants from Experiment 1 and Experiment 2 were similar in their mathematical abilities.

Strategy Descriptions

Participants reported strategies consistent with Kalamian and LeFevre (2004, Experiment 2). As shown in Table 2, on exact addition problems, participants reported using one of four strategies. Thirteen participants reported the right-to-left columnar algorithm: adding the units digits first followed by adding the tens digits (e.g., for the problem $39 + 14$, they added $9 + 4 = 13$, then $30 + 10 = 40$, and $40 + 3 = 53$; interim results are shown in italics). Nine participants reported a left-to-right algorithm: adding the tens digits first followed by adding the units digits (e.g., $30 + 10 = 40$, $9 + 4 = 13$, $40 + 13 = 53$). One participant reported adding the larger of the operands first followed by adding the decade of the smaller operand and finally adding the units of the smaller operand (e.g., $39 + 10 = 49$, $49 + 4 = 53$). One participant reported changing strategies depending on the orientation of the problem (horizontal - addition of tens first followed by addition of units, vertical - addition of units first followed by addition of tens). As can be seen from Table 2, the majority of high- (62.5%) and low- (50%) skill participants used the standard right-to-left algorithm. Presumably this was the most common approach because people learn to solve arithmetic problems in columns through schooling.

As shown in Table 3, participants reported using one of six strategies on approximate problems (e.g., $39 + 14$). Ten participants reported rounding both operands and then adding them (e.g., $40 + 10 = 50$). Eight participants reported adding the tens digits first followed by performing a rounding operation on the units digits and adjusting the answer if the sum of the units digits was five or greater (e.g., $30 + 10 = 40$, $9 + 4 > 5$ therefore $40 + 10 = 50$). Two participants reported adding the units digits first followed

by adding the tens digits (e.g., $9 + 4 > 5$, $10 + 30 + 10$ answer 50). One participant reported adding a number between the two operands twice and rounding the sum to the nearest decade (e.g., $26 + 26 = 52$, 52 rounds to 50). Two participants reported adding the tens digits first followed by scanning the units digits of the operands to determine whether an additional 10 or 20 digits would be added. If none of the operands had a unit number five or larger, only the tens were added. If one of the operands had a unit number five or larger, an additional 10 digits were added to the tens. If two of the operands had a unit number five or larger, an additional 20 digits were added to the tens. One participant reported changing strategies depending on the orientation of the problem (horizontal - rounding both operands and adding them, vertical - adding tens, followed by a rounding operation on units). As can be seen from Table 3, regardless of skill, approximate addition problems were solved with strategies that involved a combination of rounding and adding. Thus, for both exact and approximate addition problems, regardless of skill, participants performed similar computations (i.e., precise addition for exact addition, simplification for approximate addition).

In summary, the differences between exact and approximate strategies were that (a) fewer interim results needed to be maintained during subsequent calculation, and (b) fewer calculations were required. Further to the strategies reported by participants, they also made the following comments. For exact addition only, four participants (3 low-skill, 1 high-skill) reported that they visually aligned horizontal problems vertically. Four other low-skill participants commented that vertical problems were easier to solve than horizontal problems because of the columnar alignment of vertical problems.

Participants did not report using different strategies to solve problems based on

orientation (with only 2 exceptions). Thus, orientation did not appear to influence strategy selection, even though the latencies suggest that it influenced strategy execution (see below). In contrast, task influenced strategy selection in that participants simplified the approximate problems before calculating. Task, therefore, influenced both strategy selection and strategy execution.

Table 2
Experiment 2: Number of Participants at Each Skill Level Who Reported Each of Three Exact Addition Strategies

Strategies	<i>Skill</i>	
	Low	High
Units addition followed by tens addition	8	5
Tens addition followed by units addition	7	2
Larger operand + ten of smaller operand + unit of smaller operand	1	0
Horizontal: Tens addition followed by units addition	0	1
Vertical: Units addition followed by tens addition		

Table 3
Experiment 2: Number of Participants at
Each Skill Level Who Reported Each of Six Approximate Addition Strategies

Strategies	<i>Skill</i>	
	Low	High
Round both operands then add	8	2
Tens addition followed by rounding based on sum of units	3	5
Tens addition followed by rounding based on examination of operands for units digits 5 or larger	2	0
Units addition, followed by rounding based on sum of units, followed by addition of tens	1	1
Adding a number between both operands twice and rounding sum	1	0
Horizontal: Round both operands then add: Vertical: Tens addition followed by rounding based on sum of units	1	0

Response Times and Percentage of Errors

Participants completed a total of 3840 trials. Seven trials (0.18%) were excluded from the analysis because of extraneous vocalizations (e.g., coughing). Only correct trials were included in the analysis of response times ($n = 3514$). For exact addition problems, a trial was scored as correct if participants provided the exact answer to the addition problems (e.g., $39 + 14 = 53$). For approximate addition problems, a trial was scored as correct if participants provided an approximate answer within 10 of the exact answer (e.g., for $39 + 14 = 53$, acceptable approximations were 50 and 60). Response times and percentage of errors were analyzed using separate 2 (orientation: horizontal, vertical), $\times$ 2 (difficulty: carry, no-carry) $\times$ 2 (task: exact, approximate) $\times$ 2 (skill: low,

high) ANOVAs, with repeated measures on the first three factors. The results for both dependent variables are discussed together.

There were two significant main effects for response times and one significant main effect for percentage of errors. As in Experiment 1, participants solved carry problems more slowly than no-carry problems (2995 vs. 2316 ms), $F(1, 22) = 41.89$, $MSE = 469713$, $p < .05$ and they made more errors on carry than no-carry problems (11% vs. 6%), $F(1, 22) = 18.28$, $MSE = 68$, $p < .05$. The difference in errors between carry and no-carry problems was larger in Experiment 2 than Experiment 1 (5% vs. 1%). Thus, the choice task in Experiment 1 was easier than the production task in Experiment 2. Carry effects are expected because carry problems require more working memory resources than no-carry problems (DeStefano & LeFevre, 2004). Further, as in Experiment 1, and consistent with Trbovich and LeFevre (2003), participants solved horizontally presented problems more slowly than vertically presented problems (2825 vs. 2486 ms), $F(1, 22) = 26.28$, $MSE = 186933$, $p < .05$. The difference in response times between horizontal and vertical problems was larger in Experiment 2 than Experiment 1 (339 vs. 232 ms), providing further support that the production task was more difficult than the choice task.

There was one significant interaction for response times and three significant interactions for percentage of errors. Consistent with Kalaman and LeFevre (2004, Experiment 2), the interaction between difficulty and task was significant for both response times, $F(1, 22) = 15.84$, $MSE = 425415$, $CI = 276$ ms, $p < .05$, and percentage of errors, $F(1, 22) = 8.58$, $MSE = 52$, $CI = 3\%$, $p < .05$. As shown in Figure 5, response times and errors were not different for exact and approximate problems on no-carry problems. On carry problems, however, participants were significantly slower and more

error prone in the exact than in the approximate condition. Thus, exact addition was more difficult than approximate addition only when a carry operation was required. These results support the strategy descriptions because participants reported precise calculations for exact addition versus rounding for approximate addition.

Figure 5. Experiment 2. Interaction between difficulty and task.

The interaction between orientation and skill was significant for percentage of errors only, $F(1, 22) = 5.31$, $MSE = 42$, $CI = 3\%$, $p < .05$. As shown in Figure 6, low-skill participants solved both horizontal and vertical problems more slowly than high-skill participants. In contrast, low-skill participants made more errors than high-skill participants on vertical problems. Thus, only high-skill participants showed an advantage in errors on vertical problems, despite both groups showing a latency advantage. The

pattern of results for high-skill participants is consistent with Trbovich and LeFevre (2003). High-skill participants may be less error prone on vertical than horizontal problems because of more developed central executive functions (e.g., switching between tasks) than low-skill participants.

Figure 6. Experiment 2. Interaction between orientation and task.

L = low-skill, H = high-skill.

As shown in Figure 7, the interaction of skill and orientation was qualified by the three-way interaction of these variables with difficulty (errors only), $F(1, 22) = 13.66$, $MSE = 25$, $CI = 2\%$, $p < .05$. Errors were not different for high- and low-skill participants on no-carry problems. In contrast, errors were significantly different for high- and low-skill participants on carry problems. Low-skill participants solved carry problems presented vertically more quickly than carry problems presented horizontally,

but did not show any differences across orientation for errors. In contrast, high-skill participants solved carry problems presented vertically more quickly and more accurately than carry problems presented horizontally. Carry problems are more demanding of working memory resources than no-carry problems (DeStefano & LeFevre, 2004; Kalaman & LeFevre, 2004). Thus, the pattern of results shown by low-skill participants may reflect less efficient working memory for low- than high-skill participants.

Figure 7. Interaction of difficulty, orientation, and skill.

H = horizontal, V = vertical.

Discussion

In Experiment 2, the effects of orientation and skill on the performance of problems requiring exact (e.g., $24 + 32 = 56$) versus approximate calculations (e.g., $24 + 32 \approx 60$) were investigated. Participants provided verbal responses to multi-digit addition problems. Three hypotheses were made. First, an interaction between orientation and task was predicted, such that exact addition problems would be most difficult to solve when presented horizontally, whereas approximate addition problems would be most difficult to solve when presented vertically. This hypothesis was based on the assumption that horizontally presented problems would compete with exact addition for phonological resources, and vertically presented problems would compete with approximate addition for visual-spatial resources. Second, an interaction between skill and task was predicted, such that greater performance differences between exact and approximate addition would occur for low-skill participants than high-skill participants. This hypothesis was based on research suggesting a relationship between low-skill in solving basic arithmetic problems and low working memory capacity (Bull & Scerif, 2001; Geary & Widaman, 1992). Third, it was predicted that the above two-way interactions would be qualified by the three-way interaction of these variables with difficulty. Thus, the effects of orientation or skill on exact and approximate addition were expected to be greatest for carry problems than no-carry problems because the former require more central executive resources than the latter (DeStefano & LeFevre, 2004).

None of the above hypotheses were supported. The random mixing of problems may have resulted in the weak manifestation of the task variable. Kalamian and LeFevre

(2004) presented participants with exact and approximate addition problems in separate blocks. The exact and approximate addition problems in the experiments for this thesis were randomly mixed. In comparing the interaction between difficulty and task in this experiment with the same interaction found in Kalaman and LeFevre, greater differences between exact and approximate addition problems that involved carrying were found in Kalaman and LeFevre than were found in this experiment. Thus, it would appear that the random mixing of problems reduced differences between exact and approximate addition problems by making exact and approximate calculations less distinct. For example, if a participant solved three consecutive approximate addition problems followed by an exact addition problem, a more simplified calculation for the exact addition problem could have occurred (e.g., $18 + 27 = 20 + 30 - 5$) because of prior exposure to approximate addition problems. Therefore, blocking exact and approximate addition trials may be more beneficial than randomly mixing trials in order to preserve the distinct calculations associated with both types of arithmetic.

General Discussion

The goal of the present research was to investigate the effects of orientation (i.e., horizontal vs. vertical problem presentation) and skill (i.e., arithmetic fluency) on the performance of exact (e.g., $23 + 23 = 46$) and approximate (e.g., $23 + 23 \approx 50$) multi-digit addition problems. Researchers investigating differences in exact and approximate arithmetic have not manipulated orientation or compared performance across skill groups (Dehaene et al., 1999; Kalaman & LeFevre, 2004; Spelke & Tsivkin, 2001). Prior research has shown that both orientation and skill manipulations tap working memory resource usage (Geary & Widaman, 1992; Trbovich & LeFevre, 2003). According to

Trbovich and LeFevre, horizontal problems require phonological resources and vertical problems require visual-spatial resources. According to Geary and Widaman, small working memory capacity is related to difficulty in solving arithmetic problems. Further, exact addition problems require greater working memory resources than approximate addition problems (Kalamian & LeFevre, 2004). Hence, it seemed reasonable to develop hypotheses regarding potential interactions between orientation, skill, and task manipulations because all three variables require working memory resources.

Three hypotheses were developed based on the resources shared by task, orientation, and skill in working memory. First, it was predicted that orientation and task would interact such that problems requiring precise calculations (i.e., close distractors, exact addition) would be most difficult to solve when presented horizontally because a competition for phonological resources would occur. In contrast, it was predicted that problems requiring simplification (i.e., far distractors, approximate addition) would be most difficult to solve when presented vertically because a competition for visual-spatial resources would occur. Second, it was predicted that skill and task would interact, such that greater performance differences between precise and simplified calculations would be seen in people with low-skill in arithmetic problem solving because of reduced working memory capacity compared to high-skill people. Third, it was predicted that the above two-way interactions would be qualified by the three-way interactions of these variables with difficulty because carry problems require more central executive resources than no-carry problems (DeStefano & LeFevre, 2004).

Two experiments to test these hypotheses were developed. In both experiments, participants solved two-digit (e.g., $16 + 26$) addition problems. In Experiment 1,

participants were shown an addition problem and responded by choosing the answer from two possible options (i.e., the correct answer and a wrong answer at various distances from the correct answer). It was assumed that wrong answers close to the correct answer (e.g., $16 + 26 = 42$ vs. 44) would require more calculation, whereas wrong answers far from the correct answer would require less calculation (e.g., $16 + 26 = 42$ vs. 62). In Experiment 2, participants solved two-digit addition problems by providing verbal responses. If the problem required an exact answer, the participant spoke the precise answer (e.g., $16 + 26 = 42$). If the problem required an approximate answer, the participant spoke an answer closest to the precise answer (e.g., $16 + 26 \approx 40$).

Limitations of the methodologies used resulted in weak operationalization of the task variable (i.e., exact and approximate addition). In Experiment 1, participants used a common strategy to solve most problems. Across most problems, participants added the units digits of each operand and matched the sum with the units digit in the response alternatives. This common strategy was not expected, and as a result, problems with wrong answers close to the correct answer were the easiest to solve because they required calculation of the units digits only. Thus, the intended manipulation of task was unsuccessful. As mentioned in Experiment 2, the random mixing of exact and approximate addition problems may have reduced differences between both types of arithmetic by making exact and approximate calculations less distinct from each other. Further, in contrast to Kalamian and LeFevre (2004) where participants saw each addition problem presented once, participants in the experiments for this thesis saw each addition problem presented more than once (i.e., 3 repetitions per problem in each set for Experiment 1 vs. 2 repetitions per problem in each set for Experiment 2). The multiple

exposure to problems may have increased the participants' familiarity in solving problems, thereby reducing differences between exact and approximate tasks. Thus, as a result of all of these limitations, the orientation and skill manipulations failed to differentially affect performance on exact and approximate multi-digit addition problems.

Even though the hypotheses were not supported, consistencies were found between this research and research investigating the role of the central executive in solving arithmetic problems. According to Kalamian and LeFevre (2004), arithmetic problems requiring more involved calculations (e.g., carry problems such as $23 + 48$) activate more central executive resources than problems requiring more simplification (e.g., approximate problems such as $23 + 48 \approx$). The source of these findings was further clarified by the strategy reports. Participants reported that they simplified the approximate problems before they performed any calculations, and essentially changed carry problems into no-carry problems. Hence, there were no latency or error effects across difficulty for approximate problems. Further, in Experiment 1, there were latency effects across difficulty for low-skill participants and not for high-skill participants, presumably because low-skill participants are more impaired by functions associated with the central executive (i.e., arithmetic carrying) than high-skill participants. Thus, this research provides more support for the role of the central executive in arithmetic problem solving.

A discussion of consistencies between experiments may provide insight into future experimentation regarding the effects of orientation and skill on the performance of exact and approximate addition problems. Regardless of the responses requested of participants (i.e., choose an answer vs. verbally produce an answer), horizontal problems

were more difficult to solve than vertical problems, and carry problems were more difficult to solve than no-carry problems. Orientation produces consistent results because people receive more exposure to solving problems vertically than horizontally in early schooling. The difficulty manipulation produces consistent results because carry problems consume more working memory resources than no-carry problems (DeStefano & LeFevre, 2004; Kalamian & LeFevre, 2004). Thus, the task manipulation should also produce consistent results, providing that the differences between exact and approximate addition are as well defined as the differences are within the levels of the orientation and difficulty manipulations.

Future research investigating the effects of orientation and skill on the performance of multi-digit arithmetic problems should take into consideration the limitations of the present research. More importantly, the task manipulation must be well defined in order to interact with variables such as orientation and skill. One possibility to increase differences in the performance of exact and approximate addition problems may be to present different groups of people with different tasks (e.g., group 1 receives exact addition trials only and group 2 receives approximate addition trials only). In presenting different groups of people with different arithmetic tasks, the distinction between exact and approximate arithmetic tasks can be preserved without calculations overlapping with each other (as may be the case when participants receive a combination of exact and approximate addition problems). If this approach is used for designing future experiments in this line of research, participants must be carefully selected such that their early schooling in arithmetic problem solving is similar. Matching groups based on arithmetic education would ensure that differences between exact and approximate

addition calculations can not be attributed to the demographic characteristics of groups of participants.

In summary, the effects of orientation and skill on the performance of exact and approximate addition problems remain uncertain. However, there are certain distinctions between horizontal and vertical problems, such that horizontal problems are more difficult to solve than vertical problems. Dehaene et al. (1999) claimed that exact arithmetic shows language dependence, whereas approximate arithmetic shows language independence. Future tasks defining exact and approximate addition must be distinct enough to allow for adequate testing of hypotheses related to Dehaene et al.'s claim. Before designing more novel exact and approximate addition tasks, future experiments should be created by using Dehaene et al.'s methodology for training bilingual people to solve exact and approximate addition problems. This will allow for effects of orientation and skill to be investigated with tasks that already support Dehaene et al.'s claim. In closing, it is important to realize that exact and approximate addition are different from one another. Every attempt must be made to preserve these differences in order to avoid uncertainty concerning the effects of orientation and skill on the performance of exact and approximate addition problems.

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Appendix A

Addition Problems for Experiment 1 and Experiment 2

For Experiment 1 only, the close, medium, and far distractors were used.

In Set B, for all addition problems, the presentation of operands was reversed (e.g., 22 + 12 became 12 + 22).

Set A

Problem	Answer	<i>Distractor</i>		
		Close	Medium	Far
22 + 12	34	38	44	58
17 + 21	38	36	26	18
21 + 16	37	39	47	59
18 + 21	39	35	25	19
13 + 31	44	48	58	64
33 + 13	46	44	34	26
15 + 32	47	49	57	69
37 + 12	49	45	39	29
31 + 25	56	58	68	76
22 + 36	58	54	44	38
31 + 24	55	59	65	79
26 + 33	59	57	47	39
21 + 41	62	66	76	82
43 + 25	68	64	58	44
24 + 43	67	69	79	87

44 + 25	69	67	59	47
42 + 34	76	78	88	96
34 + 44	78	76	68	56
42 + 31	73	77	83	97
33 + 42	75	71	61	51
15 + 17	32	36	46	52
19 + 17	36	32	26	12
12 + 19	31	33	43	51
17 + 16	33	31	23	11
26 + 16	42	44	54	62
19 + 29	48	46	38	26
28 + 15	43	47	53	67
18 + 27	45	41	31	21
14 + 38	52	56	66	72
36 + 18	54	52	44	32
13 + 38	51	55	61	75
39 + 14	53	51	41	33
37 + 27	64	66	74	86
28 + 38	66	62	56	42
37 + 24	61	65	75	81
26 + 39	65	61	51	45
23 + 49	72	74	82	94
49 + 25	74	72	62	54

$25 + 46$	71	73	81	93
$48 + 29$	77	73	67	57

Set B

Problem	Answer	<i>Distractor</i>		
		Close	Medium	Far
$12 + 22$	34	38	44	58
$21 + 17$	38	36	26	18
$16 + 21$	37	39	47	59
$21 + 18$	39	35	25	19
$31 + 13$	44	48	58	64
$13 + 33$	46	44	34	26
$32 + 15$	47	49	57	69
$12 + 37$	49	45	39	29
$25 + 31$	56	58	68	76
$36 + 22$	58	54	44	38
$24 + 31$	55	59	65	79
$33 + 26$	59	57	47	39
$41 + 21$	62	66	76	82
$25 + 43$	68	64	58	44
$43 + 24$	67	69	79	87
$25 + 44$	69	67	59	47
$34 + 42$	76	78	88	96

44 + 34	78	76	68	56
31 + 42	73	77	83	97
42 + 33	75	71	61	51
17 + 15	32	36	46	52
17 + 19	36	32	26	12
19 + 12	31	33	43	51
16 + 17	33	31	23	11
16 + 26	42	44	54	62
29 + 19	48	46	38	26
15 + 28	43	47	53	67
27 + 18	45	41	31	21
38 + 14	52	56	66	72
18 + 36	54	52	44	32
38 + 13	51	55	61	75
14 + 39	53	51	41	33
27 + 37	64	66	74	86
38 + 28	66	62	56	42
24 + 37	61	65	75	81
39 + 26	65	61	51	45
49 + 23	72	74	82	94
25 + 49	74	72	62	54
46 + 25	71	73	81	93
29 + 48	77	73	67	57

Appendix B

Problem Examples for Strategy Description Task for Experiment 1

In each case, participants were asked “How did you solve problems like these?” and were shown Example A and Example B.

Problem Type	Example A	Example B
No-Carry Horizontal	$22 + 12$	$17 + 21$
Close Distractors	34 38	36 38
No-Carry Vertical	12 + 22	21 + 17
Close Distractors	34 38	36 38
No-Carry Horizontal	$15 + 32$	$26 + 33$
Medium Distractors	57 47	59 47
No-Carry Vertical	32 + 15	33 + 26
Medium Distractors	57 47	59 47
No-Carry Horizontal	$21 + 16$	$18 + 21$
Far Distractors	37 59	19 39
No-Carry Vertical	16 + 21	21 + 18
Far Distractors	37 59	19 39

Problem Type	Example A	Example B
Carry Horizontal Close Distractors	$12 + 19$ 31 33	$19 + 29$ 46 48
Carry Vertical Close Distractors	19 $+ 12$ 31 33	29 $+ 19$ 46 48
Carry Horizontal Medium Distractors	$17 + 16$ 33 23	$26 + 16$ 54 42
Carry Vertical Medium Distractors	16 $+ 17$ 33 23	16 $+ 26$ 54 42
Carry Horizontal Far Distractors	$28 + 15$ 43 67	$39 + 14$ 33 53
Carry Vertical Far Distractors	15 $+ 28$ 43 67	14 $+ 39$ 33 53

Appendix C

Math Background and Interests Questionnaire

1. In which province did you receive your education (or country if not Canada)?

Elementary:

Secondary:

Post-Secondary (University):

2. Age (in years):

3. Sex: male female

4. What languages do you speak?

English French Other (please specify)

5. Which do you consider your first language (i.e., the one you are the most proficient in speaking)?

English French Other (please specify)

6. If you were in French immersion in school, please answer the following questions

a. I was in French immersion from grades _____ to _____

b. I studied math in French from grades _____ to _____

7. Do you wear/have you ever worn corrective lenses (i.e., glasses or contacts)?

YES NO

If yes, for what purpose do/did you wear them (e.g., driving, reading):

8. Have you ever had a seizure?

YES NO

If yes, how frequently do you have seizures (i.e., number per week or year)

9. Have you ever had a head injury (e.g., concussion)?

YES NO

If yes, how severe was the head injury?

1	2	3	4	5	6	7
mild		moderate			severe	

10. Are you colour blind or colour weak (i.e., red & green deficiency only)?

YES NO

If yes, rate the extent of your colour blindness

1	2	3	4	5	6	7
mild		moderate			severe	

11. Please rate your visual-motor ability (e.g., catching a ball; walking a balance beam)

1	2	3	4	5	6	7
very poor		average			very good	

MATHEMATICS

1. When did you start using a calculator (e.g., Grade 6, first year university):

2. How frequently do you use a calculator to do calculations such as 6×9 ?

1	2	3	4	5	6	7
never		sometimes			always	

3. How frequently do you use a calculator to do calculations such as $34 + 97 + 18$?

1	2	3	4	5	6	7
never		sometimes			always	

4. Please rate your level of basic mathematical skill (e.g., skill at calculation):

1	2	3	4	5	6	7
very low		moderate			very high	

5. Please rate your level of mathematical skill in more complex areas of mathematics (e.g., calculus, algebra):

1	2	3	4	5	6	7
very low		moderate			very high	

6. How frequently do you AVOID situations involving mathematics?

1	2	3	4	5	6	7
almost always		sometimes			almost never	

7. How frequently do you find that situations involving mathematics make you nervous?

1	2	3	4	5	6	7
almost always		sometimes			almost never	

8. How difficult was mathematics for you in high school?

1	2	3	4	5	6	7
extremely		moderately			not at all	
difficult		difficult			difficult	

SOLVING SIMPLE ARITHMETIC PROBLEMS

1. How frequently do you solve math problems such as 3×9 by switching the numbers around so that the larger number is placed first (9×3)?

1	2	3	4	5
never	rarely	sometimes	often	always

2. How frequently do you solve math problems such as 7×6 by switching the numbers around so that the smaller number is placed first (6×7)?

1	2	3	4	5
never	rarely	sometimes	often	always

3. Overall, please estimate what proportion of the time you solve simple multiplication problems in ways *other than* by automatically remembering the answers?

1	2	3	4	5
never	rarely	sometimes	often	always

4. Overall, please estimate what proportion of the time you solve simple addition problems in ways *other than* by automatically remembering the answers?

1	2	3	4	5
never	rarely	sometimes	often	always

5. Overall, please estimate what proportion of the time you solve subtraction problems in ways *other than* by automatically remembering the answers?

1	2	3	4	5
never	rarely	sometimes	often	always

6. Overall, please estimate what proportion of the time you solve division problems in ways *other than* by automatically remembering the answers?

1	2	3	4	5
never	rarely	sometimes	often	always

Appendix D

Problem Examples for Strategy Description Task for Experiment 2

In each case, participants were asked “How did you solve problems like these?” and were shown Example A and Example B.

Problem Type	Example A	Example B
No-Carry Horizontal Exact Addition	EXACT $22 + 12$	EXACT $17 + 21$
No-Carry Vertical Exact Addition	EXACT 12 $+ 22$	EXACT 21 $+ 17$
No-Carry Horizontal Approximate Addition	APPROXIMATE $22 + 12$	APPROXIMATE $17 + 21$
No-Carry Vertical Approximate Addition	APPROXIMATE 12 $+ 22$	APPROXIMATE 21 $+ 17$
Carry Horizontal Exact Addition	EXACT $12 + 19$	EXACT $19 + 29$
Carry Vertical Exact Addition	EXACT 19 $+ 12$	EXACT 29 $+ 19$

Problem Type	Example A	Example B
Carry Horizontal Approximate Addition	APPROXIMATE $12 + 19$	APPROXIMATE $19 + 29$
Carry Vertical Approximate Addition	APPROXIMATE 19 $+ 12$	APPROXIMATE 29 $+ 19$