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Title of Thesis — Titre de la thèse

A STUDY OF THE GAIN MECHANISM OF GAS
FILLED PROPORTIONAL WIRE COUNTERS

University — Université

CARLETON UNIVERSITY

Degree for which thesis was presented — Grade pour lequel cette thèse fut présentée

DOCTOR OF PHILOSOPHY

Year this degree conferred — Année d'obtention de ce grade

1984

Name of Supervisor — Nom du directeur de thèse

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A STUDY OF THE GAIN MECHANISM OF GAS FILLED
PROPORTIONAL WIRE COUNTERS

by

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A thesis submitted to the faculty of graduate studies in
partial fulfillment of the requirements for the degree of
Doctor of Philosophy.

Carleton University

Ottawa Ontario

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ABSTRACT

Multiwire Proportional Chambers, (MWPC), are one of the most widely used devices for the detection of particles in subatomic physics. In the past, it was normally assumed that the gas avalanche gain factor in a MWPC was determined by Townsend's first ionization coefficient. The Townsend coefficient applies when the primary ionization and secondary electrons remain in equilibrium with the electric field as these electrons drift through the avalanche region toward the anode. This equilibrium is associated with uniform fields and is shown to be never exactly appropriate for wire geometry. When the electric field changes too quickly for the electrons to maintain equilibrium the gas gain remains proportional, but is determined by a more general ionization coefficient. Systematic optimization of the signal in a MWPC requires the development of new methods for dealing with this more general coefficient. This includes differentiating between the case when the Townsend coefficient is approximately valid and when the more general ionization coefficient is required.

A simple technique has been developed which permits the extraction of the ionization coefficient for a given gas mixture from experimental gain measurements made on a single wire proportional tube. In this case the gas mixture was 80% Argon and 20% Methane. This technique makes possible detailed predictions of the proportional

wire gain for a given size of anode wire and gas pressure from measurements made on other wires.

It has been experimentally demonstrated that the ionization coefficient responsible for the gas gain is never exactly equal to the Townsend coefficient, although in certain proportional chamber operating regions the difference is negligible.

By an examination of the symmetries expected for the ionization coefficients it is also possible to accurately measure the end of the proportional region in the MWPC. The effect of positive ion space charge on the gas gain is the normal process responsible for the end of the proportional region. This end is measured for two cases, the saturation of the gain for thin anode wires and the non-proportional discharge increase of the gain with thick wires. These observations may be empirically compared with earlier parallel plate observations suggesting that when the second derivative of the ionization coefficient with respect to the field is negative, space charge leads to stable saturation of the gain, but when this factor is positive, space charge tends to increase the gain. However, the observed case of a discontinuous gain increase could also be caused by other secondary processes.

ACKNOWLEDGEMENTS

I would like to thank R. K. Carnegie and C. Hargrove for their helpful guidance and assistance throughout the course of this work. I would like to thank G. Findlay and L. R. Sainsbury for their assistance in the design and setup stages of this project and R. Bruce for help in the data taking phase of the project.

I would like to thank my father for both his encouragement and his critical reading of this document and my wife for her support and encouragement.

CONTENTS

ABSTRACT	ii
----------------	----

ACKNOWLEDGEMENTS	iv
------------------------	----

<u>CHAPTER</u>	<u>page</u>
----------------	-------------

I. INTRODUCTION	1
-----------------------	---

1.1 The detector	4
------------------------	---

1.3 Primary ionization	5
------------------------------	---

1.4 Drift velocity	7
--------------------------	---

1.5 The gas gain	9
------------------------	---

1.6 Introduction to the measurements	16
--	----

1.7 Guide to the thesis	18
-------------------------------	----

II. THE THEORY OF THE AVALANCHE	20
---------------------------------------	----

2.1 Townsend coefficient	20
--------------------------------	----

2.2 Non Equilibrium Case	23
--------------------------------	----

2.3 Scaling Laws and Definition of Variables for describing Proportional Chamber Operation	25
---	----

2.4 Classical model of electrons in a gas	31
---	----

Simplification of the collision physics	32
---	----

2.5 Avalanche simulation	36
--------------------------------	----

2.6 Simulation results	41
------------------------------	----

2.7 Empirical dependence of the Townsend coefficient on the electric field	52
---	----

2.8 Penning effect	56
--------------------------	----

2.9 Empirical formulae for the wire gain	62
--	----

Townsend	62
----------------	----

Rose and Korff	64
----------------------	----

General	68
---------------	----

2.10 Second order effects	73
---------------------------------	----

Ionization by positive ion impact	76
---	----

Electrons released from the cathode	
-------------------------------------	--

by positive ions	77
------------------------	----

Electrons released by the photoelectric	
---	--

effect	79
--------------	----

Ionization by double electron impact ..	80
---	----

2.11 Space charge effects	83
---------------------------------	----

2.12 Pulse rise time dependence on z	86
--	----

2.13 Theory of the experiment	89
-------------------------------------	----

III.	EXPERIMENTAL MEASUREMENTS	92
3.1	Mechanical description	93
3.2	Electronics	96
3.3	X-Ray source	99
3.4	Data taking procedure	109
3.5	Pressure and temperature measurement	111
3.6	Correction for the time structure of the signal	114
3.7	Monitor wire	120
3.8	Verification of the absolute gain measurement	123
IV.	EXPERIMENTAL EXAMINATION OF THE VALIDITY OF THE TOWNSEND COEFFICIENT	128
4.1	Empirical evaluation of the data	130
4.2	The empirical ionization coefficient	138
4.3	Introduction to the K,s gas amplification plane	140
4.4	Expansion of $\ln G$ in the K,s plane	153
4.5	Extraction of the ionization coefficient z ...	157
4.6	Search for the Townsend coefficient	161
4.7	Measurement of z by small pressure differences	168
4.8	Summary	171
V.	THE AVALANCHE AND SPACE CHARGE EFFECTS	173
5.1	Loss of proportionality in the Townsend region	176
5.2	Space charge effect in the non-Townsend region	180
5.3	Empirical treatment of the Space charge effect	189
	Comparison to parallel plate phenomenology	193
	Empirical effect of the charge density	199
5.4	Summary	208
VI.	EPILOGUE	210
VII.	APPENDIX	216
	A) Simulation of the parallel plate case	216
	B) Gain fluctuations	222

LIST OF FIGURES

FIGURE		page
1	The MWPC and Proportional Tube	2
2	Charged particle energy loss in passing through a gas	6
3	Current versus voltage characteristics of a proportional tube	10
4	Collision probabilities versus electron energy	40
5	Townsend coefficient versus E/P	42
6	Energy frequency distributions	44
7	Relative number of each type of collision versus E/P .	46
8	Energy Sharing of the collision processes	47
9	Drift velocity versus E/P	51
10	Average electron energy versus E/P	54
11	Eta versus E/P	57
12	Electron motion in an Electric field	65
13	Relative size of the first derivative of the ionization coefficient versus E	71
14	The Townsend coefficient versus Electric Field	72
15	Mechanical apparatus	94
16	Block diagram of the electronics	97
17	Typical Fe55 spectrum from the qVt	100
18	Pressure dependence of the Fe55 count rate	105
19	Comparison of Barocel and Kulite Pressure Transducers	113

20	Calculated output pulse shape	118
21	Time constant correction	119
22	$\ln G/K$ versus X for the monitor wire	122
23	$\ln G/K$ versus X for the 100 and 400 micron radius wires	126
24	$\ln G/K$ versus X for the total data sample	131
25	$\ln G/K$ versus X subject to a gain cut	132
26	Restricted values of $\ln G/K$ versus X	135
27	Gas amplification versus K	141
28	Gas amplification versus s	144
29	Lines of equal gain in the K, s plane	147
30	z versus X from the fit in the nonTownsend region .	158
31	z versus s from both regions	162
32	dz/ds versus s	166
33	z versus X from pressure differences	170
34	Space charge saturation of the gain in the Townsend region	177
35	Fe55 spectra in the space charge region	178
36	Deviation of data from fit	182
37	Avalanche size versus pressure	186
38	Fe55 spectra in the nonTownsend saturation region .	188
39	Radial charge density versus s	202
40	Radial charge density versus s in the nonTownsend region	203
41	Error in the measurement of the Townsend ionization coefficient from the parallel plate experiment	221

CHAPTER ONE

INTRODUCTION

The original proportional counters consisted of a wire strung down the center of a gas filled metal tube as shown in figure 1a. The central anode wire was held at a positive voltage relative to the surrounding tube, the cathode. Inside the tube the electric field varies as the inverse of the distance from the anode center. Some of the gas atoms become ionized when high energy charged particles or photons pass through the counter. The electrons thus freed, migrate toward the anode wire under the influence of the electric field. During the migration the electrons gain energy from the field and lose it colliding with gas atoms or molecules. With a suitable voltage difference between the anode and the cathode, it is possible that in the high fields near the anode surface, the electrons can gain enough energy to cause further ionization. This causes an electron avalanche, greatly increasing the number of electrons. This charge then generates a detectable signal. In some cases, the signal is proportional to the amount of original ionization.

Figure 1

Schematic diagram of a proportional tube and a simple
Multiwire Proportional Chamber.

The Multiwire Proportional Counter, (MWPC), is an extension of the same principles. Figure 1b illustrates one of the simpler MWPC configurations. The MWPC, as its name suggests, is a detector which consists of more than one proportional wire. Each of the anode wires functions in a manner similar to the single wire of a proportional tube.

Over the last decade there has been a great proliferation in the number of applications for Multiwire Proportional Counters. This has increased the need for fully understanding their properties. Recent developments in the computer modelling of the electrostatics of MWPC's [1], have reduced many of the problems of understanding and predicting their characteristics to that of understanding the characteristics of a single wire proportional tube. Attempts to apply present theoretical approaches to make precision predictions of the initial signal rise time and shape for a wire gas counter [2] showed the need for a more critical experimental examination of the theoretical techniques.

This thesis extends the experimental approach and theoretical methods for the study of the gas gain mechanism of a single wire proportional chamber. In order to place this work in context, it is necessary to briefly review the main physics factors affecting proportional chamber operation and performance.

1.1 The Detector

The *raison d'être* of the gas filled detector or MWPC is the detection and classification of particles from some kind of "event". The nature and complexity of the possible events are diverse and dependent on the individual branch of physics. In High Energy Physics the detector is normally designed to reconstruct the trajectory and or energy deposition of one or more charged particles. This is to aid the identification of the mass, charge and momentum of the particles surviving some kind of interaction. A measurement of the track or path followed by the particle may aid in vertex reconstruction or, if curving in a magnetic field, momentum determination. Measurement of the rate of energy deposition, dE/dx , can help determine the particle's velocity. Knowledge of both velocity and momentum determine a particle's mass. Since mass is quantized one may identify a particle without knowing the velocity to the same precision as the momentum.

The optimization of dE/dx counters has been reviewed by Walenta [3]. There are three properties of the gas composition which require optimization: the primary ionization, the electron drift characteristics, and the gas amplification. Each of these processes in part depends on the interaction of free electrons with gas atoms, but for different values of electron energy. This leads to the hope that it may be possible to optimize each separately.

1.2 Primary Ionization

The initial ionization has been discussed in detail by Allison [4]. Initially, a charged particle will pass through the gas volume of the detector leaving a track of electron-ion pairs, the primary clusters. The electrons released from the atom may have enough energy to cause further ionization increasing the size of the cluster. The position and size of each cluster is all the information available about the initial charged particle.

The average total ionization deposited per unit thickness is referred to as dE/dx and is a function of the incoming particle's velocity. Figure 2 gives a typical example of the energy loss as a charged particle travels through one centimeter of gas. Conversely a measurement of total ionization gives information on the particle velocity. There are four regions to the dE/dx curve. The first is the $1/v^2$ drop predicted by classical physics. The second is the minimum ionizing region. The relativistic increase in the incident particle's electromagnetic field causes the particle to interact with an increasing number of atoms of the gas. This increase continues into the third region or region of relativistic rise where there is a logarithmic increase in ionization. Finally, the fourth region is the fermi plateau. This is the density effect. The finite density of the gas modifies the electric field of the incident particle. This causes the ionization to

Figure 2

Standard energy loss curve for a 1 cm thickness of gas at atmospheric pressure [3]. The four regions are the initial inverse velocity squared fall, the minimum, the logarithmic rise, and the Fermi plateau respectively.

saturate. The lower frequency components of the field are the first to be screened. As they are also responsible for the smaller clusters which are most numerous, the number of clusters of ionization saturate at a rate faster than the total ionization loss.

The dE/dx curve is a statistical average quantity and has been well described in the past by average quantities such as average ionization potential. However, when considering the frequency distributions of clusters of a given size, a more careful treatment is required as discussed by Walenta [3] and Allison [4]. This is, at present, an area of active research.

1.3 Drift Velocity and Diffusion

Once the clusters have been deposited they must be transported to an area where they may be detected and measured. This process occurs in the drift region of the detector. Ideally, one would like to maintain the information about the initial position as the ionization electrons move through the drift region.

The physics processes behind the drift and diffusion of electrons have been reviewed by Palladino and Sadoulet [5,6]. Some useful calculations have been made by Shultz and Gresser [7]. In the absence of an electric field the electron moves about randomly with thermal energies. In

the presence of the low electric fields of normal drift the above authors have had success treating the net drift of the electrons as a perturbation on the random velocity of the electron. Then in the framework of classical statistical physics the energy gained from the electric field is balanced with that lost in collisions with gas atoms or molecules in order to calculate drift velocities.

Usually, low diffusion and a saturated drift velocity are desired. A saturated drift velocity is one that becomes independent of the electric field over a range of field values. The low diffusion reduces the loss of information about the original position of the cluster. The saturated drift velocity will keep the drift velocity stable in the presence of power supply instabilities and perturbation of the field in the presence of positive ions disturbing the field in the drift region. The actual value of drift velocity desired is very application dependent.

It has been found that the addition of small percentages of molecules such as carbon dioxide or methane to a noble gas base may significantly change drift velocity and diffusion. The addition of the molecules has a twofold effect. First, the increased number of possible molecular modes of dissipating energy causes significant cooling of the electrons and so reduces diffusion and increases drift velocity. This is further amplified by the second difference, namely, that the energy transfer cross section

in the heavier noble atoms has a minimum, the Ramsauer dip, around 0.3ev. In the case of the pure noble gas this is of the order of the average electron energy in normal drift conditions and so this minimum tends to further heat the electrons.

This leads to the prospect that having chosen the bulk of the gas to optimize energy loss one may still have a large control over the diffusion and drift velocity with the remaining ten or twenty percent of the gas. At present much work is being done in an attempt to accomplish this goal.

1.4 The Gas Gain

The final physical process requiring consideration is the gas gain mechanism.

The gas gain mechanism is controlled by the same basic principles as Drift and Diffusion. As the electric field is increased the average energy of the electrons increases until subsequent ionizing collisions are possible and the number of electrons begins to increase. This happens in the region of increased field near the anode wire.

For low values of the gain the final signal is proportional to the initial number of electrons. As the gain increases space charge and other secondary processes destroy proportionality. Traditionally, if one plots

Figure 3

Generalized graph of the gas amplification versus voltage for a hypothetical proportional wire tube. The five regions correspond to: 1) where only part of the primary ionization is collected, 2) Region where all the initial ionization is collected but amplification is negligible, 3) the proportional region, 4) the semi proportional region, and 5) the Geiger region.

signal versus counter voltage for a proportional tube there are five regions as shown in figure 3. The first corresponds to the increase in signal as voltage is increased, not due to amplification but due to increased collection of the ionization as opposed to recombination effects. The second region is the saturated region of an ionization chamber; essentially one hundred percent of the ionization is collected and there is no amplification. Region three is the proportional region. The initial ionization is amplified but the output signal remains proportional to the initial ionization. One then enters the semi-proportional region where space charge begins to destroy proportionality. Finally, region five, is the Geiger region where the signal becomes completely independent of the initial ionization. The relative size of each region is variable and may in fact be indefinable; for example, amplification may begin before one starts to collect one hundred percent of the ionization.

In the historical development of the MWPC, the first MWPC's were, in fact, not proportional at all but operated in the semi-proportional region. This is because as they evolved from spark chambers, only position measurement was desired. The semi-proportional region was optimal for position measurement due to its large clear signal in certain circumstances. Because of the more controlled avalanche in the semi-proportional region as opposed to the

spark region, the first MWPC's had better rate characteristics. The continued evolution of the MWPC has introduced more variability in the choice of the operating region on the signal versus voltage curve.

Position measurement, whether by detecting induced signals on a subdivided cathode or drift time measurement, generally improves with increasing amplification. Energy measurements in dE/dx require strict proportionality. Separation of two different incident particle tracks which arrive at the wire within a small time interval require fast signal rise time and no distortion of the second track signal by the positive ions of the first. The attempt to measure all of these properties on the same wire comes very close to the original desire to detect the size and original position of each of the initial clusters. This means that optimization is no longer just a matter of picking a region on the signal vs voltage curve but the ability to control the signal vs voltage curve.

It is then, in general, desirable to work with large but proportional gain, and also fast signal rise time. This requires a systematic and controllable understanding of the proportional region and what is responsible for ending it. This requires a simple flexible theoretical grasp of the situation. It is the purpose of this thesis to put experimental constraints on the present theoretical understanding of the process. Even though gain depends on

many of the same processes as drift and diffusion it is hoped that advantage may be taken of the difference in the electron energy regions so that they may be separately optimizable.

The gas gain is an exponential process that may be described by means of a parameter, z , defined by the differential equation,

$$dN = zNds.$$

1.1

This states that the infinitesimal increase in the number of electrons N in moving an infinitesimal distance ds is equal to $zNds$. The parameter z is called the ionization coefficient. This parameter is the middle ground between understanding the process at an atomic level and the totally empirical approximations to the integration of (1.1). Thus z is a macroscopic quantity somewhat analogous to temperature. It results from the statistical averaging of many microscopic or atomic processes. It is possible to relate z to basic atomic collision cross-sections. Although it is impossible to solve these relations in an analytic way, it is possible to obtain approximate solutions by numerical means. These will be treated in Chapter 2. However, the simplifying assumptions necessary result in only an approximate treatment. It is hoped that in the future these techniques may be made more accurate.

Since it has been impractical to calculate z , a number of empirical approaches have been developed to approximate the integral of equation 1.1. It is the integral of z which determines the gain. There are two general problems that are common to most of the empirical formulae. The first results from approximating z to depend only on the value of the electric field at a point and not on the way in which it changes. This is equivalent to assuming that the ionization coefficient, z is identical to the first Townsend ionization coefficient usually labelled α in the literature. This ignores the effect of the changing field on the equilibrium of the electrons. This assumption appears to be common to all empirical formulae for the gas amplification.

The second problem is associated with attempts to derive simple functional relationships for z . In this case the derived gain formulae could be considered to be non empirical. However they are no better than the empirical formulae and so are classified with them. The attempts to derive a formula are often associated with the assumption that the electrons move in a non random way that can be parameterized by a mean free path between ionization collisions. In Chapter 2 it will be seen that for wire geometry this approach is only valid for very large fields, in which case the ionization coefficient is clearly not a function of E alone.

Chapter 4 presents a measurement of the experimental operating region where the Townsend coefficient may be considered as valid for the wire proportional chamber geometry. The measurement was for the gas mixture 80% Argon and 20% Methane. This allows the identification of the region where the divergence of the ionization coefficient α from the Townsend coefficient becomes significant. This then corresponds to operation conditions where the electric field near the wire changes at a rate where the electron swarm is no longer in equilibrium with the field.

In the past, the Townsend coefficient was measured in parallel plate geometry, or constant electric field. For this reason it is considered to be a function only of the electric field at a point.

The most important secondary process is the onset of the semi-proportional region due to positive ion space charge modification of the electric field. This is the critical limitation of "large gain" strictly proportional MWPC's. The effect of space charge is application dependent, as is illustrated by the experimentally measured dependence of the gas gain on the angle between a charged track and the anode wire. Chapter 5 presents some measurements of the gain at which space charge significantly affects the gas gain. Chapter 5 also discusses some of the empirical relationships between the

functional form of z and the nature of the space charge effect.

In Chapter 2 it is shown how a knowledge of the spatial dependence of z , which gives the spatial distribution of the avalanche, allows one to look in detail at the initial rise time of the proportional tube signal. This has, in fact, two stages one dominated by the speed at which the electrons move and the second by the speed at which the positive ions move. The inability of the empirical formulae to predict the separation of these stages and the effect pressure and wire radius have on it, was one of the major motivations of this study.

1.5 Introduction to the Measurements.

In a proportional tube there are three parameters which can be varied: voltage, pressure and anode wire radii. From equation 1.1 clearly it would be advantageous if wire radii were a continuously variable parameter. This would allow z to be obtained by direct differentiation. However, having a large number of wire radii is not a practical situation. It will be shown in the next chapter how varying pressure can be equivalent to varying wire radii. This is a result of the proportional wire gain being strictly a function of the scaling variables s and K where $s = P \cdot r$ and $K = V / \ln(R_c / R_a)$. P is the pressure, V the

voltage of the anode wire relative to the cathode cylindrical tube, R_c is the cathode radius, R_a the anode radius and r is normal distance. This scaling is valid in the region where both the Townsend coefficient and the more general ionization coefficient are valid for calculating the gas gain. The symmetry is broken, however, when space charge becomes non negligible.

The gas gain of the proportional wire counter has been measured in a standard way using a monochromatic 5.9keV Fe55 X ray source to provide the initial ionization. For fixed pressure the mean pulse height in the 5.9keV photoabsorption peak was measured for a range of voltages and hence gain. This procedure was repeated for a number of different pressures. It is found that for fixed gas gain the required electric field on the anode wire surface divided by the pressure decreases as pressure increases. This allows z to be determined from the pressure dependence of the gain in regions where the gain is low enough for space charge to be negligible. It is also possible to compare the values of z so obtained at similar values of E/P but at different total gains.

1.6 Guide to Thesis

The thesis can be divided into four steps.

The first step is to review and develop the relevant theory. This involves a definition of both the Townsend coefficient and the more general ionization coefficient. From these definitions the symmetry relationships used in the experiment are extracted. For completeness some of the atomic physics on which the ionization coefficients depend are reviewed. A simulation technique is used to examine the dependence of the drift velocity and ionization coefficient in the large avalanche fields on the more fundamental atomic cross sections. The lack of measured values of diffusion and drift velocity in the high fields limits this technique. A brief discussion of the assumptions involved in some of the empirical formulae is given plus a review of some of the possible secondary processes which could be involved.

The second step is the actual experimental gain measurements themselves. Chapter 3 describes in detail the apparatus, the source and the secondary physics measurements that were made to monitor the reliability of the gain measurements, and to establish the absolute gain normalization.

The third step is to use the gain measurements to demonstrate that the Townsend coefficient is not universally valid for the description of the gain of a proportional tube. Techniques are then developed that permit the gain measurements to be parameterized. From this parameterization the ionization coefficient is then extracted. From these results it is possible to illustrate that the Townsend coefficient is never exactly appropriate for the description of the gas gain and to show the limitations of empirical formulae.

The fourth step is to show how the end of the proportional region may be identified using the above techniques for two examples. This is done by identifying the region where the symmetry predicted by the two ionization coefficients is broken. From an examination of the results it is possible to make the empirical observation that the type of space charge effect may be related to the sign of the second derivative of the ionization coefficient.

CHAPTER 2

THE THEORY OF THE GAS GAIN MECHANISM

Electron Avalanche is the term used to describe the gas gain mechanism. If a single electron or electron cluster is formed in the gas and then drifts in an external electric field, it gains energy from the field and loses it in collisions with the gas. When the field is strong enough the electron can gain enough energy to cause further ionization on a subsequent collision. These additional electrons in turn cause further ionization resulting in an exponential growth in the number of electrons present.

2.1 Townsend Coefficient

J. S. Townsend published the first experimental evidence for the hypothesis that electrons may accelerate in an electric field and ionize gas atoms or molecules [8]. His research was directed at showing that the negative ions produced by X-rays or the photoelectric effect had the same properties as those that they could free by impact with the gas. Or in short, that they were all the same particle, the electron. He introduced the differential equation,

$$dN = \alpha N dr,$$

2.1

where dN is the increase in the number of electrons, N , as the electron swarm drifts a distance dr . The equation is based on the assumption that the electrons freed by the ionizing collision must have the same characteristics as the original electrons. This means that they must have the same probability of causing further ionization. As it is now accepted that the electrons are the same particle independent of source, the only requirement is that the electrons be part of the same energy distribution.

In a constant electric field at constant pressure, temperature and gas composition, α is found to be a constant. (2.1) is then a statement that the number of electrons grows exponentially with distance,

$$N = N_0 \exp \alpha r.$$

2.2

When one hypothesizes the same differential equation, (2.1), for the gas gain at the anode wire of a MWPC, α is no longer a constant because α depends on E and E is varying. There are then different ways of defining α . One may define α by requiring that the differential equation exactly describe the wire gain. In this case α would have to take into account all the geometric properties of the MWPC and all possible second order processes such as space charge, photon effects, etc. However, having α depend on so many special circumstances causes it to cease to be a useful parameter.

In order to make α as useful as possible it is restricted to be strictly independent of the number of electrons N or the derivatives of the electric field.

Choosing it independent of the number of electrons excludes the second order processes such as space charge or ionization by double electron impact from preliminary discussion. These terms depend on electron densities and including them introduces dependency on initial ionization distributions, among other factors.

Requiring α to be independent of the derivatives of the electric field allows it (once measured or calculated as a function of E) to be applied to the widest variety of cases. This is done by simple integration of (2.1) to obtain the gain, G ,

$$G = \exp \int \alpha dr . \quad 2.3$$

Thus α may be compared to measurements made between parallel plates as done originally by Townsend. For this reason this definition of α is considered to be equivalent to Townsend's coefficient.

α may be considered the mean number of ion pairs formed by an electron per cm of drift in the direction of the electric field. However, considering α to be the inverse of a mean free path between ionizing collisions, as is sometimes done in the literature [9], tends to be misleading. α is a macroscopic quantity in the same way

that temperature and pressure are. It is the result of averaging many microscopic collisions. The actual motion of the electron is almost a random walk. The number of non-ionizing collisions always outnumber the ionizing collisions.

2.2 Non Equilibrium Case "thin" Wires

As experimentally determined in Chapter 4, the gas amplification in a proportional chamber is not determined exactly by the Townsend coefficient. Therefore, it is necessary to introduce a more general ionization coefficient κ that depends on the way in which the electric field changes. Although dependence on the derivatives of the field is permitted, κ is still restricted to be strictly independent of the number of electrons present.

κ corresponds to the situation where the electric field is changing too rapidly for the electrons to be in equilibrium with the field. This is the case for the operation of a proportional chamber with "thin" wires. is also valid if the electrons are in equilibrium with the field. This is the trivial statement that $\kappa = \alpha$ when the electrons are in equilibrium.

Appendix A discusses the problem of electrons originating at the cathode with small initial energy coming to equilibrium with the field. In this case the effect was noticeable due to the large difference between the electron's initial energy and the average equilibrium energy.

In the case of a variable electric field near the anode wire the problem is initially not so severe. The electron is continually drifting into regions of higher electric field. However, the field changes by only small amounts as the electron moves small distances. Thus if the electron swarm can be considered to be in equilibrium with the field at a position r then when it drifts a distance $-dr$ toward the anode the entire swarm must change its energy only by a small amount on average to remain in equilibrium with the field.

Therefore whether or not the electron can stay in equilibrium with the field and thus have a gain dependent on α rather than the more general coefficient κ depends on how fast the electric field changes and on how fast the electron energy distribution changes as a function of E .

As illustrated in the experimental results of Chapter 4, the transition between the two cases is a very gradual one. The difficulty in defining a sharp border between the two regions may be responsible for the fact that it has

been ignored in the past.

The next section treats the symmetry characteristics of the two coefficients. At the same time a preferred set of variables is introduced.

2.3 Scaling Laws and Definition of Variables for Describing Proportional Chamber Operation

This section introduces the geometric, electric field and other variables associated with the calculations of the gas gain in wire proportional chambers. It is possible to absorb the way in which the gas amplification scales with Temperature and Pressure into the geometric dependence. This results in a simplified list of variables. This list will introduce most of the notation used in this thesis.

The two ionization coefficients α and κ have already been introduced. Both are based on the hypothesis that ionization is caused by collision of electrons with gas atoms or molecules. α corresponds to the case when the electrons are in equilibrium with the field and κ when they are not. Both are dependent on the way in which the electron gains energy from the field between collisions and the way in which it loses energy in collisions.

The probability of a collision is given by,

$$1/\ell = N_m \sigma, \quad 2.4$$

where σ is the total collision cross section, N_m is the number of molecules per unit volume, and ℓ is the mean distance between collisions. Since σ is the fundamental parameter one could write,

$$\sigma = 1/\ell N_m. \quad 2.5$$

This suggests introducing a transformation and defining distance in such a way as to absorb the N_m , for example defining $N_m r$ as determining the unit of distance.

For an ideal gas $N_m = P/RT$ where P is the pressure and T is the temperature. For fixed temperature P is proportional to N_m and thus alternatively one may define Pr as a distance. The new distance parameter is defined to be

$$s = Pr, \quad 2.6$$

in cm*torr as this is compatible with most of the literature concerning α . However, $N_m r$ is probably superior in that it explicitly reminds one that temperature is also involved.

$T_0 = 0$ degrees centigrade, 273 degrees Kelvin is used as the constant temperature. All data points were scaled by multiplying the pressure by T_0/T .

As the kinetic energy of the gas atoms or molecules has negligible influence on the physics of the avalanche region, one may ignore the way in which the temperature

changes the molecular kinetic energy. This is not the case for electrons drifting in very low fields. Thus both temperature and pressure dependence is as described by the ideal gas law.

The electron kinetic energy is

$$\epsilon = 0.5mv^2, \quad 2.7$$

where m is the electron mass and v is the velocity. It is possible to leave the velocity of the electron and hence its energy unchanged by the transformation of the distance by transforming the time to,

$$u = Pt. \quad 2.8$$

This gives,

$$\begin{aligned} \bar{v} &= d\bar{r}/dt \\ &= d\bar{s}/du \end{aligned} \quad 2.9$$

This shows that distance scaling implies time scaling.

The energy gained by the electrons in moving between two points in the absence of a collision is determined by the difference in the scalar potential $V(s)$ at the two points. To define a field from the scalar potential one has,

$$\begin{aligned} \bar{X} &= -\bar{\nabla}_s V(s), \\ &= -(1/P) \nabla_r V(s), \\ &= E/P, \end{aligned} \quad 2.10$$

where E is the normal electric field.

For the case of cylindrical geometry the electric field is proportional to $1/r$ or

$$E = K/r, \quad 2.11$$

or

$$X = K/s, \quad 2.12$$

K the constant of proportionality. In the case of the long proportional tube, the electric field proportionality constant,

$$K = V/\ln(r_c/r_a), \quad 2.13$$

where V is the voltage applied between the anode and the cathode, r_c the cathode radius and r_a the anode radius.

Notice also that $r_c/r_a = s_c/s_a$. For the MWPC the field is usually more complicated but K may be expressed as

$$\begin{aligned} K &= E_a r_a, \\ &= X_a s_a, \end{aligned} \quad 2.14$$

where E_a is the electric field at the surface of the anode.

This assumes that in the avalanche region near the wire the field may be approximated by a $1/s$ dependence.

As the ionization coefficient is the number of ionizations per unit distance, it is transformed when distance is transformed. Letting z equal the transformed ionization coefficient, the differential equation determining growth is

$$dN = zNds, \quad 1.1$$

as given in Chapter 1.

From $zds = Kdr$ it follows that

$$z = K/P. \quad 2.15$$

As the electric field and its derivatives are determined by K and s it follows that the ionization coefficient z is also a function of K and s . The gain, G , is then given by

$$\begin{aligned} \ln G &= \int K(K,s)dr \\ &= \int z(K,s)ds \\ &= \ln G(K, s_a). \end{aligned} \quad 2.16$$

where $s_a = Pr_a$. Thus the wire gain is a function of only K and s_a , the product of pressure and wire radius. This is the general scaling law of wire chamber gain with respect to pressure and wire radius. Francis [10,11] refers to this as a Similarity Relationship.

Not all processes obey this scaling law or similarity relationship. For example, space charge distortion of the electric field due to a build up of positive ions will break the scaling. The electric field from a point charge falls off as $1/r^2$ and not as $1/r$.

It is an important result that the gain is a function of K and s_a whether the electrons are in equilibrium with the field or not. When considering design criteria this reduces the problem of considering the three variables K, P, r down to just K and s_a , reducing the problem from a three dimensional one to one of two dimensions.

A further reduction is possible for the equilibrium case when $\kappa = \alpha$ is applicable. When this is valid z is a function of $X=K/s$ and not of K and s separately. In this case,

$$s = K/X$$

$$ds = -(K/X^2)dX,$$

and so

$$\ln G = \int z(X) ds,$$

$$= -K \int z(X)/X^2 dX$$

$$= KY(X_a). \quad 2.17$$

Y has been introduced for partial consistency with Charles [12],

$$Y = \ln G/K. \quad 2.18$$

In the general case the gas amplification may be considered to be a scalar function in the plane given by the variables K and s_a . In the case where the electrons may be assumed to be in equilibrium with the field the two dimensional dependence may be further reduced to the equivalent of a one dimensional dependence.

The ionization coefficient may be obtained from the measured values of the gas gain by differentiation,

$$z = -d \ln G / ds. \quad 2.19$$

If the electrons are in equilibrium and $z = \alpha/P$ then

$$\alpha/P = X^2 dY/dX. \quad 2.20$$

Up to this point the dependence of the ionization coefficient, z on the gas mixture has been ignored. As a guide to the readers intuition this subject will now be discussed. As the discussion is meant to be qualitative rather than quantitative it will be done only in reference to the Townsend coefficient.

2.4 Classical Theory of Electrons in Gases

V. Palladino and B. Sadoulet have reviewed the applications of the classical theory of electrons in gases to Multiwire Proportional and Drift Chambers [5]. For a range of values of the electric field where the drift of the electron may be treated as a perturbation of the overall motion of the electron they have calculated α for pure argon. Their calculated values agree fairly well with experimental values.

Palladino and Sadoulet begin with the Boltzmann equation and balance the energy gained from the external electric field with that lost in collisions in order to calculate the velocity distribution of the electrons. They solve for the distribution by approximating it by the first two terms of a Legendre series. This is an approximation which breaks down for large values of the electric field. It is possible to construct a simplified approximately "classical" model of the electron gas atom interactions

which is equivalent to some of the simplifying steps they make in their calculations. This permits an intuitive look at the physics involved in determining α .

2.4a Simplification of the Collision Physics

The classical picture of an electron colliding with an atom is that of two solid spheres colliding. The probability of the collision is determined by the cross-sectional size of the atom. Since the mass of the electron, m , is much smaller than the mass of the atom, M , the fraction of electrons scattered per unit solid angle is constant. However, in the region of the present interest the electrons have kinetic energies of the order of the atomic energy levels of the atom and therefore the scattering is very non classical [13]. The scattering is pronounced in the forward direction with maxima and minima as a function of angle. Thus the collision between electrons and real atoms is not as randomizing as in the classical picture.

Effectively this complication is avoided by approximating the real gas by what amounts to an equivalent "classical gas". Instead of using the actual electron collision cross sections one may deal with an averaged quantity called the momentum transfer cross section,

$$Q(\epsilon) = 2\pi \int_0^\pi \sigma(\epsilon, \theta) (1 - \cos\theta) \sin\theta d\theta . \quad 2.21$$

The actual total cross section would be,

$$q(\epsilon) = 2\pi \int_0^\pi \sigma(\epsilon, \theta) \sin\theta d\theta, \quad 2.22$$

where σ is the cross section per unit solid angle. If σ was independent of θ , the two cross sections would be the same, as is the case with the classical picture of colliding spheres. We must retain the nonclassical energy dependence, however.

Due to the pronounced forward scattering, the actual number of collisions is much larger than the momentum transfer cross section implies. The momentum transfer cross section corresponds to an averaging of the momentum lost in a collision along the direction of the incoming electron, or an averaging of $p(1-\cos\theta)$. Q is thus more of a randomizing probability than a collision probability.

A second useful result of the integral determining Q is that to first order in m/M , the energy loss of the electron in an elastic collision [14] is,

$$\Delta\epsilon = 2\epsilon (m/M)(1 - \cos\theta). \quad 2.23$$

So the average energy loss for fixed ϵ is

$$\langle \Delta\epsilon \rangle = 2\epsilon (m/M) Q(\epsilon). \quad 2.24$$

If only elastic scattering is present one may consider a classical system where the collision frequency is determined by $Q(\epsilon)$ with an average energy loss of $2\epsilon m/M$ per collision.

When treating excitation collisions, whether molecular or electronic, and ionization collisions, Palladino and Sadoulet use the normal collision cross section. This is because the energy loss is usually much greater than in elastic collisions and there are, in general, fewer excitation collisions than elastic collisions. The effect excitation and ionization collisions have on the energy distribution is then much more important than the effect they have in randomizing the electron motion. Thus ignoring the angular effect of the excitation collisions should introduce only a small error. Although this is true in the energy region considered by Palladino and Sadoulet, it is not true for the high fields of the avalanche region. It is possible to adjust the elastic momentum transfer cross-section to compensate for any error introduced.

Adjusting one cross section to compensate for another is an odd sort of statement to interpret physically. It comes from the difficulty in obtaining measured values of cross sections and their angular dependence for all the gases of interest over the energy regions of interest. It is common to infer values of the momentum transfer cross section from measured values of drift velocity and diffusion [5,6,7,15]. One may numerically solve the Boltzmann equation to calculate drift velocity and diffusion, given the momentum transfer cross section as input. One may then treat Q as a parameter which is varied

until the best agreement is obtained between experiment and calculation.

This value of Q can then be considered a measurement of the momentum transfer cross section. It, however, contains all the errors due to the simplifications and approximations made in the calculation. The Q obtained in this way is useful though, in predicting the effect of varying gas concentrations in a gas mixture as the errors in Q cancel the errors in the calculation.

Work is still being done in this area with regard to optimizing the drift velocity and diffusion characteristics of MWPC's. However, little is being done with regard to the avalanche sector. Here the errors due to ignoring the angular dependence of the excitation collisions are larger, while at the same time the energy loss due to elastic collisions begins to become negligible. Thus it may be possible to redefine Q as just a randomizing parameter and use it explicitly to correct for lack of knowledge of the angular dependence of the excitation collisions.

A second problem in the avalanche region is the unreliability of the expansion of the electron velocity distribution using only the first two terms of a Legendre series. This is because the electron drift velocity can no longer be considered just a perturbation on the overall electron velocity. The absence of a good analytic

technique implies that this area deserves some theoretical consideration.

In the absence of an improved technique one may experiment with a simulation model based upon the above classical gas model. The model is a simplification and improvement of earlier work of the author [2]. It allows some intuitive discussion of the functional relationship between the Townsend coefficient and the electric field. However, one requires a larger body of experimental data than is presently available before one can discuss its usefulness in predicting the effect of mixing gases. For that reason the discussion is restricted to pure argon, a gas which is probably the simplest to treat and for the same reason unusable in MWPC's as will be discussed below.

2.5 Avalanche Simulation

To optimize the intuitive benefits the electron will be moved through the "classical" gas described above in small time steps. Only a single electron is considered. The electron at time t is at position,

$$\vec{r}(t) = (x, y, z)$$

with velocity

$$\vec{v}(t) = (v_x, v_y, v_z)$$

in the presence of an electric field,

$$\vec{E}(r) = (E_x, E_y, E_z).$$

The charge on the electron is treated as being equal to unity. Assuming the electron does not undergo a collision, then, in time $t+dt$, one has,

$$\begin{aligned}\bar{r}(t+dt) &= \bar{r}(t) + \bar{v}(t)dt \\ \bar{v}(t+dt) &= \bar{v}(t) + (1/m)\bar{E}(r)dt,\end{aligned}$$

for m = the electron mass.

Let Q equal the sum of the elastic, excitation and ionization cross sections, then the probability of no collisions occurring as the electron moves a distance dr is,

$$p(dr) = \exp(-NQdr),$$

where N is the number of argon atoms per unit volume. In terms of time, dt ,

$$p(dt) = \exp(-NQvdt),$$

where v is the electron speed. The probability of one or more collisions in time dt is then,

$$P(dt) = 1 - p(dt),$$

or,

$$P(dt) = 1 - \exp(-NQvdt). \quad 2.25$$

The electron is drifted a time dt in the electric field. Then a random number between 0 and 1 is generated. If it is less than $P(dt)$ for that electron energy, a collision has occurred at time $t+dt$. This introduces two approximations. The first is the time of the collision. The second is it excludes the possibility of more than one collision in the time dt . Both these errors are negligible

for $P(dt)$ small enough. How one defines negligible is, of course, application dependent.

After each elastic collision $2E_m/M$ is removed from the electron energy and 11.5ev after each excitation. After an ionizing collision 15.75ev is removed and then half the remaining energy. This is meant to represent even sharing of the energy between the two electrons. This does not happen in practice as one electron usually has more of the energy but it is the most democratic way of dividing the energy and is in keeping with the physical meaning of the differential equation

$$dN = \alpha N dr.$$

In order to avoid repeated multiplications the units are chosen such that the above equations of motion become,

$$\bar{r}(t+dt) = \bar{r}(t) + \bar{v}(t)$$

$$\bar{v}(t+dt) = \bar{v}(t) + \bar{E}(r)$$

and

$$E = v^2.$$

2.26

To accomplish this, set $dt = 10$ femtoseconds $= 1$ and $0.5m = 1$.

This sets the unit of distance to be 5.931 nanometers and the electric field in units of 0.5volts/5.931 nanometers.

In order that the simulation results may be compared to the results of the calculation of Palladino and Sadoulet, their values of the cross sections [16] are used to determine P . For the momentum transfer cross section:

$$\begin{aligned}
 11.5 < \epsilon < 100 \text{ ev} \quad \sigma &= 1.52\text{E-}15 \sqrt{\epsilon/11.5} \text{ cm}^2 \\
 1.15 < \epsilon < 11.5 \text{ ev} \quad \sigma &= 1.52\text{E-}15 \epsilon/11.5 \text{ cm}^2 \\
 0.30 < \epsilon < 1.15 \text{ ev} \quad \sigma &= 1.46\text{E-}17 + 1.9\text{E-}16 (\epsilon - 0.3)^2 \text{ cm}^2,
 \end{aligned}$$

the excitation cross section

$$\sigma = 9\text{E-}17 \sqrt{\epsilon/11.5} (\epsilon/11.5 - 1) \text{ cm}^2,$$

The ionization cross section,

$$\sigma = 3\text{E-}16 \ln(\epsilon/15.7) \text{ cm}^2. \quad 2.27$$

To avoid repeated calculation of P, the cross sections are approximated by a series of step functions. This allows access to it by an array index found by converting the velocity squared into an integer. Setting $0.5m=1$ corresponds to setting the energy bin step size to 1ev which is appropriate to the 0 to 100ev energy range of interest. To have a finer step size merely requires changing the value of 0.5m.

One of the results of this step size is that the existence of the Ramsauer dip at 0.3ev in the elastic cross section is practically ignored by approximating the entire cross section between 0 and 1 by a constant. In this case, this constant is the value given by Palladino and Sadoulet for 0.5ev. It will be seen that this is justified by the fact that its structure has little effect on the avalanche region although it has a large effect on the drift velocity of argon for low fields.

Figure 4

The probability of a collision per unit time versus the electron energy for pressure of 76 and 760 Torr respectively. The solid line represents the probability of an elastic collision, the dashed excitation, and the dash-dot an ionization.

Figure 4 shows the relative contributions of the three types of collisions to P as a function of electron energy. Figure 4a gives P for a temperature of 0 degrees Centigrade and a pressure of 760.0 mm of Hg, figure 4b for 76.0 mm of Hg. Although dt is fixed at 10fsec one is still free to vary P by varying N or in this case the pressure for fixed temperature. This allows easy control of the approximation associated with only allowing one collision per step size.

One notices immediately that above 60ev the probability of an excitation collision is greater than an elastic collision as modelled by the momentum transfer cross section. Thus one may no longer ignore its effect on randomizing the electron motion. This will be discussed in more detail later.

2.6 Simulation Results

Before discussing some of the intuitive aspects of the avalanche, the simulated values of α and the drift velocity will be compared to experimental values.

For fixed values of the electric field the number of ionizing collisions as the electron moves are counted. α is defined as the average number per cm. Figure 5 gives the results of the model compared to the calculated points of Palladino and Sadoulet [5] and the experimental points

Figure 5

Comparison of z vs X from the four sources. The simulation points are calculated at $P=760$ Torr using the first model, that which assumes random direction after all collisions and the second model which assumes random directions only after elastic collisions.

for pure argon given by Druyvesteyn and Penning [17]. For E/P between 20 and 200 the agreement between all three is very good.

The agreement between the calculations of Palladino and Sadoulet and the experimental points actually improves around $E/P = 300$ volts/cm/torr where their expansion as a Legendre series breaks down. This is probably a coincidence as the overall agreement is much better than one might expect given the simple parameterization of the excitation and ionization cross sections.

The simulated values of α increase faster than the experimental values for high E/P . This has a number of possible explanations associated with the approximations of the model.

The distribution of electron energy as a function of E/P is shown in figure 6. For large E/P , the electrons begin to have significant probability of attaining higher energies. As shown in figure 4a, $P(dt)$ increases with energy. Most of this increase is associated with the excitation cross section. This means that the probability of two collisions happening in time dt is no longer negligible. In order to investigate this effect the simulation was run at one tenth the pressure and the probabilities of figure 4b. This had the effect of increasing the average number of excitative collisions, or

Figure 6

Energy distributions of the electrons in a uniform electric field. The distributions were simulated over a period of 150nsec.

of effectively increasing the excitation cross section. This resulted in a 7% drop in α for $E/P = 1500$ volts/cm/torr. At the same time it reduced the speed of the calculation by almost a factor of ten. It is probably possible to artificially adjust for this by increasing the excitative cross section.

Another related effect that has been ignored is the assignment of random outgoing electron directions after all three types of collisions. This is justified in the case of the elastic collisions by the definition of the momentum transfer cross section but not a priori for the other collisions. In their calculations Palladino and Sadoulet make the assumption that the number of excitations and ionizations must be negligible compared to elastic collisions. When this is the case, the inelastic cross section angular dependencies do not affect the overall motion of the electron. However, as shown in figure 7, at high E/P the relative number of inelastic collisions approaches the number of elastic collisions. In this situation one may no longer ignore their angular dependence.

The possible effects of this phenomena were investigated by going to the other extreme of leaving the electron direction unchanged after an excitation or ionizing collision. Random directions were only assigned after elastic collisions. Figure 8 shows that this has the

Figure 7

Relative number of each type of collision in % as a function of E/P . The solid line corresponds to random directions after all collisions, the dotted only after elastic collisions.

Figure 8

Relative energy sharing of the competing processes. The dotted line shows the effect of changing the way in which the electron direction is decided after a collision.

effect of increasing the fraction of energy that goes into the electron temperature. This is caused by decreasing the random part of the electron motion.

Figure 5 shows the effect this has on α . For the lower values of α there is no change. For the higher values it reduces the value. The points remain larger than the experimental values. This shows that it would be possible to achieve agreement by further reducing the momentum transfer cross section for high values of electron energy.

It is important to notice the separation of the different regions of physics. For low E/P the momentum transfer cross section is responsible for determining both the energy loss of the electron and the degree of randomization of its motion. There is a transition region where the excitation collisions begin to dominate the energy loss of the electron while having negligible effect on its randomization. Then there is the high E/P region where the momentum transfer cross section has no relevant effect on the energy loss of the electron and the angular dependence of the excitation collisions becomes important.

It becomes reasonable to redefine the momentum transfer cross section to include the total cross section instead of just the elastic cross section. For this to be valid the above high and low E/P regions must be distinct.

While the momentum transfer cross section still influences the energy loss of the electron the relative number of all other types of collisions must be negligible and vice versa.

Instead of cross sections for ionization and excitation one works with a fraction of the total collisions that are ionizations and a fraction that are excitations. As the real number of collisions is larger than that predicted by the momentum transfer cross section it is conceivable that the fraction of one type of collision could be greater than one.

This leaves three continuous parameters which are related to more fundamental cross sections but lend themselves to simpler evaluation. Clearly as α is only one continuous parameter there would be an infinite number of ways in which to select the collision parameters to model α . As shown, reducing the momentum transfer cross section at high energies or increasing the fraction of excitation collisions has the same effect on α . Further experimental constraints are needed.

These constraints could come from direct measurement of the probabilities or other dependent parameters such as electron drift velocity.

The simulation model has been compared to existing measurements of drift velocity in argon, figure 9 and the agreement is good. The experimental points are only available for $E/P < 5$ volts/cm/torr. This means they do not help resolve the high E/P discrepancies.

It should be noted that it is impractical to calculate drift velocity for small E/P with our approach due to its small value. The smaller the drift velocity the longer time one must drift to achieve a reasonable distance of the order of a cm, in which to define it. The simulation is subject to the same errors due to longitudinal diffusion as a real experiment. Thus to accumulate reasonable statistics the expense of computing tends to be proportional to $1/w$, w the drift velocity.

The drift velocity is an increasing function of E/P in figure 9 and attains relatively large values. It should be mentioned that although many of the gases used in MWPC's have saturated drift velocities for values of E/P in the region of drift it should not be expected that this saturation would continue into the large fields of the avalanche region. In general, the values of diffusion and drift velocity for these large fields are completely unknown for practical gases.

Figure 9

Comparison of the electron drift velocity from experiment and from the simulation. The agreement is satisfactory over the range of available measurements. The graph also illustrates the effect of changing the direction assignment.

This is unfortunate as drift velocity would make a useful constraint on the collision probabilities. Figure 9 shows the drift velocity calculated using the same parameters as above for the two extremes of handling the direction after excitation and ionization. It may be observed that whereas only selecting directions at random after an elastic collision reduced α slightly, it causes a large increase in drift velocity.

Thus measurements of both α and w would allow one to distinguish between the case of decreasing the momentum transfer cross section from that of increasing the excitation cross section. Such a distinction is necessary for predicting the effect of mixing gases.

The errors due to the above two causes would approximately halve the discrepancy between the experimental and model points at high E/P . Another possible source of error is the experiment itself. This has been discussed in appendix A as a useful application of the above approach.

2.7 Empirical Dependence of The Townsend Coefficient on E

One can identify three main regions of the curve of α versus E , shown in figure 5. The first is the experimentally unmeasurable low field region, $E/P <$

5volts/cm/torr. There is then a rapid exponential rise in α as a function of E and finally a relatively flat approximately linear dependence of α on E for high field values. These three regions are paralleled in a curve of average electron energy versus electric field, figure 10. There is the initial rapid increase followed by a region of slow increase followed by another region of rapid increase. An examination of the frequency distributions of figure 6 illustrate how these three regions come about.

Initially, the probability of an electron attaining the excitation energy is negligible and there are thus practically no excitation collisions and fewer ionizations. In this region α is experimentally unmeasurable as is evident from the lack of gain in the drift region of normal MWPC's.

As the electric field increases, the mean electron energy increases and excitations begin to occur. As the energy lost per excitation collision is much larger than that due to an elastic collision they have a noticeable cooling effect even when they are four orders of magnitude more rare than elastic collisions as illustrated by figures 7 and 8. This tends to sharpen the edge of the energy distribution and slow the rate with which the mean energy increases with electric field as seen in figure 10. This tends to make the ionization collisions still negligible.

Figure 10

Average electron energy versus E/P . Notice the initial rise, plateau and final further rise in the average electron energy with field. It is probable that in a region of variable electric field electron equilibrium would be easier to maintain in the plateau region.

With further increase in the field one approaches a region where the electron energy distribution has a non negligible overlap with the ionization region. This is a small, approximately exponential tail whose position shifts with increasing E and gives the exponential increase in α .

The exponential increase dies off as the mean electron energy becomes of the order of the ionization energy. Then the number of ionizations is determined by the relative sharing of the available energy between excitations and ionizations. In the region above the exponential rise, it begins to make sense to talk about the average energy required to produce an ion pair and conservation of energy.

The electron cannot lose any more energy in collisions than it had when it started plus that available from the field. This is important in considering the functional dependence of α on E and in limiting what may happen in situations when the electrons are not in equilibrium with the field. This has led, historically, to the introduction of the parameter $\eta = \alpha/E$, the average number of ionizations per electron volt obtained from the field. $1/\eta$ could be considered the mean work required to produce an ion pair in the avalanche.

Figure 8 reveals that η has a maximum for argon as the fraction of energy that goes into ionization. Initially increases to a maximum around $E/P = 200$ volts/cm/torr and

then decreases as the amount of energy going to heat the electron energy distribution increases.

It is the energy that goes to heating the electron cloud that distinguishes the avalanche from the case of primary particles depositing energy as they move through a gas volume and creating an average number of ion pairs per unit energy loss. For many gases this is approximately constant for a large range of incident particle energies. Whereas α is not really constant for any range of E/P .

In attempting to modify the shape of the α vs E curve one can consider introducing competing energy processes. One of the most effective processes for increasing α seems to be the Penning effect.

2.8 Penning Effect

The Penning effect is a way of increasing the amount of energy that goes to ionization by introducing a two step process where one atom is excited by the electron and then ionizes by colliding with another atom whose ionization energy is less than the excitation energy of the first.

The classic examples of the Penning Effect are for interactions between noble atoms and noble atoms. The Argon Neon inter-reaction was studied at some length by Kruithof and Penning [19] using the parallel plate

Figure 11

η versus E/P for the data of References 17 and 19. Notice the large increase in the ionization coefficient for the small admixture of Argon in Neon. This is caused by the Penning effect.

measurements discussed in appendix A. Figure 11 shows their values of η for Neon, Argon and Ne+0.1%Argon.

One can see immediately that there is a substantial increase in the fraction of energy available that goes to ionization. Penning attributes this to the ionization of Argon atoms by collision with metastable Neon atoms. This means that for a diagram equivalent to figure 8 a given fraction of the energy going to exciting neon is transformed into energy causing ionization of the Argon.

The maximum value of η for the mixture is $3.71\text{E}-2$ number/volt at $E/P = 3.0$ volts/cm/torr. $1/\eta = 27$ so on average one ion pair is produced for every 27ev available from the electric field. For pure Neon and Argon the maximum η are $1.493\text{E}-2$ and $2.23\text{E}-2$ respectively, so the minimum energy per ion pair would be 67ev and 45ev respectively. Once again it should be pointed out that these numbers should not be directly compared to the values of the mean energy lost per ion pair created by high energy particles travelling through the gas. This is because in the case of the avalanche, the number also includes the energy required to bring the electrons into equilibrium with the field.

A second observation to be made from figure 11 is that in the region of low E/P , the value of η for pure argon is less than for pure neon. At the same time the ionization

energy of argon is 15.7 eV compared to 21.6 eV for neon. This is contrary to what one might naively expect. The momentum transfer cross section for neon is less than for argon, classically speaking neon is a smaller atom. This reduces both the randomization of the electron motion and the energy lost due to elastic collisions. Hence the electrons have a greater mean energy in pure neon than in pure argon for the same field strength. This is why the maximum "Penning effect" occurs for the mixture of Neon and 0.1% Argon. Kruithof and Penning measured η for mixtures ranging from 1E-4% to 10% argon. They found that the effect initially increases with argon percentage and then drops. The initial increase is due to the increase in argon atoms available to collide with the metastable neon atoms. The drop is due to the cooling effect of the argon as compared to neon.

This cooling effect appears to be unavoidable in practical MWPC gas mixtures. As seen below, a MWPC cannot work with pure noble gases. They require the addition of more complicated molecules to quench secondary effects. These molecules tend to present many more modes of energy dissipation and thus dominate the cooling of the electron population at low E/P. As they are added to be effective quenchers of photons they are also effective at deexciting atoms without necessarily becoming ionized.

However, one may reverse the above discussion and instead of considering the argon to be "cooling" with respect to neon, one may consider the neon to be "heating" with respect to argon. Thus even for their mixture of neon and 10% argon Kruithof and Penning measure values of η above those for pure neon at low E/P. Thus while the "Penning effect" may decrease in importance, the effect of neon on increasing electron temperature may become important.

This has been applied to a practical MWPC gas by Fuzesy et al [20]. They tested various mixtures of argon xenon and carbon dioxide. The carbon dioxide content was fixed at 7% while the others were varied. The ionization energy of xenon is 12.1ev which is above 11.7ev, the first excitation energy of argon, thus the Penning Effect should not have a significant presence. The effect of argon raising the electron temperature relative to xenon will be present. They observe an increase in wire gain at fixed voltage as the xenon concentration is increased up to about 9% and then a decrease in the gain as the xenon content is further increased. They also observe an increase in the level of proportional gain achievable prior to saturation. This may suggest a flattening of the α vs E curve for the argon xenon mixtures which will be discussed in the section reviewing space charge.

This section began by stating that the Penning effect could be used to modify the α versus E curve. Strictly speaking, this is not true as the Penning effect is a two stage process. However, normally the excited atom will not move significantly from the position where the electron excited it before it ionizes the second atom or molecule. This means that the two stage process may be considered to happen at a point in space and so not modify the gain differential equation.

That the two interactions may be treated as occurring at a point in space does not mean they occur at a point in time. This means that they could affect the time structure of the MWPC signal and can be subject to scaling violations. This is because at atmospheric pressure the time at which the electrons move and ionize the gas is much less than the average time between collisions of molecules with molecules, a few hundred psec, which is less than the time taken for atoms to radiate, typically of the order of 10nsec. Thus at atmospheric pressure it is not necessary that an atom be excited to a metastable state in order to ionize another atom by collision before it radiates. If the second variety of atom is only a few percent of the total gas, then the average time between collisions of the second gas with the first becomes of the order of the mean time to radiate. Thus the process becomes sensitive to the time variations cause by changing pressure and do not allow

α/P to be defined as strictly a function of E/P , if these processes are allowed in the definition of α .

Kruithof and Penning worked between $1/100$ and $1/1000$ of atmospheric pressure and even smaller partial pressures, of argon. As a consequence most neon atoms excited to radiative states would de-excite before collisions with argon were probable. Thus they required neon to be in a longer lived metastable state. As mentioned above the added quenchers of practical MWPC gases would be expected to be very efficient at removing metastable atoms.

2.9 Empirical Formulae For Wire Gain

Many authors have proposed empirical formulae for the gas gain. A useful summary is given by Charles [12] and they are reviewed by Paladino and Sadoulet [5]

2.9.1 Townsend's Formula

One of the first empirical formula for was derived by Townsend [21] based on simple assumptions. The first is that the collision cross section was a constant independent of the electron velocity. This was a reasonable assumption at the time. However, theory and experiment have shown it not to be the case. The second assumption was that the

electron had a velocity distribution predominantly in the direction of the field with a negligible random component. This ignores the possibility that collisions would randomize the motion, or convert energy gained from the field into velocity components not in the field direction.

He comments [21]:

"The number of collisions per centimeter depends both on the velocity of agitation and the velocity due to the electric force, except when the latter velocity is very large, and in that case the number of collisions C per centimeter is constant, and the number of paths that exceed a certain value x may easily be expressed in terms of the mean free path ℓ ."

Thus he effectively assumes that the electrons move only in the direction of the field losing almost all their energy in each collision. Put in its historical perspective this is not such a bad assumption. At that time it was impossible to say whether the collisions between electrons and molecules should be treated as between "hard" or "soft" spheres; "soft" being the limit where the electron loses almost all its energy during each collision.

Then if the electron must attain the ionization energy V to ionize the gas it must travel at least a distance,

$$x = V/E.$$

The probability of travelling greater than a distance x before a collision is

$$\exp(-x/\ell).$$

So if each time the electron has a free path exceeding V/E

an ionization occurs, the number per cm or α would be,

$$\begin{aligned}\alpha &= C \exp(-V/\ell E), \\ &= C \exp(-CV/E),\end{aligned}\quad 2.28$$

where $C=1/\ell$ the mean number of collisions per cm.

As shown in figure 12, the overall motion of the electron is in the direction of the field, however the actual integrated distance travelled by the electron is much greater than that given by a straight line. As shown in figure 4 the cross section is also not constant. Thus the above derivation is not justified; however, it is simple and so of some use.

In a MWPC if the field is approximately given by $E=K/r$ near the anode wire, for K a constant, then the gain is given by,

$$\ln G = (K/V) \exp(-CV/E_a), \quad 2.29$$

where E_a is the electric field at the surface of the anode. This assumes that it is legitimate to integrate α from the surface of the wire to infinity, where infinity is a region of negligible gain.

2.9.2 The Formula of Rose and Korff

Ross and Korff [22] derived one of the simplest formula for the gain of the anode wire and hence are widely quoted although their actual result is one of the poorest

$E/P=10$ $E/P=100$ $E/P=500$ $E/P=1500$

Figure 12

Examples of the electron motion in a uniform field. As the field increases the electrons move in a more and more determined way. The electric field was 10, 100, 500, and 1500 volts/cm/torr respectively in the diagrams.

of the empirical formulae. Their approach is interesting in that it deals specifically with the avalanche near the wire. They introduce the ionization coefficient $\kappa(r)$ to determine the gain. Unlike Townsend's coefficient α they allow κ to depend on the history of the field and not just the value of the field at a point. Hence does not assume any state of equilibrium with the field. In the end they make simplifying assumptions which reduce them to a dependence on E only at a point while continuing to ignore any question of equilibrium. For historical continuity the use of κ is retained for those cases where the electron is not in equilibrium with the field.

Ross and Korff retain Townsend's assumption that the random motion of the electron is negligible and the electron moves only along the direction of the field. The electron drift velocity is then equal to its average velocity. At this time, 1941, it was generally accepted that the electron velocity was predominantly random for drift in uniform fields and some attempts at investigating its velocity distribution had been made. Thus Rose and Korff's assumption that the electron only moved along the electric field lines is much less defensible.

Having made this assumption they conclude that κ would be the average of the inverse of the mean free path for ionization,

$$\kappa(r) = [1/l(E)]_{\text{average}}.$$

Using the experimental result that the cross section for ionization may be expressed as,

$$\sigma = a(\epsilon - V),$$

where V is the ionization energy, " a ", a constant depending on the gas and σ the ionization cross section. They define their energy ϵ such that $V=0$ and so have

$$1/l(\epsilon) = aN\epsilon,$$

where N is the average number of molecules per unit volume. It turns out that the cross section does not rise as fast as linearly, but that this is still a good threshold parameterization. This gives

$$\kappa(r) = aN\langle\epsilon\rangle,$$

where $\langle\epsilon\rangle$ is the average energy. Actually it is really the average energy above the ionization energy, a distinction they tend to lose. They then derive that the average energy is proportional to the square root of the field,

$$\langle\epsilon\rangle = (\phi(0)E/aN)^{0.5},$$

and so,

$$\kappa(r) = (\phi(0)aNE)^{0.5},$$

where $\phi(0)$ is a dimensionless correction term assumed near one. Their derivation of $\langle\epsilon\rangle$ amounts to not much more than dimensional predictions.

Simplifying the constants gives

$$\kappa(r) = AE^{0.5}.$$

2.30

Their κ may be identified with α as κ now only depends on the value of the field at a point. In order to integrate

this parameterization of α they must introduce a cut as the integral diverges. Thus the gain is given by

$$\ln G = 2A(E_a)^{0.5} - C \quad 2.31$$

where C is an arbitrary constant.

The difficulty in defining this constant or an equivalent threshold voltage was later discussed by Rose and Ramsey [23].

2.9.3 General Formulae

The above two formulae are actually derived formulae and not strictly empirical formulae. However, their derivations involve assumptions and approximations so severe that they amount to not much more than dimensional arguments. Other authors have tried to calculate a formula for α using specific distribution functions for the electron energies [17,24]. Basically, without arbitrarily adjustable parameters, it is hard to get a good fit.

In order to generalize the discussion of the empirical formulae the above two formulae are generalized to

$$\alpha = AE^\mu \quad 2.32$$

and

$$\alpha = B \exp(-CE^{-\nu}), \quad 2.33$$

where μ and ν are typically 1 or 1/2 for a number of formulae found in the literature [12].

The first formula is integrable for all values of μ , while the second is integrable to a closed form only for ν equal to 1 or 1/2.

Taking the logarithm of both sides of (2.32) gives,

$$\ln(\alpha) = \ln(A) + \mu \ln(E). \quad 2.34$$

Thus a log log plot of α vs E should give a straight line. As shown in figures 5, 11, 14, this is really never true. It is possible to approximate parts of the curve with a line. This is often very useful in examining the effects of small changes. If this is done for an interval centered at E , it is found that μ is generally a decreasing function of E . μ would be large for the initial rapid increase in α and then decrease as the curve starts to flatten out. Integrating (2.32) for cylindrical geometry gives,

$$\ln G = AK^\mu r^{1-\mu} / (1-\mu) \Big|_\infty^{r_a}, \mu \neq 1.$$

This is convergent as $r \rightarrow \infty$ for $\mu > 1$, which is the case for low values of E . It will generally remain preferable to introduce a cut in the integral and add a constant of integration.

Taking the logarithm of 2.33 gives,

$$\ln(\alpha) = \ln(B) - CE^{-\nu}. \quad 2.35$$

This formula does not require that the log log plot be a straight line. It does require that a semilog plot of α vs $E^{-\nu}$ be a straight line. ν equal to 0.5 corresponds to Ward's formula [12] and integrates to ,

$$\ln G = (2BK/C^2)(1+C/E_a^{0.5})\exp(-C/E_a^{0.5}). \quad 2.35$$

Again best results would come from placing a cut in the integral. Ward's formula works reasonably well as ν is often closer to 0.5 than 1.

In considering ν to be a variable it is of interest to introduce the variable,

$$\Delta = (d\alpha/dE)/\alpha, \quad 2.37$$

where Δ/p is again a function of E/p . It will be seen that Δ is of interest in discussing space charge. Differentiating (2.33) yields,

$$d\alpha/dE = \nu CE^{-(\nu+1)} B \exp(-CE^{-\nu}),$$

and so

$$\Delta = \nu CE^{-(1+\nu)}, \quad 2.38$$

or

$$\ln(\Delta) = \ln(\nu C) - (1+\nu)\ln(E). \quad 2.39$$

Thus a log log plot of Δ versus E should be a straight line. Figure 13 gives the results from the simulation model. The derivative was defined by simple subtraction of neighboring points. Despite the crude differentiation a reasonably straight line results. Fitting a line to the data gives $\nu = 0.43$, which is close to 0.5. Fitting a line to $\ln \Delta$ vs $E^{-.43}$ gives $C=4.6E2$ and $B=7.2E4$ using $\nu=0.5$ gave $C=8.5E2$ and $B=4.2E4$. These constants are fitting parameters, attempts to relate them to more fundamental physical quantities is, in general, futile. Figure 14 gives the final fit to the simulation model points for

Figure 13.

Graph of Δ versus E . Graph includes the straight line fit of $\ln \Delta$ to $\ln E$. It is seen that the straight line approximation is reasonably accurate. The fit gave $v=0.43$ and $C=459$.

Figure 14

α versus E for the simulation data with the fit using $\nu=0.43$. The fit gave $C=458$ and $A=71,532$.

$v=0.43$. This demonstrates that it is possible to parameterize the numerical calculations over a wide range of electric field values.

Repeating this analysis for the data of figure 11 gives $v=0.38$ for neon, 0.36 for neon + 0.1% argon and 0.47 for argon. A value of 0.27 was obtained for smaller percentages of argon. Thus the use of an empirical formula can serve to simplify the use of data. Their constants are usually nothing more than arbitrary parameters. Their arbitrary nature makes it difficult to determine whether in fact, discussing the gain in terms of α is valid. Any second order processes that begin to modify the gain tend to become absorbed into the empirical parameters without being noticed.

2.10 Second Order Effects

This section discusses four second order processes that can be responsible for unstable operation or general breakdown in MWPC's. The four are, ionization by positive ion impact, the emission of electrons from the cathode during the neutralization of positive ions, the emission of electrons from the cathode by photoelectric effect, and ionization by double electron impact.

A general formalism is introduced for processes which depend on the number of electrons squared. This then leads to a general functional form for the breakdown which the individual examples follow. Since all four processes can be described by similar functional forms it can be difficult to isolate which processes may actually be dominating. However, all are undesirable and so important with regard to choosing MWPC gas composition.

Of the four processes, ionization by positive ion impact and ionization by double electron impact occur at the same time as the first avalanche, while the other two cause time delayed separate avalanches. Thus the first two change the actual pulse height, the others add spurious signals.

Second order processes are sometimes called β processes after Townsend's second ionization coefficient β . Townsend originally introduced β to describe ionization by positive ion impact.

In the past, most discussion of secondary effects has been with regard to parallel plate geometry. In extending these discussions to cylindrical geometry it is necessary to make some simplifying assumptions about the functional form of the effects in some cases.

It is usually difficult to isolate secondary phenomena due to their similar effect on the gain. This is illustrated by hypothesizing a general form for secondary phenomena and showing its similarity for specific phenomena. A second term is added to the gain differential equation,

$$dN = \alpha(r)Ndr + \beta(r)N^2dr. \quad 2.40$$

In order to separate variables it is assumed that β scales with α in a way such that,

$$b = \beta(r)/\alpha(r)$$

is independent of r . As one is, in general, only interested in threshold behavior this assumption is usually valid. For the parallel plate case α and β are constants and it is explicitly true. Separating variables yields

$$dN/(N + bN^2) = \alpha(r)dr$$

and so

$$N = N_0 \exp(\int \alpha dr) / [1 - bN_0 \exp(\int \alpha dr)]. \quad 2.41$$

As expected $N \rightarrow N_0 \exp(\int \alpha dr)$ as $\beta \rightarrow 0$.

For $b < 0$ the β process limits the gain so that as $\int \alpha dr$ gets large $N \rightarrow -1/b = -\alpha/\beta$.

For $b > 0$ the gain diverges for

$$1/b = N_0 \exp(\int \alpha dr).$$

The divergent case is the usual one, with a rapid transition from β being negligible to breakdown.

2.10.1 Ionization by Positive Ion Impact

The following treatment of ionization by positive ion impact follows that of Townsend [27,28]. Let β = the number of ionizing collisions by positive ions per cm. Let N_0 = the number of initial electrons which it is assumed arrive from a region where there is no amplification. This region is considered to be given by r equal to infinity. In practice infinity is usually less than a millimeter. Let N = the final number of electrons. Let q electrons be produced between $r = \infty$ and $r = z$, then $N - N_0 - q$ are produced between $r = z$ and $r = r_a$. The number produced between z and $z + dr$ is

$$\begin{aligned} dq &= (N_0 + q)\alpha(r)dr + (N - N_0 - q)\beta(r)dr \\ &= (\alpha - \beta)qdr + N_0(\alpha - \beta)dr + \beta(r)Ndr. \end{aligned}$$

Let

$$b = \beta / (\alpha - \beta),$$

and assume b is independent of r . Then

$$dq / [q + (bN + N_0)] = (\alpha - \beta)dr,$$

$$q + bN + N_0 = A \exp[\int (\alpha - \beta)dr],$$

where A is a constant of integration subject to $q=0$ at $r=\infty$, thus

$$A = bN + N_0.$$

This gives

$$N = N_0 + q = (N_0 + bN) \exp[\int (\alpha - \beta)dr] - bN,$$

using,

$$N = N_0 + q \text{ for } z = r_a.$$

Thus

$$N = N_0 \exp[\int(\alpha-\beta)dr] / [1 - b(\exp[\int(\alpha-\beta)dr] - 1)]. \quad 2.42$$

It is seen that this is almost identical to equation 2.41 and in fact, as $b \ll 1$ the two are identical for all practical purposes.

The assumption that b is independent of r is somewhat plausible if, for the short distance that it matters, both α and β may be parameterized as being proportional to E . In this case both the E and hence r dependence would factor out. However, as the positive ions have approximately the same mass as the neutral atoms or molecules around them, they lose on average, half their energy in each collision with a neutral atom and so it is very hard for them to accumulate much of the energy they gain from the field. For normal MWPC operation, ionization by positive ion impact is negligible.

2.10.2 Electrons Released from the Cathode by Positive Ions

Townsend considered the effect of electrons being released as the positive ions are neutralized by the cathode [29,30]. Let b represent the efficiency of this process, i.e. if n positive ions are neutralized at the cathode bn electrons are released. In general, b is a small number, of the order of $1.0E-3$ even for noble ions striking "reasonable" surfaces. b is much smaller for

molecules that have other modes of energy dissipation. This is one of the reasons that they must be added to the gas mixture. The positive ions accompanying the initial ionization are ignored for convenience's sake. They are, in general, negligible and do not always strike the same cathode as the bulk of those generated by the avalanche.

Thus if there are N_0 initial electrons,

$$N_0 [\exp(\int \alpha dr) - 1]$$

positive ions are released in the initial avalanche causing

$$b N_0 [\exp(\int \alpha dr) - 1]$$

secondary electrons to arrive at the avalanche region at a later time. This causes

$$b^2 N_0 [\exp(\int \alpha dr) - 1]^2$$

new positive ions and subsequently

$$N_0 b^2 [\exp(\int \alpha dr) - 1]^2$$

electrons. Thus the final number of electrons would be,

$$N = N_0 \exp(\int \alpha dr) + N_0 \exp(\int \alpha dr) b [\exp(\int \alpha dr) - 1]$$

$$+ N_0 \exp(\int \alpha dr) b^2 [\exp(\int \alpha dr) - 1]^2 + \dots$$

$$= N_0 \exp(\int \alpha dr) / [1 - b(\exp(\int \alpha dr) - 1)] \quad 2.43$$

which diverges when

$$[\exp(\int \alpha dr) - 1] = 1/b.$$

It is seen that this result is identical in form to equation 2.41. Here b is independent of r and so does not require the same approximation as above. It should be noted that b is not independent of the counter voltage. As the detector voltage is increased to increase the gain, the

electric field on the cathode is also increased. This will increase the probability of extracting electrons and hence b .

2.10.3 Electrons Released by Photoelectric Effect

It is possible for photons produced by atoms excited in the avalanche to release electrons from the cathode by the photoelectric effect. Let $\gamma(r)$ be the average number of atoms excited by one electron per cm of drift. Redefine b to be the efficiency for the excited atom to produce a photon which survives to reach the cathode and extract an electron. Then the number of photons produced by N electrons drifting a distance dr is

$$dN_Y = \gamma(r)N(r)dr,$$

where

$$N(r) = N_0 \exp(\int^r \alpha dr'),$$

and so

$$N_Y = N_0 \int \gamma(r) [\exp \int^r \alpha(r') dr'] dr,$$

or

$$N_Y = N_0 I,$$

where I represents the value of the integral. Then by arguments similar to above,

$$N = N_0 \exp(\int \alpha dr) / [1 - bI]. \quad 2.44$$

This has the same divergent form and so is difficult to distinguish from the above process. The integral I is

dominated by the latter stages of the avalanche when the number of electrons is large. Therefore for a fixed gain it depends on the relative number of excitations to ionizations. As seen in figure 7 this can be approximately constant for a large range of Electric field values.

This process and the freeing of electrons by positive ions make it impossible to run a MWPC with pure argon. It is necessary to add another gas like Methane to absorb the photons before they reach the cathode or to de-excite the argon atom by collision before it radiates the photon.

2.10.4 Ionization by Double Electron Impact

Ionization by double electron impact is the case of one electron exciting an atom and a second electron ionizing it before it may de-excite by radiation or collision with another molecule. This is a rare effect for normal MWPC operation, as it depends on electron densities in almost the same way as space charge effects. Thus space charge effects have usually destroyed proportional gain before ionization by double electron impact is non negligible. It is possible that this effect is present at very low pressure where space charge can be reduced. A possible example of this is given in Chapter 5.

The probability of ionizing an excited atom has a lower threshold and can in general be an order of magnitude higher than a normal atom [25,26]. However the density of excited states is, in general, very small. The process is interesting in that its threshold behavior is expected to be similar to the above three processes. However, it is limited by the conservation of energy and so is not divergent.

As discussed in the section on the Penning effect a 20ev electron has a velocity 1000-times greater than that of a thermal hydrogen atom and thus much greater than any other atom or molecule. The electrons may undergo thousands of collisions before an excited atom may de-excite. The avalanche then may be effectively over before a significant number of excited atoms are removed by any means other than electron impact.

This process is geometry dependent because it depends on the densities of the electrons and the excited states, not just their total number. In order to examine its differential characteristics it is necessary to make some simplifying assumptions. It will be assumed that the electrons are uniformly distributed over a thin disk. The size of the disk is absorbed into the definition of β allowing densities to be treated in a one dimensional sense. β is defined by setting

$$\beta N(dN_{\gamma}/dr)dr,$$

as the rate at which excited atoms are ionized by electrons. As before N_Y is the number of excited states and dN_Y/dr their density. The equation giving the number of excited states must be modified to account for their removal due to ionization. Thus,

$$dN_Y = NY(r)dr - \beta N(r)(dN_Y/dr)dr$$

$$dN_Y/dr = YN(r)/[1 + \beta N(r)]$$

The gain equation becomes,

$$\begin{aligned} dN &= \alpha(r)N(r)dr + \beta N(dN_Y/dr)dr, \\ &= \alpha(r)N(r)dr + \beta Y N^2/[1 + \beta N(r)]dr. \end{aligned}$$

In the region where this is a small effect βN is small compared to $\beta Y N^2$ and the equation reduces to the same form as the general secondary process equation.

For a general solution if it may be assumed that β and γ/α are approximately independent of r , then

$$dN/[N + \frac{\beta Y}{\alpha} N^2/(1 + \beta N)] = \alpha dr$$

letting

$$b = \beta(1 + \gamma/\alpha),$$

gives

$$(1 + \beta N)/(N + bN^2)dN = \alpha dr$$

or

$$N/[1 + bN]^{\frac{\gamma}{\alpha+\gamma}} = N_0 \exp(\int \alpha dr).$$

This expression is not divergent since for $bN \gg 1$,

$$N = b^{\frac{\gamma}{\alpha+\gamma}} [N_0 \exp(\int \alpha dr)]^{\frac{\alpha+\gamma}{\alpha}}$$

This is no longer linear in N_0 but not divergent. This process is limited by the conservation of energy as was the Penning effect. In some ways it resembles an inefficient density dependent Penning effect.

In order of magnitude terms, β would depend on the cross section for ionizing an excited atom multiplied by the ratio of true electron velocity divided by drift velocity. It would be negligible except at low pressures where the space charge effect is reduced. However, in this region the time scale on which the electrons have collisions will approach the decay time of the atoms and this process will no longer scale in a simple way.

2.11 Space Charge Effects

Space charge effects are different from the above secondary processes in that they do not scale with pressure and distance. Space charge increases as pressure increases.

To see this consider Poisson's equation:

$$\nabla_r^2 V = -\rho'/\epsilon_0$$

where ρ' is the charge density. Rewriting this in terms of the scaled distance s in cm^2torr gives

$$(1/P^2) \nabla_r^2 V = -P\rho'/\epsilon_0 P^3.$$

Or

$$\nabla_S^2 V = -P\rho/\epsilon_0$$

where now ρ is the charge per unit volume in terms of s .

$$\nabla_S^2 V = -Pen/\epsilon_0$$

where e is the charge on the ions and n is the net number of ions per $(\text{cm}^3 \text{ torr})$. n is then the number of positive ions minus the number of negative ions.

As discussed above it is the energy available to the electron in terms of the distance between collisions that is important and this is seen to depend on the product of pressure, charge and number density. Thus for the purposes of discussing space charge effects on the gain it would be possible to define an effective charge q on the ions according to

$$q = Pe.$$

Holding all other things equal, space charge would increase linearly with pressure as discussed in Chapter five.

It is instructive from the point of view of understanding past literature, to consider the case of space charge for a parallel plate chamber with constant voltage between the plates. Following Von Engel [31] assume that it is permissible to integrate α in the presence of a space charge distortion of the field. Thus the gain to second order is given by

$$\begin{aligned} \ln(G) &= \int_0^d \alpha dr \\ &= \int_0^d (\alpha_0 + \alpha/dE \delta E + 0.5 d^2 \alpha / dE^2 (\delta E)^2) dx, \end{aligned}$$

Where α_0 , $d\alpha/dE$ and $d^2\alpha/dE^2$ are constants. Thus

$$\begin{aligned}\ln(G) &= \alpha_0 d + d\alpha/dE \int \delta E dx + 0.5 d^2\alpha/dE^2 \int (\delta E)^2 dx, \\ &= \alpha_0 d + d\alpha/dE \cdot \delta V + 0.5 d^2\alpha/dE^2 \int (\delta E)^2 dx.\end{aligned}$$

It has been assumed that the voltage difference between the plates has been held constant and so $\delta V=0$ and the first order term vanishes. As the integral of $(\delta E)^2$ is a positive quantity, whether the space charge effect increases or decreases the gain depends on the sign of $d^2\alpha/dE^2$. Thus in the past, much of the breakdown or sparking criteria has been discussed in terms of $d^2\alpha/dE^2$ and the first derivative has been ignored,

This is not the case for cylindrical geometry where to first order,

$$\begin{aligned}\ln(G) &= \int (\alpha(r) + d\alpha/dE \cdot \delta E) dr \\ &= \int \alpha(r) [1 + \Delta \delta E] dr,\end{aligned}$$

Where $\alpha(r)$ assumes those values it would in the absence of space charge. Δ is as defined in equation 2.37.

$$\Delta = (1/\alpha) d\alpha/dE.$$

Δ represents in some sense the fractional increase or decrease in α due to the space charge effect. It is a more intuitive parameter than the straight first derivative of α . As seen in figure 13, it is decreasing function of E .

2

Chapter 5 treats two practical examples of space charge distortion of the gain. In the light of these examples it appears as if the second order term dominates. The results will be seen to be compatible with the assumption that the optimal operational point is that where the second derivative vanishes. However, a larger number of examples are needed to resolve the situation.

2.12 Pulse Rise Time Dependence on the Ionization Coefficient

The pulse that comes out of a MWPC is caused by the charges induced on the anode and cathode by the free ions moving in the gas volume. As the MWPC acts as a capacitor, the signal from the anode is equal to the negative of the cathode signal. The signal begins as the avalanche begins. As the growing cloud of electrons moves toward the anode away from the positive ion cloud, there is a net positive charge induced on the anode. This is seen as a negative signal moving from the anode. At the same time a net negative charge is induced on the cathode. As this negative charge is drawn from the outside of the chamber, it appears to be an outgoing positive signal.

This is the first of two stages of the pulse development. This first stage is due to the electron motion, which is much faster than the positive ion motion. The second stage is due to the positive ions. The avalanche stops when the electrons reach the anode wire. Part of the charge of the electrons is neutralized by the positive charge induced on the anode during the first stage. The rest takes the place of the charge induced by the remaining free positive ions. As the positive ions drift toward the cathode the amount of negative charge induced on the cathode increases as that on the anode decreases. This second stage is the dominant process in producing the MWPC signal. It is also the slowest and will be treated in section 3.6 as a necessary correction to the gain measurements.

As the first stage of the signal is only a few percent of the total and happens quite quickly, it is difficult to measure directly. However, since α determines the spatial extent of the avalanche it also determines what fraction of the total charge will be a result of the motion of the electrons and what part is a result of the motion of the positive ions.

In the old ionization chambers or the present multi-step chamber using parallel plate geometry, these two stages are again present. However, their relative importance is reversed, i. e. the useful part of the

signal is the first stage or the fast rise due to the electron motion. The tail due to the positive ion drift is, in general, ignored.

For the constant electric field in a parallel plate chamber α is a constant and as given above the gain is,

$$G = \exp(\alpha d)$$

for d the plate separation. If the effect of the movement of the positive ions is ignored, then for an electron starting at the cathode there will be,

$$Q_I = (1/\alpha d) \exp(\alpha d), \quad \exp(\alpha d) \gg 1, \quad 2.45$$

electrons induced on the cathode as part of the first stage.

For a gain of $1.0E5$ this means that approximately 8.7% of the total signal is controlled by the speed at which the electron cloud drifts. The remainder is in the tail and depends on the motion of the positive ions.

If the usual proportional tube is considered, then for a single electron coming from the region of no gain,

$$Q_I = (1/\ln(r_c/r_a)) \int \alpha(r) \ln(r/r_a) \exp(\int^r \alpha(r') dr') dr, \quad 2.46$$

electrons will be induced on the cathode as a result of this first stage of the avalanche.

As an aside, it is because contributions to the integral are negligible except for very near the anode wire, that the first stage of the signal is so fast. The

speed of this mechanism would constitute the physical limit to separating the single clusters produced by an incident ionizing particle. It suggests that the actual limit which physics places on the measurement of individual primary clusters is of the order of a few tens of picoseconds. In reality the statistical fluctuations of the avalanche process will smear out the time structure of the avalanche. This is a result of the deterministic nature of the differential equation using α being only approximately valid.

2.13 Theory of the Experiment

This Chapter has outlined some of the possible mechanisms that can affect the gain of a MWPC. It has been seen that the simplest behavior is that where the gain is strictly determined by the integration of the Townsend coefficient α , section 2.1. A simple simulation method for calculating α for high values of the electric field for a simple gas like argon has been outlined, section 2.5. All of the empirical formulae for the gas amplification of a MWPC rest on the assumption that the gain is given by the integration of the Townsend coefficient. Testing the validity of this assumption is the experimental focus of this thesis.

As it is impossible to achieve reasonable gain using pure argon due to cathode emission of secondary electrons, it has been decided to work with 80% argon and 20% methane. The methane has been added as an effective absorber of the ultra violet photons emitted by the argon [32] and as its ionization energy 14.5ev is slightly less than argon it effectively removes the argon ions from the gas. The methane ionization energy is at the same time large enough that one does not have to worry about the Penning effect as it is above the lower and most probable excitation levels of argon.

Using this gas mixture a series of gas amplification measurements were performed. From these data points, a scatter plot of $\ln G/K$ versus E/P revealed the existence of a region compatible with the assumption that $\ln G/K$ is a function of E/P . This relation is required by the assumption that the Townsend coefficient is valid, as seen in section 2.3. The same plot also revealed the existence of a region where it was not valid. Further investigation revealed that this region is compatible with the assumptions associated with the non equilibrium coefficient κ defined in section 2.2. This illustrates the reason that the empirical formulae are effective in some cases, while not effective in others.

Using the fact that space charge effects do not scale with pressure as described in section 2.11, it is possible to identify the regions where the scaling laws of section 2.3 are violated as being caused by space charge modification of the electric field.

CHAPTER 3

EXPERIMENTAL MEASUREMENTS

This chapter describes the apparatus and techniques used to measure the gas amplification in 80% Argon and 20% Methane. The technique used to measure the gain was very standard. Simple proportional tubes described in section 3.1 were mounted in a general test facility of the laboratory. These tubes provided the standard cylindrical geometry used in the measurement. A monoenergetic Fe55 X-ray source was used as the source of primary ionization. By measurement of the total charge after gas amplification it was possible to deduce the final absolute gain of the system.

Section 3.3 describes some of the physical principles associated with the X-ray source that were used to confirm that the measurement was, in fact, the amplification of the original X-ray ionization. That the absolute gain was indeed measured was confirmed using the principles of section 2.3.

The final result of this chapter is a measurement of the gain as a function of voltage for four different wire radii over a range of different pressure values. These data points are then used in Chapter 4 to illustrate over what range the gas amplification may be given by the Townsend coefficient and hence parameterized by the standard empirical formulae. It gives the experimental evidence proving that the Townsend coefficient is not universally applicable.

3.1 Mechanical Description

The proportional counters used for measuring the gas amplification were mounted inside a pressure vessel as shown in figure 15. The pressure vessel consisted of a 96cm long half inch thick steel tube with an eight and three quarter inch inner diameter. At the ends of the tube were double welded steel flanges. The steel flanges were also of one half inch steel and extended the outer cross sectional diameter to eleven inches. These were machined flat to present a surface for an O-ring seal. All electronics connections to the test proportional chambers are accomplished through the end plates. Thus the end plate would be designed for each experiment. All gas handling, temperature and pressure monitoring are handled by fittings to the body of the tube.

Figure 15

The experimental apparatus.

There were five proportional tubes used in this experiment. Each was 40cm long and constructed from 6.0cm inner diameter aluminum tube. To permit easy circulation of the gas, the ends were left open except for a lucite support for the central anode wire. Gain measurements were made with wires referenced in the thesis as being of 10, 12.5, 100, and 400 micron radius. The actual measured values of the radius used in the calculations were 9.62, 12.3, 100, and 406 microns. The fifth tube mounted on the second end plate had a 62.5 micron radius wire and was used exclusively as a monitor for changes in gas composition over the period of measurements. The monitor will be discussed more fully below. The 10 micron radius wire was of gold plated tungsten. The other wires were of stainless steel. This caused inferior operation for the case of 12.5 micron radius wire but was quite satisfactory for the thicker wires. This is due to a reduction in sensitivity to surface irregularities as the wire radius increases.

The two end plates were made of three eighth inch aluminum plate with machined O-ring grooves. Each had two high voltage feed throughs, one in the center, the other 3cm above center. Each was attached to the steel tank by means of eight bolts. The proportional tubes were supported on G-10 rods which screwed into the end plate.

The Fe55 source was mounted on the outside of the center of the aluminum tubes. There was a small hole through which the X-rays passed into the gas volume of the counter. This is discussed further in section 3.3.

3.2 Electronics

The electronics used to make the measurements are shown in Figure 16. Both the anode and cathode were connected through high voltage feed throughs. For all measurements discussed here, the cathode was at negative high voltage and the anode at ground. The low pass filter was introduced just before the cathode in order to reduce external noise. The current from the high voltage supply was monitored to check for breakdown in the chamber or at the connectors. This was done on the power supply side of the filter.

At the end plate, the anode went directly into an Ortec 120-2b preamplifier for measurements of low proportional wire gas gain and into an Ortec 113 preamplifier for high gas gain. The input of the Ortec 113 preamplifier was modified with a 1 megohm resistor to ground to maintain the anode wire at ground.

Figure 16

Block diagram showing the electronics.

The Berkeley Nucleonics PB-4 Pulse Generator was connected to the test input of the preamplifiers in order to monitor and calibrate the electronics gain of the entire system. The calibration was done in the standard fashion. The pulser fed a voltage into the preamp with a 50nsec rise time and a 200 microsecond fall time. For the case of the Ortec 120 preamp, 1mV corresponded to injecting 1fC into the preamp while the exponential decay back to baseline occurred with a time constant much larger than the Ortec 450 Research amplifier differentiating time constant.

The pulser was also used to monitor the electronics gain stability and repeatability to within 5% over the many orders of magnitude that the gas gain was to be measured. The monitoring of the electronics gain involved assuring that the pulser calibration after each spectrum was in agreement with that expected from the Ortec 450 amplifier gain settings. Both were subject to human error. The Ortec 450 fine gain adjustment was continuously variable and required an operator to accurately record its setting. The pulse generator had a pedestal of a few millivolts subject to a continuously variable voltage setting with a slight misadjustment of the zero. These two factors dominated the electronics error.

The output of the preamp went to the normal input of an Ortec 450 research amplifier. The research amplifier had a discretely variable coarse gain between 5 and 2000 and a continuously variable fine gain between 0.5 and 1.5. The integration and differentiation times were separately variable between 0.1 and 10 microseconds. For optimal noise considerations, they were fixed at 1.5 microseconds for the measurements. There were three outputs from the amplifier, the fast bipolar went to the discriminator, the delayed unipolar to the LeCroy qVt multichannel analyser and the delayed bipolar to a Tektronix oscilloscope for signal observation.

The qVt was externally triggered by the gate generator. The qVt was controlled and read out by a PDP-11 computer through a CAMAC interface. The data points were subsequently transferred to a Vax computer for off-line analysis.

3.3 X-Ray Source

Measurements of the gas gain require a reliable source which provides a known number of initial electrons. A simple and distinct point was provided by the center of the main peak produced by the Fe55 5.9keV X-ray in Argon. Figure 17 gives a typical Fe55 spectrum in Argon.

Figure 17

Typical Fe55 spectrum taken using the 100 micron wire. The pressure was 61 Torr and the cathode voltage was -1.1 kvolts. The results of the fit are also shown.

This peak position could be extracted to an accuracy exceeding that of the electronics calibration by means of a simple Gaussian fit. The fit to the Fe55 data was done in three steps to ensure reliability. During the data-taking procedure the operator recorded an approximate estimate of the position and standard deviation of the two peaks characteristic of the Fe55 spectrum. A separate fit to each peak was done to the data within approximately one standard deviation of the estimate. Then the procedure was repeated using as a starting point the assumption that the channel with the most counts closely approximated the center of the main peak. From the best of these two fits an estimate of the center of the main peak was extracted. Using this new value the fit was repeated using data within one standard deviation with the standard deviation fixed at the fitted value. The final step was done to avoid any bias due to possible peak asymmetry.

The Fe55 nucleus decays by electron capture or

with a half life of 2.7 years. The decay is most likely to involve a K shell electron. The escaping neutrino carries off the difference in energy between initial and final states. The result is an excited neutral Manganese atom with a single K shell vacancy.

The excited atom may then radiate by means of an L shell electron falling down to fill the K shell. The energy difference between the two states is radiated off as a photon or X-ray. The K shell energy of Manganese is 6.542keV [33]. In order to conserve angular momentum the dominant reaction is for the p wave electron of the L shell to fall into the empty s wave state. For the L shell p wave the $j=1/2$ state has energy 0.656keV and the $j=3/2$ state 0.644keV. So the two transitions result in either a 5.886keV or 5.898keV photon. There are twice as many $j=3/2$ electrons as $j=1/2$ so there would be approximately twice as many 5.898keV photons as 5.886keV, for a weighted mean energy of 5.894keV. As will be seen, due to statistical fluctuations in both the primary ionization and the avalanche it is impossible to distinguish between these two cases with a proportional chamber. Thus the Fe55 is considered to be a source of monoenergetic 5.9keV photons. In the old X-ray notation the $j=1/2$ and $j=3/2$ states of the L shell are termed L_{II} and L_{III} respectively.

The count rate of the proportional tube depends on the absorption probability of the X-ray in the filling gas. This involves both pressure and geometry concerns. In deciding what count rate is desirable, a fast rate leads to faster and more efficient data acquisition, while too rapid a rate leads to distortion of the resolution.

In order to adjust the count rate a collimator was made by drilling out the center of a brass screw. If the source is assumed to be a point at one end of the collimator emitting X-rays isotropically in space, then the number of X-rays emerging from the collimator would be,

$$N = N_0 [1 - \ell / (\ell^2 + x^2)^{1/2}]$$

Where ℓ is the collimator length and x the internal radius of the hole, 0.8mm. The count rate was measured for $\ell = 16.3\text{mm}$ and then increased to the final value by reducing ℓ to 10.5mm.

In deciding the final value of the count rate, it was necessary to take into account its pressure variation. If the X-ray does not interact in the gas it must travel the length of the collimator, ℓ , and the diameter of the tube, 6cm, without being absorbed by the Argon. The X-ray absorption depends almost exclusively on the Argon concentration and not on the Methane concentration because the 5.9keV is close to the Argon K-shell energy. The X-ray mean free path in Argon at atmospheric pressure and temperature is about 2cm; for Methane it is about 2m [34]. Thus for 80% Argon the count rate would be given by,

$$R = N_0 [1 - \ell / (\ell^2 + x^2)^{1/2}] [\exp\{-k(\ell - x_0)P\}] [1 - \exp\{-k(6.01 + x_0)P\}]$$

where

$$k = 0.80/\lambda$$

for P in atmospheres and λ the mean free path at

atmospheric pressure. x_0 is a correction that must be introduced to account for the fact that some of the X-rays that convert in the gas inside the collimator, cause ionization that drifts to the anode. In practice this leads to more of a smear that contributes to the background, as well as the main peak, but for simplicity we describe it by defining x_0 as the distance into the collimator where converting X-rays contribute to the main peak.

Figure 18 shows some of the experimental values of the count rate plus a fit to the above formula. The fitted value of x_0 was 6.4mm and λ was 2.1cm. The points below 100 Torr were taken with the 16.3mm long collimator and have been corrected according to the formula for collimator attenuation. The points above 500 Torr have larger errors due to manual time measurement. The other points were taken using a PDP-11 computer with an internal clock. The formula gives a reasonable fit considering the crude nature of the x_0 parameter.

The fact that the rate is 100 times more sensitive to Argon concentration than to Methane concentration, means that it could be used to measure Argon partial pressure independent of the total gas pressure provided one worked on the lower part of the rate curve. This was not done in the present experiment but could prove useful in a future experiment demanding more stability. That the Argon K shell absorption is the dominant effect is also

Figure 18

Plot showing the increase in count rate with pressure. The first two points were taken with the 16.3mm collimator and are adjusted to correspond with the other points.

corroborated by the fact that this assumption leads to an accurate description of the observed spectra.

The discussion is now restricted to the 5.9kev photon's effect on the 80% Argon and 20% Methane mixture of our proportional tubes. The dominant process is then for the photon to be absorbed by an Argon atom with a K shell electron being emitted from the Argon. Argon has a K shell energy of 3.200kev compared to Carbon's 0.284kev [33].

When the photon ejects the electron from the K shell the electron has 2.7kev in kinetic energy, the difference between the photon and K shell energy. The Argon $j=1/2$ and $3/2$ L shell p wave energies are 0.246 and 0.244 kev respectively [33]. Thus in the same manner discussed above it is possible to radiate a photon of 2.954kev or 2.956kev which is below the Argon K absorption edge and so has a high probability of escaping from the proportional tube. A second atomic de-excitation process is an internal rearrangement of the atomic orbitals that results in the emission of an Auger electron while the K shell is filled.

These two processes result in two peaks in the pulse height spectrum of the proportional tube. The largest is the Photo absorption peak corresponding to the deposition of almost the full 5.9kev in the chamber (Auger de-excitation); the second is the escape peak representing the case when the 2.954 or 2.956kev photon is radiated and

escapes. Thus the escape peak corresponds to about $1/2$ the energy and hence $1/2$ the initial number of ionization electrons as the main photoabsorption peak.

In the absence of any background, the fraction of the total number of counts in the escape peak is called the Fluorescence Yield. Sauli [35] quotes a value of 15% for Argon based on an earlier article [36] which quotes experimental values from 7% to 15%. The Fluorescence Yield increases with atomic number and a rough interpolation of the data of [36] seems to favour a value of approximately 12.5%. Curran [37] quotes proportional counter measurements yielding a value of $11 \pm 1\%$. The Fluorescence yield is difficult to measure without paying special attention to background and collimator design. Without any special attention paid to the background the present data gives $10 \pm 3\%$.

The average number of electrons before gas amplification in each peak depends on the way in which the emitted electrons are thermalized and the excited and ionized Argon atom is neutralized. The common way to consider the thermalization of the electrons is the concept of the mean energy required to produce an ion pair, w . Sauli [38] gives $w=26\text{ev}$ for Argon and $w=28\text{ev}$ for Methane. With these figures 5.9kev would correspond to an average of 223 electrons. The numbers are only approximate but are sufficient for calculating the chamber gain.

The concept of a mean work to produce an ion pair is expected to be fairly accurate for the thermalization of the one or two 2.7keV electrons corresponding to the bulk of the escape or main peak energy. However, how the .25keV or .5keV that remains in the excited Argon ion is dissipated, is less clear. Our data cannot resolve this difference because in both extremes the main peak is twice the escape peak.

These numbers are also subject to statistical fluctuations. The standard deviation of these fluctuations δN is usually described by the Fano factor such that

$$(\delta N/N_0)^2 = F/N_0,$$

where F is the Fano factor and N_0 is the mean number of electrons. Recent measurements of F for Argon Methane mixtures are not completely consistent. Kase et al [39] give 0.18 ± 0.01 for Argon plus 10% Methane using alpha particles, Hurst et al [40] give $0.21 \pm .10$ for the 5.9keV Fe55 X-rays and $0.31 \pm .10$ for 2.6keV Ar37 X-rays using the same gas mixture. This makes it difficult to make absolute gain fluctuation measurements. The simple model of Appendix B predicts that the standard deviation of the total pulse height δP is given by,

$$\begin{aligned} (\delta P/P)^2 &= (\delta N/N_0)^2 + 1/N_0 \\ &= (1+F)/N_0. \end{aligned}$$

or

$$\delta P/P = 7.3\%$$

for $F=0.18$ and $N_0=223$. This small width makes the Fe55 spectra appropriate for gain measurements.

In practice the simple model of Appendix B is only approximate and the statistical fluctuations tend to be somewhat less for thinner wires due to the breakdown of the assumptions on which integration of the Townsend coefficient is based.

3.4 Data Taking Procedure

The data taking procedure involved a large amount of human interaction with the apparatus. This contributes to the errors involved in the final data points.

The procedure was controlled by a data acquisition program that ran on the PDP-11. The program was based on a check list principle. A gain curve is defined as a series of gain measurements at different voltages made at a fixed pressure. The operator would manually select the pressure. With the gas inside the pressure vessel fixed the operator then selected the voltages at which the gain would be measured. The amplifier gains and discriminator threshold were then adjusted to obtain a reasonably placed Fe55 spectrum, typically with the photopeak near channel 600 on the 1024 channel qVt analyser. The qVt was then started by the PDP-11 on command of the operator. Reasonable spectra

could be accumulated in a couple of minutes. During this time the operator was required to go through the check list at the prompt of the computer.

The check list began with the monitor proportional counter and amplifier settings including those which were held constant throughout the measurements, but could be accidentally varied such as time constants. All answers were recorded in the data file so that it would be possible to confirm the settings at a later time.

For all the Fe55 data pulse height spectra the following key variables were recorded:

- Cathode Voltage
- Amplifier Course Gain
- Amplifier Fine Gain
- Channel of Noise Cut
- Estimates for fitting of:
 - Photo Peak Channel number
 - Escape Peak Channel number
 - FWHM of Photo Peak
 - FWHM of Escape Peak
- Pressure
- Temperature
- Start time of the qVt
- Stop time of the qVt

After the qVt was read out it was stored and the pulser was turned on and the channel number of the pulser was recorded for at least three pulse voltages.

All of the information was stored in a data file that was transferred to the Vax computer. On the Vax all the files corresponding to a fixed pressure were appended into one file. Thus each gain curve was stored in a single

file. A Fortran program was used to fit the histograms and print the resultant values of peak position, gain and other relevant parameters in a more compact file. From this stage a number of scanning or monitoring programs were used to spot the more serious typing errors. These were corrected and final condensed files were produced for later use.

This procedure was followed to obtain gain measurements for the four different wire radii as a function of voltage and pressure. Figures 27, 28, and 29 give the final values of the gain measurements..

3.5 Pressure and Temperature Measurement

Temperature and pressure are both important as they affect the gas density and must be corrected for. Pressure is the most important as it is also a parameter in the study.

Both temperature and pressure were manually recorded. At the request of an on-line data acquisition program the present temperature and pressure were typed manually to the output file. Thus the possibility of human error cannot be ignored. In a log book the approximate value of the pressure was recorded for each series of gain measurements as a method of checking for large errors.

The pressure was measured using a Datametrics Barocel model 590-3000T-2Q1-V1X-4D pressure transducer. The transducer had an operating range from 0 to 3000 torr with an output from 0 to 1 volt. It operated with 1/3 of a mV equal to 1 torr with a specified accuracy of 0.05% of the reading. The transducer was based on a metal diaphragm that moved with respect to the gas pressure and so measured absolute gas pressure independent of gas composition. For each data point the pressure was read out four times spread over the course of the time the individual spectrum was taken. Major deviations from the log book value of the pressure were found by an external scan of the preliminary data.

To monitor possible finer deviations the pressure vessel had a Kulite pressure transducer, model number XCQ-093-50A installed inside it. It had a specified sensitivity of 0.0277 mV/Torr. As the transducer output voltage was recorded by the program and not the converted pressure, the difference in the sensitivity of the two transducers made subconscious correlation of an error unlikely. The Kulite pressure was recorded once per spectrum as a monitor to the effect of human error on the pressure measurement. Figure 19 shows a scatter plot of the resultant pressures. Four points have been excluded from the figure. Of the 250 data points where the Kulite pressure transducer was in use only these 4 differed by

Figure 19

Comparison of the Barocel 590 and Kulite Pressure Transducers. The consistency between the two readings permits a monitor of the data taking procedure.

more than 50 torr.

Inside the tank there was an Analog Devices AD590L temperature transducer. This was also recorded four times per Fe55 spectrum. At the end of the histogram accumulation, the air temperature was also read from a thermometer outside the tank. This permits only an approximate check on the temperature inside the tank as the two need not be the same. It takes time before changes in the lab temperature are conducted through to the inside of the sealed vessel.

3.6 Correction For the Time Structure of the Signal

As discussed in section 2.12 the signal from the proportional wire consists of two stages, the first due to the motion of the electrons and the second due to the motion of the positive ions. For the purposes of absolute gain measurements, the first stage is negligible. Thus in this section it is only necessary to discuss the effect of the movement of the positive ions on the time structure of the anode pulse. It is assumed the positive ions are produced in the avalanche process very near the anode wire. The output signal ends when these positive ions reach the cathode. This is expected to take of the order of 1 to 10 msec for a 3cm anode cathode separation. This signal is measured using the 1.5 microsecond time constants of the

Ortec 450 Research Amplifier. Thus it is necessary to correct for the fact that only part of the signal produced by the motion of the positive ions is sampled.

It is assumed that the rate of drift of the positive ions may be described in terms of a mobility constant μ , such that the drift velocity of the positive ions may be expressed as

$$\begin{aligned} dr/dt &= \mu E/p \\ &= \mu K/rp. \end{aligned}$$

In the literature the pressure is often assumed to be constant and absorbed into the definition of μ .

Integrating the above equation gives,

$$r(t) = (r_a^2 + 2\mu Kt/p)^{0.5},$$

or

$$r(t)/r_a = (1 + t/t_0)^{0.5}$$

using

$$t_0 = r_a^2 p / 2\mu K.$$

The charge induced on the cathode, and consequently released from the anode, when a charge Q_0 is at a position $r(t)$ is given by

$$Q(t) = Q_0 \ln(r(t)/r_a) / \ln(r_c/r_a),$$

for cylindrical geometry or

$$\begin{aligned} Q(t) &= 0.5Q_0 \ln(1 + t/t_0) / \ln(r_c/r_a) \\ &= Q_0 C \ln(1 + t/t_0). \end{aligned}$$

The most usual applications relating to MWPC's t_0 is of the order of 1-2nsec. In the present application t_0 varies between 0.34nsec and 307nsec assuming $\mu=147\text{mm Torr/Vmsec}$ [41] for Methane ions drifting in 80% Argon and 20% Methane.

The two extreme values of t_0 correspond to the 10 and 400micron radius wires respectively. It is of interest to consider the expected position of the positive ions after 1.5microseconds of drift. For the $t_0 = .34\text{nsec}$ case the ions are at $r=0.67\text{mm}$ and 52% of the total charge has been induced on the cathode. For the $t_0 = 307\text{nsec}$ case, r is 0.97mm and only 21% of the charge is induced on the cathode. So naively one would expect to measure about 52% of the total charge in one case and 21% in the other. This effect must be compensated for, if one is to compare the gain measurements for the two cases and all the other cases in between.

As it turns out the naive idea is fairly accurate but since the more detailed calculation is quite standard it will be discussed briefly, see for example, Breyer and Cimermann [42]. They have calculated the correction for the active filter used in the Ortec 450, for values of τ/t_0 from 60 to 1600 where τ is the amplifier time constant. The correction was required for values of τ/t_0 from 5 to 4400 and so their calculation was repeated.

Given the response of the amplifier to an impulse input it is possible to calculate it for general input. Thus the calculation of the output pulse requires numerically evaluating the convolution,

$$O(t) = Q_0 C \frac{e^2}{2} \int_0^t \ln(1+t'/t_0) \exp[-(t-t')/\tau] \{ \sin[(t-t')/\tau] + \cos[(t-t')/\tau] - 1 \} dt'.$$

Figure 20 gives an example of the expected amplifier output signal. The qVt samples the maximum of the pulse and this normalized by C is plotted in figure 21. The maximum of the output pulse, O_m , is then a measure of the fraction of the total charge that is measured. Our points agree with those of Breyer and Cimerman [42] to within 0.1%.

Figure 21 compares the calculated values with the naive assumption that all the charge up to $t = \tau$ is measured and none after. This corresponds to $O_m/C = \ln(1 + \tau/t_0)$ and effectively ignores the effect of the actual pulse shape as given by the convolution. Notice that the point where the signal actually stops is t_f given by $r(t_f) = r_c$ and then $1/C$ is equal to $\ln(1 + t_f/t_0)$ or $t_f = t_0(r_c/r_a)^2$.

All of the above is, of course, based on the assumption that the positive ion drift velocity is described by a mobility constant valid over a large range of field values.

Figure 20

The calculated output pulse shape for the Ortec 450 amplifier. Calculations were for the case $\tau/t_0 = 1000.0$

Figure 21

Correction to the measured value of the charge to obtain the total charge, using both the calculated and naive approach.

It is interesting to observe that in considering the gas gain to be a function of K/s one may write,

$$\begin{aligned}\tau/t_0 &= \tau 2\mu K/r_a^2 p \\ &= (2\mu\tau/r_a)(K/s),\end{aligned}$$

thus if there is an error in μ it will cause a systematic shift that is constant for constant wire radius, however it changes for the different wires. It is hoped that this effect would be no more than 1% and this seems to be confirmed in section 3.8.

3.7 Monitor Wire

While measuring the wire gas gain of each sample counter as a function of cathode voltage, the gas gain of a second tube held at fixed voltage at the other end of the pressure vessel was also measured. This was designed to monitor any sharp deviations in the gas composition over the span of time the measurements were made.

This experiment must measure the gas gain over many orders of magnitude for a number of wire radii and pressure values. Thus, it is important to ensure that the running conditions remain stable over the period of days on which the measurements are taken.

In practice it is difficult to define a standard point at which to compare all measurements made at different pressures. In theory this would be possible if all processes other than those described by the Townsend coefficient could be turned off. It would then be possible to define a standard point independent of pressure. From Section 2.3 one has for this case that $\ln G/K$ is strictly a function of X_a , where X_a is the electric field on the surface of the wire divided by the pressure, $X_a = K/s_a$. Hence for a fixed ratio of K/s_a or given fixed geometry and Voltage/pressure, $\ln G/K$ is a constant. However, if one were to vary the pressure by a factor of 10, then the voltage and hence $\ln G$ must also vary by a factor of 10 in order to keep the quantity $\ln G/K$ a constant. This would mean that if the lowest gain one could measure was 10, as is the case in the present experiment, then one would also have to deal with gains of the order of 10^{10} , assuming no space charge or other distortions.

However, as only general trends over days are of interest, it is only necessary to monitor for any sharp deviations from smoothness in the $\ln G/K$ vs X_a curve. This is a bootstrap technique as it begins by assuming no knowledge of the gain process and only that the results have a self consistency. Figure 22 shows the monitor wire gas gain values accumulated over the life of the experiment and they confirm this general consistency.

Figure 22

Graph of $\ln G/K$ versus X_a for the monitor wire. This was used to test for the general repeatability of the data over the run of the experiment.

The monitor data points were obtained in a manner similar to the main wire data. However, only one voltage point was taken for each gain curve on the other wire.

A simple preamp based on the TRA1000 chip was used in connection with a Tennelec TC203BLR linear amplifier. The electronics were actually more general than was necessary as most of the data points were taken with an amplifier gain of between 100 and 200 and a fixed gain would have been adequate. In fact a fixed gain amplifier may have even been preferable as it would have permitted less opportunity for human error.

The chapter now concludes by showing how an analysis similar to the above can be used to look for deviations from the absolute gain measurement.

3.8 Verification of The Absolute Gain Measurement

One of the more interesting ways to summarize the errors involved in measuring the gas amplification is to consider the data accumulated on the thicker wires in a manner similar to the discussion of the monitor wire.

There are two general kinds of error. The first consists of the random elements involved in measuring temperature and pressure and the human contributions. The second consists of the systematic errors. These cause a

bias in the data in a particular way. Of the systematic errors there are three important possible contributors. There is the fact that the system has been calibrated assuming that a 1mV pulse fed into the test input of the Ortec 120 preamp simulates a charge of 1fC. An error in this would contribute a multiplicative error to the gain measurement. The second is the assumption that the 5.9kev photopeak has an average initial ionization of 223 electrons, independent of pressure. This contributes an error identical to the calibration error. The third is a systematic error in the way in which the measurements are corrected for the finite amplifier time constants. This, unlike the above two, will be different for different wires.

Little can be done to improve on the random errors after the data points have been taken. The systematic errors can be corrected if detected, thus they deserve further examination.

Assume that there is a multiplicative systematic error in the gain measurement, E. The experimental measurement of $Y = \ln(G)/K$ would no longer be a strict function of X_a or K/s even if the Townsend coefficient were valid in describing the physics of the gas avalanche. Instead,

$$Y(\text{experimental}) = \ln(EG)/K \\ = \ln(E)/K + Y(K/s),$$

and so a fit of the experimental data would exhibit a

systematic error as a function of $1/K$. E could then be treated as a fitting parameter and the deviations minimized with respect to it.

Figure 23 shows the experimental values of $\ln G/K$ as a function of X_a using the 100 and 400 micron radius wires for X_a less than 100 volts/cm/torr and gain values below 6000. These are the data points most likely to be determined by the Townsend coefficient and as can be seen it seems to follow the assumption that $\ln G/K$ is a function of X_a reasonably well.

Restricting consideration to the 100 micron radius wire data, it is found that the fit to E can be interpreted as corresponding to 226 initial electrons as opposed to the initial assumption of 223. This is, however, still within the errors in the values of the mean work required to create an electron ion pair, w used in the calculation. Using this value and fitting to a simple quadratic, gives a standard deviation in values about the fit of 3.0% associated with the individual measurements. This is less than the 5% fluctuation between the pulser check on the electronics and the amplifier gain settings. This indicates that the electronics were actually more stable than indicated by the pulser.

Figure 23.

Graph of $\ln G/K$ as a function of X_a or K/s using data from the 100 and 400 micron radius wires.

This result should not be taken as an accurate measure of the initial number of electrons, as it absorbs other possible sources of systematic error, notably the time constant correction.

Seventy experimental points from the 100 micron wire data were used in the above fit. Thirty-two of the points using the 400 micron radius wire also satisfied the above cut. Fitting these and considering the error to be dominated by N_0 yielded a value of 228 initial electrons. This is a 1% error most likely associated with the time constant correction.

Using $N_0 = 226$ and looking at the deviations from the quadratic fit, give standard deviations of 3.2% for the total sample, 3.0% for the 100 micron wire data and 3.5% for the 400 micron wire data. Thus it is possible to conclude that the gas amplification on the wire has been measured to within 3.0%, with a possible further systematic effect contributing another 1%.

CHAPTER 4
EXPERIMENTAL EXAMINATION OF THE REGION OF
VALIDITY OF THE TOWNSEND COEFFICIENT

In this chapter an examination is made of the experimental evidence for whether or not the gas amplification in a wire proportional counter may be described by an ionization coefficient that depends only on the value of the electric field at a point. This is what has been defined as the Townsend coefficient. This chapter examines only the low gain data. Discussion of the high gain space charge saturated region has been deferred to the next chapter.

The various empirical formulae that have been developed in the past all rely on the assumption that the gain is determined by the Townsend coefficient. Thus it is natural to examine to what degree an empirical parameterization of the data tests this hypothesis. This permits the conclusion that there exists a region where the assumption breaks down.

More detailed analysis shows that there is, in fact, no region where the ionization coefficient is exactly constant with respect to constant values of E/p . There are, however, still two proportional regions of MWPC behavior. The first is the region approximately determined by the Townsend coefficient. The second is the region where the electrons are no longer in equilibrium with the electric field but the gain is still described by the ionization coefficient α . In this region the gas amplification still retains a symmetry with respect to K and s but is no longer described by a simple one dimensional dependence. Recall from section 2.3 that K is the product of the electric field on the surface of the wire and the wire radius. In other words it is the constant of proportionality between the electric field and inverse radial distance. s is the product of distance and pressure.

The experimental evidence that in the second region the gain does follow the assumption of depending only on K and s , proves the symmetry expected from the discussion of section 2.3. This symmetry allows the consolidation of many proportional chamber design considerations.

One of the most striking results of the present investigation is that what is here defined as the approximate Townsend region is, in fact, smaller than originally expected. In many of the MWPC's constructed at

Carleton University the anode wire has a 10 micron radius. For this wire the gas amplification may not be properly defined by integration of an ionization coefficient assumed to depend only on the value of the electric field at a point under normal conditions. By normal conditions it is meant around atmospheric pressure and the present gas mixture.

In Figure 23 only the data points most likely to be determined by the Townsend coefficient were shown. This corresponds to the simplest assumptions that one may make about the gain mechanism. In Figure 24 the full set of measurements for the 80% argon and 20% methane gas mixture is shown. It is immediately apparent that for low values of E/p the data appear to be a simple function of E/p , however, the functional relationship is lost at larger values. This is a sign that processes other than those described strictly by the Townsend coefficient are at work.

4.1 Empirical Evaluation of The Data

To ignore space charge experimentally requires constraining the gas amplification to moderate values. Figure 25 shows the experimental values of $\ln G/K$ versus X subject to a gain cut of twenty thousand.

Figure 24

Graph of $\ln(G)/K$ as a function of X or K/s for the experimental data sample. This plot includes the experimental values without any cut on the gas amplification, thus it includes those modified by space charge as well as those due to breakdown of electron equilibrium.

Figure 25

Graph of $\ln G/K$ vs X as in Figure 24 except with a cut of twenty thousand on the gas amplification. This is to remove those points that deviate due to space charge saturation. The cut is essentially arbitrary. The remaining deviation at high X is due to the dependence of the ionization coefficient on dX/ds .

Ignoring for the present the question of what is the process determining the gain, it is important to make the empirical observation that $\ln G/K$ is approximately a function of X . In this sense a totally empirical parameterization of the gain is possible. This allows, for example, simple estimation of approximate operating voltages for MWPC's in the design stage. This, in itself, is invaluable in evaluating the interplay of the electric field in drift regions with that of the amplification region, quantities often determined by the same externally applied voltages.

In order to avoid any theoretical bias the data points were parameterized first by three second order polynomials, each for the specific range of X values given below,

$$Y_1(X) = -6.29 \cdot 10^{-3} + 2.53 \cdot 10^{-4} X - 3.20 \cdot 10^{-7} X^2, \quad 40 < X < 100, \quad 4.1$$

$$Y_2(X) = -5.21 \cdot 10^{-3} + 2.50 \cdot 10^{-4} X - 3.85 \cdot 10^{-7} X^2, \quad 100 < X < 200, \quad 4.2$$

$$Y_3(X) = 4.41 \cdot 10^{-3} + 1.52 \cdot 10^{-4} X - 1.27 \cdot 10^{-7} X^2, \quad 200 < X < 400. \quad 4.3$$

The standard deviation between the measured gain and the gain predicted from the fit normalized by the gain was calculated. This was to provide easy comparison to the errors in the gain measurement although the fit was a least squares fit to $\ln G/K$. The two values are similar as to first order,

$$\Delta \ln(G) = \Delta G/G.$$

The results were,

4%, $40 < X < 100$,

8%, $100 < X < 200$,

10%, $200 < X < 400$.

Using the same fitted parameters but including individual gain measurements up to 10^5 the calculated standard deviation became 5%, 11% and 22% respectively. For $X > 200$, the standard deviation is significantly larger than the experimental error on the individual points. The random experimental error is given in chapter 3 as 3% with a possible systematic error of an additional 1%.

This is an indication that $\ln G/K$ is not strictly a function of X , over the full X range. For $X < 100$ the relationship is satisfactory, for $100 < X < 200$ there is some doubt, and for $200 < X$ $\ln G/K$ is clearly not a function of X alone.

The fit to the quadratic polynomials are shown in figure 26a including all points up to a gain of 10^5 . The improvement in representation using the logarithmic axis for X shown in figure 26a suggests fitting the data as a quadratic function of $\ln(X)$. Such a fit for $X < 400$ gives:

$$Y_4(X) = -3.33 \cdot 10^{-3} - 9.10 \cdot 10^{-3} \ln(X) + 2.88 \cdot 10^{-3} [\ln(X)]^2. \quad 4.4$$

The fit used a gain cut of $2 \cdot 10^4$ and is shown in figure 26b for all data up to a gain of 10^5 . The total standard deviation of the points up to 10^5 was 16%. This fit

Figure 26a

lnG/K versus X including the fit to the three polynomials given in 4.1, 4.2 and 4.3. Plot includes all points up to a gain of 10^5 . The logarithmic X axis is chosen as it empirically reduces the steepness of the function.

Figure 26b

Data of figure 26a with the fit to a single quadratic in $\ln(X)$ as described by 4.4.

deteriorates for low X . However, the advantage of the logarithmic fit would be in attempting to extrapolate beyond the data points. The polynomials give a better fit in their limited range, but as they do not match the rapid initial rise they tend to give poor extrapolation.

At this point it should be noted that the 16% standard deviation assumes the errors result from random statistics. As the errors on the points are expected to be no more than 4%, it is clear there are additional systematic contributions associated with the fit. In this case the maximum deviation should be considered. These are +53% and -66% for equation 4.4 and +54% and -84% for equation 4.3. In terms of a multiplicative factor this means that the gain is only parameterized to within a factor of 2.

The fit to the three quadratics represents the limit that bounds all the present empirical formulae for the gas amplification in MWPC's as discussed in section 2.9. The differences in the various formulae only amount to different choices of parameterization. They all have in common the assumption that the avalanche depends on an ionization coefficient that depends only on the value of the electric field at a point and not on its past history. From this assumption it follows that $\ln G/K$ must be strictly a function of X . As the above data points deviate from this assumption one may then conclude that the constraint on the ionization coefficient is not justified.

The fits can then only be used to give an empirical ionization coefficient, one which is not based on the principles of section 2.1 or 2.2.

4.2 The Empirical Ionization Coefficient

It is of interest to take time to see how such an empirical ionization coefficient may be extracted. This coefficient is interesting as it is the assumption on which the empirical formulae developed over the years are based as outlined in section 2.3 and 2.9. These formulae allow easy manipulation of general design parameters.

As explained in section 2.3 it is possible to extract an ionization coefficient from the experimental curve of

$$\ln G/K = Y(X_a) . \quad 2.17$$

The subscript a refers to the value of the field at the surface of the anode and is usually dropped for convenience. The integral of the ionization coefficient is represented by $\ln G$ and hence $\ln G/K$.

Charles [12] has extracted an effective ionization coefficient from an empirical parameterization of this curve, using equation 2.20,

$$z = X^2 dY/dX \quad 2.20$$

Using the parameterizations of equations 4.1 to 4.3 this gives for z the following,

$$z(X) = 2.53 \cdot 10^{-4} X^2 - 6.40 \cdot 10^{-7} X^3, \quad 40 < X < 100,$$

$$z(X) = 2.50 \cdot 10^{-4} X^2 - 7.70 \cdot 10^{-7} X^3, \quad 100 < X < 200,$$

$$z(X) = 1.52 \cdot 10^{-4} X^2 - 2.54 \cdot 10^{-7} X^3, \quad 200 < X < 400.$$

Using the fit to the quadratic in $\ln(X)$ gives,

$$z(X) = -9.10 \cdot 10^{-3} X + 5.76 \cdot 10^{-3} X \ln X, \quad 40 < X < 400.$$

Charles identified this coefficient as being the Townsend coefficient evaluated at X_a . However, in the region where Y or $\ln G/K$ is not properly a function of X , the derivative does not properly exist. By differentiating the above empirical parameterization one obtains only an empirical average ionization coefficient. The reason that this coefficient has been confused with the Townsend coefficient in the past was due to the lack of sufficient points to observe different gains for fixed X .

In order to make a proper analysis it is necessary to incorporate only the assumptions involved in z , the general ionization coefficient which includes dependence on the past history of the field and then to identify those areas where it is equal to the Townsend coefficient. This involves considering the K and s symmetry separately from any possible K/s symmetry.

4.3 Introduction to the K, s Gas Amplification Plane

The common way of displaying gain curves is to plot gain versus voltage for data taken at constant pressure. In terms of present parameters the equivalent approach would be to plot gain versus K for lines of equal s , examples of which are shown in figures 27. Figures 27a, b, c correspond to gain measurements made with the 10 micron, 100 micron, and 400 micron wire radii respectively. The inverse method would be to reverse K and s and plot gain versus s for lines of constant K as in figures 28a, b, c.

Experimentally, the first case corresponds to fixing pressure and measuring gain as a function of K or voltage, as is usually done. The second would represent fixing the counter voltage and measuring the gain as the pressure is increased. The second is much less practical than the first and so is rarely done.

As explained in section 2.3, it is possible that K and s are the fundamental parameters on which the proportional gain in a normal MWPC depends, once the gas type is fixed. This is the reason that K and s are being used for the figures instead of the equivalent voltage and pressure for fixed wire radii. That K and s are indeed the fundamental parameters will be demonstrated in this chapter. This demonstration requires interpolation between data points in a way as free of empirical bias as possible.

Figure 27a

Data from the 10 micron wire. The lines are lines of equal pressure or s . The symbols represent the same points as in figure 28. The lines represent the fit to the data as described in the text.

Figure 27b

The data taken with the 100 micron wire.

Figure 27c

The 400 micron wire data.

Figure 28a

The same data as in figure 27a for the 10 micron wire. In this case the gain is plotted versus s and the lines are lines of constant K or voltage.

Figure 28b

The data taken with the 100 micron wire.

Figure 28c

The 400 micron wire data.

Figure 29a

This is an example of the data taken with the 10 micron wire expressed in terms of the K, s plane. The solid lines represent contours of equal gain in logarithmic steps from 10 to 10^6 . The dotted lines are lines of equal X for $X=200, 300, 400, 500, \text{ and } 600$ volts/cm/torr.

Figure 29b

This is an example of the data taken with the 100 micron wire expressed in terms of the K,s plane. The solid lines represent contours of equal gain in logarithmic steps from 10 to 10^6 . The dotted lines are lines of equal X for $X=50, 100, 150, 200$, and 250 volts/cm/torr.

Figure 29c

This is an example of the data taken with the 400 micron wire expressed in terms of the K,s plane. The solid lines represent contours of equal gain in logarithmic steps from 10 to 10^6 . The dotted lines are lines of equal X for X=50, 60, 70, 80, and 90 volts/cm/torr.

One of the best ways to interpolate between a set of points, while retaining any information available about underlying analytic structure is to fit to polynomials. This was done to each of the gain curves of figure 27. These lines are shown in the figure. From these lines it is possible to determine the value of K for each s that corresponds to a given value of gas amplification. Thus for fixed gas amplification one may think of s as being a function of K . These lines of constant gain are shown in figure 29a, b, c for the 10, 100, and 400 micron wire radii respectively. From this curve it is possible to fix K and obtain the lines of gain versus s shown in figure 28.

As an aside, it should be observed that figure 29 also shows the lines of constant X or E/P . Naively one might expect that as one increased the pressure for fixed wire radius the gas amplification might scale with E/P such that for the same gain one would require the same value of E/P at the surface of the wire. This would mean that the lines of constant gain would also be lines of constant E/P . Clearly this is not even remotely true of any of the lines shown in each of figure 29a, b, c.

As an example of this one may consider the constant gain = 10^4 line of figure 29a. For $s=0.2$ cm*torr, $P=200$ torr and K is about 150 volts; for $s=1.2$ cm*torr, $P=1200$ torr and K is only 275 volts, thus while the pressure has increased by a factor of 6 the voltage has less than

doubled. After it has been proven in the next section, 4.4, that the curves of one wire predict the gains of others figures 29a, b, c become much more powerful. This is because ignoring space charge, the measurements at $s=7$ cm*torr or $P=700$ torr on the 100 micron wire give the expected gain at $P=7000$ torr for the 10 micron wire.

In extracting an ionization coefficient from the gain curves it is necessary to differentiate $\ln G$ with respect to s along the lines of constant K shown in figure 28. This has already become quite a cumbersome project. In the above approach each of the many lines required, generate more parameters. Without empirical constraints these parameters are difficult to relate to each other. In section 4.6 it is necessary to consider the rate of change of the ionization coefficient along lines of constant X , this would in the above approach generate even more parameters. This is a consequence of taking a one dimensional approach to a two dimensional problem.

The preferred solution would be to fit the gain, or actually the natural logarithm of the gain, to a two dimensional function. This would permit the calculation of general derivatives without the introduction of new parameters. In originally choosing to use polynomials to interpolate between the data points, the goal was to preserve any underlying information about the analytic structure of the gain. In moving to a two dimensional

representation of the data the same goals are still in play. This leads to the conclusion that a general two dimensional Taylor's expansion would be the best approach. In order to treat the various first and second mixed partial derivatives as functions of K and s themselves, it is necessary to include many more terms in the expansion than are really required to parameterize the gain. However, the number will be considerably reduced from the above one dimensional approach and keeping them in order will be essentially trivial.

Before attempting this it is instructive to consider, in a intuitive fashion, the dependence of gain on K and s . For fixed s , the gain decreases as K decreases as this corresponds to decreasing the counter voltage. For K equal to zero the amplification would be one, although no ionization would be collected. For fixed K the gain is a monotonically decreasing function of s or conversely monotonically increasing as s decreases. This depends on the definition requiring the initial ionization to originate in a region of no amplification. In practice, if the voltage on a proportional tube is held constant and the pressure reduced in steps to zero, the gain increases to a maximum and then starts to decrease due to the absence of atoms to ionize. In this case the ionization does not begin in a region free of amplification. The gain increase as s decreases is also bounded as although the energy

available to the electron from the field becomes infinite as s approaches zero, the mean distance, in $\text{cm} \cdot \text{Torr}$, between collisions is approximately constant and so there is not enough gas to ionize as the electron energy increases. This increase will be dramatic enough to appear singular, possibly as a singularity at some non physical negative s value.

Thus in choosing to parameterize $\ln G$ in the K, s plane there are no singularities in the region containing the experimental data and so a simple Taylor's expansion is valid. It is, however, preferable to consider possible singularities. It is interesting to observe that working in the K, s plane is equivalent to working in the K, X plane, where $X = K/s$. In this representation one would expect no singular behavior for K and X bounded.

4.4 Expansion of $\ln G$ in the K, s Plane

The next two sections are concerned primarily with the differential characteristics of the gain. It is, therefore, useful to expand in terms of powers of K and s . For the reasons discussed above, negative powers of s are used as well as positive powers of both K and s . It is convenient to define $H = \ln G$ at this point. The introduction of a special symbol for the logarithm of the gain simplifies the labelling of the expansion

coefficients. In the region where the Townsend coefficient is valid, from section 2.3 one expects,

$$\begin{aligned}\alpha/P &= -\partial H/\partial s, \\ &= z(K/s),\end{aligned}\tag{2.19}$$

and so it is useful to have coefficients that are easily identifiable with K/s . For this reason a term in $\ln(s)$ is also included in the expansion.

Note that in section 2.3 the derivative was defined with respect to s for constant K . This is identical to the partial derivative with respect to s in the present coordinate frame. In another coordinate frame this would be defined as the directional derivative with respect to s along the direction of constant K .

Thus, the experimental data points were fitted to,

$$H(K,s) = \sum_{i=0}^n \sum_{j=-\ell}^m K^i h_{ij} s^j + \sum_{i=0}^n K^i h'_i \ln(s).$$

In a more compact form,

$$H(K,s) = U^i H_{ij} V^j,\tag{4.5}$$

for

$$\bar{U} = [1, K^1, K^2, \dots, K^n]\tag{4.6}$$

and

$$\bar{V} = [1, s, \ln(s), s^{-1}, s^{-2}, \dots, s^{-\ell}, s^2, s^3, \dots, s^m].\tag{4.7}$$

The general ionization coefficient $z = \kappa/p$ is then,

$$\begin{aligned}z &= -\partial H/\partial s \\ &= -U^i H_{ij} dV^j/ds.\end{aligned}\tag{4.8}$$

Notice that,

$$U^i dv^{i+1}/ds = (K/s)^i,$$

4.9

for $i=1, \dots, n$. Thus in the limit of perfect Townsend theory where z is the Townsend coefficient, the H_{i+1} terms determine the Townsend coefficient dependence and the first column of H the H_{i0} determine the constant of integration.

H is then an $(n+1)$ by $(l+m+2)$ matrix to be found. This was done by minimizing the squares of the deviations of the experimental points from the fitted points. This involves solving $(n+1)(l+m+2)$ independent linear equations. These equations may be expressed in matrix form with a symmetric matrix. This may then be inverted and the coefficients H_{ij} can be found. However, it was found preferable to fit in two stages. First, all the coefficients that would be zero in the Townsend limit were fixed at zero and the remaining coefficients fitted. This was then used as a starting point to solve for the other coefficients iteratively using the Relaxation method; see, for example, Burden [43].

It was found that the two step process, although slower, gave more recognizable coefficients that explicitly exhibit the expected K/s symmetry in the Townsend region. When the fit was done in one step the resulting standard deviation was slightly better and the overall differential characteristics exhibited approximately the same K/s symmetry, but involved all the coefficients. This is because in the way that the H_{ij} are defined there are too

many parameters. For example, for $\ell=3$, $m=2$ and $n=4$, H has 35 coefficients, only 5 of which parameterize the Townsend coefficient. There are not enough data points to fully constrain all the coefficients without the experimental errors affecting the exact analytical form.

Using this fitting procedure, it is possible to confirm the general K,s symmetry that is expected from the discussion of section 2.3. This is done by fitting H to the data of one wire and examining whether this fit describes the data of another wire. To avoid any questions about extrapolating the fit, it was required that the regions of K,s overlap. This region is defined by $0.56 < s < 13.9 \text{ cm} \cdot \text{torr}$ and a gain of less than $2 \cdot 10^4$. The fit was done using $\ell=3$, $m=2$ and $n=4$ to 132 experimental points taken with the 100 micron wire. This means that 35 coefficients were used to parameterize 132 points. Ideally one would like to parameterize the gain with an infinite number of parameters and so have complete knowledge of all derivatives to all orders. However, further increasing the number of parameters merely causes the experimental fluctuations to be fit, yielding no information about the functional nature of the gain mechanism. The standard deviation of this fit to the 100 micron wire points was 4.4%. Thirty points from the 10 micron wire overlapped, in the K,s plane with the 100 micron wire fit and were described with a 4.1% deviation; twenty-two 400 micron

wire points had a 4.6% standard deviation. It is clear that the agreement is within the systematic errors of the experiment.

Using this fit one could regenerate the lines of figures 27 to 29, however, their real purpose is to permit a simple discussion of some of the analytical properties of the gain function. The simplicity of this will be seen in the next two sections. Naturally for a fixed wire radius and fixed gas type this kind of expansion can be used to describe both the proportional and non proportional region, however in the non proportional region it is not expected that the measurements made on one wire should predict those of another. The border between the two regions will be discussed in the next Chapter.

4.5 Extraction of the Ionization Coefficient z

Extraction of the ionization Coefficient z is now straight forward using,

$$z = -U_i H_{ij} \, dV^j / ds. \quad 4.8$$

z is then itself a scalar function on the K, s plane. This means that z may not be represented as a function of any one variable. An example of this is shown in figure 30a where z is shown as a function of X calculated along lines of constant s . The lines of constant s are those at which the data points were taken for the 10 micron wire. The

Graph of z along the lines of constant s , for the particular s values listed in the text. The lines represent the actual value of z that is calculated. As this quantity gives the pressure dependence, it can be seen that pressure dependence cannot be parameterized as a function of X alone.

lowest and highest values of s were excluded and so the eight lines of figure 30a correspond to $s = 0.26, 0.402, 0.536, 0.672, 0.805, 0.941, 1.07, \text{ and } 1.21 \text{ cm} \cdot \text{torr}$. It should be recalled that constant s corresponds to constant pressure, the normal condition for gain curves. It is interesting to observe that for large s which corresponds to low X the lines collapse down to the expected one dimensional ionization coefficient.

It is interesting to compare the value of z calculated from the full derivative with that of z calculated using only those coefficients that are explicitly a function of K/s or X . Let this latter quantity be z' ,

$$z'(X) = -H_{01} - H_{12}X + H_{23}X^2 + 2H_{34}X^3 + 3H_{45}X^4. \quad 4.10$$

The coefficients of z' were,

$$\begin{aligned} H_{01} &= 4.79, & H_{12} &= -7.13 \cdot 10^{-2}, & H_{23} &= -1.48 \cdot 10^{-4} \\ H_{34} &= 7.64 \cdot 10^{-8}, & H_{45} &= -6.52 \cdot 10^{-12}. \end{aligned}$$

In figure 30b the values of z' have been added to the data of figure 30a. As well, one of the empirical ionization coefficients extracted in section 4.2 is plotted for comparison.

As expected the agreement is good for low X and poor for large X . The spread in values of z for fixed X increases rapidly with X for X greater than 150 volts/cm/torr. Furthermore, the data used in these determinations of z were restricted to gains below $3 \cdot 10^4$.

Figure 30b

Dotted lines are as in figure 30a. The upper solid line is the value of z' obtained from those coefficients that are expressly a function of K/s . The lower dashed line is the value of z inferred from the empirical parameterizations of equations 4.1-4.3.

Inclusion of higher gain values would further increase the spread in values of the ionization coefficient for fixed X .

In the region where the agreement is good, $X < 150$, it is reasonable to assume that the z calculated above approximates the Townsend coefficient; however, where the agreement is poor the coefficients cannot be interpreted as having this meaning.

In order to more precisely examine where the Townsend coefficient might apply it is possible to look at z along lines of constant X in the K, s plane. If z is independent of X it will be equal to the Townsend coefficient. In figure 31, z is shown versus s for various values of the field X for the three different wires. It is clear that for the 10 micron wire the lines are not constant and that for the thicker wires they become more constant as s increases and X decreases. To be more quantitative requires a more precise study of the derivative of these lines.

4.6 Search for the Townsend Coefficient

To examine any possible identity of the measured ionization coefficient with the Townsend coefficient requires the calculation of the rate of change of z along the lines of equal X in the K, s plane. This requires the calculation of a directional derivative. This is a

Figure 31

Here z is shown for various lines of constant electric field over pressure as a function of s . The $X=300$ to 600 volts/cm/torr lines are from the fit to the 10 micron wire, $X=100$ to 200 the 100 micron wire, and the $X=60$ volts/cm/torr line is from the fit to the 400 micron wire. The lines are not constant for small s and become constant for large s . This is evidence that the 10 micron wire does not operate in the Townsend region.

consequence of the present choice of the coordinate frame. The K, s coordinates allow for the trivial differentiation with respect to s along lines of constant K . To choose X as a coordinate axis would simplify differentiation along the lines of constant X at the expense of calculating z . Thus it is necessary to define the direction.

Defining the direction is trivial as all lines of constant X begin at the origin. Thus for given K, s the direction is defined by the ordered pair (k, s) . Thus a unit vector in this direction would be,

$$\hat{v} = (K, s) / (K^2 + s^2)^{.5}.$$

The derivative in this direction is then

$$\begin{aligned} dz/dv &= \hat{v} \cdot \nabla z \\ &= [K \partial z / \partial K + s \partial z / \partial s] / (K^2 + s^2)^{.5}. \end{aligned}$$

The partial derivatives are given by

$$\begin{aligned} \partial z / \partial K &= -dU^i / dK H_{ij} dv^j / ds, \\ \partial z / \partial s &= -U^i H_{ij} d^2 v^j / ds^2. \end{aligned}$$

The normalization factor $(K^2 + s^2)^{.5}$ is not open to physical interpretation due to the mixed units introduced by the choice of dv . This problem may be eliminated by considering the rate of change of s in the same direction,

$$\begin{aligned} \partial s / \partial v &= \hat{v} \cdot \nabla s \\ &= s / (K^2 + s^2)^{.5}. \end{aligned}$$

Thus

$$dz/ds = [K \partial z / \partial K + s \partial z / \partial s] / s,$$

where the derivative is defined along the direction of

constant X . This is an important distinction.

A practical problem in calculating dz/ds comes from the fact that $K\partial z/\partial K$ is very nearly equal to $-s\partial z/\partial s$. This is to be expected as in the Townsend region the two exactly cancel since,

$$K(\partial/\partial K)(K/s)^n = n(K/s)^n,$$

and

$$s(\partial/\partial s)(K/s)^n = -n(K/s)^n.$$

Thus dz/ds is given only by those coefficients that have been treated in a perturbative way in the two dimensional fitting procedure for the matrix H .

To consider the difference between $z(K,s)$ and the value of the Townsend coefficient, z' , measured in a uniform field of K/s , one has,

$$z'(K/s) - z(K,s) = \int_v^\infty dz(K,s)/dv \, dv$$

for

$$v = (K,s) \text{ and } X = K/s.$$

This is because as $v \rightarrow \infty$, $K,s \rightarrow \infty$ such that $K/s = X$, a constant, thus $K/s^2 \rightarrow 0$ or the uniform field case.

Equivalently one has

$$z'(X) - z(K,s) = \int_{s_a}^\infty dz(Xs',s')/ds' \, ds'. \quad 4.11$$

From this the difference between the gain as calculated by Townsend coefficient and that experimentally measured would be:

$$\Delta \ln G = \int_{s_a}^\infty [z'(X) - z(K,s)] ds$$

$$= \int_{s_a}^{\infty} \int_s^{\infty} (dz/ds') ds' ds . \quad 4.12$$

Figure 32 is a log-log plot showing the values of dz/ds versus s for the same lines of constant X as shown in figure 31. As in figure 31 the data for X between 300 and 600 volts/cm/torr is from the fit to the 10 micron wire, between 100 and 200 the 100 micron wire and the $X=60$ volts/cm/torr line is from the fit to the 400 micron wire. dz/ds is a scalar function of both X and s or equivalently K and s . For fixed X it is expected to be a decreasing function of s . The lines of figure 31 and 32 have been plotted for gains between 100 and $2 \cdot 10^4$. For the thick wires the values of dz/ds becomes very small and are no longer reliable as is seen by the fact that the $X=200$ line is not strictly decreasing.

To perform the integral of equation 4.11 it is necessary to integrate dz/ds to infinity. As seen in figure 32, dz/ds is only known for fixed X for a limited range of s values. As these lines are the derivative of the lines of figure 31, to integrate only over the given range is equivalent to subtracting the values of z at the two ends of each of the curves of figure 31. For the $X=300, 400, 500$, and 600 lines this gives 1.1, 2.7, 5.4, and 8.1 respectively, as the change in z .

Figure 32

dz/ds as a function of s for the lines of constant X given in figure 31.

Over the range of values plotted the values of dz/ds appear quite linear in the log-log scale. This implies that dz/ds may be parameterized by a function of the form,

$$dz/ds = Cs^{-B},$$

where C and B would be functions of X . Fitting the lines one obtains $B=1.9$ for $X=300$ and 400 volts/cm/torr and $B=2.0$ for $X=500$ and 600 . There is at present no theoretical reason to assume that B should have a value of about 2. Similarly there is no experimental evidence that it should remain about two outside the region bounded by the data. With these two provisos, it is interesting to make the empirical assumption that B is equal to 2 independent of s and X , C remains a function of X . From these assumptions one has,

$$dz/ds = C(X)/s^2, \quad 4.13$$

$$C(X) = s^2 dz/ds, \quad 4.14$$

from this 4.11 becomes

$$\begin{aligned} z'(X) - z(K,s) &= C(X)/s \\ &= s dz/ds. \end{aligned} \quad 4.15$$

The actual fitted values of C were .95, 1.8, 2.5, and 3.3 for X from 300 to 600. Ideally one could now calculate the true value of the Townsend coefficient from the data using,

$$z'(X) = z(K,s) + s dz/ds. \quad 4.16$$

This gives values of 8, 13, 18, and 27 number/cm/torr for X from 300 to 600, however as these values have no bearing on the gas amplification of a MWPC their usefulness is limited.

The final empirical statement is that taken as a percent change in the ionization coefficient the difference between the real ionization coefficient and the Townsend coefficient drops below 10% for $X < 150$ volts/cm/torr. Thus this is approximately the upper limit of where the empirical formulae for the gas amplification may be considered to be valid for 80% Argon and 20% Methane.

4.7 Measurement of z by Small Pressure Differences

This section presents an example of an alternative method of calculating z . With the present data this approach is not very accurate. In fact, even if it was more accurate it requires a larger amount of data than the above approach anyway. However, it is interesting as it indicates another use for the ionization coefficient, z .

It has been seen that the wire gain may be described within a factor of two using a totally empirical ionization coefficient. In terms of understanding effects requiring higher precision or a greater knowledge of the spatial extent of the avalanche, it is necessary to have a detailed knowledge of the true ionization coefficient. Understanding the effect of small pressure variations for a fixed anode wire is one such effect.

Section 2.3 gives

$$z = -d\ln G/ds, \quad 2.17$$

Provided $\ln G$ is a fairly smooth function it is possible to approximate z by

$$z = [\ln G(P) - \ln G(P + \Delta P)]/(r_a \Delta P), \quad 4.17$$

or equivalently

$$\Delta \ln G = z r_a \Delta P, \quad 4.18$$

where,

$$r_a \Delta p = \Delta s.$$

The results of calculating z using 4.17 are shown in figure 33 for the values of pressure given in Table 4.1. The errors shown are dominated by the experimental error in calculating the difference in $\ln G$.

Table 4.1

r microns	P Torr	P + Δp
100	56	61
100	140	146
100	278	283
100	485	514
100	667	693
400	169	279

Due to the pressure wire radii symmetry established in this Chapter a small change in pressure can be thought of as equivalent to a small change in wire radius. The fact that this is proportional to the actual ionization coefficient and not an empirical ionization coefficient

Figure 33

The points are values of z as calculated by approximating $d[\ln(G)]/ds$ by subtracting points of fixed proportional tube voltage at small pressure difference and subject to a gain cut of 20000.

underlines the importance of knowing the true ionization coefficient.

This illustrates the importance of the technique outlined in this chapter for dealing with the more general ionization coefficient. If in the design stage of a particle detector, it is necessary to know exactly how the apparatus will behave under variation in atmospheric pressure for different wire radii, the more general coefficient is required.

4.8 Summary

For proportional chambers operating at atmospheric pressure it has been shown that the Townsend coefficient will approximate satisfactorily the gas amplification below the onset of space charge effects for all wires of radii greater than about 50 microns. This corresponds to assuming that X should be below about 150 volts/cm/torr. However, as the departure from the Townsend approximation is a gradual function of s it can be assumed that for some applications an effective ionization coefficient will describe the gain for thinner wires. The accuracy with which it describes the gain depends on the range in gain values to which it has been fitted and is to describe.

For more detailed calculations such as small pressure and temperature fluctuations it is still necessary to have a knowledge of the more general ionization coefficient α .

It is now possible to use the symmetry relationships established in this chapter to identify the space charge modification of the gas amplification. In the region where the Townsend coefficient is approximately applicable, this may be identified as a deviation from the gain as predicted by the Townsend coefficient in the large gain region. In the non Townsend region it is more difficult to study this effect. However, it is possible to identify it using the K, α symmetry breaking characteristics of space charge.

CHAPTER 5

THE AVALANCHE AND SPACE CHARGE
EFFECTS

This chapter will measure two additional non-proportional effects that contribute when wire counters are operated at high gas gain. These effects can be identified by the onset of an operating region where the measured gas amplification differs from that expected, neglecting these new contributions. In the region, $X < 150$ volts/cm/torr, of approximate validity of the Townsend coefficient, the expected gain is given by simple relations, and the identification of the operating conditions at which the measured gain differs is relatively easy. In the nonequilibrium region, $X > 150$ volts/cm/torr, the breakdown of the proportional region can be identified, using the techniques developed in Chapter 4, by a divergence from the K , symmetry which defines the region.

For the specific example for the nonequilibrium region given here, the divergence from proportional amplification may be attributed to the space charge saturation of the signal. This divergence is identified by the increase in

space charge saturation as pressure is increased for fixed K and s . The space charge effect in proportional chambers refers to the build up of a sufficient number of electron ion pairs in the gas such that the local electric field is modified.

In the second example for which the end of the proportional region occurs in the Townsend region, the transition is not continuous and the present experiment cannot distinguish whether it is caused by space charge effects or other secondary effects.

Empirically, the effects observed are quite different in the two regions. For the 10 micron radius wire, operating in the region where the electrons are not in equilibrium with the field, space charge is seen to lead to a stable attenuation of the expected gain. In the case of the 100 micron radius wire, operated in the approximately Townsend region, an instability in the gain mechanism is observed. There appears to be a sharp transition from a proportional output pulse to a regime in which the output pulse is very much larger. The large gain increase process could be due to either the onset of limited streamer or Geiger pulse formation. However, the 100 micron wire measurements are insufficient to establish a particular mechanism as the origin of the large pulse effect.

The two observed effects happen to be correlated with the two different X operating regions corresponding to the Townsend and non Townsend formalism describing the moderate gain data. However from the measurements made in this thesis it is not possible to make any theoretical conclusion regarding this empirical correlation.

However, a first approach to understanding the difference between the two cases can be obtained from a naive extrapolation from the space charge effect in a parallel-plate case [31]. This leads to the observation that in the region where the second derivative of $z = \alpha/P$ with respect to $\beta = E/P$,

$$d^2 z / dX^2$$

is positive, there should be an instability as is the case with the thick wire and in the region where $d^2 z / dX^2$ is negative, a stable saturation. The parallel plate analogy suggests that the optimal operational point with regard to maximum proportional gas gain is the point where $d^2 z / dX^2$ vanishes. It should be noted that the parallel-plate case assumes a steady state discharge such that the space charge does not change with time. This is clearly not valid for the present case, a cylindrical proportional chamber detecting individual particles. However, there are conditions where the approximations may be justified as will be discussed.

5.1 The Loss of Proportionality in the Townsend Region

The analysis presented in chapter 4 shows that the gain measurements made using the 100 micron radius wire correspond to the chamber operating in the region where the Townsend coefficient is approximately valid. In the Townsend region the expected gain is given by the simple parameterizations of $Y = \ln G/K$, given in the previous chapter,

$$Y(X) = -3.33 \cdot 10^{-3} - 9.10 \cdot 10^{-3} \ln(X) + 2.88 \cdot 10^{-3} [\ln(X)]^2 \quad 4.4$$

Figure 34 shows a plot of the deviation of the measured gas amplification from the expected amplification, using both the main and escape peak for X values near $X=110$. The data points were taken using the 100 micron radius wire at 702 Torr. Recall that all pressures are normalized to zero degrees centigrade. Thus 702 Torr at 0 degrees C corresponds to 761 Torr at 23 degrees C, in terms of equivalent molecular density.

Figure 35 gives the experimental qVt spectra for the middle four data points of figure 34. The data have been normalized to a data acquisition time of 100 seconds. Histograms 35a and 35b, taken at cathode voltages of -4300 and -4400 volts, lie to the left of the dashed line on figure 34. The other two, 35c and 35d, at voltages of -4430 and -4470 volts lie to the right of the dashed line. Recall that the data taking procedure involved adjusting the amplifier gain such that the main peak was centered in

Figure 34

The attenuation of the measured gain relative to that expected using the fit of equation 4.4. The measurements are from the 100 micron wire at atmospheric temperature and pressure. When reduced to zero degrees centigrade, the pressure becomes 702 Torr.

Figure 35

Fe55 spectra corresponding to four of the points in figure 34. The voltages on the counter were 4300, 4400, 4430 and 4470 volts respectively, the third, fourth, fifth and sixth points of figure 34.

the qVt range.

It can be seen by comparing peak areas that the main peak remains approximately intact up to about the dashed line of figure 34. This corresponds to about 1.5×10^8 electrons in the final avalanche. In figure 35c the area is reduced by about a factor of four after an increase of only 30 volts. What is actually occurring is that between $V=4400$ and 4500 volts most of the events that would normally go into the main peak are now going into the overflow channel, i.e. the photop~~peak~~ events now initiate much larger output pulses.

The increase in background caused by the cathode emission processes described in section 2.10 make it difficult to continue to measure the gain using the escape peak. However it appears that the escape peak remains intact until the gas gain is about double that required to send the main peak into overflow. This indicates that whatever process is responsible for the discontinuous change in gain depends on the total charge in the avalanche and not the gain specifically.

It becomes difficult to measure the absolute gain of the wire in the presence of the background and distortion of the peaks. However, it remains relatively clear at what total avalanche charge the discontinuity occurs. Even those events which might be attributed to cathode noise

seem to have a distinct discontinuous transition between gain mechanisms.

It will be seen that this total charge corresponds to a larger value of proportional gain than measured for the 10 micron wire in the next section, or a much higher value of proportional gain than is usual for the thin wires normally used in MWPC. The dominant effect at the end of the proportional region is not the attenuation of the signal but the discontinuous increase in the signal in a fashion similar to the limited streamer or pseudo-Geiger mechanism.

5.2 Space Charge Effect in the Non-Townsend Region

The identification of loss of proportionality in the non-Townsend region is more difficult to extract from the data. It requires application of the more detailed procedures developed in section 4.4. This is possible because the loss of proportionality is dominated by space charge effects in this case.

In the operating region where the Townsend coefficient is valid, the end of the proportional region may be identified using the data of only one wire, because by changing the pressure it is possible to measure the gain for fixed X on the wire surface for different values of the

final total charge. However, in the non-Townsend region, the gain depends on both K and s in a two dimensional way. In order to identify space charge saturation, it is necessary to measure the gain in the same K and s region with two different wires, because, as explained in section 2.11, the space charge effect destroys the K, s symmetry in a manner dependent on the pressure. For fixed K and s , linearly decreasing pressure is effectively the same as linearly decreasing the space charge due to the ions.

In Chapter 4 only low gain data, $G < 2 \cdot 10^4$, was considered when expanding the data in terms of the coefficients H_{ij} . In this chapter, it is the high gain regions that are of interest. A fit was done to the experimental data of the 100 micron wire between $p = 0.56$ and $s = 1.39 \text{ cm} \cdot \text{Torr}$ for gains between $2.0 \cdot 10^4$ and $1.0 \cdot 10^6$. Figure 36a gives the deviations of the data points from this fit, and also displays the deviations from a similar fit, to all points of gain less than $2 \cdot 10^4$. For the purposes of this section the fit is merely a technique for interpolating between data points, while smoothing out the statistical fluctuations. Thus the division of the data into two regions is essentially arbitrary, but convenient for distinguishing between the region discussed here and the region discussed in chapter 4.

Figure 36a

Graph of the deviation of the gain measured using the 100 micron wire from the fit to the same data as a function of the experimental gas amplification.

3 3

DF / DE

The standard deviation of the individual data points compared to the fit increases with gain. The average agreement of the data with the fit is 2% for the low gain region and 5% for the high gain data. The higher gain data might exhibit space charge effects, as well as other possible secondary effects not easily described by the H_{ij} parameterization. However, the deviations remain within the statistical limits expected for the individual data points.

Figure 36b compares the experimental gain measurements from the 10 micron wire for s between 0.67 and 1.34 cm*torr with the 100 micron wire measurements at the same K and s values. It can be seen that the agreement is good for low values of the gain as discussed in chapter 4; however, for the large values of the gain, the measured 10 micron wire gain decreases rapidly from the value expected from the 100 micron wire data. Figure 36c provides an enlarged view of the region where the K, s gain symmetry breaks down.

From figures 36c and 37 it can be seen that the onset of space charge attenuation is a function of s . Figure 37 gives the value of the total number of electrons in the avalanche at which space charge attenuates the gain by a factor of 0.5, versus s . The solid line is the total charge for the 10 micron wire in the presence of space charge, the dashed line represents the 100 micron wire at one tenth the pressure, or effectively what the avalanche

Figure 36b

Graph of the attenuation from the expected gain as the electric field on the anode surface is increased. The measurements are from the 10 micron wire, while the expected gain is calculated using the fit to the 100 micron wire for the same range of s values.

Figure 36c

Amplified version of the deviation of figure 36b.
 Only those points of gain greater than 10000 are shown.

Figure 37

Total number of electrons in the avalanche versus s for the wire radius fixed. The solid line is as extracted from the fit to the 10 micron wire, while the dashed line is from the fit to the 100 micron wire. The K values have been selected so that the deviation in gain is 100%. Thus space charge has attenuated the gain by a factor of 2.

would be without space charge, twice the solid line. Since the attenuation data are obtained from the 10 micron wire, the s dependence corresponds experimentally to a pressure dependence of the magnitude of space charge induced gain attenuation.

It is quite significant to note that as s increases one can obtain larger gains for a given value of space charge attenuation. For example, from section 2.11, for s fixed one has that space charge increases as pressure increases, one would expect to get more space charge attenuation for the case of a 10 micron wire at 1400 torr than a 20 micron wire at 700 torr, both having the same s value. Figure 37 indicates that one can obtain higher gain for fixed space charge attenuation for the 10 micron wire at 1400 torr than for the same wire at 700 torr. Clearly a 20 micron wire at 700 torr would be superior to both.

Figure 38 provides some examples of the experimental Fe55 spectra from the 10 micron radius wire illustrating the effect of space charge. Two observations are apparent. First, the onset of space charge is a gradual continuous process, in contrast to the discontinuous behavior of the 100 micron wire at atmospheric pressure. This is demonstrated by the normal appearance of the second spectrum, even with an attenuation factor of 0.3. Second, it should be observed that the final effect is a controlled broadening and merging of the two Fe55 peaks, resulting in

Figure 38

Fe55 spectra corresponding to the last four experimental points of the $s=0.672$ data points of figure 36c. The attenuation due to space charge was .95, .27, .05 and .007 respectively.

a stable semiproportional signal. This effect was not observed for the case of the 100 micron wire at atmospheric pressure as noted above. The gas amplification of the 10 micron wire passes in a continuous fashion from the proportional region to the semiproportional region. This is in agreement with the intuitive description of figure 3 in chapter 1.

The broadening and merging of the peaks is probably heavily influenced by the large spread in initial photon conversion points inside the proportional tube. The ionization drifts over a distance that varies between 0 and 3cm before being amplified. Thus the electron clouds arrive at the anode with different initial sizes due to diffusion. This is likely to cause broadening by influencing the way that space charge modifies the gain.

5.3 Empirical Treatment of The Space Charge Effect

Section 5.1 and 5.2 have demonstrated an important experimental technique. They have illustrated that by using the symmetry expected from the simple models of gas amplification, the point at which these models break down may be measured.

In order to tie these techniques back to the nature of the discussions of chapter 2 a totally empirical look at the the way various variables can influence space charge will be taken. With further theoretical developments these empirical estimations could be subjected to experimental tests in a manner similar to the tests of this Thesis. Thus, this section does not have the concrete nature of the measurements described in the first half of the chapter. It is meant to be a guide as to what aspects of geometry and gas composition warrant further study.

The dynamics of the avalanche, in the presence of a non negligible space charge effect, is significantly more complex than that described by the simple integration of an ionization coefficient, which depends only on the radial distance. The electric field from the space charge cloud destroys the radial symmetry of the problem. The z and θ components of the field vanish only along a radial line down the center of the charge distribution. This causes electrons that drift toward the wire to be focused toward the center of the positive ion charge distribution. Thus, in practice, an exact solution to the problem is impossible.

Evans discusses a similar problem for the case of electrical breakdown along particle trajectories in a wide gap spark chamber [44]. In this case, individual clusters of ionization are deposited along a track inclined at an

angle to the uniform electric field. As each ionization cluster undergoes an avalanche, the space charge of one avalanche begins to attract the electrons of another. This causes the electrons to move at an angle to the electric field. Thus, when breakdown occurs, it is along a direction between that of the original track and the electric field. Evans discusses how this case may be treated numerically, if the primary ionization coefficient, electron drift velocity, and diffusion coefficient are known. He works under the assumption that the ionization coefficient is the Townsend coefficient, and that the change in gain may be given by the change in field.

For the avalanche at the surface of a wire, the present measurements give the ionization coefficient; however, they yield no information about the electron drift velocity or the transverse and longitudinal diffusion. In this case, the electric field is much larger than in the case of a parallel plate spark chamber. In many instances, the drift velocity and diffusion, can plateau for a range of values of X characteristic of normal drift chambers. From the results of the simulation of Chapter 2, though, it should not be expected that the values of these parameters remain plateaued. Extrapolation from low values of the field into the avalanche region, would not be expected, therefore, to be very reliable.

In the spark chamber or parallel plate example, it is possible to take into account both the electron and positive ion densities and ignore charges induced on the electrodes, as the avalanche occurs for the most part in a region away from the electrodes. In the case of a wire, most of the charge is very near the surface of the anode wire. Thus the electrons play a larger role as image charges held at the surface of the wire after arrival than as a free moving cloud. This is true even in the case of the Fe55 source. Although the ionization may be assumed to be initially localized at a point with respect to the volume of the counter, it actually begins with the dimensions of the order of 10's of microns and quickly diffuses to the order of 100's of microns. As a result, the electrons arrive at the anode wire over a time interval of one or two nanoseconds. With the 10 micron anode wire most of the positive ion space charge is confined to a few microns from the anode wire, and the later arriving electrons would be expected to pass through this region in a few picoseconds. Thus as the avalanche builds it does so by electrons moving from a region where there is no space charge, through a region of positive charge to the anode. The incremental charge of the next group of electrons is always small compared to the charge of the positive ions. For the 100 micron wire this distinction is no longer clear. It might be expected, that as the wire radius increases, there is a gradual return to the parallel plate

geometry of the spark chamber.

5.3a Comparison to Parallel Plate Phenomenology

In section 2.11, the effect of the space charge on the gain in a parallel plate discharge chamber was discussed. In that example it was assumed that the space charge did not change in time, that is, a steady state discharge was in effect. This caused the first order term, which was proportional to δV to vanish, as the plates were assumed to be at fixed voltage. This is not necessarily valid for the case where the space charge cloud is growing in the region where the electrons are drifting, as the voltage at a particular point can change. Thus $\int \delta X ds$ does not have to vanish.

However, by assuming that $\int \delta X ds$ vanishes, and then carrying out the simple integration of an ionization coefficient, it is possible to come up with a result analogous to the parallel plate result. Since z is not a constant in the wire case, the simple expansion

$$z = z_0 + \frac{dz}{dX} \delta X + 0.5 \frac{d^2 z}{dX^2} (\delta X)^2,$$

is not as useful in this situation where the field changes over a wide range. The analogous thing to do is to approximate z by a quadratic in terms of X ,

$$z = z_0 + z_1 X + z_2 X^2 \quad . \quad 5.1$$

In this case, it is necessary to introduce a cut when

integrating to calculate the gain. As this does not add to the discussion, it will be ignored. Then if X represents the unperturbed field, and $X + \delta X$ the field in the presence of space charge, then the gain would be given by,

$$H = \int z_0 ds + \int z_1 (X + \delta X) ds + \int z_2 (X + \delta X)^2 ds ,$$

$$= H_0 + z_1 \int \delta X ds + z_2 (2 \int X \delta X ds + \int (\delta X)^2 ds), \quad 5.2$$

where H_0 is $\ln G$ in the absence of space charge. If the first order term vanishes the change in gain is then proportional to z_2 , where if z was given exactly by the quadratic

$$z_2 = 0.5 d^2 z / dX^2 . \quad 5.3$$

This case differs from the uniform field case in that z does not multiply a necessarily positive quantity. The question is not really very important since the change in the electric field variable δX depends on s and u , and so the integrals may not be evaluated explicitly anyway.

Recall that $u = Pt$, the scaled time variable which matches s the scaled distance variable and X the scaled field.

These ideas may now be applied qualitatively to a thin wire proportional chamber. In the limit of thin wires, where the space charge is very near the surface of the wire, and the ionization passes through the space charge region in small quantities, then it may be assumed that during the passing of each subsequent quantity of ionization the space charge changes very little. In this limit, the first order term vanishes, making it permissible

to use the second order term to consider the change in the amplification of the ionization cluster by cluster. This returns, then, to the intuitive discussion of section 2.11.

Assuming the coefficient multiplying z_2 is positive, then the way in which space charge affects the gain depends on the sign of z_2 . If negative, the space charge leads to controlled attenuation of the signal; if positive, it leads to an increase in the gain followed by probable breakdown. The point where z_2 vanishes would be optimal from the point of view of avoiding space charge effects.

There is only empirical justification for the above analysis. This justification comes from the observation that, for the 10 micron wire, there was a steady attenuation of the signal as space charge increased, while the 100 micron wire data led to the expected breakdown. This is correlated with the observation of much higher space charge free amplification factors for the 100 micron wire.

The second derivative, $d^2 z/dx^2$ may be only approximately defined using the fits of Chapter 4. For example, the fit of the total wire data to a quadratic in $\ln X$ leads to the result that,

$$d^2 z/dX^2 = 5.67 \cdot 10^{-3} / X,$$

which is always positive and nonzero. However, the slightly more accurate fits to quadratics in X itself over

limited ranges do allow for the second derivative to vanish. The points where the second derivative vanished are:

$$X=132 \quad \text{for} \quad 40 < X < 100$$

$$X=108 \quad \text{for} \quad 100 < X < 200.$$

As the value of 132 is outside the range of the data to which that fit was done, it will be assumed that $X=108$ volts/cm/torr is the more accurate number. For $X < 108$, the second derivative is positive; for $X > 108$, it is negative.

The experimental observations are then well described by assuming the above dependence on the second derivative, including the fact that the largest space charge free gain is in the area where the second derivative vanishes. The fourth data point of figure 34, corresponding to the 4400 volt histogram of figure 35 had a value of X at the surface of the wire of 110 volts/cm/torr. Thus near the surface the second derivative vanished and was positive in the region away from the wire. The remaining instability can then be explained by the fact that the second derivative is positive in the region away from the wire where the space charge increases the gain. In agreement with the above phenomenology, this is the region where the largest space charge free gas amplification was observed.

In the example of space charge in the non-Townsend region as shown in figures 36, 37 and 38, the values of X at the wire surface ranged between 220 and 425. Thus all the examples operated in a region where the second derivative was negative and as expected space charge led to a stable attenuation of the signal. As s increased X decreased and so the highest amplification with the absence of space charge increased as X decreased, again in agreement with the above phenomenology.

This leads to the naive conclusion that, once the pressure has been chosen, the optimal operational point with respect to avoiding space charge would be at a wire radius such that X is equal to about 110 at the surface of the wire for the desired gas amplification. Using equation 4.2 one has $\ln G/K = 0.0176$ inverse volts when $X=110$ volts/cm/torr. Using $K=110s$, this gives

$$\ln G = 1.94s$$

or

$$s = 0.52 \ln G.$$

5.4

The present experiment does not constrain the optimal value of $\ln G$. As space charge depends separately on the variables K , s , and P , therefore to constrain $\ln G$ for most normal cases requires many more measurements to be made.

Clearly it would be preferable to have a better theoretical treatment, one which would constrain the optimal value of G . In the absence of this constraint and as the above

phenomenology is only approximate anyway, one may proceed by guessing the gain. $G = 10^5$ gives $s=5.9$ cm*torr and $K=653$ volts while $G = 10^4$ gives $s=4.8$ and $K=522$ volts. Thus an order of magnitude change in G only changes the optimal value of s by 20%. From the present data there is no reason to expect this rule of thumb to be better than 20% anyway; so this is probably quite acceptable.

There are two main reasons why this optimal point is not used in many cases. Increasing the wire radius increases the value of K or voltage necessary to achieve a given gain. In many applications of MWPC, it is difficult to increase the voltage without having external breakdown problems or internal problems with high cathode fields. The second problem with the thicker wires is the discontinuity in the gain mechanism which depends on the amount of primary ionization. This can be troublesome in the case of detecting relativistic particles where the statistical fluctuation in the primary ionization is much greater. This means that while working in a region where the average event behaves in a stable fashion there will always be a strong probability of a heavily ionizing event leading to Geiger-like behaviour.

A third less major reason for avoiding thick wires is in the total number of photons produced in the avalanche. The total number of photons produced in an avalanche, from section 2.10.3, would be,

$$N_Y = N_O \int \gamma(r) [\exp \int \alpha(r') dr'] dr,$$

The integral is dominated by the final stages of the avalanche. In this case it is possible to approximate $\gamma(r)$ as being proportional to $\alpha(r)$, or

$$\gamma(r) = C\alpha(r),$$

and so

$$\begin{aligned} N_Y &= CN_O \int \alpha(r) [\exp \int \alpha(r') dr'] dr, \\ &= CN_O G, \end{aligned}$$

where G is the final gain. This is equivalent to the approximation that the number of excitation collisions is proportional to the number of ionization collisions. As seen in figure 7, this is only approximately true and in the approximation C decreases as X increases. Thus C would increase as wire radius increases. Thus for a given size of avalanche there are more photons capable of producing secondary effects in the case of the thicker wire. This is, in fact, an alternative possible cause of the instability for the thick wire.

Thus, the actual optimal point is in practice application dependent.

5.3b Empirical Effect of the Charge Density

This section introduces a second phenomenological aspect of the space charge mechanism which leads to the result that the space charge effect is reduced as the wire radius increases, or equivalently as the parameter s_a

increases for fixed pressure. This involves considering the three dimensional aspects of the avalanche or more precisely the decrease in the number of positive ions per unit volume, the charge density, as s_a increases.

The charge density is determined in an approximate way by three factors, the linear radial charge density, the spread of the charge along the wire in the direction parallel to the wire's axis, and the angular spread around the wire.

The radial charge density incorporates the total charge or total avalanche size. The radial charge distribution, in the absence of space charge effects is given by

$$\begin{aligned}\rho(s) &= -N_0 (dH/ds) \exp H(s) , \\ &= N_0 z \exp \int z ds , \\ &= N_0 z G(s). \end{aligned} \quad 5.5$$

This is easily evaluated in either the Townsend or non-Townsend regions using the techniques of Chapter 4. When space charge is included, this is no longer valid due to the added pressure dependence. For the data displayed in figure 34, the first four data points have relatively little space charge distortion and so may be approximated using the Townsend coefficient in equation 5.5. This has been done in figure 39. It is not possible to predict anything about the radial charge density after the discontinuous change in the gain.

In the non-Townsend region one cannot extract the true value of the radial charge density in the saturated region due to the added pressure dependence. However, it is possible to examine the qualitative features of the radial charge distribution. This is done by defining an effective ionization coefficient and evaluating (5.5). Figure 40 compares the results using the fit to the 10 and 100 micron wires for comparable s and K values. The curves correspond to space charge gain attenuation factors of .27, .05, and 0.007 for the 10 micron radius wire. Initially, as s decreases approaching the wire, the charge density at the lower pressure rises faster than at the higher pressure, because the solid line is in the presence of space charge distortion while the other is not. The dotted line levels off at not quite 10 times the charge density of the solid line. This might indicate that the 100 micron wire charge distribution is now also undergoing some saturation effects.

The fact that the dotted line then starts to increase rapidly near the surface could indicate the presence of a further second order process. As the ion densities are larger than normal, this could represent ionization by double electron impact. The example is too isolated, however, to permit anything other than speculation.

Figure 39

Plot showing the expected radial charge distributions for the first four points of figure 34.

Figure 40

Plot showing the expected radial charge distributions for the last three points of figure 38. The dotted lines are those expected using the fit to the 100 micron wire, while the solid lines use the fit to the 10 micron wire data.

This probably means that in defining the space charge attenuation using the approach of section 5.2, it is not valid to extrapolate past an attenuation factor of 0.1.

The radial charge density may be considered to be experimentally determined from the fits to the measured gain data. Next let us consider how the longitudinal and angular spread of the avalanche can lead to a decrease in total charge density as s increases. This occurs as a result of the decrease in the focusing of the charge by the electric field lines, as the wire radius is increased.

To intuitively understand the focusing effect of the field it is helpful to consider the difference between the parallel plate case and the wire case. In a parallel plate, the electron cloud diffuses in the plane perpendicular to the field as it drifts. Thus, in this plane, it is natural to expect a two-dimensional Gaussian distribution of charge. Near the wire, however, the field lines converge toward the centre of the wire. As the electrons follow the field lines, they then converge toward the centre of the wire. This then causes their density to increase. Intuitively, this may be seen by considering the following deterministic model. Assume that away from the wire, the electrons diffuse in a Gaussian way to a standard deviation of 100 microns; they then approach the 10 micron wire. In the coordinate parallel to the wire, the electrons continue to diffuse and finish with a standard

deviation greater than 100 microns. However, in the coordinate perpendicular to the wire axis, they may occupy no more than the wire circumference. In fact, the electrons do not spread around the wire [47]. As they drift toward the wire following the field lines, the actual angular spread around the wire is dominated by the transverse diffusion in the last few microns. This happens in the high fields where the diffusion parameters are not known.

As the positive ion cloud is left in the wake of the electron cloud, its density then suffers from the same focusing as the electrons. In the absence of calculating the exact dynamics of the space charge cloud, it is possible to attempt to consider empirically the threshold effects in the above framework.

The charge density at the surface of the wire depends on the radial charge density given experimentally above, the diffusion along the direction of the wire, and the angular diffusion perpendicular to the wire axis.

The diffusion along the direction of the wire is probably dominated by the diffusion over the total drift of the electrons, between zero and three centimeters in the present case. The diffusion along the direction of the wire would be approximately proportional to the square root of the distance between the anode and the point where the

ionization originates in $\text{cm} \cdot \text{Torr}$. If this distance is fixed in terms of centimeters, then the diffusion in terms of $\text{cm} \cdot \text{Torr}$ increases with the square root of the pressure.

Intuitively, this can be confusing as in terms of normal distance, that is, cm increasing pressure decreases diffusion. However, it decreases it in terms of cm , but it increases it in terms of the average distance between molecules.

Only the diffusion in terms of θ , which depends on the transverse diffusion in the very high fields near the wire is not known. It is possible that this component of the diffusion becomes greater as the wire gets thinner. This is because the amount of spatial diffusion needed to get a given angular size decreases as the radius decreases. However, as s decreases, X increases, making it possible that the transverse diffusion also decreases. Thus in the absence of a measured value it will be assumed that the two effects cancel, and that the angular spread is constant, and thus that the spatial extent is proportional to s .

Thus it appears that both components describing the transverse size of the electron charge distribution will increase due to transverse diffusion effects as s_a is increased. This means that for equal gains, the net charge density is decreased. This is now applied to the present experimental examples.

First, it can help explain why higher space charge free gain is achievable with the 100 micron wire at atmospheric pressure, and not with the 10 micron wire at atmospheric pressure, independent of the value of d^2z/dx^2 . Note that the diffusion along the direction of the wire would be approximately the same in both cases. However, around the wire the diffusion should be approximately a factor of 10 greater for the 100 micron wire. The net result being that one would require a factor of 10 increase in radial charge density to achieve the same peak value of space charge density. This is approximately the case at the two thresholds, however it does not explain why the 10 micron wire has a semi proportional region and the other does not.

It is interesting to consider what effect variable pressure would have on δX , the change in the electric field due to the positive ions. Section 2.11 gives that δX depends on the product of pressure and charge density. For the comparison between the 10 and 100 micron wires of figure 40, the value of s_a is the same, so only the square root of pressure dependence of the diffusion parallel to the wire axis should affect the transverse charge density. Thus δX should increase as the square root of P . As stated above, there seems to be almost a factor of 10 difference in the radial charge density. This would indicate that the effect of the space charge depends on pressure in a linear

way. If the change in the field δX depends on the square root of the pressure, then it follows that the modification of the gain would depend on $(\delta X)^2$, and so the second order term would dominate. This supports the phenomenology of section 5.3a.

5.4 Summary

Sections 5.1 and 5.2 illustrate simple techniques for measuring the limits of proportional operation of the gas amplification of MWPC.

From these measurements a few empirical observations may be made. First, the comparison of the 10 and 100 micron wires shows that for K and s fixed space charge modification of the gain increases as r decreases or equivalently as P increases. This does not mean that for fixed wire radius, space charge modification increases with pressure. In fact experimentally, the space charge modification is reduced for the 10 micron wire, as the pressure is increased from 1 to 2 atmospheres. However, at the same time the dominant area of the z versus X curve is becoming more linear. This linearity can be expected to reduce space charge effects.

The analysis in this chapter shows that it is neither the charge density nor the functional form of z alone that determines the way in which space charge modifies the gain. As the charge density depends on the source of primary ionization and other geometrical factors not easily varied, it is useful to associate an empirical rule with the functional form of z .

The data points given here are compatible with the empirical rule that space charge leads to a stable attenuation of the gain in the region where d^2z/dX^2 is negative. The space charge effect decreases as d^2z/dX^2 increases to zero. As d^2z/dX^2 changes sign, and then increases, space charge can lead to a nonproportional increase in chamber gain.

CHAPTER 6

EPILOGUE

The general processes that control the operation of MWPC have been known for many years. This has led to a qualitative but not quantitative understanding of their behavior. It has been the purpose of this thesis to introduce a more systematic approach to the study of gas amplification that may one day lead to a fuller understanding of the operating characteristics of gas filled proportional counters. Even without a complete understanding, a more quantitative understanding can permit gas counter optimization for some cases.

The basic technique used in the thesis centers around demonstrating how measurements made under operating conditions that may be considered "abnormal", thick wires at low pressure, may lead to a better understanding of "normal" operating conditions, thin wires at atmospheric pressure. This involves having a theoretical basis for why the two cases should be similar and what can cause them to differ, then producing accurate measurements that verify under what conditions they are similar or different. This permits the explicit separation of the effects of different processes.

To demonstrate this approach, the simple models for predicting gas amplification were examined and their conditions of validity extracted. It was shown how space charge effects could break the simple symmetry of these models in a way different from other secondary processes. From this difference it is possible to extract the effect of the space charge.

The simplest model of the gain mechanism relies on the parameterization of the gain process by an ionization coefficient that is a function of the value of the electric field at a point. To be universally applicable, this coefficient should be equal to the Townsend coefficient as measured in a uniform electric field. For the uniform field, the electrons may be assumed to be in some form of equilibrium with the field. This leads to the conclusion that the gain for a wire is given by

$$\ln G = KY(X_a) , \quad 2.17$$

for X_a equal to E/P at the anode wire surface and K equal to $E_a r_a$, the product of the electric field at the wire surface and the wire radius. This formula is the basis of all the traditional empirical formulae for the gas amplification in MWPC. However, in the avalanche near the surface of the wire, the field is not uniform and the electrons would not necessarily be expected to be in equilibrium with the field. In this case the gain is only given by

$$\ln G = \ln G(K, s_a) ,$$

2.16

for s_a equal to Pr_a .

Both these formulae deal with the same process, ionization by electron impact. They ignore all secondary effects and so describe only proportional gain.

To experimentally verify these formulae required a wide dynamic range of experimental gain measurements. The technique for measuring the gas amplification was described in Chapter 3. The technique used was very much the standard one. However, the technique for insuring that the final measurements are, in fact, absolute gain measurements is very important. In particular the correction for the finite amplifier time constants, although standard, is crucial.

As equation 2.17 is simpler to use than 2.16, the first step was to determine over what range it gives a reasonable representation of the gain. This was found to be for values of the electric field at the surface of the anode divided by the pressure, X_a less than about 150 volts/cm/Torr. It was also noted that most proportional and drift chambers used in physics experiments do not in fact operate in this region. Above this value of X it must be assumed that the electrons are no longer in equilibrium with the field and 2.16 must be used. In this latter case the ionization coefficient and hence gain is not just a

function of X but would also depend on the way in which the field changes with distance

$$dX/ds = -X/s$$

as well as the way in which the average electron energy changes with field,

$$d\langle\epsilon\rangle/dX = F(X).$$

Although the average electron equilibrium energy is not known, it is possible that it has a plateau region similar to figure 10, which ends around 150 volts/cm/Torr.

Having identified where equation 2.17 ceases to be valid, it was then necessary to develop an analysis procedure for dealing with equation 2.16, the two dimensional expansion in the K,s plane given by equation 4.5. From this it was possible to show that equation 2.16, does in fact, accurately describe all data expected to be in the proportional region.

The verification that equation 2.16 is valid in the proportional region is important in two respects. It allows the quantitative extrapolation and prediction of the gas amplification factor over a wide dynamic range of chamber voltages, wire radii and gas parameters. Note that most wire chambers, in fact, operate in this higher X , and therefore, non-Townsend proportional region. Secondly, it allows the identification and isolation of other non proportional effects. This is due to the scaling laws that are its consequence. In section 2.11 space charge was

shown to violate this scaling, while the other second order processes described in section 2.10 approximately obey the scaling. Thus the breakdown of 2.16 was used as a measure of the onset of the positive ion space charge cloud modifying the gas gain. This was done by comparison of the "abnormal" example of a 100 micron radius wire operating between one tenth and one fifth of atmospheric pressure, with the "normal" case of a 10 micron wire operating between one and two times atmospheric pressure.

The results indicated the expected dependence of the space charge effect on both the total charge present and the three dimensional spreading of this charge. This is in contrast with any possible simple dependence on the gain or total charge alone.

Incorporating arguments about the three dimensional nature of the space charge effects seemed to indicate a dependence on the second derivative of the ionization coefficient with respect to the field and the square of the space charge change in field. This is in empirical agreement with earlier parallel plate phenomenology giving a dependence on the sign of d^2z/dX^2 . This predicts a saturation of the gain if this quantity is negative and an increase if positive. Although both effects were observed, the case of gain increase was discontinuous and not separable from other secondary processes due to a lack of data.

Other effects found in MWPC are open to similar kinds of study. This thesis has emphasized the importance of the spatial scaling variable $s=rP$ for the study of gas amplification. As spatial scaling implies time scaling, spatial scaling can act as a microscope into the time structure of the avalanche. From lower pressure measurements in the same K and s region where ionization by double electron impact was suspected, it may be possible to distinguish this contribution from ionization by positive ion impact or space charge effect. Similarly gain measurements in the region of the high gain discontinuity for s_a equal to $7 \text{ cm} \cdot \text{Torr}$ might distinguish whether the gain discontinuity is space charge or photon triggered.

Parallel measurements with different gases may provide additional answers. For example, changing the concentration of a fixed gas constituent can give information about the effect its more fundamental cross sections have on the ionization coefficient, space charge, loss of electron equilibrium and the importance of photon effects.

The ultimate goal remains the possibility that through further theoretical and experimental work all the phenomena observed in MWPC may be related to the more basic atomic processes discussed in sections 2.4 to 2.8.

APPENDIX A

SIMULATION OF THE PARALLEL PLATE MEASUREMENT OF α

In Chapter 2 the simulation calculations of α in Argon are compared to the experimental values of Druyvestyn and Penning [17]. Their measurements were made in parallel plate geometry. The possibility of systematic errors in their measurements will now be examined.

The experimental values of Druyvestyn and Penning [17] are accumulated from the earlier work of Kruithof and Penning [18,19]. The earlier papers actually only publish values of α/P up to $E/P = 430$ volts/cm/torr. It is assumed that the last 4 points have been added by Penning privately, but are from the same source. They measure α in parallel plate geometry using electrons emitted from the cathode by an external Ultra Violet mercury lamp. They must introduce a correction δ due to the electrons initial low velocity.

The correction is defined by replacing the simple equation for the gain,

$$G = \exp(\alpha d),$$

with the equation,

$$G = \exp[\alpha(d-\delta)],$$

where d is the gap distance.

Their actual gain equation is more complicated due to the existence of secondary cathode phenomena. This will be discussed below.

The correction δ intuitively corresponds to assuming that the gain takes place over a slightly reduced gap distance. This is to allow the electrons to come into equilibrium with the field.

To see that the form of the correction is justified consider the following. Since the electrons are not initially at equilibrium with the field, the gain would then depend on x , the distance from the cathode. It is assumed that the avalanche growth is given by,

$$dN = \kappa(x)dx,$$

where κ is initially less than α due to the small initial energy. Once the electrons are in equilibrium with the field κ would become equal to α and independent of x .

Then,

$$\begin{aligned} \ln(d) &= \int_0^d \kappa(x) dx, \\ &= \int_0^d \alpha dx - \int_0^d (\alpha - \kappa) dx \\ &= \alpha d - \int_0^{x_0} (\alpha - \kappa) dx \\ &= \alpha(d - \delta). \end{aligned}$$

Where it is assumed $\kappa = \alpha$ for $x > x_0$ and.

$$\delta = (1/\alpha) \int_0^{x_0} (\alpha - \kappa) dx$$

Provided one measures the gain for a number of gap distances d_i such that $d_i > x_0$ for all i one may obtain α and δ from a linear fit of $\ln(G)$ to distance.

Experimentally there are a number of other parameters, like the number of initial electrons, which depends on the cathode field, the rate of electrons generated by positive ions and avalanche photons hitting the cathode also dependent on the field, etc. Thus defining δ is not so easy. In their first paper [18], Kruithof and Penning have set $\delta = 15.7/E$. This comes from assuming the electron begins with energy zero and must travel a distance $15.7/E$ before ionization becomes possible. The distance required to obtain the ionization energy is not the distance at which the electrons may be considered in equilibrium with the field. As it will be seen, something more of the order of the distance required to gain the average equilibrium energy from the field may be more appropriate.

In their later paper [19] dealing with mixtures of Argon and Neon, Kruithoff and Penning have chosen to treat δ as a fitting parameter and have obtained it as a function of E/P . However, the data given by Druyvestyn and Penning [17] agree with [18] and so are probably derived assuming $\delta = 15.7/E$.

To consider the effect this has on the measured value we begin by starting the electrons at $x=0$ with $\epsilon=lev$. The electrons are drifted in a uniform field in the x direction. The average number of ionizations $n(x)$ up to the point x is then found. The uncorrected value of α would then be,

$$\alpha = n(x)/x.$$

Two cases of the correction are considered.

$$\alpha = n(x)/(x-\delta).$$

The first using $\delta = 15.7/E$ and the second from fitting the data above a certain cut to a line to extract δ and using it to correct point by point.

The cut is a difficult thing to define as one should not expect the point x_0 to correspond to δ . In general, the point where α and κ become equal is greater than δ . This is because δ assumes there is no ionization until x_0 and then after x_0 ionization continues as if equilibrium has been established. In practice it is impossible to define a point where equilibrium has been established. This problem can be eliminated by calculating the gain for large distances. This leads to large gains which make similar experiments impossible. Kruithof and Penning for the most part have worked at gains of less than 100. This is because of the unstable nature of counters filled with pure Argon.

Figure 41 shows the results for two values of E/P . It is seen that the uncorrected data are always less than the true value as one would expect. The simple correction $\delta = 15.7/E$ underestimates the data for high E/P . This is a consequence of the high mean electron energy at high E/P . At $E/P=100$ the simple correction overestimates the value. This is due to the mean energy of the electron being less than 15.7ev as seen in figure 10. Although no ionizations can happen until the electron has passed through a voltage drop of at least 15.7ev, few are expected. Thus the simple correction adds more ionizations than would be predicted by κ .

When expressed as a voltage, i.e. δE , the points seem to have corrections very near the mean energy of the electrons at equilibrium.

Figure 41

Deviation of the calculated values of α/p from the expected true value for a constant field of 100 and 1000 volts/cm/Torr. The expected value of α/P is that obtained by extrapolating to higher gain than would be accessible to experiment. For $E/P = 100$ the average electron energy is less than 15.7ev while for $E/P = 1000$ it is greater. Thus a simple correction of 15.7ev is not really appropriate. The squares are the uncorrected data, circles are the simple correction, and the triangles represent the correction from the fit.

APPENDIX B

GAIN FLUCTUATIONS

For the sake of completeness a small section on the expected statistical fluctuations in the gain is included. As expected, these fluctuations are most serious when measuring small numbers of initial electrons. This is not the case for the measurements of this thesis. It is, however, very important in very refined dE/dx measurements and in particular cluster counting, as it becomes very difficult to determine clusters of size one from clusters of size two, due to the fluctuations in the gain. In fact, one may never measure all clusters of size one just due to the probability of small gain.

A large body of experimental and theoretical work has been done on the statistical fluctuations of the avalanche process. As this work is not in complete agreement [45] we will not attempt to review it here.

The differential equation

$$dN = \alpha dr$$

as originally proposed by Townsend does not correspond to

the true physical situation as it does not allow for any statistical fluctuations in the gain. It assumes a completely deterministic growth in the number of electrons.

Briefly, the avalanche is not strictly a Poisson process, as the probability of having collisions does depend upon how much time has elapsed since the last collision. Most of the more sophisticated models attempt to take this into account by various means such as approximating the electron distribution as being made up of "slow" and "fast" electrons.

These effects are not taken into account by the above differential equation. It is possible to build a statistical model based upon the same physical assumptions as the Townsend equation. This follows the standard example of a linear birth process given in many text books, for example, Cox and Miller [46].

Let $N(r)$ be generalized to be a random variable representing the number of electrons present in a cloud centered at r . r remains the deterministic variable along which the electrons drift due to the drift velocity w . Suppose there are initially N_0 electrons, let the probability that $N(r)$ is equal to i be given by,

$$P(i, r).$$

Now we proceed as usual. The probability of an ionization in the interval $(r, r+dr)$ conditional on $N(r)=i$ is

$$i\alpha dr + O(dr^2),$$

assuming the independent superposition of i Poisson processes each of rate α . Then the probability that there are i electrons at $r+dr$ would be given by the probability of i electrons at r and no ionizations occur in the interval $(r, r+dr)$ plus the probability that there are $i-1$ electrons at r and one ionization occurs in the interval $(r, r+dr)$. Ignoring the probability that there might have been two ionizations in the interval which is negligible as dr is an infinitesimal distance. In symbols,

$$P(i, r+dr) = P(i, r)(1 - \alpha dr) + P(i-1, r)\alpha(i-1)dr.$$

Re-ordering,

$$[P(i, r+dr) - P(i, r)]/dr = -\alpha i P(i, r) + \alpha(i-1)P(i-1, r).$$

Hence,

$$dP(i, r)/dr = -\alpha i P(i, r) + \alpha(i-1)P(i-1, r).$$

This we solve subject to the boundary conditions that we have N_0 electrons at position $r=\infty$ so $P(i, \infty) = \delta_{iN_0}$.

Introducing the generating function

$$G(z, r) = \sum_{i=0}^{\infty} P(i, r)z^i,$$

we notice $G(1, r)=1$ and

$$\delta G(z, r)/\delta z = \sum_{i=0}^{\infty} i P(i, r)z^{i-1}.$$

So multiplying the above by z^i and summing yields

$$\delta G(z, r)/\delta r = -\alpha(r)z(1-z)\delta G(z, r)/\delta z$$

subject to

$$G(z,0) = z^{N_0}.$$

The solution of which is

$$\begin{aligned} G(z,r) &= (z \exp(-\int \alpha dr) / [1 - z \exp(-\int \alpha dr)])^{N_0}, \\ &= [az / (1 - bz)]^{N_0}, \end{aligned}$$

where for simplicity we let

$$1/a = \exp(\int \alpha dr)$$

and

$$b = 1 - a.$$

From the definition of G we have,

$$P(n,r) = (1/n!) \delta^n G(0,r) / \delta z^n.$$

For $N_0 = 1$ this gives,

$$P(n,r) = ab^{n-1}.$$

This is in direct analogy with a simple coin flipping model where the probability of a head is "a", and $P(n)$ is the probability that the first head is on the n th toss. The general solution is the negative binomial distribution giving the probability that the m th head is on the n th toss. As expected the average gain in this model is

$$1/a = \exp(\int \alpha dr),$$

in complete agreement with the deterministic model.

After routine manipulation we may calculate the standard deviation

$$\sigma = \sqrt{N_0} b/a$$

The expected number is N_0/a so that the relative standard deviation is

$$b/\sqrt{N_0}.$$

For routine operating conditions the gain is much larger than 1 so b is approximately 1 and the relative standard deviation is

$$1/\sqrt{N_0},$$

which for single electrons is 1.

Converting from discrete to continuous variables we find the general probability of obtaining a signal of charge Q to be

$$P(Q) = a^{N_0} Q^{N_0-1} \exp(-Qa)/(N_0-1)! .$$

This distribution, while simple to derive, does not take into account the fact that the electrons are not in equilibrium with the field in all cases. Thus it is most accurate for the thicker wires where the Townsend coefficient is valid as discussed in Chapter 4. As the Townsend coefficient ceases to be valid for thinner wires the statistical fluctuations are reduced.

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