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Diagonal Tension Buckling of Stiffened Aluminum and GLARE Cylinders

By

**Yosha Mendis, B.Eng. Aerospace Engineering
Ryerson University, Toronto, 2000**

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ABSTRACT

Fibre-Metal Laminates (FMLs) are hybrid materials that are currently being investigated in Canada for applications in primary aircraft structures. As part of a larger ongoing collaborative project between the National Research Council – Institute for Aerospace Research – Structures Materials Propulsion Laboratory (NRC-IAR-SMPL), Bombardier Aerospace and Carleton University, this investigation studied the diagonal tension behaviour in curved webs made from an FML termed GLARE (GLASS Reinforced aluminum). For this study, a stiffened cylindrical specimen was designed (IAR cylinder) using the methodology developed by the National Advisory Committee for Aeronautics (NACA) for aluminum alloys. Varying thickness of GLARE 3, GLARE 4, and 2024-T3 aluminum were used for the cylinder web. An added objective was to determine if the NACA-based diagonal tension analysis method can accurately predict the stresses and failures of an orthotropic material in a curved web system.

This thesis summarizes the diagonal tension theory, design of the test specimen, development of the numerical simulation methodology, its calibration using experimental results from NACA, and its application to the FML specimens.

The numerical simulation using the code, LS-DYNA, showed very good agreement with NACA experiment. As a result the same modeling procedure was used to model and provide predictions for the IAR cylinder. The predictions will be compared to experimental test results that will be obtained by NRC later this year.

Non scholae sed vitae discimus!

Dedicated to my father, Shanti,
who has taught me his belief, through example,
that learning is a life long process.
Thank you for your tireless hours of editing,
nagging and of course, love!

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UNITS

The U.S system of units is used in place of the International System (SI) throughout the thesis due to the recognition of its use within the Aerospace industry in North America.

U.S. Units	SI Units
1 in	25.4 mm
1 ft (12 in)	0.305 m
1 lbf	4.45 N
1 kip (1000 lbf)	4450 N
1 psi	6895 Pa
1 ksi (1000 psi)	6.895 MPa

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NOMENCLATURE

A_{ST}	area of stringer, in ²
A_{RG}	area of frame, in ²
C_2	stress concentration factor developed for flat panels to compensate for the non uniform stress distribution
d	distance between frames, in
E	Young's modulus, psi
h	distance between stringers, in
k	diagonal tension factor
k_s	shear buckling stress coefficient
R	radius of cylinder, in
t	web thickness, in
t_{RG}	thickness of frame, in
t_{ST}	thickness of stringer, in
Z	curvature parameter
α	angle of diagonal tension, deg.
α_{PDT}	pure diagonal tension angle, deg.
Δ	correction for allowable ultimate shear stress in a curved web
ϵ	web strain
ϵ_x	flange strain (defined for flat panels)
ϵ_y	stringer strain (defined for flat panels)
ϵ_{RG}	frame strain

ϵ_{ST}	stringer strain
μ	Poisson's ratio
σ_{RG}	frame stress, psi
σ_{ST}	Stringer stress, psi
σ_{RG_max}	maximum frame stress, psi
σ_{ST_max}	maximum stringer stress, psi
τ	nominal applied shear stress, psi
τ_{all}	allowable ultimate shear stress in a curved web, psi
τ_{CR}	critical shear stress corrected for plasticity, psi
$\tau_{CR_elastic}$	critical shear stress in the elastic range, psi
τ_{DT}	component of applied shear force carried by diagonal tension, psi
τ_{max}	maximum shear stress, psi
τ_S	component of applied shear force carried by web shear, true shear, psi

1.0. INTRODUCTION

One of the primary objectives in designing and building aircraft structures is to use materials that result in an optimal strength-to-weight ratio while meeting the necessary certification criteria and safety standards.

Over the past 80 years, aluminum has been extensively used in conventional airframes due to its low material costs, light weight, and ease of repair and maintenance; however, metals are susceptible to fatigue and corrosion. This weakness was highlighted in the Aloha Airlines disaster (NTSB 1989) that initiated a number of studies of aging aircraft problems, one of which was another study of fatigue damage in riveted metallic structures. Aluminum's disadvantages resulted in the investigation of new materials.

This investigation into new materials led to the development of the polymer matrix composite materials (composites) for aerospace applications in the 1970's. In many cases, composites provide higher strength-to-weight and stiffness-to-weight ratios and better fatigue damage tolerance than metals. Composites have found applications in structures such as the empennage and floor beams of the Boeing 777, the aft pressure bulkhead of the Gulfstream G-IV, and the fan blades of the GE 90 Engine. However, use of composites is limited due to disadvantages such as low impact resistance. Also, although raw material costs are continuously decreasing, the manufacturing and finishing processes are very labour intensive, which keeps the product costs high. Additionally, quality control and *in-situ* inspection methods are either ineffective or provide very limited results. For example, for composites, Eddy current and C-scan inspections that

are useful on metals, show only a general area of damage. Repair of damage in composites introduces new challenges. Mechanically fastened and bonded patches are the two common methods used for repair. Riveting or bolting procedures, although used, are not desirable as they introduce discontinuous fibres and stress concentrations. Bonding, while producing a more effective repair, requires specialized environmental controls and highly qualified personnel (Clark 2002).

While the majority of researchers were investigating composite materials, a smaller group concentrated their efforts on developing a material that combined the advantages of the metals and composite materials without their respective disadvantages (Asundi and Choi 1997). This resulted in the development of a class of hybrid materials termed Fibre Metal Laminates (FMLs).

The most mature FML variant is GLARE (GLAss REinforced aluminum), a laminate consisting of alternating layers of thin aluminum and glass fibre/adhesive layers in unidirectional or cross-ply arrangement.

FMLs have strength comparable to monolithic aluminum, and their use can result in weight savings of up to thirty percent. Additionally, FMLs have better fatigue properties, impact resistance, and durability. Unlike current composite materials, inspection and repair techniques for FMLs are similar to the existing techniques for conventional aluminum structures. Currently, FMLs are used in aircraft secondary and tertiary structures such as cargo bay floors, access hatches, and wing flaps. Success of FMLs in the secondary structures led to the application of GLARE for the Airbus A380 fuselage structure.

Currently in Canada, the National Research Council – Institute for Aerospace Research – Structures Materials Propulsion Laboratory (NRC-IAR-SMPL), Bombardier Aerospace and Carleton University are studying the feasibility of FMLs for primary structural applications in an extensive collaborative study.

One phase of the collaborative study is the investigation of the diagonal tension properties of GLARE. The concept of diagonal tension, initially studied by Herbert Wagner (Wagner 1931a, b, c), hypothesized that, even in a buckled state, an aircraft structure is capable of carrying a shear load through diagonal tension bands. This led to a design philosophy in which an aircraft structure is optimized by allowing specific areas of fuselage or wing spars to undergo elastic buckling. While visible diagonal tension bands develop (Figure 1.1), there are no permanent deformations when the loads are removed.

Figure 1.1: Diagonal Tension in a Boeing B-52 Stratofortress (Benak)

In the mid 1930's, Paul Kuhn of the National Advisory Committee for Aeronautics (NACA) used Wagner's analytical theories of diagonal tension to develop a semi-empirical design procedure to verify the suitability of aluminum as an aircraft web material (Kuhn *et al.* 1952a). Many organizations have further developed the NACA diagonal tension theory for various isotropic materials and manufacturing methods. The applicability of the NACA analytical theory to orthotropic material such as GLARE is in the early stages of development for flat stiffened web systems (Jodoin 2002); however, validity of the theory as applied to stiffened curved web systems is still unknown.

The intention of this task of the collaborative study was to follow the previous studies conducted by NACA during the early 1950's (Clark and Moore 1952; Kuhn *et al.* 1952 a, b) and to investigate the diagonal tension properties of GLARE and aluminum in curved web systems. To achieve this objective, the NACA based analytical method was used to design a stiffened cylinder, while the finite element (FE) code LS-DYNA was used for the buckling analyses.

This thesis presents the analysis and results carried out in the design of a cylindrical test specimen following the methods developed by NACA. A literature review provides the development of the NACA based Incomplete Diagonal Tension Theory and the advances of FMLs. This is followed by a definition of the research objectives, a description of the IAR specimen design and experimental set-up. The finite element analysis approach, simulation and results are followed by conclusions and recommendations for future work.

The simulation results will be compared to the experimental test results that will be obtained by NRC-IAR-SMPL (SMPL) later this year.

2.0. DEVELOPMENT OF DIAGONAL TENSION

Currently, the most widely used design and analysis method for stiffened shear panels is the *theory of diagonal tension* as presented by Paul Kuhn (Kuhn *et al.* 1952a) in the early 1950s. This method assumes that the stringers and frames provide rigid boundaries against local buckling and, as a result, web bays buckle first. The various formulae used in this method are based on a large number of experiments performed on flat panels by NACA. The theory, with a few modifications, was also applied to curved panels. This application was then validated by a number of experiments with stiffened and unstiffened cylinders.

The following section explains the diagonal tension theory of isotropic curved panels and includes suggested modifications for investigating *incomplete diagonal tension* of hybrid materials.

2.1. SHEAR RESISTANT WEBS

Early research into light weight structures was strongly influenced by civil engineering and shipbuilding design principles. Metallic airframes were often designed with *shear resistant webs*, assuming that the web carried the entire shear loads while compressive or tensile stresses were carried by flanges. From the design perspective, the theoretical load at which the web buckled was considered to be the structural failure load, and these structures were designed to withstand loads only up to the buckling limit of the web material.

The resulting web thickness was much greater than an optimal-weight design since the initial buckling typically occurs at a fraction of the ultimate load. Stiffeners were used to improve the buckling load of these webs. However, this solution was not a structurally or economically advantageous design.

2.2. THEORY OF PURE DIAGONAL TENSION

In the late 1920's, German Professor Herbert Wagner realized that the existing aircraft design methods were too conservative. While working for the Rohrbach Metal Airplane Company, Wagner developed the concept of *pure diagonal tension (PDT)*. This theory states that an initially buckled web only limits the compressive load carrying capacity of the web material. This means that the structure with transverse stiffeners is still capable of withstanding shear loads (Wagner 1931 a, b, c). The existing English reports that present the proof of PDT are poor translations from the original German. As a result, the presentation of the theory is not coherent. However, reports such as Kuhn's summary (Kuhn 1933; Kuhn 1940) and other texts (Megson 1990) provide a better description of the theory proposed by Wagner.

The principles of PDT were developed using a single frame flat web panel. PDT theory states that when a shear load, P , is applied to a truss frame (Figure 2.1a), the strut A is in tension while the strut B is in compression. If A and B are thin columns, strut B will buckle as shown in Figure 2.1b. According to Wagner, when the shear load is further increased, tension in strut A will carry 100% of the increased shear and the load carried by strut B will be negligible.

Figure 2.1: Principles of Diagonal Tension (Kuhn 1933)

If a very thin web panel replaces the struts, a similar reasoning can be applied. The compressive stresses in direction of strut *B* will cause the web to fold into a large number of narrow diagonal buckles inclined at an angle α to the frame (Figure 2.1c). These buckles will behave as a series of tension diagonals that carry the shear load in the web. Such a web panel is the basic cell of a *Wagner beam*. If the web panel is square, the folds will form at an angle of approximately 45° . If the web panel is rectangular, the direction of the folds may vary; however, PDT theory (Jodoin 2002) has shown that the angle is still approximately 45° , with the assumption that the web panel edge supports are perfectly stiff.

Practical experience showed that designs based on PDT theory are conservative, mainly because the web continues to carry shear loads. Up to the point of buckling, the web is in a state of pure shear that is equivalent to tensile and compressive stresses of equal magnitude at 45° to the horizontal and vertical directions. In contrast to his PDT theory,

Wagner found that some compressive stresses still continued to exist in the direction of strut B , (Figure 2.1) beyond the buckling point (Kuhn 1933).

The relative effect of the compressive stresses decreases after buckling as the shear force increases. The finite strength of the material is the determining factor for failure, which occurs before the diagonal-tension field is fully developed. Therefore, the state of pure diagonal tension is a theoretical limiting case that is approached asymptotically but never reached in actual structures.

As a result, for an optimum design of a thin web panel, an incomplete diagonal tension theory, a hybrid of the shear resistant panel theory and the pure diagonal tension panel theory, must be considered.

2.3. BEYOND PURE DIAGONAL TENSION

Following the flat web experiments, Wagner (1936) postulated that the compressive stresses remain constant at the buckling stress value, beyond the buckling point, for both flat and curved panels. Under this revised theory, stresses greater than the buckling stress are converted into diagonal tension while the buckling stress is carried by the sheet as true shear. This was stated as:

$$\tau_{DT} = \tau - \tau_{CR} \quad (2.1a)$$

$$= \tau \left(1 - \frac{\tau_{CR}}{\tau} \right) \quad (2.1b)$$

where τ is the nominal applied shear stress, τ_{DT} is the part of the applied shear stress that is carrying the diagonal tension and τ_{CR} is the critical stress at buckling. The second form of the equation (Equation 2.1b) shows that diagonal tension is a function of the inverse of the loading ratio (τ/τ_{cr}). The equation was found to be inadequate when compared to experiments, since the panel folds were overly deep soon after initial buckling, followed very quickly by permanent buckling and failure.

2.4. INCOMPLETE DIAGONAL TENSION

Schapitz (1937) continued Wagner's work to develop one of the first theories of *incomplete diagonal tension* (IDT). He considered a web-stringer flat panel loaded by combined normal and shear stresses, assumed a stress distribution within the panel, and left one basic parameter free in order to explain the anomalies found between the experiments and the theory. Schapitz applied similar assumptions for curved panels and left two parameters free to fit numerical results to experimental results. Like Wagner, Schapitz made the erroneous assumption that the compressive diagonal stress continued to be constant after buckling. Due to this assumption, Schapitz's attempts to correlate IDT theory to experiments were also unsuccessful.

In the early 1930's Paul Kuhn, a researcher at NACA, began an extensive study of diagonal tension. By the early 1940's, he had determined that Wagner's and Schapitz's methods were too conservative when applied to aircraft design (Kuhn 1940; Kuhn 1939). Kuhn verified that the buckled web of an aircraft was capable of carrying compressive loads but this ability was reduced as shear forces were increased. He continued to

conduct research on flat and curved web systems for another decade and finally summarized the theory (Kuhn 1952a) and the supporting experiments (Kuhn 1952b) in May 1952. This “new” theory of IDT, assumes that the total shear force in the web can be divided into a shear force carried by diagonal tension (τ_{DT}) and a shear force carried in shear by the web (τ_s).

$$\tau = \tau_s + \tau_{DT} \quad (2.2)$$

Although Kuhn’s theory was originally developed for flat panels, the theory was later applied to curved web systems by using empirical relations and making various assumptions. The following section outlines the assumptions made and the resulting theoretical procedure.

2.4.1 Principles of Incomplete Diagonal Tension for Curved Web Systems

Analysis for curved web systems initiated by considering a polygonal cylinder (Figure 2.2a). The theory of IDT developed for flat panels could be applied, with minor modifications, to a polygonal cylinder subjected to torque loads. If the cylinder were built with circular web sections and attached to polygonal frames (Figure 2.2b), the web would initially “flatten” after buckling. As the torque load increased further, the cylinder would approach the shape of a polygonal cylinder. Therefore, in the limit, the theory derived for flat panels may be applicable, but not for the intermediate stages. In a fuselage, the frames are circular, not polygonal (Figure 2.2c). Therefore, unlike in a flat panel, all the tension diagonals of one bay cannot lie in one plane even when the diagonal tension is fully developed.

Figure 2.2: Diagonal Tension in Curved Webs (Kuhn 1952a)

In order to derive a theory of incomplete diagonal tension in circular cylinders with minimal modifications to the flat panel theory, Kuhn started with Wagner's basic relations for two cases:

- a. cylinders with long panels in the axial direction ($d > 2h$, see Figure 2.3a) between two closely spaced stringers ($h < R/3$)
- b. cylinders with long panels in the circumferential direction ($d/h > 2$, see Figure 2.3b) between two closely spaced frames ($d < R/3$)

Figure 2.3: Panel Types (Kuhn 1952a)

In both cases, d specifies the distance between two frames, h specifies the distance between two stringers and R , the radius of the cylinder. In case (a), a major portion of the tension diagonals lies in the surface planes of the “polygonalized” cylinder while in case (b), a majority of the tension diagonals lies on a hyperboloid of revolution.

The remainder of the section develops the theory for panels of case (a).

Referring to equation (2.2), IDT is based on the assumption that the web-shear, τ , can be divided into a *true shear*, τ_s , portion and a *diagonal tension* portion, τ_{DT} as given by:

$$\tau_s = (1-k)\tau \quad (2.3)$$

$$\tau_{DT} = k\tau \quad (2.4)$$

where k is the *diagonal tension factor*, which gives the percentage of complete diagonal tension in the web. A value of $k = 0$ defines initial buckling of the web and $k = 1$ defines a web in pure diagonal tension. Figure 2.4 shows the state of stress in the web for the limiting and the general intermediate case. The angle from the neutral axis, α , is defined as the *angle of diagonal tension* and measures the angle of the web buckles.

Figure 2.4: Stress Systems in the Webs (Kuhn 1952a)

For a specified web system at a given torque load, the diagonal tension factor, k , is specified by fitting a generalized formula to experimental data as:

$$k = \tanh \left[\left(0.5 + 300 \frac{d}{h R} \right) \log \left(\frac{\tau}{\tau_{cr}} \right) \right] \quad (2.5)$$

with the following supplementary rules:

- (a) If $h > d$, replace d/h by h/d
- (b) If d/h (or h/d) is larger than 2, use 2, due to the asymptotical nature of the relationship as shown graphically in Figure 2.5.

Figure 2.5: Diagonal Tension Factor, k (Kuhn 1952a)

In equation (2.5), the ratio τ/τ_{cr} is called the *loading ratio*, t is the thickness of the web, and τ_{cr} is the critical shear stress, at which web buckling first occurs.

The above figure implies that it is impossible to reach a pure diagonal tension state. The applied shear stress, τ , must be several hundred times greater than the critical shear stress to achieve a high level of diagonal tension. Most aircraft structures are estimated to fail when a loading ratio between sixty and eighty is reached. As a corollary, under shear loading the failure will generally occur in a state of incomplete diagonal tension.

In the elastic range, the critical shear stress is computed using:

$$\tau_{cr \text{ elastic}} = \frac{k_s \pi^2 E h^2}{12 R^2 Z^2} \quad (2.6)$$

This equation contains three constants: E , the Young's modulus of the material, Z , the curvature parameter and k_s , the shear buckling stress coefficient. The curvature parameter is found from the following equation:

$$Z = \frac{h^2 \sqrt{1 - \mu^2}}{R t} \quad (2.7)$$

where μ , is the Poisson's ratio. Curvature parameters for aircraft fuselages range from eight to thirty-five (compiled using Niu 1988).

Shear buckling stress coefficient k_s is based on the assumption of simply supported edges and obtained using the following figure:

Figure 2.6: Shear Buckling Stress Coefficient for Simply Supported System (Kuhn 1952a)

Experiments have shown that empirical modifications were necessary to properly predict stresses within the web. One such relation was a plasticity correction for the critical shear stress. Empirical data was collected to correlate the elastic critical shear stress to plastic critical shear stress, as shown in Figure 2.7. In essence, this figure is used to correct for plasticity for both 2024-T3 aluminum (24S-T3) and Alclad 7075-T6 aluminum (75S-T6) alloys.

Figure 2.7: Plasticity correction for critical shear (Kuhn 1952a)

The theory for critical shear stress assumes that the edges of the webs are simply supported. However, during manufacturing, the web edges are riveted to the stringers and frames, resulting in an edge condition that falls between the assumed simply supported and clamped states. Results of experiments by Batdorf *et al.* (1947 a) and Kuhn and Griffith (1947) confirmed good agreement with theoretical and experimental values. Kuhn and Griffin (1947) hypothesized that a reduction factor may be necessary in the following two cases:

- i. when the thickness of the stringers or frames is less than the thickness of the web.
- ii. when the web is very thin (i.e. $t < 0.020\text{in}$).

However, in their report, Kuhn and Griffin (1947) concluded that available data were insufficient to present even an approximate reduction factor.

The buckle pattern that occurs immediately after initial buckling does not consist of a pattern of parallel folds; this pattern is only approached asymptotically. As a result, the defined angle of diagonal tension, α , is used to measure the web buckles. The angle α , adjusts itself so that the total strain energy in the web becomes a minimum. Due to the “coupling” between diagonal tension and shear in the case of IDT, the angle α cannot be calculated directly. In stiffened flat panels, the fundamental formula derived for the angle is:

$$\tan^2 \alpha = \frac{\epsilon - \epsilon_x}{\epsilon - \epsilon_y} \quad (2.8)$$

where ϵ is the web strain, ϵ_x flange strain and ϵ_y stringer strain (Kuhn 1952a).

Kuhn applied the above formula to cylinders using the following simplification: if the panel was cut along one long edge and both curved edges were flattened out, the long cut edge would be separated from the stringer by a distance, Δ . This distance is equal to the difference between the length of the arc and the length of the chord and is approximated by the equation:

$$\Lambda = \frac{1}{24} \frac{h^3}{R^2} \quad (2.9)$$

The same curved web configuration would have resulted if the flat panel had been compressed by an amount Λ . The change from a circular section to a polygonal section as the diagonal tension develops is therefore equal to the compressive strain, Λ/h in the frames.

As a result, the term ϵ_y can be replaced by:

$$\epsilon_y = \epsilon_{RG} - \frac{1}{24} \left(\frac{h}{R} \right)^2 \quad (2.10)$$

where, ϵ_{RG} is the strain in the frames.

The coefficient $1/24$ is a theoretical value for fully developed diagonal tension and should be less than $1/24$ in the incomplete diagonal tension region. Test observations have shown that the flattening proceeds very rapidly and the corresponding calculations concluded that the best agreement with the test data was obtained by using the full value of the coefficient (Kuhn and Griffith 1947). The formula for the diagonal tension angle then became:

$$\tan^2 \alpha = \frac{\epsilon - \epsilon_{ST}}{\epsilon - \epsilon_{RG} + \frac{1}{24} \left(\frac{h}{R} \right)^2} \quad (2.11)$$

where, ϵ is the strain in the sheet along the direction of the diagonal tension, ϵ_{ST} the strain in the stringer given by:

$$\epsilon_{ST} = \frac{\sigma_{ST}}{E} \quad (2.12)$$

and ϵ_{RG} the strain in the frame given by:

$$\epsilon_{RG} = \frac{\sigma_{RG}}{E} \quad (2.13)$$

σ_{ST} and σ_{RG} are the stringer and frame stresses as calculated below (equations 2.15, 2.16).

Due to the empirical factors contained in the development and for practical purposes, the formula of the diagonal tension angle is assumed to be valid if $\frac{d}{h} > 1.0$ and $h < \frac{R}{3}$.

The strain in the web is evaluated by:

$$\epsilon = \frac{\tau}{E} \left[\frac{2k}{\sin 2\alpha} + \sin 2\alpha (1-k)(1+\mu) \right] \quad (2.14)$$

The stringer stress and the frame stress is calculated by:

$$\sigma_{ST} = - \frac{k \tau \cot \alpha}{\frac{A_{ST}}{ht} + 0.5(1-k)} \quad (2.15)$$

$$\sigma_{RG} = - \frac{k \tau \tan \alpha}{\frac{A_{RG}}{ht} + 0.5(1-k)} \quad (2.16)$$

where, A_{RG} , is the frame cross-sectional area, and A_{ST} , is the stringer cross-sectional area.

Since the formulae for the web strain, stringer strain and frame strain are inter-dependent, the equations must be solved by successive iterations. In this process, a value for α is initially estimated. The strains in the stiffeners are then calculated using the estimated angle of diagonal tension. The resulting values are used to calculate an improved value of α using the formula for the diagonal tension angle. The iterations proceed until the calculated value of α is sufficiently close to the assumed value. Calculations have shown that no more than three cycles of the iteration process are necessary.

A first estimate of the diagonal tension angle is obtained by:

$$\alpha \approx \alpha_{PDT} \frac{\alpha}{\alpha_{PDT}} \quad (2.17)$$

where α_{PDT} is the web buckle angle if the web was in a state of pure diagonal tension and the value is obtained from Figure 2.8.

Figure 2.8: Angle of Pure Diagonal Tension (Kuhn 1952a)

where, R_R and R_S are defined as:

$$R_R = \frac{d t}{A_{RG}} \quad (2.18)$$

$$R_S = \frac{h t}{A_{ST}} \quad (2.18)$$

Using the diagonal tension factor, the ratio α/α_{PDT} is obtained from Figure 2.9.

Figure 2.9: Correction factor for Angle of Diagonal Tension (Kuhn 1952a)

The above curve represents the average of the scatter band obtained by plotting the ratio α/α_{PDT} for a number of webs with different dimensions.

2.4.2. Strength Analysis of Web

The shear stress for an aluminum 2024-T3 curved web failure is given directly by Figure 2.10, assuming that the diagonal tension is uniformly distributed.

Figure 2.10 Basic 24S-T3 Aluminum Allowable Values for τ_{max} (Kuhn 1952a)

For flat panels, a stress-concentration factor C_2 was developed to compensate for the non-uniform stress distribution by Wagner's theory of Flange Flexibility Effects (Kuhn 1952a). For the curved web system, a factor such as C_2 , was not developed and was assumed to be zero. In order to account for the error introduced by this assumption, an empirical reduction factor was developed. By analogy with flat panels it was assumed that the reduction factor should depend primarily on the bending stiffness of the stringers and frames; however, the experimental results were in much better agreement when the stiffening ratios involved the areas as parameters. Therefore, the allowable ultimate value, τ_{all} , for the shear stress in a curved web was given by the empirical expression:

$$\tau_{all} = \tau_{max} (0.65 + \Delta) \quad (2.20)$$

where, Δ , is defined by:

$$\Delta = 0.3 \tanh \frac{A_{RG}}{dt} + 0.1 \tanh \frac{A_{ST}}{ht} \quad (2.21)$$

Kuhn (1956) notes that τ_{all} may exceed τ_{max} because the term Δ may exceed a value of 0.35 if the stringers and frames are "heavy". This is because a grid system of stringers and frames can absorb some shear analogous to the portal frame effect (Kuhn 1952a) in the flat panels.

2.4.3. Strength Analysis of Stiffeners (Stringers and Frames)

Empirical corrections were necessary to account for the maximum stress developed in the stiffeners, and for the stress allowable due to localized buckling effects. The maximum stress in the web stiffeners was determined by the ratio of the bay width, d , and the bay height, h , plotted for different values of the diagonal tension factor, k , developed in the web (Figure 2.11).

Figure 2.11: Ratio of Maximum Stress to Average Stress in Stiffeners (Kuhn 1952a)

The following equation was derived from the graph, Figure 2.11:

$$\frac{\sigma_{stiff_max}}{\sigma_{stiff}} = 1 + (1 - k) \left(0.78 - 0.64 \frac{h}{d} \right) \quad (2.22)$$

where, σ_{stiff_max} is the maximum stringer stress σ_{ST_max} , or the maximum frame stress σ_{RG_max} , and σ_{stiff} is the stringer stress σ_{ST} , or the frame stress σ_{RG} .

The most common failure mechanism for a stiffener is forced crippling. The leg of the stiffener that is attached to the web suffers severe deformation as it is being forced to accommodate the shear buckles of the web (Figure 2.12). The extra deformation increases the detrimental effect of the compressive stress acting on the stiffener. The “free” leg also shows a corresponding wave pattern, but its amplitude is negligible compared to that of the attached leg.

Figure 2.12: Failure of Stiffeners by Forced Crippling (Kuhn 1952a)

Since the wavelength of the web wrinkles is small, the maximum value of the stiffener stress (σ_{stiff_max}), which occurs at mid-height, is used as the limiting case for this type of failure. NACA determined empirical equations for two material types to predict the allowable limit to prevent forced crippling:

For stiffeners made from 2024-T3 (24S-T3) aluminum alloys:

$$\sigma_{stiff_max} = (26 \text{ ksi}) k^{\frac{2}{3}} \left(\frac{t_{stiff}}{t} \right) \quad (2.23)$$

For stiffeners made from 7075-T6 (75S-T6) aluminum alloys:

$$\sigma_{stiff_max} = (32.5 \text{ ksi}) k^{\frac{2}{3}} \left(\frac{t_{stiff}}{t} \right) \quad (2.24)$$

where t_{stiff} is the stringer web thickness, t_{ST} or the frame web thickness, t_{RG} .

2.5. BEYOND INCOMPLETE DIAGONAL TENSION

Following NACA's investigations, Grumman Aircraft Engineering Corporation (Ratay 1968) and the Boeing Super-Sonic Transport project (Barevics *et al.* 1972) made efforts to analyze the diagonal tension properties of flat-web panels (Jodoin 2002). Both projects attempted to extend Kuhn's methodology to different materials and construction, but did not cover curved web panels.

In 1985, Northrop Corporation studied the existing data on buckled stiffened panels and concluded that there were limitations in analytical and experimental data with regard to composite panels and metal curved panels under shear or compression loading. Northrop then developed and reported on efforts made to automate and modify the NACA methodology for post-buckled composite and metal curved panels loaded in shear and compression (Deo 1985). Experiments were used to validate the analytical method

developed. The fundamental objectives were to develop techniques to predict the initial buckling load, ultimate failure load, and failure mode. Northrop's modifications to the NACA method are as follows:

2.5.1. Diagonal Tension Angle, α

For curved panels, Northrop made an initial assumption of 30° for the diagonal tension angle. The "new" value of α was calculated by the equation:

$$\alpha = \tan^{-1} \left(\frac{\varepsilon - \varepsilon_s}{\varepsilon - \varepsilon_r - R_f} \right)^{\frac{1}{2}} \quad (2.25)$$

where, ε , web strain in the diagonal tension direction, and ε_s and ε_r , strains in the stringer and frame that is attached to the web, respectively. R_f is the curvature parameter given by:

$$R_f = \frac{1}{24} \left(\frac{h_s}{R} \right)^2 \quad \text{if } h_r > h_s \quad (2.26a)$$

$$= \frac{1}{8} \left(\frac{h_r}{R} \right)^2 \tan^2 \alpha \quad \text{if } h_s > h_r \quad (2.26b)$$

where, R is the panel radius and h_s and h_r stringer and frame spacing as shown in Figure 2.13.

Figure 2.13: Diagonal Tension in Curved Webs (Kuhn 1952a)

Calculations of the web, stringer, and frame strains incorporate the anisotropic nature of the composite as follows:

$$\varepsilon = \frac{\tau}{E_{w\alpha}} \left[\frac{2k}{\sin 2\alpha} + \frac{E_{w\alpha}}{2G_{rs}} (1-k) \sin 2\alpha \right] \quad (2.27)$$

$$\varepsilon_s = \frac{-k \tau \cot \alpha}{\left[\frac{\overline{EA}_s}{h_s t_w} + 0.5(1-k)E_{ws} \right]} \quad (2.28)$$

$$\varepsilon_r = \frac{-k \tau \tan \alpha}{\left[\frac{\overline{EA}_r}{h_r t_w} + 0.5(1-k)E_{wr} \right]} \quad (2.29)$$

where, t_w is the web thickness and G_{rs} is the web shear modulus. $E_{w\alpha}$, E_{ws} , and E_{wr} are web moduli in the direction of the tension field, stringers and frames respectively. A_s and A_r are the stringer and frame cross-sectional areas respectively. $\overline{EA}_s$ and $\overline{EA}_r$ are the effective axial stiffnesses of the stringers and frames calculated by:

$$\overline{EA}_s = EA_s \frac{EI_s}{EI_s} \quad (2.30)$$

$$\text{and } \overline{EA}_r = EA_r \frac{EI_r}{EI_r} \quad (2.31)$$

where, E is the Young's modulus of the material, EI the bending stiffness about the stiffener (s , stringer; r , ring or frame) neutral axis, and $\overline{EI}$ bending stiffness about the web mid-surface.

In general, the calculated diagonal tension angle, α , will not equal the initially assumed value of 30° . Therefore, α is used as the next estimate and computations of equations (2.27 to 2.31) are repeated until the process converges. Once the diagonal tension angle has been determined, the margins of safety are computed.

2.5.2. Stringer and Frame Margins of Safety

Northrop identified the following three failure mode checks for the stringers and frames:

$$\text{i. For stringer and frame stability (i.e. no column failure)} \quad \varepsilon_{sB} > \varepsilon_{s \text{ ave}}$$

$$\varepsilon_{rB} > \varepsilon_{r \text{ ave}}$$

where ε_{sB} , and ε_{rB} are critical stringer and frame strains corresponding to the bending stiffness required for stiffener stability (i.e. Euler buckling strains for the stringer and the frame respectively) calculated by:

$$\varepsilon_{sB} = \frac{4\pi^2 EI_s}{E_{xs} A_s h_r^2} \quad \text{and} \quad \varepsilon_{rB} = \frac{4\pi^2 EI_r}{E_{xr} A_r h_s^2} \quad (2.32a, 2.32b)$$

where E_{xs} and E_{xr} are the compression moduli of stringers and frames in the loading direction. The stringer and frame strains averaged over the cross section lengths are $\varepsilon_{s\text{ ave}}$ and $\varepsilon_{r\text{ ave}}$ respectively. They are given by:

$$\varepsilon_{s\text{ ave}} = \varepsilon_s \frac{\overline{EA_s}}{\overline{EA_s}} \text{ and } \varepsilon_{r\text{ ave}} = \varepsilon_r \frac{\overline{EA_r}}{\overline{EA_r}} \quad (2.33a, 2.33b)$$

- ii. For stability of the total panel (i.e. prevent buckling of the web as a whole prior to the formation of the diagonal tension)
- $$EI_s > E_s t_w^3 \left(\frac{3h_s}{h_r} - 2 \right) h_s$$
- $$EI_r > E_r t_w^3 \left(\frac{3h_r}{h_s} - 2 \right) h_r$$

- iii. To prevent forced crippling of stiffeners

$$\varepsilon_{os} > \varepsilon_{s\text{ max}}$$

$$\varepsilon_{or} > \varepsilon_{r\text{ max}}$$

where $\varepsilon_{r\text{ max}}$ and $\varepsilon_{s\text{ max}}$ are the maximum strains in the legs attached to the web, given by:

$$\varepsilon_{s\text{ max}} = \varepsilon_s \left[1 + 0.0775(1 - k) \left(1 - 0.8 \frac{h_r}{h_s} \right) \right] \text{ if } h_s > h_r \quad (2.34a)$$

$$= \varepsilon_s \left[1 + 0.0775(1 - k) \left(1 - 0.8 \frac{h_s}{h_r} \right) \right] \text{ if } h_s < h_r \quad (2.34b)$$

$$\varepsilon_{r\text{ max}} = \varepsilon_r \left[1 + 0.0775(1 - k) \left(1 - 0.8 \frac{h_r}{h_s} \right) \right] \text{ if } h_s > h_r \quad (2.34c)$$

$$= \varepsilon_r \left[1 + 0.0775(1 - k) \left(1 - 0.8 \frac{h_s}{h_r} \right) \right] \text{ if } h_s < h_r \quad (2.34d)$$

where, k the diagonal tension factor and ε_{os} and ε_{or} are the stringer and frame forced crippling respectively, given by:

$$\varepsilon_{os} = 0.00058 \left[\left(\frac{\varepsilon_{all} E_{cs}}{1000} \right)^{\frac{2}{5}} k^{\frac{2}{3}} \left(\frac{t_{us}}{t_w} \right)^{\frac{1}{3}} \right] \quad (2.35a)$$

$$\varepsilon_{or} = 0.00058 \left[\left(\frac{\varepsilon_{all} E_{cr}}{1000} \right)^{\frac{2}{5}} k^{\frac{2}{3}} \left(\frac{t_{ur}}{t_w} \right)^{\frac{1}{3}} \right] \quad (2.35b)$$

The laminate allowable strain is ε_{all} and the modulus of the stringer and frame attached to the web are, E_{cs} and E_{cr} respectively. The thickness of the stringer and the frame leg attached to the web are, t_{us} and t_{ur} , respectively.

2.5.3. Experimental Validation

Northrop compared results of the modified semi-empirical NACA method to the experimental results of a total of seven panel configurations with different stiffener shapes, web thicknesses, web laminate orientations and stiffener spacing (Table 2.1).

Table 2.1: Comparison of Semi-Empirical Analysis with Experimental Data for Composite Shear Panels (Doe *et al.* 1985)

Investigator	h_s [in]	h_r/h_s	Stiffener Shape	EA_s [ksi]	τ_{ult}/τ_{cr}	P_{anl}/P_{exp}
Northrop/ Navy	10	1.5	Hat	2.5	5	0.93
McDonnell/ Navy	6	2.67	Hat	1.8	9.2	0.91
Lockheed/ Navy	6	3.75	I	1.5	6	1.08
Grumman/ Navy	7	3.43	Hat	2.9	6	1.025
Northrop/ IR&D	13	1.15	Hat	2.9	10	1.05
Northrop/ IR&D	13	1.15	I	2.9	10	0.91
Northrop/ IR&D	9	1.66	Hat	2.9	7	0.80

τ_{ult} – Ultimate failure stress

τ_{cr} – Buckling stress

P_{anl} – Analytical failure load

P_{exp} – Experimental failure load

The analytical method predicted stringer forced crippling as the failure mode, while experiments showed that the failure mode was separation of the stringers from the panel web. However, as shown in Table 2.1 there was good agreement between predicted failure loads and the experimentally obtained values despite the different failure modes.

2.6. DIAGONAL TENSION IN FIBRE METAL LAMINATES

Most diagonal tension investigations have studied isotropic materials, such as aluminum, and anisotropic materials, such as composites. Recently, the Structural Laminates Company and the Technical University of Delft have attempted to identify diagonal tension properties for FMLs. The following section highlights the development of FMLs, their properties, and summarizes the diagonal tension investigations of curved FML panels.

3.0. FIBRE METAL LAMINATES

FMLs are advanced hybrid materials that were originally developed for aircraft wing skins, but have found use in other aerospace applications due to their excellent fatigue, impact and corrosion properties.

This section will trace the history of FMLs, their properties, discuss the role of FMLs in the aerospace industry and identify diagonal tension investigations for stiffened web systems.

3.1. DEVELOPMENT OF FMLS

The concept of FMLs originated in the 1950's at Fokker in the Netherlands. The company moved away from the concept of building integrally machined structures due to economical considerations and experimented with laminated bonded metal structures to tailor the local strength. The bonded laminate metal structure was found to be successful in preventing the rapid fatigue crack growth that occurs in monolithic materials. Studies of metal and composite combinations in England and the United States prompted the addition of fibres to the adhesive of the metal laminates. Initially, Nylon fibres were applied as a weave and carbon fibres embedded in bond lines. In addition to the slower crack growth, an unexpected benefit was that the fatigue cracks in the FML structure was confined to the outer aluminum layer. Studies by ALCOA (Aluminum Company Of America) in the 1960's confirmed that FMLs could serve as a superior alternative to aluminum as an aircraft material, in certain airframe applications.

By the early 1980s the first commercially available FML's, based on Aramid fibres, were introduced at Delft University as a semi-finished sheet product (Vlot *et al.* 1999). The laminate named ARALL (Aramid Reinforced Aluminum Laminate) used unidirectional fibre layers because the laminate was optimized for wing skins. Basic descriptions of ARALL variants are shown in Table 3.1.

Table 3.1: Commercial forms of ARALL laminates (Lawcock *et al.* 1995a)

Designation	Metal Alloy	Description	Fibre	Post-cure Stretch
ARALL 1	7075-T6	120 ⁰ C (250 ⁰ F) cure, UD fibres	Twaron aramid	0.4% permanent stretch
ARALL 2	2024-T3	120 ⁰ C (250 ⁰ F) cure, UD fibres	Twaron aramid	with or without 0.4% stretch
ARALL 3	7475-T76	120 ⁰ C (250 ⁰ F) cure, UD fibres	Twaron aramid	0.4% permanent stretch

*Note: 1. UD – UniDirectional.

2. Twaron is a trade name of AKZO.

ARALL characteristics were tested using a full-scale Fokker-50 wing panel. This test proved that a weight saving of twenty percent was possible with FML materials in comparison to traditional materials. Subsequently, ARALL was used in the production of the C17 cargo door, which demonstrated a twenty-six percent weight saving when compared to the aluminum counterpart. However, the mass production of ARALL doors was not economical due to a complex manufacturing process that included stretch forming, roll forming, routing, bonding and finally riveting the different components (Vlot and Gunnink – Eds. 2001).

In 1987 the second crop of laminates (Figure 3.1) was generated as an improvement over ARALL. This new laminate replaced the aramid fibres with high strength glass fibres and was named GLARE (GLASS REinforced) (Roebroeks 1994).

Figure 3.1: Build up of a cross-ply GLARE laminate (Asundi and Choi 1997)

In 1991, a partnership between AKZO and ALCOA was formed to produce and commercialize GLARE. As a result of the agreement between the two companies, Structural Laminates Company (SLC) was founded. It was determined that SLC would produce the material while the research and development would be continued at Delft University.

The success of GLARE when compared to ARALL is due to the improved bonding characteristics between the adhesive and the glass fibres (Asundi and Choi 1997). In addition, the glass fibres do not share the tendency of aramid fibres to suffer microbuckling under compressive stresses. As such GLARE is not limited, even in its as-cured condition, to tension dominated applications. More importantly, due to the glass fibre's higher tensile strength, GLARE can be commercially developed in both the

unidirectional and biaxial variants (Krishnakumar 1993). The improved bi-axial properties allow for applications in areas subjected to combined loading such as pressurized fuselage skin.

Currently, there are six different standard GLARE grades as shown in Table 3.2. All variants are based on unidirectional S-glass fibres that are reinforced by FM 94 adhesive. A coding system based on the orientation and number of pre-impregnated fibre (prepreg) layers between the aluminum sheets defines the grade and sub-set of each variant of GLARE. For example, GLARE 4B-3/2 refers to a GLARE laminate consisting of 3 aluminum layers, 2 fibre layers, with a 90/0/90 fibre orientation within each layer.

**Table 3.2: Commercial standard forms of GLARE laminates
(Vlot & Gunnink – Eds. 2001)**

Variant	Sub Set	Metal Alloy	Prepreg Orientation in each Fibre Layer*	Main Beneficial Characteristics
GLARE 1	-	7475-T761 t = 0.3 mm	0/0	Fatigue, strength, yield stress
GLARE 2	A	2024-T3 t = 0.3 mm	0/0	Fatigue, strength
	B		90/90	Fatigue, strength
GLARE 3	-	2024-T3 t = 0.3 mm	0/90	Fatigue, impact
GLARE 4	A	2024-T3 t = 0.3 mm	0/90/0	Fatigue, strength in 0° dir.
	B		90/0/90	Fatigue, strength in 90° dir
GLARE 5	-	2024-T3 t = 0.3 mm	0/90/90/0	Impact
GLARE 6	A	2024-T3 t = 0.3 mm	+45/-45	Shear, off-axis properties
	B		-45/+45	Shear, off-axis properties

*Note: 1. All aluminum rolling directions in standard laminates are in the same orientation; the rolling direction is defined as 0°, the transverse rolling direction is defined as 90°.

2. The number of orientations in the column is equal to the number of prepreg layers (each nominally 0.127 mm thick) in each fibre layer.

GLARE has significantly better fatigue properties, durability and impact resistance when compared to monolithic aluminum. The manufacturing, inspection and repair techniques are similar to the existing techniques for conventional aluminum structures. The combination of the previous mentioned characteristics has resulted in GLARE being used in aircraft bulkheads and cargo bay floors. A number of other structures such as GLARE fuselage panels for Airbus A380 and A310 aircraft are currently under development.

The following section discusses the manufacture of FMLs and provides a comparison of the significant properties between ARALL and GLARE. This is followed by a brief discussion of the current aircraft applications of FMLs.

3.2. MANUFACTURE OF FML

FMLs are manufactured by bonding layers of unclad aluminum alloy sheets with preimpregnated materials (prepreg) in an autoclave or a press. ASTM Standard D-3933-80 specifies the standardized pre-treatment of aluminum alloys used for FMLs. (Krishnakumar 1993; Lawcock *et al.* 1995a).

Once manufactured, FMLs can be handled and fabricated (i.e. cut, sawed, and milled) using conventional workshop practices. However, a very sharp cutting tool edge must be used, as even a slightly blunt edge will pull the fibres out of the adhesive rather than cutting them. The laminate must also be backed when drilling, to prevent delamination near the hole. Either riveting or adhesive bonding may be used to join FMLs; however, adhesively-bonded joints show a greater strength and durability over riveted joints since

the strength in the rivet joint is reduced by the presence of the rivet holes (Lawcock *et al.* 1995a).

3.3. FML PROPERTIES

A main advantage of FMLs over aluminum is the improved fatigue behaviour. Fatigue initiation only occurs in the outer aluminum layers of GLARE and exhibits crack growth rates 10-100 times slower than monolithic aluminum (Vlot *et al.* 1999). Although cracks in GLARE initiate at approximately the same number of cycles as in monolithic aluminum (Krishnakumar 1993), GLARE's much slower crack growth rate when compared to monolithic aluminum, translates to a longer life span for the material. The reason for this behaviour is the *fibre crack bridging mechanism*, where the fibres remain intact to bridge a crack that initiates in the aluminum ply of a GLARE laminate. Through-thickness cracks are delayed by the interlaminar adhesive (Lawcock *et al.* 1995b). This dissipates a majority of the crack energy such that the speed of its propagation is reduced.

The durability of GLARE is dependent on the resistance of the fibres and the adhesive matrix, to environmental conditions. The prepreg layers are protected from the environment by the outer aluminum sheets and as a result, FMLs are inherently more durable than both metallic sheets and pure composites (Vlot *et al.* 1999).

An advantage of aluminum is its high resistance to impact damage. Aluminum barely suffers any damage from low energy impacts and any minor damage is directly visible as

a dent on the surface of the metal. Therefore, the outer layer of aluminum on the FMLs is highly advantageous as it provides easy detection of any impact damage in the form of visible indentations (Vogeleang and Vlot 2000). Composites, however, may not show visible dents, yet may have internal damage such as delaminations that are not readily detectable. Tests have shown that after impact, the GLARE fibres remained unbroken while a crack was produced in the aluminum layer farthest from the point of impact, parallel to the fibre orientation. At higher impact energies, internal fibre failure will not usually occur without a visual indication of the failure in the outer aluminum layers (Vlot *et al.* 1995).

Unlike in composites, impact damage, corrosion and fatigue cracking in FMLs is easily detectable by visual inspections, since these forms initiate at the surface layers. Therefore, non-destructive inspection techniques used on aluminum alloys such as eddy current and dye penetrant can be used to inspect the FMLs.

The only form of damage that is not detectable in GLARE by conventional techniques is the de-bonding between layers. It is reported that the *Fokker Bondtester* has been successfully used to detect and locate the depth of delamination in ARALL laminates. The Bondtester probe uses a piezo-electric crystal to scan the laminate surface area and cause any delaminated sections of the structure below the crystal to resonate (Speijer 1989). Currently research is still in progress to develop a reliable method of detecting delamination.

Repair techniques such as riveting and bonding used for conventional aluminum alloys can also be successfully used for repair of FMLs (Krishnakumar 1993). FMLs can also be used as repair materials for current aircraft structures and were found to be effective crack stoppers. It has been shown that the relatively low modulus GLARE 1 and 2 perform better than the very stiff and expensive boron fibres in high-altitude cruise (i.e. low temperature) crack patching applications. This is due to a very large mismatch in thermal expansion coefficients that exist between boron and the aluminum fuselage skin (Vogelesang and Vlot 2000). Currently, experimental GLARE repair patches are applied in the C5 Galaxy of the U.S Air Force (Figure 3.2).

Figure 3.2: Bonded GLARE Patch on the Crown of C5 Galaxy (Vlot *et al.* 1999)

3.4. FML APPLICATIONS

Initially, the majority of the studies on ARALL concentrated on wing applications; however, once Messerschmitt-Bölkow-Blohm GmbH (MBB) joined the research efforts, Delft University considered ARALL for possible fuselage skin application. Initial fuselage construction studies showed negative results. ARALL indicated only an eight

percent weight saving while the cost increased approximately eight to ten times. Tests to determine ARALL's feasibility as an aircraft skin showed that under fuselage fatigue loading conditions, the aramid fibres around the fatigue cracks failed. Without intact fibres, the fibre crack bridging mechanism fails and the crack growth is not contained. The fibre failure was believed to be due to a poor bonding between the fibres and the adhesive. The aramid fibres are thought to lie loosely in a tube within the adhesive and, when loaded, the fibres would slip out of the tube without deforming the adhesive. If compression loads were present, the fibres would then damage and break (Vlot and Gunnink – Eds. 2001).

GLARE, however, bonds much better with the adhesive; it demonstrates impressive fatigue and impact properties and has commercial biaxial variants. As a result, much of the attention was turned toward GLARE fuselage applications. Initial results showed an approximate weight saving of twenty-six percent that translates to a cost savings of 280 dollars per kilogram weight saved (Vlot and Gunnink – Eds. 2001). In a fatigue-critical area, a GLARE structure was estimated to carry twenty to twenty-five percent higher loads than the current aluminum structure. GLARE 3 is recommended for the upper part of the fuselage where the load in the two directions is approximately a one-to-one ratio due to the combination of pressure load and the bending loads on the fuselage under its own weight. GLARE 4 is recommended for locations where the load in one direction is twice as high as in the other direction.

In 1996, Delft University, SLC, and NLR Dutch Aerospace Laboratory, in collaboration with Airbus Company, started a program to develop GLARE for the A3XX/A380

fuselage. Previously, the largest drawback for using GLARE in aircraft fuselage applications was the high cost when compared to aluminum. Assembling the layers with a built-in curvature followed by bonding of the doublers and stringers in one process step followed by an autoclave cure, resulted in a cost breakthrough. Traditional assembly was a multi-step process where flat panels were rolled into the curvature and milled to thickness. In the last step of the process, doublers, stringers and frames were riveted.

The first primary GLARE structure was tested on a German Luftwaffe A310 aircraft on October 04, 1999 (Burchell 2002). In May 2001 Airbus signed a contract with Fokker to supply GLARE fuselage shells for the A380 aircraft. Besides the skin, the shells include doublers and stringers. Each A380 will include approximately 380 square meters of GLARE, located forward and aft of the centre section of the aircraft. Dependent on service experience, Airbus plans to include GLARE in the centre fuselage section of the next A380 version, which may be a stretched, or freighter version (Dornheim 2001).

So far, the use of GLARE in fuselage structures has been based on the material's fatigue, impact, fire resistance, and repair capabilities. A critical component of the material's future feasibility for primary aircraft structures is its diagonal tension capabilities.

3.5. DIAGONAL TENSION IN GLARE

In 1997, SLC released a document describing stability aspects of stiffened GLARE under compressive and shearing loads (Wimersma *et al.* 1997). This included an analytical modification of the NACA method for GLARE material. The basic issues of shear buckling are discussed in the following sections.

3.5.1. Initial Buckling

In the modified method, the shear buckling stress is found by:

$$\tau_{cr} = 1.584 \times 10^{-3} k_s \frac{\pi^2 D}{b^2 t} \quad (3.1)$$

where, k_s is a non-dimensional buckling coefficient which depends on the panel edge conditions. In standard design practices, the most conservative approach is to consider the panel edges to be simply supported. The panel length, width, and thickness are given be a , b , and t , respectively. D is the bending stiffness of the panel.

Since the bending stiffness of GLARE varies in the longitudinal (L) and longitudinal transverse (LT) directions, D for most practical GLARE configurations is calculated by:

$$D = \sqrt{D_{11} D_{22}} \quad (3.2)$$

where, D_{11} and D_{22} refers to the L and LT direction, respectively

3.5.2. Post Buckling

The various SLC formulae involved for web, stringer, and frame stresses are very similar to NACA methods (Kuhn 1952a). The modifications for GLARE are due to the directional dependence of the elastic moduli that appear in the formula.

3.5.3. Failure

The failure modes, such as web failure, forced crippling or column buckling of the stringers, are similar to that of the NACA method. However, the calculations are different as presented below:

Web

Web failure is analyzed in three different ways. The first check is for general web failure in shear:

$$\tau_{all} = \frac{\sigma_{ult}}{2} \quad (3.3)$$

where, τ_{all} is the allowable shear stress of the web and σ_{ult} is the ultimate tensile stress in the L-direction.

The second criterion checks for web shear failure at the stringer-web rivet line by:

$$\tau_{all} = C_{\tau} \sigma_{ult} \quad (3.4)$$

$$C_{\tau} = 10^{-3} e^{\left(0.6625k^2 - 0.741k + 6.096\right)}, \quad \text{for } 0 \leq k \leq 0.5 \quad (3.5)$$

$$C_{\tau} = -0.0316k + 0.3776, \quad \text{for } 0.5 \leq k \leq 1 \quad (3.6)$$

where, k is the diagonal tension factor.

The third check for web tension failure at the stringer-web rivet line is by the blunt-notch strength of the skin material. In practice, the angle of diagonal tension is approximately 45° . As a result the blunt-notch strength at 45° is used for the check:

$$\sigma_{ult} = \sigma_{bn, 45deg} < \text{web stress in diagonal tension direction} \quad (3.7)$$

Stringers

The stringers, made of aluminum, are designed against forced crippling. The allowable forced crippling stress is calculated by:

$$\sigma_{fc} = \frac{\sigma_{0.2, st}}{\sqrt{\frac{\sigma_{0.2, st}}{E_{1, st}} + 0.002}} k^{\frac{2}{3}} \left(\frac{t_{st}}{t_{web}} \right)^{\frac{1}{3}} \quad (3.8)$$

where, t_{st} and t_{web} are the thickness of the stringers and web respectively.

The stringers will be loaded in compression in a post-buckled shear panel. As a result, very slender stringer-skin combinations are checked for column buckling. The column

buckling stress of the stringers is calculated with the Euler-Johnson method as outlined in Bruhn (1973).

3.5.4. Experimental and Finite Element Analysis

In 2000, an experimental program was initiated to validate the modified NACA method (Vlot and Gunnink – Eds. 2001). Two 1m x 1m flat, stiffened panels were constructed using GLARE 3-5/4 (S1) (0.118in, 3.0mm) and GLARE 4 – 5/4 (S2) (0.138in, 3.5mm) webs. They were tested using the picture frame method. The dimensions of the test panels were taken to be representative of the future generation Ultra-High Capacity Aircraft (UHCA) fuselage. Due to the lack of a suitable test facility, curved panels were not considered for the first set of tests.

Predictions of possible failure modes were made using the modified NACA method hand calculations and the finite element code STAGS. The hand calculations predicted panel failure due to skin rupture at the stringer rivet line. The FE analysis showed that the panels were prone to overall-type buckling behaviour – i.e. the stringers buckled along with the skin. This is a violation of NACA theory's basic assumption that the skin buckles locally such that the stringers carry the load through compression.

Unfortunately, experiments confirmed that the S1 and S2 panels were prone to overall buckling behaviour. SLC believes that the cause for the buckling mode was the large skin thickness and small stiffener spacings, which caused the initial buckling load of each

web bay to be higher than the overall buckling load of the panel. However, the experimental results verified the FE models.

During the same year, plasticity effects of GLARE panels loaded in pure shear were investigated using simply supported boundary condition (Vlot and Gunnink – Eds. 2001). These programs concluded that GLARE could be treated as an isotropic material, until the proportionality limit of the material was reached. Once the material entered a fully plastic state, the direction of the principal stresses may have a large impact on the load carrying capacity of the material. This is believed to be due to the orthotropic load carrying behaviour of the glass fibres.

Most of these investigations focused on the stiffened flat panel system. As a result, there is a need to investigate the post-buckling behaviour of GLARE in a curved web system.

4.0. THESIS OBJECTIVES

The objectives of this thesis are to evaluate the diagonal tension properties of GLARE in a stiffened curved web system, and to determine if the NACA-based diagonal tension analysis method can accurately predict the stresses and failures of an orthotropic material in a curved web system.

The initial approach to achieve these objectives consisted of four stages. The first stage was to design a standardized stiffened cylinder using the NACA diagonal tension analysis method. This specimen was to be designed to ensure a web and/or stringer failure. The second stage was to assist SMPL in preparing detail drawings of the cylinder and loading rig. Assistance was also provided in sourcing the various cylinder and loading rig components. Subsequently, ten specimens of varying thickness and grades of GLARE 3, GLARE 4 and 2024-T3 Aluminum were to be constructed. The third stage was to load the specimens to failure in torsion, with proper instrumentation, using the Multi-Actuating Test Platform (MAT-P) at SMPL. Finally, the test results were to be compared to the design predictions and previous NACA experimental results to determine if the NACA-based diagonal tension analysis method could accurately predict the stresses and failures of GLARE.

However, the focus of the research was changed prior to the third stage due to delays in the preparation of detailed drawings which, in turn, delayed the manufacture of the specimens and the loading rig.

As a result, the third stage was changed from a large-scale experiment to a simulation of the experiment. A finite element simulation of the stiffened cylinders in torsion was developed using the software LS-DYNA at Carleton University. Simulations were developed to analyze the elastic, buckling and post buckling ranges of stiffened cylinders. The LS-DYNA simulations were calibrated by modelling the aluminum test specimens in NACA-TN-2821 (Clark & Moore 1952) and comparing the FE results to the NACA experimental results. The initial simulation results for aluminum cylinders were also used to develop an instrumentation plan for the full-scale tests, in collaboration with the IAR design team. The final simulations approximated the behaviour of the GLARE specimens.

The initial buckling, post buckling and failure data of the simulations will be used by IAR to define test parameters. A combination of the simulation results and NACA-based analytical results will provide a theoretical framework for the comparison with the SMPL experimental results.

The following sections outline the work completed to fulfill the thesis objectives. Section 5 provides a detailed description of the stiffened cylinder. Section 6 describes the manufacturing and loading arrangements for the test specimens. Section 7 details the finite element software and development of the simulations. Sections 8 and 9 present the result and analysis of cylinder buckling, and conclusions while Section 10 provides recommendations for future work.

5.0. SPECIMEN DESIGN

Due to the lack of original NACA engineering drawings, the current cylinder design was developed by examining the photographs and sketches presented in the NACA Technical Notes (TN) and by using basic aircraft structural design methods. The cylinder design was based on the analysis methods derived by NACA as presented in Section 2.

All equations and empirical graphs were converted to Excel graphs and calculations files to automate the numerical process. The Excel calculations were checked and presented using MathCAD. A set of detailed MathCAD calculations and a set of Excel results can be found in Appendices A to E. These calculations were also presented in a design report that was approved by Bombardier Aerospace and SMPL (Mendis *et al.* to be published).

5.1. NACA DESIGN COMPARISONS

For the initial design of the current cylinder, the following NACA designs were considered:

- a. the double internal/external stringer and internal frame structure used in NACA TN 2662 (Kuhn 1952b)
- b. the external stringer internal frame design presented by NACA TN 2821 (Clark and Moore 1952).

The characteristics of the two configurations can be seen in Table 5.1.

Table 5.1: NACA Specimen Configurations

	NACA TN 2662	NACA TN 2821
Stringer x-section	L or Zed (Figure 5.1)	Hat (Figure 5.2)
Number of stringers	12	0, 8 or 16
Stringer arrangement	Inside and outside of cylinder	Outside cylinder
Stringer attachment	Double (Figure 5.1)	Single (Figure 5.2)
Frame x-section	Zed	Square
Cylinder length	61 in or 79 in	36 in
Frame arrangement	Inside cylinder	Inside cylinder
Frame attachment	Fixed to cylinder web or "floating"	Fixed to cylinder web

Figure 5.1: Example of NACA TN 2662 Cylinder and Double Stringer Configuration (Kuhn 1952b) **Figure 5.2: Example of NACA TN 2821 Cylinder and Stringer Configuration (Clark and Moore 1952)**

The main difference between the two approaches was in the arrangement of the stringers. Cylinder configuration from NACA TN 2662 (Kuhn 1952b) used the unconventional arrangement of double stringers to ensure low bending stresses. NACA TN 2821 (Clark and Moore 1952) used stringers on the outside of the cylinder for ease of manufacture.

Possible failure modes were identified in order to establish the final design, which is discussed in detail in section 5.3.

5.2. FAILURE CRITERIA

The NACA technical notes (Kuhn 1952b; Clark and Moore 1952) present possible cylinder failure modes, namely, web failure, stringer failure, frame failure and fastener failure. These failure modes are discussed below.

1. Web Failure

- Web failure was defined as the failure of the web in shear when the nominal shear stress exceeds the allowable shear stress limit for the web, which is the gross stress in the panel.

2. Stringer Failure Modes

Possible stringer failure mechanisms are as follows:

- an axially loaded fixed end column, free to twist about the shear centre.

- an axially loaded simply supported column, where the web forces the stringer to twist about an axis through the line of attachment that is between the stringer and the web.
- an axially loaded fixed end column, where the web forces the stringer to twist about an axis through the line of attachment that is between the stringer and the web.

3. *Frame Failure Modes*

- The primary mode of failure for the frame is forced crippling shown in Figure 2.12. Forced crippling is a localized buckling failure of a stiffener that is connected directly to the web. The maximum stiffener stress is checked against the allowable forced crippling limit.

4. *Fastener Failure*

Fastener failure could be attributed to one of the following failure mechanisms (Bruhn 1973):

- Bearing failure – The applied loads may cause the rivet to press against the plate wall. If the load per unit area is high enough, the rivet will push out the adjacent material causing an elongated hole.
- Prying failure – Tensile forces on the rivets may “pop off” the rivet heads or pull the web up and around the rivet heads. These loads are not determinable by any known method. Bruhn (1973) proposed a criterion where the tensile strength of

the rivet/web combination is proportional to the ultimate tensile strength and the thickness of the material. This equation gives the tensile strength necessary to keep the folds of the web from lifting off the stiffeners.

- Shear failure – Load may be transferred to the rivet by the bearing of the plate (i.e. web) on the rivet. The load will be then transferred along the rivet and resisted by bearing of the opposite plate (i.e. frame). Since the plate bearing forces do not align, the forces will shear. As a result, ultimate shear strength of rivets must be checked.

A primary design consideration was that web and/or stringer failures precede the other cylinder failure modes. Different combinations of stringers and frames were analyzed to ensure that the desired critical failure was in the web. The design was checked to preclude undesired modes of failure. However, it was noted that in the case of the NACA TN 2821 design (Clark and Moore 1952), stringer failures if any, would not provide useful information since failure does not simulate real fuselage stringer conditions. This is because the failure modes are not representative of a real fuselage when the stringers are placed on the outside of the cylinder (Figure 5.2).

5.3. DESIGN DEVELOPMENT

After analyzing the specimen designs presented in NACA TN 2662 and NACA TN 2821, it was concluded that the current cylinder would emulate the sixteen-stringer design (Figure 5.3) reported in NACA TN 2821 (Clark and Moore 1973) and tested by Aluminum Company of America (ACA, now known as ALCOA). The ACA design was chosen mainly due to the practical ease of applying the load and the ease of manufacture.

Figure 5.3: Cylinder Configuration Outlined in NACA TN 2821 (Clark and Moore 1973) – Test Specimen 9

The initial layout for the cylinder is shown in Figure 5.4. The cylinder has a diameter of 30in and a length of 36in. It contains three frame bays that are 10in, 12in and 10in wide. Frames and stringers border a panel shown by the crosshatched section. The test section was considered to be the 12in bay while the 10in bays were used to ensure that a uniform load was distributed on to the test section.

Figure 5.4: Initial Cylinder Layout
Dimension in inches (typical)

For standardization, ease of manufacture and economical considerations, it was concluded that one size of stringer and frame should be selected for all cylinders. The only variables for each of the test cylinders were the web material and web thickness as listed in Table 5.2.

Table 5.2: Web Thickness Used for Testing

Material	Thickness [in]
Aluminum 2024-T3	0.055
GLARE 3 3/1	0.033
GLARE 3 3/2	0.055
GLARE 4 3/2	0.065

Failure analysis was carried out for the stiffener combinations with the thickest GLARE web size (GLARE 4 3/2, $t = 0.065$ in). The most suitable stringer design was a hat section of aluminum 7075-T6 as shown in Figure 5.5.

Figure 5.5: Stringer Cross-Section

The dimensions of the frames, which met the allowable margins of safety were 1.5 x 0.75 x 0.19in aluminum 7075-T6 angle sections. The frames are directly attached to the web and stringers as shown in Figure 5.6.

Figure 5.6: Frame Cross-section

The cylinder is manufactured by rolling 95 x 36in sheets of GLARE or aluminum. The sheet edges are joined by a longitudinal butt-splice as shown in Figure 5.7.

Figure 5.7: Longitudinal Splice Configuration

The stringers, frames, longitudinal splice and the rolled aluminum ring will be fastened to the cylinder web using solid aluminum universal head rivets.

Initially, the cylinder was to be loaded through a 2in deep AISI 1020 load interface flange, as shown in Figure 5.8, to ensure uniform torque over the complete cylinder surface.

Figure 5.8: Load Interface Flange and Cylinder - Initial Concept

The initial test specimen concept was refined through several iterations to accommodate practical considerations. During the detail design stage it was found that the 2in deep

load interface flange as in Figure 5.8 was insufficient to provide a proper load transfer. To ensure a smooth load transfer, the end bays of a specimen were shortened to 8in and the extra length was used to accommodate a steel ring (Figure 5.9). The first IAR aluminum baseline test specimen is shown in Figure 5.10.

The load interface now consists of a steel plate, steel ring and a rolled aluminum ring. One side of the steel plate is attached to the loading frame while the other end is bolted to the steel ring. The steel ring is bolted to the 6061-T3 aluminum ring with a 4 x 0.25in cross-section, which in turn is riveted to the cylinder web. The rolled aluminum ring provides a manufacturing skeleton for the cylinder web, and reduces the stiffness difference between the steel ring and the cylinder. The aluminum ring is fastened to the steel ring using 3/8in diameter bolts while the steel ring is attached to the steel plate using 1/2in diameter bolts.

NOT TO SCALE

Figure 5.9: Load Interface and the Cylinder - Final Concept

Figure 5.10: Aluminum IAR Cylinder for Test 1

A cutaway of the final cylindrical specimen is shown in Figure 5.11:

Figure 5.11: Cylinder Specimen – Final Configuration

6.0. EXPERIMENTAL SETUP

An experimental test program at the SMPL will be used to validate the NACA method as well as the finite element cylindrical model. This section will describe the loading structure, and the software and hardware used to gather the data.

6.1. MULTI-ACTUATING (MAT) TEST PLATFORM

Tests for this program will be conducted using the MAT Platform (Figure 6.1). This large-scale test facility was developed by SMPL to accommodate different structural tests. It has been used previously to test the diagonal tension properties of stiffened FML flat beams (Jodoin 2002).

Figure 6.1: MAT Platform Incorporating 3 Test Specimens

The test structure is a large self-reacting steel space frame. As shown in Figure 6.1, the MAT Platform can house several different specimens and has the capability to add extra mounts, hard-points, beams and reinforcements to accommodate customer's test specifications. Reinforced clamp-on-blocks are attached to structural members of the MAT Platform to provide extra stiffness and strength to the test frame as well as to provide attachment points. Plexiglas and Lexan shields are placed to ensure the safety of the personnel and equipment from flying debris, in case of catastrophic failure.

6.2. LOADING STRUCTURE

The design team at SMPL's Large Scale Testing Facility used the available descriptions of the NACA TN 2821 loading structure (Figure 6.2) as the starting point to develop the current loading structure (Figure 6.3).

Figure 6.2: NACA Loading Structure (Clark and Moore 1952)

In the NACA loading structure (Figure 6.2), the cylinder was bolted to frames 1 and 2. Frame 1 was rigidly fixed to the floor to resist rotation of one end of the cylinder about its

longitudinal axis. Frame 2 was rotated counter-clockwise using screws which loaded the cylinder in torsion. At the ends of the loading screws, spherically seated nuts were used to allow the frame to rotate freely. The forces at the ends of the frame were determined by an aluminum dynamometer with a capacity of 5000lbs, that provided a maximum torque capacity of approximately 60000ft-lbs. In all tests the torque was applied in increments.

A majority of the cylinders with stringers were loaded more than once. The first loading caused the buckling of the least stable panels. However, this loading did not buckle all of the panels, even in the cylinders containing panels of the same dimensions. Critical loads for the remaining panels were determined by subsequent loading cycles.

The design of the SMPL loading rig was based on the analytical and simulation predicted loads for initial buckling and failure as well as the predicted longitudinal, compressive and rotational displacements. As Figure 6.3 shows, the cylinder is vertically situated. One end of the cylinder is bolted to the reaction plate, which is rigidly fixed to the MAT Platform using four clamp-on-loading blocks. The other end of the cylinder is bolted to the loading frame. The loading frame mass is counterbalanced by an assembly consisting of an I-beam, spring, swivel and a steel plate, that is placed inside the cylinder. The loading frame rests on this assembly and is allowed to move freely without applying extra loads on the cylinder.

The cylinder is loaded using two 35 kip actuators attached at the end of the loading frame. The actuators are counterbalanced by springs attached to a welded assembly that is clamped on to the MAT-Platform.

Figure 6.3: NRC Loading Structure

6.3. INSTRUMENTATION

The cylinder instrumentation will comprise of twenty-four strain gauges and eight string pots. During the test, the data will be fed into two computers; one to manage the test control and the second to manage the data acquisition.

6.3.1. Strain Gauges

Twenty-four strain gauges are attached to various locations on the test specimen (Figure 6.4). Two rosette strain gauges (R1 and R2, in figure) will be attached to the cylinder surface of the critical panel, directly opposite to the panel containing the longitudinal splice. Two more rosettes will be attached to the web surface of the central bay containing the butt splice. The rosettes will be paired and set at back-to-back locations. Based on the stiffened flat panel tests, rosettes CEA-13-125UR-350 will be used for the aluminum cylinders and CEA-06-125UR-350 for all FML cylinders. Data obtained from the rosettes will be used to determine the onset of initial web buckling, the diagonal tension angle and the web stresses.

Figure 6.4: Strain Gauge Locations

Uniaxial strain gauges (U) will be placed on the underside of the frames (Figure 6.5) and on the upper surface of the stringers (Figure 6.6) surrounding critical panel and the central panel containing the butt splice. Uniaxials U1 and U2 will be used to measure the

frame strains while uniaxials U3 and U4 will measure the stringer strains. Uniaxials U5 to U12 will be used to measure the strain distribution at the load interface (Figure 6.4). Based on the availability of the number of signals on the data acquisition system, uniaxials U_x will be used to monitor the distortion of the stiffeners. Based on the stiffened flat panel tests, uniaxial CEA-13-125UN-350 gauges will be used for all cylinders.

Figure 6.5: Strain Gauge Locations on Frame

Figure 6.6: Strain Gauge Locations on Stringer

6.3.2. String Pots

Four linear position transducers, string pots (SP) as shown in Figure 6.7, will be used to determine the total angle of twist of the cylinder. String pots will be mounted on three fixed supports, each at 120° spacing from another support. A stainless steel cable, which is pulled out of the unit, will be attached to a cylinder component (Figure 6.8). The

attachment location was based on the finite element analysis results. As the cylinder moves, the extension of the cable rotates an internal capstan and a precision sensor to produce an electrical output signal proportional to the cable extension. An internal spring maintains tension on the cable and serves as the retraction mechanism. String pots 2, 3 and 4 are attached to the top of the cylinder while string pot 1 is attached to the cylinder bottom, parallel to string pot 2. The angle of twist will be obtained by comparing the displacement of string pot 1 to string pots 2, 3 and 4.

Figure 6.7: CeleSCO PTX101-0020-111-4100 String Pot

Figure 6.8: Placement of String Pots to Measure Rotation – Top View

Another four string pots will be used to measure the longitudinal displacement and warp of the cylinder (Figure 6.9) by comparing the data from string pot 5 to string pots 6, 7 and 8.

Figure 6.9: Placement of String Pots to Measure Displacement and Warp

7.0. FINITE ELEMENT ANALYSIS

Finite element (FE) methods are used to discretize and analyze structural systems by numerical methods. The main advantage is the ability to cost effectively analyze complex structures and multiple design parameters.

The FE method was used in this work to verify the behaviour of cylindrical specimens predicted by the NACA methodology. The analysis was initially attempted using the finite element code, ABAQUS. This program was not user friendly to the inexperienced. It was not possible to obtain technical support to resolve issues regarding the loading of the cylinder and other related difficulties, which occurred with increasing frequency.

As a result, it was decided to use the LS-DYNA FE code. The pre/post processor FEMB (Finite Element Model Builder) was used to define the geometry, mesh and loading conditions. Initially the ACA aluminum specimens were simulated and the results were compared to the experimental data presented in NACA-TN-2821 (Clark and Moore 1952). This verified the FE modelling method. Once the ACA specimens were satisfactorily simulated predictions were made for the cylinder designed as part of this work.

The following sub-section discusses the important aspects of the LS-DYNA software. Sections 7.2 and 7.3 provide a description of the test program, and the steps necessary to create the complete simulation.

7.1. LS-DYNA SOFTWARE

LS-DYNA is a vectorized explicit 3-D finite element program, which is capable of large deformation static and dynamic analysis of inelastic materials. Critical aspects of modelling in LS-DYNA such as implicit vs. explicit solvers, contact algorithm and hourglassing are discussed below.

7.1.1. Implicit vs. Explicit

In LS-DYNA numerical integration can be performed either implicitly or explicitly. Implicit solvers are recommended for low frequency, static, quasi-static or dynamic problems. One such application is metal forming model featuring spring-back behaviour. The main advantage of this integration approach is a reduced number of required load or time steps, usually 100 to 10000 times fewer than for an explicit solver (Hallquist 1998).

Explicit integration is the default solver in LS-DYNA. This method is generally used for modelling high frequency events. It is ideal for simulating non-linear material behaviour and high speed computing. In addition, the explicit solver offers many other advantages over implicit method. An explicit solution does not require a stiffness matrix, thus saving on storage requirements. Complex geometries can be simulated with many elements undergoing large deformations. Also, for a given time step, an explicit code requires fewer computations per time step than the implicit code. This has been found to be an advantage in car crash and airbag simulations, as the explicit method has been shown to be faster, more accurate and more versatile when compared to the implicit method. The explicit method is recommended for simulations of entire systems because of speed,

convergence and storage considerations. Therefore, the explicit method was considered suitable for the current simulations.

7.1.2. Contact

A challenge in FE modelling is to define the relationship between adjacent component interfaces. One option in LS-DYNA is to use the `*COINCIDENT_NODES` module. In this case, the coinciding nodes on adjacent interfaces are combined to one existing node. This requires the interfacing components to have identical meshes. While this method ensures proper movement of adjoining interfaces, it does not allow for proper definition of 3D component thicknesses. However, LS-DYNA provides a solution to modelling the interfaces between 3D components with the contact algorithms.

Current simulations use the contact algorithm, `*TIED_SURFACE_TO_SURFACE` to simulate a uniform load transfer from the load interface to the complete surface of the cylinder web. In this type of contact, one component is defined as the master while the corresponding mating component is defined as the slave. Sets of elements are connected using a tied interface that constrains rotations and displacements of the slave surface with respect to the master surface. It is recommended that constraints be imposed only on slave nodes and the more coarsely meshed interface be designated as the master (Hallquist 1998). In the current FE model, the mating elements of the load interface were defined as the master while the cylinder web was defined as the slave.

An advantage of tied contact algorithms is that it does not need to match nodes across interfaces of merged parts as shown in Figure 7.1.

Figure 7.1: Tied Interface Used for Mesh Transition (Hallquist 1998)

However, ideally, each master node should coincide with a slave node to ensure complete displacement compatibility along the interface. Master nodes that do not coincide with a slave node may penetrate through the slave surface. In most FE models, coincident master/slave nodes are impractical. For the current model it was feasible to ensure that each node of the load flange coincided with a node of the cylinder.

The tied interface contact is implemented by looping through and independently updating each tied interface at each time step.

Also, `*AUTOMATIC_CONTACT` type was defined for the overall model to ensure that nodes did not penetrate adjoining components.

7.1.3. Hourglassing

Hourglass modes are observed as severe deformations of the entire mesh. In most cases the shells deform according to an hourglass shape as shown in Figure 7.2.

Figure 7.2: One hourglass deformation mode (Hallquist 1998)

Hourglass modes are orthogonal to real deformation in the element and as a result there are no internal forces to resist the deformations. This is a result of insufficient integration points in the finite elements. It creates physically impossible mathematical states such as zero energy modes and element shapes without any stiffness.

Hourglassing can be resisted either by viscous or stiffness damping, or set by a control factor. However, hourglass modes may be prevented by refining the mesh, creating uniform meshes or by distributing the loading.

In the current simulations, hourglassing was only encountered at the end of each run. This was because the model “snapped back” when releasing the cylinder load at the end of the loading period. Therefore, the last step of each simulation was disregarded.

7.2. MODEL FORMULATION

The FE models were developed to accurately represent the main physical test specimen components. For numerical analysis the various cylinder components were defined into parts and each of these parts was divided into a number of smaller elements. Elements were then assigned a specific material and section properties and joined together with nodes. Nodes do not have a mass or stiffness but provide links between the elements. The collection of elements is referred to as a mesh.

Computational time required is related to the number of elements. It is common practice to model a portion of the cylinder by utilizing symmetry, which reduces the computational time. Due to the complex boundary and loading conditions, it was not possible to obtain a representative physical system. As a result, it was decided that the longer computational time for the full model was acceptable.

A complete FE model contains the following parameters (Brauer 1993):

- Elements
- Loading
- Materials
- Boundary Conditions

These parameters are discussed in the following sections.

7.2.1. Elements

The complete cylindrical model contains a combination of elements. Thin-shell elements were used for the cylinder load interface, web, and stringers. The feasibility of thick-shell elements for the cylinder web and frames was also investigated. Solid-brick elements were used to model the cylinder frames.

7.2.1.1. Thin-Shell Elements

The Belytschko-Lin-Tsay shell elements consist of four nodes and are used mainly due to their high computational efficiency and accuracy when compared to the alternative Hughes-Liu shell elements. The computational efficiency is due to two mathematical simplifications (Hallquist 1998):

1. The embedding of the coordinate system in the element avoids the complexities of non-linear mechanics by deforming with the element. As a result element strain is restricted to small values but not the out of plane deformations.
2. Evaluations are based on either the velocity strain or the rate of deformation.

Thin-shell elements only have one layer of nodes. As a result, nodes must be defined as being part of master and slave surfaces simultaneously. This may cause numerical problems, such as hourglassing, that can be avoided if thick-shell or solid-brick elements are used. However, hourglassing was not a factor for the current use of thin shell elements.

7.2.1.2. Thick-Shell Elements

Thick-shell elements are geometrically identical to 8-node solid bricks, which have four nodes on the top and bottom surfaces. These elements minimize the problems associated with thin-shell elements when defining contact interfaces. Each group of four nodes may be assigned to a separate contact segment. Therefore, one node is not assigned to two different contact segments simultaneously.

In thick-shells a reference surface is located midway between the upper and lower surfaces of the element (Hallquist 1998). Similar to thin-shells, a co-ordinate system, which deforms with the element is present on the reference surface. Thick-shell elements, however, require the use of hourglass control similar to other elements that use single-point integration in LS-DYNA.

Thick-shell elements were considered for use in the cylinder web. However, compared to a model using thin-shell web, the thick-shell model required twice the computational time without an appreciable increase in the solution accuracy.

7.2.1.3. Solid-Brick Elements

The solid-brick elements are similar to the thick-shell elements described above. However, the solid-brick elements do not use a reference surface. Instead a single-point integration scheme is used. This significantly reduces computation time, but creates the requirement for hourglass control (Hallquist 1998).

7.2.1.4. Mesh Sensitivity

The number of elements and nodes were varied to study the effect of mesh density on an unstiffened aluminum cylinder simulation results. The mesh was defined by specifying the number of nodes along the cylinder circumference and length. Following node configurations (element dimensions) were considered:

1. 16 x 18 (5.89 x 2.00 in)
2. 32 x 36 (2.94 x 1.00 in)
3. 64 x 72 (1.47 x 0.50 in)
4. 80 x 72 (1.18 x 0.50 in)
5. 96 x 72 (0.98 x 0.50 in)

The elastic range and the initial buckling point were compared to the NACA experimental results to determine simulation accuracy.

As seen in Figure 7.3 and 7.4 as the number of nodes and elements were increased the simulation accuracy also increased. However, beyond the third node configuration the increased accuracy is minimal compared to the increase in the number of problem cycles and the CPU time per zone cycles. As a result, a mesh containing 64 nodes along the cylinder circumference and 72 nodes along the cylinder length was used for the simulations.

Figure 7.3: Effect of Mesh Density

7.2.2. Loading Criteria

Initially, the cylinder was loaded by simulating a crude T-section flange interface. Although the model displayed the expected buckling and the diagonal tension bands, a less cumbersome loading method that would take advantage of the preprocessor loading module, was investigated. This resulted in using the `*LOAD_NODE` option, which applies a concentrated load to a node using the default global co-ordinate system or a local user defined co-ordinate system. In this case, the cylinder was loaded using the global co-ordinate system by applying a load at each of the outer nodes on the upper flange as shown in Figure 7.4. The *follower force* option was activated to allow the load to rotate with each node as the cylinder deforms.

Figure 7.4: Cylinder Loading

A load vs. time relationship was defined to apply the cylinder load. The load limit was based on the buckling loads of the ACA experimental cylinders. Load application time was determined by trial and error. If the load is applied too fast, the simulation may be affected by its inertial loads, resulting in a higher buckling load. If the load is applied over a very long time period, the computational time will increase without a significant improvement in the result. A trade-off study resulted in the definition of the following applied load vs. time relationship (Figure 7.5):

Figure 7.5: Load Curve for Cylinder Simulations

The load axis was normalized to extend from 0 $\rightarrow$ 1. Both axes were controlled using scale factors. For example, if simulation did not show buckling with a given load scale factor, the values were adjusted correspondingly and the simulation was repeated.

7.2.3. Materials

The cylinder web, stringers, and frames were assigned properties of aluminum as stated in NACA-TN-2821 (Clark and Moore 1952) while the load interface was assigned properties of steel.

Initially, the LS_DYNA *MAT_ELASTIC (Type 1) material model (Hallquist 2001b) was used to define the material behaviour of all components of the cylinder, in order to capture initial buckling behaviour.

Later, the *MAT_PIECEWISE_LINEAR_PLASTICITY (Type 24.1) material model (Hallquist 2001b) was used in the stringers and cylinder web to capture the initial and post-buckling effects. In this model, the stress-strain curve for the material is divided into discrete linear portions as defined by the user in load curve and entered into the code.

The material properties for aluminum and steel are summarized in Table 7.1. Data from aluminum coupon tests carried out at NRC were used to construct the stress-strain curve shown in Figure 7.6. The test data were validated using sheet manufacturer's specifications. The material property data for steel were obtained from the Matweb database.

Table 7.1: Aluminum and Steel Material Properties

		Aluminum	Steel
Density, ρ	lb/in ³	0.100	0.284
Young's Modulus, E	ksi	10500	29000
Poisson's Ratio, μ		0.32	0.29
Yield Stress	ksi	41.34	47.86

The specimens designed for the current study required the simulation of GLARE for the cylinder webs while the stringers and frames were simulated using aluminum properties.

GLARE was initially modelled using the *MAT_ORTHOTROPIC_ELASTIC (Type 2.1) (Hallquist 2001b) material model to trouble-shoot the material model as well as to predict the initial buckling point and the beginning of the post buckling region. Once this was successfully obtained, the simulation using *MAT_NON-LINEAR_ORTHOTROPIC (Type 40.1) (Hallquist 2001b) material model was attempted to define the plastic region of GLARE. In this material model, it is possible to define the plastic region of the material by entering into the code, the stress-strain curves for the various GLARE variants as measured in NRC coupon tests.

However the simulations were unsuccessful as the models became unstable within the first compiled run. Later, the LS-DYNA code developers confirmed that the Type 40.1 material model is inherently unstable for all elements types with the exception of modelling flat thin-shelled element. As such, Type 40.1 is not recommended for the current simulations, and the Type 24.1 was used instead to approximate GLARE behaviour. Although Type 24.1 assumes isotropic behaviour of a material, it is believed

that good definition of the stress-strain curves for the GLARE variants allowed this approximation. GLARE property data, to be published in the near future, was measured at NRC and is as follows:

Table 7.2: GLARE Material Properties

		GLARE 3-2/1	GLARE 3-3/2	GLARE 4-3/2
Density, ρ	lb/in ³	0.092	0.090	0.087
Elastic Modulus, E1 (E_x)	ksi	8800	8440	8351
Elastic Modulus, E2 (E_y)	ksi	8800	8440	7427
Shear Modulus, G	ksi	3380	3380	3380
Poisson's Ratio, μ		0.297	0.288	0.289
Yield Stress, L	ksi	45.7	44.2	45.2
Yield Stress, LT	ksi	41.6	41.0	37.0

Figure 7.6: Stress-Strain Data for Cylinder Materials

7.2.4. Boundary Conditions

The model boundary conditions were based on the examination of the experimental setup of the ACA and NRC tests. In the ACA loading fixture (Figure 6.2), the bottom flange of the cylinder was bolted to a steel frame that was anchored to the floor to prevent rotation. In the SMPL loading fixture (Figure 6.3), the bottom flange will be rigidly fixed to the MAT-P via the reaction plate. Restricting motion in all degrees of freedoms of motion simulated constraints of both ACA and NRC experimental setup.

The upper cylinder flange (load interface) in the ACA tests was bolted to the moveable frame of the torsion-loading fixture. In the NRC loading fixture the upper flange will be bolted to a loading frame to which the actuators will be attached. The FE model boundary conditions simulated the experiments by restricting the z-translational and z-rotational motion of the outer nodes. Due to the simplification of the experimental setup in the FE approach, the z-translational restriction ensures that the cylinder does not longitudinally collapse upon initial buckling. However, in actual test conditions, the experimental set-up ensures that the cylinder does not collapse on initial buckling. The z-rotational restriction prevents the nodes from rotating along its longitudinal axis.

The upper and lower cylinder flanges were attached to the cylinder web using *TIED_SURFACE_TO_SURFACE contact (Hallquist 2001a). The flanges were defined as master components while the cylinder web was defined as the slave component.

The stringers and frames were attached to the cylinder web using the *CONSTRAINED_SPOT_WELD option (Hallquist 2001a). Spot welds are massless,

rigid beams that connect the non-contiguous nodal points of the nodal pairs. The spot welds must be connected to nodes belonging to elements with rotary inertias such as beams or shells (Hallquist 1998).

The option `*CONSTRAINED_NODAL_RIGID_BODY` (Nodal rigids) was used to attach the cylinder web to the load interfaces as well as the stringer, frame and cylinder web overlap connections. Nodal rigids are also massless beams. However, unlike spot welds, the nodal rigids can join two or more coincident and non-contiguous nodes (Hallquist 1998).

7.3. TEST PROGRAM

The development of the stiffened cylinder was a gradual process. Successive models were increased in complexity. This approach corresponds to the ACA test matrix and has allowed the calibration of the FE model.

In all ACA cylinders, the web thickness was 0.0325in with a tolerance of +0.0015in and -0.0005in. The stringers were of hat configuration that was formed from 2024-T3 aluminum sheets with the dimension shown in Figure 5.2. The frames were 0.5in square, aluminum 2024-T4 bars.

The initial objective of the simulations was to accurately predict the load vs. angle of twist of the cylinders tested by the ACA (Clark and Moore 1952). The simplest model, an unstiffened cylinder shown in Figure 7.7, was defined using three one-layer shell element components: upper and lower load interfaces and a web.

Figure 7.7: Initial Cylinder Model

Next, eight shell element stringers were added to the model (Figure 7.8). In order to simplify the modelled stringer mesh, the stringer fillets were removed and the cross-section as shown in Figure 7.8 was modelled.

Figure 7.8: Cylinder with Eight Stringers

The next step was to model three solid element frames, upper and lower flanges and the cylinder web (Figure 7.9).

Figure 7.9: Cylinder with Three Frames

Then the eight stringers and three frames were combined to one model to simulate the simplest fully stiffened curved web system (Figure 7.10).

Figure 7.10: Cylinder with Eight Stringers and Three Frames

Finally, different variations were simulated. The complete test matrix is summarized below:

Table 7.3: Model Test Matrix (adapted from Clark and Moore 1952)

Test Case	ACA Specimen Number	# of Frames	# of Stringers
A-1	1	0	0
A-2	4	3	0
A-3	7	4	0
A-4	10	5	0
B-1	2	0	8
B-2	5	3	8
B-3	8	4	8
B-4	11	5	8
C-1	3	0	16
C-2	6	3	16
C-3	9	4	16
C-4	12	5	16

8.0. RESULTS AND ANALYSIS OF CYLINDER BUCKLING

The finite element model described in Section 7 was used to obtain simulated buckling data for ACA cylinder web and stringers for different test cases. The goal of simulating the ACA cylinders was to verify the FE model by predicting the buckling behaviour of each cylinder.

This section will describe the data collection processes and analysis of the ACA tests and the FE models. Then, a comparison is made between the ACA experimental results, current simulation results and NACA method's analytical results. Finally, the simulation results for the IAR cylinders are compared to the NACA method analytical results.

8.1. ACA DATA COLLECTION

ACA obtained the experimental results from a combination of:

- i. visual observation of the behaviour of the cylinders
- ii. qualitative determination of buckling and ultimate torques
- iii. measurement of overall twist and of strains in the web and stringers

The twist was determined by using a 10in level bar as shown in Figure 8.1. The bar was equipped with a 45-second bubble level and a micrometer screw graduated in 0.001in intervals that was sensitive to changes in slope of approximately 1 part in 50000. Once a load was applied, the test was paused while the micrometer was turned to re-center the bubble. Micrometer readings were then used to calculate the angle of twist.

Figure 8.1: Ten-inch Level Bar for Measuring Twist (Clark and Moore 1952)

The centre of rotation of the cylinder was not fixed; therefore, transverse bending as well as torque could be applied to the cylinder. Examining the only available picture of the ACA equipment (Figure 8.1), it appears that reference bars 1, 2 and 3 were used to maintain a constant cylinder position relative to the floor in order to ensure a pure torque loading. Reference bar 1 was mounted on the central longitudinal axis of the cylinder to serve as a reference for maintaining a constant position relative to the floor. Reference bar 2 was mounted to the cylinder and reference bar 1 while reference bar 3 was mounted on the floor and reference bar 1 (Figure 8.2). A constant reading on the dial indicators of reference bars 2 and 3 confirmed that only pure torque was applied to the cylinder.

Figure 8.2: Schematic of the Possible ACA Loading Rig

Clark and Moore (1952) reported that it was not possible to predict the initial buckling point from visual observation. Therefore, the buckling torques were determined by closely monitoring the dynamometer deflections to capture the maximum values. This had to be closely monitored since the dynamometer torque readings fell off as soon as initial buckling occurred.

Web strains were measured by Baldwin-Southwark SR-4 type R-1 wire resistance strain rosettes. The average compressive strains in the stringers were made using 10-inch Whittemore strain gauges. Locations of the web strain gauges were not reported. A sketch showed the 2 strain gauges mounted on the stringer, starting from one end of the cylinder. However, base location relative to the cylinder top or bottom was not specified.

The data was analyzed by plotting torque vs. twist curves for the various specimens. Since panels of different widths did not buckle simultaneously in a cylinder, the buckling torque was determined by averaging the torques of the individual panels observed at buckling. Clark and Moore (1952) observed that once the first panel buckled, some of

the remaining panels became unstable at a lower value of torque in the subsequent loading cycles. The critical torque for each of these panels was assumed to be the highest load withstood by the panel before buckling. The experimental torque vs. twist curves for a given configuration was not necessarily the result of one loading cycle. It was stated that in some cases, different portions of the curve were determined from different loading cycles. Finally, Clark and Moore (1952) concluded that the final loading torque-twist relationship was not affected by the previous loadings cycles.

8.2. COMPARISON OF EXPERIMENTS WITH SIMULATIONS

This section analyzes the elastic, buckling and post-buckling behaviour for a variety of frame and stringer configurations. Data obtained from ACA experimental results (from now on to be referred to as experimental) and the simulation results using LS-DYNA, were used for a comparative analysis. For the cylinders with 8 and 16 stringers an additional set of data for comparison was obtained by the NACA analytical method.

8.2.1. Cylinder Stiffened by Frames

Figure 8.3 shows the graphs of torque vs. angle of twist for experimental and simulation results, for cylinders with varying number of frames. The ACA's objective of testing this configuration, is believed to be, to obtain the initial buckling point of the panels in each frame bay.

The buckling of each frame bay displayed the classical buckling characteristics of thin shells. In the experiments, an increasing load was applied until initial buckling of panels in a frame bay occurred. Once the frame bay buckled, extra frames were added to stiffen the buckled area. Then the cylinder was re-loaded to observe the buckling characteristics of the remaining frame bays. From Figure 8.3, it is seen that the slope(s) of the experimental and simulation results, in the elastic regions were identical. Also, it is noted that as the panel size decreased, the buckling load increased. The buckling loads for the simulations were within 15% of the experimental results for all configurations as shown in Table 8.1.

Figure 8.3: Torque vs. Angle of Twist for Cylinder with 0 Stringers and Varying Number of Frames

Table 8.1: Comparison of the Experimental and Simulation Initial Buckling Loads

Test Case	Initial Buckling		
	Experiment	Simulation	Difference
	[ft-lb]	[ft-lb]	[%]
A-1	9650	10008	4
A-2	14050	12746	9
A-3	16050	13974	13
	17200	15107	12
A-4	17950	16334	9
	21950	18883	14

8.2.2. Cylinders Stiffened with Frames and Stringers

For the cylinder with stringers, Clark and Moore (1952) define four distinct stages in the behaviour of the cylinder in the elastic, buckling and post-buckling range(s) as shown in Figure 8.4:

1. The range of shear-resistant action before buckling, where the relation between torque and angle of twist is linear.
2. The buckling range, in which there is a marked decrease in torsional stiffness. The range is defined up to the point where the web is capable of transmitting shear by diagonal tension.
3. The tension field range in which the shear is carried principally by diagonal tension in the web. This approximates to a linear relation between torque and twist.

Figure 8.4: Stages of Cylinder Behaviour illustrated with an ACA Specimen B3 with 4 Frames and 8 Stringers

4. Ultimate failure, which occurs by the collapse of stringers, for the ACA cylinders.

Stages 1 and 2: Shear Resistant and Buckling Range

As shown in Figures 8.5 and 8.6, the simulation results mirror the experimental results of the first stage of buckling but differ in the second. This is attributed to the loading conditions as explained below.

In the experiments, loading was under displacement control. As the angle of twist (displacement) increased, the cylinder load also increased linearly until initial buckling of the primary panels (largest panels of the cylinder) occurred. Upon initial buckling, the cylinder experienced a loss of torsional stiffness, which was observed as a decrease in the load while the angle of twist increased.

The simulations, however, used the applied load as a control parameter. As the load was increased, the angle of twist increased linearly up to the initial buckling point, comparing well with the experimental data. The initial buckling load was simulated to within 20% of the experimental values for all cases except C-4, as shown in Table 8.2. Since the applied load was the controlling parameter, the simulation was driven to higher load values, and could not capture the load decrease seen in the experiments. Instead a “plateau” bounds the buckling range. As seen in Figure 8.5 and 8.6, the plateau ends when the diagonal tension begins to form. It is believed that if the simulation load was applied in displacement control, the buckling range as observed in the experiments would have been captured. Attempts were made to simulate the cylinders in LS-DYNA using displacement control. The simulations were unsuccessful due to difficulties encountered in manipulating the loading module, which could not be resolved. However, it is common practice for large-scale structural tests to use load control. As such the SMPL

tests will use load control; therefore, the predictions generated in this work, will be directly comparable to the SMPL experimental data.

Stage 3: Tension Field Range

In the tension field range, a linear relationship between the load and the angle of twist is seen (Figure 8.5 and 8.6). However, a change of slope is observed at the initial buckling point of the secondary cylinder panels (smaller panels).

The tension field range of a majority of the simulations compared well with the experimental results. While the buckling loads of the smaller cylinder panels agreed well with the experiment (Table 8.2), in some cases the values for the angle of twist were found to deviate.

The number of frames in the specimen is believed to have an effect on the difference between the simulation and experimental angle of twist values. Figures 8.5 and 8.6 shows that as the numbers of frames increased the simulation accuracy also increased and the plateau length decreased. The simulation for specimens with the single frame bay (Specimen B-1 and C-1) and equal sized panels, (Specimen B-2 and C-2) may not be accurately predicted under ideal conditions because, the buckling occurs simultaneously across all the panels. Under these conditions, the load distributes uniformly across the panels. Therefore, the simulations plateau after buckling until the web is able to carry the load in diagonal tension. The proportional decrease in the plateau length to the increase in the number of frames may be attributed to the decrease of the secondary cylinder panel sizes. As the secondary panel sizes decrease, the buckling loads of the panels increase

and are closer to the buckling values of the primary panels. As a result, LS-DYNA has less time to oscillate between stage 2 and stage 3 before the load is picked up by diagonal tension.

In the specimens with 8 stringers (B-1 to B-4), the approximate linear relationship between torque and angle of twist beyond the buckling range of the secondary panels are simulated well. As shown in Figure 8.5, slopes of the lines in this range are similar or are exactly equal to those from experimental results. For the specimens containing 16 stringers (C-1 to C-4), the tension field range deviated from the experimental values but still increased linearly and was similar to that of the NACA analytical method. However, as the number of frames increased to 4 and 5, a change in the slope of the simulation tension field range occurred. This reduction of stiffness was thought to be due to a local buckling some where in the specimen. However, a study of the LS-DYNA displacement contours did not provide an explanation for this behaviour.

Stage 4: Failure

All cylinders containing stringers experienced failure by the collapse of these stringers in both the experiments and the simulations. The NACA analytical method however, predicted web failure of the cylinder. This was because the analytical method only accounted for failure of the stringers by the force-crippling mode and so the failure by collapse of the stringers could not be predicted analytically.

Figure 8.5: Torque vs. Angle of Twist for Cylinder with 8 Stringers and Varying Number of Frames

Figure 8.6: Torque vs. Angle of Twist for Cylinder with 16 Stringers and Varying Number of Frames

Table 8.2: Comparison of the Experimental and Simulation Initial Buckling and Failure Loads

Test Case	Panel Width	Initial Buckling Results			Ultimate Failure Results		
		Experiment	Simulation	Difference	Experiment	Simulation	Difference
	[in]	[ft-lb]	[ft-lb]	[%]	[ft-lb]	[ft-lb]	[%]
B-1	32	13100	11865	9	17200	22912	33
B-2	16	15900	14918	6	27700	24549	11
B-3	12	18200	20016	10	34000	33802	1
	10	19800	20961	6	-	-	-
B-4	9	19850	22031	11	40000	39341	2
	6	23500	22975	2	-	-	-
C-1	32	17900	14050	20	18800	-	-
C-2	16	23600	28011	19	44200	42174	4.6
C-3	12	21800	20011	8	53600	48154	10.2
	10	24650	21716	12	-	-	-
C-4	9	27500	35879	30	57600	54448	5
	6	36050	37453	3	-	-	-

8.3. DETAILED ANALYSIS OF STIFFENED SYSTEM

The following section presents a detailed analysis for a stiffened cylinder containing 4 frames and a varying number of stringers. This configuration was chosen to analyze in detail since the IAR cylinder is based on a variation of this design. Each analysis case includes images of displacement contour plots of major buckling behaviour points.

8.3.1. Case A-3: 4 Frames, No Stringers

Figure 8.7 shows the graph of torque vs. angle of twist for experimental and simulation results for a cylinder containing 4 frames. Similar to the experiments, only the initial buckling values for each shell size were obtained from the LS-DYNA simulation.

Initial buckling of the 12in and 10in frame bays in the simulated cylinder occurred within 14% of the experimental buckling loads.

**Figure 8.7: Torque vs. Angle of Twist for
ACA Cylinder A-3 with 4 Frames**

8.3.2. Case B-3: 4 Frames, 8 Stringers

Figure 8.8 shows the graphs of torque vs. angle of twist for experimental, simulation and NACA analytical method results, for a stiffened cylinder containing 4 frames and 8 stringers. It is observed that the slopes of the elastic regions and the tension field regions for the experimental and simulation methods are identical while the slope estimated by the analytical method in the elastic region is lower.

Initial buckling of the simulated cylinder occurred in the test section within 10% of the experimental initial buckling load. The analytical initial buckling load occurring in the same frame bay was found to be within 8% of the experimental load. As previously mentioned, the simulation buckling range was not captured as defined by the experiments. However, the tension field range starts at the same angle of twist as in the experiment and shows excellent agreement. Finally, the simulated failure occurs within 1% of the experimental load. Comparison of the simulation output to the ACA photographs shows agreement that the failure mode was the collapse of the stringers.

The NACA analytical method analyzes individual panels of 10in and 12in. It should be noted that the experimental and simulation results include the combined effects of these different frame bay widths. The analytical method is not capable of predicting this combined effect. As a result, the initial buckling point is not directly comparable to the experimental and simulation results. Although the analytical tension field range is also comparable to the experimental and simulation values, it is expected to be conservative since the NACA method is a design tool.

The initial buckling of the 10in panels in the simulation occurred at a load 6% higher than the experimental initial buckling load. Analytical secondary buckling loads was found to be 14% and 8% higher than the experimental and simulation buckling loads, respectively.

Although analytical method values for the torque and angles of twist did not compare well with the experimental and simulation values, it is seen that the slopes of the tension field range are similar.

Figure 8.8: Torque vs. Angle of Twist for ACA Cylinder B-3 with 4 Frames and 8 Stringers

Displacement Plots for ACA Aluminum Cylinders

Pt. 1: Initial Buckling

Pt. 2: Buckling Range

Pt. 3: Tension Field Range

Pt. 4: Failure

8.3.3. Case C-3: 4 Frames, 16 Stringers

Figure 8.9 compares the torque and the angle of twist values obtained from experimental, simulation and NACA analytical method for a stiffened cylinder containing 4 frames and 16 stringers. All three sets of results were similar for the elastic region.

Initial buckling of the simulated cylinder occurred in the 12in panel at a torque that was 8% lower than the experimental load while the analytical initial buckling load was found to be 12% higher than the experimental load. Similar to case B-3, the buckling range for this simulation showed a load plateau. The simulation predicts the beginning of the tension field range at a twist angle of approximately 0.01 rad, which is very similar to the experiment. The increasing stiffness in this range is observed by the expected linear relationship up to 0.02 rad. However, the slope in this range is lower than the experiment but similar to the NACA analytical prediction. Beyond 0.02 rad, the simulation predicts significantly lower stiffness than either the experimental or the NACA analysis. The reason for this discrepancy is unexplained. However, the failure occurs within 10% and the failure modes are identical, collapse of the stringers.

The buckling of the 10in panel in the simulated cylinder occurred at a load 12% lower than the experimental while the analytical buckling load for the 10in panel was found to be 0.3% and 13% higher than the experimental and simulation buckling loads, respectively.

Displacement Plots for ACA Aluminum Cylinders

Figure 8.9: Torque vs. Angle of Twist for ACA Cylinder C-3 with 4 Frames and 16 Stringers

Pt. 1: Initial Buckling

Pt. 2: Start of Tension Field Range

Pt. 3: Tension Field Range

Pt. 4: Failure

8.4. PREDICTION OF GLARE CYLINDRICAL SPECIMEN BUCKLING

The simulations of buckling of NACA cylinders showed good agreement with experimental results, in terms of initial buckling, final failure loads and failure modes. In the post-buckling (diagonal tension) range there were discrepancies observed between the experiment and simulation for stiffness, which are currently unexplained. Overall, the quality of the simulations was considered to be adequate and the same modelling approach was used for the IAR cylinder.

Buckling of the IAR cylinder was simulated for webs of aluminum, GLARE 3 2/1 and GLARE 3 3/2. In order to compare the buckling behaviour of aluminum and GLARE cylinders, additional IAR aluminum cylinders were simulated for each of the GLARE variant thicknesses. The results and analysis are presented below for the 12in test sections.

8.4.1. Test Case 1 – GLARE 3 2/1

Figure 8.10 shows the predictions for an IAR cylinder with 0.033in thick web. The predictions for GLARE 3 2/1 using two material model simulations are compared to the aluminum simulation data and the NACA analytical results.

The GLARE 3 2/1 and aluminum IAR cylinder simulations predict the expected linear shear resistant range. The initial buckling point of this cylinder is well established as the Type 2.1 (Orthotropic elastic) and Type 24.1 (Linear isotropic) material models predict a coincident buckling point, at a torque of 50400 ft-lb. As expected the initial buckling of

the aluminum cylinder was found to be 5% higher than for the GLARE 3 2/1 initial buckling point. Although the slope of the linear shear resistant range is similar for all modes of analysis, the overall results of the NACA analytical method are approximately 170% less than that obtained from the simulations. The cause for this discrepancy is unexplained as it was expected that the NACA analytical method and the simulation would predict a similar initial buckling point. However, comparing the NACA aluminum results to the similar ACA experimental and analytical results, it is believed that the NACA analytical method is more reliable. The NACA analytical and simulation results will be confirmed by the tests at IAR later this year.

A plateau, similar to the NACA cylinder simulations, again bounds the buckling range. The orthotropic elastic material model predicted a buckling range displacement similar to that of aluminum cylinder simulation. However, the linear isotropic material model simulated the buckling range displacement to be 35% less.

The tension field range shows the expected linear relationship between the torque and angle of twist. The initial buckling load of the secondary panels, which is the beginning of the tension field range, was simulated to within 1% between the two material models and 5% less than the aluminum cylinder.

The slope(s) of the tension field range are similar for the experimental and the simulation results using orthotropic elastic material model. However, the simulation using linear isotropic material model shows a tension field range that has a much lower slope. This may be due to the use of an isotropic model to define an orthotropic material such as

GLARE. The stress-strain curve defined for GLARE 3 2/1 in the isotropic material model used with one element layer in effect simulates “smeared properties” of a GLARE web.

The NACA analytical results for the aluminum cylinder show slopes similar to that of the aluminum cylinder simulation in the tension field range. The GLARE 3 2/1 analytical results show a tension field range that has a higher slope when compared to the simulation results.

Failure was predicted only by using the linear isotropic material model. Similar to the aluminum cylinders, the failure mode of the GLARE simulation was the collapse of the stringers.

Displacement Contours for GLARE 3 2/1 IAR Cylinders

Figure 8.10: Torque vs. Angle of Twist for 0.033in Web Thick IAR Cylinder

Pt. 1: Initial Buckling

Pt. 2: Tension Field Range

Pt. 3: Failure

8.4.2. Test Case 2 – GLARE 3 3/2

Figure 8.11 shows the predictions for an IAR cylinder with 0.055in thick web. As for the GLARE 3 2/1 cylinders, predictions for GLARE 3 3/2 were made using two material model simulations and compared to the aluminum simulation data and the NACA analytical results.

The GLARE 3 3/2 and aluminum IAR cylinder simulations again show the expected linear shear resistant range. The initial buckling point, however, is not as clearly established. The orthotropic elastic material model predicts the initial buckling point to be at a load of 46700 ft-lb. Linear isotropic material model show a slope decrease of the linear shear resistant range at 46700 ft-lb but predicts the initial buckling to occur at 74000 ft-lb.

Possible causes for the slope decrease was thought to be local buckling or yielding. As a result displacement contour plots were analyzed to determine if any local buckling occurred at or near the change in slope. The images of the buckling behaviour indicate a “rippling effect” in the cylinder web (Figure 8.11, Pt. 1) that initiated the cylinder web buckles in the orthotropic elastic material model, and a softening of the shear resistant range in the Linear isotropic material model. It is believed that the “rippling effect” was initiated by the imperfections of the FE model. A cylindrical configuration is meshed by approximating the arc into discrete linear elements. The approximation may introduce imperfections in all FE models. However, in the current test case, the web is 67% thicker than the cylinder webs used for the mesh sensitivity study (section 7.2.1.4). Therefore, the introduced imperfections could have a greater effect in a thicker web. A possible

solution would be to re-mesh the cylinder such that element size to thickness ratio is constant for all models such that the effect of thickness will be removed.

Although the web was expected to initially buckle within the elastic region, possible local or global buckling was investigated. This is due to the fact that the decrease in slope was similar to the GLARE yielding behaviour as shown in Figure 7.6. Consequently, the principal stresses of the cylinder, more specifically of the web, were investigated. However, the contour plots of the stress in the elements showed that the stresses in the web were well below the yielding stress prior to the 74000 ft-lb initial buckling torque.

The buckling behaviour of the 0.055in thick aluminum cylinder parallels that of the GLARE 3 3/2 cylinder simulation using the Linear isotropic material model. The initial buckling of the aluminum cylinder was observed at a load 14% higher than GLARE 3 3/2. The NACA analytical method determined the initial buckling load of the test section 12% lower in GLARE cylinders and 8% lower in the aluminum cylinders as compared to the respective simulations.

A plateau in all three simulations again defines the buckling range and the tension field range shows the expected linear relationship between the torque and angle of twist. The initial buckling load of the 10in GLARE 3 3/2 panel, was simulated to be 7% higher than the NACA analytical method and 14% less than the aluminum cylinder simulation.

Failure was predicted only by using the linear isotropic material model. Similar to the aluminum cylinders, the failure mode of the GLARE simulation was the collapse of the stringers.

**Figure 8.11: Torque vs. Angle of Twist for
0.055in Web Thick IAR Cylinder**

Displacement Contours for GLARE 3 3/2 IAR Cylinders

Pt. 1: Initial Buckling

**Pt. 2: Tension Field
Range**

Pt. 3: Failure

8.5. GLARE MODELLING OPTIONS

The GLARE simulations show that the elastic orthotropic material model can accurately simulate the linear shear resistant range, initial buckling and the buckling range. To obtain the buckling behaviour in the plastic region it was necessary to model the GLARE cylinders using an isotropic material model. This is not entirely accurate, as the stiffness values in the orthogonal directions are equal but the off-axis stiffness is lower. It is expected that a non-linear orthotropic material model would capture the full buckling range of the GLARE cylinders. As discussed in section 7.2.3, the LS-DYNA code developers have yet to produce a stable model that will simulate the plastic region of an orthotropic material.

There are other possible methods to simulate GLARE. It may be modelled as discrete layers instead of one layer of elements with a defined thickness. Thick-shell elements may also be used to define each layer. However, these methods add to the complexity of the model and therefore increase the run time. Also, they were considered to be beyond the scope of this work.

8.6. LESSONS LEARNED USING LS-DYNA

A major component of the current work was the understanding and use of LS-DYNA software. The following describes some of the critical aspects in forming the complete model.

Once a component is meshed, it is necessary to complete a node coincident check. A coincident check, confirms that two nodes do not exist within a specified separation distance. If extra nodes are found, the check provides the option to merge and delete them. For example, a cylinder mesh creates two sets of nodes at the cylinder seam that are less than 0.01 mm apart. If these nodes are not merged and deleted, a closed component cannot be modelled.

The IAR specimen was designed such that the stringers and frames are to be riveted to the cylinder web. As such, it was not feasible for the component attachments to be modelled using contact. Instead, the stringer-web and frame web attachments were simulated using spot welds (Figure 8.12). Spot welds are rigid massless beams that connect two non-contiguous nodes. The spot welds can be defined in a part and copied as any other element. Once spot welds are copied, a node coincident check is required in order to combine the spot weld and component nodes.

Figure 8.12: Top View of Cylinder Highlighting Spot Welds

Nodal rigids are used to attach three or more coincident or non-contiguous nodes. In the current simulation, nodal rigids are used to attach the stringer, cylinder web and the

loading interface (Figure 8.13) as well as the stringer, cylinder web and frames. Unlike spot welds, nodal rigids cannot be copied and connected to the parts by the node coincident check. Each nodal rigid must be defined by choosing a specific node connection path. If nodal rigids are used in component with contact constraints, the contact module must be defined prior to the definition of the nodal rigids.

Figure 8.13: Top View of Cylinder Highlighting Nodal Rigids

The cylinder was loaded in torsion using the *follower force* option in the *LOAD_NODE module. This allowed a concentrated nodal force to be applied to an individual node. The follower force acting on a plane was defined by nodes M_1 , M_2 and M_3 (Figure 8.14). A positive force acts normal to the plane defined by the nodes. In the current model node M_1 was defined along the edge of the load interface flange and M_2 and M_3 was defined along the longitudinal axis. The load curve was defined by inputting a load vs. time relationship in to the *LOAD_CURVE module.

Figure 8.14: Nodes for Defining a Follower Force

The *MAT_ELASTIC_ORTHOTROPIC material model was defined to be locally orthotropic with the material axes determined by a point (x_p, y_p, z_p) in space and the global location of the element centre; the a-direction as shown in Figure 8.15.

Figure 8.15: Definition of the Principal Material Axes

As shown in the above figure, a discrete point for the local material axis and the centre of each element needed to be defined to use the *MAT_ELASTIC_ORTHOTROPIC material model. The most logical approach was to define the cylinder centre as the discrete point for the local material axis. In order to define the centre of each element as a vector with the centre of the cylinder as a reference point (vector 'a', in Figure 8.15), each horizontal strip of cylinder web elements was classified as a LS-DYNA "part"

(Figure 8.16). Also, an individual material card was defined for each strip of element since the coordinates of element midpoint varied.

Figure 8.16: Section of LS-DYNA Cylinder Web Element Configuration (each coloured strip represents a LS-DYNA “part”)

Over time it was found that the repeated use of a FEMB file caused the models to “corrupt”. In such a corrupt file, the LS-DYNA compiling run returned an error that pointed to a non-existent node or caused a series of unsolvable errors. A cause for the error could not be found even after a line-by-line comparison of a “corrupt” and “running” input deck. A solution to the problem was to re-build the models from the start; however, this became a very time consuming exercise.

9.0. CONCLUSIONS

A cylindrical specimen was designed using the NACA analytical method and analyzed using the FE code LS-DYNA in order to gain a better understanding of the diagonal tension properties of GLARE and aluminum in curved web systems. Conclusions drawn from this project are as follows:

On the NACA aluminum cylinder analysis:

1. The FE program predicted the elastic region prior to buckling very well in all simulations. The initial buckling point was simulated to within 20% of the experimental results. The buckling range was not captured as defined by the experimental program due to simulation control parameters. However, excellent agreement was established in the diagonal tension field range as well as in simulating the failure modes.
2. The NACA analytical method reasonably predicted the elastic region in all simulations. However, it provided very conservative predictions for the other regions.

On the IAR cylinder analysis:

1. GLARE simulations confirmed that diagonal tension will develop in a similar manner to aluminum cylinders

2. The initial buckling for GLARE 3 2/1 was determined to be at a load 5% lower than a 0.033in aluminum cylinder while the initial buckling load for GLARE 3 3/2 was at a load 14% lower than a 0.055in aluminum cylinder.
3. Investigation into LS-DYNA material models concluded that the current version of FE code does not support the modelling of plastic orthotropic materials as the current Type 40.1 model is unstable. However, Type 2.1 material model was partially successful in approximating the buckling behaviour of the GLARE cylinders. The post-buckling behaviour was simulated using Type 24.1 material model. Failure was determined to be the collapse of the stringers.
4. The NACA analytical method predicted the buckling loads of GLARE 3 3/2 and 0.055in thick aluminum cylinders to within 15% of the simulations. Although, the NACA analytical method predicted similar slopes for the buckling behaviour of the GLARE 3 2/1 and 0.033in aluminum, the buckling loads are 170% less than the simulation results.

As the non-linear orthotropic material model simulations were unsuccessful, there is not high confidence that the predictions for the GLARE cylinder are accurate. The FE analysis may have to be calibrated by cylinder tests as per the original objective until LS-DYNA developers have corrected the non-linear orthotropic material model.

10.0. RECOMMENDATIONS FOR FUTURE WORK

The following are recommended for future investigations:

- An investigation of model symmetry to simulate a portion of the cylinder, to reduce the computation times. The challenge in such an attempt would be to define the loading and boundary constraints to obtain a representative physical system.
- Use of a multi-layered element approach to simulate GLARE properties without the need for the non-linear orthotropic material model.
- Modelling of the longitudinal seam to determine the influence of the splice to the test section. An initial approach may be to reinforce a cylinder section with extra elements.
- Replace the FE control parameters to displacement control. This may capture the buckling range as defined by the experiments.

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APPENDIX A: SAMPLE DIAGONAL TENSION CALCULATIONS FOR FML CYLINDER

This example calculates diagonal tension properties for a cylinder similar to that tested on NACA TN 2821.

- GLARE 4 3/2 ($t \sim 0.065$) cylinder of diameter of 30 in and a length of 36 in
- main bay of 12 in and two adjacent bays of 10 in each
- 16 hat shaped stringers and 4 "L" shaped frames

Figure A.1: Cylinder Layout

Figure A-2: Stringer Cross Section

Figure A-3: Frame Cross Section

The above stringer is approximated by the following for calculation purposes:

Figure A-5: Approximated Stringer Cross Section

$$A_{st} := 0.287 \text{ in}^2 \quad I_{st} := 0.0585 \text{ in}^4 \quad \rho_{st} := 0.452 \text{ in}$$

Cylinder Geometry:

$R := 15 \text{ in}$	<< radius of curvature, in
$t := 0.065 \text{ in}$	<< thickness of skin (web), in
$h := 5.89 \text{ in}$	<< distance between stringers, in
$d := 12 \text{ in}$	<< distance between frames, in
$T := 1680 \cdot 10^3 \cdot \text{in} \cdot \text{lbf}$	<< Torsion applied, in-lbf
$T = 140000 \text{ ft} \cdot \text{lbf}$	<< Torsion applied, ft-lbf

Stringer Geometry:

$t_{st} := 0.060 \text{ in}$	<< thickness of stringer, in
$A_{st} := 0.287 \text{ in}^2$	<< area of stringer, sq. in
$y_{st} := 0.693 \text{ in}$	<< distance to the stringer neutral axis, in
$I_{xx_st} := 0.0585 \text{ in}^4$	<< stringer moment of inertia, in ⁴

Frame Geometry:

$\text{trg} := 0.19\text{in}$	<< thickness of frame, in
$\text{Arg} := 0.386 \cdot \text{in}^2$	<< area of ring, sq. in
$\text{y_rg} := 0.579\text{in}$	<< distance to the frame neutral axis, in
$\text{prg} := 0.468\text{in}$	<< frame radius of gyration
$\text{Ixx_rg} := 0.0843\text{in}^4$	<< frame moment of inertia, in ⁴

Material Properties:

$\mu := 0.32$	<< Poisson's ratio
$G := 3.38 \cdot 10^6 \text{psi}$	<< Shear modulus, psi
$E := 8.8 \cdot 10^6 \text{psi}$	<< Young's modulus, GLARE, psi
$E_{\text{al}} := 10.4 \cdot 10^6 \text{psi}$	<< Young's modulus, Aluminum, psi
$\Phi := 0.5$	<< angle subtended by 2 stringers ~ 0.5 rad
$\sigma_{\text{cy_2024}} := 40000 \text{psi}$	<< compressive yield stress for Al 2024-T3, psi
$\sigma_{\text{cy_7075}} := 67000 \text{psi}$	<< compressive yield stress for Al 7075-T6, psi

MathCAD Conversion Definitions

$\text{cotd}(\alpha) := \cot(\alpha \cdot \text{deg})$	$\text{sind}(\alpha) := \sin(\alpha \cdot \text{deg})$
$\text{tand}(\alpha) := \tan(\alpha \cdot \text{deg})$	$\text{tanhd}(\beta) := \tanh(\beta \cdot \text{deg})$
$\text{tand}(\beta) := \tan(\beta \cdot \text{deg})$	

A.1 Critical Shear Stress, τ_{cr}

The critical shear stress is the stress at which web buckling first occurs. Based on the assumption of simply supported edges, the value is computed in the elastic range, using the following formula:

$$\tau_{cr_elastic} := \frac{\pi^2 \cdot E \cdot h^2 \cdot k_s}{12R^2 \cdot Z^2} \quad \ll \text{ [Kuhn 1952a] } \\ \text{Section 10.1, Pg. 78}$$

The equation contains two constants: the curvature parameter, Z and the shear buckling stress coefficient.

The curvature parameter is a function of the radius of curvature, thickness of web, distance between stringers and the Poisson's ratio. The value can be obtained by the following equation:

$$Z := \frac{h^2 \cdot \sqrt{1 - \mu^2}}{R \cdot t} \quad \ll \text{ [Kuhn 1952a] } \\ \text{Fig. 30a, Pg. 125}$$

$$Z = 33.71$$

The shear buckling stress coefficient is a function of the panel dimensions (i.e. aspect ratio d/h) and the curvature parameter Z. The value is obtained by using figure 30 in reference [Kuhn 1952a] .

$$\text{with } \frac{d}{h} = 2.04 \quad \text{if } \frac{d}{h} \text{ greater than 2 use 2}$$

$$k_s := 13$$

From the above constants, the elastic shear buckling stress for the web is found to be:

$$\tau_{cr_elastic} := \frac{k_s \cdot \pi^2 \cdot E \cdot h^2}{12R^2 \cdot Z^2}$$

$$\tau_{cr_elastic} = 12.77 \times 10^3 \text{ psi}$$

The critical shear value is corrected for plasticity by using figure 12c in reference [Kuhn 1952a]. Figure 12c was converted to the following best fit equation (**Appendix D , Section D.1, Figure D.1.2**):

$$\tau_{cr_elastic} := \frac{\tau_{cr_elastic}}{1000 \cdot \text{psi}}$$

Note: converting $\tau_{cr_elastic}$ to ksi to accommodate the units of the best-fit equation

$$A := -2 \cdot 10^{-6}$$

$$B := 0.0005$$

$$C := -0.0393$$

$$D := 1.6209$$

$$F := -1.7267$$

if $\tau_{cr_elastic} > 16 \text{ ksi}$

$$\tau_{cr_ksi} := A \cdot \tau_{cr_elastic}^4 + B \cdot \tau_{cr_elastic}^3 + C \cdot \tau_{cr_elastic}^2 + D \cdot \tau_{cr_elastic} + F$$

if $\tau_{cr_elastic} < 16 \text{ ksi}$

$$\tau_{cr_ksi} := \tau_{cr_elastic}$$

Therefore, the corrected critical shear value is:

$$\tau_{cr_ksi} = 12.77$$

$$\tau_{cr_psi} := \tau_{cr_ksi} \cdot 1000 \text{ psi}$$

Note: converting τ_{cr} from ksi to psi

$$\tau_{cr} := \tau_{cr_psi}$$

$$\tau_{cr} = 12.77 \times 10^3 \text{ psi}$$

<< τ_{cr} corrected for plasticity

A.2 Nominal Shear Stress, τ

The shear stress due to the applied torque load, T, is calculated as follows:

$$\tau := \frac{T}{2 \cdot \pi \cdot R^2 \cdot \left(1 - \frac{\Phi^2}{6}\right) \cdot t}$$

<< [Kuhn 1952a]
Section 10.2, Pg. 78

$$\tau = 19.08 \times 10^3 \text{ psi}$$

The applied torque, T, at which the web fails was calculated through several iteration cycles. The starting point was at the torque where the web buckles and k, the diagonal tension factor (reference A.4) was zero. This calculation was continued through increments of 10% for k until the web failed. The sample calculation was based on this failing torque.

A.3 Loading Ratio

Using the nominal shear stress and the critical shear stress the loading ratio is calculated as follows:

$$\frac{\tau}{\tau_{cr}} = 1.49$$

A.4 Diagonal-Tension Factor, k

The diagonal tension factor gives the percentage of complete diagonal tension in the web. For example $k = 0$ for initial buckling of the web and $k = 1$ for complete diagonal tension. The loading ratio, panel geometry, and the radius of curvature is used to calculate the diagonal tension factor as follows:

$$k := \tanh \left[\left(0.5 + 300 \cdot \frac{d}{h} \cdot \frac{t}{R} \right) \cdot \log \left(\frac{\tau}{\tau_{cr}} \right) \right] \quad \ll \text{[Kuhn 1952a]} \\ \text{Pg. 69, Eqn. 50}$$

$$k = 0.5$$

A.5 Diagonal Tension Angle, α

Diagonal tension angle is the angle from the neutral axis and defines the angle of the "web buckles". This value depends on the distance between stringers, radius of curvature and the strains of the web, stringer and frame as follows.

Note: the equation is valid only if $h < \frac{1}{3} \cdot R$

$$(\tan \alpha)^2 := \frac{(\varepsilon - \varepsilon_{st})}{\varepsilon - \varepsilon_{rg} + \frac{1}{24} \cdot \left(\frac{h}{R} \right)^2} \quad \ll \text{[Kuhn 1952a]} \\ \text{Pg. 64, Eqn. 44}$$

The stringer and frame strains also depend on the diagonal tension angle. Therefore, the angle is obtained through an iterative process.

A.5.1 Pure Diagonal Tension Angle, α PDT

NACA method suggests using the value of the pure diagonal tension, α PDT as a first approximation to the diagonal tension angle. It is obtained from figure 28a in reference [Kuhn 1952a]. The figure requires the following constants:

$$R_r := \frac{d \cdot t}{A_{rg}}$$

$$R_s := \frac{h \cdot t}{A_{st}}$$

$$R_r = 2.02$$

$$R_s = 1.33$$

<< [Kuhn 1952a]
Pg. 65, Eqn. 44a
Pg. 124, Fig. 28a

*For ease of determination
this Figure was reproduced.
(Appendix D , Section
E.2, Figure E.2.2)

$$\frac{1 + R_s}{1 + R_r} = 0.77 \quad \frac{\frac{h}{R} \cdot \sqrt{\frac{E}{\tau}}}{\sqrt{1 + R_r}} = 4.85$$

$$\alpha_{PDT} := 37.3$$

The first approximation can be further improved by correcting the pure diagonal tension angle using figure 32 in reference [Kuhn 1952a]. This curve represents the average of

the scatter band obtained by plotting the ratio $\frac{\alpha}{\alpha_{PDT}}$ for a number of webs with a variety of proportions.

The curve in figure 32 was converted into the following best fit equation. (Appendix D , Section D.3, Figure D.3.2):

$$\text{corr} := 3.124 \cdot k^5 - (9.7762 \cdot k^4) + 11.737 \cdot k^3 - (7.0488 \cdot k^2) + 2.6315 \cdot k + 0.3408$$

$$\text{corr} = 0.85$$

The first approximation for the diagonal tension angle is obtained as follows:

$$\text{corr} = 0.85$$

$$\alpha := \text{corr} \cdot \alpha_{PDT}$$

$$\alpha = 31.63 \quad \text{deg}$$

First Iteration Cycle

A.5.2 Stringer Stress and Strain

The first approximation for the diagonal tension angle is used to calculate the average stress in the stringer as follows:

$$\sigma_{st} := \frac{(-k \cdot \tau) \cdot \cotd(\alpha)}{\left[\left(\frac{A_{st}}{h \cdot t} \right) + 0.5 \cdot (1 - k) \right]}$$

$$\sigma_{st} = -15.49 \times 10^3 \text{ psi}$$

<< [Kuhn 1952a]
Pg. 69, Eqn 51

Note: negative sign
indicates compression

The stringer strain is related to the stringer stress as follows:

$$\epsilon_{st} := \frac{\sigma_{st}}{E_{al}}$$

$$\epsilon_{st} = -1.489 \times 10^{-3}$$

<< [Kuhn 1952a]
Pg. 69, Eqn 51

A.5.3 Frame Stress and Strain

The average stress through the floating frame is found using the following equation:

$$\sigma_{rg} := \frac{-k \cdot \tau \cdot \tand(\alpha)}{\left(\frac{A_{rg}}{d \cdot t} \right) + 0.5 \cdot (1 - k)}$$

$$\sigma_{rg} = -7.89 \times 10^3 \text{ psi}$$

<< [Kuhn 1952a]
Pg. 69, Eqn 52

The frame strain is related to the frame stress as follows:

$$\epsilon_{rg} := \frac{\sigma_{rg}}{E_{al}}$$

$$\epsilon_{rg} = -7.58 \times 10^{-4}$$

<< [Kuhn 1952a]
Pg. 69, Eqn 52

A.5.4 Web Strain

The web strain is determined by the following equation:

$$\varepsilon := \frac{(\lambda \cdot \tau)}{E}$$

<< [Kuhn 1952a]
Pg. 20, Eqn 30d

where λ is a function of the diagonal tension factor, Poisson's ratio and the diagonal tension angle and related through the following equation

$$\lambda := \frac{2 \cdot k}{\sin(2 \cdot \alpha)} + \sin(2 \cdot \alpha) \cdot (1 - k) \cdot (1 + \mu)$$

$$\lambda = 1.71$$

The web strain is determined to be:

$$\varepsilon := \frac{\tau \cdot \lambda}{E}$$

$$\varepsilon = 3.71 \times 10^{-3}$$

The stringer, frame and web strains are used to recalculate the diagonal tension angle.

$$\alpha := \beta$$

$$\beta := \text{atan} \left[\sqrt{\frac{(\varepsilon - \varepsilon_{st})}{\varepsilon - \varepsilon_{fg} + \frac{1}{24} \cdot \left(\frac{h}{R}\right)^2}} \right]$$

<< [Kuhn 1952a]
Pg. 64, Eqn. 44

solved for α .

$$\beta = 34.63 \text{ deg}$$

The computed value and the initial approximation has not converged. Therefore, the iteration is continued.

Second Iteration Cycle

For next the iteration, the diagonal tension angle is approximated as:

$$\alpha := \text{atan} \left[\tan(\beta) + \frac{1}{4} \cdot (\tan(\alpha) - \tan(\beta)) \right]$$

$$\alpha = 33.9 \text{ deg}$$

Web strain is recalculated using the second approximation of α :

$$\lambda := \frac{2 \cdot k}{\sin(2 \cdot \alpha)} + \sin(2 \cdot \alpha) \cdot (1 - k) \cdot (1 + \mu)$$

$$\lambda = 1.69$$

$$\varepsilon := \frac{(\lambda \cdot \tau)}{E}$$

$$\varepsilon = 3.67 \times 10^{-3}$$

Stringer stress and strain recalculated using the second approximation for α :

$$\sigma_{st} := \frac{(-k \cdot \tau) \cdot \cot(\alpha)}{\left[\left(\frac{A_{st}}{h \cdot t} \right) + 0.5 \cdot (1 - k) \right]}$$

$$\sigma_{st} = -14.2 \times 10^3 \text{ psi}$$

$$\varepsilon_{st} := \frac{\sigma_{st}}{E_{al}}$$

$$\varepsilon_{st} = -1.37 \times 10^{-3}$$

Frame stress and strain recalculated using the second approximation for α :

$$\sigma_{rg} := \frac{-k \cdot \tau \cdot \tan(\alpha)}{\left(\frac{\text{Arg}}{d \cdot t}\right) + 0.5 \cdot (1 - k)}$$

$$\sigma_{rg} = -8.6 \times 10^3 \text{ psi}$$

$$\varepsilon_{rg} := \frac{\sigma_{rg}}{E_{al}}$$

$$\varepsilon_{rg} = -8.27 \times 10^{-4}$$

The stringer, frame and web strains are used to recalculate the diagonal tension angle.

$$\alpha := \beta$$

$$\beta := \text{atan} \left[\sqrt{\frac{(\varepsilon - \varepsilon_{st})}{\varepsilon - \varepsilon_{rg} + \frac{1}{24} \cdot \left(\frac{h}{R}\right)^2}} \right]$$

$$\beta = 34.17 \text{ deg}$$

$$\alpha := \beta$$

The computed value of the diagonal tension angle agrees with the first cycle value. Therefore, further iteration is not required.

The final value of α is used to calculate the web strain, stringer stress and strain and frame stress and strain:

$$\lambda := \frac{2 \cdot k}{\sin(2 \cdot \alpha)} + \sin(2 \cdot \alpha) \cdot (1 - k) \cdot (1 + \mu)$$

$$\lambda = 1.69$$

$$\varepsilon := \frac{(\lambda \cdot \tau)}{E}$$

$$\varepsilon = 3.66 \times 10^{-3}$$

$$\sigma_{st} := \frac{(-k \cdot \tau) \cdot \cot(\alpha)}{\left[\left(\frac{A_{st}}{h \cdot t} \right) + 0.5 \cdot (1 - k) \right]}$$

$$\sigma_{st} = -14.06 \times 10^3 \text{ psi}$$

$$\epsilon_{st} := \frac{\sigma_{st}}{E_{al}}$$

$$\epsilon_{st} = -1.35 \times 10^{-3}$$

$$\sigma_{rg} := \frac{-k \cdot \tau \cdot \tan(\alpha)}{\left(\frac{A_{rg}}{d \cdot t} \right) + 0.5 \cdot (1 - k)}$$

$$\sigma_{rg} = -8.7 \times 10^3 \text{ psi}$$

$$\epsilon_{rg} := \frac{\sigma_{rg}}{E_{al}}$$

$$\epsilon_{rg} = -8.36 \times 10^{-4}$$

A.6 Allowable Shear Stress, τ_{all}

The allowable ultimate value for the shear stress in a curved web is given by the following empirical formula:

$$\tau_{all} := \tau_{all} \cdot (0.65 + \Delta)$$

<< [Kuhn 1952a]
Pg. 72, Eqn. 53

The value of τ_{all} is dependent on the diagonal tension factor and the diagonal tension angle and is determined by Figure 19 in reference [Kuhn 1952a]. For ease of determination figure 19 was reproduced and is shown in **Appendix D, Section D.4, Figure D.4.2** and converted to the following best-fit equation:

$$\tau_{all} := (89188 \cdot k^6 - 306294 \cdot k^5 + 414627 \cdot k^4 - 286399 \cdot k^3 + 114882 \cdot k^2 - 34262 \cdot k + 27500) \cdot \text{psi}$$

$$\tau_{all} = 21.03 \times 10^3 \text{ psi}$$

with $k = 0.5$ and $\alpha_{PDT} = 37.3$ $\tau_{all} = 21.03 \times 10^3$ psi

The value of Δ is obtained by the following equation:

$$\Delta := \tanh\left(\frac{Arg}{d \cdot t}\right) \cdot 0.3 + \tanh\left(\frac{Ast}{h \cdot t}\right) \cdot 0.1$$

<< [Kuhn 1952a]
Pg. 72, Eqn. 54

$$\Delta = 0.20$$

The allowable shear stress is:

$$\tau_{all} := \tau_{all} \cdot (0.65 + \Delta)$$

$$\tau_{all} = 17.89 \times 10^3 \text{ psi}$$

A check is now made to see if the web has failed:

$$MS := \frac{\tau_{all}}{\tau} - 1$$

$$MS = -0.06$$

The web has failed by shear.

A.7 Strength Checks for Stringers

The stringer strength limits will be checked using a finite elements model.

NACA [Kuhn 1952a] suggests checking stringers against forced crippling.

Forced Crippling

Forced crippling is a localized buckling failure of a stiffener that is connected directly to the web. The strength against forced crippling is checked in the same manner as for uprights in the flat panels. The maximum frame stress, σ_{rg_max} , and the allowable force crippling limit σ_{o_fc} must be determined.

The **maximum frame stress**, σ_{rg_max} is a function of the diagonal tension factor, k , and the ratio d/h . The value is determined by computing the $\sigma_{max/s}$ ratio using figure 15 in reference [Kuhn 1952a]. The lines in Figure 15 was converted to a best fit equation and used as below. (**Appendix D, Section D.5, Figure D.5.2**).

With $\frac{h}{d} = 0.49$ and $k = 0.5$

$$\frac{\sigma_{\max}}{\sigma} := \text{ratio}$$

<< [Kuhn 1952a]
Pg. 80, Section 10.8

$$\text{ratio} := 1 + (1 - k) \cdot \left(0.78 - .64 \cdot \frac{h}{d} \right)$$

<< Appendix D
Section D.5

$$\text{ratio} = 1.23$$

$$\sigma_{\text{rg_max}} := \sigma_{\text{rg}} \cdot \text{ratio}$$

$$\sigma_{\text{rg_max}} = -10.72 \times 10^3 \text{ psi}$$

The **allowable force crippling limit** for aluminum 2024-T3 (NACA symbol: 24S-T3) alloy and a 7075-T6 (NACA symbol: 75S-T6) alloy is defined by NACA [Kuhn 1952a] as follows:

for a 2024-T3 (NACA designation - 24S-T3) alloy, single upright

$$\sigma_{\text{oc_fc}} := -26 \cdot 10^3 \text{ psi} \cdot k^{\frac{2}{3}} \cdot \left(\frac{\text{trg}}{t} \right)^{\frac{1}{3}}$$

<< [Kuhn 1952a]
Pg. 47, Eqn. 37a

$$\sigma_{\text{oc_fc}} = -23.42 \times 10^3 \text{ psi}$$

Margin of safety:

$$\text{MS} := \frac{\sigma_{\text{rg_max}}}{\sigma_{\text{oc_fc}}}$$

$$\text{MS} = 0.46$$

Frame within limits of safety.

for a 7075-T6 (NACA designation - 75S-T6) alloy, single upright

$$\sigma_{fc} := -32.5 \cdot 10^3 \text{ psi} \cdot k^{\frac{2}{3}} \cdot \left(\frac{t_{st}}{t} \right)^{\frac{1}{3}}$$

<< [Kuhn 1952a]
Pg. 47, Eqn. 37b

$$\sigma_{fc} = -19.93 \times 10^3 \text{ psi}$$

Margin of safety

$$MS := \frac{\sigma_{rg_max}}{\sigma_{fc}}$$

$$MS = 0.54$$

A.8 Strength Checks for Frames

Forced Crippling

NACA [Kuhn 1952a] suggests checking a continuous frame against failure by force crippling.

The **maximum frame stress**, σ_{rg_max} is a function of the diagonal tension factor, k , and the ratio d/h . The value is determined by computing the σ_{max}/σ ratio using figure 15 in reference [Kuhn 1952a]. The lines in Figure 15 was converted to a best fit equation and used as below. (**Appendix D, Section D.5, Figure D.5.2**).

$$\text{With } \frac{h}{d} = 0.49 \quad \text{and} \quad k = 0.5$$

<< [Kuhn 1952a]
Pg. 75

$$\frac{\sigma_{max}}{\sigma} := \text{ratio}$$

$$\text{ratio} := 1 + (1 - k) \cdot \left(0.78 - .64 \cdot \frac{h}{d} \right)$$

<< **Appendix D**
Section D.5

$$\text{ratio} = 1.23$$

$$\sigma_{rg_max} := \sigma_{rg} \cdot \text{ratio}$$

$$\sigma_{rg_max} = -10.72 \times 10^3 \text{ psi}$$

The allowable force crippling limit for 24S-T3 alloy and a 75S-T6 alloy is defined by NACA [Kuhn 1952a] as follows:

for a 2024-T3 (NACA designation - 24S-T3) alloy, single upright

$$\sigma_{o_fc} := -26 \cdot 10^3 \text{ psi} \cdot k^{\frac{2}{3}} \cdot \left(\frac{\text{trg}}{t} \right)^{\frac{1}{3}}$$

<< [Kuhn 1952a]
Pg. 47, Eqn. 37a

$$\sigma_{o_fc} = -23.42 \times 10^3 \text{ psi}$$

Margin of safety:

$$MS := \frac{\sigma_{o_fc}}{\sigma_{rg_max}} - 1$$

$$MS = 1.19$$

for a 7075-T6 (NACA designation - 75S-T6) alloy, single upright

$$\sigma_{o_fc} := -32.5 \cdot 10^3 \text{ psi} \cdot k^{\frac{2}{3}} \cdot \left(\frac{\text{trg}}{t} \right)^{\frac{1}{3}}$$

<< [Kuhn 1952a]
Pg. 47, Eqn. 37b

$$\sigma_{o_fc} = -29.27 \times 10^3 \text{ psi}$$

Margin of safety

$$MS := \frac{\sigma_{o_fc}}{\sigma_{rg_max}} - 1$$

$$MS = 1.73$$

A.9 General Instability

A check against overall instability of the combination of frames and stringers is suggested by using the empirical criteria in figure 34 [Kuhn 1952a]. The allowables in this figure is based on pure torsion tests. This check is analogous to the column stability check for the uprights of a flat web beam. However, for curved webs, the skin does not contribute in preventing instability. The structure is checked by:

$$\frac{\tau}{\tau_{inst}} \leq 1$$

For ease of determination figure 34 was reproduced and is shown in **Appendix D, Section, D.6, Figure D.6.1**.

$$x := \frac{(\rho_{st} \cdot \rho_{rg})^{\frac{7}{8}}}{(d \cdot h)^{\frac{1}{2}} \cdot R^{\frac{3}{4}}} \cdot 10^4$$

$$x = 40.09$$

from figure D.6.1:

$$\tau_{inst} := (2.7907 \cdot x - 2.1258) \cdot 1000 \text{ psi}$$

$$\tau_{inst} = 109742.67 \text{ psi}$$

Therefore the margin of safety against collapse is:

$$MS := \frac{\tau}{\tau_{inst}}$$

$$MS = 0.17$$

Excel Calculation Procedure

Torque	θ	Norm τ	Z	Ks	τ_{cr}	τ/τ_{cr}	k
93700	0.4938	12.77	33.72	13	12.76	1.00	0.0004
100820	0.5314	13.74	33.72	13	12.76	1.08	0.100
108650	0.5726	14.81	33.72	13	12.76	1.16	0.200
117470	0.6191	16.01	33.72	13	12.76	1.25	0.300
127700	0.6730	17.40	33.72	13	12.76	1.36	0.400
133501	0.7036	18.19	33.72	13	12.76	1.43	0.450
140000	0.7379	19.08	33.72	13	12.76	1.49	0.500
155525	0.8197	21.19	33.72	13	12.76	1.66	0.600
176650	0.9310	24.07	33.72	13	12.76	1.89	0.700

First approximation for a PDT

$(1+R_s)/(1+R_r)$	x	α PDT	Corr Factor	α	σ_{st}	ϵ_{st}	σ_{rg}	ϵ_{rg}
0.77	5.93	35.3	0.34	12.07	-0.02	-2.05E-06	0.00	-1.18E-07
0.77	5.72	35.7	0.54	19.44	-3.25	-3.12E-04	-0.51	-4.93E-05
0.77	5.51	36.1	0.66	23.99	-5.79	-5.55E-04	-1.47	-1.41E-04
0.77	5.30	36.5	0.74	27.05	-8.56	-8.19E-04	-2.90	-2.78E-04
0.77	5.08	36.9	0.80	29.47	-11.75	-1.12E-03	-4.95	-4.74E-04
0.77	4.97	37.9	0.82	31.22	-13.18	-1.26E-03	-6.44	-6.17E-04
0.77	4.85	37.3	0.85	31.64	-15.50	-1.48E-03	-7.89	-7.56E-04
0.77	4.60	37.8	0.89	33.77	-20.04	-1.92E-03	-12.24	-1.17E-03
0.77	4.32	38.3	0.93	35.72	-26.07	-2.50E-03	-18.79	-1.80E-03

First Iteration

α	Norm τ	k	λ	ϵ	β
12.07	12.77	0.00	0.54	7.86E-04	18.29
19.44	13.74	0.10	1.06	1.66E-03	26.22
23.99	14.81	0.20	1.32	2.23E-03	29.35
27.05	16.01	0.30	1.49	2.71E-03	31.48
29.47	17.40	0.40	1.61	3.19E-03	33.18
31.22	18.19	0.45	1.66	3.43E-03	33.80
31.64	19.08	0.50	1.71	3.71E-03	34.62
33.77	21.19	0.60	1.79	4.30E-03	35.87
35.72	24.07	0.70	1.85	5.07E-03	37.03

Second Iteration

α	λ	ϵ	σ_{st}	ϵ_{st}	σ_{rg}	ϵ_{rg}	β
16.77	0.73	1.06E-03	-0.02	-1.46E-06	0.00	-1.66E-07	20.64
24.59	1.16	1.82E-03	-2.51	-2.40E-04	-0.67	-6.39E-05	26.46
28.06	1.36	2.29E-03	-4.84	-4.63E-04	-1.76	-1.69E-04	29.09
30.41	1.49	2.72E-03	-7.45	-7.13E-04	-3.34	-3.19E-04	31.05
32.28	1.60	3.17E-03	-10.51	-1.01E-03	-5.53	-5.30E-04	32.70
33.17	1.65	3.41E-03	-12.23	-1.17E-03	-6.95	-6.65E-04	33.44
33.90	1.69	3.67E-03	-14.22	-1.36E-03	-8.61	-8.24E-04	34.16
35.35	1.77	4.26E-03	-18.89	-1.81E-03	-12.99	-1.24E-03	35.50
36.70	1.84	5.03E-03	-25.14	-2.41E-03	-19.48	-1.87E-03	36.77

Third Iteration

α	λ	ϵ	σ_{st}	ϵ_{st}	σ_{rg}	ϵ_{rg}	β
19.69	0.84	1.22E-03	-0.01	-1.23E-06	0.00	-1.97E-07	20.63
26.00	1.19	1.86E-03	-2.36	-2.26E-04	-0.71	-6.81E-05	26.37
28.84	1.37	2.30E-03	-4.68	-4.49E-04	-1.82	-1.75E-04	29.02
30.89	1.50	2.72E-03	-7.30	-6.99E-04	-3.40	-3.26E-04	30.99
32.60	1.60	3.16E-03	-10.38	-9.94E-04	-5.60	-5.36E-04	32.66
33.37	1.65	3.40E-03	-12.13	-1.16E-03	-7.00	-6.70E-04	33.40
34.10	1.69	3.66E-03	-14.11	-1.35E-03	-8.67	-8.30E-04	34.13
35.46	1.77	4.26E-03	-18.81	-1.80E-03	-13.04	-1.25E-03	35.48
36.75	1.84	5.03E-03	-25.10	-2.40E-03	-19.52	-1.87E-03	36.76

Web Strength Check

Δ	k	α PDT	τ^* all	τ all [ksi]	τ [ksi]	MS < 1
0.20	0.00	35.3	27.48	23.39	12.77	0.55
0.20	0.10	35.7	24.97	21.25	13.74	0.65
0.20	0.20	36.1	23.52	20.01	14.81	0.74
0.20	0.30	36.5	22.51	19.15	16.01	0.84
0.20	0.40	36.9	21.69	18.46	17.40	0.94
0.20	0.45	37.9	21.34	18.16	18.19	1.00
0.20	0.50	37.3	21.02	17.89	19.08	1.07
0.20	0.60	37.8	20.52	17.46	21.19	1.21
0.20	0.70	38.3	20.14	17.14	24.07	1.40

APPENDIX B: SAMPLE DIAGONAL TENSION CALCULATIONS USING NACA VALUES

This example applies the methodology used to analyze the FML cylinders to a cylinder from NACA-TN-2821, in order to compare the theoretical values, to NACA experimental values. The process is used to validate the calculation procedure.

For the example, specimen 9 from the NACA-TN-2821 was chosen. The cylinder had 16 hat section stringers and 4 square tube frames. The analyzed bay had the stringers 5.89 in apart and the frames 12 in apart. The analysis was made for the test load that produced web failure

Cylinder Geometry:

$R := 15.04\text{in}$	<< radius of curvature, in
$t := 0.0315\text{in}$	<< thickness of skin (web), in
$h := 5.89\text{in}$	<< distance between stringers, in
$d := 12\text{in}$	<< distance between frames, in
$T := 720 \cdot 10^3 \cdot \text{in} \cdot \text{lbf}$	<< Torsion applied, in-lbf
$T = 6 \times 10^4 \text{ft} \cdot \text{lbf}$	<< Torsion applied, ft-lbf

Stringer Geometry:

$t_{st} := 0.0315\text{in}$	<< thickness of stringer, in
$A_{st} := 0.0697\text{in}^2$	<< area of stringer, sq. in

Frame Geometry:

$t_{rg} := 0.5\text{in}$	<< thickness of frame, in
$A_{rg} := 0.25\text{in}^2$	<< area of ring, sq. in

Material Properties:

$\mu := 0.32$	<< Poisson's ratio
$G := 4 \cdot 10^6 \text{psi}$	<< shear modulus, psi
$E := 10.4 \cdot 10^6 \text{psi}$	<< Young's modulus, Aluminum, psi
$E_{al} := 10.4 \cdot 10^6 \text{psi}$	

$\Phi := 0.5$ << angle subtended by 2 stringers ~ 0.5 rad
 $\sigma_{cy_2024} := 40000 \text{ psi}$ << compressive yield stress for Al 2024-T3, psi
 $\sigma_{cy_7075} := 67000 \text{ psi}$ << compressive yield stress for Al 7075-T6, psi

MathCAD Conversion Definitions

$\text{cotd}(\alpha) := \cot(\alpha \cdot \text{deg})$ $\text{sind}(\alpha) := \sin(\alpha \cdot \text{deg})$
 $\text{tand}(\alpha) := \tan(\alpha \cdot \text{deg})$ $\text{tanhd}(\beta) := \tanh(\beta \cdot \text{deg})$
 $\text{tand}(\beta) := \tan(\beta \cdot \text{deg})$

B.1 Critical Shear Stress, τ_{cr}

The critical shear stress is the stress at which web buckling first occurs. Based on the assumption of simply supported edges, this value is computed in the elastic range, using the following formula:

$$\tau_{cr_elastic} := \frac{\pi^2 \cdot E \cdot h^2 \cdot k_s}{12 R^2 \cdot Z^2} \quad \text{<< [Kuhn 1952a] Section 10.1, Pg. 78}$$

The equation contains two constants: the curvature parameter, Z and the shear buckling stress coefficient.

The curvature parameter is a function of the radius of curvature, thickness of web, distance between stringers and the Poisson's ratio. The value can be obtained by the following equation:

$$Z := \frac{h^2 \cdot \sqrt{1 - \mu^2}}{R \cdot t} \quad \text{<< [Kuhn 1952a] Fig. 30a, Pg. 125}$$

$$Z = 69.38$$

The shear buckling stress coefficient is a function of the panel dimensions (i.e. aspect ratio d/h) and the curvature parameter Z. The value is obtained by using figure 30 in reference [Kuhn 1952a].

$$\text{with } \frac{d}{h} = 2.037 \times 10^0 \text{ if } \frac{d}{h} \text{ greater than 2 use 2}$$

$$k_s := 25$$

From the above constants, the elastic shear buckling stress for the web is found to be:

$$\tau_{cr_elastic} := \frac{k_s \cdot \pi^2 \cdot E \cdot h^2}{12 R^2 \cdot Z^2}$$

$$\tau_{cr_elastic} = 6.81 \times 10^3 \text{ psi}$$

The critical shear value is corrected for plasticity by using figure 12c in reference [Kuhn 1952a]. Figure 12c was converted to the following best fit equation (Appendix D, Section D.1, Figure D.1.2):

$$\tau_{cr_elastic} := \frac{\tau_{cr_elastic}}{1000 \cdot \text{psi}}$$

Note: converting $\tau_{cr_elastic}$ to ksi to accommodate the units of the best-fit equation

$$A := -2 \cdot 10^{-6}$$

$$B := 0.0005$$

$$C := -0.0393$$

$$D := 1.6209$$

$$F := -1.7267$$

if $\tau_{cr_elastic} > 16 \text{ ksi}$

$$\tau_{cr_ksi} := A \cdot \tau_{cr_elastic}^4 + B \cdot \tau_{cr_elastic}^3 + C \cdot \tau_{cr_elastic}^2 + D \cdot \tau_{cr_elastic} + F$$

if $\tau_{cr_elastic} < 16 \text{ ksi}$

$$\tau_{cr_ksi} := \tau_{cr_elastic}$$

Therefore, the corrected critical shear value is:

$$\tau_{cr_ksi} = 6.814 \times 10^0$$

$$\tau_{cr_psi} := \tau_{cr_ksi} \cdot 1000 \text{ psi}$$

Note: converting τ_{cr} from ksi to psi

$$\tau_{cr} := \tau_{cr_psi}$$

$$\tau_{cr} = 6.81 \times 10^3 \text{ psi}$$

<< τ_{cr} corrected for plasticity

B.2 Nominal Shear Stress, τ

The shear stress due to the applied torque load, T, is calculated as follows:

$$\tau := \frac{T}{2 \cdot \pi \cdot R^2 \cdot \left(1 - \frac{\Phi^2}{6}\right) \cdot t}$$

<< [Kuhn 1952a]
Section 10.2, Pg. 78

$$\tau = 16.78 \times 10^3 \text{ psi}$$

The applied torque, T , at which the web fails was calculated through several iteration cycles. The starting point was at the torque where the web buckles and k , the diagonal tension factor (reference A.4) was zero. This calculation was continued through increments of 10% for k until the web failed. The sample calculation was based on this failing torque.

B.3 Loading Ratio

Using the nominal shear stress and the critical shear stress the loading ratio is calculated as follows:

$$\frac{\tau}{\tau_{cr}} = 2.46$$

B.4 Diagonal-Tension Factor, k

The diagonal tension factor gives the percentage of complete diagonal tension in the web. For example $k = 0$ for initial buckling of the web and $k = 1$ for complete diagonal tension. The loading ratio, panel geometry, and the radius of curvature is used to calculate the diagonal tension factor as follows:

$$k := \tanh \left[\left(0.5 + 300 \cdot \frac{d}{h} \cdot \frac{t}{R} \right) \cdot \log \left(\frac{\tau}{\tau_{cr}} \right) \right] \quad \ll \text{[Kuhn 1952a]} \\ \text{Pg. 69, Eqn. 50}$$

$$k = 0.6$$

B.5 Diagonal Tension Angle, α

Diagonal tension angle is the angle from the neutral axis and defines the angle of the "web buckles". This value depends on the distance between stringers, radius of curvature and the strains of the web, stringer and frame as follows.

Note: the equation is valid only if $h < \frac{1}{3} \cdot R$

$$(\tan \alpha)^2 := \frac{(\varepsilon - \varepsilon_{st})}{\varepsilon - \varepsilon_{rg} + \frac{1}{24} \cdot \left(\frac{h}{R} \right)^2} \quad \ll \text{[Kuhn 1952a]} \\ \text{Pg. 64, Eqn. 44}$$

The stringer and frame strains also depends on the diagonal tension angle. Therefore, the angle is obtained through an iterative process.

B.6.1 Pure Diagonal Tension Angle, α_{PDT}

NACA method suggests using the value of the pure diagonal tension, α_{PDT} as a first approximation to the diagonal tension angle. It is obtained from figure 28a in reference [Kuhn 1952a]. The figure requires the following constants:

$$R_r := \frac{d \cdot t}{A_{rg}}$$

$$R_r = 1.51$$

$$R_s := \frac{h \cdot t}{A_{st}}$$

$$R_s = 2.66$$

<< [Kuhn 1952a]
Pg. 65, Eqn. 44a
Pg. 124, Fig. 28a

$$\frac{1 + R_s}{1 + R_r} = 1.46$$

$$\frac{\frac{h}{R} \cdot \sqrt{\frac{E}{\tau}}}{\sqrt{1 + R_r}} = 6.15$$

*For ease of determination
this Figure was reproduced.
(Appendix D, Section D.2,
Figure D.2.2)

$$\alpha_{PDT} := 40.2$$

The first approximation can be further improved by correcting the pure diagonal tension angle by using figure 32 in reference [Kuhn 1952a]. This curve represents the average of

the scatter band obtained by plotting the ratio $\frac{\alpha}{\alpha_{PDT}}$ for a number of webs with a variety of proportions.

The curve in figure 32 was converted to the following best fit equation (Appendix D, Section D.3, Figure D.3.2):

$$\text{corr} := 3.124 \cdot k^5 - (9.7762 \cdot k^4) + 11.737 \cdot k^3 - (7.0488 \cdot k^2) + 2.6315 \cdot k + 0.3408$$

$$\text{corr} = 0.89$$

The first approximation for the diagonal tension angle is obtained as follows:

$$\text{corr} = 0.89$$

$$\alpha := \text{corr} \cdot \alpha_{PDT}$$

$$\alpha = 35.9 \text{ deg}$$

First Iteration Cycle

B.6.2 Stringer Stress and Strain

The first approximation for the diagonal tension angle is used to calculate the average stress in the stringer as follows:

$$\sigma_{st} := \frac{(-k \cdot \tau) \cdot \cotd(\alpha)}{\left[\left(\frac{A_{st}}{h \cdot t} \right) + 0.5 \cdot (1 - k) \right]}$$

$$\sigma_{st} = -24.26 \times 10^3 \text{ psi}$$

<< [Kuhn 1952a]
Pg. 69, Eqn 51

Note: negative sign indicates
compression

The stringer strain is related to the stringer stress as follows:

$$\epsilon_{st} := \frac{\sigma_{st}}{E_{al}}$$

$$\epsilon_{st} = -2.33 \times 10^{-3}$$

<< [Kuhn 1952a]
Pg. 69, Eqn 51

B.6.3 Frame Stress and Strain

The average stress through the floating frame is found using the following equation:

$$\sigma_{rg} := \frac{-k \cdot \tau \cdot \tand(\alpha)}{\left(\frac{A_{rg}}{d \cdot t} \right) + 0.5 \cdot (1 - k)}$$

$$\sigma_{rg} = -8.52 \times 10^3 \text{ psi}$$

<< [Kuhn 1952a]
Pg. 69, Eqn 52

<< NACA's example used
Notched Zed section
frames. This adds an
extra term to the
denominator.
i.e. $0.5(1-k)$

The frame strain is related to the frame stress as follows:

$$\epsilon_{rg} := \frac{\sigma_{rg}}{E_{al}}$$

$$\epsilon_{rg} = -8.19 \times 10^{-4}$$

<< [Kuhn 1952a]
Pg. 69, Eqn 52

B.6.4 Web Strain

The web strain is determined by the following equation:

$$\varepsilon := \frac{(\lambda \cdot \tau)}{E}$$

<< [Kuhn 1952a]
Pg. 20, Eqn 30d

where λ is a function of the diagonal tension factor, Poisson's ratio and the diagonal tension angle using the following equation:

$$\lambda := \frac{2 \cdot k}{\sin(2 \cdot \alpha)} + \sin(2 \cdot \alpha) \cdot (1 - k) \cdot (1 + \mu)$$

$$\lambda = 1.77$$

The web strain is determined to be:

$$\varepsilon := \frac{\tau \cdot \lambda}{E}$$

$$\varepsilon = 2.85 \times 10^{-3}$$

The stringer, frame and web strains are used to recalculate the diagonal tension angle.

$$\alpha := \beta$$

$$\beta := \text{atan} \left[\sqrt{\frac{(\varepsilon - \varepsilon_{st})}{\varepsilon - \varepsilon_{rg} + \frac{1}{24} \cdot \left(\frac{h}{R}\right)^2}} \right]$$

<< [Kuhn 1952a]
Pg. 64, Eqn. 44
solved for α .

$$\beta = 35.7 \text{ deg}$$

The computed value and the initial approximation has not converged. Therefore, the iteration is continued.

Second Iteration Cycle

For next iteration, the diagonal tension angle is approximated as:

$$\alpha := \text{atan} \left[\tan(\beta) + \frac{1}{4} \cdot (\tan(\alpha) - \tan(\beta)) \right]$$

$$\alpha = 35.7 \text{ deg}$$

Web strain is recalculated using the second approximation of α :

$$\lambda := \frac{2 \cdot k}{\sin(2 \cdot \alpha)} + \sin(2 \cdot \alpha) \cdot (1 - k) \cdot (1 + \mu)$$

$$\lambda = 1.77$$

$$\varepsilon := \frac{(\lambda \cdot \tau)}{E}$$

$$\varepsilon = 2.85 \times 10^{-3}$$

Stringer stress and strain recalculated using the second approximation for α :

$$\sigma_{st} := \frac{(-k \cdot \tau) \cdot \cot(\alpha)}{\left[\left(\frac{A_{st}}{h \cdot t} \right) + 0.5 \cdot (1 - k) \right]}$$

$$\sigma_{st} = -24.45 \times 10^3 \text{ psi}$$

$$\varepsilon_{st} := \frac{\sigma_{st}}{E_{al}}$$

$$\varepsilon_{st} = -2.35 \times 10^{-3}$$

Frame stress and strain recalculated using the second approximation for α :

$$\sigma_{rg} := \frac{-k \cdot \tau \cdot \tan(\alpha)}{\left(\frac{A_{rg}}{d \cdot t} \right) + 0.5 \cdot (1 - k)}$$

$$\sigma_{rg} = -8.46 \times 10^3 \text{ psi}$$

$$\epsilon_{rg} := \frac{\sigma_{rg}}{Ea}$$

$$\epsilon_{rg} = -8.13 \times 10^{-4}$$

The stringer, frame and web strains are used to recalculate the diagonal tension angle.

$$\alpha := \beta$$

$$\beta := \text{atan} \left[\frac{(\epsilon - \epsilon_{st})}{\sqrt{\epsilon - \epsilon_{rg} + \frac{1}{24} \cdot \left(\frac{h}{R} \right)^2}} \right]$$

$$\beta = 35.7 \text{ deg}$$

$$\alpha := \beta$$

The computed value of the diagonal tension angle agrees with the first cycle value. Therefore, further iteration is not required.

The final value of α is used to calculate the web strain, stringer stress and strain and frame stress and strain:

$$\lambda := \frac{2 \cdot k}{\sin(2 \cdot \alpha)} + \sin(2 \cdot \alpha) \cdot (1 - k) \cdot (1 + \mu)$$

$$\lambda = 1.77$$

$$\epsilon := \frac{(\lambda \cdot \tau)}{E}$$

$$\epsilon = 2.85 \times 10^{-3}$$

$$\sigma_{st} := \frac{(-k \cdot \tau) \cdot \cot(\alpha)}{\left[\left(\frac{A_{st}}{h \cdot t} \right) + 0.5 \cdot (1 - k) \right]}$$

$$\sigma_{st} = -24.46 \times 10^3 \text{ psi}$$

$$\epsilon_{st} := \frac{\sigma_{st}}{E_{al}}$$

$$\epsilon_{st} = -2.35 \times 10^{-3}$$

$$\sigma_{rg} := \frac{-k \cdot \tau \cdot \tan(\alpha)}{\left(\frac{Arg}{d \cdot t}\right) + .5 \cdot (1 - k)}$$

$$\sigma_{rg} = -8.5 \times 10^3 \text{ psi}$$

$$\epsilon_{rg} := \frac{\sigma_{rg}}{E_{al}}$$

$$\epsilon_{rg} = -8.13 \times 10^{-4}$$

B.7 Allowable Shear Stress, τ_{all}

The allowable ultimate value for the shear stress in a curved web is given by the following empirical formula:

$$\tau_{all} := \tau_{all} \cdot (0.65 + \Delta)$$

<< [Kuhn 1952a]
Pg. 72, Eqn. 53

The value of τ_{all} is depended on the diagonal tension factor and the diagonal tension angle and is determined by Figure 19 [Kuhn 1952a]. For ease of determination figure 19 was reproduced and is shown in **Appendix D , Section D.4, Figure D.4.2** and converted to the following best-fit equation:

$$\tau_{all} := (89188 \cdot k^6 - 306294 \cdot k^5 + 414627 \cdot k^4 - 286399 \cdot k^3 + 114882 \cdot k^2 - 34262 \cdot k + 27500) \cdot \text{psi}$$

$$\tau_{all} = 20.51 \times 10^3 \text{ psi}$$

$$\text{with } k = 0.6 \quad \text{and} \quad \alpha_{PDT} = 40.2 \quad \tau_{all} = 20.51 \times 10^3 \text{ psi}$$

The value of Δ is obtained by the following equation:

$$\Delta := \tanh\left(\frac{Arg}{d \cdot t}\right) \cdot 0.3 + \tanh\left(\frac{Ast}{h \cdot t}\right) \cdot 0.1$$

<< [Kuhn 1952a]
Pg. 72, Eqn. 54

$$\Delta = 0.21$$

The allowable shear stress is:

$$\tau_{all} := \tau_{all} \cdot (0.65 + \Delta)$$

$$\tau_{all} = 17.63 \times 10^3 \text{ psi}$$

A check is now made to see if the web has failed:

$$MS := \frac{\tau}{\tau_{all}}$$

$$MS = 0.95$$

The web is close to failure.

B.8 Strength Checks for Frames

NACA [Kuhn 1952a] suggests checking stringers against forced crippling.

Forced Crippling

Forced crippling is a localized buckling failure of a stiffener that is connected directly to the web. The strength against forced crippling is checked in the same manner as for uprights in the flat panels. The maximum frame stress, σ_{rg_max} , and the allowable force crippling limit σ_{o_fc} must be determined.

The **maximum frame stress**, σ_{rg_max} is a function of the diagonal tension factor, k , and the the ratio d/h . The value is determined by computing the σ_{max}/s ratio using figure 15 in reference[Kuhn 1952a]. The lines in Figure 15 was converted to a best fit equation and used as below. (**Appendix D, Section D.5, Figure E.5.2**).

$$\text{With } \frac{h}{d} = 0.49 \quad \text{and} \quad k = 6.023 \times 10^{-1}$$

$$\frac{\sigma_{max}}{\sigma} := \text{ratio}$$

<< [Kuhn 1952a]
Pg. 80, Section 10.8

$$\text{ratio} := 1 + (1 - k) \cdot \left(0.78 - .64 \cdot \frac{h}{d} \right)$$

<< **Appendix D**
Section D.5

$$\text{ratio} = 1.19$$

$$\sigma_{rg_max} := \sigma_{rg} \cdot \text{ratio}$$

$$\sigma_{rg_max} = -10.02 \times 10^3 \text{ psi}$$

The **allowable force crippling limit** for aluminum 2024-T3 (NACA symbol: 24S-T3) alloy and a 7075-T6 (NACA symbol: 75S-T6) alloy is defined by NACA [Kuhn 1952a] as follows:

for a 2024-T3 (NACA designation - 24S-T3) alloy, single upright

$$\sigma_{o_fc} := -26 \cdot 10^3 \text{ psi} \cdot k^{\frac{2}{3}} \cdot \left(\frac{trg}{t} \right)^{\frac{1}{3}}$$

<< [Kuhn 1952a]
Pg. 47, Eqn. 37a

$$\sigma_{o_fc} = -46.6 \times 10^3 \text{ psi}$$

Margin of safety:

$$MS := \frac{\sigma_{rg_max}}{\sigma_{o_fc}}$$

$$MS = 0.21$$

Frame within limits of safety.

for a 7075-T6 (NACA designation - 75S-T6) alloy, single upright

$$\sigma_{o_fc} := -32.5 \cdot 10^3 \text{ psi} \cdot k^{\frac{2}{3}} \cdot \left(\frac{tst}{t} \right)^{\frac{1}{3}}$$

<< [Kuhn 1952a]
Pg. 47, Eqn. 37b

$$\sigma_{o_fc} = -23.18 \times 10^3 \text{ psi}$$

Margin of safety

$$MS := \frac{\sigma_{rg_max}}{\sigma_{o_fc}}$$

$$MS = 0.43$$

APPENDIX C: SAMPLE CALCULATIONS FOR AL AND FML CYLINDER FASTENER LOADS

This example calculates the fastener loads for connections between:

- Aluminum ($t \sim 0.055$) cylinder to stringers and frames
- GLARE 4 3/2 ($t \sim 0.065$) cylinder to stringers and frames
- Steel Load Interface (Flange) to Aluminum cylinder
- Steel Load Interface (Flange) to GLARE cylinder

C.1. Rivet calculations for stringer and frame connections for Al Cylinder

$$\tau := 19856 \frac{\text{lb}}{\text{in}^2} \quad \ll \text{Shear stress}$$

$$\alpha := 36.2 \text{deg} \quad \ll \text{Diagonal tension angle}$$

$$k := 0.5 \quad \ll \text{Diagonal tension factor}$$

$$F_{bry_Al} := 124000 \frac{\text{lb}}{\text{in}^2} \quad \ll \text{Bearing strength for AL 2024-T3}$$

$$F_{tu_Al} := 70300 \frac{\text{lb}}{\text{in}^2} \quad \ll \text{Ultimate tensile strength for AL 2024-T3}$$

$$k_{bry} := 1.24 \quad \ll \text{Material factor for AL 2024-T3}$$

$$t := 0.055 \text{in} \quad \ll \text{Web thickness}$$

$$D := \frac{5}{32} \text{ in}$$

<< Rivet diameter

$$d_{\text{pitch_min}} := 0.5625 \text{ in}$$

<< Min. pitch distance for $\frac{5}{32}$ in rivet

$$d_{\text{st}} := 36 \text{ in}$$

<< Length of stringer

$$d_{\text{fr}} := 30 \cdot \pi \text{ in}$$

<< Diameter of frame

$$L_{\text{stringer}} := \tau \cdot t \cdot \left[1 + k \cdot \left(\frac{1}{\cos(\alpha)} - 1 \right) \right]$$

<< [Bruhn 1973]
Pg. C11.36
Eqn. 96

$$L_{\text{stringer}} = 1.223 \times 10^3 \frac{\text{lb}}{\text{in}}$$

$$L_{\text{frame}} := \tau \cdot t \cdot \left[1 + k \cdot \left(\frac{1}{\sin(\alpha)} - 1 \right) \right]$$

<< [Bruhn 1973]
Pg. C11.36
Eqn. 97

$$L_{\text{frame}} = 1.471 \times 10^3 \frac{\text{lb}}{\text{in}}$$

$$e := 2 \cdot D$$

$$e = 0.313 \text{ in}$$

$$\frac{e}{D} = 2$$

$$P_{\text{st}} := L_{\text{stringer}} \cdot d_{\text{pitch_min}}$$

$$P_{\text{fr}} := L_{\text{frame}} \cdot d_{\text{pitch_min}}$$

$$P_{\text{st}} = 687.77 \text{ lb}$$

$$P_{\text{fr}} = 827.203 \text{ lb}$$

$$N_{\text{riv_st}} := \frac{d_{\text{st}}}{d_{\text{pitch_min}}}$$

$$N_{\text{riv_fr}} := \frac{d_{\text{fr}}}{d_{\text{pitch_min}}}$$

$$N_{\text{riv_st}} = 64$$

$$N_{\text{riv_fr}} = 167.55$$

Bearing Loads

$$P_{u_br} := k_{bry} \cdot F_{bry_Al} \cdot D \cdot t$$

<< [Bruhn 1973]

$$P_{u_br} = 1.321 \times 10^3 \text{ lb}$$

Pg. D1.6

Eqn. 5

$$MS_{st} := \frac{P_{u_br}}{P_{st}} - 1$$

$$MS_{fr} := \frac{P_{u_br}}{P_{fr}} - 1$$

$$MS_{st} = 0.92$$

$$MS_{fr} = 0.6$$

Prying Loads

$$P_{tu} := 0.22 \cdot t \cdot F_{tu_Al} \cdot d_{pitch_min}$$

<< [Bruhn 1973]

$$P_{tu} = 478.479 \text{ lb}$$

Pg. C11.36

Eqn. 98

$$P_t := (d_{pitch_min} - D) \cdot t \cdot F_{tu_Al}$$

$$P_t = 1.571 \times 10^3 \text{ lb}$$

$$MS_{tu} := \frac{P_t}{P_{tu}} - 1$$

$$MS_{tu} = 2.28$$

C.1.1 Rivet calculations for stringer and frame connections for GLARE

$$\tau := 18191.71 \frac{\text{lb}}{\text{in}^2}$$

<<Shear stress

$$\alpha := 37.9 \text{ deg}$$

<<Diagonal tension angle

$$k := 0.45$$

<<Diagonal tension factor

$$F_{bry_g} := 90000 \frac{\text{lb}}{\text{in}^2}$$

<<Bearing strength for GLARE 4 3/2

$$F_{tu_g4} := 148953.8 \frac{\text{lb}}{\text{in}^2}$$

<<Ultimate tensile strength for GLARE 4 3/2

$$kbry := 1.24$$

<<Material factor for AL 2024-T3

$$t := 0.065\text{in}$$

<< Web thickness

$$D := \frac{5}{32}\text{in}$$

<< Rivet diameter

$$d_pitch_min := 0.5625\text{in}$$

<< Min. pitch distance for $\frac{5}{32}$ in rivet

$$d_st := 36\text{in}$$

<< Length of stringer

$$d_fr := 30 \cdot \pi\text{in}$$

<< Diameter of frame

$$L_stringer := \tau \cdot t \cdot \left[1 + k \cdot \left(\frac{1}{\cos(\alpha)} - 1 \right) \right]$$

<< [Bruhn 1973]
Pg. C11.36
Eqn. 96

$$L_stringer = 1.325 \times 10^3 \frac{\text{lb}}{\text{in}}$$

$$L_frame := \tau \cdot t \cdot \left[1 + k \cdot \left(\frac{1}{\sin(\alpha)} - 1 \right) \right]$$

<< [Bruhn 1973]
Pg. C11.36
Eqn. 97

$$L_frame = 1.517 \times 10^3 \frac{\text{lb}}{\text{in}}$$

$$e := 2 \cdot D$$

$$e = 0.313\text{in}$$

$$\frac{e}{D} = 2$$

$$P_st := L_stringer \cdot d_pitch_min$$

$$P_fr := L_frame \cdot d_pitch_min$$

$$P_st = 745.14\text{lb}$$

$$P_fr = 853.074\text{lb}$$

$$N_riv_st := \frac{d_st}{d_pitch_min}$$

$$N_riv_fr := \frac{d_fr}{d_pitch_min}$$

$$N_riv_st = 64$$

$$N_riv_fr = 167.55$$

Bearing Loads

$$P_{u_br} := k_{bry} \cdot F_{bry_g} \cdot D \cdot t$$

$$P_{u_br} = 1.133 \times 10^3 \text{ lb}$$

$$MS_{st} := \frac{P_{u_br}}{P_{st}} - 1$$

$$MS_{st} = 0.52$$

$$MS_{fr} := \frac{P_{u_br}}{P_{fr}} - 1$$

$$MS_{fr} = 0.33$$

<< [Bruhn 1973]
Pg. D1.6
Eqn. 5

Prying Loads

$$P_{tu} := 0.22 \cdot t \cdot F_{tu_g4} \cdot d_{pitch_min}$$

$$P_{tu} = 1.198 \times 10^3 \text{ lb}$$

$$P_t := (d_{pitch_min} - D) \cdot t \cdot F_{tu_g4}$$

$$P_t = 3.933 \times 10^3 \text{ lb}$$

$$MS_{tu} := \frac{P_t}{P_{tu}} - 1$$

$$MS_{tu} = 2.28$$

<< [Bruhn 1973]
Pg. C11.36
Eqn. 98

C.2. Cylinder to load interface calculations for Al cylinder

Connection between the flange and the Aluminum cylinder

$$Rf_out := 15\text{in}$$

<< Flange outer radius

$$Rf_in := 14.5\text{in}$$

<< Flange inner radius

$$D := \frac{1}{4}\text{in}$$

<< Diameter of bolt

$$Fsu := 40600 \frac{\text{lb}}{\text{in}^2}$$

<< Ultimate shearing strength for GLARE 4 3/2

$$tf := 0.5\text{in}$$

<< Flange thickness

$$\tau := 18191.71 \cdot \frac{\text{lb}}{\text{in}^2}$$

<< Torque at web failure

$$t := 0.055\text{in}$$

<< Web thickness

$$e := 2.25 \cdot D$$

<< edge distance

$$e = 0.56\text{in}$$

$$P := 4 \cdot D$$

<< Pitch distance

$$P = 1\text{in}$$

$$L_b_tot := \tau \cdot 2\text{in} \cdot (t + tf)$$

<< Total load on bolts

$$L_b_tot = 2.02 \times 10^4 \text{lb}$$

$$L_b := \tau \cdot \pi \cdot \frac{D^2}{4}$$

<< Load per bolt

$$L_b = 892.98\text{lb}$$

$$N_b := \frac{L_b_tot}{L_b}$$

<< Number of bolts

$$N_b = 22.613$$

Failure by Shear Tear Out:

$$A_s := 2 \cdot \left(e - \frac{D}{2} \cdot \cos \left(40 \cdot \frac{\pi}{180} \right) \right) \cdot t$$

<< Shear Area

$$A_s = 0.05 \text{ in}^2$$

$$P_{u_sh} := F_{su} \cdot A_s$$

<< [Bruhn 1973]

Pg. D1.6

Eqn. 3

$$P_{u_sh} = 2.08 \times 10^3 \text{ lb}$$

Margin of Safety:

$$MS := \frac{P_{u_sh}}{L_b} - 1$$

$$MS = 1.33$$

Bearing Strength

$$P_{u_bry} := F_{bry_Al} \cdot t \cdot D$$

<< [Bruhn 1973]

Pg. D1.6

Eqn. 5

$$P_{u_bry} = 1.7 \times 10^3 \text{ lb}$$

Margin of Safety:

$$MS := \frac{P_{u_bry}}{L_b} - 1$$

$$MS = 0.91$$

C.2.1 Cylinder to load interface calculations for FML cylinder

Connection between the flange and the GLARE cylinder

$$R_{f_out} := 15 \text{ in}$$

<< Flange outer radius

$$R_{f_in} := 14.5 \text{ in}$$

<< Flange inner radius

$$D := \frac{1}{4} \text{ in}$$

<< Diameter of bolt

$$F_{su} := 40600 \frac{\text{lb}}{\text{in}^2}$$

<< Ultimate shearing strength for GLARE 4 3/2

$$t_f := 0.5 \text{ in}$$

<< Flange thickness

$$\tau := 18191.71 \cdot \frac{\text{lb}}{\text{in}^2}$$

<< Torque at web failure

$$t := 0.055 \text{ in}$$

<< Web thickness

$$e := 2.25 \cdot D$$

<< edge distance

$$e = 0.56 \text{ in}$$

$$P := 4 \cdot D$$

<< Pitch distance

$$P = 1 \text{ in}$$

$$L_{b_tot} := \tau \cdot 2 \text{ in} \cdot (t + t_f)$$

<< Total load on bolts

$$L_{b_tot} = 2.02 \times 10^4 \text{ lb}$$

$$L_b := \tau \cdot \pi \cdot \frac{D^2}{4}$$

<< Load per bolt

$$L_b = 892.98 \text{ lb}$$

$$N_b := \frac{L_{b_tot}}{L_b}$$

<< Number of bolts

$$N_b = 22.613$$

Failure by Shear Tear Out:

$$A_s := 2 \cdot \left(e - \frac{D}{2} \cdot \cos \left(40 \cdot \frac{\pi}{180} \right) \right) \cdot t$$

<< Shear Area

$$A_s = 0.05 \text{ in}^2$$

$$P_{u_sh} := F_{su} \cdot A_s$$

<< [Bruhn 1973]

Pg. D1.6

$$P_{u_sh} = 2.08 \times 10^3 \text{ lb}$$

Eqn. 3

Margin of Safety:

$$MS := \frac{Pu_{sh}}{L_b} - 1$$

$$MS = 1.33$$

Bearing Strength

$$Pu_{bry} := F_{bry_Al} \cdot t \cdot D$$

$$Pu_{bry} = 1.7 \times 10^3 \text{ lb}$$

<< [Bruhn 1973]

Pg. D1.6

Eqn. 5

Margin of Safety:

$$MS := \frac{Pu_{bry}}{L_b} - 1$$

$$MS = 0.91$$

C.3. Calculation Method for Lap Joint:

$$\begin{aligned}\tau_{\text{per area}} &= \text{shear} && [\text{lb in}^2] \\ \tau &= \text{shear on splice area} && [\text{lb}] \\ A &= \text{effective area} && [\text{in}^2] \\ t &= \text{thickness} && [\text{in}] \\ L &= \text{cylinder length} && [\text{in}]\end{aligned}$$

$$\tau = \frac{T}{2 A t}$$

$$\begin{aligned}A &= \pi r_{\text{cl}}^2 \\ &= \pi (0.0275 + 15) \\ &= 710.22 [\text{in}^2]\end{aligned}$$

$$T = 165000 \text{ ft lb} = 1\,980\,000 [\text{in lb}]$$

$$\begin{aligned}\tau_{\text{per area}} &= \frac{T}{2 A t} \\ &= \frac{1\,980\,000}{2(710.22)(0.055)} \\ &= 25\,344.26 [\text{lb in}^2]\end{aligned}$$

Shear acting on splice area

$$\begin{aligned}\tau &= L t \tau_{\text{per area}} \\ &= 36(0.055)(25\,344.26) \\ &= 50\,181.63 \text{ lb}\end{aligned}$$

C.3.1 Safety check by the required number of Rivets

of Rivets to prevent shear failure:

$$\begin{aligned}
 \# &= \frac{\tau_{\text{splice}}}{\text{shear limit}} \\
 &= \frac{2 \times 50181.63}{596} \\
 &= 168.2 \text{ rivets} \\
 &\approx \text{minimum 163 rivets required}
 \end{aligned}$$

of Rivets to prevent bearing failure:

$$\begin{aligned}
 \# &= \frac{\tau_{\text{splice}}}{\text{bearing limit}} \\
 &= \frac{2 \times 50181.63}{481} \\
 &= 206.6 \text{ rivets} \\
 &\approx \text{minimum 207 rivets required}
 \end{aligned}$$

APPENDIX E: EMPIRICAL DATA CURVE FITTING

D.1: Plasticity Correction for 2024-T3 Aluminum

τ_{elastic}	τ_{cr}
0.00	0.00
5	5.00
10	10.00
15	15.00
15.83	15.83
20.0	18.33
23.33	20.00
30	22.50
35	23.75
42.08	25.00
45	25.42
50	26.25
55	26.67

Figure D.1.1: Plasticity Correction for 2024-T3 Aluminum graph [Kuhn 1952a] and selected data points for best-fit curve

Figure D.1.2: Best -fit curve for the 2024-T3 Aluminum Plasticity Correction

D.2: Angle of Pure Diagonal Tension

Figure D.2.1: Angle of Pure Diagonal Tension [Kuhn 1952a] and selected data points used for best-fit curve

Figure D.2.2: Best -fit curves for the Angle of Pure Diagonal Tension

D.3: Correction Factor for Angle of Diagonal Tension

k	α/α_{PDT}
0.02	0.40
0.10	0.54
0.24	0.70
0.4	0.80
0.5	0.85
0.6	0.89
0.9	0.99
1	1.01

Figure D.3.1: Correction Factor for Angle of Pure Diagonal Tension [Kuhn 1952a] and selected data points used for best-fit curve.

Figure D.3.2: Correction Factor for Angle of Pure Diagonal Tension

D.4: Basic Allowable Values of $\tau_{\max}$ for 24S-T3 Aluminum Alloy

	k	τ_{all}^*
$\alpha = 20$	0.3	21.1
	1.0	13.95
$\alpha = 25$	0.3	22
	1.0	16.5
$\alpha = 30$	0.3	22.5
	1.0	18.7
$\alpha = 35$	0.3	23
	1.0	20.25
$\alpha = 40$	0.3	23.2
	1.0	21.25
$\alpha = 45$	0.3	23.5
	1.0	21.5

Figure D.4.1: Basic Allowable Values of $\tau_{\max}$ [Kuhn 1952a] and selected data points used for best-fit curve.

Figure D.4.2: Best fit curves for Basic Allowable Values of $\tau_{\max}$

D.5: Ratio of Maximum Stress to Average Stress in Web Stiffeners

k	d/h	$\sigma_{u\max}/\sigma_u$
0	0.0	1.78
	0.4	1.52
0.2	0.0	1.62
	0.4	1.42
0.4	0.0	1.46
	0.4	1.32
0.6	0.0	1.32
	0.4	1.21
0.8	0.0	1.16
	0.4	1.11

Figure D.5.1: Ratio of Maximum Stress to Average Stress in Web Stiffeners graph [Kuhn 1952a] and selected data points used for best-fit curve.

Figure D.5.2: Best -fit curves for the Ratio of Maximum Stress to Average Stress in Web Stiffeners

D.6: Empirical Criterion for General Instability Failures of Stiffened 24S-T3 Aluminum Alloy Cylinders Subjected to Torsion

X-axis	Y-axis
0.00	1
0.50	2
2.00	7
3.00	10
4.50	14
6.50	16
7.50	20

Figure D.6.1: Graph of General Instability Failures of Stiffened 24S-T3 Aluminium Alloy Cylinders subjected to Pure Torsion [Kuhn 1952a] and selected data

Figure D.6.2: Best-Fit Curve for Graph of General Instability Failures of Stiffened 24S-T3 Aluminium Alloy Cylinders subjected to Pure Torsion