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**FURTHER DEVELOPMENTS IN THE MODELLING OF THE
PERFORMANCE OF PNEUMATIC TIRES OVER DEFORMABLE TERRAIN**

by

John A. Jastremski, B.Eng.

**A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of
Master of Engineering**

**Ottawa-Carleton Institute
for Mechanical and Aerospace Engineering
Department of Mechanical and Aerospace Engineering
Carleton University
Ottawa, Ontario
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**"FURTHER DEVELOPMENTS IN THE MODELLING OF THE PERFORMANCE
OF PNEUMATIC TIRES OVER DEFORMABLE TERRAIN"**

submitted by John A. Jastremski
in partial fulfilment of the requirements for
the degree of Master of Engineering

Thesis Supervisor

Chair, Department of Mechanical
and Aerospace Engineering

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ABSTRACT

Tire-terrain interaction is one of the main topics in off road vehicle mobility studies. The performance of a tire is usually characterized by its thrust, motion resistance, sinkage, and slip. Accurate predictions of these parameters are of interest to all designers and users of off road vehicles.

A model of tire-terrain interaction based on M.G. Bekker's approach is developed in this thesis. The tire model is constructed using physical tire dimensions and information contained in the so called generalized deflection chart readily available from tire manufacturers. The terrain is represented by well known semi-empirical equations. To improve the predictions of tire performance an approach in which the normal pressure distribution is a function of slip is incorporated into the model. The following procedure is used:

- An approach based on the nominal pressure beneath the tire is used to determine if the tire is operating in the rigid or elastic mode.
- If the tire is found to be operating in the rigid mode then the tire is modelled as a rigid rim.
- If the tire is found to be operating in the elastic mode then the tire is represented by a rigid rim with a larger diameter (new effective diameter). A procedure is developed to determine the value of the new effective diameter.

The predictions based on the new model are compared with existing models and with data obtained in a laboratory soil bin using full size tires. Predictions using the new model agree much better with experimental data than those obtained using previous models.

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List of Symbols

a_d	tire deflection
A_c	projected tire-terrain contact area
b	smallest dimension in tire-terrain contact patch
B_t	tire tread width
c	cohesion of soil
C_1	modified pressure distribution parameter
C_2	modified pressure distribution parameter
DBP	drawbar pull
F_c	cable force on forced-slip tire tester
F_t	towing force on forced-slip tire tester
i	slip
j	shear displacement
K	shear deformation modulus
K_c	cohesive modulus of terrain deformation
K_ϕ	frictional modulus of terrain deformation
n	exponent of terrain deformation
N_L	normal load
N_1	lug area / carcass area ratio

P	normal pressure
P_c	pressure at carcass
P_g	ground pressure
P_L	pressure at lug tips
r	tire radius
R	new effective radius
R_m	motion resistance
R_1	radius of slip pulley
R_2	rolling radius of a tire
T	thrust
V_j	shear velocity
W	vertical reaction of the terrain
Z	sinkage
Θ_1	initial contact angle
Θ_M	angle corresponding to maximum normal pressure
σ	normal pressure beneath wheel
τ	shear stress
ϕ	angle of shearing resistance
ω	angular velocity of wheel
ϵ	overall error in convergence criterion

Chapter I

Introduction

Since its invention nearly 5000 years ago, the wheel has played a major role in off-road transport. One of the fundamental problems in off-road vehicle mobility studies is the mechanics of wheel-soil interaction. In the mid 1950's M.G. Bekker established useful principles of land locomotion, many of which are still in wide use today. This particular area of study has become known as terramechanics.

Tire-terrain interaction is a very complex problem. One of the main objectives, in attempting to solve this problem, is to develop a relationship and understanding between design parameters and the performance of a tire on a given terrain. Once this has been achieved, a solid foundation will be available which will enable designers to select optimum running gear for specific operational requirements. The performance of a driven wheel is usually characterized by its thrust, motion resistance, sinkage, slip, input torque, and angular speed. Accurate predictions of these parameters are of interest to all designers and users of off road vehicles.

In the following sections existing tire-terrain models will be reviewed and

evaluated. As a result of the evaluation, a new tire-terrain interaction model is proposed to improve the prediction of tire performance over deformable terrain. A new procedure is developed to determine the operating mode (rigid or elastic) of a pneumatic tire. A new scheme for representing a tire in the elastic operating mode is proposed. In the new model, the normal pressure distribution is also considered as a function of slip. The predictions based on the new model will be compared with experimental data obtained in a laboratory soil bin using full size tires. It will be shown that performance predictions based on the new model agree much better with experimental data than those obtained using existing models.

1.1 Review of tire-terrain modelling

In most tire-terrain models, tire performance predictions are based on the normal pressure and shear stress distributions on the tire-terrain interface. Over the years many tire-terrain interaction models have been developed [5,6,17,18,21]. In 1993 Song-Tao Chen, as part of his Ph.D. thesis [17], provided a comprehensive review of the state of the art in the study of tire-terrain interaction. The following sections will therefore only outline the main features of some of the better known models.

1.1.1 The Bekker Model

In the early 1950's M.G. Bekker proposed several tire models. Since in many cases a pneumatic tire operating over a soft deformable surface behaves like a rigid rim, one of his first approaches started with an analysis of a rigid tire over a deformable

terrain. He assumed that the normal pressure acting on the wheel rim was equal to the pressure under a plate at the same depth. Based on this assumption he developed a method for predicting the motion resistance and sinkage of a wheel. The following equation was developed to represent the pressure (P) under a plate of width (b), at a sinkage (Z).

$$P = (K_c / b + K_\phi) Z^n \quad (1.1)$$

K_c , K_ϕ , and n are terrain parameters. Equation 1.1 was then used to predict the normal pressure exerted on the wheel by the terrain. The normal pressure distribution curve starts at zero where the wheel enters the terrain, and reaches its maximum at the bottom dead centre of the wheel. After incorporating equation 1.1 into the following equations, which describe the equilibrium of a towed wheel (Figure 1.1),

$$R_m = b \int_0^{\theta_1} (P) r \sin \theta d\theta \quad (1.2)$$

$$W = b \int_0^{\theta_1} (P) r \cos \theta d\theta \quad (1.3)$$

expressions for motion resistance (R_m) and sinkage (Z) were proposed [2].

$$R_m = b \left[(K_c / b + K_\phi) \frac{Z^{n+1}}{n+1} \right] \quad (1.4)$$

$$Z = \left[\frac{3W}{b(3-n)(K_c / b + K_\phi) \sqrt{D}} \right]^{(2/(2n+1))} \quad (1.5)$$

Bekker pointed out that acceptable predictions could be obtained using equations 1.4 and 1.5 for moderate sinkages ($Z < (\text{Diameter}/6)$).

Figure 1.1 A simplified schematic of a rigid tire

Bekker also pointed out that if the terrain is firm enough, and the sum of the inflation pressure (P_i) and the pressure produced by the stiffness of the carcass (P_c) is less than the pressure the terrain can support at the lowest point of the tire circumference, the tire would be flattened and could no longer be modelled as a rigid rim. In this situation he assumed the tire had the shape shown in Figure 1.2b and hence considered it to be operating in the elastic mode. Bekker thus developed equation 1.6 to define the "critical inflation pressure" at which a tire becomes an elastic one.

$$P_i = \frac{W(n+1)}{b \left[3W/(3-n) b k \sqrt{D} \right]^{1/(2n+1)} \sqrt{D} - \left[3W/(3-n) b k \sqrt{D} \right]^{2/(2n+1)}} - P_c \quad (1.6)$$

Bekker later proposed that the sum of P_i and P_c could be represented by the "average ground pressure", P_g . The average ground pressure of a specific tire, at a given normal load and inflation pressure, is equal to the normal load carried by the tire divided by its projected ground contact area on a hard surface. This average ground pressure can be determined from the so-called "generalized deflection chart", generally available from tire manufacturers. A sample generalized deflection chart can be seen in Figure 1.3.

To predict the thrust developed by the wheel, Bekker proposed a method to characterize the shearing characteristics of a terrain by using a shear ring apparatus [2]. This apparatus was used to obtain shearing torque vs. displacement curves at various normal loads. Equation 1.7 used to fit the shearing torque-displacement data and predict the shear stress at the tire-terrain interface is a simplified equation proposed by Janosi and Hanamoto [2]. This equation is based on the Mohr-Coulomb criterion of soil failure.

Figure 1.2 Modelling of rigid and elastic modes by Bekker

29.5-25 Super Hard Rock Log DL-3 tire
 Goodyear Tire and Rubber Company, Akron, Ohio
 Lab data 4-28-82

Figure 1.3 A generalized deflection chart

$$\tau = (c + P \tan \phi) (1 - e^{-i x / K}) \quad (1.7)$$

The first part of equation 1.7 is the basic Coulomb equation, where c is the apparent cohesion of the material and generally referred to as the cohesive portion of the shear strength. $(P \tan \phi)$ represents the frictional portion of the shear strength, where P is the normal pressure, and ϕ is the angle of shearing resistance of the material. K is referred to as the shear deformation modulus, i is the slip and x denotes the distance between the front edge of the contact area and the point in question. The thrust developed by the running gear of a vehicle can be predicted by integrating the shear stress over the projected contact area.

$$T = b \int_0^{0_1} (\tau) r \cos \theta d \theta \quad (1.8)$$

A latest version of this equation, based on the thrust developed by a tracked vehicle, is as follows [32]

$$T = (Ac + W \tan \phi) (1 - (K / i l) [1 - \exp(-i l / K)]) \quad (1.9)$$

where T is the thrust generated by the ground at slip i . A is the ground contact area of the tire or track, and l is the length of A measured along the direction of motion. W is the load and K , c , and ϕ are terrain parameters described above

1.1.2 The Wong-Reece Model

In the late 1960's Wong and Reece developed a model of rigid wheel-soil interaction. Their model followed the Bekker approach. An improved method [11,13] was developed to predict the shear displacement along a rigid rim, based on the consideration of the slip velocity of a point on the rim. The shear velocity at a point on the rigid rim was given by equation 1.10.

$$V_j = r\omega(1 - (1-i)\cos\theta) \quad (1.10)$$

This resulted in an equation for the shear displacement j .

$$j = \int_0^t V_j dt = \int_0^{\theta_1} r\omega[1 - (1-i)\cos\theta] \frac{d\theta}{\omega} \quad (1.11)$$

Integrating the above equation results in equation 1.12.

$$j = r [(\theta_1 - \theta) - (1-i) (\sin \theta_1 - \sin \theta)] \quad (1.12)$$

This equation for the shear displacement was incorporated into equation 1.7 for predicting the shear stress on the wheel-soil interface.

By the late 1960's Wong and Reece [11,13,16] had shown that, motion resistance was a function of slip and, that the actual normal pressure distribution beneath a wheel has its peak well forward of bottom dead centre and moved forward with increasing slip. They proposed a method to improve the predictions of the normal pressure beneath a wheel when working on sand. They divided the normal pressure distribution into a front and a rear region with the transition point being the peak pressure value and used separate equations to model each region. An equation derived from experimental data was used

to determine the relation between the location of this peak pressure and the slip. Equations 1.2 and 1.8 were then used to determine the motion resistance and the thrust of the wheel. Their model gave better predictions of wheel performance than the previous theory.

1.1.3 The Yong Model

In the late 1970's Yong developed a new model to study the strain and stress fields in the soil beneath a tire. He modelled the tire as a linearly elastic body and used piecewise linear elastic finite elements to model the terrain. Normal and shear stress data were used as inputs and a modified Hertz theory was used to predict the length of the contact patch between the two elastic bodies. This model improved contact geometry predictions but it should be pointed out that if the normal and shear stress distributions at the tire-soil surface are known, the performance of the tire is completely defined [12].

1.1.4 The Wong-Preston-Thomas Model

In 1986 Wong and Preston-Thomas developed another model of tire-terrain interaction [18]. Their approach was similar to Bekker's model and assumed a tire was operating in either a rigid or elastic mode. They improved operating mode predictions with a method based on the nominal pressure beneath the wheel [18]. The nominal pressure (P_n) is described by equation 1.13,

$$P_n = (N1) P_L + (1 - N1) P_C \quad (1.13)$$

where $N1$ is the ratio of lug area to carcass area, and P_L and P_C are the pressures acting

on the lugs and carcass respectively. They proposed that a "transition sinkage" be determined. This transition sinkage is found by equating the nominal pressure with the average ground pressure. They then calculated the vertical reaction of the terrain, assuming that the tire had remained rigid. If this reaction was greater than the normal load, the tire was assumed not to have sunk to the transition sinkage and thus had remained rigid (Figure 1.2a). If the vertical reaction was less than the normal load, the tire was assumed to have a certain amount of deflection and thus could no longer be modelled as a rigid rim. In this case the tire was determined to be elastic and was assumed to have the shape shown in Figure 1.2b, with the nominal pressure beneath the flattened portion of the tire remaining at the average ground pressure.

Once the operating mode had been determined the pressure-sinkage relation derived by Bekker (equation 1.1) was used to predict the normal pressure on the tire-terrain contact surface, and then equation 1.2 was used to predict the motion resistance of the wheel.

Wong and Preston-Thomas used equation 1.8 to predict the thrust developed by a tire but proposed an improvement in the prediction of shear stress when working with a frictional terrain. Equation 1.7, as mentioned above, was derived from shear tests under constant normal pressures. Shear stress-shear displacement curves for three constant normal pressures can be seen in Figure 1.4a. It was shown by Wong and Preston-Thomas that since the normal pressure under a wheel varies continuously, the frictional portion of shear stress should follow the path shown by Figure 1.4b [18]. They developed a procedure which divided the contact region into small segments. Each segment analyzed

(a)

(b)

Figure 1.4 Illustration of re-shearing concept for frictional component of shear stress

is assumed to be under a constant normal pressure. The increase in shear displacement over the segment is described by,

$$\Delta j_k = j_k - j_{k-1} \quad (1.14)$$

or

$$\Delta j_k = \Delta l_k - (1-i) \Delta x_k \quad (1.15)$$

where

$$\Delta x_k = r (\cos \theta_k - \cos \theta_{k-1}) \quad (1.16)$$

$$\Delta l_k = r (\theta_k - \theta_{k-1}) \quad (1.17)$$

The x and l can be seen in Figure 1.5.

The frictional shear stress at the leading edge of the segment in question is assumed to be τ_{k-1} , with an apparent shear displacement of j_{k-1} .

$$j_{k-1} = -K \ln \left[1 - \frac{\tau_{k-1}}{P_k \tan \phi} \right] \quad (1.18)$$

At the trailing edge of the segment the frictional shear stress can then be written as

$$\tau_k = P_k \tan \phi \left(1 - e^{\frac{-(j_{k-1} - \Delta j_k)}{K}} \right) \quad (1.19)$$

It has been shown that this path dependent method improves shear stress predictions when operating over frictional terrains such as dry sand [17,18].

Figure 1.5 Schematic of repetitive shearing analysis

1.1.5 The Nakashima-Wong Model

In an attempt to improve the prediction of tire-terrain contact geometry, Nakashima and Wong developed a finite element tire model [5]. Due to the proprietary nature of the mechanical properties of the tire (such as the rubber compounds and belts) the model was based on readily available information from the "generalized deflection chart". A back analysis procedure was used to determine two equivalent Young's moduli for the tread and the sidewall. This procedure was based on the load-deflection and contact area-deflection relations. The latest version [5] is a complete three dimensional finite element tire model. Results published show that a reasonable tire model can be constructed using this method.

1.1.6 The Chen Model

Chen incorporated the Nakashima-Wong finite element tire model into a tire-terrain model [17]. In his model the terrain was modelled using Bekker's semi-empirical normal pressure and shear stress equations. A work equivalence scheme was developed in an attempt to model the effects of tire slip-sinkage. The work equivalence scheme states "that the work done by a distributed load along the predicted slip-sinkage is equal to the work done by a distributed load along a geometrically compatible slip-sinkage" [17]. In his scheme the tire is assumed to rotate about its point of entry to the terrain as it experiences slip-sinkage. Thus a compatible slip-sinkage can be determined strictly from the geometry and the rotation. An equation was developed to predict the slip-sinkage based on data obtained during shear tests conducted at various normal loads.

This equation is used to determine the predicted slip-sinkage. The procedure starts by assuming a trial sinkage (Z) and determining the normal pressure distribution and the predicted slip-sinkage. Using the predicted slip-sinkage and the work equivalence theorem the extent of the tire rotation is determined. The normal pressure distribution found at the assumed sinkage is kept and rotated along with the tire. The vertical equilibrium is checked. If the system is not in vertical equilibrium the procedure is repeated with a new assumed sinkage (Z) until vertical equilibrium is achieved.

1.2 Evaluation of existing models

All models mentioned have been steady state models in which dynamic inertia effects have been neglected. In these models, the terrain is most commonly represented with semi-empirical equations which describe the normal pressure and shear stress at the tire-terrain interface. The tire can be modelled either using a finite element approach or a practical approach based on the Bekker model.

As an evaluation the Nakashima-Wong finite element tire model was constructed using experimental data obtained in the laboratory. Using the procedure outlined in reference [5] a model of the Michelin 12.5/75R20 was constructed. These results (Figures 1.6-1.10) show good agreement between measured and fitted values. Chen had developed two tire-terrain models with separate tire representations. One incorporated the previously mentioned FEM tire model and the other incorporated a tire model based on Bekker's original practical approach. Models representing the new tire to be tested were constructed and run. Results obtained using both models are quite similar for tires with

Michelin 12.5/75R20 XL

on Hard Ground, Inflation Pres. 69 kPa

Figure 1.6 Experimental vs. fitted relationships for the FEM tire model

Michelin 12.5/75R20 XL

on Hard Ground, Inflation Pres. 138 kPa

Figure 1.7 Experimental vs. fitted relationships for the FEM tire model

Michelin 12.5/75R20 XL

on Hard Ground, Inflation Pres. 207 kPa

Figure 1.8 Experimental vs. fitted relationships for the FEM tire model

Michelin 12.5/75R20 XL

on Hard Ground, Inflation Pres. 276 kPa

Figure 1.9 Experimental vs. fitted relationships for the FEM tire model

Michelin 12.5/75R20 XL

on Hard Ground, Inflation Pres. 345 kPa

Figure 1.10 Experimental vs. fitted relationships for the FEM tire model

Figure 3.3 Block diagram of the data acquisition system for the static tire test

Michelin
12.5/75R20 XL

Terrain : Sand

26

Normal load : 4.43 kN

Inflation pressure : 345 kPa (50 psi)

Figure 1.15 Measured vs. simulated tire performance using existing models

Normal load : 9.23 kN

Inflation pressure : 138 kPa (20 psi)

high inflation pressures (Figures 1.11 - 1.20). The Chen models predicted the thrust and motion resistance reasonably well but over predicted the sinkage in all cases. This appears to be due to a misrepresentation of the normal pressure beneath the wheel. Wong and Reece have shown [11] that the normal pressure in the front region of the tire-terrain contact area can be represented quite well by the Bekker pressure-sinkage equation 1.1. Chen's scheme uses equation 1.1 to predict a normal pressure distribution over the entire contact area beneath the tire, at a particular sinkage. It then allows the tire to rotate about its point of entry to the terrain resulting in an increased sinkage and contact area. The normal pressure around the rim found previously is kept and rotated with the tire. The contact area is not modified to incorporate the additional sinkage. This results in an under prediction of the normal pressure and an over prediction of tire sinkage.

If a practical approach based on Bekker's model is to be followed then one of the criteria for determining the operating mode of the tire described previously can be used. In Chen's practical tire model the operating mode of the tire was determined using the approach based on the nominal pressure developed by Wong and Preston-Thomas. This approach was also used to predict the operating mode for the tire tested in this study. All but one load-inflation pressure combination were predicted to be operating in the rigid mode contrary to observations made in the laboratory. Therefore, the method used to determine the operating mode described previously does not appear to be adequate.

A commonly used elastic tire model is the one proposed by Wong and Preston-Thomas in which the tire assumes the flattened shape shown in Figure 1.2b. Many have shown that this is not necessarily the case [2,3]. Smith and Freitag [2] have shown that

Load 13.3 kN (3000 lb)

Inflation pressure 133 kPa (19 psi)

Figure 1. 21 Deflection shape of a 11.00 - 20 powered and towed 12 - PR tire on sand [2]

a deformed tire operating on sand to have the shape shown in Figure 1.21. McKibben [2] has shown that when an elastic tire is operating over a soft terrain the loading surface resembles a rigid tire with an increased diameter. In Wong and Preston-Thomas's model the normal pressure is found using Bekker's pressure-sinkage equation which results in a constant pressure under the flattened region. This pressure distribution is not necessarily consistent with published experimental data [2].

1.3 Scope of the present work

Based on the above evaluations the following will be examined and explored in this thesis.

- 1) An improved tire-terrain model which incorporates a more realistic normal pressure distribution beneath the tire will be developed. This model will follow a practical approach based on Bekker's model.
- 2) A re-evaluation of the criterion for determining the operating mode of the tire will be made in an attempt to improve predictions.
- 3) A model will be developed to improve performance predictions of a tire operating in the elastic mode. This model will provide a more realistic representation of the tire-terrain contact geometry.
- 4) Predictions made with the new model will be compared with experimental data and predictions made by previous models.

Chapter II

Terrain Modelling

To develop a realistic tire-terrain interaction model, terrain behaviour must be appropriately characterized. Currently, there are two principal methods for measuring and characterizing terrain conditions: one is based on the cone penetrometer technique and the other is based on the beavometer technique.

2.1 The Cone Penetrometer

The cone penetrometer was originally developed by the U.S. Army Corps of Engineers Waterways Experiment Station (WES) in 1947. The cone penetrometer is a small hand held device consisting of a rod, cone, and force measuring device. The cone penetrometer measures the resistance to penetration per unit cone base area, referred to as the cone index. Shear and compressive characteristics of the terrain are lumped into the cone index. The cone penetrometer technique was originally designed as a quick and simple device to determine if a particular vehicle could be mobile over a specific terrain on a "go or no - go" basis. This technique has been investigated by many organizations, and as pointed out by Reece and Peca [9] the cone index is useful for defining properties

of clay but is inadequate for defining the properties of sand.

2.2 The Bevameter

The bevameter was pioneered by M.G. Bekker in the mid 1950's. It was designed to subject the terrain to loading similar to that imposed by a vehicle. A typical vehicle exerts a normal force and a shear force on the ground through its running gear. This normal load causes the running gear to sink, resulting in a motion resistance. The torque applied to the running gear results in a shearing at the vehicle-terrain interface. This shearing results in the development of a thrust and an associated slip.

To simulate this vehicle action the bevameter method is composed of two separate tests. The first is a plate penetration test. In this test a plate which simulates the contact area of the tire is penetrated into the terrain and the pressure-sinkage relationship is measured. The second is a shear test. In this test a shear plate or ring is used to determine the characteristics of shearing between the running gear and the terrain. This test results in a shear stress-shear displacement curve as well as the shear strength constants for the terrain.

2.2.1 The pressure-sinkage relation

For terrain that is considered homogeneous throughout the area of interest, Bekker [22] proposed the following equation

$$P = (K_c / b + K_\phi) Z^n \quad (2.1)$$

where P is the pressure, Z is the sinkage, b is the smaller dimension (or the radius) of the

Figure 2.1 Schematic of the bevameter taken from reference [17]

Figure 2.2 Block diagram of the data acquisition system for the bevameter

contact patch, the K_c , K_ϕ , and the n are terrain parameters.

The values of K_c , K_ϕ , and n can be determined from a minimum of two tests using two different plate sizes. Using logarithms the pressure sinkage relation can be written as follows

$$\ln P = \ln (K_c / b + K_\phi) + n \ln Z \quad . \quad (2.2)$$

A weighted least squares method is used to derive the desired parameters. The weighted error function is given by

$$F = \sum P^2 [\ln P - \ln (K_{eq}) - n \ln Z]^2 \quad (2.3)$$

where $K_{eq} = K_c/b + K_\phi$.

To minimize the value of the function, the partial derivatives of F with respect to n and K_{eq} are taken and set to zero. Solving both equations simultaneously leads to the following expressions

$$n = \frac{(\sum P^2) (\sum P^2 \ln P \ln Z) - (\sum P^2 \ln P) (\sum P^2 \ln Z)}{(\sum P^2) (\sum P^2 (\ln Z)^2) - (\sum P^2 \ln Z)^2} \quad (2.4)$$

$$\ln K_{eq} = \frac{\sum P^2 \ln P - n \sum P^2 \ln Z}{\sum P^2} \quad . \quad (2.5)$$

The Bekker equation requires that there be one unique value of n for a specific terrain. The n values calculated for each plate are averaged to produce one single n value. The remaining two terrain parameters are then calculated using information gathered from both plates used in the test.

$$K_c = \frac{(K_{eq})_{b=b1} - (K_{eq})_{b=b2} (b1) (b2)}{b2-b1} \quad (2.6)$$

$$K_\phi = (K_{eq})_{b=b1} - \left[\frac{(K_{eq})_{b=b1} - (K_{eq})_{b=b2}}{b2-b1} \right] b2 \quad (2.7)$$

For this study a 5 cm, 7.5 cm, and a 10 cm diameter plate were used during the test. A sample of the data obtained during these tests can be seen in Figures 2.3 and 2.4.

The pressure sinkage parameters of the sand in the laboratory bin, obtained using the bevameter, can be found in Table 2.1. These values were used in the tire performance prediction model.

Table 2.1 Pressure-Sinkage Parameters

n	K_c kN/m ⁽ⁿ⁺¹⁾	K_ϕ kN/m ⁽ⁿ⁺²⁾	goodness of fit %
0.97	-17.53	1968.10	92.80

2.2.2 The shear stress-shear displacement relationship

The most commonly used semi-empirical equation to describe the shear stress - shear displacement relationship of soils, such as sand, is as follows

$$\tau = \tau_{\max} (1 - e^{-i/K}) \quad (2.8)$$

Bevometer

Pressure-sinkage test

Figure 2.3 Sample of measured pressure-sinkage data

Bevometer

Pressure-sinkage test

Figure 2.4 Sample of measured pressure-sinkage data

where τ is the shear stress, j is the shear displacement, $\tau_{\max}$ is the maximum shear stress under a particular load, and K is referred to as the "shear deformation modulus".

To determine the required parameter K , the function F below is minimized using a weighted least squares method,

$$F = \sum (1 - \tau_m / \tau_{\max})^2 [\ln(1 - \tau_m / \tau_{\max}) + j/K]^2 \quad (2.9)$$

where $(1 - \tau_m / \tau_{\max})^2$ is the weighting factor and τ_m is the measured stress.

To obtain a value that minimizes the function F , the partial derivative of the function with respect to K is taken and set equal to zero. Solving for K one obtains equation 2.10.

$$K = - \frac{\sum w_i (j^2)}{\sum w_i (j) \ln(1 - \frac{\tau_m}{\tau_{\max}})} \quad (2.10)$$

To solve for an appropriate value of $\tau_{\max}$ the following iterative procedure is used.

- 1) An initial value of $\tau_{\max}$ is selected
- 2) K is calculated using the above equation
- 3) A goodness of fit is calculated using equation 2.11

where τ_m is the measured value and τ is the calculated one using equation 2.8:

$$e = 1 - \left[\frac{\frac{\sqrt{\sum (\tau_m - \tau)^2}}{N-2}}{\frac{\sum \tau_m}{N}} \right] \quad (2.11)$$

- 4) The goodness of fit is checked to see if its maximum value has been reached. If it has not been reached the $\tau_{\max}$ value is changed accordingly and the procedure is repeated.

The maximum shear strength is assumed to observe the following Mohr-Coulomb failure criterion,

$$\tau_{\max} = c + P \tan \phi \quad (2.12)$$

where c is the cohesion and ϕ is the angle of shearing resistance, and P is the normal pressure.

For this study a shear ring with grousers and a ring with a rubber shearing surface were used to determine the internal and rubber-terrain shearing characteristics. The values of the shear parameters obtained for the sand in the laboratory sand bin can be found in Tables 2.2 and 2.3. A sample of the data gathered from the shear test can be seen in Figures 2.5-2.10.

Table 2.2 Rubber-Terrain Shearing Parameters

C (kPa)	PHI (DEG)	K (cm)	goodness of fit (%)
0.34	27.59	1.94	97.55

Bevometer

Internal shear test on sand

Figure 2.5 Sample of measured shear stress vs. shear displacement data for a normal pressure of 4.37 kPa

Bevometer

Internal shear test on sand

Figure 2.6 Sample of measured shear stress vs. shear displacement data for a normal pressure of 14.87 kPa

Bevameter

Internal shear test on sand

Figure 2.7 Sample of measured shear stress vs. shear displacement data for a normal pressure of 27.22 kPa

Bevometer

Rubber shear test on sand

Figure 2.8 Sample of measured shear stress vs. shear displacement data for a normal pressure of 4.37 kPa

Bevameter

Rubber shear test

on sand

Figure 2.9 Sample of measured shear stress vs. shear displacement data for a normal pressure of 14.87 kPa

Bevameter

Rubber shear test on sand

Figure 2.10 Sample of measured shear stress vs. shear displacement data for a normal pressure of 27.22 kPa

Table 2.3 Internal Shearing Parameters

C (kPa)	PHI (DEG)	K (cm)	goodness of fit (%)
0.87	27.24	2.99	95.40

2.2.3 Slip sinkage and repetitive loading

The following relationship characterizing slip-sinkage was proposed by Chen [17].

$$Z_s = (A_s + P B_s) (1 - e^{-t/K_s}) \quad (2.13)$$

This equation is used to fit data obtained during the shear ring test mentioned earlier. The physical significance of A_s , B_s , and K_s are not apparent. Although equation 2.13 fits experimental data quite well it was shown by Chen that the equation fails to describe the physical process of slip-sinkage adequately [17]. By taking the second derivative of equation 2.13 with respect to time it can be shown that the vertical acceleration is always negative. Since the shear ring starts from rest there must be some initial acceleration. The slip-sinkage parameters obtained for the sand in the laboratory sand bin can be found in Table 2.4. These parameters were used when running the two Chen models.

Table 2.4 Slip Sinkage Parameters

slip-sinkage type	A_s (m)	B_s (m/kPa)	K_s (m)
Internal	0.01626	0.00201	0.10990
Rubber	-0.00317	0.00233	0.13538

A terrain element can be subject to repetitive loading of consecutive wheels of a multi-axle wheeled vehicle. If this is to be modelled, the response of the terrain to repetitive loading must be measured. Based on experimental observations Wong proposed the following equation.

$$P = P_u - k_s (Z_u - Z) \quad (2.14)$$

P_u and Z_u are the pressure and sinkage when the unloading of the terrain begins. The k_s is a pressure-sinkage parameter derived from experimental data, and represents the slope of an unloading-loading curve. Since a single tire was used in this study the response of the terrain to repetitive loading was not measured.

Chapter III

Tire Modelling

To develop a realistic tire-terrain interaction model, the tire must be adequately represented. A brief review of tire modelling will be given and the tire model used in this study will be described. In order to model a tire certain physical tire data must be known. Due to the proprietary nature of the tire's construction and materials, most tire models rely on tire dimensions and information contained in the generalized deflection chart. The generalized deflection chart describes the relationship between tire deflection, contact area, inflation pressure, and normal load. This chapter will present tire data as well as outline the procedures and equipment used to perform these measurements.

3.1 The tire model used in this study

There are two main approaches to modelling a tire. The first and most common is a practical approach based on Bekker's model. The second is an approach which incorporates the finite element method. In the first approach a tire that experiences little or no deformation is represented as a rigid rim. If a tire experiences significant deformation it can no longer be represented as a rigid rim. In this case the tire is considered to be operating in the elastic mode. Various representations of a tire with

considerable deformation have been published. The most common elastic tire model has been described in section 1.1.4. Later in this thesis an improved elastic tire model will be developed.

Lately attempts have been made to incorporate the finite element method into tire modelling. One such approach has been outlined in section 1.1.4. The finite element tire model has not been investigated further in this study due to the high computing power required.

In this study an approach based on Bekker's model is used. This approach uses the method developed by Wong and Preston-Thomas outlined in section 1.1.4 to determine if the tire is in the rigid or elastic operating mode. If the tire is found to be operating in the rigid mode then it is represented as rigid rim. If the tire is found to be operating in the elastic mode then it is represented as a rigid rim with a larger diameter. The development of this proposed elastic tire model will be described in Chapter IV. To improve this common practical approach to tire modelling the effect of the tire lugs was included. For soft soils such as sand this approach considers the spaces between the lugs to be filled with terrain material. Work done by many in this field has supported this approach [12,28]. The normal pressure, a function of sinkage, will be higher on the lugs due to the lug height (shown in Figure 3.1). If N_1 is defined as the ratio of lug area over carcass area then the pressure applied on the lugs will act on an effective area of $(N_1)(B_t)$, where B_t is the tire tread width. This results in an effective carcass area of $(1-N_1)(B_t)$. Wong has shown that for conventional lugged tires the spacing between lugs is such that the terrain could not fail in the same way as that in front of a cutting blade,

Figure 3.1 Illustration of lug / carcass geometry

thus indicating that shearing across the lug tips should not be neglected [12]. For lugged portions of the tire, rubber-terrain shearing characteristics are used, while internal shearing characteristics are used for the area between the lugs.

3.2 Dimensions of the tire used in this study

The tire dimensions were measured using standard measuring devices. All measured dimensions for the Michelin 12.5/75R20 XL used in this study can be found in Table 3.1.

Table 3.1 Physical tire dimensions (Michelin 12.5/75R20 XL)

	dimensions
Tire diameter	0.977 m
Tread width	0.292 m
Lug height	0.012 m
Lug width	0.050 m
Lug tip area/carcass area	0.51

3.3 The generalized deflection chart

The relationships between tire deflection, contact area, inflation pressure, and normal load for the tire used in this study were obtained by performing a number of "static tire tests" on hard ground and are described in the form of a chart commonly known as the generalized deflection chart (Figure 1.3). An improved procedure was developed to obtain a more reliable print of the tire contact area at the various desired normal loads.

The static tire tester (Figure 3.2) is used to gather the data required to assemble the generalized deflection chart. A tire can be modelled using the previously mentioned physical tire dimensions and the information recorded during these static tire tests. The test assembly consists of a frame and a moveable platform. The tire to be tested is placed above the moveable platform. Before the tire is put into position the area expected to be in contact with the platform is painted with black ink. A potentiometer is set up to record the displacement of the platform and, a load cell is placed under the platform to measure the applied load. A piece of paper is placed on the platform under the tire. The platform is then raised and the load and the deflection are recorded at predetermined load intervals using the data acquisition system outlined in Figure 3.3. Once the desired load or deflection has been reached the platform is lowered and the print of the tire's contact area can be removed and measured. The print is checked to ensure that all the marks made by the lugs on the longitudinal extremities of the print do not show the complete lug area. If this is the case a good area approximation is obtainable. The end of the contact region can not be readily identified if a full lug mark is visible since the amount of space

Figure 3.2 Schematic of the static tire tester taken from reference [17]

Figure 3.3 Block diagram of the data acquisition system for the static tire test

between lugs, at the longitudinal extremity in the contact patch, cannot be measured. In this instance the tire must be repainted, rotated slightly, and then retested. This is to ensure that the ink stain at the longitudinal boundary of the print is caused by the lugs. Once a proper print is obtained the tire is repainted and the next test can be started.

For this research five tests were done for each inflation pressure-load combination. Once all the desired tire prints were available, improved methods to measure the contact area were examined. A planimeter in the Carleton Geography Department was modified to facilitate the measuring of the large tire prints and then used to determine the contact areas. All results were averaged and are shown in Table 3.2.

Table 3.2 Generalized deflection table - Michelin 12.5/75R20 XL

inflation pressure	Deflection (cm)	Load (kN)	Area (cm²)
69 kPa	1.896	2.225	179.93
	3.623	4.450	517.26
	5.119	6.675	726.09
	6.447	8.900	947.36
	7.917	11.125	1085.95
	9.169	13.350	1214.12
138 kPa	1.239	2.225	145.82
	2.423	4.450	262.89
	3.477	6.675	424.44
	4.413	8.900	604.21
	5.306	11.125	749.58
	6.098	13.350	865.65

inflation pressure	Deflection (cm)	Load (kN)	Area (cm²)
207 kPa	1.021	2.225	127.81
	1.963	4.450	211.79
	2.770	6.675	304.42
	3.528	8.900	414.85
	4.191	11.125	556.91
	4.851	13.350	649.45
276 kPa	0.894	2.225	102.68
	1.640	4.450	182.92
	2.311	6.675	251.47
	2.939	8.900	343.35
	3.494	11.125	443.03
	4.059	13.350	525.61
345 kPa	0.776	2.225	100.92
	1.435	4.450	165.86
	2.008	6.675	233.76
	2.533	8.900	297.14
	3.030	11.125	372.77
	3.494	13.350	445.50

Chapter IV

The Tire-Terrain Interaction Model

Published experimental data [1,3,4,11,13,20] has shown that the pressure distribution beneath a wheel on deformable terrain has certain characteristics. These characteristics can be seen in Figures 4.1 - 4.2 and include the following.

- The pressure starts at zero where the tire enters the terrain, and increases to a maximum pressure with a distribution similar to that predicted using Bekker's pressure-sinkage equation 1.1.
- The location of this maximum pressure is ahead of bottom dead centre and moves forward with increasing slip or skid.
- A decrease to zero pressure, from the point of maximum pressure to the point where the tire exits the terrain.

Using a tire-terrain interaction model which simply employs a pressure sinkage relation such as equation 1.1 results in a specific normal pressure distribution under the wheel and hence a particular motion resistance and sinkage regardless of the slip value. It has been shown that the normal pressure distribution, motion resistance, and sinkage are functions of slip [11,13,16].

(a)

(b)

Figure 4.1 Measured normal pressure distribution under a tire [3] (a), and under a rigid wheel [1] (b)

Figure 4.2 Normal and shear stress distributions under a rigid wheel on sand (a) 3.1% slip and (b) 22.1% slip [13]

4.1 A modified pressure distribution under a rigid tire

In the late 1960's Wong and Reece [11] developed a method based on experimental data, to predict the normal pressure distribution beneath a rigid wheel when working on sand. They demonstrated that the pressure beneath a rigid wheel exhibited the characteristics shown in Figure 4.3 and could be modelled by dividing the distribution into a front and rear region. The front region was modelled using equation 4.1, while the rear region was modelled using equation 4.2.

$$\sigma_r(\theta) = (K_1 + K_2 b) (r/b)^n [(\cos \theta - \cos \theta_1)]^n \quad (4.1)$$

$$\sigma_r(\theta) = (K_1 + K_2 b) (r/b)^n [\cos(\theta_1 - \frac{\theta}{\theta_M}(\theta_1 - \theta_M)) - \cos \theta_1]^n \quad (4.2)$$

K_1 , K_2 , and n are terrain constants. The r and b are the radius of the wheel and the smaller dimension in the wheel-soil contact patch. The θ_1 , and θ_M represent the point of entry to the terrain and the location of the peak normal pressure respectively.

Using several sets of experimental data published by Onafeko, Hegedus, and Sela [11], Wong showed that the location of the point of maximum radial stress could be expressed by the following equation when working in sand.

$$\theta_M = (C_1 + C_2 \pm) \theta_1 \quad (4.3)$$

C_1 and C_2 are coefficients related to the terrain and are determined experimentally. The C coefficients given by Wong can be seen in Table 4.1.

$$\frac{\theta_R - \theta_2}{\theta_M - \theta_2} = \frac{\theta_1 - \theta_F}{\theta_1 - \theta_M}$$

Figure 4.3 Illustration of the characteristics of the radial stress distribution on sand of a driven wheel [11]

Table 4.1 Coefficients C_1 and C_2 taken from reference [11]

Soil	C_1	C_2	Data from
compact sand	0.43	0.32	Onafeko
loose sand	0.18	0.32	Onafeko
sand	0.285	0.32	Hegedus
dry sand	0.38	0.41	Sela

It should be noted that the values of C_1 and C_2 given in Table 4.1 were obtained using steel wheels [11]. Since the publishing of Wong's paper, additional experimental data on pneumatic tires [3,4] has been obtained. They indicate that the characteristics of the pressure distribution under rigid steel rims mentioned above, are similar to those under pneumatic tires, indicating that the general approach proposed by Wong may be applicable to modelling tires in either the rigid or elastic mode.

Since no experimental pressure distribution data for any tires tested in the sand at Carleton were available it was decided to use a back propagation method to determine the two coefficients C_1 and C_2 used in equation 4.3. Using Wong's proposed equations 4.1 and 4.2 and the vertical equilibrium equation 4.4, the coefficients C_1 and C_2 in equation 4.3 were determined by iteration, until the closest match between the predicted sinkage and the measured tire sinkage was obtained throughout the available slip range.

$$W = xb \left[\int_0^{\pi} p'(\theta) \cos \theta d\theta - \int_0^{\pi} \tau(\theta) \sin \theta d\theta \right] \quad (4.4)$$

P is the normal pressure and τ is the shear stress. A Michelin 12.5/75R20 in rigid mode was used to gather the sinkage data. A C_1 value of 0 and a C_2 value of 1.5 were obtained and a sample of the predicted and measured sinkages can be seen in Figure 4.4. A sample of the predicted normal pressure distribution curves at normal loads of 4.43 kN and 9.23 kN can be seen in Figures 4.5 - 4.7. All normal pressure distributions displayed the expected and desired characteristics outlined earlier.

4.2 The prediction of the operating mode of a tire

The model based on the normal pressure beneath a tire outlined in Chapter I had been run to obtain operating mode prediction. Some inflation pressure-load combinations observed to be elastic were predicted rigid. Based on those results the prediction model was closely examined. The "transition sinkage" was defined as the sinkage where the pressure exerted by the terrain on the tire equals the average ground pressure, which is taken from the "generalized deflection chart". At this sinkage the vertical reaction of the terrain is calculated assuming the tire is rigid. If this reaction is greater than the normal load the tire is assumed to have remained rigid. If the reaction is less than the normal load the tire is assumed to be operating in the elastic mode.

It is a common approach in off-road vehicle studies to set the variable b in equation 1.1 to the wheel width [1]. In some cases the width of the wheel might not be the smaller dimension in the contact patch. For our particular tire operating in the rigid mode the tire width does not become the smaller dimension in the contact patch until the

MICHELIN 12.5/75R20

rigid mode, 9.23kN load, on sand

Figure 4.4 Measured vs. predicted tire sinkage

MICHELIN 12.5/75R20

Modified Pressure Distribution 50% slip

Figure 4.5 Predicted pressure distribution under a tire with a 4.43 kN load.

MICHELIN 12.5/75R20

Modified Pressure Distribution 10% slip

Figure 4.6 Predicted pressure distribution under a tire with a 9.23 kN load.

MICHELIN 12.5/75R20

Modified Pressure Distribution 35% slip

Figure 4.7 Predicted pressure distributions under a tire with a 9.23 kN load.

tire has sunk to a depth of 8 cm. The procedure was modified to calculate, and then use, the smaller dimension in the contact patch each time the equation was used.

4.3 The development of an elastic tire model

In the elastic tire model described in Chapter I (Figure 1.2b), the assumption that a tire operating in the elastic mode over a deformable terrain has a flat surface may not be realistic. Krick [3] used plaster of Paris to obtain a variety of pneumatic tire imprints. He found that the loading surface did not have a flat region. Smith and Freitag [2] have published the deflection shape of a powered tire operating on sand (Figure 1.21). This deflection shape also indicates that the existing representation of a deformed tire may not be adequate. McKibben has shown experimentally "that a pneumatic tire only forms a flatter and larger loading surface than a steel wheel of the same diameter and has a curvature larger than the wheel radius....it might be possible to replace the flat elastic tire surface by the rigid arc of a larger wheel" [2].

The previous elastic tire model assumed the pressure under the flattened portion of the tire to be equal to the average ground pressure P_g mentioned earlier. Krick [3] has shown experimentally that the characteristics of the pressure distribution under a rigid rim are similar to those under a pneumatic tire. Based on the above analysis it was felt that the current elastic tire model needed improvement. Modelling the loading surface of an deformed tire with the arc of a larger rigid wheel was investigated further.

To keep consistency between all models it was decided that all information required to construct the elastic tire model be taken from the physical tire dimension table and the "generalized deflection chart".

In modelling the elastic tire the following assumptions were made. A) The deflection of an elastic tire remains constant for all values of slip. B) The tire remains rigid until the "transition sinkage" has been reached, at which time it starts to experience some deflection. C) The elastic tire on a soft soil does not deflect more than the "critical deflection value" which is the value measured on hard ground for the same load and inflation pressure combination. D) A tire will only sink past the "transition sinkage" once its deflection has reached the "critical deflection value". The "transition sinkage" was found using the average ground pressure obtained during the static tire test. Since the average ground pressure is the normal load on a tire over its corresponding contact area, when the tire is at the "critical deflection value" assumption (D) above seems reasonable.

The procedure used to determine the effective diameter of the rigid wheel that represents the elastic tire is outlined below. A schematic indicating the variables can be seen in Figure 4.8. Once the rigid tire reaches the transition sinkage it may experience deflection. If deflection occurs the centre of the tire will lower with respect to the original terrain surface. The new arc between the bottom dead centre of the wheel (point B Figure 4.8), and the new point of entry of the terrain (point A Figure 4.8) coincides with part of the circumference of a larger wheel, with its centre located on the same

vertical axis (Figure 4.9). Using these geometric constraints a relationship was developed relating the original radius, sinkage, and deflection to a new effective radius (R).

The geometry as seen in Figure 4.9 allows for two equations.

$$R = r + x \quad (4.5)$$

$$R^2 = (x + (r - Z))^2 + r^2 - (r - a_d - Z)^2 \quad (4.6)$$

R is the new effective radius, r is the original tire radius, a_d is the deflection of the original tire, Z is the sinkage of the tire, and x is the distance between original tire centre and new effective tire centre. Let $d1 = r - a_d - Z$, $d2 = r - Z$, and $d3 = \sqrt{(r^2) - (d1^2)}$.

Combining both equations results in

$$r^2 + 2rx + x^2 = x^2 + 2xd2 + d2^2 + r^2 - d1^2 \quad (4.7)$$

Then solving for x gives

$$x = \frac{d2^2 - d1^2}{2(r - d2)} \quad (4.8)$$

Therefore given r, a_d , and Z, the value of x can be determined.

Using the method outlined earlier in section 4.2 the "transition sinkage" of a tire with a given load and inflation pressure can be found. At this point the tire has not experienced any deflection yet ($a_d = 0$). Calculating the vertical reaction of the terrain will result in a value lower than the normal load on the tire indicating that the tire-terrain system is not in vertical equilibrium. Using equation 4.8 an iterative procedure can be used to increment the deflection (a_d), determine the new effective radius, and check for vertical equilibrium. Once the vertical equilibrium has been reached a deflection value

Previous elastic tire model

Proposed elastic tire model

Figure 4.8 Schematic of the new modelling concept for a tire operating in the elastic mode

$$1) \quad R = r + x$$

$$2) \quad R^2 = (x + (r - Z))^2 + r^2 - (r - a_d - Z)^2$$

Figure 4.9 Schematic of the geometric relation between the original tire diameter and the new effective diameter

(a_d) has been determined. This value (a_d) is only valid while the tire is operating with the same normal load and inflation pressure. Since the pressure distribution under the tire is a function of slip the sinkage which is dependant on the normal pressure will also be a function of slip. This change in sinkage with a change in slip will also result in a change in the effective radius. To determine the effective diameter for the same tire with the same operating conditions at another slip value, the deflection value is set to the value (a_d) determined previously. An iterative procedure can be used to increment the sinkage (Z) , determine the new effective diameter using equation 4.8, and then check for vertical equilibrium. This will result in a sinkage and new effective diameter for each slip value tested. Predictions using the previously mentioned practical rigid analysis and the modified pressure distribution can be performed. If the deflection value (a_d) reaches the "critical deflection value" obtained from the "generalized deflection chart" during the first part of this procedure, it is kept at that value and the sinkage (Z) is substituted into the incremental procedure. Block diagrams of this procedure and the entire tire-terrain modelling procedure can be seen in Figures 4.10 - 4.12.

PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

PRECISIONSM RESOLUTION TARGETS

Figure 4.10 First stage of the elastic tire modelling flow chart

Figure 4.11 Second stage of the elastic tire modelling flow chart

Table 4.2 Elastic model effective diameters

Slip %	4.43 KN at 69 kPa	7.45 KN at 69 kPa	9.23 KN at 69 kPa	9.23 KN at 138 kPa
2	1.41 m	1.63 m	1.62 m	1.19 m
5	1.41 m	1.62 m	1.61 m	1.19 m
10	1.39 m	1.60 m	1.59 m	1.18 m
20	1.36 m	1.55 m	1.53 m	1.16 m
35	1.29 m	1.44 m	1.41 m	1.12 m
50	1.18 m	1.26 m	1.23 m	1.07 m

Figure 4.12 Flow chart of the new tire-terrain interaction model

Chapter V

Experimental Evaluation of Various Tire-Terrain Interaction Models

The predictions by the new tire-terrain model was compared with those of a previous model and with experimental data. The apparatus used to collect the tire performance data, was the forced-slip tire tester located in the Transport Technology Research Laboratory at Carleton University. The apparatus will be described and experimental and predicted performance data will be presented.

5.1 The forced-slip tire tester

The main components of the forced-slip tire tester (Figure 5.1) are:

- a horizontal motion carriage
- a frame and test bed
- a double parallelogram mechanism
- a slip pulley chain system
- a data acquisition system

The tire is mounted in the horizontal motion carriage. The horizontal motion carriage is on tracks which allows it to travel horizontally over the test bed. The slip

Figure 5.1 Schematic of the forced-slip tire tester taken from reference [17]

pulley and chain drive system force the tire to slip as the motion carriage moves forward thus resulting in a thrust being developed at the tire-terrain surface. Horizontal forces in the system are measured through two load cells. The first load cell is located at the front of the carriage which tows the tire, and the second is attached to the cable which supplies the force to the slip pulley drive system. These forces as well as the vertical sinkage are all recorded by means of a data acquisition system (Figure 5.2) and then processed. For this research tests were run at inflation pressures from 69 - 345 kPa (10 - 50 psi) with 69 kPa increments. Slip values tested where 2, 5, 10, 20, 35, 50 %, and normal loads used where 4.43, 7.45, and 9.23 kN. Three test runs were done for each inflation pressure - slip - normal load combination.

To process the experimental data into meaningful results an analysis of the forces must be done. Sprocket 2, (Figure 5.3), and the tire are mounted on the same shaft, while the slip pulley and sprocket 1 are mounted on the other shaft. Both sprockets are linked by a chain which transmits the driving torque.

The slip of a wheel is defined by,

$$i = \left(1 - \frac{V}{r\omega} \right) \times 100 \% \quad (5.1)$$

where V is the linear speed of the tire centre, ω is the angular speed of the tire, and r is the rolling radius of the free-rolling tire. Using the variables in Figure 5.3 the slip can be expressed as

$$i = \left(1 - \frac{R_1 \theta_1}{R_2 \theta_2} \right) \times 100 \% \quad (5.2)$$

Since the sprockets are linked together by a chain the following relationship exists.

Figure 5.2 Block diagram of the data acquisition system for the forced slip tire tester

Figure 5.3 Free body diagram of the forced-slip tire tester

$$\theta_1 r_1 = \theta_2 r_2 \quad (5.3)$$

This then results in a slip defined by the following equation.

$$i = 1 - \left[\frac{R_1 r_2}{R_2 r_1} \right] \quad (5.4)$$

If the tester is considered as a free body (Figure 5.3) in equilibrium then the following equation exists.

$$F_t + T - R_m - F_c = 0 \quad (5.5)$$

The drawbar pull can then be written as

$$T - R_m = DB = F_c - F_t \quad (5.6)$$

where DB represents the drawbar pull.

Using the moment equilibrium results in equation 5.7.

$$T = \frac{F_c R_1}{r_1} + \frac{r_2}{R_2} \quad (5.7)$$

$$T = F_c (1 - i) \quad (5.8)$$

Therefore the motion resistance can be written as equation 5.9.

$$R_m = T - DB \quad (5.9)$$

The above equations were developed using the geometry of a rigid wheel. Therefore the thrust value is exact for a rigid wheel on hard ground where R_m is approximately zero.

A test was done to determine the force required to overcome the friction in the

bearings and the chain drive system. The carriage was raised and the force (F_c in Figure 5.3) required to maintain a constant angular velocity of the tire was measured. A relatively small force of approximately 6.5 N considered negligible was measured, thus the efficiency of the chain drive system was not included in the above calculations.

5.2 Experimental tire performance data

A sample of the results obtained from the dynamic tire test can be seen in Figures 5.4 - 5.9. As can be seen, a small start up time is required to bring the system into steady state, at which time the data becomes useable. During the test some dynamic effects were noticed. These effects are referred to as the slip-stick phenomena between tire and terrain. This slip-stick phenomena results in fluctuations in the data readings. Many tests were done and the slip-stick effect was observed to increase as the slip value increased. To filter out these effects the experimental data is averaged out in the steady state region. The tabulated tire performance data can be seen below in Tables 5.1 - 5.12.

5.3 Experimental determination of the operating mode.

During the experiments, the actual operating modes of the tire were determined by observation. For each load-inflation pressure combination the front view and side view of the tire were observed to identify the deformation of the carcass. If the tire experienced minimal deflection and seemed to have a circular circumference, it was considered to be operating in the rigid mode. If the tire had considerable deformation and

Dynamic Tire Tester Run
 4.43 kN load
 69 kPa inflation pressure
 20% slip on sand

Tire rotation angle (rad)

Figure 5.4 Sample of data obtained during the dynamic tire test

Dynamic Tire Tester Run
4.43 kN load
345 kPa inflation pressure
20% slip on sand

Tire rotation angle (rad)

Figure 5.5 Sample of data obtained during the dynamic tire test

Figure 5.6 Sample of data obtained during the dynamic tire test

Dynamic Tire Tester Run
4.43 kN load
345 kPa inflation pressure
50% slip on sand

Tire rotation angle (rad)

Figure 5.7 Sample of data obtained during the dynamic tire test

Dynamic Tire Tester Run
9.23 kN load
69 kPa inflation pressure
35% slip on sand

Figure 5.8 Sample of data obtained during the dynamic tire test

Dynamic Tire Tester Run
9.23 kN load
276 kPa inflation pressure
35% slip on sand

Figure 5.9 Sample of data obtained during the dynamic tire test

no longer seemed to have a circular circumference it was considered to be operating in the elastic mode. Several observations were photographed and a sample of these can be seen in Figures 5.10 - 5.12. Experimental observations and predictions are outlined below.

Table 5.1 Operating modes of the Michelin 12.5/75R20 XL

Normal Load (kN)	Inflation Pressure (kPa)	Observed Operating Mode	Predicted Operating Mode
4.43	69 and less	elastic	elastic
4.43	138 and higher	rigid	rigid
7.45	69 and less	elastic	elastic
7.45	138 and higher	rigid	rigid
9.23	138 and less	elastic	elastic
9.23	207 and higher	rigid	rigid

Table 5.2 Elastic model data (all sinkages at 2% slip)

	4.43 kN at 69 kPa	7.45 kN at 69 kPa	9.23 kN at 69 kPa	9.23 kN at 138 kPa
$Z_{\text{transition}}$	5.10 cm	5.57 cm	5.69 cm	8.53 cm
$Z_{\text{predicted}}$	5.10 cm	6.85 cm	7.87 cm	8.53 cm
$a_{\text{predicted}}$	2.60 cm	5.70 cm	6.70 cm	2.30 cm
a_{critical}	3.62 cm	5.70 cm	6.70 cm	4.55 cm

Figure 5.10 Photographs showing the tire deflection
at 69 kPa

Figure 5.11 Photographs showing the tire deflection at 138 kPa

Figure 5.12 Photographs showing the tire deflection
at 173 kPa

5.4 Experimental evaluation of the proposed tire-terrain model

Comparisons of tire performance using the new model, the finite element model and the practical model mentioned in Chapter one, and the measured data are tabulated in Tables 5.3 - 5.14 and graphically shown in Figures 5.13 - 5.25. These Figures include the results for the tire operating in both the rigid and elastic modes.

Chen has shown in his thesis [17] that a point-by-point comparison may not reflect completely the correlation of the two curves. To select a standard way of comparing curves an expression for a regression coefficient is introduced,

$$R_c = 1 - \sqrt{\frac{\sum (y_p - y_m)^2}{\sum y_m^2}} \quad (5.10)$$

where y_p and y_m are the predicted and measured values respectively. It should be noted that regression coefficients are only meaningful in relative terms and should not be compared with the coefficients of other parameters since the magnitudes used in the calculation of regression coefficients for other parameters may not be similar. The tabulated regression coefficients can be seen in Tables 5.15 - 5.18.

Table 5.3 Measured and predicted thrust (kN) for the Michelin 12.5/75R20 XL under a 4.43 kN load

Slip	Measured	Practical model	FEM model	New model
69 kPa				
2	0.310	0.395		0.397
5	0.301	0.603		0.590
10	0.687	0.888	0.411	0.845
20	1.220	1.277	1.160	1.204
35	1.511	1.575	1.598	1.438
50	1.680	1.709	1.749	1.500
138 kPa				
2	0.362	0.395	-.074	0.406
5	0.428	0.603	0.215	0.569
10	0.750	0.888	0.638	0.817
20	1.161	1.277	1.178	1.137
35	1.450	1.575	1.540	1.358
50	1.681	1.709	1.709	1.416
207 kPa				
2	0.428	0.395	0.048	0.406
5	0.491	0.603	0.312	0.569
10	0.772	0.888	0.679	0.817
20	1.161	1.277	1.165	1.137
35	1.455	1.575	1.517	1.358
50	1.679	1.709	1.688	1.416
276 kPa				
2	0.440	0.395	0.106	0.406
5	0.526	0.603	0.357	0.569
10	0.763	0.888	0.696	0.817

Slip	Measured	Practical model	FEM model	New model
20	1.161	1.277	1.160	1.137
35	1.491	1.575	1.518	1.358
50	1.683	1.709	1.688	1.416
345 kPa				
2	0.460	0.395	0.136	0.406
5	0.527	0.603	0.375	0.569
10	0.808	0.888	0.708	0.817
20	1.161	1.277	1.157	1.137
35	1.509	1.575		1.358
50	1.674	1.709	1.690	1.416

Table 5.4 Measured and predicted motion resistance (kN) for the Michelin 12.5/75R20 XL under a 4.43 kN load

Slip	Measured	Practical model	FEM model	New model
69 kPa				
2	0.602	0.723		0.646
5	0.606	0.791		0.646
10	0.600	0.872	0.473	0.678
20	0.801	0.988	0.732	0.762
35	1.111	1.111	.920	0.955
50	1.250	1.200	1.120	1.317
138 kPa				
2	0.839	0.723	0.320	0.799
5	0.840	0.791	.459	0.798
10	0.801	0.872	.621	0.852
20	0.970	0.988	.812	0.941
35	1.183	1.111	1.040	1.128
50	1.319	1.200	1.165	1.451
207 kPa				
2	0.860	0.723	0.410	0.799
5	0.860	0.791	0.518	0.798
10	0.831	0.872	0.696	0.852
20	1.013	0.988	0.866	0.941
35	1.241	1.111	1.098	1.128
50	1.366	1.200	1.210	1.451
276 kPa				
2	0.892	0.723	0.478	0.799
5	0.890	0.791	0.558	0.798
10	0.839	0.872	0.719	0.852

Slip	Measured	Practical model	FEM model	New model
20	1.060	0.988	0.915	0.941
35	1.291	1.111	1.129	1.128
50	1.437	1.200	1.228	1.451
345 kPa				
2	0.900	0.723	0.517	0.799
5	0.901	0.791	0.594	0.798
10	0.835	0.872	0.740	0.852
20	1.076	0.988	0.946	0.941
35	1.313	1.111		1.128
50	1.437	1.200	1.226	1.451

Table 5.5 Measured and predicted drawbar pull (kN) for the Michelin 12.5/75R20 XL under a 4.43 kN load

Slip	Measured	Practical model	FEM model	New model
69 kPa				
2	-.292	-.328		-.267
5	-.305	-.188		-.056
10	0.087	0.016	-.062	0.167
20	0.420	0.289	0.428	0.442
35	0.402	0.464	0.678	0.483
50	0.431	0.509	0.629	0.183
138 kPa				
2	-.477	-.328	-.246	-.393
5	-.412	-.188	-.244	-.229
10	-.051	0.016	0.017	-.035
20	0.191	0.289	0.366	0.196
35	0.267	0.464	0.500	0.230
50	0.361	0.509	0.544	-.035
207 kPa				
2	-.432	-.328	-.3613	-.393
5	-.369	-.188	-.206	-.229
10	-.058	0.016	-.017	-.035
20	0.148	0.289	0.299	0.196
35	0.214	0.464	0.419	0.230
50	0.313	0.509	0.478	-.035
276 kPa				
2	-.452	-.328	-.372	-.393
5	-.364	-.188	-.201	-.229
10	-.076	0.016	-.023	-.035

Slip	Measured	Practical model	FEM model	New model
20	0.101	0.289	0.245	0.196
35	0.201	0.464	0.389	0.230
50	0.246	0.509	0.460	-.035
345 kPa				
2	-.440	-.328	-.381	-.393
5	-.374	-.188	-.219	-.229
10	-.027	0.016	-.032	-.035
20	0.085	0.289	0.211	0.196
35	0.196	0.464		0.230
50	0.237	0.509	0.464	-.035

Table 5.6 Measured and predicted sinkage (cm) for the Michelin 12.5/75R20 XL under a 4.43 kN load

Slip	Measured	Practical model	FEM model	New model
69 kPa				
2	5.0	6.3		5.1
5	5.0	6.8		5.1
10	5.0	7.6	5.1	5.3
20	5.6	9.0	6.7	5.7
35	7.1	10.4	8.3	6.8
50	9.0	11.7	10.0	9.6
138 kPa				
2	5.4	6.3	4.4	5.6
5	5.4	6.8	4.4	5.6
10	5.4	7.6	5.4	5.8
20	6.2	9.0	7.2	6.2
35	8.0	10.4	9.1	7.3
50	9.5	11.7	10.8	10.1
207 kPa				
2	5.8	6.3	4.1	5.6
5	5.8	6.8	4.8	5.6
10	5.4	7.6	6.0	5.8
20	6.5	9.0	7.6	6.2
35	8.1	10.4	9.6	7.3
50	9.8	11.7	11.2	10.1
276 kPa				
2	5.8	6.3	4.5	5.6
5	5.6	6.8	5.2	5.6
10	5.8	7.6	6.3	5.8

Slip	Measured	Practical model	FEM model	New model
20	6.5	9.0	8.0	6.2
35	8.0	10.4	10.1	7.3
50	9.8	11.7	11.5	10.1
345 kPa				
2	6.0	6.3	4.9	5.6
5	5.8	6.8	5.4	5.6
10	5.9	7.6	6.4	5.8
20	6.5	9.0	8.2	6.2
35	8.4	10.4		7.3
50	9.8	11.7	11.4	10.1

Table 5.7 Measured and predicted thrust (kN) for the Michelin 12.5/75R20 XL under a 7.45 kN load

	69 kPa	69 kPa	138 kPa	138 kPa	276 kPa	276 kPa
Slip	Measured	New model	Measured	New model	Measured	New model
2	0.150	0.835	1.001	0.977	0.951	0.977
5	0.191	1.256	1.101	1.293	1.010	1.293
10	1.299	1.725	1.702	1.651	1.610	1.651
20	2.350	2.285	1.947	2.128	2.002	2.128
35	3.001	2.626	2.509	2.468	2.491	2.468
50	3.299	2.710	2.871	2.573	2.809	2.573

Table 5.8 Measured and predicted motion resistance (kN) for the Michelin 12.5/75R20 XL under a 7.45 kN load

	69 kPa	69 kPa	138 kPa	138 kPa	276 kPa	276 kPa
Slip	Measured	New model	Measured	New model	Measured	New model
2	0.680	1.141	1.334	1.618	1.550	1.618
5	0.719	1.172	1.322	1.657	1.548	1.657
10	0.848	1.234	1.609	1.687	1.640	1.687
20	1.101	1.395	1.501	1.876	1.831	1.876
35	1.862	1.781	2.101	2.258	2.142	2.258
50	2.280	2.526	2.390	2.991	2.349	2.991

Table 5.9 Measured and predicted drawbar pull (kN) for the Michelin 12.5/75R20 XL under a 7.45 kN load

	69 kPa	69 kPa	138 kPa	138 kPa	276 kPa	276 kPa
Slip	Measured	New model	Measured	New model	Measured	New model
2	-.530	-.306	-.333	-.641	-.599	-.641
5	-.528	.084	-.221	-.364	-.538	-.364
10	0.451	0.491	0.093	-.036	-.030	-.036
20	1.249	0.890	0.446	0.252	0.171	0.252
35	1.139	0.845	0.408	0.210	0.349	0.210
50	1.019	0.181	0.481	-.418	0.460	-.418

Table 5.10 Measured and predicted sinkage (cm) for the Michelin 12.5/75R20 XL under a 7.45 kN load

	69 kPa	69 kPa	138 kPa	138 kPa	276 kPa	276 kPa
Slip	Measured	New model	Measured	New model	Measured	New model
2	7.0	6.9	7.3	7.9	7.7	7.9
5	6.9	6.9	7.2	8.0	7.7	8.0
10	7.1	7.1	7.6	8.1	8.0	8.1
20	7.4	7.6	8.7	8.7	8.5	8.7
35	8.2	9.3	10.7	10.3	9.8	10.3
50	12.3	13.4	12.3	14.6	13.1	14.6

Table 5.11 Measured and predicted thrust (kN) for the Michelin 12.5/75R20 XL under a 9.23 kN load

Slip	Measured	Practical model	FEM model	New model
69 kPa				
2	0.534			1.169
5	0.602			1.695
10	1.375			2.277
20	2.750		2.236	2.949
35	3.120		3.581	3.366
50	3.410		3.934	3.316
138 kPa				
2	0.571	1.196		1.276
5	1.009	1.625		1.699
10	1.450	2.178	0.566	2.194
20	2.201	2.879	2.652	2.840
35	2.808	3.359	3.486	3.227
50	3.198	3.559	3.723	3.257
207 kPa				
2	0.883	1.196		1.358
5	1.009	1.625	-.209	1.771
10	1.420	2.178	1.250	2.239
20	2.232	2.879	2.696	2.789
35	2.795	2.359	3.629	3.164
50	3.067	3.559	3.419	3.194
276 kPa				
2	0.995	1.196	-.478	1.358
5	1.063	1.625	0.379	1.771
10	1.549	2.178	1.558	2.239

Slip	Measured	Practical model	FEM model	New model
20	2.300	2.879	2.728	2.789
35	2.820	3.359	3.375	3.164
50	3.013	3.559	3.575	3.194
345 kPa				
2	1.030	1.196	-.624	1.358
5	1.100	1.625	0.709	1.771
10	1.600	2.178	1.671	2.239
20	2.270	2.879	2.720	2.789
35	2.817	3.359	3.348	3.164
50	2.946	3.559	3.581	3.194

Table 5.12 Measured and predicted motion resistance (kN) for the Michelin 12.5/75R20 XL under a 9.23 kN load

Slip	Measured	Practical model	FEM model	New model
69 kPa				
2	0.960			1.524
5	1.001			1.564
10	1.080			1.646
20	1.102		1.519	1.865
35	1.380		2.305	2.389
50	1.971		2.605	3.421
138 kPa				
2	1.501	2.272		1.872
5	1.490	2.389		1.917
10	1.482	2.559	1.223	2.006
20	1.681	2.800	1.969	2.237
35	2.271	3.044	2.571	2.770
50	2.730	3.219	2.991	3.788
207 kPa				
2	1.871	2.272		2.108
5	1.803	2.389	1.008	2.196
10	1.821	2.559	1.656	2.277
20	1.990	2.800	2.236	2.482
35	2.480	3.044	3.187	3.009
50	2.820	3.219	2.995	3.973
276 kPa				
2	1.871	2.272	0.970	2.108
5	1.863	2.389	1.487	2.196
10	1.940	2.559	1.808	2.277

Slip	Measured	Practical model	FEM model	New model
20	2.220	2.800	2.348	2.482
35	2.591	3.044	2.888	3.009
50	2.901	3.219	3.281	3.973
345 kPa				
2	1.880	2.272	1.323	2.108
5	1.901	2.389	1.597	2.196
10	2.071	2.559	1.929	2.277
20	2.286	2.800	2.456	2.482
35	2.732	3.044	2.980	3.009
50	2.799	3.219	3.289	3.973

Table 5.13 Measured and predicted drawbar pull (kN) for the Michelin 12.5/75R20 XL under a 9.23 kN load

Slip	Measured	Practical model	FEM model	New model
69 kPa				
2	-.426			-.355
5	-.397			0.131
10	0.295			0.631
20	1.648		0.717	1.084
35	1.740		1.276	0.977
50	1.489		1.329	-.1042
138 kPa				
2	-.928	-1.075		-.599
5	-.481	-.763		-.218
10	-.028	-.380	-.657	0.188
20	0.520	0.079	0.683	0.603
35	0.537	0.315	0.915	0.457
50	0.468	0.340	0.732	-.531
207 kPa				
2	-.986	-1.075		-.750
5	-.794	-.763	-1.217	-.425
10	-.398	-.380	-.406	-.038
20	0.242	0.079	0.460	0.305
35	0.315	0.315	0.442	0.155
50	0.247	0.340	0.424	-.779
276 kPa				
2	-.874	-1.075	-1.448	-.750
5	-.794	-.763	-1.108	-.425
10	-.391	-.380	-.250	-.038

Slip	Measured	Practical model	FEM model	New model
20	0.080	0.079	0.380	0.305
35	0.229	0.315	0.487	0.155
50	0.112	0.340	0.294	-.779
345 kPa				
2	-.850	-1.075	-1.947	-.750
5	-.799	-.763	-.888	-.425
10	-.471	-.380	-.258	-.038
20	-.016	0.079	0.264	0.305
35	0.085	0.3158	0.368	0.155
50	0.147	0.340	0.292	-.779

Table 5.14 Measured and predicted sinkage (cm) for the Michelin 12.5/75R20 XL under a 9.23 kN load

Slip	Measured	Practical model	FEM model	New model
69 kPa				
2	8.0			7.8
5	8.8			8.0
10	9.0			8.2
20	9.8		9.6	8.8
35	11.5		13.3	10.7
50	14.0		15.2	15.7
138 kPa				
2	8.0	11.4		8.5
5	8.0	12.2		8.5
10	7.5	13.3	7.9	8.9
20	8.9	15.3	11.0	9.5
35	10.8	17.6	14.2	11.4
50	14.0	19.3	16.6	16.5
207 kPa				
2	8.7	11.4		9.0
5	9.0	12.2	6.8	9.2
10	9.0	13.3	9.0	9.4
20	9.7	15.3	11.8	10.0
35	11.6	17.6	17.5	11.9
50	14.4	19.3	14.7	16.9
276 kPa				
2	9.0	11.4	7.6	9.0
5	9.1	12.2	8.0	9.2
10	8.8	13.3	9.5	9.4

Slip	Measured	Practical model	FEM model	New model
20	10.2	15.3	12.2	10.0
35	12.1	17.6	15.4	11.9
50	15.0	19.3	18.2	16.9
345 kPa				
2	9.3	11.4	7.2	9.0
5	9.1	12.2	8.3	9.2
10	9.1	13.3	10.0	9.4
20	10.2	15.3	12.9	10.0
35	12.2	17.6	16.1	11.9
50	14.8	19.3	18.5	16.9

Michelin

12.5/75R20 XL

Terrain : Sand

122

Normal load : 4.43 kN

Inflation pressure : 207 kPa (30 psi)

Figure 5.15 Measured vs. simulated tire performance using the new model

Michelin
12.5/75R20 XL

Terrain : Sand

123

Normal load : 4.43 kN

Inflation pressure : 276 kPa (40 psi)

Figure 5.15 Measured vs. simulated tire performance using the new model

Michelin
12.5/75R20 XL

Terrain : Sand

124

Normal load : 4.43 kN

Inflation pressure : 345 kPa (50 psi)

Figure 5.17 Measured vs. simulated tire performance using the new model

Michelin
12.5/75R20 XL

Terrain : Sand

125

Normal load : 7.45 kN

Inflation pressure : 69 kPa (10 psi)

Figure 5.18 Measured vs. simulated tire performance using the new model

Michelin
12.5/75R20 XL

Terrain : Sand

126

Normal load : 7.45 kN

Inflation pressure : 138 kPa (20 psi)

Figure 5.19 Measured vs. simulated tire performance using the new model

Michelin
12.5/75R20 XL

Terrain : Sand

127

Normal load : 7.45 kN

Inflation pressure : 276 kPa (40 psi)

Figure 5.20 Measured vs. simulated tire performance using the new model

Table 5.15 Regression coefficients for fitted performance curves

Normal load 4.43 kN					
Tire Inflation Pressure	Tractive Performance	Regression Coefficients			
		Practical Model	FEM Model	New Model	Best Model
69 kPa (10 psi)	Thrust	.82	.89	.85	FEM
	Resistance	.81	.87	.91	New
	Drawbar Pull	.61	.56	.56	Practical
	Sinkage	.58	.88	.95	New
138 kPa (20 psi)	Thrust	.89	.81	.88	Practical
	Resistance	.92	.71	.93	New
	Drawbar Pull	.52	.04	.44	Practical
	Sinkage	.70	.85	.94	New
207 kPa (30 psi)	Thrust	.91	.84	.89	Practical
	Resistance	.92	.71	.93	New
	Drawbar Pull	.41	.49	.45	FEM
	Sinkage	.73	.82	.94	New
276 kPa (40 psi)	Thrust	.92	.86	.89	Practical
	Resistance	.86	.77	.91	New
	Drawbar Pull	.31	.47	.52	New
	Sinkage	.74	.80	.95	New
345 kPa (50 psi)	Thrust	.93	.86	.88	Practical
	Resistance	.86	.79	.90	New
	Drawbar Pull	.26	.53	.50	FEM
	Sinkage	.76	.85	.93	New

Table 5.16 Regression coefficients for fitted performance curves

Normal load 9.23 kN					
Tire Inflation Pressure	Tractive Performance	Regression Coefficients			
		Practical Model	FEM Model	New Model	Best Model
69 kPa (10 psi)	Thrust		.85	.72	FEM
	Resistance		.58	.24	FEM
	Drawbar Pull		.63	.30	FEM
	Sinkage		.91	.90	FEM
138 kPa (20 psi)	Thrust	.70	.74	.72	FEM
	Resistance	.50	.88	.68	FEM
	Drawbar Pull	.49	.42	.19	Practical
	Sinkage	.44	.80	.87	New
207 kPa (30 psi)	Thrust	.72	.69	.73	New
	Resistance	.72	.79	.73	FEM
	Drawbar Pull	.85	.29	.16	Practical
	Sinkage	.61	.74	.90	New
276 kPa (40 psi)	Thrust	.75	.64	.76	New
	Resistance	.78	.80	.77	FEM
	Drawbar Pull	.75	.38	.17	Practical
	Sinkage	.61	.80	.92	New
345 kPa (50 psi)	Thrust	.75	.62	.77	New
	Resistance	.81	.85	.77	FEM
	Drawbar Pull	.69	.06	.10	Practical
	Sinkage	.60	.76	.92	New

**Table 5.17 Regression coefficients for fitted performance curves
with the values at 50% slip neglected**

Normal load 4.43 kN					
Tire Inflation Pressure	Tractive Performance	Regression Coefficients			
		Practical Model	FEM Model	New Model	Best Model
69 kPa (10 psi)	Thrust	.78	.86	.83	FEM
	Resistance	.77	.86	.89	New
	Drawbar Pull	.56	.56	.62	New
	Sinkage	.53	.87	.96	New
138 kPa (20 psi)	Thrust	.86	.76	.91	New
	Resistance	.92	.67	.95	New
	Drawbar Pull	.33	.42	.71	New
	Sinkage	.67	.85	.94	New
207 kPa (30 psi)	Thrust	.89	.80	.94	New
	Resistance	.91	.72	.93	New
	Drawbar Pull	.42	.50	.75	New
	Sinkage	.71	.81	.93	New
276 kPa (40 psi)	Thrust	.90	.82	.93	New
	Resistance	.88	.74	.89	New
	Drawbar Pull	.39	.54	.72	New
	Sinkage	.71	.79	.94	New
345 kPa (50 psi)	Thrust	.92	.83	.91	Practical
	Resistance	.87	.77	.88	New
	Drawbar Pull	.35	.66	.69	New
	Sinkage	.74	.86	.92	New

**Table 5.16 Regression coefficients for fitted performance curves
with the values at 50% slip neglected**

Normal load 9.23 kN					
Tire Inflation Pressure	Tractive Performance	Regression Coefficients			
		Practical Model	FEM Model	New Model	Best Model
69 kPa (10 psi)	Thrust		.84	.64	FEM
	Resistance		.50	.22	FEM
	Drawbar Pull		.57	.53	FEM
	Sinkage		.91	.92	New
138 kPa (20 psi)	Thrust	.64	.70	.64	FEM
	Resistance	.40	.87	.76	FEM
	Drawbar Pull	.47	.42	.62	New
	Sinkage	.37	.79	.91	New
207 kPa (30 psi)	Thrust	.67	.62	.66	Practical
	Resistance	.68	.75	.81	New
	Drawbar Pull	.86	.29	.57	Practical
	Sinkage	.58	.69	.97	New
276 kPa (40 psi)	Thrust	.71	.58	.71	Practical - New
	Resistance	.75	.78	.85	New
	Drawbar Pull	.83	.39	.55	Practical
	Sinkage	.57	.81	.97	New
345 kPa (50 psi)	Thrust	.73	.57	.73	New
	Resistance	.80	.85	.89	New
	Drawbar Pull	.72	.05	.47	Practical
	Sinkage	.56	.76	.98	New

Chapter VI

Discussion and Conclusions

6.1 Discussion

The performance predicted using the new tire model is generally in good agreement with measured data. The comparisons of performance curves in Tables 5.15 and 5.16 indicate that the new model predicts the performance of the tire best in 18 out of 40 cases with the FEM and practical models trailing with 12 out of 40 cases and 10 out of 40 respectively. In comparing the new model with the FEM model the new model predicts tire performance better in 24 out of 40 cases. In comparing the new model with the practical model the new model predicts the tire performance better in 27 out 40 cases. Even though the FEM model seems to have the best predictions at low inflation pressure it should be pointed out that at these low inflation pressures the FEM model is unable to predict the tire performance at the low slip values. A regression coefficient was calculated based on the points available but in some cases the FEM model only provided performance predictions for 3 of the 6 desired slip values.

Use of the method proposed by Wong for rigid wheels, which takes into account the effects of slip on the normal pressure distribution and the experimentally determined

coefficients C_1 and C_2 has resulted in an improved predictions of pressure distribution and tire performance. Based on the encouraging performance predictions at various loads it is felt that this approach could be successfully applied to performance prediction of similar tires over similar terrains. However, to apply this methodology to different tire-terrain systems more experimental work or further analytical studies will have to be done to determine the appropriate values of the coefficients C_1 and C_2 which identify the location of the maximum normal pressure.

As can be seen in Tables 5.3 - 5.14 the predictions obtained using the elastic tire model are better than if the rigid tire model had been used. This indicates that the current scheme to determine the operating mode has resulted in improved performance predictions and appears to be adequate for the range of parameters tested.

It appears that representing an elastic tire as a rigid wheel with a larger radius is reasonable. Published results have shown that the pressure distribution beneath a rigid and an elastic tire are very similar and that the contact surface of an elastic tire on soft soil is not necessarily flat. Using this simplification, the elastic wheel can fit into the framework of the current rigid tire model. Even though the predictions are encouraging it should be pointed out that the actual deformation of the tire will be different from that predicted by the new model. Due to the belts and cords in the actual tire a deformation will exist past the edges of the contact area. Also when a tire experiences significant deformation a bulge appears as shown in Figure 5.10 effectively increasing the tire contact width. This bulge appears to be at its maximum at the bottom dead centre and decreases as it goes around the rim. Due to these simplifications the model over predicts

the tire sinkage. Due to the cohesionless nature of the soil used in this study and the inability to measure the rut depth the sinkage of the tire was measured by monitoring the vertical movement of the carriage. It should be pointed out that in the cases when the inflation pressure of the tire is low this sinkage measurement includes the deformation of the tire. This indicates that at low inflation pressures the measured sinkage is also likely high.

With the high computing power required to run the finite element tire model, it is felt that it will still be some time before it could be widely used in practice. In Chapter I it was shown that for a tire with high inflation pressure, performance predictions using either the FEM model or the practical model were quite similar. The FEM tire model might improve predictions around the elastic-rigid transition. Even if it provides an improved prediction, the computing power and time required to run such a model might not make it a cost-effective option in many cases.

It is well known that the increase in sinkage with increasing slip has been attributed to the slip-sinkage phenomenon. In this study the normal pressure was considered a function of slip. Using this normal pressure an increase in sinkage was predicted with an increasing in slip. This modified pressure distribution then seems to have indirectly incorporated the effects of slip-sinkage.

To improve the new model proposed the effect of the "elastic rebound of the terrain" at the rear part of the contact area should be taken into account. This "elastic rebound of the terrain" could be attributed to actual elastic recovery of the terrain or to the soil flow caused by the slip of the tire. It has been pointed out by Wong [11], that

if the wheel is not very wide and the slip value is not very high, the effect of the elastic rebound of the terrain can often be neglected. In the new model developed, the thrust and motion resistance predictions get worse with increasing slip. Incorporating the effect of the elastic rebound of the terrain would improve predictions at higher slip values by extending the tire-terrain contact surface. Increasing the contact area would increase the shearing surface of the tire thus increasing the thrust. This increase in contact area would also result in a decrease in the motion resistance, since the added horizontal component of the normal pressure would be facing the direction of wheel travel. Tables 5.17 and 5.18 show that if the tire performance predictions at 50% slip are neglected the number of cases in which the new model gives the best predictions goes up substantially. In a direct comparison with the FEM model the new model predicts better in 34 out of 40 cases. In a direct comparison with the practical model the new model also predicts better in 34 out 40 cases.

6.2 Conclusions

Based on the results of this study the following conclusions were drawn.

- 1). Taking into account the shift of the location of the peak normal pressure and the implementation of the new procedure for predicting the normal pressure distribution, the new model provides a significant improvement in the prediction of the overall performance of a tire on sand.

- 2). The performance of a tire in the elastic operating mode with significant deflection can be predicted with reasonable accuracy by considering it equivalent to a rigid wheel with larger effective radius. The proposed procedure for determining the effective radius of an equivalent rigid wheel, representing the elastic tire, appears to work well.
- 3). Performance predicted using the new tire-terrain interaction model proposed in this thesis is in closer agreement with the measured data than that predicted using previously developed models.
- 4). Reasonable operating mode predictions for the Michelin 12.5/75R20 can be obtained with the method based on the nominal pressure outlined in Section 4.2.
- 5). The improved tire performance predictions based on a slip dependent normal pressure distribution partially attributed to the slip-sinkage phenomenon, indicate that slip-sinkage should not be neglected when predicting tire performance on frictional terrains.
- 6). The bevameter technique provides a reasonable method for characterizing the behaviour of a frictional terrain. The pressure-sinkage and shear stress-shear displacement relationships measured with the bevameter (outlined in Chapter II) provide an adequate representation of terrain behaviour for tire performance studies.

6.3 Recommendations

Based on the above study the following recommendations can be made.

- 1). Further study on the mechanism of slip-sinkage.
- 2). Further study of the effects of slip on the normal pressure distribution on the tire-terrain interface by either:
 - a) measuring the pressure distribution under a variety of tires on various soils, and developing an empirical procedure for predicting the location of the peak normal pressure.
 - b) an analytical method for predicting the shift of the location of the peak normal pressure with slip, possibly based on the theory of plastic equilibrium of soil mechanics.
- 3) Taking into account the effects of "elastic rebound of the terrain". This will result in a more realistic model and will improve the accuracy of predictions.

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Appendix A1

A1 -Tire performance prediction code


```

C   WHICH REPLACES THE BECKER PRESSURE-SENSAGE
C   EQUATION
C
C   IT ALSO INCORPORATES REPETITIVE SHEARING WHICH
C   IS APPLICABLE FOR SANDY TERRAIN
C
C
C   INTEGER FPL,FPLC,FPLF,FPC,FPLAQ,Y,
1   FPLF,FPLC,FPLF,FPCF
REAL ALPHA,PO,KC,HT,TEST,X(1),Z(1),BL,BLUG,FJ,TRC
REAL FPL,FPLC,FPLF,FPC,FACT(1),L1
1
PHOM,T,ALPHA1,B,RAA,ANG,ZC,TOT,LOAD,X,CT,FIL,KT,
1
2
X(1),X(1),Z(1),ALPHA1,X(1),Z(1),FJ,AL1
REAL A,B,X,B,X,P,W,TH,RES,SD,Z(1),AS,B,X,K,X,AL,ZE,TR,MO,M,L1
1
2,TR,TRC,TRHS
CHARACTER*7 MODE
REAL FJ,FJ,FJ,FJ
1
2 FPL,FPLC,FPLF,FPC,FACT1
EXTERNAL FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ
C
C   OPENING THE FILE
C   TYPICALLY READ FROM DATAIN
C
C
C
C
C   READ(X,KC,KON
C   READ(X,DET,BLUG,HT,P
C   READ(X,LOAD,PO,ALPHA
C   READ(X,C,FPL,CT,TR,ST
C   READ(X,AS,B,X
C   READ(X,ZE,FACT,FACT1,L1
C
C   ZE IS EDGEAGE, FACT IS C1, FACT1 IS C2, AND L1 IS AN
C   EXTRA VARIABLE THAT CAN BE PASSED TO THE INTEGRATOR IF
C   DESIRED
C
C   THE INPUT EDGEAGE ZE IS SET TO ZT
C
C
C   ZT-ZE
C
C
C   CALCULATE COMMONLY USED VALUES IN SUBROUTINE VALUES
C
C   CALL VALUES(ZL,D,ZT,X(1),BL,ST,BLUG,FJ,KC,
1   KON,HT,LSC,KC,PC)
C
C
C   CALCULATE THE TERRAIN VERTICAL REACTION FORCES
C   USING VERTICAL
C
C
C
C   CALL VERTICAL (FPL,FPL,FPLC,FPLF,FPLF,FPC,FPCF,
1   KCON,KC,KRAA,HT,CT,C,FPL,FJ,FJ,FJ,
2   FPL,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,
3   FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,
4   FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,
5   FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,
6   FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ)
C
C
C   HERE THE FORCES DUE TO SHEAR ARE CALCULATED INCORPORATING
C   REPETITIVE SHEARING, FOR LUGS AND CARCASS RESPECTIVELY
C
C
C   CALL RESHEAR (RAA,ZT,XC,BE,ANG,ZC,FJ,KT,TR,ST,FJ,L1,FTR)
C
C   CALL RESHEAR (RAA,ZT,XC,BE,KON,KC,FJ,TRC,ST,FJ
1   ,HT,AT,TRC)
C
C
C   RED=FPL,FPLC,FPLF,FPLC
C   TRHS=TR+FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ,FJ
C
C
C   WRITE OUTPUT FILE
C
C   WRITE(4,*) ' THE FOLLOWING IS THE OUTPUT FROM THE '
C   WRITE(4,*) ' PROGRAM SLIPST ON TIRE PERFORMANCE '
C   WRITE(4,*)
C   WRITE(4,*) ' THE TOTAL EDGEAGE IS 'ZT,' METERS'
C   WRITE(4,*)
C   WRITE(4,*) ' RES FORCES ARE '
C   WRITE(4,*) FPL,FPLC,FPLF,FJ,FJ,FJ
C   WRITE(4,*) ' THE THRUST IS 'TRHS,' KN'
C   WRITE(4,*) ' THE RESISTANCE IS 'RES,' KN'
C   WRITE(4,*)
C   WRITE(4,*) ' LOAD IS 'LOAD,' KN ' AND SLIP IS 'L1,'
C   WRITE(4,*)
C   WRITE(4,*) ' RADIUS OF WHEEL IS 'RAA,' METERS'
C   WRITE(4,*) ' THE CONSTANTS KC,KON ARE 'KC,KON
C   WRITE(4,*)
C   WRITE(4,*) ' THE RESHEARING RESIST. AND THRUST ARE BELOW '
C   WRITE(4,*) ' THE RESHEAR LUG FORCE INCREASING PRESSURE IS '
1   ' TR,' KN'
C
C   WRITE(4,*) ' THE RESHEAR CARC FORCE INCREASING PRESSURE IS '
1   ' TRC,' KN'
C
C   WRITE(4,*)
C   WRITE(4,*)
C   WRITE(4,*) ' FLAGS BELOW SHOULD BE 0 FOR SUCCESS INT.'
C   WRITE(4,*) FPL,FPLC,FPLF,FJ,FJ,FJ
C   WRITE(4,*) FPL,FPLC,FPLF,FJ,FJ,FJ
C   WRITE(4,*) ' THE ENTRY ANGLE LUGS IS 'ALPHA1,' RADIAN'
C   WRITE(4,*) ' THE ENTRY ANGLE FOR THE CARCASS IS 'AI,' RADIAN'
C   WRITE(4,*)
C   WRITE(4,*) ' THE SHEAR ZONE SEPARATION ANGLE AT LUGS IS '
1   ' L1,' RADIAN'
C   WRITE(4,*) ' THE SHEAR ZONE SEPARATION ANGLE AT CARCASS IS '
1   ' L2,' RADIAN'
C   WRITE(4,*)
C   WRITE(4,*) ' THE EQUIVALENT GROUND PRESSURE IS 'PO,' KPA'
C   WRITE(4,*)
C   WRITE(4,*) ' VALUES FACT,FACT1,L1 FROM INPUT ARE BELOW '
C   WRITE(4,*) FACT,FACT1,L1
C   WRITE(4,*)
C   STOP
C   END
C
C
C   ALL SUBROUTINES AND FUNCTIONS BELOW
C
C
C
C   SUBROUTINE VALUES(ZL,D,ZT,X(1),BL,ST,BLUG,FJ,KC,
1   KON,HT,LSC,KC,PC)
C
C   THIS CALCULATES VALUES NEEDED TO CALC FORCES
C
C   REAL L2,L,D,ZT,BL,ST,BLUG,FJ,KC,
1   KON,HT,LSC,KC,PC
C
C
C   AT THIS ZT THE FJUGS AND PCARC ARE CALCULATED
C
C
C   L2=SQRT((D/2)**2-(D/2-ZT)**2)
C   BL=(D/2)*BLUG/L2
C   PL=(KC/BL+KON/ZT)**2
C
C
C   IF (ZT .GT. HT) THEN
C   L2=SQRT((D/2-HT)**2-(D/2-HT-ZT+HT)**2)
C   BL=(D/2)*BLUG/L2
C   PL=(KC/BL+KON/ZT)**2
C
C
C   ELSE
C   PL=0
C
C   ENDIF
C
C   RETURN
C   END
C
C
C

```



```

      REAL FFL,FLC,FSL,FSC,FACT1,1,
1  PNOBT,ALPHA1,RAA*PIEC,PTOT,LOAD,X,CT,FU,KT,
2  KUP,T,KL,LCALPHA1,KDC,ET,SC,PC,KL,MLA1,
      REAL AB,DT,BCP,W,TR,RE,LS1,3
      CHARACTER*1 MODE
      REAL FFI,F2,F3,F4,FACT1
2  FFI,FLC,FSL,FSC,TR,TRC,TRIS
      REAL ZTRANS,AA,C1,C2,RAD,DET,DIAM,RS
      EXTERNAL FFI,F2,F3,F4,F5,F1,F3,F4,F5
C
C  OPENING THE FILES
C  TYPICALLY READ FROM DATA IN
C
      READ(1)'KCK,KAM
      READ(1)'DET,ST,BLUG,FL,KC,
      READ(1)'LOAD,PC,ALPHA
      READ(1)'X,CL,PC,CT,PT,KT
      READ(1)'FACT,FACT1,1,ZTRANS,AA
C
C
C  THIS IS THE SLIP
C
C  ALPHA2=1/100
C
C
      CALL VALRES(2,1,DET,ST,BLUG,FL,KC,
1  KAM,HT,LC,BC,PC)
C
      ZT=ZTRANS
      RAD=1/10
C
C  HERE THE ITERATIVE LOOP STARTS TO MARK W LOW IN THE
C  THE EXACT BEHAVIOR
C
C  DETAILER: BEHAV VARIABLES
C
C
C  DO 30 Y=1,120
C
C
      CI=(RAD-AA-ET)
      C=RAD-ET
      DET=(C**2)*(C**2)*(C**2)*(RAD-CI)
      RAA=RAD-DET
      WRITE(4,*) 'THE NEW EFFECTIVE DIAMETER IS 'RAA*2
      D=RAA*2
C
      CALL VALRES(2,1,DET,ST,BLUG,FL,KC,
1  KAM,HT,LC,BC,PC)
C
C
C  CALCULATE THE VERTICAL FORCES
C  USING VERTICAL
C
C
      CALL VERTICAL (FFI,FFI,FLC,FLC,FSL,FSL,FSC,FSC,
1  NO,DE,CE,LE,RAA,ST,CT,CL,PC,PT,KT,
2  KUP,KT,LCALPHA1,KDC,PC,KL,MLA1,KT,
3  ML,AL,LS1,FFI,FFI,FFI,FLC,FLC,FLC,
4  FSL,FSL,FSC,FSC,FACT,FACT1)
C
C
C  HERE THE SHEAR FORCES ARE CALCULATED INCORPORATING THE
C  REPETITIVE BEHAVIOR
C
C
      CALL BEHEAR (RAA,ET,KCL,JE,JE,DET,KT,TR,
1  ST,PC,LS,TRC)
C
      CALL BEHEAR (RAA,ET,KDC,CE,CE,DET,FLC,TRC,
1  LS,PC,LS,TRC)
C
C  HERE THE VERTICAL FORCES ARE SUMMED TO SEE IF EQUILIBRIUM
C  HAS BEEN REACHED
C

```

```

C  PTOT=FFI+FLC+FSL+TR+TRC+FFI+FLC+FSL+FSC
      TEST=(0.0001)-(PTOT)
      WRITE(4,*) 'PTOT',PTOT
C
C  IF THIS IS TRUE EXIT LOOP
C
      T=ABS(TEST)
      IF (T .LE. 0.001) GO TO 30
C
C  IF NOT READY TO EXIT LOOP THEN IT
C  IS TIME TO MODIFY ET
C
      IF (TEST .GT. 0.000) THEN
        ZT=ZT+0.0005
      ELSE
        ZT=ZT-0.0005
      ENDIF
      WRITE(4,*) 'THE TIE BEHAVIOR IS 'ZT
20  CONTINUE
      WRITE(4,*) 'TOO MANY STEPS SOMETHING WRONG '
30  CONTINUE
C
      STOP
      END

```

END

3 1 - 0 5 - 9 5

FIN