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**AN ANALYSIS METHOD FOR THE
VISCOUS /INVISCID FLOW AROUND
AIRFOILS WITH EXTENSIVE TRAILING
EDGE SEPARATION INCLUDING SURFACE
ROUGHNESS EFFECTS**

**Sean Adam Bittle
B.E.Sc. (Mechanical)**

*A Thesis submitted to the Faculty of Graduate
Studies and Research in partial fulfillment
of the requirements for the degree
of Master of Engineering*

**DEPARTMENT OF MECHANICAL AND AERONAUTICAL ENGINEERING
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OTTAWA, ONTARIO
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The undersigned recommend to the Faculty of Graduate Studies and Research acceptance of the thesis;

**An Analysis Method for the Viscous-Inviscid Flow
around Airfoils with Extensive Trailing Edge Separation
including Surface Roughness Effects.**

submitted by Sean Adam Bittle, B.E.Sc, in partial fulfilment of the requirements for the degree Master of Aeronautical Engineering.

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Abstract

A computational method is developed which can solve the flow field around airfoil sections with extensive trailing edge separation. The algorithm involves a coupled, iterative solution between potential flow and boundary layer methods and is designed to treat single element airfoils in a steady incompressible flow field.

The separated flow region is modelled in the potential flow analysis using constant strength free vortex sheets, requiring an inner iteration to establish their force-free shape. A semi-empirical approach is used to determine the length of the wake sheets. Boundary layer displacement effects are modelled using the transpiration (i.e. *source distribution*) technique. The Karman-Tsien compressibility correction is applied.

The laminar boundary layer development is calculated by Thwaites method. Transition is predicted using the $H-R_s$ scheme as well as a forced transition criterion. Head's entrainment method is used to predict the development of the turbulent boundary layer. Surface roughness effects are included in the turbulent boundary layer analysis.

The iterative procedure involves passing the potential flow pressure distribution to the integral boundary layer analysis which supplies the separation points and source distribution for the next iteration.

Calculated and experimental results are compared for a series of airfoils. Comprehensive computations were carried out in order to assess the stability and convergence characteristics of the potential flow separated wake model. Calculated and experimental results are compared for a number of laminar and turbulent boundary layer test cases.

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In conclusion, I would like to thank my family; Mom, Dad, Kevin, Trilby and Andre, who from the beginning gave their complete support, knowing it was possible for me to succeed.

*Aerodynamics is traditionally the
subject of specialists¹*

¹ John E. Allen, *Aerodynamics; The Science of Air in Motion*, 2nd Ed., Grenada Publishing, 1982 (Preface)

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Nomenclature

A	-	Set of airfoil influence coefficients given by equation (4.7b), also denotes overall Matrix of airfoil and wake influence coeffs.
B	-	Upper wake influence coefficients given by equation 5.6a.
C	-	Lower wake influence coefficients given by equation 5.6b.
$x_k(1-4)$	-	Coefficients used in equations 4.4 and 7.3 given by equations 4.5a-d.
$x_{kc}(1-2)$	-	Coefficients used in equation 5.2 given by equations 5.3a,b.
x, y	-	Cartesian coordinates, parallel and normal, respectively, to section chordline.
r, θ	-	Polar coordinates, see figures 5 and 6.
s	-	Distance measured along the airfoil
l	-	Panel length.
u, v	-	Velocity components induced by a source or vortex sheet tangential and normal, respectively, to itself.
U_∞	-	Free stream velocity
V_i	-	Local velocity at the outer edge of the boundary layer.
C_o	-	Chord length of airfoil.

P	-	Static Pressure.
q	-	Boundary layer source strength/unit length, $\frac{d}{dx}(\rho V_\infty \delta^*)$.
M	-	Free stream mach number.
k	-	Roughness height.
N	-	Number of panels comprising airfoil.
NPU	-	Number of panels comprising upper wake surface.
NPL	-	Number of panels comprising lower wake surface.
ΔS	-	Boundary layer step size

Greek Symbols

γ	-	Vortex Strength.
α	-	Airfoil angle of attack.
$\Delta\beta$	-	Local wake surface correction angle.
Θ	-	Boundary layer momentum thickness $\int_0^{\delta} \frac{V}{V_{\infty}} (1 - \frac{V}{V_{\infty}}) dy$
δ^*	-	Boundary layer displacement thickness $\int_0^{\delta} (1 - \frac{V}{V_{\infty}}) dy$
ω	-	Convergence criterion.
δ	-	Boundary layer thickness.
ν	-	Kinematic viscosity.
μ	-	Dynamic viscosity.
ρ	-	Density.
τ	-	Shear stress.
τ_w	-	Local wall shear stress.
Γ	-	Circulation around airfoil section
ξ	-	Point relaxation factor.
ψ	-	Stream function.
ϕ	-	Potential function, also angle between x-axis and vortex sheet.
U_r	-	Friction Velocity, $\sqrt{\frac{\tau_w}{\rho}}$

Dimensionless Quantities

C_f	-	Local skin friction coefficient, $\frac{\tau_w}{\frac{1}{2}\rho U^2}$.
H	-	Shape factor, $\frac{\delta^*}{\theta}$.
$H_{\delta-\delta^*}$	-	Shape factor, $\frac{(\delta-\delta^*)}{\theta}$.
R_c	-	Chord Reynolds number, $\frac{U_\infty C_a}{\nu}$.
R_s	-	Local Reynolds number, $\frac{U_{\infty s}}{\nu}$.
R_θ	-	Momentum thickness Reynolds Number, $\frac{U_\infty \theta}{\nu}$.
l	-	Thwaites alternative shear stress parameter, $\frac{1}{2} C_f R_\theta$.
ϵ	-	Thwaites dimensionless pressure gradient parameter, $\frac{\theta^2}{\nu} \frac{\partial V_\infty}{\partial x}$.
$\frac{U_r K}{\nu}$	-	Dimensionless roughness height.
$\frac{\Delta u_1}{U_r}$	-	Roughness contribution to law of the wall.
$\frac{\Delta u_2}{U_r}$	-	Pressure gradient contribution to velocity defect law.
C_p	-	Pressure coefficient, $\frac{(P-P_\infty)}{\frac{1}{2}\rho U_\infty^2}$.
λ	-	Roughness density parameter.
J	-	Pressure gradient parameter, $\sqrt{\frac{2}{C_f}} \left[\frac{(H-1)}{H} \right]$.
C_L	-	Section lift coefficient.
C_D	-	Section pressure drag coefficient.
$C_{M_{L.E.}}$	-	Section pitching moment coefficient about airfoil leading edge.
$C_{M_{\frac{1}{4}}}$	-	Section pitching moment coefficient about airfoil quarter chord point.

Subscripts

- i - Refers to airfoils or wake control points.
- m - Refers to airfoil surface junction points.
- l - Refers to lower wake surface junction points.
- u - Refers to upper wake surface junction points.
- L - Refers to lower wake.
- U - Refers to upper wake.
- e - Refers to edge of boundary layer.
- ∞ - Refers to undisturbed free stream condition.
- w - Refers to wall condition.
- n - Refers to normal direction to any panel.
- t - Refers to tangential direction to any panel.

Superscripts

- γ - Denotes quantity resulting from linear vortex distribution.
- q - Denotes quantity resulting from linear source distribution.
- c - Denotes quantity resulting from constant vortex distribution.

Chapter 1

INTRODUCTION

1.1 Background

An aircraft wing is said to *stall* when the airflow over the wing separates or breaks away from its upper surface. This occurs when the angle of attack of the wing exceeds a certain value. Stall places an upper limit on the lift coefficient that a wing can develop. Furthermore, the aerodynamic drag force increases drastically when stall occurs.

Wing stall has an important influence on aircraft design and performance. Landing and take-off speeds are governed by wing area and by the lift coefficient that the wing can develop without stalling. Prediction of wing stall has consequently been a long time goal of aerodynamic theory.

A complete three-dimensional study of the problem of wing stall is a formidable, if not impossible, undertaking at present. In a discussion by *McCullough and Gault* (1951), they point out that the variables in the stall of a complete wing are many and include such factors as airfoil shape, wing plan form, wing twist, fuselage and engine nacelle interference, free stream turbulence and Reynolds number. A related natural problem influencing wing stall is the accumulation of hoar frost on aircraft surfaces. The formation of frost can seriously alter the surface roughness characteristics of an aircraft wing. Physically, the added roughness increases the rate at which the boundary layer thickens, inducing flow separation. Frost accumulation poses the greatest threat at take-off since unpredicted flow separation can lead to significant loss of lift with possible catastrophic consequences.

Since the stall of a wing is the result of viscous effects, a complete understanding of it depends on an understanding of the mechanics of boundary layer separation, a flow phenomenon which has not been fully explained to date. Considerable insight into the problem of wing stall may however be gained by modelling the process of boundary layer

separation and its inherent aerodynamic effects in a two-dimensional flow field. A study of the stalling characteristics of airfoil¹ sections is, therefore, an important phase in the overall problem of wing stalling and can greatly assist the Aerodynamicist in the selection of airfoil sections for specific aviation applications.

1.2 Numerical and Experimental Approaches

Traditionally, the aerospace industry has placed heavy emphasis on experimental methods to determine the aerodynamic performance of airfoils. Considering an experimental approach, models must be designed, constructed and be compatible with the available wind tunnel testing facilities. Further, the wind tunnel must be capable of producing the required free stream conditions in the test section. Difficulties in interpretation of results can arise for tests involving scaled models since inaccuracies are introduced due to Reynolds number effects. This has traditionally led to risks in airfoil application and to considerable conservatism in design to reduce the risk. In addition, wind tunnels require large amounts of manpower and, sometimes, of energy for their operation; cost of experimental work is therefore high and testing must be kept to minimum.

With the advent of the digital computer, computational approaches to solving many fluid flow problems have become available. Although experimental testing continues to be extremely important, especially when the flows are very complex, the trend in the aerospace industry is clearly towards greater reliance on computer based prediction in design [Anderson et. al. 1984]. The trend can be largely explained by economics; over the years computer speed has increased much more rapidly than computer costs. The relative cost of computing a given flow field has decreased by almost three orders in magnitude during the past twenty years, and this trend is expected to continue in the near future [Anderson et. al. 1984].

The experimental approach to the study of airfoil stall has the capability of produc-

¹ For the purpose of this report the term *airfoil* refers to a two-dimensional lifting surface of non-zero thickness.

ing the most realistic results. Unfortunately, the generalizations one can make from the experimental data for a single airfoil shape are limited. The development of the boundary layer depends strongly on the imposed pressure gradient which is the result of the shape and orientation of the airfoil. An experimental study therefore is severely limited unless one is prepared to test a large number of configurations at great expense.

In contrast, a computational approach to the study of airfoil stall would allow for the analysis of arbitrary shapes and would be relatively inexpensive. All computational approaches however, have inherent drawbacks and limitations which include; computer storage, speed and our inability to mathematically model certain complex flow phenomena. Some of the advantages and disadvantages of the two approaches are summarized in table I.

1.3 Description of the Real Flow

The steady subsonic flow over a single element airfoil divides itself neatly into two regions; a thin viscous layer (boundary layer) next to the airfoil surface, where friction plays an important role, and the inviscid main flow outside the viscous layer. The main flow is almost irrotational since the shear is so low everywhere that viscous stresses impart a negligible rotation to the fluid. Furthermore, the region is almost incompressible since we are assuming the mach number is low enough to make compressibility negligible (i.e. $M < 0.3$). Thus the main flow is nearly ideal and can be solved by inviscid flow methods.

The boundary layer, which is confined to a region close to the airfoil surface, is predominantly influenced by viscosity. Fluid particles very close to the surface encounter molecular forces and adhere to it so that the fluid velocity increases rapidly away from the surface (thus creating viscous forces), until the main flow is reached. Prandtl pointed out that if the boundary layer remained sufficiently thin, its effect on the inviscid flow would be negligible, and the total flow field could be analysed as two separate, but coupled problems.

He further suggested that the boundary layer displacement effect could be accounted

Introduction

for in the inviscid analysis by adding the boundary layer displacement thickness, δ^* , to the airfoil surface.

The above matching of viscous and inviscid solution is, however, valid only for streamlined bodies without flow separation. The phenomenon of flow separation occurs when the flow in the boundary layer is reversed under the influence of a strong adverse (rising) pressure gradient, imposed on the boundary layer by the main inviscid flow. When separation occurs, the region where viscous effects are important grows to a size comparable to the airfoil chord, so that the whole concept of a streamline flow with a thin boundary layer is not applicable. Clancy (1975) notes that the most obvious aerodynamic effects of flow separation are:

- (a) The breakdown of the streamline flow.
- (b) The sudden loss of the adverse pressure gradient which led to the flow separation.
- (c) A large increase in the form drag of the airfoil, because of the drastic change in the effective shape of the airfoil.
- (d) A sudden loss in the lift developed by the airfoil.

Obviously, some type of separation model must be incorporated in the inviscid-viscous flow analysis, if calculations are to be carried out for cases involving extensive flow separation. This approach forms the basis of the present work. The steady, subsonic flow around a single element airfoil with trailing edge separation is illustrated in figure 1.

1.4 Objectives of Thesis

The objective of this research was the development of a computational algorithm for predicting the aerodynamic characteristics of airfoils with extensive areas of separated flow. Fundamental to this goal is the implementation of a separation model, incorporated into the potential flow analysis. The complete analysis is based on an iterative, coupled, potential flow—boundary layer interaction.

The emphasis of the research has not been on developing a single complete analysis

program. Instead, two independent programs (potential flow and boundary layer analysis codes) have been developed which can be externally coupled with a third service program to provide the full inviscid-viscous flow analysis. This approach was adopted in order to reduce development and ease future modification efforts.

Implicit to the research was the development of a useful engineering tool, rather than proposing a new state of the art analysis method. Therefore, whenever possible an effort was made to draw on the successful work of others in order to further reduce the development effort.

1.5 Scope of Thesis

The present computational analysis considers only single element airfoil sections in a steady, subsonic flow field. The separation model implemented in the present analysis was originally proposed by *Dvorak and Maskew* (1977) and is limited to the modelling of the trailing edge separation only. Two programs previously developed at Carleton University to determine the pure potential flow for a single element airfoil and the rough wall turbulent boundary layer parameters, were extended and used as the basic components in the inviscid and viscous flow analysis cases, respectively.

Numerical stability and convergence tests were conducted in order to assess the influence of various parameters on the potential flow separation model. Numerical and experimental results are compared for a series of airfoils with the separation points known. The boundary layer analysis was run against various test cases in order to validate its limitations and prediction capabilities. A sample inviscid-viscous analysis was carried out, for a series of airfoils, in order to assess the prediction capabilities of the complete analysis package. The number of inviscid-viscous iterations was an input parameter in the current study; convergence characteristics have not been investigated.

Chapter 2

HISTORICAL REVIEW

2.1 Introduction

The first theoretical treatment of fluid motion is credited to Leonard Euler, who in 1755 derived the equations of motion for a non-viscous, frictionless fluid. Even today, these equations remain essentially unchanged and form the basis of ideal-fluid theory, or *hydrodynamics*. Unfortunately, many of the theoretical results obtained from hydrodynamic theory stand in glaring contradiction to certain real flow phenomena. This was perhaps best illustrated by Jean d'Alembert who in 1752 published his famous paradox, showing that any body immersed in an inviscid flow would experience zero drag. Experiments, of course, showed that bodies immersed in real flows did experience a drag force, and in the case of bluff bodies the drag force is relatively large. Faced with the need to solve real flow problems, engineers developed the science of hydraulics, which abandoned theory entirely and relied totally on experimental measurements. The first real step towards unifying these two radically different branches of fluid mechanics occurred in 1904 when Ludwig Prandtl introduced the concept of a thin *boundary layer*, and showed the behavior of the fluid in this boundary layer could cause separation of the flow from the body. The resulting viscous-inviscid matching procedure has made viscous flow problems amenable to approximate analysis. The following is a brief review of inviscid flow theory, panel methods and viscous flow theory.

2.2 Inviscid Flow Theory

The simplest type of continuum flow is characterized as being incompressible (constant density), inviscid (neglects viscous effects) and two-dimensional. If the flow is irrotational as well (i.e. $\vec{V} = \nabla\phi$), the equation of mass conservation (continuity), can be expressed

as Laplace's equation:

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (2.1)$$

The solution of this equation, with correct boundary conditions, will completely define the velocity field at any point in the flow. The u and v components of velocity (parallel to the x and y axes respectively) are related to the stream function, ψ , and the potential function, ϕ , as shown below;

$$u = \frac{\partial \phi}{\partial x} = + \frac{\partial \psi}{\partial y} \quad (2.2)$$

$$v = \frac{\partial \phi}{\partial y} = - \frac{\partial \psi}{\partial x} \quad (2.3)$$

All points having the same value of the velocity potential lie on an imaginary *equipotential surface* which is everywhere at right angles to the streamline passing through it.

In order to determine the pressure variation throughout the flow field Bernoulli's equation can be applied, provided the flow remains irrotational. Inviscid flows past solid bodies must satisfy certain *boundary conditions* listed by Clancy (pg. 139—140, 1975):

- 1) Far from the body, the effect of its presence is not felt, so the velocity is simply equal to the free stream value.
- 2) Physically, there can be no flow across the solid boundary of the body. Thus the boundary must be a streamline of the flow.
- 3) There can be no singularities¹ within the region occupied by the fluid. Thus all the singularities must be located within or on the surface of the solid body.
- 4) Since there is no fluid crossing the boundary of the solid (flow tangency condition), continuity demands that, if the body is closed, there can be no fluid created or destroyed within the closed streamline that represents it.

A variety of techniques exist for solving the differential equations of incompressible two-dimensional steady flow. *Direct Methods* involve finding analytic solutions of

¹ The term *singularity* is used to describe sources, vortices, doublets and other fundamental solutions of Laplace's equation that are singular at some point, necessarily outside the flow field.

Laplace's equation which satisfy specified boundary conditions. This method is difficult to apply in practice and solutions exist in only the simplest cases. Other techniques include, indirect (*hit and miss*), graphical and analogue methods. The interested reader is directed to Clancy (1975) and Prandtl-Tsien (1934) for a more rigorous discussion of the solution techniques mentioned above.

An important class of solutions relies on the so-called *superposition technique*. Since Laplace's equation is linear² and homogeneous³, simple solutions of the the governing equation can be added or superimposed in order to create more complicated ones. In recent years powerful numerical procedures, *panel methods* have evolved from the superposition principle. A brief outline of the fundamental principles of panel methods is given below.

3.2.1 Panel Methods

All panel methods have their mathematical foundation in the previously described *superposition technique* of solving Laplace's equation. It is a numerical procedure, in which airfoils of non-zero thickness are represented by singularities distributed around the solid body in the flow field. The name itself is derived from the fact that the airfoil surface is approximated by a collection of *junction points* joined by straight line panels forming a closed polygon, as depicted in figure 2. Panel methods differ not only in the type of singularity (source or vortex) chosen, but also in the functional variation of the singularity distributed over the airfoil surface.

Fundamentally, Kind (1973) points out, if an airfoil contour is to be a streamline the singularity distribution must satisfy a Fredholm integral equation⁴ at all points on the contour. The integral equation is, however, difficult to evaluate even for simple forms

² A P.D.E. is said to be linear if it is of the first degree in the field variable (i.e. ϕ) and its partial derivatives.

³ A linear P.D.E. is said to be homogeneous if every term in the equation contains either the field variable (i.e. ϕ) or one of its derivatives.

⁴ The Fredholm integral equation represents the boundary condition of zero internal tangential velocity at an arbitrary point on the airfoil geometry.

of singularity strengths, unless the surface on which the singularities are distributed is a straight line. In the numerical solution, the airfoil contour is divided into intervals (panels) and the total integral taken as the algebraic sum of the integrals over each panel.

Historically, Prager (1928) was the first to formulate a general boundary condition equation for a continuous distribution of vortices on the surface of a single element airfoil. However, the classic panel method was devised by Smith and Hess (1966), the so-called Douglas-Neuman method was based on a constant strength of sources and vortices on planar panels. Moran (1984) points out, this numerical procedure made it possible for the first time to analyse flows past bodies of realistic geometry.

Some current panel methods developed by Kind (1973) and others (Dvorak et. al. (1974)) use plane panels with a linear vorticity distribution on each panel. Kind (1973) points out that this model comes quite close to retaining the essential physical requirements without being unduly complex. These panel methods replace the original contour by a polygon. To make the polygon at least approximate a streamline, it is required that the vorticity be continuous over the polygon⁵. This requirement is satisfied in Kind's approach. Kind notes that the attainment of a given level of accuracy requires the use of far fewer vortex panels than with the discrete vortex or uniform vorticity panel methods. This translates into savings in computer execution time and memory storage space.

In the present work, Kind's program for single element airfoils was extended to deal with airfoils with extensive trailing edge flow separation.

2.3 Viscous Flow Theory

The most fundamental physical-mathematical formulation for viscous fluid flow was first derived by C.L.M.H. Navier in 1823 and Sir George G. Stokes in 1845. The classic Navier-Stokes equations express the conservation of momentum principle for viscous fluids. They have an apparent but deceptive simplicity when written vectorially for the

⁵ The vorticity is in fact discontinuous at junction points unless the adjacent panels are co-planar.

incompressible, constant viscosity case; viz:

$$\rho \frac{D\mathbf{V}}{Dt} = -\nabla P + \mu \nabla^2 \mathbf{V} \quad (2.4)$$

Where the vector $\mathbf{V}$ is the velocity with components $u_i (i=1, 2, 3)$, $\frac{D}{Dt}$ represents the substantial time derivative and the symbol ∇^2 denotes the Laplace operator.

These equations are however very difficult to solve and to date only about seventy particular analytical solutions have been found, [White, (1974)]. Fortunately, as previously outlined, Prandtl showed that the flow about a solid surface could be divided into two regions: A thin layer (boundary layer) next to the solid surface where friction plays an important role and the inviscid region outside the layer where frictional effects could be neglected. He derived a simplified form of the Navier-Stokes equation, namely the boundary layer equations, which can be used in thin viscous flow layers. The boundary layer equations are derived by performing an order of magnitude analysis on the full non-dimensionalized Navier-Stokes equations. Second and higher order terms are neglected, based on the thinness assumption⁶, in order to simplify the original equations.

For a steady, thin, incompressible, two-dimensional flow, where the curvature of the surface is small, the boundary layer equations for laminar and turbulent flows take the form:

Laminar

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} \quad (2.5)$$

$$\frac{1}{\rho} \frac{\partial P}{\partial y} = 0$$

Turbulent

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{\partial}{\partial y} \left(\frac{\mu}{\rho} \frac{\partial u}{\partial y} - u'v' \right) \quad (2.6)$$

$$\frac{1}{\rho} \frac{\partial P}{\partial y} = 0$$

⁶ The validity of the thinness assumption depends on the Reynolds number, R_e . For the thinness assumption to hold the Reynolds number, R_e , must be large (say ≥ 100).

The x and y cartesian coordinates refer to the streamwise and normal directions respectively. The overbar denotes time-averaged quantities and the prime denotes turbulent fluctuation quantities: u and v are the velocity components in the x and y directions, respectively.

2.3.1 Integral/Differential Methods

A variety of laminar and/or turbulent boundary layer analysis methods exist, and generally fall under two broad categories, namely integral and differential methods. All integral methods, whether for laminar or turbulent flows are based on Von Karman momentum integral equation. Von Karman showed that a useful equation could be obtained by integrating the boundary layer equations across the layer. The resulting ordinary differential equation, shown below, relates a number of important boundary layer parameters.

$$\frac{\partial \Theta}{\partial x} = \frac{C_f}{2} - (H + 2) \frac{\Theta}{V_\infty} \frac{\partial V_\infty}{\partial x} \quad (2.7)$$

where

C_f = local skin friction coefficient

V_∞ = streamwise velocity at the outer edge of the boundary layer

$\Theta = \int_0^\delta \frac{V}{V_\infty} \left(1 - \frac{V}{V_\infty}\right) dy \equiv$ momentum thickness

$\delta^* = \int_0^\delta \left(1 - \frac{V}{V_\infty}\right) dy \equiv$ displacement thickness

$H = \frac{\delta^*}{\Theta} \equiv$ shape factor

One of the simplest and most accurate laminar boundary layer integral prediction methods was developed by Thwaites (1949). In his method, Thwaites assumed a one-parameter family of velocity profiles which he then used to solve the momentum integral equation. Other laminar flow integral techniques include the methods of Von Karman (1921), Pohlhausen (1921) and Curle (1967).

The unsteady eddying and random velocity fluctuations present in turbulent flows give rise to additional Reynolds stress terms in the time-averaged Navier-Stokes and boundary

layer equations (the $u'v'$ terms in equation (2.6)). However, these Reynolds stress terms result in there being more unknowns than equations. This is commonly referred to, in turbulence modelling, as the *closure problem*. Closure is generally achieved by assuming that some type of relationship exists between parameters such that the number of unknowns matches the number of equations. Relationships are developed from a combination of experimental data and/or theoretical hypotheses regarding the turbulence structure in the flow. Both integral and differential turbulence models are generally classified according to the type of assumptions used to achieve closure.

One of the simplest integral methods for turbulent boundary layers was developed by Head (1958). Head argued that the rate at which the turbulent boundary layer entrains free stream fluid should be a function of the velocity profile shape. He developed an entrainment equation which takes the form;

$$\frac{1}{V_*} \frac{\partial}{\partial z} (V_* \Theta H^*) = F(H^*) \quad (2.8)$$

and

$$H^* = G(H) \quad (2.9)$$

The functions F and G are both obtained from experimental results. To calculate the turbulent boundary layer development, equation 2.8 and the momentum integral equation are integrated in steps along the layer using F , G and a skin friction correlation $C_f = C_f(H, \Theta)$.

Differential methods solve numerically the full partial differential boundary layer equations (2.5, 2.6). They allow essentially exact solutions for the laminar boundary layer development to be obtained for any free stream conditions. In the case of turbulent flows, the turbulence model used to achieve closure is, however, not an exact representation of reality and even differential methods are therefore not exact. Yet differential methods allow a detailed picture to be obtained for the parameters (i.e. velocity profile) across the layer. This however is at the expense of greater computational complexity and cost. Numerically, *finite difference*, and more recently *finite element* techniques have been used to solve the governing differential equations of the fluid motion. An important

development in testing of turbulent layer calculation methods was a conference held at Stanford University in 1968. The conference served several purposes, namely;

- (i) Evaluate the prediction capabilities of the various turbulent analysis methods that are presently available.
- (ii) It pointed out the importance of testing the methods against a wide range of flows.
- (iii) It identified a hierarchy of flows of increasing difficulty to be used for testing calculation methods. In addition, the proceedings of the Stanford conference are an excellent source of information on the subject of turbulent boundary layers.

Integral methods were selected to carry out the required viscous flow analysis in the present study. They are relatively simple to implement and are computationally less expensive in comparison to differential methods, as Dvorak (1977) points out, integral methods are typically about one hundred times faster than finite difference methods and, for conventional boundary layers are equally accurate. Furthermore, they yield all the information required for the present purposes, namely skin friction, displacement thickness and separation positions.

Chapter 3

AIRFOIL SECTION STALL

3.1 Introduction

The study of the flow separation from two-dimensional airfoils and its inherent aerodynamic effects has captured the interests of many Aerodynamicists. In a discussion by Henderson (1978), he pointed out that the full computational analysis of airfoil sections has traditionally been limited by our inability to accurately predict the effects of separation on section forces and moments. The boundary layer on the airfoil surface will not always be thin enough for only a pure potential flow panel method analysis to give accurate results. At low angles of attack, where there is no separation, coupled inviscid-viscous analysis methods are generally adequate. However, when the angle of attack increases and flow separation begins to occur, predicted and measured results diverge. Obviously, as noted by Henderson, the aerodynamic influence of separation must be included, in some fashion, in order to predict correctly the various aerodynamic parameters. Figure 3 indicates the relative importance of a separation model on the predicted lift curve for a GA(W)-1 airfoil.

Over the past ten years a number of iterative, coupled, viscous-inviscid computational analysis programs have been developed for predicting the aerodynamic characteristics of airfoils up to and beyond stall. A number of these methods are briefly described below. In addition, the characteristic types of airfoil stall are discussed.

3.2 Previous Work : Separation Models

The first successful model for trailing edge separation was developed by Jacob (1967). To obtain a constant pressure region over the separated airfoil element, Jacob employed a

tailored source distribution in the separated flow region. This modelling resulted in a jet wake issuing from the trailing edge of the separated element. In general, Jacob's method predicted the upstream pressure distribution satisfactorily. However, the base pressure level was not consistent with experimental results [Dvorak and Maskew, 1977].

More recently several iterative, coupled viscous-inviscid programs have been developed for predicting the aerodynamic characteristics of stalled airfoils. The potential flow/boundary layer method developed by Barnwell (1981) utilizes a finite difference transonic potential flow method and a moment-of-momentum integral boundary layer calculation. It is designed to treat airfoils with substantial compressibility effects. Carlson (1980) developed a similar transonic, separated flow program utilizing a finite difference full potential code coupled with a Thwaites laminar boundary layer routine and a Nash-McDonald integral turbulent boundary layer method. Both programs are intended to model both subsonic and transonic trailing edge separation.

In contrast, the models developed by Dvorak and Maskew (1977), Henderson (1978), and Gilmer and Bristow (1982) are strictly valid for airfoils in a subsonic flow field. Dvorak and Maskew's code couples a separate inviscid vortex panel method and an integral boundary layer analysis numerically and solves iteratively for an effective inviscid streamline shape until the lift converges to within a specified tolerance. Free, constant strength, vortex sheets are used to construct the wake once separation is found. The length of the wake is prescribed using an empirical correlation curve developed by Dvorak and Maskew. The major assumption introduced in the model is that the pressure on the airfoil surface downstream of the separation point is constant. In the separated region, the effective streamline of the flow corresponds to the separated wake boundary, which is determined iteratively to satisfy the boundary condition that the shear layer is force free.

The coupling procedure implemented by Gilmer and Bristow (1982) involves a simultaneous matching of the inviscid and viscous solutions. According to Ramsey (1984), at the beginning of each iteration cycle, first order relationships are developed between perturbations to the unknowns (which represent the effective inviscid streamline geometry) and the resulting changes in inviscid and viscous quantities. A set of linear algebraic

equations results in which the solution establishes corrections to the original unknowns.

Mani (1983) has extended the separation model proposed by Dvorak and Maskew (1977) to compute the flow field around multi-element airfoil configurations. Dvorak and Choi (1983) have also modified the original incompressible flow separation model proposed by Dvorak and Maskew to handle transonic flow.

In a recent study by Blascovich (1983) the separation models developed by Dvorak and Maskew (1977) and by Barnwell (1981) were assessed to be the two most accurate production codes in comparative studies of five such programs¹ [Ramsey, 1984]. The subsonic flow separated wake model originally proposed by Dvorak and Maskew (1977) was implemented in the present study.

3.3 Characteristic Types of Airfoil Stall

The separation model implemented in the present study is restricted to modelling only trailing edge separation up to and beyond stall. However, three characteristic types of airfoil flow separation have been identified and are illustrated in figure 4. In a study by McCullough and Gault (1951), airfoil stalling is classified under three categories;

Trailing edge stall:

Separation first occurs at the airfoil trailing edge and moves forward with increasing angle of attack.

Leading edge stall:

Characterized by abrupt flow separation near the airfoil leading edge, generally without subsequent reattachment. It is generally associated with airfoils of moderate thickness (symmetrical sections with thickness/chord rates of approximately 0.09 to 0.015).

Thin airfoil stall:

Thin airfoil stall is generally preceded by a laminar separation bubble at the leading

¹ Three of the five codes were developed by: Dvorak and Maskew (1977), Barnwell (1981) and Carlson (1980).

edge. (Flow separation with subsequent reattachment at a point which moves progressively rearward with increasing angle of attack). This type of stall occurs on all sharp edge airfoils and some thin, rounded edge airfoils (thickness/chord ratios of 0.09 and less).

Of special interest is trailing edge separation/stall which is characteristic of most thick airfoil sections (thickness/chord ratios of approximately 0.15 and greater). McCullough and Gault point out, the lift curve generally shows a smooth, continuous variation from zero lift to well beyond stall. In addition the peak of the lift curve is rounded and loss of lift after stall is gradual. The pressure distribution in the separated flow region is generally constant.

Chapter 4

INVISCID FLOW ANALYSIS

4.1 Overview (Attached Flow)

The numerical procedure for solving the inviscid attached flow over a single element airfoil of arbitrary shape consists of replacing the airfoil contour by a number of plane uniform two-dimensional vortex sheets, whose strength varies linearly between junction points. In general the boundary condition of flow tangency on the airfoil surface is governed by the integral equation;

$$\int_c A\gamma(s)ds + U_\infty \vec{n} = 0 \quad (4.1)$$

where the influence coefficient, A , is the normal velocity component induced per unit vorticity at any point for which the boundary condition is being enforced. The symbol, c , represents the integration path around the airfoil surface. The numerical formulation of the panel method, given below, follows closely the derivation presented by Kind (1973).

4.2 Numerical Formulation

We require expressions for the velocity components $u_{i,m}^{\gamma}$ and $v_{i,m}^{\gamma}$ parallel and normal, respectively, to sheet m at any control point i . Consider figure 5; an element ds of the sheet will induce velocities at point i .

$$du = -\frac{\gamma d\theta}{2\pi} \quad (4.2a)$$

$$dv = -\frac{\gamma}{2\pi} \frac{dr}{r} \quad (4.2b)$$

The vortex strength $\gamma(s)$ is assumed to vary linearly between junction points, giving;

$$\gamma(s) = \left[\frac{\gamma_{m+1} - \gamma_m}{l_m} \right] s + \gamma_m \quad (4.3)$$

substituting equation (4.3) in equation (4.2) and integrating, yields

$$u_{i,m}^\gamma = zk(1)_{i,m} \gamma_m + zk(2)_{i,m} \gamma_{m+1} \quad (4.4a)$$

$$v_{i,m}^\gamma = zk(3)_{i,m} \gamma_m + zk(4)_{i,m} \gamma_{m+1} \quad (4.4b)$$

where the superscript, γ , designates a linear vorticity variation between junction points, and

$$zk(1)_{i,m} = \frac{1}{2\pi l_m} \left[(\theta_{i,m} - \theta_{i,m+1})(l_m - r_{i,m} \cos \theta_{i,m}) \right] - \frac{1}{2\pi l_m} \left[\ln \left(\frac{\sin \theta_{i,m+1}}{\sin \theta_{i,m}} \right) + r_{i,m} \sin \theta_{i,m} \right] \quad (4.5a)$$

$$zk(2)_{i,m} = \frac{1}{2\pi l_m} \left[(\theta_{i,m} - \theta_{i,m+1}) r_{i,m} \cos \theta_{i,m} \right] + \frac{1}{2\pi l_m} \left[\ln \left(\frac{\sin \theta_{i,m+1}}{\sin \theta_{i,m}} \right) r_{i,m} \sin \theta_{i,m} \right] \quad (4.5b)$$

$$zk(3)_{i,m} = \frac{1}{2\pi l_m} \left[\ln \left(\frac{r_{i,m}}{r_{i,m+1}} \right) (l_m - r_{i,m} \cos \theta_{i,m}) \right] + \frac{1}{2\pi l_m} \left[(\theta_{i,m} - \theta_{i,m+1})(r_{i,m} \sin \theta_{i,m}) + l_m \right] \quad (4.5c)$$

$$zk(4)_{i,m} = \frac{1}{2\pi l_m} \left[\ln \left(\frac{r_{i,m}}{r_{i,m+1}} \right) (r_{i,m} \cos \theta_{i,m}) \right] - \frac{1}{2\pi l_m} \left[(\theta_{i,m} - \theta_{i,m+1})(r_{i,m} \sin \theta_{i,m}) - l_m \right] \quad (4.5d)$$

Using this result, it can be shown that there is no velocity singularity at the junction points if γ is made continuous at these points and the adjacent panels are co-planar.

The boundary condition of flow tangency (i.e. $V_n = 0$) to any vortex panel i , when applied at the control point i , yields the equation:

$$\sum_{m=1}^N u_{i,m}^\gamma \sin(\phi_m - \phi_i) + \sum_{m=1}^N v_{i,m}^\gamma \cos(\phi_m - \phi_i) = -U_\infty \sin(\alpha - \phi_i) \quad (4.6)$$

The nomenclature is defined in figure 2. The terms on the left hand of equation (4.6) represent the velocity normal to panel i at control point i , resulting from the vorticity

distribution on panel m . The term on the right hand side of equation (4.6) represents the component of free stream velocity, U_∞ , normal to panel i at control point i . The sign convention assumes the inward normal direction to any panel is positive.

Applying equation (4.6) to each control point yields a system of linear algebraic equations, giving N equations in $(N + 1)$ unknowns, $\gamma_1, \gamma_2, \dots, \gamma_{N+1}$:

$$\sum_{m=1}^{N+1} A_{i,m} \gamma_m = -U_\infty \sin(\alpha - \phi_i) \quad i = 1, 2, \dots, N \quad (4.7a)$$

where

$$A_{i,m} = zk(1)_{i,m} \sin(\phi_m - \phi_i) + zk(2)_{i,m-1} \sin(\phi_{m-1} - \phi_i) \\ + zk(3)_{i,m} \cos(\phi_m - \phi_i) + zk(4)_{i,m-1} \cos(\phi_{m-1} - \phi_i) \quad (4.7b)$$

with

$$zk(2)_{i,0} = zk(4)_{i,0} = 0$$

and

$$zk(1)_{i,N+1} = zk(3)_{i,N+1} = 0$$

Since the boundary condition is only applied at a finite number of points, the true airfoil contour and the polygon of the vortex sheets can only approximately coincide with a streamline of flow yielded by the solution. The resulting set of algebraic equations can be represented as a matrix:

$$[A][\gamma] = [-U_\infty \sin(\alpha - \phi_i)] \quad (4.8)$$

One additional equation is required in addition to the set represented by equation (4.8) in order to obtain a unique solution. A special auxiliary condition is applied at the trailing edge of the airfoil, which supplies the $(N + 1)^{th}$ equation. This condition is known as the Kutta condition and is discussed later. The solution of the set of $N + 1$ equations gives us the unknown junction point vorticity values. Using these results in equation (4.4), the resultant velocities (parallel and normal to the airfoil surface) can be determined at

each control point. In addition, since the velocity inside the contour must be zero, the local strength, γ , of the vortex sheet must be equal to the local tangential velocity on the airfoil surface. Bernoulli's equation can then be used to determine the static pressure at the airfoil control and/or junction points.

4.3 The Kutta Condition

The Kutta condition is used to obtain a unique and realistic solution for the governing integral equation (4.1). It is applied to ensure a smooth flow over the trailing edge of the airfoil. Various forms of the Kutta condition exist and two versions are available for use in the program, namely;

$$\gamma_1 = \gamma_{n+1} = 0 \quad (4.9a)$$

[Stagnation point at the trailing edge]

$$\gamma_1 = -\gamma_{n+1} \quad (4.9b)$$

[Equal velocity on the upper and lower surfaces at the trailing edge]

Both versions give virtually identical pressure distributions except for cusped trailing edge airfoils, where, in the latter approach, the vortex strength tended to be extremely large. Since two equations are required to implement equation (4.9a), one equation from the matrix must be deleted. This means the boundary condition of-zero normal velocity is applied at one less control point at the surface.

The relaxation of the zero normal velocity condition should be chosen in an area where the variation in pressure is typically small. This is usually the case on the lower (pressure) surface of the airfoil for positive angles of attack. A numerical search technique is employed in order to determine this position, (numerically designated, NA) if the trailing edge stagnation point Kutta condition is implemented.

Chapter 5

INVISCID SEPARATION MODEL

5.1 Overview

The inviscid (attached flow) calculation procedure developed in the previous chapter is extended to treat single element airfoils with extensive trailing edge separation. The separated wake model originally proposed by Dvorak and Maskew (1977) is incorporated for this purpose. The model is based on the premise that the effects of a separated wake on an airfoil pressure distribution are known if the streamline displacement caused by the wake is known.

Specifically, the separation model uses a pair of constant strength vortex sheets to represent the shear layers which bound the separated region in the potential flow analysis. The basic problem is one of computing, in general, the shape of the wake boundaries in order to obtain the correct constant base pressure level in the separated region. Physically, the wake boundaries are free and as such cannot support any forces. The condition that the sheets be force free is obtained in an iterative cycle in which the sheets are moved until the force free positions are found. The governing integral equation (4.1) of flow tangency on the airfoil surface is modified to include the influence of the free shear layers, giving:

$$\underbrace{\int_c A\gamma(s)ds}_{\text{airfoil}} + \underbrace{\gamma_u \int_u Bds}_{\text{wake boundary (u)}} + \underbrace{\gamma_l \int_l Cds}_{\text{wake boundary (l)}} + \underbrace{U_\infty \vec{n}}_{\text{freestream}} = V_N \quad (5.1)$$

Where the constant values of the strengths of the upper and lower free vorticity sheets are used. The influence coefficient functions, A , B and C represent the normal velocity component due to a unit point vortex at the point associated with an element ds . The integration paths c , u and l are the airfoil contour, and the upper and lower free shear layers, respectively. The unknowns are the vorticity strengths on the airfoil, and of the upper and lower free vortex sheets represented by $\gamma(s)$, γ_u and γ_l , respectively. The

airfoil vorticity strengths are a function of position, whereas the remaining two (γ_u , γ_l) are constants. The right hand side of equation (5.1), V_N , is zero for the initial potential flow solution.

The numerical formulation for incorporating the separation model in the previously described method is developed below. Implicit to the airfoil separation model is the condition that, for positive angles of attack the lower surface flow separates at the airfoil trailing edge. The upper surface separation point is a variable and is constrained only to fall on an existing airfoil junction point. A modified technique is adopted for a bluff body analysis, allowing for a variable lower surface separation point, constrained as before to fall on an existing junction point. A separation model code designation has been adopted and will be used throughout this thesis;

- (i) *Airfoil separation model* designates a variable upper surface separation point, but a fixed trailing edge lower surface separation point.
- (ii) *Bluff body separation model* designates variable upper and lower surface separation points.

5.2 Numerical Formulation

In order to incorporate the wake surfaces in the potential flow analysis, expressions for the velocity components $u_{i,k}^c$ and $v_{i,k}^c$ parallel and normal, respectively to a wake sheet k are required¹. Integration of equations (4.2a, b) assuming the vorticity distribution on the free shear layers is constant between junction points, yields the expressions,

$$u_{i,k}^c = xkc(1)\gamma_k \quad (5.2a)$$

$$v_{i,k}^c = xkc(2)\gamma_k \quad (5.2b)$$

¹ The superscript, c , designates a constant vorticity between junction points. The subscript, k , designates either the upper wake panels ($k = u$) or the lower wake panels ($k = l$)

where

$$zk(1) = \frac{1}{2\pi}(\theta_{i,k} - \theta_{i,k+1}) \quad (5.3a)$$

$$zk(2) = \frac{1}{2\pi} \ln \left[\frac{r_{i,k}}{r_{i,k+1}} \right] \quad (5.3b)$$

A new boundary condition equation, for flow tangency can be developed by algebraically superimposing the influence of the upper and lower free shear layers on equation 4.6, yielding;

$$\begin{aligned} & \sum_{m=1}^N u_{i,m}^{\gamma} \sin(\phi_m - \phi_i) + \sum_{m=1}^N v_{i,m}^{\gamma} \cos(\phi_m - \phi_i) \\ & + \sum_{a=1}^{NPU} u_{i,a}^{\epsilon} \sin(\phi_a - \phi_i) + \sum_{a=1}^{NPU} v_{i,a}^{\epsilon} \cos(\phi_a - \phi_i) \\ & + \sum_{l=1}^{NPL} u_{i,l}^{\epsilon} \sin(\phi_l - \phi_i) + \sum_{l=1}^{NPL} v_{i,l}^{\epsilon} \cos(\phi_l - \phi_i) \\ & = -U_{\infty} \sin(\alpha - \phi_i) \end{aligned} \quad (5.4)$$

The nomenclature is shown in figure (6). Application of equation (5.4) results in N equations in $N + 4$ and $N + 5$ unknowns for the airfoil and bluff body separation models, respectively.

Simplifying equation (5.4), and using the results from section 4.2, yields;

$$\sum_{m=1}^N A_{i,m} \gamma_m + \gamma_a \sum_{a=1}^{NPU} B_{i,a} + \gamma_l \sum_{l=1}^{NPL} C_{i,l} = -U_{\infty} \sin(\alpha - \phi_i) \quad (5.5)$$

where

$i = 1, \dots, N + 4$: airfoil separation model

$i = 1, \dots, N + 5$: bluff body separation model

$A_{i,m}$ is the set of airfoil influence coefficients, given by equation (4.7b) and shown below for reference.

$$\begin{aligned} A_{i,m} = & zk(1)_{i,m} \sin(\phi_m - \phi_i) + zk(2)_{i,m-1} \sin(\phi_{m-1} - \phi_i) \\ & + zk(3)_{i,m} \cos(\phi_m - \phi_i) + zk(4)_{i,m-1} \cos(\phi_{m-1} - \phi_i) \end{aligned} \quad (4.7b)$$

with the conditions

$$zk(2)_{i,0} = zk(4)_{i,0} = 0$$

$$zk(2)_{i,NPUS} = zk(4)_{i,NPUS} = 0$$

$$zk(2)_{i,NPLS} = zk(4)_{i,NPLS} = 0$$

$$zk(1)_{i,NPLSS} = zk(3)_{i,NPLSS} = 0$$

$$zk(1)_{i,NPT} = zk(3)_{i,NPT} = 0$$

The subscripts (*NPUS*, *NPLS*, *NPLSS*, *NPT*) used in the above conditions are defined in figures 7b and 8b. They represent the numerical designation of various junction points associated with the airfoil and bluff body separation models, respectively.

The upper and lower wake influence coefficients, $B_{i,u}$ and $C_{i,l}$ respectively are given by

$$B_{i,u} = zkc(1) \sin(\phi_u - \phi_i) + zkc(2) \cos(\phi_u - \phi_i) \quad (5.6a)$$

$$C_{i,l} = zkc(2) \sin(\phi_l - \phi_i) + zkc(2) \cos(\phi_l - \phi_i) \quad (5.6b)$$

The constant strength upper and lower wake surface vorticity values γ_U and γ_L are computationally designated γ_{NPT1} and γ_{NPT2} respectively in the program.

The assembled matrix equation takes the form;

$$[A][\gamma] = [BR] \quad (5.7)$$

where $[A]$ is the set of airfoil, upper and lower wake influence coefficients, which take the form;

$$[A] = \begin{bmatrix} A_{1,1} & A_{1,2} & \dots & A_{1,N+2} & B_{1,NPT1} & C_{1,NPT2} \\ A_{2,1} & & & & & \vdots \\ \vdots & & & & & \vdots \\ A_{N,1} & \dots & \dots & A_{N,N+2} & B_{N,NPT1} & C_{N,NPT2} \end{bmatrix} \quad (5.8)$$

$[\gamma]$ is the set of unknown airfoil junction point and free wake sheet vortex strengths.

given by;

$$[\gamma] = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_{n+2} \\ \gamma_{NPT1} \\ \gamma_{NPT2} \end{bmatrix} \quad (5.9)$$

and BR is the onset term represented in the initial potential flow analysis by the free stream velocity component normal to any airfoil control point, given by;

$$[BR] = -U_\infty \begin{bmatrix} \sin(\alpha - \phi_1) \\ \sin(\alpha - \phi_2) \\ \vdots \\ \sin(\alpha - \phi_{n+2}) \end{bmatrix} \quad (5.10)$$

A number of additional equations or auxiliary conditions are required in addition to the set represented by equation (5.7) in order to obtain a unique solution and are discussed below.

5.2.1 Auxiliary Conditions

Three auxiliary conditions are applied to the set of equations (5.7) developed in the previous section. The first condition is related to the Kutta condition, and specifies that the vorticity values at the upper and lower airfoil separation points are equal but of opposite sign. Even with extensive areas of separated flow it is physically impossible to have a discontinuity in the pressure distribution around an airfoil section. Furthermore, the constant pressure value in the separated flow region must be an inherent part of the solution to the boundary value problem in order to obtain a unique solution. Physically this modified Kutta condition ensures that the constant pressure value in the separated flow region is numerically equal to the pressure at the section separation points. Computationally, the upper and lower surface separation points are designated $NPUS$ and NPT , respectively for the airfoil separation model. However, for the bluff body separation model, the variable lower surface separation point is designated $NPLSS$. One equation is required to implement this modified Kutta condition in the coefficient matrix:

Airfoil model:

$$\gamma_{NPUS} = -\gamma_{NPT} \quad (5.10a)$$

Bluff body separation model:

$$\gamma_{NPUS} = -\gamma_{NPLSS} \quad (5.10b)$$

The second condition is related to the first and insures the upper and lower free wake sheet vorticity values, γ_{NPT1} and γ_{NPT2} , are equal to their corresponding airfoil separation point vorticity values. Two conditions are required to implement this condition;

Airfoil separation model:

$$\gamma_{NPUS} = \gamma_{NPT1} \quad (5.11a)$$

$$\gamma_{NPT} = \gamma_{NPT2} \quad (5.11b)$$

Bluff body separation model:

$$\gamma_{NPUS} = \gamma_{NPT1} \quad (5.12a)$$

$$\gamma_{NPLSS} = \gamma_{NPT2} \quad (5.12b)$$

The third condition concerns the vorticity distribution on the airfoil/bluff body surface in the separated flow region. This distribution is constrained to start and finish with zero vorticity. (Vorticity values in between are obtained as part of the solution, but are generally very small in the final converged solution.) Two equations are required to implement this condition in the matrix;

Airfoil separation model:

$$\gamma_1 = 0 \quad (5.13a)$$

$$\gamma_{NPUS} = 0 \quad (5.13b)$$

Bluff body separation model:

$$\gamma_{NPLS} = 0 \quad (5.14a)$$

$$\gamma_{NPUS} = 0 \quad (5.14b)$$

The airfoil separation model provides N equations in $N + 4$ unknowns. The auxiliary conditions require the use of five equations, and therefore, one boundary equation must be deleted. This means the boundary condition of zero normal velocity is applied at one less control point. The resulting square matrix, being linear in the unknowns, is amenable to solution. In contrast, the bluff body separation model provides N equations in $N + 5$ unknowns. Since the auxiliary conditions provide five equations, the resulting square matrix is as well amenable to solution. Figures 7(a, b and c) indicate the auxiliary conditions, numbering system and matrix formulation associated with the airfoil separation model. The bluff body separation model has a modified numbering system and matrix formulation as shown in figures 8(a, b and c)

5.2.2 Force Free Wake Boundary Conditions

The boundary condition that the wake sheets be force free is satisfied in an iterative cycle which involves the potential flow analysis only. Since the initial guess at the wake shape and position is not correct, the wake geometry is not a streamline and the normal velocities on the wake panels will not be zero. In order to relax the wake geometry into its force free position (i.e. $V_{n_i} = 0$), segments of the sheets are aligned with the local flow directions.

Specifically, using the previous vorticity distribution, external normal and tangential velocities are calculated at the panel control points on the free vortex sheets. Local surface angle corrections are defined for each wake panel, using the calculated local normal and tangential velocities as follows:

$$\Delta\beta_i = \tan^{-1} \left(\frac{V_{n_i}}{V_{t_i}} \right) \quad (5.15)$$

The new wake shape is then determined by a piecewise integration, starting at the airfoil separation points. In the integration process, each wake panel is rotated by this angle to force the wake geometry to more closely approximate a streamline. The upper and lower sheet downstream end points, which were coincident in the initial wake, are allowed to move independently in subsequent iterations. At each iteration, the upper and lower wake

surface influence coefficients ($B_{i,NPT1}$, $C_{i,NPT2}$) are recalculated, entered into the overall coefficient matrix and a new potential flow solution obtained.

A relative convergence criterion, based on the percent change in base pressure level between successive iterations, is employed to limit the number of wake iterations. However if desired, a fixed number of wake iterations can be input which overrides the convergence criterion. A point relaxation technique is applied to the local surface correction angles in order to speed the rate of convergence of the wake iterations.

5.3 Initial Wake Shape

Dvorak and Maskew's technique of establishing the initial free shear layer shapes has been adopted for use in the present study. Specifically the upper and lower sheets are represented by parabolic curves passing from the separation points to a common point downstream. As discussed by Dvorak and Maskew, the slope at the upstream end is the mean between the free stream direction, α , and the local surface slope. The common point downstream is positioned on the mean wake line, distance WL downstream from the wake midpoint as illustrated in figure 9.

5.3.1 Wake Length

In an experimental study carried out by Seetharam and Wentz (1975) a number of important features of the flow in wakes were observed. As expected they noted a constant total pressure value in the separated flow region of the airfoil. Also noted was a progressive growth of the width of the wake with increasing longitudinal station and a progressive reduction of static pressure gradients in the wake (Seetharam and Wentz, 1975).

Theoretically the wake which forms as a result of boundary layer separation extends to infinity. However an examination of the wake velocities by Seetharam and Wentz indicated the region of flow reversal (and hence de-energized flow) terminated within a relatively short distance downstream of the airfoil trailing edge (i.e. $0.1c - 0.2c$). They refer to the point at which the reversed flow terminated (characterized by a zero velocity point in the

velocity profile) as the *recombination point*. In addition the flow direction in the wake past the recombination point was measured to be in the free stream direction.

Preliminary calculations indicated that the results were sensitive to the length of free vortex sheets. In order to establish the correct length of the wake, the semi-empirical approach, developed by Dvorak and Maskew, was adopted for use in the present study. Dvorak and Maskew noted that good correlation with experimental results were obtained only with relatively short wakes, i.e., wakes extending $0.1c$ to $0.2c$ beyond the trailing edge. Such a model appears reasonable in light of the experimental evidence (as discussed above): the separated wake does, in fact, close quickly downstream of the trailing edge, as a result of the strong entrainment process brought about by the rotation in the free shear layers (Dvorak and Maskew, 1977).

Specifically, their approach is based on a *wake length factor* correlated with the airfoil thickness/ chord ratio. The wake length is obtained by multiplying the calculated wake height (figure 9) by the wake length factor. As indicated in figure 9, the wake height is a function of the free stream direction and the relative position of the point of separation on the airfoil upper surface. Therefore the choice of these parameters to establish the length of the wake seems reasonable: the position of separation on the airfoil upper surface is determined by among other factors the imposed inviscid adverse pressure gradient which itself is a direct consequence of the airfoil angle of attack. Dvorak and Maskew present a tentative curve which indicated a possible correlation of wake length factor with airfoil thickness/chord ratio. This curve is used to determine the proper wake length and is presented in figure 10.

Chapter 6

VISCOUS FLOW ANALYSIS

The classic treatment of viscous effects on airfoils has been traditionally based on Prandtl's concept of a thin boundary layer. Methods to calculate the development of a boundary layer are generally classified as *integral* or *differential*. In general, the choice of the method depends to a great extent on the nature of the problem and the information required from the analysis. In the present study and many other practical engineering applications, we are only interested in the distribution of the displacement thickness, δ^* , (in order to model the displacement effect of the boundary layer), the skin friction coefficient, C_f , for drag estimation and location of separation. Since integral methods supply these parameters, they offer the most economical approach both in terms of computer time and computational complexity. The integral methods selected in the present study represent a compromise between simplicity and accuracy.

6.1 Laminar Boundary Layer

Thwaites's (1949) integral method is used to predict the growth of the laminar boundary layer. It is an approximate method based on the momentum integral equation.

In solving the momentum integral equation given below

$$\frac{d\Theta}{dx} = \frac{C_f}{2} - \frac{\Theta}{V_\infty} \frac{dV_\infty}{dx} [H + 2] \quad (6.1)$$

Thwaites multiplied through by R_Θ to get

$$\frac{V_\infty \Theta}{\nu} \frac{d\Theta}{dx} + \frac{\Theta^2}{\nu} (2 + H) \frac{dV_\infty}{dx} = l \quad (6.2)$$

where, l , is an alternative shear stress parameter defined by

$$l = \frac{1}{2} C_f R_\Theta \quad (6.3)$$

Thwaites then defined a dimensionless pressure gradient parameter

$$\epsilon = \frac{\Theta^2}{\nu} \frac{\partial V_e}{\partial x} \quad (6.4)$$

and rewrote equation (6.2) as

$$V_e \frac{\partial}{\partial x} \left(\frac{\Theta^2}{\nu} \right) = 2[l - \epsilon(H + 2)] \quad (6.5)$$

Equation (6.5) becomes an ordinary differential equation (in ϵ , and can be integrated) if we assume that l and H can be expressed as functions of ϵ only. Based on this assumption, equation (6.5) can be rewritten

$$V_e \frac{\partial}{\partial x} \left(\frac{\Theta^2}{\nu} \right) = F(\epsilon) \quad (6.6)$$

In particular Thwaites obtained the three functions $l(\epsilon)$, $H(\epsilon)$ and $F(\epsilon)$ by plotting l , H and F against ϵ for a range of exact solutions. Thwaites then established the following relationship for F :

$$F(\epsilon) \simeq 0.45 - 6\epsilon. \quad (6.7)$$

This allows equation (6.6) to be integrated explicitly, giving

$$\frac{V_e^6 \Theta^2}{\nu} = 0.45 \int_0^x V_e^6 dx + \left(\frac{V_e^6 \Theta^2}{\nu} \right)_{x=0} \quad (6.8)$$

Having found $\Theta(x)$ from equation (6.8) the other boundary layer parameters are calculated from

$$\epsilon = \frac{\Theta^2}{\nu} \frac{\partial V_e}{\partial x} \quad (6.9)$$

$$\tau_w = \frac{\mu V_e}{\Theta} l(\epsilon) \quad (6.10)$$

$$\delta^* = \Theta H(\epsilon) \quad (6.11)$$

The parameters $l(\epsilon)$ and $H(\epsilon)$ are calculated from the correlation formulas given by Cebeci and Bradshaw (1977):

$$\begin{aligned} l(\epsilon) &= 0.22 + 1.57\epsilon - 1.8\epsilon^2 \quad \text{for } 0 < \epsilon < 0.1 \\ &= 0.22 + \frac{1.402\epsilon}{\epsilon + 0.107} \quad \text{for } -0.1 < \epsilon < 0 \end{aligned} \quad (6.12)$$

and

$$\begin{aligned} H(\epsilon) &= 2.61 - 3.75\epsilon + 5.24\epsilon^2 \quad \text{for } 0 < \epsilon < 0.1 \\ &= 2.088 + \frac{0.0731}{\epsilon + 0.14} \quad \text{for } -0.1 < \epsilon < 0 \end{aligned} \quad (6.13)$$

Since Thwaites method requires the solution of a first order ordinary differential equation (6.8), it requires one initial condition to start the calculation procedure. This condition is obtained from a stagnation point flow analysis, given by Moran (1984) as:

$$\Theta(0) = \sqrt{\left(\frac{0.075\nu}{V_0}\right)} \quad (6.14)$$

with

$$\frac{\partial V_s}{\partial s} = \frac{V_{s,ADJ} - V_{s,STAG}}{\Delta s}$$

where $V_{s,STAG}$ and $V_{s,ADJ}$ are the free stream velocities at the stagnation and adjacent step points, respectively. Δs is the streamline distance between the two step points

Numerically, equation 6.8 is integrated using the trapezoidal rule given by

$$\int_a^b f(x)dx \sim \frac{1}{2}[f(a) + f(b)](b - a) \quad (6.15)$$

The boundary layer development is first calculated on the upper airfoil surface. Thwaites method is used until transition or laminar separation occurs.

6.2 Boundary Layer Transition

When the laminar boundary layer reaches a certain critical Reynolds number it is no longer stable with respect to small disturbances. These small disturbances grow until the boundary layer reaches the transition Reynolds number which indicates the start of the disorderly state known as turbulence. The transition process is very complex and is influenced by many factors. Moran (1984) notes that some of the more important factors are the pressure gradient imposed on the boundary layer by the inviscid flow and surface roughness. Since there is no complete exact analysis of the transition process, empirical correlations are used to predict its occurrence. A variety of correlations are available and

include the techniques developed by Granville (1953), Michel (1951), in addition to e^9 and $H - R_x$ methods [Wazzan et. al., 1981].

Wazzan et. al. (1981) note that accumulated data indicates that the e^9 method is the best all-round technique that now exists for predicting boundary layer transition. The e^9 method predicts transition by predicting the growth of the Tollmein—Schlichting waves through an amplification ratio of about e^9 , using the Orr—Sommerfeld equation. Since the $H - R_x$ method is extremely easy to use and is rooted in the e^9 method, it has been implemented in the present analysis.

Specifically, the H represents the shape factor and R_x , the local Reynolds number, based on the distance x measured in the direction of the flow along the airfoil surface. Wazzan et. al. (1981) developed a correlation for transition based on the e^9 method, which indicates that the transition point is primarily a function of H [Wazzan et. al. (1981)]. Their results are shown in figure (11). The calculation procedure involves carrying out the laminar flow analysis along the airfoil surface and numerically plotting H against R_x . The plot will always start beneath the e^9 locus of figure (11). When the curve crosses the e^9 locus, transition is considered to occur.

Wazzan et. al. (1981) developed an equation that fits the e^9 locus well and is given by

$$\log[R_x(e^9)] = -40.4557 + 64.8066H - 26.7536H^2 + 3.3819H^3 \quad 2.1 < H < 2.8 \quad (6.16)$$

A second transition criterion is implemented in the present study. Moran (1984) notes that the boundary layer will not remain laminar much past the minimum pressure point in that an adverse pressure gradient substantially decreases stability of the laminar boundary layer. Specifically, the program performs a search and determines the minimum pressure point on the upper airfoil surface. When this point is reached, the turbulent flow calculations begin. The lower airfoil surface transition point is a variable in the second method and must be judiciously chosen by the user.

6.3 Turbulent Boundary Layer

A turbulent boundary computer code, previously developed at Carleton University, was adopted for use in the present viscous flow analysis. The solution procedure, originally proposed by Dvorak (1969), predicts the development of a turbulent boundary layer over rough surfaces in pressure gradients. A brief outline is presented below, however, a complete discussion of the solution scheme is given by Dvorak (1969).

Following the discussion of Choo (1977), roughness has a considerable direct effect on the mean velocity distribution in the wall region of the boundary layer. This effect is accounted for in a modified skin friction law which is used in conjunction with the momentum integral equation and an auxiliary equation for the determination of the shape factor, H , to solve for the turbulent boundary layer parameters. Head's (1958) entrainment equation for smooth surfaces is particularly suitable for adaption to the rough wall boundary layer calculation, since it does not depend on conditions close to the wall where the roughness effects are concentrated. Head suggested that the rate of entrainment of fluid into the turbulent boundary layer was a function of the boundary layer thickness, of the velocity outside the boundary layer and of the velocity distribution in the outer portion of the layer. It can thus be used without modification to provide the auxiliary equation for the shape factor, H , for boundary layers developing over rough surfaces.

Head's entrainment equation, given by equation (6.17) is solved simultaneously with the momentum integral equation given in equation (6.18).

$$\frac{d}{dx}(\delta - \delta^*) = F(H_{\delta-\delta^*}) - \frac{(\delta - \delta^*)}{V_e} \frac{dV_e}{dx} \quad (6.17)$$

$$\frac{d\Theta}{dx} = \frac{C_f}{2} - (H + 2) \left(\frac{\Theta}{V_e} \right) \frac{dV_e}{dx} \quad (6.18)$$

where

$$H_{\delta-\delta^*} = \frac{\delta - \delta^*}{\Theta} = G(H)$$

$$F(H_{\delta-\delta^*}) = \exp[-3.512 - 0.617 \ln(H_{\delta-\delta^*} - 3.0)]$$

and

$$G(H) = 3.3 + \exp[0.4667 - 2.722 \ln(H - 0.6792)]$$

for $H \leq 1.6$

or

$$G(H) = 3.3 + \exp[0.4383 - 3.064 \ln(H - 0.6792)]$$

for $H > 1.6$

6.3.1. Skin Friction Law for Roughened Surfaces

The skin friction relations developed by Dvorak (1968) and summarized below, represent a complete expression for the prediction of the skin friction coefficient on smooth, transitionally rough, or fully rough surfaces in zero, favourable, or adverse pressure gradients.

The skin friction law is written in the form,

$$\sqrt{\frac{2}{C_f}} = 5.6 \log_{10} \left(\frac{V_* \delta^*}{\nu} \right) + 4.8 - \frac{\Delta U_1}{U_*} + \frac{\Delta U_2}{U_*} \quad (6.19)$$

The function $\Delta U_2/U_*$ is the pressure gradient contribution to the velocity defect law and is defined by the simple relationships which follow [Dvorak, 1969]:

$$\frac{\Delta U_2}{U_*} = 1.253(J - 6.7) \quad (6.20a)$$

for $J \geq 6.7$ (adverse pressure gradient) and

$$\frac{\Delta U_2}{U_*} = 0.404(J - 6.7) \quad (6.20b)$$

for $J < 6.7$ (favourable pressure gradient).

Where J is defined as a pressure gradient parameter and is given by

$$J = \sqrt{\frac{2}{C_f}} \left[\frac{(H - 1)}{H} \right] \quad (6.21)$$

The function $\Delta U_1/U_*$ is the roughness contribution to the law of the wall and is defined below for the various regimes;

Fully Rough Regime:

$$\frac{\Delta U_1}{U_r} = 5.6 \log_{10} \frac{U_r k}{\nu} + 17.35(1.625 \log_{10} \lambda - 1) \quad (6.22a)$$

for $\lambda \leq 5$

or

$$\frac{\Delta U_1}{U_r} = 5.6 \log_{10} \frac{U_r k}{\nu} - 5.95(1.103 \log_{10} \lambda - 1) \quad (6.22b)$$

for $\lambda > 5$

The extension of the correlation for $\Delta U_1/U_r$ into the transitional roughness regime, involves determining the logarithmic polynomial which has the correct value of $\Delta U_1/U_r$ and its first derivative for two values of $U_r k/\nu$, [Dvorak, 1969]. The two values of $U_r k/\nu$ are, respectively, that below which the surface is considered aerodynamically smooth (assumed by Dvorak to be $U_r k/\nu \approx 5$), and that above which the surface is considered fully rough (assumed by Dvorak to be $U_r k/\nu \approx 50$).

In the present calculation, the logarithmic polynomial takes the form,

$$\frac{\Delta U_1}{U_r} = C_0 + C_1 \ln \left(\frac{U_r k}{\nu} \right) + C_2 \ln^2 \left(\frac{U_r k}{\nu} \right) + C_3 \ln^3 \left(\frac{U_r k}{\nu} \right) \quad (6.22c)$$

The skin friction coefficient is then evaluated at each step of the calculation by solving equation (6.19) using a Newton-Raphson iteration.

6.3.2. Calculation Procedure

The first order partial differential equations (6.17) and (6.18) are numerically solved, simultaneously, using the fourth order Runge-Kutta formula given below.

$$y_1 = y_0 + \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4] \quad (6.23)$$

where

$$k_1 = \Delta_S f(x_0, y_0)$$

$$k_2 = \Delta_S f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_1\right)$$

$$k_3 = \Delta_S f\left(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}k_2\right)$$

$$k_4 = \Delta_S f(x_0 + h, y_0 + k_3)$$

and

$$\Delta s \equiv \text{step size.}$$

In order to start the turbulent flow analysis initial values of H , Θ and C_f are required. In the present study, these values are inputted and are not obtained from the laminar flow analysis. In addition, the roughness density parameter, λ , and non-dimensional roughness height k/c are required. The Reynolds number, $U_\infty c_0/\nu$, is assumed known as is the inviscid flow pressure distribution. The turbulent boundary layer analysis is carried out until the airfoil trailing edge is reached or turbulent separation is predicted.

6.4 Boundary Layer Separation

Thwaites's method predicts laminar separation will occur when the parameter $l(\epsilon)$ vanishes, since l is defined in equation (6.3) to be proportional to the wall shear stress. According to Moran (1984), laminar separation will occur when $\epsilon < -0.0842$. This criterion is used in the present code.

Unfortunately a similar criterion cannot be used with Head's method to predict the point of turbulent boundary layer separation. However, according to Moran (1984), one may predict turbulent separation by examining the development of the shape factor, H . Specifically the shape factor is defined as the ratio of the displacement thickness to the momentum thickness $H = \frac{\delta^*}{\theta}$. Since both these parameters are only functions of the velocity profile, their ratio, H , is a useful measure of the fullness of the velocity profile. Physically as the point of separation is approached the velocity profile becomes increasingly thin. Consequently the displacement thickness must grow rapidly in order to account for the increasing mass deficit in the boundary layer. In comparison, the displacement thickness grows at a greater rate than the momentum thickness near separation. This is due in part to the strong momentum transport in turbulent boundary layers. In the present study, the program declares that separation occurs when the shape factor exceeds three, (i.e. $H > 3.0$). As discussed by Moran (1984), this criterion is usually adequate, since the shape factor tends to increase rapidly near the point of separation.

Chapter 7

INVISCID - VISCOUS FLOW INTERACTION

7.1 Overview

After calculating the airfoil boundary layer development based on the pure inviscid pressure distribution, it is possible to correct the next inviscid solution for the presence of the boundary layer. In carrying out the boundary layer analysis, the distribution of the displacement thickness, δ^* , can be determined. Fundamentally this parameter is seen to be a measure of the amount by which the boundary layer is deficient in mass flow as compared to the free stream. The name itself results from the fact that it is the distance the external inviscid flow streamlines are displaced by the boundary layer, as shown in figure 12.

Two techniques exist for modelling the displacement effect caused by the boundary layer in the inviscid flow analysis. The first method involves displacing the airfoil surface normally outward by an amount equal to the boundary layer displacement thickness, δ^* . An inviscid calculation of the flow field, over the modified airfoil geometry, would produce an improved velocity distribution beyond the edge of the boundary layer. The displaced surface technique is illustrated in figure 13. A second and preferred approach, (implemented in the present study), uses a surface source distribution to model the viscous/inviscid flow interaction. The source distribution is chosen such that the mass flux from the sources displaces the external flow streamlines by an amount δ^* from the surface of the airfoil. Seebohm et. al. (1975) and Duncan et. al. (1969) give a complete discussion on the two methods.

The source distribution technique offers several major advantages over the displaced surface method previously described. First, as discussed by Kind and Dvorak (1979), the matrix of airfoil aerodynamic influence coefficients remains the same for all iterations.

with attendant saving in computer time. Since the source strengths are known before every inviscid flow calculation, these terms can be included on the right hand side of equation (5.5) as an additional onset flow term. Furthermore the displaced surface technique tends to produce peculiar surface geometries, particularly in the trailing edge region. This is a serious problem as solutions are sensitive to trailing edge geometry because the Kutta condition must be applied there. These difficulties are completely avoided if the surface source distribution technique is employed. Following the discussion of Choo (1977), the new boundary condition is derived by maintaining zero normal velocity at each control point, just inside the airfoil contour. However, just outside the contour the normal velocity is equal to the local source strength q . To illustrate this, consider a single source panel; at the control point, the normal velocity due to the source panel is $q/2$ outward from the panel. This is as well the magnitude of the normal velocity inward due to the symmetry of the source flow. Let the resultant normal velocity due to all remaining source and vortex panels including the local source panel equal $q/2$ normal outward at the local control point. Superimposing the two flows results in a reinforcement of the outward normal velocity to give a resultant of magnitude q outwards and a cancellation of the inward normal velocity, giving zero internal normal velocity.

7.2 Numerical Formulation

The source distribution is placed on the planar panels which are used to represent the airfoil section. The source strength is assumed to vary linearly between junction points and is continuous at the junction points, [Kind and Dvorak, 1979]. The boundary layer analysis supplies the required junction point source strength, given by:

$$q = \frac{d}{ds} (\rho V_\infty \delta^*) \quad (7.1)$$

where ρ is the fluid density, s denotes the arc length, V_∞ is the local inviscid flow velocity and δ^* is the local boundary layer displacement thickness.

Numerically we require expressions for the velocity components $u_{i,m}^q$ and $v_{i,m}^q$ parallel and normal, respectively, to sheet m at control point i . Specifically an element ds of the

sheet will induce velocities at point i ,

$$du = -\frac{q}{2\pi} \frac{dr}{r} \quad (7.2a)$$

$$dv = \frac{q}{2\pi} d\theta \quad (7.2b)$$

Integrating equations 7.2(a and b) assuming a linear source variation between junction points yields;

$$u_{i,m}^q = zk(3)q_m + zk(4)q_{m+1} \quad (7.3a)$$

$$v_{i,m}^q = -[zk(1)q_m + zk(2)q_{m+1}] \quad (7.3b)$$

where $zk(1)$ through $zk(4)$ are given by equations (4.5a—d).

A new boundary condition, (flow tangency) equation can be developed by algebraically superimposing the influence of the source panels on equation 5.5, yielding

$$\begin{aligned} & \sum_{m=1}^N A_{i,m} \gamma_m + \gamma_\infty \sum_{n=1}^{NPU} B_{i,n} + \gamma_l \sum_{l=1}^{NPL} C_{i,l} + \\ & \sum_{m=1}^N v_{i,m}^q \cos(\phi_m - \phi_i) + \sum_{m=1}^N u_{i,m}^q \sin(\phi_m - \phi_i) + U_\infty \sin(\alpha - \phi_i) \\ & = -q_i \end{aligned} \quad (7.4)$$

Since the source strengths to be used in the next iteration are known, keeping the unknowns on the left hand side, equation 7.4 becomes;

$$\begin{aligned} & \sum_{m=1}^N A_{i,m} \gamma_m + \gamma_\infty \sum_{n=1}^{NPU} B_{i,n} + \gamma_l \sum_{l=1}^{NPL} C_{i,l} \\ & = - \left[\sum_{m=1}^N v_{i,m}^q \cos(\phi_m - \phi_i) + \sum_{m=1}^N u_{i,m}^q \sin(\phi_m - \phi_i) + U_\infty \sin(\alpha - \phi_i) \right] \\ & - q_i \end{aligned} \quad (7.5)$$

where

$i = 1, \dots, N + 4$: Airfoil separation model

$i = 1, \dots, N + 5$: Bluff body separation model

The influence coefficients $A_{i,m}$, $B_{i,n}$ and $C_{i,l}$ are given by equations (4.7b), (5.6a) and (5.6b), respectively.

7.2.1 The Modified Kutta Condition

The original inviscid attached Kutta condition flow (equation 4.9a or 4.9b) must be modified in order to include the influence of the added source distribution. A modified Kutta condition, used by Choo (1977) and originally developed by Woodward and Dvorak (1974) was implemented in the present study. In general, Woodward and Dvorak applied equal pressures on the upper and lower trailing edge surfaces. As discussed by Woodward and Dvorak (1974), with the source strengths known *a priori*, the trailing edge vortex strength is chosen to achieve zero outward normal velocity on both surfaces. For a more detailed discussion of the method see Choo (1977) or Woodward and Dvorak (1974).

If separation has occurred, the source strengths in the separated region are set to zero.

Chapter 8

CALCULATION OF THE AERODYNAMIC PARAMETERS

8.1 Calculation of Pressures

Two techniques exist for calculating the inviscid pressure distribution around the airfoil surface. The first method involves determining the resultant control point velocities by adding to the free stream, the velocities induced by the source and vortex panels. The second, and preferred technique is based on the premise that the velocity inside the airfoil contour must be zero; hence the local source and vortex strengths must equal the local external normal and tangential velocities respectively. Since the inviscid solution provides the local junction point vortex strengths, these results, in conjunction with the local source strengths, can be used to determine the static pressure variation on the airfoil surface. Both techniques were implemented in the present study for comparative purposes. However, due to certain inherent inaccuracies involved in the first control point method, all quoted results are based on the latter technique.

The static pressures are calculated according to Bernoulli's equation, given below in non-dimensional form:

$$C_p = \frac{P - P_\infty}{\frac{1}{2} \rho U_\infty^2} \quad (8.1)$$

Equation (8.1) can be rewritten in terms of local source and vortex junction point strengths as;

$$C_{P_m} = \frac{\frac{1}{2} \rho [U_\infty^2 - \gamma_m^2 - q_m^2]}{\frac{1}{2} \rho U_\infty^2} \quad (8.2)$$

In the potential flow analysis, all velocities are non-dimensionalized with respect to the free stream, U_∞ . The pressure coefficient expression, given by equation (8.2) then becomes;

$$C_{P_m} = 1 - \gamma_m^2 - q_m^2 \quad (8.3)$$

However for the pure potential flow solution

$$q_m = 0$$

Hence equation (8.3) reduces to

$$C_{P_m} = 1 - \gamma_m^2 \quad (8.4)$$

8.1.1 Base Pressure Level

It is well known that in two-dimensional subsonic flow the pressure is constant in the wake over the separating airfoil. In the present study the pressure in the constant pressure region of the separated wake is set equal to the pressure at the upper airfoil surface separation point. The correct separation pressure is obtained when the wake iteration cycle has converged. The junction point pressure distribution in the separated region of the airfoil is given by:

$$C_{P_{BASE}} = 1 - (\gamma_{NPUS})^2 \quad (8.5)$$

8.2.1 Compressibility Effects

The general nature of the flow pattern past a two-dimensional airfoil remains the same as long as the flow remains entirely subsonic, but the magnitude of the flow parameters can be affected by compressibility effects [Clancy, 1975]. The general effect of compressibility is to increase the lift, even in coefficient form. Compressibility effects are introduced in the present potential flow analysis using the Karman—Tsien pressure correction formula, as given by Shapiro (1953).

$$C_{P_c} = \frac{C_{P_i}}{\sqrt{1 - M_\infty^2} + \frac{M_\infty^2}{1 + \sqrt{1 - M_\infty^2}} \left(\frac{C_{P_i}}{2} \right)} \quad (8.6)$$

where, subscripts *c* and *i* denote compressible and incompressible respectively.

If compressibility effects are to be included in the analysis, the free stream mach number, M_∞ , must be known. Equation (8.6) is then used to calculate modified control

and junction point pressure coefficients. These modified C_P values are then used to determine the integrated aerodynamic parameters C_L , C_M and C_D .

8.2 Calculation of C_L , C_M and C_D

In general the lift, pitching moment and pressure drag coefficients are determined by integrating the static pressure distribution around the airfoil contour. It can be shown that the lift and pressure drag coefficients are given by

$$C_L = C_y \cos \alpha - C_x \sin \alpha \quad (8.7)$$

$$C_D = C_x \cos \alpha + C_y \sin \alpha \quad (8.8)$$

where

$$C_y = \oint C_P d\left(\frac{z}{c}\right) \quad (8.9)$$

$$C_x = - \oint C_P d\left(\frac{y}{c}\right) \quad (8.10)$$

Numerically equations (8.9) and (8.10) are given by;

$$C_y = \sum_{i=1,3,\dots}^N [F_i \cos \theta_i + F_{i+1} \cos \theta_{i+1}] \quad (8.11)$$

$$C_x = - \sum_{i=1,3,\dots}^N [F_i \sin \theta_i + F_{i+1} \sin \theta_{i+1}] \quad (8.12)$$

where F_i and F_{i+1} are the integral of the pressure coefficient over two adjacent panels i and $i+1$. To evaluate the integrals F_i and F_{i+1} , a quadratic curve is fitted to the pressure coefficients at three consecutive junction points, m , $m+1$ and $m+2$ as shown in figure 14. The s axis represents the arc length along the airfoil surface.

The equation for the quadratic curve is

$$C_P(s) = as^2 + bs + c \quad (8.13)$$

where the coefficients a , b and c are given by

$$a = \frac{[C_P(m+2) - C_P(m) - b(s_3 - s_1)]}{[s_3^2 - s_1^2]} \quad (8.13a)$$

$$b = \frac{C_P(m)[s_3^2 - s_2^2] - C_P(m+1)[s_3^2 - s_1^2] + C_P(m+2)[s_2^2 - s_1^2]}{[s_2^2(s_3 - s_1) + s_1^2(s_2 - s_3) + s_3^2(s_1 - s_2)]} \quad (8.13b)$$

$$c = C_P(m+1) - a(s_2^2) - b(s_2) \quad (8.13c)$$

and

$$s_1 = s_3$$

$$s_2 = s_1 + l(m)$$

$$s_3 = s_2 + l(m+1)$$

where $s_3 = 0$ for first airfoil panel.

The integrals F_i and F_{i+1} can now be evaluated by integrating the function $C_P(s)$, over the intervals

$$F_i = \int_{s_1}^{s_2} C_P(s) ds \quad (8.14)$$

$$F_{i+1} = \int_{s_2}^{s_3} C_P(s) ds \quad (8.15)$$

The resulting equations are

$$F_i = \frac{a}{3}[s_2^3 - s_1^3] + \frac{b}{2}[s_2^2 - s_1^2] + c[s_2 - s_1] \quad (8.16)$$

$$F_{i+1} = \frac{a}{3}[s_3^3 - s_2^3] + \frac{b}{2}[s_3^2 - s_2^2] + c[s_3 - s_2] \quad (8.17)$$

The program automatically non-dimensionalizes and translates the airfoil contour, such that the airfoil leading edge is at $x/c = 0.0$ and the trailing edge at $x/c = 1.0$. Therefore the leading edge and quarter-chord pitching-moment coefficients are given by;

$$C_{M_{L.E.}} = - \sum_{i=1}^N \partial C_y(XI) + \sum_{i=1}^N \partial C_z(YI) \quad (8.18)$$

$$C_{M_q} = - \sum_{i=1}^N \partial C_y[XI - 0.25] + \sum_{i=1}^N \partial C_z(YI) \quad (8.19)$$

where

$XI \equiv x$ control point coordinate

$YI \equiv y$ control point coordinate

and nose up is considered positive.

An alternative method for determining the lift coefficient for the pure potential flow analysis (for airfoils with a closed trailing edge) is obtained by determining the total circulation around the airfoil.

$$\Gamma = \sum_{m=1}^N \frac{\gamma_m + \gamma_{m+1}}{2} (l_m) \quad (8.20)$$

where l_m is the length of the panel m . The lift coefficient can be determined from

$$C_L = \frac{2\Gamma}{U_\infty C_0} \quad (8.21)$$

Chapter 9

MAIN FEATURES OF THE PROGRAMS

As discussed in the introduction, the emphasis of the research was not on developing a single complete analysis program. Instead, independent inviscid and viscous flow codes were developed, which are externally coupled with a third service program to provide the full potential flow-boundary layer separated flow analysis. The inviscid flow analysis carried out by the *stalled airfoil modelling program (STAMP)*, the viscous flow solution performed by the *boundary layer analysis program (BLAP)* and the *initial wake modelling program (IWMP)* are all written in Fortran 77. A modular approach was adopted in writing each program to ease future modifications. Specifically *STAMP* consists of twenty six subroutines, *BLAP* consists of twenty three subroutines and *IWMP* of only four. The three codes are all interactive and prompt the user for various parameters. A brief discussion on each program is given below.

9.1 The Stalled Airfoil Modelling Program

The stalled airfoil modelling program, carries out the inviscid attached or separated flow analysis, up to and beyond stall, for a single element airfoil or bluff body. A variety of output formats are available, ranging from a brief summary of aerodynamic parameters to a full output file containing all junction, control point airfoil and wake parameters. If desired, the program will perform a numerical check to determine if any wake panels have overlapped. If this has occurred the panels past the intersection point are omitted from further calculation. The program automatically translates and non-dimensionalizes all x and y coordinates such that the leading edge is at $x/c = 0.0$ and the trailing edge is at $x/c = 1.0$.

Appendix A provides a complete user's manual for the stalled airfoil modelling pro-

gram. In addition, Appendix B provides a dictionary of variable names while Appendix C provides a complete description of the subroutines which comprise the stalled airfoil modelling program.

9.1.1 Convergence Criterion

Initially, the free shear layer streamlines are not known and so the correct shape of the wake boundaries must be obtained iteratively. The iteration is terminated when a certain convergence criterion is satisfied. The two most commonly used types of convergence criteria are absolute and relative criterion. In the present analysis, a relative convergence criterion is used which takes the form

$$\left| \frac{z_i^{(I)} - z_i^{(I-1)}}{z_i^{(I)}} \right| \times 100 \leq \omega$$

This type of criteria has been found to be the safest choice if the magnitudes of z_i are not known before hand [Hornbeck, 1974], and corresponds to specifying the maximum allowable percentage change in each unknown on two successive iterations.

The separated flow base pressure was selected as the convergence parameter, z_i . This aerodynamic parameter was selected since it has a wider range of application than global integrated parameters such as the lift coefficient. In addition the base pressure was found to be more sensitive to changes in the wake shape than the lift coefficient. The default convergence criterion (ω) is set at 0.25 with a maximum of twenty iterations. This criterion must be met in two successive iterations in order to assure convergence.

In order to speed the rate of convergence, a point relaxation method is implemented for correcting each wake panel orientation angle after each iteration. The fundamental operational equation for this point relaxation is

$$\phi_i^{(I)} = \phi_i^{(I-1)} + \xi(\phi_i^{(I)*} - \phi_i^{(I-1)})$$

where ϕ_i represents the local panel orientation angle, (I) is the current iteration value and $(I-1)$ is the preceding value. The angle $\phi_i^{(I)*}$ represents the current corrected panel angle. The quantity ξ is a pure number in the range $0 < \xi < 2$ where

$$0 < \xi < 1 \quad \text{under relaxation}$$

$$1 < \xi < 2 \text{ over relaxation}$$

Over-relaxation is generally used to speed convergence while under-relaxation is employed to make a non-convergent iterative process converge [Hornbeck, 1974]. In the present formulation of the program the relaxation factor is an input variable. Generally over-relaxation is used with the relaxation factor in the range 1.3-1.8.

9.1.2 Sequence of Calculations

The calculation sequence for the stalled airfoil modelling program including boundary layer correction and compressibility effects is outlined below. A flow chart of the calculation sequence is illustrated in figure 15.

- (1) All input data is read in, program flags are set and various program constants are determined (*NPUSS*, *NPUS*, *NPUS2* etc).
- (2) All airfoil and wake junction point coordinates are non-dimensionalized and translated such that the leading edge is at $x/c = 0.0$ and the trailing edge is at $x/c = 1.0$.
- (3) The airfoil and wake geometry parameters are determined. This includes panel lengths, orientation angles and control points.
- (4) The matrix $[A]$ of airfoil influence coefficients is formed.
- (5) Wake iteration loop entered. Wake coefficients are formed and entered into matrix $[A]$. This sequence is repeated for each iteration.
- (6) Boundary and auxiliary conditions are implemented in matrix $[A]$. Onset flow vector (including source terms) are formed and entered in array $[BR]$ (shown in figures 7c and 8c for the airfoil and bluff body separation models, respectively)
- (7) Matrix $[A]$ and array $[BR]$ are entered into two holding arrays $[AN]$ and $[BB]$ for the next iteration.
- (8) The $[A]$ matrix is then inverted and pre-multiplied to the $[BR]$ onset vector in order to obtain the unknown junction point vortex strengths
- (9) The control point normal and tangential velocities are determined on the airfoil and

wake panels, (external velocities).

- (10) Control and junction point pressure coefficients are determined. Correction for compressibility effects are calculated.
- (11) Aerodynamic parameters are then integrated using the corrected airfoil pressure coefficients.
- (12) Convergence criterion for wake iterations is checked, if met in two successive iterations, control is transferred out of iteration loop.
- (13) A new wake shape is determined by a piecewise integration, starting at the separation points. The upper and lower sheet downstream end points, which were coincident in the initial wake, are allowed to move independently in subsequent iterations.
- (14) If desired, program check to see if any wake panels have overlapped, if so the wake is truncated at the cross over point.
- (15) Holding matrix arrays [AN], [BB] are re-entered back into primary matrix arrays [A] and [BR].
- (16) Operations 5 — 16 are repeated until convergence is achieved.
- (17) If desired a boundary layer data file is generated. (This file contains the inviscid junction point velocity distribution, surface distance measured from the airfoil leading edge and initial conditions. See Appendix D for complete documentation.)

9.1.3 Matrix Solver

A library matrix routine performs the direct inversion of the influence coefficient matrix [A]. The IMSL routine *LEQTIF* performs Gaussian elimination (Crout algorithm) with equilibrium and partial pivoting

9.1.4 Stagnation Point Search Technique

The boundary layer analysis begins at the airfoil stagnation point for both the upper and lower surface analysis. *STAMP* performs a junction point pressure coefficient search

in order to determine the maximum positive pressure coefficient (ideally $C_P = 1.0$). This point is taken as the airfoil stagnation point.

9.2 The Boundary Layer Analysis Program

The boundary layer analysis program (BLAP) calculates the boundary layer growth on both the upper and lower airfoil surface. The program's primary function is to supply the airfoil separation points and the required source distribution for use in STAMP. The analysis is first carried out on the upper airfoil surface, followed by the lower surface. However, for testing purposes, just a laminar and/or a turbulent analysis can be carried out on a single surface. The boundary layer step size $\Delta s/c$ is a variable, inputted by the user. The sequence of calculations for the boundary layer analysis on the upper and lower airfoil surface is given below. A flow chart of the calculation sequence, for BLAP, is shown in figure 16.

Appendix D provides a complete user's manual for the boundary layer analysis program. In addition, Appendix E provides a dictionary of variable names while Appendix F provides a complete description of the subroutines, which comprise the boundary layer analysis program.

9.2.1 Sequence of Calculations

- (1) All data is read into the program, flags are set and various program constants are determined.
- (2) The velocity gradients are determined at each incremented airfoil position.
- (3) The laminar analysis begins and continues until one of the following things occur:
 - (a) The last upper or lower surface airfoil point is reached. The laminar analysis then begins on the lower airfoil surface for upper surface case.
 - (b) Laminar separation is predicted (i.e. $\epsilon < -0.0842$). The program determines the x cartesian coordinate corresponding to the separation point and outputs various

messages. The laminar analysis then begins on the lower airfoil surface

- (c) Transition to turbulence is predicted either by the $H - R_s$ method or the alternative criterion.
- (4) Turbulent analysis begins. The turbulent analysis continues until one of two things happen.
 - (a) Last upper or lower airfoil surface point is reached. For upper surface case, laminar calculations begin on lower surface
 - (b) Turbulent separation, ($H > 3.0$) occurs on the upper or lower airfoil surface. The x cartesian coordinate at the separation point is determined. For the upper surface case, laminar calculations begin on the lower airfoil surface.
- (5) After the full analysis has been carried out on the airfoil upper and lower surfaces, the program determines the source strengths at the airfoil junction points. It then arranges them in the proper counter-clockwise order such that the first source strength corresponds to the trailing edge airfoil junction point. If separation has been predicted, the junction point source strengths are set to zero in the separation flow region. This data is then stored in a output data file for later use in the stalled airfoil modelling program.

9.2.2 Velocity Interpolation

The inviscid potential flow velocity distribution along the airfoil upper and lower surfaces (at the junction points) is supplied by the stalled airfoil modelling program. *BLAP* is set up to handle a constant streamwise step size, $\Delta s/c^1$. This allows the user to carry out a more accurate analysis by choosing a smaller step size. In order for the program to carry out the required step by step calculation, it must interpolate for the velocity and velocity gradient at each incremented step point. This is done by fitting a parabola to junction point velocities at three successive points (bounding the point in

¹ The s of the boundary layer analysis is not a cartesian coordinate, but a measure of distance along the curved airfoil surface, with $s = 0.0$ located at the stagnation point

question). This requires that the coefficients a , b and c be determined from the quadratic equation

$$V(s) = as^2 + bs + c \quad (9.1)$$

Hence for any value of s (in the applicable range), the interpolated velocity value can be determined. The velocity gradient is found by differentiating equation (9.1) with respect to s and evaluating the result at the point under study

9.3 Initial Wake Modelling Program

The initial wake modelling program is a service code which is used to generate the initial wake streamline. As was previously described, the upper and lower wake sheets are initially represented by parabolic curves passing from the separation points to a common point downstream. The program prepares the required input data file for *STAMP*, if a separated wake, potential flow analysis is to be carried out. Appendix G provides a complete user's manual for the initial wake modelling program. In addition, Appendix H contains a set of sample input data files while Appendix I contains a set of sample output data files. The stalled airfoil modelling program listing, the boundary layer analysis program listing and the initial wake modelling program listing are given in Appendices J, K and L respectively.

A block diagram, showing the viscous-inviscid interaction is shown in figure 17.

Chapter 10

DISCUSSION AND RESULTS

The discussion of the results follows the development and testing of each computational method which comprises the full inviscid-viscous analysis and is divided into four parts.

- (1) Stability and convergence characteristics of inviscid separation model.
- (2) Inviscid flow results (attached and separated flow with separation points assumed known)
- (3) Viscous flow test case results.
- (4) Full inviscid-viscous flow results.

10.1 Stability and Convergence Studies

A comprehensive numerical stability and convergence study was carried out in order to investigate the influence of various parameters on the inviscid separation model solution. This study considered a circular cylinder (bluff body) and a NASA GA(W)-1 airfoil. In particular the upper and lower surface separation points for the circular cylinder were set at $\Theta_{sep} = 107^\circ$ and the angle of attack, α , set at 0° . The upper surface separation point for the GA(W)-1 airfoil was set at $x/c = 0.5$ with the lower surface separation point fixed at the airfoil trailing edge. In addition the airfoil angle of attack was set at $\alpha = 20.05^\circ$.

The effects of varying the number of vortex panels used to model the airfoil contour is shown in figures 19(a-b). The solution is seen to be unstable when less than twenty panels are used to model the airfoil contour. The solution increases in accuracy as the number of panels increases from twenty. However, an almost constant level of accuracy is obtained when using forty or more vortex airfoil panels. The percent change in the lift

coefficient from twenty to forty panels is approximately 7% whereas from forty to fifty panels the percent change is only 0.1%. In all test cases approximately thirty eight vortex panels were used to model the airfoil contour. In addition, every attempt was made to ensure a smooth spacing between junction points, with a high density of panels at the leading edge where the pressure distribution varies rapidly (no particular spacing formula was used to distribute the junction points). The influence of the number of panels used to model the free shear layers of the wake for the airfoil and bluff body are shown in figures 20(a-b) and 21(a-b), respectively. The study was carried out using an equal number of panels on the upper and lower wake surfaces for both configurations. In both cases, using more than four panels per surface resulted in essentially a constant level of accuracy. The percent change in the lift coefficient is seen to be 2.5%, when the number of wake panels is increased from one to four, whereas the percent change is only 0.4% when the number of wake panels is increased from four to nine.

The importance of using the correct wake length factor, WLF , (used to determine the length of the free wake vortex sheets) is indicated in figures 22(a-b) and 23(a-b) for the airfoil and circular cylinder respectively. Considering the influence of the wake length factor on the airfoil solution (figures 22a-b), its greatest influence is in the range $0.5 \leq WLF \leq 1.75$. The percent change in the airfoil lift coefficient is 41% in this range. The effect of the wake length factor begins to diminish steadily for values greater than 1.75. In contrast, as indicated by figures 23(a-b), the effect of the wake length factor on the circular cylinder solution is felt over a wide range of values. The base pressure level and pressure drag coefficient can be seen to decrease in value for increasing values of the WLF . Obviously this parameter represents the empirical link in the method and must be carefully selected by the user. In the present study, the wake length factor for each airfoil section was determined from Dvorak's correlation curve, shown in figure (10). This curve is however based on a fully coupled inviscid-viscous solution and could, therefore, be fine tuned in order to provide better results for a simple inviscid separation model airfoil analysis.

The influence of the overall solution criterion on the various aerodynamic parameters,

for the airfoil and bluff body configurations are illustrated in figures 24(a-b) and 25(a-b), respectively. The results indicate that for a convergence criterion less than 0.25%, the aerodynamic parameters remain essentially constant. In light of these results, the default convergence value is set at 0.25% in the present version of the program for both separation models. Figure 26(a-b) indicates the influence of the number of wake iterations on the airfoil separation model aerodynamic parameters. If the convergence criterion is bypassed and a fixed number of iterations is inputted, it can be seen that after approximately ten iterations the aerodynamic parameters remain essentially constant.

In formulating the present separation model, the boundary layer separation points are constrained to fall on existing airfoil junction points. The influence on the solution due to the placement of the upper surface separation point is shown in figures 27(a-b). As indicated (and expected), a forward movement of the upper surface separation point decreases the lift coefficient while increasing the pressure drag coefficient and base pressure level for a given angle of attack. Obviously the constraint that the separation point falls on an existing junction point represents a limitation of the current method.

The inviscid numerical stability and convergence studies were concluded by investigating the effect of varying the value of the point relaxation factor, on the various aerodynamic parameters. Figures 28(a-b) and 29(a-b) indicate the results for the airfoil and bluff body configurations. As anticipated, the relaxation factor does increase the rate of convergence, while not influencing, to any appreciable extent the various aerodynamic parameters for either configuration. The initial and final converged wake geometry, for the GA(W) 1 airfoil and circular cylinder are shown in figures 30(a) and 30(b), respectively. These converged wake geometries were each obtained after nine wake iterations.

10.2 Inviscid Flow Results (Separation Points known)

The ability of the stalled airfoil modelling program to predict the inviscid attached or separated flow field around single element airfoils is discussed in this section. The analysis does not include boundary layer correction but does include compressibility effects, if the

free stream mach number is known. The airfoil or bluff body surface separation points (for each separated flow angle of attack) were determined from the available experimental pressure distribution results. Numerical and experimental results are compared for four different airfoils, over the angle of attack range from zero through stall. The NASA GA(W)-1, NACA 4412, NACA 63₃ - 018 and NACA 63₁ - 012 airfoils shown in figures 18(a-d), represent a wide variety of leading edge radii, maximum thickness and camber variation for the present analysis. A bluff body (circular cylinder) separated flow analysis is as well carried out with the separation points determined from available experimental data. Table II indicates the flow conditions and source of the experimental data for each configuration.

The inviscid separated flow analysis for each airfoil was carried out using the results obtained from the stability and convergence studies. Approximately thirty eight vortex panels were used to model each airfoil contour. The number of panels used to model the upper and lower free shear layers varied according to the relative size of the wake; such that the approximate non-dimensional length of each panel was $l/c \approx 0.1$. For all the airfoil separated flow incidence cases, the lower surface separation point was fixed at the airfoil trailing edge. The approximate non-dimensional distance between the upper airfoil surface junction points was $0.05z/c$. This allowed the upper surface separation point to be placed with a maximum possible error of $\pm 0.025z/c$. The default wake iteration convergence criterion (0.25%) was used for all airfoil separated flow incidence cases. In addition, an over-relaxation factor, in the range 1.3 - 1.8, was used.

The wake length factor for each airfoil, was determined from figure (10); no attempt was made to correct the curve for the inviscid separated flow analysis along. For comparison, the attached potential flow solution at each separated flow incidence case (for each airfoil), is included in order to indicate the large change in pressures due to the addition of the separation model. In order to test the pure inviscid attached flow analysis, a test case was performed using a Joukowski airfoil, in which the exact solution was known. Figure (31) compares the exact and predicted pressure distribution for a 10° angle of attack. The predicted lift coefficient, using twenty-four vortex panels, was 0.71% below the exact value

of 1.93.

10.2.1 NASA GA(W)-1 Airfoil

The computational method was first applied to the NASA GA(W)-1 airfoil. This thick ($t/c=0.17$) airfoil stalls by means of the progression of the separation point from the trailing edge forward. Compressibility effects were introduced only for the separated flow analysis. Figures 32(a-g) indicate the pressure distribution for the range of incidences, $8.0^\circ \leq \alpha \leq 21.14^\circ$. In general, the method predicts the attached flow pressure distribution ($\alpha = 8.0^\circ$) quite satisfactorily, even without boundary layer correction as indicated in figure 32(a). However, the numerical pressures at the trailing edge do deviate slightly from the experimental values.

The separated flow analysis was carried out for the incidence cases in the range $12.0^\circ \leq \alpha \leq 21.14^\circ$ as shown in figures 32(b-g). For each incidence case, IWMP was used to generate the initial starting wake. Convergence of the wake geometry was generally achieved after 8 - 12 iterations. The predicted inviscid separated flow base pressure level is not in good agreement with the corresponding experimental values. In all cases the predicted level is greater (is less negative) than the experimental level. As discussed earlier, in the present formulation of the inviscid separation model the pressure distribution in the trailing edge separated flow region is assumed constant. This does in fact represent a simplification of the real flow field in that the constant pressure region extending back to the airfoil trailing edge is followed by a short recompression region. A closer modelling of the recompression zone, possibly by introducing a graded vorticity model over the recompression region (replacing the present constant strength vorticity model) could be implemented to improve the predicted inviscid base pressure level. The sharp pressure recovery *kink*, which occurs just before the upper surface separation point is reached, is unrealistic and would probably disappear in a fully coupled inviscid-viscous analysis. For all incident cases, the method over predicted the peak leading edge C_p value in comparison with the experimental results, indicating the need of a viscous flow analysis.

Figure (33) compares the variation of the predicted and experimental values of the

lift and quarter-chord pitching moment coefficients at a Reynolds number of 6.3×10^6 . As indicated, at low angles of attack ($\alpha < 8^\circ$) the numerical attached potential flow and experimental results are in excellent agreement. However, at greater angles of attack ($\alpha > 8^\circ$) there is substantial deviation between the attached potential flow and experimental lift and pitching moment curves. The results obtained by introducing the separation model are in good agreement with the experimental values of lift and pitching moment coefficients, and represent a vast improvement over the pure attached potential flow results at these incidence cases.

10.2.2 NACA 4412 Airfoil

Figures 34(a-g) illustrate the analysis of the NACA 4412 airfoil. This airfoil section stalls in a similar fashion to the NASA GA(W)-1. Compressibility effects were not taken into account in the analysis. Figure 34(a) compares the attached potential flow solution and experimental results for an angle of 8° . As indicated, the pure inviscid analysis fails to accurately predict the correct experimental pressure distribution, again indicating the need for a fully coupled inviscid-viscous analysis.

Figures 34(b-g) compare the separation model and experiment results ($Re = 3.4 \times 10^6$) for a range of incidence cases. As noted for the GA(W)-1 airfoil, the separation model fails to accurately predict the experimental base pressure for all separated flow angles of attack. However, the addition of the separation model represents a substantial improvement, especially at high angles of attack, in comparison with the superimposed pure inviscid pressure distribution.

The calculated (attached and separated flow) and experimental lift and pitching moment coefficients are compared in figure (35) for a range of incidences. In general, the separation model over predicted the the experimental lift coefficient at each separated flow incidence case, but does give a much improved estimate of the lift in comparison to the pure inviscid attached flow results. In general, it was observed that as the angle of attack increased (hence a forward movement of the upper surface separation point) the number of wake iterations to convergence decreased (i.e. $\alpha = 12^\circ$ -13 iterations, $\alpha = 30^\circ$ -8

iterations) This would seem to indicate that the separated flow model is sensitive to the position and relative length of the wake, which for the present analysis (i.e. only modelling trailing edge separation) is directly related to the airfoil angle of attack. Further tests are suggested.

10.2.3 NACA 63₃ - 018 Airfoil

The NACA 63₃ - 018 symmetric airfoil section, shown in figure 18(c), stalls similarly to the previous two airfoil sections. Compressibility effects were introduced into the separated flow analysis using a free stream mach number of $M = 0.17$. The results for the attached and separated flow cases are shown in figures 36(a-f). The attached potential flow calculated pressure distribution, for an incidence of 6.0° shown in figure 36(a) is in excellent agreement with the corresponding experiment values.

The results obtained using the separation model are shown in figures 36(b-f). The sharp pressure recovery just before the point of separation, as discussed for the GA(W) 1 airfoil, is present for this airfoil as well, for angles of attack greater than 12° . The lift coefficient distribution is shown in figure (37). Similar to the NACA 4412 airfoil, the separation model over predicts the lift for the separated flow incidences cases, indicating the need for a fully coupled inviscid-viscous analysis. In general 8 - 13 iterations were required to relax the wake geometry into its force free position. Figure 36(f) is of academic interest and shows the pressure distribution for the NACA 63₃ - 018 airfoil at 90° to the free stream. The calculated lift and pressure drag coefficients are 0.2 and 1.99 respectively.

10.2.4 NACA 63₁ - 012 airfoil

The final airfoil considered for the inviscid attached and separated flow analysis was the NACA 63₁ - 012, this section is experimentally known to stall by sudden bursting of a short separation bubble. Compressibility effects were introduced for the separated flow analysis using a free stream mach number of 0.17. Experimentally, incipient separation occurred at the trailing edge at $\alpha = 12^\circ$ and bubble bursting occurred between 13.5° and 14° angle of attack. Once it was determined that there was no reattachment to the

airfoil the separated flow analysis was carried out in a similar fashion to the previous three airfoils. The upper surface separation point was set at $x/c = 0.01$ for all separated flow incidence cases as shown in figures 38(b-d). The inviscid attached flow pressure distribution is shown in figure 38(a). Poor results were obtained for the 14.8° and 17.8° incidence cases. However, these results do indicate the influence of the angle of attack, for a fixed upper surface separation point, on the resulting pressure distribution. Approximately nine iterations were required for each incidence case in order to relax the wake geometry.

10.2.5 Circular Cylinder

The bluff body separated flow analysis was carried out using a two-dimensional circular cylinder. Three different separation points were considered, which corresponded to three different Reynolds numbers. Figures 39(a-c) indicate the calculated and experimental pressure distributions for the three separation points ($\theta_{sep} = 107^\circ, 110^\circ, 120^\circ$). The inviscid attached flow pressure distribution as determined by the program is included for comparison. In all three cases, the separation model fails to predict the correct base pressure level, in comparison with the experimental results. In addition, for $\theta_{sep} = 120^\circ$ and $\theta_{sep} = 110^\circ$, the method fails to predict the position and value of the peak negative pressure coefficient value. In general, seven iterations were required to relax the bluff body wake geometry into its force free position. Compressibility effects were not included in the analysis. Although the separation model is an improvement over the pure inviscid solution, obviously a viscous analysis must be included in order to close the gap between the experimental and numerical pressure distributions.

10.3 Viscous Flow Results

10.3.1 Laminar Flow Test Case

An exact laminar flow test case developed by Curle (1958) was used in order to verify the prediction capabilities of the laminar boundary layer analysis component of the viscous

flow code. The theoretical velocity variation for the test case is given by

$$\frac{U}{U_{\infty}} = \xi(1 - \xi^2) \quad (10.1)$$

where

$$\xi = x/c$$

Figures 40(a-c) compare the predicted and exact values of various boundary layer parameters. As indicated, the predicted values are in general in good agreement with the corresponding exact values. A boundary layer step size of $\Delta S/C_o = 0.01$ was used.

10.3.2 Turbulent Flow Test Case

Numerical studies were carried out on the rough wall turbulent boundary layer analysis code, in order to investigate the influence of various parameters on the turbulent boundary layer development. For these studies the inviscid velocity variation over the upper surface of the NASA GA(W)-1 airfoil at an incidence of 12° was used. The turbulent flow analysis began at the maximum leading edge velocity value.

Figure (41) indicates the effect of varying the boundary layer step size, $\Delta S/C_o$, on the predicted position of turbulent boundary layer separation. As shown, the predicted position of separation converges for step sizes smaller than $\Delta S/C_o = 0.01$. At this step size, the ratio of the boundary layer thickness, δ , at the mid-chord position, to the step size is approximately 1.2. This parameter is calculated and included at each test point in figure (41) for comparative purposes. In light of the above result, a boundary layer step size of 0.01 was used for all turbulent flow test cases. The influence of the roughness density value, λ^1 on the development of various boundary layer parameters is shown in figures 42(a-c). A constant value of the non-dimensional roughness height, k/C_o , was used in the analysis. As indicated, when the roughness density is increased from $\lambda = 0.5$ to $\lambda = 4.0$ the boundary layer becomes increasingly thick, leading to early flow separation.

¹ λ is the ratio of frontal area of roughness elements divided by airfoil surface area

In a similar fashion, the influence of varying the non dimensional roughness height k/C_o , on the development of the various boundary layer parameters is shown in figure 43(a-c). A constant roughness density ($\lambda = 2$) was used for this study. As expected, decreasing the non dimensional roughness height, decreases the roughness influence on the boundary layer development. A smooth surface is essentially obtained when using a non dimensional roughness height of $k/C_o = 0.00001$. Decreasing the non dimensional roughness height value further, results in essentially a single curve for each boundary layer parameter as indicated in the figures.

Several experimental investigations have previously been performed in order to study the influence of surface roughness on the development of a turbulent boundary layer. Comparison with the experiments of Scottron and Power (1965) were made in order to demonstrate the rough surface analysis capability of the present turbulent flow model. Specifically, two comparisons were made and are shown in figures (44-45). As indicated in these figures, the agreement between calculated and experimental results are satisfactory, even for figure (45), in which the experimental data was taken in a strong adverse pressure gradient. Furthermore, the present calculated results are in excellent agreement with Dvorak's (1969) corresponding calculated values.

The rough wall turbulent boundary layer analysis code was further tested against two flows taken from the Stanford conference. The two flows are identified below for reference.

- (a) ID2700, Herring and Norbury — Favourable pressure gradient equilibrium boundary layer. (See fig.46)
- (b) ID2400 — Bradshaw relaxing flow. (See fig.47)

Figure 46 compares the predicted and experimental boundary layer parameters for the Herring and Norbury favourable pressure gradient layer. As shown, the predicted results correlate satisfactorily with the corresponding experimental results. The predicted results, are however, based on a rough surface combination of λ and k/c . Therefore, the predicted results could possibly be improved by considering a fully smooth surface.

Figure 47 compares the predicted and experimental results for Bradshaw's relaxing

flow. As indicated, and expected, the results are poor. This is due in part to the fact Head's entrainment method fails to take into account the upstream history effects of the flow.

10.4 Inviscid-Viscous Flow Results

The full inviscid-viscous analysis was performed on the NASA GA(W)-1, NACA 4412 and NACA 63₃-018 airfoils. As was previously discussed, for positive angles of attack these airfoils stall by a gradual forward (upstream) movement of the upper surface separation point and therefore can be modelled with the present analysis code. The analysis of each airfoil was carried out for a range of increasing incidence cases. Each new incidence case started with the previous solution and wake model (poor results were obtained when only one high incidence case was considered). The non-dimensional roughness height and roughness density parameter were selected such that the influence of the roughness was minimal. Compressibility effects were introduced when the free stream Mach number was known. Only two viscous flow iterations were performed for any one incidence case. Convergence of the inviscid-viscous iteration loop was not investigated. In the present study, the viscous flow analysis supplied the source distribution and position of boundary layer separation.

For comparison, the inviscid separated flow solution, for each airfoil incidence case, is included in order to indicate the relative change in the pressure distribution due to the modelled displacement effect of the boundary layer.

The $H - R_s$ method was used to predict the point of transition for the NASA GA(W)-1 airfoil at each incidence case. In general, the method predicted the point of transition on the upper airfoil surface at or very near (within one step point) the point of peak negative pressure. The lower surface transition point was predicted, for all incidence cases, at approximately the mid-chord position. The $H - R_s$ method proved unreliable in predicting transition for the NACA 4412 and NACA 63₃-018 airfoils. It was observed that the extremely favourable and adverse pressure gradients near the leading edge of

these airfoils pushed the shape factor outside the applicable range of the $H - R_s$ method. The alternative transition criterion was therefore used for these airfoils. The lower surface transition point was fixed at $S/C_o = 0.15$ for all tests. This position was selected in order to trigger transition quickly on the lower surface and to avoid any interference with the leading-edge stagnation point.

Figures (48 - 50) show the pressure distribution for the NASA GA(W)-1, NACA 63₃ - 018 and NACA 4412 airfoils. In general, the GA(W)-1 and NACA 4412 airfoils required from 10 - 12 wake iterations in order to meet the default convergence criterion for each inviscid solution at each incidence case. A fixed number of wake iterations (5) were used for the NACA 63₃ - 018 airfoil at each incidence case.

In reference to figures (48-50), the predicted and experimental results are in excellent agreement for all incidence cases considering only two inviscid-viscous iterations were performed. The sharp pressure recovery, just before separation, which was predicted by the inviscid separated flow analysis is essentially eliminated with the addition of the source terms. In addition, the predicted base pressure levels, for the three airfoils, are in good agreement with their corresponding experimental values.

In the present study, the upper surface source term grew rapidly at the point of separation. No attempt however was made to limit this growth or match this term with the corresponding lower surface trailing edge separation point source term. Hence, although the separation point vortex strengths were constrained to be equal, the upper and lower surface separation point pressure values were no longer equal.

The viscous flow analysis, in general, predicted the point of separation very well. However, as previously noted, only two viscous flow iterations were performed without concern for achieving any type of convergence. Obviously, in light of the predicted results, the possibility of over-predicting the point of separation exists. A second, related, consideration is the inviscid solution constraint, that the boundary layer separation point must fall on an existing airfoil junction point. For all the airfoil incidence cases, the actual predicted point of boundary layer separation had to be modified, to varying degrees, to accommodate this inherent limitation.

Discussion and Results

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In conclusion, figure (51) indicates the improvement in the predicted lift and pitching moment coefficients for the NACA 4412 airfoil.

Chapter 11

CONCLUSIONS

11.1 Conclusions

A practical engineering tool for analysing the incompressible flow field around single airfoils, with extensive trailing edge separation has been developed, which is applicable to a wide range of airfoil shapes. The major observations made during this research are outlined below.

- (i) The separation model, originally proposed by Dvorak and Maskew (1977) was successfully incorporated into the previously developed attached inviscid flow scheme for single element airfoils.
- (ii) The stability and convergence characteristics of the inviscid separation model were demonstrated.
- (iii) The number of wake iterations and the wake length factor were found to be important parameters in the separated potential flow solution.
- (iv) The basic inviscid separated flow analysis method (with the separation points known) predicted both the pressure and lift variation quite well, for a wide variety of airfoil sections and demonstrated a substantial improvement over the pure attached flow results.
- (v) The modified skin friction law for rough surfaces in turbulent flow was shown to predict reasonably well the experimental results for the non-zero pressure gradient cases tested.
- (vi) The roughness density parameter and non-dimensional roughness height, were found to be important parameters which influence the growth of the turbulent boundary layer.
- (vii) The inviscid-viscous flow interaction capability of the complete algorithm was suc-

cessively demonstrated for a number of single element airfoils.

(viii) Post stall behavior is best predicted for the trailing-edge type of separation.

(ix) Leading-edge and thin airfoil stall prediction could be considered by the implementation of a laminar separation bubble bursting criterion.

Chapter 12

LIMITATIONS OF THE PROGRAMS

12.1 Limitations of the Programs

Certain limitations are associated with each component of the complete analysis method and should be noted when using the algorithm.

- (i) The upper surface boundary layer separation point must fall on an existing airfoil or bluff body junction point.
- (ii) Only single element airfoils can be considered. A maximum of 90 airfoil and wake junction points are allowed for defining the complete configurations.
- (iii) An empirical correlation must be used to determine the correct length of the wake.
- (iv) The method is limited to modelling only a subsonic flow field and it cannot handle strong compressibility effects.
- (v) Only trailing edge separation can be predicted and modelled.
- (vi) A maximum of 400 boundary layer step points are allowed for analysing both the upper and lower airfoil surfaces in the viscous flow analysis.
- (vii) The inviscid-viscous flow analysis is not fully coupled.

Chapter 13

FUTURE WORK

13.1 Recommendations for Future Work

In this chapter, a number of suggestions for future work are presented and discussed.

- (i) Develop a fully coupled inviscid-viscous separated flow analysis program.
- (ii) Extend the present inviscid separated flow model to handle multi-element airfoil configurations.
- (iii) Evaluation of a graded wake vorticity model.
- (iv) Carry out a full inviscid-viscous separated flow analysis for a wide range of airfoil sections.
- (v) The establishment of a more reliable form of the modified separated flow Kutta condition for the inviscid-viscous flow analysis.
- (vi) Implement the option of using an airfoil panel selection criterion based on an equal angular displacement between junction points (i.e. use of a cosine law distribution of points).
- (vii) Modify the initial wake modelling program to generate the wake boundaries based on an equal distance between wake junction points.
- (viii) Implement a new transition criterion, (replacing the $H - R_\tau$ method) which is more applicable to the present type of analysis.
- (ix) Implement laminar separation bubble bursting criterion allowing for the prediction of thin airfoil stall.
- (x) The implementation of Head's improved entrainment method [Head and Patel, 1970] in the present rough wall turbulent flow analysis code.

- (xi) Towards investigating the problem of the aerodynamic effect of hoar frost on aircraft wings, equivalent roughness heights and density characteristics for hoar frost are required. These characteristics could be established through an experimental program.

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TABLES

Approach	Advantages	Disadvantages
<i>Experimental</i>	1) Capable of being most realistic	1) Equipment required 2) Scaling problems 3) Tunnel corrections 4) Measurement difficulties 5) Operating costs
<i>Computational</i>	1) No restriction to Linearity 2) Complicated Physics can be treated 3) Time evolution of flow can be obtained	1) Truncation error 2) Boundary Condition problems 3) Computer costs

Table-I: Comparison of Experimental and Computational Approaches.

[Anderson et. al., 1984 (1)]

Section	Reynolds No.	Mach No.	Source of Data
NASA GA(W)-1	6.3×10^6	0.15	McGhee, R.J. & Beasley, W.D. (1973).
NACA 4412	3.4×10^6	≈ 0	Pinkerton, R.M. (1938)
NACA 63 ₃ - 018	5.8×10^6	0.17	McCullough, G.B. & Gault, D.E. (1951)
NACA 63 ₁ - 012	5.8×10^6	0.17	McCullough, G.B. & Gault, D.E. (1951)
Cylinder	0.44×10^6	≈ 0	Celik, I. & Patel, V.C. (1982)
Cylinder	30×10^6	≈ 0	Data Item, Engineering Sources, Data Unit, ESDU 80025, (Sept. 1981)

Table II Summary of Airfoil Sections, Conditions
Considered and Source of Experimental Data

TABLE III. NASA GA(W)-1 AIRFOIL COORDINATES

X/C	(Y/C) _{upper}	(Y/C) _{lower}
0.000	0.000	0.000
0.005	0.02035	-0.01444
0.0125	0.03069	-0.02052
0.0250	0.04165	-0.02691
0.0375	0.04974	-0.03191
0.0500	0.05600	-0.03569
0.1000	0.07309	-0.04700
0.1500	0.08413	-0.05426
0.2000	0.09209	-0.05926
0.2500	0.09778	-0.06265
0.3000	0.10169	-0.06448
0.3500	0.10409	-0.06517
0.4000	0.10500	-0.06483
0.4500	0.10456	-0.06344
0.5000	0.10269	-0.06091
0.5500	0.09917	-0.05683
0.6000	0.09374	-0.05061
0.6500	0.08604	-0.04265
0.7000	0.07639	-0.03383
0.7250	0.07096	-0.02930
0.7500	0.06517	-0.02461
0.7750	0.05913	-0.02030
0.8000	0.05291	-0.01587
0.8250	0.04644	-0.01191
0.8500	0.03983	-0.00852
0.8750	0.03313	-0.00565
0.9000	0.02639	-0.00352
0.9250	0.01965	-0.00248
0.9500	0.01287	-0.00257
0.9750	0.00604	-0.00396
1.0000	-0.00074	-0.00783

TABLE IV.- NACA 4412 AIRFOIL COORDINATES

X/C	(Y/C) _{upper}	(Y/C) _{lower}
0.000	0.000	0.000
0.0125	0.0244	-0.0143
0.0250	0.0339	-0.0195
0.0500	0.0473	-0.0249
0.0750	0.0576	-0.0274
0.1000	0.0659	-0.0286
0.1500	0.0789	-0.0288
0.2000	0.0880	-0.0274
0.2500	0.0941	-0.0250
0.3000	0.0976	-0.0226
0.4000	0.0980	-0.0180
0.5000	0.0919	-0.0140
0.6000	0.0814	-0.0100
0.7000	0.0669	-0.0065
0.8000	0.0489	-0.0039
0.9000	0.0271	-0.0022
0.9500	0.0147	-0.0016
1.0000	0.0013	-0.0013

TABLE V.- NACA 63₃-018 AIRFOIL COORDINATES

X/C	(Y/C) _{upper}	(Y/C) _{lower}
0.000	0.000	0.000
0.0050	0.01404	-0.01404
0.0075	0.01713	-0.01713
0.0125	0.02217	-0.02217
0.0250	0.03104	-0.03104
0.0500	0.04362	-0.04362
0.0750	0.05308	-0.05308
0.1000	0.06068	-0.06068
0.1500	0.07225	-0.07225
0.2000	0.08048	-0.08048
0.2500	0.08600	-0.08600
0.3000	0.08913	-0.08913
0.3500	0.09000	-0.09000
0.4000	0.08845	-0.08845
0.4500	0.08482	-0.08482
0.5000	0.07942	-0.07942
0.5500	0.07256	-0.07256
0.6000	0.06455	-0.06455
0.6500	0.05567	-0.05567
0.7000	0.04622	-0.04622
0.7500	0.03650	-0.03650
0.8000	0.02691	-0.02691
0.8500	0.01787	-0.01787
0.9000	0.00985	-0.00985
0.9500	0.00348	-0.00348
1.0000	0.0000	0.000

TABLE VI.- NACA 63₁-012 AIRFOIL COORDINATES

x/c	$(y/c)_{upper}$	$(y/c)_{lower}$
0.000	0.000	0.000
0.0050	0.00985	-0.00985
0.0075	0.01194	-0.01194
0.0125	0.01519	-0.01519
0.0250	0.02102	-0.02102
0.0500	0.02925	-0.02925
0.0750	0.03542	-0.03542
0.1000	0.04039	-0.04039
0.1500	0.04799	-0.04799
0.2000	0.05342	-0.05342
0.2500	0.05712	-0.05712
0.3000	0.05930	-0.05930
0.3500	0.06000	-0.06000
0.4000	0.05920	-0.05920
0.4500	0.05704	-0.05704
0.5000	0.05370	-0.05370
0.5500	0.04935	-0.04935
0.6000	0.04420	-0.04420
0.6500	0.03840	-0.03840
0.7000	0.03210	-0.03210
0.7500	0.02556	-0.02556
0.8000	0.01902	-0.01902
0.8500	0.01274	-0.01274
0.9000	0.00707	-0.00707
0.9500	0.00250	-0.00250
1.0000	0.000	0.000

FIGURES

Fig.1 Flow over a single element airfoil with trailing-edge separation.

Fig.2 Approximate representation of airfoil section.

Fig.3 Relative importance of separation modelling.
(Henderson, 1978)

Fig.4 Characteristic types of separated flow on airfoil sections.

Fig.5 Velocity induced at control point 1 by vortex sheet m .

Fig.6 Airfoil separation model nomenclature.

Fig.8a

Fig.8b

Fig.8a,b Bluff body separation model vorticity, auxiliary conditions and numbering system.

	1	A	A_1	0			γ_1	BR1
	2	A	A_2	0			γ_2	BR2
NPUSS	3	0	0	1	0 0 0 0 0 0 0 0 0 0 0 0 0 0	$\vdots$	0	
NPUS	4	:	0			0	:	
	5	:	0			0	:	
	6		0			0		
	7		0			0		
	8		0			0		
	9		0			0		
NPLSS	10	0	0	0	0 0 0 0 0 0 0 0 0 1 0 0 0 0	$\vdots$	0	
NPLS	11		0			0		
	12		0			0		
NPT	13	0	0	0	1 0 0 0 0 0 0 0 1 0 0 0 0 0	$\vdots$	0	
NPT1	14	0	0	0	-1 0 0 0 0 0 0 0 0 0 0 0 1 0	$\vdots$	0	
NPT2	15	0	0	0	0 0 0 0 0 0 0 0 -1 0 0 0 0 1	γ_5	0	

Fig.8c Matrix formulation; Bluff body separation model.

Fig.9 Initial wake geometry.

Fig.10 Wake length correlation curve.
(Dvorak and Maskew, 1977)

Fig.11 Critical and transition Reynolds number for adiabatic and heated wedge flows in water. (Wazzan, et. al., 1981)

Fig.12 Definition of the boundary layer displacement thickness

Fig.13 Analysis of displacement effect by altering the body surface shape.

Fig.14 Quadratic curve used for integrating pressure coefficients.

Fig.15

Stalled airfoil modelling program (STAMP)
calculation procedure.

Fig.15 Continued

Fig.15 Continued

Fig.15 Concluded.

Fig.16 . Boundary layer analysis program (BLAP) calculation procedure.

Fig.16 Continued

Fig.16 Concluded.

Fig.17 Concluded.

Fig. 18a Section shape for NASA GA(W)-1 airfoil.

Fig. 18b Section shape for NACA-4412 airfoil.

Fig. 18c Section shape for NACA 63-018 airfoil.

Fig. 18d Section shape for NACA 63₁-012 airfoil.

Fig. 29 Variation of aerodynamic parameters with the relaxation factor for a circular cylinder at 0.0° angle of attack.

Fig.20 Variation of aerodynamic parameters with the number of wake panels for the GA(W)-1 airfoil at 20.05° angle of attack.

Fig.21a

Fig.21b

Fig.21 Variation of aerodynamic parameters with the number of wake panels for a circular cylinder at 0.0° angle of attack.

Fig. 22 Variation of aerodynamic parameters with the wake length factor for the BAW-11 airfoil at 10.05° angle of attack.

Fig. 23 Variation of aerodynamic parameters with the wake length factor for a circular cylinder at 90° angle of attack.

Fig. 24 Variation of aerodynamic parameters with the overall solution criterion for the GA(W)-1 airfoil at 20.05° angle of attack.

Fig. 25 Variation of aerodynamic parameters with the overall solution criterion for a circular cylinder at 0.0° angle of attack.

Fig.27 Variation of aerodynamic parameters with the location of the upper surface separation point for the GA(W)-1 airfoil at 20.05° angle of attack.

Fig. 28 Variation of aerodynamic parameters with the relaxation factor for the GA(W)-1 airfoil at 20.05° angle of attack.

Fig.29 Variation of aerodynamic parameters with the relaxation factor for a circular cylinder at 0.0° angle of attack.

Fig.30a Position of initial and final converged wake geometry for the GA(W)-1 airfoil at 21.14° angle of attack.

Fig.30b Position of initial and final converged wake geometry for a circular cylinder at 0.0° angle of attack.

Fig. 31 Comparison of exact and predicted chordwise pressure distribution for a Joukowski airfoil at 10.0° angle of attack; 13.7% thickness, 5.2% camber.

Fig. 32 • Comparison of experimental and predicted (separation points known) chordwise pressure distributions for the GA(W)-1 airfoil.

Fig. 32 Continued.

NASA GA(W)-1 Airfoil

Incidence = 16.34°

Fig.32 Continued

NASA GA(W)-1 Airfoil

(a) Incidence = 18.25°

Fig.32 Continued

NASA GA(W)-1 Airfoil

e Incidence = 19.06°

Fig.32 Continued

f) Incidence = 20.05°

Fig.32 Continued

(g) Incidence = 21.14°

Fig.32 Concluded.

Fig.33 Comparison of experimental and predicted (separation points known) aerodynamic characteristics for the GA(W)-1 airfoil.

(a) Incidence $\alpha = 8.0^\circ$

Fig. 34

Comparison of experimental and predicted (separation points known) chordwise pressure distributions for the NACA-4412 airfoil.

(b) Incidence = 12.0°

Fig.34 Continued

NACA 4412 Airfoil

for incidence = 16.0°

Fig. 34 Continued

NACA 4412 Airfoil

(d) Incidence = 18.0°

Fig.34 Continued

NACA 4412 Airfoil

(e) Incidence = 20.0°

Fig. 34 Continued

NACA 4412 Airfoil

(°) Incidence = 24.0°

Fig.34 continued

NACA 4412 Airfoil

(g) Incidence = 30.0°

Fig.34 Concluded.

Fig. 35 Comparison of experimental and predicted (separation points known) aerodynamic characteristics for the NACA-4412 airfoil.

(a) Incidence = 6.0°

Fig. 36

Comparison of experimental and predicted (separation points known) chordwise pressure distributions for the NACA 633-018 airfoil.

NACA 633-018 Airfoil

Incidence = 12.0°

Fig.36 Continued

NACA 63₃-018 Airfoil (c) Incidence = 14.0°

Fig.36 Continued

NACA 63₃-018 Airfoil
 α Incidence = 16.0°

Fig. 36. Continued.

NACA 63₃-018 Airfoil (e) Incidence = 18.0°

Fig.36 Continued

NACA 63₃-018 Airfoil

(f) Incidence = 90.0°

Fig.36 Concluded.

Fig. 37 Comparison of experimental and predicted (separation points known) lift curve for the NACA 63-018 airfoil.

(a) Incidence = 12.8°

Fig.38

Comparison of experimental and predicted (separation points known) chordwise pressure distributions for the NACA 63₁-012 airfoil.

NACA 63₁-012 Airfoil

Fig.38 Continued

NACA 631-012 Airfoil

(c) Incidence = 17.8°

Fig.38 Continued

NACA 63₁-012 Airfoil

(d) Incidence = 20.8°

Fig.38 Concluded.

Fig. 39 Comparison of experimental and predicted (separation points known) pressure distributions for a circular cylinder at 0.0° angle of attack

(b) $\theta_{Sep} = 120.0^\circ$
Fig.39 Continued

(c) $\theta_{sep} = 107.0^\circ$

Fig. 39 Concluded.

Fig.40

Comparison of exact and predicted values of various parameters for the laminar boundary layer test case.

(b)

Fig.40 Continued

(c)

Fig.40 Concluded.

Fig. 41 Variation of predicted upper surface separation point with the boundary layer step size for the GA(W) 1 airfoil at $\alpha=1.0^\circ$ angle of attack.

Fig. 42 Variation of predicted turbulent boundary layer parameters with roughness density parameter for the (A) foil at 12.0° angle of attack.

(b)

Fig. 42 Continued

Fig.43 Variation of predicted turbulent boundary layer parameters with non-dimensional roughness height for the GA(W)-1 airfoil at 12.0° angle of attack.

Fig. 43 continued

Fig. 43 Concluded.

Fig.44 Comparison of experimental and predicted turbulent boundary layer parameters for a rough surface (Scottson and Power, data-A).

Fig.45 Comparison of experimental and predicted turbulent boundary layer parameters for a rough surface (Scottson and Power, data-E).

Fig.46 Comparison of experimental and predicted turbulent boundary layer parameters for Herring and Norbury's equilibrium boundary layer.

3

Fig.47 Comparison of experimental and predicted turbulent boundary layer parameters for Bradshaw's relaxing flow boundary layer.

Fig.48

Comparison of experimental and predicted (inviscid-viscous) chord-wise pressure distributions for the GA(W)-1 airfoil.

NASA GA(W)-1 Airfoil

(b) Incidence = 18.25°

Fig.48 Continued

NASA GA(W)-1 Airfoil

(c) Incidence = 20.05°

Fig.48 Concluded.

Fig.49 Comparison of experimental and predicted (inviscid-viscous) cord-wise pressure distributions for the NACA 633-018 airfoil.

NACA 63₃-018 Airfoil

(b) Incidence = 16.0°

Fig.49 Continued

NACA 63₃-018 Airfoil

(c) Incidence = 18.0°

Fig.49 Concluded.

Fig.50 Comparison of experimental and predicted (inviscid-viscous) chord-wise pressure distributions for the NACA 4412 airfoil.

NACA 4412 Airfoil

(b) Incidence = 18.0°

Fig.50 Continued

(c) Incidence = 20.0°

Fig.50 Continued

NACA 4412 Airfoil

(d) Incidence = 24.0°

Fig.50 Concluded.

Fig.51 Comparison of experimental and predicted (inviscid-viscous) aerodynamic parameters for the NACA 4412 airfoil.

APPENDICES

APPENDIX A

A user's manual for the stalled airfoil modelling program (STAMP).

A-1: Introduction

STAMP is a potential flow panel method computer program which models the inviscid flow field around single element airfoils with extensive areas of separated flow. In order to solve the governing Laplace equation, the surface vortex singularity method has been adopted. The program models the solid surface of the airfoil section configuration in terms of contiguous straight-edged panels. On the airfoil surface, the vortex singularities are distributed over the panels with a linear variation. The free shear layers of the separated flow region are modelled in terms of constant strength vortex singularity panels. The wake surfaces require an inner iteration in order to establish their free shape.

Boundary layer displacement effects are modelled in the analysis using the transpiration (i.e. source distribution) method. Compressibility effects can be introduced using the Karman—Tsien formula. The panel method formulation gives rise to a set of simultaneous linear equations (flow tangency condition) of the form:

$$[A][\gamma] = [BR]$$

where $[A]$ is the overall influence coefficient matrix which includes the influence of the upper and lower free shear layers. $[\gamma]$ represents the vector of unknowns (vortex junction point strengths). The right side of the equation represents the known onset terms. In the initial potential flow analysis, $[BR]$ is the normal component of the undisturbed free stream velocity. Following a boundary layer analysis, $[BR]$ includes the displacement effect represented by linear distribution of sources around the airfoil surface. In the separated flow region of the airfoil, the sources are set to zero.

A-2: Program Formulation

STAMP is written in Fortran IV and was implemented on the CP-6 computer system at Carleton University, Ottawa. A modular approach was adopted in formulating the program for ease of modification and debugging. All primary variables are entered in common blocks so that each subroutine has access to them. All input formats are in general format (G) to facilitate ease of data preparation. The program is partially interactive in nature, allowing the user to choose the type of analysis and set various parameters. Program flags (POSTN) are used throughout the program to control the execution of various statements.

A complete inviscid-viscous analysis can be carried out by externally coupling the results of STAMP with the boundary layer analysis program (BLAP). STAMP has been set up to be completely independent of the boundary layer analysis program. If this type of analysis is to be carried out, the user must know the location of the airfoil separation points. The initial wake modelling program (IWMP) is a service program which must be used to generate the required initial wake shape geometry. In addition, it prepares an input data file for the separated flow analysis in STAMP.

STAMP has two primary subroutines *FAMOD* and *FSMOD* which control the type of analysis to be carried out. Subroutine *FAMOD* implements the attached potential flow case, while subroutine *FSMOD* handles the separated potential flow analysis. There are an additional 21 secondary subroutines which interact with the primary subroutines.

A complete listing of program variable names and their definition is included in appendix (B). In addition, appendix (C) gives a complete list and description of each subroutine which comprises STAMP.

A-3: Types of Analysis

In general, STAMP is restricted to the analysis of incompressible, steady flow around single element two-dimensional airfoils.

STAMP can handle both inviscid attached or separated potential flows. If an attached flow analysis is to be carried out (without boundary layer displacement correction), two types of Kutta condition are available for implementation¹.

- (i) Stagnation point at the airfoil trailing edge.
- (ii) Equal but opposite velocities on the upper and lower surface airfoil trailing edge.

The separated potential flow is divided into two types of analysis.

- (i) Airfoils with the upper surface separation point a variable and the lower surface separation point fixed at the airfoil trailing edge.
- (ii) Body configurations where both the upper and lower surface separation points are variables. (This type of analysis is typical of bluff body separation).

Boundary layer displacement effects can be implemented for both the attached and separated flow cases. A boundary layer analysis must be carried out, which provides the required source strength distributions. Compressibility effects can be introduced if desired.

A-4: Preparing Data for STAMP

In general, the input data necessary to carry out a full separated potential flow analysis (with boundary layer correction), will come from three sources.

- (i) Interactive input data, which are inputted as the program is running.
- (ii) Data file prepared by the initial wake modelling program which contains the airfoil and wake junction points.
- (iii) Data file prepared by the boundary layer analysis program which prepares the junction point source strength distribution.

¹ If boundary layer correction is desired for the attached flow case, a special form of the Kutta condition is used—see section(7.2.1)

A-4.1: Interactive Input Data

Once the program is running, STAMP will prompt the user for a number of parameters. The user should try to avoid typing more than one return after responding to each prompt. The following is a list of interactive variables (in order) with a brief description of each, used for the separated potential flow analysis.

1) NAM1, NAM2, NAM3 - airfoil section name

The airfoil or bluff body section name can be any string of characters, up to a maximum length of twelve. The name is used for later identification.

2). ALPHAA - airfoil angle of attack

Enter the aircraft angle of attack (in degrees) which corresponds to the present incidence case (range 0 - -90°). The angle of attack is the angle between the direction of the flow and the x-axis. At 0° angle of attack, the flow is in the positive x direction.

3). Boundary Layer Effects—POSTN(9)

ENTER-0 Boundary layer displacement effects are to be neglected.

ENTER-1 Boundary layer displacement effects are to be included.

Note: A data file containing the proper source distribution must be available for this case.

4). Compressibility Effects—POSTN(12)

ENTER-0 Compressibility effects are not to be taken into account.

ENTER-1 Compressibility effects are to be accounted for.

5). Free Stream Mach Number—AMACH

If compressibility effects are to be accounted for the user must enter the mach number.

6). Type of Flow Analysis—POSTN(1)

ENTER-0 If the flow is not separated from the airfoil. This will invoke the attached flow analysis.

ENTER-1 Flow is separated from the airfoil.

7). Type of Separation Analysis—POSTN(8)

ENTER-0 The separation on the upper and lower surface is a variable (i.e. bluff body analysis).

ENTER-1 The separation on the upper surface is a variable but lower surface separation point is fixed at the trailing edge.

8). Type of Solution Criterion—POSTN(16)

ENTER-0 If a fixed number of wake iterations is to be performed.

ENTER-1 If full wake convergence is desired.

9). Convergence Criterion—CRT1

Default convergence criteria is set at a 0.25% change of base pressure between two successive wake iterations. This criteria can be changed if desired. Maximum number of wake iterations is set at 20.

10). Number of Wake Iterations—POSTN(2) Enter the fixed number of wake iterations that are to be performed (postn(15)=?).

11). Relaxing Factor—Relax

Enter the desired over relaxation factor.

12). Output File—POSTN(3)

ENTER-0 For only the converged solution results (this will cause the program to output only the final results).

ENTER-1 For all wake shape iteration results.

13). Wake Coordinates, Output File—POSTN(11)

ENTER-1 If the wake junction and control point coordinates and parameters are to be included in the output file.

ENTER-0 If they are not to be included.

14). Check for Overlapping Wake Panels—POSTN(17)

ENTER-1 Perform check to see if any wake panels have overlapped. If any panels have overlapped the wake is truncated at the intersection point.

ENTER-0 Program will not perform the check.

15). Airfoil Junction Point Parameters, Output file—POSTN(6)

ENTER-1 If the parameters of each junction point are to be included in the output file (i.e. x/c , y/c vortex strength, C_p etc.)

ENTER-0 If parameters are not to be included.

16). Airfoil Control Point Parameter, Output File—POSTN(10)

ENTER-1 If the parameters of each control point are to be included in the output file.

ENTER-0 If the parameters are not to be included.

17). Display Influence Coefficient Matrix, Output File—POSTN(5)

ENTER-0 If the influence coefficient matrix is not to be outputted in the output file.

ENTER-1 If it is to be included in the output file. (This should only be done for debugging purposes)

18). Converged Wake Data Input File—POSTN(13)

If a full inviscid/viscous analysis is being carried out, the user can output a converged wake input data file. This file contains the converged wake junction points, airfoil points and all data necessary to run STAMP at the next incidence case.

ENTER-1 If a file is desired.

ENTER-0 No file is desired.

19). Boundary Layer Data File—POSTN(14)

ENTER-1 If a data file is desired for use in the boundary layer analysis program.

ENTER-0 If file is not desired.

A-4.2 Data File Prepared by IWMP

The initial wake modelling program generates a data file in the correct format for use in the STAMP. This file contains the following parameters.

First Input Data File Line

N, CHORD, WLF

- N** -Actual number of airfoil panels used to model the solid surface of the airfoil.
- CHORD** -Chord length of the airfoil.
- WLF** -Wake Length Factor.(see fig.10)

Second Input Data File Line

NPU, NPL

- NPU** -Number of panels used to model upper wake surface.
- NPL** -Number of panels used to model lower wake surface.

Following Lines—Airfoil and wake junction points

$X(I), Y(I)$

The $X(I), Y(I)$ coordinates of the airfoil section junction points are first inputted in a counter-clockwise fashion starting at the airfoil trailing edge (upper surface). The upper surface separation point is repeated twice in the data file. If the lower surface separation point is not at the trailing edge, then a similar arrangement is made (i.e. lower surface separation coordinates are repeated twice).

After the last airfoil junction point, the first upper surface wake junction points follow. The first point corresponds to the upper surface separation point and must be numerically equal to it. After the last upper surface wake junction point, the lower surface junction points are inputted. In a similar fashion, the first point corresponds to the lower surface separation point and must be numerically equal to it.

The input data file should take the following form for the fixed trailing edge lower

surface separation point case.

	<i>N</i> <i>NPU</i> <i>X</i> (1), <i>X</i> (2), : : <i>X</i> (<i>NPUS</i>)	<i>CHORD</i> <i>NPL</i> <i>Y</i> (1), <i>Y</i> (2), : : <i>Y</i> (<i>NPUS</i>)	<i>WLF</i> ← first airfoil junction point ← upper airfoil surface junction point, repeated twice in data file ← <i>X</i> (<i>NPUS</i>) = <i>X</i> (<i>NPUS</i>); <i>Y</i> (<i>NPUS</i>) = <i>Y</i> (<i>NPUS</i>)
Airfoil surface junction point	<i>X</i> (<i>NPUS</i>)	<i>Y</i> (<i>NPUS</i>)	
	<i>X</i> (<i>NPT</i>)	<i>Y</i> (<i>NPT</i>)	← last airfoil junction point equal to first first airfoil point
	<i>X</i> (<i>NUS</i>)	<i>Y</i> (<i>NUS</i>)	← first upper surface wake junction point must be equal to <i>X</i> (<i>NPUS</i>), <i>Y</i> (<i>NPUS</i>)
upper wake surface junction points	<i>X</i> (<i>NUF</i>) <i>X</i> (<i>NLS</i>)	<i>Y</i> (<i>NUF</i>) <i>Y</i> (<i>NLS</i>)	← last upper surface wake point ← first lower surface wake junction point. Must be equal to <i>X</i> (<i>NPT</i>), <i>Y</i> (<i>NPT</i>)
lower wake surface junction points	<i>X</i> (<i>NLF</i>)	<i>Y</i> (<i>NLF</i>)	last lower surface wake point

A-4.3 Source Distribution Input Data File

The input source distribution data file is prepared by the boundary layer analysis program. The junction point source strengths are read in a counter-clockwise fashion starting at the trailing edge. If separation has been predicted by BLAP, the junction point source strengths are set to zero in the separated region.

A-5: Running STAMP

In order to run the program, type the following commands

```
!FORTRAN STAMP OVER *1
!LINK *1 OVER *STAMP (UN=IMSL$IMSL)
!SET 4 DATASOR.
!SET 5 DATAIR
!SET 6 OUT1
!SET 7 DATBL
!SET 8 DATC
!*STAMP
```

where

DATASOR	:Junction point source distribution file
DATAIR	:Is the name of the data file containing airfoil and wake junction points.
OUT1	:Is the file which will contain the results of the analysis.
DATBL	:File containing data for the boundary layer analysis.
DATC	:Converged wake input data file for next incidence case.

APPENDIX B

Dictionary of variable names for the stalled airfoil modelling program
(STAMP).

	<i>Computer Symbol</i>	<i>Mathematical Symbol</i>	<i>Definition</i>
1.	<i>A</i>	$A_{i,m}$	Influence Coefficient.
2.	<i>BR</i>	γ	Vortex strength after Matrix has been solved initially, represents onset terms.
3.	<i>XI</i>	$z_{control}$	z cartesian coordinate of control point.
4.	<i>YI</i>	$y_{control}$	y cartesian coordinate of control points.
5.	<i>XL</i>	l_m	Panel length.
6.	<i>PH</i>	θ	Panel orientation angle.
7.	<i>X</i>	z	z cartesian coordinate of junction points.
8.	<i>Y</i>	y	y cartesian coordinate of junction points.
9.	<i>VNN</i>	V_n	Normal velocity at control points.
10.	<i>VTAN</i>	V_T	Tangential velocity at control points.
11.	<i>CP</i>	C_P	Pressure coefficient at control points.
12.	<i>PC</i>	$C_{P,corr}$	Corrected pressure coefficient (Karman-Tsien) at control points.
13.	<i>CPJ</i>	C_P	Pressure coefficient at junction points.
14.	<i>PCJ</i>	$C_{P,corr}$	Pressure coefficient at junction points (Karman-Tsien).
15.	<i>QSJ</i>	q	Source strengths at junction points.
16.	<i>XK(1 - 4)</i>	$zk(1 - 4)$	Engineering coefficients. Linear Strength

			varying vortex panels.
17.	$XKC(1-2)$	$zkc(1-2)$	Engineering coefficients, constant strength vortex panels.
18.	CL	C_L	Lift coefficient.
19.	CD	C_D	Pressure drag coefficient.
20.	$CM1$	$C_{M_{L.E}}$	Pitching moment about leading edge.
21.	$CM2$	$C_{M_{\frac{1}{4}}}$	Pitching moment about quarter-chord point.
22.	$CHORD$	$Chord$	Chord length of airfoil.
23.	$AMACH$	M	Mach number.
24.	$CLGAM$	C_L	Lift coefficient based on integration of vortex strengths.
25.	N		Actual number of panels used to model airfoil points.
26.	NPT		Total number of airfoil points.
27.	NUS		First junction panel point upper surface wake.
28.	NUF		Last junction panel point upper surface wake.
29.	NLS		First junction panel point lower surface wake.
30.	NLF		Last junction panel point lower surface wake.
31.	NFN		Total number of junction points (airfoil + wake) to be read in.
32.	NPU		Number of panels used to model upper wake.
33.	NPL		Number of panels used to model lower wake.
34.	PI	π	3.14159.
35.	$NPTI$		Counter to implement boundary condition upper

			surface separation point, (upper wake surfaces).
36.	<i>NPT2</i>		Same as above at lower surface separation point.
37.	<i>NPUS</i>		Counter—first point inside upper surface separated region (see fig. 7b, 8b).
38.	<i>NPUS</i>		Counter—first point outside upper surface separated region (see fig. 7b, 8b).
39.	<i>NPUS2</i>		Counter—second point inside separated region.
40.	<i>NPLSS</i>		Counter—first point outside lower surface separated region (see fig. 8b).
41.	<i>NPLS</i>		Counter—first point inside lower surface separated region (see fig. 8b).
42.	<i>NPLS2</i>		Counter—second point inside lower surface separated region.
43.	<i>WLF</i>		Wake length factor.
44.	<i>ALPHAA</i>	α	Angle of attack (degrees).
45.	<i>ALPHA</i>	α	Angle of attack (rads).
46.	<i>RELAX</i>	ξ	Relaxation factor.
47.	<i>NA</i>		Counter—which equation is to be deleted from the lower surface in order to implement a boundary condition (see fig. 7b).
48.	<i>CRT1</i>	ω	Convergence criteria for wake iterations —default—0.25.
49.	<i>CPB</i>	C_{P1-1}	Previous base pressure coefficient used

in wake iterations.

AN

After the influence coefficients have been formed, they are entered into [AN] so they will not have to be formed for the next iteration.

BB

After the onset terms have been formed in array *BR*, they are entered in *BB* array for the next iteration.

Stalled Airfoil Modelling Program Flag Designation

Flag	Description
POSTN (1)=0	Flow is not separated from airfoil.
POSTN (1)=1	Flow is separated from airfoil.
POSTN (2)=?	Number of wake shape iteration to be performed.
POSTN (3)=0	Output file containing only converged results.
POSTN (3)=1	Output file containing all iteration results.
POSTN (4)=0	Kutta condition—equal velocities at trailing edge.
POSTN (4)=1	Kutta condition—stagnation point at trailing edge.
POSTN (5)=0	Matrix will not be printed in output file.
POSTN (5)=1	Matrix will be printed in output file
POSTN (6)=0	Junction point parameters will not be printed in output file.
POSTN (6)=1	Junction point parameters will be printed in output file.
POSTN (7)	Counter.
POSTN (8)=0	Lower surface separation point is at some point on the pressure surface.
POSTN (8)=1	Lower surface separation point set at trailing edge.
POSTN (9)=0	Pure inviscid solution.
POSTN (9)=1	Model displacement effect with source terms.
POSTN (10)=0	Output file will not contain control point parameters.
POSTN (10)=1	Output file will contain control point parameters.

POSTN (11)=0 Output file will not contain parameters of the wake surfaces.

POSTN (11)=1 Output file will contain parameters of the wake surfaces.

POSTN (12)=0 Compressibility effects will not be taken into account.

POSTN (12)=1 Compressibility effects will be taken into account.

POSTN (13)=1 Output file containing the airfoil coordinates; converged wake geometry, and all other data necessary to run the program will be outputted.

POSTN (13)=0 will not be printed.

POSTN (14)=1 Output file for use in the boundary layer program will be generated.

POSTN (14)=0 No file will be generated.

POSTN (15)=0 Input number of wake iterations.

POSTN (15)=1 Full convergence of the wake.

POSTN (16)=0 Default criteria.

POSTN (16)=1 Input a new convergence criteria.

POSTN (17)=0 Program will not check to determine if wake panels are crossed.

POSTN (17)=1 Will check to determine if any of the wake panels are crossed. If so, the wake panels are truncated at the intersection point.

APPENDIX C

Description of each subroutine which comprises the stalled airfoil modelling program.

Subroutine	Function
INPUT	This subroutine controls the input of all data necessary to run the program. It is partially interactive, prompting the user for various input data.
FAMOD	This subroutine carries out the analysis necessary to find the pressure coefficient distribution about the body for the attached flow case. It calls eight other subroutines.
KUTTA1	This subroutine establishes the kutta condition which forces the trailing edge of the airfoil to be a stagnation point.
KUTTA2	The subroutine establishes the kutta condition which forces the trailing edge velocities leaving the airfoil to be equal.
KUTTA3	The subroutine establishes the modified kutta condition for the attached flow case if boundary layer effects are to be modelled using the source strengths at the trailing edge, equal pressures are established there.
FSMOD	This is the controlling subroutine if flow is separated from the airfoil, and the separated wake model is to be used. It calls fourteen other subroutines in order to determine the various aerodynamic parameters.
AIRCOE	This subroutine generates the airfoil geometry influence coefficients. The influence coefficients are based on a linear variation of vorticity on each panel.
WAKITER	This subroutine determines the new wake shape in order to satisfy the

force free wake condition. The new wake shape is determined by a piecewise integration, starting at the separation point locations.

WAKCOE

This subroutine generates the influence coefficients for the upper and lower wake panels, assuming they are of constant strength.

LIFT

This subroutine integrates the bound vorticity around the airfoil, upper and lower wake contours. The total circulation and hence lift coefficient for the separated wake model is then determined.

AERO

This subroutine determines four aerodynamic parameters. They are the lift, pressure drag and pitching moment (leading edge and quarter-chord point) coefficients. The pressure coefficient distribution about the airfoil is integrated in three ways 1). *Fit a quadratic curve through 3 control point values of pressure coefficient and then integrate for the parameters.* (2). *Fit a linear line through adjoining junction point values of C_p and then integrate for the parameters.* (3). *The third method fits a parabola through three junction point values of C_p and then integrates for the parameters.*

VELOCITY

This subroutine calculates both the normal and external tangential velocities on the airfoil surface, upper and lower wake surfaces.

PRESCOE

This subroutine determines the pressure coefficients about the airfoil for the separated flow analysis.

CPCOMP

This subroutine uses the Karman—Tsien pressure coefficient correction formula in order to account for the effects of compressibility.

BOUND1

This subroutine implements the auxiliary conditions for the airfoil separated wake model. The vorticity distribution is constrained to start and finish with zero vorticity in the separated region. The vorticity value at the upper separation point is set equal but opposite to the lower surface (trailing edge) separation point. The upper wake surface is constrained to have the same vorticity value as the upper surface

separation point. The lower surface wake is constrained to have the same vorticity value as the lower surface separation point value.

BOUND2	This subroutine implements the same auxiliary conditions as above, only for the bluff body analysis.
RHS	This subroutine calculates the right hand side of the matrix for either the pure potential separated wake model or for the separated wake model with the boundary layer source distribution included.
TRANSX	This subroutine translates all x coordinates such that the leading edge is at $x = 0.0$. All x, y coordinates are then non-dimensionalized.
XKS	This subroutine calculates $zk(1)$ to $zk(4)$ which represent the engineering variables. The influence coefficients, $(A_{i,m})$ consist of simple combinations of these coefficients.
XKCONS	This subroutine calculates $zkc(1)$ and $zkc(2)$ which represent the engineering variables for the constant strength wake panels.
GEOMAIR	This subroutine calculates the coordinates of the control points, the panel angle $PH(I)$ and the length of each panel on the airfoil surface.
GEOMWAK	Similar to GEOMAIR but for panels used to model the upper and lower wake panels.
OUTPUT	This subroutine controls all the outputs of the program.
TABLE	This subroutine generates a table of various aerodynamic parameters.
AUX	This subroutine controls the output of the velocities U/U_{REF} , x coordinate and linear surface distance along the airfoil (starting at the stagnation point for both the upper and lower airfoil surfaces) along with all the other parameters necessary to run the boundary layer program.

APPENDIX D

A user's manual for the boundary layer analysis program *BLAP*.

D-1: Introduction

The boundary layer analysis program *BLAP* calculates the development of a two-dimensional boundary layer on the upper and lower surfaces of an airfoil. The pressure distribution needed to perform the analysis (in the form of a velocity variation) is generated by the stalled airfoil modelling program (*STAMP*). The primary function of the program is to calculate the separation points and determine the required junction point source strength distribution for use in *STAMP*. Integral boundary layer methods are used to calculate the growth of the layer.

The laminar boundary layer development is predicted using Thwaites method. The program calculates the development of the two-dimensional turbulent boundary layer over rough surfaces in pressure gradients using the method of Dvorak (1969). Transition is predicted using the $H - R_s$ method. The program is interactive and prompts the user for various input parameters.

D-2: Program Formulation

BLAP is written in standard Fortran and was implemented on the CP6 computer system at Carleton University, Ottawa. Similar to the stalled airfoil modelling program, a modular approach was adopted in formulating the program for ease of modification and debugging. Extensive use is made of common block storage for all primary variables. The program is partially interactive allowing the user to set various parameters as the program is running. Flags are used throughout the program to control the execution of various statements.

BLAP is composed of one main program which carries out the laminar-transition-turbulent analysis and calls twenty-two other subroutine to perform various support calculations. The program calculates the boundary layer growth along the airfoil surface. The x of the boundary analysis is not a cartesian coordinate, but is a measure along the airfoil surface starting at the stagnation point.

A complete listing of the program variable names and their definition are included in appendix (E). In addition, appendix (F) gives a complete list and description of each subroutine in BLAP.

D-3: Preparing Data for BLAP

In general the input data necessary to carry out a full laminar, transition turbulent analysis on both the upper and lower airfoil surfaces will come from two sources.

- (i) Interactive input data, which is inputted as the program is running.
- (i) Data prepared by the stalled airfoil modelling program (*STAMP*).

D-3.1: Interactive Input Data

The following is a list of the interactive variables (in order), with a brief description of each.

1). *FLAG (9)-Change in Current Type of Analysis*

The type of analysis (i.e. laminar only, turbulent only or full analysis) is set in the input data file from *STAMP*. This flag allows you to change the analysis.

ENTER 0 -No, do not change the type of analysis.

ENTER 1 -Yes, change the type of analysis.

2). *FLAG (4)-Type of Analysis (only if FLAG (9)=1)*

ENTER 0 -If laminar analysis is only desired.

ENTER 1 -If turbulent analysis is only desired.

ENTER 2 -Full laminar-transition-turbulent analysis is desired.

3). FLAG (10)-Starting Condition Laminar Analysis

ENTER 0 -If laminar analysis is to begin as flat plate flow.

ENTER 1 -If laminar analysis is to begin at stagnation point.

4). FLAG (8)-Calculation on one or both Airfoil Surfaces

ENTER 0 -Perform the boundary layer analysis on only one surface.

ENTER 1 -Perform the boundary layer analysis on both the upper and lower airfoil surfaces.

5). FLAG (3)-Select Transition Criterion

ENTER 0 -Use the $H - R_{\theta}$ method to predict transition on the upper and lower airfoil surfaces.

ENTER 1 -Fix transition point at the upper surface peak velocity point, lower surface transition point as input parameter.

6). FLAG (5)-Junction Point Source Distribution File

ENTER 0 -Do not generate a source file.

ENTER 1 -If a junction point source distribution file is desired for use in STAMP.
(Note: Analysis must be carried out on both upper and lower airfoil surface).

D-3.2: Data File Prepared by STAMP

The stalled airfoil modelling program generates a Data file in the correct format for use in the boundary layer analysis program. Each file contains the following parameters.

First Line

NAM1, NAM2, NAM3

NAM1, NAM2, NAM3 -Airfoil section name (up to 12 characters). Use for identifica-

tion.

*Second Line***ALPHAA, FLAG (4)****ALPHAA** -Airfoil angle of attack (degrees) use for reference.**FLAG (4)** -Type of analysis.

0 - Laminar analysis only.

1 - Turbulent analysis only.

2 - Full laminar ~~turbulent~~ analysis.*Third line***CHORD, UREF, STEPX****CHORD** -Chord length of the airfoil.**UREF** -Free stream or reference velocity used for reference.**STEPX** -The desired boundary layer step size.*Fourth Line***ATHETA, AHCONV, CFO****ATHETA** -Initial starting value of the momentum thickness parameter (non-dimensional Θ/C_o).**AHCONV** -Initial starting value of shape factor H for the turbulent analysis.**CFO** -Initial guess of the shear stress coefficient. Used to start turbulent calculations.*Fifth Line***RDEN, RHT, REYNO****RDEN** -Roughness density parameter, λ , surface area divided by frontal area of roughness.**RHT** -Non-dimensional roughness height, (k/C_o) .

REYNO - Reynolds number based on *UREF* and chord length of airfoil.

Sixth line

NTOT, NUP, NLOW

NTOT - Total number of junction velocity points to be read in.

NUP - Number of junction points on the upper airfoil surface including the stagnation point.

NLOW - Number of junction points on the lower airfoil surface including the stagnation point.

Remaining Lines

XCART, XC, UC

XCART - z coordinate (non-dimensional) corresponding to each junction point value of velocity.

XC - Represents the surface distance along the airfoil, starting at the stagnation point, measured between junction points.

UC - Junction point non-dimensional velocities, starting at the stagnation point.

Variables *XCART*, *XC* and *UC* are read in starting at the stagnation point and moving in sequence along the upper airfoil surface to the trailing edge. After the last point is reached on the upper airfoil, the lower surface points are read in starting at the stagnation point.

The input data file will take the following form for an upper and lower airfoil analysis.

	<i>NAM1</i>	<i>NAM2</i>	<i>NAM3</i>	
	<i>ALPHA</i>	<i>FLAG(4)</i>		
	<i>CHORD</i>	<i>UREF</i>	<i>STEPX</i>	
	<i>ATHETA</i>	<i>AHCONV</i>	<i>CFO</i>	
	<i>RDEN</i>	<i>RHT</i>	<i>REYNO</i>	
	<i>NTOT,</i>	<i>NUP,</i>	<i>NLOW,</i>	
	<i>XCART(1),</i>	<i>XC(1),</i>	<i>UC(1),</i>	← stagnation point
	<i>XCART(2),</i>	<i>XC(2),</i>	<i>UC(2),</i>	
Data upper	:	:	:	
airfoil surface	:	:	:	
	:	:	:	
	<i>XCART(NUP)</i>	<i>XC(NUP)</i>	<i>UC(NUP)</i>	← Last point upper airfoil surface
	<i>XCART(NUP + 1)</i>	<i>XC(NUP + 1)</i>	<i>UC(NUP + 1)</i>	← stagnation point
Data lower	:	:	:	
airfoil surface	:	:	:	
	<i>XCART(NTOT)</i>	<i>XC(NTOT)</i>	<i>UC(NTOT)</i>	← last point lower airfoil surface

D-4: Running BLAP

In order to run the program, type the following commands.

!FORTRAN BLAP OVER *2

!LINK *2 OVER *BLAP

!SET 5 DAT

!SET 6 OUT

!SET 4 DATSOR

!BLAP

where

DAT	- Data file prepared by STAMP
OUT	- Output file containing all boundary parameters.
DATSOR	- Junction point source strength distribution, arranged in proper order for direct use in STAMP.

In order to review results of the boundary layer analysis type

!COPY OUT

In order to obtain a hard copy of the results type

!COPY OUT TO LPOME351

APPENDIX E

Definition of variable names for the boundary layer analysis program
(BLAP).

	Computer Symbol	Mathematical Symbol	Definition
1.	UC	V_j	Junction point velocities read in from STAMP (non-dimensional).
2.	XC	S/C_o	Surface distance measured along airfoil surface corresponding to junction point velocities
3.	XCART	z	z cartesian coordinate corresponding to junction point velocities.
4.	U	V_i	Interpolated non-dimensional velocities along airfoil surface.
5.	X	S/C_o	Increasing stepped distance along airfoil surfaces corresponding to interpolated velocities $U(I)$.
6.	DU	$\frac{\partial V_i}{\partial s}$	Derivative of velocity at each interpolated data point.
7.	DX	$\Delta S/C_o$	Step size between interpolated data points.
8.	THETA	Θ/c	Momentum thickness non-dimensionalized by the chord.
9.	HCONV	$H = \delta^*/\Theta$	Boundary layer shape factor.
10.	HDELS	$H_{\delta-\delta^*}$	Modified boundary layer shape factor.
11.	DSTAR	δ^*	Displacement thickness.
12.	DELTA	δ	Boundary layer thickness.

13.	DELS	$\delta - \delta^*$	Boundary layer thickness-displacement thickness.
14.	Q	$q = \frac{\partial}{\partial s}(\rho V_s \delta^*)$	Source strength at incremented surface distances along airfoil.
15.	source	$q_{junction}$	Source strengths at junction points.
16.	REX	$R_x = x V_s / \nu$	Local Reynolds number.
17.	REYNO	$R = C_0 U_\infty / \nu$	Global Reynolds number.
18.	CHORD	CHORD	Chord length of airfoil.
19.	UREF	U_∞	Reference velocity.
20.	VIS	ν	Kinematic viscosity.
21.	CFI	C_f	Shear stress coefficient.
22.	UKNOV	$U_r K / \nu$	Dimensionless roughness height.
23.	RDEN	λ	Roughness density Parameter, Surface area divided by frontal area of roughness.
24.	RHT	k/c	Non-dimensional roughness height.
25.	CFO	$C_{f,initial}$	Initial guess of shear stress coefficient.
26.	RETH	R_θ	Reynolds number based on momentum thickness.
27.	CSCR	q	Source terms arranged in proper order for use in STAMP.
28.	ATHETA	$\Theta_{initial}$	Initial guess of the momentum thickness.
29.	AHCONV	$H_{initial}$	Initial guess of the shape factor.
30.	SSEP(1)		Linear distance along upper surface from the stagnation point. Set at trailing edge until changed.
31.	SSEP(2)		same as above; but lower surface.

32.	<i>XSEP(1)</i>	$(x/c)_{sep.}$	<i>x</i> - cartesian upper surface separation point.
33.	<i>XSEP(2)</i>	$(x/c)_{sep.}$	Same as above; but lower surface.
34.	<i>NUP</i>		Number of junction points on upper airfoil surface including stagnation point.
35.	<i>NLOW</i>		Same as above but on lower surface.
36.	<i>NTOT</i>		Total number of points, $NTOT = NUP + NLOW$.
37.	<i>NCLS</i>		First point (stagnation point) lower surface ($NUP + 1$).
38.	<i>NPU</i>		Number of fine incremented points on upper surface.
39.	<i>NPL</i>		Same as above, but for lower surface.
40.	<i>STEPX</i>	Δs	Step size along surface.
41.	<i>NPTU</i>		Number of fine increment points on the upper surface.
42.	<i>NPTL</i>		Same as above, but for lower surface.
43.	<i>NPT</i>		Total number of fine increment points. $NPT = NPTU + NPTL$.
44.	<i>NLS</i>		First point lower surface (Stagnation Point).
45.	<i>ITRANS</i>		Counter for upper or lower surface transition point.
46.	<i>ISEPU</i>		Counter at upper surface transition point.
47.	<i>ISEPL</i>		Counter at lower surface transition point.
48.	<i>c</i>		Thwaites pressure gradient parameter.
49.	<i>XTP(1)</i>		Lower surface forced transition point.

Boundary Layer Analysis Program Flag Definition

Flag	Definition
FLAG (1)=0	Program performs only laminar calculations until transition or separation or trailing edge.
FLAG (1)=1	Program performs turbulent calculations until trailing edge or separation.
FLAG (2)=0	Calculations upper surface.
FLAG (2)=1	Calculations lower surface.
FLAG(3)=0	Use $H - R_s$ method to predict transition on the upper and lower surfaces.
FLAG(3)=1	Use alternative method to predict transition.
FLAG(4)=0	Laminar calculations only.
FLAG(4)=1	Turbulent calculations only.
FLAG(4)=2	Laminar-transition-turbulent calculations.
FLAG(5)=0	No source distribution is generated for the airfoil program.
FLAG(5)=1	Source distribution file is generated for the airfoil program.
FLAG(6)=0	No separation upper surface (integer counter).
FLAG(6)=1	Separation upper surface (integer counter).
FLAG(7)=0	Same as above, but for lower surface.
FLAG(7)=1	Same as above, but for lower surface.
FLAG(8)=0	Calculations performed only on the upper surface.

FLAG(8)=1	Calculations performed on both surfaces.
FLAG(9)=0	No change to flag (4).
FLAG(9)=1	Allows user to change flag (4).
FLAG(10)=0	Initial condition, laminar flow, flat plate analysis.
FLAG(10)=1	Initial condition, laminar flow, stagnation point.

APPENDIX F

Description of each subroutine which comprises the boundary layer program (BLAP).

Subroutine	Function
CFOO	This function returns the value of C_f , which is obtained through a Newton—Raphson iteration.
FNOO	This function returns the value of C_f for the input values of the C_f guessed, H , Θ , roughness height and roughness density.
FNO	This function returns the value of the derivative of the function FNOO to be used in the Newton—Raphson iteration towards C_f .
FN1	This function returns $RETAH$, the value of $U_\tau k/\nu$ at the upper limit of the transitionally rough range; it represents fig. 3 of Dvorak's (1969) paper.
FN2	This function returns the value of the law of the wall shift. It represents fig. 1 of Dvorak's (1969) paper.
FN3	This function subroutine returns the value of $\Delta U_2/U_\tau$.
FN4	This function subroutine returns the value of the derivative of $\Delta U_1/U_\tau$ at the upper limit of the transitionally rough region.
FN5	This function returns the value of the derivative of $\Delta U_1/U_\tau$ at the upper limit of the transitionally rough region for FN2.
FN6	This function returns the function value of H_{s-s^*} , based on equations A2 and A3 of Dvorak's (1969) paper.
FN7	This function returns the function value of H , given the input value of H_{s-s^*} , based on equations A2 and A3 of Dvorak's (1969) paper.
GJORD	This subroutine find the coefficients for the log polynomial in the tran-

sitionally rough range using a Gauss—Jordon elimination.

RUNGEK

This subroutine contains the fourth order Runge—Kutta integration of the momentum integral equation and Head's entrainment equation in order to get the boundary layer parameter at the next station.

INPUT

This subroutine controls the reading in, of all the required data necessary to run the program.

INPUTX

This subroutine divides up the upper and lower airfoil surface into small increments in order to run the program.

INTERVEL

This subroutine determines the velocity at the finely incremented positions.

VELOCITY

This subroutine forms the quadratic coefficients *A*, *B* and *C*. It determines the point velocity and derivatives of velocity at each of the incremented points along the airfoil surface.

OUT1

This subroutine outputs various boundary layer parameters.

THWATS

This subroutine contains the Thwaites correlation formulas for the laminar boundary layer analysis.

DISPLACE

This subroutine determines the source distribution for use in the stalled airfoil modelling program.

XPOSED

This subroutine determines the airfoil upper and lower surface separation points in the cartesian coordinate system.

LAMSEP

This subroutine controls the output of certain statements if laminar separation has occurred on either airfoil surface.

TURBSEP

This subroutine controls the output of certain statements if turbulent separation has occurred on either airfoil surface.

APPENDIX G

User's manual for the initial wake modelling program (IWMP).

G-1: Introduction

IWMP is a service program which generates the initial wake streamline boundaries. The upper and lower wake sheets are represented by parabolic curves passing from the separation points to a common point downstream. A semi-empirical approach is used to determine the length of the wake. This is determined through the correct choice of the wake length factor which is determined from curve (figure 10).

G-2: Preparing Data for IWMP

The data necessary to run *IWMP* comes from two sources;

- i) Interactive input data, which is inputted as the program is running.
- ii) Data file prepared by hand which contains the airfoil junction points.

G-2.1: Interactive Input Data

- <i>NI</i>	- Input option for the type of initial wake model.
<i>ENTER 0</i>	Airfoil initial wake model—lower surface separation point is fixed at the trailing edge
<i>ENTER 1</i>	Bluff body initial wake model—lower surface separation point is variable
<i>ALPHA</i>	- The airfoil angle of attack (inputted in degrees).
<i>WLF</i>	- Wake length factor— determined from curve (10).
<i>NPU</i>	- The number of wake panels to be used to model the upper wake

surface.

NPL - The number of wake panels to be used to model the lower wake surface.

NK - If a separate input data file is to be generated for STAMP.

ENTER 0 No.

ENTER 1 Yes.

G-2.2: Input File Prepared by Hand

First Line

NPT - The total number of junction points used to model the airfoil.

Following Lines

X(I), Y(I) - The x, y coordinates of the airfoil section junction points are inputted in a counter-clockwise fashion starting at the airfoil trailing edge. The last airfoil junction point must correspond to the first point.

Last Line

XSU, YSU, XSL, YSL - The upper and lower airfoil surface separation points. These two points must fall on airfoil junction points.

A typical input data file will take the form.

		<i>NPT</i>	
	<i>z</i> (1),	<i>y</i> (1),	← First airfoil junction point.
	<i>z</i> (2),	<i>y</i> (2),	
Airfoil junction	:	:	
points	:	:	
	<i>z</i> (<i>NPT</i>),	<i>y</i> (<i>NPT</i>),	← Last airfoil junction point.
	<u><i>X</i><i>SU</i>, <i>Y</i><i>SU</i></u>	<u><i>X</i><i>SL</i>, <i>Y</i><i>SL</i></u>	
	Upper surface	Lower surface	
	sep. coords.	sep. coords.	

G-3: Running IWMP

In order to run the program, type the following commands.

!FORTRAN IWMP OVER *3

!LINK *3 OVER 'IWMP

!SET 5 DAT1

!SET 6 OUT

!SET 3 DAT2

!IWMP

where

- DAT1** - Input file containing airfoil junction points.
- DAT2** - output file generated by IWMP which is the input data file for STAMP.
- OUT** - output file containing the results.

G-4: Subroutine description : IWMP

Subroutine	Function
INPUT	This subroutine controls the reading in of all data necessary to run the program.
INWAKE	This subroutine performs all the necessary calculations in order to give an initial guess estimate of the wake sheet.
OUTPUT	This subroutine controls the writing of both the output file for use in the stalled airfoil modelling program as well as the standard output file.

APPENDIX H-1

Sample Input Data File for IWMP (GA(W)-1 Airfoil Junction Points)

53

1.000	-0.00074
0.9500	0.01287
0.9000	0.02639
0.8500	0.03983
0.8000	0.05291
0.7500	0.06517
0.7000	0.07639
0.6500	0.08604
0.6000	0.09374
0.5500	0.09917
0.5000	0.10289
0.4500	0.10456
0.4000	0.10500
0.3500	0.10409
0.3000	0.10189
0.2500	0.09778
0.2000	0.09209
0.1500	0.08413
0.1000	0.07309
0.0750	0.06561
0.0500	0.05600

0.0375 0.04974
0.0250 0.04165
0.0125 0.03069
0.0050 0.02035
0.0020 0.01300
0.0000 0.00000
0.0020 -0.00974
0.0050 -0.01444
0.0125 -0.02052
0.0250 -0.02691
0.0375 -0.03191
0.0500 -0.03569
0.0750 -0.04209
0.1000 -0.04700
0.1500 -0.05426
0.2000 -0.05926
0.2500 -0.06265
0.3000 -0.06448
0.3500 -0.06517
0.4000 -0.06483
0.4500 -0.06344
0.5000 -0.06091
0.5500 -0.05683
0.6000 -0.05061
0.6500 -0.04265
0.7000 -0.03383
0.7500 -0.02461
0.8000 -0.01587
0.8500 -0.00852
0.9000 -0.00352

Appendix H: Sample Input Data Files

218

0.9500 -0.00257

1.0000 -0.00783

0.5500 0.09917 1.0000 -0.00783

APPENDIX H-2

Airfoil , Wake Junction Points, Sample Input Data File for STAMP (Generated by IWMP)

52 1.00000 2.00

8 6

1.00000 -.00074

.95000 .01287

.90000 .02639

.85000 .03983

.80000 .05291

.75000 .06517

.70000 .07639

.65000 .08604

.60000 .09374

.55000 .09917

.50000 .09917

.45000 .10269

.40000 .10456

.35000 .10500

.30000 .10409

.25000 .10169

.20000 .09778

.15000 .09209

.10000 .08413

.10000	.07309
.07500	.06561
.05000	.05600
.03750	.04974
.02500	.04165
.01250	.03069
.00500	.02035
.00200	.01300
.00000	.00000
.00200	-.00974
.00500	-.01444
.01250	-.02052
.02500	-.02691
.03750	-.03191
.05000	-.03569
.07500	-.04209
.10000	-.04700
.15000	-.05426
.20000	-.05926
.25000	-.06265
.30000	-.06448
.35000	-.06517
.40000	-.06483
.45000	-.06344
.50000	-.06091
.55000	-.05683
.60000	-.05061
.65000	-.04265
.70000	-.03383
.75000	-.02461

Appendix H: Sample Input Data Files

221

.80000	-.01587
.85000	-.00852
.90000	-.00352
.95000	-.00257
1.00000	-.00783
.55000	.09917
.63728	.10917
.72456	.11821
.81185	.12628
.89913	.13338
.98641	.13951
1.07369	.14468
1.16098	.14889
1.24826	.15212
1.00000	-.00783
1.04138	.00798
1.08275	.02813
1.12413	.05262
1.16550	.08146
1.20688	.11462
1.24826	.15212

APPENDIX H-3

Sample Input Data File For STAMP (Source Distribution Determined by BLAP)

.00000000
.00000000
.00000000
.00000000
.00000000
.00000000
.00000000
.00000000
.00000000
.00000000
.39858062
.07497133
.04227896
.03448305
.02497213
.02550180
.02506260
.02508421
.03032074
.03044606
.03409351
.02579415

.01829626
.01279209
.02790691
02254732
.01043537
.00272531
00207233
00178461
00134186
00113381
00123557
00085961
.00069191
.00108191
00097110
.00057426
.00066537
.00058588
.00057546
.00047462
.00081062
.00039644
.00100541
.00116833
.00245455
.00228502
.00248288
.00242120
.00214319
.00158306

Appendix H: Sample Input Data Files

224

- 00019247

- 00206724

APPENDIX H-4

Sample Input Data File For BLAP (Generated by STAMP)

CA(V)-1

18 25 2

1.0000 100.0000 .0200

0002000 1.40000 .0050000

1 000 .000055 4500000.

54 34 20

.07500 .00000 .00000

.06000 .02581 .13331

.03750 .03887 .32298

.02500 .05233 .58592

.01250 .06637 1 .11330

.00500 .07602 1 .85485

.00200 .08160 2 .31631

.00000 .09154 2 .63609

.00200 .10469 2 .85211

.00500 .11263 3 .00843

.01250 .12541 2 .91175

.02500 .14203 2 .70895

.03750 .15992 2 .58882

.06000 .17090 2 .43390

.07500 .19768 2 .22092

.10000 .22378 2 .08054

15000	27498	1	89212
20000	32561	1	77529
25000	37593	1	68462
30000	42609	1	61129
35000	47614	1	55063
40000	52615	1	49093
45000	57615	1	43555
50000	62619	1	37677
55000	67631	1	30417
60000	72661	1	12115
65000	77720	1	12115
70000	82812	1	12115
75000	87936	1	12115
80000	93084	1	12115
85000	98253	1	12115
90000	1.03430	1	12115
95000	1.08610	1	12115
1.00000	1.13792	1	12115
07500	00000	00000	
10000	02548	23837	
15000	07600	41800	
20000	12625	53083	
25000	17637	61498	
30000	22640	67625	
35000	27640	72745	
40000	32641	77198	
45000	37642	81477	
50000	42649	84201	
55000	47665	87625	
60000	52704	88364	

65000	.57767	.87481
70000	.62844	.86400
75000	.67928	.84583
80000	.73004	.82831
85000	.78058	.81623
90000	.83083	.81194
95000	.88084	.83515
1 00000	.93111	1.12115

APPENDIX I-1 Output File (STAMP)

RESULTS FROM AIRFOIL ANALYSIS

VARIOUS PARAMETERS

```

1] THE AIRFOIL SECTION NAME IS: GA(W)-1
2] THE CHORD LENGTH OF THE AIRFOIL IS: 1.0000
3] THE AIRFOIL ANGLE OF ATTACK (DEG) IS: 18.2500
4] THE LIFT COEFFICIENT BASED ON CIRCULATION: 1.5381
5] THE NUM. OF PANELS USED TO MODEL THE AIRFOIL: 52
6] THE # OF PANELS USED TO MODEL THE UPPER WAKE: 8
7] THE # OF PANELS USED TO MODEL THE LOWER WAKE: 8
8] THE AIRFOIL UPPER SURFACE SEP. PT. (X,Y): .5500 .0992
9] THE AIRFOIL LOWER SURFACE SEP. PT. (X,Y): 1.0000 -.0078
10] THE TOT. # OF WAKE SHAPE ITER. TO BE PERFORMED: 20
11] THE RELAXATION FACTOR USED FOR THE WAKE IT.: 1.50
12] THE NUMBER OF ITERATIONS PERFORMED: 10
13] THE WAKE LENGTH FACTOR IS: 2.00
14] THE OVERALL SOLUTION CRITERIA: .5000
15] THE FREE STREAM MACH NUMBER IS: .1500
  
```

VARIOUS AERODYNAMIC PARAMETERS				
Type of Int.	C _L	C _d (pres)	C _m (l.a.)	C _m (c/4)
3 Con. Pts.	1.5628	.0938	-.4346	-.0583
2 Jun. Pts.	1.6742	.1259	-.5086	-.1012
3 Jun. Pts.	1.6823	.1289	-.5149	-.1054

JUNCTION POINT PARAMETERS: CO-ORDINATES X,Y
SOURCE AND VORTEX STRENGTHS, C_p (INCOMP), C_p (COMPRES)

I	X/C	Y/C	SOR	GAM	CP(IN)	CP(COM)
1	1.00000	-.00074	.00000	.00000	-.60154	-.61052
2	.95000	.01287	.00000	-.00234	-.60154	-.61052
3	.90000	.02639	.00000	.00096	-.60154	-.61052
4	.85000	.03983	.00000	.00007	-.60154	-.61052
5	.80000	.05291	.00000	-.00030	-.60154	-.61052
6	.75000	.06517	.00000	-.00050	-.60154	-.61052
7	.70000	.07639	.00000	-.00106	-.60154	-.61052
8	.65000	.08604	.00000	-.00096	-.60154	-.61052
9	.60000	.09374	.00000	-.00393	-.60154	-.61052
10	.55000	.09917	.00000	.00000	-.60154	-.61052
11	.55000	.09917	.39858	-1.20111	-.60154	-.61052

12	60000	10289	07497	-1.11378	-24809	-24925
13	45000	10454	04228	-1.27743	-63381	-64320
14	40000	10500	03448	-1.35833	-84828	-86011
15	35000	10409	02497	-1.43323	-1.05478	-1.07332
16	30000	10169	02550	-1.50413	-1.26304	-1.28680
17	25000	09778	02508	-1.57984	-1.49590	-1.52908
18	20000	09209	02508	-1.66983	-1.78931	-1.82850
19	15000	08413	03032	-1.77888	-2.16483	-2.21688
20	10000	07309	03045	-1.84707	-2.79200	-2.86980
21	07500	06581	03409	-2.06889	-3.28022	-3.38122
22	05000	05800	02579	-2.25158	-4.07017	-4.21491
23	03750	04974	01830	-2.38811	-4.70339	-4.88878
24	02500	04185	01279	-2.50103	-5.25530	-5.48023
25	01250	03089	02791	-2.66888	-6.12382	-6.41750
26	00500	02035	02255	-2.71814	-6.38882	-6.70711
27	00200	01300	01044	-2.55897	-5.53820	-5.78488
28	00000	00000	00273	-2.32330	-4.39773	-4.58287
29	00200	00974	00207	-1.99248	-2.98988	-3.06579
30	00500	01444	00178	-1.65797	-1.42728	-1.45548
31	01250	02052	00134	-88785	21207	21424
32	02500	02891	00113	-41449	82819	83372
33	03750	03191	00124	-17413	98968	97538
34	05000	03589	00086	-00678	99997	1.00565
35	07500	04209	00089	19932	98027	98586
36	10000	04700	00108	33215	88968	89530
37	15000	05428	00097	49889	75330	75865
38	20000	05928	00057	80080	63928	64424
39	25000	06285	00067	67901	53894	54343
40	30000	06448	00059	73802	45828	46231
41	35000	06517	00058	78435	38479	38834
42	40000	06483	00047	82891	31822	31928
43	45000	06344	00081	86988	24365	24610
44	50000	06091	00040	89356	20158	20363
45	55000	05883	00101	92807	13889	14017
46	60000	05081	00117	93380	12801	12938
47	65000	04285	00245	92337	14739	14895
48	70000	03383	00228	91233	16784	16940
49	75000	02481	00248	89358	20752	20359
50	80000	01587	00242	87823	23224	23455
51	85000	00852	00214	86510	25191	25412
52	90000	00352	00158	86280	25558	25813
53	95000	00257	00019	88809	20952	21188
54	1.00000	00783	00207	1.20111	44288	44888

CONTROL POINT CO-ORDINATES, X, Y AND PRESSURE
COEFFICIENTS BASED ON TANGENTIAL VELOCITY AT THE CON. PTS.

I	X/C	Y/C	VP	VT	CP (CON)	IL
1	.97500	.00808	- .11458E-05	.00507	- .41090	.05182
2	.92500	.01943	- .70985E-06	-.00194	- .41088	.05180
3	.87500	.03311	- .85724E-06	.00028	- .41088	.05177
4	.82500	.04637	- .85970E-06	.00015	- .41088	.05188
5	.77500	.05904	- .83298E-06	-.00003	- .41088	.05148
6	.72500	.07078	- .83444E-06	-.00039	- .41088	.05124
7	.67500	.08121	- .83828E-06	-.00086	- .41088	.05092
8	.62500	.08989	- .72125E-06	-.00250	- .41088	.05069
9	.57500	.09848	- .19829E-06	-.00772	- .41094	.05029
10	.57500	.09848	- .19829E-06	-.00772	- .41094	.05029

11	.52500	.10093	-.23678E+00	-1.10930	-.29037	.05012
12	.47500	.10343	-.58626E-01	-1.20145	-.45320	.05003
13	.42500	.10478	-.38382E-01	-1.31317	-.73727	.05000
14	.37500	.10454	-.29729E-01	-1.38912	-.94821	.05001
15	.32500	.10289	-.25239E-01	-1.48237	-1.15979	.05008
16	.27500	.09973	-.25284E-01	-1.53410	-1.38031	.05015
17	.22500	.09493	-.25075E-01	-1.61497	-1.64230	.05032
18	.17500	.08811	-.27704E-01	-1.71210	-1.97601	.05063
19	.12500	.07861	-.30385E-01	-1.83952	-2.44542	.05120
20	.08750	.06935	-.32270E-01	-1.99779	-3.07916	.02610
21	.06250	.06080	-.29945E-01	-2.13032	-3.85367	.02678
22	.04375	.05287	-.22047E-01	-2.30577	-4.47703	.01398
23	.03125	.04569	-.15546E-01	-2.41478	-5.02564	.01489
24	.01875	.03617	-.20351E-01	-2.52800	-5.62847	.01882
25	.00875	.02552	-.25228E-01	-2.62789	-6.18297	.01277
26	.00350	.01667	-.16492E-01	-2.61795	-6.12610	.00794
27	.00100	.00650	-.65818E-02	-2.35323	-4.71202	.01315
28	.00100	-.00487	-.24002E-02	-2.08208	-3.43887	.00994
29	.00350	-.01209	-.19306E-02	-1.76194	-2.15444	.00558
30	.00875	-.01748	-.15453E-02	-1.17954	-.39674	.00985
31	.01875	-.02372	-.12391E-02	-.63002	.60788	.01404
32	.03125	-.02941	-.11858E-02	-.29225	.92024	.01348
33	.04375	-.03380	-.10482E-02	-.09016	.99756	.01308
34	.06250	-.03889	-.77611E-03	.09707	.99826	.02581
35	.08750	-.04455	-.88723E-03	.26292	.93853	.02548
36	.12500	-.05063	-.10288E-02	.41180	.83596	.05052
37	.17500	-.05676	-.77286E-03	.54468	.70852	.05025
38	.22500	-.06096	-.61993E-03	.63554	.60086	.05011
39	.27500	-.06357	-.62562E-03	.70339	.50956	.05003
40	.32500	-.06482	-.58073E-03	.75819	.43202	.05000
41	.37500	-.06500	-.52485E-03	.80138	.36114	.05000
42	.42500	-.06414	-.64229E-03	.84236	.29327	.05002
43	.47500	-.06218	-.33805E-02	.87699	.23320	.05006
44	.52500	-.05887	-.70067E-03	.90680	.17957	.05017
45	.57500	-.05372	-.10847E-02	.92585	.14431	.05039
46	.62500	-.04663	-.18113E-02	.92502	.14586	.05063
47	.67500	-.03824	-.23696E-02	.91811	.16243	.05077
48	.72500	-.02922	-.23838E-02	.90247	.18746	.05084
49	.77500	-.02024	-.24518E-02	.88824	.21676	.05078
50	.82500	-.01220	-.22820E-02	.87377	.23890	.05054
51	.87500	-.00602	-.18628E-02	.86958	.24627	.05025
52	.92500	-.00305	-.69611E-03	.88748	.21456	.05001
53	.97500	-.00520	-.11297E-02	1.03389	-.06675	.05028

UPPER WAKE CONTROL POINTS, X AND Y; NORMAL AND
EXTERNAL TANGENTIAL VELOCITIES AND PANEL LENGTHS.

I	I/C	Y/C	VN	VT	XL
55	.59359	.10455	.20332E-03	1.20684	.08785
56	.68071	.11549	.53996E-04	1.20209	.08775
57	.76773	.12645	.63179E-04	1.20024	.08768
58	.85474	.13672	.11065E-03	1.19882	.08757
59	.94182	.14550	.19130E-03	1.19700	.08749
60	1.02905	.15171	.29847E-03	1.19392	.08743
61	1.11639	.15316	.38812E-03	1.18611	.08739
62	1.20224	.14095	.10321E-02	1.10144	.08734

LOWER WAKE CONTROL POINTS, X AND Y; NORMAL AND

EXTERNAL TANGENTIAL VELOCITIES AND PANEL LENGTHS

I	I/C	Y/C	VH	VT	IL
64	1.02207	- .00507	- .20607E-03	1.18384	.04430
65	1.08680	- .00013	- .27700E-03	1.18068	.04802
66	1.11282	.00868	- .32147E-03	1.18110	.04808
67	1.16022	.02284	- .29501E-03	1.18012	.05042
68	1.20883	.04038	.73388E-04	1.18158	.05303
69	1.25440	.08821	.13880E-02	1.18442	.05584

UPPER WAKE JUNCTION POINTS, (X,Y)

LOWER WAKE JUNCTION POINTS, (X,Y)

I	I/C	Y/C	I	I/C	Y/C
55	.55000	.08817	64	1.00000	- .00783
56	.83719	.10983	65	1.04414	- .00410
57	.72423	.12105	66	1.08848	.00385
58	.81122	.13185	67	1.13617	.01527
59	.89825	.14158	68	1.18427	.03040
60	.98538	.14841	69	1.23340	.05038
61	1.07270	.15400	70	1.27940	.08204
62	1.16008	.15232			
63	1.24441	.12868			

APPENDIX I-2 Output File (BLAP)

AIRFOIL BOUNDARY LAYER ANALYSIS

VARIOUS PARAMETERS

(1) THE AIRFOIL SECTION NAME IS:GA(W)-1
(2) THE AIRFOIL ANGLE OF ATTACK (DEG) IS: 18.25
(3) THE CHORD LENGTH OF THE AIRFOIL IS: 1.00
(4) THE FREE STREAM VELOCITY IS:100.00
(5) THE BOUNDARY LAYER STEP SIZE IS: .0200
(6) THE KINEMATIC VISCOSITY IS: .22222E-06
(7) THE REYNOLDS NUMBER BASED ON THE CHORD IS: 4500000.0
(8) THE NON-DIMENSIONAL SURFACE FINISHNESS HEIGHT, K/L: .0000055
(9) THE ROUGHNESS DENSITY PARAMETER IS: 1.000

BOUNDARY LAYER PARAMETERS UPPER AIRFOIL SURFACE

LAMINAR FLOW REGION

1	S/C	U/Uref	DX/DX	DELTA/R/C	THETA/R/C	H/L	Rex	Reth	CF	U/alpha/MU	HUFLS	DELS	SOK
1	.000	.0000	5.1451	.0001340	.0000568	2.358	.00000E+00	.00000E+00	.00000E+00	.25633E-02	.0006919		
2	.020	.1033	5.1451	.0001340	.0000568	2.358	.92971E+04	.26406E+02	.24814E-01	.0004919			
3	.040	.3359	12.1927	.0001148	.0000547	2.136	.60466E+05	.82628E+02	.10384E-01	.0012692			
4	.060	.8420	38.4149	.0000715	.0000348	2.034	.22734E+06	.13189E+03	.71252E-02	.0010496			
5	.080	2.1815	83.6137	.0000456	.0000216	2.113	.78536E+06	.21199E+03	.41716E-02	.0019672			
6	.100	2.8021	13.8568	.0000589	.0000237	2.485	.12610E+07	.29914E+03	.18744E-02	.0017811			
7	.120	3.0035	-9.9478	.0000869	.0000306	2.834	.16219E+07	.41425E+03	.5.72125E-03	.0047867			

THE W-Rex METHOD HAS PREDICTED TRANSITION ON THE
UPPER AIRFOIL SURFACE

TURBULENT FLOW REGION

1	S/C	U/Uref	DX/DX	DELTA/R/C	THETA/R/C	H/L	Rex	Reth	CF	U/alpha/MU	HUFLS	DELS	SOK
7	.120	3.0035	-9.948	.0002800	.0002000	1.400	.16219E+07	2703.	.19113E-02	.22980E+02	7.197	.0014391	.033790
8	.140	2.7300	-10.608	.0003963	.0002742	1.445	.17199E+07	3368.	.17397E-02	.19927E+02	6.601	.0018100	.012042
9	.160	2.5307	-11.311	.0005765	.0003813	1.512	.18365E+07	4377.	.15031E-02	.17407E+02	5.930	.0022413	.019435
10	.180	2.3497	-8.581	.0008906	.0005493	1.621	.19033E+07	5808.	.12119E-02	.14316E+02	5.164	.0028367	.031103
11	.200	2.2072	-5.850	.0012605	.0007168	1.711	.19865E+07	7318.	.10175E-02	.12323E+02	4.711	.0034709	.034486

12	.220	2.0990	-4.971	.0016106	.0009133	1.763	.20780F+07	8627.	.91690F-03	.11123E+00	4.512	.004120H	.07991R
13	.240	2.0084	-4.092	.0020090	.0011067	1.815	.21691E+07	10002.	.87797E-03	.10114E+00	4.350	.0048145	.032714
14	.260	1.9353	-3.213	.0024266	.0013063	1.858	.22643E+07	11327.	.76029E-03	.93191E+01	4.219	.0055372	.033069
15	.280	1.8794	-2.511	.0028117	.0014960	1.879	.23680E+07	12652.	.72125E-03	.88455E+01	4.187	.0062647	.029400
16	.300	1.8312	-2.310	.0031415	.0016691	1.882	.24721E+07	13755.	.71240E-03	.85336E+01	4.182	.0069804	.021414
17	.320	1.7870	-2.110	.0034974	.0018508	1.890	.25732E+07	14881.	.69595E-03	.82705E+01	4.165	.0077080	.024878
18	.340	1.7468	-1.910	.0038717	.0020384	1.899	.26726E+07	16023.	.67793E-03	.79795E+01	4.144	.0084464	.015664
19	.360	1.7106	-1.709	.0042545	.0022290	1.909	.27711E+07	17158.	.66108E-03	.76971E+01	4.124	.0091939	.035731
20	.380	1.6782	-1.567	.0046332	.0024188	1.915	.28697E+07	18267.	.64773E-03	.74748E+01	4.111	.0099476	.074897
21	.400	1.6479	-1.467	.0050179	.0026108	1.922	.29682E+07	19360.	.63225E-03	.72687E+01	4.098	.0106980	.024671
22	.420	1.6195	-1.367	.0054194	.0028080	1.930	.30609E+07	20464.	.62156E-03	.70663E+01	4.082	.0114625	.025400
23	.440	1.5932	-1.267	.0058321	.0030088	1.938	.31545E+07	21571.	.60793E-03	.68747E+01	4.066	.0122348	.025735
24	.460	1.5688	-1.168	.0062493	.0032114	1.946	.32475E+07	22671.	.59540E-03	.66995E+01	4.052	.0130127	.025673
25	.480	1.5459	-1.070	.0066612	.0034133	1.952	.33391E+07	23744.	.58494E-03	.65411E+01	4.041	.0137934	.024816
26	.500	1.5216	-1.196	.0071774	.0036432	1.970	.34236E+07	24946.	.56276E-03	.63172E+01	4.010	.0146087	.031019
27	.520	1.4980	-1.161	.0077392	.0038859	1.992	.35054E+07	26196.	.53806E-03	.60813E+01	3.977	.0154469	.036620
28	.540	1.4752	-1.127	.0083531	.0041422	2.017	.35846E+07	27497.	.51117E-03	.58169E+01	3.937	.0163074	.036476
29	.560	1.4530	-1.092	.0090244	.0044124	2.045	.36615E+07	28850.	.48737E-03	.55844E+01	3.897	.0171946	.039505
30	.580	1.4315	-1.059	.0097596	.0046972	2.078	.37363E+07	30258.	.45189E-03	.53557E+01	3.855	.0181092	.042940
31	.600	1.4092	-1.168	.0105483	.0049975	2.115	.38050E+07	31692.	.41987E-03	.50536E+01	3.813	.0190535	.046113
32	.620	1.3848	-1.278	.0116976	.0053605	2.182	.38635E+07	33404.	.36934E-03	.46375E+01	3.745	.0200748	.065266
33	.640	1.3581	-1.387	.0133499	.0058044	2.300	.39114E+07	35474.	.29785E-03	.41021E+01	3.653	.0212056	.096613
34	.660	1.3293	-1.496	.0160964	.0063595	2.531	.39480E+07	38042.	.19822E-03	.32753E+01	3.535	.0224799	.163297
35	.680	1.2845	-5.186	.0236920	.0071006	3.337	.39307E+07	41044.	.12336E-03	.24969E+01	3.378	.0239833	.431811

***** TURBULENT SEPARATION *****

HE TURBULENT BOUNDARY LAYER HAS SEPARATED FROM

HE UPPER AIRFOIL SURFACE

HE X-CARTESIAN UPPER SURF. SEP. PT. IS: .55367

BOUNDARY LAYER PARAMETERS LOWER AIRFOIL SURFACE

MINAR FLOW REGION

I	S/C	U/Uref	TU/US	DELTA S/C	THETA/C	H/C	Re _x	Re _{th}	C _f	SOK
9	.006	.0000	8.2472	.0001060	.0000450	2.358	.00000E+00	.00000E+00	.32391E-02	.0006919
10	.020	.1955	8.2472	.0001060	.0000450	2.358	.17592E+05	.39542E+02	.16571E-01	.0010361
11	.040	.3299	5.1948	.0001358	.0000578	2.349	.59380E+05	.87797E+02	.77304E-02	.0012033
12	.060	.4033	2.1424	.0001630	.0000663	2.461	.10888E+06	.12023E+03	.47114E-02	.0010477
13	.080	.4280	2.4838	.0001981	.0000845	2.144	.15409E+06	.16275E+03	.41022E-02	.0009519
14	.100	.4754	2.2581	.0002000	.0000845	2.365	.21395E+06	.18089E+03	.35488E-02	.0005149
15	.120	.5183	2.0323	.0002072	.0000862	2.379	.27991E+06	.20117E+03	.31661E-02	.0007641
16	.140	.5567	1.8066	.0002125	.0000889	2.391	.35074E+06	.22245E+03	.28151E-02	.0005965
17	.160	.5906	1.5809	.0002214	.0000922	2.403	.47574E+06	.24497E+03	.25172E-02	.0006244
18	.180	.6198	1.3105	.0002328	.0000961	2.421	.50201E+06	.26814E+03	.22394E-02	.0006745
19	.200	.6452	1.2302	.0002427	.0001004	2.417	.58063E+06	.29151E+03	.20721E-02	.0006153

70	.220	.6690	1.1500	.0002517	.0001042	2.416	.66228E+06	.31363E+03	.19291E-02	.0005901
71	.240	.6912	1.0697	.0002603	.0001077	2.417	.74646E+06	.33497E+03	.18034E-02	.0005769
72	.260	.7118	.9895	.0002688	.0001111	2.420	.83276E+06	.35586E+03	.16909E-02	.0005717
73	.280	.7307	.9053	.0002776	.0001145	2.425	.92070E+06	.37672E+03	.15868E-02	.0005763
74	.300	.7487	.8914	.0002846	.0001177	2.418	.10107E+07	.39652E+03	.15216E-02	.0005087
75	.320	.7664	.8775	.0002906	.0001204	2.412	.11046E+07	.41338E+03	.14639E-02	.0004814
76	.340	.7838	.8636	.0002958	.0001229	2.408	.11992E+07	.43332E+03	.14119E-02	.0004602
77	.360	.8009	.8497	.0003006	.0001250	2.405	.12975E+07	.45054E+03	.13647E-02	.0004436
78	.380	.8165	.8347	.0003167	.0001275	2.484	.13962E+07	.46849E+03	.11672E-02	.0008907
79	.400	.8267	.8200	.0003249	.0001320	2.461	.14881E+07	.49119E+03	.11544E-02	.0005007
80	.420	.8381	.8053	.0003305	.0001355	2.438	.15840E+07	.51116E+03	.11460E-02	.0004186
81	.440	.8505	.7906	.0003339	.0001381	2.417	.16841E+07	.52871E+03	.11428E-02	.0003503
82	.460	.8641	.7759	.0003354	.0001399	2.397	.17887E+07	.54411E+03	.11420E-02	.0002937
83	.480	.8772	.7622	.0003366	.0001417	2.516	.18949E+07	.55942E+03	.92788E-03	.0011489
84	.500	.8817	.7585	.0003763	.0001474	2.553	.19838E+07	.58475E+03	.83418E-03	.0009483
85	.520	.8836	.7498	.0003999	.0001539	2.598	.20676E+07	.61206E+03	.73314E-03	.0010807
86	.540	.8829	.7409	.0004291	.0001615	2.657	.21454E+07	.64158E+03	.62829E-03	.0012740

THE H-REX METHOD HAS PREDICTED TRANSITION ON THE
LOWER AIRFOIL SURFACE

TURBULENT FLOW REGION

I	S/C	U/Uref	DU/DS	DESTAR/C	THETA/C	H/L	Re _x	Relin	CF	ULAW3K/NU	HDEL'S	DFLS	SUK
86	.540	.8829	-.099	.0002800	.0002000	1.400	.71454E+07	.795	.18292E-02	.95615E+01	7.197	.0014393	-.005309
87	.560	.8796	-.228	.0003368	.0002391	1.408	.72167E+07	.947	.36562E-02	.93085E+01	7.075	.0016917	.002452
88	.580	.8743	-.142	.0003950	.0002793	1.414	.72819E+07	1.099	.34799E-02	.90260E+01	6.995	.0019538	.002455
89	.600	.8709	-.200	.0004482	.0003168	1.415	.73514E+07	1.242	.33689E-02	.88463E+01	6.990	.0022146	.002247
90	.620	.8663	-.268	.0005028	.0003553	1.415	.74169E+07	1.385	.32724E-02	.86726E+01	6.980	.0024798	.002265
91	.640	.8605	-.317	.0005599	.0003952	1.417	.74783E+07	1530	.31838E-02	.84977E+01	6.959	.0027503	.002312
92	.660	.8536	-.375	.0006204	.0004371	1.419	.75352E+07	1.679	.30995E-02	.83170E+01	6.927	.0030277	.002385
93	.680	.8455	-.397	.0006851	.0004815	1.423	.75874E+07	1.832	.30169E-02	.81278E+01	6.881	.0033133	.002487
94	.700	.8380	-.355	.0007525	.0005276	1.426	.76398E+07	1.990	.29390E-02	.79510E+01	6.834	.0036054	.002567
95	.720	.8313	-.313	.0008180	.0005727	1.428	.76936E+07	2142	.28771E-02	.78040E+01	6.807	.0038983	.002471
96	.740	.8255	-.271	.0008812	.0006166	1.429	.77489E+07	2290	.28277E-02	.76825E+01	6.797	.0041906	.002371
97	.760	.8205	-.229	.0009418	.0006591	1.429	.78061E+07	2433	.27888E-02	.75832E+01	6.799	.0044811	.002364
98	.780	.8163	-.187	.0009993	.0006998	1.428	.78653E+07	2571	.27588E-02	.75040E+01	6.814	.0047686	.002149
99	.800	.8113	-.148	.0010532	.0007386	1.426	.79206E+07	2697	.27393E-02	.74311E+01	6.839	.0050519	.001937
00	.820	.8105	.071	.0011036	.0007754	1.423	.79908E+07	2828	.27207E-02	.73989E+01	6.874	.0053302	.001999
01	.840	.8141	.291	.0011289	.0007980	1.415	.80755E+07	2924	.27405E-02	.74389E+01	6.989	.0055771	.001731
02	.860	.8221	.510	.0011299	.0008063	1.401	.81817E+07	2983	.27925E-02	.76034E+01	7.178	.0057871	.000491
03	.880	.8345	.729	.0011090	.0008011	1.384	.83048E+07	3008	.28704E-02	.78249E+01	7.436	.0059572	-.000170
04	.900	.8513	.948	.0010703	.0007841	1.365	.84478E+07	3004	.29680E-02	.81167E+01	7.761	.0060858	-.000716
05	.920	.8725	1.167	.0010184	.0007575	1.344	.86120E+07	2974	.30443E-02	.84246E+01	8.150	.0061732	-.001136
06	.931	.8861	1.289	.0009560	.0007327	1.323	.87128E+07	2882	.30927E-02	.86341E+01	8.406	.0062200	-.002062

HE X-CARTESIAN LOWER SURF, SEP. PT. IS: 1.000000

APPENDIX J-1

Stalled Airfoil Modelling Program

A-1

ST.A.M.P.

Stalled Airfoil Modelling Program

This program models the flow about single element two-dimensional airfoils or other bluff bodies up to and beyond stall. It is an iterative procedure between potential flow and boundary layer solutions. The airfoil geometry is modeled using vortex sheets with a linear variation of vorticity. The separated region is modeled in the potential flow analysis using free vortex sheets of constant strength which require an inner iteration to establish their shapes. Compressibility effects can be taken accounted (using the "Karman-Tsien" formula) if the free stream mach number is known. The "Transpiration" or source distribution method is used to model the displacement thickness effect of the boundary layer.

The program itself is divided into two major sub-programs. If the flow is attached to the airfoil subroutine "FANOD" is called. If the flow is separated from the airfoil then subroutine "FSNOD" is called. In either case, if desired, the effect of the boundary layer can be incorporated, using the "Source Distribution" method.

Both these subroutines call various other subroutines to perform other calculations.

A separate program "BLAP" is used to determine the separation points and the required source distribution.

A separate program "IWMP" is used to generate the initial co-ordinates of the upper and lower wake boundaries.

One IMSL library subroutine "LEQTIF" is used for the matrix inversion.

The program is interactive in nature, and prompts the user for various input parameters.

```

*      *****
*      MAIN PROGRAM
*      *****

      IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

      COMMON A(90,90),BB(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
*      PH(90),XL(90),VNH(90),VTAN(90),CP(90),QSJ(90),POSTN(20),
*      XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
*      CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
*      N,NH,NPT,NUS,NUF,NUFF,NLB,NLF,NLFF,I,M,PI,CHORD,NFN,
*      NPT1,NPT2,NAM1,NAM2,NAM3,CLCAM,NPUSS,NPUS2,NPUS,WLF,CPB,
*      NPLS,NPLS2,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

      INTEGER POSTN
      REAL NAM1,NAM2,NAM3

*      -----
*      INITIALIZE ALL OPTION INDICATORS
*      -----
      DO 1 I=1,20
      POSTN(I)=0.0
1 CONTINUE
      PI=3.1415926536

*      -----
*      THE NEXT STATEMENT CALLS SUBROUTINE "INPUT", WHICH
*      WHICH READS IN ALL THE NECESSARY DATA.
*      -----
      CALL INPUT
      OUTPUT,'The calculations have begun.'
      OUTPUT,' '

*      -----
*      IF THE FLOW IS SEPARATED, SUBROUTINE "FEMOD" IS CALLED
*      -----
      IF(POSTN(1).EQ.1)CALL FEMOD

*      -----
*      IF THE FLOW IS ATTACHED, SUBROUTINE "FAMOD" IS CALLED
*      -----
      IF(POSTN(1).EQ.0)CALL FAMOD
      IF(POSTN(1).EQ.0)CALL OUTPUT

*      -----
*      IF A BOUNDARY LAYER INPUT FILE IS TO BE GENERATED, THE
*      NEXT STATEMENT CALLS SUBROUTINE "AUX" WHICH GENERATES
*      THIS FILE.
*      -----
      IF(POSTN(14).EQ.1)CALL AUX
      OUTPUT,'The calculations have finished.'

      STOP

      END

*      *****
*      SUBROUTINE INPUT
*      *****

```

```

..... A-2 .....
*
*   This subroutine controls the reading in of data into
*   the program. It is partially interactive prompting the
*   user for various input parameters.
*
.....

      IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

      COMMON A(90,90),BR(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
      * PH(90),XL(90),VNN(90),VTAN(90),CP(90),QBJ(90),POSTN(20),
      * XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
      * CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
      * N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
      * NPT1,NPT2,NAM1,NAM2,NAM3,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPB,
      * NPLS,NPLSS,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

      INTEGER POSTN
      REAL NAM1,NAM2,NAM3

      -----
      *
      *   THE NEXT STATEMENT ALLOWS THE USER TO INPUT THE SECTION
      *   NAME OF THE AIRFOIL FOR LATER IDENTIFICATION.
      *
      *   OUTPUT, 'Enter the airfoil section name (up to 12 char )'
      *   READ(2,30)NAM1,NAM2,NAM3
30  FORMAT(3A4)

      *
      *   -----
      *   READ IN THE AIRFOIL'S ANGLE OF ATTACK (DEGREES)
      *   -----
      *
      *   OUTPUT, 'Enter the airfoil angle of attack (degrees)'
      *   READ(2,1)ALPHAA
      *   FORMAT(1G)
1  ALPHA=(ALPHAA*PI)/180

      *
      *   -----
      *   THE NEXT STATEMENT ALLOWS THE USER TO DECIDE IF THE
      *   ANALYSIS IS TO INCLUDE BOUNDARY LAYER EFFECTS. THE
      *   EFFECTS OF THE BOUNDARY LAYER ARE HANDLED BY THE
      *   "TRANSPARATION" METHOD.
      *
      *   OUTPUT, 'Are boundary layer effects to be incorporated'
      *   OUTPUT, 'in the airfoil solution?'
      *   OUTPUT, 'ENTER 0-NO, (Pure inviscid solution)'
      *   OUTPUT, 'ENTER 1-YES, (Boundary layer correction included)'
      *   READ(2,8)POSTN(9)
8  FORMAT(1G)

      *
      *   -----
      *   IF THE MACH NUMBER IS KNOWN COMPRESSIBILITY EFFECTS
      *   ARE ACCOUNTED FOR USING THE "KARMAN-TSIEN" FORMULA
      *   -----
      *
      *   OUTPUT, 'Are compressibility effects to be taken into account'
      *   OUTPUT, 'using the "Karmen-Tsien" equation?'
      *   OUTPUT, 'NOTE: The free stream mach number must be known.'
      *   OUTPUT, 'ENTER 0-NO, ENTER 1-YES.'

```

```

      READ(2,71)POSTN(12)
71  FORMAT(1G)
      IF(POSTN(12) EQ. 0) THEN
        OUTPUT, 'Enter the free stream mach number'
        READ(2,77)AMACH
77  FORMAT(1G)
      END IF

      -----
      INPUT FROM USER TO DETERMINE IF THE FLOW IS SEPARATED
      -----
      OUTPUT, 'Is the flow separated from the airfoil?'
      OUTPUT, 'ENTER 0-NO AND 1-YES'
      READ(2,2)POSTN(1)
2  FORMAT(G)

      -----
      INPUT FROM THE USER TO SPECIFY IF THE SEPARATION
      POINT IS AT THE TRAILING EDGE OR ON THE PRESSURE
      SURFACE OF THE AIRFOIL.
      -----
      IF(POSTN(1) EQ. 1) THEN
        OUTPUT, 'Does the pressure surface separation point'
        OUTPUT, 'occur at the trailing edge or on the pressure'
        OUTPUT, 'surface?'
        OUTPUT, 'ENTER 1- separation at trailing edge'
        OUTPUT, 'ENTER 0-separation on pressure surface(i.e. bluff body)'
        READ(2,6)POSTN(8)
6  FORMAT(1G)
      END IF

      -----
      THE NEXT STATEMENTS ALLOW THE USER TO DETERMINE THE
      KUTTA CONDITION TO BE IMPLIMENTED AT THE TRAILING EDGE
      FOR THE UNSEPARATED AIRFOIL POTENTIAL FLOW ANALYSIS.
      -----
      IF(POSTN(9) EQ. 0 AND POSTN(1) NE. 1) THEN
        OUTPUT, 'Select the "KUTTA CONDITION" to be implimented for'
        OUTPUT, 'the unseparated airfoil potential flow analysis.'
        OUTPUT,
        OUTPUT, 'ENTER 1 for the airfoil to have a stagnation'
        OUTPUT, 'point at its trailing edge.'
        OUTPUT, 'ENTER 0 for the airfoil to have equal velocities'
        OUTPUT, 'at its trailing edge.'
        READ(2,9)POSTN(4)
9  FORMAT(I1)
      END IF

      -----
      READ IN THE ACTUAL NUMBER OF PANELS, THE CHORD LENGTH
      AND THE WAKE LENGTH FACTOR.
      -----
      READ(5,3)N,CHORD,WLF
3  FORMAT(3G)
      IF(POSTN(1) EQ. 1) THEN
        IF(POSTN(8) EQ. 1) NPT=N+2
        IF(POSTN(8) EQ. 0) NPT=N+3
      ELSE

```

```

      NPT=N+1
      -----
      * VARIOUS "DUMPY" VARIABLES ARE DEFINED FOR THE PURE
      * INVISCID ATTACHED POTENTIAL FLOW ANALYSIS
      * -----
      NPUS=N+2
      NPUS2=N+2
      NPLS=N+2
      NPLS2=N+2
      END IF

      -----
      * IF THE FLOW IS SEPARATED FROM THE AIRFOIL THE NUMBER OF
      * WAKE PANELS ON THE UPPER AND LOWER SURFACES ARE READ IN
      * -----
      IF(POSTN(1) EQ.1)THEN
        READ(5,70)NPU,NPL
70  FORMAT(2G)
      END IF

      -----
      * IF BOUNDARY LAYER EFFECTS ARE TO BE INCLUDED, THE
      * NEXT SET OF STATEMENTS READS IN THE SOURCE STRENGTHS
      * AT THE JUNCTION POINTS AS DETERMINED FROM THE
      * BOUNDARY LAYER ANALYSIS. THE SOURCE STRENGTHS ARE
      * READ IN COUNTER-CLOCKWISE STARTING AT THE TRAILING
      * EDGE. THIS IS ONLY FOR THE ATTACHED FLOW CALCULATIONS.
      * -----
      IF(POSTN(9) EQ.1)THEN
        READ(4,12)(QSJ(I),I=1,NPT)
12  FORMAT(1G)
      END IF

      -----
      * THE NEXT STATEMENT ALLOWS THE USER TO SELECT
      * THE NUMBER OF WAKE SHAPE ITERATIONS WHICH WILL
      * BE PERFORMED, OR THE OVERALL SOLUTION ACCURACY
      * -----
      IF(POSTN(1) EQ.1)THEN
        OUTPUT,'Do you want full convergence of the solution?'
        OUTPUT,'ENTER 0-NO, 1-YES.'
        READ(2,7)POSTN(15)
7  FORMAT(1G)

      -----
      * THE USER CAN NOW SELECT A NEW SOLUTION CRITERIA IF
      * DESIRED. IF NOT, THE SOLUTION IS ASSUMED CONVERGED
      * IF THE PERCENT CHANGE BETWEEN SUCCESSIVE ITERATIONS
      * OF THE BASE PRESSURE IS LESS THEN 0.25%.
      * -----
      IF(POSTN(15) EQ.1)THEN
        OUTPUT,'The default convergence criteria is set at 0.25%'
        OUTPUT,'change between two successive iterations in the'
        OUTPUT,'base pressure.'
        OUTPUT,'Is a new convergence criteria desired?'
        OUTPUT,'ENTER 0-NO, 1-YES'
        READ(2,10)POSTN(16)

```

```

10 FORMAT(1G)
   IF(POSTN(16).EQ.1)THEN
      OUTPUT,'Enter the new base pressure convergence criteria.'
      READ(2,11)CRT1
11  FORMAT(1G)
      ELSE
      CRT1= 0.250
      END IF
      POSTN(2)=20
      END IF

      IF(POSTN(15).EQ.0)THEN
      OUTPUT,'Enter the number of wake iterations to be performed.'
      READ(2,72)POSTN(2)
72  FORMAT(1G)
      END IF

```

```

-----
THE USER NOW INPUTS THE RELAXATION FACTOR TO BE USED
-----

```

```

OUTPUT,'Enter the relaxation factor for the wake iteration.'
OUTPUT,'Select a number between 0.0 and 2.0.'
OUTPUT,'Between 0 and 1 UNDER-RELAXATION'
OUTPUT,'Between 1 and 2 OVER-RELAXATION'
READ(2,73)RELAX
73  FORMAT(1G)

```

```

-----
THE NEXT STATEMENT ALLOWS THE USER TO DECIDE IF HE/
SHE WANTS A COMPLETE OUTPUT DATA FILE, INCLUDING ALL
WAKE ITERATION PARAMETERS.
-----

```

```

OUTPUT,'Select the type of output file required.'
OUTPUT,'ENTER-0 for only the final converged solution results.'
OUTPUT,'ENTER-1 for all wake shape iteration results.'
READ(2,5)POSTN(3)
5  FORMAT(1G)
OUTPUT,'Do you want the coordinates and parameters'
OUTPUT,'of the upper and lower wake in the output file?'
OUTPUT,'ENTER 0-NO, 1-YES'
READ(2,67)POSTN(11)
67  FORMAT(1G)

```

```

-----
THE NEXT FLAG ALLOWS THE USER TO DECIDE IF THEY WANT THE
PROGRAM TO CHECK IF THE WAKE PANELS ARE CROSSED AT ANY
POINT AND IF SO TO TRUNCATE THE WAKE AFTER THE CROSS
OVER POINT
-----

```

```

OUTPUT,'Do you want the program to perform a check to determine'
OUTPUT,'if any of the wake panels have overlapped and if so'
OUTPUT,'to take corrective action.'
OUTPUT,'ENTER 0-NO, 1-YES'
READ(2,69)POSTN(17)
69  FORMAT(1G)

```

```

-----
VARIOUS INDEX COUNTERS FOR THE UPPER AND LOWER WAKE
-----

```

```

*   GEOMETRY ARE DETERMINED IN THE FOLLOWING STATEMENTS
*   -----
NUS=NPT+1
NUF=NUS+NPU
NLS=NUF+1
NLF=NLS+NPL
NFN= NLF

*   -----
*   READ IN THE COORDINATES OF THE AIRFOIL AND
*   UPPER AND LOWER WAKE SURFACES.
*   -----
      READ(5,4)(X(I),Y(I),I=1,NFN)
4  FORMAT(2G)
      END IF

*   -----
*   READ IN THE CO-ORDINATES OF THE AIRFOIL
*   -----
      IF(POSTN(1).EQ.0)THEN
      READ(5,75)(X(I),Y(I),I=1,NPT)
75  FORMAT(2G)
      END IF

*   -----
*   INPUT FROM THE USER TO DECIDE ON THE PARAMETERS TO BE
*   DISPLAYED IN THE OUTPUT FILE.
*   -----
      OUTPUT,'Do you want the output file to contain'
      OUTPUT,'the junction point parameters.(i.e. X,Y,CP)?'
      OUTPUT,'ENTER 0-NO, 1-YES'
      READ(2,74)POSTN(6)
74  FORMAT(1G)
      OUTPUT,'Do you want the output file to contain'
      OUTPUT,'the control point parameters?'
      OUTPUT,'ENTER 0-NO, 1-YES'
      READ(2,68)POSTN(10)
68  FORMAT(1G)

*   -----
*   THE NEXT STATEMENT ALLOWS THE USER TO DISPLAY THE
*   MATRIX WHICH IS TO BE INVERTED, IF SO WISHED
*   -----
      OUTPUT,'Do you want to display the matrix to be inverted?'
      OUTPUT,'ENTER 0-NO, 1-YES'
      READ(2,76)POSTN(5)
76  FORMAT(1G)

*   -----
*   AN OUTPUT FILE CONTAINING THE NUMBER AIRFOIL PANELS
*   CHORD LENGTH, NUMBER OF UPPER AND LOWER WAKE PANELS
*   AIRFOIL AND CONVERGED WAKE JUNCTION POINTS CAN BE
*   GENERATED IF DESIRED.
*   -----
      IF(POSTN(1).EQ.1)THEN
      OUTPUT,'Do you want the program to generate an output file'
      OUTPUT,'containing the airfoil and converged wake junction'
      OUTPUT,'points?'

```



```

      OUTPUT, 'ENTER O-NO, 1-YES'
      READ(2,80)POSTN(13)
80  FORMAT(G)
      END IF

```

```

* -----
* AN OUTPUT FILE CONTAINING THE DATA NECESSARY TO RUN
* THE BOUNDARY LAYER PROGRAM CAN BE GENERATED IF
* DESIRED
* -----

```

```

      OUTPUT, 'Do you wish an output file for use with the'
      OUTPUT, 'boundary layer program?'
      OUTPUT, 'ENTER O-NO, 1-YES'
      READ(2,81)POSTN(14)
81  FORMAT(1G)

```

```

      RETURN

```

```

      END

```

```

* .....
* SUBROUTINE FAMOD
* .....

```

```

* ..... A-3 .....
*
*      This subroutine carries out the calculations
*      (assuming the flow is attached) in order to find the
*      pressure coefficients and aerodynamic parameters
*      around the airfoil at the stated angle of attack.
*      Boundary layer effects can be incorporated in the
*      solution if desired.
*      Two forms of the kutta condition are available for
*      implementation for the pure potential flow model
*      If boundary layer effects are incorporated a modified
*      kutta condition is used.
*
*      It calls eight other subroutines;
*
* .....!.....

```

```

      IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)
      REAL WKAREA(72)

```

```

      COMMON A(90,90),BR(90,1),BS(90,1),AN(90,90),XI(90),YI(90),
*      PH(90),XL(90),VNN(90),VTAN(90),CP(90),QSJ(90),POSTN(20),
*      IX(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
*      CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
*      N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
*      NPT1,NPT2,NAM1,NAM2,NAMS,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPB,
*      NPLS,NPLS8,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

```

```

      INTEGER POSTN
      REAL NAM1,NAM2,NAMS

```

```

* -----
* THE NEXT SET OF STATEMENTS DETERMINE THE NUMBER OF PANELS
* ON THE AIRFOIL PRESSURE SURFACE (UNDER SURFACE) IN ORDER

```

- TO DETERMINE THE CONTROL POINT WHERE THE CONDITION OF
- ZERO NORMAL VELOCITY MAYBE RELAXED, IN ORDER TO IMPLIMENT
- ONE OF THE AUXILLARY CONDITIONS.

```

-----
DO 1 I=2,NPT
IF(X(I) LE. X(I-1))THEN
CONTINUE
ELSE
NUP=I-2
NLP=N-NUP
GO TO 2
END IF
1 CONTINUE
2 CONTINUE
NA=(NPT-NLP/2)+2

```

- THE NEXT STATEMENT CALLS THE SUBROUTINE "TRANSX"
- WHICH TRANSLATES THE X CO-ORDINATES SUCH THAT
- THE LEADING EDGE IS AT X=0.0 AND ALL CO-ORDINATES
- ARE NON-DIMENSIONALIZED

```

-----
CALL TRANSX

```

- THE FOLLOWING STATEMENT CALLS THE SUBROUTINE GEOMAIR
- WHICH CALCULATES THE CONTROL POINTS, THE ANGLES PH(I)
- ,AND THE LENGTH OF EACH PANEL ON THE AIRFOIL.

```

-----
CALL GEOMAIR

```

- THE FOLLOWING STATEMENTS COMPUTE THE INFLUENCE
- COEFFICIENTS FOR THE VORTEX PANELS AND SETS UP THE
- MATRIX OF BOUNDARY CONDITION EQUATIONS.
- THE VARIABLES XK(1) TO XK(4) ARE THE ENGINEERING
- VARIABLES A,B,C, AND D RESPECTIVELY. THEY ARE
- EVALUATED BY THE SUBROUTINE "XKS".

```

-----
NN=N+1
DO 3 I=1,N
DO 4 M=1,N
CALL XKS

IF(M EQ.1)THEN
A(I,M)=XK(1)*SIN(PH(M)-PH(I))+XK(3)*COS(PH(M)-PH(I))
ELSE
A(I,M) = XK(1)*SIN(PH(M)-PH(I))+XK20*SIN(PH(M-1)-
PH(I))+XK(3)*COS(PH(M)-PH(I))+XK40*
COS(PH(M-1)-PH(I)),
END IF
IF(M EQ. N)THEN
A(I,NN)=XK(2)*SIN(PH(M)-PH(I))+XK(4)*COS(PH(M)-PH(I))
END IF
20 XK20 = XK(2)
XK40 = XK(4)
4 CONTINUE
3 CONTINUE

```

```

* -----
* THE RIGHT HAND SIDE OF THE MATRIX IS NOW COMPUTED WITH
* NO CORRECTION FOR THE BOUNDARY LAYER EFFECTS. THIS IS
* THE PURE INVISCID SOLUTION.
* -----
IF(POSTN(9).EQ.0)THEN
  NR=N
  DO 6 I=1,NR
    BR(I,1) = SIN(PH(I)-ALPHA)
6  CONTINUE

* -----
* THE NEXT STATEMENTS CALL EITHER SUBROUTINE KUTTA1 OR
* SUBROUTINE KUTTA2 DEPENDING ON THE TYPE OF KUTTA
* CONDITION WHICH WAS SELECTED TO BE IMPLIMENTED.
* -----
IF(POSTN(4).EQ.1)CALL KUTTA1
IF(POSTN(4).EQ.0)CALL KUTTA2

ELSE

* -----
* IF BOUNDARY LAYER EFFECTS ARE TO BE INCLUDED IN THE
* SOLUTION CONTROL IS TRANSFERED TO THE NEXT STATEMENTS
* THE SOURCE DISTRIBUTION WHICH ACCOUNTS FOR THE BOUND-
* ARY LAYER EFFECT ARE ADDED TO THE RIGHT HAND SIDE.
* -----
DO 11 I=1,N
  QC=(QSJ(I)+QSJ(I+1))/2
  RS=-1*(QC*SIN(ALPHA-PH(I)))
  DO 12 M=1,N
    CALL XRS
    VQ=-(XK(1)*QSJ(M)+XK(2)*QSJ(M+1))
    UQ= XK(3)*QSJ(M)+XK(4)*QSJ(M+1)
    RS=RS-(VQ*COS(PH(M)-PH(I))+UQ*SIN(PH(M)-PH(I)))
12 CONTINUE
    BR(I,1)=RS
11 CONTINUE

* -----
* THE NEXT STATEMENT CALLS THE SUBROUTINE KUTTA3 WHICH
* IMPLIMENTS THE MODIFIED KUTTA CONDITION FOR THE
* VISCOUS-INVISCID SOLUTION, USING THE SOURCE STRENGTHS
* AT THE TRAILING EDGE.
* -----
CALL KUTTA3

END IF

* -----
* A LIBRARY SUBROUTINE IS CALLED TO SOLVE THE BOUNDARY
* CONDITION EQUATIONS. THE ROUTINE RETURNS THE COEFFICIENTS
* OF THE INVERSE MATRIX AND THE SOLUTION OF THE EQUATIONS
* -----
M2 = 1
IA = 90
M3 = 2
NP=NN

```

```
CALL LEQT1F(A,M2,NP,IA,BR,M3,WAREA,IER)
```

```
-----
THE FOLLOWING STATEMENTS INTEGRATE THE BOUND VORTICITY
AROUND THE AIRFOIL CONTOUR TO DETERMINE THE TOTAL
CIRCULATION AROUND THE AIRFOIL
-----
```

```
GAM = 0.0
DO 8 I=1,N
GAM=GAM+XL(I)*(BR(I,1)+BR(I+1,1))
8 CONTINUE
```

```
-----
LIFT COEFFICIENT BASED ON THE BOUND CIRCULATION
-----
```

```
CLGAM = -GAM/CHORD
```

```
-----
CALCULATION OF THE VELOCITIES AT THE AIRFOIL CONTROL POINTS
-----
```

```
DO 9 I=1, N
```

```
-----
FREE STREAM TANGENTIAL VELOCITY AT A PANEL
-----
```

```
V = COS(ALPHA-PH(I))
```

```
-----
FREE STREAM NORMAL VELOCITY AT A PANEL
-----
```

```
VN = SIN(ALPHA-PH(I))
```

```
DO 10 M=1, N
```

```
CALL XKS
```

```
V=V+(BR(M,1)*XK(1)+BR(M+1,1)*XK(2))*COS(PH(M)-PH(I))-
(BR(M,1)*XK(3)+BR(M+1,1)*XK(4))*SIN(PH(M)-PH(I))
```

```
VN=VN+(BR(M,1)*XK(1)+BR(M+1,1)*XK(2))*SIN(PH(M)-PH(I))+
(BR(M,1)*XK(3)+BR(M+1,1)*XK(4))*COS(PH(M)-PH(I))
```

```
-----
BOUNDARY LAYER SOURCE EFFECTS ARE INCLUDED IN THE
NEXT SET OF STATEMENTS
-----
```

```
IF(POSTN(9) EQ 1) THEN
VQ=-(XK(1)*QSJ(M)+XK(2)*QSJ(M+1))
UQ= XK(3)*QSJ(M)+XK(4)*QSJ(M+1)
VN=VN+UQ*SIN(PH(M)-PH(I))+VQ*COS(PH(M)-PH(I))
V = V +UQ*COS(PH(M)-PH(I))-VQ*SIN(PH(M)-PH(I))
END IF
```

```
10 CONTINUE
```

```
-----
SUMMATION OF THE VELOCITIES PRODUCED AT EACH PANEL CON PT
-----
```

```
VNM(I) = VN
VTAM(I) = V
```

```

* -----
*   DETERMINATION OF THE CONTROL POINT PRESSURE COEFFICIENTS
* -----
  IF(POSTN(9).EQ.0)CP(I)=1.0-VTAN(I)**2
  IF(POSTN(9).EQ.1)CP(I)=1.0-(VTAN(I)**2+VNN(I)**2)
9  CONTINUE

* -----
*   DETERMINATION OF THE PRESSURE COEFFICIENT VALUES AT
*   THE JUCTION POINTS BASED ON THE VORTICITY VALUES.
* -----
  DO 14 I=1,NPT
  IF(POSTN(9).EQ.0)CPJ(I)=1.0-BR(I,1)**2
  IF(POSTN(9).EQ.1)CPJ(I)=1.0-(BR(I,1)**2+QSJ(I)**2)
14 CONTINUE

* -----
*   IF THE EFFECTS OF COMPRESSIBILITY ARE TO BE TAKEN INTO
*   ACCOUNT, THE NEXT STATEMENT CALLS THE SUBROUTINE
*   "CPCOMP".
* -----
  IF(POSTN(12).EQ.1)THEN
  CALL CPCOMP
  ELSE
  DO 7 I=1,NPT
  PCJ(I)=CPJ(I)
  IF(I.LT.NPT)PC(I)=CP(I)
7  CONTINUE
  END IF

* -----
*   THE NEXT STATEMENT ENTERS THE JUNCTION POINT PARAMETERS
*   INTO A SECOND ARRAY FOR USE IN THE "AERO" SUBROUTINE.
* -----
  DO 15 I=1,NPT
  PJ(I)=PCJ(I)
  IF(I.LE.N)THEN
  SL(I)=XL(I)
  PPH(I)=PH(I)
  KXI(I)=XI(I)
  YYI(I)=YI(I)
  END IF
15 CONTINUE

* -----
*   THE NEXT STATEMENT CALLS THE SUBROUTINE "AERO" WHICH
*   DETERMINES VARIOUS AERODYNAMIC PARAMETERS.
* -----
  CALL AERO

  RETURN

  END

* *****
*   SUBROUTINE KUTTA1
*   *****

```

```

***** A-4 *****
*
*   This subroutine establishes the kutta condition which
*   forces the trailing edge of the airfoil to be a
*   stagnation point
*
*****

```

```

      IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

```

```

      COMMON A(90,90),BR(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
*           PH(90),XL(90),VNN(90),VTAN(90),CP(90),QBJ(90),POSTN(20),
*           XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
*           CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
*           N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
*           NPT1,NPT2,NAM1,NAM2,NAM3,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPB,
*           NPLS,NPLS8,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

```

```

      INTEGER POSTN
      REAL NAM1,NAM2,NAM3

```

```

      DO 3 I=1,NN
      A(NA,I)=0.0
      A(I,1)=0.0
      A(I,NN)=0.0
      A(NN,I)=0.0
3 CONTINUE
      BR(NA,1)=0.0
      BR(NN,1)=0.0
      A(NA,1)=1.0
      A(NN,NN)=1.0

```

```

      RETURN

```

```

      END

```

```

*   *****
*   SUBROUTINE KUTTA2
*   *****

```

```

***** A-5 *****
*
*   This subroutine establishes the kutta condition which
*   forces the trailing edge velocities to be equal
*
*****

```

```

      IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

```

```

      COMMON A(90,90),BR(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
*           PH(90),XL(90),VNN(90),VTAN(90),CP(90),QBJ(90),POSTN(20),
*           XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
*           CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
*           N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
*           NPT1,NPT2,NAM1,NAM2,NAM3,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPB,
*           NPLS,NPLS8,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

```

```

      INTEGER POSTN
      REAL NAM1,NAM2,NAM3

```

```

A(NN,1)=1.0
A(NN,NN)=1.0
DO 1 I=2,N
A(NN,I)=0.0
1 CONTINUE
BR(NN,1)=0.0

```

```

RETURN

```

```

END

```

```

. ....
. SUBROUTINE KUTTA3
. ....

```

```

..... A-6 .....
.
. This subroutine establishes the modified kutta con-
. dition for the viscous-inviscid solution Using the
. source strengths at the trailing edge equal pressures are
. are established at the trailing edge
.
.....

```

```

IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

```

```

COMMON A(90,90),BR(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
. PH(90),XL(90),VNN(90),VTAN(90),CP(90),QSJ(90),POSTN(20),
. XK(4),XKC(2),I(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
. CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
. N,NM,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
. NPT1,NPT2,NAM1,NAM2,NAM3,CLCAM,NPUSS,NPUS2,NPUS,WLF,CPS,
. NPLS,NPLSS,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

```

```

INTEGER POSTN
REAL NAM1,NAM2,NAM3

```

```

DO 3 I=1,NN
A(I,1)=0.0
A(NA,I)=0.0
A(NN,I)=0.0
A(I,NN)=0.0
3 CONTINUE

```

```

ANG1=PI-PH(1)
ANG2=PH(N)
SLOPE=ABS(ANG1-ANG2)
BETA=ATAN(SLOPE)

```

```

BR(NA,1)=(QSJ(NN)-QSJ(1)*COS(BETA))/SIN(BETA)
BR(NN,1)=(-QSJ(1)+QSJ(NN)*COS(BETA))/SIN(BETA)
A(NA,1)=1.0
A(NN,NN)=1.0
4 CONTINUE
RETURN

```

```

END

```

```

* *****
* SUBROUTINE FSMOD
* *****

***** A-7 *****
*
* This subroutine carries out the calculations
* necessary to find the pressure coefficient distribution
* and various aerodynamic parameters (i.e. lift coe)
* around the airfoil at the stated angle of attack,
* including the effects of the separated wake model.
* The effects of the boundary layer maybe incorporated
* if desired.
*
* This subroutine calls fourteen other subroutines;
*
*****

IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)
REAL WKAREA(72)

COMMON A(90,90),BR(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
* PH(90),XL(90),VNN(90),VTAN(90),CP(90),QBJ(90),POSTN(20),
* XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
* CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
* N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
* NPT1,NPT2,NAM1,NAM2,NAM3,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPB,
* NPLS,NPLSS,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

INTEGER POSTN
REAL NAM1,NAM2,NAM3

* -----
* VARIOUS CONSTANTS USED IN THE PROGRAM ARE NOW
* ARE NOW DEFINED.
* -----

IF(POSTN(8).EQ.1)NN=N+1
IF(POSTN(8).EQ.0)NN=N+2
NUFF=NUF-1
NLFF=NLF-1
NPT1=NPT+1
NPT2=NPT+2

* -----
* DETERMINATION OF THE JUNCTION POINTS FALLING BOTH
* INSIDE AND OUTSIDE OF THE INTERSECTION POINT OF THE
* AIRFOIL SURFACE AND THE WAKE STREAMLINE
* -----

POSTN(7)=0.0
DO 9 I=1,NN
IF(X(I+1).EQ.X(I).AND.Y(I+1).EQ.Y(I))THEN
IF(POSTN(8).EQ.1)THEN
NPUSS=I
NPUS=I+1
NPUS2=I-1
NPLS=N+2
NPLSS=N+2

```


Appendix J: STAMP

250

```

NPLS2=N+2
GO TO 8
ELSE
IF(POSTN(7).LT.1)THEN
NPUS8=I
NPUS=I+1
NPUS2=I-1
POSTN(7)=1.0
ELSE
NPLS=I+1
NPLSS=I
NPLS2=I-1
GO TO 8
END IF
END IF
ELSE
CONTINUE
END IF
9 CONTINUE
8 CONTINUE
POSTN(7)=0.0

```

* THE NEXT SET OF STATEMENTS DETERMINE THE NUMBER OF PANELS
* ON THE AIRFOIL PRESSURE SURFACE (UNDER SURFACE) IN ORDER
* TO DETERMINE THE CONTROL POINT WHERE THE CONDITION OF
* ZERO NORMAL VELOCITY MAYBE RELAXED, IN ORDER TO IMPLIMENT
* ONE OF THE AUXILLARY CONDITIONS.
* -----

```

DO 6 I=2,NPT
IF(X(I) LE X(I-1))THEN
CONTINUE
ELSE
NUP=I-2
NLP=N-NUP
GO TO 7
END IF
6 CONTINUE
7 CONTINUE
NA=(NPT-NLP/2.0)+2

```

* IF SELECTED THE NEXT STATEMENT CALLS THE SUBROUTINE
* "TRANSX" WHICH TRANSLATES THE X CO-ORDINATES SUCH
* THAT THE LEADING EDGE IS AT X=0.0. IT ALSO NON-DIM-
* ENSIONALIZES ALL THE CO-ORDINATES.
* -----

CALL TRANSX

* THE FOLLOWING STATEMENTS CALLS THE SUBROUTINE GEOMAIR
* WHICH CALCULATES THE CONTROL POINTS ON THE AIRFOIL,
* THE ANGLE PH(I), AND THE LENGTH OF EACH VORTEX PANEL.
* -----

CALL GEOMAIR

* THE FOLLOWING STATEMENT CALLS THE SUBROUTINE GEOMWAK
* WHICH CALCULATES THE CONTROL POINTS ON THE UPPER AND

```

* LOWER WAKE SURFACES, THE ANGLES PH(I) AND THE LENGTH
* OF EACH WAKE PANEL.
* -----
CALL GEONWAK

* -----
* THE STATEMENT CALLS THE SUBROUTINE "AIRCOE" IN ORDER TO
* FORM THE AIRFOIL INFLUENCE COEFFICIENTS
* -----
CALL AIRCOE

OUTPUT, 'The wake iterations have now begun'
OUTPUT, '

* -----
* THE NEXT DO LOOP SETS UP THE ITERATIVE CYCLE IN
* ORDER TO RELAX THE WAKE SHAPE INTO A FORCE FREE
* POSITION
* -----
DO 5 JI = 1, POSTN(2)
OUTPUT, '
OUTPUT, 'The iteration cycle now being performed', JI
OUTPUT, '

* -----
* THE NEXT STATEMENT CALLS THE SUBROUTINE "WAKCOE"
* WHICH FORMS AND SUMS THE INFLUENCE COEFFICIENTS
* FOR THE UPPER AND LOWER WAKE VORTEX PANELS
* -----
CALL WAKCOE

* -----
* THE NEXT STATEMENT CALLS THE SUBROUTINE BOUNDARY WHICH
* IMPLEMENTS THE AUXILIARY CONDITIONS FOR THE SEPARATED
* WAKE FLOW MODEL
* -----
IF(POSTN(8) EQ 1)CALL BOUND1
IF(POSTN(8) EQ 0)CALL BOUND2

* -----
* ENTER THE NORMAL VELOCITY INFLUENCE COEFFICIENT ARRAY INTO
* A SECOND ARRAY CALLED AN(I,M) FOR LATER USE.
* -----
DO 1 I=1,NPT2
DO 2 M=1,NPT2
AN(I,M)=A(I,M)
2 CONTINUE
1 CONTINUE

DO 3 I=1,NPT2
BB(I,1)=BR(I,1)
3 CONTINUE

* -----
* A LIBRARY SUBROUTINE IS CALLED TO SOLVE THE BOUNDARY
* CONDITION EQUATIONS. THE ROUTINE RETURNS THE COEFFICIENTS
* OF THE INVERSE MATRIX AND THE SOLUTION OF THE EQUATIONS.
* -----
M2 = 1
IA = 90
M3 = 2

```

NP=NPT+2

CALL LEQ1F(A,M2,NP,IA,BB,M3,WKAREA,IER)

THE FOLLOWING STATEMENT CALLS THE SUBROUTINE "LIFT" WHICH
INTEGRATES THE BOUND VORTICITY AROUND THE AIRFOIL AND WAKE
CONTOUR IN ORDER TO DETERMINE THE LIFT DEVELOPED

IF(POSTN(17) EQ 1 OR POSTN(3) EQ 1 OR J1 EQ POSTN(2)) THEN
CALL LIFT
END IF

THE NEXT STATEMENT CALLS THE SUBROUTINE "VELOCITY" IN
ORDER TO CALCULATE THE NORMAL AND EXTERNAL TANGENTIAL
VELOCITIES AT THE AIRFOIL AND WAKE CONTROL POINTS

CALL VELOCITY

THE NEXT STATEMENT CALLS THE SUBROUTINE PRESOC
IN ORDER TO DETERMINE THE PRESSURE COEFFICIENTS

CALL PRESOC

THE NEXT STATEMENT CALLS THE SUBROUTINE "AERO" WHICH
DETERMINES THE LIFT AND PRESSURE DRAG ON THE BODY

IF(POSTN(17) EQ 1 OR POSTN(3) EQ 1 OR J1 EQ POSTN(2)) THEN
CALL AERO
END IF

THE NEXT STATEMENT CALLS THE SUBROUTINE "OUTPUT" IF
THE USER WISHES TO HAVE A COMPLETE LISTING OF THE
ITERATIONS

IF(POSTN(3) EQ 1) CALL OUTPUT
IF(POSTN(3) EQ 0 AND J1 EQ POSTN(2) AND POSTN(16) EQ 0)
CALL OUTPUT

THE NEXT STATEMENT TRANSFERS CONTROL OUT OF THE WAKE
ITERATION LOOP IF THE "BASE" PRESSURE PERCENT. CHANGE
IS LESS THEN "CRT1" BETWEEN TWO SUCCESSIVE ITERATIONS

IF(J1 GT 1) THEN
CHECK=(ABS(PCJ(1)-CPB)/ABS(PCJ(1)))*100.0
OUTPUT, 'The percent change in base pressure between'
OUTPUT, 'successive iterations is: ', CHECK
IF(CHECK LT CRT1) THEN
IF(POSTN(17) EQ 0) THEN
POSTN(17)=1.0
ELSE
OUTPUT, 'The wake geometry has converged to a stable solution.'

```

IF(POSTN(3) EQ 0)CALL OUTPUT
GO TO 4
END IF
ELSE
IF(POSTN(17) EQ 1)POSTN(17)=0
END IF

```

```

-----
THE NEXT STATEMENT SETS THE CURRENT VALUE OF THE
"BASE" PRESSURE EQUAL TO CPB FOR COMPARISON IN THE
NEXT ITERATION
-----

```

```

END IF
CPB=PCJ(1)

```

```

-----
THE NEXT STATEMENT CALLS THE SUBROUTINE "WAKITER"
WHICH DETERMINES THE NEW IMPROVED WAKE SHAPE
-----

```

```

CALL WAKITER

```

```

5 CONTINUE

```

```

4 CONTINUE

```

```

RETURN

```

```

END

```

```

.....
SUBROUTINE AIRCOE
.....

```

```

..... A-8 .....
*
* This subroutine generates the influence coefficient-
* ents for the airfoil panels based on the assumption
* that they have a linear variation of vorticity.
* This subroutine calls a separate routine "XKS"
* to evaluate parts of the influence coefficients
*
*.....

```

```

IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

```

```

COMMON A(90,90),BR(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
* PH(90),XL(90),VNN(90),VTAN(90),CP(90),QSJ(90),POSTN(20),
* XI(4),XIC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
* CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
* N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
* NPT1,NPT2,NAM1,NAM2,NAM3,CLGAN,NPUSS,NPUS2,NPUS,WLF,CPB,~
* NPLS,NPLSS,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

```

```

INTEGER POSTN
REAL NAM1,NAM2,NAM3

```

```

-----
THE INFLUENCE COEFFICIENTS FOR THE AIRFOIL ARE NOW FORMED
-----

```

```

DO 19 I=1,NN
IF(I.EQ.NPUSS.OR.I.EQ.NPLSS)GO TO 81

```

```

DO 20 M=1,NN
IF(M EQ NPUS8 OR M EQ NPLS8)GO TO 22
CALL XKS
IF(M EQ 1 OR M EQ NPUS OR M EQ NPLS)THEN
A(I,M)=XK(1)*SIN(PH(M)-PH(I))+XK(3)*COS(PH(M)-PH(I))
ELSE
A(I,M) = XK(1)*SIN(PH(M)-PH(I))+XK20*SIN(PH(M-1)-
PH(I))+XK(3)*COS(PH(M)-PH(I))+XK40*
COS(PH(M-1)-PH(I))
END IF
IF(M EQ NPLS2 OR M EQ NN)THEN
A(I,M+1)=XK(2)*SIN(PH(M)-PH(I))+XK(4)*COS(PH(M)-PH(I))
END IF
XK20=XK(2)
XK40=XK(4)
22 CONTINUE
20 CONTINUE
81 CONTINUE
19 CONTINUE

```

RETURN

END

```

* .....
* SUBROUTINE WAKITER
* .....

```

```

* ..... A-9 .....
*
* This subroutine determines the new wake shape
* in order to satisfy the force free wake condition
* The new wake shape is determined by a piecewise int-
* egration, starting at the separation point locations
*
* This subroutine calls two other routines,
* -GEOMWAK to determine various geometric wake para-
* meters
* -WAKCOE to form and sum the upper and lower wake
* wake influence coefficients
*
* .....

```

IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

```

COMMON A(90,90),BR(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
* PH(90),XL(90),VNN(90),VTAN(90),CP(90),QSJ(90),POSTN(20),
* XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
* CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
* N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFM,
* NPT1,NPT2,NAM1,NAM2,NAM3,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPS,
* NPLS,NPLS8,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

```

```

INTEGER POSTN
REAL NAM1,NAM2,NAM3

```

```

* -----
* THE NEXT SET OF STATEMENTS SET UP THE ITERATIVE LOOP
* IN ORDER TO RELAX THE WAKE INTO A FORCE FREE POSITION.

```

```

      NS=NUS
      NF=NUFF
4    DO 1 I=NS,NF

```

```

      THE NEXT SET OF STATEMENTS PERFORM THE PIECEWISE INTE-
      GRATION

```

```

      DEL=ATAN2(VNM(I),VTAN(I))
      ANG1=PH(I)+DEL
      ANG2= PH(I)+RELAX*(ANG1-PH(I))
      X(I+1)=XL(I)*COS(ANG2)+X(I)
      Y(I+1)=XL(I)*SIN(ANG2)+Y(I)
1    CONTINUE

```

```

      THE NEXT STATEMENT CHANGES THE OBJECTS IN DO LOOP
      1 SO THAT THE LOWER WAKE IS RELAXED

```

```

      IF(NF EQ NUFF)THEN
      NS=NLS
      NF=NLFF
      GO TO 4
      END IF

```

```

      THE NEXT GROUP OF STATEMENTS DETERMINE IF ANY OF THE
      PANELS HAVE OVERLAPPED ONE-ANOTHER IF THIS IS THE
      CASE THOSE PANELS PAST THE CROSS OVER POINTS ARE
      OMITTED FROM THE CALCULATIONS

```

```

      IF(POSTN(17) EQ 1)THEN
      NU=0
      NL=0
      DO 5 I=NUS,NUFF
      NU=NU+1
      SLOP1=(Y(I+1)-Y(I))/(X(I+1)-X(I))
      CONST1=Y(I)-SLOP1*X(I)
      DO 6 M=NLS,NLFF
      NL=NL+1
      SLOP2=(Y(M+1)-Y(M))/(X(M+1)-X(M))
      CONST2=Y(M)-SLOP2*X(M)

```

```

      THE NEXT STATEMENT PROTECTS AGAINST PARALLEL LINES

```

```

      IF(SLOP1 EQ SLOP2)GO TO 7
      XINTER=(CONST2-CONST1)/(SLOP1-SLOP2)
      YINTER=SLOP1*XINTER+CONST1

```

```

      THE NEXT STATEMENT DETERMINES IF THE PANELS ARE CROSSED

```

```

      IF(XINTER GE X(I) .AND. XINTER LE X(I+1) .AND. YINTER GE Y(M)
      .AND. YINTER LE Y(M+1))THEN
      OUTPUT,
      OUTPUT,'The wake panels have crossed one another at'

```

```

OUTPUT , 'some point Corrective action has been taken.'
OUTPUT ,
IF (NU NE NPU) THEN
NC=NPU-NU
NPU=NU
NUF=NUB+NPU
NUFF=NUF-1
N1=NLS+NL

```

```

-----
THE NEXT STATEMENT RENUMBERS THE LOWER WAKE PANELS
-----

```

```

DO 9 J=NLS,N1
X(J-NC)=X(J)
Y(J-NC)=Y(J)
9 CONTINUE
ELSE
CONTINUE
END IF
IF (NL NE NPL OR NU NE NPU) THEN
NPL=NL
NLS=NUF+1
NLF=NLS+NPL
NLF=NLF-1
ELSE
CONTINUE
END IF

X(NUF)=XINTER
Y(NUF)=YINTER
X(NLF)=XINTER
Y(NLF)=YINTER
GO TO 8
ELSE
CONTINUE
END IF
7 CONTINUE
6 CONTINUE
NL=0
5 CONTINUE
8 CONTINUE
END IF

```

```

-----
THE NEXT STATEMENT CALLS THE SUBROUTINE "GEOMWAK"
WHICH CALCULATES THE CONTROL POINTS, PH(I), AND
LENGTH OF EACH OF THE NEW WAKE PANELS.
-----

```

```

CALL GEOMWAK

```

```

-----
ENTER THE AIRFOIL INFLUENCE COEFFICIENTS INTO ARRAY
-----

```

```

DO 12 I=1,NPT2
DO 10 M=1,NPT2
A(I,M)=AN(I,M)
10 CONTINUE
12 CONTINUE

```

```

* -----
*   ENTER THE RIGHT HAND SIDE OF THE MATRIX AGAIN
* -----
DO 11 I=1,NPT2
  BR(I,1)=BB(I,1)
11 CONTINUE

  RETURN

  END

* .....
*   SUBROUTINE WAKCOE
* .....

* ..... A-10 .....
*
*   This subroutine generates the influence coefficient-
*   ents for the upper and lower wake panels assuming they
*   are of constant strength
*   This subroutine calls a separate routine "XKONS"
*   to evaluate parts of the influence coefficients
* .....

  IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

  COMMON A(90,90),BR(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
*   PH(90),XL(90),VWN(90),VTAN(90),CP(90),QSJ(90),POSTN(20),
*   XI(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
*   CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
*   N,NM,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
*   NPT1,NPT2,NAM1,NAM2,NAM3,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPB,
*   NPLS,NPLSS,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

  INTEGER POSTN
  REAL NAM1,NAM2,NAM3

* -----
*   SUMATION OF THE INFLUENCE COEFFICIENTS ON THE UPPER
*   WAKE PANELS, ASSUMING THEY ARE OF CONSTANT STRENGTH
* -----
DO 15 I=1,NM
  SUMU=0.0
  IF(I.EQ.NPUSS.OR.I.EQ.NPLSS)GO TO 79
  DO 16 M=NUS,NUFF
    CALL XKONS
    SUMU=SUMU+XKC(1)*SIN(PH(M)-PH(I))+XKC(2)
*   *COS(PH(M)-PH(I))
16 CONTINUE
79 CONTINUE
  A(I,NPT1)=SUMU
15 CONTINUE

* -----
*   SUMATION OF THE INFLUENCE COEFFICIENTS ON THE LOWER
*   WAKE PANELS, ASSUMING THEY ARE OF CONSTANT STRENGTH
* -----
DO 18 I=1,NM

```



```

SUML=0.0
IF(I EQ NPUSS OR I EQ NPLSS)GO TO 80
DO 99 M=NLS,NLFF
CALL XKCONS
SUML=SUML+XKC(1)*SIN(PH(M)-PH(I))+XKC(2)
      *COS(PH(M)-PH(I))
99 CONTINUE
80 CONTINUE
A(I,NPT2)=SUML
18 CONTINUE

```

```

RETURN

```

```

END

```

```

.....
SUBROUTINE LIFT
.....

```

```

..... A-11 .....
.
. This subroutine integrates the bound vorticity
. around the airfoil, upper and lower wake contours in-
. order to determine the total circulation and lift about
. the separated airfoil geometry
.
.....

```

```

IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

```

```

COMMON A(90,90),BR(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
. PH(90),XL(90),VNM(90),VTAN(90),CP(90),QSJ(90),POSTN(20),
. XK(4),XKC(2),I(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
. CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
. N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
. NPT1,NPT2,NAM1,NAM2,NAM3,CLCAM,NPUSS,NPUS2,NPUS,WLF,CPB,
. NPLS,NPLSS,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

```

```

INTEGER POSTN
REAL NAM1,NAM2,NAM3

```

```

CAM=0.0
DO 77 I=1,NN
IF(I EQ NPUSS OR I EQ NPLSS)GO TO 60
CAM=CAM+XL(I)*(BR(I,1)+BR(I+1,1))
60 CONTINUE
77 CONTINUE
BRU=BR(NPT1,1)
DO 61 I=NUS,NUFF
CAM=CAM+XL(I)*BRU*2
61 CONTINUE
BRL=BR(NPT2,1)
DO 62 I=NLS,NLFF
CAM=CAM+XL(I)*BRL*2
62 CONTINUE

```

```

-----
LIFT COEFFICIENT BASED ON THE BOUND CIRCULATION
-----

```

```
CLGAM=-CAM/CHORD
```

```
RETURN
```

```
END
```

```

* .....
* SUBROUTINE AERO
* .....

```

```

..... A-12 .....
*
* This subroutine determines four aerodynamic par-
* ameters. They are the lift, pressure drag and pitching
* moment (about the L E , and quarter chord point) coe-
* The pressure coefficient distribution is integrated
* in three different ways in order to determine the par-
* ameters
* The first method fits a parabola through control
* point values of the pressure coefficient and then inte-
* grates for the parameters.
* The second method fits a straight line through
* adjoining junction points and then integrates for the
* parameters
* The third method fits a parabola through three jun-
* ction points and then integrates for the parameters
* .....

```

```
IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)
```

```

COMMON A(90,90),BR(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
* PH(90),XL(90),VNN(90),VTAN(90),CP(90),QSJ(90),POSTN(20),
* XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
* CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
* N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
* NPT1,NPT2,NAM1,NAM2,NAM3,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPB,
* NPLS,NPLSS,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

```

```

INTEGER POSTN
REAL NAM1,NAM2,NAM3

```

```

* -----
* THE FIRST METHOD FITS A PARABOLA THROUGH THREE CONTROL
* POINT VALUES OF CP AND THEN INTEGRATES FOR THE PARA-
* METERS.
* -----

```

```

CX=0.0
CY=0.0
CM=0.0
CM4=0.0
SC3=XL(1)/2.0
IF(POSTN(1).EQ.1)NP=NN-1
IF(POSTN(1).EQ.0)NP=N-1
DO 1 I=2,NP
IF(I.EQ.NPUSS.OR.I.EQ.NPLSS)GO TO 2
JJ=I+1
J=I-1
IF(I.EQ.NPUS2.OR.I.EQ.NPLS2)JJ=I+2

```

```

IF(I.EQ.NPUS.OR.I.EQ.NPLS)J=I-2
SC1=SC3
SJ2=SC1+XL(J)/2.0
SC3=8J2+XL(I)/2.0
SJ4=SC3+XL(I)/2.0
SC5=8J4+XL(JJ)/2.0

```

```

-----
* DETERMINATION OF THE COEFFICIENTS A1,B1 AND C1 FOR THE
* QUADRATIC EQUATION.

```

```

B1=(PC(J)*(SC5**2-SC3**2)+PC(JJ)*(SC3**2-SC1**2)-PC(I)*
1 (SC5**2-SC1**2))/(SC3**2*(SC5-SC1)+SC1**2*(SC3-SC5)+
1 SC5**2*(SC1-SC3))

```

```

A1=(PC(JJ)-PC(J)-B1*(SC5-SC1))/(SC5**2-SC1**2)

```

```

C1=PC(I)-A1*SC3**2-B1*SC3

```

```

-----
* THE FUNCTIONAL VALUE OF THE PRESSURE INTEGRAL IS USED
* TO EVALUATE THE ACTUAL X AND Y FORCE COMPONENTS.
* THESE FORCE "COEFFICIENTS" ARE THEN USED TO DETER-
* THE LIFT,PRESSURE DRAG, LEADING EDGE AND QUARTER
* CHORD PITCHING MOMENT COEFFICIENTS.

```

```

IF(I.EQ.2)THEN

```

```

FUN1=A1/3*(SJ2**3)+B1/2*(SJ2**2)+C1*SJ2

```

```

DCX=(-1)*FUN1*SIN(PH(1))

```

```

DCY=FUN1*COS(PH(1))

```

```

CX=DCX+CX

```

```

CY=DCY+CY

```

```

CM=DCX*YI(1)-DCY*XI(1)+CM

```

```

CM4=(-1)*DCY*(XI(1)-.25)+DCX*YI(1)+CM4

```

```

END IF

```

```

FUN1=A1/3*(SJ4**3-SJ2**3)+B1/2*(SJ4**2-SJ2**2)

```

```

1 +C1*(SJ4-SJ2)

```

```

DCX=(-1)*FUN1*SIN(PH(I))

```

```

DCY=FUN1*COS(PH(I))

```

```

CX=DCX+CX

```

```

CY=DCY+CY

```

```

CM=(-1)*DCY*XI(I)+DCX*YI(I)+CM

```

```

CM4=(-1)*DCY*(XI(I)-0.25)+DCX*YI(I)+CM4

```

```

IF(I.EQ.NP)THEN

```

```

SJ6=SJ4+XL(I+1)

```

```

FUN1=A1/3*(SJ6**3-SJ4**3)+B1/2*(SJ6**2-SJ4**2)

```

```

1 +C1*(SJ6-SJ4)

```

```

DCX=(-1)*FUN1*SIN(PH(I+1))

```

```

DCY=FUN1*COS(PH(I+1))

```

```

CX=DCX+CX

```

```

CY=DCY+CY

```

```

CM=(-1)*DCY*XI(I+1)+DCX*YI(I+1)+CM

```

```

CM4=(-1)*DCY*(XI(I+1)-0.25)+DCX*YI(I+1)+CM4

```

```

END IF

```

```

2 CONTINUE

```

```

1 CONTINUE

```

```

-----
* THE LIFT AND PRESSURE DRAG COEFFICIENTS ARE NOW

```

```

*   DETERMINED FROM THE X AND Y FORCE COEFFICIENTS
*   -----
CL(1)=CY*COS(ALPHA)-CX*SIN(ALPHA)
CD(1)=CX*COS(ALPHA)+CY*SIN(ALPHA)
CM1(1)=CM
CM2(1)=CM4

*   -----
*   THE NEXT SET OF STATEMENTS INTEGRATES THE PRESSURE
*   COEFFICIENTS AROUND THE AIRFOIL IN ORDER TO DETER-
*   THE LIFT COEFFICIENT AND PRESSURE DRAG COEFFICIENT
*   A LINEAR VARIATION OF CP IS ASSUMED BETWEEN JUNC-
*   TION POINTS.
*   -----
CX=0.0
CY=0.0
CM=0.0
CM4=0.0
SJ2=0.0
DO 3 I=1,N
  SJ1=SJ2
  SJ2=SJ1+SL(I)

*   -----
*   DETERMINATION OF THE LINEAR COEFFICIENTS A2 AND B2
*   -----
A2=(PJ(I+1)-PJ(I))/(SJ2-SJ1)
B2=PJ(I)-A2*SJ1

*   -----
*   DETERMINATION OF THE INTEGRATED PRESSURE FUNCTION.
*   -----
FUN2=(A2/2)*(SJ2**2-SJ1**2)+B2*(SJ2-SJ1)

*   -----
*   DETERMINATION OF THE INCREMENT OF X AND Y FORCE
*   COEFFICIENT (DCX,DCY) AND THE SUMATION OF X AND
*   Y FORCE COEFFICIENTS (CX,CY).
*   -----
DCX=(-1)*FUN2*SIN(PPH(I))
DCY=FUN2*COS(PPH(I))
CX=CX+DCX
CY=CY+DCY
CM=(-1)*DCY*XXI(I)+DCX*YYI(I)+CM
CM4=(-1)*DCY*(XXI(I)-0.25)+DCX*YYI(I)+CM4
4 CONTINUE
3 CONTINUE

*   -----
*   DETERMINATION OF THE LIFT AND PRESSURE DRAG COEFF-
*   ICIENTS AS FOUND FROM CX AND CY.
*   -----
CL(2)=CY*COS(ALPHA)-CX*SIN(ALPHA)
CD(2)=CX*COS(ALPHA)+CY*SIN(ALPHA)
CM1(2)=CM
CM2(2)=CM4

*   -----
*   THE NEXT METHOD FITS A PARABOLA THROUGH THREE JUNCT-

```

```

*   ION POINT VALUES OF CP AND INTEGRATES FOR THE PAR-
*   AMETERS OVER TWO PANELS.
*   -----
      CX=0 0
      CY=0 0
      CM3=0 0
      CM4=0 0
      SJ3=0 0
      I3=1
6    I1=I3
      I2=I1+1
      I3=I2+1
      IF(I1 GE (N+1))GO TO 5
      SJ1=SJ3
      SJ2=SJ1+SL(I1)
      SJ3=SJ2+SL(I2)
*   -----
*   DETERMINATION OF THE COEFFICIENTS A3,B3 AND C3 FOR THE
*   QUADRATIC EQUATION.
*   -----
      B3=(PJ(I1)*(SJ3**2-SJ2**2)-PJ(I2)*(SJ3**2-SJ1**2)+
1     PJ(I3)*(SJ2**2-SJ1**2))/(SJ2**2*(SJ3-SJ1)+SJ1**2
1     *(SJ2-SJ3)+SJ3**2*(SJ1-SJ2))

      A3=(PJ(I3)-PJ(I1)-B3*(SJ3-SJ1))/(SJ3**2-SJ1**2)

      C3=PJ(I2)-A3*SJ2**2-B3*SJ2
*   -----
*   EVALUATION OF THE PRESSURE COEFFICIENT INT-
*   GRAL OVER TWO AJJOINING PANELS.
*   -----
      FUN4=(A3/3)*(SJ3**3-SJ2**3)+(B3/2)*(SJ3**2-SJ2**2)+
1     C3*(SJ3-SJ2)
      FUN3=(A3/3)*(SJ2**3-SJ1**3)+(B3/2)*(SJ2**2-SJ1**2)+
1     C3*(SJ2-SJ1)
*   -----
*   DETERMINATION OF THE THREE PARAMETERS FOR TWO
*   AJJOINING PANELS USING CX AND CY, (THE X AND Y
*   FORCE COEFFICIENTS).
*   -----
      DCX=(-1)*FUN3*SIN(PPH(I1))
      DCY=FUN3*COS(PPH(I1))
      CX=DCX+CX
      CY=DCY+CY
      CM3=(-1)*DCY*XXI(I1)+DCX*YYI(I1)+CM3
      CM4=(-1)*DCY*(XXI(I1)-0.25)+DCX*YYI(I1)+CM4
      DCX=(-1)*FUN4*SIN(PPH(I2))
      DCY=FUN4*COS(PPH(I2))
      CX=CX+DCX
      CY=CY+DCY
      CM3=(-1)*DCY*XXI(I2)+DCX*YYI(I2)+CM3
      CM4=(-1)*DCY*(XXI(I2)-0.25)+DCX*YYI(I2)+CM4
      IF((I3+1).EQ.(N+1))THEN
*   -----
*   THE NEXT STATEMENTS ARE USED IF THERE IS AND

```

* ODD NUMBER OF PANELS USED TO MODEL THE AIRFOIL.

```

* -----
I1=I2
I2=I1+1
I3=I1+2
SJ1=8J2
SJ2=8J3
SJ3=8J2+8L(I2)

B3=(PJ(I1)*(SJ3**2-SJ2**2)-PJ(I2)*(SJ3**2-SJ1**2)+
1 PJ(I3)*(SJ2**2-SJ1**2))/(SJ2**2*(SJ3-SJ1)+8J1**2
1 *(SJ2-SJ3)+SJ3**2*(SJ1-SJ2))
A3=(PJ(I3)-PJ(I1)-B3*(SJ3-SJ1))/(8J3**2-SJ1**2)
C3=PJ(I2)-A3*8J2**2-B3*8J2
10 CONTINUE
FUN4=(A3/3)*(SJ3**3-SJ2**3)+(B3/2)*(SJ3**2-SJ2**2)+
1 C3*(SJ3-SJ2)
DCX=(-1)*FUN4*SIN(PPH(I2))
DCY=FUN4*COS(PPH(I2))
CX=CX+DCX
CY=CY+DCY
CM3=(-1)*DCY*XXI(I2)+DCX*YYI(I2)+CM3
CM4=(-1)*DCY*(XXI(I2)-0.25)+DCX*YYI(I2)+CM4
7 CONTINUE
GO TO 5
END IF
GO TO 6
6 CONTINUE

```

* -----
* THE EVALUATION OF THE LIFT AND PRESSURE DRAG COE-
* FFICIENTS FROM THE INTEGRATED VALUES OF CX AND CY.
* -----

```

CL(3)=CY*COS(ALPHA)-CX*SIN(ALPHA)
CD(3)=CX*COS(ALPHA)+CY*SIN(ALPHA)
CM1(3)=CM3
CM2(3)=CM4

```

RETURN

END

```

* *****
* SUBROUTINE VELOCITY
* *****

```

***** A-13 *****

* This subroutine calculates both the normal and
* external tangential velocities on the airfoil sur-
* face, upper wake surface and lower wake surface.

* This subroutine calls two other routines;
* -XKS to find parts of the influence coefficients
* for the linearly varying vortex sheets.
* -XKCONS to find parts of the influence coeffi-
* ents for the constant strength wake vor-
* tex sheets.

4

OF / DE

4

```

.....

      IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

      COMMON A(90,90),BR(90,1),BS(90,1),AN(90,90),XI(90),YI(90),
      *      PH(90),XL(90),VNN(90),VTAN(90),CP(90),QSJ(90),POSTN(20),
      *      XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPE(90),
      *      CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
      *      N,NN,NPT,NUS,NUP,NUPF,NLS,NLF,NLFF,I,M,PI,CHORD,NFW,
      *      NPT1,NPT2,NAM1,NAM2,NAMS,CLGAN,NPUSS,NPUS2,NPUS,WLF,CPB,
      *      NPLS,NPLS3,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

      INTEGER POSTN
      REAL NAM1,NAM2,NAM3

      -----
      *      CALCULATION OF THE VELOCITIES AT THE CONTROL POINTS
      *      -----
      NS=1
      NF=NN
45  DO 38 I=NS,NF
      IF(I.EQ.NPUSS.OR.I.EQ.NPLS3)THEN
      VNN(I)=VNN(I-1)
      VTAN(I)=VTAN(I-1)
      GO TO 53
      END IF

      *      -----
      *      FREE STREAM TANGENTIAL VELOCITY AT A PANEL
      *      -----
      VT=COS(ALPHA-PH(I))

      *      -----
      *      FREE STREAM NORMAL VELOCITY AT A PANEL
      *      -----
      VN=SIN(ALPHA-PH(I))

      *      -----
      *      CALCULATION OF THE EXTERNAL TANGENTIAL AND NORMAL VELOCITY
      *      INDUCED BY THE VORTEX PANELS AT THE CONTROL POINTS.
      *      -----
      DO 39 M=1,NN
      IF(M.EQ.NPUSS.OR.M.EQ.NPLS3)GO TO 69
      CALL XKS
      VT=VT+(BR(M,1)*XK(1)+BR(M+1,1)*XK(2))*COS(PH(M)-PH(I))-
      *      (BR(M,1)*XK(3)+BR(M+1,1)*XK(4))*SIN(PH(M)-PH(I))

      VN=VN+(BR(M,1)*XK(1)+BR(M+1,1)*XK(2))*SIN(PH(M)-PH(I))+
      *      (BR(M,1)*XK(3)+BR(M+1,1)*XK(4))*COS(PH(M)-PH(I))

      IF(POSTN(9).EQ.1)THEN

      *      -----
      *      BOUNDARY LAYER SOURCE EFFECTS ARE INCLUDED IN THE
      *      NEXT SET OF STATEMENTS.
      *      -----
      VQ=-(XK(1)*QSJ(M)+XK(2)*QSJ(M+1))
      UQ= XK(3)*QSJ(M)+XK(4)*QSJ(M+1)

```



```

VN=VN+UQ*SIN(PH(M)-PH(I))+VQ*COS(PH(M)-PH(I))
VT=VT+UQ*COS(PH(M)-PH(I))-VQ*SIN(PH(M)-PH(I))
END IF

```

```

69 CONTINUE
39 CONTINUE

```

```

* -----
* SUMATION FOR THE UPPER WAKE INFLUENCE ON THE VELOCITIES
* -----

```

```

SUT=0.0
SUN=0.0
DO 41 M=NUS,NUFF
CALL XKONS
SUT=SUT+(XKC(1)*COS(PH(M)-PH(I))-
* XKC(2)*SIN(PH(M)-PH(I)))

```

```

SUN=SUN+(XKC(1)*SIN(PH(M)-PH(I))+
* XKC(2)*COS(PH(M)-PH(I)))
41 CONTINUE
VT=VT+SUT*BR(NPT1,1)
VN=VN+SUN*BR(NPT1,1)

```

```

* -----
* SUMATION FOR THE LOWER WAKE INFLUENCE ON THE VELOCITIES
* -----

```

```

SLT=0.0
SLN=0.0
DO 42 M=NLS,NLFF
CALL XKONS
SLT=SLT+(XKC(1)*COS(PH(M)-PH(I))-
* XKC(2)*SIN(PH(M)-PH(I)))
SLN=SLN+(XKC(1)*SIN(PH(M)-PH(I))+
* XKC(2)*COS(PH(M)-PH(I)))

```

```

42 CONTINUE
VT=VT+BR(NPT2,1)*SLT
VN=VN+BR(NPT2,1)*SLN
VNM(I)=VN
VTAN(I)=VT
53 CONTINUE
38 CONTINUE

```

```

* -----
* THE NEXT STATEMENTS CHANGE THE VARIABLES IN DO LOOP 38
* SO THAT THE VELOCITIES INDUCED ON THE UPPER AND LOWER
* WAKE SURFACES ARE CALCULATED.
* -----

```

```

IF(NF.EQ.NLFF)GO TO 101
IF(NF.EQ.NN)THEN
NS=NUS
NF=NUFF
GO TO 45
ELSE
NS=NLS
NF=NLFF
GO TO 45
END IF

```

```
101 CONTINUE
```

```
RETURN
```

```
END
```

```
*****  
SUBROUTINE PRESOE  
*****
```

```
***** A-14 *****
```

```
*****  
* This subroutine determines the pressure coefficients *  
* about the airfoil. It as well determines the pressure *  
* coefficients in the separated region. *  
*****
```

```
IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)
```

```
COMMON A(90,90),BR(90,1),BS(90,1),AN(90,90),XI(90),YI(90),  
* PE(90),XL(90),VHN(90),VTAN(90),CP(90),QSJ(90),POSTN(20),  
* XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),  
* CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XII(90),YII(90),  
* N,NM,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,MFN,  
* NPT1,NPT2,NAM1,NAM2,NAM3,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPS,  
* NPLS,NPLS2,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1
```

```
INTEGER POSTN  
REAL NAM1,NAM2,NAM3
```

```
-----  
* DETERMINATION OF THE MEAN EXTERIOR TANGENTIAL *  
* UPPER AND LOWER WAKE VELOCITIES *  
-----
```

```
SUMU=0.0  
DO 1 I=NUS,NUFF  
SUMU=SUMU+VTAN(I)  
1 CONTINUE  
SUML=0.0  
DO 2 I=NLS,NLFF  
SUML=SUML+VTAN(I)  
2 CONTINUE  
VUM1=SUMU/NPU  
VLM1=SUML/NPL
```

```
-----  
* THE VELOCITY ON THE INTERIOR OF THE UPPER WAKE *  
-----
```

```
VUIN=VUM1-ABS(BR(NPT1,1))  
VLIN=VLM1-ABS(BR(NPT2,1))
```

```
-----  
* THE MEAN VELOCITY OF THE UPPER AND LOWER WAKES *  
-----
```

```
VUM2=(VUM1+VUIN)/2  
VLM2=(VLM1+VLIN)/2  
VAV=(VUM2+VLM2)/2
```

```

* -----
* THE PRESSURE DROP ACROSS THE UPPER AND LOWER WAKES
* -----
DELE=VAV*BR(NPT1,1)*2.0

* -----
* DETERMINATION OF THE AIRFOIL PRESSURE COEFFICIENTS
* ABOUT THE AIRFOIL SURFACE.
* -----
DO 3 I=1,NPT
IF(I.EQ.NPUSS.OR.I.EQ.NPLSS)THEN
CP(I)=CP(I-1)
GO TO 5
END IF
IF(I.LT.NPUSS.OR.I.GT.NPLSS)THEN
CP(I)=1.0-(VTAN(I)**2.0+VNN(I)**2)*DELE
ELSE
CP(I)=1.0-VTAN(I)**2.0-VNN(I)**2.0
END IF
5 CONTINUE
3 CONTINUE

* -----
* PRESSURE COEFFICIENTS AT THE JUNCTION POINTS BASED
* ON THE VORTICITY VALUES AT THE JUNCTION POINTS.
* -----
IF(POSTN(9).EQ.0)THEN
DO 6 I=1,NPT
IF(I.LE.NPUS.OR.I.GE.NPLSS)THEN
CPJ(I)=1.0-BR(NPT1,1)**2.0
ELSE
CPJ(I)=1.0-BR(I,1)**2.0
END IF
6 CONTINUE
END IF
IF(POSTN(9).EQ.1)THEN
DO 7 I=1,NPT
IF(I.LE.NPUS.OR.I.GE.NPLSS)THEN
CPJ(I)=1.0-BR(NPT1,1)**2.0-QSJ(NPUS)**2.0
ELSE
CPJ(I)=1.0-BR(I,1)**2.0-QSJ(I)**2.0
END IF
7 CONTINUE
END IF

* -----
* IF COMPRESSIBILITY EFFECTS ARE TO BE TAKEN INTO
* ACCOUNT THE NEXT STATEMENT CALLS SUBROUTINE "CPCOMP".
* -----
IF(POSTN(12).EQ.1)THEN
CALL CPCOMP
ELSE
DO 4 I=1,NPT
PCJ(I)=CPJ(I)
IF(I.LT.NPT)PC(I)=CP(I)
4 CONTINUE
END IF

```

• THE NEXT SET OF STATEMENTS RE-ORGANIZE THE JUNCTION
 • POINT NUMBERING ORDER FOR USE IN SUBROUTINE "AERO".
 • -----

```

J1=1
DO 8 I=1,NPT
IF(I.EQ.NPUSS.OR.I.EQ.NPLS2)GO TO 9
IF(I.LE.NN)THEN
SL(J1)=XL(I)
PPH(J1)=PH(I)
XXI(J1)=XI(I)
YYI(J1)=YI(I)
END IF
PJ(J1)=PCJ(I)
J1=J1+1
9 CONTINUE
8 CONTINUE

```

RETURN

END

•
 • SUBROUTINE CPCOMP
 •

..... A-15

• This subroutine takes into account the effects of
 • compressibility using the "Karmen-Tsien" pressure cor-
 • rection formula. A new Cp distribution is determined.
 •
 •

IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

```

COMMON A(90,90),BB(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
• PH(90),XL(90),VNN(90),VTAN(90),CP(90),QSJ(90),POSTN(20),
• XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
• CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
• N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,N,PI,CHORD,NFN,
• NPT1,NPT2,NAM1,NAM2,NAM3,CLGAM,NPUSS,NPUSS2,NPUS,WLF,CPS,
• NPLS,NPLS2,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

```

```

INTEGER POSTN
REAL NAM1,NAM2,NAM3

```

```

CONST1=SQRT(1-AMACH**2)
CONST2=1+CONST1
CONST3=(AMACH**2)/CONST2

```

```

DO 1 I=1,NPT
PCJ(I)=CPJ(I)/(CONST1+CONST3*CPJ(I)/2)
IF(I.LT.NPT)PC(I)=CP(I)/(CONST1+CONST3*CP(I)/2)
1 CONTINUE

```

RETURN

END

```

* .....
* SUBROUTINE BOUND1
* .....

```

```

* ..... A-16 .....
*
* This subroutine implements the auxillary conditions
* for the separated wake model. The vorticity distribution
* is constrained to start and finish with zero vorticity in
* the separated region. The vorticity value at the upper
* separation point is equal but opposite to the trailing
* edge value. The upper wake surface have the same vorticity
* icity value as the upper separation point. The lower wake
* surface has the same vorticity value as the trailing edge
* value.
*
* .....

```

```

IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

```

```

COMMON A(90,90),BB(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
* PH(90),XL(90),VNN(90),VTAN(90),CP(90),QSJ(90),POSTN(20),
* XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
* CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
* N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
* NPT1,NPT2,NAM1,NAM2,NAM3,CLCAN,NPUSS,NPUS2,NPUS,WLF,CPE,
* NPLS,NPLS2,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

```

```

INTEGER POSTN
REAL NAM1,NAM2,NAM3

```

```

* -----
* THE NEXT STATEMENTS IMPLIMENT ALL THE AUXILLARY CONDITIONS
* -----

```

```

DO 3 I=1,NPT2
A(NA,I)=0.0
A(I,1)=0.0
A(NPUSS,I)=0.0
A(I,NPUSS)=0.0
A(NPT,I)=0.0
A(NPT1,I)=0.0
A(NPT2,I)=0.0
3 CONTINUE

```

```

A(NA,1)=1.0
A(NPUSS,NPUSS)=1.0
A(NPT,NPUS)=1.0
A(NPT,NPT)=1.0
A(NPT1,NPUS)=-1.0
A(NPT1,NPT1)=1.0
A(NPT2,NPT2)=1.0
A(NPT2,NPT)=-1.0

```

```

* -----
* SUBROUTINE "RHS" IS CALLED TO DETERMINE THE RIGHT
* HAND SIDE OF THE MATRIX.
* -----

```

```
IF(JI.EQ.1)CALL RES
```

```
RETURN
```

```
END
```

```
*****  
SUBROUTINE BOUND2  
*****
```

```
***** A-17 *****  
*  
* This subroutine implements the auxiliary conditions  
* for the separated wake model, assuming that the separat-  
* ion point is located on the under "pressure" surface. The  
* vorticity distribution is constrained to start and finish  
* with zero vorticity in the separated region.  
* The vorticity value at the upper airfoil surface  
* separation point is equal but opposite to the lower sur-  
* face value. The upper wake surface have the same vorticity  
* icity value as the upper separation point. The lower wake  
* surface has the same vorticity value as the lower surface  
* value.  
*  
*****
```

```
IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)
```

```
COMMON A(90,90),BR(90,1),BB(90,1),AN(90,90),XI(90),YI(90),  
* PH(90),XL(90),VNH(90),VTAN(90),CP(90),QSJ(90),POSTN(20),  
* XX(4),XKC(2),I(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),  
* CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),  
* N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,  
* NPT1,NPT2,NAM1,NAM2,NAMS,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPS,  
* NPLS,NPLS2,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1
```

```
INTEGER POSTN  
REAL NAM1,NAM2,NAM3
```

```
-----  
* THE NEXT SET OF STATEMENTS IMPLMENTS ALL THE  
* BOUNDARY AND AUXILIARY CONDITIONS.  
*-----
```

```
DO 1 I=1,NPT2  
A(NPLS,I)=0.0  
A(I,NPLS)=0.0  
A(I,NPUSS)=0.0  
A(NPUSS,I)=0.0  
A(NPT,I)=0.0  
A(NPT1,I)=0.0  
A(NPT2,I)=0.0  
1 CONTINUE  
  
A(NPLS,NPLS)=1.0  
A(NPUSS,NPUSS)=1.0  
A(NPT,NPUS)=1.0  
A(NPT,NPLS)=1.0  
A(NPT1,NPUS)=-1.0  
A(NPT1,NPT1)=1.0
```

```

      A(NPT2,NPLSS)=-1.0
      A(NPT2,NPT2)=1.0
* -----
*   SUBROUTINE "RES" IS CALLED TO DETERMINE THE RIGHT
*   HAND SIDE OF THE MATRIX.
* -----
      IF(J1.EQ.1)CALL RES

      RETURN

      END

*   *****
*   SUBROUTINE RES
*   *****

*   ***** A-18 *****
*   This subroutine calculates the right hand side of
*   the matrix for either the pure potential flow model or the
*   model with the boundary layer source distribution included.
*   *****

      IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

      COMMON A(90,90),BR(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
*   PH(90),XL(90),VNN(90),VTAN(90),CP(90),QBJ(90),POSTN(20),
*   XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
*   CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
*   N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
*   NPT1,NPT2,NAM1,NAM2,NAMS,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPS,
*   NPLS,NPLSS,NPLS2,ALPHA,NPU,NPL,RELAX,J1,NA,AMACH,CRT1

      INTEGER POSTN
      REAL NAM1,NAM2,NAMS

* -----
*   COMPUTATION OF THE RIGHT HAND SIDE OF THE MATRIX
* -----
      IF(POSTN(9).EQ.0)THEN

* -----
*   THE RIGHT HAND SIDE OF THE MATRIX IS NOW FORMED FOR THE
*   PURE INVISCID SEPARATION MODEL.
* -----
        DO 4 I=1,NN
          IF(I.EQ.NPUSS.OR.I.EQ.NPLSS)GO TO 5
          BR(I,1)=SIN(PH(I)-ALPHA)
        5 CONTINUE
        4 CONTINUE
        ELSE /

* -----
*   IF BOUNDARY LAYER EFFECTS ARE TO BE INCLUDED IN THE
*   SOLUTION CONTROL IS TRANSFERRED TO THE NEXT STATEMENTS.
*   THE SOURCE DISTRIBUTION WHICH ACCOUNTS FOR THE BOUND-
*   ARY LAYER EFFECT ARE ADDED TO THE RIGHT HAND SIDE.

```

```

* -----
DO 11 I=1,NM
IF (I.EQ.NPUSS.OR.I.EQ.NPLSS)GO TO 6
QC=(QSJ(I)+QSJ(I+1))/2
RS=-1*(QC*SIN(ALPHA-PH(I)))
DO 12 M=1,NM
IF (M.EQ.NPUSS.OR.M.EQ.NPLSS)GO TO 7
CALL XRS
VQ=(-1)*(XK(1)*QSJ(M)+XK(2)*QSJ(M+1))
UQ= XK(3)*QSJ(M)+XK(4)*QSJ(M+1)
RS=RS-(VQ*COS(PH(M)-PH(I))+UQ*SIN(PH(M)-PH(I)))
7 CONTINUE
12 CONTINUE
BR(I,1)=RS
6 CONTINUE
11 CONTINUE
END IF

* -----
* IF THE SEPARATION POINT IS AT THE TRAILING EDGE THE
* NEXT SET OF STATEMENTS ARE USED ON THE R.H.S.
* -----
IF (POSTN(8).EQ.1) THEN
BR(NA,1)=0.0
BR(NPUSS,1)=0.0
BR(NPT,1)=0.0
BR(NPT1,1)=0.0
BR(NPT2,1)=0.0

ELSE

* -----
* THE NEXT SET OF STATEMENTS MODIFY THE R.H.S IF
* THE LOWER SURFACE SEPARATION POINT IS NOT AT THE
* TRAILING EDGE.
* -----
BR(NPUSS,1)=0.0
BR(NPLSS,1)=0.0
BR(NPT,1)=0.0
BR(NPT1,1)=0.0
BR(NPT2,1)=0.0
END IF

RETURN
END

* *****
* SUBROUTINE TRANSX
* *****

***** A-19 *****
*
* This subroutine translates all the x co-ordinates such
* that the leading edge x co-ordinate is at X=0.0. In add-
* ition all co-ordinates are non-dimensionalized using the
* chord length.
*

```



```

.....
      IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

      COMMON A(90,90),BB(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
      *      PH(90),XL(90),VHN(90),VTAN(90),CP(90),QBJ(90),POSTN(20),
      *      XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPE(90),
      *      CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
      *      N,NH,NPT,NUS,NUP,NUPF,NLS,NLF,NLFF,I,M,PI,CHORD,NFM,
      *      NPT1,NPT2,NAM1,NAM2,NAM3,CLGAM,NPUS,NPUS2,NPUS,WLF,CPS,
      *      NPLS,NPLS2,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

      INTEGER POSTN
      REAL NAM1,NAM2,NAM3

      *
      * -----
      * DETERMINE THE CHORD LENGTH OF THE AIRFOIL AND THE
      * X CO-ORDINATE OF THE LEADING EDGE POINT.
      * -----
      DO 1 I=2,NPT
      IF (X(I).LE.X(I-1))THEN
      CONTINUE
      ELSE
      XLE=X(I-1)
      GO TO 2
      END IF
1 CONTINUE
2 CONTINUE

      *
      * -----
      * TRANSLATE ALL X CO-ORDINATES SO THAT THE LEADING
      * EDGE IS AT X=0.0, ALSO ALL CO-ORDINATES ARE NON-
      * DIMENSIONALIZED USING THE CHORD LENGTH.
      * -----
      IF (XLE.LT.0.0)XLE=(-1)*XLE
      IF (POSTN(1).EQ.0)NQ=NPT
      IF (POSTN(1).EQ.1)NQ=NFM
      DO 3 I=1,NQ
      X(I)=X(I)+XLE
      X(I)=X(I)/CHORD
      Y(I)=Y(I)/CHORD
3 CONTINUE
      CHORD=CHORD/CHORD

      RETURN

      END

      *
      * .....
      * SUBROUTINE XKS
      * .....

..... A-20 .....
      *
      * This subroutine calculates XK(1) to XK(4) which
      * represent the engineering variables A,B,C, and D.
      * The influence coefficients consist of simple
      * combinations of these coefficients.
      *

```

.....

```
IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)
```

```
COMMON A(90,90),BR(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
* PH(90),XL(90),VNH(90),VTAN(90),CP(90),QBJ(90),POSTN(20),
* XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
* CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
* N,NM,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
* NPT1,NPT2,NAM1,NAM2,NAM3,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPB,
* NPLS,NPLS8,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1
```

```
INTEGER POSTN
REAL NAM1,NAM2,NAM3
```

```
-----
RADIUS TO CONTROL POINT FROM ONE END OF PANEL
-----
```

```
XX = XI(I)-X(M)
YY = YI(I)-Y(M)
RA = SQRT (XX*XX+YY*YY)
```

```
-----
RADIUS TO CONTROL POINT FROM OTHER END OF PANEL
-----
```

```
XX1= XI(I)-X(M+1)
YY1= YI(I)-Y(M+1)
RB = SQRT (XX1*XX1+YY1*YY1)
```

```
-----
ANGLE BETWEEN PANEL AND RADII TO CONTROL POINT
-----
```

```
THA = ATAN2(YY,XX)-PH(M)
THB = ATAN2(YY1,XX1)-PH(M)
```

```
-----
THE NEXT STATEMENT CAUSES THE CALCULATION ON THE
OUTSIDE OF THE CONTOUR.
-----
```

```
IF (ABS(ABS(THA-THB)-PI) .LE. 0.00001) THA =2.0*PI;
THB =-PI
```

```
-----
THE NEXT TWO STATEMENTS ENSURE THAT "THA" AND "THB"
ALWAYS HAVE THE SAME SIGN SO THAT THEIR DIFFERENCE
IS ALWAYS LESS THAN PI.
-----
```

```
IF (THA .LT. 0.0 .AND. THB .GT. 0.0) THB = THB-2.*PI
IF (THA .GT. 0.0 .AND. THB .LT. 0.0) THA = THA-2.*PI
```

```
C1 = RA*SIN(THA)/(2.*PI*XL(M))
IF (ABS(SIN(THA)) .LE. 0.001 .OR. ABS(SIN(THB)) .LE. 0.001)
* CS = 0.0 ; GO TO 1
C5 = C1*ALOG(SIN(THB)/SIN(THA))
1 C4 = C1*(THB-THA)
C6 = RA*COS(THA)/XL(M)
C2 = 1.0-C6
C3 = C6/(2.0*PI)
```

C7 = ALOG(RA/RB)

XX(1) = C2*(THA-THB)/(2.0*PI)-C5

XX(2) = C3*(THA-THB)+C5

XX(3) = C7*C2/(2.0*PI)-C4+1.0/(2.0*PI)

XX(4) = C3*C7 +C4-1.0/(2.0*PI)

RETURN

END

```
*
* *****
* SUBROUTINE XCONS
* *****
*
```

```
* ***** A-21 *****
*
* This subroutine calculates XKC(1) and XKC(2)
* which represent the engineering variables A and B
* when considering simple constant strength vortex
* sheets.
* *****
```

IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

```
COMMON A(90,90),BR(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
* PH(90),XL(90),VNN(90),VTAN(90),CP(90),QSJ(90),POSTN(20),
* XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
* CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
* N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
* NPT1,NPT2,NAM1,NAM2,NAM3,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPB,
* NPLS,NPLSS,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1
```

```
INTEGER POSTN
REAL NAM1,NAM2,NAM3
```

```
*
* -----
* RADIUS TO CONTROL POINT FROM ONE END OF PANEL
* -----
```

```
XX = XI(I)-X(M)
YY = YI(I)-Y(M)
RA = SQRT (XX*XX+YY*YY)
```

```
*
* -----
* RADIUS TO CONTROL POINT FROM OTHER END OF PANEL
* -----
```

```
XX1= XI(I)-X(M+1)
YY1= YI(I)-Y(M+1)
RB = SQRT (XX1*XX1+YY1*YY1)
```

```
*
* -----
* ANGLES BETWEEN PANEL AND RADII TO CONTROL POINT
* -----
```

```
THA = ATAN2(YY,XX)-PH(M)
THB = ATAN2(YY1,XX1)-PH(M)
```

```
*
* -----
* THE NEXT STATEMENTS CAUSE THE CALCULATION ON THE
```

```

*   OUTSIDE OF THE UPPER AND LOWER WAKE CONTOURS.
*   -----
*   IF(M.LE.NUF)THEN
*   IF (ABS(ABS(THA-THB)-PI) .LE. 0.00001) THA=0.0;
*   THB=PI
*   ELSE
*   IF (ABS(ABS(THA-THB)-PI) .LE. 0.00001) THA=2.0*PI;
*   THB=PI
*   END IF
*
*   -----
*   THE NEXT TWO STATEMENTS ENSURE THAT "THA" AND "THB"
*   ALWAYS HAVE THE SAME SIGN SO THAT THEIR DIFFERENCE
*   IS ALWAYS LESS THAN PI.
*   -----
*   IF (THA .LT. 0.0 .AND. THB .GT. 0.0) THB = THB-2.*PI
*   IF (THA .GT. 0.0 .AND. THB .LT. 0.0) THA = THA-2.*PI
*
*   XKC(1)=(THA-THB)*1/(2*PI)
*   XKC(2)=ALOG(RA/RB)*1/(2*PI)
*
*   RETURN
*
*   END

```

```

*   *****
*   SUBROUTINE GEDMAIR
*   *****

```

```

***** A-22 *****
*
*   This subroutine calculates the co-ordinates of
*   the control points, the angles PH(I), and the
*   length of each panel on the airfoil surface.
*
*****

```

```

IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

```

```

COMMON A(90,90),BR(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
*   PH(90),XL(90),VNN(90),VTAN(90),CP(90),QSJ(90),POSTN(20),
*   XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
*   CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
*   N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
*   NPT1,NPT2,NAM1,NAM2,NAM3,CLGAN,NPUSS,NPUS2,NPUS,WLF,CPB,
*   NPLS,NPLSS,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

```

```

INTEGER POSTN
REAL NAM1,NAM2,NAM3

```

```

*   -----
*   THE FOLLOWING STATEMENTS CALCULATE CO-ORDINATES OF THE
*   CONTROL POINTS, THE ANGLES PH(I), AND THE LENGTH OF
*   EACH PANEL ON THE AIRFOIL SURFACE.
*   -----
*   IF(POSTN(1).EQ.0)NQ=N
*   IF(POSTN(1).EQ.1)NQ=NN
*   DO 10 I=1,NQ

```

```

      IF(I.EQ.NPUSS.OR.I.EQ.NPLSS) GO TO 66
      XI(I)=0.5*(X(I)+X(I+1))
      YI(I)=0.5*(Y(I)+Y(I+1))
      XX=X(I+1)-X(I)
      YY=Y(I+1)-Y(I)
      PH(I)=ATAN2(YY,XX)
      XL(I)=SQRT(XX**2+YY**2)
66  CONTINUE
10  CONTINUE
      XI(NPUSS)=XI(NPUS2)
      YI(NPUSS)=YI(NPUS2)
      PH(NPUSS)=PH(NPUS2)
      XL(NPUSS)=XL(NPUS2)
      XI(NPLSS)=XI(NPLS2)
      YI(NPLSS)=YI(NPLS2)
      PH(NPLSS)=PH(NPLS2)
      XL(NPLSS)=XL(NPLS2)

```

```

      RETURN

```

```

      END

```

```

*      *****
*      SUBROUTINE GEONWAK
*      *****

```

```

***** A-23 *****
*
*      This subroutine calculates the co-ordinates of the
*      upper and lower wake control points, the angles PH(I),
*      and the length of each panel used to model the wake.
*
*****

```

```

      IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

```

```

      COMMON A(90,90),BR(90,1),BB(90,4),AN(90,90),XI(90),YI(90),
*      PH(90),XL(90),VNN(90),VWAN(90),CP(90),QSI(90),POSTN(20),
*      XL(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
*      CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
*      N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
*      NPT1,NPT2,NAM1,NAM2,NAM3,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPE,
*      NPLS,NPLSS,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

```

```

      INTEGER POSTN
      REAL NAM1,NAM2,NAM3

```

```

*      -----
*      THE FOLLOWING STATEMENTS CALCULATE CO-ORDINATES OF THE
*      CONTROL POINTS, THE ANGLES PH(I), AND THE LENGTH OF
*      EACH PANEL ON THE UPPER WAKE BOUNDARY.
*      -----

```

```

      DO 12 I=NUS,NUFF
      XI(I)=0.5*(X(I)+X(I+1))
      YI(I)=0.5*(Y(I)+Y(I+1))
      XX=X(I+1)-X(I)
      YY=Y(I+1)-Y(I)
      PH(I)=ATAN2(YY,XX)
      XL(I)=SQRT(XX**2+YY**2)

```

```

12 CONTINUE
  XI(NUF)=0.0
  YI(NUF)=0.0
  PH(NUF)=0.0
  XL(NUF)=0.0

```

```

* -----
* THE FOLLOWING STATEMENTS CALCULATE CO-ORDINATES OF THE
* CONTROL POINTS, THE ANGLES PH(I), AND THE LENGTH OF
* EACH PANEL ON THE LOWER WAKE BOUNDARY.
* -----

```

```

DO 13 I=NLS,NLFF
  XI(I)=0.5*(X(I+1)+X(I))
  YI(I)=0.5*(Y(I+1)+Y(I))
  XX=X(I+1)-X(I)
  YY=Y(I+1)-Y(I)
  PH(I)=ATAN2(YY,XX)
  XL(I)=SQRT(XX**2+YY**2)
13 CONTINUE

```

```

  RETURN

```

```

  END

```

```

* *****
* SUBROUTINE OUTPUT
* *****

```

```

***** A-24 *****
*
* This subroutine controls all the outputs of the
* program.
*
*****

```

```

IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

```

```

COMMON A(90,90),BB(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
* PH(90),XL(90),VNN(90),VTAN(90),CP(90),QSJ(90),POSTN(20),
* XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
* CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
* N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
* NPT1,NPT2,NAM1,NAM2,NAM3,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPB,
* NPLS,NPLSS,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

```

```

INTEGER POSTN
REAL NAM1,NAM2,NAM3
JJI=JI-1
IF (POSTN(1).EQ.1) THEN
  NC=N+1
  NPT3=NPT+2
ELSE
  NC=N
  NPT3=NPT
END IF

```

```

ALPHAA=ALPHA*180/PI

```

```

WRITE(6,1)

```

```

1  FORMAT(//,20X,'RESULTS FROM AIRFOIL ANALYSIS')
   WRITE (6,2)
2  FORMAT(20X,'-----')

   WRITE(6,3)
3  FORMAT(//,'VARIOUS PARAMETERS')
   WRITE(6,4)
4  FORMAT('-----')

   WRITE(6,5)NAM1,NAM2,NAM3
5  FORMAT('/[1] THE AIRFOIL SECTION NAME IS:',3A4)
   WRITE(6,6)CHORD
6  FORMAT('/[2] THE CHORD LENGTH OF THE AIRFOIL IS:',F8.4)
   WRITE(6,7)ALPHA
7  FORMAT('/[3] THE AIRFOIL ANGLE OF ATTACK (DEG) IS:',F8.4)
   WRITE(6,11)CLGAM
11 FORMAT('/[4] THE LIFT COEFFICIENT BASED ON CIRCULATION:',F8.4)
   WRITE(6,8)N
8  FORMAT('/[5] THE NUM. OF PANELS USED TO MODEL THE AIRFOIL:',I2)
   IF(POSTN(1).EQ.0.AND.POSTN(12).EQ.1)THEN
   WRITE(6,60)AMACH
60 FORMAT('/[6] THE FREE STREAM MACH NUMBER IS:',F6.4)
   END IF
   IF(POSTN(1).EQ.1)THEN
   WRITE(6,9)NPU
9  FORMAT('/[6] THE # OF PANELS USED TO MODEL THE UPPER WAKE:',I2)
   WRITE(6,10)NPL
10 FORMAT('/[7] THE # OF PANELS USED TO MODEL THE LOWER WAKE:',I2)
   WRITE(6,35)X(NUS),Y(NUS)

35 FORMAT('/[8] THE AIRFOIL UPPER SURFACE SEP. PT. (X,Y):',2F8.4)
   WRITE(6,36)X(NLS),Y(NLS)
36 FORMAT('/[9] THE AIRFOIL LOWER SURFACE SEP. PT. (X,Y):',2F8.4)
   WRITE(6,38)POSTN(2)
38 FORMAT('/[10] THE TOT.# OF WAKE SHAPE ITER. TO BE PERFORMED:',I2)
   WRITE(6,37)RELAX
37 FORMAT('/[11] THE RELAXATION FACTOR USED FOR THE WAKE IT:',F4.2)
   WRITE(6,39)JJ1
39 FORMAT('/[12] THE NUMBER OF ITERATIONS PERFORMED:',I2)
   WRITE(6,67)WLF
67 FORMAT('/[13] THE WAKE LENGTH FACTOR IS:',F4.2)
   WRITE(6,68)CRT1
68 FORMAT('/[14] THE OVERALL SOLUTION CRITERIA:',F6.4)
   IF(POSTN(12).EQ.1)THEN
   WRITE(6,43)AMACH
43 FORMAT('/[15] THE FREE STREAM MACH NUMBER IS:',F6.4)
   END IF
   END IF

* -----
* THE NEXT STATEMENT CALLS THE SUBROUTINE "TABLE" WHICH
* PRODUCES A TABLE CONTAINING THE LIFT,PRESSURE DRAG
* AND PITCHING MOMENT (L.E.) AS DETERMINED BY THE VAR-
* IOUS INTEGRATION METHODS.
* -----
CALL TABLE

IF(POSTN(6).EQ.0)GO TO 50
IF(POSTN(12).EQ.0.AND.POSTN(9).EQ.0)THEN

```

```

WRITE(6,12)
WRITE(6,53)
WRITE(6,13)
WRITE(6,53)
12 FORMAT(/, 'JUNCTION POINT PARAMETERS: CO-ORDINATES X,Y ',/,
  *'VORTEX STRENGTHS, PRESSURE COEFFICIENTS')
13 FORMAT(/, ' I X/C Y/C GAM CP')
END IF
IF(POSTN(9).EQ.1.AND.POSTN(12).EQ.0)THEN
WRITE(6,44)
WRITE(6,58)
WRITE(6,45)
WRITE(6,58)
44 FORMAT(/, 'JUNCTION POINT PARAMETERS: CO-ORDINATES X,Y ',/,
  *'SOURCE AND VORTEX STRENGTHS, PRESSURE COEFFICIENTS')
45 FORMAT(/, ' I X/C Y/C SOR GAM
1 CP(IN)')
END IF
IF(POSTN(9).EQ.0.AND.POSTN(12).EQ.1)THEN
WRITE(6,46)
WRITE(6,58)
WRITE(6,47)
WRITE(6,58)
46 FORMAT(/, 'JUNCTION POINT PARAMETERS: CO-ORDINATES X,Y ',/,
  *'VORTEX STRENGTHS, Cp (INCOMPRESSIBLE), Cp (COMPRESSIBLE)')
47 FORMAT(/, ' I X/C Y/C GAM CP(IN)
1 CP(COM)')
END IF
IF(POSTN(9).EQ.1.AND.POSTN(12).EQ.1)THEN
WRITE(6,48)
WRITE(6,54)
WRITE(6,49)
WRITE(6,54)
48 FORMAT(/, 'JUNCTION POINT PARAMETERS: CO-ORDINATES X,Y ',/,
  *'SOURCE AND VORTEX STRENGTHS, Cp (INCOMP), Cp (COMPRES)')
49 FORMAT(/, ' I X/C Y/C SOR GAM
1 CP(IN) CP(COM)')
END IF
DO 14 I=1,NPT
IF(POSTN(9).EQ.0.AND.POSTN(12).EQ.0)THEN
WRITE(6,15)I,X(I),Y(I),BR(I,1),CPJ(I)
15 FORMAT(1X,12,7X,F9.5,3X,F9.5,3X,F9.5,3X,F9.5)
IF(I.EQ.NPT)WRITE(6,53)
END IF
IF(POSTN(9).EQ.1.AND.POSTN(12).EQ.0)THEN
WRITE(6,55)I,X(I),Y(I),QSJ(I),BR(I,1),CPJ(I)
55 FORMAT(1X,12,7X,F9.5,3X,F9.5,3X,F9.5,3X,F9.5,3X,F9.5)
IF(I.EQ.NPT)WRITE(6,58)
END IF
IF(POSTN(9).EQ.0.AND.POSTN(12).EQ.1)THEN
WRITE(6,56)I,X(I),Y(I),BR(I,1),CPJ(I),PCJ(I)
56 FORMAT(1X,12,7X,F9.5,3X,F9.5,3X,F9.5,3X,F9.5,3X,F9.5)
IF(I.EQ.NPT)WRITE(6,58)
END IF
IF(POSTN(9).EQ.1.AND.POSTN(12).EQ.1)THEN
WRITE(6,57)I,X(I),Y(I),QSJ(I),BR(I,1),CPJ(I),PCJ(I)
57 FORMAT(1X,12,7X,F9.5,3X,F9.5,3X,F9.5,3X,F9.5,3X,F9.5,3X,F9.5)
IF(I.EQ.NPT)WRITE(6,54)

```



```

      END IF
14  CONTINUE
50  CONTINUE
      IF(POSTN(10).EQ.0)GO TO 51
      WRITE(6,31)
      WRITE(6,61)
      IF(POSTN(12).EQ.0)WRITE(6,32)
      IF(POSTN(12).EQ.1)WRITE(6,69)
      WRITE(6,61)
31  FORMAT(/,'CONTROL POINT CO-ORDINATES, X,Y AND PRESSURE',/,
      *'COEFFICIENTS BASED ON TANGENTIAL VELOCITY AT THE CON. PTS. ')
32  FORMAT(/,' I      X/C      Y/C      VN      VT
      *CP(IN)      XL')
59  FORMAT(/,' I      X/C      Y/C      VN      VT
      *CP(COM)      XL')
      DO 34 I=1,NC
      WRITE(6,33)I,XI(I),YI(I),VNN(I),VTAN(I),PC(I),XL(I)
33  FORMAT(1X,I2,1X,F9.5,2X,F9.5,2X,E12.5,3X,F9.5,3X,F9.5,2X,F8.6)
      IF(I.EQ.NC)WRITE(6,61)
34  CONTINUE
51  CONTINUE
      IF(POSTN(1).EQ.1)THEN
      IF(POSTN(11).EQ.0)GO TO 52
      WRITE(6,16)
      WRITE(6,63)
      WRITE(6,17)
      WRITE(6,63)
16  FORMAT(/,'UPPER WAKE CONTROL POINTS, X AND Y; NORMAL AND ',/,
      *'EXTERNAL TANGENTIAL VELOCITIES AND PANEL LENGTHS. ')
17  FORMAT(/,' I      X/C      Y/C      VN      VT
      *XL')
      DO 18 I=NUS,NUFF
      WRITE(6,19)I,XI(I),YI(I),VNN(I),VTAN(I),XL(I)
19  FORMAT(1X,I2,1X,F9.5,2X,F9.5,2X,E12.5,2X,F9.5,2X,F9.5)
18  CONTINUE
      WRITE(6,66)
      WRITE(6,20)
      WRITE(6,63)
      WRITE(6,21)
      WRITE(6,63)
20  FORMAT(/,'LOWER WAKE CONTROL POINTS, X AND Y; NORMAL AND ',/,
      *'EXTERNAL TANGENTIAL VELOCITIES AND PANEL LENGTHS. ')
21  FORMAT(/,' I      X/C      Y/C      VN      VT
      *XL')
      DO 22 I=NLS,NLFF
      WRITE(6,23)I,XI(I),YI(I),VNN(I),VTAN(I),XL(I)
23  FORMAT(1X,I2,1X,F9.5,2X,F9.5,2X,E12.5,2X,F9.5,2X,F9.5)
22  CONTINUE
      WRITE(6,66)
      WRITE(6,30)
      WRITE(6,64)
      WRITE(6,42)
      WRITE(6,64)
30  FORMAT(/,'UPPER WAKE JUNCTION POINTS, (X,Y)      LOWER WAKE JUNCTION
      * POINTS, (X,Y)')
42  FORMAT(/,' I      X/C      Y/C      I      X/C
      *      Y/C')
      DO 40 I=NUS,NUF
      II=I+NPU+1

```

```

      IF(NPU.EQ.NPL)WRITE(6,41)I,X(I),Y(I),II,X(II),Y(II)
      IF(NPU.NE.NPL)THEN
      IF(II.LE.NLF)WRITE(6,41)I,X(I),Y(I),II,X(II),Y(II)
      IF(II.GT.NLF)WRITE(6,62)I,X(I),Y(I)
      END IF
41  FORMAT(3X,12,5X,F9.5,3X,F9.5,9X,12,5X,F9.5,3X,F9.5)
62  FORMAT(3X,12,5X,F9.5,3X,F9.5)
40  CONTINUE
      WRITE(6,66)
63  CONTINUE
      END IF

```

```

      IF(POSTN(5).EQ.1)THEN
      WRITE(6,28)
      WRITE(6,29),((AN(I,M),M=1,NPTS),I=1,NPTS),BB(I,1)
28  FORMAT(/,'THE MATRIX TO BE INVERTED, WITH R.N.S. ')
29  FORMAT(/,10E12.4,F9.5)

```

```

      END IF

```

```

*-----
*   IF DESIRED, THE NEXT STATEMENTS GENERATE AN OUTPUT FILE
*   OF THE AIRFOIL AND "CONVERGED" WAKE GEOMETRY FOR USE IN
*   IN THIS PROGRAM IN THE PROPER FORMAT. USE A "SET 8" FORMAT
*-----

```

```

      IF(POSTN(1).EQ.1)THEN
      IF(POSTN(13).EQ.1)THEN
      WRITE(6,80)N,CHORD,WLF
80  FORMAT(12,2X,F4.2,2X,F4.2)
      WRITE(6,81)NPU,NPL
81  FORMAT(12,2X,12)
      WRITE(6,82),(X(I),Y(I),I=1,NFN)
82  FORMAT(F8.5,2X,F8.5)
      END IF
      END IF

```

```

63  FORMAT(/,'-----
*--')
64  FORMAT(/,'-----
1-----')
68  FORMAT(/,'-----
*-----')
61  FORMAT(/,'-----
*-----')
63  FORMAT(/,'-----
*-----')
66  FORMAT('-----
*-----')
64  FORMAT(/,'-----
*-----')
66  FORMAT('-----
*-----')

```

```

      RETURN

```

```

      END

```

```

*-----
*   SUBROUTINE TABLE

```

```

* .....
* ..... A-25 .....
* This subroutine generates a table of various aerodynamic parameters.
* .....
*
      IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

      COMMON A(90,90),BB(90,1),BB(90,1),AN(90,90),XI(90),YI(90),
*          PH(90),XL(90),VNN(90),VTAN(90),CP(90),QBJ(90),POSTN(20),
*          XK(4),XKC(2),I(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
*          CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
*          N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
*          NPT1,NPT2,NAM1,NAM2,NAM3,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPB,
*          NPLS,NPLSS,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

      INTEGER POSTN
      REAL NAM1,NAM2,NAM3

      WRITE(6,1)
1  FORMAT(/'+-----+
1'-----+')
      WRITE(6,2)
2  FORMAT('|                                     VARIOUS AERODYNAMIC PARAMETERS
1'-----+')
      WRITE(6,10)
10 FORMAT(''+-----+-----+-----+-----+
1'-----+')
      WRITE(6,3)
3  FORMAT('|Type of Int. |      Cl      | Cd (pres) | Cm (l.e.) | Cm'
1' (c/4) |')
      WRITE(6,8)
8  FORMAT(''+-----+-----+-----+-----+
1'-----+')
      WRITE(6,4)
4  FORMAT('|                                     |                                     |
1'-----+')
      WRITE(6,5)CL(1),CD(1),CM1(1),CM2(1)
5  FORMAT('| 3 Con. Pts. |' 2X,F6.4,3X,'|',3X,F6.4,2X,'|',3X,
1F6.4,3X,'|',3X,F6.4,2X,'|')
      WRITE(6,4)
      WRITE(6,8)
      WRITE(6,4)
      WRITE(6,6)CL(2),CD(2),CM1(2),CM2(2)
6  FORMAT('| 2 Jun. Pts. |' 2X,F6.4,3X,'|',3X,F6.4,2X,'|',3X,
1F6.4,3X,'|',3X,F6.4,2X,'|')
      WRITE(6,4)
      WRITE(6,8)
      WRITE(6,4)
      WRITE(6,7)CL(3),CD(3),CM1(3),CM2(3)
7  FORMAT('| 3 Jun. Pts. |' 2X,F6.4,3X,'|',3X,F6.4,2X,'|',3X,
1F6.4,3X,'|',3X,F6.4,2X,'|')
      WRITE(6,4)
      WRITE(6,8)

```

RETURN

END

*
* SUBROUTINE AUX
*

* A-26
*
* This subroutine controls the output of the velocities
* U/Uref and their corresponding X positions (starting at
* the stagnation point for both the upper and lower airfoil
* surfaces) to a separate output file. This file is to be
* used by the program BOUNDARY_LAYER.
*
*

IMPLICIT INTEGER (I-N), REAL (A-H,O-Z)

DIMENSION VUS(90),VLS(90),SUS(90),SLS(90),XCU(35),XCL(35)

COMMON A(90,90),BR(90,1),BS(90,1),AN(90,90),XI(90),YI(90),
* PH(90),XL(90),VNN(90),VTAN(90),CP(90),QSJ(90),POSTN(20),
* XK(4),XKC(2),X(90),Y(90),CPJ(90),PJ(90),SL(90),PPH(90),
* CL(3),CD(3),CM1(3),CM2(3),PC(90),PCJ(90),XXI(90),YYI(90),
* N,NN,NPT,NUS,NUF,NUFF,NLS,NLF,NLFF,I,M,PI,CHORD,NFN,
* NPT1,NPT2,NAM1,NAM2,NAM3,CLGAM,NPUSS,NPUS2,NPUS,WLF,CPB,
* NPLS,NPLS2,NPLS2,ALPHA,NPU,NPL,RELAX,JI,NA,AMACH,CRT1

INTEGER POSTN

REAL NAM1,NAM2,NAM3

* -----
* DETERMINATION OF THE AIRFOIL STAGNATION POINT.
* -----

CPMAX=0.0
NQ=NPT-1
DO 1 I=2,NQ
IF(PCJ(I).GE.CPMAX)THEN
CPMAX=PCJ(I)
NSTAG=I
END IF
1 CONTINUE
NUJ=NSTAG
ALPHA=(ALPHA+180)/PI

* -----
* DETERMINATION OF THE TOTAL DISTANCE ON THE UPPER
* AIRFOIL SURFACE FROM THE STAGNATION POINT.
* -----

DUST=0.0
DO 2 I=1,NSTAG-1
IF(I.EQ.NPUSS.OR.I.EQ.NPLS2)GO TO 3
DUST=DUST+XL(I)
3 CONTINUE
2 CONTINUE

* -----

* DETERMINATION OF THE UPPER SURFACE POINT VELOCITIES

```

* -----
DUS=0.0
JA=0
DO 6 I=1,NUJ
IF(I.EQ.NPUSS.OR.I.EQ.NPLSS)GO TO 7
IF(POSTN(1).EQ.1)JA=JA+1
J=NUJ-JA
IF(I.EQ.1)JJA=J
IF(PCJ(I).GT.1.0)PCJ(I)=1.0
VUS(J)=SQRT(1.0-PCJ(I))
SUS(J)=(DUS-DUS)/CHORD
XCU(J)=X(I)
DUS=DUS+XL(I)
IF(POSTN(1).EQ.0)JA=JA+1
7 CONTINUE
6 CONTINUE
VUS(1)=0.00

```

* DETERMINATION OF THE LOWER SURFACE POINT VELOCITIES

```

* -----
SLD=0.0
JB=0
DO 8 I=NUJ,NPT
IF(I.EQ.NPUSS.OR.I.EQ.NPLSS)GO TO 9
JB=JB+1
IF(PCJ(I).GT.1.0)PCJ(I)=1.0
VLS(JB)=SQRT(1.0-PCJ(I))
SLS(JB)=SLD
XCL(JB)=X(I)
SLD=SLD+XL(I)
IF(I.EQ.NPT)JJB=JB
9 CONTINUE
8 CONTINUE
VLS(1)=0.00

```

* VARIOUS PARAMETERS NEEDED TO RUN THE BOUNDARY LAYER PROGRAM ARE NOW INPUTTED.

```

* -----
OUTPUT,
OUTPUT, 'Various parameters needed to run the Boundary'
OUTPUT, 'Layer program are now inputted.'
OUTPUT,
OUTPUT, 'Enter the number corresponding to the type of'
OUTPUT, 'boundary layer analysis desired.'
OUTPUT, 'Enter 0 LAMINAR ANALYSIS ONLY'
OUTPUT, 'Enter 1 TURBULENT ANALYSIS ONLY'
OUTPUT, 'Enter 2 FULL LAMINAR-TURBULENT ANALYSIS'
READ(2,10)FLAG4
OUTPUT, 'Input Nconv(1)'
READ(2,10)NCONV
OUTPUT, 'Input Theta(1)'
READ(2,10)THETA
OUTPUT, 'Input Lambda'
READ(2,10)RDEN
OUTPUT, 'Input roughness height'

```

```

      READ(2,10)RHT
      OUTPUT,'Input initial Cf guess'
      READ(2,10)CFO
      OUTPUT,'Input Reynolds number'
      READ(2,10)REYNO
      OUTPUT,'Input reference velocity Uref'
      READ(2,10)UREF
      OUTPUT,'Enter the desired step size for the'
      OUTPUT,'boundary layer analysis.'
      READ(2,10)STEPX
10  FORMAT(1G)

```

```

-----
*   THE OUTPUT FILE FOR THE BOUNDARY LAYER PROGRAM IS
*   NOW GENERATED, USING A [SET 7] OUTPUT FORMAT.
-----

```

```

      JJT=JJA+JJB
      WRITE(7,16)NAM1,NAM2,NAM3
16  FORMAT(3A4)
      WRITE(7,17)ALPHAA,FLAG4
17  FORMAT(F8.2,2X,I1)
      WRITE(7,18)CHORD,UREF,STEPX
18  FORMAT(F8.4,2X,F8.4,2X,F8.4)
      WRITE(7,11)THETA,HCONV,CFO
11  FORMAT(F8.7,2X,F8.5,2X,F8.7)
      WRITE(7,12)RDEN,RHT,REYNO
12  FORMAT(F9.3,2X,F8.6,2X,F10.0)
      WRITE(7,13)JJT,JJA,JJB
13  FORMAT(I3,2X,I2,2X,I2)
      WRITE(7,14),(XCU(I),SUB(I),VUS(I),I=1,JJA)
14  FORMAT(3(F8.5))
      WRITE(7,15),(XCL(I),SLS(I),VLS(I),I=1,JJB)
15  FORMAT(3(F8.5))

```

```

      RETURN

```

```

      END

```

APPENDIX K-1

Boundary Layer Analysis Program Listing

```

..... B-1 .....
      B.L.A.P.

      BOUNDARY LAYER ANALYSIS PROGRAM


      This program calculates the development of a two-dimensional
boundary layer over the upper and lower surface of an airfoil
or bluff body. The pressure distribution used in the analysis
(in the form of a velocity variation) is generated from the
STAMP Program. The results of the analysis (i.e. separation
points on the upper and lower surface and source strength
distribution) are then fed back into STAMP in order to account
for the effects of the boundary layer. The source distribution
method is used in order to account for the effects of the
boundary layer displacement thickness.


      The laminar boundary layer development is predicted using
the method of Thwaites.


      Transition of the boundary layer from laminar to turbulent
can be implemented in two ways.

          [1] Using the H-Rex method of predicting transition
          [2] Upper surface - peak negative Cp point of transition
              Lower surface - into linear airfoil distance to
                  point of forced transition.


      The program calculates the development of the two-dimen-
sional turbulent boundary layer over rough surfaces in pres-
sure gradients using the method of Dvorak(1969).


      The program is interactive and prompts the user for various
input parameters.
.....

*****
MAIN PROGRAM
*****

IMPLICIT INTEGER(I-N), REAL(A-H,O-Z)

```

```

COMMON UC(100),XC(100),XCART(100),U(400),X(400),THETA(400),
* DELS(400),NDELS(400),NCONV(400),DSTAR(400),REX(400),
* DELTA(400),DU(400),DX(400),CFI(400),UKGDU(400),
* SOURCE(400),FLAG(10),XTP(2),XCTP(2),CBOR(100),Q(400),
* RETH(400),SSEP(2),ISEP(2),
* NTOT,NUP,NLOW,NAM1,NAM2,NAM3,CORRD,UREF,RDEN,RNT,REYNO,
* ALPHAA,STEPX,VIS,NPU,NPL,NPTU,NPTL,NLS,NLSS,I,J,PI,
* LAMBDA,L,ITRANS,ISEPU,ISEPL,CFO,NCLS,NPT,ATHETA,ANCONV

INTEGER FLAG
REAL NAM1,NAM2,NAM3,LAMBDA,L

* -----
* INITIALIZE ALL THE OPTION INDICATORS.
* -----
DO 1 I=1,12
  FLAG(I)=0.0
1 CONTINUE

* -----
* THE NEXT STATEMENT CALLS THE SUBROUTINE "INPUT" WHICH READS
* IN ALL THE REQUIRED DATA TO RUN THE PROGRAM.
* -----
CALL INPUT

* -----
* THE NEXT STATEMENT CALLS THE SUBROUTINE "INTERX" WHICH
* DIVIDES UP THE UPPER AND LOWER SURFACE INTO SMALL INCREMENTS.
* -----
CALL INTERX

* -----
* THE NEXT STATEMENT CALLS THE SUBROUTINE "INTERVEL" WHICH
* CALCULATES THE FINELY INCREMENTED VALUES OF VELOCITY AND
* DERIVATIVE OF VELOCITY AT EACH STEP.
* -----
CALL INTERVEL

* -----
* UNTIL CHANGED, THE UPPER AND LOWER SURFACE SEPARATION POINTS
* ARE SET AT THE AIRFOIL TRAILING EDGE.
* -----
ISEPU=NPTU
ISEPL=NPT
SSEP(1)=XC(NUP)
SSEP(2)=XC(NTOT)
ISEP(1)=XCART(NUP)
ISEP(2)=XCART(NTOT)

* -----
* DEFINITION OF VARIOUS CONSTANTS(Pi & kinematic Viscosity)
* -----
VIS=1/REYNO
PI=3.141592654

* -----
* THE NEXT STATEMENT CALLS THE SUBROUTINE "OUT1" WHICH
* OUTPUTS VARIOUS PARAMETERS OF THE BOUNDARY LAYER ANALYSIS
* -----

```



```

CALL OUT1

WRITE(6,16)
WRITE(6,16)
15 FORMAT(//,32X,'BOUNDARY LAYER PARAMETERS UPPER AIRFOIL SURFACE')
16 FORMAT(32X,'-----')

IF(FLAG(4).EQ.0)FLAG(1)=0
IF(FLAG(4).EQ.1)FLAG(1)=1
IF(FLAG(4).EQ.2)FLAG(1)=0

*
* AN OVERALL DO LOOP IS SET UP WHICH CALCULATES THE DEVELOPMENT OF BOTH THE LAMINAR AND TURBULENT BOUNDARY LAYER.
* THE UPPER AIRFOIL SURFACE IS FIRST CONSIDERED AND THEN
* "NS" AND "NF" (the initial and limit values) ARE CHANGED
* SO THAT THE CALCULATIONS ARE PERFORMED ON THE LOWER AIR-
* FOIL SURFACE, IF DESIRED.
*
*-----
NS=1
NF=NPTU
3 DO 2 I=NS,NF
  II=I+1
  IF(FLAG(4).EQ.1)GO TO 6
  IF(FLAG(1).EQ.0)THEN

*****
*
* LAMINAR FLOW REGION
* (Thwaites' Method)
*
*****

    IF(I.EQ.1.OR.I.EQ.NLS)THEN
      WRITE(6,17)
      WRITE(6,18)
17 FORMAT(//,'LAMINAR FLOW REGION')
18 FORMAT('-----')
      WRITE(6,19)
      WRITE(6,20)
      WRITE(6,19)
19 FORMAT(/,'-----')
      1-----
20 FORMAT(/,' I      S/C      U/Uref      DU/DS      DELSTAR/C      THETA/
1C H/C      Rex      Reth      Cf      SON ')
      END IF

      IF(I.EQ.1)THEN
        IF(FLAG(10).EQ.0)THETA(1)=0.0
        IF(FLAG(10).EQ.1)THETA(1)=SQRT(0.075/(REYNO*DU(1)))
      END IF
      IF(I.EQ.NLS)THEN
        IF(FLAG(10).EQ.0)THETA(NLS)=0.0
        IF(FLAG(10).EQ.1)THETA(NLS)=SQRT(0.075/(REYNO*DU(NLS)))
      END IF

      LAMBDA=THETA(I)**2.0*DU(I)*REYNO

```

```

* THE NEXT SET OF STATEMENTS CHECK FOR LAMINAR SEPARATION
* -----
IF (LAMBDA.LT.-0.0842) THEN
CALL LAMSEP
GO TO 100
END IF

* -----
* THE NEXT STATEMENT CALLS SUBROUTINE "THWATS" WHICH
* DETERMINES THE VALUES OF THWAITES PARAMETERS.
* -----
CALL THWATS

RETH(I)=U(I)*THETA(I)*REYNO
DSTAR(I)=HCONV(I)*THETA(I)

IF (I.EQ.1 OR I.EQ.NLS) THEN
CFI(I)=(2.0*L)/(REYNO*THETA(I))
REX(I)=0.0
Q(I)=DSTAR(I)*DU(I)
GO TO 91
END IF
90 CONTINUE
CFI(I)=(2.0*L)/(REYNO*THETA(I)*U(I))
REX(I)=U(I)*X(I)*REYNO
Q(I)=(U(I)*DSTAR(I)-U(I-1)*DSTAR(I-1))/STEPX
91 CONTINUE
WRITE(6,23) I,X(I),U(I),DU(I),DSTAR(I),THETA(I),HCONV(I),
1 REX(I),RETH(I),CFI(I),Q(I)
23 FORMAT(I3,5X,F4.3,4X,F6.4,5X,F8.4,4X,F8.7,2X,F8.7,2X,F6.3,2X,
1 E10.5,2X,E10.5,2X,E10.5,2X,F9.7)

IF (FLAG(2).EQ.0 AND II.GT.NPTU) THEN
WRITE(6,19)
GO TO 100
END IF
IF (FLAG(2).EQ.1 AND II.GT.NPT) THEN
WRITE(6,19)
GO TO 100
END IF

* -----
* THE TRAPIZODIAL RULE IS NOW USED TO INTEGRATE THE DIFF-
* ERENTIAL EQUATION FROM "I" TO "I+1".
* -----
SUM=0.225*(U(II)**5+U(I)**5)*(X(II)-X(I))/REYNO
THETA(II)=SQRT(((THETA(I)**2)*(U(I)**6)+SUM)/(U(II)**6))
IF (II.EQ.2) THETA(2)=THETA(1)
IF (II.EQ.NLSS) THETA(NLSS)=THETA(NLS)

*****
*
* TEST FOR TRANSITION
*
* (Upper surface peak Cp)
* (Lower surface forced transition point)
*
*****

```

```

IF(FLAG(3).EQ.1.AND.FLAG(2).EQ.0.AND.I.EQ.ITRANS)THEN
  OUTPUT,'The point of forced transition (based on maximum peak'
  OUTPUT,'velocity) on the upper airfoil surface has been reached.'
  OUTPUT,
  OUTPUT,'Turbulent calculations have begun.'
  OUTPUT,
  WRITE(6,19)
  WRITE(6,24)
24 FORMAT(/,'THE POINT OF FORCED TRANSITION (UPPER SURFACE) HAS',/,
1 'BEEN REACHED, TURBULENT CALCULATIONS HAVE BEGUN.')
  ITRANS=I
  FLAG(1)=1.0
  GO TO 6
END IF

```

```

IF(FLAG(3).EQ.1.AND.FLAG(2).EQ.1.AND.X(I).GE.XTP(1))THEN
  OUTPUT,'The point of forced transition has been reached on the'
  OUTPUT,'lower airfoil surface.'
  OUTPUT,
  OUTPUT,'Turbulent calculations have begun.'
  OUTPUT,
  WRITE(6,19)
  WRITE(6,25)
25 FORMAT(/,'THE POINT OF FORCED TRANSITION (LOWER SURFACE) HAS',/,
1 'BEEN REACHED, TURBULENT CALCULATIONS HAVE BEGUN.')
  ITRANS=I
  FLAG(1)=1.0
  GO TO 6
END IF

```

```

*****
*                                     *
*               TEST FOR TRANSITION   *
*                                     *
*            (Using H-Rex Method)     *
*                                     *
*****

```

```

IF(FLAG(3).EQ.0)THEN
  IF(HCONV(I).GT.2.1.AND.HCONV(I).LT.3.1)THEN
    RLOG= -40.4557+(64.8066*HCONV(I))-(26.7538*(HCONV(I)**2.0))
    *      +(3.3819*(HCONV(I)**3.0))
    RE9=10.0**RLOG
  END IF
  IF(HCONV(I).GE.1.9.AND.HCONV(I).LE.2.1)THEN
    RLOG= -40.4557+(64.8066*2.1)-(26.7538*(2.1**2.0))
    *      +(3.3819*(2.1**3.0))
    RE9=10.0**RLOG
  END IF
  IF(HCONV(I).LT.1.9.OR.HCONV(I).GT.3.1)THEN
    OUTPUT,'The shape factor H is outside the applicable range of'
    OUTPUT,'the H-Rex method for predicting the point of transition'
    WRITE(6,19)
    WRITE(6,28)
28 FORMAT(/,'SHAPE FACTOR H IS OUTSIDE THE APPLICABLE RANGE OF',/,
    * 'H-Rex METHOD---STOP')
    GO TO 100
  
```

```

      END IF
      IF (REX(1) GE. RE9) THEN
      IF (FLAG(2) EQ. 0) THEN
      OUTPUT, '
      OUTPUT, 'The H-Rex method has predicted transition on the upper'
      OUTPUT, 'airfoil surface. The turbulent calculations have begun'
      OUTPUT, '
      WRITE(6,19)
      WRITE(6,26)
26  FORMAT(/, 'THE H-Rex METHOD HAS PREDICTED TRANSITION ON THE',/,
1    'UPPER AIRFOIL SURFACE')
      END IF
      IF (FLAG(2) EQ. 1) THEN
      OUTPUT, '
      OUTPUT, 'The H-Rex method has predicted transition on the lower'
      OUTPUT, 'airfoil surface. The turbulent calculations have begun'
      OUTPUT, '
      WRITE(6,19)
      WRITE(6,27)
27  FORMAT(/, 'THE H-Rex METHOD HAS PREDICTED TRANSITION ON THE',/,
1    'LOWER AIRFOIL SURFACE')
      END IF
      IF (FLAG(4) EQ. 0) GO TO 100
      ITRANS=1
      FLAG(1)=1.0
      GO TO 6
      END IF
      END IF
      END IF
6  CONTINUE

```

```

*****
*                                     *
*          ROUGH WALL TURBULENT FLOW REGION          *
*          [Dvoraks' Method (1969)]                  *
*                                     *
*****

```

```

      IF (FLAG(4) EQ. 0) GO TO 5
      IF (FLAG(1) EQ. 1) THEN
      IF (I EQ. ITRANS OR I EQ. NS) THEN
      WRITE(6,29)
      WRITE(6,30)
29  FORMAT(/, 'TURBULENT FLOW REGION')
30  FORMAT('-----')
      WRITE(6,31)
      WRITE(6,32)
31  FORMAT(/, '-----')
      *
      *
32  FORMAT(/, ' I      S/C    U/Uref    DU/DS  DESTAR/C  THETA/C  H/C
*      Rex      Reth      CF      Utau*K/NU  HDELS  DELS  SOR')
      WRITE(6,31)
      END IF

```

```

*
*  CALCULATE THE INITIAL VALUE (for turbulent calculations)
*  OF H DELTA-DELTASTAR, EQUAL TO G(Hconv).
*
-----

```

```

      IF(I.EQ.ITRANS.OR.I.EQ.NS)THEN
      IF(AHCONV.GT.1.6)GO TO 33
      HDELS(I)=3.3*EXP(0.4667-2.722*LOG(AHCONV-0.6796))
      GO TO 34
33 HDELS(I)=3.3*EXP(0.4383-3.064*LOG(AHCONV-0.6796))
34 CONTINUE

      U1=U(I)
      THETA(I)=ATHETA
      HCONV(I)=AHCONV
      DELS(I)=HDELS(I)*THETA(I)
      JJJ=0
      CF01=CF0

      *-----
      * TO FIND CF (SKIN FRICTION COEFFICIENT) AT THE FIRST TURBULENT
      * DATA POINT.
      *-----
      CFINTL=CF00(CF01,AHCONV,ATHETA,VIS,U1,RNT,RDEN,JJJ)
      CFI(I)=CFINTL
      UKONU(I)=SQRT(CFI(I)/2.0)*RNT*U(I)*REYNO
      REX(I)=U(I)*X(I)*REYNO
      RETH(I)=U(I)*THETA(I)*REYNO
      DSTAR(I)=HCONV(I)*THETA(I)
      IF(FLAG(4).EQ.2)THEN
      Q(I)=(U(I)*DSTAR(I)-U(I-1)*DSTAR(I-1))/STEPX
      END IF
      IF(FLAG(4).EQ.1)THEN
      Q(I)=ABS(DSTAR(I)*DU(I))
      END IF
      END IF

      WRITE(6,35)I,X(I),U(I),DU(I),DSTAR(I),THETA(I),HCONV(I),
      *          REX(I),RETH(I),CFI(I),UKONU(I),HDELS(I),DELS(I),Q(I)
35  FORMAT(I3,2X,F6.3,2X,F6.4,2X,F7.3,2X,F8.7,2X,F8.7,2X,F6.3,2X,
      *          E10.6,1X,F7.0,1X,E10.6,1X,E10.6,2X,F6.3,2X,F8.7,2X,F8.6)

      *-----
      * THE "OLD" OR INITIAL VALUES TO BE USED TO FIND THE VALUES
      * AT THE NEXT STATION ARE TAKEN AS THE VALUES AT THE PRESENT
      * STATION
      *-----
      CF01=CFI(I)
      HCONV0=HCONV(I)
      THETA0=THETA(I)
      U1=U(I)
      U2=U(I+1)
      HDELS0=HDELS(I)
      DELS0=DELS(I)
      DUP=DU(I)
      DXP=DX(I)

      *-----
      * NUMERICAL CHECK TO DETERMINE IF THE TURBULENT BOUNDARY
      * LAYER HAS SEPARATED FROM EITHER AIRFOIL SURFACE.
      *-----
      IF(HCONV(I).GT.3.0)THEN
      CALL TURBSEP
      GO TO 100
      END IF

```

```

* -----
* THE SUBROUTINE RUNGEK IS CALLED TO INTEGRATE THE DIFFERENTIAL
* EQUATIONS TO FIND VALUES FOR HCONV, THETA, DELTA-DELTASTAR,
* H DELTA-DELTASTAR, CF AND SOURCE STRENGTH AT THE NEXT STATION.
* -----
* CALL RUNGEK(CF01, HCONVO, THETA0, VIS, U1, U2, RBT, RDN, JJJ, HDELSO,
* DELSO, DUP, HCONVN, THETAN, DELSN, HDELSN, CFN, DXP)
*
* -----
* UPDATING OF INITIAL VALUES
* -----
HCONVO=HCONVN
THETA0=THETAN
HDELSO=HDELSN
DELSO=DELSN
HCONV(I+1)=HCONVN
THETA(I+1)=THETAN
DELS(I+1)=DELSN
HDELS(I+1)=HDELSN
CFI(I+1)=CFN
UKONU(I+1)=SQRT(CFI(I+1)/2.0)*RBT*REYNO*U(I+1)
DSTAR(I+1)=HCONV(I+1)*THETA(I+1)
REX(I+1)=(U(I+1)*X(I+1))/VIS
RETH(I+1)=(U(I+1)*THETA(I+1))/VIS
Q(I+1)=(U(I+1)*DSTAR(I+1)-U(I)*DSTAR(I))/STEPX
END IF
5 CONTINUE
IF(I.EQ.NF)GO TO 99
2 CONTINUE
99 WRITE(6,31)
100 CONTINUE

* -----
* THE NEXT STATEMENT CALLS THE SUBROUTINE "XPOSEP" WHICH
* DETERMINES THE CARTESIAN "X" POSITION OF THE UPPER AND
* LOWER SURFACE SEPARATION POINTS.
* -----
CALL XPOSEP

* -----
* IF ONLY ONE SURFACE OR ONE TEST RUN IS TO BE PERFORMED
* THE NEXT STATEMENT TRANSFERS CONTROL.
* -----
IF(FLAG(8).EQ.0)GO TO 14

* -----
* THE SET OF STATEMENTS CHANGES THE OBJECT OF DO LOOP 2 IN
* ORDER FOR THE PROGRAM TO CALCULATE THE DEVELOPMENT OF THE
* LOWER SURFACE BOUNDARY LAYER.
* -----
IF(NF.EQ.NPT)GO TO 14
IF(NS.EQ.1.AND.NF.EQ.NPTU)THEN
NS=NLS
NF=NPT
CF01=CF0
IF(FLAG(4).EQ.0)FLAG(1)=0
IF(FLAG(4).EQ.1)FLAG(1)=1

```

```

      IF (FLAG(4).EQ.2) FLAG(1)=0
      FLAG(2)=1.0
      WRITE(6,38)
      WRITE(6,39)
38  FORMAT(//,40X,'BOUNDARY LAYER PARAMETERS LOWER AIRFOIL SURFACE')
39  FORMAT(40X,'-----')
      GO TO 3
      ELSE
      GO TO 14
      END IF
14  CONTINUE

```

```

*-----*
* THE NEXT STATEMENT CALLS THE SUBROUTINE DISPLACE IN ORDER
* TO GENERATE A FILE OF THE SOURCE STRENGTHS AT THE AIRFOIL
* JUNCTION POINTS.
*-----*

```

```

      IF (FLAG(5).EQ.1) THEN
      CALL DISPLACE
      END IF

```

```

      STOP

```

```

      END

```

```

***** B-2 *****
*
* FUNCTION SUBROUTINE CF00
*
* This function returns the value of Cf, which is obtained
* through a Newton-Raphson iteration
*
*****

```

```

      FUNCTION CF00(CFI,HCONVP,THETAP,VIS,UP,RHT,RDEN,JJJ)
      CF0=CFI
      IF (CF0.LT.0.0) CF0=0.000001
10  A1=FNO0(CF0,HCONVP,THETAP,VIS,UP,RHT,RDEN,JJJ)
      A2=FNO(CF0,HCONVP,THETAP,VIS,UP,RHT,RDEN,JJJ)
      A3=CF0-A1
      A4=A3/A2
      CF11=CF0-A4

```

```

*
* CF222=CF11
* IF (CF222.LT.0.0) CF222=0.0
* WRITE(3,30), SQRT(CF222/2.0)*RHT*UP/VIS
* 30  FORMAT(' Utah*k/NU =',F15.9)
      IF (ABS((CF0-CF11)/CF0).LE.0.001) GO TO 20
      CF0=CF11
      GO TO 10
20  CF00=CF11
      IF (CF00.LT.0.0) CF00=0.000001

```

```

      RETURN

```

```

      END

```

```

***** B-3 *****
*

```

```
***** B-4 *****
*
*      FUNCTION SUBROUTINE FNO
*
*      This funtion returns the value of the derivative of the
*      function FNOO to be used in the Neuton-Raphson iteration
*      toward Cf.
*
*****
```

```

***** B-5 *****
*
*           FUNCTION SUBROUTINE FN1
*
* This function returns RETAH, the value of UTAH*k/NU at the upper
* limit of the transitionally rough range; it represents fig.3

```


* of Dvorak's paper. *

.....*

```

      FUNCTION FN1(RDEN)
      IF (RDEN.LT.3.25) GO TO 10
      IF (RDEN.LT.3.75) GO TO 20
      IF (RDEN.LT.9.40) GO TO 30
      IF (RDEN.LT.25.0) GO TO 40
      IF (RDEN.GT.25.0) GO TO 50
      FN1=0.0
      RETURN
10    FN1=70.0
      RETURN
20    FN1=70.0-80.0*(RDEN-3.25)
      RETURN
30    FN1=30.0
      RETURN
40    FN1=30.0+2.564*(RDEN-9.4)
      RETURN
50    FN1=70.0

      RETURN
END

```

```

..... B-6 .....
*
*      FUNCTION SUBROUTINE FN2
*
*      This function returns the value of the law of wall shift
*      ( $\Delta U_1$ )/UTAH; it represents fig.1 of Dvorak's paper.
*
*      RETAH is the input value of  $(U \lambda) \cdot k/v$ 
*
* .....

```

```

      FUNCTION FN2(RETAH, RETAHU, UTAH, RDEN, UTAHU, JJJ, UTAH1)
      REAL KK, KKH, K1, KU1, KO
      DIMENSION C(4,9)
      COMMON XX(4)

*      -----
*      IF UTAH*k/NU <= 5.0 THEN FLOW IS AERODYNAMICALLY SMOOTH, AND
*      FN2=0
*      -----
      IF (RETAH.LE.5.0) GO TO 10

*      -----
*      RETAHU IS THE UPPER LIMIT OF UTAH*k/NU FOR TRANSITIONALLY ROUGH
*      SURFACES. IF OUR VALUE OF UTAH*k/NU IS LARGER THAN THIS THE
*      THERE IS NO NEED TO FIT THE LARGE POLYNOMIAL NECESSARY FOR THE
*      TRANSITIONALLY ROUGH REGIME.
*      -----
      IF (RETAH.LT.RETAHU) GO TO 30

*      -----
*      FOR THE FULLY ROUGH CASE CALCULATE ( $\Delta U_1$ )/UTAH FOR LAMBDA

```

```

*   GREATER THEN OR LESS THAN 5, ACCORDING TO FIG.2 OF DVORAK'S
*   PAPER.
*   -----
      IF(RDEN.LE.5.0) GO TO 20
      FN2=5.6+0.434294*LOG(RETAN)-5.95*(1.103+0.434294*LOG(RDEN)-1.0)
      RETURN
10  FN2=0.0
      RETURN
20  FN2=5.6+0.434294*LOG(RETAN)+17.35*(1.625+0.434294*LOG(RDEN)-1.0)
      RETURN

*   -----
*   CHECK TO SEE IF THE TRANSITION REGION HAS BEEN CALCULATED BEFORE
*   -----
30  IF(JJJ.GT.0) GO TO 65

*   -----
*   CALCULATE THE TRANSITION REGION BY FITTING A POLYNOMIAL BETWEEN
*   THE SMOOTH AND FULLY ROUGH REGIONS.
*   -----
      KK=LOG(5.0)
      KKU=LOG(RETANU)
      K1=1.0/UTAH1
      KU1=1.0/UTAHU
      C(1,1)=1.0
      C(1,2)=KK
      C(1,3)=KK**2
      C(1,4)=KK**3
      C(1,5)=0.0
      C(2,1)=0.0
      C(2,2)=K1
      C(2,3)=2.0*K1*KK
      C(2,4)=3.0*K1*KK**2
      C(2,5)=0.0
      C(3,1)=0.0
      C(3,2)=KU1
      C(3,3)=2.0*KU1*KKU
      C(3,4)=3.0*KU1*KKU**2
      C(3,5)=FN4(RETANU,RDEN)
      C(4,1)=1.0
      C(4,2)=KKU
      C(4,3)=KKU**2
      C(4,4)=KKU**3
      C(4,5)=FN5(RETANU,RDEN)
      WRITE(3,39)
* 39  FORMAT(10X,' THE COEFFICIENT MATRIX')
*   DO 40 I=1,4
* 40  WRITE (3,41), (C(I,J),J=1,5)
* 41  FORMAT(10X,5F12.6)
      EPS=0.00001
      N=4
      CALL GJORD(C,N,EPS)
      JJJ=JJJ+1
65  KO=LOG(RETAN)
      FN2=XX(1)+XX(3)*KO+XX(3)*KO**2+XX(4)*KO**3
*   WRITE (3,70) , (XX(I),I=1,4)
* 70  FORMAT(2X,'X(1)=' ,F12.6,' X(2)=' ,F12.6,' X(3)=' ,F12.6,
*   1      X(4)=' ,F12.6)

```

```

*   WRITE (3,80), FN2
* 80   FORMAT(10X,'FN2=',F12.6)

      RETURN
      END

```

```

***** B-7 *****
*
*   FUNCTION SUBROUTINE FN3
*
*   This function returns the value of (delta U2)/(UTAH)
*
*****

```

```

      FUNCTION FN3(H,ACF)
2   G=SQRT(2.0/ACF)*((H-1.0)/H)
      IF(G.LT.6.7) THEN
        FN3=0.404*(G-6.7)
      ELSE
        FN3=1.253*(G-6.7)
      END IF
1   CONTINUE
      RETURN
      END

```

```

***** B-8 *****
*
*   FUNCTION SUBROUTINE FN4
*
*   This function returns the value of the derivative of
*   (delta U1)/(U lambda) at the upper limit of the transitionally
*   rough region.
*
*****

```

```

      FUNCTION FN4(RETANU,RDEN)
      EPS=0.01
      RET1=RETANU+EPS
      RET2=RETANU-EPS
      FN4=(FN5(RET1,RDEN)-FN5(RET2,RDEN))/(2.0*EPS)

      RETURN
      END

```

```

***** B-9 *****
*
*   FUNCTION SUBROUTINE FN5
*
*   This function returns the value of (delta U1)/(UTAH) at
*   the upper limit of the transitionally rough region, for FN2
*
*****

```

```

      FUNCTION FN5(RETANU,RDEN)
      IF (RDEN.LE.5.0) GO TO 20

```

```

      FN5=5.6*0.434294*LOG(RETANU)-5.95*(1.103*0.434294*LOG(RDEN)-1.0)
      RETURN
20    FN5=5.6*0.434294*LOG(RETANU)+17.35*(1.625*0.434294*LOG(RDEN)-1.0)

      RETURN

      END

```

```

***** B-10 *****
*
*      FUNCTION SUBROUTINE FN6
*
* This function returns the value of F(H delta-deltastar), equation
* A1 of Dvorak's paper.
*
*****

```

```

      FUNCTION FN6(HDELS)
      FN6=EXP(-3.512-0.617*LOG(HDELS-3.0))

      RETURN

      END

```

```

***** B-11 *****
*
*      FUNTION SUBROUTINE FN7
*
* This function returns the value of Hconv, given the input
* value of H delta-deltastar, based on equations A2 and A3 of
* Dvorak's paper.
*
*****

```

```

      FUNCTION FN7(HDELS)
      IF (HDELS.LE.3.305) HDELS=3.305
      IF (HDELS.GT.5.30) GO TO 10
      FN7=0.6798*EXP((0.4383-LOG(HDELS-3.3))/3.064)
      RETURN
10    FN7=0.6798*EXP((0.4667-LOG(HDELS-3.3))/2.722)

      RETURN

      END

```

```

***** B-12 *****
*
*      SUBROUTINE GJORD
*
* This subroutine finds the coefficients for the log polynomial
* in the transitionally rough range, using a Gauss-Jordan
* elimination.
*
*****

```

```

      SUBROUTINE GJORD(A,N,EPS)
      DIMENSION A(4,9)

```

```

COMMON XX(4)
DO 2 J=6,9
DO 2 I=1,4
IF((J-5).EQ.1) GO TO 3
A(I,J)=0.0
GO TO 2
3 A(I,J)=1.0
2 CONTINUE
* DO 112 I=1,4
* 112 PRINT 113, (A(I,J), J=1,9)
* 113 FORMAT(10X,9F12.6)
.N=N+1
NPLUSM=N+M
DETER=1.0
DO 9 K=1,N
DETER=DETER+A(K,K)
IF (ABS(A(K,K)).GT.EPS) GO TO 5
PRINT 101
101 FORMAT(10X,' PIVOT TO SMALL')
RETURN
5 KP1=K+1
DO 6 J=KP1,NPLUSM
6 A(K,J)=A(K,J)/A(K,K)
A(K,K)=1.0
DO 9 I=1,N
IF(I.EQ.K) GO TO 9
IF(A(I,K).EQ.0.0) GO TO 9
DO 8 J=KP1,NPLUSM
8 A(I,J)=A(I,J)-A(I,K)*A(K,J)
A(I,K)=0.0
9 CONTINUE
DO 10 I=1,N
10 XX(I)=A(I,5)
* DO 102 I=1,4
* 102 PRINT 103, (A(I,J), J=1,9)
* 103 FORMAT(10X,9F12.6)

RETURN

END

```

```

***** B-13 *****
*
* SUBROUTINE RUNGEK
*
* This subroutine contains the Runge-Kutta integration of the
* momentum integral equation and Head's entrainment equation
* to get the values of Hconv, theta, delta-deltastar, Hdelta-
* deltastar, and Cf at the next station.
*
*****

```

```

SUBROUTINE RUNGEK(CFO,HCONVO,THETA0,VIS,U1,U2,RHT,RDEN,JJJ,HDEL50,
1DEL50,DU,HCONVM,THETAN,DELSN,HDELSN,CFN,DX)
UI=(U1+U2)/2.0
DIHETO=(CFO/2.0-(HCONVO*2.0)*(THETA0/U1)*DU)*DX
DDEL50=(FNG(HDEL50)-(DELSN*DU/UI))*DX

```

```

THETA1=THETA0+0.50*DTHETO
DELS1=DELS0+0.50*DDELS0
HDELS1=DELS1/THETA1
HCONV1=FN7(HDELS1)
IF (HCONV1 GE 3.0) THEN
  CFN=0.0
  HDELSN=HDELS1
  DELSN=DELS1
  THETAN=THETA1
  HCONVN=HCONV1
  GO TO 1
END IF
CF1=CF00(CF0,HCONV1,THETA1,VIS,U1,RHT,RDEN,JJJ)
DTHET1=(CF1/2.0-(HCONV1+2.0)*(THETA1/UI)*DU)*DX
DDELS1=(FN6(HDELS1)-(DELS1*DU/UI))*DX

THETA2=THETA0+0.5*DTHET1
DELS2=DELS0+0.5*DDELS1
HDELS2=DELS2/THETA2
HCONV2=FN7(HDELS2)
IF (HCONV2 GE 3.0) THEN
  CFN=CF1
  HDELSN=HDELS2
  DELSN=DELS2
  THETAN=THETA2
  HCONVN=HCONV2
  GO TO 1
END IF
CF2=CF00(CF1,HCONV2,THETA2,VIS,U1,RHT,RDEN,JJJ)
DTHET2=(CF2/2.0-(HCONV2+2.0)*(THETA2/UI)*DU)*DX
DDELS2=(FN6(HDELS2)-(DELS2*DU/UI))*DX

THETA3=THETA0+DTHET2
DELS3=DELS0+DDELS2
HDELS3=DELS3/THETA3
HCONV3=FN7(HDELS3)
IF (HCONV3 GE 3.0) THEN
  CFN=CF2
  HDELSN=HDELS3
  DELSN=DELS3
  THETAN=THETA3
  HCONVN=HCONV3
  GO TO 1
END IF
CF3=CF00(CF2,HCONV3,THETA3,VIS,U2,RHT,RDEN,JJJ)
DTHET3=(CF3/2.0-(HCONV3+2.0)*(THETA3/U2)*DU)*DX
DDELS3=(FN6(HDELS3)-(DELS3*DU/U2))*DX

THETAN=THETA0+(DTHETO+2.0*DTHET1+2.0*DTHET2+DTHET3)/6.0
DELSN=DELS0+(DDELS0+2.0*DDELS1+2.0*DDELS2+DDELS3)/6.0
HDELSN=DELSN/THETAN
HCONVN=FN7(HDELSN)
IF (HCONVN GE 3.0) THEN
  CFN=CF3
  GO TO 1
END IF
CFN=CF00(CF3,HCONVN,THETAN,VIS,U2,RHT,RDEN,JJJ)
1 CONTINUE

```

RETURN

END

```

      *****
      SUBROUTINE INPUT
      *****

      ***** B-14 *****
      *
      *   This subroutine controls the reading in of all the required *
      *   data necessary to run the boundary layer program.          *
      *
      *****

      IMPLICIT INTEGER(I-N), REAL(A-H,O-Z)

      COMMON UC(100), XC(100), XCART(100), U(400), X(400), THETA(400),
      * DELS(400), HDELS(400), HCONV(400), DSTAR(400), REX(400),
      * DELTA(400), DU(400), DX(400), CFI(400), UKONU(400),
      * SOURCE(400), FLAG(10), XTP(2), XCTP(2), CSOR(100), Q(400),
      * REYH(400), SSEP(2), XSEP(2),
      * NTOT, NUP, NLOW, NAM1, NAM2, NAM3, CHORD, UREF, RDEN, RHT, REYNO,
      * ALPHAA, STEPX, VIS, NPU, NPL, NPTU, NPTL, NLS, NLSS, I, J, PI,
      * LAMBDA, L, ITRANS, ISEPU, ISEPL, CPO, NCLS, NPT, ATHETA, AHCONV

      INTEGER FLAG
      REAL NAM1, NAM2, NAM3, LAMBDA, L

      READ(5,1) NAM1, NAM2, NAM3
      1 FORMAT(3A4)
      READ(5,2) ALPHAA, FLAG(4)
      2 FORMAT(2G)
      READ(5,3) CHORD, UREF, STEPX
      3 FORMAT(3G)
      READ(5,4) ATHETA, AHCONV, CPO
      4 FORMAT(3G)
      READ(5,5) RDEN, RHT, REYNO
      5 FORMAT(3G)
      READ(5,6) NTOT, NUP, NLOW
      6 FORMAT(3G)
      READ(5,7), (XCART(I), XC(I), UC(I), I=1, NTOT)
      7 FORMAT(3G)

      OUTPUT, 'The current type of boundary-layer analysis is.'
      OUTPUT,
      IF(FLAG(4).EQ.0) OUTPUT, 'Laminar flow analysis only.'
      IF(FLAG(4).EQ.1) OUTPUT, 'Turbulent flow analysis only.'
      IF(FLAG(4).EQ.2) OUTPUT, 'Full Laminar-Turbulent flow analysis.'
      OUTPUT,
      OUTPUT, 'Do you want to change the type of analysis?'
      OUTPUT, 'ENTER 0-NO, 1-YES'
      READ(2,13) FLAG(9)
      13 FORMAT(1G)
      IF(FLAG(9).EQ.1) THEN
      OUTPUT,
      OUTPUT, 'Enter the number corresponding to the type of'
      OUTPUT, 'boundary-layer analysis desired.'

```

```

OUTPUT, 'Enter 0 Laminar analysis only.'
OUTPUT, 'Enter 1 Turbulent analysis only.'
OUTPUT, 'Enter 2 Full Laminar-Turbulent analysis.'
READ(2,14)FLAG(4)
14 FORMAT(1G)
END IF

IF(FLAG(4).EQ.0.OR.FLAG(4).EQ.2)THEN
OUTPUT, 'Enter the number code corresponding to the type of'
OUTPUT, 'initial condition for the laminar flow calculations.'
OUTPUT, 'ENTER 1 Analysis begins at a stagnation point.'
OUTPUT, 'ENTER 0 Flat plate analysis'
OUTPUT, '      (i.e. initial cond. zero thick. bound. layer)'
READ(2,15)FLAG(10)
15 FORMAT(1G)
END IF

OUTPUT, 'Do you want the program to perform the boundary layer'
OUTPUT, 'analysis on both the upper and lower airfoil surfaces?'
OUTPUT, 'ENTER 0-NO, 1-YES'
READ(2,12)FLAG(8)
12 FORMAT(1G)

IF(FLAG(4).EQ.2.OR.FLAG(4).EQ.0)THEN
OUTPUT, 'Select the transition criterion.'
OUTPUT, '-----'
OUTPUT, 'Two options are available for implementation.'
OUTPUT, 'ENTER 0 Use the E-Rex method of predicting transition'
OUTPUT, 'ENTER 1 Fix the upper surface transition point'
OUTPUT, '      at the peak velocity position.'
OUTPUT, '-----'
READ(2,8)FLAG(3)
8 FORMAT(I1)
END IF
IF(FLAG(3).EQ.1.AND.FLAG(8).EQ.1)THEN
OUTPUT, 'Enter the surface linear distance length'
OUTPUT, '(for the transition point) on the lower airfoil surface'
READ(2,9)XTP(1)
9 FORMAT(1G)
END IF

* -----
* IF A SOURCE DISTRIBUTION IS DESIRED FOR USE WITH
* "CAMP" THEN THE NEXT STATEMENT SETS THE FLAG.
* -----
IF(FLAG(8).EQ.1)THEN
OUTPUT, 'Do you want a source strength distribution (at the'
OUTPUT, 'airfoil junction points) for use in the STAM_Program'
OUTPUT, 'ENTER 0-NO, 1-YES'
READ(2,11)FLAG(5)
11 FORMAT(I1)
END IF

```


RETURN

END

```
*
* *****
* SUBROUTINE INTERX
* *****
```

```
*****|***** B-15 *****
*
* This subroutine divides up the upper and lower airfoil
* surface into small increments in order to run the program.
*
* *****|*****
```

IMPLICIT INTEGER(I-N), REAL(A-H, O-Z)

```
COMMON UC(100), XC(100), XCART(100), U(400), X(400), THETA(400),
* DELS(400), HDELS(400), HCONV(400), DSTAR(400), REX(400),
* DELTA(400), DU(400), DX(400), CFI(400), UKONU(400),
* SOURCE(400), FLAG(10), XTP(2), XCTP(2), CSQR(100), Q(400),
* RETH(400), SSEP(2), XSEP(2),
* NTOT, NUP, NLOW, NAM1, NAM2, NAM3, CHORD, UREF, RDEN, RHT, REYNO,
* ALPHAA, STEPX, VIS, NPU, NPL, NPTU, NPTL, NLS, NLSS, I, J, PI,
* LAMBDA, L, ITRANS, ISEPU, ISEPL, CFO, NCLS, NPT, ATHETA, AHCONV
```

INTEGER FLAG

REAL NAM1, NAM2, NAM3, LAMBDA, L

```
*
* -----
* IF BOTH AIRFOIL SURFACES ARE TO BE ANALYZED THE NEXT
* SET OF STATEMENTS GENERATES VARIOUS CONSTANTS NEEDED.
* -----
```

```
IF (FLAG(8) .EQ. 1) THEN
21 NCLS=NUP+1
NPU=((XC(NUP)-XC(1))/STEPX)+1.0
NPTU=NPU+1
NPL=((XC(NTOT)-XC(NCLS))/STEPX)+1.0
NPTL=NPL+1
NLS=NPTU+1
NLSS=NLS+1
NPT=NPTU+NPTL
IF (NPT .GE. 400) THEN
OUTPUT, '***** WARNING! *****'
OUTPUT, 'The number of surface points exceeds the current'
OUTPUT, 'maximum limit of 400 for the upper and lower'
OUTPUT, 'surface combined.'
OUTPUT, 'The current number of airfoil increment points is:', NPT
OUTPUT, 'ENTER A NEW SMALLER STEP SIZE'
READ(2, 20) STEPX
20 FORMAT(1G)
GO TO 21
END IF

X(1)=XC(1)
X(NLS)=XC(NCLS)
```

```

      DO 1 I=1,NPU
        II=I+1
        X(II)=I*STEPX+X(1)
        IF(X(II).GT.XC(NUP))X(II)=XC(NUP)
1     CONTINUE

      DO 2 I=1,NPL
        II=I+NLS
        X(II)=I*STEPX+X(NLS)
        IF(X(II).GT.XC(NTOT))X(II)=XC(NTOT)
2     CONTINUE
      NK=NPT-1
      DO 3 I=1,NK
        DX(I)=STEPX
3     CONTINUE
      END IF

```

```

*-----
* IF ONLY ONE SURFACE IS TO BE CONSIDERED THE NEXT SET
* OF STATEMENTS GENERATE THE REQUIRED CONSTANTS.
*-----
      IF(FLAG(8).EQ.0)THEN
23    NPU=((XC(NUP)-XC(1))/STEPX)+1.0
        NPTU=NPU+1
        NPT=NPTU
        IF(NPT.GE.400)THEN
          OUTPUT,
          OUTPUT, '***** WARNING! *****'
          OUTPUT,
          OUTPUT, 'The number of surface points exceeds the current'
          OUTPUT, 'maximum limit of 400 for the single surface'
          OUTPUT, 'analysis.'
          OUTPUT, 'The current number of airfoil increment points is:',NPT
          OUTPUT, 'ENTER A NEW SMALLER STEP SIZE'
          READ(2,22)STEPX
22    FORMAT(1G)
          GO TO 23
        END IF

```

```

      X(1)=XC(1)
      DO 4 I=1,NPU
        II=I+1
        X(II)=I*STEPX+X(1)
        IF(X(II).GT.XC(NUP))X(II)=XC(NUP)
4     CONTINUE

      NK=NPT-1
      DO 5 I=1,NK
        DX(I)=STEPX
5     CONTINUE

```

```

*-----
* SINCE THE LOWER SURFACE IS NOT BEING CONSIDERED,
* A SET OF "DUMMY" VARIABLES ARE GENERATED.
*-----

```

```

      NCLS=NPT+2
      NPL=NPT+2
      NPIL=NPT+2

```

```
NLS=NPT+2
NLSS=NPT+2
```

```
END IF
```

```
RETURN
```

```
END
```

```
* *****
* SUBROUTINE INTERVEL
* *****
```

```
***** B-16 *****
```

```
* This subroutine fits a parabola to three course velocity
* data points in order to determine the value of the velocity
* at the finely incremented positions. In addition the derivat-
* ive of the velocity is as well determined at each point.
*
* *****
```

```
IMPLICIT INTEGER(I-N), REAL(A-H, O-Z)
```

```
COMMON UC(100), XC(100), XCART(100), U(400), X(400), THETA(400),
* DELS(400), HDELS(400), HCONV(400), DSTAR(400), REX(400),
* DELTA(400), DU(400), DX(400), CFI(400), UKONU(400),
* SOURCE(400), FLAG(10), XTP(2), XCTP(2), CSOR(100), Q(400),
* RETH(400), SSEP(2), ISEP(2),
* NTOT, NUP, NLOW, NAM1, NAM2, NAM3, CHORD, UREF, RDEN, RHT, REYNO,
* ALPHAA, STEPX, VIS, NPU, NPL, NPTU, NPTL, NLS, NLSS, I, J, PI,
* LAMBDA, L, ITRANS, ISEPU, ISEPL, CFO, NCLS, NPT, ATHTA, ANCONV
```

```
INTEGER FLAG
REAL NAM1, NAM2, NAM3, LAMBDA, L
```

```
U(1)=UC(1)
J=3
NJ=NUP-1
```

```
DO 1 I=2, NPTU
2 IF(X(I) LE XC(J) OR J EQ NJ) THEN
CALL VELCOE
ELSE
J=J+2
IF(J GT NUP) J=NUP
GO TO 2
END IF
1 CONTINUE
IF(FLAG(8) EQ 1) THEN
U(NLS)=UC(NCLS)
J=NUP+3
NJ=NTOT-1
DO 3 I=NLSS, NPT
4 IF(X(I) LE XC(J) OR J EQ NJ) THEN
CALL VELCOE
ELSE
J=J+2
IF(J GT NTOT) J=NTOT
```

```

      GO TO 4
      END IF
3     CONTINUE
      END IF

```

```

* -----
*     THE NEXT STATEMENT DETERMINES THE MAXIMUM VELOCITY
*     ON THE UPPER SURFACE AND ITS POSITION.
* -----

```

```

      IF (FLAG(3) .EQ. 1) THEN
        DO 5 I=2, NPTU
          IF (U(I) .LT. U(I-1)) THEN
            ITRANS=I-1
            GO TO 6
          END IF
        5 CONTINUE
        6 CONTINUE
      END IF
      DU(1)=DU(2)
      DU(NLS)=DU(NLS+1)

```

```

      RETURN

```

```

      END

```

```

* .....
*     SUBROUTINE VELCOE
* .....

```

```

* ..... B-17 .....

```

```

*     This subroutine determines the coefficients of the quadratic
*     equation and then determines the point velocity and derivative
*     of the velocity at the incremented points on the airfoil sur-
*     face.

```

```

      IMPLICIT INTEGER(I-N), REAL(A-H,O-Z)

```

```

      COMMON UC(100), XC(100), XCART(100), U(400), X(400), THETA(400),
*     DELS(400), HDELS(400), HCONV(400), DSTAR(400), REX(400),
*     DELTA(400), DU(400), DX(400), CFI(400), UKONU(400),
*     SOURCE(400), FLAG(10), XTP(2), XCTP(2), CSOR(100), Q(400),
*     RETH(400), SSEP(2), XSEP(2),
*     NTOT, NUP, NLOW, NAM1, NAM2, NAM3, CHORD, UREF, RDEN, RHT, REYN0,
*     ALPHAA, STEPX, VIS, NPU, NPL, NPTU, NPTL, NLS, NLSS, I, J, PI,
*     LAMBDA, L, ITRANS, ISEPU, ISEPL, CFO, NCLS, NPT, ATHETA, ANCONV

```

```

      INTEGER FLAG
      REAL NAM1, NAM2, NAM3, LAMBDA, L

```

```

      J1=J-2
      J2=J-1
      J3=J

```

```

* -----
*     DETERMINATION OF THE QUADRATIC COEFFICIENTS A,B,C
* -----

```

```

      B1=(UC(J1)*(XC(J3)**2-XC(J2)**2)-UC(J2)*(XC(J3)**2-
      *   XC(J1)**2)+UC(J3)*(XC(J2)**2-XC(J1)**2))/(XC(J2)**2*
      *   (XC(J3)-XC(J1))+XC(J1)**2*(XC(J2)-XC(J3))+XC(J3)**2*
      *   (XC(J1)-XC(J2)))

      A1=(UC(J3)-UC(J1)-B1*(XC(J3)-XC(J1)))/(XC(J3)**2-XC(J1)**2)

      C1=UC(J2)-A1*XC(J2)**2-B1*XC(J2)

      * -----
      * DETERMINATION OF THE VELOCITY AND VELOCITY GRADIENT
      * -----
      U(I)=A1*X(I)**2+B1*X(I)+C1
      DU(I)=2.0*A1*X(I)+B1

      IF(J3.EQ.3)THEN
      IF(X(I).LT.XC(J2).AND.X(I).GT.XC(J1))THEN
      CONST1=(XC(J2)-X(I))/(XC(J2)-XC(J1))
      U(I)=UC(J2)-CONST1*(UC(J2)-UC(J1))
      DU(I)=(U(I)-U(I-1))/STEPX
      END IF
      IF(X(I).LT.XC(J3).AND.X(I).GT.XC(J2))THEN
      CONST1=(XC(J3)-X(I))/(XC(J3)-XC(J2))
      U(I)=UC(J3)-CONST1*(UC(J3)-UC(J2))
      DU(I)=(U(I)-U(I-1))/STEPX
      END IF
      END IF

      RETURN

      END

      * *****
      * SUBROUTINE OUT1
      * *****

      * ***** B-18 *****
      *
      * This subroutine outputs various boundary layer parameters.
      *
      * *****

      IMPLICIT INTEGER(I-N),REAL(A-H,O-Z)

      COMMON UC(100),XC(100),XCART(100),U(400),X(400),THETA(400),
      * DELS(400),HDELS(400),HCONV(400),DSTAR(400),REX(400),
      * DELTA(400),DU(400),DX(400),CFI(400),UKONU(400),
      * SOURCE(400),FLAG(10),XTP(2),XCTP(2),CSOR(100),Q(400),
      * RETH(400),SSEP(2),ISEP(2),
      * NTOT,NUP,NLOW,NAM1,NAM2,NAM3,CHORD,UREF,RDEN,RHT,REYNO,
      * ALPHAA,STEPX,VIS,NPU,NPL,NPTU,NPTL,NLS,NLSS,I,J,PI,
      * LAMBDA,L,ITRANS,ISEPU,ISEPL,CFO,NCLS,NPT,ATHETA,ABCONV

      INTEGER FLAG
      REAL NAM1,NAM2,NAM3,LAMBDA,L

      WRITE(6,1)
      WRITE(6,2)
      WRITE(6,3)

```

```

WRITE(6,4)
WRITE(6,5)NAM1,NAM2,NAM3
WRITE(6,6)ALPHAA
WRITE(6,7)CHORD
WRITE(6,8)UREF
WRITE(6,9)STEPX
WRITE(6,10)VIS
WRITE(6,11)REYNO
IF(FLAG(4) EQ 1 OR FLAG(4) EQ 2) THEN
WRITE(6,12)RNT
WRITE(6,13)RDEN
END IF

1 FORMAT(45X,'AIRFOIL BOUNDARY LAYER ANALYSIS')
2 FORMAT(45X,'-----')
3 FORMAT(//,'VARIOUS PARAMETERS')
4 FORMAT('-----')
5 FORMAT(//,'[1] THE AIRFOIL SECTION NAME IS:',F3A4)
6 FORMAT(' [2] THE AIRFOIL ANGLE OF ATTACK (DEG) IS:',F6.2)
7 FORMAT(' [3] THE CHORD LENGTH OF THE AIRFOIL IS:',F6.2)
8 FORMAT(' [4] THE FREE STREAM VELOCITY IS:',F6.2)
9 FORMAT(' [5] THE BOUNDARY LAYER STEP SIZE IS:',F6.4)
10 FORMAT(' [6] THE KINEMATIC VISCOSITY IS:',E10.5)
11 FORMAT(' [7] THE REYNOLDS NUMBER BASED ON THE CHORD IS:',F10.1)
12 FORMAT(' [8] THE NON-DIMENSIONAL ROUGHNESS HEIGHT, K/C:',F7.6)
13 FORMAT(' [9] THE ROUGHNESS DENSITY PARAMETER IS:',F6.3)

RETURN

END

* *****
* SUBROUTINE THWATS
* *****

***** B-19 *****
*
* This subroutine contains and determines Thwaites' correl-
* ation formulas.
*
*****

IMPLICIT INTEGER(I-N), REAL(A-H,O-Z)

COMMON UC(100), XC(100), XCART(100), U(400), X(400), THETA(400),
* DELS(400), HDELS(400), HCONV(400), DSTAR(400), REX(400),
* DELTA(400), DU(400), DX(400), CFI(400), UKONU(400),
* SOURCE(400), FLAG(10), XTP(2), XCTP(2), CSOR(100), Q(400),
* RETH(400), SSEP(2), XSEP(2),
* NTOI, NUP, NLOW, NAM1, NAM2, NAM3, CHORD, UREF, RDEN, RNT, REYNO,
* ALPHAA, STEPX, VIS, NPU, NPL, NPTU, NPTL, NLS, NLSS, I, J, PI,
* LAMBDA, L, ITRANS, ISEPU, ISEPL, CFO, NCLS, NPT, ATHETA, ANCONV

INTEGER FLAG
REAL NAM1, NAM2, NAM3, LAMBDA, L

IF(LAMBDA LT 0.0) GO TO 1
L=0.22+LAMBDA*(1.57-1.8*LAMBDA)
HCONV(I)=2.61-LAMBDA*(3.75-5.24*LAMBDA)

```

```

GO TO 2
1 L=0.22+1.402*LAMBDA+0.018*LAMBDA/(0.107+LAMBDA)
  HCONV(1)=2.088+0.0731/(0.14+LAMBDA)
2 CONTINUE
  RETURN
  END

```

```

* *****
* SUBROUTINE DISPLACE
* *****

```

```

***** B-20 *****
*
* This subroutine determines the sources distribution to be
* used in the STAM_Program.
*
*****

```

```

IMPLICIT INTEGER(I-N),REAL(A-H,O-Z)

```

```

COMMON UC(100),XC(100),XCART(100),U(400),X(400),THETA(400),
* DELS(400),HDELS(400),HCONV(400),DSTAR(400),REX(400),
* DELTA(400),DU(400),DX(400),CFI(400),UKONU(400),
* SOURCE(400),FLAG(10),XTP(2),ICTP(2),CSOR(100),Q(400),
* BETH(400),SSEP(2),ISEP(2),
* NTOT,NUP,NLOW,NAM1,NAM2,NAM3,CHORD,UREF,RDEN,RHT,REYNO,
* ALPHAA,STEPX,VIS,NPU,NPL,NPTU,NPTL,NLS,NLSS,I,J,PI,
* LAMBDA,L,ITRANS,ISEPU,ISEPL,CFO,NCLS,NPT,ATHETA,ARCONV

```

```

INTEGER FLAG
REAL NAM1,NAM2,NAM3,LAMBDA,L

```

```

* -----
* THE NEXT SET OF STATEMENTS DETERMINE THE SOURCE STRENGTHS
* AT EACH OF THE AIRFOIL JUNCTION POINTS ON THE UPPER
* AND LOWER AIRFOIL SURFACES.
* -----

```

```

SOURCE(1)=Q(1)
DO 5 J=2,NUP
DO 6 I=2,NPTU
  IF(I.LE.ISEPU)THEN
    IF(X(I).GE.XC(J))THEN
      CONST1=(X(I)-XC(J))/(X(I)-X(I-1))
      SOURCE(J)=Q(I)-CONST1*(Q(I)-Q(I-1))
    GO TO 7
  END IF
  ELSE

```

```

* -----
* THE NEXT SET OF STATEMENTS SET THE SOURCE STRENGTHS TO
* ZERO IN THE SEPARATED FLOW REGION OF THE AIRFOIL.
* -----

```

```

DO 13 K=J,NUP
  SOURCE(K)=0.0
13 CONTINUE
GO TO 14
END IF
6 CONTINUE

```

```

7 CONTINUE
6 CONTINUE
14 CONTINUE

```

```

      NKK=NUP+2
      SOURCE(NCLS)=Q(1)
      DO 8 J=NKK,NTOT
      DO 9 I=NLS,NPT
      IF(I.LE.ISEPL)THEN
      IF(X(I).GE.XC(J))THEN
      CONST1=(X(I)-XC(J))/(X(I)-X(I-1))
      SOURCE(J)=Q(I)-CONST1*(Q(I)-Q(I-1))
      GO TO 10
      END IF
      ELSE

```

```

* -----
* THE NEXT SET OF STATEMENTS SET THE SOURCE STRENGTHS
* TO ZERO IN THE LOWER SURFACE SEPARATED FLOW REGION.
* -----

```

```

      DO 15 K=J,NTOT
      SOURCE(K)=0.0
15 CONTINUE
      GO TO 30
      END IF
      9 CONTINUE
10 CONTINUE
      8 CONTINUE
30 CONTINUE

```

```

* -----
* THE NEXT SET OF STATEMENTS REORGANIZE THE SOURCE DISTR-
* IBUTION FOR USE IN THE "STAM_PROGRAM".
* -----

```

```

* -----
* CASE(A): SEPARATION DOES NOT OCCUR ON EITHER AIRFOIL SURFACE
* -----

```

```

      IF(FLAG(6).EQ.0.AND.FLAG(7).EQ.0)THEN
      J=NUP
      DO 11 I=1,NUP
      CSOR(I)=SOURCE(J)
      J=J-1
11 CONTINUE

```

```

      NT=NTOT-1
      DO 12 I=NCLS,NT
      CSOR(I)=SOURCE(I+1)
12 CONTINUE
      END IF

```

```

* -----
* CASE(B): SEPARATION FROM THE UPPER AIRFOIL SURFACE ONLY.
* -----

```

```

      IF(FLAG(6).EQ.1.AND.FLAG(7).EQ.0)THEN
      CSOR(1)=0.0
      J=NUP
      DO 16 I=2,NCLS
      CSOR(I)=SOURCE(J)
      J=J-1

```



```

16 CONTINUE
   NP=NUP+2
   NT=NTOT
   DO 17 I=NP,NT
     CSOR(I)=SOURCE(I)
17 CONTINUE
   END IF

* -----
* CASE(C) SEPARATION FROM BOTH AIRFOIL SURFACES
* -----
   IF (FLAG(6).EQ.1.AND.FLAG(7).EQ.1) THEN
     CSOR(1)=0.0
     J=NUP
     DO 18 I=2,NCLS
       CSOR(I)=SOURCE(J)
       J=J-1
18 CONTINUE
     NT=NTOT+1
     NP=NUP+2
     CSOR(NT)=0.0
     DO 19 I=NP,NTOT
       CSOR(I)=SOURCE(I)
19 CONTINUE
     END IF

* -----
* THE NEXT SET OF STATEMENTS GENERATES THE DATA FILE
* CONTAINING THE AIRFOIL JUNCTION POINT SOURCE STRENGTHS
* (IN A COUNTER-CLOCK WISE ORDER STARTING AT THE TRAILING
* EDGE) FOR USE IN THE AIRFOIL ANALYSIS PROGRAM "CAMP".
* -----
   WRITE(4,20), (CSOR(I), I=1,NT)
20 FORMAT(F10.8)

   RETURN

   END

* .....
* SUBROUTINE XPOSEP
* .....

* ..... B-21 .....
*
* This subroutine determines the airfoil upper and lower
* surface separation points in the cartesian co-ordinate system.
* .....

IMPLICIT INTEGER(I-N), REAL(A-H,O-Z)

COMMON UC(100), IC(100), ICART(100), U(400), X(400), THETA(400),
* DELS(400), HDELS(400), HCONV(400), DSTAR(400), REI(400),
* DELTA(400), DU(400), DX(400), CFI(400), UKONU(400),
* SOURCE(400), FLAG(10), XTP(2), XCTP(2), CSOR(100), Q(400),
* RETN(400), SSEP(2), ISEP(2),
* NTOT, NUP, NLOW, NAM1, NAM2, NAM3, CHORD, UREF, RDEN, RHT, REYNO,
* ALPHA, STEPX, VIS, NPU, NPL, NPTU, NPTL, NLS, NLSS, I, J, PI,

```

```

      *      LAMBDA, L, ITRANS, ISEPU, ISEPL, CFO, NCLS, NPT, ATHETA, ARCONV

```

```

      INTEGER FLAG
      REAL NAM1, NAM2, NAM3, LAMBDA, L

```

```

      *      -----
      *      THE UPPER AIRFOIL SURFACE IS CONSIDERED FIRST.
      *      -----

```

```

      IF (FLAG(2).EQ.0) THEN
      IF (ISEPU.EQ.NPTU) GO TO 1
      DO 2 I=1, NUP
      IF (SSEP(1).LE.XC(I)) THEN
      CONST1=(XC(I)-SSEP(1))/(XC(I)-XC(I-1))
      XSEP(1)=XCART(I)-CONST1*(XCART(I)-XCART(I-1))
      GO TO 1
      END IF
2 CONTINUE
1 CONTINUE
      WRITE(6,5) XSEP(1)
5 FORMAT(//, 'THE X-CARTESIAN UPPER SURF. SEP. PT. IS: ', F8.5)
      END IF

```

```

      *      -----
      *      THE LOWER AIRFOIL SURFACE IS CONSIDERED NEXT.
      *      -----

```

```

      IF (FLAG(2).EQ.1) THEN
      IF (ISEPL.EQ.NPT) GO TO 3
      DO 4 I=NCLS, NTOT
      IF (SSEP(2).LE.XC(I)) THEN
      CONST2=(XC(I)-SSEP(2))/(XC(I)-XC(I-1))
      XSEP(2)=XCART(I)-CONST2*(XCART(I)-XCART(I-1))
      GO TO 3
      END IF
4 CONTINUE
3 CONTINUE
      WRITE(6,6) XSEP(2)
6 FORMAT(//, 'THE X-CARTESIAN LOWER SURF. SEP. PT. IS: ', F8.6)
      END IF

```

```

      RETURN

```

```

      END

```

```

      *      *****
      *      SUBROUTINE LAMSEP
      *      *****

```

```

      *      ***** B-22 *****
      *
      *      This subroutine controls the output of certain statements
      *      if Laminar separation has occurred on either airfoil surface.
      *
      *      *****

```

```

      IMPLICIT INTEGER(I-N), REAL(A-H, O-Z)

```

```

      *      COMMON UC(100), XC(100), XCART(100), U(400), X(400), THETA(400),
      *      DELS(400), HDELS(400), HCONV(400), DSTAR(400), REX(400),

```

```

      DELTA(400), DU(400), DX(400), CFI(400), UKONU(400),
      SOURCE(400), FLAG(10), XTP(2), XCTP(2), CSOR(100), Q(400),
      RETH(400), SSEP(2), XSEP(2),
      NTOT, NUP, NLOW, NAM1, NAM2, NAM3, CHORD, UREF, RDEN, RHT, REYNO,
      ALPHAA, STEPX, VIS, NPU, NPL, NPTU, NPTL, NLS, NLSS, I, J, PI,
      LAMBDA, L, ITRANS, ISEPU, ISEPL, CFO, NCLS, NPT, ATHETA, ANCONV

      INTEGER FLAG
      REAL NAM1, NAM2, NAM3, LAMBDA, L

      IF (FLAG(2).EQ.0) THEN
        OUTPUT,
        OUTPUT, 'Laminar separation has occurred on the'
        OUTPUT, 'airfoil upper surface.'
        OUTPUT,
        WRITE(6,19)
        WRITE(6,1)
        WRITE(6,21)
21  FORMAT(/, 'LAMINAR SEPARATION HAS OCCURED ON THE UPPER SURFACE')
        SSEP(1)=X(I)
        ISEPU=I
        END IF

      IF (FLAG(2).EQ.1) THEN
        OUTPUT,
        OUTPUT, 'Laminar separation has occurred on the'
        OUTPUT, 'airfoil lower surface.'
        OUTPUT,
        WRITE(6,19)
        WRITE(6,1)
        WRITE(6,22)
        SSEP(2)=X(I)
        ISEPL=I
22  FORMAT(/, 'LAMINAR SEPARATION HAS OCCURRED ON THE LOWER SURFACE')
        END IF
19  FORMAT(/, '-----')
1  FORMAT(//, 30X, 10* ' ', 'LAMINAR SEPARATION', 10* ' ')

      RETURN

      END

      .....
      SUBROUTINE TURBSEP
      .....

      ..... B-23 .....
      *
      * This subroutine controls the output of certain statements
      * if Turbulent separation has occurred on either airfoil surface.
      *
      .....

      IMPLICIT INTEGER(I-N), REAL(A-H, O-Z)

      COMMON UC(100), IC(100), ICART(100), U(400), X(400), THETA(400),
      * DELS(400), HDELS(400), HCONV(400), DSTAR(400), REI(400),
      * DELTA(400), DU(400), DX(400), CFI(400), UKONU(400),

```

```

*      SOURCE(400), FLAG(10), XTP(2), XCTP(2), CSOR(100), Q(400),
*      RETH(400), SSEP(2), XSEP(2),
*      NTOT, NUP, NLOW, NAM1, NAM2, NAM3, CHORD, UREF, RDEM, RHT, REYNO,
*      ALPHA, STEPX, VIS, NPU, NPL, NPTU, NPTL, NLS, NLSS, I, J, PI,
*      LAMBDA, L, ITRANS, ISEPU, ISEPL, CFO, NCLS, NPT, ATHETA, ANCONV

INTEGER FLAG
REAL NAM1, NAM2, NAM3, LAMBDA, L

IF (FLAG(2) .EQ. 0) THEN
  OUTPUT, 'The turbulent boundary layer has separated from the'
  OUTPUT, 'upper airfoil surface.'
  OUTPUT,
  WRITE(6, 31)
  WRITE(6, 1)
  WRITE(6, 36)
36 FORMAT(/, 'THE TURBULENT BOUNDARY LAYER HAS SEPARATED FROM', /,
*      'THE UPPER AIRFOIL SURFACE')
  SSEP(1) = X(I)
  ISEPU = I
  FLAG(6) = 1
  END IF

IF (FLAG(2) .EQ. 1) THEN
  OUTPUT,
  OUTPUT, 'The turbulent boundary layer has separated from the'
  OUTPUT, 'lower airfoil surface.'
  OUTPUT,
  WRITE(6, 31)
  WRITE(6, 1)
  WRITE(6, 37)
37 FORMAT(/, 'THE TURBULENT BOUNDARY LAYER HAS SEPARATED FROM', /,
*      'THE LOWER AIRFOIL SURFACE')
  SSEP(2) = X(I)
  ISEPL = I
  FLAG(7) = 1
  END IF
31 FORMAT(/, '-----'
*      '-----')
1 FORMAT(//, 30X, 10* ' ', 'TURBULENT SEPARATION', 10* ' ')

RETURN

END

```

APPENDIX L-1

Initial Wake Modelling Program

```
..... C-1 .....
*
*
*           I.W.M.P.
*
*       INITIAL WAKE MODELLING PROGRAM
*
*
*   This program gives an initial "guesstimate" of the wake shape
*   streamline boundary contour. The upper and lower free shear
*   layers are initially represented by two parabolic curves passing
*   from the upper and lower separation points, converging to a
*   common downstream point.
*
*   The program is interactive in nature and asks the user for
*   various input parameters. As well a separate data file is
*   used to input the number of airfoil points, co-ordinate points
*   of the airfoil and the separation points.
*
*
*       DATA FILE INPUT VARIABLES
*       -----
*
*   [1] NPT-----The number of airfoil co-ordinate points
*   [2] XN(I),Y(I)--The airfoil co-ordinate points
*   [3] XSU,YSU-----The upper surface separation points
*   [4] XSL,YSL-----The lower surface separation co-ordinates
*
*
*       INTERACTIVE INPUT VARIABLES
*       -----
*
*   [1] ALPHA----The airfoil angle of attack
*   [2] WLF-----Wake length factor
*   [3] NPU-----Number of panels to be used on the upper wake
*   [4] NPL-----Number of panels to be used on the lower wake
*   [5] N1 -----Input option for either bluff body configura-
*                  tion (i.e. lower surface separation point not
*                  at trailing edge.) or airfoil configuration.
*
*
* .....
```

```

*   MAIN PROGRAM
*   *****

COMMON X(90),Y(90),XN(90),XWU(30),YWU(30),XWL(30),
*   XC(90),YC(90),XWLC(30),YWLC(30),XWUC(30),
*   YWL(30),YWUC(30),
*   NPT,NPL,PI,XSU,YSU,XSL,YSL,ALPHA,NPUS,NPLS,
*   NPU,CHORDX,NK,NP2,NP3,ALPHAA,AL,BL,CL,WL,
*   SL,SU,SM,AU,BU,CU,WAKEH,WLF,N,XLL,YLL,N1

*   -----
*   THE NEXT STATEMENT CALLS THE SUBROUTINE "INPUT"
*   WHICH READS IN ALL THE DATA
*   -----
CALL INPUT

*   -----
*   THE NEXT STATEMENT CALLS THE SUBROUTINE "INWAKE"
*   WHICH DOES ALL THE CALCULATIONS.
*   -----
CALL INWAKE

*   -----
*   THE NEXT STATEMENT CALLS THE SUBROUTINE "OUTPUT"
*   WHICH GENERATES THE OUTPUT FILE.
*   -----
CALL OUTPUT

STOP

END

*   *****
*   SUBROUTINE INPUT
*   *****

***** C-2 *****
*   This subroutine controls the reading in of all the necessary
*   data.
*   *****

COMMON X(90),Y(90),XN(90),XWU(30),YWU(30),XWL(30),
*   XC(90),YC(90),XWLC(30),YWLC(30),XWUC(30),
*   YWL(30),YWUC(30),
*   NPT,NPL,PI,XSU,YSU,XSL,YSL,ALPHA,NPUS,NPLS,
*   NPU,CHORDX,NK,NP2,NP3,ALPHAA,AL,BL,CL,WL,
*   SL,SU,SM,AU,BU,CU,WAKEH,WLF,N,XLL,YLL,N1

*   -----
*   THE NEXT STATEMENT ALLOWS THE USER TO DECIDE IF
*   THE INITIAL WAKE MODEL IS FOR AN AIRFOIL OR BLUFF
*   BODY (i.e. circular cylinder).
*   -----
OUTPUT,'Input option for the type of initial wake model.'
OUTPUT,'Enter 0 AIRFOIL initial wake model.'
OUTPUT,'Enter 1 BLUFF BODY initial wake model.'
READ(2,5)N1

```

```

5 FORMAT(1G)

* -----
* READ IN THE AIRFOIL CONTOUR CO-ORDINATES
* -----
  READ(5,1)NPT
1 FORMAT(G)
  READ(5,2)(XN(I),Y(I),I=1,NPT)
2 FORMAT(2G)

* -----
* READ IN THE SEPARATION POINTS
* -----
  READ(5,3)XSU,YSU,XSL,YSL
3 FORMAT(4G)

* -----
* READ IN THE ANGLE OF ATTACK
* -----
  PI=3.141592654
  OUTPUT,'Enter the angle of attack (degrees). '
  READ(2,4),ALPHA
4 FORMAT(G)
  ALPHA=(ALPHA*PI)/180

* -----
* READ IN THE WAKE LENGTH FACTOR--WLF
* -----
  OUTPUT,'Enter the wake length factor. '
  READ(2,6)WLF
6 FORMAT(1G)

* -----
* DETERMINATION OF THE WAKE PANEL CO-ORDINATES
* -----
  OUTPUT,'Enter the number of panels on the upper wake surface '
  READ(2,7)NPU
7 FORMAT(G)
  OUTPUT,'Enter the number of panels on the lower wake surface. '
  READ(2,8)NPL
8 FORMAT(G)
  OUTPUT,'Do you want a separate data file to be generated. '
  OUTPUT,'for use with the Wake Modelling Potential Flow Program '
  OUTPUT,'ENTER 1-YES, AND 0-NO. '
  READ(2,56),NK
56 FORMAT(1G)

  RETURN

  END

* *****
* SUBROUTINE INWAKE
* *****

***** C-3 *****
* This subroutine does all the necessary calculations.

```

```

COMMON X(90),Y(90),XN(90),XWU(30),YWU(30),XWL(30),
      . XC(90),YC(90),XWLC(30),YWLC(30),XWUC(30),
      . YWL(30),YWUC(30),
      . NPT,NPL,PI,XSU,YSU,XSL,YSL,ALPHA,NPUS,NPLS,
      . NPU,CHORDX,NK,NP2,NP3,ALPHAA,AL,BL,CL,WL,
      . SL,SU,SM,AU,BU,CU,WAKEN,WLF,N,XLL,YLL,N1

```

```

-----
DETERMINE THE CHORD LENGTH OF THE AIRFOIL
-----

```

```

XTE=XN(1)
DO 10 I=2,NPT
  IF(XN(I) LE XN(I-1))THEN
    CONTINUE
  ELSE
    ILE=I-1
    XLE=XN(I-1)
    CHORDX=XTE-XLE
    GO TO 13
  END IF
10 CONTINUE
13 CONTINUE

```

```

-----
NORMALIZE ALL X CO-ORDINATES SO THAT
THE LEADING EDGE IS AT X=0
-----

```

```

IF(XLE LT 0.0)XLE=-XLE
DO 11 I=1,NPT
  X(I)=XN(I)+XLE
  XC(I)=X(I)/CHORDX
  YC(I)=Y(I)/CHORDX
11 CONTINUE
XSUM=XSU+XLE
XSLM=XSL+XLE

```

```

-----
TOTAL NUMBER OF PANELS USED TO REPRESENT THE AIRFOIL-N
-----

```

```

N=NPT-1

```

```

-----
DETERMINATION OF THE PANEL NUMBER AND LOCAL SURFACE SLOPE
ON THE UPPER "SUCTION" SURFACE OF THE AIRFOIL
-----

```

```

DO 5 I=1,ILE
  IF(X(I) LE XSUM)THEN
    NPUS=I-1
    DX1=X(I)-X(I-1)
    DY1=Y(I)-Y(I-1)
    DX2=X(I+1)-X(I)
    DY2=Y(I+1)-Y(I)
    SLOP1=DY1/DX1
    SLOP2=DY2/DX2
    SLOPU=(SLOP1+SLOP2)/2.0

```



```

      GO TO 55
      ELSE
      CONTINUE
      END IF
      5 CONTINUE
      55 CONTINUE

```

```

* -----
* DETERMINATION OF THE PANEL NUMBER AND LOCAL SURFACE
* SLOPE ON THE SUCTION OR "PRESSURE" SURFACE
* -----

```

```

      DO 50 I=ILE,NPT
      IF(X(I) GE XSLM) THEN
      NPLS=I-1
      DX1=X(I)-X(I-1)
      DY1=Y(I)-Y(I-1)
      DX2=X(I+1)-X(I)
      DY2=Y(I+1)-Y(I)
      SLOP1=DY1/DX1
      SLOP2=DY2/DX2
      SLOPL=(SLOP1+SLOP2)/2.0
      GO TO 51
      ELSE
      CONTINUE
      END IF
      50 CONTINUE
      51 CONTINUE

```

```

* -----
* DETERMINATION OF THE INITIAL SLOPE OF THE UPPER SEPARATION SHEET
* THE INITIAL SLOPE OF THE LOWER SEPARATION SHEET, AND THE SLOPE
* OF THE MEAN BISECTING LINE OF THE WAKE.
* -----

```

```

      IF(N1.EQ.0) THEN
      SU=(SLOPU+TAN(ALPHA))/2.0
      SL=TAN(ALPHA)
      ELSE
      SU=(SLOPU+TAN(ALPHA))/2.0
      SL=(SLOPL+TAN(ALPHA))/2.0
      END IF
      SM=(SU+SL)/2

```

```

* -----
* CO-ORDINATES OF THE MIDPOINT OF THE SEPARATED REGION
* -----

```

```

      XMW=(XSUM+XSLM)/2
      YMW=(YSU+YSL)/2

```

```

* -----
* DETERMINATION OF THE WAKE HEIGHT
* -----

```

```

      DX1=XSLM-XSUM
      IF(DX1.LT.0.0) DX1=-DX1
      Y1=DX1*TAN(ALPHA)
      Y2=YSU-YSL
      IF(Y2.LT.0.0) Y2=-Y2
      R=Y2+Y1
      WAKEH=R/COS(ALPHA)

```

```

.-----
. DETERMINATION OF THE WAKE LENGTH
.-----
WL=WAKEN*WLF

.-----
. DETERMINE THE DOWNSTREAM INTERSECTING POINT OF THE UPPER
. AND LOWER PARABOLIC SHEETS
.-----
XM=XW+WL*COS(ATAN(SM))
YM=YW+WL*SIN(ATAN(SM))
XLU=XM-XSUM
XLL=XM-XSLM
YLL=YM-YSL

.-----
. DETERMINATION OF THE UPPER PARABOLIC COEFFICIENTS-AU,BU,CU
.-----
AU=(YM-YSU-SU*(XSUM-XM))/(XM-XSUM)**2
BU=SU-2*AU*XSUM
CU=YSU-AU*XSUM**2-BU*XSUM

.-----
. DETERMINATION OF THE LOWER PARABOLIC COEFFICIENTS-AL,BL,CL
.-----
IF(XLL GE YLL)THEN
AL=(YM-YSL-SL*(XSLM-XM))/(XM-XSLM)**2
BL=SL-2*AL*XSLM
CL=YSL-AL*XSLM**2-BL*XSLM
ELSE
T=1/SL
AL=(XM-XSLM+T*(YSL-YM))/(YM-YSL)**2
BL=T-2*AL*YSL
CL=XSLM-AL*YSL**2-BL*YSL
END IF

.-----
. INCREMENTAL VALUE OF "X" AND "Y" FOR THE UPPER SHEET
.-----
DXU=XLU/NPU
XWU(1)=XSUM
YWU(1)=YSU
NP2=NPU+1
DO 9 I=2,NP2
XWU(I)=XWU(I-1)+DXU
YWU(I)=AU*XWU(I)**2+BU*XWU(I)+CU
9 CONTINUE

.-----
. INCREMENTAL VALUE OF "X" AND "Y" FOR THE LOWER SHEET
.-----
XWL(1)=XSLM
YWL(1)=YSL
NP3=NPL+1
IF(XLL GE YLL)THEN
DXL=XLL/NPL
DO 12 I=2,NP3
XWL(I)=XWL(I-1)+DXL
YWL(I)=AL*XWL(I)**2+BL*XWL(I)+CL

```

```

12 CONTINUE
   ELSE
     DYL=YLL/NPL
     DO 36 I=2,NP3
       YWL(I)=YWL(I-1)+DYL
       XWL(I)=AL*YWL(I)**2+BL*YWL(I)+CL
36 CONTINUE
   END IF

```

```

* -----
*   NON-DIMENSIONALIZE ALL THE OUTPUT PARAMETERS
* -----

```

```

   DO 15 I=1,NP2
     XWUC(I)=XWU(I)/CHORDX
     YWUC(I)=YWU(I)/CHORDX
15 CONTINUE
   DO 16 I=1,NP3
     XWLC(I)=XWL(I)/CHORDX
     YWLC(I)=YWL(I)/CHORDX
16 CONTINUE

```

```

   RETURN

```

```

   END

```

```

* *****
*   SUBROUTINE OUTPUT
* *****

```

```

***** C-4 *****
*
*   This subroutine controls the writing out of all the results.
*
*****

```

```

COMMON X(90),Y(90),XN(90),XWU(30),YWU(30),XWL(30),
*       IC(90),YC(90),XWLC(30),YWLC(30),XWUC(30),
*       YWL(30),YWUC(30),
*       NPT,NPL,PI,XSU,YSU,XSL,YSL,ALPHA,NPUS,NPLS,
*       NPU,CHORDX,NK,NP2,NP3,ALPHA,AL,BL,CL,WL,
*       SL,SU,SM,AU,BU,CU,WAKEH,WLF,N,XLL,YLL,N1

```

```

WRITE(6,17)

```

```

17 FORMAT(10X,'OUTPUT RESULTS OF THE INITIAL WAKE PROGRAM')

```

```

WRITE(6,18)

```

```

18 FORMAT(10X,'*****')

```

```

WRITE(6,19)CHORDX

```

```

19 FORMAT(/,'THE AIRFOIL CHORD LENGTH IS',F8.4)

```

```

WRITE(6,44)ALPHA

```

```

44 FORMAT(/,'THE AIRFOIL ANGLE OF ATTACK IS',F8.4)

```

```

WRITE(6,20)AU,BU,CU

```

```

20 FORMAT(/,'THE COE. OF THE UPPER ARC:AU=',F8.5,2X,'BU=',F8.5

```

```

1 2X,'CU=',F8.5)

```

```

IF(XLL.GE.YLL)THEN

```

```

WRITE(6,40)

```

```

40 FORMAT('COE. FOR THE LOWER SHEET BASED ON Y=AX...')

```

```

ELSE

```

```

WRITE(6,41)

```

```

41 FORMAT('COE. FOR THE LOWER SHEET BASED ON X=AY...')

```

```

      END IF
      WRITE(6,21)AL,BL,CL
21  FORMAT('THE COE. OF THE LOWER ARC AL=',F9 5,2X,'BL=',F9 5
1    ,2X,'CL=',F9 5)
      WRITE(6,22)
22  FORMAT(/13X,'AIRFOIL DIMENSIONAL AND NON-DIMENSIONAL CO-OD')
      WRITE(6,23)
23  FORMAT(/10X,'I',10X,'X',10X,'Y',10X,'X\C',10X,'Y\C')
      DO 25 I=1,NPT
      WRITE(6,24)I,X(I),Y(I),XC(I),YC(I)
24  FORMAT(10X,I2,6X,F7 4,4X,F7 4,5X,F7 5,6X,F7 5)
25  CONTINUE
      WRITE(6,26)
26  FORMAT(/10X,'WAKE CO-ORDINATES')
      WRITE(6,27)
27  FORMAT(10X,'.....')
      WRITE(6,28)
28  FORMAT(/1X,'X-UPPER',2X,'Y-UPPER',3X,'X/C-UPPER',2X,
1    ,'Y/C-UPPER')
      DO 29 I=1,NP2
      WRITE(6,30)XWU(I),YWU(I),XWUC(I),YWUC(I)
30  FORMAT(F7 4,2X,F7 4,5X,F7 5,3X,F7 5)
29  CONTINUE
      WRITE(6,31)
31  FORMAT(/1X,'X-LOWER',2X,'Y-LOWER',3X,'X/C-LOWER',2X,
1    ,'Y/C-LOWER')
      DO 32 I=1,NP3
      WRITE(6,33)XWL(I),YWL(I),XWLC(I),YWLC(I)
33  FORMAT(F7 4,2X,F7 4,5X,F7 5,3X,F7 5)
32  CONTINUE
      WRITE(6,34)NPUS
34  FORMAT(/'THE UPPER SEPARATION PANEL NUMBER IS ',I4)
      WRITE(6,46)NPLS
46  FORMAT('THE LOWER SEPARATION PANEL NUMBER IS ',I4)
      WRITE(6,35)WAKEN,WL
35  FORMAT('THE WAKE HEIGHT=',F8 4,2X,'THE WAKE LENGTH=',F8 4)
      WRITE(6,43)WLF
43  FORMAT('THE WAKE LENGTH FACTOR IS',F8 4)
      WRITE(6,46)SU,SL,SM
46  FORMAT(/'UPPER SEP SLOPE',F8 4,2X,'LOWER SEP SLOPE',
1    ,F8 4,2X,'MID POINT SLOPE',F8 4)
      IF(NK EQ 1)THEN
      -----
      *   THE NEXT SET OF STATEMENTS GENERATE THE FILE IN THE
      *   PROPER FORMAT AND ORDER FOR USE WITH THE AIRFOIL
      *   PROGRAM   THIS IS GENERATED USING A SET [7] FORMAT
      *   -----
      N=NPT-1
      WRITE(3,66)N,CHORDX,WLF
      66  FORMAT(I4,1X,F8 5,1X,F4 2)
      WRITE(3,67)NPU,NPL
      67  FORMAT(I3,1X,I3)
      DO 47 I=1,NPT
      IF(X(I) EQ XWU(1) AND Y(I) EQ YWU(1))THEN
      WRITE(3,101)XWU(1),YWU(1)
101  FORMAT(F8 5,1X,F8 5)
      END IF

```

```
      IF (N1.EQ.1) THEN
      IF (X(I).EQ.XWL(1).AND.Y(I).EQ.YWL(1)) THEN
      WRITE(3,1)XWL(1),YWL(1)
1    FORMAT(F8.5,1X,F8.5)
      END IF
      END IF
      WRITE(3,102)X(I),Y(I)
102  FORMAT(F8.5,1X,F8.5)
47  CONTINUE
      WRITE(3,48).(XWU(I),YWU(I),I=1,NP2)
48  FORMAT(F8.5,1X,F8.5)
      WRITE(3,49).(XWL(I),YWL(I),I=1,NP3)
49  FORMAT(F8.5,1X,F8.5)
      END IF

      RETURN

      END
```

END

1 | 7 | 0 | 4 | 8 | 6

FIN