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**A Linear Elastic Fracture Mechanics Study
of the Size Effect on Fatigue Behaviour**

By Waryoba, Daudi C. Rigenda.

B.Sc. Eng., University of Dar es Salaam, TANZANIA, 1987

*A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of*

Master of Engineering

**Ottawa Carleton Institute of
Mechanical and Aeronautical Engineering
Carleton University
Ottawa, Ontario, CANADA.**

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Acceptance Sheet

The undersigned recommend to the Faculty of Graduate Studies and Research acceptance of the thesis, *A Linear Elastic Fracture Mechanics Study of the Size Effect on Fatigue Behaviour*, submitted by Waryoba Daudi C.R., in partial fulfillment of the requirements for the degree of Master of Engineering.

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Dedicated to my Parents.

A b s t r a c t

Experimental results show that the bending endurance limits of plain and notched specimens decrease as specimen size increases. Several theories have been suggested to account for this effect, but, although Fatigue is now understood to be a *Fracture Mechanics* phenomenon, there has been no attempt to explain the size effect using a Fracture Mechanics approach.

In this thesis, specimen size effect in Aluminium alloy 7075-T6 is analysed by *Linear Elastic Fracture Mechanics* (L.E.F.M) approach. The analysis is performed on smooth round specimens and on edge notched plates subjected to bending and axial loads, using analytical and numerical methods. For the numerical approach, two-dimensional finite element and boundary integral element programs were used. The results were then compared with published data, and they both show that stress gradient is indeed the governing factor. Finally, concluding remarks and recommendation for further analysis are also included.

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NOMENCLATURE¹

a	length of a crack, constant of material having a dimension of length.
a_f	final crack size.
a_o	initial crack size .
A	material constant (in Peterson's formula, $A = 1$).
A_K	material constant in equation 4.7.
b	Weibull slope or shape parameter. distance from the notch (in the direction of the crack front).
B_K	material constant in equation 4.7.
C	numerical proportionality factor given by elasticity theory, material constant in equation 4.7.
C_o	material constant in equation 4.7.
d	material constant in equation 4.7.
D	diameter, critical or smallest diameter of specimen, distance from the neutral axis to the extreme fibre for other shapes.
E	Young's modulus (modulus of elasticity).
f, F	function.
F_E	correction factor to account for elliptical crack fronts.
F_S	free surface correction factor.
F_W	finite width correction factor.
F_o	factor to account for non-uniform opening stress.
$F(N)$	Fraction failed in time or cycles N .

¹In some cases, particular symbols have more than one meaning in different parts of this thesis, although this should not cause any serious ambiguity, some are redefined within the contexts of particular pieces of analysis.

h	critical depth, depth below the surface where fracture starts.
K	stress intensity factor.
K_c	critical stress intensity factor (for unstable crack growth).
K_F	fatigue notch reduction factor.
K_I	mode I (opening mode) stress intensity factor.
K_{IC}	plane strain fracture toughness.
K_{Ic}	effective fracture toughness for surface or elliptical shaped crack.
K_N	Neuber technical factor.
K_{max}	maximum stress intensity factor.
K_{th}	threshold stress intensity factor.
K_T	theoretical stress concentration factor.
M	bending moment.
m	material constant in equation 4.7.
n	material constant in equation 4.7.
N	number of cycles.
N_a	Characteristic life (time or cycles when 63.2 % have failed).
N_f	number of cycles at failure.
N_o	number of cycles at $a = a_o$, i.e number of cycles during initiation phase.
q	notch sensitivity factor, material constant in equation 4.7.
r	radius of the profile of a specimen, in.
r_p	reversed plastic zone size.
R	radius, stress ratio.
p	material constant in equation 4.7.

S_e	endurance limit of direct stress specimen.
S'_e	endurance limit of bending specimen.
t	thickness of the plate, depth of the notch.
W	width of specimen.
ΔK	stress intensity factor range.
ΔK_0	threshold stress intensity factor range corresponding to $R = 0$.
$\Delta\sigma$	stress range.
ω	included angle of notch in radians, for notches in the form of v-grooves.
ν	poisson ratio.
ψ	relative stress gradient.
σ'	intrinsic fatigue strength.
σ	net section stress or surface stress determined by conventional engineering methods.
σ_a	the value of the applied stress at $b = a$.
σ_b	bending stress as obtained from the conventional elasticity formula $\sigma_b = My/I$), fatigue limit of small diameter specimens in rotating bending.
σ_{b_1}	normal stress in the finite element model of the structural detail where the crack will be inserted.
$\sigma_{b_{max}}$	maximum bending stress at outer fiber.
σ_B	fatigue limit for larger diameters in rotating bending.
σ_{RBD}	fatigue limit in rotating-bending of specimens with diameter D .
σ_{TC}	fatigue limit in tension-compression of standard laboratory specimens of the same material.

- σ_y stress in the y-direction,
0.3 % yield strength in equation 5.7.
- σ_{ys} yield strength.
- σ_∞ remote applied stress.
- ρ a material constant having dimension of length, and
representing half the width of the elementary block.
- ρ notch root radius.
- α term depending only on the state of stress and particular
angular orientation relative to the cracktip.
- λ ratio of $\frac{a}{D}$ or $\frac{a}{W}$.
- Φ geometrical factor = $f(a, W, D)$.
- $\Phi(\lambda)$ magnification or geometrical factor = $f(\lambda)$.
- Γ strain energy release rate.

Chapter 1

OBJECTIVE OF THIS THESIS

During the first half of the industrial era, structural failures were numerous and alarming. Boiler explosions, bridge collapses, railroad accidents, etc, became common problems. Apparently all were catastrophic fracture failures, and many were caused by a structural flaw and a slowly propagating crack, i.e fatigue failures. Researches were then directed on how to prevent these premature structural failures. Ever since then, an extremely large number of proposals for estimation of the fatigue strength have been forwarded. Many proposed formulas use fatigue strength reduction factor, K_F , as a correction factor in determining safe load.

However, most of these investigations were carried out using small laboratory specimens, typically in the region of about 8 mm (approximately 0.3 in) diameter. But in design it is often more important to know how to estimate the fatigue strength of larger parts with large diameters and notch radii, much larger than those commonly used in tests with laboratory specimens. It was soon learned that fatigue strength obtained from small specimens was higher than the corresponding value for large specimens of the same material having similar geometry. This brought a new era in fatigue research. The question then was how to use extensive results of tests

on small laboratory specimens for predicting the fatigue of much larger pieces.

Though many investigations have been done, there are still contradictory conclusions and difficulties in application of the results in the design of large components. Moreover, there is also a strong movement today to give a probabilistic interpretation to size effect in design [Shigley and Mitchell, 1983, Mischke, 1979, Haugen and Wirsching, 1975]. However, as we will see in section 2.2, this may lead to erroneous conclusions. This suggested a need to examine the problem from another direction, different from the usual classical analysis of notch factor and notch sensitivity. The objective of this thesis is, therefore, to try to explain size effect from a *Linear Elastic Fracture Mechanics* (L.E.F.M) point of view. A literature search did not find any report on an attempt to explain size effect using Fracture Mechanics concepts.

L.E.F.M assumes that fracture is caused by a structural flaw or a crack. A crack may grow, starting from a flaw or a stress concentration, and develop slowly in size due to repeated service loading. Crack advance results in a lower residual strength. To prevent failure, structural strength should not drop below a certain safe value. This means that cracks must be prevented from growing to a critical size at which catastrophic fracture will occur. In order to determine which size of crack is admissible, one must be able to calculate how the structural strength is affected by cracks (as a function of their size); and in order to determine the safe operational life, one must be able to calculate the time in which a crack grows to a critical size.

Fracture Mechanics is one of the methods that can be used to answer the above

questions. *Linear Elastic Fracture Mechanics* (L.E.F.M) is used when fracture is accompanied by small scale yielding (brittle fracture), i.e small fracture energy. Conversely, if fracture is accompanied by excessive plastic deformation (ductile fracture), then *Elastic-Plastic Fracture Mechanics* (E.P.F.M) is used.

Chapter 2

REVIEW OF THEORIES OF SIZE EFFECT

The tendency of a machine component to exhibit a lower fatigue strength, due to the dimensions, the shape, and the method of loading, is called *Size Effect*. This behaviour has been observed on various steels and non-ferrous alloys. Experimental results have shown that fatigue strength of plain specimens is independent of specimen size in direct stress tests [Phillips and Heywood, 1939]¹. However, there is a remarkable size effect when notched specimens are used. This effect has also been observed on plain specimens tested in bending [Moore and Morkovin, 1942, Grover *et al*, 1956] or torsion tests [Morrison, 1939]. Although the bending strength of a material is lower than the tensile strength, experimental results show that rotating bending endurance limit is higher than the endurance limit in reversed direct stress [Forrest and Tapsell, 1954]. Experiments on steel indicate that up to about 13 mm (0.5 in) diameter the bending or torsion strength decrease with size by about 15 percent, after which it appears to vary little up to about 50 mm (2 in) diameter [Lipson and Juvinall, 1963]. Figure A.1 illustrates this effect.

¹Figures in the brackets refer to references

Several theories have so far been suggested to account for the size effect in engineering materials. The following is a review of some of the explanations proposed by various authors.

2.1 Inherent flaw theory

It is apparent that all materials contain inherent flaws of one type or another distributed at random within a material, each having a characteristic length a [Heywood, 1952]. Inherent flaw theory is a probabilistic theory where failure is governed by the maximum stress and the probability of a flaw being present in the region of maximum stress [Frost *et al*, 1974]. Thus, there is a higher probability of failure in a plain specimen under uniform tension than at a root of a sharp notch under the same load. This may imply that Inherent flaw theory can be correlated with the stress gradient theory to be discussed in Section 2.5.

However, influence of internal flaws will depend on their size in relation to the region of high stress. That is, introduction of a mechanical notch of about the same size as a flaw will make little difference to the fatigue strength. Fatigue strength of cast iron (with graphite flakes acting as internal flaws) is almost completely insensitive to small external notches. The inherent flaw approach leads to the following equations,

$$K_F = \frac{K_T}{1 + 2\sqrt{a/\rho}} \quad [\text{Frost } et \text{ al, 1974}] \quad (2.1)$$

for notched specimens;

and

$$\sigma_b = \left(1 + 2\sqrt{a/R}\right) \sigma_B \quad [\text{Heywood, 1962}] \quad (2.2)$$

for plain round specimens.

2.2 Statistical Theories

Statistical theories assume that failure is initiated from some type of flaw, such as inclusions, grain-boundaries, etc., or dislocations which are randomly distributed throughout the material. A cumulative distribution function, which has shown to give a better correlation with the experimental data, was proposed by Weibull, W. [Weibull, 1961b].

$$F(N) = 1 - \exp \left[- \left(\frac{N - N_o}{N_a - N_o} \right)^b \right] \quad (2.3)$$

where N_o = Minimum time or cycles to failure.

The above is a "three-parameter Weibull function", which allows a simple graphical method to fit the distribution and hence to estimate the unknowns b , N_o , and N_a . On the other hand, the two-parameter Weibull model assumes the minimum life, N_o , of a population to be zero, therein leading to the following function.

$$F(N) = 1 - \exp \left[-(N/N_a)^b \right]$$

To estimate the probability that the life of a specimen reaches a certain value requires the lives of a batch of specimens (population) to be determined at the required stress level. However, although it is a simple matter to obtain these data,

it is not equally simple to obtain data from which to estimate the probability that a certain stress level will give failure in a given number of cycles, because it is impossible to subject each specimen of a batch to differing stress levels which will result in each specimen having the required life.

If failure is given on the basis of the statistical distribution of flaws, it is expected that this theory would predict the presence of a size effect with plain specimens loaded axially, depending on the volume of material under test. However, since a size effect has not been observed for steel specimens tested in axial loading, then statistical theories can not be used to explain size effect.

2.3 Elementary block concept

In [Neuber, 1946], Neuber suggested that there may be an elementary structural unit of length ρ , measured inwards from the notch root on the plane of minimum cross-section area over which the average cyclic stress must exceed the plain fatigue limit of the material before cracking can occur. That is, fatigue failure is governed by the average stress over an elementary block of finite size.

He suggested the following relationship to account for the effect of inelasticity for bodies having sharp notches.

$$K_N = 1 + \frac{K_T - 1}{1 + \frac{\pi}{\pi - \omega} \sqrt{\rho/\rho}} \quad (2.4)$$

Dimensions in Neuber's equation. Note: $\omega = 0$ for holes and for notches with parallel sides.

His factor has been used quite extensively and many other theories base their predictions on it. The material constant ρ has been shown to be a function of the tensile strength and is regarded to be related to the grain size [Kaary and Dolan, 1953]. Figure A.2 shows values of ρ for steels and aluminium alloys.

Kuhn, P. [Kuhn, 1958] found that the value of the Neuber constant ρ is about the same for Al 2024-T6 (Aluminium-Copper alloy) and for Al 7075-T6 (Aluminium-Zinc-Magnesium alloy), and that a value of $\rho = 0.02$ in (≈ 0.5 mm) gave reasonable agreement with the experimental results for both materials.

2.4 Empirical formula

From the general trend in experimental results, Heywood [Heywood, 1962] suggested the following empirical formula as an approximation of fatigue limit in bending.

$$\sigma_b = \left(1 + \frac{0.014}{0.1 + D^2} \right) \sigma_B \quad (2.5)$$

which gives fairly good results. For the hypothetical limiting case, when $D = 0$, the formula reduces to

$$\sigma_b = 1.14 \sigma_B$$

However, the relationship can only be used for smooth round specimens, it can not be used to predict fatigue strength of notched components in bending. It also predict a maximum size effect of about 14% which is not always the case.

2.5 Stress Gradient effect

If we consider the nature of the stress distribution on specimens tested in tension, bending and torsion, it will be noted that stress distribution is uniform over the test section in plain specimens tested in direct stress; while in bending or torsional load, there is a stress gradient over the test section. Similarly, for a notched specimen, there is a stress gradient across the critical cross section of the test piece due to the presence of the notch. This may signify that size effect is a consequence of the limiting of the maximum stress to a small volume of material. That is, under a constant bending load test, the average stress over a finite volume of a material increases as the size of specimen increases. Thus, as the specimen size increases, the stress gradient decreases, the average stress increases, and the fatigue strength decreases. Figures A.3 and A.4 show schematic representations of the effect of stress gradient on the average stress over the critical depth, h or a .

For notched specimens, if the lower fibres remote from the notch root are regarded as giving support to the higher stressed fibres at the notch root, by restricting their deformation, then the steeper the stress gradient the greater the supporting effect. If, the severity of the stress gradient, as well as the value of the maximum stress range has an influence on the notched fatigue limit, it follows that the fatigue limits of

batches of notched specimens all containing notches of the same stress concentration, K_T , values but of different geometries will depend on the stress gradients at the notch roots; the steeper the stress gradient, the higher the notched fatigue limit.

Taking the case of the bending test of round specimens, applying the well-known theoretical formula of stress gradient

$$\frac{d\sigma_b}{dy} = C \frac{\sigma_{max}}{R} \quad (2.6)$$

Peterson, Heywood, and Buch came up with the following empirical relationship

$$\sigma_b = \frac{\sigma_U}{1 - 2a/D} \quad (2.7)$$

where

- σ_B - intrinsic fatigue limit.
- = bending fatigue limit for larger diameter specimen ... [Heywood, 1962]
- = endurance limit for direct stress specimen ... [Peterson, 1959, Buch, 1974].
- a = h , depth below the surface ... [Peterson, 1959, Buch, 1974].

It can be noted that the formula breaks down in a limiting case of exceedingly small diameters, i.e when $D = 2a$. The formula is also limited to smooth round specimens. Furthermore, since the value of ' a ' and ' h ' are nearly the same, while the fatigue limit for larger diameters in bending is lower than the endurance limit in tension, then, the fatigue limit in bending computed using Heywood's formula would surely results to a lower value than the one computed using either Peterson's or Buch's proposal.

Chapter 3

LITERATURE SURVEY

There are numerous references in the literature on the effect of specimen size on fatigue limit. A few of these will be presented here.

In the development of his technical factor, Neuber¹ recognized that, as the notch radius approaches the dimensions of the order of the metallurgical grain size, the behaviour of materials departs from the homogeneous and isotropic behaviour assumed in the theory of elasticity. To account for this effect, he assumed that stresses within some finite volume of material are undetermined and that the actual maximum stresses at a sharp discontinuity approach a limiting value which would be computed if the notch radius were equal to the radius of this volume. His factor takes into account the size effect by incorporating a critical dimension of the notch, the notch radius.

Neuber also introduced the dependence of the notch sensitivity factor, q , on the specimen geometry.

¹See page 7, equation 2.4

$$\begin{aligned}
 q &= \frac{K_F - 1}{K_T - 1} \\
 &= \frac{1}{1 + \sqrt{\dot{\rho}/\varrho}}
 \end{aligned}
 \tag{3.1}$$

However, there is some discrepancy between the above formula and the experimental results obtained by Hyler, W.S *et al* [Hyler *et al*, 1954]. Figure A.5 shows the comparison between the two results.

Moore and Morkovin [Moore and Morkovin, 1942], in their progressive report on the effect of specimen size on fatigue strength of heat-treated steels, observed that S.A.E 1020 exhibits a maximum size effect of about 25 % of the endurance limit for 2 in. diameter specimens and a constant endurance for $\frac{1}{2}$ in. and larger diameters. Their results indicated that the endurance limit decreases with increasing size but tends to approach a lower limit, the "endurance limit of large specimens". Beyond 0.5 in. there seemed to be no indication of any further drop in endurance limit with further increase in size. Figures A.6 and A.7 show this trend.

In their third progressive report, Moore and Morkovin [Moore and Morkovin, 1944], considered inherent defects as the weakening factor. Their observations on hot-rolled S.A.E 1020 and S.A.E 1035 steels show that inherent defects may be regarded as the possible explanation of size effect. Applying Neuber's formula, they came up with the following empirical formula for estimating fatigue notch factor.

$$K'_F = 1 + \frac{K'_T - 1}{1 + \sqrt{\dot{\rho}/\varrho_D}}
 \tag{3.2}$$

where

$$K'_F = \frac{\text{unnotched fatigue strength for material devoid of defects}}{\text{fatigue strength of plain material with defects}}$$

K'_T - theoretical stress concentration factor for semicircular notch or defect of constant radius ρ_D (assumed to be 0.006 in for steel) i.e a small 'defect' is assumed to be equivalent in weakening the material under repeated stress to a semicircular notch with radius ρ_D .

However, in the discussion of the same report, Peterson could not agree with this observation. He explained that inherent defects may be a possible explanation in the heat-treated pieces but can not be applied to other steels. Instead he suggested another formula, based on Neuber's analysis, for calculating bending endurance limit of plain specimen. In his analysis, he assumed that failure occurs when the endurance limit is reached at a distance h beneath the surface (see figure A.8).

$$S'_e = \frac{S_e}{1 - 2h/D} \quad (3.3)$$

In [Kuguel, 1961], Kuguel R. explore size effect from the concept of highly stressed volume (HSV). His analysis was based on the fact that a decrease in fatigue strength accompanies an increase in the volume of material subjected to at least 95% of the maximum stress. Figure A.9 shows the variation of fatigue strength with the volume of stressed material.

Based on that concept he then came up with the following relationship between

the theoretical stress concentration factor, K_T , and the fatigue notch reduction factor, K_F .

$$\begin{aligned}
 K_F/K_T &= \left(0.8 \sqrt{\frac{r_n^3}{r_s D^2}}\right)^{0.034} \\
 &= (0.8)^{0.034} \left(\frac{r_n^{0.051}}{r_s^{0.017} D^{0.034}}\right) \quad (3.4)
 \end{aligned}$$

$\neq f(\text{material})$

subscripts

n - for notched specimens

s - for smooth specimens

However, the major drawback of this concept is the difficulty to determine the "highly stressed volume" in the vicinity of a discontinuity or area of stress concentration.

Findley W. N. [Findley, 1972] explained size effect based on the assumption that cyclic slip associated with the initiation of fatigue occurs over the entire area of the active plane in the group of the neighbouring crystals involved. It was further assumed that in order for such slip to occur, the shear stress on the active plane must reach at least a certain value, characteristic of the material, over the entire active plane. He then proposed the following formula for bending endurance strength, which is similar to the one given by Peterson.

$$\sigma = \frac{\sigma'}{1 - 2h/D} \quad (3.5)$$

Similarly, Buch A. [Buch, 1974], in explaining size effects in fatigue of aircraft materials, suggested a formula for endurance limit in bending similar to that of Peterson.

$$\sigma_{RB} = \frac{\sigma_{TC}}{1 - 2h/D} \quad (3.6)$$

However, he differed from Peterson in the expression for fatigue notch factor, K_F . He criticized the one-parameter formulas proposed by Neuber, Peterson, and Stieler; and suggested the following two-parameter formula.

$$K_F = \frac{K_T}{A} \left(1 - \frac{2.1h}{\varrho + \varrho_0} \right) \quad (3.7)$$

where $\varrho_0 = \frac{6.3h}{(3-A)}$ constant to avoid stress gradient becoming

infinite when the radius approaches zero, $\lim_{\varrho \rightarrow 0}$.

The two parameters, h and A , take into account the so called "support effect"; h takes into account the stress gradient support effect and A the cyclical strain hardening support effect. Table 3.1 shows different formulations of notch sensitivity factor, q .

The above are a few suggestions made by different authors on how to account

for the size effect. Though the authors have differed in their proposals, one thing is clear; stress gradient is a major factor in causing size effect and most of these proposals are based on Neuber's analysis. Furthermore, it is intriguing to note that although fatigue is now understood to be a Fracture Mechanics phenomenon, the stress gradient problem has not been examined in this light.

Equation	Material Parameter	References
$q = \frac{K_T - 1}{K_F - 1} = \frac{1}{1 + \sqrt{\dot{\rho}/\rho}}$	$\dot{\rho} = f(\sigma_B)$ $\dot{\rho} = 0.02\text{in. for Al alloys}$	[Kuhn and Hardrath, 1952]
$q = \frac{K_T - 1}{K_F - 1} = \frac{1}{1 + a/\rho}$	$a = 0.0025\text{in. for temp. steels}$ $a = 0.010\text{in. for norm. steels}$ $a = 0.026\text{in. for Al alloys}$	[Peterson, 1959]
$\frac{K_F}{K_T} = \frac{1}{1 + 2\sqrt{a/\rho}}$	$a = f(\sigma_B)$	[Heywood, 1962]
$\frac{K_F}{K_T} = \frac{1}{A} \left(1 - \frac{2.1h}{\rho + \rho_0} \right)$	$A, h = \text{const.}$ $h = 0.006\text{in. for Al alloys}$	[Buch, 1974]

Table 3.1: *Different formulations of notch sensitivity factor.*

Chapter 4

PROCEDURE

4.1 Fracture Mechanics: An Overview

Fatigue failure of engineering components and structures is due to fracture by crack propagation, and the way a crack propagates depends on the load applied to it. There are three distinct modes of fracture [Hertzberg, 1983], these are:

Mode I , which is often referred to as the *tensile opening mode*, is associated with loading which is symmetrical about the plane of the crack and tends to separate the crack faces.

Mode II , which is often referred to as the *shearing mode*, is associated with in-plane loading which is anti-symmetrical about the plane of the crack.

Mode III , which is often referred to as *tearing mode*, is due to out-of-plane shearing forces.

Mode I loading is the most common type of fracture, and in this thesis we will only consider this type.

In many fracture problems, elastic behaviour of the material is assumed and Linear Elastic Fracture Mechanics (LEFM) is used. However, there are situations where a significant amount of plastic deformation occurs in the vicinity of the crack tip, and Elastic-Plastic Fracture Mechanics must be used. In Linear Elastic Fracture Mechanics, the quantitative measure of the severity of a growing crack is given by the *stress intensity factor*, K , which characterizes the *intensity of the stress field* in a small region surrounding the crack front. This parameter is a function of the applied stress, the crack size, and the geometrical configuration of both the crack and the body in which it is embedded.

As the crack increases, the stress intensity factor increases, and the propagation rate, $\frac{da}{dN}$, increases. There are two extremes of the stress intensity factor (see figure A.10); one is the critical stress intensity factor that causes failure, called *fracture toughness*, K_C for plane stress or K_{IC} for plane strain, and the other is the lower limit below which fatigue damage is highly unlikely, called *threshold stress intensity factor*, K_{IA} .

Crack propagation may be either *stable* or *unstable*. Unstable or fast fracture frequently results in catastrophic failures and is sometimes referred to as *brittle fracture*, as there is usually very little plastic deformation of the material in the vicinity of the crack. Nevertheless, under fatigue conditions, stable crack growth is also unavoidable and may eventually lead to fast fracture. Thus, in order to determine the useful working life of a component, one has to be able to predict rates of crack growth.

4.2 Procedure

In this analysis, we will first determine the initial crack size, a_0 , which is the intrinsic crack length. We will then examine the effect of size on fatigue behaviour of smooth round specimens having surface cracks loaded in rotating bending. Since the analysis can be performed in closed form, TK-Solver Plus will be used in this case. Thereafter, we will explore the same effect on single edge-notched plates having surface cracks under axial loads. As there is no close-form solution for this case, a numerical solution using a finite element method will be used. We will also attempt to compare the analytical solution proposed by Albrecht and Yamada [Albrecht and Yamada, 1977] with the numerical solution. Finally, a boundary integral equation method will be used to analyse the size effect in single edge-notched plates loaded in bending. Figure A.11 shows the procedure carried out.

The analysis will be based on Linear Elastic Fracture Mechanics with the following conditions applied:

- Crack is sharp.
- Small scale yielding, i.e reversed plastic zone size is small compared to the crack length, $r_p \ll a$.
- Pre-existence of flaw, thus the useful life of the component is given by the number of cycles the macrocrack took to propagate from its initial size to failure.
- Nominal applied stress should not exceed the yield stress.

Material Properties	Alloy Al 7075-T6
Ultimate Tensile Strength (UTS)	87.0 ksi
Yield Strength (YS)	79.2 ksi
Reduction of Area (RA)	16.6 %
Elongation	12.2 %
Modulus of Elasticity (E)	10.5×10^6 psi

Table 4.1: *Material properties for Al 7075-T6.*

[Hyler *et al*, 1954]

- Assumed that crack propagation is controlled by plane strain conditions along the crack front.
- Assumed that cyclic loading is constant, and crack closure in the plane strain region is either non-existent or negligible.

Table 4.1 gives the properties of the Aluminium alloy 7075-T6 used in this thesis.

4.3 Governing Equations

In general, the stress intensity factor is defined as

$$K = \sigma \Phi \sqrt{\pi a} \quad (4.1)$$

The stress intensity factor range is defined as

$$\begin{aligned} \Delta K &= K_{max} - K_{min} \\ \text{and stress range, as } \Delta \sigma &= \sigma_{max} - \sigma_{min} \end{aligned} \quad (4.2)$$

The stress ratio is defined as

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

The analysis was performed for $R = 0$ (i.e. $\sigma_{min} = 0$) and crack-closure effect was not considered.

Fracture toughness, K_c , can be estimated from the following empirical relationship proposed by Irwin [Irwin, 1967]

$$K_c^2 = K_{IC}^2 (1 + 1.4 \beta_{IC}^2) \quad (4.3)$$

$$\text{where } \beta_{IC} = \frac{1}{t} \left(\frac{K_{IC}}{\sigma_{ys}} \right)^2$$

t — thickness of specimen

or from the following relation [Forman *et al*, 1986] which is a modification of the relationship proposed by Vroman [Vroman, 1983]

$$\frac{K_c}{K_{IC}} = 1 + B_K e^{\omega} \quad (4.4)$$

where

$$\omega = \left(A_K \frac{t}{t_o} \right)^2$$

$$t_o = 2.5 \left(\frac{K_{IC}}{\sigma_{ys}} \right)^2$$

See table 4.2 for the values of A_K , and B_K .

Many researchers have expressed fatigue propagation rate with a number of empirical and theoretical laws, but the stress intensity factor has been shown to be a key parameter controlling crack propagation. Paris [Paris, 1962] has expressed crack growth rates in terms of the stress intensity factor range, ΔK , in the form

$$\frac{da}{dN} = C \Delta K^n \quad (4.5)$$

whereas, Forman *et al* [Forman and Hu, 1984] have proposed crack propagation rates in terms of ΔK , K_C , and R in the form

$$\frac{da}{dN} = \frac{C \Delta K^n}{(1 - R) K_c - \Delta K} \quad (4.6)$$

In this thesis, the following modified Forman's equation [Forman *et al*, 1986] was used in the calculation of crack growth rate.

$$\frac{da}{dN} = \frac{C (1 - R)^m \Delta K^n (\Delta K - \Delta K_{th})^p}{((1 - R) K_c - \Delta K)^q} \quad (4.7)$$

Putting $m = p = q = 0$ results in the most frequently used Paris equation,
 whereas with $m = p = 0, q = 1$ results in Forman's equation.

Equation 4.7 incorporates both:

Instability condition, when $(1 - R) K_c - \Delta K \leq 0$

and

Threshold condition, when $\Delta K - \Delta K_{th} \leq 0$

The fatigue threshold, ΔK_{th} , is calculated using the following equation

$$\Delta K_{th} = (1 - C_o R)^d \Delta K_o \quad (4.8)$$

Putting $C_o = 1$ results in Klesnil and Lukas equation [Klesnil and Lukas, 1972],
 whereas with $d = 1$ results in Barsom equation [Barsom, 1964].

See Table 4.2 for the values of m , n , p , q , d , C_o , ΔK_o , and K_c for Aluminium alloy 7075-T6.

Number of stress cycles to failure, during the propagation phase, can be obtained by integrating equation 4.7. If Φ in equation 4.1 is assumed to be independent of the crack length, this will lead to:

$$\begin{aligned} \Delta N &= N_f - N_o \\ &= \int_{a_o}^{a_f} f(a) da \end{aligned} \quad (4.9)$$

$$\text{where } f(a) = \frac{((1 - R) K_c - \Delta K)^q}{C (1 - R)^m \Delta K^n (\Delta K - \Delta K_{th})^p}$$

In actual fact, Φ varies with crack length; therefore integration has to be performed using one of the numerical methods, e.g Simpson rule. In that case, equation 4.9 can be approximated as

$$\Delta N = \left(f_o + f_n + 4 \sum_{i=1, \text{ odd}}^{n-1} f_i + 2 \sum_{i=2, \text{ even}}^{n-2} f_i \right) \frac{\Delta a}{3} \quad (4.10)$$

where $f_o = f(a) |_{a=a_o}$

$f_n = f(a) |_{a=a_f}$

$\Delta a = \frac{a_f - a_o}{n}$

n - number of integration intervals

Material Constants	Alloy Al 7075-T6
K_{Ic}	42 ksi $\sqrt{\text{in}}$
K_{IC}	27 ksi $\sqrt{\text{in}}$
A_K	0.75
B_K	1.25
C	2.75×10^{-8}
n	2.836
m	0
$p = q$	1
$C_o = d$	1
ΔK_o	2.5 ksi $\sqrt{\text{in}}$

Table 4.2: Values of constants in the Forman equation for Al 7075-T6.

[Forman et al, 1986]

Chapter 5

DETERMINATION OF INITIAL CRACK SIZE

The initial crack size, a_o , has to be established before hand; because fatigue limit and number of cycles to failure depend, among other factors, on its value. In the determination of the initial crack size, the distinction between short, long, and non-propagating fatigue cracks should be clearly understood.

It is well documented that cracks (short cracks) do grow below the threshold stress intensity factor, K_{Ih} , when high stresses are applied. The small crack problem in aluminium alloys was first addressed by Pearson [Pearson, 1975]. He observed that, for several precipitation hardened alloys, cracks of the order of, or less than, the grain size tended to grow faster than large cracks driven by corresponding stress intensities. As such, a crack may be considered to be born as soon as the slip band is created. Thus, threshold stress intensity factor, K_{Ih} , is no longer a lower bound solution, below which the crack will not grow, but an upper bound solution, above which the crack will propagate to failure.

Miller [Miller, 1984] has outlined three phases of crack growth namely:

- i) A short crack growth regime, which is dependent on stress level and the microstructure.
- ii) A longer crack growth regime, which is applicable to long cracks (as defined by LEFM)
- iii) A transition phase, in which the growth rate is the sum of (i) and (ii).

Should (i) and (ii) not overlap, then non-propagating cracks occur. This implies that, the critical stage is not related to the birth of a crack, but to the period before the attainment of a minimum crack growth rate. Figures A.12 and A.13 show the behaviour of long, short, and non-propagating cracks.

Comparison of the relationship proposed by Hertzberg [Hertzberg, 1983] to account for the effect of the plastic zone size

$$K_{eff} \approx \Phi \sigma \sqrt{a + r_p} \quad (5.1)$$

and that proposed by Haddad *et al* [Haddad *et al*, 1979]

$$\Delta K = \Delta \sigma \sqrt{\pi(a + a_o)} \quad (5.2)$$

reveals the close relationship between the initial crack size, a_o , and the plastic zone size, r_p .

According to L.E.F.M definition, a crack is long if it satisfies the condition, $a \gg r_p$. Under uniaxial loading, the reversed plastic zone size, r_p , can be approximated

as

$$r_p \approx \frac{1}{2\pi} \left(\frac{K}{\sigma_{ys}} \right)^2 \quad \text{plane stress} \quad (5.3)$$

or

$$r_p \approx \frac{1}{6\pi} \left(\frac{K}{\sigma_{ys}} \right)^2 \quad \text{plane strain} \quad (5.4)$$

Since we know that the plastic zone increases as stress intensity factor increases, and assuming the maximum size occurs when

$$K = K_{IC} \text{ (or } K_c) \quad (5.5)$$

we can then estimate the initial crack size as

$$a_o \approx \frac{1}{6\pi} \left(\frac{K}{\sigma_{ys}} \right)^2 \quad \text{assuming plane strain prevails} \quad (5.6)$$

For Al 7075-T6, $K_{IC} = 25 \text{ ksi} \sqrt{\text{in}}$. Thus,

$$a_o \approx \frac{1}{6\pi} \left(\frac{25}{79} \right)^2 [\text{in}]$$

$$a_o \approx 5.313 \times 10^{-3} \text{ in}$$

The above value corresponds to the one given by Pearson [Pearson, 1975], who did experiments on two commercial aluminium alloys (Al-Cu-Mg-Si-Mn and Al-Zn-Mg-Cu-Cr) in a cantilever machine to observe the initiation of fatigue cracks at a plane polished surface and the subsequent growth of very short cracks. He found that the growth for short cracks tended towards that predicted from long crack [$> 0.25 \text{ mm}$ (0.01 in.)] for crack depths greater than about 0.13 mm (0.005 in.). It is also in agreement with material constant given by Buch [Buch, 1974] (as a critical depth, h) for aluminium alloys.

On the other hand, Lankford and Davidson [Lankford *et al*, 1976, Lankford and Davidson, 1977] on their work on cracktip plasticity in aluminium alloys by carrying out electron channeling experiments, found that the maximum plastic zone size is related to the stress intensity (near-zero stress ratio) by

$$r_p = \alpha \left(\frac{K_{max}}{\sigma_y} \right)^2 \quad (5.7)$$

For Al 7075-T6 $\alpha \approx 0.7$

In [Lankford, 1982], Lankford, in his investigation of small fatigue cracks in Al 7075-T6 (having a 0.3% yield strength of 515 MPa), observed that the behaviour of large and small cracks begins to coincide around a ΔK of $5 \text{ MPa} \sqrt{\text{m}}$. This suggests that the maximum plastic zone size in Al 7075-T6 can be approximated as

$$\begin{aligned} r_p &\approx 0.7 \left(\frac{5}{515} \right)^2 [\text{m}] \\ &\approx 65.98 \mu\text{m} \end{aligned} \quad (5.8)$$

which is in the order of the size of the grain. Lankford [Lankford, 1982] reported a maximum grain size of $80 \mu\text{m}$.

It is of interest to note that twice that value yields

$$\begin{aligned} 2r_p &\approx 2(65.98) = 131.96 \mu\text{m} \\ &\approx 0.0052 \text{ in} \end{aligned}$$

which also corresponds to the values given by Pearson, and Bush.

However, Findley [Findley, 1972], on explaining size effect, suggested that multiple slip required by the constraints associated with polycrystalline material requires

slip on at least five independent slip systems; and thus, the plastic region would have to be at least two or three grains deep in order to develop these constraints. Approximating the initial crack size as being two grains deep, and taking the size of the grain as being equivalent to the calculated plastic zone size, yields

$$\begin{aligned}a_o &\approx 2(65.98) = 131.96 \mu\text{m} \\&\approx 0.0052 \text{ in} \\&\approx 2r_p \text{ as before}\end{aligned}$$

whereas, assuming it to be three grains deep gives

$$\begin{aligned}a_o &\approx 3(65.98) = 197.94 \mu\text{m} \\&\approx 0.008 \text{ in}\end{aligned}$$

Note that if we had assumed the grain size as being twice the computed plastic region, the above would have yielded

$$\begin{aligned}a_o &\approx 2(131.96) = 263.92 \mu\text{m} \\&\approx 0.01 \text{ in} \\&\text{or} \\a_o &\approx 3(131.96) = 395.88 \mu\text{m} \\&\approx 0.016 \text{ in}\end{aligned}$$

which correspond to Neuber's material constant, ρ for aluminium alloys. On the other hand, using the value of the grain size obtained by Lankford [Lankford, 1982] would have yielded

$$a_o \approx 2(80) = 160 \mu\text{m}$$

$$\approx 0.006 \text{ in}$$

or

$$a_o \approx 3(80) = 240 \text{ } \mu\text{m}$$

$$\approx 0.01 \text{ in}$$

Other authors [Haddad *et al*, 1979, Taylor, 1986], have expressed the initial crack size as a function of the grain size, d . It has been postulated that the initial crack size can be estimated as

$$a_o \approx 10d$$

or

$$a_o \approx 10r_p \quad \text{whichever is larger.}$$

where d - characteristic microstructural dimension which is usually equal to the grain size but becomes carbide lath width or precipitate spacing in microstructures dominated by fine precipitates.

Now if we take $d = 80 \text{ } \mu\text{m}$ (Lankford observation [Lankford, 1982])

and $r_p \approx 66 \text{ } \mu\text{m}$ (from equation 5.8)

then initial crack size can be approximated as

$$a_o \approx 10(80) \text{ } \mu\text{m}$$

$$\approx 0.8 \text{ mm (0.03 in)}$$

or

$$a_o \approx 10(66) \text{ } \mu\text{m}$$

$$\approx 0.66 \text{ mm (0.026 in)}$$

Both values are around the critical depth, h , for aluminium alloys given by Peter-

son [Peterson, 1959].

As there is no definite answer to which initial crack size should be used, three values of initial crack sizes ($a_0 = 0.008, 0.01, \text{ and } 0.02 \text{ in}$) were chosen; and were used to evaluate the number of cycles to failure, and the propagation rates at different specimen diameters. The results were then compared with the experimental results, and the initial crack size which gives a curve that lies along the experimental curve was then chosen.

Chapter 6

ROTATING BENDING WITH SMOOTH ROUND BARS

For solid cylinders under rotating bending, the stress intensity factor is given by

$$K = \sigma_{b_{max}} \Phi(\lambda) \sqrt{\pi a} \quad (6.1)$$

Crack model: A solid cylinder under Rotating bending.

According to Forman and Shivakumar [Forman and Shivakumar, 1986], the magnification factor for the above model is given by

$$\Phi(\lambda) = g(\lambda) [0.923 + 0.199 Y^4] \quad (6.2)$$

$$\text{where } g(\lambda) = 0.92 (2/\pi) \sec \beta [(\tan \beta)/\beta]^{\frac{1}{2}}$$

$$Y = 1 - \sin \beta$$

$$\beta = \lambda \frac{\pi}{2}$$

In the modified Forman equation 4.7 (page 23), the value of p and q given in reference [Forman *et al*, 1986] was 0.5 for both; however, in order to fit the experimental data of [Hyler *et al*, 1954], their values had to be changed to 1.

6.1 Procedure

In the study of the effect of specimen size on fatigue behaviour of smooth round specimens in bending, six different specimen diameters were selected, 0.125 in., 0.25 in., 0.5 in., 1.0 in., 1.75 in., and 2.0 in., so that the results can be compared with the experimental results of [Hyler *et al*, 1954].

The following procedure was used to determine number of stress cycles to failure at different maximum bending stresses.

Step 1: Obtain material properties and constants, e.g plain strain fracture tough-

ness, K_{IC} , threshold stress intensity factor, K_{IA} , initial crack size, a_o , etc.

Then for each of the above diameters

Step 2: Select applied stresses such that $\sigma_{max} < \sigma_y$.

and for each of the above stresses

Step 3: Calculate critical crack length, a_f .

Step 4: Select integration steps, n , between a_o and a_f . Then using TK solver
Plus, calculate number of stress cycles during crack propagation by integrating
equation 4.9.

Step 5: Repeat steps 3–4 for each of the above stresses.

Step 6: Repeat steps 2–4 for each of the above diameters.

Step 7: Get results, compare with experimental results.

Figure A.14 shows the procedure that was used to determine the SN-curves. The same procedure was used to evaluate stress cycles at different crack lengths. In this analysis, plane strain condition was assumed to prevail throughout the crack propagation. In that case, the critical crack length was obtained by substituting K_{IC} in equation 6.1.

In the determination of the crack propagation rate, $\frac{da}{dN}$, the following algorithm was used.

Step 1 3: Same as in the determination of number of stress cycles to failure.

Step 4: Select crack sizes between a_o and a_f

Then for each of the above crack sizes

Step 5: Calculate stress intensity factor range, ΔK .

Step 6: Using equation 4.7, calculate crack propagation rate, $\frac{da}{dN}$.

Step 7: Repeat steps 5 and 6 for each of the above crack lengths.

Step 8: Repeat steps 3-6 for each of the above stresses.

Step 9: Repeat steps 2-6 for each of the above diameters.

Figure A.15 shows the procedure used to determine propagation rate at different crack lengths and specimen diameters, and it was also employed to determine strain energy release rates.

Strain Intensity factor range, in this case, was defined as

$$\Delta \epsilon = \frac{\Delta K}{E^*} \quad (6.3)$$

where

$$E^* = \begin{cases} E & \text{Plane stress} \\ \frac{E}{1 - \nu^2} & \text{Plane strain} \end{cases}$$

Strain energy release rate was defined as

$$\Gamma = \frac{K^2}{E^*} \quad (6.4)$$

or

$$\Gamma = \frac{\kappa + 1}{8\mu} K^2$$

where

$$\kappa = \begin{cases} 3 - 4\nu & \text{Plane stress} \\ \frac{3 - \nu}{1 + \nu} & \text{Plane strain} \end{cases}$$

$$\mu = \frac{E}{2(1 + \nu)}$$

6.2 Results and Discussion

1. The results of this work show that fatigue limit depends on the assumed initial crack size—the depth at which a microcrack developing in a plain specimen starts to grow as a macrocrack. The longer the initial crack size, the lower the fatigue limit, the faster the propagation rate to failure, the shorter the useful life of the component.

Figures A.16 through A.20 show that for a better agreement with experimental results, the value of this changeover depth varies from 0.008 inch (≈ 0.2 mm) to 0.1 inch (≈ 0.25 mm), within an accuracy of about 80 percent.

2. It has been verified that the value of the initial crack size is in the order of the maximum plastic zone size, and can be compared to the values of material constants given by Neuber (ρ), Peterson (a), and Buch (h).
3. Figure A.16 shows predicted and experimental S-N curves for a smooth round specimen of 0.125 inch diameter at three different initial crack sizes, a_o , of 0.008, 0.01, and 0.02 inch. As can be seen, to fit the experimental curve, the value of the initial crack size should be within the range 0.008 inch to 0.01 inch.
4. Figure A.17 shows predicted and experimental S-N curves for a smooth round specimen of 0.25 inch diameter. Here again the results show that the value of the initial crack size lies within the range 0.008 inch to 0.01 inch.
5. Figure A.18 shows predicted and experimental S-N curves for a smooth round specimen of 0.5 inch diameter. The figure shows the experimental curve to be eclipsed within the two predicted curves, verifying that the value of the initial crack size lies within the range 0.008 inch to 0.01 inch.
6. Figure A.19 shows predicted and experimental S-N curves for a smooth round specimen of 1.0 inch diameter. In this case, the results show that the initial

crack size should be smaller than 0.01 inch but still in the range of 0.008 ± 0.001 inch.

7. Figure A.20 shows predicted and experimental S-N curves for a smooth round specimen of 1.75 inch diameter. In this case, the results show that the initial crack size should be 0.08 inch or smaller.
8. Another feature observed from figures A.16–A.20 is the dependence of the component's life on the initial crack size. It can be seen that as the initial flaw size increases, specimen's life decreases.
9. Figures A.21 and A.22 show predicted S-N curves for smooth round specimens of 0.125, 0.5, and 1.0 inch diameters, respectively, at initial flaw sizes of 0.008 inch and 0.01 inch. Both curves show that size effect occurs at long life, beyond 3×10^6 cycles.
10. Figure A.23 shows relative endurance limit versus specimen diameter. The figure shows a drop of the endurance limit with size of about 0.05%, which is significantly small. The small predicted size effect can be explained from the fact that the difference of the relative stress gradient, χ , of these specimens was also small, in the range $0.0225\text{--}0.315 \text{ in}^{-1}$. Moreover, the difference in cross-section that a macrocrack has to traverse makes little difference to the total number of cycles to failure.

11. Figures A.24 and A.25 show experimental S-N curves for smooth and notched round specimens of 0.125, 0.25, 0.5, 1.0 and 1.75 inch diameters, respectively.
12. Comparison of figures A.21 and A.22 with A.24 shows some discrepancy between the predicted and the experimental results of [Hyler *et al*, 1954]. Predicted results shows a marginal size effect at long life, whereas, for the same specimen sizes, experimental results show a noticeable and a peculiar trend in size effect. Experimental results show that fatigue limit decreases with size for diameters from 0.125 inch to 0.5 inch, thereafter it increases for all diameters greater than 0.5inch. The reason for such behaviour is unknown.
13. Figure A.26 shows crack length versus stress cycles for three smooth round specimens of diameters, 0.25, 0.5, and 1.75 inch, at a maximum bending stress of 25 ksi.

Figure A.27 shows crack length versus stress cycles for three smooth round specimens of diameters, 0.25, 0.5, and 1.75 inch, at a maximum bending stress of 50 ksi.

Figure A.28 shows crack length versus stress cycles for smooth round specimen of diameter 0.25 inch, at two maximum bending stresses of 25 ksi and 50 ksi.

All three figures (A.26-A.28) show the dependence of the component's life on the component's size and the applied load. Results show that component's life

decreases as the component size increases and as applied load increases. These same results were obtained by Weibull [Weibull, 1961a] when investigating the effect of size and stress history on fatigue crack initiation and propagation.

14. Figure A.29 shows how the rationalized stress intensity factor, $\frac{\Delta K}{\sigma}$, varies with crack ratio, $\frac{a}{D}$. From this figure, we can see that the stress intensity factor increases as the crack length increases, as well as the specimen size increases.

Figure A.30 shows the propagation rate, $\frac{da}{dN}$, versus crack ratio, $\frac{a}{D}$. It can be seen that the propagation rate increases as the crack length increases, as well as the specimen size increases.

Figure A.31 shows the propagation rate, $\frac{da}{dN}$, versus stress intensity factor range, ΔK , strain intensity factor range, $\frac{\Delta K}{E}$, and strain energy release rate, Γ , for three diameters 0.25, 0.5, and 1.75 inch. The figure shows that propagation rate increases as stress intensity factor range, strain intensity factor range, and strain energy release rate increase; and that the stress intensity factor range is the controlling parameter for fatigue crack propagation.

From these figures we can deduce that the decrease of the component's life with size is due to the increase of the propagation rate as specimen size increases. The increase of the propagation rate with size, and a subsequent decrease of the component's life with size, is due to the increase of the stress intensity factor range with size.

It should be understood that since the state of stress was assumed to be plane strain, then all specimen were exhibiting the same ΔK at the point of failure. However, the state of stress varies with specimen size; small diameters experiencing plane stress and plane strain for larger diameters. This results in a higher ΔK_c for a smaller specimen than for a larger one, thereby decreasing it's rate of crack propagation at fast fracture region (from equation 4.7).

15. Figure A.32 shows variation of $\Phi(\lambda)$ as a function of relative crack depth, $\frac{a}{D}$, for a solid cylinder loaded in bending (from [Forman and Shivakumar, 1986]).

In a hypothetical sense, we can see that, ignoring other weakening factors such as environment effect, surface effect, crack-closure effect, etc, fatigue limit can be expressed in terms of threshold conditions by the following simple formula

$$\sigma_L = \frac{K_{th}}{\Phi(\lambda) \sqrt{\pi a_o}}$$

From this figure we can see that within the range

$$0 \leq \lambda \leq 0.15, \quad \Phi(\lambda) \text{ decreases with } \lambda$$

$$\text{while for } \lambda > 0.15, \quad \Phi(\lambda) \text{ increases with } \lambda$$

Since K_{th} and a_o are material constants, then fatigue limit is inversely proportional to $\Phi(\lambda)$

$$\sigma_L \propto \frac{1}{\Phi(\lambda)}$$

Taking the initial crack size as $a_o = 0.01$ in., then fatigue limit will decrease with size for all diameters greater than 0.07 in.

From this analysis, we have observed that fatigue life decreases as specimen size and applied load increase; and crack propagation rate increases as crack length and specimen size increase. We can then deduce that the decrease of specimen's fatigue life as specimen size increases is associated with the increase of the propagation rate with size, i.e the bigger the specimen, the higher the propagation rate, the shorter the useful life.

We can also say that the decrease of the fatigue limit with size is also attributed to the increase of the propagation rate with size. Since we know that all specimens exhibit the same threshold stress intensity factor and initial crack propagation rate regardless of their size, then from equation 4.7, to keep the propagation rate constant, the observed increase of the propagation rate with size should consequently result in a decrease of stress intensity factor range required to propagate a flaw from its initial size. This, of course, results in a decrease of the fatigue limit with size.

Chapter 7

SPECIMEN SIZE EFFECT IN NOTCHED SPECIMENS

7.1 Edge Notched Plates under Axial load

It is a fact that in most fatigue behaviour analyses based on Linear Elastic Fracture Mechanics, such as prediction of fatigue life or crack propagation rate, the foremost parameter to be determined is the Stress Intensity Factor (SIF) at different crack lengths.

Unfortunately, closed form analytical solutions for stress intensity factor exist only for a limited number of idealized geometries. Numerical solutions have to be employed for all practical problems like cracks in structural details. Among the numerical techniques available, the two most convenient and most extensively used methods for accurate evaluation of stress intensity factor are the Finite Element Method (F.E.M) and the Boundary Integral Equation (B.I.E) method (sometimes referred to as Boundary Element Method (B.E.M)).

Computation of stress intensity factor by finite element method becomes costly and time consuming when several cases have to be analysed. In that case boundary integral equation method becomes favourable. There exist various approximate calibrations of the mode I stress intensity factor K_I proposed for a crack emanating from a notch root. These approximate solutions are broadly presented by Skorupa *et al* [Skorupa *et al*, 1987].

In this thesis, two of these methods were used; the one by Albrecht and Yamada [Albrecht and Yamada, 1977], and the other one proposed by Jergus [Jergus, 1978]. Both are valid for all crack lengths, i.e short and long cracks. The Albrecht and Yamada method requires the determination of the correction factor that accounts for the non-uniformity of the stress field at a structural detail. That is, expressing the stress intensity factor as

$$K = F(a) \sigma \sqrt{\pi a} \quad (7.1)$$

the value of $F(a)$ is then given by

$$F(a) = F_E F_S F_W F_\sigma \quad (7.2)$$

If the stress distribution in an uncracked body is given in function form (see figure A.33), then Albrecht and Yamada expressed F_σ as

$$F_\sigma = \sqrt{\pi a} \left[\sigma_a - \frac{2}{\pi} \int_0^a \arcsin\left(\frac{b}{a}\right) \frac{d\sigma}{db} db \right] \quad (7.3)$$

If the stress distribution is given in discrete form rather than in function form (see figure A.34), F_σ is expressed as

$$F_s = \frac{2}{\pi} \sum_{i=1}^n \frac{\sigma_{b_i}}{\sigma} \left[\arcsin\left(\frac{b_{i+1}}{a}\right) - \arcsin\left(\frac{b_i}{a}\right) \right] \quad (7.4)$$

Jergus presented K_I in the following form

$$K_I = 1.122\sigma \sqrt{\pi(a + \acute{e})} \quad (7.5)$$

where $\acute{e}$ - is the notch contribution to the crack size given by

$$\acute{e} = t \left\{ 1 - \exp \left[-4 \left(1 + \sqrt{t/\rho} \right) a/\sqrt{t\rho} \right] \right\}$$

For finite width geometries, the Jergus concept has to be adjusted by F_W .

The finite element method requires discretisation of the entire domain or the symmetrical part of the domain. Figure A.35 shows the discretised region and boundary conditions for a single edge-notched plate under a remote axial stress, σ_∞ . Figures A.36 -A.40 show different meshes used in the finite element program. It is worthwhile to note that the stress intensity factor results are sensitive to element size, thus a slight change of mesh design at the cracktip would naturally cause a relatively large change of the results, especially for very short crack lengths which are still within the influence of the notch. However, virtual crack extension approach with quarter-point elements gave results which were less mesh-sensitive.

The finite element program by Fawkes and Owen [Owen and Fawkes, 1983] was used in this thesis. To determine its accuracy for short cracks, an unnotched model of width $W = 100$ mm, crack length $a = 3$ mm, and under an axial load of $\sigma_\infty = 100$ MPa was used. The actual solution for this model is $K_I = 345.34$ MPa $\sqrt{\text{mm}}$.

Finite element with virtual crack extension method ($\Delta a = 0.001$ mm) yields $K_I = 336.3$ MPa $\sqrt{\text{mm}}$, an error of about -2.6% .

The two methods, Albrecht *et al.* and Jergéus, were also compared with finite element results. The method by Albrecht *et al.* gave better approximations for all crack lengths; however, for long cracks, both methods gave far lower approximations and are inferior to the usual LEFM method [Tada *et al.*, 1973]. It was observed that for all crack ratios $a/W > 0.4$, the long crack L.E.F.M method gave better approximation than both of them.

In this study of the effect of specimen size on notched plates subjected to axial load, six different models were used, each having different stress distribution characteristic and different relative stress gradient, χ , but having the same stress concentration factor, K_T . This was achieved by changing the notch root radius and / or the notch depth. In this way, it was possible to determine the effect of the stress gradient without the influence of the stress concentration factor.

The six models consist of the following sizes

- Width $W = 100$ mm, with relative stress gradient $\chi = 2.275 \text{ mm}^{-1}$
- Width $W = 50$ mm, with $\chi = 1.502 \text{ mm}^{-1}$
- Width $W = 50$ mm, with $\chi = 1.665 \text{ mm}^{-1}$
- Width $W = 50$ mm, with $\chi = 3.018 \text{ mm}^{-1}$
- Width $W = 75$ mm, with $\chi = 1.945 \text{ mm}^{-1}$

- Width $W = 25$ mm, with $\chi = 1.39 \text{ mm}^{-1}$

7.1.1 Procedure

The procedure used to determine number of cycles to failure and the propagation rate is similar to the one used in the closed form solution for smooth round specimens, however, in this case stress intensity factor at different crack lengths has to be determined by the method of Albrecht and Yamada or by numerical solution using finite element method.

Step 1: Model specimens in such a way that all have the same stress concentration factor but different stress gradients.

Step 2: Obtain material properties and constants, e.g plane strain fracture toughness, K_{IC} , threshold stress intensity factor, K_{th} , initial crack size, a_o , etc.

For each of the above six models

Step 3: Select applied stresses such that $\sigma_{max} < \sigma_{ys}$.

and for each of the above stresses

Step 4: Determine critical crack length, a_f , by trial and error method.

Step 5: Select crack sizes at equal intervals between a_o and a_f . Then for each of the above crack sizes, calculate stress intensity factor range, ΔK .

Step 6: Select integration steps between a_o and a_f , then using Simpson rule, calculate number of stress cycles during crack propagation from equation 4.11.

Step 7: Repeat steps 3-4 for each of the above stresses.

Step 8: Repeat steps 2-4 for each of the above specimens.

Step 9: Get results, compare with experimental results.

In the case of the determination of the crack propagation rate, the same procedure was used.

Step 1-4: Same as above.

Step 5: Select crack sizes between a_o and a_f

Then for each of the above crack sizes

Step 6: Calculate stress intensity factor range, ΔK .

Step 7: Using equation 4.7, calculate crack propagation rate, $\frac{da}{dN}$.

Step 8: Repeat steps 6 and 7 for each of the above crack lengths.

Step 9: Repeat steps 4-7 for each of the above stresses.

Step 10: Repeat steps 3-7 for each of the above models.

7.2 Edge Notched Plates under Bending load

One of the bright features of all boundary integral equation methods is that instead of attempting to find an approximate solution for the governing differential equation throughout the relevant solution domain, as is done in the finite element method, the equation is converted into an integral form, normally involving only integrals over the boundary of the domain. Thus, only the boundary has to be discretised in order to carry out the integrations, and a two-dimensional plane problem is thereby effectively reduced to a one-dimensional line integration.

In the case of bending of edge-notched plates, the boundary integral equation method was preferred over the finite element method for two reasons. The amount of labour required in the preparation of the input data and the amount of computation required are substantially lower for the former than for the latter method. The accuracy of the B.I.E was determined as before and gave an error of only -1.5% as compared to that of F.E.M of -2.6% . Figure A.41 shows the discretised region and boundary conditions for a single edge-notched plate under a remote bending moment, M . Figures A.42 A.44 show different meshes used in the boundary integral equation program.

Three models were used in this case, and as in tension, each having different stress distribution characteristic and different relative stress gradient, χ , but having the same stress concentration factor, K_T . The three models were the same models used in axial load. Table 7.1 shows the comparison of the relative stress gradients and the stress concentration factors for the three models loaded in bending and tension.

To obtain values of stress intensity factor at different crack lengths with changing applied load, the best method was to determine first the modified magnification factor given by

$$\psi(\lambda) = \frac{K_I}{\sigma_{b_{max}}} \quad (7.6)$$

where

$$\psi(\lambda) = \Phi(\lambda)\sqrt{\pi a}$$

$$\lambda = \frac{a}{W}$$

Table 7.2 shows the values of $\psi(\lambda)$ for three models loaded in tension and bending.

The characteristic curves of $\psi(\lambda)$ versus λ or $\psi(\lambda)$ versus a , were then used to evaluate stress intensity factor, crack propagation rate and stress cycles (life). Same procedure as in section 7.1.1 was used to determine number of cycles to failure and crack propagation rates.

Width (mm)	Axial load		Bending load	
	K_T	$\chi \text{ mm}^{-1}$	K_T	$\chi \text{ mm}^{-1}$
50	7.2	1.502	5.1	1.6
75	7.2	1.945	5.1	2.412
25	7.2	1.39	5.1	1.833

Table 7.1: Comparison of the relative stress gradients for three models loaded in tension and bending.

7.3 Results and Discussion

Study of the effect of size on fatigue behaviour using notched plates has revealed the strong influence of the stress gradient on this behaviour. It has been shown that, as the stress gradient increases, the useful life and fatigue limit of the component increases regardless of size or method of loading.

1. Figures A.45 and A.46 show the stress distribution characteristic for edge notched plates, in tension and bending respectively, having the same stress concentration factor, K_T , but different relative stress gradients, ψ .

Figure A.47 and A.48 shows the effect of the stress gradient on the fatigue life of edge notched plates in axial and bending loads respectively. From the two figures we can conclude that the useful life of the machine component depends on the stress gradient at the notch root, the steeper the stress gradient the longer the useful life regardless of its size.

From the four figures (A.45–A.48), we can also verify that fatigue limits of batches of notched specimens, all containing notches of the same stress concentration factor but of different geometries, depend on the stress gradients at the notch roots; the steeper the stress gradient, the higher the notched fatigue limit regardless of their sizes. This indicates the strong influence of the stress gradient on the fatigue behaviour.

2. Figures A.49 and A.50 show characteristic curves of modified magnification factor, $\psi(\lambda)$, versus crack ratio, $\frac{a}{W}$, and $\psi(\lambda)$ versus crack length, a . From these figures we can deduce that magnification factor, which is the weakening factor, increases as flaw size increases and as the stress gradient decreases.
3. Figure A.51 shows effect of the stress gradient on the fatigue life of edge notched plates in axial and bending loads having different relative stress gradients and stress concentration factors. From this figure we can conclude that if two components are loaded in such a manner that both experience the same stress distribution characteristic, i.e same stress gradient and stress concentration factor at notch roots, then both will have the same fatigue limit and useful life, regardless of their sizes and method of loading. However, the stress gradient is the dominant factor.
4. Figure A.52 shows the effect of the stress gradient on the fatigue limit of edge notched plates subjected to axial and bending loads. In this case, we can also conclude that the higher the stress gradient, the higher the fatigue limit regardless of specimen size and method of loading.
5. Figure A.53 shows propagation rate versus stress intensity factor range for both edge notched plates and smooth round specimens subjected to bending and axial loads.
6. It has been also noted that size effect is more pronounced in notched components than in smooth specimens. This can be explained by comparing stress

gradients from both cases when equally loaded in bending or tension.

7. Both the results of this thesis and experimental results show that at constant stress amplitude, fatigue limit decreases as specimen size increases. This can be explained by the fact that as the specimen size increases, the stress gradient decreases, crack propagation increases, consequently decreasing the fatigue limit.

Since we have observed an increase of crack propagation rate with size and the decrease of stress intensity factor with stress gradient, we can therefore clarify that, the decrease of the fatigue life and fatigue limit with size is a consequence of the increase of the crack propagation rate with size, which is attributed to the decrease of the stress gradient with size.

Width (W, mm)	Crack length (a, mm)	λ (a/W)	Magnification factor, $\psi(\lambda)$		
			TC	D2	D1
50	0.1	0.002	.46047E+01	.45220E+01	.46191E+01
	0.3	0.006	.59992E+01	.58211E+01	.59776E+01
	2.	0.04	.69236E+01	.66043E+01	.69636E+01
	5.	0.1	.10434E+02	.99263E+01	.10429E+02
	10.	0.2	.12660E+02	.11936E+02	.12708E+02
	15.	0.3	.15602E+02	.14678E+02	.15695E+02
	20.	0.4	.19690E+02	.18479E+02	.19849E+02
	25.	0.5	.25766E+02	.23699E+02	.26485E+02
75	0.5	0.0067	.52670E+01	.50794E+01	.52001E+01
	2.	0.0267	.72139E+01	.68935E+01	.71741E+01
	5.	0.0667	.83027E+01	.79057E+01	.82723E+01
	10.	0.1333	.96546E+01	.91832E+01	.96258E+01
	15.	0.2	.11129E+02	.10551E+02	.11122E+02
	20.	0.2667	.12845E+02	.12146E+02	.12862E+02
	25.	0.3333	.14912E+02	.14096E+02	.14962E+02
	30.	0.4	.17526E+02	.16546E+02	.17606E+02
100	35.	0.4667	.20957E+02	.19736E+02	.21053E+02
	0.3	0.01	.51534E+01	.50584E+01	.51104E+01
	0.5	0.0167	.59318E+01	.58227E+01	.58798E+01
	1.	0.0333	.70784E+01	.68845E+01	.70151E+01
	2.	0.0667	.8462E+01	.79596E+01	.82956E+01
	3.	0.1	.90378E+01	.86852E+01	.91211E+01
	5.	0.1667	.10483E+02	.98491E+01	.10583E+02
	7.	0.2333	.11975E+02	.11220E+02	.12124E+02
	9.	0.3	.13631E+02	.12776E+02	.13895E+02
	10.	0.3333	.14683E+02	.13741E+02	.14982E+02
	15.	0.5	.22607E+02	.20991E+02	.23228E+02

where TC - Traction formula

D2 - Two-point displacement formula

D1 - One-point displacement formula

Table 7.2: Values of the magnification factor $\psi(\lambda)$ for edge notched specimens.

Chapter 8

CONCLUSION AND RECOMMENDATIONS

Stress gradient supporting effect had been postulated as a hypothetical explanation for the effect of specimen size on fatigue behaviour. The results of this thesis have verified that, indeed, the stress gradient supporting effect does contribute to a larger extent to this phenomenon. We have been able to demonstrate that size effect is due to the interaction of the stress gradient on crack growth. The higher the stress gradient, the higher the crack propagation rate, the lower the fatigue limit and life of the machine component.

The decrease of fatigue limit with increase of specimen size is explained by the fact that as the specimen size increases, the stress gradient decreases, thereby increasing the crack propagation rate which lowers the fatigue limit. We have verified that the fatigue limit and the life of the machine component depends on the stress gradient at the critical cross section and not on the size or type of the applied load. In other words, fatigue limit is better related to the stress gradient than to specimen size, and thus we should be stressing on the effect of stress gradient on fatigue behavior

instead of the effect of size.

In this thesis we have assumed system isotropy and homogeneity; we have ignored other weakening factors such as environmental effects, surface-effects, which have an impact on the crack propagation; and we did not include plasticity effect because we assumed small scale yielding. However, in practical problems, machine components are subjected to all of the above effects. Thus, the problem should further be examined by incorporating these effects. Another difficulty, which is also a practical one, is that of determining the shape of the growing crack. Stress intensity factor depends on the shape of the crack, a change in the crack shape will undoubtedly result in a change in the stress intensity factor which eventually affects the propagation rate. We have assumed a straight sharp crack throughout the entire life of the component, but in actual fact that is not the case. It may be necessary to analyse a series of crack shapes for growing sizes of crack.

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Figure A.1: *Effect of size on the endurance limits for steels.* Figure A.2: *Neuber constants for Steels and Aluminium [Kuhn, 1964]*

Figure A.3: A simple representation of the effect of stress gradient.

Figure A.4: Critical depth below the surface.

Figure A.5: Comparison of the experimental and the theoretical results for 7075-T6 round specimens with grooves. Vertical lines represent observed experimental points.

[Moore and Morkovin, 1942]

Figure A.6: Endurance limit-Size diagram.

[Moore and Morkovin, 1942]

Figure A.7: Relative endurance limits: Shaded area shows the "scatter" of test results.

Figure A.8: Gradient effect in notched and unnotched pieces.

Figure A.9: Relation between Fatigue strength and the Highly stressed volume.

Figure A.10: A schematic representation of the crack propagation curve.

Figure A.11: Representation of the procedure carried out.

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Figure A.12: Schematic of the behaviour of long, short and non-propagating fatigue cracks [Lankford, 1982]

[Lankford and Davidson, 1984]

Figure A.13: *Compilation of growth of many fatigue microcracks in PA7075 Al, versus large cracks and early microcrack data of Pearson on DTD 5050.*

Figure A.14: Algorithm for determination of SN-curve by LEFM approach

Figure A.15: Algorithm for determination of crack propagation by LEFM approach

SN-CURVE FOR AL 7075-T6 (EFFECT OF INITIAL CRACK SIZE)

Figure A.16: Effect of initial crack size—Predicted results on smooth round specimens.

SN-CURVE FOR AL 7075-T6 (EFFECT OF INITIAL CRACK SIZE)

Figure A.17: Effect of initial crack size—Predicted results on smooth round specimens.

SN-CURVE FOR AL 7075-T6 (EFFECT OF INITIAL CRACK SIZE)

Figure A.18: *Effect of initial crack size—Predicted results on smooth round specimens.*

SN-CURVE FOR AL 7075-T6 (EFFECT OF INITIAL CRACK SIZE)

Figure A.19: Effect of initial crack size—Predicted results on smooth round specimens.

SN-CURVE FOR AL 7075-T6 (EFFECT OF INITIAL CRACK SIZE)

Figure A.20: Effect of initial crack size—Predicted results on smooth round specimens.

SN-CURVE FOR AL 7075-T6 (EFFECT OF SPECIMEN SIZE)

Figure A.21: *Effect of specimen size—Predicted results on smooth round specimens.*

SN-CURVE FOR AL 7075-T6 (EFFECT OF SPECIMEN SIZE)

Figure A.22: Effect of specimen size-Predicted results on smooth round specimens.

Figure A.23: *Relative endurance limit versus specimen diameter—Predicted results on smooth round specimens.*

SN-CURVE FOR AL 7075-T6 (EFFECT OF SPECIMEN SIZE)

Figure A.24: Effect of specimen size: Experimental results on Smooth specimens.

SN-CURVE FOR AL 7075-T6 (EFFECT OF SPECIMEN SIZE)

Figure A.25: Effect of specimen size: Experimental results on Semi-circular notched specimens.

CRACK LENGTH VS. NUMBER OF CYCLES FOR AL 7075-T6

Figure A.26: Crack length versus stress cycles.

Figure A.27: Crack length versus stress cycles.

Figure A.28: Crack length versus stress cycles.

Figure A.29: SIF ratio, $\frac{\Delta K}{\sigma}$, versus crack ratio, $\frac{a}{D}$.

Figure A.30: Propagation rate, $\frac{da}{dN}$, versus crack ratio, $\frac{a}{D}$.

PROPAGATION RATE CURVES FOR A1 7075-T6

Figure A.31: Propagation rate, $\frac{da}{dN}$, versus stress intensity factor range, ΔK , strain intensity factor range, $\Delta \epsilon$, and Strain energy release rate, Γ .

Figure A.32: Variation of $\Phi(\lambda)$ as a function of relative crack depth for a solid cylinder loaded in bending [Forman and Shivakumar, 1986]

Figure A.33: Definition of a and b Figure A.34: Definition of σ and σ_b

Figure A.35: FEM discretised region.

Figure A.36: *FEM mesh no.1: Single edge-notched plate, $t/W = 0.4$, $\rho = 7.5$ mm, $W = 50$ mm, $\omega = 33.7^\circ$*

Figure A.37: FEM mesh no.2: Single edge-notched plate, $t/W = 0.3$, $\rho = 5.0$ mm,
 $W = 50$ mm, $\omega = 49.9^\circ$

Figure A.38: *FEM mesh no.3: Single edge-notched plate, $t/W = 0.15$, $\rho = 2.5$ mm, $W = 50$ mm, $\omega = 32.3^\circ$*

Figure A.39: *FEM mesh no.4: Single edge-notched plate, $t/W = 0.2$, $\rho = 4.0$ mm, $W = 75$ mm, $\omega = 53.1^\circ$*

Figure A.40: *FEM mesh no.5: Single edge-notched plate, $t/W = 0.5$, $\rho = 7.5$ mm, $W = 25$ mm, $\omega = 33.7^\circ$*

Figure A.41: *BIE discretised region.*

MESH No. 1: SINGLE EDGE NOTCHED PLATE $t/W=0.4$

Figure A.42: *BIE mesh no.1: Single edge-notched plate, $t/W = 0.4$, $q = 7.5$ mm, $W = 50$ mm, $\omega = 33.7^\circ$*

MESH No. 2: SINGLE EDGE NOTCHED PLATE $t/W=0.2$

Figure A.43: *BIE mesh no.2: Single edge-notched plate, $t/W = 0.2$, $q = 4.0$ mm, $W = 75$ mm, $\omega = 53.1^\circ$*

MESH No. 3: SINGLE EDGE NOTCHED PLATE $t/W=0.5$

Figure A.44: *BIE mesh no.3: Single edge-notched plate, $t/W = 0.5$, $\rho = 7.5$ mm, $W = 25$ mm, $\omega = 33.7^\circ$*

STRESS DISTRIBUTION CHARACTERISTIC (AXIAL LOAD)

Figure A.45: Stress distribution characteristic of edge notched plates (Axial load).

Figure A.46: *Stress distribution characteristic of edge notched plates (Bending load).*

SN-CURVE FOR A1 7075-T6 (AXIAL LOADING)

Figure A.47: Stress-Life curve: Edge notched plates (Axial load).

Figure A.48: Stress-Life curve: Edge notched plates (Bending load).

Figure A.49: Magnification factor $[\psi(\lambda)]$ vs. λ for edge notched plates under bending load.

Figure A.50: Magnification factor $[\psi(\lambda)]$ vs. crack length (a) for edge notched plates under bending load.

EFFECT OF STRESS GRADIENT ON SN-CURVE

Figure A.51: Effect of stress gradient on Stress-Life curve: Edge notched plates.

EFFECT OF STRESS GRADIENT ON FATIGUE LIMIT

Figure A.52: *Effect of stress gradient on Fatigue Limit: Edge notched plates.*

Figure A.53: Propagation rate curve of Al 7075-T6.

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