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Preliminary Design and Analysis
Of Propellers

by

Paul Vyriotes

A thesis submitted to
the Faculty Of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of

Master of Engineering

Department of Mechanical and Aeronautical Engineering

Carleton University

Ottawa, Ontario

April 1, 1991

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Preliminary Design and Analysis
Of Propellers

submitted by Paul Vyriotes, B.A.Sc.
in partial fulfilment of the requirements for
the degree of Master of Engineering

Thesis Supervisor

Chair, Department of Mechanical and Aeronautical Engineering

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ABSTRACT

This thesis describes a hybrid lifting line/strip theory computer program written in standard FORTRAN capable of performing comprehensive preliminary design calculations for propellers. Given a matrix of airfoil polar data, the program either accepts or generates propeller geometry in the form of chord, thickness to chord ratio and camber for several radial stations; the configuration is then tested at several advance ratios and Mach numbers to generate propeller performance charts.

The program is based upon Lerbs' lifting line theory for moderately loaded propellers. The method accounts for arbitrary circulation distributions, non-uniform inflow and a finite hub diameter. Prediction of thrust and circulation distribution for heavily loaded propellers closely matches that of Theodorsen. Theodorsen's theory is *theoretically* valid for heavily loaded propellers, which may suggest that Lerbs' *moderately* loaded assumption will not be too restrictive for conventional propellers. Good correlation with experimental data is also demonstrated.

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Paul Vyriotes

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Nomenclature

A	Area of propeller disk
AF	Activity factor = $\frac{100,000}{16} \int_{x_h}^1 \left(\frac{l}{D}\right) x^3 dx$
c	Chord
C_D	Drag coefficient
C_L	Lift coefficient
C_P	Power coefficient = $power / (\rho n^3 D^5)$
C_T	Thrust coefficient = $thrust / (\rho n^2 D^4)$
D	Diameter of propeller
F	Specified shape of the circulation distribution
G	Non-dimensional circulation = $\Gamma / (\pi D v)$
G_m	m^{th} coefficient of the sine series expansion for G
K	Theodorsen's $K(x)$
K_P	Power coefficient ¹ = $power / (\frac{1}{2} \rho v^3 A)$
K_T	Thrust coefficient = $thrust / (\frac{1}{2} \rho v^2 A)$
k_0	$r \tan \beta_i$
g	Number of blades
h_m	m^{th} coefficient of the "H-factors"
i	Induction factor
I_m	Modified Bessel function of the first kind of order m or the m^{th} coefficient of the even Fourier series expansion for the induction factor, i .
J	Advance coefficient = $v / (nD)$

¹ There are two different definitions for the thrust and power coefficients. However, this need not cause inconsistency as two distinct symbols are used to represent them

J_m	Modified Bessel function of the second kind of order m
l	Chord
n	Propeller rotational frequency.
r	Radius to an arbitrary section on the propeller
R	Radius to the propeller tip from the centerline
t	Thickness of blade section
u	Induced velocity non-dimensionalized with respect to forward speed, v
v	Airplane forward speed
V	Total velocity at a propeller section
w_a	Axially induced interference velocity
w_d	Velocity deduction non-dimensionalized with respect to forward speed, v
w_t	Tangentially induced interference velocity
$\overline{w}$	$\frac{dw}{dr} dr$
w^*	Apparent rearward displacement velocity of the trailing vortex sheet
x or x	Non-dimensional radius= r/R
x_h	Non-dimensional hub radius

Greek Symbols

α	Angle of attack
β	Angle with respect to the propeller disk formed by the vector addition of the onset velocity and the rotational velocity at a propeller section. $\beta = \tan^{-1}(v(1 - w_d)/(\omega r))$
β_i	Same as above except that the induced velocities are considered.
γ	Specific heat ratio for air

Γ	Circulation. $\Gamma = \int_{line} \mathbf{v} \cdot d\mathbf{s}$
η	Efficiency
λ	$\lambda = J/\pi$
ϕ	Angular coordinate along propeller blade
ρ	Density of fluid
ψ	Section blade angle with respect to the propeller disk
ω	Rotational speed of propeller in rad/s or vorticity

Subscripts and Superscripts

a	Axial
D	Design
e	External
h or H	Hub
i	Induced or internal
t	Tangential

Mathematical Symbols

Z	A dummy argument
$\underline{Z}$	Vector
$\overline{Z}$	$\frac{\partial Z}{\partial x} dx$, where x is an arbitrary argument.
$\overline{\nabla}$	$\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$
$\hat{Z}$	A unit, dimensionless vector
$\cdot$	Vector dot product
$\times$	Vector cross product

Chapter 1

Introduction

In the aerodynamic design of propellers there are two general approaches :

- i) Design the propeller for a specified distribution of circulation using a previously tested family of airfoils for building blocks (lifting line theory).
- ii) Specify both radial and chordwise pressure distribution and find the corresponding blade shape (lifting surface theory).

Both methods have their advantages and disadvantages. A very attractive method of design would be to use a sufficiently complex lifting line approach to outline a preliminary design for the propeller and then to use a lifting surface theory to verify some of the design points, make some small geometric modifications and get some flow details. It is desirable that the programs be sufficiently complex so that the hub and propeller be designed to complement each other. This approach is attractive because lifting line theories are several orders of magnitude cheaper than lifting surface theories and can quickly and quite accurately tell the designer what the propeller should look like. Furthermore, risk is kept to a minimum because the sections have already been tried and tested. In a later design stage, lifting surface theory can be used to check and modify the results of the lifting line calculations. In fact this two-step lifting-line/lifting-surface method is precisely the approach today's aircraft propeller manufacturers use. For example, all modern production aircraft propellers are built using a well defined family of 2-D airfoil sections such as ARA-D, NACA-16, NACA-65, HS-1 and HS-2. Noise, structural and flutter properties must also be examined in later design stages.

This paper describes a lifting line program¹ which performs a three-step design process. In the first step, the optimum circulation distribution is determined from a minimum number of input parameters including hub diameter and radial variation of axial onset flow. If desired, blade geometry can also be determined at this stage. The next step is optional. It allows the designer to modify the circulation distribution obtained in step 1 (to reduce tip loading for example) and, if desired, to get correspondingly new blade geometry. In the third step, given the blade geometry, a propeller performance chart is obtained for a given forward speed and axial onset flow.

Fundamentally, the method solves for the induced velocities due to a variable pitch helical wake. Appropriately, then, this thesis begins with a review of classical vortex theory followed by a more rigorous treatment of the propeller wake problem and the numerical methods utilized. Following this, the current computer code is subjected to a validation procedure. Validation begins by carrying out comparisons with other methods, self-check processes within the current code and by comparisons with other implementations of the theory. Also, an in-depth analysis of the effects of non-uniform inflow and propeller hubs is performed to expose some potential weaknesses in classical wake theories , which assume uniform inflow and zero hub diameter.

¹ Lifting surface corrections may be added later.

Chapter 2

Review Of Vortex Theory

Vortex theory is used to define the *unique* relationship between the induced velocity field of a fluid and its distribution of vorticity. The velocity field is considered irrotational outside of the vortices, but not at the vortices themselves, which are idealized to exist as lines or surfaces in physical space. Vortex theory is nothing more than a reformulation of incompressible, non-viscous flow theory. The justification in its extensive use in fluid mechanics is the simplification it offers to solving complex flow fields. This simplicity is due to the ease with which the induced velocity due to an arbitrary line vortex is solved (with computers) via the Biot-Savart integral and the linear superposition principle of vortices.

2.1 Concept Of Vorticity

Consider a straight circular cylinder of infinite length and radius r_0 in an unbounded, ideal fluid as shown in figure 1. Recall that vorticity, ω , is defined as $\omega = \nabla \times \mathbf{V}$. Assume ω has the constant magnitude ω_0 throughout the cylinder and a value of 0 outside the cylinder. In other words, assume the fluid is rotational *only* inside the cylinder. Applying Stoke's circulation theorem to the system and selecting concentric circles centered at $r = 0$ for the integration path yields

$$\int v_i ds = \omega_0 \pi r_i^2, \quad r_i < r_0 ; \quad \int v_e ds = \omega_0 \pi r_0^2, \quad r_e > r_0.$$

From symmetry, v_i and v_e are constant at a given radius, so the circulation can also be written as

$$2\pi r_i v_i = \omega_0 \pi r_i^2, \quad 2\pi r_e v_e = \omega_0 \pi r_0^2 (= \Gamma).$$

Which gives

$$v_i = \frac{1}{2} \omega_0 r_i, \quad r_i < r_0 ; \quad v_e = \frac{1}{2} \omega_0 r_0^2 / r_e, \quad r_e > r_0.$$

The v_i , v_e are defined as induced velocities. Γ here is also known by the well known aerodynamic term of “circulation”. Hence, the presence of the cylindrical vortex implies the coexistence of a unique velocity field known as the *induced velocity field*. Although the velocity field is conventionally referred to as *induced*, it is more precise to consider the velocity field as *coexisting* with the vorticity as one can not exist by itself. Note that the velocity a line vortex induces upon its centre is zero. Rewriting v_e as $v_e = \Gamma/(2\pi r_e)$, it can be seen at once that the velocity induced at r_e is independent of r_0 provided that $r_e > r_0$ and $\Gamma = \text{const}$.

Several convenient abstractions have been introduced in vortex theory to facilitate efficiency in problem definition and solution. The most notable one is that the velocity field coexisting with a cylindrical vortex is well approximated by assuming a line vortex ($r_0 \rightarrow 0$) while keeping $\Gamma = \text{const}$ for sufficiently small r_0 . Naturally, the diameter of the vortex is never zero as that would imply extremely high velocities near $r_0 = 0$ and hence negative pressures. The static pressure can only take positive values. The minimum radius is given by $r_0 > (\Gamma/2\pi)(\rho/P_\infty)^{0.5}$ [17]. Typical values for an aircraft with a propeller diameter of 3m travelling at $100 \frac{m}{s}$ are: $\Gamma \approx 9 \frac{m^2}{s}$, $\rho \approx 1.2 \frac{Kg}{m^3}$, $P_\infty \approx 101 KPa$. These give $r_0 \approx 5mm$ or 0.16% propeller diameter, which is indeed a small fraction of the propeller diameter. The value for r_0 will always be small for conventional aircraft due to the dominance of the $1/P_\infty$ term. Note that the line vortex assumption leads to $v = \infty$ at $r = 0$. This is merely a consequence of the *line* assumption. As shown above, the velocity at the centre of a cylindrical vortex is by definition zero.

2.1.1 Superposition Of Vortices

One of the attractions of vortex theory is the superposition principle. Consider two line vortices of equal strength but opposite signs as shown in figure 2. In the absence of vortex 2, vortex 1 induces a velocity given by $\underline{v}_1 = \Gamma/(2\pi r_1)e^{i(\theta_1 + \frac{\pi}{2})}$ and vortex 2 induces a velocity given by $\underline{v}_2 = -\Gamma/(2\pi r_2)e^{i(\theta_2 + \frac{\pi}{2})}$. The total velocity at a point z is given by $\underline{v}_{tot} = \underline{v}_1 + \underline{v}_2$. It can be shown that the sources will also be rotating about a common centre due to the influence of each other. But, the details

of this complication are not important to the discussion.

2.1.2 The Biot–Savart Law

In the previous sections, the velocity induced by an infinite line vortex (two dimensional) was determined. In reality, most vortex systems, such as the trailing vortex sheet of the propeller in figure 3, are three dimensional in nature. Consider the bold helical vortex line in figure 3. It has an infinitesimal circulation $\bar{\Gamma} = (\partial\Gamma/\partial r)dr$ associated with it. As mentioned earlier, the vortex line must necessarily coexist with a unique velocity field. Reference [17] gives this coexistence relationship as

$$\bar{\mathbf{w}}_P = \frac{\bar{\Gamma}}{4\pi} \int_{line} \frac{d\mathbf{l} \times \mathbf{S}}{S^3}$$

where the whole of space is occupied by fluid. A propeller surface is also present, but this need not cause problems as the physical effects of the propeller (ie. impermeable, rigid surface) can be duplicated by a suitable distribution of vorticity.

2.1.3 Formation Of the Propeller Vortex System

The *induced* velocity field generated by a propeller trailing vortex sheet results in modifications to the onset flow magnitude and direction. This effect is important if an accurate strip theory analysis of the propeller is desired, and becomes increasingly important as loading or circulation is increased since this increases the induced velocity (see Biot–Savart equation). The vortex sheet considered in the following is assumed to be semi-infinite and steady.

Consider a propeller section advancing through the air with positive thrust (figure 4). At the hub and tip, the boundary condition is $\Gamma = 0$. As $\bar{L} = \rho V \Gamma$, where V is the total onset velocity at a radius, it follows that $\bar{L}$ is also zero at the hub and tip. Positive lift along the propeller radius implies the existence of a higher pressure on one side of the blade and a lower pressure on the other side of the blade. Fluid flows from higher pressure towards lower pressure; therefore, fluid

on the “pressure” side of the blade will have a flow component toward the tip and fluid on the “suction” side of the blade will have a flow component toward the root. At the hub, however, the physical boundary dictates zero radial flow. The situation is depicted in figure 5. The flow impinges on the leading edge, separates into an upper and a lower stream, and then reunites at the trailing edge. Clearly, at a point E of the trailing edge, the two reuniting streams have different velocities. However, the pressure must be continuous, which by Bernoulli's Theorem¹, implies the *speeds* at point E are equal. Noting that $\omega = \nabla \times \mathbf{V} \neq 0$ across the discontinuity, it can be seen that a trailing vortex sheet is shed from the trailing edge (figure 3). In reality, the trailing vortex sheet will not remain rigid. It will roll up into g concentrated helical tip vortices — one for each propeller blade — and one hub vortex at a sufficient distance downstream (figure 6). It is difficult to predict how quickly the trailing vortex sheet will roll up. A computational code which will permit such a calculation will be both complicated and computationally expensive. Furthermore, the availability of extensive experimental data on propeller wake development is limited.

In spite of the *apparent* dissimilarities between the idealized (figure 3) and the actual trailing vortex sheet, it has been found satisfactory to assume the idealized trailing vortex depicted in figure 3. This is because the Biot-Savart integral gives almost identical results for the induced velocity *at the propeller blade* for both vortex distributions. To explain this intuitively, first consider the far wake at a distance D and a point on the propeller P . For the idealized wake, the point d induces a velocity $v_P(d)$ at P . However, since point d is far ($D/(2R) \gg 1$) from the propeller, it would induce almost the same velocity at P if the same vortex were at point d' . Furthermore, $v_P(d)$ would be essentially constant across the propeller radius (ie. $v_P(d) \approx v'_P(d)$). It follows that the wake in the far field is well approximated by concentrated line vortices. Close to the propeller disk, however, detail is important. For moderately loaded propellers, *free vortex* sheets are shed

¹ $\frac{P}{\rho} + \frac{1}{2}v^2 = \text{const}$

from the trailing edge (ie. the sheets follow the path of the resultant total velocity vectors at the lifting line), hence the geometry of the sheets is well defined and the Biot-Savart integral can be utilized. Provided that this sheet remains sufficiently stable over a few propeller diameters, very accurate values for the induced velocities can be obtained. Reference [3] states that the contribution of the vortex wake beyond the third rotation to the induced velocity at the propeller, is negligible. In view of the excellent predictive capability of moderate loading ideal wake methods, this must indeed be the case. It is desirable to assume the trailing vortex form of figure 3 because this greatly reduces the numerical calculation burden.

In the case of *heavy* loading, there is no accurate wake method because the exact structure of the wake immediately behind the propeller is not known a priori. However, this need not cause alarm as essentially all propellers fall into the moderately loaded category under normal cruise conditions, even the "SR-1, SR-2, SR-3" (references [1],[23],[2]) series of propellers fall into the moderately loaded categories at cruise conditions. A rather remarkable observation. A propeller is considered heavily loaded if the induced velocities at the propeller exceed 30% of the forward speed. Propellers are invariably heavily loaded at take-off. Helicopters in hover are also heavily loaded. Hence, the current program cannot be used to make performance calculations for these cases.

2.2 The Lifting-Line Assumption

Recall that for an aerofoil in two dimensional flow (figure 4) the section lift is given by $L = \frac{1}{2}\rho V^2 l C_L(\alpha) dr$ and the axial and tangential induced velocities, w_a, t , equal zero. For a lifting wing, however, a trailing vortex sheet is necessarily present. This implies the existence of an induced velocity field. Generally, the streamlines of the flow are almost perpendicular to the propeller radial axis except near at the tips. Hence, if induced velocity at a particular radial station is constant across the chord (ie. assume the chord is sufficiently small), the lift for the inner sections can be given by $L' = \frac{1}{2}\rho V_{tot}^2 l C_L(\alpha) dr$, where the C_L versus α relationship is that established

by 2-D testing and analysis. Note that V_{tot} now includes contributions from the induced velocities. Provided that the region over which the streamlines deviate significantly from the freestream direction remains small, the error in assuming the previously stated formula across the span will remain small. In other words, “tip relief” is ignored. Secondly, recall that the vortex field of an aerofoil consists of both the trailing vortex sheet and a *bound vorticity* distribution representing the blades. It can be shown that the bound vorticity contributes little to the induced velocity field if the aspect ratio is “large” and can usually be ignored in preliminary calculations. The above three assumptions encompass the *lifting line* approximation. Provided that the aspect ratio is greater than about 4, experiments have shown that the lifting line assumptions gives remarkably accurate predictions for wings (reference [17] Chapter 9). It is expected that propellers should have a similar threshold.

Chapter 3

Theory

3.1 Introduction

The Lerbs' lifting line theory is based upon the vortex theory of propellers. The major assumptions are as follows:

- i) Slipstream contraction is neglected
- ii) Rigid semi-infinite wake
- iii) Centrifugal forces are negligible
- iv) Assumes all onset flow in the axial direction
- v) Hub is a circular cylinder which extends from $-\infty$ to $+\infty$
- vi) Induced velocity is constant along the chord

i) and iii) are necessary considerations for "heavy" loading. Theodorsen's theory predicts a slipstream contraction of only a few percent for even the most heavily loaded conventional propellers at cruise conditions. This suggests that at cruise conditions slipstream contraction can be neglected — with minor error. Furthermore, under high loading many of the wake assumptions are less likely to hold.

For wide chord propellers, assumption vi) breaks down. It has been found that the deviation of propeller performance predictions from theory for moderately loaded wide chord — typically marine — propellers has been more due to flow curvature effects as opposed to slipstream contraction (reference [16], round table discussion). Flow curvature can be compensated for by modifications to the camber of the airfoil sections in the input data files using "correction factors" such as those in reference [27]. Such correction factors were not included in the program as the author preferred to keep the program as analytical as possible.

The current theory assumes a semi-infinite, rigid wake. Realistically, the wake begins to dissipate and roll up into g concentrated helical tip vortices and one concentrated hub vortex immediately after being shed from the propeller blade. The wake is anything but rigid after a few revolutions. Nevertheless, rigid wake theories give very good predictions for cruise conditions. At low advance ratios, however, the predictions break down because of the proximity of the distorted wake to the propeller disk. Also, propellers are invariably heavily loaded at very low advance ratios. Theodorsen's theory predicts large slipstream contraction for small ratios of advance, which suggests that the "free vortex sheet" assumption of Lerbs may not be valid.

Points iv) and v) are complementary. Normally, there exists a radial component of velocity at the propeller disk due to hub contour. How this will affect the accuracy of predictions clearly depends upon how strong the radial component is. No general guidelines have been established yet. In chapter 5, the 30 degree swept SR-1 propeller will be analyzed by the program for two different Mach numbers. Sweep has been partially accounted for by defining section planform data along streamlines. For example, in figure 7 the chord at radial station x has been defined as c_s and not c . The good correlation of theory with experiment for the SR-1 propeller at fully subsonic ($M_\infty = 0.6$) operation was initially surprising to the author.

3.2 Velocity Field Of Symmetrical Vortex Lines

The starting point for the current method is the analytical solution for the velocity field of g helical, symmetrically spaced and identical line vortices of constant pitch from the law of Biot-Savart. Along a lifting line, the tangential and axial induced velocities at radial station r due to g symmetrically spaced vortices at radial station r_0 is given by,

$$\begin{aligned}\bar{w}_{a,tot} &= \bar{w}_{ai} + \bar{w}_{ae} \\ \bar{w}_{t,tot} &= \bar{w}_{ti} + \bar{w}_{te}\end{aligned}\tag{1}$$

where,

$$\begin{aligned}
 \bar{w}_{ai} &= \frac{g\bar{\Gamma}}{4\pi k_0} \left\{ 1 - 2g \frac{r_0}{k_0} \sum_{n=1}^{\infty} n I_{ng} \left(\frac{ng}{k_0} r \right) K'_{ng} \left(\frac{ng}{k_0} r_0 \right) \right\} \\
 \bar{w}_{ae} &= -\frac{g^2 \bar{\Gamma} r_0}{2\pi k_0^2} \sum_{n=1}^{\infty} n K_{ng} \left(\frac{ng}{k_0} r \right) I'_{ng} \left(\frac{ng}{k_0} r_0 \right) \\
 \bar{w}_{ti} &= \frac{g^2 \bar{\Gamma} r_0}{2\pi r k_0} \sum_{n=1}^{\infty} n I_{ng} \left(\frac{ng}{k_0} r \right) K'_{ng} \left(\frac{ng}{k_0} r_0 \right) \\
 \bar{w}_{te} &= \frac{g\bar{\Gamma}}{4\pi r} \left\{ 1 + 2g \frac{r_0}{k_0} \sum_{n=1}^{\infty} n K_{ng} \left(\frac{ng}{k_0} r \right) I'_{ng} \left(\frac{ng}{k_0} r_0 \right) \right\} \\
 \bar{w}_r &= 0
 \end{aligned} \tag{2}$$

The subscripts i and e refer to points where $r < r_0$ and $r > r_0$ respectively. In other words the i subscript refers to points *inside* of the r_0 station, which is the radial location of the g circumferentially spaced line vortices in question, and the subscript e refers to points *outside* of the r_0 station. The distinction between variables with a "0" subscript and variables without the "0" subscript, β_i and β_{i0} for example, is that variables with the "0" subscript refer to conditions at $r = r_0$. A development for equations (2) can be found in reference [16].

I and K are modified Bessel functions of the first and second kind respectively. The primes represent differentiation with respect to the argument. To facilitate computation, I and K are replaced by Nicholson's asymptotic expansions. This leads to negligible error ([16]) and avoids the problem of calculating the solutions to I and K , which is a computationally expensive task. Currently, there is a more accurate expansion in reference [18]. But, considering the good agreement with Brockett and Caster and the approximate nature of the lifting line approximation, it has been deemed unessential to add these modifications to the program. Using Nicholson's asymptotic expansions, the modified Bessel functions of the first and second kind, respectively, are written as follows:

$$\begin{aligned}
 I_n(ny) &= \left(\frac{1}{2\pi n \sqrt{1+y^2}} \right)^{0.5} e^{nY} \\
 K_n(ny) &= \left(\frac{\pi}{2n \sqrt{1+y^2}} \right)^{0.5} e^{-nY}
 \end{aligned} \tag{3}$$

where,

$$Y = \sqrt{1+y^2} - \frac{1}{2} \log_e \frac{\sqrt{1+y^2}+1}{\sqrt{1+y^2}-1} \quad (4)$$

$$y = \frac{r}{k_0} = \frac{x}{x_0 \tan \beta_{i0}}$$

With these approximations, the induced velocities can be written as

$$\bar{w}_{ai} = \frac{g\bar{\Gamma}}{k_0 4\pi} (1 + B_2), \quad \bar{w}_{ae} = \frac{-g\bar{\Gamma}}{k_0 4\pi} B_1$$

$$\bar{w}_{ti} = -\frac{g\bar{\Gamma}}{r 4\pi} B_2, \quad \bar{w}_{te} = \frac{g\bar{\Gamma}}{r 4\pi} (1 + B_1) \quad (5)$$

where,

$$\bar{\Gamma} = \frac{\partial \Gamma}{\partial r_0} dr_0 \quad (6)$$

$$B_{1,2} = \left(\frac{1+y_0^2}{1+y^2} \right)^{0.25} \left(\frac{1}{e^{gA_{1,2}} - 1} \mp \frac{y_0^2}{2g(1+y_0^2)^{1.5}} \log_e \left(1 + \frac{1}{e^{gA_{1,2}} - 1} \right) \right) \quad (7)$$

$$A_{1,2} = \pm \left(\sqrt{1+y^2} - \sqrt{1+y_0^2} \right) \mp \frac{1}{2} \log_e \frac{(\sqrt{1+y_0^2}-1)(\sqrt{1+y^2}+1)}{(\sqrt{1+y_0^2})(\sqrt{1+y^2}-1)},$$

$$(or A_1 = -A_2) \quad (8)$$

$$y_0 = \frac{r_0}{k_0} = \frac{1}{\tan \beta_{i0}} \quad (9)$$

The subscript, “1,2”, in $B_{1,2}$ of equations (7) for example, is a succinct presentation of the equations for B_1 and B_2 . In this thesis, wherever subscripts are separated by commas it is to succinctly represent several equations of very similar structure.

However, as the origin of a line vortex is approached, $r \rightarrow r_0$, the velocities approach ∞ in the order¹ of $1/(r-r_0)$. It is convenient to define axial and tangential induction factors, i_a and i_t , to facilitate numerical computation and avoid this singularity.

$$\bar{w}_{a,t} = \frac{\bar{\Gamma}}{4\pi(r-r_0)} i_{a,t} \quad (10)$$

The utility of this approach will become much more evident later on when the velocities are expressed as series representations of “H-factors” and coefficients of

¹ See 1.1

the sine series representation for the circulation. Note that since $\bar{w}_{a,t} \rightarrow \infty$ of order $1/(r - r_0)$ as $r \rightarrow r_0$, the induction factors must approach a finite value as $r \rightarrow r_0$. Using (1), (5) and (10) the induction factors are written:

$$\begin{aligned} i_{ai} &= g \frac{x}{x_0 \tan \beta_{i0}} \left(\frac{x_0}{x} - 1 \right) (1 + B_2) \\ i_{ae} &= -g \frac{x}{x_0 \tan \beta_{i0}} \left(\frac{x_0}{x} - 1 \right) B_1 \\ i_{ie} &= g \left(\frac{x_0}{x} - 1 \right) B_2 \\ i_{ii} &= -g \left(\frac{x_0}{x} - 1 \right) (1 + B_1) \end{aligned} \tag{11}$$

3.2.1 Some Numerical Limits For the Induction Factors

There are some cases for which special care must be taken to evaluate the induction factors correctly due to the fact that computers have finite precision. For example, in determining A_1 a $y_0 = \epsilon = \text{small}$ will lead to $\sqrt{1 + y_0^2} = \sqrt{1 + \epsilon^2} \approx \sqrt{1}$, which is 1. This in turn leads to the undefined quantity $\log_e(0)$. Initially, the program used to “bomb” periodically. Round off errors such as that mentioned above were found to be the culprit. In the following subsections limits for all the possible singularities will be specified. No derivations will be provided as the limits are easily obtained using methods no more advanced than that of introductory calculus courses.

3.2.1.1 $\beta_{i0} \rightarrow (\frac{\pi}{2})^\mp$

As $\beta_{i0} \rightarrow (\frac{\pi}{2})^\mp$, $\tan(\beta_{i0}) \rightarrow \mp\infty$. Most typically, this occurs close to the hub where the trailing vortex filaments are more closely aligned with the free stream. Recall that the distinction between β_i and β_{i0} is that the β_i is the trailing vortex pitch at an arbitrary station, r , and that β_{i0} is the pitch at station r_0 . r_0 is the radial location of the g circumferentially spaced helical line vortices for which the induced velocity contribution through the propeller blade is being calculated. The

limits for $\beta_{i0} \rightarrow (\frac{\pi}{2})^\mp$, are as follows

$$\begin{aligned} i_{ai} &\rightarrow 0^\mp \\ i_{ae} &\rightarrow 0^\mp \\ i_{ti} &\rightarrow g \left(\frac{x_0}{x} - 1 \right) \frac{1}{(\frac{x_0}{x})^g - 1} \\ i_{te} &\rightarrow -g \left(\frac{x_0}{x} - 1 \right) \left(\frac{1}{(\frac{x_0}{x})^{-g} - 1} + 1 \right). \end{aligned}$$

The program utilizes the above limits when $|\beta_{i0} - \pi/2| < 0.0001$.

3.2.1.2 $\tan(\beta_{i0}) \rightarrow 0^\mp$

As $\beta_{i0} \rightarrow 0^\mp$, the induction factors simplify to

$$\begin{aligned} i_{ai} &= g \left(\frac{(x_0/x) - 1}{(x_0/x) \tan \beta_{i0}} \right) \\ i_{ae} &= -i_{ai} B_1 \\ i_{te} &= -g (x_0/x - 1) \\ i_{ti} &= -i_{te} B_2 \end{aligned}$$

In the program, if $|\beta_{i0}| < .009$ the above limits are used. The $B_{1,2}$ are very small so that $i_{ae}, i_{ti} \approx 0$. However, $i_{ai} \rightarrow \infty^\mp$ as $\beta_{i0} \rightarrow 0^\mp$. This leads to an infinite axially induced velocity in the propeller slipstream if $\bar{\Gamma} \neq 0$ in equation (10).

Recall that the propeller trailing vortex sheet is semi-infinite. This implies that an infinite amount of energy has been supplied to the wake. Now, setting $\beta_{i0} = 0$ is equivalent to taking the semi-infinite vortex filament at station x_0 — which has an infinite amount of energy — and compressing it into a single ring. Hence the infinite energy concentration and consequently velocity at the propeller disk if $\bar{\Gamma} \neq 0$. In reality, even if $\bar{\Gamma} \neq 0$, the energy would not be infinite due viscous dissipation.

This example clearly demonstrates a weakness with infinite wake theories. If the propeller advance ratio is sufficiently small, not accounting for the wake dissipation results in erroneously large induced velocities. It is suspected that for low advance ratios wake analysis procedures can be improved by integrating over a limited length

of the wake, a length which is not yet known specifically. This would require numerical integration.

3.2.1.3 $x_0/x \rightarrow 0^+$

This limit determines the influence of the hub upon the rest of the propeller for zero hub diameter. Only external induction factors are meaningful. These are as follows

$$i_{ae} \rightarrow 0$$

$$i_{te} \rightarrow g$$

3.2.1.4 $x_0/x \rightarrow \infty^+$

For this case, the induction factors determine propeller influence upon a hub of diameter zero. Only the internal induction factors are meaningful. These are as follows

$$i_{ai} \rightarrow g / \tan \beta_{i0}$$

$$i_{ti} \rightarrow 0$$

3.2.1.5 $x_0/x \rightarrow 1$

The limits are as follows

$$i_{ai} \rightarrow \cos \beta_{i0}$$

$$i_{ti} \rightarrow \sin \beta_{i0}$$

3.3 Determination Of The Boundary Condition For A Finite Hub

For the case of zero hub diameter, it is a well established fact the circulation approaches zero smoothly towards the hub. In the following, it will be argued that this is also the case for non- zero hub diameter. No *general* proof of this is known to the author, but it can be shown that this must be so for the ideal lightly loaded case. Further arguments that $\Gamma \rightarrow 0$ as $x \rightarrow x_h$ are provided through experimental evidence by Pankhurst, et.al [19], and physical arguments.

Proof That $\Gamma_H = 0$ For the Optimum Case:

If optimum flow exists, it is a well established fact the induced velocity must be at right angles to the resultant velocity at a lifting line and the trailing vortex sheet will form a helical surface of constant pitch (references [10], [26]). For example, in figure 4 the resultant of the vector addition of w_a to w_t will be perpendicular to V .

$$w_t \cos \beta_i + w_a \sin \beta_i = 0 \quad (\text{normality})$$

$$k_0 = r_0 \tan(\beta_{i0}) = r \tan(\beta_i) \quad (\text{constant pitch})$$

Furthermore, the trailing vortex sheets are aligned with the resultant velocity assuming "free vortex sheets". The normality condition can be rewritten as

$$w_t + w_a \tan \beta_i = w_t + w_a \left(\frac{k_0}{r} \right)$$

Now, from the analytical equations for $\bar{w}_a$ and $\bar{w}_t$

$$\begin{aligned} w_a(x) &= \int_r^R \bar{w}_{ae} + \int_{r_h}^r \bar{w}_{ai} \\ w_t(x) &= \int_r^R \bar{w}_{te} + \int_{r_h}^r \bar{w}_{ti} \end{aligned}$$

Let

$$\begin{aligned} A_1 &= \sum_{n=1}^{\infty} n I_{ng} \left(\frac{ng}{k_0} r \right) K'_{ng} \left(\frac{ng}{k_0} r_0 \right) \\ A_2 &= \sum_{n=1}^{\infty} n I'_{ng} \left(\frac{ng}{k_0} r_0 \right) K_{ng} \left(\frac{ng}{k_0} r \right) \end{aligned}$$

Then

$$\begin{aligned} w_a(x) &= \int_{r_h}^r \frac{g(d\Gamma/dr_0)}{4\pi k_0} dr_0 - \int_{r_h}^r \frac{g(d\Gamma/dr_0)}{4\pi k_0} \left(2g \frac{r_0}{k_0} A_1 \right) dr_0 - \int_r^R \frac{g^2(d\Gamma/dr_0)r_0}{2\pi k_0^2} A_2 dr_0 \\ w_t(x) &= \int_{r_h}^r \frac{g^2 r_0 (d\Gamma/dr_0)}{2\pi k_0 r} A_1 dr_0 + \int_r^R \frac{g(d\Gamma/dr_0)}{4\pi r} dr_0 + \int_r^R \frac{g^2 r_0 (d\Gamma/dr_0)}{2\pi k_0 r} A_2 dr_0 \end{aligned}$$

Now determining the sum $w_t + w_a(\frac{k_0}{r})$ gives

$$w_t + w_a\left(\frac{k_0}{r}\right) = \int_{r_h}^R \frac{g(d\Gamma/dr_0)}{4\pi r} dr_0 = \frac{g}{4\pi r} (\Gamma_{tip} - \Gamma_{hub})$$

Which must equal zero to satisfy the normality condition. Since $\Gamma_{tip} = 0$, it follows that Γ_{hub} must also be zero. Hence, for the optimum case there is an equalizing of pressures from the pressure side of one blade to the suction side of the neighboring one, which results in the circulation smoothly going to zero at the hub.

Fluid travels from high pressure towards low pressure. At the hub, this flow results in an equalization of pressures between the “pressure” side of the blade and the “suction” side of the blade and hence zero circulation at the hub. There is no immediately apparent reason to suspect that this will not be so for off optimum conditions. Hence, it can be concluded that $\Gamma_h \rightarrow 0$ smoothly.

3.4 Induced Velocity At A Lifting Line Due To A Trailing Vortex Sheet

This section outlines a method by which the total induced velocity at discrete points along the propeller lifting line can be found for an arbitrary helical vortex sheet. It is important to note that the only assumption about the onset flow is radial symmetry with all velocity vectors perpendicular to the propeller disk. Total tangential and axial induced velocities along a lifting line at radius r are obtained by integrating the elemental induced components at r due to the trailing vortex sheet.

$$w_a(r) = \int_{r_h}^R \frac{dw_a(r)}{dr_0} dr_0 = \int_{r_h}^R \bar{w}_a(r)$$

Introducing $G = \Gamma/(\pi Dv)$ into (10) and using non-dimensional radii, the velocity can be written

$$\frac{w_a(r)}{v} = \frac{1}{2} \int_{x_h}^1 \frac{dG}{dx_0} \frac{1}{(x - x_0)} i_a(x) dx_0 \quad (12)$$

Introduce a new variable ϕ such that $\phi = 0$ at $x = x_h$ and $\phi = \pi$ at $x = 1$. Recall that $x = r/R$ and $x_h = r_h/R$.

$$x = \frac{1}{2}(1 + x_h) - \frac{1}{2}(1 - x_h) \cos \phi \quad (13)$$

Since circulation is zero at $\phi = 0$ and $\phi = \pi$, an odd Fourier series seems an

appropriate representation for G

$$G(\phi_0) = \sum_{m=1}^{\infty} G_m \sin(m\phi_0), \quad G_m = \text{const.} \quad (14)$$

Hence,

$$\frac{dG}{dx_0} = \frac{\partial \phi_0}{\partial x} \frac{\partial G}{\partial \phi_0} = \frac{2}{(1-x_h) \sin \phi_0} \sum_{m=1}^{\infty} m G_m \cos(m\phi_0) \quad (15)$$

Substitution of (15) and (13) into (12) yields

$$\frac{w_a}{v} = \frac{1}{(1-x_h)} \sum_{m=1}^{\infty} m G_m \int_0^{\pi} \frac{i_a \cos(m\phi_0)}{\cos \phi_0 - \cos \phi} d\phi_0 = \frac{1}{1-x_h} \sum_{m=1}^{\infty} m G_m h_m^a(\phi). \quad (16)$$

Similarly,

$$\frac{w_a}{v} = \frac{1}{1-x_h} \sum_{m=1}^{\infty} m G_m h_m^a(\phi) \quad (17)$$

where

$$h_m^{a,t}(\phi) = \int_0^{\pi} \frac{i_{a,t} \cos(m\phi_0)}{\cos \phi_0 - \cos \phi} d\phi_0 \quad (18)$$

The induction factors, $i_{a,t}$, and the “H-factors”, $h_m^{a,t}$, depend both upon the point at which velocity is desired, ϕ , and upon ϕ_0 which is the point inducing the velocity at ϕ . ie

$$i_{a,t} = i_{a,t}(\phi, \phi_0) \quad (19)$$

It has been demonstrated (references [16],[25]) that i is well represented by an even Fourier series

$$i_{a,t}(\phi, \phi_0) = \sum_{n=0}^{\infty} I_n^{a,t}(\phi) \cos(n\phi_0) \quad (20).$$

Substituting the above equation into equation (18), analytical expressions for the $h_m^{a,t}$ can be determined. This involves working out solutions for improper integrals, the solutions for which are well known (reference [9]), as

$$h_m^{a,t} = \frac{\pi}{\sin \phi} \left[\sin(m\phi) \sum_{n=0}^m I_n^{a,t}(\phi) \cos(n\phi) + \cos(m\phi) \sum_{n=m+1}^{\infty} I_n^{a,t}(\phi) \sin(n\phi) \right] \quad (21).$$

For the case of $\phi = 0, \pi$, the $1/\sin(\phi)$ term goes to infinity. However, limiting values for the $h_m^{a,t}$ exist and are given by

$$\begin{aligned} h_m^{a,t}(0) &= \pi \left[m \sum_{n=0}^m I_n^{a,t}(0) + \sum_{n=m+1}^{\infty} I_n^{a,t}(0) \right] \\ h_m^{a,t}(\pi) &= -\pi \cos(m\pi) \left[m \sum_{n=0}^m I_n^{a,t}(\pi) \cos(n\pi) + \sum_{n=m+1}^{\infty} I_n^{a,t}(\pi) \cos(n\pi) \right] \end{aligned} \quad (22)$$

3.4.1 Calculation Of Induced Velocity Given G and β_i

In the previous sections, all the necessary tools have been specified to determine the induced velocity components due to a trailing vortex sheet at a point on a lifting line once $G(x)$ and $\beta_i(x)$ are specified.

Consider the circulation distribution, $G(x)$, and the shape of the trailing vortex sheet, $\beta_i(x)$, given in figures 8 and 9 respectively. These values are tabulated at equal angular intervals in **Output B1** of Appendix B, and can be used by the reader to perform a simple — but perhaps tedious — hand calculation of the induced velocities if desired. In figures 8 and 9, the inputs are: g, x_h, J, C_P and circulation shape ($= F(x)$). Velocity deduction, w_d , was set equal to zero. The outputs are: $G(x) = \text{const} * F(x)$ (figure 8), β_i (figure 9), and induced velocities (figure 10).

Recall equation (17) for the axial induced velocity expressed as an infinite series

$$\frac{w_a(x)}{v} = \frac{1}{1 - x_h} \sum_{m=1}^{\infty} m G_m h_m^a(\phi).$$

This series is rapidly convergent so very few terms are needed to obtain good accuracy. Thus, let $\infty \rightarrow M$ so that

$$\frac{w_a(x)}{v} = \frac{1}{1 - x_h} \sum_{m=1}^M m G_m h_m^a(\phi). \quad (23)$$

A good value for M depends upon how smooth $G(x)$ and $h_m^a(\phi)$ are, but it has been found that $M = 5$ almost invariably gives excellent values. In the program, M can be changed through NUM (see nomenclature). The author has yet to experience significant numerical inaccuracy in assuming a NUM ($= M + 1$) of 6 for a conventional $G(x)$. Figure 11 shows a sample of the effect of varying NUM on the prediction of the optimum circulation distribution for a 4 bladed propeller at $J = 1.309$ and $w^* = .10$.

First, it is necessary to get the unknown G_m (equation (14)). This requires solving the system of M linear equations for the M unknown constants, $G_1, G_2, \dots, G_M$.

$$G(\phi_1) = G_1 \sin(\phi_1) + G_2 \sin(2\phi_1) + \dots + G_M \sin(M\phi_1)$$

$$G(\phi_2) = G_1 \sin(\phi_2) + G_2 \sin(2\phi_2) + \dots + G_M \sin(M\phi_2)$$

$$G(\phi_3) = G_1 \sin(\phi_3) + G_2 \sin(2\phi_3) + \dots + G_M \sin(M\phi_3)$$

...

...

$$G(\phi_M) = G_1 \sin(\phi_M) + G_2 \sin(2\phi_M) + \dots + G_M \sin(M\phi_M).$$

In the above, $G(\phi_i)$, and ϕ_i are known and the G_i need to be solved for. The above system of equations is derived from equation (14) by letting $\infty \rightarrow M$ and substituting $\phi_i = i\pi/(M+1)$, which gives M equations for the unknown G_i ($i = 1, 2, \dots, M$).

Next, the unknown $h_m^a(\phi_j)$ need to be determined using equations (21) and (22). These equations are solved immediately once the $I_n^a(\phi_j)$ are known. Recall equation (20) for the induction factors, which is slightly modified here by the addition of the subscripts j .

$$i_{a,t}(\phi_j, \phi_0) = \sum_{n=0}^{\infty} I_n^{a,t}(\phi_j) \cos(n\phi_0)$$

This gives the induction factor at a *fixed* station ϕ_j attributable to an *arbitrary* station ϕ_0 . This series is rapidly convergent so let $\infty \rightarrow M$, which gives

$$i_{a,t}(\phi_j, \phi_0) = \sum_{n=0}^M I_n^{a,t}(\phi_j) \cos(n\phi_0).$$

For brevity, consider the axial component only. The induction factor is given by,

$$i_{ai} = g \frac{x_j}{x_0 \tan \beta_{i0}} \left(\frac{x_0}{x_j} - 1 \right) (1 + B_2) \quad x_0/x_j > 1$$

$$i_{ae} = -g \frac{x_j}{x_0 \tan \beta_{i0}} \left(\frac{x_0}{x_j} - 1 \right) B_1, \quad x_0/x_j < 1$$

where the $B_{1,2}$ are given by equations (7) and the subscript j has been added. Induction factors are strictly a function of wake geometry. They depend upon the number of blades, the spatial variables, x_j and x_0 , and the pitch distribution, β_i . In calculating the induction factor, it is only the pitch distribution which is unknown.

It's specification enables the following system of linear equations to be solved for the $I_n^a(\phi_j)$:

$$\begin{aligned}
 i_a(\phi_j, 0) &= I_0^a(\phi_j) + I_1^a(\phi_j) + I_2^a(\phi_j) + \dots + I_N^a(\phi_j) \\
 i_a(\phi_j, \phi_1) &= I_0^a(\phi_j) + I_1^a(\phi_j) \cos(\phi_1) + I_2^a(\phi_j) \cos(2\phi_1) + \dots + I_N^a(\phi_j) \cos(N\phi_1) \\
 i_a(\phi_j, \phi_2) &= I_0^a(\phi_j) + I_1^a(\phi_j) \cos(\phi_2) + I_2^a(\phi_j) \cos(2\phi_2) + \dots + I_N^a(\phi_j) \cos(N\phi_2) \\
 &\dots \\
 &\dots \\
 i_a(\phi_j, \phi_M) &= I_0^a(\phi_j) + I_1^a(\phi_j) \cos(\phi_M) + I_2^a(\phi_j) \cos(2\phi_M) + \dots + I_N^a(\phi_j) \cos(N\phi_M) \\
 i_a(\phi_j, \pi) &= I_0^a(\phi_j) - I_1^a(\phi_j) + I_2^a(\phi_j) + \dots + (-1)^N I_N^a(\phi_j)
 \end{aligned}$$

This system must be solved for each ϕ_j . For an iterative procedure, this matrix may need to be solved hundreds of times. Fortunately, the square matrix remains the same for all calculations and only the column vectors change. Performing the LU factorization upon the matrix and storing it significantly speeds up the computational process. This fact is utilized for other matrix operations. The calculations for the tangential component are *exactly* analogous.

The first step in the calculation is to specify the ϕ_j , N , M , ϕ_1 , $\phi_2, \dots, \phi_M$. These are specified as follows:

$$N = M + 1, \quad \phi_j = \phi_1, \phi_2, \dots, \phi_M$$

and $\phi_j = j\pi/N$. The induced velocities can now be calculated by substituting into (23). Referring to figure 4, the pitch, β_i , is given by

$$\tan \beta_i = \frac{1 - w_d + w_a/v}{x/\lambda - w_t/v}. \quad (24)$$

The w_d is a velocity deduction term which accounts for deviations from free stream velocity due to, for example, area ruling. Recall that the calculation outlined above was performed with an assumed β_i . The induced velocities, as calculated above, can now be substituted into equation (24) to reverify the assumed β_i . If the β_i are not equal, the new estimate of β_i is used to perform another iteration. If the information of figures 8 and 9 was used, the distributions of β_i will be almost identical. In most

calculations, however, the distribution of β_i will not be known a priori and must be determined by iteration. A good initial estimate for β_i is $\beta_i = \beta$.

The procedure just outlined for the calculation of induced velocities given the circulation and $\beta_i(x)$ distribution forms the focal point of the propeller analysis program. Furthermore, it is the analytical expressions for the induction factors, as deduced from the law of Biot-Savart [16], that makes this method feasible.

Chapter 4

Practical Overview

What follows is a brief description of the propeller analysis program. To date, the code has run on an IBM compatible personal computer, Honeywell CP-6, and an IBM 3090 mainframe. The program was deliberately written using the most basic level of FORTRAN, so it should ideally run on any computer which has a FORTRAN compiler with minimal effort. Core memory requirements for running the code vary from a minimum of 200K bytes to several megabytes depending upon the size of the matrix of sectional polars. A high resolution data bank with several Mach numbers, Reynolds numbers, thickness/chords, and cambers can be several megabytes in length. There is no virtual memory requirement. Execution time for the code is influenced by many factors such as the stability of the problem, computation level (CODE=1,2,3, or 4) and hardware. For example, a representative CODE 4 type problem takes approximately 0.4 seconds CPU time on the Carleton CP-6, 0.1 seconds CPU on an IBM 3090, and 20 seconds on a 286 based PC. In the generation of propeller performance maps where a few thousand points are needed, PC based calculations can be time consuming.

The code has four fundamental levels of computation depending upon whether CODE=1,2,3, or 4 (see appendix A). These are, respectively:

- i) Given¹ the "displacement velocity", $\bar{w}$, determine the optimum $G(x)$, C_T and C_P , and the induced velocities, $u_{a,i}(x)$. The optimum condition is based upon the rigid, fixed pitch wake, uniform inflow velocity and zero thrust deduction² criterion of Betz [10], [26]. Thus, this calculation should agree with that of Theodorsen [26].

¹ g and λ are fundamental parameters to all four levels

² ie. ignore body interference

- ii) This calculation is fundamentally equivalent to i) except that one of the loading coefficients, C_T or C_P , is given instead of the displacement velocity.
- iii) Given a shape for the circulation $F(x)$ (see figure 12), velocity profile, thrust deduction profile, and C_T or C_P , determine the circulation $kF(x)$ ($k = \text{const}$), $w_{a,t}(x)$ and the unknown loading coefficient, C_P or C_T .
- iv) Given the onset velocity profile, thrust deduction profile, polar curves, planform data³, some atmospheric variables, and a range of blade angle and advance ratio, determine $G(x)$, $w_{a,t}(x)$, C_T and C_P . For this level, several other aerodynamic details such as lift distribution, Mach and Reynolds number distribution and stall warnings are printed. Furthermore, Mach and Reynolds number dependencies can be accounted for by including them in the polar data bank. For more details, it is best to consult some of the outputs in appendix B. A description of terms in the input and output files can be found in Appendix A.

The latter 3 levels involve iteration until a specified tolerance is achieved. In other words, the user chooses the level of precision for the numerical results. In the first three levels, it is also possible to get planform data, which can subsequently be used to determine a performance map using level 4. Polar data is specified by giving lift coefficient versus angle of attack for several profiles, which are defined by Mach number, Reynolds number, thickness to chord and camber. It is up to the user to determine whether an extensive data file covering a wide range of Mach and Reynolds numbers is necessary. For this program, in spite of the lifting-line/stable-vortex assumptions, it has been found that including an extensive data bank can greatly improve accuracy — especially in efficiency and C_P calculations. The profile data sets are assumed to be ordered from lowest Mach number, Reynolds number, camber and thickness to chord to highest Mach number, Reynolds number, camber and thickness to chord. Camber is certainly important in the program as it specifies the design lift coefficient of a section, however the magnitude of *camber number* is

³ These should be referenced along streamlines

not important. Camber is merely a label so the user does not have to worry about the details of its definition, which do not appear to be universal. In determining planform data, for example, once the section Reynolds number, Mach number, thickness to chord and desired C_L have been found, the program checks the NCAMB (see appendix A, CARD L) available cambers and selects the one with the lowest C_D .

The underlying principles of the above calculations have been explained in Chapter 3. Hopefully, the program will appear very user friendly. The input file description is rather lengthy, but that is simply because of all the possible input scenarios. Input files may vary in length from as small as INPUT A1 to dozens of pages depending, for example, upon how large a polar data bank one considers necessary. There is not a single case where all 29 input data cards will be necessary.

4.1 Determination Of Thrust and Power With Viscous Corrections

Viscosity is not accounted for rigorously in the code; however, it is accounted for implicitly by considering airfoil section viscous drag coefficient data, C_D , in the polar data banks. Viscous drag has a strong influence on propeller performance (figure 13).

Referring to figure 4, the elemental thrust at a propeller section is given by

$$dT = dL \cos \beta_i - dD \sin \beta_i. \quad (1)$$

From the definition of circulation, Γ

$$dL = \rho V \Gamma g \quad (2)$$

where g = number of propeller blades. From the definition of lift coefficient, $C_L = L / (0.5 \rho A V^2)$,

$$dL = \frac{1}{2} g C_L \rho V^2 c dr. \quad (3)$$

Using the definition $G = \Gamma/(\pi Dv)$, after some rearranging (3) can be rewritten as

$$dL = \rho V g G v (2A) dx, \quad (4)$$

where $A = \pi R^2$ and $x = r/R$. Referring to figure 4, $\cos \beta_i$ can be written as

$$\cos \beta_i = \frac{\omega r - w_t}{V} = \frac{v}{V} \left(\frac{x}{\lambda} - \frac{w_t}{v} \right). \quad (5)$$

Combining (4) and (5) gives

$$dL \cos \beta_i = \left(\frac{1}{2} \rho A v^2 \right) 4gG \left(\frac{x}{\lambda} - \frac{w_t}{v} \right) dx. \quad (6)$$

The $dL \cos \beta_i$ term gives the inviscid thrust contribution due to sectional circulation. In the presence of viscous drag, the thrust is modified by the $-dD \sin \beta_i$ term in equation (1). From the definition of drag coefficient,

$$dD = \frac{1}{2} g \rho V^2 C_L c dr. \quad (7)$$

Utilizing definitions specified in this section, this can be rewritten as

$$dD = \left(\frac{1}{2} \rho A v^2 \right) \left(\frac{V}{v} \right) \left(4gG \frac{C_D}{C_L} \right) dx \quad (8)$$

Referring to figure 4, $\sin \beta_i$ can be written as

$$\sin \beta_i = \frac{v - v_d + w_a}{V}, \quad (9)$$

where v_d is the velocity deduction. The v_d term is not shown in figure 4. Combining (8) and (9) gives

$$dD \sin \beta_i = \left(\frac{1}{2} \rho A v^2 \right) \left(4gG \frac{C_D}{C_L} \right) \left(1 - w_d + \frac{w_a}{v} \right) dx \quad (10)$$

Combining (6) and (10), equation (1) is rewritten as

$$dT = 4gG \left(\frac{1}{2} \rho A v^2 \right) \left(\left(\frac{x}{\lambda} - \frac{w_t}{v} \right) - \frac{C_D}{C_L} \left(1 - w_d + \frac{w_a}{v} \right) \right). \quad (11)$$

From (5) and (9),

$$\tan \beta_i = \frac{1 - w_d + w_a/v}{x/\lambda - w_t/v}. \quad (12)$$

Substituting (12) into (11) and integrating, the thrust coefficient can be written as

$$K_T = 4g \int_{x_h}^1 G \left(\frac{x}{\lambda} - \frac{w_t}{v} \right) (1 - \epsilon \tan \beta_i) dx, \quad (13)$$

where $\epsilon = C_D/C_L$ and $K_T = T/(\frac{1}{2}\rho A v^2)$. Equation (13) relates thrust coefficient to the distribution of G , induced velocities, β_i , and profile drag distribution.

From figure 4, the elemental power is given as

$$dP = (dL \sin \beta_i + dD \cos \beta_i) \omega r. \quad (14)$$

The mechanics of reducing this term into an expression for the power coefficient in terms of the distribution of circulation, induced velocities, β_i , and viscous drag is analogous to that for the thrust coefficient. All the necessary definitions and equations to do this have been presented above. Omitting the algebraic details, the equation for power coefficient is written as

$$K_P = \frac{4g}{\lambda} \int_{x_h}^1 G x \left(1 + \frac{w_a}{v} \right) \left(1 + \frac{\epsilon}{\tan \beta_i} \right) dx, \quad (15)$$

Figures 8, 9, and 10 give circulation, wake pitch and induced velocities respectively for a six bladed propeller with $x_h = 0$, $J = 1.0177$, $C_P = 0.23358$, and $\epsilon = 0$. Utilizing these three figures, a numerical integration can be performed on equation (15). This will yield $C_P = 0.23358 (= K_P \pi J^3 / 8)$.

4.2 Determination Of Propeller Performance From Geometry and Operating Conditions

The most practical, complex, and computationally intensive aspect of the current code is the determination of propeller performance given geometry and operating conditions. Chapter 3 contains the theoretical fundamentals of the method. This section will demonstrate how the theoretical fundamentals are utilized to determine propeller performance. First, define solidity, σ , as

$$\sigma = \frac{gc}{\pi D}. \quad (16)$$

Recall

$$L = \rho V \Gamma = \pi D v (\rho g V G), \quad (17)$$

where L is the lift per unit length. Eliminating L between (17) and (18) gives

$$C_L \sigma = 2gGv/V. \quad (19)$$

Using equation (5) to eliminate the v/V term in (19) gives

$$C_L \sigma = \frac{2gG \cos \beta_i}{(x/\lambda - w_t/v)} \quad (20)$$

which can be rewritten as,

$$\frac{x}{\lambda} C_L \sigma = 2gG \cos \beta_i + C_L \sigma \left(\frac{w_t}{v} \right). \quad (21)$$

Substituting the expressions for G and w_t/v from equations (14) of Chapter 3 and (17) of Chapter 3, respectively, into (21) gives

$$\frac{\sigma x}{\lambda} C_L(\alpha) = \sum_{m=1}^M G_m \left[2g \sin(m\phi) \cos(\beta_i) + \frac{m h_m^t}{(1-x_h)} \sigma C_L(\alpha) \right]. \quad (22)$$

C_L has been written as $C_L(\alpha)$ in equation (22) to explicitly show the dependence of the equation to angle of attack. Substituting the expressions for induced velocity of equation (23) from Chapter 3 into equation (12) above gives

$$\tan \beta_i = \frac{\left(1 - w_d + \frac{1}{1-x_h} \sum_{m=1}^M m G_m h_m^a \right)}{\left(\frac{x}{\lambda} - \frac{1}{1-x_h} \sum_{m=1}^M m G_m h_m^t \right)}. \quad (23)$$

From figure 4, the angle of attack, α , is given by

$$\alpha = \psi - \beta_i \quad (24),$$

where ψ is the propeller geometric blade angle with respect to the propeller disk plane.

Solving for propeller performance involves an iterative calculation between equations (22), (23), and (24). For the first iteration, the induced velocities are assumed to be zero, which implicitly initiates β_i , α , and C_L . Next, the $h_m^{a,t}$ are calculated

using the procedure of Chapter 3. This allows equation (22) to be solved for G_m . Using the G_m and $h_m^{a,t}$ calculated above, solve equation (23) for the β_i . Finally, use the results of equation (23) to solve equation (24) for α . The above procedure is repeated for the M radial stations, $\phi_j = \phi_1, \phi_2, \dots, \phi_M$, where $\phi_j = j\pi/(M+1)$. If the difference in between initial and final values of α at any of the M radial stations is greater than 0.05 degrees, the iteration is repeated. The starting values for the next iteration are determined from the most recent values for induced velocities, and $\alpha = \alpha_{old} + k(\alpha_{new} - \alpha_{old})$, where $k = 0.1 \rightarrow 0.4$. For conditions close to stalling, k must be kept small to keep the procedure stable. For conditions well removed from stall, the maximum value for k is used to speed convergence.

Chapter 5

Examples

What follows is a brief demonstration of the propeller analysis program. Five examples are presented in the following five sections. The computer outputs for the first four examples could be found in appendix B under Example A, Example B, Example C, and Example D respectively. These examples demonstrate the four fundamental levels of computation depending upon whether CODE=1,2,3, or 4.

The aim of this chapter is to provide both a validation and illustration of the code. Validation begins by carrying out a comparison with Theodorsen's method [26], comparisons with other implementations of the method, experimental comparisons and a self-check procedure. Experimental comparisons span the range of advance ratios of 0.4 to 4.0, which represents typical take-off to typical dive speed. Also, comparisons are performed with wind tunnel data for the 30 degree swept SR1 propeller at a forward speed of Mach 0.6.

5.1 Comparison With Theodorsen's Theory

Example A, appendix B shows the comparison between Theodorsen's theory and Lerbs' theory in predicting Theodorsen's $K(x)$ at

$\lambda_\alpha = v(1 + \bar{w}_\infty)/(\omega R_\infty) = 0.5$, where λ_∞ refers to the wake at infinity. It is important to make two important distinctions before proceeding with the comparison:

- 1) Lerbs refers to the non-dimensional displacement velocity, $\bar{w}$, immediately behind the propeller disk while Theodorsen refers to displacement velocity at infinity and hence gets a value approximately twice that of Lerbs' value.
- 2) The above distinction also holds for λ .

To distinguish between these two different definitions, the subscript ∞ will be used for variables referring to the wake at infinity. If the ∞ subscript is not present, the variables are referring to the wake immediately behind the propeller disk. To determine Theodorsen's $K(x)$ as outlined by Theodorsen, the charts in reference [5] — which are believed to be more accurate than those of [26] — were used. To determine $K(x)$ from Lerbs' $G(x)$, Theodorsen's definition for $K(x)$ will first be stated:

$$K(x) = \frac{g\Gamma\omega}{2\pi(v + w_\infty)w_\infty}.$$

Substituting $G = \Gamma/(\pi Dv)$ gives

$$K(x) = \frac{gGRv\omega}{(v + w_\infty)w_\infty} = gG \frac{v}{\lambda_\infty w_\infty} \frac{R}{R_\infty}$$

Now, Theodorsen's $K(x)$ is dependent only upon the wake pitch at infinity, λ_∞ . In figure 14, a plot of Theodorsen's $K(x)$ is presented for $\lambda_\infty = 0.5$, so it is desired to find a set of $\bar{w}$ and λ such that $\lambda_\infty = 0.5 = \lambda(1 + 2\bar{w})$. Glancing at INPUT A1, it can be seen that $J = 1.309 (= \pi\lambda)$ and $\bar{w} = 0.1$, which indeed give $\lambda_\infty = 0.5$ if slipstream contraction can be neglected.

Note that there are two inputs, INPUT A1 and INPUT A2, and the corresponding outputs. Results for INPUT A1 are plotted along with the values given by Theodorsen. INPUT A1 specifies $\lambda_\infty = 0.5$, while INPUT A2 corrects the λ to account for the slipstream contraction Theodorsen predicts. It is interesting to note that this has little effect on the final results. But, it should also be noted that the slipstream contraction itself is very small for this example.

5.2 Comparison Of Three Numerical Codes

Caster, et al [8] and Brockett [7] have individually computerized portions of Lerbs' theory. This section compares some hydrodynamic predictions of the current program with their results in figures 8, 9, and 10. It can be seen that all three versions give near identical results. The discrepancy in the prediction of the tangent of the trailing vortex sheet, $\tan \beta_{i0}$, near the hub is due to the fact that β_{i0} is large.

The differences in β_{i0} are negligibly small, which is the number of importance in practical calculations such as angle of attack. Small differences also occur in the prediction of induced velocities. These may be due, for example, to different approximations for the Bessel functions, different integration schemes and differences in convergence criteria. However, from a practical standpoint, these differences are immaterial.

5.3 A Comparison With Experimental Results

Example C of appendix B demonstrates how well and with what detail the program can predict propeller performance beginning with the geometry and polar curves. The results are presented in figures 15 and 16. In [19], propeller test data is given along with blade geometry and zero lift angle of attack. Polar data, however, were not given. Due to the difficulty of determining a reasonably accurate shape of the lift curve slope near the stall (figure 17), the calculations were confined to the operating ranges which had a *linear* lift curve slope (ie. $C_L < 1$). The one exception is the calculation for $J = 2.5$ (please see the corresponding output file in appendix B, output C2), where $C_L \approx 1.3$ near the tip.

The greatest discrepancy occurs for $J = 3$ in figure 15. However, upon looking at the experimental points for that case, it can be seen that a *possible* experimental or recording error has occurred. All the experimental thrust gradings in figures 15 and 15, except for $J = 3$, go to zero at the tip. The $J = 3$ curve, however, seems to be displaced upwards by ≈ 0.4 . If all the experimental points for $J = 3$ are shifted downwards by this amount, the correlation will be much better. The best recommendation the author can give is to discard the data for $J = 3$ until this apparent inconsistency is resolved.

Lift curve slopes were estimated by back calculation from the given experimental thrust grading curves. Slopes were assumed to be straight lines, which makes aerodynamic predictions near the stall invalid (eg. $J = 1.99$). The back calculation procedure is rather crude, so it is possible that “rule-of-thumb” methods, such as

those found in [6] and [11], may have yielded more accurate lift curve slopes. Other unquantified error sources are possible geometrical modifications to the blades due to centrifugal and aerodynamic forces, and turbulence field amplification in the presence of the propeller. Reference [15] gives some very interesting turbulence field measurements in the neighbourhood of a model propeller. Nevertheless, it is felt that the indicated correlation of theoretical with experimental results is sufficient to demonstrate the validity of the program and the built-in theoretical approximations. It is especially interesting to note the agreement between thrust coefficients (C_T) and the prediction of negative thrust elements near the root for $J = 1.20$ of figure 16.

Skin friction drag coefficient was not specified in the paper, so a constant value of $C_D = 0.014$ was used. This was determined by matching efficiencies. C_D has little effect on C_T and thrust grading so this will have little effect on the indicated results. Furthermore, $C_D = 0.014$, although large by wind tunnel test standards, may in fact be quite a realistic value considering the fact that section Reynolds numbers were low and the construction techniques and materials may have encouraged large C_D values. Furthermore, large C_D values tend to lower lift curve slopes, C_{L_α} , which seems to be consistent with the relatively low $C_{L_\alpha} \approx 4.9$ per radian obtained by back calculation.

5.4 A Complete Preliminary Design Loop

In Example D of Appendix B, a propeller is designed according to the three step method outlined in the introduction. It is desired to design a propeller with some of the main features of the Hamilton Standard 4 bladed, .300 integrated design C_L and 100 activity factor (AF). Uniform inflow and zero thrust deduction will be assumed. While it is certainly not expected that the program will match the Hamilton Standard map exactly, close agreement is expected. Judging from the sample Hamilton Standard propeller map of figure 18, it seems that $J = 2$ and $C_P = .21$ is a good approximation for the propeller design J and C_P . These were the

values used in STEP 1 of the design process, which may in fact be quite erroneous. The input file is displayed in INPUT D1 of appendix B, with the corresponding output in OUTPUT D1. The circulation for this output, which is based upon the optimum criteria of Betz, is displayed in figure 12.

Suppose that the designer considers the loading near the tip too large or too abrupt for practical reasons such as excessive bending moments, noise, cavitation and compressibility constraints. Thus, CODE=3 is specified and a desired modified circulation *shape*, $F(x)$, is input. This input shape is plotted on figure 12. It is important to note that this *shape* does not guarantee $C_p = .21$. In fact, the main purpose of this step is to find the constant multiplier, k , for $F(x)$ which will yield the design C_p . After one iteration, the constant is determined to be $k = .84146$, and the desired circulation distribution, $G(x) = kF(x)$ is plotted on figure 12. The ideal efficiency for the optimum case is .947, which is lowered to .944 by modifying the circulation distribution. Thus, it can be seen that the performance penalty in *ideal* efficiency for deviating from the classically optimum condition is indeed small. This calculation is carried one step further to determine a planform for the propeller.

Note that the polar data bank (INPUT D2 of appendix B) has 2 polar curves which differ only in the thickness to chords. This may seem pointless at first, but there is a good reason for having this seemingly redundant input. The algorithm for determining the chord doesn't permit the thickness to chord to *exceed* the maximum thickness to chord in the polar data bank. Thus, for example, one doesn't have to worry about the program specifying a thickness/chord which is unrealistically large near the root where thickness is large due to structural considerations and low aerodynamic loading dictates a small chord. The program first internally calculates the thickness/chord then checks to see if it exceeds the largest thickness/chord in the data bank. If it does, then the chord is extended and the lift coefficient accordingly lowered such that thickness/chord=maximum thickness/chord in the polar data bank. A look at output D2 will indeed show that the largest thickness/chord is

0.25, which corresponds to the maximum in the data file.

If the program is working "bug free", then it should be possible to run the program using CODE=4 and getting the same results as for CODE=3. In other words, it should be possible to begin with the planform data and duplicate the aerodynamic results which were used to build the planform. This has been done using a .05 degree convergence tolerance — which translates into a .005 C_L tolerance — in angle of attack and the results are presented in figures 19, 20, and 21. Figures 19, 20, and 21 represent comparisons of circulation distribution, pitch distribution and induced velocity distribution respectively. As expected, the lines lie on top of each other within visual resolution.

Finally, the generated planform data was used to determine a performance map for the propeller. The results are presented graphically, superimposed on the Hamilton Standard map in figure 18. In the results presented, $C_D = 0$ was arbitrarily assumed. Typically, adding skin friction drag reduces efficiency by 3 to 5 percent (see figure 13 for example), which would bring the lines of constant efficiency of the two maps very close to each other. Several factors are responsible for the differences in blade angle some of which are: different lift curve slopes, the $C_D = 0$ assumption, differences in blade geometry, and general theoretical limitations. Other unknowns are turbulence and structural modifications under loading, which were mentioned in the previous section. Nevertheless, an encouragingly familiar looking performance map is obtained.

It is interesting to note that some of the operating points in the outputs for this example, which are presented in figure 18, predict a negative thrust. These results should be valid for sufficiently small negative thrust, but at large negative thrust the results should begin to break down due to strong induced velocities and radial components of flow. The range of validity of negative thrust prediction can be determined by comparison with experimental data, which has not been done yet. However, it is expected that the conclusion of such a study will merely demonstrate that the range of validity of thrust reversal prediction will be too small to be of

practical significance.

At small thrust, C_L is small so small changes in incidence should result in large relative changes of C_T or C_P . The indicated results used an incidence tolerance of 0.1 degrees as a convergence criteria, which should be kept in mind if prediction of lightly loaded conditions are required. If desired, incidence tolerance can be modified by change of a *single* constant.

5.5 Application To A High Speed Swept Propeller

The method has been applied to the NASA/United Technologies SR1, 8 bladed, 200 activity factor, 30 degree swept propeller [1]. The method does not rigorously account for sweep, however classical sweep corrections were included in the propeller polar data bank. The propeller uses NACA series 16/65 sections, for which section data was obtained from [5]. Section planform data is defined along streamlines (figure 7).

The results of the analysis are presented on figure 22. The correlation looks encouraging, however this does not necessarily imply that aerodynamic details such as lift distribution will be equally well predicted. An accurate representation of the propeller radial loading distribution is necessary for analysis such as propeller noise, flutter and strength requirements. For $J < 2.6$ the flow is transonic, which the current method is unsuitable for, so any comparison in that region should be avoided.

Recall that the method assumes zero radial velocity components and a cylindrical hub. However, the SR1 propeller has non-zero radial velocity components and a non-cylindrical hub. In the science of CFD (computational fluid dynamics) simplifying assumptions are invariably essential to allow solution of the governing fluid flow equations, the most fundamental being the Navier–Stokes equations. Experimental comparison to wind tunnel data is, to a large part, a check upon one's assumptions. Fair correlation of SR1 experimental data with the current code suggests that the current mathematical model takes into account most of the *important*

flow parameters.

5.6 Concluding Remarks

Quite often, it is possible to have a computer code that is of little practical use, — for example it may be overly susceptible to crashing, be too imprecise (“magic numbers”, for example, may abound), and may be effectively irreparable if it is poorly structured and too many bugs are present — but nevertheless works misleadingly well for a few problems. It is worthy to note that the author selected the current examples *without* any such bias. For instance, in example 3 it is possible to select lift curve slopes which will cause the predicted thrust grading, dC_T/dx^2 , to line up with the experimental thrust grading almost exactly — but to be objective, the lift curve slopes were determined in an unbiased, analytical manner.

There are no “magic numbers” in the current code. Originally, the author was considering adding some to account for *effective sweep*¹, and to include lifting surface effects through modification factors² to the camber of the aerofoils such as those found in [27]. These were omitted to keep the program analytical. Moreover, it would be a fairly simple matter for the user to include these factors by modifications to the polar data input.

¹ A straight bladed propeller may have effective sweep due to strong radial components of velocity in the neighbourhood of the hub.

² For an aeroplane propeller, where the chord is small ($AR > 5$), these factors are negligible. This is not so for marine propellers.

Chapter 6

Effects Of Hub and Non-uniform Inflow On Propeller Performance¹

A hub is a necessary element of any propeller. Most obviously, it forms the link between the thrust generating propeller blades and the power source. The presence of a hub or any nearby body, however, tends to distort the oncoming air so that the propeller blades are no longer operating in a uniform flow field. Improved propeller performance, possibly better than that for uniform inflow, can be achieved by carefully contouring the hub to achieve some specified onset flow. Some of the objectives to be achieved are, for example:

- i) Higher velocities near the hub to augment ram pressure rise and hence gas turbine efficiency.
- ii) Higher velocities in the outer regions where the profiles are more efficient.
- iii) Lower velocities in the neighbourhood of the root to reduce possible choking problems.
- iv) Lower velocities in the tip regions to reduce compressibility noise, structural weight, and moments.
- v) Minimal circumferential velocity gradients.

The relative importance of the individual constraints are problem dependent and, as can be observed, often conflicting.

In the current study, a complete analysis of propeller hub integration will not be attempted. Rather, a demonstration of the effect of using classical wake theories ([26], [10]) to design a propeller will be demonstrated.

¹ Presented at the 8th Canadian Symposium On Fluid Dynamics.[28]

Classical wake theories assume zero hub diameter, uniform onset flow and an ideal circulation distribution, $G(x)$. Lerbs' theory, however, is valid for a non-uniform onset flow, finite hub diameter and an arbitrary distribution of circulation. To the author's knowledge, no study has been made of the *precise* effects of using classical wake theories in propeller design. In this study, 4 propellers will be designed and analyzed for a uniform onset flow, ideal circulation at $J=2.0$ and $C_P = .2141$, and two different distributions of lift coefficient. Following this, a propeller in a radially symmetric non-uniform flow field will be optimized for isolated propeller efficiency. It will be assumed that all sections have the same polar curve, which is shown in figure 23.

In designing a planform using the current method circulation, $G(x)$, $C_L(x)$ and thickness ($t(x)$) must be known. In the current code $C_L(x)$ and $t(x)$ are specified by the user, and $G(x)$ is calculated. Circulation distribution gives a relation of $C_L(x)l(x)$ along the blade, where l =chord.

$$C_L(x)l(x) = 2\pi DG(x)\frac{v}{V}$$

where D =diameter, v =onset velocity, V =total velocity. Given C_L , chord is determined, which in turn can be used to determine thickness to chord ratio from the specified thickness distribution. Camber is determined by searching the polar data bank for the polar curve which gives the lowest C_D for the specified C_L . Finally, $C_L(x)$ gives the angle of attack via the polar data bank, which is added to the trailing vortex pitch angle to give blade twist distribution. Hence, the propeller planform is fully specified.

6.1 Effects Of Hub On Propeller Performance

figure 24 shows two planforms. One has a design circulation as specified via the Theodorsen method — which spans the range of $x=0$ to $x=1$ in figure 25 — and the other by Lerbs' theory. Both have the same thickness and specified $C_L(x)$. The original planform for the Theodorsen design extends from $x=0$ to 1, but is "cut off"

at $x = 0.25$, which represents 19.6% of the propeller disk area. The curves of figure 25 labelled "Lerbs" and "Theodorsen" were built to give a $C_P = 0.2141$ at $J = 2.0$. As the Theodorsen curve is then cut off at $x_h = 0.25$ it is reasonable to suspect that this will effectively lead to a lower C_P due to the fact that part of the thrust generating surface is removed. However, before succumbing to this conclusion it would be prudent to test the planform designed with Theodorsen's theory using the current computer program. This has been done using the planform of figure 24 and the resulting circulation is labelled as "Theodorsen's resultant" in figure 25 with a resultant $C_P = 0.2075$, which represents a loss of 3.3%.

This relatively small loss may be surprising initially — as it was to the author — until a more concrete description of the physical situation is presented. Plotting

$$\frac{\int_0^x G(x)x \, dx}{\int_0^1 G(x)x \, dx}$$

versus x gives a very good approximation for the cumulative percent thrust versus x for light to moderate loading. This relation is plotted in figure 25 for $J = 2.0$ and $C_P = 0.2141$. As can be seen, only 2.4% of the thrust is contributed by the sections within $x = 0.25$. The remaining loss ($3.3 - 2.5 = 0.8\%$) is attributable to the fact that the zero circulation boundary condition at the hub must be satisfied and $\partial G(x)/\partial x$ must be continuous.

The circulation as set out by Lerbs is higher than that of Theodorsen for $x \approx 0.488$. For $x \approx 0.6$, there is negligible difference between the Theodorsen circulation and the "cut-off" Theodorsen circulation distributions. Furthermore, the sections beyond $x = 0.6$ contribute 66.5% to the total thrust². Thus the introduction of a hub with $x_h = 0.25$ causes negligible effect on the outer and most important part of the propeller.

The efficiencies corresponding to the design point of the planforms of figure 24 (circulations shown on figure 25) are 88.7% for both "Theodorsen's resultant" and Lerbs' propellers. However, the power for "Theodorsen's resultant" is slightly lower

² This slightly value is higher for the circulation as set out by Lerbs' theory for a propeller with non-zero hub diameter.

than that of Lerbs' so the efficiency for the Theodorsen propeller should be lowered slightly upon raising C_P from 0.2075 to 0.2141. Furthermore, imposing the hub boundary condition, $G(x = x_h) = 0$, results in lower operating lift coefficients in the neighbourhood of the hub, which for this problem results in higher C_D/C_L . This lowers the efficiency of the sections close to the hub. For example, in the propeller designed via the Theodorsen method (figure 24), the specified C_L at $x = 0.2786$ was 0.3 but apparently turned out to be 0.2106 when analyzed by the program. Nevertheless, the Theodorsen propeller leads to a loading near the hub in excess of that for minimum induced drag due to an excessively large chord.

The performance maps of the propellers in figure 24 are shown on figure 26 superimposed upon each other. The efficiencies are approximately 0.5% lower for the Theodorsen propeller. Part of this loss is attributable to a non-ideal circulation distribution and part is attributable to lower efficiencies of the sections near the hub. It is suspected that the dominant contribution to this loss is due to the higher C_D/C_L — ie. skin friction losses dominate induced drag losses — near the hub in view of the small effect upon efficiency demonstrated in Example D of Chapter 3 due to a moderate deviation from the optimum circulation. However, only the efficiency contours for the Lerbs' propeller are presented to avoid an *unnecessarily* cluttered map.

To test the effects of C_L distribution on the propeller, the above procedure was repeated with a different C_L distribution, which is shown along with the resulting planform in figure 27³. In figure 28, the performance curves of the 2 planforms, with the efficiency contours omitted, are presented. For this map, the efficiency contours are similar to those of figure 26, except that the efficiency should be slightly lower due to higher values of C_D/C_L near the hub. Specifically, the efficiency of the propeller designed via the Lerbs' method — with the difference in planform being due to different specifications for the C_L distribution — is lowered by approximately 0.3% with the resulting power coefficients (figure 29) largely unchanged.

³ The chord is certainly of an impractical shape, but this is merely an exercise. One of the objectives in selecting a C_L distribution may be to facilitate a practical chord distribution.

The above discussion should not lead one to believe that the hub has little effect on free-running propellers. The fact that the hub region contributes little to thrust does not imply that the hub cannot contribute significantly to losses. A poorly designed hub *can* result in significant propeller performance losses due to flow separation and — possibly most importantly — may result in secondary losses. Some of the secondary losses are:

- Possible increase in extent of turbulent boundary layer over the hub.
- Reduction in *ram pressure recovery*⁴ which reduces engine (due to lower carburetor pressure) or gas turbine efficiency.

From reference [5], an 8% loss in ram pressure recovery may result in a loss of 1% in SFC⁵ and a 2% loss in shaft horsepower. As ram pressure recovery $\approx (V/v)^2 = ((v+w)/v)^2 = (1+\bar{w})^2 \approx (1+2\bar{w})$ for small $\bar{w}$, it can be seen that if $\bar{w}=0.04$ an 8% increase in ram pressure is possible. Furthermore, $\bar{w} = 0.04$ is a realistic number. Only losses in isolated propeller efficiency will be considered here and it should be pointed out that this will tend to **understate** the overall performance penalties.

Consider the propeller designed via the Lerbs' circulation specification in figure 24. For this planform, the section profiles close to the hub are made elliptic, which results in a $C_D/C_L=1.0$ at $x = 0.2786$ blending to $C_D/C_L = 0.029$ at $x = 0.3598$. "Testing" this propeller results in the performance map of figure 30. The efficiency contours of the elliptic hub propeller are indicated by the dashed lines, which are superimposed upon the efficiency contours from figure 26. It can be seen that the propeller with the streamline sections at the root is more efficient over the entire operating range except for the intersection of the $\eta = 80\%$ lines, which intersect near $J = 1.5$. This is an erroneous result, which is due to the fact that the calculation involves an iterative procedure that is completed once a specified

⁴ Ram pressure recovery is defined as $(P_{TOT_INLET} - P_\infty)/(P_0 - P_\infty)$, where P_∞ =free stream static pressure and P_0 =free stream total pressure.

⁵ Specific Fuel Consumption.

tolerance is achieved. Naturally, this error could have been reduced to well under 1% by specifying a tighter tolerance on the outputs. However, the author does not feel that it would be justified to attach such a high resolution to a lifting-line/strip theory/potential flow result.

For this propeller, the design point efficiency is 86% versus 89% for the propeller with the same planform, but streamline, low drag aerofoils at the root sections. Furthermore, the deterioration in efficiency due to inefficient root sections increases with increasing advance ratio, which vindicates the experimental results of [22]. Along $\beta = 46$, for example, at $J = 1.8$ efficiency for the streamline root (η_1) is 82.1% and for the elliptic root (η_2) it is 81.4%. For $J = 2.4$, this changes to $\eta_1 = 88.4$ and $\eta_2 = 85.4$.

6.2 Effects Of Non-uniform Inflow On Propeller Performance

In this section, the Theodorsen propeller of figure 24 will be used on a NACA E type cowling. This results in a nonuniform inflow to the propeller, the profile of which is shown in figure 31. Specifying this onset velocity profile⁶ — which is assumed to be radially symmetric — the isolated performance of the propeller is obtained (figure 32). It can be seen that the power coefficient is lower for all indicated points. Peak efficiency is reduced to approximately 87% from approximately 89% for the uniform inflow case. The lower efficiency is partly due to the fact that the propeller essentially operates at a higher local advance ratio due to the fact that the onset flow speed is greater than that of the free stream. At higher advance ratios, the efficiency is lowered because tangentially induced velocity are increased. figure 33 illustrates this relationship for constant thrust. The efficiencies are those for an ideal circulation distribution. It is interesting to note that there is a non-zero ideal advance ratio if drag is present. This represents an important consideration

⁶ The presence of the propeller typically has negligible effect on the onset flow for airplane propellers. For marine propellers, this weak coupling principle may break down if the propeller operates within a significant portion of the hull boundary layer (reference [13])

in propeller design. If drag is not present, the ideal advance ratio $\rightarrow 0$ with the resulting efficiency being equal to that obtained via momentum theory.

Lower isolated propeller efficiency does not necessarily mean that the nacelle/propeller *system* will have a lower efficiency. The loss in isolated efficiency may be compensated for by the beneficial effects of increased total pressure behind the propeller or beneficial boundary layer effects may result from the acceleration and possible contraction of the flow across the propeller disk. These beneficial (or detrimental) effects attributable to potential flow effects form what is termed the "Thrust deduction", which is an optional input in CARD D of Appendix B.

For a propeller operating in nonuniform inflow, the ideal circulation distribution is different from that of classical theory and somewhat more complicated because thrust deduction and nonuniform inflow must now be considered. Currently, the program does not perform such an idealized calculation. There are two reasons for this. Firstly, in the process of developing this software, the author realized after several tests that the classically optimum design, which seeks to minimize induced drag, is not necessarily⁷ the best design. To get maximum efficiency, for example, it is total drag that must be minimized, which in general is significantly larger than induced drag. Secondly, the classically optimum circulation for a uniform inflow can be used as a good starting point in propeller design. Once the classically optimum circulation is obtained, it is slightly decreased at the sections with an increase in onset velocity over that of the free stream (compare figure 34 with figure 25 and note figure 31 as an example) and vice versa for the sections operating at reduced velocities. After several runs, a maximum efficiency is found, which in general will be a small improvement over that of the classical circulation. This was done to optimize a propeller for operation in the radially symmetric flow field specified in figure 31. After several runs, the best circulation shape was found to be that of figure 34, which is labelled as "optimum". The curve labelled "resultant" is the resultant circulation of the propeller designed for a uniform onset flow with the

⁷ Possibly rarely.

design circulation given by “Lerbs”. This calculation is then carried one step further to determine a planform and subsequently a performance map. The performance map is shown in figure 35 superimposed upon the map for the Theodorsen propeller of figure 24 operating in the nonuniform flow field. The optimization process has resulted in a very small improvement in efficiency.

6.3 Concluding Remarks

In spite of the fact that the hub does not contribute significantly to thrust, it *can* contribute significantly to drag. Furthermore, inefficient sections in the neighborhood of the hub may lead to secondary losses such as increased skin friction on the nacelle or lower engine efficiency due to lower ram pressure or lower carburetor pressure.

It has been demonstrated that the induced drag due to moderate deviation from the classically optimum circulation distribution results in a very small loss in efficiency. Thus, one can make modifications to the circulation distribution to achieve certain objectives⁸ with minimal — but not trivial — worry about performance penalties. In fact, skin friction drag effects may favour a circulation distribution quite different from that for minimum induced drag. Moreover, body and engine performance effects are an important factor in net propeller efficiency.

To get a propeller that is truly optimum, one must optimize the entire *system*, which includes the nacelle, propeller, and engine ram pressure effects. To be useful to this task, a propeller code must be capable of handling an arbitrary distribution of circulation and non-uniform inflow. Classical theory is not well adapted to this task because it merely minimizes induced drag. The current code is capable of such tasks except that the flow is assumed to be radially symmetric. Nevertheless, for small variation of the velocity along a circumference, the circumferentially averaged velocity will give good results. This is because the propeller tends to smooth out circumferential velocity gradients so that the flow in the slipstream has smaller

⁸ See first paragraph of this chapter for examples.

velocity gradients along a circumference. In regions of a circumference where the onset velocity is lower than the average for that circumference, the propeller section will experience a higher C_L , which will result in an augmentation of circulation and hence induced velocity. The converse holds in regions of a circumference where the onset velocity is higher than average. Hence, circumferential velocity gradients tend to be smoothed out by the action of the propeller. Without the radial symmetry stipulation, the code would necessarily have to be a non-steady one, which is a much more computationally demanding problem. The current program is quite *fast*. For example, the power coefficients and associated aerodynamic details of figure 22 were computed in approximately 6 minutes of computing time on a Honeywell CP6 computer. This integrated design approach may sound like a very complex problem, but it is greatly simplified by the fact that the propeller and onset flow problems are very weakly coupled and hence can be solved separately.

Chapter 7

Conclusions

An intermediate level propeller analysis method based upon the lifting-line theory of Lerbs for moderately loaded propellers has been presented and compared to both experimental data and other implementations of the theory. The current method also has an inverse design capability, which has been proof tested in chapter 5. These comparisons have proved the mathematical correctness of the program and good correlation with experimental data suggests that the simplifying assumptions inherent in the model result in a fair representation of aerodynamic details at the idealized lifting lines.

The main difference between classical methods ([26], [10]) and the current method is in the treatment of the hub, allowance of an arbitrary distribution of circulation, and allowance of a radially varying onset flow. An entire chapter has been devoted to these issues (chapter 6). Designing an ideal propeller of non-zero hub with $\Gamma_h \neq 0$, as one would do if the design point circulation were based upon Goldstein's or Theodorsen's theories, results into a loading distribution which is slightly higher at the root than that due to an optimum propeller design based upon the more rigorous method of Lerbs. Nevertheless, the isolated performance of the two propellers would be nearly identical if the root sections have favourable lift to drag characteristics. For very poor sectional root characteristics, the propeller design based upon $\Gamma_h \neq 0$ is expected to show substantial performance degradation at high ratios of advance J .

Allowance of an arbitrary distribution of circulation allows one to tailor a propeller loading distribution such that, for example, noise or wing/body interference can be reduced. Modest deviations of the circulation distribution from that of the optimum result in minor performance losses. Hence, moderately tailoring a circu-

lation distribution to reduce noise or wing/body interaction will result in small, possibly nil, performance loss.

Propellers are rarely in a uniform onset flow due to the presence of interfering bodies. Propeller/body interaction in the current method is assessed by allowing both onset velocity and thrust deduction to vary in the radial direction. Accounting for flow non-uniformities can result in relatively large changes in isolated propeller efficiency. But including buoyancy effects through the thrust deduction will generally bring efficiencies closer to those expected in a uniform flow.

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Figure 1 — Line vortex model.

Figure 2 — Superposition of two line vortices.

Figure 3 — Propeller trailing vortex sheet.

Figure 4 — Velocity/lift relationship at a propeller section.

Figure 5 — Pressure and suction sides on a propeller.

Figure 6 — Rollup of trailing vortex sheet.

Figure 7 — Blade geometry along streamlines.

Figure 8 — Comparison of three different circulation predictions ($g = 6$, $x_h = .176$, $J = 1.0177$, $C_{P_{design}} = .23358$).

Figure 9 — Comparison of three different pitch predictions.

Figure 10 — Velocity comparisons.

Figure 11 — Effect of varying NUM upon circulation distribution.

Figure 12 — Modifying the classical, optimum circulation.

Figure 13 — Effect of C_D/C_L on efficiency.

Figure 14 — Comparison of Lerbs' and Theodorsen's circulation ($g = 4$, $\lambda_\infty = v/(\omega R_\infty) = 0.5$).

Figure 15 — Prediction of thrust grading for a four bladed propeller at $\beta_{0.7} = 60$.

Figure 16 — Prediction of thrust grading for a four bladed propeller at $\beta_{0.7} = 25$.

Figure 17 — Typical C_L versus α curve.

Figure 18 — Performance map comparison of a sample propeller design with a Hamilton Standard 4 bladed, 100 AF, $C_{L_i} = .3$ propeller.

Figure 19 — Prediction of $G(x)$ given blade data (CODE=4).
The blade data were designed by specifying a design $G(x)$ (CODE=3).

Figure 20 — Prediction of $\tan \beta_i$ given blade data (CODE=4).
The blade data were designed by specifying a design $G(x)$ (CODE=3).

Figure 21 — Prediction of $u_{a,i}$ given blade data (CODE=4).
The blade data were designed by specifying a design $G(x)$ (CODE=3).

Figure 22 — Performance map prediction for the SR1 30 degree swept propeller ($M=0.6$).

Figure 23 — Single ARA-D polar curve used in Hub effect study.

Figure 24 — Comparison of planforms for designs with identical thickness distributions and constant C_L , but different circulation distributions.

Figure 25 — Comparison of current and classical circulation distributions for planform design.

Figure 26 — Performance maps for propellers with $C_L = \text{const} = 0.3$.

Figure 27 — Comparison of planforms for designs with identical thickness and C_L distributions ($C_L \neq \text{const}$), but different circulation distributions.

Figure 28 — Performance maps for propellers with $C_L \neq \text{const.}$

Figure 29 — Performance map comparison between propellers designed with $C_L \neq \text{const}$ and $C_L = \text{const}$ using Lerbs' circulation.

Figure 30 — Effect of poorly designed hub on propeller performance.

Figure 31 — Velocity inflow profile due to the presence of an NACA D type cowl.

Figure 34 — Effect of non-uniform inflow on design point circulation.

Figure 35 — Performance comparison with propeller optimized for non-uniform inflow.

Appendix A

Input File Description

CARD A :

FORMAT(4(I4),2(F10.0))

VARIABLES : CODE,NUM,NBLADE,NGAUSS,LAM,X_H

DESCRIPTION :

CODE= 1,2,3, or 4.

CODE=1 specifies that the optimum circulation given DISVEL is to be found.

CODE=2 specifies that the optimum circulation given thrust (CT) or power (CP) coefficient is to be found.

CODE=3 -- find circulation given the shape of the circulation distribution.

CODE=4 -- given propeller geometry and the polar curves of the sections, find the operating characteristics.

NUM= Any number between 3 and 12. In general, the greater NUM is, the more reliable the numerical results will be. It is suspected, however, that instabilities may occur if NUM>12.

NBLADE= Number of blades.

NGAUSS= 5,7 or 9 is the order of integration used in the procedures. NGAUSS= 7 has been found to be satisfactory in all the test cases thus far, however, if this is suspect try NGAUSS=9.

LAM= Advance ratio=SPEED/(DIA*(revolution rate)).

X_H= Non-dimensional hub radius.

CARD B: If CODE=4, go to CARD C.

If CODE=1 { FORMAT(F10.0)

VARIABLES : DISVEL }

If CODE=2 { FORMAT(I4,2(F10.0))

VARIABLES : COEFF,C_GIVE,C_TOL }

If CODE=3 { FORMAT(I4,F10.0)

VARIABLES : COEFF,C_GIVE }

DESCRIPTION :

DISVEL=Non-dimensional 'displacement velocity' of vortex sheet.

COEFF= 1 or 2 depending upon whether thrust (CT) or power (CP) coefficient respectively is specified.

C_GIVE= Value of CT or CP.

C_TOL= Error bound of CT (COEFF=1) or CP (COEFF=2).

CARD C: If (CODE=1) or (CODE=2), uniform flow assumed. Therefore, go to CARD E.

FORMAT(A10)

VARIABLES : PROFLE

DESCRIPTION :

PROFLE= 'UNIFORM ' or 'NONUNIFORM' depending upon whether flow is uniform or not.

CARD D: NUM+1 cards required. If PROFILE='UNIFORM ' go to CARD E

FORMAT(3(F10.0))

VARIABLES : XX(I),VELDED(I),THRDED(I) I=0,NUM

DESCRIPTION :

XX(I)= $0.5 \cdot (1 + X_H - (1 - X_H) \cdot \cos(\pi \cdot I / \text{NUM}))$ = Non-dimensional radius.

VELDED(I)=Velocity deduction at radial station XX(I) or in other words the velocity at station XX(I) is equal to $(1 - \text{VELDED}(I)) \cdot (\text{free stream velocity})$.

THRDED(I)=Thrust deduction at station XX(I)= fraction by which the thrust element at station XX(I) must be reduced by to account for body interference. Setting this to zero will give the isolated propeller thrust.

CARD E: If CODE=4 go to CARD I since drag data is automatically retrieved from the polar curves.

FORMAT(I4)

VARIABLES : NDRAG

DESCRIPTION :

NDRAG=Number of drag data points given. If NDRAG=1, skin friction drag coefficient is considered to be constant across the blade. NDRAG >=1

CARD F: NDRAG cards required

FORMAT(A10,2(F10.0))

VARIABLES : DCHAR,XDRAG(I),DDRAG(I) I=1,NDRAG

DESCRIPTION:

DCHAR='DRAG '

XDRAG(I)=Non-dimensional radius. It is assumed that $XDRAG(I+1) > XDRAG(I)$. Linear interpolation is used to interpolate between points.

DDRAG(I)=Skin friction drag to lift coefficient (CD/CL) at station XDRAG(I).

CARD G: If (CODE.NE.3) go to CARD I

FORMAT(F15.0)

VARIABLES : KTOL

DESCRIPTION :

KTOL=Tolerance to which the multiplication factor for the circulation is to be determined. In this mode (ie CODE=3) a desired shape for the circulation distribution along with power or thrust coefficient is specified and the program then finds the factor by which the circulation shape must be multiplied to achieve the desired CT or CP. The factor is determined to a tolerance of KTOL.

CARD H: Several cards necessary

FORMAT(2(F15.0))

VARIABLES : X_SPEC(I),F_SPEC(I)

DESCRIPTION :

X_SPEC(I)=Non-dimensional radius.

F_SPEC(I)=Value of circulation profile at station
X_SPEC(I).

NOTES :

- DO NOT specify shape at hub or tip as program automatically assumes these to be zero.
- Assumes X_SPEC(I+1) > X_SPEC(I).
- Quadratic interpolation used to find circulation at intermediate values of X.
- END OF INPUT DATA CARD SPECIFIED BY SETTING X_SPEC(I)=999.999.

CARD I:

FORMAT(10(A8))

VARIABLES : TITLE

DESCRIPTION :

TITLE=Program title for the problem can be up to 80 characters long.

CARD J: if CODE=4 go to CARD K

FORMAT(I4)

VARIABLES : IEND

DESCRIPTION :

IEND= 0 if geometric data generation is desired.

= 1 if geometric data generation is not desired.

CARD K: If (IEND=1) and (CODE.NE.4), stop here.

FORMAT(2(F15.0))

VARIABLES : AMIN,AMAX

DESCRIPTION :

AMIN=Upper bound for the minimum input value for angle of attack.

AMAX=Lower bound for the maximum input value for angle of attack.

NOTES :

Polar data for each 2D section MUST span the range of incidence from at least lower than AMIN to at least larger than AMAX, otherwise an error message will be printed and the program halted.

CARD L:

FORMAT(5(I4))

VARIABLES : NREY,NMACH,NTBYC,NCAMB,NUM3

DESCRIPTION :

NREY=Number of different Reynold's numbers for which the polar data bank is fully specified.

NMACH=Analogous to NREY.

NTBYC=Number of different thickness to chord values in the polar data bank.

NCAMB=Analogous to NTBYC.

NUM3=This number specifies the order of the interpolating function (see NOTES below).

NOTES :

- To keep the data requirements reasonable, the following limits have been set,
 - NMACH,NREY $\leq$ 5
 - NTBYC,NCAMB $\leq$ 8
 - NUM3 $\leq$ 7
- Note that a full data bank may end up being a formidable size ie $5*5*8*8 = 1600$ data files or polar curves. However, it is unlikely that a full data bank will ever be necessary.
- CD(X) and CL(X) are cast into the form

$$f(X)=a_0(k)+a_1(k)*\sin(r*x)+a_2(k)*\sin(2*r*x)+...+a_{NUM3}(k)*\sin(NUM3*r*x).$$
 where,
 - $f(X)=CD$ if $k=2$
 - $=CL$ if $k=1$,
 - $X=$ Angle of incidence,
 - $x=X-AMIN$,
 - $r=45$ degrees.

CARD M:

```

FORMAT(5(F15.0))
VARIABLES : REY(I)  I=1,NREY
DESCRIPTION :
      REY(I)=Reynold's number. REY(I+1) > REY(I).
```

CARDS N, O, P, : Exactly analogous to CARD M except that REY is replaced with MACH, TBYC and CAMBER respectively.

```
DO 10 IREY=1,NREY
```

CARD Q:

```

FORMAT(A5,F15.0)
VARIABLES : WORD,REY(IREY)
DESCRIPTION :
      WORD='REY '
      REY(IREY)=PREVIOUSLY specified Reynold's number.
```

```
DO 20 IMACH=1,NMACH
```

CARD R:

```

FORMAT(A5,F15.0)
VARIABLES : WORD,MACH(IMACH)
DESCRIPTION :
      WORD='MACH '
      MACH(IMACH)=PREVIOUSLY specified mach number.
```

```
DO 30 ITBYC=1,NTBYC
```

CARD S:

```

FORMAT(A5,F15.0)
VARIABLES : WORD,TBYC(ITBYC)
DESCRIPTION :
```

WORD='TBYC '
 TBYC(ITBYC)=PREVIOUSLY defined thickness to chord.

DO 40 ICMBER=1,NCMBER

CARD T:
 FORMAT(A5,F15.5)
 VARIABLES : WORD,CAMBER(ICMBER)
 DESCRIPTION:
 WORD='CAMB '
 CAMBER(ICMBER)=PREVIOUSLY defined camber.

CARD U: Several cards necessary.
 FORMAT(A5,3(F15.0))
 VARIABLES : WORD,A1(I),A2(I,1),A2(I,2)
 DESCRIPTION :
 WORD='DATA ' if card specifies a point on the
 polar curve.
 ='ENDAT' to signify end of input for the
 polar curve.
 A1(I)=Angle of incidence IN RADIANS. $A1(I+1) >$
 $A1(I)$ and $A1(1) \leq AMIN$, $A1(MAX) \geq AMAX$ (see CARD K)
 $A2(I,K)=CL$ or CD depending upon whether $K=1$ or $K=2$
 respectively.

NOTES :
 - Linear interpolation is used to determine CL and
 CD at equally spaced values before the sine
 series representation described in the NOTES
 section of CARD L is determined.

40 CONTINUE
 30 CONTINUE
 20 CONTINUE
 10 CONTINUE

CARD V: If (CODE=4) go to CARD X.
 FORMAT(I4)
 VARIABLES : NTHICK
 DESCRIPTION :
 NTHICK=Number of thicknesses to be read.

CARD W: NTHICK cards required.
 FORMAT(3(F15.0))
 VARIABLES : XDATA(I),THICK(I),XCRUS(I)
 DESCRIPTION :
 $XDATA(I)$ =Non-dimensional radius. $XDATA(I+1) > XDATA(I)$.
 $THICK(I)$ =Thickness at $XDATA(I)$ in meters.
 $XCRUS(I)$ =Design section lift coefficient.

CARD X:
 FORMAT(5(F15.0))
 VARIABLES : SPEED,DIA,GAM,TEMP,VISCOS
 DESCRIPTION :
 SPEED=Forward speed in m/s.

DIA=Diameter of propeller in m.
 GAM=Specific heat ratio (usually about 1.4 for air).
 TEMP=Temperature in KELVIN.
 VISCOS=Kinematic viscosity in m*m/s.

CARD Y: If (CODE.NE.4) input complete --- STOP

FORMAT(2(F15.0))

VARIABLES : XPSI,PSI_07

DESCRIPTION :

XPSI=X position to which all blade angle settings are referenced.

PSI_07=Blade angle setting IN RADIANS referenced to the propeller disk at XPSI to which the data in CARD Z are referenced to.

CARD Z: NUM-1 cards required

FORMAT(5(F15.0))

VARIABLES : XDATA(I),GCHORD(I),GCAMBER(I),GTBYC(I),PSI(I)

I=1,NUM-1

DESCRIPTION :

XDATA(I)= $0.5*(1+X_H-(1-X_H)*\cos(\pi*I/NUM))$.

GCHORD(I)=Chord at station I in m.

GCAMBER(I)=Camber at station I.

GTBYC(I)=Thickness to chord.

PSI(I)=Blade angle setting at station I in radians, referenced to the propeller disk.

CARD AA:

FORMAT(2(I5))

VARIABLES : NLAM,NPSI

DESCRIPTION :

NLAM=Number of different advance ratios for which performance is to be determined.

NPSI=Number of different blade angle settings for which performance is to be determined.

CARD AB:

FORMAT(2(F15.0))

VARIABLES : LAMMIN,LAMINC

DESCRIPTION :

LAMMIN=Starting value advance ratio.

LAMINC=Value by which LAMMIN is increased after each pass.

NOTES :

- Performance is calculated at NLAM advance coefficients beginning with LAMMIN and ending with LAMMIN+(NLAM*LAMINC).

CARD AC:

FORMAT(5(F15.0))

VARIABLES : BANGLE(I) I=1,NPSI

DESCRIPTION :

BANGLE(I)=Blade angle setting at station XPSI.

Description Of Terms In the Output File

GENERAL: ALL TERMS ARE DIMENSIONLESS AND ALL ANGLES IN RADIANS.

ADVANCE RATIO: $V/(ND)$.

ALPHA: Angle of incidence of the airfoil section.

BLADE ANGLE: Angle of blade at radius of reference with respect to propeller disk.

BIO: Angle of trailing vortex wake with respect to the propeller disk.

CL: Section lift coefficient.

CP: $(\text{Power input})/(\text{density} \cdot N^3 \cdot D^5)$.

CT: $(\text{Isolated propeller thrust})/(\text{density} \cdot N^2 \cdot D^4)$.

DISP. VEL: w/V . Where, w = apparent rearward displacement velocity of the resulting rigid helical trailing vortex sheet for the uniform onset flow, optimum circulation problem (ie. classical wake).

EFFICIENCY: Refers to isolated propeller efficiency if the thrust deduction is zero. If thrust deduction is specified, then interference due to potential effects is accounted for.

G: Lerb's non-dimensional circulation.

G FOURIER: Coefficients of the sine series representation for G.

MACH: Mach number, including interference velocities, at a specified section. Sound speed determined from specified input temperature and specific heat ratio.

POWER COEFF: See CP.

PSI: Blade angle of section with respect to the propeller disk.

REY: Section Reynold's number based on local chord and resultant total velocity vector.

TANBI: Tangent of BIO.

THRUST COEFF: See CT.

UA and UT: Non-dimensional axial and tangential interference velocities respectively. Velocities non-dimensionalized with respect to forward speed.

Appendix B

Example A

Input A1: Comparison With Theodorsen -- No Contraction

```
1 8 4 7 1.3090 .000
  .10
1
DRAG .300 .00000
SAMPLE TO COMPARE CIRCULATION OF LERBS AND THEODORSEN (CD = 0)
1
```

Output A1: Comparison With Theodorsen -- No Contraction

```
*****
RESULTS OF THE PROPELLER LIFTING LINE CALCULATIONS
*****

PROBLEM TITLE : SAMPLE TO COMPARE CIRCULATION OF LERBS AND THEODORSEN
*****
```

PROBLEM TYPE : OPTIMUM PROPELLER DESIGN

FUNDAMENTAL INPUT DATA :

```
DISP. VEL. = .10000
ADVANCE RATIO= 1.30900
BLADE NUMBER = 4.00000
HUB DIAMETER = .00000
```

```
(DRAG)/(LIFT) COEFFICIENT DATA
X      CD/CL
.30000 .00000
```

```
*****
RESULTS OF THE NUMERICAL ANALYSIS :
*****
```

```
X      G  G FOURIER  TANBI  UA  UT
```


.00000	.00000	.00000	66.10322	.00000	-.01513
.03806	.00028	.01049	12.04234	.00068	.00825
.14645	.00280	-.00410	3.12970	.00926	.02899
.30866	.00728	-.00070	1.48493	.03120	.04633
.50000	.01118	-.00019	.91667	.05434	.04981
.69134	.01267	-.00001	.66296	.06947	.04605
.85355	.01104	.00002	.53697	.07762	.04168
.96194	.00645	.00001	.47647	.08150	.03883
1.00000	.00000	.00000	.45849	.08293	.03801

DISP. VEL. = .10000
 THRUST COEFF = .12353
 POWER COEFF = .17767
 EFFICIENCY = .90909

***** END OF LIFTING LINE ANALYSIS *****

Input A2: Comparison With Theodorsen — Contraction Effect

1 8 4 7 1.2884 .000
 .10
 1
 DRAG .300 .00000
 SAMPLE TO COMPARE CIRCULATION OF LERBS AND THEODORSEN (CD = 0)
 1

Output A2: Comparison With Theodorsen — Contraction Effect

RESULTS OF THE PROPELLER LIFTING LINE CALCULATIONS

PROBLEM TITLE : SAMPLE TO COMPARE CIRCULATION OF LERBS AND THEODORSEN

PROBLEM TYPE : OPTIMUM PROPELLER DESIGN

FUNDAMENTAL INPUT DATA :

DISP. VEL. = .10000
 ADVANCE RATIO = 1.28840
 BLADE NUMBER = 4.00000
 HUB DIAMETER = .00000

(DRAG)/(LIFT) COEFFICIENT DATA
 X CD/CL
 .30000 .00000

RESULTS OF THE NUMERICAL ANALYSIS :

X	G	G FOURIER	TANBI	UA	UT
.00000	.00000	.00000	65.28834	.00000	-.01532
.03806	.00028	.01050	11.85283	.00071	.00838
.14645	.00282	-.00409	3.08045	.00953	.02937
.30866	.00730	-.00071	1.46156	.03189	.04660
.50000	.01119	-.00020	.90224	.05513	.04974
.69134	.01266	-.00001	.65253	.07014	.04577
.85355	.01102	.00002	.52852	.07817	.04131
.96194	.00644	.00001	.46897	.08197	.03844
1.00000	.00000	.00000	.45127	.08339	.03762

DISP. VEL. = .10000
 THRUST COEFF = .12159
 POWER COEFF = .17232
 EFFICIENCY = .90909

***** END OF LIFTING LINE ANALYSIS *****

Example B: Comparison With Brockett

Input B1:

3 10 6 7 1.0177 .176
 2 .233580

NONUNIFORM

0.176	.5642	.1630
0.196	.5534	.1630
0.255	.5008	.1630
0.346	.3954	.1630
0.461	.2796	.1630
0.588	.2132	.1630
0.715	.1821	.1630
0.830	.1538	.1630
0.921	.1339	.1630
0.980	.1252	.1630
1.000	.1224	.1630

1
 DRAG .300 .05000

.0001	
.196	.0072
.250	.0126
.300	.0151
.400	.0172
.500	.0179
.600	.0181
.700	.0179
.800	.0168
.900	.0136
.950	.0103
.980	.0068
.999.999	.0000

SAMPLE RUN TO CHECK RESULTS WITH THOSE OF BROCKETT

1

Output B1:

NUMBER OF ITERATIONS = 3

RESULTS OF THE PROPELLER LIFTING LINE CALCULATIONS

PROBLEM TITLE : SAMPLE RUN TO CHECK RESULTS WITH THOSE OF BROCKETT

PROBLEM TYPE : OFF DESIGN PROPELLER CALCULATION

FUNDAMENTAL INPUT DATA :

POWER COEFF = .23358
 ADVANCE RATIO = 1.01770
 BLADE NUMBER = 6.00000
 HUB DIAMETER = .17600

SPECIFIED SHAPE OF THE CIRCULATION DISTRIBUTION :

X	CIRC
.19600	.00720
.25000	.01260
.30000	.01510
.40000	.01720
.50000	.01790
.60000	.01810
.70000	.01790
.80000	.01680
.90000	.01360
.95000	.01030
.98000	.00680

NONUNIFORM FLOW SPECIFIED

X	WELDED	THREADED
.17600	.56420	.16300
.19616	.55340	.16300
.25468	.50080	.16300
.34583	.39540	.16300
.46068	.27960	.16300
.58800	.21320	.16300
.71531	.18210	.16300
.83017	.15380	.16300
.92131	.13390	.16300
.97984	.12520	.16300
1.00000	.12240	.16300

(DRAG)/(LIFT) COEFFICIENT DATA

X	CD/CL
.30000	.05000

RESULTS OF THE NUMERICAL ANALYSIS :

X	G	G FOURIER	TANBI	UA	UT
.17600	.00000	.00000	1.54743	.08057	.20961

.1816	.00719	.01935	1.29583	.09282	.18928
.25468	.01283	.00026	1.01569	.12538	.17127
.34583	.01841	.00128	.82845	.15480	.15091
.46068	.01765	.00009	.87406	.15929	.11705
.58800	.01796	-.00008	.55080	.16166	.09316
.71531	.01765	-.00005	.46146	.16497	.07819
.83017	.01593	.00005	.40434	.16300	.06675
.92131	.01223	.00003	.36882	.15599	.05770
.97984	.00677	.00003	.34733	.15662	.05511
1.00000	.00000	.00000	.34348	.16318	.05685

THRUST COEFF = .16989
 POWER COEFF = .23358
 EFFICIENCY = .74019

***** END OF LIFTING LINE ANALYSIS *****

Example C: Comparison With Experimental Results

Input C1:

4	6	4	7	.2500					
UNIFORM									
PERFORMANCE CURVE FOR PANKHURST'S EXPERIMENT (SLOPE EST.)									
				-.06981	.26180				
1	1	8	1	3					
1600000	.00000								
	.40000								
	.10000				.11000	.12759	.14100	.17241	.21400
.23793		.31000							
	1.00000								
REY	1600000	.00000							
MACH		.40000							
TBYC		.10000							
CAMB		1.00000							
DATA		-.10000		-.18154		.01400			
DATA		.30000		1.77050		.01400			
ENDAT		.00000		.00000		.00000			
TBYC		.11000							
CAMB		1.00000							
DATA		-.10000		-.13859		.01400			
DATA		.30000		1.81341		.01400			
ENDAT		.00000		.00000		.00000			
TBYC		.12759							
CAMB		1.00000							
DATA		-.10000		-.07862		.01400			
DATA		.30000		1.87538		.01400			
ENDAT		.00000		.00000		.00000			
TBYC		.14100							
CAMB		1.00000							
DATA		-.10000		-.04490		.01400			
DATA		.30000		1.90710		.01400			
ENDAT		.00000		.00000		.00000			
TBYC		.17241							
CAMB		1.00000							
DATA		-.10000		.08436		.01400			
DATA		.30000		2.03636		.01400			
ENDAT		.00000		.00000		.00000			
TBYC		.21400							
CAMB		1.00000							
DATA		-.10000		.04450		.01400			
DATA		.30000		1.99650		.01400			
ENDAT		.00000		.00000		.00000			
TBYC		.23793							
CAMB		1.00000							
DATA		-.10000		-.09621		.01400			

DATA	.30000	1.85579	.01400		
ENDAT	.00000	.00000	.00000		
TBYC	.31000				
CAMB	1.00000				
DATA	-.10000	-.50508	.01400		
DATA	.30000	1.44690	.01400		
ENDAT	.00000	.00000	.00000		
	50.0	.762	1.4	288.	.000015
	.70000	.84648			
	.30024	.08179	.10000	.23793	1.20672
	.43750	.08668	.10000	.17241	1.05627
	.62500	.08513	.10000	.12759	.91961
	.81250	.07043	.10000	.11000	.81402
	.94976	.02562	.10000	.10000	.76288
4	1				
	2.0000	.5000			
	1.04720				

Output C1:

RESULTS OF THE PROPELLER LIFTING LINE CALCULATIONS

PROBLEM TITLE : PERFORMANCE CURVE FOR PANKHURST'S EXPERIMENT (SLOPE EST.)

UNIFORM FLOW SPECIFIED

FUNDAMENTAL INPUT DATA :

BLADE NUMBER = 4.00000
HUB DIAMETER = .25000

FORWARD SPEED : 50.00000
DIAMETER : .76200
KINEMATIC VISCOSITY: .000015000
SOUND SPEED : 340.23331
MACH NUMBER : .14696
BLADE GEOMETRY :

X	CHORD	CAMBER	T/C	PSI
.30024	.08179	.10000	.23793	1.20672
.43750	.08668	.10000	.17241	1.05627
.62500	.08513	.10000	.12759	.91961
.81250	.07043	.10000	.11000	.81402
.94976	.02562	.10000	.10000	.76288

BLADE ANGLE AT 70.0 % CHORD = .84648
ACTIVITY FACTOR = 113.86

RESULTS OF THE NUMERICAL ANALYSIS :

BLADE ANGLE = 1.04720
 ADVANCE RATIO = 2.00000

 CT = .42086
 CP = 1.13624
 EFFICIENCY = .74078

X	G	G FOURIER	BIO	UA	UT
.25000	.00000	.00000	1.37824	.05799	.18642
.30024	.01613	.03395	1.31167	.06764	.19015
.43750	.02741	-.00161	1.11770	.08716	.15792
.62500	.03363	-.00027	.92710	.11609	.14361
.81250	.03241	.00128	.80719	.16664	.15811
.94976	.01670	-.00058	.67188	.12005	.07628
1.00000	.00000	.00000	.58863	.04250	.00917

X	REY	MACH	CL	ALPHA	PSI
.30024	299896.7	.16165	.85897	.09577	1.40744
.43750	346720.0	.17635	1.25201	.13929	1.25699
.62500	391598.0	.20280	1.35484	.19323	1.12033
.81250	375626.7	.23513	1.36289	.20766	1.01474
.94976	152012.0	.26159	1.72757	.29172	.96360

NUMBER OF ITERATIONS = 13

.....

BLADE ANGLE = 1.04720
 ADVANCE RATIO = 2.50000

 CT = .35864
 CP = 1.14559
 EFFICIENCY = .78266

X	G	G FOURIER	BIO	UA	UT
.25000	.00000	.00000	1.41138	.03595	.14760
.30024	.01196	.02319	1.35251	.04263	.14771
.43750	.01950	-.00027	1.18033	.05341	.11673
.62500	.02269	.00009	.99855	.06666	.09795
.81250	.02139	.00082	.87751	.09691	.10867
.94976	.01100	-.00041	.74692	.07000	.05590
1.00000	.00000	.00000	.68657	.02311	.00831

X	REY	MACH	CL	ALPHA	PSI
.30024	288991.3	.15578	.65961	.05493	1.40744
.43750	323605.3	.16459	.94633	.07666	1.25699
.62500	357546.0	.18517	1.00548	.12178	1.12033
.81250	333368.7	.20868	1.01898	.13723	1.01474
.94976	131516.0	.22632	1.31572	.20668	.96360

NUMBER OF ITERATIONS = 12

.....

BLADE ANGLE = 1.04720
 ADVANCE RATIO = 3.00000

 CT = .29531

CP = 1.09785
EFFICIENCY = .80697

X	G	G FOURIER	BIO	UA	UT
.25000	.00000	.00000	1.43500	.02374	.12191
.30024	.00921	.01608	1.38107	.02880	.11972
.43750	.01426	.00061	1.22696	.03494	.08923
.62500	.01545	.00033	1.05544	.03902	.06626
.81250	.01412	.00053	.93642	.05645	.07299
.94976	.00723	-.00030	.83036	.04081	.03932
1.00000	.00000	.00000	.77112	.01201	.00587

X	REV	MACH	CL	ALPHA	PSI
.30024	283538.7	.15284	.52055	.02637	1.40744
.43750	312048.0	.15871	.71895	.03003	1.25699
.62500	334844.7	.17341	.72796	.06469	1.12033
.81250	305196.7	.19105	.73149	.07832	1.01474
.94976	119560.0	.20574	.95659	.13324	.96360

NUMBER OF ITERATIONS = 10

BLADE ANGLE = 1.04720
ADVANCE RATIO = 3.50000

CT = .23016
CP = .98947
EFFICIENCY = .81413

X	G	G FOURIER	BIO	UA	UT
.25000	.00000	.00000	1.45219	.01668	.10325
.30024	.00722	.01104	1.40301	.02051	.09942
.43750	.01052	.00120	1.26303	.02381	.06933
.62500	.01031	.00051	1.10096	.02236	.04288
.81250	.00898	.00031	.98525	.03141	.04583
.94976	.00461	-.00022	.89220	.02322	.02646
1.00000	.00000	.00000	.84505	.00641	.00462

X	REV	MACH	CL	ALPHA	PSI
.30024	278086.0	.14990	.41349	.00443	1.40744
.43750	353269.3	.15578	.54301	-.00604	1.25699
.62500	323494.0	.16753	.50600	.01937	1.12033
.81250	286415.3	.17929	.49335	.02949	1.01474
.94976	111020.0	.19105	.65477	.07140	.96360

NUMBER OF ITERATIONS = 10

BLADE ANGLE = 1.04720
ADVANCE RATIO = 4.00000

CT = .16196
CP = .81522
EFFICIENCY = .79467

X	G	G FOURIER	BIO	UA	UT
.25000	.00000	.00000	1.46537	.01220	.08924
.30024	.00573	.00726	1.41994	.01517	.08410
.43750	.00772	.00163	1.29145	.01669	.05430
.62500	.00846	.00064	1.13790	.01170	.02485
.81250	.00514	.00015	1.02607	.01501	.02433
.94976	.00265	-.00016	.94509	.01196	.01580
1.00000	.00000	.00000	.90671	.00329	.00337

X	REY	MACH	CL	ALPHA	PSI
.30024	278086.0	.14990	.33088	-.01251	1.40744
.43750	300490.7	.15284	.40431	-.03446	1.25699
.62500	312143.3	.16165	.32574	-.01757	1.12033
.81250	277024.7	.17341	.29422	-.01133	1.01474
.94976	105896.0	.18223	.39688	.01851	.96360

NUMBER OF ITERATIONS = 10

.....

Example D: Example Of A Complete Design Loop

Input D1: Determination Of Betz' Optimum Circulation

```

2  6  4  7    2.00    .15
2    .21000
1
DRAG      .50000      .00000
ROUGH COMPARISON WITH HAMILTON STANDARD (CD=0)
1

```

Output D1: Solution For Betz' Optimum Circulation

NUMBER OF ITERATIONS = 3

RESULTS OF THE PROPELLER LIFTING LINE CALCULATIONS

PROBLEM TITLE : SAMPLE TO COMPARE A PRELIMINARY DESIGN TO HAMILTON STANDARD DATA (CD EQUAL 0)

PROBLEM TYPE : OPTIMUM PROPELLER DESIGN

FUNDAMENTAL INPUT DATA :

```

POWER COEFF =    .21000
ADVANCE RATIO=  2.00000
BLADE NUMBER =  4.00000
HUB DIAMETER =  .15000

```

(DRAG)/(LIFT) COEFFICIENT DATA

X	CD/CL
.20000	.00000
.30000	.00000
.60000	.00000
.90000	.00000

RESULTS OF THE NUMERICAL ANALYSIS :

X	G	G FOURIER	TANBI	UA	UT
.15000	.00000	.00000	4.47192	.00255	.01143
.20694	.00174	.00621	3.24840	.00484	.01573
.36250	.00416	-.00141	1.85440	.01260	.02336
.57500	.00635	-.00014	1.16908	.02363	.02762
.78750	.00659	-.00001	.85361	.03235	.02761

.94306	.00420	.00000	.71281	.03708	.02643
1.00000	.00000	.00000	.67230	.03860	.02595

DISP. VEL.	=	.05592
THRUST COEFF	=	.09943
POWER COEFF	=	.20998
EFFICIENCY	=	.94704

***** END OF LIFTING LINE ANALYSIS *****

Input D2: File Used To Generate Planform Data
For A User Specified Circulation Distribution

```

3 6 4 7 2.00 .15
2 .21000
UNIFORM
1
DRAG .50000 .00000
.00100
.20700 .02240 .00000
.36300 .05350 .00000
.57500 .08500 .00000
.78800 .08500 .00000
.94300 .02600 .00000
999.999
ROUGH COMPARISON WITH HAMILTON STANDARD (CD=0)
0
-.06981 .26180
1 1 2 1 3
1600000.00000
.40000
.08000 .25000
1.00000
REY 1600000.00000
MACH .40000
TBYC .08000
CAMB 1.00000
DATA -.07850 .00000 .00000
DATA .00000 .48900 .00000
DATA .06500 .92800 .00000
DATA .13960 1.35600 .00000
DATA .20940 1.53900 .00000
DATA .27930 1.46600 .00000
ENDAT .00000 .00000 .00000
TBYC .25000
CAMB 1.00000
DATA -.07850 .00000 .00000
DATA .00000 .48900 .00000
DATA .06500 .92800 .00000
DATA .13960 1.35600 .00000
DATA .20940 1.53900 .00000
DATA .27930 1.46600 .00000
ENDAT .00000 .00000 .00000
6
.20700 .03936 .30000
.36300 .01140 .30000
.57500 .00687 .30000
.75000 .00520 .30000
.78800 .00374 .30000
.94300 .00208 .30000
100. 3. 1.4 288. .0000172

```

Output D2: Solution For Planform Data¹

NUMBER OF ITERATIONS = 1

RESULTS OF THE PROPELLER LIFTING LINE CALCULATIONS

PROBLEM TITLE : ROUGH COMPARISON WITH HAMILTON STANDARD (CD=0)

PROBLEM TYPE : OFF DESIGN PROPELLER CALCULATION

FUNDAMENTAL INPUT DATA :

POWER COEFF = .21000
 ADVANCE RATIO = 2.00000
 BLADE NUMBER = 4.00000
 HUB DIAMETER = .15000

SPECIFIED SHAPE OF THE CIRCULATION DISTRIBUTION :

X CIRC

.20700 .00224
 .36300 .00535
 .57500 .00850
 .78800 .00850
 .94300 .00260

UNIFORM FLOW SPECIFIED

(DRAG)/(LIFT) COEFFICIENT DATA

X CD/CL
 .50000 .00000

RESULTS OF THE NUMERICAL ANALYSIS :

X	G	G FOURIER	TANBI	UA	UT
.15000	.00000	.00000	4.33879	.00215	.00465
.20694	.00188	.00643	3.26634	.00822	.01700
.36250	.00450	-.00085	1.86139	.01411	.02460
.57500	.00715	-.00103	1.18007	.02817	.03193
.78750	.00716	.00068	.86819	.04296	.03570
.94306	.00219	-.00030	.67246	-.00193	-.00285
1.00000	.00000	.00000	.59297	-.05009	-.03114

THRUST COEFF = .09913
 POWER COEFF = .21000
 EFFICIENCY = .94410

¹ $C_L \neq .3$ at hub because such a C_L would have resulted in too large a thickness to chord

***** END OF LIFTING LINE ANALYSIS *****

RESULTS OF THE PROPELLER STRIP THEORY CALCULATIONS

FORWARD SPEED : 100.00000
 DIAMETER : 3.00000
 KINEMATIC VISCOSITY: .00002
 SOUND SPEED : 340.23331
 MACH NUMBER : .29392

GEOMETRICAL PARAMETERS :

X POSITION MAX_CL	CAMBER	CHORD	THICKNESS	T/C	TWIST
.20700	1.00000	.15744	.03936	.25000	1.22921
1.54349					
.36300	1.00000	.24648	.01140	.04625	1.04557
1.54349					
.57500	1.00000	.33430	.00687	.02055	.83614
1.54349					
.75000	1.00000	.31023	.00520	.01676	.70672
1.54349					
.78800	1.00000	.28289	.00374	.01322	.68285
1.54349					
.94300	1.00000	.07707	.00208	.02699	.56038
1.54349					

ACTIVITY FACTOR = 105.74

Input D3: Input To Ensure That the Aerodynamic
 Parameters Of Output D2 Can Be Duplicated By
 Beginning With the Planform Data

4 6 4 7 2.00 .15
 UNIFORM
 ROUGH COMPARISON WITH HAMILTON STANDARD (CD=0)
 -.06981 .26180
 1 1 2 1 3
 1600000.00000
 .40000
 .08000 .25000
 1.00000
 REY 1600000.00000
 MACH .40000
 TBYC .08000
 CAMB 1.00000
 DATA -.07850 .00000 .00000
 DATA .00000 .48900 .00000
 DATA .06500 .92800 .00000
 DATA .13960 1.35600 .00000
 DATA .20940 1.53900 .00000
 DATA .27930 1.46600 .00000
 ENDAT .00000 .00000 .00000
 TBYC .25000
 CAMB 1.00000
 DATA -.07850 .00000 .00000
 DATA .00000 .48900 .00000
 DATA .06500 .92800 .00000
 DATA .13960 1.35600 .00000

DATA	.20940	1.53900	.00000		
DATA	.27930	1.46400	.00000		
ENDAT	.00000	.00000	.00000		
	100.	3.	1.4	288.	.0000172
	.75000	.70672			
	.20700	.15744	1.00000	.25000	1.22921
	.36300	.24648	1.00000	.04625	1.04557
	.57500	.33430	1.00000	.02055	.83614
	.78800	.28289	1.00000	.01322	.68285
	.94300	.07707	1.00000	.02699	.56038
1	1				
	2.0000	.2000			
	.70672				

Output D3: Solution To Input D3

RESULTS OF THE PROPELLER LIFTING LINE CALCULATIONS

PROBLEM TITLE : ROUGH COMPARISON WITH HAMILTON STANDARD (CD=0)

UNIFORM FLOW SPECIFIED

FUNDAMENTAL INPUT DATA :

BLADE NUMBER = 4.00000
HUB DIAMETER = .15000

FORWARD SPEED : 100.00000
DIAMETER : 3.00000
KINEMATIC VISCOSITY: .000017200
SOUND SPEED : 340.23331
MACH NUMBER : .29392
BLADE GEOMETRY :

X	CHORD	CAMBER	T/C	PSI
.20700	.15744	.10000	.25000	1.22921
.36300	.24648	.10000	.04625	1.04557
.57500	.33430	.10000	.02055	.83614
.78800	.28289	.10000	.01322	.68285
.94300	.07707	.10000	.02699	.56038

BLADE ANGLE AT 75.0 % CHORD = .70672
ACTIVITY FACTOR = 104.72

RESULTS OF THE NUMERICAL ANALYSIS :

BLADE ANGLE = .70672
ADVANCE RATIO = 2.00000

CT = .09945
 CP = .21071
 EFFICIENCY = .94397

X	G	G FOURIER	BIO	UA	UT
.15000	.00000	.00000	1.34491	.00229	.00528
.20700	.00191	.00645	1.27364	.00634	.01739
.36300	.00453	-.00085	1.07698	.01422	.02477
.57500	.00717	-.00102	.86762	.02828	.03200
.78800	.00716	.00067	.71434	.04296	.03563
.94300	.00221	-.00030	.59161	-.00155	-.00255
1.00000	.00000	.00000	.53581	-.04918	-.03059

X	REY	MACH	CL	ALPHA	PSI
.20700	961116.3	.30861	.21663	-.04443	1.22921
.36300	1647976.8	.33800	.30014	-.03141	1.04557
.57500	2604430.1	.39385	.29968	-.03148	.83614
.78800	2615087.8	.46733	.29960	-.03149	.68285
.94300	802065.7	.52611	.30127	-.03123	.56038

NUMBER OF ITERATIONS = 13

*****/*****

**Output D4: Propeller Performance Map Calculation
For Design Produced By Input D2 — Input Essen-
tially Identical To That Of D3.**

RESULTS OF THE PROPELLER LIFTING LINE CALCULATIONS

PROBLEM TITLE : ROUGH COMPARISON WITH HAMILTON STANDARD (CD=0)

UNIFORM FLOW SPECIFIED

FUNDAMENTAL INPUT DATA :

BLADE NUMBER = 4.00000
HUB DIAMETER = .15000

FORWARD SPEED : 100.00000
DIAMETER : 3.00000
KINEMATIC VISCOSITY: .000017200
SOUND SPEED : 340.23331
MACH NUMBER : .29392
BLADE GEOMETRY :

X	CHORD	CAMBER	T/C	PSI
.20700	.15744	.10000	.25000	1.22921
.36300	.24648	.10000	.04625	1.04557
.57500	.33430	.10000	.02055	.83614
.78800	.28289	.10000	.01322	.68285
.94300	.07707	.10000	.02699	.56038

BLADE ANGLE AT 75.0 % CHORD = .70672
ACTIVITY FACTOR = 104.72

RESULTS OF THE NUMERICAL ANALYSIS :

BLADE ANGLE = .71600
ADVANCE RATIO = 1.60000

CT = .21755
CP = .40416
EFFICIENCY = .86128

X	G	G FOURIER	BIO	UA	UT
.15000	.00000	.00000	1.30944	.01311	.02354
.20700	.00502	.01801	1.23638	.02207	.04517
.36300	.01215	-.00302	1.01562	.04660	.06447
.57500	.01988	-.00241	.80803	.09140	.08709
.78800	.01998	.00151	.65796	.12752	.08950
.94300	.00763	-.00054	.50991	.03035	.00892

1.00000 .00000 .00000 .43288 -.07371 -.04099

X	REY	MACH	CL	ALPHA	PSI
.20700	988576.7	.31743	.55454	.00811	1.23849
.36300	1762618.6	.36152	.75448	.03823	1.05485
.57500	2934842.9	.44381	.74272	.03739	.84542
.78800	3026265.0	.54081	.72213	.03417	.69213
.94300	945451.7	.62016	.88455	.05975	.56966

NUMBER OF ITERATIONS = 11

BLADE ANGLE = .71600
ADVANCE RATIO = 1.80000

CT = .16301
CP = .33137
EFFICIENCY = .90178

X	G	G FOURIER	BIO	UA	UT
.15000	.00000	.00000	1.33095	.00710	.01551
.20700	.00348	.01210	1.25579	.01341	.03186
.36300	.00828	-.00187	1.04991	.02849	.04446
.57500	.01336	-.00167	.84097	.05675	.05920
.78800	.01338	.00107	.68881	.08173	.06264
.94300	.00486	-.00041	.55212	.01286	.00374
1.00000	.00000	.00000	.48526	-.06079	-.03577

X	REY	MACH	CL	ALPHA	PSI
.20700	970269.8	.31155	.39081	-.01730	1.23849
.36300	1690967.4	.34682	.53408	.00494	1.05485
.57500	2740482.5	.41442	.53095	.00445	.84542
.78800	2796005.8	.49966	.52366	.00332	.69213
.94300	864797.1	.56726	.61528	.01754	.56966

NUMBER OF ITERATIONS = 11

BLADE ANGLE = .71600
ADVANCE RATIO = 2.00000

CT = .11148
CP = .23781
EFFICIENCY = .93759

X	G	G FOURIER	BIO	UA	UT
.15000	.00000	.00000	1.34883	.00345	.00916
.20700	.00222	.00726	1.27738	.00765	.02112
.36300	.00511	-.00094	1.08020	.01606	.02799
.57500	.00803	-.00107	.87133	.03152	.03579
.78800	.00801	.00073	.71813	.04751	.03955
.94300	.00258	-.00031	.59292	.00045	-.00153
1.00000	.00000	.00000	.53517	-.05010	-.03138

X	REY	MACH	CL	ALPHA	PSI
---	-----	------	----	-------	-----

.20700	981116.3	.30861	.25211	-.03889	1.23849
.36300	1647976.8	.33800	.33905	-.02535	1.05485
.57500	2604430.1	.39385	.33545	-.02591	.84542
.78800	2615087.8	.46733	.33491	-.02600	.69213
.94300	802065.7	.52611	.35253	-.02326	.56966

NUMBER OF ITERATIONS = 13

BLADE ANGLE = .71600
 ADVANCE RATIO = 2.20000

 CT = .05479
 CP = .12440
 EFFICIENCY = .96893

X	G	G FOURIER	BIO	UA	UT
.15000	.00000	.00000	1.36393	.00123	.00407
.20700	.00118	.00328	1.29560	.00385	.01243
.36300	.00250	-.00018	1.10669	.00745	.01443
.57500	.00363	-.00058	.89901	.01298	.01597
.78800	.00359	.00045	.74564	.02163	.01948
.94300	.00071	-.00023	.63182	-.00824	-.00653
1.00000	.00000	.00000	.58277	-.04074	-.02733

X	REY	MACH	CL	ALPHA	PSI
.20700	951962.8	.30567	.13544	-.05711	1.23849
.36300	1604986.0	.32919	.16917	-.05184	1.05485
.57500	2507249.9	.37915	.15794	-.05359	.84542
.78800	2467063.9	.44087	.15847	-.05351	.69213
.94300	748295.9	.49084	.10311	-.06216	.56966

NUMBER OF ITERATIONS = 14

BLADE ANGLE = .71600
 ADVANCE RATIO = 2.40000

 CT = -.00442
 CP = -.01030
 EFFICIENCY = 1.03108

X	G	G FOURIER	BIO	UA	UT
.15000	.00000	.00000	1.37673	-.00010	-.00017
.20700	.00031	-.00007	1.31119	.00131	.00521
.36300	.00030	.00046	1.12999	.00134	.00302
.57500	-.00007	-.00016	.92422	-.00096	-.00115
.78800	-.00013	.00022	.77137	.00166	.00172
.94300	-.00086	-.00016	.66873	-.01442	-.01130
1.00000	.00000	.00000	.62796	-.03256	-.02356

X	REY	MACH	CL	ALPHA	PSI
.20700	942809.3	.30273	.03566	-.07270	1.23849
.36300	1576325.5	.32331	.02005	-.07514	1.05485
.57500	2429505.8	.36739	-.00342	-.07880	.84542
.78800	2351934.3	.42030	-.00623	-.07924	.69213
.94300	712449.4	.46733	-.13324	-.09907	.56966

NUMBER OF ITERATIONS = 15

BLADE ANGLE = .80327
 ADVANCE RATIO = 1.60000

CT = .30615
 CP = .60635
 EFFICIENCY = .80785

X	G	G FOURIER	BIO	UA	UT
.15000	.00000	.00000	1.34714	.02934	.06039
.20700	.00801	.02623	1.26641	.03820	.08068
.36300	.01783	-.00415	1.04599	.06802	.09464
.57500	.02857	-.00305	.84302	.12710	.12546
.78800	.02882	.00219	.69406	.17914	.13082
.94300	.01141	-.00071	.52100	.05304	.01398
1.00000	.00000	.00000	.42376	-.08971	-.05447

X	REY	MACH	CL	ALPHA	PSI
.20700	988576.7	.31743	.88201	.05935	1.32576
.36300	1762618.6	.36152	1.10532	.09613	1.14212
.57500	2915406.9	.44087	1.06745	.08967	.93269
.78800	3026265.0	.54081	1.04173	.08534	.77940
.94300	945451.7	.62016	1.31780	.13593	.65693

NUMBER OF ITERATIONS = 14

BLADE ANGLE = .80327
 ADVANCE RATIO = 1.80000

CT = .26516
 CP = .56462
 EFFICIENCY = .84534

X	G	G FOURIER	BIO	UA	UT
.15000	.00000	.00000	1.36833	.02032	.05235
.20700	.00645	.02006	1.29127	.02729	.06726
.36300	.01392	-.00288	1.08041	.04749	.07498
.57500	.02176	-.00225	.87596	.08929	.09668
.78800	.02178	.00166	.72466	.12807	.10212
.94300	.00855	-.00055	.56482	.03427	.01125
1.00000	.00000	.00000	.47830	-.07120	-.04615

X	REY	MACH	CL	ALPHA	PSI
.20700	970269.8	.31155	.72416	.03449	1.32576
.36300	1690967.4	.34682	.89682	.06171	1.14212
.57500	2740482.5	.41442	.86554	.05373	.93269
.78800	2796005.8	.49966	.85300	.05474	.77940
.94300	864797.1	.56726	1.08187	.09211	.65693

NUMBER OF ITERATIONS = 11

BLADE ANGLE = .80327
 ADVANCE RATIO = 2.00000

 CT = .22052
 CP = .50186
 EFFICIENCY = .87881

X	G	G FOURIER	BIO	UA	UT
.15000	.00000	.00000	1.38604	.01455	.04601
.20700	.00518	.01494	1.31261	.01993	.05648
.36300	.01065	-.00181	1.11043	.03313	.05847
.57500	.01611	-.00161	.90608	.06130	.07209
.78800	.01598	.00126	.75358	.08952	.07724
.94300	.00612	-.00044	.60508	.01958	.00726
1.00000	.00000	.00000	.52888	-.05866	-.03996

X	REY	MACH	CL	ALPHA	PSI
.20700	961116.3	.30861	.58701	.01315	1.32576
.36300	1647976.8	.33800	.70625	.03169	1.14212
.57500	2604430.1	.29385	.67365	.02661	.93269
.78800	2615087.8	.45733	.66856	.02582	.77940
.94300	802065.7	.52611	.83475	.05185	.65693

NUMBER OF ITERATIONS = 12

BLADE ANGLE = .80327
 ADVANCE RATIO = 2.20000

 CT = .17379
 CP = .42079
 EFFICIENCY = .90860

X	G	G FOURIER	BIO	UA	UT
.15000	.00000	.00000	1.40120	.01065	.04113
.20700	.00410	.01069	1.33062	.01473	.04782
.36300	.00793	-.00093	1.13660	.02292	.04471
.57500	.01143	-.00108	.93337	.04039	.05126
.78800	.01118	.00095	.78062	.06002	.05574
.94300	.00406	-.00034	.64317	.00826	.00268
1.00000	.00000	.00000	.57609	-.04943	-.03535

X	REY	MACH	CL	ALPHA	PSI
.20700	951962.8	.30567	.47032	-.00486	1.32576
.36300	1604986.0	.32919	.53784	.00552	1.14212
.57500	2507249.9	.37915	.49786	-.00068	.93269
.78800	2467063.9	.44087	.49440	-.00122	.77940
.94300	752776.7	.49378	.59092	.01376	.65693

NUMBER OF ITERATIONS = 12

BLADE ANGLE = .80327
 ADVANCE RATIO = 2.40000

 CT = .12506

CP = .32093
EFFICIENCY = .93527

X	G	G FOURIER	BIO	UA	UT
.15000	.00000	.00000	1.41407	.00796	.03707
.20700	.00321	.00711	1.34621	.01098	.04057
.36300	.00563	-.00018	1.15974	.01546	.03307
.57500	.00750	-.00065	.95831	.02444	.03332
.78800	.00715	.00069	.80597	.03702	.03683
.94300	.00233	-.00027	.67940	-.00019	-.00186
1.00000	.00000	.00000	.62077	-.04157	-.03134

X	REY	MACH	CL	ALPHA	PSI
.20700	942809.3	.30273	.37056	-.02045	1.32576
.36300	1576325.5	.32331	.38878	-.01762	1.14212
.57500	2429505.8	.36739	.33736	-.02562	.93269
.78800	2351934.3	.42030	.33122	-.02657	.77940
.94300	712449.4	.46733	.35759	-.02247	.65693

NUMBER OF ITERATIONS = 13

BLADE ANGLE = .62873
ADVANCE RATIO = 1.60000

CT = .12007
CP = .20883
EFFICIENCY = .91996

X	G	G FOURIER	BIO	UA	UT
.15000	.00000	.00000	1.27250	-.00200	-.01233
.20700	.00200	.00959	1.19499	.00583	.01008
.36300	.00631	-.00180	.98465	.02437	.03333
.57500	.01101	-.00181	.77227	.05303	.04869
.78800	.01096	.00089	.62137	.07320	.04927
.94300	.00357	-.00039	.49612	.00414	-.00043
1.00000	.00000	.00000	.43774	-.06559	-.03302

X	REY	MACH	CL	ALPHA	PSI
.20700	988576.7	.31743	.22086	-.04377	1.15122
.36300	1748288.4	.35858	.39232	-.01707	.96758
.57500	2915406.9	.44087	.41131	-.01412	.75815
.78800	3026265.0	.54081	.39590	-.01651	.60486
.94300	945451.7	.62016	.41378	-.01373	.48239

NUMBER OF ITERATIONS = 12

BLADE ANGLE = .62873
ADVANCE RATIO = 1.80000

CT = .05895
CP = .11039
EFFICIENCY = .96121

X	G	G FOURIER	BIO	UA	UT
.15000	.00000	.00000	1.29412	-.00554	-.02059
.20700	.00050	.00409	1.22076	-.00083	-.00324
.36300	.00261	-.00084	1.01955	.00883	.01367
.57500	.00494	-.00112	.80575	.02223	.02246
.78800	.00495	.00061	.65255	.03328	.02464
.94300	.00107	-.00028	.53950	-.00940	-.00637
1.00000	.00000	.00000	.49032	-.05290	-.02892

X	REY	MACH	CL	ALPHA	PSI
.20700	970269.8	.31155	.05584	-.06954	1.15122
.36300	1690967.4	.34682	.16832	-.05197	.96758
.57500	2740482.5	.41442	.19628	-.04760	.75815
.78800	2796005.8	.49966	.19384	-.04799	.60486
.94300	860316.3	.56432	.13544	-.05711	.48239

NUMBER OF ITERATIONS = 14

BLADE ANGLE = .62873
ADVANCE RATIO = 2.00000

CT = -.00530
CP = -.01030
EFFICIENCY = 1.02988

X	G	G FOURIER	BIO	UA	UT
.15000	.00000	.00000	1.31198	-.00727	-.02721
.20700	-.00072	-.00037	1.24229	-.00484	-.01389
.36300	-.00040	-.00007	1.05003	-.00145	-.00229
.57500	.00000	-.00055	.83647	.00029	.00048
.78800	.00009	.00021	.68261	.00395	.00333
.94300	-.00095	-.00019	.58103	-.01829	-.01191
1.00000	.00000	.00000	.54085	-.04163	-.02493

X	REY	MACH	CL	ALPHA	PSI
.20700	961116.3	.30861	-.08196	-.09107	1.15122
.36300	1647976.8	.33800	-.02676	-.08245	.96758
.57500	2604430.1	.39385	-.00034	-.07832	.75815
.78800	2615087.8	.46733	.00335	-.07775	.60486
.94300	797584.9	.52317	-.13045	-.09864	.48239

NUMBER OF ITERATIONS = 15

BLADE ANGLE = .62873
ADVANCE RATIO = 2.20000

CT = -.07292
CP = -.15460
EFFICIENCY = 1.03763

X	G	G FOURIER	BIO	UA	UT
.15000	.00000	.00000	1.32694	-.00795	-.03263
.20700	-.00173	-.00407	1.26048	-.00721	-.02261
.36300	-.00290	.00056	1.07666	-.00842	-.01556

.57500	-.00411	-.00008	.86446	-.01572	-.01838
.78800	-.00395	-.00005	.71050	-.01810	-.01546
.94300	-.00262	-.00012	.62039	-.02416	-.01712
1.00000	.00000	.00000	.58907	-.03172	-.02107

X	REY	MACH	CL	ALPHA	PSI
.20700	951962.8	.30567	-.19861	-.10926	1.15122
.36300	1604986.0	.32919	-.19744	-.10908	.96758
.57500	2507249.9	.37915	-.17964	-.10631	.75815
.78800	2467063.9	.44087	-.17538	-.10564	.60486
.94300	748295.9	.49084	-.38353	-.13800	.48239

NUMBER OF ITERATIONS = 15

BLADE ANGLE = .62873
ADVANCE RATIO = 2.40000

CT = -.14433
CP = -.32414
EFFICIENCY = 1.06868

X	G	G FOURIER	BIO	UA	UT
.15000	.00000	.00000	1.33960	-.00801	-.03717
.20700	-.00258	-.00723	1.27605	-.00855	-.02993
.36300	-.00503	.00110	1.10004	-.01324	-.02685
.57500	-.00761	.00033	.88990	-.02766	-.03485
.78800	-.00738	-.00026	.73655	-.03502	-.03231
.94300	-.00405	-.00006	.65780	-.02808	-.02209
1.00000	.00000	.00000	.63479	-.02314	-.01741

X	REY	MACH	CL	ALPHA	PSI
.20700	942809.3	.30273	-.29873	-.12483	1.15122
.36300	1576325.5	.32331	-.34783	-.13246	.96758
.57500	2429505.8	.36739	-.34323	-.13175	.75815
.78800	2351934.3	.42030	-.34290	-.13169	.60486
.94300	712449.4	.46733	-.62420	-.17541	.48239

NUMBER OF ITERATIONS = 16

Appendix C

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