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Application of Adaptive Filtering to Direct-Sequence Spread-Spectrum Code Synchronization

by

Mohamed G. El-Tarhuni, B.Sc., M.Sc.

**A Thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy**

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in partial fulfillment of the requirements for
the degree of Doctor of Philosophy

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Abstract

Direct-Sequence Spread-Spectrum (DS/SS) systems offer several attractive features such as interference rejection, multipath diversity, multiple access capability, and accurate ranging. Recently, a greater interest has been devoted to the use of DS/SS systems in many mobile radio applications including digital cellular systems and indoor wireless networks due to the expected increase in capacity and an increased demand for new services. This interest motivates growing research on code synchronization for SS systems as a result of the critical role that it plays in the operation of SS systems. This thesis introduces an application of adaptive filtering for code synchronization in DS/SS systems.

In the proposed scheme, an adaptive filter is used to carry both functions of code synchronization: Acquisition and Tracking. The filter coefficients are adjusted toward their optimal value by using the Least-Mean-Square (LMS) algorithm, and the filter tap weights are used to estimate the delay offset between the incoming SS signal and a locally generated PN sequence. Theoretical and simulation results for both acquisition and tracking modes are presented in this thesis.

The proposed acquisition system is classified as a *hybrid* acquisition scheme since several code phases are concurrently tested for acquisition. It is shown that the proposed acquisition scheme has similar improvements in acquisition time to that offered by conventional hybrid schemes but at a reduced hardware complexity. The proposed scheme has also a better acquisition performance than a partial matched filter system in low SNR conditions. It is also shown that the proposed scheme is robust to fading. The proposed acquisition scheme suffers only a slight degradation (about 0.5 to 3 dB) in SNR, when operating in slow frequency selective and frequency non-selective Rayleigh fading. The adaptive nature of the system is exploited to acquire and track the strongest received path.

The tracking performance is modelled and analyzed using Markov chain theory, and good tracking is demonstrated. Both the mean hold-in time and the mean false alarm penalty time are derived. It is shown that the hold-in time is robust to small fluctuations in received SNR as opposed to conventional delay-lock loops which exhibit dramatic reductions in the hold-in time with small change in SNR. It is also shown that the residual tracking error performance of the proposed tracking scheme outperforms the maximum likelihood estimator by approximately 4 dB. Tracking of fast changing delays is also investigated. The BER performance of a DS/SS system utilizing this tracking system is simulated in a relatively fast changing environment, and the degradation from the ideal perfectly synchronized system is shown to be about 1 dB.

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List of Abbreviations

ACQ	- acquisition state
AWGN	- additive white Gaussian noise
BER	- bit error rate (probability of bit error)
BPSK	- binary phase shift keying
CDMA	- code division multiple access
CMF	- chip matched filter
dB	- decibels
DLL	- delay locked loop
DS/SS	- direct-sequence spread spectrum
FA	- false alarm state
FIR	- finite impulse response
FH	- frequency hopping
H_0	- out-of-phase hypothesis
H_1	- in-phase hypothesis
i.i.d.	- independent identically distributed
IIR	- infinite impulse response
LMS	- least mean-square
MDP	- multipath delay profile
MF	- matched filter
MLE	- maximum likelihood estimator
MMSE	- minimum mean-square error
MSE	- mean-square error
PDF	- probability density function
PN	- pseudonoise
RASE	- rapid acquisition by sequential estimation
RARASE	- recursive-aided RASE
RLS	- recursive least squares
SAW	- surface acoustic wave

SLS	- search/lock strategy
SPRT	- sequential probability ratio test
SS	- spread spectrum
STTA	- short term time average
TDL	- tau-dither loop
TDMA	- time division multiple access
WSS	- wide sense stationary

List of Symbols

A	- maximum number of STTA tests for convergence per cell
$a(t)$	- PN spreading signal
$\{a\}$	- ± 1 valued PN sequence
B	- number of threshold crossing required to declare convergence
$b(t)$	- information signal
$\{b\}$	- ± 1 valued information data
$\hat{b}(i)$	- estimate of i th data bit
C	- covariance matrix of the filter coefficients
$\{c\}$	- reference sequence used during tracking obtained by interpolating the spreading sequence by N_s samples
D	- transition matrix for the tracking process
$d(n)$	- reference desired signal for the adaptive filter
$e(n)$	- error signal at the adaptive filter output
$E\{T_{acq}\}$	- mean acquisition time
f_c	- carrier frequency
f_s	- sampling frequency
$f_{e H_i}(\vartheta)$	- PDF of the error signal under H_i
$f_{y H_i}(\vartheta)$	- PDF of filter output under H_i
$f_{\Lambda_s H_i}(\vartheta)$	- PDF of the STTA under H_i
$f(w_\alpha)$	- PDF of w_α
$f(w_0 < w_\alpha, w_1 < w_\alpha, \dots, w_{M-1} < w_\alpha w_\alpha)$	- conditional joint PDF of $w_j < w_\alpha \ j = 0, 1, \dots, M-1; j \neq \alpha$
$H(Z)$	- transfer function between different states of the transition diagram used to represent the acquisition process
$H(1)$	- $H(Z)$ when $Z = 1$
$H'(1)$	- $dH(Z) / dZ$ when $Z = 1$
$h(t)$	- low-pass equivalent impulse response of the multipath channel
I	- identity matrix
I_A	- identity matrix representing the absorbing states
J_{min}	- minimum mean-square error (MMSE)

J_{ss}	- steady state MSE
$J(\underline{w})$	- cost function
K_a	- adaptation period in chip durations
K_d	- decision period in chip durations
K_e	- examination period in chip durations
K_p	- false alarm penalty time during acquisition in chip durations
k	- an integer representing the random delay τ
$\hat{k}$	- estimate of k
L	- PN code length
L_p	- maximum number of resolvable paths for a frequency selective fading channel
M	- number of adaptive filter coefficients
M_c	- center tap number
m_α	- mean value of tap number α
N	- correlation period for the MLE
N_s	- number of samples per chip
$N(j)$	- AWGN samples
$n(t)$	- baseband AWGN
$n_p(t)$	- bandpass White Gaussian noise
P	- average received signal power
$\underline{P}$	- cross-correlation vector between filter input and reference signal
$\hat{\underline{P}}$	- LMS estimate of cross-correlation vector
$P_{ACQ}(n)$	- acquisition probability in the n th step
$P_{ACQ}(Z)$	- acquisition time generating function
P_c	- probability of correct decision about the position of the peak of filter taps
P_D	- probability of detection
$P_{D,ov}$	- overall detection probability
$P_{j,ACQ}(Z)$	- transfer function between state j and the ACQ state
$P_{j,q}(Z)$	- transfer function between state j and the s_q state
P_{Fi}	- in-phase false alarm probability
P_{Fo}	- out-of-phase false alarm probability
P_M	- probability of missing acquisition

P_ϵ	- error probability ($P_\epsilon = 1 - P_c$)
$P_{\eta H_i}$	- probability that the STTA is less than the threshold η under H_i
p	- transition probability between SLS states indicating in-lock
$\tilde{p}$	- transition probability between SLS states indicating out-of-lock ($\tilde{p} = 1 - p$)
q	- number of cells required to represent the state transition diagram during acquisition mode
R	- autocorrelation matrix of the input vector
$\hat{R}$	- LMS estimate of the autocorrelation matrix, R
R_b	- autocorrelation matrix of the information data $\{b\}$
$R_{b\hat{b}}$	- cross-correlation matrix between $\{b\}$ and $\{\hat{b}\}$
R_c	- autocorrelation matrix of the sequence $\{c\}$
$r(t)$	- received SS signal
r_{jk}	- j/k th element in R
S	- averaging window of the STTA
s_i	- i th cell in the acquisition state transition diagram
SNR_c	- signal-to-noise ratio per chip
SNR_s	- signal-to-noise ratio per sample
$\bar{T}_{abs}$	- vector of mean absorption time
$\underline{T}$	- vector of mean transition times between states during tracking
T_b	- data bit duration
T_c	- chip duration
$\bar{T}_{ij}$	- mean time to reach state j starting from state i during tracking
T_{LL}	- mean time to loss-of-lock (mean hold-in time)
T_m	- maximum delay spread of the fading channel
T_p	- mean time to clear a false alarm (mean penalty time)
$var\{T_{acq}\}$	- variance of the acquisition time
$\underline{w}$	- FIR filter tap-weight vector
$\underline{w}_{opt}$	- optimum tap-weight vector
$\underline{x}(n)$	- adaptive filter input vector
$x(n)$	- samples of $x(t)$
$x(t)$	- received baseband SS signal
$y(n)$	- adaptive filter output

Z	- unit delay
α	- phase offset between spreading codes ($\alpha = \hat{k} - k$)
α_i	- decision statistic for the MLE
Y_l	- amplitude of l th received path
Δ	- matrix of transition probabilities from transient to absorbing states during tracking
$\delta(k)$	- Dirac delta function
ε	- error in delay estimate during tracking
η	- threshold value during acquisition
η_t	- threshold value during tracking
η_{opt}	- optimum threshold value during acquisition
Θ_l	- random phase of l th received path
φ	- random carrier phase
Φ	- matrix of transition probabilities between transient states during tracking
Ψ	- an all zero matrix used in the canonical form of the transition matrix during tracking
ψ_{T_b}	- rectangular pulse of duration T_b
Λ_S	- short term time average (STTA) of the squared error signal $e(n)$
λ_i	- i th eigenvalue of the autocorrelation matrix R
λ_{av}	- average eigenvalue of R
λ_{max}	- maximum eigenvalue of R
μ	- LMS step size parameter
σ_n^2	- AWGN variance
σ_s^2	- energy per sample
σ_w^2	- variance of filter taps
σ_x^2	- variance of the input signal $x(n)$
σ_y^2	- variance of filter output
τ	- random time delay to be estimated by the system
$\hat{\tau}$	- system's estimate of τ
τ_l	- delay of l th received path
τ_{mse}	- time constant of an exponential approximating the LMS MSE-learning curve
$\underline{\Omega}$	- M -by-1 vector with zero elements except element number $\hat{k} - k$ which

	equals to one
$\tilde{\underline{\Omega}}$	- M -by-1 vector with zero elements except elements corresponding to multipath delays
ξ^2	- noncentrality parameter of non-central chi-squared random variable
$\underline{\nabla}$	- gradient vector
$\hat{\underline{\nabla}}$	- LMS estimate of gradient vector
$E\{ \}$	- ensemble average
$\lceil z \rceil$	- ceiling operator (smallest integer larger than or equal to z)
$\lfloor z \rfloor$	- floor operator (largest integer smaller than or equal to z)
$()^{-1}$	- matrix inverse operator
$()^T$	- matrix transpose operator
$ $	- magnitude operator
$m!$	- factorial of $m = m(m-1)(m-2)...3.2.1$
$\binom{m}{j}$	- Binomial factor = $\frac{m!}{j!(m-j)!}$
$I_j(z)$	- j th order modified Bessel function of the first kind
$\Gamma(p)$	- Gamma function
$Q(\cdot)$	- Q -function
d / dZ	- first order derivative
d^2 / dZ^2	- second order derivative

Chapter 1 Introduction

1.1 Background

Recent research in synchronization techniques for spread spectrum (SS) systems is motivated by the growing interest in using SS for commercial digital cellular mobile radio systems as well as in many wireless data networks [1]-[4]. The development of reliable and fast synchronization techniques for SS signals has received considerable attention in spread spectrum communication systems research ([5][6] and references therein). If code synchronization is not established accurately then not only that none of the claimed advantages of SS, such as interference rejection, multiple access, multipath diversity, etc., are obtained, but also reliable exchange of information even in the absence of receiver noise becomes very difficult if not impossible. Therefore, fast code synchronization is desirable to reduce the overhead time needed in accessing the system; this is particularly important in simplex applications and packet-switching data networks. In the proposed CDMA digital cellular systems, code synchronization has a critical role in the operation of such systems since the efficiency of the access channels may also suffer which may result in capacity loss.

In a spread spectrum system, the transmission bandwidth used by the signal is usually much larger than the bandwidth of the information data; a pseudo-random (PN) code is used to spread the information signal power over a bandwidth much wider than the required minimum signal bandwidth. At the receiver, the same PN code is used to despread the received signal to its original information bandwidth. This is followed by data detection. This processing results in rejection of any interfering signals that may exist in the same transmission bandwidth. However, due to the noise-like correlation characteristic of the PN code, the despreading code must be perfectly synchronized to the code in the incoming signal in order to ensure complete removal of the spreading code from the information data.

Code synchronization means that the time delay between the incoming code and the locally generated code is made to be very small compared to the time duration of a code symbol, which is known as a chip interval. Code synchronization must be maintained by the system for the duration of communication. The time delay, which the receiver must estimate to synchronize, is a random parameter which is usually assumed to be uniformly distributed over the time uncertainty region. Several sources contribute to the randomness of the time delay estimation process including initial random phase offset between the transmitter's and receiver's codes, random channel variations, interference effects, AWGN, unstable clock generators, Doppler variations, and more importantly the distance dependent time delays.

Code synchronization is usually developed over two steps, namely acquisition and tracking [5]-[8]. Acquisition is the process by which the two codes are coarsely aligned such that their phase offset is less than one chip duration; while tracking is used to minimize the delay offset and to maintain the time alignment of the two codes.

Several performance measures could be used to assess the efficiency of a certain code synchronization scheme. For instance, performance of acquisition systems is usually measured by the statistics of the time elapsed prior to acquisition. In addition, depending on the intended application, the mean acquisition time or the probability of achieving acquisition within a specific time limit may be used to compare different acquisition schemes. Another factor which need to be considered is the hardware complexity of the system since some schemes achieve fast acquisition by expanding on the hardware requirements. For tracking systems, the mean time to lose lock and the variance of the estimation error are usually important.

Code acquisition schemes can be classified as either active serial search, passive matched filters, or sequential estimation methods. In active serial search techniques (Figure 1-1) the time uncertainty region is quantized with a fraction of a chip interval. A locally generated PN code, with its phase set at the start of the quantized uncertainty region, is correlated with the incoming signal. The correlator output is compared to a

threshold after a fixed examination period which is generally much longer than the chip duration. If the threshold is not exceeded, the local code phase is advanced to the next cell and the process is repeated until the threshold is crossed. Once the correct cell is detected, the tracking circuit is initiated either directly or after a verification mode which is used to reduce the effects of false alarm errors. Serial search schemes are generally simple and reliable under small signal-to-noise ratio conditions because they incorporate the processing gain in the acquisition process, but their acquisition time performance is relatively poor especially if the uncertainty region is large. Their performance could be improved by increasing the search rate of the system through parallel or hybrid searching strategies. In certain situations, where the probability distribution of the delay offset is not uniform, other searching strategies may improve the performance.

In parallel acquisition schemes (Figure 1-2) all the cells are tested simultaneously and the cell that results in a maximum correlation is assumed tentatively as the correct phase offset. Further verification of this cell is used to minimize false alarm errors. Parallel systems have fast acquisition time performance but their hardware requirements are usually prohibitively complex, particularly if the uncertainty region is large. On the other hand, a hybrid acquisition system tries to compromise the acquisition speed with hardware complexity. This is achieved by dividing the uncertainty region into disjoint intervals each with more than one cell. The system starts searching the intervals sequentially while the cells within each interval are tested concurrently.

Passive acquisition systems provide fast acquisition performance by using matched filters. The receiver does not generate a replica of the PN code but it uses a filter with an impulse response matched to the PN code. The output of the matched filter (MF) will have a peak at the correct time instant which corresponds to the delay offset between the two codes. Nevertheless, this technique is only used for systems with short codes due to the implementation complexity encountered in designing matched filters with long periods. Some variation of the MF technique may be used as an acquisition aiding subsystem in which a synchronization preamble is appended to the spreading code. A filter matched to

Figure 1-1: Serial search system with 1/2-chip step size.

Figure 1-2: Parallel acquisition system.

the preamble is used to detect the code starting time.

Acquisition by sequential estimation [9] is based on estimating the present state of the incoming PN code. The local code generator is initially loaded with this estimate and a decision is made to check for acquisition after a correlation period. If acquisition is not detected, the estimation process is repeated. Sequential estimation methods provide fast acquisition in moderate to high signal-to-noise ratio conditions, however, for small signal-to-noise ratios the estimation process is unreliable and the acquisition time can be very high. Also this scheme is extremely vulnerable to interfering signals since it will try to acquire them. It is therefore not useful for multiple-access applications where other users share the same channel at the same time.

For tracking of PN signals a delay-locked loop (DLL) is usually employed as shown in Figure 1-3. Two identical codes with a relative delay offset of one chip or less are independently correlated with the incoming signal. The difference of the correlators outputs is used as a control signal to adjust the clock rate of the local codes to keep the error signal within the pull-in range of the DLL. Another tracking loop is known as the tau-dither loop (TDL) where a single correlator is used. A low frequency square wave is used to dither the timing of the local code generator which results in amplitude modulation of the correlator output. The amplitude shift is used as an error signal to drive the clock generator of the local PN code [8].

In this research, a new technique for code synchronization in DS/SS systems is presented. The proposed scheme treats the synchronization problem as an adaptive filtering application. Both code acquisition and tracking are accomplished by using the same hardware. The major contributions of the research are summarized in the next section.

1.2 Thesis Contributions

As mentioned in the previous section, synchronization is essential in spread spectrum systems. Most of the previous work in the acquisition problem concentrate on correlation techniques or sequential estimation methods. In this thesis, we propose to treat the

Figure 1-3: Delay-lock loop with envelope correlation.

synchronization problem as an adaptive filtering application. An adaptive filter in a transversal format is proposed to function as both acquisition and tracking circuit. The filter coefficients (taps) are adapted according to a certain algorithm and timing information are extracted from these coefficients. As before, code synchronization is evolved over two steps, acquisition and tracking.

The acquisition mode of operation is described and analyzed under the AWGN channel model which is a commonly used assumption for analyzing acquisition systems [6][7][10]-[16]. The mean acquisition time is derived for the proposed system by using flow graph techniques. Analytical expressions for the detection and false alarm probabilities are also presented. Sensitivity of acquisition performance to several parameters of the system and comparison with a partial matched filter based acquisition technique with the same number of taps are also included. The results show that the proposed scheme has similar gain in acquisition time to that offered by other hybrid schemes but at a reduced hardware complexity. It is also shown that the proposed technique outperforms the partial matched filter based system in noisy conditions.

Mean acquisition time performance for the proposed system in frequency selective and flat slow Rayleigh fading channels is investigated using Monte Carlo simulation. For frequency selective fading, two scenarios for the multipath delay profile (MDP) are considered; namely, uniform MDP and exponential MDP. It is concluded that the system is robust to fading since only a small degradation is observed in signal-to-noise ratio (about 0.5 to 3 dB)¹. Furthermore, the adaptive nature of the proposed scheme is exploited to acquire the strongest received path.

The same adaptive filter used for acquisition is proposed to function as a tracking circuit for the spread spectrum signal. This results in significant simplification of the receiver structure and the hardware requirement. Detailed description of the ability of the system to track the incoming signal in presence of data and AWGN is presented. Expressions for the mean hold-in time and mean time to clear a false alarm (mean penalty

1. In [17] a 9 to 10 dB degradation has been reported for a parallel acquisition scheme.

time) are derived. The ability of the system to operate under fast changing conditions and/or poor clock stability is demonstrated through computer simulations. Also, the bit-error-rate (BER) performance of a BPSK DS/SS system using the adaptive system to track and demodulate the data is simulated. Finally, the tracking error performance of the system is compared to a conventional maximum likelihood estimator (MLE).

It is shown that the system can keep track of the randomly changing delay for long periods of time, and, in case of a false acquisition, it can clear the false alarm rapidly. It is noticed that the proposed scheme is less sensitive to small variations in SNR as opposed to systems based on DLL. In addition, the tracking system is shown to provide a very good performance under fast changing conditions. It also outperforms the MLE in the sense of producing a smaller residual tracking error. Finally, the degradation in BER is found to be less than 1 dB from the ideal perfectly synchronized (zero residual tracking error) system.

1.3 Thesis Organization

Chapter 2 presents an overview of previous research in the area of code synchronization of SS signals. Different techniques are described as well as a general description of the proposed synchronization system.

In Chapter 3, a detailed description of the proposed acquisition scheme is presented. The optimum tap-weight vector of the adaptive filter is derived. A Gaussian approximation to the probability density function of the tap weights is also discussed in this chapter.

Chapter 4 describes the analysis for the acquisition system in AWGN. The acquisition process is modelled as a Markov process for which flow graph theory is used to develop a state transition diagram. The derivation of the mean acquisition time is followed by investigations into the system's false alarm and detection probabilities. Simulation results are also presented to validate the developed statistical models to characterize different parameters. The sensitivity of the acquisition time performance to variations in adaptation algorithm step size and threshold setting is discussed. The system is also compared to a matched filter based acquisition scheme. Acquisition performance under frequency

selective and flat Rayleigh fading conditions is included in this chapter.

Chapter 5 investigates on the tracking mode of operation. The mean hold-in time and mean penalty time are derived under AWGN channel model. The effect of data-decision errors on the tracking performance is investigated. Then, tracking of linearly and randomly fast changing delays are presented followed by a performance comparison of the tracking error with a MLE testing the same window of delays. Finally, the BER performance for the system is simulated.

Chapter 6 presents some concluding remarks and future work to follow this research is suggested.

Chapter 2 Synchronization of DS/SS Systems

2.1 Related Research on Synchronization of SS Signals

Spread spectrum systems have received considerable attention in military applications due to their attractive features such as anti-jam capability, low probability of intercept, and accurate ranging. Recently, it was recommended to use SS techniques in commercial systems such as digital cellular radio systems where it is expected to provide larger capacity improvement than the capacity obtained using TDMA systems [1]. Also, SS is a prime candidate for use in PCS and indoor wireless systems [2][18][19]. All SS applications require code synchronization prior to data detection. There have been extensive research activities in developing fast, reliable, and simple synchronization schemes for SS signals. In what follows, we present an overview of major research on synchronization of DS/SS systems reported in literature.¹

One of the early work on code acquisition using a sliding correlator was conducted by Sage [7]. Both coherent and noncoherent serial search acquisition systems were studied in AWGN channels. Expressions for the false alarm and detection probabilities were derived, however, no analysis for the acquisition time was presented.

Holmes and Chen in [11] analyzed a single dwell serial search acquisition system with noncoherent detection. The time uncertainty region was divided into cells which were searched serially and the step size for the search update was assumed to be a fraction of a chip. A flow graph technique was used to model the acquisition process as a Markov chain which was simplified to obtain an acquisition time generating function. The mean and variance of the acquisition time were analyzed as a function of the false alarm and detection probabilities. Doppler effects were also considered in an approximate analysis. A double dwell time system, in which a short time correlation was initially used to increase

1. Only DS/SS synchronization is considered in this thesis since it is the commonly suggested method for cellular applications. The other SS system in the form of Frequency Hopping (FH) is out of the scope of this thesis.

the cell search rate, followed by a longer correlation time to verify the more probable cells, was introduced to reduce the acquisition time. Their technique could be altered to suit other systems, however, a considerable amount of algebraic manipulations will be needed.

In [12], Hopkins considered the complete problem of code synchronization. He used Markov chain theory to derive the transition matrix that defines the acquisition and tracking processes. Frequency offset errors due to Doppler were found to either increase or decrease the search rate depending on the sign of the Doppler offset and the direction of local code update (advancing or retarding phase). Also, it was shown that the mean acquisition time and the mean time to lose lock could be improved by using different dwell times and different thresholds for the acquisition and tracking modes, respectively.

Another measure of performance for the single dwell serial search acquisition system was introduced by DiCarlo and Weber [13]. The probability of successful synchronization was derived as a function of the detection probability, false alarm probability, detector integration time, and false alarm penalty time. The acquisition time generating function was arranged in such a way to permit identification of the probability of successful acquisition terms. The false alarm penalty time was found to have a strong effect on performance optimization.

In [14] DiCarlo and Weber considered a multiple dwell serial search system. Variable examination intervals were used to allow fast dismissal of incorrect cells. This was accomplished by testing a cell for a number of dwells of an increasing period provided that all the previous tests have indicated synchronization. The mean and variance of the acquisition time were derived. It was shown that significant improvement over a single dwell system is gained by using a double dwell system. A diminishing gain is noticed as the number of dwells were increased.

Polydoros and Weber [20] developed a simple yet generalized analysis for fixed dwell serial search code acquisition systems. Their scheme provides a simpler way of analysis than that introduced in [11]. A flow graph representation of the acquisition process based on symbolic notation was presented to obtain the generating function. Thus, by proper

definition of the flow graph branches according to the system in hand, it is possible to use the developed expressions, with little or no modifications, to obtain different performance measures of the acquisition system.¹ Polydoros and Weber then applied their analysis to a noncoherent matched filter code acquisition scheme [15]. A high rejection rate for the incorrect cells was obtained (a fraction of a chip interval), while a verification mode based on majority voting was used to minimize the false alarm rate. The mean acquisition time was found to depend on the matched filter length. It was also concluded that an optimum threshold value which minimizes the mean acquisition time exists for each value of signal-to-noise ratio. A similar conclusion is reached in our analysis as explained in Chapter 4.

Due to the relatively large acquisition time of serial search acquisition systems, Milstein et al. [21] proposed a parallel processing acquisition scheme that uses multiple surface acoustic wave (SAW) convolvers. The spreading sequence is divided into several subsequences which are used as references for the convolvers. The search rate is reduced by a factor proportional to the number of chips per subsequence times the number of convolvers used. It was shown that acquisition is possible within a period twice the number of chips in a subsequence. However, a large amount of hardware is required.

Su [16] discussed several parallel search acquisition algorithms that use matched filters. A parallel search with fixed dwell time detector, based on the maximum likelihood principle, is used. Another variation of the above algorithm that compares the decision statistic with a threshold prior to starting verification is also presented. Moreover, parallel algorithms with Wald's sequential detection were analyzed.

Sourour and Gupta [17][22] proposed an acquisition technique that uses a bank of noncoherent matched filters to improve the performance under fading channels. The code period was divided into disjoint subsequences and two filters were matched to each subsequence. First, under AWGN channel, the mean acquisition time was derived along with the false alarm and detection probabilities. Next, the performance over non-selective, slow Rayleigh fading channels was obtained by averaging the AWGN channel results over

1. The unified approach is the basis of our analysis for the proposed adaptive synchronization system.

the fading distribution. Finally, for fast fading in which the channel changes within the length of a matched filter period, the performance measures were re-evaluated. It is interesting to note that, under non-fading and slowly fading conditions with small signal-to-noise ratio, it was found that it is better to increase the filter lengths and decrease the number of parallel matched filters. In contrast, under fast fading, more parallelism is better. Also, it was noted that in general fading degrades code acquisition systems performance significantly (about 9 to 10 dB in SNR).

As a result of the increasing interest in using CDMA systems for commercial applications, several papers that address the acquisition problem in the presence of multiple access interference were presented. Rick and Milstein [23] studied the reverse link acquisition performance for a DS-CDMA system. A noncoherent parallel acquisition scheme was investigated for a chip synchronous CDMA system with random sequences and perfect power control conditions. Also, no Doppler, fading, or data was assumed to be present in the received signal. The false alarm probability was used to assess the performance as a function of the observation interval and the number of active users in the system. It was concluded that the acquisition system performance may be the limiting factor for the CDMA system capacity rather than the bit-error rate (BER) performance. This is similar to the conclusion obtained by Madhow and Pursley [24] using asymptotic analysis for a matched filter system.

Recently, some parallel acquisition schemes were suggested [25]-[28]. In [25], a differentially coherent system was shown to yield 4 to 5 dB improvement over noncoherent detection. Optimal and suboptimal estimators based on sequential probability ratio test (SPRT) were derived in [26] and [27]. In [28], a partial code correlation and majority decision rule scheme was proposed to operate under fast fading channels.

Between the two extremes, serial and parallel schemes, other methods based on hybrid strategies were studied by Baum and Veeravalli [29]. These systems reduce the hardware complexity incurred by parallel systems while keeping the acquisition time performance at an acceptable level. Several cells were examined for acquisition simultaneously and SPRT

was used to increase the rejection rate of the incorrect intervals. The mean acquisition time was derived as a function of the false alarm and detection probabilities which were evaluated by computer simulation.

Rapid acquisition by sequential estimation (RASE) was originally developed by Ward [9]. It is based on estimating, from the received signal, the initial loading of the shift register used to generate the local PN code. The system then correlates the two codes for a period of time to check if the codes are synchronized or not. If not, a new estimate is made and the process is repeated. This technique provides fast acquisition for moderate input signal-to-noise ratio, however, the performance degrades significantly under interfering conditions. Thus, it is not suitable for multiple access applications.

Several variations of the RASE scheme were introduced [30][31]. These schemes try to improve the initial phase estimation by using error correcting logic which results in a significant increase in hardware. In [32], a recursive-aided RASE (RARASE) technique that discards initial phase estimates which appear to suffer from errors without attempting to verify them. This improves the system performance without increasing the hardware required. However, the basic limitations of sequential estimation methods are still the same, i.e., susceptibility to interference.

Based on the previous survey of SS acquisition techniques, it is observed that most of the schemes, serial or parallel, rely either on correlation principles or sequential estimation. In this research, we consider a new strategy to solve the SS synchronization problem which is based on adaptive filtering techniques. A finite impulse response (FIR) adaptive filter is proposed to operate as a synchronization circuit. Before we present a brief description of the proposed system, some applications of adaptive filtering in SS systems are mentioned.

2.2 Applications of Adaptive Filters in SS Systems

An FIR adaptive filter is basically a filter with a time varying impulse response. The time variation is obtained by adjusting the filter taps in a certain way. Adaptive filters have been widely used in different areas due to their ability to adjust to their operation environment.

For a brief review of adaptive filters see Appendix A.

In SS systems, adaptive filters were used to suppress narrow-band interference in DS/SS techniques [33]-[38]. An adaptive filter is used to estimate the narrow-band interference, which is highly correlated, from the received signal. The interference estimate is subtracted from the received signal which is then applied to the correlator for data detection. The same idea could be interpreted as using a filter with zeros at the interfering signals frequencies. These systems improve the performance if the processing gain is small to provide sufficient protection against the strong narrow-band interference.

A Near-Far multi-user receiver for DS-CDMA systems which uses an FIR adaptive filter has been suggested in [39]. It has been shown that, with or without power control, the adaptive filter receiver provides larger system capacity than conventional matched filter receivers. Furthermore, the adaptive receiver does not need any information regarding the spreading sequences of other users and their received powers. A similar system was also described in [40].

In [41], an FIR adaptive filter is used for code acquisition. The adaptive filter has a number of taps equals to the code length and the input to the filter is sampled at the chip rate while the output is sampled at the bit rate. A maximum likelihood decision about the delay is obtained from the tap-weight vector after convergence. The system has good acquisition performance; however, under severe interference conditions, the adaptation time required by the system before making a reliable decision about the delay estimate is very long which may increase the acquisition time performance. Also, the system requires that users access the system one at a time.

The proposed structure in this research is different from that suggested in [41] in several aspects. First, a smaller filter length is used in this study as opposed to the filter in [41] which has a number of taps equals to the code length. Secondly, the decision strategies used to detect acquisition and to extract timing information are different. Finally, the proposed scheme uses the same filter for code tracking as well while the system in [41] is used for code acquisition only. This thesis presents analytical expressions for the mean

acquisition time, detection probability, and false alarm probability for a typical AWGN environment while [41] restricts itself to finding an upper bound for the probability of not achieving acquisition in a Near-Far environment.

2.3 Adaptive Filter Based Code Synchronization Scheme

In this thesis, we investigate SS code synchronization using an adaptive filter in a transversal configuration. The proposed scheme has an improved acquisition time performance over serial search techniques; it also has less hardware complexity than other hybrid systems. Moreover, the same filter is used for code tracking and as a result the SS system will have a simpler structure than other conventional SS systems which use a separate code tracking loop. The proposed tracking system has a very good performance and less sensitivity to small variations in SNR compared to conventional DLLs.

A simplified block diagram for the proposed synchronization system is shown in Figure 2-1. The adaptive filter has M taps which are adjusted to minimize the mean-square error (MSE) between the received SS signal and a locally generated replica of the spreading PN code. Initially, the adaptive filter operates in acquisition mode in which the time uncertainty region is divided into intervals each of M chips duration. For each interval, the filter coefficients are adapted toward their optimum value (in the MSE sense) and, after an adaptation period, convergence in the MSE is checked. If convergence is detected, the delay offset is estimated from the tap weights, as will be explained in Chapter 3, otherwise the next interval is tested and the adaptation process is repeated.

Once acquisition is declared, the system switches to tracking mode in which the received signal is interpolated to increase the time resolution of the adaptive filter. The local code phase is continuously adjusted to maintain the peak of the filter coefficients at the center tap while the MSE is maintained below a preset threshold value. If tracking has been initiated due to a false alarm error, the tracking algorithm will sense the increase in the MSE and restart the acquisition process.

The mean acquisition time performance of the proposed system will be evaluated

analytically. Also the detection and false alarm probabilities will be determined. Acquisition under fading conditions is investigated using Monte Carlo simulation. Tracking performance is then presented.

Figure 2-1: A simplified block diagram for the adaptive filter based code synchronization system.

Chapter 3 Adaptive Filter Based DS/SS Code Acquisition System

3.1 Introduction

In this chapter, the proposed code acquisition system is described in details. The principles of the proposed acquisition system are discussed and a decision statistic used to detect acquisition is defined. The optimum Wiener solution for the adaptive filter coefficients is derived and their joint probability density function at steady state and LMS algorithm adaptation is approximated. The mean vector and covariance matrix of the tap weights are also derived.

3.2 Acquisition System Description

The adaptive acquisition system block diagram is shown in Figure 3-1. The system consists of a filter with M adaptive taps, an adaptation algorithm to adjust the filter coefficients iteratively, a pseudo-noise (PN) code generator, and other blocks to check for acquisition. The received SS signal, $r(t)$, is

$$r(t) = \sqrt{2P}b(t-\tau)a(t-\tau)\cos(2\pi f_c t + \phi) + n_p(t) \quad (3.1)$$

where P is the power of the received signal, $b(t)$ is the information data, $a(t)$ is the PN spreading signal, f_c and ϕ are the frequency and random phase of the carrier signal, respectively, and τ is the random time delay that must be estimated by the synchronizer. It is assumed that τ is uniformly distributed over the spreading code length. The receiver's thermal noise, $n_p(t)$, is modelled as an additive white Gaussian noise (AWGN). The information data is

$$b(t) = \sum_j b_j \psi_{T_b}(t - jT_b) \quad (3.2)$$

where $\{b_k\}$ is a sequence of ± 1 with equal probability, and $\psi_{T_b}(t)$ is a shaping rectangular pulse defined as

$$\psi_{T_b}(t) = \begin{cases} 1 & ; 0 \leq t < T_b \\ 0 & ; \text{elsewhere} \end{cases} \quad (3.3)$$

where T_b is the symbol duration. The spreading waveform, $a(t)$, is given by

$$a(t) = \sum_i a(i) \psi_{T_c}(t - iT_c) \quad (3.4)$$

where $\{a\}$ is the spreading sequence such that

$$\{a\} = (\dots, a(-2), a(-1), a(0), a(1), a(2), \dots)$$

with $a(i) \in \{-1, 1\}$, T_c is the chip duration, and the sequence is assumed to be periodic with period L , i.e., $a(i \pm L) = a(i)$, for all i . Also, it is assumed that the system processing gain equals to the code length, i.e., $T_b / T_c = L$.

It has been assumed, as in [10][23][26][29], that the receiver is perfectly synchronized with the RF carrier phase. Even though, from a practical point of view, this might be a strong assumption to make, it simplifies the analysis and has been widely used in the literature. The subject of adaptive equalization for noncoherent or differentially coherent detection did not receive much attention in the literature because of the analytical complexity introduced by the nonlinear detector. This nonlinearity precludes the use of commonly used adaptation algorithms to adapt the filter taps such as the simple LMS algorithm. Recently, equalization techniques for noncoherent receivers have been proposed in [42]. It has been shown that, if the receiver neglects the carrier phase completely, the equalizer coefficients will adjust themselves to compensate for the carrier phase and the carrier phase does not have to be recovered provided that the phase variations are slow enough to allow the LMS algorithm to track channel variations. It has also been proposed to use a combination of a linear equalizer with a modified LMS algorithm and a differentially coherent receiver to cancel the phase error [43]. In short, although a coherent system is difficult to implement in practice, it enables a tractable analysis for the proposed system. Also, further research to develop noncoherent or differentially coherent adaptive acquisition systems is needed.

Figure 3-1: Block diagram for the proposed adaptive acquisition system.

Let us also assume, without loss of generality, that the received signal power is normalized to unity. The equivalent baseband signal, $x(t)$, is then given by

$$x(t) = b(t - \tau)a(t - \tau) + n(t) \quad (3.5)$$

where $n(t)$ is a zero mean AWGN with two sided power spectral density $N_0 / 2$. The baseband signal, $x(t)$, is passed through a chip Matched Filter (CMF), whose output is sampled at a rate of f_s , such that

$$f_s = 1/T_c \quad (3.6)$$

The samples of the CMF output, $x(n)$, are applied to the FIR filter whose output at the n th time instant is given by

$$y(n) = \underline{w}^T(n)\underline{x}(n) \quad (3.7)$$

where $\underline{w}(n)$ is the tap-weight vector of the filter coefficients at the n th time instant represented as

$$\underline{w}(n) = [w_0(n) \ w_1(n) \ \dots w_{M-1}(n)]^T \quad (3.8)$$

with the subscripts $0, 1, \dots, M-1$ denote the tap number. Also, $\underline{x}(n)$ is the filter input vector at the n th instant given by

$$\underline{x}(n) = [x(n) \ x(n-1) \ \dots x(n-M+1)]^T \quad (3.9)$$

which consists of the present sample and past $M-1$ samples of the received signal. The superscript T denotes matrix transposition operation.

At every sampling instant, a vector of the received signal samples is used to produce an output, $y(n)$, that represents an estimate of the desired signal, $d(n)$. The desired signal is chosen as the locally generated despreading sequence at a certain phase, i.e., $d(n) = a(n - \hat{\tau})$ where $\hat{\tau}$ can assume any integer value between 0 and $(L-1)T_c$. Initially, the phase of the local sequence is set at the beginning of the time uncertainty region, i.e., $\hat{\tau} = 0$. The error signal, $e(n)$, is defined as the difference between the filter output and the desired signal,

$$e(n) = d(n) - y(n) \quad (3.10)$$

As time progresses, the filter coefficients are adjusted according to a certain algorithm and the filter output is produced using the updated tap weights. This process continues for a period of time which has been assumed to be long enough for the coefficients to converge near their optimum value. After the end of this adaptation period, a decision period is started in which the error signal is checked for convergence towards the minimum mean-squared error (MMSE) obtained by this filter with coefficient vector set to the optimum Wiener value [44].

If convergence is detected, i.e., the MSE is less than a preset threshold η , acquisition is declared, and the time delay between the received SS signal and the local despreading sequence will be within the time span of the FIR filter during the acquisition phase of M chips. The exact value of the delay is estimated from the weight distribution of the filter coefficients $\underline{w}(n)$, which, as will be explained later, is peaked at the tap with the correct delay estimate [45]. On the other hand, if no convergence in the MSE is detected, the phase of the locally generated sequence, $d(n)$, is adjusted by M chips, the tap weights assume their initial values, and the test for convergence is repeated.

The time uncertainty region is divided into q disjoint intervals or cells, such that

$$q = \left\lceil \frac{L}{M} \right\rceil \quad (3.11)$$

where $\lceil z \rceil$ is the smallest integer greater than or equal to z . There are, at least, $q - 1$ intervals each of width M chips, while the last interval has a width of M chips or less. The system tests the cells for acquisition one at a time. The time required to test a cell equals to the adaptation time plus the decision time. Convergence in the MSE is then used to decide either to accept the cell currently under test as the acquisition cell or to advance the local code to test the next cell.

The FIR filter coefficients are adapted toward their optimum value, in the MMSE sense, during every test for acquisition. The LMS adaptation algorithm is used to update the coefficients as described in Appendix A. The algorithm adaptation step size, μ , is chosen to guarantee both convergence in the MSE and convergence in the filter coefficients

to the optimum Wiener value. Numerical optimization of μ to minimize the mean acquisition time of the system is used as explained in Chapter 4.

Since the LMS algorithm uses an instantaneous estimate of the gradient vector instead of the exact gradient vector to update the filter coefficients, the instantaneous values of the estimation error, $e(n)$, may have large variations, and hence do not constitute a reliable decision statistic for the acquisition process. Figure 3-2 shows such variations observed in the squared error, $e^2(n)$, when the delay offset between the received and locally generated codes is: (a) less than or equal to the time span of the filter, i.e., M chips, which is called the in-phase hypothesis (H_1); (b) greater than the time span of the filter, which is termed the out-of-phase hypothesis (H_0). Note that, under the in-phase hypothesis where synchronization is possible, the instantaneous values of the squared error around iteration number 170 are large. Similarly, under the out-of-phase hypothesis, the squared error has very small values around iteration number 50 while no convergence in the MSE is detected; this may result in a large false alarm probability.

To overcome this difficulty, a short term time average (STTA) of the squared error over a window of S samples is used to remove these large fluctuations. During the decision interval, every S samples of the squared error are averaged and their average value, Λ_S , is compared to a threshold, η , to check for the convergence in the MSE. The STTA is given by

$$\Lambda_S = \frac{1}{S} \sum_{n=1}^S e^2(n) \quad (3.12)$$

A prototype of the short term time average signals are shown in Figure 3-3.

3.3 Optimum Tap-Weight Vector Distribution of the FIR Filter

The optimum tap-weight vector of the FIR filter can be evaluated from the autocorrelation matrix, $\mathbf{R}$, of the tap-input vector $\mathbf{x}(n)$, and the cross-correlation vector, $\mathbf{P}$, between the tap-input vector, $\mathbf{x}(n)$, and the desired signal $d(n)$ [44][46]. In this section, these

(a)

(b)

Figure 3-2: Variations in the instantaneous squared error: (a) in-phase; (b) out-of-phase.

(a)

(b)

Figure 3-3: STTA of the squared error signal for $S = 30$ chips, $M = 15$ taps, $L = 511$ chips (a) in-phase; (b) out-of-phase.

correlation functions and the optimum filter coefficients are determined.

To simplify the analysis, several assumptions are made as follows:

1. The spreading sequence, $\{a\}$, is modelled as a random process consists of independent and identically distributed (i.i.d.) random variables which can take the values of ± 1 with equal probability. This implies that

$$E\{a(i)\} = 0 \quad (3.13)$$

$$E\{a(i)a(j)\} = \begin{cases} 0 & ; i \neq j \\ 1 & ; i = j \end{cases} \quad (3.14)$$

2. The spreading sequence is independent of the samples of the AWGN, that is

$$E\{a(i)n(j)\} = 0 \quad ; \text{ all } i, j \quad (3.15)$$

3. We assume that there is no information data transmitted during acquisition. This situation occurs when the system allows an acquisition period prior to commencing data transmission or if a separate pilot signal is available for acquisition.
4. It is also assumed that the system is chip synchronous, i.e., τ is an integer value over the code period LT_c . This assumption has been made to simplify the analysis; some comments on the effect of the chip asynchronous case will be presented later. To emphasize that τ is an integer we replace it by k , with $k \in [0, 1, \dots, L-1]$.

The received baseband signal, $x(t)$, can be expressed as

$$x(t) = \sum_{j=-\infty}^{\infty} a(j)\psi_{T_c}(t - (k + j)T_c) + n(t) \quad (3.16)$$

which, with $j = l - k$, can be written as

$$x(t) = \sum_{l=-\infty}^{\infty} a(l - k)\psi_{T_c}(t - lT_c) + n(t) \quad (3.17)$$

where k is the unknown phase (in chips) of the received sequence. The CMF output at the

j th instant is given by

$$x(j) = a(j-k) + N(j) \quad (3.18)$$

where $\{N(j)\}$ is a sequence of i.i.d. Gaussian random variables with zero mean and variance $\sigma_n^2 = N_0/2T_c$. The autocorrelation matrix, $\mathbf{R}$, is given by [44]

$$\mathbf{R} = E\{\underline{x}(n)\underline{x}^T(n)\} \quad (3.19)$$

which is an M -by- M matrix whose elements are evaluated as

$$\begin{aligned} r_{jm} &= r(j-m) = E\{x(n-j)x(n-m)\} \\ &= E[a(n-j-k) + N(n-j)][a(n-m-k) + N(n-m)] \end{aligned} \quad (3.20)$$

for $j, m = 0, 1, \dots, M-1$. In (3.20), it has been assumed that $\{x\}$ is a Wide Sense Stationary (WSS) random process. By incorporating the previous assumptions about the spreading sequence and the noise samples we get

$$r_{jm} = \begin{cases} 1 + \sigma_n^2 & ; j = m \\ 0 & ; j \neq m \end{cases} \quad (3.21)$$

In a matrix form, $\mathbf{R}$ is a diagonal matrix which can be written as

$$\mathbf{R} = (1 + \sigma_n^2)\mathbf{I} \quad (3.22)$$

where $\mathbf{I}$ is the M -by- M identity matrix. Thus, $\{x\}$ is a sequence of i.i.d. Gaussian random variables with zero mean and variance $\sigma_x^2 = 1 + \sigma_n^2$.

The cross-correlation vector between $\underline{x}(n)$ and the desired signal $d(n)$ is given by [44]

$$\underline{P} = E\{\underline{x}(n)d(n)\} = [p(0) \ p(-1) \ \dots p(-M+1)]^T \quad (3.23)$$

The desired signal is the same as the locally generated spreading sequence delayed by a certain number of chips, $\hat{k}$, which is assumed to be an integer number of chips, i.e., $\hat{k} = 0, 1, \dots, L-1$. Therefore, the desired signal is

$$d(t) = \sum_{j=-\infty}^{\infty} d(j)\psi_{T_c}(t-jT_c) \quad (3.24)$$

where

$$d(j) = a(j - \hat{k}) \quad (3.25)$$

The elements of the cross-correlation column vector are

$$\begin{aligned} p(-j) &= E\{x(n-j)d(n)\} \quad ; j = 0, 1, \dots, M-1 \\ &= E\{[a(n-j-k) + N(n-j)]a(n-\hat{k})\} \end{aligned} \quad (3.26)$$

which can be evaluated, from the previous assumptions, to get

$$p(-j) = \begin{cases} 1 & ; j = \hat{k} - k \\ 0 & ; \text{elsewhere} \end{cases} \quad (3.27)$$

for $j = 0, 1, \dots, M-1$. The cross-correlation vector, $\underline{P}$, can be written as

$$\underline{P} = \underline{\Omega} \quad (3.28)$$

where $\underline{\Omega}$ is an M -by-1 vector whose elements are all zeros except element number $\hat{k} - k$ which has a value of one.

The optimum tap-weight vector, $\underline{w}_{opt}$, is related to the autocorrelation matrix, $\underline{R}$, and the cross-correlation vector, $\underline{P}$, according to the Wiener filter theory [44][46] as follows

$$\underline{w}_{opt} = \underline{R}^{-1} \underline{P} \quad (3.29)$$

where $\underline{R}^{-1}$ is the inverse of $\underline{R}$. The optimum tap-weight vector, during the acquisition process, is evaluated under two possible scenarios, the in-phase hypothesis and the out-of-phase hypothesis.

3.3.1 In-Phase Hypothesis (H_1)

Under the in-phase hypothesis, H_1 , it is assumed that the phase offset between the incoming SS signal and the locally generated despreading sequence is within the time span of the FIR filter during the acquisition mode, which is M chips; this implies that $0 \leq \alpha \leq M-1$ where $\alpha = \hat{k} - k$. Under this condition, convergence in the MSE is detected and, at the optimum, the elements of the cross-correlation vector will be

$$p(-j) = \begin{cases} 1 & ; j = \alpha \\ 0 & ; \text{elsewhere} \end{cases} \quad (3.30)$$

where $j = 0, 1, \dots, M-1$. In a vector form, we can write (3.30) as

$$\underline{P} = \underline{\Omega} \quad (3.31)$$

and the optimum filter coefficients are

$$\underline{w}_{opt} = (1 + \sigma_n^2)^{-1} \underline{I} \underline{P} \quad (3.32)$$

By using (3.31) into (3.32), we get

$$\underline{w}_{opt} = \frac{1}{1 + \sigma_n^2} \underline{\Omega} \quad (3.33)$$

Thus, under hypothesis H_1 , the position of the peak of the tap-weight vector indicates the required delay estimate. By defining the signal-to-noise ratio per chip as

$$SNR_c = \frac{1}{\sigma_n^2} \quad (3.34)$$

we can write the optimum tap vector as

$$\underline{w}_{opt} = \frac{SNR_c}{SNR_c + 1} \underline{\Omega} \quad (3.35)$$

3.3.2 Out-of-Phase Hypothesis (H_0)

Under hypothesis H_0 , the phase offset, α , between the two signals will be out of the span of the FIR filter, i.e., $-L + 1 \leq \alpha < 0$ or $M \leq \alpha < L$, where L is the code length. The cross-correlation vector, $\underline{P}$, will be equal to zero

$$\underline{P} = \mathbf{0} \quad (3.36)$$

and, therefore, the tap-weight vector will also be zero

$$\underline{w}_{opt} = \mathbf{0} \quad (3.37)$$

where $\mathbf{0}$ is the M -by-1 null vector. This means that the filter can do nothing to minimize the MSE, and no convergence in the MSE can be detected as an indication of acquisition unless there is a false alarm error. A false alarm may be caused by the residual cross-correlation between shifted versions of the locally generated PN sequence or due to the inadequate smoothing of the short term time average to remove the large fluctuations in

the decision statistic. Eventually, the tracking mode is initiated, either due to a correct detection of acquisition or due to a false alarm, to keep the two signals in time alignment or to clear the false alarm error.

3.4 Probability Density Function of the LMS Adaptive Filter Coefficients

The LMS algorithm is used to adjust the tap weights of the adaptive filter toward their optimum Wiener solution. In doing so, the filter coefficients exhibit random variations during the transient state of the adaptation process, and also after reaching the steady state condition. Hence, the tap weights could be modelled as a random process with a certain probability distribution function that describes the behavior of the coefficients as a function of time. In general, determination of the joint probability density function (PDF) for the filter coefficients is not required, and it is sufficient to find the tap-weight vector mean and covariance matrix for evaluating the MSE behavior of the adaptive filter [47][48]. However, in the system under consideration, additional statistical information about the tap-weight vector is needed. This is caused by the dependence of the false alarm probability on the joint PDF of the tap weights, as will be explained latter.

In [49], Bershad and Qu evaluated the joint PDF for the tap weights from their conditional characteristic function. In their analysis, the tap-input vector is assumed to be a zero mean Gaussian sequence, which is independent from iteration to iteration. It was found [49], analytically and using simulation, that the tap weights are jointly Gaussian at steady state but not during the transient period.

Based on these results, and by invoking the central limit theorem it is possible to approximate the weights by a jointly Gaussian PDF [50][34]. In the following sections, we present the tap-weight vector mean and covariance matrix under both the in-phase and out-of-phase hypotheses. These parameters are required in order to evaluate the system's in-phase false alarm probability.

3.4.1 Mean Value for the Tap-Weight Vector

In this section, we will show that the ensemble average of the tap-weight vector of the adaptive FIR filter using the LMS algorithm converges to the optimum Wiener filter as the number of iterations approaches infinity. Similar to [51], the algorithm for adjusting the coefficients may be expressed as

$$\underline{w}(n+1) = \underline{w}(n) + \mu \underline{x}(n)e(n) \quad (3.38)$$

and using the definition of the error signal, see equation (3.10), we get

$$\begin{aligned} \underline{w}(n+1) &= \underline{w}(n) + \mu \underline{x}(n)[d(n) - \underline{x}^T(n)\underline{w}(n)] \\ &= \underline{w}(n) + \mu [d(n)\underline{x}(n) - \underline{x}(n)\underline{x}^T(n)\underline{w}(n)] \end{aligned} \quad (3.39)$$

By assuming that the data and the filter coefficients at time n are uncorrelated [52], and by taking the expected value for both sides of equation (3.39) to obtain

$$E[\underline{w}(n+1)] = (\mathbf{I} - \mu \mathbf{R})E[\underline{w}(n)] + \mu \underline{P} \quad (3.40)$$

which is further expressed as

$$E[\underline{w}(n+1)] = (1 - \mu(\sigma_n^2 + 1))E[\underline{w}(n)] + \mu \underline{P} \quad (3.41)$$

Assuming that the initial mean tap-weight vector is zero, i.e. $E[\underline{w}(0)] = \mathbf{0}$, the above equation can be expressed as

$$E[\underline{w}(n)] = \mu \underline{P} \sum_{i=0}^{n-1} (1 - \mu(\sigma_n^2 + 1))^{n-i-1} \quad (3.42)$$

The summation in (3.42) can be simplified, using the general formula [53]

$$\sum_{i=0}^{n-1} \rho^i = \frac{\rho^n - 1}{\rho - 1} \quad ; \quad \rho \neq 1 \quad (3.43)$$

to obtain

$$\sum_{i=0}^{n-1} (1 - \mu(\sigma_n^2 + 1))^{n-i-1} = \frac{1 - [1 - \mu(\sigma_n^2 + 1)]^n}{\mu(\sigma_n^2 + 1)} \quad (3.44)$$

Thus, (3.42) can be written as

$$E[\underline{w}(n)] = \frac{1}{\sigma_n^2 + 1} (1 - [1 - \mu(\sigma_n^2 + 1)]^n) \underline{P} \quad (3.45)$$

which, as $n \rightarrow \infty$ and $0 < \mu < 2/(1 + \sigma_n^2)$, converges to the optimum tap-weight vector given by (3.35) or (3.37). To summarize, we can write

$$E\{\underline{w}\} = \begin{cases} \frac{SNR_c}{SNR_c + 1} \underline{\Omega} & ; H_1 \\ 0 & ; H_0 \end{cases} \quad (3.46)$$

3.4.2 Covariance Matrix for the Tap-Weight Vector

The covariance matrix of the filter coefficients, assumed to be real-valued, is defined as

$$C = E\{(\underline{w} - \underline{w}_{opt})(\underline{w} - \underline{w}_{opt})^T\} \quad (3.47)$$

where we have used the previous derivation for the mean of the filter coefficients. At steady state, the covariance matrix for the adaptive filter coefficients, which uses the LMS algorithm, has been shown to be [49][46]

$$C \approx \frac{\mu}{2} J_{min} I \quad (3.48)$$

under the assumption that $\frac{\mu}{2} \lambda_i \ll 1$, where λ_i 's are the eigenvalues of the autocorrelation matrix, $\mathbf{R}$. J_{min} is the minimum mean-squared error (MMSE) given by [44][46]

$$J_{min} = 1 - \underline{w}_{opt}^T \underline{P} \quad (3.49)$$

where, as before, it is assumed that the desired signal variance is unity.

Equation (3.48) indicates that the filter coefficients are uncorrelated random variables, and since all but one have zero mean, they are also orthogonal. Using the derived equations of the optimum filter coefficients and the cross-correlation vector, the MMSE can be written as

$$J_{min} = \begin{cases} \frac{1}{SNR_c + 1} & ; H_1 \\ 1 & ; H_0 \end{cases} \quad (3.50)$$

The variance of the filter coefficients is given by

$$\sigma_{w_j}^2 = \begin{cases} \frac{\mu}{2SNR_c + 1} & ; H_1 \\ \frac{\mu}{2} & ; H_0 \end{cases} \quad (3.51)$$

for $j = 0, 1, 2, \dots, M - 1$. Note that, under H_0 , the variance of the filter coefficients does not depend on the signal-to-noise ratio and only a function of the step size of the adaptation algorithm. However, under H_1 , as the SNR_c increases, the variance of the filter coefficients decreases. As the SNR_c decreases, the difference in the MMSE diminishes and its difficult to distinguish between the two hypotheses, and all coefficients will have the same variance which is approximately equals to $\mu/2$.

Computer simulations were performed to validate the previous approximations. The results, which are not included here, indicate that the Gaussian approximation is quite accurate. Moreover, the mean vector given by (3.46) and the covariance matrix approximated by (3.48) were found to agree very well with the simulation. Since the tap weights are assumed to be jointly Gaussian, their mean vector and covariance matrix are sufficient to evaluate their joint PDF. These results will be used in the next chapter to evaluate the probability of false alarm under the in-phase hypothesis which occurs when the peak of the tap-weight vector is located at one of the taps other than the correct tap, α .

3.5 Summary

The acquisition mode of the proposed adaptive synchronization system has been described. The optimum MMSE solution for the adaptive acquisition system was evaluated, under both the in-phase and out-of-phase hypotheses, for an AWGN channel model. Finally, the joint probability density function of the filter coefficients, their mean vector, and covariance matrix were evaluated. These results will be used in the next chapter to evaluate the acquisition performance of the proposed system.

Chapter 4 Performance Analysis for the Adaptive Acquisition System

4.1 Introduction

The performance of the proposed acquisition system is presented in this chapter. The average time spent by the system searching for the correct delay offset is defined as the mean acquisition time. Minimization of the mean acquisition time is an essential objective of acquisition techniques. A flow graph technique is used to model the proposed acquisition system, and using flow graph reduction methods a mathematical formula for the mean acquisition time is derived. Furthermore, the false alarm and detection probabilities are analyzed. These probabilities depend on the received signal-to-noise ratio as well as on other parameters of the system such as LMS step size parameter and threshold value. The sensitivity of the acquisition time to variations in these parameters is discussed. Numerical results for the performance of the acquisition system under AWGN channel model are presented and discussed. Then, the performance under frequency selective and frequency non-selective Rayleigh fading is investigated. Most of the results obtained in this chapter have been reported in [54], [55], and [56].

4.2 Mean Acquisition Time Analysis

The acquisition system described in Chapter 3 is analyzed by using a flow graph technique originally proposed in [20] and [8]. The acquisition process is modelled as a Markov process which can be described by a finite state transition diagram. The generating function of the acquisition time is then derived from the developed state diagram and the statistics of the acquisition time are obtained.

The actual delay, τ , between the incoming SS signal and the locally generated code sequence is assumed to be uniformly distributed over the code period of LT_c chips. The number of states required to model the acquisition process is

$$q = \lceil L/M \rceil \quad (4.1)$$

where M is the adaptive filter length. Each state represents M chips. Let these states be numbered as $s_1, s_2, \dots, s_q$, among them only one state, assumed to be s_q , is under the in-phase hypothesis, H_1 . All the other $q - 1$ states are under the out-of-phase hypothesis, H_0 . Two extra states, denoted as ACQ and FA, are used to indicate the acquisition and false alarm conditions, respectively. As before, we assume that τ to be an integer multiple of T_c , i.e., the system is chip synchronous. Therefore, $\hat{\tau} - \tau$ is also an integer, where $\hat{\tau}$ represents the phase of the local sequence with reference to a certain initial phase.

For each state under test, the filter coefficients are adapted for a time period of $K_a T_c$ which represents the time required for the coefficients to converge to their optimum value. Then, a decision period of $K_d T_c$ is started to obtain A decisions about the MSE convergence according to (3.12). If B or more tests indicate convergence, i.e., $\Lambda_S \leq \eta$, acquisition is declared and tracking is initiated. This decision is either correct with probability P_D if the correct cell, s_q , is under test and the peak of the tap-weight vector is located at the correct phase offset $\hat{\tau} - \tau$, or false with probability P_{Fi} or P_{Fo} . The probability P_{Fi} denotes the in-phase false alarm probability which occurs when s_q is under test while the tap-weight peak is not located at tap number $\hat{\tau} - \tau$. If acquisition has been declared while a cell other than s_q is being tested, then an out-of-phase false alarm error, P_{Fo} , has occurred. The false alarm penalty time, $K_p T_c$, accounts for the time required to clear the false alarm error and to restart acquisition. We should mention that K_a, K_d , and K_p are assumed to be dimensionless integers. Upon clearing the false alarm error, the acquisition process is restarted from the state next to the one at which the false alarm has occurred. On the other hand, a miss of acquisition occurs with probability $P_M = 1 - P_D - P_{Fi}$ if the system fails to detect B threshold crossings of the short term time average while s_q is under test.

A portion of the state transition diagram that represents the acquisition system is

shown in Figure 4-1. The states are arranged in a circular arrangement, with the i th state corresponding to the i th cell. There is only one absorbing state which is the detection state ACQ. Transition between the different states is described by the transfer functions defined as

$$H_1(Z) = P_D Z^{(K_a + K_d)T_c} \quad (4.2)$$

$$H_0(Z) = (1 - P_D - P_{Fi}) Z^{(K_a + K_d)T_c} \quad (4.3)$$

$$H_3(Z) = P_{Fo} Z^{(K_a + K_d)T_c} \quad (4.4)$$

$$H_4(Z) = Z^{K_p T_c} \quad (4.5)$$

$$H_5(Z) = (1 - P_{Fo}) Z^{(K_a + K_d)T_c} \quad (4.6)$$

$$H_6(Z) = P_{Fi} Z^{(K_a + K_d)T_c} \quad (4.7)$$

where Z is the unit delay operator.

The state transition diagram is simplified by introducing two transfer functions defined as

$$\begin{aligned} H_2(Z) &= H_0(Z) + H_6(Z)H_4(Z) \\ &= (1 - P_D - P_{Fi}) Z^{(K_a + K_d)T_c} + P_{Fi} Z^{(K_a + K_d + K_p)T_c} \end{aligned} \quad (4.8)$$

and

$$\begin{aligned} H_e(Z) &= H_3(Z) + H_5(Z)H_4(Z) \\ &= P_{Fo} Z^{(K_a + K_d)T_c} + (1 - P_{Fo}) Z^{(K_a + K_d + K_p)T_c} \end{aligned} \quad (4.9)$$

Using these equivalent transfer functions, the flow graph of Figure 4-1 may be reduced to part (a) of Figure 4-2, which is further simplified as shown in part (b) of the same figure.

The simplified flow graph is used to evaluate the probability of acquisition, $P_{ACQ}(n)$, which is defined as the probability of starting from some initial state with a given a priori probability, the process will reach the ACQ state in n steps. The generating function of the acquisition time is defined as the Z -transform of the acquisition probability [8], i.e.,

Figure 4-1: A portion of the state transition diagram for the adaptive acquisition system.

(a)

(b)

Figure 4-2: Simplified flow graph diagram for the acquisition system in Fig. 4-1.

$$P_{ACQ}(Z) = \sum_{n=0}^{\infty} P_{ACQ}(n)Z^n \quad (4.10)$$

Assuming that the system can start searching at any state with equal probability of $\frac{1}{q}$, we have

$$P_{ACQ}(Z) = \frac{1}{q} \sum_{j=1}^q P_{j,ACQ}(Z) \quad (4.11)$$

where $P_{j,ACQ}(Z)$ is the transfer function between the j th state and the ACQ state. Equation (4.11) can be written as

$$P_{ACQ}(Z) = \frac{H_1(Z)}{q} \sum_{j=1}^q P_{j,q}(Z) \quad (4.12)$$

where $P_{j,q}(Z)$ is the transfer function between the j th state and s_q . From the simplified flow graph in Figure 4-2, we have

$$P_{j,q}(Z) = \frac{H_e^{q-j}(Z)}{1 - H_2(Z)H_e^{q-1}(Z)} \quad (4.13)$$

By using (4.13) into (4.12), we get

$$P_{ACQ}(Z) = \frac{1}{q} \frac{H_1(Z)}{1 - H_2(Z)H_e^{q-1}(Z)} \sum_{j=1}^q H_e^{q-j}(Z) \quad (4.14)$$

which, after some algebra, reduces to

$$P_{ACQ}(Z) = \frac{1}{q} \frac{H_1(Z)\{1 - H_e^q(Z)\}}{\{1 - H_2(Z)H_e^{q-1}(Z)\}\{1 - H_e(Z)\}} \quad (4.15)$$

The overall detection probability, $P_{D,ov}$ is defined as the probability of reaching acquisition at any time, which is evaluated from $P_{ACQ}(Z)$ as

$$P_{D,ov} = P_{ACQ}(Z)|_{Z=1} \quad (4.16)$$

Also, the mean and variance of the acquisition time are given by [8]

$$E\{T_{acq}\} = \left. \frac{dP_{ACQ}(Z)}{dZ} \right|_{Z=1} \quad (4.17)$$

$$var\{T_{acq}\} = \left. \left\{ \frac{d^2P_{ACQ}(Z)}{dZ^2} + \frac{dP_{ACQ}(Z)}{dZ} - \left[\frac{dP_{ACQ}(Z)}{dZ} \right]^2 \right\} \right|_{Z=1} \quad (4.18)$$

By incorporating (4.15) into (4.17), we get [20]

$$E\{T_{acq}\} = \frac{P_{D,ov}}{H_1(1)} \left\{ H_1'(1) + \frac{qH_e'(1)H_e^{q-2}(1)}{1-H_e^q(1)} [-H_1(1)H_e(1) + P_{D,ov} \right. \\ \left. (qH_e(1) + \frac{1}{H_e^{q-2}(1)} + \frac{H_2'(1)}{H_e'(1)}(1-H_e(1))(H_e(1)-H_2(1))) \right] \} \quad (4.19)$$

for $H_e(1) < 1$, and

$$E\{T_{acq}\} = \frac{1}{H_1(1)} \left[H_1'(1) + H_2'(1) + (q-1)H_e'(1) \left(1 - \frac{H_1(1)}{2} \right) \right] \quad (4.20)$$

for $H_e(1) = 1$. The prime sign denotes differentiation with respect to Z .

For the system under consideration, in which there is only one absorbing state namely ACQ, (4.16) may be rewritten using (4.2)-(4.9) as

$$P_{D,ov} = \frac{H_1(1)}{1-H_2(1)}$$

which reduces to $P_{D,ov} = 1$ since $H_1(1) + H_2(1) = 1$. Similarly, by noting that $H_e(1) = 1$, the mean acquisition time is obtained from (4.20) as

$$E\{T_{acq}\} = \frac{1}{P_D} \left\{ \left[1 + (q-1) \frac{2-P_D}{2} \right] (K_a + K_d) + \right. \\ \left. + \left[P_{Fi} + (q-1)P_{Fo} \frac{2-P_D}{2} \right] K_p \right\} T_c \quad (4.21)$$

From the previous analysis, it is obvious that the mean acquisition time is determined by the detection and false alarm probabilities of the system. Determination of these probabilities is discussed in the next section.

4.3 System Probabilities

In order to evaluate the mean acquisition time performance, the false alarm and detection probabilities are needed. In general, analytical evaluation of these probabilities is a complicated problem, and several assumptions and simplifications about the system's structure and operation are required in order to make the problem analytically tractable. In the following sections, statistical models for the estimation error and the short term time average (STTA) are derived under both H_1 and H_0 . These models, along with the approximation for the PDF of the filter coefficients described in section (3.4), are used to determine the required probabilities.

4.3.1 Statistical Models

As described in Chapter 3, the adaptive filter input, $\underline{x}(n)$, is a sequence of i.i.d. Gaussian random variables with zero mean and variance

$$\sigma_x^2 = 1 + \sigma_n^2 \quad (4.22)$$

The adaptive filter output, $y(n)$, is related to $\underline{x}(n)$ by the convolution sum as follows

$$y(n) = \sum_{j=0}^{M-1} w_j(n)x(n-j) \quad (4.23)$$

The following assumptions about the FIR filter tap weights $\underline{w}(n)$ and input sequence $\underline{x}(n)$ are made¹:

- (1) $\underline{w}(n)$ and $\underline{x}(n)$ are independent random variables.
- (2) $w_j(n)$, $j = 0, 1, 2, \dots, M-1$ are i.i.d. Gaussian random variables with mean value m_α given by (3.35) or (3.37) and variance σ_w^2 given by (3.51).

At steady state, assuming the filter taps have converged to the optimum solution, the filter output is a zero mean Gaussian random variable with a variance of

1. These assumptions are similar to the independence theorem described in [44][47].

$$\sigma_y^2 = \begin{cases} \frac{\mu M SNR_c + 1}{2 SNR_c} & ; H_0 \\ \frac{\mu M}{2 SNR_c} + \frac{SNR_c}{SNR_c + 1} & ; H_1 \end{cases} \quad (4.24)$$

where (3.46), (3.51), and (4.22) have been used to derive this result. Figure 4-3 and Figure 4-4 illustrate histograms obtained by computer simulation for the filter output for a SNR_c of 0 dB and -5 dB under H_1 and H_0 , respectively. Also, Gaussian PDFs with variances according to (4.24) are included to show their good agreement with the simulation results.

The estimation error signal, $e(n)$, is formed as the difference between the filter output and the desired signal, $d(n)$, at the n th time instant, i.e.,

$$e(n) = d(n) - y(n) \quad (4.25)$$

Since $d(n)$ is either +1 or -1 with equal probability and $y(n)$ is Gaussian with zero-mean, $e(n)$ will have a Gaussian PDF centered around +1 if $d(n) = +1$, or at -1 if $d(n) = -1$. If the variance of $y(n)$ is large, the two Gaussian PDFs will overlap to produce a zero-mean Gaussian PDF. However, for a small variance of $y(n)$, a different distribution is obtained for the error signal due to the narrower overlapping distributions. These observations are confirmed by computer simulation as shown in Figure 4-5 and Figure 4-6. As before, theoretical distributions are plotted in the same figures to show their match to the simulation results. A different interpretation to the previous observations is obtained by noting that, under H_1 , $y(n)$ and $d(n)$ are highly correlated and the subtraction process results in a zero mean Gaussian PDF, provided that the SNR_c is large; however, for small SNR_c , the filter output does not resemble the desired signal perfectly and the error signal will have a non-zero mean value. Similarly, under H_0 , $d(n)$ and $y(n)$ are not correlated and $e(n)$ will have a Gaussian distribution with a mean value of either +1 or -1 regardless of the SNR_c .

Figure 4-3: Histograms for the adaptive filter output under H_1 ; (a) $SNR_c = 0$ dB, (b) $SNR_c = -5$ dB.

Figure 4-4: Histograms for the adaptive filter output under H_0 ; (a) $SNR_c = 0$ dB, (b) $SNR_c = -5$ dB.

Figure 4-5: Histograms for the error signal under H_1 ; (a) $SNR_c = 0$ dB, (b) $SNR_c = -5$ dB.

(a)

(b)

Figure 4-6: Histograms for the error signal under H_0 ; (a) $SNR_c = 0$ dB, (b) $SNR_c = -5$ dB.

To ensure that the filter is operating near steady state, a long enough transition period is chosen before decisions about acquisition are made. This enables us to approximate the second order moment of the error signal by the steady state mean square error obtained by the LMS algorithm. This has been shown in [44] to be

$$J_{ss} = \frac{J_{min}}{1 - \sum_{i=1}^M \frac{\mu \lambda_i}{2 - \mu \lambda_i}} \quad (4.26)$$

where J_{min} is the MMSE, and λ_i 's are the eigenvalues of the autocorrelation matrix of the input signal. For the system under consideration, we have

$$\lambda_i = 1 + \sigma_n^2 \quad ; \quad i = 1, 2, \dots, M \quad (4.27)$$

and (4.26) can be simplified to

$$J_{ss} = \begin{cases} \frac{2SNR_c - \mu(SNR_c + 1)}{2SNR_c - \mu(SNR_c + 1)(M + 1)} & ; H_0 \\ \frac{2SNR_c - \mu(SNR_c + 1)}{2SNR_c - \mu(SNR_c + 1)(M + 1)(SNR_c + 1)} \cdot 1 & ; H_1 \end{cases} \quad (4.28)$$

where we have used equations (4.27), (3.34) and (3.50) to obtain the above result.

From the previous discussion, the mean and variance of the error signal are found to be

$$E\{e(n)\} = \begin{cases} \pm 1 & ; H_0 \\ 0 & ; H_1 \text{ large } SNR_c \\ \pm J_{ss} & ; H_1 \text{ small } SNR_c \end{cases} \quad (4.29)$$

$$\sigma_e^2 = \begin{cases} J_{ss} - 1 & ; H_0 \\ J_{ss} & ; H_1 \text{ large } SNR_c \\ J_{ss}(1 - J_{ss}) & ; H_1 \text{ small } SNR_c \end{cases} \quad (4.30)$$

where the mean value under H_1 with small SNR_c was obtained through computer simulation. Note that, a zero mean approximation of the error signal under H_1 could be used for all values of SNR_c , however, as will be demonstrated later, the accuracy of this

model tends to deteriorate as the SNR_c decreases.

The decision statistic used by the system to check for acquisition is the short term time average, Λ_S , which was defined as

$$\Lambda_S = \frac{1}{S} \sum_{i=1}^S e^2(i) \quad (4.31)$$

It is the average of the square of S Gaussian random variables, which, depending on the hypothesis under test and the SNR_c value, can be modelled as a random variable with a chi-squared distribution with S degrees of freedom [57]. To elaborate, under H_0 , Λ_S will have a non-central chi-squared distribution with S degrees of freedom and non-centrality parameter of

$$\xi^2 = \frac{1}{S} \sum_{i=1}^S E^2\{e(i)\} = 1 \quad (4.32)$$

The probability density function of Λ_S is given by

$$f_{\Lambda_S|H_0}(\vartheta) = \begin{cases} \frac{1}{2\sigma_e^2/S} (\vartheta/\xi^2)^{\frac{S-2}{4}} e^{-\frac{\xi^2+\vartheta}{2\sigma_e^2/S}} I_{\frac{S}{2}-1}\left(\sqrt{\vartheta}\frac{\xi}{\sigma_e^2/S}\right) & ; \vartheta \geq 0 \\ 0 & ; \vartheta < 0 \end{cases} \quad (4.33)$$

where $I_j(z)$ is the j th order modified Bessel function of the first kind. On the other hand, under H_1 , for the large SNR_c case where the error signal has a zero mean, Λ_S will have a centralized chi-squared distribution with S degrees of freedom. The PDF of Λ_S is given by

$$f_{\Lambda_S|H_1}(\vartheta) = \begin{cases} \frac{1}{(\sigma_e^2/S)^S 2^{S/2} \Gamma(S/2)} \vartheta^{S/2-1} e^{-\frac{\vartheta}{2\sigma_e^2/S}} & ; \vartheta \geq 0 \\ 0 & ; \vartheta < 0 \end{cases} \quad (4.34)$$

where $\Gamma(p)$ is the gamma function defined as

$$\Gamma(p) = \int_0^{\infty} t^{p-1} e^{-t} dt \quad ; \quad p > 0 \quad (4.35)$$

Under H_1 and small SNR_c , Λ_S has a non-central chi-squared distribution with S degrees of freedom and non-centrality parameter

$$\xi^2 = J_{ss}^2 \quad (4.36)$$

The PDF of Λ_S is the same as that given by (4.33) with ξ^2 given by (4.36). Note that the variance of the error signal used in the previous distributions is chosen from (4.30) according to the condition under consideration. Also, using the zero mean Gaussian approximation for the error signal under H_1 and all values of SNR_c , results in Λ_S having a PDF given by (4.34) with the error variance equals to J_{ss} .

Histograms of Λ_S obtained from computer simulation are presented in Figure 4-7 and Figure 4-8. Figure 4-7a shows the histogram of Λ_S , under H_1 for a $SNR_c = 0$ dB, along with the PDFs of a non-central chi-squared random variable and the central chi-squared PDF obtained from the zero mean approximation of the error signal. It is obvious that both models have good agreement with the histogram; which indicates the accuracy of the approximation under large SNR_c . The histogram for Λ_S , under H_1 when the $SNR_c = -5$ dB, is shown in Figure 4-7b. The non-central chi-squared model is shown to match the simulation results, however, the central chi-squared approximation seems to over bound the histogram. This will also be shown in the acquisition time performance results. The histogram, shown in Figure 4-8a, for Λ_S under H_0 with a $SNR_c = 0$ dB is compared to the PDF of a non-central chi-squared random variable, using (4.33) with the values of σ_e^2 and ξ^2 obtained from (4.30) and (4.32), respectively. In part (b) of the same figure we show the results for a $SNR_c = -5$ dB. These plots indicate that the proposed statistical models for Λ_S follow the simulation results accurately.

Figure 4-7: Histograms for the STTA under H_1 ; (a) $SNR_c = 0$ dB, (b) $SNR_c = -5$ dB.

Figure 4-8: Histograms for the STTA under H_0 ; (a) $SNR_c = 0$ dB, (b) $SNR_c = -5$ dB.

The probability density functions of the STTA under different hypotheses will be used in the next section to evaluate the probability of detection, the in-phase false alarm probability, and the out-of-phase false alarm probability. However, for the detection and in-phase false alarm probabilities, we need to find the probability of finding the peak of the filter taps at one of the positions other than the correct tap number when the last threshold crossing occurs. This will be evaluated in the next section.

4.3.2 Error Probability in Peak Location

In section (3.4), the filter coefficients were shown to be independent identically distributed Gaussian random variables all with zero mean but the tap at which the peak is located has a mean value of

$$m_\alpha = w_{\alpha, opt} = \frac{SNR_c}{1 + SNR_c} \quad (4.37)$$

The variance is the same for all the random variables which equals to

$$\sigma_w^2 = \frac{\mu/2}{1 + SNR_c} \quad (4.38)$$

The probability that the correct tap is larger than all the other $M-1$ taps may be expressed as follows

$$P_c = \int_{-\infty}^{\infty} f(w_0 < w_\alpha, w_1 < w_\alpha, \dots, w_{M-1} < w_\alpha | w_\alpha) f(w_\alpha) dw_\alpha \quad (4.39)$$

where $f(w_0 < w_\alpha, w_1 < w_\alpha, \dots, w_{M-1} < w_\alpha | w_\alpha)$ is the joint probability that $w_j < w_\alpha$ for $j = 0, 1, 2, \dots, M-1; j \neq \alpha$, conditioned on w_α . Due to the independence of the filter coefficients, their joint PDF factors into the product of their marginal probabilities to get

$$f(w_0 < w_\alpha, w_1 < w_\alpha, \dots, w_{M-1} < w_\alpha | w_\alpha) = [f(w_j < w_\alpha | w_\alpha)]^{M-1} \quad (4.40)$$

where

$$p(w_j < w_\alpha | w_\alpha) = \int_{-\infty}^{w_\alpha} \frac{1}{\sqrt{2\pi}\sigma_w} e^{-\frac{w_j^2}{2\sigma_w^2}} dw_j \quad (4.41)$$

which reduces to

$$p(w_j < w_\alpha | w_\alpha) = 1 - Q\left(\frac{w_\alpha}{\sigma_w}\right) \quad (4.42)$$

where $Q(\cdot)$ is the Q -function defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt \quad (4.43)$$

Hence, the probability of correct decision is

$$P_c = \int_{-\infty}^{\infty} \left[1 - Q\left(\frac{w_\alpha}{\sigma_w}\right)\right]^{M-1} \frac{1}{\sqrt{2\pi}\sigma_w^2} e^{-\frac{(w_\alpha - m_\alpha)^2}{2\sigma_w^2}} dw_\alpha \quad (4.44)$$

and the probability of error is simply

$$P_\varepsilon = 1 - P_c \quad (4.45)$$

Note that there is no closed form solution to (4.44) and numerical integration is needed to evaluate this equation.

4.3.3 Detection and False Alarm Probabilities

The detection and false alarm probabilities are determined from the probability that the decision statistic, i.e., the STTA, is less than a threshold value depending upon the hypothesis under test. Let $P_{\eta|H_i}$ be the probability that the STTA is less than or equal to a threshold value η under hypothesis H_i , $i = 0, 1$, i.e.,

$$\begin{aligned} P_{\eta|H_i} &= \text{probability}(\Lambda_S \leq \eta | H_i) \\ &= \int_{-\infty}^{\eta} f_{\Lambda_S|H_i}(\vartheta) d\vartheta \end{aligned} \quad (4.46)$$

Substituting (4.33) into (4.46) we get the probability of threshold crossing under H_0 or

under H_1 for the small SNR_c case, depending on the values used for the error variance and the non-centrality parameter.¹ On the other hand, using (4.34) into (4.46) will result in the probability of threshold crossing under H_1 , for the large SNR_c case and also for the zero mean approximation of the error signal, which may be expressed, using tables of integration [53], as

$$P_{\eta H_1} = 1 - e^{-\frac{\eta}{2\sigma_e^2/S}} \sum_{k=0}^{\frac{S}{2}-1} \frac{1}{k!} \left(\frac{\eta}{2\sigma_e^2/S} \right)^k \quad (4.47)$$

where S is assumed to be an even integer value. $P_{\eta H_i}$ is used to determine the false alarm and detection probabilities, P_D , P_{Fi} , and P_{Fo} .

Recall that the detection probability is the probability that B out of A tests of the STTA are less than or equal to η and the peak of the filter coefficients is located at tap number α . Since the decisions are independent the probability of detection is given by the cumulative binomial distribution as

$$P_D = \sum_{j=B}^A \binom{A}{j} (1 - P_\varepsilon) P_{\eta H_1}^j (1 - P_{\eta H_1})^{A-j} \quad (4.48)$$

while the in-phase false alarm probability is given by

$$P_{Fi} = P_\varepsilon / (1 - P_\varepsilon) P_D \quad (4.49)$$

and finally, the out-of-phase false alarm probability is

$$P_{Fo} = \sum_{j=B}^A \binom{A}{j} P_{\eta H_0}^j (1 - P_{\eta H_0})^{A-j} \quad (4.50)$$

These probabilities are then substituted in (4.21) to obtain the mean acquisition time.

4.4 Numerical Results and Discussions

To see how the previous analysis can be used to determine the acquisition time

1. see equations (4.30), (4.32), and (4.36).

performance for the adaptive system, numerical results will be presented. After selecting the system parameters, the effects of the LMS step size parameter and threshold value on the detection and false alarm probabilities are investigated. Numerical optimization of these parameters is used to minimize the mean acquisition time. The influence of the number of filter taps on the acquisition time is presented. Finally, the sensitivity of the acquisition time performance to the variations of the step size and threshold setting from their optimum values is discussed.

4.4.1 System Parameters

For the rest of this analysis, unless stated otherwise, it is assumed that the system has the following parameters

1. Code length $L = 127$ chips.
2. Number of adaptive filter coefficients $M = 16$ taps.
3. Maximum number of tests per cell $A = 10$.
4. Required number of threshold crossings to declare acquisition $B = 3$.
5. STTA window $S = 10$ chips.
6. Adaptation period is $K_a = 2/\mu\lambda_{av}$ where λ_{av} is the average eigenvalue of the autocorrelation matrix of the filter input. This period is selected as an approximation to the MSE time constant of the LMS algorithm [44][46].
7. Decision period is $K_d = AS$.
8. False alarm penalty time $K_p = q(K_a + K_d)$, where q is defined in (4.1).

4.4.2 Effects of LMS Step Size Parameter on the PDF of the Decision Statistic and System Probabilities

The steady state mean-square error produced by the LMS algorithm is determined by the step size parameter used to adjust the filter taps. A large step size results in a fast initial convergence and consequently a small transition period to reach steady state; however, a larger steady state MSE is observed. The value of the adaptation step size must be carefully selected such that the limits of stability of the LMS algorithm, as described in

Appendix A, are not violated. Provided that these limits are satisfied, the step size parameter has a large influence on the probability density function of the short term time average of the squared error.

The variations observed on the PDF of the STTA are shown in Figure 4-9 for a SNR_c of 0 dB and -5 dB, respectively. For the first case, $SNR_c = 0$ dB, a change of μ from 1×10^{-3} to 5×10^{-3} results in a large variation to both PDFs which may cause a dramatic change in the false alarm and detection probabilities. For instance, for a threshold value of 0.5, the reduction in the area under the tail of $f_{\Lambda_s|H_1}(\vartheta)$, which determines the detection probability, is quite large. There is also a slight increase in the area under the tail of $f_{\Lambda_s|H_0}(\vartheta)$ which increases the out-of-phase false alarm probability. The previous effects are more profound for the second case where the SNR_c equals to -5 dB.

Figure 4-10 to Figure 4-12 show the detection probability, in-phase false alarm probability, and out-of-phase false alarm probability, respectively, as a function of the LMS step size μ with the threshold value η as a parameter for a SNR_c of 0 dB and -5 dB. The detection probability deteriorates as μ increases with a decreasing threshold η as shown in Figure 4-10. However, for small values of μ , the detection probability can be made to approach unity. Figure 4-11 indicates that, the in-phase false alarm probability starts to increase with increasing μ because it is dominated by the error probability P_ε , however, as μ increases, the reduction in the detection probability starts to reduce the in-phase false alarm probability. The out-of-phase false alarm probability is largely effected by the threshold value as shown in Figure 4-12. It is also increasing with μ but, for large μ , it saturates at a level determined by the SNR_c and the threshold setting. For instance, when $\mu \geq 1 \times 10^{-2}$ and SNR_c of 0 dB, P_{Fo} is approximately fixed at 1×10^{-4} , 2×10^{-3} , and 2×10^{-2} for threshold values of 0.6, 0.7, and 0.8 respectively.

Figure 4-13 shows the probability of error in selecting the correct filter tap, P_ε , as a function of the step size μ with the SNR_c as a parameter. The influence of μ is quite large and a small change in μ causes a large change in the error probability. For example,

Figure 4-9: Variations of the decision statistic (STTA) PDF with the LMS step size parameter, (a) $SNR_c = 0$ dB, (b) $SNR_c = -5$ dB.

(a)

(b)

Figure 4-10: Detection Probability verses the step size parameter; (a) $SNR_c = 0$ dB, (b) $SNR_c = -5$ dB.

Figure 4-11: In-phase false alarm probability versus the step size parameter; (a) $SNR_c = 0$ dB, (b) $SNR_c = -5$ dB.

(a)

(b)

Figure 4-12: Out-of-phase false alarm probability versus the step size parameter; (a) $SNR_c = 0$ dB,
(b) $SNR_c = -5$ dB

reducing μ from 2×10^{-2} to 1×10^{-2} improves the error probability from 1×10^{-6} to 1×10^{-10} at SNR_c of 0 dB.

An important conclusion from these observations is that the step size parameter and the threshold value must be carefully selected to ensure good system performance. One possibility is to optimize both the step size parameter and the threshold value such that the detection probability is maximized under the constraint that the false alarm probability does not exceed a certain value. However, this criterion may not lead to the minimum mean acquisition time performance. For example, a small value of μ may be needed to meet design specifications which may increase the acquisition time due to the increased transition period required for the LMS algorithm to reach steady state.

Since the mean acquisition time is one of the most important characteristics for assessing different code acquisition systems; it is preferable to optimize μ and η , for every SNR_c , to minimize the mean acquisition time. In Figure 4-14, the optimum step size parameter, in a minimum mean acquisition time sense, is plotted versus the SNR_c for different number of filter taps. It is observed that, the optimum step size that will result in a minimum acquisition time is, on a logarithmic scale, linearly increasing with the SNR_c . Therefore, the adaptive system has to have a variable step size parameter in order to ensure optimum performance. Note that, a variable step size parameter was previously suggested to improve the convergence speed of the simple LMS algorithm [58][59].

4.4.3 Mean Acquisition Time Performance

The mean acquisition time for the proposed adaptive system is evaluated using the previously developed expressions with the step size parameter chosen according to the above mentioned criterion. The optimized mean acquisition time is illustrated in Figure 4-15 where, as expected, the performance improves with increasing SNR_c . Since the proposed system tests M chips at a time, it has an improvement in the mean acquisition

Figure 4-13: Probability of error in selecting the correct filter tap.

Figure 4-14: Optimum LMS step size parameter.

time of M to $2M$ times over conventional 1/2-chip serial-search schemes. For example, a 1/2-chip serial search system will have a mean acquisition time of about 16000 chips (detection probability = 1, false alarm probability = 0, dwell time = 127 chips) while the proposed system has a mean acquisition time of 700 chips ($M = 16$, $S = 10$, $SNR_c = 0$ dB); an improvement factor of about 23 times. Even though it is not fair to compare the proposed system to the conventional 1/2-chip serial-search scheme because of their difference in structure, this comparison is useful in obtaining a qualitative comparison with other hybrid acquisition schemes. Since hybrid acquisition schemes which test $2M$ cells at a time also have a $2M$ improvement over 1/2-chip serial-search, however $2M$ correlators and decision devices are needed. In contrast, the proposed system requires a single M -tap adaptive filter to attain such gain. Note that, the proposed scheme has a small computational complexity since it uses the LMS algorithm to update the filter coefficients.

Increasing the number of filter taps results in a smaller acquisition time because the number of cells simultaneously searched is increased as shown in Figure 4-16. Note that, for all values of SNR_c , there is an improvement in the mean acquisition time performance. However, a diminishing gain is obtained as the filter length is increased. The mean acquisition time saturates at a higher level as the SNR_c decreases; about an order of magnitude between 0 dB and -12 dB. This is primarily caused by the larger transition period required to reach steady state which is a result of the large reduction in μ for small SNR_c . Thus, the acquisition performance could be improved by reducing the transition period required by the algorithm to reach steady state. The RLS algorithm has faster convergence rate than that of the LMS algorithm, however, under small signal-to-noise ratio, they converge at the same rate.

In Figure 4-17, the exact mean acquisition time is compared to the approximations obtained by modeling the PDF of the STTA under H_1 by a central chi-squared distribution. This approximation provides tight results for large SNR_c but it becomes loose as the SNR_c decreases, particularly for a filter with a large number of taps. Nevertheless, the

Figure 4-15: Optimized mean acquisition time performance: $L = 127$, $B = 3$, $A = 10$, $S = 3$.

Figure 4-16: Mean acquisition time for different filter lengths.

approximation is useful in obtaining a closed form solution for the out-of-phase false alarm probability which is given by using (4.47) into (4.50).

In order to ensure the optimum performance of the acquisition system, the SNR_c should be estimated by the receiver and the corresponding values of μ and η are used during the acquisition process. An error in the estimation of the SNR_c causes a degradation in the system performance since the algorithm will use non-optimal values for both μ and η . The sensitivity of the mean acquisition time to the estimation error is shown in Figure 4-18 for two values of the SNR_c . For example, a 3 dB error in estimating the SNR_c will cause the mean acquisition time to be double that of the minimum mean acquisition time obtained by an optimized system. We should also emphasize that the step size should always be selected to satisfy the stability limits required by the LMS algorithm (see Appendix A).

It is well known that acquisition techniques based on matched filters (MF) have a very fast acquisition time. However, their hardware complexity is usually large especially for long code lengths. Although the code length used in the example is not large and a full code MF is not difficult to build, we choose to compare the performance of the proposed adaptive filter based acquisition system to a system based on a partially MF similar to that described in [5] in order to have the same number of taps. The filter is matched to an M segment of the spreading code. The received signal is passed through the MF. If the output of the MF exceeds a threshold value, a verification period is started to validate the decision. During the verification mode, the taps of the MF are changed to match that of the next segment of the spreading code and A disjoint decisions are made. Acquisition is declared if B or more decisions exceed the threshold value; otherwise the process is repeated.

In our comparison, the penalty time is the same as that used for the adaptive filter. The verification mode parameters (A,B) are chosen as $(4,2)$ since it was reported that these values provide good performance for the matched filter [5][17]. Simulation results for the partial MF system is shown in Figure 4-19 along with the adaptive filter performance for a

Figure 4-17: Exact and approximate mean acquisition time.

Figure 4-18: Sensitivity of the acquisition time performance to the SNR_c estimation error.

Figure 4-19: Simulation results of the mean acquisition time for the partially MF and the adaptive filter based acquisition schemes for $M = 16$.

16 tap adaptive filter. The MF has better performance at larger values of SNR_c because of the large detection probability while the adaptive filter operates better in noisy conditions which is the dominant case in fading and multi-user environments. This is because of the ability of the adaptive filter to adapt to its environment while the MF uses a fixed tap setting. Another advantage for the adaptive filter system stems from the ability to use the same adaptive filter for code tracking, therefore, a significant simplification in receiver structure is gained.

4.5 Acquisition Performance Under Frequency Selective and Frequency Non-Selective Rayleigh Fading Channels

In this section, the optimum tap-weight vector for the adaptive filter is derived under fading conditions usually encountered when the signal is transmitted over mobile radio channels. We also present simulation results for the mean acquisition time performance of the proposed acquisition system under both frequency selective and frequency non-selective slow Rayleigh fading channels. For the frequency selective case, exponential and uniform multipath delay profiles are examined.

4.5.1 Multipath Fading Channel Model

In multipath fading channels, if an impulse is transmitted through the channel, the received signal is composed of several replica or echoes of the transmitted signal each delayed and attenuated independently. The maximum time over which the received signal power is non-negligible is usually termed the excess multipath delay spread. This is proportional to the inverse of the channel coherence bandwidth, where the channel coherence bandwidth is defined as the width of the frequency range over which the channel frequency response is highly correlated. If the transmission bandwidth of the spread spectrum signal exceeds the coherence bandwidth, the multipath components can be resolved into a discrete number of paths and the channel is called frequency selective. On the other hand, when the transmission bandwidth is less than the coherence bandwidth

of the channel, the paths are not resolvable and it is called frequency non-selective or flat fading channel.

The maximum number of resolvable paths is usually given by [57]

$$L_p = \left\lfloor \frac{T_m}{T_c} \right\rfloor + 1 \quad (4.51)$$

where T_m is the maximum delay spread. The fading channel is modelled as a tapped delay line with a tap spacing of one chip (T_c) as shown in Figure 4-20. Each tap is multiplied by an independent complex Gaussian random variable with amplitude Υ_l which has a Rayleigh distribution and phase Θ_l which is uniformly distributed over the interval $[0, 2\pi]$. The low-pass equivalent impulse response of the passband channel can be represented as

$$h(t) = \sum_{l=1}^{L_p} \Upsilon_l \delta(t - \tau_l) e^{j\Theta_l} \quad (4.52)$$

where τ_l is the propagation delay for the l th path. The average power in each resolvable path is given by the second moment of its amplitude. If the paths have equal power, the channel is said to have a uniform multipath delay profile (MDP). On the other hand, when the average power decreases exponentially with increasing path number, the MDP is said to be exponential with a certain decay rate.

If channel parameters are assumed to vary with time, the channel will have a time varying impulse response. The rate of change of channel parameters is usually measured by the Doppler rate which is proportional to the inverse of the coherence time of the channel. The coherence time is the maximum time over which the channel parameters are assumed to be highly correlated. A fast fading channel is a channel whose parameters change at a rate that is comparable to the symbol rate; otherwise it is known as a slow fading channel.

The received baseband signal can be expressed as

Figure 4-20: Tapped delay line frequency selective fading channel model.

$$x(t) = \sum_{k=1}^{L_p} Y_k a(t - \tau_k) e^{j\Theta_k} + n(t) \quad (4.53)$$

whose autocorrelation matrix, $\mathbf{R}$, has the km th element given by

$$\begin{aligned} r_{km} &= E\{x(n-k)x(n-m)\} \\ &= \sum_{l=1}^{L_p} \sum_{j=1}^{L_p} E\{Y_l Y_j\} E\{a(n-\tau_l-k)a(n-\tau_j-m)\} + \sigma_n^2 \delta(k-m) \end{aligned} \quad (4.54)$$

The diagonal elements of $\mathbf{R}$ ($k=m$) are easily obtained as

$$r_{kk} = \sum_{l=1}^{L_p} E\{Y_l^2\} + \sigma_n^2 \quad (4.55)$$

which, if the total received power from all paths is normalized to unity, reduces to $1 + \sigma_n^2$.

The rest of the elements of $\mathbf{R}$ depend on the correlation between different paths' amplitudes and the difference between paths' delays. For independent identically distributed (i.i.d) amplitudes, these terms may be shown to be upper bounded by

$$r_{km} \leq L_p E^2\{Y_1\} \quad (4.56)$$

for $k-m = |\tau_l - \tau_j|$, $k, m = 0, 1, \dots, M-1$, $k \neq m$ and $l, j = 1, 2, \dots, L_p$, $l \neq j$, and $r_{km} = 0$ otherwise. The autocorrelation matrix may be approximated, for small values of mean path strength, by

$$\mathbf{R} \approx (1 + \sigma^2) \mathbf{I} \quad (4.57)$$

where $\mathbf{I}$ is the M -by- M identity matrix.

The elements of the M -by-1 cross-correlation vector are given by

$$\begin{aligned} p(-k) &= E\{x(n-k)d(n)\} \\ &= \sum_{l=1}^{L_p} E\{Y_l\} E\{a(n-\tau_l-k)a(n-\hat{\tau})\} \end{aligned} \quad (4.58)$$

Therefore, for a random sequence model, the cross-correlation vector would have several peaks each corresponding to one resolvable path provided that the total delay offset

between the incoming paths and the reference signal is less than the filter length M . On the other hand, if the total delay offset is greater than M , the cross-correlation vector is zero. The total delay offset is composed of initial delay offset plus the channel delay spread. In reality, the received paths may be located at different cells and the system will have a higher probability of acquisition since there are more cells corresponding to the in-phase condition (H_1).

The optimum tap-weight vector is given by

$$\underline{w}_{opt} = \begin{cases} (1 + \sigma_n^2) \tilde{\underline{\Omega}} & ; H_1 \\ 0 & ; H_0 \end{cases} \quad (4.59)$$

where $\tilde{\underline{\Omega}}$ is an M -by-1 vector whose elements are all zero except those corresponding to the delay offset between the incoming paths and the locally generated code which are given by the average received signal strength from each resolvable path.

Figure 4-21 shows a typical snapshot of the optimum tap-weight vector for a 16-tap adaptive filter operating in a 3-path Rayleigh fading channel with uniform MDP and a chip signal-to-noise ratio of -6 dB. The delays are 7, 9, and 12 chips for the first, second, and third paths, respectively. Simulation results are also included to confirm their agreement with the theoretical results. Note that tap number 16 corresponds to a delay of one chip and tap number 15 corresponds to a delay of two chips and so on. In the next section, we present the mean acquisition time performance for the adaptive system.

4.5.2 Simulation Results for the Mean Acquisition Time Under Fading Conditions

In our simulations, we consider both frequency selective and flat fading channels. The fading is assumed to be a Rayleigh distributed random process. A slow fading process which remains constant during an acquisition experiment but is independent from one experiment to another is assumed. For each experiment, the delays were randomly generated with a uniform distribution over the code length with a maximum delay spread

of 6 chip durations. It has also been assumed that all paths are situated within the same cell, i.e., there is only one state that may lead to acquisition. Although this assumption does not affect the results significantly, it does simplify the simulation. For the frequency selective case, the number of resolvable paths is assumed to be fixed at 3. The number of adaptive filter taps, M , is 16, while all other parameters are the same as in section (4.4.1). The LMS step size and the threshold values are the same as those used in the AWGN channel case. The decision rule for the delay estimate is also the same, i.e. choose the tap number at which the peak of filter taps is located.

Figure 4-22 shows the mean acquisition time performance for flat fading, frequency selective fading with a 0.5 decay rate exponential MDP, and frequency selective fading with uniform MDP. Also, the performance under AWGN channel is included. We can see that, for a fixed acquisition time, flat fading degrades the performance by about 3 dB while frequency selective fading has a 0.5 to 2 dB degradation depending on the MDP. The degradation is far less than that reported in [17] for a parallel acquisition system which was about 9 to 10 dB. Note that uniform MDP is better than exponential MDP due to the larger inherent selection diversity. The ability of the proposed system to combat fading in general could be attributed to the adaptive nature of the system and the decision strategy which maximizes the detection probability. Although, for data detection, a *linear* adaptive filter may not operate successfully in a fading environment [57], for code acquisition it provides good performance due to the expected large minimum MSE.

The ability of the system to acquire to the strongest received path is demonstrated in Figure 4-23. A simulation experiment which has 200 snapshots of the channel is performed. The channel has three paths with uniform MDP. The strongest received path number is shown in the upper part of the figure. For instance, during the first 23 snapshots, path number two has the strongest signal, then path number one becomes larger for the next 20 snapshots, and finally path number three is the strongest for the rest of the experiment. The lower part of the figure shows the path number whose delay is being estimated by the adaptive system. As we can see, most of the time, the system tends to estimate the delay of

the strongest path; there are occasions where it estimates the other paths since there strength is sometimes comparable to the strongest path. Acquisition miss and false alarm errors result in a broken line in the lower part of the figure.

4.6 Summary

Flow graph techniques have been used to model the acquisition process and to derive the mean acquisition time. Computer simulation results have been used to confirm some of the theoretical statistical models derived for the error signal and the STTA. Also, the detection and false alarm probabilities have been derived. The mean acquisition time performance for the proposed adaptive system was presented for AWGN, frequency selective fading, and frequency non-selective fading channels. It has been shown that the system has similar gains in acquisition time to that offered by other hybrid techniques over serial search schemes. The proposed system has also been compared to a system based on a partially matched filter. Finally, it has been shown using computer simulation that fading does not degrade the acquisition performance of the proposed system significantly.

Figure 4-21: Theoretical and simulated optimum tap-weight vector for a 16-tap filter.

Figure 4-22: Mean acquisition time performance for a 16-tap adaptive filter under AWGN, flat fading, and frequency selective fading channels.

Figure 4-23: A simulation experiment demonstrating the ability of the system to acquire to the strongest available path. $M = 16$, $L_p = 3$, uniform MDP Rayleigh fading.

Chapter 5 Adaptive Filter Based DS/SS Code Tracking System

5.1 Introduction

The Tracking mode of the proposed synchronization system is described in this chapter. The same adaptive filter used for code acquisition is used for code tracking. The optimum tap-weight vector for the adaptive filter is derived and expressions for the mean hold-in time and mean penalty time are presented. Numerical results are also included. Then, other performance measures for the tracking loop are considered, namely tracking of time varying delays and residual tracking error energy. The residual tracking error produced by proposed system is shown to outperform the maximum likelihood estimator. Finally, the BER performance of a DS/SS system using the proposed tracking system is simulated. the results indicate that the degradation suffered by the system compared to a perfectly synchronized system is less than 1 dB for a relatively fast varying channel. Most of the results presented in this chapter have been reported in [60], [61], and [62].

5.2 Description of the Tracking System

The block diagram of the proposed DS/SS tracking system is shown in Figure 5-1. The same M -tap FIR adaptive filter used for code acquisition is utilized for code tracking in addition to other blocks used for data estimation and interpolation. The baseband signal, $x(t)$ given by (3.5) is sampled at a rate of $1 / T_s = N_s / T_c$, where N_s is an integer greater than one. These samples are applied, after a delay of one bit duration,¹ to the adaptive filter input given by

$$x(j) = b\left(\left\lfloor \frac{j - N_s \tau}{N_s L} \right\rfloor\right) c(j - \lfloor N_s \tau \rfloor) + N(j) \quad (5.1)$$

where $\lfloor z \rfloor$ is the smallest integer less than or equals to z , $\{N\}$ are the WGN samples with

1. This delay is introduced to facilitate operation in presence of random data as will be explained later.

zero mean and variance σ_n^2 , $b(i)$ is the i th data bit, $c(j)$ is the j th element in the sequence $\{c\}$ constructed by repeating every element in the spreading sequence, $\{a\}$, by a factor of N_s , i.e., $c(j) = a(n)$; $j = nN_s, nN_s + 1, \dots, (n+1)N_s - 1$, where n can assume any integer value. The adaptive filter coefficients are adjusted by using the LMS algorithm to minimize the MSE between the filter output, $y(n)$, and a desired response, $d(n)$, which is chosen as the interpolated locally generated despread sequence multiplied by the estimate of the information data, i.e.,

$$d(n) = \hat{b}\left(\left\lfloor \frac{n - M_c - N_s \hat{\tau}}{N_s L} \right\rfloor\right) c(n - \lfloor N_s \hat{\tau} \rfloor - M_c) \quad (5.2)$$

where $\hat{\tau}$ is the delay estimate, M_c is the center tap number, $M_c = \left\lfloor \frac{M-1}{2} \right\rfloor$, and $\hat{b}(i)$ is the estimate of the i th data bit obtained as shown in Figure 5-1. Note that the system needs the data estimate, $\hat{b}(i)$, in order to ensure convergence of the adaptive filter (similar to the decision directed mode in equalization). To elaborate, suppose that the i th data bit is received, then the input to the filter is $b(i-1)$ times the oversampled spreading code. The correlator output at the end of the $(i-1)$ th interval yields the estimate of the data bit $\hat{b}(i-1)$ which, with high probability, will be the same as the actual transmitted data bit. To ensure convergence in the MSE the data estimate is multiplied by the reference sequence. It will be shown later that the system also works well even in presence of decision errors.

As before, the time average of the squared error over a window of S samples is used as a decision statistic to test for convergence in the MSE according to (3.12). The filter coefficients are adapted for a period of time after which Λ_S is checked for convergence A times. If B or more tests out of A indicate convergence, i.e. $\Lambda_S \leq \eta_t$, where η_t is the threshold value during tracking, a *hit* is declared and a state counter is decreased by one (a minimum count of one is assumed); otherwise a *miss* is declared and the state counter is

increased by one. After a *hit*, the phase of the local sequence is delayed or advanced by a certain number of samples to keep the peak of filter taps located at the center tap M_c . This process is repeated to keep track of the variations in the delay offset until the state counter reaches a certain number (in the numerical results a count of four is used) where an out-of-lock condition is declared and acquisition is re-initiated.

The optimum tap-weight vector of the FIR adaptive filter is still obtained from (3.29) but the autocorrelation matrix, $\mathbf{R}$, of the tap-input vector, $\underline{x}(n)$, and the cross-correlation vector, $\underline{P}$, between $\underline{x}(n)$ and $d(n)$ need to be re-evaluated. $\mathbf{R}$ is an M -by- M matrix whose elements can be obtained from (5.1) as

$$r_{mk} = \mathbf{R}_b \left(\left\lfloor \frac{n-m-N_s\tau}{N_sL} \right\rfloor - \left\lfloor \frac{n-k-N_s\tau}{N_sL} \right\rfloor \right) \mathbf{R}_c(m-k) + \sigma_n^2 \delta(m-k) \quad (5.3)$$

for $m, k = 0, 1, \dots, M-1$; where $\delta(m)$ is the Dirac delta function, $\mathbf{R}_b$ is the autocorrelation matrix of the information data and $\mathbf{R}_c$ is the autocorrelation matrix of the sequence $\{c\}$ given by

$$\mathbf{R}_c(m) = \begin{cases} \left(1 - \frac{|m|}{N_s}\right) \sigma_s^2 & ; |m| \leq N_s \\ 0 & ; |m| > N_s \end{cases} \quad (5.4)$$

where σ_s^2 is the energy per sample. Note that during tracking where $M \ll L$, the time shift between the samples in (5.3) is very small compared to the bit duration, T_b , and hence it is possible to assume that the information data autocorrelation function, $\mathbf{R}_b$, is unity. Thus, the elements of autocorrelation matrix, $\mathbf{R}$, can be expressed as

$$r_{mk} = \begin{cases} \sigma_s^2 + \sigma_n^2 & ; m = k \\ \left(1 - \frac{|m-k|}{N_s}\right) \sigma_s^2 & ; |m-k| \leq N_s \\ 0 & ; |m-k| > N_s \end{cases} \quad (5.5)$$

The M -by-1 cross-correlation vector is defined as

Figure 5-1: Tracking system block diagram.

$$\underline{P} = E\{\underline{x}(n)d(n)\} = [p(0) \ p(-1) \ \dots \ p(-M+1)]^T \quad (5.6)$$

whose elements are given by

$$\begin{aligned} p(-j) &= E\{x(n-j)d(n)\} \\ &= \hat{\mathbf{R}}_b \left(\left\lfloor \frac{n-M_c-N_s\hat{\tau}}{N_sL} \right\rfloor - \left\lfloor \frac{n-j-N_s\tau}{N_sL} \right\rfloor \right) \mathbf{R}_c(\varepsilon + M_c - j) \end{aligned} \quad (5.7)$$

where $j = 0, 1, \dots, M-1$, $\varepsilon = \lfloor N_s\hat{\tau} \rfloor - \lfloor N_s\tau \rfloor$ is the delay estimate error and $\hat{\mathbf{R}}_b$ is the cross-correlation matrix between $b(i)$ and $\hat{b}(i)$. By assuming that data decisions are correct, i.e., $\hat{b}(i) = b(i)$, and ε is small compared to L , we can simplify (5.7) to get

$$p(-j) = \mathbf{R}_c(\varepsilon + M_c - j) \quad (5.8)$$

Now, the optimum tap-weight vector is obtained from (5.3) and (5.8). For $\varepsilon = 0$, it will have a peak value located at the center tap M_c , and as ε changes, the peak moves around the center tap. Thus, by readjusting the phase of the local sequence, $d(n)$, to maintain the peak at M_c , the time delay estimation error, ε , is minimized and variations in the delay offset are tracked. However, if $|\varepsilon| \geq (M_c + N_s)$, then all the cross-correlation elements in $\underline{P}$ will be zero which corresponds to an out-of-sync condition since the optimum tap-weight vector, $\underline{w}_{opt}$, is zero. A simple expression for the tap-weight vector may be obtained by approximating the autocorrelation matrix of the sequence $\{c\}$ by that of a random sequence, i.e.,

$$\mathbf{R}_c(m) = \begin{cases} \sigma_s^2 & ; m = 0 \\ 0 & ; m \neq 0 \end{cases} \quad (5.9)$$

which results in an optimum tap-weight vector of

$$\underline{w}_{opt} = \begin{cases} \frac{SNR_s}{SNR_s + 1} \underline{\Omega} & ; H_1 \\ \mathbf{0} & ; H_0 \end{cases} \quad (5.10)$$

where SNR_s is the signal-to-noise ratio per sample defined as $SNR_s = \sigma_s^2 / \sigma_n^2$, and $\underline{\Omega}$ is an M -by-1 vector whose elements are all zero except element number $\varepsilon + M_c$ which equals to one. In (5.10), H_1 denotes the in-sync condition ($|\varepsilon| < (M_c + N_s)$) while H_0 represents the out-of-sync condition.

In the next section we model the tracking process by a Markov chain from which the mean time to loss-of-lock and the mean time to clear a false alarm are evaluated as a function of the transition probabilities. These probabilities are determined from the probability density function of the decision statistic.

5.3 Mean Hold-in Time and Mean Penalty Time Analysis

As mentioned earlier, a state counter is used to monitor the status of the tracking process in a fashion similar to the Search/Lock Strategy (SLS) described in [8][12]. The tracking process is best described by a finite state Markov chain as depicted in Figure 5-2. The counter will start at state number 1 and after a period of time, $K_e T_c$, (an adaptation period plus a decision period) the counter is either increased to state 2 with a probability $\bar{p}$ or it stays in state 1 with probability p ($p = 1 - \bar{p}$). Once at state 2, the counter can go, after a period of $K_e T_c$, to state 3 or return to state 1 with probability $\bar{p}$ or p , respectively. Eventually, the counter will reach the absorbing state 4 which represents the out-of-lock condition. Absorption time is the time required to reach the absorbing state 4 from any transient state. Starting from state 1, absorption time can be interpreted as time to loss-of-lock or time to clear a false alarm, depending on what has caused the initiation of the tracking mode (a true detection or a false alarm).

A Markov chain is usually analyzed by obtaining its transition matrix. This matrix is a square matrix whose k th element represents the probability of going from state k to state l in one step. It is beneficial to put the transition matrix in its *canonical* form [8] which, for the system under consideration, can be shown to be

Figure 5-2: State transition diagram representing the verification mode.

$$D = \begin{array}{c} 4 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \vdots \\ \vdots \\ \vdots \\ \vdots \end{array} \begin{array}{ccc} 1 & 2 & 3 \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & p & \tilde{p} \\ 0 & p & 0 \\ \tilde{p} & 0 & p \end{bmatrix} \end{array} = \begin{bmatrix} I_A & \Psi \\ \Delta & \Phi \end{bmatrix} \quad (5.11)$$

where I_A is an identity matrix representing the absorbing states, Ψ is a matrix of all zeros, Φ contains the transition probabilities of the transient states, and Δ gives the transition probabilities from transient states to absorbing states. The mean absorption time is given by the vector [8][12]

$$\bar{T}_{abs} = \begin{bmatrix} \bar{T}_{14} \\ \bar{T}_{24} \\ \bar{T}_{34} \end{bmatrix} = [I - \Phi]^{-1} \underline{T} \quad (5.12)$$

where I is the identity matrix with same dimensions as Φ and $\underline{T}$ is a vector representing the transition times between states. In the system at hand, all elements of $\underline{T}$ equal to $K_e T_c$. Using (5.11) into (5.12), the mean time to reach state 4 from state 1 is given by

$$\bar{T}_{14} = \frac{1 + 2(1-p)^2}{(1-p)^3} K_e T_c \quad (5.13)$$

where p is the probability that the decision statistic, Λ_s , is less than the threshold, η_t , B or more times. This probability has two interpretations; under the in-phase hypothesis, H_1 , p is the probability of correct decision and (5.13) yields the mean time to loss-of-lock, T_{LL} , while, under H_0 , p gives the error probability and (5.13) represents the mean time to clear a false alarm, T_p .

To determine p , the probability density function (PDF) of Λ_s is needed under both hypotheses which is determined by the PDF of the error signal, $e(n)$. In Chapter 4, it has been shown that $e(n)$ is Gaussian with a mean m_i and variance σ_i^2 given by

$$m_i = \begin{cases} \pm \sigma_s & ; H_0 \\ \pm J_{ss} & ; H_1 \end{cases} \quad (5.14)$$

$$\sigma_i^2 = \begin{cases} J_{ss} - \sigma_s^2 & ; H_0 \\ J_{ss}(1 - J_{ss}) & ; H_1 \end{cases} \quad (5.15)$$

where J_{ss} is the steady state MSE produced by the LMS algorithm given by

$$J_{ss} = \frac{J_{min}}{1 - \sum_{i=0}^{M-1} \frac{\mu \lambda_i}{2 - \mu \lambda_i}} \quad (5.16)$$

where $J_{min} = \sigma_s^2 - \underline{w}_{opt}^T \underline{P}$ is the minimum MSE (MMSE) and λ_i 's are the eigenvalues of the autocorrelation matrix $\mathbf{R}$. Note that J_{min} has a value of σ_s^2 under hypothesis H_0 since the cross-correlation vector is zero.

Referring to (3.12), the decision statistic, Λ_S , is the sum of the square of S independent non-zero mean Gaussian random variables, and, hence, it has a non-central chi-squared distribution with S degrees of freedom given by (4.33). The non-centrality parameter is

$$\xi_i^2 = \frac{1}{S} \sum_{i=1}^S m_i^2 = \begin{cases} \sigma_s^2 & ; H_0 \\ J_{ss}^2 & ; H_1 \end{cases} \quad (5.17)$$

Now, the problem is reduced to a form similar to that during acquisition which was analyzed in Chapter 4. Equations (4.46) and (4.48) can be used to obtain the threshold crossing probability and p , respectively. Note that P_ϵ , in (4.48), is set to zero since during tracking we should expect the position of the peak to continuously move around the center tap and there is no meaning to including the in-phase false alarm probability in the analysis. These probabilities are substituted into (5.13) to obtain T_{LL} and T_p .

5.3.1 Numerical Results

In this section we present some numerical results of the performance of the proposed tracking scheme used in a DS/SS system with a PN code period $L = 127$ and a chip rate of 1.2288 Mcps. The number of samples per chip, N_s , and the examination period, $K_e T_c$, are chosen to be 5 and 1.63 milliseconds respectively. The averaging window S , the number of adaptive filter taps M , the decision rule parameters A and B , the LMS step size μ , and the threshold value η_t , are varied to see their effect on the mean hold-in time, T_{LL} , and the mean penalty time, T_p .

Figure 5-3 and Figure 5-4 show the effect of the averaging window on the hold-in time (mean time to loss-of-lock) and the penalty time, respectively. Large improvement in the hold-in time is observed as the signal-to-noise ratio per chip, SNR_c , is increased. Notice that, at small SNR_c , the non-centrality parameter, ξ_i^2 , is large and a smaller averaging window provides better T_{LL} performance by virtue of an increased threshold crossing probability. However, for large SNR_c , ξ_i^2 is small and a larger value of S is preferred so as to increase the concentration of the decision statistic PDF and thus increasing the threshold crossing probability. It is interesting to note that, for $S = 4$, T_{LL} is reduced from one hour to one minute with a change in SNR_c of more than 10 dB while a similar reduction for the system described in [12] occurs with a change in SNR_c of less than 2 dB. The mean false alarm penalty time performance is shown in Figure 5-4. The minimum penalty time is determined by the Search/Lock Strategy (for the system under consideration the minimum penalty time is $3K_e T_c$). Notice that a larger averaging window results in smaller penalty time owing to the reduction in the number of false alarm errors.

The impact of changing the number of filter taps, M , on tracking system performance is investigated in Figure 5-5 and Figure 5-6. Notice that, for small SNR_c , a filter with fewer number of taps has better hold-in time performance than a longer filter. This conclusion seems to contradict the logical instinct that a longer filter covers more time delay and thus

Figure 5-3: Effect of averaging window on mean time to loss-of-lock: $M = 17$ taps.

Figure 5-4: Effect of averaging window on mean time to clear a false alarm: $M = 17$ taps.

Figure 5-5: Variation of the mean time to loss-of-lock with the number of filter taps.

Figure 5-6: Variation of the mean penalty time with the number of filter taps.

capable of tracking large delay offsets. To explain why this is not the case, we observe that a filter with small number of taps produces a smaller steady state MSE, see (5.16), than a longer filter; and thus the threshold crossing probability is larger (longer hold-in time). The situation is reversed for large SNR_c , i.e., a longer filter is better, since the large reduction in the MMSE with increasing filter length overcomes the increase in the steady state MSE. Similar conclusions regarding the penalty time may be drawn from Figure 5-6.

The effect of the decision rule parameters (A, B) on the performance is shown in Figure 5-7 and Figure 5-8. Although a decision rule that maximizes the detection probability has better hold-in time performance than that obtained from a majority decision rule, the penalty time is degraded accordingly.

Figure 5-9 to Figure 5-12 show the effects of the LMS step size and the threshold value on the tracking performance. Figure 5-9 and Figure 5-10 show that a smaller step size is favorable for both performance measures, however, the ability to track fast varying channels is reduced. Finally, the conclusion obtained from Figure 5-11 and Figure 5-12 agrees with the commonly used practice in tracking loops [5][12] which is to change the threshold setting such that the detection probability is maximized even though a larger false alarm probability is incurred.

The above results were obtained by assuming that correct data decisions have been made by the detector to facilitate convergence of the taps to the optimal solution. Decision errors may cause the system to adapt the filter taps in a wrong direction and therefore may result in an out-of-phase hypothesis which could lead to out-of-sync condition even though the delay offset is within the filter span. The behavior of the tracking system with detected data being used in the reference sequence is investigated by computer simulation. An adaptive filter with 17 taps is assumed with a STTA window of 10 samples. Initially, The codes have been perfectly synchronized and the filter taps have been set to their optimum value. The actual delay has been changed every 16 bit intervals in a random-walk model with positive or negative increment of one sample duration. The filter taps have been adapted for an examination period equivalent to 16 bits (1 bit = 5×127 samples)

Figure 5-7: Variation of the mean time to loss-of-lock with the decision parameters.

Figure 5-8: Variation of the mean penalty time with the decision parameters.

Figure 5-9: Variation of the mean time to loss-of-lock with the LMS step size.

Figure 5-10: Variation of the mean penalty time with the LMS step size.

Figure 5-11: Effect of threshold setting on the mean time to loss-of-lock.

Figure 5-12: Effect of threshold setting on the penalty time.

and a decision is made on the delay estimate at the end of the examination period. The information data used during this examination period have been obtained by correlating the received signal with the de-spreading code as set by the previous examination period.

Figure 5-13 shows two typical experiments for the actual varying delay and the estimated delay for a SNR_c of -17 dB. As we can see, with the delay varies by more than 14 chips, the system successfully tracks the variation of the delay though with occasional slips possibly due to data decision errors. From (a), for instance, it is shown that the system tracks the varying delay for a period of time equivalent to about 88,000 bits. With some errors deliberately introduced, the system's ability to recover after a decision error is shown in Figure 5-14. Here the tap weights are shown to collapse during the error period and then build up to their optimal values after the error is cleared. Although it seems that the system is more sensitive to burst errors especially if they occur at the end of the decision period, the SLS is very effective in alleviating such problem. Note that, for independent error events, the probability of an error at any particular bit within the examination interval is very small (the BER raised to the number of bits per examination interval).

5.4 Tracking Performance Under Time Varying Conditions

In this section we evaluate, using computer simulation, the tracking performance of the proposed scheme under different time varying conditions. First, we demonstrate the ability of the system to successfully estimate a delay changing in a linear or a random-walk fashion. Then, the residual tracking error (MSE in time delay estimation) for the adaptive filter tracking system is shown to outperform the maximum likelihood estimator. Finally, the bit-error rate (BER) for a DS/SS system using the proposed tracking system is simulated.

5.4.1 Tracking of a Varying Delay

To demonstrate the tracking ability of the system, two scenarios are investigated. First, the

Figure 5-13: Two examples showing tracking behavior in decision directed mode. (a) actual and estimated delay for experiment no. 1, (b) actual and estimated delay for experiment no. 2: $M = 17$ taps, $N_s = 5$, $S = 10$, and $SNR_c = -17$ dB.

Figure 5-14: Effect of decision errors on the tap-weight vector: $M = 17$, $SNR_c = -10$ dB. Tap number 9 corresponds to delay offset = 0.

delay is assumed to be linearly increasing with on sample duration increment. The other scenario assumes a random-walk with reflecting boundaries model [63] for the changing delay with a positive or negative sample duration increment and the reflecting boundaries are located at $\pm M_c$ (the center tap number). The delay is assumed to change after a time period which equals to 1000 and 2000 samples for the linear and random-walk models, respectively.

The tracking performance for the linearly changing delay is shown in Figure 5-15 for different lengths of the adaptive filter. It is clear that the system tracks the changing delay successfully with a very small offset (about one sample). Note that, since the phase of the local PN code has not been changed to keep the peak at the center tap, as it should be, tracking is lost as the delay increases beyond the time spanned by the filter. An example of the tracking performance under the random-walk model for a filter with 17 taps is shown in Figure 5-16. As we can see, the delay estimate matches perfectly the actual delay most of the time except for a few number of changes where the estimate is offset by one sample duration.

In order to have a practical insight into the operating conditions that may lead to the above scenarios, a brief discussion is presented. First, it is easy to show that, for a system with a chip rate of 1.2288 Mcps and $N_s = 5$ samples/chip, a delay change of one sample duration is equivalent to a mobile speed of about 175 km/h. This speed is faster than that usually encountered in typical mobile radio applications. Next, the assumption that the delay is changing every 1000 or 2000 samples implies a clocking system stability of one part per thousand or one part per two thousands. This is a quite poor clocking system and in practice a stability of better than 10 parts per million is not uncommon. Hence, despite the severe operating conditions, fast moving receiver and poor clocking stability, the tracking system performance is still satisfactory.

Figure 5-15: Tracking of a linearly varying delay using an adaptive filter with M taps.

Figure 5-16: Adaptive filter tracking of a delay changing in a random-walk model.

5.4.2 Residual Tracking Error

The normalized residual tracking error is defined as the difference between the actual time delay, τ , and the delay estimate, $\hat{\tau}$, normalized by the chip duration. Since each tap of the FIR filter covers a time of T_s seconds, the minimum error that could be resolved by the filter is $\pm T_s$. The energy in the tracking error is usually of practical interest and it is always desirable to minimize this parameter.

The maximum likelihood estimator (MLE) is based on selecting the delay estimate, $\hat{\tau}$, as the value that maximizes the probability density function (PDF) of the received signal parameterized by the unknown delay, τ . It is possible to show that (see [64]) the MLE for the delay is obtained by correlating the incoming signal with all possible phase shifted versions of the PN code and the phase that results in a maximum output is selected as the delay estimate. The decision statistic is defined as

$$\alpha_i = \sum_{n=i}^{N+i-1} x(n)a(n-i)\hat{b}(n-i) \quad (5.18)$$

where, N is the correlation period (in our simulations, N equals to one code period) and i is the phase under test. In order to get a fair comparison between the adaptive filter tracking system and the MLE, the MLE is made to check M phases with an increment of T_s seconds.

Both the adaptive filter tracking system and the MLE are simulated for a DS/SS system with a PN code period $L = 127$ and the number of samples per chip $N_s = 5$. The number of filter taps is used as a parameter and the residual MSE is plotted versus the signal-to-noise ratio per chip. As shown in Figure 5-17, the MLE has a better performance for small values of SNR_c (large MSE), however the adaptive filter rapidly outperforms the MLE for the practical range of MSE. The improvements in SNR_c , at a MSE of -15 dB, are approximately 1.5, 2.5, and 4 dB for M equals to 5, 9, and 39, respectively. Note that the larger M is the larger the improvement, however, for a fixed value of SNR_c , the smaller M

is the smaller the MSE. Hence a filter with a small number of taps is better during tracking as opposed to the conclusions made during acquisition.

In order to explain the superiority of the adaptive filter tracking system over the MLE, the PDFs for the normalized tracking error, ε , are shown in Figure 5-18 for both estimators. As we can see, ε in the adaptive filter system has a uniform PDF at small SNR_c while for the MLE it has a somewhat peaked PDF, consequently, the MLE has a smaller MSE for small SNR_c . As the SNR_c is increased, the PDFs tend to follow an exponential distribution with a faster decay rate for the adaptive filter than the MLE case. Thus, the MSE for the adaptive filter decreases rapidly with increasing SNR_c as opposed to the slower decrease for the MLE.

One final comment regarding the adaptive filter tracking system is that the LMS algorithm used to adapt the filter taps has an adaptation step size of 1×10^{-4} . Although the MSE in the tracking error may be further reduced by reducing the LMS step size, the ability of the system to track fast varying channels is simultaneously reduced.

5.4.3 Simulation of the BER Performance

During the tracking mode of operation, the receiver uses the synchronized replica of the spreading code to despread the signal, and subsequently to detect the information data. One of the most important characteristics that reflect the reliability of the communications system is the bit-error-rate (BER). Theoretically, under ideal conditions, the BER of a BPSK DS/SS system operating in AWGN channel is easily derived [65][57]. However, in order to see the effect of the imperfect code synchronization in BER performance, we will present some simulation results for the proposed tracking system operating under time varying conditions. The simulation starts by assuming that correct acquisition has already been established, then the tracking mode is used to track a delay changing in a random-walk model. The locally generated code is used by a correlator receiver to detect the information data and the BER is estimated using Monte Carlo techniques.

Figure 5-17: MSE tracking performance for the proposed adaptive filter scheme and the MLE.

Figure 5-18: Probability density function for the tracking error for: (a) Adaptive filter; (b) MLE. For both estimators: $M = 17$ and $N_s = 5$.

Figure 5-19 shows the BER for the adaptive system under two cases. In the first case, the delay changes every 8 to 10 bit intervals, while in the second case, the delay changes every 30 to 34 bit intervals. These cases represent a relatively fast varying conditions compared to what is usually encountered in practice where the channel is assumed to be fixed for more than a packet duration (typically a packet contains hundreds of bits). The size of the delay change is one sample duration. As we can see, for the second case, the system has a very good performance since the *SNR* degradation from that of a perfectly synchronized system is less than 1 dB. The first case shows an irreducible BER of about 2×10^{-3} which indicates that the delay is changing at a rate faster than the tracking capability of the system.

5.5 Summary

The tracking mode of operation of the proposed adaptive system is described and analyzed. The mean hold-in time and the mean penalty time are derived. Numerical results for the effect of several system parameters are also included. The MSE tracking performance is simulated and compared to that of a MLE. Finally, the BER performance of a BPSK DS/SS system using the proposed tracking system is simulated under fast varying channel conditions.

Figure 5-19: BER performance of a BPSK DS/SS system incorporating the proposed adaptive tracking system in fast varying channel conditions.

Chapter 6 Conclusions and Future Work

Code synchronization is essential in spread spectrum systems since a lack of synchronization means no useful information could be communicated through the system. The synchronization process is usually divided into two stages, acquisition and tracking. After a brief review of several code acquisition and tracking schemes, a new system employing adaptive filtering techniques has been introduced. The proposed system is intended to execute both acquisition and tracking functions using the same hardware structure.

In Chapter 3, we have introduced the proposed synchronization scheme and derived the decision statistic used to indicate acquisition. The optimum tap-weight vector for the adaptive filter has also been obtained.

Chapter 4 presents the mean acquisition time analysis of the system based on flow graph techniques. Theoretical and simulation results for the mean acquisition time under AWGN, frequency selective, and frequency non-selective Rayleigh fading channels have been presented in this chapter.

The tracking mode of operation has been described and analyzed in Chapter 5. The mean hold-in time and mean false alarm penalty time have been derived. Numerical results have also been presented. Effect of data decision errors on the tracking behavior has been investigated. The ability of the system to track fast changing delays and the residual tracking error have been simulated. BER performance for a DS/SS system using the adaptive tracking scheme has also been simulated.

Several conclusions can be drawn from this research. The most important conclusion is the ability of the system to perform both code acquisition and tracking using the same hardware structure. This eliminates the cost of having a separate acquisition circuit and tracking loop and therefore reduces the receiver cost.

The proposed system is classified as a hybrid acquisition system since more than one cell is tested for acquisition simultaneously. Thus, there is an inherent improvement in the

mean acquisition time performance over that of single-cell serial-search schemes. For instance, the proposed system has an improvement factor of twice the number of filter taps over 1/2-chip serial-search system. Notice that this improvement is obtained for an adaptive filter with coherent and chip synchronous scenarios, and the 1/2-chip serial-search system with perfect conditions (unity detection probability and zero false alarm probability). It should be noted that the proposed acquisition system has similar acquisition time performance to that of other hybrid schemes that test the same number of cells at a time. However, the proposed system has simple hardware requirement, an M -tap adaptive filter with LMS adaptation, while other hybrid systems need $2M$ correlators and decision devices to test the $2M$ cells. It has also been shown that the proposed scheme has better performance, under small SNR conditions, than a system based on a partial matched filter of the same length.

It has been concluded that the LMS step size and the threshold value should be optimized to minimize the mean acquisition time rather than to optimize the detection or false alarm probabilities. The mean acquisition time performance is shown to be robust to small errors in estimating the signal-to-noise ratio. The mean acquisition time performance has been improved as the filter length increases because of the reduction in the number of cells to be searched (increased parallelism). The mean acquisition time is noticed to saturate at a higher level as the number of taps increases for the small SNR case. This is mainly due to the smaller step size parameter needed, and consequently a larger transition period required to reach steady state may balance the gains obtained through the large searching window.

It has also been shown that fading does not degrade the acquisition performance of the proposed system significantly; a degradation of less than 3 dB in SNR has been noticed. This degradation is quite small compared to that reported in [17][22] where fading has about 10 dB degradation. Frequency selective fading is less harmful than flat fading due to the inherent diversity gain in the multipath signal. It has also been shown that the tap-weight vector for the adaptive filter has distinct peaks corresponding to the resolvable

paths. As the paths fade, the system acquires to the new strongest path.

The tracking performance of the proposed system has been assessed by evaluating the mean hold-in time (mean time to loss-of-lock) and the mean penalty time (mean time to clear a false alarm). It has been shown that the system has long hold-in time and short penalty time. The mean hold-in time has been shown to decrease from one hour to one minute with a 10 dB reduction in SNR. This variation is very small compared to the system described in [12] where a similar variation has been reported with a change of less than 2 dB in SNR.

It has also been shown that a filter with a smaller length is preferred in order to increase the mean hold-in time. Furthermore, a shorter filter will produce a smaller tracking error. This suggests that a filter with a varying length might be used in order to have fast acquisition and long tracking (use a longer filter during acquisition and reduce the filter length during tracking). The tracking system has been shown, using computer simulation, to outperform a tracking system based on a maximum likelihood estimator (MLE). The proposed tracking system needs smaller SNR to produce the same residual tracking error as the MLE; a SNR improvement of about 4 dB.

Tracking of a linearly changing delay and a delay changing in a random-walk fashion have been investigated. It has been shown that the system can track these variations successfully under operating conditions that correspond to fast moving terminals and/or poor clocking systems. The BER of a DS/SS system using the tracking system has been found to degrade by about 1 dB from the ideal synchronization case in a relatively fast changing environment.

Finally, the adaptive nature of the proposed system makes it attractive for use in diverse applications such as in indoor wireless systems and digital cellular radio where the communications environment is continuously varying due to terminal movement and random activation of other users. This feature is especially important in fading and/or multi-user environments where acquisition time might be a limiting factor. As we mention previously, fading has small degradation in the proposed adaptive system performance

while the effect of multiple access interference needs to be investigated.

6.1 Suggestions for Further Research

The following are some suggestions for further work stemming from this research.

In this research, it has been assumed for the analysis that the carrier phase is known (a coherent system). This assumption was made to simplify the analysis but it is difficult to implement in practice. The phase of the carrier is usually random and time varying especially in fading environments, and some measures need to be taking to ensure proper operation of the system. It has been mentioned in this report that if the phase changes slowly, the adaptive filter can compensate for the random carrier phase and there is no need to estimate it at all. However, if the phase changes more rapidly, a noncoherent or a differentially coherent structure could be used. It is desirable to see how such systems can be analyzed and implemented since the conventional adaptation algorithms are not readily applicable [42][43].

A chip synchronous system has been assumed in this study. In reality the delay is not an integer multiple of the chip period and the effect of the fractional delay on the acquisition performance need to be examined. We anticipate that the presence of a fractional delay offset (less than a chip interval) will cause more than one tap to have a large value since the actual position of the filter peak should occur between the taps. One possibility to obtain the fractional delay offset is to interpolate between the delays corresponding to the two large taps. Alternatively, one may increase the time resolution of the system by increasing the sampling rate and, therefore, can estimate the delay to within a sampling interval.

The LMS algorithm has been used to adapt the filter taps toward their optimum value. It has been noted that the convergence rate of the LMS algorithm is slow especially in small SNR conditions. This may have a significant contribution to the acquisition time. It is worth investigating other adaptation algorithms or techniques to speed the convergence of the system. It is possible to use the RLS algorithm but the improvement may not justify the

increased complexity.

The analysis presented in this research have assumed an AWGN environment while computer simulations have been used to obtain the acquisition performance in fading conditions. It is interesting to derive analytical results for the mean acquisition time, mean hold-in time, and mean penalty time under fading channels.

Finally, a single user system has been investigated in this study. The effects of multiple access interference in a CDMA system and the near-far problem worth more investigations. We expect that the proposed system should have some resistance to the near-far problem usually encountered in DS/SS systems since it is based on the MMSE criterion which has been shown to provide near-far immunity in a system with a full code length filter [39].

Appendix A Review of Adaptive Filtering Theory

A.1 Introduction

Adaptive filters have received considerable attention due to their applicability in many different areas such as echo cancelling, adaptive arrays, communication channels equalization, radar, and biomedical engineering. This is caused by their ability to operate in unknown environments and also to track the changes in the input signal characteristics. The theory of adaptive filters is well established [44][46] and only a brief overview is presented here.

A.2 Principles of Adaptive Filters

An adaptive filter is essentially a digital filter with adjustable coefficients or taps according to a certain criterion. The filter is completely described by the set of coefficients which determine the discrete impulse response of the filter or equivalently its transfer function. A block diagram of an adaptive filter is shown in Figure A-1. It consists of a digital filter with M taps and an adaptive algorithm to modify the coefficients of the filter. The filter is usually realized in a transversal or finite impulse response (FIR) structure. Other forms are possible such as the infinite impulse response (IIR) or the lattice structure, however, the FIR structure is commonly used because of its simplicity and guaranteed stability.

Two discrete-time signals, $x(n)$ and $d(n)$, are applied simultaneously to the adaptive filter. The signal $x(n)$ is the primary input to be processed by the filter to produce an output, $y(n)$, that is similar in some sense to the desired signal, $d(n)$. The filter output is given by

$$y(n) = \sum_{k=0}^{M-1} w_k(n)x(n-k) = \underline{w}^T(n)\underline{x}(n) \quad (\text{A.1})$$

where $\underline{w}(n)$ and $\underline{x}(n)$, the weight vector and the input signal vector, respectively, are given by

Figure A-1: Block diagram of an adaptive filter.

$$\underline{w}(n) = \begin{bmatrix} w_0(n) \\ w_1(n) \\ \vdots \\ w_{M-1}(n) \end{bmatrix}, \quad \underline{x}(n) = \begin{bmatrix} x(n) \\ x(n-1) \\ \vdots \\ x(n-M+1) \end{bmatrix} \quad (\text{A.2})$$

The filter coefficients are adjusted to minimize some cost function of the error signal produced by the filter which is defined as

$$e(n) = d(n) - y(n) = d(n) - \underline{w}^T(n) \underline{x}(n) \quad (\text{A.3})$$

One of the most popular cost functions is the mean-square error (MSE) criterion, defined by

$$J(\underline{w}) = E\{e^2(n)\} \quad (\text{A.4})$$

where $E\{\cdot\}$ denotes the statistical expectation operation. The square of the error signal defined in (A.3) is

$$\begin{aligned} e^2(n) &= (d(n) - \underline{w}^T \underline{x}(n))(d(n) - \underline{w}^T \underline{x}(n))^T \\ &= d^2(n) - 2d(n)\underline{x}^T(n)\underline{w} + \underline{w}^T \underline{x}(n)\underline{x}^T(n)\underline{w} \end{aligned} \quad (\text{A.5})$$

Assuming that the input vector, $\underline{x}(n)$, and the output, $y(n)$, are jointly stationary, the mean-square error (MSE) is given by

$$\begin{aligned} J(\underline{w}) &= E\{d^2(n)\} - 2E\{d(n)\underline{x}^T(n)\}\underline{w} + \underline{w}^T E\{\underline{x}(n)\underline{x}^T(n)\}\underline{w} \\ &= \sigma_d^2 - 2\underline{P}^T \underline{w} + \underline{w}^T \underline{R} \underline{w} \end{aligned} \quad (\text{A.6})$$

where $\sigma_d^2 = E\{d^2(n)\}$ is the variance of the desired signal,

$$\underline{P} = E\{d(n)\underline{x}^T(n)\}$$

is the M -by-1 cross-correlation vector and

$$\underline{R} = E\{\underline{x}(n)\underline{x}^T(n)\}$$

is the M -by- M autocorrelation matrix of the input vector. The MSE is, therefore, a quadratic function with a minimum J_{min} which is called the minimum mean-square error (MMSE). The gradient vector of the MSE with respect to $\underline{w}$ is

$$\begin{aligned}\underline{\nabla}(J) &= \frac{dJ}{d\underline{w}} = -2\underline{P} + 2\underline{R}\underline{w} \\ &= -2E\{e(n)\underline{x}(n)\}\end{aligned}\tag{A.7}$$

At the minimum point of the MSE, the gradient is zero and the filter weight vector has its optimum value given by

$$\underline{w}_{opt} = \underline{R}^{-1}\underline{P}\tag{A.8}$$

The MMSE produced by this filter is

$$J_{min} = \sigma_d^2 - \underline{P}^T \underline{w}_{opt}\tag{A.9}$$

Equation (A.8) is known as the Wiener-Hopf equation which requires the knowledge of both $\underline{R}$ and $\underline{P}$ apriori. It also involves the undesirable matrix inversion operation. Furthermore, for nonstationary signals, both $\underline{R}$ and $\underline{P}$ are time varying and, therefor, $\underline{w}_{opt}$ is not fixed. Adaptive algorithms are used to adjust the filter taps to approximate the optimum value given by $\underline{w}_{opt}$ without having to compute $\underline{R}$ and $\underline{P}$ explicitly or perform a matrix inversion.

A.3 The LMS Adaptive Algorithm

Several techniques have been proposed to solve the problem of finding the minimum of the MSE performance surface. One of the oldest methods is known as the method of steepest descent which is used to adjust the filter coefficients recursively towards their optimum value. Starting from an arbitrary value, $\underline{w}(0)$, which could represent our best guess of the optimum solution, the algorithm computes the gradient vector given by (A.7) and changes the tap-weight vector in an opposite direction to the computed gradient. The coefficients update equation is given by

$$\underline{w}(n+1) = \underline{w}(n) - \frac{1}{2}\mu \underline{\nabla}(J)\tag{A.10}$$

where μ is a positive real-valued constant known as the step size parameter which controls the stability and rate of convergence. Equation (A.10) can be rewritten as

$$\underline{w}(n+1) = \underline{w}(n) + \mu E\{e(n)\underline{x}(n)\} \quad (\text{A.11})$$

The steepest descent algorithm still requires the knowledge of $\underline{R}$ and $\underline{P}$. However, by using an instantaneous estimate of $\underline{R}$ and $\underline{P}$ we obtain an estimate of the gradient vector given by

$$\hat{\underline{V}}(J) = -2\hat{\underline{P}} + 2\hat{\underline{R}}\underline{w} \quad (\text{A.12})$$

where $\hat{\underline{P}} = d(n)\underline{x}^T(n)$ and $\hat{\underline{R}} = \underline{x}(n)\underline{x}^T(n)$. Incorporating (A.12) into (A.10), we obtain the simple Least Mean-Square (LMS) algorithm such that

$$\underline{w}(n+1) = \underline{w}(n) + \mu e(n)\underline{x}(n) \quad (\text{A.13})$$

The LMS algorithm does not require the knowledge of $\underline{R}$ and $\underline{P}$, but only their instantaneous estimates. The coefficients obtained by the LMS algorithm are estimates of the optimum Wiener filter which improves gradually with time, but it will never reach the optimum value due to the noisy gradient used to adjust the coefficients.

There are two forms of convergence which need to be considered:

(1) convergence in the mean

$$E\{\underline{w}(n)\} \rightarrow \underline{w}_{opt} \quad \text{as } n \rightarrow \infty$$

(2) convergence in the mean-square

$$E\{e^2(n)\} < \infty \quad \text{as } n \rightarrow \infty$$

The step size, μ , must be selected to ensure both forms of convergence. The limits that must be satisfied by μ are

(1) convergence in the mean

$$0 < \mu < \frac{2}{\lambda_{max}} \quad (\text{A.14})$$

(2) convergence in the mean-square

$$\begin{aligned} \text{(i)} \quad & 0 < \mu < \frac{2}{\lambda_{max}} \\ \text{(ii)} \quad & \sum_{j=1}^M \frac{\mu \lambda_j}{2 - \mu \lambda_j} < 1 \end{aligned} \quad (\text{A.15})$$

where $\lambda_j, j = 1, 2, \dots, M$, are the eigenvalues of the autocorrelation matrix $\mathbf{R}$.

The mean-squared error produced by the LMS algorithm has the final value

$$J_{ss} = \frac{J_{min}}{1 - \sum_{j=1}^M \frac{\mu \lambda_j}{2 - \mu \lambda_j}} \quad (\text{A.16})$$

which is always larger than the MMSE.

The learning curve is defined as the plot of the mean-squared error of the LMS algorithm with time. It shows how the filter performance is improved in the MSE sense as time progresses. It may be approximated by a single exponential with a time constant given by

$$\tau_{mse} = \frac{1}{2\mu\lambda_{av}} \quad (\text{A.17})$$

where λ_{av} is the average eigenvalue of $\mathbf{R}$. The time to reach steady state by the exponential function is usually approximated as four times the time constant.

It should be noted that the step size has two conflicting effects. Reducing μ results in minimization of the steady state error which means a smaller difference between the steady state MSE and the MMSE. However, longer time is needed for the filter taps to converge to their optimum value. The opposite is true, that is, increasing μ improves the convergence rate but the MSE misadjustment at steady state is large. Hence, careful attention has to be given to the selection of μ .

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